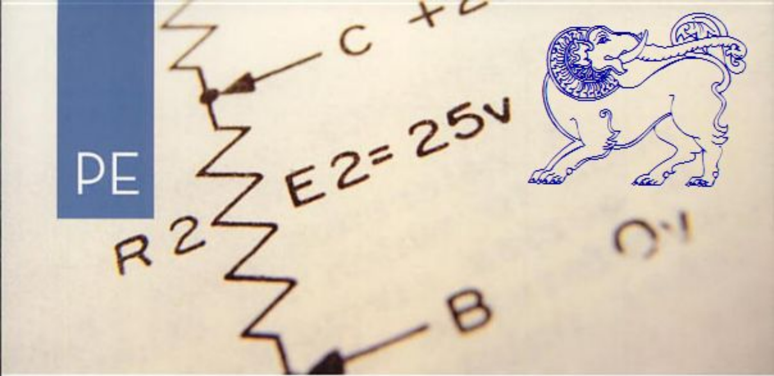
PE



ELECTRICAL ENGINEERING

PROBLEMS & SOLUTIONS

Ninth Edition

Lincoln D. Jones, MS, PE Elec. Eng.

- Ideal review for the breadth/depth exam
- Over 200 problems & solutions
- Tips & techniques for passing the exam on the first try

PE EXAM PREPARATION

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Introduction

Donald G. Newnan

OUTLINE

HOW TO USE THIS BOOK

Electrical Engineering: Problems & Solutions and its companion texts form a three-step approach to preparing for the Principles and Practice of Engineering (PE) exam:

- *Electrical Engineering: License Review* contains the conceptual review of electrical engineering topics for the exam, including key terms, equations, analytical methods and reference data. Because it does not contain problems and solutions, the book can be brought into the open-book PE exam as one of your references.
- *Electrical Engineering: Problems & Solutions* provides problems for you to solve in order to test your understanding of concepts and techniques. Ideally, you should solve these problems after completing your conceptual review. Then, compare your solution to the detailed solutions provided, to get a sense of how well you have mastered the content and what topics you may want to review further.
- *Electrical & Computer Engineering: Sample Exam* provides complete morning and afternoon exam sections so that you can simulate the experience of taking the PE test within its actual time constraints and with questions that match the test format. Take the sample exam after you're satisfied with your review of concepts and problem-solving techniques, to test your readiness for the real exam.

BECOMING A PROFESSIONAL ENGINEER

To achieve registration as a professional engineer there are four distinct steps: (1) education, (2) the Fundamentals of Engineering/Engineer-In-Training (FE/EIT) exam, (3) professional experience, and (4) the professional engineer (PE) exam, more formally known as the Principles and Practice of Engineering Exam. These steps are described in the following sections.

Education

The obvious appropriate education is a B.S. degree in electrical engineering from an accredited college or university. This is not an absolute requirement. Alternative, but less acceptable, education is a B.S. degree in something other than electrical engineering, or a degree from a non-accredited institution, or four years of education but no degree.

Fundamentals of Engineering (FE/EIT) Exam

Most people are required to take and pass this eight-hour multiple-choice examination. Different states call it by different names (Fundamentals of Engineering, E.I.T., or Intern Engineer), but the exam is the same in all states. It is prepared and graded by the National Council of Examiners for Engineering and Surveying (NCEES). Review materials for this exam are found in other Engineering Press books such as *Fundamentals of Engineering: FE Exam Preparation*.

Experience

Typically one must have four years of acceptable experience before being permitted to take the Professional Engineer exam (California requires only two years). Both the length and character of the experience will be examined. It may, of course, take more than four years to acquire four years of acceptable experience.

Professional Engineer Exam

The second national exam is called Principles and Practice of Engineering by NCEES, but just about everyone else calls it the Professional Engineer or P.E. exam. All states, plus Guam, the District of Columbia, and Puerto Rico, use the same NCEES exam.

ELECTRICAL ENGINEERING PROFESSIONAL ENGINEER EXAM

The reason for passing laws regulating the practice of civil engineering is to protect the public from incompetent practitioners. Most states require engineers working on projects involving public safety to be registered, or to work under the supervision of a registered professional engineer. In addition, many private companies encourage or require engineers in their employ to pursue registration as a matter of professional development. Engineers in private practice, who wish to consult or serve as expert witnesses, typically also must be registered. There is no national registration law; registration is based on individual state laws and is administered by boards of registration in each of the states. A listing of the state boards is in Table 1.1.

Table 1.1 State boards of registration for engineers

State/Territory	Web site	Telephone
AK	www.dced.state.ak.us/occ/pael.htm	(907) 465-1676
AL	www.bels.state.al.us	(334) 242-5568
AR	www.state.ar.us/pels	(501) 682-2824
AZ	www.state.ar.us/pels	(602) 364-4930
CA	dca.ca.gov/pels/contacts/htm	(916) 263-2230
CO	dova.state.co.us/engineers _surveyors	(303) 894-7788
CT	state.ct.us/dcp	(806) 713-6145
DC		(202) 442-4320
DE	www.dape.org	(302) 368-6708
FL	www.fbpe.org	(850) 521-0500
GA	www.sos.state.ga.us/plb/pels/	(478) 207-1450
GU	www.guam-peals.org	(671) 646-3138
HI	www.hawaii.gov/dcca/pbl	(808) 586-2702
IA	www.ia.us/government/com	(515) 281-4126
ID	www.state.id.us/ipels/index.htm	(208) 334-3860
IL	www.kpr.state.il.us	(217) 785-0877
IN	www.in.gov/pla/bandc/engineers	(317) 232-2980
KS	www.accesskansas.org/ksbtp	(785) 296-3053
KY	www.kybocls.state.ky.us	(502) 573-2680
LA	www.lapels.com	(225) 925-6291
MA	www.state.ma.us/reg	(617) 727-9957
MD	www.dllr.state.md.us	(410) 230-6322
ME	www.professionals.maineusa.com	(207) 287-3236
MI	www.michigan.gov/cis/0,1607.7-154-10557_12992_14016—.00.htm.us	(517) 241-9253
MN	www.aclslagid.state.mn.us	(651) 296-2388
MO	www.pr.mo.gov/apelsla.asp	(573) 751-0047
MP		(011)-(670) 234-5897
MS	www.pepls.state.ms.us	(601) 359-6160
MT	www.discoveringmontana.com/dli/bsb/license/bsd_board/pel_board/board_page.htm	(406) 841-2367
NC	www.ncbels.org	(919) 881-4000
ND	www.ndpelsboard.org/	(701) 258-0786
NE	www.ea.state.ne.us	(402) 471-2021
NH	www.state.nh.us/jtboard/home.htm	(603) 271-2219
NJ	www.state.nj.us	(973) 504-6460
NM	www.state.nm.us/pepsboard	(518) 827-7561
NV	www.boe.state.nv.us	(775) 688-1231
NY	www.op.nysed.gov	(518) 474-3846 3817 × 140
OH	www.ohiopeps.org	(614) 466-3651
OK	www.pels.state.ok.us/	(405) 521-2874
OR	www.osbeels.org	(503) 362-2666
PA	www.dos.state.pa.us/eng	(717) 783-7049
PR	P.O. Box 3271, San Juan 00904	(787) 722-2122
RI	www.bdp.state.ri.us	(401) 222-2565
SC	www.lln.state.sc.us/POL/Engineers	(803) 896-4422

(*Continued*)

Table 1.1 State boards of registration for engineers *(Continued)*

State/Territory	Web site	Telephone
SD	www.state.sd.us/dol/boards/engineer	(605) 394-2510
TN	www.state.tn.us/commerce/boards/ae	(615) 741-3221
TX	www.tbpe.state.tx.us	(512) 440-7723
UT	www.dopl.utah.gov	(801) 530-6632
VA	www.state.va.us/dopr	(804) 367-8514
VI	www.dlca.gov.vi/pro-aels.html	(340) 773-2226
VT	www.vtprofessionals.org	(802) 828-3256
WA	www.dol.wa.gov/engineers/engfront.htm	(360) 664-1595
WI	www.drl.state.wi.us	(608) 261-7096
WV	www.wvpebd.org	(304) 558-3554
WY	www.wrds.uwyo.edu/wrds/borpe/borpe.html	(307) 777-6155

Examination Development

Initially the states wrote their own examinations, but beginning in 1966 the NCEES took over the task for some of the states. Now the NCEES exams are used by all states. This greatly eases the ability of an engineer to move from one state to another and achieve registration in the new state.

The development of the engineering exams is the responsibility of the NCEES Committee on Examinations for Professional Engineers. The committee is composed of people from industry, consulting, and education, plus consultants and subject matter experts. The starting point for the exam is a task analysis survey, which NCEES does at roughly 5- to 10-year intervals. People in industry, consulting, and education are surveyed to determine what civil engineers do and what knowledge is needed. From this NCEES develops what it calls a "matrix of knowledge" that forms the basis for the exam structure described in the next section.

The actual exam questions are prepared by the NCEES committee members, subject matter experts, and other volunteers. All people participating must hold professional registration. Using workshop meetings and correspondence by mail, the questions are written and circulated for review. The problems relate to current professional situations. They are structured to quickly orient one to the requirements, so that the examinee can judge whether he or she can successfully solve it. Although based on an understanding of engineering fundamentals, the problems require the application of practical professional judgment and insight.

Examination Structure

The morning breadth exam consists of 40 multiple-choice questions covering the following areas of electrical engineering (relative exam weight for each topic is shown in parentheses):

- Basic electrical engineering (45%). This area encompasses economics, ethics, professional practice, safety, electric circuits, electric and magnetic theory and applications, and digital logic.
- Electronics, electronic circuits and components (20%).
- Controls and communication systems (15%)
- Power (20%)

You will have four hours to complete the breadth exam.

The afternoon depth exam is actually three exams; you choose the depth exam you wish to take. The depth exams are:

- Computers
- Electronics, Controls, and Communications
- Power

Clearly, you should choose the exam that best matches your training and professional practice. You will have four hours to answer the 40 multiple-choice questions that make up the depth exam.

Both the breadth and depth questions include four possible answers (A, B, C, D) and are objectively scored by computer.

For more information on the topics and subtopics and their relative weights on the breadth and depth portions, visit the NCEES Web site at www.ncees.org.

Exam Dates

The National Council of Examiners for Engineering and Surveying (NCEES) prepares Professional Engineer exams for use on a Friday in April and October of each year. Some state boards administer the exam twice a year in their state, whereas others offer the exam once a year. The scheduled exam dates are:

	April	October
2005	15	28
2006	21	27
2007	20	26
2008	11	24

People seeking to take a particular exam must apply to the state board several months in advance.

Exam Procedure

Before the morning four-hour session begins, the proctors pass out an exam booklet and solutions pamphlet to each examinee.

The solution pamphlet contains grid sheets on right-hand pages. Only the work on these grid sheets will be graded. The left-hand pages are blank and are to be used for scratch paper. The scratchwork will not be considered in the scoring.

If you finish more than 30 minutes early, you may turn in the booklets and leave. In the last 30 minutes, however, you must remain to the end to ensure a quiet environment for all those still working and the orderly collection of materials.

The afternoon session will begin following a one-hour lunch break. The afternoon exam booklet will be distributed along with an answer sheet.

Preparing for and Taking the Exam

Give yourself time to prepare for the exam in a calm and unhurried way. Many candidates like to begin several months before the actual exam. Target a number of hours per day or week that you will study, and reserve blocks of time for doing so. Creating a review schedule on a topic-by-topic basis is a good idea. Remember to allow time for both reviewing concepts and solving practice problems.

In addition to review work that you do on your own, you may want to join a study group or take a review course. A group study environment might help you stay committed to a study plan and schedule. Group members can create additional practice problems for one another and share tips and tricks.

You may want to prioritize the time you spend reviewing specific topics according to their relative weight on the exam, as identified by NCEES, or by your areas of relative strength and weakness.

People familiar with the psychology of exam taking have several suggestions for people as they prepare to take an exam.

1. Exam taking involves really, two skills. One is the skill of illustrating knowledge that you know. The other is the skill of exam taking. The first may be enhanced by a systematic review of the technical material. Exam-taking skills, on the other hand, may be improved by practice with similar problems presented in the exam format.
2. Since there is no deduction for guessing on the multiple choice problems, answers should be given for all of them. Even when one is going to guess, a logical approach is to attempt to first eliminate one or two of the four alternatives. If this can be done, the chance of selecting a correct answer obviously improves from 1 in 4 to 1 in 3 or 1 in 2.
3. Plan ahead with a strategy. Which is your strongest area? Can you expect to see several problems in this area? What about your second strongest area? What will you do if you still must find problems in other areas?
4. Plan ahead with a time allocation. Compute how much time you will allow for each of the subject areas in the breadth exam and the relevant topics in the depth exam. You might allocate a little less time per problem for those areas in which you are most proficient, leaving a little more time in subjects that are difficult for you. Your time plan should include a reserve block for especially difficult problems, for checking your scoring sheet, and to make last-minute guesses on problems you did not work. Your strategy might also include time allotments for two passes through the exam—the first to work all problems for which answers are obvious to you, and the second to return to the more complex, time-consuming problems and the ones at which you might need to guess. A time plan gives you the confidence of being in control and keeps you from making the serious mistake of misallocation of time in the exam.
5. Read all four multiple-choice answers before making a selection. An answer in a multiple-choice question is sometimes a plausible decoy—not the best answer.
6. Do not change an answer unless you are absolutely certain you have made a mistake. Your first reaction is likely to be correct.
7. Do not sit next to a friend, a window, or other potential distractions.

Exam Day Preparations

The exam day will be a stressful and tiring one. This will be no day to have unpleasant surprises. For this reason we suggest that an advance visit be made to the examination site. Try to determine such items as

1. How much time should I allow for travel to the exam on that day? Plan to arrive about 15 minutes early. That way you will have ample time, but not

too much time. Arriving too early, and mingling with others who also are anxious, will increase your anxiety and nervousness.

2. Where will I park?
3. How does the exam site look? Will I have ample workspace? Where will I stack my reference materials? Will it be overly bright (sunglasses), cold (sweater), or noisy (earplugs)? Would a cushion make the chair more comfortable?
4. Where are the drinking fountain, lavatory facilities, pay phone?
5. What about food? Should I take something along for energy in the exam? A bag lunch during the break probably makes sense.

What to Take to the Exam

The NCEES guidelines say you may bring only the following reference materials and aids into the examination room for your personal use:

1. Handbooks and textbooks, including the applicable design standards.
2. Bound reference materials, provided the materials remain bound during the entire examination. The NCEES defines "bound" as books or materials fastened securely in their covers by fasteners that penetrate all papers. Examples are ring binders, spiral binders and notebooks, plastic snap binders, brads, screw posts, and so on.
3. Battery-operated, silent, nonprinting, noncommunicating calculators. Beginning with the April 2004 exam, NCEES has implemented a more stringent policy regarding permitted calculators. For more details, see the NCEES website (www.ncees.org), which includes the policy and a list of permitted calculators. You also need to determine whether or not your state permits preprogrammed calculators. Bring extra batteries for your calculator just in case; many people feel that bringing a second calculator is also a very good idea.

At one time NCEES had a rule barring "review publications directed principally toward sample questions and their solutions" in the exam room. This set the stage for restricting some kinds of publications from the exam. *State boards may adopt the NCEES guidelines, or adopt either more or less restrictive rules*. Thus an important step in preparing for the exam is to know what will—and will not—be permitted. We suggest that if possible you obtain a written copy of your state's policy for the specific exam you will be taking. Occasionally there has been confusion at individual examination sites, so a copy of the exact applicable policy will not only allow you to carefully and correctly prepare your materials, but will also ensure that the exam proctors will allow all proper materials that you bring to the exam.

As a general rule we recommend that you plan well in advance what books and materials you want to take to the exam. Then they should be obtained promptly so you use the same materials in your review that you will have in the exam.

License Review Books

The review books you use to prepare for the exam are good choices to bring to the exam itself. After weeks or months of studying, you will be very familiar with their organization and content, so you'll be able to quickly locate the material you want to reference during the exam. Keep in mind the caveat just discussed—some state boards will not permit you to bring in review books that consist largely of sample questions and answers.

Textbooks

If you still have your university textbooks, they are the ones you should use in the exam, unless they are too out of date. To a great extent the books will be like old friends with familiar notation.

Bound Reference Materials

The NCEES guidelines suggest that you can take any reference materials you wish, so long as you prepare them properly. You could, for example, prepare several volumes of bound reference materials, with each volume intended to cover a particular category of problem. Maybe the most efficient way to use this book would be to cut it up and insert portions of it in your individually prepared bound materials. Use tabs so that specific material can be located quickly. If you do a careful and systematic review of civil engineering, and prepare a lot of well-organized materials, you just may find that you are so well prepared that you will not have left anything of value at home.

Other Items

In addition to the reference materials just mentioned, you should consider bringing the following to the exam:

- *Clock*—You must have a time plan and a clock or wristwatch.
- *Exam assignment paperwork*—Take along the letter assigning you to the exam at the specified location. To prove you are the correct person, also bring something with your name and picture.
- *Items suggested by advance visit*—If you visit the exam site, you probably will discover an item or two that you need to add to your list.
- *Clothes*—Plan to wear comfortable clothes. You probably will do better if you are slightly cool.
- *Box for everything*—You need to be able to carry all your materials to the exam and have them conveniently organized at your side. Probably a cardboard box is the answer.

CHAPTER 1

Fundamental Concepts and Techniques

OUTLINE

PROBLEMS

1.1 Determine the node voltages V_1, V_2, and V_3 in Exhibit 1.1a by setting up the proper nodal equations in matrix form, manipulating the matrices to form the matrix solution for the voltages.

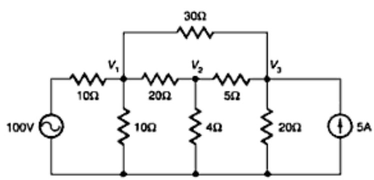
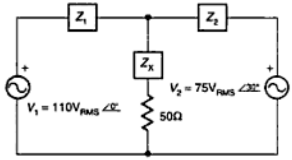


Exhibit 1.1a

1.2 The 50 Ω load is to receive maximum power from the system in Exhibit 1.2a. A small series reactance, Z_x is to be placed in series with the load to accomplish this. Find the proper element for Z_x and calculate the power into the 50 Ω load when this element is placed in the circuit.



(Assume $Z_1 = 60 + j80$, $Z2 = 30 - j50$, $f = 60\text{Hz}$)

Exhibit 1.2a

1.3 Find the current through the 3 ohm resistor for the circuit in Exhibit 1.3 with a dependent voltage source. Also find the power taken from the source. Check the powers being dissipated in the resistors: Do they equal the power taken from the source?

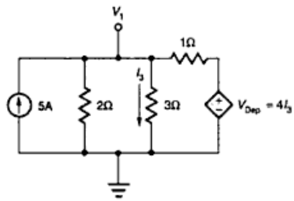


Exhibit 1.3

1.4 Determine the value of $i(t)$, as shown in Exhibit 1.4a, in the steady state.

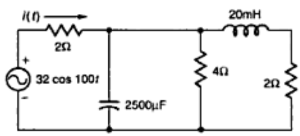


Exhibit 1.4a

1.5 Find the phasor current I and the phasor voltage drop across each element in Exhibit 1.5, in polar form, using the source as the reference for angles.

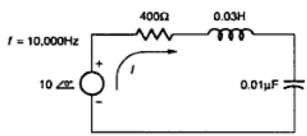


Exhibit 1.5

1.6 The frequency of each source is 1000 Hz with the phase relations shown in Exhibit 1.6a. Replace this circuit by its Norton phasor equivalent.

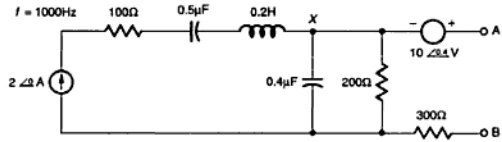


Exhibit 1.6a

1.7 A 2.0 μf capacitor is charged so that it has 100 volts across its terminals. The terminals are suddenly connected through a negligible resistance to the terminals of a 4.0 μf capacitor having no initial charge.
   a. What are the final steady state voltages across each capacitor?
   b. What are the initial stored energies of each capacitor?
   c. What are the final steady state stored energies of each capacitor?

1.8 A 100 ohm resistor is placed in series with the 4.0 μf capacitor in Problem 1.7 so that the charging current will flow through the resistor. The same 2.0 μf capacitor charged to 100 volts is connected to the combination of the 4.0 μf capacitor and 100 ohm resistor in series with the 4.0 μf capacitor having no initial charge.
   a. What is the time constant of the circuit?
   b. What are the final steady state voltages across each capacitor?
   c. What are the final steady state stored energies of each capacitor?
   d. How do you explain or account for the results of the energies calculated in (c) above in light of the different resistor power losses?

1.9 The circuit shown in Exhibit 1.9a is in a steady state.
   a. What current is drawn from the power source in this initial steady state condition?
   b. What is the analytical expression for the current drawn from the power source after closing switch *S*?

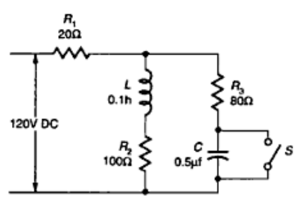


Exhibit 1.9a

1.10 The switch in Exhibit 1.10 was closed sufficiently long ago such that the current i (through the source) has reached steady state. The switch is then opened at time $t = t(0)$.

a. Find the current i just before the switch is opened, that is, at $t = t(0^-)$

b. Find the current i just after the switch is opened, that is, at $t = t(0^+)$

c. Find the current i as a function of time after the switch is opened; that is, find $i(t - t_0)$, where $t_0 = t(0)$.

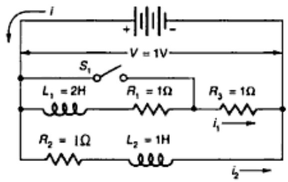


Exhibit 1.10

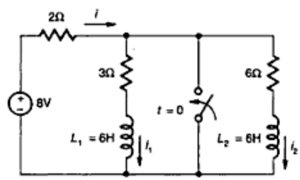


Exhibit 1.10a

1.11 For the circuit shown in Exhibit 1.10a, determine $i_1(0^+)$, $i_2(0^+)$, energy stored in L_1 and L_2 at $t = 0$, $i_1(t)$ and $i_2(t)$ for $t > 0$.

1.12 For the circuit shown in Exhibit 1.12a, find and sketch $i_L(t)$.

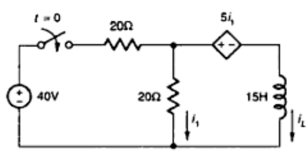


Exhibit 1.12a

1.13 Find the voltage, v, as shown in Exhibit 1.13a. The circuit is initially unenergized.

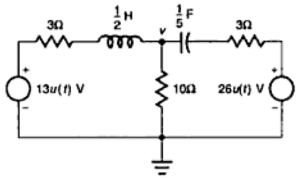


Exhibit 1.13a

1.14 In Exhibit 1.14, V_S is a 60 Hz source. The goal is to maximize the power delivered to the 108 Ω resistor. The 75 Ω resistance is internal to the source. X_1 and X_2 are reactive elements (capacitors or inductors) to be added. Find appropriate values for these elements so that the power to the load is maximum.

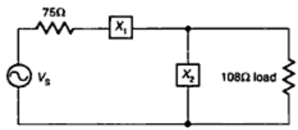


Exhibit 1.14

1.15 The current flowing in the 200 Ω load in Exhibit 1.15a is 165° out of phase with the 20 volt source. Find the coupling coefficient k and the current $i(t)$.

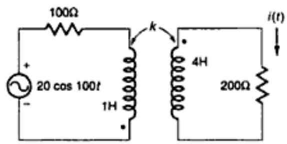


Exhibit 1.15a

1.16 Find the Thevenin equivalent circuit for the circuit in Exhibit 1.16a at terminals a and b. All resistor values are in ohms.

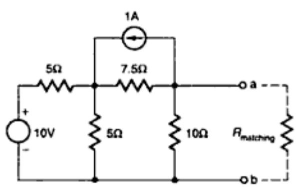


Exhibit 1.16a

1.17 Given the circuit in Exhibit 1.17a, find

a. n for maximum power in the 2-ohm resistor.

b. the power in the 2 ohm resistor if $n = 4$.

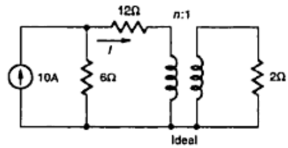


Exhibit 1.17a

The following data applies to problems 1.18 through 1.25. The expression for a wave is given as

$$f(t) = 10.0 \sin \omega t + 2.0 \cos(3\omega t + 90°).$$

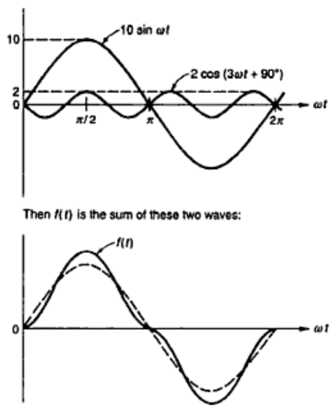


Exhibit 1.18 A sketch of the wave for problems 1.18 through 1.25

1.18 The dc component of the wave, $f(t)$, is

a. 0.0	b. 1.0
c. 1.5	d. 2.0

1.19 The half-period average of the wave, $f(t)$, is

a. 2.0	b. 3.28
c. 5.96	d. 6.36

1.20 The rms of effective value of the wave, $f(t)$, is

a. 7.07 b. 7.21
c. 7.63 d. 8.02

1.21 The maximum or peak value of the wave, $f(t)$ is

a. 10.0 b. 11.5
c. 12.0 d. 14.0

1.22 If the wave, $f(t)$, represents a current in amperes that is flowing through a resistor having a resistance of 3.0 ohms, the power loss in watts is

a. 110 b. 156
c. 187 d. 221

1.23 If the wave, $f(t)$, represents a current in amperes that is flowing through an inductor having an inductance of 0.1 henry, the rms or effective voltage in volts across the inductor is

a. 0.33 b. 0.67
c. 0.83 d. 1.00

1.24 If the wave, $f(t)$, represents a current in amperes having an angular velocity of 6000 radians per second, passing through a capacitor having negligible losses, with no initial charge, and a capacitance of 0.002 farads, the rms of effective voltage in volts across the capacitor is

a. 0.59 b. 0.74
c. 1.00 d. 5.90

1.25 An alternating current voltmeter consists of a series connection of an ideal half-wave diode and a D'Arsonval meter. The meter is calibrated to read the rms value of an applied voltage. When the waveform sketched in Exhibit 1.25a is applied, the meter reads 80 volts. What is the peak value of the applied waveform?

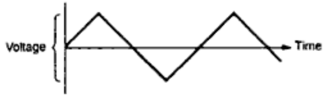


Exhibit 1.25a

1.26 A 50 micro-ampere meter movement with a 2k ohm internal resistance is to be used with a shunt arrangement so as to have full-scale ranges of 10mA, 100mA, 500mA, and 10amps (Exhibit 1.26a). The maximum voltage drop across the input terminals for a full-scale deflection should not exceed 250 millivolts.

Determine the resistor sizes needed and state any assumptions made.

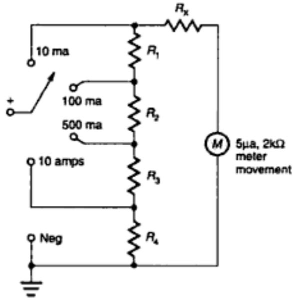


Exhibit 1.26a

1.27 An amplifier of voltage gain equal to 70dB is to be built by cascading transistor stages that have a gain-bandwidth product of 3×10^8 radians/sec. Find the maximum bandwidth that can be achieved using these stages, and the total number of stages required.

1.28 Refer to the circuit in Exhibit 1.28a. (a) If the inductance L is 10.0 micro-henries and the frequency of the applied voltage is 9.55 MHz, determine the value of the reactance of C so that the circuit will be series resonant. (b) What is the impedance looking into the circuit under these conditions?

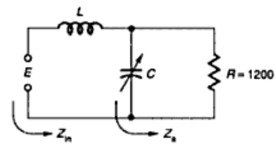


Exhibit 1.28a

1.29 For the circuit in Exhibit 1.29a, find $H(s) = V_2(s)/V_s$ and all its critical frequencies. Then find an expression for $|H(\omega)|$ and determine $|H(\omega)|_{max}$.

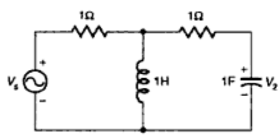


Exhibit 1.29a

1.30 Determine the transimpedance $V_0(S)/I_1(S)$ for the network in Exhibit 1.30.

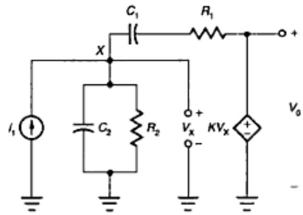


Exhibit 1.30

1.31 The network shown in Exhibit 1.31a represents an oscilloscope probe connected to an oscilloscope. The components C_2 and R_2 represent the input circuitry of the oscilloscope, and C_1 and R_1 represent the probe.

a. Find the transfer function $V_0/V_1(s)$.
b. Find a relationship among the components that makes the natural response equal to zero for all time.
c. Suppose the excitation, $V_1(t)$, is a unit step of voltage. Sketch $V_0(t)$ if

(1) $\frac{C_1}{C_1+C_2} = \frac{R_2}{R_1+R_2}$ (2) $\frac{C_1}{C_1+C_2} \rangle \frac{R_2}{R_1+R_2}$ (3) $\frac{C_1}{C_1+C_2} \langle \frac{R_2}{R_1+R_2}$

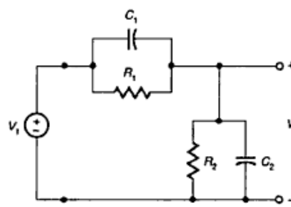


Exhibit 1.31a

The following practice problems have relatively short solutions. When comparing answers, choose the nearest value to your calculated one. If your solution is a graphical one, remember that voltage and current do not have to be plotted to the same scale and, of course, use a reasonably large plot.

When solving problems in the actual exam, a number of tips may help you eliminate certain answer choices and improve your odds. For example, always glance at the answer selections to see the form of the answer. Are the answers separated widely? If so, two or three digits may be sufficient. Is only the magnitude wanted? If so, you can carry along any vector notation only as far as it is needed. Is it easier to solve graphically?

1.32 For a parallel plate capacitor separated by an air gap of 1 cm and with an applied dc voltage across the plates of 500 volts, determine the force on an electron mass of 18.2×10^{-31} kg inserted in the space. The mass of an electron is 9.1×10^{-31} kg.

a. 3.2×10^{-14} N b. 1.6×10^{-14} N
c. 9.1×10^{-31} N d. 1.6×10^{-19} N

1.33 Assume a point charge of 0.3×10^{-3} C at an origin. What is the magnitude of the electric field intensity at a point located 2 meters in the x-direction, 3 meters in the y-direction, and 4 meters in the z-direction away from the origin?

a. 500 kV/m b. 5 kV/m
c. 93 kV/m d. 9.3 MV/m

1.34 An infinite sheet of charge, with a positive charge density, σ, has an electric field of $\mathbf{E} = \sigma/(2\varepsilon_o)_{ax}$ for $x > b$. If a second sheet of charge with a charge density of $-\sigma$ is placed at a (see Exhibit 1.34), what is the electric field for $a < x < b$?

a. $\sigma/\varepsilon_{o\ ax}$ b. 0
c. $-\sigma/\varepsilon_{o\ ax}$ d. $\sigma/2\varepsilon_{o\ ax}$

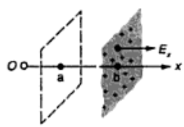


Exhibit 1.34

1.35 Two equal charges of 10 micro-Columbs are located one meter apart on a horizontal line, and another charge of 5 micro-Coulombs is placed one meter below the first charge (forming a right triangle). What is the magnitude of the force on the 5 micro-Coulomb charge?

a. 0.09×10^{6} N b. 12.6×10^{4} N
c. 6.39×104 N d. 63×10^{-2} N

1.36 For a coil of 100 turns wound around a toroidal core of iron with a relative permeability of 1000, find the current necessary to produce a magnetic flux density of 0.5 Tesla in the core. The dimensions of the core are given in Exhibit 1.36.

a. 390 A b. 39 A
c. 1.2 A d. 12.2 A

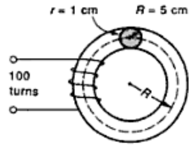


Exhibit 1.36

1.37 Two long straight wires, bundled together, have a magnetic flux density around them. One wire carries a current of 5 amperes and the other carries a current of 1 ampere in the opposite direction. Determine the magnitude of the flux density measured at a point 0.2 meters away (*i.e.*, normal to the wires).

a. $2\pi \times 10^{-6}$ T b. $4\pi \times 10^{-6}$ T
c. 4×10^{-6} T d. 6×10^{-6} T

1.38 Given a cast steel core of toroid with an air gap of 1 mm cut into it, determine the current necessary in a 600-turn coil wound around the core to produce a flux across the air gap of 0.6 mWb. Assume it is known from a magnetization curve that cast steel requires an H of 400 A-t/m for a flux density of 0.6 Tesla. Refer to Exhibit 1.38 and neglect any fringing in the air gap.

a. 0.6 A b. 1.4 A
c. 0.4 A d. 1.0 A

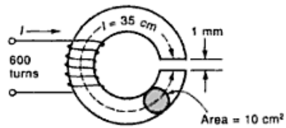


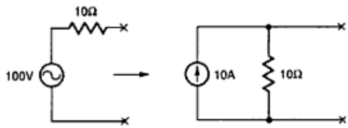
Exhibit 1.38

1.39 A circuit lifting magnet for a crane is to be designed so that with a flux density of 30,000 lines per square inch in each air gap, the length of each air gap is 0.5 inch. Leakage and saturation effects are such that the magnetomotive force for the air gaps is 0.85 of the magnetomotive force for the complete magnetic circuit. The mean length per turn of winding is 24 inches and the magnet is to operate with an applied terminal voltage of 70 volts with the winding at a temperature of 60°C.

a. What is the pull or tractive force in pounds per square inch for the magnet?
b. What size wire should be used for the winding?

SOLUTIONS

1.1 To simplify the equations we may first replace the voltage source with its series resistance with a Norton equivalent circuit as shown in Exhibit 1.1b.



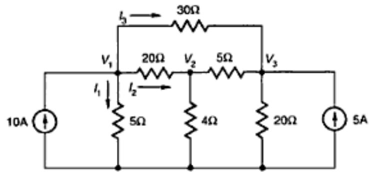


Exhibit 1.1b

Note that the 10 Ω resistor from the source is now in parallel with the 10 Ω resistor of the network, making it effectively a 5 Ω parallel branch. The nodal equations are

$$10 = V_1\left(\frac{1}{5}+\frac{2}{20}+\frac{1}{30}\right) - V_2\left(\frac{1}{20}\right) - V_3\left(\frac{1}{30}\right)$$

$$0 = -V_1\left(\frac{1}{20}\right) + V_2\left(\frac{1}{20}+\frac{1}{4}+\frac{1}{5}\right) - V_3\left(\frac{1}{5}\right)$$

$$5 = -V_1\left(\frac{1}{30}\right) - V_2\left(\frac{1}{5}\right) + V_3\left(\frac{1}{20}+\frac{1}{5}+\frac{1}{30}\right).$$

One would normally solve these equations by use of Cramer's Rule; however, the problem specifies the use of matrices. The matrix form is

$$[I] = [Y][V]$$

Where

$$\begin{bmatrix} 10 \\ 0 \\ 5 \end{bmatrix} = \begin{bmatrix} 0.2833 & -0.05 & -0.0333 \\ -0.05 & +0.5 & -0.2 \\ -0.0333 & -0.2 & 0.2833 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}.$$

The matrix solution is

$$[V] = [Y^{-1}][I]$$

where

$$[Y^{-1}] = \frac{\begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}}{|Y|}$$

with A_{ij} = Signed minor of Y_{ij} (cofactor) and $|Y|$ the determinant of the Y matrix.

$$|Y| = 0.0269$$

So

$$Y^{-1} = \frac{\begin{bmatrix} +\begin{vmatrix} .5 & -.2 \\ -.2 & .2833 \end{vmatrix} & -\begin{vmatrix} -.05 & -.0333 \\ -.2 & .2833 \end{vmatrix} & +\begin{vmatrix} -.05 & -.0333 \\ .5 & -.2 \end{vmatrix} \\ -\begin{vmatrix} -.05 & -.2 \\ -.0333 & .2833 \end{vmatrix} & +\begin{vmatrix} .2833 & -.0333 \\ -.0333 & .2833 \end{vmatrix} & -\begin{vmatrix} .2833 & -.0333 \\ -.05 & -.2 \end{vmatrix} \\ +\begin{vmatrix} -.05 & .5 \\ -.0333 & -.2 \end{vmatrix} & -\begin{vmatrix} .2833 & -.05 \\ -.0333 & -.2 \end{vmatrix} & +\begin{vmatrix} .2833 & -0.5 \\ -.05 & .5 \end{vmatrix} \end{bmatrix}}{0.0269}$$

$$Y^{-1} = \begin{bmatrix} 3.7807 & .7732 & .9926 \\ .7732 & 2.9442 & 2.1673 \\ .9926 & 2.1673 & 5.1747 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \left[Y^{-1}\right] \begin{bmatrix} 10 \\ 0 \\ 5 \end{bmatrix}.$$

Multiplying row by column,

$$V_1 = 10(3.7807) + 0(0.7732) + 5(0.9926) = 42.77 \text{ volts}$$
$$V_2 = 10(0.7732) + 0(2.9442) + 5(2.1673) = 18.57 \text{ volts}$$
$$V_3 = 10(0.9926) + 0(2.1673) + 5(5.1747) = 35.80 \text{ volts.}$$

Check node no. 1

$$I_1 = \frac{42.8}{5} = 8.56 \text{ Amps} \quad I_2 = \frac{42.8 - 18.57}{20} = 1.21 \text{ Amps}$$

$$I_3 = \frac{42.8 - 35.8}{30} = 0.23 \text{ Amps}$$

(Reference: Lipshutz, *Theory and Problems of Linear Algebra*, Shaum's Outline Series, McGraw-Hill, Chapter 8.)

1.2 This problem may be simplified by first finding a Thevenin equivalent circuit for the two sources (Exhibit 1.2b).

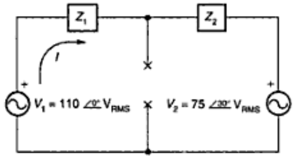


Exhibit 1.2b

$$V_{xx} = V_1 - IZ_1 = V_{OC} \quad \text{and} \quad I = \frac{V_1 - V_2}{Z_1 + Z_2}$$

$$I = \frac{110\angle 0° - 75\angle 30°}{60 + j80 + 30 - j50} = \frac{110 + j0 - (64.95 + j37.5)}{90 + j30}$$

$$I = \frac{58.61\angle -39.78°}{94.87\angle 18.43°} = 0.62\angle -58.2° \text{ Amps Rms}$$

Then

$$\begin{aligned} V_{OC} &= 110\angle 0° - (0.62\angle -58.2°)(60 + j80) \\ &= 110\angle 0° - (0.62\angle -58.2°)(100\angle 5.31°) \\ &= 110\angle 0° - 62\angle -5.1° \\ &= 48.8\angle 6.4° V_{RMS} = V_{Thev} = V_{OC} \end{aligned}$$

To find the Thevenin impedance, we look into terminals *xx* with all sources disabled.

$$\begin{aligned} Z_{eq} = Z_{Thev} &= \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(60 + j80)(30 - j50)}{90 + j30} \\ &= \frac{(100\angle 53.1°)(58.3\angle -59°)}{94.9\angle 18.4°} \\ &= 61.46\angle -24.3° = 56 - j25.3 \end{aligned}$$

The circuit now looks like Exhibit 1.2c.

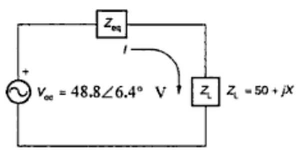


Exhibit 1.2c

The phase angle of the source is only a reference to the phase of the original source and may be ignored while finding jX. To maximize power, $Z_L = Z^*_{Thev}$, that is, the load impedance should be the complex conjugate of the source. Since we cannot make the real parts equal, $56 \neq 50$, we must recognize that the maximum current through R_L will deliver maximum power.

$$I = \frac{V_{OC}}{Z_{eq} + Z_L}$$
$$Z_{eq} + Z_L = 56 - j25.3 + 50 + jX$$

The denominator will be minimum when $|-j25.3| = |+jX|$.

