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Electrical Technology

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1.1 Introduction

The study of the electrical engineering, basically involves the analysis of the energy transfer from one form to another. An electrical machine, deals with the energy transfer either from mechanical to electrical form or from electrical to mechanical form. This process is called **electromechanical energy conversion.**

An electrical machine which converts mechanical energy into an electrical energy is called an **electric generator**. While an electrical machine which converts an electrical energy into the mechanical energy is called an **electrical motor.**

Such electrical machines may be related to an electrical energy of an alternating type called **a.c. machines** or may be related to an electrical energy of direct type called **d.c. machines.**

The d.c. machines are classified as d.c. generators and d.c. motors. The construction of a d.c. machine basically remains same whether it is a generator or a motor. In this chapter constructional features of a d.c. machine, working principle and types of d.c. generator are discussed. But before beginning the study of the d.c. machines, it is necessary to revise the basic concepts of magnetism and electromagnetism.

1.2 Revision of Magnetism

Magnetism is a property by virtue of which a piece of solid body attracts iron pieces and pieces of some other metals. Such a piece of solid body is called a **natural magnet.** The two ends of a magnet are called its **poles**. When such a magnet is suspended freely by a piece of a silk fiber, it turns and adjusts itself in the direction of North and South of the earth. The end adjusting itself in the direction of North is called **N pole** while other is called **S pole**. When such two magnets are brought near each other, their behaviour is governed by some laws called **laws of magnetism.**

1.2.1 Laws of Magnetism

Law 1 : It states that 'like' magnetic poles repel and 'unlike' poles attract each other.

When the two magnets are brought near each other, such that two like poles i.e. N and N or S and S are facing each other, then the two magnets experience a force of repulsion. As against this, if two unlike poles i.e. N and S or S and N are facing towards each other, then they experience a force of attraction and try to attract each other.

Law 2 : This law is experimentally proved by Scientist Coulomb and hence also known as **Coulomb's Law.**

The force (F) exerted by one pole on the other pole is,

1. Directly proportional to the product of the pole strengths.
2. Inversely proportional to the square of the distance between them and
3. Dependent on the nature of medium surrounding the poles.

Mathematically this law can be expressed as,

$$F \propto \frac{M_1 M_2}{d^2}$$

where M_1 and M_2 are the pole strengths of the poles while 'd' is the distance between the two poles.

$$\boxed{F = \frac{K M_1 M_2}{d^2}}$$

where K is constant which depends on the nature of the surrounding.

1.2.2 Magnetic Field and Flux

The region around a magnet within which the influence of the magnet can be experienced is called its **magnetic field.** The presence of magnetic field is represented by imaginary lines around a magnet. These are called magnetic lines of force.

The total number of lines of force existing in a particular magnetic field is called **magnetic flux,** denoted by a symbol 'ϕ'. It is measured in a unit weber.

$$\boxed{1 \text{ weber} = 10^8 \text{ lines of force.}}$$

> **Key Point :** *The lines of force never intersect each other and are like stretched rubber bands and always try to contract in length.*

These properties of lines of force play an important role in the understanding of the working principle of the d.c. machines.

The lines of flux have a fixed direction. These flux lines start at N-pole and terminate at S-pole, **external** to the magnet. While the direction of flux lines is from S-pole to N-pole, **internal** to the magnet. The distribution and direction of such flux lines for a bar magnet is shown in the Fig. 1.1.

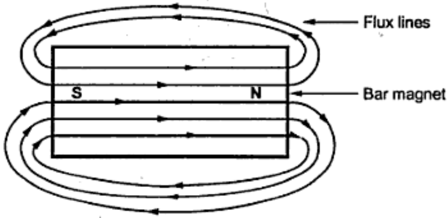


Fig. 1.1 Direction of the flux lines

These lines always form a closed loop and never intersect each other.

1.3 Revision of Electromagnetism

When a conductor carries a current, it creates a magnetic field around it. The direction of such magnetic field depends on the direction of the current passing through the conductor. So electric current and magnetism are very closely related to each other. This relationship plays an important role in the d.c. machines.

Let us see in brief, the rule to determine the direction of the flux produced by a current carrying conductor.

1.3.1 Right Hand Thumb Rule

It states that "Hold the current carrying conductor in the right hand such that the thumb is pointing in the direction of current and parallel to the conductor, then curled fingers point in the direction of the magnetic field or flux around it".

The Fig. 1.2 explains the rule.

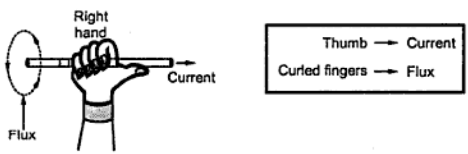
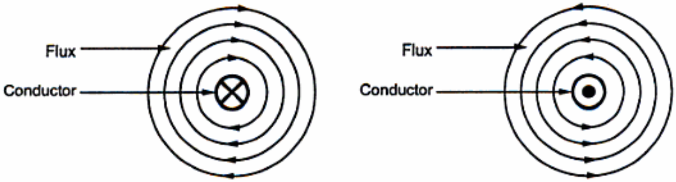


Fig. 1.2 Right hand thumb rule

Conventionally, such conductors are observed, assuming them to be placed perpendicular to the plane of the paper. So current moving away from the observer is denoted by a 'cross' while current coming towards the observer is denoted by a 'dot'. If now right hand is adjusted in such a way, that the thumb is pointing in the direction of current denoted as 'cross' i.e. going into the paper, then curled fingers indicate the



(a) Current moving away from observer (b) Current moving towards observer

Fig. 1.3

direction of flux as clockwise, as shown in the Fig. 1.3 (a). While if thumb of right hand is adjusted in the direction of current shown as 'dot' i.e. coming out of paper, then curled fingers indicate the direction of flux as anticlockwise as shown in the Fig. 1.3 (b).

1.3.2 Magnetic Field due to Circular Conductor

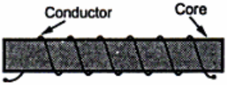


Fig. 1.4 Solenoid

Consider an arrangement in which a long conductor is wound with number of turns on a core, close together to form a coil. This is called a **solenoid** as shown in the Fig. 1.4. When such a conductor carries a current, the magnetic field gets produced around the core.

Identifying the direction of flux and hence identifying the two ends of the core as N pole or S pole is important in understanding the principle of d.c. machine. The right hand thumb rule can be modified for such case as stated below,

The right hand thumb rule : Hold the solenoid in the right hand such that curled fingers point in the direction of the current through the curled conductor, then the outstretched thumb along the axis of the solenoid points to the North pole of the solenoid or points in the direction of flux lines **inside** the core.

This is represented in the Fig. 1.5.

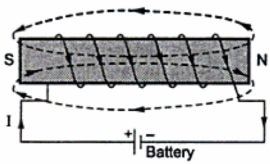


Fig. 1.5 Direction of flux around circular conductor

Key Point : *The direction of flux can be reversed either by changing direction of current through the conductor by reversing the polarities of the supply or by changing the direction of winding of the conductors around the core.*

With this background, let us start the detail study of a d.c. generator.

1.4 Principle of Operation of a D.C. Generator

All the generators work on a principle of dynamically induced e.m.f. This principle is nothing but the Faraday's law of electromagnetic induction . It states that, 'Whenever the number of magnetic lines of force i.e. flux linking with a conductor or a coil changes, an electromotive force is set up in that conductor or coil.' The change in flux associated with the conductor can exist only when there exists a relative motion between a conductor and the flux. The relative motion can be achieved by rotating conductor with respect to flux or by rotating flux with respect to a conductor. So a voltage gets generated in a conductor, as long as there exists a relative motion between conductor and the flux.

Such an induced e.m.f. which is due to physical movement of coil or conductor with respect to flux or movement of flux with respect to coil or conductor is called **dynamically induced e.m.f.**

Key Point : *So a generating action requires following basic components to exist, i) The conductor or a coil ii) The flux iii) The relative motion between conductor and flux.*

In a practical generator, the conductors are rotated to cut the magnetic flux, keeping flux stationary. To have a large voltage as the output, the number of conductors are connected together in a specific manner, to form a winding. This winding is called **armature winding** of a d.c. machine. The part on which this winding is kept is called **armature** of a d.c. machine. To have the rotation of conductors, the conductors placed on the armature are rotated with the help of some external device. Such an external device is called a **prime mover**. The commonly used prime movers are diesel engines, steam engines, steam turbines, water turbines etc. The necessary magnetic flux is produced by current carrying winding which is called **field winding**. The direction of the induced e.m.f. can be obtained by using Fleming's right hand rule.

1.5 Fleming's Right Hand Rule

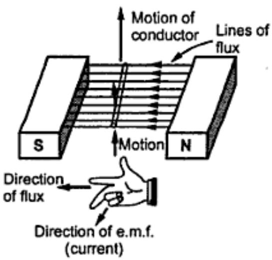


Fig. 1.6 Fleming's right hand rule

If three fingers of a right hand, namely thumb, index finger and middle finger are outstretched so that everyone of them is at right angles with the remaining two, and if in this position index finger is made to point in the direction of lines of flux, thumb in the direction of the relative motion of the conductor with respect to flux then the outstretched middle finger gives the direction of the e.m.f. induced in the conductor. Visually the rule can be represented as shown in the Fig. 1.6.

This rule mainly gives direction of current which induced e.m.f. in conductor will set up when closed path is provided to it.

Verify the direction of the current through conductor in the four cases shown in the Fig. 1.7 by using Fleming's right hand rule.

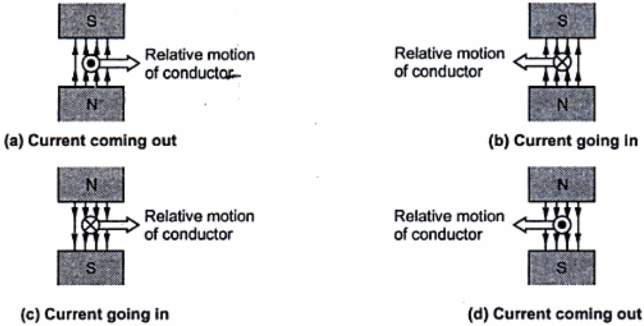


Fig. 1.7 Fleming's right hand rule

Key Point : *It can be observed from the Fig. 1.7 that if the direction of relative motion of conductor is reversed keeping flux direction same or if flux direction is reversed keeping direction of relative motion of conductor same then the direction of induced e.m.f. and hence direction of current it sets up in an external circuit gets reversed.*

The magnitude of the induced e.m.f. is given by,

$$\boxed{E = B \times l \times v}$$

where l = Active length of conductor in m.

v = Relative velocity component of conductor in m/s in the direction perpendicular to direction of the flux.

The active length means the length of conductor which is under the influence of magnetic field. In all the cases above, direction of motion of conductor is perpendicular to the plane of the flux.

But if it is not perpendicular then the component of velocity which is perpendicular to the plane of the flux, is only responsible for inducing e.m.f. in the conductor. This is shown in the Fig. 1.8 (a). In this Fig. 1.8 (a), though the velocity is v, its component v′ which is perpendicular to the flux lines is only responsible for the induced e.m.f.

If the plane of the rotation of conductor is parallel to the plane of the flux, there will not be any cutting of flux and hence there cannot be any induced e.m.f. in the conductor. This is shown in the Fig. 1.8 (b).

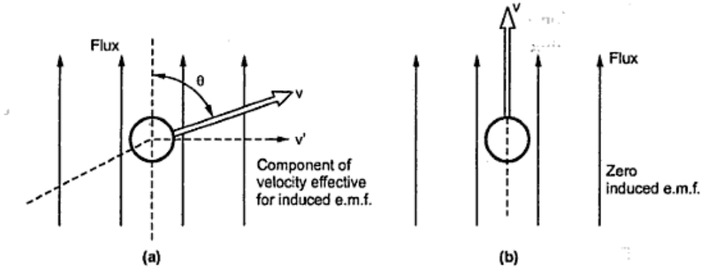


Fig. 1.8

> **Key Point**: *So to have an induced e.m.f. in the conductor not only the relative motion between the conductor and the flux is necessary but plane of rotation and plane of flux* ***should not*** *be parallel to each other.*

If angle between the plane of rotation and the plane of the flux is 'θ' as measured from the axis of the plane of flux then the induced e.m.f. is given by,

$$\boxed{E = B\,l\,(v \sin\theta) \text{ volts}}$$

Where v sinθ is the component of velocity which is perpendicular to the plane of flux and hence responsible for the induced e.m.f. This is shown in the Fig. 1.9.

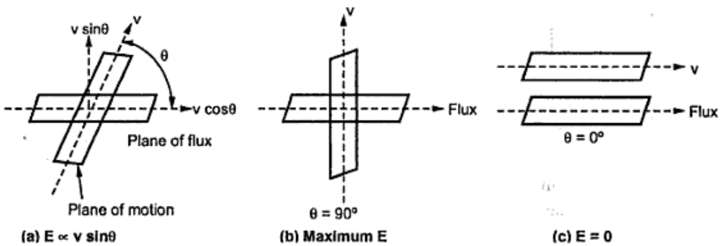


Fig. 1.9

From the equation of the induced e.m.f., it can be seen that the basic nature of the induced e.m.f. in a d.c. generator is purely sinusoidal i.e. alternating. To have d.c. voltage, a device is used in a d.c. generator to convert the alternating e.m.f. to unidirectional e.m.f. This device is called **commutator**. An alternator is a machine which produces an alternating e.m.f. is without a **commutator**. So an alternator with a commutator is the basic d.c. generator. Practically there is a difference between the construction of an alternator and a d.c. generator though the basic principle of working is same.

1.6 Single Turn Alternator

It consists of a permanent magnet with two poles. A single turn rectangular coil is kept in the vicinity of the permanent magnet. The coil is made up of conducting material like copper or aluminium.

The coil is made up of the two conductors namely a-b and c-d. Such two conductors are connected at one end to form a coil as shown in the Fig. 1.10.

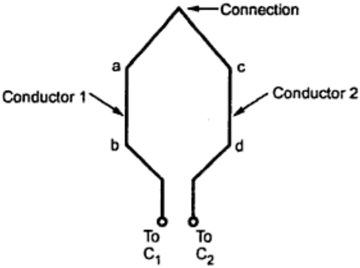


Fig. 1.10 Single turn coil

The remaining two ends are to be connected to the rings mounted on the shaft, called slip rings C_1 and C_2. Slip rings also rotate along with armature of a machine. The two brushes P and Q are resting on the slip rings, just making a contact with slip rings. The brushes P and Q are stationary. The slip ring and brush assembly is required to collect the e.m.f. induced in the rotating coil and make it available to the stationary external resistance. The overall construction is shown in the Fig. 1.11.

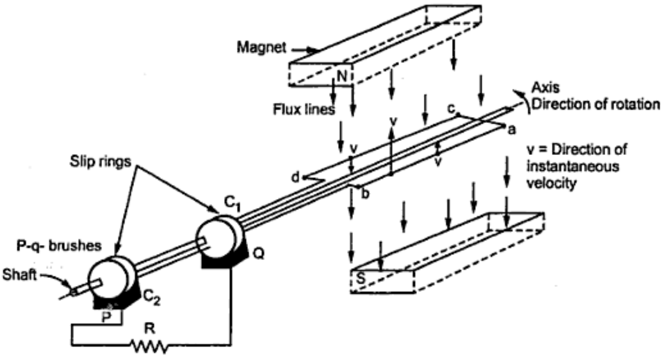


Fig. 1.11 Single turn alternator

The coil is rotated in an anticlockwise direction. While rotating, the conductors ab and cd cut the lines of flux of the permanent magnet. Due to Faraday's law of electromagnetic induction, e.m.f. gets induced in the conductors. This e.m.f. drives a current through resistance R connected across the brushes P and Q. The magnitude of the induced e.m.f. depends on the position of the coil in the magnetic field. Let us see the relation between magnitude of the induced e.m.f. and various positions of the coil. Consider different instants and positions.

Instant 1 : Let the initial position of the coil be as shown in the Fig. 1.11. The plane of the coil is perpendicular to the direction of the magnetic field. The instantaneous component of velocity of conductors ab and cd, is parallel to the magnetic field as shown and there cannot be the cutting of the flux lines by the conductors. Hence no e.m.f. will be generated in the conductors ab and cd and no current will flow through the external resistance R. This position can be represented by considering the front view of the Fig. 1.11 as shown in the Fig. 1.12 (a).

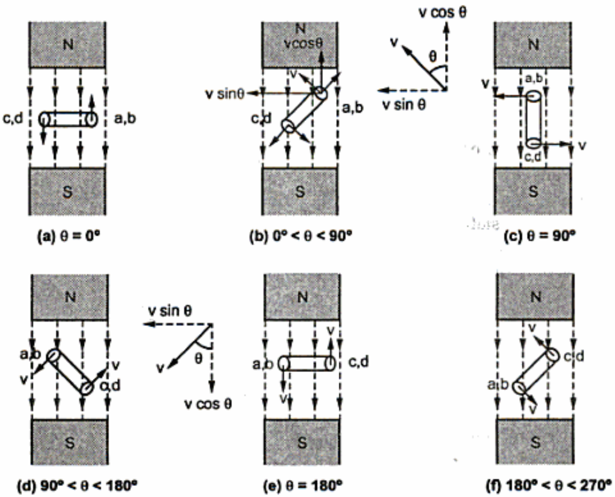


Fig. 1.12 The different instants of induced e.m.f.

Instant 2 : When the coil is rotated in anticlockwise direction through some angle θ, then the velocity will have two components vsinθ perpendicular to flux lines and vsinθ parallel to the flux lines. Due to vsinθ component, there will be cutting of the flux and proportionally there will be as induced e.m.f. in the conductor ab and cd. This e.m.f. will drive a current through the external resistance R. This is shown in the Fig. 1.12 (b).

Instant 3 : As angle 'θ' increases, the component of velocity acting perpendicular to flux lines increases, hence induced e.m.f. also increases. At $\theta = 90°$, the plane of the coil is parallel to the plane of the magnetic field while the component of velocity cutting the lines of flux is at its maximum. So induced e.m.f. in this position, is at its maximum value. This is shown in the Fig. 1.12 (c).

So as θ increases from 0° to 90°, e.m.f. induced in the conductors increases gradually from 0 to maximum value. The current through external resistance R also varies according to the induced e.m.f.

Instant 4 : As the coil continues to rotate further from $\theta = 90°$ to 180°, the component of velocity, perpendicular to magnetic field starts decreasing hence gradually decreasing the magnitude of the induced e.m.f. This is shown in the Fig. 1.12 (d).

Instant 5 : In this position, the velocity component is fully parallel to the lines of flux similar to the instant 1. Hence there is no cutting of flux and hence no induced e.m.f. in both the conductors. Hence current through external circuit is also zero.

Instant 6 : As the coil rotates beyond $\theta = 180°$, the conductor ab uptill now cutting flux lines in one particular direction reverses the direction of cutting the flux lines. Similar is the behaviour of conductor cd. So direction of induced e.m.f. in conductor ab is opposite to the direction of induced e.m.f. in it for the rotation of $\theta = 0°$ to 180°. Similarly the direction of induced e.m.f. in conductor cd also reverses. This change in direction of induced e.m.f. occurs because the direction of rotation of conductors ab and cd reverses with respect to the field as θ varies from 180° to 360°. This process continues as coil rotates further. At $\theta = 270°$ again the induced e.m.f. achieves its maximum value but the direction of this e.m.f. in both the conductors is opposite to the previous maximum position i.e. $\theta = 90°$. From $\theta = 270°$ to 360°, induced e.m.f. decreases without change in direction and at $\theta = 360°$, coil achieves the starting position with zero induced e.m.f.

Key Point: *In general the variations in the magnitude of the induced e.m.f. in a single conductor are alternating in nature as θ varies from 0° to 360°.*

It completes positive half cycle when θ varies from 0° to 180° while it completes negative half cycle when θ varies from 180° to 360°. One such cycle of an alternating induced e.m.f. is shown in the Fig. 1.13.

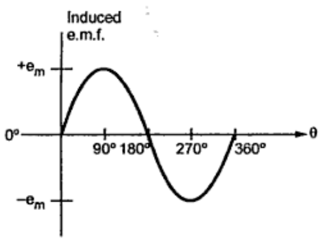


Fig. 1.13 Graphical representation of the induced e.m.f.

It is clear from the above discussion that the induced e.m.f. in a conductor is an alternating in nature. This is true in case of d.c. generator too. In d.c. generator, such alternating induced e.m.f.

is required to be rectified to get unidirectional d.c. e.m.f. This is possible by replacing slip rings by a device called commutator.

Key Point : *A commutator converts internally generated alternating e.m.f. to an unidirectional e.m.f. In an alternator, such a commutator is absent as an alternator is meant for producing an alternating e.m.f.*

The action of commutator is discussed later.

Let us discuss now the construction of a practical D.C. generator, including the function, choice of material and the method of construction of each part.

1.7 Construction of a Practical D.C. Machine

As stated earlier, whether a machine is **d.c. generator or a motor** the construction basically remains the same as shown in the Fig. 1.14.

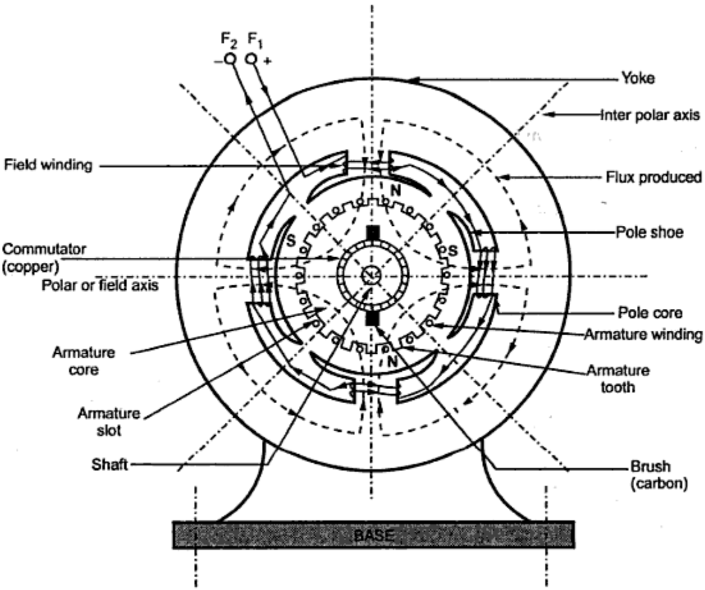


Fig. 1.14 A cross-section of typical d.c. machine

It consists of the following parts :

1.7.1 Yoke

a) Functions :

1. It serves the purpose of outermost cover of the d.c. machine. So that the insulating materials get protected from harmful atmospheric elements like moisture, dust and various gases like SO_2 , acidic fumes etc.

2. It provides mechanical support to the poles.

3. It forms a part of the magnetic circuit. It provides a path of low reluctance for magnetic flux. The low reluctance path is important to avoid wastage of power to provide same flux. Large current and hence the power is necessary if the path has high reluctance, to produce the same flux.

b) Choice of material : To provide low reluctance path, it must be made up of some magnetic material. It is prepared by using cast iron because it is cheapest. For large machines rolled steel, cast steel, silicon steel is used which provides high permeability i.e. low reluctance and gives good mechanical strength.

1.7.2 Poles

Each pole is divided into two parts namely, I) Pole core and II) Pole shoe.

This is shown in the Fig. 1.15.

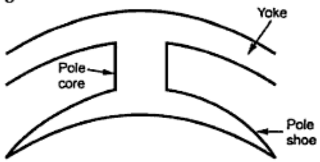


Fig. 1.15 Pole structure

a) Functions of pole core and pole shoe :

1. Pole core basically carries a field winding which is necessary to produce the flux.
2. It directs the flux produced through air gap to armature core, to the next pole.
3. Pole shoe enlarges the area of armature core to come across the flux, which is necessary to produce larger induced e.m.f. To achieve this, pole shoe has been given a particular shape.

b) Choice of material : It is made up of magnetic material like cast iron or cast steel.

As it requires a definite shape and size, laminated construction is used. The laminations of required size and shape are stamped together to get a pole which is then bolted to the yoke.

1.7.3 Field Winding (F1 - F2)

The field winding is wound on the pole core with a definite direction.

a) Functions : To carry current due to which pole core, on which the field winding is placed behaves as an electromagnet, producing necessary flux.

As it helps in producing the magnetic field i.e. exciting the pole as an electromagnet it is called **Field winding or Exciting winding.**

b) Choice of material : It has to carry current hence obviously made up of some conducting material. So aluminium or copper is the choice. But field coils are required to take any type of shape and bend about pole core and copper has good pliability i.e. it can bend easily. So copper is the proper choice.

> **Key Point :** *Field winding is divided into various coils called field coils. These are connected in series with each other and wound in such a direction around pole cores, such that alternate 'N' and 'S' poles are formed.*

By using right hand thumb rule for current carrying circular conductor, it can be easily determined that how a particular core is going to behave as 'N' or 'S' for a particular winding direction around it. The direction of winding and flux can be observed in the Fig. 1.10.

1.7.4 Armature

It is further divided into two parts namely,

I) Armature core and II) Armature winding

I) Armature core : Armature core is cylindrical in shape mounted on the shaft. It consists of slots on its periphery and the air ducts to permit the air flow through armature which serves cooling purpose.

a) Functions :

1. Armature core provides house for armature winding i.e. armature conductors.
2. To provide a path of low reluctance to the magnetic flux produced by the field winding.

b) Choice of material : As it has to provide a low reluctance path to the flux, it is made up of magnetic material like cast iron or cast steel.

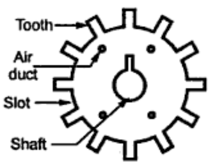


Fig. 1.16 Single circular lamination of armature core

It is made up of laminated construction to keep eddy current loss as low as possible. A single circular lamination used for the construction of the armature core is shown in the Fig. 1.16.

II) Armature winding : Armature winding is nothing but the interconnection of the armature conductors, placed in the slots provided on the armature core periphery. When the armature is rotated, in case of generator, magnetic flux gets cut by armature conductors and e.m.f. gets induced in them.

a) Functions :

1. Generation of e.m.f. takes place in the armature winding in case of generators.
2. To carry the current supplied in case of d.c. motors.
3. To do the useful work in the external circuit.

b) Choice of material : As armature winding carries entire current which depends on external load, it has to be made up of conducting material, which is copper.

Armature winding is generally former wound. The conductors are placed in the armature slots which are lined with tough insulating material.

1.7.5 Commutator

We have seen earlier that the basic nature of e.m.f. induced in the armature conductors is alternating. This needs rectification in case of d.c. generator, which is possible by a device called commutator.

a) Functions :

1. To facilitate the collection of current from the armature conductors.
2. To convert internally developed alternating e.m.f. to unidirectional (d.c.) e.m.f.
3. To produce unidirectional torque in case of motors.

b) Choice of material : As it collects current from armature, it is also made up of copper segments.

It is cylindrical in shape and is made up of wedge shaped segments of hard drawn, high conductivity copper. These segments are insulated from each other by thin layer of mica. Each commutator segment is connected to the armature conductor by means of copper lug or strip. This connection is shown in the Fig. 1.17.

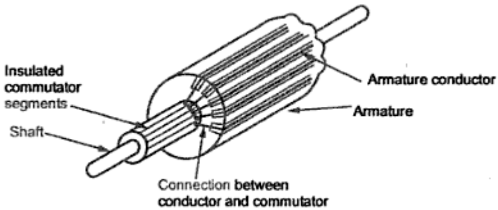


Fig. 1.17 Commutator

1.7.6 Brushes and Brush Gear

Brushes are stationary and resting on the surface of the commutator.

a) Function : To collect current from commutator and make it available to the stationary external circuit.

b) Choice of material : Brushes are normally made up of soft material like carbon.

Brushes are rectangular in shape. They are housed in brush holders, which are usually of box type. The brushes are made to press on the commutator surface by means of a spring, whose tension can be adjusted with the help of lever. A flexible copper conductor called **pig tail** is used to connect the brush to the external circuit. To avoid wear and tear of commutator, the brushes are made up of soft material like carbon.

1.7.7 Bearings

Ball-bearings are usually used as they are more reliable. For heavy duty machines, roller bearings are preferred.

1.8 Types of Armature Winding

We have seen that there are number of armature conductors, which are connected in specific manner as per the requirement, which is called **armature winding.** According to the way of connecting the conductors, armature winding has basically two types namely,

a) Lap winding b) Wave winding

1.8.1 Lap Winding

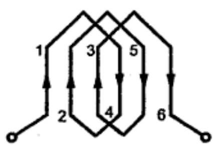


Fig. 1.18 Lap winding

In this case, if connection is started from conductor in slot 1 then connections overlap each other as winding proceeds, till starting point is reached again.

Developed view of part of the armature winding in lap fashion is shown in the Fig. 1.18.

As seen from the Fig. 1.18, there is overlapping of coils while proceeding.

Key Point : *Due to such connection, the **total number of conductors get divided into 'P' number of parallel paths, where P = number of poles in the machine.***

Large number of parallel paths indicate high current capacity of machine hence lap winding is preferred for high current rating generators.

1.8.2 Wave Winding

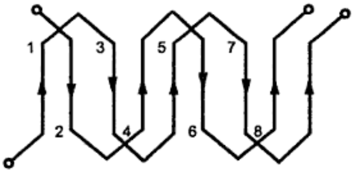


Fig. 1.19 Wave winding

In this type of connection, winding always travels ahead avoiding overlapping. It travels like a progressive wave hence called wave winding. To get an idea of wave winding a part of armature winding in wave fashion is shown in the Fig. 1.19.

Both coils starting from slot 1 and slot 2 are progressing in wave fashion.

Key Point : *Due to this type of connection, the* ***total number of conductors get divided into two number of parallel paths always, irrespective of number of poles of the machine.*** *As number of parallel paths are less, it is preferrable for low current, high voltage capacity generators.*

The number of parallel paths in which armature conductors are divided due to lap or wave fashion of connection is denoted as A. So **A = P** for **lap** connection and **A = 2** for **wave** connection.

1.8.3 Comparison of Lap and Wave Type Winding

Sr. No	Lap winding	Wave winding
1.	Number of parallel paths (A) = poles (P)	Number of parallel paths (A) = 2 (always)
2.	Number of brush sets required is equal to number of poles.	Number of brush sets required is always equal to two.
3.	Preferable for high current, low voltage capacity generators.	Preferable for high voltage, low current capacity generators.
4.	Normally used for generators of capacity more than 500 A.	Preferred for generators of capacity less than 500 A.

1.9 E.M.F. Equation of D.C. Generator

Let P = Number of poles of the generator

ϕ = Flux produced by each pole in webers (Wb)

N = Speed of armature in r.p.m.

Z = Total number of armature conductors

A = Number of parallel paths in which the 'Z' number of conductors are divided

So $A = P$ for lap type of winding

$A = 2$ for wave type of winding

Now e.m.f. gets induced in the conductor according to Faraday's law of electromagnetic induction. Hence average value of e.m.f. induced in each armature conductor is,

$$e = \text{Rate of cutting the flux} = \frac{d\phi}{dt}$$

Now consider one revolution of conductor. In one revolution, conductor will cut total flux produced by all the poles i.e. $\phi \times P$. While time required to complete one revolution is $\frac{60}{N}$ seconds as speed is N r.p.m.

$$\therefore \quad e = \frac{\phi P}{\frac{60}{N}} = \phi P \frac{N}{60}$$

This is the e.m.f. induced in one conductor. Now the conductors in one parallel path are always in series. There are total Z conductors with A parallel paths, hence $\frac{Z}{A}$ number of conductors are always in series and e.m.f. remains same across all the parallel paths.

∴ Total e.m.f. can be expressed as,

$$E = \phi P \frac{N}{60} \times \frac{Z}{A} \quad \text{volts}$$

This is nothing but the e.m.f. equation of a d.c. generator.

So

$$\boxed{E = \frac{\phi PNZ}{60\,A} \quad \text{e.m.f. equation}}$$

$$E = \frac{\phi NZ}{60} \quad \text{for lap type as } A = P$$

$$E = \frac{\phi PNZ}{120} \quad \text{for wave type as } A = 2$$

Example 1.1 : *A 4 pole, lap wound, d.c. generator has a useful flux of 0.07 Wb per pole. Calculate the generated e.m.f. when it is rotated at a speed of 900 r.p.m. with the help of prime mover. Armature consists of 440 number of conductors. Also calculate the generated e.m.f. if lap wound armature is replaced by wave wound armature.*

Solution : $P = 4 \quad Z = 440 \quad \phi = 0.07$ Wb and $N = 900$ r.p.m.

$$E = \frac{\phi PNZ}{60A}$$

i) For lap wound, $A = P = 4$

$$\therefore \quad E = \frac{\phi NZ}{60} = \frac{0.07 \times 900 \times 440}{60} = \mathbf{462\ V}$$

ii) For wave wound $A = 2$

$$\therefore \quad E = \frac{\phi P N Z}{120} = \frac{0.07 \times 900 \times 4 \times 440}{120} = 924 \text{ V}$$

1.10 Winding Terminologies

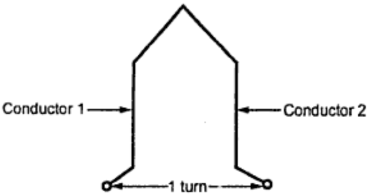


Fig. 1.20 Single turn

a) Conductor : It is the actual armature conductor which is under the influence of the magnetic field, placed in the armature slot.

b) Turn : The two conductors placed in different slots when connected together, forms a turn. While describing armature winding the number of turns may be specified from which, the number of conductors can be decided.

$$Z = 2 \times \text{Number of turns.}$$

c) Coil : For simplicity of connections, the turns are grouped together to form a coil. If coil contains only one turn it is called single turn coil while coil more than one turn is called multiturn coil.

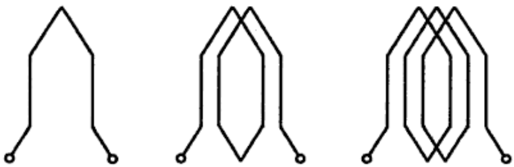


Fig. 1.21 Armature coils

Hence if number of coils, along with number of turns per coil are specified, it is possible to determine the total number of turns and hence total number of armature conductors 'Z' required to calculate generated e.m.f.

Example 1.2 : *A 4 pole, lap wound, d.c. generators has 42 coils with 8 turns per coils. It is driven at 1120 r.p.m. If useful flux per pole is 21 mWb, calculate the generated e.m.f. Find the speed at which it is to be driven to generate the same e.m.f. as calculated above, with wave wound armature.*

Solution : P = 4 $\phi = 21$ mWb $= 21 \times 10^{-3}$ Wb N = 1120 r.p.m.

Coils = 42 and turns / coil = 8

Total turns = coils × turns / coil = 42 × 8 = 336

Z = 2 × total turns = 2 × 336 = 672

i) For lap wound, A = P

$$\therefore \quad E = \frac{\phi N Z}{60} = \frac{21 \times 10^{-3} \times 1120 \times 672}{60} = \mathbf{263.424\ V}$$

ii) For wave wound, A = 2

and E = 263.424 V

$$\therefore \quad E = \frac{\phi P N Z}{120}$$

$$\therefore \quad 263.424 = \frac{21 \times 10^{-3} \times 4 \times N \times 672}{120}$$

$$N = \mathbf{560\ r.p.m.}$$

1.11 Single Layer and Double Layer Winding

Basically there are two physical types of the windings. These are i) Single layer winding ii) Double layer winding. The sequential arrangement of coils around the armature is different for both these types of windings.

