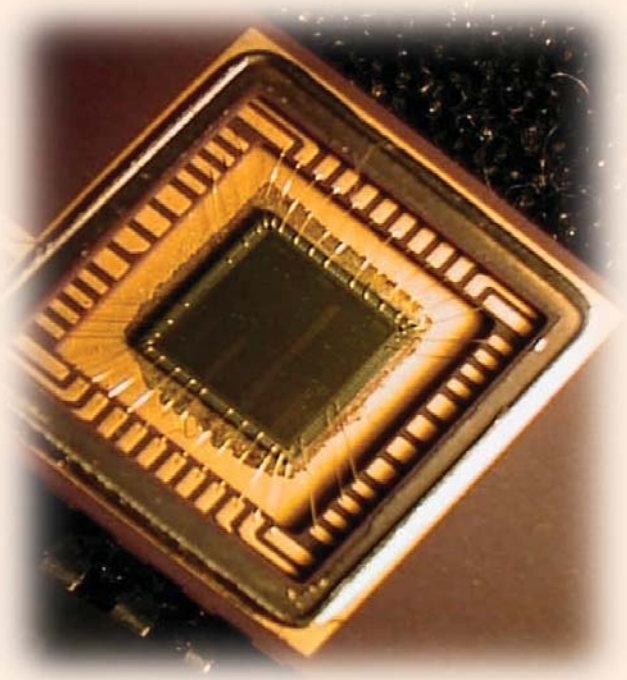
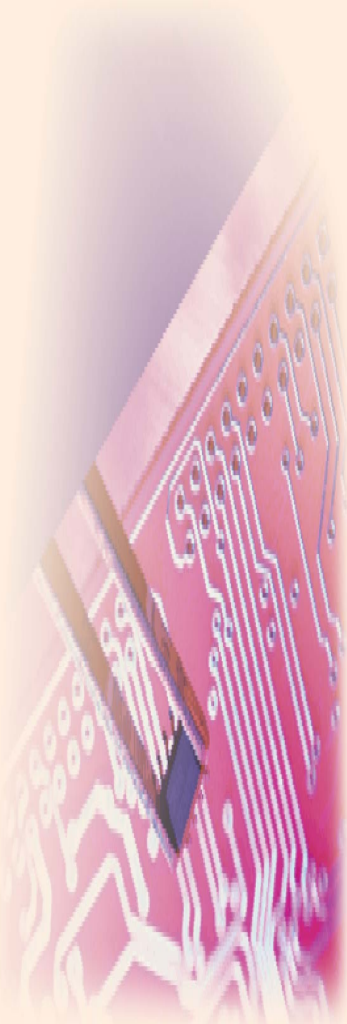
FIRST MULTICOLOUR EDITION

A TEXTBOOK OF

ELECTRICAL TECHNOLOGY

ALL VOLUME
BASIC ELECTRICAL ENGINEERING
IN
S.I. SYSTEM OF UNITS





A TEXTBOOK OF
ELECTRICAL TECHNOLOGY

VOLUME I
BASIC ELECTRICAL ENGINEERING

IN
S.I. SYSTEM OF UNITS

(Including rationalized M.K.S.A. System)
For the Examinations of B.E. (Common Course to All Branches), B.Tech., B.Sc. (Engg), Sec. A & B of AMIE(I), A.M.I.E.E. (London), I.E.R.E. (London), Grade I.E.T.E., Diploma and other Competitive Examinations

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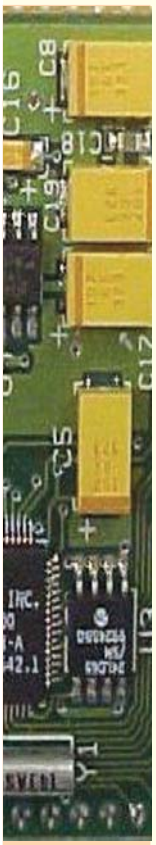
Revised by :

S.G. TARNEKAR
B.E. (Hons), M.Tech., (El. Machines)
Ph.D. (Electrical Power Systems)
Former Professor & Head, Electrical Engineering Department
Visvesvaraya National Institute of Technology, Nagpur

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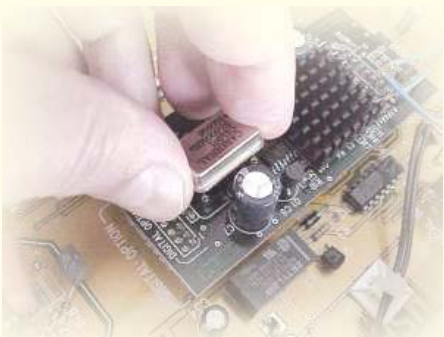
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Preface to the Twenty-Third Revised Multicoloured Edition

Authors feel happy to present to their esteemed readers this revised first **multicoloured** edition of Vol. I of "A Textbook of Electrical Technology". To provide a comprehensive treatment of topics in ''Basic Electrical Engineering'' both for electrical as well as non-electrical students pursuing their studies in civil, mechanical, mining, textile, chemical, industrial, environmental, aerospace, electronic and computer engineering, information technology both at the Degree and Diploma level. Based on the suggestions received from our esteemed readers, both from India and abroad, the scope of the book has been enlarged according to their requirements.

Establishment of Technological Universities have taken place in recent past. This resulted into a pool of expert manpower within a large area. Unification of syllabi has taken place and the question papers set during the last 4-5 years have a wider variety and are of more inquisitive nature. Solutions to these with brief logical reasonings have been added for the benefit of our student community.

Many universities include a brief coverage on methods of "Electrical Power Generation", in their first and basic paper on this subject. Hence, this revision includes an introductory chapter on this topic.

It is earnestly hoped that with these extensive additions and revisions, this revised edition will prove even more useful to our numerous readers in developing more confidence while appearing at national competitive examinations.

I would like to thank my Publishers particularly Mr. Ravindra Kumar Gupta, M.D. and Mr. Bhagirath Kaushik, Regional Manager (Western India) of S. Chand & Company Ltd., for the personal interest they look in the publishing of this revised and enlarged edition.

Our student-friends, teacher-colleagues, Booksellers and University authorities have been showing immense faith and affection in our book, which is acknowledged with modesty and regards. We are sure that this revised edition will satisfy their needs to a still greater extent and serve its cause more effectively.

S.G. TARNEKAR



VOLUME – I

BASIC ELECTRICAL ENGINEERING

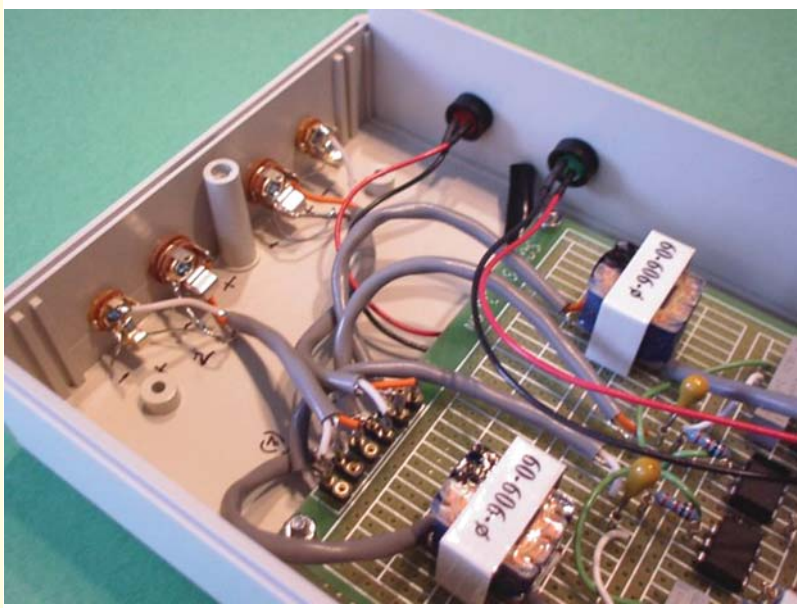
Contents

CHAPTER 1

ELECTRIC CURRENT AND OHM'S LAW

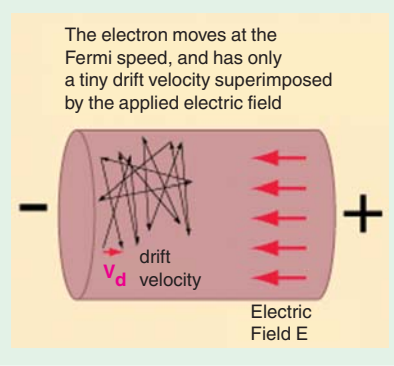
Learning Objectives

- Electron Drift Velocity
- Charge Velocity and Velocity of Field Propagation
- The Idea of Electric Potential Resistance
- Unit of Resistance
- Law of Resistance
- Units of Resistivity Conductance and Conductivity
- Temperature Coefficient of Resistance
- Value of α at Different Temperatures
- Variation of Resistivity with Temperature
- Ohm's Law
- Resistance in Series
- Voltage Divider Rule
- Resistance in Parallel
- Types of Resistors
- Nonlinear Resistors
- Varistor
- Short and Open Circuits
- 'Shorts' in a Series Circuit
- 'Opens' in Series Circuit
- 'Open's in a Parallel Circuit
- 'Shorts' in Parallel Circuits
- Division of Current in Parallel Circuits
- Equivalent Resistance
- Duality Between Series and Parallel Circuits
- Relative Potential
- Voltage Divider Circuits



Ohm's law defines the relationship between voltage, resistance and current. This law is widely employed while designing electronic circuits

1.1. Electron Drift Velocity



Suppose that in a conductor, the number of free electrons available per m^3 of the conductor material is n and let their axial drift velocity be v metres/second. In time dt, distance travelled would be $v \times dt$. If A is area of cross-section of the conductor, then the volume is $vAdt$ and the number of electrons contained in this volume is $vA\,dt$. Obviously, all these electrons will cross the conductor cross-section in time dt. If e is the charge of each electron, then total charge which crosses the section in time dt is $dq = nAev\,dt$.

Since current is the rate of flow of charge, it is given as

$$i = \frac{dq}{dt} = \frac{nAev\,dt}{dt} \quad \therefore\ i = nAev$$

Current density, $J = i/A = ne\,v$ ampere/metre^2

Assuming a normal current density $J = 1.55 \times 10^6$ A/m^2, $n = 10^{29}$ for a copper conductor and $e = 1.6 \times 10^{-19}$ coulomb, we get

$$1.55 \times 10^6 = 10^{29} \times 1.6 \times 10^{-19} \times v \quad \therefore v = 9.7 \times 10^{-5}\ \text{m/s} = 0.58\ \text{cm/min}$$

It is seen that contrary to the common but mistaken view, the electron drift velocity is rather very slow and is independent of the current flowing and the area of the conductor.

N.B. Current density *i.e.*, the current per unit area, is a vector quantity. It is denoted by the symbol $\vec{J}$.

Therefore, in vector notation, the relationship between current I and $\vec{J}$ is :

$$I = \vec{J} . \vec{a} \qquad \text{[where } \vec{a} \text{ is the vector notation for area '}a\text{']}$$

For extending the scope of the above relationship, so that it becomes applicable for area of any shape, we write :

$$I = J . da$$

The magnitude of the current density can, therefore, be written as $J \cdot \alpha$.

Example 1.1. *A conductor material has a free-electron density of 10^{24} electrons per metre^3. When a voltage is applied, a constant drift velocity of 1.5×10^{-2} metre/second is attained by the electrons. If the cross-sectional area of the material is 1 cm^2, calculate the magnitude of the current. Electronic charge is 1.6×10^{-19} coulomb.* **(Electrical Engg. Aligarh Muslim University)**

Solution. The magnitude of the current is

$$i = nAev \text{ amperes}$$

Here, $n = 10^{24}$; $A = 1\ \text{cm}^2 = 10^{-4}\ \text{m}^2$

$e = 1.6 \times 10^{-19}$ C ; $v = 1.5 \times 10^{-2}$ m/s

$\therefore$ $i = 10^{24} \times 10^{-4} \times 1.6 \times 10^{-19} \times 1.5 \times 10^{-2} =$ **0.24 A**

1.2. Charge Velocity and Velocity of Field Propagation

The speed with which charge drifts in a conductor is called the ***velocity of charge***. As seen from above, its value is quite low, typically fraction of a metre per second.

However, the *speed* with which the effect of e.m.f. is experienced at all parts of the conductor resulting in the flow of current is called the ***velocity of propagation of electrical field***. It is independent of current and voltage and has high but constant value of nearly 3×10^8 m/s.

Example 1.2. *Find the velocity of charge leading to 1 A current which flows in a copper conductor of cross-section 1 cm^2 and length 10 km. Free electron density of copper = 8.5 × 10^{28} per m^3. How long will it take the electric charge to travel from one end of the conductor to the other?*

Solution. $i = neA\nu$ or $\nu = i/neA$

$\therefore \quad \nu = 1/(8.5 \times 10^{28} \times 1.6 \times 10^{-19} \times 1 \times 10^{-4}) = 7.35 \times 10^{-7}$ m/s = **0.735 μm/s**

Time taken by the charge to travel conductor length of 10 km is

$$t = \frac{\text{distance}}{\text{velocity}} = \frac{10 \times 10^3}{7.35 \times 10^{-7}} = 1.36 \times 10^{10} \text{ s}$$

Now, 1 year = 365 × 24 × 3600 = 31,536,000 s

$$t = 1.36 \times 10^{10}/31{,}536{,}000 = \textbf{431 years}$$

1.3. The Idea of Electric Potential

In Fig. 1.1, a simple voltaic cell is shown. It consists of copper plate (known as anode) and a zinc rod (*i.e.* cathode) immersed in dilute sulphuric acid (H_2SO_4) contained in a suitable vessel. The chemical action taking place within the cell causes the electrons to be removed from copper plate and to be deposited on the zinc rod at the same time. This transfer of electrons is accomplished through the agency of the diluted H_2SO_4 which is known as the electrolyte. The result is that zinc rod becomes negative due to the deposition of electrons on it and the copper plate becomes positive due to the removal of electrons from it. The large number of electrons collected on the zinc rod is being attracted by anode but is prevented from returning to it by the force set up by the chemical action within the cell.

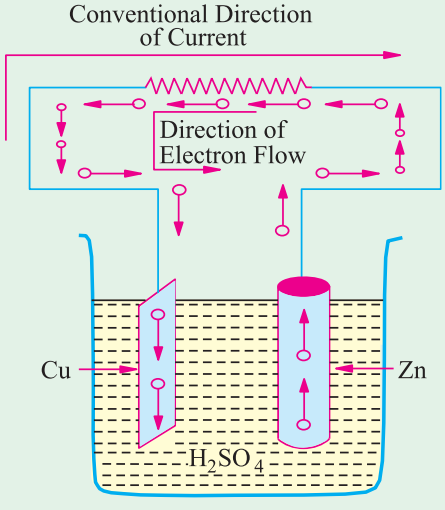


Fig. 1.1.

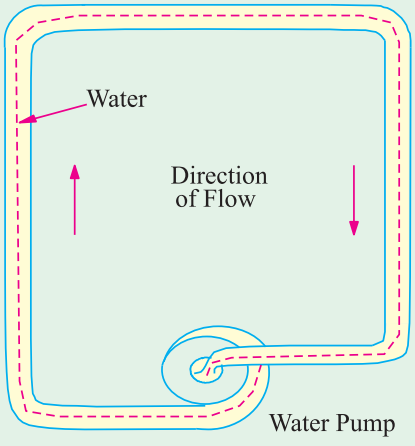


Fig. 1.2

But if the two electrodes are joined by a wire ***externally***, then electrons rush to the anode thereby equalizing the charges of the two electrodes. However, due to the continuity of chemical action, a continuous difference in the number of electrons on the two electrodes is maintained which keeps up a continuous flow of current through the external circuit. The action of an electric cell is similar to that of a water pump which, while working, maintains a continuous flow of water *i.e.,* water current through the pipe (Fig. 1.2).

It should be particularly noted that the direction of ***electronic*** current is from zinc to copper in the external circuit. However, the direction of ***conventional*** current (which is given by the direction

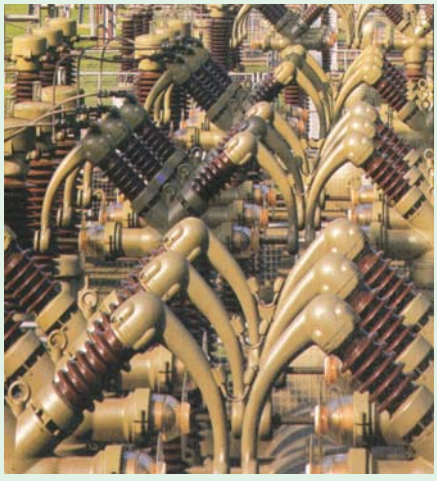
of flow of positive charge) is from copper to zinc. In the present case, there is no flow of positive charge as such from one electrode to another. But we can look upon the arrival of electrons on copper plate (with subsequent decrease in its positive charge) as equivalent to an actual departure of positive charge from it.

When zinc is negatively charged, it is said to be at negative potential with respect to the electrolyte, whereas anode is said to be at positive potential relative to the electrolyte. Between themselves, copper plate is assumed to be at a higher potential than the zinc rod. The difference in potential is continuously maintained by the chemical action going on in the cell which supplies energy to establish this potential difference.

1.4. Resistance

It may be defined as the property of a substance due to which it opposes (or restricts) the flow of electricity (*i.e.*, electrons) through it.

Metals (as a class), acids and salts solutions are good conductors of electricity. Amongst pure metals, silver, copper and aluminium are very good conductors in the given order.* This, as discussed earlier, is due to the presence of a large number of free or loosely-attached electrons in their atoms. These vagrant electrons assume a directed motion on the application of an electric potential difference. These electrons while flowing pass ***through*** the molecules or the atoms of the conductor, collide and other atoms and electrons, thereby producing heat.



Cables are often covered with materials that do not carry electric current easily

Those substances which offer relatively greater difficulty or hindrance to the passage of these electrons are said to be relatively poor conductors of electricity like bakelite, mica, glass, rubber, p.v.c. (polyvinyl chloride) and dry wood etc. Amongst good insulators can be included fibrous substances such as paper and cotton when dry, mineral oils free from acids and water, ceramics like hard porcelain and asbestos and many other plastics besides p.v.c. It is helpful to remember that electric friction is similar to friction in Mechanics.

1.5. The Unit of Resistance

The practical unit of resistance is ohm.** A conductor is said to have a resistance of one ohm if it permits one ampere current to flow through it when one volt is impressed across its terminals.

For insulators whose resistances are very high, a much bigger unit is used *i.e.,* mega-ohm = 10^6 ohm (the prefix 'mega' or mego meaning a million) or kilo-ohm = 10^3 ohm (kilo means thousand). In the case of very small resistances, smaller units like milli-ohm = 10^{-3} ohm or micro-ohm = 10^{-6} ohm are used. The symbol for ohm is Ω



George Simon Ohm

* However, for the same resistance per unit length, cross-sectional area of aluminium conductor has to be 1.6 times that of the copper conductor but it weighs only half as much. Hence, it is used where economy of weight is more important than economy of space.

** After George Simon Ohm (1787-1854), a German mathematician who in about 1827 formulated the law known after his name as Ohm's Law.

Table 1.1. Multiples and Sub-multiples of Ohm

Prefix	*Its meaning*	*Abbreviation*	*Equal to*
Mega-	One million	M Ω	10^6 Ω
Kilo-	One thousand	k Ω	10^3 Ω
Centi-	One hundredth	–	–
Milli-	One thousandth	m Ω	10^{-3} Ω
Micro-	One millionth	μ Ω	10^{-6} Ω

1.6. Laws of Resistance

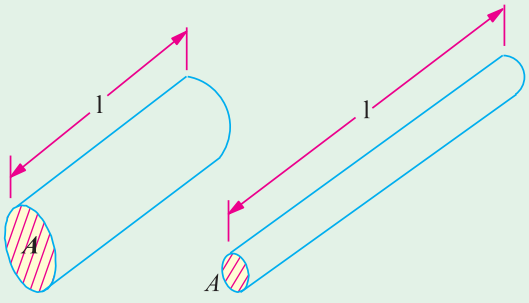
The resistance R offered by a conductor depends on the following factors :

(i) It varies directly as its length, l.

(ii) It varies inversely as the cross-section A of the conductor.

(iii) It depends on the nature of the material.

(iv) It also depends on the temperature of the conductor.



Smaller l — Larger A — Low R

Larger l — Smaller A — Greater R

Fig. 1.3.

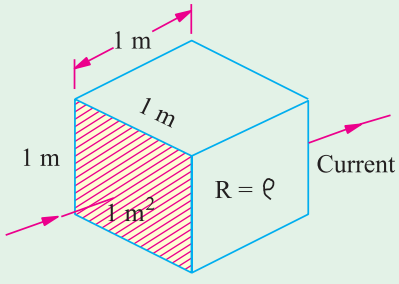


Fig. 1.4

Neglecting the last factor for the time being, we can say that

$$R \propto \frac{l}{A} \quad \text{or} \quad R = \rho \frac{l}{A} \qquad \text{...(i)}$$

where ρ is a constant depending on the nature of the material of the conductor and is known as its ***specific resistance*** **or** ***resistivity***.

If in Eq. ***(i)***, we put

l = 1 metre and A = 1 metre^2, then $R = \rho$ (Fig. 1.4)

Hence, specific resistance of a material may be defined as ***the resistance between the opposite faces of a metre cube of that material***.

1.7. Units of Resistivity

From Eq. ***(i)***, we have $\rho = \frac{AR}{l}$

In the S.I. system of units,

$$\rho = \frac{A \text{ metre}^2 \times R \text{ ohm}}{l \text{ metre}} = \frac{AR}{l} \text{ ohm-metre}$$

Hence, the unit of resistivity is ohm-metre (Ω-m).

It may, however, be noted that resistivity is sometimes expressed as so many ohm per m^3. Although, it is incorrect to say so but it means the same thing as ohm-metre.

If l is in centimetres and A in cm^2, then ρ is in ohm-centimetre (Ω-cm).

Values of resistivity and temperature coefficients for various materials are given in Table 1.2. The resistivities of commercial materials may differ by several per cent due to impurities etc.

Table 1.2. Resistivities and Temperature Coefficients

Material	*Resistivity in ohm-metre at 20°C ($\times 10^{-8}$)*	*Temperature coefficient at 20°C ($\times 10^{-4}$)*
Aluminium, commercial	2.8	40.3
Brass	6 – 8	20
Carbon	3000 – 7000	–5
Constantan or Eureka	49	+0.1 to -0.4
Copper (annealed)	1.72	39.3
German Silver (84% Cu; 12% Ni; 4% Zn)	20.2	2.7
Gold	2.44	36.5
Iron	9.8	65
Manganin (84% Cu ; 12% Mn ; 4% Ni)	44 – 48	0.15
Mercury	95.8	8.9
Nichrome (60% Cu ; 25% Fe ; 15% Cr)	108.5	1.5
Nickel	7.8	54
Platinum	9 – 15.5	36.7
Silver	1.64	38
Tungsten	5.5	47
Amber	5×10^{14}	
Bakelite	10^{10}	
Glass	$10^{10} – 10^{12}$	
Mica	10^{15}	
Rubber	10^{16}	
Shellac	10^{14}	
Sulphur	10^{15}	

Example 1.3. *A coil consists of 2000 turns of copper wire having a cross-sectional area of 0.8 mm^2. The mean length per turn is 80 cm and the resistivity of copper is 0.02 μΩ-m. Find the resistance of the coil and power absorbed by the coil when connected across 110 V d.c. supply.*

(F.Y. Engg. Pune Univ. May 1990)



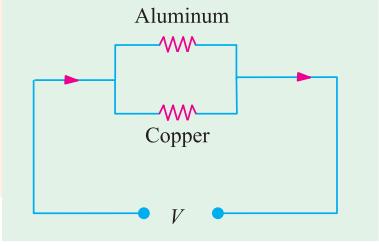
Solution. Length of the coil, $l = 0.8 \times 2000 = 1600$ m ; $A = 0.8$ mm^2 $= 0.8 \times 10^{-6}$ m^2.

$$R = \rho \frac{l}{A} = 0.02 \times 10^{-6} \times 1600/0.8 \times 10^{-6} = \mathbf{40\ \Omega}$$

Power absorbed $= V^2 / R = 110^2/40 = \mathbf{302.5\ W}$

Example 1.4. *An aluminium wire 7.5 m long is connected in a parallel with a copper wire 6 m long. When a current of 5 A is passed through the combination, it is found that the current in the aluminium wire is 3 A. The diameter of the aluminium wire is 1 mm. Determine the diameter of the copper wire. Resistivity of copper is 0.017 μΩ-m ; that of the aluminium is 0.028 μΩ-m.*

(F.Y. Engg. Pune Univ. May 1991)



Solution. Let the subscript 1 represent aluminium and subscript 2 represent copper.

$$R_1 = \rho \frac{l_1}{a_1} \text{ and } R_2 = \rho_2 \frac{l_2}{a_2} \qquad \therefore \frac{R_2}{R_1} = \frac{\rho_2}{\rho_1} \cdot \frac{l_2}{l_1} \cdot \frac{a_1}{a_2}$$

$$\therefore \quad a_2 = a_1 \cdot \frac{R_1}{R_2} \cdot \frac{\rho_2}{\rho_1} \cdot \frac{l_2}{l_1} \qquad \text{...(i)}$$

Now $I_1 = 3$ A ; $I_2 = 5 - 3 = 2$ A.

If V is the common voltage across the parallel combination of aluminium and copper wires, then

$$V = I_1 R_1 = I_2 R_2 \quad \therefore \quad R_1/R_2 = I_2/I_1 = 2/3$$

$$a_1 = \frac{\pi d^2}{4} = \frac{\pi \times 1^2}{4} = \frac{\pi}{4} \text{ mm}^2$$

Substituting the given values in Eq. (*i*), we get

$$a_2 = \frac{\pi}{4} \times \frac{2}{3} \times \frac{0.017}{0.028} \times \frac{6}{7.5} = 0.2544 \text{ m}^2$$

$$\therefore \quad \pi \times d_2^2/4 = 0.2544 \quad \text{or} \quad d_2 = \mathbf{0.569\ mm}$$

Example 1.5. **(*a*)** *A rectangular carbon block has dimensions 1.0 cm × 1.0 cm × 50 cm.* **(*i*)** *What is the resistance measured between the two square ends ?* **(*ii*)** *between two opposing rectangular faces / Resistivity of carbon at 20°C is 3.5 × 10^{-5} Ω-m.*

(*b*) *A current of 5 A exists in a 10-Ω resistance for 4 minutes* **(*i*)** *how many coulombs and* **(*ii*)** *how many electrons pass through any section of the resistor in this time ? Charge of the electron = 1.6 × 10^{-19} C.*

(M.S. Univ. Baroda)

Solution.

(*a*) (*i*) $R = \rho\, l/A$

Here, $A = 1 \times 1 = 1$ cm^2 $= 10^{-4}$ m^2 ; $l = 0.5$ m

$\therefore$ $R = 3.5 \times 10^{-5} \times 0.5/10^{-4} = \mathbf{0.175\ \Omega}$

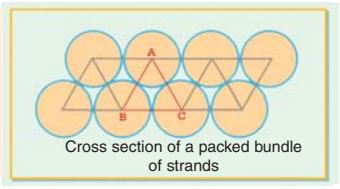
(*ii*) Here, $l = 1$ cm; $A = 1 \times 50 = 50$ cm^2 $= 5 \times 10^{-3}$ m^2

$R = 3.5 \times 10^{-5} \times 10^{-2}/5 \times 10^{-3} = \mathbf{7 \times 10^{-5}\ \Omega}$

(b) (i) $Q = It = 5 \times (4 \times 60) = \mathbf{1200\ C}$

(ii) $n = \dfrac{Q}{e} = \dfrac{1200}{1.6 \times 10^{-19}} = \mathbf{75 \times 10^{20}}$

Example 1.6. *Calculate the resistance of 1 km long cable composed of 19 stands of similar copper conductors, each strand being 1.32 mm in diameter. Allow 5% increase in length for the 'lay' (twist) of each strand in completed cable. Resistivity of copper may be taken as 1.72 × 10^{-8} Ω-m.*



Cross section of a packed bundle of strands

Solution. Allowing for twist, the length of the stands.

$= 1000\ \text{m} + 5\%\ \text{of}\ 1000\ \text{m} = 1050\ \text{m}$

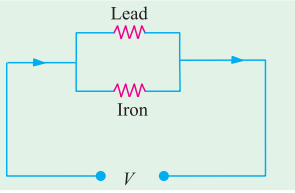
Area of cross-section of 19 strands of copper conductors is

$19 \times \pi \times d^2/4 = 19\ \pi \times (1.32 \times 10^{-3})^2/4\ \text{m}^2$

Now, $R = \rho \dfrac{l}{A} = \dfrac{1.72 \times 10^{-8} \times 1050 \times 4}{19\pi \times 1.32^2 \times 10^{-6}} = \mathbf{0.694\ \Omega}$

Example 1.7. *A lead wire and an iron wire are connected in parallel. Their respective specific resistances are in the ratio 49 : 24. The former carries 80 percent more current than the latter and the latter is 47 percent longer than the former. Determine the ratio of their cross sectional areas.*

(Elect. Engg. Nagpur Univ. 1993)



Solution. Let suffix 1 represent lead and suffix 2 represent iron. We are given that

$\rho_1/\rho_2 = 49/24$; if $i_2 = 1$, $i_1 = 1.8$; if $l_1 = 1$, $l_2 = 1.47$

Now, $R_1 = \rho_1 \dfrac{l_1}{A_1}$ and $R_2 = \rho_2 \dfrac{l_2}{A_2}$

Since the two wires are in parallel, $i_1 = V/R_1$ and $i_2 = V/R_2$

$\therefore \quad \dfrac{i_2}{i_1} = \dfrac{R_1}{R_2} = \dfrac{\rho_1 l_1}{A_1} \times \dfrac{A_2}{\rho_2 l_2}$

$\therefore \quad \dfrac{A_2}{A_1} = \dfrac{i_2}{i_1} \times \dfrac{\rho_2 l_2}{\rho_1 l_1} = \dfrac{1}{1.8} \times \dfrac{24}{49} \times 1.47 = \mathbf{0.4}$

Example 1.8. *A piece of silver wire has a resistance of 1 Ω. What will be the resistance of manganin wire of one-third the length and one-third the diameter, if the specific resistance of manganin is 30 times that of silver.*

(Electrical Engineering-I, Delhi Univ.)

Solution. For silver wire, $R_1 = \dfrac{l_1}{A_1}$; For manganin wire, $R = \rho_2 \dfrac{l_2}{A_2}$

$\therefore \quad \dfrac{R_2}{R_1} = \dfrac{\rho_2}{\rho_1} \times \dfrac{l_2}{l_1} \times \dfrac{A_1}{A_2}$

Now $A_1 = \pi d_1^2/4$ and $A_2 = \pi\ d_2^2/4 \qquad \therefore\ A_1/A_2 = d_1^2/d_2^2$

$\therefore \quad \dfrac{R_2}{R_1} = \dfrac{\rho_2}{\rho_1} \times \dfrac{l_2}{l_1} \times \left(\dfrac{d_1}{d_2}\right)^2$

$R_1 = 1\ \Omega$; $l_2/l_1 = 1/3$, $(d_1/d_2)^2 = (3/1)^2 = 9$; $\rho_2/\rho_1 = 30$

$\therefore \quad R_2 = 1 \times 30 \times (1/3) \times 9 = \mathbf{90\ \Omega}$

Example 1.9. *The resistivity of a ferric-chromium-aluminium alloy is 51×10^{-8} Ω-m. A sheet of the material is 15 cm long, 6 cm wide and 0.014 cm thick. Determine resistance between* ***(a)*** *opposite ends and* ***(b)*** *opposite sides.* **(Electric Circuits, Allahabad Univ.)**

Solution. ***(a)*** As seen from Fig. 1.5 (a) in this case,

$l = 15 \text{ cm} = 0.15 \text{ cm}$

$A = 6 \times 0.014 = 0.084 \text{ cm}^2$

$= 0.084 \times 10^{-4} \text{ m}^2$

$$R = \rho \frac{l}{A} = \frac{51 \times 10^{-8} \times 0.15}{0.084 \times 10^{-4}}$$

$= \mathbf{9.1 \times 10^{-3}\ \Omega}$

(b) As seen from Fig. 1.5 (b) here

$l = 0.014 \text{ cm} = 14 \times 10^{-5} \text{ m}$

$A = 15 \times 6 = 90 \text{ cm}^2 = 9 \times 10^{-3} \text{ m}^2$

$\therefore \quad R = 51 \times 10^{-8} \times 14 \times 10^{-5} / 9 \times 10^{-3} = \mathbf{79.3 \times 10^{-10}\ \Omega}$

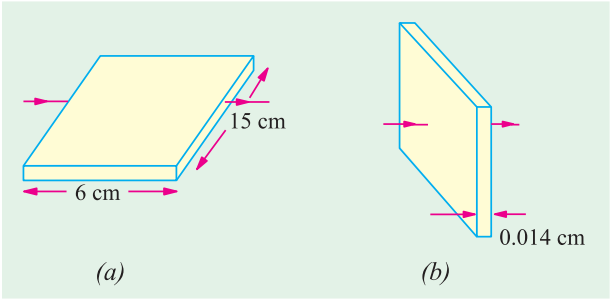


Fig. 1.5

Example 1.10. *The resistance of the wire used for telephone is 35 Ω per kilometre when the weight of the wire is 5 kg per kilometre. If the specific resistance of the material is 1.95×10^{-8} Ω-m, what is the cross-sectional area of the wire ? What will be the resistance of a loop to a subscriber 8 km from the exchange if wire of the same material but weighing 20 kg per kilometre is used ?*

Solution. Here $R = 35\ \Omega$; $l = 1 \text{ km} = 1000 \text{ m}$; $\rho = 1.95 \times 10^{-8}$ Ω-m

Now, $R = \rho \frac{l}{A}$ or $A = \frac{\rho l}{R}$ $\therefore$ $A = \frac{1.95 \times 10^{-8} \times 1000}{35} = \mathbf{55.7 \times 10^{-8}\ m^2}$

If the second case, if the wire is of the material but weighs 20 kg/km, then its cross-section must be greater than that in the first case.

Cross-section in the second case $= \frac{20}{5} \times 55.7 \times 10^{-8} = 222.8 \times 10^{-8} \text{ m}^2$

Length of wire $= 2 \times 8 = 16 \text{ km} = 16000 \text{ m}$ $\therefore$ $R = \rho \frac{l}{A} = \frac{1.95 \times 10^{-8} \times 16000}{222.8 \times 10^{-8}} = \mathbf{140.1\ \Omega}$

Tutorial Problems No. 1.1

1. Calculate the resistance of 100 m length of a wire having a uniform cross-sectional area of 0.1 mm^2 if the wire is made of manganin having a resistivity of 50×10^{-8} Ω-m.
   If the wire is drawn out to three times its original length, by how many times would you expect its resistance to be increased ? **[500 Ω; 9 times]**
2. A cube of a material of side 1 cm has a resistance of 0.001 Ω between its opposite faces. If the same volume of the material has a length of 8 cm and a uniform cross-section, what will be the resistance of this length ? **[0.064 Ω]**
3. A lead wire and an iron wire are connected in parallel. Their respective specific resistances are in the ratio 49 : 24. The former carries 80 per cent more current than the latter and the latter is 47 per cent longer than the former. Determine the ratio of their cross-sectional area. **[2.5 : 1]**
4. A rectangular metal strip has the following dimensions :
   $x = 10 \text{ cm}, y = 0.5 \text{ cm}, z = 0.2 \text{ cm}$
   Determine the ratio of resistances R_x, R_y, and R_z between the respective pairs of opposite faces.
   [$R_x : R_y : R_z$: 10,000 : 25 : 4] *(Elect. Engg. A.M.Ae. S.I.)*
5. The resistance of a conductor 1 mm^2 in cross-section and 20 m long is 0.346 Ω. Determine the specific resistance of the conducting material. **[1.73×10^{-8} Ω-m] *(Elect. Circuits-1, Bangalore Univ. 1991)***
6. When a current of 2 A flows for 3 micro-seconds in a coper wire, estimate the number of electrons crossing the cross-section of the wire. ***(Bombay University, 2000)***
   Hint : With 2 A for 3 μ Sec, charge transferred = 6 μ-coulombs
   Number of electrons crossed $= 6 \times 10^{-6} / (1.6 \times 10^{-19}) = 3.75 \times 10^{+13}$

1.8. Conductance and Conductivity

Conductance (G) is reciprocal of resistance*. Whereas resistance of a conductor measures the ***opposition*** which it offers to the flow of current, the conductance measures the ***inducement*** which it offers to its flow.

From Eq. **(*i*)** of Art. 1.6, $R = \rho \frac{l}{A}$ or $G = \frac{1}{\rho} \cdot \frac{A}{l} = \frac{\sigma A}{l}$

where σ is called the ***conductivity or specific conductance*** of a conductor. The unit of conductance is siemens (S). Earlier, this unit was called mho.

It is seen from the above equation that the conductivity of a material is given by

$$\sigma = G \frac{l}{A} = \frac{G \text{ siemens} \times l \text{ metre}}{A \text{ metre}^2} = G \frac{l}{A} \text{ siemens/metre}$$

Hence, the unit of conductivity is siemens/metre (S/m).

1.9. Effect of Temperature on Resistance

The effect of rise in temperature is :

(*i*) to ***increase*** the resistance of pure metals. The increase is large and fairly regular for normal ranges of temperature. The temperature/resistance graph is a straight line (Fig. 1.6). As would be presently clarified, metals have a positive temperature co-efficient of resistance.

(*ii*) to ***increase*** the resistance of alloys, though in their case, the increase is relatively small and irregular. For some high-resistance alloys like Eureka (60% Cu and 40% Ni) and manganin, the increase in resistance is (or can be made) negligible over a considerable range of temperature.

(*iii*) to ***decrease*** the resistance of electrolytes, insulators (such as paper, rubber, glass, mica etc.) and partial conductors such as carbon. Hence, insulators are said to possess a ***negative*** temperature-coefficient of resistance.

1.10. Temperature Coefficient of Resistance

Let a metallic conductor having a resistance of R_0 at 0°C be heated of t°C and let its resistance at this temperature be R_t. Then, considering normal ranges of temperature, it is found that the increase in resistance $\Delta R = R_t - R_0$ depends

(*i*) directly on its initial resistance

(*ii*) directly on the rise in temperature

(*iii*) on the nature of the material of the conductor.

or $$R_t - R_0 \propto R \times t \quad \text{or} \quad R_t - R_0 = \alpha R_0 t \qquad \text{...(i)}$$

where α (alpha) is a constant and is known as the *temperature coefficient of resistance* of the conductor.

Rearranging Eq. **(*i*)**, we get $\alpha = \frac{R_t - R_0}{R_0 \times t} = \frac{\Delta R}{R_0 \times t}$

If $R_0 = 1\ \Omega, t = 1°C$, then $\alpha = \Delta R = R_t - R_0$

Hence, the temperature-coefficient of a material may be defined as :

the increase in resistance per ohm original resistance per °C rise in temperature.

From Eq. **(*i*)**, we find that $R_t = R_0 (1 + \alpha t)$ ...**(*ii*)**

---

* In a.c. circuits, it has a slightly different meaning.

It should be remembered that the above equation holds good for both rise as well as fall in temperature. As temperature of a conductor is decreased, its resistance is also decreased. In Fig. 1.6 is shown the temperature/resistance graph for copper and is practically a straight line. If this line is extended backwards, it would cut the temperature axis at a point where temperature is – 234.5°C (a number quite easy to remember). It means that theoretically, the resistance of copper conductor will become zero at this point though as shown by solid line, in practice, the curve departs from a straight line at very low temperatures. From the two similar triangles of Fig. 1.6 it is seen that :

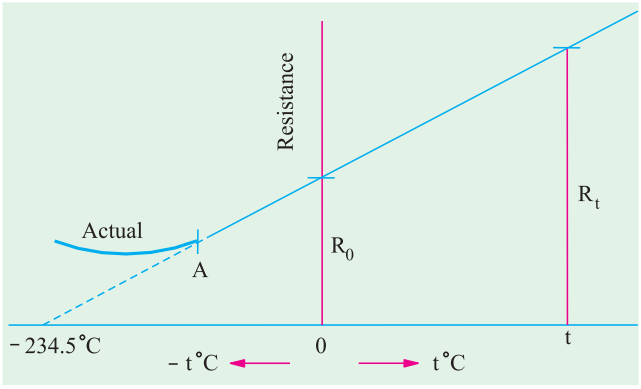


Fig. 1.6

$$\frac{R_t}{R_0} = \frac{t + 234.5}{234.5} = \left(1 + \frac{t}{234.5}\right)$$

$$\therefore \quad R_t = R_0 \left(1 + \frac{t}{234.5}\right) \text{ or } R_t = R_0 (1 + \alpha t) \text{ where } \alpha = 1/234.5 \text{ for copper.}$$

1.11. Value of α at Different Temperatures

So far we did not make any distinction between values of α at different temperatures. But it is found that value of α itself is not constant but depends on the initial temperature on which the increment in resistance is based. When the increment is based on the resistance measured at 0°C, then α has the value of α_0. At any other initial temperature t°C, value of α is α_t and so on. It should be remembered that, for any conductor, α_0 has the maximum value.

Suppose a conductor of resistance R_0 at 0°C (point A in Fig. 1.7) is heated to t°C (point B). Its resistance R_t after heating is given by

$$R_t = R_0 (1 + \alpha_0 t) \qquad \text{...(i)}$$

where α_0 is the temperature-coefficient at 0°C.

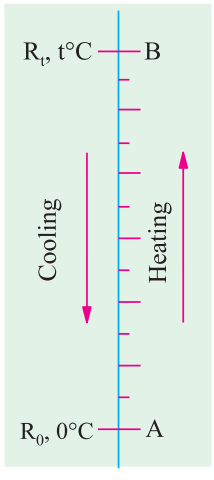


Fig. 1.7

Now, suppose that we have a conductor of resistance R_t at temperature t°C. Let this conductor be ***cooled*** from t°C to 0°C. Obviously, now the initial point is B and the final point is A. The final resistance R_0 is given in terms of the initial resistance by the following equation

$$R_0 = R_t [1 + \alpha_t (-t)] = R_t (1 - \alpha_t . t) \qquad \text{...(ii)}$$

From Eq. **(ii)** above, we have $\alpha_t = \dfrac{R_t - R_0}{R_t \times t}$

Substituting the value of R_t from Eq. (*i*), we get

$$\alpha_t = \frac{R_0 (1 + \alpha_0 t) - R_0}{R_0 (1 + \alpha_0 t) \times t} = \frac{\alpha_0}{1 + \alpha_0 t} \quad \therefore \ \alpha_t = \frac{\alpha_0}{1 + \alpha_0 t} \qquad \text{...(iii)}$$

In general, let α_1= tempt. coeff. at t_1°C ; α_2 = tempt. coeff. at t_2°C. Then from Eq. **(iii)** above, we get

$$\alpha_1 = \frac{\alpha_0}{1+\alpha_0 t_1} \quad \text{or} \quad \frac{1}{\alpha_1} = \frac{1+\alpha_0 t_1}{\alpha_0}$$

Similarly, $$\frac{1}{\alpha_2} = \frac{1+\alpha_0 t_2}{\alpha_0}$$

Subtracting one from the other, we get

$$\frac{1}{\alpha_2} - \frac{1}{\alpha_1} = (t_2 - t_1) \text{ or } \frac{1}{\alpha_2} = \frac{1}{\alpha_1} + (t_2 - t_1) \text{ or } \alpha_2 = \frac{1}{1/\alpha_1 + (t_2 - t_1)}$$

Values of α for copper at different temperatures are given in Table No. 1.3.

Table 1.3. Different values of α for copper

Tempt. in °C	0	5	10	20	30	40	50
α	0.00427	0.00418	0.00409	0.00393	0.00378	0.00364	0.00352

In view of the dependence of α on the initial temperature, we may define ***the temperature coefficient of resistance at a given temperature as the charge in resistance per ohm per degree centigrade change in temperature from the given temperature.***

In case R_0 is not given, the relation between the known resistance R_1 at t_1°C and the unknown resistance R_2 at t_2°C can be found as follows :

$$R_2 = R_0 (1 + \alpha_0 t_2) \quad \text{and} \quad R_1 = R_0 (1 + \alpha_0 t_1)$$

$$\therefore \quad \frac{R_2}{R_1} = \frac{1+\alpha_0 t_2}{1+\alpha_0 t_1} \qquad \text{...(iv)}$$

The above expression can be simplified by a little approximation as follows :

$$\frac{R_2}{R_1} = (1 + \alpha_0 t_2)(1 + \alpha_0 t_1)^{-1}$$

$$= (1 + \alpha_0 t_2)(1 - \alpha_0 t_1) \qquad \text{[Using Binomial Theorem for expansion and}$$

$$= 1 + \alpha_0 (t_2 - t_1) \qquad \text{neglecting squares and higher powers of } (\alpha_0 t_1)\text{]}$$

$$\therefore \quad R_2 = R_1 [1 + \alpha_0 (t_2 - t_1)] \qquad \text{[Neglecting product } (\alpha_0^2 t_1 t_2)\text{]}$$

For more accurate calculations, Eq. ***(iv)*** should, however, be used.

1.12. Variations of Resistivity with Temperature

Not only resistance but specific resistance or resistivity of metallic conductors also increases with rise in temperature and ***vice-versa***.

As seen from Fig. 1.8 the resistivities of metals vary linearly with temperature over a significant range of temperature-the variation becoming non-linear both at very high and at very low temperatures. Let, for any metallic conductor,

ρ_1 = resistivity at t_1°C

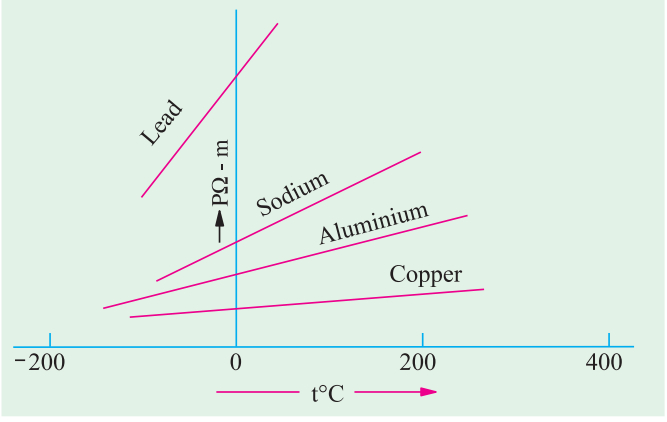


Fig. 1.8

ρ_2 = resistivity at t_2°C

m = Slope of the linear part of the curve

Then, it is seen that

$$m = \frac{\rho_2 - \rho_1}{t_2 - t_1}$$

or $\rho_2 = \rho_1 + m\,(t_2 - t_1)$ or $_2 \quad _1 \; 1 \quad \frac{m}{1}(t_2 \quad t_1)$

The ratio of m/ρ_1 is called the ***temperature coefficient of resistivity*** at temperature t_1°C. It may be defined as numerically equal to the fractional change in ρ_1 per °C change in the temperature from t_1°C. It is almost equal to the temperature-coefficient of resistance α_1. Hence, putting $\alpha_1 = m/\rho_1$, we get

$$\rho_2 = \rho_1 [1 + \alpha_1 (t_2 - t_1)] \quad \text{or} \quad \text{simply as } \rho_t = \rho_0 (1 + \alpha_0 t)$$

Note. It has been found that although temperature is the most significant factor influencing the resistivity of metals, other factors like pressure and tension also affect resistivity to some extent. For most metals except lithium and calcium, increase in pressure leads to decrease in resistivity. However, resistivity increases with increase in tension.

Example 1.11. *A copper conductor has its specific resistance of 1.6×10^{-6} ohm-cm at 0°C and a resistance temperature coefficient of 1/254.5 per °C at 20°C. Find **(i)** the specific resistance and **(ii)** the resistance - temperature coefficient at 60°C.* **(F.Y. Engg. Pune Univ. Nov.)**

Solution. $\alpha_{20} = \dfrac{\alpha_0}{1 + \alpha_0 \times 20}$ or $\dfrac{1}{254.5} = \dfrac{\alpha_0}{1 + \alpha_0 \times 20}$ $\therefore \quad \alpha_0 = \dfrac{1}{234.5}$ per °C

(i) $\rho_{60} = \rho_0 (1 + \alpha_0 \times 60) = 1.6 \times 10^{-6} (1 + 60/234.5) =$ **2.01×10^{-6} Ω-cm**

(ii) $\alpha_{60} = \dfrac{\alpha_0}{1 + \alpha_0 \times 60} = \dfrac{1/234.5}{1 + (60/234.5)} =$ **$\dfrac{1}{294.5}$ per°C**

Example 1.12. *A platinum coil has a resistance of 3.146 Ω at 40°C and 3.767 Ω at 100°C. Find the resistance at 0°C and the temperature-coefficient of resistance at 40°C.*

(Electrical Science-II, Allahabad Univ.)

Solution. $R_{100} = R_0 (1 + 100\,\alpha_0)$ ...**(i)**

$R_{40} = R_0 (1 + 40\,\alpha_0)$ ...**(ii)**

$\therefore \quad \dfrac{3.767}{3.146} = \dfrac{1 + 100\,\alpha_0}{1 + 40\,\alpha_0}$ or $\alpha_0 = 0.00379$ or 1/264 per°C

From **(i)**, we have $3.767 = R_0 (1 + 100 \times 0.00379)$ $\therefore$ $R_0 =$ **2.732 Ω**

Now, $\alpha_{40} = \dfrac{\alpha_0}{1 + 40\,\alpha_0} = \dfrac{0.00379}{1 + 40 \times 0.00379} =$ **$\dfrac{1}{304}$ per°C**

Example 1.13. *A potential difference of 250 V is applied to a field winding at 15°C and the current is 5 A. What will be the mean temperature of the winding when current has fallen to 3.91 A, applied voltage being constant. Assume $\alpha_{15} = 1/254.5$.* **(Elect. Engg. Pune Univ.)**

Solution. Let R_1 = winding resistance at 15°C; R_2 = winding resistance at unknown mean temperature t_2°C.

$\therefore \quad R_1 = 250/5 = 50\ \Omega$; $R_2 = 250/3.91 = 63.94\ \Omega$

Now $R_2 = R_1 [1 + \alpha_{15} (t_2 - t_1)]$ $\therefore$ $63.94 = 50\left[1 + \dfrac{1}{254.5}(t_2 - 15)\right]$

$\therefore \quad t_2 =$ **86°C**

Example 1.14. *Two coils connected in series have resistances of 600 Ω and 300 Ω with tempt. coeff. of 0.1% and 0.4% respectively at 20°C. Find the resistance of the combination at a tempt. of 50°C. What is the effective tempt. coeff. of combination ?*

Solution. Resistance of 600 Ω resistor at 50°C is = 600 [1 + 0.001 (50 −20)] = 618 Ω

Similarly, resistance of 300 Ω resistor at 50°C is = 300 [1 + 0.004 (50 −20)] = 336 Ω

Hence, total resistance of combination at 50°C is = 618 + 336 = **954 Ω**

Let β = resistance-temperature coefficient at 20°C

Now, combination resistance at 20°C = 900 Ω

Combination resistance at 50°C = 954 Ω

$$\therefore \quad 954 = 900\,[1 + \beta\,(50 - 20)] \quad \therefore\ \beta = \mathbf{0.002}$$

Example 1.15. *Two wires A and B are connected in series at 0°C and resistance of B is 3.5 times that of A. The resistance temperature coefficient of A is 0.4% and that of the combination is 0.1%. Find the resistance temperature coefficient of B.* **(Elect. Technology, Hyderabad Univ.)**

Solution. A simple technique which gives quick results in such questions is illustrated by the diagram of Fig. 1.9. It is seen that $R_B/R_A = 0.003/(0.001 - \alpha)$

or $\quad 3.5 = 0.003/(0.001 - \alpha)$

or $\quad \alpha = 0.000143°C^{-1}$ or **0.0143 %**

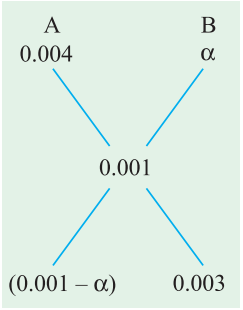


Fig. 1.9

Example 1.16. *Two materials A and B have resistance temperature coefficients of 0.004 and 0.0004 respectively at a given temperature. In what proportion must A and B be joined in series to produce a circuit having a temperature coefficient of 0.001 ?*

(Elect. Technology, Indore Univ.)

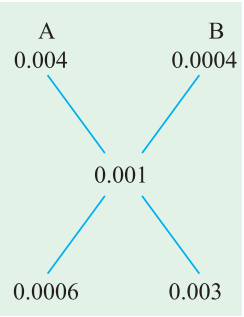


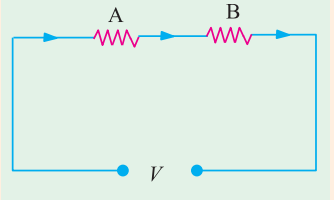
Fig. 1.10

Solution. Let R_A and R_B be the resistances of the two wires of materials *A* and *B* which are to be connected in series. Their ratio may be found by the simple technique shown in Fig. 1.10.

$$\frac{R_B}{R_A} = \frac{0.003}{0.0006} = 5$$

Hence, R_B must be **5 times R_A.**

Example 1.17. *A resistor of 80 Ω resistance, having a temperature coefficient of 0.0021 per degree C is to be constructed. Wires of two materials of suitable cross-sectional area are available. For material A, the resistance is 80 ohm per 100 metres and the temperature coefficient is 0.003 per degree C. For material B, the corresponding figures are 60 ohm per metre and 0.0015 per degree C. Calculate suitable lengths of wires of materials A and B to be connected in series to construct the required resistor. All data are referred to the same temperature.*



Solution. Let R_a and R_b be the resistances of suitable lengths of materials *A* and *B* respectively which when joined in series will have a combined temperature coeff. of 0.0021. Hence, combination resistance at any given temperature is $(R_a + R_b)$. Suppose we heat these materials through *t*°C.

When heated, resistance of *A* increases from R_a to $R_a\,(1 + 0.003\,t)$. Similarly, resistance of *B* increases from R_b to $R_b\,(1 + 0.0015\,t)$.

$\therefore$ combination resistance after being heated through *t*°C

$$= R_a\,(1 + 0.003\,t) + R_b\,(1 + 0.0015\,t)$$

The combination α being given, value of combination resistance can be also found directly as

$$= (R_a + R_b)(1 + 0.0021\,t)$$

$$\therefore \quad (R_a + R_b)(1 + 0.0021\,t) = R_a(1 + 0.003\,t) + R_b(1 + 0.0015\,t)$$

Simplifying the above, we get $\dfrac{R_b}{R_a} = \dfrac{3}{2}$ ...(*i*)

Now $R_a + R_b = 80\ \Omega$ ...(*ii*)

Substituting the value of R_b from (*i*) into (*ii*) we get

$$R_a + \frac{3}{2}R_a = 80 \quad \text{or} \quad R_a = 32\ \Omega \quad \text{and} \quad R_b = 48\ \Omega$$

If L_a and L_b are the required lengths in metres, then

$$L_a = (100/80) \times 32 = \mathbf{40\ m} \quad \text{and} \quad L_b = (100/60) \times 48 = \mathbf{80\ m}$$

Example 1.18. *A coil has a resistance of 18 Ω when its mean temperature is 20°C and of 20 Ω when its mean temperature is 50°C. Find its mean temperature rise when its resistance is 21 Ω and the surrounding temperature is 15° C.* **(Elect. Technology, Allahabad Univ.)**

Solution. Let R_0 be the resistance of the coil and α_0 its tempt. coefficient at 0°C.

Then, $18 = R_0(1 + \alpha_0 \times 20)$ and $20 = R_0(1 + 50\,\alpha_0)$

Dividing one by the other, we get

$$\frac{20}{18} = \frac{1 + 50\,\alpha_0}{1 + 20\,\alpha_0} \qquad \therefore\ \alpha_0 = \frac{1}{250}\ \text{per°C}$$

If t°C is the temperature of the coil when its resistance is 21 Ω, then,

$$21 = R_0(1 + t/250)$$

Dividing this equation by the above equation, we have

$$\frac{21}{18} = \frac{R_0(1 + t/250)}{R_0(1 + 20\,\alpha_0)}; \quad t = 65°\text{C};\ \text{temp. rise} = 65 - 15 = \mathbf{50°C}$$

Example 1.19. *The coil of a relay takes a current of 0.12 A when it is at the room temperature of 15°C and connected across a 60-V supply. If the minimum operating current of the relay is 0.1 A, calculate the temperature above which the relay will fail to operate when connected to the same supply. Resistance-temperature coefficient of the coil material is 0.0043 per°C at 6°C.*

Solution. Resistance of the relay coil at 15°C is $R_{15} = 60/0.12 = 500\ \Omega$.

Let t°C be the temperature at which the minimum operating current of 0.1 A flows in the relay coil. Then, $R_1 = 60/0.1 = 600\ \Omega$.

Now $R_{15} = R_0(1 + 15\,\alpha_0) = R_0(1 + 15 \times 0.0043)$ and $R_t = R_0(1 + 0.0043\,t)$

$$\therefore \quad \frac{R_t}{R_{15}} = \frac{1 + 0.0043\,t}{1.0654} \quad \text{or} \quad \frac{600}{500} = \frac{1 + 0.0043\,t}{1.0645} \quad \therefore\ t = \mathbf{65.4°C}$$

If the temperature rises above this value, then due to increase in resistance, the relay coil will draw a current less than 0.1 A and, therefore, will fail to operate.

Example 1.20. *Two conductors, one of copper and the other of iron, are connected in parallel and carry equal currents at 25°C. What proportion of current will pass through each if the temperature is raised to 100°C ? The temperature coefficients of resistance at 0°C are 0.0043/°C and 0.0063/°C for copper and iron respectively.* **(Principles of Elect. Engg. Delhi Univ.)**

Solution. Since the copper and iron conductors carry equal currents at 25°C, their resistances are the same at that temperature. Let each be R ohm.

For copper, $R_{100} = R_1 = R\,[1 + 0.0043\,(100 - 25)] = 1.3225\,R$

For iron, $R_{100} = R_2 = R\,[1 + 0.0063\,(100 - 25)] = 1.4725\,R$

If I is the current at 100°C, then as per current divider rule, current in the copper conductor is

$$I_1 = I\frac{R_2}{R_1+R_2} = I\frac{1.4725\,R}{1.3225\,R+1.4725\,R} = 0.5268\,I$$

$$I_2 = I\frac{R_2}{R_1+R_2} = I\frac{1.3225\,R}{2.795\,R} = 0.4732\,I$$

Hence, copper conductor will carry **52.68%** of the total current and iron conductor will carry the balance *i.e.* **47.32%.**

Example 1.21. *The filament of a 240 V metal-filament lamp is to be constructed from a wire having a diameter of 0.02 mm and a resistivity at 20°C of 4.3 μΩ-cm. If α = 0.005/°C, what length of filament is necessary if the lamp is to dissipate 60 watts at a filament tempt. of 2420°C ?*

Solution. Electric power generated = I^2R watts = V^2/R watts

$\therefore \quad V^2/R = 60 \quad \text{or} \quad 240^2/R = 60$

Resistance at 2420°C $\quad R_{2420} = \dfrac{240\times240}{60} = 960\ \Omega$

Now $\quad R_{2420} = R_{20}\,[1 + (2420 - 20)\times 0.005]$

or $\quad 960 = R_{20}\,(1 + 12)$

$\therefore \quad R_{20} = 960/13\ \Omega$

Now $\quad \rho_{20} = 4.3\times 10^{-6}\ \Omega\text{-cm} \quad \text{and} \quad A = \dfrac{\pi(0.002)^2}{4}\ \text{cm}^2$

$\therefore \quad l = \dfrac{A\times R_{20}}{\rho_{20}} = \dfrac{\pi\,(0.002)^2\times 960}{4\times13\times4.3\times10^{-6}} = \mathbf{54\ cm}$

Example 1.22. *A semi-circular ring of copper has an inner radius 6 cm, radial thickness 3 cm and an axial thickness 4 cm. Find the resistance of the ring at 50°C between its two end-faces. Assume specific resistance of Cu at 20°C = 1.724 × 10^{-6} ohm-cm and resistance tempt. coeff. of Cu at 0°C = 0.0043/°C.*

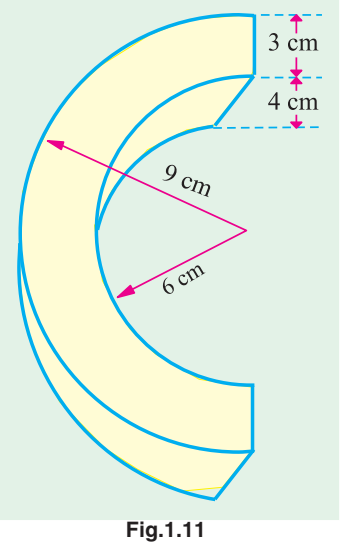


Fig.1.11

Solution. The semi-circular ring is shown in Fig. 1.11.

Mean radius of ring = (6 + 9)/2 = 7.5 cm

Mean length between end faces = 7.5 π cm = 23.56 cm

Cross-section of the ring = 3 × 4 = 12 cm^2

Now $\quad \alpha_0 = 0.0043/°\text{C};\ \alpha_{20} = \dfrac{0.0043}{1+20\times0.0043} = 0.00396$

$\rho_{50} = \rho_{20}\,[1 + \alpha_0\,(50 - 20)]$

$= 1.724\times 10^{-6}\,(1 + 30\times 0.00396) = 1.93\times 10^{-6}\ \Omega\text{-cm}$

$R_{50} = \dfrac{\rho_{50}\times l}{A} = \dfrac{1.93\times10^{-6}\times23.56}{12} = \mathbf{3.79\times10^{-6}\ \Omega}$

Tutorial Problems No. 1.2

1. It is found that the resistance of a coil of wire increases from 40 ohm at 15°C to 50 ohm at 60°C. Calculate the resistance temperature coefficient at 0°C of the conductor material.
[1/165 per °C] (*Elect. Technology, Indore Univ.*)
2. A tungsten lamp filament has a temperature of 2,050°C and a resistance of 500 Ω when taking normal working current. Calculate the resistance of the filament when it has a temperature of 25°C. Temperature coefficient at 0°C is 0.005/°C. **[50 Ω] (*Elect. Technology, Indore Univ.*)**

3. An armature has a resistance of 0.2 Ω at 150°C and the armature Cu loss is to be limited to 600 watts with a temperature rise of 55°C. If α_0 for Cu is 0.0043/°C, what is the maximum current that can be passed through the armature ? **[50.8 A]**

4. A d.c. shunt motor after running for several hours on constant voltage mains of 400 V takes a field current of 1.6 A. If the temperature rise is known to be 40°C, what value of extra circuit resistance is required to adjust the field current to 1.6 A when starting from cold at 20°C ? Temperature coefficient = 0.0043/°C at 20°C. **[36.69 Ω]**

5. In a test to determine the resistance of a single-core cable, an applied voltage of 2.5 V was necessary to produce a current of 2 A in it at 15°C.

(*a*) Calculate the cable resistance at 55°C if the temperature coefficient of resistance of copper at 0°C is 1/235 per°C.

(*b*) If the cable under working conditions carries a current of 10 A at this temperature, calculate the power dissipated in the cable. **[(*a*) 1.45 Ω (*b*) 145 W]**

6. An electric radiator is required to dissipate 1 kW when connected to a 230 V supply. If the coils of the radiator are of wire 0.5 mm in diameter having resistivity of 60 μΩ-cm, calculate the necessary length of the wire. **[1732 cm]**

7. An electric heating element to dissipate 450 watts on 250 V mains is to be made from nichrome ribbon of width 1 mm and thickness 0.05 mm. Calculate the length of the ribbon required (the resistivity of nichrome is 110×10^{-8} Ω-m). **[631 m]**

8. When burning normally, the temperature of the filament in a 230 V, 150 W gas-filled tungsten lamp is 2,750°C. Assuming a room temperature of 16°C, calculate (*a*) the normal current taken by the lamp (*b*) the current taken at the moment of switching on. Temperature coefficient of tungsten is 0.0047 Ω/Ω°C at 0°C. **[(*a*) 0.652 A (*b*) 8.45 A] (*Elect. Engg. Madras Univ.*)**

9. An aluminium wire 5 m long and 2 mm diameter is connected in parallel with a wire 3 m long. The total current is 4 A and that in the aluminium wire is 2.5 A. Find the diameter of the copper wire. The respective resistivities of copper and aluminium are 1.7 and 2.6 μΩ-m. **[0.97 mm]**

10. The field winding of d.c. motor connected across 230 V supply takes 1.15 A at room temp. of 20°C. After working for some hours the current falls to 0.26 A, the supply voltage remaining constant. Calculate the final working temperature of field winding. Resistance temperature coefficient of copper at 20°C is 1/254.5. **[70.4°C] (*Elect. Engg. Pune Univ.*)**

11. It is required to construct a resistance of 100 Ω having a temperature coefficient of 0.001 per°C. Wires of two materials of suitable cross-sectional area are available. For material *A*, the resistance is 97 Ω per 100 metres and for material *B*, the resistance is 40 Ω per 100 metres. The temperature coefficient of resistance for material *A* is 0.003 per °C and for material *B* is 0.0005 per °C. Determine suitable lengths of wires of materials *A* and *B*. **[A : 19.4 m, B : 200 m]**

12. The resistance of the shunt winding of a d.c. machine is measured before and after a run of several hours. The average values are 55 ohms and 63 ohms. Calculate the rise in temperature of the winding. (Temperature coefficient of resistance of copper is 0.00428 ohm per ohm per °C). **[36°C] (*London Univ.*)**

13. A piece of resistance wire, 15.6 m long and of cross-sectional area 12 mm^2 at a temperature of 0°C, passes a current of 7.9 A when connected to d.c. supply at 240 V. Calculate (*a*) resistivity of the wire (*b*) the current which will flow when the temperature rises to 55°C. The temperature coefficient of the resistance wire is 0.00029 Ω/Ω/°C. **[(*a*) 23.37 μΩ-m (*b*) 7.78 A] (*London Univ.*)**

14. A coil is connected to a constant d.c. supply of 100 V. At start, when it was at the room temperature of 25°C, it drew a current of 13 A. After sometime, its temperature was 70°C and the current reduced to 8.5 A. Find the current it will draw when its temperature increases further to 80°C. Also, find the temperature coefficient of resistance of the coil material at 25°C. **[7.9 A; 0.01176°C^{-1}] (*F.Y. Engg. Univ.*)**

15. The resistance of the filed coils with copper conductors of a dynamo is 120 Ω at 25°C. After working for 6 hours on full load, the resistance of the coil increases to 140 Ω. Calculate the mean temperature rise of the field coil. Take the temperature coefficient of the conductor material as 0.0042 at 0°C. **[43.8°C] (*Elements of Elec. Engg. Banglore Univ.*)**

1.13. Ohm's Law

This law applies to electric to electric conduction through good conductors and may be stated as follows :

The ratio of potential difference (V) between any two points on a conductor to the current (I) flowing between them, is constant, provided the temperature of the conductor does not change.

In other words, $\frac{V}{I} = \text{constant}$ or $\frac{V}{I} = R$

where R is the resistance of the conductor between the two points considered.

Put in another way, it simply means that provided R is kept constant, current is directly proportional to the potential difference across the ends of a conductor. However, this linear relationship between V and I does not apply to all non-metallic conductors. For example, for silicon carbide, the relationship is given by $V = KI^m$ where K and m are constants and m is less than unity. It also does not apply to non-linear devices such as Zener diodes and voltage-regulator (VR) tubes.

Example 1.23. *A coil of copper wire has resistance of 90 Ω at 20°C and is connected to a 230-V supply. By how much must the voltage be increased in order to maintain the current consant if the temperature of the coil rises to 60°C ? Take the temperature coefficient of resistance of copper as 0.00428 from 0°C.*

Solution. As seen from Art. 1.10

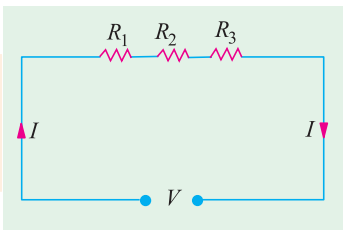
$$\frac{R_{60}}{R_{20}} = \frac{1 + 60 \times 0.00428}{1 + 20 \times 0.00428} \quad \therefore R_{60} = 90 \times 1.2568/1.0856 = 104.2\ \Omega$$

Now, current at 20°C = 230/90 = 23/9 A

Since the wire resistance has become 104.2 Ω at 60°C, the new voltage required for keeping the current constant at its previous value = 104.2 × 23/9 = 266.3 V

∴ increase in voltage required = 266.3 −230 = **36.3 V**

Example 1.24. *Three resistors are connected in series across a 12-V battery. The first resistor has a value of 1 Ω, second has a voltage drop of 4 V and the third has a power dissipation of 12 W. Calculate the value of the circuit current.*



Solution. Let the two unknown resistors be R_2 and R_3 and I the circuit current

$\therefore \quad I^2 R_3 = 12$ and $IR_3 = 4 \quad \therefore R_3 = \frac{3}{4} R_2^2$. Also, $I = \frac{4}{R_2}$

Now, $I(1 + R_2 + R_3) = 12$

Substituting the values of I and R_3, we get

$$\frac{4}{R_2}\left(1 + R_2 + \frac{3}{4} R_2^2\right) = 12 \quad \text{or} \quad 3R_2^{\,2} - 8R_2 + 4 = 0$$

$$\therefore \quad R_2 = \frac{8 \pm \sqrt{64 - 48}}{6} \quad \therefore R_2 = 2\ \Omega \quad \text{or} \quad \frac{2}{3}\Omega$$

$$\therefore \quad R_3 = \frac{3}{4} R_2^2 = \frac{3}{4} \times 2^2 = 3\ \Omega \quad \text{or} \quad \frac{3}{4}\left(\frac{2}{3}\right)^2 = \frac{1}{3}\Omega$$

$$\therefore \quad I = \frac{12}{1 + 2 + 3} = \mathbf{2\ A} \quad \text{or} \quad I = \frac{12}{1 + (2/3) + (1/3)} = \mathbf{6\ A}$$

1.14. Resistance in Series

When some conductors having resistances R_1, R_2 and R_3 etc. are joined end-on-end as in Fig. 1.12, they are said to be connected in series. It can be proved that the equivalent resistance or total resistance between points A and D is equal to the sum of the three individual resistances. Being a series circuit, it should be remembered that **(*i*)** current is the same through all the three conductors

(ii) but voltage drop across each is different due to its different resistance and is given by Ohm's Law and ***(iii)*** sum of the three voltage drops is equal to the voltage applied across the three conductors. There is a progressive fall in potential as we go from point A to D as shown in Fig. 1.13.

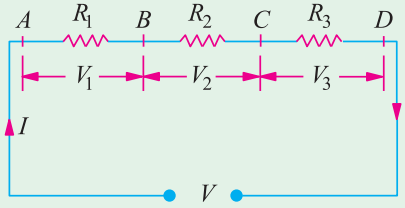


Fig. 1.12

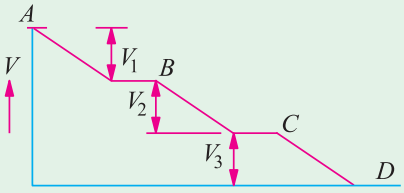


Fig. 1.13

$\therefore \quad V = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3$ —Ohm's Law

But $V = IR$

where R is the equivalent resistance of the series combination.

$\therefore \quad IR = IR_1 + IR_2 + IR_3 \quad$ or $\quad R = R_1 + R_2 + R_3$

Also $\dfrac{1}{G} = \dfrac{1}{G_1} + \dfrac{1}{G_2} + \dfrac{1}{G_3}$

As seen from above, the main characteristics of a series circuit are :

1. same current flows through all parts of the circuit.
2. different resistors have their individual voltage drops.
3. voltage drops are additive.
4. applied voltage equals the sum of different voltage drops.
5. resistances are additive.
6. powers are additive.

1.15. Voltage Divider Rule

Since in a series circuit, same current flows through each of the given resistors, voltage drop varies directly with its resistance. In Fig. 1.14 is shown a 24-V battery connected across a series combination of three resistors.

Total resistance $R = R_1 + R_2 + R_3 = 12\ \Omega$

According to Voltage Divider Rule, various voltage drops are :

$$V_1 = V \cdot \frac{R_1}{R} = 24 \times \frac{2}{12} = 4\ \text{V}$$

$$V_2 = V \cdot \frac{R_2}{R} = 24 \times \frac{4}{12} = 8\ \text{V}$$

$$V_3 = V \cdot \frac{R_3}{R} = 24 \times \frac{6}{12} = 12\ \text{V}$$

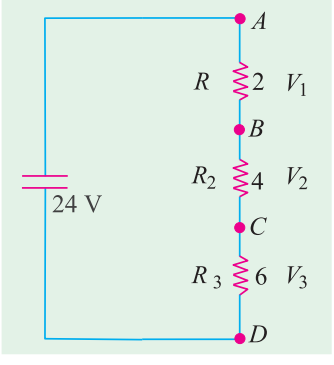


Fig.1.14

1.16. Resistances in Parallel

Three resistances, as joined in Fig. 1.15 are said to be connected in parallel. In this case ***(i)*** p.d. across all resistances is the same ***(ii)*** current in each resistor is different and is given by Ohm's Law and ***(iii)*** the total current is the sum of the three separate currents.

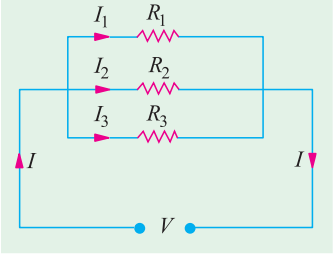


Fig.1.15

$$I = I_1 + I_2 + I_3 = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

Now, $I = \frac{V}{R}$ where V is the applied voltage.

R = equivalent resistance of the parallel combination.

$$\therefore \quad \frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \quad \text{or} \quad \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Also $G = G_1 + G_2 + G_3$

The main characteristics of a parallel circuit are :

1. same voltage acts across all parts of the circuit
2. different resistors have their individual current.
3. branch currents are additive.
4. conductances are additive.
5. powers are additive.

Example 1.25. *What is the value of the unknown resistor R in Fig. 1.16 if the voltage drop across the 500 Ω resistor is 2.5 volts ? All resistances are in ohm.* **(Elect. Technology, Indore Univ.)**

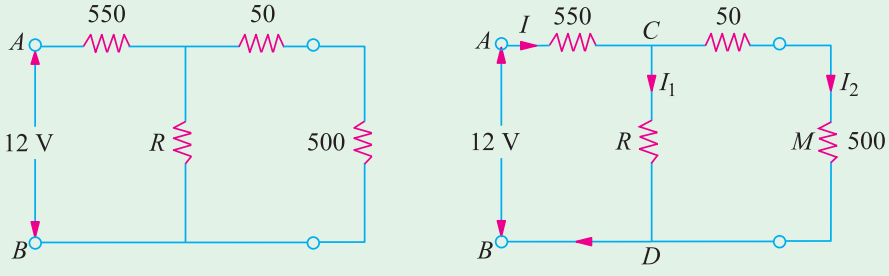


Fig. 1.16

Solution. By direct proportion, drop on 50 Ω resistance = 2.5 × 50/500 = 0.25 V

Drop across CMD or CD = 2.5 + 0.25 = 2.75 V

Drop across 550 Ω resistance = 12 − 2.75 = 9.25 V

I = 9.25/550 = 0.0168 A, I_2 = 2.5/500 = 0.005 A

I_1 = 0.0168 − 0.005 = 0.0118 A

$\therefore$ 0.0118 = 2.75/R; R = **233 Ω**

Example 1.26. *Calculate the effective resistance of the following combination of resistances and the voltage drop across each resistance when a P.D. of 60 V is applied between points A and B.*

Solution. Resistance between A and C (Fig. 1.17).

= 6 || 3 = 2 Ω

Resistance of branch ACD = 18 + 2 = 20 Ω

Now, there are two parallel paths between points A and D of resistances 20 Ω and 5 Ω

Hence, resistance between A and D = 20 || 5 = 4 Ω

∴Resistance between A and B = 4 + 8 = 12 Ω

Total circuit current = 60/12 = 5 A

Current through 5 Ω resistance = $5 \times \frac{20}{25} = 4$ A

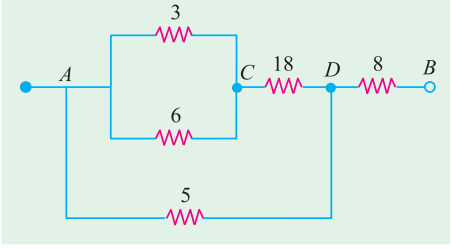


Fig. 1.17

—Art. 1.25

Current in branch ACD $= 5 \times \frac{5}{25} = 1$ A

$\therefore$ P.D. across 3 Ω and 6 Ω resistors $= 1 \times 2 =$ **2 V**

P.D. across 18 Ω resistors $= 1 \times 18 =$ **18 V**

P.D. across 5 Ω resistors $= 4 \times 5 =$ **20 V**

P.D. across 8 Ω resistors $= 5 \times 8 =$ **40 V**

Example 1.27. *A circuit consists of four 100-W lamps connected in parallel across a 230-V supply. Inadvertently, a voltmeter has been connected in series with the lamps. The resistance of the voltmeter is 1500 Ω and that of the lamps under the conditions stated is six times their value then burning normally. What will be the reading of the voltmeter ?*

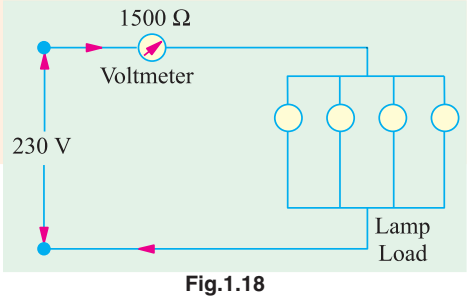


Fig.1.18

Solution. The circuit is shown in Fig. 1.18. The wattage of a lamp is given by :

$$W = I^2 R = V^2/R$$

$\therefore \quad 100 = 230^2/R \quad \therefore \quad R = 529\ \Omega$

Resistance of each lamp under stated condition is $= 6 \times 529 = 3174\ \Omega$

Equivalent resistance of these four lamps connected in parallel $= 3174/4 = 793.5\ \Omega$

This resistance is connected in series with the voltmeter of 1500 Ω resistance.

$\therefore$total circuit resistance $= 1500 + 793.5 = 2293.5\ \Omega$

$\therefore$ circuit current $= 230/2293.5$ A

According to Ohm's law, voltage drop across the voltmeter $= 1500 \times 230/2293.5 =$ **150 V (approx)**

Example 1.28. *Determine the value of R and current through it in Fig. 1.19, if current through branch AO is zero.* **(Elect. Engg. & Electronics, Bangalore Univ.)**

Solution. The given circuit can be redrawn as shown in Fig. 1.19 (*b*). As seen, it is nothing else but Wheatstone bridge circuit. As is well-known, when current through branch *AO* becomes zero, the bridge is said to be balanced. In that case, products of the resistances of opposite arms of the bridge become equal.

$\therefore \quad 4 \times 1.5 = R \times 1;\ R = 6\ \Omega$

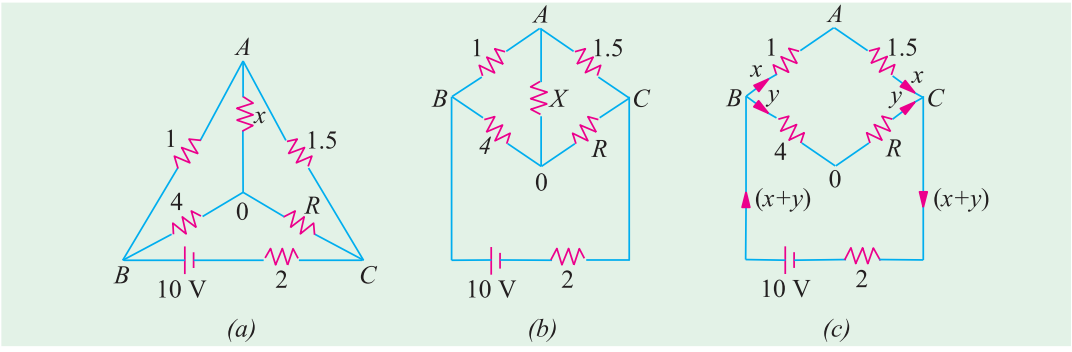


Fig.1.19

Under condition of balance, it makes no difference if resistance X is removed thereby giving us the circuit of Fig. 1.19 (*c*). Now, there are two parallel paths between points B and C of resistances $(1 + 1.5) = 2.5\ \Omega$ and $(4 + 6) = 10\ \Omega$. $R_{BC} = 10 \parallel 2.5 = 2\ \Omega$.

Total circuit resistance $= 2 + 2 = 4\ \Omega$. Total circuit current $= 10/4 = 2.5\ A$

This current gets divided into two parts at point B. Current through R is

$$y = 2.5 \times 2.5/12.5 =$$ **0.5 A**

Example 1.29. *In the unbalanced bridge circuit of Fig. 1.20 (a), find the potential difference that exists across the open switch S. Also, find the current which will flow through the switch when it is closed.*

Solution. With switch open, there are two parallel branches across the 15-V supply. Branch *ABC* has a resistance of (3 + 12) = 15 Ω and branch *ABC* has a resistance of (6 + 4) = 10 Ω. Obviously, each branch has 15 V applied across it.

$$V_B = 12 \times 15/15 = 12 \text{ V}; \; V_D = 4 \times 15/(6+4) = 6 \text{ V}$$

$\therefore$ p.d. across points *B* and *D* = $V_B - V_D = 12 - 6 = 6$ V

When *S* is closed, the circuit becomes as shown in Fig. 1.20 (*b*) where points *B* and *D* become electrically connected together.

$R_{AB} = 3 \parallel 6 = 2\ \Omega$ and $R_{BC} = 4 \parallel 12 = 3\ \Omega$

$R_{AC} = 2 + 3 = 5\ \Omega$; $I = 15/5 = 3$ A

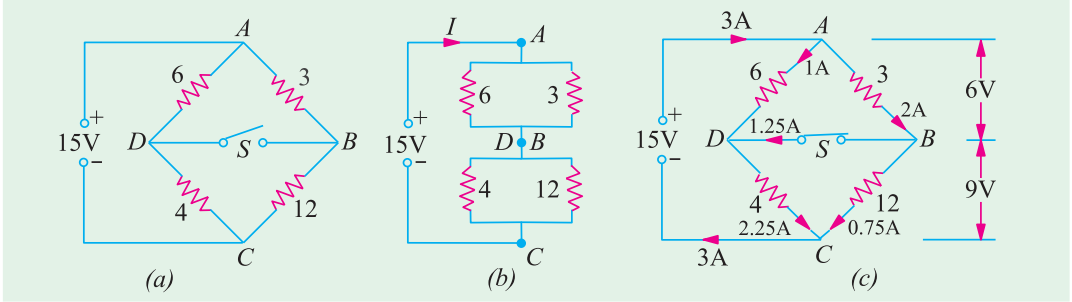


Fig. 1.20

Current through arm *AB* = 3 × 6/9 = 2 A. The voltage drop over arm *AB* = 3 × 2 = 6 V. Hence, drop over arm *BC* = 15 − 6 = 9 V. Current through *BC* = 9/12 = 0.75 A. It is obvious that at point *B*, the incoming current is 2 A, out of which 0.75 A flows along *BC*, whereas remaining 2 − 0.75 = 1.25 A passes through the switch.

As a check, it may be noted that current through *AD* = 6/6 = 1 A. At point *D*, this current is joined by 1.25 A coming through the switch. Hence, current through *DC* = 1.25 + 1 = 2.25 A. This fact can be further verified by the fact that there is a voltage drop of 9 V across 4 Ω resistor thereby giving a current of 9/4 = 2.25 A.

Example 1.30. *A 50-ohm resistor is in parallel with 100-ohm resistor. Current in 50-ohm resistor is 7.2 A. How will you add a third resistor and what will be its value of the line-current is to be its value if the line-current is to be 12.1 amp ?* **[Nagpur Univ., Nov. 1997]**

Solution. Source voltage = 50 × 7.2 = 360 V, Current through 100–ohm resistor = 3.6 A

Total current through these two resistors in parallel = 10.8 A

For the total line current to be 12.1 A, third resistor must be connected in parallel, as the third branch, for carrying (12.1 − 10.8) = 1.3 A. If *R* is this resistor *R* = 360/1.3 = 277 ohms

Example 1.31. *In the circuit shown in Fig. 1.21, calculate the value of the unknown resistance R and the current flowing through it when the current in branch OC is zero.*

[Nagpur Univ., April 1996]

Solution. If current through *R*-ohm resistor is *I* amp, *AO* branch carries the same current, since, current through the branch *CO* is zero. This also means that the nodes *C* and *O* are at the equal potential. Then, equating voltage-drops, we have $V_{AO} = V_{AC}$.

This means branch *AC* carries a current of 4*I*.

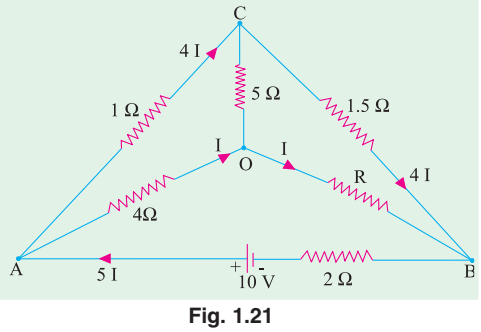


Fig. 1.21

This is current of 4 I also flows through the branch CB. Equating the voltage-drops in branches OB and CB,

$1.5 \times 4I = R\,I$, giving $R = 6\ \Omega$

At node A, applying KCL, a current of 5 I flows through the branch BA from B to A. Applying KVL around the loop $BAOB$, $I = 0.5$ Amp.

Example 1.32. *Find the values of R and V_s in Fig. 1.22. Also find the power supplied by the source.* **[Nagpur University, April 1998]**

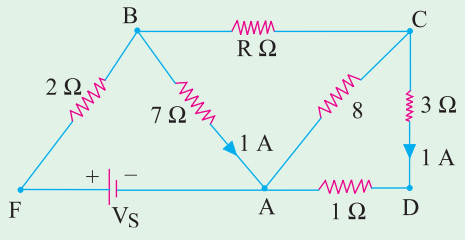


Fig. 1.22

Solution. Name the nodes as marked on Fig. 1.22. Treat node A as the reference node, so that $V_A = 0$. Since path ADC carries 1 A with a total of 4 ohms resistance, $V_C = +4$ V.

Since $V_{CA} + 4$, $I_{CA} = 4/8 = 0.5$ amp from C to A. Applying KCL at node C, $I_{BC} = 1.5$ A from B to C. Along the path BA, 1 A flows through 7–ohm resistor.

$V_B = +7$ Volts. $V_{BC} = 7 - 4 = +3$.

This drives a current of 1.5 amp, through R ohms. Thus $R = 3/1.5 = 2$ ohms.

Applying KCL at node B, $I_{FB} = 2.5$ A from F to B.

$V_{FB} = 2 \times 2.5 = 5$ volts, F being higher than B from the view-point of Potential. Since V_B has already been evaluated as + 7 volts, $V + 12$ volts (w.r. to A). Thus, the source voltage $V_s = 12$ volts.

Example 1.33. *In Fig. 1.23 (a), if all the resistances are of 6 ohms, calculate the equivalent resistance between any two diagonal points.* **[Nagpur Univ. April 1998]**

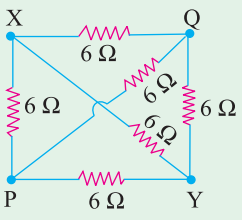


Fig. 1.23 (*a*)

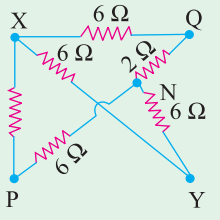


Fig. 1.23 (*b*)

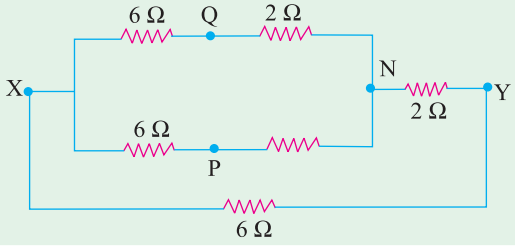


Fig. 1.23 (*c*)

Solution. If X-Y are treated as the concerned diagonal points, for evaluating equivalent resistance offered by the circuit, there are two ways of transforming this circuit, as discussed below :

Method 1 : Delta to Star conversion applicable to the delta of PQY introducing an additional node N as the star-point. Delta with 6 ohms at each side is converted as 2 ohms as each leg of the star-equivalent. This is shown in Fig. 1.23 (b), which is further simplified in Fig. 1.23 (c). After handling series-parallel combinations of resistances,

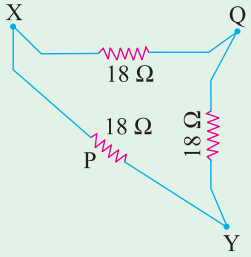


Fig. 1.23 (*d*)

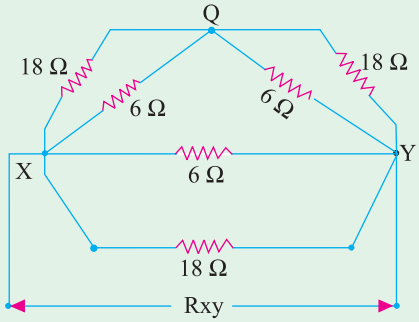


Fig. 1.23 (*e*)

Total resistance between X and Y terminals in Fig. 1.23 (c) comes out to be 3 ohms.

Method 2 : Star to Delta conversion with P as the star-point and XYQ to be the three points of concerned converted delta. With star-elements of 6 ohms each, equivalent delta-elements will be 18 ohms, as Fig. 1.23 (*d*). This is included while redrawing the circuit as in Fig. 1.23 (*e*).

After simplifying, the series-parallel combination results into the final answer as $R_{XY} = 3$ ohms.

Example 1.34. *For the given circuit find the current I_A and I_B.* **[Bombay Univ. 1991]**

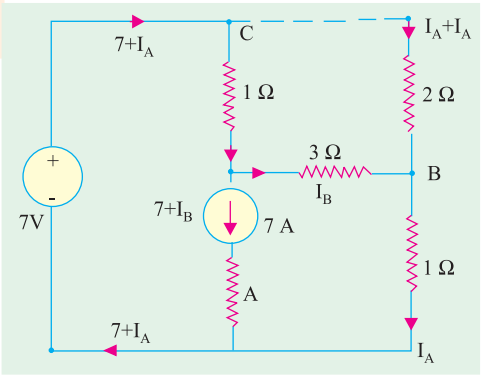


Fig. 1.24

Solution. Nodes *A, B, C, D* and reference node 0 are marked on the same diagram.

I_A and I_B are to be found.

Apply *KCL* at node *A*. From *C* to *A*, current = $7 + I_B$

At node 0, *KCL* is applied, which gives a current of $7 + I_A$ through the 7 volt voltage source. Applying *KCL* at node *B* gives a current $I_A - I_B$ through 2-ohm resistor in branch *CB*. Finally, at node *A, KCL* is applied. This gives a current of $7 + I_B$ through 1-ohm resistor in branch *CA*.

Around the Loop *OCBO*, $2\,(I_A - I_B) + 1.\,I_A = 7$

Around the Loop *CABC*, $1\,(7 + I_B) + 3\,I_B - 2\,(I_A - I_B) = 0$

After rearranging the terms, $3\,I_A - 2\,I_B = 7$, $2\,I_A + 6\,I_B = -7$

This gives $I_A = 2$ amp, $I_B = -0.5$ amp.

This means that I_B is 0.5 amp from *B* to *A*.

Example 1.35. *Find R_{AB} in the circuit, given in Fig. 1.25.* **[Bombay Univ. 2001]**

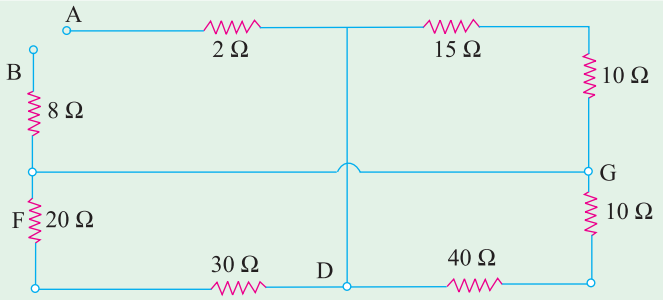


Fig. 1.25 (*a*)

Solution. Mark additional nodes on the diagram, *C, D, F, G*, as shown. Redraw the figure as in 1.25 (*b*), and simplify the circuit, to evaluate R_{AB}, which comes out to be 22.5 ohms.

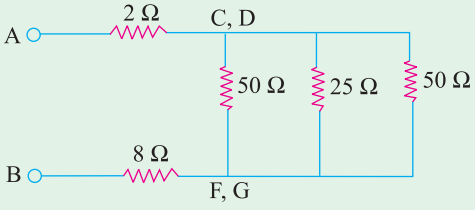


Fig. 1.25 (*b*)

Example 1.36. *Find current through 4 resistance.* **[Bombay Univ. 2001]**

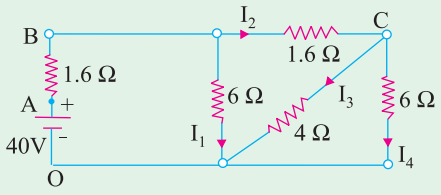


Fig. 1.26

Solution. Simplifying the series-parallel combinations, and solving the circuit, the source current is 10 amp. With respect to 0, $V_A = 40$, $V_B = 40 - 16 = 24$ volts.

I_1 = 4 amp, hence $I_2 = 6$ amp

$V_C = V_B - I_2 \times 1.6 = 24 - 9.6 = 14.4$ volts

$I_3 = 14.4/4 = 3.6$ amp, which is the required answer. Further $I_4 = 24$ amp.

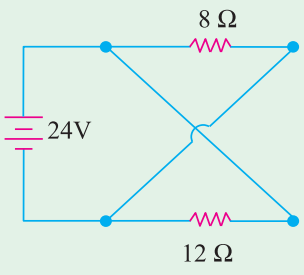


Fig. 1.27

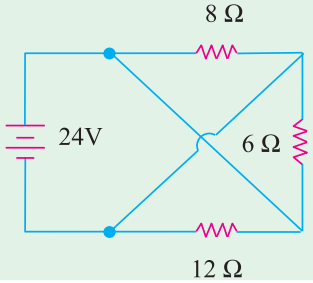


Fig. 1.28

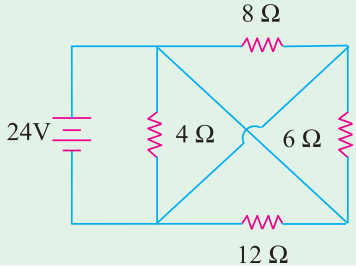


Fig. 1.29

Tutorial Problems No. 1.3

1. Find the current supplied by the battery in the circuit of Fig. 1.27. **[5 A]**
2. Compute total circuit resistance and battery current in Fig. 1.28. **[8/3 Ω, 9 A]**
3. Calculate battery current and equivalent resistance of the network shown in Fig. 1.29. **[15 A; 8/5 Ω]**
4. Find the equivalent resistance of the network of Fig. 1.30 between terminals *A* and *B*. All resistance values are in ohms. **[6 Ω]**
5. What is the equivalent resistance of the circuit of Fig. 1.31 between terminals *A* and *B* ? All resistances are in ohms. **[4 Ω]**
6. Compute the value of battery current *I* in Fig. 1.32. All resistances are in ohm. **[6 A]**

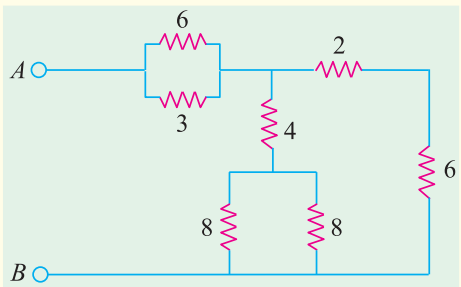


Fig. 1.30

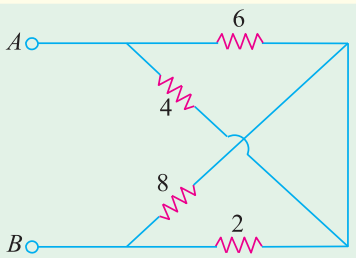


Fig. 1.31

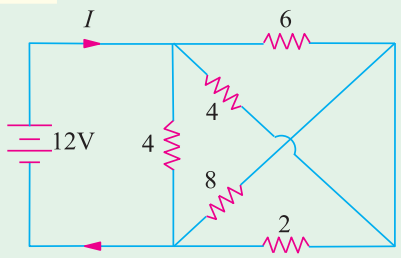


Fig. 1.32

7. Calculate the value of current I supplied by the voltage source in Fig. 1.33. All resistance values are in ohms. **(Hint :** Voltage across each resistor is 6 V) **[1 A]**

8. Compute the equivalent resistance of the circuit of Fig. 1.34 (*a*) between points (*i*) *ab* (*ii*) *ac* and (*iii*) *bc*. All resistances values are in ohm. **[(*i*) 6 Ω, (*ii*) 4.5 Ω, (*iii*) 4.5 Ω]**

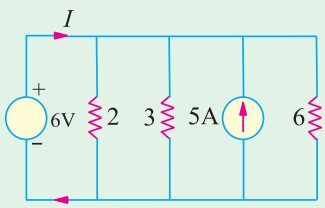


Fig. 1.33

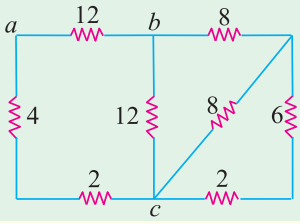


Fig. 1.34

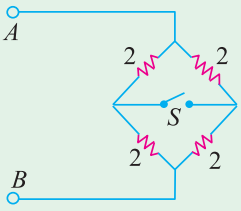


Fig. 1.35

9. In the circuit of Fig. 1.35, find the resistance between terminals A and B when switch is (*a*) open and (*b*) closed. Why are the two values equal ? **[(*a*) 2 Ω (*b*) 2 Ω]**

10. The total current drawn by a circuit consisting of three resistors connected in parallel is 12 A. The voltage drop across the first resistor is 12 V, the value of second resistor is 3 Ω and the power dissipation of the third resistor is 24 W. What are the resistances of the first and third resistors ? **[2Ω; 6Ω]**

11. Three parallel connected resistors when connected across a d.c. voltage source dissipate a total power of 72 W. The total current drawn is 6 A, the current flowing through the first resistor is 3 A and the second and third resistors have equal value. What are the resistances of the three resistors ? **[4 Ω; 8 Ω; 8 Ω]**

12. A bulb rated 110 V, 60 watts is connected with another bulb rated 110-V, 100 W across a 220 V mains. Calculate the resistance which should be joined in parallel with the first bulb so that both the bulbs may take their rated power. **[302.5 Ω]**

13. Two coils connected in parallel across 100 V supply mains take 10 A from the line. The power dissipated in one coil is 600 W. What is the resistance of the other coil ? **[25 Ω]**

14. An electric lamp whose resistance, when in use, is 2 Ω is connected to the terminals of a dry cell whose e.m.f. is 1.5 V. If the current through the lamp is 0.5 A, calculate the internal resistance of the cell and the potential difference between the terminals of the lamp. If two such cells are connected in parallel, find the resistance which must be connected in series with the arrangement to keep the current the same as before.
[1 Ω ; 1 V ; 0.5 Ω] (*Elect. Technology, Indore Univ.*)

15. Determine the current by the source in the circuit shown below. **(*Bombay Univ. 2001*)**

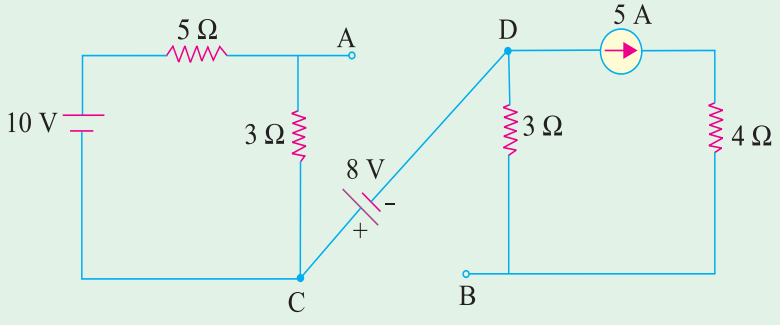


Fig. 1.36. (*a*)

Hint. Series-parallel combinations of resistors have to be dealt with. This leads to the source current of 28.463 amp.

16. Find the voltage of point A with respect to point B in the Fig. 1.36 (b). Is it positive with respect to B ? ***(Bombay University, 2000)***

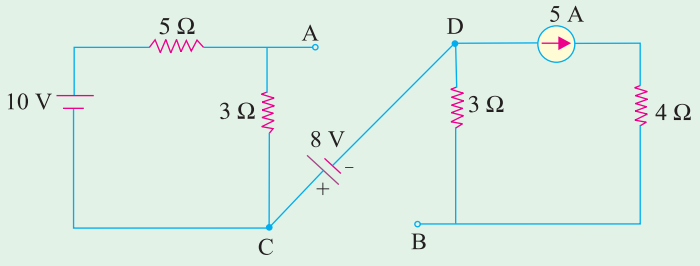


Fig. 1.36 (*b*)

Hint. If $V_A = 0, V_C = -1.25 \times 3 = -3.75$ V

$V_D = -3.75 - 8 = -11.75$ V

$V_B = V_D + 15 = +3.25$ volts

Thus, the potential of point A with respect to B is -3.25 V.

1.17. Types of Resistors

(*a*) *Carbon Composition*

It is a combination of carbon particles and a binding resin with different proportions for providing desired resistance. Attached to the ends of the resistive element are metal caps which have axial leads of tinned copper wire for soldering the resistor into a circuit. The resistor is enclosed in a plastic case to prevent the entry of moisture and other harmful elements from outside. Billions of carbon composition resistors are used in the electronic industry every year. They are available in power ratings of 1/8, 1/4, 1/2, 1 and 2 W, in voltage ratings of 250, 350 and 500 V. They have low failure rates when properly used.

Such resistors have a tendency to produce electric noise due to the current passing from one carbon particle to another. This noise appears in the form of a hiss in a loudspeaker connected to a hi-fi system and can overcome very weak signals. That is why carbon composition resistors are used where performance requirements are not demanding and where low cost in the main consideration. Hence, they are extensively used in entertainment electronics although better resistors are used in critical circuits.

(*b*) *Deposited Carbon*

Deposited carbon resistors consist of ceramic rods which have a carbon film deposited on them. They are made by placing a ceramic rod in a methane-filled flask and heating it until, by a gas-cracking process, a carbon film is deposited on them. A helix-grinding process forms the resistive path. As compared to carbon composition resistors, these resistors offer a major improvement in lower current noise and in closer tolerance. These resistors are being replaced by metal film and metal glaze resistors.

(*c*) *High-Voltage Ink Film*

These resistors consist of a ceramic base on which a special resistive ink is laid down in a helical band. These resistors are capable of withstanding high voltages and find extensive use in cathode-ray circuits, in radar and in medical electronics. Their resistances range from 1 kΩ to 100,000 MΩ with voltage range upto 1000 kV.

(*d*) *Metal Film*

Metal film resistors are made by depositing vaporized metal in vacuum on a ceramic-core rod. The resistive path is helix-ground as in the case of deposited carbon resistors. Metal film resistors have excellent tolerance and temperature coefficient and are extrememly reliable. Hence, they are very suitable for numerous high grade applications as in low-level stages of certain instruments although they are much more costlier.

(e) *Metal Glaze*

A metal glaze resistor consists of a metal glass mixture which is applied as a thick film to a ceramic substrate and then fired to form a film. The value of resistance depends on the amount of metal in the mixture. With helix-grinding, the resistance can be made to vary from 1 Ω to many mega-ohms.

Another category of metal glaze resistors consists of a tinned oxide film on a glass substrate.

(f) *Wire-wound*

Wire-wound resistors are different from all other types in the sense that no film or resistive coating is used in their construction. They consist of a ceramic-core wound with a drawn wire having accurately-controlled characteristics. Different wire alloys are used for providing different resistance ranges. These resistors have highest stability and highest power rating.

Because of their bulk, high-power ratings and high cost, they are not suitable for low-cost or high-density, limited-space applications. The completed wire-wound resistor is coated with an insulating material such as baked enamel.

(g) *Cermet (Ceramic Metal)*

The cermet resistors are made by firing certain metals blended with ceramics on a ceramic substrate. The value of resistance depends on the type of mix and its thickness. These resistors have very accurate resistance values and show high stability even under extreme temperatures. Usually, they are produced as small rectangles having leads for being attached to printed circuit boards (*PCB*).

1.18. Nonlinear Resistors

Those elements whose $V-I$ curves are not straight lines are called nonlinear elements because their resistances are nonlinear resistances. Their $V-I$ characteristics can be represented by a suitable equation.

Examples of nonlinear elements are filaments of incandescent lamps, diodes, thermistors and varistors. A varistor is a special resistor made of carborundum crystals held together by a binder. Fig. 1.37 (*a*) shows how current through a varistor increase rapidly when the applied voltage increases beyond a certain amount (nearly 100 V in the present case).

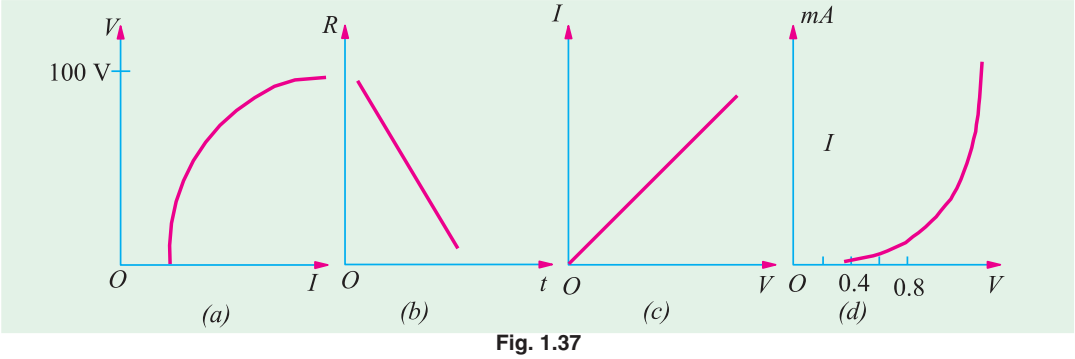


Fig. 1.37

There is a corresponding rapid decrease in resistance when the current increases. Hence, varistors are generally used to provide over-voltage protection in certain circuits.

A thermistor is made of metallic oxides in a suitable binder and has a large negative coefficient of resistance *i.e.,* its resistance decreases with increase in temperature as shown in Fig. 1.30 (*b*). Fig. 1.30 (*c*) shows how the resistance of an incandescent lamp increases with voltage whereas Fig. 1.30 (*d*) shows the *V-I* characteristics of a typical silicon diode. For a germanium diode, current is related to its voltage by the relation.

$$I = I_o\,(e^{V/0.026} - 1)$$

1.19. Varistor (Nonlinear Resistor)

It is a voltage-dependent metal-oxide material whose resistance decreases sharply with increasing voltage. The relationship between the current flowing through a varistor and the voltage applied across it is given by the relation : $i = ke^{\eta}$ where i = instantaneous current, e is the instantaneous voltage and η is a constant whose value depends on the metal oxides used. The value of η for silicon-carbide-based varistors lies between 2 and 6 whereas zinc-oxide-based varistors have a value ranging from 25 to 50.

The zinc-oxide-based varistors are primarily used for protecting solid-state power supplies from low and medium surge voltage in the supply line. Silicon-carbide varistors provide protection against high-voltage surges caused by lightning and by the discharge of electromagnetic energy stored in the magnetic fields of large coils.

1.20. Short and Open Circuits

When two points of circuit are connected together by a thick metallic wire (Fig. 1.38), they are said to be *short-circuited.* Since 'short' has practically zero resistance, it gives rise to two important facts :

(*i*) no voltage can exist across it because $V = IR = I \times 0 = 0$

(*ii*) current through it (called short-circuit current) is very large (theoretically, infinity)

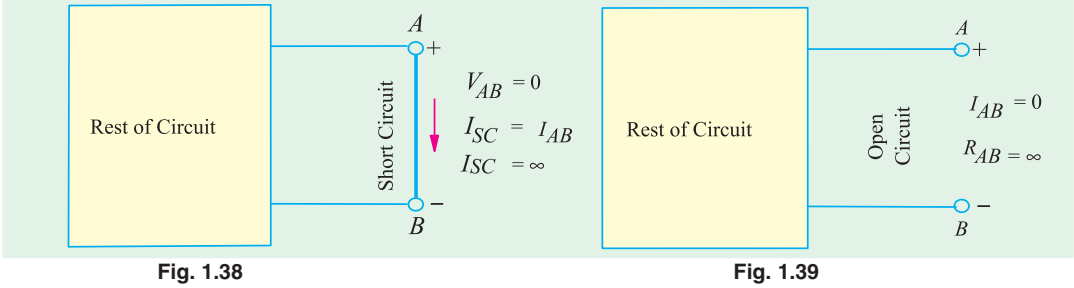


Fig. 1.38 Fig. 1.39

Two points are said to be open-circuited when there is no direct connection between them (Fig. 1.39). Obviously, an 'open' represents a break in the continuity of the circuit. Due to this break

(*i*) resistance between the two points is infinite.

(*ii*) there is no flow of current between the two points.

1.21. 'Shorts' in a Series Circuit

Since a dead (or solid) short has almost zero resistance, it causes the problem of excessive current which, in turn, causes power dissipation to increase many times and circuit components to burn out.

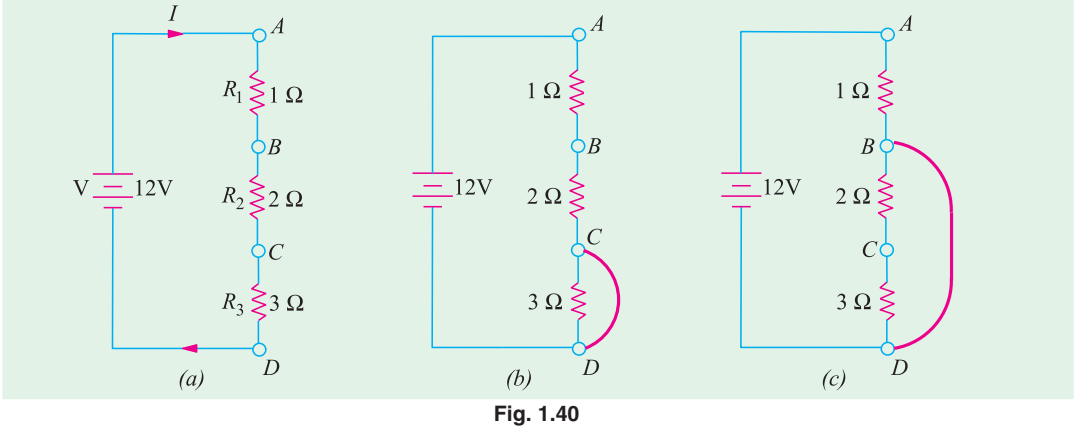


Fig. 1.40

In Fig. 1.40 (*a*) is shown a normal series circuit where

$$V = 12 \text{ V}, R = R_1 + R_2 + R_3 = 6\ \Omega$$
$$I = V/R = 12/6 = 2 \text{ A}, P = I^2R = 2^2 \times 6 = 24 \text{ W}$$

In Fig. 1.40 (*b*), 3-Ω resistor has been shorted out by a resistanceless copper wire so that $R_{CD} = 0$. Now, total circuit resistance $R = 1 + 2 + 0 = 3\ \Omega$. Hence, $I = 12/3 = 4$ A and $P = 4^2 \times 3 = 48$ W.

Fig. 1.40 (*c*) shows the situation where both 2 Ω and 3 Ω resistors have been shorted out of the circuit. In this case,

$$R = 1\ \Omega, I = 12/1 = 12 \text{ A} \quad \text{and} \quad P = 12^2 \times 1 = 144 \text{ W}$$

Because of this excessive current (6 times the normal value), connecting wires and other circuit components can become hot enough to ignite and burn out.

1.22. 'Opens' in a Series Circuit

In a normal series circuit like the one shown in Fig. 1.41 (*a*), there exists a current flow and the voltage drops across different resistors are proportional to their resistances. If the circuit becomes 'open' anywhere, following two effects are produced :

(i) since 'open' offers infinite resistance, circuit current becomes zero. Consequently, there is no voltage drop across R_1 and R_2.

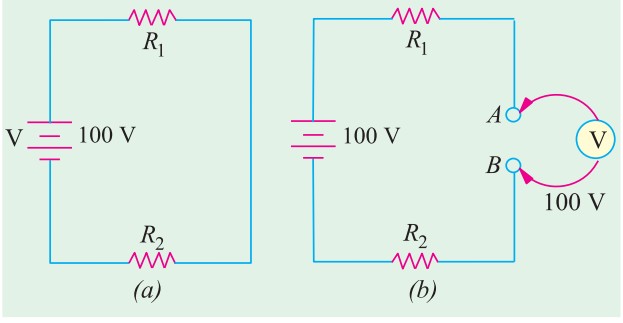


Fig. 1.41

(ii) whole of the applied voltage (i.e. 100 V in this case) is felt across the 'open' i.e. across terminals A and B [Fig. 1.41 (b)].

The reason for this is that R_1 and R_2 become negligible as compared to the infinite resistance of the 'open' which has practicallly whole of the applied voltage dropped across it (as per Voltage Divider Rule of art. 1.15). Hence, voltmeter in Fig. 1.41 (*b*) will read nearly 100 *V i.e.* the supply voltage.

1.23. 'Opens' in a Parallel Circuit

Since an 'open' offers infinite resistance, there would be no current in that part of the circuit where it occurs. In a parallel circuit, an 'open' can occur either in the main line or in any parallel branch.

As shown in Fig. 1.42 (*a*), an open in the main line prevents flow of current *to* ***all branches***. Hence, neither of the two bulbs glows. However, full applied voltage (*i.e.* 220 V in this case) ***is available across the open***.

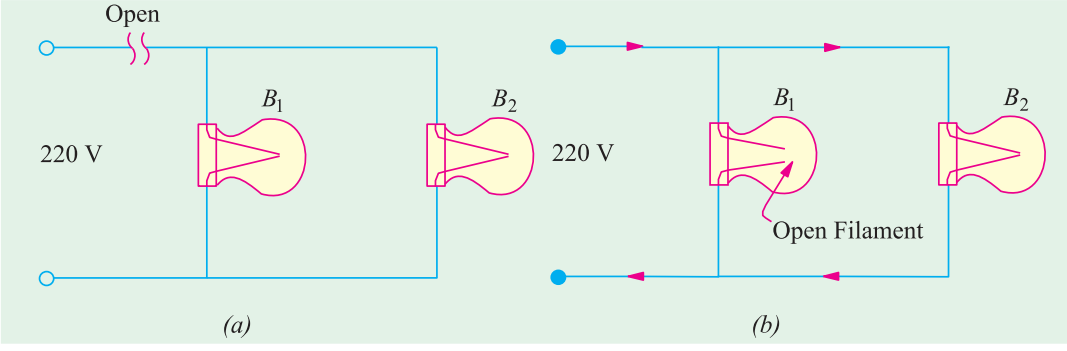


Fig. 1.42

In this Fig. 1.42 (*b*), 'open' has occurred in branch circuits of B_1. Since there is no current in this branch, B_1 will not glow. However, as the other bulb remains connected across the voltage supply, it would keep operating normality.

It may be noted that if a voltmeter is connected across the open bulb, it will read full supply voltage of 220 V.

1.24. 'Shorts' in Parallel Circuits

Suppose a 'short' is placed across R_3 (Fig. 1.43). It becomes directly connected across the battery and draws almost infinite current because not only its own resistance but that of the connecting wires AC and BD is negligible. Due to this excessive current, the wires may get hot enough to burn out unless the circuit is protected by a fuse.

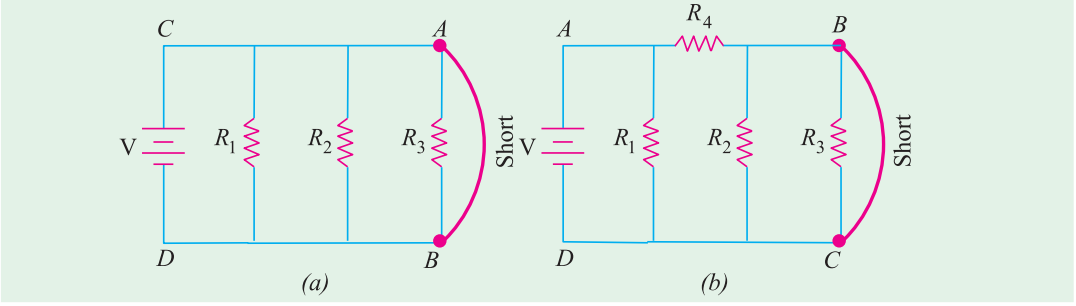


Fig. 1.43

Following points about the circuit of Fig. 1.43 (*a*) are worth noting.

1. not only is R_3 short-circuited but both R_1 and R_2 are also shorted out *i.e.* ***short across one branch means short across all branches***.
2. there is no current in shorted resistors. If there were three bulbs, they will not glow.
3. the shorted components are not damaged, For example, if we had three bulbs in Fig. 1.43 (*a*), they would glow again when circuit is restored to normal conditions by removing the short-circuited.

It may, however, be noted from Fig. 1.43 (*b*) that a short-circuit across R_3 may short out R_2 but not R_1 since it is protected by R_4.

1.25. Division of Current in Parallel Circuits

In Fig. 1.44, two resistances are joined in parallel across a voltage V. The current in each branch, as given in Ohm's law, is

$$I_1 = V/R_1 \text{ and } I_2 = V/R_2$$

$$\therefore \quad \frac{I_1}{I_2} = \frac{R_1}{R_2}$$

$$\text{As} \quad \frac{I}{R_1} = G_1 \text{ and } \frac{I}{R_2} = G_2$$

$$\therefore \quad \frac{I_1}{I_2} = \frac{G_1}{G_2}$$

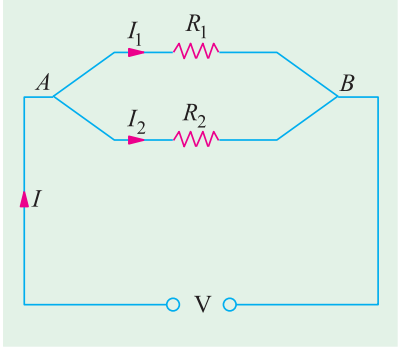


Fig. 1.44

Hence, the division of current in the branches of a parallel circuit is directly proportional to the conductance of the branches or inversely proportional to their resistances. We may also express the branch currents in terms of the total circuit current thus :

Now $$I_1 + I_2 = I; \quad \therefore I_2 = I - I_1 \quad \therefore \frac{I_1}{I - I_1} = \frac{R_2}{R_1} \quad \text{or} \quad I_1 R_1 = R_2 (I - I_1)$$

$$\therefore \quad I_1 = I \frac{R_1}{R_1 - R_2} = I \frac{G_1}{G_1 + G_2} \text{ and } I_2 = I \frac{R_1}{R_1 - R_2} = I \cdot \frac{G_1}{G_1 + G_2}$$

This Current Divider Rule has direct application in solving electric circuits by Norton's theorem (Art. 2.25).

Take the case of three resistors in parallel connected across a voltage V (Fig. 1.45). Total current is $I = I_1 + I_2 + I_3$. Let the equivalent resistance be R. Then

$$V = IR$$

Also $$V = I_1 R_1 \quad \therefore \; IR = I_1 R_1$$

or $$\frac{I}{I_1} = \frac{R_1}{R} \quad \text{or} \quad I_1 = IR/R_1 \quad ...(i)$$

Now $$\frac{I}{R} = \frac{I}{R_1} + \frac{I}{R_2} + \frac{I}{R_3}$$

$$R = \frac{R_1 R_2 R_3}{R_2 R_3 + R_2 R_3 + R_1 R_2}$$

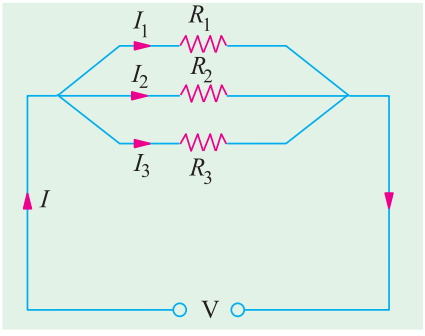


Fig. 1.45

From **(*i*)** above, $$I_1 = I\left(\frac{R_2 R_3}{R_1 R_2 + R_2 R_3 + R_3 R_1}\right) = I \cdot \frac{G_1}{G_1 + G_2 + G_3}$$

Similarly, $$I_2 = I \frac{R_1 R_3}{R_1 R_2 \quad R_2 R_3 \quad R_3 R_1} \quad I \cdot \frac{G_2}{G_1 \quad G_2 \quad G_3}$$

$$I_3 = I \frac{R_1 R_2}{R_1 R_2 \quad R_2 R_3 \quad R_3 R_1} \quad I \cdot \frac{G_3}{G_1 \quad G_2 \quad G_3}$$

Example 1.37. *A resistance of 10 Ω is connected in series with two resistances each of 15 Ω arranged in parallel. What resistance must be shunted across this parallel combination so that the total current taken shall be 1.5 A with 20 V applied ?*

(Elements of Elect. Engg.-1; Banglore Univ.)

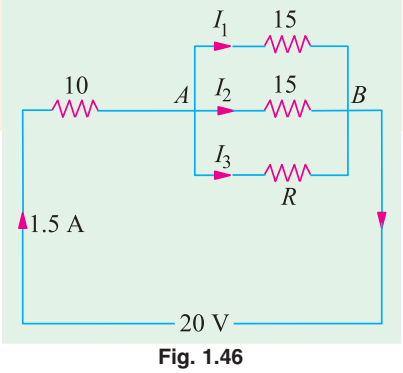


Fig. 1.46

Solution. The circuit connections are shown in Fig. 1.46.

Drop across 10-Ω resistor = $1.5 \times 10 = 15$ V

Drop across parallel combination, $V_{AB} = 20 - 15 = 5$ V

Hence, voltage across each parallel resistance is 5 V.

$$I_1 = 5/15 = 1/3 \text{ A}, \; I_2 = 5/15 = 1/3 \text{ A}$$

$$I_3 = 1.5 - (1/3 + 1/3) = 5/6 \text{ A}$$

$$\therefore \quad I_3 R = 5 \quad \text{or} \quad (5/6) R = 5 \quad \text{or} \quad R = \mathbf{6\ \Omega}$$

Example 1.38. *If 20 V be applied across AB shown in Fig. 1.40, calculate the total current, the power dissipated in each resistor and the value of the series resistance to have the total current.*

(Elect. Science-II, Allahabad Univ. 1992)

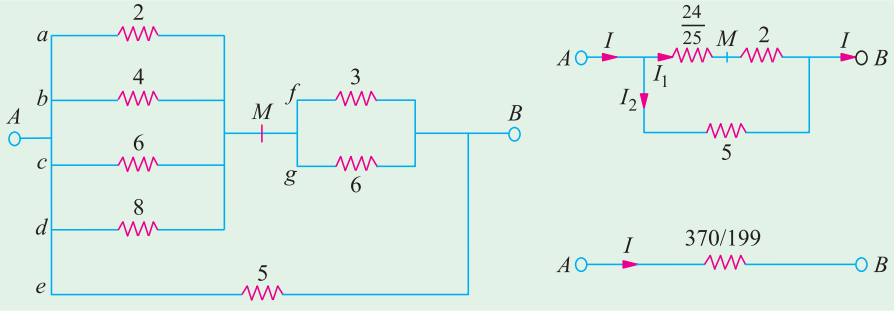


Fig. 1.47

Solution. As seen from Fig. 1.47. $R_{AB} = 370/199\ \Omega$

Hence, total current $= 20 \div 370/199 = \mathbf{10.76\ A}$

$I_1 = 10.76 \times 5(5 + 74.25) = 6.76\ A$; $I_2 = 10.76 - 6.76 = 4$ A

$I_f = 6.76 \times 6/9 = 4.51$ A; $I_g = 6.76 - 4.51 = 2.25$ A

Voltag drop across A and M, $V_{AM} = 6.76 \times 24/25 = 6.48$ V

$I_a = V_{AM}/2 = 6.48/2 = 3.24$ A; $I_b = 6.48/4 = 1.62$ A; $I_c = 6.48/6 = 1.08$ A

$I_d = 6.48/8 = 0.81$ A, $I_e = 20/5 = 4$ A

Power Dissipation

$P_a = I_a^2 R_a = 3.24^2 \times 2 = \mathbf{21\ W}$, $P_b = 1.62^2 \times 4 = \mathbf{10.4\ W}$, $P_c = 1.08^2 \times 6 = \mathbf{7\ W}$

$P_d = 0.81^2 \times 8 = \mathbf{5.25\ W}$, $P_e = 4^2 \times 5 = \mathbf{80\ W}$, $P_f = 4.51^2 \times 3 = \mathbf{61\ W}$

$P_g = 2.25^2 \times 6 = \mathbf{30.4\ W}$

The series resistance required is $\mathbf{370/199\ \Omega}$

Incidentally, total power dissipated $= I^2 R_{AB} = 10.76^2 \times 370/199 = 215.3$ W (as a check).

Example 1.39. *Calculate the values of different currents for the circuit shown in Fig. 1.48. What is the total circuit conductance ? and resistance ?*

Solution. As seen, $I = I_1 + I_2 + I_3$. The current division takes place at point B.

As seen from Art. 1.25.

$$I = I \cdot \frac{G_1}{G_1 + G_2 + G_3}$$

$$= 12 \times \frac{0.1}{0.6} = \mathbf{2\ A}$$

$I_2 = 12 \times 0.2/0.6 = \mathbf{4\ A}$

$I_3 = 12 \times 0.3/0.6 = \mathbf{6\ A}$

$G_{BC} = 0.1 + 0.2 + 0.3 = 0.6$ S

$$\frac{1}{G_{AC}} = \frac{1}{G_{AB}} + \frac{1}{G_{BC}} = \frac{1}{0.4} + \frac{1}{0.6} = \frac{25}{6}\ \text{S}^{-1}$$

$\therefore\ R_{AC} = 1/G_{AC} = \mathbf{25/6\ \Omega}$

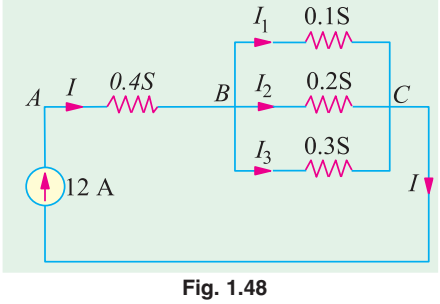


Fig. 1.48

Example 1.40. *Compute the values of three branch currents for the circuits of Fig. 1.49 (a). What is the potential difference between points A and B ?*

Solution. The two given current sources may be combined together as shown in Fig. 1.49 (*b*). Net current = 25 – 6 = 19 A because the two currents flow in opposite directions.

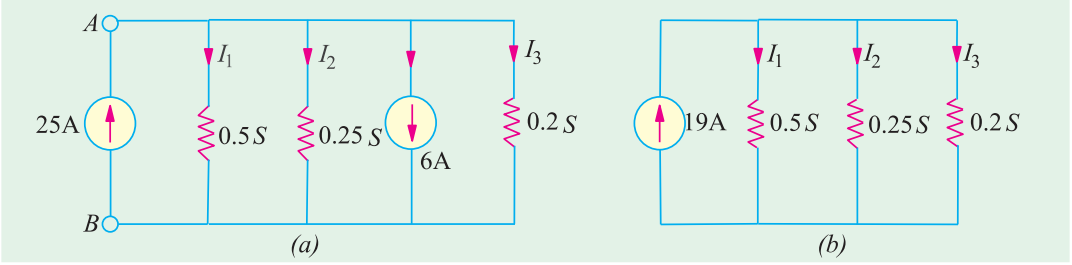


Fig. 1.49

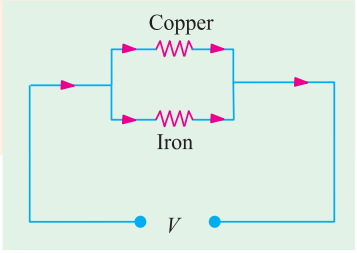
Now, $G = 0.5 + 0.25 + 0.2 = 0.95$ S; $I_1 = I\frac{G_1}{G} = 19 \times \frac{0.5}{0.95} = \mathbf{10\ A}$

$I_2 = I\frac{G_2}{G} = 19 \times \frac{0.5}{0.95} = \mathbf{5\ A}$; $I_3 = I\frac{G_3}{G} = 19 \times \frac{0.2}{0.95} = \mathbf{4\ A}$

$$V_{AB} = I_1 R_1 = \frac{I_1}{G_1} = \frac{I_2}{G_2} = \frac{I_3}{G_3} \therefore V_{AB} = \frac{10}{0.5} = \mathbf{20\ A}$$

The same voltage acts across the three conductances.

Example 1.41. *Two conductors, one of copper and the other of iron, are connected in parallel and at 20°C carry equal currents. What proportion of current will pass through each if the temperature is raised to 100°C ? Assume α for copper as 0.0042 and for iron as 0.006 per °C at 20°C. Find also the values of temperature coefficients at 100°C.* **(Electrical Engg. Madras Univ.)**



Solution. Since they carry equal current at 20°C, the two conductors have the same resistance at 20°C *i.e.* R_{20}. As temperature is raised, their resistances increase through unequally.

For Cu, $R_{100} = R_{20}(1 + 80 \times 0.0042) = 1.336\, R_{20}$

For iron $R'_{100} = R_{20}(1 + 80 \times 0.006) = 1.48\, R_{20}$

As seen from Art. 1.25, current through Cu conductor is

$$I_1 = I \times \frac{R'_{100}}{R_{100} + R'_{100}} = I \times \frac{1.48\, R_{20}}{2.816\, R_{20}} = 0.5256\, I \text{ or } \mathbf{52.56\% \text{ of } I}$$

Hence, current through Cu conductor is 52.56 per cent of the total current. Obviously, the remaining current *i.e.* 47.44 per cent passes through iron.

Or current through iron conductor is

$$I_2 = I. \frac{R'_{100}}{R_{100} + R'_{100}} = I \times \frac{1.336\, R_{20}}{2.816\, R_{20}} = 0.4744\, I \text{ or } \mathbf{47.44\% \text{ of } I}$$

For Cu, $\alpha_{100} = \dfrac{1}{(1/0.0042) + 80} = \mathbf{0.00314°C^{-1}}$

For iron, $\alpha_{100} = \dfrac{1}{(1/0.006) + 80} = \mathbf{0.0040°C^{-1}}$

Example 1.42. *A battery of unknown e.m.f. is connected across resistances as shown in Fig. 1.50. The voltage drop across the 8 Ω resistor is 20 V. What will be the current reading in the ammeter ? What is the e.m.f. of the battery ?* **(Basic Elect. Engg.; Bangladesh Univ., 1990)**

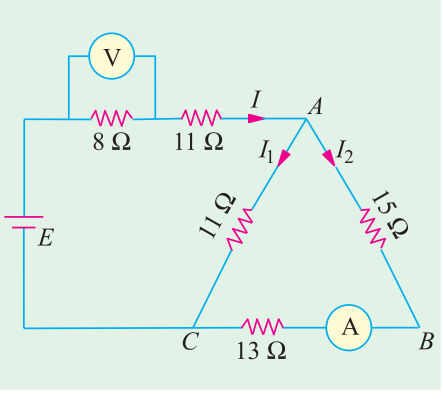


Fig. 1.50

Solution. Current through 8 Ω resistance = 20/8 = 2.5 A

This current is divided into two parts at point *A*; one part going along path *AC* and the other along path *ABC* which has a resistance of 28 Ω

$$I_2 = 2.5 \times \frac{11}{(11 + 28)} = 0.7$$

Hence, ammeter reads **0.7 A.**

Resistance between *A* and *C* = (28 × 11/39) ohm.

Total circuit resistance = 8 + 11 + (308/39) = 1049/39 Ω

$\therefore$ $E = 2.5 \times 1049/39 = \mathbf{67.3\ V}$

1.26. Equivalent Resistance

The equivalent resistance of a circuit (or network) between its any two points (or terminals) is given by that ***single*** resistance which can replace the entire given circuit between ***these two points***. It should be noted that resistance is always between two ***given*** points of a circuit and can have different

values for different point-pairs as illustrated by Example 1.42. it can usually be found by using series and parallel laws of resistances. Concept of equivalent resistance is essential for understanding network theorems like Thevenin's theorem and Norton's theorem etc. discussed in Chapter 2.

Example 1.43. *Find the equivalent resistance of the circuit given in Fig. 1.51 (a) between the following points* **(i)** *A and B* **(ii)** *C and D* **(iii)** *E and F* **(iv)** *A and F and* **(v)** *A and C. Numbers represent resistances in ohm.*

Solution. (*i*) Resistance Between *A* and *B*

In this case, the entire circuit to the right side of *AB* is in parallel with 1 Ω resistance connected directly across points *A* and *B*.

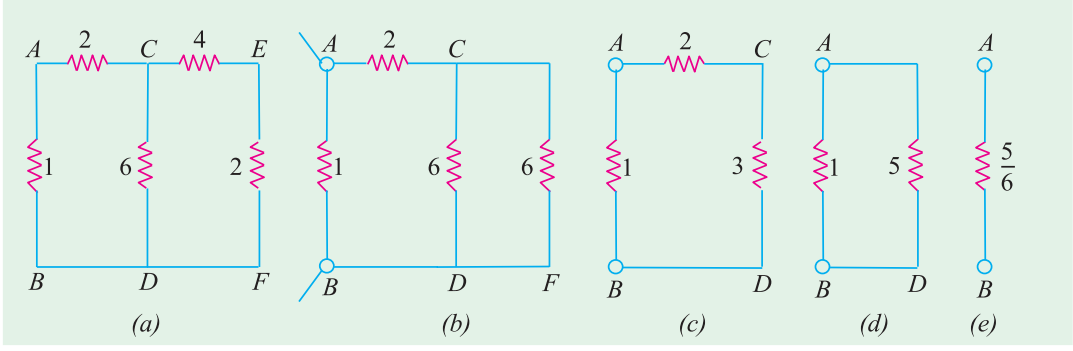


Fig. 1.51

As seen, there are two parallel paths across points *C* and *D*; one having a resistance of 6 Ω and the other of (4 + 2) = 6 Ω. As shown in Fig. 1.51 (*c*), the combined resistance between *C* and *D* is = 6 || 6 = 3 Ω Further simplifications are shown in Fig. 1.51 (*d*) and (*e*). As seen, R_{AD} = **5/6 Ω**

(*ii*) Resistance between *C* and *D*

As seen from Fig. 1.51 (*a*), there are three parallel paths between *C* and *D* **(*i*)** *CD* itself of 6 Ω **(*ii*)** *CEFD* of (4 + 2) = 6 Ω and **(*iii*)** *CABD* of (2 + 1) = 3 Ω It has been shown separately in Fig. 1.52 (*a*). The equivalent resistance R_{CD} = 3 || 6 || 6 = 1.5 Ω as shown in Fig. 1.52 (*b*).

(*iii*) Resistance between *E* and *F*

In this case, the circuit to the left side of *EF* is in parallel with the 2 Ω resistance connected directly across *E* and *F*. This circuit consists of a 4 Ω resistance connected in series with a parallel

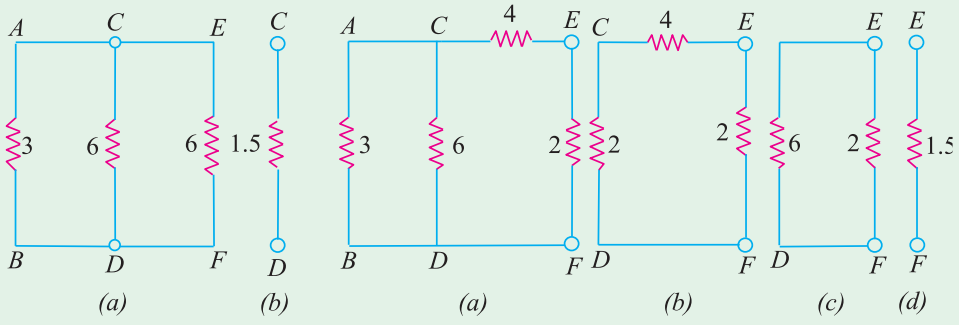


Fig. 1.52 **Fig. 1.53**

circuit of 6 || (2 + 1) = 2 Ω resistance. After various simplifications as shown in Fig. 1.53, R_{EF} = 2 || 6 = **1.5 Ω**

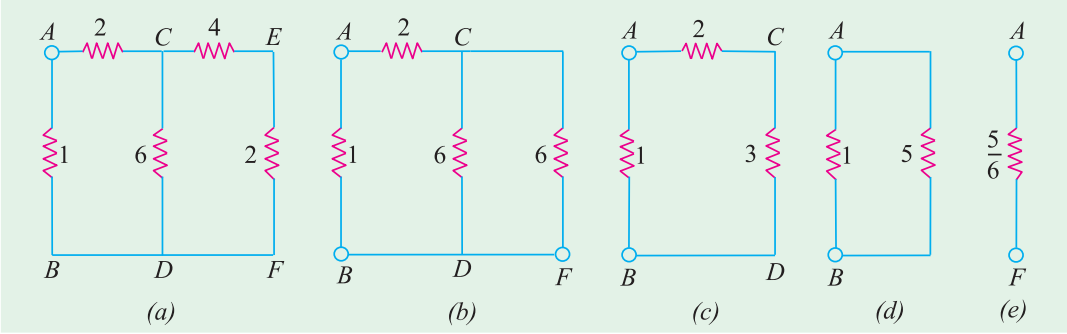


Fig. 1.54

(iv) Resistance Between *A* and *F*

As we go from *A* and *F*, there are two possible routes to begin with : one along *ABDF* and the other along *AC*. At point *C*, there are again two alternatives, one along *CDF* and the other along *CEF*.

As seen from Fig. 1.54 (*b*), $R_{CD} = 6 \parallel 6 = 3\ \Omega$. Further simplification of the original circuit as shown in Fig. 1.54 (*c*), (*d*) and (*e*) gives $R_{AF} =$ **5/6 Ω**

(v) Resistance Between *A* and *C*

In this case, there are two parallel paths between *A* and *C* ; one is directly from *A* to *C* and the other is along *ABD*. At *D*, there are again two parallel paths to *C*; one is directly along *DC* and the other is along *DFEC*.

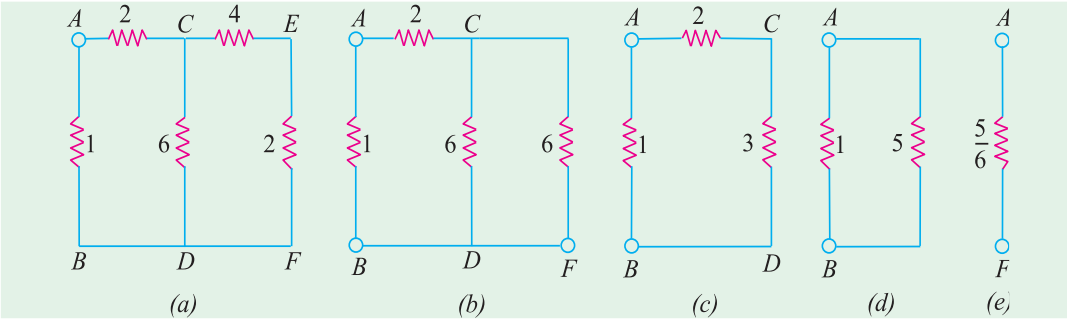


Fig. 1.55

As seen from Fig. 1.55 (*b*), $R_{CD} = 6 \parallel 6 = 3\ \Omega$ Again, from Fig. 1.55 (*d*), $R_{AC} = 2 \parallel 4 =$ **4/3 Ω**

Example 1.44. *Two resistors of values 1 kΩ and 4 Ω are connected in series across a constant voltage supply of 100 V. A voltmeter having an internal resistance of 12 kΩ is connected across the 4 kΩ resistor. Draw the circuit and calculate*

(a) *true voltage across 4 kΩ resistor before the voltmeter was connected.*

(b) *actual voltage across 4kΩ resistor after the voltmeter is connected and the voltage recorded by the voltmeter.*

(c) *change in supply current when voltmeter is connected.*

(d) *percentage error in voltage across 4 kΩ resistor.*

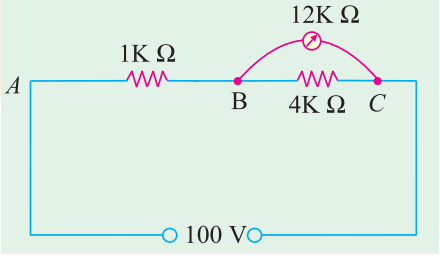


Fig. 1.56

Solution. (a) True voltage drop across 4 kΩ resistor as found by voltage-divider rule is $100 \times 4/5 = 80$ V

Current from the supply = $100/(4 + 1) = 20$ mA

(b) In Fig. 1.56, voltmeter has been joined across the 4 kΩ resistor. The equivalent resistance between *B* and *C* = $4 \times 12/16 = 3$ kΩ

Drop across B and $C = 100 \times 3/(3+1) = 75$ V.

(c) Resistance between A and $C = 3 + 1 = 4$ kΩ

New supply current $= 100/4 = 25$ mA

$\therefore$ increase in current $= 25 - 20 = 5$ mA

(d) Percentage error in voltage $= \dfrac{\text{actual voltage} - \text{true voltage}}{\text{true voltage}} = \dfrac{(75-80)}{80} \times 100 = -6.25\%$

The reduction in the value of voltage being measured in called voltmeter loading effect because voltmeter loads down the circuit element across which it is connected. Smaller the voltmeter resistance as compared to the resistance across which it is connected, greater the loading effect and, hence, greater the error in the voltage reading. Loading effect cannot be avoided but can be minimized by selecting a voltmeter of resistance much greater than that of the network across which it is connected.

Example 1.45. *In the circuit of Fig. 1.57, find the value of supply voltage V so that 20-Ω resistor can dissipate 180 W.*

Solution. $I_4^2 \times 20 = 180$ W; $I_4 = 3$ A

Since 15 Ω and 20 Ω are in parallel,

$I_3 \times 15 = 3 \times 20 \quad \therefore I_3 = 4$ A

$I_2 = I_3 + I_4 = 4 + 3 = 7$ A

Now, resistance of the circuit to the right of point A is

$= 10 + 15 \times 20/35 = 130/7$ Ω

$\therefore \quad I_1 \times 25 = 7 \times 130/7$

$\therefore \quad I_1 = 26/5$ A $= 5.2$ A

$\therefore \quad I = I_1 + I_2 = 5.2 + 7 = 12.2$ A

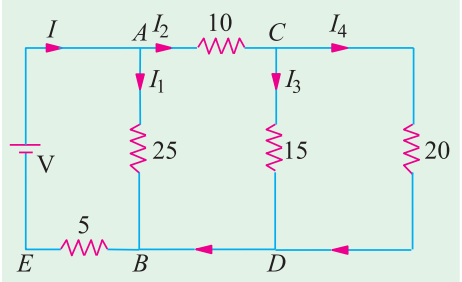


Fig. 1.57

Total circuit resistance

$R_{AE} = 5 + 25 \parallel 130/7 = 955/61$ Ω

$\therefore \quad V = I \,.\, R_{AE} = 12.2 \times 955/61 = \mathbf{191\ V}$

Example 1.46. *For the simple ladder network shown in Fig. 1.58, find the input voltage V_i which produces a current of 0.25 A in the 3 Ω resistor. All resistances are in ohm.*

Solution. We will assume a current of 1 A in the 3 Ω resistor. The voltage necessary to produce 1 A bears the same ratio to 1 A as V_i does to 0.25 A because of the linearity of the network. It is known as Current Assumption technique.

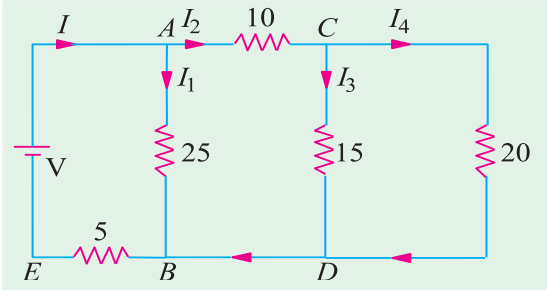


Fig. 1.58

Since $R_{cdef} = R_{cf} = 6$ Ω

Hence, $I_{cf} = 1$ A

and $V_{cf} = V_{cdef} = 1 \times 6 = 6$ V.

Also, $I_{bc} = 1 + 1 = 2$ A

$V_{bg} = V_{bb} + V_{ef} = 2 \times 5 + 6 = 16$ V

$I_{bg} = 16/8 = 2$ A

$I_{ab} = I_{bc} + b_{bg} = 2 + 2 = 4$ A

$V_i = V_{ab} + V_{bg} + V_{gh}$

$= 4 \times 7 + 16 + 4 \times 9 = 80$ V

Taking the proportion, we get

$$\frac{80}{1} = \frac{V_i}{0.25} \quad \therefore V_i = 80 \times 0.25 = \mathbf{20\ V}$$

Example 1.47. *In this circuit of Fig. 1.59, find the value R_1 and R_2 so that $I_2 = I_1/n$ and the input resistance as seen from points A and B is R ohm.*

Solution. As seen, the current through R_2 in $(I_1 - I_2)$. Hence, p.d. across points C and D is

$$R_2 (I_1 - I_2) = (R_1 + R)\, I_2 \text{ or } R_2 I_1 = (R_1 + R_2 + R)\, I_2$$

$$\therefore \quad \frac{I_1}{I_2} = \frac{R_1 + R_2 + R}{R_2} = n \qquad \text{...(i)}$$

The input resistance of the circuit as viewed from terminals A and B is required to be R.

$$\therefore \quad R = R_1 + R_2 \,\|\, (R_1 + R)$$

$$= R_1 + \frac{R_1 + R}{n} \qquad \text{...using Eq. (i)}$$

$$R\,(n-1) = R_1\,(n+1)$$

$$\therefore \quad R_1 = \frac{n-1}{n+1} R \quad \text{and} \quad R_2 = \frac{R_1 + R}{(n-1)} = \frac{2n}{n^2 - 1} R$$

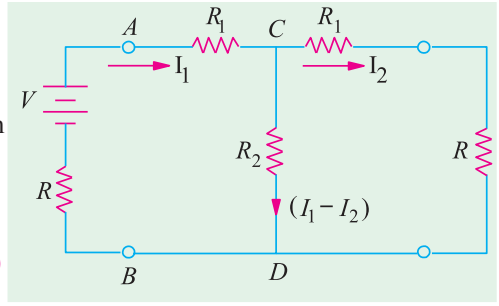


Fig. 1.59

1.27. Duality Between Series and Parallel Circuits

There is a certain peculiar pattern of relationship between series and parallel circuits. For example, in a series circuit, current is the same whereas in a parallel circuit, voltage is the same. Also, in a series circuit, individual voltages are added and in a parallel circuit, individual currents are added. It is seen that while comparing series and parallel circuits, voltage takes the place of current and current takes the place of voltage. Such a pattern is known as "duality" and the two circuits are said to be duals of each other.

As arranged in Table 1.4 the equations involving voltage, current and resistance in a series circuit have a corresponding dual counterparts in terms of current, voltage and conductance for a parallel circuit.

Table 1.4

Series Circuit	Parallel Circuit
$I_1 = I_2 = I_3 = ...$	$V_1 = V_2 = V_3 = ...$
$V_T = V_1 + V_2 + V_3 + ...$	$I_1 = I_1 + I_2 + I_3 + ...$
$R_T = R_1 + R_2 + R_3 + ...$	$G_T = G_1 + G_2 + G_3 + ...$
$I = \frac{V_1}{R_1} = \frac{V_2}{R_2} = \frac{V_3}{R_3} = ...$	$V = \frac{I_1}{G_1} = \frac{I_2}{G_2} = \frac{I_3}{G_3} = ...$
Voltage Divider Rule $V_1 = V_T \frac{R_1}{R_T}, V_2 = V_T \frac{R_2}{R_T}$	Current Divider Rule $I_1 = I_T \frac{G_1}{G_T}, I_2 = I_T \frac{G_2}{G_T}$

Tutorial Problems No. 1.4

1. Using the current-divider rule, find the ratio I_L/I_S in the circuit shown in Fig. 1.60. **[0.25]**
2. Find the values of variables indicated in the circuit of Fig. 1.61. All resistances are in ohms. **[(a) 40 V (b) 21 V; 15 V (c) –5 A; 3 A]**

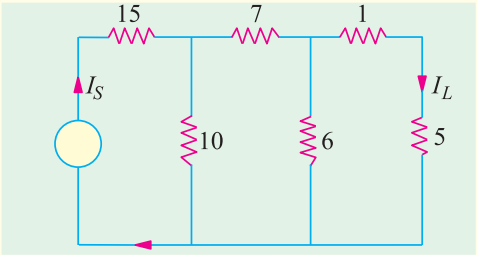


Fig. 1.60

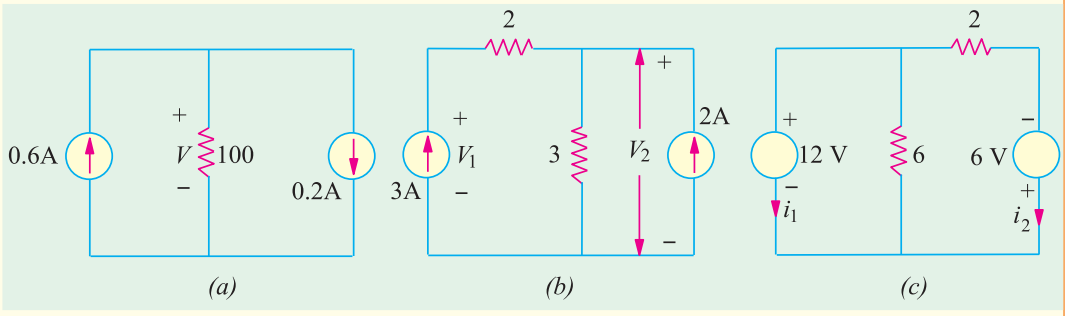


Fig. 1.61

3. An ohmeter is used for measuring the resistance of a circuit between its two terminals. What would be the reading of such an instrument used for the circuit of Fig. 1.62 at point (*a*) *AB* (*b*) *AC* and (*c*) *BC* ? All resistances are in ohm.

[(*a*) 25 Ω (*b*) 24 Ω (*c*) 9 Ω]

4. Find the current and power supplied by the battery to the circuit of Fig. 1.63 (*i*) under normal conditions and (*ii*) when a 'short' occurs across terminals *A* and *B*. All resistances are in kilo-ohm.

[(*i*) 2 mA; 24 m W; (*ii*) 34 mA; 36 mW]

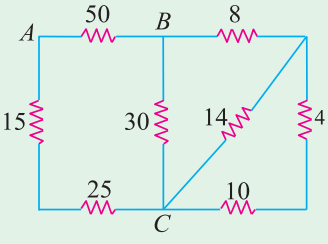


Fig. 1.62

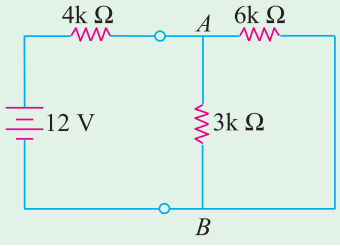


Fig. 1.63

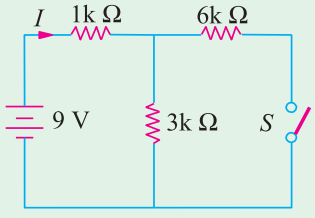


Fig. 1.64

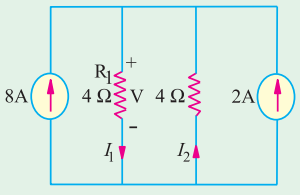


Fig. 1.65

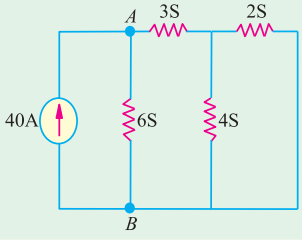


Fig. 1.66

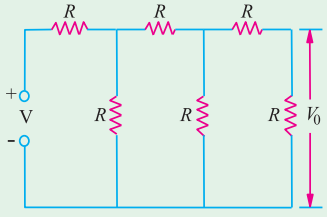


Fig. 1.67

5. Compute the values of battery current *I* and voltage drop across 6 kΩ resistor of Fig. 1.64 when switch *S* is **(*a*)** closed and **(*b*)** open. All resistance values are in kilo-ohm.

[(*a*) 3mA; 6 V; (*b*) 2.25 mA; 0V]

6. For the parallel circuit of Fig. 1.65 calculate (*i*) *V* (*ii*) I_1 (*iii*) I_2. **[(*i*) 20 V; (*ii*) 5 A; (*iii*) −5 A]**

7. Find the voltage across terminals *A* and *B* of the circuit shown in Fig. 1.66. All conductances are in siemens (*S*). **[5 V]**

8. Prove that the output voltage V_0 in the circuit of Fig. 1.67 is *V*/13.

9. A fault has occurred in the circuit of Fig. 1.68. One resistor has burnt out and has become an open. Which is the resistor if current supplied by the battery is 6 A ? All resistances are in ohm. **[4 Ω]**

10. In Fig. 1.69 if resistance between terminals *A* and *B* measures 1000 Ω, which resistor is open-circuited. All conductance values are in milli-siemens (mS). **[0.8 mS]**

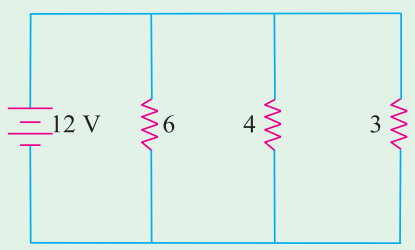


Fig. 1.68

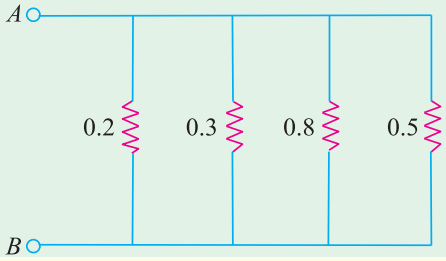


Fig. 1.69

11. In the circuit of Fig. 1.70, find current (*a*) I and (*b*) I_1. **[(*a*) 2 A; (*b*) 0.5 A]**

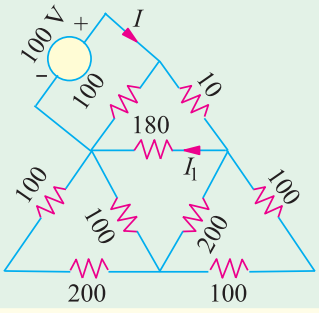


Fig. 1.70

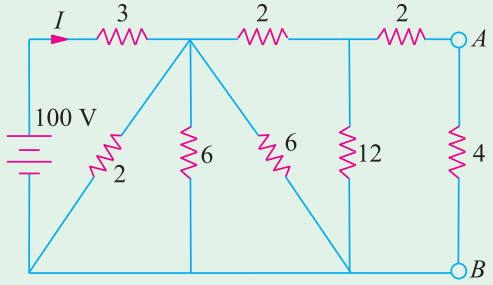


Fig. 1.71

12. Deduce the current I in the circuit of Fig. 1.71. All resistances are in ohms. **[25 A]**

13. Two resistors of 100 Ω and 200 Ω are connected in series across a 4-V cell of negligible internal resistance. A voltmeter of 200 Ω resistance is used to measures P.D. across each. What will the voltage be in each case ? **[1 V across 100 Ω ; 2 V across 200 Ω]**

14. Using series–parallel combination laws, find the resistance between terminals A and B of the network shown in Fig. 1.72. **[4 *R*]**

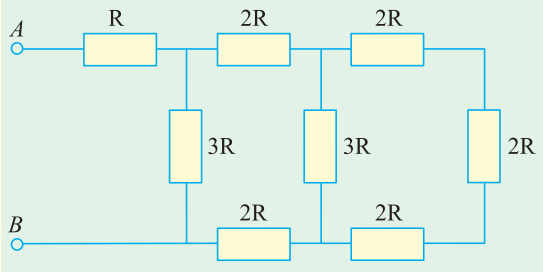


Fig. 1.72

15. A resistance coil AB of 100 Ω resistance is to be used as a potentiometer and is connected to a supply at 230 V. Find, by calculation, the position of a tapping point C between A and B such that a current of 2 A will flow in a resistance of 50 Ω connected across A and C.

[43.4 Ω from *A* to *C*] (*London Univ.*)

16. In the circuit shown in Fig. 1.73, calculate (*a*) current I (*b*) current I_1 and (*c*) V_{AB}. All resistances are in ohms. **[(*a*) 4 A (*b*) 0.25 *A* (*c*) 4 *V*]**

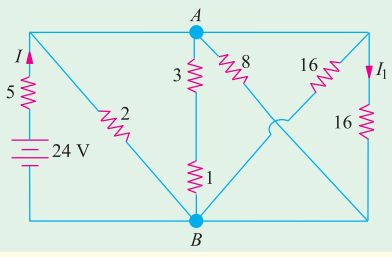


Fig. 1.73

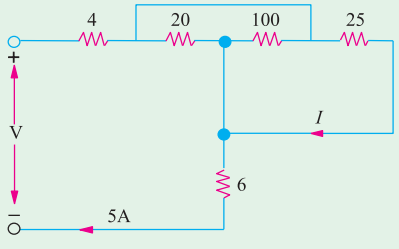


Fig. 1.74

17. In the circuit given in Fig. 1.74, calculate (*a*) current through the 25 Ω resistor (*b*) supply voltage *V*. All resistances are in ohms. **[(*a*) 2 A (*b*) 100 V]**

18. Using series and parallel combinations for the electrical network of Fig. 1.75, calculate (*a*) current flowing in branch *AF* (*b*) p.d. across branch *CD*. All resistances are in ohms. **[(*a*) 2 A (*b*) 1.25 V]**

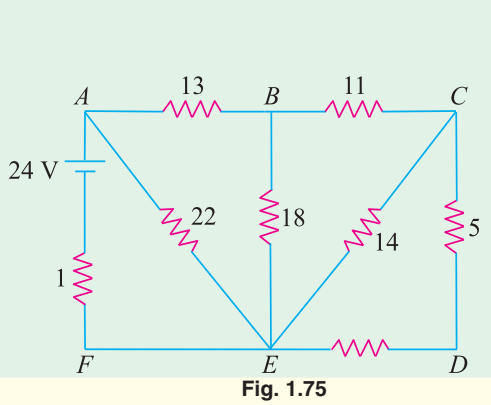


Fig. 1.75

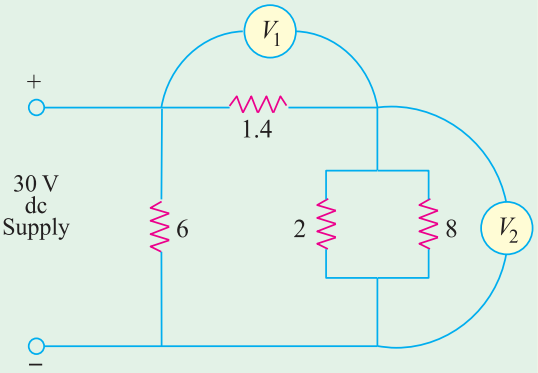


Fig. 1.76

19. Neglecting the current taken by voltmeters V_1 and V_2 in Fig. 1.76, calculate (*a*) total current taken from the supply (*b*) reading on voltmeter V_1 and (*c*) reading on voltmeter V_2.

[(*a*) 15 A (*b*) 14 V (*c*) 16 V]

20. Find the equivalent resistance between terminals *A* and *B* of the circuit shown in Fig. 1.77. Also, find the value of currents I_1, I_2, and I_3. All resistances are in ohm.

[8 Ω ; I_1 = 2 *A*; I_2 = 0.6 *A*; I_3 = 0.4 *A*]

21. In Fig. 1.78, the 10 Ω resistor dissipates 360 W. What is the voltage drop across the 5 Ω resistor ? **[30 V]**

22. In Fig. 1.79, the power dissipated in the 10 Ω resistor is 250 W. What is the total power dissipated in the circuit ? **[850 W]**

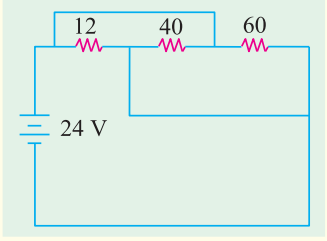


Fig. 1.77

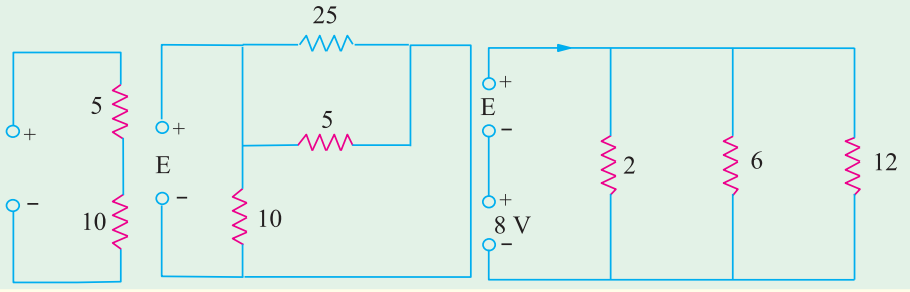


Fig. 1.78 Fig. 1.79 Fig. 1.80

23. What is the value of *E* in the circuit of Fig. 1.80 ? All resistances are in ohms. **[4 V]**

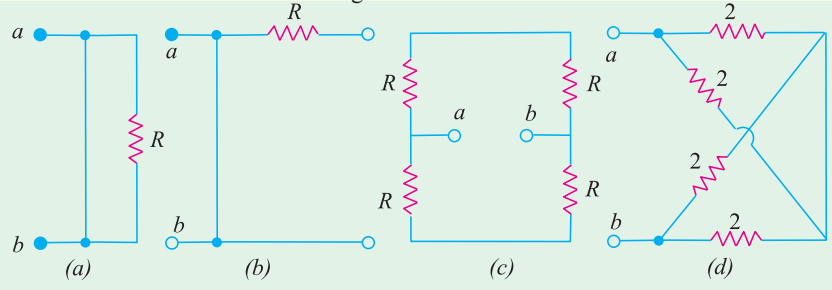


Fig. 1.81

24. Find the equivalent resistance R_{a-b} at the terminals a –b of the networks shown in Fig. 1.81.

[(*a*) 0 (*b*) 0 (*c*) *R* (*d*) 2 Ω]

25. Find the equivalent resistance between terminals a and b of the circuit shown in Fig. 1.82 (a). Each resistance has a value of 1 Ω **[5/11 Ω]**

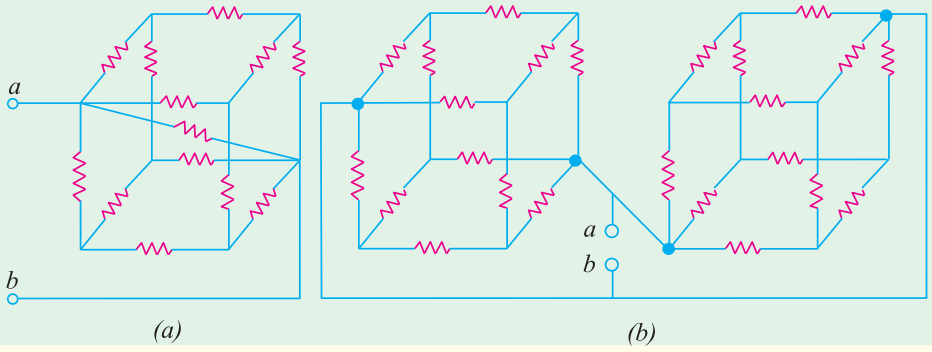


Fig. 1.82

26. Find the equivalent resistance between terminals a and b of the circuit shown in Fig. 1.82 (b). Each resistor has a value of 1 Ω **[5/12 Ω]**

27. Two resistors of value 1000 Ω and 4000 Ω are connected in series across a constant voltage supply of 150 V. Find (a) p.d. across 4000 ohm resistor (b) calculate the change in supply current and the reading on a voltmeter of 12,000 Ω resistance when it is connected across the larger resistor.

[(*a*) 120 V (*b*) 7.5 mA; 112.5 V]

1.28. Relative Potential

It is the voltage of one point in a circuit with respect to that of another point (usually called the reference or common point).

Consider the circuit of Fig. 1.83 (a) where the most negative end-point C has been taken as the reference. With respect to point C, both points A and B are positive though A is more positive than B. The voltage of point B with respect to that of C *i.e.* $V_{BC} = +30$ V.

Similarly, $V_{AC} = +(20 + 30) = +50$ V.

In Fig. 1.83 (b), the most positive end point A has been taken as the reference point. With respect to A, both B and C are negative though C is more negative than B.

$V_{BA} = -20$ V, $V_{CA} = -(20 + 30) = -50$ V

In Fig. 1.83 (c), mid-point B has been taken as the reference point. With respect to B, A is at positive potential whereas C is at a negative potential.

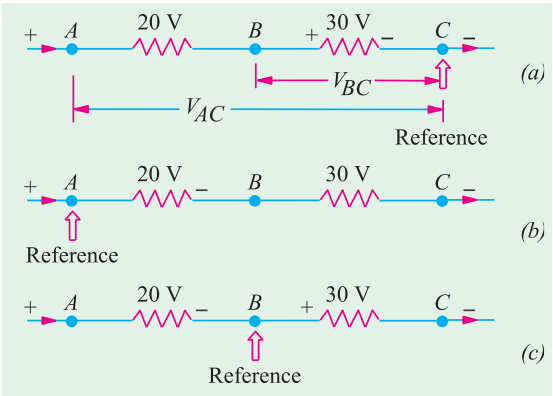


Fig. 1.83

Hence, $V_{AB} = +20$ V and $V_{CB} = -30$ V (of course, $V_{BC} = +30$ V)

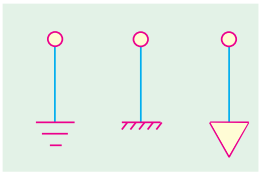


Fig. 1.84

It may be noted that ***any point*** in the circuit can be chosen as the reference point to suit our requirements. This point is often called ***ground*** or *earth* because originally it meant a point in a circuit which was ***actually*** connected to earth either for safety in power systems or for efficient radio reception and transmission. Although, this meaning still exists, yet it has become usual today for **'*ground*'** to mean any point in the circuit which is connected to a large metallic object such as the metal chassis of a transmit-

ter, the aluminium chassis of a receiver, a wide strip of copper plating on a printed circuit board, frame or cabinet which supports the whole equipment. Sometimes, reference point is also called ***common*** point. The main advantage of using a ground system is to simplify our circuitry by saving on the amount of wiring because ground is used as the return path for may circuits. The three commonly-used symbols for ***ground*** are shown in Fig. 1.84.

Example 1.48. *In Fig. 1.85, calculate the values of* ***(i)*** V_{AF} ***(ii)*** V_{EA} *and* ***(iii)*** V_{FB}.

Solution. It should be noted that V_{AF} stands for the potential of point A with respect to point F. The easiest way of finding it is to start from the reference point F and go to point A along any available path and calculate the algebraic sum of the voltages met on the way. Starting from point F as we go to point A, we come across different battery voltages. Taking the sign convention given in Art. 1.28, we get

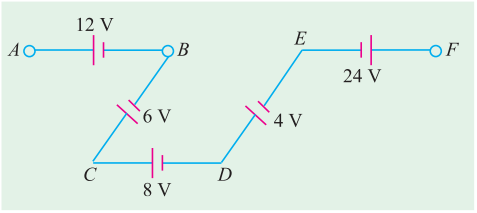


Fig. 1.85

(i) $V_{AF} = -24 + 4 + 8 - 6 + 12 = -6$ V

The negative sign shows that point A is negative with respect to point F by 6 V.

(ii) Similarly, $V_{EA} = -12 + 6 - 8 - 4 = -18$ V

(iii) Starting from point B, we get $V_{FB} = 6 - 8 - 4 + 24 = 18$ V.

Since the result is positive it means that point F is at a higher potential than point B by 18 V.

Example 1.49. *In Fig. 1.86 compute the relative potentials of points A, B, C, D and E which* ***(i)*** *point A is grounded and* ***(ii)*** *point D is grounded. Does it affect the circuit operation or potential difference between any pair of points ?*

Solution. As seen, the two batteries have been connected in *series opposition.* Hence, net circuit voltage = 34 – 10 = 24 V

Total circuit resistance = 6 + 4 + 2 = 12 Ω

Hence, the circuit current = 24/12 = 2 A

Drop across 2 Ω resistor = 2 × 2 = 4 V, Drop across 4 Ω resistor = 2 × 4 = 8 V

Drop across 6 Ω resistor = 2 × 6 = 12 V

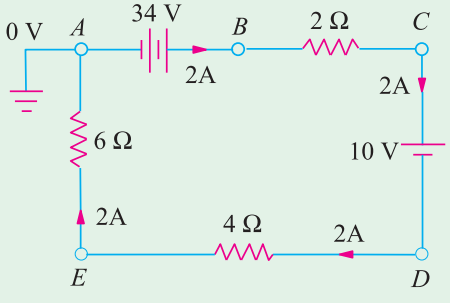


Fig. 1.86

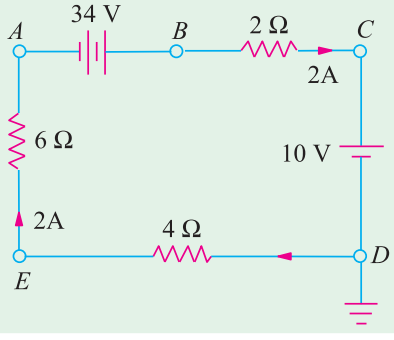


Fig. 1.87

(i) Since point B is directly connected to the positive terminal of the battery whose negative terminal is earthed, hence $V_B = +34$ **V.**

Since there is a fall of 4 V across 2 Ω resistor, $V_C = 34 - 4 =$ **30 V**

As we go from point C to D *i.e.* from positive terminal of 10-V battery to its negative terminal, there is a *decrease* in potential of 10 V. Hence, $V_D = 30 - 10 =$ **20** *i.e.* point D is 20 V above the ground A.

Similarly, $V_E = V_D$ – voltage fall across 4 Ω resistor = 20 – 8 = **+ 12 V**

Also $V_A = V_E$ – fall across 6 Ω resistor = 12 (2 × 6) = **0 V**

(ii) In Fig. 1.87, point *D* has been taken as the ground. Starting from point *D*, as we go to *E* there is a fall of 8 V. Hence, V_E = **–8 V.** Similarly, $V_A = -(8 + 12)$ = **–20 V.**

As we go from *A* to *B*, there is a sudden increase of 34 V because we are going from negative terminal of the battery to its positive terminal.

$\therefore$ $V_B = -20 + 34$ = **+ 14 V**

$V_C = V_B$ – voltage fall across 2 Ω resistor = 14 – 4 = **+ 10 V.**

It should be so because *C* is connected directly to the positive terminal of the 10 V battery.

Choice of a reference point does not in any way affect the operation of a circuit. Moreover, it also does not change the voltage across any resistor or between any *pair* of points (as shown below) because the ground current $i_g = 0$.

Reference Point *A*

$V_{CA} = V_C - V_A = 30 - 0 = +30$ V; $V_{CE} = V_C - V_E = 30 - 12 = +18$ V

$V_{BD} = V_B - V_D = 34 - 20 = +14$ V

Reference Point *D*

$V_{CA} = V_C - V_A = 10 - (-20) = +30$ V; $V_{CE} = V_C - V_E = 10 - (-8) = +18$ V

$V_{BD} = V_B - V_D = 14 - 0$ = **+ 14 V**

Example 1.50. *Find the voltage V in Fig. 1.88 (a). All resistances are in ohms.*

Solution. The given circuit can be simplified to the final form shown in Fig. 1.88 (*d*). As seen, current supplied by the battery is 1 A. At point *A* in Fig. 1.88 (*b*), this current is divided into two equal parts of 0.5 A each.

Obviously, voltage *V* represents the potential of point *B* with respect to the negative terminal of the battery. Point *B* is above the ground by an amount equal to the voltage drop across the series combination of (40 + 50) = 90 Ω

$V = 0.5 \times 90 = 45$ V.

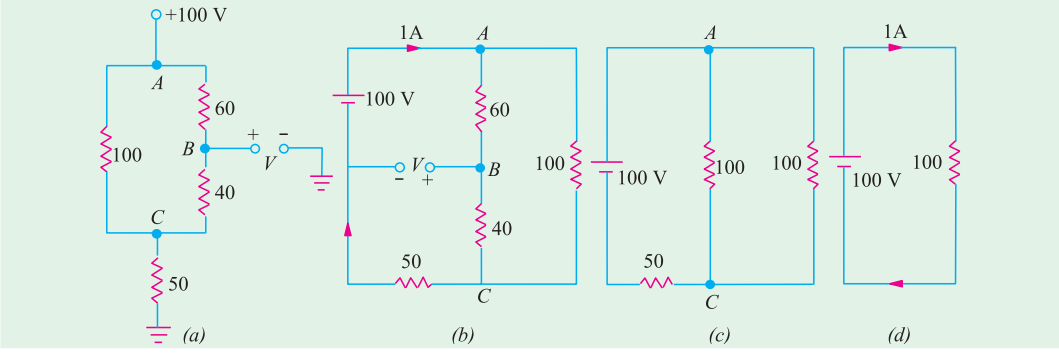


Fig. 1.88

1.29. Voltage Divider Circuit

A voltage divider circuit (also called potential divider) is a series network which is used to feed other networks with a number of different voltages and derived from a single input voltage source.

Fig. 1.89 (*a*) shows a simple voltage divider circuit which provides two output voltages V_1 and V_2. Since no load is connected across the output terminals, it is called an ***unloaded*** voltage divider.

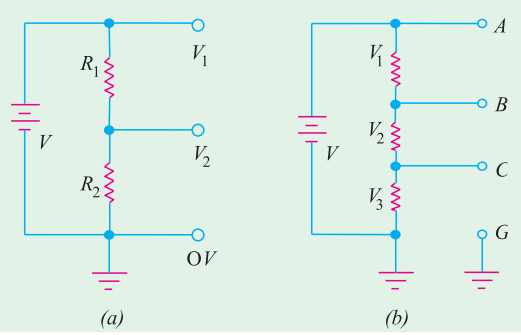


Fig. 1.89

As seen from Art. 1.15.

$$V_1 = V\frac{R_1}{R_1 + R_2} \text{ and } V_2 = V\cdot\frac{R_2}{R_1 + R_2}$$

The ratio V_2/V is also known as voltage-ratio ***transfer function***.