Then

$$X = \omega L = 25.3$$
$$L = \frac{25.3}{\omega} \quad \text{but} \quad \omega = 2\pi(60) = 377$$
$$L = \frac{25.3}{377} = 67.2 \text{ millihenrys}$$
$$|I| = \frac{48.8}{50+56} = \frac{48.8}{106} = 0.46 \text{ Amps rms}$$
$$\text{So Power} = I^2 R_L$$
$$P = (0.46)^2(50) = 10.6 \text{ watts}$$

1.3 Sum the currents at voltage node 1.

$$-5 + V_1/2 + V_1/3 + (V_1 - V_{dep})/1 = 0$$
$$\times 6{:}\ -5 \times 6 + 3V_1 + 2V_1 + 6V_1 - 6(4V_1/3)$$
$$30 = 3V_1;\ V_1 = 10 \text{ V};\ I_\alpha = 10/3 = 3.33 \text{ A}$$

Power taken from current source:

$$P_{CS} = VI = 10 \times 5 = 50 \text{ watts}$$

Power dissipated in resistors:

$$P_1 = I_1^2 R_1,$$

where

$$I_1 = (V_1 - V_{dep})/1 = (10 - 4 \times 3.33)/1 = -3.33 \text{ A}$$
$$P_1 = (-3.33)^2 \times 1 = 11.09 \text{ watts}$$
$$P_2 = V_1^2/R_2 = 10^2/2 = 50 \text{ watts}$$
$$P_3 = V_1^2/R_3 = 10^2/3 = 33.3 \text{ watts}$$
$$P_{Tot} = 11.09 + 50 + 33.3 = 94.4 \text{ watts}$$

Therefore the power supplied by the dependent source is

$$94.4 - 50 = 44.4 \text{ watts.}$$

1.4 Check:

$$P_{dep} = V_{dep} \times I_1 = (4 \times I_3)I_1 = (4 \times 3.33) \times 3.33 = 44.4 \text{ watts}$$

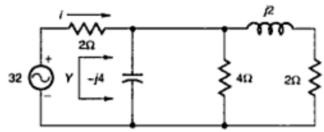


Exhibit 1.4b

$$Y = \frac{j}{4} + \frac{1}{4} + \frac{1}{2(1+j)} = \frac{1}{4}(1+j) + \frac{1}{2}\left(\frac{1}{1+j}\right)$$

$$= \frac{1}{4}\left[\frac{(1+j)^2+2}{1+j}\right] = \frac{1}{4}\left[\frac{1+j2-1+2}{1+j}\right]$$

$$= \frac{1}{2}\left[\frac{1+j}{1+j}\right] = \frac{1}{2}$$

$$Z = \frac{1}{Y} = 2\Omega$$

$$I = \frac{32}{Z+2} = \frac{32}{4} = 8\,\text{A}$$

$i(t) = 8 \cos 100t$ A

1.5

$$Z_R = 400\ \Omega$$

$$Z_L = j(20{,}000\pi)(.03) = j1885.0\ \Omega$$

$$Z_C = \frac{-j}{(20{,}000\pi)(0.01\times 10^{-6})} = -j1591.5\Omega$$

KVL: $-10 + 400I + j1885I - j1591.5I = 0$

$$I = \frac{10\angle 0^\circ}{400 + j293.5} = 0.0202\angle -36.3^\circ\,\text{A}$$

$V_R = 400I = 8.08\angle -36.3^\circ$ V

$V_L = j1885\ I$

$V_L = 37.99\angle 53.7^\circ$

$V_C = j1591.5\ I = 32.08\ \angle -126.3^\circ$ V

$V_{Source} = 10\ \angle 0$ V

1.6

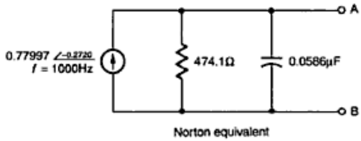


Exhibit 1.6b

$$Y_{0.4\mu F} = j(2000\pi)(0.4 \times 10^{-6}) = j0.0025133$$

$$\text{KCL Node } x: -2 + j0.0025133V_{x0} + \frac{1}{200}V_{x0} + \frac{1}{300}(V_{x0} + 10\angle 0.4) = 0$$

$$V_{x0} = \frac{1.96934\angle -0.0065913}{0.008704\angle 0.29292} = 226.257\angle -0.29951$$

Short circuit,

$$I_{AB} = \frac{1}{300}[(216.184 - j66.7576) - 10\angle 0.4]$$
$$I_{AB} = 0.77997\angle -0.27197 \text{ A}$$

Replacing each source by its internal impedance,

$$Z_{AB} = \frac{1}{0.005 + j0.0025133} + 300 = 459.66 - j80.254$$
$$Z_{AB} = 466.61\angle -0.17285\ \Omega$$

$$Y_{AB} = \frac{1}{Z_{AB}} = 0.0021431\angle 0.17285 = 0.0021093 + j0.0003686$$

$$R = \frac{1}{0.002193} = 474.1\Omega;\quad C = \frac{0.003686}{2000\pi} = 0.0586\,\mu\text{F}.$$

1.7 a. $Q = CE = 2 \times 10^{-6} \times 100 = 200 \times 10^{-6}$ coulomb

Capacitors are suddenly connected together, assuming no circuit resistance and the 4μFd capacitor has $Q_2 = 0$. The capacitors are in parallel across V so electrons will flow until equilibrium is reached. The total charge $Q = 200 \times 10^{-6}$ coulomb remains in the system.

So $$V = \frac{Q}{C_1 + C_2} = \frac{200 \times 10^{-6}}{(2+4) \times 10^{-6}} = 33.3 \text{ Volts.}$$

The voltage across each capacitor is 33.3 volts.

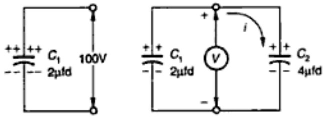


Exhibit 1.7a

b. $$W = \int \frac{1}{C} dq = \frac{1}{2}\frac{Q^2}{C} \text{ and initially } 0$$

$Q_1 = 200 \times 10^{-6}$ coulomb, so

$$W_1 = \frac{1}{2}\frac{(200\times10^{-6})^2}{2\times10^{-6}} = \frac{(2\times10^{-4})^2}{4\times10^{-6}}$$

$$W_1 = \frac{1\times10^{-8}}{10^{-6}} = 1\times10^{-2} \text{ joules}$$

energy initially in C_1.

Q_2 was zero initially, so $W_2 = 0$.

c. Final energy

$$Q_1 = 2 \times 10^{-6} \times 33.3 = 66.6 \times 10^{-6} \text{ coulomb}$$

$$W_1 = \frac{1}{2}\frac{(66.6\times10^{-6})^2}{2\times10^{-6}} = \frac{4.45\times10^{-10}}{4\times10^{-6}}$$

$$= 1.11 \times 10^{-3} \text{ joules in } C_1$$

$$Q_2 = 4 \times 10^{-6} \times 33.3 = 133.3 \times 10^{-6}$$

$$W_2 = \frac{1}{2}\frac{(133.3\times10^{-6})^2}{4\times10^{-6}} = \frac{1.76\times10^{-8}}{8\times10^{-6}}$$

$$= 2.22 \times 10^{-3} \text{ joules in } C_2$$

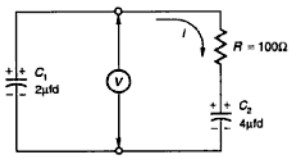


Exhibit 1.7b

Time constant $t = RC$

$$Ri + \frac{1}{C_1}\int i dt + \frac{1}{C_2}\int i dt = 0$$

i steady state = 0, so i transient is the solution of

$$R\frac{di}{dt}+\frac{1}{C_1}i+\frac{1}{C_2}i=0 \quad R\frac{di}{dt}+\left(\frac{C_1+C_2}{C_1C_2}\right)i=0$$

$$\text{So } i = \text{K}e^{-\left(\frac{C_1+C_2}{C_1C_2R}\right)t} \quad \text{where} \quad \frac{C_1+C_2}{C_1C_2R}=\frac{2+4}{2\times 4R}=\frac{3}{4R}$$

$$i=\frac{VC_1}{R}e^{-\left(\frac{C_1C_2}{C_1C_2R}\right)t} \quad \text{so} \quad \frac{t}{RC}=\frac{t}{100\times\frac{4}{3}}=1.$$

1.8 a. Time constant $t = RC = 100 \times 10^{-6}$

$= 133 \times 10^{-6}$ seconds

b. $E_{C_1} = E_{C_2} = 33.3$ Volts, same as part (a).

c. $W_1=\frac{1}{2}\frac{(66.6\times10^{-6})^2}{2\times10^{-6}}=1.11\times10^{-3}$ joules

$W_2=\frac{1}{2}\frac{(133.3\times10^{-6})^2}{4\times10^{-6}}=2.22\times10^{-3}$ joules

d. The circuit resistance determines the peak discharge or charge current, so there is I^2R loss in parts (a), (b), and (c) even if the resistance of the copper bar seems negligible. This accounts for the loss in energy.

1.9 a. With the switch open, and with the statement that the circuit is in steady state (to a dc source), one may make the assumption that the current through the inductor is no longer changing. The voltage across this element will then be zero. The voltage across the capacitor has built up to its steady state value and therefore no current will be flowing in this branch.

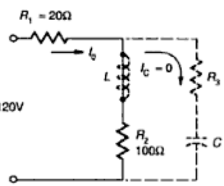


Exhibit 1.9b

$$I_0=\frac{E}{R_1+R_2}=\frac{120}{120}=1\text{ A}$$

b. After the switch is closed, the equivalent circuit will be as follows (with an initial inductor current as found in part (a):

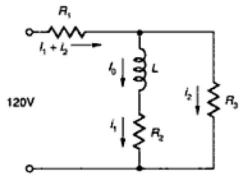


Exhibit 1.9c

Loop (differential) equations are

(1) $E = R_1(i_1 + i_2) + L\dfrac{di}{dt} + R_2 i_1$

(2) $E = R_1(i_1 + i_2) + R_3 i_2$

with an initial condition of $i_1(0) = I_0$.

Using Laplace transforms to solve,

(1) $\dfrac{E}{s} = R_1(I_1 + I_2) + L[sI_1 - i_1(0)] + R_2 I_1$

(2) $\dfrac{E}{s} = R_1(I_1 + I_2) + R_3 I_2$

Rearranging terms,

(1) $\dfrac{E}{s} + Li_1(0) = [(R_1 + R_2) + Ls]I_1 + R_1 I_2$

$$\frac{120}{s} + 0.1 = (120 + 0.1s)I_1 + 20I_2$$

(2) $\dfrac{120}{s} = (20)I_1 + (100)I_2$

Solving for I_1 and I_2,

$$I_1 = \frac{\begin{vmatrix} \left(\frac{120}{s}+0.1\right) & 20 \\ \left(\frac{120}{s}\right) & 100 \end{vmatrix}}{\begin{vmatrix} (120+0.1s) & 20 \\ 20 & 100 \end{vmatrix}} = \frac{\left(\frac{120}{s}+0.1\right)100 - \left(\frac{120}{s}\right)20}{(120+0.1s)100 - 20^2}$$

$$= \frac{96 + 0.1s}{s(116 + 0.15)}$$

$$I_2 = \frac{\begin{vmatrix} (120+0.1s) & \left(\frac{120}{s}+0.1\right) \\ 20 & \left(\frac{120}{s}\right) \end{vmatrix}}{(120+0.1s)100 - 20^2} = \frac{120 + 0.1s}{s(116 + 0.1s}.$$

But

$$I_{\text{Source}} = I_1 + I_2$$

$$\therefore I_s = \frac{96+0.1s+120+0.1s}{s(116+0.1s)} = \frac{216+0.2s}{s(116+0.1s)}$$

$$= \frac{216}{116}\left[\frac{1+\frac{0.2}{216}s}{s\left(1+\frac{0.1}{116}s\right)}\right] = 1.86\left[\frac{1+\tau_1 s}{s(1+\tau_2 s)}\right]$$

where

$$\tau_1 = 0.000927$$
$$\tau_2 = 0.0008625$$

$$\therefore i_s(t) = \ell^{-1} = [I_s] = 1.86\left[1-\left(1-\frac{\tau_1}{\tau_2}\right)e^{-\frac{t}{\tau_2}}\right]$$

$$= 1.86\left(1+0.072e^{-\frac{t}{\tau_2}}\right).$$

1.10 a. At $t = t(0^-)$ only R_3 and R_2 determine the magnitude of i.

$$R_{\text{total}} = \frac{1\times 1}{1+1} = 0.5 \text{ ohm}$$

$$i = \frac{E}{R_{\text{total}}} = \frac{1}{0.5} = 2\text{amp}$$

b. At $t = t(0^+)$, because the current in an inductance will not change instantaneously, the current in L_1 will remain at 0. Only R_2 will determine the magnitude of i.

$$i = \frac{E}{R_2} = \frac{1}{1} = 1\text{amp}$$

c. At $(t - t_0)$ the R_2 and L_2 branch of the circuit is in steady state and is not considered. Therefore for the L_1, R_1 and R_3 branch,

$$i = i_{\text{transient}} + i_{R_2}$$

$$i_{\text{transient}} = \frac{E}{R}\left(1-e^{-\frac{Rt}{L_1}}\right)$$

$$i_{R_2} = 1 \text{ amp (See above (b))}$$
$$R = R_1 + R_3 = 1 + 1 = 2 \text{ ohms.}$$

Therefore,

$$i = \frac{1}{2} - \frac{1}{2}e^{-t} + 1 = 1.5 - 0.5e^{-t}.$$

Proof: at $t = 0$, $e^{-t} = 1.0$ and $i = 1.5 - 0.5 \times 1.0 = 1$ amp

Note: Part (c) can be also worked by using the classic solutions of

$$L_1 \frac{di_1}{dt} + (R_1 + R_3)i_1 = 1$$

$$L_2 \frac{di_2}{dt} + R_2 i_2 = 1$$

$i = i_1 + i_2 = 1.5 - 0.5e^{-t}$ by applying the proper boundary conditions.

1.11 The (dc steady state) circuit for $t < 0$ is shown.

So $$i(o^-) = \frac{8}{2 + \frac{3\times6}{3+6}} = 2A$$

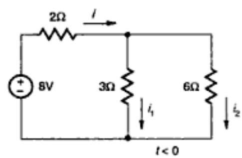


Exhibit 1.11.

By the current divider rule,

$$i_1(0^-) = 2 \times \frac{6}{9} = \frac{4}{3}\,\text{A}$$

and $$i_2(0^-) = 2 \times \frac{3}{9} = \frac{2}{3}\,\text{A}$$

Because currents in inductors cannot change instantaneously,

$$i_1(0^-) = i_1(0^+) = \frac{4}{3}\,\text{A} \quad \text{and} \quad i_2(0^-) = i_2(0^+) = \frac{2}{3}\,\text{A}.$$

Energy stored in L_1, at $t = 0$ is

$$\frac{1}{2}L_1 i_1^2(0^+) = \frac{1}{2}\times 6\times\left(\frac{4}{3}\right)^2 = \frac{16}{3}J.$$

Similarly, energy stored in L_2 at $t = 0$ is

$$\frac{1}{2}L_2 i_2^2(0^+) = \frac{1}{2}\times 6\times\left(\frac{2}{3}\right)^2 = \frac{4}{2}J.$$

At $t = 0$ the switch is closed. The stored energies in L_1 and L_2 flow through the closed switch and get dissipated in the $3\,\Omega$ and $6\,\Omega$ resistances.

For $$t > 0: i_1(t) = i_1(0^+)e^{-\frac{3\Omega}{6H}t} = \frac{4}{3}e^{-\frac{t}{2}}\text{A}.$$

and $$i_2(t) = i_2(0^+)e^{-\frac{6\Omega}{6H}t} = \frac{2}{3}e^{-t}\text{A}.$$

1.12 One may replace the circuit to the left of the inductor with its Thevenin equivalent for $t \geq 0$ by

$$V_{oc} = V_{\text{Thev}} = \frac{40\times 20}{20+20} - 5\times\frac{40}{20+20} = 15\text{ V (open circuit voltage)}.$$

To find R_{eq} "kill" the 40 V source and replace the inductor with a 1A source.

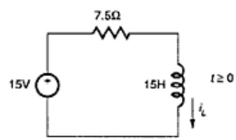


Exhibit 1.12b

Then

$$R_{eq} = \frac{V_0}{1\text{A}} = 1\text{A}\times\frac{20\times 20}{20+20} - 5\frac{20}{20+20} = 7.5\Omega.$$

The circuit then becomes as shown in Exhibit 1.12b.

$$\text{Forced } i_L = i_{Lf} = \frac{15}{7.5} = 2\,\text{A}$$

$$\text{Natural } i_L = i_{Ln} = Ae^{-\frac{t}{L/R}} = Ae^{-\frac{t}{2}}$$

$$i_L = Ae^{-\frac{t}{2}} + 2$$

$$i_L(0) = 0 = A + 2;\ A = -2$$

$$\therefore i_L = 2\left(1 - e^{-\frac{t}{2}}\right) \text{amps } t \geq 0$$

$$= 0 \quad t < 0$$

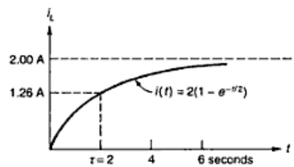


Exhibit 1.12c

1.13

$$(3+10)i_1 + \frac{1}{2}\frac{di_1}{dt} - 10i_2 = 13$$

$$-10i_1 + (3+10)i_2 + 5\int i_2 dt = -26$$

$$10i_1 = 13i_2 + 5\int i_2 dt + 26$$

$$i_1 = 1.3i_2 + 5\int i_2 dt + 2.6$$

$$13i_1 = 16.9i_2 + 6.5\int i_2 dt + 33.8$$

$$\frac{di_1}{dt} = 1.3\frac{di_2}{dt} + 0.5i_2$$

$$\frac{1}{2}\frac{di_1}{dt} = 0.65\frac{di_2}{dt} + 0.25i_2$$

$$1.69i_2 + 6.5\int i_2 dt + 33.8 + 0.65\frac{di_2}{dt} + 0.25i_2 - 10i_2 = 13$$

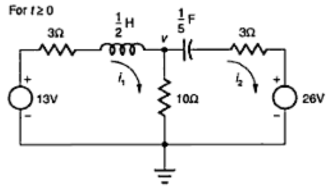


Exhibit 1.13b

$$0.65\frac{di_2}{dt}+7.15i_2+6.5\int i_2 dt=-20.8$$

$$\frac{di_2}{dt}+11i_2+10\int i_2 dt=32$$

$$\frac{d^2 i_2}{dt^2}+11\frac{di_2}{dt}+10i=0$$

The Laplace transform equivalent is:

$$i_1(t)=1.3Ae^{-10t}+1.3Be^{-t}+.5A\int e^{-10t}dt+.5B\int e^{-t}dt+2.6$$

$$1.25\ Ae^{-10}+0.8\ Be^{-t}+C+2.6$$

$$\lim_{t\to 0} i_1(t)=1 \quad \therefore\ C=-1.6$$

$$i_1(t)=1.25\ Ae^{-10t}+0.8\ Be^{-t}+1$$

$$i_1(0)=0=1.25\ A+0.8\ B+1$$

$$i_2(0)=-2=A+B$$

$$A+B=-2\ 1.25\ A+0.8\ B=-1$$

$$1.25A+1.25B=-2.5$$

$$-1.25A-0.8\ B=+1.0 \quad \therefore\ B=-\frac{10}{3} \quad A=\frac{4}{3}$$

$$i_1(t)=\frac{5}{3}e^{-10t}-\frac{8}{3}e^{-t}+1$$

$$i_1(t)=\frac{4}{3}e^{-10t}+\frac{10}{3}e^{-t}$$

$$v(t)=10(i_1-i_2)=10\left(\frac{5}{3}e^{-10t}-\frac{8}{3}e^{-t}+1-\frac{4}{3}e^{-10t}+\frac{10}{3}e^{-t}\right)$$

$$=10\left(\frac{1}{3}e^{-10t}+\frac{2}{3}e^{-t}+1\right)$$

$$=\frac{10}{3}e^{-10t}+\frac{20}{3}e^{-t}+10$$

1.14 Assume that X_1 is part of the source impedance and X_2 will be part of the load impedance. Then maximum power will be delivered to the load when $Z_s = Z_s^*$ (the load is the complex conjugate of the source impedance). Assume X_1 to be inductive so

$$Z_s = 75 + jX_1.$$

Then the parallel load impedance is capacitive,

$$\frac{108(-jX_2)}{108 - jX_2} = \frac{(108)\left(\frac{1}{j\omega C}\right)}{108 + \frac{1}{j\omega C}}\left(\frac{108 - \frac{1}{j\omega C}}{108 - \frac{1}{j\omega C}}\right)$$

$$= \frac{108}{1+\omega^2(108)^2C^2} - \frac{j\omega(108)^2 C}{1+\omega^2(108)^2C^2}.$$

For the conjugate match, the real part of the load must be equal to the imaginary part, thus

$$75 = \frac{(108)}{1+\omega^2(108)^2C^2}$$

$$\omega = 2\pi 60 = 377$$

$$75(1 + 1.66' 10^9 C^2) = 108$$

$$C^2 = \frac{0.440}{1.66' \, 10^9} = 265' 10^{-12}$$

$$C = 16.3\ \mu\text{f}$$

Then for complete matching, the reactive parts must cancel each other. For the load we have

$$\frac{-j\omega(108)^2 C}{1+\omega^2(108)^2C^2} = \frac{-j71.7}{1+0.439} = -j49.8.$$

Then

$$|jX_1| = |-j49.8|$$

$$\omega L = 49.8 \; Z = Z_L^* = (75 + j49.8)\Omega$$

$$L = \frac{49.8}{377} = 0.132 \text{ henrys.}$$

An equally valid solution can be found by assuming the load to be inductive and the source capacitive.

1.15

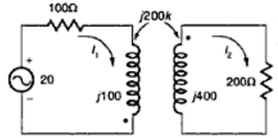


Exhibit 1.15b

$$M = k\sqrt{L_1 L_2} = k\sqrt{1\times 4} = 2k$$
$$(100 + j100)I_1 + j200kI_2 = 20$$
$$j200kI_1 + (200 + j400)I_2 = 0$$
$$(1 + j)I_1 + j2kI_2 = 0.2$$
$$jkI_1 + (1 + j2)I_2 = 0$$

$$I_2 = \frac{\begin{vmatrix} 1+j & 0.2 \\ jk & 0 \end{vmatrix}}{\begin{vmatrix} 1+j & j2k \\ jk & 1+j2 \end{vmatrix}} = \frac{-j0.2k}{1+j+j^2-2+2k^2}$$

$$= \frac{j0.2k}{1-2k^2-j^3} = \frac{0.2k\angle 90^\circ}{\sqrt{(1-2k^2)^2+9}\angle\tan^{-1}\left(\frac{-3}{1-2k^2}\right)}$$

$$90^\circ - \tan^{-1}\left(\frac{-3}{1-2k^2}\right) = 165^\circ$$

$$\tan^{-1}\left(\frac{-3}{1-2k^2}\right) = -75^\circ$$

$$\frac{-3}{1-2k^2} = \tan(-75^\circ) = -(2+\sqrt{3})$$

$$\frac{3}{1-2k^2} = 2+\sqrt{3} \quad 1-2k^2 = \frac{3}{2+\sqrt{3}}$$

$$2k^2 = 1 - \frac{3}{2+\sqrt{3}} \quad k^2 = \frac{1}{2} - \frac{2}{2+\sqrt{3}}$$

$$k = \sqrt{\frac{1}{2} - \frac{\frac{3}{2}}{2+\sqrt{3}}} = 0.313$$

$$I = I_2 = \frac{(0.2)\times(0.313)\angle 90^\circ}{\sqrt{(1-2(.313)^2+9}\angle -75^\circ}$$
$$= 0.0202\angle 165^\circ \text{ A}$$
$$i(t) = 20.2\cos(100t + 165^\circ) \text{ mA}$$

1.16 Convert the current source in parallel with the 7.5 Ω resistor to a voltage source in series to get the circuit in Exhibit 1.16b.

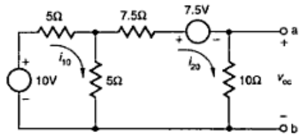


Exhibit 1.16b

$$10i_{10} - 5i_{20} = 10$$
$$-5i_{10} + 22.5i_{20} = -7.5$$

$$i_{20} = \frac{\begin{vmatrix} 10 & 10 \\ -5 & -7.5 \end{vmatrix}}{\begin{vmatrix} 10 & -5 \\ -5 & 22.5 \end{vmatrix}} = \frac{-75+50}{225-25} = \frac{-25}{200}$$

$$i_{20} = -\frac{1}{8}\text{A} \quad v_{oc} = 10i_{20} = -\frac{5}{4}V = v_{Thev}$$

Short the terminals a and b to get the circuit in Exhibit 1.16c.

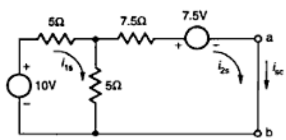


Exhibit 1.16c

$$10i_{15} - 5i_{23} = 10$$
$$-5i_{23} + 12.5i_{25} = -75$$

$$i_{23} = \frac{\begin{vmatrix} 10 & 10 \\ -5 & -7.5 \end{vmatrix}}{\begin{vmatrix} 10 & -5 \\ -5 & 12.5 \end{vmatrix}} = \frac{-75+50}{125-25} = -\frac{1}{4}\text{A} = i_{sc}$$

$$R_{eq} = R_{Thev} = \frac{v_{oc}}{i_{sc}} - \frac{-5/4}{-1/4} = 5\Omega$$

∴ The Thevenin equivalent is as shown in Exhibit 1.16d.

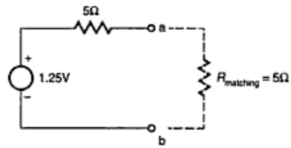


Exhibit 1.16d

1.17 a. Find the Thevenin equivalent of the circuit to the left of the transformer (Exhibit 1.17b).

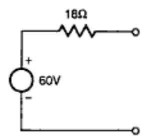


Exhibit 1.17b

To obtain the maximum power from this circuit the load across it should be 18 Ω. The impedance seen looking into the n turn side of the transformer is $4^2(2) = 32\,\Omega$. The impendance seen looking into the n turn side of the transformer is n^2 (2).

$$n^2(2) = 18$$
$$n = 3$$

For $n = 3$, Power in 2 Ω = 50 w

b. If $n = 4$, the impedance seen looking into the transformer in 2 Ω is the same as power into transformer.

Using current division,

$$I = 10\frac{6}{6+44} = \frac{6}{5}\text{A}$$
$$P = I^2(32) = 46.08 \text{ w},$$

or calculating power in secondary,

$$P = \left(\frac{6}{5}\times 4\right)^2 (2) = 46.08 \text{ w}.$$

1.18 a. dc component $= a_o = \frac{1}{2\pi}\int_0^{2\pi} f(t)d(\omega t)$

$$= \frac{1}{2\pi}\int_0^{2\pi} 10\sin\omega t d(\omega t) + \frac{1}{2\pi}\int_0^{2\pi} 2\cos(3\omega t + 90^\circ)\, d(wt)$$
$$= 0 + 0 = 0$$

1.19 c. The half-period average of the wave, $f(t)$, is

$$\text{d.c half-period average} = \frac{a_1}{2} = \frac{1}{\pi}\int_0^{\pi} f(t)d(\omega t)$$
$$= \frac{1}{\pi}\left[\int_0^{\pi} 10\sin\omega t d(\omega t) + \frac{1}{3}\int_0^{\pi} 2\cos(3\omega t + 90^\circ)d(\omega t)\right]$$
$$= \frac{1}{\pi}[+10 + -2/3 - 2/3] = \frac{1}{\pi}[20 - 4/3] = \frac{18.67}{\pi}$$
$$= 5.95.$$

Note that there is one more negative than positive third harmonic loop per half-cycle.

1.20 b. The rms or effective value of the wave, $f(t)$, is

$$\text{effective value of fundamental} = F_1 = \frac{10}{\sqrt{2}}$$

$$\text{effective value of 3rd harmonic} = F_3 = \frac{2}{\sqrt{2}}$$

$$F_{\text{TOT(effective)}} = \sqrt{F_1^2 + F_3^2} = \sqrt{\frac{100}{2} + \frac{4}{2}} = 7.21.$$

1.21 c. Note from the sketch the fundamental and the 3rd harmonic peaks occur in phase, thus $f(t)_{\text{peak}}$ is merely: $f(t)_{\text{peak}} = 10 + 2 = 12$.

1.22 b. From Problem 1.20,

$$I_{\text{eff}} = \sqrt{I_1^2 + I_3^2} = 7.21 \text{ amperes.}$$

Then

$$P = I^2_{\text{eff}}R = (7.21)^2(3.00) = 156 \text{ watts.}$$

1.23 c. The effective voltage across the inductor is

$$E_{eff} = \sqrt{E_1^2 + E_3^2} \quad \text{where} \quad E_1 = I_1 X_{L1} = \left(\frac{10}{\sqrt{2}}\right)(\omega L) = \frac{1.0}{\sqrt{2}}\omega$$

and

$$E_3 = I_3 X_{L3} = \left(\frac{2}{\sqrt{2}}\right)(3\omega L) = \frac{0.6}{\sqrt{2}}\omega$$

$$\therefore E_{eff} = \sqrt{\frac{\omega^2}{2} + \frac{.36\omega^2}{2}} = \frac{\omega}{\sqrt{2}}\sqrt{1+.36} = 0.83\omega.$$

1.24 a. For $\omega = 6000$ radians/second,

$$E_1 = X_{c1} I_1 = \left(\frac{1}{\omega C}\right)\left(\frac{10}{\sqrt{2}}\right) = \frac{10}{(6\times10^3)(2\times10^{-3})\sqrt{2}} = \frac{10}{12\sqrt{2}}$$

$$E_3 = X_{c3} I_3 = \left(\frac{1}{3\omega C}\right)\left(\frac{2}{\sqrt{2}}\right) = \frac{2}{3(6\times10^3)(2\times10^{-3})} = \frac{2}{36\sqrt{2}}$$

$$\therefore E_{eff} = \sqrt{\left(\frac{10}{12\sqrt{2}}\right)^2 + \left(\frac{2}{36\sqrt{2}}\right)^2} = 0.59.$$

1.25 The voltage as seen by the D'Arsonval meter (assuming a lossless diode rectifier) from a pure sine wave: $e = E_{max} \sin\omega t$

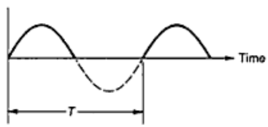


Exhibit 1.25b

Then:

$$E_{avg} = \frac{1}{T}\int_0^{\frac{1}{2}T} E_{max} \sin\omega t d(\omega t) + \frac{1}{T}\int_{\frac{1}{2}T}^{T} 0 d(\omega t).$$

Actual voltage read would be $E_{max}\ \pi$, but the meter is calibrated to read

$$\frac{E_{max}}{\sqrt{2}} \text{ (for an rms value).}$$

For the wave shape given,

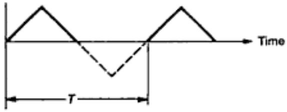


Exhibit 1.25c

$$e = \frac{E_{max}}{\frac{1}{4}t}t = \frac{4E_{max}}{T}t \quad \text{for} \quad 0 \le t \le \frac{1}{4}T$$

Then

$$E_{avg} = \frac{2}{T}\int_0^{\frac{1}{4}T} \frac{4E_{max}}{T} t\,dt = \frac{1}{4}E_{max}$$

But the meter reads (for a pure sine wave) $E_{mater} = \frac{\pi}{\sqrt{2}}E_{avg}$. Then, for the saw-tooth wave, as given

$$80x \text{ volts} = \frac{\pi}{\sqrt{2}}E_{avg} = \frac{\pi}{\sqrt{2}}\left(\frac{1}{4}E_{max}\right)$$

$$E_{max} = \left(\frac{\sqrt{2}}{\pi}\right)(4)(80) = 143.7 \text{ volts.}$$

X_{Note} : 80 volts is not the true rms voltage of the given wave form.

1.26 R_x can be determined from the voltage limit. At 0.250 volts, the current through the meter must be 50μa when using the 10ma setting.

Then

$$\frac{0.25}{R_x + 2k} = 50 \times 10^{-6}, \quad R_x = 3\text{k}\Omega\,.$$

Now the current through the shunt string of $R_1 + R_2 + R_3 + R_4$ must be 10ma – 50μA when the voltage is 0.250 volts.

$$10 \times 10^{-3} - 50 \times 10^{-6} = 9.95\text{mA}$$

This is only 0.5% less than 10mA.

The accuracy of our computations will not be seriously affected by ignoring this small amount when calculating the value of

$$R_1 + R_2 + R_3 + R_4.$$

Then

$$R_1 + R_2 + R_3 + R_4 = \frac{0.25}{10\text{mA}} = 25\Omega.$$

Now with the switch in the 100ma position, the equivalent circuit looks like Exhibit 1.26b.

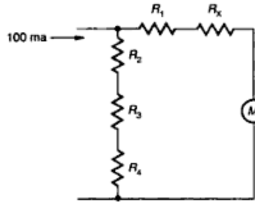


Exhibit 1.26b

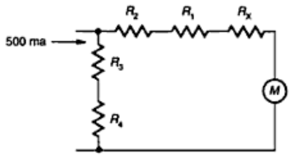


Exhibit 1.26c

Assume $R_1 \cdot R_2 + 2k$ since $R_1 + R_2 + R_3 + R_4 = 25$ and $R_x + 2k = 5k$. This is at worst a 0.5% error if $R_2 + R_3 + R_4 = 0$.

$$\text{Then} \quad R_2 + R_3 + R_4 \cong \frac{0.25}{100 \times 10^{-3}} = 2.5\Omega .$$

$$\text{So} \quad R_1 = 25 - 2.5 = 22.5\,\Omega.$$

Now with the switch in the 500 mA position, the circuit looks like Exhibit 1.26c.

Again, the error of neglecting $R_2 + R_1 \cdot R_x + 2K$ is less than 0.5% so

$$R_3 + R_4 \cong \frac{0.25}{500 \times 10^{-3}} = 0.5\Omega$$

and $\quad R_2 = 2.5 - 0.5 = 2\,\Omega.$

Finally, the switch at the position for 10 amps is shown in Exhibit 1.26d.

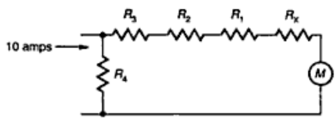


Exhibit 1.26d

As before,

$$R_3 + R_4 \cdot R_s + 2K.$$

$$\text{So} \quad R_4 \equiv \frac{0.25}{10} = 0.025\Omega$$

and $$R_3 = 0.5 - 0.025 = 0.475\,\Omega.$$

The absolute value of these resistors could be calculated without these simplifying assumptions by writing simultaneous equations for both parallel branches of the circuit in each case. Because resistors are seldom available in better than 1% tolerances, or at best 1/2%, the added labor for this additional accuracy is not warranted.

1.27 A voltage gain of 70dB,

$$20\log\frac{v_0}{v_i} = 70; \quad \log\frac{v_0}{v_i} = 35$$

$$\frac{v_0}{v_i} = 3162$$

for optimum bandwidth, the individual stage gain should be 1.65 (see *Modern Electronic Circuit Design* by David Comer, Addison-Wesley, 1976).

Then

$$1.65^n = 3162$$
$$n\log 1.65 = \log 3162$$
$$0.22n = 3.5$$
$$n = 16.09.$$

Because an integral number of stages is required, we select 17. This leads to an individual stage bandwidth of

$$BW = \frac{3\times10^8}{1.65} = 182\times10^6\,\text{Rad/sec.}$$

Bandwidth shrinkage due to cascading is

$$BW_{\text{overall}} = (\text{Bandwidth of single stage})\left(\sqrt{2^{\frac{1}{n}} - 1}\right)$$

$$= 182\times10^6\left(\sqrt{2^{\frac{1}{17}} - 1}\right)$$

$$BW = 37.1\times10^6\,Rad/\text{sec} \Rightarrow \frac{37.1\times10^6}{2\pi} = 5.91\times10^6\,\text{Hz.}$$

1.28 a. Solve the circuit by the admittance method.

$$Y_a = \frac{1}{Z_a} = \frac{1}{1200} + jY_{ca} = 0.833\times10^{-3} + jY_{ca}$$

$$Z_a = \frac{1}{0.833\times10^{-3} + jY_{ca}} = \frac{0.833\times10^{-3}}{D} - \frac{jY_{ca}}{D} = R_a - jX_{ca}$$

where denominator = D after rationalizing the fraction with the

conjugate $D = 0.693\times10^{-6} + Y_{ca}^2$
using $(a-b)(a+b) = a^2 - b^2$.

The new equivalent circuit is shown in Exhibit 1.28b.

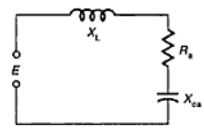


Exhibit 1.28b

At resonance,

$$X_{ca} = X_L$$

$$X_L = 2\pi fL = (6.28)(9.55\times10^6)(10\times10^{-6}) = 599.7 \text{ ohms.}$$

Thus $\dfrac{Y_{ca}}{D} = 599.7 = X_{ca}$ based on the principles of resonance.

If $\dfrac{Y_{ca}}{D} = 599.7$, then $Y_{ca} = 599.7\left(0.693\times10^{-6} + Y_{ca}^2\right)$

$$599.7Y_{ca}^2 - Y_{ca} + 4.156\times10^{-4} = 0\,.$$

Solving the second-degree equation by using the standard formula we obtain

$$Y_{ca} = \frac{1\pm\sqrt{1-4(4.156\times10^{-4})(599.7)}}{2\times599.7}$$

$$= \frac{1\pm\sqrt{1-0.9979}}{2\times599.7} = \frac{1}{1199.4} = 0.833\times10^{-3}$$

where the radical was approximated to be zero. Therefore the reactance requested is

$$X_{ca} = \frac{1}{Y_{ca}} = 1{,}199.4 \text{ ohms.}$$

b. $Z_{in} = R_a$, as in resonance the only effective part of impedance is the real part of the complex expression.

$$Z_{in} = R_a = \frac{0.833\times10^{-3}}{D} = \frac{0.833\times10^{-3}}{0.693\times10^{-6}+Y_{ca}^2}$$

$$= \frac{0.833\times10^{-3}}{0.693\times10^{-6}+(0.833\times10^{-3})^2}$$

$$Z_{in} = 0.6 \times 10^3 = 600 \text{ ohms.}$$

1.29

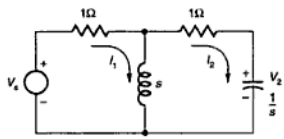


Exhibit 1.29b

$$V_2(s) = \frac{I_2(s)}{s}$$

$$(s + 1)I_1 - sI_2 = V_s$$

$$-sI_1 + \left(s+1+\frac{1}{s}\right)I_2 = 0$$

$$-s^2I_1 + (s^2 + s + 1)I_2 = 0$$

$$I_2 = \frac{\begin{vmatrix} s+1 & V_s \\ -s^2 & 0 \end{vmatrix}}{\begin{vmatrix} s+1 & -s \\ -s^2 & s^2+s+1 \end{vmatrix}} = \frac{s^2V_s}{s^3+2s^2+2s+1-s^3}$$

$$I_2 = \frac{s^2V_s}{2s^2+2s+1}$$

$$V_2(s) = \frac{sV_s}{2s^2+2s+1}$$

$$\text{H}(s) = \frac{s}{2s^2+2s+1}$$

The critical frequencies are

$$\text{Zeros: } s = 0,\ s \to \infty$$
$$\text{Poles: } 2s^2 + 2s + 1 = 0$$
$$s = \frac{-2 \pm \sqrt{4-8}}{4} = \frac{-2 \pm j2}{4}$$
$$s = -0.5 \pm j0.5$$
$$\mathrm{H}(\omega) = \frac{j\omega}{-2\omega^2 + j2\omega + 1} = \frac{j\omega}{1 - 2\omega^2 + j2\omega}$$
$$|\mathrm{H}(\omega)| = \frac{\omega}{\sqrt{(1-2\omega^2)^2 + 4\omega^2}} = \frac{\omega}{\sqrt{1+4\omega^4}}$$

$|\mathrm{H}(\omega)|_{max}$ occurs when $\mathrm{H}(\omega)$ is real. $\mathrm{H}(\omega)$ is real when $1 - 2\omega^2 = 0$,

$$\omega = \frac{1}{\sqrt{2}}$$
$$|\mathrm{H}(\omega)|_{max} = \frac{j\left(\frac{1}{\sqrt{2}}\right)}{j^2\left(\frac{1}{\sqrt{2}}\right)} = \frac{1}{2} = H\left(\frac{1}{\sqrt{2}}\right).$$

1.30 Node Equation at X:

$$-I_1 + \frac{V_x(s)}{\frac{R_2}{sC_2R_2+1}} + \frac{V_x(s) - KV_x(s)}{\frac{1}{sC_1} + R_1} = 0$$
$$V_x = I_1 \frac{R_2(sC_1R_1 + 1)}{s^2C_1C_2R_1R_2 + s(C_1R_1 + C_2R_2 + (1-K)C_1R_2) + 1}$$
$$V_0(s) = KV_x(s)$$
$$\frac{V_0}{I_1} = \frac{K}{C_2}\left[\frac{s + \frac{1}{R_1C_1}}{s^2 + s\frac{C_1R_1 + C_2R_2 + (1-K)C_1R_2}{C_1C_2R_1R_2} + \frac{1}{C_1C_2R_1R_2}}\right]$$

1.31 a. Use a voltage divider.

$$\frac{V_0}{V_1}(s) = \frac{\frac{R_2}{sC_2R_2+1}}{\frac{R_1}{sC_1R_1+1} + \frac{R_2}{sC_2R_2+1}}$$
$$\frac{V_0}{V_1} = \frac{C_1}{C_1 + C_2}\left[\frac{s + \frac{1}{C_1R_2}}{s + \frac{R_1+R_2}{R_1+R_2(C_1+C_2)}}\right]$$

b. To make the natural response zero, eliminate the pole in $\frac{V_0}{V_1}$ by causing it to cancel with the zero.