1.11.1 Single Layer Winding

In this type of winding, the complete slot is containing only one coil side of a coil. This type of winding is not normally used for machines having commutators. It is shown in the Fig. 1.22.

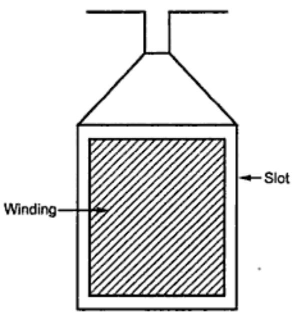


Fig. 1.22 Single layer winding

In single layer windings permit the use of semi enclosed and closed types of slots. Also the coils can be pushed through the slots from one end of the core and are connected during the process of windings at the other end. Here the insulation can be properly applied and consolidated which is advantageous in large output machines with high voltage.

The single layer windings used in high voltage machines use small groups of concentrically placed coils. The interlinking between these coils is in such a way so as to minimize the space taken up outside the slot and in the overhang connections.

1.11.2 Double Layer Winding

It is shown in the Fig. 1.23. It consists of identical coils with one coilside of each coil in top half of the slot and the other coilside in bottom half of another slot which is nearly one pole pitch away.

In the Fig. 1.23 (a) there are two coilsides per slot while in (b) there are eight coilsides per slot. Each layer may contain more than one coil side if large number of coils are required. For placing double layer windings, usually open slots are used.

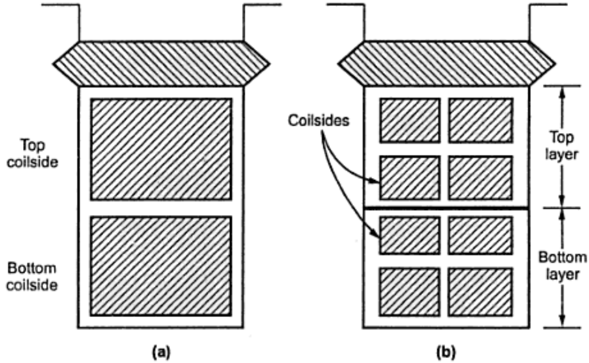


Fig. 1.23 Double layer winding

Advantages of double layer winding

The double layer winding has following advantages,

1) It provides neat arrangement as all coils are identical.
2) Greater flexibility can be achieved with double layer winding as coil span can be easily selected.

1.12 Closed and Open Windings

Armature windings are classifed into two different types namely i) Closed type winding ii) Open type winding.

1.12.1 Closed Type Winding

In this type of winding, a closed path is formed around the armature. The starting point of the winding is reached again after passing through all the turns. The current passing through closed type of winding is through brushes placed on commutator. The commutator segments are connected to various armature coils.

The armature current gets divided into different parallel paths. The current flowing through the coil changes continuously but from brush side the winding view remains same and polarity is maintained which is in effect due to use of commutator segments.

The closed type of winding is normally used in a.c. and d.c. commutator machines. This type of winding is usually double layer.

1.12.2 Open Type Winding

In case of a.c. machines, commutator is not used and hence closed winding is not required to be used. In such cases open type winding is used. The armature is left open at one or more points.

The ends of each section of the winding can be brought at the terminals to do the required type of interconnection externally. The open type of winding is preferred over closed type as it gives better flexibility in design and freedom of connections.

These type of windings are either single layer type or double layer type and are mainly used in induction machines and synchronous machines.

1.13 Action of Commutator

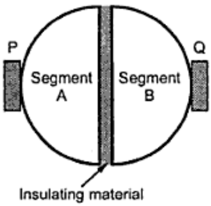


Fig. 1.24 Split ring

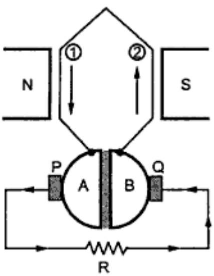


Fig. 1.25 Action of split ring

It is seen that, the e.m.f. induced in the conductors is always sinusoidal and commutator converts this sinusoidal e.m.f. to unidirectional e.m.f. Let us see, how it happens.

For simplicity of understanding the commutator action, consider commutator in its simplest form. Commutator is divided into number of copper segments insulated from each other. In its simplest form, it is assumed to be divided into two segments, each is nothing but the half of the entire commutator drum, separated by insulating material. So in its simplest form it is a ring with two halves separated by insulation as shown in the Fig. 1.24.

Such a ring is called **split ring**. The brushes P and Q are stationary and pressed on the surface of split ring. Split ring is mounted on the shaft and rotates as armature rotates.

Consider a single turn generator with conductors (1) and (2). These armature conductors are connected to the two segments of split ring. The external resistance R is connected across brushes P and Q.

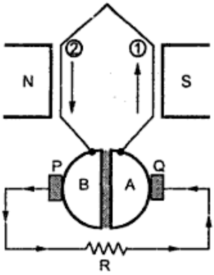


Fig. 1.26

Under instant 1, the current flowing through resistance R is flowing from left to right as shown in the Fig. 1.25. This is by assuming the direction of current through conductor (1) downwards which is under N pole and through conductor (2) upwards which is under S pole, at the instant considered. At this instant brush P behaves as positive and brush Q as negative.

After a next half revolution, we have seen that direction of e.m.f. in the individual conductors reverses. Hence conductor (1) now will carry a current which will be upwards and due to half revolution it will be under S pole. Similarly conductor (2) individually will carry a current downwards now, and will be under N pole as shown in the Fig. 1.26

Now split ring i.e. commutator is mounted on shaft and rotates with armature. So when conductors will reverse their positions, the split ring sections will also reverse their positions as shown in the Fig. 1.26. But brushes P and Q are stationary and tapping the current from the commutator segments which are in contact with them.

Hence under instant 2, segment B will be in contact with brush P and segment A will be in contact with brush Q. Due to this, current through resistance R maintains its direction from left to right as shown in the Fig. 1.26. Brush P remains positive and Q remains as negative.

The Fig. 1.27 shows the waveforms of current in the individual conductor and current in external resistance R. Effectively one brush always taps those conductors carrying current in one particular direction and other brush always taps those conductors carrying a current which is in 180° opposite direction to the conductors under brush one. So one brush remains always positive and other always negative, and the load current is unidirectional.

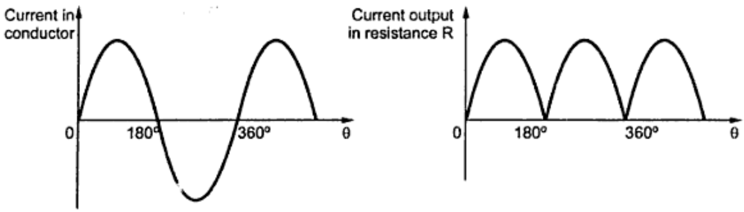


Fig. 1.27 Waveforms of current

Key Point: *In the limiting case, number of segments of a commutator is equal to number of armature coils in a practical generator. Due to this, commutating action is very fast and almost straightline i.e. pure d.c. can be obtained across the load.*

In a practical d.c. generator, the small poles in addition to the main poles, fixed to the yoke in between the main poles are used to improve the commutation. These poles are called **interpoles.**

1.14 Symbolic Representation of D.C. Generator

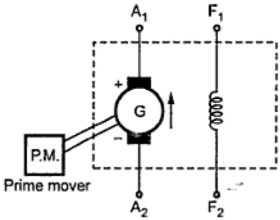


Fig. 1.28 Symbolic representation of D.C. generator

The armature is denoted by a circle with two brushes. Mechanically it is connected to another device called prime mover. The two ends of armature are denoted as A_1 - A_2. The field winding is shown near armature and the two ends are denoted as F_1 - F_2. The representation of field vary little bit, depending on the type of generator.

The symbolic representation is shown in the Fig. 1.28. Many times an arrow (↑) is indicated near armature. This arrow denotes the direction of current which induced e.m.f. will set up, when connected to an external load.

Key Point: *Every practical generator needs a **prime mover** to rotate its armature. Hence to avoid complexity of the diagram, prime mover need not be included in the symbolic representation of generator.*

1.15 Types of Generators

The magnetic field required for the operation of a d.c. generator is produced by an electromagnet. This electromagnet carries a field winding which produces required magnetic flux when current is passed through it.

Key Point: *The field winding is also called **exciting winding** and current carried by the field winding is called an **exciting current.***

Thus supplying current to the field winding is called excitation and the way of supplying the exciting current is called **method of excitation.**

There are two methods of excitation used for d.c. generators,

1. Separate excitation
2. Self excitation

Depending on the method of excitation used, the d.c. generators are classified as,

1. Separately excited generator
2. Self excited generator

In **separately excited generator,** a separate external d.c. supply is used to provide exciting current through the field winding.

The d.c. generator produces d.c. voltage. If this generated voltage itself is used to excite the field winding of the same d.c. generator, it is called **self excited generator.** The d.c. voltage is produced in the armature winding of a d.c. generator, which is used to excite the field winding of the same generator. Depending on how electrically the armature winding is connected to the field winding, the self excited d.c. generators are classified as,

a) Shunt generators b) Series generators c) Compound generators.

In shunt the two windings, field and armature are in parallel while in series type the two windings are in series. In compound type the part of the field winding is in parallel while other part in series with the armature winding.

The compound generators are further classified as long shunt and short shunt compound generators. The overall classification of d.c. generators is shown in the Fig. 1.29.

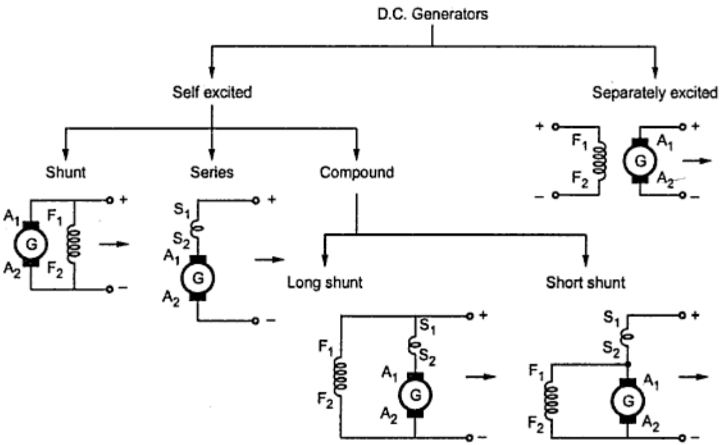


Fig. 1.29 Types of d.c. generators

1.16 Separately Excited Generator

When the field winding is supplied from external, separate d.c. supply i.e. excitation of field winding is separate then the generator is called separately excited generator. Schematic representation of this type is shown in the Fig. 1.30.

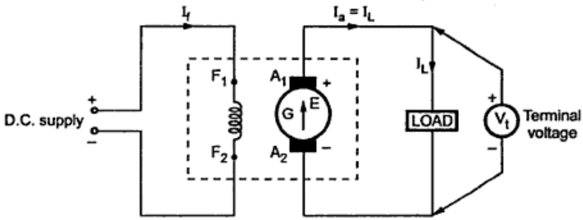


Fig. 1.30 Separately excited generator

The field winding of this type of generator has large number of turns of thin wire. So length of such winding is more with less cross-sectional area. So resistance of this field winding is high in order to limit the field current.

1.16.1 Voltage and Current Relations

The field winding is excited separately, so the field current depends on supply voltage and resistance of the field winding.

For armature side, we can see that it is supplying a load, demanding a load current of I_L at a voltage of V_t which is called **terminal voltage.**

Now $\quad I_a = I_L$

The internally induced e.m.f. E is supplying the voltage of the load hence terminal voltage V_t is a part of E. But E is not equal to V_t while supplying a load. This is because when armature current I_a flows through armature winding, due to armature winding resistance R_a ohms, there is a voltage drop across armature winding equal to $I_a R_a$ volts. The induced e.m.f. has to supply this drop, along with the terminal voltage V_t. To keep $I_a R_a$ drop to minimum, the resistance R_a is designed to be very very small. In addition to this drop, there is some voltage drop at the contacts of the brush called brush contact drop. But this drop is negligible and hence generally neglected. So in all, induced e.m.f. E has three components namely,

i) Terminal voltage V_t ii) Armature resistance drop $I_a R_a$ iii) Brush contact drop V_{brush}

So voltage equation for separately excited generator can be written as,

$$E = V_t + I_a R_a + V_{brush}$$

Where $E = \frac{\phi P N Z}{60 A}$

Generally V_{brush} is neglected as is negligible compared to other voltages.

1.17 Self Excited Generator

When the field winding is supplied from the armature of the generator itself then it is said to be self excited generator. Now without generated e.m.f., field cannot be excited in such generator and without excitation there cannot be generated e.m.f. So one may obviously wonder, how this type of generator works. The answer to this is residual magnetism possessed by the field poles, under normal condition.

Practically though the generator is not working, without any current through field winding, the field poles possess some magnetic flux. This is called **residual flux** and the property is called **residual magnetism.** Thus when the generator is started, due to such residual flux, it develops a small e.m.f. which now drives a small current through the field winding. This tends to increase the flux produced. This inturn increases the induced e.m.f. This further increases the field current and the flux. The process is cumulative and continues till the generator develops rated voltage across its armature. This is **voltage building process** in self excited generators.

Based on how field winding is connected to the armature to derive its excitation, this type is further divided into following three types :

i) Shunt generator ii) Series generator iii) Compound generator

Let us see the connection diagrams and voltage, current relations for these types of generators.

1.18 Shunt Generator

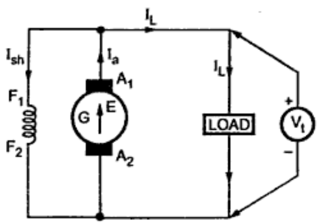


Fig. 1.31 Shunt generator

When the field winding is connected in parallel with the armature and the combination across the load then the generator is called **shunt generator.**

The field winding has large number of turns of thin wire so it has high resistance. Let R_{sh} be the resistance of the field winding.

1.18.1 Voltage and Current Relations

From the Fig. 1.31, we can write

$$I_a = I_L + I_{sh}$$

Now voltage across load is V_t which is same across field winding as both are in parallel with each other.

$$\therefore \quad I_{sh} = \frac{V_t}{R_{sh}}$$

While induced e.m.f. E, still requires to supply voltage drop I_aR_a and brush contact drop.

$$\therefore \quad E = V_t + I_aR_a + V_{brush}$$

where $E = \frac{\phi P N Z}{60 A}$

In practice, brush contact drop can be neglected.

1.19 Series Generator

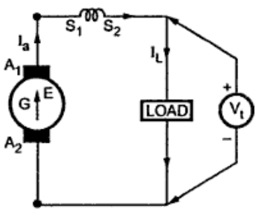


Fig. 1.32 Series generator

When the field winding is connected in series with the armature winding while supplying the load then the generator is called **series generator**. It is shown in the Fig. 1.32.

Field winding, in this case is denoted as S_1 and S_2. The resistance of series field winding is very small and hence naturally it has less number of turns of thick cross-section wire as shown in the Fig. 1.32.

Let R_{se} be the resistance of the series field winding.

1.19.1 Voltage and Current Relations

As all armature, field and load are in series they carry the same current.

$$\therefore \quad I_a = I_{se} = I_L$$

where I_{se} = Current through series field winding.

Now in addition to drop I_aR_a , induced e.m.f. has to supply voltage drop across series field winding too. This is $I_{se} R_{se}$ i.e. I_aR_{se} as $I_a = I_{se}$. So voltage equation can be written as,

$$\therefore \quad E = V_t + I_a R_a + I_a R_{se} + V_{brush}$$

$\therefore \quad E = V_t + I_a (R_a + R_{se}) + V_{brush}$

Where $\quad E = \frac{\phi P N Z}{60 A}$

1.20 Compound Generator

In this type, the part of the field winding is connected in parallel with armature and part in series with the armature. Both series and shunt field windings are mounted on the same poles. Depending upon the connection of shunt and series field winding, compound generator is further classified as : i) Long shunt compound generator, ii) Short shunt compound generator.

1.20.1 Long Shunt Compound Generator

In this type, shunt field winding is connected across the series combination of armature and series field winding as shown in the Fig. 1.33.

Voltage and current relations are as follows :

From the Fig. 1.33, $I_a = I_{se}$

and $\boxed{I_a = I_{sh} + I_L}$

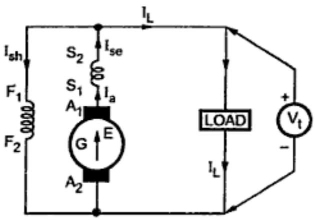


Fig. 1.33 Long shunt compound generator

Voltage across shunt field winding is V_t .

$\therefore \quad \boxed{I_{sh} = \frac{V_t}{R_{sh}}}$

Where R_{sh} = Resistance of shunt field winding

And voltage equation is,

$$\boxed{E = V_t + I_a R_a + I_a R_{se} + V_{brush}}$$

Where R_{se} = Resistance of series field winding

1.20.2 Short Shunt Compound Generator

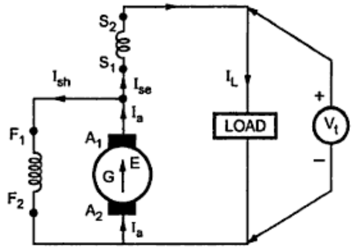


Fig. 1.34 Short shunt compound generator

In this type, shunt field winding is connected, only across the armature, excluding series field winding as shown in the Fig. 1.34.

Voltage and current relations are as follows :

For the Fig. 1.34, $I_a = I_{se} + I_{sh}$

and $I_{se} = I_L$

$\therefore$ $$\boxed{I_a = I_L + I_{sh}}$$

The drop across shunt field winding is drop across the armature only and not the total V_t, in this case. So drop across shunt field winding is $E - I_a R_a$.

$\therefore$ $$\boxed{I_{sh} = \frac{E - I_a R_a}{R_{sh}}}$$

Now the voltage equation is $E = V_t + I_a R_a + I_{se} R_{se} + V_{brush}$

Now $$I_{se} = I_L$$

$\therefore$ $$E = V_t + I_a R_a + I_L R_{se} + V_{brush}$$

Neglecting V_{brush}, we can write,

$$E = V_t + I_a R_a + I_L R_{se}$$

$\therefore$ $$E - I_a R_a = V_t + I_L R_{se}$$

$\therefore$ $$\boxed{I_{sh} = \frac{V_t + I_L R_{se}}{R_{sh}}}$$

Any of the two above expressions of I_{sh} can be used, depending on the quantities known while solving the problems.

1.20.3 Cumulative and Differential Compound Generator

It is mentioned earlier that the two windings, shunt and series field are wound on the same poles. Depending on the direction of winding on the pole, two fluxes produced by shunt and series field may help or may oppose each other. This fact decides whether generator is cumulative or differential compound. If the two fluxes help each other as shown in Fig. 1.35 the generator is called **cumulative compound generator**.

$$\phi_T = \phi_{sh} + \phi_{se}$$

Where ϕ_{sh} = Flux produced by shunt.

ϕ_{se} = Flux produced by series, field windings.

If the two windings are wound in such a direction that the fluxes produced by them oppose each other then the generator is called **differential compound generator.** This is shown in the Fig. 1.36.

$$\phi_T = \phi_{sh} + \phi_{se}$$

Where ϕ_{sh} = Flux produced by shunt field winding.

ϕ_{se} = Flux produced by series field winding.

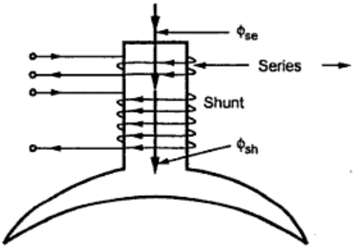


Fig. 1.35 Cumulative compound generator

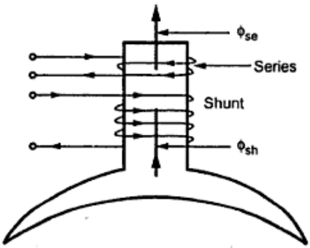


Fig. 1.36 Differential compound generator

Example 1.3 : *A d.c. shunt generator has shunt field winding resistance of 100 Ω. It is supplying a load of 5 kW at a voltage of 250 V. If its armature resistance is 0.22 Ω, calculate the induced e.m.f. of generator.*

Solution : Consider shunt generator as shown in the Fig. 1.37.

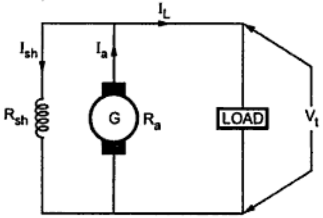


Fig. 1.37

$$I_a = I_L + I_{sh}$$

$$I_{sh} = \frac{V_t}{R_{sh}}$$

Now $V_t = 250$ V

and $R_{sh} = 100\ \Omega$

$\therefore$ $I_{sh} = \frac{250}{100} = 2.5$ A

Load power = 5 kW.

$\therefore$ $P = V_t \times I_L$

$\therefore \quad I_L = \frac{P}{V_t} = \frac{5 \times 10^3}{250} = 20 \text{ A}$

$\therefore \quad I_a = I_L + I_{sh} = 20 + 2.5 = 22.5 \text{ A}$

Now $\quad E = V_t + I_a R_a$ (neglect V_{brush})

$\therefore \quad E = 250 + 22.5 \times 0.22 = \mathbf{254.95 \text{ V}}$

This is the induced e.m.f. to supply the given load.

Example 1.4 : *A 250 V, 10 kW, separately excited generator has an induced e.m.f. of 255 V at full load. If the brush drop is 2 V per brush, calculate the armature resistance of the generator.*

Solution : Consider separately excited generator as shown in the Fig. 1.38.

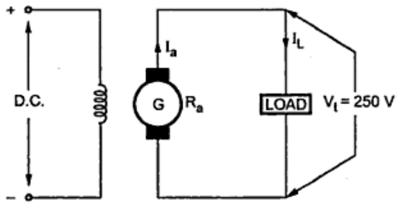


Fig. 1.38

$I_a = I_L$

Note that 250 V, 10 kW generator means the full load capacity of generator is to supply 10 kW load at a terminal voltage $V_t = 250$ V.

$\therefore \quad V_t = 250 \text{ V and } P = 10 \text{ kW}$

and $\quad P = V_t \times I_L$

$\therefore \quad I_L = \frac{10 \times 10^3}{250} = 40 \text{ A}$

$\therefore \quad I_a = I_L = 40 \text{ A}$ ... As separately excited

Now $\quad E = V_t + I_a R_a + V_{brush}$

Now there are two brushes and brush drop is 2 V/brush.

$\therefore \quad V_{brush} = 2 \times 2 = 4 \text{ V}$

$\therefore \quad E = 250 + 40 \times R_a + 4$

But $\quad E = 255 \text{ V on full load}$

$$\therefore \quad 255 = 250 + 40\, R_a + 4$$

$$\therefore \quad R_a = 0.025\ \Omega$$

Example 1.5 : *A d.c. series generator has armature resistance of 0.5 Ω and series field resistance of 0.03 Ω. It drives a load of 50 A. If it has 6 turns/coil and total 540 coils on the armature and is driven at 1500 r.p.m., calculate the terminal voltage at the load. Assume 4 poles, lap type winding, flux per pole as 2 mWb and total brush drop as 2 V.*

Solution : Consider the series generator as shown in Fig. 1.39

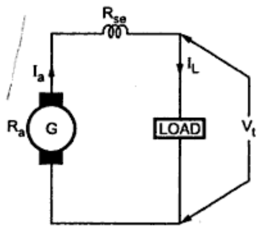


Fig. 1.39

$$R_a = 0.5\ \Omega,\ R_{se} = 0.03\ \Omega$$

$$V_{brush} = 2\ V$$

$$N = 1500\ \text{r.p.m.}$$

Total coils are 540 with 6 turns/coil.

$$\therefore \quad \text{Total turns} = 540 \times 6 = 3240$$

$$\therefore \text{Total conductors } Z = 2 \times \text{turns}$$

$$= 2 \times 3240 = 6480$$

$$\therefore \quad E = \frac{\phi P N Z}{60\, A}$$

For lap type, $A = P$

and $\phi = 2\ \text{mWb} = 2 \times 10^{-3}\ \text{Wb}$

$$\therefore \quad E = \frac{2 \times 10^{-3} \times 1500 \times 6480}{60}$$

$$= 324\ V$$

$$E = V_t + I_a(R_a + R_{se}) + V_{brush} \quad \text{... Total } V_{brush} \text{ given}$$

Where $I_a = I_L = 50$ A

$\therefore \quad 324 = V_t + 50\,(0.5 + 0.03) + 2$

$\therefore \quad V_t = $ **295.5 V**

Example 1.6 : *A short shunt compound d.c. generator supplies a current of 75 A at a voltage of 225 V. Calculate the generated voltage if the resistance of armature, shunt field and series field windings are 0.04 Ω , 90 Ω and 0.02 Ω respectively.*

Solution : Consider a short shunt generator as shown in the Fig. 1.40.

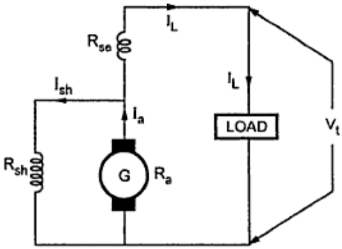


Fig. 1.40

$R_a = 0.04\ \Omega, \quad R_{sh} = 90\ \Omega, \quad R_{se} = 0.02\ \Omega$

$V_t = 225$ V

$I_L = 75$ A

$I_a = I_L + I_{sh}$

Now $\quad E = V_t + I_a R_a + I_L R_{se}$

and drop across armature terminals is,

$$E - I_a R_a = V_t + I_L R_{se}$$

$$= 225 + 75 \times 0.02 = 226.5 \text{ V}$$

$$\therefore \quad I_{sh} = \frac{E - I_a R_a}{R_{sh}} = \frac{V_t + I_L R_{se}}{R_{sh}}$$

$$= \frac{226.5}{90} = 2.5167 \text{ A}$$

$\therefore \quad I_a = I_L + I_{sh} = 75 + 2.5167 = 77.5167$ A

$\therefore \quad E = V_t + I_a R_a + I_L R_{se}$

$= 225 + 77.5167 \times 0.04 + 75 \times 0.02 =$ **229.6 V**

1.21 Characteristics of D.C. Generators

The d.c. generators have following characteristics in general,

1) Magnetization characteristics 2) Load characteristics

1.21.1 Magnetization Characteristics

This characteristics is the graph of generated no load voltage E against the field current I_f , when speed of generator is maintained constant. As it is plotted without load with open output terminals it is also called **No load characteristics** or **Open circuit characteristics.**

E_0 Vs I_f is magnetization characteristics

where E_0 = No load induced e.m.f.

But for generator,

$$E = \frac{\phi P N Z}{60 A}$$

$$\therefore \quad E \propto \phi \quad \text{with } \frac{PNZ}{60A} \text{ constant}$$

$$\therefore \quad \boxed{E \propto I_f} \quad \text{as } \phi \propto I_f$$

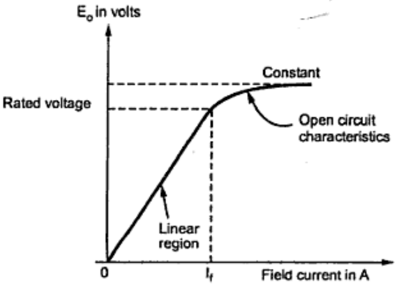


Fig. 1.41(a) Magnetization characteristics

Thus induced e.m.f. increases directly as I_f increases. But after certain I_f core gets saturated and flux ϕ also remains constant though I_f increases. Hence after saturation, voltage also remains constant.

Key Point: *Thus characteristics is linear till saturation and after that bends such that voltage remains constant though I_f increases.*

The characteristics is shown in the Fig. 1.41 (a).

Now the induced e.m.f. also varies with speed.

Actually, $E \propto N \phi$

So if magnetization characteristics for various speeds are plotted we will get family of parallel characteristics as shown in the Fig. 1.41(b). For lower speeds, generated voltages are less so characteristics for lower speeds are below the characteristics for higher speeds.

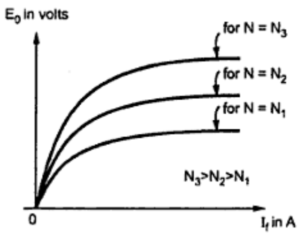


Fig. 1.41(b)

1.21.2 Load Characteristics

These are further divided into two categories,

1) External characteristics

2) Internal characteristics

The **external characteristics** is the graph of the terminal voltage V_t against load current I_L.

The **internal characteristics** is the graph of the generated induced e.m.f. E against the armature current I_a.

> **Key Point**: *While plotting both the characteristics, the speed N of the generator is maintained constant.*

Note : In most of the cases, the shunt field current is very small as compared with load current I_L. Hence in practice, the internal characteristics shows the graph of induced e.m.f. E against load current I_L , instead of I_a , neglecting I_{sh}.

1.22 Characteristics of Separately Excited D.C. Generators

The characteristics of separately excited d.c. generator are divided into two types,

1) Magnetization and 2) Load characteristics

1.22.1 Magnetization or Open Circuit Characteristics

The arrangement to obtain this characteristics is shown in the Fig. 1.42.

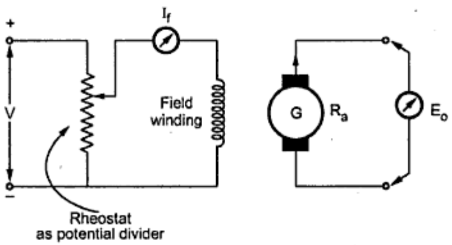


Fig. 1.42 Obtaining O.C.C. of separately excited generator

The rheostat as a potential divider is used to control the field current and the flux. It is varied from zero and is measured on ammeter connected.

$$E_o = \frac{\phi \, PNZ}{60 \, A}$$

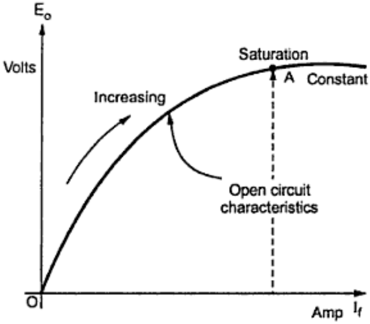


Fig. 1.43 Open circuit characteristics

As I_f is varied, then ϕ changes and hence induced e.m.f. E_o also varies. It is measured on voltmeter connected across armature. No load is connected to machine, hence characteristics are also called **no load characteristics** which is graph of E_o against field current I_f as shown in the Fig. 1.43 As I_f increases, flux ϕ increases and E_o increases. After point A, saturation occurs when ϕ becomes constant and hence E_o saturates.

1.22.2 Load Saturation Curve

This is the graph of terminal voltage V_t against field current I_f. When generator is loaded, armature current I_a flows and armature reaction exists. Due to this, terminal voltage V_t is less than the no load rated voltage. On no load, current I_a is zero and armature reaction is absent. Hence less number of ampere turns are required to produce rated voltage E_o.

These ampere-turns are equal to OB as shown in the Fig. 1.44. On load, to produce same voltage more field ampere-turns are required due to demagnetising effect of armature reaction. These are equal to BC as shown in the Fig. 1.44. Similarly there is drop $I_a R_a$ across armature resistance. Hence terminal voltage $V = E - I_a R_a$. This graph OR is also shown in the Fig. 1.44. The triangle PQR is called **drop reaction triangle**. Thus OP is no load saturation curve, OQ is the graph of generated voltage on load and OR is the graph of terminal voltage on load.

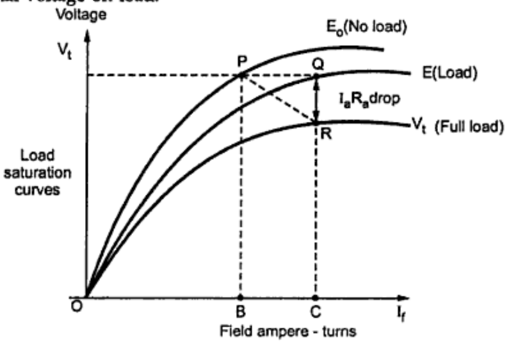


Fig. 1.44 Load saturation curves

1.22.3 Internal and External Characteristics

Let E_o be the no load rated voltage which drops to E due to armature reaction on load and further drops to V_t due to armature resistance drop $I_a R_a$ on load.

The graph of V_t against load current I_L is called **external characteristics** while the graph of E against load current I_L is called **internal characteristics**. These are shown in the Fig. 1.45 for separately excited d.c. generator. The graphs are to be plotted for constant field current. In case of separately excited d.c. generator induced e.m.f. is totally dependent on flux ϕ i.e. field curent I_f. Hence to have control over the field current, in case of separately excited d.c. generators **field regulator** is necessary.

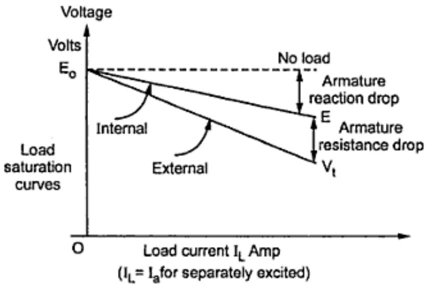


Fig. 1.45 Load characteristics

1.23 Load Characteristics of D.C. Shunt Generator

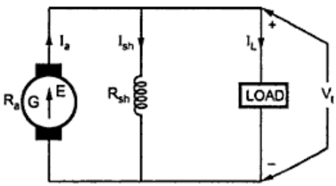


Fig.1.46

Consider the d.c. shunt generator shown in the Fig. 1.46. The internal characteristics is E Vs I_L while the external characteristics is V_t against I_L.

Let us see the nature of these two characteristics.

1.23.1 Internal Characteristics

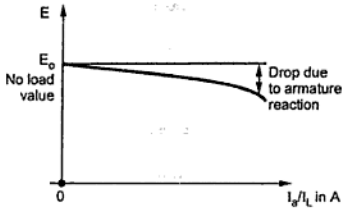


Fig. 1.47 Internal characteristics

Ideally the induced e.m.f. is not dependent on the load current I_L or armature current I_a. But as load current increases, the armature current I_a increases to supply load demand. As I_a increases, armature flux increases.

Key Point: *The effect of flux produced by armature on the main flux produced by the field winding is called an armature reaction.*

Due to the armature reaction, main flux pattern gets distorted. Hence lesser flux gets linked with the armature conductors. This reduces the induced e.m.f.

Key Point: *Thus the armature reaction decreases the generated e.m.f.*

This is shown in the Fig. 1.47

1.23.2 External Characteristics

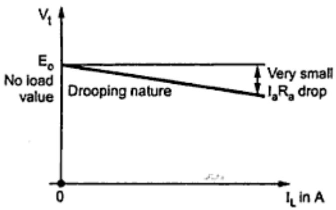


Fig. 1.48 (a) External characteristics

For d.c. shunt generator we know that,

$E = V_t + I_a R_a$ neglecting other drops. So as load current I_L increases, I_a increases. Thus the drop I_a R_a increases and terminal voltage $V_t = E - I_a R_a$ decreases. But the value of armature resistance is very small, the drop in terminal voltage as I_L changes from no load to full load is very small. This is shown in the Fig. 1.48. Hence d.c. shunt generator is called **constant voltage generator.**

Why shunt generator load characteristics turns back when overloaded?