As seen, $\frac{V_2}{V} = \frac{R_2}{R_1 + R_2} = \frac{1}{1 + R_1/R_2}$

The voltage divider of Fig. 1.89 (*b*) can be used to get six different voltages :

$$V_{CG} = V_3,\ V_{BC} = V_2,\ V_{AB} = V_1,\ V_{BG} = (V_2 + V_3),\ V_{AC} = (V_1 + V_2) \text{ and } V_{AG} = V$$

Example 1.51. *Find the values of different voltages that can be obtained from a 12-V battery with the help of voltage divider circuit of Fig. 1.90.*

Solution.

$R = R_1 + R_2 + R_3 = 4 + 3 + 1 = 8\ \Omega$

Drop across $R_1 = 12 \times 4/8 = 6$ V

$\therefore\ V_B = 12 - 6 = 6$ V above ground

Drop across $R_2 = 12 \times 3/8 = 4.5$ V

$\therefore\ V_C = V_B - 4.5 = 6 - 4.5 = 1.5$

Drop across $R_3 = 12 \times 1/8 = 1.5$ V

Different available load voltages are :

(*i*) $V_{AB} = V_A - V_B = 12 - 6 =$ **6 V**

(*ii*) $V_{AC} = 12 - 1.5 =$ **10.5 V**

(*iii*) $V_{AD} =$ **12 V**

(*iv*) $V_{BC} = 6 - 1.5 =$ **4.5 V**

(*v*) $V_{CD} =$ **1.5 V**

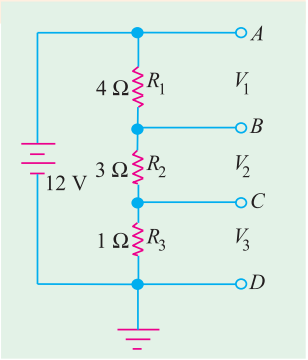


Fig. 1.90

Example 1.52. *What are the output voltages of the unloaded voltage divider shown in Fig. 1.91 ? What is the direction of current through AB ?*

Solution. It may be remembered that both V_1 and V_2 are with respect to the ground.

$R = 6 + 4 + 2 = 12\ \Omega$

$\therefore\ V_1 =$ drop across R_2

$= 24 \times 4/12 =$ **+ 8 V**

$V_2 =$ drop across $R_3 = -24 \times 2/12 =$ **– 4 V**

It should be noted that point *B* is at negative potential with respect to the ground.

Current flows from *A* to *B i.e.* from a point at a higher potential to a point at a lower potential.

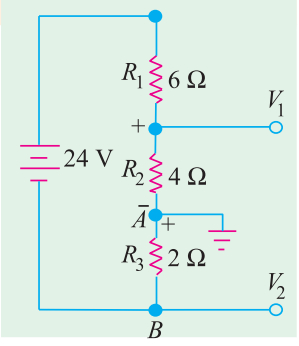


Fig. 1.91

Example 1.53. *Calculate the potentials of point A, B, C and D in Fig. 1.92. What would be the new potential values if connections of 6-V battery are reversed ? All resistances are in ohm.*

Solution. Since the two batteries are connected in additive series, total voltage around the circuit is = 12 + 6 = 18 V. The drops across the three resistors as found by the voltage divider rule as shown in Fig. 1.92 (*a*) which also indicates their proper polarities. The potential of any point in the circuit can be found by starting from the ground point *G* (assumed to be at 0V) and going to the point either in clockwise direction or counter-clockwise direction. While going around the circuit, the rise in potential would be taken as positive and the fall in potential as negative. (Art. 2.3). Suppose we start from point *G* and proceed in the clockwise direction to point *A*. The only potential met on the way is the battery voltage which is taken as positive because there is a rise of potential since we are going from its negative to positive terminal. Hence, V_A is + 12 V.

$V_B = 12 - 3 = 9$ V; $V_C = 12 - 3 - 6 = 3$ V

Similarly, $V_D = 12 - 3 - 6 - 9 = -6$ V.

It is also obvious that point D must be at -6 V because it is directly connected to the negative terminal of the 6-V battery.

We would also find the potentials of various points by starting from point G and going in the counter-clockwise direction. For example, $V_B = -6 + 9 + 6 = 9$ V as before.

The connections of the 6 –V battery have been reversed in Fig. 1.92 (*b*). Now, the net voltage around the circuit is 12 –6 = 6 V. The drop over the 1 Ω resistor is = 6 × 1/(1 + 2 + 3) = 1 V; Drop over 2 Ω resistor is = 6 × 2/6 = 2 V. Obviously, $V_A = +12$ V, $V_B = 12 - 1 = 11$ V, $V_C = 12 - 1 - 2 = 9$ V. Similarly, $V_D = 12 - 1 - 2 - 3 = +6$ V.

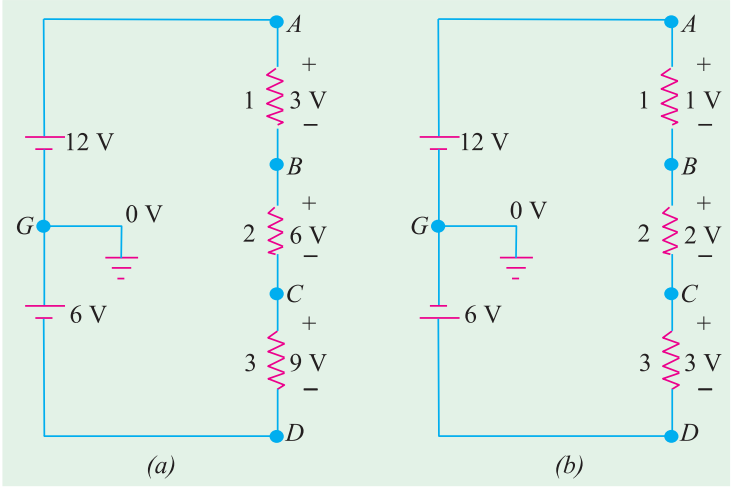


Fig. 1.92

Example 1.54. *Using minimum number of components, design a voltage divider which can deliver 1 W at 100 V, 2 W at –50 V and 1.6 W at –80 V. The voltage source has an internal resistance of 200 Ω and supplies a current of 100 mA. What is the open-circuit voltage of the voltage source ? All resistances are in ohm.*

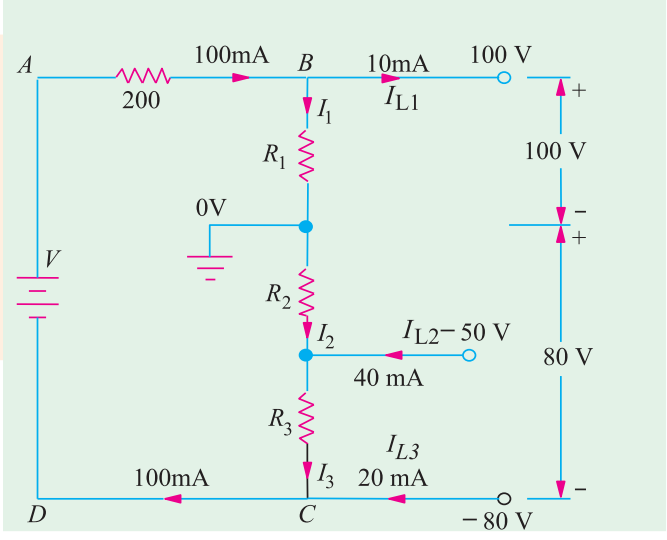


Fig. 1.93

Solution. From the given load conditions, the load currents are as follows :

$I_{L1} = 1/100 = 10$ mA,

$I_{L2} = 2/50 = 40$ mA,

$I_{L3} = 1.6/80 = 20$ mA

For economising the number of components, the internal resistance of 200 Ω can be used as the series dropping resistance. The suitable circuit and the ground connection are shown in Fig. 1.93.

Applying Kirchhoff's laws to the closed circuit *ABCDA*, we have

$V - 200 \times 100 \times 10^{-3} - 100 - 80 = 0$ or $V = 200$ V

$I_1 = 100 - 10 = 90$ mA $\quad \therefore R_1 = 100$ V/90 mA = **1.11 kΩ**

$I_3 = 100 - 20 = 80$ mA; voltage drop across $R_3 = -50 - (-80) = 30$ V

$\therefore \quad R_3 = 30\ V/80$ mA = 375 Ω

$I_2 + 40 = 80 \quad \therefore I_2 = 40$ mA; $R_2 = 50$ V/40 mA = **1.25 kΩ**

Example 1.55. *Fig. 1.94 shows a transistor with proper voltages established across its base, collector are emitter for proper function. Assume that there is a voltage drop V_{BE} across the base-emitter junction of 0.6 V and collector current I_C is equal to collector current I_E. Calculate* ***(a)*** *V_1* ***(b)*** *V_2 and V_B* ***(c)*** *V_4 and V_E* ***(d)*** *I_E and I_C* ***(e)*** *V_3* ***(f)*** *V_C* ***(g)*** *V_{CE}. All resistances are given in kilo-ohm.*

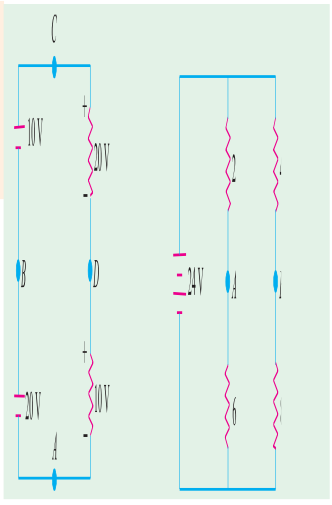


Fig. 1.94

Solution. ***(a)*** The 250 k and 50 k resistors form a voltage-divider bias network across 20 V supply.

$\therefore \quad V_1 = 20 \times 250/300 =$ **16.7 V**

(b) $V_2 = 20 - 16.7 =$ **3.3 V**

The voltage of point B with respect to ground is $V_2 =$ **3.3 V**

(c) $V_E = V_2 - V_{BE} = 3.3 - 0.6 = 2.7$ V. Also $V_4 =$ **2.7 V**

(d) $I_E = V_4/2 = 2.7\ V/2$ k $=$ **1.35 mA.** It also equals I_C.

(e) $V_3 =$ drop across collector resistor $= 1.35$ mA $\times$ 8 k $= 10.8$ V

(f) Potential of point C is $V_C = 20 - 10.8 =$ **9.2 V**

(g) $V_{CE} = V_C - V_E = 9.2 - 2.7 =$ **6.5 V**

Tutorial Problems No. 1.5

1. A direct - current circuit comprises two resistors, 'A' of value 25 ohms, and 'B' of unknown value, connected in parallel, together with a third resistor 'C' of value 5 ohms connected in series with the parallel group. The potential difference across C is found to 90 V. It the total power in the circuit is 4320 W, calculate :
(*i*) the value of resistor B, (*ii*) the voltage applied to the ends of the whole circuit,
(*iii*) the current in each resistor.
(Mumbai University 2002) (Nagpur University, Summer 2002)

2. A current of 5 A flows through a non inductive resistance connected in series with a choke coil when supplied at 250 V, 50 Hz. If voltage across resistance is 125 V and across coil is 200 V calculate :
(*i*) impedance, resistance and reactance of coil
(*ii*) power in coil
(*iii*) total power consumed in the circuit
(*iv*) draw phasor diagram.
(Pune University 2002) (Nagpur University, Winter 2003)

3. Define temp. coefficient of resistance.

Prove $\alpha_{t_1} = \dfrac{\alpha_0}{(1 + \alpha_0 t_1)}$

where α_0 = temp. coeff. of resistance at 0^oC. ***(Gujrat University, Summer 2003)***

4. A resistance wire 10 m long and cross section area 10 mm^2 at 0^oC passes a current of 10 A, when connected to a d.c. supply of 200 volts.
Calculate :
(*a*) resistivity of the material
(*b*) current which will flow through the wire when the temp. rises to 50^oC. Given $\alpha_0 = 0.0003$ *per* ^oC.
(Mumbai University, 2003) (Gujrat University, Summer 2003)

5. Why domestic appliances are connected in parallel ? Give comparison with series ckt.
(B.P.T.U., Orissa 2003) (Gujrat University,Summer 2003)

6. Two wires A and B made up of same material, wire B has twice the length of wire A and having half the diameter to that of A. Calculate the ratio R_B/R_A. ***(Gujrat University,Summer 2003)***

7. A resistor of 12 Ω is connected in series with a combination of 15 Ω and 20 Ω resistor in parallel. When a voltage of 120 V is applied across the whole circuit find the current taken from the supply.
(V.T.U., Belgaum, Karnataka University, Summer 2002)

8. A network is arranged as shown in Fig. 1.95. Determine the value of currents in each resistor.
(V.T.U., Belgaum, Karnataka University, Summer 2002)

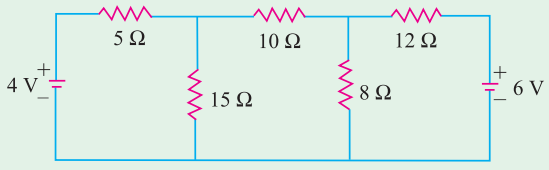


Fig. 1.95

9. A resistance of 100Ω is connected in series with 100μF capacitor across 200V, 60Hz supply. Find the impedance, current and power factor.
(V.T.U., Belgaum, Karnataka University, Summer 2002)

10. An EMF whose instantaneous value is 100sin (314t – π/4) volts is applied to a circuit and the current flowing through it is 20sin (314t – 1.5708) Amperes. Find the frequency and the values of circuit elements, assuming a series combination of circuit elements.
(V.T.U.,Belgaum, Karnataka University, Wimter 2003)

11. An inductive coil draws a current of 2A, when connected to a 230V, 50Hz supply. The power taken by the coil is 100 watts. Calculate the resistance and inductance of the coil.
(Pune University, 2003) (V.T.U.,Belgaum University, Winter 2003)

12. Find the resistance between the terminals A and B for the network shown in Fig.1.96.
(Pune University, 2003) (V.T.U.,Belgaum University, Winter 2003)

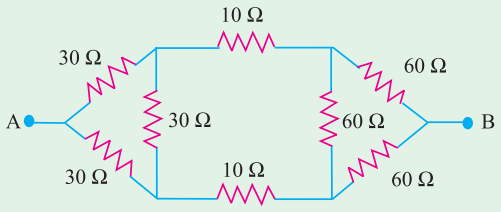


Fig. 1.96

13. A network is arranged as shown in Fig 1.97 Determine the current in each resistanc using loop current method. ***(V.T.U., Belgaum, Karnataka University, Winter 2003)***

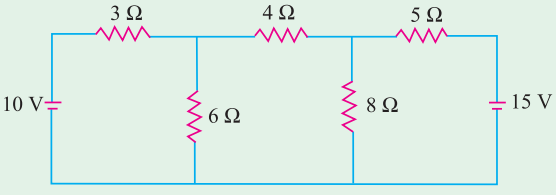


Fig. 1.97

14. A resistor of 12Ω is connected in series with a combination of 15Ω and 20Ω resistor in parallel. When a voltage of 120V is applied across the whole circuit. Find the current taken from the supply.
(V.T.U., Belgaum, Karnataka University, Winter 2004)

15. Four wires a,b,c and d are connected at a common point. The currents flowing in a,b and c towards the common point are $i_a = 6sin\left(wt + \frac{\pi}{3}\right)$, $i_b = 5cos\left(wt + \frac{\pi}{3}\right)$ and $i_c = 3cos\left(wt + \frac{2\pi}{3}\right)$. Determine the current in the fourth wire. ***(V.T.U., Belgaum, Karnataka University, Winter 2004)***

16. Two resistors $R_1 = 2500\Omega$ and $R_2 = 4000\Omega$ are in series across a 100V supply. The voltage drop across R_1 and R_2 are successively mesured by a voltmeter having a resistance of 50,000Ω Find the sum of the two readings. ***(V.T.U., Belgaum, Karnataka University, Winter 2004)***

17. Explain 'resistance', 'reactance' and 'impedance'. ***(RGPV, Bhopal December 2002)***

18. A 4 ohm resistor is connected to a 10 mH inductor across a 100 V, 50 Hz voltage source. Find input current, voltage drops across resistor and inductor, power factor of the circuit and the real power consumed in the circuit. ***(Mumbai University 2002)*** ***(RGPV, Bhopal December 2003)***

19. Define and explain the terms MMF, Reluctance, Permeance, flux density and fringing. ***(RGPV, Bhopal December 2003)***

20. Find the value of resistance (R), if source current is 6 amp and source voltage is 66 V is shown in Fig.1.98 ***(Pune University 2003)*** ***(Nagpur University, Winter 2002)***

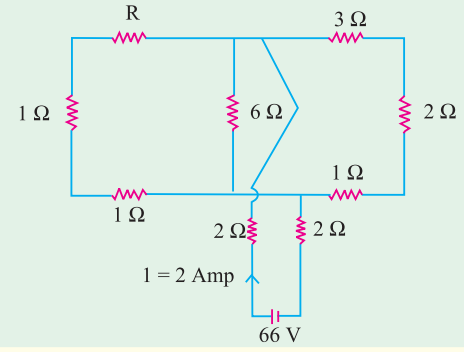


Fig. 1.98

21. Determine a non-negative value of R such that the power consumed by the 2-Ω resistor in the Fig.1.99 is shown maximum. ***(Pune University 2003)(Engineering Services Examination 2003)***

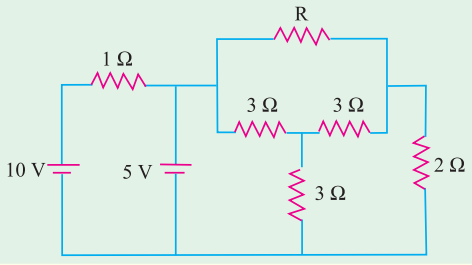


Fig. 1.99

OBJECTIVE TESTS –1

1. A 100 μA ammeter has an internal resistance of 100 Ω For extending its range to measure 500 μA, the shunt required is of resistance (in Ω)

(*a*) 20.0 (*b*) 22.22

(*c*) 25.0 (*d*) 50.0

(GATE 2001)

2. Resistances R_1 and R_2 have, respectively, nominal values of 10Ω and 5Ω, and tolerances of ± 5% and ± 10%. The range of values for the parallel combination of R_1 and R_2 is

(*a*) 3.077 Ω to 3.636 Ω

(*b*) 2.805 Ω to 3.371 Ω

(*c*) 3.237 Ω to 3.678 Ω

(*d*) 3.192 Ω to 3.435 Ω

(GATE 2001)

3. The open circuit impedance of a certain length of a loss-less line is 100 Ω The short circuit impedance of the same line is also 100 Ω The characteristic impedance of the line is

(*a*) 100 $\sqrt{2}$ Ω (*b*) 50 Ω

(*c*) $\frac{100}{\sqrt{2}}$ Ω (*d*) 100 Ω

(ESE 2001)

4. The current in the given circuit with a dependent voltage source is

(a) 10A (b) 12 A

(c) 14 A (d) 16 A

(ESE 2001)

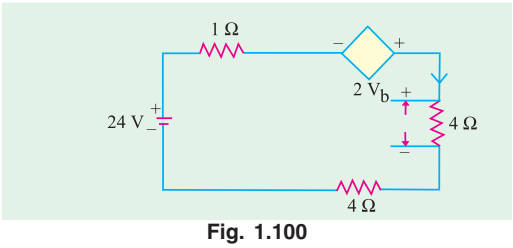


Fig. 1.100

5. The value of resistance 'R' shown in the given Fig. 1.101 is

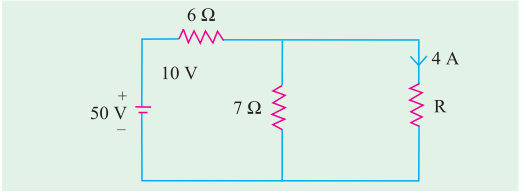


Fig. 1.101

(*a*) 3.5 Ω (*b*) 2.5 Ω
(*c*) 1 Ω (*d*) 4.5 Ω

(ESE 2001)

6. For the circuit shown in the given Fig. 1.102 the current I is given by

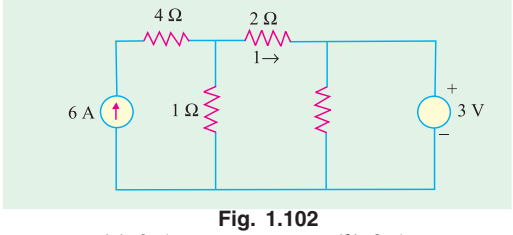


Fig. 1.102

(*a*) 3 A (*b*) 2 A
(*c*) 1 A (*d*) zero

(Pune University 2003) (ESE 2001)

7. The value of V in the circuit shown in the given Fig. 1.103 is

(Mumbai University 2003)

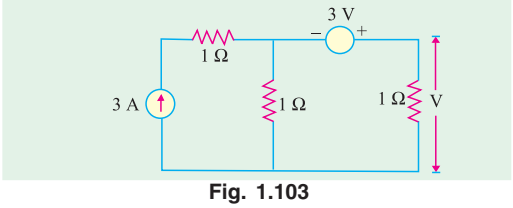


Fig. 1.103

(*a*) 1 V (*b*) 2 V
(*c*) 3 V (*d*) 4 V

(GATE 2003) (ESE 2001)

8. In the circuit shown in Fig. 1.104, the value of V_s is 0, when I = 4A. The value of I when V_s = 16V, is

(*a*) 6 A (*b*) 8 A
(*c*) 10 A (*d*) 12 A

(GATE 2003) (ESE 2003)

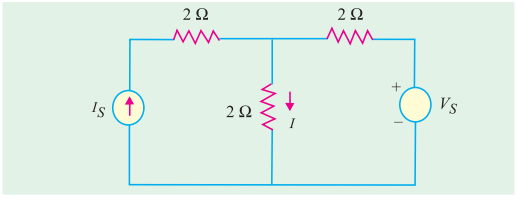


Fig. 1.104

9. The linear network as in Fig. 1.105 has only resistors. If I_1 = 8A and I_2 = 12 A; V is found to be 80 V. V = 0 when I_1 = –8A and I_2 = 4A. Then the value of V when I_1 = I_2 = 10 A, is

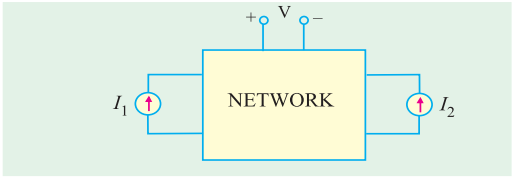


Fig. 1.105

(*a*) 25 V (*b*) 50
(*c*) 75 V (*d*) 100 V

(GATE 2003) (ESE 2003)

10. In Fig. 1.106, the value of R is

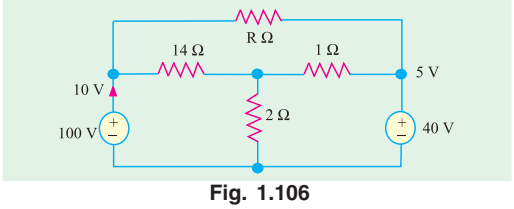


Fig. 1.106

(*a*) 10 Ω (*b*) 18 Ω
(*c*) 24 Ω (*d*) 12 Ω

(GATE 2003)

11. In the circuit shown in Fig. 1.107, the switch S is closed at time t = 0. The voltage across the inductance at $t = 0^+$, is

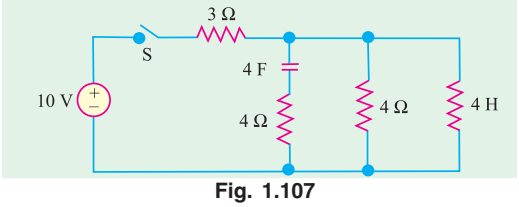


Fig. 1.107

(*a*) 2 V (*b*) 4 V
(*c*) – 6 V (*d*) 8 V

(GATE 2003)

12. The *rms* value of the resultant current in a wire which carries a dc current of 10 A and a sinusoidal alternating current of peak value 20 A is

(*a*) 14.1 A (*b*) 17.3 A
(*c*) 22.4 A (*d*) 30.0 A

(GATE 2004)

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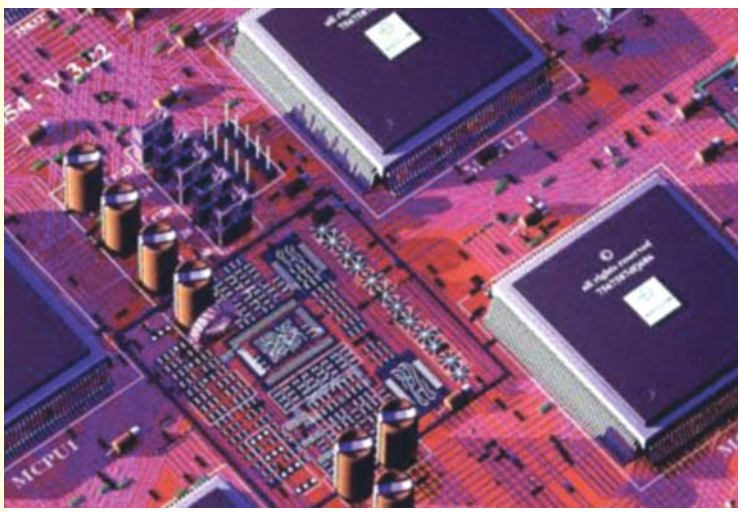
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CHAPTER 2

DC NETWORK THEOREMS

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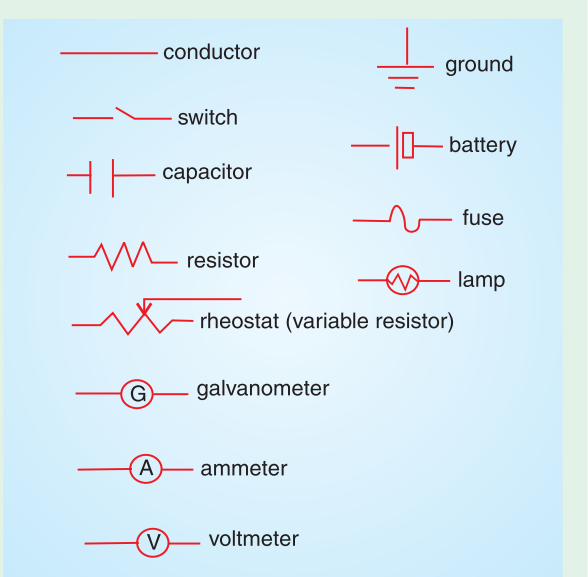


Network theorems help to determine the unknown values of current, resistance and voltage etc, in electric networks

2.1. Electric Circuits and Network Theorems

There are certain theorems, which when applied to the solutions of electric networks, wither simplify the network itself or render their analytical solution very easy. These theorems can also be applied to an a.c. system, with the only difference that impedances replace the ohmic resistance of d.c. system. Different electric circuits (according to their properties) are defined below :

1. **Circuit.** A circuit is a closed conducting path through which an electric current either flows or is intended flow.
2. **Parameters.** The various elements of an electric circuit are called its parameters like resistance, inductance and capacitance. These parameters may be ***lumped or distributed.***
3. **Liner Circuit.** A linear circuit is one whose parameters are constant *i.e.* they do not change with voltage or current.
4. **Non-linear Circuit.** It is that circuit whose parameters change with voltage or current.
5. **Bilateral Circuit.** A bilateral circuit is one whose properties or characteristics are the same in either direction. The usual transmission line is bilateral, because it can be made to perform its function equally well in either direction.
6. **Unilateral Circuit.** It is that circuit whose properties or characteristics change with the direction of its operation. A diode rectifier is a unilateral circuit, because it cannot perform rectification in both directions.
7. **Electric Network.** A combination of various electric elements, connected in any manner whatsoever, is called an electric network.
8. **Passive Network** is one which contains no source of e.m.f. in it.
9. **Active Network** is one which contains one or more than one source of e.m.f.
10. **Node** is a junction in a circuit where two or more circuit elements are connected together.
11. **Branch** is that part of a network which lies between two junctions.
12. **Loop.** It is a close path in a circuit in which no element or node is encountered more than once.
13. **Mesh.** It is a loop that contains no other loop within it. For example, the circuit of Fig. 2.1 (*a*) has even branches, six nodes, three loops and two meshes whereas the circuit of Fig. 2.1 (*b*) has four branches, two nodes, six loops and three meshes.



Standard symbols

It should be noted that, unless stated otherwise, an electric network would be assumed passive in the following treatment.

We will now discuss the various network theorems which are of great help in solving complicated networks. Incidentally, a network is said to be completely

solved or analyzed when all voltages and all currents in its different elements are determined.

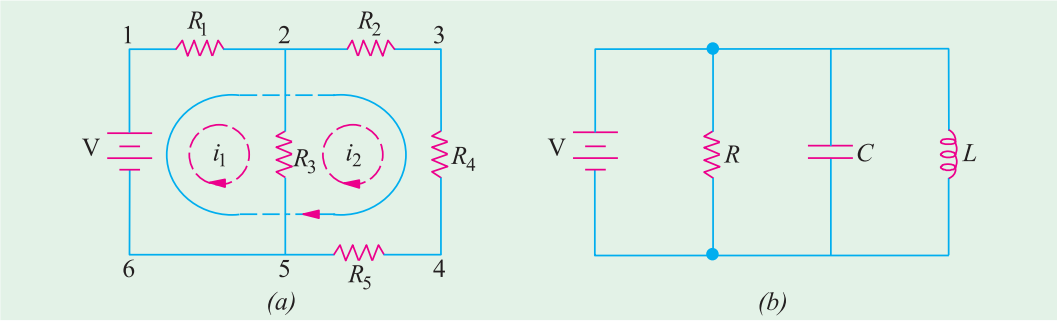


Fig. 2.1

There are two general approaches to network analysis :

(*i*) Direct Method

Here, the network is left in its original form while determining its different voltages and currents. Such methods are usually restricted to fairly simple circuits and include Kirchhoff's laws, Loop analysis, Nodal analysis, superposition theorem, Compensation theorem and Reciprocity theorem etc.

(*ii*) Network Reduction Method

Here, the original network is converted into a much simpler equivalent circuit for rapid calculation of different quantities. This method can be applied to simple as well as complicated networks. Examples of this method are : Delta/Star and Star/Delta conversions. Thevenin's theorem and Norton's Theorem etc.


Kirchhoff

2.2. Kirchhoff's Laws *

These laws are more comprehensive than Ohm's law and are used for solving electrical networks which may not be readily solved by the latter. Kirchhoff's laws, two in number, are particularly useful **(*a*)** in determining the equivalent resistance of a complicated network of conductors and **(*b*)** for calculating the currents flowing in the various conductors. The two-laws are :

1. Kirchhoff's Point Law or Current Law (KCL)

It states as follows :

in any electrical network, the algebraic sum of the currents meeting at a point (or junction) is zero.

Put in another way, it simply means that the total current *leaving* a junction is equal to the total current ***entering*** that junction. It is obviously true because there is no accumulation of charge at the junction of the network.

Consider the case of a few conductors meeting at a point *A* as in Fig. 2.2 (*a*). Some conductors have currents leading to point *A*, whereas some have currents leading away from point *A*. Assuming the incoming currents to be positive and the outgoing currents negative, we have

$$I_1 + (-I_2) + (-I_3) + (+ I_4) + (-I_5) = 0$$

or $$I_1 + I_4 - I_2 - I_3 - I_5 = 0 \quad \text{or} \quad I_1 + I_4 = I_2 + I_3 + I_5$$

or **incoming currents = outgoing currents**

* After Gustave Robert Kirchhoff (1824-1887), an outstanding German Physicist.

Similarly, in Fig. 2.2 (*b*) for node *A*

$+ I + (-I_1) + (-I_2) + (-I_3) + (-I_4) = 0$ or $I = I_1 + I_2 + I_3 + I_4$

We can express the above conclusion thus : $\Sigma I = 0$ ....at a junction

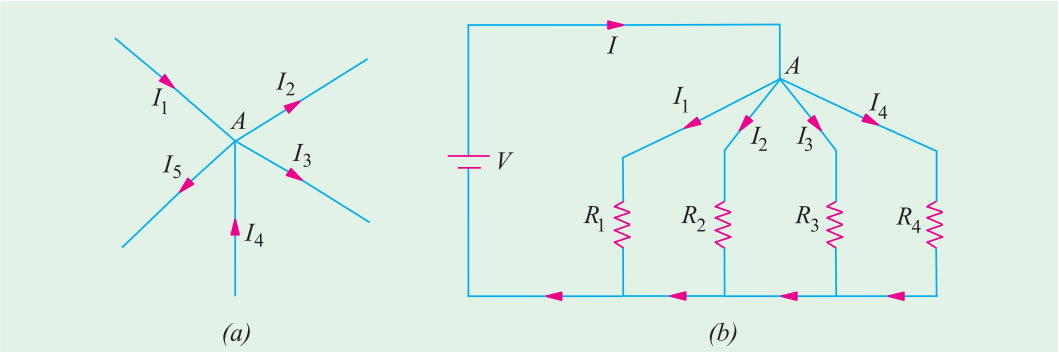


Fig. 2.2

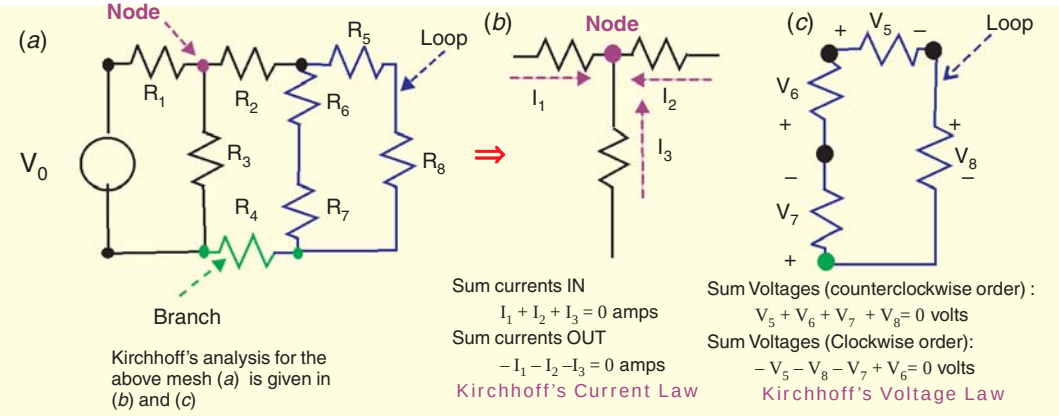
2. Kirchhoff's Mesh Law or Voltage Law (KVL)

It states as follows :

The algebraic sum of the products of currents and resistances in each of the conductors in any closed path (or mesh) in a network plus the algebraic sum of the e.m.fs. in that path is zero.

In other words, $\Sigma IR + \Sigma e.m.f. = 0$ ...round a mesh

It should be noted that algebraic sum is the sum which takes into account the polarities of the voltage drops.



The basis of this law is this : If we start from a particular junction and go round the mesh till we come back to the starting point, then we must be at the same potential with which we started. Hence, it means that all the sources of e.m.f. met on the way must necessarily be equal to the voltage drops in the resistances, every voltage being given its proper sign, plus or minus.

2.3. Determination of Voltage Sign

In applying Kirchhoff's laws to specific problems, particular attention should be paid to the algebraic signs of voltage drops and e.m.fs., otherwise results will come out to be wrong. Following sign conventions is suggested :

(*a*) Sign of Battery E.M.F.

A *rise* in voltage should be given a + ve sign and a ***fall*** in voltage a –ve sign. Keeping this in

mind, it is clear that as we go from the –ve terminal of a battery to its +ve terminal (Fig. 2.3), there is a *rise* in potential, hence this voltage should be given a + ve sign. If, on the other hand, we go from +ve terminal to –ve terminal, then there is a *fall* in potential, hence this voltage should be preceded

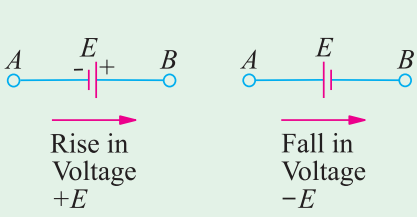


Fig. 2.3

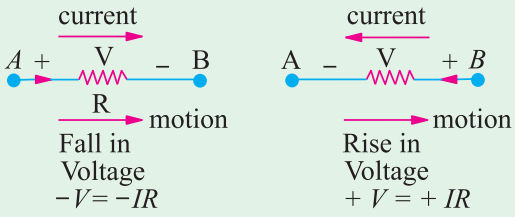


Fig. 2.4

by a –ve sign. ***It is important to note that the sign of the battery e.m.f. is independent of the direction of the current through that branch.***

(*b*) Sign of *IR* Drop

Now, take the case of a resistor (Fig. 2.4). If we go through a resistor in the *same* direction as the current, then there is a fall in potential because current flows from a higher to a lower potential. Hence, this voltage fall should be taken –ve. However, if we go in a direction opposite to that of the current, then there is a *rise* in voltage. Hence, this voltage rise should be given a positive sign.

It is clear that the sign of voltage drop across a resistor depends on the direction of current through that resistor but is independent of the polarity of any other source of e.m.f. in the circuit under consideration.

Consider the closed path *ABCDA* in Fig. 2.5. As we travel around the mesh in the clockwise direction, different voltage drops will have the following signs :

I_1R_2 is – ve (fall in potential)
I_2R_2 is – ve (fall in potential)
I_3R_3 is + ve (rise in potential)
I_4R_4 is – ve (fall in potential)
E_2 is – ve (fall in potential)
E_1 is + ve (rise in potential)

Using Kirchhoff's voltage law, we get

$$-I_1R_1 - I_2R_2 - I_3R_3 - I_4R_4 - E_2 + E_1 = 0$$

or $$I_1R_1 + I_2R_2 - I_3R_3 + I_4R_4 = E_1 - E_2$$

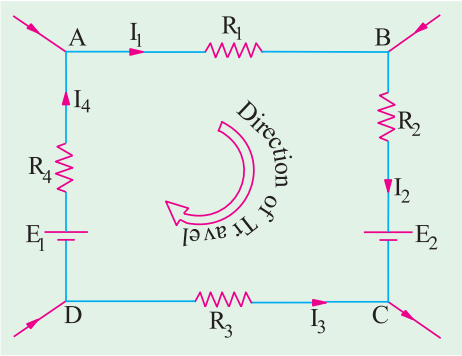


Fig. 2.5

2.4. Assumed Direction of Current

In applying Kirchhoff's laws to electrical networks, the question of assuming proper direction of current usually arises. The direction of current flow may be assumed either clockwise or anticlockwise. If the assumed direction of current is not the actual direction, then on solving the quesiton, this current will be found to have a minus sign. If the answer is positive, then assumed direction is the same as actual direction (Example 2.10). ***However, the important point is that once a particular direction has been assumed, the same should be used throughout the solution of the question.***

Note. It should be noted that Kirchhoff's laws are applicable both to d.c. and a.c. voltages and currents. However, in the case of alternating currents and voltages, any e.m.f. of self-inductance or that existing across a capacitor should be also taken into account (See Example 2.14).

2.5. Solving Simultaneous Equations

Electric circuit analysis with the help of Kirchhoff's laws usually involves solution of two or three simultaneous equations. These equations can be solved by a systematic elimination of the variables but the procedure is often lengthy and laborious and hence more liable to error. Determinants and Cramer's rule provide a simple and straight method for solving network equations through manipulation of their coefficients. Of course, if the number of simultaneous equations happens to be very large, use of a digital computer can make the task easy.

2.6. Determinants

The symbol $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ is called a determinant of the second order (or 2 × 2 determinant) because it contains two rows (*ab* and *cd*) and two columns (*ac* and *bd*). The numbers *a*, *b*, *c* and *d* are called the elements or constituents of the determinant. Their number in the present case is $2^2 = 4$.

The evaluation of such a determinant is accomplished by cross-multiplicaiton is illustrated below :

$$\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

The above result for a second order determinant can be remembered as

upper left times lower right* minus *upper right times lower left

The symbol $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ represents a third-order determinant having $3^2 = 9$ elements. It may be evaluated (or expanded) as under :

1. Multiply each element of the first row (or alternatively, first column) by a determinant obtained by omitting the row and column in which it occurs. (It is called minor determinant or just minor as shown in Fig. 2.6).

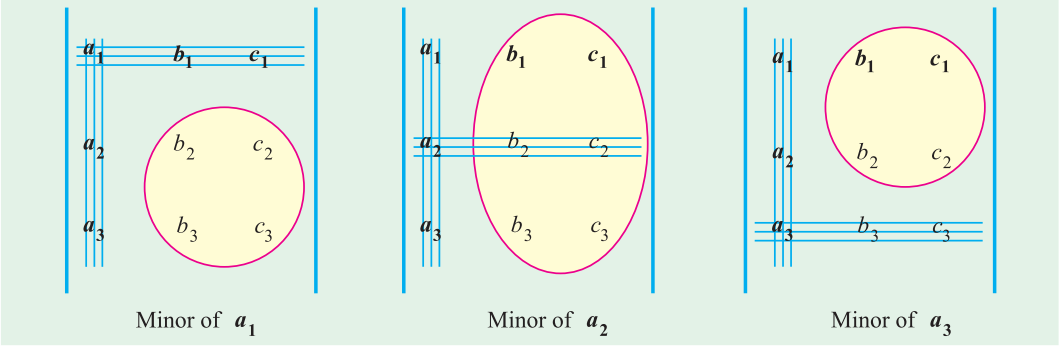


Fig. 2.6

2. Prefix + and –sing alternately to the terms so obtained.
3. Add up all these terms together to get the value of the given determinant.

Considering the first column, minors of various elements are as shown in Fig. 2.6.

Expanding in terms of first column, we get

$$\Delta = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$$

$$= a_1 (b_2c_3 - b_3c_2) - a_2 (b_1c_3 - b_3c_1) + a_3 (b_1c_2 - b_2c_1) \quad ...(i)$$

Expanding in terms of the first row, we get

$$\Delta = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

$$= a_1 (b_2c_3 - b_3c_2) - b_1 (a_2c_3 - a_3c_2) + c_1 (a_2b_3 - a_3b_2)$$

which will be found to be the same as above.

Example 2.1. *Evaluate the determinant* $\begin{vmatrix} 7 & -3 & -4 \\ -3 & 6 & -2 \\ -4 & -2 & 11 \end{vmatrix}$

Solution. We will expand with the help of 1st column.

$$D = 7 \begin{vmatrix} 6 & -2 \\ -2 & 11 \end{vmatrix} - (-3) \begin{vmatrix} -3 & -4 \\ -2 & 11 \end{vmatrix} + (-4) \begin{vmatrix} -3 & -4 \\ 6 & -2 \end{vmatrix}$$

$$= 7 [(6 \times 11) - (-2 \times -2)] + 3 [(-3 \times 11) - (-4 \times -2)] - 4 [(-3 \times -2) - (-4 \times 6)]$$

$$= 7 (66 - 4) + 3 (-33 - 8) - 4 (6 + 24) = \mathbf{191}$$

2.7. Solving Equations with Two Unknowns

Suppose the two given simultaneous equations are

$$ax + by = c$$
$$dx + ey = f$$

Here, the two unknown are x and y, a, b, d and e are coefficients of these unknowns whereas c and f are constants. The procedure for solving these equations by the method of determinants is as follows :

1. Write the two equations in the matrix form as $\begin{bmatrix} a & b \\ d & e \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c \\ f \end{bmatrix}$
2. The ***common*** determinant is given as $\Delta = \begin{bmatrix} a & b \\ d & e \end{bmatrix} = ae - bd$
3. For finding the determinant for x, replace the coefficients of x in the original matrix by the constants so that we get determinant Δ_1 given by $\Delta_1 = \begin{vmatrix} c & b \\ f & e \end{vmatrix} = (ce - bf)$
4. For finding the determinant for y, replace coefficients of y by the constants so that we get $\Delta_2 = \begin{vmatrix} a & c \\ d & f \end{vmatrix} = (af - cd)$
5. Apply Cramer's rule to get the value of x and y

$$x = \frac{\Delta_1}{\Delta} = \frac{ce - bf}{ae - bd} \quad \text{and} \quad y = \frac{\Delta_2}{\Delta} = \frac{af - cd}{ae - bd}$$

Example 2.2. *Solve the following two simultaneous equations by the method of determinants :*

$$4i_1 - 3i_2 = 1$$
$$3i_1 - 5i_2 = 2$$

Solution. The matrix form of the equations is $\begin{bmatrix} 4 & -3 \\ 3 & -5 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$$\Delta = \begin{vmatrix} 4 & -3 \\ 3 & -5 \end{vmatrix} = (4 \times -5) - (-3 \times 3) = -11$$

$$\Delta_1 = \begin{vmatrix} 1 & -3 \\ 2 & -5 \end{vmatrix} = (1 \times -5) - (-3 \times 2) = 1$$

$$\Delta_2 = \begin{vmatrix} 4 & 1 \\ 3 & 2 \end{vmatrix} = (4 \times 2) - (1 \times 3) = 5$$

$$\therefore \qquad i_1 = \frac{\Delta_1}{\Delta} = \frac{1}{-11} = -\frac{1}{11}; \quad i_2 = \frac{\Delta_2}{\Delta} = -\frac{5}{11}$$

2.8. Solving Equations With Three Unknowns

Let the three simultaneous equations be as under :

$$\begin{aligned} ax + by + cz &= d \\ ex + fy + gz &= h \\ jx + ky + lz &= m \end{aligned}$$

The above equations can be put in the matrix form as under :

$$\begin{bmatrix} a & b & c \\ e & f & g \\ j & k & l \end{bmatrix} \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d \\ h \\ m \end{bmatrix}$$

The value of common determinant is given by

$$\Delta = \begin{vmatrix} a & b & c \\ e & f & g \\ j & k & l \end{vmatrix} = a\,(fl - gk) - e(bl - ck) + j(bg - cf)$$

The determinant for x can be found by replacing coefficients of x in the original matrix by the constants.

$$\therefore \qquad \Delta_1 = \begin{vmatrix} d & b & c \\ h & f & g \\ m & k & l \end{vmatrix} = d\,(fl - gk) - h(bl - ck) + m(bg - cf)$$

Similarly, determinant for y is given by replacing coefficients of y with the three constants.

$$\Delta_2 = \begin{vmatrix} a & d & c \\ e & h & g \\ j & m & l \end{vmatrix} = a\,(hl - mg) - e(dl - mc) + j\,(dg - hc)$$

In the same way, determinant for z is given by

$$\Delta_3 = \begin{vmatrix} a & b & d \\ e & f & h \\ j & k & m \end{vmatrix} = a\,(fm - hk) - e(bm - dk) + j\,(bh - df)$$

As per Cramer's rule $\qquad x = \dfrac{\Delta_1}{\Delta}, \; y = \dfrac{\Delta_2}{\Delta}, \; z = \dfrac{\Delta_3}{\Delta}$

Example 2.3. *Solve the following three simultaneous equations by the use of determinants and Cramer's rule*

$$\begin{aligned} i_1 + 3i_2 + 4i_3 &= 14 \\ i_1 + 2i_2 + i_3 &= 7 \\ 2i_1 + i_2 + 2i_3 &= 2 \end{aligned}$$

Solution. As explained earlier, the above equations can be written in the form

$$\begin{bmatrix} 1 & 3 & 4 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 14 \\ 7 \\ 2 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 1 & 3 & 4 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix} = 1(4-1) - 1\,(6-4) + (3-8) = -9$$

$$\Delta_1 = \begin{bmatrix} 14 & 3 & 4 \\ 7 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix} = 14\,(4-1) - 7\,(6-4) + 2(3-8) = 18$$

$$\Delta_2 = \begin{bmatrix} 1 & 14 & 4 \\ 1 & 7 & 1 \\ 2 & 2 & 2 \end{bmatrix} = 1(14-2) - 1(28-8) + 2(14-28) = -36$$

$$\Delta_3 = \begin{bmatrix} 1 & 3 & 14 \\ 1 & 2 & 7 \\ 2 & 1 & 2 \end{bmatrix} = 1(4-7) - 1(6-14) + 2(21-28) = -9$$

According to Cramer's rule,

$$i_1 = \frac{\Delta_1}{\Delta} = \frac{18}{9} = 2\text{A}\ ;\ i_2 = \frac{\Delta_2}{\Delta} = \frac{36}{9} = 4\text{A}\ ;\ i_3 = \frac{\Delta_3}{\Delta} = \frac{9}{9} = 1\text{A}$$

Example 2.4. *What is the voltage V_s across the open switch in the circuit of Fig. 2.7 ?*

Solution. We will apply *KVL* to find V_s. Starting from point *A* in the clockwise direction and using the sign convention given in Art. 2.3, we have

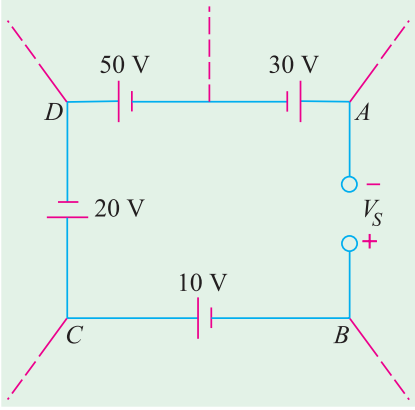


Fig. 2.7

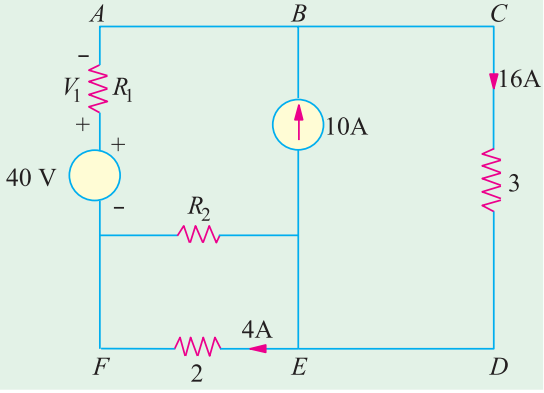


Fig. 2.8

$$+V_s + 10 - 20 - 50 + 30 = 0 \quad \therefore\ V_s = 30\ \text{V}$$

Example 2.5. *Find the unknown voltage V_1 in the circuit of Fig. 2.8.*

Solution. Initially, one may not be clear regarding the solution of this question. One may think of Kirchhoff's laws or mesh analysis etc. But a little thought will show that the question can be solved by the simple application of Kirchhoff's voltage law. Taking the outer closed loop ***ABCDEFA*** and applying ***KVL*** to it, we get

$$-16 \times 3 - 4 \times 2 + 40 - V_1 = 0\ ; \quad \therefore\ V_1 = -16\ \text{V}$$

The negative sign shows there is a fall in potential.

Example 2.6. *Using Kirchhoff's Current Law and Ohm's Law, find the magnitude and polarity of voltge V in Fig. 2.9 (a). Directions of the two current sources are as shown.*

Solution. Let us arbitrarily choose the directions of I_1, I_2 and I_3 and polarity of *V* as shown in Fig. 2.9.(*b*). We will use the sign convention for currents as given in Art. 2.3. Applying KCL to node *A*, we have

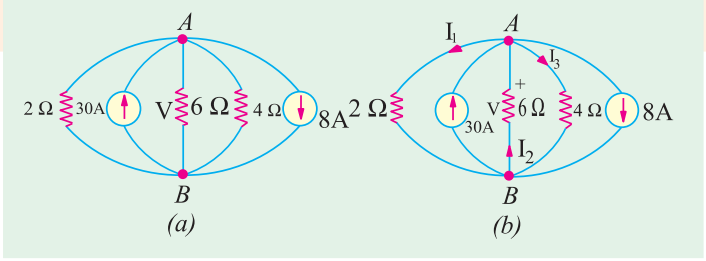


Fig. 2.9

$$-I_1 + 30 + I_2 - I_3 - 8 = 0$$

or $$I_1 - I_2 + I_3 = 22 \quad ...(i)$$

Applying Ohm's law to the three resistive branches in Fig. 2.9 (*b*), we have

$$I_1 = \frac{V}{2}, I_3 = \frac{V}{4}, I_2 = -\frac{V}{6}$$ (Please note the –ve sign.)

Substituting these values in (*i*) above, we get

$$\frac{V}{2} - \left(\frac{-V}{6}\right) + \frac{V}{4} = 22 \quad \text{or} \quad V = \mathbf{24\ V}$$

$\therefore$ $I_1 = V/2 = 24/2 = 12$ A, $I_2 = -24/6 = -4$ A, $I_3 = 24/4 = 6$ A

The negative sign of I_2 indicates that actual direction of its flow is opposite to that shown in Fig. 2.9 (*b*). Actually, I_2, flows from *A* to *B* and not from *B* to *A* as shown.

Incidentally, it may be noted that all currents are outgoing except 30A which is an incoming current.

Example 2.7. *For the circuit shown in Fig. 2.10, find V_{CE} and V_{AG}.*

(F.Y. Engg. Pune Univ.)

Solution. Consider the two battery circuits of Fig. 2.10 separately. Current in the 20 V battery circuit *ABCD* is 20 (6 + 5 + 9) = 1A. Similarly, current in the 40 V battery curcuit *EFGH* is = 40/(5 + 8 + 7) = 2A. Voltage drops over different resistors can be found by using Ohm's law.

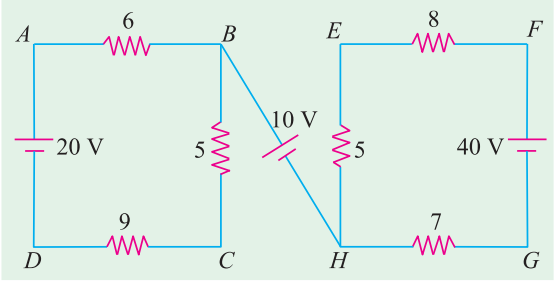


Fig. 2.10

For finding V_{CE} *i.e.* voltage of point *C* with respect to point *E*, we will start from point *E* and go to *C* via points *H* and *B*. We will find the algebraic sum of the voltage drops met on the way from point *E* to *C*. Sign convention of the voltage drops and battery e.m.fs. would be the same as discussed in Art. 2.3.

$\therefore$ $V_{CE} = (-5 \times 2) + (10) - (5 \times 1) = -5$V

The negative sign shows that point *C* is negative with respect to point *E*.

$$V_{AG} = (7 \times 2) + (10) + (6 \times 1) = 30 \text{ V}.$$

The positive sign shows that point *A* is at a positive potential of 30 V with respect to point *G*.

Example 2.8. *Determine the currents in the unbalanced bridge circuit of Fig. 2.11 below. Also, determine the p.d. across BD and the resistance from B to D.*

Solution. Assumed current directions are as shown in Fig. 2.11.

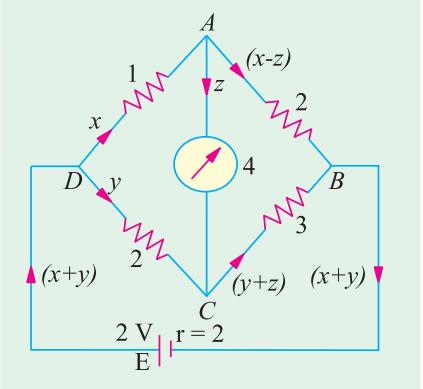


Fig. 2.11

Applying Kirchhoff's Second Law to circuit *DACD*, we get

$-x - 4z + 2y = 0$ or $x - 2y + 4z = 0$ ...(1)

Circuit *ABCA* gives

$-2(x - z) + 3(y + z) + 4z = 0$ or $2x - 3y - 9z = 0$ ...(2)

Circuit *DABED* gives

$-x - 2(x - z) - 2(x + y) + 2 = 0$ or $5x + 2y - 2z = 2$ ...**(3)**

Multiplying **(1)** by 2 and subtracting **(2)** from it, we get

$-y + 17z = 0$ ...**(4)**

Similarly, multiplying (1) by 5 and subtracitng (3) from it, we have

$-12y + 22z = -2$ or $-6y + 11z = -1$ ...**(5)**

Eliminating y from **(4)** and **(5)**, we have $91z = 1$ or $z = 1/91$ A

From (4); $y = 17/91$ A. Putting these values of y and z in **(1)**, we get $x = 30/91$ A

Current in $DA = x =$ **30/91 A** Current in $DC = y =$ **17/91 A**

Current in $AB = x - z = \frac{30}{91} - \frac{1}{91} = \mathbf{\frac{29}{91}}$ **A**

Current in $CB = y - z = \frac{17}{91} - \frac{1}{91} = \mathbf{\frac{18}{91}}$ **A**

Current in external circuit $= x + y = \frac{30}{91} + \frac{17}{91} = \mathbf{\frac{47}{91}}$ **A**

Current in $AC = z =$ **1/91 A**

Internal voltage drop in the cell $= 2(x + y) = 2 \times 47/91 =$ **94/91 V**

$\therefore$ P.D. across points D and $B = 2 - \frac{94}{91} = \mathbf{\frac{88}{91}}$ **V** *

Equivalent resistance of the bridge between points D and B

$$= \frac{\text{p.d. between points } B \text{ and } D}{\text{current between points } B \text{ and } D} = \frac{88/91}{47/91} = \frac{88}{47} = \textbf{1.87 Ω (approx)}$$

Solution By Determinants

The matrix from the three simultaneous equations **(1)**, **(2)** and **(3)** is

$$\begin{bmatrix} 1 & -2 & 4 \\ 2 & -3 & -9 \\ 5 & 2 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 1 & -2 & 4 \\ 2 & -3 & -9 \\ 5 & 2 & -2 \end{bmatrix} = 1(6+18) - 2(4-8) + 5(18+12) = 182$$

$$\Delta_1 = \begin{bmatrix} 0 & -2 & 4 \\ 0 & -3 & -9 \\ 2 & 2 & -2 \end{bmatrix} = 0(6+18) - 0(4-8) + 2(18+12) = 60$$

$$\Delta_2 = \begin{bmatrix} 1 & 0 & 4 \\ 2 & 0 & -9 \\ 5 & 2 & -2 \end{bmatrix} = 34, \Delta_3 = \begin{bmatrix} 1 & -2 & 0 \\ 2 & -3 & 0 \\ 5 & 2 & 2 \end{bmatrix} = 2$$

$\therefore$ $x = \frac{\Delta_1}{\Delta} = \frac{60}{182} = \mathbf{\frac{30}{91}}$ **A,** $y = \frac{34}{182} = \mathbf{\frac{17}{91}}$ **A,** $z = \frac{2}{182} = \mathbf{\frac{1}{91}}$ **A**

Example 2.9. *Determine the branch currents in the network of Fig. 2.12 when the value of each branch resistance is one ohm.* **(Elect. Technology, Allahabad Univ. 1992)**

Solution. Let the current directions be as shown in Fig. 2.12.

Apply Kirchhoff's Second law to the closed circuit *ABDA*, we get

$5 - x - z + y = 0$ or $x - y + z = 5$ ...***(i)***

* *P.D. between D and B = drop across DC + drop across CB = 2 × 17/91 + 3 × 18/91 =* **88/91 V.**

Similarly, circuit *BCDB* gives

$$-(x-z)+5+(y+z)=0$$

or $$x-y-3z=5 \quad \text{...(ii)}$$

Lastly, from circuit *ADCEA*, we get

$$-y-(y+z)+10-(x+y)=0$$

or $$x+3y+z=10 \quad \text{...(iii)}$$

From Eq. (*i*) and (*ii*), we get, $z = 0$

Substituting $z = 0$ either in Eq. (*i*) or (*ii*) and in Eq. (*iii*), we get

$$x-y=5 \quad \text{...(iv)}$$

$$x+3y=10 \quad \text{...(v)}$$

Subtracting Eq. **(*v*)** from **(*iv*)**, we get

$-4y = -5$ or $y = 5/4 = 1.24$ A

Eq. (*iv*) gives $x = 25/4$ A $= 6.25$ A

Current in branch *AB* = current in branch *BC* = **6.25 A**

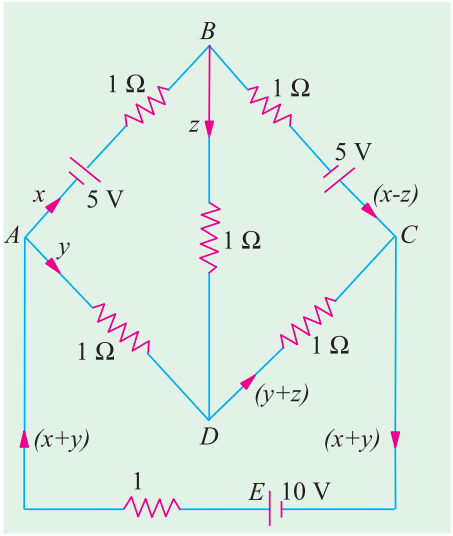


Fig. 2.12

Current in branch *BD* = 0; current in branch *AD* = current in branch *DC* = **1.25 A;** current in branch *CEA* = 6.25 + 1.25 = **7.5 A.**

Example 2.10. *State and explain Kirchhoff's laws. Determine the current supplied by the battery in the circuit shown in Fig. 2.12 A.* **(Elect. Engg. I, Bombay Univ.)**

Solution. Let the current distribution be as shown in the figure. Considering the close circuit *ABCA* and applying Kirchhoff's Second Law, we have

$$-100x-300z+500y=0$$

or $$x-5y+3z=0 \quad \text{....(i)}$$

Similarly, considering the closed loop *BCDB*, we have

$$-300z-100(y+z)+500(x-z)=0$$

or $$5x-y-9z=0 \quad \text{...(ii)}$$

Taking the circuit *ABDEA*, we get

$$-100x-500(x-z)+100-100(x+y)=0$$

or $$7x+y-5z=1 \quad \text{...(iii)}$$

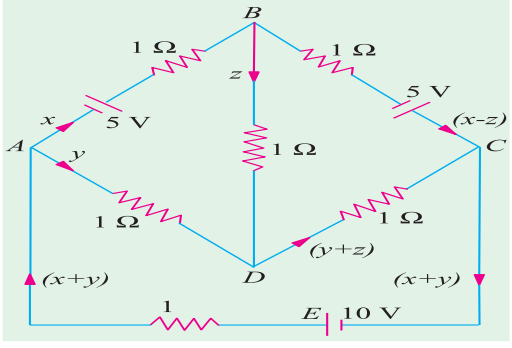


Fig. 2.12 A

The value of *x*, *y* and *z* may be found by solving the above three simultaneous equations or by the method of determinants as given below :

Putting the above three equations in the matrix form, we have

$$\begin{bmatrix} 1 & -5 & 3 \\ 5 & -1 & -9 \\ 7 & 1 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 1 & -5 & 3 \\ 5 & -1 & -9 \\ 7 & 1 & -5 \end{bmatrix} = 240, \quad \Delta_1 = \begin{bmatrix} 0 & -5 & 3 \\ 0 & -1 & -9 \\ 1 & 1 & -5 \end{bmatrix} = 48$$

$$\Delta_2 = \begin{bmatrix} 1 & 0 & 3 \\ 5 & 0 & -9 \\ 7 & 1 & -5 \end{bmatrix} = 24, \quad \Delta_3 = \begin{bmatrix} 1 & -5 & 0 \\ 5 & -1 & 0 \\ 7 & 1 & 1 \end{bmatrix} = 24$$

$\therefore \quad x = \frac{48}{240} = \frac{1}{5}$ **A**; $y = \frac{24}{240} = \frac{1}{10}$ **A**; $z = \frac{24}{240} = \frac{1}{10}$ **A**

Current supplied by the battery is $x + y = 1/5 + 1/10 =$ **3/10 A.**

Example 2.11. *Two batteries A and B are connected in parallel and load of 10 Ω is connected across their terminals. A has an e.m.f. of 12 V and an internal resistance of 2 Ω ; B has an e.m.f. of 8 V and an internal resistance of 1 Ω. Use Kirchhoff's laws to determine the values and directions of the currents flowing in each of the batteries and in the external resistance. Also determine the potential difference across the external resistance.*

(F.Y. Engg. Pune Univ.)

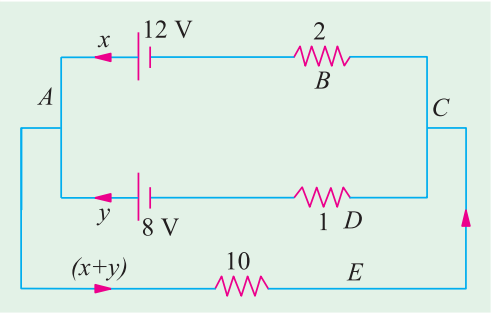


Fig. 2.13

Solution. Applying KVL to the closed circuit *ABCDA* of Fig. 2.13, we get

$$-12 + 2x - 1y + 8 = 0 \quad \text{or} \quad 2x - y = 4 \qquad \ldots(i)$$

Similarly, from the closed circuit *ADCEA*, we get

$$-8 + 1y + 10\,(x + y) = 0 \quad \text{or} \quad 10x + 11y = 8 \qquad \ldots(ii)$$

From Eq. **(*i*)** and **(*ii*),** we get

$x =$ **1.625 A** and $y =$ **–0.75 A**

The negative sign of y shows that the current is flowing into the 8-V battery and not out of it. In other words, it is a charging current and not a discharging current.

Current flowing in the external resistance $= x + y = 1.625 - 0.75 =$ **0.875 A**

P.D. across the external resistance $= 10 \times 0.875 =$ **8.75 V**

Note. To confirm the correctness of the answer, the simple check is to find the value of the external voltage available across point *A* and *C* with the help of the two parallel branches. If the value of the voltage comes out to be the same, then the answer is correct, otherwise it is wrong. For example, $V_{CBA} = -2 \times 1.625 + 12 = 8.75$ V. From the second branch $V_{CDA} = 1 \times 0.75 + 8 = 8.75$ V. Hence, the answer found above is correct.

Example 2.12. *Determine the current x in the 4-Ω resistance of the circuit shown in Fig. 2.13 (A).*

Solution. The given circuit is redrawn with assumed distribution of currents in Fig. 2.13 A (*b*). Applying KVL to different closed loops, we get

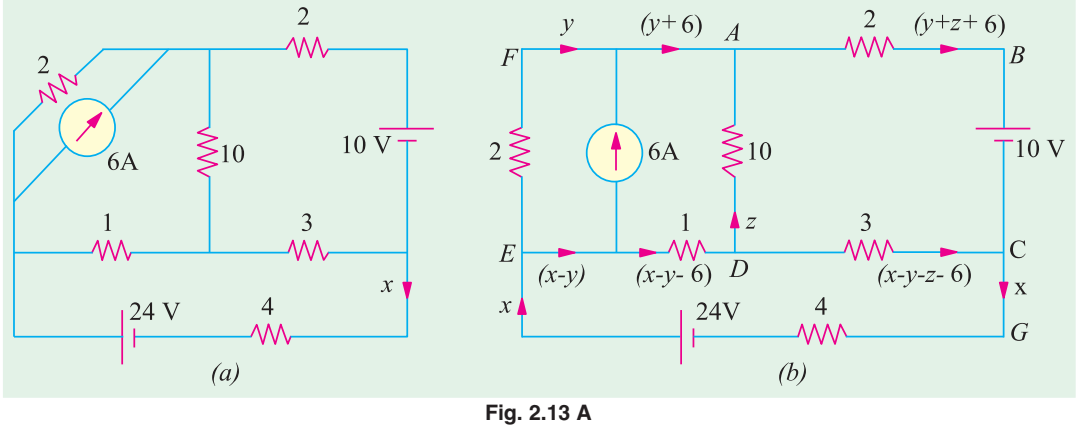


Fig. 2.13 A

Circuit EFADE

$$-2y + 10z + (x - y - 6) = 0 \quad \text{or} \quad x - 3y + 10z = 6 \qquad ...(i)$$

Circuit ABCDA

$$2(y + z + 6) - 10 + 3(x - y - z - 6) - 10z = 0 \quad \text{or} \quad 3x - 5y - 14z = 40 \qquad ...(ii)$$

Circuit EDCGE

$$-(x - y - 6) - 3(x - y - z - 6) - 4x + 24 = 0 \quad \text{or} \quad 8x - 4y - 3z = 48 \qquad ...(iii)$$

From above equations we get x = **4.1 A**

Example 2.13. *Applying Kirchhoff's laws to different loops in Fig. 2.14, find the values of V_1 and V_2.*

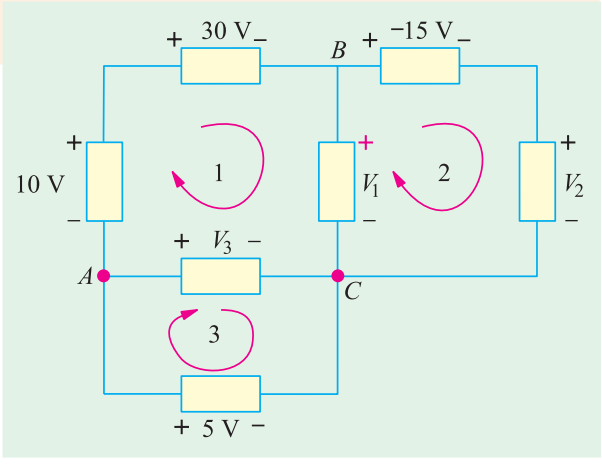


Fig. 2.14

Solution. Starting from point A and applying Kirchhoff's voltage law to loop No.3, we get

$$-V_3 + 5 = 0 \quad \text{or} \quad V_3 = 5 \text{ V}$$

Starting from point A and applying Kirchhoff's voltage law to loop No. 1, we get

$$10 - 30 - V_1 + 5 = 0 \quad \text{or} \quad V_1 = \mathbf{-15 \text{ V}}$$

The negative sign of V_1 denotes that its polarity is opposite to that shown in the figure.

Starting from point B in loop No. 3, we get

$$-(-15) - V_2 + (-15) = 0 \quad \text{or} \quad V_2 = \mathbf{0}$$

Example 2.14. *In the network of Fig. 2.15, the different currents and voltages are as under :*

$i_2 = 5e^{-2t}$, $i_4 = 3 \sin t$ *and* $v_3 = 4e^{-2t}$

Using KCL, find voltage v_1.

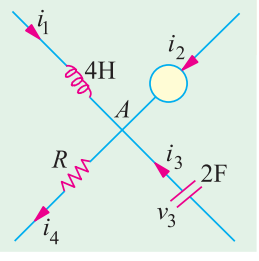


Fig. 2.15

Solution. According to *KCL*, the algebraic sum of the currents meeting at juncion A is zero *i.e.*

$$i_1 + i_2 + i_3 + (-i_4) = 0$$
$$i_1 + i_2 + i_3 - i_4 = 0 \qquad ...(i)$$

Now, current through a capacitor is given by $i = C\, dv/dt$

$$\therefore \quad i_3 = C \frac{dv_3}{dt} \quad \frac{2d(4e^{2t})}{dt} \quad 16e^{2t}$$

Substituting this value in Eq **(*i*)** above, we get

$$i_1 + 5e^{-2t} - 16e^{-2t} - 3 \sin t = 0$$

or

$$i_1 = 3 \sin t + 11e^{-2t}$$

The voltage v_1 developed across the coil is

$$v_1 = L \frac{di_1}{dt} = 4. \frac{d}{dt}(3 \sin t + 11e^{-2t})$$
$$= 4(3 \cos t - 22e^{-2t}) = 12 \cos t - 88e^{-2t}$$

Example 2.15. *In the network shown in Fig. 2.16, $v_1 = 4V$, $v_4 = 4 \cos 2t$ and $i_3 = 2e^{-t/3}$. Determine i_2.*

Solution. Applying *KVL* to closed mesh *ABCDA*, we get

$$-v_1 - v_2 + v_3 + v_4 = 0$$

Now $$v_3 = L\frac{di_3}{dt} = 6.\frac{d}{dt}(2e^{-t/3})$$

$$= -4e^{-t/3}$$

$$\therefore \quad -4 - v_2 - 4e^{-t/3} + 4\cos 2t = 0$$

or $$v_2 = 4\cos 2t - 4e^{-t/3} - 4$$

Now, $$i_2 = C\frac{dv_2}{dt} = 8\frac{d}{dt}(4\cos 2t - 4e^{-t/3} - 4)$$

$$\therefore \quad i_2 = 8\left(-8\sin 2t + \frac{4}{3}e^{-t/3}\right) = -64\sin 2t + \frac{32}{3}e^{-t/3}$$

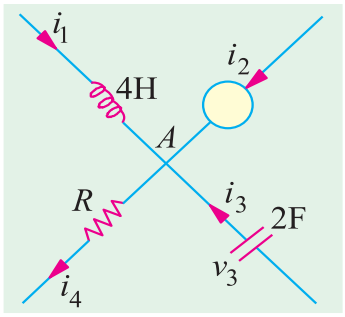


Fig. 2.16

Example 2.16. *Use nodal analysis to determine the voltage across 5 Ω resistance and the current in the 12 V source.* **[Bombay University 2001]**

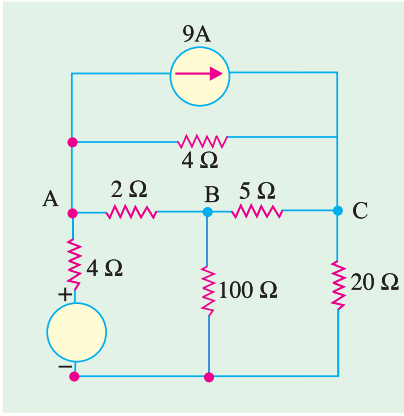


Fig. 2.17 (*a*)

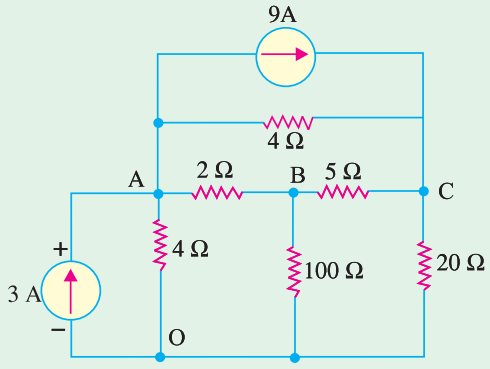


Fig. 2.17 (*b*)

Solution. Transform the 12-volt and 4-ohm resistor into current-source and parallel resistor.

Mark the nodes *O*, *A*, *B* and *C* on the diagram. Self-and mutual conductance terms are to be wirtten down next.

At *A*, G_{aa} = 1/4 + 1/2 + 1/4 = 1 mho

At *B*, G_{bb} = 1/2 + 1/5 + 1/100 = 0.71 mho

At *C*, G_{cc} = 1/4 + 1/5 + 1/20 = 0/50 mho

Between *A* and *B*, G_{ab} = 0.5 mho,

Between *B* and *C*, G_{be} = 0.2 mho,

Between *A* and *C*, G_{ac} = 0.25 mho.

Current Source matrix : At node *A*, 3 amp incoming and 9 amp outgoing currents give a net outgoing current of 6 amp. At node *C*, incoming current = 9 amp. At node *B*, no current source is connected. Hence, the current-source matrix is : $\begin{bmatrix} -6 \\ 0 \\ 9 \end{bmatrix}$

The potentials of three nodes to be found are : V_A, V_B, V_C

$$\begin{bmatrix} 1 & -0.5 & -0.25 \\ -0.5 & 0.71 & -0.20 \\ -0.25 & -0.20 & 0.5 \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} -6 \\ 0 \\ 9 \end{bmatrix}$$

For evaluating V_A, V_B, V_C, following steps are required.

$$\Delta = \begin{vmatrix} 1 & -0.5 & -0.25 \\ -0.5 & 0.71 & -0.20 \\ -0.25 & -0.20 & 0.5 \end{vmatrix} = 1\times(0.710.5 - 0.04) + 0.5\,(-0.25 - 0.05) - 0.25\,(0.1 + 0.71\times 0.25)$$

$$= 0.315 - 0.15 - 0.069375 = 0.095625$$

$$\Delta_a = \begin{vmatrix} -6 & -0.5 & -0.25 \\ -0.5 & 0.71 & -0.20 \\ 9 & -0.20 & +0.5 \end{vmatrix} = +0.6075$$

$$\Delta_b = \begin{vmatrix} 1 & -6 & -0.25 \\ -0.5 & 0 & -0.20 \\ -0.25 & 9 & 0.50 \end{vmatrix} = 1.125$$

$$\Delta_c = \begin{vmatrix} 1 & -0.5 & -6 \\ -0.5 & 0.71 & 0 \\ -0.25 & -0.20 & 9 \end{vmatrix} = 2.2475$$

$V_A = \Delta_a/\Delta = +0/6075/0.095625 = 6.353$ volts

$V_B = \Delta_b/\Delta = 1.125/0.095625 = 11.765$ volts

$V_C = \Delta_c/\Delta = 2.475/0.95625 = 25.882$ volts

Hence, voltage across 5-ohm resistor = $V_C - V_B = 14.18$ volts. Obviously. *B* is positive w.r. to *A*. From these node potentials, current through 100-ohm resistor is 0.118 amp; ***(i)*** current through 20 ohm resistor is 1.294 amp.

(ii) Current through 5-ohm resistor = 14.18/5 = 2.836 amp.

(iii) Current through 4-ohm resistor between *C* and *A* = 19.53/4 = 4.883 amp

Check : Apply *KCL* at node *C*

Incoming current = 9 amp, from the source.

Outgoing currents as calculated in ***(i)***, ***(ii)*** and ***(iii)*** above = 1.294 + 2.836 + 4.883 ≅ 9 amp

(iv) Current through 2-ohm resistor = $(V_B - V_A)/2 = 2.706$ amp, from *B* to *A*.

(v) Current in *A*-*O* branch = 6.353/4 = 1.588 amp

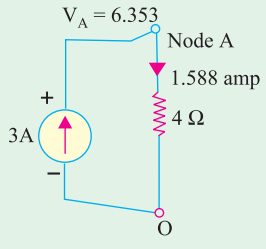


Fig. 2.17 (*c*) Equivalent

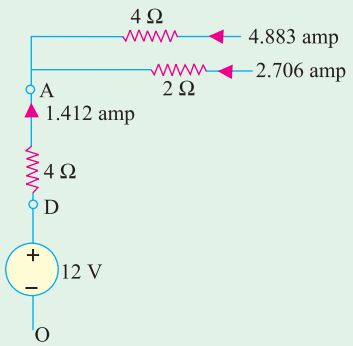


Fig. 2.17 (*d*) Actual elements

In Fig. 2.17 (*c*), the transformed equivalent circuit is shown. The 3-amp current source (*O* to *A*) and the current of 1.588 amp in *A*-*O* branch have to be interpreted with reference to the actual circuit, shown in Fig. 2.17 (*d*), where in a node *D* exists at a potential of 12 volts w.r. to the reference node. The 4-ohm resistor between *D* and *A* carries an upward current of {(12 –6.353)/4 =} 1.412 amp, which is nothing but 3 amp into the node and 1.588 amp away from the node, as in Fig. 2.17 (*c*), at node *A*. The current in the 12-V source is thus 1.412 amp.

Check. *KCL* at node *A* should give a check that incoming currents should add-up to 9 amp.

1.412 + 2.706 + 4.883 ≅ 9 amp

Example 2.17. *Determine current in 5-Ω resistor by any one method.*
(Bombay University 2001)

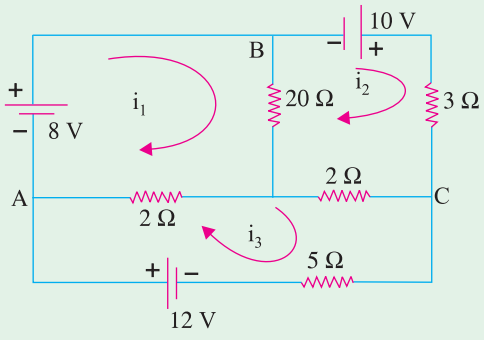


Fig. 2.18 (*a*)

Soltuion (*A*). Matrix-method for Mesh analysis can be used. Mark three loops as shown, in Fig. 2.18 (*a*). Resistance-matrix should be evaluated for current in 5-ohm resistor. Only, i_3 is to be found.

$R_{11} = 3, R_{22} = 6, R_{33} = 9 \qquad R_{12} = 1, R_{23} = 2, R_{13} = 2$

Voltage-source will be a column matrix with entries serially as : + 8 Volts, + 10 Volts, + 12 Volts.

$$\Delta = \begin{vmatrix} 3 & -1 & -2 \\ -1 & 6 & -2 \\ -2 & -2 & 9 \end{vmatrix} = 3\times(54-4)+1(-9-4)-2(2+12) = 109$$

$$\Delta_3 = \begin{vmatrix} 3 & -1 & 8 \\ -1 & 6 & 10 \\ -2 & -2 & 12 \end{vmatrix} = 396$$

$$i_3 = \Delta_3/\Delta = 396/109 = 3.633 \text{ amp.}$$

Solution (*B*). Alternatively, Thevenin's theorem can be applied.

For this, detach the 5-ohm resistor from its position, Evaluate V_{TH} at the terminals *X*-*Y* in Fig. 2.18 (*b*) and de-activating the source, calculate the value of R_{TH} as shown in Fig. 2.18 (*c*).

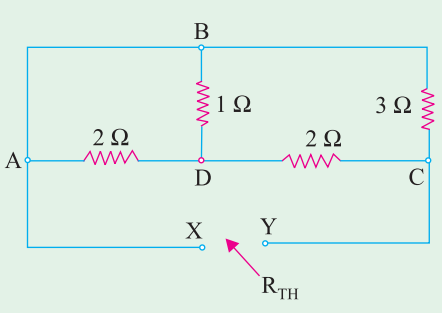


Fig. 2.18 (*b*)

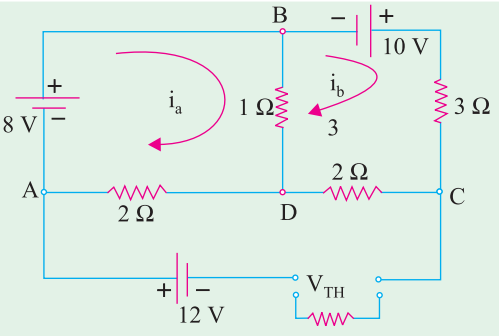


Fig. 2.18 (*c*)

By observation, Resistance-elements of 2 × 2 matrix have to be noted.

$$R_{aa} = 3, R_{bb} = 5, R_{ab} = 1$$

$$\begin{vmatrix} 3 & -1 \\ -1 & 6 \end{vmatrix} \begin{vmatrix} i_a \\ i_b \end{vmatrix} = \begin{vmatrix} +8 \\ +10 \end{vmatrix}$$

$$i_a = \begin{vmatrix} 8 & -1 \\ 10 & 6 \end{vmatrix} \div \begin{vmatrix} 3 & -1 \\ -1 & 6 \end{vmatrix} = 58/17 = 3.412 \text{ amp}$$

$$i_b = \begin{vmatrix} 3 & 8 \\ -1 & 10 \end{vmatrix} \div (17) = 38/17 = 2.2353 \text{ amp}$$

$$V_{XY} = V_{TH} = 12 + 2\boldsymbol{i}_a + 2\boldsymbol{i}_b = 23.3 \text{ Volts},$$

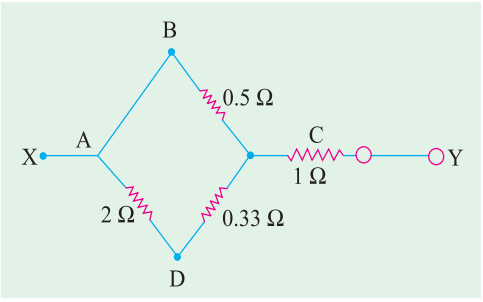


Fig. 2.18 (*d*)

with y positive w.r. to X. R_{TH} can be evaluated from Fig. 2.18 (*c*), after transforming delta configuration at nodes *B-D-C* to its equivalent star, as shown in Fig. 2.18 (*d*)

Further simplification results into :

$R_{XY} = R_{TH} = 1.412$ ohms

Hence, Load Current $= V_{TH}/(R_L + R_{TH}) = 23.3/6.412 = 3.643$ amp.

This agrees with result obtained earlier.

Example 2.18 (*a*). *Determine the voltages 1 and 2 of the network in Fig. 2.19 (a) by nodal analysis.* **(Bombay University, 2001)**

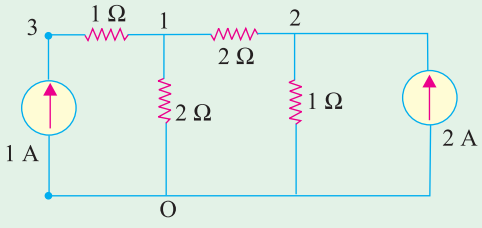


Fig. 2.19 (*a*)

Solution. Write the conductance matrix for the network, with nodes numbered as 1, 2, 4 as shown.

$$g_{11} = 1 + 0.5 + 0.5 = 2 \text{ mho}, \quad g_{22} = 1 + 0.5 = 1.5 \text{ mho},$$

$$g_{33} = 1 \text{ mho}, g_{12} = 0.5 \text{ mho}, g_{23} = 0, g_{13} = 1 \text{ mho}$$

$$\Delta = \begin{vmatrix} 2 & -0.5 & -1 \\ -0.5 & 1.5 & 0 \\ -1 & 0 & 1.0 \end{vmatrix} = 1.25, \quad \Delta_1 = \begin{vmatrix} 0 & -0.5 & -1 \\ 2 & 1.5 & 0 \\ 1 & 0 & 1 \end{vmatrix} = 2.5$$

$$\Delta_2 = \begin{vmatrix} 2 & 0 & -1 \\ -0.5 & 2 & 0 \\ -1 & 1 & 1.0 \end{vmatrix} = 2.5$$

This gives $V_1 = \Delta_1/\Delta = 2.50/1.25 = 2$ Volts

And $V_2 = \Delta_2/\Delta = 2.50/1.25 = 2$ Volts

It means that the 2-ohm resistor between nodes 1 and 2 does not carry current.

Example 2.18 (*b*). *In the circuit of Fig. 2.19 (b), find current through 1-Ω resistor using both THEVENIN's theorem and SUPERPOSITION theorem.*

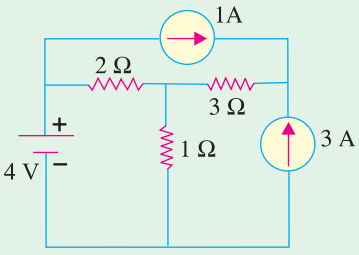


Fig. 2.19 (*b*)

Solution. ***(i)*** By Thevenin's Theorem :

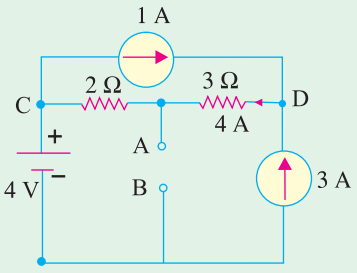


Fig. 2.19 (*c*)

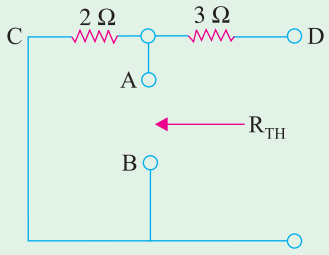


Fig. 2.19 (*d*)

Take $V_B = 0$. Then $V_A = 4 + 8 = 12$, since from *D* to *C*, a current of 4 A must flow, as shown in Fig. 2.19 (*b*), applying *KCL* ot Node *D*.

$$V_{TH} = V_{AB} = 12 \text{ volts}$$

From Fig. 2.19 (*d*), $R_{TH} = 2$ ohms

$$IL = 12/(2 + 1) = 4 \text{ amp}$$

(ii) By Superposition Theorem : One source acts at a time. Current through *A*-*B* (1 ohm) is to be calculated due to each source and finally all these contributions added.

Due to 4-V source :

1-ohm resistor carries a current of 4/3 amp from *A* to *B*, as shown in Fig. 2.19 (*e*).

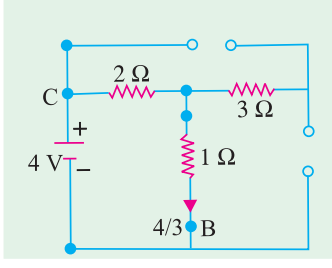


Fig. 2.19 (*e*). 4-V Source acts

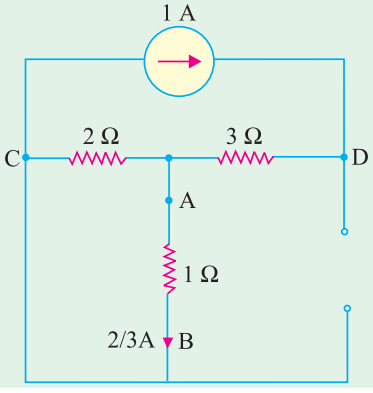


Fig. 2.19 (*f*). 1-A Source acts

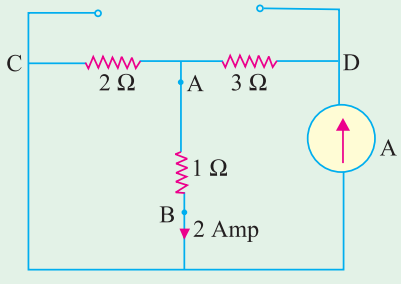


Fig. 2.19 (*g*). 3-A Source acts

Due ot 1-A source : 2/3 Amp from *A* to *B*, as shown in Fig. 2.19 (*f*)

Due to 3-A source : 2 Amp from *A* to *B* as shown in Fig. 2.19 (*g*)

Total current = 4 amp from *A* to *B*.

2.9. Independent and Dependent Sources

Those voltage or current sources, which do not depend on any other quantity in the circuit, are called independent sources. An independent d.c. voltage source is shown in Fig. 2.20 (*a*) whereas a time-varying voltage source is shown in Fig. 2.20 (*b*). The positive sign shows that terminal *A* is positive with respect to terminal *B*. In other words, potential of terminal *A* is *v* volts higher than that of terminal *B*.

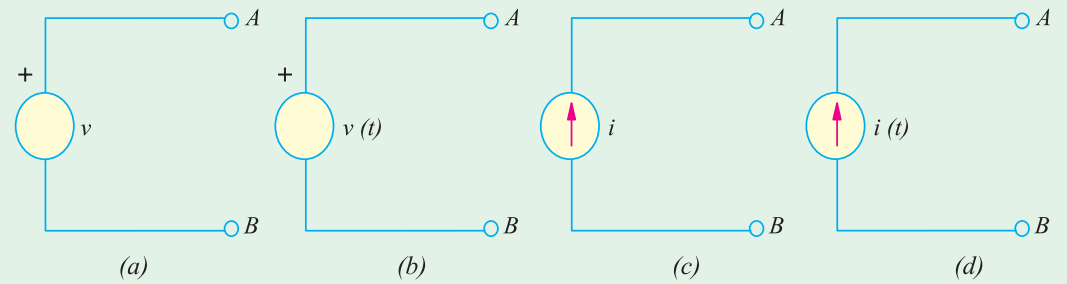


Fig. 2.20

$$\Delta_2 = \begin{vmatrix} 2 & -0.5 & -1 \\ -0.5 & 1.5 & 0 \\ -1 & 0 & 1.0 \end{vmatrix} = 1.25, \quad \Delta_1 = \begin{vmatrix} 0 & -0.5 & -1 \\ 2 & 1.5 & 0 \\ 1 & 0 & 1 \end{vmatrix} = 2.5$$

Similarly, Fig. 2.20 (*c*) shows an ideal constant current source whereas Fig. 2.20 (*d*) depicts a time-varying current source. The arrow shows the direction of flow of the current at any moment under consideration.

A dependent voltage or current source is one which depends on some other quantity in the circuit which may be either a voltage or a current. Such a source is represented by a diamond-shaped symbol as shown in Fig. 2.21 so as not to confuse it with an independent source. There are four possible dependent sources :

1. Voltage-dependent voltage source [Fig. 2.21 (*a*)]
2. Current-dependent voltage source [Fig. 2.21 (*b*)]
3. Voltage-dependent current source [Fig. 2.21 (*c*)]
4. Current-dependent current source [Fig. 2.21 (*d*)]

Such sources can also be either constant sources or time-varying sources. Such sources are often met in electronic circuits. As seen above, the voltage or current source is dependent on the and is

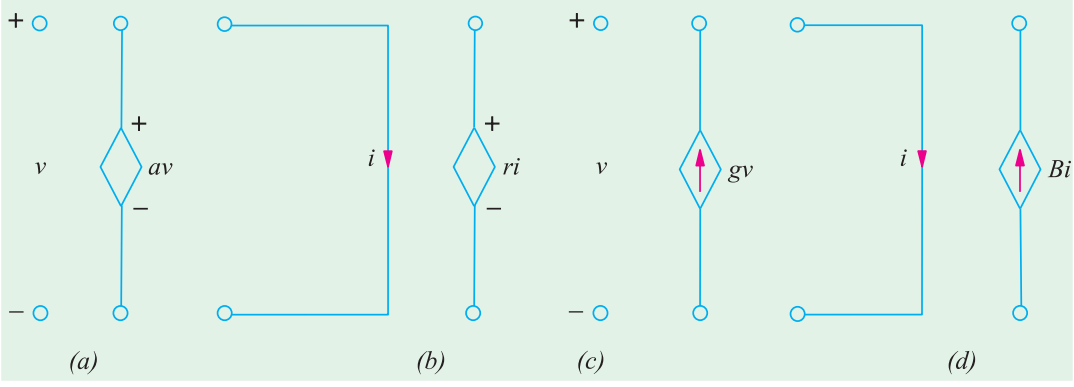


Fig. 2.21

proportional to another current or voltage. The constants of proportionality are written as *a*, *r*, *g* and β. The constants *a* and β have no unis, *r* has the unit of ohms and *g* has the unit of siemens.

Independent sources actually exist as physical entities such as a battery, a *d.c.* generator and an alternator etc. But dependent sources are parts of *models* that are used to represent electrical properties of electronic devices such as operational amplifiers and transistors etc.

Example 2.19. *Using Kirchhoff's current law, find the values of the currents i_1 and i_2 in the circuit of Fig. 2.22 (a) which contains a current-dependent current source. All resistances are in ohms.*

Solution. Applying *KCL* to node *A*, we get

$$2 - i_1 + 4\,i_1 - i_2 = 0 \quad \text{or} \quad -3i_1 + i_2 = 2$$

By Ohm's law, $i_1 = v/3$ and $i_2 = v/2$

Substituting these values above, we get

$$-3(v/3) + v/2 = 2 \quad \text{or} \quad v = -4 \text{ V}$$

$$\therefore \quad i_1 = -4/3 \text{ A and } i_2 = -4/2 = -2 \text{ A}$$

The value of the dependent current source is $= 4i_1 = 4 \times (-4/3) =$ **$-16/3$ A.**

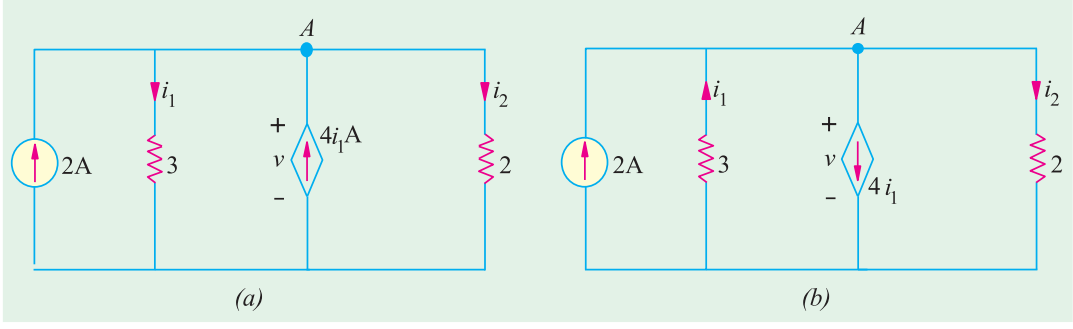


Fig. 2.22

Since i_1 and i_2 come out to be negative, it means that they flow upwards as shown in Fig. 2.22(*b*) and not downwards as presumed. Similarly, the current of the dependent source flows downwards as shown in Fig. 2.22 (*b*). It may also be noted that the sum of the upwards currents equals that of the downward currents.

Example 2.20. *By applying Kirchhoff's current law, obtain the values of v, i_1 and i_2 in the circuit of Fig. 2.23 (a) which contains a voltage-dependent current source. Resistance values are in ohms.*

Solution. Applying *KCL* to node *A* of the circuit, we get

$$2 - i_1 + 4v - i_2 = 0 \quad \text{or} \quad i_1 + i_2 - 4v = 2$$

Now, $\quad i_1 = v/3 \quad \text{and} \quad i_2 = v/6$

$$\therefore \quad \frac{v}{3} + \frac{v}{6} - 4v = 2 \quad \text{or} \quad v = \frac{-4}{7} \text{ V}$$

$$\therefore \quad i_1 = \frac{4}{21} A \text{ and } i_2 \quad \frac{2}{21} \text{A and } 4v \quad 4 \quad \frac{4}{7} \quad \frac{\mathbf{16}}{\mathbf{7}} \mathbf{V}$$

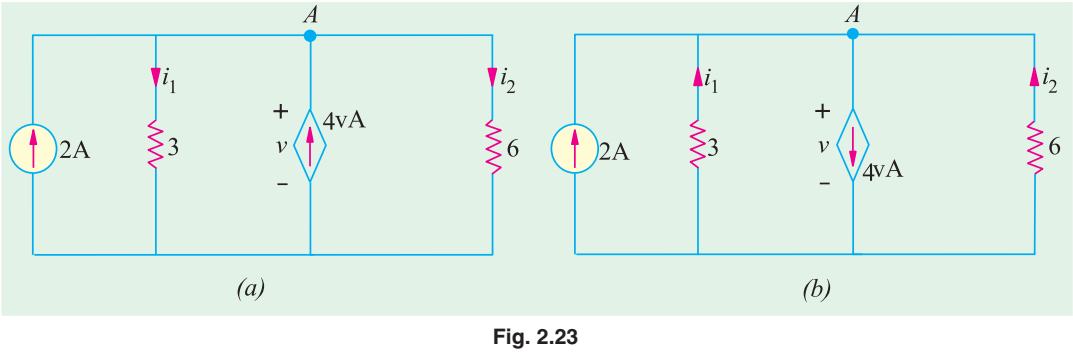


Fig. 2.23

Since i_1 and i_2 come out to be negative and value of current source is also negative, their directions of flow are opposite to those presumed in Fig. 2.23 (*a*). Actual current directions are shown in Fig. 2.23 (*b*).

Example 2.21. *Apply Kirchhoff's voltage law, to find the values of current i and the voltage drops v_1 and v_2 in the circuit of Fig. 2.24 which contains a current-dependent voltage source. What is the voltage of the dependent source ? All resistance values are in ohms.*

Solution. Applying *KVL* to the circuit of Fig. 2.24 and starting from point *A*, we get

$$-v_1 + 4i - v_2 + 6 = 0 \quad \text{or} \quad v_1 - 4i + v_2 = 6$$

Now, $v_1 = 2i$ and $v_2 = 4i$

$\therefore \quad 2i - 4i + 4i = 6 \quad \text{or} \quad i = 3\text{A}$

$\therefore \quad v_1 = 2 \times 3 = 6\text{V}$ and $v_2 = 4 \times 3 = \mathbf{12\ V}$

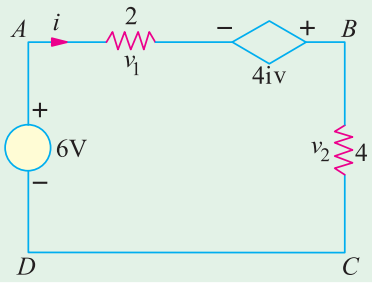


Fig. 2.24

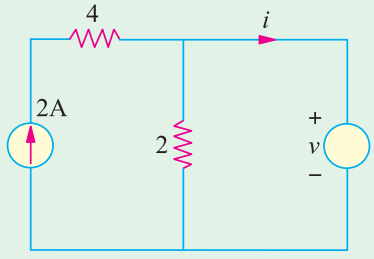


Fig. 2.25

Voltage of the dependent source = $4i = 4 \times 4 = \mathbf{12\ V}$

Example 2.22. *In the circuit shown in Fig. 2.25, apply KCL to find the value of i for the case when* ***(a)*** *v = 2 V,* ***(b)*** *v = 4 V* ***(c)*** *v = 6 V. The resistor values are in ohms.*

Solution. ***(a)*** When $v = 2$ V, current through 2 Ω resistor which is connected in parallel with the 2 *v* source = 2/2 = 1A. Since the source current is 2 A, $i = 2 - 1 = \mathbf{1\ A.}$

(b) When $v = 4$V, current through the 2Ω resistor = 4/2 = **2 A.** Hence $i = 2 - 2 = \mathbf{0\ A.}$

(c) When $v = 6$ V, current through the 2Ω resistor = 6/2 = 3 A. Since current source can supply only 2 A, the balance of 1 A is supplied by the voltage source. Hence, $i = -1$ A *i.e.* it flows in a direction opposite to that shown in Fig. 2.25.

Example 2.23. *In the circuit of Fig. 2.26, apply KCL to find the value of current i when* ***(a)*** *K = 2* ***(b)*** *K = 3 and* ***(c)*** *K = 4. Both resistances are in ohms.*

Solution. Since 6 Ω and 3 Ω resistors are connected in parallel across the 24-V battery, $i_1 = 24/6 = 4$ A.

Applying *KCL* to node *A*, we get $i - 4 + 4\text{K} - 8 = 0$ or $i = 12 - 4\text{K}$.

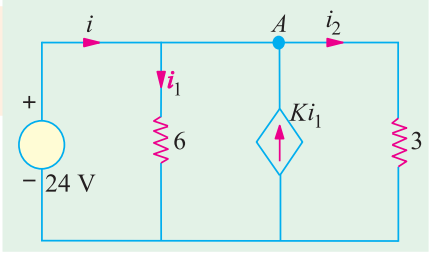


Fig. 2.26

(a) When $K = 2$, $i = 12 - 4 \times 2 = \mathbf{4\ A}$

(b) When $K = 3$, $i = 12 - 4 \times 3 = \mathbf{0\ A}$

(c) When $K = 4$, $i = 12 - 4 \times 4 = \mathbf{-4\ A}$

It means that current *i* flows in the opposite direciton.

Example 2.24. *Find the current i and also the power and voltage of the dependent source in Fig. 2.72 (a). All resistances are in ohms.*

Solution. The two current sources can be combined into a single source of 8 –6 = 2 A. The two parallel 4 Ω resistances when combined have a value of 2 Ω which, being in series with the 10 Ω resistance, gives the branch resistance of 10 + 2 = 12 Ω. This 12 Ω resistance when combined with the other 12 Ω resistance gives a combination resistance of 6 Ω. The simplified circuit is shown in Fig. 2.27 (*b.*)

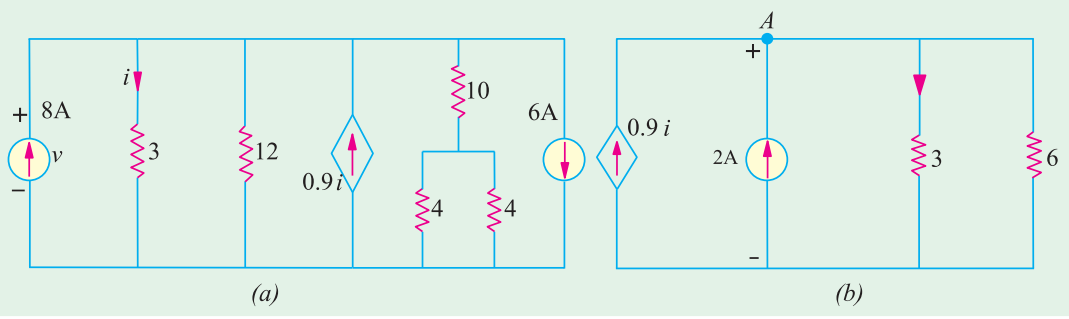


Fig. 2.27

Applying *KCL* to node A, we get

$$0.9i + 2 - i - V/6 = 0 \quad \text{or} \quad 0.6i = 12 - v$$

Also $v = 3i \quad \therefore \; i = 10/3$ A. Hence, $v =$ **10 V.**

The power furnished by the current source = $v \times 0.9\, i = 10 \times 0.9\,(10/3) =$ **30 W.**

Example 2.25. *By using voltage-divider rule, calculate the voltages v_x and v_y in the network shown in Fig. 2.28.*

Solution. As seen, 12 V drop in over the series combination of 1, 2 and 3 Ω resistors. As per voltage-divider rule v_x = drop over 3 Ω = 12 × 3/6 = 6 V.

The voltage of the dependent source = 12 × 6 = 72 V.

The voltage v_y equals the drop across 8 Ω resistor connected across the voltage source of 72 V.

Again using voltge-divider rule, drop over 8 Ω resistor = 72 × 8/12 = 48 V.

Hence, $v_y = -48$ V. The negative sign has been given because positive and negative signs of v_y are actually opposite to those shown in Fig. 2.28.

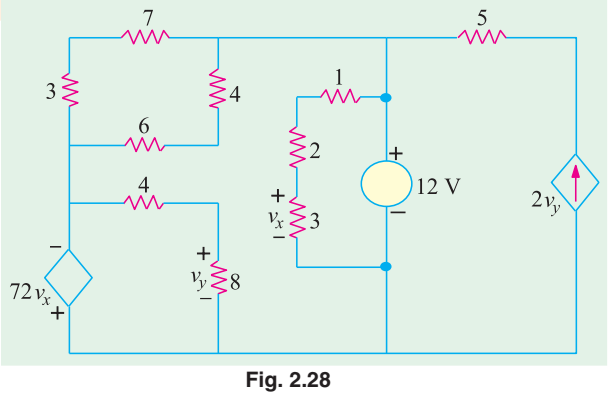


Fig. 2.28

Example 2.26. *Use KCL to find the value of v in the circuit of Fig. 2.29.*

Solution. Let us start from ground and go to point *a* and find the value of voltage v_a. Obviously, $5 + v = v_a$ or $v = v_a - 5$. Applying *KCL* to point, we get

$6 - 2\,v + (5 - v_a)/1 = 0$ or $6 - 2\,(v_a - 5) + (5 - v_a) = 0$ or $v_a = 7$ V

Hence, $v = v_a - 5 = 7 - 5 = 2$ V. Since it turns out to be positive, its sign as indicated in the figure is correct.

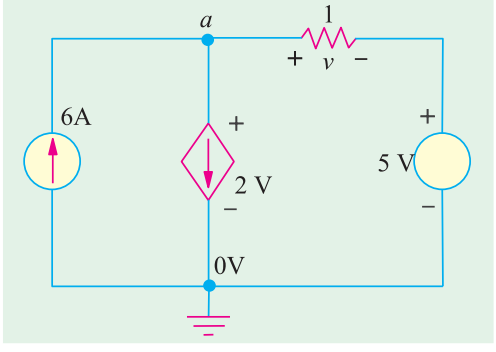


Fig. 2.29

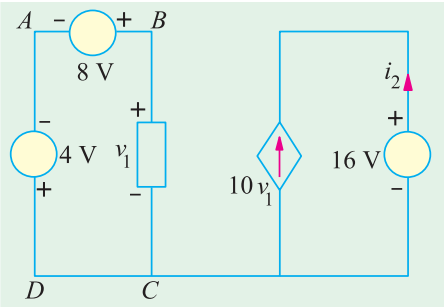


Fig. 2.30

Example 2.27. (*a*) Basic Electric Circuits by Cunninghan. *Find the value of current i_2 supplied by the voltage-controlled current source (VCCS) shown in Fig. 2.30.*

Solution. Applying *KVL* to the closed circuit *ABCD*, we have $-4 + 8 - v_1 = 0 \quad \therefore \; v_1 = 4$ V

The current supplied by *VCCS* is $10\, v_1 = 10 \times 4 = 40$ A. Since i_2 flows in an opposite direction to this current, hence $i_2 = -40$ **A.**

Example 2.27. (*b*). *Find the voltage drop v_2 across the current-controlled voltage source (CCVS) shown in Fig. 2.28.*

Solution. Applying *KCL* to point *A*, we have $2 + 6 - i_1 = 0$ or $i_1 = 8$ A.

Application of *KVL* to the closed circuit on the right hand side gives $5\, i_1 - v_2 = 0$ or $v_2 = 5\, i_1 = 5 \times 8 = 40$ V.

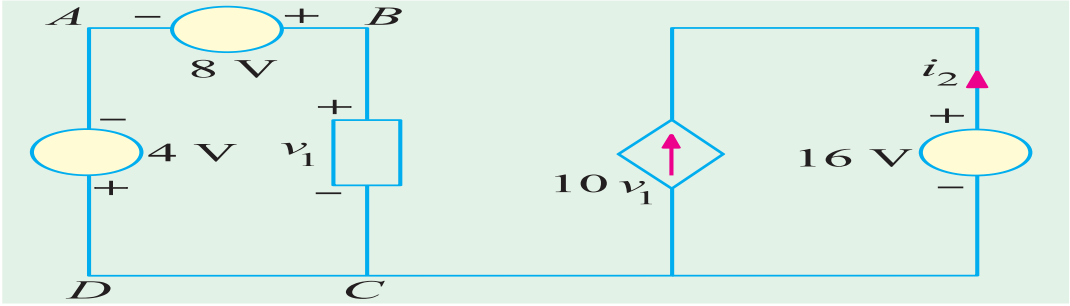


Fig. 2.31

Fig. 2.32

Example 2.28. *Find the values of i_1, v_1, v_x and v_{ab} in the network of Fig. 2.32 with its terminals a and b open.*

Solution. It is obvious that $i_1 = 4$ A. Applying *KVL* to the left-hand closed circuit, we get $-40 + 20 - v_1 = 0$ or $v_1 = -20$ V.

Similarly, applying *KVL* to the second closed loop, we get

$v_1 - v_x + 4v_1 - 50 = 0$ or $v_x = 5\, v_1 - 50 = -5 \times 20 - 50 = -150$ V

Again applying *KVL* to the right-hand side circuit containing v_{ab}, we get

$50 - 4v_1 - 10\, v_{ab} = 0 \quad$ or $\quad v_{ab} = 50 - 4\,(-20) - 10 = 120$ V

Example 2.29 (*a*). *Find the current i in the circuit of Fig. 2.33. All resistances are in ohms.*

Solution. The equivalent resitance of the two parallel paths across point *a* is $3 \parallel (4 + 2) = 2\ \Omega$. Now, applying *KVL* to the closed loop, we get $24 - v - 2v - 2i = 0$. Since $v = 2i$, we get $24 - 2i - 2(2i) - 2i = 0$ or $i = 3$ A.

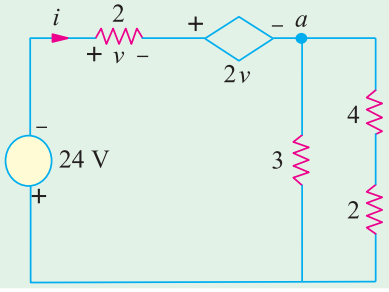


Fig. 2.33

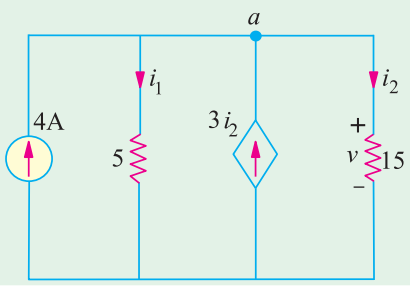


Fig. 2.34

Example 2.29. (*b*) *Determine the value of current i_2 and voltage drop v across 15 Ω resistor in Fig. 2.34.*

Solution. It will be seen that the dependent current source is related to i_2. Applying *KCL* to node *a*, we get $4 - i + 3i_2 - i_2 = 0$ or $4 - i_1 + 3\,i_2 = 0$.

Applying ohm's law, we get $i_1 = v/5$ and $i_2 = v/15$.

Substituting these values in the above equation, we get $4 - (v/5) + 2\,(v/15) = 0$ or $v = 60$ V and $i_2 = 4$ A.

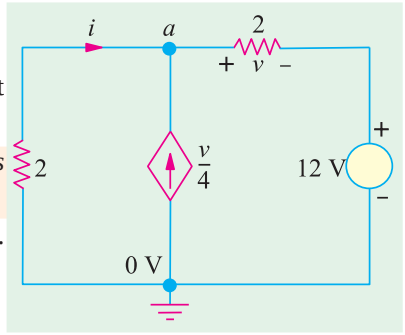


Fig. 2.35

Example 2.29 (*c*). *In the circuit of Fig. 2.35, find the values of i and v. All resistances are in ohms.*

Solution. It may be noted that $12 + v = v_a$ or $v = v_a - 12$. Applying *KCL* to node *a*, we get

$$\frac{0 - v_a}{2} + \frac{v}{4} - \frac{v_a - 12}{2} = 0 \quad \text{or} \quad v_a = 4 \text{ V}$$

Hence, $v = 4 - 12 = -8$ V. The negative sign shows that its polarity is opposite to that shown in Fig. 2.35. The current flowing from the point *a* to ground is $4/2 = 2$ A. Hence, $i = -2$ A.

Tutorial Problems No. 2.1

1. Apply *KCL* to find the value of *I* in Fig. 2.36. **[8 A]**

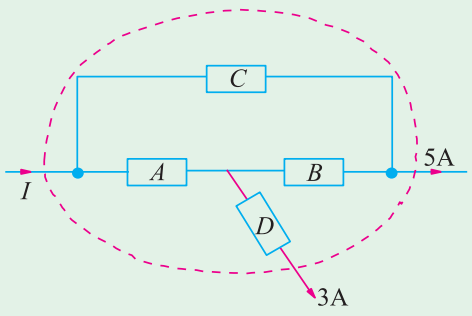


Fig. 2.36

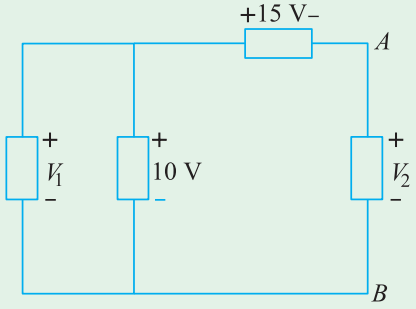


Fig. 2.37

2. Applying Kirchhoff's voltage law, find V_1 and V_2 in Fig. 2.37. **[V_1 = 10 V; V_2 = 5 V]**

3. Find the values of currents I_2 and I_4 in the network of Fig. 2.38. **[I_2 = 4 A ; I_4 = 5 A]**

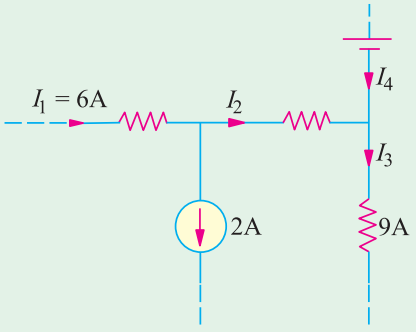


Fig. 2.38

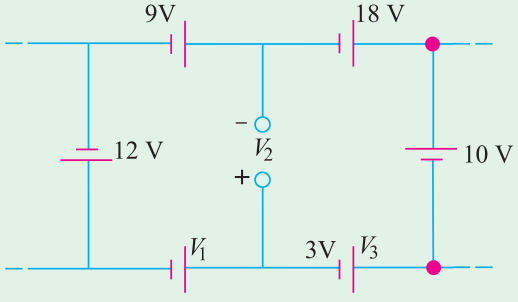


Fig. 2.39

4. Use Kirchhoff's law, to find the values of voltages V_1 and V_2 in the network shown in Fig. 2.39. **[V_1 = 2 V ; V_2 = 5 V]**

5. Find the unknown currents in the circuits shown in Fig. 2.40 (*a*). **[I_1 = 2 A ; I_2 = 7 A]**

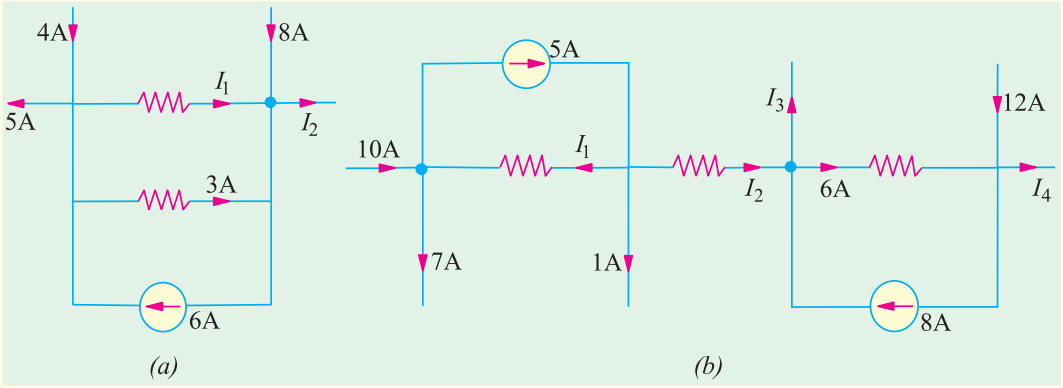


Fig. 2.40

6. Using Kirchhoff's current law, find the values of the unknown currents in Fig. 2.40 (*b*).
[I_1 = 2 A; I_2 = 2 A ; I_3 = 4 A ; I_4 = 10 A]

7. In Fig. 2.41, the potential of point *A* is –30 V. Using Kirchhoff's voltage law, find (*a*) value of V and (*b*) power dissipated by 5 Ω resistance. All resistances are in ohms. **[100 V; 500 W]**

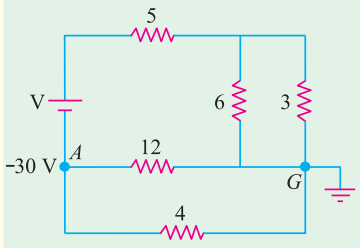


Fig. 2.41

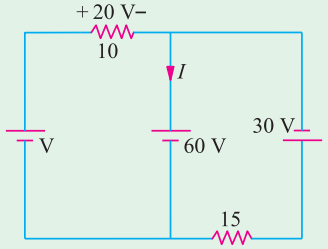


Fig. 2.42

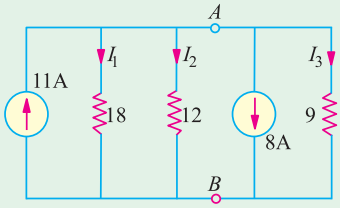


Fig. 2.43

8. Using *KVL* and *KCL*, find the values of *V* and *I* in Fig. 2.42. All resistances are in ohms.
[80 V; –4 A]

9. Using *KCL*, find the values V_{AB}, I_1, I_2 and I_3 in the circuit of Fig. 2.43. All resistances are in ohms.
[V_{AB} = 12 V ; I_1 = 2/3 A; I_2 = 1 A; I_3 = 4/3 A]

10. A bridge network *ABCD* is arranged as follows :
Resistances between terminals *A*-*B*, *B*-*C*, *C*-*D*, *D*-*A*, and *B*-*D* are 10, 20, 15, 5 and 40 ohms respectively. A 20 V battery of negligible internal resistance is connected between terminals *A* and *C*. Determine the current in each resistor.
[*AB* = 0.645 A; *BC* = 0.678 A; *AD* = 1.025 A; *DB* = 0.033 A; *DC* = 0.992 A]

11. Two batteries *A* and *B* are connected in parallel and a load of 10 Ω is connected across their terminals. A has an e.m.f. of 12 V and an internal resistance of 2 Ω ; *B* has an e.m.f. of 8 V and an internal resistance of 1 Ω. Use Kirchhoff's laws to determine the values and directions of the currents flowing in each of the batteries and in the external resistance. Also determine the p.d. across the external resistance. **[I_A = 1.625 A (discharge), I_B = 0.75 A (charge) ; 0.875 A; 8.75 V]**

12. The four arms of a Wheatstone bridge have the following resistances ; *AB* = 100, *BC* = 10, *CD* = 4, *DA* = 50 ohms.
A galvanometer of 20 ohms resistance is connected across *BD*. Calculate the current through the galvanometer when a potential difference of 10 volts is maintained across *AC*.
[0.00513 A] [*Elect. Tech. Lond. Univ.*]

13. Find the voltage V_{da} in the network shown in Fig. 2.44 (*a*) if *R* is 10 Ω and (*b*) 20 Ω
[(*a*) 5 V (*b*) 5 V]

14. In the network of Fig. 2.44 (*b*), calculate the voltage between points *a* and *b* *i.e.* V_{ab}.
[30 V] *(Elect. Engg. I, Bombay Univ.)*

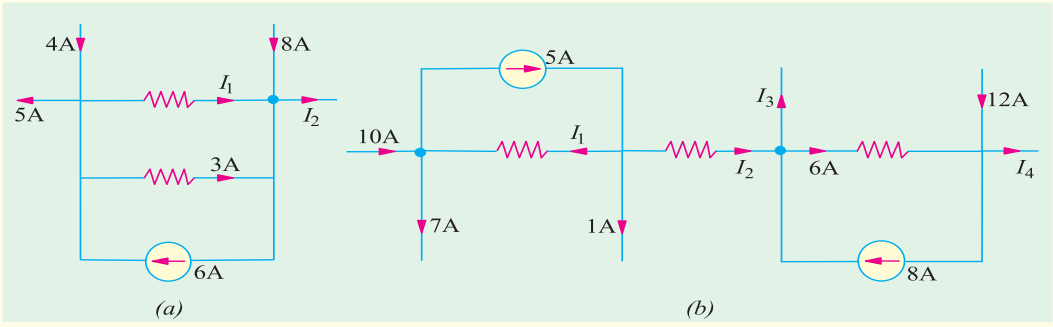


Fig. 2.44

[**Hint :** In the above two cases, the two closed loops are independent and no current passes between them].

15. A battery having an *E.M.F.* of 110 V and an internal resistance of 0.2 Ω is connected in parallel with another battery having an *E.M.F.* of 100 V and internal resistance 0.25 Ω. The two batteries in parallel are placed in series with a regulating resistance of 5 Ω and connected across 200 V mains. Calculate the magnitude and direction of the current in each battery and the total current taken from the supply mains.

[I_A = 11.96 (discharge); I_B = 30.43 A (charge) : 18.47 A] (*Elect Technology, Sumbhal Univ.*)

16. Three batteries *P, Q* and *R* consisting of 50, 55 and 60 cells in series respectively supply in parallel a common load of 100 A. Each cell has a e.m.f of 2 V and an internal resistance of 0.005 Ω. Determine the current supplied by each battery and the load voltage.

[1.2 A; 35.4 A : 65.8 A : 100.3 V] (*Basic Electricity, Bombay Univ.*)

17. Two storage batteries are connected in parallel to supply a load having a resistance of 0.1 Ω. The open-circut e.m.f. of one battery (*A*) is 12.1 V and that of the other battery (*B*) is 11.8 V. The internal resistances are 0.03 Ω and 0.04 Ω respectively. Calculate (*i*) the current supplied at the lead (*ii*) the current in each battery (*iii*) the terminal voltage of each battery.

[(*i*) 102.2 A (*ii*) 62.7 A (*A*). 39.5 A (*B*) (*iii*) 10.22 V] (*London Univ.*)

18. Two storage batteries, *A* and *B*, are connected in parallel to supply a load the resistance of which is 1.2 Ω. Calculate (*i*) the current in this lood and (*ii*) the current supplied by each battery if the open-circuit e.m.f. of *A* is 12.5 V and that of *B* is 12.8 V, the internal resistance of *A* being 0.05 Ω an that of *B* 0.08 Ω

[(*i*) 10.25 A (*ii*) 4 (A), 6.25 A (B)] (*London Univ.*)

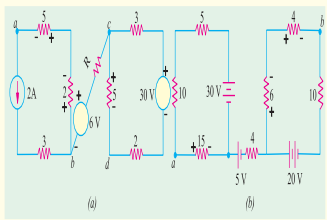


Fig. 2.45

19. The circuit of Fig. 2.45 contains a voltage-dependent voltage source. Find the current supplied by the battery and power supplied by the voltage source. Both resistances are in ohms. **[8 A ; 1920 W]**

20. Find the equivalent resistance between terminals *a* and *b* of the network shown in Fig. 2.46. **[2 Ω]**

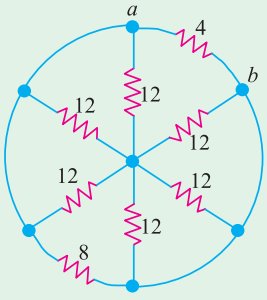


Fig. 2.46

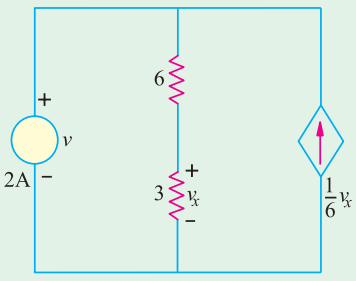


Fig. 2.47

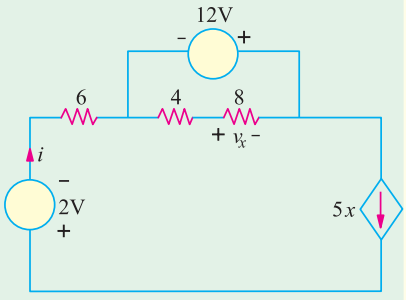


Fig. 2.48

21. Find the value of the voltage v in the network of Fig. 2.47. **[36 V]**

22. Determine the current i for the network shown in Fig. 2.48. **[–40 A]**

23. State and explain Kirchhoff's current law. Determine the value of R_S and R_P in the network of Fig. 2.49 if $V_2 = V_1/2$ and the equivalent resistance of the network between the terminals A and B is 100 Ω.

[R_S = 100/3 Ω R_P = 400/3 Ω] (*Elect. Engg. I, Bombay Univ.*)

24. Four resistance each of R ohms and two resistances each of S ohms are connected (as shown in Fig. 2.50) to four terminasl AB and CD. A p.d. of V volts is applied across the terminals AB and a resistance of Z ohm is connected across the terminals CD. Find the value of Z in terms of S and R in order that the current at AB may be V/Z.

Find also the relationship that must hold between R and S in order that the p.d. at the points EF be $V/2$. **[$Z = \sqrt{R\,(R + 2S)}$; $S = 4R$]**

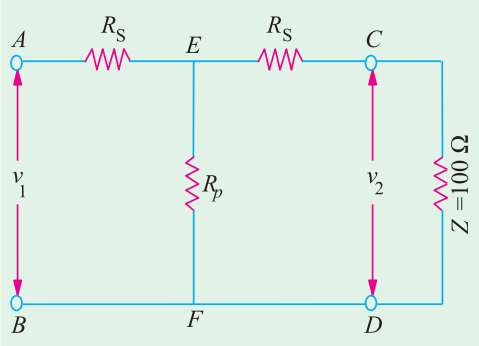


Fig. 2.49

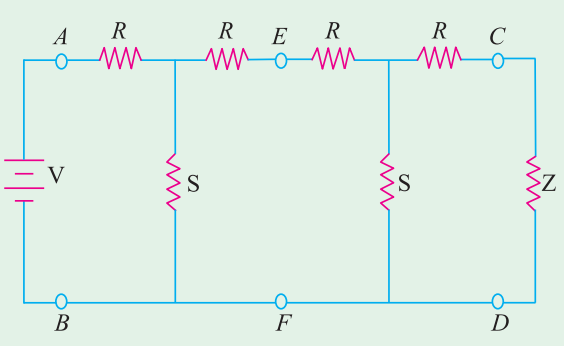


Fig. 2.50

2.10. Maxwell's Loop Curent Method

This method which is particularly well-suited to coupled circuit solutions employs a system of ***loop*** or ***mesh*** currents instead of ***branch*** currents (as in Kirchhoff's laws). Here, the currents in different meshes are assigned continuous paths so that they do not split at a junction into branch currents. This method eliminates a great deal of tedious work involved in the branch-current method and is best suited when energy sources are voltage sources rather than current sources. Basically, this method consists of writing loop voltage equations by Kirchhoff's voltage law in terms of unknown loop currents. As will be seen later, the number of independent equations to be solved reduces from b by Kirchhoff's laws to $b-(j-1)$ for the loop current method where b is the number of branches and j is the number of junctions in a given network.

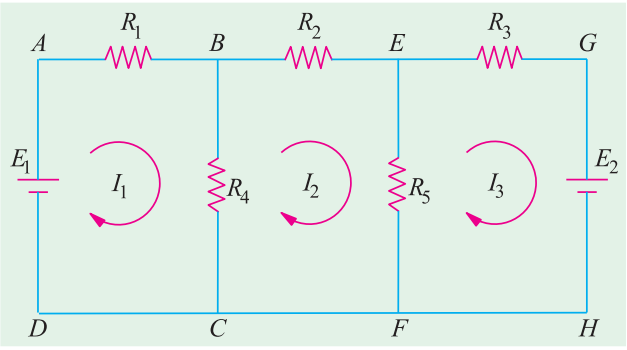


Fig. 2.51

Fig. 2.51 shows two batteries E_1 and E_2 connected in a network consisting of five resistors. Let the loop currents for the three meshes be I_1, I_2 and I_3. It is obvious that current through R_4 (when considered as a part of the first loop) is $(I_1 - I_2)$ and that through R_S is $(I_2 - I_3)$. However, when R_4 is considered part of the second loop, current through it is $(I_2 - I_1)$. Similarly, when R_5 is considered part of the third loop, current through it is $(I_3 - I_2)$. Applying Kirchhoff's voltage law to the three loops, we get,

$$E_1 - I_1R_1 - R_4\,(I_1 - I_2) = 0 \quad \text{or} \quad I_1\,(R_1 + R_4) - I_2\,R_4 - E_1 = 0 \qquad \text{...loop 1}$$

Similarly, $-I_2R_2 - R_5 (I_2 - I_3) - R_4 (I_2 - I_1) = 0$

or $I_2 R_4 - I_2 (R_2 + R_4 + R_5) + I_3R_5 = 0$ ...**loop 2**

Also $-I_3R_3 - E_2 - R_5 (I_3 - I_2) = 0$ or $I_2R_5 - I_3 (R_3 + R_5) - E_2 = 0$ ...**loop 3**

The above three equations can be solved not only to find loop currents but branch currents as well.

2.11. Mesh Analysis Using Matrix Form

Consider the network of Fig. 2.52, which contains resistances and independent voltage sources and has three meshes. Let the three mesh currents be designated as I_1, I_2 and I_3 and all the three may be assumed to flow in the clockwise direction for obtaining symmetry in mesh equations.

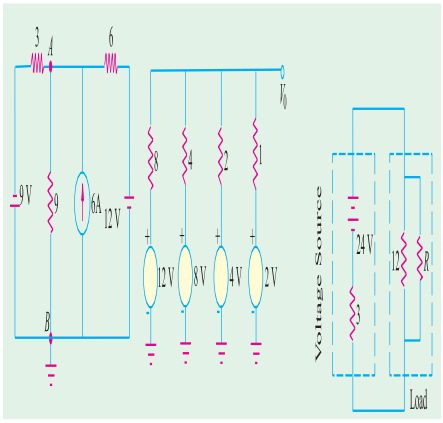


Fig. 2.52

Applying *KVL* to mesh **(*i*)**, we have

$E_1 - I_1R_1 - R_3 (I_1 - I_3) - R_2 (I_1 - I_2) = 0$

or $(R_1 + R_2 + R_3) I_1 - R_2I_2 - R_3I_3 = E_1$ ...**(*i*)**

Similarly, from mesh **(*ii*)**, we have

$E_2 - R_2 (I_2 - I_1) - R_5 (I_2 - I_3) - I_2R_4 = 0$

or $-R_2I_1 + (R_2 + R_4 + R_5) I_2 - R_5I_3 = E_2$ ...**(*ii*)**

Applying *KVL* to mesh **(*iii*)**, we have

$E_3 - I_3R_7 - R_5 (I_3 - I_2) - R_3 (I_3 - I_1) - I_3 R_6 = 0$

or $-R_3I_1 - R_5I_2 + (R_3 + R_5 + R_6 + R_7) I_3 = E_3$ ...**(*iii*)**

It should be noted that signs of different items in the above three equations have been so changed as to make the items containing self resistances positive (please see further).

The matrix equivalent of the above three equations is

$$\begin{bmatrix} +(R_1 + R_2 + R_3) & -R_2 & -R_3 \\ -R_2 & +(R_2 + R_4 + R_5) & -R_5 \\ -R_3 & -R_5 & +(R_3 + R_5 + R_6 + R_7) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

It would be seen that the first item is the first row *i.e.* $(R_1 + R_2 + R_3)$ represents the self resistance of mesh **(*i*)** which equals the sum of all resistance in mesh **(*i*)**. Similarly, the second item in the first row represents the mutual resistance between meshes **(*i*)** and **(*ii*)** *i.e.* the sum of the resistances common to mesh **(*i*)** and **(*ii*)**. Similarly, the third item in the first row represents the mutual-resistance of the mesh **(*i*)** and mesh **(*ii*)**.

The item E_1, in general, represents the algebraic sum of the voltages of all the voltage sources acting around mesh **(*i*)**. Similar is the case with E_2 and E_3. The sign of the *e.m.f*'s is the same as discussed in Art. 2.3 *i.e.* while going along the current, if we pass from negative to the positive terminal of a battery, then its *e.m.f.* is taken positive. If it is the other way around, then battery *e.m.f.* is taken negative.

In general, let

R_{11} = self-resistance of mesh **(*i*)**

R_{22} = self-resistance of mesh **(*ii*)** *i.e.* sum of all resistances in mesh **(*ii*)**

R_{33} = Self-resistance of mesh **(*iii*)** *i.e.* sum of all resistances in mesh **(*iii*)**

$R_{12} = R_{21}$ = –[Sum of all the resistances common to meshes **(*i*)** and **(*ii*)**] *

$R_{23} = R_{32}$ = –[Sum of all the resistances common to meshes **(*ii*)** and **(*iii*)**]*

* Although, it is easier to take all loop currents in one direction (Usually clockwise), the choice of direcion for any loop current is arbitrary and may be chosen independently of the direction of the other loop currents.

$R_{31} = R_{13}$ = –[Sum of all the resistances common to meshes ***(i)*** and ***(iii)***] *

Using these symbols, the generalized form of the above matrix equivalent can be written as

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

If there are m independent meshes in any liner network, then the mesh equations can be written in the matrix form as under :

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} & \dots & R_{1m} \\ R_{21} & R_{22} & R_{23} & \dots & R_{2m} \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ R_{31} & R_{32} & R_{33} & \dots & R_{3m} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \dots \\ \dots \\ I_m \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \dots \\ \dots \\ E_m \end{bmatrix}$$

The above equations can be written in a more compact form as $[R_m]\,[I_m] = [E_m]$. It is known as Ohm's law in matrix form.

In the end, it may be pointed out that the directions of mesh currents can be selected arbitrarily. If we assume each mesh current to flow in the clockwise direction, then

(i) All self-resistances will always be postive and ***(ii)*** all mutual resistances will always be negative. We will adapt this sign convention in the solved examples to follow.

The above main advantage of the generalized form of all mesh equations is that they can be easily remembered because of their symmetry. Moreover, for any given network, these can be written by inspection and then solved by the use of determinants. It eliminates the tedium of deriving simultaneous equations.

Example. 2.30. *Write the impedance matrix of the network shown in Fig. 2.53 and find the value of current I_3.* **(Network Analysis A.M.I.E. Sec. B.W. 1980)**

Solution. Different items of the mesh-resistance matrix $[R_m]$ are as under :

$$R_{11} = 1 + 3 + 2 = 6\ \Omega\,;\ R_{22} = 2 + 1 + 4 = 7\ \Omega\,;\ R_{33} = 3 + 2 + 1 = 6\ \Omega\,;$$

$$R_{12} = R_{21} = -2\ \Omega\,;\ R_{23} = R_{32} = -1\ \Omega\,;\ R_{13} = R_{31} = -3\ \Omega\,;$$

$$E_1 = +5\ \text{V}\,;\ E_2 = 0\,;\ E_3 = 0.$$

The mesh equations in the matrix form are

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \text{ or } \begin{bmatrix} 6 & -2 & -3 \\ -2 & 7 & -1 \\ -3 & -1 & 6 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 6 & -2 & -3 \\ -2 & 7 & -1 \\ -3 & -1 & 6 \end{bmatrix} = 6(42-1) + 2(-12-3) - 3(2+21) = 147$$

$$\Delta_3 = \begin{bmatrix} 6 & -2 & 5 \\ -2 & 7 & 0 \\ -3 & -1 & 0 \end{bmatrix} = 6 + 2(5) - 3(-35) = 121$$

$$I_3 = \Delta_3/\Delta = \frac{121}{147} = 0.823\ \text{A}$$

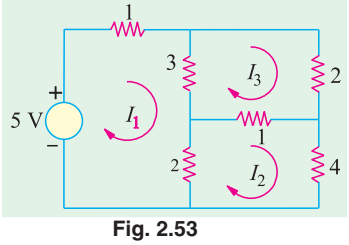


Fig. 2.53

* In general, if the two currents through the common resistance flow in the same direction, then the mutual resistance is taken as negative. One the other hand, if the two currents flow in the same direction, mutual resistance is taken as positive.

Example 2.31. *Determine the current supplied by each battery in the circuit shown in Fig. 2.54.* **(Electrical Engg. Aligarh Univ.)**

Solution. Since there are three meshes, let the three loop currents be shown in Fig. 2.51.

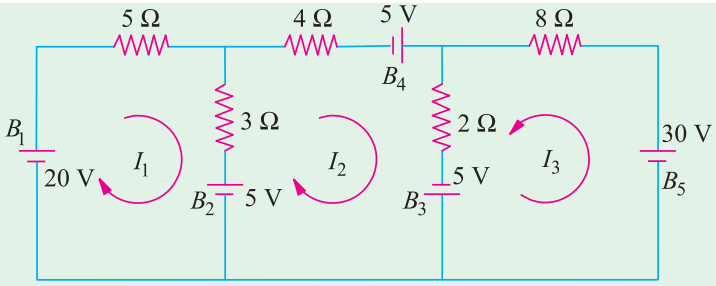


Fig. 2.54

For loop 1 we get

$20 - 5I_1 - 3(I_1 - I_2) - 5 = 0$ or $8I_1 - 3I_2 = 15$ ...(*i*)

For loop 2 we have

$-4I_2 + 5 - 2(I_2 - I_3) + 5 + 5 - 3(I_2 - I_1) = 0$ or $3I_1 - 9I_2 + 2I_3 = -15$ ...(*ii*)

Similarly, for loop 3, we get

$-8I_3 - 30 - 5 - 2(I_3 - I_2) = 0$ or $2I_2 - 10I_3 = 35$ ...(*iii*)

Eliminating I_1 from (*i*) and (*ii*), we get $63I_2 - 16I_3 = 165$ ...(*iv*)

Similarly, for I_2 from (*iii*) and (*iv*), we have $I_2 = 542/299$ A

From (*iv*), $I_3 = -1875/598$ A

Substituting the value of I_2 in (*i*), we get $I_1 = 765/299$ A

Since I_3 turns out to be negative, actual directions of flow of loop currents are as shown in Fig. 2.55.

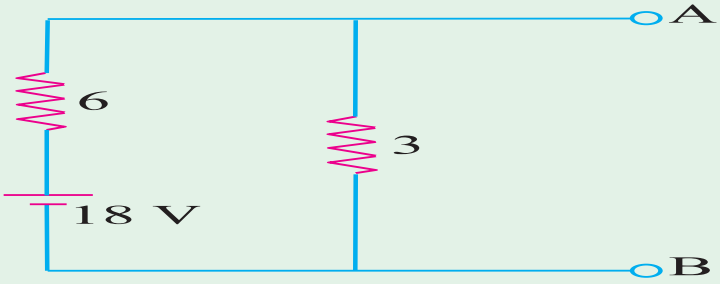


Fig. 2.55

Discharge current of B_1 = **765/299A**

Charging current of B_2 = $I_1 - I_2$ = **220/299 A**

Discharge current of B_3 = $I_2 + I_3$ = **2965/598 A**

Discharge current of B_4 = I_2 = **545/299 A**; Discharge current of B_5 = **1875/598 A**

Solution by Using Mesh Resistance Matrix.

The different items of the mesh-resistance matrix $[R_m]$ are as under :

$R_{11} = 5 + 3 = 8\ \Omega$; $R_{22} = 4 + 2 + 3 = 9\ \Omega$; $R_{33} = 8 + 2 = 10\ \Omega$

$R_{12} = R_{21} = -3\ \Omega$; $R_{13} = R_{31} = 0$; $R_{23} = R_{32} = -2\ \Omega$

E_1 = algebraic sum of the voltages around mesh (i) = $20 - 5 = 15$ V

$E_2 = 5 + 5 + 5 = 15$ V ; $E_3 = -30 - 5 = -35$ V

Hence, the mesh equations in the matrix form are

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \text{ or } \begin{bmatrix} 8 & -3 & 0 \\ -3 & 9 & -2 \\ 0 & -2 & 10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 15 \\ 15 \\ -35 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 8 & -3 & 0 \\ -3 & 9 & -2 \\ 0 & -2 & 10 \end{vmatrix} = 8(90-4) + 3(-30) = 598$$

$$\Delta_1 = \begin{vmatrix} 15 & -3 & 0 \\ 15 & 9 & -2 \\ -35 & -2 & 10 \end{vmatrix} = 15(90-4) - 15(-30) - 35\,(6) = 1530$$

$$\Delta_2 = \begin{vmatrix} 8 & 15 & 0 \\ -3 & 15 & -2 \\ 0 & -35 & 10 \end{vmatrix} = 8(150-70) + 3(150+0) = 1090$$

$$\Delta_3 = \begin{vmatrix} 8 & -3 & 15 \\ -3 & 9 & 15 \\ 0 & -2 & -35 \end{vmatrix} = 8(-315+30) + 3(105+30) = -1875$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{1530}{598} = \frac{765}{299}\text{ A};\ I_2 = \frac{\Delta_2}{\Delta} = \frac{1090}{598} = \frac{545}{299}\text{ A};\ I_3 = \frac{\Delta_3}{\Delta} = \frac{-1875}{598}\text{ A}$$

Example 2.32. *Determine the current in the 4-Ω branch in the circuit shown in Fig. 2.56.*

(Elect. Technology, Nagpur Univ.)

Solution. The three loop currents are as shown in Fig. 2.53 (*b*).

For loop 1, we have

$-1\,(I_1 - I_2) - 3\,(I_1 - I_3) - 4I_1 + 24 = 0$ or $8I_1 - I_2 - 3I_3 = 24$ ...(*i*)

For loop 2, we have

$12 - 2I_2 - 12\,(I_2 - I_3) - 1\,(I_2 - I_1) = 0$ or $I_1 - 15I_2 + 12I_3 = -12$ ...(*ii*)

Similarly, for loop 3, we get

$-12\,(I_3 - I_2) - 2I_3 - 10 - 3\,(I_3 - I_1) = 0$ or $3I_1 + 12I_2 - 17I_3 = 10$ ...(*iii*)

Eliminating I_2 from Eq. (*i*) and (*ii*) above, we get, $119I_1 - 57I_3 = 372$ ...(*iv*)

Similarly, eliminating I_2 from Eq. (*ii*) and (*iii*), we get, $57I_1 - 111I_3 = 6$ ...(*v*)

From (*iv*) and (*v*) we have,

$$I_1 = 40{,}950/9{,}960 = \mathbf{4.1\ A}$$

Solution by Determinants

The three equations as found above are

$$\begin{aligned} 8I_1 - I_2 - 3I_3 &= 24 \\ I_1 - 15I_2 + 12I_3 &= -12 \\ 3I_1 + 12I_2 - 17I_3 &= 10 \end{aligned}$$

Their matrix form is $\begin{bmatrix} 8 & -1 & -3 \\ 1 & -15 & 12 \\ 3 & 12 & -17 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 24 \\ -12 \\ 10 \end{bmatrix}$

$$\Delta = \begin{bmatrix} 8 & -1 & -3 \\ 1 & -15 & 12 \\ 3 & 12 & -17 \end{bmatrix} = 664,\quad \Delta_1 = \begin{bmatrix} 24 & -1 & -3 \\ -12 & -15 & 12 \\ 10 & 12 & -17 \end{bmatrix} = 2730$$

$\therefore\quad I_1 = \Delta_1/\Delta = 2730/664 = \mathbf{4.1\ A}$

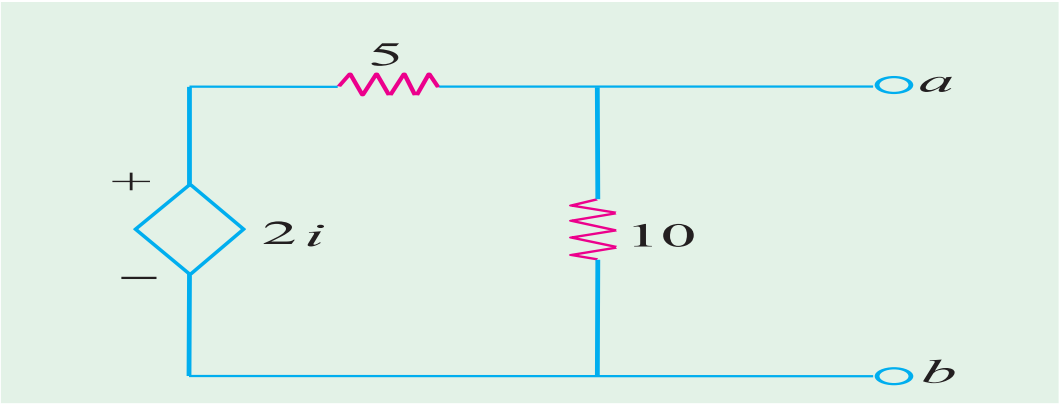


Fig. 2.56

Solution by Using Mesh Resistance Matrix

For the network of Fig. 2.53 (*b*), values of self resistances, mutual resistances and e.m.f's can be written by more inspection of Fig. 2.53.

$R_{11} = 3 + 1 + 4 = 8\ \Omega$; $R_{22} = 2 + 12 + 1 = 15\ \Omega$; $R_{33} = 2 + 3 + 12 = 17\ \Omega$

$R_{12} = R_{21} = -1$; $R_{23} = R_{32} = -12$; $R_{13} = R_{31} = -3$

$E_1 = 24$ V ; $E_2 = 12$ V ; $E_3 = -10$ V

The matrix form of the above three equations can be written by inspection of the given network as under :-

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \text{ or } \begin{bmatrix} 8 & -1 & -3 \\ -1 & 15 & -12 \\ -3 & -12 & 17 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 24 \\ 12 \\ -10 \end{bmatrix}$$

$$\Delta = 8\,(255 - 144) + 1(-17 - 36) - 3\,(12 + 45) = 664$$

$$\Delta_1 = \begin{bmatrix} 24 & -1 & -3 \\ 12 & 15 & -12 \\ -10 & -12 & 17 \end{bmatrix} = 24\,(255 - 144) - 12(-17 - 36) - 10(12 + 45) = 2730$$

$$\therefore \quad I_1 = \frac{\Delta_1}{\Delta} = \frac{2730}{664} = \mathbf{4.1\ A}$$

It is the same answer as found above.

Tutorial Problems No. 2.2

1. Find the ammeter current in Fig. 2.57 by using loop analysis.

[1/7 A] (*Network Theory Indore Univ. 1981*)

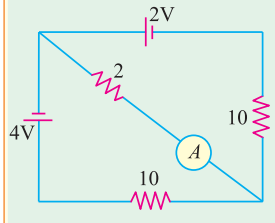


Fig. 2.57

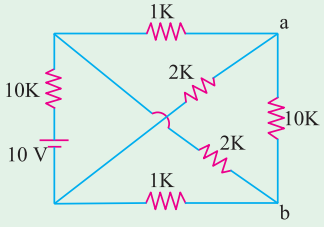


Fig. 2.58

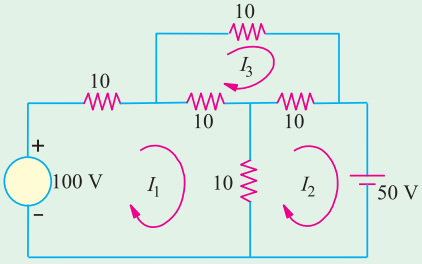


Fig. 2.59

2. Using mesh analysis, determine the voltage across the 10 kΩ resistor at terminals *a-b* of the circuit shown in Fig. 2.58. **[2.65 V] (*Elect. Technology, Indore Univ.*)**

3. Apply loop current method to find loop currents I_1, I_2 and I_3 in the circuit of Fig. 2.59.

[I_1 = 3.75 A, I_2 = 0, I_3 = 1.25 A]

2.12. Nodal Analysis With Sources

The node-equation method is based directly on Kirchhoff's current law unlike loop-current method which is based on Kirchhoff's voltage law. However, like loop current method, nodal method also has the advantage that a minimum number of equations need be written to determine the unknown quantities. Moreover, it is particularly suited for networks having many parallel circuits with common ground connected such as electronic circuits.

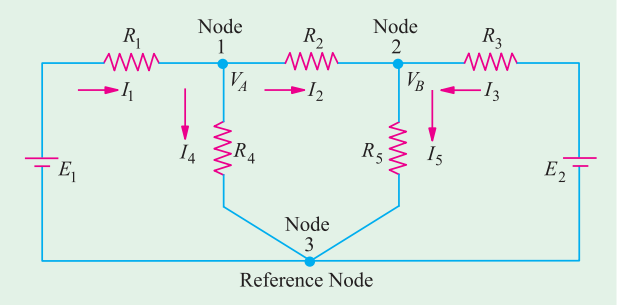


Fig. 2.60

For the application of this method, every junction in the network where three or more branches meet is regarded a node. One of these is regarded as the reference node or datum node or zero-potential node. Hence the number of simultaneous equations to be solved becomes $(n-1)$ where n is the number of independent nodes. These node equations often become simplified if all voltage sources are converted into current sources (Art. 2.12).

(*i*) First Case

Consider the circuit of Fig. 2.60 which has three nodes. One of these *i.e.* node 3 has been taken in as the reference node. V_A represents the potential of node 1 with reference to the datum node 3. Similarly, V_B is the potential difference between node 2 and node 3. Let the current directions which have been chosen arbitrary be as shown.

For node 1, the following current equation can be written with the help of *KCL*.

$$I_1 = I_4 + I_2$$

Now $I_1R_1 = E_1 - V_A \quad \therefore\ I_1 = (E_1 - V_A)/R_1$ ...(*i*)

Obviously, $I_4 = V_A/R_4$ Also, $I_2R_2 = V_A - V_B \quad (\because\ V_A > V_B)$

$\therefore\quad I_2 = (V_A - V_B)/R_2$

Substituting these values in Eq. (*i*) above, we get,

$$\frac{E_1 - V_A}{R_1} = \frac{V_A}{R_4} + \frac{V_A - V_B}{R_2}$$

Simplifying the above, we have

$$V_A\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4}\right) - \frac{V_B}{R_2} - \frac{E_1}{R_1} = 0 \qquad \text{...(ii)}$$

The current equation for node 2 is $I_5 = I_2 + I_3$

or $$\frac{V_B}{R_5} = \frac{V_A - V_B}{R_2} + \frac{E_2 - V_B}{R_3} \qquad \text{...(iii)}$$

or $$V_B\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5}\right) - \frac{V_A}{R_2} - \frac{E_2}{R_3} = 0 \qquad \text{...(iv)}$$

Though the above nodal equations (*ii*) and (*iii*) seem to be complicated, they employ a very simple and systematic arrangement of terms which can be written simply by inspection. Eq. (*ii*) at node 1 is represented by

1. The product of node potential V_A and $(1/R_1 + 1/R_2 + 1/R_4)$ *i.e.* the sum of the reciprocals of the branch resistance connected to this node.
2. ***Minus*** the ratio of adjacent potential V_B and the interconnecting resistance R_2.
3. ***Minus*** ratio of adjacent battery (or generator) voltage E_1 and interconnecting resistance R_1.
4. All the above set to zero.

Same is the case with Eq. ***(iii)*** which applies to node 2.

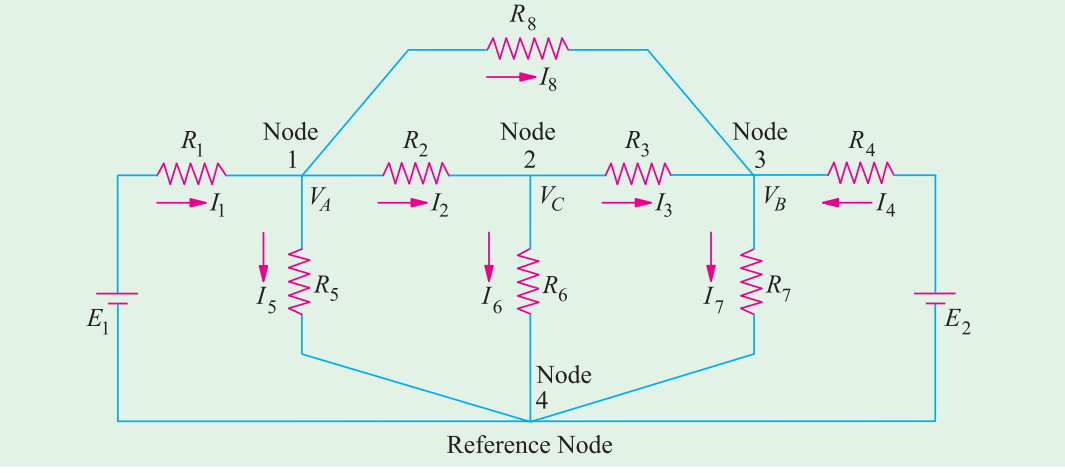


Fig. 2.61

Using conductances instead of resistances, the above two equations may be written as

$$V_A (G_1 + G_2 + G_4) - V_B G_2 - E_1 G_1 = 0 \quad \text{...(iv)}$$

$$V_B (G_2 + G_3 + G_5) - V_A G_2 - E_2 G_3 = 0 \quad \text{...(v)}$$

To emphasize the procedure given above, consider the circuit of Fig. 2.61.

The three node equations are

$$V_A\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_5}+\frac{1}{R_8}\right)-\frac{V_C}{R_2}-\frac{V_B}{R_8}-\frac{E_1}{R_1}=0 \quad \text{(node 1)}$$

$$V_C\left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_6}\right)-\frac{V_A}{R_2}-\frac{V_B}{R_3}=0 \quad \text{(node 2)}$$

$$V_B\left(\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_7}+\frac{1}{R_8}\right)-\frac{V_C}{R_3}-\frac{V_A}{R_8}-\frac{E_4}{R_4}=0 \quad \text{(node 3)}$$

After finding different node voltages, various currents can be calculated by using Ohm's law.

(ii) Second Case

Now, consider the case when a third battery of e.m.f. E_3 is connected between nodes 1 and 2 as shown in Fig. 2.62.

It must be noted that as we travel from node 1 to node 2, we go from the -ve terminal of E_3 to its +ve terminal. Hence, according to the sign convention given in Art. 2.3, E_3 must be taken as *positive*. However, if we travel from node 2 to node 1, we go from the +ve to the -ve terminal

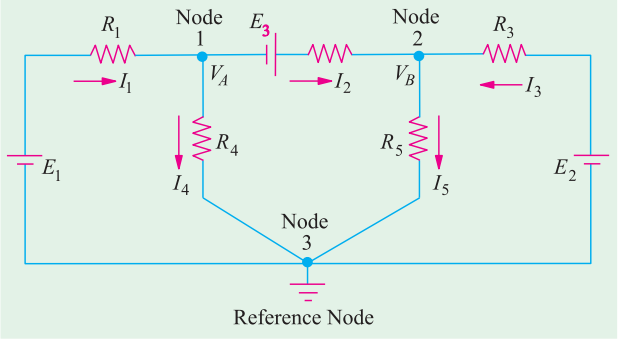


Fig. 2.62

of E_3. Hence, ***when viewed from node 2, E_3 is taken negative.***

For node 1

$$I_1 - I_4 - I_2 = 0 \text{ or } I_1 = I_4 + I_1 \text{ –as per KCL}$$

Now, $$I_1 = \frac{E_1 - V_A}{R_1};\ I_2 = \frac{V_A + E_3 - V_B}{R_2};\ I_4 = \frac{V_A}{R_4}$$

$\therefore$ $$\frac{E_1 - V_A}{R_1} = \frac{V_A}{R_4} + \frac{V_A + E_3 - V_B}{R_2}$$

or $$V_A\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) - \frac{E_1}{R_1} - \frac{V_B}{R_2} + \frac{E_2}{R_2} = 0 \qquad \text{...(i)}$$

It is exactly the same expression as given under the First Case discussed above except for the additional term involving E_3. This additional term is taken as $+E_3/R_2$ (and not as $-E_3/R_2$) because this third battery is so connected that when viewed from mode 1, it represents a *rise* in voltage. Had it been connected the other way around, the additional term would have been taken as $-E_3/R_2$.

For node 2

$$I_2 + I_3 - I_5 = 0 \quad \text{or} \quad I_2 + I_3 = I_5 \quad \text{–as per } KCL$$

Now, as before, $$I_2 = \frac{V_A + E_3 - V_B}{R_2},\ I_3 = \frac{E_2 - V_B}{R_3},\ I_5 = \frac{V_B}{R_5}$$

$\therefore$ $$\frac{V_A + E_3 - V_B}{R_2} + \frac{E_2 - V_B}{R_3} = \frac{V_B}{R_5}$$

On simplifying, we get $$V_B\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5}\right) - \frac{E_2}{R_3} - \frac{V_A}{R_2} - \frac{E_3}{R_2} = 0 \qquad \text{...(ii)}$$

As seen, the additional terms is $-E_3/R_2$ (and not $+E_3/R_2$) because as viewed from this node, E_3 represents a ***fall*** in potential.

It is worth repeating that the additional term in the above Eq. **(*i*)** and **(*ii*)** can be either $+E_3/R_2$ or $-E_3/R_2$ depending on whether it represents a rise or fall of potential when viewed from ***the node under consideration.***

Example 2.33. *Using Node voltage method, find the current in the 3Ω resistance for the network shown in Fig. 2.63.* **(Elect. Tech. Osmania Univ.)**

Solution. As shown in the figure node 2 has been taken as the reference node. We will now find the value of node voltage V_1. Using the technique developed in Art. 2.10, we get

$$V_1\left(\frac{1}{5} + \frac{1}{2} + \frac{1}{2}\right) - \frac{4}{2} - \left(\frac{4+2}{5}\right) = 0$$

The reason for adding the two battery voltages of 2 V and 4 V is because they are connected in additive series. Simplifying above, we get V_1 = 8/3 V. The current flowing through the 3 Ω resistance towards node 1 is = $\frac{6 - (8/3)}{(3+2)} = \frac{2}{3}$ A

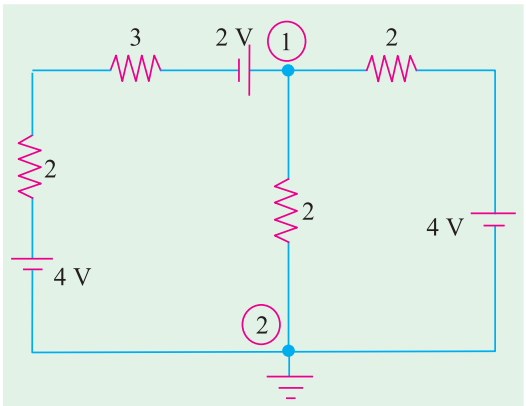


Fig. 2.63

Alternatively

$$\frac{6 - V_1}{5} + \frac{4}{2} - \frac{V_1}{2} = 0$$

$$12 - 2V_1 + 20 - 5V_1 = 0$$

$$7V_1 = 32$$

Also $$\frac{6-V_1}{5}+\frac{4-V_1}{2} = \frac{V_1}{2}$$

$$12 - 2V_1 + 20 - 5V_1 = 5V_1$$

$$12V_1 = 32;\ V_1 = 8/3$$

Example 2.34. *Frame and solve the node equations of the network of Fig. 2.64. Hence, find the total power consumed by the passive elements of the network.* **(Elect. Circuits Nagpur Univ.)**

Solution. The node equation for node 1 is

$$V_1\left(1+1+\frac{1}{0.5}\right)-\frac{V_2}{0.5}-\frac{15}{1}=0$$

or $4V_1 - 2V_2 = 15$ ...(*i*)

Similarly, for node 2, we have

$$V_1\left(1+\frac{1}{2}+\frac{1}{0.5}\right)-\frac{V_2}{0.5}-\frac{20}{1}=0$$

or $4V_1 - 7V_2 = -40$ ...(*ii*)

$\therefore$ $V_2 = 11$ volt and $V_1 = 37/4$ volt

Now,

$$I_1 = \frac{15-37/4}{I} = \frac{23}{4}\text{ A} = 5.75\text{ A};\ I_2 = \frac{11-37/4}{0.5} = 3.5\text{ A}$$

$$I_4 = 5.75 + 3.5 = 9.25\text{ A};\ I_3 = \frac{20-11}{1} = 9\text{ A};\ I_5 = 9 - 3.5 = 5.5\text{ A}$$

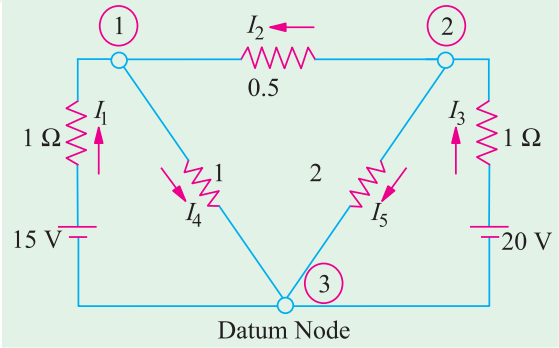


Fig. 2.64

The passive elements of the network are its five resistances. Total power consumed by them is $= 5.75^2 \times 1 + 3.5^2 \times 0.5 + 9^2 \times 1 + 9.25^2 \times 1 + 5.5^2 \times 2 =$ **266.25**

Example 2.35. *Find the branch currents in the circuit of Fig. 2.65 by using* ***(i)*** *nodal analysis and* ***(ii)*** *loop analysis.*

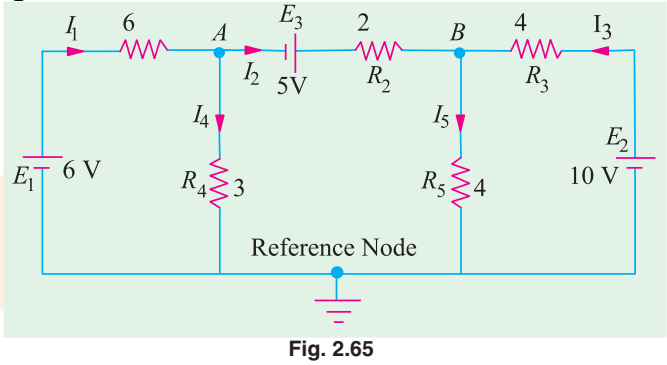


Fig. 2.65

Solution. *(i)* Nodal Method

The equation for node *A* can be written by inspection as explained in Art. 2-12.

$$V_A\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_4}\right)-\frac{E_1}{R_1}-\frac{V_B}{R_2}+\frac{E_3}{R_2}=0$$

Substituting the given data, we get,

$$V_A\left(\frac{1}{6}+\frac{1}{2}+\frac{1}{3}\right)-\frac{6}{6}-\frac{V_B}{2}+\frac{5}{2}=0 \quad \text{or } 2V_A - V_B = -3 \qquad \text{...(}i\text{)}$$

For node *B*, the equation becomes

$$V_B\left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_5}\right)-\frac{E_2}{R_3}-\frac{V_A}{R_2}-\frac{E_3}{R_2}=0$$

$\therefore$ $$V_B\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{4}\right)-\frac{10}{4}-\frac{V_A}{2}-\frac{5}{2}=0 \quad \therefore \quad V_B-\frac{V_A}{2}=5 \qquad \text{...(}ii\text{)}$$

From Eq. ***(i)*** and ***(ii)***, we get,

$V_A = \frac{4}{3} V, V_B = \frac{17}{3} V$

$I_1 = \frac{E_1 \quad V_A}{R_1} \quad \frac{6 \quad 4/3}{6} \quad \frac{7}{9}$ **A**

$I_2 = \frac{V_A \quad E_3 \quad V_B}{R_2} \quad \frac{(4/3) \quad 5 \quad (17/3)}{2} \quad \frac{1}{3}$ **A**

$I_3 = \frac{E_2 \quad V_B}{R_3} \quad \frac{10 \quad 17/3}{4} \quad \frac{13}{12}$ **A**

$I_4 = \frac{V_A}{R_4} \quad \frac{4/3}{3} \quad \frac{4}{9}$ **A,** $I_5 \quad \frac{V_B}{R_5} \quad \frac{17/3}{4} \quad \frac{17}{12}$ **A**

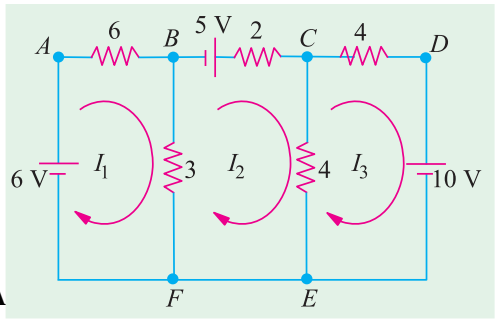


Fig. 2.66

(*ii*) Loop Current Method

Let the direction of flow of the three loop currents be as shown in Fig. 2.66.

Loop ABFA :

$$-6I_1 - 3(I_1 - I_2) + 6 = 0$$

or $$3I_1 - I_2 = 2 \quad ...(i)$$

Loop BCEFB :

$$+ 5 - 2I_2 - 4(I_2 - I_3) - 3 (I_2 - I_1) = 0$$

or $$3I_1 - 9I_2 + 4I_3 = - 5 \quad ...(ii)$$

Loop CDEC :

$$-4I_3 - 10 - 4 (I_3 - I_2) = 0 \quad \text{or} \quad 2I_2 - 4I_3 = 5 \quad ...(iii)$$

The matrix form of the above three simultaneous equations is

$$\begin{bmatrix} 3 & -1 & 0 \\ 3 & -9 & 4 \\ 0 & 2 & -4 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -5 \\ 5 \end{bmatrix}; \Delta = \begin{bmatrix} 3 & -1 & 0 \\ 3 & -9 & 4 \\ 0 & 2 & -4 \end{bmatrix} = 84 - 12 - 0 = 72$$

$$\Delta_1 = \begin{vmatrix} 2 & 1 & 0 \\ -5 & -9 & 4 \\ 5 & 2 & -4 \end{vmatrix} = 56; \Delta_2 = \begin{vmatrix} 3 & 2 & 0 \\ 3 & -5 & 4 \\ 0 & 5 & -4 \end{vmatrix} = 24; \Delta_3 = \begin{vmatrix} 3 & -1 & 2 \\ 3 & -9 & -5 \\ 0 & 2 & 5 \end{vmatrix} = -78$$

$\therefore \quad I_1 = \Delta_1/\Delta = 56/72 = 7/9$ A; $I_2 = \Delta_2/\Delta = 24/72 = 1/3$ A

$I_3 = \Delta_3/\Delta = -78/72 =$ **−13/12 A**

The negative sign of I_3 shows that it is flowing in a direction opposite to that shown in Fig. 2.64 *i.e.* it flows in the CCW direction. The actual directions are as shown in Fig. 2.67.

The various branch currents are as under :

$I_{AB} \quad I_1 \quad$ **7/9 A;** $I_{BF} \quad I_1 \quad I_2 \quad \frac{7}{9} \quad \frac{1}{3} \quad \frac{4}{9}$ **A**

$I_{BC} \quad I_2 \quad \frac{1}{3}$ **A;** $I_{CE} \quad I_2 \quad I_3 \quad \frac{1}{3} \quad \frac{13}{12} \quad \frac{17}{12}$ **A**

$I_{DC} \quad I_3 \quad \frac{13}{12}$ **A**

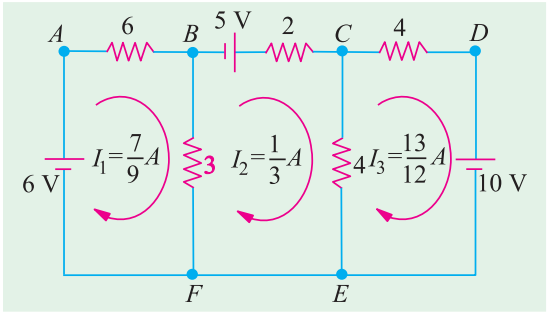


Fig. 2.67

Solution by Using Mesh Resistance Matrix

From inspection of Fig. 2.67, we have

$R_{11} = 9; \; R_{22} = 9; R_{33} = 8$

$R_{12} = R_{21} = -3 \; \Omega ; R_{23} = R_{32} = -4 \; \Omega ; R_{13} = R_{31} = 0 \; \Omega$

$E_1 = 6$ V : $E_2 = 5$ V; $E_3 = -10$ V

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \text{ or } \begin{bmatrix} 9 & -3 & 0 \\ -3 & 9 & -4 \\ 0 & -4 & 8 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \\ -10 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 9 & -3 & 0 \\ -3 & 9 & -4 \\ 0 & -4 & 8 \end{vmatrix} = 9(72-16) + 3(-24) = 432$$

$$\Delta_1 = \begin{vmatrix} 6 & -3 & 0 \\ 5 & 9 & -4 \\ -10 & -4 & 8 \end{vmatrix} = 6(72-16) - 5(-24) - 10(12) = 336$$

$$\Delta_2 = \begin{vmatrix} 9 & 6 & 0 \\ -3 & 5 & -4 \\ 0 & -10 & 8 \end{vmatrix} = 9(40-40) + 3(48) = 144$$

$$\Delta_3 = \begin{vmatrix} 9 & -3 & 6 \\ -3 & 9 & 5 \\ 0 & -4 & -10 \end{vmatrix} = 9(-90+90) - 3(30+24) = -468$$

$$I_1 = \Delta_1/\Delta = 336/432 = 7/9 \text{ A}$$

$$I_2 = \Delta_2/\Delta = 144/432 = 1/3 \text{ A}$$

$$I_3 = \Delta_3/\Delta = -468/432 = -13/12 \text{ A}$$

These are the same values as found above.

2.13. Nodal Analysis with Current Sources

Consider the network of Fig. 2.68 (*a*) which has two current sources and three nodes out of which 1 and 2 are independent ones whereas No. 3 is the reference node.

The given circuit has been redrawn for ease of understanding and is shown in Fig. 2.68 (*b*). The current directions have been taken on the assumption that

1. both V_1 and V_2 are positive with respect to the reference node. That is why their respective curents flow from nodes 1 and 2 to node 3.
2. V_1 is positive with respect to V_2 because current has been shown flowing from node 1 to node 2.

A positive result will confirm out assumption whereas a negative one will indicate that actual direction is opposite to that assumed.

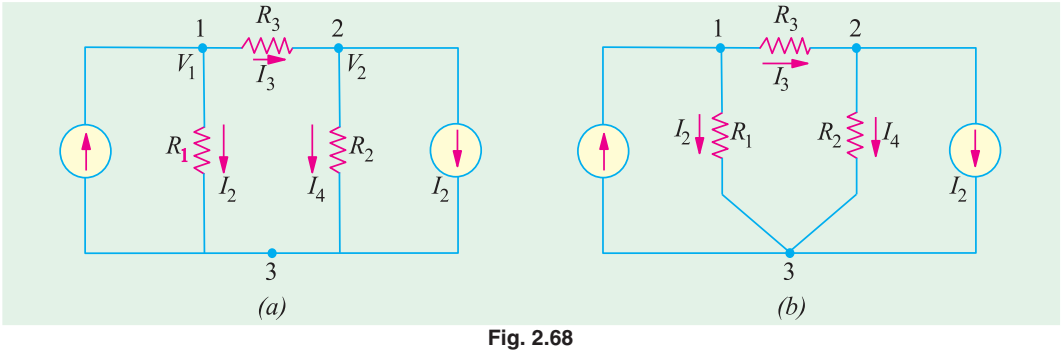


Fig. 2.68

We will now apply *KCL* to each node and use Ohm's law to express branch currents in terms of node voltages and resistances.

Node 1

$$I_1 - I_2 - I_3 = 0 \quad \text{or} \quad I_1 = I_2 + I_3$$

Now $I_2 = \frac{V_1}{R_1}$ and $I_3 = \frac{V_1 - V_2}{R_3}$

$\therefore$ $I_1 = \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_3}$ or $V_1\left(\frac{1}{R_1} + \frac{1}{R_3}\right) - \frac{V_2}{R_3} = I_1$ ...(*i*)

Node 2

$I_3 - I_2 - I_4 = 0$ or $I_3 = I_2 + I_4$

Now, $I_4 = \frac{V_2}{R_2}$ and $I_3 = \frac{V_1 - V_2}{-R_3}$ –as before

$\therefore$ $\frac{V_1 - V_2}{R_3} = I_2 + \frac{V_2}{R_2}$ or $V_2\left(\frac{1}{R_2} + \frac{1}{R_3}\right) - \frac{V_1}{R_3} = -I_1$ ...(*ii*)

The above two equations can also be written by simple inspection. For example, Eq. (*i*) is represented by

1. ***product*** of potential V_1 and $(1/R_1 + 1/R_3)$ *i.e.* sum of the reciprocals of the branch resistances connected to this node.

2. ***minus*** the ratio of adjoining potential V_2 and the interconnecting resistance R_3.

3. all the above equated to the current supplied by the current source connected to this node.

This current is taken *positive* if flowing *into* the node and negative if flowing *out* of it (as per sign convention of Art. 2.3). Same remarks apply to Eq. (*ii*) where I_2 has been taken negative because it flows *away* from node 2.