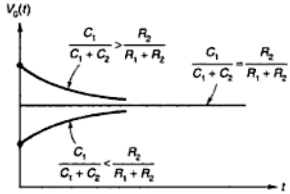


Exhibit 1.31b

$$\frac{1}{C_1R_1} = \frac{R_1+R_2}{R_1R_2(C_1+C_2)}$$

Thus $\frac{C_1+C_2}{C_1} = \frac{R_1+R_2}{R_2}$ or $1+\frac{C_2}{C_1} = 1+\frac{R_1}{R_2}$ or $\frac{C_2}{C_1} = \frac{R_1}{R_2}$.

c. If $V_1(t) = \text{Unit Step}, \quad V_1(s) = \frac{1}{s}$

$$V_0(s) = \frac{C_1}{C_1+C_2} \times \left[\frac{s+\frac{1}{C_1R_1}}{s\left[s+\frac{R_1+R_2}{R_1R_2(C_1+C_2)}\right]}\right] = \frac{K_1}{s} + \frac{K_2}{s+\frac{R_1+R_2}{R_1R_2(C_1+C_2)}}$$

$$K_1 = \frac{R_1}{R_1+R_2} \quad \text{and} \quad K_2 = \frac{C_1}{C_1+C_2} - \frac{R_2}{R_1+R_2}$$

$$v_0(t) = \frac{R_2}{R_1+R_2} + \left[\frac{C_1}{C_1+C_2} - \frac{R_2}{R_1+R_2}\right] e^{\frac{-t}{\tau}}, \; t>0$$

where $\tau = \frac{R_1R_2(C_1+C_2)}{R_1+R_2}$.

(1) if $\frac{C_1}{C_1+C_2} = \frac{R_2}{R_1+R_2}$ then $v_0(t) = \frac{R_2}{R_1+R_2} = \frac{C_1}{C_1+C_2}$

(2) and (3)

$$v_0(t=0^+) = \frac{C_1}{C_1+C_2}$$

$$v_0(t\to\infty) = \frac{R_2}{R_1+R_2}$$

1.32 b. The "mass" of 2 electrons has a charge $Q = 3.2 \times 10^{-19}$ C. Thus the electric field is

$$E = \frac{500}{0.01} = 50 \times 10^3 \text{ V/m}.$$

The force is then

$$F = QE = (3.2 \times 10^{-19}) \times (50 \times 10^3) = 1.6 \times 10^{-14} \text{ N}.$$

1.33 c. The magnitude of the length of the resultant vector, R, in the x, y, z plane is

$$R = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{29}$$

The magnitude of the electric field, E, is

$$E = \frac{Q}{4\pi\varepsilon R^2} = \frac{(0.3 \times 10^{-3})}{4\pi(8.85 \times 10^{-12})29}$$
$$= 93{,}000 \text{ V/m}.$$

1.34 c. On the b plane (for that plane alone), $E^+ = (-\sigma/2\varepsilon)_{ax}$, and for the negatively charged plane at the a plane (again for that plane alone, but acting to the right of a), is the same as before.

Therefore,

$$E = E^+ + E^- = \left(\frac{-\sigma}{\varepsilon_0}\right)_{ax} \text{ V/m}$$

1.35 d. From a sketch of the vectors in Exhibit 1.35, the value of E_{net} is 12.6×10^4 V/m.

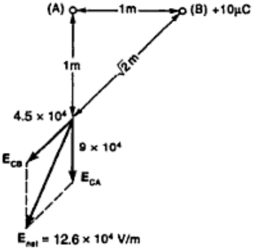


Exhibit 1.35

F is then found to be

$$E = \frac{10\times10^{-6}}{4\pi(8.85\times10^{-12})r^2}$$

$$F = Q_cE_{net} = 5\times10^{-6}\times12.6\times10^4 = 63\times10^{-2}\ \text{N}$$

1.36 c.

The cross-sectional area of the iron core is $\pi r^2 = \pi\times10^{-4}\,\text{m}^2$.

The path length is: $\ell = 2\pi R = 2\pi(5\times10^{-2}) = 0.1\pi\text{m}$.

The permeability is $\mu = \mu_0\mu_r = 4\pi(10^{-7})\times10^3 = 4\pi\times10^{-4}$.

Hence, $\text{H} = \frac{B}{\mu} = \frac{0.5}{4\pi\times10^{-4}} = 390$ A • t/m. and because H = mmf/length,

$\text{mmf} = \text{H}\times\text{length} = 390\times0.1\pi = 122.5\ \text{A}-\text{t}$.

Thus, $I = \text{mmf/turns} = 122.5/100 = 1.225$ A.

1.37 c. Assume the two wires are bundled close together (with respect to the 0.2 meter position). Then the net current to the right is: $I_{net} = 5 - 1 = 4$ A. The flux density is given by,

$$B = \frac{(\mu I)}{2\pi r} = \frac{(4\pi\times10^{-7})(4)}{(2\pi\times0.2)} = 4\times10^{-6}\ \text{Tesla.}$$

1.38 d. The flux through the air gap and the core must be the same. Because there is no fringing, the effective cross-sectional gap area is the same as the core. Thus the B's are the same.

For $B_{caststeel} = 0.6$ Tesla, H = 400 A • t/m = (mmf)(length), so

$\text{mmf}_{caststeel} = 400\times0.35 = 140$ A • t;

Because $\text{H}_{airgap} = \frac{B}{\mu_0} = \frac{46}{(4\pi\times10^{-7})} = 0.48\times10^6$

and $\text{mmf}_{airgap} = H\times\text{length} = 0.48\times10^3$.

The total $\text{mmf} = \text{mmf}_{caststeel} + \text{mmf}_{airgap}$

$= 140 + 480 = 620$ A • t, $I = \text{mmf}/t = \frac{620}{600} = 1.03$ A.

1.39 a. Assume a dc magnet

Force in dynes: $F = \frac{B^2A}{8\pi}$ Maxwell's equation

Force in lbs/sq. in. $= \frac{B^2}{72\times10^6} = \frac{(30{,}000)^2}{72\times10^6}$

(1) Force = 12.5 lbs/sq. in. pull

(2) $B = 30{,}000$ lines/sq. in., $L = 0.5$ in air gap.

$$\text{mmf}_{\text{airgap}} = 0.85 \text{ total NI}$$
$$L_{\text{mean}} \text{ of coil} = 24 \text{ inches}$$
$$V = 70 \text{ volts on coil}$$
$$H_{\text{airgap}} = 0.313 \text{ B NI per inch for air gap}$$

$$NI_{\text{for airgap}} = 0.313 \times 30{,}000 \times 0.5 \times 2 = 9{,}400 \text{ amp - turns for air gap}$$

$$= \frac{9400}{0.85} = 11{,}050 \text{ ampere turns for air gap}$$

$$NI_{\text{total}} = \frac{9{,}400}{0.85} = 11{,}050 \text{ ampere turns for air gap and magnetic ckt.}$$

Because the coil dimensions are not given, assume 400 square inches of radiating surface and allow 0.7 watts per square inch dissipation at 60°C.

$$\text{Watts} = 400 \times 0.7 = 280 \text{ watts dissipated}$$

$$\text{and } I = \frac{280 \text{ watts}}{70 \text{ volts}} = 4 \text{ Amperes coil current}$$

$$N = \frac{NI}{I} = \frac{1050}{4} = 2763 \text{ turns on coil}$$

$$\text{Wire length} = \frac{2763 \times 24 \text{ inches}}{12 \text{ inches/foot}} = 5526 \text{ feet}$$

$$R_{\text{wire}} = \frac{70 \text{ volts}}{4 \text{ amperes}} = 17.5 \text{ ohms}$$

$$R_{\text{wire}} = \frac{\rho \text{ length}}{\text{area}} \quad \text{where} \quad \rho = 12 \text{ at } 60°\text{C}$$

$$A_{\text{cir. mills}} = \frac{12 \times 5526}{17.5} = 3880 \text{ cir. mills}$$

No. 14 AWG Magnet wire has 4,107 cir. mills and 2.525 ohms/1000′.

No. 15 AWG Magnet wire has 3,257 cir mills and 3.184 ohms/1000′.

Choose No. 14 AWG Magnet wire.

RECOMMENDED REFERENCES

Bobrow, *Elementary Linear Circuits Analysis*, 2nd edition, Holt, Rinehart and Winston, 1987.

Jones, Lincoln and Howard Smolleck, *Electrical Engineering: FE Exam Preparation*, 3rd Edition, Kaplan AEC.

Nilsson, *Electric Circuits*, Prentice-Hall, any edition.

CHAPTER 2

Machines

OUTLINE

PROBLEMS

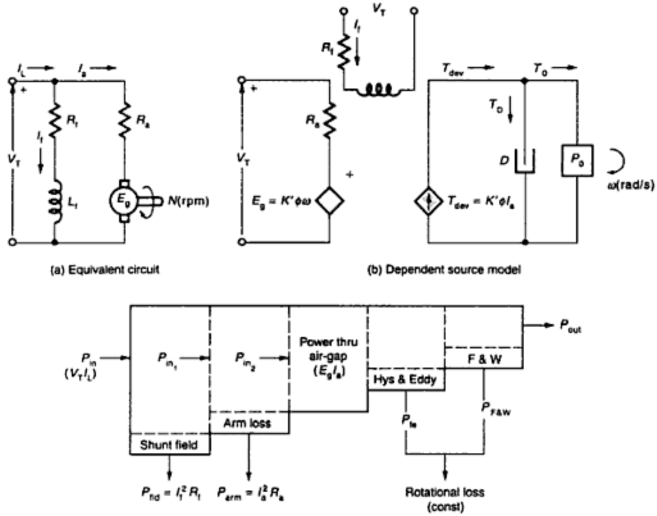
Note: Although several of the following problems are much longer than might be expected on the examination, it is suggested that the reader follow them through where appropriate.

2.1 An older dc shunt motor (Exhibit 2.1a) is intended to be used to drive a load whose output power requirement varies between 5 and 15 hp but will only tolerate a small speed variation. The name plate rating of the machine is given as 15 hp, 230 volts, 57.1 A., 1400 rpm. The field circuit resistance is 115Ω and its armature resistance is 0.13Ω. No data or test results are available on its no-load characteristics.

The machine needs to be analyzed as to its suitability for the speed requirements (that is, speed regulation) and its efficiency over the various load requirements. It has already been decided that the accuracy of the analysis does not require taking into account any effect that armature reaction might produce. The data needed for making a judgment on using this machine are

a. No-load and 5 hp line currents.
b. No-load and 5 hp speeds.
c. Efficiency at both 5 and 15 hp.

The following data should be used for Problems 2.2 through 2.11. A 50 kVA transformer rated at 2300/230 volts at 60 Hz is to be tested in a laboratory so that its characteristics can be determined. The standard test requires both an open and short circuit test; the results of the tests are shown in Exhibit 2.2.
The open circuit test measures core loss with negligible copper loss. The short circuit test measures the copper loss with negligible core loss.
For future and present requirements, the parameters of an equivalent circuit need to be found. In addition, several values of efficiency and voltage regulation are to



(a) Equivalent circuit

(b) Dependent source model

(c) Power flow diagram for a dc shunt motor

Exhibit 2.1

be determined (the assumption is that the coils are designed such that $I_1^2 R_1 = I_2^2 R_2$).

2.2 Determine the coil resistance of the high windings side.

a. 35.4 Ω b. 0.605 Ω
c. 0.0065 Ω d. 5.29 Ω

Open Circuit Test (Core Loss)			Short Circuit Test (Cu Loss)		
I	E	W	I	E	W
6.5 A	230 V	187 watts	21.7 A	115 V	570 watts

Exhibit 2.2

2.3 Determine the copper power loss.

a. 187 W b. 757 W
c. 570 W d. 1.5 kW

2.4 Determine the core loss.

a. 187 W b. 757 W
c. 570 W d. 1.5 kW

2.5 Determine the efficiency of the transformer at full load (assume unity power factor).

a. 0.985 b. 0.996
c. 0.990 d. 0.636

2.6 Determine the efficiency of the transformer at half load (assume unity power factor).

a. 0.985 b. 0.996
c. 0.990 d. 0.636

2.7 Find the percent voltage regulation of the transformer for unity power factor.

a. 98.22% b. 1.175%
c. 96.44% d. −0.62%

2.8 Find the percent voltage regulation of the transformer for a 0.8 lagging power factor.

a. 98.29% b. 3.56%
c. 96.44% d. −2.62%

2.9 Find the percent voltage regulation of the transformer for a 0.8 leading power factor.

a. 98.29% b. 3.56%
c. 96.44% d. −2.62

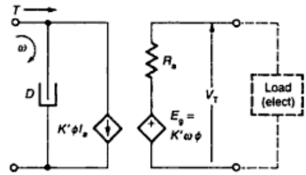
2.10 Determine the no-load standby current when the transformer is connected to its rated source on the high side.

a. 6.5 A b. 65 A
c. 0.65 A d. 31.6 A

2.11 Determine the high side of the steady state current when the transformer is connected to its rated voltage source on the high side and the low side is inadvertently connected to a load of 0.0394 Ω pure resistance.

a. 6.4 A b. 65 A
c. 31.6 A d. 316 A

2.12 For a perfectly operating dc generator, the input mechanical power source may be considered to have constant speed over a range of torque requirements from no-load to full-load. However, the field current may be considered constant for a particular application. The generator is rated at 15 kW, 240 volts, 62.5 amperes, at a rated speed of 1200 rpm. The armature resistance is known to be 0.2 Ω.



(a) Generator model

Exhibit 2.12

For the stated conditions determine

a. The no-load terminal voltage (neglect any armature reaction effects).
b. The percent voltage regulation and the input torque taken from the mechanical power source (neglect the rotational losses).
c. The percent voltage regulation and the input torque taken from the mechanical power source assuming the rotational torque loss is 5.0 N-m.

Now assume the speed control mechanism is faulty and while the speed at no-load is still 1,200 rpm, the terminal voltage drops because the load resistance is unchanged, and the speed drops to 1000 rpm. Determine

d. The terminal voltage and the new power output.
e. The new input torque taken from the mechanical power source (again, neglect the rotational losses).

2.13 Two single-phase, 120 volt, 60 Hz motors are being considered for a particular application requiring one motor of 1/4 hp and the other motor 1/2 hp. These two motors are located at some distance from a power source so that a low line current is important such that the line voltage drop is negligible. The lower hp one will be a standard split-phase induction motor. Because of the low current constraint, the other motors will be a special type that has a slightly leading power factor (a built-in capacitor in series with one of the windings). The data in Exhibit 2.13a have been obtained for these two motors.

Motor	Power Out (Horsepower)	Motor Efficiency	Motor Power Factor
"A"	1/4	60%	0.7 lagging
"B"	1/2	70%	0.95 lagging

Exhibit 2.13a

To make an engineering judgment on the suitability of using these two motors, it will be necessary to find the total power needed, the combined line current, and the combined power factor of these two motors operating in parallel. In addition, it will be necessary to have a complete, carefully labeled phasor diagram indicating the line voltage (reference), and the individual and total line currents.

2.14 In an emergency, a dc motor must be used as a generator (Exhibit 2.14a). The motor is a cumulative-compound motor; its efficiency is 90%, and its positive and negative terminals are marked.

The cumulative-compound characteristic must be maintained when it is used as a generator; the rotation must be kept in the same direction and the rpm will be the same. The positive terminal must remain the positive in the operation as a generator. The interpoles must aid commutation in both motor and generator mode. The machine is to deliver the same power to the line in the generator operation as it took from the line in the motor operation, and the losses in the machine are the same in both cases.

a. Should the shunt-winding connections be changed?
b. Should the compound-winding connections be changed?
c. Should the interpole-winding connections be changed?
d. Should the field resistance be changed?
e. Has the load on the mechanical clutch of the machine (Exhibit 2.14b) increased or decreased when the operation changed from motor to generator if it was running at full capacity as a motor?
f. How much does the electrical power transferred at the line connection increase or decrease if the mechanical torque is maintained on the shaft?

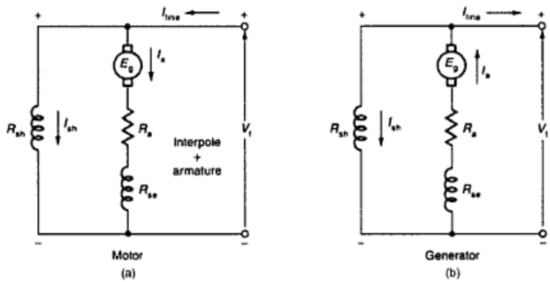


Exhibit 2.14

2.15 A 10 kVA, 240 ac voltage source sends power through two ideal transformers that are separated by a short transmission line to a single-phase induction motor load whose impedance is $4 + j3$ ohms (Exhibit 2.15). The first transformer is a 1:2 step-up and the second is a 2:1 step-down; the equivalent series impedance of the interconnecting transmission line is $2 + j2$ ohms. The operator is considering purchasing an ac capacitor to connect across the motor load to reduce some of the line loss.

Analyze the system using per-unit values to determine the various voltages, currents, and power taken from the source both before and after the capacitor is added. Also, calculate the kVAR rating for the capacitor (to correct the load power factor to unity).

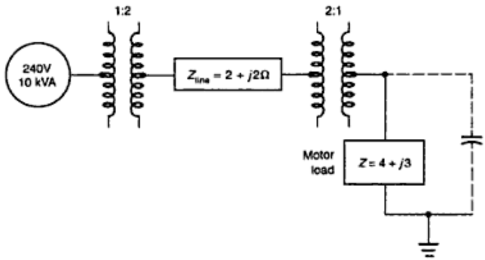


Exhibit 2.15

2.16 An emergency 120/208-volt, 3-phase, 4-wire, 60 Hz generator supplies an external circuit. The load on the external circuit consists of 9000 watts of incandescent light connected between line and neutral and evenly distributed among the 3 phases, and a 10-HP, 3-phase air conditioner motor of 83% efficiency and 0.707 power factor.

a. Show the phasor diagram of the currents and voltages on the load side of the generator.
b. Determine the microfarads of the capacitor to be connected to the generator in order to reduce the generator load current to 105% of that which would flow if the power factor = 1.
c. Determine the size of the conduit and wire to be used as a feeder and the required fuse size to protect the feeder when it is run between the generator and its distribution board in the next room, if the load is continuous, and the capacitor calculated in (b) is connected.

2.17 Two Y-connected induction motors are fed by a 4160 V, line-to-line, 3-phase 60 Hz motor-control center 20 feet away. Motor #1 drives a 600 HP compressor. The efficiency of the motor is 90%, and its power factor is 0.5. Instruments of motor #2 indicate 1730 kW, 277 amps.

a. Show the phasor diagram of the loads, kW and kVA.
b. Determine the capacity in microfarads of a wye-connected capacitor bank that is required to correct the power factor of the total load to 0.966.
c. If a synchronous motor is installed in place of motor #2 and used instead of the capacitor bank to achieve the same overall power factor (0.966), what must its power factor be?

2.18 A 12,500 kVA, 6600 volt, 3600 r.p.m., 60 Hz., 3-phase, Y-connected alternator has magnetization and short-circuit characteristics curves as shown in Exhibit 2.18a.

Determine the percentage voltage regulation for a 0.707 lagging power factor. Let the ac armature resistance be 0.5 ohms and make (and state) any reasonable assumption necessary for your solution.

2.19 A 10 kVA, 480-120 V, single-phase transformer is to be connected as an autotransformer to connect a 480 V source to a 600 V load. Determine the maximum kVA load that the autotransformer can supply without exceeding the winding current ratings.

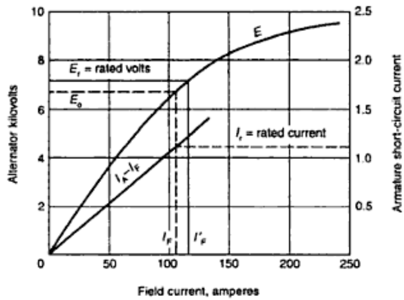


Exhibit 2.18a

2.20 A 3-phase 132-13.2 kV, 15 MVA transformer is supplying a 12 MVA 3-phase load at 13 kV. The primary windings are connected in delta and the secondary in wye. Determine the primary and secondary winding and line currents.

2.21 A 3-phase, 60 Hz, two pole, 4.16 kV, wye-connected synchronous generator is to be applied as an emergency power source to supply 500 kW at a power factor of 0.90 lagging. The stator resistance is 0.30 ohm/phase. Rotational losses in the machine are estimated to be 15 kW. Determine the required horsepower and torque of the prime mover.

2.22 A 3-phase synchronous motor draws 250 A at a line-line voltage of 4000 V and 1.0 power factor from a large system. The machine is wye connected and has a synchronous reactance of 10 ohms/phase. Stator resistance is negligible. If the field current is held constant and the mechanical load is gradually increased, determine the maximum horsepower that can be delivered without pulling out of synchronism.

2.23 A 3-phase, 100 hp, 460 V, delta-connected induction motor has a locked rotor code letter F. The motor is started with a wye-delta controller. Estimate the largest starting current that the motor will draw from the supply system.

2.24 A 3-phase induction motor is supplied from a 50 Hz system. It runs at 950 rpm at full load. Determine the number of poles, the slip at full load, and frequency of the rotor current at full load.

2.25 Two 3-phase transformers are to be operated in parallel. One transformer is rated 7500 kVA and has an impedance of 6.5%. The second transformer is rated 5000 kVA and has an impedance of 5.5%. Both transformers have the same voltage rating and are set at the same voltage tap. Estimate the maximum kVA load that the transformers can supply without exceeding the rating of either transformer. The transformer impedances can be assumed to be purely inductive.

2.26 A single-phase, 25 kVA, 2400-240 V transformer has an equivalent series impedance of 3.45 + *j*5.75 ohms referred to the high voltage side. The transformer is supplying rated kVA at 240 V and 85% lagging power factor. Determine the percent voltage regulation.

2.27 The transformer of Problem 2.26 has a no-load loss of 500 W. For the same loading condition given in Problem 2.26, determine the transformer efficiency.

2.28 A 460 V, 100 hp, wye-connected induction motor has a locked rotor impedance of 0.12 + *j*0.35 ohm/phase. If the motor is started with an autotransformer set at the 65% voltage tap, determine the current drawn from the supply system at start.

2.29 A wye-connected synchronous generator is supplying 4000 A at a line-line voltage of 12 kV and unity power factor to a large system. The synchronous reactance is 2.0 ohms/phase and stator resistance is negligible. If the prime mover set point is unchanged and the field current is increased by 15%, determine the new stator current and power factor.

2.30 A 200 V series dc motor has an armature resistance of 0.2 ohm and a field resistance of 0.7 ohms. When operating at rated voltage, the motor draws 50 A. The rotational losses at this operating condition are 180 W. Determine the shaft output horsepower and the efficiency.

2.31 A separately excited dc generator is operating at its full load rating of 150 V, 30 A, and 1800 rpm. When the load is removed the output voltage is 170 V. Determine the voltage regulation, armature resistance, and the full load developed torque.

2.32 A 3-phase, 200 V, four pole, wye-connected, 60 Hz squirrel cage induction motor has a per phase equivalent circuit shown in Exhibit 2.32. The circuit parameters are

$R_1 = 0.15$ ohm $\quad R_2 = 0.10$ ohm $\quad X_1 = X_2 = 0.50$ ohm $\quad X_m = 20$ ohms

Neglect magnetic and mechanical losses. For a slip of 0.035 determine the output torque.

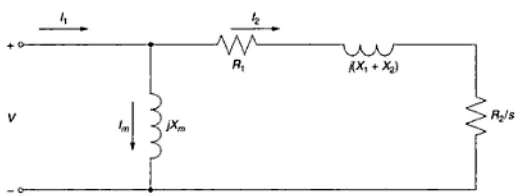


Exhibit 2.32

2.33 If the motor of Problem 2.32 is started at rated voltage, determine the starting current and starting torque.

SOLUTIONS

2.1 For the conditions stated (disregard armature reaction since data is unavailable on no-load characteristics) you can make a simplified equivalent circuit or a dependent source type model, as shown in Exhibit 2.1. The figure also shows a power flow diagram to help organize your solution process.

The following calculations are obvious for full load conditions using the equivalent circuit diagram. The shunt field loss (that will be constant) will be

$$I_f = \frac{V_T}{R_f} = \frac{230}{115} = 2\text{ A}$$

$$P_{fld} = I_f^2 R_f = (2)^2 \times 115 = 460 \text{ watts.}$$

The armature current loss (that is a function of the load) will be

$$I_a = I_L - I_f = 57.1 - 2.0 = 55.1 \text{ A}$$

$$P_{arm} = I_a^2 R_a = (55.1)^2 \times 0.13 = 395 \text{ watts.}$$

The power transfer across the air gap ($E_g\, I_a$) will be

$$P_{in2} = P_{in} - (P_{fld} + P_{arm}) = 230 \times 57.1 - (460 + 395) = 12{,}278 \text{ watts.}$$

The rotational losses (F & W plus iron loss) will be

$$P_{rot} = P_{in2} - P_0 = 12{,}278 - (15 \times 746) = 1088 \text{ watts.}$$

The speed and machine-flux-constant relationship is

$$N = \frac{V_t - I_a R_a}{\mathrm{K}\theta} = 1400 \text{ rpm} = \frac{230 - 55.1 \times 0.13}{K\theta}; K\theta = \frac{222.8}{1400}$$
$$= 0.159 \text{ V/rpm.}$$

The following calculations are needed for no-load conditions.

$$P_{in1} = P_{arm} + P_{rot} + P_0; \quad 230 I_a = I_a^2 \times 0.13 + 1088 + 0$$

The no-load armature current equation and solution is

$$I_a^2 - \left(\frac{230}{0.13}\right) I_a + \left(\frac{1088}{0.13}\right) = 0; \quad \text{from} \quad I_a - \frac{b}{2} \pm \frac{1}{2}\sqrt{b^2 - 4ac}$$

$$I_a = -\left(\frac{-1769}{2}\right) \pm \frac{1}{2}\sqrt{(-1769)^2 - 4 \times 8369}$$

$$= 1764 \text{ A} \quad \text{or} \quad 4.7 \text{ A} \leftarrow \text{Choose}$$

(Alternative method - assumes $P_{rot} > P_{arm} \approx \frac{P_{rot}}{V_a} = \frac{1088}{230} = 4.73$ A.)

The no-load speed is

$$N_{NL} = \frac{V_T - I_a R_a}{K\theta} = \frac{230 - 4.7 \times 0.13}{0.159} = 1443 \text{ rpm.}$$

The following calculations are needed for a five hp output. The armature current equations and solutions will be

$$P_{in1} = P_{arm} + P_{rot} + P_0; \quad 230 I_a = I_a^2 \times 0.13 + 1088 + 5 \times 746$$

$$I_a^2 - \left(\frac{230}{0.13}\right) I_a + \left(\frac{4818}{0.13}\right) = 0$$

$$I_a = -\left(\frac{-1769}{2}\right) \pm \frac{1}{2}\sqrt{(1769)^2 - 4 \times 37{,}062}; I_a = 1748$$

or 21.2 A ← Choose

The speed is

$$N_{5hp} = \frac{V_T - I_a R_a}{K\theta} = \frac{230 - 212 \times 0.13}{0.159} = 1425 \text{ rpm.}$$

The efficiency at 5 hp out is

$$\text{Eff (at 5hp)} = \frac{P_0}{P_{in}} = \frac{P_0}{P_0 + \text{ all losses}} = \frac{5 \times 746}{5 \times 746 + 460 + 584 + 1088} = 0.699.$$

The efficiency at 15 hp out is

$$\text{Eff (at 15hp)} = \frac{P_0}{P_{in}} = \frac{P_0}{P_0 + \text{all losses}} = \frac{15 \times 746}{15 \times 746 + 460 + 395 + 1088} = 0.852.$$

Exhibit 2.1b Results of the preceding calculations, assuming rotational losses are constant

Power Out	Speed	%Speed Regulation	Arm Current	%Efficiency
15 hp	1400 rpm	3.2%	55.1 A	85.2%
5 hp	1425 rpm	1.8%	21.2 A	69.6%
0 hp	143 rpm	0	4.7 A	0

Following is an alternate method of solution using the dependent source model in Exhibit 2.1b at full load (1400 rpm or 146.5 rad/sec).

The speed and machine-flux constant at 15 hp using rad/sec for speed is

$$\omega(\text{rad/s}) = \frac{V_T - I_a R_a}{K'\Phi} = \frac{E_g}{K'\Phi};\ K'\Phi = \frac{E_g}{\omega} = \frac{222.8}{146.5} = 1.521$$

for developed torque

$$T_{dev} = K'\Phi I_a = 1.521 \times 55.1 = 83.8 \text{ N-m.}$$

Torque needed for a 15 hp output is

$$P_0 = \text{hp} \times 746 = 15 \times 746 = 11{,}190;\ T_0 = \frac{11{,}190}{146.5} = 76.38 \text{ N-m.}$$

Torque needed for rotational losses is

$$T_D = T_{dev} - T_0 = 83.81 - 76.38 = 7.43 \text{ N-m.}$$

Torques required for a 5 hp output is

$$T_0 = 5 \times \frac{746}{\omega} = \frac{3730}{\omega} \text{ N} - \text{m}\ .$$

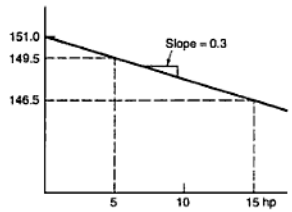


Exhibit 2.1c

Total developed torque needed is

$$T_{dev} = T_D + T_0 = 7.43 + \frac{3730}{\omega} \text{ N} - \text{m}\ .$$

For the speed at 5 hp, assume a linear relationship between no-load and full-load speed.

$$y = mx + b;\ w\omega = m \times hp \times \omega_0 = -0.3 \times 5 + 151 = 149.5$$

$$\therefore T_{dev} = 7.43 + \frac{3730}{149.5} = 32.4 \text{ N} - \text{m}$$

Armature current needed for a 5 hp output is

$$T_{dev} = K'\Phi I_a; \quad I_a = \frac{T_{dev}}{K'\Phi} = 21.3 \text{ A.}$$

These results agree (within the approximations used) for both methods. From the table of results, a judgmental engineering decision may be made as to the suitability of using this particular motor.

2.2 b. The coils are designed so that

$$I_1^2 R_1 = I_2^2 R_2$$

where I_1 and R_1 are the high-voltage coil current and ac resistance. I_2 and R_2 are the low-voltage coil current and resistance.

$$\frac{E_1}{E_2} = \frac{N_1}{N_2} = \frac{I_2}{I_1}$$

where $$\frac{N_1}{N_2} = a \text{ (turns ratio).}$$

Since

$$W_{culoss} = I_1^2 R_1 + I_2^2 R_2 = 570 \text{ watts}$$

and

$$I_1^2 R_1 = I_2^2 R_2 \text{ for good transformer design}$$
$$570 = 2I_1^2 R_1 = 2I_2^2 R_2$$
$$I_1 = \frac{50{,}000}{2300} = 21.7 \text{ amps (Rated)}$$
$$I_2 = \frac{50{,}000}{230} = 217 \text{ amps (Rated)}$$
$$\therefore R_1 = \frac{570}{2(21.7)^2} = 0.605\Omega \text{ (High Side)}$$
$$R_2 = \frac{570}{2(217)^2} = 0.00605\Omega \text{ (Low Side).}$$

2.3 c. Copper loss $= I_1^2 R_1 + I_2^2 R_2 = 570$ watts

2.4 a. Core loss = 187 watts neglecting the no load exciting current copper loss which would amount to $(6.5)^2$ (0.00605) = 0.255 watt

2.5 a. Full load efficiency $= \dfrac{\text{output}}{(\text{output} + \text{cu loss} + \text{core loss})}$

Assume power factor =1

$$\text{Efficiency} = \frac{50,000}{50,000+570+187} = \frac{50,000}{50,757} = 0.985$$

2.6 c. Half load efficiency

Assume power factor = 1

$$\text{Efficiency} = \frac{25,000}{25,000+\frac{570}{4}+187} = \frac{25,000}{25,329.5} = 0.99$$

2.7 b. $\text{Voltage Regulation} = \frac{(\text{No Load Voltage} - \text{Full Load Voltage})}{(\text{Full Load Voltage})}$

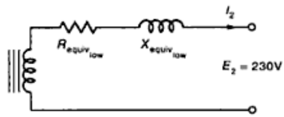


Exhibit 2.7

Convert the equivalent transformer circuit, referring it to the low-voltage coil. Assume the low-voltage coil has constant voltage E_2 = 230 volts.

$$R_{\text{Equiv}_{\text{Low}}} = \frac{\text{Power}}{I_2^2} = \frac{570}{(217)^2} = 0.0121 \text{ ohm}$$

$$Z_{\text{Equiv}_{\text{High}}} = \frac{115}{21.7} = 5.3\Omega$$

$$Z_{\text{Equiv}_{\text{Low}}} = \frac{Z_{\text{Equiv}_{\text{High}}}}{a^2} = \frac{5.3}{100} = 0.053\Omega$$

$$X_{\text{Equiv}_{\text{Low}}} = \sqrt{\left(Z^2_{\text{Equiv}_{\text{Low}}} - R^2_{\text{Equiv}_{\text{Low}}}\right)} = \sqrt{(0.053)^2 - (0.0121)^2}$$

$$= 0.0516 \text{ ohms}$$

Assume rated current I_2 = 217 amps (at unity power factor).

$$\frac{\overline{E_1}}{a} = E_2 + I_2\left(R_{\text{Equiv}_{\text{Low}}} + jX_{\text{Equiv}_{\text{Low}}}\right)$$

$$= 230 + 217(0.0121 + j0.053)$$

$$= 230 + 2.63 + j11.5$$

$$= 232.63 + j11.5$$

$$= 232.7\angle 2.84°$$

$$\text{Voltage Regulation} = \frac{232.7 - 230}{230} = \frac{2.70}{230}$$

$$= 0.01175 \quad \text{or} \quad 1.175 \text{ percent}$$

2.8 b. Find voltage regulation for P.f. = 0.8 lag.

Assume rated current $I_2 = 217$ amps. Draw the phasor diagram.

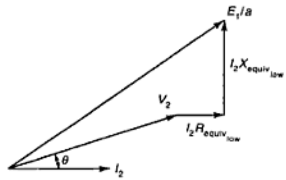


Exhibit 2.8

$$\frac{\overline{E_1}}{a} = V_2(\cos\theta + j\sin\theta) + I_2R_{\text{Equiv}_{\text{Low}}} + jI_2X_{\text{Equiv}_{\text{Low}}}$$
$$= 230(0.8 + j0.6) + 217(0.0121 + j0.053)$$
$$= 184 + j138 + 2.63 + j11.5$$
$$= 186.63 + j149.5$$
$$= 238.2\angle 38.7°$$

$$\text{Voltage regulation} = \frac{238.2 - 230}{230} = \frac{8.2}{230} = 0.0356 \quad \text{or} \quad 3.56\%$$

2.9 d. Voltage regulation for power factor = 0.8 lead.

Assume $I_2 = 217$ amperes, rated current.

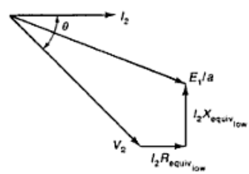


Exhibit 2.9

$$\frac{\overline{E}_1}{a} = V_2(\cos\theta + j\sin\theta) + I_2R_{\text{Equiv}_{\text{Low}}} + jI_2X_{\text{Equiv}_{\text{Low}}}$$
$$= 230(0.8 - j0.6) + 2.63 + j11.5$$
$$= 184 = j138 + 2.63 + j11.5$$
$$= 186.63 - j126.5$$
$$= 224\angle 34.2°$$

$$\text{Voltage regulation} = \frac{224 - 230}{230} = -0.0262 \quad \text{or} \quad -2.62\%$$

2.10 c. $$Z_{\text{core-hi-side}} = \frac{V_1}{I_1} = \frac{10V_2}{0.1I_2} = \frac{10 \times 230}{0.65} = 3{,}538\Omega$$

$$I_{\text{core-hi-side}} = \frac{V_1}{Z_1} = \frac{2300}{3538} = 0.65 \text{ A (neglecting Cu Loss.)}$$

2.11 d. $$Z_{\text{Copper-hi-side}} = \frac{115}{21.7} = 5.29\Omega \text{ (neglecting core loss)}$$

$$R_{\text{Eq-hi-side}} = \frac{W}{I_1^2} = \frac{570}{(21.7)^2} = 1.21\Omega$$

$$\cos\phi = \frac{W}{VA} = \frac{570}{115 \times 21.7} = 0.228,\ \phi = \cos^{-1}(0.228) = 76.8^\circ$$

$$X_{\text{Eq-hi-side}} = Z\sin 76.8^\circ = 5.29 \times 0.974 = 5.15$$

$$R_{\text{Load-hi-side}} = a^2 R_L = 10^2 \times 0.0394 = 3.94\Omega$$

$$\text{new } Z_{\text{hi-side}} = 1.21 + j5.15 + 3.94 = 5.15 + j5.15$$

$$= \sqrt{2} \times 5.15\angle + 45^\circ$$

$$I_{\text{hi-side}} = \frac{V}{Z_{\text{new}}} = \frac{2300 \text{ v}}{\sqrt{2} \times 5.15\angle + 45^\circ} = 316 \text{ A}\angle - 45^\circ$$

(neglecting core loss current)

2.12 a. The no-load terminal voltage (since there is no speed change) is E_g (may be given as $K'\Phi\omega$). It remains the same for a generator, and is given as

$$E_g = V_T + I_a R_a = 240 + 62.5 \times 0.2 = 252.5 \text{ volts.}$$

b. The percent voltage regulation is

$$\%\text{V.R.} = (V_{\text{no-load}} + V_{\text{full-load}}) \times 100/\text{V}_{\text{full-load}}$$
$$= (252.5 - 240) \times 100/240 = 5.2\%.$$

The input torque is found by $T\omega = E_g I_a = 252.2 \times 62.5 = 15{,}781$ watts, thus,

$$T = E_g I_a/\omega = 15{,}781/[(1200 \times 2\pi)/60] = 125.6 \text{ N-m.}$$

c. The voltage regulation is the same (5.2%) but the new required torque is increased by the amount of rotational loss torque,

$$T' = T + 5.0 = 125.6 + 5 = 130.6 \text{ N-m,}$$

and the new power input requirement is

$$P_{\text{in}} = T \times \omega = 130.6 \times (1{,}200 \times 2\pi/60) = 17.93 \text{ kW.}$$

d. E_g is directly proportional to speed for a constant field,

$$E_g' = E_g(1{,}000/1{,}200) = 210.4 \text{ volts.}$$

The load current then becomes $E_g/(R_a + R_l)$, where the original load resistance was 240/62.5 = 3.84 Ω,

$$I_a = 210.4/0.2 + 3.84) = 52.8\ \text{A}.$$
$$V_T = E_g - I_aR_a = 210.4 - 52.8 \times .2 = 200\ \text{V}.$$
$$P_{out} = 200 \times 52.8 = 10.56\ \text{kW}.$$

e. The new required input torque is easily found as

$$T = P_{in}/\omega = (210.4 \times 52.8)/(1000 \times 2\pi/60) = 11{,}109/104.6$$
$$= 106.1\ \text{N-m}.$$

2.13 Using the data given in Exhibit 2.13a,

Output Motor A = $VI\cos\Theta \times$ Eff.