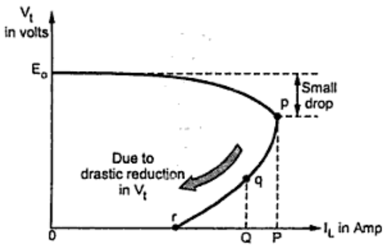


Fig. 1.48 (b) External characteristics

Consider the external characteristics of d.c. shunt generator as shown in the Fig. 1.48 (b). From E_o to point p, the load resistance decreases hence load current increases and due to I_aR_a and armature reaction, the voltage decreases by small amount. Hence characteristics is drooping in nature.

But if the load resistance is reduced beyond point p i.e load I_L is increased beyond P then it increases momentarily. This is very large current and generator gets overloaded. Due to such a large current the armature reaction is severe and drop I_aR_a is also large due to which the voltage V_t drastically reduces. This causes the current to decreases from P to Q, rather than increasing. Thus on the curve pqr, the voltage goes on reducing rapidly and at point r becomes zero. Thus beyond point p, if the generator is loaded, the load characteristics turns back till the generator gets short circuited and the curve meets x axis at point r where voltage is zero. At this point, small E is present due to residual magnetism.

Key Point: *In portion E_0 to p, the effect of decrease in load resistance dominates the effect of change in V_t as change in V_t is very small. While in portion 'pqr', the drastic reduction in V_t dominates the effect of decrease in load resistance hence curve turns back, reducing the load current.*

1.24 Load Characteristics of D.C. Series Generator

Consider a series generator shown in the Fig. 1.49.

In case of series generator,

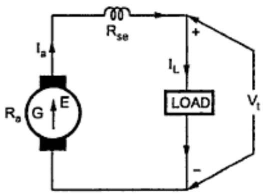


Fig. 1.49

$$I_a = I_{se} = I_L$$

As load current increases, I_{se} increases. The flux ϕ is directly proportional to I_{se}. So flux also increases. The induced e.m.f. E is proportional to flux hence induced e.m.f. also increases. Thus the characteristics of E against i.e. internal characteristics is of increasing nature. As I_a increases, armature reaction increases but its effect is negligible compared to increase in E. But for high load current, saturation occurs and flux remains constant. In such case, due to the armature reaction E starts decreasing as shown by dotted line in the Fig. 1.50.

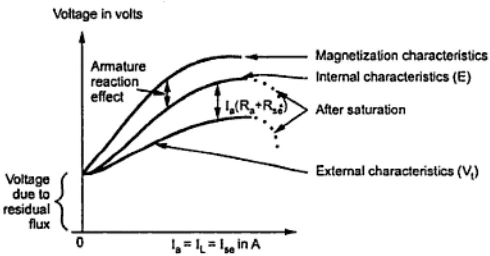


Fig. 1.50 Characteristics of d.c. series generator

Now as $I_L = I_a$ increases, thus the drop $I_a (R_a + R_{se})$ increases.

$$V_t = E - I_a (R_a + R_{se})$$

Thus the external characteristics is also of rising nature as E increases but it will be below internal characteristics due to drop $I_a (R_a + R_{se})$. This is shown in the Fig. 1.50.

In self excited series generator, open circuit characteristics cannot be obtained. In open circuit, $I_a = I_L = 0$ hence induced e.m.f. is zero. Thus open circuit characteristics is possible only by separately exciting the field winding. It is also shown in the Fig. 1.50.

In practice when there is no load $I_L = 0$, then there exists certain induced e.m.f. due to residual flux retained by the field winding.

Key Point: *Hence for zero load current, there is a voltage present and the internal and external characteristics do not pass through origin as shown in the Fig. 1.50.*

1.25 Load Characteristics of D.C. Compound Generator

The characteristics depends on whether generator is cumulatively compound or differentially compound generator. In cumulatively compound, $\phi_T = \phi_{sh} + \phi_{se}$. As load current increases, I_a increases hence I_{se} also increases producing more flux. Thus induced e.m.f. increases and terminal voltage also increases. But as I_a increases, the various voltage drops and armature reaction drop also increases. Hence there is drop in the terminal voltage.

If drop in V_t due to increasing I_L is more dominating than increase in V_t due to increase in flux then generator is called **under compounded** and its characteristics is dropping in nature, as shown in the Fig. 1.51.

If drop in V_t due to armature reaction and other drops is much less than increase in V_t due to increase in flux then generator is called **over compounded** and its characteristics is rising in nature, as shown in the Fig. 1.51. If the effects of the two are such that on full load current V_t is same as no load induced e.m.f. i.e. the effects are neutralising each other on full load then generator is called **flat compounded or level compounded.** Its characteristics is shown in the Fig. 1.51.

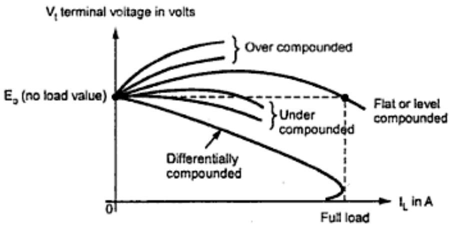


Fig. 1.51 Characteristics of compound generator

In differentially compound, $\phi_T = \phi_{sh} \sim \phi_{se}$. The net flux is difference between the two. As I_L increases, ϕ_{sh} is almost constant but ϕ_{se} increases rapidly. Hence the resultant flux ϕ_T reduces. Hence the induced e.m.f. E and hence the terminal voltage also decreases drastically. There is drop due to armature resistance, series field resistance, armature reaction due to which terminal voltage drops further. Thus we get the characteristics of such differentially compound generator as shown in the Fig. 1.51.

1.26 Voltage Building in Self Excited Generator

For every generator which is used as a self excited generator there must exist some residual magnetic flux. When armature rotates, conductors cut this small residual flux to produce the e.m.f. E_r. This e.m.f. drives small current through field winding. Thus field current I_f now produces more flux which is greater than residual flux. Hence more e.m.f. gets induced. This further drives more current through field to produce more flux. This process is cumulative and continues till rated voltage gets build up. This is shown in the Fig. 1.52

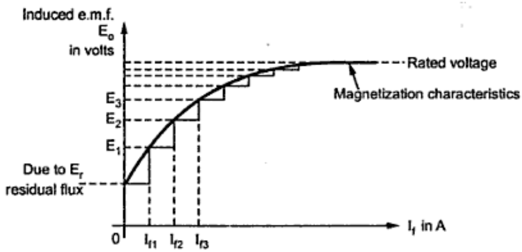


Fig. 1.52 Voltage building in self excited generator

1.26.1 Causes of Failure to Excite Self Excited Generator

The causes of failure, the method of detection and the corresponding remedy are given in the Table 1.1.

Sr. No.	Cause	Method of detection	Remedy
1.	Absence of residual magnetism due to ageing.	Zero reading on voltmeter after rotating armature.	Operate the generator as separately excited first and then as a self excited.
2.	Wrong field winding connections. Due to this, flux gets produced in opposite direction to residual flux. So residual flux cancels the main flux.	Voltmeter reading decreases rather than increasing as generator is started.	Interchange the field connections.
3.	Field resistance is more than the critical resistance.	Voltmeter shows zero reading.	Reduce the resistance of field circuit using proper field divertor.
4.	Generator is driven in opposite direction.	This wipes out the residual flux and fails to excite.	Drive the generator in proper direction.

Table 1.1

1.27 Critical Field Resistance in D.C. Shunt Generator

Consider the magnetization characteristics of a d.c. shunt generator shown in the Fig. 1.53

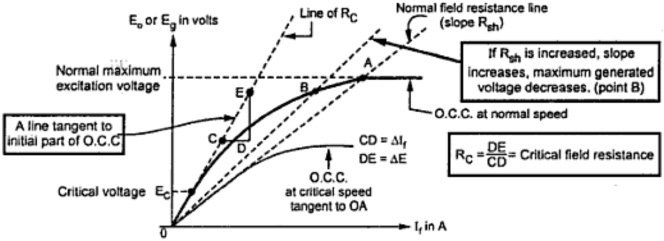


Fig. 1.53 Concept of critical resistance

The Fig. 1.53 shows that generator voltage builds in step till point A. This point is intersection of field resistance line with the open circuit characteristics (O.C.C.). The voltage corresponding to point A is the maximum voltage it can generate. If the slope of field resistance line is reduced by decreasing the field resistance, the maximum voltage generator can build will be higher than that corresponding to point A. Similarly if the slope of field resistance line is increased by increasing the field resistance, the maximum voltage generator can build will be less than that corresponding to point A i.e. corresponding to point B.

If now the slope of the field resistance line is increased in such a way that it becomes tangential to the lower part of the open circuit characteristics. The voltage corresponding to this point is E_C. This voltage is just sufficient to drive the current through field resistance so that cumulative process of building the voltage starts. This value of field resistance is called the **critical resistance** denoted as R_C, of the shunt field circuit at given speed.

Key Point : *If field circuit resistance is more than* R_C *at start then induced e.m.f. fail to drive current through field circuit and generator fails to excite at given speed.*

Thus we can define critical resistance as that resistance of the field circuit at a given speed at which generator just excites and starts voltage building while beyond this value generator fails to excite.

The critical resistance is the slope of the critical resistance line.

$$\therefore \quad R_C = \frac{\Delta E}{\Delta I_f} = \frac{DE}{CD} = \tan\theta$$

Similar to the critical resistance there is a concept of critical speed N_C. We know that $E \propto N$. As speed decreases the induced e.m.f. decreases and we get O.C.C. below the O.C.C. at normal speed. If we go on reducing the speed, at a particular speed we will get O.C.C. just tangential to normal field resistance line.

Key Point : *This speed at which the machine just excites for the given field circuit resistance is called the* ***critical speed*** *of a shunt generator denoted as* N_C.

1.27.1 Practical Determination of R_C

Generally data for plotting the open circuit characteristics is given. Plot the characteristics on the graph paper to the scale.

Draw the tangent, to the initial part of this O.C.C. then the slope of this line is the critical resistance for the speed at which the data is given.

Key Point: *If speed changes, then the O.C.C. changes hence the value of R_C changes.*

Now if R_C is asked at speed N_2, while data for O.C.C. is given at N_1. It is known that,

$$E_o \propto N$$

$$\therefore \quad \frac{E_{o1}}{E_{o2}} = \frac{N_1}{N_2}$$

$$\therefore \quad \boxed{E_{o2} = \frac{N_2}{N_1} E_{o1}}$$

Key Point: *Generate the data for O.C.C. at new speed and repeat the procedure to obtain* R_C.

➠ **Example 1.7 :** *The data for open circuit characteristics of a d.c. shunt generator driven at rated speed is given as,*

I_f	A	0.5	1.0	1.5	2.0	2.5	3.0	3.5
E_o	V	60	120	138	145	149	151	152

If resistance of field circuit is adjusted to 53 Ω calculate the open circuit voltage and load current when the terminal voltage is 100 V. Neglect the armature reaction and assume $R_a = 0.1\ \Omega$. *Use graph paper.*

Solution : Draw the open circuit characteristics on the graph paper from the given data.

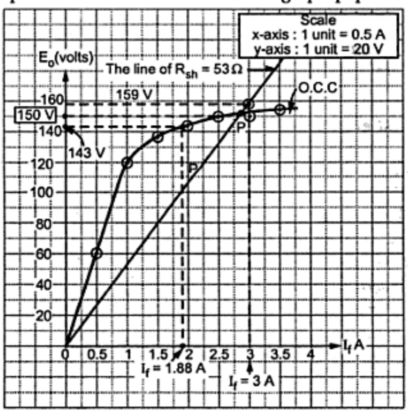


Fig. 1.54

Draw the line corresponding to $R_{sh} = 53\ \Omega$. To draw this line, consider an equation of line y = mx which passes through origin.

Now $y = E_o$, $x = I_f$ and m = slope = R_{sh}

$$\therefore \quad E_o = R_{sh}\, I_f$$

One point on this line is (0,0).

For second point on line consider $I_f = 3$ A say, then

$$E_o = R_{sh} \times 3 = 53 \times 3 = 159 \text{ V}$$

So second point is (3, 159).

Draw the line on graph paper passing through (0, 0) and (3, 159), till it intersects O.C.C. at P, as shown. The induced e.m.f. corresponding to point P is 150 V.

Thus machine has open circuit voltage of **150 V**. The terminal voltage $V_t = 100$ V. For a shunt generator,

$$I_{sh} = \frac{V_t}{R_{sh}} = \frac{100}{53} = 1.8867 \text{ A} = I_f$$

and $$E_o = V_t + I_a R_a$$

From graph, $$E_o = 143 \text{ V for } I_{sh} = I_f = 1.8867 \text{ A}$$

$$\therefore \quad 143 = 100 + I_a \times 0.1$$

$$\therefore \quad I_a = 430 \text{ A}$$

Now $$I_a = I_L + I_{sh} \qquad \text{... For shunt generator}$$

$$\therefore \quad I_L = I_a - I_{sh} = 430 - 1.8867 = \mathbf{428.1133\ A}$$

This is the load current when $V_t = 100$ V.

1.27.2 Critical Speed N_C

It is known that as speed changes, the open circuit characteristics also changes, similarly for different shunt field resistances, the corresponding lines are also different.

> **Key Point** : *The speed for which the given field resistance acts as critical resistance is called the critical speed, denoted as N_C.*

Thus if the line is drawn representing given R_{sh} then O.C.C. drawn for such a speed to which this line is tangential to the initial portion, is nothing but the critical speed N_C.

Graphically critical speed can be obtained for given R_{sh}. The steps are,

1. Drawn O.C.C. for given speed N_1.
2. Draw a line tangential to this O.C.C. say OA.
3. Draw a line representing the given R_{sh} say OP.
4. Select any field current say point R.
5. Draw vertical line from R to intersect OA at S and OP at T.
6. Then the critical speed N_C is,

While $\frac{RT}{RS} = \frac{N_C}{N_1}$ i.e. $N_C = N_1 \frac{RT}{RS}$

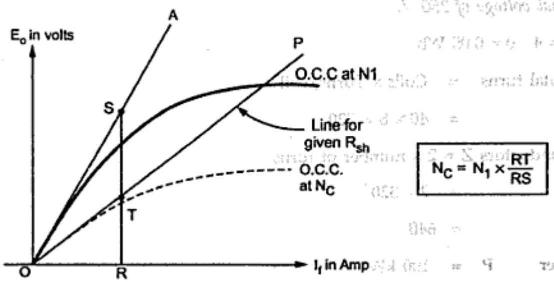


Fig. 1.55 Determine cirtical speed

1.28 Applications of Various Types of Generators

Separately excited generators :

As a separate supply is required to excite field, the use is restricted to some special applications like electro-plating, electro-refining of materials etc.

Shunt generators :

Commonly used in battery charging and ordinary lighting purposes.

Series generators :

Commonly used as boosters on d.c. feeders, as a constant current generators for welding generator and arc lamps.

Cumulatively compound generators :

These are used for domestic lighting purposes and to transmit energy over long distance.

Differential compound generators :

The use of this type of generators is very rare and it is used for special application like electric arc welding.

Examples with Solutions

➡ **Example 1.8 :** *The armature of a 4 pole, lap wound d.c. shunt generator has 40 coils with 8 turns per coil. Its shunt field resistance is 70 Ω and armature resistance of 0.03 Ω. If the flux per pole is 0.05 Wb, find the speed of the machine when supplying 100 kW at a terminal voltage of 250 V.*

Solution : $P = 4,\ \phi = 0.05$ Wb

$$\text{Total turns} = \text{Coils} \times \text{Turns/coil} = 40 \times 8 = 320$$

$$\therefore \text{Total conductors } Z = 2 \times \text{number of turns} = 2 \times 320 = 640$$

$$\text{Load power } P = 100 \text{ kW} = V_t \times I_L$$

$$\therefore \quad 100 \times 10^3 = 250 \times I_L \text{ as } V_t = 250 \text{ V}$$

$$\therefore \quad I_L = 400 \text{ A}$$

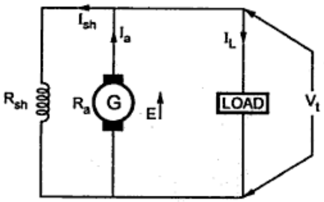


Fig. 1.56

Consider d.c. shunt generator as shown in the Fig. 1.56

$$I_{sh} = \frac{V_t}{R_{sh}} = \frac{250}{70} = 3.333 \text{ A}$$

$$I_a = I_L + I_{sh} = 400 + 3.333 = 403.333 \text{ A}$$

Now $E = V_t + I_a R_a$

$= 250 + 403.333 \times 0.03$

$= 262.1$ V

and $E = \frac{\phi P N Z}{60 A}$ (A = P = 4 as lap)

$\therefore \quad 262.1 = \frac{0.05 \times 4 \times N \times 640}{60 \times 4}$

$\therefore \quad N = 491.437$ **r.p.m.**

Example 1.9 : *A long shunt d.c. compound generator drives 20 lamps, all are connected in parallel. Terminal voltage is 550 V with each lamp resistance as 500 Ω. If R_{sh} = 25 Ω, R_a = 0.06 Ω and R_{se} = 0.04 Ω, calculate the armature current and the generated e.m.f.*

Solution : Consider the arrangement as shown in the Fig. 1.57.

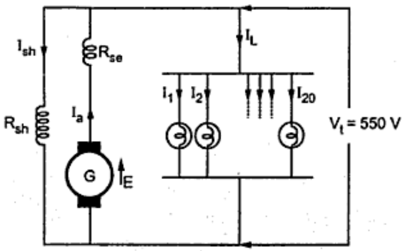


Fig. 1.57

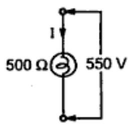


Fig. 1.58

As all lamps are in parallel, the voltage across all of them is same which is terminal voltage of generator V_t = 550 V. Consider only one lamp as shown in the Fig. 1.58.

So current drawn by each lamp is

$$I = \frac{V_t}{R_{lamp}} = \frac{550}{500} = 1.1 \text{ A}$$

Such 20 lamps are used as a load.

$\therefore \quad I_L = 20 \times I_{lamp} = 20 \times 1.1 = 22$ A

Now $I_{sh} = \frac{V_t}{R_{sh}} = \frac{550}{25} = 22\text{ A}$

$\therefore$ $I_a = I_L + I_{sh} = 44\text{ A}$

$\therefore$ $E = V_t + I_a R_a + I_a . R_{se} = 550 + 44 \times 0.06 + 44 \times 0.04$

$= 554.4\text{ V}$

Example 1.10 : *A 4 pole, lap wound 750 r.p.m. d.c. shunt generator has an armature resistance of 0.4 Ω and field resistance of 200 Ω. The armature has 720 conductors and the flux per pole is 30 mWb. If the load resistance is 15 Ω, determine the terminal voltage.*

Solution : Consider the generator as shown in the Fig. 1.59.

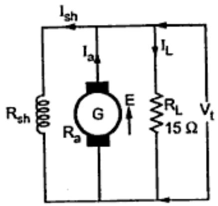


Fig. 1.59

$P = 4, \quad A = P = 4$

$N = 750\text{ r.p.m.}$

$\phi = 30\text{ mWb} = 30 \times 10^{-3}\text{ Wb}, \quad Z = 720$

$\therefore$ $E = \frac{\phi P N Z}{60\, A}$

$= \frac{30 \times 10^{-3} \times 4 \times 750 \times 720}{60 \times 4}$

$= 270\text{ V}$

$E = V_t + I_a R_a$

Now $V_t = I_L \times R_L$ i.e. $I_L = \frac{V_t}{R_L}$

and $I_{sh} = \frac{V_t}{R_{sh}}$

$$I_a = I_L + I_{sh}$$
$$= \frac{V_t}{R_L} + \frac{V_t}{R_{sh}}$$

Substituting in voltage equation,

$$E = V_t + \left[\frac{V_t}{R_L} + \frac{V_t}{R_{sh}}\right] R_a$$

$\therefore$ $$270 = V_t + \left[\frac{V_t}{15} + \frac{V_t}{200}\right] 0.4$$

$\therefore$ $$270 = 1.0286\ V_t$$

$\therefore$ $$V_t = \mathbf{262.4757\ V}$$

Example 1.11 : *A 20 kW, 200 V shunt generator has an armature resistance of 0.05 Ω and a shunt field resistance of 200 Ω. Calculate the power developed in the armature when it delivers rated output.* **[May-2004 (Set-1), Dec.-2004 (Set-1)]**

Solution : The generator is shown in the Fig. 1.60

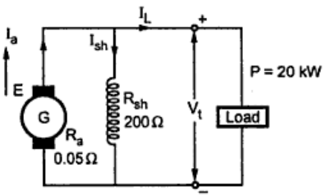


Fig. 1.60

$$I_a = I_{sh} + I_L$$
$$V_t = 200\ V$$
$$P = V_t\ I_L$$

$\therefore$ $$20 \times 10^3 = 200 \times I_L$$

$\therefore$ $$I_L = 100\ A$$

and $$I_{sh} = \frac{V_t}{R_{sh}} = \frac{200}{200} = 1\ A$$

$\therefore$ $$I_a = I_{sh} + I_L = 101\ A$$

Now $$E = V_t + I_a R_a = 200 + 101 \times 0.05 = 202.05\ V$$

Thus the power developed in the armature is

$$P_a = E \times I_a = 205.05 \times 101$$

$$= \mathbf{20.71005\ kW}$$

Example 1.12 : *The open circuit characteristic of a shunt generator at 800 r.p.m. gives :*

Field current (A)	0	0.5	1.0	2.0	3.0	4.0	5.0
Induced e.m.f. (V)	10	50.0	100.0	175.0	220.0	245.0	262.0

Find graphically the critical resistance of shunt field circuit. If the field circuit resistance is changed to 75 Ω, what will be the critical speed for the machine to build up?

[May-2004 (Set-4)]

Solution : From the given data, draw the open circuit characteristics, (O.C.C.) on the graph, to the scale as shown in the Fig. 1.61

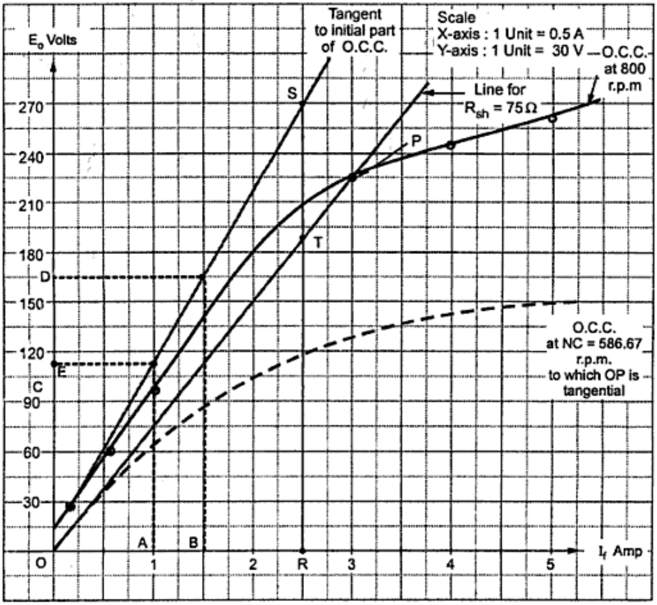


Fig. 1.61

Draw the tangent to the inital part of this O.C.C.

The slope of this tangent line i.e. $\tan\theta$ is the critical resistance R_C of the shunt field resistance.

$$\text{Slope of tangent line} = \frac{CD}{AB} = \frac{54}{0.5} = 108\ \Omega$$

$$\therefore \quad R_C = \text{Critical field resistance} = \mathbf{108\ \Omega\ at\ 800\ r.p.m.}$$

Now draw a line for $R_{sh} = 75\ \Omega$ so slope of the line is 75 and hence the equation is,

$$E_o = R_{sh}\, I_f = 75\, I_f$$

$$\therefore \quad E_o = 75 \times 3 = 225\ V \text{ at } I_f = 3\ A$$

So plotting the point P(3,225) and joining to origin we get line corresponding to $R_{sh} = 75\ \Omega$, which is line OP.

Take any covenient point R on x axis say at $I_f = 2.5$ A. Draw vertical till it intersects critical R_C line and $R_{sh} = 75\ \Omega$ line at points S and T respectively.

$$\text{Then} \quad \frac{RT}{RS} = \frac{N_C}{800} \text{ i.e. } \frac{198}{270} = \frac{N_C}{800}$$

$$\therefore \quad N_C = \mathbf{586.67\ r.p.m.} \quad \text{...Critical speed for } R_{sh} = 75\ \Omega$$

Example 1.13 : *A 4 pole, long shunt, lap wound generator supplies 25 kW at a terminal voltage of 500 V. The armature resistance is 0.03 Ω, series field resistance is 0.04 Ω and shunt field resistance is 200 Ω. The brush drop may be taken as 1 V. Determine the e.m.f. generated.* **[Dec.-2003 (Set-2)]**

Solution : The generator is shown in the Fig. 1.62

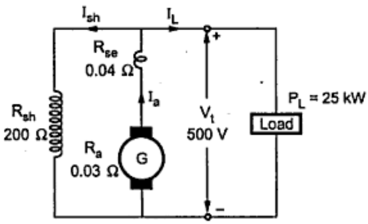


Fig. 1.62

$$V_{brush} = 1\ V$$

$$P = 4, \text{ lap i.e. } A = P$$

$$P_L = \text{load power} = V_t I_L,$$

$$\therefore \quad 25 \times 10^3 = 500 \times I_L$$

$$\therefore \quad I_L = 50\ A$$

Same voltage V_t is across R_{sh}.

$$\therefore \quad I_{sh} = \frac{V_t}{R_{sh}} = \frac{500}{200} = 2.5\ A$$

$$\therefore \quad I_a = I_L + I_{sh} = 50 + 2.5 = 52.5\ A$$

Same I_a flows through R_{se}.

$$\therefore \quad E = V_t + I_a R_a + I_a R_{se} + V_{brush}$$

$$= 500 + 52.5 \times 0.03 + 52.5 \times 0.04 + 1 \times 2 \quad \text{...2 brushes}$$

$$= 505.675 \text{ V} \quad \text{...Generated e.m.f.}$$

Example 1.14 : *The magnetization curve of a d.c. shunt generator running at 1000 r.p.m. is as follows:*

Field Amperes :	*(A) 0.25*	*0.5*	*1.0*	*1.5*	*2.0*	*2.5*	*3.0*
EMF Volts :	*(V) 36.0*	*72.0*	*138.0*	*188.0*	*225.0*	*250.0*	*270.0*

Find (i) The value of field resistance to give 240 V on no-load (ii) The speed at which the generator just fails to build up. **[Dec.-2003 (Set-3)]**

Solution : Construct the open circuit characteristics to the scale on the graph paper, as shown in the Fig. 1.63.

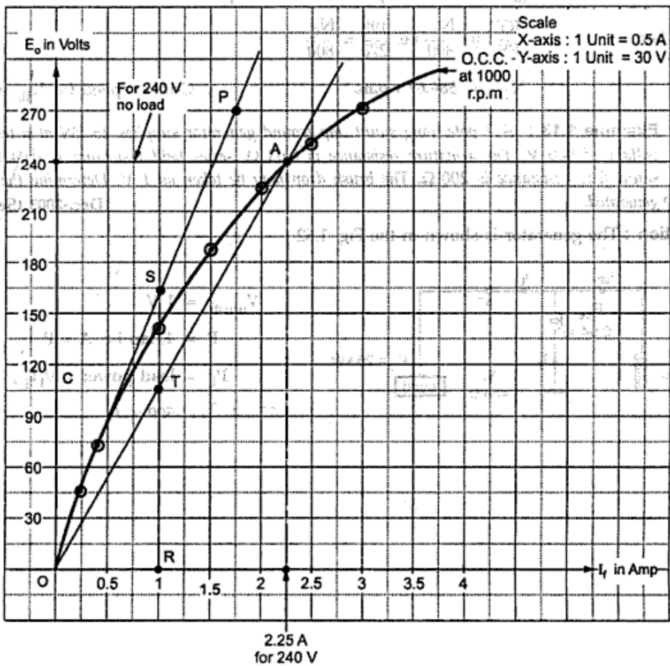


Fig. 1.63

i) On no load, E_0 = 240 V. Draw horizontal line from 240 V, to intersect O.C.C. at A. Then corresponding I_f on x-axis is **2.25 A**. The slope of line OA is corresponding R_{sh}

$$\therefore \quad R_{sh} = \frac{240}{2.25} = \mathbf{106.67\ \Omega}$$

ii) Draw a line OP tangential to the O.C.C. at N_1 = 1000 r.p.m. Select I_f = 1 A i.e. point R. Draw vertical from R to intersect OP at S and OA at T. Thus the speed at which the generator just fails to build up is its critical speed N_C.

$$\therefore \quad \frac{RT}{RS} = \frac{N_C}{N_1}$$

$$\therefore \quad N_C = \frac{RT}{RS} \times N_1 = \frac{105}{159} \times 1000 = \mathbf{660.38\ r.p.m.}$$

Example 1.15 : *The armature of 6 pole d.c. generator has a wave winding containing 664 conductors. Calculate the generated e.m.f. when flux per pole is 0.06 weber and the speed is 250 r.p.m. At what speed must the armature be driven to generate an e.m.f. of 250 V if the flux per pole is reduced to 0.058 weber ?*

[June-2003 (Set-1), Dec.-2004 (Set-2), March-2006(Set-1)]

Solution : P = 6, wave so A = 2, Z = 664, ϕ = 0.06 Wb, N_1 = 250 r.p.m.

$$E_1 = \frac{\phi_1 PZN_1}{60\,A} = \frac{0.06 \times 6 \times 664 \times 250}{60 \times 2}$$

$$= \mathbf{498\ V} \qquad \text{...Generated e.m.f.}$$

New flux ϕ_2 = 0.058 Wb and E_2 = 250 V

$$\therefore \quad E_2 = \frac{\phi_2 PZN_2}{60\,A}$$

$$\therefore \quad 250 = \frac{0.058 \times 6 \times 664 \times N_2}{60 \times 2}$$

$$\therefore \quad N_2 = \mathbf{129.83\ r.p.m.} \qquad \text{...New speed}$$

Example 1.16 : *A short shunt compound generator delivers a load current of 30 A at 220 V and has armature, series field and shunt field resistances or 0.05 Ω, 0.03 Ω and 200 Ω respectively. Calculate the induced e.m.f. and the armature current. Allow 1.0 V per brush for contact drop.* **[June-2003 (Set-3)]**

Solution : $I_L = 30$ A, $V_t = 220$ V, $R_a = 0.05\ \Omega$, $R_{se} = 0.03\ \Omega$, $R_{sh} = 200\ \Omega$.

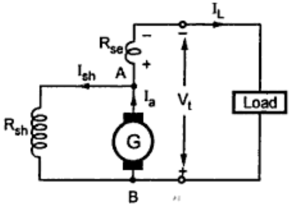


Fig. 1.64

$$V_{AB} = \text{Drop across shunt field}$$

$$= E - I_a R_a - V_{brush}$$

Also $$V_{AB} = V_t + I_L R_{se}$$

$$= 220 + 30 \times 0.03$$

$$= 220.9 \text{ V}$$

$\therefore$ $$220.9 = E - I_a R_a - 2 \quad \ldots V_{brush} = 1 \text{ V/brush}$$

Now $$I_{sh} = \frac{V_{AB}}{R_{sh}}$$

$$= \frac{220.9}{200} = 1.1045 \text{ A}$$

$\therefore$ $$I_a = I_{sh} + I_L = 1.1045 + 30 = \mathbf{31.1045\ A}$$

$\therefore$ $$E = 220.9 + 31.1045 \times 0.05 + 2 = \mathbf{224.4552\ V}$$

Example 1.17 : *The magnetization characteristics of a shunt generator at 1,000 r.p.m. is as follows :*

OC Volts	(A)	62.5	107.5	155.0	196.5	231.0	256.0	275.0	287.5
Field Amperes	(V)	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0

Estimate the no-load terminal voltage of the machine when run at 800 r.p.m. with 30 ohms field circuit resistance. **[June-2003 (Set-4), Dec.-2003 (Set-3)]**

Solution : The table is given for $N_1 = 1000$ r.p.m. while E_0 required for $N_2 = 800$ r.p.m, so obtain new table for O.C.C. using $E \propto N$.

$\therefore$ $$\frac{E_1}{E_2} = \frac{N_1}{N_2}$$

$\therefore$ $$E_2 = \frac{N_2}{N_1} \quad E_1 = \frac{800}{1000} \times E_1 = 0.8\ E_1$$

O.C. Volts	50	86	124	157.2	184.8	204.8	220	230
I_F Amp.	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0

Obtain the O.C.C. with new table to the scale on the graph paper as shown in the Fig. 1.65

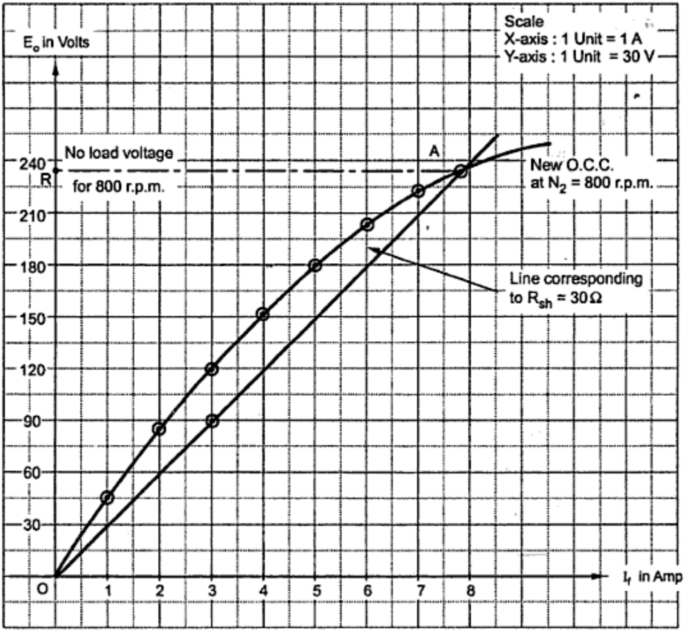


Fig. 1.65

Now R_{sh} = 30 Ω, so equation of corresponding line is $E_0 = R_{sh}\ I_f$

So at I_f = 3 A, $E_0 = 30 \times 3 = 90$ V. So two points are (0,0) and (3, 90). Line joining these two points is line for R_{sh} = 30 Ω, which intersects O.C.C. at A as shown. The corresponding E_0 = OR = **234 V** from graph is the no load voltage of machine at 800 r.p.m.