In terms of branch conductances, the above two equations can be put as

$V_1 (G_1 + G_3) - V_2 G_3 = I_1$ and $V_2 (G_2 + G_3) - V_1 G_3 = -I_2$

Example 2.36. *Use nodal analysis method to find currents in the various resistors of the circuit shown in Fig. 2.69 (a).*

Solution. The given circuit is redrawn in Fig. 2.66 (*b*) with its different nodes marked 1, 2, 3 and 4, the last one being taken as the reference or datum node. The different node-voltage equations are as under :

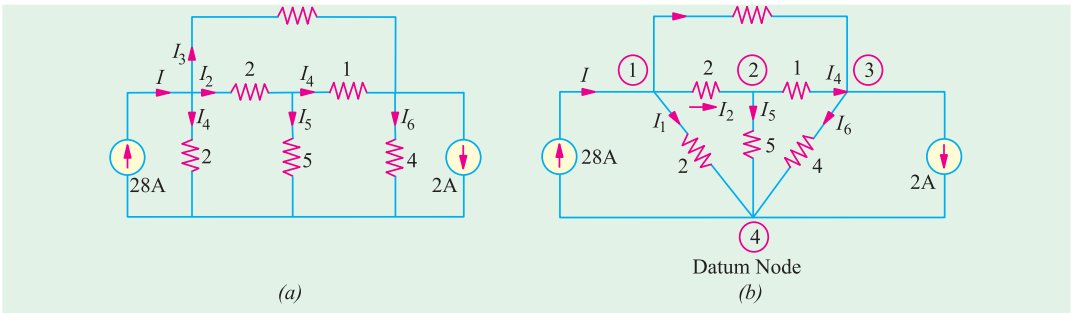


Fig. 2.69

Node 1 $V_1\left(\frac{1}{2} + \frac{1}{2} + \frac{1}{10}\right) - \frac{V_2}{2} - \frac{V_3}{10} = 8$

or $11V_1 - 5V_2 - V_3 - 280 = 0$ ...(*i*)

Node 2 $V_2\left(\frac{1}{2} + \frac{1}{5} + 1\right) - \frac{V_1}{2} - \frac{V_3}{1} = 0$

or $5V_1 - 17\,V_2 + 10\,V_3 = 0$ ...(*ii*)

Node 3 $V_3\left(\frac{1}{4} + 1 + \frac{1}{10}\right) - \frac{V_2}{1} - \frac{V_1}{10} = -2$

or $V_1 + 10\,V_2 - 13.5\,V_3 - 20 = 0$ ...(*iii*)

The matrix form of the above three equations is

$$\begin{bmatrix} 11 & -5 & -1 \\ 5 & -17 & 10 \\ 1 & 10 & -13.5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 280 \\ 0 \\ 20 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 11 & -5 & -1 \\ 5 & -17 & 10 \\ 1 & 10 & -13.5 \end{vmatrix} = 1424.5 - 387.5 - 67 = 970$$

$$\Delta_1 = \begin{vmatrix} 280 & -5 & -1 \\ 0 & -17 & 10 \\ 20 & 10 & -13.5 \end{vmatrix} = 34,920, \quad \Delta_2 = \begin{vmatrix} 11 & 280 & -1 \\ 5 & 0 & 10 \\ 1 & 20 & -13.5 \end{vmatrix} = 19,400$$

$$\Delta_3 = \begin{vmatrix} 11 & -5 & 280 \\ 5 & -17 & 0 \\ 1 & 10 & 20 \end{vmatrix} = 15,520$$

$$V_1 = \frac{\Delta_1}{\Delta} = \frac{34,920}{970} = 36 \text{ V}, V_2 = \frac{\Delta_2}{\Delta} = \frac{19,400}{970} = 20 \text{ V}, V_3 = \frac{\Delta_3}{\Delta} = \frac{15,520}{970} = 16 \text{ V}$$

It is obvious that all nodes are at a higher potential with respect to the datum node. The various currents shown in Fig. 2.69 (*b*) can now be found easily.

$$I_1 = V_1/2 = 36/2 = \mathbf{18\ A}$$
$$I_2 = (V_1 - V_2)/2 = (36 - 20)/2 = \mathbf{8\ A}$$
$$I_3 = (V_1 - V_3)/10 = (36 - 16)/10 = \mathbf{2\ A}$$

It is seen that total current, as expected, is 18 + 8 + 2 = **28 A**

$$I_4 = (V_2 - V_3)/1 = (20 - 16)/1 = \mathbf{4\ A}$$
$$I_5 = V_2/5 = 20/5 = \mathbf{4\ A}, I_6 = V_3/4 = 16/4 = \mathbf{4\ A}$$

Example 2.37. *Using nodal analysis, find the different branch currents in the circuit of Fig. 2.70 (a). All branch conductances are in siemens (i.e. mho).*

Solution. Let the various branch currents be as shown in Fig. 2.70 (*b*). Using the procedure detailed in Art. 2.11, we have

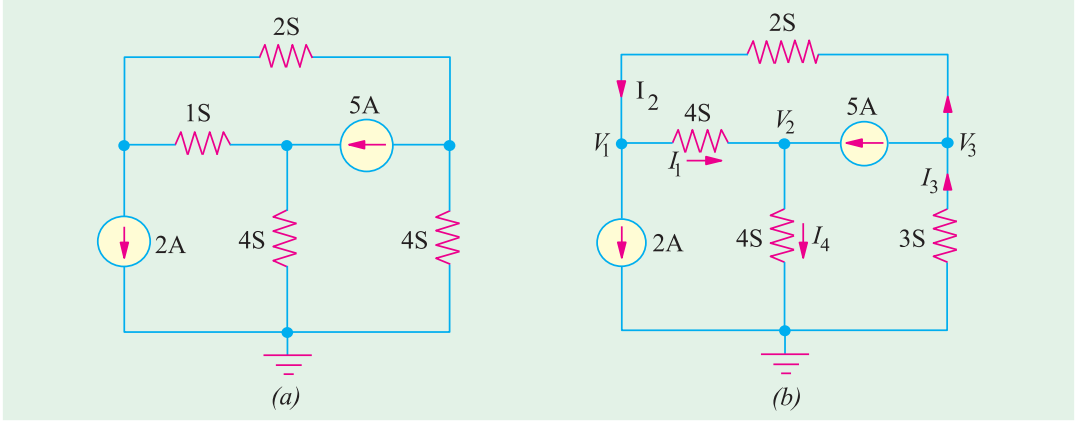


Fig. 2.70

First Node

$V_1 (1 + 2) - V_2 \times 1 - V_3 \times 2 = -2$ or $3V_1 - V_2 - 2V_3 = -2$ ...(*i*)

Second Node

$V_2 (1 + 4) - V_1 \times 1 = 5$ or $V_1 - 5V_2 = -5$ ...(*ii*)

Third Node

$V_3 (2 + 3) - V_1 \times 2 = -5$ or $2V_1 - 5V_3 = 5$ ...(*iii*)

Solving for the different voltages, we have

$$V_1 = -\frac{3}{2}\text{ V}, V_2 = \frac{7}{10}\text{ V and } V_3 = -\frac{8}{5}\text{ V}$$

$$I_1 = (V_1 - V_2) \times 1 = (-1.5 - 0.7) \times 1 = \mathbf{-2.2\ A}$$

$$I_2 = (V_3 - V_1) \times 2 = [-1.6 - (-1.5)] \times 2 = \mathbf{-0.2\ A}$$

$$I_4 = V_2 \times 4 = 4 \times (7/10) = \mathbf{2.8\ A}$$

$$I_3 = 2 + 2.8 = \mathbf{4.8\ A}$$

As seen, I_1 and I_2 flow in directions opposite to those originally assumed (Fig. 2.71).

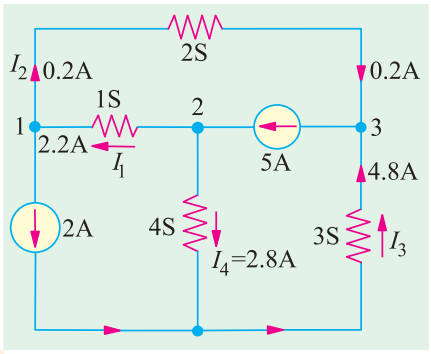


Fig. 2.71

Example 2.38. *Find the current I in Fig. 2.72 (a) by changing the two voltage sources into their equivalent current sources and then using Nodal method. All resistances are in ohms.*

Solution. The two voltage sources have been converted into their equivalent current sources in Fig. 2.72 (*b*). The circuit has been redrawn as shown in Fig. 2.72 (*c*) where node No. 4 has been

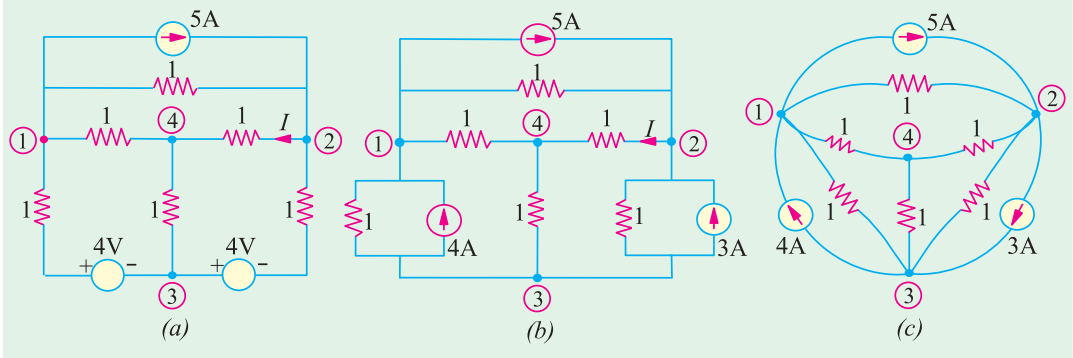


Fig. 2.72

taken as the reference node or common ground for all other nodes. We will apply *KCL* to the three nodes and taken currents coming towards the nodes as positive and those going away from them as negative. For example, current going away from node No. 1 is $(V_1 - V_2)/1$ and hence would be taken as negative. Since 4 A current is coming towards node No. 1, it would be taken as positive but 5 A current would be taken as negative.

Node 1 : $$-\frac{(V_1 - 0)}{1} - \frac{(V_1 - V_2)}{1} - \frac{(V_1 - V_3)}{1} - 5 + 4 = 0$$

or $$3V_1 - V_2 - V_3 = -1 \qquad \text{...(i)}$$

Node 2 : $$-\frac{(V_2 - 0)}{1} - \frac{(V_2 - V_3)}{1} - \frac{(V_2 - V_1)}{1} + 5 - 3 = 0$$

or $$V_1 - 3V_2 + V_3 = -2 \qquad \text{...(ii)}$$

Node 3 : $$-\frac{(V_3 - 0)}{1} - \frac{(V_3 - V_1)}{1} - \frac{(V_3 - V_2)}{1} - 4 + 3 = 0$$

or $$V_1 + V_2 - 3V_3 = 1 \qquad \text{...(iii)}$$

The matrix form of the above three equations is

$$\begin{bmatrix} 3 & -1 & -1 \\ 1 & -3 & 1 \\ 1 & 1 & -3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 3 & -1 & -1 \\ 1 & -3 & 1 \\ 1 & 1 & -3 \end{vmatrix} = 3(9-1) - 1(3+1) + 1(-1-3) = 16$$

$$\Delta_2 = \begin{vmatrix} 3 & -1 & -1 \\ 1 & -2 & 1 \\ 1 & 1 & -3 \end{vmatrix} = 3(6-1) - 1(3+1) + 1(-1-2) = 8$$

$\therefore \quad V_2 = \Delta_2/\Delta = 8/16 = 0.5$ V

$\therefore \quad I = V_2/1 = 0.5$ A

Example 2.39. *Use Nodal analysis to determine the value of current i in the network of Fig. 2.73.*

Solution. We will apply *KCL* to the two nodes 1 and 2. Equating the incoming currents at node 1 to the outgoing currents, we have

$$6 = \frac{V_1 - V_2}{4} + \frac{V_1}{8} + 3i$$

As seen. $i = V_1/8$. Hence, the above equation becomes

$$6 = \frac{V_1 - V_2}{4} + \frac{V_1}{8} + 3\frac{V_1}{8}$$

or $3V_1 - V_2 = 24$

Similarly, applying *KCL* to node No. 2, we get

$$\frac{V_1 - V_2}{4} + 3i = \frac{V_2}{6} \text{ or } \frac{V_1 - V_2}{4} + 3\frac{V_1}{8} = \frac{V_2}{6} \text{ or } 3V_1 = 2V_2$$

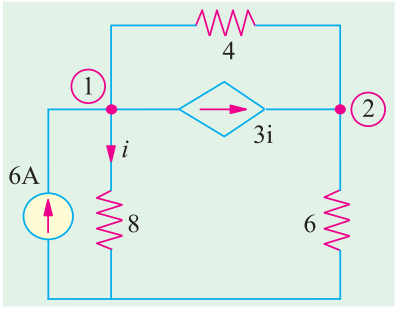


Fig. 2.73

From the above two equations, we get

$V_1 = 16$ V $\quad \therefore \; i = 16/8 =$ **2 A.**

Example 2.40. *Using Nodal analysis, find the node voltages V_1 and V_2 in Fig. 2.74.*

Solution. Applying *KCL* to node 1, we get

$$8 - 1 - \frac{V_1}{3} - \frac{(V_1 - V_2)}{6} = 0$$

or $3V_1 - V_2 = 42$ ...(*i*)

Similarly, applying *KCL* to node 2, we get

$$1 + \frac{(V_1 - V_2)}{6} - \frac{V_2}{15} - \frac{V_2}{10} = 0$$

or $V_1 - 2V_2 = -6$ ...(*ii*)

Solving for V_1 and V_2 from Eqn. (*i*) and (*ii*), we get

$V_1 = 18$ V and $V_2 = 12$ V.

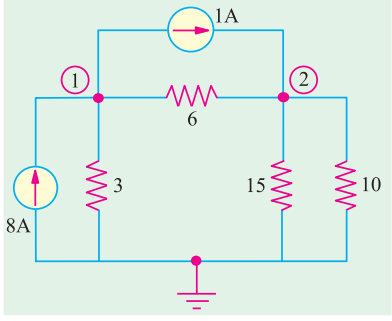


Fig. 2.74

2.14. Source Conversion

A given voltage source with a series resistance can be converted into (or replaced by) and equivalent current source with a parallel resistance. Conversely, a current source with a parallel resistance can be converted into a vaoltage source with a series resistance. Suppose, we want to convert the voltage source of Fig. 2.75 (*a*) into an equivalent current source. First, we will find the value of current supplied by the source when a 'short' is put across in termials *A* and *B* as shown in Fig. 2.75 (*b*). This current is $I = V/R$.

A current source supplying this current *I* and having the same resistance *R* connected in ***parallel*** with it represents the equivalent source. It is shown in Fig. 2.75 (*c*). Similarly, a current source of *I* and a parallel resistance *R* can be converted into a voltage source of voltage $V = IR$ and a resistance

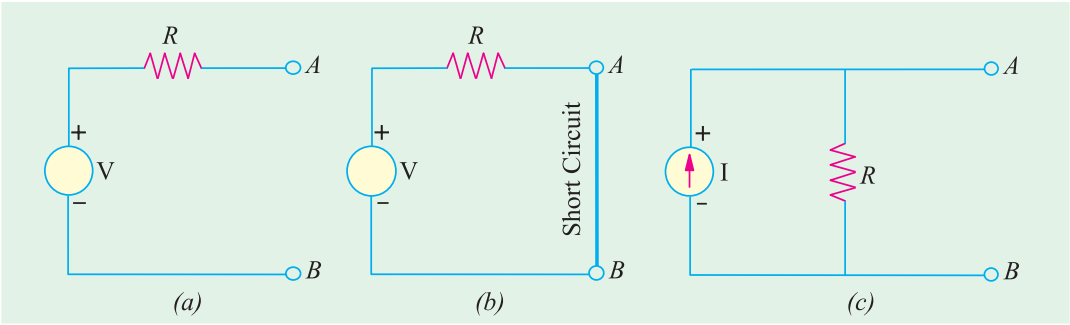


Fig. 2.75

R in series with it. It should be kept in mind that a voltage source-series resistance combination is equivalent to (or replaceable by) a current source-parallel resistance combination if, and only if their

1. respective open-circuit voltages are equal, and
2. respective short-circuit currents are equal.

For example, in Fig. 2.75 (*a*), voltage across terminals A and B when they are open (*i.e.* open-circuit voltage V_{OC}) is V itself because there is no drop across R. Short-circuit current across $AB = I = V/R$.

Now, take the circuit of Fig. 2.75 (*c*). The open-circuit voltage across AB = drop across $R = IR = V$. If a short is placed across AB, whole of I passes through it because R is completely shorted out.

Example 2.41. *Convert the voltage source of Fig. 2.73 (a) into an equivalent current source.*

Solution. As shown in Fig 2.76 (*b*), current obtained by putting a short across terminals A and B is 10/5 = 2 A.

Hence, the equivalent current source is as shown in Fig. 2.76 (*c*).

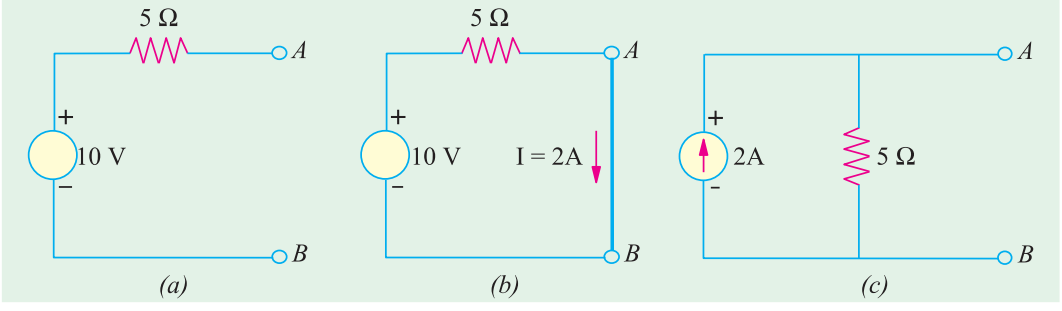


Fig. 2.76

Example 2.42. *Find the equivalent voltage source for the current source in Fig. 2.77 (a).*

Solution. The open-circuit voltage across terminals A and B in Fig. 2.77 (*a*) is

$$V_{OC} = \text{drop across } R$$
$$= 5 \times 2 = 10 \text{ V}$$

Hence, voltage source has a voltage of **10 V** and the same resistance of 2 Ωthrough connected in series [Fig. 2.77 (*b*)].

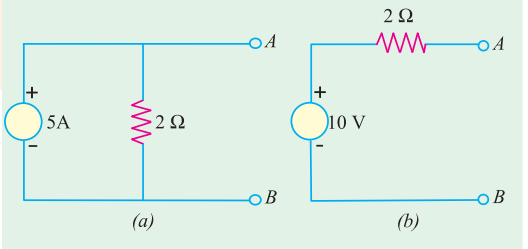


Fig. 2.77

Example 2.43. *Use Source Conversion technique to find the load current I in the circuit of Fig. 2.78 (a).*

Solution. As shown in Fig. 2.78 (*b*). 6-V voltage source with a series resistance of 3 Ω has been converted into an equivalent 2 A current source with 3 Ω resistance in parallel.

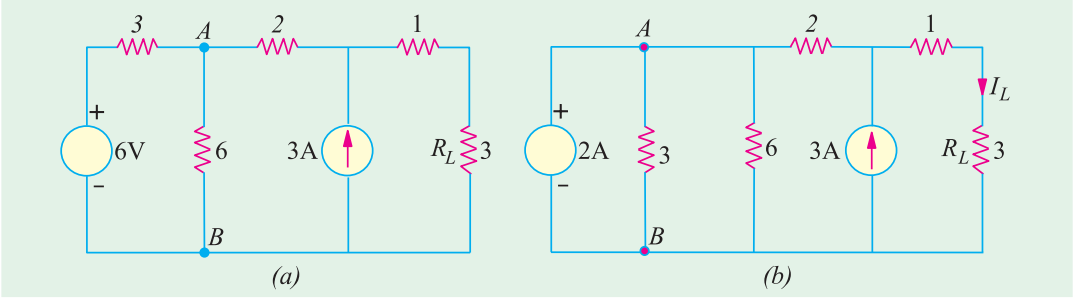


Fig. 2.78

The two parallel resistances of 3 Ω and 6 Ω can be combined into a single resistance of 2 Ω as shown in Fig. 2.79. (*a*)

The two current sources cannot be combined together because of the 2 Ω resistance present between points *A* and *C*. To remove this hurdle, we convert the 2 A current source into the equivalent 4 V voltage source as shown in Fig. 2.79 (*b*). Now, this 4 V voltage source with a series resistance of (2 + 2) = 4 Ω can again be converted into the equivalent current source as shown in Fig. 2.80 (*a*). Now, the two current sources can be combined into a single 4-A source as shown in Fig. 2.80 (*b*).

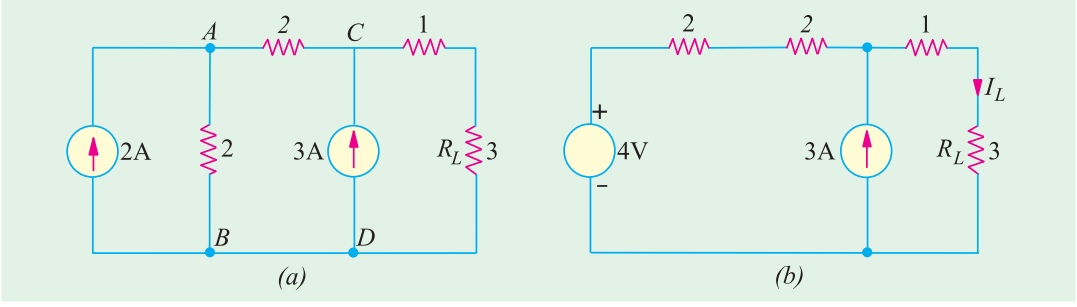


Fig. 2.79

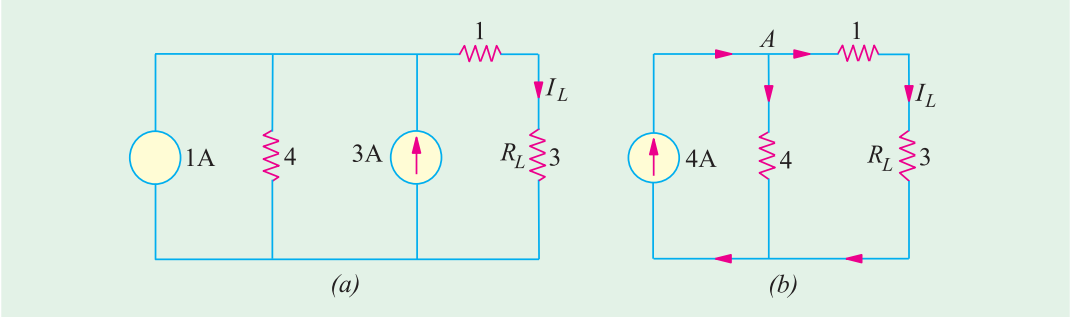


Fig. 2.80

The 4-A current is divided into two equal parts at point *A* because each of the two parallel paths has a resistance of 4 Ω. Hence I_1 = **2 A.**

Example 2.44. *Calculate the direction and magnitude of the current through the 5 Ω resistor between points A and B of Fig. 2.81 (a) by using nodal voltage method.*

Solution. The first thing is to convert the voltage source into the current sources as shown in Fig. 2.81 (*b*). Next, the two parallel resistances of 4 Ω each can be combined to give a single resistance of 2 Ω [Fig. 2.82 (*a*)]. Let the current directions be as indicated.

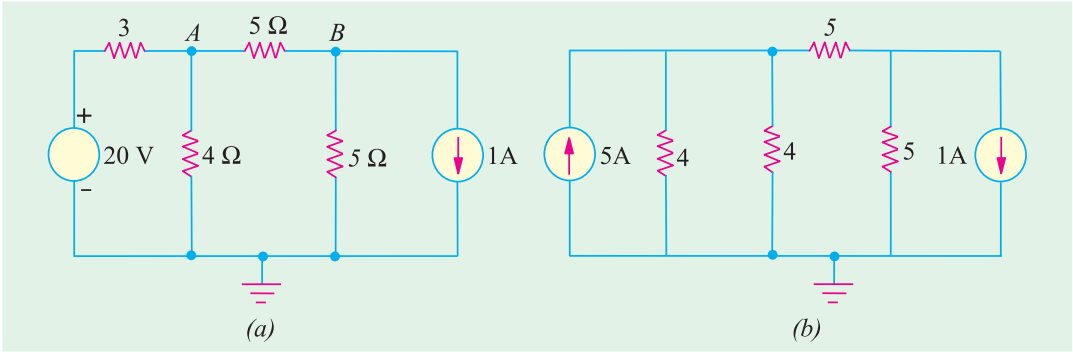


Fig. 2.81

Applying the nodal rule to nodes 1 and 2, we get

Node 1

$$V_1\left(\frac{1}{2}+\frac{1}{5}\right)-\frac{V_2}{5} = 5 \quad \text{or} \quad 7V_1 - 2V_2 = 50 \qquad \text{...(i)}$$

Node 2

$$V_2\left(\frac{1}{5}+\frac{1}{5}\right)-\frac{V_1}{5} = -1 \quad \text{or} \quad V_1 - 2V_2 = 5 \qquad \text{...(ii)}$$

Solving for V_1 and V_2, we get $V_1 = \frac{15}{2}$ V and $V_2 = \frac{5}{4}$ V.

$$I_2 = \frac{V_1 \quad V_2}{5} \quad \frac{15/2 \quad 5/4}{5} \quad \textbf{1.25 A}$$

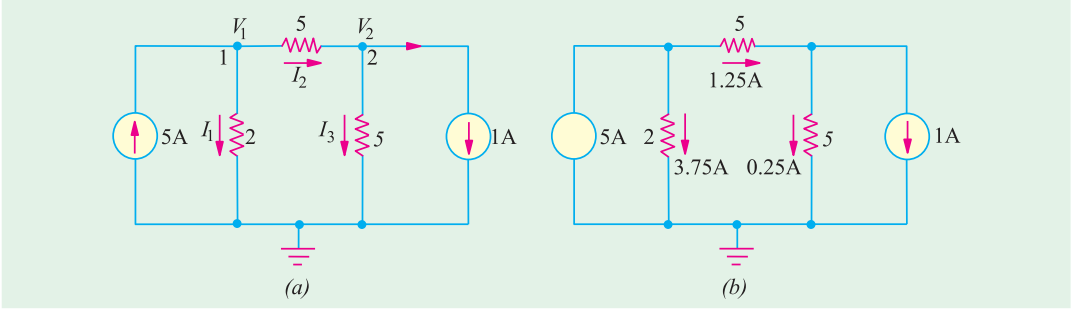


Fig. 2.82

Similarly, $I_1 = V_1/2 = 15/4 = 3.75$ A; $I_3 = V_2/5 = 5/20 = 0.25$ A.

The actual current distribution becomes as shown in Fig. 2.79 (*b*).

Example 2.45. *Replace the given network by a single current source in parallel with a resistance.*

[Bombay University 2001]

Solution. The equivalence is expected for a load connected to the right-side of terminals *A* and *B*. In this case, the voltage-source has no resistive element in series. While handling such cases, the 3-ohm resistor has to be kept aside, treating it as an independent and separate loop. This voltage source will circulate a current of 20/3 amp in the resistor, and will not appear in the calculations.

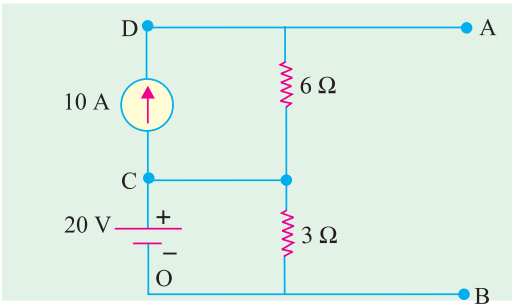


Fig. 2.83 (*a*)

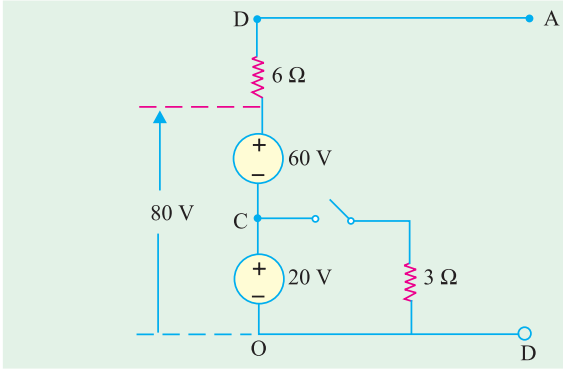


Fig. 2.83 (*b*)

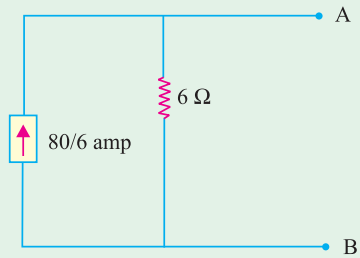


Fig. 2.83 (*c*)

This step does not affect the circuit connected to *A-B*.

Further steps are shown in Fig. 2.83 (*b*) and (*c*)

Tutorial Problems No. 2.3

1. Using Maxwell's loop current method, calculate the output voltage V_o for the circuits shown in Fig. 2.84. **[(*a*) 4 V (*b*) - 150/7 V (*c*) $V_o = 0$ (*d*) $V_o = 0$]**

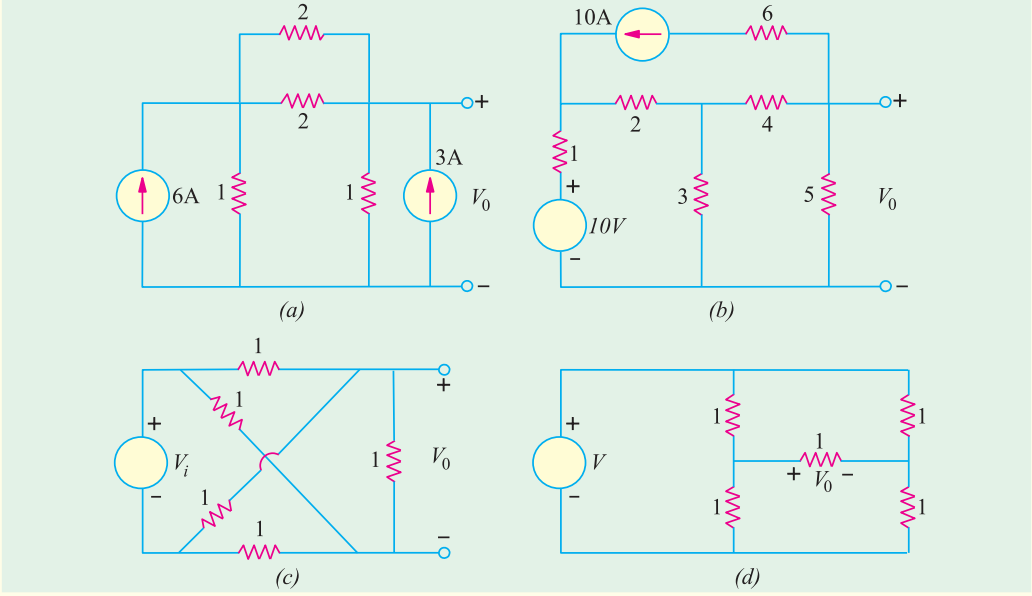


Fig. 2.84

2. Using nodal voltage method, find the magnitude and direction of current I in the network of Fig. 2.85.

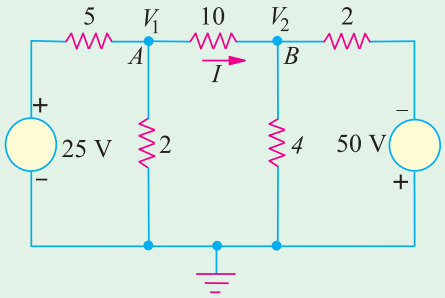


Fig. 2.85

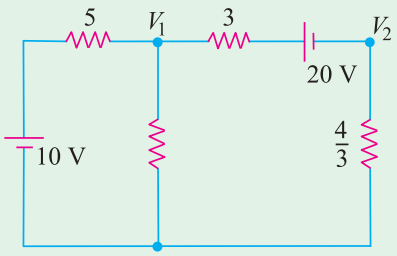
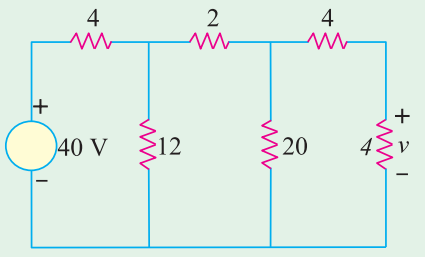
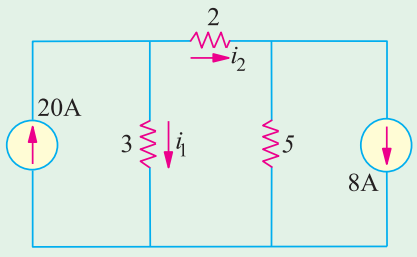


Fig. 2.86

3. By using repeated source transformations, find the value of voltage v in Fig. 2.87 (*a*). **[8 V]**



(*a*)



(*b*)

Fig. 2.87

4. Use source transformation technique to find the current flowing through the 2 Ω resistor in Fig. 2.87 (*b*). **[10 A]**

5. With the help of nodal analysis, calculate the values of nodal voltages V_1 and V_2 in the circuit of Fig. 2.86. **[7.1 V; –3.96 V]**

6. Use nodal analysis to find various branch currents in the circuit of Fig. 2.88.

[**Hint :** Check by source conversion.] **[I_{ac} = 2 A; I_{ab} = 5 A, I_{bc} = 0]**

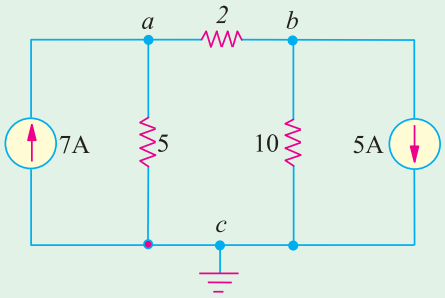


Fig. 2.88

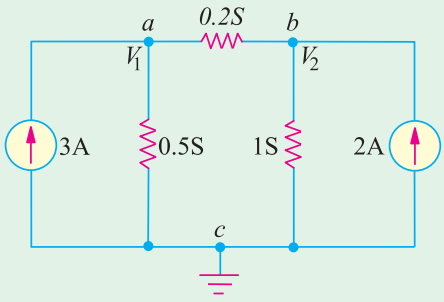


Fig. 2.89

7. With the help of nodal analysis, find V_1 and V_2 and various branch currents in the network of Fig. 2.85. **[5 V, 2.5 V; I_{ac} = 2.5 A; I_{ab} = 0.5 A; I_{bc} = 2.5 A]**

8. By applying nodal analysis to the circuit of Fig. 2.90, find I_{ab}, I_{bd} and I_{bc}. All resistance values are in ohms. **[$I_{ab} = \frac{22}{21}$ A, $I_{bd} = \frac{10}{7}$ A, $I_{bc} = \frac{-8}{21}$ A]**

[**Hint. :** It would be helpful to convert resistance into conductances.]

9. Using nodal voltage method, compute the power dissipated in the 9-Ω resistor of Fig. 2.91. **[81 W]**

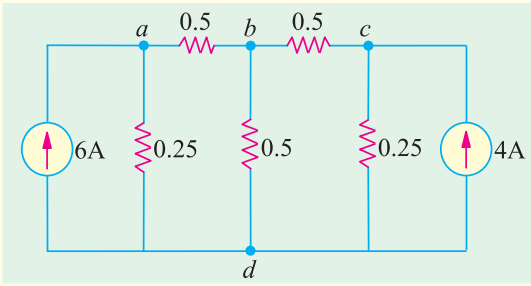


Fig. 2.90

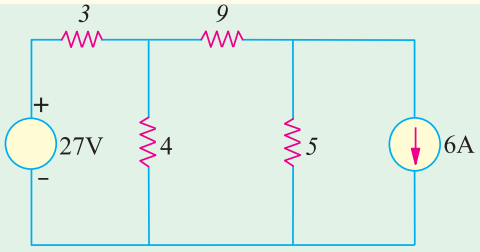


Fig. 2.91

10. Write equilibrium equations for the network in Fig. 2.92 on nodal basis and obtain the voltage V_1, V_2 and V_3. All resistors in the network are of 1 Ω **[*Network Theory and Fields, Madras Univ.*]**

11. By applying nodal method of network analysis, find current in the 15 Ω resistor of the network shown in Fig. 2.93. **[3.5 A] [*Elect. Technology-1, Gwalior Univ.*]**

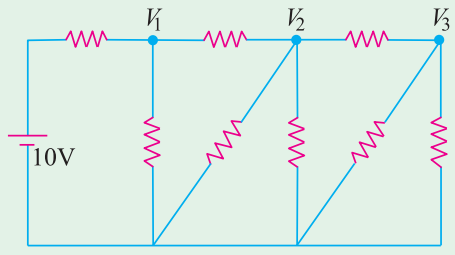


Fig. 2.92

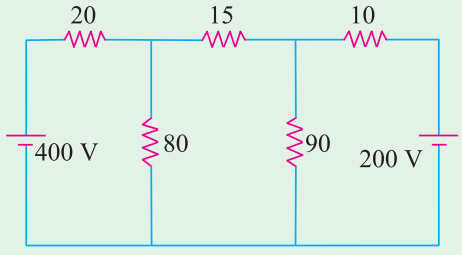


Fig. 2.93

2.15. Ideal Constant-Voltage Source

It is that voltage source (or generator) whose output voltage remains absolutely constant whatever the change in load current. Such a voltage source must possess ***zero internal resistance so that internal voltage drop in the source is zero***. In that case, output voltage provided by the source would remain constant ***irrespective of the amount of current drawn from it***. In practice, none such ideal constant-voltage source can be obtained. However, smaller the internal resistance r of a voltage source, closer it comes to the ideal sources described above.

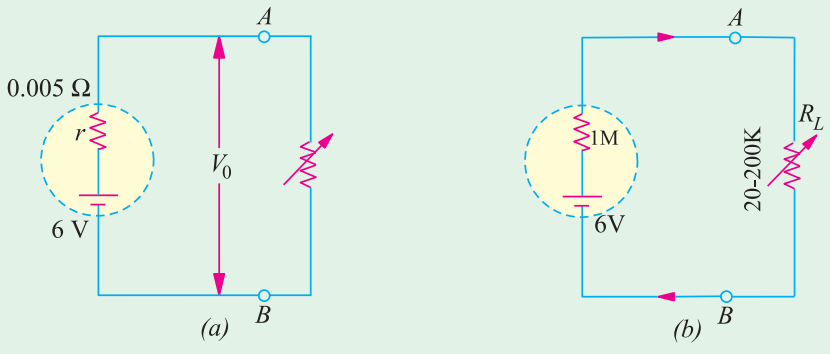


Fig. 2.94

Suppose, a 6-V battery has an internal resistance of 0.005 Ω [Fig. 2.94 (*a*)]. When it supplies no current *i.e.* it is on no-load, V_o = 6 V *i.e.* output voltage provided by it at its output terminals *A* and *B*

is 6 V. If load current increases to 100 A, internal drop = 100 × 0.005 = 0.5 V. Hence, V_o = 6 –0.5 = 5.5 V.

Obviously an output voltage of 5.5 –6 V can be considered constant as compared to wide variations in load current from 0 A ot 100 A.

2.16. Ideal Constant-Current Source

It is that voltage source whose internal resistance is infinity. In practice, it is approached by a source which posses very high resistance as compared to that of the external load resistance. As shown in Fig. 2.94 (*b*), let the 6-V battery or voltage source have an internal resistance of 1 M Ω and let the load resistance vary from 20 K to 200 K. The current supplied by the source varies from 6.1/1.02 = 5.9 μ A to 6/1.2 = 5 μ A. As seen, even when load resistance increases 10 times, current decreases by 0.9 μA. Hence, the source can be considered, for all practical purposes, to be a constant-current source.

2.17. Superposition Theorem

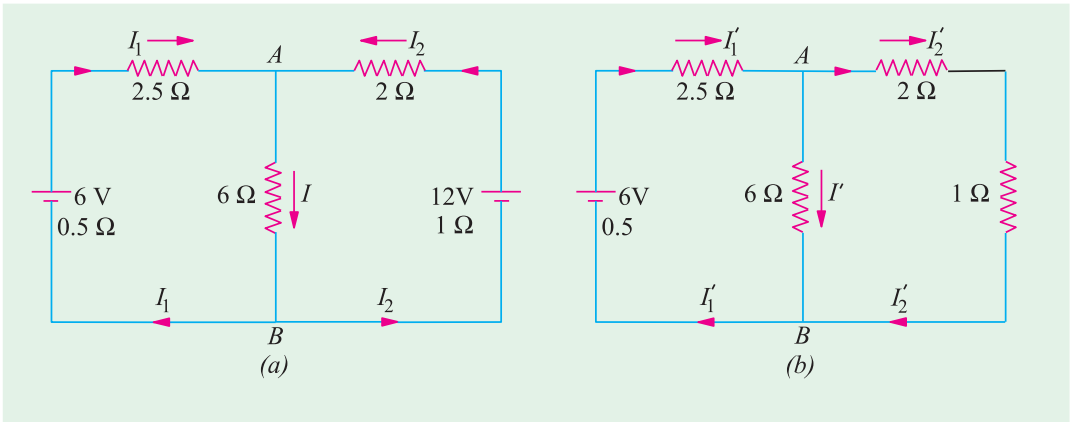


Fig. 2.95

According to this theorem, if there are a number of e.m.fs. acting simultaneously in any linear bilateral network, then each e.m.f. acts independently of the others *i.e.* as if the other e.m.fs. did not exist. The value of current in any conductor is the algebraic sum of the currents due to each e.m.f. Similarly, voltage across any conductor is the algebraic sum of the voltages which each e.m.f would have produced while acting singly. In other words, current in or voltage across, any conductor of the network is obtained by superimposing the currents and voltages due to each e.m.f. in the network. It is important to keep in mind that this theorem is applicable only to ***linear*** networks where current is ***linearly*** related to voltage as per Ohm's law.

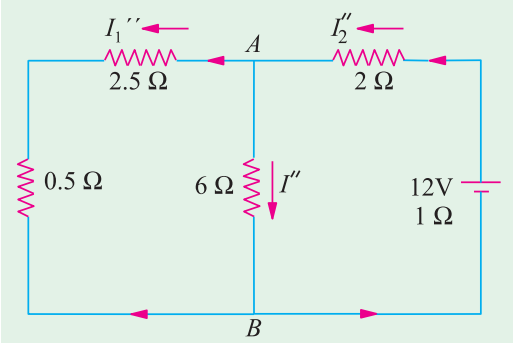


Fig. 2.96

Hence, this theorem may be stated as follows :

In a network of linear resistances containing more than one generator (or source of e.m.f.), the current which flows at any point is the sum of all the currents which would flow at that point if each generator where considered separately and all the other generators replaced for the time being by resistances equal to their internal resistances.

Explanation

In Fig. 2.95 (*a*) I_1, I_2 and I represent the values of

currents which are due to the simultaneous action of the two sources of e.m.f. in the network. In Fig. 2.95 (*b*) are shown the current values which would have been obtained if left-hand side battery had acted alone. Similarly, Fig. 2.96 represents conditions obtained when right-hand side battery acts alone. By combining the current values of Fig. 2.95 (*b*) and 2.96 the actual values of Fig. 2.95 (*a*) can be obtained.

Obviously, $I_1 = I_1' - I_1'', I_2 = I_2'' - I_2', I = I' + I''$.

Example 2.46. *In Fig. 2.95 (a) let battery e.m.fs. be 6 V and 12 V, their internal resistances 0.5 Ω and 1 Ω. The values of other resistances are as indicated. Find the different currents flowing in the branches and voltage across 60-ohm resistor.*

Solution. In Fig. 2.95 (*b*), 12-volt battery has been removed though its internal resistance of 1 Ω remains. The various currents can be found by applying Ohm's Law.

It is seen that there are two parallel paths between points *A* and *B*, having resistances of 6 Ω and (2 + 1) = 3 Ω.

$\therefore$ equivalent resistance $= 3 \parallel 6 = 2\ \Omega$

Total resistance $= 0.5 + 2.5 + 2 = 5\ \Omega \quad \therefore\ I_1' = 6/5 =$ **1.2 A.**

This current divides at point *A* inversely in the ratio of the resistances of the two parallel paths.

$\therefore \quad I' = 1.2 \times (3/9) = 0.4$ A. Similarly, $I_2' = 1.2 \times (6/9) = 0.8$ A

In Fig. 2.96, 6 volt battery has been removed but not its internal resistance. The various currents and their directions are as shown.

The equivalent resistance to the left to points *A* and *B* is $= 3 \parallel 6 = 2\ \Omega$

$\therefore$ total resistance $= 1 + 2 + 2 = 5\ \Omega \quad \therefore\ I_2'' = 12/5 = 2.4$ A

At point *A*, this current is divided into two parts,

$I_1'' = 2.4 \times 3/9 = 0.8$ A, $\quad I_1'' = 2.4 \times 6/9 = 1.6$ A

The actual current values of Fig. 2.95 (*a*) can be obtained by superposition of these two sets of current values.

$\therefore \quad I_1 = I_1' - I_1'' = 1.2 - 1.6 =$ **–0.4 A** (it is a charging current)

$I_2 = I_2'' - I_2' = 2.4 - 0.8 =$ **1.6 A**

$I = I' + I'' = 0.4 + 0.8 =$ **1.2 A**

Voltage drop across 6-ohm resistor = $6 \times 1.2 =$ **7.2 V**

Example 2.47. *By using Superposition Theorem, find the current in resistance R shown in Fig. 2.97 (a)*

$R_1 = 0.005\ \Omega,\ R_2 = 0.004\ \Omega,\ R = 1\ \Omega,\ E_1 = 2.05$ V, $E_2 = 2.15$ V

Internal resistances of cells are negligible. **(Electronic Circuits, Allahabad Univ. 1992)**

Solution. In Fig. 2.97 (*b*), E_2 has been removed. Resistances of 1 Ω and 0.04 Ω are in parallel across poins *A* and *C*. $R_{AC} = 1 \parallel 0.04 = 1 \times 0.04/1.04 = 0.038\ \Omega$. This resistance is in series with 0.05 Ω. Hence, total resistance offered to battery $E_1 = 0.05 + 0.038 = 0.088\ \Omega$. $I = 2.05/0.088 = 23.3$ A. Current through 1-Ω resistance, $I_1 = 23.3 \times 0.04/1.04 = 0.896$ A from *C* to *A*.

When E_1 is removed, circuit becomes as shown in Fig. 2.97 (*c*). Combined resistance of paths *CBA* and *CDA* is $= 1 \parallel 0.05 = 1 \times 0.05/1.05 = 0.048\ \Omega$. Total resistance offered to E_2 is $= 0.04 + 0.048 = 0.088\ \Omega$. Current $I = 2.15/0.088 = 24.4$ A. Again, $I_2 = 24.4 \times 0.05/1.05 = 1.16$ A.

To current through 1-Ω resistance when both batteries are present

$= I_1 + I_2 = 0.896 + 1.16 =$ **2.056 A.**

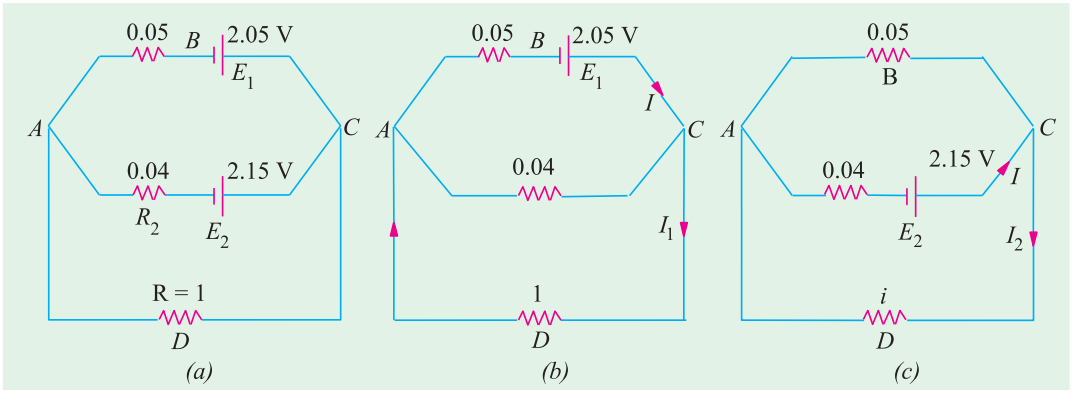


Fig. 2.97

Example 2.48. *Use Superposition theorem to find current I in the circuit shown in Fig. 2.98 (a). All resistances are in ohms.* **(Basic Circuit Analysis Osmania Univ. Jan/Feb 1992)**

Solution. In Fig. 2.98 (*b*), the voltage source has been replaced by a short and the 40 A current sources by an open. Using the current-divider rule, we get $I_1 = 120 \times 50/200 = 30$ A.

In Fig. 2.98 (*c*), only 40 A current source has been considered. Again, using current-divider rule $I_2 = 40 \times 150/200 = 30$ A.

In Fig. 2.98 (*d*), only voltage source has been considered. Using Ohm's law,

$$I_3 = 10/200 = 0.05 \text{ A}.$$

Since I_1 and I_2 cancel out, $I = I_3 = 0.005$ A.

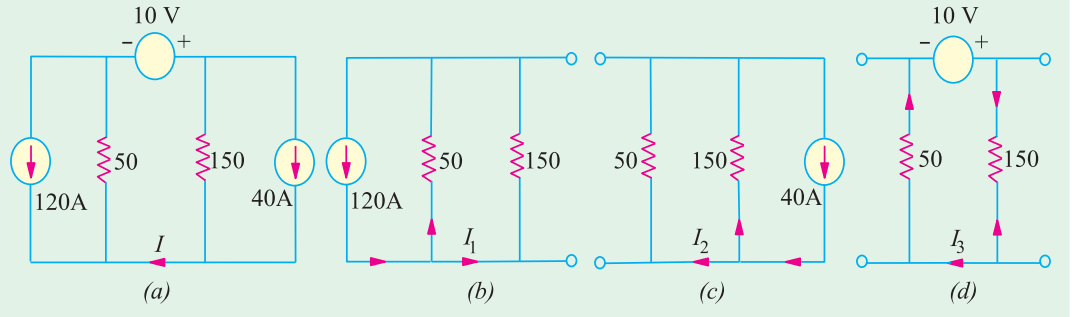


Fig. 2.98

Example 2.49. *Use superposition theorem to determine the voltage v in the network of Fig. 2.99(a).*

Solution. As seen, there are three independent sources and one dependent source. We will find the value of *v* produced by each of the three independent sources when acting alone and add the three values to find *v*. It should be noted that unlike independent source, a dependent source connot be set to zero *i.e.* it cannot be 'killed' or deactivated.

Let us find the value of v_1 due to 30 V source only. For this purpose we will replace current source by an open circuit and the 20 V source by a short circuit as shown in Fig. 2.99 (*b*). Applying KCL to node 1, we get

$$\frac{(30 - v_1)}{6} - \frac{v_1}{3} + \frac{(v_1/3 - v_1)}{2} = 0 \quad \text{or} \quad v_1 = 6 \text{ V}$$

Let us now keep 5 A source alive and 'kill' the other two independent sources. Again applying *KCL* to node 1, we get, from Fig. 2.99 (*c*).

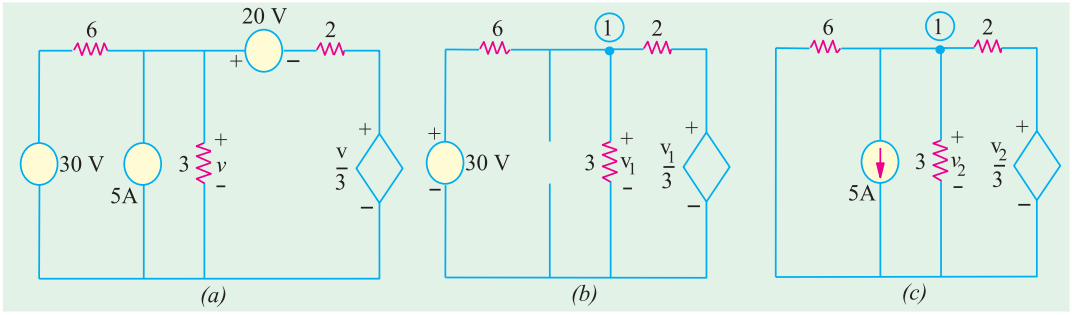


Fig. 2.99

$$\frac{v_2}{6} - 5 - \frac{v_2}{3} + \frac{(v_2/3 - v_2)}{2} = 0 \quad \text{or} \quad v_2 = -6 \text{ V}$$

Let us now 'kill' 30 V source and 5 A source and find v_3 due to 20 V source only. The two parallel resistances of 6 Ω and 3 Ω can be combined into a single resistance of 2 Ω Assuming a circulating current of i and applying *KVL* to the indicated circuit, we get, from Fig. 2.100.

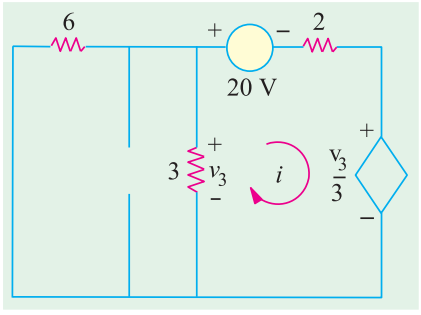


Fig. 2.100

$$-2i - 20 - 2i - \frac{1}{3}(-2i) = 0 \quad \text{or} \quad i = 6 \text{ A}$$

Hence, according to Ohm's law, the component of v that corresponds to 20 V source is $v_3 = 2 \times 6 = 12$ V. $\therefore v = v_1 + v_2 + v_3 = 6 - 6 + 12 = 12$ V.

Example 2.50. *Using Superposition theorem, find the current through the 40 W resistor of the circuit shown in Fig. 2.101 (a).* **(F.Y. Engg. Pune Univ. May 1990)**

Solution. We will first consider when 50 V battery acts alone and afterwards when 10-V battery is alone in the circuit. When 10-V battery is replaced by short-circuit, the circuit becomes as shown in Fig. 2.101 (*b*). It will be seen that the right-hand side 5 Ω resistor becomes connected in parallel with 40 Ω resistor giving a combined resistance of 5 || 40 = 4.44 Ω as shown in Fig. 101 (*c*). This 4.44 Ω resistance is in series with the left-hand side resistor of 5 Ω giving a total resistance of (5 + 4.44) = 9.44 Ω As seen there are two resistances of 20 Ω and 9.44 Ω connected in parallel. In Fig. 2.101 (*c*) current $I = 50/9.44 = 5.296$ A.

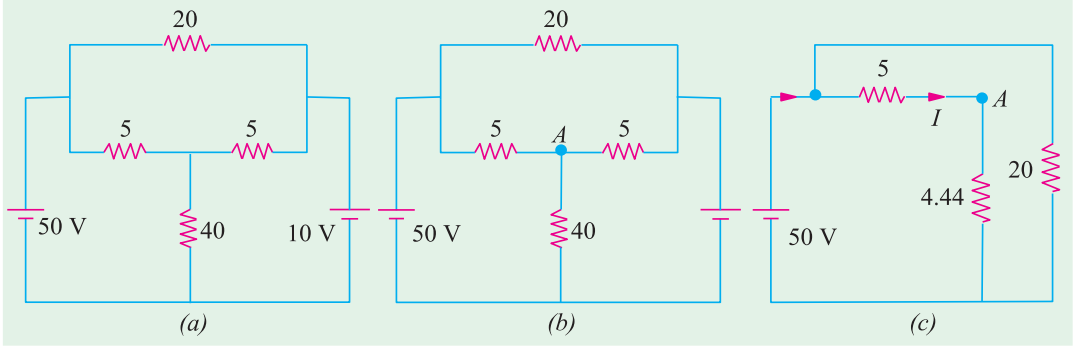


Fig. 2.101

At point *A* in Fig. 2.101 (*b*) there are two resistances of 5 Ω and 40 Ω connected in parallel, hence, current I divides between them as per the current-divider rule. If I_1 is the current flowing through the 40 Ω resistor, then

$$I_1 = I \frac{5}{5 \quad 40} \quad 5.296 \quad \frac{5}{45} \quad \textbf{0.589 A.}$$

In Fig. 2.102 (*a*), 10 V battery acts alone because 50-V battery has been removed and replaced by a short-circuit.

As in the previous case, there are two parallel branches of resistances 20 Ω and 9.44 Ω across the 10-V battery. Current I through 9.44 Ω branch is I = 10/9.44 = 1.059 A. This current divides at point B between 5 Ω resistor and 40 Ω resistor. Current through 40 Ω resistor I_2 = 1.059 × 5/45 = 0.118 A.

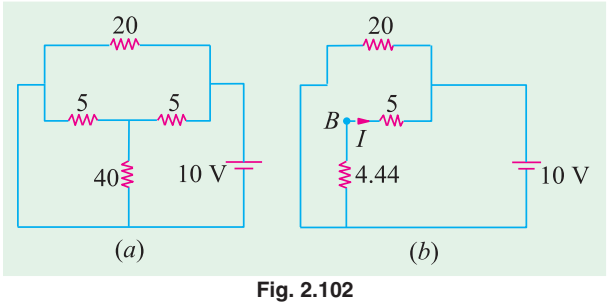


Fig. 2.102

According to the Superposition theorem, total current through 40 Ω resistance is

= $I_1 + I_2$ = 0.589 + 0.118 = **0.707 A.**

Example 2.51. *Solve for the power delivered to the 10 Ω resistor in the circuit shown in Fig. 2.103 (a). All resistances are in ohms.* **(Elect. Science - I, Allahabad Univ. 1991)**

Solution. The 4-A source and its parallel resistance of 15 Ω can be converted into a voltage source of (15 × 4) = 60 V in series with a 15 Ω resistances as shown in Fig. 2.103 (*b*).

Now, we will use Superposition theorem to find current through the 10 Ω resistances.

When 60 –V Source is Removed

When 60 –V battery is removed the total resistance as seen by 2 V battery is = 1 + 10 || (15 + 5) = 7.67 Ω

The battery current = 2/7.67 A = 0.26 A. At point *A*, this current is divided into two parts. The current passing through the 10 Ω resistor from *A* to *B* is

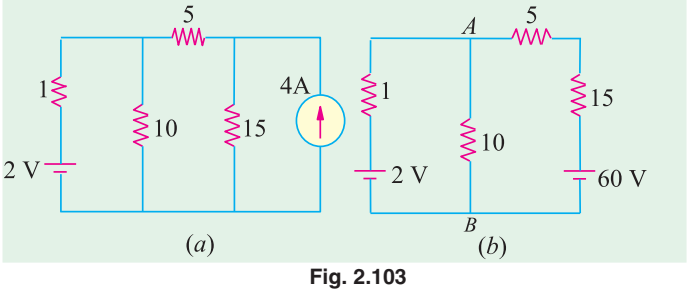


Fig. 2.103

$$I_1 = 0.26 \times (20/30) = 0.17 \text{ A}$$

When 2-V Battery is Removed

Then resistance seen by 60 V battery is = 20 + 10 || 1 = 20.9 Ω Hence, battery current = 60/20.9 = 2.87 A. This current divides at point *A*. The current flowing through 10 Ω resistor from *A* to *B* is

$$I_2 = 2.87 \times 1/(1 + 10) = 0.26 \text{ A}$$

Total current through 10 Ω resistor due to two batteries acting together is = $I_1 + I_2$ = 0.43 A.

Power delivered to the 10 Ω resistor = $0.43^2 \times 10$ = 1.85 W.

Example 2.52. *Compute the power dissipated in the 9-W resistor of Fig. 2.104 by applying the Superposition principle. The voltage and current sources should be treated as ideal sources. All resistances are in ohms.*

Solution. As explained earlier, an ideal constant-voltage sources has zero internal resistances whereas a constant-current source has an infinite internal resistance.

(*i*) When Voltage Source Acts Alone

This case is shown is in Fig. 2.104 (*b*) where constant-current source has been replaced by an open-circuit *i.e.* infinite resistance (Art. 2.16). Further circuit simplification leads to the fact that total resistances offered to voltage source is = 4 + (12 || 15) = 32/3 Ω as shown in FIg. 2.104 (*c*).

Hence current = 32 ÷ 32/3 = 3 A. At point *A* in Fig. 2.104 (*d*), this current divides into two parts. The part going alone *AB* is the one that also passes through 9 Ω resistor.

$$I' = 3 \times 12/(15 + 12) = 4/3 \text{ A}$$

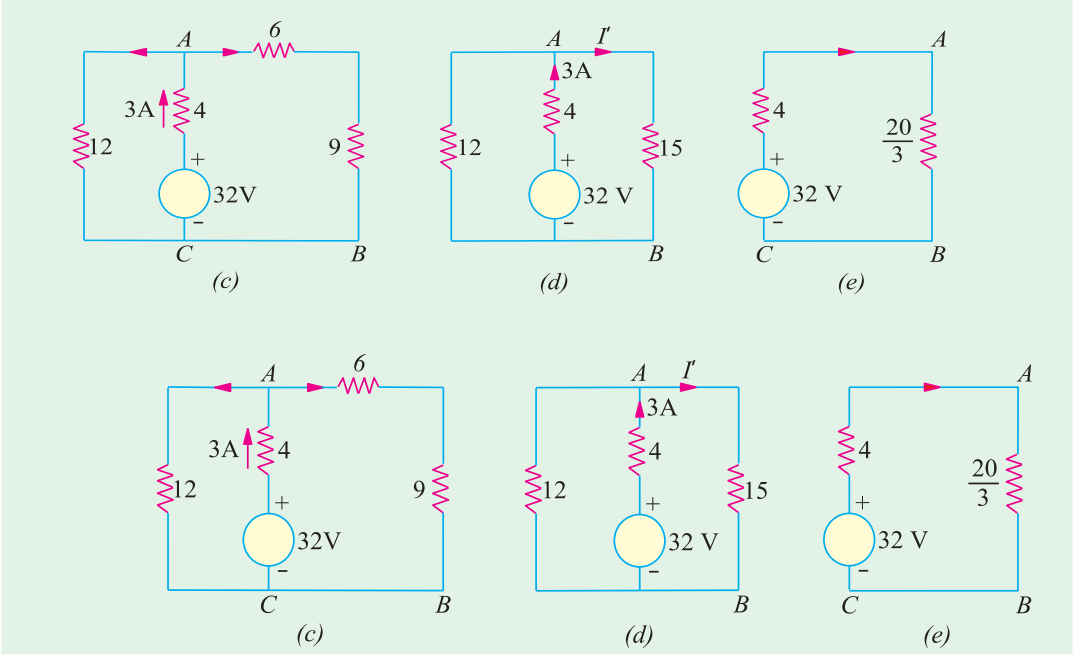


Fig. 2.104

(*ii*) When Current Source Acts Alone

As shown in Fig. 2.105 (*a*), the voltage source has been replaced by a short-circuit (Art 2.13). Further simplification gives the circuit of Fig. 2.105 (*b*).

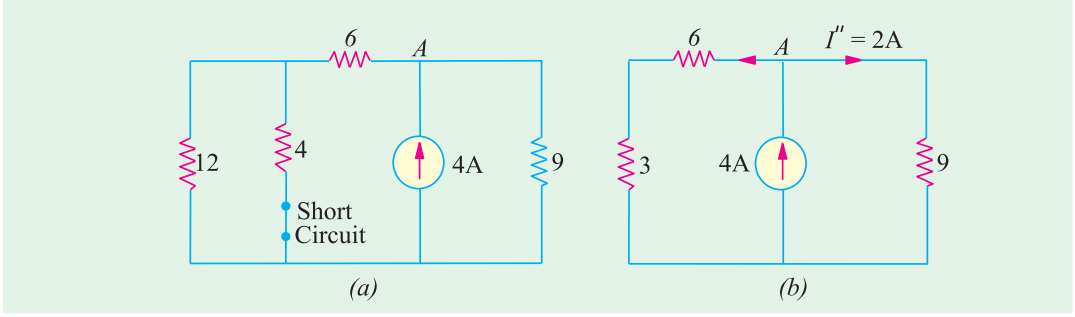


Fig. 2.105

The 4 - A current divides into two equal parts at point *A* in Fig. 2.105 (*b*). Hence $I = 4/2 = 2$ A. Since both I' and I'' flow in the *same* direction, total current through 9-Ω resistor is

$$I = I' + I'' = (4/3) + 2 = (10/3) \text{ A}$$

Power dissipated in 9 Ω resistor = $I^2 R = (10/3)^2 \times 9 =$ **100 W**

Example 2.53(*a*). *With the help of superposition theorem, obtain the value of current I and voltage V_0 in the circuit of Fig. 2.106 (a).*

Solution. We will solve this question in three steps. First, we will find the value of *I* and V_0 when current source is removed and secondly, when voltage source is removed. Thirdly, we would combine the two values of *I* and V_0 in order to get their values when both sources are present.

First Step

As shown in Fig. 2.106 (*b*), current source has been replaced by an open-circuit. Let the values of current and voltage due to 10 V source be I_1 and V_{01}. As seen $I_1 = 0$ and $V_{01} = 10$ V.

Second Step

As shown in Fig. 2.106 (*c*), the voltage source has been replaced by a short circuit. Here $I_2 = -5$ A and $V_{02} = 5 \times 10 = 50$ V.

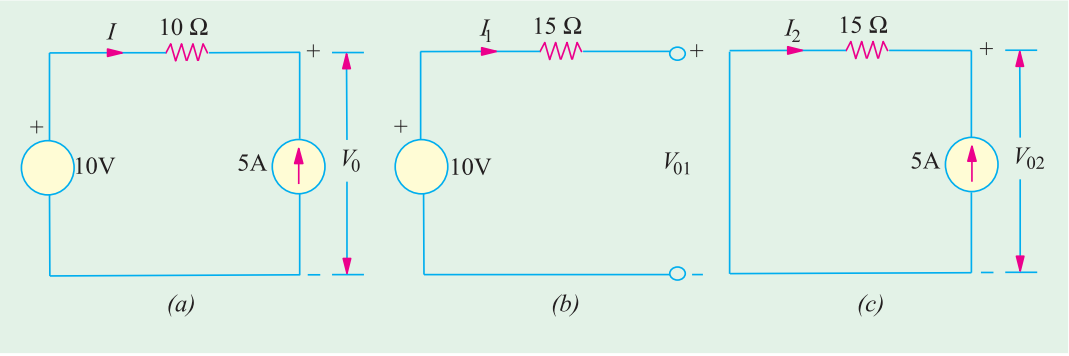


Fig. 2.106

Third Step

By applying superposition theorem, we have

$$I = I_1 + I_2 = 0 + (-5) = \mathbf{-5\ A}$$

$$V_0 = V_{01} + V_{02} = 10 + 50 = \mathbf{60\ V}$$

Example 2.53(*b*). *Using Superposition theorem, find the value of the output voltage V_0 in the circuit of Fig. 2.107.*

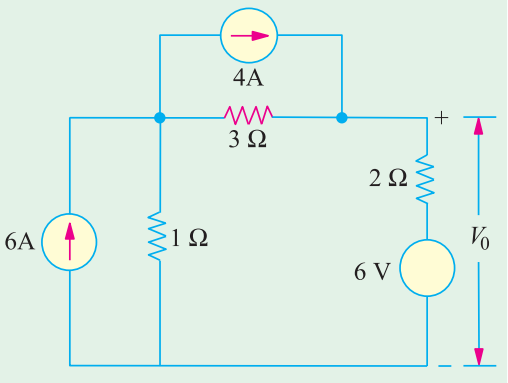


Fig. 2.107

Solution. As usual, we will break down the problem into three parts involving one source each.

(*a*) When 4 A and 6 V sources are killed*

As shown in Fig. 2.108 (*a*), 4 A source has been replaced by an open circuit and 6 V source by a short-circuit. Using the current-divider rule, we find current i_1 through the 2 Ω resistor = $6 \times 1/(1 + 2 + 3) = 1$ A $\quad \therefore\ V_{01} = 1 \times 2 = 2$ V.

(*b*) When 6 A and 6 V sources are killed

As shown in Fig. 2.108 (*b*), 6 A sources has been replaced by an open-circuit and 6 V source by a short-circuit. The current i_2 can again be found with the help of current-divider rule because there are two parallel paths across the current source. One has a resistance of 3 Ω and the other of (2 + 1) = 3 Ω. It means that current divides equally at point A.

Hence, $i_2 = 4/2 = 2$ A $\quad \therefore\ V_{02} = 2 \times 2 = 4$ V

(*c*) When 6 A and 4 A sources are killed

As shown in Fig. 2.108 (*c*), drop over 2 Ω resistor = $6 \times 2/6 = 2$ V. The potential of point *B* with respect to point *A* is = 6 –2 = + 4 V. Hence , $V_{03} = -4$ V.

* The process of setting of voltage source of zero is called *killing* the sources.

According to Superposition theorem, we have

$$V_0 = V_{01} + V_{02} + V_{03} = 2 + 4 - 4 = \mathbf{2\ V}$$

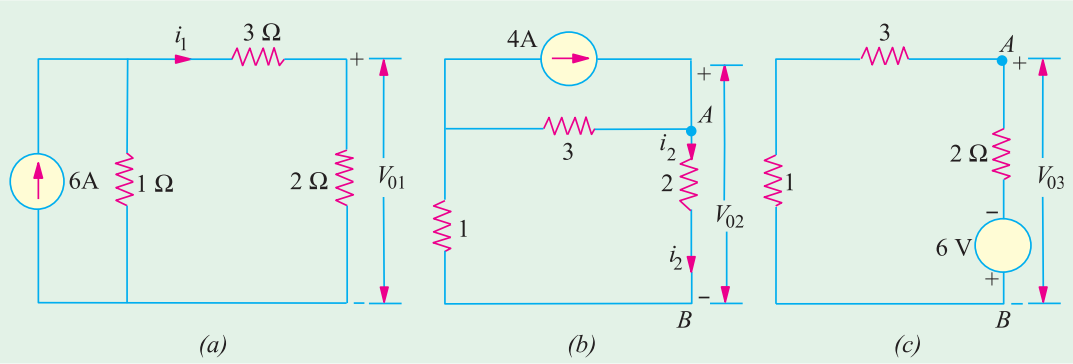


Fig. 2.108

Example 2.54. *Use Superposition theorem, to find the voltage V in Fig. 2.109 (a).*

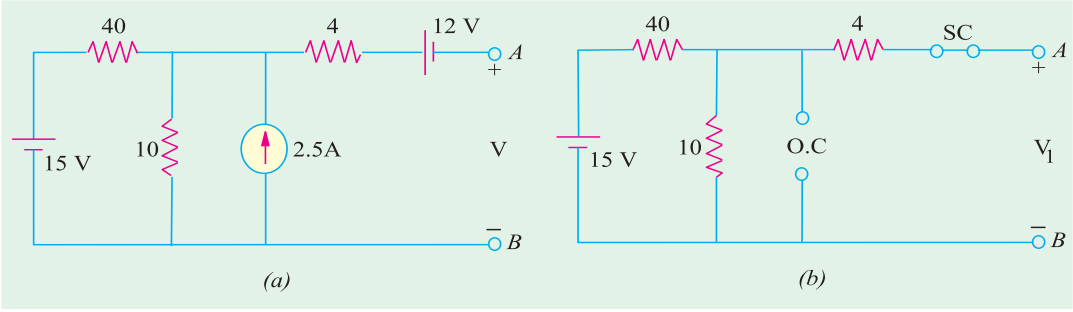


Fig. 2.109

Solution. The given circuit has been redrawn in Fig. 2.109 (*b*) with 15 - V battery acting alone while the other two sources have been killed. The 12 - V battery has been replaced by a short-circuit and the current source has been replaced by an open-circuit (O.C) (Art. 2.19). Since the output terminals are open, no current flows through the 4 Ω resistor and hence, there is no voltage drop across it. Obviously V_1 equals the voltage drop over 10 Ω resistor which can be found by using the voltage-divider rule.

$$V_1 = 15 \times 10/(40 + 10) = 3\ \text{V}$$

Fig. 2.110 (*a*) shows the circuit when current source acts alone, while two batteries have been killed. Again, there is no current through 4 Ω resistor. The two resistors of values 10 Ω and 40 Ω are

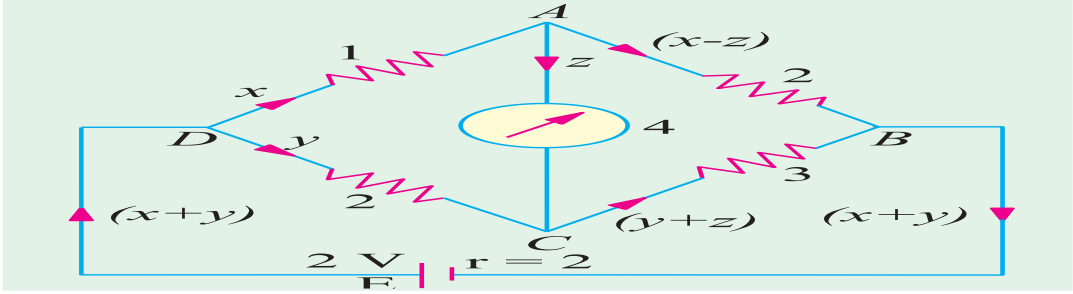


Fig. 2.110

in parallel across the current source. Their combined resistances is 10 || 40 = 8 Ω

$\therefore$ $V_2 = 8 \times 2.5 = 20$ V with point *A* positive.

Fig. 2.110 (*b*) shows the case when 12 –V battery acts alone. Here, $V_3 = -12$ V*. Minus sign has been taken because negative terminal of the battery is connected to point *A* and the positive terminal to point *B*. As per the Superposition theorem,

$$V = V_1 + V_2 + V_3 = 3 + 20 - 12 = 11 \text{ V}$$

Example 2.55. *Apply Superposition theorem to the circuit of Fig. 2.107 (a) for finding the voltage drop V across the 5 Ω resistor.*

Solution. Fig. 2.111 (*b*) shows the redrawn circuit with the voltage source acting alone while the two current sources have been 'killed' *i.e.* have been replaced by open circuits. Using voltage-divider principle, we get

$V_1 = 60 \times 5/(5 + 2 + 3) = 30$ V. It would be taken as positive, because current through the 5 Ω resistances flows from *A* to *B*, thereby making the upper end of the resistor positive and the lower end negative.

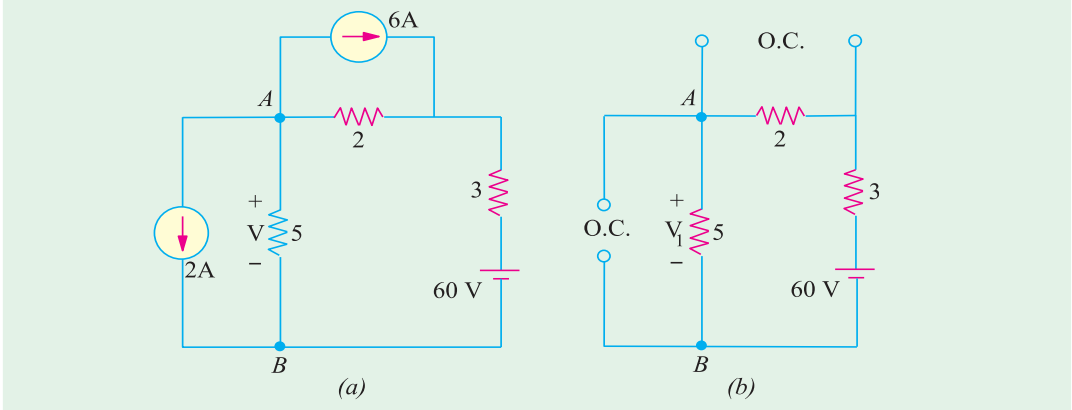


Fig. 2.111

Fig. 2.112 (*a*) shows the same circuit with the 6 A source acting alone while the two other sources have been 'killed'. It will be seen that 6 A source has to parallel circuits across it, one having a resistance of 2 Ω and the other (3 + 5) = 8 Ω. Using the current-divider rule, the current through the 5 Ω resistor = 6 × 2/(2 + 3 + 5) = 1.2 A.

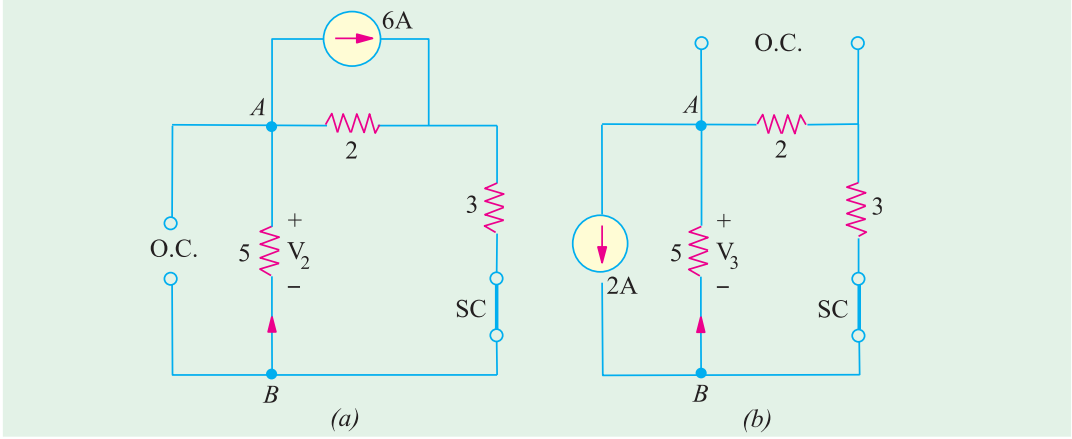


Fig. 2.112

* Because Fig. 2.110 (*b*) resembles a voltage source with an internal resistance = 4 + 10 || 40 = 12 Ω and which is an open-circuit.

$\therefore \quad V_2 = 1.2 \times 5 = 6$ V. It would be taken ***negative*** because current is flowing from B to A. *i.e.* point B is at a higher potential as compared to point A. Hence, $V_2 = -6$ V.

Fig. 2.112 (*b*) shows the case when 2-A source acts alone, while the other two sources are dead. As seen, this current divides equally at point B, because the two parallel paths have equal resistances of 5 Ω each. Hence, $V_3 = 5 \times 1 = 5$ V. It would also be taken as negative because current flows from B to A. Hence, $V_3 = -5$ V.

Using Superposition principle, we get

$$V = V_1 + V_2 + V_3 = 30 - 6 - 5 = \mathbf{19\ V}$$

Example 2.56. (*b*) *Determine using superposition theorem, the voltage across the 4 ohm resistor shown in Fig. 2.113* (*a*) **[Nagpur University, Summer 2000]**

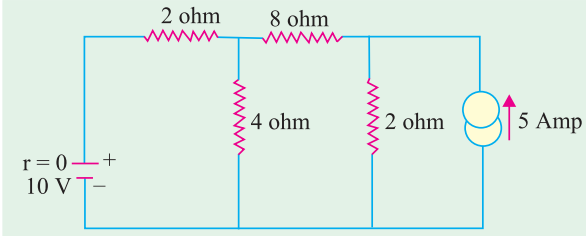


Fig. 2.113 (*a*)

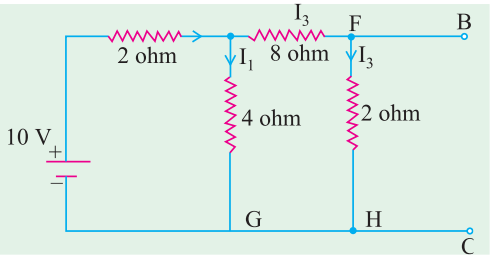


Fig. 2.113 (*b*)

Solution. Superposition theorem needs one source acting at a time.

Step I : De-acting current source.

The circuit is redrawn after this change in Fig. 2.113 (*b*)

$$I_1 = \frac{10}{2 + \dfrac{4x\,(8+2)}{4+(8+2)}} = \frac{10}{2 + \dfrac{40}{14}} = 2.059 \text{ amp}$$

$$I_2 = \frac{2.059 \quad 10}{14} \quad 1.471 \text{ amp, in downward direction}$$

Step II : De-activate the voltage source.

The circuit is redrawn after the change, in Fig. 2.113 (*c*)

With the currents marked as shown.

$I_d = 2I_c$ relating the voltage drops in Loop ADC.

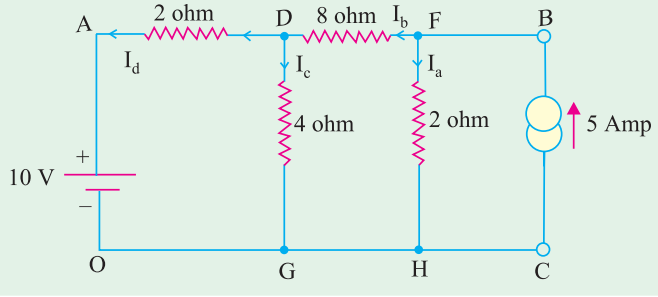


Fig. 2.113 (*c*)

Thus $I_b = 3\ I_c$.

Resistance of parallel combination of

2 and 4 ohms = $\dfrac{2 \times 4}{2+4} = 1.333\ \Omega$

Resistance for flow of I_b = 8 + 1.333 = 9.333 Ω

The 5-amp current from the sources gets divided into I_b (= 3 I_c) and I_a, at the node F.

$$I_b = 3\,I_c = \frac{2.0}{2.0 + 9.333} \times 5 = 0.8824$$

$\therefore$ I_c = 0.294 amp, in downward direction.

Step III. Apply superposition theorem, for finding the total current into the 4-ohm reistor

= Current due to Current source + Current due to Voltage source

= 0.294 + 1.471 = 1.765 amp in downward direction.

Check. In the branch *AD*,

The voltage source drives a current from *A* to *D* of 2.059 amp, and the current source drives a current of I_d (= 2I_c) which is 0.588 amp, from *D* to *A*.

The net current in branch *AD*

= 2.059 –0.588 = 1.471 amp ...eqn. ***(a)***

With respect to *O*, *A* is at a potential of + 10 volts.

Potential of *D* with respect to *O*

= (net current in resistor) × 4

= 1.765 × 4 = + 7.06 volts

Between *A* and *D*, the potential difference is (10 –7.06) volts

Hence, the current through this branch

$$= \frac{10 - 7.06}{2} = 1.47 \text{ amp from } A \text{ to } D$$ ...eqn ***(b)***

This is the same as eqn. ***(a)*** and hence checks the result, obtained previously.

Example 2.57. *Find the current flowing in the branch XY of the circuit shown in Fig. 2.114 (a) by superposition theorem.* **[Nagpur University, April 1996]**

Solution. As shown in Fig. 2.114 (*b*), one source is de-activated. Through series-parallel combinations of resistances, the currents due to this source are calculated. They are marked as on Fig. 2.114 (*b*).

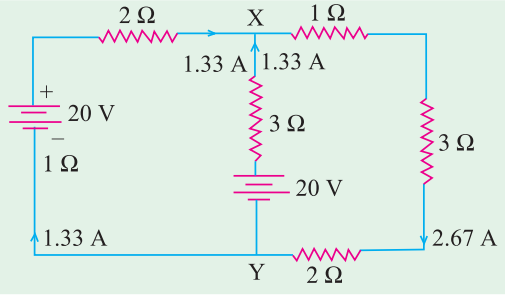


Fig. 2.114 (*a*)

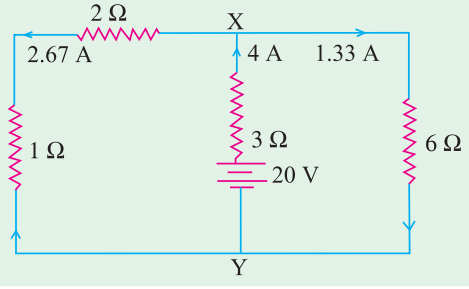


Fig. 2.114 (*b*)

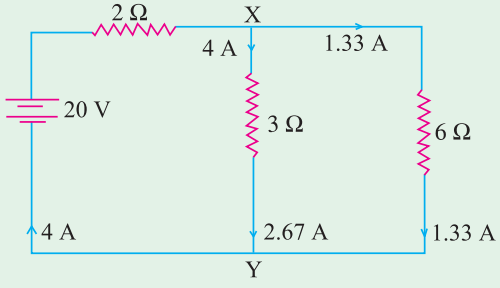


Fig. 2.114 (c)

In the next step, second source is de-activated as in Fig. 2.114 (*c*). Through simple series parallel resistances combinations, the currents due to this source are marked on the same figure.

According to the superposition theorem, the currents due to both the sources are obtained after adding the individual contributions due to the two sources, with the final results marked on Fig. 2.114 (*a*). Thus, the current through the branch *XY* is 1.33 A from *Y* to *X*.

Example 2.58. *Find the currents in all the resistors by Superposition theorem in the circuit shown in Fig. 2.115 (a). Calculate the power consumed.* **[Nagpur University, Nov. 1996]**

Solution. According to Superposition theorem, one source should be retained at a time, deactivating remaining sources. Contributions due to individual sources are finally algebraically added to get the answers required. Fig. 2.115 (*b*) shows only one source retained and the resultant currents in all branches/elements. In Fig. 2.115 (*c*), other source is shown to be in action, with concerned currents in all the elements marked.

To get the total current in any element, two component-currents in Fig. 2.115 (*b*) and Fig. 2.115 (*c*) for the element are to be algebraically added. The total currents are marked on Fig. 2.115 (*a*).

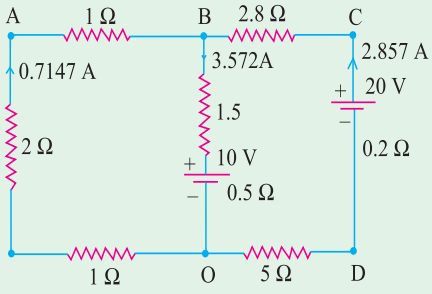


Fig. 2.115 (*a*)

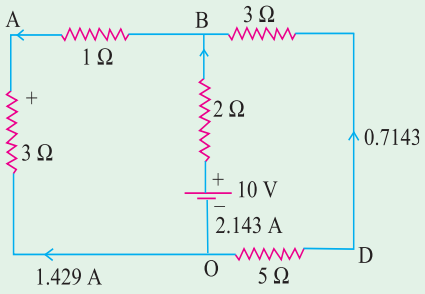


Fig. 2.115 (*b*)

All resistors are in ohms

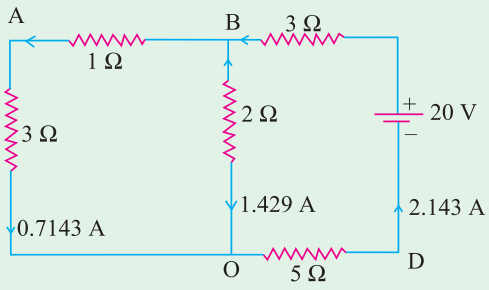


Fig. 2.115 (*c*)

Power loss calculations. ***(i)*** from power consumed by resistors :

Power = $(0.7147^2 \times 4) + (3.572^2 \times 2) + (2.875^2 \times 8) = 92.86$ watts

(ii) From Source-power.

Power = $10 \times 3.572 + 20 \times 2.857 = 92.86$ watts

Tutorial Problems No. 2.4.

1. Apply the principle of Superposition to the network shown in Fig. 2.116 to find out the current in the 10 Ω resistance. **[0.464 A]** ***(F.Y. Engg. Pune Univ.)***
2. Find the current through the 3 Ω resistance connected between *C* and *D* Fig. 2.117. **[1 A from C to D]** ***(F.Y. Engg. Pune Univ.)***

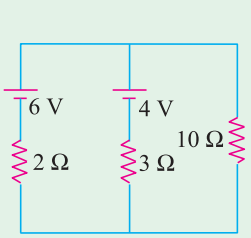


Fig. 2.116

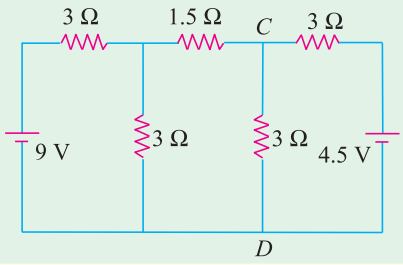


Fig. 2.117

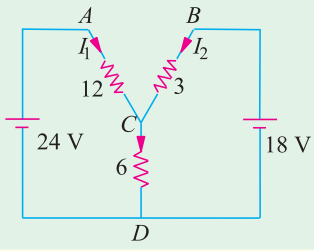


Fig. 2.118

3. Using the Superposition theorem, calculate the magnitude and direction of the current through each resistor in the circuit of Fig. 2.118. **[I_1 = 6/7 A; I_2 = 10/7 A; I_3 = 16/7 A]**

4. For the circuit shown in Fig. 2.119 find the current in R = 8 Ω resistance in the branch AB using superposition theorem. **[0.875 A] (*F.Y. Engg. Pune Univ.*)**

5. Apply superposition principle to compute current in the 2-Ω resistor of Fig. 2.120. All resistors are in ohms. **[I_{ab} = 5 A]**

6. Use Superposition theorem to calculate the voltage drop across the 3 Ω resistor of Fig. 2.121. All resistance values are in ohms. **[18 V]**

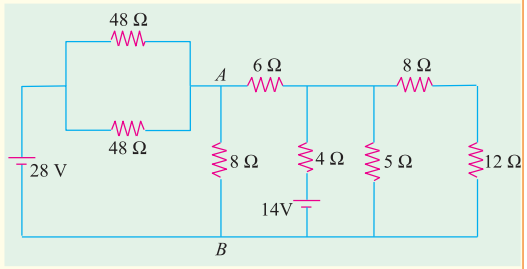


Fig. 2.119

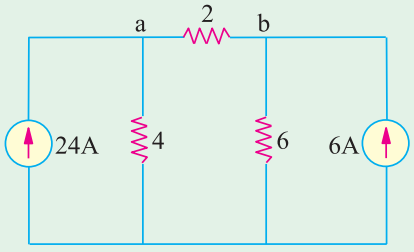


Fig. 2.120

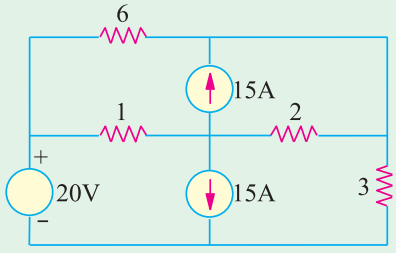


Fig. 2.121

7. With the help of Superposition theorem, compute the current I_{ab} in the circuit of Fig. 2.122. All resistances are in ohms. **[I_{ab} = −3 A]**

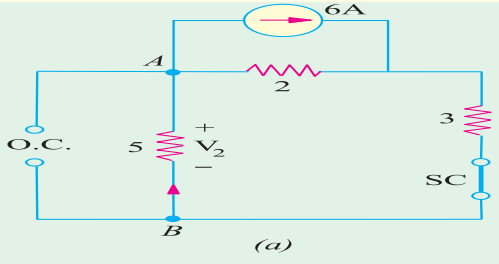


Fig. 2.122

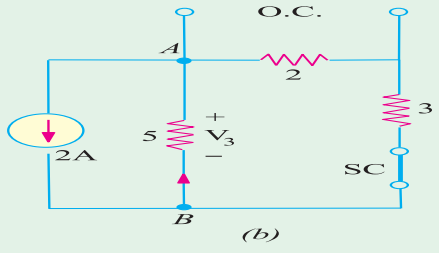


Fig. 2.123

8. Use Superposition theorem to find current I_{ab} in the circuit of Fig. 2.123. All resistances are in ohms. **[100 A]**

9. Find the current in the 15 Ω resistor of Fig. 2.124 by using Superposition principle. Numbers represent resistances in ohms. **[2.8 A]**

10. Use Superposition principle to find current in the 10-Ω resistor of Fig. 2.125. All resistances are in ohms. **[1 A]**

11. State and explain Superposition theorem. For the circuit of Fig. 2.126.

(*a*) determine currents I_1, I_2 and I_3 when switch S is in position b.

(*b*) using the results of part (*a*) and the principle of superposition, determine the same currents with switch S in position a.

[(*a*) 15 A, 10 A, 25 A (*b*) 11 A , 16 A, 27 A] (*Elect. Technology Vikram Univ.*)

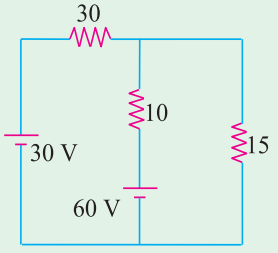


Fig. 2.124

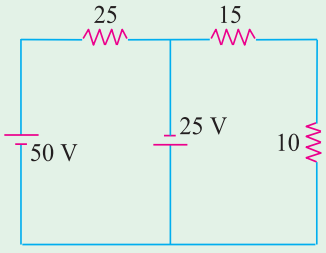


Fig. 2.125

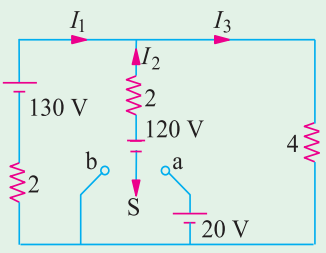
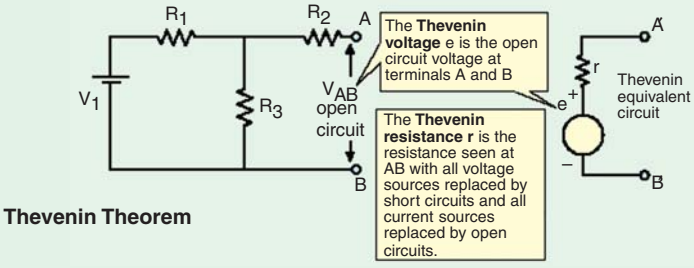


Fig. 2.126

2.18. Thevenin Theorem



Thevenin Theorem

It provides a mathematical technique for replacing a given network, as viewed from two output terminals, by ***a single voltage source with a series resistance***. It makes the solution of complicated networks (particularly, electronic networks) quite quick and easy. The application of this extremely useful theorem will be explained with the help of the following simple example.

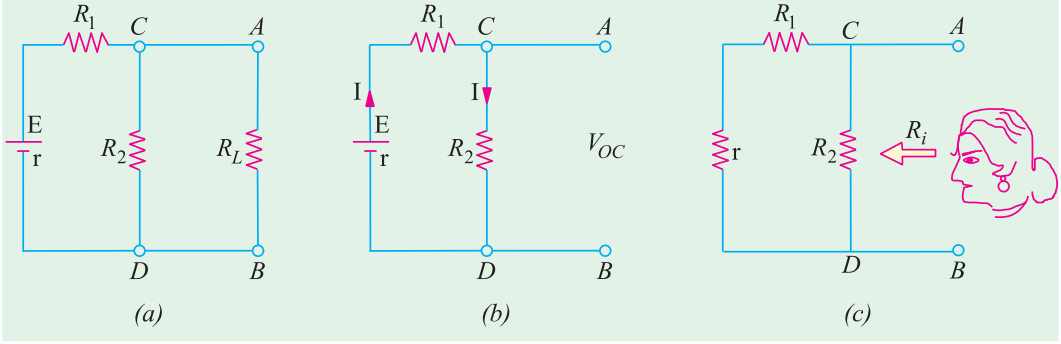


Fig. 2.127



M. L. Thevenin

Suppose, it is required to find current flowing through load resistance R_L, as shown in Fig. 2.127 (*a*). We will proceed as under :

1. Remove R_L from the circuit terminals *A* and *B* and redraw the circuit as shown in Fig. 2.127 (*b*). Obviously, the terminals have become open-circuited.
2. Calculate the open-circuit voltage V_{oc} which appears across terminals *A* and *B* when they are open *i.e.* when R_L is removed.
   As seen, V_{oc} = drop across $R_2 = IR_2$ where *I* is the circuit current when *A* and *B* are open.
   $I = \dfrac{E}{R_1 + R_2 + r}$ $\quad \therefore \; V_{oc} = IR_2 = \dfrac{ER_2}{R_1 + R_2 + r}$ [*r* is the internal resistance of battery]
   It is also called 'Thevenin voltage' V_{th}.
3. Now, imagine the battery to be removed from the circuit, leaving its internal resistance *r* behind and redraw the circuit, as shown in Fig. 2.127 (*c*). When viewed ***inwards*** from terminals *A* and *B*, the circuit consists of two parallel paths : one containing R_2 and the other containing $(R_1 + r)$. The equivalent resistance of the network, as viewed from these terminals is given as
   $$R = R_2 \parallel (R_1 + r) = \frac{R_2(R_1 + r)}{R_2 + (R_1 + r)}$$
   This resistance is also called,* Thevenin resistance R_{sh} (though, it is also sometimes written as R_i or R_0).
   Consequently, as viewed from terminals *A* and *B*, the whole network (excluding R_1) can be reduced to a single source (called Thevenin's source) whose e.m.f. equals V_∞ (or V_{sh}) and whose internal resistance equals R_{sh} (or R_i) as shown in Fig. 2.128.
4. R_L is now connected back across terminals *A* and *B* from where it was temporarily removed earlier. Current flowing through R_L is given by
   $$I = \frac{V_{th}}{R_{th} + R_L}$$

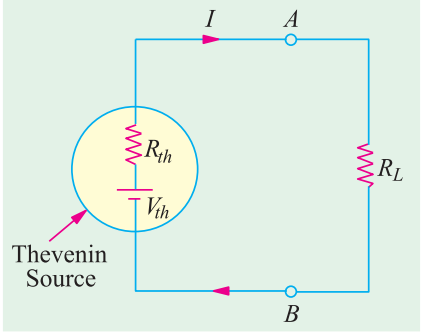


Fig. 2.128

It is clear from above that any network of resistors and voltage sources (and current sources as well) when viewed from any points *A* and *B* in the network, can be replaced by a single voltage source and a single resistance** in series with the voltage source.

After this replacement of the network by a single voltage source with a series resistance has been accomplished, it is easy to find current in any load resistance joined across terminals *A* and *B*. This theorem is valid even for those linear networks which have a nonlinear load.

Hence, Thevenin's theorem, as applied to d.c. circuits, may be stated as under :

The current flowing through a load resistance R_L connected across any two terminals A and B of a linear, active bilateral network is given by $V_{oc} \parallel (R_i + R_L)$ where V_{oc} is the open-circuit voltage (i.e. voltage across the two terminals when R_L is removed) and R_i is the internal resistance of the network as viewed back into the open-circuited network from terminals A and B with all voltage sources replaced by their internal resistance (if any) and current sources by infinite resistance.

* After the French engineer M.L. Thevenin (1857-1926) who while working in Telegraphic Department published a statement of the theorem in 1893.

** Or impedance in the case of a.c. circuits.

2.19. How to Thevenize a Given Circuit ?

1. Temporarily remove the resistance (called load resistance R_L) whose current is required.
2. Find the open-circuit voltage V_{oc} which appears across the two terminals from where resistance has been removed. It is also called Thevenin voltage V_{th}.
3. Compute the resistance of the whose network as looked into from these two terminals after all voltage sources have been removed leaving behind their internal resistances (if any) and current sources have been replaced by open-circuit *i.e.* infinite resistance. It is also called Thevenin resistance R_{th} or T_i.
4. Replace the entire network by a single Thevenin source, whose voltage is V_{th} or V_{oc} and whose internal resistance is R_{th} or R_i.
5. Connect R_L back to its terminals from where it was previously removed.
6. Finally, calculate the current flowing through R_L by using the equation,

$$I = V_{th}/(R_{th} + R_L) \quad \text{or} \quad I = V_{oc}/(R_i + R_L)$$

Example 2.59. *Convert the circuit shown in Fig. 2.129 (a), to a single voltage source in series with a single resistor.* **(AMIE Sec. B, Network Analysis Summer 1992)**

Solution. Obviously, we have to find equivalent Thevenin circuit. For this purpose, we have to calculate **(*i*)** V_{th} or V_{AB} and **(*ii*)** R_{th} or R_{AB}.

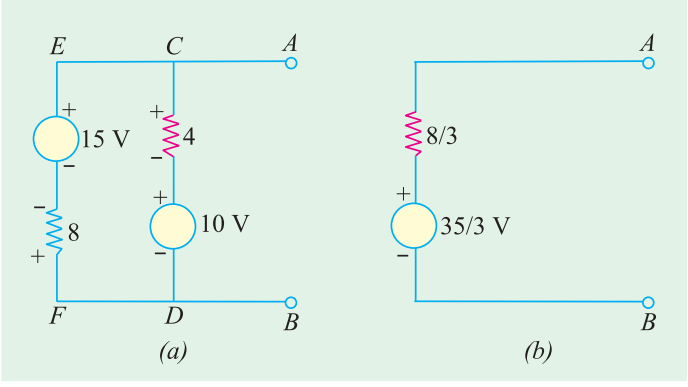


Fig. 2.129

With terminals A and B open, the two voltage sources are connected in subtractive series because they oppose each other. Net voltage around the circuit is (15 –10) = 5 V and total resistance is (8 + 4) = 12 Ω Hence circuit current is = 5/12 A. Drop across 4 Ω resistor = 4 × 5/12 = 5/3 V with the polarity as shown in Fig. 2.129 (*a*).

$$\therefore \qquad V_{AB} = V_{th} = +10 + 5/3 = 35/3 \text{ V}.$$

Incidently, we could also find V_{AB} while going along the parallel route *BFEA*.

Drop across 8 Ω resistor = 8 × 5/12 = 10/3 V. V_{AB} equal the algebraic sum of voltages met on the way from B to A. Hence, $V_{AB} = (-10/3) + 15 = 35/3$ V.

As shown in Fig. 2.129 (*b*), the single voltage source has a voltage of 35/3 V.

For finding R_{th}, we will replace the two voltage sources by short-circuits. In that case, $R_{th} = R_{AB} = 4 \parallel 8 = 8/3\ \Omega$

Example 2.60. *State Thevenin's theorem and give a proof. Apply this theorem to calculate the current through the 4 Ω resistor of the circuit of Fig. 2.130 (a).* **(A.M.I.E. Sec. B Network Analysis W.)**

Solution. As shown in Fig. 2.130 (*b*), 4 Ω resistance has been removed thereby open-circuiting the terminals A and B. We will now find V_{AB} and R_{AB} which will give us V_{th} and R_{th} respectively. The potential drop across 5 Ω resistor can be found with the help of voltage-divider rule. Its value is = 15 × 5/(5 + 10) = 5 V.

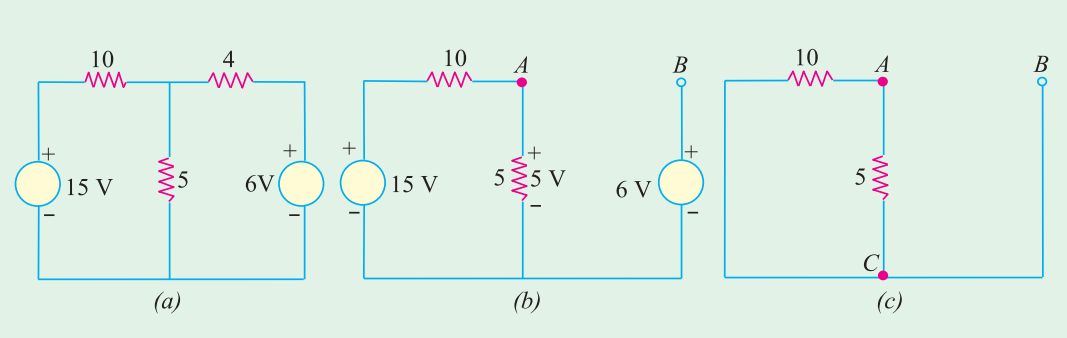


Fig. 2.130

For finding V_{AB}, we will go from point B to point A in the clockwise direction and find the algebraic sum of the voltages met on the way.

$$\therefore \qquad V_{AB} = -6 + 5 = -1 \text{ V.}$$

It means that point A is negative with respect to point E, or point B is at a higher potential than point A by one volt.

In Fig. 2.130 (c), the two voltage source have been short-circuited. The resistance of the network as viewed from points A and B is the same as viewed from points A and C.

$$\therefore \qquad R_{AB} = R_{AC} = 5 \parallel 10 = 10/3\ \Omega$$

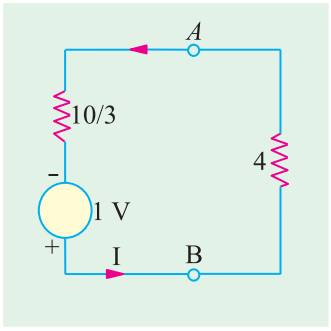


Fig. 2.131

Thevenin's equivalent source is shown in Fig. 2.131 in which 4 Ω resistor has been joined back across terminals A and B. Polarity of the voltage source is worth nothing.

$$\therefore \qquad I = \frac{1}{(10/3) + 4} = \frac{3}{22} = 0.136 \text{ A} \qquad \text{From } E \text{ to } A$$

Example 2.61. *With reference to the network of Fig. 2.132 (a), by applying Thevenin's theorem find the following :*

(i) *the equivalent e.m.f. of the network when viewed from terminals A and B.*

(ii) *the equivalent resistance of the network when looked into from terminals A and B.*

(iii) *current in the load resistance R_L of 15 Ω* **(Basic Circuit Analysis, Nagpur Univ. 1993)**

Solution. *(i)* Current in the network before load resistance is connected [Fig. 2.132 (a)]

$$= 24/(12 + 3 + 1) = 1.5 \text{ A}$$

$\therefore$ voltage across terminals $AB = V_{oc} = V_{th} = 12 \times 1.5 =$ **18 V**

Hence, so far as terminals A and B are concerned, the network has an e.m.f. of 18 volt (and not 24 V).

(ii) There are two parallel paths between points A and B. Imagine that battery of 24 V is removed but not its internal resistance. Then, resistance of the circuit as looked into from point A and B is [Fig. 2.132 (c)]

$$R_i = R_{th} = 12 \times 4/(12 + 4) = 3\ \Omega$$

(iii) When load resistance of 15 Ω is connected across the terminals, the network is reduced to the structure shown in Fig. 2.132 (d).

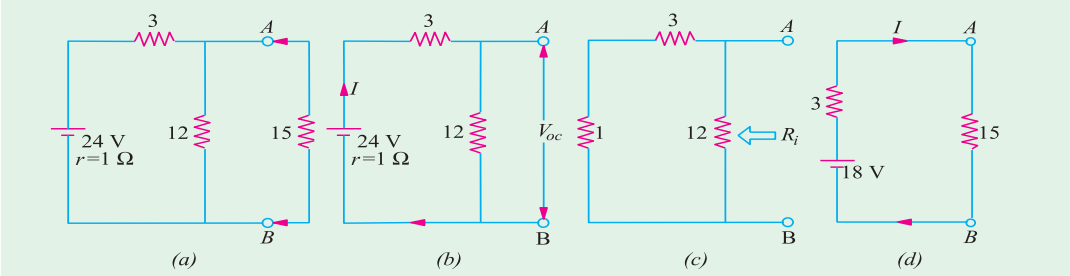


Fig. 2.132

$$I = V_{th}/(R_{th} + R_L) = 18/(15 + 3) = \mathbf{1\ A}$$

Example 2.62. *Using Thevenin theorem, calculate the current flowing through the 4 Ω resistor of Fig. 2.133 (a).*

Solution. (*i*) Finding V_{th}

If we remove the 4-Ω resistor, the circuit becomes as shown in Fig. 2.133 (*b*). Since full 10 A current passes through 2 Ω resistor, drop across it is $10 \times 2 = 20$ V. Hence, $V_B = 20$ V with respect to the common ground. The two resistors of 3 Ω and 6 Ω are connected in series across the 12 V battery. Hence, drop across 6 Ω resistor = $12 \times 6/(3 + 6) = \mathbf{8\ V}$.

$\therefore$ $V_A = 8$ V with respect to the common ground*

$\therefore$ $V_{th} = V_{BA} = V_B - V_A = 20 - 8 = 12$ V—with *B* at a higher potential

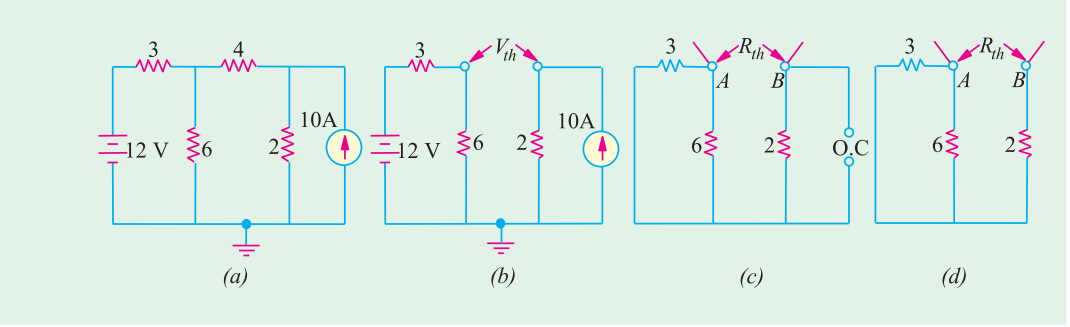


Fig. 2.133

(*ii*) Finding R_{th}

Now, we will find R_{th} *i.e.* equivalent resistance of the network as looked back into the open-circuited terminals *A* and *B*. For this purpose, we will replace both the voltage and current sources. Since voltage source has no internal resistance, it would be replaced by a short circuit *i.e.* zero resistance. However, current source would be removed and replaced by an 'open' *i.e.* infinite resistance (Art. 1.18). In that case, the circuit becomes as shown in Fig. 2.133 (*c*). As seen from Fig. 2.133 (*d*), $F_{th} = 6 \parallel 3 + 2 = 4\ \Omega$. Hence, Thevenin's equivalent circuit consists of a voltage source of 12 V and a series resistance of 4 Ω as shown in Fig. 2.134 (*a*). When 4 Ω resistor is connected across terminals *A* and *B*, as shown in Fig. 2.134 (*b*).

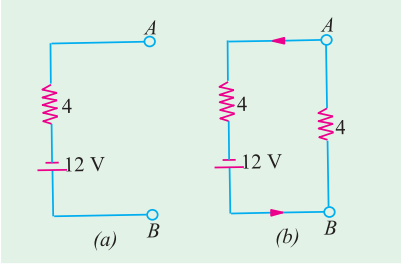


Fig. 2.134

$$I = 12/(4 + 4) = \mathbf{1.5\ A—from\ \textit{B}\ to\ \textit{A}}$$

* Also, V_A = 12 –drop across 3-Ω resistor = $12 - 12 \times 3/(6 + 3) = 12 - 4 = 8$ V

Example 2.63. *For the circuit shown in Fig. 2.135 (a), calculate the current in the 10 ohm resistance. Use Thevenin's theorem only.*

(Elect. Science-I Allahabad Univ. 1992)

Solution. When the 10 Ω resistance is removed, the circuit becomes as shown in Fig. 2.135 (*b*).

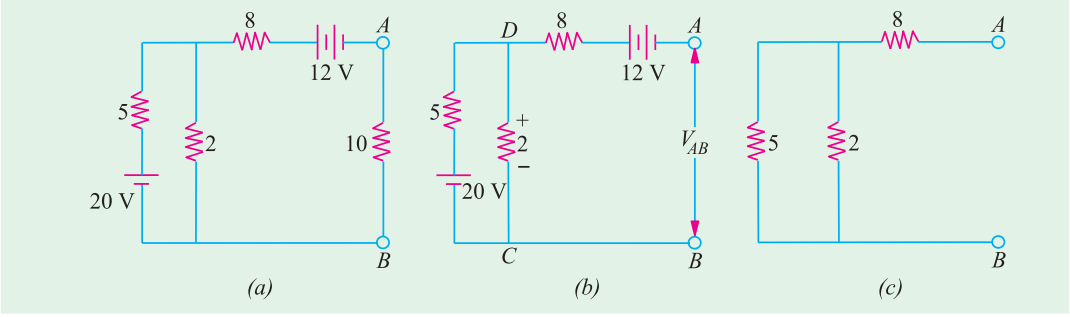


Fig. 2.135

Now, we will find the open-circuit voltage $V_{AB} = V_{th}$. For this purpose, we will go from point *B* to point *A* and find the algebraic sum of the voltages met on the way. It should be noted that with terminals *A* and *B* open, there is no voltage drop on the 8 Ω resistance. However the two resistances of 5 Ω and 2 Ω are connected in series across the 20-V battery. As per voltage-divider rule, drop on 2 Ω resistance = $20 \times 2/(2 + 5) = 5.71$ V with the polarity as shown in figure. As per the sign convention of Art.

$$V_{AB} = V_{th} = +5.71 - 12 = -6.29 \text{ V}$$

The negative sign shows that point *A* is negative with respect to point *B* or which is the same thing, point *B* is positive with respect to point *A*.

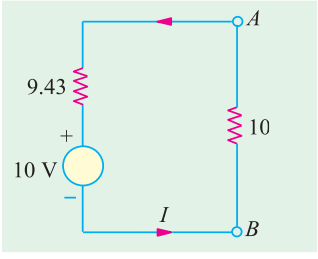


Fig. 2.136 (*a*)

For finding $R_{AB} = R_{th}$, we replace the batteries by short-circuits as shown in Fig. 2.128 (*c*).

$$\therefore \quad R_{AB} = R_{th} = 8 + 2 \parallel 5 = 9.43 \ \Omega$$

Hence, the equivalent Thevenin's source with respect to terminals *A* and *B* is as shown in Fig. 2.136. When 10 Ω resistance is reconnected across *A* and *B*, current through it is $I = 6.24/(9.43 + 10) =$ **0.32 A.**

Example 2.64. *Using Thevenin's theorem, calculate the p.d. across terminals A and B in Fig. 2.137 (a).*

Solution. (*i*) Finding V_{oc}

First step is to remove 7 Ω resistor thereby open-circuiting terminals *A* and *B* as shown in Fig. 2.137 (*b*). Obviously, there is no current through the 1 Ω resistor and hence no drop across it. Therefore $V_{AB} = V_{oc} = V_{CD}$. As seen, current *I* flows due to the combined action of the two batteries. Net voltage in the *CDFE* circuit = 18 −6= 12 V. Total resistance = 6 + 3 = 9 Ω Hence, $I = 12/9 = 4/3$ A

$$V_{CD} = 6 \text{ V} + \text{drop across } 3\ \Omega \text{ resistor} = 6 + (4/3) \times 3 = 10 \text{ V}^*$$

$$\therefore \quad V_{oc} = V_{th} = 10 \text{ V}.$$

(*ii*) Finding R_i or R_{th}

As shown in Fig. 2.137 (*c*), the two batteries have been replaced by short-circuits (SC) since their internal resistances are zero. As seen, $R_i = R_{th} = 1 + 3 \parallel 6 = 3\ \Omega$ The Thevenin's equivalent circuit is as shown in Fig. 2.137 (*d*) where the 7 Ω resistance has been reconnected across terminals *A* and *B*.

---

* Also, V_{CD} = 18-drop across 6 Ω resistor = $18 - (4/3) \times 6 = 10$ V

The p.d. across this resistor can be found with the help of Voltage Divider Rule (Art. 1.15).

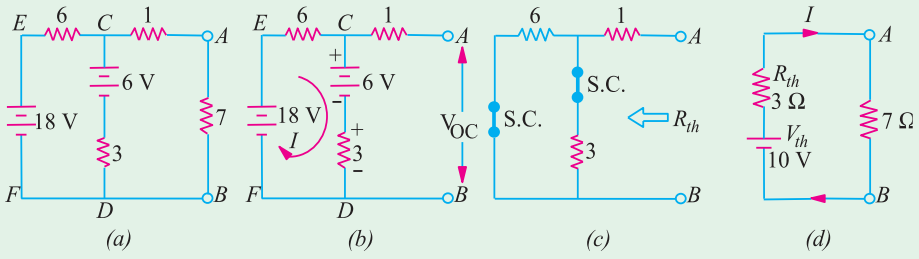


Fig. 2.137

Example 2.65. *Use Thevenin's theorem to find the current in a resistance load connected between the terminals A and B of the network shown in Fig. 2.138 (a) if the load is (a) 2 Ω (b) 1 Ω.*

(Elect. Technology, Gwalior Univ.)

Solution. For finding open-circuit voltage V_{oc} or V_{th} across terminals *A* and *B*, we must first find current I_2 flowing through branch *CD*. Using Maxwell's loop current method (Art. 2.11), we have from Fig. 2.131 (*a*).

$$-2I_1 - 4(I_1 - I_2) + 8 = 0 \quad \text{or} \quad 3I_1 - 2I_2 = 4$$

Also $$-2I_2 - 2I_2 - 4 - 4(I_2 - I_1) = 0 \quad \text{or} \quad I_1 - 2I_2 = 1$$

From these two equations, we get $I_2 = 0.25$ A

As we go from point *D* to *C*, voltage rise $= 4 + 2 \times 0.25 = 4.5$ V

Hence, $V_{CD} = 4.5$ or $V_{AB} = V_{th} = 4.5$ V. Also, it may be noted that point *A* is positive with respect to point *B*.

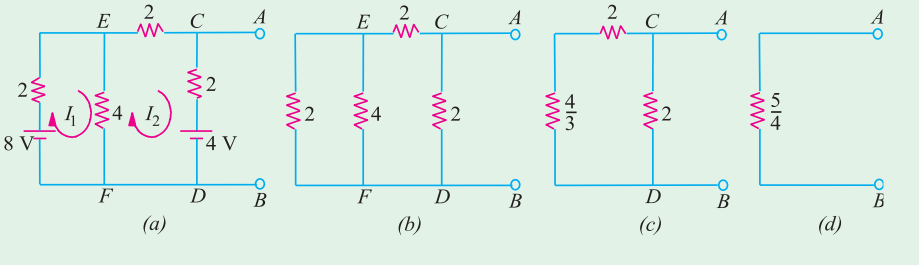


Fig. 2.138

In Fig. 2.138 (*b*), both batteries have been removed. By applying laws of series and parallel combination of resistances, we get $R_i = R_{th} = 5/4\ \Omega = 1.25\ \Omega$.

(*i*) When $R_L = 2\ \Omega$; $I = 4.5/(2 + 1.25) =$ **1.38 A**

(*ii*) When $R_L = 1\ \Omega$; $I = 4.5\ (1 + 1.25) =$ **2.0 A**

Note. We could also find V_{oc} and R_i by first Thevenining part of the circuit across terminals *E* and *F* and then across *A* and *B* (Ex. 2.62).

Example 2.66. *The four arms of a Wheatstone bridge have the following resistances :*

AB = 100, BC = 10, CD = 4, DA = 50 Ω. A galvanometer of 20 Ω resistance is connected across BD. Use Thevenin's theorem to compute the current through the galvanometer when a p.d. of 10 V is maintained across AC. **(Elect. Technology, Vikram Univ. of Ujjain)**

Solution. **(*i*)** When galvanometer is removed from Fig. 2.139 (*a*), we get the circuit of Fig. 2.139 (*b*).

(*ii*) Let us next find the open-circuit voltage V_{oc} (also called Thevenin voltage V_{th}) between points *B* and *D*. Remembering that *ABC* (as well as *ADC*) is a potential divider on which a voltage drop of 10 V takes place, we get

Potential of *B* w.r.t. *C* = $10 \times 10/110 = 10/11 = 0.909$ V

Potential of *D* w.r.t. *C* = $10 \times 4/54 = 20/27 = 0.741$ V

$\therefore$ p.d. between *B* and *D* is V_{oc} or $V_{th} = 0.909 - 0.741 = 0.168$ V

(*iii*) Now, remove the 10-V battery retaining its internal resistance which, in this case, happens to be zero. Hence, it amounts to short-circuiting points *A* and *C* as shown in Fig. 2.139 (*d*).

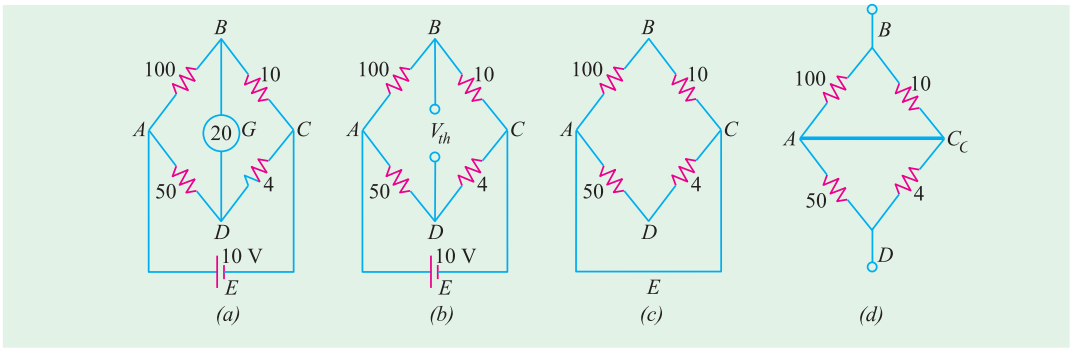


Fig. 2.139

(*iv*) Next, let us find the resistance of the whole network as viewed from points *B* and *D*. It may be easily found by noting that electrically speaking, points *A* and *C* have become one as shown in Fig. 2.140 (*a*). It is also seen that *BA* is in parallel with *BC* and *AD* is in parallel with *CD*. Hence, $R_{BD} = 10 \parallel 100 + 50 \parallel 4 = 12.79\ \Omega$.

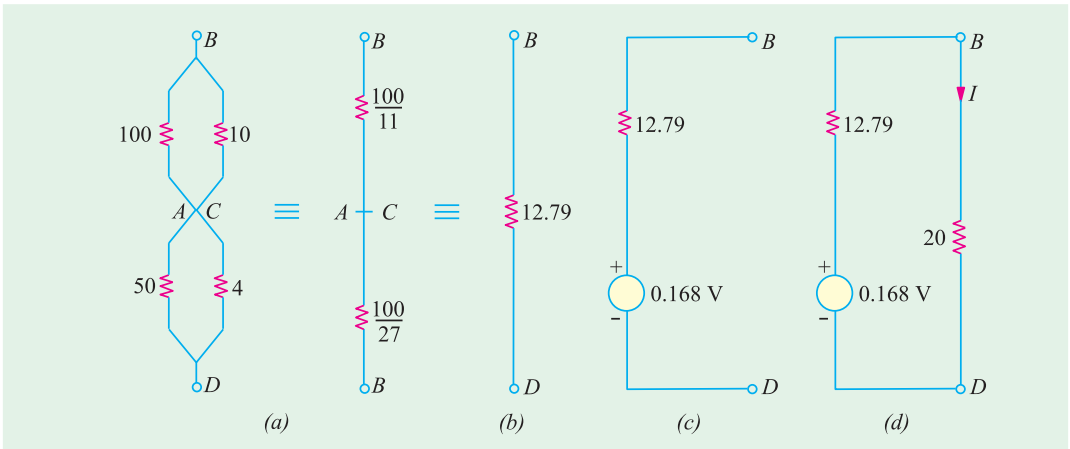


Fig. 2.140

(*v*) Now, so far as points *B* and *D* are connected, the network has a voltage source of 0.168 V and internal resistance $R_i = 12.79\ \Omega$. This Thevenin's source is shown in Fig. 2.140 (*c*).

(*vi*) Finally, let us connect the galvanometer (initially removed) to this Thevenin source and calculate the current *I* flowing through it. As seen from Fig. 2.140 (*d*).

$$I = 0.168/(12.79 + 20) = 0.005 \text{ A} = \mathbf{5\ mA}$$

Example 2.67. *Determine the current in the 1 Ω resistor across AB of network shown in Fig. 2.141 (a) using Thevenin's theorem.* **(Network Analysis, Nagpur Univ. 1993)**

Solution. The given circuit can be redrawn, as shown in Fig. 2.141 (*b*) with the 1 Ω resistor removed from terminals *A* and *B*. The current source has been converted into its equivalent voltage source as shown in Fig. 2.141 (*c*). For finding V_{th}, we will find the currents *x* and *y* in Fig. 2.141 (*c*). Applying *KVL* to the first loop, we get

$$3 - (3 + 2)\, x - 1 = 0 \quad \text{or} \quad x = 0.4 \text{ A}$$

$$\therefore \qquad V_{th} = V_{AB} = 3 - 3 \times 0.4 = 1.8 \text{ V}$$

The value of R_{th} can be found from Fig. 2.141 (*c*) by replacing the two voltage sources by short-circuits. In this case $R_{th} = 2 \parallel 3 = 1.2\ \Omega$

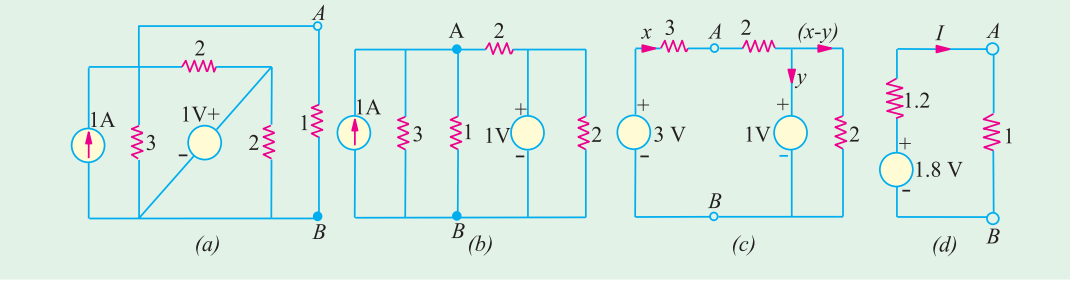


Fig. 2.141

Thevenin's equivalent circuit is shown in Fig. 2.141 (*d*). The current through the reconnected 1 Ω resistor is = 1.8/(12.1 + 1) = 0.82 A.

Example 2.68. *Find the current flowing through the 4 Ω resistor in Fig. 2.142 (a) when (i) E = 2 V and (ii) E = 12 V. All resistances are in series.*

Solution. When we remove *E* and 4 Ω resistor, the circuit becomes as shown in Fig. 2.142 (*b*). For finding R_{th} *i.e.* the circuit resistance as viewed from terminals *A* and *B*, the battery has been short-circuited, as shown. It is seen from Fig. 2.142 (*c*) that $R_{th} = R_{AB} = 15 \parallel 30 + 18 \parallel 9 = 16\ \Omega$

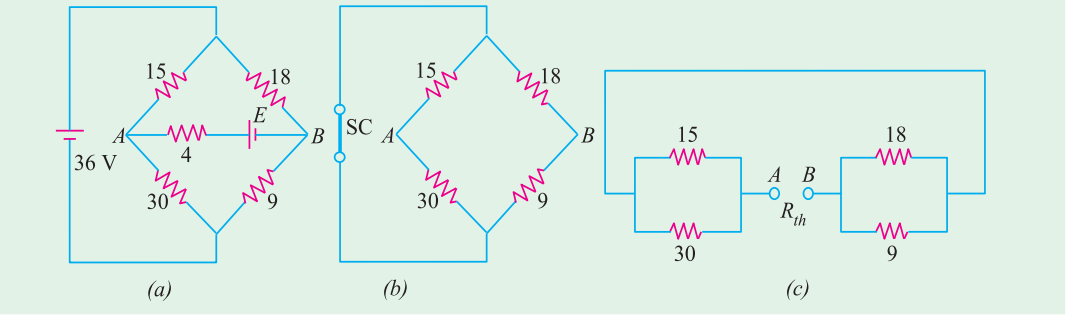


Fig. 2.142

We will find $V_{th} = V_{AB}$ with the help of Fig. 2.143 (*a*) which represents the original circuit, except with *E* and 4 Ω resistor removed. Here, the two circuits are connected in parallel across the 36 V battery. The potential of point *A* equals the drop on 30 Ω resistance, whereas potential of point *B* equals the drop across 9 Ω resistance. Using the voltage,

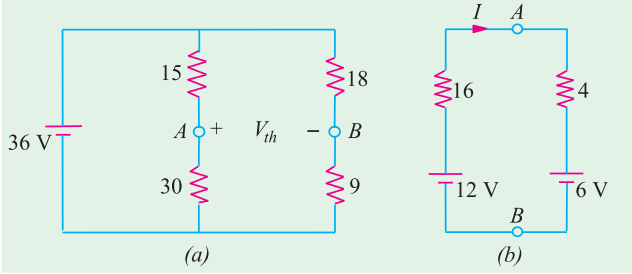


Fig. 2.143

divider rule, we have

$$V_A = 30 \times 30/45 = 24 \text{ V}$$
$$V_B = 36 \times 9/27 = 12 \text{ V}$$
$$\therefore \quad V_{AB} = V_A - V_B = 24 - 12 = 12 \text{ V}$$

In Fig. 2.143 (*b*), the series combination of E and 4 Ω resistors has been reconnected across terminals A and B of the Thevenin's equivalent circuit.

(i) $I = (12 - E)/20 = (12 - 2)/20 = 0.5$ A (*ii*) $I = (12 - 12)/20 = 0$

Example 2.69. *Calculate the value of V_{th} and R_{th} between terminals A and B of the circuit shown in Fig. 2.144 (a). All resistance values are in ohms.*

Solution. Forgetting about the terminal B for the time being, there are two parallel paths between E and F : one consisting of 12 Ω and the other of (4 + 8) = 12 Ω. Hence, $R_{EF} = 12 \parallel 12 = 6$ Ω. The source voltage of 48 V drops across two 6 Ω resistances connected in series. Hence, $V_{EF} = 24$ V. The same 24 V acts across 12 Ω resistor connected directly between E and F and across two series –connected resistance of 4 Ω and 6 Ω connected across E and F. Drop across 4 Ω resistor $= 24 \times 4/(4 + 8) = 8$ V as shown in Fig. 2.144 (*c*).

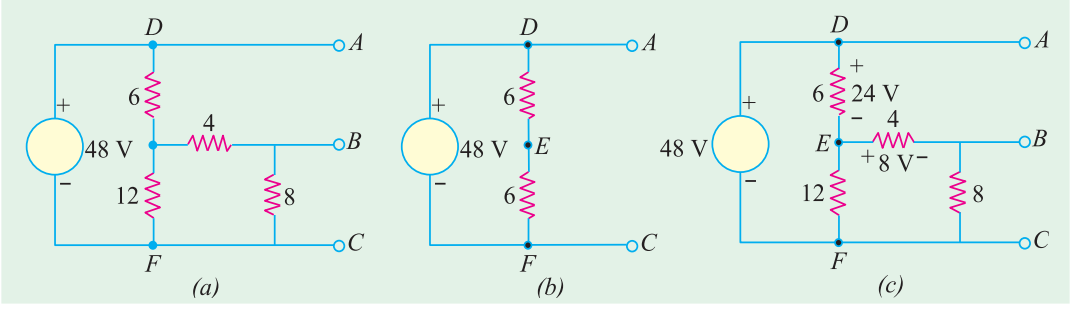


Fig. 2.144

Now, as we go from B to A via point E, there is a rise in voltage of 8 V followed by another rise in voltage of 24 V thereby giving a total voltage drop of 32 V. Hence $V_{th} = 32$ V with point A positive.

For finding R_{th}, we short-circuit the 48 V source. This short circuiting, in effect, combines the points A, D and F electrically as shown in Fig. 2.145 (*a*). As seen from Fig. 2.145 (*b*),

$$R_{th} = V_{AB} = 8 \parallel (4 + 4) = 4 \text{ Ω}$$

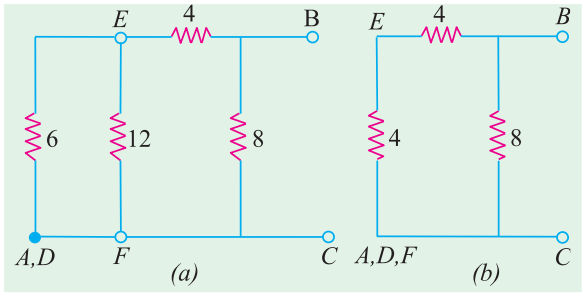


Fig. 2.145

Example 2.70. *Determine Thevenin's equivalent circuit which may be used to represent the given network (Fig. 2.146) at the terminals AB.*

(Electrical Eng.; Calcutta Univ.)

Solution. The given circuit of Fig. 2.146 (*a*) would be solved by applying Thevenin's theorem twice, first to the circuit to the left of point C and D and then to the left of points A and B. Using this technique, the network to the left of CD [Fig. 2.146 (*a*)] can be replaced by a source of voltage V_1 and series resistance R_{i1} as shown in Fig. 2.146 (*b*).

$$V_1 = \frac{12 \times 6}{(6 + 1 + 1)} = 9 \text{ volts and } R_{i1} = \frac{6 \times 2}{(6 + 2)} = 1.5 \text{ Ω}$$

Similarly, the circuit of Fig. 2.146 (*b*) reduced to that shown in Fig. 2.146 (*c*)

$$V_2 = \frac{9 \quad 6}{(6 \quad 2 \quad 1.5)} \quad \textbf{5.68} \text{ volts and } R_{i2} \quad \frac{6 \quad 3.5}{9.5} \quad \textbf{2.21}$$

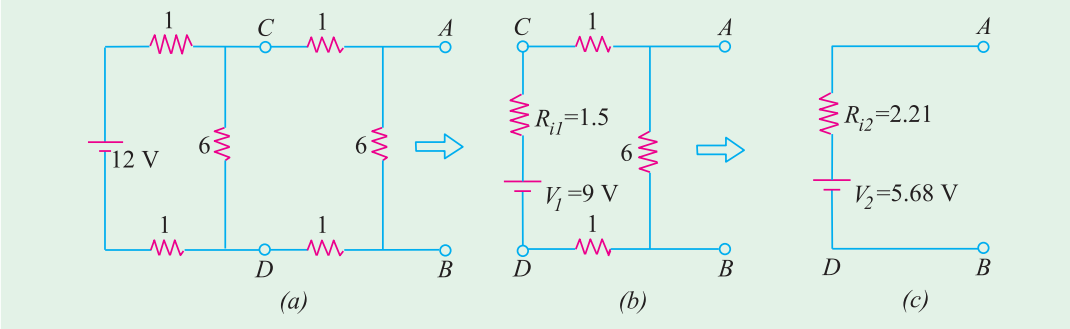


Fig. 2.146

Example 2.71. *Use Thevenin's theorem, to find the value of load resistance R_L in the circuit of Fig. 2.147 (a) which results in the production of maximum power in R_L. Also, find the value of this maximum power. All resistances are in ohms.*

Solution. We will remove the voltage and current sources as well as R_L from terminals A and B in order to find R_{th} as shown in Fig. 2.147 (b).

$$R_{th} = 4 + 6 \parallel 3 = 6\ \Omega$$

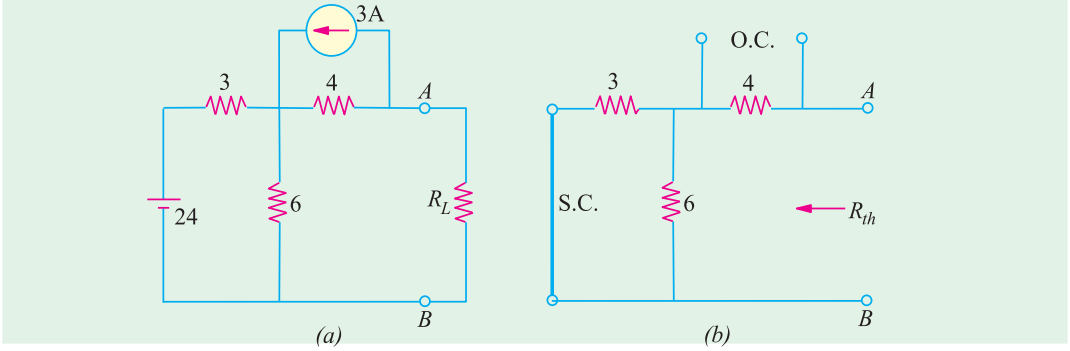


Fig. 2.147

In Fig. 2.147 (a), the current source has been converted into the equivalent voltage source for convenience. Since there is no current 4 Ω resistance (and hence no voltage drop across it), V_{th} equals the algebraic sum of battery voltage and drop across 6 Ω resistor. As we go along the path *BDCA*, we get,

$V_{th} = 24 \times 6/(6 + 3) - 12 = 4$ V

The load resistance has been reconnected to the Thevenin's equivalent circuit as shown in Fig. 2.148 (b). For maximum power transfer, $R_L = R_{th} = 6\ \Omega$

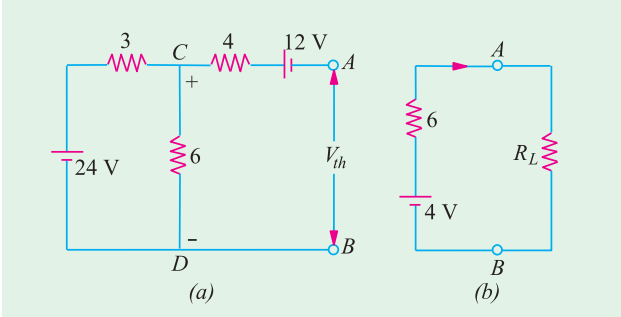


Fig. 2.148

Now, $V_L = \frac{1}{2} V_{th} \quad \frac{1}{2} \quad 4 \quad 2$ V; $P_{L\ max} \quad \frac{V_L^2}{R_L} \quad \frac{2^2}{6}$ **0.67 W**

Example 2.72. *Use Thevenin's theorem to find the current flowing through the 6 Ω resistor of the network shown in Fig. 2.149 (a). All resistances are in ohms.*

(Network Theory, Nagpur Univ. 1992)

Solution. When 6 Ω resistor is removed [Fig. 2.149 (*b*)], whole of 2 A current flows along *DC* producing a drop of (2 × 2) = 4 V with the polarity as shown. As we go along *BDCA*, the total voltage is

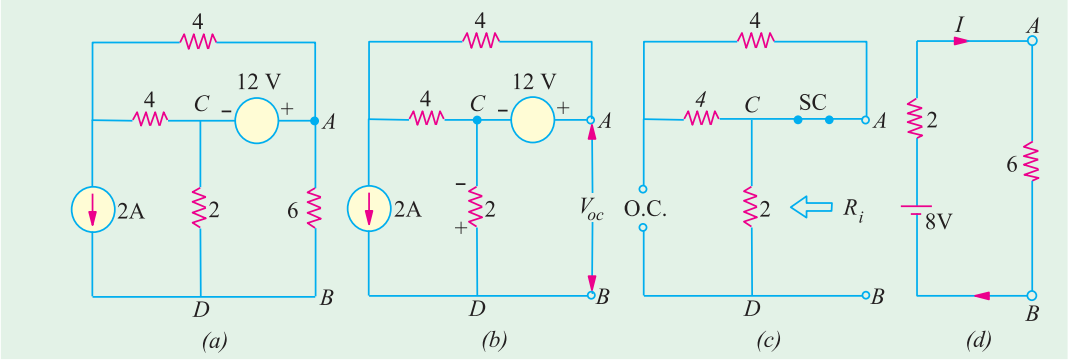


Fig. 2.149

$$= -4 + 12 = 8 \text{ V} \qquad \text{—with } A \text{ positive w.r.t. } B.$$

Hence, $V_{oc} = V_{th} = 8$ V

For finding R_i or R_{th} 18 V voltage source is replaced by a short-circuit (Art- 2.15) and the current source by an open-circuit, as shown in Fig. 2.149 (*c*). The two 4 Ω resistors are in series and are thus equivalent to an 8 Ω resistance. However, this 8 Ω resistor is in parallel with a short of 0 Ω. Hence, their equivalent value is 0 Ω. Now this 0 Ω resistance is in series with the 2 Ω resistor. Hence, $R_i = 2 + 0 = 2$ Ω. The Thevenin's equivalent circuit is shown in Fig. 2.149 (*d*).

$$\therefore \qquad I = 8/(2 + 6) = \mathbf{1 \text{ Amp}} \qquad \text{—from } A \text{ to } B$$

Example 2.73. *Find Thevenin's equivalent circuit for the network shown in Fig. 2.150 (a) for the terminal pair AB.*

Solution. It should be carefully noted that after coming to point *D*, the 6 *A* current has only one path to reach its other end *C i.e.*, through 4 Ω resistor thereby creating and *IR* drop of 6 × 4 = 24 V with polarity as shown in Fig. 2.150 (*b*). No part of it can go along *DE* or *DF* because it would not find any path back to point *C*. Similarly, current due to 18-V battery is restricted to loop *EDFE*. Drop across 6 Ω resistor = 18 × 6/(6 + 3) = 12 V. For finding V_{AB}, let us start from *A* and go to *B* via the *shortest* route *ADFB*. As seen from Fig. 2.150 (*b*), there is a *rise* of 24 V from *A* to *D* but a ***fall*** of 12 V.

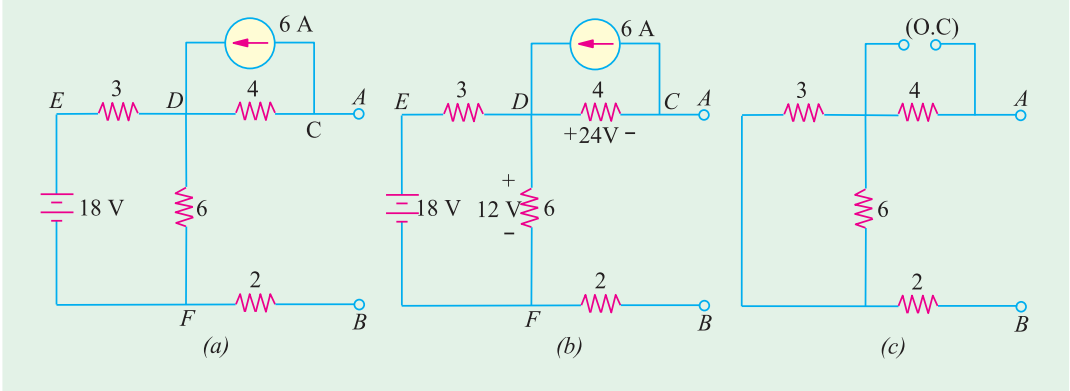


Fig. 2.150

from D to F. Hence, $V_{AB} = 24 - 12 = 12$ V with point A negative w.r.t. point B*. Hence, $V_{th} = V_{AB} = -12$ V (or $V_{BA} = 12$ V).

For finding R_{th}, 18 V battery has been replaced by a short-circuit and 6 A current source by an open-circuit, as shown in Fig. 2.150 (*c*).

As seen, $R_{th} = 4 + 6 \parallel 3 + 2$

$= 4 + 2 + 2 = 8\ \Omega$

Hence, Thevenin's equivalent circuit for terminals A and B is as shown in Fig. 2.151. It should be noted that if a load resistor is connected across AB, current through it will flow from B to A.

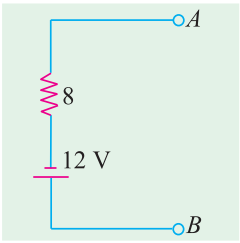


Fig. 2.151

Example 2.74. *The circuit shown in Fig. 2.152 (a) contains two voltage sources and two current sources. Calculate (a) V_{th} and (b) R_{th} between the open terminals A and B of the circuit. All resistance values are in ohms.*

Solution. It should be understood that since terminals A and B are open, 2 A current can flow only through 4 Ω and 10 Ω resistors, thus producing a drop of 20 V across the 10 Ω resistor, as shown in Fig. 2.152 (*b*). Similarly, 3 A current can flow through its own closed circuit between A and C thereby producing a drop of 24 V across 8 Ω resistor as shown in Fig. 2.152 (*b*). Also, there is no drop across 2 Ω resistor because no current flows through it.

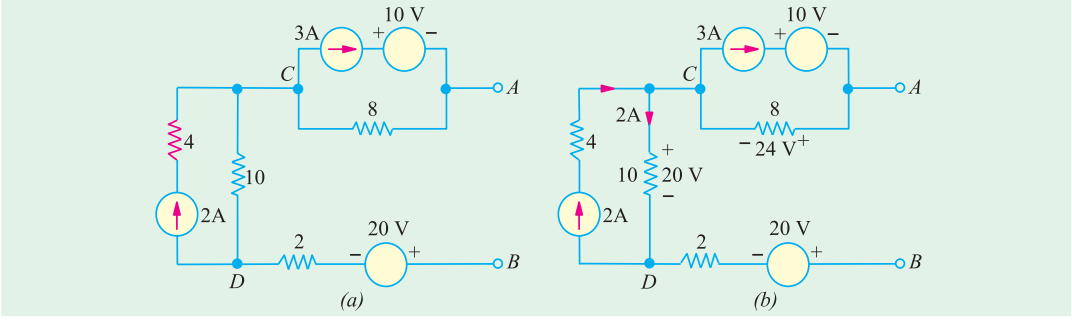


Fig. 2.152

Starting from point B and going to point A via points D and C, we get

$V_{th} = -20 + 20 + 24 = 24$ V

—with point A positive.

For finding R_{th}, we will short-circuit the voltage sources and open-circuit the current sources, as shown in Fig. 2.153. As seen, $R_{th} = R_{AB} = 8 + 10 + 2 = 20\ \Omega$.

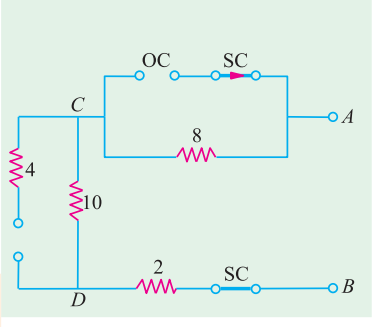


Fig. 2.153

Example 2.75. *Calculate V_{th} and R_{th} between the open terminals A and B of the circuit shown in Fig. 2.154 (a). All resistance values are in ohms.*

Solution. We will convert the 48 V voltage source with its series resistance of 12 Ω into a current source of 4 A, with a parallel resistance of 12 Ω, as shown in Fig. 2.154 (*b*).

In Fig. 2.154 (*c*), the two parallel resistance of 12 Ω each have been combined into a single resistance of 6 Ω. It is obvious that 4 A current flows through the 6 Ω resistor, thereby producing a drop of $6 \times 4 = 24$ V. Hence, $V_{th} = V_{AB} = 24$ V with terminal A negative. In other words $V_{th} = -24$ V.

If we open-circuit the 8 A source and short-circuit the 48-V source in Fig. 2.154 (*a*), $R_{th} = R_{AB} = 12 \parallel 12 = 6\ \Omega$.

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* Incidentally, had 6 A current been flowing in the opposite direction, polarity of 24 V drop would have been reversed so that V_{AB} would have equalled (24 + 12) = 36 V with A positive w.r.t. point B.

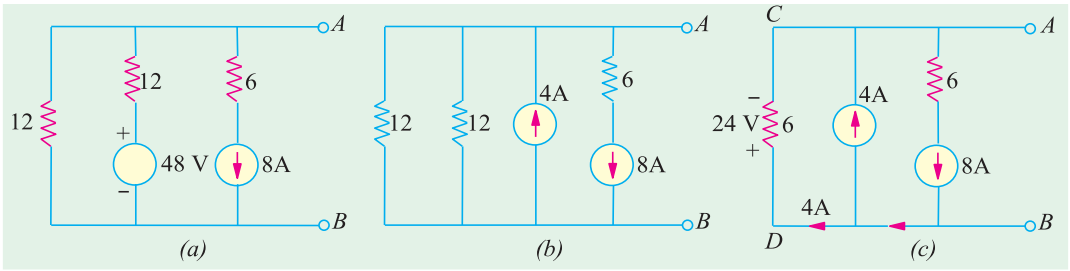


Fig. 2.154

Example 2.76. *Calculate the value of V_{th} of R_{th} between the open terminals A and B of the circuit shown in Fig. 2.155 (a). All resistance values are in ohms.*

Solution. It is seen from Fig. 2.155 (*a*) that positive end of the 24 V source has been shown connected to point *A*. It is understood that the negative terminal is connected to the ground terminal *G*. Just to make this point clear, the given circuit has been redrawn in Fig. 2.155 (*b*) as well as in Fig. 2.155 (*c*).

Let us start from the positive terminal of the battery and go to its negative terminal *G* via point *C*. We find that between points *C* and *G*, there are two parallel paths : one of 6 Ω resistance and the

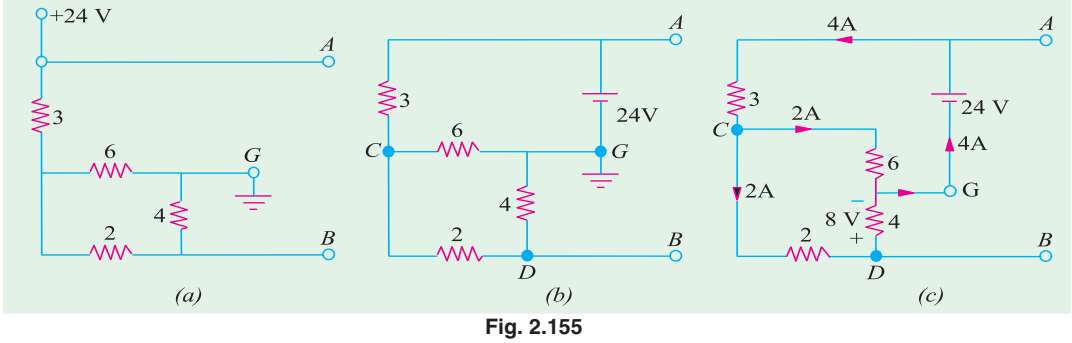


Fig. 2.155

other of (2 + 4) = 6 Ω resistance, giving a combined resistance of 6 || 6 = 3 Ω. Hence, total resistance between positive and negative terminals of the battery = 3 + 3 = 6 Ω. Hence, battery current = 24/6 = 4 A. As shown in Fig. 2.155 (*c*), this current divides equally at point *C*. Let us go from *B* to *A* via points *D* and *G* and total up the potential difference between the two, $V_{th} = V_{AB} = -8$ V + 24 V = 16 V with point *A* positive.

For finding R_{th}, let us replace the voltage source by a short-circuit, as shown in Fig. 2.156 (*a*). It connects one end each of 6 Ω resistor and 4 Ω resistor directly to point *A*, as shown in Fig. 2.156 (*b*). The resistance of branch *DCG* = 2 + 6 || 3 = 4 Ω. Hence $R_{th} = R_{AB} = 4 \parallel 4 = 2$ Ω

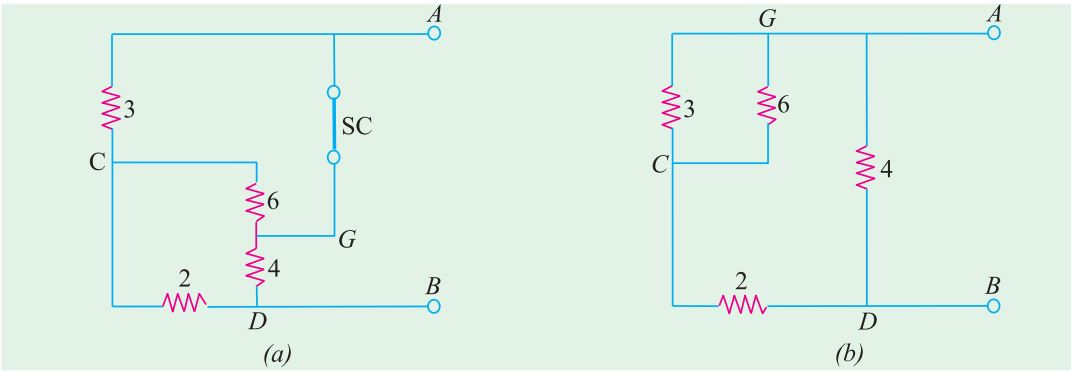


Fig. 2.156

Example 2.77. *Calculate the power which would be dissipated in the 8-Ω resistor connected across terminals A and B of Fig. 2.157 (a). All resistance values are in ohms.*

Solution. The open-circuit voltage V_{oc} (also called Thevenin's voltage V_{th}) is that which appears across terminals *A* and *B*. This equals the voltage drop across 10 Ω resistor between points *C* and *D*. Let us find this voltage. With *AB* an open-circuit, 120-V battery voltage acts on the two parallel paths *EF* and *ECDF*. Hence, current through 10 Ω resistor is

$$I = 120/(20 + 10 + 20) = 2.4 \text{ A}$$

Drop across 10-Ω resistor, $V_{th} = 10 \times 2.4 = 24$ V

Now, let us find Thevenin's resistance R_{th} *i.e.* equivalent resistance of the given circuit when looked into from terminals *A* and *B*. For this purpose, 120 V battery is removed. The results in shorting the 40-Ω resistance since internal resistance of the battery is zero as shown in Fig. 2.157 (*b*).

$$\therefore \quad R_i \text{ or } R_{th} = 16 + \frac{10 \times (20 + 20)}{10 + (20 + 20)} + 16 = 40\ \Omega$$

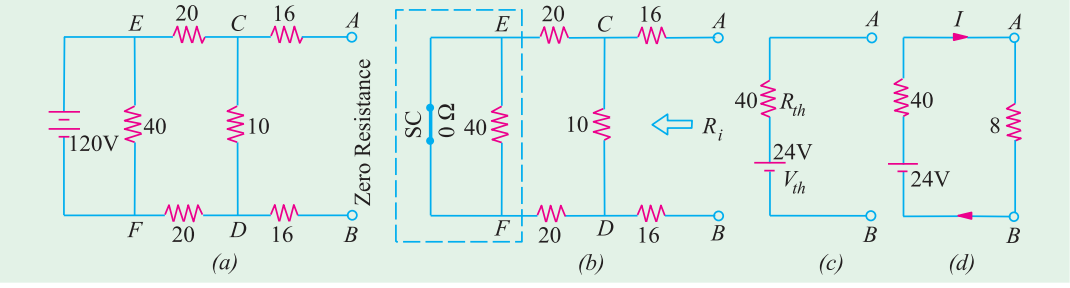


Fig. 2.157

Thevenin's equivalent circuit is shown in Fig. 2.157 (*c*). As shown in Fig. 2.157 (*d*), current through 8-Ω resistor is

$$I = 24/(40 \quad 8) \quad \frac{1}{2} \text{ A} \qquad P \quad I^2R \quad \frac{1}{2}^{2} \quad 8 \quad \mathbf{2\ W}$$

Example 2.78. *With the help of Thevenin's theorem, calculate the current flowing through the 3-Ω resistor in the network of Fig. 2.158 (a). All resistances are in ohms.*

Solution. The current source has been converted into an equivalent voltage source in Fig. 158 (*b*).

***(i)* Finding V_{oc}.** As seen from Fig. 2.158 (*c*), $V_{oc} = V_{CD}$. In closed circuit *CDFEC*, net voltage = 24 –8 = 16 *V* and total resistance = 8 + 4 + 4 = 16 Ω. Hence, current = 16/16 = 1 A.

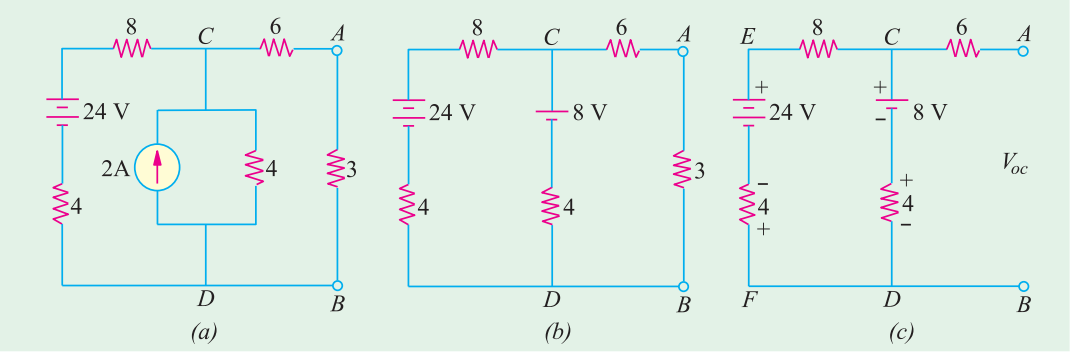


Fig. 2.158

Drop over the 4-Ω resistor in branch $CD = 4 \times 1 = 4$ V with a polarity which is in series addition with 8-V battery.

Hence, $V_{oc} = V_{th} = V_{CD} = 8 + 4 = 12$ V

(ii) Finding R_i or R_{th}. In Fig. 2.159 (*a*), the two batteries have been replaced by short-circuits because they do not have any internal resistance.

As seen, $R_i = 6 + 4 \parallel (8 + 4) = 9\ \Omega$

The Thevenin's equivalent circuit is as shown in Fig. 2.159 (*b*).

$$I = 12/(9 + 3) = \mathbf{1\ A}$$

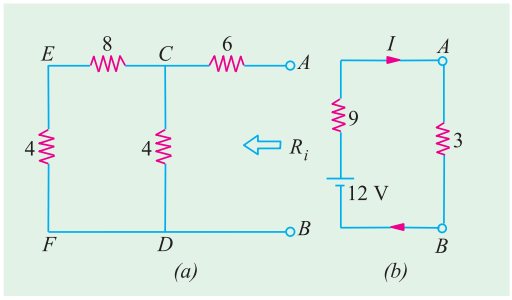


Fig. 2.159

Example 2.79. *Using Thevenin and Superposition theorems find complete solution for the network shown in Fig. 2.160 (a).*

Solution. First, we will find R_{th} across open terminals *A* and *B* and then find V_{th} due to the voltage sources only and then due to current source only and then using Superposition theorem, combine the two voltages to get the single V_{th}. After that, we will find the Thevenin equivalent.

In Fig. 2.160 (*b*), the terminals *A* and *E* have been open-circuited by removing the 10 V source and the 1 Ω resistance. Similarly, 24 V source has been replaced by a short and current source has been replaced by an infinite resistance *i.e.* by open-circuit. As seen, $R_{AB} = R_{th} = 4 \parallel 4 = 2\ \Omega$

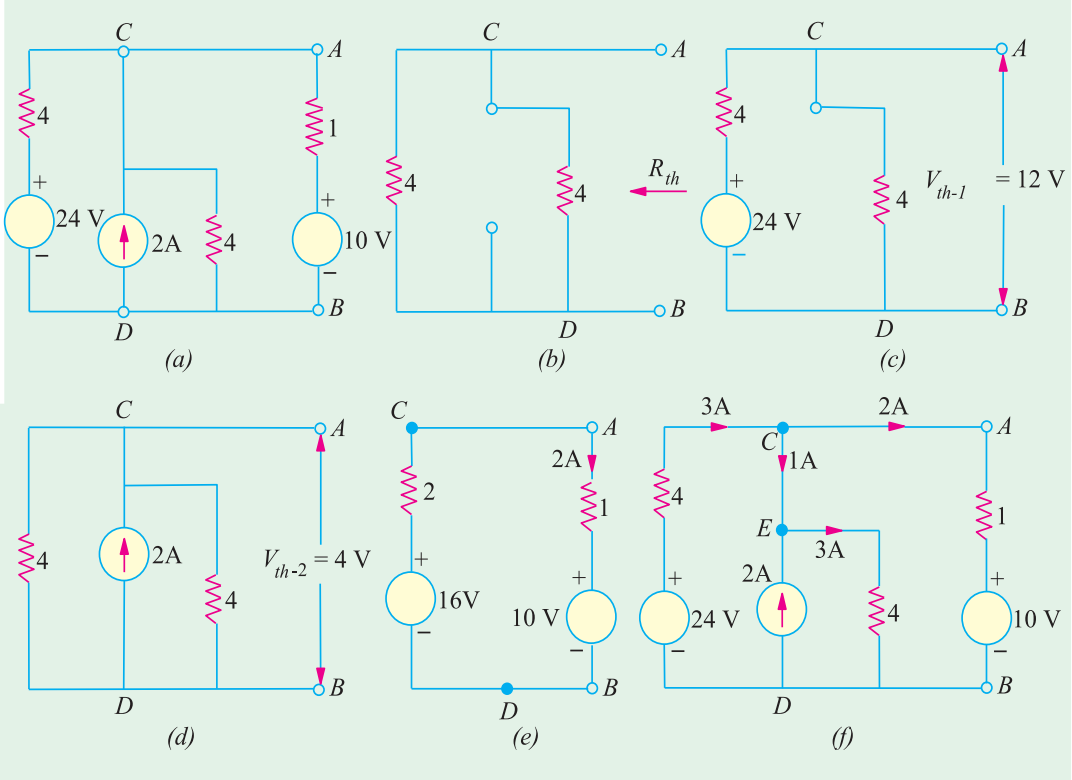


Fig. 2.160

We will now find $V_{th\,-1}$ across AB due to 24 V source only by open-circuiting the current source. Using the voltage-divider rule in Fig. 2.160 (*c*), we get $V_{AB} = V_{CD} = V_{th\,-1} = 24/2 = 12$ V.

Taking only the current source and short-circuiting the 24 V source in Fig. 2160 (*d*), we find that there is equal division of current at point C between the two 4 Ω parallel resistors. Therefore, $V_{th\,-2} = V_{AB} = V_{CD} = 1\times\ 4 = 4$ V.

Using Superposition theorem, $V_{th} = V_{th\,-1} + V_{th\,-2} = 12 + 4 = 16$ V. Hence, the Thevenin's equivalent consists of a 16 V source in series with a 2 Ω resistance as shown in Fig. 2.160 (*e*) where the branch removed earlier has been connected back across the terminals A and B. The net voltage around the circuit is = 16 –10 = 6 V and total resistance is = 2 + 1 = 3 Ω. Hence, current in the circuit is = 6/3 = 2 A. Also, $V_{AB} = V_{AD} = 16 - (2\times\ 2) = 12$ V. Alternatively, V_{AB} equals $(2\times\ 1) + 10 = 12$ V.

Since we know that $V_{AB} = V_{CD} = 12$ V, we can find other voltage drops and various circuit currents as shown in Fig. 2.160 (*f*). Current delivered by the 24-V source to the node C is $(24 - V_{CD})/4 = (24 - 12)/4 = 3$ A. Since current flowing through branch AB is 2 A, the balance of 1 A flows along CE. As seen, current flowing through the 4 Ω resistor connected across the current source is = (1 + 2) = 3 A.

Example 2.80. *Use Superposition Theorem to find I in the circuit of Fig. 2.161.*

[Nagpur Univ. Summer 2001]

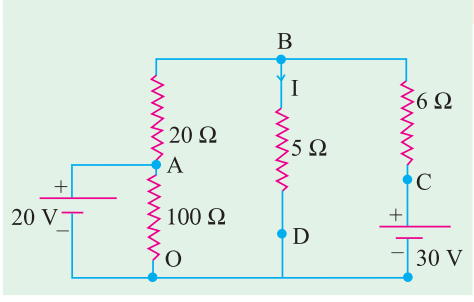


Fig. 2.161. Given Circuit

Solution. At a time, one source acts and the other is de-activated, for applying Superposition theorem. If I_1 represents the current in 5-ohm resistor due to 20-V source, and I_2 due to 30-V source,

$$I = I_1 + I_2$$

Due to 20-V source, current into node B

$$= 20/(20 + 5/6) = 0.88 \text{ amp}$$

Out of this, $I_1 = 0.88\times\ 6/11 = 0.48$ amp

Due to 30-V source, current into node B

$$= 30/(6 + 5/20) = 3 \text{ amp}$$

Out of this, $I_2 = 3\times\ 20/25 = 2.4$ amp

Hence, $I = 2.88$ amp

Alternatively, Thevenin's theorem can be applied at nodes BD after removing 5-ohms resistor from its position. Following the procedure to evaluate V_{TH} and $R_{TH,}$

Thevenin-voltage, $V_{TH} = 27.7$ Volts

and $R_{TH} = 4.62$ Ohms

Current, $I = 27.7/(4.62 + 5) = 2.88$ amp

2.20. General Instructions for Finding Thevenin Equivalent Circuit

So far, we have considered circuits which consisted of resistors and independent current or voltage sources only. However, we often come across circuits which contain both independent and dependent sources or circuits which contain only dependent sources. Procedure for finding the value of V_{th} and R_{th} in such cases is detailed below :

(*a*) When Circuit Contains Both Dependent and Independent Sources

(*i*) The open-circuit voltage V_{oc} is determined as usual with the sources activated or 'alive'.

(*ii*) A short-circuit is applied across the terminals a and b and the value of short-circuit current i_{th} is found as usual.

(*iii*) Thevenin resistance $R_{th} = v_{oc}/i_{sh}$. It is the same procedure as adopted for Norton's theorem. Solved examples 2.81 to 2.85 illustrate this procedure.

(*b*) When Circuit Contains Dependent Sources Only

(*i*) In this case, $v_{oc} = 0$

(*ii*) We connect 1 A source to the terminals *a* and *b* and calculate the value of v_{ab}.

(*iii*) $R_{th} = V_{ab}/1\ \Omega$

The above procedure is illustrated by solved examples.

Example 2.81. *Find Thevenin equivalent circuit for the network shown in Fig. 2.162 (a) which contains a current controlled voltage source (CCVS).*

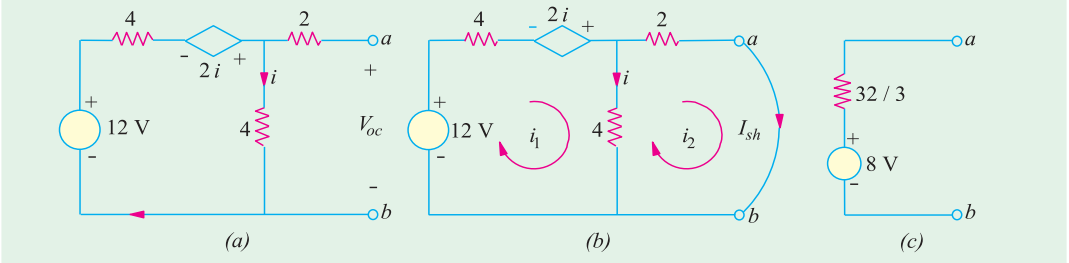


Fig. 2.162

Solution. For finding V_{oc} available across open-circuit terminals *a* and *b*, we will apply *KVL* to the closed loop.

$$\therefore \qquad 12 - 4\,i \times 2\,i - 4\,i = 0 \quad \therefore\ i = 2\ \text{A}$$

Hence, V_{oc} = drop across 4 Ω resistor = 4 × 2 = 8 V. It is so because there is no current through the 2 Ω resistor.

For finding R_{th}, we will put a short-circuit across terminals *a* and *b* and calculate I_{sh}, as shown in Fig. 2.162 (*b*). Using the two mesh currents, we have

$12 - 4\,i_1 + 2\,i - 4(i_1 - i_2) = 0$ and $-8\,i_2 - 4\,(i_2 - i_1) = 0$. Substituting $i = (i_1 - i_2)$ and Simplifying the above equations, we have

$$12 - 4\,i_1 + 2\,(i_1 - i_2) - 4\,(i_1 - i_2) = 0 \quad \text{or} \quad 3\,i_1 - i_2 = 6 \qquad \text{...(i)}$$

Similarly, from the second equation, we get $i_1 = 3\,i_2$. Hence, $i_2 = 3/4$ and $R_{th} = V_{oc}/I_{sh} = 8/(3/4) = 32/3\ \Omega$. The Thevenin equivalent circuit is as shown in Fig. 2.162 (*c*).

Example 2.82. *Find the Thevenin equivalent circuit with respect to terminals a and b of the network shown in Fig. 2.163 (a).*

Solution. It will be seen that with terminals *a* and *b* open, current through the 8 Ω resistor is $v_{ab}/4$ and potential of point *A* is the same that of point a (because there is no current through 4 Ω resistor). Applying *KVL* to the closed loop of Fig. 2.163 (*a*), we get

$$6 + (8 \times v_{ab}/4) - v_{ab} = 0 \quad \text{or} \quad v_{ab} = 12\ \text{V}$$

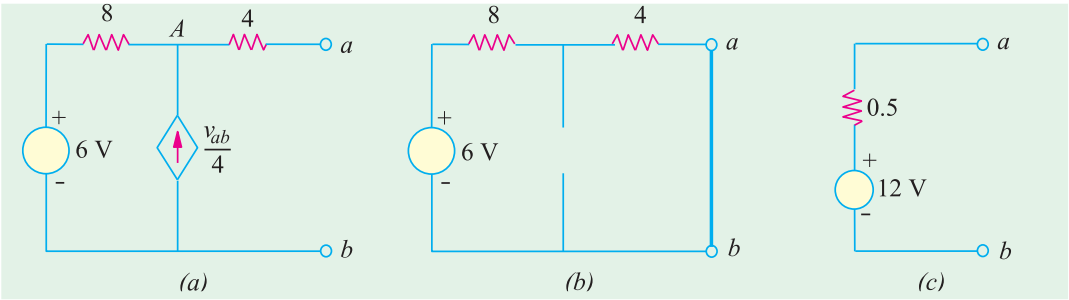


Fig. 2.163

It is also the value of the open-circuit voltage v_{oc}.

For finding short-circuit current i_{sh}, we short-circuit the terminals a and b as shown in Fig. 2.163 (b). Since with a and b short-circuited, $v_{ab} = 0$, the dependent current source also becomes zero. Hence, it is replaced by an open-circuit as shown. Going around the closed loop, we get

$$12 - i_{sh}(8 + 4) = 0 \quad \text{or} \quad i_{sh} = 6/12 = 0.5 \text{ A}$$

Hence, the Thevenin equivalent is as shown in Fig. 2.163 (c).

Example 2.83. *Find the Thevenin equivalent circuit for the network shown in Fig. 2.164 (a) which contains only a dependent source.*

Solution. Since circuit contains no independent source, $i = 0$ when terminals a and b are open. Hence, $v_{oc} = 0$. Moreover, i_{sh} is zero since $v_{oc} = 0$.

Consequently, R_{sh} cannot be found from the relation $R_{th} = v_{oc}/i_{sh}$. Hence, as per Art. 2.20, we will connect a 1 A current source to terminals a and b as shown in Fig. 2.164 (b). Then by finding the value of v_{ab}, we will be able to calculate $R_{th} = v_{ab}/1$.

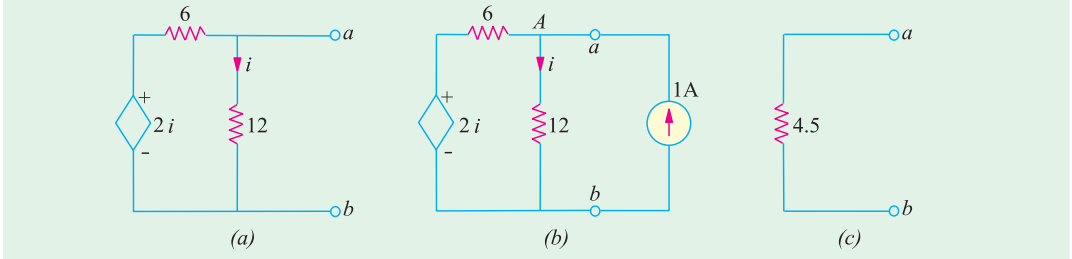


Fig. 2.164

It should be noted that potential of point A is the same as that of point a *i.e.* voltages across 12 Ω resistor is v_{ab}. Applying *KCL* to point A, we get

$$\frac{2i - v_{ab}}{6} - \frac{v_{ab}}{12} + 1 = 0 \quad \text{or} \quad 4i - 3v_{ab} = -12$$

Since $i = v_{ab}/12$, we have $4(v_{ab}/12) - 3v_{ab} = -12$ or $v_{ab} = 4.5$ V $\therefore R_{th} = v_{ab}/1 = 4.5/1 =$ **4.5 Ω**

The Thevenin equivalent circuit is shown in Fig. 2.164 (c).

Example 2.84. *Determine the Thevenins equivalent circuit as viewed from the open-circuit terminals a and b of the network shown in Fig. 2.165 (a). All resistances are in ohms.*

Solution. It would be seen from Fig. 2.165(a) that potential of node A equals the open-circuit terminal voltage v_{oc}. Also, $i = (v_s - v_{oc})/(80 + 20) = (6 - v_{oc})/100$.

Applying *KCL* to node, A we get

$$\frac{6 - V_{oc}}{100} + \frac{9 \times (6 - v_{oc})}{100} - \frac{V_{oc}}{10} = \quad \text{or} \quad V_{oc} = 3 \text{ V}$$

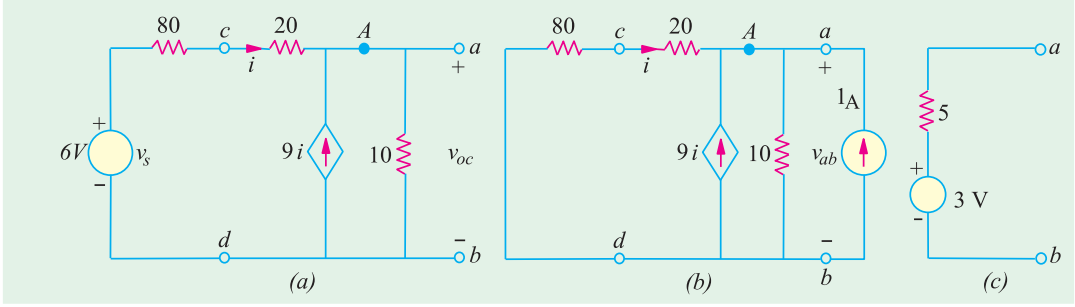


Fig. 2.165

For finding the Thevenin's resistance with respect to terminals *a* and *b*, we would first 'kill' the independent voltage source as shown in Fig. 2.165 (*b*). However, the dependent current source cannot be 'killed'. Next, we will connect a current source of 1 A at terminals *a* and *b* and find the value of v_{ab}. Then, Thevenin's resistance $R_{th} = v_{ab}/1$. It will be seen that current flowing away from node *A i.e.* from point *c* to *d* is $= v_{ab}/100$. Hence, $i = -v_{oc}/100$. Applying *KCL* to node *A*, we get

$$-\frac{v_{ab}}{100} + 9\left(-\frac{v_{ab}}{100}\right) - \frac{v_{ab}}{10} + 1 = 0 \quad \text{or} \quad v_{ab} = 5 \text{ V}$$

$\therefore$ $R_{th} = 5/1 = 5\ \Omega$. Hence, Thevenin's equivalent source is as shown in Fig. 2.165 (*c*).

Example 2.85. *Find the Thevenin's equivalent circuit with respect to terminals a and b of the network shown in Fig. 2.166 (a). All resistances are in ohms.*

Solution. It should be noted that with terminals *a* and *b* open, potential of node *A* equals v_{ab}. Moreover, $v = v_{ab}$. Applying *KCL* to node *A*, we get

$$-5 - \frac{v_{ab}}{15} + \frac{1}{10}\left[\left(\frac{v_{ab}}{3} + 150\right) - V_{ab}\right] = 0 \quad \text{or} \quad V_{ab} = 75 \text{ V}$$

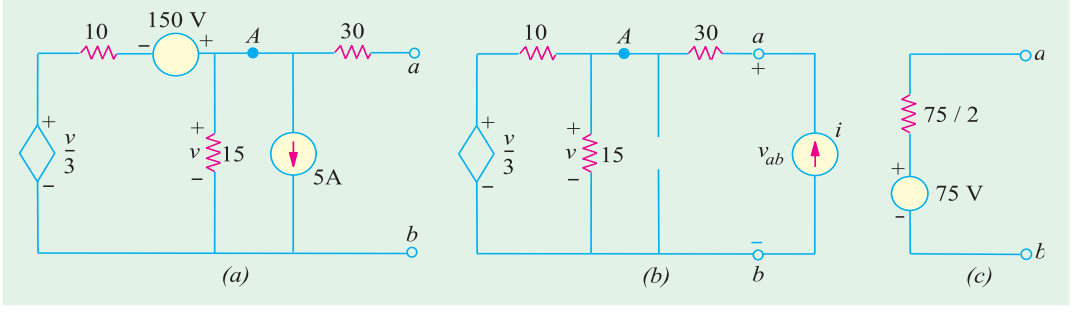


Fig. 2.166

For finding R_{th}, we will connect a current source of *iA** across terminals *a* and *b*. It should be particularly noted that in this case the potential of node *A* equals $(v_{ab} - 30\ i)$. Also, $v = (v_{ab} - 30\ i)$ = potential of node *A*, Applying *KCL* to node *A*, we get from Fig. 2.166 (*b*).

$$i = \frac{(v_{ab} - 30\ i)}{15} + \frac{1}{10}\left[\left(\frac{v_{ab} - 30\ i}{3}\right) - (v_{ab} - 30\ i)\right] = 0$$

$\therefore$ $4\ v_{ab} = 150\ i$ or $v_{ab}/i = 75/2\ \Omega$. Hence, $R_{th} = v_{ab}/i = 75/2\ \Omega$ The Thevenin's equivalent circuit is shown in Fig. 2.166 (*c*).