In this case

$$\text{HP}_{\text{OP}} = \text{Fraction of Load} \times \frac{746\ \text{watts}}{\text{HP}}$$
$$= 0.25\text{HP} \times \frac{746}{\text{HP}} = 186.5\ \text{watts}$$
$$I_{\text{Line}} = \frac{VI\cos\Theta}{V \times \text{Eff.} \times \text{Pf}} = \frac{0.25 \times 746}{120 \times 0.7 \times 0.6} = 3.7\ \text{amp}$$
$$\text{Input Motor A} = \frac{\text{Output}}{\text{Eff.}} = \frac{186.5}{0.6} = 311\ \text{watts}$$
$$E_{\text{Line}} = 120\ \text{volts}$$
$$\text{Output Motor B} = VI\cos\Theta \times \text{Eff.}$$
$$= 0.5\text{HP} \times \frac{746}{\text{HP}} = 373\ \text{watts}$$
$$I_{\text{Line}} = \frac{0.5 \times 746}{120 \times 0.95 \times 0.7} = 4.66\ \text{amp}$$
$$\text{Input Motor B} = \frac{\text{Output}}{\text{Eff.}} = \frac{373}{0.7} = 534\ \text{watts}$$

Exhibit 2.13b

Motor	Power Factor	VA	Θ	SinΘ	Input Power
A	0.7 lag	445	45°	0.715	311 watts
B	0.95 lead	562	18.2°	0.3123	534 watts

$$\text{Total power} = \text{Input power A} + \text{Input power B}$$
$$= 311 + 534 = 845\ \text{watts}$$

Motor A VARS = 318 lag $\Theta = \tan^{-1}\frac{142}{845} = 9.6°$
Motor B VARS = –176 lead
Total VARS = 142 lag $\cos\theta = \cos(9.6°) = 0.986$ lag

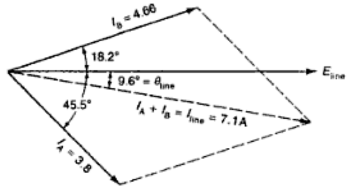


Exhibit 2.13c

$$
\begin{aligned}
\bar{I}_{\text{Line}} &= I_A(\cos 45.5° - j\sin 45.5°) + I_B(\cos 18.2° + j\sin 18.2°) \\
&= 3.7(0.7 - j0.71) + 4.66(0.95 + j0.3123) \\
&= 2.59 - j2.62 + 4.43 + j1.42 = 7.02 - j1.2 \\
I_{\text{Line}} &= 7.1 \text{ amp}
\end{aligned}
$$

$$
P_A = 311 \quad P_B = 534
$$
$$
P_{\text{Total}} = P_A + P_B = 845 \text{ watts}
$$

2.14 a. No, the shunt winding should not be changed. The problem clearly states that direction of rotation and *polarity* (positive terminal) stay the same. Therefore, regardless of whether we have a motor or generator operation, the direction of the current stays unchanged. The only drawback is the subtractive MMF of shunt and series field winding, giving a differential compounding. This matter is brought up in the next point.

b. Yes, the compound (series)-winding connections should be changed, because the armature current during generator operations is reversed and now opposes (differentially compounds) the shunt field. To still aid (cumulatively compound) the shunt field, the compound (series) winding should be reversed.

c. No, the interpole-winding connections should not be changed. Interpoles or commutating poles are narrow laminated auxiliary poles placed midway between the main poles and the plane of commutation. These interpoles are in series with the armature and are wound to oppose and nullify the armature reaction in the commutating plane. This prevents sparking that might cause flashover and also reduces iron losses in the armature teeth. Changing from motor to generator action, the polarity of the commutating pole automatically changes, with the change of the armature reaction MMF. Therefore, commutation in interpole machines is not affected by a change from motor to generator operation or a change in the direction of rotation.

d. Yes, the field (shunt) resistance should be changed. Because the generator voltage has to be larger than the terminal voltage due to the ohmic voltage drop in the armature winding, the flux has to be increased ($E_G = K\phi$ rpm). Increased ϕ means increased field current. Therefore, the shunt field resistance should be decreased.

e. The load on the mechanical clutch of the machine has increased.

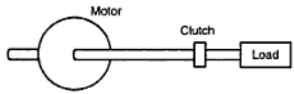


Exhibit 2.14b

$$\text{Rated Power}_{input} = \text{Losses} + \text{Load on clutch (for motor)}$$
$$\text{Rated Power}_{output} + \text{Losses} = \text{Load on clutch (for generator)}$$

Because losses remain the same in either mode of operation, the load on the clutch increases when the machine is operated as a generator.

f. Using the line connections as a reference point, due to internal losses, the power delivered to the motor shaft is 90% of the reference or input power.

Assuming that the same mechanical torque is maintained, above shaft power is now the input (prime mover) of the generator operation. Therefore, using the same 90% efficiency of the machine, the generator output referred to the original electrical power transferred at the line connections has decreased to

$$0.90 \times 0.90 = 0.81 \quad \text{or} \quad 81\%.$$

2.15 First the per-unit values need to be defined for each portion of the circuit (without the capacitor).

Generating Source Motor-load

$V_{baseG} = 240\text{ V} \rightarrow 1\text{ pu}$ ---------------------> same
$kVA_{baseK} = 10\text{ kVA} \rightarrow 1\text{ pu}$ --------------------> same
$I_{baseL} = 100000/240 = 41.67\text{A} \rightarrow 1\text{ pu}$ ---> same
$Z_{baseZ} = 240/41.67 = 5.760\ \Omega \rightarrow 1\text{ pu}$ ----> same
$Z_{motor} = (4 + j3)/5.760 = 0.6944 + j0.5208\text{ pu}$

For the transmission line,

$$V_{baseL} = 2 \times 240 = 480\text{ V} \rightarrow 1\text{ pu}$$
$$kVA_{baseL} = 10\text{ kVA} \rightarrow 1\text{ pu}$$
$$I_{baseL} = 10000/480 = 20.83 \rightarrow 1\text{ pu}$$
$$Z_{baseL} = 480/20.83 = 23.04\ \Omega \rightarrow 1\text{ pu}$$
$$Z_{line} = (2 + j2)/23.04 = 0.0868 + j0.0868\text{ pu.}$$

The total pu impedance as seen at the source is

$$Z_{Tot} = Z_{line} + Z_{motor} = 0.0868 + j0.0868 + 0.6944 + j0.5208$$
$$= 0.7812 + j0.6076 = 0.9897\angle 37.9^\circ\text{ pu}$$
$$I_{source} = 1.0\angle 0^\circ/0.9897\angle 37.9^\circ = 1.010\angle -37.9^\circ\text{ pu}$$
$$= 1.010\angle -37.9^\circ\text{ pu} \times 41.67\text{ A/pu} = 42.10\text{ A.}$$
$$P_{source} = VI\cos\Phi = 240 \times 42.10\cos 37.9^\circ = 7974\text{ watts.}$$

The motor load voltage is

$$I_{motor}Z_{motor} = 1.010 \times 0.8680 = 0.8767 \text{ pu}$$
$$V_{motor} = 0.8767 \text{ pu} \times 240 \text{ V/pu} = 210.4 \text{ V.}$$

Check.

$$P_{motor} = I^2R = 42.10^2 \times 4 = 7089 \text{ watts}$$
$$P_{line} = (I_{source}/2)^2\ R_{line} = (21.05)^2 \times 2 = 886 \text{ watts}$$

Choose capacitor such that $Z_{moto} \| Z_{capacitor}$ has only a real part,

$$Y_{motor} = 1/5\angle -36.9° = 0.16 - j0.12,$$
$$Y_{cap} = +j0.12,\ Z_{cap} = 1/Y_{cap} = 1/j0.12 = -j8.33\ \Omega$$
$$Y_{Tot} = 0.16 + j0,$$
$$Z_{Tot\ load} = 1/0.16 = 6.25\ \Omega \rightarrow 6.25/5.76 = 1.09 \text{ pu}$$

The new total pu impedance as seen at the source is

$$Z_{Tot} = 0.0868 + j0.0868 + 1.09 = 1.177 + j0.0868 \text{ pu}$$
$$= 1.180\angle 4.24°$$
$$= 1.180 \times 5.795 = 6.838\angle 4.24°\ \Omega$$
$$I_{source} = 1.0/1.180\angle 4.24° = 0.8475\angle 4.24° \text{ pu}$$
$$= 0.8475\angle -4.24° \times 41.67 \text{ A/pu} = 34.41 \text{ A}$$
$$P_{source} = VI\cos\Phi = 240 \times 35.41\cos 4.24° = 8475 \text{ watts}$$
$$P_{line} = I_{line}^{\ 2} x R_{line} = (34.41/2)^2 \times 2 = 626.9 \text{ watts}$$
$$P_{motor} = I_{motor}^{\ 2} x R_{motor} = 44.3^2 \times 4 = 7850 \text{ watts}$$

It is interesting to compare the new load voltage with the original value (210.4 V),

$$V_{motor\text{-}cap} = I(Z_{motor} \| Z_{cap}) = 34.41 \times 6.25 = 221.3 \text{ V.}$$

Not only is motor voltage higher, for a greater motor output, but the line loss power loss is less. Also, the capacitor should have a kVAR voltage rating of 240 volts (when operating with the motor unloaded), so instead of a current of 221.3/(−j8.33) = 26.56A, it should be 240/(−j8.33) = 28.8 A, giving a kVAR of 6.9 kVAR.

2.16 a. Designating: I_m = Motor current; I_{mR} = Real component of I_m; I_{mQ} = Quadrature component of I_m; I_i = Incandescent lights current; I_1 = Total load current; I_1' = Corrected total current; I_c = Capacitor current,

We obtain the phasor diagram shown in Exhibit 2.16.

$$I_i = \frac{9{,}000}{\sqrt{3} \times 208} = 25\angle 0° \text{ amps}$$

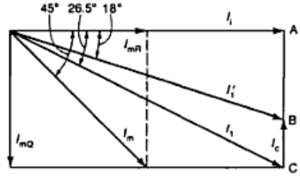


Exhibit 2.16

$$I_m = \frac{10\times 746}{\sqrt{3}\times 208\times 0.83\times 0.707} = 35.3\angle 45^\circ \text{ amps} = 25 + j25$$

Thus $I_{mR} = 25\angle 0^\circ$

$$I_{mQ} = 25\angle -90^\circ$$

$$I_1 = 25 + 25 + j25 = 50 + j25 = 56\angle 26.5^\circ \text{ amps}$$

b. If the power factor = 1, the I_1 reduces to the real component of the above current, *i.e.* $50\angle 0^\circ$. Therefore, we have to reduce $I_1 = 56\ \angle 26.5^\circ$ to 105% of 50 amps = $I_1' = 52.50$ amps at an angle to be determined.

From vector diagram (Exhibit 2.16) $I_1' = (52.50)$ ends in point B.

$$AC = 25$$

$$AB = \sqrt{52.5^2 - 50.0^2} = 16.5$$

$$BC = 25 - 16.5 = 8.5 \text{ amps} = I_c$$

$$\text{Then: } X_{C_{1\,phase}} = \frac{E}{\sqrt{3}\times I_c} = \frac{208}{\sqrt{3}\times 8.5} = 14.1 \text{ ohms}$$

$$\text{If } X_c = \frac{1}{2\pi fC}$$

$$\text{then } C = \frac{1}{377\times 14.1} = 18.7\mu\text{F/phase.}$$

$$C_{total} = 3\times 18.7 = 56.1\mu\text{F}$$

c. $$I_1' = (25 + 25) + j16.5 = 52.5\angle 18^\circ \text{ amps}$$

$$I_{m_{NEW}} = 25 + j16.5 = 30\angle 34^\circ \text{ amps}$$

Rating of wire size:

$125\%\ (I_{m_{NEW}} + I_1) = 1.25\times(30 + 25) = 68.8$ amps

From copper wire table (National Electrical Code (NEC), Article 310, Table 310-16) for 70 amps continuous current we obtain Size No. 4.

From overcurrent protection table at 30 amps (no 125% factor is needed) + 25 = 55 amps. Thus we obtain a fuse size of 120 amps.

From conduit table for 4 wires and 55 amps we obtain a conduit size of 1 1/4".

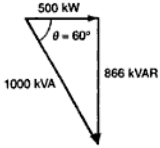


Exhibit 2.17a

2.17 a.

$$\text{Motor \#1} = \frac{600}{0.90} \times 0.746 = 497\text{kW} \cong 500\text{kW}$$

$$pf = 0.5, \quad \text{thus} \quad \theta = 60°$$

$$\text{kVA} = \frac{500\text{kW}}{\cos 60°} = \frac{500}{0.5} = 1000$$

$$\text{kVAR} = 1000 \times \sin 60° = 1000 \times 0.866 = 866$$

$$\text{Motor \#2 kVA} = \sqrt{3} \times 4160 \times 277 = 1994 \cong 2000$$

$$pf = \cos\theta = \frac{1730}{2000} = 0.866; \quad \theta = 30°$$

$$\text{kVAR} = 2000 \times \sin 30° = 2000 \times 0.5 = 1000$$

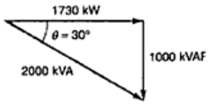


Exhibit 2.17b

b. Total load of motors #1 and #2

$$\text{kW} = 500 + 1730 = 2230$$

$$\text{kVAR} = 866 + 1000 + 1866$$

$$\text{kVA} = \sqrt{2230^2 + 1866^2} = \sqrt{8.45 \times 10^6} = 2907$$

$$\text{Actual combined } pf = \cos\theta = \frac{2230}{2907} = 0.767 \text{ lag}; \ \theta = 40°$$

$$\text{Desired combined power factor} = 0.966 \text{ lag}; \ \theta = 15°$$

$$\text{kVA new} = \frac{2230}{0.966} = 2308$$

$$\text{BC} = \text{Required leading kVAR} = 1866 - 2230 \tan 15°$$

$$= 1866 - 2230 \times 0.268 = 1866 - 598 = 1268$$

$$X_c = \frac{V^2}{RVA} = \frac{(4160)^2}{1268 \times 1000} = 13.6 \text{ ohm}$$

$$C = \frac{1}{2\pi f \times X_c} = \frac{1}{6.28 \times 60 \times 13.6} = 195\mu\text{F}$$

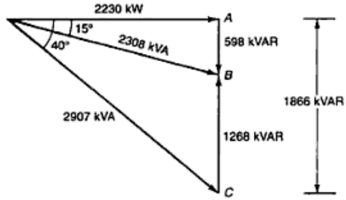


Exhibit 2.17c

c. Assuming that the synchronous motor has the same efficiency as motor #2, which it replaces,

$$\text{kVAR syn. motor} = \text{kVAR motor \#1} - \text{kVAR desired}$$
$$= AC - AB = 866 - 598 = 268.$$

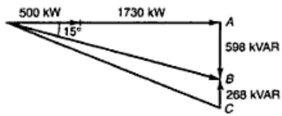


Exhibit 2.17d

The synchronous motor alone is shown in Exhibit 2.17e.

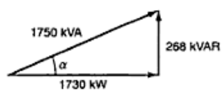


Exhibit 2.17e

$$\tan\alpha = \frac{268}{1730} = 0.155; \ \alpha = 9°$$

$$\text{kVA} = \frac{1730}{\cos 9°} = \frac{1730}{0.988} = 1750$$

$$pf = \cos 9° = 0.988$$

2.18
$$\text{Rated } I = \frac{12{,}500{,}000}{6600\sqrt{3}} = 1090 \text{ A}$$

Field current necessary to give short-circuit line current, from graph:

$$I_f(\text{dc}) = 105 \text{ A}.$$

Terminal voltage (per phase) from graph:

$$\frac{6300}{\sqrt{3}} = 3630 \text{ volts.}$$

Terminal rated voltage (per phase)

$$\frac{6600}{\sqrt{3}} = 3800 \text{ volts.}$$

(Here, one could find the synchronous reactance or impedance by taking these operating values and defining

$$X_S \approx \frac{3630 \text{ V}}{1090 \text{ A}}.$$

However, because only one condition is asked for, the synchronous voltage drop is the 3630 volts.)

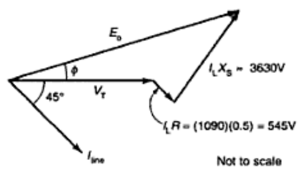


Exhibit 2.18b

$$\begin{aligned} E_0 &= V_T + R_A I_L + jX_S I_L \\ &= 3800 + 545(\cos 45° - j\sin 45°) + 3630(\cos 45° + j\sin 45°) \\ &= 6752.4 + j2181.6 = 7096\angle\phi \text{ volts} \end{aligned}$$

$$\therefore \% \text{ V.R} = \frac{E_0 - V_T}{_T}(100) = \frac{7096 - 3800}{3800}(100)$$
$$= 86.7\%$$

Note that the calculated no-load voltage is

$$7096\sqrt{3} = 12{,}290 \text{ volts,}$$

which is well beyond the range of available field current. Thus the saturated limit of no-load voltage would be between 9000 and 10,000 volts. The saturated value of synchronous reactance would be less than the value used here.

2.19 480 V winding rated current = 10,000/480 = 20.8 A
120 V winding rated current = 10,000/120 = 83.3 A

To get the specified voltages connect the transformer as in Exhibit 2.19.

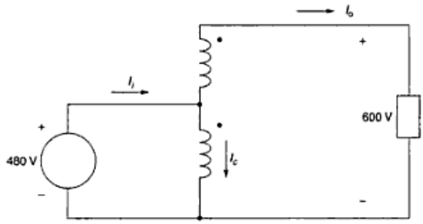


Exhibit 2.19

The largest value for I_o is 83.3 A.

The load is $S = 600 \times 83.3/1000 = 50$ kVA.

2.20 The secondary line current is $I_s = \dfrac{S}{\sqrt{3}V_S} = 12{,}000/\sqrt{3} \times 13 = 533$ A.

Because the secondary is wye connected, this is also the secondary winding current.

The secondary rated voltage is $13.2/\sqrt{3} = 7.62$ kV.

The winding ratio is $132/7.62 = 17.32$.

The primary winding current is $533/17.32 = 30.8$ A.

The primary line current is $\sqrt{3} \times 30.8 = 53.3$ A.

2.21 The stator current $I = \dfrac{P}{\sqrt{3}V\cos\theta} = \dfrac{500}{\sqrt{3} \times 4.16 \times 0.9} = 77.1$ A.

Stator copper loss $= 3I^2R = 3 \times 77.1^2 \times 0.3 = 5350$ W.

Prime mover power $= 500 + 5.35 + 15 = 520$ kW.

For a two pole 60 Hz machine synchronous speed is 3600 rpm and $\omega_s = 3600 \times 2\pi/60 = 377$ rad/second.

Prime mover torque $= 520/377 = 1.38$ kNm.

2.22 The phase to neutral supply voltage $V = 4000/\sqrt{3} = 2309.5$ V.

The excitation voltage is

$$E = V - jXI = 2309.5 - j10 \times 250 = 3403.5\angle{-47.3^\circ} \text{ V}.$$

Maximum power is found from the power-angle equation with the angle set equal to 90°.

$$P_{max} = \frac{3EV}{X} = \frac{3\times 3403.5\times 2309.5}{10} = 2{,}358{,}000 \text{ W} = 2358 \text{ kW}.$$

Convert to horsepower,

$$P_{max} = 2358/0.746 = 3160 \text{ hp}.$$

2.23 From NEC Table 430.7(B) the starting kVA/hp is in the range 5.0 – 5.59. Use 5.59 to give the largest starting current.

$$\text{Starting kVA at rated voltage} = 5.59 \times 100 = 559 \text{ kVA}.$$

$$\text{Starting current} = \frac{559{,}000}{\sqrt{3}\times 460} = 701.6 \text{ A}.$$

Using a wye-delta starter the starting current = 701.6/3 = 234 A.

2.24 Synchronous speed is $\frac{120f}{p}$ where $f = 50$ Hz and p is the number of poles. For $p = 6$, $n_s = 1000$ rpm. Because an induction motor runs at a speed slightly below synchronous speed at full load, conclude that the motor has 6 poles.

$$\text{Slip} = (n_s - n)/n_s = (1000 - 950)/1000 = 0.05$$

$$\text{Rotor current frequency} = sf = 0.05 \times 50 = 2.5 \text{ Hz}$$

2.25 Convert impedances to a common base, say 7500 kVA.

$$Z_1 = j0.065 \text{ p.u.}$$
$$Z_2 = j0.055 \times (7500/5000) = j0.0825 \text{ p.u.}$$

Equivalent circuit is shown in Exhibit 2.25.

$$I_1Z_1 = I_2Z_2$$
$$I_2 = (0.065/0.0825) \times I_1 = 0.788 \times I_1$$

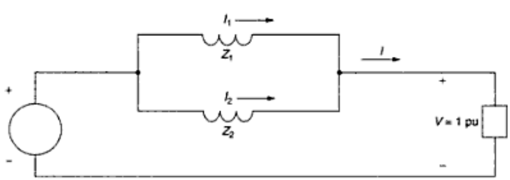


Exhibit 2.25

If T_1 is at rated load (7500 kVA), I_2 = 0.788 p.u., which is S_2 = 0 .788 × 7500 = 5910 kVA. It is overloaded.

If T_2 is at rated load I_2 = 5000/7500 = 0.667 p.u.

$$I_1 = I_2/0.788 = 0.667/0.788 = 0.846 \text{ p.u.}$$
$$S_1 = 0.846 \times 7500 = 6345 \text{ kVA}$$
$$\text{Total load} = 5000 + 6345 = 11{,}345 \text{ kVA}$$

2.26 Equivalent circuit is shown in Exhibit 2.26.

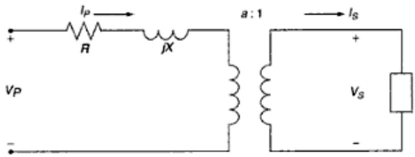


Exhibit 2.26

$$a = 2400/240 = 10$$
$$V_s = 240\angle 0^\circ \text{ V}$$
$$I_s = 25{,}000/240\ \angle{-31.79^\circ} = 104.2\angle{-31.79^\circ} \text{ A}$$
$$V_p = aV_s + I_s/a\,(R + jX) = 2400 + 10.42\ \angle{-31.79^\circ} \times (3.45 + j5.75)$$
$$V_p = 2462.3\angle 0.74^\circ \text{ V}$$

Secondary full-load voltage = 240 V

Secondary no-load voltage = 2462.3/10 = 246.2 V

$$\%\text{ voltage regulation} = \frac{246.2 - 240}{240} \times 100 = 2.58\%$$

2.27

$$P_{out} = S\cos\theta = 25{,}000 \times 0.85 = 21{,}250 \text{ W}$$
$$\text{Copper loss} = I_p^2 R = 10.42^2 \times 3.45 = 375 \text{ W}$$
$$\text{Core loss} = 500 \text{ W}$$
$$P_{in} = 21{,}250 + 375 + 500 = 22{,}125 \text{ W}$$
$$\text{Efficiency} = (P_{out}/P_{in}) \times 100 = (21{,}250/22{,}125) \times 100 = 96\%$$

2.28 See Exhibit 2.28.

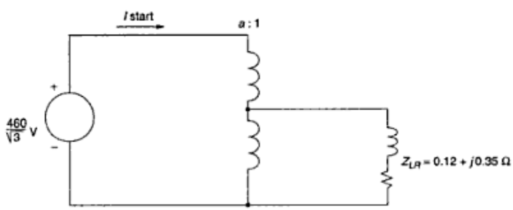


Exhibit 2.28

$$\text{Starting current at full voltage} = \frac{460/\sqrt{3}}{0.12 + j0.35} = 717.8\angle{-71.1^\circ}\ \text{A}$$

$$\text{Autotransformer ratio} = 1/0.65 = 1.538$$

$$\text{With the autotransformer, starting current} = 717.8/1.538^2 = 303.5\ \text{A}.$$

2.29 See Exhibit 2.29.

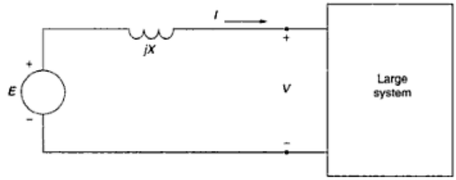


Exhibit 2.29

$$V = 12{,}000/\sqrt{3}\ \angle 0^\circ = 6928.4\angle 0^\circ\ \text{V}$$
$$I = 4000\angle 0^\circ\ \text{A}$$
$$E = V + jXI = 6928.4 + j2 \times 4000 = 10{,}583.1\angle 49.11^\circ\ \text{V}$$

Neglecting the nonlinear effects of saturation, when the field current is increased by 15% the magnitude of the excitation voltage will also increase by 15%.

$$E' = 1.15 \times 10{,}583.1 = 12{,}170.6$$

Because the prime mover does not change, the power output does not change.

$$P = \frac{3EV}{X}\sin\delta = \frac{3E'V}{X}\sin\delta'$$

$$\sin\delta' = \frac{E}{E'}\sin\delta = 0.65736$$
$$\delta' = 41.10^\circ$$

The new stator current is

$$I' = \frac{E' - V}{jX} = \frac{12{,}170.6\angle 41.10^\circ - 6928.4}{j2.0} = 4155\angle{-15.66^\circ}\text{A}.$$

The new power factor is cos 15.66° = 0.963 lagging.

2.30 See Exhibit 2.30.

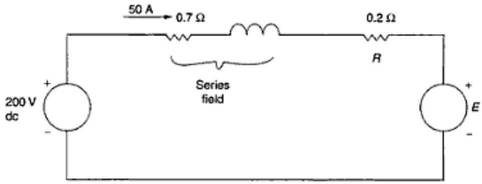


Exhibit 2.30

$$E = 200 - 50 \times (0.2 + 0.7) = 155 \text{ V}$$
$$P_{dev} = EI = 155 \times 50 = 7750 \text{ W}$$
$$P_{out} = P_{dev} - \text{rotational loss} = 7750 - 180 = 7570 \text{ W}$$
$$P_{out} \text{ in hp} = 7570/746 = 10.15 \text{ hp}$$
$$P_{in} = 200 \times 50 = 10{,}000 \text{ W}$$
$$\text{Efficiency} = 7570/10{,}000 = 0.757 \quad \text{or} \quad 75.7\%$$

2.31 See Exhibit 2.31.

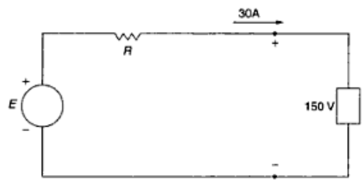


Exhibit 2.31

$$\text{Voltage regulation} = (170 - 150)/150 = 0.133 \quad \text{or} \quad 13.3\%$$
$$150 = 170 - 30 \times R$$
$$\text{Solve for } R = 0.67\ \Omega$$
$$P_{dev} = 170 \times 30 = 5100 \text{ W}$$
$$\omega = 1800 \times (2\pi 60) = 188.5 \text{ rad/sec}$$
$$T_{dev} = 5100/188.5 = 27.1 \text{ Nm}$$

2.32 $$R_2/s = 0.10/0.035 = 2.857$$

$$I_2 = \frac{115.5}{0.15 + 2.857 + j(0.50 + 0.50)} = 36.45\angle{-18.4°} \text{ A}$$

$$P_{air\ gap} = 3I_2^2\, R_2/s = 3 \times 36.45^2 \times 2.857 = 11{,}387 \text{ W}$$

For a four-pole machine synchronous speed is 1800 rpm. $\omega_{sync} = 188.5$ rad/sec

$$T_{out} = P_{air\ gap}/\omega_{sync} = 11{,}387/188.5 = 60.4 \text{ Nm}$$

2.33 At start the slip $s = 1$.

$$I_2 = \frac{115.5}{0.15 + 0.10 + j(0.50 + 0.50)} = 112.1\angle{-76.0^\circ}\text{ A}$$

$$P_{\text{air gap}} = 3I_2^2 R_2 = 3 \times 112.1^2 \times 0.10 = 3770\text{ W}$$

$$T_{\text{start}} = P_{\text{air gap}}/\omega_{\text{sync}} = 3770/188.5 = 20\text{ Nm}$$

$$I_m = 115.5/j20 = 5.80\angle{-90^\circ}\text{ A}$$

$$I_{\text{start}} = I_2 + I_m = 112.1\angle{-76.0^\circ} + 5.80\angle{-90^\circ} = 117.7\angle{-76.7^\circ}\text{ A}$$

RECOMMENDED REFERENCES

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Hambley, *Electrical Engineering Principles and Applications,* 3rd edition, Prentice Hall, 2005 (Chapters 15, 16, and 17).

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CHAPTER 3

Power Distribution

OUTLINE

PROBLEMS

3.1 A 3-phase, 60 Hz, 173 kV, 50-mile-long transmission power line delivers complex power to known load. The line has been tested for its characteristic parameters. These parameters (on a per-phase basis) are found to be

$$\text{Resistance} = 0.1 \text{ ohm/mile}$$
$$\text{Inductance} = 2.0 \text{ millihenry/mile}$$
$$\text{Capacitance} = 1.0 \times 10^{-2}\ \mu\text{F/mile}$$

For a known load at the end of the line of $75 + j30$ (three-phase, total megavolt-amperes), determine the input (sending) current and the input power necessary to support the complex load.

3.2 A 345 kV power transmission line has two bundled conductors per phase, spaced 18 inches apart horizontally (Exhibit 3.2). The conductor used in the bundle has a self GMD of 0.0403 feet and the phases are spaced horizontally 15 1/2 feet apart.

Determine the following:

a. The self GMD of the bundled conductors.

b. The mutual GMD of the line.

c. The inductive reactance per phase per mile.

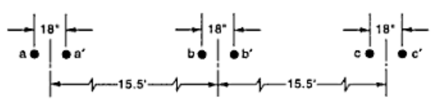


Exhibit 3.2

3.3 Power for a remote building on an industrial site is supplied through an existing buried cable from a fixed voltage 60 Hz supply.

The load in the remote building consists of lighting and induction motors. During periods of peak demand, when the cable is carrying approximately its rated current, the resulting steady-state load voltage is well below the desired value because of the characteristics of the load. A small amount of additional constant-speed motor load is anticipated in the near future.

What equipment can be installed *at the building* to improve the present situation and to permit the additional load? Explain how the equipment you recommend will improve the situation; the use of phasor diagrams is suggested.

3.4 The power distribution transmission system in Exhibit 3.4a develops a fault in one of its phases.

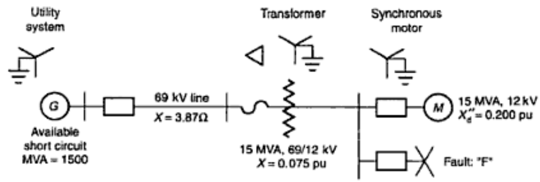


Exhibit 3.4a

Determine the fault currents at point "F" for the following conditions:

a. Assume the fault is a three-phase one.

b. Assume the fault is a single line to ground.

3.5 The voltages of an unbalanced 3-phase supply are $V_a = (200 + j0)$V, $V_b = (-j200)$V and $V_c = (-100 + j200)$V. Connected in star across this supply are three equal impedances of $(20 + j10)$ ohms. There is no connection between the star point and the supply neutral. Evaluate the symmetrical components of the A phase current and the three line currents.

3.6 The one-line diagram in Exhibit 3.6a is for a single-phase system involving a voltage source of 240 volt 10 kVA generator that supplies a load through a 1:2 step-up transformer, a relatively short transmission line with an impedance of $1 + j4$ ohms, and a 4:1 step-down transformer. The transformers are assumed to be ideal.

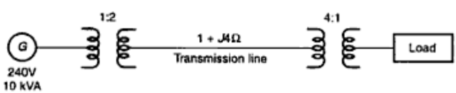


Exhibit 3.6a

a. Assume the load is known to be $1 + j1\ \Omega$. What equivalent load (in ohms) does the generator see?

b. Assume the load is known to be $1 + j1\ \Omega$. Determine the per unit values of the various system parameters along the system and determine the per unit current through the load.

c. (Optional for practice) Repeat part 2 except $Z_L = 2 + j2\ \Omega$.

3.7 Originally it is planned to furnish a plant load requirement of 1000 hp at 2200 volt, 3-phase, by induction motors operating at 80% power factor and 90% efficiency.

a. Find the line current necessary to supply this load and generator capacity.

b. Assume that rather than supplying the 1000 hp by induction motors, it is decided to produce 400 hp of this load by a synchronous motor operating in the over-excited leading mode of 85% (assume same efficiency as for an induction motor). Find the new total line current requirement and the overall power factor.

c. If, rather than installing the 400 hp synchronous motor as in Part(b), it is considered feasible to use power factor correcting capacitors for the 1000 hp motors in Part (a) to achieve the same power factor correcting as obtained in Part (b). Determine the size (kVAR) of the capacitors needed.

3.8 The peak load supplied from a substation 13.8 kV bus is 20 MW at a power factor of 0.85 lagging. Determine the Mvar size of a 3-phase capacitor bank to correct the power factor to 0.95 lagging.

3.9 A 3-phase 480 V delta-connected capacitor normally draws a line current of 200 A. If a fuse blows in one phase, determine the current flowing in the other two phases.

3.10 An induction motor and a synchronous motor are supplied from a common bus. The induction motor takes 100 kVA at 0.75 power factor lagging. The synchronous motor draws 150 kW. The field current of the synchronous motor is adjusted until the combined motor load power factor is unity. Determine the power factor of the synchronous motor.

3.11 A delta-connected 3-phase load of $60 + j30$ ohms/phase is supplied by a feeder having an impedance of $0.50 + j0.75$ ohms/phase. Determine the line-line voltage at the sending of the feeder to provide a line-line voltage of 460 V at the load.

3.12 The available fault current on the 480 V side of the transformer supplying a facility is 40,000 amps, rms symmetrical. The service entrance phase conductors are 4/0 copper installed in nonmetallic conduit. The distance between the transformer and the main panelboard is 200 feet. Determine the 3-phase fault current at the panelboard.

3.13 The ABCD constants of an overhead three-phase transmission line have been calculated to be

$$A = D = 0.90 + j0$$
$$B = 15 + j130$$
$$C = 0 + j0.002$$

The line is energized at one end at a line-line voltage of 500 kV and open circuited at the other end. Determine the line-line voltage at the open end and the charging current at the sending end.

3.14 A 100 hp squirrel cage induction motor is to be started from a 480 V bus supplied by a 250 kVA transformer with an impedance of 5%. Determine the percent voltage drop on the 480 V bus if the motor is started at full voltage. Assume that the starting current of the motor is 6 times the motor full load current.

3.15 A 3-phase overhead 12.47 kV, 60 Hz distribution feeder has the phase conductors configured as shown in Exhibit 3.15. The conductors are 266.8 kcmil ACSR. Determine the inductive reactance of the circuit in ohms per mile.

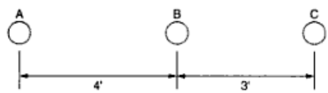


Exhibit 3.15

3.16 At an industrial plant it is planned to use 4.16 kV switchgear with a standard short circuit interrupting rating of 250 MVA. The utility supply to the plant has been selected as a 12 MVA 3-phase transformer. The plant will include on-site generation and synchronous motors, which will contribute 30 MVA to the short circuit interrupting duty.

Taking into account the manufacturing tolerance of ±7.5% for transformer impedance, determine the transformer impedance that should be specified.

3.17 The service entrance conductors for a three-phase supply to a facility have been selected as two 350 kcmil copper conductors per phase. Determine the minimum size copper grounding conductor at the service entrance panel.

3.18 Determine the required THW copper conductor and overcurrent protection device setting to supply a 50 kW, 208 V, 0.90 power factor continuous load. Ambient temperature is 30°C.

3.19 Determine the required THW copper conductor size to supply a three-phase, 200 hp, 460 V squirrel cage induction motor. Ambient temperature is 30°C.

3.20 A 13.2 kV, 480 V 3-phase transformer is equipped with fixed taps in the primary 13.2 kV windings. The tap is set at 5% below nominal. The actual voltage in the area where the transformer is installed does not vary appreciably from 12.9 kV. The voltage regulation of the transformer is estimated to be 5%. Determine the low side voltage at full load.

3.21 The equivalent sequence impedances at a 69 kV bus of a 3-phase power system have been calculated to be $Z_1 = Z_2 = j0.085$ per unit and $Z_0 = j0.20$ per unit. The impedances are on a 50 MVA 3-phase base and a line-line 69 kV voltage base. Determine the fault current in amps for a 3-phase, a phase-to-phase, and a phase-to-ground fault.

3.22 The equivalent system impedance of the supply to a 3-phase, 200 kvar, 480 V capacitor is 0.005 + j0.030 ohms/phase. Determine the percent voltage rise when the capacitor is switched on.

3.23 Determine the maximum trip setting of an inverse time circuit breaker to provide short circuit protection of a circuit supplying a 40 hp, 200 V induction motor.

3.24 Determine the required trade size rigid steel conduit to carry four 2/0 THHN copper conductors and one #2 bare stranded copper equipment grounding conductor.

3.25 A 3-phase, 480 V feeder has one 500 kcmil THW copper conductor per phase installed in nonmetallic conduit. The feeder is 200 feet long and carries a load of 300 A at 0.85 power factor lagging. Determine the percent voltage drop in the feeder.

SOLUTIONS

3.1 The equivalent circuit may be represented as Exhibit 3.1a.

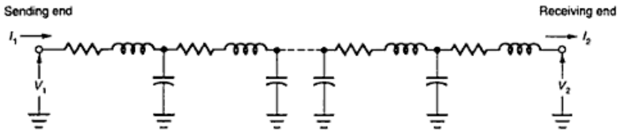


Exhibit 3.1a

A simpler approximation (on a per phase basis) is Exhibit 3.1b.

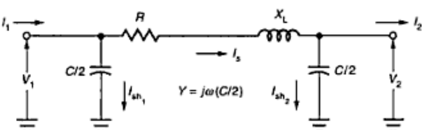


Exhibit 3.1b

The complex power, S_{pp}, (which may also be expressed as the phasor voltage times the complex conjugate of the phasor current) on a per phase basis is

$$S_{pp} = \frac{1}{3}(75 + j30) = 25 + j10 \text{ mega volt-amps.}$$

The current at the receiving end, then, is

$$I_2 = \frac{(25 + j10) \times 10^6}{\left(\frac{173}{\sqrt{3}}\right) \times 10^3} = (0.25 + j0.1) \times 10^3 \text{ A/phase.}$$

The shut and series currents are then found.

$$I_{Sh_2} = V_2 Y_2$$

where

$$\begin{aligned} Y_2 &= j\frac{1}{2}(50)(377)(1 \times 10^{-8}) \\ &= j94.25 \times 10^{-6} \\ I_{sh_2} &= \left(\frac{173}{\sqrt{3}}\right) \times 10^3 (j94.25) \times 10^{-6} \\ &= j9.425 \text{ A/phase} \\ I_{Ser} &= I_{Sh_2} + I_2 = j9.425 + 250 + j100 \\ &= 250 + j109.4 \text{ A/phase} \end{aligned}$$

And the voltage drop across the series branch may be found by first finding the series impedance.

$$\begin{aligned} Z &= R + jX_L = (50)(0.1) + j(50)(377)(2 \times 10^{-3}) \\ &= 5 + j37.7 \text{ ohms/phase} \end{aligned}$$

$$\begin{aligned} V_{drop} &= I_{Ser} Z = (250 + j109.4)(5 + j37.7) \\ &= -2.874 + j10.07 \text{ kV/phase} \end{aligned}$$

Therefore the input voltage is

$$\begin{aligned} V_1 &= V_{drop} + V_2 = -2.874 + j10.07 + \frac{173}{\sqrt{3}} \\ &= 97.13 + j10.07 \text{ kV} \end{aligned}$$

and the sending end shunt current is

$$\begin{aligned} I_{Sh_1} &= V_1 Y = (97.13 + j10.07)(j94.25 \times 10^{-6}) \\ &= 0.949 + j9.1545 \text{ A/phase.} \end{aligned}$$

Therefore the sending end current is

$$\begin{aligned} I_1 &= I_{Sh_1} + I_{Ser} = 0.949 + j9.154 + 250 + j109.4 \\ &= 250.9 + j118.55 \text{ A/phase.} \end{aligned}$$

Then one may find the sending end complex power (again complex voltage times the complex conjugate of the current) as

$$\begin{aligned} \text{Complex Power} &= (97.13 + j10.07) \times 10^3 (250.9 - j118.55) \\ &= 25.56 + j1.38 \text{ mega volt-amps/phase.} \end{aligned}$$

And, of course, the total complex power is three times this value,

$$76.68 + j4.14 \text{ mega volt-amps.}$$

3.2 a. Often self GMD of a bundled or composite conductor is called "geometric mean radius," or GMR. Self GMD may be denoted as D_s. This term includes the distances of a strand or conductor from all other strands within the same bundle plus the "distance of the strand from himself," or the self GMR of the strand.

In the present line configuration we have two strands per bundle, thus we have four distances: $D_{aa'}$, $D_{a'a}$, D_{aa}, and $D_{a'a'}$. The self GMR of a single strand is less than the actual physical radius ($R \times 0.7788$). This reduced radius is the above given self GMD of 0.0403 ft. and is available from tables. Converting all distances into feet, we obtain for one bundle

$$D_s = D_{sa} = \sqrt[4]{D_{aa'} \times D_{a'a} \times D_{aa} \times D_{a'a'}} = D_{sb} = D_{sc}.$$

Note: We extract the fourth root as we have hour distances under the radical

$$D_s = \sqrt[4]{(0.0403)^2 \times \left(\frac{18}{12}\right)^2} = \sqrt[4]{0.00366}$$
$$= 0.246 \text{ ft.}$$

b. The mutual GMD of the line, or D_{eq}, is the geometric mean of all mutual GMD values outside the bundles, *i.e.* between the three phases.

$$D_{ab} = D_{bc} = \sqrt[4]{(15.5)^2 \times 17.0 \times 14.0},$$

where $ab = 15.5'$ and $a'b' = 15.5'$

$$ab' = 15.5 + 2\frac{18}{12} \times \frac{1}{2} = 17.0'$$

$$a'b = 15.5 - 2\frac{18}{12} \times \frac{1}{2} = 14.0'$$
$$= \sqrt[4]{57{,}180} = 15.5 \text{ ft.}$$

$$D_{ac} = \sqrt[4]{(15.5 \times 2)^2 \times 32.5 \times 29.5} = \sqrt[4]{921{,}400} = 31 \text{ ft.}$$

$$D_{eq} = \sqrt[3]{D_{ab} \times D_{bc} \times D_{ac}} = \sqrt[3]{15.5 \times 15.5 \times 31.0} = 19.5 \text{ ft.}$$

c. The inductive reactance in ohm/mile or $X_L = 2\pi f \times 10^{-3} \times 0.7411 \log \frac{D_{eq}}{D_s}$.