Example 1.18 : *A 6 pole, wave wound d.c. generator, running at a speed of 300 r.p.m. generates an induced e.m.f. of 535 volts. Calculate the flux/pole, if it has 650 conductors.*

[Jan.-2003 (Set-1)]

Solution : P = 6, wave so A = 2, N = 300 r.p.m., E = 535 V, Z = 650

For generator, $$E = \frac{\phi PNZ}{60\ A}$$

$$\therefore \quad 535 = \frac{\phi 6 \times 300 \times 650}{60 \times 2}$$

$$\therefore \quad \phi = \mathbf{54.872\ mWb} \qquad \text{...Flux per pole}$$

Example 1.19 : *The armature of a 6 pole d.c. generator having 650 conductors generates an induced e.m.f. of 536.25 volts when running at a speed of 300 r.p.m., the flux per pole being 55 mWb. What is the type of the simplex winding used?* **[Jan.-2003 (Set-2)]**

Solution : P = 6, Z = 650, E = 536.25 V, N = 300 r.p.m., ϕ = 55 mWb

For generator $$E = \frac{\phi PNZ}{60A}$$

$$\therefore \quad 536.25 = \frac{55\times10^{-3}\times6\times300\times650}{60\times A}$$

$$\therefore \quad A = 2$$

As A = 2, the simplex winding used is of **wave type.**

Example 1.20 : *A certain wave-wound d.c. generator, running at a speed of 300 r.p.m. is to generate an induced e.m.f. of about 535 volts, the flux/pole being 0.055 Wb. Determine the number of poles if the number of conductors is 650.*

[Jan.-2003 (Set-3), Nov.-2006 (Set-4)]

Solution : N = 300 r.p.m., E = 535 V, ϕ = 0.055 Wb, Z = 650, A = 2 as wave

For generator, $$E = \frac{\phi PNZ}{60\ A}$$

$$\therefore \quad 535 = \frac{0.055\times P\times300\times650}{60\times2}$$

$$\therefore \quad P = 5.986 \approx \mathbf{6}$$ ...Number of poles

Example 1.21 : *A 6 pole, wave-wound d.c. generator has 650 conductors the flux per pole is 0.05 Wb. Calculate the speed at which it is to be driven to generate an e.m.f. of 550 volts.* **[Jan.-2003 (Set-4)]**

Solution : P = 6, A = 2 as wave, Z = 650, ϕ = 0.05 Wb, E = 550 V

for Generator, $$E = \frac{\phi PNZ}{60A}$$

$$\therefore \quad 550 = \frac{0.05\times6\times N\times650}{60\times2}$$

$$\therefore \quad N = \mathbf{338.461\ r.p.m.}$$ ...Speed

Example 1.22 : *A shunt generator running at a speed of 1000 r.p.m. gave the following magnetization curve :*

E.M.F., V :	95	179	224	251	272	281
Field Current, A :	1.0	2.0	3.0	4.0	5.0	6.0

If the field circuit resistance is 60 Ω, determine

i) The voltage to which the machine will build up running at the same speed.

ii) The value of the field regulating resistance if the machine is to build up to 130 V, when its field coils are grouped into two parallel circuits and generator is run at 500 r.p.m.

[Dec.-2005 (Set - 2)]

Solution : Draw the open circuit characteristics on the graph paper from the given data.

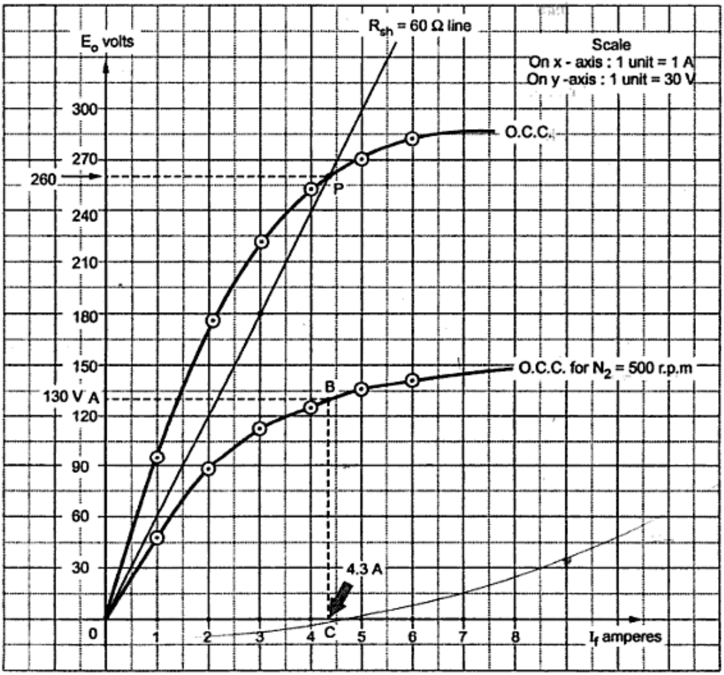


Fig. 1.66

Draw the line corresponding to $R_{sh} = 60\ \Omega$. For this, consider an equation of line y = mx, passing through the origin.

$y = E_o \qquad x = I_f \qquad \text{and} \qquad m = \text{slope} = R_{sh}$

$\therefore \qquad E_o = R_{sh}\, I_f$

Consider $I_f = 3$ A, $E_o = 60 \times 3 = 180$ V

Thus this line passes through origin and (3, 180) point.

This line intersects O.C.C. at point P as shown in the Fig. 1.66. The induced e.m.f. corresponding to point P is 260 V.

i) Thus the voltage to which the machine will build up running at the same speed is **260 V.**

ii) Now generator runs at $N_2 = 500$ r.p.m.

$$\therefore \qquad E_{o2} = \frac{N_2}{N_1} = E_{o1} = \frac{500}{1000} E_{o1} = \frac{E_{o1}}{2}$$

E_{o2} (V)	47.5	89.5	112	125.5	136	140.5
I_f (A)	1	2	3	4	5	6

Draw new O.C.C. for $N_2 = 500$ r.p.m. from above readings. Draw horizontal line of 130 V from A to meet new O.C.C. at B. Draw vertical line from B to meet I_f axis at C. The corresponding current is 4.3 A.

This is the current through each parallel path.

$\therefore \qquad I_f \text{ (total)} = 4.3 \times 2 = 8.6 \text{ A}$

$$\therefore \qquad R'_{sh} = \frac{E_o}{I_f \text{ (total)}} = \frac{130}{8.6} = 15.1162\ \Omega$$

While $\quad R_{sh} = 30 \parallel 30 = 15\ \Omega$

The original 60 Ω R_{sh} is divided into two groups i.e. each of 30 Ω and then connected in parallel.

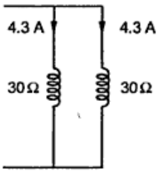


Fig. 1.67

$\therefore$ Field regulating resistance $= 15.1162 - 15 =$ **0.1162 Ω**

Example 1.23 : *The wave connected armature of a four pole d.c. generator is required to generate an e.m.f. of 520 V when driven at 600 r.p.m. Calculate the flux per pole required if the armature has 144 slots with two coil sides per slot, each coil consisting of 3 turns.*

[Nov.-2008 (Set-2)]

Solution : P = 4, E_g = 520 V, N = 600 r.p.m., Wave i.e. A = 2, Slots = 144

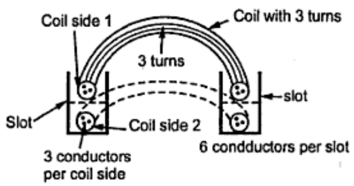


Fig. 1.68

For each coil, there is one coil side in one slot and other coil side in another slot. Such two coil sides exist in each slot as shown. Such an arrangement is called **double layer winding.** So each slot has two coil sides and each coil side has 3 conductors as each coil has 3 turns. So there are 6 conductors per slot and such 144 slots exist.

$$Z = 144 \times 6 = 864$$

$$\therefore \quad E_g = \frac{\phi PNZ}{60\ A} \quad \text{i.e.} \quad 520 = \frac{\phi \times 4 \times 600 \times 864}{60 \times 2}$$

$$\therefore \quad \phi = 0.03\ \text{Wb} = \mathbf{30\ mWb}$$

Example 1.24 : *A 4 pole generator has 48 slots and 8 conductors per slot. The useful flux per pole is 30 mWb and speed is 800 r.p.m. Find the generated e.m.f., if the machine is wave connected.* **[Nov.-2008 (Set-4)]**

Solution : P = 4, ϕ = 30 mWb, wave i.e. A = 2, N = 800 r.p.m. 48 slots, 8 conductors per slot

$$Z = \text{Slots} \times \text{Conductors / Slot} = 48 \times 8 = 384$$

$$E_g = \frac{\phi PNZ}{60\ A} = \frac{30 \times 10^{-3} \times 4 \times 800 \times 384}{60 \times 2} = \mathbf{307.2\ V}$$

Example 1.25 : *A separately excited d.c. generator has no load voltage of 120 V at a field current of 2 A , when driven at 1500 r.p.m. Assuming that it is operating on the straight line portion of its saturation curve, calculate :*

i) Generated voltage when the field current is increased to 2.5 A.

ii) Generated voltage when the speed is reduced to 1400 r.p.m. and the field current is increased to 2.84 A. **[May-2008 (Set-3)]**

Solution : For a separately excited d.c. generators,

$$E_g \propto N\phi \propto N\,I_f \qquad \text{...}\phi \propto I_f \text{ (Field current)}$$

i) $I_{f2} = 2.5$ A, speed is constant.

$$\therefore \quad \frac{E_{g1}}{E_{g2}} = \frac{I_{f1}}{I_{f2}} \quad \text{and} \quad E_{g1} = 120 \text{ V for } I_{f1} = 2 \text{ A (Given)}$$

$$\therefore \quad \frac{120}{E_{g2}} = \frac{2}{2.5}$$

$$\therefore \quad E_{g2} = \mathbf{150\ V}$$

ii) $I_{f3} = 2.84$ A, $N_3 = 1400$ r.p.m.

$$\therefore \quad \frac{E_{g1}}{E_{g3}} = \frac{N_1}{N_3} \times \frac{I_{f1}}{I_{f3}} \quad \text{i.e.} \quad \frac{120}{E_{g3}} = \frac{1500}{1400} \times \frac{2}{2.84}$$

$$\therefore \quad E_{g3} = \mathbf{159.04\ V}$$

Example 1.26 : *A series generator having combined armature and field resistance of 0.4 Ω, is running at 1000 r.p.m. and delivering 5.5 kW at a terminal voltage of 110 V. If the speed is raised to 1500 r.p.m. and load is adjusted to 10 kW, find the new current and terminal voltage. Assume the machine is working on the straight line portion of the magnetization characteristics.* **[May-2008 (Set-1, 2)]**

Solution : $N_1 = 1000$ r.p.m., $V_{t1} = 110$, $P_1 = 5.5$ kW, $R_a + R_{se} = 0.4\ \Omega$

$$\therefore \quad I_{l1} = \frac{P_1}{V_{t1}} = \frac{5.5 \times 10^3}{110} = 50 \text{ A} = I_{a1} \qquad \text{...Series machine}$$

$$\therefore \quad E_{g1} = V_{t1} + I_{a1}(R_a + R_{se}) = 110 + 50\,(0.4) = 130 \text{ V}$$

$$E_g \propto N\,\phi \propto N\,I_a \qquad \text{...}\phi \propto I_a \text{ for series}$$

$$\therefore \quad \frac{E_{g1}}{E_{g2}} = \frac{N_1}{N_2} \times \frac{I_{a1}}{I_{a2}} \quad \text{where } N_2 = 1500 \text{ r.p.m.} \qquad \text{...(1)}$$

$$E_{g2} = V_{t2} + I_{a2}(R_a + R_{se}) \qquad \text{...(2)}$$

$$\text{and } I_{a2} = \frac{P_2}{V_{t2}} \quad \text{where } P_2 = 10 \text{ kW given} \qquad \text{...(3)}$$

Using equation (3) in equation (2),

$$E_{g2} = V_{t2} + \frac{10 \times 10^3 \times 0.4}{V_{t2}} = V_{t2} + \frac{4000}{V_{t2}}$$

Using equation (3) and (4) in equation (1),

$$\frac{130}{\left[V_{t2}+\frac{4000}{V_{t2}}\right]} = \frac{1000}{1500}\times\frac{50}{\left[\frac{10\times10^3}{V_{t2}}\right]}$$

$$\therefore \quad \frac{130V_{t2}}{V_{t2}^2+4000} = 3.333\times10^{-3}\,V_{t2}$$

$$\therefore \quad V_{t2}^2 + 4000 = 390000 \quad \text{i.e.} \quad V_{t2}^2 = 35000$$

$$\therefore \quad V_{t2} = \mathbf{187.0828\ V} \qquad \text{...New terminal voltage}$$

$$\therefore \quad I_{a2} = \frac{10\times10^3}{187.0828} = \mathbf{53.4522\ A} \qquad \text{...New current}$$

Example 1.27 : *A 4 pole generator having wave wound armature winding has 48 slots with 20 conductors in each slot. What will be the voltage generated in the machine when driven at 1500 r.p.m. assuming flux per pole to be 7 mWb ?* **[Nov.-2008 (Set-2)]**

Solution : P = 4, 48 slots, 20 conductors/slots, N= 1500 r.p.m., ϕ = 7 mWb

$$A = 2 \qquad \text{...Wave wound}$$

$$Z = \text{Slots}\times[\text{Conductors/Slot}] = 48\times20 = 960$$

$$E_g = \frac{\phi PNZ}{60\,A} = \frac{7\times10^{-3}\times4\times1500\times960}{60\times2}$$

$$= \mathbf{336\ V} \qquad \text{... Voltage generated}$$

Example 1.28 : *A 4 pole machine running at 1000 r.p.m. has an armature with 90 slots having 6 conductors per slot. The flux per pole is 6 × 10^{-2} Wb. Determine the induced e.m.f. as a d.c. generator if the coils are lap connected. If the current per conductor is 50 A, determine the electrical power output of the machine.* **[Nov.-2004 (Set-3), Nov.-2003 (Set-4)]**

Solution : P = 4, N = 1000 r.p.m., Slots = 90, 6 conductors/slot,

$$\phi = 6\times10^{-2}\text{ Wb, A = P as lap connected,}$$

$$Z = \text{Slots}\times[\text{Conductors/slot}] = 90\times6 = 540$$

$$E_g = \frac{\phi PNZ}{60\,A} = \frac{60\times10^{-2}\times4\times1000\times540}{60\times4} = \mathbf{540\ V}$$

$$A = P = 4 \qquad \text{...Parallel paths}$$

Current per conductor means current per parallel path is 50 A as all the conductors in a parallel path are in series.

$\therefore$ I = [Current/ Conductor] × Parallel path = 50 × 4 = 200 A

$\therefore$ Electrical output = $E_g \times I$ = 540 × 200 = **108 kW**

Review Questions

1. *Explain with a neat sketch, the construction of a d.c. machine.*
2. *Which part of a d.c. machine is laminated ? Why ?*
3. *What is the basic nature of the induced e.m.f. in a d.c. generator ? What is the function of a commutator?*
4. *What is the difference between lap type and wave type of armature winding ?*
5. *Derive from first principles an expression for the e.m.f. of a d.c generator.*
6. *State the different types of d.c. generators and state the applications of each type.*
7. *In a particular d.c. machine, if $P = 8$, $Z = 400$, $N = 300$ r.p.m. and $\phi = 100$ mWb, calculate generated e.m.f. if winding is connected in (i) Lap fashion (ii) Wave fashion.*

   (Ans. : (i) 200 V (ii) 800 V)
8. *A 110 V, d.c. shunt generator delivers a load of 50 A. The armature resistance is 0.2 Ω and field resistance is 55 Ω. The generator is driven at 1800 r.p.m. It has 6 poles with 360 conductors connected in lap fashion. Calculate*

   i) The no load voltage (ii) The flux per pole. **(Ans. : (i) 120.4 V (ii) 0.011 Wb)**
9. *A long shunt compound generator delivers a load current of 50 A at 500 V. It has armature, series field and shunt field resistance of 0.05 Ω, 0.03 Ω and 250 Ω respectively. Calculate the generated e.m.f.* **(Ans. : 504.16 V)**
10. *A d.c. machine has 8 poles, lap connected armature with 960 conductors and flux per pole is 40 mWb. It is driven at 400 r.p.m. Calculate the generated e.m.f. If now lap connected armature is replaced by wave connected, calculate the speed at which it should be driven to generate 400 V.*

    (Ans. : 250 V, 156 r.p.m.)
11. *A 4 pole, 100 V d.c. shunt generator with lap connected armature having field and armature resistances of 50 Ω and 0.1 Ω respectively, supplied 60, 100 V, 40 W lamps. All lamps are connected in parallel. Calculate the total armature current and generated e.m.f. Assume brush drop to be 1 V/brush.* **(Ans. : 26 A, 104.6 V)**
12. *A short shunt compound d.c. generator supplied 7.5 kW at 230 V. The shunt field, series field and armature resistances are 100 Ω, 0.3 Ω and 0.4 Ω respectively. Calculate the induced e.m.f. and the load resistance.* **(Ans. : 253.8 V, 7 Ω)**
13. *A shunt generator delivers 450 A at 230 V and resistances of shunt field and armature windings are 50 Ω and 0.03 Ω respectively. Calculate the generated e.m.f.* **(Ans. : 243.638 V)**
14. *A 20 kW short shunt compound generator works on full load with terminal voltage of 250 V. Assume $R_a = 0.05\ \Omega$, $R_{se} = 0.025\ \Omega$ and $R_{sh} = 100\ \Omega$, calculate the generated e.m.f.*

    (Ans. : 256.126 V)
15. *A long shunt compound generator delivers 50 A load at 500 V. Find induced e.m.f. and the armature current. Assume $R_a = 0.05\ \Omega$, $R_{se} = 0.03\ \Omega$ and $R_{sh} = 250\ \Omega$ and brush drop of 1 V/brush. If flux per pole is 25 mWb with 250 number of armature conductors. Calculate speed of the prime mover. The machine has 6 poles with wave type armature winding.*

If same generator is connected as short shunt type, to supply same load at same voltage, at what speed it should be driven. **(Ans. : 506.16 V, 52 A, 1619.71 r.p.m., 1619.52 r.p.m.)**

16. *A separately excited generator with a speed of 1200 r.p.m., supplies a load of 200 A at 125 V. What will be its new armature current if speed is changed to 1000 r.p.m. keeping field excitation constant. Assume $R_a = 0.04\ \Omega$ and total brush drop of 2 V. Assume load resistance constant.* **(Ans. : 166.16 A)**
17. *Draw and explain magnetization characteristics of generators.*
18. *Which are the load characteristics of a generator ?*
19. *Draw and explain load characteristics of d.c. shunt generator.*
20. *Draw and explain load characteristics of d.c. series generator.*
21. *Draw and explain load characteristics of d.c. compound generator.*
22. *Explain the procedure of voltage building in self excited generator.*
23. *State the possible causes of failure of excitation of self excited generator.*
24. *How to find critical field resistance and critical speed for a generator?*
25. *Draw and explain the characteristics of separately excited d.c. generator.*

❑❑❑

2 D.C. Motors

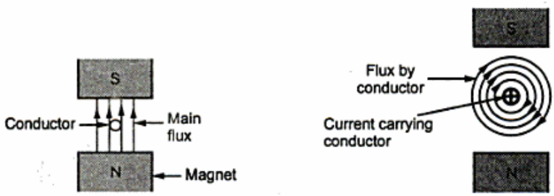
2.1 Introduction

In the last chapter we have discussed about the generators. Now let us see in detail, the various aspects of d.c. motors. A **motor** is a device which converts an electrical energy into the mechanical energy. The energy conversion process is exactly opposite to that involved in a d.c. generator. In a generator the input mechanical energy is supplied by a prime mover while in a d.c. motor, input electrical energy is supplied by a d.c. supply. The construction of a d.c. machine is same whether it is a motor or a generator, as discussed in the last chapter.

2.2 Principle of Operation of a D.C. Motor

The principle of operation of a d.c. motor can be stated in a single statement as 'when a current carrying conductor is placed in a magnetic field; it experiences a mechanical force'. In a practical d.c. motor, field winding produces a required magnetic field while armature conductors play a role of a current carrying conductors and hence armature conductors experience a force. As conductors are placed in the slots which are on the periphery, the individual force experienced by the conductors acts as a twisting or turning force on the armature which is called a **torque**. The torque is the product of force and the radius at which this force acts. So overall armature experiences a torque and starts rotating. Let us study this motoring action in detail.

Consider a single conductor placed in a magnetic field as shown in the Fig. 2.1 (a). The magnetic field is produced by a permanent magnet but in a practical d.c. motor it is produced by the field winding when it carries a current.



(a) Conductor in a magnetic field **(b) Flux produced by current carrying conductor**

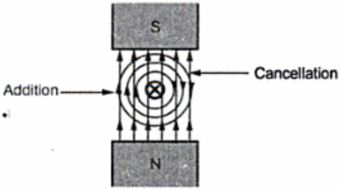
Fig. 2.1

Now this conductor is excited by a separate supply so that it carries a current in a particular direction. Consider that it carries a current away from an observer as shown in the Fig. 2.1 (b). Any current carrying conductor produces its own magnetic field around it, hence this conductor also produces its own flux, around. The direction of this flux can be determined by right hand thumb rule. For direction of current considered, the direction of flux around a conductor is clockwise. For simplicity of understanding, the main flux produced by the permanent magnet is not shown in the Fig. 2.1 (b).

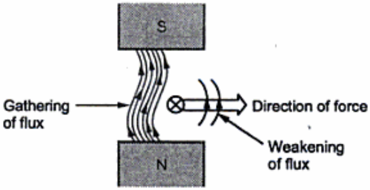
Now there are two fluxes present,

1. The flux produced by the permanent magnet called main flux.
2. The flux produced by the current carrying conductor.

These are shown in the Fig. 2.2 (a). From this, it is clear that on one side of the conductor, both the fluxes are in the same direction. In this case, on the left of the conductor there is gathering of the flux lines as two fluxes help each other. As against this, on the right of the conductor, the two fluxes are in opposite direction and hence try to cancel each other. Due to this, the density of the flux lines in this area gets weakened. So on the left, there exists high flux density area while on the right of the conductor there exists low flux density area as shown in the Fig. 2.2 (b).



(a) Interaction of two fluxes



(b) Force experienced by the conductor

Fig. 2.2

This flux distribution around the conductor acts like a stretched rubber band under tension. This exerts a mechanical force on the conductor which acts from high flux density area towards low flux density area, i.e. from left to right for the case considered as shown in the Fig. 2.2 (b).

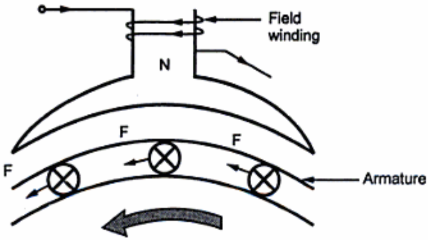


Fig. 2.3 Torque exerted on armature

Key Point : *In the practical d.c. motor, the permanent magnet is replaced by a field winding which produces the required flux called main flux and all the armature conductors, mounted on the periphery of the armature drum, get subjected to the mechanical force. Due to this, overall armature experiences a twisting force called torque and armature of the motor starts rotating.*

2.3 Direction of Rotation of Motor

The magnitude of the force experienced by the conductor in a motor is given by,

$$F = B\, l\, I \quad \text{Newtons (N)}$$

B = Flux density due to the flux produced by the field winding.

l = Active length of the conductor.

I = Magnitude of the current passing through the conductor.

The direction of such force i.e. the direction of rotation of a motor can be determined by Fleming's left hand rule. So **Fleming's right hand rule** is to determine direction of induced e.m.f. i.e. for **generating action** while **Fleming's left hand rule** is to determine direction of force experienced i.e. for **motoring action.**

2.3.1 Fleming's Left Hand Rule

The rule states that, 'Outstretch the three fingers of the left hand namely the first finger, middle finger and thumb such that they are mutually perpendicular to each other. Now point the first finger in the direction of magnetic field and the middle finger in the direction of the current then the thumb gives the direction of the force experienced by the conductor'.

The Fleming's left hand rule can be diagramatically shown as in the Fig. 2.4.

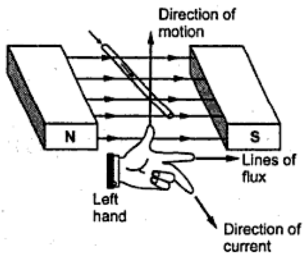


Fig. 2.4 Fleming's left hand rule

Apply the rule to crosscheck the direction of force experienced by a single conductor, placed in the magnetic field, shown in the Fig. 2.5 (a), (b), (c) and (d).

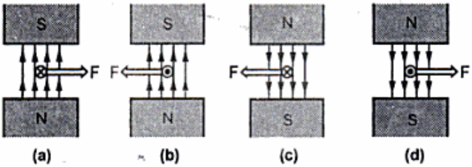


Fig. 2.5 Direction of force experienced by conductor

It can be seen from the Fig. 2.5 that if the direction of the main field in which current carrying conductor is placed, is reversed, force experienced by the conductor reverses its direction. Similarly keeping main flux direction unchanged, the direction of current passing through the conductor is reversed, the force experienced by the conductor reverses its direction. However if both the directions are reversed, the direction of the force experienced remains the same.

Key Point : *So in a practical motor, to reverse its direction of rotation, either direction of main field produced by the field winding is reversed or direction of the current passing through the armature is reversed.*

The direction of the main field can be reversed by changing the direction of current passing through the field winding, which is possible by interchanging the polarities of supply which is given to the field winding. In short, to have a motoring action two fluxes must exist, the interaction of which produces a torque.

2.4 Significance of Back E.M.F.

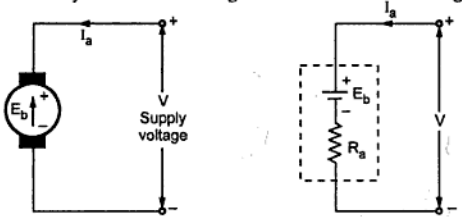
It is seen in the generating action, that when a conductor cuts the lines of flux, e.m.f. gets induced in the conductor. The question is obvious that in a d.c. motor, after a motoring action, armature starts rotating and armature conductors cut the main flux. So is there a generating action existing in a motor ? The answer to this question is 'Yes'.

After a motoring action, there exists a generating action. There is an induced e.m.f. in the rotating armature conductors according to Faraday's law of electromagnetic induction. This induced e.m.f. in the armature always acts in the opposite direction of the supply voltage. This is according to the **Lenz's law** which states that the direction of the induced e.m.f. is always so as to oppose the cause producing it. In a d.c. motor, electrical input i.e. the supply voltage is the cause and hence this induced e.m.f. opposes the supply voltage. This e.m.f. tries to set up a current through the armature which is in the opposite direction to that, which supply voltage is forcing through the conductor.

So as this e.m.f. always opposes the supply voltage, it is called **back e.m.f.** and denoted as E_b. Though it is denoted as E_b, basically it gets generated by the generating action which we have seen earlier in case of generators. So its magnitude can be determined by the e.m.f. equation which is derived earlier. So,

$$E_b = \frac{\phi P N Z}{60 A} \text{ volts}$$

where all symbols carry the same meaning as seen earlier in case of generators.



(a) Back e.m.f. in a d.c. motor **Fig. 2.6** **(b) Equivalent circuit**

This e.m.f. is shown schematically in the Fig. 2.6 (a). So if V is supply voltage in volts and R_a is the value of the armature resistance, the equivalent electric circuit can be shown as in the Fig. 2.6 (b).

2.4.1 Voltage Equation of a D.C. Motor

In case of a generator, generated e.m.f. has to supply armature resistance drop and remaining part is available across the load as a terminal voltage. But in case of d.c. motor, supply voltage V has to overcome back e.m.f. E_b which is opposing V and also various drops as armature resistance drop $I_a R_a$, brush drop etc. Infact the electrical work done in overcoming the back e.m.f. gets converted into the mechanical energy developed in the armature. Hence the voltage equation of a d.c. motor can be written as,

$$\boxed{V = E_b + I_a R_a + \text{brush drop}}$$

Neglecting the brush drop, the generalised voltage equation is,

$$V = E_b + I_a R_a$$

The back e.m.f. is always less than supply voltage ($E_b < V$). But R_a is very small hence under normal running conditions, the difference between back e.m.f. and supply voltage is very small. The net voltage across the armature is the difference between the supply voltage and back e.m.f. which decides the armature current. Hence from the voltage equation we can write,

$$\boxed{I_a = \frac{V - E_b}{R_a}}$$

Key Point : *Voltage equation gets changed a little bit depending upon the type of the motor, which is discussed later.*

➡ **Example 2.1 :** *A 220 V, d.c. motor has an armature resistance of 0.75 Ω. It is drawing an armature current of 30 A, driving a certain load. Calculate the induced e.m.f. in the motor under this condition.*

Solution : V = 200 V, I_a = 30 A, R_a = 0.75 Ω are the given values.

For a motor, $V = E_b + I_a R_a$

$\therefore \quad 220 = E_b + 30 \times 0.75$

$\therefore \quad E_b = \mathbf{197.5\ volts}$

This is the induced e.m.f. called back e.m.f. in a motor.

➡ **Example 2.2 :** *A 4 pole, d.c. motor has lap connected armature winding. The flux per pole is 30 mWb. The number of armature conductors is 250. When connected to 230 V d.c. supply it draws an armature current of 40 A. Calculate the back e.m.f. and the speed with which motor is running. Assume armature resistance is 0.6 Ω.*

Solution : P = 4, A = P = 4 as lap, V = 230 V, Z = 250

$$\phi = 30 \text{ mWb} = 30 \times 10^{-3} \text{ Wb}$$

$$I_a = 40 \text{ A}$$

From voltage equation, $V = E_b + I_a R_a$

$\therefore \quad 230 = E_b + 40 \times 0.6$

$\therefore \quad E_b = 206 \text{ V}$

And $\quad E_b = \dfrac{\phi P N Z}{60 \text{ A}}$

$\therefore \quad 206 = \dfrac{30 \times 10^{-3} \times 4 \times N \times 250}{60 \times 4}$

$\therefore \quad N = \mathbf{1648\ r.p.m.}$

2.4.2 Back E.M.F. as a Regulating Mechanism

Due to the presence of back e.mf. the d.c. motor becomes a regulating machine i.e. motor adjusts itself to draw the armature current just enough to satisfy the load demand. The basic principle of this fact is that the back e.m.f. is proportional to speed, $E_b \propto N$.

When load is suddenly put on to the motor, motor tries to slow down. So speed of the motor reduces due to which back e.m.f. also decreases. So the net voltage across the armature $(V - E_b)$ increases and motor draws more armature current. As $F = B\, l\, I$, due to increased current, force experienced by the conductors and hence the torque on the armature increases. The increase in the torque is just sufficient to satisfy increased load demand. The motor speed stops decreasing when the armature current is just enough to produce torque demanded by the new load.

When load on the motor is decreased, the speed of the motor tries to increase. Hence back e.m.f. increases. This causes $(V - E_b)$ to reduce which eventually reduces the current drawn by the armature. The motor speed stops increasing when the armature current is just enough to produce the less torque required by the new load.

Key Point *: So back e.m.f. regulates the flow of armature current and it automatically alters the armature current to meet the load requirement. This is the practical significance of the back e.m.f.*

2.5 Power Equation of a D.C. Motor

The voltage equation of a d.c. motor is given by,

$$V = E_b + I_a R_a$$

Multiplying both sides of the above equation by I_a we get,

$$\boxed{VI_a = E_b I_a + I_a^2 R_a}$$

This equation is called **power equation of a d.c. motor.**

VI_a = Net electrical power input to the armature measured in watts.

$I_a^2 R_a$ = Power loss due to the resistance of the armature called **armature copper loss.**

So difference between VI_a and $I_a^2 R_a$ i.e. input – losses gives the output of the armature.

So $E_b I_a$ is called **electrical equivalent of gross mechanical power developed by** the armature. This is denoted as P_m.

∴ Power input to the armature – Armature copper loss = Gross mechanical power developed in the armature.

2.5.1 Condition for Maximum Power

For a motor from power equation it is known that,

$$P_m = \text{Gross mechanical power developed} = E_b I_a$$

$$= VI_a - I_a^2 R_a$$

For maximum P_m, $\dfrac{dP_m}{dI_a} = 0$

$$\therefore \quad 0 = V - 2 I_a R_a$$

$$\therefore \quad \boxed{I_a = \frac{V}{2 R_a} \quad \text{i.e.} \quad I_a R_a = \frac{V}{2}}$$

Substituting in voltage equation,

$$V = E_b + I_a R_a = E_b + \frac{V}{2}$$

$$\therefore \quad \boxed{E_b = \frac{V}{2}}$$ .... Condition for maximum power

Key Point : *This is practically impossible to achieve as for this* E_b, *current required is much more than its normal rated value. Large heat will be produced and efficiency of motor will be less than 50 %.*

2.6 Torque Equation of a D.C. Motor

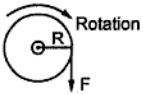


Fig. 2.7

It is seen that the turning or twisting force about an axis is called torque. Consider a wheel of radius R meters acted upon by a circumferential force F newtons as shown in the Fig. 2.7.

The wheel is rotating at a speed of N r.p.m.

Then angular speed of the wheel is,

$$\omega = \frac{2\pi N}{60} \quad \text{rad/sec}$$

So workdone in one revolution is,

$$W = F \times \text{distance travelled in one revolution}$$

$$= F \times 2\pi R \quad \text{joules}$$

And $$P = \text{Power developed} = \frac{\text{Workdone}}{\text{Time}}$$

$$= \frac{F \times 2\pi R}{\text{Time for 1 rev}} = \frac{F \times 2\pi R}{\left(\frac{60}{N}\right)} = (F \times R) \times \left(\frac{2\pi N}{60}\right)$$

∴ $$P = T \times \omega \quad \text{watts}$$

Where T = Torque in N - m

ω = Angular speed in rad/sec.

Let T_a be the gross torque developed by the armature of the motor. It is also called **armature torque**. The gross mechanical power developed in the armature is $E_b I_a$, as seen from the power equation. So if speed of the motor is N r.p.m. then,

Power in armature = Armature torque × ω

∴ $$E_b I_a = T_a \times \frac{2\pi N}{60}$$

but E_b in a motor is given by,

$$E_b = \frac{\phi P N Z}{60 A}$$

∴ $$\frac{\phi P N Z}{60 A} \times I_a = T_a \times \frac{2\pi N}{60}$$

∴ $$T_a = \frac{1}{2\pi} \phi I_a \times \frac{PZ}{A}$$

$$T_a = 0.159 \phi I_a \cdot \frac{PZ}{A} \text{ N-m}$$

This is the **torque equation** of a d.c. motor.

➡ **Example 2.3 :** *A 4 pole d.c. motor takes a 50 A armature current. The armature has lap connected 480 conductors. The flux per pole is 20 mWb. Calculate the gross torque developed by the armature of the motor.*

Solution : P = 4, A = P = 4, Z = 480

$$\phi = 20 \text{ mWb} = 20 \times 10^{-3} \text{ Wb}, I_a = 50 \text{ A}$$

Now

$$T_a = 0.159 \times \phi I_a \cdot \frac{PZ}{A} = 0.159 \times 20 \times 10^{-3} \times 50 \times \frac{4 \times 480}{4}$$

$$= \mathbf{76.394 \text{ N-m}}$$

2.6.1 Types of Torque in the Motor

Basically the torque is developed in the armature and hence gross torque produced is denoted as T_a.