2.21. Reciprocity Theorem

It can be stated in the following manner :

In any linear bilateral network, if a source of e.m.f. E in any branch produces a current I in any other branch, then the same e.m.f. E acting in the second branch would produce the same current I in the first branch.

In other words, it simply means that *E* and *I* are mutually transferrable. The ratio *E/I* is known as the ***transfer*** resistance (or impedance in a.c. systems). Another way of stating the above is that the receiving point and the sending point in a network are interchangebale. It also means that interchange of an *ideal* voltage sources and an ***ideal*** ammeter in any network will not change the ammeter reading. Same is the case with the interchange of an ideal current source and an ***ideal*** voltmeter.

---

* We could also connect a source of 1 A as done in Ex. 2.83.

Example 2.86. *In the netwrok of Fig. 2.167 (a), find* ***(a)*** *ammeter current when battery is at A and ammeter at B and* ***(b)*** *when battery is at B and ammeter at point A. Values of various resistances are as shown in diagram. Also, calculate the transfer resistance.*

Solution. ***(a)*** Equivalent resistance between points *C* and *B* in Fig. 2.167 (*a*) is

$= 12 \times 4/16 = 3\ \Omega$

$\therefore$ Total circuit reistance

$= 2 + 3 + 4 = 9\ \Omega$

$\therefore$ Battery current $= 36/9 = 4$ A

$\therefore$ Ammeter current

$= 4 \times 12/16 =$ **3 A.**

(b) Equivalent resistance between points *C* and *D* in Fig. 2.167 (*b*) is

$= 12 \times 6/18 = 4\ \Omega$

Total circuit resistance $= 4 + 3 + 1 = 8\ \Omega$

Battery current $= 36/8 = 4.5$ A

$\therefore$ Ammeter current $= 4.5 \times 12/18 =$ **3 A**

Hence, ammeter current in both cases is the same.

Transfer resistance $= 36/3 =$ **12 Ω**

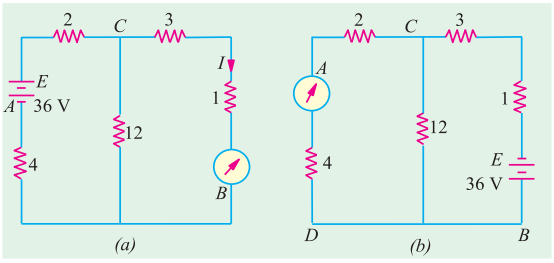


Fig. 2.167

Example 2.87. *Calculate the currents in the various branches of the network shown in Fig. 2.168 and then utilize the principle of Superposition and Reciprocity theorem together to find the value of the current in the 1-volt battery circuit when an e.m.f. of 2 votls is added in branch BD opposing the flow of original current in that branch.*

Solution. Let the currents in the various branches be as shown in the figure. Applying Kirchhoff's second law, we have

For loop *ABDA* ; $-2I_1 - 8I_3 + 6I_2 = 0$ or $I_1 - 3I_2 + 4I_3 = 0$ ...***(i)***

For loop *BCDB*, $-4(I_1 - I_3) + 5(I_2 + I_3) + 8I_3 = 0$ or $4I_1 - 5I_2 - 17I_3 = 0$ ...***(ii)***

For loop *ABCEA*, $-2I_1 - 4(I_1 - I_3) - 10(I_1 + I_2) + 1 = 0$ or $16I_1 + 10I_2 - 4I_3 = 1$ ...***(iii)***

Solving for I_1, I_2 and I_3, we get $I_1 =$ **0.494 A;** $I_2 =$ **0.0229 A;** $I_3 =$ **0.0049 A**

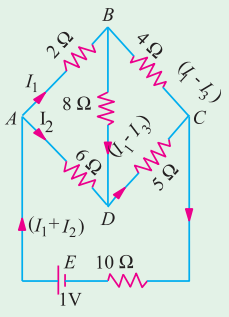


Fig. 2.168

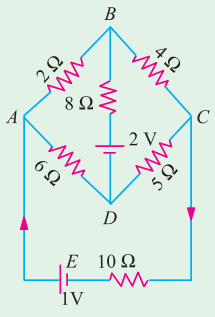


Fig. 2.169

$\therefore$ Current in the 1 volt battery circuit is $I_1 + I_2 =$ **0.0723 A.**

The new circuit having 2 - V battery connected in the branch *BD* is shown in Fig. 2.169. According to the Principle of Superposition, the new current in the 1- volt battery circuit is due to the superposition of two currents; one due to 1 - volt battery and the other due to the 2 - volt battery when each acts independently.

The current in the external circuit due to 1 - volt battery when 2 - volt battery is not there, as found above, is 0.0723 A.

Now, according to Reciprocity theorem; if 1 - volt battery were tansferred to the branch *BD* (where it produced a current of 0.0049 A), then it would produce a current of 0.0049 A in the branch *CEA* (where it was before). Hence, a battery of 2 - V would produce a current of (–2 × 0.0049) = – 0.0098 A (by proportion). The negative sign is used because the 2 - volt battery has been so connected as to oppose the current in branch *BD*,

∴ new current in branch *CEA* = 0.0723 –0.0098 = **0.0625 A**

Tutorial Problems No. 2.5

1. Calculate the current in the 8-W resistor of Fig. 2.170 by using Thevenin's theorem. What will be its value of connections of 6-V battery are reversed ? **[0.8 A ; 0 A]**

2. Use Thevenin's theorem to calculate the p.d. across terminals *A* and *B* in Fig. 2.171. **[1.5 V]**

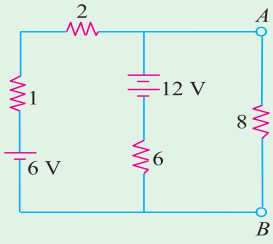


Fig. 2.170

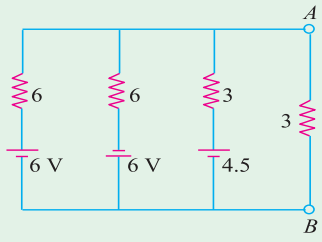


Fig. 2.171

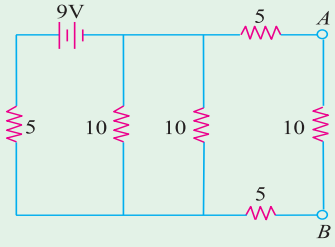


Fig. 2.172

3. Compute the current flowing through the load resistance of 10 Ω connected across terminals *A* and *B* in Fig. 2.172 by using Thevenin's theorem.

4. Find the equivalent Thevenin voltage and equivalent Thevenin resistance respectively as seen from open-circuited terminals *A* and *B* to the circuits shown in Fig. 2.173. All resistances are in ohms.

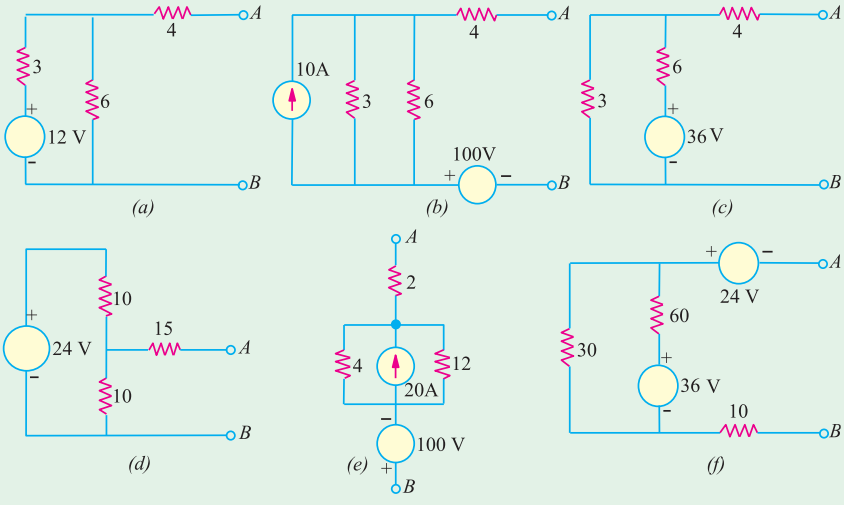


Fig. 2.173

[(*a*) 8 V, 6 Ω; (*b*) 120 V, 6 Ω; (*c*) 12 V, 6 Ω; (*d*) 12 V, 20 Ω; (*e*) –40 V, 5 Ω; (*f*) –12 V, 30 Ω]

5. Find Thevenin's equivalent of the circuits shown in Fig. 2.174 between terminals *A* and *B*.

[(*a*) $V_{th} = I\dfrac{R_1 R_2}{R_1 + R_2} + V\dfrac{R_2}{R_1 + R_2}$; $R_{th} = \dfrac{R_1 R_2}{R_1 + R_2}$ (*b*) $V_{th} = \dfrac{V_1 R_2 + V_2 R_1}{R_1 + R_2}$; $R_{th} = \dfrac{R_1 R_2}{R_1 + R_2}$

(*c*) $V_{th} = -IR$; $R_{th} = R_1$ (*d*) $V_{th} = -V_1 - IR$, $R_{th} = R$ (*e*) Not possible]

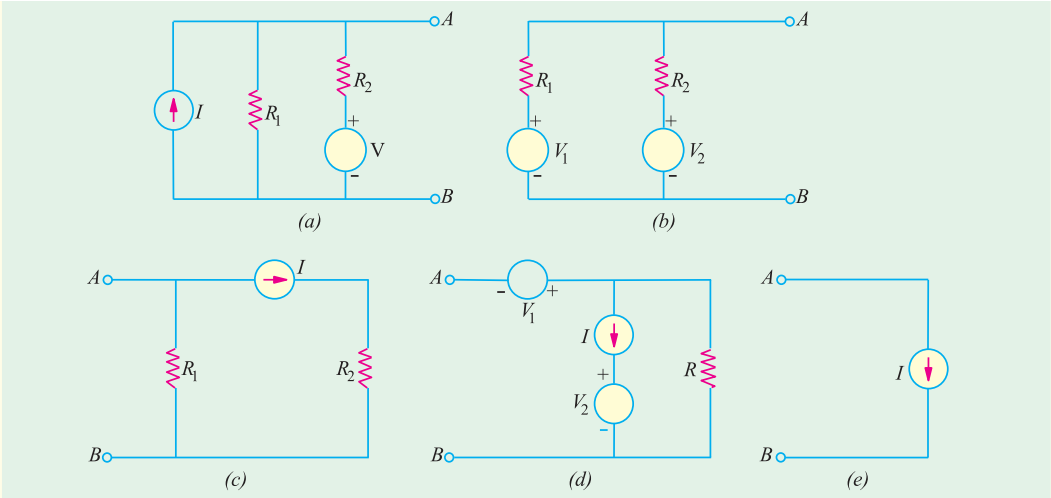


Fig. 2.174

6. The four arms of a Wheatstone bridge have the following resistances in ohms.

$$AB = 100,\ BC = 10,\ CD = 5,\ DA = 60$$

A galvanometer of 15 ohm resistance is connected across BD. Calculate the current through the galvanometer when a potential difference of 10 V is maintained across AC.

[*Elect. Engg. A.M.Ae. S.I.Dec. 1991*] **[4.88 mA]**

7. Find the Thevenin equivalent circuit for the network shown in Fig. 2.175.

[(*a*) 4 V; 8 Ω (*b*) 6 V; 3 Ω (*c*) 0V; 2/5 Ω]

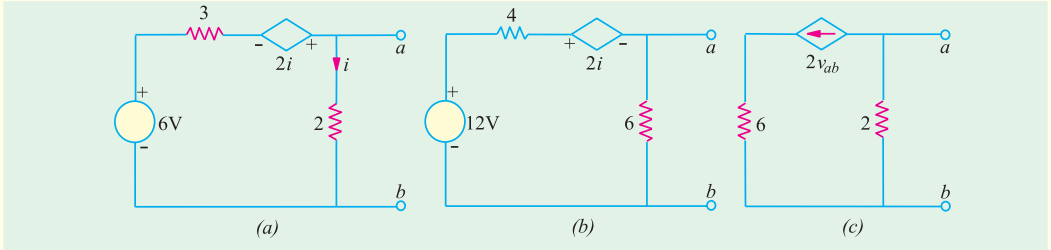


Fig. 2.175

8. Use Thevenin's theorem to find current in the branch AB of the network shown in Fig. 2.176.

[1.84 A]

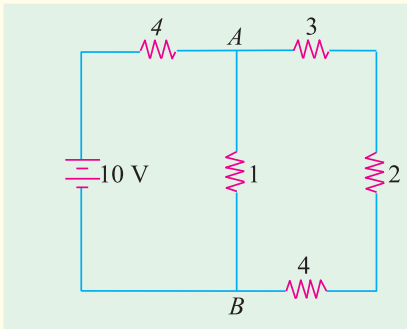


Fig. 2.176

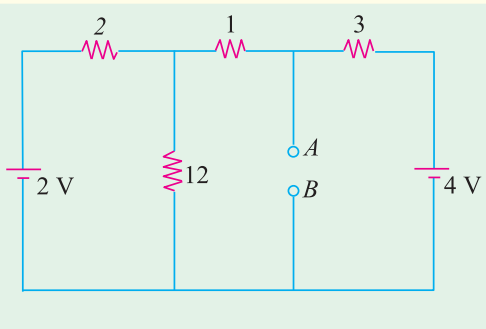


Fig. 2.177

9. In the network shown in Fig. 2.177 find the current that would flow if a 2-Ω resistor was connected between points *A* and *B* by using.

(*a*) Thevenin's theorem and (*b*) Superposition theorem. The two batteries have negligible resistance.

[0.82 A]

10. State and explain Thevenin's theorem. By applying Thevenin's theorem or otherewise, find the current through the resistance *R* and the voltage across it when connected as shown in Fig. 2.178.

[60.49 A, 600.49 V] (*Elect. and Mech. Technology, Osmania Univ.*)

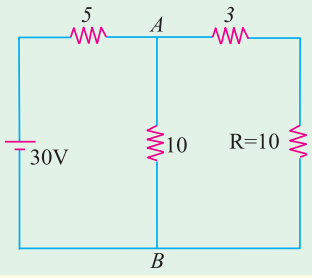


Fig. 2.178

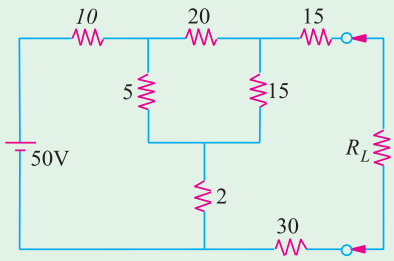


Fig. 2.179

11. State and explain Thevenin's theorem.

For the circuit shown in Fig. 2.179, determine the current through R_L when its value is 50 Ω. Find the value of R_L for which the power drawn from the source is maximum.

(Elect. Technology I, Gwalior Univ.)

12. Find the Thevenin's equivalent circuit for terminal pair *AB* for the network shown in Fig. 2.180.

[V_{th} = –16 V and R_{th} = 16 Ω]

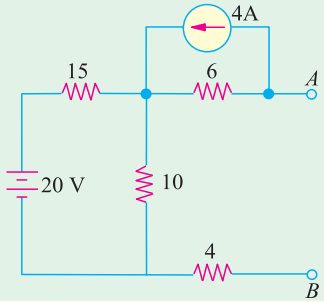


Fig. 2.180

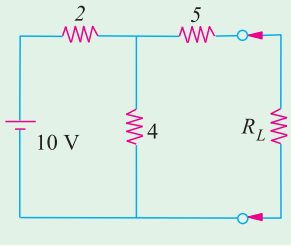


Fig. 2.181

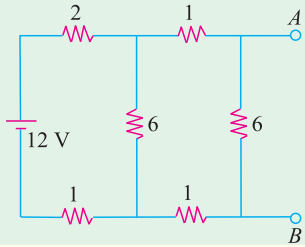


Fig. 2.182

13. For the circuit shown in Fig. 2.181, determine current through R_L when it takes values of 5 and 10 Ω.

[0.588 A, 0.408 A] (*Network Theorem and Fields, Madras Univ.*)

14. Determine Thevenin's equivalent circuit which may be used to represent the network of Fig. 2.182 at the terminals *AB*. **[V_{th} = 4.8 V, R_{th} = 2.4 Ω]**

15. For the circuit shown in Fig. 2.183 find Thevenin's equivalent circuit for terminal pair *AB*.

[6 V, 6 Ω]

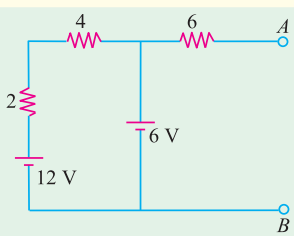


Fig. 2.183

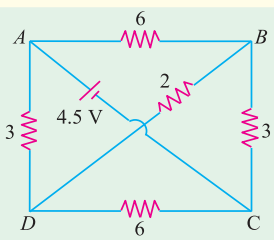


Fig. 2.184

16. *ABCD* is a rectangle whose opposite side *AB* and *DC* represent resistances of 6 Ω each, while *AD* and *BC* represent 3 Ω each. A battery of e.m.f. 4.5 V and negligible resistances is connected between diagonal points *A* and *C* and a 2 - Ω resistance between *B* and *D*. Find the magnitude and direction of the current in the 2-ohm resistor by using Thevenin's theorem. The positive terminal is connected to *A*. (Fig. 2.184) **[0.25 A from *D* to *B*] (*Basic Electricity Bombay Univ.*)**

2.22. Delta/Star* Transformation

In solving networks (having considerable number of branches) by the application of Kirchhoff's Laws, one sometimes experiences great difficulty due to a large number of simultaneous equations that have to be solved. However, such complicated network can be simplified by successively replacing delta meshes by equivalent star system and ***vice versa***.

Suppose we are given three resistances R_{12}, R_{23} and R_{31} connected in delta fashion between terminals 1, 2 and 3 as in Fig. 2.185 (*a*). So far as the respective terminals are concerned, these three given resistances can be replaced by the three resistances R_1, R_2 and R_3 connected in star as shown in Fig. 2.185 (*b*).

These two arrangements will be electrically equivalent if the resistance as measured between any pair of terminals is the same in both the arrangements. Let us find this condition.

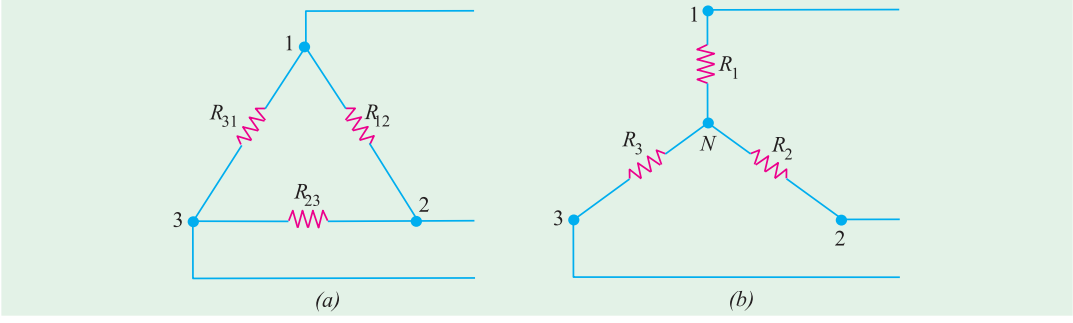


Fig. 2.185

First, take delta connection : Between terminals 1 and 2, there are two parallel paths; one having a resistance of R_{12} and the other having a resistance of $(R_{12} + R_{31})$.

$$\therefore \quad \text{Resistance between terminals 1 and 2 is} = \frac{R_{12} \times (R_{23} + R_{31})}{R_{12} + (R_{23} + R_{31})}$$

Now, take star connection : The resistance between the same terminals 1 and 2 is $(R_1 + R_2)$.

As terminal resistances have to be the same

$$\therefore \quad R_1 + R_2 = \frac{R_{12} \times (R_{23} + R_{31})}{R_{12} + R_{23} + R_{31}} \qquad \text{...(i)}$$

Similarly, for terminals 2 and 3 and terminals 3 and 1, we get

$$R_2 + R_3 = \frac{R_{23} \times (R_{31} + R_{12})}{R_{12} + R_{23} + R_{31}} \qquad \text{...(ii)}$$

and

$$R_3 + R_1 = \frac{R_{31} \times (R_{12} + R_{23})}{R_{12} + R_{23} + R_{31}} \qquad \text{...(iii)}$$

Now, subtracting ***(ii)*** from ***(i)*** and adding the result to ***(iii)***, we get

$$R_1 = \frac{R_{12}\, R_{31}}{R_{12} + R_{23} + R_{31}}; \; R_2 = \frac{R_{23}\, R_{12}}{R_{12} + R_{23} + R_{31}} \text{ and } R_3 = \frac{R_{31}\, R_{23}}{R_{12} + R_{23} + R_{31}}$$

* In Electronics, star and delta circuits are generally referred to as *T* and π circuits respectively.

How to Remember ?

It is seen from above that each numerator is the product of the two sides of the delta which meet at the point in star. Hence, it should be remembered that : ***resistance of each arm of the star is given by the product of the resistances of the two delta sides that meet at its end divided by the sum of the three delta resistances.***

2.23. Star/Delta Transformation

This tarnsformation can be easily done by using equations **(*i*)**, **(*ii*)** and **(*iii*)** given above. Multiplying **(*i*)** and **(*ii*)**, **(*ii*)** and **(*iii*)**, **(*iii*)** and **(*i*)** and adding them together and then simplifying them, we get

$$R_{12} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} = R_1 + R_2 + \frac{R_1R_2}{R_3}$$

$$R_{23} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} = R_2 + R_3 + \frac{R_2R_3}{R_1}$$

$$R_{31} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} = R_1 + R_3 + \frac{R_3R_1}{R_2}$$

How to Remember ?

The equivalent delta resistance between any two terminals is given by the sum of star resistances between those terminals plus the product of these two star resistances divide by the third star resistances.

Example 2.88. *Find the input resistance of the circuit between the points A and B of Fig 2.186(a).*
(AMIE Sec. B Network Analysis Summer 1992)

Solution. For finding R_{AB}, we will convert the delta *CDE* of Fig. 2.186 (*a*) into its equivalent star as shown in Fig. 2.186 (*b*).

$R_{CS} = 8 \times 4/18 = 16/9\ \Omega$; $R_{ES}, = 8 \times 6/18 = 24/9\ \Omega$; $R_{DS} = 6 \times 4/18 = 12/9\ \Omega$

The two parallel resistances between *S* and *B* can be reduced to a single resistance of 35/9 Ω

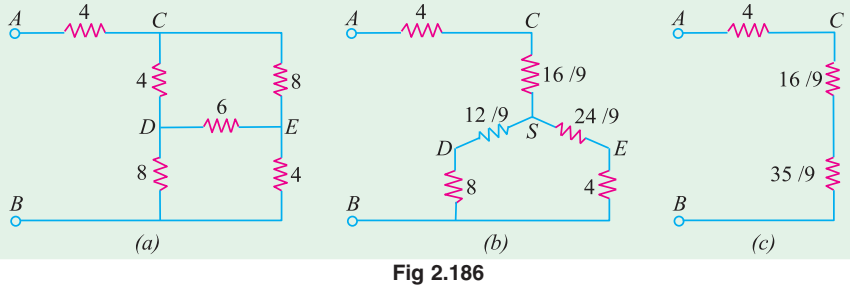


Fig 2.186

As seen from Fig. 2.186 (*c*), $R_{AB} = 4 + (16/9) + (35/9) = 87/9\ \Omega$

Example 2.89. *Calculate the equivalent resistance between the terminals A and B in the network shown in Fig. 2.187* (*a*). **(F.Y. Engg. Pune Univ.)**

Solution. The given circuit can be redrawn as shown in Fig. 2.187 (*b*). When the delta *BCD* is converted to its equivalent star, the circuit becomes as shown in Fig. 2.187 (*c*).

Each arm of the delta has a resistance of 10 Ω. Hence, each arm of the equivalent star has a resistance = 10 × 10/30 = 10/3 Ω. As seen, there are two parallel paths between points *A* and *N*, each having a resistance of (10 + 10/3) = 40/3 Ω. Their combined resistance is 20/3 Ω. Hence, $R_{AB} = (20/3) + 10/3 = 10\ \Omega$

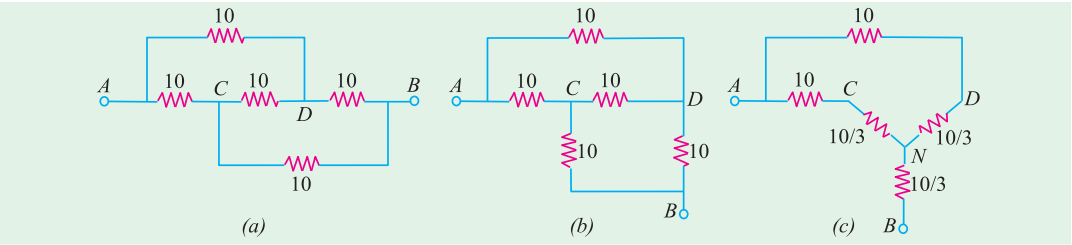


Fig. 2.187

Example 2.90. *Calculate the current flowing through the 10 Ω resistor of Fig. 2.188 (a) by using any method.* **(Network Theory, Nagpur Univ. 1993)**

Solution. It will be seen that there are two deltas in the circuit *i.e. ABC* and *DEF*. They have been converted into their equivalent stars as shown in Fig. 2.188 (*b*). Each arm of the delta *ABC* has a resistance of 12 Ω and each arm of the equivalent star has a resistance of 4 Ω. Similarly, each arm of the delta *DEF* has a resistance of 30 Ω and the equivalent star has a resistance of 10 Ω per arm.

The total circuit resistance between *A* and *F* = 4 + 48 || 24 + 10 = 30 Ω. Hence *I* = 180/30 = 6 A.

Current through 10 Ω resistor as given by current-divider rule = 6 × 48/(48 + 24) = **4 A.**

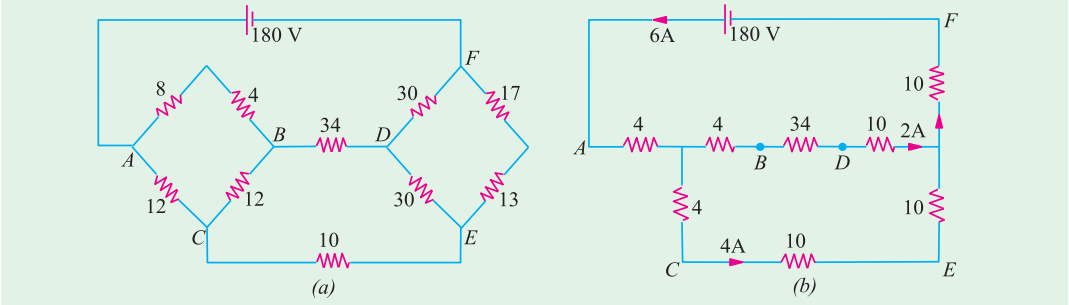


Fig. 2.188

Example 2.91. *A bridge network ABCD has arms AB, BC, CD and DA of resistances 1, 1, 2 and 1 ohm respectively. If the detector AC has a resistance of 1 ohm, determine by star/delta transformation, the network resistance as viewed from the battery terminals.*

(Basic Electricity, Bombay Univ.)

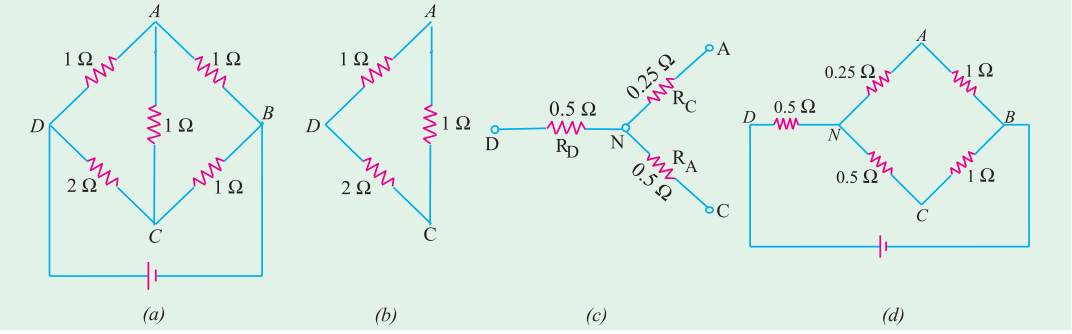


Fig. 2.189

Solution. As shown in Fig. 2.189 (*b*), delta *DAC* has been reduced to its equivalent star.

$$R_D = \frac{2\times1}{2+1+1} = 0.5\,\Omega,\quad R_A = \frac{1}{4} = 0.25\,\Omega,\quad R_C = \frac{2}{4} = 0.5\,\Omega$$

Hence, the original network of Fig. 2.189 (*a*) is reduced to the one shown in Fig. 2.189 (*d*). As seen, there are two parallel paths between points *N* and *B*, one of resistance 1.25 Ω and the other of resistance 1.5 Ω. Their combined resistance is

$$= \frac{1.25 \times 1.5}{1.25 + 1.5} = \frac{15}{22}\ \Omega$$

Total resistance of the network between points *D* and *B* is

$$= 0.5 \quad \frac{15}{22} \quad \frac{\mathbf{13}}{\mathbf{11}}\ \mathbf{\Omega}$$

Example 2.92. *A network of resistances is formed as follows as in Fig. 2.190 (a)*

AB = 9 Ω; BC = 1 Ω; CA = 1.5 Ω forming a delta and AD = 6 Ω; BD = 4 Ω and CD = 3 Ω forming a star. Compute the network resistance measured between ***(i)*** *A and B* ***(ii)*** *B and C and* ***(iii)*** *C and A.* **(Basic Electricity, Bombay Univ. 1980)**

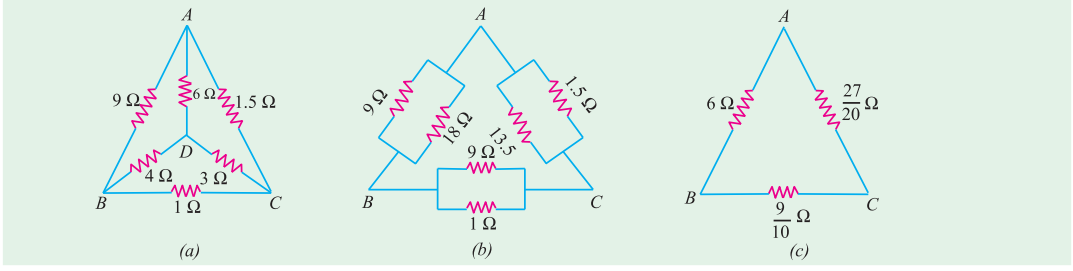


Fig. 2.190

Solution. The star of Fig. 2.190 (*a*) may be converted into the equivalent delta and combined in parallel with the given delta *ABC*. Using the rule given in Art. 2.22, the three equivalent delta resistance of the given star become as shown in Fig. 2.190 (*b*).

When combined together, the final circuit is as shown in Fig. 2.190 (*c*).

(i) As seen, there are two parallel paths across points *A* and *B*.

(*a*) one directly from *A* to *B* having a resistance of 6 Ω and

(*b*) the other *via C* having a total resistance

$$= \frac{27}{20} \quad \frac{9}{10} \quad 2.25 \qquad R_{AB} \quad \frac{6 \quad 2.25}{(6 \quad 2.25)} \quad \frac{\mathbf{18}}{\mathbf{11}}\ \mathbf{\Omega}$$

(ii) $R_{BC} = \dfrac{\frac{9}{10} \quad 6 \quad \frac{27}{20}}{\frac{9}{10} \quad 6 \quad \frac{27}{20}} \quad \dfrac{\mathbf{441}}{\mathbf{550}}\ \mathbf{\Omega}$ ***(iii)*** $R_{CA} \quad \dfrac{\frac{27}{20} \quad 6 \quad \frac{9}{10}}{\frac{9}{10} \quad 6 \quad \frac{27}{20}} \quad \dfrac{\mathbf{621}}{\mathbf{550}}\ \mathbf{\Omega}$

Example 2.93. *State Norton's theorem and find current using Norton's theorem through a load of 8 Ω in the circuit shown in Fig. 2.191(a).* **(Circuit and Field Theory, A.M.I.E. Sec. B, 1993)**

Solution. In Fig. 2.191 (*b*), load impedance has replaced by a short-circuit.

$I_{SC} = I_N = 200/2 = 100$ A.

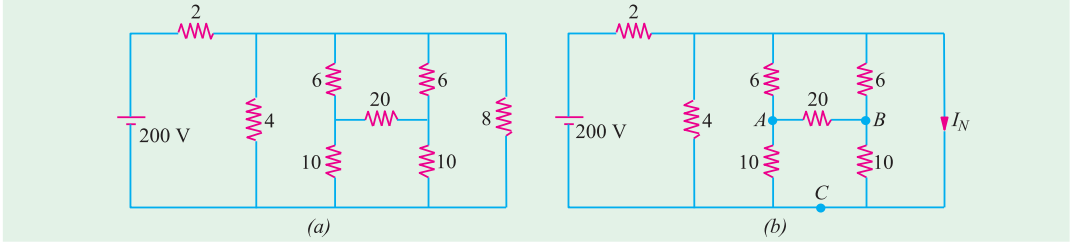


Fig. 2.191

Norton's resistance R_N can be found by looking into the open terminals of Fig. 2.191 (*a*). For this purpose ΔABC has been replaced by its equivalent Star. As seen, R_N is equal to 8/7 Ω

Hence, Norton's equivalent circuit consists of a 100 A source having a parallel resistance of 8/7Ω as shown in Fig. 2.192 (*c*). The load current I_L can be found by using the Current Divider rule.

$$I_L = 100 \times \frac{(8/7)}{8 + (8/7)} = 12.5 \text{ A}$$

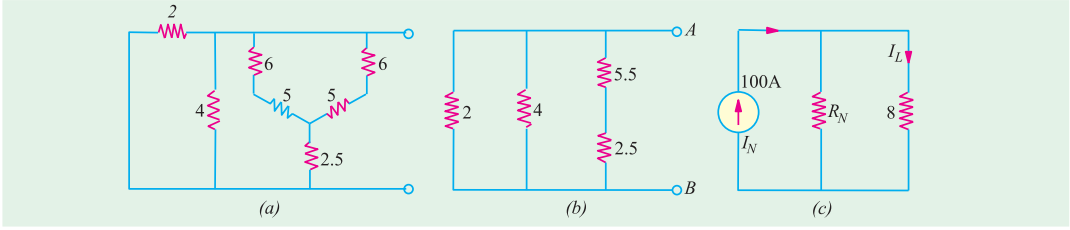


Fig. 2.192

Example 2.94. *Use delta-star conversion to find resistance between terminals 'AB' of the circuit shown in Fig. 2.193 (a). All resistances are in ohms.* **[Nagpur University April 1999]**

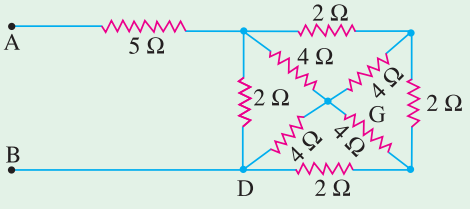


Fig. 2.193 (*a*)

Solution. First apply delta-star conversion to CGD and EDF, so as to redraw the part of the circuit with new configuration, as in Fig. 2.193 (*b*).

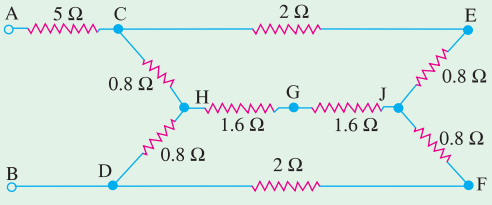


Fig. 2.193 (*b*)

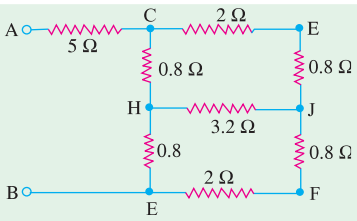


Fig. 2.193 (*c*)

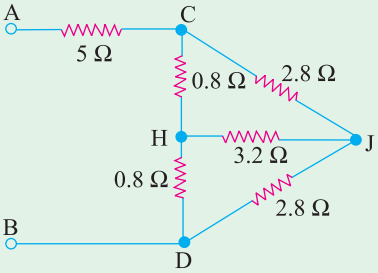


Fig. 2.193 (*d*)

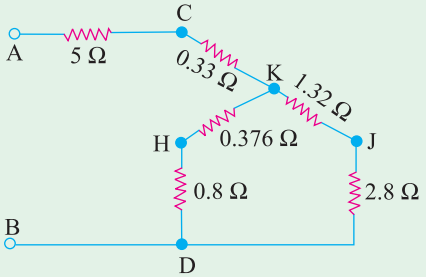


Fig. 2.193 (*e*)

Simplify to reduce the circuit to its equivalents as in Fig. 2.193 (*c*) and later as in Fig. 2.193 (*d*). Convert CHJ to its equivalent star as in Fig. 2.193 (*e*). With the help of series-parallel combinations, calculate R_{AD} as

$R_{AB} = 5.33 + (1.176 \times 4.12/5.296) = 6.245$ ohms

Note : Alternatively, after simplification as in Fig. (*d*). "CDJ – H" star-configuration can be transformed into delta. Node *H* then will not exist. The circuit has the parameters as shown in Fig. 2.193 (*f*). Now the resistance between *C* and *J* (and also between *D* and *J*) is a parallel combination of 7.2 and 2.8 ohms, which 2.016 ohms. Along *CJD*, the resistance between terminals *AB* then obtained as :

$$R_{AB} = 5.0 + (1.8 \times 4.032/5.832)$$
$$= 5.0 + 1.244 = 6.244 \text{ ohms}$$

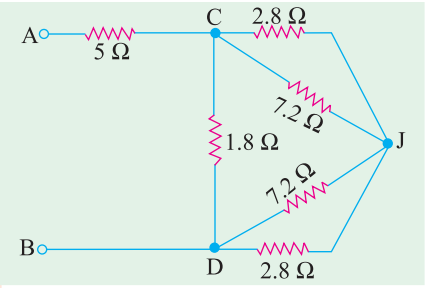


Fig. 2.193 (*f*)

Example 2.94 (*a*). *Find the resistance at the A-B terminals in the electric circuit of Fig. 2.193 (g) using Δ-Y transformation.* **[U.P. Technical University, 2001]**

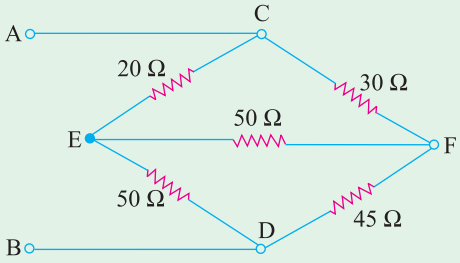


Fig. 2.193 (*g*)

Solution. Convert delta to star for nodes *C*, *E*, *F*. New node *N* is created. Using the formulae for this conversion, the resistances are evaluated as marked in Fig. 2.193 (*h*). After handling series parallel combinations for further simplifications.

$$R_{AB} = 36 \text{ ohms.}$$

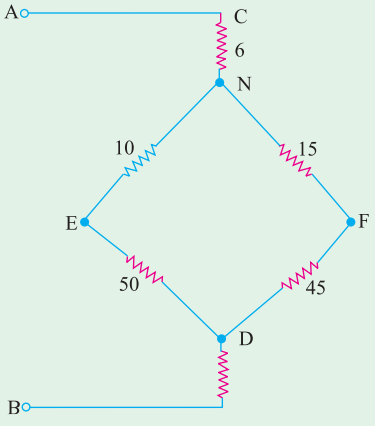


Fig. 2.193 (*h*)

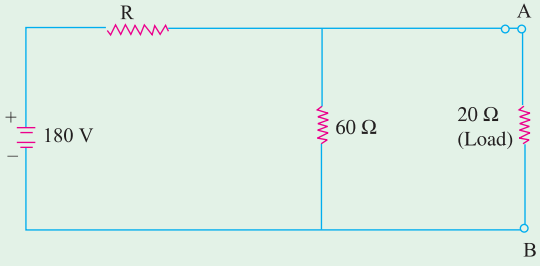


Fig. 2.193 (*i*)

Example 2.94 (*b*). *Consider the electric circuit shown in Fig. 2.193 (i)*

Determine : (*i*) *the value of R so that load of 20 ohm should draw the maximum power,* **(*ii*)** *the value of the maximum power drawn by the load.* **[U.P. Technical University, 2001]**

Solution. Maximum power transfer takes place when load resistance = Thevenin's Resistance = 20 ohms, here

$$R/60 = 20 \text{ ohms, giving } R = 30 \text{ ohms}$$
$$V_{TH} = 180 \times (60/90) = 120 \text{ volts}$$
$$\text{Current through load} = 120/40 = 3 \text{ amps}$$
$$\text{Maximum Power Load} = 180 \text{ watts}$$

Tutorial Problems No. 2.6

Delta/Star Conversion

1. Find the current in the 17 Ω resistor in the network shown in Fig. 2.194 (*a*) by using (*a*) star/delta conversion and (*b*) Thevenin's theorem. The numbers indicate the resistance of each member in ohms. **[10/3A]**
2. Convert the star circuit of Fig. 2.194 (*b*) into its equivalent delta circuit. Values shown are in ohms. Derive the formula used. ***(Elect. Technology, Indor Univ.)***

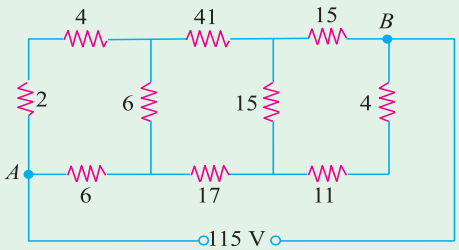


Fig. 2.194 (*a*)

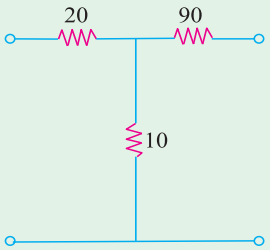


Fig. 2.194 (*b*)

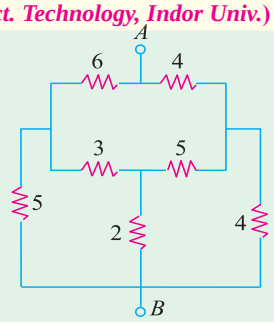


Fig. 2.195

3. Determine the resistance between points *A* and *B* in the network of Fig. 2.195. **[4.23 Ω] *(Elect. Technology, Indor Univ.)***
4. Three resistances of 20 Ω each are connected in star. Find the equivalent delta resistance. If the source of e.m.f. of 120 V is connected across any two terminals of the equivalent delta-connected resistances, find the current supplied by the source. **[60 Ω, 3A] *(Elect. Engg. Calcutta Univ.)***

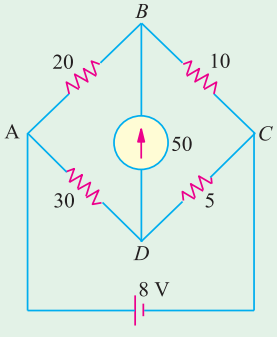
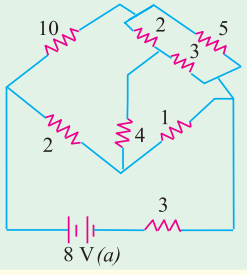
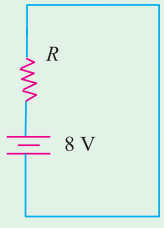


Fig. 2.196



(*a*)



(*b*)

Fig. 2.197

5. Using delta/star transformation determine the current through the galvanometer in the Wheatstone bridge of Fig. 2.196. **[0.025 A]**
6. With the aid of the delta star transformation reduce the network given in Fig. 2.197 (*a*) to the equivalent circuit shown at (*b*) **[R = 5.38 Ω]**
7. Find the equivalent resistance between points *A* and *B* of the circuit shown in Fig. 2.198. **[1.4 R]**
8. By first using a delta-star transformation on the mesh *ABCD* of the circuit shown in Fig. 2.199, prove that the current supplied by the battery is 90/83 A.

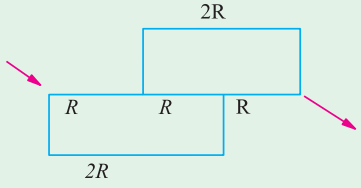


Fig. 2.198

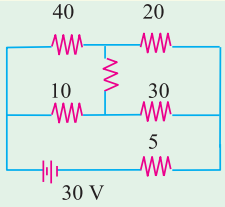


Fig. 2.199

2.24. Compensation Theorem

This theorem is particularly useful for the following two purposes :

(a) For analysing those networks where the values of the branch elements are varied and for studying the effect of tolerance on such values.

(b) For calculating the sensitivity of bridge network.

As applied to d.c. circuits, it may be stated in the following for ways :

(i) In its simplest form, this theorem asserts that ***any resistance R in a branch of a network in which a current I is flowing can be replaced, for the purposes of calculations, by a voltage equal to – IR.***

OR

(ii) If the resistance of any branch of network is changed from R to (R + ΔR) where the current flowing originally is I, the change of current at any other place in the network may be calculated by assuming that an e.m.f. – I. ΔR has been injected into the modified branch while all other sources have their e.m.f.s. suppressed and are represented by their internal resistances only.

Example 2.95. *Calculate the values of new currents in the network illustrated in Fig. 2.200 when the resistor R_3 is increased (in place of s) by 30 %.*

Solution. In the given circuit, the values of various branch currents are

$$I_1 = 75/(5 + 10) = 5 \text{ A}$$
$$I_2 = I_3 = 2.5 \text{ A}$$

Now, value of

$$R_3 = 20 + (0.3 \times 20) = 26\ \Omega$$
$$\therefore \quad \Delta R = 6\ \Omega$$
$$V = -I_3\, \Delta R$$
$$= -2.5 \times 6 = -15 \text{ V}$$

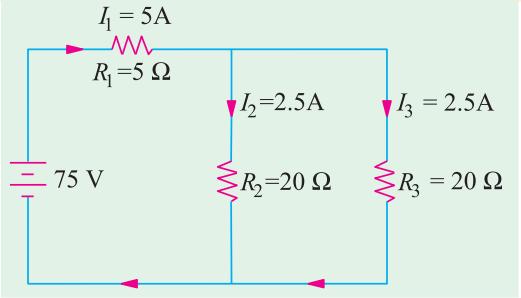


Fig. 2.200

The compensating currents produced by this voltage are as shown in Fig. 2.201 (*a*).

When these currents are added to the original currents in their respective branches the new current distribution becomes as shown in Fig. 2.201 (*b*)

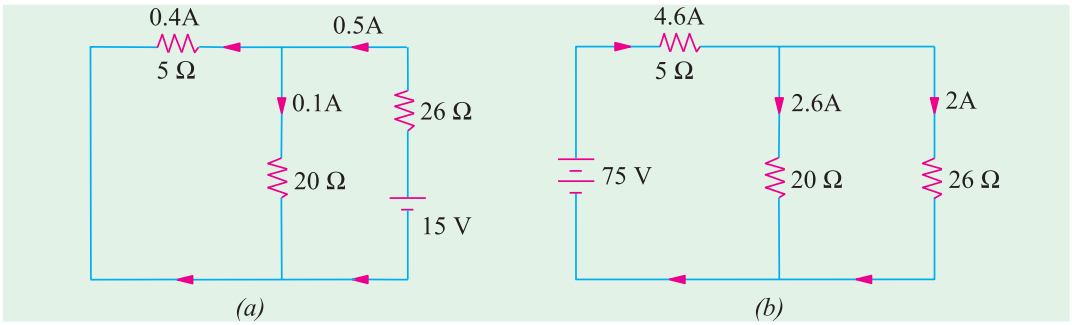


Fig. 2.201

2.25. Norton's Theorem

This theorem is an alternative to the Thevenin's theorem. In fact, it is the dual of Thevenin's theorem. Whereas Thevenin's theorem reduces a two-terminal active network of linear resistances and generators to an equivalent constant-voltage source and series resistance, Norton's theorem replaces the network by an equivalent constant-current source and a parallel resistance.

This theorem may be stated as follows :

(i) Any two-terminal active network containing voltage sources and resistance when viewed from its output terminals , is equivalent to a constant-current source and a parallel resistance. The constant current is equal to the current which would flow in a short-circuit placed across the terminals and parallel resistance is the resistance of the network when viewed from these open-circuited terminals after all voltage and current sources have been removed and replaced by their internal resistances.

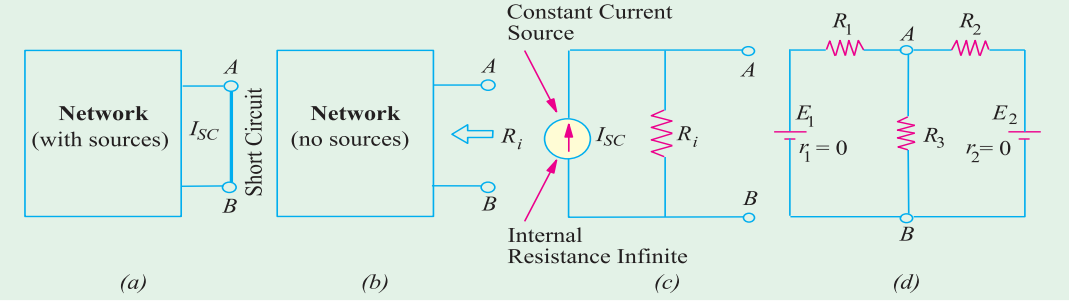


Fig. 2.202

Explanation

As seen from Fig. 2.202 (*a*), a short is placed across the terminals *A* and *B* of the network ***with all its energy sources present***. The short-circuit current I_{SC} gives the value of constant-current source.

For finding R_i, all sources have been removed as shown in Fig. 2.202 (*b*). The resistance of the network when looked into from terminals *A* and *B* gives R_i.

The Norton's* equivalent circuit is shown in Fig. 2.202 (*c*). It consists of an ideal constant-current source of infinite internal resistance (Art. 2.16) having a resistance of R_i connected in parallel with it. Solved Examples 2.96, 2.97 and 2.98 etc. illustrate this procedure.

(ii) Another useful generalized form of this theorem is as follows :

The voltage between any two points in a network is equal to I_{SC}. R_i where I_{SC} is the short-circuit current between the two points and R_i is the resistance of the network as viewed from these points with all voltage sources being replaced by their internal resistances (if any) and current sources replaced by open-circuits.

Suppose, it is required to find the voltage across resistance R_3 and hence current through it [Fig. 2.202 (*d*)]. If short-circuit is placed between *A* and *B*, then current in it due to battery of e.m.f. E_1 is E_1/R_1 and due to the other battery is E_2/R_2.

$$\therefore \quad I_{SC} = \frac{E_1}{R_1} + \frac{E_2}{R_2} = E_1 G_1 + E_2 G_2$$

where G_1 and G_2 are branch conductances.

Now, the internal resistance of the network as viewed from *A* and *B* simply consists of three resistances R_1, R_2 and R_3 connected in parallel between *A* and *B*. Please note that here load resistance ***R_3 has not been removed***. In the first method given above, it has to be removed.

$$\therefore \quad \frac{1}{R_i} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = G_1 + G_2 + G_3$$

$$\therefore \quad R_i = \frac{1}{G_1 + G_2 + G_3} \qquad \therefore \quad V_{AB} = I_{SC}.R_i = \frac{E_1 G_1 + E_2 G_2}{G_1 + G_2 + G_3}$$

Current through R_2 is $I_3 = V_{AB}/R_3$.

Solved example No. 2.96 illustrates this approach.

---

* After E.L. Norton, formerely an engineer at Bell Telephone Laboratory, U.S.A.

2.26. How To Nortonize a Given Circuit ?

This procedure is based on the first statement of the theorem given above.

1. Remove the resistance (if any) across the two given terminals and put a short-circuit across them.
2. Compute the short-circuit current I_{SC}.
3. Remove all voltage sources but retain their internal resistances, if any. Similarly, remove all current sources and replace them by open-circuits *i.e.* by infinite resistance.
4. Next, find the resistance R_1 (also called R_N) of the network as looked into from the given terminals. It is exactly the same as R_{th} (Art. 2.16).
5. The current source (I_{SC}) joined in parallel across R_i between the two terminals gives Norton's equivalent circuit.

As an example of the above procedure, please refer to Solved Example No. 2.87, 88, 90 and 91 given below.

Example 2.96. *Determine the Thevenin and Norton equivalent circuits between terminals A and B for the voltage divider circuit of Fig. 2.203 (a).*

Solution. (*a*) **Thevenin Equivalent Circuit**

Obviosuly, V_{th} = drop across $R_2 = E\dfrac{R_2}{R_1 + R_2}$

When battery is replaced by a short-circuit.

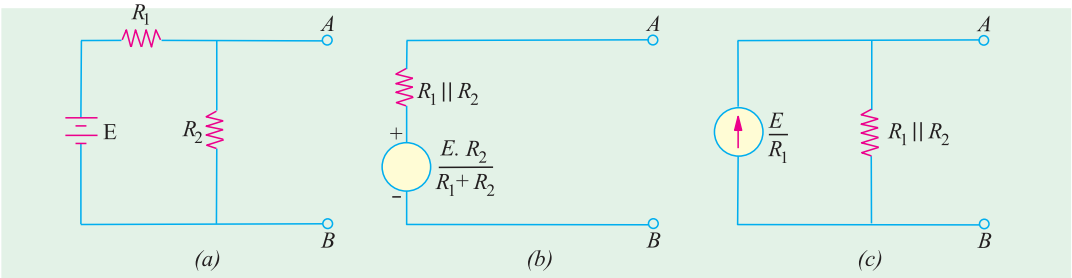


Fig. 2.203

$$R_i = R_1 \parallel R_2 = R_1 R_2/(R_1 + R_2)$$

Hence, Thevenin equivalent circuit is as shown in Fig. 2.203 (*b*).

(*b*) Norton Equivalent Circuit

A short placed across terminals *A* and *B* will short out R_2 as well. Hence, $I_{SC} = E/R_1$. The Norton equivalent resistance is exactly the same as Thevenin resistance except that it is connected in parallel with the current source as shown in Fig. 2.203 (*c*)

Example 2.97. *Using Norton's theorem, find the constant-current equivalent of the circuit shown in Fig. 2.204 (a).*

Solution. When terminals *A* and *B* are short-circuited as shown in Fig. 2.204 (*b*), total resistance of the circuit, as seen by the battery, consists of a 10 Ω resistance in series with a parallel combination of 10 Ω and 15 Ω resistances.

$\therefore$ total resistance $= 10 + \dfrac{15 \times 10}{15 + 10} = 16\ \Omega$

$\therefore$ battery current $I = 100/16 = 6.25$ A

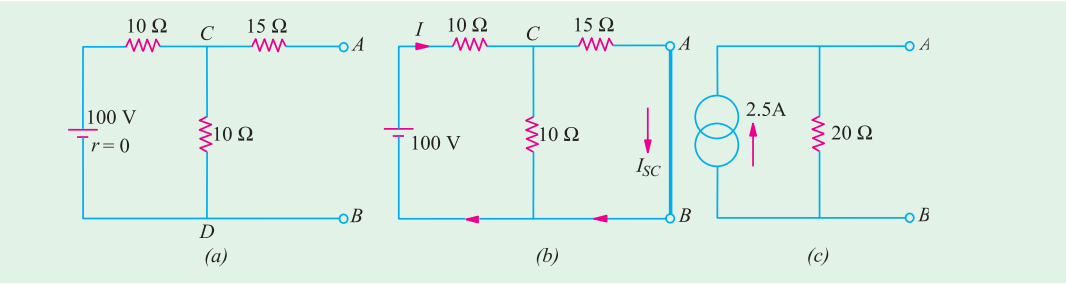


Fig. 2.204

This current is divided into two parts at point *C* of Fig. 2.204 (*b*).

Current through *A B* is $I_{SC} = 6.25 \times 10/25 = 2.5$ A

Since the battery has no internal resistance, the input resistance of the network when viewed from *A* and *B* consists of a 15 Ω resistance in series with the parallel combination of 10 Ω and 10 Ω. Hence, $R_1 = 15 + (10/2) = 20$ Ω

Hence, the equivalent constant-current source is as shown in Fig. 2.204 (*c*).

Example 2.98. *Apply Norton's theorem to calculate current flowing through* 5 – Ω *resistor of Fig. 2.05 (a).*

Solution. (*i*) Remove 5 – Ω resistor and put a short across terminals *A* and *B* as shown in Fig. 2.205 (*b*). As seen, 10 –Ω resistor also becomes short-circuited.

(*ii*) Let us now find I_{SC}. The battery sees a parallel combination of 4 Ω and 8 Ω in series with a 4 Ω resistance. Total resistance seen by the battery = 4 + 4 || 8 = 20/3 Ω. Hence, $I = 20 + 20/3 = 3$ A. This current divides at point *C* of Fig. 2.205 (*b*). Current going along path *CAB* gives I_{SC}. Its value = $3 \times 4/12 = 1$ A.

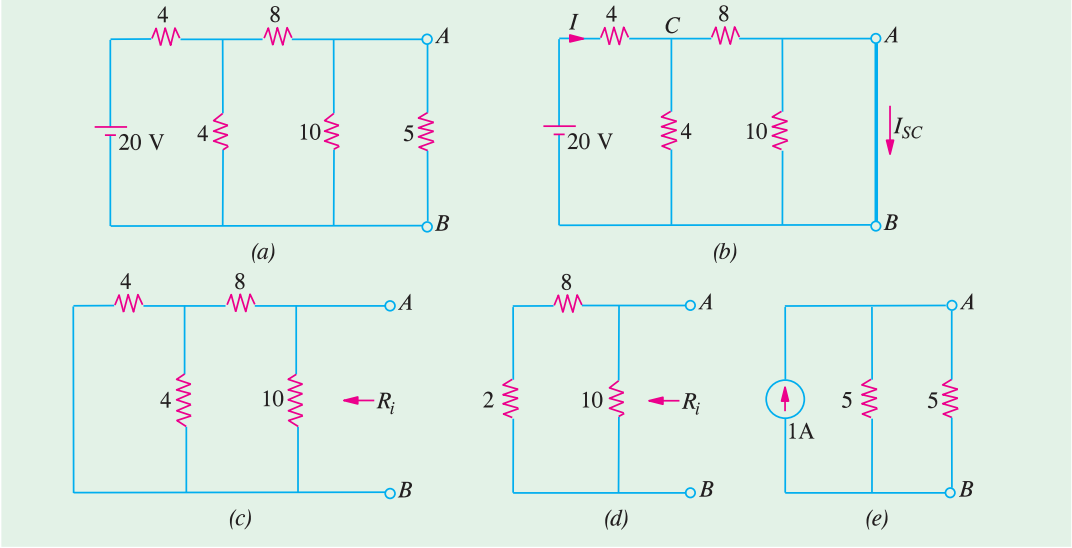


Fig. 2.205

(*iii*) In Fig. 2.205 (*c*), battery has been removed leaving behind its internal resistance which, in this case, is zero.

Resistance of the network looking into the terminals *A* and *B* in Fig. 2.205 (*d*) is

$$R_i = 10 \parallel 10 = 5 \ \Omega$$

(*iv*) Hence, Fig. 2.205 (*e*), gives the Norton's equivalent circuit.

(v) Now, join the 5 –Ω resistance back across terminals *A* and *B*. The current flowing through it, obviously, is $I_{AB} = 1 \times 5/10 =$ **0.5 A.**

Example 2.99. *Find the voltage across points A and B in the network shown in Fig. 2.206 (a) by using Norton's theorem.*

Solution. The voltage between points *A* and *B* is $V_{AB} = I_{SC} R_i$

where I_{SC} = short-circuit current between *A* and *B*

R_i = Internal resistance of the network as viewed from points *A* and *B*.

When short-circuit is placed between *A* and *B*, the current flowing in it due to 50-V battery is

$= 50/50 = 1$ A – from *A* to *B*

Current due to 100 V battery is $= 100/20 = 5$ A – from *B* to *A*

$I_{SC} = 1 - 5 = -4$ A – from *B* to *A*

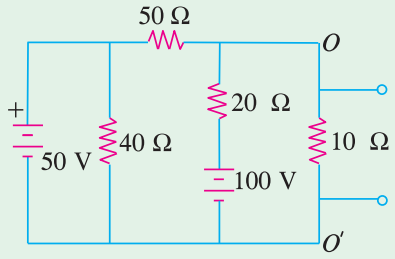


Fig. 2.206 (*a*)

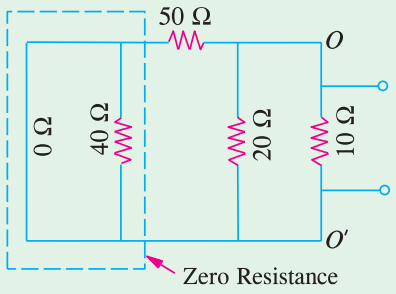


Fig. 2.206 (*b*)

Now, suppose that the two batteries are removed so that the circuit becomes as shown in Fig. 2.206 (*b*). The resistance of the network as viewed from points *A* and *B* consists of three resistances of 10 Ω, 20 Ω and 50 Ω ohm connected in parallel (as per second statement of Norton's theorem).

$$\therefore \quad \frac{1}{R_i} = \frac{1}{10} + \frac{1}{20} + \frac{1}{50}; \qquad \text{hence } R_1 = \frac{100}{17}\ \Omega$$

$$\therefore \quad V_{AB} = -4 \times 100/17 = \mathbf{-23.5\ V}$$

The negative sign merely indicates that point *B* is at a higher potential with respect to the point *A*.

Example 2.100. *Using Norton's theorem, calculate the current flowing through the 15 Ω load resistor in the circuit of Fig. 2.207 (a). All resistance values are in ohm.*

Solution. (*a*) Short-Circuit Current I_{SC}

As shown in Fig. 2.207 (*b*), terminals *A* and *B* have been shorted after removing 15 Ω resistor. We will use Superposition theorem to find I_{SC}.

(*i*) When Only Current Source is Present

In this case, 30-V battery is replaced by a short-circuit. The 4 A current divides at point *D* between parallel combination of 4 Ω and 6 Ω. Current through 6 Ω resistor is

$I_{SC}' = 4 \times 4/(4 + 6) = 1.6$ A – from *B* to *A*

(*ii*) When Only Battery is Present

In this case, current source is replaced by an open-circuit so that no current flows in the branch *CD*. The current supplied by the battery constitutes the short-circuit current

$\therefore \quad I_{sc}'' = 30/(4 + 6) =$ **3 A** – from *A* to *B*

$\therefore \quad I_{sc} = I_{sc}'' - I_{sc}' = 3 - 1.6 =$ **1.4 A** – from *A* to *B*

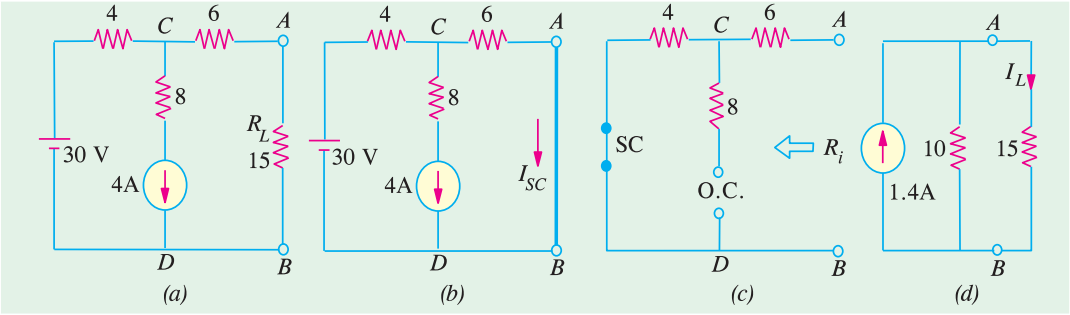


Fig. 2.207

(*b*) Norton's Parallel Resistance

As seen from Fig. 2.207 (c) $R_1 = 4 + 6 = 10\ \Omega$. The 8 Ω resistance does not come into the picture because of an open in the branch *CD*.

Fig. 2.207 (*d*) shows the Norton's equivalent circuit along with the load resistor.

$$I_L = 1.4 \times 10\ (10 + 15) = \mathbf{0.56\ A}$$

Example 2.101. *Using Norton's current-source equivalent circuit of the network shown in Fig. 2.208 (a), find the current that would flow through the resistor R_2 when it takes the values of 12, 24 and 36 Ω respectivley.* **[Elect. Circuits, South Gujarat Univ.]**

Solution. In Fig. 2.208 (*b*), terminals *A* and *B* have been short-circuited. Current in the shorted path due to E_1 is = 120/40 = 3 A from *A* to *B*. Current due to E_2 is 180/60 = 3 A from *A* to *B*. Hence I_{SC} = 6A. With batteries removed, the resistance of the network when viewed from open-circuited terminals is = 40 || 60 = 24 Ω

(i)	When R_L = 12 Ω	$I_L = 6 \times 24\ (24 + 12) = \mathbf{4\ A}$
(ii)	When R_L = 24 Ω	$I_L = 6/2 = \mathbf{3\ A.}$
(iii)	When R_L = 36 Ω	$I_L = 6 \times 24/(24 + 36) = \mathbf{2.4\ A.}$

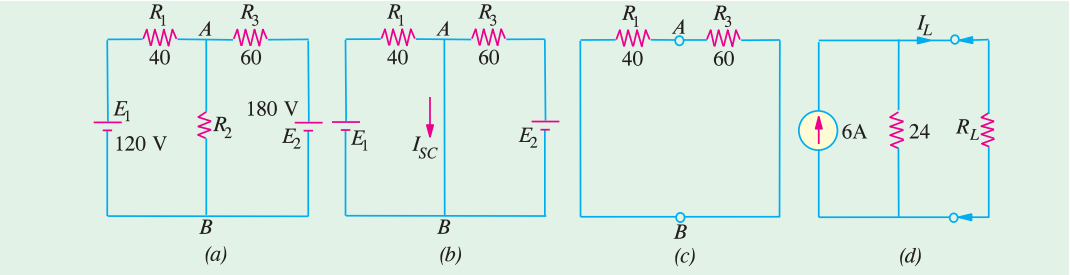
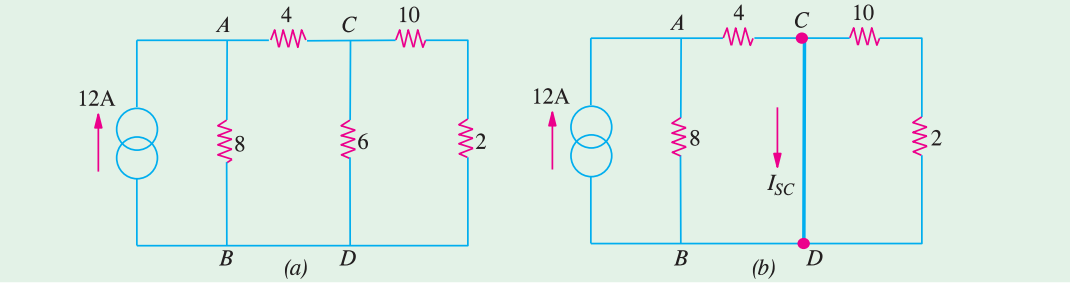


Fig. 2.208

Example 2.102. *Using Norton's theorem, calculate the current in the 6-Ω resistor in the network of Fig. 2.209 (a). All resistance are in ohms.*



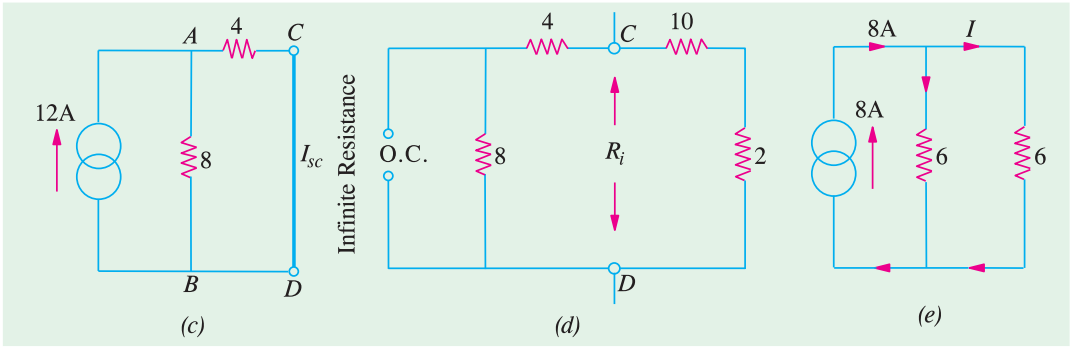


Fig. 2.209

Solution. When the branch containing 6 –Ω resistance is short-circuited, the given circuit is reduced to that shown in Fig. 2.209 (*b*) and finally to Fig. 2.209 (*c*). As seen, the 12 A current divides into two unequal parts at point *A*. The current passing through 4 Ω resistor forms the short-circuit current I_{SC}.

Resistance R_i between points *C* and *D* when they are open-circuited is

$$R_i = \frac{(4 \quad 8) \quad (10 \quad 2)}{(4 \quad 8) \quad (10 \quad 2)} \quad 6$$

It is so because the constant-current source has *infinite* resistance *i.e.*, it behaves like an open circuit as shown in Fig. 2.209 (*d*).

Hence, Norton's equivalent circuit is as shown in Fig. 2.209 (*e*). As seen current of 8 A is divided equally between the two equal resistances of 6 Ω each. Hence, current through the required 6 Ω resistor is **4 A.**

$$I_{SC} = 12 \times \frac{8}{8+4} = 8 \text{ A}$$

Example 2.103. *Using Norton's theorem, find the current which would flow in a 25 –Ω resistor connected between points N and O in Fig. 2.210 (a). All resistance values are in ohms.*

Solution. For case of understanding, the given circuit may be redrawn as shown in Fig. 2.210 (*b*). Total current in short-circuit across *ON* is equal to the sum of currents driven by different batteries through their respective resistances.

$$I_{SC} = \frac{10}{5} + \frac{20}{10} + \frac{30}{20} = 5.5 \text{ A}$$

The resistance R_i of the circuit when looked into from point *N* and *O* is

$$\frac{1}{R_i} = \frac{1}{5} + \frac{1}{10} + \frac{1}{20} = \frac{7}{20} \Omega; \quad R_i = \frac{20}{7} \Omega = 2.86 \Omega$$

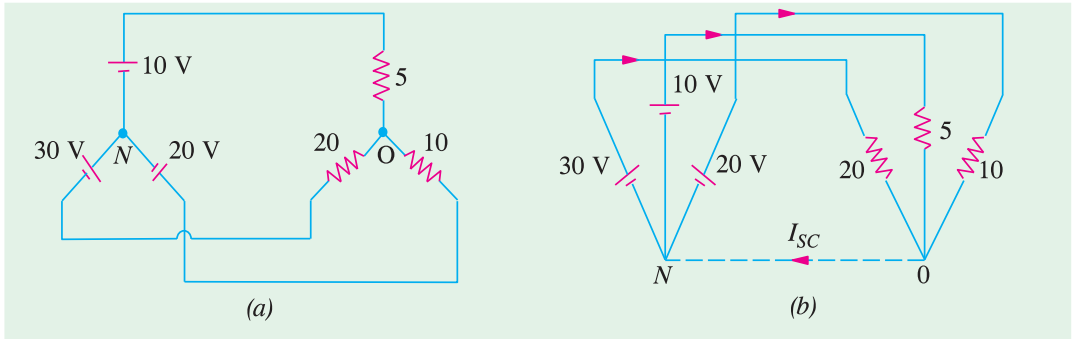


Fig. 2.210

Hence, given circuit reduces to that shown in Fig. 2.211 (*a*).

Open-circuit voltage across *NO* is = $I_{SC}R_i$ = 5.5 × 2.86 = **15.73 V**

Hence, current through 25-Ω resistor connected across *NO* is [Fig. 2.211 (*b*)]

$$I = 15.73/25 = \mathbf{0.65}\ \text{A}$$

or $$I = 5.5\ \frac{2.86}{2.86\quad 25} = \mathbf{0.56\ A.}$$

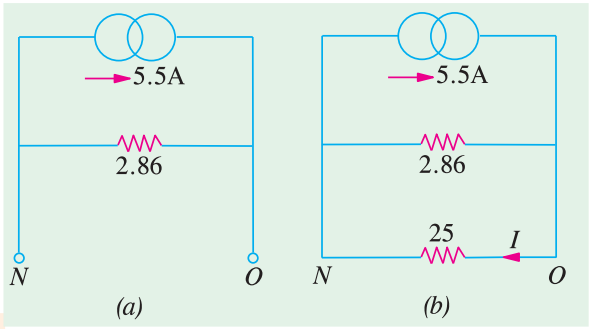


Fig. 2.211

Example 2.104. *With the help of Norton's theorem, find V_o in the circuit shown in Fig. 2.212 (a). All resistances are in ohms.*

Solution. For solving this circuit, we will Nortonise the circuit to the left to the terminals 1 –1′ and to the right of terminals 2 –2′ , as shown in Fig. 2.212 (*b*) and (*c*) respectively.

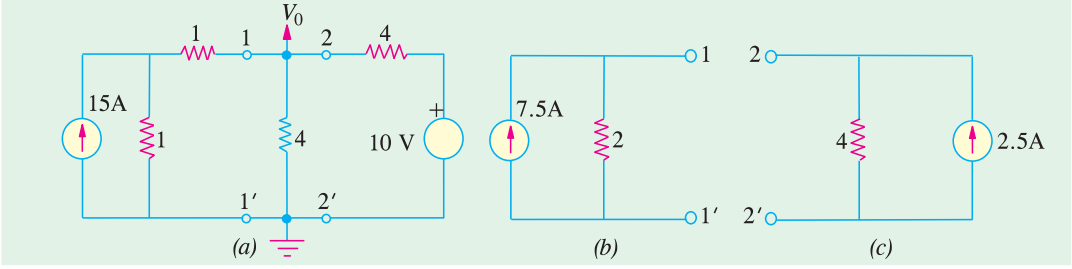


Fig. 2.212

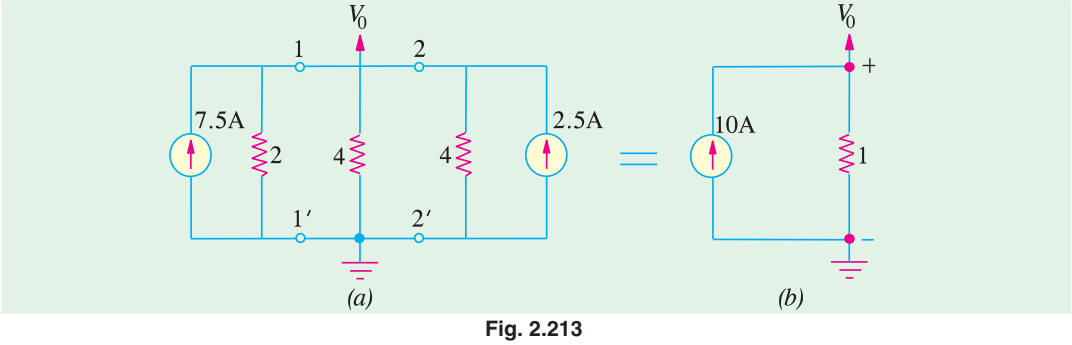


Fig. 2.213

The two equivalent Norton circuits can now be put back across terminals 1-1′ and 2-2′ , as shown in Fig. 2.213 (*a*).

The two current sources, being in parallel, can be combined into a single source of 7.5 + 2.5 = 10 A. The three resistors are in parallel and their equivalent resistances is 2 || 4 || 4 = 1 Ω. The value of V_o as seen from Fig. 2.213 (*b*) is V_o = 10 × 1 = **10 V.**

Example 2.105. *For the circuit shown in Fig. 2.214 (a), calculate the current in the 6 Ω resistance by using Norton's theorem.* **(Elect. Tech. Osmania Univ. Feb. 1992)**

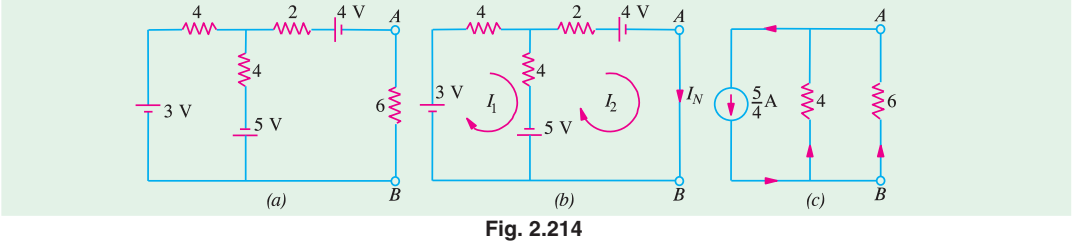


Fig. 2.214

Solution. As explained in Art. 2.19, we will replace the 6 Ω resistance by a short-circuit as shown in Fig. 2.214 (*b*). Now, we have to find the current passing through the short-circuited terminals *A* and *B*. For this purpose we will use the mesh analysis by assuming mesh currents I_1 and I_2.

From mesh ***(i)***, we get

$$3 - 4I_1 - 4(I_1 - I_2) + 5 = 0 \quad \text{or} \quad 2I_1 - I_2 = 2 \qquad ...(i)$$

From mesh (*ii*), we get

$$-2I_2 - 4 - 5 - 4(I_2 - I_1) = 0 \quad \text{or} \quad 4I_1 - 6I_2 = 9 \qquad ...(ii)$$

From ***(i)*** and ***(ii)*** above, we get $I_2 = -5/4$

The negative sign shows that the actual direction of flow of I_2 is opposite to that shown in Fig. 2.214 (*b*). Hence, $I_{sh} = I_N = I_2 = -5/4$ A *i.e.* current flows from point *B* to *A*.

After the terminals *A* and *B* are open-circuited and the three batteries are replaced by short-circuits (since their internal resistances are zero), the internal resistance of the circuit, as viewed from these terminals' is

$$R_i = R_N = 2 + 4 \parallel 4 = 4\ \Omega$$

The Norton's equivalent circuit consists of a constant current source of 5/4 A in parallel with a resistance of 4 Ω as shown in Fig. 2.214 (*c*). When 6 Ω resistance is connected across the equivalent circuit, current through it can be found by the current-divider rule (Art).

$$\text{Current through 6 } \Omega \text{ resistor} = \frac{5}{4} \times \frac{4}{10} = 0.5 \text{ from } B \text{ to } A.$$

2.27. General instructions For Finding Norton Equivalent Circuit

Procedure for finding Norton equivalent circuit of a given network has already been given in Art. That procedure applies to circuits which contain resistors and independent voltage or current sources. Similar procedures for circuits which contain both dependent and independent sources or only dependent sources are given below :

(*a*) Circuits Containing Both Dependent and Independent Sources

(i) Find the open-circuit voltage v_∞ with all the sources activated or 'alive'.

(ii) Find short-circuit current i_{sh} by short-circuiting the terminals *a* and *b* but with all sources activated.

(iii) $R_N = V_{oc}/i_{sh}$

(*b*) Circuits Containing Dependent Sources Only

(i) $i_{sh} = 0$.

(ii) Connect 1 A source to the terminals *a* and *b* calculate v_{ab}.

(iii) $R_N = v_{ab}/1$.

Example 2.106. *Find the Norton equivalent for the transistor amplifier circuit shown is Fig. 2.215 (a). All resistances are in ohms.*

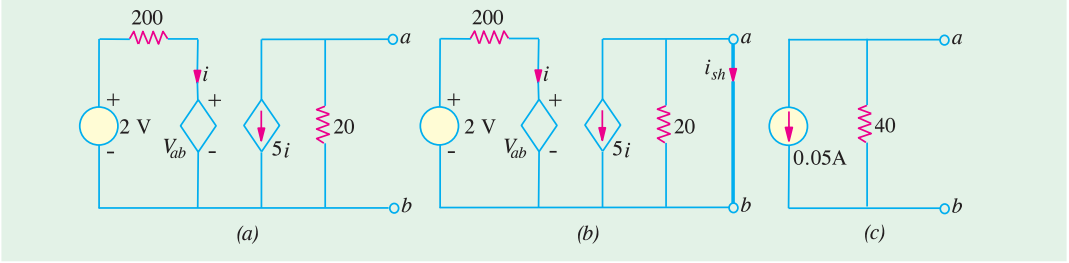


Fig. 2.215

Solution. We have to find the values of i_{sh} and R_N. It should be noted that when terminals *a* and *b* are short-circuited, $v_{ab} = 0$. Hence, in that case, we find from the left-hand portion of the circuit that $i = 2/200 = 1/100\text{A} = 0.01$ A. As seen from Fig. 2.215 (*b*), the short-circuit across terminals *a* and *b*, short circuits 20 Ω resistance also. Hence, $i_{sh} = -5\ i = -5 \times\ 0.01 = -0.05$ A.

Now, for finding R_N, we need $v_{oc} = v_{ab}$ from the left-hand portion of the Fig. 2.215 (*a*). Applying KVL to the closed circuit, we have

$$2 - 200\ i - v_{ab} = 0 \qquad \text{...(i)}$$

Now, from the right-hand portion of the circuit, we find v_{ab} = drop over 20 Ω resistance = $-20 \times 5i = -100\ i$. The negative sign is explained by the fact that currert flows from point *b* towards point *a*. Hence, $i = -v_b/100$. Substituting this value in Eqn. **(*i*)**. above, we get

$$2 - 200\ (-v_b/100) - v_{ab} = 0 \quad \text{or} \quad v_{ab} = -2 \text{ V}$$

$$\therefore \qquad R_N = v_{ab}/i_{sh} = -2/-0.05 = 40\ \Omega$$

Hence, the Norton equivalent circuit is as shown in Fig. 2.215 (*c*).

Example 2.107. *Using Norton's theorem, compute current through the 1-Ω resistor of Fig. 2.216.*

Solution. We will employ source conversion technique to simplify the given circuit. To begin with, we will convert the three voltge sources into their equivalent current sources as shown in Fig. 2.216 (*b*) and (*c*). We can combine together the two current sources on the left of *EF* but cannot combine the 2-A source across *CD* because of the 3-Ω resistance between *C* and *E*.

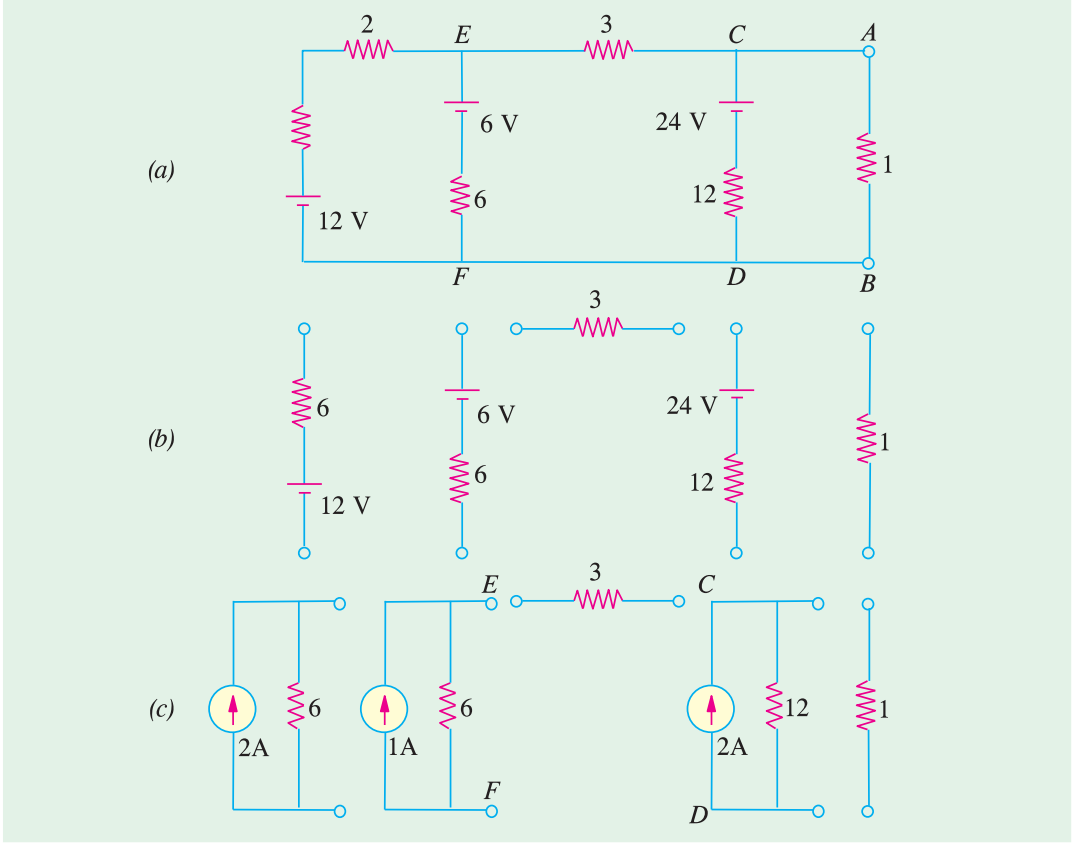


Fig. 2.216

In Fig. 2.217 (*b*), the two current sources at the left-hand side of 3 Ω resistor have been replaced by a single (2 A + 1 A) = 3 A current source having a single parallel resistance 6 || 6 = 3 Ω

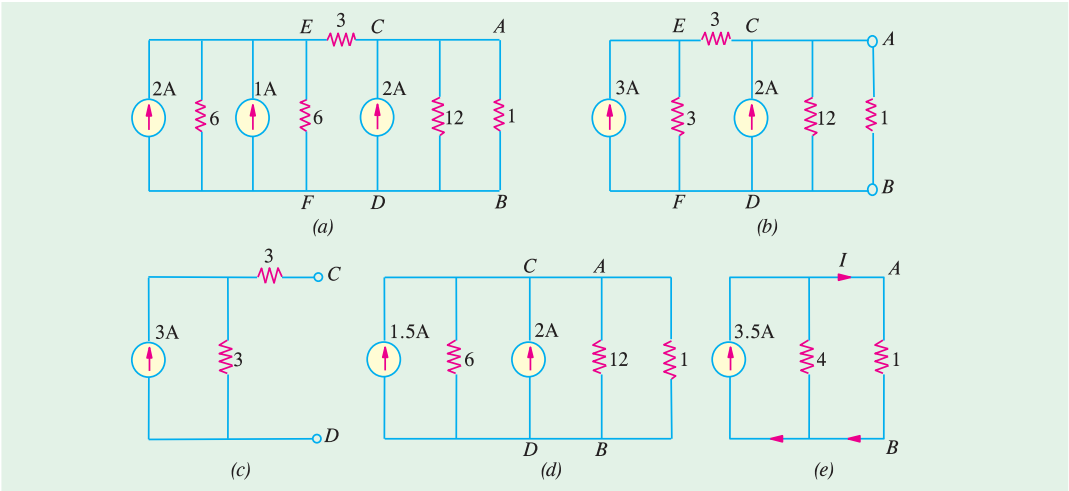


Fig. 2.217

We will now apply Norton's theorem to the circuit on the left-hand side of CD [Fig. 2.217 (*c*)] to convert it into a single current source with a single parallel resistor to replace the two 3 Ω resistors. As shown in Fig. 2.217 (*d*), it yields a 1.5 A current source in parallel with a 6 Ω resistor. This current source can now be combined with the one across *CD* as shown in Fig. 2.217 (*e*). The current through the 1-Ω resistor is

$$I = 3.5 \times 4/(4 + 1) = \mathbf{2.8\ A}$$

Example 2.108. *Obtain Thevenin's and Norton's equivalent circuits at AB shown in Fig. 2.218 (a).* **[Elect. Network, Analysis Nagpur Univ. 1993]**

Solution. ***Thevenin's Equivalent Circuit***

We will find the value of V_{th} by using two methods **(*i*)** *KVL* and **(*ii*)** mesh analysis.

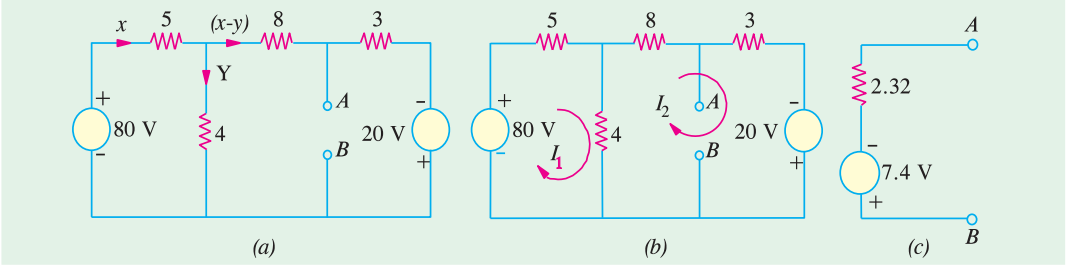


Fig. 2.218

(*a*) Using KVL

If we apply *KVL* to the first loop of Fig. 2.218 (*a*), we get

$$80 - 5x - 4y = 0 \quad \text{or} \quad 5x + 4y = 80 \qquad \text{...(}i\text{)}$$

From the second @ loop, we have

$$-11(x - y) + 20 + 4y = 0 \quad \text{or} \quad 11x - 15y = 20 \qquad \text{...(}ii\text{)}$$

From **(*i*)** and **(*ii*)**, we get $x = 10.75$ A; $y = 6.56$ A and $(x - y) = 4.2$ A.

Now, $V_{th} = V_{AB}$ *i.e.* voltage of point *A* with respect to point *B*. For finding its value, we start from point *B* and go to point *A* either *via* 3 Ω resistance or 4 Ω resistance or (5 + 8) = 13 Ω resistance and take the algebraic sum of the voltage met on the way. Taking the first route, we get

$$V_{AB} = -20 + 3(x - y) = -20 + 3 \times 4.2 = -7.4\ \text{V}$$

It shows that point *A* is negative with respect to point *B* or, which is the same thing, point *B* is positive with respect to point *A*.

(*b*) Mesh Analysis [Fig. 2.218 (*b*)]

Here, $R_{11} = 9\ ; \quad R_{22} = 15;\ R_{21} = -4$

$$\therefore \quad \begin{vmatrix} 9 & -4 \\ -4 & 15 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \end{vmatrix} = \begin{vmatrix} 80 \\ 20 \end{vmatrix};\ \Delta = 135 - 16 = 119$$

$$\Delta_1 = \begin{vmatrix} 80 & -4 \\ 20 & 15 \end{vmatrix} = 1280;\ \Delta_2 = \begin{vmatrix} 9 & 80 \\ -4 & 20 \end{vmatrix} = 500$$

$$I_1 = 1280/119 = 10.75 \text{ A}\ ;\ I_2 = 500/119 = 4.2 \text{ A}$$

Again $V_{AB} = -20 + 12.6 = -7.4$ V

Value of R_{th}

For finding R_{th}, we replace the two voltage sources by short-circuits.

$$\therefore \quad R_{th} = R_{AB} = 3 \parallel (8 + 4 \parallel 5) = 2.32\ \Omega$$

The Thevenin's equivalent circuit becomes as shown in Fig. 2.219 (*c*). It should be noted that point *B* has been kept positive with respect to point *A* in the Fig.

Example 2.109. *Find current in the 4 ohm resistor by any three methods.*

[Bombay University 2000]

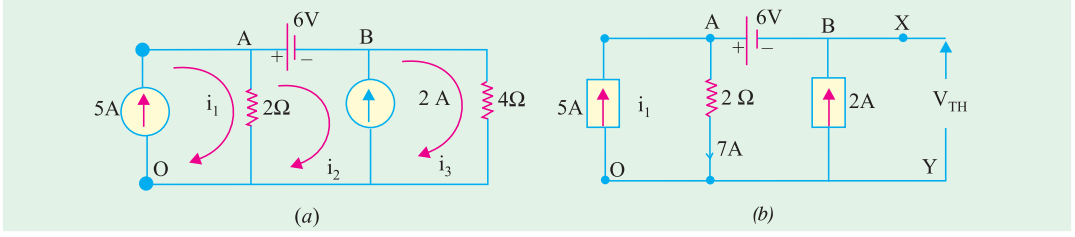


Fig. 2.219

Solution. Method 1 : Writing down circuit equations, with given conditions, and marking three clockwise loop-currents as i_1, i_2 and i_3.

$\mathbf{i_1}$ = 5 A, due to the current source of 5 Amp

$V_A - V_B$ = 6 V, due to the voltage source of 6 Volts

$\mathbf{i_3 - i_2}$ = 2 A, due to the current source of 2 Amp.

$V_A = (\mathbf{i_1 - i_2})\ 2,\ V_B = \boldsymbol{i}_3 \times 4$

With these equations, the unknowns can be evaluated.

$$2\ (i_1 - i_2) - 4\ i_3 = 6,\ 2\ (5 - i_2) - 4\ (2 + i_2) = 6$$

This gives the following values : $i_2 = -2/3$ Amp., $i_3 = 4/3$ Amp.

V_A = 34/3 volts, V_B = 16/3 volts

Method 2 : Thevenin's theorem : Redraw the circuit with modifications as in Fig. 2.219 (*b*)

$R_{TH} = +14 - 6 = 8$ V

R_{TH} = 2 ohms, looking into the circuit form *X*-*Y* terminals after de-activating the sources

$I_L = 8/(2 + 4) = 4/3$ Amp.

Method 3 : Norton's Theorem : Redraw modifying as in Fig. 2.219 (*c*)

$I_N = 2 + 2 = 4$ Amp.

This is because, *X* and *Y* are at ground potential, 2-ohm resistor has to carry 3 A and hence from 5-Amp. source, 2-Amp current is driven into *X*-*Y* nodes.

R_N = 2 ohms

Then the required current is calculated as shown in Fig. 2.219 (*d*)

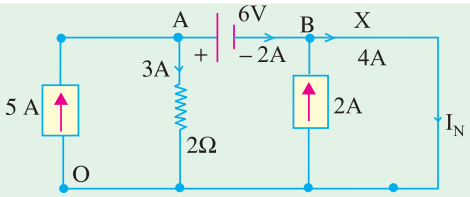


Fig. 2.219 (*c*) Evaluation of I_N

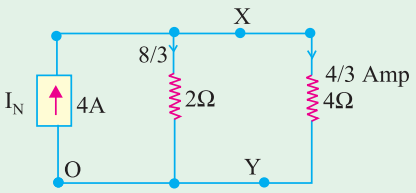


Fig. 2.219 (*d*)

Note : One more method is described. This transforms the sources such that the current through 4-ohm resistor is evaluated, as in final stage shown in Fig. 2.219 (*j*) or in Fig. 2.219 (*k*).

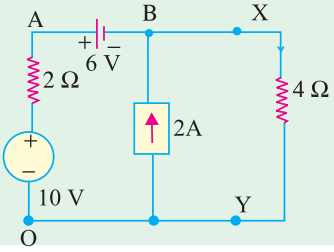


Fig. 2.219 (*e*)

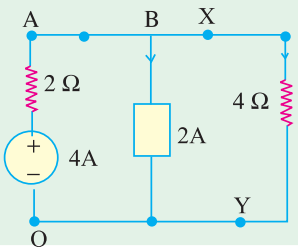


Fig. 2.219 (*f*)

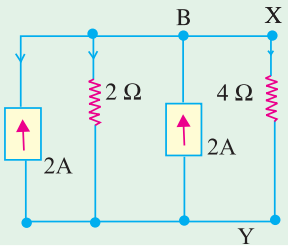


Fig. 2.219 (*h*)

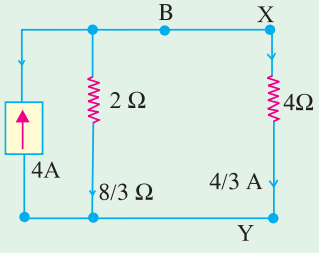


Fig. 2.219 (*j*)

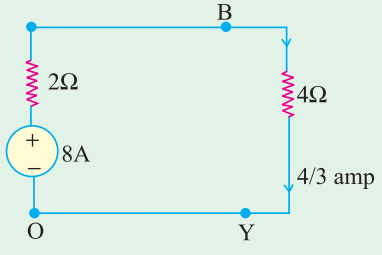


Fig. 2.219 (*k*)

Example 2.109. (*a*). *Find Mesh currents i_1 and i_2 in the electric circuit of Fig. 2.219 (m)*

[U.P. Tech. University, 2001]

Solution. Mark the nodes as shown in Fig. 2.219 (*m*).

Treat *O* as the reference node.

From the dependent current source of $3i_1$ amp between *B* and *O*,

$$i_2 - i_1 = 3i_1 \quad \text{or} \quad 4i_1 = i_2 \qquad \text{...(a)}$$

V_B is related to V_A, V_C and the voltage across resistors concerned

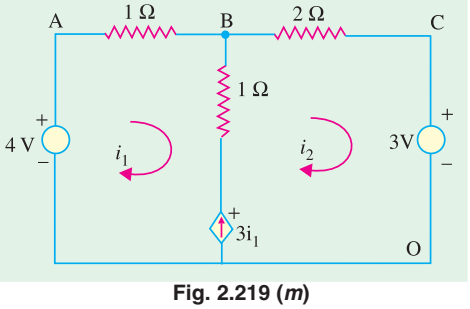


Fig. 2.219 (*m*)

$$V_B = V_A - i_1 \times 1 = 4 - i_1$$

$$V_B = V_C + i_2 \times 2 = 3 + 2i_2$$

Hence $$4 - i_1 = 3 + 2i_2 \qquad \text{...(b)}$$

From equations **(*a*)** and **(*b*)** above, $i_1 = 1/9$ amp and $i_2 = 4/9$ amp

Substituting these, $V_B = 35/9$ volts

Example 2.109 (*b*). *Determine current through 6 ohm resistance connected across A-B terminals in the electric circuit of – 2.219 (n), using Thevenin's Theorem.* **[U.P. Tech. Univ. 2001]**

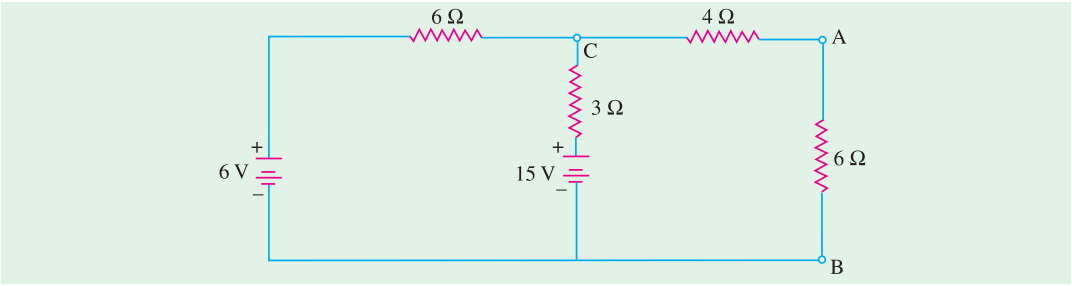


Fig. 2.219 (*n*)

Solution. Applying Thevenin's theorem, after detaching the 6-ohm resitor from terminals $A - B$,

$$V_{TH} = V_C = 15 - 1 \times 3 = 12 \text{ volts}$$
$$R_{TH} = 4 + 3/6 = 6 \text{ ohms}$$
$$I_L = 12/(6 + 6) = \text{lamp}$$

Example 2.109 (*c*). *Applying Kirchoff's Current Law, determine current I_s in the electric circuit of Fig. 2.219 (p). Take $V_o = 16$ V.* **[U.P. Tech. Univ. 2001]**

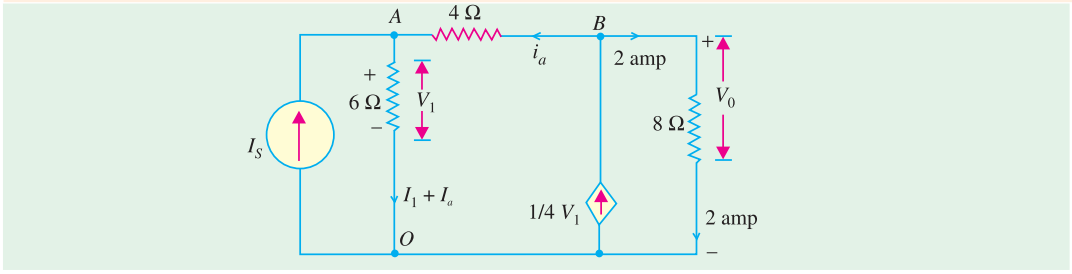


Fig. 2.219 (*p*)

Solution. Mark the nodes *A*, *B*, and *O* and the currents associated with different branches, as in Fig. 2.219 (*p*).

Since $V_0 = 16$ V, the current through 8-ohm resistor is 2 amp.

KCL at node B : $1/4\ V_1 = 2 + i_a$ ...(*a*)

KCL at node A : $I_s + i_a = V_1/6$ ...(*b*)

Further, $V_A = V_1$, $V_B = 16$, $V_B - V_1 = 4i_a$ ...(*c*)

From (*a*) and (*c*), $i_a = 1$ amp. This gives $V_1 - V_A = 12$ volts, and $I_S = 1$ amp

The magnitude of the dependent current source = 3 amp

Check : Power from 1 amp current source = $1 \times 12 = 12$ W

Power from dependent C.S. of 3 A = $3 \times 16 = 48$ W

Sum of source-output-power = 60 watts

Sum of power consumed by resistors = $2^2 \times 6 + 1^2 \times 4 + 2^2 \times 8 = 60$ watts

The power from sources equal the consumed by resistors. This confirms that the answers obtained are correct.

Norton's Equivalent Circuit

For this purpose, we will short-circuit the terminals *A* and *B* find the short-circuit currents produced by the two voltage sources. When viewed from the side of the 80-V source, a short across *AB* short-circuits everything on the right side of *AB*. Hence, the circuit becomes as shown in Fig. 2.230 (*a*). The short-circuit current I_1 can be found with the help of series-parallel circuit technique. The total resistance offered to the 80 –V source is $5 + 4 \parallel 8 = 23/3\ \Omega$

$\therefore \quad I = 80 \times 3/23 = 10.43$ A; $\quad \therefore \quad I_1 = 10.43 \times 4/12 = 3.48$ A.

When viewed from the side of the 20-V source, a short across *AB* short-circuits everything beyond *AB*. In the case, the circuit becomes as shown in Fig. 2.230 (*b*). The short circuit current flowing from *B* to *A* = 20/3 = 6.67 A.

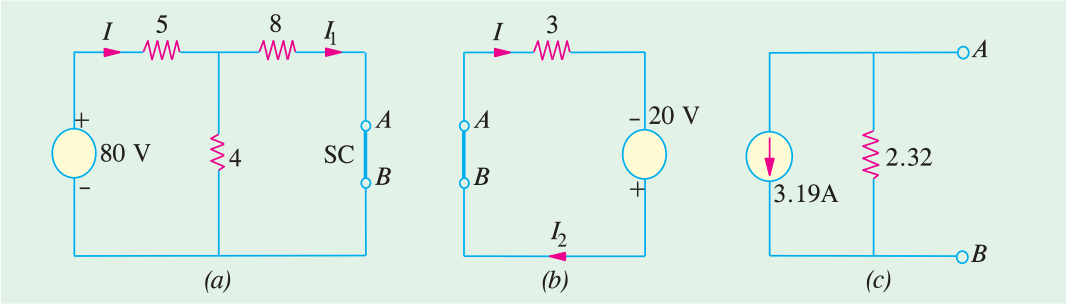


Fig. 2.220

Total short-circuit current = 6.67 – 3.48 = 3.19 A ... from *B* to *A*.

$$R_N = R_{th} = 3 \parallel (8 + 4 \parallel 5) = 2.32\ \Omega$$

Hence, the Norton's equivalent circuit becomes as shown in Fig. 2.220 (*c*).

2.28. Millman's Theorem

This theorem can be stated either in terms of voltage sources or current sources or both.

(*a*) As Applicable to Voltage Sources

This Theorem is a combination of Thevenin's and Norton's theorems. It is used for finding the common voltage across any network which contains a number of parallel voltage sources as shown in Fig. 2.221 (*a*). Then common voltage V_{AB} which appears across the output terminals *A* and *B* is affected by the voltage sources E_1, E_2 and E_3. The value of the voltage is given by

$$V_{AB} = \frac{E_1/R_1 + E_2/R_2 + E_3/R_3}{1/R_1 + 1/R_2 + 1/R_3} = \frac{I_1 + I_2 + I_3}{G_1 + G_2 + G_3} = \frac{\Sigma I}{\Sigma G}$$

This voltage represents the Thevenin's voltage V_{th}. The resistance R_{th} can be found, as usual, by replacing each voltage source by a short circuit. If there is a load resistance R_L across the terminals *A* and *B*, then load current I_L is given by

$$I_L = V_{th}/(R_{th} + R_L)$$

If as shown in Fig. 2.222 (*b*), a branch does not contain any voltage source, the same procedure is used except that the value of the voltage for that branch is equated to zero as illustrated in Example 2.210.

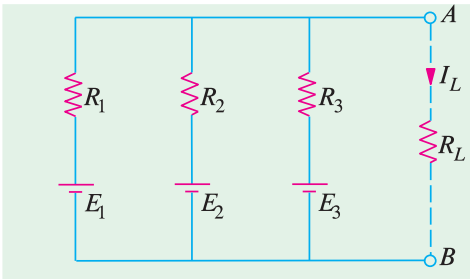


Fig. 2.221

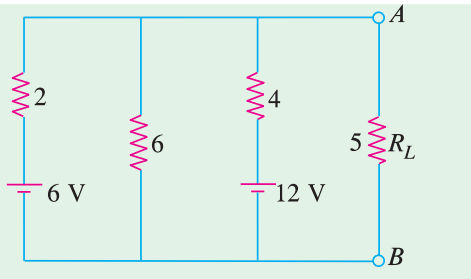


Fig. 2.222

Example 2.110. *Use Millman's theorem, to find the common voltage across terminals A and B and the load current in the circuit of Fig. 2.222.*

Solution. As per Millman's Theorem,

$$V_{AB} = \frac{6/2 + 0/6 + 12/4}{1/2 + 1/6 + 1/4} = \frac{6}{11/12} = 6.55 \text{ V}$$

$\therefore$ $V_{th} = 6.55$ V

$R_{th} = 2 \parallel 6 \parallel 4 = 12/11\ \Omega$

$$I_L = \frac{V_{th}}{R_{th} + R_L} = \frac{6.55}{(12/11) + 5} = 1.05 \text{ A}$$

(*b*) As Applicable to Current Sources

This theorem is applicable to a mixture of parallel voltage and current sources that are reduced to a single final equivalent source which is either a constant current or a constant voltage source. This theorem can be stated as follows :

Any number of constant current sources which are directly connected in parallel can be converted into a single current source whose current is the algebraic sum of the individual source currents and whose total internal resistances equals the combined individual source resistances in parallel.

Example 2.111. *Use Millman's theorem, to find the voltage across and current through the load resistor R_L in the circuit of Fig. 2.223 (a).*

Solution. First thing to do is to convert the given voltage sources into equivalent current sources. It should be kept in mind that the two batteries are connected in opposite direction. Using source conversion technique given in Art. 1.14 we get the circuit of Fig. 2.223 (*b*).

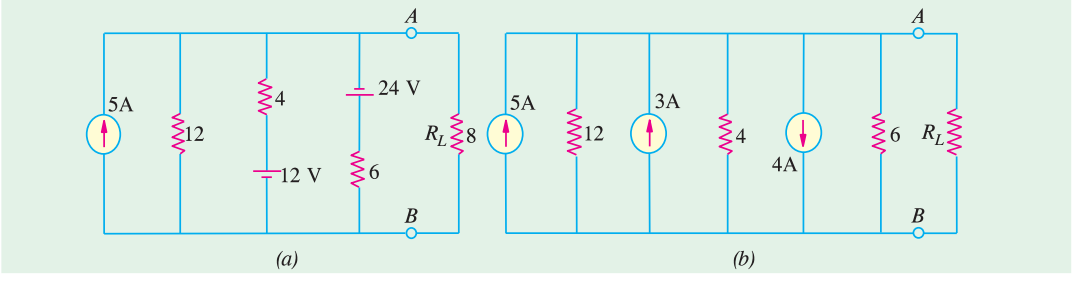


Fig. 2.223

The algebraic sum of the currents = 5 + 3 –4 = 4 A. The combined resistance is = 12 || 4 || 6 = 2 Ω. The simplified circuit is shown in the current–source form in Fig. 2.224 (*a*) or voltage source form in Fig. 2.224 (*b*).

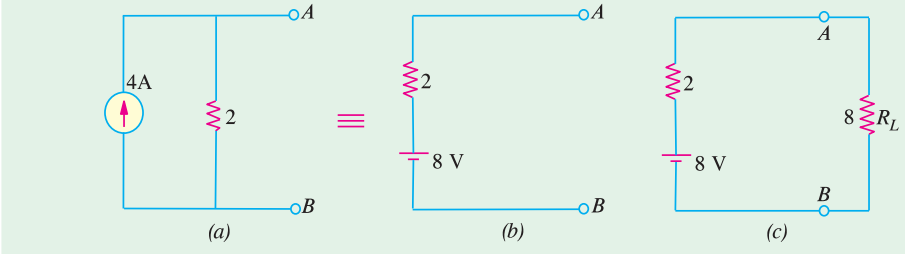


Fig. 2.224

As seen from Fig. 2.224 (*c*).

$$I_L = 8/(2 + 8) = 0.8 \text{ A} ; \quad V_L = 8 \times 0.8 = 64 \text{ V}$$

Alternatively, $V_L = 8 \times 8/(2 + 8) = 6.4$ V

Following steps are necessary when using Millman's Theorem :

1. convert all voltage sources into their equivalent current sources.
2. calculate the algebraic sum of the individual dual source currents.

3. if found necessary, convert the final current source into its equivalent voltage source.

As pointed out earlier, this theorem can also be applied to voltage sources which must be initially converted into their constant current equivalents.

2.29. Generalised Form of Millman's Theorem

This theorem is particularly useful for solving many circuits which are frequently encountered in both electronics and power applications.

Consider a number of admittances G_1, G_2, G_3... G_n which terminate at common point 0′ (Fig. 2.225). The other ends of the admittances are numbered as 1, 2, 3....*n*. Let *O* be any other point in the network. It should be clearly understood that it is not necessary to know anything about the inter-connection between point *O* and the end points 1, 2, 3...*n*. However, what is essential to know is the voltage drops from 0 to 1, 0 to 2, ... 0 to *n etc*.

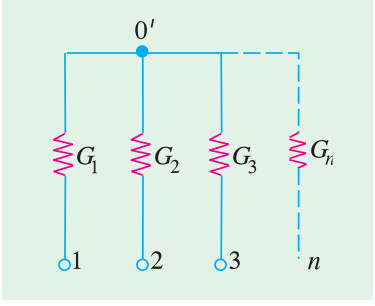


Fig. 2.225

According to this theorem, the voltage drop from 0 to 0′ (V_{oo}) is given by

$$V_{oo}' = \frac{V_{01}G_1 + V_{02}G_2 + V_{03}G_3 + ... + V_{0n}G_n}{G_1 + G_2 + G_3 + ... + G_n}$$

Proof

Voltage drop across $G_1 = V_{10}' = (V_{00}' - V_{01})$

Current through $G_1 = I_{10}' = V_{10}' \quad G_1 = (V_{00}' - V_{01})\, G_1$

Similarly, $I_{20}' = (V_{00}' - V_{02})\, G_2$

$I_{30}' = (V_{00}' - V_{03})\, G_3$

.....

.....

and $I_{n0}' = (V_{00}' - V_{0n})\, G_n$

By applying *KCL* to point 0′, we get

$$I_{10}' + I_{20}' + ... + I_{n0}' = 0$$

Substituting the values of these currents, we get

$$V_{00}' = \frac{V_{01}G_1 + V_{02}G_2 + V_{03}G_3 + ... + V_{0n}G_n}{G_1 + G_2 + G_3 + ... + G_n}$$

Precaution

It is worth repeating that only those resistances or admittances are taken into consideration which terminate at the common point. All those admittances are ignored which do not terminate at the common point even though they are connected in the circuit.

Example 2.112. *Use Millman's theorem to calculate the voltage developed across the 40 Ω resistor in the network of Fig. 2.226.*

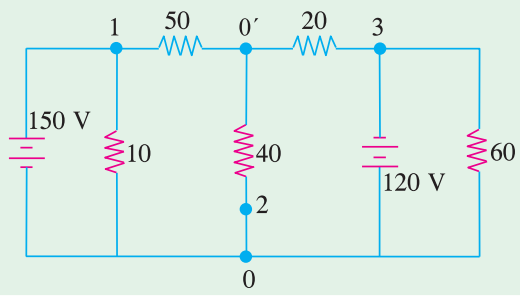


Fig. 2.226

Solution. Let the two ends of the 40 Ω resistor be marked as 0 and 0′. The end points of the three resistors terminating at the common point 0′ have been marked 1, 2 and 3. As already explained in Art. 2.29, the two resistors of values 10 Ω and 60 Ω will not come into the picture because they are not direclty connected to the common point 0′.

Here,

$$V_{01} = -150 \text{ V}; \quad V_{02} = 0; \quad V_{03} = 120 \text{ V}$$

$$G_1 = 1/50; \quad G_2 = 1/40: \quad G_3 = 1/20$$

$$\therefore \quad V_{00}' = \frac{(-150/50) + (0/40) + (120/20)}{(1/50) + (1/40) + (1/20)} = 31.6 \text{ V}$$

It shows that point 0 is at a higher potential as compared to point 0′.

Example 2.113. *Calculate the voltage across the 10 Ω resistor in the network of Fig. 2.227 by using* ***(a)*** *Millman's theorem* ***(b)*** *any other method.*

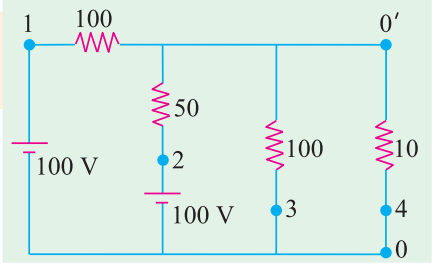


Fig. 2.227

Solution. ***(a)*** As shown in the Fig. 2.227 we are required to calculate voltage V_{00}'. The four resistances are connected to the common terminal 0′.

Let their other ends be marked as 1, 2, 3 and 4 as shown in Fig. 2.227. Now potential of point 0 with respect to point 1 is (Art. 1.25) – 100 V because (see Art. 1.25)

$$\therefore \quad V_{01} = -100 \text{ V}; \quad V_{02} = -100 \text{ V}; \quad V_{03} = 0\text{V}; \quad V_{04} = 0\text{V}.$$

$$G_1 = 1/100 = 0.01 \text{ Siemens}; \quad G_2 = 1/50 = 0.02 \text{ Siemens};$$

$$G_3 = 1/100 = 0.01 \text{ Siemens}; \quad G_4 = 1/10 = 0.1 \text{ Siemens}$$

$$\therefore \quad V_{00}' = \frac{V_{01}G_1 + V_{02}G_2 + V_{03}G_3 + V_{04}G_4}{G_1 + G_2 + G_3 + G_4}$$

$$= \frac{100 \quad 0.01 \quad (\ 100) \quad 0.02 \quad 0. \quad 0.01 \quad 0 \quad 0.1}{0.01 \quad 0.02 \quad 0.01 \quad 0.1} \quad \frac{3}{0.14} \quad \textbf{21.4 V}$$

Also, $V_{00}' = -V_{00}' = \mathbf{21.4\ V}$

(b) We could use the source conversion technique (Art. 2.14) to solve this question. As shown in Fig. 2.228 (*a*), the two voltage sources and their series resistances have been converted into current sources with their parallel resistances. The two current sources have been combined into a single resistance current source of 3 A and the three parallel resistances have been combined into a single resistance of 25 Ω. This current source has been reconverted into a voltage source of 75 V having a series resistance of 25 Ω as shown in Fig. 2.228 (*c*).

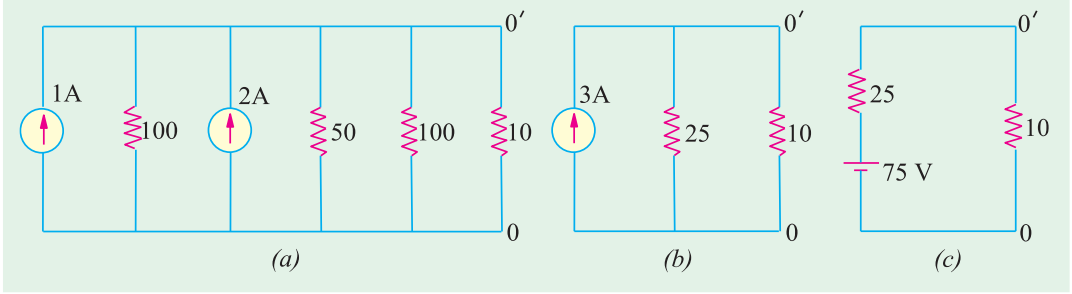


Fig. 2.228

Using the voltage divider formula (Art. 1.15), the voltage drop across 10 Ω resistance is $V_{0'0} = 75 \times 10/(10 + 25) = 21.4$ V.

Example 2.114. *In the network shown in Fig. 2.229, using Millman's theorem, or otherwise find the voltage between A and B.* **(Elect. Engg. Paper-I Indian Engg. Services 1990)**

Solution. The end points of the different admittances which are connected directly to the common point *B* have been marked as 1, 2 and 3 as shown in the Fig. 2.229. Incidentally, 40 Ω resistance will not be taken into consideration because it is not directly connected to the common point B. Here $V_{01} = V_{A1} = -50$ V ; $V_{02} = V_{A2} = 100$ V ; $V_{03} = V_{A3} = 0$ V.

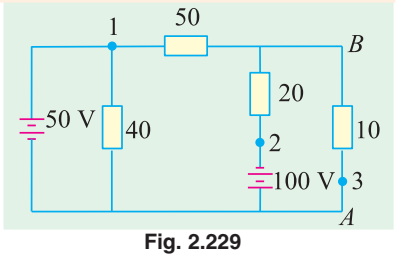


Fig. 2.229

$$\therefore V_{00}' = V_{AB} = \frac{(-50/50) + (100/20) + (0/10)}{(1/50) + (1/20) + (1/10)} = 23.5 \text{ V}$$

Since the answer comes out to be positive, it means that point *A* is at a higher potential as compared to point *B*.

The detailed reason for not taking any notice of 40 Ω resistance are given in Art. 2.29.

2.30. Maximum Power Transfer Theorem

Although applicable to all branches of electrical engineering, this theorem is particularly useful for analysing communication networks. The overall efficiency of a network supplying maximum power to any branch is 50 per cent. For this reason, the application of this theorem to power transmission and distribution networks is limited because, in their case, the goal is high efficiency and not maximum power transfer.

However, in the case of electronic and communication networks, very often, the goal is either to receive or transmit maximum power (through at reduced efficiency) specially when power involved is only a few milliwatts or microwatts. Frequently, the problem of maximum power transfer is of crucial significance in the operation of transmission lines and antennas.

As applied to d.c. networks, this theorem may be stated as follows :

A resistive load will abstract maximum power from a network when the load resistance is equal to the resistance of the network as viewed from the output terminals, with all energy sources removed leaving behind their internal resistances.

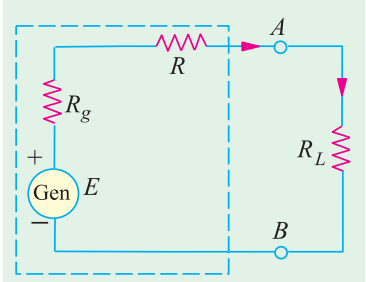


Fig. 2.230

In Fig. 2.230 (*a*), a load resistance of R_L is connected across the terminals *A* and *B* of a network which consists of a generator of e.m.f. *E* and internal resistance R_g and a series resistance *R* which, in fact, represents the lumped resistance of the connecting wires. Let $R_i = R_g + R$ = internal resistance of the network as viewed from *A* and *B*.

According to this theorem, R_L will abstract maximum power from the network when $R_L = R_i$.

Proof. Circuit current $$I = \frac{E}{R_L + R_i}$$

Power consumed by the load is

$$P_L = I^2 R_L = \frac{E^2 R_L}{(R_L + R_i)^2} \quad \text{...(i)}$$

For P_L to be maximum, $\frac{dP_L}{dR_L} = 0$.

Differentiating Eq. **(*i*)** above, we have

$$\frac{dP_L}{dR_L} = E^2\left[\frac{1}{(R_L+R_i)^2} + R_L\left(\frac{-2}{(R_L+R_i)^3}\right)\right] = E^2\left[\frac{1}{(R_L+R_i)^2} - \frac{2R_L}{(R_L+R_i)^3}\right]$$

$$\therefore \quad 0 = E^2\left[\frac{1}{(R_L+R_i)^2} - \frac{2R_L}{(R_L+R_i)^3}\right] \quad \text{or} \quad 2R_L = R_L + R_i \quad \text{or} \quad R_L = R_i$$

It is worth noting that under these conditions, the voltage across the load is hold the open-circuit voltage at the terminals *A* and *B*.

$$\therefore \quad \text{Max. power is } P_{L\,\max.} = \frac{E^2 R_L}{4R_L^2} = \frac{E^2}{4R_L} = \frac{E^2}{4R_i}$$

Let us consider an a.c. source of internal impedance $(R_1 + j X_1)$ supplying power to a load impedance $(R_L + jX_L)$. It can be proved that maximum power transfer will take place when the modules of the load impedance is equal to the modulus of the source impedance *i.e.* $|Z_L| = |Z_1|$

Where there is a completely free choice about the load, the maximum power transfer is obtained when load impedance is the complex conjugate of the source impedance. For example, if source impedance is $(R_1 + jX_1)$, then maximum transfer power occurs, when load impedance is $(R_1 - jX_1)$. It can be shown that under this condition, the load power is $= E^2/4R_1$.

Example 2.115. *In the network shown in Fig. 2.231 (a), find the value of R_L such that maximum possible power will be transferred to R_L. Find also the value of the maximum power and the power supplied by source under these conditions.* **(Elect. Engg. Paper I Indian Engg. Services)**

Solution. We will remove R_L and find the equivalent Thevenin's source for the circuit to the left of terminals *A* and *B*. As seen from Fig. 2.231 (*b*) V_{th} equals the drop across the vertical resistor of 3Ω because no current flows through 2 Ω and 1 Ω resistors. Since 15 V drops across two series resistors of 3 Ω each, $V_{th} = 15/2 = 7/5$ V. Thevenin's resistance can be found by replacing 15 V source with a short-circuit. As seen from Fig. 2.231 (*b*), $R_{th} = 2 + (3 \,||\, 3) + 1 = 4.5\ \Omega$. Maximum power transfer to the load will take place when $R_L = R_{th} = 4.5\ \Omega$

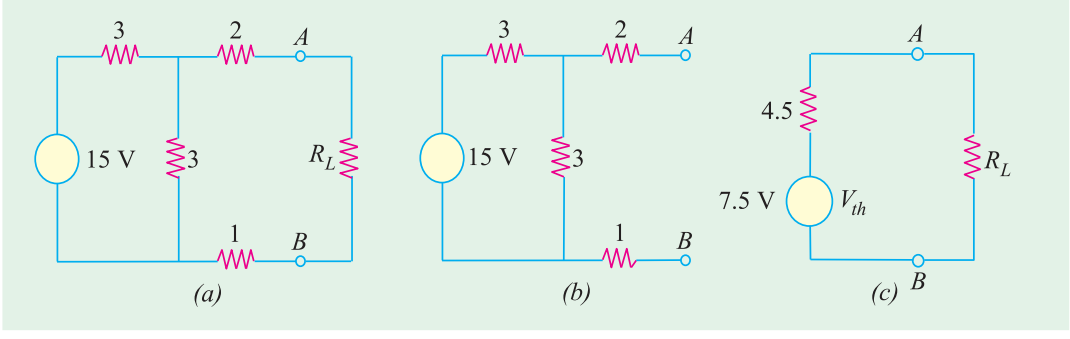


Fig. 2.231

Maximum power drawn by $R_L = V_{th}^2/4 \times R_L = 7.5^2/4 \times 4.5 = 3.125$ W.

Since same power in developed in R_{th}, power supplied by the source $= 2 \times 3.125 = 6.250$ W.

Example 2.116. *In the circuit shown in Fig. 2.232 (a) obtain the condition from maximum power transfer to the load R_L. Hence determine the maximum power transferred.*
(Elect. Science-I Allahabad Univ. 1992)

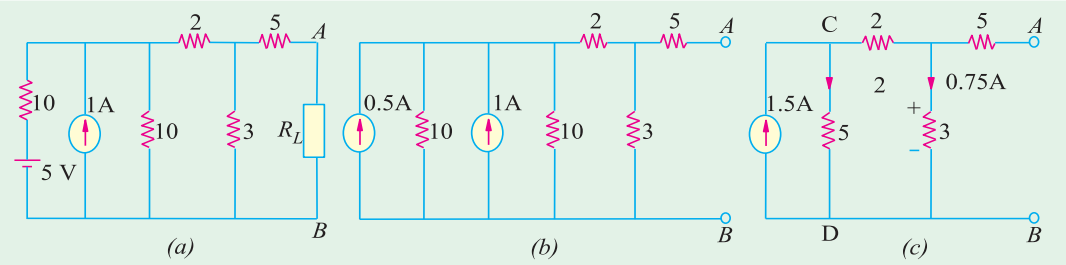


Fig. 2.232

Solution. We will find Thevenin's equivalent circuit to the left of trminals A and B for which purpose we will convert the battery source into a current source as shown in Fig. 2.232 (b). By combining the two current sources, we get the circuit of Fig. 2.232 (c). It would be seen that open circuit voltage V_{AB} equals the drop over 3 Ω resistance because there is no drop on the 5 Ω resistance connected to terminal A. Now, there are two parallel path across the current source each of resistance 5 Ω. Hence, current through 3 Ω resistance equals 1.5/2 = 0.75 A. Therefore, $V_{AB} = V_{th} = 3 \times 0.75 = 2.25$ V with point A positive with respect to point B.

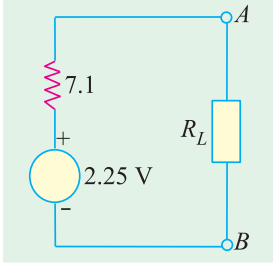


Fig. 2.233

For finding R_{AB}, current source is replaced by an infinite resistance.

$$\therefore \qquad R_{AB} = R_{th} = 5 + 3 \,||\, (2 + 5) = 7.1\ \Omega$$

The Thevenin's equivalent circuit alongwith R_L is shown in Fig. 2.233. As per Art. 2.30, the condition for *MPT* is that $R_L = 7.1\ \Omega$

Maximum power transferred = $V_{th}^2 / 4R_L = 2.25^2/4 \times 7.1 = 0.178$ W = 178 mW.

Example 2.117. *Calculate the value of R which will absorb maximum power from the circuit of Fig. 2.234 (a). Also, compute the value of maximum power.*

Solution. For finding power, it is essential to know both I and R. Hence, it is essential to find an equation relating I to R.

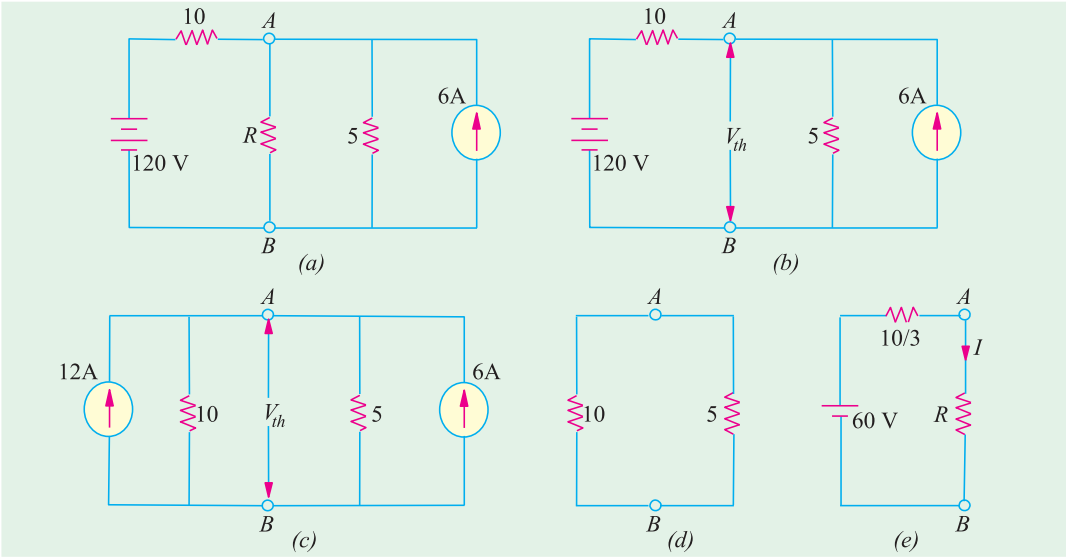


Fig. 2.234

Let us remove R and find Thevenin's voltage V_{th} across A and B as shown in Fig. 2.234 (b). It would be helpful to convert 120 V, 10-Ω source into a constant-current source as shown in Fig. 2.234 (c). Applying *KCL* to the circuit, we get

$$\frac{V_{th}}{10} + \frac{V_{th}}{5} = 12 + 6 \quad \text{or} \quad V_{th} = 60 \text{ V}$$

Now, for finding R_i and R_{th}, the two sources are reduced to zero. Voltage of the voltage-source is reduced to zero by short - circuiting it whereas current of the current source is reduced to zero by open-circuiting it. The circuit which results from such source suppression is shown in Fig. 2.234 (d). Hence, $R_i = R_{th} = 10 \parallel 5 = 10/3$ Ω. The Thevenin's equivalent circuit of the network is shown in Fig. 2.234 (e).

According to Maximum Power Transfer Theorem, R will absorb maximum power when it equals 10/3 Ω. In that case, $I = 60 \div 20/3 = 9$ A

$$P_{max} = I^2R = 9^2 \times 10/3 = \mathbf{270\ W}$$

2.31. Power Transfer Efficiency

If P_L is the power supplied to the load and P_T is the total power supplied by the voltage source, then power transfer efficiency is given by $\eta = P_L/P_T$.

Now, the generator or voltage source E supplies power to both the load resistance R_L and to the internal resistance $R_i = (R_g + R)$.

$$P_T = P_L + P_i \quad \text{or} \quad E \times I = I^2R_L + I^2R_i$$

$$\therefore \quad \eta = \frac{P_L}{P_T} = \frac{I^2R_L}{I^2R_L + I^2R_i} = \frac{R_L}{R_L + R_i} = \frac{1}{1 + (R_i / R_L)}$$

The variation of η with R_L is shown in Fig. 2.235 (a). The maximum value of η is unity when $R_L = \infty$ and has a value of 0.5 when $R_L = R_i$. It means that under maximum power transfer conditions, the power transfer efficiency is only 50%. As mentioned above, maximum power transfer condition is important in communication applications but in most power systems applications, a 50% efficiency is undesirable because of the wasted energy. Often, a compromise has to be made between the load power and the power transfer efficiency. For example, if we make $R_L = 2\ R_i$, then

$$P_L = 0.222\ E^2/R_i \quad \text{and} \quad \eta = 0.667.$$

It is seen that the load power is only 11% less than its maximum possible value, whereas the power transfer efficiency has improved from 0.5 to 0.667 *i.e.* by 33%.

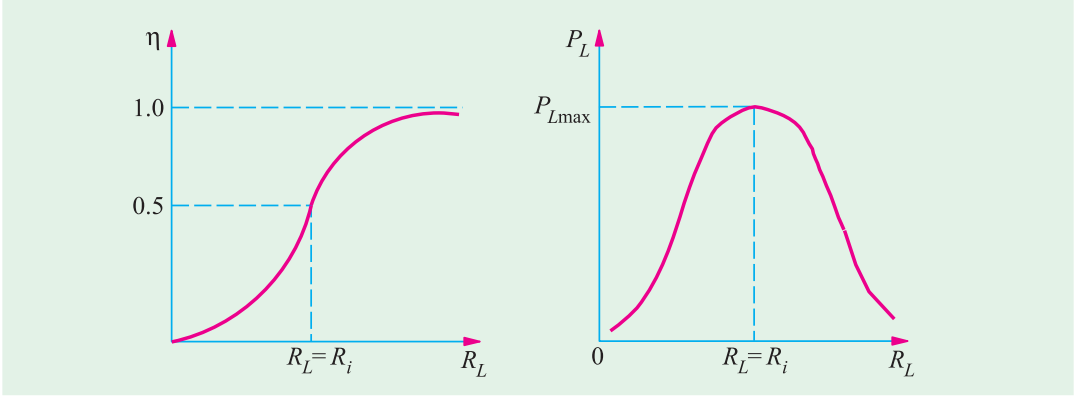


Fig. 2.235

Example 2.118. *A voltage source delivers 4 A when the load connected to it is 5 Ω and 2 A when the load becomes 20 Ω Calculate*

(a) *maximum power which the source can supply* ***(b)*** *power transfer efficiency of the source with R_L of 20 Ω* ***(c)*** *the power transfer efficiency when the source delivers 60 W.*

Solution. We can find the values of E and R_i from the two given load conditions.

(a) When $R_L = 5\ \Omega$, $I = 4$ A and $V = IR_L = 4 \times 5 = 20$ V, then $20 = E - 4R_i$ ...***(i)***

When $R_L = 20\ \Omega$, $I = 2$ A and $V = IR_L = 2 \times 20 = 40$ V $\therefore\ 40 = E - 2R_i$ ...***(ii)***

From ***(i)*** and ***(ii)***, we get, $R_i = 10\ \Omega$ and $E = 60$ V

When $R_L = R_i = 10\ \Omega$

$$P_{L\,max} = \frac{E^2}{4R_i} = \frac{60 \times 60}{4 \times 10} = 90 \text{ W}$$

(b) When $R_L = 20\ \Omega$, the power transfer efficiency is given by

$$\eta = \frac{R_L}{R_L + R_i} = \frac{20}{30} = 0.667 \quad \text{or} \quad 66.7\%$$

(c) For finding the efficiency corresponding to a load power of 60 W, we must first find the value of R_L.

Now, $$P_L = \left(\frac{E}{R_i + R_L}\right)^2 R_L$$

$\therefore$ $$60 = \frac{60^2 \times R_L}{(R_L + 10)^2} \quad \text{or} \quad R_L^2 - 40R_L + 100 = 0$$

Hence $$R_L = 37.32\ \Omega \quad \text{or} \quad 2.68\ \Omega$$

Since there are two values of R_L, there are two efficiencies corresponding to these values.

$$\eta_1 = \frac{37.32}{37.32 + 10} = 0.789 \quad \text{or} \quad 78.9\%, \quad \eta_2 = \frac{2.68}{12.68} = 0.211 \quad \text{or} \quad 21.1\%$$

It will be seen from above, the $\eta_1 + \eta_2 = 1$.

Example 2.119. *Two load resistance R_1 and R_2 dissipate the same power when connected to a voltage source having an internal resistance of R_i. Prove that* ***(a)*** $R_i^2 = R_1 R_2$ *and* ***(b)*** $\eta_1 + \eta_2 = 1$.

Solution. ***(a)*** Since both resistances dissipate the same amount of power, hence

$$P_L = \frac{E^2 R_1}{(R_1 + R_i)^2} = \frac{E^2 R_2}{(R_2 + R_i)^2}$$

Cancelling E^2 and cross-multiplying, we get

$R_1 R_2^2 + 2R_1 R_2 R_i + R_1 R_i^2 = R_2 R_1^2 + 2R_1 R_2 R_i + R_2 R_i^2$

Simplifying the above, we get, $R_i^2 = R_1 R_2$

(b) If η_1 and η_2 are the two efficiencies corresponding to the load resistances R_1 and R_2, then

$$\eta_1 + \eta_2 = \frac{R_1}{R_1 + R_i} + \frac{R_2}{R_2 + R_i} = \frac{2R_1 R_2 + R_i(R_1 + R_2)}{R_1 R_2 + R_i^2 + R_i(R_1 + R_2)}$$

Substituting $R_i^2 = R_1 R_2$, we get

$$\eta_1 + \eta_2 = \frac{2R_i^2 + R_i(R_1 + R_2)}{2R_i^2 + R_i(R_1 + R_2)} = 1$$

Example 2.120. *Determine the value of R_1 for maximum power at the load. Determine maximum power also. The network is given in the Fig. 2.236 (a).* **[Bombay University 2001]**

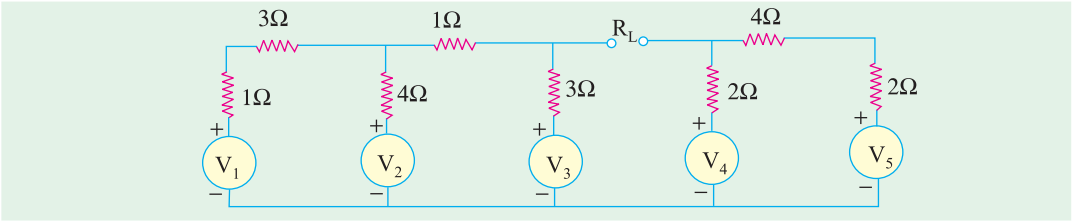


Fig. 2.236 (*a*)

Solution. This can be attempted by Thevenin's Theorem. As in the circuit, with terminals A and B kept open, from the right hand side, V_B (w.r. to reference node 0) can be calculated V_4 and V_5 will have a net voltage of 2 volts circulating a current of (2/8) = 0.25 amp in clockwise direction.

$$V_B = 10 - 0.25 \times 2 = 9.5 \text{ volts.}$$

On the Left-hand part of the circuit, two loops are there. V_A (w.r. to 0) has to be evaluated. Let the first loop (with V_1 and V_2 as the sources) carry a clockwise current of i_1 and the second loop (with V_2 and V_3 as the sources), a clockwise current of i_2. Writing the circuit equations.

$$8i - 4i_2 = +4$$
$$-4i + 8i_2 = +4$$

This gives $i_1 = 1$ amp, $i_2 = 1$ amp

Therefore, $V_A = 12 + 3 \times 1 = 15$ volts.

Thevenin −voltage, $V_{TH} = V_A - V_B = 15 - 9.5 = 5.5$ volts

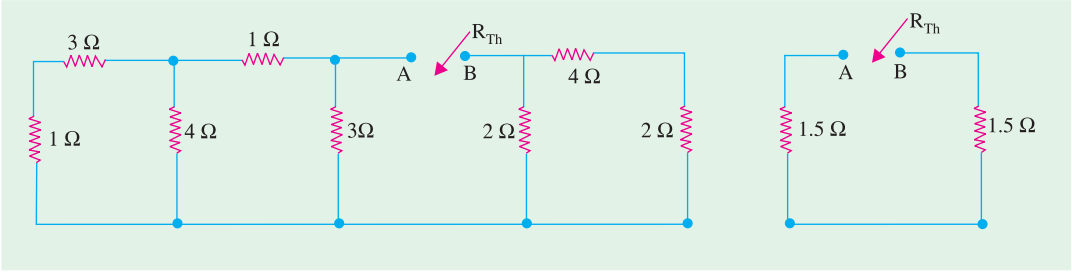


Fig. 2.236 (*b*) **Fig. 2.236 (*c*)**

Solving as shown in Fig. 2.236 (*b*) and (*c*).

$R_{TH} = 3$ ohms

For maximum power transfer, $R_L = 3$ ohms

Current = 5.5/6 = 0.9167 amp

Power transferred to load $0.9167^2 \times 3 = 2.52$ watts.

Example 2.121. *For the circuit shown below, what will be the value of R_L to get the maximum power ? What is the maximum power delivered to the load ?* **[Bombay University 2001]**

Solution. Detach R_L and apply Thevenin's Theorem.

$V_{TH} = 5.696$ volts, $R_{TH} = 11.39\ \Omega$

R_L must be 11.39 ohms for maximum power transfer.

$P_{max} = 0.712$ watt.

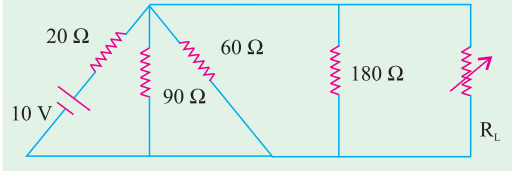


Fig. 2.237

Example 2.122. *Find the maximum power in 'R_L' which is variable in the circuit shown below in Fig. 2.238.* **[Bombay University, 2001]**

Solution. Apply Thevenin's theorem. For this R_L has to be detached from nodes A and B. Treat O as the reference node.

$V_A = 60$ V, $V_B = V_C + 2 = 50 + 2 = 52$ V

Thus, $V_{TH} = V_{AB} = 8$ volts, with A positive w.r. to B, $R_{TH} = (60//40) + (50//50) = 49$ ohms

Hence, for maximum power, $R_L = 49$ ohms

With this R_L, Current = 8/98 amp = 0.08163 amp

Power to Load = $i^2 R_L = 0.3265$ watt

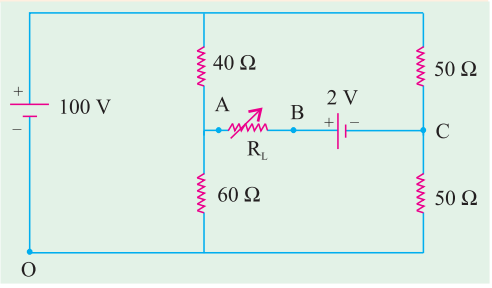


Fig. 2.238

Example 2.123. *Find V_A and V_B by "nodal analysis" for the circuit shown in Fig. 2.239 (a).* **[Bombay University]**

Solution. Let the conductance be represented by g. Let all the sources be current sources. For this, a voltage-source in series with a resistor is transformed into its equivalent current source. This is done in Fig. 2.239 (*b*).

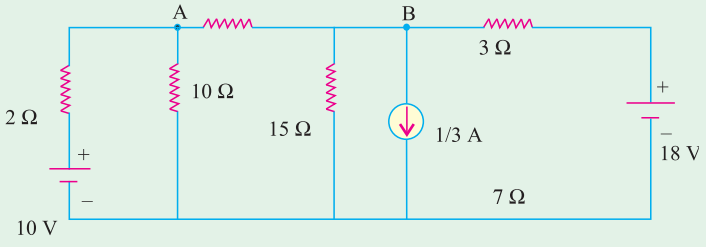


Fig. 2.239 (*a*)

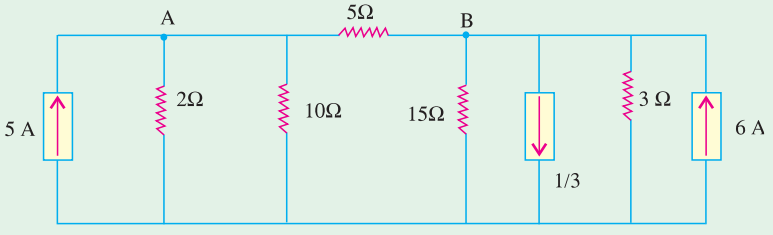


Fig. 2.239 (*b*). All Current Sources

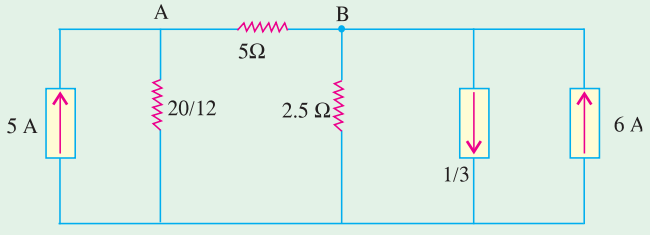


Fig. 2.239 (*c*)

Observing the circuit, $g_{11} = (1/5) + 0.6 = 0.8$, $g_{22} = 0.40 + 0.2 = 0.6$
$g_{12} = 0.2$, Current sources : + 5 amp into 'A' + 5.67 amp into 'B'

$$\Delta = \begin{bmatrix} 0.8 & -0.2 \\ -0.2 & 0.6 \end{bmatrix} = 0.44$$

$$\Delta_1 = \begin{bmatrix} 5 & -0.2 \\ 5.67 & 0.6 \end{bmatrix} = 4.134$$

$$\Delta_2 = \begin{bmatrix} 0.8 & 5 \\ -0.2 & 5.67 \end{bmatrix} = 5.526$$

$V_A = 4.134/0.44 = 9.4$ volts,
$V_B = 5.536/0.44 = 12.6$ volts.

Current in 5-ohm resistor
$= (V_B - V_A)/5 = 0.64$ amp

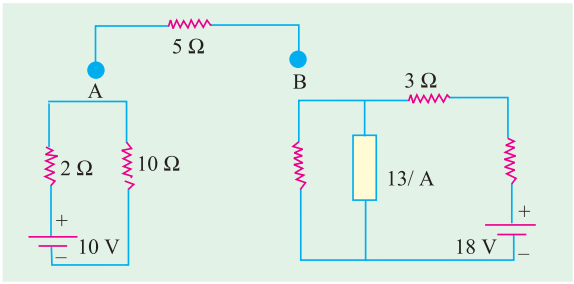


Fig. 2.239 (*d*) Thevenized Circuit

Check : Apply Thevenin's Theorem :

$V_A = 10 \times (10/12) = 8.333$ V
$V_B = (17/3) \times 2.5 = 14.167$ V
$V_{TH} = 14.167 - 8.333 = 5.834$ V
$R_{TH} = 4.167$
$I_5 = 5.834/(4.167 + 5) = 0.64$ A

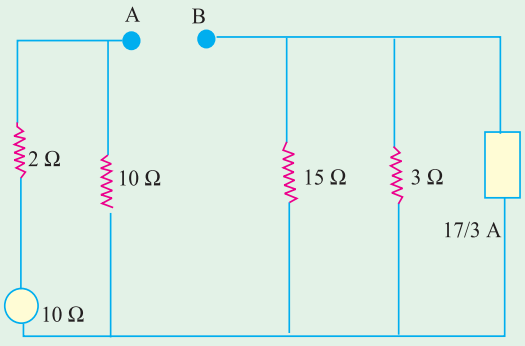


Fig. 2.239 (*e*) Right side simplified

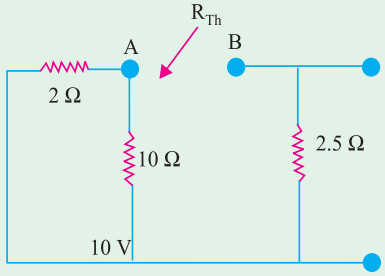


Fig. 2.239 (*f*) Evaluating R_{TH}

Example. 2.124. *Find the magnitude R_L for the maximum power transfer in the circuit shown in Fig. 2.240 (a). Also find out the maximum power.*

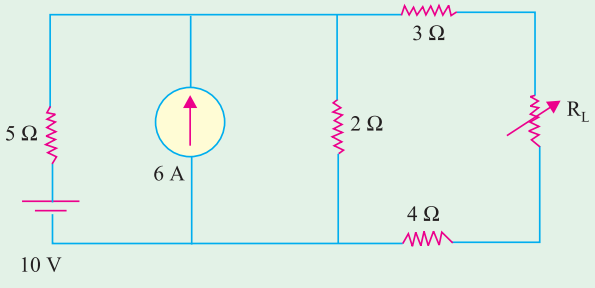


Fig. 2.240 (*a*)

Solution. Simplify by source transformations, as done in Fig. 2.240 (*b*), (*c*), (*d*)

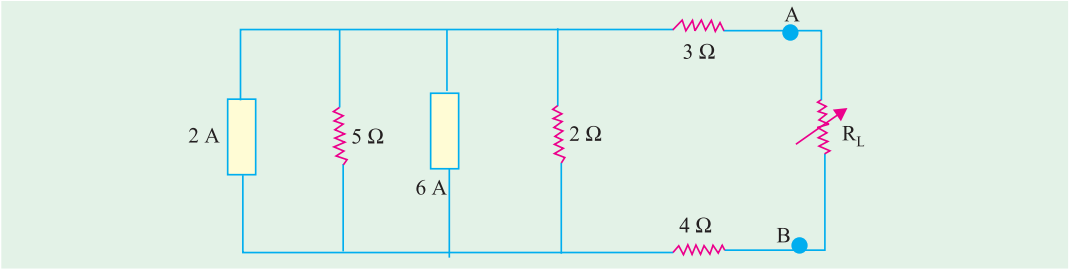


Fig. 2.240 (*b*)

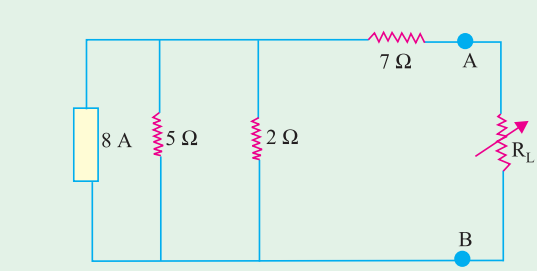


Fig. 2.240 (*c*)

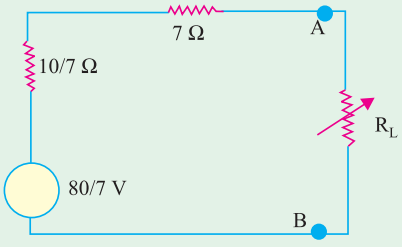


Fig. 2.240 (*d*)

For maximum power, $R_L = 7 + (10/7) = 8.43\ \Omega$

Maximum power $= [(80/7)/16.68]^2 \times 8.43 = 3.87$ watts.

Tutorial Problems No. 2.6

(*a*) Norton Theorem

1. Find the Thevenin and Norton equivalent circuits for the active network shown in Fig. 2.241 (*a*). All resistance are in ohms. [**Hint :** Use Superposition principle to find contribution of each source]

[10 V source, series resistor = 5 Ω; 2 A source, parallel resistance = 5 Ω]

2. Obtain the Thevenin and Norton equivalent circuits for the circuit shown in Fig. 2.241 (b). All resistance values are in ohms.

[15 V source, series resistance = 5 Ω; 3 A source, parallel resistance = 5 Ω]

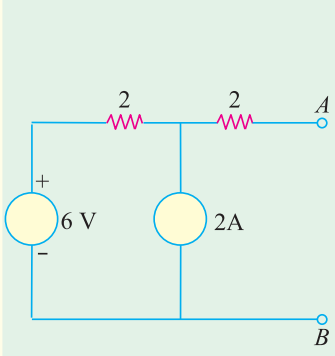


Fig. 2.241 (*a*)

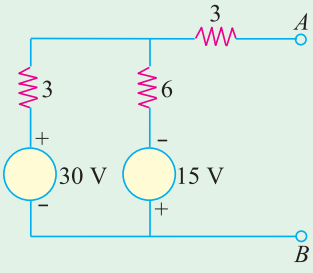


Fig. 2.241 (*b*)

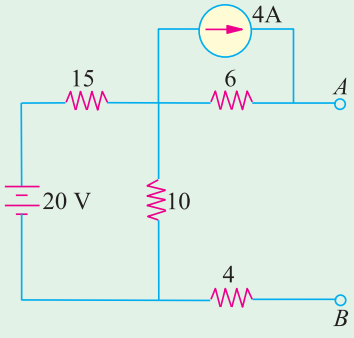


Fig. 2.241 (*c*)

3. Find the Norton equivalent circuit for the active linear network shown in Fig. 2.241 (*c*). All resistances are in ohms. **Hint :** It would be easier to first find Thevenin's equivalent circuit].

[2 A source; parallel resistance = 16 Ω]

4. Find Norton's equivalent circuit for the network shown in Fig. 2.249. Verify it through its Thevenin's equivalent circuit. **[1 A, Parallel resistance = 6 Ω]**

5. State the Tellegen's theorem and verify it by an illustration. Comment on the applicability of Tellegen's theorem on the types of networks. ***(Circuit and Field Theory, A.M.I.E. Sec. B, 1993)***

Solution. Tellegen's Theorem can be stated as under :

For a network consisting of n elements if $i_1, i_2,.....i_n$ are the currents flowing through the elements satisfying Kirchhoff's current law and $v_1, v_2.....v_n$ are the voltages across these elements satisfying Kirchhoff's law, then

$$\sum_{k=1}^{n} v_k i_k = 0$$

where v_k is the voltage across and i_k is the current through the k_{th} element. In other words, according to Tellegen's Theorem, the sum of instantaneous powers for the n branches in a network is always zero.

This theorem has wide applications. It is valid for any lumped network that contains any elements linear or non-linear, passive or active, time-variant or time-invariant.

Explanation : This theorem will be explained with the help of the simple circuit shown in Fig. 2.242. The total resistance seen by the battery is = 8 + 4 || 4 = 10 Ω.

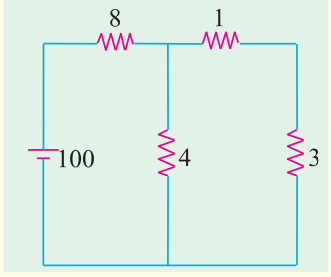


Fig. 2.242

Battery current I = 100/10 = 10 A. This current divides equally at point B,

Drop over 8 Ω resistor = 8 × 10 = 80 V

Drop over 4 Ω resistor = 4 × 5 = 20 V

Drop over 1 Ω resistor = 1 × 5 = 5 V

Drop over 3 Ω resistor = 3 × 5 = 15 V

According to Tellegen's Theorem,

= 100 × 10 −80 × 10 −20 × 5 − 5 × 5 −15 × 5 = 0

(b) Millman's Theorem

6. Use Millman's theorem, to find the potential of point A with respect to the ground in Fig. 2.243. **[V_A = 8.18 V]**

7. Using Millman's theorem, find the value of output voltage V_0 in the circuit of Fig. 2.244. All resistances are in ohms. **[4 V]**

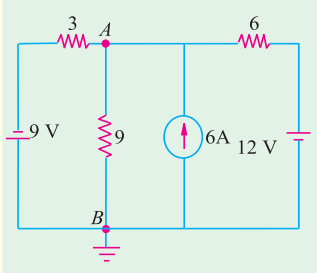


Fig. 2.243

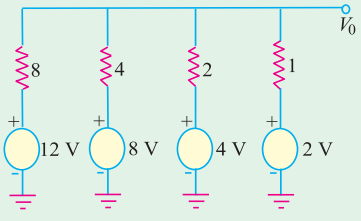


Fig. 2.244

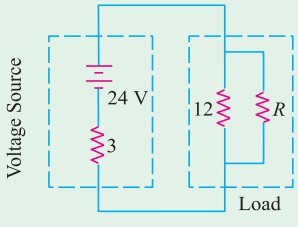


Fig. 2.245

(b) MPT Theorem

8. In Fig. 2.245 what value of R will allow maximum power transfer to the load ? Also calculate the maximum total load power. All resistances are in ohms.

[4 Ω ; 48 W]

9. Use superposition theorem to find currents in various branches of the ckt in Fig. 2.246.

(B.P.T.U., Orissa 2003) (Nagpur University, Summer 2002)

10. Find the resistance between point A and B for the circuit shown in Fig. 2.247.
(Nagpur University, Winter 2002)

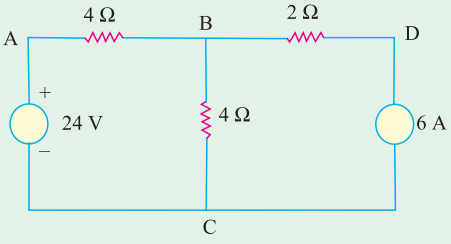


Fig. 246

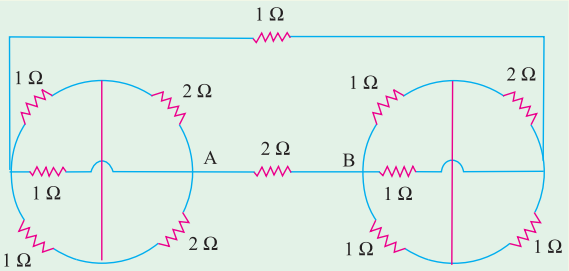


Fig. 247

11. Apply the superposition theorem and find the current through 25 ohm resistance of the circuit shown in Fig. 2.248. *(Mumbai University 2002) (Nagpur University, Summer 2003)*

12. Find the total current flowing through the circuit shown in Fig. 2.249 using stat-delta transformation if the circuit is excited by 39 volts and the value of each resistor connected in circuit is 4 ohms.
(Ravishankar University, Raipur 2003) (Nagpur University, Summer 2003)

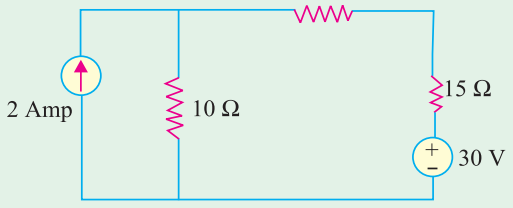


Fig. 2.248

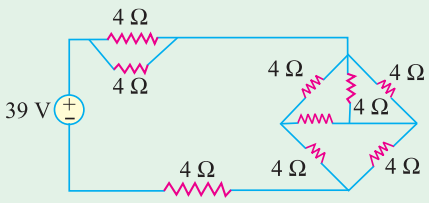


Fig. 2.249

13. Compute the power dissipated in the 9 ohm resistor in the Fig. 2.250 by applying Superposition Theorem. The voltage and current sources should be treated as ideal. All resistances are in ohm.
(Mumbai University 2003) (Nagpur University, Winter 2003)

14. Find the current in 11 ohm resistor in the Fig. 2.251 using star/delta conversion. All resistances are in ohm. *(Nagpur University, Winter 2003)*

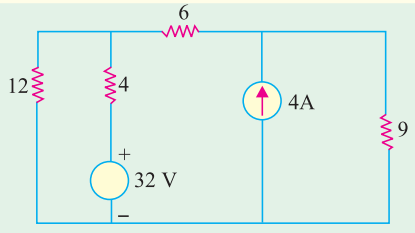


Fig. 2.250

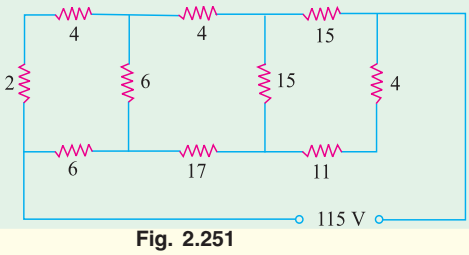


Fig. 2.251

15. Calculate current-flowing through "2 ohms" resistor in Fig. 2.252 by using Superposition theorem.
(Mumbai University 2003) (Nagpur University, Summer 2004)

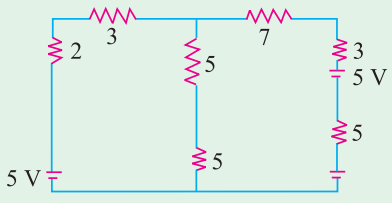


Fig. 2.252. All resistance are in ohms.

16. State and explain Superposition Theorem.
(Pune University 2003) (Nagpur University, Summer 2004)

17. A cast iron ring of 40 cm diameter is wound with a coil. The coil carries a current of 3 amp and produces a flux of 3 mwb in the air gap. The length of air gap is 2 mm. The relative permeability of the cast iron is 800. The leakage coefficient is 1.2. Calculate no. of turns of the coil.
(Nagpur University, Summer 2004)

18. Using superposition theorem, calculate the current I_{AB} in the given circuit of Fig. 2.253.
(Gujrat University, Summer 2003)

19. Using delta-star transformation, determine the current drawn from the source in the given circuit Fig.2.254. *(Gujrat University,Summer 2003)*

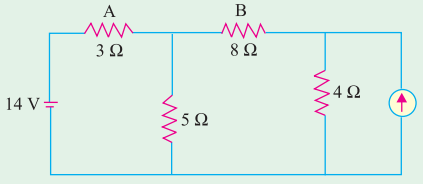


Fig. 2.253

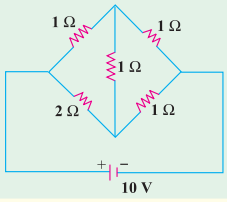


Fig. 2.254

20. State and explain Kirchhoff's laws applied to electric circuit.
(Gujrat University, Summer2003)

21. State Kirchhoff's laws. *(Madras University, April 2002)*

22. Three resistances R_{ab}, *Rbc* and *Rca* are connected in delta. Obtain expressions for their equivalent star resistances. *(V.T.U., Belgaum Karnataka University, February 2002)*

23. In the circuit, shown in Fig. 2.255 determine the value of E so that the current I = 0. Use mesh method of analysis. *(V.T.U., Belgaum Karnataka University, January/February 2004)*

24. In Fig. 2.256 derive the expressions to replace a delta connected resistances by an equivalent star connected resistances. Determine the resistance between *a* and *b*. All the resistance and 1Ω each.
(V.T.U., Belgaum Karnataka University, January/February 2004)

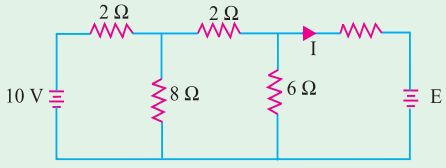


Fig. 2.255

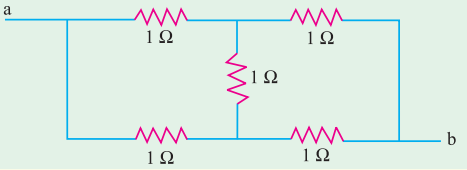


Fig. 2.256

25. Determine the values of I and R in the circuit shown in the Fig. 2.257. *(ESE 2003)*

26. In the circuit shown in the Fig. 2.258, S is closed at time $t = 0$. Determine $i_c(t)$ and the time constant.
(Pune University 2003) (ESE 2003)

27. In the circuit shown in the Fig. 2.259. S is closed at $t = 0$. Find the current $i_c(t)$ through the capacitor at $t = 0$.
(Pune University 2003) (ESE 2003)

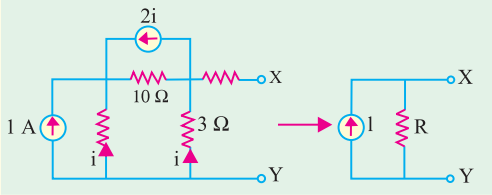


Fig. 2.257

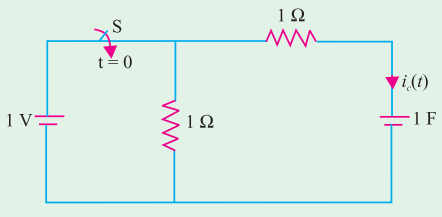


Fig. 2.258

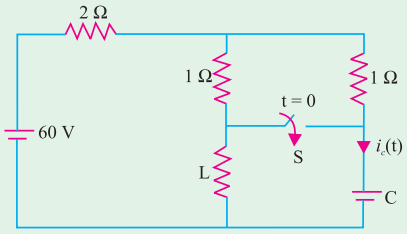


Fig. 2.259

OBJECTIVE TESTS – 2

1. Kirchhoff's current law is applicable to only
(*a*) closed loops in a network
(*b*) electronic circuits
(*c*) junctions in a network
(*d*) electric circuits.

2. Kirchhoff's voltage law is concerned with
(*a*) *IR* drops
(*b*) battery e.m.fs.
(*c*) junction voltages
(*d*) both (*a*) and (*b*)

3. According to *KVL*, the *algebraic* sum of all *IR* drops and e.m.f.s in any closed loop of a network is always
(*a*) zero
(*b*) positive
(*c*) negative
(*d*) determined by battery e.m.fs.

4. The ***algebraic*** sign of an *IR* drop is primarily dependent upon the
(*a*) amount of current flowing through it
(*b*) value of *R*
(*c*) direction of current flow
(*d*) battery connection.

5. Maxwell's loop current method of solving electrical networks
(*a*) uses branch currents
(*b*) utilizes Kirchhoff's voltage law
(*c*) is confined to single-loop circuits
(*d*) is a network reduction method.

6. Point out of the WRONG statement. In the node-voltage technique of solving networks, choice of a reference node does not
(*a*) affect the operation of the circuit
(*b*) change the voltage across any element
(*c*) alter the p.d. between any pair of nodes
(*d*) affect the voltages of various nodes.

7. For the circuit shown in the given Fig. 2.260, when the voltage E is 10 V, the current i is 1 A. If the applied woltage across terminal C-D is 100 V, the short circuit current flowing through the terminal A-B will be

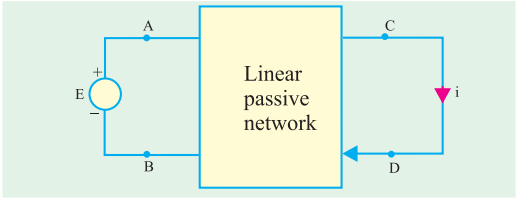


Fig. 2.260

(*a*) 0.1 A (*b*) 1 A
(*c*) 10 A (*d*) 100 A

(ESE 2001)

8. The component inductance due to the internal flux-linkage of a non-magnetic straight solid circular conductor per metre length, has a constant value, and is independent of the conductor-diameter, because
(*a*) All the internal flux due to a current remains concentrated on the peripheral region of the conductor.
(*b*) The internal magnetic flux-density along the radial distance from the centre of the conductor increases proportionately to the current enclosed
(*c*) The entire current is assumed to flow along the conductor-axis and the internal flux is distributed uniformly and concentrically
(*d*) The current in the conductor is assumed to be uniformly distributed throughout the conductor cross-section

(ESE 2003)

9. Two ac sources feed a common variable resistive load as shown n in Fig. 2.261. Under the maximum power transfer condition, the power absorbed by the load resistance R_L is

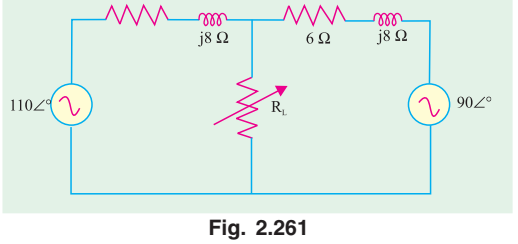


Fig. 2.261

(*a*) 2200 W (*b*) 1250W
(*c*) 1000 W (*d*) 625 W

(GATE 2003)

ANSWERS

1. *c* **2.** *d* **3.** *a* **4.** *c* **5.** *b* **6.** *d*

Top

Contents

CHAPTER 3

Learning Objectives

- Effect of Electric Current
- Joule's Law of Electric Heating
- Thermal Efficiency
- S-I. Units
- Calculation of Kilo-watt Power of a Hydroelectric Station

WORK, POWER AND ENERGY



Today, life without electricity is highly unimaginable. Electric locomotives, heaters, and fans are some of the appliances and machines which convert electricity into work and energy

3.1. Effect of Electric Current

It is a matter of common experience that a conductor, when carrying current, becomes hot after some time. As explained earlier, an electric current is just a directed flow or drift of electrons through a substance. The moving electrons as they pass through molecules of atoms of that substance, collide with other electrons. This electronic collision results in the production of heat. This explains why passage of current is always accompanied by generation of heat.

3.2. Joule's Law of Electric Heating

The amount of work required to maintain a current of I amperes through a resistance of R ohm for t second is

$$\text{W.D.} = I^2 Rt \text{ joules}$$
$$= VIt \text{ joules} \quad (\because R = V/I)$$
$$= Wt \text{ joules} \quad (\because W = VI)$$
$$= V^2t/R \text{ joules} \quad (\because I = V/R)$$

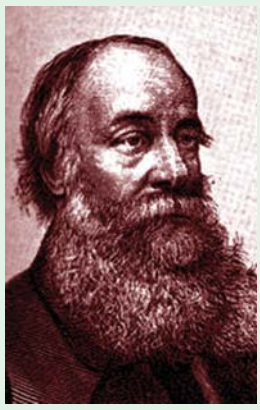
This work is converted into heat and is dissipated away. The amount of heat produced is

$$H = \frac{\text{work done}}{\text{mechanical equivalent of heat}} = \frac{W.D.}{J}$$

where $J = 4{,}186 \text{ joules/kcal} = 4{,}200 \text{ joules / kcal (approx)}$

$\therefore$ $H = I^2Rt/4{,}200 \text{ kcal} = Vlt/4{,}200 \text{ kcal}$

$= Wt/4{,}200 \text{ kcal} = V^2t/4{,}200\,R \text{ kcal}$



James Joule*

3.3. Thermal Efficiency

It is defined as the ratio of the heat actually utilized to the total heat produced electrically. Consider the case of the electric kettle used for boiling water. Out of the total heat produced **(*i*)** some goes to heat the apparatus itself *i.e.* kettle **(*ii*)** some is lost by radiation and convection etc. and **(*iii*)** the rest is utilized for heating the water. Out of these, the heat utilized for useful purpose is that in **(*iii*)**. Hence, thermal efficiency of this electric apparatus is the ratio of the heat utilized for heating the water to the total heat produced.

Hence, the relation between heat produced electrically and heat absorbed usefully becomes

$$\frac{Vlt}{J} \times \eta = ms\,(\theta_2 - \theta_1)$$

Example 3.1. *The heater element of an electric kettle has a constant resistance of 100 Ω and the applied voltage is 250 V. Calculate the time taken to raise the temperature of one litre of water from 15°C to 90°C assuming that 85% of the power input to the kettle is usefully employed. If the water equivalent of the kettle is 100 g, find how long will it take to raise a second litre of water through the same temperature range immediately after the first.*

(Electrical Engineering, Calcutta Univ.)



In an electric kettle, electric energy is converted into heat energy.

* James Joule was born in Salford, England, in 1818. He was a physicist who is credited with discovering the law of conservation of energy. Joule's name is used to describe the international unit of energy known as the joule.

Solution. Mass of water = 1000 g = 1 kg ($\because$ 1 cm^3 weight 1 gram)

Heat taken by water = $1 \times (90 - 15)$ = 75 kcal

Heat taken by the kettle = $0.1 \times (90 - 15)$ = 7.5 kcal

Total heat taken = 75 + 7.5 = 82.5 kcal

Heat produced electrically $H = I^2Rt/J$ kcal

Now, $I = 250/100 = 2/5$ A, $J = 4{,}200$ J/kcal; $H = 2.5^2 \times 100 \times t/4200$ kcal

Heat actually utilized for heating one litre of water and kettle

$$= 0.85 \times 2.5^2 \times 100 \times t/4{,}200 \text{ kcal}$$

$$\therefore \quad \frac{0.85 \times 6.25 \times 100 \times t}{4,200} = 82.5 \quad \therefore \; t = \textbf{10 min 52 second}$$

In the second case, heat would be required only for heating the water because kettle would be already hot.

$$\therefore \quad 75 = \frac{0.85 \times 6.25 \times 100 \times t}{4,200} \quad \therefore \; t = \textbf{9 min 53 second}$$

Example 3.2. *Two heater A and B are in parallel across supply voltage V. Heater A produces 500 kcal in 200 min. and B produces 1000 kcal in 10 min. The resistance of A is 10 ohm. What is the resistance of B ? If the same heaters are connected in series across the voltage V, how much heat will be prduced in kcal in 5 min ?* **(Elect. Science - II, Allahabad Univ. 1992)**

Solution. Heat produced = $\frac{V^2 t}{JR}$ kcal

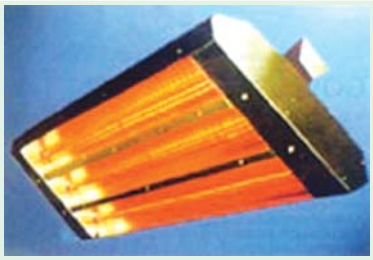
For heater *A*, $500 = \frac{V^2 \times (20 \times 60)}{10 \times J}$ ...(*i*)

For heater *B*, $1000 = \frac{V^2 \times (10 \times 60)}{R \times J}$ ...(*ii*)

From Eq. (*i*) and (*ii*), we get, $R =$ **2.5 Ω**



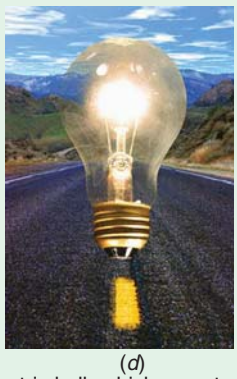
(*a*)



(*b*)



(*c*)



(*d*)

In this *a*, *b*, and *c* are heaters which convert electric energy into heat; and *d* is the electric bulb which coverts electric energy into light and heat

When the two heaters are connected in series, let H be the amount of heat produced in kcal. Since combined resistance is (10 + 2.5) = 12.5 Ω, hence

$$H = \frac{V^2 \times (5 \times 60)}{12.5 \times J} \quad \text{...(iii)}$$

Dividing Eq. **(iii)** by Eq. **(i)**, we have H = **100 kcal.**

Example 3.3. *An electric kettle needs six minutes to boil 2 kg of water from the initial temperature of 20°C. The cost of electrical energy required for this operation is 12 paise, the rate being 40 paise per kWh. Find the kW-rating and the overall efficiency of the kettle.*

(F.Y. Engg. Pune Univ.)

Solution. Input energy to the kettle = $\frac{12 \text{ paise}}{40 \text{ paise/kWh}}$ = 0.3 kWh

$$\text{Input power} = \frac{\text{energy in kWh}}{\text{Time in hours}} = \frac{0.3}{(6/60)} = 3 \text{ kW}$$

Hence, the power rating of the electric kettle is 3 kW

Energy utilised in heating the water

= mst = 2 × 1 × (100 –20) = 160 kcal = 160 /860 kWh = 0.186 kWh.

Efficiency = output/input = 0.186/0.3 = 0.62 = **62%.**

3.4. S.I. Units

1. Mass. It is quantity of matter contained in a body.

Unit of mass is kilogram (kg). Other multiples commonly used are :

1 quintal = 100 kg, 1 tonne = 10 quintals = 1000 kg

2. Force. Unit of force is newton (N). Its definition may be obtained from Newton's Second Law of Motion *i.e.* $F = ma$.

If m = 1 kg ; a = 1m/s^2, then F = 1 newton.

Hence, one newton is that force which can give an acceleration of 1 m/s^2 to a mass of 1 kg.

Gravitational unit of force is kilogram-weight (kg-wt). It may be defined as follows :

or

It is the force which can impart an acceleration of 9.8 m/s^2 to a mass of 1 kg.

It is the force which can impart an acceleration of 1 m/s^2 to a mass of 9.8 kg.

Obviously, 1 kg-wt. = 9.8 N

3. Weight. It is the force with which earth pulls a body downwards. Obviously, its units are the same as for force.

(*a*) Unit of weight is newton (N)

(*b*) Gravitational unit of weight is kg-wt.*

Note. If a body has a mass of m kg, then its weight, W = mg newtons = 9.8 newtons.

4. Work, If a force F moves a body through a distance S in its direction of application, then

Work done $W = F \times S$

(*a*) Unit of work is joule (J).

If, in the above equation, F = 1 N : S = 1 m ; then work done = 1 m.N or joule.

Hence, one joule is the work done when a force of 1 N moves a body through a distance of 1 m in the direction of its application.

(*b*) Gravitational unit of work is m-kg. wt or m-kg**.

---

* Often it is referred to as a force of 1 kg, the word 'wt' being omitted. To avoid confusion with mass of 1 kg, the *force* of 1 kg is written in engineering literature as kgf instead of kg. wt.

** Generally the work '*wt*' is omitted and the unit is simply written as m-kg.

If F = 1 kg-wt; S = 1 m; then W.D. = 1 m-kg. Wt = 1 m-kg.