The 10^{-3} factor is needed to convert the inductance L, obtained in mh/mile to h/mile to finally yield ohm/mile.

$$X_L = 0.377 \times 0.7411 \log \frac{19.5}{0.246} = 0.530 \text{ ohm/mile.}$$

3.3 The crux of the whole problem is the large portion of induction motors. The power factor of induction motors at rated load is typically from 0.70 to 0.90 with some groupings of motors resulting in even lower power factors.

For a power factor of 0.8, a motor drawing 225 kVA of power will utilize only 180 kW.

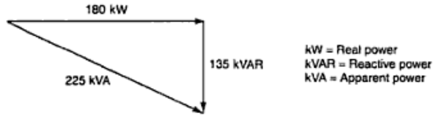


Exhibit 3.3a

If the kVA drawn in this case were equal to the real power required (kW), a 20% reduction in current would result. The reduction in current with present load would reduce the voltage drop, thus improving the voltage at the load.

Fluorescent lighting with capacitors usually has power factors from 0.95 to 0.97, therefore it is not practical to try to improve the power factor any higher.

To improve the power factor with the existing loads, capacitors should be applied. They have the characteristic of a leading power factor whereas induction motors have a lagging power factor. By adding capacitors, their leading kVAR cancels out the equivalent amount of lagging kVAR, as shown in Exhibit 3.3b.

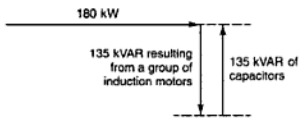


Exhibit 3.3b

The above power factor correction results in 180kVA = 180 kW or power factor = 1.0.

Another method of improving power factor is to add synchronous motors for the additional motor requirements. The synchronous motor acts like a capacitor, producing a leading power factor and leading kVAR.

Normally capacitors are the most effective in reducing system costs when located near the devices with low power factor. Here we are primarily concerned with the feeder to the building, but if there are any large induction motors or a grouping of motors, it would minimize local branch circuit voltage drop as well as the feeder, if the capacitors were located near the source of the power factor (pf).

An economic study of the situation should be made. Data from recording pf meters and kW meters should be gathered from as many places as feasible on feeder and branch circuits. Then a comparison should be done of how many and where the capacitors should be installed. The installation of the synchronous motors vs. induction motors with capacitors should be evaluated.

With the additional load, the voltage drop in the existing feeder may be too much even with unity pf. Then consideration should be given to using a boost transformer. It may well be that a combination of boosting and capacitor and synchronous motor will be the most economical solution. Boost transformers are much less in cost than regular transformers as they are just an auto-transformer. Improving the power factor beyond a certain point increases the cost disproportionately to the gain obtained, thus all alternatives should be weighed in making the ultimate decision. Using a boost transformer alone may mean that at light load an over voltage may result, which could be undesirable.

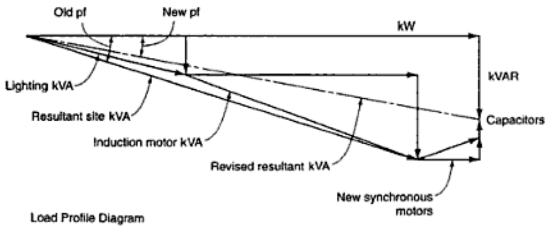


Exhibit 3.3c

3.4 kVA_{base} = 150 MVA (This value was selected to be a practical base between the two given MVA values.)

$$kV_{base} = 69 \quad \text{and} \quad 12 \text{ respectively}$$

$$Z_{base\,69} = \frac{(\text{kV})^2 \times 1000}{\text{kVA}_{base}} = \frac{(69)^2 \times 1000}{150{,}000} = 31.8 \text{ ohms}$$

$$Z_{base\,12} = \frac{(12)^2 \times 1000}{150{,}000} = 0.96 \text{ ohms}$$

$$I_{base\,69} = \frac{\text{kVA}_{base}}{\sqrt{3}\text{kV}_{base}} = \frac{150{,}000}{\sqrt{3} \times 69} = 1{,}250 \text{ amps}$$

$$I_{base\,12} = \frac{\text{kVA}_{base}}{\sqrt{3}\text{kV}_{base}} = \frac{150{,}000}{\sqrt{3} \times 12} = 7{,}230 \text{ amps}$$

$$Z_{line\,p.u.} = \frac{Z_{rated\,ohms}}{Z_{base}} = j\frac{3.87}{31.8} = j0.121 \text{ p.u.}$$

$$Z_{trans\,p.u.} = Z_{rated\,p.u.} \frac{\text{kVA}_{base}}{\text{kVA}_{rated}} = j0.075\frac{150{,}000}{15{,}000} = j0.750 \text{ p.u.}$$

$$Z_{utility} = 1.0\frac{\text{kVA}_{base}}{\text{kVA}_{sh.ckt.}} = j \times 1.0\frac{150{,}000}{1{,}500{,}000} = j0.100 \text{ p.u.}$$

$$Z_{motor\,d1'} = j0.200\frac{150{,}000}{15{,}000} = j2.00 \text{ p.u.}$$

a. Three-phase fault (Exhibit 3.4b)

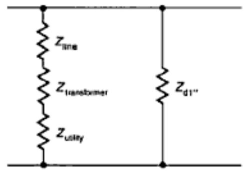


Exhibit 3.4b

$$Z_A = Z_{line} + Z_{transf} + Z_{ut} = j0.121 + j0.75 + j0.100 = j0.971$$

$$Z_B = Z_{d1'} = j2.00$$

$$Z_{eq} = \frac{j0.971 \times j2.00}{j0.971 + j0.200} = j0.655$$

$$I_{fault} = \frac{E}{Z_{eq}} = \frac{1.0}{j0.655} = -j1.525 \text{ amps}$$

$$I_{fault} \text{ 3 phase at 12 kV} = I_{fault\ p.u.} \times I_{base\ 12}$$
$$= j1.525 \times 7{,}230 = -j11{,}000 \text{ amps}$$

b. Single-phase fault

The positive sequence impedance diagram is shown in Exhibit 3.4c.

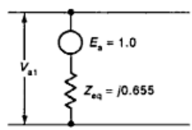


Exhibit 3.4c

The negative sequence impedance diagram is shown in Exhibit 3.4d.

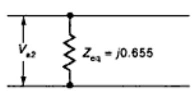


Exhibit 3.4d

The zero sequence impedance diagram is shown in Exhibit 3.4e.

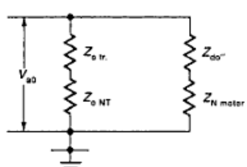


Exhibit 3.4e

69 kV system zero sequence fault currents are isolated from fault "F" by ΔY transformer.

$$Z_{0\,tr} = j0.750$$

$$Z_{motor\,d0'} = \frac{1}{2} Z_{motor\,d''} (\text{assumed}) = j1.000$$

$$Z_{0\,NT} = 3Z_{N\,motor} = 0 \ldots \text{directly connected neutrals}$$

$$Z_{0\,eq} = \frac{j0.750 \times j1.000}{j0.750 + j1.000} = j0.428$$

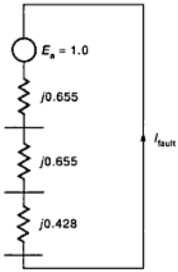


Exhibit 3.4f

The sequence network is shown in Exhibit 3.4f.

$$I_{a1} = I_{a2} = I_{a0} = \frac{E_a}{Z_1 + Z_2 + Z_0 + 3Z_N + 3Z_{fault}}$$

where $Z_N = 0$ and $Z_{fault} = 0$

thus:

$$I_{a1} = I_{a2} = I_{a0} = \frac{1.0}{j(0.655 + 0.655 + 0.428)} = -j0.580$$

$$I_{fault} = I_{fault\ p.u.} \times I_{base\ at\ 12\ KV}$$

$$= -j1.740 \times 7{,}230 = -j12{,}500 \text{ amps.}$$

3.5 The voltage components are as follows:

$$V_1 = \frac{1}{3}(V_a + aV_b + a^2V_c)$$

$$= \frac{1}{3}[200 + (-0.5 + j0.866)(-j200) + (-0.5 - j0.866)(-100 + j200)]$$

$$= \frac{1}{3}(200 + j100 + 173.2 + 50 - j100 + j86.6 + 173.2)$$

$$= \frac{1}{3}(596.4 + j86.6) = 198.8 + j28.86$$

$$V_2 = \frac{1}{3}(V_a + a^2V_b + aV_c)$$

$$= \frac{1}{3}[200 + (-0.5 - j0.866)(-j200) + (-0.5 + j0.866)(-100 + j200)]$$

$$= \frac{1}{3} = (200 + j100 - 173.2 + 50 - j100 - j86.6 - 173.2)$$

$$= (-96.4 - j86.6) = -32.13 - j28.86$$

$$V_0 = \frac{1}{3}(V_a + V_b + V_c) = \frac{1}{3}(200 + j0 - j200 - 100 + j200)$$
$$= \frac{1}{3}(100) = 33.33$$
$$V_{a1} = 198.8 + j28.86 = (20 + j10)I_{a1}$$
$$V_{a2} = -32.13 - j28.86 = (20 + j10)I_{a2}$$
$$V_{a0} = 33.33 = \infty . I_{a0},$$

since neutral is not connected, *i.e.*, there is no connection between the star point and the supply neutral.

$$I_{a1} = \frac{198.8 + j28.86}{20 + j10}$$
$$I_{a2} = \frac{-32.13 - j28.86}{20 + j10}$$
$$I_{a0} = \frac{33.33}{\infty} = 0$$
$$I_a = I_{a1} + I_{a2} + I_{a0} = \frac{1}{20 + j10}(198.8 + j28.86 - 32.13 - j28.86)$$
$$= \frac{1}{20 + j10}(166.67) = \frac{20 - j10}{500} \times 166.67$$
$$= I_a = 6.67 - j3.33 \text{ amps in line a.}$$

To obtain the other (b,c) line currents

$$I_b = a^2 I_{a1} + aI_{a2} + I_{a0}$$
$$= [(-.5 - j0.866)(198.8 + j28.86)$$
$$+ (-.5 + j0.866)(-32.13 - j28.86)]\frac{1}{20 + j10}$$
$$= (-99.4 - j172.16 - j14.43 + 24.99 + 16.65 - j27.82$$
$$+ j14.43 + 24.99)\frac{20 - j10}{500}$$
$$= (-33 - j200)\frac{20 - j10}{500} = -5.33 - j7.33 \text{ amps in line b.}$$

$$I_c = aI_{a1} + a^2 I_{a2} + I_{a0}$$
$$= [(-0.5 + j0.866)(198.8 + j28.86)$$
$$+ (-0.5 - j0.866)(-32.13 - j28.86)]\frac{1}{20 + j10}$$
$$= (-99.4 + j172.16 + j14.43 - 24.99 + 16.65$$
$$+ j27.82 - j14.43 - 24.99)\frac{20 - j10}{500}$$
$$= (-133.33 + j200)\frac{20 - j10}{500} = -1.33 + j10.67 \text{ amps in line c.}$$

as a check: $0 = I_a + I_b + I_c$ or

$$0 = 6.67 - j3.33 - 5.33 - j7.33 - 1.33 + j10.67.$$

3.6 a. The load as seen at the left of the last transformer is

$$Z = 4^2 \times 1 + j4^2 \times 1 = 16 + j16\ \Omega.$$

The impedance of the load and the transmission line as seen on the right side of the first transformer is

$$Z = (1 + 16) + j(4 + 16) = 17 + j20.$$

The impedance of the load and transmission line as seen by the generator (on the left side of the first transformer) is

$$Z = (1/2)^2 \times 17 + j(1/2)^2 \times 20 = 4.25 + j5.0 = 6.56\angle 49.6^\circ\ \Omega.$$

(The magnitude of the current through the generator is

$$I = V/Z = 240/6.56 = 36.6\text{ A.})$$

b. Of the several solutions possible, one method is to define various base quantities along the system as follows:

a. At the generator,

$$\begin{aligned} V_{B-g} &= 240\text{ V} = 1\text{ pu} \\ kVA_{B-g} &= 10\text{ kVA} = 1\text{ pu} \\ I_{B-g} &= 10{,}000/240 = 41.66\text{ pu} \\ Z_{B-g} &= 240/41.66 = 5.761\,\Omega = 1\text{ pu.} \end{aligned}$$

b. For the transmission line,

$$\begin{aligned} V_{B-tl} &= 240(2) = 480\text{ V} = 1\text{ pu} \\ kVA_{B-tl} &= 10\text{ kVA} = 1\text{ pu} \\ I_{B-tl} &= 10{,}000/480 = 20.83\text{ A} = 1\text{ pu} \\ Z_{B-tl} &= 480/20.83 = 23.04 = 1\text{ pu} \\ [Z_{tl} &= (1 + j4)/23.04) = 0.0434 + j0.1736\text{ pu}]. \end{aligned}$$

c. At the load,

$$\begin{aligned} V_{B-L} &= 480/4 = 120\text{ V} = 1\text{ pu} \\ kVA_{B-L} &= 10\text{ kVA} = 1\text{ pu} \\ I_{B-L} &= 10{,}000/120 = 83.33\text{ A} = 1\text{ pu} \\ Z_{B-L} &= 120/83.33 = 1.44 = 1\text{ pu} \\ [Z_L &= (1 + j1)/1.44 = 0.6944 + j0.6944\text{ pu}]. \end{aligned}$$

The total impedance (as seen to the left of the first transformer) is,

$$\begin{aligned} Z_{Tot} &= Z_{tl} + Z_L = 0.0434 + j0.1736 + 0.6944 + j0.6944 \\ &= 0.7378 + j0.868 = 1.139\angle 49.6^\circ\text{ pu} \end{aligned}$$

which is equivalent to

$$= 1.139 \times 5.76\angle 49.6^\circ = 6.56\ \Omega.$$

(The magnitude of the per unit current is

$$I = 1/1.139 = 0.878\text{ pu},$$

and the actual current is

$$I = 0.88 \times 41.66 = 36.6\text{ A.})$$

c. If the load is 2 + j2 then, by using u values, only Z_L needs to be recomputed.

$$Z_L = (2 + j2)/1.44 = 1.389 + j1.389 \text{ pu}$$
$$Z_{Tot} = Z_{t1} + Z_L = 0.0434 + j0.1736 + 1.389 + j1.389$$
$$= 1.432 + j1.563 \text{ pu}$$

The actual total impedance is

$$Z = 1.432 \times 5.761 + j1.563 \times 5.761 = 8.25 + j9.00 = 12.2\angle 47.5°\Omega.$$

3.7 a. Power input to motors

$$\frac{1000}{0.9} = 1111 \text{ hp} = 830 \text{ kW}$$

Generator capacity

$$\frac{830 \text{ kW}}{0.8} = 1036 \text{ kVA}$$

Current requirement

$$\frac{1036 \times 1000}{\sqrt{3} \times 2200} = 272 \text{ A}$$

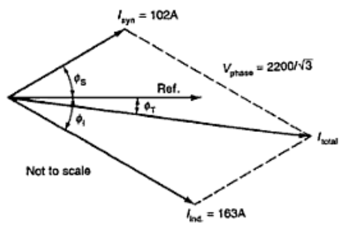


Exhibit 3.7a

b. Power requirements
Induct. motor input

$$\frac{600\text{hp}}{0.9} \times \frac{746}{1000} = 497 \text{ kW}$$

Induct. motor current

$$\frac{497 \times 1000}{\sqrt{3} \times 2200 \times 0.8} = 163 \text{ A}$$

Synch. motor input

$$\frac{400\text{ hp}}{0.9} \times \frac{746}{1000} = 332 \text{ kW}$$

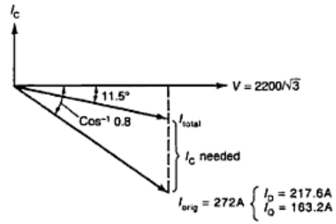


Exhibit 3.7b

Synch. motor current

$$\frac{332\times 1000}{\sqrt{3}\times 2200\times 0.85}=102\text{ A}$$

$$\begin{aligned}I_{\text{Total}} &= \sqrt{(I_I\cos\Phi_I+I_S\cos\Phi_S)^2+(I_I\sin\Phi_I+I_S\sin\Phi_S)^2}\\ &= \sqrt{(163\times 0.8+102\times 0.85)^2+(163\times 0.6-102\times 0.53)^2}\\ &= 221\angle 11.5°\text{A(lagging)}\end{aligned}$$

c.
$$I_{\text{Total}}=\frac{I_D}{\cos 11.5°}=\frac{217.6}{0.98}=222.06$$

$$I_{\text{Total}_Q}=222.06\sin 11.5°=44.3\text{ A}$$

$$\therefore\ I_C=I_{0_Q}-I_{\text{Total}_Q}=163.2-44.3=118.9\text{ A}$$

$$\therefore\ \text{kVAR}=\frac{118.9\times 2200}{1000\sqrt{3}}=151\text{ kVAR/phase}$$

3.8 Initial power factor angle = $\cos^{-1}0.85 = 31.79°$
Initial reactive power $Q = P \times \tan\theta = 20 \times \tan 31.79° = 12.4$ Mvar
New power factor angle $\theta' = \cos^{-1} 0.95 = 18.19°$
New reactive power $Q' = P \times \tan\theta' = 20 \times \tan 18.19° = 6.57$ Mvar
$Q_{\text{caps}} = 12.4 - 6.57 = 5.83$ Mvar

3.9

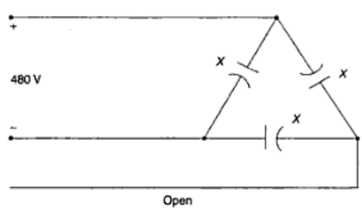


Exhibit 3.9

The normal delta current = $200/\sqrt{3} = 115.5$ A
The capacitive reactance per phase X = 480/115.5 = 4.16 Ω

With one phase open 480 V is across a reactance of X in parallel with 2X

$$X_{eq} = \frac{X2X}{X+2X} = 2/3 \times 4.16 = 2.77\Omega$$
$$I = 480/2.77 = 173 \text{ A}$$

3.10
$$\theta_{IM} = \cos^{-1}0.75 = 41.41°$$
$$Q_{IM} = 100 \times \sin 41.41° = 66.14 \text{ kvar}$$

To make the power factor unity $Q_{SM} = -Q_{IM} = -66.14$ kvar

$$\theta_{SM} = \tan^{-1}(-66.14/150) = -23.8°$$

Power factor of the synchronous motor = cos 23.8° = 0.915 leading

3.11 Convert the delta load to an equivalent wye.

$$Z_Y = (60 + j30)/3 = 20 + j10\ \Omega$$

Equivalent circuit (Exhibit 3.11)

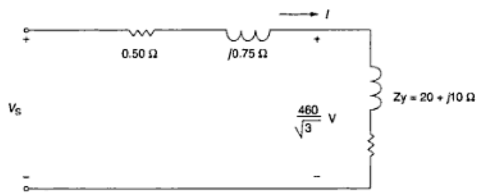


Exhibit 3.11

$$I = \frac{460/\sqrt{3}}{20+j10} = 11.9\angle{-26.6°} \text{ A}$$

$$V_s = 460/\sqrt{3} + 11.9\angle{-26.6°} \times (0.50 + j0.75) = 275\angle{1.11°} \text{ V.}$$

The line–line sending end voltage = $\sqrt{3}\times 275 = 476$ V.

3.12 The source impedance = $\dfrac{480/\sqrt{3}}{40{,}000} = 0.00693\ \Omega$
Assume the source impedance is inductive.
From NEC Table 9 the conductor impedance is

$$Z = (0.062 + j0.041) \times 200/1000 = 0.0124 + j0.0082\ \Omega$$

$$I_{fault} = \frac{480/\sqrt{3}}{j0.00693+0.0124+j0.0082} = 14{,}167\angle{-50.7°} \text{ A.}$$

3.13
$$V_S = AV_R + BI_R$$

With $I_R = 0$ (open circuit)

$$V_R = V_S/A = 500/0.90 = 556 \text{ kV}$$
$$I_S = CV_R + DI_R$$

Again with $I_R = 0$

$$I_S = CV_R = 0.002 \times 556/\sqrt{3} = 0.642 \text{ kA} \quad \text{or} \quad 642 \text{ A}$$

3.14 To take into account the motor power factor and efficiency it is reasonable to assume that the motor rated kVA = motor rated horsepower.

Using a 250 kVA base $S_{motor} = 100/250 = 0.40$ p.u.

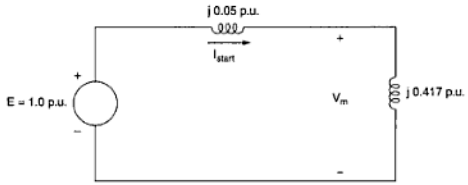


Exhibit 3.14

At rated voltage $I_{motor} = 0.40$ p.u.

$$I_{start} = 6 \times 0.40 = 2.4 \text{ p.u.}$$
$$X_{locked\ rotor} = 1/2.4 = 0.417 \text{ p.u.}$$
$$V_m = \frac{j0.417}{j(0.417 + 0.05)} = 0.89 \text{ p.u.}$$
$$\text{Voltage drop} = 1 - 0.89 = 0.11 \text{ p.u.} \quad \text{or} \quad 11\%$$

3.15 From a table of conductor characteristics the geometric mean radius = 0.0217 ft.

$$D_{eq} = \sqrt[3]{4x3x7} = 4.38 \text{ ft}$$
$$X_L = 0.1213 \ln (4.38/0.0217) = 0.644 \ \Omega/\text{mile}$$

3.16 The maximum fault contribution from the transformer is 250 – 30 = 220 MVA.

Using a 12 MVA base, the contribution = 220/12 = 18.33 p.u.

At rated voltage the transformer impedance cannot be less than 1/18.33 = 0.0546 p.u.

To account for manufacturing tolerance $X_{spec} = 0.0546 + 0.075 \times X_{spec}$

$$X_{spec} = 0.059 \text{ p.u.} \quad \text{or} \quad 5.9\%$$

3.17 Cross-section area of the phase conductors is 2 × 350 = 700 kcmil. From NEC Table 250.66 the minimum size grounding conductor is 2/0 copper.

3.18 Load current = $50{,}000/\sqrt{3} \times 208 \times 0.9 = 154$ A
Because the load is continuous, ampacity is based on 125% of this.

$$I = 1.25 \times 154 = 193 \text{ A}$$

From NEC Table 310.16 select 3/0 copper THW with an ampacity of 200 A.
Select a 200 A overcurrent protection device.

3.19 From NEC Table 430.150 motor full load current is 240 A
For conductor sizing NEC requires 125% of this.

$$1.25 \times 240 = 300 \text{ A}$$

From NEC Table 310.16, select a 350 kcmil THW copper conductor with an ampacity of 310 A.

3.20 Primary tap setting = 0.95 × 13.2 = 12.54 kV
No load ratio = 12,540/480 = 26.125

For a primary voltage of 12.9 kV the no-load secondary voltage is 12,900/26.125 = 493.8 V.
At full load the secondary voltage is 493.8/1.05 = 470 V.

3.21

$$I_{3\text{ phase}} = E/Z_1 = 1/0.085 = 11.76 \text{ p.u.}$$
$$I_{\text{base}} = 50{,}000/\sqrt{3} \times 69 = 418.4 \text{ A}$$
$$I_{3\text{ phase}} = 418.4 \times 11.76 = 4920 \text{ A}$$

Because $Z_1 = Z_2$ the phase-phase fault current is 0.87 times the three phase value.

$$I_{\text{phase-phase}} = 0.87 \times 4920 = 4280 \text{ A}$$
$$I_{\text{phase-grd}} = 3E/(Z_1 + Z_2 + Z_0) = 3/(0.085 + 0.085 + 0.20) = 8.11 \text{ p.u.}$$
$$I_{\text{phase-grd}} = 418.4 \times 8.11 = 3393 \text{ A}$$

3.22 Assume the capacitor is wye connected.

$$X_C = \frac{277^2}{200{,}000/3} = 1.151\,\Omega$$

Equivalent circuit is shown in Exhibit 3.22.

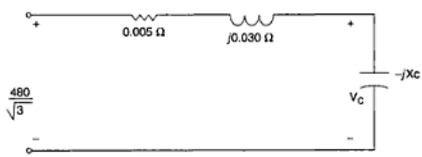


Exhibit 3.22

$$V_C = 277 \times \frac{-j1.151}{0.005 + j0.030 - j1.151} = 284.4\angle{-0.3°} \text{ V}$$

Voltage change = (284.4 − 277)/277 = 0.027 or 2.7%

CHAPTER 4

Electronics

OUTLINE

PROBLEMS

4.1 A single-stage FET power amplifier needs to be analyzed so that parameter values and specifications may be established. In addition, the FET circuit may have an interchangeable op-amp as a preamplifier for various input signal conditions.

An n-channel junction FET power amplifier circuit has the drain characteristic curves shown in Exhibit 4.1. The FET circuit eventually will be driven by an op-amp acting as preamplifier for various signal sources. The FET has

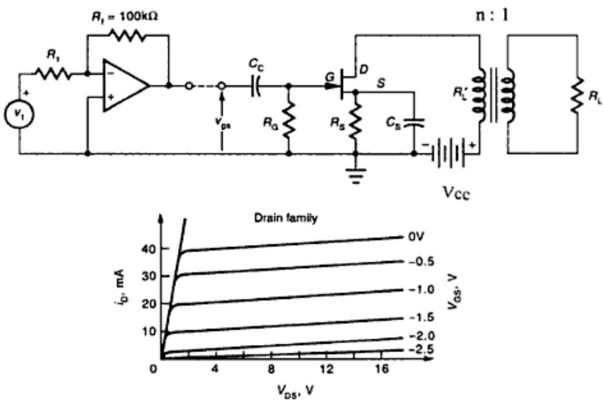


Exhibit 4.1

a maximum drain voltage upper limit of 16 volts. Assume the operating quiescent current, I_D, for the load line is 22 mA and any operation is at mid-band frequencies.

The following information is needed:

a. V_{DS} and R'_L for maximum possible output signal excursion.

b. The transformer turns ratio n for an $R_L = 8\ \Omega$ for the R'_L (as found in 1).

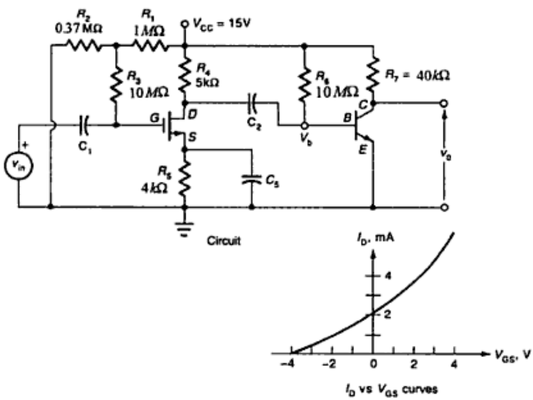
c. The approximate power output, graphically determined, when $v_{gs} = 0.5 \sin \omega t$ volts.

d. The percentage of second-harmonic distortion for i_D for the same signal as in (c).

e. With the op-amp connected and with a $V_1 = 35$ mV (rms), the value of R_1 needed to produce the signal in (c).

4.2 A transistor (BJT) amplifier is driven by a high input impedance MOSFET (p-chan, depletion mode) preamplifier. The tentative circuit design has already been proposed. However, a check of the circuit and performance specifications needs to be completed.

The component values for the circuit are given and some of the parameters of the MOSFET and BJT are known. These are as indicated in Exhibit 4.2a.



FET	BJT
$g_m = 2.5 \times 10^{-3}$ S(mho)	$r_\pi = 40$ kΩ
I_D VS V_{GS} curves (As shown.)	$\beta = 100$
	$V_{BE} = 0.7$ V

Exhibit 4.2a

This problem is an adaptation of Problem 3 as presented in James W. Morrison's *Principles and Practice of Electrical Engineers Examination P&P/E*, ARCO, 1977, pp. 113–117.

It will be necessary to determine operating points, input impedance, and gains. Specifically, the following items are needed:

a. The dc operating point, Q for the FET.

b. The dc operating point, Q for the BJT.

c. The small signal input impedance.

d. The voltage gain at mid-frequency.

4.3. Assume the BJT transistor shown in the circuit in Exhibit 4.3 has a $\beta = 100$ and that $V_{BE} = 0.7$ volts.

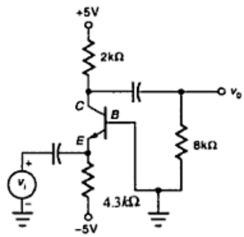


Exhibit 4.3

Find the following quantities:

a. I_C, I_E, and V_C.

b. g_m, r_π, and r_e.

c. Drawing of the small signal equivalent circuit using a T model for the BJT and calculation of the gain, v_o/v_i.

4.4 A two-stage transistor (BJT) amplifier uses constant current sources for the bias portions of the circuit; the first stage has a fixed constant current source while the second is adjustable. The component values for the circuit are as given and some of the parameters of the BJTs are known to be $|V_{BE}| = 0.65$ volts, $\beta = 100$ (since high accuracy is not expected, β is usually considered high), $V_T = 26$ mV, and all capacitors are considered large unless otherwise indicated. The circuit is as shown in Exhibit 4.4a.

The tentative circuit design has already been proposed. However, a recheck of the circuit and its performance specifications needs to be completed.

Assume the second current source (at V_4) has already been adjusted and tested to produce a constant 2.0 mA.

a. Determine the dc bias voltages at V_1.

b. Determine the dc bias voltages at V_3.

c. Determine the dc bias voltages at V_5.

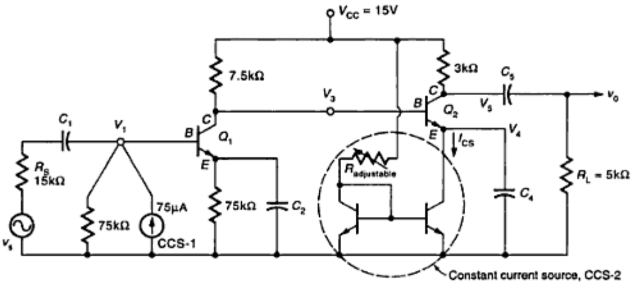


Exhibit 4.4a

d. Find the input resistance R_{in}.

e. Find the ac voltage gain from V_1 to V_5 and assume R_L is disconnected.

f. Determine the transconductance of the transistors $Q_{1,2}$.

g. Determine the output resistance at V_o (include R_L).

Now assume the simplified constant current source is made up of matched transistors with very high β's and is readjusted to give constant current source of 3.0 mA. Again, assume V_{BE} = 0.65 volts.

h. If the original setting of R_{adj} was R_x, what should the new setting be (in terms of R_x)?

i. Determine the numerical original value of $R_{adj} = R_x$ (assume V_{cc} = 15 volts).

j. Determine the dc bias voltages at V_4.

4.5 Two different MOSFETs are being considered for use in two different circuits (Exhibit 4.5). Both transistors have a V_t of 2 volts; however, each unit has a different K factor (conductivity parameter).

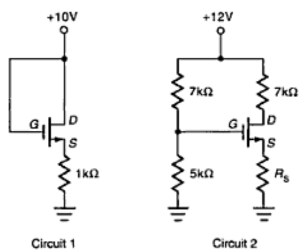


Exhibit 4.5

a. For circuit #1 the K factor is 0.25 mA/V^2 and you are to determine I_D.

b. For circuit #2 the K factor is 1.0 mA/V^2 and you are to determine R_S for a drain current of 1 mA.

4.6 A BJT transistor is being considered for use in a single-stage amplifier circuit. As part of the analysis, with only limited information on the device, you are asked to present an equivalent hybrid-π model that will "fit" into a previously designed circuit (Exhibit 4.6a). All that is known about the BJT is that β is 100, and that V_A is 100 volts (Early voltage).

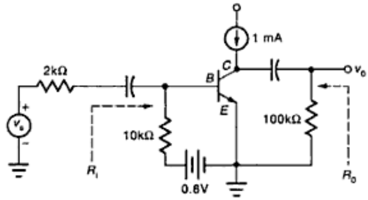


Exhibit 4.6a Previously designed amplifier circuit

a. Draw the small signal equivalent circuit using the hybrid-π model and calculate R_i and R_o.

b. Calculate the overall gain v_o/v_s.

4.7 A high gain op-amp having a unity gain bandwidth of f_t = 1 MHz is being considered in two different circuits as shown in Exhibit 4.7.

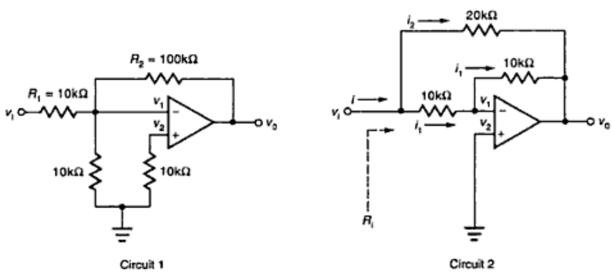


Exhibit 4.7

a. For circuit #1 you are to determine the voltage gain and the bandwidth.

b. For circuit #2 you are to determine the input resistance, R_i.

4.8 A two-stage BJT amplifier has already been designed and is shown in the circuit in Exhibit 4.8a.

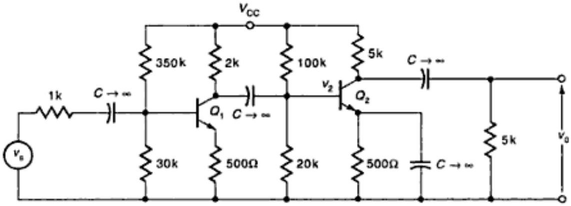


Exhibit 4.8a

The h-parameters associated with this design are given as follows:

$$h_{ie}\,(h_{11}) = 1\text{k}\,\Omega, \quad h_{fe}\,(h_{21}) = 50, \quad h_{re}\,(h_{12}) = 2 \times 10^{-4}$$
$$h_{oe}\,(h_{22}) = 20 \times 10^{-6}\ \text{mho}$$

Using an h-parameter model, you are to determine the overall mid-frequency gain, v_o/v_s. Any assumptions made should be clearly stated.

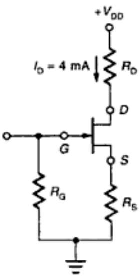


Exhibit 4.9a

4.9 The circuit shown in Exhibit 4.9a is an N-channel Junction Field-Effect Transistor with self-bias and a pinch-off voltage of –3 volts. At that value of pinch-off voltage, the current is 6 mA. The breakdown voltage for this transistor is 30 volts.

Design the circuit in Exhibit 4.9a so that the device will be biased at approximately 10V drain-to-source and have a channel current of approximately 4 mA.

4.10 A two-stage BJT has already been designed, but a complete analysis is needed for gain and losses.

Calculate the power gain of the two-stage amplifier shown in Exhibit 4.10a. Show the gain of the two individual stages, the interstage losses, and the prestage losses. The ground-based h parameters for the transistors used in both stages are

$h_{ib} = 50$ ohms; $h_{rb} = 5 \times 10^{-4}$; $h_{fb} = -0.97$ and $h_{ob} = 10 - 6$ mho.

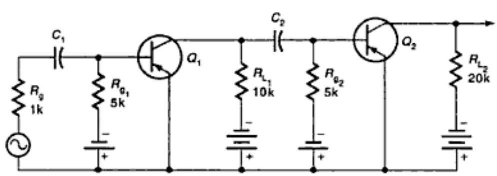


Exhibit 4.10a Assume reactance to be negligible

SOLUTIONS

4.1 a. $$V_{DS} = \frac{V_{D(\text{Upper})}}{2} = \frac{16}{2} = 8\text{ v}$$

$$R'_L \text{ (as referred to } n \text{ side of transformer)} = \frac{V_{DS(\text{Upper})} - V_{DS}}{I_D} = \frac{16-8}{22\text{ma}}$$
$$= 354\ \Omega$$

b. $$R'_L = n^2 R_L, \quad n^2 = \frac{R'_L}{R_L} = \frac{364}{8} = 45.5, \quad n = 6.7$$

c. A load line plot for a v_{gs} of 0.5V on each side of the quiescent point yields: $v_{ds(max)}$ and $v_{ds(min)}$ of 11.1 and 4.2 volts, and $i_{d(max)}$ and $i_{d(min)}$ of 31 and 12 mA

$$P_0 = \frac{\Delta v_{gs}}{2\sqrt{2}} \cdot \frac{\Delta i_d}{2\sqrt{2}} = \frac{(11-4.2)\times(31-12)\text{ma}}{8} = 16.4\text{ mW.}$$

d. $$\text{Percent distortion} = \left|\frac{\frac{(i_{d\max}+i_{d\min})}{2} - I_D}{i_{d\max} - i_{d\min}}\right| \times 100$$
$$= \left|\frac{\left(\frac{(31+12)}{2}\right)}{(31-12)}\right| \times 100 = 2.6\%$$

e. $V_{1(rms)} = 35$ mV $\cong 0.05$ (0-to-peak), $\qquad V_{gs} = 0.5$(0-to-peak)

$$\therefore \text{Op-Amp circuit gain is 10 and } \frac{R_f}{R_1} = \frac{100\times10^3}{R_1}, R_i = 10\text{ k}\Omega.$$

4.2 a. In finding the dc operating point Q for the MOSFET, the dc voltage on the gate, V_G is needed.

$$V_G = [R_2/(R_1 + R_2)]V_{CC} = 0.4/(1.1 + 0.4)]\ 15 = 4\text{ V.}$$

Then, by using the I_D vs V_{GS} characteristic curve and drawing a dc load-line on it, as shown in Exhibit 4.26 the voltage from gate to source, V_{GS}, may be found. Here $V_{GS} = -1$V. (The load-line is a plot of $V_G = V_{GS} + I_D R_S$.)

The summing voltages around the loop yield the voltage, V_S, as

$$V_S = -V_{GS} + V_G = -(-1) + 4 = 5\text{ V.}$$

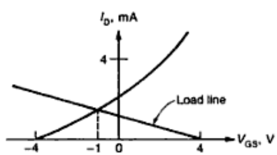


Exhibit 4.2b

Therefore the sink current, I_S, is

$$I_S = V_S/R_5 = 5/(4 \times 10^3) = 1 \text{ mA},$$

and the drain to source voltage, V_{DS}, is

$$V_{DS} = V_{CC} - I_D(R_4 + R_5) = 3.75 \text{ V}.$$

b. In finding the operating point Q for the BJT, the base current first needs to be found.

$$I_B = (V_{CC} - V_{BE})/R_6 = (15 - 0.7)/10 \times 10^6 = 1.4\ \mu\text{A}.$$

Because the collector current, I_C is βi_b, then $I_B = 140\ \mu$A, and the collector to emitter voltage, V_{CE}, is

$$V_{CE} = V_{CC} - I_C R_7 = 15 - (140 \times 10^{-6})(140 \times 10^3) = 9.4 \text{ V}.$$

c. The small signal input impedance, R_{in}, is found almost by inspection, because the MOSFET G terminal draws negligible current,

$$R_{in} = (10) \times 10^6 = 10^7\ \Omega$$

d. The voltage gain of the entire circuit (at mid-band) may be determined by drawing the equivalent small signal circuit model (where $R_A = R_4/R_6$, and $g_{m2} = \beta/r_\pi$).

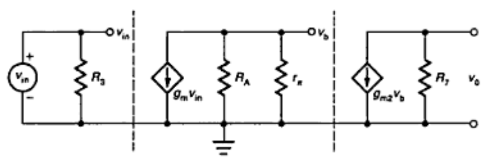


Exhibit 4.2c

Here, R_A is essentially R_4, which was 5 kΩ, and,

$$g_{m2} = 100/(40 \times 10^3) = 2.5 \times 10^{-3} \text{ S (mho)}.$$

The circuit model may be redrawn, making use of Thevenin's equivalent circuit as shown in Exhibit 4.2d.