The mechanical power developed in the armature is transmitted to the load through the shaft of the motor. It is impossible to transmit the entire power developed by the armature to the load. This is because while transmitting the power through the shaft, there is a power loss due to the friction, windage and the iron loss. The torque required to overcome these losses is called **lost torque,** denoted as T_f. These losses are also called **stray losses.**

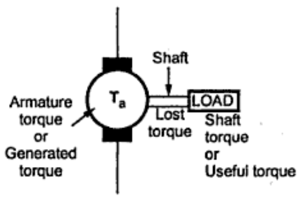


Fig. 2.8 Types of torque

The torque which is available at the shaft for doing the useful work is known as **load torque** or **shaft torque** denoted as T_{sh}.

$$\therefore \quad \boxed{T_a = T_f + T_{sh}}$$

The shaft torque magnitude is always less than the armature torque, ($T_{sh} < T_a$).

The speed of the motor remains same all along the shaft say N r.p.m. Then the product of shaft torque T_{sh} and the angular speed ω rad/sec is called power available at the shaft i.e. net output of the motor. The maximum power a motor can deliver to the load safely is called **output rating** of a motor. Generally it is expressed in H.P. It is called **H.P. rating** of a motor.

$$\boxed{\text{Net output of motor} = P_{out} = T_{sh} \times \omega}$$

2.6.2 No Load Condition of a Motor

On no load, the load requirement is absent. So $T_{sh} = 0$. This does not mean that motor is at hault. The motor can rotate at a speed say N_0 r.p.m. on no load. The motor draws an armature current of I_{a0}.

$$\boxed{I_{a0} = \frac{V - E_{b0}}{R_a}}$$

where E_{b0} is back e.m.f. on no load, proportional to speed N_0.

Now armature torque T_a for a motor is,

$$T_a \propto \phi I_a$$

As flux is present and armature current is present, hence T_{a0} i.e. armature torque exists on no load.

Now $\quad T_a = T_f + T_{sh}$

but on no load, $T_{sh} = 0$

$$\therefore \quad \boxed{T_{a0} = T_f}$$

So on no load, motor keeps on rotating at a speed of N_0 r.p.m. drawing an armature current of I_{a0}. This is just enough to produce a torque T_{a0} which satisfies the friction, windage and iron losses of the motor. On no load, speed of the motor is large hence E_{b0} is also large hence $(V - E_{b0})$ is very small hence armature current I_{a0} is also small. So **motor draws less current on no load and takes more and more current as motor load increases.**

So on no load,

Torque developed = Torque required to overcome friction, windage, iron losses.

$$\therefore \quad \boxed{\text{Power developed } (E_{b0} \times I_{a0}) = \text{ Friction, windage and, iron losses}}$$

where $\quad E_{b0}$ = Back e.m.f. on no load.

and $\quad I_{a0}$ = Armature current drawn on no load.

This component of stray losses i.e. $E_{b0} I_{a0}$ is practically assumed to be constant though the load on the motor is changed from zero to the full capacity of the motor. So T_f is practically assumed constant for all load conditions.

➡ **Example 2.4 :** *A 4 pole, lap wound d.c. motor has 540 conductors. Its speed is found to be 1000 r.p.m. when it is made to run light. The flux per pole is 25 mWb. It is connected to 230 V d.c. supply. The armature resistance is 0.8 Ω. Calculate.*
i) Induced e.m.f. ii) Armature current iii) Stray losses iv) Lost torque.

Solution : P = 4, A = P = 4

Running light means it is on no load.

$$\therefore \quad N_0 = 1000 \text{ r.p.m.}$$

$$Z = 540 \text{ and } \phi = 25 \times 10^{-3} \text{ Wb}$$

$$\therefore \quad E_{b0} = \frac{\phi P N_0 Z}{60\,A} = \frac{25\times10^{-3}\times4\times1000\times540}{60\times4} = 225\text{ V}$$

i) Induced e.m.f., E_{b0} = **225 V**

ii) From voltage equation, $V = E_b + I_a R_a$

$$\therefore \quad V = E_{b0} + I_{a0} R_a$$

$$\therefore \quad 230 = 225 + I_{a0}\times 0.8$$

$$\therefore \quad I_{a0} = \mathbf{6.25\ A}$$

iii) On no load, power developed is fully the power required to overcome stray losses.

$$\therefore \quad \text{Stray losses} = E_{b0} I_{a0} = 225\times6.25 = \mathbf{1406.25\ W}$$

iv) Lost torque $T_f = \dfrac{E_{b0} I_{a0}}{\omega_0} = \dfrac{1406.25}{\frac{2\pi N_0}{60}} = \dfrac{1406.25\times60}{2\pi\times1000} = \mathbf{13.428\ N\text{-}m.}$

2.7 Types of D.C. Motors

Similar to the d.c. generators, the d.c. motors are classified depending upon the way of connecting the field winding with the armature winding. The different types of d.c. motors are shunt motors, series motors and compound motors. The compound motors are further classified as short shunt compound and long shunt compound motors. Let us see the connection diagrams and different voltage and current relations of these types of motors.

2.8 D.C. Shunt Motor

In this type, the field winding is connected across the armature winding and the combination is connected across the supply, as shown in the Fig. 2.9.

Let R_{sh} be the resistance of shunt field winding.

R_a be the resistance of armature winding.

The value of R_a is very small while R_{sh} is quite large. Hence shunt field winding has more number of turns with less cross-sectional area.

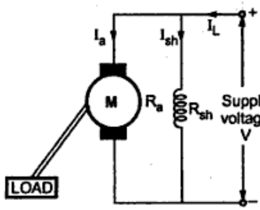


Fig. 2.9 D.C. shunt motor

2.8.1 Voltage and Current Relationship

The voltage across armature and field winding is same **equal** to the supply voltage V.

The total current drawn from the supply is denoted as line current I_L.

$$I_L = I_a + I_{sh}$$

$$I_{sh} = \frac{V}{R_{sh}}$$

and $$V = E_b + I_a R_a + V_{brush}$$

V_{brush} is generally neglected.

Now flux produced by the field winding is proportional to the current passing through it i.e. I_{sh}.

$$\boxed{\phi \propto I_{sh}}$$

Key Point : *As long as supply voltage is constant, which is generally so in practice, the flux produced is constant. Hence* ***d.c. shunt motor is called constant flux motor.***

2.9 D.C. Series Motor

In this type of motor, the series field winding is connected in series with the armature and the supply, as shown in the Fig. 2.10.

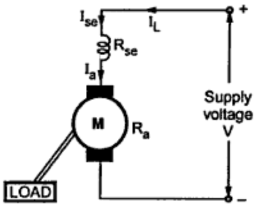


Fig. 2.10 D.C. series motor

Let R_{se} be the resistance of the series field winding. The value of R_{se} is very small and it is made of small number of turns having large cross-sectional area.

2.9.1 Voltage and Current Relationship

Let I_L be the total current drawn from the supply.

So $$I_L = I_{se} = I_a$$

and $$V = E_b + I_a R_a + I_{se} R_{se} + V_{brush}$$

$$V = E_b + I_a (R_a + R_{se}) + V_{brush}$$

Supply voltage has to overcome the drop across series field winding in addition to E_b and drop across armature winding.

Key Point : *In series motor, entire armature current is passing through the series field winding. So flux produced is proportional to the armature current.*

$$\phi \propto I_{se} \propto I_a \quad \text{for series motor}$$

2.10 D.C. Compound Motor

The compound motor consists of part of the field winding connected in series and part of the field winding connected in parallel with armature. It is further classified as long shunt compound and short shunt compound motor.

2.10.1 Long Shunt Compound Motor

In this type, the shunt field winding is connected across the combination of armature and the series field winding as shown in the Fig. 2.11.

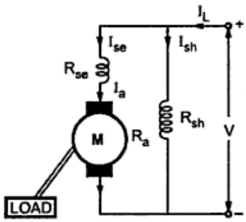


Fig. 2.11 Long shunt compound motor

Let R_{se} be the resistance of series field and R_{sh} be the resistance of shunt field winding. The total current drawn from supply is I_L.

So $\quad I_L = I_{se} + I_{sh}$

But $\quad I_{se} = I_a$

$\therefore \quad I_L = I_a + I_{sh}$

And $\quad I_{sh} = \dfrac{V}{R_{sh}}$

And $\quad V = E_b + I_a R_a + I_{se} R_{se} + V_{brush}$

But as $\quad I_{se} = I_a,$

$\therefore \quad V = E_b + I_a (R_a + R_{se}) + V_{brush}$

2.10.2 Short Shunt Compound Motor

In this type, the shunt field is connected purely in parallel with armature and the series field is connected in series with this combination shown in the Fig. 2.12.

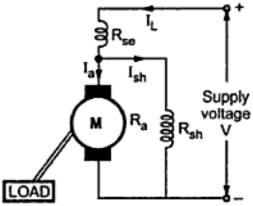


Fig. 2.12 Short shunt compound motor

$$I_L = I_{se}$$

The entire line current is passing through the series field winding.

and $$I_L = I_a + I_{sh}$$

Now the drop across the shunt field winding is to be calculated from the voltage equation.

So $$V = E_b + I_{se} R_{se} + I_a R_a + V_{brush}$$

but $$I_{se} = I_L$$

$\therefore$ $$V = E_b + I_L R_{se} + I_a R_a + V_{brush}$$

$\therefore$ Drop across shunt field winding is,

$$= V - I_L R_{se} = E_b + I_a R_a + V_{brush}$$

$\therefore$ $$\boxed{I_{sh} = \frac{V - I_L R_{se}}{R_{sh}} = \frac{E_b + I_a R_a + V_{brush}}{R_{sh}}}$$

Apart from these two, compound motor can be classified into two more types,

i) Cumulatively compound motors and ii) Differential compound motors.

Key Point : *If the two field windings are wound in such a manner that the fluxes produced by the two always help each other, the motor is called* ***cumulatively compound.*** *If the fluxes produced by the two field windings are trying to cancel each other i.e. they are in opposite direction, the motor is called* ***differential compound.***

A long shunt compound motor can be of cumulative or differential type. Similarly short shunt compound motor can be cumulative or differential type.

2.11 Torque and Speed Equations

Before analyzing the various characteristics of motors, let us revise the torque and speed equations as applied to various types of motors.

$$\therefore \quad \boxed{T \propto \phi I_a} \quad \text{from torque equation.}$$

This is because, $0.159 \frac{PZ}{A}$ is a constant for a given motor.

Now ϕ is the flux produced by the field winding and is proportional to the current passing through the field winding.

$$\phi \propto I_{field}$$

But for various types of motors, current through the field winding is different. Accordingly torque equation must be modified.

For a d.c. shunt motor, I_{sh} is constant as long as supply voltage is constant. Hence ϕ flux is also constant.

$$\therefore \quad T \propto I_a \quad \text{for shunt motors}$$

For a d.c. series motor, I_{se} is same as I_a. Hence flux ϕ is proportional to the armature current I_a.

$$\therefore \quad T \propto I_a \phi \propto I_a^2 \quad \text{for series motors.}$$

Similarly as $E_b = \frac{\phi PNZ}{60\,A}$, we can write the speed equation as,

$$E_b \propto \phi N$$

$$\therefore \quad \boxed{N \propto \frac{E_b}{\phi}}$$

But $\quad V = E_b + I_a R_a \quad$ neglecting brush drop.

$$\therefore \quad E_b = V - I_a R_a$$

$\therefore$ Speed equation becomes,

$$N \propto \frac{V - I_a R_a}{\phi}$$

So for shunt motor as flux ϕ is constant,

$$\therefore \quad N \propto V - I_a R_a$$

While for series motor, flux ϕ is proportional to I_a

$$\therefore \quad \boxed{N \propto \frac{V - I_a R_a - I_a R_{se}}{I_a}}$$

These relations play an important role in understanding the various characteristics of different types of motors.

2.11.1 Speed Regulation

The speed regulation for a d.c. motor is defined as the ratio of change in speed corresponding to no load and full load condition to speed corresponding to full load.

Mathematically it is expressed as,

$$\% \text{ speed regulation} = \frac{N_{\text{no load}} - N_{\text{full load}}}{N_{\text{full load}}} \times 100$$

2.12 D.C. Motor Characteristics

The performance of a d.c. motor under various conditions can be judged by the following characteristics.

i) Torque - armature current characteristics (T Vs I_a) :

The graph showing the relationship between the torque and the armature current is called a torque-armature current characteristic. These are also called electrical characteristics.

ii) Speed - armature current characteristics (N Vs I_a) :

The graph showing the relationship between the speed and armature current characteristics.

iii) Speed - torque characteristics (N Vs T) :

The graph showing the relationship between the speed and the torque of the motor is called speed-torque characteristics of the motor. These are also called mechanical characteristics.

The nature of these characteristics can easily be obtained by using speed and torque equations derived in section 2.11. These characteristics play a very important role in selecting a type of motor for a particular application.

2.13 Characteristics of D.C. Shunt Motor

i) Torque - armature current characteristics

For a d.c. motor $T \propto \phi I_a$

For a constant values of R_{sh} and supply voltage V, I_{sh} is also constant and hence flux is also constant.

$$\therefore \quad T_a \propto I_a$$

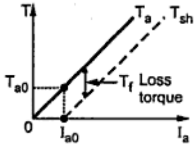


Fig. 2.13 T Vs I_a for shunt motor

The equation represents a straight line, passing through the origin, as shown in the Fig. 2.13. Torque increases linearily with armature current. It is seen earlier that armature current is decided by the load. So as load increases, armature current increases, increasing the torque developed linearly.

Now if shaft torque is plotted against armature current, it is known that shaft torque is less than the armature torque and the difference between the two is loss torque T_f as shown. On no load $T_{sh} = 0$ but armature torque is present which is just enough to overcome stray losses shown as T_{a0}. The current required is I_{a0} on no load to produce T_{a0} and hence T_{sh} graph has an intercept of I_{a0} on the current axis.

To generate high starting torque, this type of motor requires a large value of armature current at start. This may damage the motor hence d.c. shunt motors can develop moderate starting torque and hence suitable for such applications where starting torque requirement is moderate.

ii) Speed - armature current characteristics

From the speed equation we get,

$$N \propto \frac{V - I_a R_a}{\phi}$$

$$\propto V - I_a R_a \qquad \text{as } \phi \text{ is constant.}$$

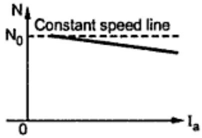


Fig. 2.14 N Vs I_a for shunt motor

So as load increases, the armature current increases and hence drop $I_a R_a$ also increases.

Hence for constant supply voltage, $V - I_a R_a$ decreases and hence speed reduces. But as R_a is very small, for change in I_a from no load to full load, drop $I_a R_a$ is very small and hence drop in speed is also not significant from no load to full load.

So the characteristics is slightly droping as shown in the Fig. 2.14.

But for all practical purposes these type of motors are considered to be a constant speed motors.

iii) Speed - torque characteristics

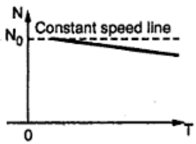


Fig. 2.15 N Vs T for shunt motor

These characteristics can be derived from the above two characteristics. This graph is similar to speed-armature current characteristics as torque is proportional to the armature current. This curve shows that the speed almost remains constant though torque changes from no load to full load conditions. This is shown in the Fig. 2.15.

2.14 Characteristics of D.C. Series Motor

i) Torque - armature current characteristics

In case of series motor the series field winding is carrying the entire armature current. So flux produced is proportional to the armature current.

$$\therefore \quad \phi \propto I_a$$

$$\text{Hence} \quad \boxed{T_a \propto \phi I_a \propto I_a^2}$$

Thus torque in case of series motor is proportional to the square of the armature current. This relation is parabolic in nature as shown in the Fig. 2.16.

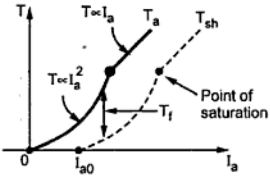


Fig. 2.16 T Vs I_a for series motor

Fig. 2.17 N Vs I_a for series motor

As load increases, armature current increases and torque produced increases proportional to the square of the armature current upto a certain limit.

As the entire I_a passes through the series field, there is a property of an electromagnet called **saturation**, may occur. Saturation means though the current through the winding increases, the flux produced remains constant. Hence after saturation the characteristics take the shape of straight line as flux becomes constant, as shown. The difference between T_a and T_{sh} is loss torque T_f which is also shown in the Fig. 2.16.

At start as $T \propto I_a^2$, these types of motors can produce high torque for small amount of armature current hence the series motors are suitable for the applications which demand high starting torque.

ii) Speed - armature current characteristics

From the speed equation we get,

$$N \propto \frac{E_b}{\phi}$$

$$\propto \frac{V - I_a R_a - I_a R_{se}}{I_a} \quad \text{as } \phi \propto I_a \text{ in case of series motor}$$

Now the values of R_a and R_{se} are so small that the effect of change in I_a on speed overrides the effect of change in $V - I_a R_a - I_a R_{se}$ on the speed.

Hence in the speed equation, $E_b \cong V$ and can be assumed constant. So speed equation reduces to,

$$N \propto \frac{1}{I_a}$$

So speed-armature current characteristics is rectangular hyperbola type as shown in the Fig. 2.17.

iii) Speed - torque characteristics

In case of series motors, $T \propto I_a^2$ and $N \propto \frac{1}{I_a}$

Hence we can write, $$N \propto \frac{1}{\sqrt{T}}$$

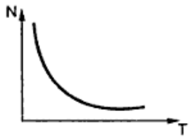


Fig. 2.18 N Vs T for series motor

Thus as torque increases when load increases, the speed decreases. On no load, torque is very less and hence speed increases to dangerously high value. Thus the nature of the speed-torque characteristics is similar to the nature of the speed-armature current characteristics.

The speed-torque characteristics of a series motor is shown in the Fig. 2.18.

2.15 Why Series Motor is Never Started on No Load ?

It is seen earlier that motor armature current is decided by the load. On light load or no load, the armature current drawn by the motor is very small.

In case of a d.c. series motor, $\phi \propto I_a$ and

on no load as I_a is small hence flux produced is also very small.

According to speed equation,

$$N \propto \frac{1}{\phi} \quad \text{as } E_b \text{ is almost constant.}$$

So on very light load or no load as flux is very small, the motor tries to run at dangerously high speed which may damage the motor mechanically. This can be seen from the speed-armature current and the speed-torque characteristics that on low armature current and low torque condition motor shows a tendency to rotate with dangerously high speed.

This is the reason why series motor should never be started on light loads or no load conditions. For this reason it is not selected for belt drives as breaking or slipping of belt causes to throw the entire load off on the motor and made to run motor with no load which is dangerous.

2.16 Characteristics of D.C. Compound Motor

Compound motor characteristics basically depends on the fact whether the motor is cumulatively compound or differential compound. All the characteristics of the compound motor are the combination of the shunt and series characteristic.

Cumulative compound motor is capable of developing large amount of torque at low speeds just like series motor. However it is not having a disadvantage of series motor even at light or no load. The shunt field winding produces the definite flux and series flux helps the shunt field flux to increase the total flux level.

So cumulative compound motor can run at a reasonable speed and will not run with dangerously high speed like series motor, on light or no load condition.

In differential compound motor, as two fluxes oppose each other, the resultant flux decreases as load increases, thus the machine runs at a higher speed with increase in the load. This property is dangerous as on full load, the motor may try to run with dangerously high speed. So differential compound motor is generally not used in practice.

The various characteristics of both the types of compound motors cumulative and the differential are shown in the Fig. 2.19 (a), (b) and (c).

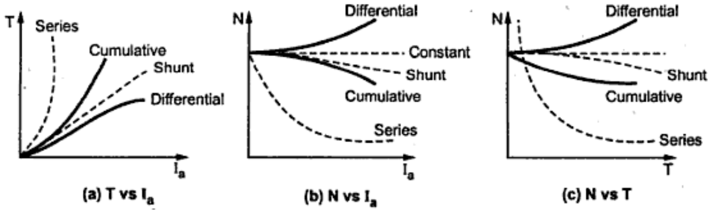


Fig. 2.19 Characteristics of d.c. compound motor

The exact shape of these characteristics depends on the relative contribution of series and shunt field windings. If the shunt field winding is more dominant then the characteristics take the shape of the shunt motor characteristics. While if the series field winding is more dominant then the characteristics take the shape of the series characteristics.

2.17 Applications of D.C. Motors

Instead of just stating the applications, the behaviour of the various characteristics like speed, starting torque etc., which makes the motor more suitable for the applications, is also stated in the Table 2.1.

Type of motor	Characteristics	Applications
Shunt	Speed is fairly constant and medium starting torque.	1) Blowers and fans 2) Centrifugal and reciprocating pumps 3) Lathe machines 4) Machine tools 5) Milling machines 6) Drilling machines.
Series	High starting torque. No load condition is dangerous. Variable speed.	1) Cranes 2) Hoists, Elevators 3) Trolleys 4) Conveyors 5) Electric locomotives.
Cumulative compound	High starting torque. No load condition is allowed.	1) Rolling mills 2) Punches 3) Shears 4) Heavy planers 5) Elevators.
Differential compound	Speed increases as load increases.	Not suitable for any practical application.

Table 2.1

Example 2.5 : *A 4 pole, 250 V, d.c. series motor has a wave connected armature with 200 conductors. The flux per pole is 25 mWb when motor is drawing 60 A from the supply. Armature resistance is 0.15 Ω while series field winding resistance is 0.2 Ω . Calculate the speed under this condition.*

Solution :

$$P = 4,\ Z = 200$$

$$A = 2,\ \phi = 25 \times 10^{-3}\ \text{Wb}$$

$$I_a = I_L = 60\ \text{A}$$

$$R_a = 0.15\ \Omega$$

$$R_{se} = 0.2\ \Omega$$

$$V = E_b + I_a R_a + I_a R_{se}$$

$$\therefore \quad 250 = E_b + 60\,(0.15 + 0.2)$$

$$\therefore \quad E_b = 229\ \text{V}$$

$$\text{Now}\quad E_b = \frac{\phi P N Z}{60\ A}$$

$$\therefore 229 = \frac{25 \times 10^{-3} \times 4\, N \times 200}{60 \times 2}$$

$$\therefore \quad \mathbf{N = 1374\ r.p.m.}$$

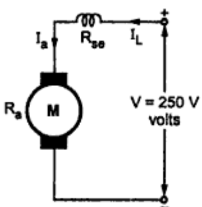


Fig. 2.20

Example 2.6 : *A 250 V, d.c. shunt motor takes a line current of 20 A. Resistance of shunt field winding is 200 Ω and resistance of the armature is 0.3 Ω. Find the armature current and the back e.m.f.*

Solution : $V = 250$ V, $I_L = 20$ A

$$R_a = 0.3\ \Omega, \quad R_{sh} = 200\ \Omega$$

$$I_L = I_a + I_{sh}$$

$$I_{sh} = \frac{V}{R_{sh}} = \frac{250}{200} = 1.25 \text{ A}$$

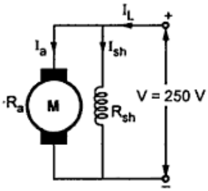


Fig. 2.21

$$\therefore \quad I_a = I_L - I_{sh} = 20 - 1.25 = \mathbf{18.75\ A}$$

Now $\quad V = E_b + I_a R_a$

$$\therefore \quad E_b = V - I_a R_a = 250 - 18.75 \times 0.3 = \mathbf{244.375\ V}$$

Example 2.7 : *A d.c. shunt motor runs at a speed of 1000 r.p.m. on no load taking a current of 6 A from the supply, when connected to 220 V d.c. supply. Its full load current is 50 A. Calculate its speed on full load. Assume $R_a = 0.3\ \Omega$ and $R_{sh} = 110\ \Omega$.*

Solution : Let no load, speed be $N_0 = 1000$ r.p.m.

$$I_{L0} = \text{Line current on no load} = 6 \text{ A}$$

$$I_{L0} = I_{a0} + I_{sh}$$

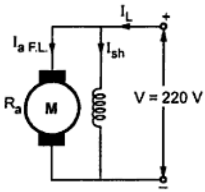


Fig. 2.22

$$I_{sh} = \frac{V}{R_{sh}} = \frac{220}{110} = 2 \text{ A}$$

$$\therefore \quad I_{a0} = I_{L0} - I_{sh} = 6 - 2 = 4 \text{ A}$$

∴ Back e.m.f. on no load E_{b0} can be determined from the voltage equation.

$$V = E_{b0} + I_{a0} R_a$$

$$\therefore \quad 220 = E_{b0} + 4 \times 0.3$$

$$\therefore \quad E_{b0} = 218.8 \text{ V}$$

On full load condition, supply voltage is constant and hence,

$$I_{sh} = \frac{V}{R_{sh}} = \frac{220}{110} = 2\ A \quad \text{(remains same)}$$

Now $I_L = I_{a\,F.L.} + I_{sh}$

$\therefore$ $50 = I_{a\,F.L.} + 2$

$\therefore$ $I_{a\,F.L.} = 48\ A$

and $V = E_{b\,F.L.} + I_{a\,F.L.} R_a$

$\therefore$ $220 = E_{b\,F.L.} + 48 \times 0.3$

$\therefore$ $E_{b\,F.L.} = 205.6\ V$

From the speed equation,

$$N \propto \frac{E_b}{\phi}$$

But ϕ is constant as I_{sh} is constant for both the load conditions.

$$\therefore \quad \frac{N_0}{N_{F.L.}} = \frac{E_{b0}}{E_{bF.L.}}$$

$$\therefore \quad N_{F.L.} = N_0 \frac{E_{b\,F.L.}}{E_{b0}} = 1000 \times \frac{205.6}{218.8} = \mathbf{939.67\ r.p.m.}$$

Example 2.8 : *A d.c. series motor is running with a speed of 800 r.p.m. while taking a current of 20 A from the supply. If the load is changed such that the current drawn by the motor is increased to 50 A, calculate the speed of the motor on new load. The armature and series field winding resistances are 0.2 Ω and 0.3 Ω respectively. Assume the flux produced is proportional to the current. Assume supply voltage as 250 V.*

Solution : For load 1, $N_1 = 800$ r.p.m., $I_1 = I_{a1} = 20\ A$

For load 2, $I_2 = I_{a2} = 50\ A$

$R_a = 0.2\ \Omega$, $R_{se} = 0.3\ \Omega$,

From voltage equation $V = E_{b1} + I_{a1} R_a + I_{se1} R_{se}$

but $I_1 = I_{a1} = I_{se1} = 20\,A$

$\therefore$ $250 = E_{b1} + 20\,(0.2 + 0.3)$

$\therefore$ $E_{b1} = 240\ V$

and $V = E_{b2} + I_{a2} R_a + I_{se2} R_{se}$

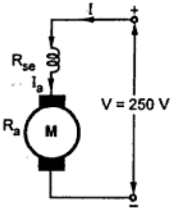


Fig. 2.23

$$\therefore \quad 250 = E_{b2} + 50\,(0.2 + 0.3)$$

$$\therefore \quad E_{b2} = 225 \text{ V}$$

From the speed equation,

$$N \propto \frac{E_b}{\phi}$$

Now $\quad \phi \propto I_{se} \propto I_a$

$$\therefore \quad \frac{N_1}{N_2} = \frac{E_{b1}}{E_{b2}} \times \frac{\phi_2}{\phi_1}$$

$$\therefore \quad \frac{N_1}{N_2} = \frac{E_{b1}}{E_{b2}} \times \frac{I_{a2}}{I_{a1}}$$

$$\therefore N_2 = N_1 \times \frac{E_{b2}}{E_{b1}} \times \frac{I_{a1}}{I_{a2}} = 800 \times \frac{225}{240} \times \frac{20}{50} = \textbf{300 r.p.m.}$$

2.18 Necessity of Starter

All the d.c. motors are basically self starting motors. Whenever the armature and the field winding of a d.c. motor receives supply, motoring action takes place. So d.c. motors do not require any additional device to start it. The device to be used as a starter conveys a wrong meaning.

Key Point : *So starter is not required to start a d.c. motor but it enables us to start the motor in a desired, safe way.*

Now at the starting instant the speed of the motor is zero, (N = 0). As speed is zero, there cannot be any back e.m.f. as $E_b \propto N$ and N is zero at start.

$$\therefore \quad E_b \text{ at start} = 0$$

The voltage equation of a d.c. motor is,

$$V = E_b + I_a R_a$$

So at start, $\quad V = I_a R_a \quad$ as $E_b = 0$

$$\therefore \quad \boxed{I_a = \frac{V}{R_a}} \quad \text{... At start}$$

Key Point : *Generally motor is switched on with normal voltage and as armature resistance is very small, the armature current at start is very high.*

Consider a motor having full load input power as 8000 watts. The motor rated voltage be 250 V and armature resistance is 0.5 Ω.

Then at start, $E_b = 0$ and motor is operated at 250 V supply, so

$$I_a = \frac{V}{R_a} = \frac{250}{0.5} = 500 \text{ A}$$

While its full load current can be calculated as,

$$I_{\text{Full load}} = \frac{\text{Power input on full load}}{\text{Supply voltage}} = \frac{8000}{250} = 32 \text{ A}$$

So at start, motor is showing a tendency to draw an armature current which is 15 to 20 times more than the full load current.

Such high current drawn by the armature at start is highly objectionable for the following reasons :

1. In a constant voltage system, such high inrush of current may cause tremendous line voltage fluctuations. This may affect the performance of the other equipments connected to the same line.
2. Such excessively high armature current, blows out the fuses.
3. If motor fails to start due to some problems with the field winding, then a large armature current flowing for a longer time may burn the insulation of the armature winding.
4. As the starting armature current is 10 to 15 times more than the full load current, the torque developed which is proportional to the I_a will also be 10 to 15 times, assuming shunt motor operation. So due to such high torque, the shaft and other accessories are thus be subjected to large mechanical stresses. These stresses may cause permanent mechanical damage to the motor.

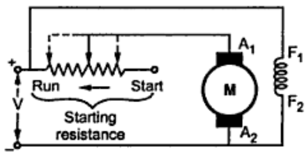


Fig. 2.24 Basic arrangement of a starter

To restrict this high starting armature current, a variable resistance is connected in series with the armature at start. This resistance is called **starter or a starting resistance.** So starter is basically a current limiting device. In the beginning the entire resistance is in the series with the armature and then gradually cut-off as motor gathers speed, producing the back e.m.f. The basic arrangement is shown in the Fig. 2.24.

In addition to the starting resistance, there are some protective devices provided in a starter. There are two types of starters used for d.c. shunt motors.

a) Three point starter

b) Four point starter

Let us see the details of three point starter.

2.19 Three Point Starter

The Fig. 2.25 shows this type of starter.

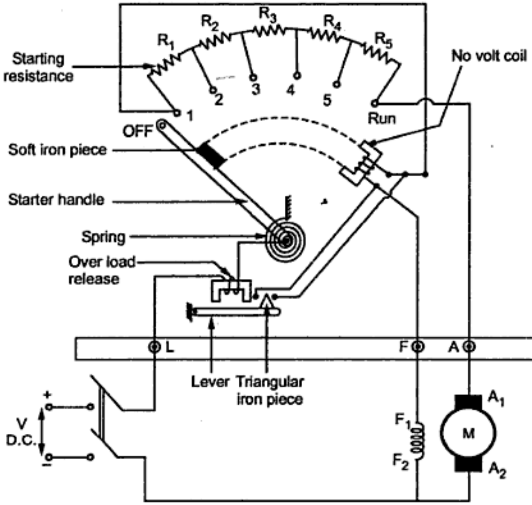


Fig. 2.25 Three point starter

The starter is basically a variable resistance, divided into number of sections. The contact points of these sections are called **studs** and brought out separately shown as OFF, 1, 2, ... upto RUN. There are three main points of this starter :

1. 'L' → Line terminal to be connected to positive of supply.
2. 'A' → To be connected to the armature winding.
3. 'F' → To be connected to the field winding.

Point 'L' is further connected to an electromagnet called **Overload Release** (OLR). The second end of 'OLR' is connected to a point where handle of the starter is pivoted. This handle is free to move from its other side against the force of the spring. This spring brings back the handle to the OFF position under the influence of its own force. Another parallel path is derived from the stud '1', given to the another electromagnet called **No Volt Coil** (NVC). The NVC is further connected to terminal 'F'. The starting resistance is entirely in series with the armature. The OLR and NVC are the two protecting devices of the starter.

Operation : Initially the handle is in the OFF position. The d.c. supply to the motor is switched on. Then handle is slowly moved against the spring force to make a contact with stud No. 1. At this point, field winding gets supply through the parallel path provided to starting resistance, through NVC. While entire starting resistance comes in series with the armature and armature current which is high at start, gets limited. As the handle is moved further, it goes on making contact with studs 2, 3, 4 etc., cutting out the starting resistance gradually from the armature circuit. Finally when the starter handle is in 'RUN' position, the entire starting resistance gets removed from the armature circuit and motor starts operating with normal speed. The handle is moved manually, and the obvious question is how handle will remain in the 'RUN' position, as long as motor is running ?

Let us see the action of NVC which will give the answer to this question along with some other functions of NVC.

2.19.1 Functions of No Volt Coil

1. The supply to the field winding is derived through NVC. So when field current flows, it magnetises the NVC. When the handle is in the 'RUN' position, soft iron piece connected to the handle gets attracted by the magnetic force produced by NVC. Design of NVC is such that it holds the handle in 'RUN' position against the force of the spring as long as supply to the motor is proper. Thus NVC holds the handle in the 'RUN' position and hence also called **hold on coil.**
2. Whenever there is supply failure or if field circuit is broken, the current through NVC gets affected. It looses its magnetism and hence not in a position to keep the soft iron piece on the handle, attracted. Under the spring force, handle comes back to OFF position, switching off the motor. So due to the combination of NVC and the spring, the starter handle always comes back to OFF position whenever there is any supply problem. The entire starting resistance comes back in series with the armature when attempt is made to start the motor everytime. This prevents the damage of the motor caused due to accidental starting.
3. NVC performs the similar action under low voltage conditions and protects the motor from such dangerous supply conditions as well.

2.19.2 Action of Over Load Release

The current through the motor is taken through the OLR, an electromagnet. Under overload condition, high current is drawn by the motor from the supply which passes through OLR. Below this magnet, there is an arm which is fixed at its fulcrum and normally resting in horizontal position. Under overloading, high current though OLR produces enough force of attraction to attract the arm upwards. Normally magnet is so designed that up to a full load value of current, the force of attraction produced is just enough to balance the gravitational force of the arm and hence not lifting it up. At the end of this arm, there is a triangular iron piece fitted. When the arm is pulled upwards the triangular piece touches to the two points which are connected to the two ends of NVC. This shorts the NVC and voltage across NVC becomes zero due to which NVC looses its magnetism. So under the spring force, handle comes back to the OFF position, disconnecting the motor from the supply. Thus motor gets saved from the overload conditions.