Hence, one m-kg is the work done by a force of one kg-wt when applied over a distance of one metre.

Obviously, 1 m-kg = 9.8 m-N or J.

5. Power. It is the rate of doing work. Its units is watt (W) which represents 1 joule per second.

$$1 \text{ W} = 1 \text{ J/s}$$

If a force of F newton moves a body with a velocity of v m./s then

$$\text{power} = F \times v \text{ watt}$$

If the velocity v is in km/s, then

$$\text{power} = F \times v \text{ kilowatt}$$

6. Kilowatt-hour (kWh) and kilocalorie (kcal)

$$1 \text{ kWh} = 1000 \times 1\frac{\text{J}}{\text{s}} \times 3600 \text{ s} = 36 \times 10^5 \text{ J}$$

$$1 \text{ kcal} = 4{,}186 \text{ J} \quad \therefore \ 1 \text{ kWh} = 36 \times 10^5/4{,}186 = 860 \text{ kcal}$$

7. Miscellaneous Units

(i) 1 watt hour (Wh) = $1\frac{\text{J}}{\text{s}} \times 3600 \text{ s} = 3600 \text{ J}$

(ii) 1 horse power (metric) = 75 m-kg/s = 75 × 9.8 = 735.5 J/s or watt

(iii) 1 kilowatt (kW) = 1000 W and 1 megawatt (MW) = 10^6 W

3.5. Calculation of Kilo-watt Power of a Hydroelectric Station

Let Q = water discharge rate in cubic metres/second (m^3/s), H = net water head in metre (m). g = 9.81, η; overall efficiency of the hydroelectric station expressed as a fraction.

Since 1 m^3 of water weighs 1000 kg., discharge rate is 1000 Q kg/s.

When this amount of water falls through a height of H metre, then energy or work available per second or available power is

$$= 1000 \, QgH \text{ J/s} \quad \text{or} \quad W = QgH \text{ kW}$$

Since the overall station efficiency is η power actually available is = 9.81 ηQH kW.

Example 3.4. *A de-icing equipment fitted to a radio aerial consists of a length of a resistance wire so arranged that when a current is passed through it, parts of the aerial become warm. The resistance wire dissipates 1250 W when 50 V is maintained across its ends. It is connected to a d.c. supply by 100 metres of this copper wire, each conductor of which has resistance of 0.006 Ω/m.*

Calculate

(a) *the current in the resistance wire*

(b) *the power lost in the copper connecting wire*

(c) *the supply voltage required to maintain 50 V across the heater itself.*

Solution. ***(a)*** Current = wattage/voltage = 1250/50 = **25 A**

(b) Resistance of one copper conductor = 0.006 × 100 = 0.6 Ω

Resistance of both copper conductors = 0.6 × 2 = 1.2 Ω

Power loss = I^2R watts = 252 × 1.2 = **750 W**

(c) Voltage drop over connecting copper wire = IR volt = 25 × 1.2 = 30 V

∴ Supply voltage required = 50 + 30 = **80 V**

Example 3.5. *A factory has a 240-V supply from which the following loads are taken :*

Lighting : Three hundred 150-W, four hundred 100 W and five hundred 60-W lamps

Heating : 100 kW

Motors : A total of 44.76 kW (60 b.h.p.) with an average efficiency of 75 percent

Misc. : Various load taking a current of 40 A.

Assuming that the lighting load is on for a period of 4 hours/day, the heating for 10 hours per day and the remainder for 2 hours/day, calculate the weekly consumption of the factory in kWh when working on a 5-day week.

What current is taken when the lighting load only is switched on ?


A factory needs electric power for lighting and running motors

Solution. The power consumed by each load can be tabulated as given below :

		Power consumed
Lighting	300×150	= 45,000 = 45 kW
	400×100	= 40,000 = 40 kW
	500×60	= 30,000 = 30 kW
		Total = 115 kW
Heating		= 100 kW
Motors		= 44.76/0.75 = 59.7 kW
Misc.		= $240 \times 40/1000$ = 9.6 kW

Similarly, the energy consumed/day can be tabulated as follows :

		Energy consumed / day
Lighting	= 115 kW × 4 hr	= 460 kWh
Heating	= 100 kW × 10 hr	= 1,000 kWh
Motors	= 59.7 kW × 2 hr	= 119.4 kWh
Misc.	= 9.6 kW × 2 hr	= 19.2 kWh
Total daily consumption		= 1,598.6 kWh
Weekly consumption		= $1,598.6 \times 5$ = **7,993 kWh**
Current taken by the lighting load alone		= $115 \times 1000/240$ = **479 A**

Example 3.6. *A Diesel-electric generating set supplies an output of 25 kW. The calorific value of the fuel oil used is 12,500 kcal/kg. If the overall efficiency of the unit is 35% **(a)** calculate the mass of oil required per hour **(b)** the electric energy generated per tonne of the fuel.*


Diesel electric generator set

Solution. Output = 25 kW, Overall η= 0.35,

Input = 25/0.35 = 71.4 kW

$\therefore$ input per hour =71.4 kWh = 71.4×860 = 61,400 kcal

Since 1 kg of fuel-oil produces 12,500 kcal

(a) $\therefore$ mass of oil required = 61,400/12,500 = **4.91 kg**

(b) 1 tonne of fuel = 1000 kg

Heat content = $1000 \times 12{,}500 = 12.5 \times 10^6$ kcal

= $12.5 \times 10^6/860$ = 14,530 kWh

Overall η = 0.35% $\therefore$ energy output = $14{,}530 \times 0.35$ = **5,088 kWh**

Example 3.7. *The effective water head for a 100 MW station is 220 metres. The station supplies full load for 12 hours a day. If the overall efficiency of the station is 86.4%, find the volume of water used.*

Solution. Energy supplied in 12 hours = 100×12 = 1200 MWh

= 12×10^5 kWh = $12 \times 10^5 \times 3^5 \times 10^5$ J = 43.2×10^{11} J

Overall η= 86.4% = 0.864 $\therefore$ Energy input = $43.2 \times 10^{11}/0.864 = 5 \times 10^{12}$ J

Suppose m kg is the mass of water used in 12 hours, then $m \times 9.81 \times 220 = 5 \times 10^{12}$

$\therefore$ m = $5 \times 10^{12}/9.81 \times 220 = 23.17 \times 10^8$ kg

Volume of water = $23.17 \times 10^8/10^3$ = **23.17×10^5 m^3**

($\because$ 1m^3 of water weighs 10^3 kg)

Example 3.8. *Calculate the current required by a 1,500 volts d.c. locomotive when drawing 100 tonne load at 45 km.p.h. with a tractive resistance of 5 kg/tonne along* ***(a)*** *level track* ***(b)*** *a gradient of 1 in 50. Assume a motor efficiency of 90 percent.*

Solution. As shown in Fig. 3.1 (*a*), in this case, force required is equal to the tractive resistance only.

(a) Force required at the rate of 5 kg-wt/tonne = 100 × 5 kg-wt. = 500 × 9.81 = 4905 N
Distance travelled/second = 45 × 1000/3600 = 12.5 m/s
Power output of the locomotive = 4905 × 12.5 J/s or watt = 61,312 W
$\eta = 0.9$ ∴ Power input = 61,312/0.9 = 68,125 W
∴ Currnet drawn = 68,125/1500 = **45.41 A**

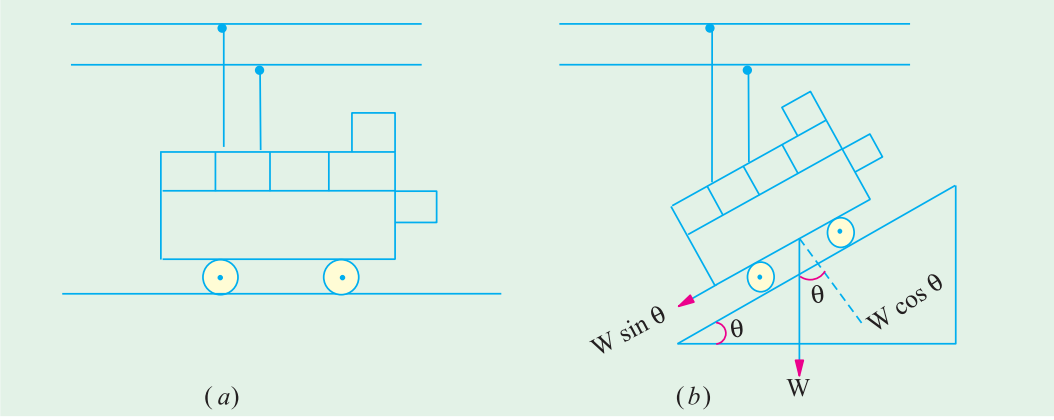


Fig. 3.1

(b) When the load is drawn along the gradient [Fig. 3.1 (*b*)], component of the weight acting downwards = 100 × 1/50 = 2 tonne-wt = 2000 kg-wt = 2000 × 9.81 = 19,620 N

Total force required = 19,620 + 4,905 = 24,525 N
Power output = force × velocity = 24,525 × 12.5 watt

Power input = 24,525 × 12.5/0.9 W ; Current drawn = $\frac{24,525 \times 12.5}{0.9 \times 1500}$ = **227 A**

Example 3.9. *A room measures 4 m × 7 m × 5 m and the air in it has to be always kept 15°C higher than that of the incoming air. The air inside has to be renewed every 35 minutes. Neglecting radiation loss, calculate the rating of the heater suitable for this purpose. Take specific heat of air as 0.24 and density as 1.27 kg/m^3.*

Solution. Volume of air to be changed per second = 4 × 7 × 5/35 = 60 = 1/15 m^3
Mass of air to be changed/second = (1/15) × 1.27 kg
Heat required/second = mass/second × sp. heat × rise in temp.
= (1.27/15) × 0.24 × 15 kcal/s = 0.305 kcal/s
= 0.305 × 4186 J/s = **1277 watt.**

Example 3.10. *A motor is being self-started against a resisting torque of 60 N-m and at each start, the engine is cranked at 75 r.p.m. for 8 seconds. For each start, energy is drawn from a lead-acid battery. If the battery has the capacity of 100 Wh, calculate the number of starts that can be made with such a battery. Assume an overall efficiency of the motor and gears as 25%.*

(Principles of Elect. Engg.-I, Jadavpur Univ.)

Solution. Angular speed $\omega = 2\pi N/60$ rad/s = $2\pi \times 75/60$ = 7.85 rad/s
Power required for rotating the engine at this angular speed is
P = torque × angular speed = ωT watt = 60 × 7.85 = **471 W**

Energy required per start is = power × time per start = 471 × 8 = 3,768 watt-s = 3,768 J
= 3,768/3600 = 1.047 Wh

Energy drawn from the battery taking into consideration the efficiency of the motor and gearing
= 1.047/0.25 = 4.188 Wh

No. of start possible with a fully-charged battery = 100/4.188 = **24 (approx.)**

Example 3.11. *Find the amount of electrical energy expended in raising the temperature of 45 litres of water by 75°C. To what height could a weight of 5 tonnes be raised with the expenditure of the same energy ? Assume efficiencies of the heating equipment and lifting equipment to be 90% and 70% respectively.* **(Elect. Engg. A.M. Ae. S.I.)**

Solution. Mass of water heated = 45 kg. Heat required = 45 × 75 = 3,375 kcal

Heat produced electrically = 3,375/0.9 = 3,750 kcal. Now, 1 kcal = 4,186 J

∴ electrical energy expended = 3,750 × 4,186 J

Energy available for lifting the load is = 0.7 × 3,750 × 4,186 J

If h metre is the height through which the load of 5 tonnes can be lifted, then potential energy of the load = mgh joules = 5 × 1000 × 9.81 h joules

∴ 5000 × 9.81 × h = 0.7 × 3,750 × 4,186 ∴ h = **224 metres**

Example 3.12. *An hydro-electric station has a turbine of efficiency 86% and a generator of efficiency 92%. The effective head of water is 150 m. Calculate the volume of water used when delivering a load of 40 MW for 6 hours. Water weighs 1000 kg/m^3.*

Solution. Energy output = 40 × 6 = 240 MWh
= 240 × 10^3 × 36 × 10^5 = 864 × 10^9 J

Overall η = 0.86 × 0.92 ∴ Energy input = $\frac{864 \times 10^9}{0.86 \times 0.92}$ = 10.92 × 10^{11} J

Since the head is 150 m and 1 m^3 of water weighs 1000 kg, energy contributed by each m^3 of water = 150 × 1000 m-kg (wt) = 150 × 1000 × 9.81 J = 147.2 × 10^4 J

∴ Volume of water for the required energy = $\frac{10.92 \times 10^{11}}{147.2 \times 10^4}$ = **74.18 × 10^4 m^3**

Example 3.13. *An hydroelectric generating station is supplied form a reservoir of capacity 6 million m^3 at a head of 170 m.*

(i) *What is the available energy in kWh if the hydraulic efficiency be 0.8 and the electrical efficiency 0.9 ?*

(ii) *Find the fall in reservoir level after a load of 12,000 kW has been supplied for 3 hours, the area of the reservoir is 2.5 km^2.*

(iii) *If the reservoir is supplied by a river at the rate of 1.2 m^3/s, what does this flow represent in kW and kWh/day ? Assume constant head and efficiency.*

Water weighs 1 tonne/m^3. **(Elect. Engineering-I, Osmania Univ.)**

Solution. ***(i)*** Wt. of water W = 6 × 10^6 × 1000 kg wt = 6 × 10^9 × 9.81 N

Water head = 170 m

Potential energy stored in this much water
= Wh = 6 × 10^9 × 9.81 × 170 J = 10^{12} J

Overall efficiency of the station = 0.8 × 0.9 = 0.71

∴ energy available = 0.72 × 10^{13} J = 72 × 10^{11}/36 × 10^5
= **2 × 10^6 kWh**

(ii) Energy supplied = 12,000 × 3 = 36,000 kWh

Energy drawn from the reservoir after taking into consideration the overall efficiency of the station = 36,000/0.72 = 5 × 10^4 kWh
= 5 × 10^4 × 36 × 10^5 = 18 × 10^{10} J

If m kg is the mass of water used in two hours, then, since water head is 170 m

$$mgh = 18 \times 10^{10}$$

or $$m \times 9.81 \times 170 = 18 \times 10^{10} \quad \therefore \; m = 1.08 \times 10^{8} \text{ kg}$$

If h metre is the fall in water level, then

$$h \times \text{area} \times \text{density} = \text{mass of water}$$

$$\therefore \quad h \times (2.5 \times 10^{6}) \times 1000 = 1.08 \times 10^{8} \quad \therefore \; h = 0.0432 \text{ m} = \mathbf{4.32 \text{ cm}}$$

(iii) Mass of water stored per second = $1.2 \times 1000 = 1200$ kg

Wt. of water stored per second = 1200×9.81 N

Power stored = $1200 \times 9.81 \times 170$ J/s = **2,000 kW**

Power actually available = $2{,}000 \times 0.72 = 1440$ kW

Energy delivered /day = $1440 \times 24 =$ **34,560 kWh**

Example 3.14. *The reservoir for a hydro-electric station is 230 m above the turbine house. The annual replenishment of the reservoir is 45×10^{10} kg. What is the energy available at the generating station bus-bars if the loss of head in the hydraulic system is 30 m and the overall efficiency of the station is 85%. Also, calculate the diameter of the steel pipes needed if a maximum demand of 45 MW is to be supplied using two pipes.*

(Power System, Allahabad Univ.)



Hydroelectric generators

Solution. Actual head available = 230 –30 = 200 m

Energy available at the turbine house = mgh

$$= 45 \times 10^{10} \times 9.81 \times 200 = 88.29 \times 10^{13} \text{ J}$$

$$= \frac{88.29 \times 10^{13}}{36 \times 10^{5}} = \mathbf{24.52 \times 10^{7} \text{ kWh}}$$

Overall $\eta = 0.85$

$\therefore$ Energy output = $24.52 \times 10^{7} \times 0.85 =$ **20.84×10^{7} kWh**

The kinetic energy of water is just equal to its loss of potential energy.

$$\frac{1}{2} mv^{2} = mgh \quad \therefore \; v = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 200} = 62.65 \text{ m/s}$$

Power available from a mass of m kg when it flows with a velocity of v m/s is

$$P = \frac{1}{2} mv^{2} = \frac{1}{2} \times m \times 62.65^{2} \text{ J/s} \quad \text{or} \quad \text{W}$$

Equating this to the maximum demand on the station, we get

$$\frac{1}{2} m \, 62.65^{2} = 45 \times 10^{6} \quad \therefore \; m = 22{,}930 \text{ kg/s}$$

If A is the total area of the pipes in m^2, then the flow of water is Av m^3/s. Mass of water flowing/second = $Av \times 10^{3}$ kg ($\therefore$ 1 m^3 of water = 1000 kg)

$$\therefore \quad A \times v \times 10^{3} = 22{,}930 \quad \text{or} \quad A = \frac{22{,}930}{62.65 \times 10^{3}} = 0.366 \text{ m}^2$$

If 'd' is the diameter of each pipe, then $\pi d^{2}/4 = 0.183 \quad \therefore \; d =$ **0.4826 m**

Example 3.15. *A large hydel power station has a head of 324 m and an average flow of 1370 cubic metres/sec. The reservoir is a lake covering an area of 6400 sq. km, Assuming an efficiency of 90% for the turbine and 95% for the generator, calculate*

(i) *the available electric power ;*

(ii) *the number of days this power could be supplied for a drop in water level by 1 metre.*

(AMIE Sec. B Power System I (E-6) Winter)

Solution. **(*i*)** Available power = 9.81 ηQH kW = (0.9 × 0.95) × 1370 × 324 = 379, 524 kW = **379.52 MW.**

(*ii*) If A is the lake area in m^2 and h metre is the fall in water level, the volume of water used is = $A \times h = m^3$. The time required to discharge this water is Ah / Q second.

Now, $A = 6400 \times 10^6$ m^2 ; $h = 1$ m; $Q = 1370$ m^3 /s.

$\therefore \quad t = 6400 \times 10^6 \times 1/1370 = 4.67 \times 10^6$ second = **540686 days**



In a hydel plant, potential energy of water is converted into kinetic energy and then into electricity.

Example 3.16. *The reservoir area of a hydro-electric generating plant is spread over an area of 4 sq km with a storage capacity of 8 million cubic-metres. The net head of water available to the turbine is 70 metres. Assuming an efficiency of 0.87 and 0.93 for water turbine and generator respectively, calculate the electrical energy generated by the plant.*

Estimate the difference in water level if a load of 30 MW is continuously supplied by the generator for 6 hours.

(Power System I-AMIE Sec. B)

Solution. Since 1 cubic metre of water weighs 1000 kg., the reservoir capacity = 8×10^6 m^3 = $8 \times 10^6 \times 1000$ kg. = 8×10^9 kg.

Wt. of water, $W = 8 \times 10^9$ kg. Wt. $8 \times 10^9 \times 9.81 = 78.48 \times 10^9$ N. Net water head = 70 m.

Potential energy stored in this much water = $Wh = 78.48 \times 10^9 \times 70 = 549.36 \times 10^{10}$ J

Overall efficiency of the generating plant = $0.87 \times 0.93 = 0.809$

Energy available = $0.809 \times 549.36 \times 10^{10}$ J = 444.4×10^{10} J

= $444.4 \times 10^{10}/36 \times 10^5$ = **12.34×10^5 kWh**

Energy supplied in 6 hours = 30 MW × 6 h = 180 MWh

= 180,000 kWh

Energy drawn from the reservoir after taking into consideration, the overall efficiency of the station = 180,000/0.809 = 224,500 kWh = $224{,}500 \times 36 \times 10^5$

= 80.8×10^{10} J

If m kg. is the mass of water used in 6 hours, then since water head is 70 m,

$mgh = 80.8 \times 10^{10}$ or $m \times 9.81 \times 70 = 80.8 \times 10^{10} \quad \therefore \quad m = 1.176 \times 10^9$ kg.

If h is the fall in water level, then $h \times$ area × density = mass of water

$\therefore \quad h \times (4 \times 10^6) \times 1000 = 1.176 \times 10^9 \quad \therefore \quad h = 0.294$ m = **29.4 cm.**

Example 3.17. *A proposed hydro-electric station has an available head of 30 m, catchment area of 50×10^6 sq.m, the rainfall for which is 120 cm per annum. If 70% of the total rainfall can be collected, calculate the power that could be generated. Assume the following efficiencies : Penstock 95%, Turbine 80% and Generator 85.* **(Elect. Engg. AMIETE Sec. A Part II)**

Solution. Volume of water available = $0.7(50 \times 10^6 \times 1.2) = 4.2 \times 10^7$ m^3

Mass of water available = $4.2 \times 10^7 \times 1000 = 4.2 \times 10^{10}$ kg

This quantity of water is available for a period of one year. Hence, quantity available per second = $4.2 \times 10^{10}/365 \times 24 \times 3600 = 1.33 \times 10^3$.

Available head = 30 m

Potential energy available = $mgh = 1.33 \times 10^3 \times 9.8 \times 30 = 391 \times 10^3$ J

Since this energy is available per second, hence power available is = 391×10^3 J/s = 391×10^3 W = 391 kW

Overall efficiency = $0.95 \times 0.80 \times 0.85 = 0.646$

The power that could be generated = 391×0.646 = **253 kW.**

Example 3.18. *In a hydro-electric generating station, the mean head (i.e. the difference of height between the mean level of the water in the lake and the generating station) is 400 metres. If the overall efficiency of the generating stations is 70%, how many litres of water are required to generate 1 kWh of electrical energy ? Take one litre of water to have a mass of 1 kg.*

(F.Y. Engg. Pune Univ.)

Solution. Output energy = 1 kWh = 36×10^5 J

Input energy = $36 \times 10^5/0.7 = 5.14 \times 10^6$ J

If m kg. water is required, then

$mgh = 5.14 \times 10^6$ or $m \times 9.81 \times 400 = 5.14 \times 10^6$, $\therefore$ **1310 kg.**

Example 3.19. *A 3-tonne electric-motor-operated vehicle is being driven at a speed of 24 km/hr upon an incline of 1 in 20. The tractive resistance may be taken as 20 kg per tonne. Assuming a motor efficiency of 85% and the mechanical efficiency between the motor and road wheels of 80%, calculate*

(a) *the output of the motor*

(b) *the current taken by motor if it gets power from a 220-V source.*

Calculate also the cost of energy for a run of 48 km, taking energy charge as 40 paise/kWh.

Solution. Different forces acting on the vehicle are shown in Fig. 3.2.

Wt. of the vehicle = $3 \times 10^3 = 3000$ kg-wt

Component of the weight of the vehicle acting downwards along the slope = $3000 \times 1/20 = 150$ kg-wt

Tractive resistance = $3 \times 20 = 60$ kg-wt

Total downward force = $150 + 60 = 210$ kg-wt

= $210 \times 9.81 = 2{,}060$ N

Distance travelled/second = $24{,}000/3600 = 20/3$ m/s

Output at road wheels = $2{,}060 \times 20/3$ watt

Mechanical efficiency = 80% or 0.8

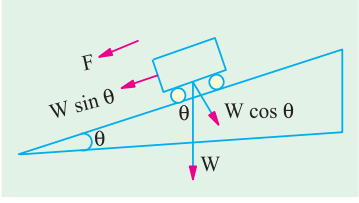


Fig. 3.2

(a) Motor output = $\dfrac{2{,}060 \times 20}{3 \times 0.8}$ = **17,167 W**

(b) Motor input = $17{,}167/0.85 = 20{,}200$ W

Current drawn = $20{,}200/220 =$ **91.7 A**

Motor power input = 20,200 W = 20.2 kW

Time for 48 km run = 2 hr.

$\therefore$ Motor energy input = $20.2 \times 2 = 40.4$ kW

Cost = Rs. $40.4 \times 0.4 =$ **Rs. 16 paise 16**

Example 3.20. *Estimate the rating of an induction furnace to melt two tonnes of zinc in one hour if it operates at an efficiency of 70%. Specific heat of zinc is 0.1. Latent heat of fusion of zinc is 26.67 kcal per kg. Melting point is 455°C. Assume the initial temperature to be 25°C.*

(Electric Drives and Utilization Punjab Univ.)

Solution. Heat required to bring 2000 kg of zinc from 25°C to the melting temperature of 455° C = $2000 \times 0.1 \times (455 - 25)$ = 86,000 kcal.

Heat of fusion or melting = mL = $2000 \times 26.67 = 53{,}340$ kcal

Total heat reqd. = $86{,}000 + 53{,}340 = 139{,}340$ kcal

Furnace input = $139{,}340/0.7 = 199{,}057$ kcal

Now, 860 kcal = 1 kWh $\therefore$ furnace input = $199.057/860 = 231.5$ kWh.

Power rating of furnace = energy input/time = 231.5 kWh/1 h = **231.5 kW.**

Example 3.21. *A pump driven by an electric motor lifts 1.5 m^3 of water per minute to a height of 40 m. The pump has an efficiency of 90% and motor has an efficiency of 85%. Determine :* ***(a)*** *the power input to the motor.* ***(b)*** *The current taken from 480 V supply.* ***(c)*** *The electric energy consumed when motor runs at this load for 4 hours. Assume mass of 1 m^3 of water to be 1000 kg.*

(Elect. Engg. Pune Univ.)

Solution. ***(a)*** Weight of the water lifted = 1.5 m^3 = $1.5 \times 1000 = 1500$ kg. Wt = $1500 \times 9.8 = 14700$ N.

Height = 40 m; time taken = 1 min. = 60 s

$\therefore$ Motor output power = $14700 \times 40/60 = 9800$ W

Combined pump and motor efficiency = 0.9×0.85

$\therefore$ Motor power input = $9800/0.9 \times 0.85 = 12810$ W = **12.81 kW.**

(*b*) Current drawn by the motor = 12810/480 = **26.7 A**

Electrical energy consumed by the motor = 12.81 kW × 4 h = **51.2 kWh.**

Example 3.22. *An electric lift is required to raise a load of 5 tonne through a height of 30 m. One quarter of electrical energy supplied to the lift is lost in the motor and gearing. Calculate the energy in kWhr supplied. If the time required to raise the load is 27 minutes, find the kW rating of the motor and the current taken by the motor, the supply voltage being 230 V d.c. Assume the efficiency of the motor at 90%.* **(Elect. Engg. A.M. Ae. S.I. June)**

Solution. Work done by the lift = $Wh = mgh = (5 \times 1000) \times 9.8 \times 30 = 1.47 \times 10^6$ J

Since 25% of the electric current input is wasted, the energy supplied to the lift is 75% of the input.

$\therefore$ input energy to the lift $= 1.47 \times 10^6/0.75 = 1.96 \times 10^6$ J

Now, 1 kWh $= 26 \times 10^5$ J

$\therefore$ energy input to the lift $= 1.96 \times 10^6/36 \times 10^5 = 0.544$ kWh

Motor energy output $= 1.96 \times 10^6$ J; $\eta = 0.9$

Motor energy input $= 1.96 \times 10^6/0.9 = 2.18 \times 10^6$ J : time taken $= 27 \times 60 = 1620$ second

Power rating of the electric motor = work done/time taken

$= 2.18 \times 10^6/1620 = 1.345 \times 10^3$ J/s = 1345 W

Current taken by the motor = 1345/230 = **5.85 A**

Example 3.23. *An electrical lift make 12 double journey per hour. A load of 5 tonnes is raised by it through a height 50 m and it returns empty. The lift takes 65 seconds to go up and 48 seconds to return. The weight of the cage is 1/2 tonne and that of the counterweight is 2.5 tonne. The efficiency of the hoist is 80 per cent that of the motor is 85 %. Calculate the hourly consumption in kWh.* **(Elect. Engg. Pune Univ.)**

Solution. The lift is shown in Fig. 3.3.

Weight raised during upward journey

= 5 + 1/2 –2.5 = 3 tonne = 3000 kg-wt

Distance travelled = 50 m

Work done during upward journey

$= 3000 \times 50 = 15 \times 10^4$ m-kg

Weight raised during downward journey

= 2.5 –0.5 = 2 tonne = 2000 kg

Similarly, work done during downward journey

$= 2000 \times 50 = 10 \times 10^4$ m-kg.

Total work done per double journey

$= 15 \times 10^4 + 10 \times 10^4 = 25 \times 10^4$ m-kg

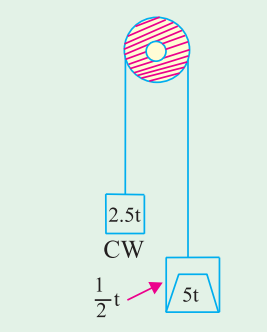


Fig. 3.3

Now, 1, m-kg = 9.8 joules

$\therefore$ Work done per double journey $= 9.8 \times 25 \times 10^4$ J $= 245 \times 10^4$ J

No. of double journey made per hour = 12

$\therefore$ work done per hour $= 12 \times 245 \times 10^4 = 294 \times 10^5$ J

Energy drawn from supply $= 294 \times 10^5/0.8 \times 0.85 = 432.3 \times 10^5$ J

Now, 1 kWh $= 36 \times 10^5$ J

$\therefore$ Energy consumption per hour $= 432.3 \times 10^5/36 \times 10^5 =$ **12 kWh**

Example 3.24. *An electric hoist makes 10 double journey per hour. In each journey, a load of 6 tonnes is raised to a height of 60 meters in 90 seconds. The hoist cage weighs 1/2 tonne and has a balance load of 3 tonnes. The efficiency of the hoist is 80 % and of the driving motor 88 %. Calculate* ***(a)*** *electric energy absorbed per double journey* ***(b)*** *hourly energy consumption in kWh* ***(c)*** *hp (British) rating of the motor required* ***(d)*** *cost of electric energy if hoist works for 4 hours/day for 30 days. Cost per kWh is 50 paise.* **(Elect. Power - 1, Bangalore Univ.)**

Solution. Wt. of cage when fully loaded = $6\frac{1}{2}$ tonne-wt.

Force exerted on upward journey = $6\frac{1}{2} - 3 = 3\frac{1}{2}$ tonne-wt.

= $3\frac{1}{2} \times 1000$ = 3,500 kg-wt.

Force exerted on downward journey = $3 - \frac{1}{2} = 2\frac{1}{2}$ tonnes-wt. = 2500 kg-wt

Distance moved = 60 m

Work done during upward journey = $3{,}500 \times 60$ m-kg

Work done during downward journey = $2{,}500 \times 60$ m-kg

Work done during each double journey = $(3{,}500 + 2{,}500) \times 60 = 36 \times 10^4$ m-kg

= $36 \times 10^4 \times 9.81 = 534 \times 10^4$ J

Overall $\eta = 0.80 \times 0.88$

$\therefore$ Energy input per double journey = $534 \times 10^4 / 0.8 \times 0.88 = 505 \times 104$ J

(*a*) Electric energy absorbed per double journey = $505 \times 10^4 / 36 \times 10^5$ = **1.402 kWh**

(*b*) Hourly consumption = 1.402×10 = **14.02 kWh**

(*c*) Before calculating the rating of the motor, maximum rate of working should be found. It is seen that maximum rate of working is required in the upward journey.

Work done = $3{,}500 \times 60 \times 9.81 = 206 \times 10^4$ J

Time taken = 90 second

$$\text{B.H.P of motor} = \frac{206 \times 10^4}{90 \times 0.8 \times 746} = \textbf{38.6(British h.p.)}$$

(*d*) Cost = $14.02 \times (30 \times 4) \times 50/100$ = **Rs. 841.2**

Example 3.25. *A current of 80 A flows for 1 hr, in a resistance across which there is a voltage of 2 V. Determine the velocity with which a weight of 1 tonne must move in order that its kinetic energy shall be equal to the energy dissipated in the resistance.*

(Elect. Engg. A.M.A.e. S.I.)

Solution. Energy dissipated in the resistance = $V\,It = 2 \times 80 \times 3600 = 576{,}000$ J

A weight of one tonne represents a mass of one tonne *i.e.*, 1000 kg. Its kinetic energy is = $(1/2) \times 1000 \times v^2 = 500\,v^2$

$\therefore$ $500\,v^2 = 576{,}000$ $\therefore$ v = 1152 m/s.

Tutorial Problems No. 3.1

1. A heater is required to give 900 cal/min on a 100 V. d.c. circuit. What length of wire is required for this heater if its resistance is 3 Ω per metre ? **[53 metres]**
2. A coil of resistance 100 Ω is immersed in a vessel containing 500 gram of water of 16° C and is connected to a 220-V electric supply. Calculate the time required to boil away all the water (1kcal = 4200 joules, latent heat of steam = 536 keal/kg). **[44 min 50 second]**
3. A resistor, immersed in oil, has 62.5 Ω resistance and is connected to a 500-V d.c. supply. Calculate
   (*a*) the current taken
   (*b*) the power in watts which expresses the rate of transfer of energy to the oil.
   (*c*) the kilowatt-hours of energy taken into the oil in 48 minutes. **[8A ; 4000 W ; 3.2 kWh]**
4. An electric kettle is marked 500-W, 230 V and is found to take 15 minutes to raise 1 kg of water from 15° C to boiling point. Calculate the percentage of energy which is employed in heating the water. **[79 per cent]**
5. An aluminium kettle weighing 2 kg holds 2 litres of water and its heater element consumes a power of 2 kW. If 40 percent of the heat supplied is wasted, find the time taken to bring the kettle of water to boiling point from an initial temperature of 20°C. (Specific heat of aluminium = 0.2 and Joule's equivalent = 4200 J/kcal.) **[11.2 min]**

6. A small electrically heated drying oven has two independent heating elements each of 1000 Ω in its heating unit. Switching is provided so that the oven temperature can be altered by rearranging the resistor connections. How many different heating positions can be obtained and what is the electrical power drawn in each arrangement from a 200 V battery of negligible resistance ?

[Three, 40, 20 and 80 W]

7. Ten electric heaters, each taking 200 W were used to dry out on site an electric machine which had been exposed to a water spray. They were used for 60 hours on a 240 V supply at a cost of twenty paise/kWh. Calculate the values of following quantities involved :
(*a*) current (*b*) power in kW (*c*) energy in kWh (*d*) cost of energy.

[(*a*) 8.33 A (*b*) 2 kW (*c*) 120 kWh (*d*) Rs. 24]

8. An electric furnace smelts 1000 kg of tin per hour. If the furnace takes 50 kW of power from the electric supply, calculate its efficiency, given : the smelting tempt. of tin = 235°C ; latent heat of fusion = 13.31 kcal/kg; initial temperature = 15ºC ; specific heat = 0.056. Take J = 4200 J/kcal.

[59.8%] *(Electrical Engg.-I, Delhi Univ.)*

9. Find the useful rating of a tin-smelting furnace in order to smelt 50 kg of tin per hour. Given : Smelting temperature of tin = 235ºC, Specific heat of tin = 0.055 kcal/kg-K. Latent heat of liquefaction = 13.31 kcal per kg. Take initial temperature of metal as 15ºC. **[1.5 kW]**

(F.Y. Engg. Pune Univ.)

10. State the relation between
(*i*) Kcal and kWh (*ii*) Horse power and watts (*iii*) kWh and joule (watt sec) (*iv*) K.E and joules.

(Gujrat University, Summer 2003)

11. The electrical load in a small workshop consists of 14 lamps, each rated at 240 V, 60 W and 3 fans each rated at 240 V, 1 kW. What is the effective resistance of the total load, total current and energy utilised if run for 8 hrs.

(Pune University 2003) (Gujrat University, Summer 2003)

OBJECTIVE TESTS – 3

1. If a 220 V heater is used on 110 V supply, heat produced by it will be —— as much.
(*a*) one-half (*b*) twice
(*c*) one-fourth (*d*) four times

2. For a given line voltage, four heating coils will produce maximum heat when connected
(*a*) all in parallel (*b*) all in series
(*c*) with two parallel pairs in series
(*d*) one pair in parallel with the other two in series

3. The electric energy required to raise the temperature of a given amount of water is 1000 kWh. If heat losses are 25%, the total heating energy required is — kWh.
(*a*) 1500 (*b*) 1250
(*c*) 1333 (*d*) 1000

4. One kWh of energy equals nearly
(*a*) 1000 W (*b*) 860 kcal
(*c*) 4186 J (*d*) 735.5 W

5. One kWh of electric energy equals
(*a*) 3600 J (*b*) 860 kcal
(*c*) 3600 W (*d*) 4186 J

6. A force of 10,000 N accelerates a body to a velocity 0.1 km/s. This power developed is —— kW
(*a*) 1,00,000 (*b*) 36,000
(*c*) 3600 (*d*) 1000

7. A 100 W light bulb burns on an average of 10 hours a day for one week. The weekly consumption of energy will be —— unit/s
(*a*) 7 (*b*) 70
(*c*) 0.7 (*d*) 0.07

(Principles of Elect. Engg. Delhi Univ.)

8. Two heaters, rated at 1000 W, 250 volts each, are connected in series across a 250 Volts 50 Hz A.C. mains. The total power drawn from the supply would be ——— watt.,
(*a*) 1000 (*b*) 500
(*c*) 250 (*d*) 2000

(Principles of Elect. Engg. Delhi Univ.)

ANSWERS

1. *c* **2.** *a* **3.** *c* **4.** *b* **5.** *b* **6.** *d* **7.** *a* **8.** *b*

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CHAPTER 4

ELECTRO-STATICS

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- Field Strength or Field Intensity or Electric Intensity (*E*)
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Lightning and thunder are created when the static electricity concentrated on the clouds suddenly discharges

4.1. Static Electricity

In the preceding chapters, we concerned ourselves exclusively with electric current *i.e.* electricity in motion. Now, we will discuss the behaviour of static electricity and the laws governing it. In fact, electrostatics is that branch of science which deals with the phenomena associated with electricity at rest.

It has been already discussed that generally an atom is electrically neutral *i.e.* in a normal atom the aggregate of positive charge of protons is exactly equal to the aggregate of negative charge of the electrons.

If, somehow, some electrons are removed from the atoms of a body, then it is left with a preponderance of positive charge. It is then said to be positively-charged. If, on the other hand, some electrons are added to it, negative charge out-balances the positive charge and the body is said to be negatively charged.

In brief, we can say that positive electrification of a body results from a deficiency of the electrons whereas negative electrification results from an excess of electrons.

The total deficiency or excess of electrons in a body is known as its charge.

4.2. Absolute and Relative Permittivity of a Medium


Charles Augustin de Coulomb*

While discussing electrostatic phenomenon, a certain property of the medium called its ***permittivity*** plays an important role. Every medium is supposed to possess two permittivities :

(i) absolute permittivity (ε) and ***(ii)*** relative permittivity (ε_r).

For measuring relative permittivity, vacuum or free space is chosen as the reference medium. It has an absolute permittivity of 8.854×10^{-12} F/m

Absolute permittivity $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m

Relative permittivity, $\varepsilon_r = 1$

Being a ratio of two similar quantities, ε_r has no units.

Now, take any other medium. If its relative permittivity, as compared to vacuum is ε_r, then its absolute permittivity is $\varepsilon = \varepsilon_0 \varepsilon_r$ F/m

If, for example, relative permittivity of mica is 5, then, its absolute permittivity is

$$\varepsilon = \varepsilon_0 \varepsilon_r = 8.854 \times 10^{-12} \times 5 = 44.27 \times 10^{-12} \text{ F/m}$$

4.3. Laws of Electrostatics

First Law. Like charges of electricity repel each other, whereas unlike charges attract each other.

Second Law. According to this law, the force exerted between two *point* charges ***(i)*** is directly proportional to the product of their strengths ***(ii)*** is inversely proportional to the square of the distance between them.

This law is known as Coulomb's Law and can be expressed mathematically as :

$$F \propto \frac{Q_1 Q_2}{d^2} \quad \text{or} \quad F = k \frac{Q_1 Q_2}{d^2}$$

In vector form, the Coulomb's law can be written as $\left.\begin{aligned} \vec{F} &= \frac{Q_1 Q_2}{d^2} \hat{d} \\ &= \frac{Q_1 Q_2}{d^2} \vec{d} \end{aligned}\right\}$

* Coulomb is better known for his law which states that the force between two point charges is proportional to each charge and inversely proportional to the square of the distance between them.

where $\hat{d}$ is the unit vector *i.e.* a vector of unit length in the direction of distance *d, i.e.* $d = \frac{d}{\hat{d}}$ (where $\vec{d}$ is the vector notation for *d*, which is a scalar notation).

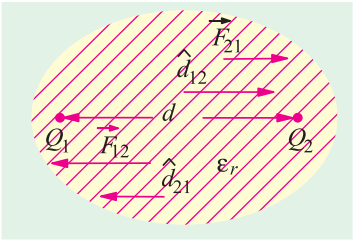


Fig. 4.1

Therefore, explicit forms of this law are :

$$\vec{F}_{21} = k\frac{Q_1Q_2}{d_{12}^2}\hat{d}_{12} = k\frac{Q_1Q_2}{d_{12}^3}\vec{d}_{12}$$

where $\vec{F}_{21}$ is the force on Q_2 due to Q_1 and $\hat{d}_{12}$ is the unit vector in direction from Q_1 to Q_2

and $\vec{F}_{12} = k\frac{Q_1Q_2}{d_{21}^2}\hat{d}_{21} = k\frac{Q_1Q_2}{d_{21}^2}\vec{d}_{21}$ where F_{12} is the force on Q_1 due to Q_2 and $\vec{d}_{21}$ is the unit vector in the direction from Q_2 to Q_1.

where *k* is the constant of proportionality, whose value depends on the system of units employed. In S.I. system, as well as M.K.S.A. system $k = 1/4\pi\varepsilon$. Hence, the above equation becomes.

$$F = \frac{Q_1Q_2}{4\pi\varepsilon d^2} = \frac{Q_1Q_2}{4\pi\varepsilon_0\varepsilon_r d^2}$$

If Q_1 and Q_2 are in colomb, *d* in metre and ε in fard/metre, then *F* is in newtons

Now $\frac{1}{4\ \ _0} \quad \frac{1}{4 \quad 8.854 \quad 10^{\ 12}} \quad 8.9878 \quad 10^9 \quad 9 \quad 10^9$ (approx.)

Hence, Coulomb's Law can be written as

$$F = 9\times10^9\frac{Q_1Q_2}{\varepsilon_r d^2} \quad \text{—in a medium}$$

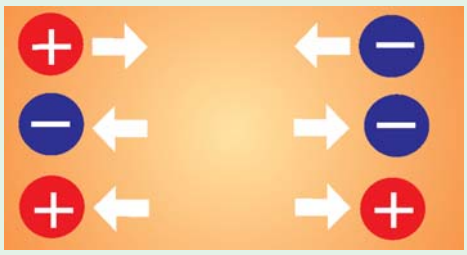
$$= 9\times10^9\frac{Q_1Q_2}{d^2} \quad \text{—in air or vacuum ...(}i\text{)}$$

If in Eq. (*i*) above

$Q_1 = Q_2 = Q$ (say), $d = 1$ metre; $F = 9\times10^9$ N

then $Q^2 = 1$ or $Q = \pm 1$ coulomb

Hence, one coulomb of charge may be defined as ***that charge (or quantity of electricity) which when placed in air (strictly vacuum) from an equal and similar charge repels it with a force of 9×10^9 N.***


Unlike charges attract and like charges repel each other

Although coulomb is found to be a unit of convenient size in dealing with electric current, yet, from the standpoint of electrostatics, it is an enormous unit. Hence, its submultiples like micro-coulomb (μ C) and micro-microcoulomb (μ μC) are generally used.

$1\ \mu\text{ C} = 10^{-6}$ C; $1\ \mu\ \mu\text{ C} = 10^{-12}$ C

It may be noted here that relative permittivity of air is one, of water 81, of paper between 2 and 3, of glass between 5 and 10 and of mica between 2.5 and 6.

Example 4.1. *Calculate the electrostatic force of repulsion between two α-particles when at a distance of 10^{-13} m from each other. Charge of an α-particles is 3.2×10^{-12} C. If mass of each particle is 6.68×10^{-27} N-m^2/kg^2.*

Solution. Here $Q_1 = Q_2 = 3.2\times10^{-19}$ C, $d = 10^{-13}$ m

$$F = 9\times10^9\times\frac{3.2\times10^{-19}\times3.2\times10^{-19}}{(10^{-13})^2} = \mathbf{9.2\times10^{-2}N}$$

The force of gravitational attraction between the two particles is given by

$$F = G\frac{m_1 m_2}{d^2} = \frac{6.67\times10^{-11}\times(6.68\times10^{-27})^2}{(10^{-13})^2} = \mathbf{2.97\times10^{-37}\ N}$$

Obviously, this force is negligible as compared to the electrostatic force between the two particles.

Example 4.2. *Calculate the distance of separation between two electrons (vacuum) for which the electric force between them is equal to the gravitation force on one of them at the earth surface.*
mass of electron = 9.1 × 11^{-31} kg, charge of electron = 1.6 × 10^{-19} C.

Solution. Gravitational force on one electron.

$$= mg \text{ newton} = 9.1\times10^{-31}\times 9.81 \text{ N}$$

Electrostatic force between the electrons

$$= 9\times10^9\frac{Q^2}{d^2} = \frac{9\times10^9\times(1.6\times10^{-19})^2}{d^2} N$$

Equating the two forces, we have

$$\frac{9\times10^9\times2.56\times10^{-38}}{d^2} = 9.1\times10^{-31}\times 9.81 \quad \therefore\ d = \mathbf{5.08\ m}$$

Example 4.3. (*a*) *Three identical point charges, each QΩ coulombs, are placed at the vertices of an equilateral triangle 10 cm apart. Calculate the force on each charge.*

(b) *Two charges Q coulomb each are placed at two opposite corners of a square. What additional charge "q" placed at each of the other two corners will reduce the resultant electric force on each of the charges Q to zero ?*

Solution. (*a*) The equilateral triangle with its three charges is shown in Fig. 4.2 (*a*). Consider the charge Q respectively. These forces are equal to each other and each is

$$F = 9\times10^9\frac{Q^2}{0.1^2} = 9\times10^{11}\ Q^2 \text{ newton}$$

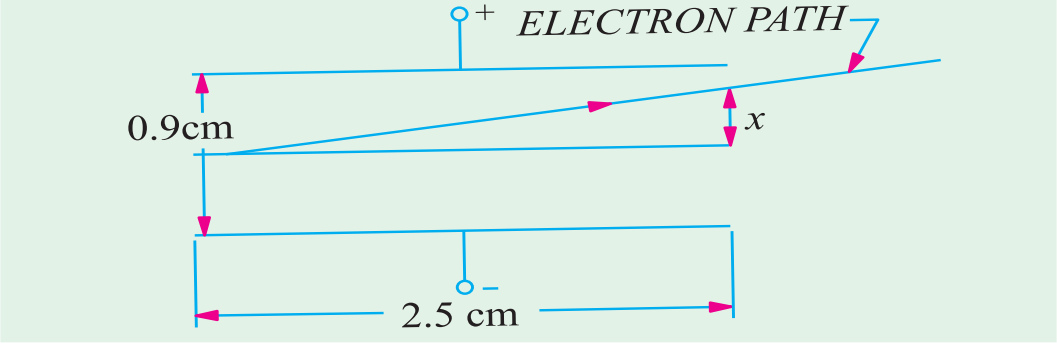


Fig. 4.2

Since the angle between these two equal forces is 60°, their resultant is

$$= 2\times F\times\cos 60°/2 = \sqrt{3}\ F = \mathbf{9\times10^{11}\times\sqrt{3}\ Q^2\ Newton}$$

The force experienced by other charges is also the same.

(b) The various charges are shown in Fig. 4.2 (*b*). The force experienced by the charge Q at point C due to the charge Q at point A acts along ACM and is

$$= 9\times10^9\frac{Q^2}{(\sqrt{2}d)^2} = 4.5\times10^9\ Q^2/d^2 \text{ newton} \qquad ...(i)$$

where d is the side of the square in metres.

If the charges q are negative, they will exert attractive forces on the charge Q at point C along CB and CD respectively. Each force is

$$= -9 \times 10^9 \frac{Qq}{d^2} \text{ newton}$$

Since these two forces are at right angles to each other, their resultant is

$$= -\sqrt{2} \times 9 \times 10^9 \frac{qQ}{d^2}$$

If net force on charge Q at point C is to be zero, then ***(i)*** must equal ***(ii)***,

$$\therefore \quad 4.5 \times 10^9 \frac{Q^2}{d^2} = -9 \times 10^9 \sqrt{2} \frac{qQ}{d^2} \quad \therefore \quad q = -\mathbf{Q/2\sqrt{2}} \text{ coulomb}$$

Example 4.4. *The small identical conducting spheres have charges of 2.0×10^{-9} C and -0.5×10^{-9} respectively. When they are placed 4 cm apart, what is the force between them ? If they are brought into contact and then separated by 4 cm, what is the force between them ?*

(Electromagnetic Theory, A.M.I.E. Sec B,)

Solution. $F = 9 \times 10^{-9} Q_1 Q_2/d^2 = 9 \times 10^{-9} \times (-0.5 \times 10^{-9})/0.04^2 = -56.25 \times 10^{-7}$ N. When two identical spheres are brought into contact with each other and then separated, each gets half of the total charge. Hence,

$$Q_1 = Q_2 = [2 \times 10^{-9} + (-0.5 \times 10^{-9})/2] = 0.75 \times 10^{-9} \text{ C}$$

When they are separated by 4 cm,

$$F = 9 \times 10^{-9} \times (0.75 \times 10^{-9})^2/0.04^2 = \mathbf{0.316 \times 10^{-5}\ N}$$

Example 4.5. *Determine resultant force on 3 μC charge due to –4μC and 10 nC charges. All these three point charges are placed on the vertices of equilateral triangle ABC of side 50 cm.*

[Bombay University, 2001]

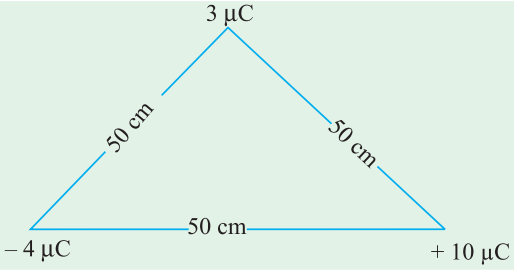


Fig. 4.3 (*a*)

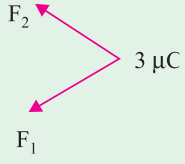


Fig. 4.3 (*b*)

Solution.

$$F_2 = \frac{Q_1 Q_2}{4 \quad _0 d^2} \qquad \frac{3 \quad 10^{6} \quad 10 \quad 10^{9}}{4 \quad 8.854 \quad 10^{12} \quad 0.50 \quad 0.50}$$

$$= 1.08 \times 10^{-3} \text{ Newton, in the direction shown}$$

Similarly, $F_1 = 0.432$ Newton, in the direction shown.

Resultant of F_1 and F_2 has to be found out.

Example 4.6. *A capacitor is composed of 2 plates separated by a sheet of insulating material 3 mm thick and of relative permitivity 4. The distance between the plates is increased to allow the insertion of a second sheet of 5 mm thick and of relative permitivity ε_r. If the equivalent capacitance is one third of the original capacitance. Find the value of ε_r.* **[Bombay University, 2001]**

Solution. $C_1 \frac{\varepsilon_0 \varepsilon_r A}{d} = k(4/3)$, where $k = \varepsilon_0 A \times 10^{+3}$

The composite capacitor [with one dielectric of $\varepsilon_{r1} = 4$ and other dielectric of ε_{r2} as relative

permitivity has a capacitance of $C/3$. Two capacitors are effectively in series. Let the second dielectric contribute a capacitor of C_2.

$$K \,.\, (4/9) = \frac{C_1 C_2}{C_1 + C_2} = \frac{K \,.\, (4/3) \,.\, C_2}{K \,.\, (4/3) + C_2}$$

This gives

$$C_2 = 2/3\ K$$

$$(2/3)\ K = \frac{\varepsilon_0 \varepsilon_{r2} A}{5 \times 10^{-3}}$$

$$E_{r2} = 10/3 \,.\, K\ 1/e_0\ A \times 10^{-3}$$

$$= 10/3\ e_0\ A \times 103\ 1/e_0\ A \times 10^{-3}$$

$$= 10/3 = \mathbf{3.33}$$

4.4. Electric Field

It is found that in the medium around a charge a force acts on a positive or negative charge when placed in that medium. If the charge is sufficiently large, then it may create such a huge stress as to cause the electrical rupture of the medium, followed by the passage of an arc discharge.

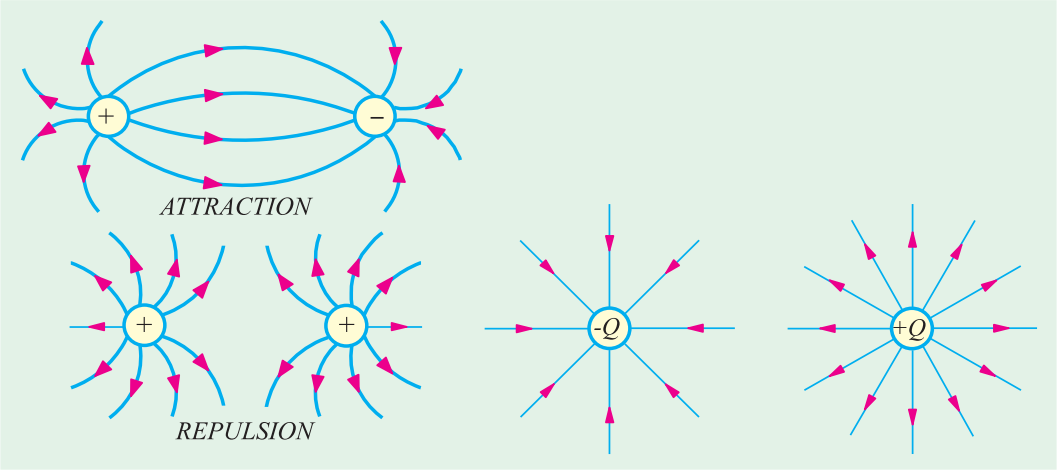


Fig. 4.4 (*a*) **Fig. 4.4 (*b*)**

The region in which the stress exists or in which electric forces act, is called an electric field or electrostatic field.

The stress is represented by imaginary lines of forces. The direction of the lines of force at any point is the direction along which a unit positive charge placed at that point would move if free to do so. It was suggested by Faraday that the electric field should be imagined to be divided into ***tubes of force*** containing a fixed number of ***lines*** of force. He assumed these tubes to the elastic and having the property of contracting longitudinally the repelling laterally. With the help of these properties, it becomes easy to explain ***(i)*** why unlike charges attract each other and try to come nearer to each other and ***(ii)*** why like charges repel each other [Fig. 4.4 (*a*)].

However, it is more common to use the term lines of force. These lines are supposed to emanate from a positive charge and end on a negative charge [Fig. 4.4 (*b*)]. These lines always leave or enter a conducting surface normally.

4.5. Electrostatic Induction

It is found that when an uncharged body is brought near a charged body, it acquires some charge. This phenomenon of an uncharged body getting charged merely by the nearness of a charged body is known as ***induction***. In Fig. 4.5, a positively-charged body A is brought close to a perfectly-insulated

uncharged body B. It is found that the end of B nearer to A gets negatively charged whereas further end becomes positively charged. The negative and positive charges of B are known as ***induced charges***. The negative charge of B is called 'bound' charge because it must remain on B so long as positive charge of A remains there. However, the positive charge on the farther end of B is called free charge. In Fig. 4.6, the body B has been earthed by a wire. The positive charge flows to earth leaving negative charge behind. If next A is removed, then this negative charge will also go to earth, leaving B uncharged. It is found that :

(i) a positive charge induces a negative charge and ***vice-versa.***

(ii) each of the induced charges is equal to the inducing charge.

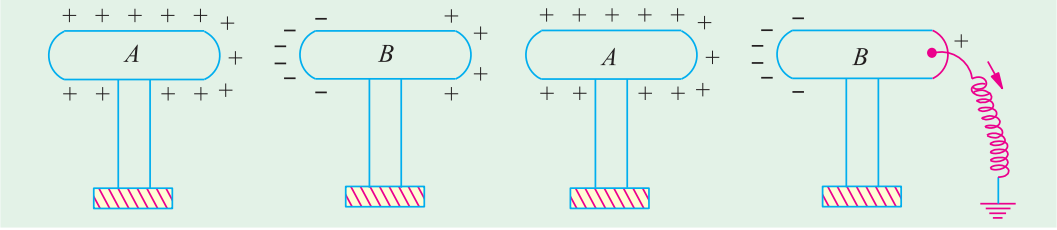


Fig. 4.5 Fig. 4.6

4.6. Electric Flux and Faraday Tubes

Consider a small closed curve in an electric field (Fig. 4.7). If we draw lines of force through each point of this closed curve, then we get a tube as shown in the figure. It is called the ***tube of the electric flux***. It may be defined as the region of space enclosed within the tubular surface formed by drawing lines of force through every point of a small closed curve in the electric field.

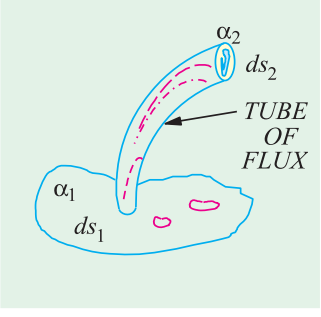
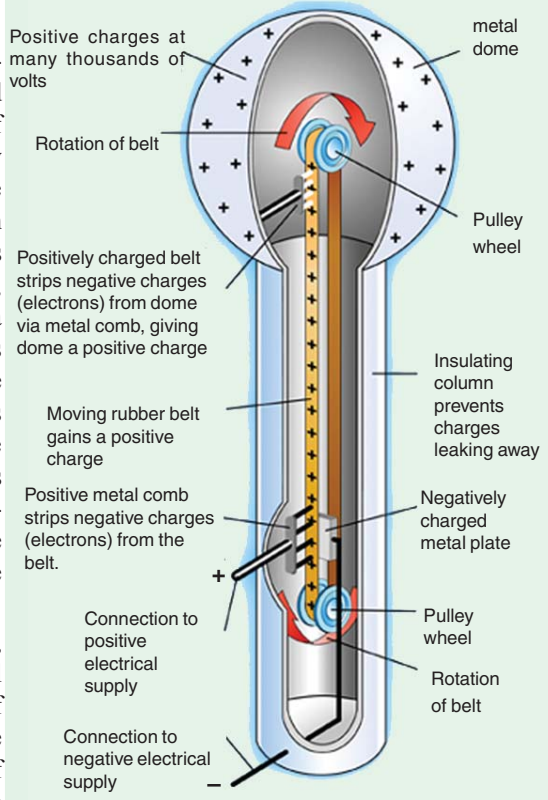


Fig. 4.7

Since lines of force end on conductors, the two ends of a flux tube will consist of small area ds_1 and ds_2 on the conductor surfaces. If surface charge densities over these areas are σ_1 and $-\sigma_2$, then charges at the two ends of the flux tube will be $\sigma_1 ds_1$ and $-\sigma_2 ds_2$. These charges are assumed to be always equal but opposite to each other. The strength of a flux tube is represented by the charge at its ends.

A ***unit tube of flux*** is one in which the

VAN DE GRAFF (ELECTROSTATIC) GENERATOR



The Van de Graff generator is able to produce very high voltages, for example, up to 50 000 volts. When someone touches the dome of the generator, it will cause hair to stand on end (since like charges repel). Touching the VDG is not dangerous since the current is very small.

end charge is one unit of charge.

In the S.I. system of units, one such tube of flux is supposed to start from a positive charge of one coulomb and terminate on a negative charge of the same amount.

A ***unit tube of flux*** is known as Faraday tube. If the charge on a conductor is $\pm Q$ coulombs, then the ***number of Faraday tubes starting or terminating on it also Q.***

The number of Faraday tubes of flux passing through a surface in an electric field is called the *electric flux* (or dielectric flux) through that surface. Electric flux is represented by the symbol ψ. Since electric flux is numerically equal to the charge, it is measured in coulombs.

Hence, $\psi = Q$ coulombs

Note. It may also be noted that 'tubes of flux' passing per unit area through a medium are also supposed to measure the 'electric displacement' of that dielectric medium. In that case, they are referred to as *lines of* ***displacement*** and are equal to ε times the lines of force (Art. 4.8). It is important to differentiate between the 'tubes of flux' and 'lines of force' and to remember that if Q is the charge, then

tubes of flux = Q and lines of force = Q/ε

4.7. Field Strength or Field Intensity or Electric Intensity (E)

Electric intensity at any point within an electric field may be defined in either of the following three ways :

(a) It is given by the force experienced by a unit positive charge placed at that point. Its direction is the direction along which the force acts.

Obviously, the unit of E is newton/coulomb (N/C).

For example, if a charge of Q coulombs placed at a particular point P within an electric field instances a force of F newton, then electric field at that point is given by

$$E = F/Q \; N/C$$

The value of E within the field due to a point charge can be found with help of Coulomb's laws. Suppose it is required to find the electric field at a point A situated at a distance of d metres from a charge of Q coulombs. Imagine a positive charge of one coulomb placed at that point (Fig. 4.8). The force experienced by this charge is

$$F = \frac{Q \times 1}{4\pi\varepsilon_0\varepsilon_r d^2} N \quad \text{or} \quad \vec{F} = \frac{Q \times 1}{4\pi\varepsilon_0\varepsilon_r d_{PA}^2} \hat{d}_{PA}$$

$$\therefore \quad E = \frac{Q \times 1}{4\pi\varepsilon_0\varepsilon_r d_{PA}^2} \text{ N/C} = 9 \times 10^9 \frac{Q}{\varepsilon_r d_{PA}^2} \text{ N/C} \quad \Big] \text{ in a medium}$$

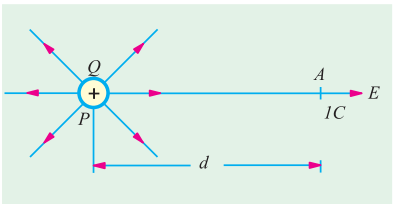


Fig. 4.8

or in vector notation,

$$\vec{E}(d) = 9 \times 10^9 \frac{Q}{\varepsilon_r d^2} \hat{d} \quad \text{where } \vec{E}(d) \text{ denotes } \vec{E} \text{ as a function of } d$$

$$= \frac{Q}{4\pi\varepsilon_0 d^2} \text{ N/C} = 9 \times 10^9 \frac{Q}{d^2} \text{ N/C} \quad \Big] \text{ in air}$$

(b) Electric intensity at a point may be defined as equal to the ***lines of force*** passing normally through a unit cross-section at that point. Suppose, there is a charge of Q coulombs. The number of

lines of force produced by it is Q/ε. If these lines fall normally on an area of A m^2 surrounding the point, then electric intensity at that point is

$$E = \frac{Q/\varepsilon}{A} = \frac{Q}{\varepsilon A}$$

Now $Q/A = D$ —the flux density over the area

$\therefore$ $E = \frac{D}{\varepsilon} = \frac{D}{\varepsilon_0 \varepsilon_r}$ —in a medium

$= \frac{D}{\varepsilon_0}$ —in air

The unit of E is volt/metre.

(c) Electric intensity at any point in an electric field is ***equal to the potential gradient at that point.***

In other words, E is equal to the rate of fall of potential in the direction of the lines of force.

$\therefore$ $E = \frac{-dV}{dx}$

Obviously, the unit of E is volt/metre.

It may be noted that E and D are vector quantities having magnitude and direction.

$\therefore$ In vector notation, $\vec{D} = \varepsilon_0 \vec{E}$

Example 4.7. *Point charges in air are located as follows :*
$+5 \times 10^{-8}$ C at (0, 0) metres, $+4 \times 10^{-8}$ C at (3, 0) metres and -6×10^{-8} C at (0, 4) metres. Find electric field intensity at (3, 4) metres.

Solution. Electric intensity at point D (3, 4) due to positive charge at point A is

$E_1 = 9 \times 10^9 \, Q/d^2 = 9 \times 10^9 \times 5 \times 10^{-8}/5^2 = 18$ V/m

As shown in Fig. 4.9, it acts along AD.

Similarly, electric intensity at point D due to positive charge at point B is $E_2 = 9 \times 10^9 \times 4 \times 10^{-8}/4^2 = 22.5$ V/m. It acts along BD.

$E_1 = 9 \times 10^9 \times 6 \times 10^{-8}/3^2 = 60$ V/m. It acts along DC.

The resultant intensity may be found by resolving E_1, E_2 and E_3 into their X-and Y-components. Now, $\tan \theta = 4/3$; $\theta = 53°8'$.

X-component $= E_1 \cos\theta - E_2 = 18 \cos 53°8' - 60$
$= -49.2$

Y-component $= E_1 \sin\theta + E_2 = 18 \sin 53°8' + 22.5 = 36.9$

$\therefore$ $E = \sqrt{(-49.2)^2 + 36.9^2} = 61.5$ V/m.

It acts along DE such that $\tan\phi = 36.9/49.2 = 0.75$.

Hence $\phi =$ **36.9°.**

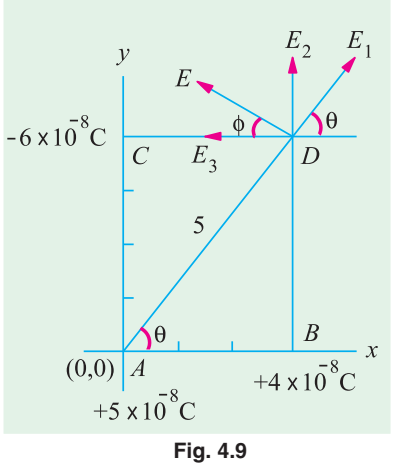


Fig. 4.9

Example 4.8. *An electron has a velocity of 1.5×10^7 m/s at right angles to the uniform electric field between two parallel deflecting plates of a cathode-ray tube. If the plates are 2.5 cm long and spaced 0.9 cm apart and p.d. between the plates is 75 V, calculate how far the electron is deflected sideways during its movement through the electric field. Assume electronic charge to be 1.6×10^{-19} coulomb and electronic mass to be 9.1×10^{-31} kg.*

Solution. The movement of the electron through the electric field is shown in Fig. 4.10. Electric intensity between the plates is $E = dV/dx = 75/0.009 = 8{,}333$ V/m.

Force on the electron is $F = QE$

$= 8{,}333 \times 1.6 \times 10^{-19} = 1.33 \times 10^{-15}$ N.

Since the deflection x is small as compared to the length of the plates, time taken by the electron to travel through the electric field is $= 0.025/1.5 \times 10^{7} = 1.667 \times 10^{-9}$ s

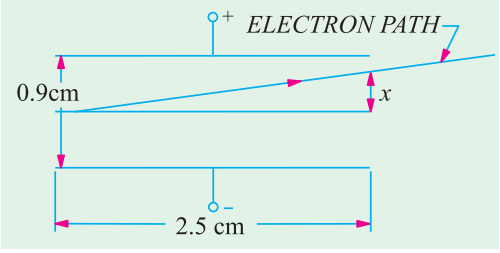
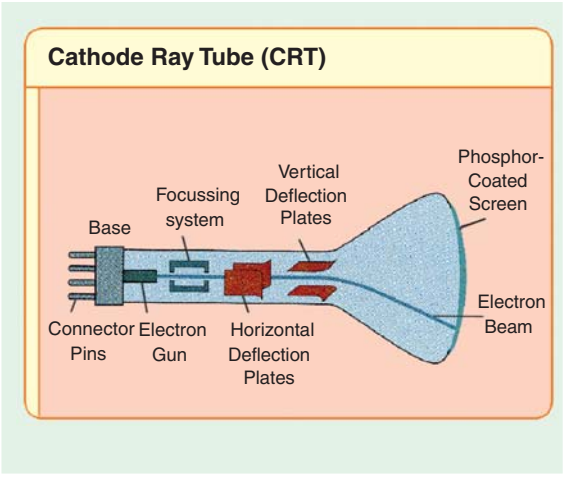


Fig. 4.10



Now, force = mass × acceleration

∴ Transverse acceleration is

$$= \frac{1.33 \times 10^{-15}}{9.1 \times 10^{-31}} = 1.44 \times 10^{15} \text{ m/s}^2$$

Final transverse velocity of the electron = acceleration × time

$= 1.44 \times 10^{-15} \times 1.667 \times 10^{-9} = 2.4 \times 10^{6}$ m/s

∴ sideways or transverse movement of the electron is

x = (average velocity) × time

$= \frac{1}{2} \times 2.4 \times 10^{6} \times 1.667 \times 10^{-9} =$ **2 mm (approx.)***

4.8. Electric Flux Density or Electric Displacement

It is given by the normal flux per unit area.

If a flux of Ψ coulombs passes normally through an area of A m^2, then flux density is

$$D = \frac{\Psi}{A} \text{ C/m}^2$$

It is related to electric field intensity by the relation

$D = \varepsilon_0 \varepsilon_r E$ ...in a medium

$= \varepsilon_0 E$ ...in free space

In other words, the product of electric intensity E at any point within a dielectric medium and the absolute permittivity ε ($= \varepsilon_0, \varepsilon_r$) at the same point is called the ***displacement*** at that point.

Like electric intensity E, electric displacement D** is also a vector quantity (see 4.7) whose direction at every point is the same as that of E but whose magnitude is $\varepsilon_0 \varepsilon_r$ times E. As E is represented by lines of force, similarly D may also be represented by lines called lines of electric

---

* The above result could be found by using the general formula

$$x = \frac{1}{2}\left(\frac{e}{m}\right)\left(\frac{V}{d}\right)\left(\frac{l}{v}\right)^2 \text{ metres}$$

where e/m = ratio of the charge and mass of the electron

V = p.d. between plates in volts; d = separation of the plates in metres

l = length of the plates in metres; v = axial velocity of the electron in m/s.

** A more general definition of displacement D is that $D = \varepsilon_0 \varepsilon_r E + P$ where P is the polarisation of the dielectric and is equal to the dipole moment per unit volume.

displacement. The tangent to these lines at any point gives the direction of D at that point and the number of lines per unit area perpendicular to their direction is numerically equal to the electric displacement at that point. Hence, the number of lines of electric displacement per unit area (D) is ε_0, ε_r times the number of lines of force per unit area at that point.

It should be noted that whereas the value of E depends on the permittivity of the surrounding medium, that of D is independent of it.

One useful property of D is that its surface integral over any closed surface equals the enclosed charge (Art. 4.9).

Let us find the value of D at a point distant r metres from a point charge of Q coulombs. Imagine a sphere of radius r metres surrounding the charge. Total flux = Q coulombs and it falls normally on a surface area of $4\pi r^2$ metres. Hence, electric flux density.

$$D = \frac{\Psi}{4\pi r^2} = \frac{Q}{4\pi r^2} \text{ coulomb/metre}^2 \text{ or } D = \frac{Q}{4\pi r^2} r = r \text{ (in vector notation)}$$

4.9. Gauss* Law

Consider a point charge Q lying at the centre of a sphere of radius r which surrounds it completely [Fig. 4.11 (*a*)]. The total number of ***tubes of flux*** originating from the charge is Q (but number of lines of force is Q/ε_0) and are normal to the surface of the sphere. The electric field E which equals $Q/4\pi\varepsilon_0 r^2$ is also normal to the surface. As said earlier, total number of lines of force passing perpendicularly through the whole surface of the sphere is

$$= E \times \text{Area} = \frac{Q}{4\pi\varepsilon_0 r^2} \times 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

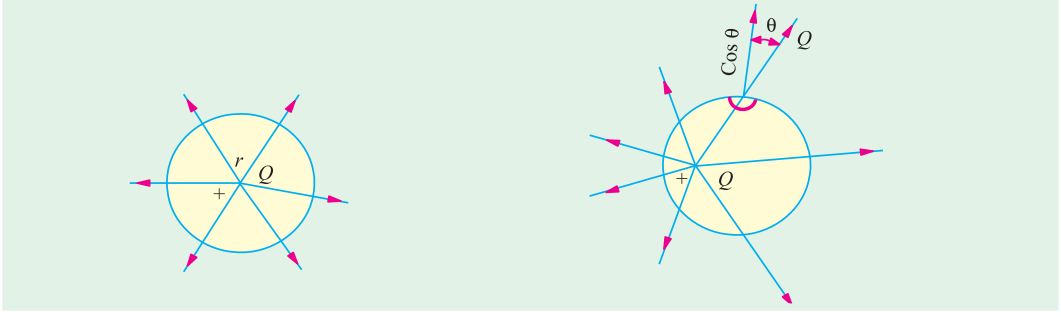


Fig. 4.11

Now, suppose we draw another sphere surrounding the charge [Fig. 4.11 (*b*)] but whose centre does not lie at the charge but elsewhere. In this case also, the number of tubes of flux emanating from the charge is Q and lines of force is $Q.\varepsilon_0$ though they are not normal to the surface. These can, however, be split up into cos θ components and sin θ components. If we add up sin θ components all over the surface, they will be equal to zero . But if add up cos θ components over the whole surface of the sphere, the normal flux will again come out to be Q (or lines of force will come out to be Q/ε_0). Hence, it shows that irrespective of where the charge Q is placed within a closed surface completely surrounding it, the total normal flux is Q and the total number of lines of force passing out normally is Q/ε_0.

In fact, as shown in Fig. 4.12, if there are placed charges of value Q_1, Q_2, $-Q_3$ inside a closed surface, the total *i.e.* net charge enclosed by the surface is $(Q_1 + Q_2 - Q_3)/\varepsilon_0$ through the closed surface.

* After the German mathematician and astronomer Karel Freidrich Gauss (1777-1855).

This is the meaning of Gauss's law which may be stated thus : the surface integral of the normal component of the electric intensity E over a closed surface is equal to $1/\varepsilon_0$ times the total charge inside it.

Mathematically, $\oint E_n ds = Q/\varepsilon_0$ (where the circle on the integral sign indicates that the surface of integration is a closed surface).

or $\oint \varepsilon_n \varepsilon_0 \, ds = Q$, *i.e.* $\oint D_n \, ds = Q$ $[\therefore D_n = \varepsilon_0 E_n]$

or $\oint \varepsilon_0 E \cos\theta \, ds = Q$, *i.e.* $\oint D \cos\theta \, ds = Q$

or $\oint \varepsilon_0 E_{ds} \cos\theta = Q$, *i.e.*, $\oint D \, ds \cos\theta = Q$

when E and D are not normal to the surface but make an angle θ with the normal (perpendicular) to the surface as shown in Fig. 4.13.

Proof. In Fig. 4.13, let a surface S completely surround a quantity of electricity or charge Q. Consider a small surface area ds subtending a small solid angle $d\omega$ at point charge Q. The field intensity at ds is $E = \dfrac{Q}{4\pi\varepsilon_0 d^2}$ where d is the distance between Q and ds.

In vector notation, $\oint \varepsilon_0 \vec{E} . \vec{ds} = Q$ *i.e.* $\oint \vec{D} \, \vec{ds} = Q = \int_v \rho \, dv$ (where ρ is the volume density of charge in the volume enclosed by closed surface S).

Thus $\oint_s \vec{D} . \vec{ds} = \int_v \rho \, dv$ is the vector statement of Gauss Law* and its alternative statement is

$\nabla D = \rho$

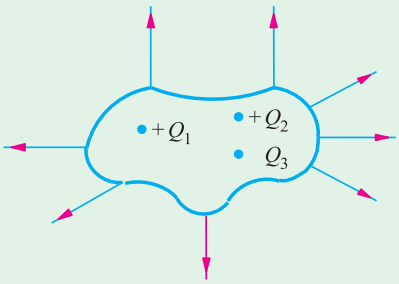


Fig. 4.12

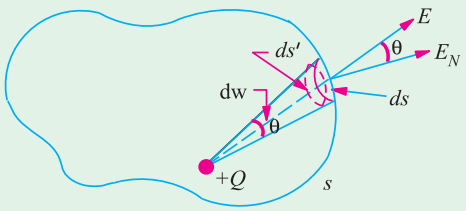


Fig. 4.13

The normal component of the intensity $E_n = E \cos\theta$

$\therefore$ No. of lines of force passing normally through the area ds is

$= E_n . d_s = E \, ds \cos\theta = E . ds$ in vector notation

Now $ds \cos\theta = ds'$ $\quad \therefore E.ds' = \dfrac{Q}{4\pi\varepsilon_0 d^2} . ds'$

Now $ds'/d^2 = d\omega$

Hence, the number of lines of force passing normally is $= \dfrac{Q}{4\pi\varepsilon_0} d\omega$

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* This results from the application of the Divergence theorem, also called the Gauss' Theorem, *viz.*, $\int_s \nabla D d\upsilon = \int D . ds$ where vector operator called 'del' is defined as

$$\nabla = \frac{\partial}{\partial x} x + \frac{\partial}{\partial y} y + \frac{\partial}{\partial z} z$$

Total number of lines of force over the whole surface

$$= \frac{Q}{4\pi\varepsilon_0}\int_s d\omega = \frac{Q}{4\pi\varepsilon_0}\times 4\pi = \frac{Q}{\varepsilon_0}$$

where sign $\oint$ denotes integration around the whole of the closed surface *i.e.* surface integral.

If the surface passes through a material medium, then the above law can be generalized to include the following :

the surface integral of the normal component of D over a closed surface equals the free charge enclosed by the surface.

As before $D = \frac{Q}{4\pi\varepsilon d^2}$. The normal component $D_n = D\cos\theta = \frac{Q}{4\pi d^2}\times\cos\theta$

Hence, the normal electric flux from area ds is

$$d\psi = D_n \times ds = \frac{Q}{4\pi d^2}\cdot\cos\theta\cdot ds = \frac{Q}{4\pi d^2}\cdot ds'$$

$$\therefore \quad d\psi = \frac{Q}{4\pi}\left(\frac{ds'}{d^2}\right) = \frac{Q}{4\pi}d\omega$$

or $$\psi = \int\frac{Q}{4\pi}.d\omega = \frac{Q}{4\pi}\int d\omega = \frac{Q}{4\pi}\times 4\pi = Q \qquad \therefore\ \Psi = Q$$

which proves the statement made above.

Hence, we may state Gauss's law in two slightly different ways.

$$\int_3 E_n\,.\,ds = \int_3 E\,.\cos\theta\,.\,ds = Q/\varepsilon_0 \quad \text{or} \quad \varepsilon_0\int_3 E_n\,.\,ds = Q$$

and $$\int_3 D_n\,.\,ds = \int_3 D_n\,.\,ds = Q$$

(vector statement is given above)

4.10. The Equations of Poisson and Laplace

These equations are useful in the solution of many problems concerning electrostatics especially the problem of space charge* present in an electronic valve. The two equations can be derived by applying Gauss's theorem. Consider the electric field set up between two charged plates P and Q [Fig. 4.14 (*a*)]. Suppose there is some electric charge present in the space between the two plates. It is, generally, known as the space charge. Let the space charge density be ρ coulomb/metre^3. It will be assumed that the space charge density varies from one point of space to the another but is uniform throughout any thin layer taken parallel to the plates P and Q. If X-axis is taken perpendicular to the plates, then ρ is assumed to depend on the value of x. It will be seen from Fig. 4.14 (*a*) that the value of electric intensity E increases with x because of the space charge.

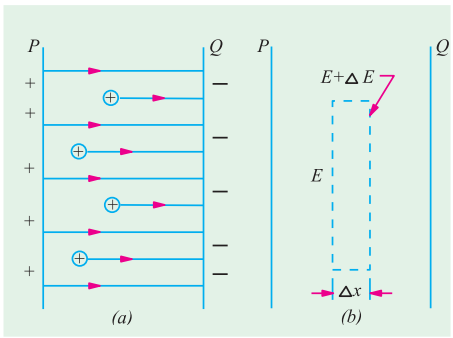


Fig. 4.14

Now, consider a thin volume element of cross-section A and thickness Δx as shown in Fig. 4.14 (*b*). The values of electric intensity at the two opposite faces of this element are E and $(E + \Delta E)$. If dE/dx represents the rate of increases of electric intensity with distance, then

$$\Delta E = \frac{\partial E}{\partial x}\times\Delta x \quad \therefore \quad E + \Delta E = E + \frac{\partial E}{\partial x}\times\Delta x$$

The surface integral of electric intensity over the right-hand face of this element is

* Such a space charge exists in the space between the cathode and anode of a vacuum tube.

$$= \left(E + \frac{\partial E}{\partial x} . \Delta x \right) A$$

The surface integral over the left-hand face of the element is $= -E \times A$

The negative sign represents the fact that E is directed inwards over this face.

The surface integral over the entire surface, *i.e.,* the closed surface of the element is $= \left(E + \frac{\partial E}{\partial x} . \Delta x \right) A - E \times A = A . \Delta x . \frac{\partial E}{\partial x}$. From symmetry it is evident that along with y and z there is no field.

Now, according to Gauss's theorem (Art. 4.9), the surface integral of electric intensity over a closed surface is equal to $1/\varepsilon_0$ time the charge within that surface.

Volume of the element, $dV = A \times \Delta x$; charge $= \rho A . \Delta x$

$$\therefore \quad A . \Delta x . \frac{\partial E}{\partial x} = \rho A . \Delta x . \frac{1}{\varepsilon_0} \quad \text{or} \quad \frac{\partial E}{\partial x} = \frac{\rho}{\varepsilon_0}$$

Now $$E = -\frac{\partial V}{\partial x} \quad \therefore \frac{\partial E}{\partial x} = \frac{\partial}{\partial x}\left(-\frac{dV}{dx}\right) = -\frac{\partial^2 V}{\partial x^2} \quad \therefore \quad \frac{\partial^2 V}{\partial x^2} = -\frac{\rho}{\varepsilon_0}$$

It is known as Poisson's equation in one dimension where potential varies with x.

When V varies with x, y and z, then $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = -\frac{\rho}{\varepsilon_0} = \nabla^2 V$ in vector notation.

If, as a special case, where space charge density is zero, then obviously,

$$\partial^2 V / \partial x^2 = 0$$

In general, we have $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$ or $\nabla^2 V = 0$ in vector notation where ∇^2 is defined (in cartesian co-ordinates) as the operation

$$\nabla^2 = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

It is known as Laplace's equation.

4.11. Electric Potential and Energy

We know that a body raised above the ground level has a certain amount of mechanical potential energy which, by definition, is given by the amount of work done in raising it to that height. If, for example, a body of 5 kg is raised against gravity through 10 m, then the potential energy of the body is $5 \times 10 = 50$ m-kg. wt. $= 50 \times 9.8 = 490$ joules. The body falls because there is attraction due to gravity and always proceeds from a place of higher potential energy to one of lower potential energy. So, we speak of gravitational potential energy or briefly 'potential' at different points in the earth's gravitational field.

Now, consider an electric field. Imagine an isolated positive charge Q placed in air (Fig. 4.15). Like earth's gravitational field, it has its own electrostatic field which theoretically extends upto infinity. If the charge X is very far away from Q, say, at infinity, then force on it is practically zero. As X is brought nearer to Q, a force of repulsion acts on it (as similar charges repel each other), hence work or energy is required to bring it to a point like A in the electric field. Hence, when at point A, charge X has

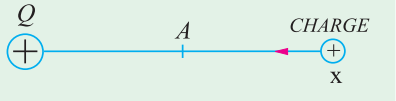


Fig. 4.15



Using Van De Graff Generator, artificial lightning can be created in the laboratory, in a miature scale.

some amount of electric potential energy. Similar other points in the field will also have some potential energy. In the gravitational field, usually 'sea level' is chosen as the place of 'zero' potential. In electric field infinity is chosen as the theoretical place of 'zero' potential although, in practice, earth is chosen as 'zero' potential, because earth is such a large conductor that its potential remains practically constant although it keeps on losing and gaining electric charge every day.

4.12. Potential and Potential Difference

As explained above, the force acting on a charge at infinity is zero, hence 'infinity' is chosen as the theoretical place of zero electric potential. Therefore, potential at any point in an electric field may be defined as

numerically equal to the work done in bringing a positive charge of one coulomb from infinity to that point against the electric field.

The unit of this potential will depend on the unit of charge taken and the work done.

If, in shifting one coulomb from infinity to a certain point in the electric field, the work done is one joule, then potential of that ponit is one volt.

Obviously, potential is work per unit charge,

$$\therefore \qquad 1 \text{ volt} = \frac{1 \text{ joule}}{1 \text{ coulomb}}$$

Similarly, potential difference (p.d.) of one volt exists between two points if one joule of work is done in shifting a charge of one coulomb from one point to the other.

4.13. Potential at a Point

Consider a positive point charge of Q coulombs placed in air. At a point x metres from it, the force on one coulomb positive charge is $Q/4\pi\varepsilon_0 x^2$ (Fig. 4.16). Suppose, this one coulomb charge is moved towards Q through a small distance dx. Then, work done is

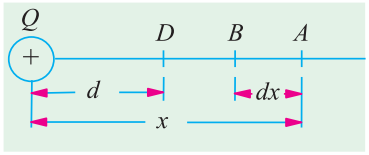


Fig. 4.16

$$dW = \frac{Q}{4\pi\varepsilon_0 r^2} \times (-dx)$$

The negative sign is taken because dx is considered along the negative direction of x.

The total work done in bringing this coulomb of positive charge from infinity to any point D which is d metres from Q is given by

$$W = -\int_{x=\infty}^{x=d} Q \cdot \frac{dx}{4\pi\varepsilon_0 x^2} = -\frac{Q}{4\pi\varepsilon_0}\int_{\infty}^{d} \frac{dx}{x^2}$$

$$= -\frac{Q}{4\pi\varepsilon_0}\left|-\frac{1}{x}\right|_{\infty}^{d} = -\frac{Q}{4\pi\varepsilon_0}\left[-\frac{1}{d}-\left(-\frac{1}{\infty}\right)\right] = \frac{Q}{4\pi\varepsilon_0 d} \text{ joules}$$

By definition, this work in joules in numerically equal to the potential of that point in volts.

$$\therefore \qquad V = \frac{Q}{4\pi\varepsilon_0 d} = 9\times10^9 \frac{Q}{d} \text{ volt} \qquad \text{—in air}$$

and

$$V = \frac{Q}{4\pi\varepsilon_0\varepsilon_r d} = 9\times10^9 \frac{Q}{\varepsilon_r d} \text{ volt} \qquad \text{—in medium}$$

We find that as d increases, V decreases till it becomes zero at infinity.

4.14. Potential of a Charged Conducting Sphere

The above formula $V = Q/4\pi\varepsilon_0\varepsilon_r d$ applies only to a charge concentrated at a point. The problem

of finding potential at a point outside a charged sphere sounds difficult, because the charge on the sphere is distributed over its entire surface and so, is not concentrated at a point. But the problem is easily solved by nothing that the lines of force of a charged sphere, like A in by noting that the lines of force of a charged sphere, like A in Fig. 4.17 spread out normally from its surface. If produced backwards, they meet at the centre of A. Hence for finding the potentials at points outside the sphere, we can imagine the charge on the sphere as concentrated at its centre O. If r is the radius of sphere in metres and Q its charge in coulomb then, potential of its surface is $Q/4\pi\ \varepsilon_0\ r$ volt and electric intensity is $Q/4\pi\varepsilon_0 r^2$. At any other point 'd' metres from the centre of the sphere, the corresponding values are $Q/4\pi\ \varepsilon_0\ d$ and $Q/4\pi\varepsilon_0\ d^2$ respectively with $d > r$ as shown in Fig. 4.18 though its starting point is coincident with that of r. The variations of the potential and electric intensity with distance for a charged sphere are shown in Fig. 4.18.

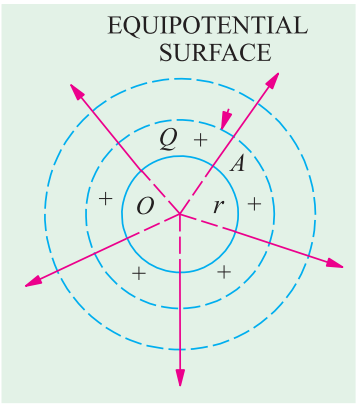


Fig. 4.17

4.15. Equipotential Surfaces

An equipotential surface is a surface in an electric field such that all points on it are at the same potential. For example, different spherical surfaces around a charged sphere are equipotential surfaces. One important property of an equipotential surface is that the direction of the electric field strength and flux density is always at right angles to the surface. Also, electric flux emerges out normal to such a surface. If, it is not so, then there would be some component of E along the surface resulting in potential difference between various points lying on it which is contrary to the definition of an equipotential surface.

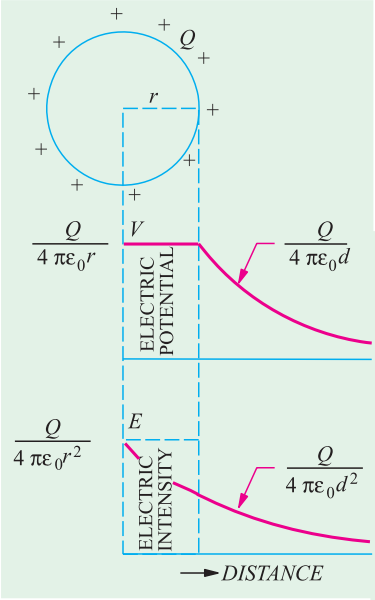


Fig. 4.18

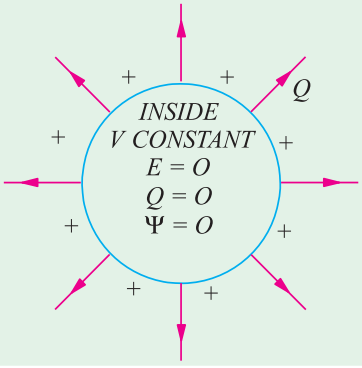


Fig. 4.19

4.16. Potential and Electric Intensity Inside a Conducting Sphere

It has been experimentally found that when charge is given to a conducting body say, a sphere then it resides entirely on its outer surface *i.e.*, within a conducting body whether hollow or solid, the charge is zero. Hence, ***(i)*** flux is zero ***(ii)*** field intensity is zero ***(iii)*** all points within the conductor are at the same potential as at its surface (Fig. 4.19).

Example 4.9. *Three concentric spheres of radii 4, 6 and 8 cm have charges of + 8, –6 and + 4 μμC respectively. What are the potentials and field strengths at points, 2, 5, 7 and 10 cm from the centre.*

Solution. As shown in Fig. 4.20, let the three spheres be marked A, B and C. It should be remembered that ***(i)*** the field intensity outside a sphere is the same as that obtained by considering the charge at its centre ***(ii)*** inside the sphere, the field strength is zero ***(iii)*** potential anywhere inside a sphere is the same as at its surface.

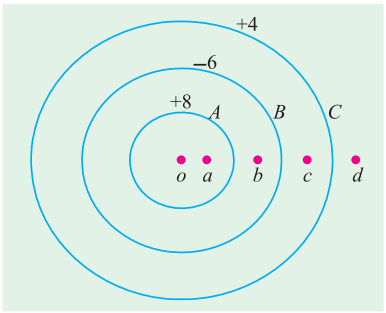


Fig. 4.20

(i) Consider point 'a' at a distance of 2 cm from the centre O. Since it is inside all the spheres, field strength at this point is zero.

Potential at 'a'

$$= \sum \frac{Q}{4\pi\varepsilon_0 d} = 9\times10^9 \sum \frac{Q}{d}$$

$$= 9\times10^9 \left(\frac{8\times10^{-12}}{0.04} - \frac{6\times10^{-12}}{0.06} + \frac{4\times10^{-12}}{0.08} \right) = \mathbf{1.35\ V}$$

(ii) Since point 'b' is outside sphere A but inside B and C.

$$\therefore \quad \text{Electrical field} = \frac{Q}{4\pi\varepsilon_0 d^2} = 9\times10^9 \frac{Q}{d} \text{ N/C}$$

$$= 9\times10^9 \times \frac{8\times10^{-12}}{0.05^2} = \mathbf{28.8\ N/C}$$

$$\text{Potential at '}b\text{'} = 9\times10^9 \times \left(\frac{8\times10^{-12}}{0.05} - \frac{6\times10^{-12}}{0.06} + \frac{4\times10^{-12}}{0.08} \right) = \mathbf{0.99\ V}$$

(iii) The field strength at point 'c' distant 7 cm from centre O

$$= 9\times10^9 \times \left[\frac{8\times10^{-12}}{0.07^2} - \frac{6\times10^{-12}}{0.07^2} \right] = \mathbf{3.67\ N/C}$$

$$\text{Potential at '}c\text{'} = 9\times10^9 \times \left[\frac{8\times10^{-12}}{0.07} - \frac{6\times10^{-12}}{0.07} + \frac{4\times10^{-12}}{0.08} \right] = \mathbf{0.71\ V}$$

(iv) Field strength at 'd' distant 10 cm from point O is

$$= 9\times10^9 \times \left[\frac{8\times10^{-12}}{0.1^2} - \frac{6\times10^{-12}}{0.1^2} + \frac{4\times10^{-12}}{0.1^2} \right] = \mathbf{5.4\ N/C}$$

$$\text{Potential at '}d\text{'} = 9\times10^9 \times \left[\frac{8\times10^{-12}}{0.1} - \frac{6\times10^{-12}}{0.1} + \frac{4\times10^{-12}}{0.1} \right] = \mathbf{0.54\ V}$$

Example 4.10. *Two positive point charges of 12×10^{-10} C and $8\cdot\times10^{-10}$ C are placed 10 cm apart. Find the work done in bringing the two charges 4 cm closer.*

Solution. Suppose the 12×10^{-10} C charge to be fixed. Now, the potential of a point 10 cm from this charge

$$= 9\times10^9 \frac{12\times10^{-10}}{0.1} = 108 \text{ V}$$

The potential of a point distant 6 cm from it

$$= 9\times10^9 \times \frac{12\times10^{-10}}{0.06} = 180 \text{ V}$$

$\therefore$ potential difference $= 180 - 108 = 72$ V

Work done $=$ charge $\times$ p.d. $= 8\times10^{-10}\times 72 = \mathbf{5.76\times10^{-8}\ joule}$

Example 4.11. *A point charge of 10^{-9} C is placed at a point A in free space. Calculate :*
(i) *the intensity of electrostatic field on the surface of sphere of radius 5 cm and centre A.*

(ii) *the difference of potential between two points 20 cm and 10 cm away from the charge at A.*
(Elements of Elect.-I, Banglore Univ. 1987)

Solution. *(i)* $E = Q/4\pi\varepsilon_0 r^2 = 10^{-9}/4\pi \times 8.854 \times 10^{-12} \times (5 \times 10^{-2})^2 =$ **3,595 V/m**

(ii) Potential of first point $= Q/4\pi\varepsilon_0 d = 10^{-9}/4\pi \times 8.854 \times 10^{-12} \times 0.2 = 45$ V

Potential of second point $= 10^{-9}/4\pi \times 8.854 \times 10^{-12} \times 0.1 = 90$ V

$\therefore$ p.d. between two points $= 90 - 45 =$ **45 V**

4.17. Potential Gradient

It is defined as the rate of change of potential with distance in the direction of electric force

i.e. $\dfrac{dV}{dx}$

Its unit is volt/metre although volt/cm is generally used in practice. Suppose in an electric field of strength E, there are two points dx metre apart. The p.d. between them is

$$dV = E \,.\, (-dx) = -E \,.\, dx \quad \therefore \; E = -\frac{dV}{dx} \qquad \ldots(i)$$

The −ve sign indicates that the electric field is directed outward, while the potential increases inward.

Hence, it means that electric intensity at a point is equal to the negative potential gradient at that point.

4.18. Breakdown Voltage and Dielectric Strength

An insulator or dielectric is a substance within which there are no mobile electrons necessary for electric conduction. However, when the voltage applied to such an insulator exceeds a certain value, then it breaks down and allows a heavy electric current (much larger than the usual leakage current) to flow through it. If the insulator is a solid medium, it gets punctured or cracked.

The disruptive or breakdown voltage of an insulator is the minimum voltage required to break it down.*

Dielectric strength of an insulator or dielectric medium is given by the ***maximum potential difference which a unit thickness of the medium can withstand without breaking down***.

In other words, the dielectric strength is given by the potential gradient necessary to cause breakdown of an insulator. Its unit is volt/metre (V/m) although it is usually expressed in kV/mm.

For example, when we say that the dielectric strength of air is 3 kV/mm, then it means that the maximum p.d. which one mm thickness of air can withstand across it without breaking down is 3 kV or 3000 volts. If the p.d. exceeds this value, then air insulation breaks down allowing large electric current to pass through.

Dielectric strength of various insulating materials is very important factor in the design of high-voltage generators, motors and transformers. Its value depends on the thickness of the insulator, temperature, moisture, content, shape and several other factors.

For example doubling the thickness of insulation does not double the safe working voltage in a machine.**

---

* Flashover is the disruptive discharge which taken places over the surface of an insulator and occurs when the air surrounding it breaks down. Disruptive conduction is luminous.

** The relation between the breakdown voltage V and the thickness of the dielectric is given approximately by the relation $V = At^{2/3}$
where A is a constant depending on the nature of the medium and also on the thickness t. The above statement is known as Baur's law.

Note. It is obvious that the electric intensity E, potential gradient and dielectric strength are dimensionally equal.

4.19. Safety Factor of a Dielectric

It is given by the ratio of the dielectric strength of the insulator and the electric field intensity established in it. If we represent the dielectric strength by E_{bd} and the actual field intensity by E, then safety factor $k = E_{bd}/E$

For example, for air $E_{bd} = 3 \times 10^6$ V/m. If we establish a field intensity of 3×10^5 V/m in it, then, $k = 3 \times 10^6 / 3 \cdot 10^5 = 10$.

4.20. Boundary Conditions

There are discontinuities in electric fields at the boundaries between conductors and dielectrics of different permittivities. The relationships existing between the electric field strengths and flux densities at the boundary are called the boundary conditions.

With reference to Fig. 4.21, first boundary conditions is that the normal component of flux density is continuous across a surface.

As shown, the electric flux approaches the boundary BB at an angle θ_1 and leaves it at θ_2. D_{1n} and D_{2n} are the normal components of D_1 and D_2. According to first boundary condition,

$$D_{1n} = D_{2n} \qquad ...(i)$$

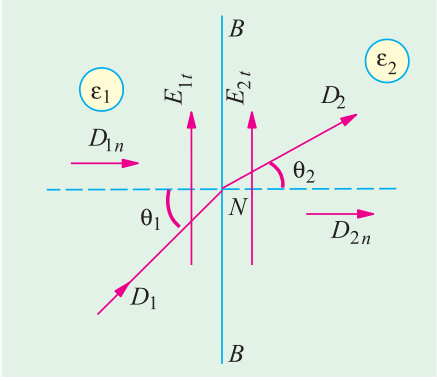


Fig. 4.21

The second boundary condition is that the tangential field strength is continuous across the boundary

$$\therefore \qquad E_{1t} = E_{2t} \qquad ...(ii)$$

In Fig. 4.21, we see that

$$D_{1n} = D_1 \cos\theta_1 \quad \text{and} \quad D_{2n} = D_2 \cos\theta_2$$

Also $E_1 = D_1/\varepsilon_1$ and $E_{1t} = D_1 \sin\theta_2/\varepsilon_1$

Similarly, $E_2 = D_2/\varepsilon_2$ and $E_{2t} = D_2 \sin\theta_2/\varepsilon_2$

$$\therefore \quad \frac{D_{1n}}{E_1 t} = \frac{\varepsilon_1}{\tan\theta_1} \quad \text{and} \quad \frac{D_{2n}}{E_{2t}} = \frac{\varepsilon_2}{\tan\theta_2}$$

Since $D_{1n} = D_{2n}$ and $E_{1t} = E_{2t}$ $\therefore$ $\dfrac{\tan\theta_1}{\tan\theta_2} = \dfrac{\varepsilon_1}{\varepsilon_2}$

This gives the law of electric flux refraction at a boundary.

It is seen that if $\varepsilon_1 > \varepsilon_2$, $\theta_1 > \theta_2$.

Table No. 4.1
Dielectric Constant and Strength
(*indicates average value)

Insulating material	*Dielectric constant or relative permittivity (e_r)*	*Dielectric Strength in kV/mm*
Air	1.0006	3.2
Asbestos*	2	2
Bakelite	5	15
Epoxy	3.3	20

Glass	5-12	12-100
Marble*	7	2
Mica	4-8	20-200
Micanite	4-5-6	25-35
Mineral Oil	2.2	10
Mylar	3	400
Nylon	4.1	16
Paper	1.8-2.6	18
Paraffin wax	1.7-2.3	30
Polyethylene	2.3	40
Polyurethane	3.6	35
Porcelain	5-6.7	15
PVC	3.7	50
Quartz	4.5-4.7	8
Rubber	2.5-4	12-20
Teflon	2	20
Vacuum	1	infinity
Wood	2.5-7	---

Example 4.12. *Find the radius of an isolated sphere capable of being charged to 1 million volt potential before sparking into the air, given that breakdown voltage of air is 30,000 V/cm.*

Solution. Let r metres be the radius of the spheres, then

$$V = \frac{Q}{4\pi\varepsilon_0 r} = 10^6 \text{ V} \qquad \text{...(i)}$$

Breakdown voltage $= 30{,}000 \text{ V/cm} = 3 \times 10^6 \text{ V/m}$

Since electric intensity equals breakdown voltage

$$\therefore \quad E = \frac{Q}{4\pi\varepsilon_0 r^2} = 3 \times 10^6 \text{ V/m} \qquad \text{...(ii)}$$

Dividing (*i*) by (*ii*), we get $r = 1/3 =$ **0.33 metre**

Example 4.13. *A parallel plate capacitor having waxes paper as the insulator has a capacitance of 3800 pF, operating voltage of 600 V and safety factor of 2.5. The waxed paper has a relative permittivity of 4.3 and breakdown voltage of 15 · 10^6 V/m. Find the spacing d between the two plates of the capacitor and the plate area.*

Solution. Breakdown voltage V_{bd} = operating voltage × safety factor = 600 · 2.5 = 1500 V

$V_{bd} = d \times E_{bd}$ or $d = 1500/15 \times 10^6 = 10^{-4}$ m = 0.1 mm

$C = \varepsilon_0 \varepsilon_r A/d$ or $A = Cd/\varepsilon_0 \varepsilon_r = 3800 \times 10^{-9} \times 10^{-4}/8.854 \times 10^{-12} \cdot 4.3 =$ **0.01 m²**

Example 4.14. *Two brass plates are arranged horizontally, one 2 cm above the other and the lower plate is earthed. The plates are charged to a difference of potential of 6,000 volts. A drop of oil with an electric charge of 1.6×10^{-19} C is in equilibrium between the plates so that it neither rises nor falls. What is the mass of the drop ?*

Solution. The electric intensity is equal to the potential gradient between the plates.

$g = 6{,}000/2 = 3{,}000 \text{ volt/cm} = 3 \times 10^5 \text{ V/m}$

$\therefore \quad E = 3 \times 10^5 \text{ V/m}$ or N/C

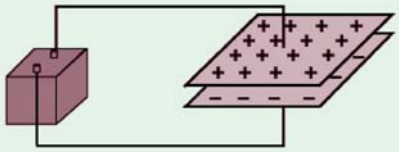
$\therefore \quad$ force on drop $= E \times Q = 3 \times 10^5 \times 1.6 \times 10^{-19} = 4.8 \times 10^{-14}$ N

Wt. of drop $= mg$ newton

$\therefore \quad m \times 9.81 = 4.8 \times 10^{-14} \quad \therefore \; m =$ **4.89×10^{-15} kg**

Example 4.15. *A parallel-plate capacitor has plates 0.15 mm apart and dielectric with relative permittivity of 3. Find the electric field intensity and the voltage between plates if the surface charge is* 5×10^{-4} *μC/cm*^{2}.

(Electrical Engineering, Calcutta Univ.)



Capacitor

A battery will transport charge from one plate to the other until the voltage produced by the charge buildup is equal to the battery voltage

Solution. The electric intensity between the plates is

$$E = \frac{D}{\varepsilon_0 \varepsilon_r} \text{ volt/metre;}$$

Now, $\sigma = 5 \times 10^{-4}\ \mu\text{ C/cm}^2 = 5 \times 10^{-6}\ \text{C/m}^2$

Since, charge density equals flux density

$$\therefore \quad E = \frac{D}{\varepsilon_0 \varepsilon_r} = \frac{5 \times 10^{-6}}{8.854 \times 10^{-12} \times 3} = 188,000\ \text{V/m} = \mathbf{188\ kV/m}$$

Now potential difference $V = E \times dx = 188{,}000 \times (0.15 \times 10^{-3}) = \mathbf{2.82\ V}$

Example 4.16. *A parallel-plate capacitor consists of two square metal plates 500 mm on a side separated by 10 mm. A slab of Teflon* ($\varepsilon_r = 2.0$) *6 mm thick is placed on the lower plate leaving an air gap 4 mm thick between it and the upper plate. If 100 V is applied across the capacitor, find the electric field* (E_0) *in the air, electric field* E_t *in Teflon, flux density* D_a *in air, flux density* D_t *in Teflon and potential difference* V_t *across Teflon slab.* **(Circuit and Field Theory, A.M.I.E. Sec. B)**

Solution.

$$C = \frac{\varepsilon_0 A}{(d_1/\varepsilon_{e1} + d_2/\varepsilon_{r2})} = \frac{8.854 \times 10^{-12} \times (0.5)^2}{(6 \times 10^{-3}/2) + (4 \times 10^{-3}/1)} = 3.16 \times 10^{-10}\ \text{F}$$

$$Q = CV = 3.16 \times 10^{-10} \times 100 = 31.6 \times 10^{-9}\ C$$

$$D = Q/A = 31.6 \times 10^{-9}/(0.5)^2 = 1.265 \times 10^{-7}\ \text{C/m}^2$$

The charge or flux density will be the same in both media *i.e.* $D_a = D_t = D$

In air, $E_0 = D/\varepsilon_0 = 1.265 \times 10^{-7}/8.854 \times 10^{-12} = 14{,}280\ \text{V/m}$

In Teflon, $E_t = D/\varepsilon_0 \varepsilon_r = 14{,}280/2 = 7{,}140\ \text{V/m}$

$$V_t = E_t \times d_t = 7{,}140 \times 6 \times 10^{-3} = \mathbf{42.8\ V}$$

Example 4.17. *Calculate the dielectric flux in micro-coulombs between two parallel plates each 35 cm square with an air gap of 1.5 mm between them, the p.d. being 3,000 V. A sheet of insulating material 1 mm thick is inserted between the plates, the permittivity of the insulating material being 6. Find out the potential gradient in the insulating material and also in air if the voltage across the plates is raised to 7,500 V.*

(Elect. Engg.-I, Nagpur Univ.)

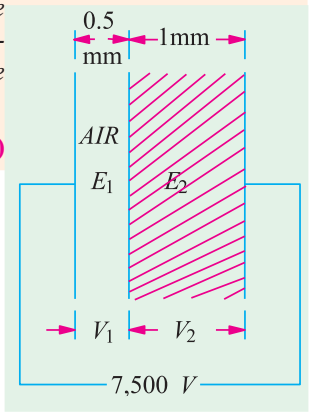


Fig. 4.22

Solution. The capacitance of the two parallel plates is

$C = \varepsilon_0 \varepsilon_r A/d$ Now, $\varepsilon_r = 1$ —for air

$A = 35 \times 35 \times 10^{-4} = 1{,}225 \times 10^{-4}\ \text{m}^2$; $d = 1.5 \times 10^{-2}\ \text{m}$

$$\therefore \quad C = \frac{8.854 \times 10^{-12} \times 1,225 \times 10^{-4}}{1.5 \times 10^{-3}}\ \text{F} = 7.22 \times 10^{-16}\ \text{F}$$

Charge $Q = CV = 7.22 \times 10^{-10} \times 3{,}000$ coulomb

Dielectric flux = $7.22 \times 3{,}000 \times 10^{-10}$ C

$= 2.166 \times 10^{-6}$ C = **2.166 μC**

With reference to Fig. 4.23, we have

$V_1 = E_1 x_1 = 0.5 \times 10^{-3} E_1$; $V_2 = 10^{-3} E_2$

Now $V = V_1 + V_2$

$\therefore \quad 7{,}500 = 0.5 \times 10^{-3} E_1 + 10^{-3} E_2$

or $E_1 + 2E_2 = 15 \times 10^6$ ...(*i*)

Also $D = \varepsilon_0 \varepsilon_{r1} E_1 = \varepsilon_0 \varepsilon_{r2} E_2 \quad \therefore E_1 = 6 E_2$ ...(*ii*)

From (*i*) and (*ii*), we obtain $E_1 = 11.25 \times 10^6$ V/m; $E_2 = 1.875 \times 10^6$ V/m

Example 4.18. *An electric field in a medium with relative permittivity of 7 passes into a medium of relative permittivity 2. If E makes an angle of 60° with the normal to the boundary in the first dielectric, what angle does the field make with the normal in the second dielectric ?*

(Elect. Engg. Nagpur Univ.)

Solution. As seen from Art. 4.19.

$$\frac{\tan \theta_1}{\tan \theta_2} = \frac{\varepsilon_1}{\varepsilon_2} \text{ or } \frac{\tan 60°}{\tan \theta_2} = \frac{7}{2} \quad \therefore \tan \theta_2 = \sqrt{3} \times 2/7 = 4.95 \text{ or } \theta_2 = 26°20'$$

Example 4.19. *Two parallel sheets of glass having a uniform air gap between their inner surfaces are sealed around their edges (Fig. 4.23). They are immersed in oil having a relative permittivity of 6 and are mounted vertically. The glass has a relative permittivity of 3. Calculate the values of electric field strength in the glass and the air when that in the oil is 1.2 kV/m. The field enters the glass at 60° to the horizontal.*

Solution. Using the law of electric flux refraction, we get (Fig. 4.23).

$$\tan \theta_2 / \tan \theta_1 = \varepsilon_2/\varepsilon_1 = \varepsilon_0 \varepsilon_{r2} / \varepsilon_0 \varepsilon_{r1} = (\varepsilon_{r2}/\varepsilon_{r1})$$

$\therefore \quad \tan \theta_2 = (6/3) \tan 60°$

$= 2 \times 1.732 = 3.464;$

$\theta_2 = 73.9°$

Similarly

$\tan \theta_3 = (\varepsilon_{r3}/\varepsilon_{r2}) \tan \theta_2 = (1/6) \tan 73.9°$

$= 0.577; \quad \therefore \theta_3 = 30°$

As shown in Art. 4.20.

$D_{1n} = D_{2n}$ or $D_1 \cos \theta_1 = D_2 \cos \theta_2$

$\therefore \quad D_2 = D_1 \times \cos \theta_1 / \cos \theta_2$ or $\varepsilon_0 \varepsilon_{r2} E_2$

$= \varepsilon_0 \varepsilon_{r1} E_1 \times \cos \theta_1 / \cos \theta_2$

$\therefore \quad 6 E_2 = 3 \times 1.2 \times 10^3 \times \cos 60° / \cos 73.9°$

$\therefore \quad E_2 = 1082$ V/m

Now, $\varepsilon_0 \varepsilon_{r3} E_3 \cos \theta_3 = \varepsilon_0 \varepsilon_{r2} E_2 \cos \theta_2$

$\therefore \quad E_3 = E_2 (\varepsilon_{r2}/\varepsilon_{r3}) \times (\cos \theta_2 / \cos \theta_3)$

$= 1082 \, (6/1) \, (\cos 73.9°/\cos 30°) =$ **2079V/m**

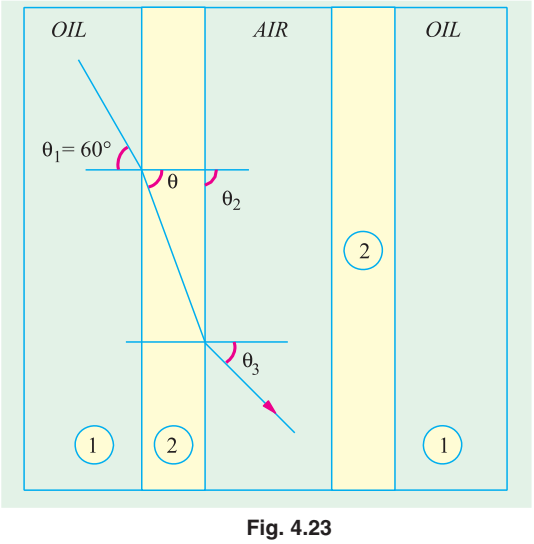


Fig. 4.23

Tutorial Problems No. 4.1

1. Two parallel metal plates of large area are spaced at a distance of 1 cm from each other in air and a p.d. of 5,000 V is maintained between them. If a sheet of glass 0.5 cm thick and having a relative permittivity of 6 is introduced between the plates, what will be the maximum electric stress and where will it occur ? **[8.57 kV/cm; in air]**

2. A capacitor, formed by two parallel plates of large area, spaced 2 cm apart in air, is connected to a 10,000 V d.c. supply. Calculate the electric stress in the air when a flat sheet of glass of thickness 1.5 cm and relative permittivity 7 is introduced between the plates. **[1.4×10^8 V/m)**

3. A capacitor is made up of two parallel circular metal discs separated by three layers of dielectric of equal thickness but having relative permittivities of 3, 4 and 5 respectively. The diameter of each disc is 25.4 cm and the distance between them is 6 cm. Calculate the potential gradient in each dielectric when a p.d. of 1,500 V is applied between the discs. **[319.2; 239.4; 191.5 kV/m]**

4. A capacitor, formed by two parallel plates of large area, spaced 2 cm apart in air, is connected to a 10,000 V d.c. supply. Calculate the electric stress in the air when a flat sheet of glass of thickness 0.5 cm and relative permittivity 5 is introduced between the plates. **[0.625×10^4 V/m]**

5. The capacitance of a capacitor formed by two parallel metal plates, each having an effective surface area of 50 cm^2 and separated by a dielectric 1 mm thick, is 0.0001 μF. The plates are charged to a p.d. of 200 V. Calculate (*a*) the charge stored (*b*) the electric flux density (*c*) the relative permittivity of the dielectric. **[(*a*) 0.02 μC (*b*) 4 μC/m^2 (*c*) 2.26]**

6. A capacitor is constructed from two parallel metallic circular plates separated by three layers of dielectric each 0.5 cm thick and having relative permittivity of 4, 6 and 8 respectively. If the metal discs are 15.25 cm in diameter, calculate the potential gradient in each dielectric when the applied voltage is 1,000 volts. ***(Elect. Engg.-I Delhi Univ.)***

7. A point electric charge of 8 μ*C* is kept at a distance of 1 metre from another point charge of −4 μC in free space. Determine the location of a point along the line joining two charges where in the electric field intensity is zero.
(Elect. Engineering, Kerala Univ.)

8. In a given R-L circuit, R = 35Ω and L = 0.1H. Find (*i*) current through the circuit (*ii*) power factor if a 50 Hz frequency, voltage V = 220∠30° is applied across the circuit.
(RGPV, Bhopal 2001)

9. Three voltage represented by $e_1 = 20 \sin \omega t$, $e_2 = 30 \sin (\omega t = 45°)$ and $e_3 = \sin (\omega t + 30°)$ are connected in series and then connected to a load of impedance $(2 + j\,3)\ \Omega$ Find the resultant current and power factor of the circuit. Draw the phasor diagram.
(B.P.T.U. Orissa 2003) (RGPV Bhopal 2001)

OBJECTIVE TESTS – 4

1. The unit of absolute permittivity of a medium is
(*a*) joule/coulomb
(*b*) newton-metre
(*c*) farad/metere
(*d*) farad/coulomb

2. If relative permittivity of mica is 5, its absolute permittivity is
(*a*) $5\,\varepsilon_0$ (*b*) $5/\varepsilon_0$
(*c*) $\varepsilon_0/5$ (*d*) 8.854×10^{-12}

3. Two similar electric charges of 1 C each are placed 1 m apart in air. Force of repulsion between them would be nearly..... newton
(*a*) 1
(*b*) 9×10^9
(*c*) $4\,\pi$
(*d*) 8.854×10^{-12}

4. Electric flux emanating from an electric charge of + *Q* coulomb is
(*a*) Q/ε_0 (*b*) Q/ε_r
(*c*) $Q/\varepsilon_0\varepsilon_r$ (*d*) *Q*

5. The unit of electric intensity is
(*a*) joule/coulomb
(*b*) newton/coulomb

(*c*) volt/metre
(*d*) both (*b*) and (*c*)

6. If D is the electric flux density, then value of electric intensity in air is
(*a*) D/ε_0 (*b*) $D/\varepsilon_0\varepsilon_r$
(*c*) dV/dt (*d*) $Q/\varepsilon A$

7. For any medium, electric flux density D is related to electric intensity E by the equation
(*a*) $D = \varepsilon_0 E$ (*b*) $D = \varepsilon_0\varepsilon_r E$
(*c*) $D = E/\varepsilon_0\varepsilon_r$ (*d*) $D = \varepsilon_0 E/\varepsilon_r$

8. Inside a conducting sphere,...remains constant
(*a*) electric flux
(*b*) electric intensity
(*c*) charge
(*d*) potential

9. The SI unit of electric intensity is
(*a*) N/m
(*b*) V/m
(*c*) N/C
(*d*) either (*b*) or (*c*)

10. According to Gauss's theorem, the surface integral of the normal component of electric flux density D over a closed surface containing charge Q is
(*a*) Q (*b*) Q/ε_0
(*c*) $\varepsilon_0 Q$ (*d*) Q^2/ε_0

11. Which of the following is zero inside a charged conducting sphere ?
(*a*) potential
(*b*) electric intensity
(*c*) both (*a*) and (*b*)
(*d*) both (*b*) and (*c*)

12. In practice, earth is chosen as a place of zero electric potential because it
(*a*) is non-conducting
(*b*) is easily available
(*c*) keeps lossing and gaining electric charge every day
(*d*) has almost constant potential.

ANSWERS

1. *c*	**2.** *a*	**3.** *b*	**4.** *d*	**5.** *d*	**6.** *a*
7. *b*	**8.** *d*	**9.** *d*	**10.** *a*	**11.** *c*	**12.** *d*

Top

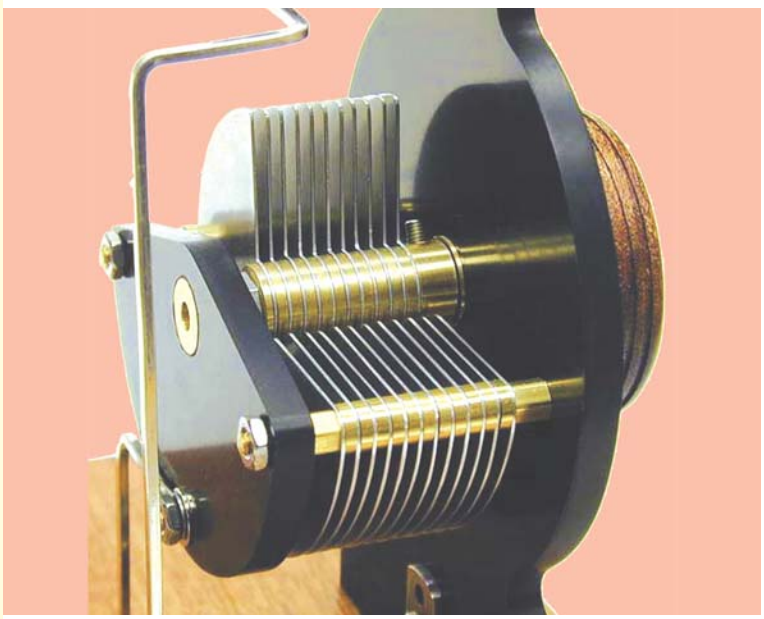
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CHAPTER 5

CAPACITANCE

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The above figure shows a variable capacitor. A capacitor stores electric charge and acts a small reservoir of energy

5.1. Capacitor

A capacitor essentially consists of two conducting surfaces separated by a layer of an insulating medium called ***dielectric***. The conducting surfaces may be in the form of either circular (or rectangular) plates or be of spherical or cylindrical shape. The purpose of a capacitor is to store electrical energy by electrostatic stress in the dielectric (the word 'condenser' is a misnomer since a capacitor does not 'condense' electricity as such, it merely stores it).



Fig. 5.1

A parallel-plate capacitor is shown in Fig. 5.1. One plate is joined to the positive end of the supply and the other to the negative end or is earthed. It is experimentally found that in the presence of an earthed plate B, plate A is capable of withholding more charge than when B is not there. When such a capacitor is put across a battery, there is a momentary flow of electrons from A to B. As negatively-charged electrons are withdrawn from A, it becomes positive and as these electrons collect on B, it becomes negative. Hence, a p.d. is established between plates A and B. The transient flow of electrons gives rise to charging current. The strength of the charging current is maximum when the two plates are uncharged but it then decreases and finally ceases when p.d. across the plates becomes slowly and slowly equal and opposite to the battery e.m.f.

5.2. Capacitance

The property of a capacitor to 'store electricity' may be called its capacitance.

As we may measure the capacity of a tank, not by the total mass or volume of water it can hold, but by the mass in kg of water required to raise its level by one metre, similarly, the capacitance of a capacitor is defined as ***"the amount of charge required to create a unit p.d. between its plates."***



A capacitor stores electricity

Suppose we give Q coulomb of charge to one of the two plate of capacitor and if a p.d. of V volts is established between the two, then its capacitance is

$$C = \frac{Q}{V} = \frac{\text{charge}}{\text{potential differnce}}$$

Hence, capacitance is the ***charge required per unit potential difference.***

By definition, the unit of capacitance is coulomb/volt which is also called ***farad*** (in honour of Michael Faraday)

$$\therefore \quad 1 \text{ farad} = 1 \text{ coulomb/volt}$$

One farad is defined as ***the capacitance of a capacitor which requires a charge of one coulomb to establish a p.d. of one volt between its plates***.

One farad is actually too large for practical purposes. Hence, much smaller units like microfarad (μF), nanofarad (nF) and micro-microfarad (μμF) or picofarad (pF) are generally employed.

$$1\ \mu F = 10^{-9}\ F;\ 1\ nF = 10^{-9}\ F\ ;\ 1\ \mu\mu F \quad \text{or} \quad pF = 10^{-12} F$$

Incidentally, capacitance is that property of a capacitor which delays and change of voltage across it.

5.3. Capacitance of an Isolated Sphere

Consider a charged sphere of radius r metres having a charge of Q coulomb placed in a medium

of relative permittivity ε_r as shown in Fig. 5.2.

It has been proved in Art 4.13 that the free surface potential V of such a sphere with respect to infinity (in practice, earth) is given by

$$V = \frac{Q}{4\pi\varepsilon_0\varepsilon_r r} \quad \therefore \quad \frac{Q}{V} = 4\pi\varepsilon_0\varepsilon_r r$$

By definition, Q/V = capacitance C

$\therefore$ $= 4\pi\varepsilon_0\varepsilon_r r$ F – in a medium

$= 4\pi\varepsilon_0 r$ F – in air

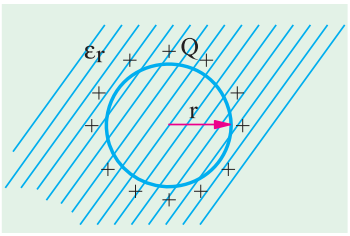


Fig. 5.2

Note : It is sometimes felt surprising that an isolated sphere can act as a capacitor because, at first sight, it appears to have one plate only. The question arises as to which is the second surface. But if we remember that the surface potential V is with reference to infinity (actually earth) then it is obvious that the other surface is earth. The capacitance $4\pi\varepsilon_0 r$ exists between the surface of the sphere and earth.

5.4. Spherical Capacitor

(*a*) When outer sphere is earthed

Consider a spherical capacitor consisting of two concentric spheres of radii 'a' and 'b' metres as shown in Fig. 5.3. Suppose, the inner sphere is given a charge of $+Q$ coulombs. It will induce a charge of $-Q$ coulombs on the inner surfaces which will go to earth. If the dielectric medium between the two spheres has a relative permittivity of ε_r, then the free surface potential of the inner sphere due to its own charge $Q/4\pi\varepsilon_0\varepsilon_r a$ volts. The potential of the inner sphere due to $-Q$ charge on the outer sphere is $-Q/4\pi\varepsilon_0\varepsilon_r b$ (remembering that potential anywhere inside a sphere is the same as that its surface).

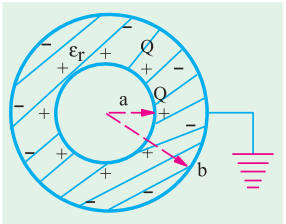


Fig. 5.3

$\therefore$ Total potential difference between two surfaces is

$$V = \frac{Q}{4\pi\varepsilon_0\varepsilon_r a} - \frac{Q}{4\pi\varepsilon_0\varepsilon_r b}$$

$$= \frac{Q}{4\pi\varepsilon_0\varepsilon_r}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{Q}{4\pi\varepsilon_0\varepsilon_r}\left(\frac{b-a}{ab}\right)$$

$$\frac{Q}{V} = \frac{4\pi\varepsilon_0\varepsilon_r ab}{b-a} \quad \therefore \quad C = 4\pi\varepsilon_0\varepsilon_r \frac{ab}{b-a} \text{ F}$$

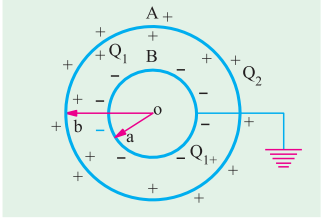


Fig. 5.4

(*b*) When inner sphere is earthed

Such a capacitor is shown in Fig. 5.4. If a charge of $+Q$ coulombs is given to the outer sphere A, it will distribute itself over both its inner and outer surfaces. Some charge Q_2 coulomb will remain on the outer surface of A because it is surrounded by earth all around. Also, some charge $+Q_1$ coulombs will shift to its inner side because there is an earthed sphere B inside A.

Obviously, $Q = Q_1 + Q_2$

The inner charge $+Q_1$ coulomb on A induces $-Q_1$ coulomb on B but the other induced charge of $+Q_1$ coulomb goes to earth.

Now, there are two capacitors connected in parallel :

(*i*) One capacitor consists of the inner surface of A and the outer surface of B. Its capacitance, as found earlier, is

$$C_1 = 4\pi\varepsilon_0\varepsilon_r \frac{ab}{b-a}$$

(*ii*) The second capacitor consists of outer surfaces of B and earth. Its capacitance is $C_2 = 4\pi\varepsilon_0 b$ –if surrounding medium is air. Total capacitance $C = C_1 + C_2$.

5.5. Parallel-plate Capacitor

(*i*) Uniform Dielectric-Medium

A parallel-plate capacitor consisting of two plates M and N each of area A m^2 separated by a thickness d metres of a medium of relative permittivity ε_r is shown in Fig. 5.5. If a charge of + Q coulomb is given to plate M, then flux passing through the medium is $\psi = Q$ coulomb. Flux density in the medium is

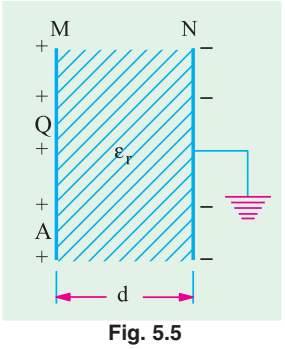


Fig. 5.5

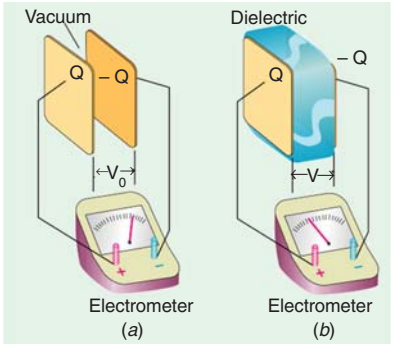
$$D = \frac{\psi}{A} = \frac{Q}{A}$$

Electric intensity $E = V/d$ and $D = \varepsilon E$

or $$\frac{Q}{A} = \varepsilon \frac{V}{d} \quad \therefore \quad \frac{Q}{V} = \frac{\varepsilon A}{d}$$

$\therefore$ $$C = \frac{\varepsilon_0 \varepsilon_r A}{d} \text{ farad} \quad \text{– in a medium} \quad \text{...(i)}$$

$$= \frac{\varepsilon_0 A}{d} \text{ farad} \quad \text{– with air as medium}$$



The figure shows how the capacitance changes when dielectric constant is changed

(*ii*) Medium Partly Air

As shown in Fig. 5.6, the medium consists partly of air and partly of parallel-sided dielectric slab of thickness t and relative permittivity ε_r. The electric flux density $D = Q/A$ is the same in both media. But electric intensities are different.

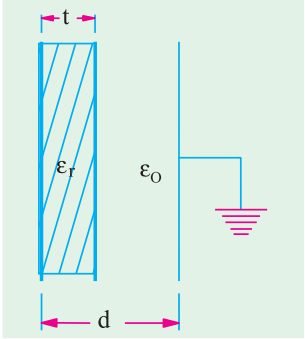


Fig. 5.6

$$E_1 = \frac{D}{\varepsilon_0 \varepsilon_r} \quad \text{... in the medium}$$

$$E_2 = \frac{D}{\varepsilon_0} \quad \text{... in air}$$

p.d. between plates, $V = E_1 \cdot t + E_2 (d - t)$

$$= \frac{D}{\varepsilon_0 \varepsilon_r} t + \frac{D}{\varepsilon_0}(d - t) = \frac{D}{\varepsilon_0}\left(\frac{t}{\varepsilon_r} + d - t\right)$$

$$= \frac{Q}{\varepsilon_0 A}[d - (t - t/\varepsilon_r)]$$

or $$\frac{Q}{V} = \frac{\varepsilon_0 A}{[d - (t - t/\varepsilon_r)]} \quad \text{or} \quad C = \frac{\varepsilon_0 A}{[d - (t - t/\varepsilon_r)]} \quad \text{...(ii)}$$

If the medium were totally air, then capacitance would have been

$$C = \varepsilon_0 A/d$$

From **(*ii*)** and **(*iii*)**, it is obvious that when a dielectric slab of thickness t and relative permittivity ε_r is introduced between the plates of an air capacitor, then its capacitance increases because as seen from **(*ii*)**, the denominator decreases. The distance between the plates is effectively reduces by $(t - t/\varepsilon_r)$. To bring the capacitance back to its original value, the capacitor plates will have to be further separated by that much distance in air. Hence, the new separation between the two plates would be $= [d + (t - t/\varepsilon_r)]$

The expression given in (*i*) above can be written as $C = \dfrac{\varepsilon_0 A}{d/\varepsilon_r}$

If the space between the plates is filled with slabs of different thickness and relative permittivities, then the above expression can be generalized into $C = \dfrac{\varepsilon_0 A}{\Sigma d/\varepsilon_r}$

The capacitance of the capacitor shown in Fig. 5.7 can be written as

$$C = \frac{\varepsilon_0 A}{\left(\dfrac{d_1}{\varepsilon_{r1}} + \dfrac{d_2}{\varepsilon_{r2}} + \dfrac{d_3}{\varepsilon_{r3}}\right)}$$

(*iii*) Composite Medium

The above expression may be derived independently as given under :

If V is the total potential difference across the capacitor plates and V_1, V_2, V_3, the potential differences across the three dielectric slabs, then

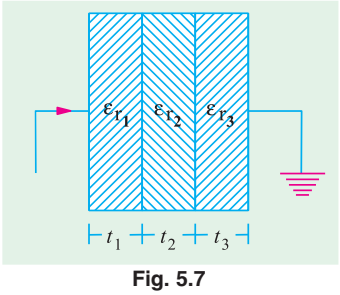


Fig. 5.7

$$V = V_1 + V_2 + V_3 = E_1 t_1 + E_2 t_2 + E_3 t_3$$

$$= \frac{D}{\varepsilon_0 \varepsilon_{r1}} . t_1 + \frac{D}{\varepsilon_0 \varepsilon_{r2}} . t_2 + \frac{D}{\varepsilon_0 \varepsilon_{r3}} . t_3$$

$$= \frac{D}{\varepsilon_0}\left(\frac{t_1}{\varepsilon_{r1}} + \frac{t_2}{\varepsilon_{r2}} + \frac{t_3}{\varepsilon_{r3}}\right) = \frac{Q}{\varepsilon_0 A}\left(\frac{t_1}{\varepsilon_{r1}} + \frac{t_2}{\varepsilon_{r2}} + \frac{t_3}{\varepsilon_{r3}}\right)$$

$$\therefore \quad C = \frac{Q}{V} = \frac{\varepsilon_0 A}{\left(\dfrac{t_1}{\varepsilon_{r1}} + \dfrac{t_2}{\varepsilon_{r2}} + \dfrac{t_3}{\varepsilon_{r3}}\right)}$$

5.6. Special Cases of Parallel-plate Capacitor

Consider the cases illustrated in Fig. 5.8.

(i) As shown in Fig. 5.8 (*a*), the dielectric is of thickness d but occupies only a part of the area. This arrangement is equal to two capacitors in parallel. Their capacitances are

$$C_1 = \frac{\varepsilon_0 A_1}{d} \quad \text{and} \quad C_2 = \frac{\varepsilon_0 \varepsilon_r A_2}{d}$$

Total capacitance of the parallel-plate capacitor is

$$C = C_1 + C_2 = \frac{\varepsilon_0 A_1}{d} + \frac{\varepsilon_0 \varepsilon_r A_2}{d}$$

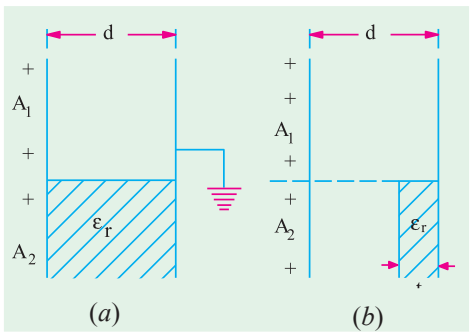


Fig. 5.8

(ii) The arrangement shown in Fig. 5.8 (*b*) consists of two capacitors connected in parallel.

(a) one capacitor having plate area A_1 and air as dielectric. Its capacitance is $C_1 = \dfrac{\varepsilon_0 A_1}{d}$

(b) the other capacitor has dielectric partly air and partly some other medium. Its capacitance is [Art 5.5 (*ii*)]. $C_2 = \dfrac{\varepsilon_0 A_2}{[d - (t - t/\varepsilon_r)}$. Total capacitance is $C = C_1 + C_2$

5.7. Multiple and Variable Capacitors

Multiple capacitors are shown in Fig. 5.9 and Fig. 5.10.

The arrangement of Fig. 5.9. is equivalent to two capacitors joined in parallel. Hence, its capacitance is double that of a single capacitor. Similarly, the arrangement of Fig. 5.10 has four times the capacitance of single capacitor.

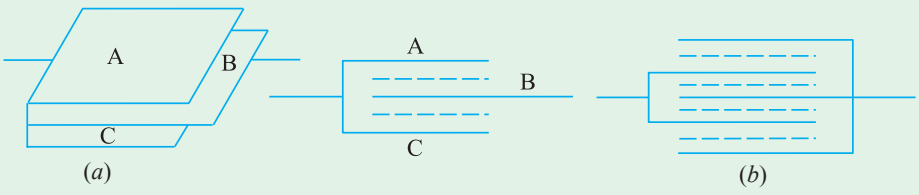


Fig. 5.9 Fig. 5.10

If one set of plates is fixed and the other is capable of rotation, then capacitance of such a multiplate capacitor can be varied. Such variable-capacitance air capacitors are widely used in radio work (Fig. 5.11). The set of fixed plates F is insulated from the other set R which can be rotated by turning the knob K. The common area between the two sets is varied by rotating K, hence the capacitance between the two is altered. Minimum capacitance is obtained when R is completely rotated out of F and maximum when R is completely rotated in *i.e.* when the two sets of plates completely overlap each other.

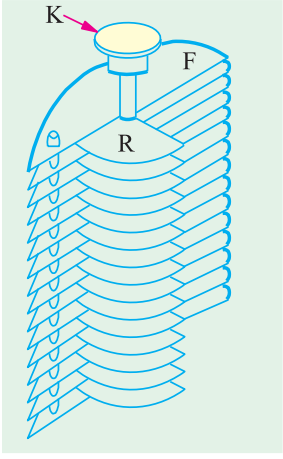


Fig. 5.11

The capacitance of such a capacitor is

$$= \frac{(n-1)\,.\,\varepsilon_0 \varepsilon_r\, A}{d}$$

where n is the number of plates which means that $(n-1)$ is the number of capacitors.

Example 5.1. *The voltage applied across a capacitor having a capacitance of 10 μ F is varied thus :*

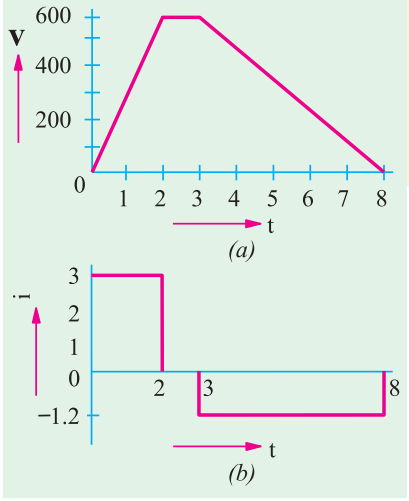


Fig. 5.12

The p.d. is increased uniformly from 0 to 600 V in seconds. It is then maintained constant at 600 V for 1 second and subsequently decreased uniformly to zero in five seconds. Plot a graph showing the variation of current during these 8 seconds. Calculate ***(a)*** *the charge* ***(b)*** *the energy stored in the capacitor when the terminal voltage is 600.*

(Principles of Elect. Engg.-I, Jadavpur Univ.)

Solution. The variation of voltage across the capacitor is as shown in Fig. 5.12 (*a*).

The charging current is given by

$$i = \frac{dq}{dt} = \frac{d}{dt}(Cv) = C\,.\,\frac{dv}{dt}$$

Charging current during the first stage

$$= 10 \times 10^{-6} \times (600/2) = 3 \times 10^{-3}\ \text{A} = 3\ \text{mA}$$

Charging current during the second stage is zero because $dv/dt = 0$ as the voltage remains constant.

Charging current through the third stage

$$= 10 \times 10^{-6} \times \left(\frac{0-600}{5}\right) = -1.2 \times 10^{-3}\ \text{A} = -1.2\ \text{mA}$$

The waveform of the charging current or capacitor current is shown in Fig. 5.12 (*b*).

(a) Charge when a steady voltage of 600 V is applied is $= 600 \times 10 \times 10^{-6} = \mathbf{6 \times 10^{-3}\ C}$

(b) Energy stored $= \frac{1}{2} C V^2 = \frac{1}{2} \times 10^{-5} \times 600^2 = \mathbf{1.8\ J}$

Example 5.2. *A voltage of V is applied to the inner sphere of a spherical capacitor, whereas the outer sphere is earthed. The inner sphere has a radius of a and the outer one of b. If b is fixed and a may be varied, prove that the maximum stress in the dielectric cannot be reduced below a value of 4 V/b.*

Solution. As seen from Art. 5.4,

$$V = \frac{Q}{4\pi\varepsilon_0\varepsilon_r}\left(\frac{1}{a}-\frac{1}{b}\right) \quad \text{...(i)}$$

As per Art. 4.15, the value of electric intensity at any radius x between the two spheres is given by $E = \dfrac{Q}{4\pi\varepsilon_0\varepsilon_r x^2}$ or $Q = 4\pi\varepsilon_0\varepsilon_r x^2 E$

Substituting this value in (i) above, we get

$$V = \frac{4\pi\varepsilon_0\varepsilon_r x^2 E}{4\pi\varepsilon_0\varepsilon_r}\left(\frac{1}{a}-\frac{1}{b}\right) \quad \text{or} \quad E = \frac{V}{(1/a-1/b)\,x^2}$$

As per Art. 5.9, the maximum value of E occurs as the surface of inner sphere *i.e.* when $x = a$

For E to be maximum or minimum, $dE/da = 0$.

$$\therefore \quad \frac{d}{da}\left(\frac{1}{a}-\frac{1}{b}\right)a^2 = 0 \quad \text{or} \quad \frac{d}{da}(a - a^2/b) = 0$$

$$\text{or} \quad 1 - 2a/b = 0 \quad \text{or} \quad a = b/2$$

$$\text{Now,} \quad E = \frac{V}{(1/a-1/b)\,x^2} \quad \therefore E_{max} = \frac{V}{(1/a-1/b)\,a^2} = \frac{V}{(a-a^2/b)}$$

$$\text{Since,} \quad a = b/2 \quad \therefore E_{max} = \frac{V}{(b/2-b^2/4b)} = \frac{4bV}{2b^2-b^2} = \frac{4bV}{b^2} = \mathbf{\frac{4V}{b}}$$

Example 5.3. *A capacitor consists of two similar square aluminium plates, each 10 cm × 10 cm mounted parallel and opposite to each other. What is their capacitance in μμ F when distance between them is 1 cm and the dielectric is air ? If the capacitor is given a charge of 500 μμ C, what will be the difference of potential between plates ? How will this be affected if the space between the plates is filled with wax which has a relative permittivity of 4 ?*

Solution. $C = \varepsilon_0 A/d$ farad

Here $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m ; $A = 10 \times 10 = 100$ cm^2 $= 10^{-2}$ m^2

$d = 1$ cm $= 10^{-2}$ m

$$\therefore \quad C = \frac{8.854\times10^{-12}\times10^{-2}}{10^{-2}} = 8.854 \times 10^{-12} \text{ F} = \mathbf{8.854\ \mu\mu F}$$

$$\text{Now} \quad C = \frac{Q}{V} \quad \therefore V = \frac{Q}{C} \quad \text{or} \quad V = \frac{500\times10^{-12}}{8.854\times10^{-12}}\frac{\text{C}}{\text{F}} = \mathbf{56.5\ volts.}$$

When wax is introduced, their capacitance is increased four times because

$$C = \varepsilon_0\varepsilon_r A/d \quad \text{F} = 4 \times 8.854 = 35.4\ \mu\mu\text{ F}$$

The p.d. will obviously decrease to one fourth value because charge remains constant.

$$\therefore \quad V = 56.5/4 = \mathbf{14.1\ volts.}$$

Example 5.4. *The capacitance of a capacitor formed by two parallel metal plates each 200 cm^2 in area separated by a dielectric 4 mm thick is 0.0004 microfarads. A p.d. of 20,000 V is applied. Calculate* **(a)** *the total charge on the plates* **(b)** *the potential gradient in V/m* **(c)** *relative permittivity of the dielectric* **(d)** *the electric flux density.* **(Elect. Engg. I Osmaina Univ.)**

Solution. $C = 4 \times 10^{-4}$ μF ; $V = 2 \times 10^4$ V

(a) $\therefore$ Total charge $Q = CV = 4 \times 10^{-4} \times 2 \times 10^4$ μC $= 8$ μC $= \mathbf{8 \times 10^{-6}\ C}$

(b) Potential gradient $= \dfrac{dV}{dx} = \dfrac{2\times10^4}{4\times10^{-3}} = \mathbf{5 \times 10^6\ V/m}$

(c) $D = Q/A = 8 \times 10^{-6}/200 \times 10^{-4} = \mathbf{4 \times 10^{-4}\ C/m^2}$

(d) $E = 5 \times 10^6$ V/m

Since $D = \varepsilon_0\varepsilon_r$ E $\quad \therefore \quad \varepsilon_r = \dfrac{D}{\varepsilon_0 \times E} = \dfrac{4\times10^{-4}}{8.854\times10^{-12}\times5\times10^6} = \mathbf{9}$

Example 5.5. *A parallel plate capacitor has 3 dielectrics with relative permittivities of 5.5, 2.2 and 1.5 respectively. The area of each plate is 100 cm^2 and thickness of each dielectric 1 mm. Calculate the stored charge in the capacitor when a potential difference of 5,000 V is applied across the composite capacitor so formed. Calculate the potential gradient developed in each dielectric of the capacitor.* **(Elect. Engg. A.M.Ae.S.I.)**

Solution. As seen from Art. 5.5,

$$C = \frac{\varepsilon_0 A}{\left(\frac{d_1}{\varepsilon_{r1}} + \frac{d_2}{\varepsilon_{r2}} + \frac{d_3}{\varepsilon_{r3}}\right)} = \frac{8.854\times10^{-12}\times(100\times10^{-4})}{\left(\frac{10^{-3}}{5.5} + \frac{10^{-3}}{2.2} + \frac{10^{-3}}{1.5}\right)} = \frac{8.854\times10^{-14}}{10^{-3}\times0.303} = 292 \text{ pF}$$

$Q = CV = 292 \times 10^{-12} \times 5000 = 146 \times 10^{-8}$ coulomb

$D = Q/A = 146 \times 10^{-8}/(100 \times 10^{-4}) = 146 \times 10^{-6}$ C/m^2

$g_1 = E_1 = D/\varepsilon_0 \varepsilon_{r1} = 146 \times 10^{-6}/8.854 \times 10^{-12} \times 5.5 = \mathbf{3 \times 10^6}$ **V/m**

$g_2 = E_2 = D/\varepsilon_0 \varepsilon_{r2} = 7.5 \times 10^6$ V/m; $g_3 = D/\varepsilon_0 \varepsilon_{r3} = \mathbf{11 \times 10^6}$ **V/m**

Example 5.6. *An air capacitor has two parallel plates 10 cm^2 in area and 0.5 cm apart. When a dielectric slab of area 10 cm^2 and thickness 0.4 cm was inserted between the plates, one of the plates has to be moved by 0.4 cm to restore the capacitance. What is the dielectric constant of the slab ?* **(Elect. Technology, Hyderabad Univ. 1992)**

Solution. The capacitance in the first case is

$$C_a = \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 \times 10 \times 10^{-4}}{0.5\times10^{-2}} = \frac{\varepsilon_0}{5}$$

The capacitor, as it becomes in the second case, is shown in Fig. 5.13. The capacitance is

$$C_m = \frac{\varepsilon_0 A}{\Sigma d/\varepsilon_r} = \frac{\varepsilon_0 \times 10^{-3}}{\left(\frac{0.5\times10^{-3}}{\varepsilon_r}\right)} = \frac{\varepsilon_0}{\left(\frac{5}{\varepsilon_r} + 4\right)}$$

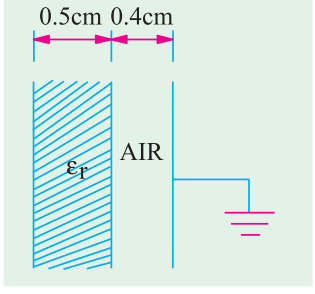


Fig. 5.13

Since, $C_a = C_m \quad \therefore \quad \frac{\varepsilon_0}{5} = \frac{\varepsilon_0}{(5/\varepsilon_r + 4)} \quad \therefore \quad \varepsilon_r = 5$

Note. We may use the relation derived in Art. 5.5 (*ii*)

Separation $= (t - t/\varepsilon_1) \quad \therefore \quad 0.4 = (0.5 - 0.5/\varepsilon_r)$ or $\varepsilon_r = \mathbf{5}$

Example 5.7. *A parallel plate capacitor of area, A, and plate separation, d, has a voltage, V_0, applied by a battery. The battery is then disconnected and a dielectric slab of permittivity ε_1 and thickness, d_1, ($d_1 < d$) is inserted.* ***(a)*** *Find the new voltage V_1 across the capacitor,* ***(b)*** *Find the capacitance C_0 before and its value C_1 after the slab is introduced.* ***(c)*** *Find the ratio V_1/V_0 and the ratio C_1/C_0 when $d_1 = d/2$ and $\varepsilon_1 = 4\,\varepsilon_0$.*

(Electromagnetic Fields and Waves AMIETE (New Scheme) June 1990)

Solution. (*b*) $C_0 = \frac{\varepsilon_0 A}{d}; \quad C_1 = \frac{A}{\left(\frac{(d-d_1)}{\varepsilon_0} + \frac{d_1}{\varepsilon_1}\right)}$

Since $d_1 = d/2$ and $\varepsilon_1 = 4\,\varepsilon_0 \therefore \quad C_1 = \frac{A}{\left(\frac{d}{2\varepsilon_0} + \frac{d}{2\times4\,\varepsilon_0}\right)} = \frac{8\,\varepsilon_0 A}{5d}$

(a) Since the capacitor charge remains the same

$$Q = C_0 V_0 = C_1 V_1 \quad \therefore \quad V_1 = V_0 \frac{C_0}{C_1} = V_0 \times \frac{\varepsilon_0 A}{d} \times \frac{5d}{8\,\varepsilon_0 A} = \frac{5V_0}{8}$$

(c) As seen from above, $V_1 = V_0 5/8$; $C_1 C_0 = \frac{8\varepsilon_0 A}{5d} \times \frac{d}{\varepsilon_0 A} = \frac{5}{8}$

Tutorial Problems No. 5.1

1. Two parallel plate capacitors have plates of an equal area, dielectrics of relative permittivities ε_{r1} and ε_{r2} and plate spacing of d_1 and d_2. Find the ratio of their capacitances if $\varepsilon_{r1}/\varepsilon_{r2} = 2$ and $d_1/d_2 = 0.25$.

[$C_1/C_2 = 8$]

2. A capacitor is made of two plates with an area of 11 cm^2 which are separated by a mica sheet 2 mm thick. If for mica $\varepsilon_r = 6$, find its capacitance. If, now, one plate of the capacitor is moved further to give an air gap 0.5 mm wide between the plates and mica, find the change in capacitance.

[29.19 pF, 11.6 pF]

3. A parallel-plate capacitor is made of two plane circular plates separated by d cm of air. When a parallel-faced plane sheet of glass 2 mm thick is placed between the plates, the capacitance of the system is increased by 50% of its initial value. What is the distance between the plates if the dielectric constant of the glass is 6 ?

[0.5×10^{-3} m]

4. A p.d. of 10 kV is applied to the terminals of a capacitor consisting of two circular plates, each having an area of 100 cm^2 separated by a dielectric 1 mm thick. If the capacitance is 3×10^{-4} μ F, calculate

(*a*) the total electric flux in coulomb

(*b*) the electric flux density and

(*c*) the relative permittivity of the dielectric.

[(*a*) 3×10^{-6}C (*b*) 3×10^{-4} μ C/m^2 (*c*) 3.39]

5. Two slabs of material of dielectric strength 4 and 6 and of thickness 2 mm and 5 mm respectively are inserted between the plates of a parallel-plate capacitor. Find by how much the distance between the plates should be changed so as to restore the potential of the capacitor to its original value.

[5.67 mm]

6. The oil dielectric to be used in a parallel-plate capacitor has a relative permittivity of 2.3 and the maximum working potential gradient in the oil is not to exceed 10^6 V/m. Calculate the approximate plate area required for a capacitance of 0.0003 μ F, the maximum working voltage being 10,000 V.

[147×10^{-3} m^2]

7. A capacitor consist of two metal plates, each 10 cm square placed parallel and 3 mm apart. The space between the plates is occupied by a plate of insulating material 3 mm thick. The capacitor is charged to 300 V.

(*a*) the metal plates are isolated from the 300 V supply and the insulating plate is removed. What is expected to happen to the voltage between the plates ?

(*b*) if the metal plates are moved to a distance of 6 mm apart, what is the further effect on the voltage between them. Assume throughout that the insulation is perfect.

[300 ε_r ; 600 ε_r ; where ε_r is the relative permittivity of the insulating material]

8. A parallel-plate capacitor has an effecting plate area of 100 cm^2 (each plate) separated by a dielectric 0.5 mm thick. Its capacitance is 442 μμ F and it is raised to a potential differences of 10 kV. Calculate from first principles

(*a*) potential gradient in the dielectric (*b*) electric flux density in the dielectric

(*c*) the relative permittivity of the dielectric material.

[(*a*) 20 kV/mm (*b*) 442 μC/m^2 (*c*) 2.5]

9. A parallel-plate capacitor with fixed dimensions has air as dielectric. It is connected to supply of p.d. V volts and then isolated. The air is then replaced by a dielectric medium of relative permittivity 6. Calculate the change in magnitude of each of the following quantities.

(*a*) the capacitance (*b*) the charge (*c*) the p.d. between the plates

(*d*) the displacement in the dielectric (*e*) the potential gradient in the dielectric.

[(*a*) 6 : 1 increase (*b*) no change (*c*) 6 : 1 decrease (*d*) no change (*e*) 6 : 1 decrease]

5.8. Cylindrical Capacitor

A single-core cable or cylindrical capacitor consisting two co-axial cylinders of radii a and b metres, is shown in Fig. 5.14. Let the charge per metre length of the cable on the outer surface of the inner cylinder be $+Q$ coulomb and on the inner surface of the outer cylinder be $-Q$ coulomb. For all practical purposes, the charge $+Q$ coulomb/metre on the surface of the inner cylinder can be supposed to be located along its axis. Let ε_r be the relative permittivity of the medium between the two cylinders. The outer cylinder is earthed.

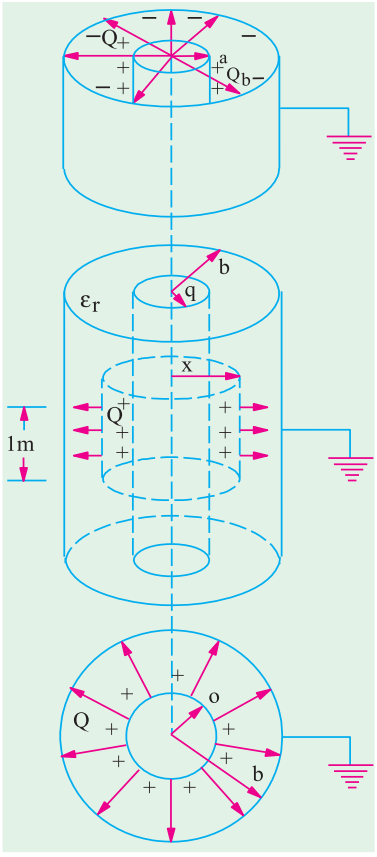


Fig. 5.14

Now, let us find the value of electric intensity at any point distant x metres from the axis of the inner cylinder. As shown in Fig. 5.15, consider an imaginary co-axial cylinder of radius x metres and length one metre between the two given cylinders. The electric field between the two cylinders is radial as shown. Total flux coming out radially from the curved surface of this imaginary cylinder is Q coulomb. Area of the curved surface $= 2\pi x \times 1 = 2\pi x$ m^2.

Hence, the value of electric flux density on the surface of the imaginary cylinder is

$$D = \frac{\text{flux in coulomb}}{\text{area in metre}^2} = \frac{\Psi}{A} = \frac{Q}{A} \text{ C/m}^2 \quad \therefore D = \frac{Q}{2\pi x} \text{ C/m}^2$$

The value of electric intensity is

$$E = \frac{D}{\varepsilon_0 \varepsilon_r} \quad \text{or} \quad E = \frac{Q}{2\pi\varepsilon_0\varepsilon_r x} \text{ V/m}$$

Now, $dV = -E\,dx$

or $$V = \int_b^a -E.dx = \int_b^a -\frac{Q\,dx}{2\pi\varepsilon_0\varepsilon_r x}$$

$$= \frac{-Q}{2\pi\varepsilon_0\varepsilon_r}\int_b^a \frac{dx}{x} = \frac{-Q}{2\pi\varepsilon_0\varepsilon_r}\left|\log x\right|_b^a$$

$$= \frac{-Q}{2\pi\varepsilon_0\varepsilon_r}(\log_e a - \log_e b) = \frac{-Q}{2\pi\varepsilon_0\varepsilon_r}\log_e\left(\frac{a}{b}\right) = \frac{Q}{2\pi\varepsilon_0\varepsilon_r}\log_e\left(\frac{a}{b}\right)$$

$$\frac{Q}{V} = \frac{2\pi\varepsilon_0\varepsilon_r}{\log_e\left(\frac{b}{a}\right)} \quad \therefore C = \frac{2\pi\varepsilon_0\varepsilon_r}{2.3\log_{10}\left(\frac{b}{a}\right)} \text{ F/m} \quad \left(\log_e\left(\frac{b}{a}\right) = 2.3\log_{10}\left(\frac{b}{a}\right)\right)$$

The capacitance of l metre length of this cable is $C = \dfrac{2\pi\varepsilon_0\varepsilon_r l}{2.3\log_{10}\left(\frac{b}{a}\right)}$ F

In case the capacitor has compound dielectric, the relation becomes

$$C = \frac{2\pi\varepsilon_0 l}{\Sigma \log_e\left(\frac{b}{a}\right)/\varepsilon_r} \text{ F}$$

The capacitance of 1 km length of the cable in μ F can be found by putting $l = 1$ km in the above expression.

$$C = \frac{2\pi\times 8.854\times 10^{-12}\times\varepsilon_r\times 1000}{2.3\log_{10}\left(\frac{b}{a}\right)} \text{ F/km} = \frac{0.024\,\varepsilon_r}{\log_{10}\left(\frac{b}{a}\right)} \text{ μ F/km}$$

5.9. Potential Gradient in a Cylindrical Capacitor

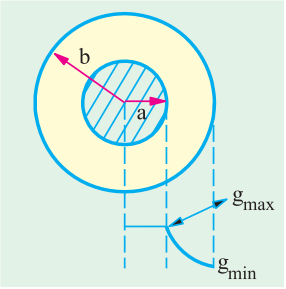


Fig. 5.15

It is seen from Art. 5.8 that in a cable capacitor

$$E = \frac{Q}{2\pi\varepsilon_0\varepsilon_r x} \text{ V/m}$$

where x is the distance from cylinder axis to the point under consideration.

Now $E = g \quad \therefore g = \dfrac{Q}{2\pi\varepsilon_0\varepsilon_r x}$ V/m ...(*i*)

From Art. 5.8, we find that $V = \dfrac{Q}{2\pi\varepsilon_0\varepsilon_r}\log_e\left(\dfrac{b}{a}\right)$ or $Q = \dfrac{2\pi\varepsilon_0\varepsilon_r V}{\log_e\left(\dfrac{b}{a}\right)}$

Substituting this value of Q in (*i*) above, we get

$$g = \frac{2\pi\varepsilon_0\varepsilon_r V}{\log_e\left(\dfrac{b}{a}\right)\times 2\pi\varepsilon_0\varepsilon_r x} \text{ V/m or } g = \frac{V}{x\log_e\left(\dfrac{b}{a}\right)} \text{ V/m or } g = \frac{V}{2.3\,x\log_{10}\left(\dfrac{b}{a}\right)} \text{ volt/metre}$$

Obviously, potential gradient varies inversely as x.

Minimum value of $x = a$, hence maximum value of potential gradient is

$$g_{max} = \frac{V}{2.3\,a\log_{10}\left(\dfrac{b}{a}\right)} \text{ V/m} \quad \text{...(ii)}$$

Similarly, $$g_{max} = \frac{V}{2.3\,b\log_{10}\left(\dfrac{b}{a}\right)} \text{ V/m}$$

Note. The above relation may be used to obtain most economical dimension while designing a cable. As seen, greater the value of permissible maximum stress E_{max}, smaller the cable may be for given value of V. However, E_{max} is dependent on the dielectric strength of the insulating material used.

If V and E_{max} are fixed, then Eq. (*ii*) above may be written as

$$E_{max} = \frac{V}{a\log h_e\left(\dfrac{b}{a}\right)} \text{ or } a\log h\left(\frac{b}{a}\right) = \frac{V}{E_{max}} \quad \therefore \frac{b}{a} = e^{k/a} \text{ or } b = a\,e^{k/a}$$

For most economical cable $db/da = 0$

$$\therefore \quad \frac{db}{da} = 0 = e^{k/a} + a\,(-k/a^2)e^{k/a} \quad \text{or} \quad a = k = V/E_{max} \text{ and } b = ae = 2.718\,a$$

Example 5.8. *A cable is 300 km long and has a conductor of 0.5 cm in diameter with an insulation covering of 0.4 cm thickness. Calculate the capacitance of the cable if relative permittivity of insulation is 4.5.* **(Elect. Engg. A.M.Ae. S.I.)**

Solution. Capacitance of a cable is $C = \dfrac{0.024\,\varepsilon_r}{\log_{10}\left(\dfrac{b}{a}\right)}$ μF/km

Here, $a = 0.5/2 = 0.25$ cm ; $b = 0.25 + 0.4 = 0.65$ cm ; $b/a = 0.65/0.25 = 2.6$; $\log_{10}^{2.6} = 0.415$

$$\therefore \quad C = \frac{0.024\times 4.5}{0.415} = 0.26$$

Total capacitance for 300 km is $= 300 \times 0.26 = 78$ μF.

Example 5.9. *In a concentric cable capacitor, the diameters of the inner and outer cylinders are 3 and 10 mm respectively. If ε_r for insulation is 3, find its capacitance per metre.*

A p.d. of 600 volts is applied between the two conductors. Calculate the values of the electric force and electric flux density : ***(a)*** *at the surface of inner conductor* ***(b)*** *at the inner surface of outer conductor.*

Solution. $a = 1.5$ mm ; $b = 5$ mm ; $\therefore b/a = 5/1.5 = 10/3$; $\log_{10}\left(\frac{10}{3}\right) = 0.523$

$$C = \frac{2\pi\varepsilon_0\varepsilon_r l}{2.3\log_{10}\left(\frac{b}{a}\right)} = \frac{2\pi\times 8.854\times 10^{-12}\times 3\times 1}{2.3\times 0.523} = 138.8\times 10^{-12}\text{ F} = \mathbf{138.8\ pF}$$

(a) $D = Q/2\pi a$

Now $Q = CV = 138.8\times 10^{-12}\times 600 = 8.33\times 10^{-9}$ C

$D = 8.33\times 10^{-8}/2\pi\times 1.5\times 10^{-3} = \mathbf{8.835\ \mu C/m^2}$

$E = D/\varepsilon_0\varepsilon_r = \mathbf{332.6\ V/m}$

(b) $D = \frac{8.33\times 10^{-8}}{2\pi\times 5\times 10^{-3}}$ C/m^2 $= \mathbf{2.65\ \mu C/m^2}$; $E = D/\varepsilon_0\varepsilon_r = \mathbf{99.82\ V/m.}$

Example 5.10. *The radius of the copper core of a single-core rubber-insulated cable is 2.25 mm. Calculate the radius of the lead sheath which covers the rubber insulation and the cable capacitance per metre. A voltage of 10 kV may be applied between the core and the lead sheath with a safety factor of 3. The rubber insulation has a relative permittivity of 4 and breakdown field strength of 18 × 10^6 V/m.*

Solution. As shown in Art. 5.9, $g_{max} = \frac{V}{2.3\,a\log_{10}\left(\frac{b}{a}\right)}$

Now, $g_{max} = E_{max} = 18\times 10^6$ V/m ; V = breakdown voltage x

Safety factor $= 10^4\times 3 = 30{,}000$ V

$$\therefore 8\times 10^6 = \frac{30{,}000}{2.3\times 2.25\times 10^{-3}\times\log_{10}\left(\frac{b}{a}\right)} \quad \therefore \frac{b}{a} = 2.1 \text{ or } b = 2.1\times 2.25 = 4.72\text{ mm}$$

$$C = \frac{2\pi\varepsilon_0\varepsilon_r l}{2.3\log_{10}\left(\frac{b}{a}\right)} = \frac{2\pi\times 8.854\times 10^{-12}\times 4\times 1}{2.3\log_{10}(2.1)} = \mathbf{3\times 10^{-9}\ F}$$

5.10. Capacitance Between Two Parallel Wires

This case is of practical importance in overhead transmission lines. The simplest system is 2-wire system (either *d.c.* or *a.c.*). In the case of *a.c.* system, if the transmission line is long and voltage high, the charging current drawn by the line due to the capacitance between conductors is appreciable and affects its performance considerably.

With reference to Fig. 5.16, let

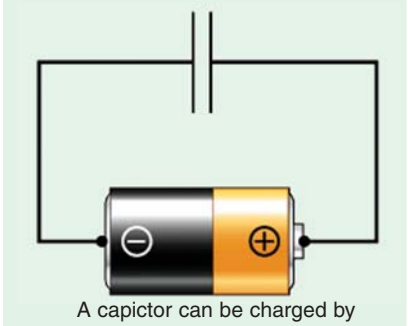
d = distance between centres of the wires A and B

r = radius of each wire ($\leq d$)

Q = charge in coulomb/metre of each wire*

Now, let us consider electric intensity at any point P between conductors A and B.

Electric intensity at P* due to charge + Q coulomb/metre on A is



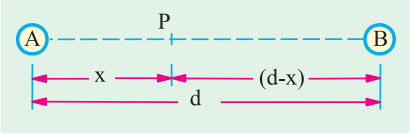


Fig. 5.16

* If charge on A is + Q, then on B will be $-Q$.

$$= \frac{Q}{2\pi\varepsilon_0\varepsilon_r x} \text{ V/m} \quad \text{... towards } B.$$

Electric intesity at P due to charge $-Q$ coulomb/metre on B is

$$= \frac{Q}{2\pi\varepsilon_0\varepsilon_r (d-x)} \text{ V/m} \quad \text{... towards } B.$$

Total electric intensity at P, $E = \frac{Q}{2\pi\varepsilon_0\varepsilon_r}\left(\frac{1}{x}+\frac{1}{d-x}\right)$

Hence, potential difference between the two wires is

$$V = \int_r^{d-r} E.dx = \frac{Q}{2\pi\varepsilon_0\varepsilon_r}\int_r^{d-r}\left(\frac{1}{x}+\frac{1}{d-x}\right)dx$$

$$V = \frac{Q}{2\pi\varepsilon_0\varepsilon_r}\left|\log_e x - \log_e (d-x)\right|_r^{d-r} = \frac{Q}{\pi\varepsilon_0\varepsilon_r}\log_e\frac{d-r}{r}$$

Now $C = Q/V \quad \therefore C = \frac{\pi\varepsilon_0\varepsilon_r}{\log_e\frac{(d-r)}{r}} = \frac{\pi\varepsilon_0\varepsilon_r}{2.3\log_{10}\frac{(d-r)}{r}} = \frac{\pi\varepsilon_0\varepsilon_r}{2.3\log_{10}\left(\frac{d}{r}\right)}$ F/m (approx.)

The capacitance for a length of l metres $C = \frac{\pi\varepsilon_0\varepsilon_r}{2.3\log_{10}\left(\frac{d}{r}\right)}$ F

The capacitance per kilometre is

$$C = \frac{\pi\times 8.854\times 10^{-12}\times\varepsilon_r\times 100\times 10^6}{2.3\log_{10}\left(\frac{d}{r}\right)} = \frac{0.0121\,\varepsilon_r}{\log_{10}\left(\frac{d}{r}\right)} = \mu \text{ F/km}$$

Example 5.11. *The conductors of a two-wire transmission line (4 km long) are spaced 45 cm between centre. If each conductor has a diameter of 1.5 cm, calculate the capacitance of the line.*

Solution. Formula used $C = \frac{\pi\varepsilon_0\varepsilon_r}{2.3\log_{10}\left(\frac{d}{r}\right)}$ F

Here $l = 4000$ metres ; $r = 1.5/2$ cm ; $d = 45$ cm ; $\varepsilon_r = 1$–for air $\quad \therefore \frac{d}{r} = \frac{45\times 2}{1.5} = 60$

$$C = \frac{\pi\times 8.854\times 10^{-12}\times 4000}{2.3\log_{10} 60} = \mathbf{0.0272\times 10^{-6}\ F}$$

[or $\quad C = 4\ \frac{0.0121}{\log_{10} 60}\ \mathbf{0.0272\mu F}$]

5.11. Capacitors in Series

With reference of Fig. 5.17, let

C_1, C_2, C_3 = Capacitances of three capacitors

V_1, V_2, V_3 = p.ds. across three capacitors.

V = applied voltage across combination

C = combined or equivalent or joining capacitance.

In series combination, charge on all capacitors is the same but p.d. across each is different.

$$\therefore \quad V = V_1 + V_2 + V_3$$

$$\text{or} \quad \frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$\text{or} \quad \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

For a changing applied voltage,

$$\frac{dV}{dt} = \frac{dV_1}{dt} + \frac{dV_2}{dt} + \frac{dV_3}{dt}$$

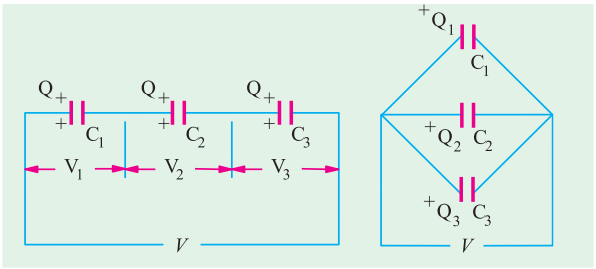


Fig. 5.17

Fig. 5.18

We can also find values of V_1, V_2 and V_3 in terms of V. Now, $Q = C_1 V_1 = C_2 V_2 = C_3 V_3 = CV$

where $$C = \frac{C_1 C_2 C_3}{C_1C_2 + C_2C_3 + C_3C_1} = \frac{C_1C_2C_3}{\Sigma C_1C_2}$$

$$\therefore \quad C_1V_1 = CV \text{ or } V_1 = V\frac{C}{C_1} = V.\frac{C_2C_3}{\Sigma C_1C_2}$$

Similarly, $$V_2 = V \cdot \frac{C_1C_3}{\Sigma C_1C_2} \text{ and } V_3 = V \cdot \frac{C_1C_2}{\Sigma C_1C_2}$$

5.12. Capacitors in Parallel

In this case, p.d. across each is the same but charge on each is different (Fig. 5.18).

$\therefore$ $Q = Q_1 + Q_2 + Q_3$ or $CV = C_1V + C_2V + C_3V$ or $C = C_1 + C_2 + C_3$

For such a combination, dV/dt is the same for all capacitors.

Example 5.12. *Find the C_{eq} of the circuit shown in Fig. 5.19. All capacitances are in μ F.*

(Basic Circuit Analysis Osmania Univ. Jan./Feb. 1992)

Solution. Capacitance between C and $D = 4 + 1 \parallel 2 = 14/3$ μ F.

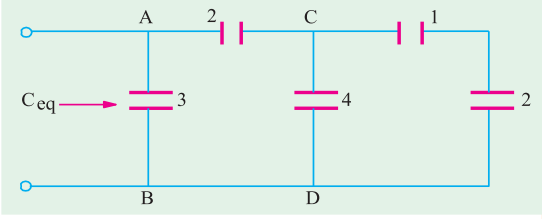


Fig. 5.19

Capacitance between A and B *i.e.* $C_{eq} = 3 + 2 \parallel 14/3 = 4.4$ μ F

Example 5.13. *Two capacitors of a capacitance 4 μF and 2 μF respectively, are joined in series with a battery of e.m.f. 100 V. The connections are broken and the like terminals of the capacitors are then joined. Find the final charge on each capacitor.*

Solution. When joined in series, let V_1 and V_2 be the voltages across the capacitors. Then as charge across each is the same.

$\therefore$ $4 \times V_1 = 2V_2$ $\therefore$ $V_2 = 2V_1$ Also $V_1 + V_2 = 100$

$\therefore$ $V_1 + 2V_1 = 100$ $\therefore$ $V_1 = 100/3$ V and $V_2 = 200/3$ V

$\therefore$ $Q_1 = Q_2 = (200/3) \times 2 = (400/3)$ μ C

$\therefore$ Total charge on both capacitors = 800/3 μ C

When joined in parallel, a redistribution of charge takes place because both capacitors are reduced to a common potential V.

Total charge = 800/3 μ C; total capacitance = 4 + 2 = 6 μ F

$$\therefore \quad V = \frac{800}{3 \times 6} = \frac{400}{9} \text{ volts}$$

Hence $Q_1 = (400/9) \times 4 = 1600/9 = \mathbf{178\ \mu C}$

$Q_2 = (400/9) \times 2 = \mathbf{800/9 = 89\ \mu C\ (approx.)}$

Example 5.14. *Three capacitors A, B, C have capacitances 10, 50 and 25 μF respectively. Calculate* ***(i)*** *charge on each when connected in parallel to a 250 V supply* ***(ii)*** *total capacitance and* ***(iii)*** *p.d. across each when connected in series.* **(Elect. Technology, Gwalior Univ.)**

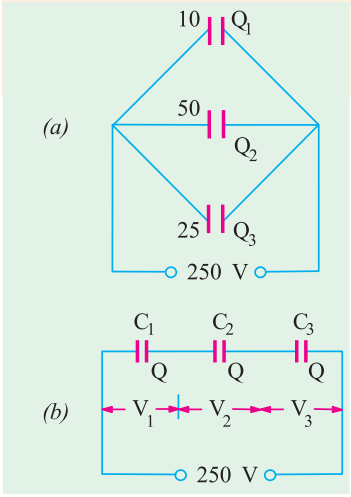


Fig. 5.20

Solution. (*i*) Parallel connection is shown in Fig. 5.20 (*a*). Each capacitor has a p.d. of 250 V across it.

$Q_1 = C_1V = 10 \times 250 = \mathbf{2500\ \mu C}$; $Q_2 = 50 \times 250 = \mathbf{12{,}500\ \mu C}$

$Q_3 = 25 \times 250 = \mathbf{6{,}750\ \mu C.}$

(ii) $C = C_1 + C_2 + C_3 = 10 + 50 + 25 = \mathbf{85\ \mu F}$

(iii) Series connection is shown in Fig. 5.20 (*b*). Here charge on each capacitor is the same and is equal to that on the equivalent single capacitor.

$1/C = 1/C_1 + 1/C_2 + 1/C_3$; $C = 25/4\ \mu F$

$Q = CV = 25 \times 250/4 = 1562.5\ \mu F$

$Q = C_1V_1$; $V_1 = 1562.5/10 = \mathbf{156.25\ V}$

$V_2 = 1562.5/25 = \mathbf{62.5\ V;}$ $V_3 = 1562.5/50 = \mathbf{31.25\ V.}$

Example 5.15. *Find the charges on capacitors in Fig. 5.21 and the p.d. across them.*

Solution. Equivalent capacitance between points *A* and *B* is

$C_2 + C_3 = 5 + 3 = 8\ \mu F$

Capacitance of the whole combination (Fig. 5.21)

$$C = \frac{8 \times 2}{8 + 2}\ 1.6\ \mu F$$

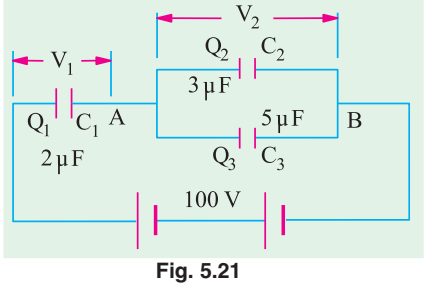


Fig. 5.21

Charge on the combination is

$Q_1 = CV = 100 \times 1.6 = \mathbf{160\ \mu C}$

$$V_1 = \frac{Q_1}{C_1} = \frac{160}{2} = 80\ V\ ;\ V_2 = 100 - 80 = \mathbf{20\ V}$$

$Q_1 = C_2V_2 = 3 \times 10^{-6} \times 20 = \mathbf{60\ \mu C}$

$Q_3 = C_3V_2 = 5 \times 10^{-6} \times 20 = \mathbf{100\ \mu C}$

Example 5.16. *Two capacitors A and B are connected in series across a 100 V supply and it is observed that the p.d.s. across them are 60 V and 40 V respectively. A capacitor of 2 μF capacitance is now connected in parallel with A and the p.d. across B rises to 90 volts. Calculate the capacitance of A and B in microfarads.*

Solution. Let C_1 and C_2 μ F be the capacitances of the two capacitors. Since they are connected in series [Fig. 5.22 (*a*)], the charge across each is the same.