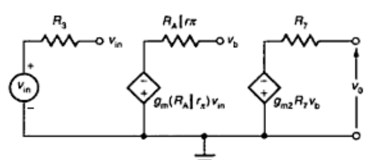


Exhibit 4.2d

The following stage parameters may then be found as:

$$v_o = -g_{m2}R - v_b = -(2.5 \times 10^{-3})(40 \times 10^3)v_b = -100v_b \text{ V}.$$
$$v_b = -g_m(R_A \| r_\pi)v_{in} = -2.5[5 \times 40)/(5+40)]v_{in}$$
$$= -2.5 \times 4.4v_{in} = 11.1v_{in} \text{ V}.$$
$$\therefore v_o = -100v_b = -(100)(-11.1v_{in}) \approx 1100\, v_{in} \text{ V}.$$

Therefore, the overall gain is approximately,

$$v_o/v_b = 1100.$$

4.3 a.

$$I_E = [-0.7 - (-5)]/4.3 = 1 \text{ mA},$$
$$I_C \cong I_E = 1 \text{ mA},$$
$$V_C = 5 - I_C/x2 = 3 \text{ V}.$$

b.

$$g_m = I_C/V_T = 1 \text{ mA}/0.025 \text{ V} \cong 40 \text{ mA/V},$$
$$r_\pi = \beta/g_m \cong 100/40 = 2.5 \text{ k}\Omega$$
$$r_e = r_\pi/(\beta+1) \cong 2.5 \text{ k}\Omega/101 = 25\ \Omega.$$

c. The equivalent circuit is as shown in Exhibit 4.3b, and the values are then

$$v_o = -\alpha i_e(2\text{k} \| 8\text{k}),$$
$$i_e = -v_i/r_e,$$
$$v_o = +\alpha(v_i/_e)(2\text{k} \| 8\text{k}),$$
$$v_o/v_i = g_m(2\text{k} \| 8\text{k}) \cong 40 \times 1.6 = 64 \text{ V/V}.$$

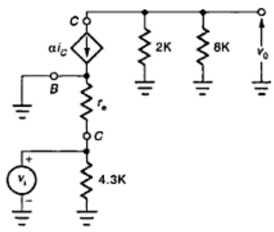


Exhibit 4.3b

4.4 a. $V_I = (75 \times 10^3)(75 \times 10^{-6}) = 5.63$ volts ($i_b \to 0$ for high β),

b. $I_C = I_E = 5.0\text{V}/7.5\text{k}\Omega = 0.667$ mA, $V_3 = 15 - 7.5 \times 0.667 = 10$ V.

(The 5V comes from $V_1 - V_{BE} - 5.63 - 0.65 \approx 5\text{V}$.)

c. $V_5 = 15 - (3 \times 10^3)(2 \times 10^{-3}) = 9$ V.

d. To find R_{in}, it is first necessary to find r_{e1} or g_{m1}:

$$r_{e1} = V_T/I_c = 26 \text{ mV}/0.667 \text{ mA} = 39\ \Omega$$
$$R_{in} = 75\text{k} \parallel r_\pi = 75\text{k}|(\beta + 1)r_{e1} = 75\text{k} \parallel 3.94\text{k} = 3.76\ \text{k}\Omega$$

e. To find the overall voltage gain, it will also be necessary to find either r_{e2} or g_{m2}.

$$r_{e2} = V_T/I_C = 26 \text{ mV}/2 \times 10^{-3} = 13\ \Omega$$
$$A = \{[-7.5\text{k} \parallel (\beta + 1)r_{e2}(\beta/(\beta + 1))]/r_{e1}\}\{[-3\text{k}/r_{e2}][\beta/(1 + \beta)]\}$$
$$= \{[-7.5 \text{ k} \parallel (101 \times 13)]/39\}(-3\text{k}/13) = 6470 \text{ V/V}.$$

f. Because the r_e's have already been found,

$$g_{m1} = 1/r_{e1} = 1/39 = 25.6 \text{ mA/V} \quad \text{and} \quad g_{m2} = 1/r_{e2} = 1/13 = 77 \text{ mA/V}.$$

g. $R_{out} = R_L \parallel 3\text{k} = 1.87\ \text{k}\Omega$.

For the adjustable constant current source, the transistors are matched and $I_{C1} = I_{C2}$, the I_{ref} is given by:

$$I_{ref} = I_{C1} + 2I_{C1}/\beta,$$
$$I_{C1} = I_{ref}/(1 + 2/\beta) = I_{C2},$$

for $\beta > 1$,

$$I_{C1} = I_{C1} = I_{ref}$$
$$= [(-V_{cc}) - V_E(\text{on})]/R_{adj}$$

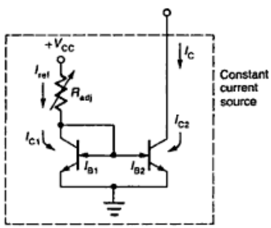


Exhibit 4.4b

h. Changing the constant current source from 2 mA to 3 mA makes $R_{adj\text{-new}} = (2/3)R_x$.

i. $I_C = 2 \text{ mA} = [-(-15) - 0.65]/R_x$, $R_x = 7.18\ \text{k}\Omega$.

j. $V_3 - V_{BE} = 10 - 0.65 = 9.35$ volts.

4.5 a. For circuit #1 the following relationships are obvious:

$$V_D = V_G = 10 \text{ V};\ V_{DS,sat} = V_{GS} - V_t;\ V_{DS} = V_{GS};\ V_{DS} > V_{DS,sat}.$$

Then,

$$I_D = K(V_{GS} - V_t)^2,$$
$$V_{GS} = V_G - V_S = 10 - 1\ k x I_D = 10 - I_D,$$
$$I_D = 0.25(10 - I_D - 2)^2,$$
$$4I_D = (8 - I_D)^2 = 64 + I_D^2 - 16I_D.$$
$$I_D = +10 +/- \sqrt{100 - 64} = 10 +/- 6.$$
$$I_D = 4 \text{ mA (the 16 mA is not acceptable)}.$$

b. For circuit #2 (with a K = 1 mA/V^2) the calculations for R_S for a drain current of 1 mA are as follows:

$$V_G = [12/(7+5)]5 = 5 \text{ V},$$
$$V_D = 12 - (7\ k\Omega)(1\ mA) = 5 \text{ V},$$
$$I_D = K(V_{GS} - V_T)^2 = 1 \text{ mA} = 1\ (V_{GS} - 2)^2,$$
$$V_{GS} = 3 \text{ V}, V_{GS} = 1 < V_t \text{ (not acceptable)},$$
$$3 = 5 - V_S \Rightarrow V_S = 2 \text{ V},$$
$$R_S = V_S/I_D = 2 \text{ V}/1 \text{ mA} = 2\ k\Omega.$$

4.6 a. The model, using the hybrid-π for the BJT, is shown in Exhibit 4.6b.

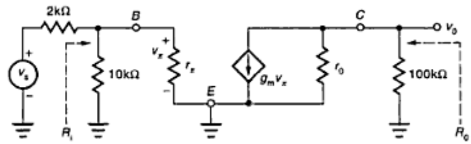


Exhibit 4.6b

R_i and R_o may be found by going through the following calculations:

$$I_C = 1 \text{ mA (constant current source)},$$
$$g_m = I_C/V_T = 1 \text{ mA}/0.0025 = 40 \text{ mA/V},$$
$$r_\pi = \beta/g_m = 100/(40 \text{ mA/V}) = 2.5\ k\Omega,$$
$$r_o = V_A/I_C = 100/1 \text{ mA} = 100\ k\Omega.$$

Then

$$r_i = 10\ k\|r_\pi = (10 \times 2.5)/(10 + 2.5) = 2\ k\Omega.$$

And for r_o set $v_s = 0$, then $v_\pi = 0$ and $g_m v_\pi = 0$, therefore $R_o = r_o \| 100k$,

$$R_o = (100 \times 100)/(100 + 100) = 50\ k\Omega.$$

b. The circuit gain may now be found as

$$v_o = -g_m v_\pi (r_o \| 100k),$$
$$v_\pi = [r_i/(r_i + 2k\Omega)]v_s = [2/(2 + 2)]v_s = (1/2)v_s,$$

then the overall gain is

$$v_o/v_s = -(1/2)g_m(r_o\|100k) = -(1/2)40(50) = -1000.$$

4.7 a. Because the difference in the input voltages v_1 and v_2 must be very small and the input current to the op-amp itself is negligible, then

$$v_1 = v_2 = 0,$$
$$v_o = -(R_2/R_1)V_1$$
$$v_o/v_1 = -(R_2/R_1 = -100/10 = -10.$$

For the bandwidth,

$$\omega_t = (1 + R_2/R_1)\omega'_b,$$
$$f'_b = f_{-3db} = f_t/)1 + R_2/R_1) = 10^6/(1 + 10) = 90.9 \text{ kHz}.$$

b. For circuit #2, the input resistance may be found as follows:

$$R_i = v_i/I,$$
$$v_1 = v_2 = 0,$$
$$i_1 = v_i/10\text{k},$$
$$v_o = v_1 - 10i_1 = 0 - 10\text{k}(v_i/10\text{k}) = -V_i,$$
$$i_2 = (v_1 - v_o)/20\text{k} = [v_1 - (-v_i)]/20\text{k} = v_i/10\text{k},$$
$$I = i_1 + i_2 = v_i/10\text{k} + v_i/10\text{k} = v_i/5\text{k},$$
$$R_i = v_i/(v_i/5\text{k}) = 5 \text{ k}\Omega.$$

4.8 Using the h parameter model will first find the voltage gain of stage 2, v_0/v_2 and the input impedance looking into Q_2. Then we can find the gain of stage 1 as loaded by stage 2.

For stage 2, the equivalent circuit in h parameter form is shown in Exhibit 4.8b.

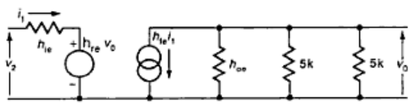


Exhibit 4.8b

Let

$$Y_L = \frac{1}{5\text{k} \| 5\text{k}} \quad Y_L = 400 \times 10^{-6} \text{ mhos}$$

(I) $$v_0 = \frac{-h_{fe}i_1}{h_{oe} + Y_L} = -119 \times 10^3 i_1$$

(II) $$v_2 = h_{ie}i_1 + h_{re}v_0$$

Then

$$v_2 = i_1 h_{ie} - \frac{i_1 h_{fe} h_{re}}{h_{22} + Y_L} = i_1\left[h_{ie} - \frac{h_{fe}h_{re}}{h_{22} + Y_L}\right]$$

$$z_{in_2} = h_{ie} - \frac{h_{fe}h_{re}}{h_{22} + Y_L} = 1000 - 119 \times 10^3 (2 \times 10^{-4}) = 976\,\Omega.$$

From (I) above, we have

$$i_1 = -\frac{v_0(h_{oe}+Y_L)}{h_{fe}} = -v_0(8.4\times10^{-6}).$$

Substituting into (II) gives

$$v_2 = h_{ie}\left[-\frac{v_0(h_{oe}+Y_L)}{h_{fe}}\right] + h_{re}v_0$$
$$= -(1k)(8.4\times10^{-6})v_0 - 2\times10^{-4}\,v_0$$
$$= -8.6\times10^{-3}v_0$$

$$\frac{v_0}{v_2} = -116.$$

Now we take an equivalent circuit for the input stage including the loading by the input to the second stage and the biasing networks.

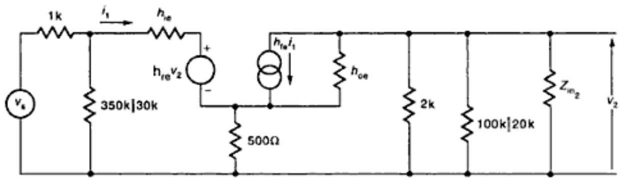


Exhibit 4.8c

We note the fact that the 2k collector load is in parallel with Z_{in2} (976Ω) and these are in parallel with the bias network for transistor #2.

Note that 100k/20k = 16.7k while 2k/976Ω = 656Ω.

Therefore assume bias network for stage #2 can be neglected.

The unbypassed emitter resistor in stage #1 presents a very high input impedance to the source.

$$(Z_{in} = h_{ie} + (h_{fe}+1)R_E).$$

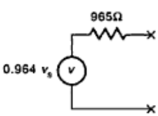


Exhibit 4.8d

However, the h_{oe} of the transistor is high compared to the 500Ω so we may assume that we can neglect h_{oe} as being high resistance [$1/h_{oe}$ = 50k] compared to either the load (656Ω) or the 500Ω R_E. Finally, we will incorporate the bias network for transistor #1 into the source by taking a Thevenin equivalent circuit.

By the previous discussion we will refer the R_E (500Ω) resistor to the input by

$$R'_E = (h_{fe}+1)\,R_E = 25.5\text{ k}\Omega.$$

Now our equivalent circuit looks like Exhibit 4.8e.

$$i_1 = \frac{0.964v_S - h_{re}v_2}{965 + h_{ie} + 25.5\text{k}} \quad v_2 = -h_{fe}i_1(656)$$

$$i_1 = \frac{-v_2}{(656)(50)} = -30.5\times10^{-6}v_2$$

$$-305\times10^{-6}v_2 = \frac{0.964v_S - 2\times10^{-4}v_2}{965 + 1\text{k} + 25.5\text{k}}$$

$$-838\times10^{-3}v_2 = 0.964v_S - 2\times10^{-4}v_2$$

$$-v_2(838\times10^{-3}) = 0.964\,v_S$$

$$\frac{v_2}{v_S} = \frac{-0.964}{838\times10^{-3}} = -1.15$$

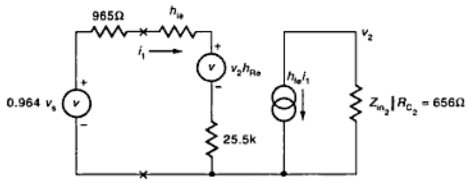


Exhibit 4.8e

Then the total gain

$$\frac{v_0}{v_S} = \frac{v_0}{v_2}\times\frac{v_2}{\gamma_S} = (-1.15)(-116) = 134.$$

The gain of this amplifier could be estimated reasonably as follows: Stage #1 has strong current-series feedback (the unbypassed emitter resistor). Thus its gain is very nearly $\frac{-R_{L_{eff}}}{R_{E_{eff}}}$ where $R_{L_{eff}}$ is the 2k collector load (in parallel with h_{ie} of Q_2) and the $R_{E_{eff}}$ is the R_E plus h_{ib}, h_{ib} is approximately $\frac{h_{ie}}{h_{fe}} = 20\Omega$.

Thus stage #1 has an approximate gain of

$$\text{Gain} = \frac{-667}{500+20} = -1.28.$$

For stage #2 the gain is approximately $\dfrac{-h_{fe}R_{L_{eff}}}{h_{11}}$ where

$R_{L_{eff}}$ is $5\text{k}\|5\text{k}$ or $2.5\text{ k}\Omega$.

Then

$$\text{Gain \#2} \approx \frac{-(2.5\text{k})(50)}{1\text{k}} = -125.$$

Finally, the voltage divider of the input stage and the source resistance

$$Z_{in}\,\#1 = h_{11} + (h_{fe}+1)\,R_E \approx 1\text{k} + (51)(5\text{k}) = 27\text{ k}.$$

The biasing network to stage #1 was found to be about 27k so that the combined parallel input impedance is about 13.5 k.

Then

$$v_i = \frac{v_S(13.5\text{k})}{13.5\text{k} + 1\text{k}} = 0.93 v_S.$$

Then the overall gain is estimated as follows:

$$G = \frac{v_o}{v_S} \approx 0.93(-125)(-1.28) = 149.$$

This is about 11% higher than found by the more accurate procedure; well within the tolerance of the known parameters.

4.9 For a junction FET

$$I_D \approx I_{\text{DSS}}\left(1 - \frac{|V_{\text{GS}}|}{|V_p|}\right)^2$$

given that

$$I_D = 4 \text{ mA}$$
$$I_{\text{DSS}} = 6 \text{ mA}$$
$$|V_P| = 3 \text{ volts}$$

$$\therefore 4 \cong 6\left(1 - \frac{|V_{\text{GS}}|}{3}\right)^2 \Rightarrow |V_{\text{GS}}| \cong 0.55 \text{ Volt.}$$

Gate leakage current I_{GSS} is typically of the order of nanoamperes. Choose $R_G = 1$ MΩ, so that the gate remains within millivolts of ground potential. Choose R_S to obtain the proper V_{GS}.

$$|V_{\text{GS}}| = 0.55 \text{ volts}, \quad I_D = 4 \text{ mA} \Rightarrow R_S = \frac{0.55}{4 \times 10^{-3}} = 140\Omega$$

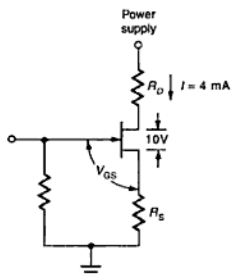


Exhibit 4.9b

Because the breakdown voltage is 30 volts, choose $V_{DD} = 24$ volts.

$$V_{DS} = V_{DD} - I_D(R_D + R_S)$$
$$10 = 24 - 4 \times 10^{-3}(R_D + R_S)$$

$$R_D + R_S = \frac{14}{4 \times 10^{-3}} = 3500\Omega$$
$$\therefore R_S = 150\ \Omega \quad \text{and} \quad R_D = 3.3\text{k}\Omega.$$

If 10% resistors are used, the available values are

$$R_S = 150\ \Omega \quad \text{and} \quad R_D = 3.3\text{k}\Omega.$$

4.10 Solution is based on the following assumptions:

1. Mid-band frequency—reactance of capacitors is negligible.
2. Power gain defined as power delivered to load divided by power delivered from source (this is power delivered to R_{L2} divided by power delivered of the voltage node at the junction of C_1 and R_{g1}).
3. Power losses are defined as

a. Pre-first stage—power lost in bias resistor R_{g1}.

b. Inter-stage—power lost in R_{L1} and R_{g2}.

c. Power losses—at signal frequency only, dc bias losses not considered.

Parameters are given in common base configuration; because transistors are operated in common emitter orientation, the parameters must be converted to common emitter form. Also, gain and impedance equations must either be derived or found in a transistor handbook.

Handbook conversion tables (see Table 4.1 in *Electrical Engineering: License Review)* give the following relationships:

$$h_{ie} = \frac{h_{ib}}{1 + h_{fb}} = 1670\Omega \quad h_{re} = \frac{h_{ib}h_{ob}}{1 + h_{fd}} = 11.7 \times 10^{-4}$$
$$h_{fe} = \frac{-h_{fd}}{1 + h_{fb}} = 32 \quad h_{oe} = \frac{h_{ob}}{1 + h_{fd}} = 33 \times 10^{-5}\text{mho}$$

and

$$\text{Eq. I.} \quad Z_{in} \cong \frac{v_1}{i_1} = \frac{h_{ie}h_{oe} - h_{fe}h_{re} + h_{ie}G_L}{h_{oe} + G_L} = h_{ie} - \frac{h_{fe}h_{re}}{h_{oe} + G_L}$$

$$\text{Eq. II.} \quad A_V \cong \frac{v_2}{v_1} = \frac{h_{fe}R_L}{h_{ie} + R_L(h_{ie}h_{oe} - h_{fe}h_{re})} = \frac{-h_{fe}}{h_{ie}G_L + (h_{ie}h_{oe} - h_{fe}h_{re})}.$$

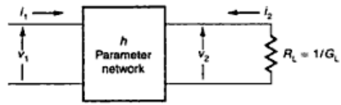


Exhibit 4.10b

The gain of each stage may be calculated by breaking the circuit as shown in Exhibit 4.10c.

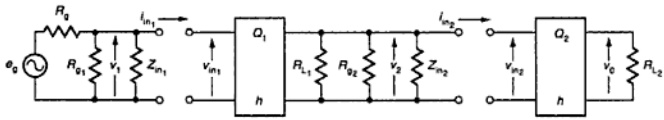


Exhibit 4.10c

Definitions:

v_1 = voltage at base of $Q_1 v_0$ = voltage across load R_{L_2}

$v_1 = v_{in_1}$

$$v_1 = e_g \frac{(R_{g1} \text{ in parallel with } Z_{in_1})}{R_{g1} + (R_{g1} \text{ in parallel with } Z_{in_1})}$$

v_2 = voltage at collector of Q_1

$v_2 = v_{in_2}$

Calculation of Stage Q_2 (Eq. II)

$$\frac{v_0}{v_{in_2}} = A_V = \frac{-h_{fe}}{h_{ie}G_L + (h_{ie}h_{oe} - h_{fe}h_{re})}$$

$$= \frac{-32}{(1670)(0.5\times10^{-4}) + (1670)(3.3\times10^{-5}) - (11.7\times10^{-4})(32)} = -320$$

To calculate gain of first stage, the impedance Z_{in_2} must first be obtained. This appears in parallel with R_{L_1} and R_{g_2}, all of which appears as a parallel load for stage Q_1. Then Z_{in_2} (from Eq. I) is given as

$$z_{in_2} = h_{ie} - \frac{h_{re}h_{fe}}{h_{oe} + G_L} = 1670 - 450 = 1220\,\Omega$$

Then the effective load for Q_1 is

$$G'_{L_1} = \frac{1}{R'_{L_1}} = \frac{1}{R_{g_2}} + \frac{1}{R_{L_1}} + \frac{1}{Z_{\text{in}_2}} = \frac{1}{5\text{K}} + \frac{1}{10\text{K}} + \frac{1}{1.22\text{K}} = \frac{1}{895}$$
$$= 1.12 \times 10^{-3} \text{mho}$$
$$R'_{L_1} = 895 \text{ ohm.}$$

Then (from Eq. II),

$$\frac{v_2}{v_{\text{in}_1}} = A_V = \frac{-h_{fe}}{h_{ie}G'_L + (h_{ie}h_{oe}h_{fe})}$$
$$= \frac{-32}{(1670)(1.12\times10^{-3}) + (1670)(3.3\times10^{-5}) - (11.7\times10^{-4})(32)}$$
$$= -16.9.$$

The total voltage gain from v_1 to v_0 is

$$\frac{v_0}{v_1} = \left(\frac{v_{\text{in}_2}}{v_1}\right)\left(\frac{v_0}{v_{\text{in}_2}}\right) = (-320)(-16.9) = 5400.$$

To find the power delivered by the source, Z_{in_1} must be calculated (Eq. I).

$$z_{\text{in}_1} = \frac{V_{\text{in}}}{i_{\text{in}_1}} = h_{ie} - \frac{h_{re}h_{fe}}{h_{oe} + G_{L_1}}$$
$$= 1670 - \frac{(11.7\times10^{-4})(32)}{(3.3\times10^{-5}) + (1.12\times10^{-3})} = 1670 - 30 = 1640\,\Omega$$

Then the power delivered by the generator is

$$P_{\text{in}} = \frac{v_1^2}{\text{Parallel combination of } R_{g1} \text{ and } Z_{\text{in}_1}} = \frac{v_1^2}{\frac{(5\times10^3)(1640)}{(5\times10^3)+1640}} = \frac{v_1^2}{1.25\times10^3}$$

and the power delivered to the load is

$$P_0 = \frac{v_0^2}{R_{L_2}} = \frac{v_0^2}{20\times10^3}.$$

Then the power gain is

$$G = \frac{P_0}{P_{\text{in}}} = \frac{\frac{v_0^2}{20\times10^3}}{\frac{v_1^2}{1.25\times10^3}} = 0.0625\left(\frac{v_0}{v_1}\right)^2 = 0.0625(5400)^2$$
$$= 1.82\times10^6$$
$$G_{db} = 10\ \log G = 62.6 \text{ db.}$$

Now consider the inter-stage power loss (loss in parallel combination of R_{L_1} and R_{g2}):

$$\frac{(R_{L_1})(R_{g_2})}{R_{L_1}+R_{g_2}} = \frac{(10\times10^3)(10\times10^3)}{(10\times10^3)+(5\times10^3)} = 3.33\times10^3\Omega$$

$$P_{\text{I.L.}} = \frac{v_2^2}{3.33\times10^3} = \frac{(16.9v_1)^2}{3.33\times10^3}$$

The total loss is then

$$P_{\text{Losses}} = \frac{v_1^2}{5\times10^3} + \frac{(16.9v_1)^2}{3.33\times10^3}$$

but $v_1 = 0.556\ e_g$

$$P_{\text{Losses}} = \frac{(0.556e_g)^2}{5\times10^3} + \frac{(16.9(0.556e_g))^2}{3.33\times10^3}$$
$$= 27.7\times10^{-3}e_g^2 \text{ watts.}$$

RECOMMENDED REFERENCES

Carlson and Gisser, *Electrical Engineering Concepts and Applications*, Addison-Wesley, 1990, p. 286.

Savant, Roden, and Carpenter, *Electronic Circuit Design*, The Benjamin/Cummings Publishing Company, 1987.

Sedra and Smith, *Microelectronic Circuits*, 2nd edition, Holt, Rinehart and Winston, 1987, pp. 350–51.

CHAPTER 5

Control Systems

OUTLINE

PROBLEMS

5.1 The following system has already been designed; however, the performance parameters need to be specified.

Determine whether the following system is stable and predict the closed loop pole location for the system for $K = 4$. Also, find the system error.

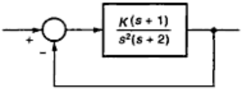


Exhibit 5.1a

5.2 The open loop transfer function for a control system is approximated by

$$G(s) = \frac{C(s)}{E(s)} = \frac{K(s-3)}{(s+0.5)(s+7)}.$$

It is desired to make the output signal (C) correspond as nearly as possible to some input signal, (R), in steady state, at the same time keeping the system stable.

a. Sketch a block diagram for a feedback control system to accomplish the given objective. Carefully label the summation polarity of all signals coming into the feedback junction summing point. (Note that the given transfer function has peculiar properties.)

b. Select a value of K which assures system stability and at the same time brings the ratio C/R in steady state as close to +1.0 as possible. (Note that the properties of $G(s)$ are such that it is advisable to make a very careful check on the requirements for closed-loop system stability.)

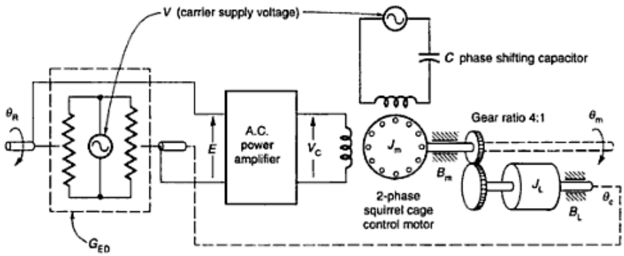


Exhibit 5.6a

a. Determine the total effective polar moment of inertia, J_{eff}, of the motor-load combination (as referred to the motor shaft when connected through the 4:1 ($a = 4$) ideal gear train.

b. Determine the transfer function of the motor-load combination, G_{M-L}. The transfer function (here, assume the power amplifier has a gain of unity, then the error voltage, E, equals amplifier control voltage, V_{cont}) is given as

$$G_{M-L} = \Theta_M/V_{cont} = \Theta_m/E = K/[s(\tau s + 1)].$$

c. Show the system block diagram (ready for simulation) with the numerical values for the transfer function(s).

d. For a unit step input of one radian applied to Θ_R and for a different power amplifier gain (other than unity), the error voltage as viewed on a strip-chart recorder is shown in Exhibit 5.6b.

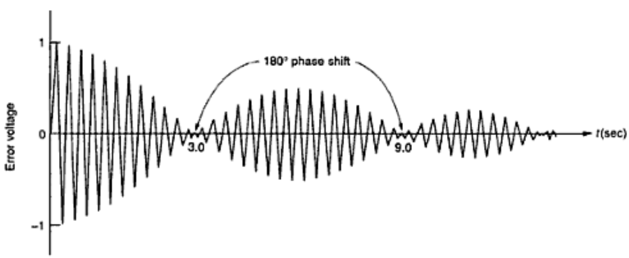


Exhibit 5.6b

Determine approximate percent overshoot and time-to-first peak for the closed loop system.

e. From your calculations for part (d) do you expect the gain was greater or less than unity? Why?

For problems 5.7 through 5.16, the positional movement of a robot arm has a theoretical open loop transfer function of

$$G(s) = C(s)/E(s) = K(s + 4)/(s(s + 1)(s + 2)).$$

5.7 If an optical sensor is used to monitor the arm position such that a closed loop system is achieved, what is the closest gain, K, that would just cause system instability (assume the electronic optical sensor unit is equivalent to unity feedback)?

a. 0 b. 2.0
c. 4.0 d. 6.0

5.8 In open loop the arm transfer function has a frequency response such that the phase shift will be −180° at a frequency nearest to what value (in rad/sec)?

a. 0 b. Infinity
c. 3.0 d. 5.0

5.9 In closed loop (with the unity feedback sensor connected) the percentage overshoot of the output is to be near 16% for a step input. What would be the approximate damping ratio, zeta, (if the system may be approximated by a second ordered one)?

a. Overdamped (nonexistent) b. 0.5
c. 0.7 d. 1.0

5.10 In closed loop (with the unity feedback sensor connected) and with an increased gain setting, it is found that the system will just break into oscillation. What is the closest natural frequency (in rad/sec) of this oscillation?

a. Eventually reaches infinity b. 1.0
c. 3.0 d. 6.0

5.11 For the system in closed loop (through the optical sensor, for unity feedback) the gain has been set such that the damping ratio (zeta) is 0.5. What percent overshoot may be expected for a step input? (Assume the system is approximated by a second-ordered one.)

a. 16% b. 25%
c. 37% d. 50%

5.12 For the system in closed loop (unity feedback) to have a damped natural frequency of 1.0 rad/sec, what is the closest gain setting for K?

a. 6.0 b. 4.0
c. 2.0 d. 0.5

5.13 The arm (in closed loop) is to follow a moving object with a slew rate (ramp) of 2.0 rad/sec. What is the closest steady state error of the arm and the object for a system gain $K = 4.0$?

a. Zero b. 0.25
c. 4.0 d. 10.0

determine the percent overshoot (to a unit step input) the system would have for this same value of K.

c. If, for the complete system, a compensator consisting of a pure integration ($1/s$) is inserted in the forward path (either just ahead or just following the subsystem), what value of K would cause the system to be marginally stable? What is the natural frequency of this marginally stable system?

5.18 The system shown in Exhibit 5.18a is to be stabilized by the addition of tachometer feedback, K_t.

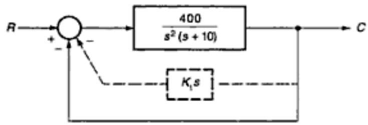


Exhibit 5.18a

a. Find the minimum value of K_t such that the system will just be stable.

b. If the value of K_t (found in Part (a)) were increased by a factor of 1.25 [*i.e.*, $1.25 \times K_t$ min], determine the approximate step response characteristics. (Hint: one method of an approximate solution is to use only a reasonably accurate root-locus sketch and then approximate with "standardized" 2nd order curves.)

SOLUTIONS

5.1 For system stability use the Routh-Hurwitz method.

$$G_{Syst} = \frac{G}{1+G} = \frac{K(s+1)}{s^2(s+2)+K(s+1)} = \frac{K(s+1)}{s^3+2s^2+Ks+K}$$

Characteristic polynomial: $s^3 + 2s^2 + Ks + K$
Routhian array:

s^3	1 K
s^2	2 K
s^1	x
s^0	y

where $x = \frac{(2)(K)-(K)}{2} = \frac{K}{2}$

$$y = \frac{\left(\frac{K}{2}\right)(K)-0}{\frac{K}{2}} = K$$

Exhibit 5.1b

The first column is positive for all positive values of K and the system is stable. The root locus is sketched by setting $G = -1$ (that is, using the basic rules of root locus, $|G| = 1$, $\phi_G = \pm n180°$ with n being any odd integer) and one obtains a sketch like that in Exhibit 5.1c.

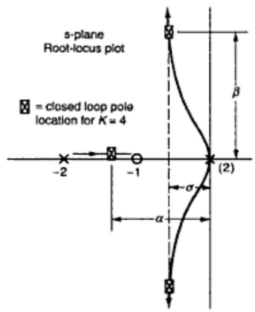


Exhibit 5.1c

The system is stable for all values of positives K's.

$$G_{Syst} = \frac{K}{s^3+2s^2+Ks+K} = \frac{4}{(s+\alpha)[(s+\sigma \pm j\beta)]}$$

with $K = 4$; $\alpha \cong 1.25$, $\sigma \cong 0.45$, $\beta \cong 1.8$

The system error is zero for both a step and ramp input, but is finite for an acceleration input (a/s^3).

$$e\Big|_{t\to 0} = \lim_{s\to 0} s\left[\frac{R}{1+G}\right] = \left[\frac{\left(\frac{a}{s^3}\right)}{1+\frac{4(s+1)}{s^2(s+2)}}\right]$$

$$= \lim_{s\to 0} s\frac{a}{s^2+\frac{4(s+1)}{s+2}} = \frac{a}{\left(\frac{4}{2}\right)} = 0.5a$$

Therefore the error is 0.5 of the acceleration input.

5.2 a. The block diagram could be given as in Exhibit 5.2a (with either positive or negative feedback).

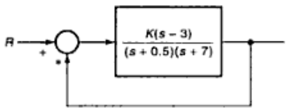


Exhibit 5.2a

* For negative (−) feedback, the system function is

$$G_{Syst} = \frac{G}{1+G} = \frac{K(s-3)}{(s+0.5)(s+7)+K(s-3)}$$

$$= \frac{K(s-3)}{s^2+(7.5+K)s+(3.5-3K)}.$$

The denominator (which determines the "character" of the response) must not have any negative factors (indicating closed loop poles in the right half plane, or an unstable system). A simple test for stability is the Routh criterion; or in this case (for a simple second-order system), all of the coefficients of the denominator polynomial must be positive. therefore,

$$G_{Syst} = \frac{G}{1+G} = \frac{K(s-3)}{(s+0.5)(s+7)+K(s-3)}$$

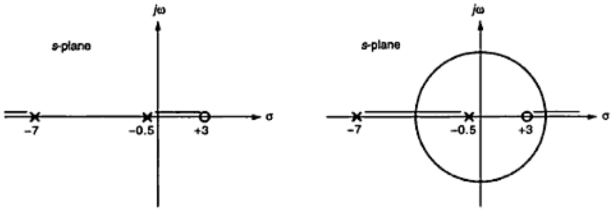
$$= \frac{K(s-3)}{s^2+(7.5+K)s+(3.5-3K)}.$$

Here, for stability, again the denominator polynomial must be positive, thus

$$3K < 3.5,\ K < 1.167.$$

Of course the root locus method of analysis may also be used as shown in Exhibit 5.26.

b. For C/R to be as close to unity as possible, it is only necessary to minimize $E = R - C$ (since it is a unity feedback system).



Root locus for negative f.b. (angles sum to ±180°)

Root locus for positive f.b. (angles sum to 0°)

Exhibit 5.2b

For negative feedback,

$$E = \frac{R}{1+G}.$$

Now assume a step input $\left(R = \frac{r_0}{s}\right)$ and that one is interested in the error after a long period of time such that the final value theorem may be applied.

$$e\Big|_{t\to\infty} = \lim_{s\to 0} sE = \lim_{s\to 0} s\left[\frac{\frac{r_0}{s}}{1+\frac{K(s-3)}{(s+0.5)(s+7)}}\right]$$

$$= \frac{r_0}{1+\frac{K(-3)}{(0.5)(7)}} = \frac{r_0}{1-0.857K}$$

To minimize this equation, the term, 1 − 0.857K, should be as large as possible. For system stability the maximum value that K can have is 1.167. If K is at this maximum value then $0.857K$ is equal to 1.0 and the term is zero. This causes e to approach infinity. If K is zero then the transfer function, g, goes to zero. Hence it is obvious that K must be between 1.167 and zero. If K were near zero, then there would be no forward path for the signal and the system would be useless. Referring to the left-hand s-plan root locus plot on the previous page, if K equals 1.167 then the close loop pole is at the origin and the system would act as an integrator (meaning that the error would go to infinity after a long period of time). Some value of K between the extremes would cause the error to be greater than the input step. One may conclude that the real path for the root locus is between the −0.5 pole and the zero origin.

This problem suggests that one must use some kind of compensation network, or, better, should review the original open loop transfer function for possible hardware changes such that the negative zero perhaps could be made positive. An ideal location of the zero would be to the left of

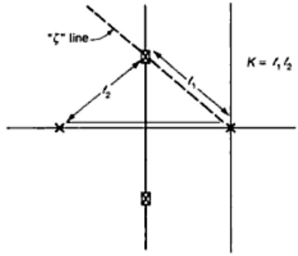


Exhibit 5.4c

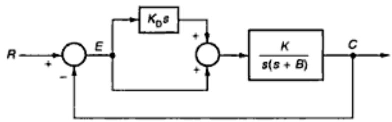


Exhibit 5.4d

The new forward transfer function then becomes

$$G_{fwd} = \frac{(1+K_D s)K}{s(s+B)}.$$

The error again may be found using the final value theorem.

$$e\big|_{t\to\infty} = \lim_{s\to 0} s\left[\frac{\frac{1}{s^2}}{1+\frac{(1+K_D s)K}{s(s+B)}}\right] \to \frac{B}{K}$$

To reduce the new error by a factor of 10, try increasing the stiffness, K, to give

$$e_{new} = 0.1 e_{old} = \frac{B}{10K}.$$

The new root locus will then have a zero located at $-1/K_D$ (yet to be determined). Because the amount of damping is to remain the same (*i.e.*, the "ζ" line to remain the same), the locus will be of the form shown in Exhibit 5.4e.

The requirement of error, being $K_{new} = 10K_{original}$, and locating the new closed loop pole of the "ζ" line will give the relationship of

$$K_{root\ locus} = \frac{\ell_1' \ell_2'}{\ell_0'} = K_D(10K) = K_D 10 \ell_1 \ell_2.$$

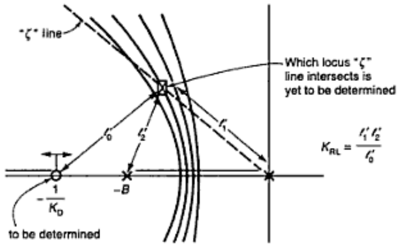


Exhibit 5.4e

Using this relationship, one may, by trial and error techniques, find the value of K_D.

However, a more straightforward technique not using root locus methods will allow us to approximate K_D directly. (This will be approximate as the zero in the closed loop response will alter the damping.)

Consider,

$$G_{Syst} = \frac{(10K)(1+K_D s)}{s^2 + Bs + K_D(10K)s + (10K)} = \frac{\omega_n^2(1+K_D s)}{s^2 + s\zeta\omega_n s + \omega_n^2}$$

where $\omega_n = \sqrt{10K}$

$$2\zeta\omega_n = B + 10KK_D$$

$$\zeta = \frac{B + 10KK_D}{2\sqrt{10K}}.$$

But the new "ζ" and the original "ζ" are required to be the same,

giving: $$B = \frac{B + 10KK_D}{\sqrt{10}}.$$

As an example, let $K = B = 2$ and "ζ" = 0.707.

Then $2 = \frac{2 + (10)(2)K_D}{\sqrt{10}}$, giving $K_D = 0.216$.

Or, the zero (for the root locus plot) is located approximately at $-1/K_D$, which gives −4.6 for the numbers previously used.

Therefore the new stiffness should be 10 times the original and the "time constant" of the derivative compensator should equal K_D.

5.5 Assume an ideal controller with torque directly proportional to error, E, ($T = KE$), and that the system is of 2nd order (Exhibit 5.5a).

$$T = KE = K(R - C)$$
$$T = (Js^2 + Bs)\Theta$$

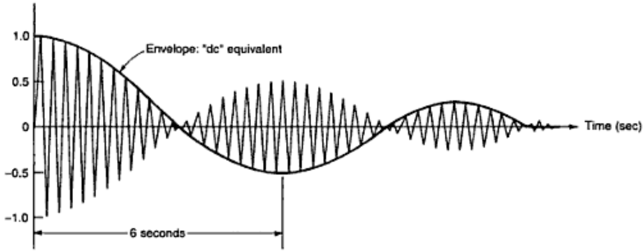


Exhibit 5.6e

The percent overshoot is over 50% and t_p is approximately six seconds.

e. From the original block diagram (with unity gain) one may easily obtain the closed loop transfer function as

$$\Theta_L/\Theta_R = 1/(3.2s^2 + 1s + 1) = 0.312/(s^2 + 0.312s + 0.312).$$

Here, $\omega_n = \sqrt{0.312} = 0.559$, $\zeta = 0.279$ (well unde damped). From standardized second ordered curves, a zeta of 0.279 gives approximately 40% of overshoot.

The percentage overshoot of approximately 50% for part (d) compared with the 40% (using the original unity gain setting) is 10% higher, therefore the new gain of part (d) must have been increased slightly.