In this starter, it can be observed that as handle is moved from different studs one by one, the part of the starting resistance which gets removed from the armature circuit, gets added to the field circuit. As the value of starting resistance is very small as compared to the field winding resistance, this hardly affects the field winding performance. But this addition of the resistance in the field circuit can be avoided by providing a brass arc or copper arc connected just below the stud, the end of which is connected to NVC, as shown in the Fig. 2.26.

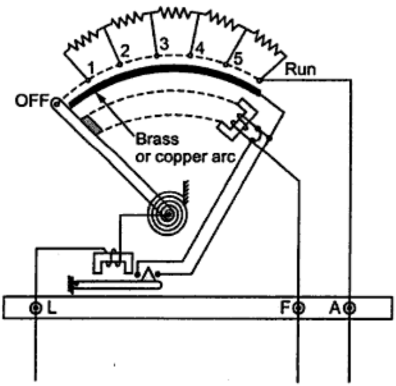


Fig. 2.26 Three point starter with brass arc

The handle moves over this arc, supplying the field current directly bypassing the starting resistance. When such an arc is provided, the connection used earlier to supply field winding, is removed.

2.19.3 Disadvantage

In this starter, the NVC and the field winding are in series. So while controlling the speed of the motor above rated, field current is reduced by adding an extra resistance in series with the field winding. Due to this, the current through NVC also reduces. Due to this, magnetism produced by NVC also reduces. This may release the handle from its RUN position switching off the motor. To avoid the dependency of NVC and the field winding, four point starter is used, in which NVC and the field winding are connected in parallel.

2.19.4 Calculation of Steps of Starter Resistance

In the operation of starter, the step is changed whenever the current falls to definite lower value. The design of starter is based on

1. The lower current limit is specified.
2. The number of section is specified.

When the lower current is specified, number of sections are selected so as to match the given upper and lower current values. While if number of sections is fixed, the lower current limit is calculated accordingly. Another important aspect of the shunt motor starter is that the resistances in the various sections existing between the studs form the **geometrical series** with a common factor as the ratio of lower current to upper current limit. Consider the shunt motor starter having three sections as shown in the Fig. 2.27

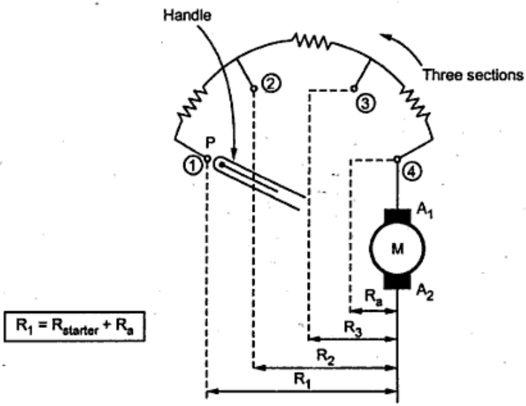


Fig. 2.27 Design of starter resistance

When point P of handle makes contact with stud 1, the armature current increases to the value,

$$\boxed{I_1 = \frac{V}{R_1}} \qquad \text{At } E_b = 0 \qquad \text{...(1)}$$

At the same time field is established so motor starts.

$$R_1 = R_{starter} + R_a$$

Key Point : *Many times I_1 is limited to 1.5 times the full load current of the motor i.e. assuming 50 % overload at start.*

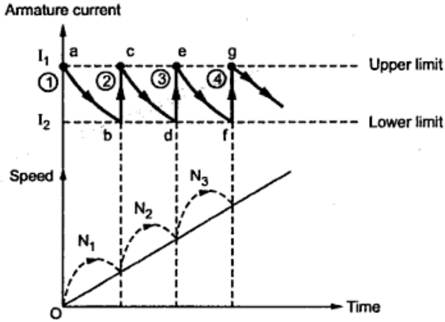


Fig. 2.28 Change in I_a and speed while starter resistance is changed

As motor starts, E_b is developed and I_1 reduces. Let I_2 be the lower limit of armature current to which it falls, due to E_b, as shown in the Fig. 2.28

This change is shown by the curve a.b. At b the current achieved is given by,

$$I_2 = \frac{V - E_{b1}}{R_1} \qquad \text{At } E_{b1} \qquad ...(2)$$

where E_{b1} = Back e.m.f. corresponding to speed N_1 achieved at 'b'.

when I_2 is achieved, handle is moved to stud 2 hence instantly the current increases to I_1 as back e.m.f. remains same as developed for N_1.

$$\therefore \qquad I_1 = \frac{V - E_{b1}}{R_2} \qquad ...(3)$$

This graph is shown as b to c in the Fig. 2.28, Taking ratio of equations (3) and (2)

$$\frac{I_1}{I_2} = \frac{R_1}{R_2} \qquad ...(4)$$

When handle is held at stud 2, then speed picks up from N_1 to N_2 and back e.m.f. changes from E_{b1} to E_{b2}. So current decreases from c to d.

$$\therefore \qquad I_2 = \frac{V - E_{b2}}{R_2} \qquad \text{At } E_{b2} \qquad ...(5)$$

At this point, handle is moved to stud 3 and current jumps to I_1 again which is given by,

$$I_1 = \frac{V - E_{b2}}{R_3} \quad \ldots(6)$$

Taking ratio of equetion (6) to (5)

$$\frac{I_1}{I_2} = \frac{R_2}{R_3} \quad \ldots(7)$$

Similarly current changes to I_2 when handle is held at stud 3 when speed changes to N_3.

$$\therefore \quad I_2 = \frac{V - E_{b3}}{R_3} \quad \text{At } E_{b3} \quad \ldots(8)$$

At this point, handle is moved to stud 4 and current jumps back to I_1 given by,

$$I_1 = \frac{V - E_{b3}}{R_a} \quad \ldots(9)$$

taking ratio of equations (9) to (8),

$$\frac{I_1}{I_2} = \frac{R_3}{R_a} \quad \ldots(10)$$

From equations (4), (7) and (10) we can write,

$$\frac{I_1}{I_2} = \frac{R_1}{R_2} = \frac{R_2}{R_3} = \frac{R_3}{R_a} = K$$

$$\therefore \quad R_3 = K\, R_a$$

$$\therefore \quad R_2 = K\, R_3 = K^2\, R_a$$

$$\therefore \quad R_1 = K\, R_2 = K^3\, R_a$$

Thus for n studs, there are (n–1) sections and resistances are calculated as,

$$R_1 = K^{n-1}\, R_a$$

$$\left(\frac{I_1}{I_2}\right)^{n-1} = \frac{R_1}{R_a}$$

When number of sections are specified then I_1 limit is given and we can calculate the R_1 as,

$$R_1 = \frac{V}{I_1}$$

As n is given and R_a is known we can calculate K from,

$$\frac{R_1}{R_a} = K^{n-1}$$

and then I_2 can be obtained as,

$$\frac{I_1}{I_2} = K$$

Once K is known and R_1 is known, all other resistances can be obtained.

Example 2.9 : *A 230 V shunt motor has an armature resistance of 0.2 Ω. The starting armature current must not exceed 50 A. If the number of sections are 5, calculate the values of resistance steps to be used in the starter.*

[May-2004 (set -1) , Dec.-2003, 2004, Nov.-2005 (set-3)]

Solution : V = 230 V, R_a = 0.2 Ω, I_1 = 50 A

Key Point : *Number of sections are 5 hence there are 6 studs i.e. n = 6.*

$$R_1 = \frac{V}{I_1} = \frac{230}{50} = 4.6\ \Omega$$

Now $$\frac{R_1}{R_a} = K^{n-1}$$

∴ $$\frac{4.6}{0.2} = K^5$$

∴ $$K = 1.8721 = \frac{I_1}{I_2}$$

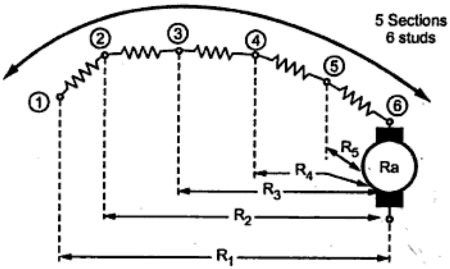


Fig. 2.29

∴ $$R_2 = \frac{R_1}{K} = 2.4571\ \Omega$$

∴ $$R_3 = \frac{R_2}{K} = 1.3125\ \Omega$$

$\therefore \quad R_4 = \frac{R_3}{K} = 0.701\ \Omega$

$\therefore \quad R_5 = \frac{R_4}{K} = 0.3744\ \Omega$

$\therefore$ Resistance of first section $= R_1 - R_2 = \mathbf{2.1429\ \Omega}$

$\therefore$ Resistance of second section $= R_2 - R_3 = \mathbf{1.446\ \Omega}$

$\therefore$ Resistance of third section $= R_3 - R_4 = \mathbf{0.6115\ \Omega}$

$\therefore$ Resistance of fourth section $= R_4 - R_5 = \mathbf{0.3266\ \Omega}$

$\therefore$ Resistance of fifth section $= R_5 - R_a = \mathbf{0.1744\ \Omega}$

2.20 Four Point Starter

The basic difference between three point and four point starter is the connection of NVC. In three point, NVC is in series with the field winding while in four point starter NVC is connected independently across the supply through the fourth terminal called 'N' in addition to the 'L', 'F' and 'A'.

Hence any change in the field current does not affect the performance of the NVC. Thus it is ensured that NVC always produce a force which is enough to hold the handle in 'RUN' position, against force of the spring, under all the operating conditions. Such a current is adjusted through NVC with the help of fixed resistance R connected in series with the NVC using fourth point 'N' as shown in Fig. 2.30.

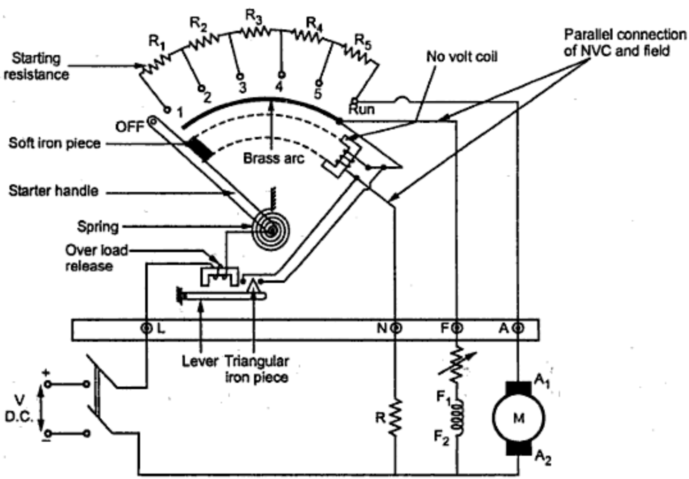


Fig. 2.30 Four point starter

2.20.1 Disadvantage

The only limitation of the four point starter is, it does not provide high speed protection to the motor. If under running condition, field gets opened, the field current reduces to zero. But there is some residual flux present and $N \propto \frac{1}{\phi}$ the motor tries to run with dangerously high speed. This is called **high speeding action** of the motor. In three point starter as NVC is in series with the field, under such field failure, NVC releases handle to the OFF position. But in four point starter NVC is connected directly across the supply and its current is maintained irrespective of the current through the field winding. Hence it always maintains handle in the RUN position, as long as supply is there. And thus it does not protect the motor from field failure conditions which result into the high speeding of the motor.

2.21 D.C. Series Motor Starter

Three point and four point starters are used for d.c. shunt motors. In case of series motors, field and armature are in series and hence starting resistance is inserted in series with the field and armature. Such a starter used to limit the starting current in case of d.c. series motors is called **two point starter.** The basic construction of two point starter is similar to that of three point starter except the fact that it has only two terminals namely Line (L) and Field (F). The F terminal is one end of the series combination of field and the armature winding. The starter is shown in the Fig. 2.31.

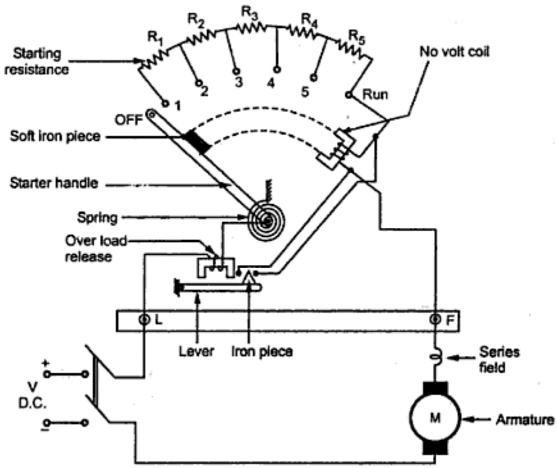


Fig. 2.31

The action of the starter is similar to that of three point starter. The handle of the starter is in OFF position. When it is moved to ON, motor gets the supply and the entire starting resistance is in series with the armature and field. It limits the starting current. The current through no volt coil energises it and when handle reaches to RUN position, the no volt coil holds the handle by attracting the soft iron piece on the handle. Hence the no volt coil is also called hold on coil.

The main problem in case of d.c. series motor is its overspeeding action when the load is less. This can be prevented using two point starter. The no volt coil is designed in such a way that it hold the handle in RUN position only when it carries sufficient current, for which motor can run safely. If there is loss of load then current drawn by the motor decreases, due to which no volt coil looses its required magnetism and releases the handle. Under spring force, handle comes back to OFF position, protecting the motor from overspeeding. Similarly if there is any supply problem such that voltage decreases suddenly then also no volt coil releases the handle and protects the motor from adverse supply conditions.

The overload condition can be prevented using overload release. When motor draws excessively high current due to overload, then current through overload magnet increases. This energises the magnet upto such an extent that it attracts the lever below it. When lever is lifted upwards, the triangular piece attached to it touches the two points, which are the two ends of no volt coil. Thus no volt coil gets shorted, loosing its magnetism and releasing the handle back to OFF position. This protects the motor from overloading conditions.

Key Point : *The starter has two points L connected to line i.e. supply and F connected to series field of the moter. Hence it is called two point starter.*

2.22 Losses in a D.C. Machine

The various losses in a d.c. machine whether it is a motor or a generator are classified into three groups as :

1. Copper losses
2. Iron or core losses
3. Mechanical losses.

2.22.1 Copper Losses

The copper losses are the losses taking place due to the current flowing in a winding. There are basically two windings in a d.c. machine namely armature winding and field winding. The copper losses are proportional to the square of the current flowing through these windings. Thus the various copper losses can be given by,

$$\boxed{\text{Armature copper loss} = I_a^2 R_a}$$

where R_a = Armature winding resistance

and I_a = Armature current

$$\text{Shunt field copper loss} = I_{sh}^2 \, R_{sh}$$

where R_{sh} = Shunt field winding resistance

and I_{sh} = Shunt field current

$$\text{Series field copper loss} = I_{se}^2 \, R_{se}$$

where R_{se} = Series field winding resistance

and I_{se} = Series field current

In a compound d.c. machine, both shunt and series field copper losses are present. In addition to the copper losses, there exists brush contact resistance drop. But this drop is usually included in the armature copper loss.

2.22.2 Iron or Core Losses

These losses are also called magnetic losses. These losses include hysteresis loss and eddy current loss.

The hysteresis loss is proportional to the frequency and the maximum flux density B_m in the air gap and is given by,

$$\text{Hysteresis loss} = \eta \, B_m^{1.6} \, f \, V \text{ watts}$$

η = Steinmetz hystersis coefficient

where V = Volume of core in m^3

f = Frequency of magnetic reversals

This loss is basically due to reversal of magnetisation of the armature core.

The eddy current loss exists due to eddy currents. When armature core rotates, it cuts the magnetic flux and e.m.f. gets induced in the core. This induced e.m.f. sets up eddy currents which cause the power loss. This loss is given by,

$$\text{Eddy current loss} = K \, B_m^2 \, f^2 \, t^2 \, V \text{ watts}$$

where K = Constant

t = Thickness of each lamination

V = Volume of core

f = Frequency of magnetic reversals

The hysteresis loss is minimised by selecting the core material having low hysteresis coefficient. While eddy current loss is minimised by selecting the laminated construction for the core.

These losses are almost constant for the d.c. machines.

2.22.3 Mechanical Losses

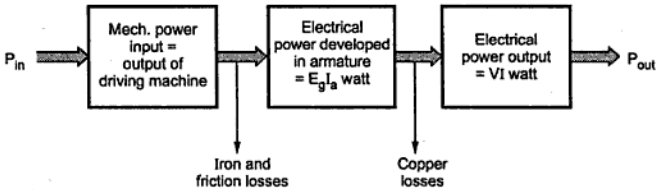
These losses consist of friction and windage losses. Some power is required to overcome mechanical friction and wind resistance at the shaft. This loss is nothing but the friction and windage loss. The mechanical losses are also constant for a d.c. machine.

The magnetic and mechanical losses together are called **stray losses.** For the shunt and compound d.c. machines where field current is constant, field copper losses are also constant. Thus stray losses along with constant field copper losses are called **constant losses.** While the armature current is dependent on the load and thus armature copper losses are called **variable losses.**

Thus for a d.c. machine,

Total losses = Constant losses + Variable losses

The power flow and energy transformation diagrams at various stages, which takes place in a d.c. machine are represented diagrammatically in Fig. 2.32 (a) and (b).



(a) Generator

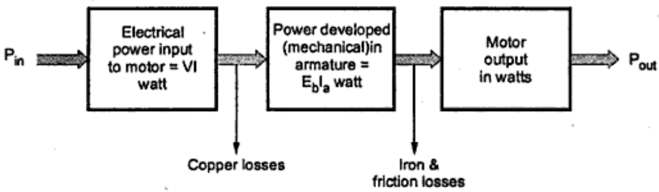


Fig. 2.32 (b) Motor

2.23 Efficiency of a D.C. Machine

For a d.c. machine, its overall efficiency is given by,

$$\% \eta = \frac{\text{Total output}}{\text{Total input}} \times 100$$

Let P_{out} = Total output of a machine

P_{in} = Total input of a machine

P_{cu} = Variable losses

P_i = Constant losses

then $P_{in} = P_{out} + P_{cu} + P_i$

$$\therefore \quad \% \eta = \frac{P_{out}}{P_{in}} \times 100 = \frac{P_{out}}{P_{out} + \text{losses}} \times 100$$

$$\therefore \quad \boxed{\% \eta = \frac{P_{out}}{P_{in} + P_{cu} + P_i} \times 100}$$

2.23.1 Condition for Maximum Efficiency

In case of a d.c. generator the output is given by,

$P_{out} = VI$

P_{cu} = Variable losses $= I_a^2 R_a = I^2 R_a$

$I_a = I$ ... Neglecting shunt field current

$$\therefore \quad \% \eta = \frac{VI}{VI + I^2 R_a + P_i} \times 100 = \frac{1}{1 + \left(\frac{I R_a}{V} + \frac{P_i}{VI}\right)} \times 100$$

The efficiency is maximum, when the denominator is minimum. According to maxima-minima theorem,

$$\frac{d}{dI}\left[1 + \left(\frac{I R_a}{V} + \frac{P_i}{VI}\right)\right] = 0$$

$$\therefore \quad \frac{R_a}{V} - \frac{P_i}{VI^2} = 0$$

$$\therefore \quad I^2 R_a - P_i = 0$$

$$\therefore \quad I^2 R_a = P_i = P_{cu}$$

Thus for the maximum efficiency, the condition is,

Variable losses = Constant losses

Let us study now the various methods of testing the d.c. motors from the losses and efficiency point of view.

2.24 Testing of D.C. Motors

The efficiency of a d.c. motor is given by,

$$\text{Efficiency} = \frac{\text{Power output}}{\text{Power input}} = \frac{\text{Power input} - \text{Losses}}{\text{Power input}}$$

The various losses taking place in a d.c. motor and efficiency can be calculated by carrying out testing of d.c. motors. There are different methods of testing d.c. motors. These methods are broadly classified as :

i) Direct method of testing

ii) Indirect method of testing

Now let us study direct method of testing d.c. motors.

2.24.1 Direct Method of Testing

In this method the d.c. motor which is to be tested is actually loaded and input and output are measured. The efficiency is given by

$$\text{Efficiency}, \eta = \frac{\text{Output}}{\text{Input}}$$

Generally this method is employed to small motors. The motor is loaded by means of a brake applied to the water cooled pulleys.

The main drawback of this method is that the accuracy in determining the mechanical power output of the motor is limited. Alternately it is difficult to provide full load for the large capacity motor.

2.24.2 Indirect Method of Testing

In these methods the motor is not loaded directly but the losses and efficiency at different loads can be estimated. Out of the different methods available for testing of d.c. motors, Swinburne's test and Hopkinson's test are commonly used in practice on shunt-motors. Since series motors cannot be started without load, the no load tests cannot be performed on d.c. series motors.

2.25 Swinburne's Test or No Load Test

This is indirect method of testing d.c. motors in which flux remains practically constant i.e. specially in case of shunt and compound motors. Without actually loading the motor the losses and hence efficiency at different loads can be found out.

The motor is run on no load at its rated voltage. At the starting some resistance is connected in series with the armature which is cut when motor attains sufficient speed. Now the speed of the motor is adjusted to the rated speed with the help of shunt field rheostat as shown in the Fig. 2.33.

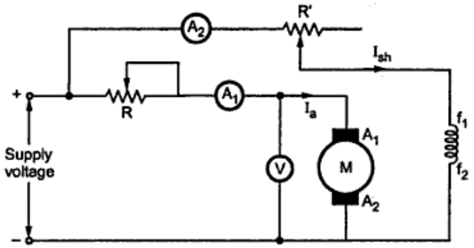


Fig. 2.33

The no load armature current I_a is measured by ammeter A_1 whereas the shunt current is measured by ammeter A_2.

If V is the supply voltage then motor input at no load will be,

$$\boxed{\text{Power input at no load} = V\,(I_a + I_{sh}) \text{ watts}}$$

There will be Cu loss in the field winding which will be given as,

$$\boxed{\text{Field copper loss} = V \times I_{sh}}$$

Let R_a be the resistance of armature,

$$\boxed{\text{Armature copper loss} = I_a^2 \cdot R_a}$$

Thus the stray losses which includes iron, friction and windage losses can be obtained as,

$$\text{Stray losses} = \text{Input at no load} - \text{Field copper losses}$$
$$- \text{No load armature copper losses}$$

$$\therefore \quad \boxed{\text{Stray losses} = V\,(I_a + I_{sh}) - (V \times I_{sh}) - (I_a^2\, R_a) = W_a}$$

In the field and armature windings there will be copper loss due to flow of current which will increase the temperature of the field and armature winding when the motor is loaded. This increase in temperature will affect their resistances.

Thus the new value of field resistance R'_{sh} and that of armature R'_a can be found by considering that rise in temperature as about 40 °C.

If α_1 = Resistance temperature co efficient of copper at room temperature

$$\boxed{R'_a = R_a\,(1 + \alpha_1 \times 40)}$$

At room temperature the shunt field winding resistance will be,

$$R_{sh} = \frac{V}{I_{sh}}$$

$$\therefore \quad \boxed{R'_{sh} = R_{sh}\,(1 + \alpha_1 \times 40)}$$

Now shunt winding current, $I'_{sh} = \dfrac{V}{R'_{sh}}$

$$\therefore \quad \boxed{\text{New field copper loss} = I'^2_{sh} \times R_{sh}}$$

Now if we want to find the efficiency of the motor at say $\frac{1}{4}$ th full load. It can be calculated as follows,

Let

$I_{F.L.}$ = Full load current of motor

W_F = Field copper loss

W = Stray losses

$$\text{Load current at } \frac{1}{4} \text{ th full load} = \frac{I_{F.L.}}{4}$$

$$\therefore \quad \text{Motor input at } \frac{1}{4} \text{ th full load} = V \times \frac{I_{F.L.}}{4} \text{ watts.}$$

$$\text{Armature current at } \frac{1}{4} \text{ th full load, } I'_a = \frac{I_{F.L.}}{4} - I'_{sh}$$

$$\text{Armature copper loss at } \frac{1}{4} \text{ th full load} = I'^2_a\, R_a = \left(\frac{I_{F.L.}}{4} - I'_{sh}\right)^2 \cdot R_a$$

$$\therefore \quad \text{Motor output at } \frac{1}{4} \text{ th full load} = \text{Motor input at } \frac{1}{4} \text{ th load} - \text{Losses}$$

$$= \left(V \times \frac{I_{F.L.}}{4}\right) - \left(\frac{I_{F.L.}}{4} - I'_{sh}\right)^2 R_a - W_F - W$$

$$= \left(V \times \frac{I_{F.L.}}{4}\right) - I'^2_a\, R_a - W_F - W$$

$$\text{Efficiency at } \frac{1}{4} \text{ th full load, } \eta = \frac{\text{Output}}{\text{Input}} = \frac{\text{Input} - \text{Losses}}{\text{Input}}$$

$$\therefore \quad \boxed{\eta = \frac{\left(V \times \frac{I_{F.L.}}{4}\right) - I'^2_a\, R_a - W_F - W}{V \times \frac{I_{F.L.}}{4}}}$$

This is the efficiency of motor when the load on motor is $\frac{1}{4}$ th of full load which can be found without loading the motor. The efficiencies at other loads can be calculated similarly.

2.25.1 Advantages

1. Since constant losses are known, the efficiency can be estimated at any load.
2. The method is convenient and economical as less power is required for testing even a large motor i.e. only no load power is to be supplied.
3. The motor is not required to be loaded i.e. only test to be carried out is the no load test.

2.25.2 Disadvantages

1. In this method, the iron losses are assumed to be constant which is not the true case as they change from no load to full load. Due to armature reaction at full load there will be distortion in flux which will increase the iron loss.

2. The only test which is carried out is the no load test. Hence it is difficult to know whether there will be satisfactory commutation at full load.

3. We have assumed that there is rise in temperature of 40 °C at full load which cannot be checked actually as we are not actually loading the motor.

4. As it is a no load test it cannot be performed on a series motor.

➠ **Example 2.10 :** *A 440 V d.c. shunt motor takes a no load current of 2.5 A. The resistance of the shunt field and the armature are 550 Ω and 1.2 Ω respectively. The full load line current is 32 A. Find the full load output and the efficiency of the motor.*

Solution : No load current I = 2.5 A,

$$\text{No load input} = V.\,I = 440 \times 2.5 = 1100\ \text{W}$$

$$I_{sh} = \frac{V}{R_{sh}} = \frac{440}{550} = 0.8\ \text{A}$$

In d.c. shunt motor, $I = I_{sh} + I_a$

$\therefore$ $I_a = I - I_{sh} = 2.5 - 0.8 = 1.7\ \text{A}$

$$\text{No load armature copper loss} = I_a^2\, R_a = (1.7)^2 \times 1.2$$

$$= 3.468\ \text{watts}$$

$$\text{Constant losses} = \text{No load input} - \text{No load armature Cu loss}$$

$$= 1100 - 3.468$$

$$= 1096.532\ \text{W}$$

Now, full load line current i.e. I = 32 A

$$I = I_{sh} + I_a$$

$$I_a = I - I_{sh} = 32 - 0.8 = 31.2 \text{ A}$$

$$\text{Full load armature copper loss} = I_a^2 \cdot R_a = (31.2)^2 \times 1.2 = 1168.128 \text{ W}$$

$$\text{Total losse} = \text{Full load armature Cu loss + Constant losses}$$

$$= 1168.128 + 1096.532 = 2264.66 \text{ W}$$

$$\text{Full load motor input} = V \cdot I = 440 \times 32 = 14080 \text{ W}$$

$$\text{Full load motor output} = \text{Input} - \text{Losses} = 14080 - 2264.66 = 11815.34 \text{ W}$$

$$\text{\% efficiency at full load} = \frac{\text{Full load Output}}{\text{Full load Input}} \times 100 = \frac{11815.34}{14080} \times 100$$

$$= 83.91$$

$\therefore$ Efficiency of motor at full load = **83.91 %**

Example 2.11 : *The no load test of a 45 kW, 230 V d.c. shunt motor gave the following results :*

Input current = 14 A

Field current = 2.55 A

Resistance of armature at 75 °C = 0.032 Ω

Brush drop = 2 V

Estimate the full load current and efficiency

Solution : No load current I = 14 A, Full load power output = 45 kW = 45000 W

$$\text{No load power input} = VI = 230 \times 14 = 3220 \text{ W}$$

$$\text{In case of shunt motors, } I = I_{sh} + I_a$$

$$I_a = I - I_{sh} = 14 - 2.55 = 11.45 \text{ A}$$

$$\text{No load armature copper loss} = I_a^2 \cdot R_a = (11.45)^2 (0.032)$$

$$= 4.19 \text{ W}$$

$$\text{Loss due to brush drop} = 2 \times 11.45 = 22.9 \text{ W}$$

$$\text{Constant losses} = \text{No load input} - \text{No load armature Cu loss} - \text{Loss due to brush drop}$$

$$= 3220 - 4.19 - 22.9 = 3192.91 \text{ W}$$

Now let I_a be the full load current in armature.

$$\therefore \text{Full load motor input current, } I = I_a + I_{sh} = (I_a + 2.55) \text{ A}$$

$$\text{Full load motor input} = V \cdot I = 230\,(I_a + 2.55) \text{ W}$$

$$\text{Motor input} = \text{Motor output + Total losses}$$

$$= \text{Motor output + Constant loss + Brush loss + Armature Cu loss}$$

$$230\,(I_a + 2.55) = 45000 + 3192.91 + 2\,I_a + I_a^2\,R_a$$

$$= 45000 + 3192.91 + 2\,I_a + I_a^2\,(0.032)$$

$$\therefore\ 0.032\,I_a^2 + 2I_a - 230\,I_a + 45000 + 3192.91 - (230)\,(2.55) = 0$$

$$\therefore\quad 0.032\,I_a^2 - 228\,I_a + 47606.41 = 0$$

$$\therefore\quad I_a^2 - 7125\,I_a + 1487700.3 = 0$$

$$\therefore\quad I_a = \frac{7125 \pm \sqrt{(7125)^2 - 4(1487700.3)}}{2}$$

$$\therefore\quad I_a = 215.30\text{ A}$$

Full load motor input current $= 215.30 + 2.55 =$ **217.85 A**

Full load motor input $= V \cdot I = (230)\,(217.85) = 50105.5$ W

Full load motor efficiency $= \dfrac{\text{Output}}{\text{Input}} \times 100 = \dfrac{45000}{50105.5} \times 100$

$= 89.81\ \%$

$\therefore$ Efficiency of motor at full load = **89.81 %**

2.26 Factors Affecting the Speed of a D.C. Motor

According to the speed equation of a d.c. motor we can write,

$$N \propto \frac{E_b}{\phi} \propto \frac{V - I_a R_a}{\phi}$$

The factors Z, P, A are constants for a d.c. motor.

But as the value of armature resistance R_a and series field resistance R_{se} is very small, the drop $I_a R_a$ and $I_a\,(R_a + R_{se})$ is very small compared to applied voltage V. Hence neglecting these voltage drops the speed equation can be modified as,

$$N \propto \frac{V}{\phi} \qquad \text{as } E_b \approx V$$

Thus the factors affecting the speed of a d.c. motor are,

1. The flux ϕ
2. The voltage across the armature
3. The applied voltage V

Depending upon these factors the various methods of speed control are,

1. Changing the flux ϕ by controlling the current through the field winding called **flux control methods.**

2. Changing the armature path resistance which in turn changes the voltage applied across the armature called **rheostatic control.**
3. Changing the applied voltage called **voltage control** method.

Before studying how these methods are used for various types of d.c. motors, let us study the ratings of a d.c. motor. These ratings decide the range in which the speed of a particular d.c. motor can be varied.

2.27 Ratings of a D.C. Motor

To change the speed as per the requirements, it is not possible to increase the voltage or currents beyond certain limit. These limits are called ratings of the motor.

The maximum voltage that can be applied to the motor, safely is called **rated voltage** or **normal voltage** of the motor. While changing the applied voltage, one should not apply the voltage more than the rated voltage of the motor.

Similarly maximum current that field winding can carry, safely is called **rated field current** of the motor. Hence while changing the flux, one should not increase field current beyond its rated value. This is important rating as far as shunt motor is concerned. In a series motor, the entire armature current flows through the series field winding. The armature current is decided by the load and it cannot be changed by changing the resistance of the armature circuit. So the maximum current that armature winding can carry safely is decided by the load called **full load current** or **full load rating** of the motor. Motor should not be loaded more than its full load capacity indicated by its full load armature current.

Exceeding the rating is dangerous from the motor point of view as due to high currents, the heat produced, which is proportional to the square of the current is very large. This may damage the windings electrically.

$$\text{Now } N \propto \frac{V}{\phi}$$

So for $V = V_{rated}$ and $\phi \propto I_f$ rated i.e. when there is no external resistance in the armature and field circuit and motor is excited by normal rated voltage, the speed obtained is called **rated speed** or **normal speed.**

$$\therefore \quad N_{rated} \propto \frac{V_{rated}}{I_{f\ rated}}$$

Key Point : *Note that the rated or normal speed is not the maximum speed with which motor can run safely but it is the speed when the electrical parameters controlling the speed are at their rated values.*

Practically a motor speed can be increased to approximately twice its normal speed safely.

Thus while controlling the speed, the voltage applied should not be more than rated voltage of a motor, the field current should not be more than its rated value and the current carried by armature should not be more than its full load value. All the ratings are provided by the manufacturer in the form of name plate of a d.c. motor. Let us study now the various methods as applied to different types of d.c. motors.

2.28 Speed Control of D.C. Shunt Motor

Out of the three methods, let us study flux control method.

2.28.1 Flux Control

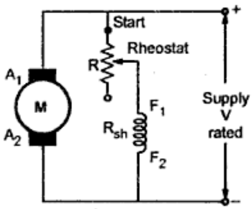


Fig. 2.34 Flux control of shunt motor

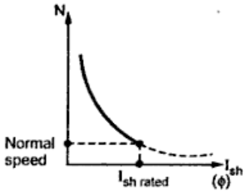


Fig. 2.35 N Vs I_{sh} (ϕ) for shunt motor

As indicated by the speed equation, the speed is inversely proportional to the flux. The flux is dependent on the current through the shunt field winding. Thus flux can be controlled by adding a rheostat (variable resistance) in series with the shunt field winding, as shown in the Fig. 2.34

At the beginning the rheostat R is kept at minimum indicated as start in the Fig. 2.35. The supply voltage is at its rated value. So current through shunt field winding is also at its rated value. Hence the speed is also rated speed also called normal speed. Then the resistance R is increased due to which shunt field current I_{sh} decreases, decreasing the flux produced. As $N \propto (1/\phi)$, the speed of the motor increases beyond its rated value.

Thus by this method, the speed control above rated value is possible. This is shown in the Fig. 2.35, by speed against field current curve. The curve shows the inverse relation between N and ϕ as its nature is rectangular hyperbola.

It is mentioned that the rated values of electrical parameters should not be exceeded but the speed which is mechanical parameter can be increased upto twice its rated value.

2.28.1.1 Advantages of Flux Control Method

1. It provides relatively smooth and easy control.

2. Speed control above rated speed is possible.

3. As the field winding resistance is high, the field current is small. Hence power loss (I_{sh}^2 R) in the external resistance is very small, which makes the method more economical and efficient.