$\therefore \quad 60\ C_1 = 40\ C_2 \quad$ or $\quad C_1/C_2 = 2/3 \qquad$ ...***(i)***

In Fig. 5.22 (*b*) is shown a capacitor of 2 μ F connected across capacitor *A*. Their combined capacitance = $(C_1 + 2)$ μ F

$\therefore \qquad (C_1 + 2)\ 10 = 90\ C_2 \quad$ or $\quad C_1/C_2 = 2/3 \qquad$ ...***(ii)***

Putting the value of $C_2 = 3C_1/2$ from ***(i)*** in ***(ii)*** we get

$$\frac{C_1 + 2}{3C_1/2} = 9 \qquad \therefore\ C_1 + 2 = 13.5\ C_1$$

or $C_1 = 2/12.4 = \mathbf{0.16\ \mu F}$ and
$C_2 = (3/2) \times 0.16 = \mathbf{0.24\ \mu F}$

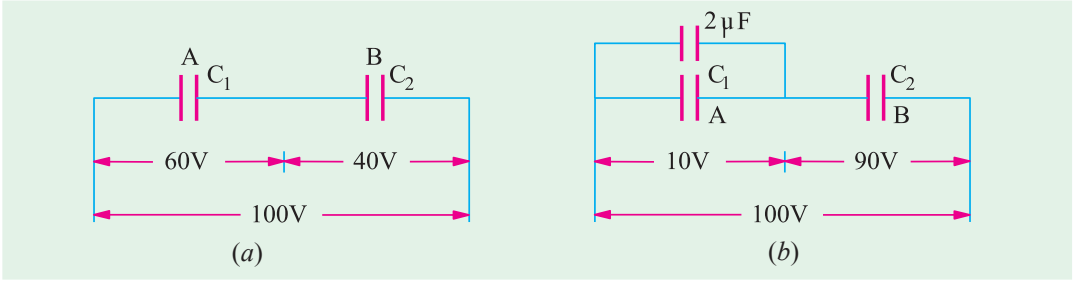


Fig. 5.22

Example 5.17. *Three capacitors of 2 μ F, 5 μ F and 10 μ F have breakdown voltage of 200 V, 500 V and 100 V respectively. The capacitors are connected in series and the applied direct voltage to the circuit is gradually increased. Which capacitor will breakdown first ? Determine the total applied voltage and total energy stored at the point of breakdown.* **[Bombay Univeristy 2001]**

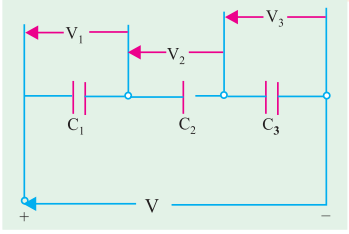


Fig. 5.23

Solution. C_1 of 2 μF, C_2 of 5 μ F, and C_3 of 10 μF are connected in series. If the equivalent single capacitor is C,

$1/C = 1/C_1 + 1/C_2 + 1/C_3$, which gives $C = 1.25$ μ F

If V is the applied voltage,

$V_1 = V \times C/C_1 = V \times (1.25 / 2)$
$= 62.5$ % of V
$V_2 = V \times (C/C_2) = C \times (1.25/5) = 25$ % of V
$V_3 = V \times (C/C_3) = V \times (1.25/10) = 12.5$ % of V

If $V_1 = 200$ volts, $V = 320$ volts and $V_2 = 80$ volts, $V_3 = 40$ volts.

It means that, first capacitor C_1 will breakdown first.

Energy stored = $1/2\ CV^2 = 1/2 \times 1.25 \times 10^{-6} \times 320 \times 320 = \mathbf{0.064\ Joule}$

Example 5.18. *A multiple plate capacitor has 10 plates, each of area 10 square cm and separation between 2 plates is 1 mm with air as dielectric. Determine the energy stored when voltage of 100 volts is applied across the capacitor.* **[Bombay University 2001]**

Solution. Number of plates, $n = 10$

$$C = \frac{(n-1)\epsilon_0}{d} = \frac{9 \times 8.854 \times 10^{-12} \times 10 \times 10^{-4}}{1 \times 10^{-3}} = 79.7 \text{ pF}$$

Energy stored $= 1/2 \times 79.7 \times 10^{-12} \times 100 \times 100 = \mathbf{0.3985\ \mu J}$

Example 5.19. *Determine the capacitance between the points A and B in figure 5.24 (a). All capacitor values are in μF.*

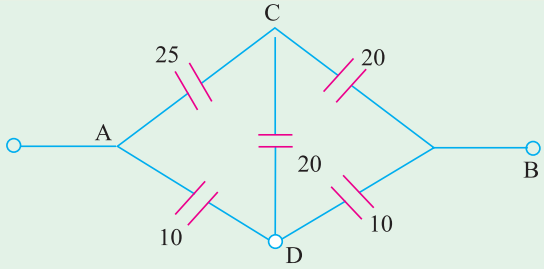


Fig. 5.24 (*a*)

Solution. Capacitances are being dealt with in this case. For simplifying this, Delta to star transformation is necessary. Formulae for this transformation are known if we are dealing with resistors or impedances. Same formulae are applicable to capacitors provided we are aware that capacitive reactance is dependent on reciprocal of capacitance.

Further steps are given below :

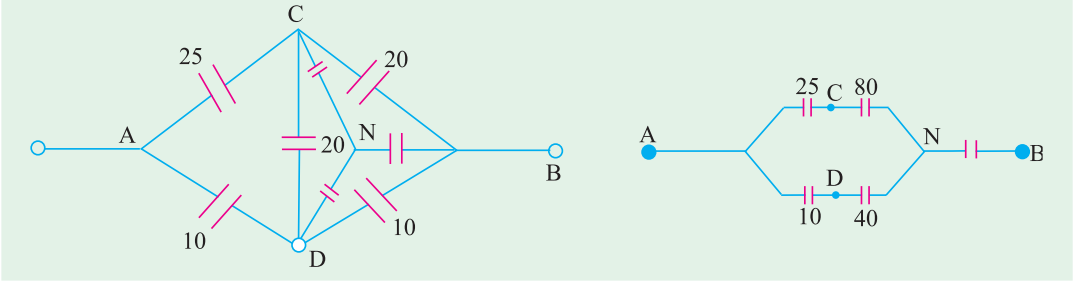


Fig. 5.24 (*b*) **Fig. 5.24 (*c*)**

Reciprocals of capacitances taken first :

Between *B*-*C* —— 0.05, Between B-D —— 0.10

Between C-D —— 0.05, Sum of these three = 0.20

For this delta, star-transformation is done :

Between N-C : $0.05 \times 0.05/0.20 = 0.0125$, its reciprocal = 80 μ F

Between N-B : $0.05 \times 0.10/0.20 = 0.025$, its reciprocal = 40 μ F

Between N-D : $0.05 \times 0.10/0.20 = 0.025$, its reciprocal = 40 μ F

This is marked on Fig. 5.24 (*c*).

With series-parallel combination of capacitances, further simplification gives the final result.

$$C_{AB} = \mathbf{16.13\ \mu F}$$

Note : Alternatively, with *ADB* as the vertices and *C* treated as the star point, star to delta transformation can be done. The results so obtained agree with previous effective capacitance of 16.14 μ F.

Example 5.20. (*a*) *A capacitor of 10 pF is connected to a voltage source of 100 V. If the distance between the capacitor plates is reduced to 50 % while it remains, connected to the 100 V supply. Find the new values of charge, energy stored and potential as well as potential gradient. Which of these quantities increased by reducing the distance and why ?*

[Bombay University 2000]

Solution.

(*i*) $C = 10$ pF	**(*ii*)** $C = 20$ pF, distance halved
Charge = 1000 p Coul	Charge = 2000 p-coul
Energy = $1/2\ CV^2 = 0.05$ μ J	Energy = 0.10 μ J

Potential gradient in the second case will be twice of earlier value.

Example 5.20 (*b*). *A capacitor 5 μ F charged to 10 V is connected with another capacitor of 10 μ F charged to 50 V, so that the capacitors have one and the same voltage after connection. What are the possible values of this common voltage ?* **[Bombay University 2000]**

Solution. The clearer procedure is discussed here.

Initial charges held by the capacitors are represented by equivalent voltage sources in Fig. 5.25 (*b*). The circuit is simplified to that in Fig. 5.25 (*c*). This is the case of C_1 and C_2 connected in series and excited by a 40-V source. If *C* is the equivalent capacitance of this series-combination,

$$1/C = 1/C_1 + C_2$$

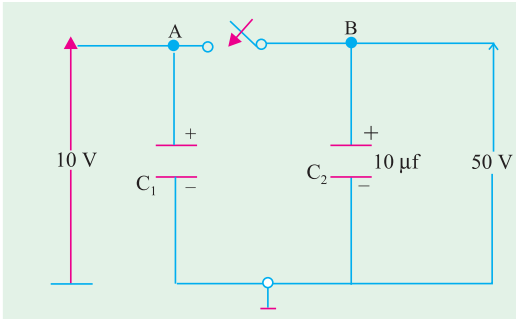


Fig. 5.25 (*a*)

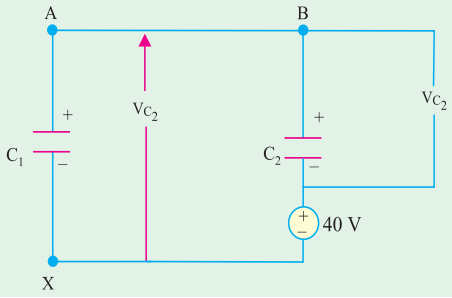


Fig. 5.25 (*c*) Simplification

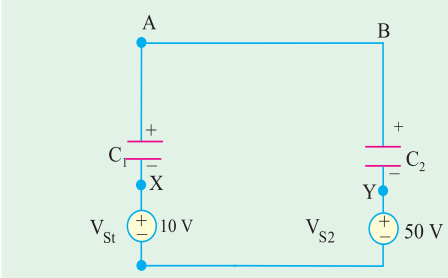


Fig. 5.25 (*b*). Initial charge represented by equiv-source

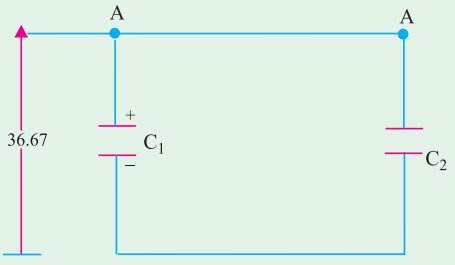


Fig. 5.25 (*d*). Final condition

This gives $C = 3.33$ μF

In Fig. (*c*), $V_{C1} = 40 \times C/C_1 = 40 \times 3.33/5 = 26.67$ volts

V_{S1} and V_{S2} are integral parts of C_1 and C_2 in Fig. 5.25 (*c*),

Voltage across $C_1 = 10 + 26.67 = 36.67$ (A w.r. to 0)

Voltage acorss $C_2 = 50 - 13.33 = 36.67$, (B w.r. to 0)

Thus, the final voltage across the capacitor is 36.67 volts.

Note : If one of the initial voltages on the capacitors happens to be the opposite to the single equivalent source voltage in Fig. 5.25 (*c*) will be 60 volts. Proceeding similarly, with proper care about signs, the final situation will be the common voltage will be 30 volts.

5.13. Cylindrical Capacitor with Compound Dielectric

Such a capacitor is shown in Fig. 5.26

Let r_1 = radius of the core

r_2 = radius of inner dielectric ε_{r1}

r_3 = radius of outer dielectric ε_{r2}

Obviously, there are two capacitors joined in series.

Now

$$C_1 = \frac{0.024\,\varepsilon_{r1}}{\log_{10}(r_2/r_1)}\ \mu\text{F/km and } C_2 = \frac{0.024\,\varepsilon_{r2}}{\log_{10}(r_3/r_2)}\ \mu\text{F/M}$$

Total capacitance of the cable is $C = \dfrac{C_1 C_2}{C_1 + C_2}$



A cyclindrical Capacitor

Now for capacitors joined in series, charge is the same.

$$\therefore \quad Q = C_1V_1 = C_2V_2$$

$$\text{or} \quad \frac{V_2}{V_1} = \frac{C_1}{C_2} = \frac{\varepsilon_{r1}\log_{10}(r_3/r_2)}{\varepsilon_{r1}\log_{10}(r_2/r_1)}$$

From this relation, V_2 and V_1 can be found,

g_{max} in inner capacitor $\dfrac{V_1}{2,3\, r_1 \log_{10}(r_2/r_1)}$

(Art. 5.9)

Similarly, g_{max} for outer capacitor $= \dfrac{V_2}{2,3\, r_2 \log_{10}(r_3/r_2)}$

$$\therefore \quad \frac{g_{max}}{g_{max}} = \frac{V_1}{2.3\, r_1 \log_{10}(r_2/r_1)} \div \frac{V_2}{2,3\, r_2 \log_{10}(r_3/r_2)}$$

$$= \frac{V_1 r_2}{V_2 r_1} \times \frac{\log_{10}(r_3/r_2)}{\log_{10}(r_2/r_1)} = \frac{C_2 r_2}{C_1 r_1} \times \frac{\log_{10}(r_3/r_2)}{\log_{10}(r_2/r_1)} \left(\therefore \frac{V_1}{V_2} = \frac{C_2}{C_1}\right)$$

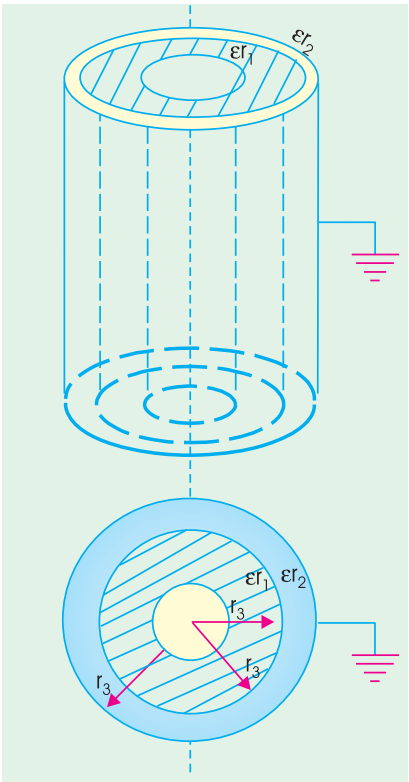


Fig. 5.26

Putting the values of C_1 and C_2, we get

$$\frac{g_{\max 1}}{g_{\max 2}} = \frac{0.024\,\varepsilon_{r2}}{\log_{10}(r_3/r_2)} \times \frac{\log_{10}(r_3/r_2)}{0.024\,\varepsilon_{r1}} = \frac{r_2}{r_1} \times \frac{\log_{10}(r_2/r_1)}{\log_{10}(r_2/r_1)} \quad \therefore \quad \frac{g_{\max 1}}{g_{\max 2}} = \frac{\varepsilon_{r2}\cdot r_2}{\varepsilon_{r1}\cdot r_1}$$

Hence, voltage gradient is inversely proportional to the permittivity and the inner radius of the insulating material.

Example 5.21. *A single-core lead-sheathed cable, with a conductor diameter of 2 cm is designed to withstand 66 kV. The dielectric consists of two layers A and B having relative permittivities of 3.5 and 3 respectively. The corresponding maximum permissible electrostatic stresses are 72 and 60 kV/cm. Find the thicknesses of the two layers.* **(Power Systems-I, M.S. Univ. Baroda)**

Solution. As seen from Art. 5.13.

$$\frac{g_{\max 1}}{g_{\max 2}} = \frac{\varepsilon_{r2}\cdot r_2}{\varepsilon_{r1}\cdot r_1} \text{ or } \frac{72}{60} = \frac{3\times r_2}{3.5\times 1} \text{ or } r_2 = 1.4 \text{ cm}$$

Now, $$g_{max} = \frac{V_1 \times \sqrt{2}}{2.3\, r_1 \log_{10} r_2/r_1}$$ ...Art. 5.9

where V_1 is the r.m.s. values of the voltage across the first dielectric.

$$\therefore \quad 72 = \frac{V_1 \times \sqrt{2}}{2.3\times 1\times \log_{10} 1.4} \text{ or } V_1 = 17.1 \text{ kV}$$

Obviously, $V_2 = 60 - 17.1 = \mathbf{48.9\ kV}$

Now, $$g_{\max 2} = \frac{V_2 \times \sqrt{2}}{2.3\, r_2 \log_{10}(r_3/r_2)} \quad \therefore \quad 60 = \frac{48.9}{2.3\times 1.4 \log_{10}(r_3/r_2)}$$

$$\therefore \quad \log_{10}(r_3/r_2) = 0.2531 = \log_{10}(1.79) \quad \therefore \quad \frac{r_3}{r_2} = 1.79 \text{ or } r_3 = 2.5 \text{ cm}$$

Thickness of first dielectric layer = 1.4 –1.0 = **0.4 cm.**

Thickness of second layer = 2.5 –1.4 = **1.1 cm.**

5.14. Insulation Resistance of a Cable Capacitor

In a cable capacitor, useful current flows along the axis of the core but there is always present some leakage of current. This leakage is radial *i.e.* at right angles to the flow of useful current. The resistance offered to this radial leakage of current is called ***insulation resistance*** of the cable. If cable length is greater, then leakage is also greater. It means that more current will leak. In other words, insulation resistance is decreased. Hence, we find that insulation resistance is inversely proportional to the cable length. This insulation resistance is not to be confused with conductor resistance which is directly proportional to the cable length.

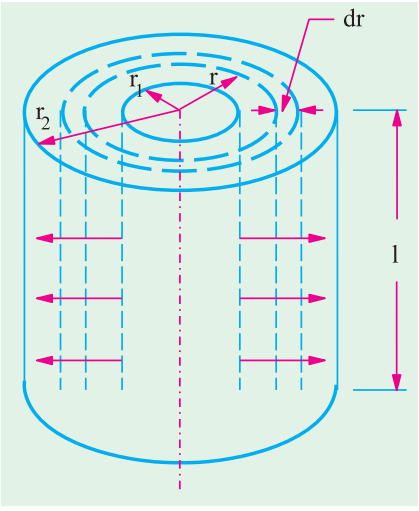


Fig. 5.27

Consider *i* metre of a single-core cable of inner-radius r_1 and outer radius r_2 (Fig. 5.27). Imagine an annular ring of radius '*r*' and radial thickness '*dr*'.

If resistivity of insulating material is ρ, then resistance of the this narrow ring is $dR = \frac{\rho\, dr}{2\pi r \times l} = \frac{\rho dr}{2\pi r l}$ ∴Insulation resistance of *l* metre length of cable is

$$\int dR = \int_{r1}^{r2} \frac{\rho dr}{2\pi\, rl} \quad or \quad R = \frac{\rho}{2\pi\, rl} \Big| \log_e (r) \Big|_{r1}^{r2}$$

$$R = \frac{\rho}{2\,\pi l} \log_e (r_2/r_1) = \frac{2.3\,\rho}{2\,\pi\, l} \log_{10} (r_2/r_1)\ \Omega$$

It should be noted

(i) *that R is inversely proportional to the cable length*

(ii) *that R depends upon the ratio r_2/r_1 and NOT on the thickness of insulator itself.*

Example 5.22. *A liquid resistor consists of two concentric metal cylinders of diameters D = 35 cm and d = 20 cm respectively with water of specific resistance ρ = 8000 Ω cm between them. The length of both cylinders is 60 cm. Calculate the resistance of the liquid resistor.*

(Elect. Engg. Aligarh Univ.,)

Solution. r_1 = 10 cm ; r_2 = 17.5 cm; $\log_{10}(1.75) = 0.243$

ρ = 8×10^3 Ω– cm; *l* = 60 cm.

Resistance of the liquid resistor $R = \frac{2.3 \times 8 \times 10^3}{2\pi \times 60} \times 0.243 = 11.85\ \Omega.$

Example 5.23. *Two underground cables having conductor resistances of 0.7 Ω and 0.5 and insulation resistance of 300 M Ω respectively are joind* ***(i)*** *in series* ***(ii)*** *in parallel. Find the resultant conductor and insulation resistance.* **(Elect. Engineering, Calcutta Univ.)**

Solution. ***(i)*** The conductor resistance will add like resistances in series. However, the leakage resistances will decrease and would be given by the reciprocal relation.

Total conductor resistance = 0.7 + 0.5 = 1.2 Ω

If *R* is the combined leakage resistance, then

$$\frac{1}{R} = \frac{1}{300} + \frac{1}{600} \qquad \therefore \quad R = 200\ M\,\Omega$$

(ii) In this case, conductor resistance is = 0.7 × 0.5/(0.7 + 0.5) = **0.3. Ω (approx)**

Insulation resistance = 300 + 600 = **900 M Ω**

Example 5.24. *The insulation resistance of a kilometre of the cable having a conductor diameter of 1.5 cm and an insulation thickness of 1.5 cm is 500 M Ω. What would be the insulation resistance if the thickness of the insulation were increased to 2.5 cm ?*

(Communication Systems, Hyderadad Univ. 1992)

Solution. The insulation resistance of a cable is

First Case $R = \dfrac{2.3\,\rho}{2\pi l}\log_{10}(r_2/r_1)$

$r_1 = 1.5/2 = 0.75$ cm ; $r_2 = 0.75 + 1.5 = 2.25$ cm

$\therefore \quad r_2/r_1 = 2.25/0.75 = 3$; $\log_{10}(3) = 0.4771 \quad \therefore \quad 500 = \dfrac{2.3\,\rho}{2\,\pi\,l} \times 0.4771$ ...(*i*)

Second Case

$r_1 = 0.75$ cm –as before $r_2 = 0.75 + 2.5 = 3.25$ cm

$r_2/r_1 = 3.25/0.75 = 4.333$; $\log_{10}(4.333) = 0.6368 \quad \therefore \quad R = \dfrac{2.4\,\rho}{2\,\pi\,l} \times 0.6368$ ...(*ii*)

Dividing Eq. ***(ii)*** by Eq. ***(i)***, we get

$$\frac{R}{500} = \frac{0.6368}{0.4771}\,;\; R = 500 \times 0.6368/0.4771 = 667.4\ \text{M}\,\Omega$$

5.15. Energy Stored in a Capacitor



Capacitors on a motherboard

Charging of a capacitor always involves some expenditure of energy by the charging agency. This energy is stored up in the electrostatic field set up in the dielectric medium. On discharging the capacitor, the field collapses and the stored energy is released.

To begin with, when the capacitor is uncharged, little work is done in transferring charge from one plate to another. But further instalments of charge have to be carried against the repulsive force due to the charge already collected on the capacitor plates. Let us find the energy spent in charging a capacitor of capacitance C to a voltage V.

Suppose at any stage of charging, the p.d. across the plates is v. By definition, it is equal to the work done in shifting one coulomb from one plate to another. If 'dq' is charge next transferred, the work done is

$$dW = v.dq$$

Now $q = Cv \quad \therefore \quad dq = C.dv \quad \therefore \quad dW = Cv.dv$

Total work done in giving V units of potential is

$$W = \int_0^v Cv.dv = C\left|\frac{v^2}{2}\right|_0^v \quad \therefore \quad W = \frac{1}{2}CV^2$$

If C is in farads and V is in volts, then $W = \dfrac{1}{2}CV^2$ joules $= \dfrac{1}{2}QV$ joules $= \dfrac{Q^2}{2\,C}$ joules

If Q is in coulombs and C is in farads, the energy stored is given in joules.

Note : As seen from above, energy stored in a capacitor is $E = \frac{1}{2} CV^2$

Now, for a capacitor of plate area A m^2 and dielectric of thickness d metre, energy per unit volume of dielectric medium.

$$\frac{1}{2}\frac{CV^2}{Ad} \quad \frac{1}{2}\frac{A}{d}\cdot\frac{V^2}{Ad} \quad \frac{1}{2} \quad \frac{V}{d}^{\,2} \quad \frac{1}{2} \quad E^2 \quad \frac{1}{2} DE \quad D^2/2 \quad \text{joules/m}^3 \ *$$

It will be noted that the formula $\frac{1}{2} DE$ is similar to the expression $\frac{1}{2}$ stress × strain which is used for calculating the mechanical energy stored per unit volume of a body subjected to elastic stress.

Example 5.25. *Since a capacitor can store charge just like a lead-acid battery, it can be used at least theoretically as an electrostatic battery. Calculate the capacitance of 12-V electrostatic battery which the same capacity as a 40 Ah, 12 V lead-acid battery.*

Solution. Capacity of the lead-acid battery = 40 Ah = 40 × 36 As = 144000 Coulomb

Energy stored in the battery = $QV = 144000 \times 12 = 1.728 \times 10^6$ J

Energy stored in an electrostatic battery = $\frac{1}{2} CV^2$

$$\therefore \quad \frac{1}{2} \times C \times 12^2 = 1.728 \times 10^6 \quad \therefore \quad C = 2.4 \times 10^4 \text{ F} = 24 \text{ kF}$$

Example 5.26. *A capacitor-type stored-energy welder is to deliver the same heat to a single weld as a conventional welder that draws 20 kVA at 0.8 pf for 0.0625 second/weld. If C = 2000 μF, find the voltage to which it is charged.* **(Power Electronics, A.M.I.E. Sec B, 1993)**

Solution. The energy supplied per weld in a conventional welder is

$$W = VA \times \cos\phi \times \text{time} = 20{,}000 \times 0.8 \times 0.0625 = 1000 \text{ J}$$

Now, energy stored in a capacitor is $(1/2)\, CV^2$

$$\therefore \quad W = \frac{1}{2} CV^2 \text{ or } V = \sqrt{\frac{2W}{C}} = \sqrt{\frac{2 \times 1000}{2000 \times 10^{-6}}} = \mathbf{1000 \text{ V}}$$

Example 5.27. *A parallel-plate capacitor is charged to 50 μC at 150 V. It is then connected to another capacitor of capacitance 4 times the capacitance of the first capacitor. Find the loss of energy.* **(Elect. Engg. Aligarh Univ.)**

Solution. $C_1 = 50/150 = 1/3$ μF ; $C_2 = 4 \times 1/3 = 4/3$ μF

Before Joining

$$E_1 = \frac{1}{2} C_1 V_1^2 = \frac{1}{2} \times \left(\frac{1}{3}\right) 10^{-6} \times 150^2 = 37.5 \times 10^{-4} \text{ J} \; ; \; E_2 = 0$$

Total energy = 37.5×10^{-4} J

After Joining

When the two capacitors are connected in parallel, the charge of 50 μ C gets redistributed and the two capacitors come to a common potential V.

$$V = \frac{\text{total charge}}{\text{total capacitance}} = \frac{50\ \mu C}{[(1/3) + (4/3)]\ \mu\text{ F}} = 30 \text{ V}$$

$$E_1 = \frac{1}{2} \times (1/3) \times 10^{-6} \times 30^2 = 1.5 \times 10^{-4} \text{ J}$$

$$E_2 = \frac{1}{2} \times (4/3) \times 10^{-6} \times 30^2 = 6.0 \times 10^{-4} \text{ J}$$

Total energy = 7.5×10^{-4} J ; Loss of energy = $(37.5 - 7.5) \times 10^{-4} = \mathbf{3 \times 10^{-2} \text{ J}}$

The energy is wasted away as heat in the conductor connecting the two capacitors.

* It is similar to the expression for the energy stored per unit volume of a magnetic field.

Example 5.28. *An air-capacitor of capacitance 0.005 μ F is connected to a direct voltage of 500 V, is disconnected and then immersed in oil with a relative permittivity of 2.5. Find the energy stored in the capacitor before and after immersion.* **(Elect. Technology : London Univ.)**

Solution. Energy before immersion is

$$E_1 = \frac{1}{2}CV^2 = \frac{1}{2}\times 0.005\times 10^{-6}\times 500^2 = \mathbf{625\times 10^{-6}\ J}$$

When immersed in oil, its capacitance is increased 2.5 times. Since charge is constant, voltage must become 2.5 times. Hence, new capacitances is 2.5 × 0.005 = 0.0125 μF and new voltage is 500/2.5 = 200 V.

$$E_2 = \frac{1}{2}\times 0.0125\times 10^{-6}\times (200)^2 = \mathbf{250\times 10^{-6}\ J}$$

Example 5.29. *A parallel-plate air capacitor is charged to 100 V. Its plate separation is 2 mm and the area of each of its plate is 120 cm^2.*

Calculate and account for the increase or decrease of stored energy when plate separation is reduced to 1 mm

***(a)** at constant voltage **(b)** at constant charge.*

Solution. Capacitance is the first case

$$C_1 = \frac{\varepsilon_0 A}{d} = \frac{8.854\times 10^{-12}\times 120\times 10^{-4}}{2\times 10^{-3}} = 53.1\times 10^{-12}\ \text{F}$$

Capacitance in the second case *i.e.* with reduced spacing

$$C_2 = \frac{8.854\times 10^{-12}\times 120\times 10^{-4}}{1\times 10^{-3}} = 106.2\times 10^{-12}\ \text{F}$$

(*a*) When Voltage is Constant

Change in stored energy $dE = \frac{1}{2}C_2V^2 \quad \frac{1}{2}C_1V^2$

$$= \frac{1}{2}\times 100^2\times (106.2-53.1)\times 10^{-12} = \mathbf{26.55\times 10^{-8}\ J}$$

This represents an increase in the energy of the capacitor. This extra work has been done by the external supply source because charge has to be given to the capacitor when its capacitance increases, voltage remaining constant.

(*b*) When Charge Remains Constant

Energy in the first case $E_1 = \frac{1}{2}\frac{Q^2}{C_1}$; Energy in the second case, $E_2 = \frac{1}{2}\frac{Q^2}{C_2}$

$\therefore$ change in energy is $dE = \frac{1}{2}Q^2\left(\frac{1}{53.1}-\frac{1}{106.2}\right)\times 10^{12}$ J

$$= \frac{1}{2}(C_1V_1)^2\left(\frac{1}{53.1}-\frac{1}{106.2}\right)\times 10^{12}\ \text{J}$$

$$= \frac{1}{2}(53.1\times 10^{-12})^2\times 10^4\times 0.0094\times 10^{12}$$

$$= \mathbf{13.3\times 10^{-8}\ joules}$$

Hence, there is a decrease in the stored energy. The reason is that charge remaining constant, when the capacitance is increased, then voltage must fall with a consequent decrease in stored energy $\left(E=\frac{1}{2}QV\right)$

Example 5.30. *A point charge of 100 μC is embedded in an extensive mass of bakelite which has a relative permittivity of 5. Calculate the total energy contained in the electric field outside a radial distance of* **(i)** *100 m* **(ii)** *10 m* **(iii)** *1 m and* **(iv)** *1 cm.*

Solution. As per the Coulomb's law, the electric field intensity at any distance x from the point charge is given by $E = Q/4\pi\varepsilon x^2$. Let us draw a spherical shell of radius x as shown in Fig. Another spherical shell of radius $(x + dx)$ has also been drawn. A differential volume of the space enclosed between the two shells is $dv = 4\pi x^2 dx$. As per Art. 5.15, the energy stored per unit volume of the electric field is $(1/2)\,DE$. Hence, differential energy contained in the small volume is

$$dW = \frac{1}{2} DE\, dv = \frac{1}{2}\varepsilon E^2\, dv = \frac{1}{2}\varepsilon\left(\frac{Q}{4\pi\varepsilon x^2}\right)^2 4\pi x^2\, dx = \frac{Q^2}{8\pi\varepsilon}\cdot\frac{dx}{x^2}$$

Total energy of the electric field extending from $x = R$ to $x = \infty$ is

$$W = \frac{Q^2}{8\pi\varepsilon}\int_R^\infty x^{-2}\, dx = \frac{Q^2}{8\pi\varepsilon R} = \frac{Q^2}{8\pi\varepsilon_0\varepsilon_r R}$$

(i) The energy contained in the electic field lying outside a radius of $R = 100$ m is

$$W = \frac{(100\times10^{-6})^2}{8\pi\times8.854\times10^{-12}\times5\times100} = \mathbf{0.90\ J}$$

(ii) For $R = 10$ m, $W = 10 \times 0.09 = \mathbf{0.09\ J}$

(iii) For $R = 1$ m, $W = 100 \times 0.09 = \mathbf{9\ J}$

(iv) For $R = 1$ cm, $W = 10{,}000 \times 0.09 = \mathbf{900\ J}$

Example 5.31. *Calculate the change in the stored energy of a parallel-plate capacitor if a dielectric slab of relative permittivity 5 is introduced between its two plates.*

Solution. Let A be the plate area, d the plate separation, E the electric field intensity and D the electric flux density of the capacitor. As per Art. 5.15, energy stored per unit volume of the field is $= (1/2)\,DE$. Since the space volume is $d \times A$, hence,

$$W_1 = \frac{1}{2} D_1E_1\times dA = \frac{1}{2}\varepsilon_0 E_1^2\times dA = \frac{1}{2}\varepsilon_0\, dA\left(\frac{V_1}{d}\right)^2$$

When the dielectric slab is introduced,

$$W_2 = \frac{1}{2} D_2E_2\times dA = \frac{1}{2}\varepsilon E_2^2\times dA = \frac{1}{2}\varepsilon_0\varepsilon_r\, dA\left(\frac{V_2}{d}\right)^2$$

$$= \frac{1}{2}\varepsilon_0\varepsilon_r\, dA\left(\frac{V_2}{\varepsilon_r d}\right)^2 = \frac{1}{2}\varepsilon_0\, dA\left(\frac{V_1}{d}\right)^2\frac{1}{\varepsilon_r} \quad\therefore\ W_2 = \frac{W_1}{\varepsilon_r}$$

It is seen that the stored energy is reduced by a factor of ε_r. Hence, change in energy is

$$dW = W_1 - W_2 = W_1\left(1-\frac{1}{\varepsilon_r}\right) = W_1\left(1-\frac{1}{5}\right) = W_1\times\frac{4}{5} \quad\therefore\ \frac{dW}{W_1} = \mathbf{0.8}$$

Example 5.32. *When a capacitor C charges through a resistor R from a d.c. source voltage E, determine the energy appearing as heat.* **[Bombay University, 2000]**

Solution. R-C series ciruit switched on to a d.c. source of voltage E, at $t = 0$, results into a current $i\,(t)$, given by

$$i\,(t) = (E/R)\, e^{-t/\tau}$$

where $t = RC$

ΔW_R = Energy appearing as heat in time Δt

$$= i^2 R \Delta t$$

ΔW_R = Energy appearing as heat in time Δt

$$= i^2 R \Delta t$$

$$W_R = R \int_0^\infty i^2 \, dt$$

$$= R (E/R)^2 \int_0^\infty (\varepsilon^{-t/\tau})^2 = \frac{1}{2} CE^2$$

Note : Energy stored by the capacitor at the end of charging process = 1/2 CE^2

Hence, energy received from the source = CF.

5.16. Force of Attraction Between Oppositely-charged Plates

In Fig. 5.28 are shown two parallel conducting plates A and B carrying constant charges of + Q and –Q coulombs respectively. Let the force of attraction between the two be F newtons. If one of the plates is pulled apart by distance dx, then work done is

$$= F \times dx \text{ joules} \qquad \text{...(i)}$$

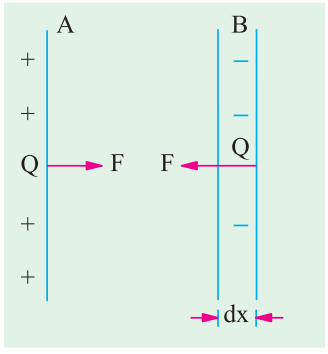


Fig. 5.28

Since the plate charges remain constant, no electrical energy comes into the arrangement during the movement dx.

∴ Work done = change in stored energy

Initial stored energy $= \frac{1}{2} \frac{Q_2}{C}$ joules

If capacitance becomes $(C - dC)$ due to the movement dx, then

Final stored energy $\frac{1}{2} \frac{Q^2}{(C \quad dC)} \quad \frac{1}{2} \cdot \frac{Q^2}{C} \frac{1}{1 \quad \frac{dC}{C}} \quad \frac{1}{2} \frac{Q^2}{C} \quad 1 \quad \frac{dC}{C}$ if $dC \quad C$

∴ Change in stored energy $= \frac{1}{2} \frac{Q^2}{C} \left(1 + \frac{dC}{C}\right) - \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} \frac{Q^2}{C^2} . dC$ ...(ii)

Equating Eq. (i) and (ii), we have $F.dx = \frac{1}{2} \frac{Q^2}{C^2} . dC$

$$F = \frac{1}{2} \frac{Q^2}{C^2} . \frac{dC}{dx} = \frac{1}{2} V^2 . \frac{dC}{dx} \qquad (\because V = Q/C)$$

Now $C = \frac{\varepsilon A}{x} \quad \therefore \quad \frac{dC}{dx} = -\frac{\varepsilon A}{x^2}$

∴ $F = -\frac{1}{2} V^2 . \frac{\varepsilon A}{x^2} = -\frac{1}{2} \varepsilon A \left(\frac{V}{x}\right)^2$ newtons $= -\frac{1}{2} \varepsilon A E^2$ newtons

This represents the force between the plates of a parallel-plate capacitor charged to a p.d. of V volts. The negative sign shows that it is a force of attraction.

Example 5.33. *A parallel-plate capacitor is made of plates 1 m square and has a separation of 1 mm. The space between the plates is filled with dielectric of* $\varepsilon_r = 25.0$. *If 1 k V potential difference is applied to the plates, find the force squeezing the plates together.*

(Electromagnetic Theory, A.M.I.E. Sec B, 1993)

Solution. As seen from Art. 5.16, $F = -(1/2)\, \varepsilon_0\, \varepsilon_r\, AE^2$ newton

Now $E = V/d = 1000/1 \times 10^{-3} = 10^{-6}$ V/m

$$\therefore \quad F = -\frac{1}{2}\varepsilon_0 \varepsilon_r AE^2 = -\frac{1}{2}\times 8.854\times 10^{-12}\times 25\times 1\times (10^6)^2 = -1.1\times 10^{-4}\ \text{N}$$

Tutorial Porblems No. 5.2

1. Find the capacitance per unit length of a cylindrical capacitor of which the two conductors have radii 2.5 and 4.5 cm and dielectric consists of two layers whose cylinder of contact is 3.5 cm in radius, the inner layer having a dielectric constant of 4 and the outer one of 6.

[440 pF/m]

2. A parallel-plate capacitor, having plates 100 cm^2 area, has three dielectrics 1 mm each and of permittivities 3, 4 and 6. If a peak voltage of 2,000 V is applied to the plates, calculate :

(*a*) potential gradient across each dielectric

(*b*) energy stored in each dielectric.

[8.89 kV/cm; 6.67 kV/cm ; 4.44 kV/cm ; 1047, 786, 524 × 10^{-7} joule]

3. The core and lead-sheath of a single-core cable are separated by a rubber covering. The cross-sectional area of the core is 16 mm^2. A voltage of 10 kV is applied to the cable. What must be the thickness of the rubber insulation if the electric field strength in it is not to exceed 6 × 10^6 V/m ?

[2.5 mm (approx)]

4. A circular conductor of 1 cm diameter is surrounded by a concentric conducting cylinder having an inner diameter of 2.5 cm. If the maximum electric stress in the dielectric is 40 kV/cm, calculate the potential difference between the conductors and also the minimum value of the electric stress.

[18.4 kV ; 16 kV/cm]

5. A multiple capacitor has parallel plates each of area 12 cm^2 and each separated by a mica sheet 0.2 mm thick. If dielectric constant for mica is 5, calculate the capacitance.

[265.6 μμF]

6. A p.d. of 10 kV is applied to the terminals of a capacitor of two circular plates each having an area of 100 sq. cm. separated by a dielectric 1 mm thick. If the capacitance is 3 × 10^{-4} microfarad, calculate the electric flux density and the relative permittivity of the dielectric.

[$D = 3\times 10^{-4}$ C/m^2, ε_r= 3.39] (*City & Guilds, London*)

7. Each electrode of a capacitor of the electrolytic type has an area of 0.02 sq. metre. The relative permittivity of the dielectric film is 2.8. If the capacitor has a capacitance of 10 μF, estimate the thickness of the dielectric film. **[4.95 × 10^{-8} m] (*I.E.E. London*)**

5.17. Current-Voltage Relationships in a Capacitor

The charge on a capacitor is given by the expression $Q = CV$. By differentiating this relation, we get

$$i = \frac{dQ}{dt} = \frac{d}{dt}(CV) = C\frac{dV}{dt}$$

Following important facts can be deduced from the above relations :

(*i*) since $Q = CV$, it means that the voltage across a capacitor is proportional to *charge,* not the *current*.

(*ii*) a capacitor has the ability to store charge and hence to provide a short of memory.

(*iii*) a capacitor can have a voltage across it even when there is *no current flowing.*

(*iv*) from $i = c\ dV/dt$, it is clear that current in the capacitor is present only when voltage on it changes with time. If $dV/dt = 0$ *i.e.* when its voltage is constant or for d.c. voltage, $i = 0$. Hence, the capacitor behaves like an *open circuit*.

(v) from $i = C\,dV/dt$, we have $dV/dt = i/C$. It shows that for a given value of (charge or discharge) current i, rate of change in voltage is inversely proportional to capacitance. Larger the value of C, slower the rate of change in capacitive voltage. Also, capacitor voltage ***cannot change instantaneously***.

(vi) the above equation can be put as $dv = \frac{i}{C}\,.\,dt$

Integrating the above, we get $\int dv = \frac{1}{C}\int i\,.\,dt$ or $dv = \frac{1}{C}\int_0^t i\,dt$

Example 5.34. *The voltage across a 5 μF capacitor changes uniformly from 10 to 70 V in 5 ms. Calculate* **(i)** *change in capacitor charge* **(ii)** *charging current.*

Solution. $Q = CV \quad \therefore dQ = C\,.\,dV$ and $i = C\,dV/dt$

(i) $dV = 70 - 10 = 60$ V, $\therefore\ dQ = 5 \times 60 =$ **300 μ C.**

(ii) $i = C\,.\,dV/dt = 5 \times 60/5 =$ **60 mA**

Example 5.35. *An uncharged capacitor of 0.01 F is charged first by a current of 2 mA for 30 seconds and then by a current of 4 mA for 30 seconds. Find the final voltage in it.*

Solution. Since the capacitor is initially uncharged, we will use the principle of Superposition.

$$V_1 = \frac{1}{0.01}\int_0^{30} 2\times10^{-3}\,.\,dt = 100\times2\times10^{-3}\times30 = 6\text{ V}$$

$$V_2 = \frac{1}{0.01}\int_0^{30} 4\times10^{-3}\,.\,dt = 100\times4\times10^{-3}\times30 = 12\text{ V};\ \therefore\ V = V_1 + V_2 = 6 + 12 = \textbf{18 V}$$

Example 5.36. *The voltage across two series-connected 10 μ F capacitors changes uniformly from 30 to 150 V in 1 ms. Calculate the rate of change of voltage for* **(i)** *each capacitor and* **(ii)** *combination.*

Solution. For series combination

$$V_1 = V\frac{C_2}{C_1 + C_2} = \frac{V}{3} \text{ and } V_2 = V\,.\,\frac{C_1}{C_1 + C_2} = \frac{2V}{3}$$

When V = 30 V $V_1 = V/3 = 30/3 = 10$ V ; $V_2 = 2V/3 = 2 \times 30/3 = 20$ V

When V = 150 V $V_1 = 150/3 = 50$ V and $V_2 = 2 \times 150/3 = 100$ V

(i) $\therefore\ \frac{dV_1}{dt} = \frac{(50-10)}{1\text{ ms}} =$ **40 kV/s** ; $\frac{dV_2}{dt} = \frac{(100-20)\text{ V}}{1\text{ ms}} =$ **80 kV/s**

(ii) $\frac{dV}{dt} = \frac{(150-30)}{1\text{ ms}} =$ **120 kV/s**

It is seen that $dV/dt = dV_1/dt + dV_2/dt$.

5.18. Charging of a Capacitor

In Fig. 5.29. (*a*) is shown an arrangement by which a capacitor C may be charged through a high resistance R from a battery of V volts. The voltage across C can be measured by a suitable voltmeter. When switch S is connected to terminal (*a*), C is charged but when it is connected to *b*, C is short circuited through R and is thus discharged. As shown in Fig. 5.29. (*b*), switch S is shifted to *a* for charging the capacitor for the battery. The voltage across C does not rise to V instantaneously but builds up slowly *i.e.* exponentially and not linearly. Charging current i_c is maximum at the start *i.e.* when C is uncharged, then it decreases exponentially and finally ceases when p.d. across capacitor plates becomes equal and opposite to the battery voltage V. At any instant during charging, let

v_c = p.d. across C; i_c = charging current

q = charge on capacitor plates

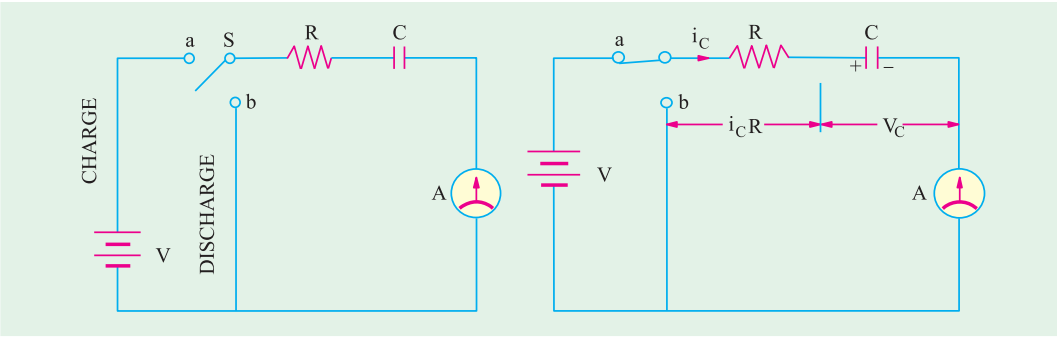


Fig. 5.29

The applied voltage V is always equal to the sum of :

(*i*) resistive drop ($i_c R$) and (*ii*) voltage across capacitor (v_c)

$$\therefore \quad V = i_c R + v_c \qquad \text{...(i)}$$

Now
$$i_c = \frac{dq}{dt} = \frac{d}{dt}(Cv_c) = C\frac{dv_c}{dt} \quad \therefore \; V = v_c + CR\frac{dv_c}{dt} \qquad \text{...(ii)}$$

or
$$-\frac{dv_c}{V - v_c} = -\frac{dt}{CR}$$

Integrating both sides, we get $\int \frac{-d\,V_c}{V - v_c} = -\frac{1}{CR}\int dt$; $\therefore \; \log_c (V - v_c) = -\frac{t}{CR} + K$ **...(iii)**

where K is the constant of integration whose value can be found from initial known conditions. We know that at the start of charging when $t = 0$, $v_c = 0$.

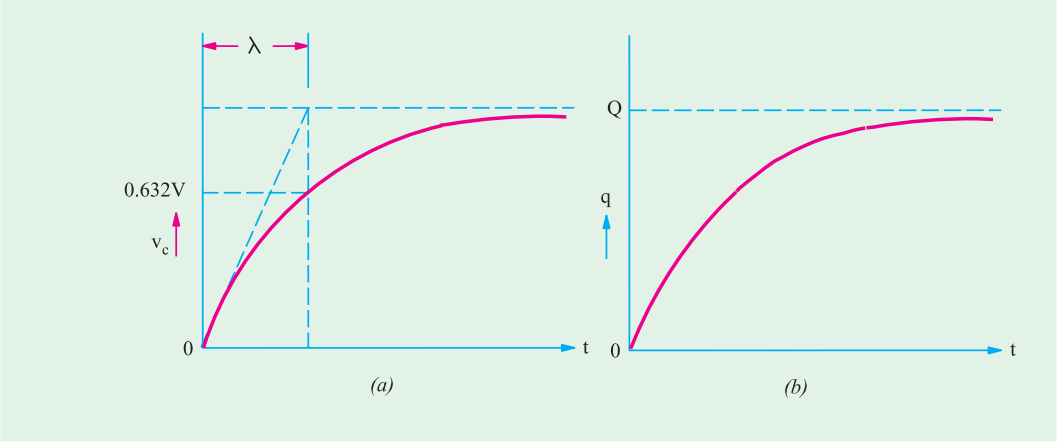
Substituting these values in **(iii)**, we get $\log_c V = K$

Hence, Eq. **(iii)** becomes $\log_e (V - v_c) = \frac{-t}{CR} + \log_e V$

or
$$\log_c \frac{V - v_c}{V} = \frac{-t}{CR} = -\frac{1}{\lambda} \quad \text{where } \lambda = CR = \text{time constant}$$

$$\therefore \quad \frac{V - v_c}{V} = e^{-t/\lambda} \quad \text{or} \quad v_c = V(1 - e^{-t/\lambda}) \qquad \text{...(iv)}$$

This gives variation with time of voltage across the capacitor plates and is shown in Fig. 5.27.(*a*)



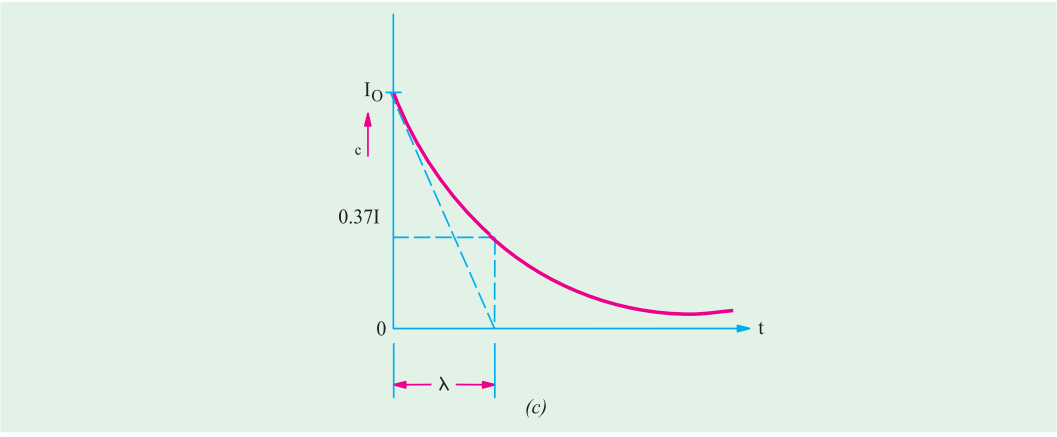


Fig. 5.30

Now $v_c = q/C$ and $V = Q/C$

Equation ***(iv)*** becomes $\frac{q}{c} = \frac{Q}{c}(1 - e^{-t/\lambda})$ $\therefore$ $q = Q(1 - e^{-t/\lambda})$ ...***(v)***

We find that increase of charge, like growth of potential, follows an exponential law in which the steady value is reached after infinite time (Fig. 5.30 *b*). Now, $i_c = dq/dt$.

Differentiating both sides of Eq. ***(v)***, we get

$$\frac{dq}{dt} = i_c = Q\frac{d}{dt}(1 - e^{-t/\lambda}) = Q\left(+\frac{1}{\lambda}e^{-t/\lambda}\right)$$

$$= \frac{Q}{\lambda}e^{-t/\lambda} = \frac{CV}{CR}e^{-t/\lambda} \qquad (\because\ Q = CV \text{ and } \lambda = CR)$$

$\therefore$ $$i_c = \frac{V}{R}.e^{-t/\lambda} \text{ or } i_c = I_o\, e^{-t/\lambda} \qquad ...(vi)$$

where I_0 = maximum current = V/R

Exponentially rising curves for v_c and q are shown in Fig. 5.30 (*a*) and (*b*) respectively. Fig. 5.30 (*c*) shows the curve for exponentially decreasing charging current. It should be particularly noted that i_c decreases in magnitude only but its direction of flow remains the same *i.e.* positive.

As charging continues, charging current decreases according to equation (*vi*) as shown in Fig. 5.30 (*c*). It becomes zero when $t = \infty$(though it is almost zero in about 5 time constants). Under steady-state conditions, the circuit appears only as a capacitor which means it acts as an open-circuit. Similarly, it can be proved that v_R decreases from its initial maximum value of V to zero exponentially as given by the relation $v_R = V\,e^{-t/\lambda}$.

5.19. Time Constant

(a) Just at the start of charging, p.d. across capacitor is zero, hence from (*ii*) putting $v_c = 0$, we get

$$V = CR\frac{dv_c}{dt}$$

$\therefore$ initial rate of rise of voltage across the capacitor is* $= \left(\frac{dv_c}{dt}\right)_{t=0} = \frac{V}{CR} = \frac{V}{\lambda}$ volt/second

If this rate of rise were maintained, then time taken to reach voltage V would have been $V + V/CR = CR$. This time is known as ***time constant*** (λ) of the circuit.

---

* It can also be found by differentiating Eq. (*iv*) with respect to time and then putting $t = 0$.

Hence, time constant of an *R-C* circuit is defined as ***the time during which voltage across capacitor would have reached its maximum value V had it maintained its initial rate of rise.***

(*b*) In equation **(*iv*)** if $t = \lambda$, then

$$v_c = V\,(1 - e^{-t/\lambda}) = V\,(1 - e^{-t/\lambda}) = V(1 - e^{-1}) = V\left(1 - \frac{1}{e}\right) = V\left(1 - \frac{1}{2.718}\right) = \mathbf{0.632\ V}$$

Hence, time constant may be defined as ***the time during which capacitor voltage actually rises to 0.632 of its final steady value.***

(*c*) From equaiton **(*vi*)**, by putting $t = \lambda$, we get

$$i_c = I_0\, e^{-\lambda/\lambda} = I_0\, e^{-1} = I_0/2.718 \cong 0.37\, I_0$$

Hence, the constant of a circuit is also the ***time during which the charging current falls to 0.37 of its initial maximum value (or falls by 0.632 of its initial value).***

5.20. Discharging of a Capacitor

As shown in Fig. 5.31 (*a*), when *S* is shifted to *b*, *C* is discharged through *R*. It will be seen that the discharging current flows in a direction opposite to that the charging current as shown in Fig. 5.31(*b*). Hence, if the direction of the charging current is taken positive, then that of the discharging current will be taken as negative. To begin with, the discharge current is maximum but then decreases exponentially till it ceases when capacitor is fully discharged.

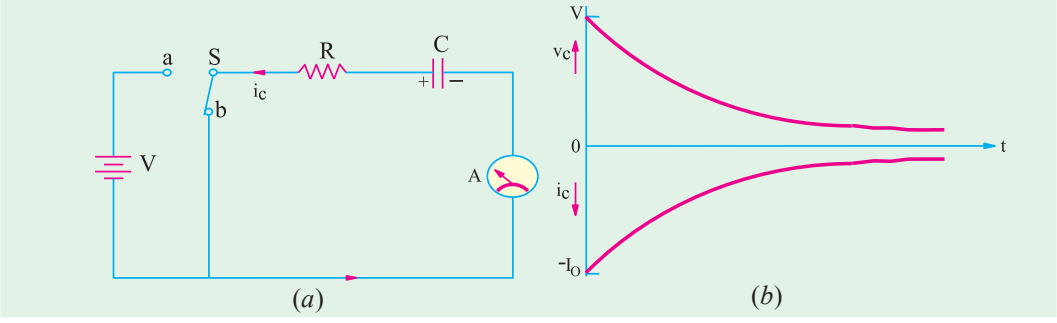


Fig. 5.31

Since battery is cut of the circuit, therefore, by putting $V = 0$ in equation **(*ii*)** of Art. 5.18, we get

$$0 = CR\frac{dv_c}{dt} \quad v_c \text{ or } v_c \quad CR\frac{dv_c}{dt} \qquad \left(\quad i_c = C\frac{dv_c}{dt} \right)$$

$$\therefore \quad \frac{dv_c}{v_c} = \frac{dt}{CR} \text{ or } \frac{dv_c}{v_c} \quad \frac{1}{CR} \quad dt \quad \log_e v_e \quad \frac{t}{CR} \quad k$$

At the start of discharge, when $t = 0$, $v_c = V$ $\therefore \log_e V = 0 + K$; or $\log_e V = K$

Putting this value above, we get

$$\log_e \quad v_c = -\frac{t}{\lambda} + \log_e V \text{ or } \log_e v_c/V = -t/\lambda$$

$$\text{or} \quad \frac{v_c}{V} = e^{-t/\lambda} \text{ or } v_c = Ve^{-t/\lambda}$$

$$\text{Similarly,} \quad q = Q\, e^{-t/\lambda} \text{ and } i_c = -I_0\, e^{-t/\lambda}$$

It can be proved that

$$v_R = -V\, e^{-t/\lambda}$$

The fall of capacitor potential and its discharging current are shown in Fig. 5.32 (*b*).

One practical application of the above charging and discharging of a capacitor is found in digital

control circuits where a square-wave input is applied across an *R-C* circuit as shown in Fig. 5.32 (*a*). The different waveforms of the current and voltages are shown in Fig. 5.32 (*b*), (*c*), (*d*), (*e*). The sharp voltage pulses of V_R are used for control circuits.

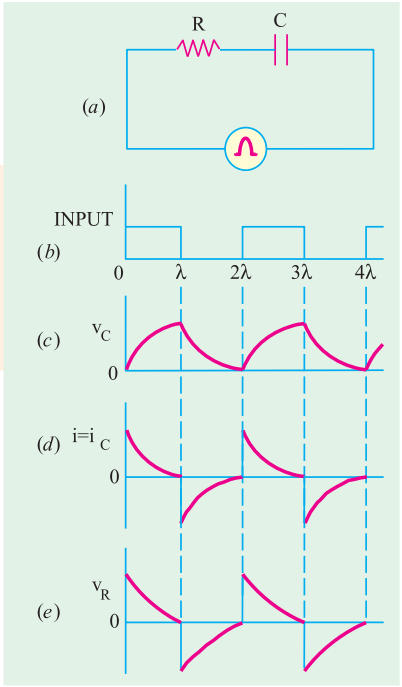


Fig. 5.32

Example 5.37. *Calculate the current in and voltage drop across each element of the circuit shown in Fig. 5.33 (a) after switch S has been closed long enough for steady-state conditions to prevail.*

Also, calculate voltage drop across the capacitor and the discharge current at the instant when S is opened.

Solution. Under steady-state conditions, the capacitor becomes fully charged and draws no current. In fact, it acts like an open circuit with the result that no current flows through the 1-Ω resistor. The steady state current I_{SS} flows through loop *ABCD* only.

Hence, $I_{SS} = 100/(6 + 4) = \mathbf{10\ A}$

Drop $V_6 = 100 \times 6/(6 + 4) = 60$ V

$V_4 = 100 \times 4/10 = \mathbf{40\ V}$

$V_1 = 0 \times 2 = \mathbf{0\ V}$

Voltage across the capacitor = drop across $B - C = \mathbf{40\ V}$

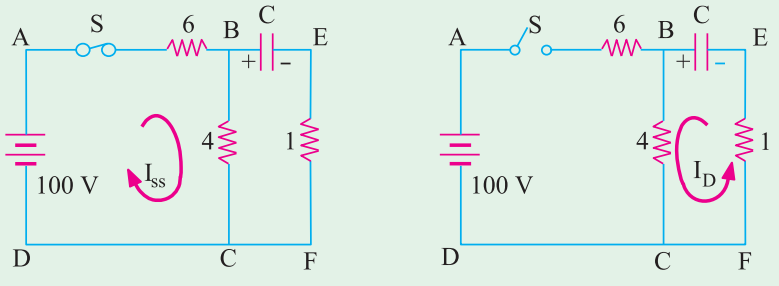


Fig. 5.33

Switch Open

When *S* is opened, the charged capacitor discharges through the loop *BCFE* as shown in Fig. 5.33 (*b*). The discharge current is given by

$$I_D = 40/(4 + 1) = \mathbf{8\ A}$$

As seen, it flows in a direction opposite to that of I_{SS}.

Example 5.38. **(*a*)** *A capacitor is charged through a large non-reactive resistance by a battery of constant voltage V. Derive an expression for the instantaneous charge on the capacitor.*

(*b*) *For the above arrangement, if the capacitor has a capacitance of 10 μF and the resistance is 1 MΩ, calculate the time taken for the capacitor to receive 90% of its final charge. Also, draw the charge/time curve.*

Solution. **(*a*)** For this part, please refer to Art. 5.18.

(*b*) $\lambda = CR = 10 \times 10^{-6} \times 1 \times 10^6 = 10\ s$; $q = 0.9\ Q$

Now, $q = Q\ (1 - e^{-t/\lambda})$ $\therefore$ $0.9\ Q = Q\ (1 - e^{-t/10})$ or $e^{t/10} = 10$

$\therefore$ $0.1\ t \log_e e = \log_e 10$ or $0.1\ t = 2.3 \log_{10} 10 = 2.3$ or $t = \mathbf{23\ s}$

The charge/time curve is similar to that shown in Fig. 5.27 (*b*).

Example 5.39. *A resistance R and a 4 μF capacitor are connected in series across a 200 V. d.c. supply. Across the capacitor is a neon lamp that strikes (glows) at 120 V. Calculate the value of R to make the lamp strike (glow) 5 seconds after the switch has been closed.*

(Electrotechnics-I.M.S. Univ. Baroda)

Solution. Obviously, the capacitor voltage has to rise 120 V in 5 seconds.

$\therefore \quad 120 = 200\,(1 - e^{-5/\lambda})$ or $e^{5/\lambda} = 2.5$ or $\lambda = 5.464$ second.

Now, $\lambda = CR \quad \therefore \; R = 5.464/4 \times 10^{-6} =$ **1.366 MΩ**

Example 5.40. *A capacitor of 0.1 μF is charged from a 100-V battery through a series resistance of 1,000 ohms. Find*

(a) *the time for the capacitor to receive 63.2 % of its final charge.*
(b) *the charge received in this time* **(c)** *the final rate of charging.*
(d) *the rate of charging when the charge is 63.2% of the final charge.*

(Elect. Engineering, Bombay Univ.)

Solution. (a) As seen from Art. 5.18 (*b*), 63.2% of charge is received in a time equal to the time constant of the circuit.

Time required $= \lambda = CR = 0.1 \times 10^{-6} \times 1000 = 0.1 \times 10^{-3} =$ **10^{-4} second**

(b) Final charge, $Q = CV = 0.1 \times 100 = 10$ μC

Charge received during this time is $= 0.632 \times 10 =$ **6.32 μ C**

(c) The rate of charging at any time is given by Eq. (*ii*) of Art. 5.18.

$$\frac{dv}{dt} = \frac{V - v}{CR}$$

Initially $v = 0$, Hence $\dfrac{dv}{dt} = \dfrac{V}{CR} = \dfrac{100}{0.1 \times 10^{-6} \times 10^{3}} =$ **10^6 V/s**

(d) Here $v = 0.632$ V $= 0.632 \times 100 = 63.2$ volts

$\therefore \quad \dfrac{dv}{dt} = \dfrac{100 - 63.2}{10^{-4}} =$ **368 kV/s**

Example 5.41. *A series combination having R = 2 MΩ and C = 0.01 μF is connected across a d.c. voltage source of 50 V. Determine*

(a) *capacitor voltage after 0.02 s, 0.04 s, 0.06 s and 1 hour*
(b) *charging current after 0.02 s, 0.04 s, 0.06 s and 0.1 s.*

Solution. $\lambda = CR = 2 \times 10^6 \times 0.01 \times 10^{-6} = 0.02$ second

$I_m = V/R = 50/2 \times 10^6 = 25$ μA.

While solving this question, it should be remembered that **(i)** in each time constant, v_c increases further by 63.2% of its *balance* value and **(ii)** in each constant, i_c decreases to 37% its previous value.

(a) (i) $t = 0.02$ s

Since, initially at $t = 0$, $v_c = 0$ V and $V_e = 50$ V, hence, in one time constant

$v_c = 0.632\,(50 - 0) =$ **31.6 V**

(ii) $t = 0.04$ s

This time equals two time-constants.

$\therefore \quad v_c = 31.6 + 0.632\,(50 - 31.6) =$ **43.2 V**

(iii) $t = 0.06$ s

This time equals three time-constants.

$\therefore \quad v_c = 43.2 + 0.632\,(50 - 43.2) =$ **47. 5 V**

Since in one hour, steady-state conditions would be established, v_c would have achieved its maximum possible value of **50 V**.

(*b*) **(*i*)** $t = 0.02$ s, $i_c = 0.37 \times 25 =$ **9.25 μA**

(*ii*) $t = 0.4$ s, $i_c = 0.37 \times 9.25 =$ **3.4 μA**

(*iii*) $t = 0.06$ s, $i_c = 0.37 \times 3.4 =$ **1.26 μA**

(*iv*) $t = 0.1$ s, This time equals 5 time constants. In this time, current falls almost to **zero** value.

Example 5.42. *A voltage as shown in Fig. 5.43 (a) is applied to a series circuit consisting of a resistance of 2 Ω in series with a pure capacitor of 100 μF. Determine the voltage across the capacitor at t = 0.5 millisecond.* **[Bombay University, 2000]**

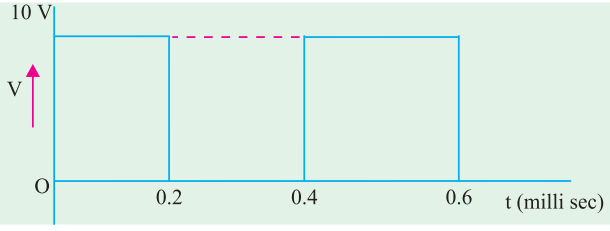


Fig. 5.34 (*a*)

Solution.

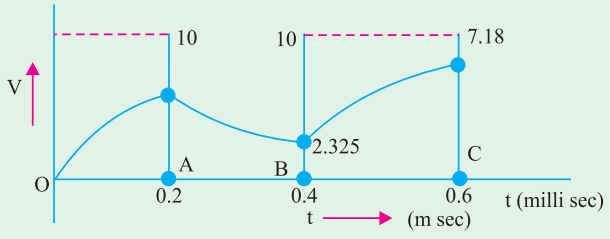


Fig. 5.34 (*b*)

$\tau = RC = 0.2$ milli-second

Between 0 and 0.2 m sec;

$v(t) = 10\,[1 - \exp(-t/\tau)]$

At $t = 0.2$, $v(t) = 6.32$ volts

Between 0.2 and 0.4 m Sec, counting time from A indicating it as t_1

$v(t_1) = 6.32 \exp(t_1/\tau)$

At point B, $t_1 = 0.2$, $V = 2.325$

Between 0.4 and 0.6 m Sec, time is counted from β with variable as t_2,

$$v(t_2) = 2.325 + (10 - 2.325)\,[1 - \exp(-t_2/\tau)]$$

At C, $t_2 = 0.2$, $V = 7.716$ volts.

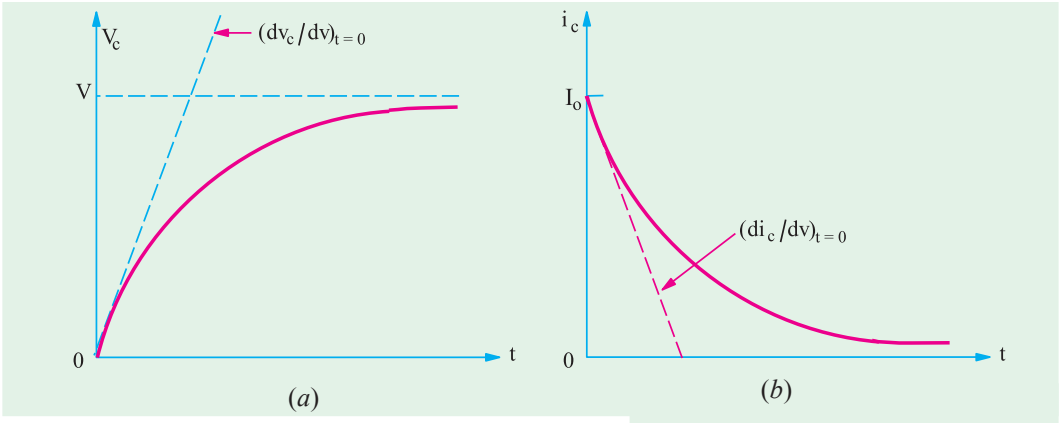
5.21. Transient Relations During Capacitor Charging Cycle

Whenever a circuit goes from one steady-state condition to another steady-state condition, it passes through a transient state which is of short duration. The first steady-state condition is called the ***initial condition*** and the second steady-state condition is called the ***final condition***. In fact, transient condition lies in between the initial and final conditions. For example, when switch S in Fig. 5.35 (a) is not connected either to a or b, the RC circuit is in its initial steady state with no current and

hence no voltage drops. When S is shifted to point a, current starts flowing through R and hence, transient voltages are developed across R and C till they achieve their final steady values. The period during which current and voltage changes take place is called ***transient condition***.

The moment switch S is shifted to point 'a' as shown in Fig. 5.35 (b), a charging current i_c is set up which starts charging C that is initially uncharged. At the beginning of the transient state, i_c is maximum because there is no potential across C to oppose the applied voltage V. It has maximum value = $V/R = I_0$. It produces maximum voltage drop across $R = i_c R = I_0 R$. Also, initially, $v_c = 0$, but as time passes, i_c decreases gradually so does v_R but v_c increases exponentially till it reaches the final steady value of V. Although V is constant, v_R and v_c are variable. However, at any time $V = v_R + v_c = i_c R + v_c$.

At the beginning of the transient state, $i_c = I_0$, $v_c = 0$ but $v_R = \text{V}$. At the end of the transient state, $i_c = 0$ hence, $v_R = 0$ but $v_c = \text{V}$.



The initial rates of change of v_c, v_R and i_c are given by

$$\left(\frac{dv_c}{dt}\right)_{t=0} = \frac{V}{\lambda} \text{ volt/second,}$$

$$\left(\frac{dv_R}{dt}\right)_{t=0} = \frac{I_0 R}{\lambda} = -\frac{V}{\lambda} \text{ volt/second}$$

$$\left(\frac{di_c}{dt}\right)_{t=0} = \frac{I_0}{\lambda} \text{ where } I_0 = \frac{V}{R}$$

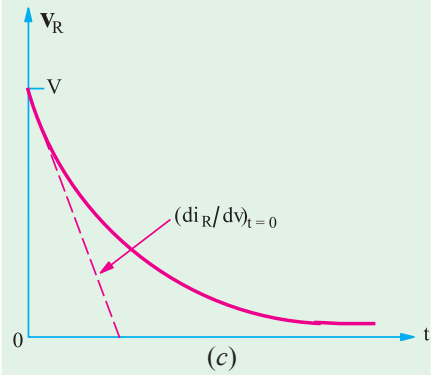


Fig. 5.35

These are the initial rates of change. However, their rate of change at any time during the charging transient are given as under :

$$\frac{dv_c}{dt} = \frac{V}{\lambda} e^{-t/\lambda}; \frac{di_c}{dt} = -\frac{dv_R}{dt} = -\frac{V}{\lambda} e^{-t/\lambda}$$

It is shown in Fig. 5.35 (c).

It should be clearly understood that a negative rate of change means a decreasing rate of change. It does not mean that the concerned quantity has reversed its direction.

5.22. Transient Relations During Capacitor Discharging Cycle

As shown in Fig. 5.36 (b), switch S has been shifted to b. Hence, the capacitor undergoes the discharge cycle. Just before the transient state starts, $i_c = 0$, $v_R = 0$ and $v_c = \text{V}$. The moment transient

state begins, i_c has maximum value and decreases exponentially to zero at the end of the transient state. So does v_c. However, during discharge, all rates of change have polarity opposite to that during charge. For example, dv_c/dt has a positive rate of change during charging and negative rate of change during discharging.

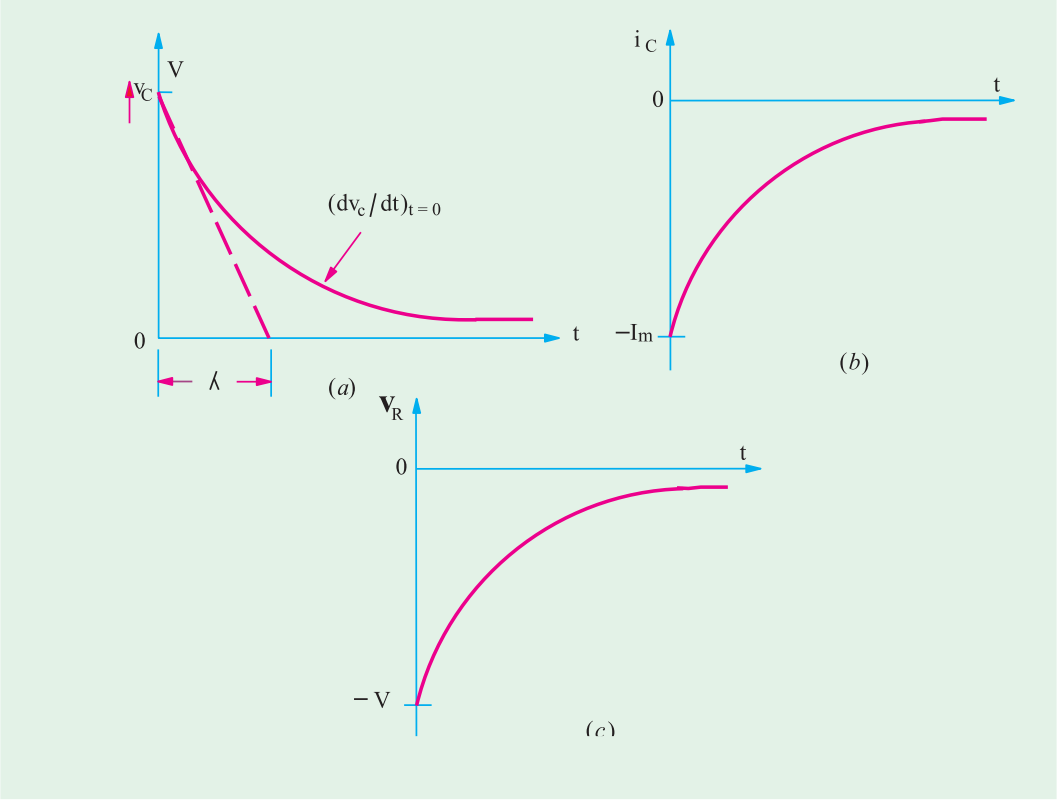


Fig. 5.36

Also, it should be noted that during discharge, v_c maintains its original polarity whereas i_c reverses its direction of flow. Consequently, during capacitor discharge, v_R also reverses its direction.

The various rates of change at *any time* during the discharge transients are as given in Art.

$$\frac{dv_c}{dt} = -\frac{V}{\lambda} e^{-t/\lambda}; \frac{di_c}{dt} = \frac{I_0}{\lambda} e^{-t/\lambda}; \frac{dv_R}{dt} = \frac{V}{\lambda} e^{-t/\lambda}$$

These are represented by the curves of Fig. 5.32.

5.23. Charging and Discharging of a capacitor with Initial Charge

In Art. 5.18, we considered the case when the capacitor was initially uncharged and hence, had no voltage across it. Let us now consider the case, when the capacitor has an initial potential of V_0 (less than V) which opposes the applied battery voltage V as shown in Fig. 5.37 (*a*).

As seen from Fig. 5.37 (*b*), the initial rate of rise of v_c is now somewhat less than when the capacitor is initially uncharged. Since the capacitor voltage rises from an initial value of v_0 to the final value of V in one time constant, its initial rate of rise is given by

$$\left(\frac{dv_c}{dt}\right)_{t=0} = \frac{V - V_0}{\lambda} = \frac{V - V_0}{RC}$$

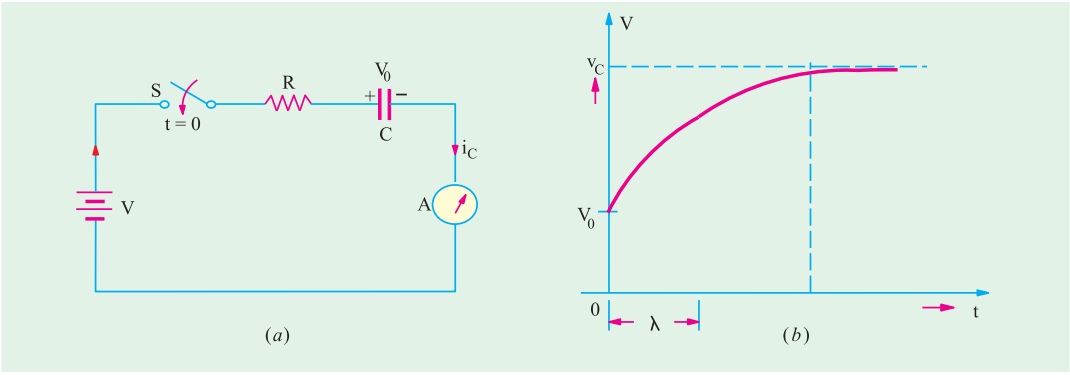


Fig. 5.37

The value of the capacitor voltage at *any time* during the charging cycle is given by

$$v_c = (V - V_0)(1 - e^{-t/\lambda}) + V_0$$

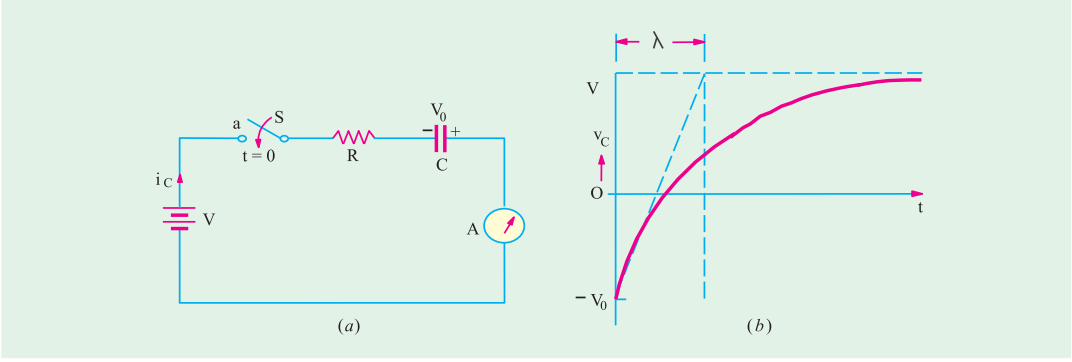


Fig. 5.38

However, as shown in Fig. 5.38 (*a*), if the initial capacitor voltage is negative with respect to the battery voltage *i.e.* the capacitor voltage is series aiding the battery voltage, rate of change of v_c is steeper than in the previous case. It is so because as shown in Fig. 5.38 (*b*), in one time period, the voltage change = $V - (-V_0) = (V + V_0)$. Hence, the initial rate of change of voltage is given by

$$\left(\frac{dv_c}{dt}\right)_{t=0} = \frac{V + V_0}{\lambda} = \frac{V + V_0}{RC}$$

The value of capacitor voltage at *any time* during the charging cycle is given by

$$v_c = (V + V_0)\ (1 - e^{-t/\lambda}) - V_0$$

The time required for the capacitor voltage to attain any value of v_c during the charging cycle is given by

$$t = \lambda\, ln\left(\frac{V - V_0}{V - v_c}\right) = RC\, ln\left(\frac{V - V_0}{V - v_c}\right) \quad \text{... when } V_0 \text{ is positive}$$

$$t = \lambda\, ln\left(\frac{V + V_0}{V - v_c}\right) = RC\, ln\left(\frac{V + V_0}{V - v_c}\right) \quad \text{... when } V_0 \text{ is negative}$$

Example 5.43. *In Fig. 5.39, the capacitor is initially uncharged and the switch S is then closed. Find the values of I, I_1, I_2 and the voltage at the point A at the start and finish of the transient state.*

Solution. At the moment of closing the switch *i.e.* at the start of the transient state, the capacitor acts as a short-circuit. Hence, there is only a resistance of 2 Ω in the circuit because 1 Ω resistance is shorted out thereby grounding point *A*. Hence, $I_1 = 0$; $I = I_2 = 12/2 = 6$A. Obviously, $V_A = 0$ V.

At the end of the transient state, the capacitor acts as an open-circuit. Hence,

$I_2 = 0$ and $I = I_1 = 12/(2 + 1)$
$= 4$ A. $V_A = 6$ V.

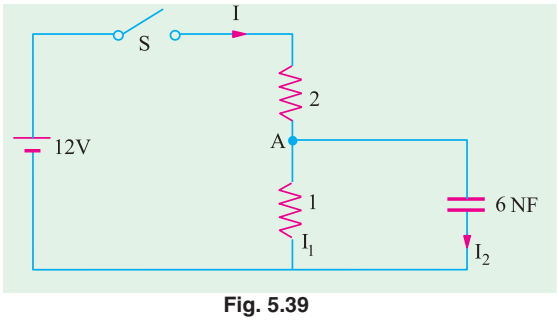


Fig. 5.39

Example 5.44. *Calculate the values of i_2, i_3, v_2, v_3, v_a, v_c and v_L of the network shown in Fig. 5.40 at the following times :*

(i) *At time, $t = 0+$ immediately after the switch S is closed ;*

(ii) *At time, $t \to \infty$ i.e. in the steady state.* **(Network Analysis AMIE Sec. B Winter 1990)**

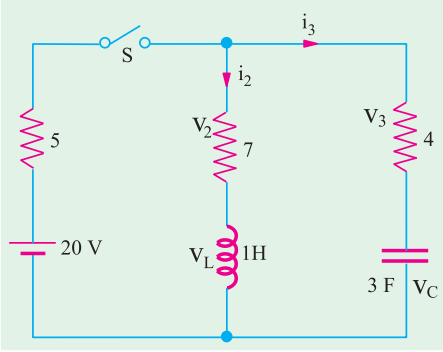


Fig. 5.40

Solution. ***(i)*** In this case the coil acts as an open circuit, hence $i_2 = 0$; $v_2 = 0$ and $v_L = 20$ V.

Since a capacitor acts as a short circuit $i_3 = 20/(5 + 4) = 9 = 20/9$ A. Hence, $v_3 = (20/9) \times 4 = 80/9$ V and $v_c = 0$.

(ii) Under steady state conditions, capacitor acts as an open circuit and coil as a short circuit. Hence, $i_2 = 20/(5 + 7) = 20/12 = 5/3$ A; $v_2 = 7 \times 5/3 = 35/3$ V; $v_L = 0$. Also $i_3 = 0$, $v_3 = 0$ but $v_c = 20$ V.

Example 5.45. *If in the RC circuit of Fig. 5.36; R = 2 M Ω, C = 5 m F and V = 100 V, calculate*

(a) *initial rate of change of capacitor voltage*

(b) *initial rate of change of capacitor current*

(c) *initial rate of change of voltage across the 2 M Ω resistor*

(d) *all of the above at t = 80 s.*

Solution. ***(a)*** $\left(\frac{dv_c}{dt}\right)_{t=0} = \frac{V}{} \quad \frac{100}{2 \quad 10^6 \quad 5 \quad 10^{-6}} \quad \frac{100}{10}$ **10 V/s**

(b) $\left(\frac{di_c}{dt}\right)_{t=0} = \frac{I_0}{} \quad \frac{V/R}{} \quad \frac{100/2 \quad 10^6}{10}$ **–5μA/s**

(c) $\left(\frac{dv_R}{dt}\right)_{t=0} = \frac{V}{} \quad \frac{100}{10}$ **–10 V/s**

(d) All the above rates of change would be zero because the transient disappears after about $5\lambda = 5 \times 10 = 50$ s.

Example 5.46. *In Fig. 5.41 (a), the capacitor C is fully discharged, since the switch is in position 2. At time t = 0, the switch is shifted to position 1 for 2 seconds. It is then returned to position 2 where it remains indefinitely. Calculate*

(a) *the maximum voltage to which the capacitor is charged when in position 1.*

(b) *charging time constant λ_1 in position 1.*

(c) *discharging time constant λ_2 in position 2.*

(d) *v_c and i_c at the end of 1 second in position 1.*

(e) *v_c and i_c at the instant the switch is shifted to positon 2 at t = 1 second.*

(f) *v_c and i_c after a lapse of 1 second when in position 2.*

(g) *sketch the waveforms for v_c and i_c for the first 2 seconds of the above switching sequence.*

Solution. **(a)** We will first find the voltage available at terminal 1. As seen the net battery voltage around the circuit = 40 –10 = 30 V. Drop across 30 K resistor = 30 × 30/(30 + 60) = 10 V. Hence, potential of terminal 1 with respect to ground G = 40 –10 = 30 V. Hence, capacitor will charge to a maximum voltage of 30 V when in position 1.

(b) Total resistance, R = [(30 K || 60 K) + 10 K] = 30 K

$\therefore$ $\lambda_1 = RC$ = 30 K × 10 μF = 0.3 *s*

(c) λ_2 = 10 K × 10 μ F = 0.1 s

(d) $v_C = V\,(1 - e^{-t/\lambda 1}) = 30\,(1 - e^{-1/0.3}) = 28.9$ V

$$i_C = \frac{V}{R}\,e^{-t/\lambda_1} = \frac{30\text{ V}}{30\text{ K}}\,e^{-1/0.03} = 1 \times 0.0361 = 0.036\text{ mA}$$

(e) v_C = 28.9 V at $t = 1^+$S at position 2 but i_C = –28.9 V/10 K = –2/89 mA at $t = 1^+$s in position 2.

(f) $v_C = 28.9\,e^{-t/\lambda_2} = 28.9\,e^{-1/0.1} = 0.0013\text{ V} = 0$ V.

$$i_C = 28.9\,e^{-t/\lambda_2} = -2.89\,e^{-1/0.1} = 0.00013\text{ mA} \cong 0.$$

The waveform of the capacitor voltage and charging current are sketched in Fig. 5.41 (*b*).

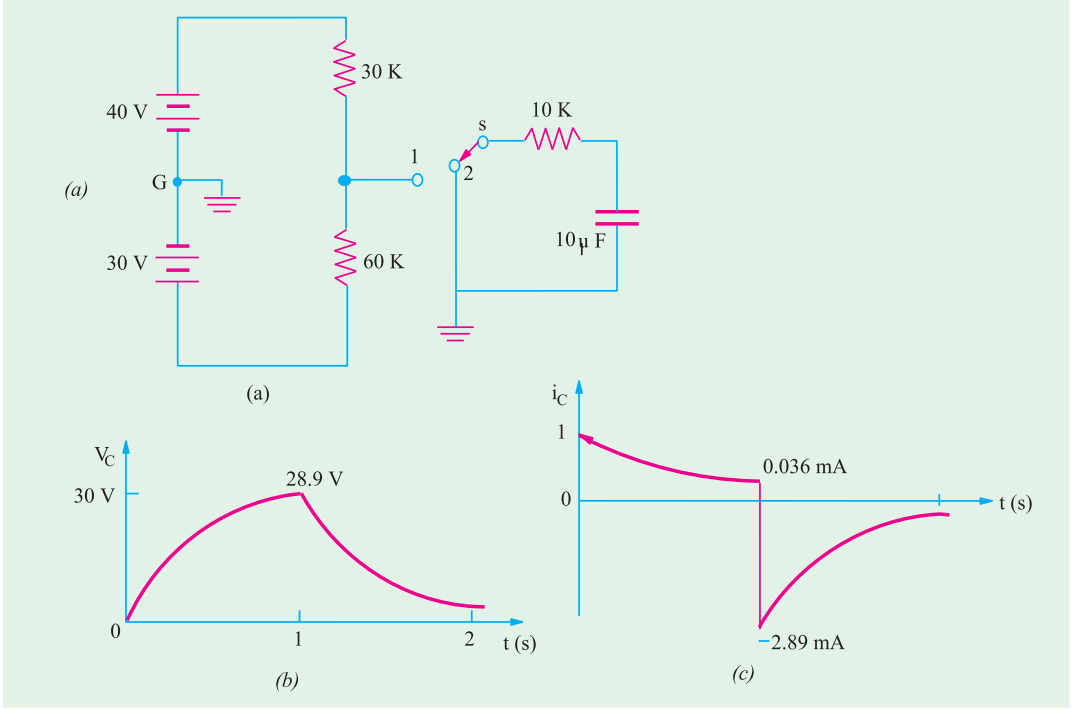


Fig. 5.41

Example 5.47. *In the RC circuit of Fig. 5.42, R = 2 M Ω and C = 5 μF, the capacitor is charged to an initial potential of 50 V. When the switch is closed at $t = 0^+$, calculate*

(a) *initial rate of change of capacitor voltage and*

(b) *capacitor voltage after a lapse of 5 times the time constant i.e. 5λ.*

If the polarity of capacitor voltage is reversed, calculate

(c) *the values of the above quantities and*

(d) *time for v_c to reach –10 V, 0 V and 95 V.*

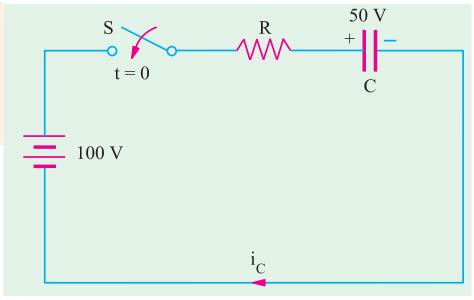


Fig. 5.42

Solution. (a) $\left(\frac{dv_c}{dt}\right)_{t=0} = \frac{V - V_0}{\lambda}$

$= \frac{V - V_0}{RC} = \frac{100 - 50}{10} =$ **5 V/s**

(b) $v_C = (V - V_0)\,(1 - e^{-t/\lambda}) + V_0$

$= (100 - 50)\,(1 - e^{-5\,\lambda/\lambda}) = 50 = 49.7 + 50 =$ **99.7 V**

(c) When $V_0 = -50$ V, $\left(\frac{dv_c}{dt}\right)_{t=0} = \frac{V - (-V_0)}{\lambda} = \frac{V + V_0}{\lambda} = \frac{150}{10} =$ **15 V/s**

$v_C = (V - V_0)\,(1 - e^{-t/\lambda}) + V_0 = [100 - (-50)]\,(1 - e^{-5}) + (-50)$

$= 150\,(1 - e^{-5}) - 50 =$ **99 V.**

(d) $t = \lambda\, ln\left(\frac{V - V_0}{V - v_c}\right) = 10\, ln\left[\frac{100 - (-50)}{100 - (-10)}\right] = 10\, ln\left(\frac{150}{110}\right) =$ **3.1 s**

$t = 10\, ln\left[\frac{100 - (-50)}{100 - (0)}\right] = 10\, ln\left(\frac{150}{100}\right) =$ **4.055 s**

$t = 10\, ln\left[\frac{100 - (-50)}{100 - 95}\right] = 10\, ln\left(\frac{150}{5}\right) =$ **34 s**

Example 5.48. *The uncharged capacitor, if it is initially switched to position 1 of the switch for 2 sec and then switched to position 2 for the next two seconds. What will be the voltage on the capacitor at the end of this period ? Sketch the variation of voltage across the capacitor.* **[Bombay University 2001]**

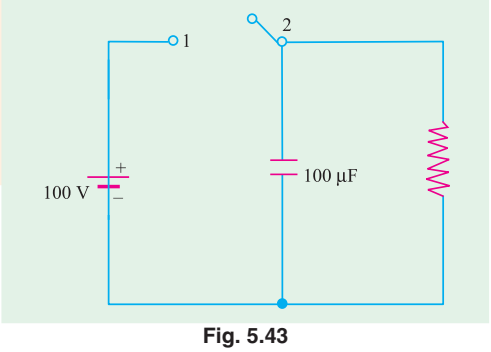


Fig. 5.43

Solution. Uncharged capacitor is switched to position 1 for 2 seconds. It will be charged to 100 volts instantaneously since resistance is not present in the charging circuit. After 2 seconds, the capacitor charged to 100 volts will get discharged through *R-C* circuit with a time constant of

$$\tau = RC = 1500 \times 10^{-3} = 1.5 \text{ sec.}$$

Counting time from instant of switching over to positon 2, the expression for voltage across the capacitor is $V(t) = 100 \exp(-t/\tau)$

After 2 seconds in this position,

$$v(t) = 100 \exp(-2/1.5) = 26/36 \text{ Volts.}$$

Example 5.49. *There are three passive elements in the circuit below and a voltage and a current are defined for each. Find the values of these six qualities at both $t = 0^-$ and $t = 0^+$.* **[Bombay University, 2001]**

Solution. Current source 4 $u(t)$ means a step function of 4 amp applied at $t = 0$. Other current source of 5 amp is operative throughout.

At $t = 0^-$, 5 amp source is operative. This unidirectional constant current establishes a steady current of 5 amp through 30-ohm resistor and 3-H inductor. Note that positive V_R means a rise from right to left.

At $t = 0$

$V_R = -150$ Volts (Since right-terminal of Resistor is + ve)

$i_L = 5$ amp

$V_L = 0$, it represents the voltage between B and O.

$i_C = 0$

$V_C = 150$ volts $= V_{BO}$ + (Voltage between A and B with due regards to sign).

$= 0 - (-150) = +150$ volts

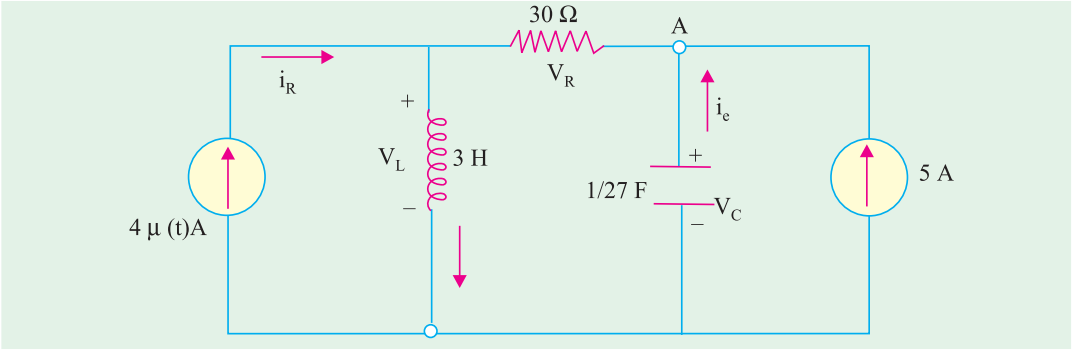


Fig. 5.44 (*a*)

At $t = O_+$, 4 amp step function becomes operative. Capacitive-voltage and Inductance-current cannot change abruptly.

Hence $i_L(0^+) = 5$ amp

$V_C(O_+) = 150$ amp

$V_C(O_+) = 150$ volts, with node A positive with respect to 0.

With these two values known, the waveforms for current sources are drawn in Fig. 5.44 (*b*).

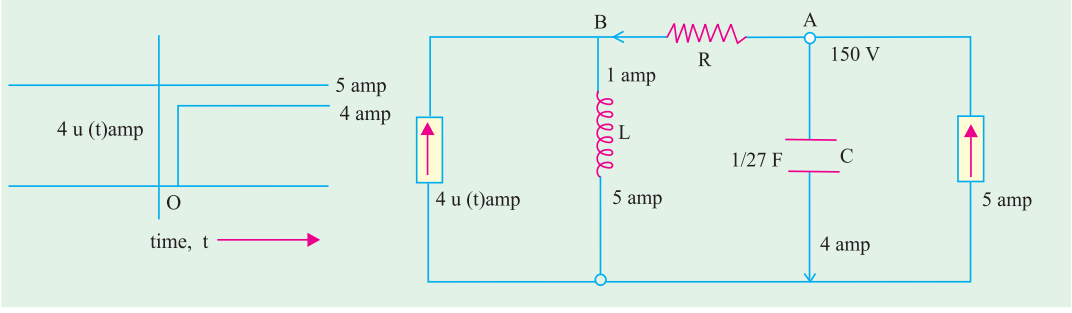


Fig. 5.44 (*b*) **Fig. 5.44 (*c*)**

Remaining four parameters are evaluated from Fig. 5.44 (*c*).

$V_L = V_B = V_A - (30 \times 1) = 120$ Volts

$i_R = 1$ amp, $V_R = -30$ Volts

$i_C = 4$ amp in downward direction.

Additional Observation. After 4 amp source is operative, final conditions (at t tending to infinity) are as follows.

Inductance carries a total direct current of 9 amp, with $V_L = 0$.

Hence, $V_B = 0$.

$i_R = 5$ amp, $V_R = -150$ volts

$V_C =$ **150 volts,** $i_C =$ **0**

Example 5.50. *The voltage as shown in Fig. 5.45 (a) is applied across –***(i)** *A resistor of 2 ohms* ***(ii)*** *A capacitor of 2 F. Find and sketch the current in each case up to 6 seconds.*

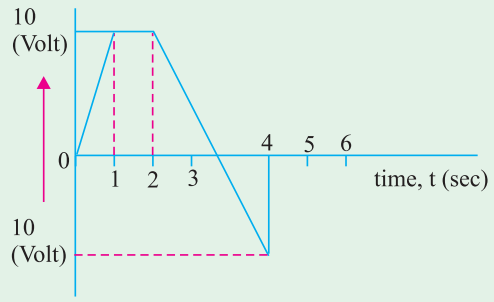


Fig. 5.45 (a)

[Bombay University 1998]

Solution.

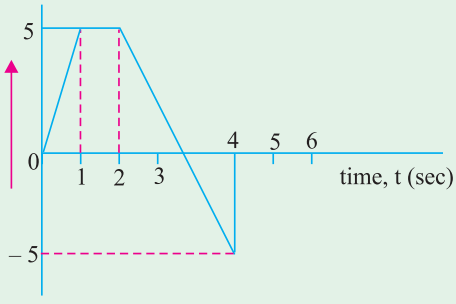


Fig. 5.45 (*b*) Current in a Resistor of 2 ohms $i_R = V(t)/2$ amp

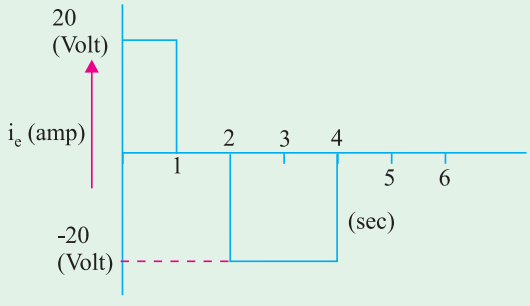


Fig. 5.45 (*c*) Current thro 2-F capacitor, $i_C = C\,(dv/dt)$

Example 5.51. *Three capacitors 2 μF, 3 μF, and 5 μF are connected in series and charged from a 900 V d.c. supply. Find the voltage across condensers. They are then disconnected from the supply and reconnected with all the + ve plates connected together and all the −ve plates connected together. Find the voltages across the combinations and the charge on each capacitor after reconnections. Assume perfect insulation.* **[Bombay University, 1998]**

Solution. The capacitors are connected in series. If C is the resultant capacitance.

$$I/C = I/C_1 + I/C_2 = I/C_3, \text{ which gives } C = (30/31)\ \mu F$$
$$V_1 = 900 \times (30/31)/2 = 435.5 \text{ volts}$$
$$V_2 = 900 \times (30/31)/3 = 290.3 \text{ volts}$$
$$V_3 = 900 \times (30/31)/5 = 174.2 \text{ volts}$$

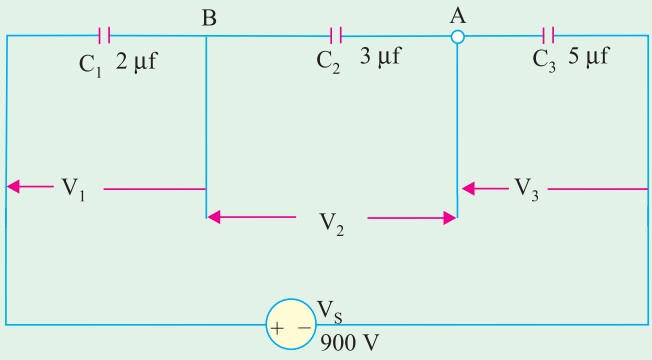


Fig. 5.46

In series connection, charge held by each capacitor is same. If it is denoted by Q.

$$Q = 435 \times 2 \times 10^{-6} = 871 \text{ µ coulombs}$$

Three capacitors hold a total charge of $(3 \times 871) = 2613$ µ coulombs

With parallel connection of these three capacitors, equivalent capacitance, $C' = C_1 + C_2 + C_3 = 10\mu F$

Since, $Q' = C', 2613 \times 10^{-6} = 10 \times 10^{-6} \times V'$

or $V' = 261$ volts.

Charge on each capacitor after reconnection is as follows :

$$Q_1' = C_1 V_1 = 2 \times 10^{-6} \times 261 = \mathbf{522\ \mu\text{-coulombs}}$$
$$Q_2' = C_2 V_1 = 3 \times 10^{-6} \times 261 = \mathbf{783\ \mu\text{-coulombs}}$$
$$Q_3' = C_3 V_2 = 5 \times 10^{-6} \times 261 = \mathbf{1305\ \mu\text{-coulombs}}$$

Tutorial Problems No. 5.3

1. For the circuit shown in Fig. 5.47 calculate (*i*) equivalent capacitance and (*ii*) voltage drop across each capacitor. All capacitance values are in µF.

[(*i*) 6 µF (*ii*) $V_{AB} = 50$ V, $V_{BC} = 40$ V]

2. In the circuit of Fig. 5.48 find (*i*) equivalent capacitance (*ii*) drop across each capacitor and (*iii*) charge on each capacitor. All capacitance values are in µF.

[(*i*) 1.82 µF (*ii*) $V_1 = 50$ V; $V_2 = V_3 = 20$ V; $V_4 = 40$ V
(*iii*) $Q_1 = 200$ µC; $Q_2 = 160$ µC; $Q_3 = 40$ µC; $Q_4 = 200$ µC]

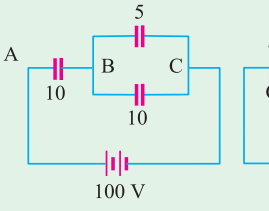


Fig. 5.47

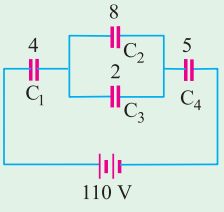


Fig. 5.48

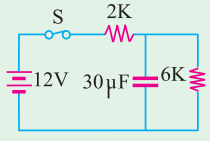


Fig. 5.49

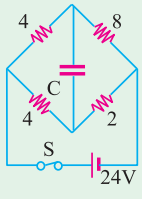


Fig. 5.50

3. With switch in Fig. 5.49 closed and steady-state conditions established, calculate ***(i)*** steady-state current ***(ii)*** voltage and charge across capacitor ***(iii)*** what would be the discharge current at the instant of opening the switch ?

[(*i*) 1.5 mA (*ii*) 9V; 270 µC (*iii*) 1.5 mA]

4. When the circuit of Fig. 5.50 is in steady state, what would be the p.d. across the capacitor ? Also, find the discharge current at the instant *S* is opened.

[8 V; 1.8 A]

5. Find the time constant of the circuit shown in Fig. 5.51. **[200 μS]**

6. A capacitor of capacitance 0.01 μF is being charged by 1000 V d.c. supply through a resistor of 0.01 megaohm. Determine the voltage to which the capacitor has been charged when the charging current has decreased to 90 % of its initial value. Find also the time taken for the current to decrease to 90% of its initial value. **[100 V, 0.1056 ms]**

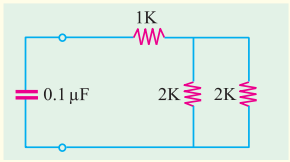


Fig. 5.51

7. An 8 μF capacitor is being charged by a 400 V supply through 0.1 mega-ohm resistor. How long will it take the capacitor to develop a p.d. of 300 V ? Also what fraction of the final energy is stored in the capacitor ? **[1.11 Second, 56.3% of full energy]**

8. An 10 μF capacitor is charged from a 200 V battery 250 times/second and completely discharged through a 5 Ω resistor during the interval between charges. Determine
(*a*) the power taken from the battery.
(*b*) the average value of the current in 5 Ω resistor. **[(*a*) 50 W (*b*) 0.5 A]**

9. When a capacitor, charged to a p.d. of 400 V, is connected to a voltmeter having a resistance of 25 MΩ, the voltmeter reading is observed to have fallen to 50 V at the end of an interval of 2 minutes. Find the capacitance of the capacitor. **[2.31 μF] *(App. Elect. London Univ.)***

10. A capacitor and a resistor are connected in series with a d.c. source of V volts. Derive an expression for the voltage across the capacitor after *'t'* seconds during discharging. ***(Gujrat University, Summer 2003)***

11. Derive an expression for the equivalent capacitance of a group of capacitors when they are connected (*i*) in parallel (*ii*) in series. ***(Gujrat University, Summer 2003)***

12. The total capacitance of two capacitors is 0.03 μ*F* when joined in series and 0.16 μ*F* when connected in parallel. Calculate the capacitance of each capacitor. ***(Gujrat University, Summer 2003)***

13. In a capacitor with two plates separated by an insulator 3mm thick and of relative permittivity of 4, the distance between the plates is increased to allow the insertion of a second insulator 5mm thick and relative permittivity E. If the capacitance so formed is one third of the original capacitance, find *E*. ***(V.TU., Belgaum Karnataka University, February 2002)***

14. Derive an expression for the capacitance of a parallel plate capacitor. ***(V.TU., Belgaum Karnataka University, Summer 2002)***

15. Three capacitors A, B and C are charged as follows
A = 10μF, 100 V B = 15μF, 150 V C = 25μF, 200 V
They are connected in parallel with terminals of like polarities together. Find the voltage across the combination. ***(V.TU., Belgaum Karnataka University, Summer 2002)***

16. Prove that average power consumed by a pure capacitance is zero. ***(V.TU., Belgaum Karnataka University, Summer 2002)***

17. Current drawn by a pure capacitor of 20μF is 1.382A from 220V AC supply. What is the supply frequency? ***(V.TU., Belgaum Karnataka University, Summer 2003)***

18. Find the equivalent capacitance between the points A and B of the network shown in fig. 1. ***(V.TU., Belgaum Karnataka University, Summer 2003)***

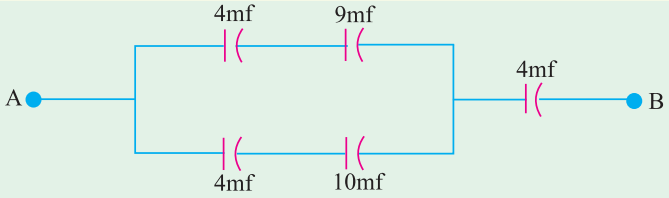


Fig. 5.52

19. Three capacitors of 1, 2 and 3 micro farads are connected in series across a supply voltage of 100V. Find the equivalent capacitance of the combination and energy stored in each capacitor. ***(Mumbai University 2003) (V.T.U. Belgaum Karnataka University, Wimter 2003)***

20. Consider a parallel plate capacitor, the space between which is filled by two dielectric of thickness d_1 and d_2 with relative permittivities $\in_1$ and $\in_2$ respectively. Derive an expression for the capacitance between the plates. ***(V.T.U. Belgaum Karnataka University, Wimter 2004)***

21. A capacitor consists of two plates of area $0.16m^2$ spaced 6mm apart. This space is filled with a layer of 1mm thick paper of relative permittivity 2, and remaining space with glass of relative permittivity 5. A dc voltage of 10kV is applied between the plates. Determine the electric field strength in each dielectric. ***(V.T.U. Belgaum Karnataka University, Wimter2004)***

22. In a give R-L circuit, R = 35Ω and L = 0.1H. Find (*i*) current through the circuit (*ii*) power factor if a 50 Hz frequency, voltage V = 220∠30° is applied across the circuit. ***(RGPV Bhopal 2001)***

23. Three voltage represented by $e_1 = 20 \sin \omega t$, $e_2 = 30 \sin (\omega t = 45°)$ and $e_3 = \sin (\omega t + 30°)$ are connected in series and then connected to a load of impedance $(2 + j\ 3)\ \Omega$ Find the resultant current and power factor of the circuit. Draw the phasor diagram. ***(Mumbai University, 2002) (RGPV Bhopal 2001)***

OBJECTIVE TESTS – 5

1. A capacitor consists of two
(*a*) insulation separated by a dielectric
(*b*) conductors separated by an insulator
(*c*) ceramic plates and one mica disc
(*d*) silver-coated insulators

2. The capacitance of a capacitor is NOT influenced by
(*a*) plate thickness
(*b*) plate area
(*c*) plate separation
(*d*) nature of the dielectric

3. A capacitor that stores a charge of 0.5 C at 10 volts has a capacitance of .....farad.
(*a*) 5 (*b*) 20
(*c*) 10 (*d*) 0.05

4. If dielectric slab of thickness 5 mm and $\varepsilon_r = 6$ is inserted between the plates of an air capacitor with plate separation of 8 mm, its capacitance is
(*a*) decreased (*b*) almost doubled
(*c*) almost halved (*d*) unaffected

5. For the circuit shown in the given figure, the current through *L* and the voltage across C_2 are respectively

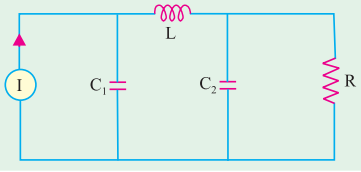


Fig. 5.53

(*a*) zero and RI
(*b*) I and zero
(*c*) zero and zero
(*d*) I and RI ***(ESE 2001)***

6. A parallel plate capacitor has an electrode area of 100 mm^2, with a spacing of 0.1 mm between the electrodes. The dielectric between the plates is air with a permittivity of 8.85×10^{-12} F/m. The charge on the capacitor is 100 V. the stored energy in the capacitor is
(*a*) 8.85 pJ (*b*) 440 pJ
(*c*) 22.1 nJ (*d*) 44.3 nJ
(GATE 2003)

7. A composite parallel plate capacitor is made up of two different dielectric materials with different thicknesses (t_1 and t_2) as shown in Fig.5.54. The two different dielectric materials are separates by a conducting foil *F*. The voltage of the conducting foil is

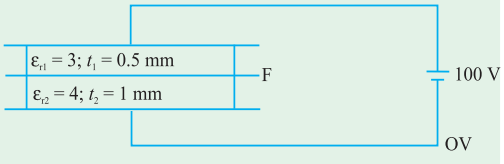


Fig. 5.54

(*a*) 52 V (*b*) 60 V
(*c*) 67 V (*d*) 33 V
(GATE 2003)

ANSWERS

1. *b* **2.** *a* **3.** *d* **4.** *b*

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CHAPTER 6

MAGNETISM AND ELECTRO-MAGNETISM

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- Susceptibility (K)
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Designing high speed magnetic levitation trains is one of the many applications of electromagnetism. Electromagnetism defines the relationship between magnetism and electricity

6.1. Absolute and Relative Permeabilities of a Medium

The phenomena of magnetism and electromagnetism are dependent upon a certain property of the medium called its permeability. Every medium is supposed to possess two permeabilities :

(i) absolute permeability (μ) and ***(ii)*** relative permeability (μ_r).

For measuring relative permeability, vacuum or free space is chosen as the reference medium. It is allotted an absolute permeability of $\mu_0 = 4\pi \times 10^{-7}$ henry/metre. Obviously, relative permeability of vacuum with reference to itself is unity. Hence, for free space,

absolute permeability $\mu_0 = 4\pi \times 10^{-7}$ H/m

relative permeability $\mu_r = 1$.

Now, take any medium other than vacuum. If its relative permeability, as compared to vacuum is μ_r, then its absolute permeability is $\mu = \mu_0 \mu_r$ H/m.

6.2. Laws of Magnetic Force

Coulomb was the first to determine experimentally the quantitative expression for the magnetic force between two *isolated* point poles. It may be noted here that, in view of the fact that magnetic poles always exist in pairs, it is impossible, in practice, to get an isolated pole. The concept of an isolated pole is purely theoretical. However, poles of a thin but long magnet may be assumed to be point poles for all practical purposes (Fig. 6.1). By using a torsion balance, he found that the force between two magnetic poles placed in a medium is

(i) directly proportional to their pole strengths

(ii) inversely proportional to the square of the distance between them and

(iii) inversely proportional to the absolute permeability of the surrounding medium.



Fig. 6.1 **Fig. 6.2**

For example, if m_1 and m_2 represent the magnetic strength of the two poles (its unit as yet being undefined), r the distance between them (Fig. 6.2) and μ the absolute permeability of the surrounding medium, then the force F is given by

$$F \propto \frac{m_1 m_2}{\mu r^2} \quad \text{or} \quad F = k\frac{m_1 m_2}{\mu r^2} \quad \text{or} \quad \vec{F} = \frac{k\, m_1 m_2}{\mu r^2}\hat{r} \quad \text{in vector from}$$

where $\hat{r}$ is a unit vector to indicate direction of r.

or $$\vec{F} = k\frac{m_1 m_2}{r^3}\vec{r} \quad \text{where } \vec{F} \text{ and } \vec{r} \text{ are vectors}$$

In the S.I. system of units, the value of the constant k is $= 1/4\pi$.

$$F = \frac{m_1 m_2}{4\pi\mu r^2}\text{ N} \quad \text{or} \quad F = \frac{m_1 m_2}{4\pi\mu_0 \mu_r r^2}\text{ N} \qquad \text{– in a medium}$$

In vector form, $$\vec{F} = \frac{m_1 m_2}{4\pi\mu r^3}\vec{r} = \frac{m_1 m_2}{4\pi\mu_0 r^2}\text{N}$$

If, in the above equation,

$$m_1 = m_2 = m \text{ (say)} ; r = 1 \text{ metre} ; F = \frac{1}{4\pi\mu_0}\text{ N}$$

Then $m^2 = 1$ or $m = \pm 1$ weber*

Hence, a unit magnetic pole may be defined as ***that pole which when placed in vacuum at a distance of one metre*** from a similar and equal pole repels it with a force of $1/4\pi\ \mu_0$ ***newtons***.**

6.3. Magnetic Field Strength (H)

Magnetic field strength at any point within a magnetic field is numerically equally to the force experienced by a *N*-pole of one weber placed at that point. Hence, unit of *H* is N/Wb.

Suppose, it is required to find the field intensity at a point *A* distant *r* metres from a pole of *m* webers. Imagine a similar pole of one weber placed at point *A*. The force experienced by this pole is

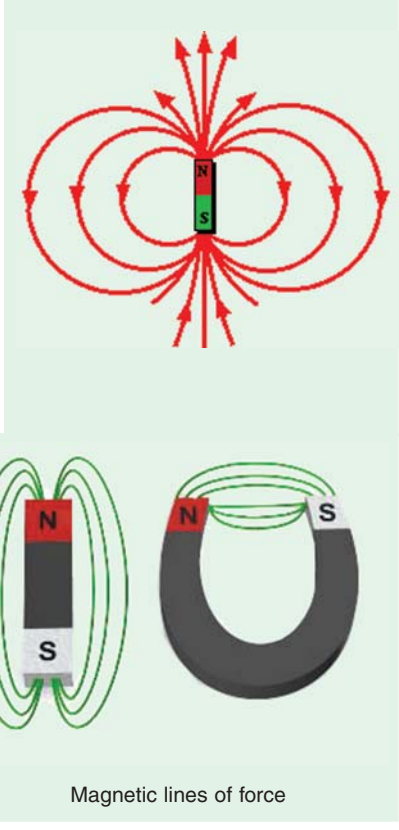
$$F = \frac{m \times 1}{4\pi\mu_0 r^2}\text{ N} \qquad \therefore\ H = \frac{m}{4\pi\mu_0 r^3}\text{ N/Wb (or A/m)}^{***}\text{ or oersted.}$$

Also, if a pole of *m* Wb is placed in a uniform field of strength *H* N/Wb, then force experienced by the pole is = *mH* newtons.

It should be noted that field strength is a vector quantity having both magnitude and direction

$$\therefore \qquad \vec{H} = \frac{m}{4\pi\mu_0 r^2} \qquad \hat{r} = \frac{m}{4\ _0 r^3}\, r$$

It would be helpful to remember that following terms are sometimes interchangeably used with field intensity : Magnetising force, strength of field, magnetic intensity and intensity of magnetic field.



Magnetic lines of force

6.4. Magnetic Potential

The magnetic potential at any point within a magnetic field is measured by the work done in shifting a *N*-pole of one weber from infinity to that point against the force of the magnetic field. It is given by

$$M = \frac{m}{4\pi\mu_0 r}\text{ J/Wb}$$

...(Art. 4.13)

It is a scalar quantity.

6.5. Flux per Unit Pole

A unit *N*-pole is supposed to radiate out a flux of one weber. Its symbol is Φ. Therefore, the flux coming out of a *N*-pole of *m* weber is given by

$$\Phi = m\text{ Wb}$$

---

* To commemorate the memory of German physicist Wilhelm Edward Weber (1804-1891).

** A unit magnetic pole is also defined as that magnetic pole which when placed at a distance of one metre from a very long straight conductor carrying a current of one ampere experiences a force of $1/2\pi$ newtons (Art. 6.18).

*** It should be noted that N/Wb is the same thing as ampere/metre (A/m) or just A/m cause 'turn' has no units

6.6. Flux Density (B)

It is given by the flux passing per unit area through a plane at right angles to the flux. It is usually designated by the capital letter B and is measured in weber/meter^2. It is a Vector Quantity.

It ΦWb is the total magnetic flux passing normally through an area of A m^2, then

$$B = \Phi/A \text{ Wb/m}^2 \text{ or tesla (T)}$$

Note. Let us find an expression for the flux density at a point distant r metres from a unit N-pole (*i.e.* a pole of strength 1 Wb.) Imagine a sphere of radius r metres drawn round the unit pole. The flux of 1 Wb radiated out by the unit pole falls normally on a surface of $4\pi r^2.m^2$. Hence

$$B = \frac{\Phi}{A} = \frac{1}{4\pi r^2} \text{ Wb/m}^2$$

6.7. Absolute Permeability (μ) and Relative Permeability (μ_r)

In Fig. 6.3 is shown a bar of a magnetic material, say, iron placed in a uniform field of strength H N/Wb. Suppose, a flux density of B Wb/m^2 is developed in the rod.

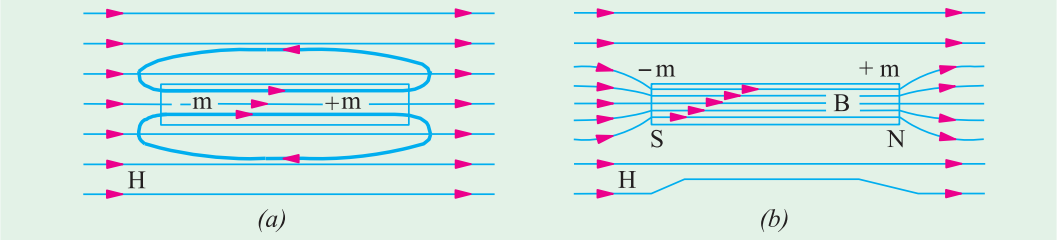


Fig. 6.3

Then, the absolute permeability of the material of the rod is defined as

$$\mu = B/H \text{ henry/metre or } B = \mu H = \mu_0 \mu_r H \text{ Wb/m}^2 \quad \text{...(i)}$$

When H is established in air (or vacuum), then corresponding flux density developed in air is

$$B_0 = \mu_0 H$$

Now, when iron rod is placed in the field, it gets magnetised by induction. If induced pole strength in the rod is m Wb, then a flux of m Wb emanates from its N-pole, re-enters its S-pole and continues from S to N-pole within the magnet. If A is the face or pole area of the magentised iron bar, the *induction* flux density in the rod is

$$B_i = m/A \text{ Wb/m}^2$$

Hence, total flux density in the iron rod consists of two parts [Fig. 6.3 (*b*)].

(*i*) B_0 –flux density in air even when rod is not present

(*ii*) B_i –induction flux density in the rod

$$B = B_0 + B_i = \mu_0 H + m/\text{A}$$

Eq. **(*i*)** above may be written as $B = \mu_r . \mu_0 H = \mu_r B_0$

$$\therefore \quad \mu_r = \frac{B}{B_0} = \frac{B(\text{material})}{B_0(\text{vacuum})} \quad \text{...for same } H$$

Hence, relative permeability of a material is equal to ***the ratio of the flux density produced in that material to the flux density produced in vacuum by the same magnetising force.***

6.8. Intensity of Magnetisation (I)

It may be defined as the induced pole strength developed per unit area of the bar. Also, it is the magnetic moment developed per unit volume of the bar.

Let m = pole strength induced in the bar in Wb

A = face or pole area of the bar in m^2

Then $I = m/A$ Wb/m^2

Hence, it is seen that intensity of magnetisation of a substance may be defined as ***the flux density produced in it due to its own induced magnetism***.

If l is the magnetic length of the bar, then the product $(m \times l)$ is known as its magnetic moment M.

$$\therefore \quad I = \frac{m}{A} = \frac{m \times l}{A \times l} = \frac{M}{V} = \text{magnetic moment/volume}$$

6.9. Susceptibility (K)

Susceptibility is defined as ***the ratio of intensity of magnetisation I to the magnetising force H.***

$$\therefore \quad K = I/H \text{ henry/metre.}$$

6.10. Relation Between B, H, I and K

It is obvious from the above discussion in Art. 6.7 that flux density B in a material is given by

$$B = B_0 + m/A = B_0 + I \qquad \therefore \; B = \mu_0 H + I$$

Now absolute permeability is $\mu = \dfrac{B}{H} = \dfrac{\mu_0 H + I}{H} = \mu_0 + \dfrac{I}{H} \quad \therefore \quad \mu = \mu_0 + K$

Also $\mu = \mu_0 \mu_r \quad \therefore \; \mu_0 \mu_r = \mu_0 + K$ or $\mu_r = 1 + K/\mu_0$

For ferro-magnetic and para-magnetic substances, K is positive and for diamagnetic substances, it is negative. For ferro-magnetic substance (like iron, nickel, cobalt and alloys like nickel-iron and cobalt-iron) μ_r is much greater than unity whereas for para-magnetic substances (like aluminium), μ_r is slightly greater than unity. For diamagnetic materials (bismuth) $\mu_r < 1$.

Example 6.1. *The magnetic susceptibility of oxygen gas at 20°C is 167×10^{-11} H/m. Calculate its absolute and relative permeabilities.*

Solution. $$\mu_r = 1 + \frac{K}{\mu_0} = 1 + \frac{167 \times 10^{-11}}{4\pi \times 10^{-7}} = \mathbf{1.00133}$$

Now, absolute permeability $\mu = \mu_0 \mu_r = 4\pi \times 10^{-7} \times 1.00133 = \mathbf{12.59 \times 10^{-7}\ H/m}$

6.11. Boundary Conditions

The case of boundary conditions between two materials of different permeabilities is similar to that discussed in Art. 4.19.

As before, the two boundary conditions are

(i) the normal component of flux density is continuous across boundary. $B_{1n} = B_{2n}$ ...(i)

(ii) the tangential component of H is continuous across boundary $H_{1t} = H_{2t}$

As proved in Art. 4.19, in a similar way, it can be shown that

$$\frac{\tan \theta_1}{\tan \theta_2} = \frac{\mu_1}{\mu_2}$$

This is called the law of magnetic flux refraction.

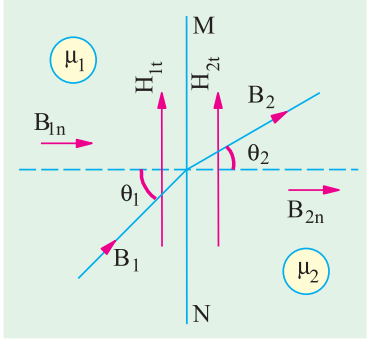


Fig. 6.4

6.12. Weber and Ewing's Molecular Theory


Fig. 6.5

This theory was first advanced by Weber in 1852 and was, later on, further developed by Ewing in 1890. The basic assumption of this theory is ***that molecules of all substances are inherently magnets in themselves, each having a N and S pole***. In an unmagnetised state, it is supposed that these small molecular

magnets lie in all sorts of haphazard manner forming more or less closed loops (Fig. 6.5). According to the laws of attraction and repulsion, these closed magnetic circuits are satisfied internally, hence there is no resultant external magnetism exhibited by the iron bar. But when such an iron bar is placed in a magnetic field or under the influence of a magnetising force, then these molecular magnets start turning round their axes and orientate themselves more or less along straight lines parallel to the direction of the magnetising force. This linear arrangement of the molecular magnets results in *N* polarity at one end of the bar and *S* polarity at the other (Fig. 6.6). As the small magnets turn more nearly in the direction of the magnetising force, it requires more and more of this force to produce a given turning moment, thus accounting for the magnetic saturation. On this theory, the hysteresis loss is supposed to be due to molecular friction of these turning magnets.

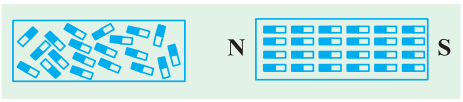
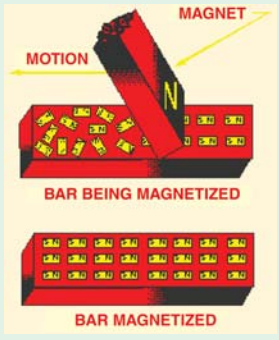
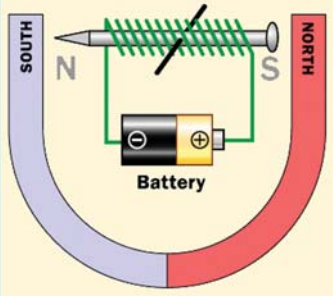


Fig. 6.6



Molecular magnets which are randomly arranged in the unmagnetised state, get oriented under the influence of an external magnetizing force

Because of the limited knowledge of molecular structure available at the time of Weber, it was not possible to explain firstly, as to why the molecules themselves are magnets and secondly, why it is impossible to magnetise certain substances like wood etc. The first objection was explained by Ampere who maintained that orbital movement of the electrons round the atom of a molecule constituted a flow of current which, due to its associated magnetic effect, made the molecule a magnet.



An iron nail converts into a magnet as soon as the external magnetizing force starts acting on it

Later on, it became difficult to explain the phenomenon of diamagnetism (shown by materials like water, quartz, silver and copper etc.) erratic behaviour of ferromagnetic (intensely magnetisable) substances like iron, steel, cobalt, nickel and some of their alloys etc. and the paramagnetic (weakly magnetisable) substances like oxygen and aluminium etc. Moreover, it was asked : if molecules of all substances are magnets, then why does not wood or air etc. become magnetised ?

All this has been explained satisfactorily by the atom-domain theory which has superseded the molecular theory. It is beyond the scope of this book to go into the details of this theory. The interested reader is advised to refer to some standard book on magnetism. However, it may just be mentioned that this theory takes into account not only the planetary motion of an electron but its rotation about its own axis as well. This latter rotation is called 'electron spin'. The gyroscopic behaviour of an electron gives rise to a magnetic moment which may be either positive or negative. A substance is ferromagnetic or diamagnetic accordingly as there is an excess of unbalanced positive spins or negative spins. Substances like wood or air are non-magnetisable because in their case, the positive and negative electron spins are equal, hence they cancel each other out.

6.13. Curie Point

As a magnetic material is heated, its molecules vibrate more violently. As a consequence, individual molecular magnets get out of alignment as the temperature is increased, thereby reducing the magnetic strength of the magnetised substance. Fig. 6.7 shows the approximate decrease of magnetic strength with rise in temperature. Obviously, it is possible to partially or even completely destroy the magnetic properties of a material by heating. The temperature at which the vibrations of the molecular magnets become so random

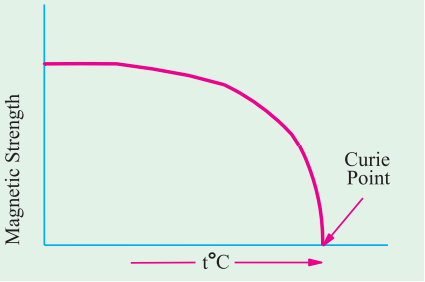


Fig. 6.7

and out of alignment as to reduce the magnetic strength to zero is called Curie point. More accurately, it is that critical temperature above which ferromagnetic material becomes paramagnetic.

ELECTROMAGNETISM

6.14 . Force on a Current-carrying Conductor Lying in a Magnetic Field

It is found that whenever a current-carrying conductor is placed in magnetic field, it experiences a force which acts in a direction perpendicular both to the direction of the current and the field. In Fig. 6.8 is shown a conductor XY lying at right angles to the uniform horizontal field of flux density B Wb/m^2 produced by two solenoids A and B. If l is the length of the conductor lying within this field and I ampere the current carried by it, then the magnitude of the force experienced by it is

$$F = BIl = \mu_0 \mu_r HIl \text{ newton}$$

Using vector notation $\vec{F} = I \vec{l} \times \vec{B}$ and $F = IlB \sin\theta$ where θ is the angle between $\vec{l}$ and $\vec{B}$ which is 90° in the present case

or $$F = Il\,B \sin 90° = Il\,B \text{ newtons} \qquad (\because \sin 90° = 1)$$

The direction of this force may be easily found by Fleming's left-hand rule.

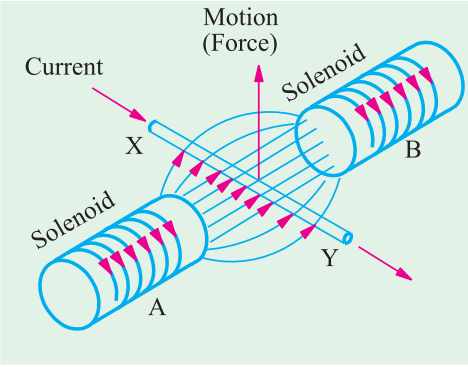


Fig. 6.8

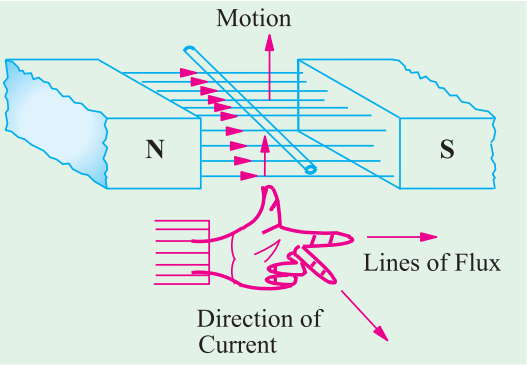


Fig. 6.9

Hold out your left hand with forefinger, second finger and thumb at right angles to one another. If the forefinger represents the direction of the field and the second finger that of the current, then thumb gives the direction of the motion. It is illustrated in Fig. 6.9.

Fig. 6.10 shows another method of finding the direction of force acting on a current carrying conductor. It is known as Flat Left Hand rule. The force acts in the direction of the thumb obviously, the direction of motor of the conductor is the same as that of the force.

It should be noted that no force is exerted on a conductor when it lies parallel to the magnetic field. In general, if the conductor lies at an angle θ with the direction of the field, then B can be resolved into two components, $B \cos\theta$ parallel to and $B \sin\theta$ perpendicular to the conductor. The former produces no effect whereas the latter is responsible for the motion observed. In that case,

$F = BIl \sin\theta$ newton, which has been expressed as cross product of vector above.*

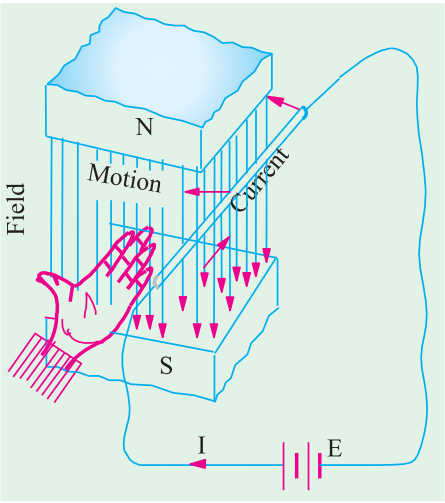


Fig. 6.10

* It is simpler to find direction of Force (Motion) through cross product of given vectors $I \vec{l}$ and $\vec{B}$.

6.15. Ampere's Work Law or Ampere's Circuital Law

The law states that m.m.f.* (magnetomotive force corresponding to e.m.f. *i.e.* electromotive force of electric field) around a closed path is equal to the current enclosed by the path. Mathematically, $\int \vec{H} \cdot d\vec{s} = I$ amperes where $\vec{H}$ is the vector representing magnetic field strength in dot product with vector $d\vec{s}$ of the enclosing path S around current I ampere and that is why line integral ($\oint$) of dot product $\vec{H} \cdot d\vec{s}$ is taken.

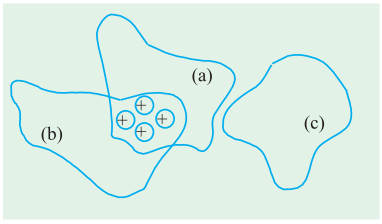


Fig. 6.11

Work law is very comprehensive and is applicable to all magnetic fields whatever the shape of enclosing path *e.g.* (*a*) and (*b*) in Fig. 6.11. Since path *c* does not enclose the conductor, the m.m.f. around it is zero.

The above work Law is used for obtaining the value of the magnetomotive force around simple idealized circuits like ***(i)*** a long straight current-carrying conductor and ***(ii)*** a long solenoid.

(*i*) Magnetomotive Force around a Long Straight Conductor

In Fig. 6.12 is shown a straight conductor which is assumed to extend to infinity in either direction. Let it carry a current of I amperes upwards. The magnetic field consists of circular lines of force having their plane perpendicular to the conductor and their centres at the centre of the conductor.

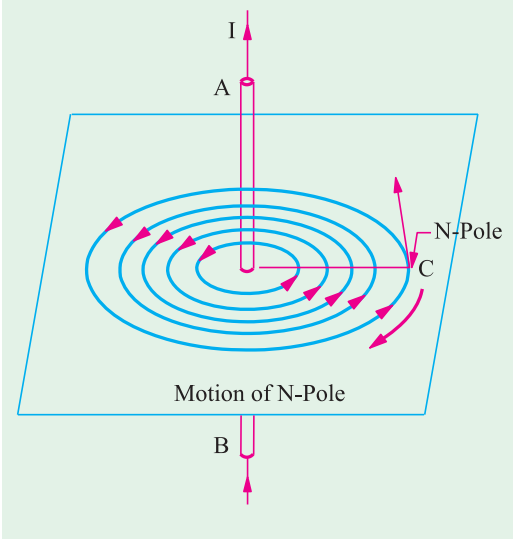


Fig. 6.12

Suppose that the field strength at point C distant r metres from the centre of the conductor is H. Then, it means that if a unit N-pole is placed at C, it will experience a force of H newtons. The direction of this force would be tangential to the circular line of force passing through C. If this unit N-pole is moved once round the conductor *against* this force, then work done *i.e.*

m.m.f. = force × distance = I

i.e. $I = H \times 2\pi r$ joules = Amperes

or $H = \dfrac{I}{2\pi r}$

$= \int \vec{H} \cdot d\vec{s}$ Joules = Amperes = I

Obviously, if there are N conductors (as shown in Fig. 6.13), then

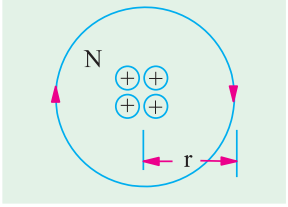


Fig. 6.13

and

$$H = \frac{NI}{2\pi r} \text{ A/m or Oersted}$$

$$B = \mu_0 \frac{NI}{2\pi r} \text{ Wb/m}^2 \text{ tesla} \quad \text{...in air}$$

$$= \frac{\mu_0 \mu_r NI}{2\pi r} \text{ Wb/m}^2 \text{ tesla} \quad \text{...in a medium}$$

** M.M.F. is not a force, but is the work done.

(ii) Magnetic Field Strength of a Long Solenoid

Let the Magnetic Field Strength along the axis of the solenoid be H. Let us assume that

(i) the value of H remains constant throughout the length l of the solenoid and

(ii) the volume of H outside the solenoid is negligible.

Suppose, a unit N-pole is placed at point A outside the solenoid and is taken once round the completed path (shown dotted in Fig. 6.14) in a direction opposite to that of H. Remembering that the force of H newtons acts on the N-pole only over the length l (it being negligible elsewhere), the work done in one round is

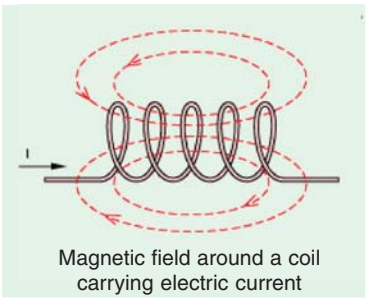
$= H \times l$ joules = Amperes

The 'ampere-turns' linked with this path are NI where N is the number of turns of the solenoid and I the current in amperes passing through it. According to Work Law

$$H \times l = NI \quad \text{or} \quad H = \frac{NI}{l} \text{ A/m or Oersted}$$

$$\text{Also } B = \frac{\mu_0 NI}{l} \text{ Wb/m}^2 \text{ or tesla ...in air}$$

$$= \frac{\mu_0 \mu_r NI}{l} \text{ Wb/m}^2 \text{ or tesla ...in a medium}$$



Magnetic field around a coil carrying electric current

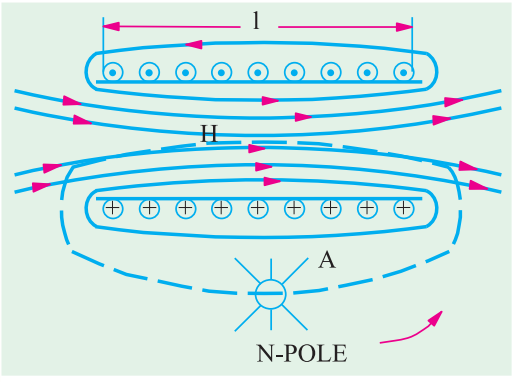


Fig. 6.14

6.16. Biot-Savart Law*

The expression for the magnetic field strength dH produced at point P by a vanishingly small length dl of a conductor carrying a current of I amperes (Fig. 6.15) is given by

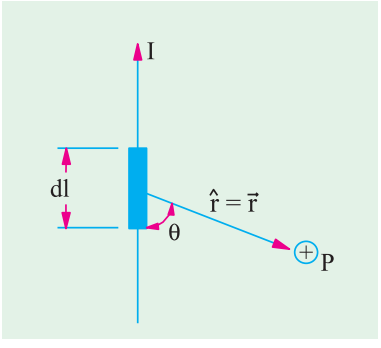


Fig. 6.15

$$dH = \frac{Idl \sin\theta}{4\pi r^2} \text{ A/m}$$

$$\text{or} \quad \vec{dH} = (Id\vec{l} \times \hat{r})/4\pi r^2 \text{ in vector form}$$

The direction of $\vec{dH}$ is perpendicular to the plane containing both '$\vec{dl}$' and $\vec{r}$ *i.e.* entering.

$$\text{or} \quad dB_0 = \frac{\mu_0 Idl}{4\pi r^2} \sin\theta \text{ Wb/m}^2$$

$$\text{and} \quad \vec{dB_0} = \frac{\mu_0 \vec{I}\, d\vec{l} \times \hat{r}}{4\pi r^2} \text{ in vector form}$$

6.17. Applications of Biot-Savart Law

(i) Magnetic Field Strength Due to a Finite Length of Wire Carrying Current

Consider a straight wire of length l carrying a steady current I. We wish to find magnetic field strength (H) at a point P which is at a distance r from the wire as shown in Fig. 6.16.

* After the French mathematician and physicist Jean Baptiste Biot (1774-1862) and Felix Savart (1791-1841) a well-known French physicist.

The magnetic field strength $d\vec{H}$ due to a small element dl of the wire shown is

$$d\vec{H} = \frac{\vec{I}\,\vec{d\,l} \times \hat{s}}{4\pi s^2} \quad \text{(By Biot-Savart Law)}$$

or $$d\vec{H} = \frac{Idl \sin\theta}{4\pi \times s^2}\hat{u}$$ (where $\hat{u}$ is unit vector perpendicular to plane containing $\vec{dl}$ and $\hat{s}$ and into the plane.)

or $$d\vec{H} = \frac{Idl \cos\phi}{4\pi s^2}\hat{u}$$ ...[$\because$ θ and ϕ are complementary angles]

The magnetic field strength due to entire length l :

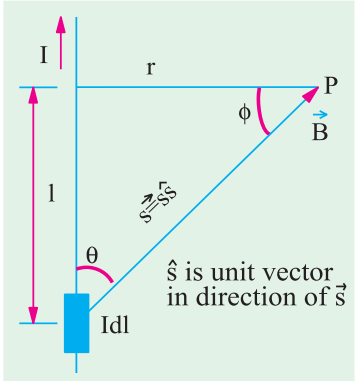


Fig. 6.16

$$\vec{H} = \frac{I}{4\pi}\left[\int_0^l \frac{\cos\phi\, dl}{s^2}\right]\hat{u}$$

$$= \frac{I}{4\pi}\left[\int_0^l \frac{r/s}{s^2}dl\right]\hat{u} \qquad \left(\cos\phi = \frac{r}{s} \text{ in Fig. 6.16}\right)$$

$$= \frac{Ir}{4\pi}\left[\int_0^l \frac{dl}{s^3}\right]\hat{u} = \frac{Ir}{4\pi}\left[\int_0^l \frac{dl}{(r^2+l^2)^{3/2}}\right]\hat{u}$$

($\because$ r is constant) ; $s = \sqrt{r^2 + l^2}$ in Fig. 6.16

$$= \frac{Ir}{4\pi r^3}\left[\int_0^l \frac{dl}{[1+(r/l)^2]3/2}\right]\hat{u} \text{ (Taking } r^3 \text{ out from denominator)}$$

To evaluate the integral most simply, make the following substitution

$$\frac{l}{r} = \tan\phi \text{ in Fig. 6.16}$$

$\therefore$ $l = r\tan\phi$ $\therefore dl = r\sec^2\phi\, d\phi$ and $1 + (r/l)^2 = 1 + \tan^2\phi = \sec^2\phi$ and limits get transformed *i.e.* become 0 to ϕ.

$$\vec{H} = \frac{Ir}{4\ r^3}\ _0\ \frac{r\sec^2}{\sec^2}d\ \ \hat{u}\ \ \frac{Ir^2}{4\ r^3}\ _0\ \cos\ \ d\ \ \hat{u}\ \ \frac{1}{4\ r}\ \sin\ \ _0\hat{u}$$

$$= \frac{I}{4\pi r}\sin\phi\,\hat{u}$$

N.B. For wire of infinite length extending it at both ends *i.e.* $-\infty$ to $+\infty$ the limits of integration would be $-\frac{\pi}{2}$ to $+\frac{\pi}{2}$, giving $\vec{H} = \frac{I}{4\pi r} \times 2\hat{u}$ or $\vec{H} = \frac{I}{2\pi r}\hat{u}$.

(*ii*) Magnetic Field Strength along the Axis of a Square Coil

This is similar to **(*i*)** above except that there are four conductors each of length say, $2a$ metres and carrying a current of I amperes as shown in Fig. 6.17. The Magnetic Field Strengths at the axial point P due to the opposite sides ab and cd are H_{ab} and H_{cd} directed at right angles to the planes containing P and ab and P and cd respectively. Now, H_{ab} and H_{cd} are numerically equal,

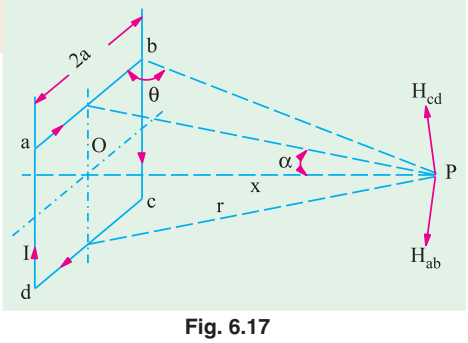


Fig. 6.17

hence their components at right angles to the axis of the coil will cancel out, but the axial components will add together. Similarly, the other two sides *da* and *bc* will also give a resultant axial component only.

As seen from Eq. ***(ii)*** above,

$$H_{ab} = \frac{I}{4\pi r}[\cos\theta - \cos(180^\circ - \theta)] = \frac{I \,.\, 2\cos\theta}{4\pi r} = \frac{I\cos\theta}{2\pi r}$$

Now $r = \sqrt{a^2 + x^2}$ $\quad\therefore\; H_{ab} = \dfrac{I\,.\cos\theta}{2\pi\sqrt{a^2 + x^2}}$

Its axial components is $H_{ab'} = H_{ab}\,.\sin\alpha = \dfrac{I\cos\theta}{2\pi\sqrt{a^2 + x^2}}\,.\sin\alpha$

All the four sides of the rectangular coil will contribute an equal amount to the resultant magnetic field at *P*. Hence, resultant magnetising force at *P* is

$$H = 4\times\frac{I\cos\theta}{2\pi\sqrt{a^2 + x^2}}\,.\sin\alpha\,,$$

Now $\cos\theta = \dfrac{a}{\sqrt{(2a^2 + x^2)}}$ and $\sin\alpha = \dfrac{a}{\sqrt{a^2 + x^2}}$

$\therefore$ $$H = \frac{2a^2\,.\,I}{\pi\,(a^2 + x^2)\,.\sqrt{x^2 + 2a^2}}\ \text{AT/m.}$$

In case, value of *H* is required at the centre *O* of the coil, then putting $x = 0$ in the above expression,

we get $$H = \frac{2a^2\,.\,I}{\pi a^2\,.\sqrt{2}\,.\,a} = \frac{\sqrt{2}\,.\,I}{\pi a}\ \text{AT/m}$$

Note. The last result can be found directly as under. As seen from Fig. 6.18, the field at point *O* due to any side is, as given by Eq. (*i*)

$$= \frac{I}{4\pi a}\int_{\pi/4}^{-\pi/4}\sin\theta\,.\,d\theta = \frac{I}{4\pi a}\Big|-\cos\theta\Big|_{45^\circ}^{-45^\circ} = \frac{I}{4\pi a}\,.\,2\cos 45^\circ = \frac{I}{4\pi a}\,.\,\frac{2}{\sqrt{2}}$$

Resultant magnetising force due to all sides is

$H = 4\times\dfrac{1}{4\pi a}\,.\,\dfrac{2}{\sqrt{2}} = \dfrac{\sqrt{2}\,I}{\pi a}$ AT/m ...as found above

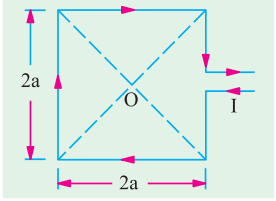


Fig. 6.18

(*iii*) Magnetising Force on the Axis of a Circular Coil

In Fig. 6.19 is shown a circular one-turn coil carrying a current of *I* amperes. The magnetising force at the axial point *P* due to a small element '*dl*' as given by Laplace's Law is

$$\left|\vec{dH}\right| = \frac{Idl}{4\pi\,(r^2 + x^2)}$$

The direction of *dH* is at right angles to the line *AP* joining point *P* to the element '*dl*'. Now, *dH* can be resolved into two components :

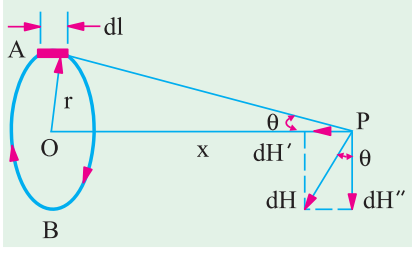


Fig. 6.19

(a) the axial component $dH' = dH\sin\theta$

(b) the vertical component $dH'' = dH\cos\theta$

Now, the vertical component $dH\cos\theta$ will be cancelled by an equal and opposite vertical component of *dH* due to element '*dl*' at point *B*. The same applies to all other diametrically opposite pairs of *dl*'s taken around the coil. Hence, the resultant magnetising force at *P* will be equal to the sum of all the axial components.

$$\therefore \quad H = \Sigma dH' = \Sigma dH \sin\theta \int dl = \Sigma \frac{I \,.\, dl \,.\, r}{4\pi (r^2 + x^2)^{3/2}} \int dl \qquad \left(\sin\theta = \frac{r}{\sqrt{r^2 + x^2}} \right)$$

$$= \frac{I \,.\, r}{4\pi (r^2 + x^2)^{3/2}} \int_0^{2\pi r} dl = \frac{I \,.\, r \,.\, 2\pi r}{4\pi (r^2 + x^2)^{3/2}} = \frac{Ir^2}{2 (r^2 + x^2)^{3/2}}$$

$$= \frac{I}{2r} \cdot \frac{r^3}{(r^2 + x^2)^{3/2}} \qquad \therefore \; H = \frac{I \sin^3\theta}{2r} \text{ AT/m}$$

or $$H = \frac{NI}{2r} \sin^3\theta \text{ AT/m} \qquad \text{–for an } N\text{-turn coil} \qquad \text{...(iii)}$$

In case the value of H is required at the centre O of the coil, then putting $\theta = 90°$ and $\sin\theta = 1$ in the above expression, we get

$$H = \frac{I}{2r} \text{ – for single-turn coil} \quad \text{or} \quad H = \frac{NI}{2r} \text{ –for } N\text{-turn coil}$$

Note. The magnetising force H at the centre of a circular coil can be directly found as follows :

With reference to the coil shown in the Fig. 6.20, the magnetising force dH produced at O due to the small element dl (as given by Laplace's law) is

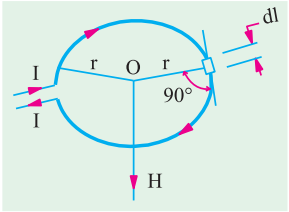


Fig. 6.20

$$dH = \frac{I \,.\, dl \sin\theta}{4\pi r^2} = \frac{I \,.\, dl}{4\pi r^2} \qquad (\because \; \sin\theta = \sin 90° = 1)$$

$$\therefore \quad \Sigma dH = \Sigma \frac{I \,.\, dl}{4\pi r^2} = \frac{I}{4\pi r^2} \Sigma dl \quad \text{or} \quad H = \frac{I \,.\, 2\pi r}{4\pi r^2} = \frac{I}{2r}$$

$$\therefore \quad H = \frac{I}{2r} \text{ AT/m –for 1-turn coil ; } \frac{NI}{2r} \text{ AT/m –for } N\text{-turn coil.}$$

(iv) Magnetising Force on the Axis of a Short Solenoid

Let a short solenoid having a length of l and radius of turns r be uniformly wound with N turns each carrying a current of I as shown in Fig. 6.21. The winding density *i.e.* number of turns per unit length of the solenoid is N/l. Hence, in a small element of length dx, there will be $N.dx/l$ turns. Obviously, a very short element of length of the solenoid can be regarded as a concentrated coil of very short axial length and having $N.dx/l$ turns. Let dH be the magnetising force contributed by the element dx at any axial point P. Then, substituting dH for H and $N.dx/l$ for N in Eq. **(iii)**, we get

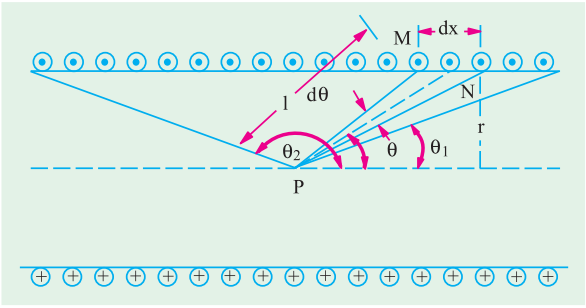


Fig. 6.21

$$dH = \frac{N \,.\, dx}{l} \cdot \frac{I}{2r} \,.\, \sin^3\theta$$

Now $$dx \,.\, \sin\theta = r \,.\, d\theta/\sin\theta^* \quad \therefore \; dx = r \,.\, d\theta/\sin^2\theta$$

Substituting this value of dx in the above equation, we get

$$dH = \frac{NI}{2l} \sin\theta \,.\, d\theta$$

Total value of the magnetising force at P due to the whole length of the solenoid may be found by integrating the above expression between proper limits.

---

* Because $l \sin\theta = r \therefore l = r/\sin\theta$. Now, $M/N = l.d\theta = r\, d\theta/\sin\theta$. Also, $MN = dx$, $\sin\theta$, hence $dx = r\, d\theta/\sin^2\theta$.

$$\therefore \quad H = \frac{NI}{2l}\int_{\theta_1}^{\theta_2} \sin\theta \, . \, d\theta = \frac{NI}{2l}\left| -\cos\theta \right|_{\theta_1}^{\theta_2}$$

$$= \frac{NI}{2l}(\cos\theta_1 - \cos\theta_2) \qquad \text{...(iv)}$$

The above expression may be used to find the value of H at any point of the axis, either inside or outside the solenoid.

(i) At mid-point, $\theta_2 = (\pi - \theta_1)$, hence $\cos\theta_2 = -\cos\theta_1$

$$\therefore \quad H = \frac{2NI}{2l}\cos\theta_1 = \frac{NI}{l}\cos\theta_1$$

Obviously, when the solenoid is very long, $\cos\theta_1$ becomes nearly unity. In that case,

$$H = \frac{NI}{l} \text{ AT/m} \qquad \text{–Art. 6.15 (ii)}$$

(ii) At any point on the axis inside a very *long* solenoid but not too close to either end, $\theta_1 \cong 0$ and $\theta_2 \cong \pi$ so that $\cos\theta_1 \cong 1$ and $\cos\theta_2 = -1$. Then, putting these values in Eq. (*iv*) above, we have

$$H \cong \frac{NI}{2l} \times 2 = \frac{NI}{l}$$

It proves that inside a very long solenoid, H is practically constant at all axial points excepts those lying too close to either end of the solenoid.

(iii) Towards either end of the solenoid, H decreases and exactly at the ends, $\theta_1 = \pi/2$ and $\theta_2 \cong \pi$, so that $\cos\theta_1 = 0$ and $\cos\theta_2 = -1$. Hence, from Eq. **(iv)** above, we get

$$H = \frac{NI}{2l}$$

In other words, value of H is decreased to half the normal value well inside the solenoid.

Example 6.2. *Calculate the magnetising force and flux density at a distance of 5 cm from a long straight circular conductor carrying a current of 250 A and placed in air. Draw a curve showing the variation of B from the conductor surface outwards if its diameter is 2 mm.*

Solution. As seen from Art. 6.15 **(i)**,

$$H = \frac{I}{2\pi r} = \frac{250}{2\pi \times 0.05} = \mathbf{795.6 \text{ AT/m}}$$

$$B = \mu_0 H = 4\pi \times 10^{-7} \times 795.6 = \mathbf{10^{-3} \text{ Wb/m}^2}$$

In general, $B = \dfrac{\mu_0 I}{2\pi r}$

Now, at the conductor surface, $r = 1 \text{ mm} = 10^{-3}$ m

$$\therefore \quad B = \frac{4\pi \times 10^{-7} \times 250}{2\pi \times 10^{-3}} = \mathbf{0.05 \text{ Wb/m}^2}$$

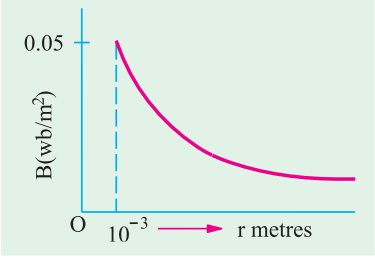


Fig. 6.22

The variation of B outside the conductor is shown in Fig. 6.22.

Example 6.3. *A wire 2.5 m long is bent (i) into a square and (ii) into a circle. If the current flowing through the wire is 100 A, find the magnetising force at the centre of the square and the centre of the circle.* **(Elec. Measurements; Nagpur Univ. 1992)**

Solution. **(i)** Each side of the square is $2a = 2.5/4 = 0.625$ m

Value of H at the centre of the square is [Art 6.17 **(ii)**]

$$= \frac{\sqrt{2}\,I}{\pi a} = \frac{\sqrt{2} \times 100}{\pi \times 0.3125} = \mathbf{144 \text{ AT/m}} \quad \textbf{(ii)} \quad 2\pi r = 2.5 \; ; \; r = 0.398 \text{ m}$$

Value of H at the centre is $= I/2r = 100/2 \times 0.398 = \mathbf{125.6 \text{ AT/m}}$

Example 6.4. *A current of 15 A is passing along a straight wire. Calculate the force on a unit magnetic pole placed 0.15 metre from the wire. If the wire is bent to form into a loop, calculate the diameter of the loop so as to produce the same force at the centre of the coil upon a unit magnetic pole when carrying a current of 15 A.* **(Elect. Engg. Calcutta Univ.)**

Solution. By the force on a unit magnetic pole is meant the magnetising force H.
For a straight conductor [Art 6.15 (*i*)] $H = I/2\pi r = 15/2\pi \times 0.15 = 50/\pi$ AT/m
Now, the magnetising force at the centre of a loop of wire is [Art. 6.17 ***(iii)***]

$$= I/2r = I/D = 15/D \text{ AT/m}$$

Since the two magnetising forces are equal

$$\therefore \quad 50/\pi = 15/D;\ D = 15\pi/50 = 0.9426 \text{ m} = \mathbf{94.26\ cm.}$$

Example. 6.5. *A single-turn circular coil of 50 m. diameter carries a direct current of 28×10^4 A. Assuming Laplace's expression for the magnetising force due to a current element, determine the magnetising force at a point on the axis of the coil and 100 m. from the coil. The relative permeability of the space surrounding the coil is unity.*

Solution. As seen from Art 6.17 (*iii*), $H = \dfrac{I}{2r} \cdot \sin^3\theta$ AT/m

Here $$\sin\theta = \frac{r}{\sqrt{r^2 + x^2}} = \frac{25}{\sqrt{25^2 + 100^2}} = 0.2425$$

$$\sin^3\theta = (0.2425)^3 = 0.01426 \quad \therefore H = \frac{28}{2}\frac{10^4}{25}\ 0.01426 \quad \mathbf{76.8\ AT/m}$$

6.18. Force Between Two Parallel Conductors

***(i)* Currents in the same direction.** In Fig. 6.23 are shown two parallel conductors P and Q carrying currents I_1 and I_2 amperes in the same direction *i.e.* upwards. The field strength in the space between the two conductors is decreased due to the two fields there being in opposition to each other. Hence, the resultant field is as shown in the figure. Obviously, the two conductors are attracted towards each other.

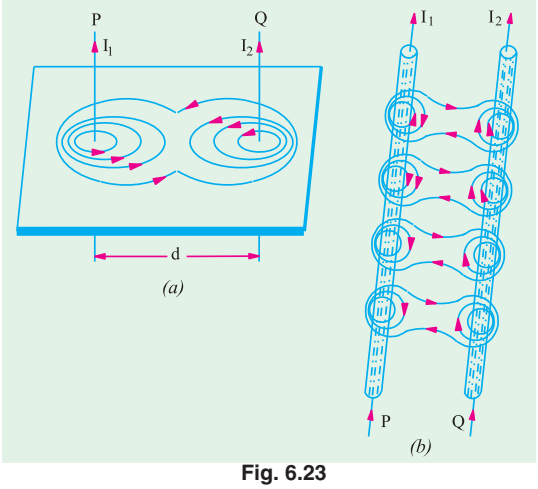


Fig. 6.23

***(ii)* Currents in opposite directions.** If, as shown in Fig. 6.24, the parallel conductors carry currents in opposite directions, then field strength is increased in the space between the two conductors due to the two fields being in the same direction there. Because of the lateral repulsion of the lines of the force, the two conductors experience a mutual force of repulsion as shown separately in Fig. 6.24 (*b*).

6.19. Magnitude of Mutual Force

It is obvious that each of the two parallel conductors lies in the magnetic field of the other conductor. For example, conductor P lies in the magnetic field of Q and Q lies in the field of P. If 'd' metres is the distance between them, then flux density at Q due to P is [Art. 6.15 ***(i)***]

$$B = \frac{\mu_0 I_1}{2\pi d} \text{ Wb/m}^2$$

If l is the length of conductor Q lying in this flux density, then force (either of attraction or repulsion) as given in Art. 6.14 is

$$F = BI_2\, l \text{ newton} \quad \text{or} \quad F = \frac{\mu_0 I_1 I_2 l}{2\pi d} \text{ N}$$

Obviously, conductor P will experience an equal force in the opposite direction.

The above facts are known as Laws of Parallel Currents and may be stated as follows :

(*i*) Two parallel conductors attract each other if currents through them flow in the *same* direction and repel each other if the currents through them flow in the *opposite* directions.

(*ii*) The force between two such parallel conductors is proportional to the product of current strengths and to the length of the conductors considered and varies inversely as the distance between them.

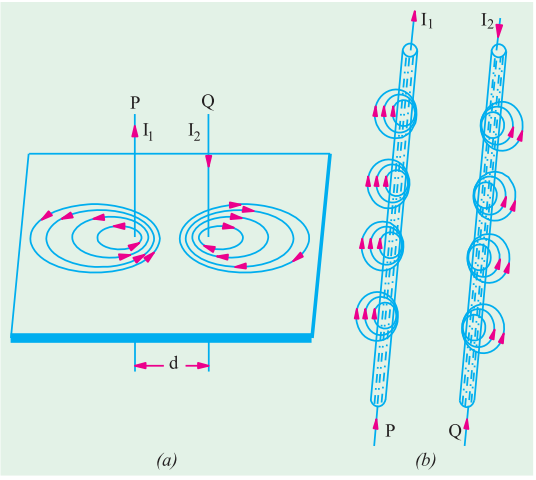


Fig. 6.24

6.20. Definition of Ampere

If has been proved in Art. 6.19 above that the force between two infinitely long parallel currently-carrying conductors is given by the expression

$$F = \frac{\mu_0 I_1 I_2 l}{2\pi d} \text{ N} \quad \text{or} \quad F = \frac{4\pi \times 10^{-7}\, I_1\, I_2\, l}{2\pi d} = 2 \times 10^{-7} \frac{I_1 I_2}{d} \text{ N}$$

The force per metre run of the conductors is

$$F = 2 \times 10^{-7} \frac{I_1 I_2}{d} \text{ N/m}$$

If $I_1 = I_2 = 1$ ampere (say) and $d = 1$ metre, then $F = 2 \times 10^{-7}$ N

Hence, we can define one ampere current as ***that current which when flowing in each of the two infinitely long parallel conductors situated in vacuum and separated 1 metre between centres, produces on each conductor a force of 2×10^{-7} N per metre length.***

Example 6.6. *Two infinite parallel conductors carry parallel currents of 10 amp. each. Find the magnitude and direction of the force between the conductors per metre length if the distance between them is 20 cm.* **(Elect. Engg. Material - II Punjab Univ. May 1990)**

Solution. $$F = 2 \times 10^{-7} \frac{10 \times 10 \times 1}{0.2} \text{ N} = \mathbf{10^{-4}\ N}$$

The direction of force will depend on whether the two currents are flowing in the same direction or in the opposite direction. As per Art. 6.19, it would be a force of attraction in the first case and that or repulsion in the second case.

Example 6.7. *Two long straight parallel wires, standing in air 2 m apart, carry currents I_1 and I_2 in the same direction. The magnetic intensity at a point midway between the wires is 7.95 AT/m. If the force on each wire per unit length is 2.4×10^{-4} N, evaluate I_1 and I_2.*

Solution. As seen from Art. 6.17, the magnetic intensity of a long straight current-carrying conductor is

$$H = \frac{I}{2\pi r} \text{ AT/m}$$

Also, it is seen from Fig. 6.23 that when the two currents flow in the same direction, net field strength midway between the two conductors is the difference of the two field strengths.

Now, $H_1 = I_1/2\pi$ and $H_2 = I_2/2\pi$ because $r = 2/1 = 2$ metre

$$\therefore \quad \frac{I_1}{2\pi} - \frac{I_2}{2\pi} = 7.95 \quad \therefore \; I_1 - I_2 = 50 \qquad ...(i)$$

Force per unit length of the conductors is $F = 2 \times 10^{-7} I_1 I_2/d$ newton

$$\therefore \quad 2.4 \times 10^{-4} = 2 \times 10^{-7} I_1 I_2/2 \quad \therefore \; I_1 I_2 = 2400 \qquad ...(ii)$$

Substituting the value of I_1 from **(*i*)** in **(*ii*)**, we get

$$(50 + I_2) I_2 = 2400 \quad \text{or} \quad I_2^2 + 50 I_2 - 2400 = 0$$

or $\quad (I_2 + 80)(I_2 - 30) = 0 \quad \therefore \; I_2 = \mathbf{30\ A}$ and $I_1 = 50 + 30 = \mathbf{80\ A}$

Tutorial Problems No. 6.1

1. The force between two long parallel conductors is 15 kg/metre. The conductor spacing is 10 cm. If one conductor carries twice the current of the other, calculate the current in each conductor.
[6,060 A; 12,120 A]
2. A wire is bent into a plane to form a square of 30 cm side and a current of 100 A is passed through it. Calculate the field strength set up at the centre of the square. **[300 AT/m]**

(Electrotechnics - I, M.S. Univ. Baroda)

MAGNETIC CIRCUIT

6.21. Magnetic Circuit

It may be defined as the route or path which is followed by magnetic flux. The law of magnetic circuit are quite similar to (but not the same as) those of the electric circuit.

Consider a solenoid or a toroidal iron ring having a magnetic path of l metre, area of cross section A m^2 and a coil of N turns carrying I amperes wound anywhere on it as in Fig. 6.25.

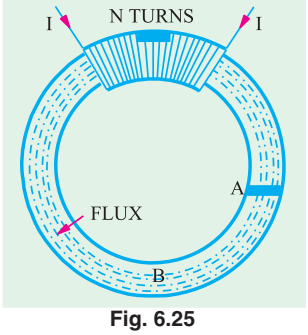


Fig. 6.25

Then, as seen from Art. 6.15, field strength inside the solenoid is

$$H = \frac{NI}{l} \text{ AT/m}$$

Now $$B = \mu_0 \mu_r H = \frac{\mu_0 \mu_r NI}{l} \text{ Wb/m}^2$$

Total flux produce $\Phi = B \times A = \dfrac{\mu_0 \mu_r A NI}{l}$ Wb

$$\therefore \quad \Phi = \frac{NI}{l/\mu_0 \mu_r A} \text{ Wb}$$

The numerator 'NI' which produces magnetization in the magnetic circuit is known as magnetomotive force (m.m.f.). Obviously, its unit is ampere-turn (AT)*. It is analogous to e.m.f. in an electric circuit.

The denominator $\dfrac{l}{\mu_0 \mu_r A}$ is called the ***reluctance*** of the circuit and is analogous to resistance in electric circuits.

$$\therefore \quad \text{flux} = \frac{\text{m.m.f.}}{\text{reluctance}} \quad \text{or} \quad \Phi = \frac{F}{S}$$

Sometimes, the above equation is called the "Ohm's Law of Magnetic Circuit" because it resembles a similar expression in electric circuits *i.e.*

* Strictly speaking, it should be only 'ampere' because turns have no unit.

$$\text{current} = \frac{\text{e.m.f.}}{\text{resistance}} \quad \text{or} \quad I = \frac{V}{R}$$

6.22. Definitions Concerning Magnetic Circuit

1. Magnetomotive force (m.m.f.). It drives or tends to drive flux through a magnetic circuit and corresponds to electromotive force (e.m.f.) in an electric circuit.

M.M.F. is equal to the work done in joules in carrying a unit magnetic pole once through the entire magnetic circuit. It is measured in ampere-turns.

In fact, as p.d. between any two points is measured by the work done in carrying a unit charge from one points to another, similarly, m.m.f. between two points is measured by the work done in joules in carrying a unit magnetic pole from one point to another.

2. Ampere-turns (AT). It is the unit of magnetometre force (m.m.f.) and is given by the product of number of turns of a magnetic circuit and the current in amperes in those turns.

3. Reluctance. It is the name given to that property of a material which opposes the creation of magnetic flux in it. It, in fact, measures the opposition offered to the passage of magnetic flux through a material and is analogous to resistance in an electric circuit even in form. Its units is **AT/Wb.***

$$\text{reluctance} = \frac{l}{\mu A} = \frac{l}{\mu_0 \mu_r A}; \quad \text{resistance} = \rho \frac{l}{A} = \frac{l}{\sigma A}$$

In other words, the reluctance of a magnetic circuit is the number of amp-turns required per weber of magnetic flux in the circuit. Since 1 AT/Wb = 1/henry, the unit of reluctance is "reciprocal henry."

4. Permeance. It is reciprocal of reluctance and implies the case or readiness with which magnetic flux is developed. It is analogous to conductance in electric circuits. It is measured in terms of Wb/AT or henry.

5. Reluctivity. It is specific reluctance and corresponds to resistivity which is 'specific resistance'.

6.23. Composite Series Magnetic Circuit

In Fig. 6.26 is shown a composite series magnetic circuit consisting of three different magnetic materials of different permeabilities and lengths and one air gap ($\mu_r = 1$). Each path will have its own reluctance. The total reluctance is the sum of individual reluctances as they are joined in series.

$$\therefore \quad \text{total reluctance} = \Sigma \frac{l}{\mu_0 \mu_r A}$$

$$= \frac{l_1}{\mu_0 \mu_{r_1} A_1} + \frac{l_2}{\mu_0 \mu_{r_2} A_2} + \frac{l_3}{\mu_0 \mu_{r_3} A_3} + \frac{l_a}{\mu_0 A_g}$$

$$\therefore \quad \text{flux } \Phi = \frac{\text{m.m.f.}}{\frac{l}{\mu_0 \mu_r A}}$$

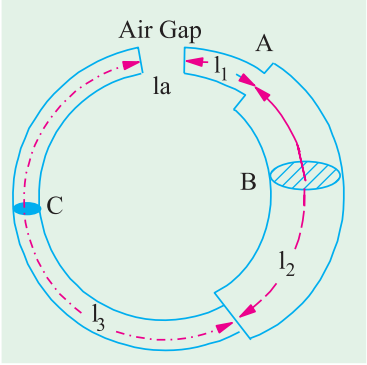


Fig. 6.26

6.24. How to Find Ampere-turns ?

It has been shown in Art. 6.15 that $H = NI/l$ AT/m or $NI = H \times l$

$\therefore$ ampere-turns AT $= H \times l$

Hence, following procedure should be adopted for calculating the total ampere turns of a composite magnetic path.

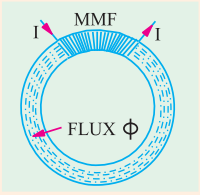
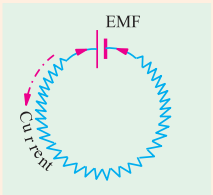
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* From the ratio $\Phi = \frac{\text{m.m.f.}}{\text{reluctance}}$, it is obvious that reluctance = m.m.f./Φ. Since m.m.f. is in ampere-turns and flux in webers, unit of reluctance is ampere-turn/weber (AT/Wb) or A/Wb.

(*i*) Find H for each portion of the composite circuit. For air, $H = B/\mu_0$, otherwise $H = B/\mu_0\mu_r$.
(*ii*) Find ampere-turns for each path separately by using the relation AT = $H \times l$.
(*iii*) Add up these ampere-turns to get the total ampere-turns for the entire circuit.

6.25. Comparison Between Magnetic and Electric Circuits.

SIMILARITIES

Magnetic Circuit	Electric Circuit
 Fig. 6.27	 Fig. 6.28
1. Flux = $\frac{\text{m.m.f.}}{\text{reluctance}}$	Current = $\frac{\text{e.m.f.}}{\text{resistance}}$
2. M.M.F. (ampere-turns)	E.M.F. (volts)
3. Flux Φ (webers)	Current I (amperes)
4. Flux density B (Wb/m^2)	Current density (A/m^2)
5. Reluctance $S = \frac{l}{\mu A}\left(= \frac{l}{\mu_0 \mu_r A}\right)$	resistance $R = \rho \frac{l}{A} = \frac{l}{\rho A}$
6. Permeance (= 1/reluctance)	Conductance (= 1/resistance)
7. Reluctivity	Resistivity
8. Permeability (= 1/reluctivity)	Conductivity (= 1/resistivity)
9. Total m.m.f. = $\Phi S_1 + \Phi S_2 + \Phi S_3 + ...$	9. Total e.m.f. = $IR_1 + IR_2 + IR_3 + ...$

DIFFERENCES

1. Strictly speaking, flux does not actually 'flow' in the sense in which an electric current flows.

2. If temperature is kept constant, then resistance of an electric circuit is constant and is independent of the current strength (or current density). On the other hand, the reluctance of a magnetic circuit does depend on flux (and hence flux density) established in it. It is so because μ (which equals the slope of B/H curve) is not constant even for a given material as it depends on the flux density B. Value of μ is large for low value of B and ***vice-versa***. Hence, reluctance is small ($S = l/\mu A$) for small values of B and large for large values of B.

3. Flow of current in an electric circuit involves continuous expenditure of energy but in a magnetic circuit, energy is needed only creating the flux initially but not for maintaining it.

6.26. Parallel Magnetic Circuits

Fig. 6.29 (*a*) shown a parallel magnetic circuit consisting of two parallel magnetic paths ACB and ADB acted upon by the same *m.m.f.* Each magnetic path has an average length of 2 ($l_1 + l_2$). The flux produced by the coil wound on the central core is divided equally at point A between the two outer parallel paths. The reluctance offered by the two parallel paths is = half the reluctance of each path.

Fig. 6.29 (*b*) shows the equivalent electrical circuit where resistance offered to the voltage source is = $R \| R = R/2$

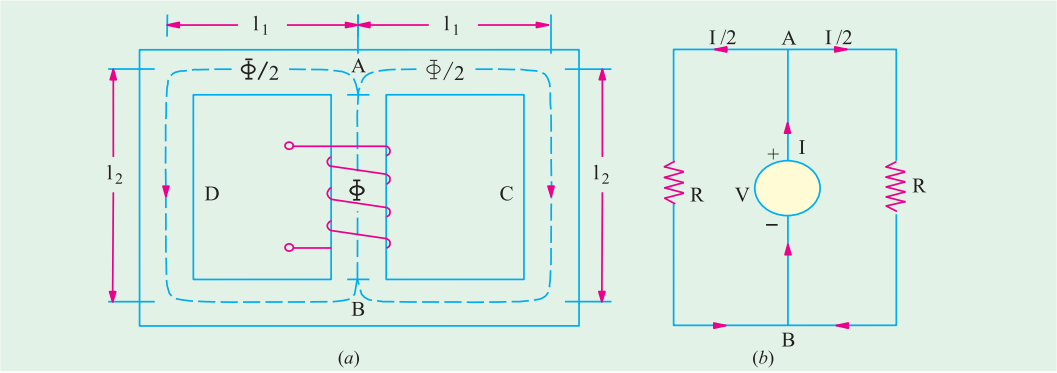


Fig. 6.29

It should be noted that reluctance offered by the central core AB has been neglected in the above treatment.

6.27. Series-Parallel Magnetic Circuits

Such a circuit is shown in Fig. 6.30 (*a*). It shows two parallel magnetic circuits *ACB* and *ACD* connected across the common magnetic path *AB* which contains an air-gap of length l_g. As usual, the flux Φ in the common core is divided equally at point *A* between the two parallel paths which have equal reluctance. The reluctance of the path *AB* consists of **(*i*)** air gap reluctance and **(*ii*)** the reluctance of the central core which comparatively negligible. Hence, the reluctance of the central core *AB* equals only the air-gap reluctance across which are connected two equal parallel reluctances. Hence, the *m.m.f.* required for this circuit would be the sum of **(*i*)** that required for the air-gap and **(*ii*)** that required for either of two paths (not both) as illustrated in Ex. 6.19, 6.20 and 6.21.