5.7 d. Use the Routh-Hurwitz method.

s^3	1	$(2+K)$
s^2	3	$4K$
1	x_1	
s^0		

$$G_{Syst} = \frac{G}{1+G} = \frac{K(s+4)}{s^3+3s^2+(2+K)s+4K}$$

$$x_1 = \frac{3(2+K)-4K}{3} > 0$$

$$\therefore 6-K>0$$

$$K<6$$

5.8 c. Use the Bode phase approximation method; sketch on semi-log paper, or use the direct calculation method with two or three "guessed" trial frequencies:

$$G = \frac{K(s+4)}{s(s+1)(s+2)} = \frac{2K(0.25s+1)}{s(s+1)(0.5s+1)}$$

$$s \to j\omega$$

$$\angle\phi = \tan^{-1} 0.25\omega - 90^\circ - \tan^{-1}\omega - \tan^{-1} 0.5\omega \Rightarrow -180^\circ$$

Try $\omega = 2$:

$$\angle\phi = 14^\circ - 90^\circ - 63.4^\circ - 26.6^\circ = -166^\circ \neq 180^\circ$$

Try $\omega = 3$:

$$\angle\phi = 36.9^\circ - 90^\circ - 71.6^\circ - 56.3^\circ = -181^\circ \approx -180^\circ$$

5.9 b. Because the system is approximated by a second-ordered one, the normalized, standardized curves give a family of plots that relate zeta to percent overshoot. Or, the damping ratio may be calculated directly.

$$\%\text{O.S.} = 100e^{-\zeta\pi/\sqrt{1-\zeta^2}}, \quad 0.16 = e^{-\zeta\pi/\sqrt{1-\zeta^2}}$$

$$\ln 0.16 = -1.833 = \frac{\zeta\pi}{\sqrt{1-\zeta^2}}, \quad (1.833)^2 = \frac{\zeta^2\pi^2}{1-\zeta^2}, \quad \zeta \cong 0.5$$

5.10 c. From the root locus plot, find the frequency where the curve just passes through the imaginary axis; or, the value may be calculated directly from the closed loop characteristic polynomial (CP) equation.

$$\begin{aligned} \text{C. P.} &= s^3 + 3s^2 + (2+K)s + 4K = 0, \quad \text{for} \quad s = 0 + j\omega \text{ (on axis)} \\ &= -j\omega\omega^2 - 3\omega^2 + j\omega(2+K) + 4K = 0 \\ &= (-3\omega^2 + 4K) + j\omega(-\omega^2 + 2 + K) = 0 \\ &= 0 + 0 = 0 \end{aligned}$$

$$\therefore\ \omega^2 = 8, \quad \omega = \sqrt{8} \approx 3$$

5.11 a. This is a "standard" graphical relationship that is given in almost any text on control systems, and is plotted for an ideal second-order curve. Or it may be calculated from

$$\begin{aligned} \%\text{O.S.} &= 100e^{-\zeta\pi/\sqrt{1-\zeta^2}} = 100e^{-.5\pi/\sqrt{1-.5^2}} \\ &= 16.3 \approx 16\%. \end{aligned}$$

5.12 d. Carefully sketch the root locus in the region near 1.0 rad/sec (at the intersection of the crude root locus and a straight line passing though the imaginary axis at 1.0).

Once the correct locus is determined, K is the product of the line lengths from the open loop poles divided by the line length of the open loop zero.

$$\begin{aligned} K &= \frac{\ell_1\ell_2\ell_3}{\ell_0} \\ &= \frac{1.1\times1.2\times1.8}{3.8} = 0.6 \end{aligned}$$

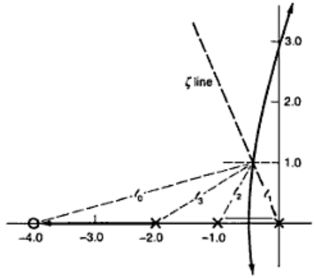


Exhibit 5.12a

5.13 b. Because this is a type I system (*i.e.*, one integration in the loop equation), the error will be finite and is found from the final value theorem.

$$E = \frac{1}{1+G}R$$

$$e(t)\Big|_{t\to\infty} = \lim_{s\to 0} s\left[\frac{1}{1+\frac{K(s+4)}{s(s+1)(s+2)}}\left(\frac{2}{s^2}\right)\right] \to \frac{2}{\frac{4(4)}{(1)(2)}} = 0.25$$

5.14 b. Because the input is an acceleration ($R(s) = 1/s^3$), the loop itself has only one integration, and the error will have to approach infinity.

5.15 c. This is somewhat lengthy to solve. One method is to use the original root locus to locate the original uncompensated zeta for a damped frequency of 1.0 (*i.e.*, a horizontal line passing through the imaginary axis at 1.0 and the root locus. Using this zeta line, locate the desired new locus location on the zeta line and a horizontal line passing through the imaginary axis at 1.5. This new locus location will be incorrect by a certain angle (*i.e.*, all correct locus locations should add to +/–n180°); find the difference between this certain angle and that of –180° (this difference will be approximately 35°). Then, by trial and error, locate the compensator pole on the real axis such that the new locus angles will sum to –180° (a location of –2.6 for this pole will satisfy the angle relationship).

5.16 a. Generally the **A** matrix may take several forms, however here it is given. The **C** matrix relating the output to the state variables may be found directly as follows:

$$G(s) = \frac{K(s+4)}{(s^3+3s^2+2s)} = \frac{(Ks^{-2}+4Ks^{-3})}{(1+3s^{-1}+2s^{-2}+0s^{-3})}$$

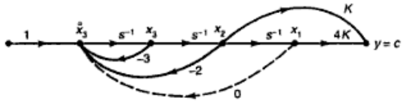


Exhibit 5.16a

$$\begin{aligned}\dot{x}_1 &= 0x_1 + 1x_2 + 0x_3\\ \dot{x}_2 &= 0x_1 + 0x_2 + 1x_3\\ \dot{x}_3 &= 0x_1 - 2x_2 - 3x_3 + 1e\\ y &= c = 4K + Kx_2 + 0x_3\end{aligned}$$

$$\dot{\underline{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} e$$

$$y = [4K \quad K \quad 0] \ \underline{x} = c$$

5.17 a. The subsystem is an ideal second-order system with poles located at $-1 + j1$ and $-j - j1$. Obviously, because the poles are located in the left hand plane of the s-plane, it is a stable system. Also, because the poles are fixed, any positive real value of K would result in a stable system.

b. For the complete system the loci of the roots would emanate from the subsystem poles by going straight up and straight down. A zeta line radiating for the origin of the s-plane at an angle of $\cos^{-1} 0.4 = 66.4°$ (from the real 180° axis) would intersect the root locus at $-1 + j2.33$. Because K is the product of the line lengths from the open loop poles to the closed loop pole, it is found to be

$$K = \ell_1 \times \ell_2 = 1.33 \times 3.33 = 4.43.$$

Again, because the new system is also an ideal second-ordered one, the percent overshoot may be calculated (or found from any standard set of second-order curves for a particular zeta) as follows:

$$\text{Percent overshoot} = 100e^{-\zeta\pi\sqrt{1-\zeta^2}} = 25.6\%$$

c. The forward path equation is merely $K/[s(s^2 + 2s + 2)]$, and since $H = 1$, the characteristic equation of the new system is

$$s^3 + 2s^2 + 2s + K.$$

A Rough-Hurwitz array yields

s^3	1	2
s^2	2	K
s^1	X	
s^0	Y	

where $X = (2 \times 2 - 1 \times K)/2$; for a marginally stable system $X = 0$, therefore $K = 4$.

Another way of finding the value of K for a marginally stable system is to allow $s \to j\omega$ (*i.e.*, the root locus crosses the $j\omega$ axis for this condition), then substitute $j\omega$ for s in the characteristic system equation. (This method also determines the natural frequency.)

$$(j\omega)^3 + 2(j\omega)^2 + 2j\omega + K = 0 = (-2\omega^2 + K) + j\omega(2 - \omega^2)$$

since both $(-2\omega^2 + K) = 0$ and $j\omega(2 - \omega^2) = 0$, therefore:

$$j\omega = 0, \quad \text{or} \quad \omega = \sqrt{2} \text{ rad/second}$$

and,

$$K = 2\omega^2 = 4.$$

5.18 a. For stability use Routh-Hurwitz criterion for the characteristic polynomial:

$$G_{Syst} = \frac{C}{R} = \frac{400}{s^2(s+10) + (1 + K_t s)400}$$

Because the equivalent block diagram may be given as Exhibit 5.18b,

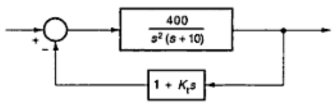


Exhibit 5.18b

$$\text{Characteristic polynomial} = s^3 + 10s^2 + 400K_t s + 400$$

Routh-Hurwitz Array:

s^3	1	$400K_t$
s^2	10	400
s^1	x_1	

$$x_1 = \frac{(10)(400K_t) - (1)(400)}{10}$$

$$\therefore K_t > 0.1 \quad \text{for} \quad x_1 > 0$$

$$(K_t \text{ minimum} = 0.1)$$

b. Plot root-locus for HG:

$$\text{let } H = 1 + K_t s = 1 + 0.125s = 0.125(8 + s)$$

$$\therefore \text{HG} = \frac{(0.125)(400)(s+8)}{s^2(s+10)} = \frac{50(s+8)}{s^2(s+10)}$$

$$\text{Here } K(\text{root} - \text{locus}) = 50.$$

Because the solution is only approximate, let $\ell_1 \approx \ell_0$

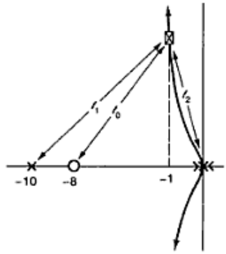


Exhibit 5.18c

and

$$\ell_2^2 = \omega_n^2.$$

Then ζ line = 80° and $\omega_n = \sqrt{50}.$

$$\zeta \approx 0.17$$

From any standard 2nd-order transient response curves, one can easily determine percent overshoot and time to first peak, t_p.

%O.S. = 60%

$t_p = 3.2/\omega_n = 0.45$ seconds.

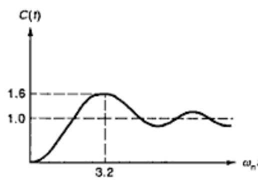


Exhibit 5.18d

6.2 A certain requirement for a magnetic circuit (an iron toroidal ring with an air gap cut into it) has a unique shape for the cross section of the iron; it is triangular in shape. Obviously the iron faces on each side of the air gap are triangular and the area of the air gap is triangular (neglecting any fringing effect). One of the design requirements is to find the amount of magnetic flux in the air gap (same as in the ring) from certain required flux densities, B, and various triangular dimensions. (Recall, the flux equation is: $\phi = B \times A$.)

The lengths of the sides of a triangle are given by the values of the variables X, Y, and Z. Then the area of the triangle can be computed from

$$\text{AREA} = \sqrt{W(W-X)(W-Y)(W-Z)} \qquad \text{where } W = \frac{X+Y+Z}{2}$$

Write a BASIC, Fortran, C, or Java computer program to do the following:

a. Input values of X, Y, and Z (in meters) and B (in Telsa) from the keyboard.
b. Compute the area of the triangle and the flux.
c. Output a heading identifying X, Y, Z and FLUX, followed by their values in an exponential format.

6.3 A tentative design for a control system to position a radar antenna does not include feedback compensation (such that minimal cost and weight factors may be achieved). However, if need be, feedback compensation could be added in by a value of K_T. Before the system is built, it is decided to test the design by computer simulation to check the performance of the system without any feedback compensation (i.e., $K_T = 0$). The tentative design block diagram is shown in Exhibit 6.3a.

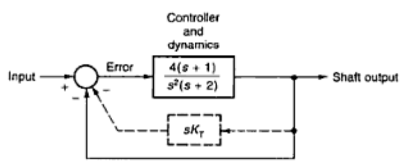


Exhibit 6.3a

Of the many high-level simulation languages available, it is decided to use CSMP with the default method of integration. Also, because the programmer is not familiar with optimization techniques of analysis for the feedback compensation, the solution for the output vs. time for a step input will be programmed for K_T equal to zero; then if the results are not satisfactory, K_T will be set in and incremented until the desired results are achieved.

6.4 An ac circuit problem (Exhibit 6.4) is to be solved for two specific conditions, thus a computer solution is appropriate.

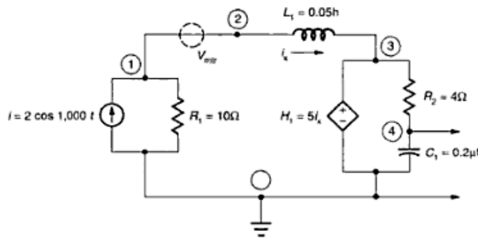


Exhibit 6.4

Using any popular packaged circuit program, state the name of the program you choose and write out your program(s) including any remark statements that would be suitable. Here, assume another person will actually run the program to obtain the solution and thus your remark statements will be needed for clarification. Your program(s) will be to find the steady-state voltage and phase angle (with respect to current source) across the capacitor for two conditions:

a. For a fixed current source of $i(t) = 2\cos 1000t$.
b. Repeat (1) except the frequency will vary from 500 to 2000 rad/sec in steps of 250 rad/sec.

6.5 A thermocouple is a sensor used for measuring temperature. It is made by joining two wires made from dissimilar materials. To measure temperature, two thermocouples are connected to a circuit as shown in Exhibit 6.5.

One thermocouple is placed in a medium with a known constant temperature T_{ref} [i.e., iced water], and the other is placed where the temperature T is to be measured. A voltage v is generated when the two temperatures

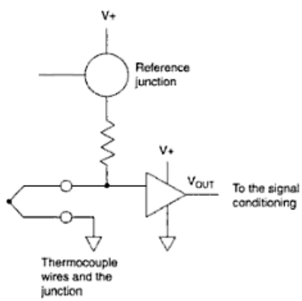


Exhibit 6.5

are not the same. The voltage v can be modeled as a function of temperature by an expression of the form

$$V = Ks(T - T_{ref})$$

where K is a constant that depends on the two materials that are utilized for the thermocouple. The following are the results from an experiment that was done for determining the constant Ks, of a certain thermocouple. Using this data, determine Ks using Matlab or a similar program, and curve fitting.

T in degrees C	25	100	200	300	400	500	600	700
v in voltage	1.11	4.03	8.16	12.62	16.54	20.9	23.7	29.17

6.6 A communication system inserts extra bits into the data transmitted by a transmission stream. The extra bits must be removed in the receiver. Conversely, the receiver circuit portion of the digitial communication system is required to remove the excess bits packed into the serial data stream at the transmitter.

Using any commercial computing package, design the receiver fsm and control logic that extracts the extra bits that were inserted at the transmitter. The resulting data stream must be an NRS [non return to zero] data stream that is a perfect replica of the transmitted stream prior to adding the required transmission bits. Describe the logic gates that make up the finite state machine.

6.7 A thermistor is used to measure the temperature of a motor from the lowest expected temperature (−10 degrees Celsius) to the highest motor operating temperature (150 degrees C). Data for the thermistor is as follows:

Temp (degrees C)	R (ohms)
−10	12.0 kohms
0.0	7.4 kohms
10.0	4.5 kohms
20.0	2.8 kohms
30.0	1.8 kohms
40.0	1.2 kohms
50.0	860 ohms
60.0	560
70.0	400
80.0	240
90.0	210
100.0	150
110.0	115
120.0	90
130.0	65
140.0	50
150.0	40

The temperature is to be displayed on a voltmeter, from 0 V to 5 V. Describe the characteristics and components of a suitable electronic interface circuit. Assume you have general-purpose commonly available operational amplifiers available for Vcc = +/−15V. Use only the standard resistance values for resistors with the standard industrial tolerance.

6.8 A black box containing a linear circuit has an on-off switch and a pair of external terminals. The voltage between the open-circuited external terminals is measured at 12 u(t) Volts when the switch is turned on. The short-circuit current is observed to be $[0.2e^{-1000t}]$ u(t) Amps when the switch is again turned on.
a. Find the current the box would deliver to a 50-ohm resistive load.
b. Find the interface voltage across the terminals.
c. What is the power dissipation in terms of watts?

6.9 A BJT transistor is operating in the active mode. The source is 24 volts and the resistors are standard values. Utilizing SPICE and the appropriate model, find the power delivered by the power source and the power absorbed by the transistor and the resistors.

6.10 An induction motor is undergoing the blocked-rotor test. The motor will have a stated line current in amps, which is drawn when the line voltage is given in volts and the total wattage will be specified in watts. For example, the typical formula for wattage is volts × amps.

Describe the formulas you would use in an interactive computer program that a technician could utilize to calculate
a. the equivalent resistance reflected to the stator per phase
b. the equivalent impedance per phase
c. the equivalent inductive reactance per phase.

If the ohms per phase for ac resistance of the stator are specified, the program should calculate
d. the resistance of the rotor per phase reflected to the stator
e. the inductive reactance of the rotor under blocked rotor conditions.

SOLUTIONS

6.1 Because the system has differentiating block, sK_t, (which is very difficult to simulate by both analog and digital methods), the block diagram is rearranged so that this block is ahead of the integrator as shown in Exhibit 6.1b.

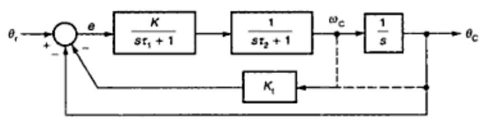


Exhibit 6.1b

a. The analog computer equivalent schematic (Exhibit 6.1c) is arranged to have a pick-off point for the system error at *e*, and the various gain controls may be easily manipulated for several different settings. Also, because all op-amps operate in the inverted mode and all loops are for negative feedback, all loops are to have an odd number of op-amps in a loop (this, of course, is not a requirement in a digital simulation program). Also, no numerical values are specified, so no provisions are made for magnitude or time scaling—which, again, is not a factor in digital simulation.

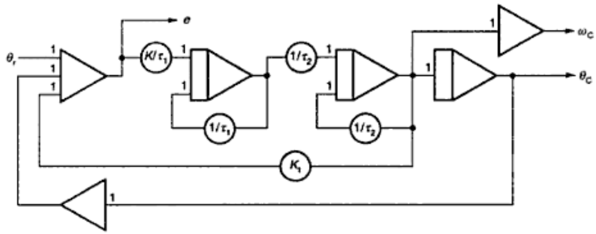


Exhibit 6.1c

b. No special circuitry is needed to obtain a step response; however, for a ramp input, an integrator block (Exhibit 6.1d) is placed ahead of the summing block.

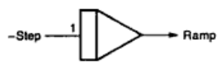


Exhibit 6.1d

c. Most packaged digital simulation programs use an automatic variable step size routine for the actual integration, however, some of the earlier ones required the programmer to enter this value. As a rule of thumb this value of delta time would be less than one-tenth of the shortest time constant in the system.

6.2 (in BASIC)

a. & b.

```
10 INPUT "INPUT X,Y,Z IN METERS, SEPARATED BY COMMAS",
   X,Y,Z
15 INPUT "INPUT FLUX DENSITY ,B (IN TESLA)", B
20 W = (X + Y + Z)/2.
30 AREA = (W*(W – X)*(W – Y)*(W – Z)^.5
36 FLUX = B*AREA
40 PRINT "SIDE X","SIDE Y","SIDE Z","AREA","FLUX"
50 PRINT USING "#.###^^^^";X,Y,Z,AREA,FLUX
60 END
```

c. *Note:* A typical output for a test input of $X = 0.002$, $Y = 0.00282$, $Z = 0.002$, and $B = 1.0$ (this portion, of course, would not be part of the exam) gives

```
RUN
INPUT X,Y,Z IN METERS, SEPARATED BY COMMAS 0.002,
   0.00282, 0.002
INPUT FLUX DENSITY, B (IN TESLA) 1.0

SIDE X      SIDE Y      SIDE W      AREA        FLUX
0.200E-02   0.282E-02   0.200E-02   0.200E-05   0.200E-05
```

6.3 Because of the possible use of the derivative function used for the feedback compensation, the block diagram is first rearranged to avoid this operation, as shown in Exhibit 6.3b.

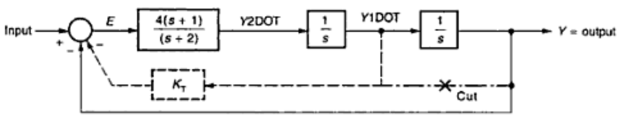


Exhibit 6.3b

Or, for programming conceptualization, the diagram is again modified as Exhibit 6.3c.

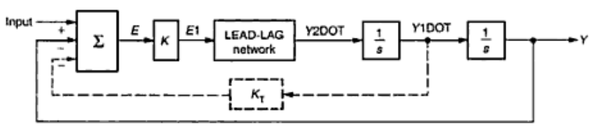


Exhibit 6.3c

It should be noted from a CSMP manual that a lead-lag network is given by

$$Y(s)/X(s) = (P_1 s + 1)/(P_2 s + 1), \quad \text{or} \quad P_2(dy/dt) + y = P_1(dx/dt) + x.$$

Then $4(s + 1)/(s + 2) = 2(s + 1)/(0.5s + 1)$.

Thus, one possible simulation program could be

```
LABEL SIMULATION OF AN ANTENNA POSITION CONTROL
SYSTEM

INITIAL
   CONSTANT K = 2.0, KT = 0.0, P1 = 1.0, P2 = 0.5
DYNAMIC
   INPUT = STEP(0.0), E = INPUT - Y - KT * Y1DOT, E1 = K * E
   Y2DOT = LEADLAG(P1,P2,E1)
   Y1DOT = INTGRL(0.0,Y2DOT)
   Y = INTGRL(0.0,Y1DOT)
   OUTPUT = Y
TERMINAL
   TIMER FINTIME = 5.0, PRTDEL = 0.05, PRTPLT OUTPUT
END
STOP
END JOB
```

The computer will then print the output for these conditions and, if the results are satisfactory, the problem is solved; if, however the results are poor, different values of *KT* may be tried. One way of obtaining the step results for these values of added feedback compensation is simply to change *KT* = 0 to several other increasing values and repeat the operation. A better way would be to initially delete *KT* from the CONSTANT line, then follow with another statement line such as

```
PARAMETER KT = (0.0, 0.2, 0.4, 0.6, 0.8, 1.0)
```

Still another way would be to do this with a FORTRAN statement along with a NOSORT notation. The FORTRAN statement could be based on a conditional requirement of some parameter of the output response and then looped until that requirement is achieved; this method depends on the sophistication of the programmer with regard to both FORTRAN and knowledge of control system optimal requirements.

6.4 One very popular digital package circuit program is PSPICE, which will be used here for demonstration.

a. Fixed parameters

```
CIRCUIT PROBLEM, ac_prob.cir

I1      0 1     AC      2a      0deg
Vmtr    1 2     0

* Vmtr is a dummy voltage source of 0 volts that reads current.

H1      3 0     Vmtr    5
R1      1 0     10ohm
R2      3 4     4ohm

* As a check for XL and XC recover from w = 1000 r/s.
```

Then simplify by use of Karnaugh map. For f_0, the Karnaugh map is

	$\bar{A}$		A	
$\bar{C}$			1	
C		1	1	1
	$\bar{B}$	B	B	$\bar{B}$

b. Therefore, $f_0 = AB + BC + AC$ (Exhibit 7.1a)

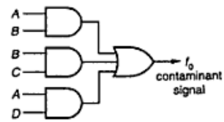


Exhibit 7.1a

For f_A the truth table is:

	$\bar{A}$		A	
$\bar{C}$		1	1	1
C	1	1		1
	$\bar{B}$	B	B	$\bar{B}$

$\therefore f_A = \bar{A}B + \bar{B}C + A\bar{C}$ (Exhibit 7.1b)

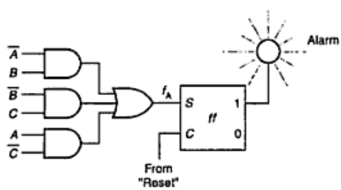


Exhibit 7.1b

One could have simplified the Karnaugh map in terms of "0's" as

$$\overline{f_0} = \bar{B}\bar{C} + \bar{A}\bar{B} + \bar{A}\bar{C}$$

$$\therefore f_0 = \overline{\bar{B}\bar{C} + \bar{A}\bar{B} + \bar{A}\bar{C}}$$

$= (B + C)\cdot(A + C)\cdot(A + B)$ to go directly to NOR logic:

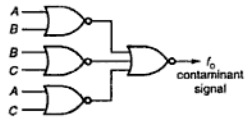


Exhibit 7.1c

and $$\overline{f_A} = ABC + \overline{A}\,\overline{B}\,\overline{C}$$
$$f_A = \overline{ABC + \overline{A}\,\overline{B}\,\overline{C}}$$
$$= (\overline{A} + \overline{B} + \overline{C}) \cdot (A + B + C)$$

c. Exhibit 7.1d shows the final logic circuit diagram for the alarm circuit.

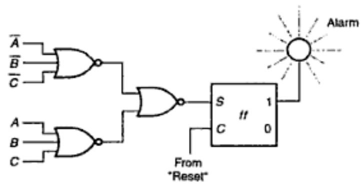


Exhibit 7.1d

7.2 a. Exhibit 7.2b shows the interconnections necessary for the equivalent 1K × 4 bit RAMS (from the 1K × 1 bit RAMS).

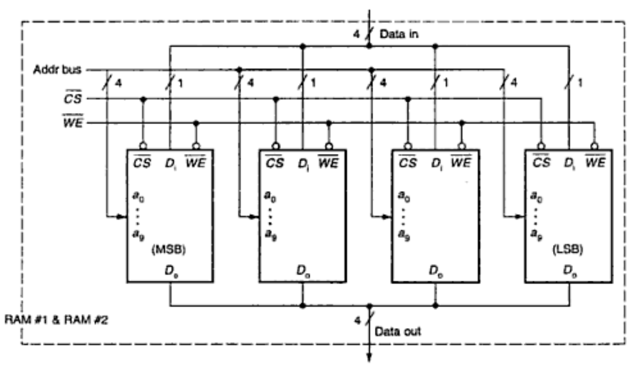


Exhibit 7.2b

b. Exhibit 7.2c shows the interconnections for the 4 full adders, the RAMs, and the 5 D-type flip flops.

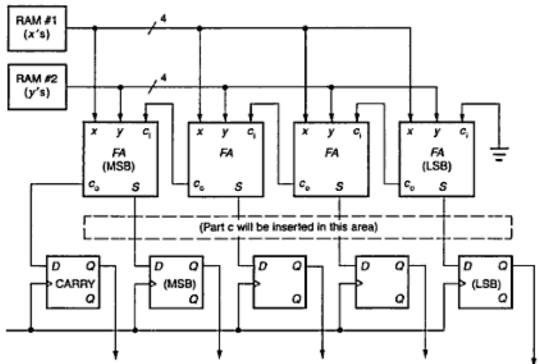


Exhibit 7.2c

c. To correct the outputs of the natural binary signals to that of binary coded decimal, it must be recalled that natural binary includes numbers between 0 and 15, while BCD numbers are limited to 0 through 9. As long as the sum does not exceed 9 the outputs are correct. However, if the sum exceeds 9, then 6 must be added to the natural binary sum to yield two BCD numbers. As an example, suppose 7 and 5 are added together, the natural binary sum is $(1100)_2$ while if we add 6 more, the new sum is $(0001\ 0010)_{BCD}$.

7 →	0111	
5 →	0101	
	1100	Sum is greater than 9!
	0110	Therefore add 6.
12 → 0001	0010	Answer in BCD.

Thus it is necessary to detect if outputs of S's of the full adder exceed 9, if so add 6 with the additional circuitry. This detection is easily accomplished if the 8 line is high AND either the 4 OR 2 line is high (and, of course, OR if the c_0 output of the MSB FA is high).

The two 7-segment driver/decoders are connected only to show a 0 or a 1 for the MSB and a 0 → 9 for the LSB.

Exhibit 7.2d shows the interconnections necessary for correcting the full adders to correctly read the BCD sums and the connections to drivers and the LED displays.

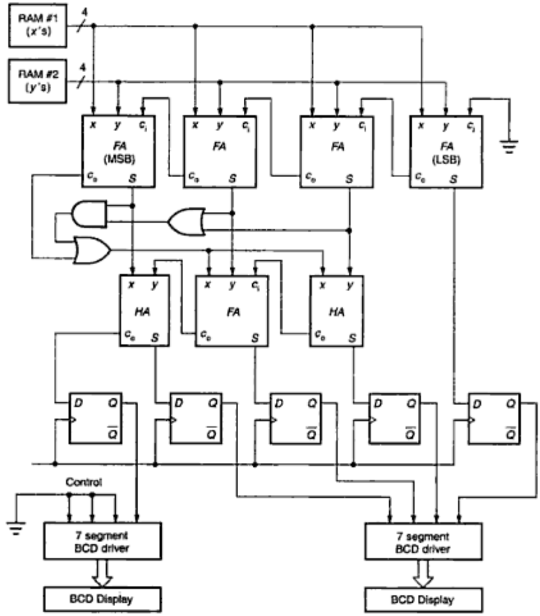


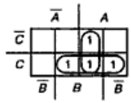
Exhibit 7.2d

7.3 b. Use the truth table to form the function on two or more 1's for the sensors.

m	A	B	C	*f*
0	0	0	0	0
1	0	0	1	0
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	1
6	1	1	0	1
7	1	1	1	1

$$f_{alarm} = \bar{A}BC + A\bar{B}C + AB\bar{C} + ABC$$

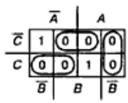
7.4 a. Form a Karnaugh map (Exhibit 7.4) for $f = \sum m(3,5,6,7)$



$$f = AB + AC + BC$$

Exhibit 7.4

7.5 c. From the same type of truth table as Problem 7.4, form the function when A, B, and C agree (i.e., terms 0 and 7). Exhibit 7.5 shows the Karnaugh map.



$$\bar{f}(\text{activate 2nd alarm}) = \bar{A}C + B\bar{C} + A\bar{B}$$
$$f(\text{activate 2nd alarm}) = \overline{\bar{A}C + B\bar{C} + A\bar{B}}$$
$$f(\text{activate 2nd alarm}) = (A + \bar{C})(\bar{B} + C)(\bar{A} + B)$$

Exhibit 7.5

7.6 b. $V_j = 2.9 = 5(1 - e^{-(1/RC)t}) = 5(1 - e^{-X}),\ x = 0.867$

$$0.867 = (1/RC)t, \quad \text{for} \quad R = 5K,\ C = 11.5 \text{ ufd}$$

7.7 b. 64 counts must occur in 50 ms.

$$64/50\text{ms} = 1.28 \times 10^3, \text{ clock} = 1.28 \text{ kHz}$$

7.8 a. Sketching a timing diagram will show that the added counter output should always start hi, otherwise the warning device will stay hi.

7.9 c. Note that $(255)_{10}$ is for full scale for an 8-bit converter. Mid range corresponds to $(127)_{10}$ which, in turn, corresponds to $(7F)_{16}$.

7.10 d. The MSB corresponds to 128 and the next MSB is 64, and 128 + 64 = 192, therefore (192/255)5.0 = 3.75 volts.

7.11 d. For the configuration of question 8 the "weight" of the two lines are equivalent to $(192)_{10}$ or 3.75 volts, therefore 3.90 – 3.75 = 0.15 volts, or (0.15/5.0)255 = 7.65 (nearest integer value = 8). Therefore the line that represents 2^3 is correct.

7.12 b.

The two four-bit solution possibility is

0000 0000
0000 0001
0000 0011
0000 0111
0000 1111
0001 1111
0011 1111
0111 1111
1111 1111
1111 1110
1111 1100
1111 1000
1111 0000
1110 0000
1100 0000
1000 0000
0000 0000

The JK flip flop characteristic equation is $Q^* = JQ' + K'Q$, and the flip flop equations follow from this. If one utilizes the D flip flop for VHDL, you can use the Q^*'s or next state values directly.

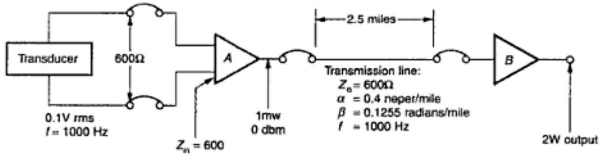


Exhibit 8.1a

transmission line, length ℓ, of 25 cm and whose characteristic impedance, Z_o, is 50Ω. The frequency of the generator is 300MHz and the phase velocity, u, on the line is 300×10^8 m/s.

8.2 The capacitance per unit length of the line is
a. 66.7 μF b. 33.4 μF
c. 50.3 μF d. 66.7 pF

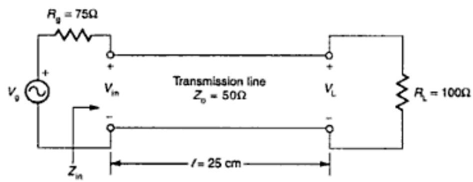


Exhibit 8.2

8.3 The inductance per unit length of the line is
a. 250 μH b. 166.7 μH
c. 25 μH d. 166.7 nH

8.4 The attenuation constant of the line is
a. 1.0 b. infinity
c. 0 d. 50

8.5 The phase constant of the line is
a. 6 rad/m b. 2π rad/m
c. 12π rad/m d. 4π rad/m

8.6 The input impedance, Z_{in}, looking toward the load is
a. 25Ω b. 250Ω
c. 2.5Ω d. 0Ω

8.7 The input voltage at Z_{in} is
a. 250 V b. 25 V
c. 50 V d. 100 V

8.8 The reflection coefficient, Γ, is
a. 0 b. 3
c. 1/3 d. 1/6

8.9 The standing wave ratio, SWR, is

a. 1 b. 1/2
c. 4 d. 2

8.10 The load voltage, V_L, is

a. $-37.5\angle 90°$ V b. $5\angle 60°$ V
c. $5\angle 90°$ V d. $50\angle 90°$ V

8.11 The average power delivered to the load is

a. 0.5 w b. 1.25 w
c. 12.5 w d. 25 w

8.12 A section of nonloaded telephone cable, located in a rural area, has the following characteristics per loop mile at a frequency of 1000 Hz:

- Series resistance, 85.8 ohms
- Inductance, 1.00 millihenry
- Capacitance, 0.062 microfarad
- Shunt conductance, 1.50 micromho.

The cable consists of 400 pairs of No. 19 AWG, is shielded, and is 3.06"o.d.

Compute the following parameters for this cable at 1000 Hz:

a. characteristic impedance
b. attenuation in decibels per mile
c. phase shift per mile
d. velocity of propagation.

8.13 Exhibit 8.13a shows a diagram of a coaxial transmission line in which the circuit elements have the values indicated, and E_g is the open circuit generator voltage, Z_0 is the characteristic impedance of all transmission lines, and L_2 is greater than one wavelength.

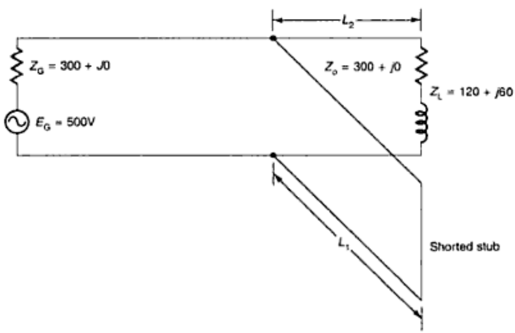


Exhibit 8.13a

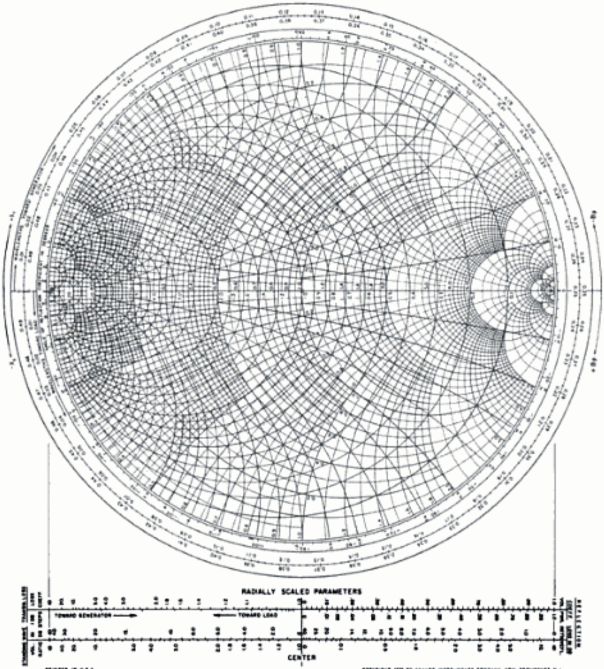


Exhibit 8.13b

a. Determine the length L_2, in terms of a distance that is greater than one wavelength, and is the proper distance so that a shorted stub attached at this point causes the line to be matched to Z_0.
b. Determine the length of the shorted stub L_1 needed to match the load to the line at this point.
c. Determine the VSWR on the unmatched portion of the line L_2.
d. Determine the power that will be supplied to the load under the matched conditions.
e. Determine the greatest voltage that will appear across the transmission line in the unmatched* section L_2.

*Note: A Smith chart is included to aid in the solution, or a straight analytical solution may be employed. "Smith" is a registered trademark of the Analog Instruments Co, and is reproduced by courtesy of Analog Instruments Co., Box 808, New Providence, NH 07974.

8.14 Two microwave stations operating at 5 GHz are 50 kilometers apart. Each has an antenna whose gain is 50 dB greater than isotropic.

If 6 watts is applied to the input of the transmitting antenna, what is the signal level at the output terminal of the receiving antenna under free-space conditions? What is the path loss?

8.15 A phased array consists of two half-wave antennas located a half-wave apart, as shown in Exhibit 8.15. Antenna currents I_A and I_B are equal, and I_A leads I_B by 90°. Maximum field strength of each individually excited antenna is 250 mv/m at a distance of 40 kM. Radiation resistance of each antenna is 73.1Ω (the same as a half-dipole).

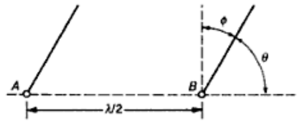


Exhibit 8.15

Determine the angle, Θ, at which the resultant field strength is maximum, and calculate the field strength of the array for Θ = 90° at a distance of 25 kM.

8.16 In the modulating circuit shown in Exhibit 8.16a, the modulating signal (see

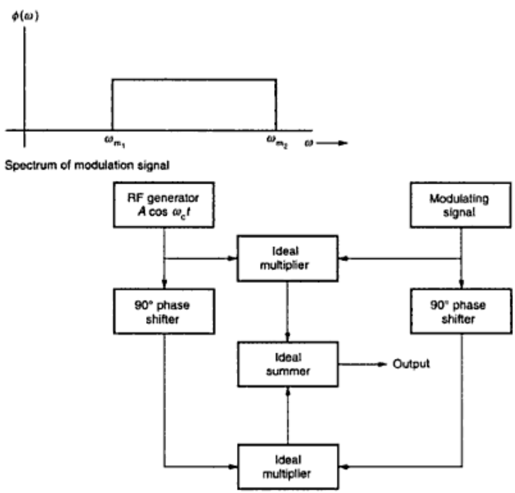


Exhibit 8.16a

spectrum sketch) is limited to angular frequencies ω where

$$\omega_{m_1} < \omega < \omega_{m_2} < \omega_c$$

and ω_m = modulating frequency; and ω_c = carrier frequency.

a. Sketch the spectrum of the output.
b. What is this type of modulated signal called?
c. Sketch another circuit that would produce the same output spectrum.

8.17 A certain transmitter has an effective radiated power of 9 kW with the carrier unmodulated and 10.125 kW when the carrier is modulated by sinusoidal signal.

a. Determine the percent modulation at 10.125 kW output.
b. Determine the total effective radiated power if in addition to the sinusoidal signal, the carrier is simultaneously modulated 40% by an audio wave.

8.18 In order to specify the communications link for a closed circuit television system, the bit rate must be known.

The monochrome television picture signal of this system requires 10 distinct levels of brightness for good resolution. This television system also includes the following parameters:

(1) Frame rate, 15 frames per second
(2) Lines per frame, 1200
(3) Discrete picture elements, 100 per line.

Determine the channel capacity in bits per second required to transmit the above signal with all levels equally probable and with all elements assumed to vary independently. List any assumptions that you make.

8.19 A series of remote stations is being planned to feed data to a large computer. The data are to be sent by the remote stations and recorded on a tape recording unit and then fed to the computer as necessary.

The data are expected to arrive from the remote stations with a Poisson distribution at an average rate of 10 transmissions from remote stations per hour. The recording time of the data varies exponentially, with a mean time of four minutes.

a. What is the average waiting time for a remote station before the data will begin to record?
b. A second tape unit including automatic switching equipment is available at a cost of $2.50 per hour. The telephone lines cost 4 cents a minute per line when used. Is the second unit economically warranted? Show sufficient calculations to justify your answer.

SOLUTIONS

8.1 a. $P_{in}\text{ level} = \frac{E^2}{R} = \frac{\left[\frac{1}{2}(0.1)\right]^2}{600} \times 10^3 = \frac{0.0167}{4}\text{ mv}$

P_{out} level is given: 0 dbm = 1 mw

$$\text{db Gain}_A = 10\ \log_{10}\frac{P_{out}}{P_{in}} = 10\ \log_{10}\frac{1}{\left(\frac{0.0167}{4}\right)} = 10\ \log_{10} 240$$

$$= 10 \times 2.38 = 23.8\text{ db}$$

b. Loss in cable: 0.4 nepers/mile × 2.5 miles = 1 neper

1 neper × 8.686 db/nepers = 8.686 dbm

P_{in} level = 0 − 8.686 = ∠8.686 dbm

$$P_{out}\text{ level} = 10\ \log\frac{2}{1\times 10^{-3}} = 10\ \log 2 \times 10^3 = 10 \times 3.3 = 33\text{ dbm}$$

$\text{db Gain}_B = 33.0 - (-8.686) = 41.686\text{ db}$

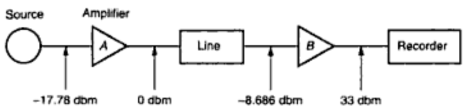


Exhibit 8.1b

c. $\text{Delay} = \frac{0.1255\text{ rad/mi} \times 2.5\text{ mi} \times 10^6\text{ microsec/sec}}{2\pi \times 1000\text{ Hz}} = 50\text{ microseconds.}$

d. The loss at 1000 Hz could be reduced the least expensively by loading (series) the telephone line; *i.e.*, inserting inductance coils at regularly spaced intervals.

e. The delay of the transmission line will increase when load coils are added, because we increased the phase shift.