4. As the field current is small, the size of the rheostat required is small.

2.28.1.2 Disadvantages of Flux Control Method

1. The speed control below normal rated speed is not possible as flux can be increased only upto its rated value.

2. As flux reduces, speed increases. But high speed affects the commutation making motor operation unstable. So there is limit to the maximum speed above normal, possible by this method.

2.28.2 Armature Voltage Control Method or Rheostatic Control

The speed is directly proportional to the voltage applied across the armature. As the supply voltage is normally constant, the voltage across the armature can be controlled by adding a variable resistance in series with the armature as shown in the Fig. 2.36.

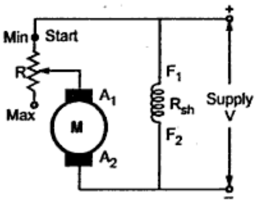


Fig. 2.36 Rheostatic control of shunt motor

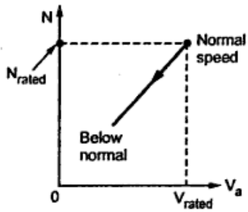


Fig. 2.37 N Vs voltage across armature

The field winding is excited by the normal voltage hence I_{sh} is rated and constant in this method. Initially the rheostat position is minimum and rated voltage gets applied across the armature. So speed is also rated. For a given load, armature current is fixed. So when extra resistance is added in the armature circuit, I_a remains same and there is voltage drop across the resistance added (I_a R). Hence voltage across the armature decreases, decreasing the speed below normal value. By varying this extra resistance, various speeds below rated value can be obtained.

So for a **constant load torque**, the speed is directly proportional to the voltage across the armature. The relationship between speed and voltage across the armature is shown in the Fig. 2.37.

2.28.2.1 Potential Divider Control

The main disadvantage of the above method is, the speed up to zero is not possible as it requires a large rheostat in series with the armature which is practically impossible. If speed control from zero to the rated speed is required, by rheostatic method then voltage across the armature can be varied by connecting rheostat in a potential divider arrangement as shown in the Fig. 2.38.

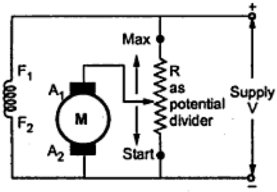


Fig. 2.38 Potential divider arrangement

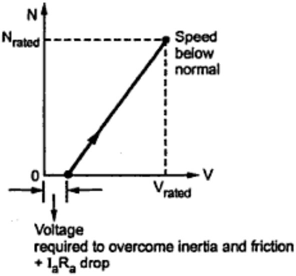


Fig. 2.39 N Vs V

When the variable rheostat position is at 'start' point shown, voltage across the armature is zero and hence speed is zero. As rheostat is moved towards 'maximum' point shown, the voltage across the armature increases, increasing the speed. At maximum point the voltage is maximum i.e. rated hence maximum speed possible is rated speed. The relationship is shown in the Fig. 2.39.

When the voltage across the armature starts increasing, as long as motor does not overcome inertial and frictional torque, the speed of the motor remains zero. The motor requires some voltage to start hence the graph of voltage and the speed does not pass through the origin as shown in the Fig. 2.39.

2.28.2.2 Advantages of Rheostatic Control

1. Easy and smooth speed control below normal is possible.
2. In potential divider arrangement, rheostat can be used as a starter.

2.28.2.3 Disadvantages of Rheostatic Control

1. As the entire armature current passes through the external resistance, there are tremendous power losses.
2. As armature current is more than field current, rheostat required is of large size and capacity.
3. Speed above rated is not possible by this method.
4. Due to large power losses, the method is expensive, wasteful and less efficient.
5. The method needs expensive heat dissipation arrangements.

2.28.3 Applied Voltage Control

Multiple voltage control : In this technique the shunt field of the motor is permanently connected to a fixed voltage supply, while the armature is supplied with various voltages by means of suitable switch gear arrangements.

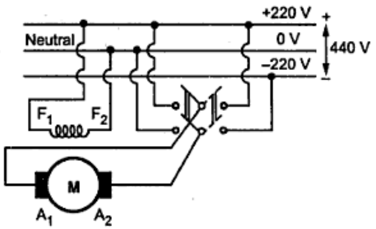


Fig. 2.40 Multiple voltage control

The Fig. 2.40 shows a control of motor by two different working voltages which can be applied to it with the help of switchgear.

In large factories, various values of armature voltages and corresponding arrangement can be used to obtain the speed control.

2.28.3.1 Advantages of Applied Voltage Control

1. Gives wide range of speed control.
2. Speed control in both directions can be achieved very easily.
3. Uniform acceleration can be obtained.

2.28.3.2 Disadvantages of Applied Voltage Control

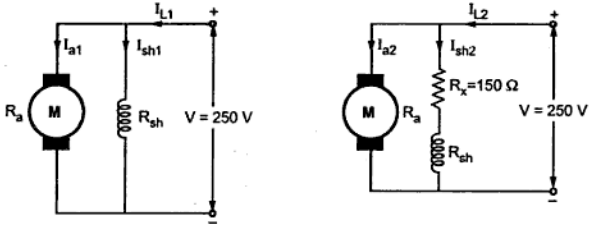
1. Arrangement is expensive as provision of various auxiliary equipments is necessary.
2. Overall efficiency is low.

• General steps to solve problems on speed control :

1. Identify the method of speed control i.e. in which winding of the motor, the external resistance is to be inserted.
2. Use the torque equation, $T \propto \phi I_a$ to determine the new armature current according to the condition of the torque given. Load condition indicates the condition of the torque.
3. Use the speed equation $N \propto \frac{E_b}{\phi}$ to find the unknown back e.m.f. or field current.
4. From the term calculated above and using voltage current relationship of the motor, the value of extra resistance to be added, can be determined. The above steps may vary little bit according to the nature of the problem but are always the base of any speed control problem.

Example 2.12 : *A 250 V d.c. shunt motor has a shunt field resistance of 200 Ω and an armature resistance of 0.3 Ω. For a given load, motor runs at 1500 r.p.m. drawing a current of 22 A from the supply. If a resistance of 150 Ω is added in series with the field winding, find the new armature current and the speed. Assume load torque constant and magnetisation curve to be linear.*

Solution : The Fig. 2.41 shows the two conditions.



(a) N_1 = 1500 r.p.m. (b) N_2 = ?

Fig. 2.41

The method is flux control method.

$$V = 250 \text{ V}, \quad R_a = 0.3\ \Omega, \quad R_{sh} = 200\ \Omega$$

In first case, $I_{L1} = 22$ A

$$I_{sh1} = \frac{V}{R_{sh}} = \frac{250}{200} = 1.25 \text{ A}$$

$\therefore$ $I_{a1} = I_{L1} - I_{sh1} = 22 - 1.25 = 20.75$ A

Now $T \propto \phi I_a \propto I_{sh} I_a$ (as $\phi \propto I_{sh}$)

$$\therefore \quad \frac{T_1}{T_2} = \frac{I_{sh1}}{I_{sh2}} \times \frac{I_{a1}}{I_{a2}}$$

As load torque is constant, $T_1 = T_2$

$$\therefore \quad I_{sh1}\, I_{a1} = I_{a2}\, I_{sh2} \qquad \ldots (1)$$

Now $$I_{sh2} = \frac{V}{R_{sh} + R_x} = \frac{250}{(200+150)} = 0.7142 \text{ A}$$

Substituting in equation (1),

$$1.25 \times 20.75 = I_{a2} \times 0.7142$$

$\therefore$ $I_{a2} = 36.3125$ A

Hence $E_{b1} = V - I_{a1} R_a = 250 - 20.75 \times 0.3 = 243.775$ V

and $E_{b2} = V - I_{a2} R_a = 250 - 36.3125 \times 0.3 = 239.1062$ V

Using speed equation $N \propto \frac{E_b}{\phi} \propto \frac{E_b}{I_{sh}}$

$$\frac{N_1}{N_2} = \frac{E_{b1}}{E_{b2}} \times \frac{I_{sh2}}{I_{sh1}}$$

$$\therefore \quad \frac{1500}{N_2} = \frac{243.775}{239.1062} \times \frac{0.7142}{0.125}$$

$$\therefore \quad N_2 = \mathbf{2575.03\ r.p.m.}$$

This shows that as flux ϕ decreases, the speed increases.

2.29 Speed Control of D.C. Series Motor

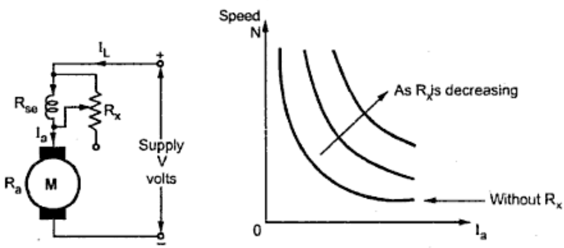
The flux produced by the winding depends on the m.m.f. i.e. magnetomotive force which is the product of current and the number of turns of the winding through which current is passing. So flux can be changed either by changing the current by adding a resistance or by changing the number of turns of the winding. Let us study the various methods based on this principle.

2.29.1 Flux Control

The various methods of flux control in a d.c. series motor are explained below :

2.29.1.1 Field Diverter Method

In this method the series field winding is shunted by a variable resistance (R_x) known as **field diverter**. The arrangement is shown in the Fig. 2.42 (a).



(a) Field diverter (b) Speed-current characteristics

Fig. 2.42

Due to the parallel path of R_x, by adjusting the value of R_x, any amount of current can be diverted through the diverter. Hence current through the field winding can be adjusted as per the requirement. Due to this, the flux gets controlled and hence the speed of the motor gets controlled.

By this method the speed of the motor can be controlled above rated value. The speed armature current characteristics with change in R_x is shown in the Fig. 2.42 (b).

2.29.1.2 Armature Diverter Method

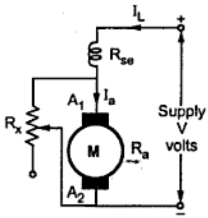


Fig. 2.43 Armature diverter

This method is used for the motor which require constant load torque. An armature of the motor is shunted with an external variable resistance (R_x) as shown in the Fig. 2.43. This resistance R_x is called **armature diverter.**

Any amount of armature current can be diverted through the diverter. Due to this, armature current reduces. But as $T \propto \phi I_a$ and load torque is constant, the flux is to be increased. So motor reacts by drawing more current from the supply. So current through field winding increases, so flux increases and speed of the motor reduces. The method is used to control the speed below the normal value.

2.29.1.3 Tapped Field Method

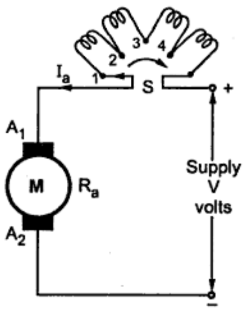


Fig. 2.44 Tapped field

In this method, flux change is achieved by changing the number of turns of the field winding. The field winding is provided with the taps as shown in the Fig. 2.44.

The selector switch 'S' is provided to select the number of turns (taps) as per the requirement. When the switch 'S' is in position 1 the entire field winding is in the circuit and motor runs with normal speed. As switch is moved from position 1 to 2 and onwards, the number of turns of the field winding in the circuit decreases. Due to this m.m.f. required to produce the flux, decreases. Due to this flux produced decreases, increasing the speed of the motor above rated value. The method is often used in electric traction.

2.29.1.4 Series - parallel Connection of Field

In this method, the field coil is divided into various parts. These parts can then be connected in series or parallel as per the requirement. The Fig. 2.45 (a) and (b) show the two parts of field coil connected in series and parallel.

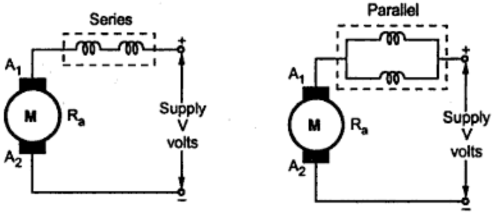
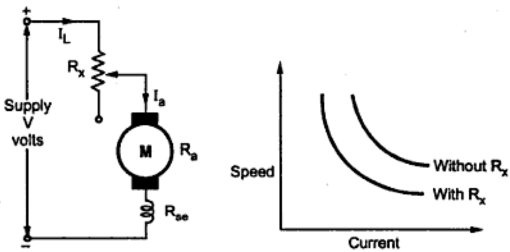


Fig. 2.45 Series-parallel grouping of field coils

For the same torque, if the field coil is arranged in series or parallel, m.m.f. produced by the coils changes, hence the flux produced also changes. Hence speed can be controlled. Some fixed speeds only can be obtained by this method. In parallel grouping, the m.m.f. produced decreases, hence higher speed can be obtained by parallel grouping. The method is generally used in case of fan motors.

2.29.2 Rheostatic Control

In this method, a variable resistance (R_x) is inserted in series with the motor circuit. As this resistance is inserted, the voltage drop across this resistance (I_a R_x) occurs. This reduces the voltage across the armature. As speed is directly proportional to the voltage across the armature, the speed reduces. The arrangement is shown in the Fig. 2.46 (a). As entire current passes through R_x, there is large power loss. The speed-armature current characteristics with change in R_x are shown in the Fig. 2.46 (b).



(a) **Fig. 2.46** **(b)**

2.29.3 Applied Voltage Control

In this method, a series motor is excited by the voltage obtained by a series generator as shown in the Fig. 2.47.

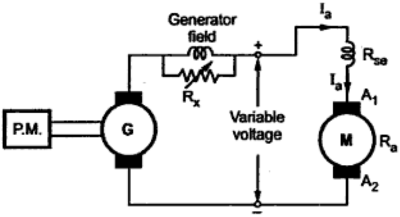


Fig. 2.47 Variable voltage control

The generator is driven by a suitable prime mover. The voltage obtained from the generator is controlled by a field divertor resistance connected across series field winding of the generator.

As $E_g \propto \phi$, the flux change is achieved, gives the variable voltage at the output terminals. Due to the change in the supply voltage, the various speeds of the d.c. series motor can be obtained.

Note : That all the advantages and disadvantages of various methods, discussed as applied to shunt motor are equally applicable to speed control of series motor.

➠ **Example 2.13 :** *A 250 V, d. c. series motor takes 30 A when running at 800 r.p.m., calculate the speed at which motor will run if field winding is shunted by a resistance equal to the field winding resistance and the load torque is increased by 50 %. Armature resistance is 0.15 Ω and series field resistance is 0.1 Ω. Assume the flux produced is proportional to the field current.*

Solution : The method is field diverter. The two conditions are shown in the Fig. 2.48 (a) and (b).

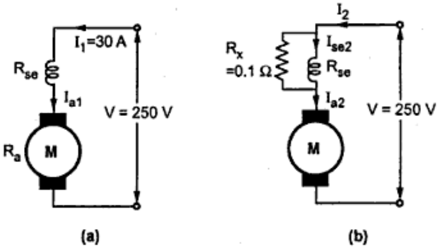


Fig. 2.48

$$V = 250 \text{ V}, \; R_a = 0.15\ \Omega, \; R_{se} = 0.1\ \Omega$$

In first case, $N_1 = 800$ r.p.m.

$$I_1 = I_{a1} = I_{se1} = 30 \text{ A}$$

$$\phi \propto I_{se}$$

According to torque equation, $T \propto \phi I_a \propto I_{se} I_a$

$$\therefore \quad \frac{T_1}{T_2} = \frac{I_{se1}}{I_{se2}} \times \frac{I_{a1}}{I_{a2}}$$

Now $\quad T_2 = T_1 + 0.5\, T_1 \quad$ ... (Increased by 50 %)

$\therefore \quad T_2 = 1.5\, T_1 \quad$ ... (1)

Let us see the current distribution in a parallel circuit. Consider two resistances R_1 and R_2 in parallel as shown in the Fig. 2.49.

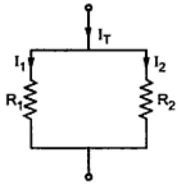


Fig. 2.49

Then $\quad I_T = I_1 + I_2$

and $\quad I_1 = I_T \times \frac{R_2}{R_1 + R_2}$

$$I_2 = I_T \times \frac{R_1}{R_1 + R_2}$$

Applying this to the Fig. 2.48 (b),

$$I_2 = I_{a2}$$

$$\therefore \quad I_{se2} = I_{a2} \times \frac{R_x}{R_x + R_{se}} = I_{a2} \times \frac{0.1}{0.1+0.1}$$

$\therefore \quad I_{se2} = 0.5\, I_{a2} \quad$ ... (2)

Substituting equations (1) and (2) in torque equation,

$$\frac{T_1}{1.5 T_1} = \frac{30}{0.5\, I_{a2}} \times \frac{30}{I_{a2}}$$

$$(I_{a2})^2 = 2700$$

$$\therefore \quad I_{a2} = 51.9615 \text{ A}$$

and $\quad I_{se2} = 0.5\, I_{a2} = 25.9807 \text{ A}$

Now $\quad E_{b1} = V - I_{a1} R_a - I_{se1} R_{se} = 250 - 30 \times 0.15 - 30 \times 0.1$

$$= 242.5 \text{ V}$$

and $\quad E_{b2} = V - I_{a2} R_a - I_{se2} R_{se} = 250 - 51.9615 \times 0.15 - 25.9807 \times 0.1$

$$= 239.607 \text{ V}$$

Use speed equation, $\quad N \propto \frac{E_b}{\phi}$

$$\therefore \quad \frac{N_1}{N_2} = \frac{E_{b1}}{E_{b2}} \times \frac{I_{se2}}{I_{se1}}$$

$$\therefore \quad \frac{800}{N_2} = \frac{242.5}{239.607} \times \frac{25.9807}{30}$$

$$\therefore \quad N_2 = \mathbf{912.744\ r.p.m.}$$

Example 2.14 : *A d.c. series motor runs at 500 r.p.m. on 220 V supply drawing a current of 50 A. The total resistance of the machine is 0.15 Ω, calculate the value of the extra resistance to be connected in series with the motor circuit that will reduce the speed to 300 r.p.m. The load torque being then half of the previous value. Assume flux proportional to the current.*

Solution : The method is rheostatic control. The two conditions are shown in the Fig. 2.50 (a), (b).

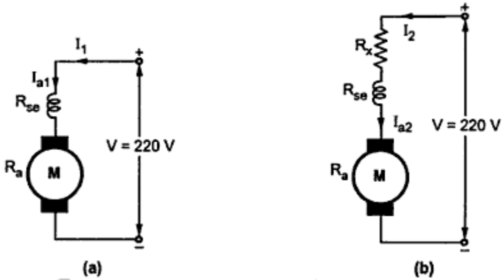


Fig. 2.50

V = 220 V, the total resistance i.e $R_e + R_{se} = 0.15\ \Omega$

In first case, $N_1 = 500$ r.p.m., $I_1 = I_{a1} = 50$ A and $T_2 = 0.5\ T_1$

In this case, the series field current is same as armature current,

$$\phi \propto I_{se} \propto I_a$$

$$\therefore \quad T \propto I_a^2$$

$$\therefore \quad \frac{T_1}{T_2} = \left(\frac{I_{a1}}{I_{a2}}\right)^2$$

$$\therefore \quad \frac{T_1}{0.5\ T_1} = \left(\frac{50}{I_{a2}}\right)^2$$

$$\therefore \quad I_{a2} = 35.355\ \text{A}$$

$$\therefore \quad E_{b1} = V - I_{a1}(R_a + R_{se}) = 220 - 50 \times 0.15 = 212.5\ \text{V}$$

and $\quad E_{b2} = V - I_{a2}(R_a + R_{se} + R_x) = 220 - 35.355\ (0.15 + R_x)$

Use speed equation, $N \propto \frac{E_b}{\phi} \propto \frac{E_b}{I_a}$

$$\therefore \quad \frac{N_1}{N_2} = \frac{E_{b1}}{E_{b2}} \times \frac{I_{a2}}{I_{a1}}$$

$$\therefore \quad \frac{500}{300} = \frac{212.5}{[220-35.355(0.15+R_x)]} \times \frac{35.355}{50}$$

$$\therefore \quad 220 - 35.355\,(0.15 + R_x) = 90.1552$$

$$\therefore \quad R_x = 3.5225\ \Omega$$

2.30 Ward - Leonard System of Speed Control

When it is desired to have wide and very sensitive speed control then this system is more generally used. The system is as shown in the Fig. 2.51.

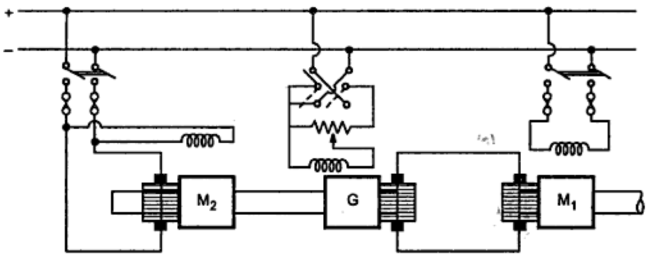


Fig. 2.51

As shown in the Fig. 2.51 M_1 is the main motor whose speed control is required. The field winding of this motor is permanently connected to d.c. supply whereas armature is supplied with variable voltage so that motor can run at any desired speed. To provide this variable voltage, motor generator set is used. It consists of either a.c. or d.c. motor directly coupled to a generator. This motor runs at an approximately constant speed.

The output of generator G is fed to motor M_1. The field circuit of this generator is separately excited from the available d.c. supply through a reversing switch and a potential divider so that its excitation can be varied from zero to maximum in both the directions. Thus generator output voltage will be varied from zero to maximum value. By reversing the direction of the field current of G with the help of reversing switch, polarity of the generated voltage will be reversed and thus change in direction of motor M_1 also will be achieved.

As this method can give unlimited speed control in either directions, this system is commonly employed for elevators, hoists and main drive in steel mills. Also this system is ideal in applications where frequent starting, stopping and reversals are required. As the generator voltage can be raised gradually from zero, the motor starts up smoothly without any extra starting equipment. Although this system is advantageous as it is giving wide range of speeds it requires two extra machines which involves high capital cost.

In modern days SCRs are used for obtaining variable d.c. voltage which will take power from a.c. mains through a transformer. Though it is not less expensive, the arrangement is neat and free from maintenance problems. It will give automatic control of speed.

The modified Ward-Leonard system is called **Ward - Leonard - Ilgner** system in which a flywheel is used in addition to motor generator set. It is used to reduce the fluctuations in power demand from the supply. When load on main motor is suddenly increased, the driving motor M_2 from motor generator set slows down. Thus inertia of flywheel is used to supply part of the overload. However when load is suddenly decreased from motor M_1 then motor M_2 from set speeds up which allows energy to store in the flywheel.

Examples with Solutions

➡ **Example 2.15 :** *A 250 V d.c. shunt motor has $R_a = 0.08\ \Omega$. When connected to 250 V d.c. supply it develops back e.m.f. of 242 V at 1500 r.p.m. Determine,*
i) Armature current ii) Armature current at start
iii) Back e.m.f. if armature current is changed to 120 A
iv) The speed of the machine if it is operated as a generator in order to deliver an armature current of 87 A at 250 V.

Solution :

$R_a = 0.08\ \Omega$, $E_{b1} = 242$ V, V = 250 V

i) $V = E_{b1} + I_{a1} R_a$

$\therefore \quad 250 = 242 + I_{a1} \times 0.08$

$\therefore \quad I_{a1} = \mathbf{100\ A}$

ii) At start, $N = 0$ hence $E_b = 0$

$$\therefore \quad I_{a(start)} = \frac{V}{R_a} = \frac{250}{0.08} = \mathbf{3125\ A}$$

iii) If $I_{a2} = 120$ A then

$$E_{b2} = V - I_{a2} R_a = 250 - 120 \times 0.08 = \mathbf{240.4\ V}$$

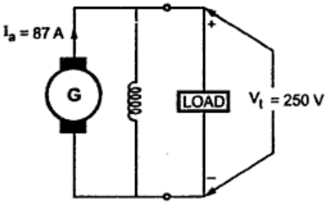


Fig. 2.52

iv) Machine is running as a generator, shown in the Fig. 2.52.

Let induced e.m.f. as a generator be E_g.

$$E_g = V_t + I_a R_a = 250 + 87 \times 0.08 = 256.96 \text{ V}$$

In both cases as a motor or generator $E \propto N \phi$

As flux is constant, $E \propto N$

$$\therefore \quad \frac{E_b}{E_g} = \frac{N_m}{N_g}$$

where N_m = Speed as a motor

N_g = Speed as a generator

$$\therefore \quad \frac{242}{256.96} = \frac{1500}{N_g}$$

$$\therefore \quad N_g = \mathbf{1592.7 \text{ r.p.m.}}$$

Example 2.16 : *A 200 V d.c. series motor drives a load at a certain speed and takes a current of 30 A. The resistance between its terminals is 1.5 Ω. Find the extra resistance to be added in series with the motor circuit to reduced the speed to 60 % of its original value. Assume that the torque produced is proportional to the cube of the speed.*

Solution : $V = 200$ V, $I_{a1} = 30$ A

Resistance across terminals $= R_a + R_{se} = 1.5\ \Omega$

$$\therefore \quad E_{b1} = V - I_{a1}(R_a + R_{se})$$

$$= 200 - 30 \times 1.5 = 155 \text{ V}$$

$$N_2 = 0.6\, N_1$$

$$\therefore \quad \frac{N_1}{N_2} = \frac{1}{0.6}$$

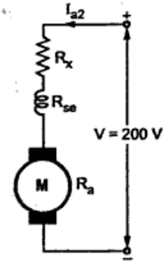


Fig. 2.53

Use torque equation,

$$T \propto \phi I_a \propto I_a^2 \quad \text{as} \ldots \phi \propto I_a$$

$$\therefore \quad \frac{T_1}{T_2} = \left(\frac{I_{a1}}{I_{a2}}\right)^2 \quad \ldots (1)$$

Also $T \propto N^3$ given, $$\frac{T_1}{T_2} = \left(\frac{N_1}{N_2}\right)^3 = \left(\frac{1}{0.6}\right)^3 \quad \ldots (2)$$

Equating equation (1) and (2), $$\left(\frac{1}{0.6}\right)^3 = \left(\frac{30}{I_{a2}}\right)^2$$

$\therefore \quad I_{a2} = 13.9427 \text{ A}$

$\therefore \quad E_{b2} = V - I_{a2}(R_a + R_{se} + R_x)$

$= 200 - 13.9427\,(1.5 + R_x) \quad \text{... (3)}$

Use speed equation, $N \propto \dfrac{E_b}{\phi} \propto \dfrac{E_b}{I_a} \quad \text{... } \phi \propto I_a$

$$\therefore \quad \frac{N_1}{N_2} = \frac{E_{b1}}{E_{b2}} \times \frac{I_{a2}}{I_{a1}}$$

$$\therefore \quad \frac{1}{0.6} = \frac{155}{E_{b2}} \times \frac{13.9427}{30}$$

$$E_{b2} = 43.22 \text{ V} \quad \text{... (4)}$$

Equating equations (3) and (4), $43.22 = 200 - 13.9427\,(1.5 + R_x)$

$$\therefore \quad R_x = 9.745\ \Omega$$

Example 2.17 : *A 230 V d.c. shunt motor takes a current of 30 A on a certain load. The armature resistance is 1 Ω and the field circuit resistance is 230 Ω. Find the resistance to be inserted in series with the armature to halve the speed if the load torque is constant.*

Solution : The two conditions are shown in the Fig. 2.54 (a) and (b).

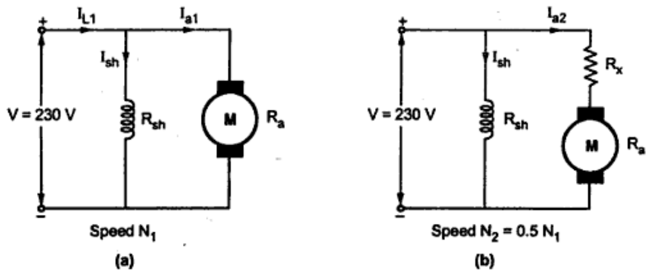


Fig. 2.54

$$I_{L1} = 30 \text{ A}$$

$$I_{sh} = \frac{V}{R_{sh}} = \frac{230}{230} = 1 \text{ A}$$

$$\therefore \quad I_{a1} = I_{L1} - I_{sh} = 30 - 1 = 29 \text{ A}$$

$$\therefore \quad E_{b1} = V - I_{a1}R_a = 230 - 29 \times 1 = 201 \text{ V}$$

$T \propto \phi I_a \propto I_a \quad \text{... } \phi \text{ is constant}$

$\therefore \quad \frac{T_1}{T_2} = \frac{I_{a1}}{I_{a2}} = 1$ ... As torque is constant

$\therefore \quad I_{a2} = I_{a1} = 29 \text{ A}$

R_x = External resistance in armature

$\therefore \quad E_{b2} = V - I_{a2}(R_a + R_x) = 230 - 29(1 + R_x)$

Now $N \propto \frac{E_b}{\phi} \propto E_b$ ... ϕ is constant

$\therefore \quad \frac{N_1}{N_2} = \frac{E_{b1}}{E_{b2}}$ ... $N_2 = 0.5 N_1$

$\therefore \quad \frac{1}{0.5} = \frac{201}{230 - 29(1 + R_x)}$

$\therefore \quad 230 - 29(1 + R_x) = 100.5$

$29(1 + R_x) = 129.5$

$\therefore \quad R_x = \mathbf{3.4655\ \Omega}$

Example 2.18 : *A d.c. series motor developing 40 Nm torque is subjected to the conditions that make field flux to decrease by 30 % and armature current to increase by 15 %. Calculate the new torque.*

Solution : Let T_1 = Initial torque = 40 Nm

ϕ_1 = Initial flux

T_2 = New torque

ϕ_2 = New flux

I_{a1} = Initial current

I_{a2} = New current

Now $\phi_2 = \phi_1 - 0.3\phi_1 = 0.7\phi_1$ ... Decrease by 30 %

and $I_{a2} = I_{a1} + 0.15\, I_{a1} = 1.15\, I_{a1}$ ... Increase by 15 %

$T \propto \phi I_a$

$\therefore \quad \frac{T_1}{T_2} = \frac{\phi_1}{\phi_2} \times \frac{I_{a1}}{I_{a2}}$

$\therefore \quad \frac{40}{T_2} = \frac{\phi_1}{0.7\phi_1} \times \frac{I_{a1}}{1.15\, I_{a1}}$

$\therefore \quad T_2 = 40 \times 0.7 \times 1.15 = \mathbf{32.2\ Nm}$

Example 2.19 : *A 250 V d.c. shunt motor runs at 1000 r.p.m. on no load and takes 5 A. The armature and shunt field resistances are 0.2 Ω and 250 Ω respectively. Calculate the speed when loaded and taking a current of 50 A. Due to armature reaction the field weakens by 3 %.*

Solution : $V = 250\text{ V},\ N_0 = 1000\text{ r.p.m.},\ I_0 = 5\text{ A},\ R_a = 0.2\ \Omega,\ R_{sh} = 250\ \Omega$

$$I_{sh} = \frac{V}{R_{sh}} = \frac{250}{250} = 1\text{ A}$$

$$\therefore \quad I_{a0} = I_0 - I_{sh} = 5 - 1 = 4\text{ A}$$

$$\therefore \quad E_{b0} = V - I_{a0}R_a = 250 - 4\times 0.2 = 249.2\text{ V}$$

$$I_L = 50\text{ A on load}$$

$$I_{sh} = \frac{V}{R_{sh}} = 1\text{ A}$$

$$\therefore \quad I_a = I_L - I_{sh} = 50 - 1 = 49\text{ A}$$

Key Point: *Note that I_{sh} remains same though flux weakens.*

$$\therefore \quad E_{b1} = V - I_a R_a = 250 - 49\times 0.2 = 240.2\text{ V} \qquad \text{... On load}$$

Now $$N \propto \frac{E_b}{\phi}$$

$$\therefore \quad \frac{N_0}{N_1} = \frac{E_{b0}}{E_{b1}} \times \frac{\phi_1}{\phi_0}$$

Now $$\phi_1 = \phi_0 - 0.03\,\phi_0 = 0.97\,\phi_0 \qquad \text{... weakens by 3 \%}$$

$$\therefore \quad \frac{1000}{N_1} = \frac{249.2}{240.2} \times 0.97$$

$$\therefore \quad N_1 = \mathbf{993.695\ r.p.m.}$$

Example 2.20 : *A 250 V d.c. shunt motor takes 4 A when running unloaded. The armature and field resistances are 0.3 Ω and 250 Ω respectively. Calculate the efficiency of the motor when on full load it takes a current of 60 A.*

Solution : No load current $= I_{L0} = 4\text{ A}$

$$I_{sh} = \frac{V}{R_{sh}} = \frac{250}{250} = 1\text{ A}$$

$$\therefore \quad I_{a0} = I_{L0} - I_{sh} = 4 - 1 = 3\text{ A}$$

$\therefore$ No. load armature copper loss $= I_{a0}^2 R_a = 3^2 \times 0.3 = 2.7\text{ W}$

No load input $= V I_{L0} = 250\times 4 = 1000\text{ W}$

$\therefore$ Constant losses = No load input – No load armature copper loss

$= 1000 - 2.7 = 997.3$ W

On full load, $I_L = 60$ A and $I_{sh} = 1$ A

$\therefore \quad I_a = I_L - I_{sh} = 59$ A

$\therefore$ Full load armature copper loss $= I_a^2 R_a = 59^2 \times 0.3 = 1044.3$ W

$\therefore$ Total loss on full load = Constant losses + $I_a^2 R_a$ loss

$= 997.3 + 1044.3 = 2041.6$ W

Total input on full load $= V I_L = 250 \times 60$

$\therefore \quad P_{in} = 15000$ W

$\therefore \quad P_{out} = P_{in}$ – Total loss $= 15000 - 2041.6 = 12958.4$ W

$$\therefore \quad \% \eta = \frac{P_{out}}{P_{in}} \times 100 = \frac{12958.4}{15000} \times 100 = \mathbf{86.389\ \%}$$

Example 2.21 : *A 400 volts shunt motor develops an output of 18.5 kW when taking 22.5 kW. Field resistance is 200 ohms and armature resistance is 0.4 ohms. What is the efficiency and power input when output is 9 kW?*

Solution : $V = 400$ V, $(P_{out})_1 = 18.5$ kW, $(P_{in})_1 = 22.5$ kW

$R_{sh} = 200\ \Omega$, $R_a = 0.4\ \Omega$, $(P_{out})_2 = 9$ kW

$(P_{in})_1 = V I_1$

$\therefore \quad 22.5 \times 10^3 = 400\ I_1$

$\therefore \quad I_1 = 56.25$ A

$$I_{sh} = \frac{V}{R_{sh}} = \frac{400}{200} = 2 \text{ A}$$

$\therefore \quad I_{a1} = I_1 - I_{sh} = 56.25 - 2 = 54.25$ A

Now the various losses in d.c. shunt motor are

$I_{a1}^2 R_a$ = Armature copper loss $= (54.25)^2 \times 0.4 = 1177.225$ W

$I_{sh}^2 R_{sh}$ = Shunt field copper loss $= (2)^2 \times 200 = 800$ W

$(P_{out})_1 = 18.5$ kW

$(P_{in})_1 - (P_{out})_1$ = Total losses = $I_{a1}^2 R_a + I_{sh}^2 R_{sh}$ + Stray and friction loss

$\therefore$ Stray and friction loss $= (P_{in})_1 - (P_{out})_1 - I_{a1}^2 R_a - I_{sh}^2 R_{sh}$

$= 22.5 \times 10^3 - 18.5 \times 10^3 - 1177.225 - 800$

$= 2022.775$ W

These losses remain constant whatever may be the load condition.