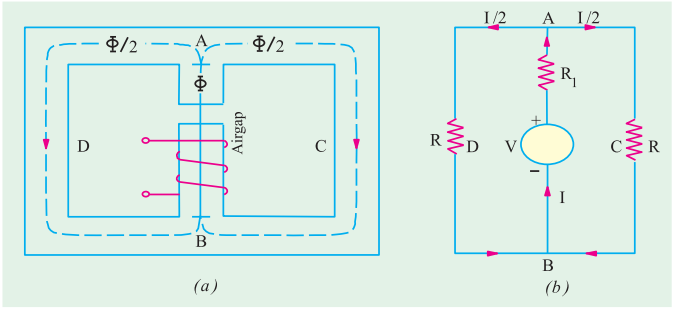


Fig. 6.30

The equivalent electrical circuit is shown in Fig. 6.30 (*b*) where the total resistance offered to the voltage source is $= R_1 + R \| R = R_1 + R/2$.

6.28. Leakage Flux and Hopkinson's Leakage Coefficient

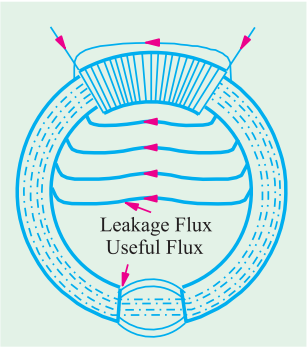


Fig. 6.31

Leakage flux is the flux which follows a path not intended for it. In Fig. 6.31 is shown an iron ring wound with a coil and having an air-gap. The flux in the air-gap is known as the useful flux because it is only this flux which can be utilized for various useful purposes.

It is found that it is impossible to confine all the flux to the iron path only, although it is usually possible to confine most of the electric current to a definite path, say a wire, by surrounding it with insulation. Unfortunately, there is no known insulator for magnetic flux. Air, which is a splendid insulator of electricity, is unluckily a fairly good magnetic conductor. Hence, as shown, some of the flux leaks through air surrounding the iron ring. The presence of leakage flux can be detected by a compass. Even in the best designed dynamos, it is found

that 15 to 20% of the total flux produced leaks away without being utilised usefully.

If, Φ_t = total flux produced ; Φ = useful flux available in the air-gap, then

$$\text{leakage coefficient } \lambda = \frac{\text{total flux}}{\text{useful flux}} \quad \text{or} \quad \lambda = \frac{\Phi_t}{\Phi}$$

In electric machines like motors and generators, magnetic leakage is undesirable, because, although it does not lower their power efficiency, yet it leads to their increased weight and cost of manufacture. Magnetic leakage can be minimised by placing the exciting coils or windings as close as possible to the air-gap or to the points in the magnetic circuit where flux is to be utilized for useful purposes.

It is also seen from Fig. 6.31 that there is fringing or spreading of lines of flux at the edges of the air-gap. This fringing increases the effective area of the air-gap.

The value of λ for modern electric machines varies between 1.1 and 1.25.

6.29. Magnetisation Curves

The approximate magnetisation curves of a few magnetic materials are shown in Fig. 6.32.

These curves can be determined by the following methods provided the materials are in the form of a ring :

(*a*) By means of a ballistic galvanometer and **(*b*)** By means of a fluxmeter.

6.30. Magnetisation Curves by Ballistic Galvanometer

In Fig. 6.33 shown the specimen ring of uniform cross-section wound uniformly with a coil *P* which is connected to a battery *B* through a reversing switch *RS*, a variable resistance R_1 and an ammeter. Another secondary coil *S* also wound over a small portion of the ring and is connected through a resistance *R* to a ballistic galvanometer *BG*.

The current through the primary *P* can be adjusted with the help of R_1. Suppose the primary current is *I*. When the primary current is reversed by means of *RS*, then flux is reversed through *S*, hence an induced e.m.f. is produced in it which sends a current through *BG*. This current is of very short duration. The first deflection or 'throw' of the *BG* is proportional to the quantity of electricity or charge passing through it so long as the time taken for this charge to flow is short as compared with the time of one oscillation.

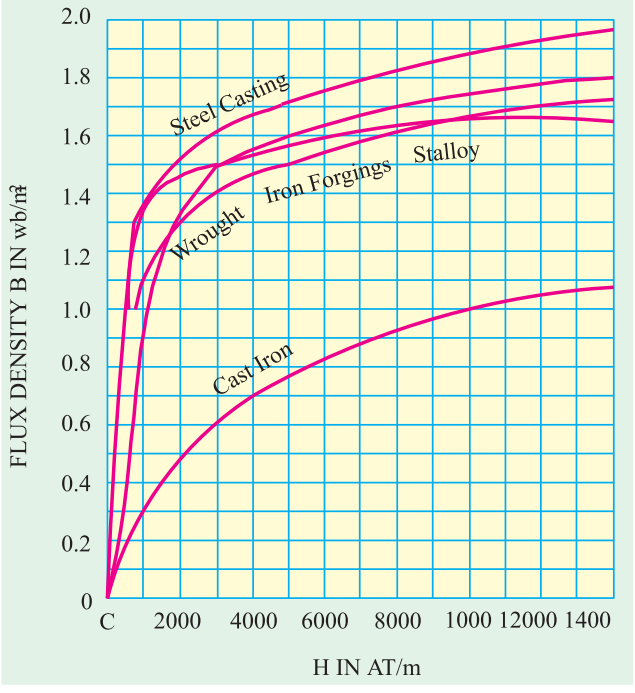


Fig. 6.32

If θ = first deflection or 'throw' of the galvanometer when primary current *I* is reversed.

k = ballistic constant of the galvanometer *i.e.* charge per unit deflection.

then, charge passing through *BG* is = $k\theta$ coulombs ...(*i*)

Let Φ= flux in Wb produced by primary current of *I* amperes ; *t* = time of reversal of flux ; then

rate of change of flux = $\frac{2\Phi}{t}$ Wb/s

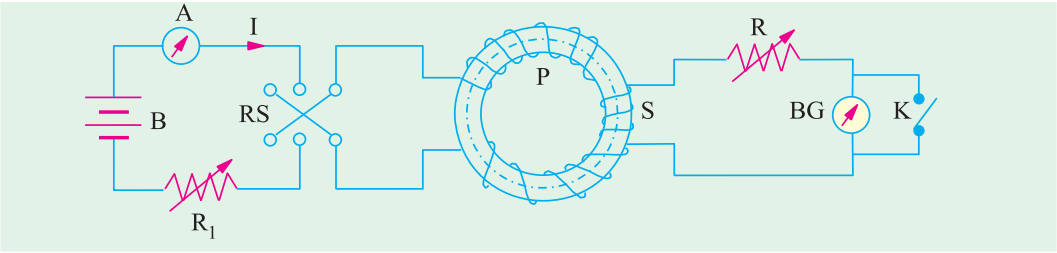


Fig. 6.33

If N_2 is the number of turns in secondary coil S, then average e.m.f. induces in it is

$$= N_2 \cdot \frac{2\Phi}{t} \text{ volt.}$$

Secondary current or current through $BG = \dfrac{2N_2\Phi}{R_s t}$ amperes

where R_s is the total resistance of the secondary circuit.

Charge flowing through BG = average current × time $= \dfrac{2N_2\Phi}{R_s t} \times t = \dfrac{2N_2\Phi}{R_s}$ coulomb ...(*ii*)

Equation (*i*) and (*ii*), we get $k\theta = \dfrac{2N_2\Phi}{R_s}$ $\therefore$ $\Phi = \dfrac{k\theta R_s}{2N_2}$ Wb

If A m^2 is the cross-sectional area of the ring, then flux density is

$$B = \frac{\Phi}{A} = \frac{k\theta R_s}{2N_2 A} \text{ Wb/m}^2$$

If N_1 is the number of primary turns and l metres the mean circumference of the ring, then, magnetising force $H = N_1 I/l$ AT/m.

The above experiment is repeated with different values of primary current and form the data so obtained, the B/H curves or magnetisation curves can be drawn.

6.31. Magnetisation Curves by Fluxmeter

In this method, the BG of Fig. 6.31 is replaced by a fluxmeter which is just a special type of ballistic galvanometer. When current through P is reversed, the flux is also reversed. The deflection of the fluxmeter is proportional to the change in flux-linkages of the secondary coil. If the flux is reversed from $+\Phi$ to $-\Phi$, the change in flux-linkages in secondary S in $= 2\Phi N_2$.

If θ = corresponding deflection of the fluxmeter

C = fluxmeter constant *i.e.* weber-turns per unit deflection.

then, change of flux-linkages in $S = C\theta$

$\therefore$ $2\Phi N_2 = C\theta$ or $\Phi = \dfrac{C\theta}{N_2}$ Wb ; $B = \dfrac{\Phi}{A} = \dfrac{C\theta}{2N_2 A}$ Wb/m^2

Example 6.8. *A fluxmeter is connected to a search-coil having 600 turns and mean area of 4 cm^2. The search coil is placed at the centre of an air-cored solenoid 1 metre long and wound with 1000 turns. When a current of 4 A is reversed, there is a deflection of 20 scale divisions on the fluxmeter. Calculate the calibration in Wb-turns per scale division.*

(Measurements-I, Nagpur Univ. 1991)

Solution. Magnetising force of the solenoid is $H = NI/l$ AT/m

$B = \mu_0 H = \mu_0 NI/l = 4\pi \times 10^{-7} \times 1000 \times 4/1 = 16\pi \times 10^{-4}$ Wb/m^2

Flux linked with the search coil is $\Phi = BA = 64\pi \times 10^{-8}$ Wb

Total change of flux-linkages on reversal

$= 2 \times 64\pi \times 10^{-8} \times 600$ Wb-turns –Art. 6.29

$= 7.68\pi \times 10^{-4}$ Wb-turns

Fluxmeter constant C is given by $= \dfrac{\text{Change in flux-linkages}}{\text{deflection produced}}$

$= 7.68\pi \times 10^{-4}/20 =$ **1.206×10^{-4} Wb-turns/division**

Example 6.9. *A ballistic galvanometer, connected to a search coil for measuring flux density in a core, gives a throw of 100 scale divisions on reversal of flux. The galvanometer coil has a resistance of 180 ohm. The galvanometer constant is 100 μC per scale division. The search coil has an area of 50 cm^2, wound with 1000 turns having a resistance of 20 ohm. Calculate the flux density in the core.* **(Elect. Instru & Measu. Nagpur Univ. 1992)**

Solution. As seen from Art. 6.28.

$$k\theta = 2N_2\Phi/R_s \quad \text{or} \quad \Phi = k\theta R_s/2N_2 \text{ Wb}$$

$$\therefore \quad BA = k\theta R_s/2N_2 \quad \text{or} \quad B = k\theta R_s/2N_2 A$$

Here $k = 100$ μC/division $= 100 \times 10^{-6} = 10^{-4}$ C/division

$\theta = 100$; $A = 50$ cm$^2 = 5 \times 10^{-3}$ m^2

$R_s = 180 + 20 = 200\ \Omega$

$\therefore \quad B = 10^{-4} \times 100 \times 200/2 \times 1000 \times 5 \times 10^{-3} =$ **0.2 Wb/m^2**

Example 6.10. *A ring sample of iron, fitted with a primary and a secondary winding is to be tested by the method of reversals to obtain its B/H curve. Give a diagram of connections explain briefly how the test could be carried out.*

In such a test, the primary winding of 400 turns carries a current of 1.8 A. On reversal, a change of 8×10^{-3} Wb-turns is recorded in the secondary winding of 10 turns. The ring is made up of 50 laminations, each 0.5 mm thick with outer and inner diameters of 25 and 23 cm respectively. Assuming uniform flux distribution, determine the values of B, H and the permeability.

Solution. Here, change of flux-linkages $= 2\Phi N_2 = 8 \times 10^{-3}$ Wb-turns

$$\therefore \quad 2\Phi \times 10 = 8 \times 10^{-3} \quad \text{or} \quad \Phi = 4 \times 10^{-4} \text{ Wb and } A = 2.5 \times 10^{-4} \text{ m}^2$$

$$\therefore \quad B = \frac{4 \times 10^{-4}}{2.5 \times 10^{-4}} = 1.6 \text{ Wb/m}^2 \; ; \; H = \frac{Nl}{l} = \frac{400 \times 1.8}{0.24\pi} = 955 \text{ AT/m}$$

Now $\mu_0 \mu_r = \dfrac{B}{H}$; $\mu_r = \dfrac{B}{\mu_0 H} = \dfrac{1.6}{4\pi \times 10^{-7} \times 955} =$ **1333**

Example 6.11. *An iron ring of 3.5 cm^2 cross-sectional area with a mean length of 100 cm is wound with a magnetising winding of 100 turns. A secondary coil of 200 turns of wire is connected to a ballistic galvanometer having a constant of 1 micro-coulomb per scale division, the total resistance of the secondary circuit being 2000 Ω. On reversing a current of 10 A in the magnetising coil, the galvanometer gave a throw of 200 scale divisions. Calculate the flux density in the specimen and the value of the permeability at this flux density.* **(Elect. Measure, A.M.I.E Sec.B. 1992)**

Solution. Reference may please be made to Art. 6.28.

Here $N_1 = 100$; $N_2 = 200$: $A = 3.5 \times 10^{-4}$ m^2 ; $l = 100$ cm $= 1$m

$k = 10^{-6}$ C/division, $\theta = 100$ divisions; $R_s = 2000\ \Omega$; $I = 10$ A

$$B = \frac{k\theta R_s}{2N_2 A} = \frac{10^{-6} \times 100 \times 2000}{2 \times 200 \times 3.5 \times 10^{-4}} = 1.43 \text{ Wb/m}^2$$

Magnetising force $H = N_1 I/l = 100 \times 10/1 = 1000$ AT/m

$$\mu = \frac{B}{H} = \frac{1.43}{1000} =$$ **1.43×10^{-3} H/m**

Note. The relative permeability is given by $\mu_r = \mu / \mu_0 = 1.43 \times 10^{-3}/4\pi \times 10^{-7} =$ **1137.**

Example 6.12. *An iron ring has a mean diameter of 0.1 m and a cross-section of* 33.5×10^{-6} m^2. *It is wound with a magnetising winding of 320 turns and the secondary winding of 220 turns. On reversing a current of 10 A in the magnetising winding, a ballistic galvanometer gives a throw of 272 scale divisions, while a Hilbert Magnetic standard with 10 turns and a flux of* 2.5×10^{-4} *gives a reading of 102 scale divisions, other conditions remaining the same. Find the relative permeability of the specimen.* **(Elect. Measu. A.M.I.E. Sec B, 1991)**

Solution. Length of the magnetic path $l = \pi D = 0.1\ \pi$ m

Magnetising Force, $H = NI/l = 320 \times 10/0.1\ \pi = 10{,}186$ AT/m

Flux density $B = \mu_0 \mu_r H = 4\pi \times 10^{-7} \times \mu_r \times 10{,}186 = 0.0128\ \mu_r$ ...(*i*)

Now, from Hilbert's Magnetic standard, we have

$$2.5 \times 10^{-4} \times 10 = K \times 102,\ K = 2.45 \times 10^{-5}$$

On reversing a current of 10 A in the magnetising winding, total change in Weber-turns is

$2\Phi N_s = 2.45 \times 10^{-5} \times 272$ or $2 \times 220 \times \Phi = 2.45 \times 10^{-5} \times 272$ or $\Phi = 1.51 \times 10^{-5}$ Wb

$\therefore\quad B = \Phi/A = 1.51 \times 10^{-5}/33.5 \times 10^{-6} = 0.45$ Wb/m^2

Substituting this value in Eq. (*i*), we have $0.0128\ \mu_r = 0.45$, $\therefore \mu_r =$ **35.1**

Example 6.13. *A laminated soft iron ring of relative permeability 1000 has a mean circumference of 800 mm and a cross-sectional area 500 mm*^{2}. *A radial air-gap of 1 mm width is cut in the ring which is wound with 1000 turns. Calculate the current required to produce an air-gap flux of 0.5 mWb if leakage factor is 1.2 and stacking factor 0.9. Neglect fringing.*

Solution. $$\text{Total AT reqd.} = \Phi_g S_g + \Phi_i S_i = \frac{\Phi_g l_g}{\mu_0 A_g} + \frac{\Phi_i l_i}{\mu_0 \mu_r A_i B}$$

Now, air-gap flux $\Phi_s = 0.5$ mWb $= 0.5 \times 10^{-3}$ Wb, $l_g = 1$ mm $= 1 \times 10^{-3}$ m; $A_g = 500$ mm^2 $= 500 \times 10^{-6}$ m^2

Flux in the iron ring, $\Phi_i = 1.2 \times 0.5 \times 10^{-3}$ Wb

Net cross-sectional area $= A_i \times$ stacking factor $= 500 \times 10^{-6} \times 0.9$ m^2

$$\therefore\quad \text{total AT reqd.} = \frac{0.5\times10^{-3}\times1\times10^{-3}}{4\pi\times10^{-7}\times500\times10^{-6}} + \frac{1.2\times0.5\times10^{-3}\times800\times10^{-3}}{4\pi\times10^{-7}\times1000\times(0.9\times500\times10^{-6})} = 1644$$

$$\therefore\quad I = 1644/1000 = \textbf{1.64 A}$$

Example 6.14. *A ring has a diameter of 21 cm and a cross-sectional area of 10 cm*^{2}. *The ring is made up of semicircular sections of cast iron and cast steel, with each joint having a reluctance equal to an air-gap of 0.2 mm. Find the ampere-turns required to produce a flux of* 8×10^{-4} *Wb. The relative permeabilities of cast steel and cast iron are 800 and 166 respectively. Neglect fringing and leakage effects.* **(Elect. Circuits, South Gujarat Univ.)**

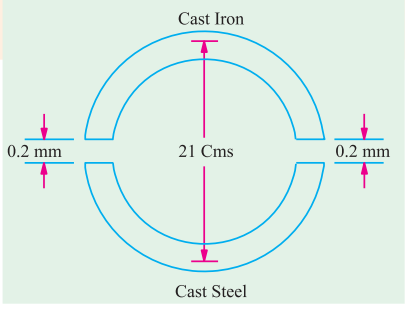


Fig. 6.34

Solution. $\Phi = 8 \times 10^{-4}$ Wb ; $A = 10$ cm$^2 = 10^{-3}$ m^2; $B = 8 \times 10^{-4}/10^{-3} = 0.8$ Wb/m^2

Air gap

$H = B/\mu_0 = 0.8/4\pi \times 10^{-7} = 6.366 \times 10^5$ AT/m

Total air-gap length $= 2 \times 0.2 = 0.4$ mm $= 4 \times 10^{-4}$ m

$\therefore\quad$ AT required $= H \times l = 6.366 \times 10^5 \times 4 \times 10^{-4} = 255$

Cast Steel Path (Fig. 6.34)

$H = B/\mu_0 \mu_r = 0.8/4\pi \times 10^{-7} \times 800 = 796$ AT/m

path $= \pi D/2 = 21\ \pi/2 = 33$ cm $= 0.33$ m

AT required $= H \times l = 796 \times 0.33 = 263$

Cast Iron Path

$H = 0.8/\pi \times 10^{-7} \times 166 = 3{,}835$ AT/m ; path $= 0.33$ m

AT required $= 3{,}835 \times 0.33 = 1265$

Total AT required $= 255 + 263 + 1265 =$ **1783.**

Example 6.15. *A mild steel ring of 30 cm mean circumference has a cross-sectional area of 6 cm^2 and has a winding of 500 turns on it. The ring is cut through at a point so as to provide an air-gap of 1 mm in the magnetic circuit. It is found that a current of 4 A in the winding, produces a flux density of 1 T in the air-gap. Find* ***(i)*** *the relative permeability of the mild steel and* ***(ii)*** *inductance of the winding.* **(F.E. Engg. Pune Univ.)**

Solution. ***(a)*** **Steel ring**

$$H = B/\mu_0 \mu_r = 1/4\pi \times 10^{-7} \times \mu_r \text{ AT/m} = 0.7957 \times 10^7/\mu_r \text{ AT/m}$$

$$\text{m.m.f.} = H \times l = (0.7957 \times 10^7/\mu_r) \times 29.9 \times 10^{-2} = 0.2379 \times 10^6/\mu_r \text{ AT}$$

(b) **Air-gap**

$$H = B/\mu_0 = 1/4\pi \times 10^{-7} = 0.7957 \times 10^6 \text{ AT/m}$$

$$\text{m.m.f. reqd.} = H \times l = 0.7957 \times 10^6 \times (1 \times 10^{-3}) = 795.7 \text{ AT}$$

$$\text{Total m.m.f.} = (0.2379 \times 10^6/\mu_r) + 795.7$$

$$\text{Total m.m.f. available} = NI = 500 \times 4 = 2000 \text{ AT}$$

(i) $\therefore \quad 2000 = (0.2379 \times 10^6/\mu_r) + 795.7 \quad \therefore \ \mu_r =$ **197.5**

(ii) Inductance of the winding $= \dfrac{N\Phi}{I} = \dfrac{NBA}{I} = \dfrac{500 \times 1 \times 6 \times 10^{-4}}{4} =$ **0.075 H**

Example 6.16. *An iron ring has a X-section of 3 cm^2 and a mean diameter of 25 cm. An air-gap of 0.4 mm has been cut across the section of the ring. The ring is wound with a coil of 200 turns through which a current of 2 A is passed. If the total magnetic flux is 0.24 mWb, find the relative permeability of iron, assuming no magnetic leakage.* **(Elect. Engg. A.M.Ae.S.I., June 1992)**

Solution. $\Phi = 0.24$ mWb; $A = 3$ cm$^2 = 3 \times 10^{-4}$ m^2;

$B = \Phi/A = 0.24 \times 10^{-3}/3 \times 10^{-4} = 0.8$ Wb/m^2

AT for iron ring $= H \times l = (B/\mu_0 \mu_r) \times l = (0.8/4\pi \times 10^{-7} \times \mu_r) \times 0.25 = 1.59 \times 10^{-5}/\mu_r$

AT for air-gap $= H \times l = (B/\mu_0) \times l = (0.8/4\pi \times 10^{-7}) \times 0.4 \times 10^{-3} = 255$

Total AT reqd. $= (1.59 \times 10^5/\mu_r) + 255$; total AT provided $= 200 \times 2 = 400$

$\therefore \quad (1.59 \times 10^5/\mu_r) + 255 = 400 \quad$ or $\quad \mu_r =$ **1096.**

Example 6.17. *A rectangular iron core is shown in Fig. 6.35. It has a mean length of magnetic path of 100 cm, cross-section of (2 cm × 2 cm), relative permeability of 1400 and an air-gap of 5 mm cut in the core. The three coils carried by the core have number of turns $N_a = 335$, $N_b = 600$ and $N_c = 600$; and the respective currents are 1.6 A, 4 A and 3 A. The directions of the currents are as shown. Find the flux in the air-gap.* **(F.Y. Engg. Pune Univ.)**

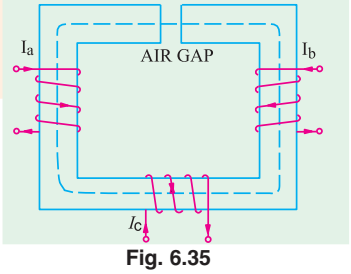


Fig. 6.35

Solution. By applying the Right-Hand Thumb rule, it is found that fluxes produced by the current I_a and I_b are directed in the clockwise direction through the iron core whereas that produced by current I_c is directed in the anticlockwise direction through the core.

$\therefore$ total m.m.f. $= N_a I_a + N_b I_b - N_c I_c = 335 \times 1.6 + 600 \times 4 - 600 \times 3 = 1136$ AT

Reluctance of the air-gap $= \dfrac{l}{\mu_0 A} = \dfrac{5 \times 10^3}{4\pi \times 10^{-7} \times 4 \times 10^{-4}} = 9.946 \times 10^6$ AT/Wb

Reluctance of the iron path $= \dfrac{l}{\mu_0 \mu_r A} = \dfrac{100 - (0.5) \times 10^{-2}}{4\pi \times 10^{-7} \times 1400 \times 4 \times 10^{-4}} = 1.414 \times 10^6$ AT/Wb

Total reluctance $= (9.946 + 1.414) \times 10^6 = 11.36 \times 10^6$ AT/Wb

The flux in the air-gap is the same as in the iron core.

Air-gap flux $= \dfrac{\text{m.m.f.}}{\text{reluctance}} = \dfrac{1136}{11.36 \times 10^6} = 100 \times 10^{-6}$ Wb $=$ **100 μWb**

Example 6.18. *A series magnetic circuit comprises of three sections* ***(i)*** *length of 80 mm with cross-sectional area 60 mm^2,* ***(ii)*** *length of 70 mm with cross-sectional area 80 mm^2 and* ***(iii)*** *and air-gap of length 0.5 mm with cross-sectional area of 60 mm^2. Sections* ***(i)*** *and* ***(ii)*** *are if a material having magnetic characteristics given by the following table.*

H (AT/m)	*100*	*210*	*340*	*500*	*800*	*1500*
B (Tesla)	*0.2*	*0.4*	*0.6*	*0.8*	*1.0*	*1.2*

Determine the current necessary in a coil of 4000 turns wound on section (ii) to produce a flux density of 0.7 Tesla in the air-gap. Neglect magnetic leakage. **(F.E. Pune Univ. May 1990)**

Solution. Section (*i*) It has the same cross-sectional area as the air-gap. Hence, it has the same flux density *i.e.* 0.7 Tsela as in the air-gap. The value of the magnetising force H corresponding to this flux density of 0.7 T as read from the B-H plot is 415 AT/m.

m.m.f. reqd $= H \times l = 415 \times (80 \times 10^{-3}) = 33.2$ AT

Section (*ii*) Since its cross-sectional area is different from that of the air-gap, its flux density would also be different even though, being a series circuit, its flux would be the same.

Air-gap flux $= B \times L = 0 \times (60 \times 10^{-6}) = 42 \times 10^{-6}$ Wb

Flux density in this section $= 42 \times 10^{-6} / 80 \times 10^{-6} = 0.525$ T

The corresponding value of the H from the given garph is 285 AT/m

m.m.f. reqd, for this section $= 285 \times (70 \times 10^{-3}) = 19.95$ AT.

Air-gap

$H = B/\mu_0 = 0.7/4\pi \times 10^{-7} = 0.557 \times 10^6$ AT/m

$\therefore$ m.m.f. reqd. $= 0.557 \times 10^{-6} \times (0.5 \times 10^{-3}) = 278.5$ AT

Total m.m.f. reqd. $= 33.2 + 19.95 + 278.5 = 331.6$

$\therefore$ $NI = 331.6$ or $I = 331.6/4000 =$ **0.083 A**

Example 6.19. *A magnetic circuit made of mild steel is arranged as shown in Fig. 6.36. The central limb is wound with 500 turns and has a cross-sectional area of 800 mm^2. Each of the outer limbs has a cross-sectional area of 500 mm^2. The air-gap has a length of 1 mm. Calculate the current rquired to set up a flux of 1.3 mWb in the central limb assuming no magnetic leakage and fringing. Mild steel required 3800 AT/m to produce flux density of 1.625 T and 850 AT/m to produce flux density of 1.3 T.* **(F.Y. Engg. Pune Univ.)**

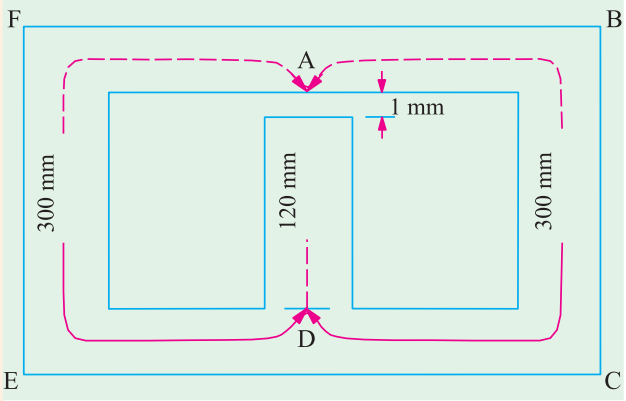


Fig. 6.36

Solution. Flux in the central limb is = 1.3 mWb = 1.3×10^{-3} Wb

Cross section $A = 800 \text{ mm}^2 = 800 \times 10^{-6} \text{ m}^2$

$\therefore \quad B = \Phi/A = 1.3 \times 10^{-6} / 800 \times 10^{-6}$

$= 1.625$ T

Corresponding value of H for this flux density is given as 3800 AT/m.

Since the length of the central limb is 120 mm. m.m.f. required is $= H \times l = 3800 \times (120 \times 10^{-3})$

$= 456$ AT/m.

Air-gap

Flux density in the air-gap is the same as that in the central limb.

$H = B/\mu_0 = 1.625/4\pi \times 10^{-7} = 0.1293 \times 10^{-7}$ AT/m

Length of the air-gap = 1 mm = 10^{-3} m

m.m.f. reqd. for the air-gap = $H \times l = 0.1293 \times 10^7 \times 10^{-3} = 1293$ AT.

The flux of the central limb divides equally at point A in figure along the two parallel path *ABCD* and *AFED*. We may consider either path, say *ABCD* and calculate the m.m.f. required for it. The same m.m.f. will also send the flux through the other parallel path *AFED*.

Flux through *ABCD* = $1.3 \times 10^{-3}/2 = 0.65 \times 10^{-3}$ Wb

Flux density $B = 0.65 \times 10^{-3}/500 \times 10^{-6} = 1.3$ T

The corresponding value of H for this value of B is given at 850 AT/m.

$\therefore$ *m.m.f.* reqd. for path *ABCD* = $H \times l = 850 \times (300 \times 10^{-3}) = 255$ AT

As said above, this, m.m.f. will also send the flux in the parallel path *AFED*.

Total m.m.f. reqd. = 456 + 1293 + 255 = 2004 AT

Since the number of turns is 500, $I = 2004/500 =$ **4A**.

Example 6.20. *A cast steel d.c. electromagnet shown in Fig. 6.37 has a coil of 1000 turns on its central limb. Determine the current that the coil should carry to produce a flux of 2.5 mWb in the air-gap. Neglect leakage. Dimensions are given in cm. The magnetisation curve for cast steel is as under :*

Flux density (Wb/m²) :	*0.2*	*0.5*	*0.7*	*1.0*	*1.2*
Amp-turns/metre :	*300*	*540*	*650*	*900*	*1150*

(Electrotechnics-I, ; M.S. Univ. Baroda)

Solution. Two points should be noted

(*i*) there are two (equal) parallel paths *ACDE* and *AGE* across the central path *AE*.

(*ii*) flux density in either parallle path is half of that in the central path because flux divides into two equal parts at point *A*.

Total m.m.f. required for the whole electromagnet is equal to the sum of the following three m.m.fs.

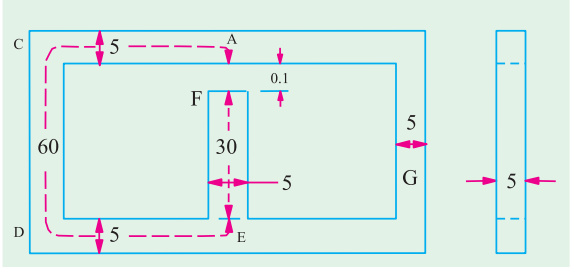


Fig. 6.37

(*i*) that required for path *EF*

(*ii*) that required for air-gap

(*iii*) that required for either of the two parallel paths ; say, path $ACDE_2$

Flux density in the central limb and air gap is

$= 2.5 \times 10^{-3} / (5 \times 5) \times 10^{-4} = 1 \text{ Wb/m}^2$

Corresponding value of H as found from the given data is 900 AT/m.

$\therefore$ AT for central limb $= 900 \times 0.3 = 270$

H in air-gap $= B/\mu_0 = 1/4\pi \times 10^{-7} = 79.56 \times 10^4$ AT/m

AT required $= 79.56 \times 10^4 \times 10^{-3} = 795.6$

Flux density in path *ACDE* is 0.5 Wb/m² for which corresponding value of H is 540 AT/m.

$\therefore$ AT required for path $ACDE = 540 \times 0.6 = 324$

Total AT required = 270 + 795.6 + 324 = 1390 ;Current required = 1390/1000 = **1.39 A**

Example 6.21. *A cast steel magnetic structure made for a bar of section 8 cm × 2 cm is shown in Fig. 6.35. Determine the current that the 500 turn-magnetising coil on the left limb should carry so that a flux of 2 mWb is produced in the right limb. Take $\mu_r = 600$ and neglect leakage.*

(Elect. Technology Allahabad Univ. 1993)

Solution. Since path C and D are in parallel with each other w.r.t. path E (Fig. 6.38), the m.m.f. across the two is the same.

$$\Phi_1 S_1 = \Phi_2 S_2$$

$$\therefore \quad \Phi_1 \times \frac{15}{\mu A} = 2 \times \frac{25}{\mu A}$$

$$\therefore \quad \Phi_1 = 10/3 \text{ mWb}$$

$$\therefore \quad \Phi = \Phi_1 + \Phi_2 = 16/3 \text{ mWb}$$

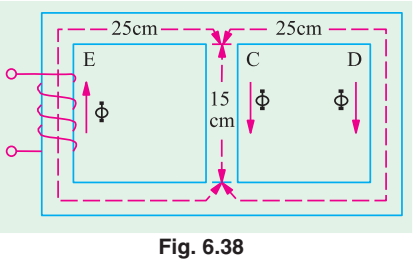


Fig. 6.38

Total AT required for the whole circuit is equal to the sum of

(*i*) that required for path E and **(*ii*)** that required for either of the two paths C or D.

$$\text{Flux density in path } E = \frac{16 \times 10^{-3}}{3 \times 4 \times 10^{-4}} = \frac{40}{3} \text{ Wb/m}^2$$

$$\text{AT reqd.} = \frac{40 \times 0.25}{3 \times 4\pi \times 10^{-7} \times 600} = 4,420$$

$$\text{Flux density in path } D = \frac{2 \times 10^{-3}}{4 \times 10^{-4}} = 5 \text{ Wb/m}^2$$

$$\text{AT reqd.} = \frac{5}{4\pi \times 10^{-7} \times 600} \times 0.25 = 1658$$

Total AT = 4,420 + 1,658 = 6,078 ;

Current needed = 6078/500 = **12.16 A**

Example 6.22. *A ring of cast steel has an external diameter of 24 cm and a square cross-section of 3 cm side. Inside and cross the ring, an ordinary steel bar 18 cm × 3 cm × 0.4 cm is fitted with negligible gap. Calculating the number of ampere-turns required to be applied to one half of the ring to produce a flux density of 1.0 weber per metre^2 in the other half. Neglect leakge. The B-H characteristics are as below :*

	For Cast Steel			*For Ordinary Plate*			
B in Wb/m^2	*1.0*	*1.1*	*1.2*	*B in Wb/m^2*	*1.2*	*1.4*	*1.45*
Amp-turn/m	*900*	*1020*	*1220*	*Amp-turn/m*	*590*	*1200*	*1650*

(Elect. Technology, Indore Univ.)

Solution. The magnetic circuit is shown in Fig. 6.39.

The m.m.f. (or AT) produced on the half A acts across the parallel magnetic circuit C and D. First, total AT across C is calculated and since these amp-turns are also applied across D, the flux density B in D can be estimated. Next, flux density in A is calculated and therefore, the AT required for this flux density. In fact, the total AT (or m.m.f.) required is the sum of that required for A and that of either for the two parallel paths C or D.

Value of flux density in C = 1.0 Wb/m^2

Mean diameter of the ring = (24 + 18)/2 = 21 cm

Mean circumference = $\pi \times 21 = 66$ cm

Length of path A or $C = 66/2 = 33$ cm $= 0.33$ m

Value of AT/m for a flux density of 1.0 Wb/m^2 as seen from the given $B.H$ characteristics = 900 AT/m

$\therefore$ Total AT for path $C = 900 \times 0.33 = 297$. The same ATs. are applied across path D.

Length of path D = 18 cm = 0.18 m $\therefore$ AT/m for path $D = 297/0.18 = 1650$

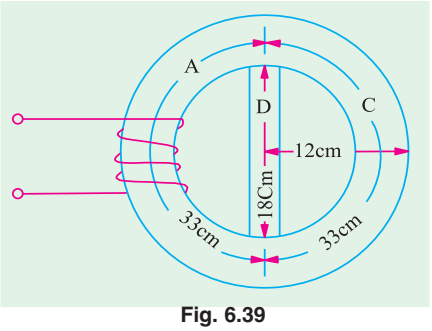


Fig. 6.39

Value of B corresponding to this AT/m from given table is = 1.45 Wb/m^2

Flux through $C = B \times A = 1.0 \times 9 \times 10^{-4} = 9 \times 10^{-4}$ Wb

Flux through $D = 1.45 \times (3 \times 0.4 \times 10^{-4}) = 1.74 \times 10^{-4}$ Wb

$\therefore$ Total flux through $A = 9 \times 10^{-4} + 1.74 \times 10^{-4} = 10.74 \times 10^{-4}$ Wb.

Flux density through $A = 10.74 \times 10^{-4}/9 \times 10^{-4} = 1.193$ Wb/m^2

No. of AT/m reqd. to produce this flux density as read from the given table = 1200 (approx.)

$\therefore$ Amp-turns required for limb $A = 1200 \times 0.33 = 396$

Total AT required = 396 + 297 = **693**

Example 6.23. *Show how the ampere-turns per pole required to produce a given flux in d.c. generator are calculated.*

Find the amp-turns per pole required to produce a flux of 40 mWb per pole in a machine with a smooth core armature and having the following dimensions :

Length of air gap = 5 mm *Area of air-gap = 500 cm^2*

Length of pole = 12 cm *Sectional area of pole core = 325 cm^2*

Relative permeability of pole core = 1,500

Length of magnetic path in yoke betwen pole = 65 cm

Cross-sectional area of yoke = 450 cm^2 ; Relative permeability of yoke = 1,200

Leakage coefficient = 1.2

The ampere-turns for the armature core may be neglected.

Solution. Air-gap $\Phi = 40$ mWb $= 4 \times 10^{-2}$ Wb ; $A = 500 \times 10^{-4} = 5 \times 10^{-3}$ m^2

$\therefore$ $B = 4 \times 10^{-2}/5 \times 10^{-2} = 0.8$ Wb/m^2 ; $H = B/\mu_0 = 0.8/4\pi \times 10^{-7} = 63.63 \times 10^{-4}$ AT/m

Air-gap length = 5×10^{-3} m ; AT reqd. = $63.63 \times 10^4 \times 5 \times 10^{-3} = 3181.5$

Pole Core

$\Phi = 1.2 \times 4 \times 10^{-2} = 4.8 \times 10^{-2}$ Wb ; $A = 325 \times 10^{-4}$ m^2

$B = 4.8 \times 10^{-2}/325 \times 10^{-4} = 1.477$ Wb/m^2

$H = B/\mu_0 \mu_r = 1.477/4\pi \times 10^{-7} \times 1{,}500 = 783$ AT/m

Pole length = 0.12 m ; AT reqd. = $783 \times 0.12 = 94$

Yoke Path

flux = half the pole flux $= 0.5 \times 4 \times 10^{-2} = 2 \times 10^{-2}$ Wb

$A = 450$ cm$^2 = 45 \times 10^{-3}$ m^2 ; $B = 2 \times 10^{-2}/45 \times 10^{-3} = 4/9$ Wb/m^2

$$H = \frac{4/9}{4\pi \times 10^{-7} \times 1{,}200} = 294.5 \text{ AT/m}$$ Yoke length = 0.65 m

At reqd = 294.5×0.65, Total AT/pole = 3181.5 + 94 + 191.4 = **3,467**

Example 6.24. *A shunt field coil is required to develop 1,500 AT with an applied voltage of 60 V. The rectangular coil is having a mean length of turn of 50 cm. Calculate the wire size. Resistivity of copper may be assumed to be 2×10^{-6} μΩ-cm at the operating temperature of the coil. Estimate also the number of turns if the coil is to be worked at a current density of 3 A/mm^2.*

(Basis Elect. Machines Nagpur Univ. 1992)

Solution. $NI = 1{,}500$ (given) or $N \cdot \frac{V}{R} = N \cdot \frac{60}{R} = 1{,}500$

$\therefore \quad R = \frac{N}{25}$ ohm Also $R = \rho \cdot \frac{l}{A} = \frac{2 \times 10^{-6} \times 50\, N}{A}$

$\therefore \quad \frac{N}{25} = \frac{10^{-4} n}{A}$ or $A = 25 \times 10^{-4}$ cm^2 or $A = 0.25$ mm^2

$\therefore \quad \frac{\pi D^2}{4} = 0/25$ or $D =$ **0.568 mm**

Current in the coil $= 3 \times 0.25 = 0.75$ A

Now, $NI = 1{,}500$; $\therefore \; N = 1{,}500/0.75 =$ **2,000**

Example 6.25. *A wooden ring has a circular cross-section of 300 sq. mm and a mean diameter of the ring is 200 mm. It is uniformly wound with 800 turns.*

Calculate :

(i) *the field strength produced in the coil by a current of 2 amperes :(assume = 1)*

(ii) *the magnetic flux density produced by this current and*

(iii) *the current required to produce a flux density of 0.02 wb/m^2.*

[Nagpur University (Summer 2000)]

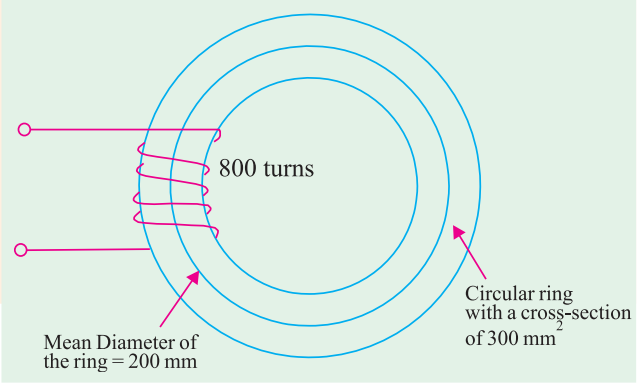


Fig. 6.40

Solution. The question assumes that the flux-path is through the ring, as shown by the dashed line, in figure, at the mean diameter, in Fig. 6.40.

With a current of 2 amp,

Coil m.m.f. $= 800 \times 2 = 1600$ AT

Mean length of path $= \pi \times 0.2$

$= 0.628$ m

(i) $H = \frac{1600}{0.628} = 2548$ amp-turns/meter

(ii) $B = \mu_0 \mu_r H = 4\pi \times 10^{-7} \times 1 \times 2548$

$= 3.20 \times 10^{-3}$ Wb/m^2

This Flux density is produced by a coil current of 2-amp

(iii) For producing a flux of 0.02 Wb/m^2, the coil current required is

$2 \times \frac{0.02}{0.0032} =$ **12.5 amp**

Example 6.26. *A magnetic core in the form of a closed circular ring has a mean length of 30 cm and a cross-sectional area of 1 cm^2. The relative permeability of iron is 2400. What direct-current will be needed in the coil of 2000 turns uniformly wound around the ring to create a flux of 0.20 mWb in iorn ? If an air-gap of 1 mm is cut through the core perpendicualr to the direction of this flux, what current will now be needed to maintain the same flux in the air gap ?*

[Nagpur University Nov. 1999]

Solution.

$$\text{Reluctance of core} = \frac{1}{\mu_0 \mu_r}\frac{L}{a} = \frac{1}{10\pi \times 10^{-7} \times 2400} \times \frac{30 \times 10^2}{1 \times 10^{-4}}$$

$$= \frac{30 \times 10^{-9}}{4\pi \times 2400 \times 1} = 995223 \text{ MKS units}$$

$$\phi = 0.2 \times 10^{-3} \text{ Wb}$$

$$\text{MMF required} = \phi \times \text{Rel}$$

$$= 0.2 \times 10^{-3} \times 995223 = 199 \text{ amp-tunrs}$$

Direct current required through the 2000 turn coil

$$= \frac{199}{2000} = 0.0995 \text{ amp}$$

Reluctance of 1 mm air gap

$$= \frac{1}{4\pi \times 10^{-7}} \times \frac{1 \times 10^{-3}}{1 \times 10^{-4}} = \frac{10^8}{4\pi} = 7961783 \text{ MKS units}$$

Addition of two reluctances

$$= 995223 + 7961783 = 8957006 \text{ MKS units}$$

MMF required to establish the given flux

$$= 0.2 \times 10^{-3} \times 8957006 = \textbf{1791 amp turns}$$

Current required through the coil

$$= \frac{1791}{2000} \quad \textbf{0.8955 amp}$$

Note : Due to the high permeability of iron, which is given here as 2400, 1 mm of air-gap length is equivalent magnetically to 2400 mm of length through the core, for comparison of mmf required.

Example 6.27. *An iron-ring of mean length 30 cm is made up of 3 pieces of cast-iron. Each piece has the same length, but their respective diameters are 4, 3 and 2.5 cm. An air-gap of length 0.5 mm is cut in the 2.5 – cm. Piece. If a coil of 1000 turns is wound on the ring, find the value of the current has to carry to produce a flux density of 0.5 Wb/m^2 in the air gap. B-H curve data of cast-iron is as follows :*

B	*(Wb/m^2) :*	*0.10*	*0.20*	*0.30*	*0.40*	*0.50*	*0.60*
H	*(AT/m) :*	*280*	*680*	*990*	*1400*	*2000*	*2800*

Permeability of free space $= 4\pi \times 10^{-7}$

Neglect Leakage and fringing effects. **[Nagpur University, November 1998]**

Solution. From the data given, plot the B-H curve for cast-iron

The magnetic circuit has four parts connected in series

Part 1. Air-gap 0.5 mm length, $B = 0.5$ wb/m^2, and

Permeability of free sapce is known

$$H_1 = B/\mu_0 = 0.5 \times 10^7/(4\pi) = 398100$$

$$\text{AT for gap} = (0.5 \times 10^{-3}) \times H_1 = 199$$

Part 2. 2.5 cm diameter, 10 – cm long, cast-iron ring portion *B* and *H* are to be related with the help of given data. In this, out of 10 cms. 0.5 mm air-gap is cut, and this portion of ring will have cast-iron length of 99.5 mm.

For $B = 0.5$ wb/m^2, $H_2 = 2000$ AT/m

$$AT_2 = 2000 \times 9.95 \times 10^{-2} = 199$$

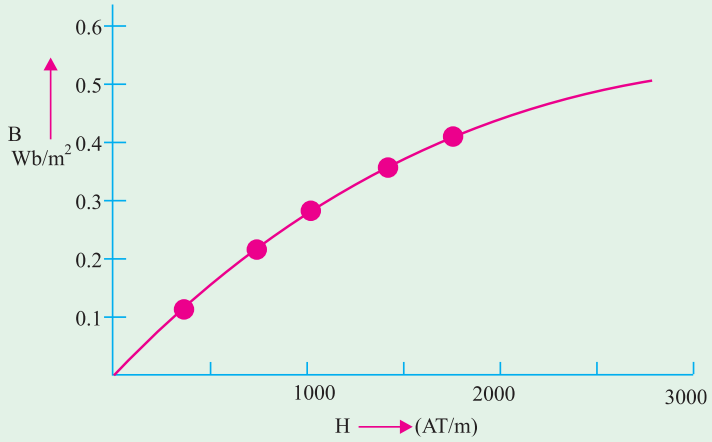


Fig. 6.41

Part 3. 3-cm diameter, 10-cm long, cast-iron ring-portion.

Here $B = 0.50 \times (2.5/3)^2 = 0.347$ Wb/m^2

For this B, H is read from B-H curve.

$H_3 = 1183$ AT/m

$AT_3 = 1183 \times 10 \times 10^{-2} = 118.3$

Part 4. 4 cm. Diameter, 10 cm long, cast-iron ring portion.

Here, $B = 0.50 \times (2.5 \times 4)^2 \times 0.195$ Wb/m^2

From, B–H curve, corresponding H is 661

$AT_4 = 661 \times 10 \times 10^{-2} = 66\ AT$

Since all these four parts in series, the total m.m.f. required is obtained by adding the above terms.

AT = 199 + 199 + 118 + 66 = 582

Coil Current = 582/1000 = 0.582 amp

Additional observations.

(a) The 2.5-cm diameter portion of the ring has $H = 2000$ for $B = 0.5$ Wb/m^2. From this, the relative permeability of cast-iron can be foud out.

$\mu_0 \mu_r = 0.5/2000$, giving $\mu_r = 199$

An air-gap of 0.5 mm is equivalent of 99.5 mm of cast-iron length. Hence, the two m.m.fs. are equal to 199 each.

(b) The common flux for this circuit is obtained from flux-density and the concerned area.

Hence $\phi = 0.5 \times (\pi/4) \times (2.5 \times 10^{-2})^2 = 0.02453 \times 10^{-2}$

$= 0.2453$ mWb

Reluctance of total magnetic circuit

$=$ m.m.f./flux $= 582/(2.453 \times 10^{-4})$

$= 2372650$ MKS units

Example 6.28. *A steel-ring of 25 cm mean diameter and of circular section 3 cm in diameter has an air-gap of 1.5 mm length. It is wound uniformly with 700 turns of wire carrying a current of 2 amp. Calculate :* ***(i)*** *Magneto motive force* ***(ii)*** *Flux density* ***(iii)*** *Magnetic flux* ***(iv)*** *Relative permeability. Neglect magnetic leakage and assume that iron path takes 35 % of total magneto motive force.* **[Nagpur University, April 1996]**

Solution. From the given data, length of mean path in the ring ($= L_m$) is to be calculated. For a mean diameter of 25 cm, with 1.5 mm of air-gap length.

$L_m = (\pi \times 0.25) - (1.5 \times 10^{-3}) = 0.7835$ m

Cross-sectional area of a 3 cm diameter ring $= 7.065 \times 10^{-4}$ sq.m.

Total m.m.f. due to coil $= 700 \times 2 = 1400$ amp-turns

Since iron-path takes 355 of the total mmf, it is 490.

Remaining mmf of 910 is consumed by the air-gap.

Corresponding H for air-gap $= 910/(1.5 \times 10^{-3}) = 606666$ amp-turns/m.

If Flux density is B_g, we have

$$B_g = \mu_0 H_g = 4\pi \times 10^{-7} \times 606666 = 0.762 \text{ Wb/m}^2$$

Iron-ring and air-gap are in series hence their flux is same. Since the two have some cross-sectional area, the flux density is also same. The ring has a mean length of 0.7835 m and needs an mmf of 490. For the ring.

$$H = 490/0.7845 = 625.4 \text{ amp-turns / m}$$

$$\mu_0 \mu_r = B/H = 0.752/625.4 = 1.218 \times 10^{-3}$$

$$\mu_r = (1.218 \times 10^{-3}) / (4\pi \times 10^{-7}) = 970$$

Flux = Flux density × Cross-sectional area $= 0.762 \times 7.065 \times 10^{-4} = 0.538$ milli-webers

Check. μ_r of 970 means that 1.5 mm of air-gap length is equivalent to $(1.5 \times 10^{-3} \times 970) = 1.455$ m of length through iron as a medium. With this equivalent.

$$\frac{\text{mmf of ring}}{\text{mmf for (ring + air-gap)}} = \frac{0.785}{0.785 + 1.455} = 0.35$$

which means that 35 % of total mmf is required for the ring

Example 6.29. ***(a)*** *Determine the amp-turns required to produce a flux of 0.38 mWb in an iron-ring of mean diameter 58 cm and cross-sectional area of 3 sq. cm. Use the following data for the ring :*

B Wb/m²	*0.5*	*1.0*	*1.2*	*1.4*
μ_r	*2500*	*2000*	*1500*	*1000*

(b) *If a saw-cut of 1mm width is made in the ring, calculate the flux density in the ring, with the mmf remaining same as in (a) above.* **[Nagpur University, Nov. 1996]**

Solution. Plot the B- μ_r curve as in Fig. 6.42

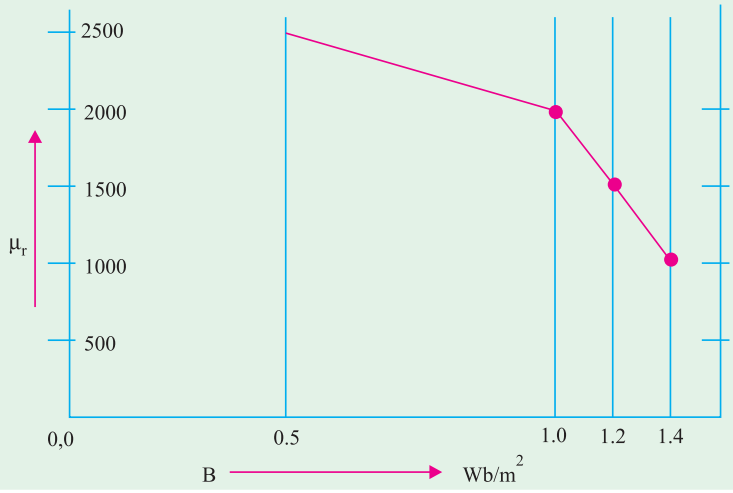


Fig. 6.42

(*a*) Cross-sectional area = 3 sq. cm = 3×10^{-4} sq. m.

Flux = 38 mWb = 0.38×10^{-3} Wb

Flux density, B = flux/area = $(0.38 \times 10^{-3})/(3 \times 10^{-4}) = 1.267$ Wb/m^2

Looking into the table relating B and μ_r, interpolation is required for evaluating μ_r for $B = 1.267$ Wb/m^2. After 1.2 Wb/m^2, μ_r decreases by 500 for a rise of 0.2 in B. Interpolation results into :

$$\mu_r = 1500 - \frac{0.067}{0.20} \times 500 = 1332$$

For mean diameter of path in the ring as 0.58 m, the length of the magnetic path in the ring is

$$l_m = p \times 0.58 = 1.8212 \text{ m}$$

Since $B = \mu_0 \mu_r H$,

$$H = 1.267/(4\pi \times 10^{-7} \times 1332) = 757$$

Hence, the required m.m.f. is

$$757 \times 1.8212 = 1378 \text{ amp-turns}$$

(*b*) If a saw-cut of 1 mm is cut in the ring, B is to be calculated with a m.m.f. of 1378. Now the magnetic circuit has two components in series : the ring with its B-μ_r curve in Fig. 6.42 and the air-gap. Since B is not known, μ_r cannot be accurately known right in the initial steps. The procedure to solve the case should be as follows :

Let B the flux density as a result of 1378 amp-turns due to the coil.

For air-gap.

$$H_g = B_g / (4\pi \times 10^{-7}) = 0.796 \times 10^6 \text{ AT/m}$$

$$AT_g = H_g \times l_g = 0.796 \times 10^6 \times 1 \times 10^{-3} \times B_g = 796\, B_g$$

Due to the air-gap, the flux-density is expected to be between 0.5 and 1 Wb/m^2, because, in (*a*) above, μ_r (for $B = 1.267$ Wb/m^2) is 1332. One mm air-gap is equivalent to 1332 mm of path added in iron-medium. This proportional increase in the reluctance of the magnetic circuit indicates that flux density should fall to a value in between 0.5 and 1 Wb/m^2.

For Iron-ring. With flux density expected to be as mentioned above, interpolation formula for μ_r can be written as :

$$\mu_r = 2500 - 500\,[(B_g - 0.50) / 0.50] = 3000 - 1000\, B_g$$

$$H_i = B_g / (\mu_0 \mu_r) = B_g / [\mu_0 (3000 - 1000\, B_g)]$$

Total m.m.f. $= AT_g + AT_i = 1378$, as previously calculated

Hence,

$$1378 = \frac{1.8212 \times B_g}{\mu_0 (3000 - 1000\, B_g)} + 796\, B_g$$

This is a quadratic equation in B_g and the solution, which gives B_g in between 0.5 & 1.0 Wb/m^2 is acceptable.

This results into $B_g = 0.925$ Wb/m^2

Corresponding $\mu_r = 3000 - 1000 \times 0.925 = 2075$

Example 6.30. *An iron-ring of mean diameter 19.1 cm and having a cross-sectional area of 4 sq. cm is required to produce a flux of 0.44 mWb. Find the coil-mmf required.*

If a saw-cut 1 mm wide is made in the ring, how many extra amp-turns are required to maintain the same flux ?

B - μ_r curve is as follows :

B (Wb/m^2)	*0.8*	*1.0*	*1.2*	*1.4*
μ_r	*2300*	*2000*	*1600*	*1100*

[Nagpur University, April 1998]

Solution. For a mean-diameter of 19.1 cm, Length of mean path, $l_m = \pi \times 0.191 = 0.6$ m

Cross-sectional area = 4 sq.cm = 4×10^{-4} m^2

Flux, $\phi = 0.44$ mWb = 0.44×10^{-3} Wb

Flux density, $B = 0.44 \times 10^{-3}/(4 \times 10^{-4}) = 1.1$ Wb/m^2

For this flux density, $\mu_r = 1800$, by simple interpolation.

$H = B/(\mu_o \mu_r) = 1.1 \times 10^7/(4\pi \times 1800) = 486.5$ amp-turns/m.

m.m.f required = $H \times l_m = 486.5 \times 0.60 = 292$

A saw-cut of 1 mm, will need extra mmf.

$H_g = B_g/\mu_o = 1.1 \times 10^7/(4\pi) = 875796$

$AT_g = H_g \times l_g = 875796 \times 1.0 \times 10^{-3} = 876$

Thus, additional mmf required due to air-gap = 876 amp-turns

Example 6.31. *A 680-turn coil is wound on the central limb of a cast steel frame as shown in Fig. 6.43 (a) with all dimensions in cms. A total flux of 1.6 mWb is required in the air-gap. Find the current in the magnetizing coil. Assume uniform flux distribution and no leakage. Data for B-H curve for cast steel is given.* **[Nagpur University, Novemeber 1997]**

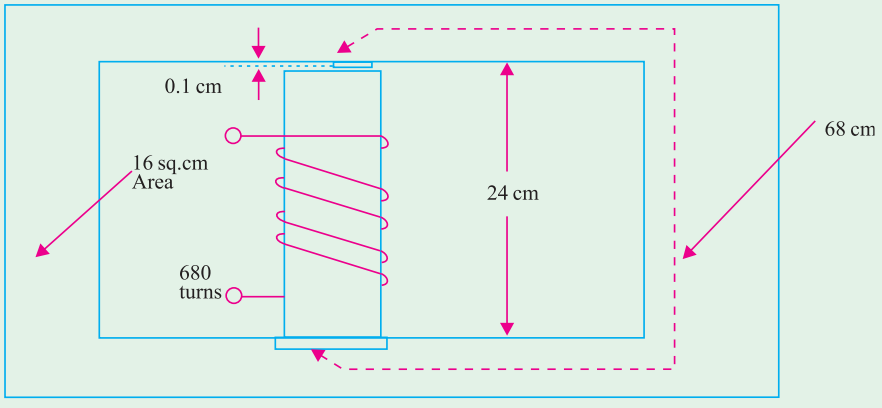


Fig. 6.43 (*a*)

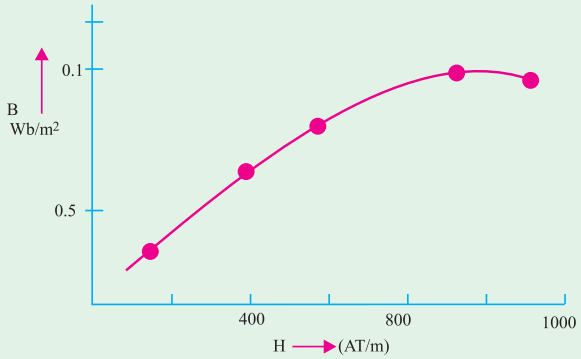


Fig. 6.43 (*b*)

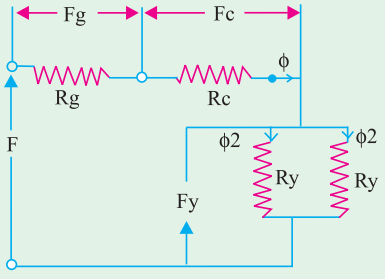


Fig. 6.43 (*c*)

Solution. ϕ = 1.6 mWb through air-gap and central limb

$\phi/2$ = 0.8 mWb through yokes

Corresponding flux densities are :

$B_g = B_c$ = 1.6 mWb/(16×10^{-4}) = 1.0 Wb/m^2

$$B_y = 0.8 \text{ m Wb}/(16 \times 10^{-4}) = 0.50 \text{ Wb/m}^2$$

MMF-Calculations :

(a) For Air gap : For B_g of 1 Wb/m^2, $H_g = 1.0/\mu_o$

$$AT_g = H_g \times l_g = [1/(4\pi \times 10^{-7})] \times (0.1 \times 10^{-2})$$
$$= 796 \text{ amp turns}$$

(b) For Central limb : $AT_c = H_c \times l_c = 900 \times 0.24 = 216$

$\therefore$ For $B_c = 1.00$, H_c from data = 900 AT/m

The yokes are working at a flux-density of 0.50 Wb/m^2. From the given data and the corresponding plot, interpolation can be done for accuracy.

$$H_y = 500 + [(0.5 - 0.45)/(0.775 - 0.45)] \times 200$$
$$= 530 \text{ AT/m}$$
$$F_y = 530 \times 0.68 = 360$$
$$\text{Total mmf required} = 796 + 216 + 360 = 1372$$
$$\text{Hence, coil-current} = 1372/680 = 2.018 \text{ A}$$

Example 6.32. *For the magnetic circuit shown in fig. 6.44 the flux in the right limb is 0.24 mWb and the number of turns wound on the central-limb is 1000. Calculate* ***(i)*** *flux in the central limb* ***(ii)*** *the current required.*

The magnetization curve for the core is given as below :

H (AT/m) :	*200*	*400*	*500*	*600*	*800*	*1060*	*1400*
B (Nb/m$_2$) :	*0.4*	*0.8*	*1.0*	*1.1*	*1.2*	*1.3*	*1.4*

Neglect Leakage and fringing. **[Rajiv Gandhi Technical University, Bhopal, Summer 2001]**

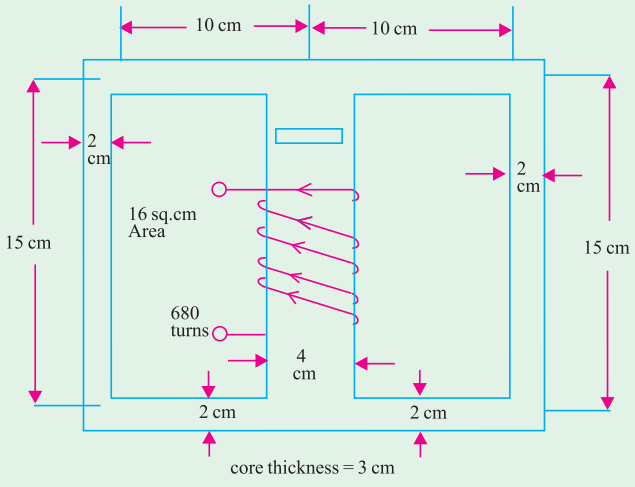


Fig. 6.44

Solution. Area of cross-section of side-limbs = 2 × 3 = 6 sq.cm

Area of cross-section of core = 3 × 4 = 12 sq.cm

Flux in side Limbs = 0.24 mWb

Flux density in side Limbs = $(0.24 \times 10^{-3})/(6 \times 10^{-4}) = 0.4$ Wb/m^2

Since the coil is wound on the central limb and the magnetic circuit is symmetrical, the flux in the central limb = 0.48 mWb. Flux density in the central limb = $(0.48 \times 10^{-3})/(12 \times 10^{-4}) = 0.4$ Wb/m^2

For the flux density of 0.40 Wb/m^2, H = 200 AT/m

Central Limb has a path length of 15 cm.

Other part carrying 0.24 mWb has a total path length of 35 cm.

Total mmf required = (200 × 0.15) + (200 × 0.35) = 100 AT

Hence, coil current = 100/1000 = 0.1 Amp.

Example 6.33. *A ring composed of three sections. The cross-sectional area is 0.001 m^2 for each section. The mean arc length are l_a = 0.3 m, l_b = 0.2 m, l_c = 0.1 m. An air-gap length of 0.1 mm is cut in the ring. Mr for sections a, b, c are 5000, 1000, and 10,000 respectively. Flux in the air gap is 7.5 × 10^{-4} Wb. Find* ***(i)*** *mmf* ***(ii)*** *exciting current if the coil has 100 turns,* ***(iii)*** *reluctances of the sections.* **[Nagpur University April 1999]**

Solution. Area = 0.001 sq.m

l_a = 0.3 m, l_b = 0.2 m, l_c = 0.1 m, $l_g = 0.1 \times 10^{-3}$ m

μ_{ra} = 5000, μ_{rb} = 1000, μ_{rc} = 10,000 $\mu_o = 4\pi \times 10^{-7}$

$\phi = 7.5 \times 10^{-4}$ Wb

(iii) Calculations of Reluctances of four parts of the magnetic circuit :

(a) Reluctance of air gap, $R_{eg} = \frac{1}{\mu_o} \times \frac{0.1 \times 10^{-3}}{0.001} = \frac{1000}{4\pi \times 0.001} = 79618$

(b) Reluctance of section '*a*' of ring

$$= R_{ea} = \frac{1}{\mu_o \mu_{ra}} \times \frac{0.3}{0.001} = \frac{10^7 \times 0.3}{4\pi \times 47770 \times 5000 \times 0.001} = 47770$$

(c) Reluctance of section '*b*' of the ring

$$= R_{eb} = \frac{1}{\mu_o \mu_{rb}} \times \frac{0.20}{0.001} = \frac{10^7}{4\pi \times 1000} \times \frac{0.10}{0.001} = 15923.6$$

(d) Reluctance of section '*c*' of the ring

$$= R_{ec} = \frac{1}{\mu_o \mu_{rc}} \times \frac{0.10}{0.001} = \frac{10^7}{4\pi \times 1000} \times \frac{0.10}{0.001} = 7961$$

Total Reluctance = $R_{eg} + R_{ea} + R_{eb} + R_{ec}$ = 294585

(i) Total mmf required = Flux × Reluctance

= $7.5 \times 10^{-4} \times 294585$ = 221 amp-turns

(ii) Current required = 221/100 = 2.21 amp

Tutorial Problems No. 62

1. An iron specimen in the form of a closed ring has a 350-turn magnetizing winding through which is passed a current of 4A. The mean length of the magnetic path is 75 cm and its cross-sectional area is 1.5 cm^2. Wound closely over the specimen is a secondary winding of 50 turns. This is connected to a ballistic glavanometer in series with the secondary coil of 9-mH mutual inductance and a limiting resistor. When the magnetising current is suddenly reversed, the galvanometer deflection is equal to that produced by the reversal of a current of 1.2 A in the primary coil of the mutual inductance. Calculate the *B* and *H* values for the iron under these conditions, deriving any formula used.

[1.44 Wb/m^2 ; 1865 AT/m] ***(London Univ.)***

2. A moving-coil ballistic galvanometer of 150 Ω gives a throw of 75 divisions when the flux through a search coil, to which it is connected, is reversed.

Find the flux density in which the reversal of the coil takes place, given that the galvanometer constant is 110 μ*C* per scale division and the search coil has 1400 turns, a mean are of 50 cm^2 and a resistance of 20 Ω **[0.1 Wb/m^2]** ***(Elect. Meas. & Measuring Inst. Gujarat Univ.)***

3. A fluxmeter is connected to a search coil having 500 turns and mean area of 5 cm^2. The search coil is placed at the centre of a solenoid one metre long wound with 800 turns. When a current of 5 A is reversed, there is a deflection of 25 scale divisions on the fluxmeter. Calculate the flux-meter constant.
[10^{-4} Wb-turn/division] (*Elect. Meas. & Measuring Inst., M.S. Univ. Baroda*)

4. An iron ring of mean length 50 cms has an air gap of 1 mm and a winding of 200 turns. If the permeability of iron is 300 when a current of 1 A flows through the coil, find the flux density.
[94.2 mWb/m^3] (*Elect. Engg. A.M.Ae.S.I.*)

5. An iron ring of mean length 100 cm with an air gap of 2 mm has a winding of 500 turns. The relative permeability of iron is 600. When a current of 3 A flows in the winding, determine the flux density. Neglect fringing. **[0.523 Wb/m^2] (*Elect. Engg. & Electronic Bangalore Univ. 1990*)**

6. A coil is wound uniformly with 300 turns over a steel ring of relative permeability 900, having a mean circumference of 40 mm and cross-sectional area of 50 mm^2. If a current of 25 amps is passed through the coil, find (*i*) m.m.f. (*ii*) reluctance of the ring and (*iii*) flux.
[(*i*) 7500 AT (*ii*) 0.7×10^6 AT/Wb (*iii*) 10.7 mWb]
(*Elect. Engg. & Electronics Bangalore Univ.*)

7. A specimen ring of transformer stampings has a mean circumference of 40 cm and is wound with a coil of 1,000 turns. When the currents through the coil are 0.25 A, 1 A and 4 A the flux densities in the stampings are 1.08, 1.36 and 1.64 Wb/m^2 respectively. Calcualte the relative permeability for each current and explain the differences in the values obtained. **[1,375,434,131]**

8. A magnetic circuit consists of an iron ring of mean circumference 80 cm with cross-sectional area 12 cm^2 throughout. A current of 2A in the magnetising coil of 200 turns produces a total flux of 1.2 mWb in the iron. Calculate :
(*a*) the flux density in the iron
(*b*) the absolute and relative permeabilities of iron
(*c*) the reluctance of the circuit
[1 Wb/m^2 ; 0.002, 1,590 ; 3.33×10^5 AT/Wb]

9. A coil of 500 turns and resistance 20 Ω is wound uniformly on an iron ring of mean circumference 50 cm and cross-sectional area 4 cm^2. It is connected to a 24-V d.c. supply. Under these conditions, the relative permeability of iron is 800. Calculate the values of :
(*a*) the magnetomotive force of the coil (*b*) the magnetizing force
(*c*) the total flux in the iron (*d*) the reluctance of the ring
[(*a*) 600 AT (*b*) 1,200 AT/m (*c*) 0.483 mWb (*d*) 1.24×10^6 AT/Wb]

10. A series magnetic circuit has an iron path of length 50 cm and an air-gap of length 1 mm. The cross-sectional area of the iron is 6 cm^2 and the exciting coil has 400 turns. Determine the current required to produce a flux of 0.9 mWb in the circuit. The following points are taken from the magnetisation characteristic :

Flux density (Wb/m^2) :	1.2	1.35	1.45	1.55	
Magnetizing force (AT/m) :	500	1,000	2,000	4,500	**[6.35 A]**

11. An iron-ring of mean length 30 cm is made of three pieces of cast iron, each has the same length but their respective diameters are 4, 3 and 2.5 cm. An air-gap of length 0.5 mm is cut in the 2.5 cm piece. If a coil of 1,000 turns is wound on the ring, find the value of the current it has to carry to produce a flux density of 0.5 Wb/m^2 in the air gap. *B*/*H* characteristic of cast-iron may be drawn from the following :

B (Wb/m^2) :	0.1	0.2	0.3	0.4	0.5	0.6	
(AT/m) :	280	620	990	1,400	2,000	2,8000	**[0.58 A]**

Permeability of free space = $4\pi \times 10^7$ H/m. Neglect leakage and fringing.

12. The length of the magnetic circuit of a relay is 25 cm and the cross-sectional area is 6.25 cm^2. The length of the air-gap in the operated position of the relay is 0.2 mm. Calculate the magnetomotive force required to produce a flux of 1.25 mWb in the air gap. The relative permeability of magnetic material at this flux density is 200. Calculate also the reluctance of the magnetic circuit when the

relay is in the unoperated position, the air-gap then being 8 mm long (assume μ_r, remains constant).

[2307 AT, 1.18 × 10^7 AT/Wb]

13. For the magnetic circuit shown in Fig. 6.45, all dimensions are in cm and all the air-gaps are 0.5 mm wide. Net thickness of the core is 3.75 cm throughout. The turns are arranged on the centre limb as shown.

Calculate the m.m.f. required to produce a flux of 1.7 mWb in the centre limb. Neglect the leakage and fringing. The magnetisation data for the material is as follows :

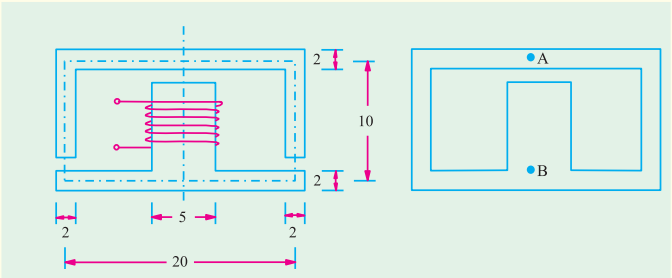


Fig. 6.45

Fig. 6.46

H (AT/m) :	400	440	500	600	800
B (Wb/m^2) :	0.8	0.9	1.0	1.1	1.2

[1,052 AT]

14. In the magnetic circuit shown in Fig. 6.46 a coil of 500 turns is wound on the centre limb. The magnetic paths A to B by way of the outer limbs have a mean length of 100 cm each and an effective cross-sectional area of 2.5 cm^2. The centre limb is 25 cm long and 5cm^2 cross-sectional area. The air-gap is 0.8 cm long. A current of 9.2 A through the coil is found to produce a flux of 0.3 mWb.

15. The magnetic circuit of a choke is shown in Fig. 6.47. It is designed so that the flux in the central core is 0.003 Wb. The cross-section is square and a coil of 500 turns is wound on the central core. Calculate the exciting current. Neglect leakage and assume the flux to be uniformly distributed along the mean path shown dotted. Dimensions are in cm.

The characteristics of magnetic circuit are as given below :

B (Wb/m^2) :	0.38	0.67	1.07	1.2	1.26
H (AT/m) :	100	200	600	1000	1400

(Elect. Technology I. Gwalior Univ.)

16. A 680-turn coil is wound on the central limb of the cast steel sheet frame as shown in Fig. 6.48 where dimensions are in cm. A total flux of 1.6 mWb is required to be in the gap. Find the current required in the magnetising coil. Assume gap density is uniform and all lines pass straight across the gap. Following data is given :

H (AT/m) :	300	500	700	900	1100
B (Wb/m^2) :	0.2	0.45	0.775	1.0	1.13

(Elect. Technology ; Indore Univ.)

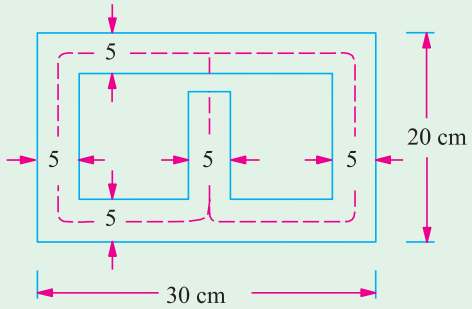


Fig. 6.47

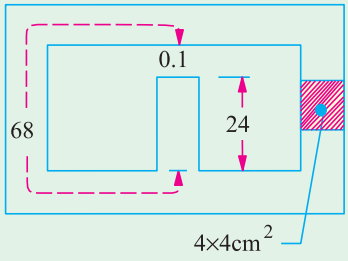


Fig. 6.48

17. In the magnetic circuit of Fig. 6.49, the core is composed of annealed sheet steel for which a stacking factor of 0.9 should be assumed. The core is 5 cm thick. When Φ_A = 0.002 Wb, Φ_B = 0.0008 Wb and Φ_C = 0.0012 Wb. How many amperes much each coil carry and in what direction ? Use of the

following magnetisation curves can be made for solving the problem.

B (Wb/m^2) :	0.2	0.4	0.6	0.8	1.0	1.4	1.6	1.8
H (AT/m^2) :	50	100	130	200	320	1200	3800	10,000

(Elect. Technology, Vikram Univ.)

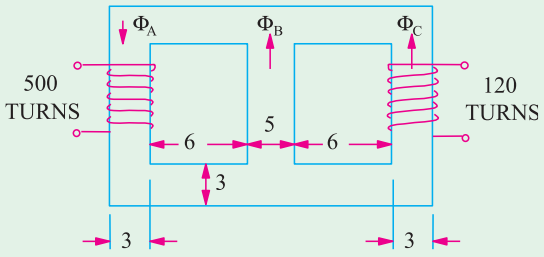


Fig. 6.49

18. A magnetic circuit with a uniform cross-sectional area of 6 cm^2 consists of a steel ring with a mean magnetic length of 80 cm and an air gap of 2 mm. The magnetising winding has 540 ampere-turns. Estimate the magnetic flux produced in the gap. The relevant points on the magnetization curve of cast steel are :

B (Wb/m^2) :	0.12	0.14	0.16	0.18	0.20
H (AT/m) :	200	230	260	290	320

[0.1128 m Wb] *(City & Guilds, London)*

19. Explain the terms related to magnetic circuits :
(*i*) reluctance (*ii*) flux density (*iii*) magnetomotive force *(Nagpur University, Summer 2002)*

20. A metal ring of mean diameter of 80 cm is made out of two semi-circular pieces of cast iron and cast steel separated at junctions by pieces of copper each of 1 mm thickness. If the ring is uniformly wound with 1000 turns, calculate the value of current required to produce a flux density of 0.85 wb/nV in the ring.
Given that relative permeability of cast iron as 200, that of cast steel is 1200 and for copper, $\mu_r = 1$. *(Nagpur University, Summer 2002)*

21. A 1154 turns coil is wound on the central limb of the case steel frame shown in Fig. 6.50. A total flux of 1.6 mwb is required in the air gap. Find the current required through the gap. Assume that the gap density is uniform and there is no leakage. Frame dimensions are given in cm. Take permeability of cast steel as 1,200. *(Nagpur University, Winter 2002)*

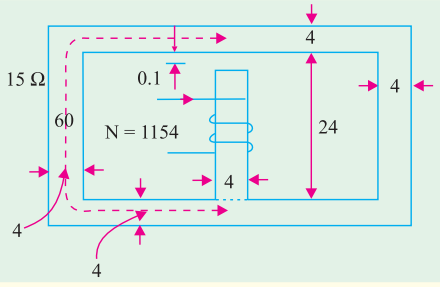


Fig. 6.50

22. Explain the terms related to magnetic circuits :
(*i*) Reluctance (*ii*) Flux density (*iii*) Coercive force (*iv*) Magnetomotive force (*v*) Residual flux. *(Nagpur University, Summer 2003)*

23. Compare electric and magnetic circuit by their similarities and dissimilarities. *(Nagpur University, Winter 2003)*

24. Compare electric and magnetic circuits with respect to their similarities and dissimilarities. *(Nagpur University, Summer 2004)*

25. A steel wire of 25 cm mean diameter and circular cross section 3 cm in diameter has an airgap of 1 mm wide. It is wound with a coil of 700 turns carrying a current of 2 A.
Calculate : (*i*) m.m.f. (*ii*) Flux density (*iii*) Reluctance (*iv*) Relative permeability.
Assume that iron path take 30% of total m.m.f. ***(Gujrat University,Summer 2003)***

26. What is a search coil in magnetic measurements? ***(Anna University, April 2002)***

27. Name the magnestic squares used to find iron loss. ***(Anna University, April 2002)***

28. What is a magnetic circuit? A magnetic circuit is made up of 3 limbs A, B and C in prallel. The reluctances of the magnetic paths of A, B and C in AT/mWb are 312, 632.6 and 520 respectively. An exciting coil of 680 turns is wound on limb B. Find the exciting current to produce of flux of lmwb in the limb A. ***(V.T.U., Belgaum Karnataka University, February 2002)***

29. An iron ring of 300cm mean circumference with a cross section of $5cm^2$ is wound uniformly with 350 turns of wire. Find the current required to produce a flux of 0.5 Mwb in iron. Take relative permeability of iron as 400. ***(V.T.U. Belgaum Karnataka University, July/August 2002)***

30. What is Biot-Savart law? Explain briefly. Find the magnetic field due to a small circular loop carrying current I at distances from loop that are large compared with its dimensions.
(Agra Univ. 1978 Supp.)

31. Magnetic potential ***(Mumbai University, 2002) (RGPV, Bhopal 2001)***

32. Flux density ***(Pune University,2002) (RGPV, Bhopal 2001)***

33. Susceptibility ***(Mumbai University, 2002) (RGPV, Bhopal 2001)***

34. Define mm*f*, flux, reluctance, absolute and relative permeabilities with reference to magnetic circuits.
(U.P. Technical University 2003) (RGPV, Bhopal 2002)

35. Discuss B-H curve of a ferro-magnetic material and explain the following.
(*i*) Magnetic saturation (*ii*) Hysteresis (*iii*) Residual magnetism (*iv*) Coercive force
(RGPV, Bhopal 2002)

36. What is meant by leakage and fringing? Define leakage coefficient.
(RGPV, Bhopal 2002)

37. Define the following terms (any five) :
(*i*) MMF (*ii*) Reluctance (*iii*) Permeance (*iv*) Magnetisation curve (*v*) flux density
(*vi*) Magnetizing force (*vii*) Susceptibility (*viii*) Relative permeability (*ix*) Magnetic potential
(RGPV, Bhopal 2002)

38. Distinguish between leakage and fringing of flux. ***(RGPV, Bhopal 2002)***

39. Explain fringing of magnetic flux, magnetic leakage, staturation of ferowegnetic materials, B-H Curve, hysteresis and eddy current losses. ***(RGPV, Bhopal 2003)***

OBJECTIVE TESTS – 6

1. Relative permeability of vacuum is
(*a*) $4\pi \times 10^{-7}$ H/m (*b*) 1 H/m
(*c*) 1 (*d*) 1/4 π

2. Unit of magnetic flux is
(*a*) weber (*b*) ampere-turn
(*c*) tesla (*d*) coulomb

3. Point out the WRONG statement.
The magnetising force at the centre of a circular coil varies.
(*a*) directly as the number of its turns
(*b*) directly as the current
(*c*) directly as its radius
(*d*) inversely as its radius

4. A pole of driving point admittance function implies
(*a*) zero current for a finite value of driving voltage
(*b*) zero voltage for a finite value of driving current
(*c*) an open circuit condition
(*d*) None of (*a*), (*b*) and (*c*) mentioned in the question ***(ESE 2001)***

ANSWERS

1. *c* **2.** *a* **3.** *a*

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Contents

CHAPTER 7

ELECTRO-MAGNETIC INDUCTION

Learning Objectives

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- Faraday's Laws of Electromagnetic Induction
- Direction of Induced E.M.F. and Current
- Lenz's Law
- Induced E.M.F.
- Dynamically-induced E.M.F.
- Statically-induced E.M.F.
- Self-Inductance
- Coefficient of Self-Inductance (*L*)
- Mutual Inductance
- Coefficient of Mutual Inductance (*M*)
- Coefficient of Coupling
- Inductances in Series
- Inductances in Parallel



The above figure shows the picture of a hydro-electric generator. Electric generators, motors, transformers, etc., work based on the principle of electromagnetic induction

7.1. Relation Between Magnetism and Electricity

It is well known that whenever an electric current flows through a conductor, a magnetic field is immediately brought into existence in the space surrounding the conductor. It can be said that when electrons are in motion, they produce a magnetic field. The converse of this is also true *i.e.* when a magnetic field embracing a conductor moves *relative* to the conductor, it produces a flow of electrons in the conductor. This phenomenon whereby an e.m.f. and hence current (*i.e.* flow of electrons) is induced in any conductor which is cut across or is cut by a magnetic flux is known as ***electromagnetic induction***. The historical background of this phenomenon is this :

After the discovery (by Oersted) that electric current produces a magnetic field, scientists began to search for the converse phenomenon from about 1821 onwards. The problem they put to themselves was how to 'convert' magnetism into electricity. It is recorded that Michael Faraday* was in the habit of walking about with magnets in his pockets so as to constantly remind him of the problem. After nine years of continuous research and experimentation, he succeeded in producing electricity by 'converting magnetism'. In 1831, he formulated basic laws underlying the phenomenon of electromagnetic induction (known after his name), upon which is based the operation of most of the commercial apparatus like motors, generators and transformers etc.

7.2. Production of Induced E.M.F. and Current

In Fig. 7.1 is shown an insulated coil whose terminals are connected to a sensitive galvanometer *G*. It is placed close to a stationary bar magnet initially at position *AB* (shown dotted). As seen, some flux from the *N*-pole of the magnet is linked with or threads through the coil but, as yet, there is no deflection of the galvanometer. Now, suppose that the magnet is ***suddenly*** brought closer to the coil in position *CD* (see figure). Then, it is found that there is a jerk or a sudden but a momentary deflection

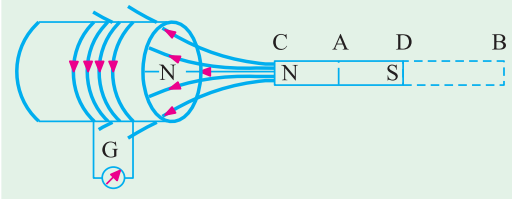


Fig. 7.1.

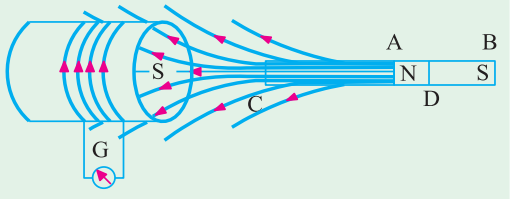


Fig. 7.2.

in the galvanometer and that this ***lasts so long as the magnet is in motion relative to the coil, not otherwise***. The deflection is reduced to zero when the magnet becomes again stationary at its new position *CD*. It should be noted that due to the approach of the magnet, flux linked with the coil is increased.

Next, the magnet is ***suddenly*** withdrawn away from the coil as in Fig. 7.2. It is found that again there is a ***momentary*** deflection in the galvanometer and it persists so long as the magnet is in motion, not when it becomes stationary. It is important to note that this deflection is in a direction opposite to that of Fig. 7.1. Obviously, due to the withdrawal of the magnet, flux linked with the coil is decreased.

The deflection of the galvanometer indicates the production of e.m.f. in the coil. The only cause of the production can be the sudden approach or withdrawal of the magnet from the coil. It is found that the actual cause of this e.m.f. is the change of flux linking with the coil. This e.m.f. exists so long as the change in flux exists. Stationary flux, however strong, will never induce any e.m.f. in a stationary conductor. In fact, the same results can be obtained by keeping the bar magnet stationary and moving the coil suddenly away or towards the magnet.

* Michael Faraday (1791-1867), an English physicist and chemist.

The direction of this electromagnetically-induced e.m.f. is as shown in the two figures given on back page.

The production of this electromagnetically-induced e.m.f. is further illustrated by considering a conductor *AB* lying within a magnetic field and connected to a galvanometer as shown in Fig. 7.3. It is found that whenever this conductor is moved up or down, ***a momentary*** deflection is produced in the galvanometer. It means that some transient e.m.f. is induced in *AB*. The magnitude of this induced e.m.f. (and hence the amount of deflection in the galvanometer) ***depends on the quickness of the movement of AB.***

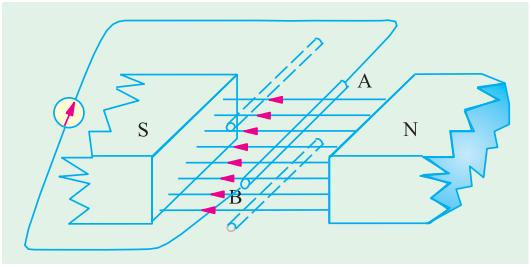


Fig. 7.3

From this experiment we conclude that whenever a conductor cuts or *shears* the magnetic flux, an e.m.f. is always induced in it.

It is also found that if the conductor is moved parallel to the direction of the flux so that it does not cut it, then no e.m.f. is induced in it.

7.3. Faraday's Laws of Electromagnetic Induction

Faraday summed up the above facts into two laws known as Faraday's Laws of Electromagnetic Induction.

First Law. It states :

Whenever the magnetic flux linked with a circuit changes, an e.m.f. is always induced in it.

or

Whenever a conductor cuts magnetic flux, an e.m.f. is induced in that conductor.

Second Law. It states :

The magnitude of the induced e.m.f. is equal to the rate of change of flux-linkages.

Explanation. Suppose a coil has N turns and flux through it changes from an initial value of Φ_1 webers to the final value of Φ_2 webers in time t seconds. Then, remembering that by flux-linkages mean the product of number of turns and the flux linked with the coil, we have

Initial flux linkages $= N\Phi_1$, add Final flux linkages $= N\Phi_2$

$$\therefore \quad \text{induced } e.m.f. \quad e = \frac{N\Phi_2 - N\Phi_1}{t} \text{ Wb/s or volt or } e = N\frac{\Phi_2 - \Phi_1}{t} \text{ volt}$$

Putting the above expression in its differential form, we get

$$e = \frac{d}{dt}(N\Phi) = N\frac{d}{dt} \text{ volt}$$

Usually, a minus sign is given to the right-hand side expression to signify the fact that the induced e.m.f. sets up current in such a direction that magnetic effect produced by it opposes the very cause producing it (Art. 7.5).

$$e = -N\frac{d\Phi}{dt} \text{ volt}$$

Example 7.1. *The field coils of a 6-pole d.c. generator each having 500 turns, are connected in series. When the field is excited, there is a magnetic flux of 0.02 Wb/pole. If the field circuit is opened in 0.02 second and residual magnetism is 0.002 Wb/pole, calculate the average voltage which is induced across the field terminals. In which direction is this voltage directed relative to the direction of the current.*

Solution. Total number of turns, $N = 6 \times 500 = 3000$

Total initial flux $= 6 \times 0.02 = 0.12$ Wb

Total residual flux $= 6 \times 0.002 = 0.012$ Wb

Change in flux, $d\Phi = 0.12 - 0.012 = 0.108$ Wb

Time of opening the circuit, dt = 0.02 second

$\therefore$ Induced e.m.f. = $N\frac{d\Phi}{dt}$ volt $= 3000 \times \frac{0.108}{0.02}$ = **16,200 V**

The direction of this induced e.m.f. is the same as the initial direction of the exciting current.

Example 7.2. *A coil of resistance 100 Ω is placed in a magnetic field of 1 mWb. The coil has 100 turns and a galvanometer of 400 Ω resistance is connected in series with it. Find the average e.m.f. and the current if the coil is moved in 1/10th second from the given field to a field of 0.2 mWb.*

Solution. Induced e.m.f. = $N \cdot \frac{d\Phi}{dt}$ volt

Here $d\Phi = 1 - 0.2 = 0.8$ mWb $= 0.8 \times 10^{-3}$ Wb

$dt = 1/10 = 0.1$ second ; $N = 100$

$e = 100 \times 0.8 \times 10^{-3}/0.1 =$ **0.8 V**

Total circuit resistance = 100 + 400 = 500 Ω

$\therefore$ Current induced = $0.8/500 = 1.6 \times 10^{-3}$ A = **1.6 mA**

Example 7.3. *The time variation of the flux linked with a coil of 500 turns during a complete cycle is as follows :*

$\Phi = 0.04\,(1 - 4\,t/T)$ Weber $\quad 0 < t < T/2$

$\Phi = 0.04\,(4t/T - 3)$ Weber $\quad T/2 < t < T$

where T represents time period and equals 0.04 second. Sketch the waveforms of the flux and induced e.m.f. and also determine the maximum value of the induced e.m.f..

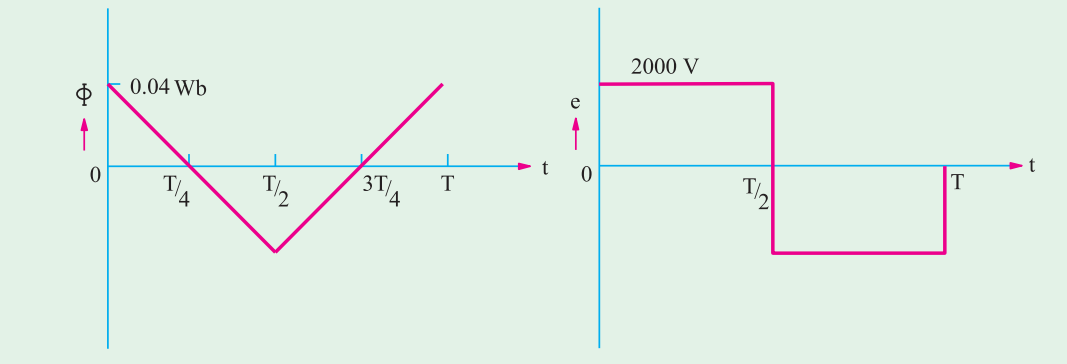


Fig. 7.4.

Solution. The variation of flux is linear as seen from the following table.

t (second) :	0	$T/4$	$T/2$	$3T/4$	T
F (Weber) :	0.04	0	−0.04	0	0.04

The induced e.m.f. is given by $e = -Nd\,\Phi/dt$

From $t = 0$ to $t = T/2$, $d\Phi/dt = -0.04 \times 4/T = -4$ Wb/s $\quad \therefore e = -500\,(-4) = 2000$ V

From $t = T/2$ to $t = T$, $d\Phi/dt = 0.04 \times 4/T = 4$ Wb/s $\quad \therefore e = -500 \times 4 = -2000$ V.

The waveforms are selected in Fig. 7.4.

7.4. Direction of induced e.m.f. and currents

There exists a definite relation between the direction of the induced current, the direction of the flux and the direction of motion of the conductor. The direction of the induced current may be found easily by applying either Fleming's Right-hand Rule or Flat-hand rule or Lenz's Law. Fleming's rule

(Fig. 7.5) is used where induced e.m.f. is due to flux-cutting (*i.e.*, dynamically induced e.m.f.) and Lenz's when it is used to change by flux-linkages (*i.e.*, statically induced e.m.f.).

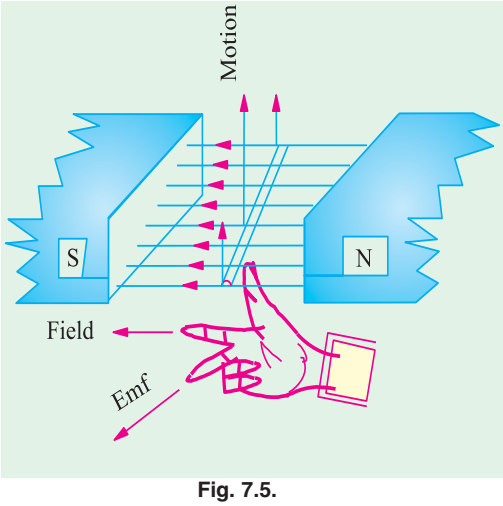


Fig. 7.5.

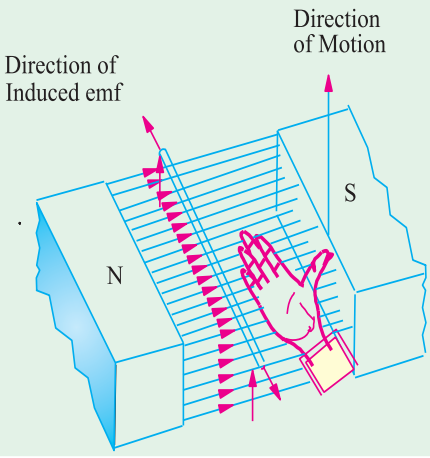


Fig. 7.6.

Fig. 7.6 shows another way of finding the direction of the induced e.m.f. It is known as Right Flat-hand rule. Here, the front side of the hand is held perpendicular to the incident flux with the thumb pointing in the direction of the motion of the conductor. The direction of the fingers give the direction of the induced e.m.f. and current.

7.5. Lenz's Law

The direction of the induced current may also be found by this law which was formulated by Lenz* in 1835. This law states, in effect, that electromagnetically induced current always flows in such direction that the action of the magnetic field set up by it tends to oppose the very cause which produces it.

This statement will be clarified with reference to Fig. 7.1 and 7.2. It is found that when N-pole of the bar magnet approaches the coil, the induced current set up by induced e.m.f. flows in the ***anti-clockwise*** direction in the coil as seen from the magnet side. The result is that face of the coil becomes a N-pole and so tends to oppose the onward approach of the *N*-Pole of the magnet (like poles repel each other). The mechanical energy spent in overcoming this repulsive force is converted into electrical energy which appears in the coil.

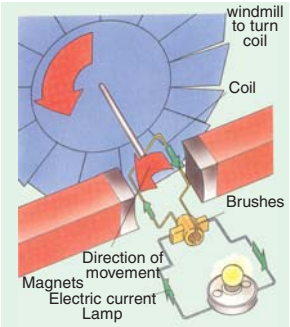
When the magnet is withdrawn as in Fig. 7.2, the induced current flows in the clockwise direction thus making the face of the coil (facing the magnet) a *S*-pole. Therefore, the *N*-pole of the magnet has to withdrawn against this attractive force of the *S*-pole of coil. Again, the mechanical energy required to overcome this force of attraction is converted into electric energy.

It can be shown that Lenz's law is a direct consequence of Law of Conservation of Energy. Imagine for a moment that when *N*-pole of the magnet (Fig. 7.1) approaches the coil, induced current flows in such a direction as to make the coil face a *S*-pole. Then, due to inherent attraction between unlike poles, the magnet would be automatically pulled towards the coil without the expenditure of any mechanical energy. It means that we would be able to create electric energy out of nothing, which is denied by the inviolable Law of Conservation of Energy. In fact, to maintain the sanctity of this law, it is imperative for the induced current to flow in such a direction that the magnetic effect produced by it tends to oppose the very cause which produces it. In the present case, it is relative motion of the magnet with magnet with respect to the coil which is the cause of the production of the induced current. Hence, the induced current always flows in such a direction to oppose this relative motion *i.e.*, the approach or withdrawal of the magnet.

* After the Russian born geologist and physicist Heinrich Friedrich Emil Lenz (1808 - 1865).

7.6. Induced e.m.f.

Induced e.m.f. can be either **(*i*) dynamically induced** or **(*ii*) statically induced.** In the first case, usually the field is stationary and conductors cut across it (as in d.c. generators). But in the second case, usually the conductors or the coil remains stationary and flux linked with it is changed by simply increasing or decreasing the current producing this flux (as in transformers).



The principle of electric generation

7.7. Dynamically induced e.m.f.

In Fig. 7.7. a conductor *A* is shown in cross-section, lying m^2 within a uniform magnetic field of flux density *B* Wb/m^2. The arrow attached to *A* shows its direction of motion. Consider the conditions shown in Fig. 7.7 (*a*) when *A* cuts across at right angles to the flux. Suppose '*l*' is its length lying within the field and let it move a distance *dx* in time *dt*. Then area swept by it is = *ldx*. Hence, flux cut = $l.dx \times B$ webers.

Change in flux = *Bldx* weber

Time taken = *dt* second

Hence, according to Faraday's Laws (Art. 7.3.) the e.m.f. induced in it (known as dynamically induced e.m.f.) is

rate of change of flux linkages = $\frac{Bldx}{dt} = Bl\frac{dx}{dt} = Blv$ volt where $\frac{dx}{dt}$ = velocity

If the conductor *A* moves at an angle θ with the direction of flux [Fig. 7.7 (*b*)] then the induced e.m.f. is $e = Bl\upsilon \sin\theta$ volts = $l\upsilon \times B$ (*i.e.* as cross product vector υ and B).

The direction of the induced e.m.f. is given by Fleming's Right-hand rule (Art. 7.5) or Flat-hand rule and most easily by vector cross product given above.

It should be noted that generators work on the production of dynamically induced e.m.f. in the conductors housed in a revolving armature lying within a strong magnetic field.

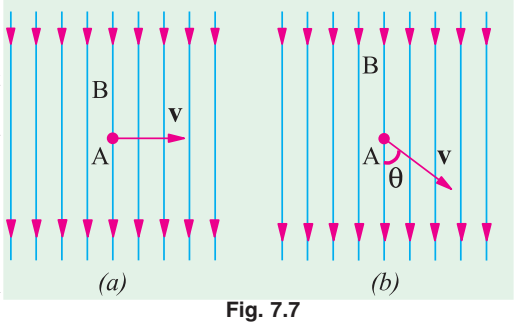


Fig. 7.7

Example 7.4. *A conductor of length 1 metre moves at right angles to a uniform magnetic field of flux density 1.5 Wb/m^2 with a velocity of 50 metre/second. Calculate the e.m.f. induced in it. Find also the value of induced e.m.f. when the conductor moves at an angle of 30° to the direction of the field.*

Solution. Here $B = 1.5$ Wb/m^2 $l = 1$ m $\upsilon = 50$ m/s ; $e = ?$

Now $e = Bl\upsilon = 1.5 \times 1 \times 50 =$ **75 V.**

In the second case $\theta = 30°$ $\therefore \sin 30° = 0.5$ $\therefore e = 75 \times 0.5 =$ **37.5 V**

Example 7.5. *A square coil of 10 cm side and with 100 turns is rotated at a uniform speed of 500 rpm about an axis at right angle to a uniform field of 0.5 Wb/m^2. Calculate the instantaneous value of induced e.m.f. when the plane of the coil is **(i)** at right angle to the plane of the field. **(ii)** in the plane of the field. **(iii)** at 45° with the field direction.* **(Elect. Engg. A.M.Ae. S.I. Dec. 1991)**

Solution. As seen from Art. 12.2, e.m.f. induced in the coil would be zero when its plane is at right angles to the plane of the field, even though it will have maximum flux linked with it. However, the coil will have maximum e.m.f. induced in it when its plane lies parallel to the plane of the field even though it will have minimum flux linked with it. In general, the value of the induced e.m.f. is given by $e = \omega N\Phi_m \sin\theta = E_m \sin\theta$ where θ is the angle between the axis of zero *e.m.f.* and the plane of the coil.

Here, $f = 500/60 = 25/3$ r.p.s ; $N = 100$; $B = 0.5$ Wb/ m^2 ; $A = (10 \times 10) \times 10^{-4} = 10^{-2}$ m^2.

$\therefore$ $E_m = 2\pi fNBA = 2\pi (25/3) \times 100 \times 0.5 \times 10^{-2} = 26.2$ V (*i*) since $\theta = 0$; $\sin\theta = 0$; therefore, $e = 0$. (*ii*) Here, $\theta = 90°$; $e = E_m \sin 90° = 26.2 \times 1 = 26.2$ V (*iii*) $\sin 45° = 1/\sqrt{2}$; $e = 26.2 \times 1/\sqrt{2} =$ **18.5 V**

Example 7.6. *A conducting rod AB (Fig. 7.8) makes contact with metal rails AD and BC which are 50 cm apart in a uniform magnetic field of B = 1.0 Wb/m^2 perpendicular to the plane ABCD. The total resistance (assumed constant) of the circuit ABCD is 0.4 Ω*

(a) *What is the direction and magnitude of the e.m.f. induced in the rod when it is moved to the left with a velocity of 8 m/s ?*

(b) *What force is required to keep the rod in motion ?*

(c) *Compare the rate at which mechanical work is done by the force F with the rate of development of electric power in the circuit.*

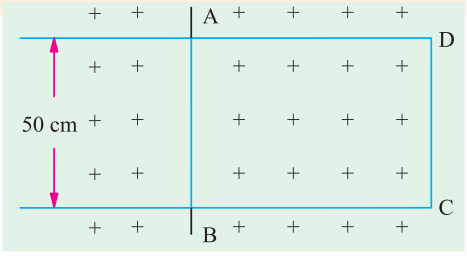


Fig. 7.8

Solution. **(*a*)** Since *AB* moves to the left, direction of the induced current, as found by applying Fleming's Right-hand rule is from *A* to *B*. Magnitude of the induced e.m.f. is given by

$e = \beta l \upsilon$ volt $= 1 \times 0.5 \times 8 =$ **4 volt**

(*b*) Current through $AB = 4/0.4 = 10$ A

Force on *AB i.e.* $F = BIl = 1 \times 10 \times 0.5 =$ **5 N**

The direction of this force, as found by applying Fleming's left-hand rule, is to the right.

(*c*) Rate of doing mechanical work $= F \times \upsilon = 5 \times 8 =$ **40 J/s or W**

Electric power produced $= e\,i = 4 \times 10 = 40$ W

From the above, it is obvious that the mechanical work done in moving the conductor against force *F* is converted into electric energy.

Example 7.7 *In a 4-pole dynamo, the flux/pole is 15 mWb. Calculate the average e.m.f. induced in one of the armature conductors, if armature is driven at 600 r.p.m.*

Solution. It should be noted that each time the conductor passes under a pole (whether *N* or *S*) it cuts a flux of 15 mWb. Hence, the flux cut in one revolution is $15 \times 4 = 60$ mWb. Since conductor is rotating at 600/60 = 10 r.p.s. time taken for one revolution is 1/10 = 0.1 second.

$$\therefore \quad \text{average e.m.f. generated} = N\frac{d\Phi}{dt} \text{ volt}$$

$N = 1$; $\quad d\Phi = 60$ mWb $= 6 \times 10^{-2}$ Wb ; $dt = 0.1$ second

$$\therefore \quad e = 1 \times 6 \times 10^{-2}/0.1 = \textbf{0.6 V}$$

Tutorial Problems No. 7.1

1. A conductor of active length 30 cm carries a current of 100 A and lies at right angles to a magnetic field of strength 0.4 Wb/m^2. Calculate the force in newtons exerted on it. If the force causes the conductor to move at a velocity of 10 m/s, calculate (*a*) the e.m.f. induced in it and (*b*) the power in watts developed by it. **[12 N; 1.2 V, 120 W]**
2. A straight horizontal wire carries a steady current of 150 A and is situated in a uniform magnetic field of 0.6 Wb/m^2 acting vertically downwards. Determine the magnitude of the force in kg/metre length of conductor and the direction in which it works. **[9.175 kg/m horizontally]**
3. A conductor, 10 cm in length, moves with a uniform velocity of 2 m/s at right angles to itself and to a uniform magnetic field having a flux density of 1 Wb/m^2. Calculate the induced e.m.f. between the ends of the conductor. **[0.2 V]**

7.8. Statically Induced E.M.F.

It can be further sub-divided into ***(a) mutually induced e.m.f.*** **and** ***(b) self-induced e.m.f.***

(*a*) Mutually-induced e.m.f. Consider two coils *A* and *B* lying close to each other (Fig. 7.9).

Coil *A* is joined to a battery, a switch and a variable resistance *R* whereas coil *B* is connected

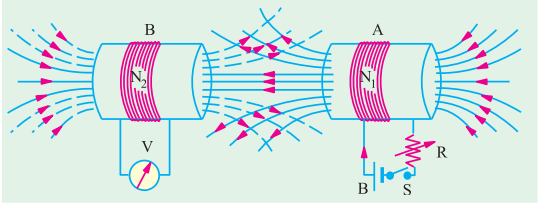


Fig. 7.9

to a sensitive voltmeter *V*. When current through *A* is established by closing the switch, its magnetic field is set up which partly links with or threads through the coil *B*. As current through *A* is changed, the flux linked with *B* is also changed. Hence, mutually induced e.m.f. is produced in *B* whose magnitude is given by Faraday's Laws (Art. 7.3) and direction by Lenz's Law (Art. 7.5).

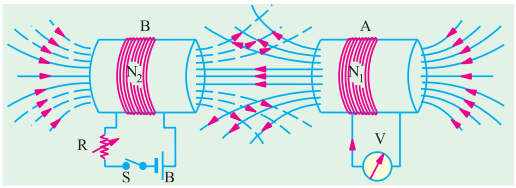


Fig. 7.10

If, now, battery is connected to *B* and the voltmeter across *A* (Fig. 7.10), then the situation is reversed and now a change of current in *B* will produce mutually-induced e.m.f. in *A*.

It is obvious that in the examples considered above, there is no movement of any conductor, the flux variations being brought about by variations in current strength only. Such an **e.m.f.** induced in one coil by the influence of the other coil is called (statically but) mutually induced e.m.f.

(*b*) Self-induced e.m.f. This is the e.m.f. induced in a coil due to ***the change of its own flux linked with it.*** If current through the coil (Fig. 7.11) is changed, then the flux linked with its own turns will also change, which will produce in it what is called *self-induced* e.m.f. The direction of this induced e.m.f. (as given by Lenz's law) would be such as to oppose any change of flux which is, in fact, the very cause of its production. Hence, it is also known as the opposing or counter e.m.f. of self-induction.

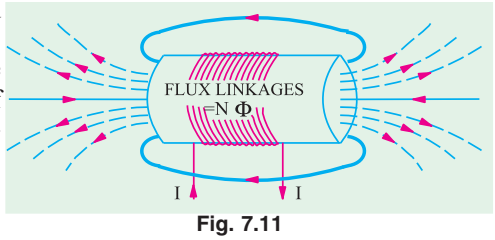


Fig. 7.11

7.9. Self-inductance

Imagine a coil of wire similar to the one shown in Fig. 7.11 connected to a battery through a rheostat. It is found that whenever an effort is made to increase current (and hence flux) through it, it is always opposed by the instantaneous production of counter e.m.f. of self-induction. Energy required to overcome this opposition is supplied by the battery. As will be fully explained later on, this energy is stored in the additional flux produced.

If, now an effort is made to decrease the current (and hence the flux), then again it is delayed due to the production of self-induced e.m.f., this time in the opposite direction. This property of the coil due to which it opposes any increase or decrease or current of flux through it, is known as ***self-inductance***. It is quantitatively measured in terms of coefficient of self induction *L*. This property is analogous to inertia in a material body. We know by experience that initially it is difficult to set a heavy body into motion, but once in motion, it is equally difficult to stop it. Similarly, in a coil having large self-induction, it is initially difficult to establish a current through it, but once established, it is equally difficult to withdraw it. Hence, self-induction is sometimes analogously called ***electrical inertia or electromagnetic inertia.***

7.10. Coefficient of Self-induction (L)

It may be defined in any one of the three ways given below :

(*i*) First Method for L

The coefficient of self-induction of a coil is defined as

the weber-turns per ampere in the coil

By 'weber-turns' is meant the product of flux in webers and the number of turns with which the flux is linked. In other words, it is the flux-linkages of the coil.

Consider a solenoid having *N* turns and carrying a current of *I* amperes. If the flux produced is Φ webers, the weber-turns are $N\Phi$. Hence, weber-turns per ampere are $N\Phi/I$.

By definition, $L = \frac{N\Phi}{I}$. The unit of self-induction is henry*.

* After the American scientist Joseph Henry (1797 - 1878), a company of Faraday.

If in the above relation, $N\Phi$ = 1 Wb-turn, I = 1 ampere, then L = 1 henry (H)

Hence a coil is said to have a self-inductance of one henry if a current of 1 ampere when flowing through it produced flux-linkages of 1 Wb-turn in it.

Therefore, the above relation becomes $L = \dfrac{N\Phi}{I}$ henry

Example 7.8. *The field winding of a d.c. electromagnet is wound with 960 turns and has resistance of 50 Ω when the exciting voltages is 230 V, the magnetic flux linking the coil is 0.005 Wb. Calculate the self-inductance of the coil and the energy stored in the magnetic field.*

Solution. Formula used : $L = \dfrac{N\Phi}{I}$ H

Current through coil = 230/50 = 4.6 A $\quad \Phi$ = 0.005 Wb ; N = 960

$L = \dfrac{960 \times 0.005}{4.6}$ = **1.0435 H**. Energy stored = $\dfrac{1}{2} L I^2 = 1\dfrac{1}{2} \times 1.0435 \times 4.6^2$ = **11.04 J**

Second Method for L

We have seen in Art. 6.20 that flux produced in a solenoid is

$$\Phi = \frac{NI}{l/\mu_0\mu_r A} \quad \therefore \frac{\Phi}{I} = \frac{N}{l/\mu_0\mu_r A} \quad \text{Now } L = N\frac{\Phi}{I} = N \cdot \frac{N}{l/\mu_0\mu_r A} H$$

$$\therefore \quad L = \frac{N^2}{l/\mu_0\mu_r A} = \frac{N^2}{S} \text{H} \quad \text{or} \quad L = \frac{\mu_0\mu_r A N^2}{l} \text{H}$$

It gives the value of self-induction in terms of the dimensions of the solenoid*.

Example 7.9. *An iron ring 30 cm mean diameter is made of square of iron of 2 cm × 2 cm cross-section and is uniformly wound with 400 turns of wire of 2 mm² cross-section. Calculate the value of the self-inductance of the coil. Assume μ_r = 800.* **(Elect. Technology. I, Gwalior Univ.)**

Solution. $L = \mu_0 \mu_r AN^2/l$. Here $N = 400$; $A = 2 \times 2 = 4$ cm^2 $= 4 \times 10^{-4}$ m^2 ; $l = 0.3\,\pi$ m ; $\mu_r = 800$

$\therefore \quad L = 4\pi \times 10^{-7} \times 800 \times 4 \times 10^{-4} \, (400)^2/_{0.3\,\pi}$ = **68.3 mH**

Note. The cross-section of the wire is not relevant to the given question.

Third Method for L

It will be seen from Art. 7.10 (i) above that $L = \dfrac{N\Phi}{I} \quad \therefore \; N\Phi = LI$ or $-N\Phi = -L\,I$

Differentiating both sides, we get $-\dfrac{d}{dt}(N\Phi) = -L \cdot \dfrac{dI}{dt}$ (assuming L to be constant) ;

$$-N \cdot \frac{d\Phi}{dt} = -L \cdot \frac{dI}{dt}$$

As seen from Art. 7.3, $-N \cdot \dfrac{d\Phi}{dt}$ = self-induced e.m.f. $\quad \therefore e_L = -L\dfrac{dI}{dt}$

If $\dfrac{dI}{dt}$ = 1 ampere/second and e_L = 1 volt, then L = 1 H

Hence, a coil has a self-inductance of one henry if one volt is induced in it when current through it changes at the rate of one ampere/second.

Example 7.10. *If a coil of 150 turns is linked with a flux of 0.01 Wb when carrying current of 10 A, calculate the inductance of the coil. If this current is uniformly reversed in 0.01 second, calculating the induced electromotive force.*

Solution. $L = N\Phi/I = 150 \times 0.01/10$ = **0.15 H**

Now, $e_L = L\,dI/dt$; $dI = -10 - (-10) = 20$ A

$\therefore \quad e_L = 0.15 \times 20/0.01$ = **300 V**

* In practice, the inductance of a short solenoid is given by $L = K\mu_0\,\mu_r AN^2/l$ where K is Nagaoka's constant.

Example 7.11. *An iron rod, 2 cm in diameter and 20 cm long is bent into a closed ring and is wound with 3000 turns of wire. It is found that when a current of 0.5 A is passed through this coil, the flux density in the coil is 0.5 Wb/m^2. Assuming that all the flux is linked with every turn of the coil, what is* ***(a)*** *the B/H ratio for the iron* ***(b)*** *the inductance of the coil ? What voltage would be developed across the coil if the current through the coil is interrupted and the flux in the iron falls to 10 % of its former value in 0.001 second ?* **(Principle of Elect. Engg. Jadavpur Univ.)**

Solution. $H = NI/l = 3000 \times 0.2 = 7500$ AT/m $B = 0.5$ Wb/m^2

(a) Now, $\frac{B}{H} = \frac{0.5}{7500} = \mathbf{6.67 \times 10^{-5}\ H/m}$. Also $\mu_r = B/\mu_a H = 6.67 \times 10^{-5}/4\pi \times 10^{-7} = 53$

(b) $L = \frac{N\Phi}{I} = \frac{300 \times \pi \times (0.02)^2 \times 0.5}{4 \times 0.5} = \mathbf{0.94\ H}$

$e_L = N \frac{N\Phi}{dt}$ volt ; $d\Phi$ = 90 % of original flux = $\frac{0.9 \times \pi \times (0.02)^2 \times 0.5}{4} = 0.45\ \pi \times 10^{-4}$ Wb

$dt = 0.001$ second $\therefore\ e_L = 3000 \times 0.45\pi \times 10^{-4}/0.001 = \mathbf{424\ V}$

Example 7.12. *A circuit has 1000 turns enclosing a magnetic circuit 20 cm^2 in section. With 4 A, the flux density is 1.0 Wb/m^2 and with 9A, it is 1.4 Wbm^2. Find the mean value of the inductance between these current limits and the induced e.m.f. if the current falls from 9 A to 4 A in 0.05 seconds.* **(Elect. Engineering-1, Delhi Univ.)**

Solution. $L = N \frac{d\Phi}{dI} = N \frac{d}{dI}(BA) = NA \frac{dB}{dI}$ henry $= 1000 \times 20 \times 10^{-4}\ (1.4 - 1)/(9 - 4) = \mathbf{0.16\ H}$

Now, $e_L = L.dI/dt$; $d\,I = (9 - 4) = 5$ A, $dt = 0.05$ s $\therefore\ e_L = 0.16 \times 5/0.05 = \mathbf{16\ V}$

Example 7.13. *A direct current of one ampere is passed through a coil of 5000 turns and produces a flux of 0.1mWb. Assuming that whole of this flux threads all the turns, what is the inductance of the coil ? What would be the voltage developed across the coil if the current were interrupted in 10^{-3} second ? What would be the maximum voltage developed across the coil if a capacitor of 10μ F were connected across the switch breaking the d.c. supply ?*

Solution. $L = N\Phi/I = 5000 \times 10^{-4} = \mathbf{0.5\ H}$; Induced e.m.f. $= L\,.\ \frac{dI}{dt} = \frac{0.5 \times 1}{10^{-3}} = \mathbf{500\ V}$

The energy stored in the coil is $= \frac{1}{2} L I^2 = \frac{1}{2} \times 0.5 \times I^2 = 0.25$ J

When the capacitor is connected, then the voltage developed would be equal to the p.d. developed across the capacitor plates due to the energy stored in the coil. If V is the value of the voltage,

then $\frac{1}{2} CV^2 \quad \frac{1}{2} \quad LI^2 ; \frac{1}{2} \quad 10 \quad 10^{\ 6} V^2 = 0.25$ or V = **224 volt**

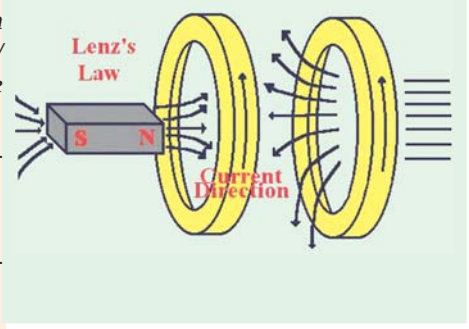
Example 7.14. ***(a)*** *A coil of 1000 turns is wound on a torroidal magnetic core having a reluctance of 10^4 AT/Wb. When the coil current is 5 A and is increasing at the rate of 200 A/s, determine.*

(i) *energy stored in the magnetic circuit* ***(ii)*** *voltage applied across the coil*

Assume coil resistance as zero.

(b) *How are your answers affected if the coil resistance is 2 Ω*

(Elect. Technology, Hyderabad Univ. 1991)



Solution. ***(a)*** $L = N^2/S = 1000^2/10^6 = 1$ H

(*i*) Energy stored $= \frac{1}{2}LI^2 = \frac{1}{2}\times 1\times 5^2 = 12.5$ **J**

(*ii*) Voltage applied across coil= self-induced e.m.f. in the coil = $L.dI/dt = 1 \times 200 =$ **200 V**

(*b*) Though there would be additional energy loss of $5^2 \times 2 = 50$ W over the coil resistance, energy stored in the coil would remain the same. However, voltage across the coil would increase by an amount = $5 \times 2 = 10$ V *i.e.*, now its value would be **210 V**.

7.11. Mutual Inductance

In Art. 7.8 (Fig. 7.9) we have that any change of current in coil *A* is always accompanied by the production of mutually-induced e.m.f. in coil *B*. Mutual inductance may, therefore, be defined as the ability of one coil (or circuit) to produce an e.m.f. in a nearby coil by induction when the current in the first coil changes. This action being reciprocal, the second coil can also induce an e.m.f. in the first when current in the second coil changes. This ability of reciprocal induction is measured in terms of the coefficient of mutual induction *M*.

Example 7.15. *A single element has the current and voltage functions graphed in figure 7.12. (a) and (b). Determine the element.* **[Bombay University 2001]**

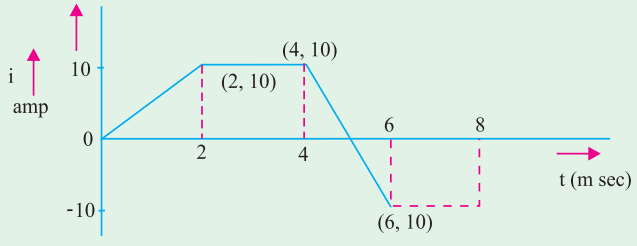


Fig. 7.12 (*a*)

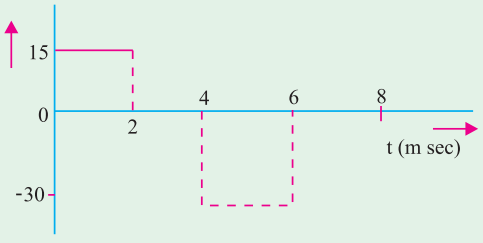


Fig. 7.12 (*b*)

Solution. Observations from the graph are tabulated below.

Sr. No.	Between time	*di/dt* amp/sec	*V*	*L*	
1	0 - 2 m Sec	5000	15	15/5000	= 3mH
2	2 - 4 m Sec	0	0	–	
3	4 - 6 m Sec	– 10,000	– 30	– 30 / (– 10,000)	= 3 mH
4	6 - 8 m Sec	0	0	–	

The element is a 3-mH inductor.

7.12. Coefficient of Mutual Inductance (M)

It can also be defined in three ways as given below :

(*i*) First Method for M

Let there be two magnetically-coupled coils having N_1 and N_2 turns respectively (Fig. 7.9). Coefficient of mutual inductance between the two coils is defined as

the weber-turns in one coil due to one ampere current in the other.

Let a current I_1 ampere when flowing in the first coil produce a flux Φ_1 webers in it. ***It is supposed that whole of this flux links with the turns of the second coil****. Then, flux-linkages *i.e.*, webers-turns in the *second* coil for unit current in the *first* coil are $N_2 \Phi_1/I_1$. Hence, by definition

$$M = \frac{N_2 \Phi_1}{I_1}$$

If weber-turns in *second* coil due to one ampere current in the *first* coil *i.e.* $N_2 \Phi_1/I_1 = 1$ then, as seen from above, $M = 1$H.

Hence, two coils are said to have a mutual inductance of 1 henry is one ampere current when flowing in one coil produces flux-linkages of one Wb-turn in the other.

Example 7.16. *Two identical coils X and Y of 1,000 turns each lie in parallel planes such that 80% of flux produced by one coil links with the other. If a current of 5 A flowing in X produces a flux of 0.5 mWb in it, find the mutual inductance between X and Y.* **(Elect. Engg. A,M.Ae.S.I.)**

Solution. Formula used $M = \frac{N_2 \Phi_1}{I_1}$ H ; Flux produced in $X = 0.5$ mWb $= 0.5 \times 10^{-3}$ Wb

Flux linked with $Y = 0.5 \times 10^{-3} \times 0.8 = 0.4 \times 10^{-3}$ Wb ; $M = \frac{1000 \times 0.4 \times 10^{-3}}{5} =$ **0.08 H**

Example 7.17. *A long single layer solenoid has an effective diameter of 10 cm and is wound with 2500 AT/metre. There is a small concentrated coil having its plane lying in the centre cross-sectional plane of the solenoid. Calculate the mutual inductance between the two coils in each case if the concentrated coil has 120 turns on an effective diameter of* ***(a)*** *8 cm and* ***(b)*** *12 cm.*

(Elect. Science - II Allahabad Univ. 1992)

Solution. The two cases ***(a)*** and ***(b)*** are shown in Fig. 7.13 (*a*) and (*b*) respectively.

(a) Let I_1 be the current flowing through the solenoid. Then

$$B = \mu_0 H \times \mu_0 N I_1/l = 2500\ \mu_0 I_1 \text{ Wb/m}^2 \quad \ldots l = 1 \text{ m}$$

Area of search coil $A_1 = \frac{\pi}{4} \times 8^2 \times 10^{-4} = 16\pi \times 10^{-4}$ m^2

Flux linked with search coil is

$$\Phi = BA_1 = 2500\ \mu_0 I_1 \times 16\pi \times 10^{-4} = 15.79\ I_1 \times 10^{-6} \text{ Wb}$$

$$\therefore \quad M = \frac{N_2 \Phi_1}{I_1} = \frac{120 \times 15.39 I_1 \times 10^6}{I_1} = \mathbf{1.895 \times 10^{-3}\ H}$$

(b) Since the field strength outside the solenoid is negligible, the effective area of the search coil, in this case, equals the area of the long solenoid.

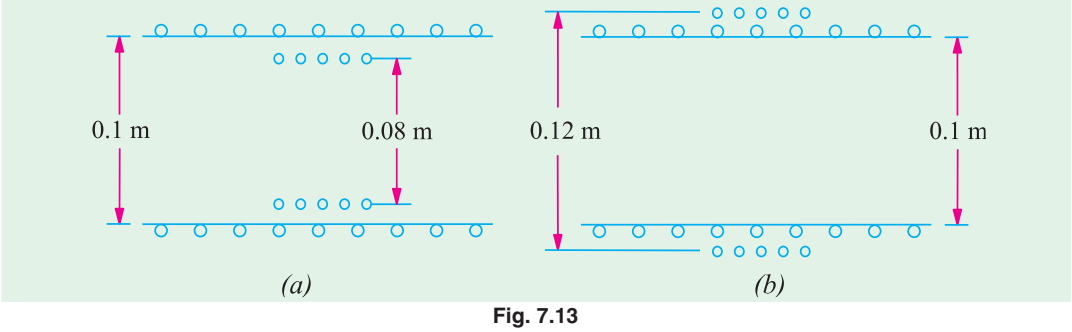


Fig. 7.13

$$A_2 = \frac{\pi}{4} \times 10^2 \times 10^{-4} = \frac{\pi}{4} 10^{-2} \text{m}^2 ;$$

* If whole of this flux does not link with turns of the second coil, then only that part of the flux which is actually linked is taken instead. (Ex. 7.13 and 7.17). In general, $M = N_2 \Phi_2/I_1$.

$$\Phi = BA_2 = 2500\,\mu_0 I_1 \times \frac{\pi}{4} \times 10^{-2} = 24.68\,I_1 \times 10^{-6}\ \text{Wb}$$

$$M = \frac{120 \times 24.68\,I_1 \times 10^{-6}}{I_1} = \mathbf{2.962 \times 10^{-3}\ H}$$

Example 7.18. *A flux of 0.5 mWb is produced by a coil of 900 turns wound on a ring with a current of 3 A in it. Calculate* ***(i)*** *the inductance of the coil* ***(ii)*** *the e.m.f. induced in the coil when a current of 5 A is switched off, assuming the current to fall to zero in 1 milli second and* ***(iii)*** *the mutual inductance between the coils, if a second coil of 600 turns is uniformly wound over the first coil.*

(F. E. Pune Univ.)

Solution. ***(i)*** Inductance of the first coil $= \dfrac{N\Phi}{I} = \dfrac{900 \times 0.5 \times 10^{-3}}{3} = \mathbf{0.15\ H}$

(ii) e.m.f. induced $e_1 = L\dfrac{di}{dt} = 0.15 \times \dfrac{(5-0)}{1 \times 10^{-3}} = \mathbf{750\ V}$

(iii) $M\dfrac{N_2\Phi_1}{I_1} = \dfrac{600 \times 0.5 \times 10^{-3}}{3} = \mathbf{0.1\ H}$

(*ii*) Second Method for M

We will now deduce an expression for coefficient of mutual inductance in terms of the dimensions of the two coils.

Flux in the first coil $\Phi_1 = \dfrac{N_2 I_1}{l/\mu_0\mu_r A}$ Wb ; Flux/ampere $= \dfrac{\Phi_1}{I_1} = \dfrac{N_1}{l/\mu_0\mu_r A}$

Assuming that whole of this flux (it usually is some percentage of it) is linked with the other coil having N_2 turns, the weber-turns in it due to the flux/ampere in the first coil is

$$M = \frac{N_2\Phi_1}{I_1} = \frac{N_2 N_1}{l/\mu_0\mu_r A} \quad \therefore\ M = \frac{\mu_0\mu_r A\, N_1 N_2}{l}\ \text{H}$$

Also $$M = \frac{N_1 N_2}{l/\mu_0\mu_r A} = \frac{N_1 N_2}{\text{reluctance}} = \frac{N_1 N_2}{S}\ \text{H}$$

Example 7.19. *If a coil of 150 turns is linked with a flux of 0.01 Wb when carrying a current of 10 A ; calculate the inductance of the coil. If this current is uniformly reversed in 0.1 second, calculate the induced e.m.f. If a second coil of 100 turns is uniformly wound over the first coil, find the mutual inductance between the coils.* **(F. E. Pune Univ.)**

Solution.

$$L_1 = N_1\Phi_1/I_1 = 150 \times 0.01/10 = \mathbf{0.15\ H}$$

$$e = L \times di/dt = 0.15 \times [10 - (-10)]/0.1 = 1 = \mathbf{30\ V}$$

$$M = N_2\Phi/I_1 = 100 \times 0.01/10 = \mathbf{0.1\ H}$$

(*iii*) Third Method for M

As seen from Art. 7.12 (*i*) $M = \dfrac{N_2\Phi_1}{I_1} \quad \therefore N_2\Phi_1 = MI_1$ or $-N_2\Phi_1 = -MI_1$

Differentiating both sides, we get : $-\dfrac{d}{dt}(N_2\Phi_1) = -M\dfrac{dI_1}{dt}$ (assuming M to be constant)

Now, $-\dfrac{d}{dt}(N_2\Phi_1)$ = mutually-induced e.m.f. in the second coil = $e_M \quad \therefore e_M = -M\dfrac{dI_1}{dt}$

If $dI_1/dt = 1$ A/s ; $e_M = 1$ volt, then $M = \mathbf{1\ H}$.

Hence, two coils are said to have a mutual inductance of one henry if current changing at the rate of 1 ampere/second in one coil induces an e.m.f. of one volt in the other.

Example 7.20. *Two coils having 30 and 600 turns respectively are wound side-by-side on a closed iron circuit of area of cross-section 100 sq.cm. and mean length 200 cm. Estimate the mutual inductance between the coils if the relative permeability of the iron is 2000. If a current of zero ampere grows to 20 A in a time of 0.02 second in the first coil, find the e.m.f. induced in the second coil.* **(Elect. Engg. I, JNT Univ., Warangal)**

Solution. Formula used : $M = \dfrac{N_1 N_2}{l/\mu_0\mu_r A}$ **H**, $N_1 = 30$; $N_2 = 600$; $A = 100 \times 10^{-4} = 10^{-2}\text{m}^2$, $l = 2\text{m}$

$$\therefore \quad M = \mu_0\mu_r A\, N_1 N_2/l = 4\pi \times 10^{-7} \times 2000 \times 10^{-2} \times 30 \times 600/2 = \mathbf{0.226\ H}$$

$$dI_1 = 20 - 0 = 20\text{ A} ; dt = 0.02\text{ s} ; e_M = MdI_1/dt = 0.226 \times 20/0.2 = \mathbf{226\ V}$$

Example 7.21. *Two coils A and B each having 1200 turns are placed near each other. When coil B is open-circuited and coil A carries a current of 5 A, the flux produced by coil A is 0.2 Wb and 30% of this flux links with all the turns of coil B. Determine the voltage induced in coil B on open-circuit when the current in the coil A is changing at the rate of 2 A/s.*

Solution. Coefficient of mutual induction between the two coils is $M = N_2\Phi_2/I_1$

Flux linked with coil B is 30 per cent of 0.2 Wb *i.e.* 0.06 Wb

$$\therefore \quad M = 1200 \times 0.06/5 = 14.4\text{ H}$$

Mutually-induced e.m.f. in coil B is $e_M = MdI_1/dt = 14.4 \times 2 = \mathbf{28.8\ V}$

Example 7.22. *Two coils are wound side by side on a paper-tube former. An e.m.f. of 0.25 V is induced in coil A when the flux linking it changes at the rate of 10^3 Wb/s. A current of 2 A in coil B causes a flux of 10^{-5} Wb to link coil A. What is the mutual inductance between the coils ?*

(Elect. Engg-I, Bombay Univ.)

Solution. Induced e.m.f. in coil A is $e = N_1 \dfrac{d\Phi}{dt}$ where N_1 is the number of turns of coil A.

$$\therefore \quad 0.25 = N_1 \times 10^{-3} \quad \therefore \ N_1 = 250$$

Now, flux linkages in coil A due to 2 A current in coil $B = 250 \times 10^{-5}$

$$\therefore \quad M = \frac{\text{flux linkages in coil } A}{\text{current in coil } B} = 250 \times 10^{-5}/2 = \mathbf{1.25\ mH}$$

7.13. Coefficient of Coupling

Consider two magnetically-coupled coils A and B having N_1 and N_2 turns respectively. Their individual coefficients of self-induction are,

$$L_1 = \frac{N_1^2}{l/\mu_0\mu_r A} \quad \text{and} \quad L_2 = \frac{N_2^2}{l/\mu_0\mu_r A}$$

The flux Φ_1 produced in A due to a current I_1 ampere is $\Phi_1 = \dfrac{N_1 I_1}{l/\mu_0\mu_r A}$

Suppose a fraction k_1 of this flux *i.e.* $k_1\Phi_1$ is linked with coil B.

Then $$M = \frac{k_1\Phi_1 \times N_2}{I_1} \text{ where } k_1 \leqslant 1.$$

Substituting the value of Φ_1, we have, $M = k_1 \times \dfrac{N_1 N_2}{l/\mu_0\mu_r A}$ ...(*i*)

Similarly, the flux Φ_2 produced in B due to I_2 ampere in it is $\Phi_2 = \dfrac{N_2 I_2}{l/\mu_0\mu_r A}$

Suppose a fraction k_2 of this flux *i.e.* $k_2\Phi_2$ is linked with A.

Then $$M = \frac{k_2\Phi_2 \times N_1}{I_2} = k_2 \frac{N_1 N_2}{l/\mu_0\mu_r A}$$ ...(*ii*)

Multiplying Eq. (*i*) and (*ii*), we get

$$M^2 = k_1k_2 \frac{N_1^2}{l/\mu_0\mu_r A} \times \frac{N_2^2}{l/\mu_0\mu_r A} \quad \text{or} \quad M^2 = k_1k_2L_1L_2$$

Putting $\sqrt{k_1k_2} = k$, we have $M = k\sqrt{L_1L_2}$ or $k = \dfrac{M}{\sqrt{L_1L_2}}$

The constant k is called the ***coefficient of coupling*** and may be defined as the ratio of ***mutual inductance actually present between the two coils to the maximum possible value.*** If the flux due to one coil completely links with the other, then value of k is unity. If the flux of one coil does not at all link with the other, then $k = 0$. In the first case, when $k = 1$, coils are said to be tightly coupled and when $k = 0$, the coils are magnetically isolated from each other.

Example 7.23. *Two identical 750 turn coils A and B lie in parallel planes. A current changing at the rate of 1500 A/s in A induces an e.m.f. of 11.25 V in B. Calculate the mutual inductance of the arrangement. If the self-inductance of each coil is 15 mH, calculate the flux produced in coil A per ampere and the percentage of this flux which links the turns of B.*

Solution. Now, $e_M = MdI_1/dt$ ...Art. 7.12

$$M = \frac{e_M}{dI_1/dt} = \frac{11.25}{1500} = 7.5 \times 10^{-3}\ \text{H} = \mathbf{7.5\ mH}$$

Now, $L_1 = \dfrac{N_1\Phi_1}{I_1}$ $\therefore$ $\dfrac{\Phi_1}{I_1} = \dfrac{L_1}{N_1} = \dfrac{15\times10^{-3}}{750} = \mathbf{2\times10^{-5}\ Wb/A}$ ...Art. 7.10

Now, $k = \dfrac{M}{\sqrt{L_1L_2}} = \dfrac{7.5\ \ 10^{-3}}{\sqrt{L^2}} \quad \dfrac{7.5\ \ 10^{-3}}{15\ \ 10^{-3}} = 0.5 = \mathbf{50\%}\ (\because L_1 = L_2 = L)$ ...Art. 7.13

Example 7.24. *Two coils, A of 12,500 turns and B of 16,000 turns, lie in parallel planes so that 60 % of flux produced in A links coil B. It is found that a current of 5A in A produces a flux of 0.6 mWb while the same current in B produces 0.8 mWb. Determine* ***(i)*** *mutual inductance and* ***(ii)*** *coupling coefficient.*

Solution. ***(i)*** Flux/ampere in $A = 0.6/5 = 0.12$ mWb

Flux linked with $B = 0.12 \times 0.6 = 0.072$ mWb

$\therefore$ $M = 0.072 \times 10^{-3} \times 16{,}000 = \mathbf{1.15\ H}$

Now, $L_1 = \dfrac{12{,}500\times0.6}{5} = 150 \times 10^{-3}$ H ; $L_2 = \dfrac{16{,}000\times0.8}{5} = 256 \times 10^{-3}$ H

(ii) $k = M/\sqrt{L_1L_2} = 1.15/\sqrt{1.5\times2.56} = \mathbf{0.586}$

Note. We could find k in another way also. Value of $k_1 = 0.6$, that of k_2 could also be found, then $k = \sqrt{k_1k_2}$.

Example 7.25. *Two magnetically-coupled coils have a mutual inductance of 32 mH. What is the average e.m.f. induced in one, if the current through the other changes from 3 to 15 mA in 0.004 second ? Given that one coil has twice the number of turns in the other, calculate the inductance of each coil. Neglect leakage.*

Solution. $M = 32 \times 10^{-3}$ H ; $dI_1 = 15 - 3 = 12$ mA $= 12 \times 10^{-3}$ A ; $dt = 0.004$ second

$$\text{Average e.m.f. induced} = M\frac{dI_1}{dt} = \frac{32\times10^{-3}\times12\times10^{-3}}{0.004} = \mathbf{96\times10^{-3}\ V}$$

Now $L_1 = \mu_0N^2A/l = k\,N^2$ where $k = \mu_0A/l$ (taking $\mu_r = 1$)

$$L_2 = \frac{(2N)^2\mu_0A}{2\,l} = 2kN^2 \ ; \ \frac{L_2}{L_1} = \frac{2kN^2}{kN^2} = 2 \quad \therefore\ L_2 = 2L_1$$

Now $M = \sqrt{L_1L_2} = \sqrt{2L_1\times L_1} = 32$, $L_1 = 32/\sqrt{2} = \mathbf{16/\sqrt{2}mH}$; $L_2 = 2\times16/\sqrt{2} = \mathbf{32\sqrt{2}\ mH}$

Example 7.26. *Two coils, A and B, have self inductances of 120 μH and 300 μH respectively. A current of 1 A through coil A produces flux linkages of 100 μWb turns in coil B. Calculate* ***(i)*** *the mutual inductance between the coils* ***(ii)*** *the coupling coefficient and* ***(iii)*** *the average e.m.f. induced in coil B if a current of 1 A in coil A is reversed at a uniform rate in 0.1 sec.*

(F. E. Pune Univ.)

Solution. ***(i)*** $M = \dfrac{\text{flux-linkages of coil } B}{\text{current in coil } A} = \dfrac{100\times10^{-6}}{1} = \mathbf{100\ \mu H}$

(ii) $M = k\sqrt{L_1L_2} \quad \therefore k = \dfrac{M}{\sqrt{L_1L_2}} = \dfrac{100\times10^{-6}}{\sqrt{120\times10^{-6}\times300\times10^{-6}}} = \mathbf{0.527}$

(iii) $e_2 = M \times di/dt = (100 \times 10^{-6}) \times 2/0.1 = 0.002$ V or **2 mV.**

7.14. Inductances in Series

(*i*) Let the two coils be so joined in series that their fluxes (or m.m.fs) are additive *i.e.*, in the same direction (Fig. 7.14).

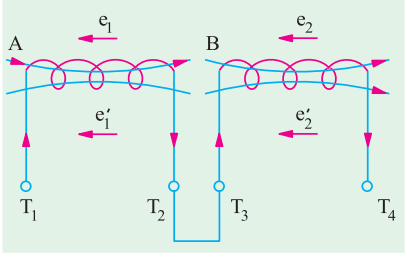


Fig. 7.14

Let M = coefficient of mutual inductance
L_1 = coefficient of self-inductance of 1st coil
L_2 = coefficient of self-inductance of 2nd coil.

Then, self induced e.m.f. in A is $= e_1 = -L_1.\dfrac{di}{dt}$

Mutually-induced e.m.f. in A due to change of current in B is $e' = -M.\dfrac{di}{dt}$

Self-induced e.m.f. in B is $= e_2 = -L_2.\dfrac{di}{dt}$

Mutually-induced e.m.f. in B due to change of current in A is $= e_2' = -M.\dfrac{di}{dt}$

(All have -ve sign, because both self and mutally induced e.m.fs. are in opposition to the applied e.m.f.). Total induced e.m.f. in the combination $= -\dfrac{di}{dt}(L_1 + L_2 + 2M)$ ...***(i)***

If L is the equivalent inductance then total induced e.m.f. in that single coil would have been

$$= -L\frac{di}{dt} \qquad \text{...}\textbf{(ii)}$$

Equating ***(i)*** and ***(ii)*** above, we have $L = L_1 + L_2 + 2M$

(ii) When the coils are so joined that their fluxes are in opposite directions (Fig. 7.15).

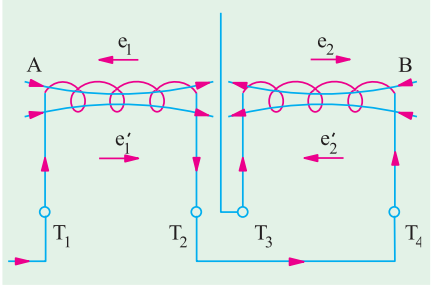


Fig. 7.15

As before $e_1 = -L_1\dfrac{di}{dt}$

$e_1' = +M.\dfrac{di}{dt}$ (mark this direction)

$e_2 = -L_2\dfrac{di}{dt}$ and $e_2' = +M.\dfrac{di}{dt}$

Total induced e.m.f. $= -\dfrac{di}{dt}(L_1 + L_2 - 2M)$

$\therefore$ Equivalent inductance

In general, we have :

$L = L_1 + L_2 - 2M$ ... if m.m.fs are additive

and

$L = L_1 + L_2 + 2M$ ... if m.m.fs. are subtractive

Example 7.27. *Two coils with a coefficient of coupling of 0.5 between them, are connected in series so as to magnetise* ***(a)*** *in the same direction* ***(b)*** *in the opposite direction. The corresponding values of total inductances are for* ***(a)*** *1.9 H and for* ***(b)*** *0.7 H. Find the self-inductances of the two coils and the mutual inductance between them.*

Solution. ***(a)*** $L = L_1 + L_2 + 2M$ or $1.9 = L_1 + L_2 + 2M$ ...***(i)***

(b) Here $L = L_1 + L_2 - 2M$ or $0.7 = L_1 + L_2 - 2M$ ...***(ii)***

Subtracting ***(ii)*** from ***(i)***, we get

$1.2 = 4M \quad \therefore \quad M = 0.3$ H

Putting this value in ***(i)*** above, we get $L_1 + L_2 = 1.3$ H ...***(iii)***

We know that, in general, $M = k\sqrt{L_1 L_2}$

$\therefore \quad \sqrt{L_1 L_2} = \dfrac{M}{k} = \dfrac{0.3}{0.5} = 0.6 \quad \therefore \; L_1 L_2 = \mathbf{0.36}$

From ***(iii)***, we get $(L_1 + L_2)^2 - 4L_1 L_2 = (L_1 - L_2)^2$

$\therefore \quad (L_1 - L_2)^2 = 0.25$ or $L_1 - L_2 = 0.5$ ...***(iv)***

From ***(iii)*** and ***(iv)***, we get $L_1 = \mathbf{0.9\ H}$ and $L_2 = \mathbf{0.4\ H}$

Example 7.28. *The combined inductance of two coils connected in series is 0.6 H or 0.1 H depending on the relative directions of the currents in the coils. If one of the coils when isolated has a self-inductance of 0.2 H, calculate* ***(a)*** *mutual inductance and* ***(b)*** *coupling coefficient.*

(Elect. Technology, Univ. of Indore)

Solution. ***(i)*** $L = L_1 + L_2 + 2M$ or $0.6 = L_1 + L_2 + 2M$ ...***(i)***

and $0.1 = L_1 + L_2 - 2M$ ...***(ii)***

(a) From (*i*) and (*ii*) we get, $M = \mathbf{0.125\ H}$

Let $L_1 = 0.2$ H, then substituting this value in (*i*) above, we get $L_2 = 0.15$ H

(b) Coupling coefficient $k = M/\sqrt{L_1 L_2} = 0.125/\sqrt{0.2 \times 0.15} = \mathbf{0.72}$

Example 7.29. *Two similar coils have a coupling coefficient of 0.25. When they are connected in series cumulatively, the total inductance is 80 mH. Calculate the self inductance of each coil. Also calculate the total inductance when the coils are connected in series differentially.*

(F. E. Pune Univ.)

Solution. If each coil has an inductance of L henry, then $L_1 = L_2 = L$; $M = k\sqrt{L_1 L_2} = k\sqrt{L \times L} = kL$

When connected in series comulatively, the total inductance of the coils is

$= L_1 + L_2 + 2M = 2L + 2M = 2L + 2kL = 2L\,(1 + 0.25) = 2.5L$

$\therefore \quad 2.5\,L = 80$ or $L = \mathbf{32\ mH}$

When connected in series differentially, the total inductance of the coils is

$= L_1 + L_2 - 2M = 2L - 2M = 2L - 2kL = 2L\,(1 - k) = 2L\,(1 - 0.25)$

$\therefore \quad 2L \times 0.75 = 2 \times 32 \times 0.75 = \mathbf{48\ mH}$.

Example 7.30. *Two coils with terminals T_1, T_2 and T_3, T_4 respectively are placed side by side. When measured separately, the inductance of the first coil is 1200 mH and that of the second is 800 mH. With T_2 joined to T_3, the inductance between T_1 and T_4 is 2500 mH. What is the mutual inductance between the two coils ? Also, determine the inductance between T_1 and T_3 when T_2 is joined to T_4.*

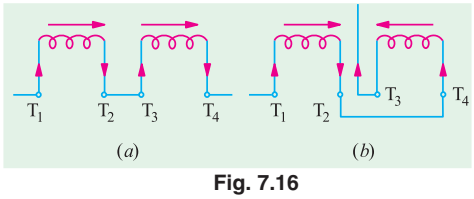


Fig. 7.16

(Electrical Circuit, Nagpur Univ. 1991)

Solution. $L_1 = 1200$ mH, $L_2 = 800$ mH

Fig. 7.16 (*a*) shows additive series.

$\therefore \quad L = L_1 + L_2 + 2M$

or $2500 = 1200 + 800 + 2M$; $M = \mathbf{250\ mH}$

Fig. 7.16 (*b*) shows the case of subtractive or opposing series.

Here, $L = L_1 + L_2 - 2M = 1200 + 800 - 2 \times 250 =$ **1500 mH**

Example 7.31. *The total inductance of two coils, A and B, when connected in series, is 0.5 H or 0.2 H, depending on the relative directions of the current in the coils. Coil A, when isolated from coil B, has a self-inductance of 0.2 H. Calculate*

(a) *the mutual inductance between the two coils*

(b) *the self-inductance of coil B*

(c) *the coupling factor between the coils.*

(d) *the two possible values of the induced e.m.f. in coil A when the current is decreasing at 1000 A per second in the series circuit.* **(Elect. Technology, Hyderabad Univ. 1992)**

Solution. ***(a)*** Combined inductance is given by $L = L_1 + L_2 \pm 2M$

$\therefore$ $0.5 = L_1 + L_2 + 2M$ ...(*i*), $0.2 = L_1 + L_2 - 2M$ ...(***ii***)

Subtracting ***(ii)*** from ***(i)***, we have $4M = 0.3$ or $M =$ **0.075 H**

(b) Adding ***(i)*** and ***(ii)*** we have $0.7 = 2 \times 0.2 + 2L_2 =$ **0.15 H**

(c) Coupling factor or coefficient is $k = M/\sqrt{L_1 L_2}$ $0.075/\sqrt{0.2 \quad 0.15} = 0.433$ or **43.4%**

(d) $e_1 = L_1 \dfrac{di}{dt} \pm M \dfrac{di}{dt}$

$\therefore$ $e_1 = (0.2 + 0.075) \times 1000 =$ **275 V** ...'cumulative connection'

$= (0.2 - 0.075) \times 1000 =$ **125 V** ...'differential connection'

Example 7.32. *Find the equivalent inductance L_{AB} in Fig. 7.17*

(Bombay University, 2001)

Solution. Series Parallel combination of Inductors has to be dealt with. Note that there is no mutual coupling between coils.

$L_{AB} = 0.5 + [0.6 \times 0.3/(0.3 + 0.3)] = 0.7$ H

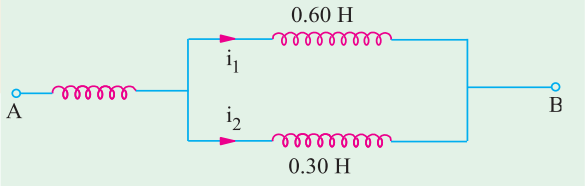


Fig. 7.17

7.15. Inductance in Parallel

In Fig. 7.18, two inductances of values L_1 and L_2 henry are connected in parallel. Let the coefficient of mutual inductance between the two be M. Let i be the main supply current and i_1 and i_2 be the branch currents

Obviously, $i = i_1 + i_2$

$\therefore$ $\dfrac{di}{dt} = \dfrac{di_1}{dt} + \dfrac{di_2}{dt}$ ...(***i***)

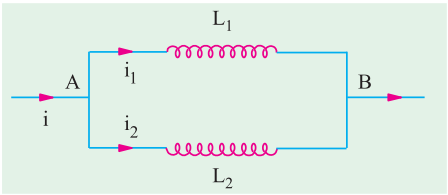


Fig. 7.18

In each coil, both self and mutually induced e.m.fs. are produced. Since the coils are in parallel, these e.m.fs. are equal. For a case when self-induced e.m.f., we get

$$e = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} \quad \therefore \quad L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

or $\dfrac{di_1}{dt}(L_1 - M) = \dfrac{di_2}{dt}(L_2 - M) \quad \therefore \quad \dfrac{di_1}{dt} = \left(\dfrac{L_2 - M}{L_1 - M}\right)\dfrac{di_2}{dt}$ ...(***ii***)

Hence, (*i*) above becomes $\dfrac{di}{dt} = \left[\left(\dfrac{L_2 - M}{L_1 - M}\right) + 1\right]\dfrac{di_2}{dt}$ ...(***iii***)

If L is the equivalent inductance, then $e = L.\dfrac{di}{dt}$ = induced e.m.f. in the parallel combination

= induced e.m.f. in any one coil = $L_1.\dfrac{di_1}{dt} \quad M\dfrac{di_2}{dt}$

$$\therefore \quad \frac{di}{dt} = \frac{1}{L}\left(L_1\frac{di_1}{dt} + M\frac{di_2}{dt}\right) \qquad ...(iv)$$

Substituting the value of di_1/dt from (*ii*) in (*iv*), we get $\frac{di}{dt} = \frac{1}{L}\left[L_1\left(\frac{L_2 - M}{L_1 - M}\right) + M\right]\frac{di_2}{dt}$ ...(*v*)

Hence, equating (*iii*) to (*iv*), we have $\frac{L_2 - M}{L_1 - M} + 1 = \frac{1}{L}\left[L_1\left(\frac{L_2 - M}{L_1 - M}\right) + M\right]$

or $$\frac{L_1 + L_2 - 2M}{L_1 - M} = \frac{1}{L}\left(\frac{L_1L_2 - M^2}{L_1 - M}\right)$$

$$\therefore \quad L = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}$$ when mutual field assists the separate fields.

Similarly, $$L = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M}$$ when the two fields oppose each other.

Example 7.33. *Two coils of inductances 4 and 6 henry are connected in parallel. If their mutual inductance is 3 henry, calculate the equivalent inductance of the combination if (i) mutual inductance assists the self-inductance (ii) mutual inductance opposes the self-inductance.*

Solution. (*i*) $$L = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M} = \frac{4 \times 6 - 3^2}{4 + 6 - 2 \times 3} = \frac{15}{4} = \mathbf{3.75\ H}$$

(*ii*) $$L = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M} = \frac{24 - 9}{16} = \frac{15}{16} = \mathbf{0.94\ H\ (approx.)}$$

Tutorial Problems No. 7.2

1. Two coils are wound close together on the same paxolin tube. Current is passed through the first coil and is varied at a uniform rate of 500 mA per second, inducing an e.m.f. of 0.1 V in the second coil. The second coil has 100 turns. Calculate the number of turns in the first coil if its inductance is 0.4 H. **[200 turns]**
2. Two coils have 50 and 500 turns respectively are wound side by side on a closed iron circuit of section 50 cm^2 and mean length 120 cm. Estimate the mutual inductance between the coils if the permeability of iron is 1000. Also, find the self-inductance of each coil. If the current in one coil grows steadily from zero to 5A in 0.01 second, find the e.m.f. induced in the other coil. **[M = 0.131 H, L_1 = 0.0131 H, L_2 = 1.21 H, E = 65.4 V]**
3. An iron-cored choke is designed to have an inductance of 20 H when operating at a flux density of 1 Wb/m^2, the corresponding relative permeability of iron core is 4000. Determine the number of turns in the winding, given that the magnetic flux path has a mean length of 22 cm in the iron core and of 1 mm in air-gap that its cross-section is 10 cm^2. Neglect leakage and fringing. **[4100]**
4. A non-magnetic ring having a mean diameter of 30 cm and a cross-sectional area of 4 cm^2 is uniformly wound with two coils *A* and *B*, one over the other. *A* has 90 turns and *B* has 240 turns. Calculate from first principles the mutual inductance between the coils.
   Also, calculate the e.m.f. induced in *B* when a current of 6 A in *A* is reversed in 0.02 second. **[11.52 μH, 6.912 mV]**
5. Two coils *A* and *B*, of 600 and 100 turns respectively are wound uniformly around a wooden ring having a mean circumference of 30 cm. The cross-sectional area of the ring is 4 cm^2. Calculate (*a*) the mutual inductance of the coils and (*b*) the e.m.f. induced in coil *B* when a current of 2 A in coil *A* is reversed in 0.01 second. **[(*a*) 100.5 μH (*b*) 40.2 mV]**
6. A coil consists of 1,000 turns of wire uniformly wound on a non-magnetic ring of mean diameter 40 cm and cross-sectional area 20 cm^2.
   Calculate (*a*) the inductance of the coil (*b*) the energy stored in the magnetic field when the coil is carrying a current of 15 A (*c*) the e.m.f. induced in the coil if this current is completely interrupted in 0.01 second. **[(*a*) 2mH (*b*) 0.225 joule (*c*) 3V]**
7. A coil of 50 turns having a mean diameter of 3 cm is placed co-axially at the centre of a solenoid 60 cm long, wound with 2,500 turns and carrying a current of 2 A. Determine mutual inductance of the arrangement. **[0.185 mH]**
8. A coil having a resistance of 2 Ω and an inductance of 0.5 H has a current passed through it which

varies in the following manner ; (*a*) a uniform change from zero to 50 A in 1 second (*b*) constant at 50 A for 1 second (*c*) a uniform change from 50 A to zero in 2 seconds. Plot the current graph to a time base. Tabulate the p.d. applied to the coil during each of the above periods and plot the graph of p.d. to a time base. **[(*a*) 25 to 125 V (*b*) 100 V (*c*) 87.5 V to –12.5 V]**

9. A primary coil having an inductance of 100 μH is connected in series with a secondary coil of 240 μH and the total inductance of the combination is measured as 146 μH. Determine the coefficient of coupling.

10. Find the total inductance measured from A-B terminals, in Fig. 7.19. **[62.6%] (*Circuit Theory, Jadavpur Univ.*)**

[**Hint :** $L = 100 + 50 - (2 \times 60) = 30$ μH, due to opposite senses of currents with respect to dot-markings.]

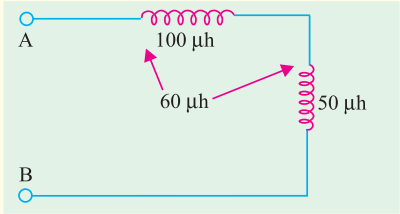


Fig. 7.19

11. Given that relative permeablility of cast iron as 200, that of cast steel is 1200 and for Copper $\mu_0 = 1$. ***(Nagpur University, Summer 2003)***

12. State Faraday's laws of electromagnetic induction. Distinguish between statically induced emf and dynamically induced emf with examples.
(V.T.U., Belgaum Karnataka University, February 2002)

13. State : (*i*) Flemming's right hand rule, and (*ii*) Fleming's left hand rule. Mention their applications. ***(V.T.U., Belgaum Karnataka University, Winter 2003)***

14. Define : (*i*) Self inductance, and (*ii*) Mutual inductance. Mention their units and formula to calculate each of them. Derive an expression for the energy stored in an inductor of self inductance 'L' henry carrying the current of 'I' amperes.
(V.T.U., Belgaum Karnataka University, Winter 2003)

15. State and explain Faraday's laws of electro magnetic induction, Lenz's Law. Fleming's right hand rule and Fleming's left hand rule. ***(V.T.U., Belgaum Karnataka University, Summer 2003)***

16. A coil of 300 turns wound on a core of non magnetic material has an inductance of 10mH. Calculate (*i*) the flux produced by a current of 5A (*ii*) the average value of the emf induced when a current of 5Amps is reversed in 8 mills seconds. ***(V.T.U., Belgaum Karnataka University, Summer 2003)***

OBJECTIVE TESTS – 7

1. According to Faraday's Laws of Electromagnetic Induction, an e.m.f. is induced in a conductor whenever it
(*a*) lies in a magnetic field
(*b*) cuts magnetic flux
(*c*) moves parallel to the direction of the magnetic field
(*d*) lies perpendicular to the magnetic flux.

2. A pole of driving point admittance function implies
(*a*) zero current for a finite value of driving voltage
(*b*) zero voltage for a finite value of driving current
(*c*) an open circuit condition
(*d*) None of (*a*), (*b*) and (*c*) mentioned in the question ***(ESE 2001)***

3. The inductance of a long solenoid of length 1000 mm wound uniformly with 3000 turns on a cylindrical paper tube of 60 mm diameter is
(a) 3.2 μH (b) 3.2 mH
(c) 32.0 mH (d) 3.2 H
(GATE 2004)

4. A moving iron ammeter produced a full scale torque of 240 μNm with a deflection of 120^0 at a current of 10 A. The rate of change of self inductance (μH/radian) of the instrument at full scale is
(a) 2.0 μH/radian
(b) 4.8 μH/radian
(c) 12.0 μH/radian
(d) 114.6 μH/radian
(GATE 2004)

5. The self-inductance of a long cylindrical conductor due to its internal flux linkages is k H/m. If the diameter of the conductor is doubled, then the selfinductance of the conductor due its internal flux linkages would be
(*a*) 0.5 K H/m (*b*) K H/m
(*c*) 1.414 K H/m (*d*) 4 K H/m
(GATE)

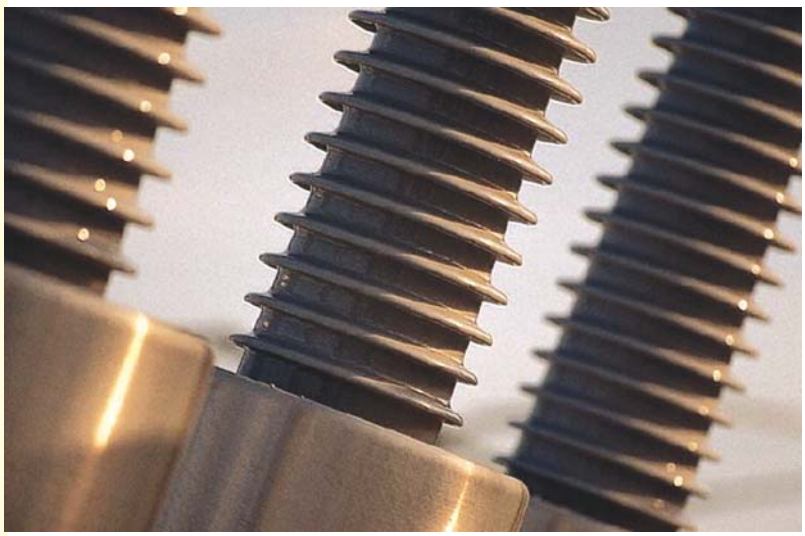
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CHAPTER 8

MAGNETIC HYSTERESIS

Learning Objectives

- Magnetic Hysteresis
- Area of Hysteresis Loop
- Properties and Application of Ferromagnetic Materials
- Permanent Magnet Materials
- Steinmetz Hysteresis Law
- Energy Stored in Magnetic Field
- Rate of Change of Stored Energy
- Energy Stored per Unit Volume
- Lifting Power of Magnet
- Rise of Current in Inductive Circuit
- Decay of Current in Inductive Circuit
- Details of Transient Current Rise in *R-L* Circuit
- Details of Transient Current Decay in *R-L* Circuit
- Automobile Ignition System



Magnetic hysteresis is one of the important considerations in choosing and designing the cores of transformers and other electric machines

8.1. Magnetic Hysteresis

It may be defined as the lagging of magnetisation or induction flux density (B) behind the magnetising force (H). Alternatively, it may be defined as that quality of a magnetic substance, due to which energy is dissipated in it, on the reversal of its magnetism.

Let us take an unmagnetised bar of iron AB and magnetise it by placing it within the field of a solenoid (Fig. 8.1). The field H (= NI/l) produced by the solenoid is called the magnetising force. The value of H can be increased or decreased by increasing or decreasing current through the coil. Let H be increased in steps from zero up to a certain maximum value and the corresponding values of flux density (B) be noted. If we plot the relation between H and B, a curve like OA, as shown in Fig. 8.2, is obtained. The material becomes magnetically saturated for $H = OM$ and has at that time a maximum flux density of B_{max} established through it.

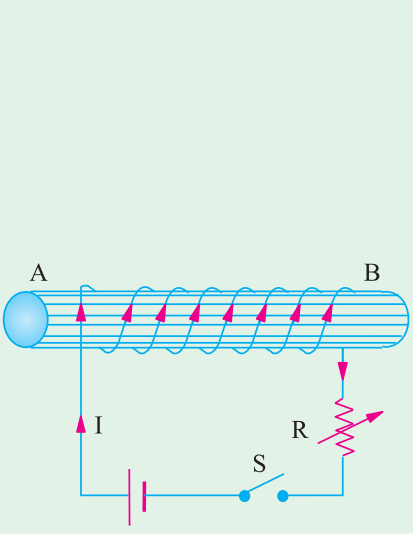


Fig. 8.1

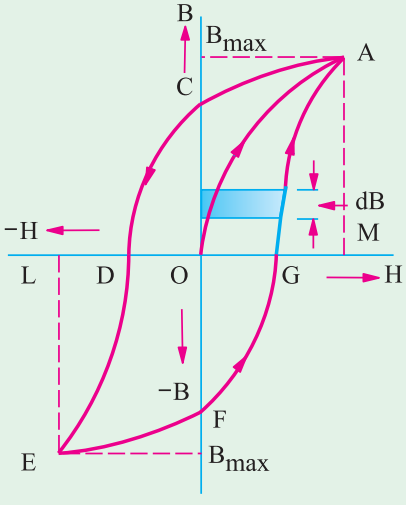


Fig. 8.2

If H is now decreased gradually (by decreasing solenoid current), flux density B will not decrease along AO, as might be expected, but will decrease less rapidly along AC. When H is zero, B is not but has a definite value $B_r = OC$. It means that on removing the magnetising force H, the iron bar is not completely demagnetised. This value of B (= OC) measures the ***retentivity or remanence*** of the material and is called the remanent or residual flux density B_r.

To demagnetise the iron bar, we have to apply the magnetising force in the reverse direction. When H is reversed (by reversing current through the solenoid), then B is reduced to zero at point D where $H = OD$. This value of H required to wipe off residual magnetism is known as ***coercive force*** (H_c) and is a measure of the ***coercivity*** of the material *i.e.* its 'tenacity' with which it holds on to its magnetism.

If, after the magnetisation has been reduced to zero, value of H is further increased in the 'negative' *i.e.* reversed direction, the iron bar again reaches a state of magnetic saturation, represented by point L. By taking H back from its value corresponding to negative saturation, (= OL) to its value for positive saturation (= OM), a similar curve $EFGA$ is obtained. If we again start from G, the same curve $GACDEFG$ is obtained once again.*

---

* In fact, when H is varied a number of times between fixed positive and negative maxima, the size of the loop becomes smaller and smaller till the material is cyclically magnetised. A material is said to be cyclically magnetised when for each increasing (or decreasing) value of H, B has the same value in successive cycles.

It is seen that B always lag behind H. The two never attain zero value simultaneously. This lagging of B behind H is given the name 'hystereis' which literally means ***'to lag behind'***. The closed loop $ACDEFGA$ which is obtained when iron bar is taken through one complete cycle of magnetisation is known as 'hypothesis loop'.

By one cycle of magnetisation of a magnetic material is meant its being carried through one reversal of magnetisation, as shown in Fig. 8.3.

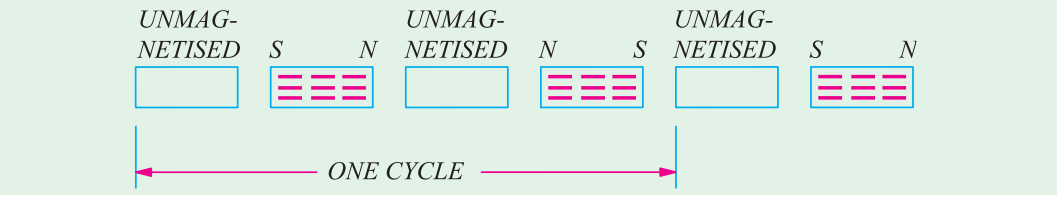


Fig. 8.3

8.2. Area of Hysteresis Loop

Just as the area of an indicator diagram measures the energy made available in a machine, when taken through one cycle of operation, so also the area of the hysteresis loop represents the net energy spent in taking the iron bar through one cycle of magnetisation.

According to ***Weber's Molecular Theory*** of magnetism, when a magnetic material is magnetised, its molecules are forced along a straight line. So, energy is spent in this process. Now, if iron has no retentivity, then energy spent in straightening the molecules could be recovered by reducing H to zero in the same way as the energy stored up in a spring can be recovered by allowing the spring to release its energy by driving some kind of load. Hence, in the case of magnetisation of a material of ***high retentivity***, all the energy put into it originally for straightening the molecules is not recovered when H is reduced to zero. We will now proceed to find this loss of energy per cycle of magnetisation.

Let l = mean length of the iron bar ; A = its area of cross-section; N = No. of turns of wire of the solenoid.

If B is the flux density at any instant, then $\Phi = BA$.

When current through the solenoid changes, then flux also changes and so produces an induced e.m.f. whose value is

$$e = N\frac{d\Phi}{dt}\text{ volt} = N\frac{d}{dt}(BA) = NA\frac{dB}{dt}\text{ volt} \qquad \text{(neglecting -ve sign)}$$

Now $$H = \frac{NI}{l} \quad \text{or} \quad I = \frac{Hl}{N}$$

The power or rate of expenditure of energy in maintaining the current 'I' against induced e.m.f. 'e' is

$$= e\,I \text{ watt} = \frac{Hl}{N} \times NA\frac{dB}{dt} = AlH\frac{dB}{dt} \text{ watt}$$

Energy spent in time 'dt' $= Al.H\frac{dB}{dt} \times dt = Al.H.dB$ joule

Total net work done for one cycle of magnetisation is $W = Al \oint H\,dB$ joule

where $\oint$ stands for integration over the whole cycle. Now, '$H\,dB$' represents the shaded area in Fig. 8.2. Hence, $\oint HdB$ = area of the loop *i.e.* the area between the B/H curve and the B-axis

$\therefore$ work done/cycle = $A_l \times$ (area of the loop) joule. Now Al = volume of the material

$\therefore$ net work done/cycle/m^3 = (loop area) joule, or W_h = (Area of B/H loop) joule m^3/cycle

Precaution

Scale of B and H should be taken into consideration while calculating the actual loop area.

For example, if the scales are, 1 cm = x AT/m –for H and 1 cm = y Wb/m^2 –for B

then $W_h = xy$ (area of B/H loop) joule/m^3/cycle

In the above expression, loop area has to be in cm^2.

As seen from above, hysteresis loop measures the energy dissipated due to hysteresis which appears in the form of heat and so raises the temperature of that portion of the magnetic circuit which is subjected to magnetic reversal. The shape of the hysteresis loop depends on the nature of the magnetic material (Fig. 8.4).

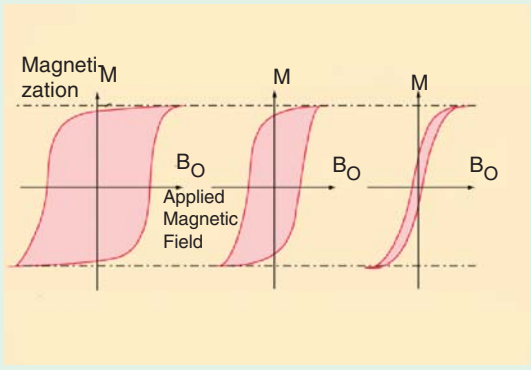


Fig. 8.4

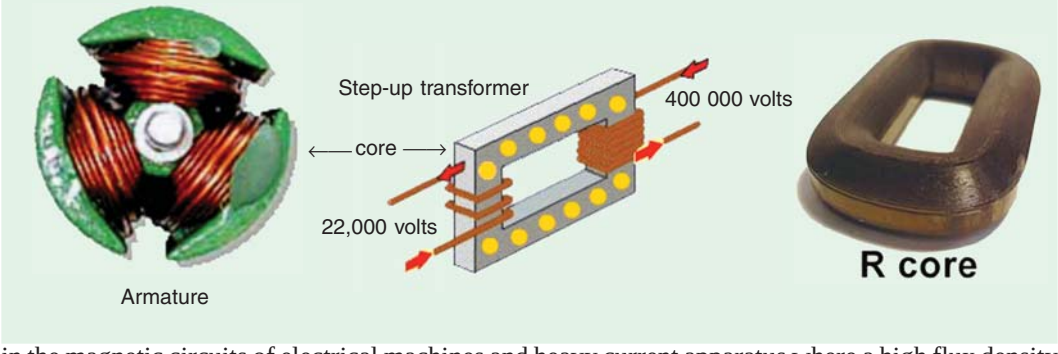
Loop 1 is for hard steel. Due to its high retentivity and collectivity, it is well suited for making permanent magnets. But due to large hysteresis loss (as shown by large loop area) it is not suitable for rapid reversals of magnetisation. Certain alloys of aluminium, nickel and steel called Alnico alloys have been found extremely suitable for making permanent magnets.

Loop 2 is for wrought iron and cast steel. It shows that these materials have high permeability and fairly good coercivity, hence making them suitable for cores of electromagnets.

Loop 3 is for alloyed sheet steel and it shows high permeability and low hysteresis loss. Hence, such materials are most suited for making armature and transformer cores which are subjected to rapid reversals of magnetisation.

8.3. Properties and Applications of Ferromagnetic Materials

Ferromagnetic materials having low retentivities are widely used in power and communication apparatus. Since silicon iron has high permeability and saturation flux density, it is extensively used



in the magnetic circuits of electrical machines and heavy current apparatus where a high flux density is desirable in order to limit the cross-sectional area and, therefore, the weight and cost. Thin silicon-iron laminations (clamped together but insulated from each other by varnish, paper or their own surface scale) are used in the construction of transformer and armature cores where it is essential to minimize hysteresis and eddy-current losses.

In field systems (where flux remains constant), a little residual magnetism is desirable. For such systems, high permeability and high saturation flux density are the only important requirements which are adequately met by fabricated rolled steel or cast or forged steel.

Frequencies used in line communication extend up to 10 MHz whereas those used in radio vary from about 100 kHz to 10 GHz. Hence, such material which have high permeability and low losses are very desirable. For these applications, nickel-iron alloys containing up to 80 per cent of nickel and a small percentage of molybdenum or copper, cold rolled and annealed are very suitable.

8.4. Permanent Magnet Materials

Permanent magnets find wide application in electrical measuring instruments, magnetos, mag-

netic chucks and moving-coil loudspeakers etc. In permanent magnets, high retentivity as well as high coercivity are most desirable in order to resist demagnetisation. In fact, the product $B_r H_c$ is the best criterion for the merit of a permanent magnet. The material commonly used for such purposes are carbon-free iron-nickel-aluminium copper-cobalt alloys which are made anisotropic by heating to a very high temperature and then cooling in a strong magnetic field. This alloy possesses $B_r H_c$ value of about 40,000 J/m^3 as compared with 2,500 J/m^3 for chromium-steel.

Example 8.1. *The hysteresis loop of a sample of sheet steel subjected to a maximum flux density of 1.3 Wb/m^2 has an area of 93 cm^2, the scales being 1 cm = 0.1 Wb/m^2 and 1 cm = 50 AT/m. Calculate the hysteresis loss in watts when 1500 cm^3 of the same material is subjected to an alternating flux density of 1.3 Wb/m^2 peak value of a frequency of 65 Hz.*

(Electromechanics, Allahabad Univ, 1992)

Solution.
Loss = xy (area of B/H loop) J/m^3/cycle
= $0.1 \times 50 \times 93 = 465$ J/m^3/cycle
Volume = 1500 cm^3 = 15×10^{-4} m^3 ; No. of reversals/second = 65
$\therefore$ W_h = $465 \times 15 \times 10^{-4} \times 65$ J/s = **45.3 W**

Note. The given value of B_{max} = 1.3 Wb/m^2 is not required for solution.

Example 8.2. *Calculate the hourly loss of energy in kWh in a specimen of iron, the hysteresis loop of which is equivalent in area to 250 J/m^3. Frequency 50 Hz ; specific gravity of iron 7.5 ; weight of specimen 10 kg.* **(Electrical Engg. Materials, Nagpur Univ. 1991)**

Solution.
Hysteresis loss = 250 J/m^3/cycle, Mass of iron = 10 kg
Volume of iron specimen = $10/7.5 \times 10^3$ m^3 = $10^{-2}/7.5$ m^3
No. of cycles of reversals/hr = $60 \times 50 = 3000$
$\therefore$ loss/hour = $250 \times (10^{-2}/7.5) \times 3000 = 1000$ J = $1000/36 \times 10^5$ = **27.8×10^{-5} kWh**

Example 8.3. *The hysteresis loop for a certain magnetic material is drawn to the following scales : 1 cm = 200 AT/m and 1 cm = 0.1 Wb/m^2. The area of the loop is 48 cm^2. Assuming the density of the material to be 7.8×10^3 kg/m^3, calculate the hysteresis loss in watt/kg at 50 Hz.*

(Elect. Circuits & Fields, Gujarat Univ.)

Solution.
Hysteresis loss = xy (area of B/H loop) J/m^3/cycle
Now, 1 cm = 200 AT/m ; 1 cm = 0.1 Wb/m^2
$\therefore$ x = 200, y = 0.1, area of loop = 48 cm^2
$\therefore$ loss = $200 \times 0.1 \times 48 = 960$ J/m^3/cycles, Density = 7.8×10^2 kg/m^2
Volume of 1 kg of material = mass/density = $1/7.8 \times 10^3$ m^3
$\therefore$ loss = $960 \times 1/7.8 \times 10^3$ J/cycle No. of reversals/second = 50
$\therefore$ loss = $960 \times 50 \times 10^{-3}/7.8 = 6.15$ J/s or watt
$\therefore$ hysteresis loss = **6.15 watt/kg.**

Example 8.4. *Determine the hysteresis loss in an iron core weighing 50 kg having a density of 7.8×10^3 kg/m^3 when the area of the hysteresis loop is 150 cm^2, frequency is 50 Hz and scales on X and Y axes are : 1 cm = 30 AT/cm and 1 cm = 0.2 Wb/m^2 respectively.*

(Elements of Elect. Engg-1, Bangalore Univ.)