8.2 d. $Z_o = \sqrt{L/C}$; $m = 1/\sqrt{LC}$; $Z_o m = 1/C$.

$C = 1/(Z_o\mu) = 1/(50 \times 3 \times 10^8) = 0.06667 \times 10^{-9} = 66.67$ pF.

8.3 d. $Z_o^2 C = L$; $L = 25000 \times 0.06667 \times 10^{-9} = 166.67$ nH.

8.4 c. $\alpha = 0$ (since the line is lossless).

8.5 b. $\beta = \omega\sqrt{LC} = \omega/\mu$; $\omega = 2\pi f$; $\beta = (2\pi \times 300 \times 10^6)/(3 \times 10^8) = 2\pi$ rad/m.

8.6 a. $\lambda = C/f = (3 \times 10^8)/(300 \times 10^6) = 1$ m; $\ell = 25\text{cm} = \lambda/4$.

The line is a quarter wave length long, therefore: $Z_{in} = Z_o^2/R_L = 25\Omega$.

b. The length of the shorted stub L_1 is such that reactance of type opposite to line is required.

On the Smith chart start at U_1 ($Y\gamma_{stub}/Y_0 = \infty$), move CCW to U_2 where the imaginary component of $G + jB$ is opposite that of the stub location but equal in magnitude.

Initial location	0°
Final location	270°
Difference	270°

The length of the stub: 270°/2 = 135°

c. To determine the VSWR on the unmatched portion of the line L_2, from point C on the Smith chart, we obtain

$$N = 2.6$$

This value can also be obtained as follows:

$$\rho = \frac{Z_L - Z_o}{Z_L + Z_o} = \frac{120 + 6j0 - 300 - j0}{120 + j60 + 300 + j0} = \frac{-180 + j60}{420 + j60} \times \frac{420 - j60}{420 - j60}$$

$$= 0.4 + j0.2$$

$$|\rho| = \sqrt{\overline{0.4}^2 + \overline{0.2}^2} = 0.446$$

$$\text{VSWR} = N = \frac{1 + |\rho|}{1 - |\rho|} = \frac{1 + 0.446}{1 - 0.446} = \frac{1.446}{0.554} = 2.6 \text{ q.e.d.}$$

d.

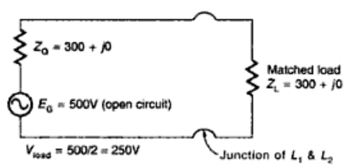


Exhibit 8.13c

The power then will be supplied to the load under the matched condition.

$$P_{load} = \frac{E^2}{R} = \frac{(250)^2}{300} = 208 \text{ watts}$$

e. $$V_{max} = V_{load}(1 + |\rho|) = 250 \times 1.446 = 360 \text{ volts}$$

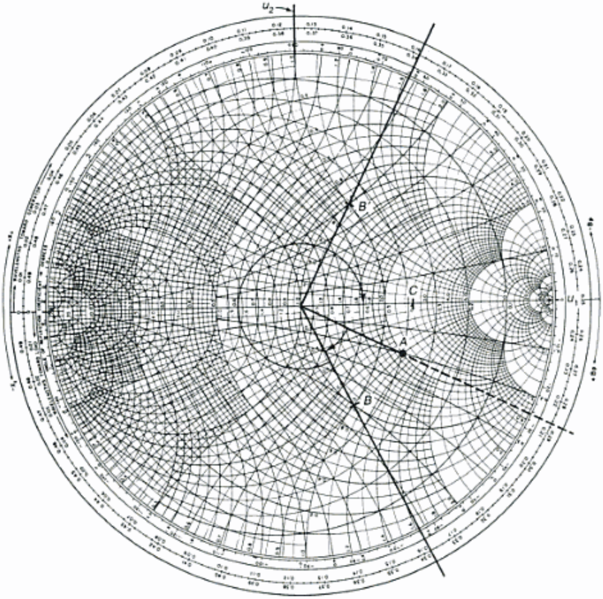


Exhibit 8.13d

8.14 $G = 50 \text{ dB} = 10^5, f = 5 \times 10^9 \text{ Hz}, P_T = 6 \text{ watts}$

$$\lambda = \frac{v}{f} = \frac{3\times10^8}{5\times10^9} = 6\times10^{-2}\,\text{m}$$

$$A = \frac{\lambda^2 G}{4\pi} = \frac{(6\times10^{-2})^2 10^5}{4\pi} = 28.65 \text{ m}^2 \text{ effective antenna area}$$

$$\text{watts/m}^2 \text{ @ } 50\,\text{kM} = \frac{P_T G}{\text{surface area of sphere}} = \frac{6\times10^5}{4\pi(50\times10^3)^2}$$

$$= 1.91\times10^{-5} \text{ watts/m}^2$$

$$P_{\text{RCVR}} = A[\text{w/m}^2 \text{ @ } 50\text{kM}] = 28.65 \times 1.91 \times 10^{-5} = 5.47 \times 10^{-4} \text{ watt}$$

$$\text{Path Loss} = -10\log \text{G} = -10\log\frac{4\pi}{\lambda^2}(4\pi r^2) = -10\log\frac{4\pi r^2}{\lambda}$$

$$= -20\log\frac{4\pi\times50\times10^3}{6\times10^{-2}} = -20\log 1.05\times10^7$$

$$= -140\log 1.05 = 140.4 \text{ dB}$$

8.15 The rms value of the resultant electric field is given by the formula

$$\varepsilon_r = \varepsilon_{rms}\cos\left(\pi n\sin\phi + \frac{\delta}{2}\right)$$

where

δ is the phase angle between I_A and I_B, and is positive when I_A leads I_B.
n is the distance between A and B in wavelengths, and is usually fractional.

Angle ϕ is measured from the normal to the line of the antennas.

Thus,

$$\varepsilon_r = \varepsilon_{rms}\cos\left(\frac{\pi}{2}\sin\phi + \frac{\pi}{4}\right).$$

For maximum ε_r,

$$\cos\left(\frac{\pi}{2}\sin\phi + \frac{\pi}{4}\right) = 1$$

or

$$\left(\frac{\pi}{2}\sin\phi + \frac{\pi}{4}\right) = 0$$

$$\sin\phi = 0.5, \quad \phi = 30°$$

$$\therefore \Theta = 60°.$$

At $\Theta = 90°$

$$\cos\left(\frac{\pi}{2}\sin 0° + \frac{\pi}{4}\right) = \cos\frac{\pi}{4} = 0.707.$$

The field strength of the array at 25 kM is

$$\varepsilon_i = 0.707 \times 250\frac{mv}{m}\left[\frac{40}{25}\right]^2 = 452.5 \text{ mv/m}.$$

8.16 a. The in-phase input is as follows:

$$(A\cos\omega_C t)(\phi\cos\omega_m t) = \frac{A\phi}{2}[\cos(\omega_C - \omega_m)t - \cos(\omega_C + \omega_m)t].$$

The 90° phase shift is as follows:

$$(A\sin\omega_c t)(\phi\sin\omega_m t) = \frac{A\phi}{2}[\cos(\omega_C - \omega_m)t + \cos(\omega_C + \omega_m)t].$$

The sum of above two expressions is

$$A\phi\cos(\omega_C - \omega_m)t.$$

b. Denoting the audio wave frequency with ω_2, we obtain

$$v_0(t) = A(1+0.5\cos\omega_1 t + 0.4\cos\omega_2 t)\cos\omega_0 t$$
$$= A\cos\omega_0 t + \frac{0.5A}{2}\cos(\omega_0+\omega_1)t + \frac{0.5A}{2}\cos(\omega_0-\omega_1)t$$
$$+\frac{0.4A}{2}\cos(\omega_0+\omega_2)t + \frac{0.4A}{2}\cos(\omega_0-\omega_2)t.$$

Then the effective radiated power is

$$P = \frac{A^2 + 2\left(\frac{0.5A}{2}\right)^2 + \left(\frac{0.4A}{2}\right)^2}{2} = 10.125 + 9\left[2\left(\frac{0.4}{2}\right)^2\right]$$
$$= 10.125 + 0.72 = 10.845 \text{ kW}.$$

The frequency spectrum is as shown in Exhibit 8.17.

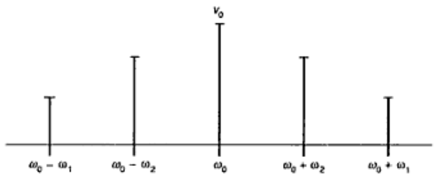


Exhibit 8.17

8.18 The number of different possible pictures is:

$$P = 10 \times 10 \ldots \times 10 = 10^{1200\times100} = 10^{1.2\times10^5}$$

Probability of each element or picture is

$$= \frac{1}{10^{1.2\times10^5}}.$$

The channel capacity is defined as

$C = \lim_{T\to\infty} \frac{1}{T}\log_2 M(T)$ where $M(T)$ is the total number of messages in T seconds.

$= S\log_2 P$ bit/sec where S is the signaling speed.

$S = 15$

$C = 15\log_2 10^{1.2\times10^5}$

$= 1.8\times10^6 \log_2 10$

$= 6.0\times10^6$ bit/sec

Assumption: It is assumed that the signal-to-noise ratio is large or the error probability is small so the channel is deemed noiseless.

8.19 a. This is the single-stage, single-server queuing model with Poisson arrivals and exponential service. Any basic operations research text gives the desired queue equations (*e.g.* Sasieni, Yaspan, and Friedman: *Operations Research-Methods and Problems*, John Wiley, pp. 126–138).

mean arrival rate $\lambda = 10$ transmissions/hour
mean service time $= 1/\mu = 4$ minutes $= 1/5$ hour
mean service rate $\mu = 15$ recordings/hour.

Average waiting time of an arrival

$$E(w) = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{10}{15(15-10)} = \frac{10}{75}\text{ hour } = 8 \text{ minutes}$$

b. This is the single-stage, two-server queuing situation. The various expectations for this model may also be obtained from a basic operations research (or queuing) text.

Before proceeding, a quick check can be made. We know a second recorder will substantially reduce (but not eliminate) waiting time. If the elimination of waiting time would not justify the second recorder then we need not bother to make the exact computation. Instead, we could simply conclude the second recorder is not economically warranted.

Hourly saving (assuming elimination of waiting time)

= mean arrival rate × mean waiting time reduction × line charge
$= \lambda[E(w_1) - E(w_2)] \times 0.04 = (10)(8-0)(0.04) = \$3.20$

Hourly cost = \$2.50

Thus we have been unable to show that the second recorder is uneconomical at zero waiting time. We must, therefore, proceed to compute the expectation of average waiting time.

$$P_0 = \frac{1}{\left[\sum_{n=0}^{k-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right] + \frac{1}{k!}\left(\frac{\lambda}{\mu}\right)^k \frac{k\mu}{k\mu-1}}$$

For two service facilities

$$k=2 \quad \lambda=10 \quad \mu=15$$

$$P_0 = \frac{1}{\frac{\lambda}{\mu} + \frac{1}{2}\left(\frac{\lambda}{\mu}\right)^2 \frac{2\mu}{2\mu-\lambda}} = \frac{1}{\frac{10}{15}+\frac{100}{450}\frac{30}{20}} = \frac{1}{1} = 1.$$

Average waiting time of an arrival

$$E(w) = \frac{\mu\left(\frac{\lambda}{\mu}\right)^k}{(k-1)!(k\mu-\lambda)^2} P_0 = \frac{15\cdot\left(\frac{10}{15}\right)^2}{(30-10)^2}(1) = \frac{1}{60} \text{ hour}$$

$$= 1 \text{ minute.}$$

$$\text{Hourly saving of second recorder} = \lambda[E(w_1) - E(w_2)] \times 0.04$$

$$= (10)(8.0 - 1.0)(0.04) = \$2.80.$$

Thus the second recorder is economically warranted, for the hourly savings exceeds the hourly cost.

RECOMMENDED REFERENCES

Sasieni, Yaspan, and Friedman, *Operations Research-Methods and Problems,* John Wiley, 1964, pp. 126–138.

Schwartz. *Information Transmission, Modulation and Noise*, McGraw-Hill, 1990, pp. 106, 107.

Stevenson, William D. Jr., *Elements of Power System Analysis,* McGraw-Hill, 1982, pp. 100–106. Also see the 1994 revised edition referenced at the end of Chapter 3.

CHAPTER 9

Biomedical Instrumentation and Safety

OUTLINE

PROBLEMS

9.1 The output of a pair of electrocardiograph skin electrodes is to be amplified with the differential instrumentation amplifier shown in Exhibit 9.1. The minimum peak differential signal amplitude expected from the electrodes is 0.5mV. The maximum peak differential signal amplitude expected from the electrodes is 4.0 mV. The minimum peak signal amplitude desired from the differential amplifier output is 0.5V.

The gain of the output amplifier stage from V_c and V_d to V_o is desired to be 40 times the gain of the output amplifier state from V^- and V^+ to V_c and V_d.

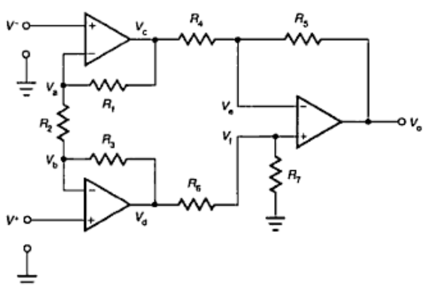


Exhibit 9.1

The differential gain of the entire amplifier (G_D) is the differential gain of the output stage (G_{Dout}) times the differential gain of the input stage (G_{Din}), or $G_D = G_{Dout} \cdot G_{Din}$.

a. If R_2 is 100 kΩ, R_4 is 10 kΩ, and R_6 is 10 kΩ, what are the values of R_1, R_3, R_5, and R_7? Assume ideal op-amps.

b. If the op-amps are capable of producing an output voltage within 1 V of the power supply voltage, what should the power supply voltage be so that the maximum peak signal amplitude from the differential amplifier output is not clipped?

9.2 For the differential instrumentation amplifier shown in Exhibit 9.1

a. What is the best CMRR that can be achieved using ±10% tolerance resistors, if all seven resistors were desired to be identical? Assume ideal op-amps.

b. What is the minimum resistor tolerance necessary to achieve a CMRR of at least 100 dB? Assume ideal op-amps.

9.3 Exhibit 9.2 shows a microshock circuit.

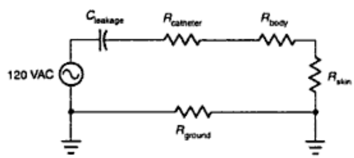


Exhibit 9.2

a. If an isolation amplifier is inserted between the pressure transducer and the power supply, what is the minimum value of the series isolation resistance necessary to reduce the leakage current to one tenth of the maximum safe current limit?

b. The engineer installing the isolation amplifier is concerned about saline solution from the catheter being spilled on the isolation amplifier circuitry and thus bridging the isolation protection with a low impedance leakage path. The isolation amplifier is 10 mm wide, and 20 mm long from the input terminals to the output terminals.

A column of saline solution has cross-sectional resistance per length of about 200 Ω mm^2/mm for each mm of column length per each mm^2 of column surface area. Thus, the total resistance of a saline column is (200 Ω mm) · (saline column length in mm)/(saline column cross-sectional area in mm^2). How deep would a layer of saline solution spanning the isolation amplifier have to be for the leakage current to reach the maximum safe microshock current limit?

c. The engineer installing the isolation amplifier decides to encapsulate the amplifier with a plastic that has a primarily dielectric insulation characteristic. The leakage current through the effective encapsulation capacitor between the input and output of the isolation amplifier is to be no greater that the leakage current through the isolation amplifier. What is the maximum value of the encapsulation capacitor?

APPENDIX A

Engineering Economics

PROBLEMS

A.1 A loan was made 2½ years ago at 8% simple annual interest. The principal amount of the loan has just been repaid along with \$600 of interest. The principal amount of the loan was closest to

a. \$300 c. \$4000
b. \$3000 d. \$5000

A.2 A \$1000 loan was made at 10% simple annual interest. It will take how many years for the amount of the loan and interest to equal \$1700?

a. 6 years c. 8 years
b. 7 years d. 9 years

A.3 A retirement fund earns 8% interest, compounded quarterly. If \$400 is deposited every three months for 25 years, the amount in the fund at the end of 25 years is nearest to

a. \$50,000 c. \$100,000
b. \$75,000 d. \$125,000

A.4 For some interest rate *i*, and some number of interest periods *n*, the uniform series capital recovery factor is 0.2091 and the sinking fund factor is 0.1941. The interest rate *i* must be closest to

a. 1½% c. 3%
b. 2% d. 4%

A.5 The repair costs for some handheld equipment is estimated to be \$120 the first year, increasing by \$30 per year in subsequent years. The amount a person will need to deposit into a bank account paying 4% interest to provide for the repair costs for the next five years is nearest to

a. \$500 c. \$700
b. \$600 d. \$800

A.15 If 10% nominal annual interest is compounded daily, the effective annual interest rate is nearest to
a. 10.00% c. 10.50%
b. 10.38% d. 10.75%

A.16 If 10% nominal annual interest is compounded continuously, the effective annual interest rate is nearest to
a. 10.00% c. 10.50%
b. 10.38% d. 10.75%

A.17 If the quarterly effective interest rate is 5½% with continuous compounding, the nominal interest rate is nearest to
a. 5.5% c. 16.5%
b. 11.0% d. 21.4%

A.18 A continuously compounded loan has what effective interest rate if the nominal interest rate is 25%?
a. $e^{1.25}$ c. $e^{0.25} - 1$
b. $e^{0.25}$ d. ln(1.25)

A.19 A continuously compounded loan has what *nominal interest rate* if the *effective interest rate* is 25%?
a. $e^{1.25}$ c. ln(1.25)
b. $e^{0.25}$ d. $\log_{10}(1.25)$

A.20 An individual wishes to deposit a certain quantity of money now so that he will have $500 at the end of five years. With interest at 4% per year, compound semiannually, the amount of the deposit is nearest to
a. $340 c. $410
b. $400 d. $608

A.21 A steam boiler is purchased on the basis of guaranteed performance. A test indicates that the operating cost will be $300 more per year than the manufacturer guaranteed. If the expected life of the boiler is 20 years, and money is worth 8%, the amount the purchaser should deduct from the purchase price to compensate for the extra operating cost is nearest to
a. $2950 c. $4100
b. $3320 d. $5520

A.22 A consulting engineer bought a fax machine with one year's free maintenance. In the second year the maintenance is estimated at $20. In subsequent years the maintenance cost will increase $20 per year (that is, third year maintenance will be $40, fourth year maintenance will be $60, and so forth). The amount that must be set aside now at 6% interest to pay the maintenance costs on the fax machine for the first six years of ownership is nearest to
a. $101 c. $229
b. $164 d. $284

A.23 An investor is considering buying a 20-year corporate bond. The bond has a face value of $1000 and pays 6% interest per year in two semiannual payments. Thus the purchaser of the bond will receive $30 every six months, and in addition he will receive $1000 at the end of 20 years, along with the last $30 interest payment. If the investor believes he should receive 8% annual interest, compounded semiannually, the amount he is willing to pay for the bond value of closest to

a. $500 c. $800
b. $600 d. $900

A.24 Annual maintenance costs for a particular section of highway pavement are $2000. The placement of a new surface would reduce the annual maintenance cost to $500 per year for the first five years and to $1000 per year for the next five years. The annual maintenance after 10 years would again be $2000. If maintenance costs are the only saving, the maximum investment that can be justified for the new surface, with interest at 4%, is closest to

a. $5,500 c. $10,000
b. $7,170 d. $10,340

A.25 A project has an initial cost of $10,000, uniform annual benefits of $2400, and a salvage value of $3000 at the end of its 10-year useful life. At 12% interest the net present worth of the project is closest to

a. $2,500 c. $4,500
b. $3,500 d. $5,500

A.26 A person borrows $5000 at an interest rate of 18%, compounded monthly. Monthly payments of $167.10 are agreed upon. The length of the loan is closest to

a. 12 months c. 24 months
b. 20 months d. 40 months

A.27 A machine costing $2000 to buy and $300 per year to operate will save labor expenses of $650 per year for eight years. The machine will be purchased if its salvage value at the end of eight years is sufficiently large to make the investment economically attractive. If an interest rate of 10% is used, the minimum salvage value must be closest to

a. $100 c. $300
b. $200 d. $400

A.28 The amount of money deposited 50 years ago at 8% interest that would now provide a perpetual payment of $10,000 per year is nearest to

a. $3,000 c. $50,000
b. $8,000 d. $70,000

A.29 An industrial firm must pay a local jurisdiction the cost to expand its sewage treatment plant. In addition, the firm must pay $12,000 annually toward the plant operating costs. The industrial firm will pay sufficient money into a fund that earns 5% per year, to pay its share of the plant operating costs forever. The amount to be paid to the fund is nearest to

a. $15,000 c. $160,000
b. $60,000 d. $240,000

A.30 At an interest rate of 2% per month, money will double in value in how many months?

a. 20 months
b. 35 months
c. 50 months
d. 65 months

A.31 A woman deposited $10,000 into an account at her credit union. The money was left on deposit for 80 months. During the first 50 months the woman earned 12% interest, compounded monthly. The credit union then changed its interest policy so that the woman earned 8% interest compounded quarterly during the next 30 months. The amount of money in the account at the end of 80 months is nearest to

a. $10,000
b. $15,000
c. $20,000
d. $25,000

A.32 An engineer deposited $200 quarterly in her savings account for three years at 6% interest, compounded quarterly. Then for five years she made no deposits or withdrawals. The amount in the account after eight years is closest to

a. $1200
b. $1800
c. $2400
d. $3600

A.33 A sum of money, Q, will be received six years from now. At 6% annual interest the present worth now of Q is $60. At this same interest rate the value of Q 10 years from now is closest to

a. $60
b. $77
c. $90
d. $107

A.34 If $200 is deposited in a savings account at the beginning of each of 15 years and the account earns interest at 6%, compounded annually, the value of the account at the end of 15 years will be most nearly

a. $4500
b. $4700
c. $4900
d. $5100

A.35 The maintenance expense on a piece of machinery is estimated as follows:

Year	1	2	3	4
Maintenance	$150	$300	$450	$600

If interest is 8%, the equivalent uniform annual maintenance cost is closest to

a. $250
b. $300
c. $350
d. $400

A.36 A payment of $12,000 six years from now is equivalent, at 10% interest, to an annual payment for eight years starting at the end of this year. The annual payment is closest to

a. $1000
b. $1200
c. $1400
d. $1600

A.37 A manufacturer purchased $15,000 worth of equipment with a useful life of six years and a $2000 salvage value at the end of the six years. Assuming a 12% interest rate, the equivalent uniform annual cost is nearest to
a. $1500 c. $3500
b. $2500 d. $4500

A.38 Consider a machine as follows:

Initial cost: $80,000

End-of-useful-life salvage value: $20,000

Annual operating cost: $18,000

Useful life: 20 years

Based on 10% interest, the equivalent uniform annual cost for the machine is closest to
a. $21,000 c. $25,000
b. $23,000 d. $27,000

A.39 Consider a machine as follows:

Initial cost: $80,000

Annual operating cost: $18,000

Useful life: 20 years

What must be the salvage value of the machine at the end of 20 years for the machine to have an equivalent uniform annual cost of $27,000? Assume a 10% interest rate. The salvage value is closest to
a. $10,000 c. $40,000
b. $20,000 d. $50,000

A.40 Twenty-five thousand dollars is deposited in a savings account that pays 5% interest, compounded semiannually. Equal annual withdrawals are to be made from the account beginning one year from now and continuing forever. The maximum amount of the equal annual withdrawals is closest to
a. $625 c. $1250
b. $1000 d. $1265

A.41 An investor is considering investing $10,000 in a piece of land. The property taxes are $100 per year. The lowest selling price the investor must receive if she wishes to earn a 10% interest rate after keeping the land for 10 years is
a. $21,000 c. $27,000
b. $23,000 d. $31,000

A.42 The rate of return of a $10,000 investment that will yield $1000 per year for 20 years is closest to
a. 1% c. 8%
b. 4% d. 12%

A.43 An engineer invested $10,000 in a company. In return he received $600 per year for six years and his $10,000 investment back at the end of the six years. His rate of return on the investment was closest to

a. 6% c. 12%
b. 10% d. 15%

A.44 An engineer made 10 annual end-of-year purchases of $1000 of common stock. At the end of the tenth year, just after the last purchase, the engineer sold all the stock for $12,000. The rate of return received on the investment is closest to

a. 2% c. 8%
b. 4% d. 10%

A.45 A company is considering buying a new piece of machinery.

Initial cost: $80,000

End-of-useful-life salvage value: $20,000

Annual operating cost: $18,000

Useful life: 20 years

The machine will produce an annual savings in material of $25,700. What is the before-tax rate of return if the machine is installed? The rate of return is closest to

a. 6% c. 10%
b. 8% d. 15%

A.46 Consider the following situation: Invest $100 now and receive two payments of $102.15—one at the end of Year 3, and one at the end of Year 6. The rate of return is nearest to

a. 8% c. 18%
b. 12% d. 22%

A.47 Two mutually exclusive alternatives are being considered:

Year	A	B
0	−$2500	−$6000
1	+746	+1664
2	+746	+1664
3	+746	+1664
4	+746	+1664
5	+746	+1664

The rate of return on the difference between the alternatives is closest to

a. 6% c. 10%
b. 8% d. 12%

A.53 Given two machines:

	A	B
Initial cost	$55,000	$75,000
Total annual costs	$16,200	$12,450

With interest at 10% per year, at what service life do these two machines have the same equivalent uniform annual cost? The service life is closest to

a. 5 years
b. 6 years
c. 7 years
d. 8 years

A.54 A machine part that is operating in a corrosive atmosphere is made of low-carbon steel. It costs $350 installed, and lasts six years. If the part is treated for corrosion resistance it will cost $700 installed. How long must the treated part last to be as economic as the untreated part, if money is worth 6%?

a. 8 years
b. 11 years
c. 15 years
d. 17 years

A.55 A firm has determined the two best paints for its machinery are Tuff-Coat at $45 per gallon and Quick at $22 per gallon. The Quick paint is expected to prevent rust for five years. Both paints take $40 of labor per gallon to apply, and both cover the same area. If a 12% interest rate is used, how long must the Tuff-Coat paint prevent rust to justify its use?

a. 5 years
b. 6 years
c. 7 years
d. 8 years

A.56 Two alternatives are being considered:

	A	B
Cost	$1000	$2000
Useful life in years	10	10
End-of-useful-life salvage value	100	400

The net annual benefit of *A* is $150. If interest is 8%, what must be the net annual benefit of *B* for the two alternatives to be equally desirable? The net annual benefit of *B* must be closest to

a. $150
b. $200
c. $225
d. $275

A.57 Which one of the following is *NOT* a method of depreciating plant equipment for accounting and engineering economics purposes?

a. double-entry method
b. modified accelerated cost recovery system
c. sum-of-years-digits method
d. straight-line method

A.66 An individual bought a one-year savings certificate for $10,000, and it pays 6%. He has a taxable income that puts him at the 28% incremental income tax rate. His after-tax rate of return on this investment is closest to

a. 2% c. 4%
b. 3% d. 5%

A.67 A tool costing $300 has no salvage value. Its resulting before-tax cash flow is shown in the following partially completed cash flow table.

Year	Before-Tax Cash Flow	Effect on SOYD Deprec	Effect on Taxable Income	Income Taxes	After-Tax Cash Flow
0	−$300				
1	+100				
2	+150				
3	+200				

The tool is to be depreciated over three years using sum-of-years-digits depreciation. The income tax rate is 50%. The after-tax rate of return is nearest to

a. 8% c. 12%
b. 10% d. 15%

A.68 An engineer is considering the purchase of an annuity that will pay $1000 per year for 10 years. The engineer feels he should obtain a 5% rate of return on the annuity after considering the effect of an estimated 6% inflation per year. The amount he would be willing to pay to purchase the annuity is closest to

a. $1500 c. $4500
b. $3000 d. $6000

A.69 An automobile costs $20,000 today. You can earn 12% tax free on an "auto purchase account." If you expect the cost of the auto to increase by 10% per year, the amount you would need to deposit in the account to provide for the purchase of the auto five years from now is closest to

a. $12,000 c. $16,000
b. $14,000 d. $18,000

A.70 An engineer purchases a building lot for $40,000 cash and plans to sell it after five years. If he wants an 18% before-tax rate of return, after taking the 6% annual inflation rate into account, the selling price must be nearest to

a. $100,000 c. $150,000
b. $125,000 d. $175,000

A.71 A piece of equipment with a list price of $450 can actually be purchased for either $400 cash or $50 immediately plus four additional annual payments of $115.25. All values are in dollars of current purchasing power. If the typical customer considered a 5% interest rate appropriate, the inflation rate at which the two purchase alternatives are equivalent is nearest to

a. 5% c. 8%
b. 6% d. 10%

A.72 A man wants to determine whether to invest $1000 in a friend's speculative venture. He will do so if he thinks he can get this money back. The probabilities of the various outcomes at the end of one year are:

Result	Probability
$2000 (double his money)	0.3
1500	0.1
1000	0.2
500	0.3
0 (lose everything)	0.1

His expected outcome if he invests the $1000 is closest to

a. $800 c. $1000
b. $900 d. $1100

A.73 The amount you would be willing to pay for an insurance policy protecting you against a one in twenty chance of losing $10,000 three years from now, if interest is 10%, is closest to

a. $175 c. $1000
b. $350 d. $1500

SOLUTIONS

A.1 b.

$$F = P + Pin$$
$$600 + P = P + P(0.08)(2.50)$$
$$P = [600]/[0.08(2.50)] = \$3000$$

A.2 b.

$$F = P + Pin$$
$$1700 = 1000 + 1000(0.10)(n)$$
$$n = [700]/[1000(0.10)] = 7 \text{ years}$$

A.3 d.

$$F = A(F/A,i,n) = 400(F/A,2\%,100)$$
$$= 400(312.33) = \$124,890$$

A.4 a. The relationship between the capital recovery factor and the sinking fund factor is $(A/P,i,n) = (A/F,i,n) + i$. Substituting the values in the problem

$$0.2091 = 0.1941 + i$$
$$i = 0.2091 - 0.1941 = 0.015 = 1\frac{1}{2}\%$$

A.5 d.

$$P = A(P/A,i,n) + G(P/G,i,n)$$
$$= 120(P/A,4\%,5) + 30(P/G,4\%,5)$$
$$= 120(4.452) + 30(8.555) = \$791$$

A.6 d.

A.7 c.

At i = 1%/month: $F = 1000(1 + 0.01)^{12} = \$1126.83$

At i = 12%/year: $F = 1000(1 + 0.12)^{1} = \$1120.00$

Saving in interest charges = 1126.83 − 1120.00 = $6.83

A.8 a.

$$P = Fe^{-rn} = 10{,}000e^{-0.09(10)} = 4066$$

A.9 c. The nominal interest rate is the annual interest rate ignoring the effect of any compounding. Nominal interest rate = 1½% × 12 = 18%.

A.10 d. The interest paid per year = 330 × 4 = 1320. The nominal annual interest rate = 1320/10,000 = 0.132 = 13.2%.

A.11 d.

$$i_{\text{eff}} = (1 + r/m)^m - 1 = (1 + 0.03)^6 - 1 = 0.194 = 19.4\%$$

A.12 d.

$$i_{\text{eff}} = (1 + r/m)^m - 1$$
$$r/m = (1 + i_{\text{eff}})^{1/m} - 1 = (1 + 0.1956)^{1/12} = 0.015$$
$$r = 0.015(m) = 0.015 \times 12 = 0.18 = 18\%$$

A.13 a.

$$i_{\text{eff}} = 20.52/300 = 0.0684 = 6.84\%$$

A.14 d.

$$i_{\text{eff}} = (1 + r/m)^m - 1$$
$$0.12 = (1 + r/12)^{12} - 1$$
$$(1.12)^{1/12} = (1 + r/12)$$
$$1.00949 = (1 + r/12)$$
$$r = 0.00949 \times 12 = 0.1138 = 11.38\%$$

A.15 c.

$$i_{eff} = (1 + r/m)^m - 1 = (1 + 0.10/365)^{365} - 1 = 0.1052 = 10.52\%$$

A.16 c.

$$i_{eff} = e^r - 1$$

where r = nominal annual interest rate

$$i_{eff} = e^{0.10} - 1 = 0.10517 = 10.52\%$$

A.17 d. For 3 months: $i_{eff} = e^r - 1$; $0.055 = e^r - 1$
The rate per quarter year is $r = \ln(1.055) = 0.05354$; $r = 4 \times 0.05354$
$= 0.214 = 21.4\%$ per year.

A.18 c.

$$i_{eff} = e^r - 1 = e^{0.25} - 1$$

A.19 c.

$$i_{eff} = e^r - 1 = 0.25$$
$$e^r = 1.25$$
$$\ln(e^r) = \ln(1.25)$$
$$r = \ln(1.25)$$

A.20 c.

$$P = F(P/F,i,n) = 500(P/F,2\%,10) = 500(0.8203) = \$410$$

A.21 a.

$$P = 300(P/A,8\%,20) = 300(9.818) = \$2945$$

A.22 c. Using single payment present worth factors:

$$P = 20(P/F,6\%,2) + 40(P/F,6\%,3) + 60(P/F,6\%,4)$$
$$+ 80(P/F,6\%,5) + 100(P/F,6\%,6) = \$229$$

Alternate solution using the gradient present worth factor:

$$P = 20(P/G,6\%,6) = 20(11.459) = \$229$$

A.23 c.

$$PW = 30(P/A,4\%,40) + 1000(P/F,4\%,40)$$
$$= 30(19.793) + 1000(0.2083) = \$802$$

A.24 d. Benefits are \$1500 per year for the first five years and \$1000 per year for the subsequent five years.

As Exhibit A.24 indicates, the benefits may be considered as \$1000 per year for ten years, plus an additional \$500 benefit in each of the first five years.

$$\begin{aligned}\text{Maximum investment} &= \text{Present worth of benefits}\\ &= 1000(P/A, 4\%, 10) + 500(P/A, 4\%, 5)\\ &= 1000(8.111) + 500(4.452) = \$10{,}337\end{aligned}$$

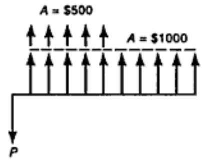


Exhibit A.24

A.25 c.

$$\begin{aligned}\text{NPW} &= \text{PW of benefits} - \text{PW of cost}\\ &= 2400(P/A, 12\%, 10) + 3000(P/F, 12\%, 10) - 10{,}000 = \$4526\end{aligned}$$

A.26 d.

$$\begin{aligned}\text{PW of benefits} &= \text{PW of cost}\\ 5000 &= 167.10(P/A, 1.5\%, n)\\ (P/A, 1.5\%, n) &= 5000/167.10 = 29.92\end{aligned}$$

From the $1\frac{1}{2}\%$ interest table, $n = 40$.

A.27 c.

$$\begin{aligned}\text{NPW} &= \text{PW of benefits} - \text{PW of cost} = 0\\ &= (650 - 300)(P/A, 10\%, 8) + S_8(P/F, 10\%, 8) - 2000 = 0\\ S_8 &= 132.75/0.4665 = \$285\end{aligned}$$

A.28 a. The amount of money needed now to begin the perpetual payments is $P' = A/i = 10{,}000/0.08 = 125{,}000$. From this we can compute the amount of money, P, that would need to have been deposited 50 years ago:

$$P = 125{,}000(P/F, 8\%, 50) = 125{,}000(0.0213) = \$2663$$

A.29 d.

$$P = A/i = 12{,}000/0.05 = \$240{,}000$$

A.30 b.

$$\begin{aligned}2 &= 1(F/P, i, n)\\ (F/P, 2\%, n) &= 2\end{aligned}$$

From the 2% interest table, n = about 35 months.

A.31 c. At end of 50 months

$$F = 10{,}000(F/P, 1\%, 50) = 10{,}000(1.645) = \$16{,}450$$

At end of 80 months

$$F = 16{,}450(F/P, 2\%, 10) = 16{,}450(1.219) = \$20{,}053$$

A.32 d.

$$\begin{aligned} FW &= 200(F/A,1.5\%,12)(F/P,1.5\%,20) \\ &= 200(13.041)(1.347) = \$3513 \end{aligned}$$

A.33 d. The present sum $P = 60$ is equivalent to Q six years hence at 6% interest. The future sum F may be calculated by either of two methods:

$$F = Q(F/P,6\%,4) \quad \text{and} \quad Q = 60(F/P,6\%,6)$$
$$F = P(F/P,6\%,10)$$

Since P is known, the second equation may be solved directly.

$$F = P(F/P,6\%,10) = 60(1.791) = \$107$$

A.34 c.

$$F' = A(F/A, i, n) = 200(F/A, 6\%, 15) = 200(23.276) = \$4655.20$$
$$F = F'(F/P, i, n) = 4655.20(F/P, 6\%, 1) = 4655.20(1.06) = \$4935$$

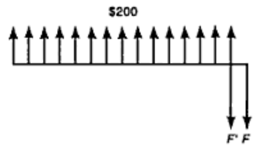


Exhibit A.34

A.35 c.

$$EUAC = 150 + 150(A/G,8\%,4) = 150 + 150(1.404) = \$361$$

A.36 b.

$$\begin{aligned} \text{Annual payment} &= 12{,}000(P/F,10\%,6)(A/P,10\%,8) \\ &= 12{,}000(0.5645)(0.1874) = \$1269 \end{aligned}$$

A.37 c.

$$\begin{aligned} EUAC &= 15{,}000(A/P,12\%,6) - 2000(A/F,12\%,6) \\ &= 15{,}000(0.2432) - 2000(0.1232) = \$3402 \end{aligned}$$

A.38 d.

$$\begin{aligned} EUAC &= 80{,}000(A/P,10\%,20) - 20{,}000(A/F,10\%,20) \\ &\quad + \text{annual operating cost} \\ &= 80{,}000\ (0.1175) - 20{,}000\ (0.0175) + 18{,}000 \\ &= 9400 - 350 + 18{,}000 = \$27{,}050 \end{aligned}$$

A.39 b.

$$\begin{aligned} EUAC &= EUAB \\ 27{,}000 &= 80{,}000(A/P,10\%,20) + 18{,}000 - S(A/F,10\%,20) \\ &= 80{,}000(0.1175) + 18{,}000 - S(0.0175) \\ S &= (27{,}400 - 27{,}000)/0.0175 = \$22{,}857 \end{aligned}$$

A.40 d. The general equation for an infinite life, $P = A/i$, must be used to solve the problem.

$$i_{eff} = (1 + 0.025)^2 - 1 = 0.050625$$

The maximum annual withdrawal will be $A = Pi = 25{,}000(0.050625) = \$1266$.

A.41 c.

$$\text{Minimum sale price} = 10{,}000(F/P, 10\%, 10) + 100(F/A, 10\%, 10)$$
$$= 10{,}000(2.594) + 100(15.937) = \$27{,}530$$

A.42 c.

$$\text{NPW} = 1000(P/A, i, 20) - 10{,}000 = 0$$
$$(P/A, i, 20) = 10{,}000/1000 = 10$$

From interest tables: $6\% < i < 8\%$.

A.43 a. The rate of return was $= 600/10{,}000 = 0.06 = 6\%$.

A.44 b.

$$F = A(F/A, i, n)$$
$$12{,}000 = 1000(F/A, i, 10)$$
$$(F/A, i, 10) = 12{,}000/1000 = 12$$

In the 4% interest table: $(F/A, 4\%, 10) = 12.006$, so $i = 4\%$.

A.45 b.

PW of cost = PW of benefits

$$80{,}000 = (25{,}700 - 18{,}000)(P/A, i, 20) + (20{,}000(P/F, i, 20)$$

Try $i = 8\%$.

$$80{,}000 = 7709(9.818) + 20{,}000(0.2145) = 79{,}889$$

Therefore, the rate of return is very close to 8%.

A.46 c.

PW of cost = PW of benefits

$$100 = 102.15(P/F, i, 3) + 102.15(P/F, i, 6)$$

Solve by trial and error:

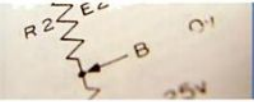
Try $i = 12\%$.

$$100 = 102.15(0.7118) + 102.15(0.5066) = 124.46$$

The PW of benefits exceeds the PW of cost. This indicates that the interest rate i is too low. Try $i = 18\%$.

$$100 = 102.15(0.6086) + 102.15(0.3704) = 100.00$$

Therefore, the rate of return is 18%.



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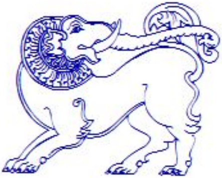

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