Now in second case $(P_{out})_2 = 9$ kW

Hence gross mechanical power developed is

$$P_m = (P_{out})_2 + \text{Stray and friction loss}$$
$$= 9\times10^3 + 2022.775$$
$$= 11022.775 \text{ W}$$

Now $P_m = E_{b2}\, I_{a2}$

and $E_{b2} = V - I_{a2}\, R_a$

$\therefore$ $P_m = (V - I_{a2}\, R_a)\, I_{a2}$

$\therefore$ $11022.775 = (400 - I_{a2} \times 0.4)\, I_{a2}$

$\therefore$ $0.4\,(I_{a2})^2 - 400\, I_{a2} + 11022.775 = 0$

$$\therefore \quad I_{a2} = \frac{400 \pm \sqrt{(400)^2 - 4\times0.4\times11022.775}}{2\times0.4}$$
$$= 28.36 \text{ A},\ 971.63 \text{ A}$$

Neglecting higher value, as output in second case is less than in first case.

$\therefore$ $I_{a2} = 28.36 \text{ A}$

$\therefore$ $I_{a2}^2\, R_a = (28.36)^2 \times (0.4) = 321.715 \text{ W}$

$I_{sh}^2\, R_{sh} = (2)^2 \times (200)$ I_{sh} is same.

$= 800 \text{ W}$

$\therefore$ $(P_{in})_2 = (P_{out})_2 + I_{a2}^2\, R_a + I_{sh}^2\, R_{sh} + (\text{Stray and friction})$

$$= 9\times10^3 + 321.715 + 800 + 2022.775$$
$$= 12144.491 \text{ W}$$
$$= 12.144 \text{ kW}$$

This is the power input when output is 9 kW.

$$\%\,\eta = \frac{(P_{out})_2}{(P_{in})_2}\times100$$
$$= \frac{9\times10^3}{12144.491}\times100$$
$$= \mathbf{74.107\ \%}$$

Example 2.22 : *A 250 V d.c. shunt machine has line current of 80 A. It has armature and field resistances of 0.1 Ω and 125 Ω respectively. calculate power developed in armature when running as (i) Generator and (ii) Motor.* **[Dec.-2004, June-2003]**

Solution : $V = 250$ V, $I_L = 80$ A, $R_a = 0.1\ \Omega$, $R_{sh} = 125\ \Omega$

For shunt machine,

$$I_{sh} = \frac{V}{R_{sh}} = \frac{250}{125} = 2 \text{ A}$$

a) For a generator,

$$I_a = I_L + I_{sh} = 80 + 2 = 82 \text{ A}$$

$$\therefore \quad E = V + I_a R_a = 250 + 82 \times 0.1 = 258.2 \text{ V}$$

$$\therefore \quad P_a = \text{Power developed in armature} = EI_a$$

$$= 258.2 \times 82 = \mathbf{21.1724 \text{ kW}}$$

b) For a motor

$$I_L = I_a + I_{sh} \text{ i.e. } I_a = 80 - 2 = 78 \text{ A}$$

$$\therefore \quad E_b = V - I_a R_a = 250 - 78 \times 0.1 = 242.2 \text{ V}$$

$$\therefore \quad P_a = \text{Power developed in armature} = E_b I_a$$

$$= 242.2 \times 78 = \mathbf{18.8916 \text{ kW}}$$

Example 2.23 : *A 200 V d.c. series motor runs at 1000 r.p.m. when operating at its full load current of 30 A. The motor resistance is 0.5 Ω and the magnetic circuit can be assumed unsaturated. What will be the speed if*
i) The load torque is increased by 44 % and ii) the motor current is 20 A ?
[Dec.-2004, May-2004]

Solution : The moter is shown in the Fig. 2.55.

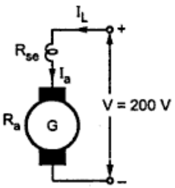


Fig. 2.55

$$I_L = I_{se} = 30 \text{ A} = (I_a)_{FL}$$

$$R_a + R_{se} = 0.5 \ \Omega$$

i) T_1 increased by 44 %

$$\therefore T_1 = T_{FL} + 0.44 \ T_{FL} = 1.44 \ T_{FL}$$

N_{FL} = 1000 r.p.m., N_1 = speed at T_1

$$T \propto \phi I_a \propto I_{se} I_a \propto I_a^2$$

$$\therefore \frac{T_{FL}}{T_1} = \left(\frac{I_{aFL}}{I_1}\right)^2$$

$$\therefore \quad \frac{1}{1.44} = \left(\frac{30}{I_1}\right)^2 \text{ i.e. } I_1 = \mathbf{36 \text{ A}} \qquad \text{...New } I_a$$

$$E_{bFL} = V - I_{aFL}(R_a + R_{se}) = 200 - 30 \times 0.5 = 185 \text{ V}$$

And $$E_{b1} = V - I_1 (R_a + R_{se}) = 200 - 36 \times 0.5 = 182 \text{ V}$$

And $$N \propto \frac{E_b}{\phi} \propto \frac{E_b}{I_{se}} \propto \frac{E_b}{I_a}$$

$$\therefore \quad \frac{N_{FL}}{N_1} = \frac{E_{bFL}}{E_{b1}} \times \frac{I_1}{I_{aFL}}$$

$\therefore \quad \frac{1000}{N_1} = \frac{185}{182} \times \frac{36}{30}$

$\therefore \quad N_1 = \mathbf{819.82\ r.p.m.}$ ...New speed

ii) Motor current $I_2 = 20\ A = I_{a2}$

and $E_{b2} = V - I_{a2}(R_a + R_{se}) = 200 - 20 \times 0.5 = 190\ V$

$\therefore \quad N \propto \frac{E_b}{I_a}$

$\therefore \quad \frac{N_{FL}}{N_2} = \frac{E_{bFL}}{E_{b2}} \times \frac{I_{a2}}{I_{aFL}}$

$\therefore \quad \frac{1000}{N_2} = \frac{185}{190} \times \frac{20}{30}$

$\therefore \quad N_2 = \mathbf{1540.54\ r.p.m.}$

Example 2.24 : *A 200 V D.C. shunt machine has armature and field resistances 0.2 Ω and 200 Ω respectively. The line current is 40 A. Find i) Output as generator ii) Input as motor iii) Power developed in armature and iv) Copper losses in both the cases.*

[May-2004, March-2006 (Set-2)]

Solution : The two conditions of machine are shown in the Fig. 2.56

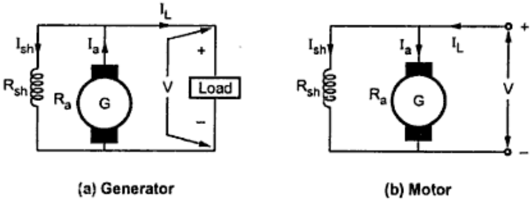


Fig. 2.56

Case a : As a generator

$$P_{out} = V \times I_L = 200 \times 40 = \mathbf{8\ kW}$$

$$I_{sh} = \frac{V}{R_{sh}} = \frac{200}{200} = 1\ A$$

$$I_a = I_L + I_{sh} = 40 + 1 = 41\ A$$

$$E = V + I_a R_a = 200 + 41 \times 0.2 = 208.2\ V$$

$$P_a = E \times I_a = 208.2 \times 41 = \mathbf{8.5362\ kW}$$

$$\therefore \quad P_{cu} = \text{Copper losses} = I_a^2 R_a + I_{sh}^2 R_{sh}$$

$$= (41)^2 \times 0.2 + (1)^2 \times 200 = \mathbf{536.2\ W}$$

Case b : As a motor

$$P_{in} = V \times I_L = 200 \times 40 = \mathbf{8\ kW}$$

$$I_{sh} = \frac{V}{R_{sh}} = 1\ A$$

$$I_a = I_L - I_{sh} = 40 - 1 = 39\ A$$

$$E_b = V - I_a R_a = 200 - 39 \times 0.2 = 192.2\ V$$

$$\therefore \quad P_a = E_b I_a = 192.2 \times 39 = \mathbf{7.4958\ kW}$$

$$P_{cu} = I_a{}^2 R_a + I_{sh}{}^2 R_{sh} = 39^2 \times 0.2 + (1)^2 \times 200 = \mathbf{504.2\ W}$$

Example 2.25 : *A 4-pole series motor has 944 wave-connected armature conductors. At a certain load the flux per pole is 34.6 mWb and the total mechanical torque developed is 209 Nm. Calculate the line current taken by the motor and the speed at which it will run with an applied voltage of 500 V. Total motor resistance is 3 Ω.*

[May-2004 (Set 2), June-2003 (Set-3)]

Solution : $P = 4$, $Z = 944$, $A = 2$ as wave, $\phi = 34.6$ mWb, $T_a = 209$ Nm

$$V = 500\ V \text{ and } R_a + R_{se} = 3\ \Omega$$

$$T_a = 0.159\ \phi I_a \frac{P_Z}{A}$$

$$\therefore \quad 209 = 0.159 \times 34.6 \times 10^{-3} \times I_a \times \frac{4 \times 944}{2}$$

$$\therefore \quad I_a = \mathbf{20.122\ A} = \text{Line current}$$

$$E_b = V - I_a\ (R_a + R_{se}) = 500 - 20.122 \times 3 = 439.634\ V$$

$$\text{But} \quad E_b = \frac{\phi PZN}{60\,A}$$

$$\therefore \quad 439.634 = \frac{34.6 \times 10^{-3} \times 4 \times N \times 944}{60 \times 2}$$

$$\therefore \quad N = \mathbf{403.798\ r.p.m.}$$

Example 2.26 : *A long shunt dynamo running at 1000 r.p.m. supplies 22 kW at a terminal voltage 220 V the resistance of armature, shunt field and series field are 0.05, 110 and 0.06 ohms respectively. The overall efficiency at above load is 88 %. Find :*
i) Copper loss ii) Iron and friction losses. **[May-2004 (Set-3)]**

Solution : $P = 22$ kW, $V_t = 220$ V, $R_a = 0.05\ \Omega$, $R_{sh} = 110\ \Omega$, $R_{se} = 0.06\ \Omega$

$$I_{sh} = \frac{V_t}{R_{sh}} = \frac{220}{110} = 2\ A$$

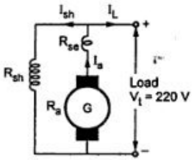


Fig. 2.57

Dynamo means generator

$$I_L = \frac{P}{V_t} = \frac{22\times10^3}{220} = 100 \text{ A}$$

$$I_a = I_L + I_{sh} = 102 \text{ A}$$

$$I_{se} = I_a = 102 \text{ A}$$

P_{out} = Power supplied to load

= 22 kW

$\%\eta$ = 88 %

$$= \left[\frac{P_{out}}{P_{out} + \text{Losses}}\right]\times 100$$

$$\therefore \quad 0.88 = \frac{22\times10^3}{22\times10^3 + \text{Losses}}$$

$\therefore$ Total losses = Copper losses + Iron and friction losses

= 25000 – 22000 = 3 kW

$$\text{Total copper losses} = I_a^2 R_a + I_{se}^2 R_{se} + I_{sh}^2 R_{sh}$$

$$= 102^2 \times 0.05 + 102^2 \times 0.06 + 2^2 \times 110 = \mathbf{1584.44\ W}$$

$\therefore$ Iron and friction losses = $3\times10^3 - 1584.4$ = **1415.56 W**

Example 2.27 : *A 460 V d.c. series motor runs at 1000 r.p.m. taking a current of 40 A. Calculate the speed and percentage change in torque if the load is reduced so that the motor is taking 30 A. Total resistance of the armature and field circuits is 0.8 Ω. Assume flux is proportional to the field current.* **[May-2004 (Set-3), May-2005 (Set-3, 4)]**

Solution : V = 460 V, N_1 = 1000 r.p.m., I_{a1} = 40 A, I_{a2} = 30 A, $R_a + R_{se}$ = 0.8 Ω

$$\therefore \quad E_{b1} = V - I_{a1}(R_a + R_{se}) = 460 - 40\times0.8 = 428 \text{ V}$$

$$E_{b1} = V - I_{a2}(R_a + R_{se}) = 460 - 30\times0.8 = 436 \text{ V}$$

$$N \propto \frac{E_b}{\phi} \propto \frac{E_b}{I_{se}} \propto \frac{E_b}{I_a} \qquad \ldots I_a = I_{se}$$

$$\therefore \quad \frac{N_1}{N_2} = \frac{E_{b1}}{E_{b2}} \times \frac{I_{a2}}{I_{a1}}$$

$$\therefore \quad \frac{1000}{N_2} = \frac{428}{436} \times \frac{30}{40}$$

$\therefore \quad N_2 = \mathbf{1358.255\ r.p.m.}$ ...New speed

And $\quad T \propto \phi I_a \propto I_a^2$

$$\therefore \quad \frac{T_1}{T_2} = \left(\frac{I_{a1}}{I_{a2}}\right)^2 = \left(\frac{40}{30}\right)^2 = 1.777$$

$\therefore \quad T_2 = \mathbf{0.5625}\ T_1 = 56.25\ \%$ of T_1.

Thus torque reduces to 56.25 % of original torque.

$$\%\text{ change in torque} = \frac{T_1 - T_2}{T_1} \times 100$$

$$= \frac{T_1 - 0.5625\,T_1}{T_1} \times 100 = \mathbf{43.75\ \%}$$

Example 2.28 : *A 400 V shunt generator has full load current of 200 A, its armature resistance of 0.06 Ω., field resistance 100 Ω; the stray losses are 2000 W. Find the horse power of its prime-mover when it is delivering full load and find the load for which the efficiency of the generator is maximum.* **[Dec.-2003 (Set-1)]**

Solution : $V = 400$ V, $I_L = 200$ A, $R_a = 0.06\ \Omega$, $R_{sh} = 100\ \Omega$, stray loss = 2 kW

$$I_{sh} = \frac{V}{R_{sh}} = \frac{400}{100} = 4\text{ A}$$

$\therefore \quad I_a = I_L + I_{sh} = 200 + 4 = 204$ A ...As generator

Field cu loss $= I_{sh}^2 R_{sh} = (4)^2 \times 100 = 1600$ W = constant

$\therefore$ Constant loss = Field cu loss + Stray loss = 3600 W

$P_{in} = P_{out}$ + Total losses

$= V \times I_L$ + Armature copper losses + Constant loss

$= (400 \times 200) + I_a^2 R_a + 3600 = 80000 + (204)^2 \times 0.06 + 3600$

$= 86096.96$ W

This must be input to generator i.e. output of prime mover.

$$\therefore \quad \text{horse power rating} = \frac{86096.96}{735.5}$$

$= \mathbf{117.059\ h.P.}$

For maximum efficiency,

Armature copper losses = Constant losses

$\therefore \quad I_a^2 R_a$ = Stray losses + Field copper losses

$\therefore \quad I_a^2 \times 0.06 = 2000 + (4)^2 \times 100 = 3600$

$\therefore \quad I_a = \sqrt{\frac{3600}{0.06}} = 244.948 \text{ A}$

$\therefore \quad I_L = I_a - I_{sh} = 244.948 - 4 = \mathbf{240.948 \text{ A}}$

This is the load at which %η is maximum.

Example 2.29 : *A 220 V D.C. shunt motor with constant field drives a load whose torque is proportional to the speed. When running at 1000 r.p.m. it takes 30 A. Find the speed at which it will run if a 10 Ω resistance is connected in series with its armature. The resistance of armature may be neglected.* **[Dec.-2003 (Set-2)]**

Solution :

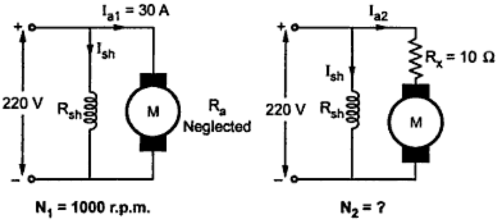


Fig. 2.58

$$E_{b1} = V - I_{a1} R_a = 220 - 0 = 220 \text{ V} \qquad \ldots R_a \text{ neglected}$$

$$T \propto \phi I_{a1} \propto I_a \text{ while } T \propto N \text{ (given)}$$

$$\therefore \quad \frac{T_1}{T_2} = \frac{I_{a1}}{I_{a2}} = \frac{N_1}{N_2}$$

$$\therefore \quad I_{a2} = \frac{N_2}{N_1} \times I_{a1} = \frac{30}{1000} N_2 = 0.03 N_2 \qquad \ldots(1)$$

Also
$$N \propto \frac{E_b}{\phi} \propto E_b \qquad \ldots\text{Flux is constant}$$

$$\therefore \quad \frac{N_1}{N_2} = \frac{E_{b1}}{E_{b2}} \text{ and } E_{b2} = V - I_{a2} R_x \qquad \ldots R_a = 0\ \Omega$$

$$\therefore \quad \frac{1000}{N_2} = \frac{220}{220 - I_{a2} \times 10} \qquad \ldots(2)$$

Using equation (1) in (2),

$$\therefore \quad 1000[220 - 0.03 N_2 \times 10] = 220 N_2$$

$$\therefore \quad 220 - 0.3 N_2 = 0.22 N_2$$

$$\therefore \quad N_2 = \frac{220}{0.52} = \mathbf{423.076 \text{ r.p.m.}}$$

Example 2.30 : *250 V D.C. shunt motor takes 41 A at full load. Resistances of motor armature and shunt field windings are 0.1 Ω and 250 Ω respectively. Find the back e.m.f. on full load. What will be its generated e.m.f., if working as generator and supplying 41 A to load at terminal voltage of 250 V ?* **[Dec.-2003, March-2006 (Set-3), Nov.-2004 (Set-4)]**

Solution : $V = 250$ V, $I_L = 41$ A, $R_a = 0.1\ \Omega$, $R_{sh} = 250\ \Omega$

$$I_{sh} = \frac{V}{R_{sh}} = \frac{250}{250} = 1 \text{ A}$$

$$\therefore \quad I_a = I_L - I_{sh} = 41 - 1 = 40 \text{ A}$$

$$\therefore \quad E_b = V - I_a R_a = 250 - 40 \times 0.1 = \mathbf{246\ V} \quad \text{...Back e.m.f.}$$

As a generator : $V_t = 250$ V, $I_L = 41$ A

$$\therefore \quad I_{sh} = \frac{V_t}{R_{sh}} = 1 \text{ A}$$

But $$I_a = I_L + I_{sh} = 41 + 1 = 42 \text{ A}$$

$$\therefore \quad E_b = V_t + I_a R_a = 250 + 42 \times 0.1 = \mathbf{254.2\ V}$$

Example 2.31 : *Find the torque in N-m exerted by a 4-pole series motor whose armature has 1200 conductors connected up in a 2-circuit winding. The motor current is 10 A and the flux per pole is 0.02 Wb.* **[June-2003 (Set-1), Nov.-2005 (Set-4)]**

Solution : $P = 4$, $Z = 1200$, 2 circuit winding i.e. $A = 2$, $\phi = 0.02$ Wb, $I_a = 10$ A

$$T_a = 0.159\ \phi I_a \frac{PZ}{A} = 0.159 \times 0.02 \times 10 \times \frac{4 \times 1200}{2}$$

$$= \mathbf{76.32\ Nm}$$

Example 2.32 : *A 6-pole, 250 V series motor is wave connected. There are 240 slots and each slot has 4 conductors. The flux per pole is 1.75×10^{-2} Wb when the motor is taking 80 A. The field resistance is 0.05 Ω, the armature resistance is 0.1 Ω and the iron and frictional loss is 0.1 kW. Calculate i) Speed ii) Bhp and iii) Shaft torque.* **[June-2003 (Set-2)]**

Solution : $P = 6$, $V = 250$ V, $\phi = 1.75 \times 10^{-2}$ Wb, $R_{se} = 0.05\ \Omega$, $R_a = 0.1\ \Omega$

$$I_a = 80 \text{ A and iron + friction loss} = 0.1 \text{ kW}$$

$$Z = \text{Total slots} \times (\text{conductors / slot}) = 240 \times 4 = 960$$

$$E_b = V - I_a (R_a + R_{se}) = 250 - 80 \times (0.05 + 0.1) = 238 \text{ V}$$

But $$E_b = \frac{\phi PNZ}{60\,A}$$

$$238 = \frac{1.75\times 10^{-2}\times 6\times N\times 960}{60\times 2}$$ ...A = 2 as wave

i) $$N = \frac{238\times 120}{1.75\times 10^{-2}\times 6\times 960} = \mathbf{283.33\ r.p.m.}$$

ii) $$P_{in} = V\times I_L = 250\times 80 = 20\ kW$$

$$\text{Armature cu loss} = I_a{}^2 R_a = (80)^2\times 0.1 = 640\ W$$

$$\text{Field cu loss} = I_a{}^2 R_{se} = (80)^2\times 0.05 = 320\ W$$

$$\text{Total losses} = 640 + 320 + (\text{Iron} + \text{Friction})$$

$$= 640 + 320 + 0.1\times 10^3 = 1060\ W$$

$$\therefore \quad P_{out} = P_{in} - \text{Total losses} = 18.940\ kW$$

$$\therefore \quad bhp = \frac{P_{out}\ \text{in W}}{735.5} = \mathbf{25.75\ bhp}$$

iii) $$T_{sh} = \frac{P_{out}}{\omega} = \frac{P_{out}}{\frac{2\pi N}{60}} = \frac{18.940\times 10^3\times 60}{2\pi\times 283.33}$$

$$= \mathbf{638.343\ Nm}$$ ...Shaft torque

Example 2.33 : *The armature of a 6-pole, lap-wound d.c. shunt motor takes a current of 350 A at a speed of 400 r.p.m. The flux/pole is 75 mWb and the number of armature conductors is 1200. Calculate the brake horsepower if 3 % of the torque is lost in friction, windage and iron losses.* **[Jan.-2003 (Set-1)]**

Solution : P = 6, A = P as lap, I_a = 350 A, N = 400 r.p.m., ϕ = 75 mWb, Z = 1200

$$T_a = 0.159\,\phi I_a\frac{PZ}{A} = 0.159\times 75\times 10^{-3}\times 350\times\frac{6\times 1200}{6}$$

$$= 5008.5\ Nm$$

$$T_{sh} = T_a - T_{LOST} = T_a - (3\%T_a)$$

$$\therefore \quad P_{out} = T_{sh}\times\omega = 4858.245\times\frac{2\pi N}{60} = 203.5016\ kW$$

$$= 0.97\ T_a = 4858.245\ Nm$$

$$\therefore \quad bhp = \frac{P_{out}\ \text{in W}}{735.5} = \frac{203.5016\times 10^3}{735.5} = \mathbf{276.684\ bhp}$$

Example 2.34 : *A series motor of resistance 1 ohm between terminals runs at 1000 r.p.m. at 250 V with a current of 20 A. Find the speed at which it will run when connected in series with a 6 Ω resistance and taking the same current at the same supply voltage.*

[Dec.-2004 (Set-4)]

Solution : $R_a + R_{se} = 1\ \Omega$, $N_1 = 1000$ r.p.m., $V = 250$ V, $I_{a1} = 20$ A

$$E_{b1} = V - I_{a1}(R_a + R_{se}) = 250 - 20 \times 1 = 230 \text{ V}$$

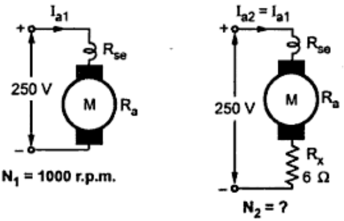


Fig. 2.59

$$E_{b2} = V - I_{a2}(R_a + R_{se} + R_x) = 250 - 20 \times 6 = 130 \text{ V}$$

$$N \propto \frac{E_b}{\phi} \propto \frac{E_b}{I_{se}} \propto \frac{E_b}{I_a} \qquad \ldots I_{se} = I_a$$

$$\therefore \quad \frac{N_1}{N_2} = \frac{E_{b1}}{E_{b2}} \times \frac{I_{a2}}{I_{a1}} \qquad \because I_{a2} = I_{a1}$$

$$\therefore \quad \frac{1000}{N_2} = \frac{230}{130}$$

$$\therefore \quad N_2 = \mathbf{565.2173 \text{ r.p.m.}}$$

Example 2.35 : *A 4 pole series wound fan motor draws an armature current of 50 Amps, when running at 2000 r.p.m. on a 230 V d.c. supply with four field coils connected in series. The four field coils are then reconnected in two parallel groups of two coils in series. Assuming flux/pole to be proportional to the exciting current and load torque proportional to the square of the speed. Find the new speed and armature current.*

Solution : $P = 4$, $I_{a1} = 50$ A, $N_1 = 2000$ r.p.m., $V = 230$ V

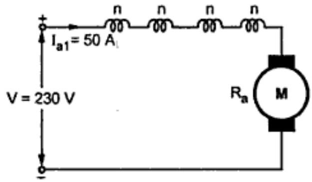


Fig. 2.60

The field coil is divided into 4 groups each of say 'n' turns. In first case, the coils are in series as shown in the Fig. 2.60

So flux ϕ_1 produced in this case is proportional to total ampere turns produced by field coils.

$$\therefore \quad \phi_1 \propto I_{a1} \times (4\ n) \propto 50 \times 4n \propto 200\ n \quad \text{... (1)}$$

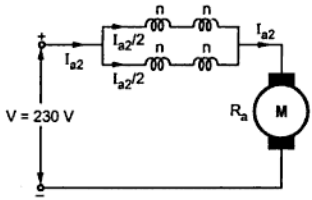


Fig. 2.61

Now the coils are reconnected in two parallel groups of two coils in series. This is shown in the Fig. 2.61. As coil group resistances are equal, the current I_{a2} will split into two equal parts as $I_{a2}/2$. Now ϕ_2 will be proportional to the total ampere turns.

$$\therefore \quad \phi_2 \propto \left[\frac{I_{a2}}{2} \times 2\,n + \frac{I_{a2}}{2} \times 2\,n\right] \propto 2\ n\ I_{a2} \quad \text{... (2)}$$

Dividing equation (1) and (2),

$$\frac{\phi_1}{\phi_2} = \frac{200\ n}{2\ n\ I_{a2}} = \frac{100}{I_{a2}} \quad \text{... (3)}$$

Now $\quad T \propto \phi I_a$

$$\therefore \quad \frac{T_1}{T_2} = \frac{\phi_1}{\phi_2} \times \frac{I_{a1}}{I_{a2}} \quad \text{... (4)}$$

and $\quad T \propto N^2 \quad$ ... Given

$$\therefore \quad \frac{T_1}{T_2} = \left(\frac{N_1}{N_2}\right)^2 \quad \text{... (5)}$$

$$\therefore \quad \left(\frac{N_1}{N_2}\right)^2 = \frac{\phi_1}{\phi_2} \times \frac{I_{a1}}{I_{a2}} \quad \text{... (6)}$$

Now $\quad N \propto \dfrac{E_b}{\phi}$

$\therefore \quad \frac{N_1}{N_2} = \frac{E_{b1}}{E_{b2}} \times \frac{\phi_2}{\phi_1}$

But as resistances are not given, the drops across windings can be neglected, as practically the drops are very small hence $E_{b1} = E_{b2}$.

$\therefore \quad \frac{N_1}{N_2} = \frac{\phi_2}{\phi_1}$ ... (7)

Substitute equation (7) in (6),

$$\left(\frac{\phi_2}{\phi_1}\right)^2 = \frac{\phi_1}{\phi_2} \times \frac{I_{a1}}{I_{a2}}$$

$\therefore \quad \left(\frac{\phi_2}{\phi_1}\right)^3 = \frac{I_{a1}}{I_{a2}}$ ... (8)

Using equation (3) in (8),

$\therefore \quad \left(\frac{I_{a2}}{100}\right)^3 = \frac{50}{I_{a2}}$

$\therefore \quad (I_{a2})^4 = 50 \times (100)^3$

$\therefore \quad I_{a2} = \mathbf{84.089\ A}$ ... New armature current

Now $\quad \left(\frac{N_1}{N_2}\right)^2 = \frac{\phi_1}{\phi_2} \times \frac{I_{a1}}{I_{a2}}$

$\left(\frac{N_1}{N_2}\right)^2 = \frac{N_2}{N_1} \times \frac{I_{a1}}{I_{a2}}$ ... From (7)

$\therefore \quad \left(\frac{N_1}{N_2}\right)^3 = \frac{I_{a1}}{I_{a2}}$

$\therefore \quad \left(\frac{2000}{N_2}\right)^3 = \frac{50}{84.089}$

$\therefore \quad N_2 = \mathbf{2378.408\ r.p.m.}$ ... New speed of motor

Example 2.36 : *A 20 kW, 250 V d.c. shunt generator has armature and field resistancse of 0.04 Ω and 200 Ω respectively. Determine the total power developed when working i) As generator delivering 20 kW output and ii) As a motor taking 20 kW input.*

[March-2006 (Set-2), May-2004 (Set-4), Dec.-2004 (Set-2)]

Solution : Case I : Working as a generator

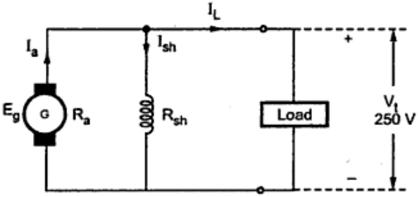


Fig. 2.62 (a)

$$P_{out} = 20 \text{ kW} = V_t \times I_L$$

$$\therefore I_L = \frac{20 \times 10^3}{250} = 80 \text{ A}$$

$$I_{sh} = \frac{V_t}{R_{sh}} = \frac{250}{200} = 1.25 \text{ A}$$

$$\therefore I_a = I_L + I_{sh} = 81.25 \text{ A}$$

$$\therefore \quad E_g = V_t + I_a R_a = 250 + (81.25 \times 0.04) = 253.25 \text{ V}$$

$$\therefore \quad P_{dev} = E_g \times I_a = 253.25 \times 81.25 = \mathbf{20.5765 \text{ kW}}$$

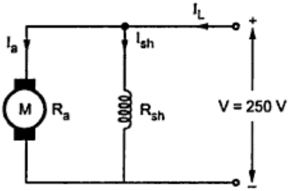


Fig. 2.62 (b)

Case II : Working as a motor

$$P_{in} = 20 \text{ kW} = V \times I_L$$

$$\therefore \quad I_L = \frac{20 \times 10^3}{250} = 80 \text{ A}$$

$$I_{sh} = \frac{V}{R_{sh}} = \frac{250}{200} = 1.25 \text{ A}$$

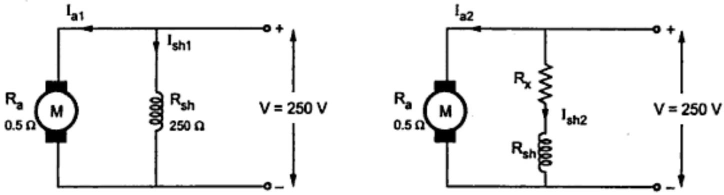
$$\therefore \quad I_a = I_L - I_{sh} = 78.75 \text{ A}$$

$$\therefore \quad E_b = V - I_a R_a = 250 - (78.75 \times 0.04) = 246.85 \text{ V}$$

$$\therefore \quad P_m = E_b \times I_a = 246.85 \times 78.75 = \mathbf{19.4394 \text{ kW}}$$

Example 2.37 : *A 250 V d.c. shunt motor has an armature resistance of 0.5 Ω and field resistance of 250 Ω. When delivering a load of constant torque at 600 r.p.m., the armature current is 20 A. If it is desired to raise the speed from 600 r.p.m. to 800 r.p.m., what resistance should be inserted in the shunt field circuit ? Assume unsaturated magnetic circuit.* **[May-2004 (Set-1)]**

Solution :



(a) N_1 = 600 r.p.m. (b) N_2 = 800 r.p.m.

Fig. 2.63

$$I_{a1} = 20\text{ A}, N_1 = 600\text{ r.p.m.}, R_a = 0.5\ \Omega$$

$$I_{sh1} = \frac{V}{R_{sh}} = \frac{250}{250} = 1\text{ A}$$

$$T \propto \phi I_a \propto I_{sh} I_a \qquad \ldots\ \phi \propto I_{sh}$$

$$\therefore \quad \frac{T_1}{T_2} = \frac{I_{sh1} I_{a1}}{I_{sh2} I_{a2}} = 1 \qquad \ldots\ T_1 = T_2 \text{ constant}$$

$$\therefore \quad 1 \times 20 = I_{sh2} I_{a2} \qquad \ldots(1)$$

Now $$E_{b1} = V - I_{a1} R_a = 250 - 20 \times 0.5 = 240\text{ V}$$

$$E_{b2} = V - I_{a2} R_a = 250 - 0.5 I_{a2}$$

$$N \propto \frac{E_b}{\phi} \propto \frac{E_b}{I_{sh}}$$

$$\therefore \quad \frac{N_1}{N_2} = \frac{E_{b1}}{E_{b2}} \times \frac{I_{sh2}}{I_{sh1}} \quad \text{i.e.} \quad \frac{600}{800} = \frac{240}{[250 - 0.5 I_{a2}]} \times \frac{I_{sh2}}{1}$$

From equation (1), $I_{a2} = \frac{20}{I_{sh2}}$ and using above,

$$\frac{600}{800} = \frac{240}{\left[250 - \frac{0.5 \times 20}{I_{sh2}}\right]} \times \frac{I_{sh2}}{1}$$

$$\therefore \quad 320\ I_{sh2}^2 - 250\ I_{sh2} + 10 = 0$$

Solving, $$I_{sh2} = 0.7389\text{ A} \qquad \ldots \text{ Neglecting lower value}$$

But, $$I_{sh2} = \frac{V}{R_{sh} + R_x} \quad \text{i.e.} \quad 0.7389 = \frac{250}{250 + R_x}$$

$$\therefore \quad R_x = 88.3407\ \Omega \qquad \ldots \text{ Additional resistance in field}$$

Example 2.38 : *A series motor takes 20 A at 400 V to drive a fan at 200 r.p.m. Its resistance is 1 Ω. If the torque required to drive the fan varies as the square of the speed, find the necessary applied voltage and current to drive the fan at 300 r.p.m.*

[Nov.-2006 (Set-3)]

Solution : $V_1 = 400\text{ V}, I_{a1} = 20\text{ A}, N_1 = 200\text{ r.p.m.}, N_2 = 300\text{ r.p.m.}$

$$R_t = R_a + R_{se} = 1\ \Omega, \quad T \propto N^2$$

$$T \propto \phi I_a \propto I_a^2 \qquad \ldots\ \phi \propto I_a \text{ for series}$$

and $$T \propto N^2 \qquad \ldots \text{ Given}$$