Solution.
Hysteresis loss = xy (area of B/H loop) J/m^3/cycle
1 cm = 30 AT/cm = 3000 AT/m ; 1 cm = 0.2 Wb/m^2
x = 3000, y = 0.2, A = 150 cm^2
$\therefore$ loss = $3000 \times 0.2 \times 150 = 90{,}000$ J/m^3/cycle
Volume of 50 kg of iron = $m/\rho = 50/7.8 \times 10^{-3} = 6.4 \times 10^{-3}$ m^3
$\therefore$ loss = $90{,}000 \times 6.4 \times 10^{-3} \times 50 = 28{,}800$ J/s or watts = **28.8 kW**

Example 8.5. *In a transformer core of volume 0.16 m^3, the total iron loss was found to be 2,170 W at 50 Hz. The hysteresis loop of the core material, taken to the same maximum flux density, had an area of 9.0 cm^2 when drawn to scales of 1 cm = 0.1 Wb/m^2 and 1 cm = 250 AT/m. Calculate the total iron loss in the transformer core if it is energised to the same maximum flux density but at a frequency of 60 Hz.*

Solution. $W_h = xy \times$ (area of hysteresis loop) where x and y are the scale factors.
$W_h = 9 \times 0.1 \times 250 = 225$ J/m^3/cycle

At 50 Hz

Hysteresis loss = $225 \times 0.16 \times 50 = 1{,}800$ W ; Eddy-current loss = 2,170 –1800 = 370 W

At 60 Hz

Hysteresis loss = $1800 \times 60/50 = 2{,}160$ W ; Eddy-current loss = $370 \times (60/50)^2 = 533$ W

Total iron loss = 2,160 + 533 = **2,693 W**

Tutorial Problems No. 8.1

1. The area of a hysteresis loop of a material is 30 cm^2. The scales of the co-ordinates are : 1 cm = 0.4 Wb/m^2 and 1 cm = 400 AT/m. Determine the hysteresis power loss if 1.2×10^{-3} m^3 of the material is subjected to alternating flux density at 50 Hz. **[288 W] (*Elect. Engg., Aligarh Univ*)**
2. Calculate the loss of energy caused by hysteresis in one hour in 50 kg of iron when subjected to cyclic magnetic changes. The frequency is 25 Hz, the area of the hysteresis loop represents 240 joules/m^3 and the density of iron is 7800 kg/m^3. **[138,240] (*Principles of Elect. Engg. I, Jadvapur Univ.*)**
3. The hysteresis loop of a specimen weighing 12 kg is equivalent to 300 joules/m^3. Find the loss of energy per hour at 50 Hz. Density of iron is 7500 kg/m^3. **[86,400] (*Electrotechnics – I, Gawahati Univ.*)**
4. The area of the hysteresis loop for a steel specimen is 3.84 cm^2. If the ordinates are to the scales : 1 cm = 400 AT/m and 1 cm = 0.5 Wb/m^2, determine the power loss due to hysteresis in 1,200 cm^3 of the steel if it is magnetised from a supply having a frequency of 50 Hz. **[46.08 W]**
5. The armature of a 4-pole d.c. motor has a volume of 0.012 m^3. In a test on the steel iron used in the armature carried out to the same value of maximum flux density as exists in the armature, the area of the hysteresis loop obtained represented a loss of 200 J/m^3. Determine the hysteresis loss in watts when the armature rotates at a speed of 900 r.p.m. **[72 W]**
6. In a magnetisation test on a sample of iron, the following values were obtained.

H (AT/m)	1,900	2,000	3,000	4,000	4,500	3,000	1,000	0	–1,000	–1,900
B (Wb/m^2)	0	0.2	0.58	0.7	0.73	0.72	0.63	0.54	0.38	0

Draw the hysteresis loop and find the loss in watts if the volume of iron is 0.1 m^3 and frequency is 50 Hz. **[22 kW]**

8.5. Steinmetz Hysteresis Law

It was experimentally found by Steinmetz that hysteresis loss per m^3 per cycle of magnetisation of a magnetic meterial depends on **(*i*)** the maximum flux density established in it *i.e.* B_{max} and **(*ii*)** the magnetic quality of the material.

$\therefore$ Hysteresis loss $W_h \propto B_{max}^{1.6}$ joule/m^3/cycle $= \eta B_{max}^{1.6}$ joule/m^3 cycle

where η is a constant depending on the nature of the magnetic material and is known as **Steinmetz hysteresis coefficient.** The index 1.6 is empirical and holds good if the value of B_{max} lies between 0.1 and 1.2 Wb/m^2. If B_{max} is either lesser than 0.1 Wb/m^2 or greater than 1.2 Wbm^2, the index is greater than 1.6.

$\therefore$ $$W_h = \eta B_{max}^{1.6} fV \text{ J/s or watt}$$

where f is frequency of reversals of magnetisation and V is the volume of the magnetic material.

The armatures of electric motors and generators and transformer cores etc. which are subjected to rapid reversals of magnetisation should, obviously, be made of substances having low hysteresis coefficient in order to reduce the hysteresis loss.

Example 8.6. *A cylinder of iron of volume 8×10^{-3} m^3 revolves for 20 minutes at a speed of 3,000 r.p.m in a two-pole field of flux density 0.8 Wb.m^2. If the hysteresis coefficient of iron is 753.6 joule/m^3, specific heat of iron is 0.11, the loss due to eddy current is equal to that due to hysteresis and 25% of the heat produced is lost by radiation, find the temperature rise of iron. Take density of iron as 7.8×10^3 kg/m^3.* **(Elect. Engineering-I, Osmania Univ.)**

Solution. An armature revolving in a multipolar field undergoes one magnetic reversal after passing under a pair of poles. In other words, number of magnetic reversals in the same as the number of *pair* of poles. If P is the number of poles, the magnetic reversals in one revolution are $P/2$. If speed of armature rotation is N r.p.m, then number of revolutions/second = $N/60$.

$$\text{No. of reversals/second} = \text{reversals in one revolutions} \times \text{No. of revolutions/second}$$
$$= \frac{P}{2} \times \frac{N}{60} = \frac{PN}{120} \text{ reversals/second}$$

Here $N = 3{,}000$ r.p.m ; $P = 2$ $\therefore f = \dfrac{3{,}000 \times 2}{120} = 50$ reversals/second

According to Steinmetz's hysteresis law, $W_h = \eta B_{max}^{1.6} f V$ watt

Note that f here stands for magnetic reversals/second and not for mechanical frequency armature rotation.

$$W_h = 753.6 \times (0.8)^{1.6} \times 50 \times 8 \times 10^{-3} = 211 \text{ J/s}$$

Loss in 20 minutes = $211 \times 1{,}200 = 253.2 \times 10^3$ J

Eddy current loss = 253.2×10^3 J; Total loss = 506.4×10^3 J

Heat produced = $506.4 \times 10^3/4200 = 120.57$ kcal ; Heat utilized = $120.57 \times 0.75 = 90.43$ kcal

Heat absorbed by iron = $(8 \times 10^{-3} \times 7.8 \times 10^3) \times 0.11\, t$ kcal

$\therefore$ $(8 \times 10^{-3} \times 7.8 \times 10^3) \times 0.11 \times t = 90.43$ $\therefore$ $t =$ **13.17°C**

Example 8.7. *The area of the hysteresis loop obtained with a certain specimen of iron was 9.3 cm^2. The coordinates were such that 1 cm = 1,000 AT/m and 1 cm = 0.2 Wb/m^2. Calculate* ***(a)*** *the hysteresis loss per m^3 per cycle and* ***(b)*** *the hysteresis loss per m^3 at a frequency of 50 Hz if the maximum flux density were 1.5 Wb/m^2* ***(c)*** *calculate the hysteresis loss per m^3 for a maximum flux density of 1.2 Wb/m^2 and a frequency of 30 Hz, assuming the loss to be proportional to $B_{max}^{1.6}$.*

(Elect. Technology, Allahabad Univ. 1991)

Solution. ***(a)*** $W_h = xy \times$ (area of B/H loop)

$= 1{,}000 \times 0.2 \times 9.3 =$ **1860 J/m^2/cycle**

(b) $W_h = 1{,}860 \times 50$ J/s/m^3 = **93,000 W/m^3**

(c) $W_h = B_{max}^{1.8} f V$ W For a given specimen, $W_h \propto B_{max}^{1.8} f$

In (*b*) above, $93{,}000 \propto 1.5^{1.8} \times 50$ and $W_h \propto 1.2^{1.8} \times 30$

$$\therefore \quad \frac{W_h}{93{,}000} = \left(\frac{1.2}{1.5}\right)^{1.8} \times \frac{30}{50}; \; W_h = 93{,}000 \times 0.669 \times 0.6 = \mathbf{37.360}$$

Example 8.8. *Calculate the loss of energy caused by hysteresis in one hour in 50 kg of iron if the peak density reached is 1.3 Wb/m^2 and the frequency is 25 Hz. Assume Steinmetz coefficient as 628 J/m^3 and density of iron as 7.8 × 10^3 kg/m^3.*

What will be the area of B/H curve of this specimen if 1 cm = 12.4 AT/m and 1 cm = 0.1 Wb/m^2.

(Elect. Engg. ; Madras Univ.)

Solution. $W_h = \eta B_{max}^{1.6} f V$ watt ; volume $V = \dfrac{50}{7.8 \times 10^3} = 6.41 \times 10^{-3}$ m^3

$\therefore$ $W_h = 628 \times 1.3^{1.6} \times 25 \times 6.41 \times 10^{-3} = 152$ J/s

Loss in one hour = $153 \times 3{,}600 = 551{,}300$ J

As per Steinmetz law, hysteresis loss = $\eta B_{max}^{1.6}$ J/m^3/cycle

Also, hysteresis loss = xy (area of B/H loop)

Equating the two, we get

$$628 \times 1.3^{1.6} = 12.5 \times 0.1 \times \text{loop area}$$

$\therefore$ loop area = $628 \times 1.3^{1.6}/1.25 =$ **764.3 cm^2**

Tutorial Problems No. 8.2

1. In a certain transformer, the hysteresis loss is 300 W when the maximum flux density is 0.9 Wb/m^2 and the frequency 50 Hz. What would be the hysteresis loss if the maximum flux density were increased to 1.1 Wb/m^2 and the frequency reduced to 40 Hz. Assume the hysteresis loss over this range to be proportional to $B_{max}^{1.7}$ **[337 W]**
2. In a transformer, the hysteresis loss is 160 W when the value of B_{max} = 1.1 Wb/m^2 and when supply frequency is 60 Hz. What would be the loss when the value of B_{max} is reduced to 0.9 Wb/m^2 and the supply frequency is reduced to 50 Hz. **[97 W] (*Elect. Engg. II, Bangalore Univ.*)**

8.6. Energy Stored in a Magnetic Field

For establishing a magnetic field, energy must be spent, though no energy is required to ***maintain*** it. Take the example of the exciting coils of an electromagnet. The energy supplied to it is spent in two ways **(*i*)** part of it goes to meet I^2R loss and is lost once for all **(*ii*)** part of it goes to create flux and is stored in the magnetic field as potential energy and is similar to the potential energy of a raised weight. When a weight W is raised through a height of h, the potential energy stored in it is W_h. Work is done in ***raising*** this weight but once raised to a certain height, no further expenditure of energy is required to ***maintain*** it at that position. This mechanical potential energy can be recovered, so can be the electrical energy stored in the magnetic field.

When current through an inductive coil is gradually changed from zero to maximum value I, then every change of it is opposed by the self-induced e.m.f. produced due to this change. Energy is needed to overcome this opposition. This energy is stored in the magnetic field of the coil and is, later on, recovered when that field collapses. The value of this stored energy may be found in the following two ways :

(*i*) First Method. Let, at any instant,

i = instantaneous value of current ; e = induced e.m.f. at that instant = $L.di/dt$

Then, work done in time dt in overcoming this opposition is

$$dW = ei\,dt = L.\frac{di}{dt} \times i \times dt = Li\,di$$

Total work done in establishing the maximum steady current of I is

$$\int_0^W dW = \int_0^I L.i.di = LI^2 \quad \text{or} \quad W = \frac{1}{2} L\,I^2$$

This work is stored as the energy of the magnetic field $\therefore E = \frac{1}{2} L\,I^2$ joules

(*ii*) Second Method

If current grows uniformly from zero value to its maximum steady value I, then average current is $I/2$. If L is the inductance of the circuit, then self-idcued e.m.f. is $e = LI/t$ where 't' is the time for the current change from zero to I.

$\therefore$ Average power absorbed = induced e.m.f. × average current

$$= L\frac{1}{t} \times \frac{1}{2} I = \frac{1}{2} \frac{L\,I^2}{t}$$

Total energy absorbed = power × time = $\frac{1}{2} \frac{L\,I^2}{t} \times t = \frac{1}{2} L\,I^2$

$\therefore$ energy stored $E = \frac{1}{2} L\,I^2$ joule

It may be noted that in the case of series-aiding coils, energy stored is

$$E = \frac{1}{2}(L_1 + L_2 + 2M)\,I^2 = \frac{1}{2} L_1 I^2 + \frac{1}{2} L_2 I^2 + M\,I^2$$

Similarly, for series-opposing coils, $E = \frac{1}{2} L_1 I^2 + \frac{1}{2} L_2 I^2 - M\,I^2$

Example 8.9. *Reluctance of a magnetic circuit is known to be 10^5 AT/Wb and excitation coil has 200 turns. Current in the coil is changing uniformly at 200 A/s. Calculate* ***(a)*** *inductance of the coil* ***(b)*** *voltage induced across the coil and* ***(c)*** *energy stored in the coil when instantaneous current at t = 1 second is 1 A. Neglect resistance of the coil.* **(Elect. Technology, Univ. of Indore, 1987)**

Solution. ***(a)*** $L = N^2/S = 200^2/10^5 =$ **0.4 H**

(b) $e_L = L\, dI/dt = 0.4 \times 200 =$ **80 V**

(c) $E = \frac{1}{2} L I^2 = 0.5 \times 0.4 \times I^2 =$ **0.2 J**

Example 8.10. *An iron ring of 20 cm mean diameter having a cross-section of 100 cm^2 is wound with 400 turns of wire. Calculate the exciting current required to establish a flux density of 1 Wb/m^2 if the relative permeability of iron is 1000. What is the value of energy stored ?*

(Elect. Engg-I, Nagpur Univ. 1992)

Solution. $B = \mu_0 \mu_r NI/l$ Wb/m^2

$\therefore$ $1 = 4\pi \times 10^{-7} \times 1000 \times 400\, I/0.2\pi$ or $I =$ **1.25 A**

Now, $L = \mu_0 \mu_r AN^2/l = 4\pi \times 10^{-7} \times 10^3 \times (100 \times 10^{-4}) \times (400)^2/0.2\pi = 32.$H

$E = \frac{1}{2} LI^2 = \frac{1}{2} \times 3.2 \times 1.25^2 =$ **2.5 J**

8.7. Rate of Change of Stored Energy

As seen from Art. 8.6, $E = \frac{1}{2} L I^2$. The rate of change of energy can be found by differentiating the above equation

$$\frac{dE}{dt} = \frac{1}{2}\left[L.2.I.\frac{dI}{dt} + I^2 \frac{dL}{dt}\right] = LI.\frac{dI}{dt} = \frac{1}{2} I^2 \frac{dL}{dt}$$

Example 8.11. *A relay (Fig. 8.5) has a coil of 1000 turns and an air-gap of area 10 cm^2 and length 1.0 mm. Calculate the rate of change of stored energy in the air-gap of the relay when*

(i) *armature is stationary at 1.0 mm from the core and current is 10 mA but is increasing at the rate of 25 A/s.*

(ii) *current is constant at 20 mA but inductance is changing at the rate of 100 H/s.*

Solution. $L = \frac{\mu_0 N^2 A}{l_g}$

$$= \frac{4\pi \times 10^{-7} \times (10^3)^2 \times 10 \times 10^{-4}}{1 \times 10^{-3}} = 1.26 \text{ H}$$

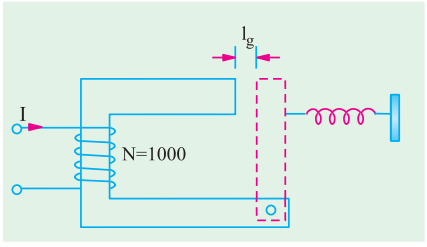


Fig. 8.5

(i) Here, $dI/dt = 25$ A/s, $dL/dt = 0$ because armature is stationary.

$\therefore$ $\frac{dE}{dt} = L\, I \frac{dI}{dt} = 1.26 \times 10 \times 10^{-3} \times 15 =$ **0.315 W**

(ii) Here, $dL/dt = 100$ H/s; $dI/dt = 0$ because current is constant.

$\therefore$ $\frac{dE}{dt} = \frac{1}{2} I^2 \frac{dL}{dt} = \frac{1}{2} (20 \times 10^{-3})^2 \times 100 =$ **0.02 W**

8.8. Energy Stored Per Unit Volume

It has already been shown that the energy stored in a magnetic field of length l metre and of cross-section A m^2 is $E = \frac{1}{2} L I^2$ joule or $E = \frac{1}{2} \times \frac{\mu_0 \mu_r AN^2}{l} . I^2$ joule

Now $H = \frac{NI}{l}$ $\therefore$ $E = \left(\frac{NI}{l}\right)^2 \times \frac{1}{2}\mu_0\mu_r\, Al = \frac{1}{2}\mu_0\,\mu_r\, H^2 \times Al$ joule

Now, Al = volume of the magnetic field in m^3

$\therefore$ energy stored/m^3 = $\frac{1}{2}\mu_0\mu_r H^2 = \frac{1}{2}BH$ joule ($\because$ $\mu_0\,\mu_r\, H = B$)

$= \frac{B^2}{2\,\mu_0\,\mu_r}$ joule –in a medium

or $= \frac{B^2}{2\,\mu_0}$ joule –in air

8.9. Lifting Power of a Magnet

In Fig. 8.6 let, P = pulling force in newtons between two poles and A = pole area in m^2

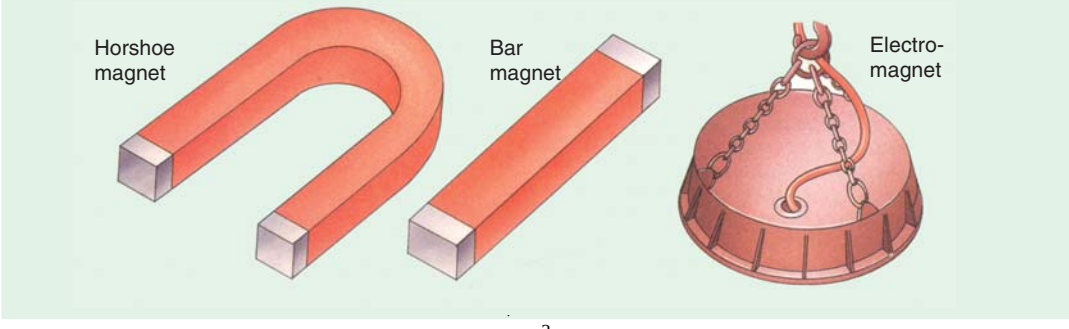
If one of the poles (say, upper one) is pulled apart against this attractive force through a distance of dx metres, then work done = $P \times dx$ joule ...(*i*)

This work goes to provide energy for the additional volume of the magnetic field so created.

Additional volume of the magnetic field created is

$= A \times dx\ m^3$

Rate of energy requirement is = $\frac{B^2}{2\,\mu_0}$ joule/m^3



$\therefore$ energy required for the new volume = $\frac{B^2}{2\,\mu_0} \times A\,dx$ ...(*ii*)

Equating (*i*) and (*ii*), we get,

$P.dx = \frac{B^2 \times A.dx}{2\,\mu_0}$

$\therefore$ $P = \frac{B^2 A}{2\,\mu_0}\ \text{N} = 4,00,000\,\frac{B^2}{A}\ \text{N}$

or $P = \frac{B^2 A}{2\,\mu_0}\ \text{N/m}^2 = 4,00,000\ B^2\ \text{N/m}^2$

Also $P = \frac{B^2 A}{9.81 \times 2\,\mu_0} = \frac{B^2\ A}{19.62\,\mu_0}$ kg-wt

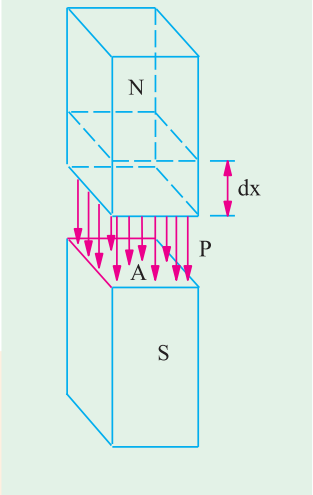


Fig. 8.6

Example 8.12. *A horse-shoe magnet is formed out of a bar of wrought iron 45.7 cm long, having a corss-section of 6.45 cm^2. Exciting coils of 500 turns are placed on each limb and connected in series. Find the exciting current necessary for the magnet to lift a load of 68 kg assuming that the load has negligible reluctance and makes close contact with the magnet. Relative permeability of iron = 700.*

(Elect. Engg. A.M.Ae. S.I., June, 1992)

Solution. Horse-shoe magnet is shown in Fig. 8.7.

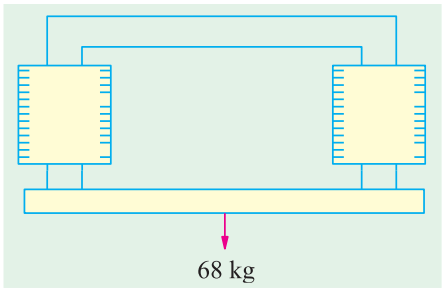


Fig. 8.7

Force of attraction of each pole = 68/2 = 34 kg = 34 × 9.81 = 333.5 N

$A = 6.45 \text{ cm}^2 = 6.45 \times 10^{-4} \text{ m}^3$

Since $F = \frac{B^2 A}{2\mu_0}$ N

$\therefore \quad 333.5 = \frac{B^2 \quad 6.45 \quad 10^{\ 4}}{2 \quad 4 \quad 10^{\ 7}}$

$B \quad \sqrt{1.3} \quad 1.14 \text{ Wb/m}^2$

and $H = B/\mu_0 \mu_r = 1.14/4\pi \times 10^{-7} \times 700 = 1296$ AT/m

Length of the plate = 45.7 cm = 0.457 m

$\therefore$ AT required = 1296 × 0.457 = 592.6

No. of turns = 500 × 2 = 1000 $\therefore$ current required = 592.6/1000 = **0.593 A**

Example 8.13. *The pole face area of an electromagnet is 0.5 m^2/pole. It has to lift an iron ingot weighing 1000 kg. If the pole faces are parallel to the surface of the ingot at a distance of 1 millimetre, determine the coil m.m.f. required. Assume permeability of iron to be infinity at the permeability of free space is $4\pi \times 10^{-7}$ H/m.* **(Elect. Technology, Univ. of Indore)**

Solution. Since iron has a permeability of infinity, it offers zero reluctance to the magnetic flux.

Force at two poles = $2 \times B^2A/2\mu_0 = B^2A/\mu_0$

$\therefore \quad B^2 \times 0.5/4\pi \times 10^{-7} = 1000 \times 9.8$, $B = 0.157$ Wb/m^2

$\therefore \quad H = 0.157/4\pi \times 10^{-7} = 125 \times 10^3$ AT/m, $l = 2 \times 1 = 2$ mm $= 2 \times 10^{-3}$ m

$\therefore$ AT required = $125 \times 10^3 \times 2 \times 10^{-3}$ = **250.**

Example 8.14. *A soft iron ring having a mean circumference of 40 cm and cross-sectional area of 3 cm^2 has two radial saw cuts made at diametrically opposite points. A brass plate 0.5 mm thick is inserted in each gap. The ring is wound with 800 turns. Calculate the magnetic leakage and fringing. Assume the following data for soft iron :*

B (Wb/m^2) :	*0.76*	*1.13*	*1.31*	*1.41*	*1.5*
H (AT/m) :	*50*	*100*	*150*	*200*	*250*

(Elect. Engineering-I, Delhi Univ.)

Solution. It should be noted that brass is a non-magnetic material.

Force at one separation = $B^2A/2\mu_0$ newton.

Force at both separations = B^2A/μ_0 newton.

Now F = 12 kg wt = 12 × 9.81 = 117.7 N

$\therefore \quad 117.7 = B^2 \times 3 \times 10^{-4}/4\pi \times 10^{-7}$; $B = 0.7$ Wb/m^2

If B/H curve is drawn, it will be found that for $B = 0.7$ Wb/m^2, value of $H = 45$ AT/m.

Now, length of iron path = 40 cm = 0.4 m. AT required for iron path = 45 × 0.4 = 18

Value of H in the non-magnetic brass plates = $B/\mu_0 = 0.7/4\pi \times 10^{-7}$ = 557,042 AT/m

Total thickness of brass plates = 0.5 × 2 = 1 mm

AT required = $557{,}042 \times 1 \times 10^{-3}$ = 557, Total AT needed = 18 + 557 = 575

$\therefore$ magnetising current required = 557/800 = **0.72 A**

Example 8.15. *The arm of a d.c. shunt motor starter is held in the 'ON' position by an electro-magnet having a pole face area of 4 cm^2 and air gap of 0.6 mm. The torque exerted by the spring is 12 N-m and effective radius at which the force is exerted is 15 cm. What is the minimum number of AT required to keep the arm in the 'ON' position ?*

Solution. The arm is shown in Fig. 8.8

Let F be the force in newtons exerted by the two poles of the electromagnet.

Torque = Force × radius

$\therefore$ 12 = $F \times 0.15$; $F = 80$ N

Force per pole = 80/2 = 40 N

Now $F = \dfrac{B^2 A}{2\mu_0} N$ $\therefore$ $40 = \dfrac{B^2 \times 4 \times 10^{-4}}{2 \times 4\pi \times 10^{-7}}$

$\therefore$ B = 0.5 Wb/m^2 $\therefore H = 0.5/\pi \times 10^{-7}$ AT/m

Total air-gap = $2 \times 0.6 \times 10^{-3} = 1.2 \times 10^{-3}$ m

$\therefore$ AT reqd. = $Hl = \dfrac{0.5 \times 1.2 \times 10^{-3}}{4\pi \times 10^{-7}} = 477$

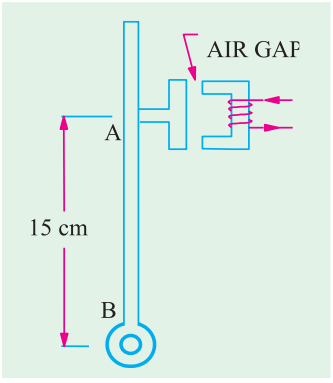


Fig. 8.8

Example 8.16. *The following particulars are taken from the magnetic circuit of a relay; Mean length of iron circuit = 20 cm; length of air gap = 2 mm, number of turns on core = 8000, current through coil = 50 mA, relative permeability of iron = 500. Neglecting leakage, what is the flux density in the air-gap ? If the area of the core is 0.5 cm^2, what is the pull exerted on the armature ?*

Solution. Flux $\Phi = \dfrac{NI}{\Sigma l / \mu_0 \mu_r A}$.

Now, m.m.f = $NI = 8000 \times 50 \times 10^{-3} = 400$ AT

Total circuit reluctance = $\dfrac{l}{\Sigma \mu_0 \mu_r A}$ AT/Wb or H^{-1}

$= \dfrac{0.2}{500 \times 4\pi \times 10^{-7} \times 0.5\ 10^{-4}} + \dfrac{2 \times 10^{-3}}{4\pi \times 10^{-7} \times 0.5 \times 10^{-4}}$

$\Phi = \dfrac{400}{12 \times 10^7 / \phi}$ Wb ; Flux density $B = \dfrac{\Phi}{A} = \dfrac{400\pi}{12 \times 10^7 \times 0.5 \times 10^{-4}} = \mathbf{0.21\ Wb/m^2}$

The pull on the armature = $\dfrac{B^2 A}{2\mu_0} N = \dfrac{0.21^2 \times 0.5 \times 10^{-4}}{2 \times 4\pi \times 10^{-7}} = 0.87$ N

Tutorial Problems No. 8.2

1. An air-cored solenoid has a length of 50 cm and a diameter of 2 cm. Calculate its inductance if it has 1,000 turns and also find the energy stored in it if the current rises from zero to 5 A.
[0.7 mH; 8.7 mJ] (*Elect. Engg. and Electronic Bangalore Univ. 1998*)
2. An air-cored solenoid 1 m in length and 10 cm in diameter has 5000 turns. Calculate (*i*) the self inductance (*ii*) the energy stored in the magnetic field when a current of 2 A flows in the solenoid.
[(*i*) 0.2468 H (*ii*) 0.4936 J] (*F.E. Pune Univ.*)
3. Determine the force required to separate two magnetic surfaces with contact area of 100 cm^2 if the magnetic flux density across the surface is 0.1 Wb/m^2. Derive formula used, if any.
[39.8 N] (*Elect. Engg. A.M.Ae.S.I.*)
4. In a telephone receiver, the size of each of the two poles is 1.2 cm × 0.2 cm and the flux between each pole and the diaphragm is 3×10^{-6} Wb; with what force is the diaphragm attracted to the poles ?
[0.125 N] (*Elect. Engg. A.M.Ae.S.I. June 1991*)
5. A lifting magnet is required to raise a load of 1,000 kg with a factor of safety of 1.5. If the flux density across the pole faces is 0.8 Wb/m^2, calculate the area of each pole. **[577 cm^2]**
6. Magnetic material having a surface of 100 cm^2 are in contact with each other. They are in a magnetic circuit of flux. 0.01 Wb uniformly distributed across the surface. Calculate the force required to detach the two surfaces. **[3,978 N] (*Elect. Engg. Kerala Univ.*)**

7. A steel ring having a mean diameter of 35 cm and a cross-sectional area of 2.4 cm^2 is broken by a parallel-sided air-gap of length 1.2 cm. Short pole pieces of negligible reluctance extend the effective cross-sectional area of the air-gap to 12 cm^2. Taking the relative permeability of steel as 700 and neglecting leakage, determine (*a*) the current necessary in 300 turns of wire wound on the ring to produce a flux density in the air-gap of 0.25 Wb/m^2 (*b*) the tractive force between the poles.

[(*a*) 13.16 A (*b*) 29.9 N]

8. A cast iron ring having a mean circumference of 40 cm and a cross-sectional area of 3 cm^2 has two radial saw-cuts at diametrically opposite points. A brass plate is inserted in each gap (thickness 0.5 mm). If the ring is wound with 800 turns, calculate the magnetising current to exert a total pull of 3 kg between the two halves. Neglect any magnetic leakage and fringing and assume the magnetic data for the cast iron to be :

B (Wb/m^2) :	0.2	0.3	0.4	0.5	
H (AT) :	850	1150	1500	2000	**[1.04 A]**

9. A magnetic circuit in the form of an inverted U has an air-gap between each pole and the armature of 0.05 cm. The cross-section of the magnetic circuit is 5 cm^2. Neglecting magnetic leakage and fringing, calculate the necessary exciting ampere-turns in order that the armature may exert a pull of 15 kg. The ampere-turns for the iron portion of the magnetic circuit may be taken as 20 percent of those required for the double air-gap.

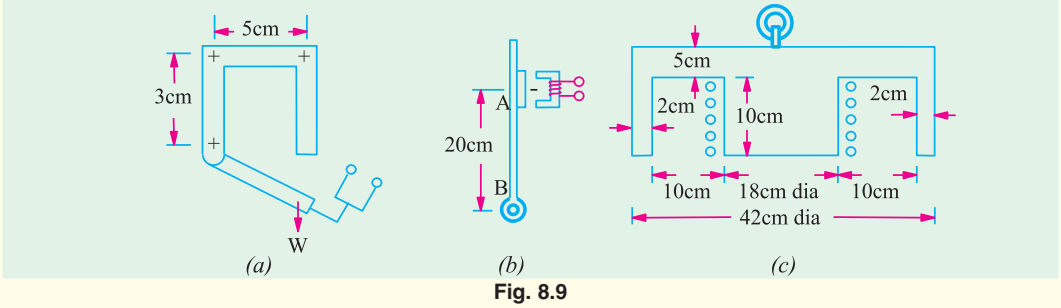


Fig. 8.9

10. In Fig. 8.9 (*a*) is shown the overload trip for a shunt motor starter. The force required to lift the armature is equivalent to a weight of W = 0.8165 kg positioned as shown. The air-gaps in the magnetic circuit are equivalent to a single gap of 0.5 cm. The cross-sectional area of the circuit is 1.5 cm^2 throughout and the magnetisation curve is as follows :

H (AT/m) :	1000	2000	3000	4000
B (Wb/m^2) :	0.3	0.5	0.62	0.68

Calculate the number of turns required if the trip is to operate when 80 A passes through the coil.

[21 turns]

11. The armature of a d.c. motor starter is held in the 'ON' position by means of an electromagnet [Fig 8.9. (*b*)]. A spiral spring exerts a mean counter torque of 8 N-m on the armature in this position after making allowance for the weight of the starter arm. The length between the centre of the armature and the pivot on the starter arms is 20 cm and cross sectional area of each pole face of the electromagnet 3.5 cm^2.

Find the minimum number of AT required on the electromagnet to keep the arm in the 'ON' position when the air-gap between the armature and the electromagnet is 0.5 mm. (Neglect the AT needed for the iron of the electromagnet). **[301 AT]**

12. A cylindrical lifting magnet of the form shown in Fig. 8.9 (*c*) has a winding of 200 turns which carries a current of 5 A. Calculate the maximum lifting force which could be exerted by the magnet on a flat iron sheet 5 cm thick. Why would this value not be realized in practice ? The relative permeability of the iron can be taken as 500. **[698 N]**

8.10. Rise of current in an Inductive Circuit

In Fig. 8.10 is shown a resistance of R in series with a coil of self-inductance L henry, the two being put across a battery of V volt. The R-L combination becomes connected to battery when switch

S is connected to terminal '*a*' and is short-circuited when *S* is connected to '*b*'. The inductive coil is assumed to be resistanceless, its actual small resistance being included in *R*.

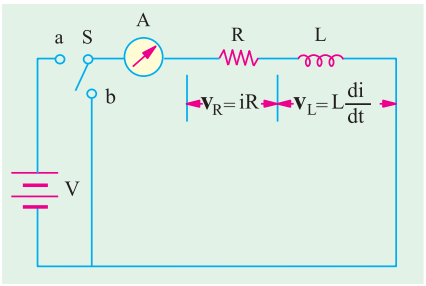


Fig. 8.10

When *S* is connected to '*a*' the *R-L* combination is suddenly put across the voltage of *V* volt. Let us take the instant of closing *S* as the starting zero time. It is easily explained by recalling that the coil possesses electrical inertia *i.e.* self-inductance and hence, due to the production of the counter e.m.f. of self-inductance, delays the instantaneous full establishment of current through it.

We will now investigate the growth of current *i* through such an inductive circuit.

The applied voltage *V* must, at any instant, supply not only the ohmic drop *iR* over the resistance *R* but must also overcome the e.m.f. of self inductance *i.e.* L*di*/*dt*.

$$\therefore \quad V = v_R + v_L = iR + \frac{di}{dt}$$

$$\text{or} \quad (V - iR) = L\frac{di}{dt} \quad \therefore \quad \frac{di}{V - iR} = \frac{dt}{L}. \qquad \text{...(i)}$$

Multiplying both sides by (–*R*), we get $(-R)\frac{di}{(V - iR)} = -\frac{R}{L}dt$

Integrating both sides, we get $\int \frac{(-R)\,di}{(V - iR)} = \int dt \quad \therefore \quad \log_e^{V - iR} = -\frac{R}{L}t + K$ ...(ii)

where *e* is the Napierian logarithmic base = 2.718 and *K* is constant of integration whose value can be found from the initial known conditions.

To begin with, when $t = 0$, $i = 0$, hence putting these values in **(ii)** above, we get

$$\log_e^V = K$$

Substituting this value of *K* in the above given equation, we have

$$\log_e^{V - iR} = \frac{R}{L}t + \log_e^V \quad \text{or} \quad \log_e^{V - iR} - \log_e^V = -\frac{R}{L}t$$

$$\text{or} \quad \log_e \frac{V - iR}{V} = -\frac{R}{L}t = -\frac{1}{\lambda} \quad \text{where } L/R = \lambda \text{ 'time constant'}$$

$$\therefore \quad \frac{V - iR}{V} = e^{-t/\lambda} \quad \text{or} \quad i = \frac{V}{R}(1 - e^{-t/\lambda})$$

Now, *V*/*R* represents the maximum steady value of current I_m that would eventually be established through the *R-L* circuit.

$$\therefore \quad i = I_m (1 - e^{-t/\lambda}) \qquad \text{...(iii)}$$

This is an exponential equation whose graph is shown in Fig. 8.11. It is seen from it that current rise is rapid at first and then decreases until at $t = \infty$, it becomes zero. Theoretically, current does not reach its maximum steady value I_m until infinite time. However, in practice, it reaches this value in a relatively short time of about 5λ.

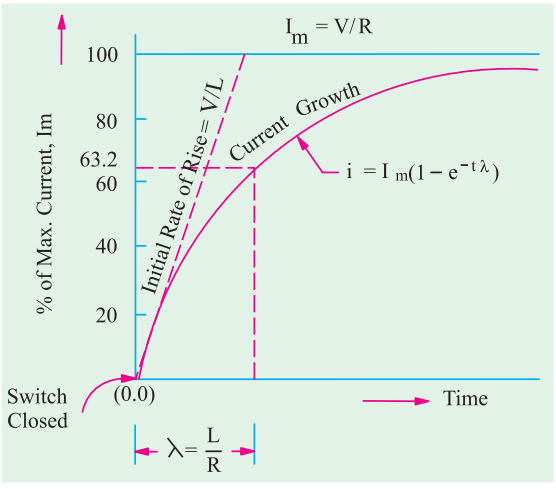


Fig. 8.11

The rate of rise of current *di*/*dt* at any stage can be found by differentiating Eq. (*ii*) above w.r.t. time. However, the ***initial*** rate of rise of

current can be obtained by putting $t = 0$ and $i = 0$ in (i)* above.

$$\therefore \quad V = 0 \times R + L\frac{di}{dt} \quad \text{or} \quad \frac{di}{dt} = \frac{V}{L}$$

The constant $\lambda = L/R$ is known as the ***time-constant*** of the circuit. It can be variously defined as :

(i) It is the ***time*** during which current would have reached its maximum value of I_m ($= V/R$) had it maintained its initial rate of rise.

$$\text{Time taken} = \frac{I_m}{\text{initial rate of rise}} = \frac{V/R}{V/L} = \frac{L}{R}$$

But actually the current takes more time because its rate of rise decreases gradually. In actual practice, in a time equal to the time constant, it merely reaches 0.632 of its maximum values as shown below :

Putting $t = L/R = \lambda$ in Eq. (iii) above, we get

$$i = I_m(1 - e^{-\lambda/\lambda}) = I_m\left(1 - \frac{1}{e}\right) = I_m\left(1 - \frac{1}{2.718}\right) = 0.632\, I_m$$

(ii) Hence, the time-constant λ of an R-L circuit may also be defined as the time during which the current ***actually*** rises to 0.632 of its maximum steady value (Fig. 8.11).

This delayed rise of current in an inductive circuit is utilized in providing time lag in the operation of electric relays and trip coils etc.

8.11. Decay of Current in an Inductive Circuit

When the switch S (Fig. 8.10) is connected to point 'b', the R-L circuit is short-circuited. It is found that the current does not cease immediately, as it would do in a non-inductive circuit, but continues to flow and is reduced to zero only after an appreciable time has elapsed since the instant of short-circuit.

The equation for decay of current with time is found by putting $V = 0$ in Eq. (i) of Art. 8.10

$$0 = iR + L\frac{di}{dt} \quad \text{or} \quad \frac{di}{i} = -\frac{R}{L}dt$$

Integrating both sides, we have

$$\int \frac{di}{i} = -\frac{R}{L}\int dt \quad \therefore \quad \log i = \frac{R}{L}t + K \qquad \text{...(i)}$$

Now, at the instant of switching off current, $i = I_m$ and if time is counted from this instant, then $t = 0$

$$\therefore \qquad \log_e I_m = 0 + K$$

Putting the value of K in Eq (i) above, we get,

$$\log_e i = -\frac{t}{\lambda} = \log_e I_m$$

$$\therefore \quad \log_e i/I_m = -\frac{t}{\lambda}$$

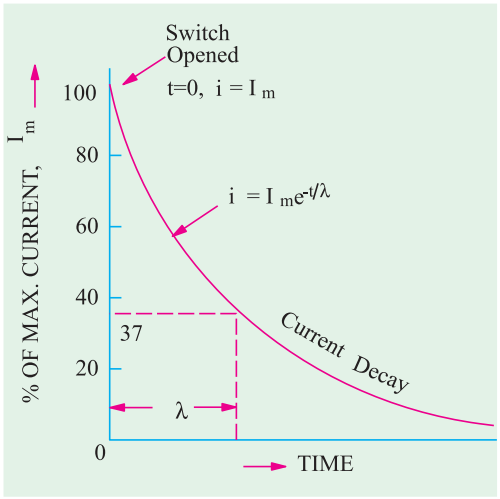


Fig. 8.12

* Initial value of di/dt can also be found by differentiating Eq. (iii) and putting $t = 0$ in it. In fact, the three qunatities V, L, R give the following various combinations :
$V/R = I_m$ – the maximum final steady current.
V/L = initial rate of rise of current.
L/R = time constant of the circuit.
The first rule of switching is that the current flowing through an inductance cannot change instantaneously.
The second rule of switching is that the voltage across a capacitor cannot change instantaneously.

$$\therefore \quad \frac{i}{I_m} = e^{-t/\lambda}$$

or $$i = I_m e^{-t/\lambda} \qquad \text{...(ii)}$$

It is decaying exponential function and is plotted in Fig. 8.12. It can be shown again that theoretically, current should take infinite time to reach zero value although, in actual practice, it does so in a relatively short time of about 5λ.

Again, putting $t = \lambda$ in Eq. (*ii*) above, we get

$$i = \frac{I_m}{e} = \frac{I_m}{2.178} = 0.37\ I_m.$$

Hence, time constant (λ) of an *R-L* circuit may also be defined as the time during which current falls to 0.37 or 37% of its maximum steady value while decaying (Fig. 8.12).

Example 8.17. *A coil having an effective resistance of 20 Ω and an inductance of 5 H. is suddenly connected across a 50-V dc supply. What is the rate at which energy is stored in the field of the coil when current is* **(a)** *0.5 A* **(b)** *1.0 A and* **(c)** *steady ? Also find the induced EMF in the coil under the above conditions.*

Solution. **(a)** Power input = 50 × 0.5 = 25 W

Power wasted as heat = $i^2R = 0.5^2 \times 25 = 6.25$ W. Hence, rate of energy storage in the coil field is 25 –6.25 = 18.75 W or J/s. **(b)** Power input = 50 × 1 = 50 W

Power lost as heat = $1^2 \times 25 = 25$ W. ∴Rate of energy storage in field = 50 –25 = 25 W or J/s.

(c) Steady value of current = 50/25 = 2 A. Power input = 50 × 2 = 100 W

Power lost as heat = $2^2 \times 25 = 100$ W

Rate of energy storage in field = 100 –100 = 0; Now, $V = iR + e_L = V - iR$

(a) $e_L = 50 - 0.5 \times 25 = 37.5$ V **(b)** $e_L = 50 - 1 \times 25 = 25$ V **(c)** $e_L = 50 - 2 \times 25 = 0$ V.

Example 8.18. *A coil having a resistance of 10 Ω and an inductance of 4 H is switched across a 20-W dc source. Calculate* **(a)** *time required by the current to reach 50% of its final steady value and* **(b)** *value of the current after 0.5 second.*

Solution. The rise of current through an inductive circuit is given by the equation $i = I\,(1 - e^{-t/\lambda})$. It may be written as

$$e^{-t/\lambda} = \frac{I - i}{I} \quad \text{or} \quad \frac{I}{e^{t/h}} = \frac{I - i}{I} \quad \text{or} \quad e^{e/h} = \frac{I}{I - i}$$

Taking logs of both sides, we have

$$\frac{t}{\lambda} . \log^e = \log^{I/(I-i)} = I_n \frac{I}{(I - i)}$$

$$\therefore \quad \frac{Rt}{L} = R_n \frac{I}{(I - i)} \quad \text{or} \quad t = \frac{L}{R} l_n \frac{I}{I - i}$$

(a) Now, $I = V/R = 20/10 = 2$ A

$$\therefore \quad t = \frac{4}{10} l_n \frac{2}{(2 - 1)} = \frac{4}{10} \times 0.693 = 0.2777 \text{ s.}$$

(b) $\lambda = L/R = 4/10 = 0.4$ s and $t = 0.5$ s

$$\therefore \quad i = 2\ (1 - e^{-t/0.4})$$

Example 8.19. *With reference to the circuit shown in Fig. 8.13, calculate :*
(i) *the current taken from the d.c. supply at the instant of closing the switch*
(ii) *the rate of increase of current in the coil at the instant of switch*
(iii) *the supply and coil currents after the switch has been closed for a long time*
(iv) *the maximum energy stored in the coil*
(v) *the e.m.f. induced in the coil when the switch is opened.*

Solution. **(i)** When switch *S* is closed (Fig. 8.13), the supply d.c. voltage of 120 V is applied

across both arms. The current in R_2 will immediately become 120/30 = 4 A. However, due to high inductance of the second arm, there would be no instantaneous flow of current in it. Hence current taken from the supply at the instant of switching on will be **4 A.**

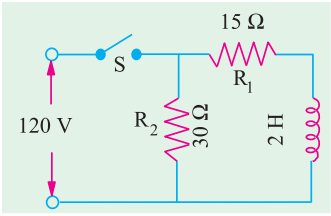


Fig. 8.13

(ii) Since at the ***instant*** of switching on, there is no current through the inductor arm, no potential drop will develop across R_1. The whole of the supply voltage will be applied acorss the inductor. If di/dt is the rate of increase of current through the inductor at the instant of switching on, the back e.m.f. produced in it is $L.\,di/dt$. This e.m.f. is equal and opposite to the applied voltage.

$120 = L\,di/dt$ or $di/dt = 120/2 =$ **60 A/s**

(iii) When switch has been closed for a sufficiently long time, current through the inductor arm reaches a steady value = $120/R_1 = 120/15 =$ **8 A**

Current through $R_2 = 120/30 = 4$ A ; Supply current = 8 + 4 = **12 A**

(iv) Maximum energy stored in the inductor arm

$$= \frac{1}{2} LI^2 = \frac{1}{2} \times 2 \times 8^2 = \textbf{64 J}$$

(v) When switch is opened, current through the inductor arm cannot change immediately because of high self-inductance of the inductor. Hence, inductance current remains at 8 A. But the current through R_2 can change immediately. After the switch is opened, the inductor current path lies through R_1 and R_2. Hence, e.m.f. inducted in the inductor at the instant of switching off is = 8 × (30 + 15) = **360 V.**

Example 8.20. *A coil has a time constant of 1 second and an inductance of 8 H. If the coil is connected to a 100 V d.c. source, determine :*

(i) *the rate of rise of current at the instant of switching* ***(ii)*** *the steady value of the current and* ***(iii)*** *the time taken by the current to reach 60% of the steady value of the current.*

(Electrotechnics-I, M.S. Univ. Baroda)

Solution. $\lambda = L/R$; $R = L/\lambda = 8/1 = 8$ ohm

(i) Initial $di/dt = V/L = 100/8 =$ **12.5 A/s** ***(ii)*** $I_M = V/R = 100/8 =$ **12.5 A**

(iii) Here, i = 60% of 12.5 = 7.5 A

Now, $i = I_m\,(1 - e^{-t/\lambda})$ $\therefore$ $7.5 = 12.5\,(1 - e^{-t/1})$; $t =$ **0.915 second**

Example 8. 21. *A d.c. voltage of 80 V is applied to a circuit containing a resistance of 80 Ω in series with an inductance of 20 H. Calculate the growth of current at the instant* ***(i)*** *of completing the circuit* ***(ii)*** *when the current is 0.5 A and* ***(iii)*** *when the current is 1 A.*

(Circuit Theory, Jadavpur Univ.)

Solution. The voltage equation for an *R-L* circuit is

$$V = iR + L\frac{di}{dt} \quad \text{or} \quad L\frac{di}{dt} = V - iR \quad \text{or} \quad \frac{di}{dt} = \frac{1}{L}(V - iR)$$

(i) when $i = 0;\ \frac{di}{dt} = \frac{1}{L}(V - 0 \times R) = \frac{V}{L} = \frac{80}{20} =$ **4 A/s**

(ii) when $i = 0.5\ A;\ \frac{di}{dt} = \frac{80 - 0.5 \times 80}{20} =$ **2 A/s**

(iii) when $i = 1\ A;\ \frac{di}{dt} = \frac{80 - 80 \times 1}{20} =$ **0.**

In other words, the current has become steady at 1 ampere.

Example 8.22. *The two circuits of Fig. 8.14 have the same time constant of 0.005 second. With the same d.c. voltage applied to the two circuits, it is found that the steady state current of circuit (a) is 2000 times the initial current of circuit (b). Find R_1, L_1 and C.* **(Elect. Engg.-I, Bombay Univ.)**

Solution. The time constant of circuit 8.14 (*a*) is $\lambda = L_1/R_1$ second, and that of circuit 8.14 (*b*) is $\lambda = CR_2$ second.

$$\therefore \quad L_1/R_1 = 0.005$$
$$C \times 2 \times 10^6 = 0.005, \; C = 0.0025 \times 10^{-6} = \mathbf{0.0025\ \mu F}$$

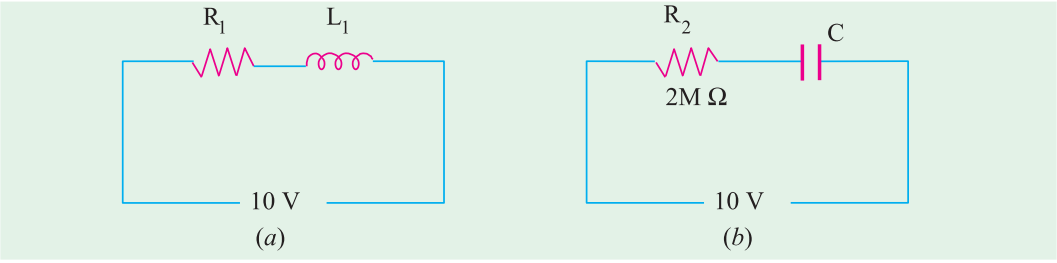


Fig. 8.14

Steady-state current of circuit 8.14 (*a*) is $= V/R_1 = 10/R_1$ amperes.

Initial current of circuit 8.14 (*b*) $= V/R_2 = 10/2 \times 10^6 = 5 \times 10^{-6} A$*

Now $10/R_1 = 2000 \times 5 \times 10^{-6} \therefore R_1 = 1000\ \Omega$

Also $L_1/R_1 = 0.005 \quad \therefore \; L_1 = 1000 \times 0.005 = \mathbf{5\ H}$

Example 8.23. *A constant voltage is applied to a series R-L circuit at t = 0 by closing a switch. The voltage across L is 25 V at t = 0 and drops to 5 V at t = 0.025 second. If L = 2 H, what must be the value of R ?* **(Elect. Engg.-I Bombay Univ.)**

Solution. At $t = 0$, $i = 0$, hence there is no iR drop and the applied voltage must equal the back e.m.f. in the coil. Hence, the voltage across L at $t = 0$ represents the applied voltage.

At $t = 0.025$ second, voltage across L is 5 V, hence voltage across

$R = 25 - 5 = 20$ V $\quad \therefore \; iR = 20$ V – at $t = 0.025$ second. Now $i = I_m (I - e^{-t/\lambda})$

Here $I_m = 25/R$ ampere, $t = 0.025$ second $\quad \therefore \; i = \dfrac{25}{R}(1 - e^{-0.025/\lambda})$

$$R \times \frac{25}{R}(1 - e^{-0.025/\lambda}) = 20 \quad \text{or} \quad e^{0.025/\lambda} = 5 \quad \therefore \; 0.025/\lambda = 2.3 \log_{10} 5 = 1.6077$$

$\therefore \quad \lambda = 0.025/1.6077 \quad$ Now $\lambda = L/R = 2/R \quad \therefore \; 2/R = 0.025/1.6077 \quad \therefore \; \mathbf{R = 128.56\ \Omega}$

Example 8.24. *A circuit of resistance R ohms and inductance L henries has a direct voltage of 230 V applied to it. 0.3 second second after switching on, the current in the circuit was found to be 5 A. After the current had reached its final steady value, the circuit was suddenly short-circuited. The current was again found to be 5 A at 0.3 second second after short-circuiting the coil. Find the value of R and L.* **(Basic Electricity, Bombay Univ.)**

Solution. For growth ; $\quad 5 = I_m (1 - e^{-0.3/\lambda})$ ...(*i*)

For decay; $\quad 5 = I_m e^{-0.3/\lambda}$

Equating the two, we get, $I_m e^{-0.3/\lambda} = (1 - e^{-0.3/\lambda}) I_m$

or $\quad 2 e^{-0.3/\lambda} = 1 \quad \therefore \; e^{-0.3/\lambda} = 0.5$ or $\lambda = 0.4328$

Putting this value in (*i*), we get,

$$5 = I_m = e^{0.3/0.4328} \quad \text{or} \quad I_m = 5 e^{+0.3/0.4328} = 5 \times 2 = 10 \text{ A.}$$

Now, $\quad I_m = V/R \quad \therefore \; 10 = 230/R \quad$ or $\quad R = 230/10 = 23\ \Omega$ (approx.)

As $\quad \lambda = L/R = 0.4328 \; ; \; L = 0.4328 \times 23 = \mathbf{9.95\ H}$

---

* Because just at the time of starting the current, there is no potential drop across C so that the applied voltage is dropped across R_2. Hence, the initial charging current $= V/R_2$.

Example 8.25. *A relay has a coil resistance of 20 Ω and an inductance of 0.5 H. It is energized by a direct voltage pulse which rises from 0-10 V instantaneously, remains constant for 0.25 second and then falls instantaneously to zero. If the relay contacts close when the current is 200 mA (increasing) and open when it is 100 mA (decreasing), find the total time during which the contacts are closed.*

Solution. The time constant of the relay coil is

$$\lambda = L/R = 0.5/20 = 0.025 \text{ second}$$

Now, the voltage pulse remains constant at 10 V for 0.25 second which is long enough for the relay coil current to reach its steady value of $V/R = 10/20 = 0.5$ A

Let us now find the value of time required by the relay coil current to reach a value of 200 mA = 0.2 A. Now $i = I_m (1 - e^{-t/\lambda})$ $\quad \therefore \; 0.2 = 0.5 (1 - e^{-t/0.025}) \; \therefore \; e^{40/t} = 5/3$

$\therefore \quad t = 0.01276$ second

Hence, relay contacts close at $t = 0.01276$ second and will remain closed till current falls to 100 mA. Let us find the time required by the current to fall from 0.5 A to 0.1 A.

At the end of the voltage pulse, the relay current decays according to the relation

$$i = I_m e^{-t/\lambda} \qquad \therefore \quad 0.1 = 0.5 \, e^{-t/0.025} \qquad \therefore \; e^{40\,t} = 5$$

$\therefore \quad t = 0.04025$ second after the end of the voltage pulse.

Hence, the time for which contacts remain closed is

$$= (0.25 - 0.01276) + 0.04025 \text{ second} = \textbf{277.5 milli-second (approx)}$$

8.12. Details of Transient Current Rise in an R-L Circuit

As shown in Fig. 8.15 (*a*), when switch *S* is shifted to position *a*, the *R-L* circuit is suddenly energised by *V*. Since a coil opposes any change in current, the initial value of current is zero at $t = 0$ and but then it rises exponentially, although its rate of rise keeps decreasing. After some time, it reaches a maximum value of I_m when it becomes constant *i.e.* its rate of rise becomes zero. Hence, just at the start of the transient state, $i = 0$, $V_R = 0$ and $V_L = V$ with its polarity opposite to that of battery voltage as shown in Fig. 8.15 (*a*). Both *i* and V_R rise exponentially during the transient state, as shown in Fig. 8.15 (*b*) and (*c*) respectively. However V_L decreases exponentially to zero from its initial maximum value of $V = I_m R$. It does not become negative during the transient rise of current through the circuit.

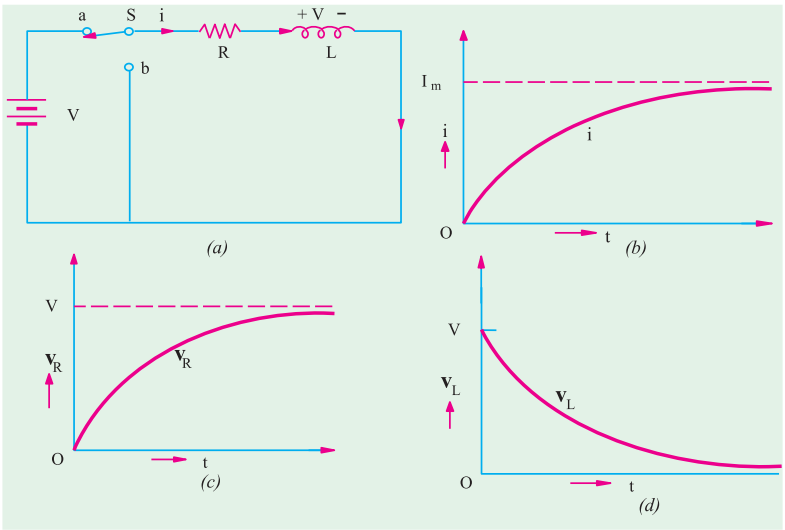


Fig. 8.15

Hence, during the transient rise of current, the following equations hold good :

$$i = \frac{V}{R}(1 - e^{-t/\lambda}) = I_m (1 - e^{-t/\lambda}) \; ; \; V_R = iR = V (1 - e^{-t/\lambda}) = I_m R (1 - e^{-t/\lambda}); \; v_L = Ve^{-t/\lambda}$$

If *S* remains at '*a*' long enough, *i* reaches a steady value of I_m and V_R equals $I_m R$ but since $di/dt = 0$, $v_L = 0$.

Example 8.26. *A voltage as shown in Fig. 8.16 is applied to an inductor of 0.2 H, find the current in the inductor at t = 2 sec.*

Solution. $L(di/dt) = 2$ volts, for t between 0 and 1 sec, and corresponding value of $di/dt = 2/0.2 = 10$ amp/sec, uniform during this period.

After $t > 1$ voltage is zero, hence $di/dt = 0$

Current variation is marked on the same diagram.

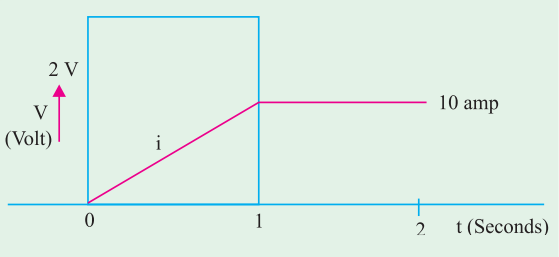


Fig. 8.16

8.13. Details of Transient Current Decay in an R-L Circuit

Now, let us consider the conditions during the transient decay of current when S is shifted to point 'b'. Just at the start of the decay condition, the following values exist in the circuit.

$i = I_m = V/R$, $v_R = I_m R = V$ and since initial di/dt is maximum, $v_L = -V = -I_m R$.

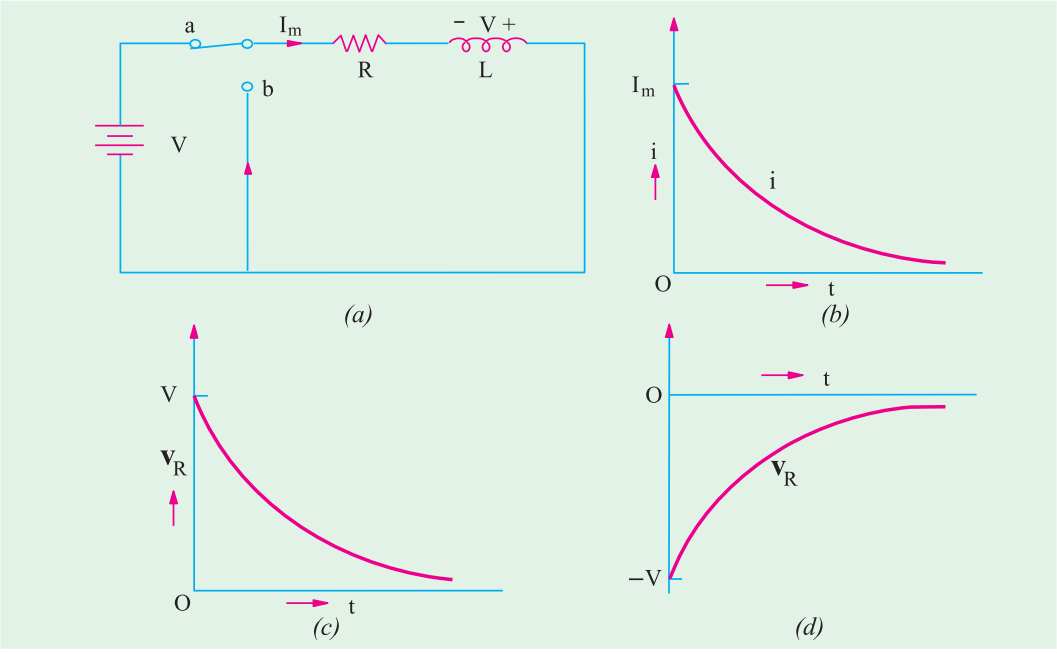


Fig. 8.17

The change in the polarity of voltage across the coil in Fig. 8.17 (*a*) is worth noting. Due to its property of self-induction, the coil will not allow the circuit current to die immediately, but only gradually. In fact, by reversing the sign of its voltage, the coil tends to maintain the flow of currrent in the original direction. Hence, as the decay continues i decreases exponentially from its maximum value to zero, as shown in Fig. 8.17 (*b*). Similarly, v_R decreases exponentially from its maximum value to zero, as shown in Fig. 8.17 (*c*). However, v_L is reversed in polarity and decreases exponentially from its initial value of $-V$ to zero as shown in Fig. 8.17 (*d*).

During the transient decay of current and voltage, the following relations hold good :

$$i = i_L = I_m e^{-t/\lambda} = \frac{V}{R} e^{-t/\lambda}$$

$$v_R = Ve^{-t/\lambda} = I_m R e^{-t/\lambda}$$

$$v_L = -V e^{-t/\lambda} = -I_m R e^{-t/\lambda}$$

8.14. Automobile Ignition System

Practical application of mutual induction is found in the single-spark petrol-engine ignition system extensively employed in automobiles and air-engines. Fig. 8.18 shows the circuit diagram of such a system as applied to a 4-cylinder automobile engine.

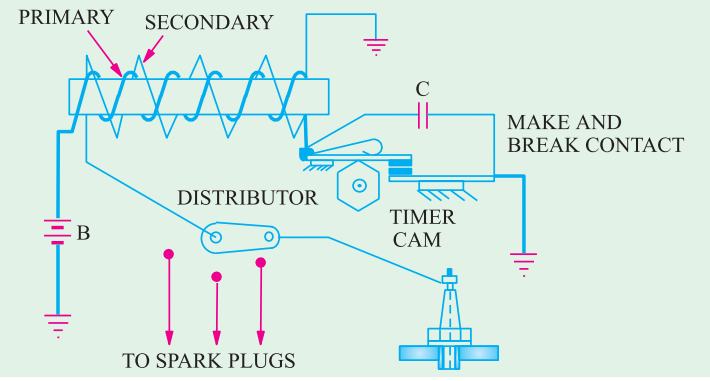


Fig. 8.18

It has a spark coil (or induction coil) which consists of a primary winding (of a few turns) and a secondary winding (of a large number of turns) wound on a common iron core (for increasing mutual induction). The primary circuit (containing battery *B*) includes a 'make and break contact' actuated by a timer cam. The secondary circuit includes the rotating blade of the distributor and the spark gap in the spark plug as shown in Fig. 8.17. The timer cam and the distributor are mounted on the same shaft and are geared to rotate at exactly half the speed of the engine shaft. It means that in the case of automobile engines (which are four-cycle engines) each cylinder is fired only once for every two revolutions of the engine shaft.

Working

When timer cam rotates, it alternately closes and opens the primary circuit. During the time primary circuit is closed, current through it rises exponentially after the manner shown in Fig. 8.11 and so does the magnetic field of the primary winding. When the cam suddenly opens the primary circuit, the magnetic field collapses rapidly thereby producing a very large e.m.f. in secondary by mutual induction. During the time this large e.m.f. exists, the distributor blade rotates and connects the secondary winding across the proper plug and so the secondary circuit is completed except for the spart gap in the spark plug. However, the induced e.m.f. is large enough to make the current jump across the gap thus producing a spark which ignites the explosive mixture in the engine cylinder.

The function of capacitor *C* connected across the 'make and break' contact is two-fold :

(i) to make the break rapid so that large e.m.f. is induced in secondary and

(ii) to reduce sparking and burning at the 'make-and-break' contact thereby prolonging their life.

Tutorial Problems No. 8.3

1. A relay has a resistance of 300 Ω and is switched on to a 110 V d.c. supply. If the current reaches 63.2 per cent of its final steady value in 0.002 second, determine

(*a*) the time-constant of the circuit (*b*) the inductance of the circuit

(*c*) the final steady value of the circuit (*d*) the initial rate of rise of current.

[(*a*) 0.002 second (*b*) 0.6 H (*c*) 0.366 A (*d*) 183 A/second]

2. A coil with a self-inductance of 2.4 H and resistance 12 Ω is suddenly switched across a 120-V d.c. supply of negligible internal resistance. Determine the time constant of the coil, the instantaneous value of the current after 0.1 second, the final steady value of the current and the time taken for the current to reach 5 A. **[(*a*) 0.2 second; 3.94 A; 10 A; 0.139 second]**

3. A circuit whose resistance is 20 Ω and inductance 10 H has a steady voltage of 100 V suddenly applied to it. For the instant 0.5 second after the voltage is applied, determine (*a*) the total power input to the circuit (*b*) the power dissipated in the resistance. Explain the reason for the difference between (*a*) and (*b*). **[(*a*) 316 W (*b*) 200 W]**

4. A lighting circuit is operated by a relay of which the coil has a resistance of 5 Ω and an inductance of 0.5 H. The relay coil is supplied from a 6-V d.c. source through a push-button switch. The relay operates when the current in the relay coil attains a value of 500 mA. Find the time interval between the pressing of the push-button and the closing of lighting circuit. **[53.8 ms]**

5. The field winding of a separately-excited d.c. generator has an inductance of 60 H and a resistance of 30 Ω. A discharge resistance of 50 Ω is permanently connected in parallel with the winding which is excited from a 200-V supply. Find the value of decay current 0.6 second after the supply has been switched off. **[3.0 A]**

6. The field winding of a dynamo may be taken to have a constant inductance of 120 H and an effective resistance of 30 Ω. When it is carrying a current of 5 A, the supply is interrupted and a resistance of 50 Ω is connected across the winding. How long will it take for the current to fall to 1-0 ? **[2.415 s]**

7. A 200-V d.c. supply is suddenly switched to a relay coil which has a time constant of 3 milli-second. If the current in the coil reaches 0.2 A after 3 milli-second, determine the final steady value of the current and the resistance of the coil. **[0.316 A; 632 Ω; 1.896 H]**

8. Explain the terms related to magnetic circuits :
(i) coercive force (ii) residual flux. ***(Nagpur University, Summer 2002)***

9. The B-H characteristic of cast iron may be drawn from the following :-

$B(Wb/m^2)$	H(AT/m)
0.1	280
0.2	620
0.3	990
0.4	1400
0.5	2000
0.6	2800

(Nagpur University, Winter 2003)

10. Derive an expression for the energy stored in the magnetic field of a coil of an inductance L henry. ***(Gujrat University, June/July 2003)***

11. A cast steel ring has a circular cross-section of 3 cm in diameter and a mean circumference of 80 cm. A 1 mm air gap is cut out in the ring which is wound with a coil of 600 turns. Estimate the current required to establish a flux of 0.75 mWb in the air gap. Neglect fringing and leakage. The magnetization data of the material is as under :
(R.G.P.V. Bhopal University, June 2004)

H (At/m)	B (T)
200	0.10
400	0.32
600	0.60
800	0.90
1000	1.08
1200	1.18
1400	1.27
1600	1.32
1800	1.36

12. What is the difference between B.H. curve and hysteresis loop? ***(Anna University, April 2002)***

13. Explain with neat diagram how can you obtain B.H. curve and hystersis loop of ring specimens. ***(Anna University, April 2002)***

14. Derive an expression for energy stored in an inductance. ***(V.T.U., Belgaum Karnataka University, Summer 2002)***

15. Derive an expression for the energy stored in a magnetic circuit. ***(V.T.U., Belgaum Karnataka University, January/February 2003)***

16. What is hysteresis and eddy current losses? What are the undesirable effects of eddy currents and hysteresis loss? How can they be minimised? Mention some applications of eddy currents. ***(U.P. Technical University, 2002) (RGPV Bhopal June 2002)***

OBJECTIVE TESTS – 8

1. Permanent magnets are normally made of
(*a*) aluminium (*b*) wrought iron
(*c*) cast iron (*d*) alnico alloys

2. A coil of 1000 turns is wound on a core. A current of 1 A flowing through the coil creates a core flux of 1 mWb. The energy stored in the magnetic field is
(*a*) 0.25 J (*b*) 0.5 J
(*c*) 1 J (*d*) 2 J
(ESE 2003)

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CHAPTER 9

ELECTRO-CHEMICAL POWER SOURCES

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- High Temperature Batteries
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- Fuel Cells



Electrochemical cells convert chemical energy into electrical energy

9.1. Faraday's Laws of Electrolysis


Michael Faraday

From his experiments, Faraday deduced two fundamental laws which govern the phenomenon of electrolysis. These are :

(*i*) First Law. The mass of ions liberated at an electrode is directly proportional to the quantity of electricity *i.e.* charge which passes through the electrolyte.

(*ii*) Second Law. The masses of ions of different substances liberated by the same quantity of electricity are proportional to their chemical equivalent weights.

Explanation of the First Law

If m = mass of the ions liberated, Q = quantity of electricity = $I \times t$ where I is the current and t is the time, then according to the first law $m \propto Q$ or $m = Z\,Q$ or $m = Z\,It$

where Z is a constant and is known as the electrochemical equivalent (*E.C.E.*) of the substance.

If Q = 1 coulomb *i.e.* I = 1 ampere and t = 1 second, then $m = Z$.

Hence, ***E.C.E.* of a substance is *equal to the mass of its ions liberated by the passage of one ampere current for one second through its electrolytic solution or by the passage of a charge of one coulomb.***

In fact, the constant Z is composite and it depends on the valency and atomic weight of the substance concerned. Its value is given by $Z = \left(\frac{1}{F} \cdot \frac{a}{v}\right)$ where a is the atomic weight, v the valency and F is Faraday's constant. It is so because m is proportional to atomic weight, since each ion carries a definite charge. Obviously, the charge carried by an ion is proportional to its valency. Now, consider the molecules of sulphuric acid and copper sulphate. The sulphion SO_4^{--} in the acid molecule is combined with two positive hydrogen ions, whereas in $CuSO_4$ molecule, it is combined only with one positive (bivalent) Cu^{++} ion. It is seen that a copper ion being bivalent carries twice the charge of a hydrogen ion which is univalent (monovalent). It means that in order to transfer a given quantity of electricity, only one-half as many bivalent copper ions as univalent hydrogen ions will be required. In other words, greater is the valency of an ion, smaller is the number of ions needed to carry a given quantity of electricity or charge which means that the mass of an ion liberated is inversely proportional to its valency.

$$\therefore \qquad m = \left(\frac{1}{F} \cdot \frac{a}{v}\right) It = \left(\frac{1}{F} \cdot \frac{a}{v}\right) Q = \frac{E}{F} \cdot Q$$

where E is the chemical equivalent weight (= a/v).

The constant F is known as Faraday's constant. The value of Faraday's constant can be found thus. It is found that one coulomb liberates 0.001118 gram of silver. Moreover, silver is univalent and its atomic weight is 107.88. Hence, substituting these values above, we find that

$$0.001118 = \frac{1}{F} \cdot 107.88 \times 1$$

$\therefore$ F = 107.88 / 0.001118 = 96,500 coulomb, which corresponds to 96,500/3600 = 26.8 Ah

Faraday's constant is defined as ***the charge required to liberate one gram-equivalent of any substance.***

For all substances, $\frac{\text{chemical equivalent } (E)}{\text{electrochemical equivalent } (Z)}$ = Faraday's constant (F) = 96,500 coulomb or $F = E/Z$

Explanation of the Second Law

Suppose an electric current is passed for the same time through acidulated water, solution of $CuSO_4$ and $AgNO_3$, then for every 1.0078 (or 1.008) gram of hydrogen evolved, 107.88 gram of silver and 31.54 gram of Cu are liberated. The values 107.88 and 31.54 represent the equivalent weights* of silver and copper respectively *i.e.* their atomic weights (as referred to hydrogen) divided by their respective valencies.

Example 9.1. *Calculate the time taken to deposit a coating of nickel 0.05 cm thick on a metal surface by means of a current of 8 A per cm^2 of surface. Nickel is a divalent metal of atomic weight 59 and of density 9 gram/cm^3. Silver has an atomic weight of 108 and an E.C.E. of 1.118 mg/C.*

Solution. Wt. of nickel to be deposited per cm^2 of surface = $1 \times 0.05 \times 9 = 0.45$ g

Now $$\frac{E.C.E. \text{ of Ni}}{E.C.E. \text{ of Ag}} = \frac{\text{chemical equivalent of Ni}}{\text{chemical equivalent of Ag}}$$

$$\therefore \quad E.C.E. \text{ of } Ni = 1.118 \times 10^{-3} \times \frac{(59/2)}{108} = 0.0003053 \text{ g/C}$$

(chemical equivalent = atomic wt./valency)

Now $m = ZIt$ $\therefore$ $0.45 = 0.0003053 \times 8 \times t$; $t = 184$ second = **3 min. 4 second**

Exmaple 9.2. *If 18.258 gm of nickel are deposited by 100 amp flowing for 10 minutes, how much copper would be deposited by 50 amp for 6 minutes : Atomic weight of nickel = 58.6 and that of copper 63.18. Valency of both is 2.* **(Electric Power AMIE Summer 1991)**

Solution. From Faraday's first law, we get $m = ZIt = m\left(\frac{1}{F} \cdot \frac{a}{v}\right) It.$

If m_1 is the mass of nickel deposited and m_2 that of copper, then

$$m_1 = 18.258 = \left(\frac{1}{F} \cdot \frac{58.6}{2}\right) \times 100 \ (10 \times 60), \left(\frac{1}{F} \cdot \frac{63.18}{2}\right) \times 50 \times (6 \times 60)$$

$$\therefore \quad \frac{m_2}{18.258} = \frac{31.59}{29.3} \times \frac{18,000}{60,000} \qquad \therefore \ m_2 = \mathbf{5.905 \text{ gm}}$$

Example 9.3. *The cylindrical surface of a shaft of diameter 12 cm and length 24 cm is to be repaired by electrodeposition of 0.1 cm thick nickel on it. Calculate the time taken if the current used is 100 A. The following data may be used :*

Specific gravity of nickel = 8.9 ; Atomic weight of nickel = 58.7 (divalent) ; E.C.E. of silver = 1.2 mg/C; Atomic weight of silver = 107.9. **(Elect. Engg. A.M.Ae. S.I. June, 1991)**

Solution. Curved surface of the salt = $\pi D \times l = \pi \times 12 \times 24$ cm^2

Thickness of nickel layer = 0.1 cm

Volume of nickel to be deposited = $12\pi \times 24 \times 0.1 = 90.5$ cm^3

Mass of nickel deposited = $90.5 \times 8.9 = 805.4$ g

$$\text{Chemical equivalent of } Ni = \frac{\text{atomic weight}}{\text{valency}} = \frac{58.7}{2} = 29.35$$

---

* The electro-chemical equivalents and chemical equivalents of different substance are inter-related thus :

$$\frac{E.C.E. \text{ of } A}{E.C.E. \text{ of } B} = \frac{\text{chemical equivalent of } A}{\text{chemical equivalent of } B}$$

Further, if m_1 and m_2 are masses of ions deposited at or liberated from an electrode, E_1 and E_2 their chemical equivalents and Z_1 and Z_2 their electrochemical equivalent weights, then

$$m_1/m_2 = E_1/E_2 = Z_1/Z_2$$

Now $\frac{E.C.E.\text{ of Ni}}{E.C.E.\text{ of Ag}} = \frac{\text{chemical equivalent of Ni}}{\text{chemical equivalent of Ag}}$

$\therefore \quad \frac{E.C.E.\text{ of Ni}}{1.12} = \frac{29.35}{107.9}$

$\therefore \quad E.C.E.\text{of Ni} = 1.12 \times 29.35/107.9 = 0.305$ mg/C

Now $m = ZIt$

$\therefore \quad 805.4 = 0.305 \times 10^{-3} \times 100 \times t \quad \therefore \quad t = 26{,}406$ second or **7 hr, 20 min 7 s**

Example 9.4. *The worn-out part of a circular shaft 15 cm in diameter and 30 cm long is to be repaired by depositing on it 0.15 cm of Nickel by an electro-depositing process. Estimate the qunatity of electricity required and the time if the current density is to be 25 mA/cm^2. The current efficiency of the process may be taken as 95 per cent. Take E.C.E. for nickel as 0.3043 mg/coulomb and the density of nickel as 8.9 g/cm^3.* **(Elect. Power-I, Bangalore Univ.)**

Solution. Curved surface area of shaft $= \pi D \times l = \pi \times 15 \times 30 = 1414$ cm^2.

Thickness of nickel layer $= 0.15$ cm

Volume of nickel to be deposited $= 1414 \times 0.15 = 212$ cm^3

Mass of nickel to be deposited $= 212 \times 8.9 = 1887$ gram

Now, $m = ZQ$; $Q = m/Z = 1887/0.343 \times 10^{-3} =$ **62×10^5 C**

Now, current density $= 15 \times 10^{-3}$ A/cm^2; $A = 1414$ cm^2

$I = 25 \times 10^{-3} \times 1414 = 35.35$ A

Since $Q = It \quad \therefore \quad t = 62 \times 10^5/35.35 = 1.7 \times 10^5$ s = **47.2 hr.**

Example 9.5. *A refining plant employs 1000 cells for copper refining. A current of 5000 A is used and the voltage per cell is 0.25 volt. If the plant works for 100 hours/week, determine the annual output of refined copper and the energy consumption in kWh per tonne. The electrochemical equivalent of copper is 1.1844 kg/1000 Ah.*

(Electric Drives and Utilization, Punjab Univ. Jan. 1991)

Solution. Total cell voltage = 0.25 × 1000 = 250 V; $I = 5000$ A; plant working time = 100 hour/week = 100 × 52 = 5200 hour/year; $Z = 1.1844$ kg/1000 Ah; 1 Ah = 1 × 60 × 60 = 3600 C;

$\therefore \quad Z = 1.1844$ kg/1000 × 3600 = 0.329×10^{-6} kg/C.

According to Faraday's law of Electrolysis, the amount of refined copper produced per year is

$m = Z\,I\,t = 0.329 \times 10^{-6} \times 5000 \times (5200 \times 3600) = 3079$ kg = **3.079 tonne.**

Hence, annual output of refined copper = 3.079 tonne

Energy consumed per year = 250 × 5000 × 5200/1000 = 6500 kWh

This is the energy consumed for refining 3.079 tonne of copper

$\therefore$ Energy consumed per tonne = 6500/3.079 = **2110 kWh/tonne.**

Example 9.6. *A sheet of iron having a total surface area of 0.36 m^2 is to be electroplated with copper to a thickness of 0.0254 mm. What quantity of electricity will be required ? The iron will be made the cathode and immersed, together with an anode of pure copper, in a solution of copper sulphate.*

(*Assume the mass density of copper = 8.96×10^3 kg m^{-3}; E.C.E. of copper —32.9×10^{-8} kg C^1 Current density = 300 Am^{-2}*) **(AMIE Sec. B Utilisation of Electric Power Summer 1992)**

Solution. Area over which copper is to be deposited = 0.36 m^2

Thickness of the deposited copper = 0.0254×10^{-3} m

Volume of deposited copper $= 0.36 \times 0.0254 \times 10^{-3} = 9.144 \times 10^{-6}$ m^3

Mass of copper deposited = volume × density

$= 9.144 \times 10^{-6} \times 8.96 \times 10^3 = 0.0819$ kg

Now, $m = Z\,Q \quad \therefore \quad Q = m/Z = 0.0819/32.9 \times 10^{-8} =$ **248936 C**

Tutorial Problems No. 9.1

1. A steady current was passed for 10 minutes through an ammeter in series with a silver voltameter and 3.489 grams of silver were deposited. The reading of the ammeter was 5A. Calculate the percentage error. Electrochemical equivalent of silver = 1.1138 mg/C.

[3.85 %] (*City and Guilds, London*)

2. Calculate the ampere-hours required to deposit a coating of silver 0.05 mm thick on a sphere of 5 cm radius. Assume electrochemical equivalent of silver = 0.001118 and density of silver to be 10.5 g cm^3. **[*Utilization of Elect. Power, A.M.I.E. Summer*]**

9.2. Polarisation of Back E.M.F.

Let us consider the case of two platinum electrodes dipped in dilute sulphuric acid solution. When a small potential difference is applied across the electrodes, no current is found to flow. When, however, the applied voltage is increased, a time comes when a temporary flow of current takes place. The H^+ ions move towards the cathodes and O^{--} ions move towards the anode and are absorbed there. These absorbed ions have a tendency to go back into the electolytic solution, thereby leaving them as oppositely-charged electrodes. This tendency produces an e.m.f. that is in opposition to the applied voltage which is consequently reduced.

This opposing e.m.f. which is produced in an electrolyte due to the absorption of gaseous ions by the electrolyte from the two electrodes is known as the back e.m.f. of electolysis or polarisation.

The value of this back e.m.f. is different from different electrolytes. The minimum voltage required to decompose an electrolyte is called the *decomposition* voltage for that electrolyte.

9.3. Value of Back E.M.F.

For producing electrolysis, it is necessary that the applied voltage must be greater than the back e.m.f. of electrolysis for that electrolyte. The value of this back e.m.f. of electrolysis can be found thus :

Let us, for example, find the decomposition voltage of water. We will assume that the energy required to separate water into its constituents (*i.e.* oxygen and hydrogen) is equal to the energy liberated when hydrogen and oxygen combine to form water. Let H kcal be the amount of heat energy absorbed when 9 kg of water are decomposed into 1 kg of hydrogen and 8 kg of oxygen. If the electrochemical equivalent of hydrogen is Z kg/coulomb, then the passage of q coulomb liberates Zq kg of hydrogen. Now, H is the heat energy required to release 1 kg of hydrogen, hence for releasing Zq kg of hydrogen, heat energy required is HZq kcal to $JHZq$ joules. If E is the decomposition voltage, then energy spent in circulating q coulomb of charge is Eq joule. Equating the two amounts of energies, we have

$$Eq = JHZq \text{ or } E = JHZ$$

where J is 4200 joule/kcal.

The e.m.f. of a cell can be calculated by determining the two electrode potentials. The electrode potential is calculated on the assumption that the electrical energy comes entirely from the heat of the reactions of the constituents. Let us take a zinc electrode. Suppose it is given that 1 kg of zinc when dissolved liberates 540 kcal of heat and that the electrochemical equivalent of zinc is 0.338×10^{-6} kg/coulomb. As calculated above,

$$E = JHZ = 4200 \times 540 \times 0.338 \times 10^{-6} = 0.76 \text{ volt}$$

The electrode potentials are usually referred to in terms of the potential of a standard hydrogen electrode *i.e.* an electrode of hydrogen gas at normal atmospheric pressure and in contact with a normal acid solution. In table No. 9.1 are given the elctrode potentials of various elements as referred to the standard hydrogen electrode. The elements are assumed to be in normal solution and at atmospheric pressure.

In the case of Daniel cell having copper and zinc electrodes, the copper electrode potential with respect to hydrogen ion is + 0.345 V and that of the zinc electrode is –0.758 V. Hence, the cell e.m.f. is 0.345 –(–0.758) = 1.103 volt. The e.m.f. of other primary cells can be found in a similar way.

Table No. 9.1

Electrode	*Potential (volt)*	*Electrode*	*Potential (volt)*
Cadmium	–0.398	Mercury	+ 0.799
Copper	+ 0.345	Nickel	–0.231
Hydrogen	0	Potassium	–2.922
Iron	–0.441	Silver	+ 0.80
Lead	–0.122	Zinc	–0.758

Example 9.7. *Calculate the weight of zinc and MnO_2 required to produce I ampere-hour in a leclanche cell.*

Atomic weights : Mn, 55 ; O, 16 ; Zn, 65. E.C.E. of hydrogen = 1.04 × 10^{-8} kg/C.

Solution. 1 ampere-hour = 3600 A-s = 3600 C

Wt. of hydrogen liberated = $Zq = 1.04 \times 10^{-8} \times 3600 = 37.44 \times 10^{-6}$ kg

Now, the chemical reactions in the cell are

$$Zn + 2NH_4Cl = ZnCl_2 + 2NH_3 + H_2$$

It is seen that 1 ***atom*** of zinc is used up in liberating ***two atoms*** of hydrogen. In other words, to produce 2 kg of hydrogen, 65 kg of zinc will have to go into chemical combination.

∴ Zinc required to produce 37.44×10^{-6} kg of hydrogen = $37.44 \times 10^{-6} \times 65/2$

$$= 1.217 \times 10^{-3} \text{ kg}$$

The hydrogen liberated combines with manganese dioxide as under :

$$2MnO_2 + H_2 = H_2O + Mn_2O_3$$

Atomic weight of $2\ MnO_2 = 2(55 + 16 \times 2) = 174$

It is seen that 174 kg of MnO_2 combine with 2 kg of hydrogen, hence Wt. of MnO_2 needed to combine with 37.44×10^{-6} kg of hydrogen = $37.44 \times 10^{-6} \times 174/2 = 3.258 \times 10^{-3}$ kg.

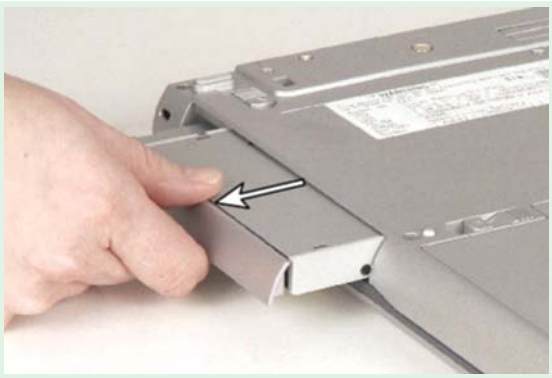
Hence, for 1 ampere-hour, **1.217×10^{-3} kg** of zinc and **3.258×10^{-3} kg** of MnO_2 are needed.



Primary battery

9.4. Primary and Secondary Batteries

An electric battery consists of a number of electrochemical cells, connected either in series or parallel. A cell, which is the basic unit of a battery, may be defined as a power generating device, which is capable of converting stored chemical energy into electrical energy. If the stored energy is inherently present in the chemical substances, it is called a primary cell or a non-rechargeable cell. Accordingly, the battery made of these cells is called primary battery. The examples of primary cells are Leclanche cell, zinc-chlorine cell, alkaline-manganese cell and metal air cells etc.


Secondary battery

If, on the other hand, energy is induced in the chemical substances by applying an external source, it is called a secondary cell or rechargeable cell. A battery made out of these cells is called a secondary battery or storage battery or rechargeable battery. Examples of secondary cells are lead-acid cell, nickel-cadmium cell, nickel-iron cell, nickel-zinc cell, nickel-hydrogen cell, silver-zinc cell and high temperature cells like lithium-chlorine cell, lithium-sulphur cell, sodium-sulphur cell etc.

9.5. Classification of Secondary Batteries Based on Their Use

Various types of secondary batteries can be grouped in to the following categories as per their use :

1. *Automotive Batteries or SLI Batteries or Portable Batteries*

These are used for starting, lighting and ignition (*SLI*) in internal-combustion-engined vehicles. Examples are; lead-acid batteries, nickel-cadmium batteries etc.

2. *Vehicle Traction Batteries* or *Motive Power Batteries* or *Industrial Batteries*

These are used as a motive power source for a wide variety of vehicles. Lead-acid batteries, nickel-iron batteries, silver-zinc batteries have been used for this purpose. A number of advance batteries including high-temperature batteries are under development for electric vehicle (*EV*) use. These high-temperature batteries like sodium-sulphur and lithium-iron sulphide have energy densities in the range of 100-120 Wh/kg.

3. *Stationary Batteries.*

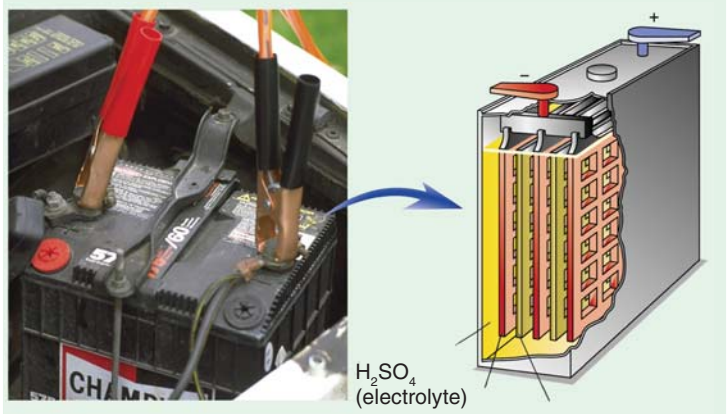
These fall into two groups ***(a)*** standby power system which is used intermittently and ***(b)*** load-levelling system which stores energy when demand is low and, later on, uses it to meet peak demand.

9.6. Classification of Lead Storage Batteries

Lead storage batteries may be classified according to the service which they provide.

1. *SLI Batteries*

The primary purpose of these batteries is to supply power for engine starting, lighting and ignition (*SLI*) of vehicles propelled by IC engines such as automobiles, buses, lorries and other heavy road vehicles and motor cycles etc. Usually, these batteries provide 12 V and consist of six series-connected lead-acid cells with capacity of the other of 100 Ah. Their present-day energy density is about 45 Wh/kg and 75 Wh/dm^3.



Lead storage batteries

These days 'maintenance-free' (*MF*) *SLI* batteries have been designed, which do not require the addition of water throughout their normal service life of 2-5 years. *MF* versions of the *SLI* batteries are constructed of such material that no gassing occurs during charging. In *MF* batteries, the electrolyte is either absorbed within the microporous separators and the plates or is immobilized with suitable gelling agents.

These days the *SLI* battereis are charged from an alternator (*AC* generator) and not from dynamo (*DC* generator). The alternating current produced by the alternator is converted into direct current by

a full-wave bridge rectifier, which uses semi-conductor diodes. In this arrangement, no cutout is needed and the transistorised voltage controller regulates the alternator output to suit the electrical load and the state of charge of the battery. The battery is charged under constant-voltage conditions.

2. *Vehicle Traction Batteries*

The recent universal concern over the levels of toxic gases (particularly in urban areas) emitted by the *IC* engines has revived interest in electric traction. There has been great development in the use of battery-powered vehicles, primarily industrial trucks and commercial road vehicles of various types like 'milk floats' (*i.e.* bottled-milk delivery trucks), fork lift trucks, mining, airport tractors, aircraft service vehicles, electric cars and, more recently, in robotics and guided vehicles.

Traction batteries are of higher quality than *SLI* batteries. They provide constant output voltage, high volumetric capacity, good resistance to vibration and a long service life. They can withstand prolonged and deep discharges followed by deep recharges usually on a daily basis. The voltage of traction batteries varies from 12 V to 240 V and they have a cycle life of 1000-1500 cycles.

A number of advanced batteries and under development for *EV* use **(*i*)** room temperature batteries like zinc-nickel oxide battery (75 Wh/kg) and zinc-chlorine hydrate battery (80 Wh/kg) and **(*ii*)** high-temperature batteries like sodium-sulphur battery (120 Wh/kg) and lithium-iron sulphide battery (100 Wh/kg).

3. *Stationary Batteries*

Their use falls into two groups :

(*a*) as standby power system and **(*b*)** as load-levelling system.

In the standby applications, the battery is used to power essential equipment or to provide alarms or emergency lighting, in case of break-down in the main power supply. Standby applications have increased in recent years with increasing demand for uninterruptable power systems (*UPS*) and a tremendous growth in new telecommunication networks. The *UPS* provides 'clean' a.c supply free of sage or surges in the line voltage, frequency variations, spikes and transients to modern computer and electronic equipment. Banks of sealed lead-acid (*SLA*) standby batteries have been recently used in telecommunication systems and for *UPS* applications.

Recently, advanced lead-acid batteries have been used for load-levelling purpose in the electric generating plants. A 100 M Wh lead-acid battery load-levelling system could occupy a building two and a half storey high and an area of about 250,000 m^2.

9.7. Parts of a Lead-acid Battery

A battery consists of a number of cells and each cell of the battery-consists of (*a*) positive and negative plants **(*b*)** separators and **(*c*)** electrolyte, all contained in one of the many compartments of the battery container.* Different parts of a lead-acid battery are as under :

(*i*) *Plates.* A plate consists of a lattice type of grid of cast antimonial lead alloy which is covered with *active* material (Art. 9.8). The grid not only serves as a support for the fragile active material but also conducts electric current. Grid for the positive and negative plates are often of the same design although negative plate grids are made somewhat lighter. As discussed in Art. 9.10, positive plates are usually Plante plates whereas negative plates are generally of Faure or pasted type.

(*ii*) *Separators.* These are thin sheets of a porous material placed between the positive and negative plates for preventing contact between them and thus avoiding internal short-circuiting of the battery. A separator must, however, be sufficiently porous to allow diffusion or circulation of electrolyte between the plates. These are made of especially-treated cedar wood, glass wool mat, microporous rubber (mipor), microporous plastics (plastipore, miplast) and perforated p.v.c. as shown in Fig. 9.1. In addition to good porosity, a separator must possess high electrical resistance and mechanical strength.

* The most common form of lead-acid cell used for marine applications is the tubular cell which consists of 'armoured' tubular positive plate of standard flat negative plate.

(iii) Electrolyte. It is dilute sulphuric acid which fills the cell compartment to immerse the plates completely.

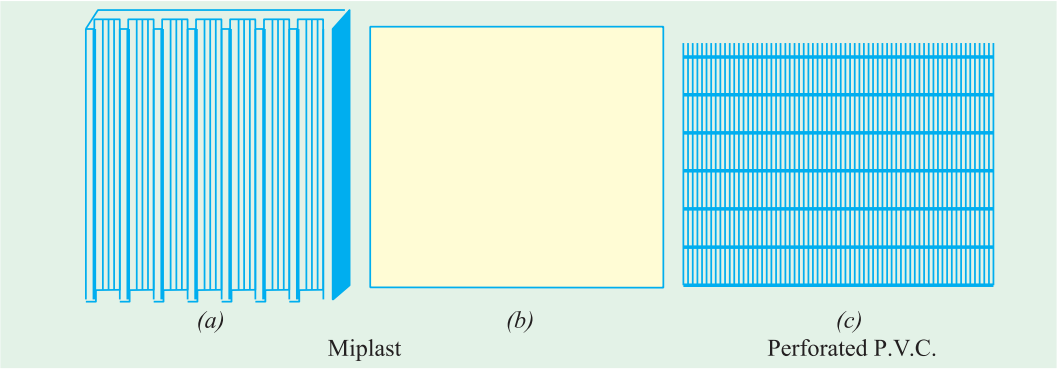


Fig. 9.1

(iv) Container. It may be made of vulcanised rubber or moulded hard rubber (ebonite), moulded plastic, ceramics, glass or celluloid. The vulcanised rubber containers are used for car service, while glass containers are superior for lighting plants and wireless sets. Celluloid containers are mostly used for portbable wireless set batteries. A single monoblock type container with 6 compartments generally used for starting batteries is shown in Fig. 9.2. Full details of a Russian 12-CAM-28 lead-acid battery parts are shown in Fig. 9.3. Details of some of these parts are as follows:

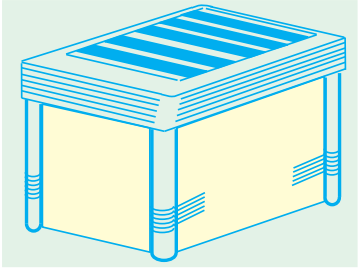


Fig. 9.2

(a) Bottom Grooved Support Blocks. These are raised ribs, either fitted in the bottom of the container or made with the container itself. Their function is to

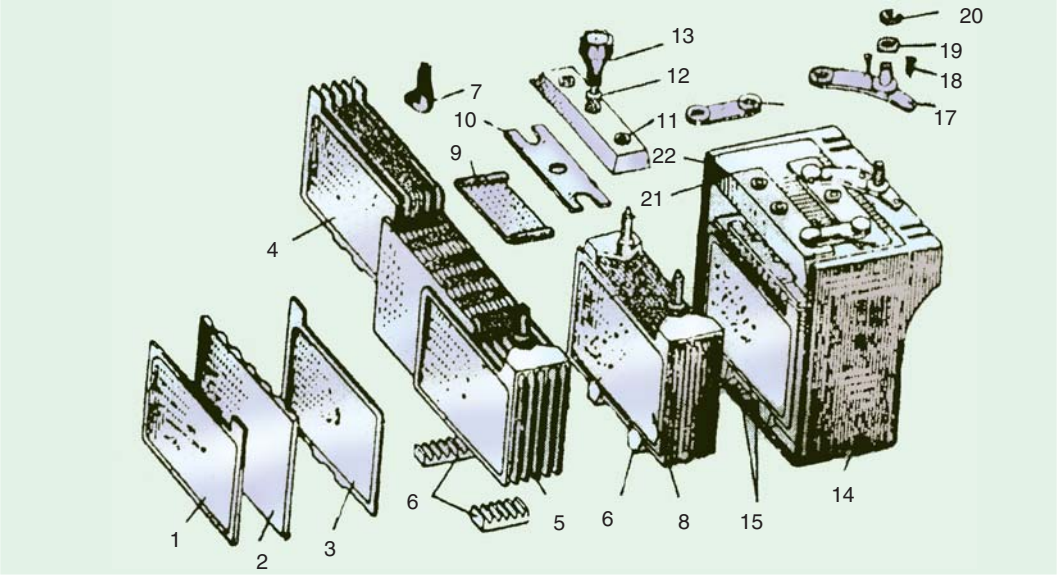


Fig. 9.3. (Courtesy MIR Publishers. Moscow)

1. –ve plate 2. separator 3. + ve plate. 4. + ve group 5. –ve group 6. –ve group grooved support block 7. lug 8. plate group 9. guard screen 10. guard plate 11. cell cover 12. plug washer 13. vent plug 14. monoblock jar 15. supporting prisms of + ve group 16. inter-cell connector 17. terminal lug 18. screw 19. washer 20. nut 21. rubber packing 22. sealing compound.

support the plates and hold them in position and at the same time protect them from short-circuits that would otherwise occur as a result of fall of the active material from the plates onto the bottom of the container.

(*b*) *Connecting Bar.* It is the lead alloy link which joins the cells together in series connecting the positive pillar of one cell to the negative pillar of the next one.

(*c*) *Terminal Post* or *Pillar*. It is the upward extension from each connecting bar which passes through the cell cover for cable connections to the outside circuits. For easy indentification, the negative terminal post is smaller in diameter than the positive terminal post.

(*d*) *Vent Plugs* or *Filler Caps.* These are made of polystyrene or rubber and are usually screwed in the cover. Their function is to prevent escape of electrolyte but allow the free exit of the gas. These can be easily removed for topping up or taking hydrometer readings.

(*e*) *External Connecting Straps.* These are the antimonial lead alloy flat bars which connect the positive terminal post of one cell to the negative of the next across the top of the cover. These are of very solid construction especially in starting batteries because they have to carry very heavy currents.

9.8. Active materials of a Lead-acid Cell

Those substances of the cell which take active part in chemical combination and hence absorb or porduce electricity during charging or discharging, are known as ***active materials*** of the cell.

The active materials of a lead-acid cell are :

1. *Lead peroxide* (PbO_2) for + ve plate **2. *Sponge Lead*** (Pb) for –ve plate **3. *Dilute Sulphuric Acid*** (H_2SO_4) as electrolyte.

1. Lead Peroxide

It is a combination of lead and oxygen, is dark chocolate brown in colour and is quite hard but brittle substance. It is made up of one atom of lead (Pb) and two atoms of oxygen (O_2) and its chemical formula is PbO_2. As said earlier, it forms the positive active material.

2. Sponge Lead

It is pure lead in soft sponge or porous condition. Its chemical formula is Pb and forms the negative active material.

3. Dilute Sulphuric Acid

It is approximately 3 parts water and one part sulphuric acid. The chemical formula of the acid is H_2SO_4. The positive and negative plates are immersed in this solution which is known as electrolyte. It is this medium through which the current produces chemical changes.

Hence, the lead-acid cell depends for its action on the presence of two plates covered with PbO_2 and Pb in a solution of dilute H_2SO_4 of specific gravity 1.21 or nearabout.

Lead in the form of PbO_2 or sponge Pb has very little mechanical strength, hence it is supported by plates of pure lead. Those plates covered with or otherwise supporting PbO_2 are known as + ve plates and those supporting sponge lead are called –ve plates. The + ve and –ve plates are arranged alternately and are connected to two common +ve and –ve terminals. These plates are assembled in a suitable jar or container to make a complete cell as discussed in Art. 9.4 above.

9.9. Chemical changes

(*i*) DISCHARGING (Fig. 9.4)

When the cell is fully charge, its positive plate or anode is PbO_2 (dark chocolate brown) and the negative plate or cathode is Pb (slate grey). When the cell discharges *i.e.* it sends current through the external load, then H_2SO_4 is dissociated into positive H_2 and negative SO_4 ions. As the current within the cell is flowing from cathode to anode, H_2 ions move to anode and SO_4 ions move to the cathode.

At anode (PbO_2), H_2 combines with the oxygen of PbO_2 and H_2SO_4 attacks lead to form $PbSO_4$.

$$PbO_2 + H_2 + H_2SO_4 \longrightarrow PbSO_4 + 2H_2O$$

At the cathode (Pb), SO_4 combines with it to form $PbSO_4$

$$Pb + SO_4 \longrightarrow PbSO_4$$

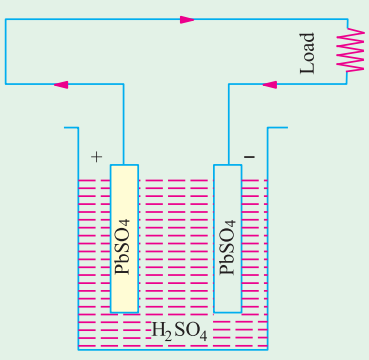


Fig. 9.4

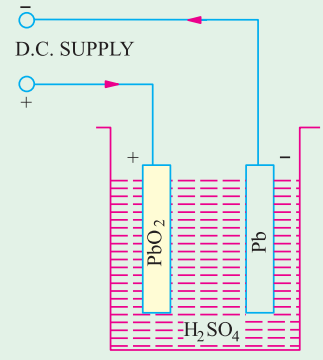


Fig. 9.5

It will be noted that during discharging :

(i) Both anode and cathode become $PbSO_4$ which is somewhat whitish in colour.

(ii) Due to formation of water, specific gravity of the acid decreases.

(iii) Voltage of the cell decreases. (iv) The cell gives out energy.

(ii) CHARGING (Fig. 9.5)

When the cell is recharged, the H_2 ions move to cathode and SO_4 ions go to anode and the following changes take place :

At Cathode $PbSO_4 + H_2 \longrightarrow Pb + H_2SO_4$

At Anode $PbSO_4 + 2H_2O \longrightarrow PbO_2 + 2H_2SO_4$

Hence, the anode and cathode again become PbO_2 and Pb respectively.

(i) The anode becomes dark chocolate brown in colour (PbO_2) and cathode becomes grey metallic lead (Pb).

(ii) Due to consumption of water, specific gravity of H_2SO_4 is increased.

(iii) There is arise in voltage. (iv) Energy is absorbed by the cell.

The charging and discharging of the cell can be represented by a single reversible equation given below :

$$\underset{PbO_2 + 2H_2SO_4 +}{\text{Pos. Plate}} \quad \underset{Pb}{\text{Neg. Plate}} \quad \underset{\text{Charge}}{\overset{\text{Discharge}}{\rightleftharpoons}} \quad \underset{PbSO_4 + 2H_2O}{\text{Pos. Plate}} \quad \underset{+ PbSO_4}{\text{Neg. Plate}}$$

For discharge, the equation should be read from left to right and for charge from right to left.

Example 9.8. *Estimate the necessary weight of active material in the positive and negative plates of a lead-acid secondary cell per ampere-hour output (atomic weight of lead 207, valency 2, E.C.E. of hydrogen 0.0104 × 10^{-6} kg/C).*

Solution. Wt. of hydrogen evolved per ampere-hour = $0.0104 \times 10^{-6} \times 3{,}600$

$= 37.44 \times 10^{-6}$ kg

During discharge, reaction at cathode is $Pb + H_2SO_4 = PbSO_4 + H_2$

As seen, 207 kg of lead react chemically to liberate 2 kg of hydrogen.

Hence, weight of Pb needed per ampere-hour = $37.44 \times 10^{-6} \times 207/2 = 3.876 \times 10^{-3}$ kg

At anode the reaction is : $PbO_2 + H_2 \longrightarrow PbO + H_2O$

Atomic weight of $PbO_2 = 207 + 32 = 239$

$\therefore$Wt. of PbO_2 going into combination per ampere-hour = $37.44 \times 10^{-6} \times 239/2 =$ **4.474×10^{-3} kg**

Therefore, quantity of active material required per ampere-hour is : lead **3.876×10^{-3} kg** and lead peroxide **4.474×10^{-3} kg.**

9.10. Formation of Plates of Lead-acid Cells

There are, in general, two methods of producing the active materials of the cells and attaching them to lead plates. These are known after the names of their inventors. **(1) Plante** plates or formed plates **(2) Faure** plates or pasted plates.

9.11. Plante Process

In this process, two sheets of lead are taken and immersed in dilute H_2SO_4. When a current is passed into this lead-acid cell from a dynamo or some other external source of supply, then due to electrolysis, hydrogen and oxygen are evolved. At anode, oxygen attacks lead converting it into PbO_2 whereas cathode is unaffected because hydrogen can form on compound with Pb.

If the cell is now discharged (or current is reversed through it), the peroxide-coated plate becomes cathode, so hydrogen forms on it and combines with the oxygen of PbO_2 to form water thus :

$$PbO_2 + 2H_2 \longrightarrow Pb + 2H_2O$$

At the same time, oxygen goes to anode (the plate previously unattacked) which is lead and reacts to form PbO_2. Hence, the anode becomes covered with a thin film of PbO_2.

By continuous reversal of the current or by charging and discharging the above electrolytic cell, the thin film of PbO_2 will become thicker and thicker and the polarity of the cell will take increasingly longer time to reverse. Two lead plates after being subjected to hundreds of reversals will acquire a skin of PbO_2 thick enough to possess sufficiently high capacity. This process of making positive plates is known as ***formation***. The negative plates are also made by the same process. They are turned from positive to negative plates by reversing the current through them until whole PbO_2 is converted into sponge lead. Although Plante positives are very commonly used for stationary work, Plante negatives have been completely replaced by the Faure or pasted type plates as discussed in Art. 9.13. However, owing to the length of time required and enormous expenditure of electrical energy, this process is commercially impracticable. The process of formation can be accelerated by forming agents such as acetic, nitric or hydrochloric acid or their salts but still this method is expensive and slow and plates are heavy.

9.12. Structure of Plante Plates

It is seen that since active material on a Plante plate consists of a thin layer of PbO_2 formed on and from the surface of the lead plate, it must be made of large superficial area in order to get an appreciable volume of it. An ordinary lead plate subjected to the forming process as discussed above will have very small capacity. Its superficial area and hence its capacity, can be increased by grooving or laminating. Fig. 9.6 shows a Plante positive plate which consists of a pure lead grid with finely laminated surfaces. The construction of these plates consists of a large number of thin vertical laminations which are strengthened at intervals by horizontal binding ribs. This results in an increase of the superifical area 10 to 12 times that possessed by a plain lead sheet of the same overall dimensions. The above design makes possible the expansion of the plate structure to accommodate the increase in mass and the value of the active material (PbO_2) which takes place when the cell goes through a series of chemical changes during each cycle of charge or discharge. The expansions of the plate structure takes place downwards where there is room left for such purpose. Usually, a Plante positive plate expands by about 10% or so of its length during the course of its useful life.

Another type of Plante positive plate is the 'rosette' plate which consists of a perforated cast grid or framework of lead alloy with 5 to 12 per cent of antimony holding rosettes or spirals of corrugated pure lead tape. The rosettes (Fig. 9.7) provide the active material of the positive plate and, during formation, they expand in the holes of the grid which are countersunk on both sides of the grid. The advantages of such plates are that the lead-antimony grid is itself unaffected by the chemical action and the complete plate is exceptionally strong.

Other things being equal, the life of a Plante plate is in direct proportion to the weight of lead metal in it, because as the original layer of PbO_2 slowly crumbles away during the routing charging

and discharging of the cell, fresh active material is formed out of the underlying lead metal. Hence, the capacity of such a plate lasts as long as the plate itself. In this respect, Plante plate is superior to the Faure or pasted plate.

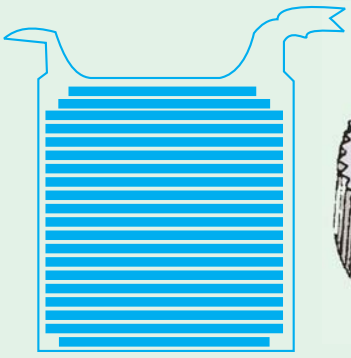


Fig. 9.6



Fig. 9.7

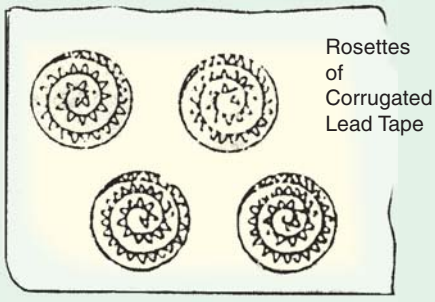


Fig 9.8

9.13. Faure Process

In the making of Faure plates, the active material is mechanically applied instead of being electrochemically produced out of lead plate itself as in Plante process. The active material which is in the form of red lead (Pb_3O_4) or litharge PbO or the mixture of the two in various proportions, is pressed into the interstices of a thin lead grid or lattice work of intersecting ribs which also serves as conductor of current. The plates after being thus pasted are allowed to dry and harden, are then assembled in weak solution of H_2SO_4 of specific gravity 1.1 to 1.2 and are formed by passing an electric current between them. If plates are meant to be positive, they are connected up as anodes, if negative, then as cathodes. The oxygen evolved at the anode converts the lead oxide (Pb_3O_4) into peroxide (PbO_2) and at cathode the hydrogen reduces PbO to sponge lead by abstracting the oxygen.

9.14. Positive Pasted Plates

Formation of positive plate involves converting lead oxide into PbO_2. A high lead oxide like Pb_3O_4 is used for economy both in current and time, although in practice, a mixture of Pb_3O_4 and PbO is taken-the latter being added to assist in the setting or cementation of the plate.

9.15. Negative Pasted Plates

Faure process is much better adopted for making a negative rather than a positive plate. The negative material *i.e.* sponge lead is quite tough instead of being hard and brittle like PbO_2 and, moreover, it undergoes a comparatively negligible change in volume during the charging and discharging of the cell. Hence, it has no tendency to disintegrate or shed out of the grid although it does tend to lose its porosity and become dense and so lose capacity. Hence, in the manufacture of the pasted negatives, a small percentage of certain substances like powdered pumic or graphite or magnesium sulphate or barium sulphate is added to increase the porosity of the material. If properly handled, a paste made with H_2SO_4, glycerine and PbO (or mixture of PbO and Pb_3O_4) results in a very good negative, because glycerine is carbonised during formation and so helps in keeping the paste porous.

Faure plates are in more general use because they are cheaper and have a high (capacity/weight) ratio than Plante plates. Because of the lightness and high capacity/weight ratio, such plates are used

practically for all kinds of portable service like electric vehicles, train lighting, car-lighting and strating etc. But their life is shorted as compared to Plante plates.

9.16. Structure of Faure Plates

Usually, the problem of Faure type grid is relatively simple as compared to the Plante type. In the case of Faure plates, the grid serves simply as a support for the active material and a conductor for the current and as a means for distributing the current evenly over the active material. Unlike Plante plates, it is not called upon to serve as a kind of reservoir from which fresh active material is continuosly being formed for replacing that which is lost in the wear and tear of service. Hence, this makes possible the use of an alloy of lead and antimony which, as pointed out earlier, resists the attack of acid and 'forming' effect of current more effectively than pure lead and is additionally much harder and stiffer.

Because of the hardening effect of antimony, it is possible to construct very thin light plates which possess sufficient rigidity to withstand the expensive action of the positive active material. Simplest type of grid consists of a meshwork of veritcal and horizontal ribs intersecting each other thereby forming a number of rectangular spaces in which the paste can be pressed and allowed to set. Such a thin grid has the disadvantage that there is not much to 'key' in the paste and due to a great shock or vibration the pellets are easily 'started' and so fall out.

A much better support to the active material can be given by the construction illustrated in Fig. 9.9 which is known as 'basket' type or screened grid. The paste instead of being is isolated pellets forms a continuous sheet contained and supported by the horizontal ribs of the gird. With this arrangement the material can be very effectively keyed in.

Another type of grid structure used in pasted plates is shown in Fig. 9.10.

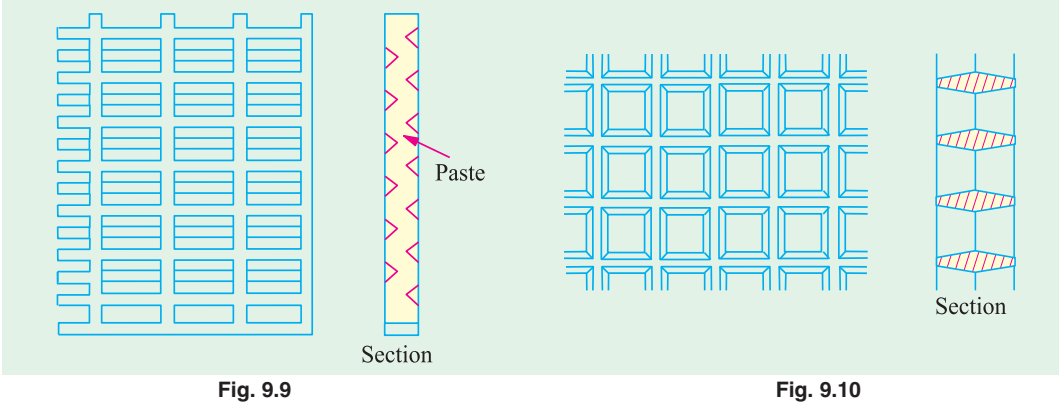


Fig. 9.9

Fig. 9.10

9.17. Comparison : Plante and Faure Plates

1. Plante plates have a longer life and can withstand rapid discharging (as in traction work) better than Faure's.
2. They are less liable to disintegration when in use then Faure's plates.
3. They are heavier and more expensive than Faure plates.
4. Plante plates have less capacity-to-weight ratio, values being 12 to 21 Ah per kg of plate, the corresponding values for Faure plate being 65 to 90 Ah/kg.

9.18. Internal Resistance and Capacity of a Cell

The secondary cell possesses internal resistance due to which some voltage is lost in the form of potential drop across it when current is flowing. Hence, the internal resistance of the cell has to be kept to the minimum.

One obvious way to lessen internal resistance is to increase the size of the plates. However, there

is a limit to this because the cell will become too big to handle. Hence, in practice, it is usual to multiply the number of plate inside the cell and to join all the negative plates together and all the positives ones together as shown in Fig. 9.11.

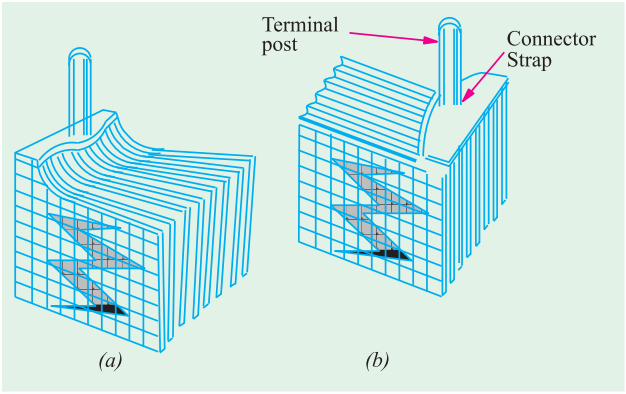


Fig. 9.11

The effect is equivalent to joining many cells in parallel. At the same time, the length of the electrolyte between the electrodes is decreased with a consequent reduction in the internal resistance.

The 'capacity' of a cell is given by the product of current in amperes and the time in hours during which the cell can supply current until its e.m.f. falls to 1.8 volt. It is expressed in ampere-hour (Ah).

The interlacing of plates not only decreases the internal resistance but additionally increases the capacity of the cell also. There is always one more negative plate than the positive paltes *i.e.* there is a negative plate at both ends. This gives not only more mechanical strength but also assures that both sides of a positive plate are used.

Since in this arrangement, the plates are quite close to each other, something must be done to make sure that a positive plate does not touch the negative plate otherwise an internal short-circuit will take place. The separation between the two plates is achieved by using separators which, in the case of small cells, are made of treated cedar wood, glass, wool mat, microporous rubber and mocroporous plastic and in the case of large stationary cells, they are in the form of glass rods.

9.19. Two Efficiencies of the Cell

The efficiency of a cell can be considered in two ways :

1. ***The quantity or ampere-hour (Ah) efficiency***
2. ***The energy or watt-hour (Wh) efficiency***

The Ah efficiency does not take into account the varying voltages of charge and discharge. The Wh efficiency does so and is always less than Ah efficiency because average p.d. during discharging is less than that during charging. Usually, during discharge the e.m.f. falls from about 2.1 V to 1.8 V whereas during charge it rises from 1.8 volt to about 2.5 V.

$$\text{Ah efficiency} = \frac{\text{amp-hour discharge}}{\text{amp-hour charge}}$$

The Ah efficiency of a lead-acid cell is normally between 90 to 95%, meaning that about 100 Ah must be put back into the cell for every 90-95 Ah taken out of it. Because of gassing which takes place during the charge, the Ah available for delivery from the battery decreases. It also decreases **(*i*)** due to self-discharge of the plates caused due to local reactions and **(*ii*)** due to leakage of current because of faulty insulation between the cells of the battery.

The Wh efficiency varies between 72-80%.

If Ah efficiency is given, Wh efficiency can be found from the following relation :

$$\text{Wh efficiency} = \text{Ah efficiency} \times \frac{\text{average volts on discharge}}{\text{average volts on charge}}$$

From the above, it is clear that anything that increases the charge volts or reduces the discharge volts will decrease Wh efficiency. Because high charge and discharge rates will do this, hence it is advisable to avoid these.

9.20. Electrical Characteristics of the Lead-acid Cell

The three important features of an accumulator, of interests to an engineer, are **(1)** voltage **(2)** capacity and **(3)** efficiency.

1. Voltage

The open-circuit voltage of a fully-charged cell is approximately 2.2 volt. This value is not fixed but depends on **(*a*)** length of time since it was last charged **(*b*)** specific gravity-voltage increasing with increase in sp. gravity and *vice versa*. If sp. gravity comes near to density of water *i.e.* 1.00 then voltage of the cell will disappear altogether **(*c*)** temperature-voltage increases, though not much, with increase in temperature.

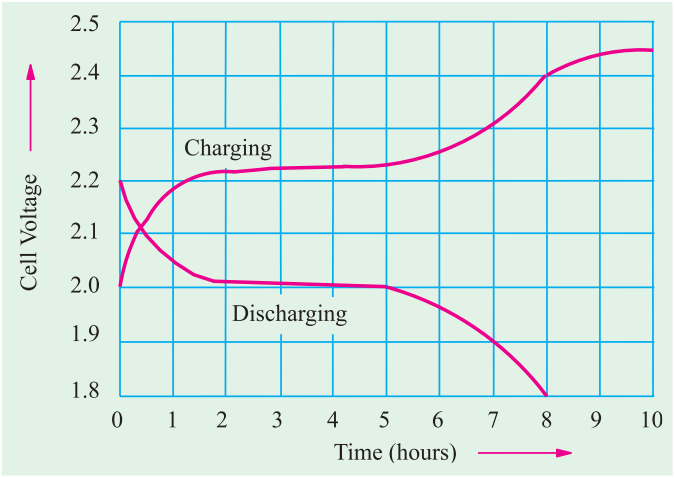


Fig. 9.12

The variations in the terminal p.d. of a cell on charge and discharge are shown in Fig. 9.12. The voltage-fall depends on the rate of discharge. Rates of discharge are generally specified by the number of hours during which the cell will sustain the rate in question before falling to 1.8 V. The voltage falls rapidly in the beginning (rate of fall depending on the rate of discharge), then vary slowly up to 1.85 and again suddenly to 1.8 V.

The voltage should not be allowed to fall to lower than 1.8 V, otherwise hard insoluble lead sulphate is formed on the plate which increases the internal resistance of the cell.

The general form of the voltage-time curves corresponding to 1-, 3- 50 and 10- hour rates of corresponding to the steady currents which would discharge the cell in the above mentioned times (in hour). It will be seen that both the terminal voltage and the rate at which the voltage and the rate at which the voltage falls, depend on the rate of discharge. The more rapid fall in voltage at higher rates of discharge is due to the rapid increase in the internal resistance of the cell.

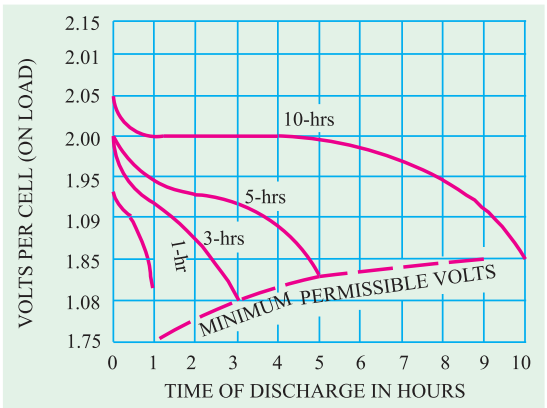


Fig. 9.13

During charging, the p.d. increases (Fig. 9.12). The curve is similar to the discharge curve reversed but is everywhere higher due to the increased density of H_2SO_4 in the pores of the positive plate.

2. Capacity

It is measured in amp-hours (Ah). The capacity is always given at a specified rate of discharge (10-hour discharge in U.K., 8-hour discharge in U.S.A.). However, motor-cycle battery capacity is based on a 20-hour rate (at 30° C). The capacity depends upon the following :

(*a*) *Rate of discharge.* The capacity of a cell, as measured in Ah, depends on the discharge rate. It decreases with increased rate of discharge. Rapid rate of discharge means greater fall in p.d. of the cell due to internal resistance of the cell. Moreover, with rapid discharge the weakening of the acid in the pores of the plates is also greater. Hence, the chemical change produced at the plates by 1 ampere for 10 hours is not the same as produced by 2 A for 5 hours or 4 A for 2.5 hours. It is found that a cell having a 100 Ah capacity at 10 hour discharge rate, has its capacity reduced to 82.5 Ah at 5-hour rate and 50 Ah at 1-hour rate. The variation of capacity with discharge rate is shown in Fig. 9.14.

(*b*) *Temperature.* At high temperature,

(*i*) chemical reactions within the cell take place more vigorously.

(*ii*) the resistance of the acid is decreased and

(*iii*) there is a battery diffusion of the electrolyte.

Hence, high temperature increases the capacity of the lead-acid cell. Apparently, it is better to operate the battery at a high temperature. However, at high temperatures :

(*a*) the acid attacks the antimony-lead alloy grid, terminal posts and wooden separators.

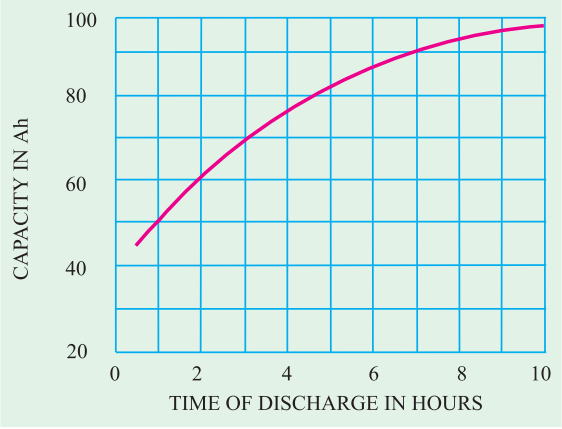


Fig. 9.14

(*b*) the paste is rapidly changed into lead sulphate. Sulphation is always accompanied by expansion of paste particularly at the positive plates and results in buckling and cracking of the grid.

Hence, it is not advisable to work batteries above 40° C.

As temperature is lowered, the speed of chemical reactions is decreased. Moreover, cell resistance also increases. Consequently, the capacity of the cell decreases with decrease in temperature till at freezing point the capacity is reduced to zero even though the battery otherwise be fully charged.

(*c*) *Density of electrolyte.* As the density of electrolyte affects the internal resistance and the vigour of chemical reaction, it has an important effect on the capacity. Capacity increases with the density.

(*d*) *Quantity of active material.* Since production of electricity depends on chemical action taking place within the cells, it is obvious that the capacity of the battery must depend directly upon the kind and amount of the active material employed. Consider the following calculations:

The gram-equivalent of lead is 103.6 gram and Faraday's constant is 96,500 coulombs which is = 96,500/3600 = 26.8 Ah. Hence, during the delivery of one Ah by the cell, the quantity of lead expended to form lead sulphate at the negative plate is 103.6/26.8 = 3.86 gram.

Similarly, it can be calculated that, at the same time, 4.46 gram of PbO_2 would be converted into lead sulphate at the positive plate while 3.66 gram of acid would be expended to form 0.672 gram of water. It is obvious that for obtaining a cell of a greater capacity, it is necessary to provide the plates with larger amounts of active material.

3. Efficiency

It has already been discussed in Art. 9.19

9.21. Battery Ratings

Following standards have been adopted, both by industry and government organisations to get a fair picture of battery quantity :

1. Ampere-hour Capacity

It is a function of the total plate area *i.e.* size of the individual plate multiplied by the number of plates. For measuring this capacity, the battery is discharged continuously for 20 hours and its current output supplied to a standard load is measured. Suppose that a battery delivers 4A current for 20 hours. Hence, its rating is 80 Ah which is stamped on the battery case.

2. Reserve Capacity

It is one of the newly-developed rating standards and is more realistic because it provides a double-check on the Ah figures. The capacity is given by the number of minutes a battery will tolerate a 25 A drain without dropping below 10.5 V. Higher this rating, better the battery.

3. Zero Cranking Power

It was the first cold weather rating and is applicable in relation to crafts which ply in freezing weather. This zero-degree performance test gives a valuable insight into battery quality. Large batteries are tested at a 300 A drain with battery chilled to –18°C (0°F) whereas smaller sizes are tested at half this value.

This test consists of two part ***(a)*** the battery is first chilled to –18°C (0°F) and the load applied for 5 second. Meanwhile, the voltage output is recorded. It is the first part of the zero-cranking-power rating. ***(b)*** The test is continued further till voltage drops to 5 V. The number of minutes it takes to reduce the voltage to 5 V forms the second half of the rating. Higher both the digits, better the battery quality.

4. Cold Cranking Power

This simple rating is applied to all 12-V storage batteries regardless of their size. The battery is loaded at –18°C (0°F) till the total voltage drops to 7.2 V. The output current in amperes is measured for 30 seconds. Higher the output, better the battery.

Example 9.9. *An alkaline cell is discharged at a steady current of 4 A for 12 hours, the average terminals voltage being 1.2 V. To restore it to its original state of charge, a steady current of 3 A for 20 hours is required, the average terminal voltage being 1.44 V. Calculate the ampere-hour (Ah) efficiency and Wh efficiency in this particular case.*

(Principles of Elect. Engg.-I, Jadavpur Univ.)

Solution. As discussed in Art. 9.19

$$\text{Ah efficiency} = \frac{\text{Ah of discharge}}{\text{Ah of charge}} = \frac{12 \times 4}{20 \times 3} = \mathbf{0.8 \quad or \quad 80\%}$$

$$\text{Wh efficiency} = \text{Ah effi.} \times \frac{\text{Av. volts on discharge}}{\text{Av. volts on charge}} = \frac{0.8 \times 1.2}{1.44} = \mathbf{0.667 \quad or \quad 66.7\%}$$

Example 9.10. *A discharged battery is charged at 8 A for 2 hours after which it is discharged through a resistor of R Ω. If discharge period is 6 hours and the terminal voltage remains fixed at 12 V, find the value of R assuming the Ah efficiency of the battery as 80%.*

Solution. Input amp-hours $= 8 \times 2 = 16$

Efficiency $= 0.8 \quad \therefore$ Ah output $16 \times 0.8 = 12.8$

Discharge current $= 12.8/6$ A $\quad \therefore \quad R = \dfrac{12}{12.8/6} = \dfrac{6 \times 12}{12.8} = \mathbf{5.6\ \Omega}$

9.22. Indications of a Full-charged Cell

The indications of a fully-charged cell are :

(i) gassing (ii) voltage (iii) specific gravity and (iv) colour of the plates.

(i) Gassing

When the cell is fully charged, it freely gives off hydrogen at cathode and oxygen at the anode, the process being known as "Gassing". Gassing at ***both*** plates indicates that the current is no longer doing any useful work and hence should be stopped. Moreover, when the cell is fully charged, the electrolyte assumes a milky appearance.

(ii) Voltage

The voltage ceases to rise when the cell becomes fully-charged. The value of the voltage of a fully-charged cell is a variable quantity being affected by the rate of charging, the temperature and specific gravity of the electrolyte etc. The approximate value of the e.m.f. is 2.1 V or so.

(iii) Specific Gravity of the Electrolyte

A third indication of the state of charge of a battery is given by the specific gravity of the electrolyte. We have seen from the chemical equations of Art. 9.9, that during discharging , the density of electrolyte decreases due to other production of water, whereas it increases during charging due to the

absorption of water. The value of density when the cell is fully charged is 1.21 and 1.18 when discharged up to 1.8 V. Specific gravity can be measured with a suitable hydrometer which consists of a float, a chamber for the electrolyte and a squeeze bulb.

(*iv*) Colour

The colour plates, on full charge is deep chocolate brown for positive plate and clear slate gray for negative plate and the cell looks quite brisk and alive.

9.23. Applications of Lead-acid Batteries

Storage batteries are these days used for a great variety and range of purposes, some of which are summarised below :

1. In Central Stations for supplying the whole load during light load periods, also to assist the generating plant during peak load periods, for providing reserve emergency supply during periods of plant breakdown and finally, to store energy at times when load is light for use at time when load is at its peak value.

2. In private generating plants both for industrial and domestic use, for much the same purpose as in Central Stations.

3. In sub-stations, they assist in maintaining the declared voltage by meeting a part of the demand and so reducing the load on and the voltage drop in, the feeder during peak-load periods.

4. As a power source for industrial and mining battery locomotives and for road vehicles like cars and trucks.

5. As a power source for submarines when submerged.

6. Marine applications include emergency or stand-by duties in case of failure of ship's electric supply, normal operations where batteries are subjected to regular cycles of charge and discharge and for supplying low-voltage current to bells, telephones, indicators and warning systems etc.

7. For petrol motor-car starting and ignition etc.

8. As a low voltage supply for operating purposes in many different ways such as high-tension switchgear, automatic telephone exchange and repeater stations, broadcasting stations and for wireless receiving sets.

9.24. Voltage Regulators

As explained in Art. 9.20, the voltage of a battery varies over a considerable range while under discharge. Hence, it is necessary to find some means to control the battery voltage upto the end so as to confine variations within reasonable limits – these limits being supplied by the battery.

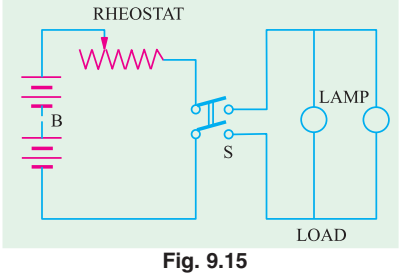


Fig. 9.15

The voltage control systems may be hand-operated or automatic. The simplest form of hand-operated control consists of a rheostat having a sufficient number of steps so that assistance can be inserted in the circuit when battery is fully charged and gradually cut out as the discharge continues, as shown in Fig. 9.15.

The above system can be designed for automatic operation as shown in Fig. 9.16. A rise in voltage results in the release of pressure on the carbon block rheostat, thereby increasing its resistance whereas a fall in voltage results in increasing the pressure on the block thereby decreasing its resistance. By this automatic variation of control resistance, variations in battery voltage are automatically controlled.

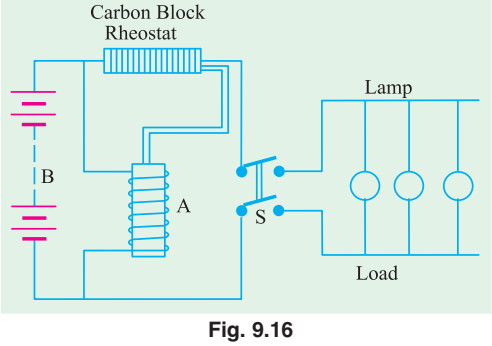


Fig. 9.16

9.25. End-cell Control System

The use of rheostat for controlling the battery

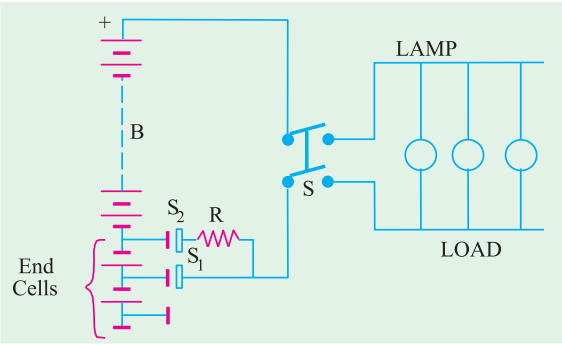


Fig. 9.17

voltage is objectionable especially in large-capacity installations where the I^2R loss would be considerable. Hence other more economical systems have been developed and put into use. One such system is the end-cell control system. It consists of suitable regulator switches which cut one or more of a selected number of cells out of the circuit when the battery is fully charged and into the circuit again as the discharge continues. To make the process of cutting cells in and out at the battery circuit simple, the group of cells selected for this control is situated at one end of the battery where from it derives the name ***end cell.*** By moving the contact arm of the switch to the left or right, cells are cut in or out of the discharge circuit and so the voltage is varied accordingly.

For making the end-cell switch operate without opening the circuit or shortcircuiting the cells during its passage from one cell to another, an auxiliary constraint S_2 is employed. S_2 prevents the circuit from being open entirely but has sufficient resistance R between it and the main contact arm S_1 to prevent any objectionably large current to flow on short-circuit. The above mechanism usually incorporates devices for preventing the stoppage of the switch in the short-circuit position.

9.26. Number of End-cells

For maintaining a supply voltage of V volts from a battery of lead-acid cells when the latter are approaching their discharge voltage of 1.83 (depending on the discharge rate), the number of cells required is $V/1.83$. When the battery is fully charged with each cell having an e.m.f. of 2.1 V, then the number of cells required is $V/2.1$. Hence, the number of end-cells required is $(V/1.83 - V/2.1)$. These are connected to a regulating switch which adds them in series with the battery one or two at a time, as the discharge proceeds.

9.27. Charging Systems

In various installations, batteries are kept floating on the line and are so connected that they are being charged when load demands are light and automatically discharged during peak periods when load demands are heavy or when the usual power supply fails or is disconnected. In some other installations, the battery is connected to the feeder circuit as and when desired, allowed to discharge to a certain point, then removed and re-charged for further requirements.

For batteries other than the 'floating' and 'system-governed' type, following two general methods (though there are some variations of these) are employed.

(i) The Constant-current System and ***(ii) The Constant-voltage System.***

9.28. Constant-current System

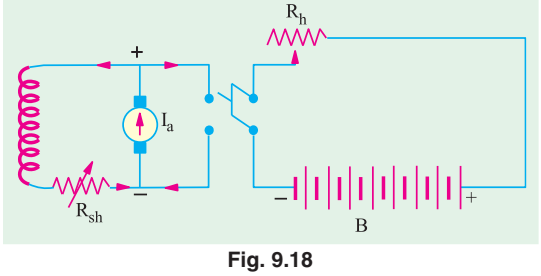


Fig. 9.18

In this method, the charging current is kept constant by varying the supply voltage to overcome the increased back e.m.f. of cells. If a charging booster (which is just a shunt dynamo directly driven by a motor) is used, the current supplied by it can be kept constant by adjusting its excitations. If charged on a d.c. supply, the current is controlled by varying the rheostat connected in the circuit. The value of charging current should be so chosen that there would be no

excessive gassing during final stages of charging and, also, the cell temperature does not exceed 45°C. This method takes a comparatively longer time.

9.29. Constant-voltage System

In this method, the voltage is kept constant but it results in very large charging current in the beginning when the back e.m.f. of the cells is low and a small current when their back e.m.f. increases on being charged.

With this method, time of charging is almost reduced to half. It increases the capacity by approximately 20% but reduces the efficiency by 10% or so.

Calculations

When a secondary cell or a battery of such cells is being charged, then the emf of the cells acts in opposition to the applied voltage. If V is the supply voltage which sends a charging current of I against the back e.m.f. E_b, then input is VI but the power spent in overcoming the opposition (Fig. 9.19) is E_bI. This power E_bI is converted into the chemical energy which is stored in the cell. The charging current can be found from the following equation :

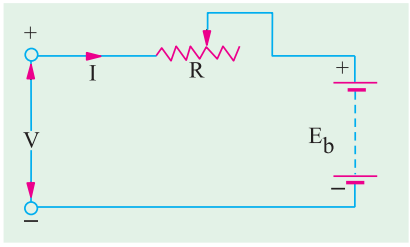


Fig. 9.19

$$I = \frac{V - E_b}{R}$$

where R = total circuit resistance including internal resistance of the battery

I = charging current

By varying R, the charging current can be kept constant throughout.

Example 9.11. *A battery of accumulators of e.m.f. 50 volt and internal resistance 2 Ω is charged on 100 volt direct means. What series resistance will be required to give a charging current of 2 A?*

If the price of energy is 50 paise per kWh, what will it cost to charge the battery for 8 hours and what percentage of energy supplied will be used in the form of heat ?

Solution. Applied voltage = 100 V; Back e.m.f. of the battery = 50 V

Net charging voltage = 100 –50 = 50 V

Let R be the required resistance, then 2 = $50/(R + 2)$; $R = 46/2 = 23$ Ω

Input for eight hours = $100 \times 2 \times 8 = 1600$ Wh = 1.6 kWh

Cost = 50×1.5 = **80 paise;** Power wasted on total resistance = $25 \times 2^2 = 100$ W

Total input = $100 \times 2 = 200$ W : Percentage waste = $100 \times 100/200$ = **50 %**

Example 9.12. *A 6-cell, 12-V battery is to be charged at a constant rate of 10 A from a 24-V d.c. supply. If the e.m.f. of each cell at the beginning and end of the charge is 1.9 V and 2.4 V, what should be the value of maximum resistance to be connected in series with the battery. Resistance of the battery is negligible.*

Solution. Beginning of Charging

Total back e.m.f. of battery = $6 \times 1.9 = 11.4$ volt

Net driving voltage = 24 –11.4 = 12.6 V ; $R_{max} = 12.6/10$ = **1.26 Ω**

End of Charging

Back e.m.f. of battery = $6 \times 2.4 = 14.4$ volt

Net driving voltage = 24 –14.4 = 9.6 V; $R_{min} = 9.6/10$ = **0.96 Ω**

Example 9. 13. *Thirty accumulators have to be charged from their initial voltage of 1.8 V using a direct current supply of 36 volt. Each cell has an internal resistance of 0.02 Ω and can be charged at 5 amperes. Sketch a circuit by which this can be done, calculating the value of any resistance or resistances used. What will be the current taken from the mains towards the end of the charging period when the voltage has risen to 2.1 volt per cell ?*

Solution. Since the supply voltage (36 V) is less than the back e.m.f. of the 30 cell battery (54 V), hence the cells are divided into two equal groups and placed in parallel across the supply for charging as shown in Fig. 9.20. It would be economical to use a separate resistance R in series with each group.

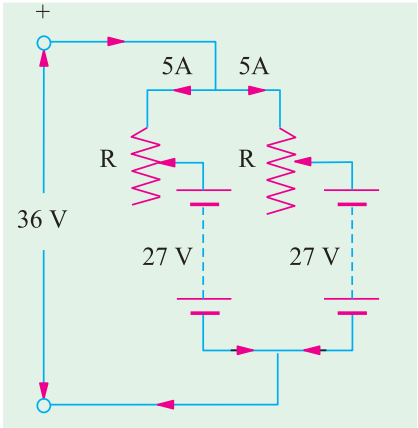


Fig. 9.20

Here $V = 36$ V, $E_b = 15 \times 1.8 = 27$ V

Internal resistance of each parallel group

$= 15 \times 0.02 = 0.3\ \Omega$

Charging current = 5A $\therefore 5 = \dfrac{36 - 27}{R + 0.3}$

$R = \mathbf{1.5\ \Omega}$

Now, when the voltage per cell becomes 2.1 V, then back e.m.f. of each parallel group = 15 × 2.1 = 31.5 V

$\therefore$ Charging current $= \dfrac{36 - 31.5}{1.5 + 0.3} = \mathbf{2.5\ A}$

Example 9.14. *A battery of 60 cells is charged from a supply of 250 V. Each cell has an e.m.f. of 2 volts at the start of charge and 2.5 V at the end. If internal resistance of each cell is 0.1 Ω and if there is an external resistance of 19 Ω in the circuit, calculate* ***(a)*** *the initial charging current* ***(b)*** *the final charging current and* ***(c)*** *the additional resistance which must be added to give a finishing charge of 2 A rate.*

Solution. ***(a)*** Supply voltage $V = 250$ V

Back e.m.f. of the battery E_b at start = 60 × 2 = 120 V and at the end = 60 × 2.5 = 150 V

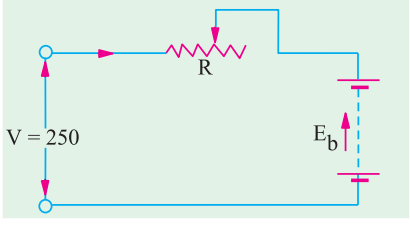


Fig. 9.21

Internal resistance of the battery $= 60 \times 0.1 = 6\ \Omega$

Total circuit resistance $= 19 + 6 = 25\ \Omega$

(a) Net charging voltage at start $= 250 - 120 = 130$ V

$\therefore$ Initial charging current $= 130/25 = \mathbf{5.2\ A}$

(b) Final charging current $= 100/25 = \mathbf{4\ A}$

(c) Let R be the external resistance, then

$2 = \dfrac{100}{R + 6} \quad \therefore\ R = 88/2 = 44\ \Omega$

$\therefore$ Additional resistance required = 44 – 19 = **25 Ω**

Example 9.15. *Two hundred and twenty lamps of 100 W each are to be run on a battery supply at 110 V. The cells of the battery when fully charged have an e.m.f. of 2.1 V each and when discharged 1.83 V each. If the internal resistance per cell is 0.00015 Ω* ***(i)*** *find the number of cells in the battery and* ***(ii)*** *the number of end cells. Take the resistance of the connecting wires as 0.005 Ω*

Solution. Current drawn by lamps = 220 × 100/110 = 200 A

Voltage drop on the resistance of the connecting wires = 0.005 × 200 = 1.0 V

Battery supply voltage = 110 + 1 = 111 V

Terminal voltage/cell when fully charged and supplying the load

= 2.1 –(200 × 0.00015) = 2.08 V

Terminal voltage/cell when discharged

= 1.83 –(200 × 0.00015) = 1.8 V

(*i*) No. of cells in the battery = 111/2.08

= 53.4 say, **54**

(*ii*) No. of cells required when discharged

= 111/1.8 = 62

Hence, number of end cells = 62 –54 = **8**

The connections are shown in Fig. 9.22.

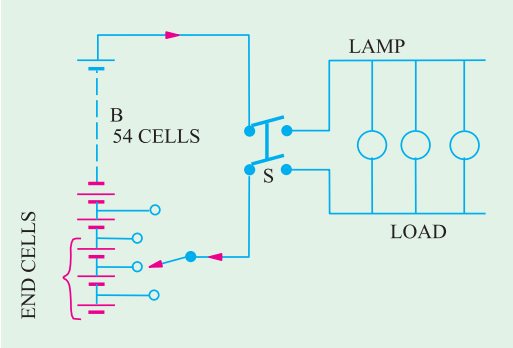


Fig. 9.22

Example 9.16. *A storage battery consists of 55 series-connected cells each of internal resistance 0.001 Ω and e.m.f. 2.1 V. Each cell consists of 21 plates, ten positive and eleven negative, each plate measuring 20 × 25 cm. If full-load current per cell is 0.01 A per cm^2 of positive plate surface, find* **(*i*)** *full-load terminal voltage of the battery and* **(*ii*)** *power wasted in the battery if the connectors have a total resistance of 0.025 Ω*

Solution. Since both sides of a positive plate are utilized, the area of both sides will be taken into consideration.

Total area (both sides) of ten positive plates = 2 × 20 × 25 × 10 = 10,000 cm^2

Full load current = 10,000 × 0.01 = 100 A

Voltage drop in battery and across connectors = 100 [(55 × 0.001) + 0.025] = 8 V

Battery e.m.f. = 55 × 1.2 = 115.5 V

(*i*) Battery terminal voltage on full-load = 115.5 –8 = **107.5 V**

(*ii*) Total resistance = (55 × 0.001) + 0.025 = 0.08 Ω; Power loss = 100^2 × 0.08 = **800 W.**

Example 9.17. *A charging booster (shunt generator) is to charge a storage battery of 100 cells each of internal resistance 0.001 Ω. Terminal p.d. of each cell at completion of charge is 2.55 V. Calculate the e.m.f. which the booster must generate to give a charging current of 20 A at the end of charge. The armature and shunt field resistances of the generator are 0.2 and 258 Ω respectively and the resistance of the cable connectors is 0.05 Ω*

Solution. Terminal p.d. per cell = 2.55 volt

The charging voltage across the battery must be capable of overcoming the back e.m.f. and also to supply the voltage drop across the internal resistance of the battery.

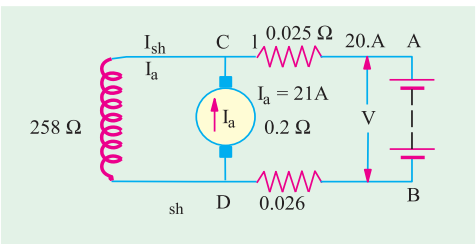


Fig. 9.23

Back e.m.f. = 100 × 2.55 = 255 V

Voltage drop on internal resistance

= 100 × 0.001 × 20 = 2 V

∴ P.D. across points *A* and *B* = 255 + 2 = 257 V

P.D. across terminals *C* and *D* of the generator

= 257 + (20 × 0.05) = 258 V

∴ I_{sh} = 258/258 = 1A; I_a = 20 + 1 = 21 A

∴ I_aR_a = 21 × 0.2 = 4.2 V

∴ Generated e.m.f. = 258 + 4.2 = **262.2 V**

Tutorial Problems No. 9.2

1. A 60-cell storage battery having a capacity of 360 Ah takes 8 hours when charged by a dc generator at a voltage of 220 V. Calculate the charging current and the range of the rheostat required to ensure a constant charging current. The emf of each cell is 1.8 V at the beginning of charging and 2.7 V at the end of the charging. Ignore the internal resistance of the cell.

[45 A; 2.45 to 1.29 Ω]

2. A storage battery consists of 55 series connected cells each of internal resistance 0.001 Ω and e.m.f. 2.1 V. Each cell consists of 21 plates, ten positive and eleven negative, each plate measuring 20 × 25 cm. If full-load current per cell is 0.01 A per cm^2 of positive plate surface, find (*i*) full-load terminal voltage of the battery and (*ii*) power wasted in the battery if the connectors have a total resistance of 0.025 Ω

[(*i*) 107.5 V (*ii*) 800 W]

9.30. Trickle Charging

When a storage battery is kept entirely as an emergency reserve, it is very essential that it should be found fully charged and ready for service when an emergency arises. Due to leakage action and other open-circuit losses, the battery deteriorates even when idle or on open-circuit. Hence, to keep it fresh, the battery is kept on a trickle charge. The rate of trickle charge is small and is just sufficient to balance the open-circuit losses. For example, a standby battery for station bus-bars capable of giving 2000 A for 1 hour or 400 Ah at the 10-hr rate, will be having a normal charging rate of 555 A, but a continuous 'trickle' charge of 1 A or so will keep the cells fully charged (without any gassing) and in perfect condition. When during an emergency, the battery gets discharged, it is recharged at its normal charging rate and then is kept on a continuous trickle charge.

9.31. Sulphation-Causes and Cure

If a cell is left incompletely charged or is not fully charged periodically, then the lead sulphate formed during discharge, is not converted back into PbO_2 and Pb. Some of the unreduced $PbSO_4$ which is left, gets deposited on the plates which are then said to be sulphated. $PbSO_4$ is in the form of minute crystals which gradually increase in size if not reduced by thoroughly charging the cells. It increases the internal resistance of the cell thereby reducing its efficiency and capacity. Sulphation also sets in if the battery is overcharged or left discharged for a long time.

Sulphated cells can be cured by giving them successive overcharges, for which purpose they are cut out of the battery during discharge, so that they can get two charges with no intervening discharge. The other method, in which sulphated cells need not be cut out of the battery, is to continue charging them with a ***'milking booster'*** even after the battery as a whole has been charged. A milking booster is a motor-driven low-voltage dynamo which can be connected directly across the terminals of the sulphated cells.

9.32. Maintenance of Lead-acid Cells

The following important points should be kept in mind for keeping the battery in good condition:

1. Discharging should not be prolonged after the minimum value of the voltage for the particular rate of discharge is reached.
2. It should not be left in discharged condition for long.
3. The level of the electrolyte should always be 10 to 15 mm above the top of the plates which must not be left exposed to air. Evaporation of electrolyte should be made up by adding distilled water occasionally.
4. Since acid does not vaporise, none should be added.

5. Vent openings in the filling plug should be kept open to prevent gases formed within from building a high pressure.
6. The acid and corrosion on the battery top should be washed off with a cloth moistened with baking soda or ammonia and water.
7. The battery terminals and metal supports should be cleaned down to bare metal and covered with vaseline or petroleum jelly.

9.33. Main Operated Battery Chargers

A battery charger is an electrical device that is used for putting energy into a battery. The battery charger changes the a.c. from the power line into d.c. suitable for charger. However, d.c. generator and alternators are also used as charging sources for secondary batteries.

In general, a mains-operated battery charger consists of the following elements :

1. A step-down transformer for reducing the high a.c. mains voltage to a low a.c. voltage.
2. A half-wave or full-wave rectifier for converting alternating current into direct current.
3. A charger-current limiting element for preventing the flow of excessive charging current into the battery under charge.
4. A device for preventing the reversal of current *i.e.* discharging of the battery through the charging source when the source voltage happens to fall below the battery voltage.

In addition to the above, a battery charger may also have circuitry to monitor the battery voltage and automatically adjust the charging current. It may also terminate the charging process when the battery becomes fully charged. However, in many cases, the charging process is not totally terminated but only the charging rate is reduced so as to keep the battery on trickle charging. These requirements have been illustrated in Fig. 9.24.

Most of the modern battery chargers are fully protected against the following eventualities :

(*a*) They are able to operate into a short-circuit.
(*b*) They are not damaged by a reverse-connected battery.
(*c*) They are operated into a totally flat battery.
(*d*) They can be regulated both for current and voltage.

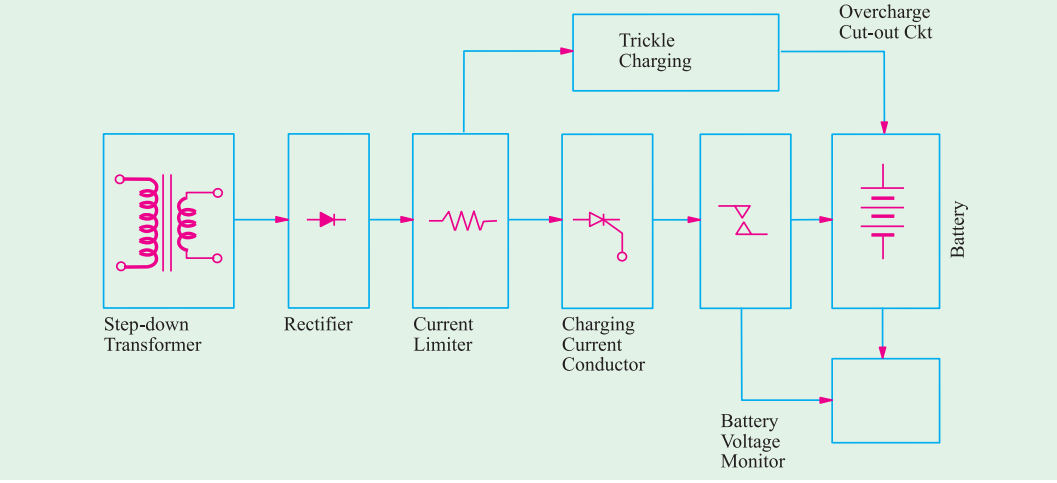


Fig. 9.24

9.34. Car Battery Charger Using SCR

Fig. 9.25 shows the circuitry of a very simple lead-acid battery charger which has been provided

protection from load short-circuit and from reverse battery polarity. The *SCR* is used as a half-wave rectifier as well as switching element to terminate the high-current charging process when battery gets fully-charged.

Working

The *SCR* acts as a half-wave rectifier during only the positive half-cycles of the secondary voltage when point *M* in Fig. 9.25 is at a positive potential. The *SCR* does not conduct during the negative half-cycle of the secondary voltage when point *M* achieves negative potential. When *M* is at positive potential, the *SCR* is triggered into conduction because of the small gate current I_g passing via R_1 and diode D_1. In this way, the charging current *I* after passing through R_5 enters the battery which is being charged.

In the initial state, when the battery voltge is low, the potential of point *A* is also low (remember that R_3, R_4 and preset resistor R_6 are connected across the battery via R_5) which means that the forward bias on the base of transistor *T* is not sufficient to make it conduct and thereby stop the conduction of *SCR*. Hence, *SCR* keeps conducting , consequently, keeps charging the battery through the current limiting resistor R_5.

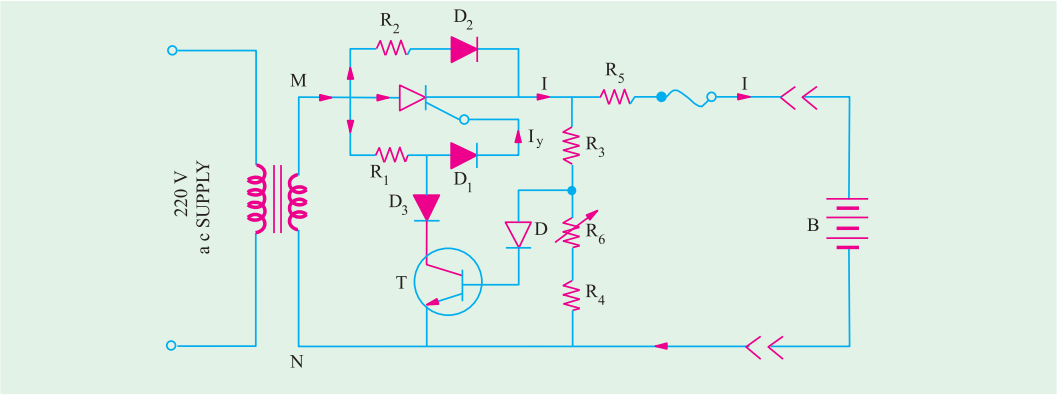


Fig. 9.25

As the battery gets progressively charged, its voltage rises and when it becomes fully charged, the potential of point *A* increases thereby increasing the forward bias of *T* which starts conducting. In that case, *T* bypasses the triggering gate current of the *SCR* via R_1 and D_3. Since the *SCR* can no longer be triggered, the charging process stops. However, a small trickle charging current keeps flowing via. R_2 and D_2. The function of diode D_2 is to prevent reverse flow of the current through the battery when point *M* has negative potential during the negative cycle of the secondary voltage. The value of trickle charging current is determined by R_2 because R_5 has a fixed but small value. The resistor R_5 also limits the flow of excessive charging current when the charger is connected to a completely dead battery.

The charger described above is not suitable for fast charging because it utilizes half-wave rectification. Most of the mains-operated chargers working on a single-phase supply use a full-wave rectifier consisting of a center-tapped tarnsformer and two diodes or a bridge circuit using for diodes.

9.35. Automobile Battery Charger Using Full-wave Rectifier

The battery charger shown in Fig. 9.26, is used to recharge run-down lead-acid batteries in automobiles without removing them from their original mountings and without any need for constant attention. When the battery is fully charged, the circuit automatically switches from charging current to trickle charging and an indicator lamp lights up to provide a visual indication of this condition.

As shown in Fig. 9.26, diodes D_1, and D_2 form a full-wave rectifier to provide pulsating direct current for charging the battery. The battery is charged through the *SCR* which is also used as switch to terminate the charging process when the battery becomes fully charged. The two transistors T_1 and T_2 together form an electronic switch that has two stable states *i.e.* the *ON* state in which T_1 and T_2 conduct and the OFF state in which T_1 and T_2 do not conduct. The ON-OFF state of this switch is decided by the battery voltage and setting of the "current adjust" potentiometer R_6.

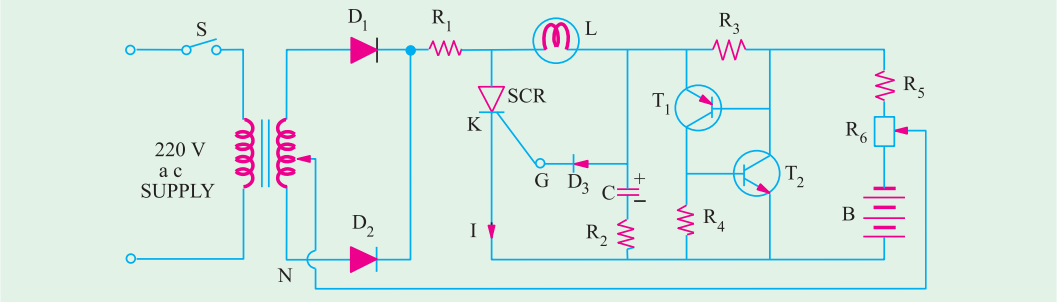


Fig. 9.26

Working

When switch *S* is turned on, the full-wave rectified output of D_1 and D_2 charges capacitor *C* through R_1, lamp *L* and R_2. In a very short time, capacitor voltage rises high enough to make diode D_3 conduct the gate current thereby triggering *SCR* into conduction during each half-cycle of the output voltage. Hence, full charging current is passed through the cathode *K* of the *SCR* to the positive terminal of the battery whose negative terminal is connected directly to the center tap of the step-down transformer. Resistor R_1 limits the charging current to a safe value in order to protect the rectifier diodes D_1 and D_2 in case the load happens to be a "dead" battery.

When the battery is being charged and has low voltage, the two transistors T_1 and T_2 remain in the non-conducting state. However, when the battery voltage rises and finally the battery becomes fully-charged, the two transistors T_1 and T_2 (which form a regenerative switch) are triggered into conduction at a point set by R_6. In this way, T_1 and T_2 provide a low-impedance discharge path for *C*. Hence, *C* discharges through R_2 and the T_1 –T_2 switch, thereby cutting off the gate current of the *SCR* which stops conducting thereby terminating the battery charge. Thereafter, small trickle charge current keeps on flowing into the battery via *L* and the regenerative switch formed by T_1 and T_2. A glowing lamp *L* indicates that the battery is under trickle charging.

Fig. 9.27 shows the same circuit as shown in Fig. 9.26 except that the two-diode full-wave rectifier has been replaced by a full-wave bridge rectifier using four diodes.

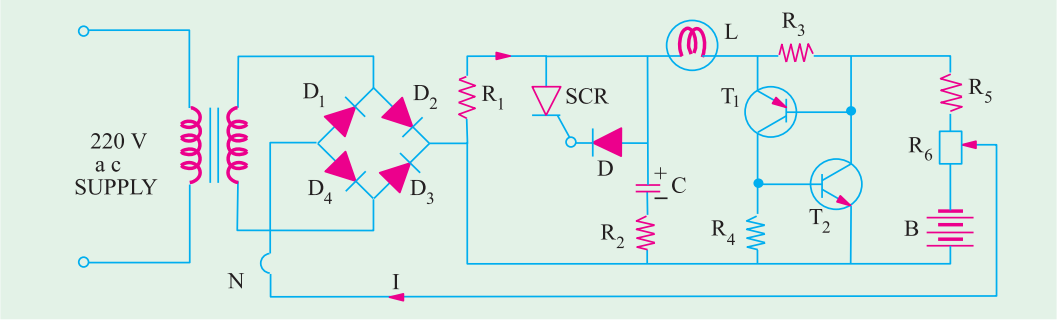


Fig. 9.27

9.36. Static Uninterruptable Power Systems (UPS)

The function of a *UPS* is to ensure absolute continuity of power to the computerised control systems thereby protecting critical equipment from electrical supply failure. A *UPS* makes it possible to provide a 'clean' reliable supply of alternating current free of sags of surges in the line voltage, frequency variation, spikes and transients. *UPS* systems achieve this by rectifying the standard mains supply, using the direct current to charge the standby battery and to produce 'clean' alternating current by passing through an inverter and filter system.

Components of a *UPS* System

The essential components of a *UPS* system as shown in Fig. 9.28 are as under :

1. A rectifier and thyristor-controlled battery charger which converts the *AC* input into regulated *DC* output and keeps the standby battery fully charged.
2. A standby battery which provides *DC* input power to inverter during voltage drops or on failure of the normal mains *AC* supply.
3. An inverter which converts *DC* to clean *AC* thus providing precisely regulated output voltage and frequency to the load as shown.

Working

As shown in Fig. 9.28 the main flow of energy is from the rectifier to the inverter with the standby battery kept on 'float'. If the supply voltage falls below a certain level or fails completely, the battery output to the inverter maintains a clean a.c. supply. When the mains power supply is resorted, the main energy flow against starts from the rectifier to the inverter but, in addition, the rectifier recharges the battery. When the standby battery gets fully charged, the charging current is automatically throttled back due to steep rise in the back e.m.f. of the battery . An automatic/manual bypass switch is used to connect the load either directly to the mains a.c. supply or to the inverter a.c. supply.

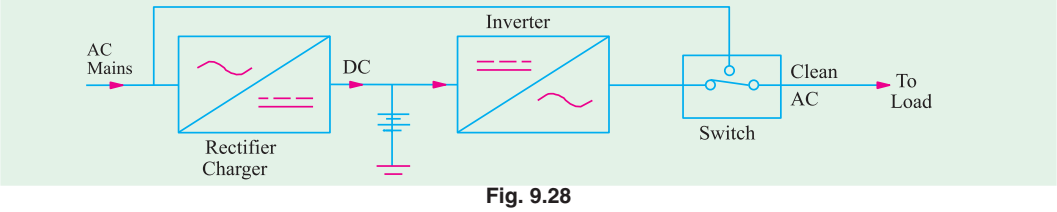


Fig. 9.28

Depending on the application, the voltage of the *UPS* standby battteries may be anywhere between 12 V and 400 V. Typical values are 24 V, 48 V, 110 V and 220 V with currents ranging from a few amperes to 2000 A. Fig. 9.28 shows Everon 4-kVA on-line *UPS* system which works on 170 V-270 V a.c. input and provides an a.c. output voltage of 230 V at 50 Hz frequency with a voltage stability of ± 2% and frequency stability of ± 1%. It has zero change over time and has audio beeper which indicates mains fail and battery discharge. It provides 100% protection against line noise, spikes, surges and radio frequency interference. It is manufactured by Everon Electro Systems Pvt. Ltd. New Delhi.

9.37. Alkaline Batteries

Such batteries are ideally suited for portable work. Like lead-acid cells, the alkaline cells also consist of positive and negative plates immersed in an electrolyte. The plates and the electrolyte are placed in a suitable-container.

The two types of alkaline batteries which are in general use are :

(i) nickel-iron type of Edison type.

(ii) nickel-cadmium type of Jungnor type which is commercially known as NIFE battery.

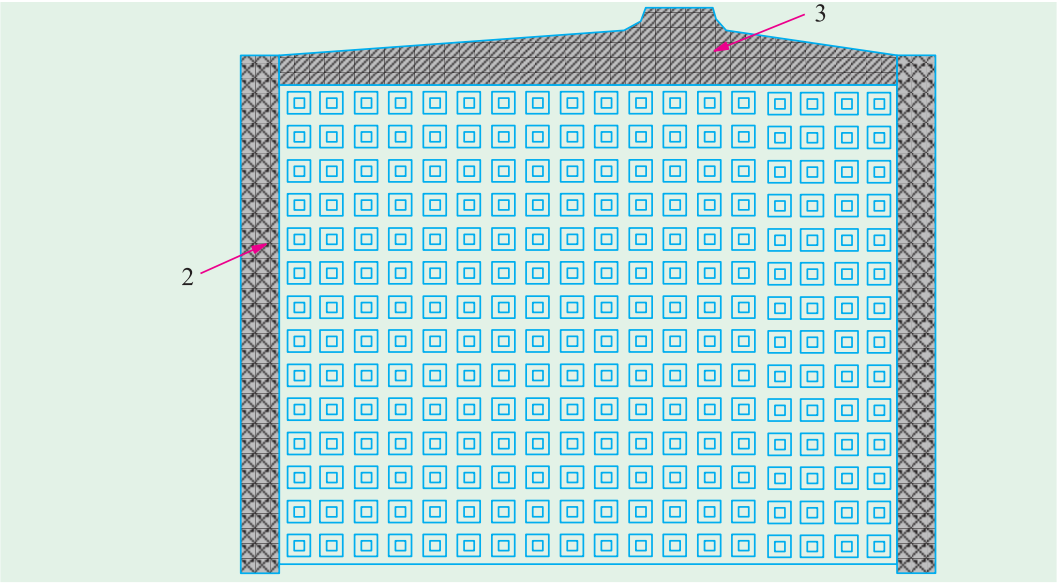


Fig. 9.29

Another alkaline battery which differs from the above only in the mechanical details of its plates is known as Alkum battery which uses nickel hydroxide and graphite in the positive plates and a powdered alloy of iron and chromium in the negative plates.

Silver-zinc type of alkaline batteries are also made whose active material for the positive plate is silver oxide (Ag_2O) and for negative plate is zinc oxide and zinc powder. The electrodes or plates of the alkaline cells are designed to be either of the enclosed-pocket type or open-pocket type. In the case of enclosed-pocket type plates, the active material is inside perforated metal envelopes whereas in the other type, the active material is outside directly in contact with the electrolyte. As shown in Fig. 9.29, the active material of the enclosed-pocket type plates is enclosed in nickel-plated perforated steel pockets or packs which are pressed into the steel frames of the plates.

The open-pocket type plates are made of the following three materials :

(i) metal-ceramic plate-the frame of the plate is a nickel-plated steel grid with the active material pressed in under a pressure of 800 to 1900 kg/cm^2.

(ii) foil plate-the base of such a plate is a thin nickel foil coated with a layer of nickel suspension deposited by a spray technique.

(iii) pressed plates–the base member of these plates is a nickel–plated pressed steel grid. The active material is pressed into them at a pressure of about 400 kg/cm^2.

9.38. Nickel-iron and Edison Batteries

There is revived interest in the nickel-iron battery because it seems to be one of the few systems which may be developed into a high-energy density battery for electric vehicles. Since long the two main designs for this battery have been the tubular positive type and the flat pocket plate type although cells with sintered type negative are also being manufactured.

The active materials in a nickel-iron cell are :

(i) Nickel hydroxide $Ni(OH)_4$ or apple green nickel peroxide NiO_2 for the positive plate. About 17 per cent of graphite is added to increase conductivity. It also contains an activating additive barium hydroxide which is about 2 per cent of the active material. This additive increases the service life of the plates.

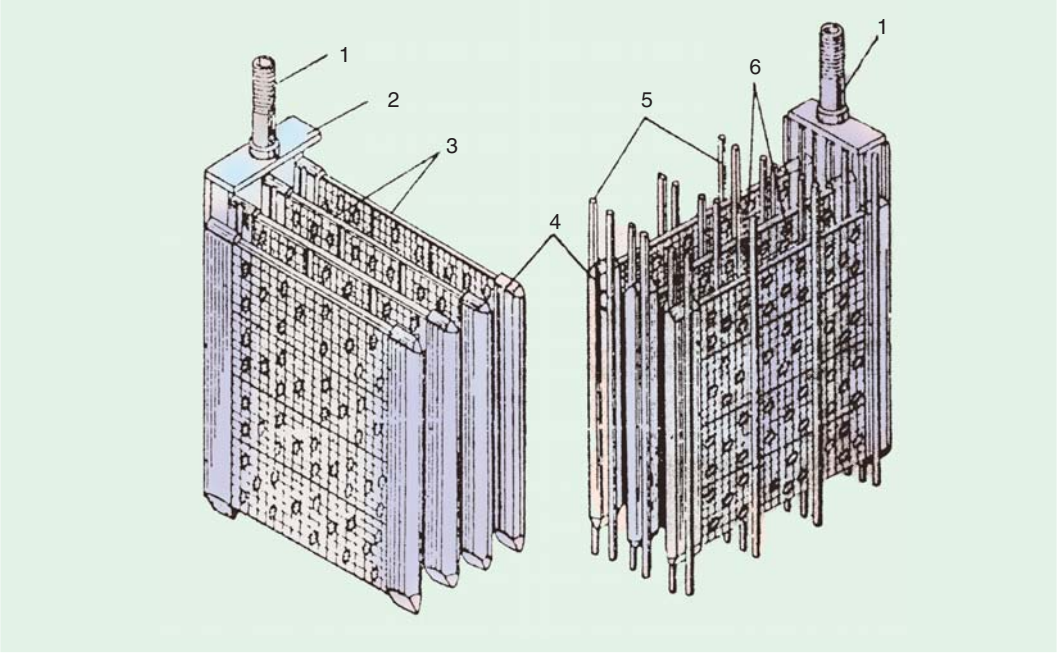


Fig. 9.30 Plates groups of an alkaline cell

(*a*) + ve group (*b*) – ve group, 1–terminal post 2–connecting strap, 3–plates. 4–plates side members. 5–ebonite spacer sticks. 6–pockets.

(ii) powdered iron and its oxides for the negative plate. Small quantities of nickel sulphate and ferrous sulphide are added to improve the performance of the coil.

(iii) the electrolyte is 21 per cent solution of caustic potash KOH (potassium hydrate) to which is added a small quantity of lithium hydrate LiOH for increasing the capacity of the cell.

As shown in Fig. 9.30, plates of the same polarity with their pockets filled, are assembled into cells groups for which purpose they are welded to a common strap having a threaded post.

The number of negative plates is one more than the positive plates. The extreme negative plates are electrically connected to the container. Ebonite separating sticks are placed between the positive and negative plates to prevent any short-circuiting.

The steel containers of the batteries are press-formed from steel and the joints are welded. The body and the cover are nickel-plated and have a dull finish. However, it should be kept in mind that since these containers are electrically alive, no loose wires should touch them owing to the danger of severe sparking from short-circuits.

9.39. Chemical Changes

The exact nature of the chemical changes taken place in such a cell is not clearly understood because the exact formula for the nickel oxide is not yet well established but the action of the cell can be understood by assuming the peroxide NiO_2 or its hydrated form $Ni(OH)_4$.

First, let us assume that at positive plate, nickel oxide is in its hydrate form $Ni(OH)_4$. During discharge, electrolyte KOH splits up into positive K ions and negative OH ions. The K ions go to anode and reduce $Ni(OH)_4$ to $NI(OH)_2$. The OH ions travel towards the cathode and oxidise iron. During charging, just the opposite reactions take place *i.e.* K ions go to cathode and OH ions go to anode. The chemical reactions can be written thus :

$$KOH \longrightarrow K + OH$$

During discharge

Positive plate : $Ni(OH)_4 + 2\,K \longrightarrow Ni(OH)_2 + 2\,KOH$

Negative plate : $Fe + 2\,OH \longrightarrow Fe(OH)_2$

During Charging

Positive plate : $Ni(OH)_2 + 2OH \longrightarrow Ni(OH)_4$

Negative plate : $Fe(OH)_2 + 2K \longrightarrow Fe + 2\,KOH$

The charging and discharging can be represented by a single reversible equation thus :

$$\underset{\text{Pos. Plate}}{Ni(OH)_4} + \underset{\text{Neg. Plate}}{KOH} + Fe \underset{\text{Charge}}{\overset{\text{Discharge}}{\rightleftharpoons}} \underset{\text{Pos. Plate}}{Ni(OH)_2} + KOH + \underset{\text{Neg. Plate}}{Fe(OH)_2}$$

It will be observed from the above equation that as no water is formed, there is no overall change in the strength of the electrolyte. Its function is merely to serve as a conductor of as a vehicle for the transfer of OH ions from one plate to another. Hence, the specific gravity of the electrolyte remains practically constant, both during charging and discharging. That is why only a small amount of electrolyte is required which fact enables the cells to be small in bulk.

Note. If, however, we assume the nickel oxide to be in the form NiO_2, then the above reactions can be represented by the following reversible equation :

$$\underset{\text{+ ve Plate}}{6NiO_2} + 8KOH + \underset{\text{–ve Plate}}{3Fe} \underset{\text{Charge}}{\overset{\text{Discharge}}{\rightleftharpoons}} \underset{\text{+ve Plate}}{2Ni_3O_4} + 8KOH + \underset{\text{–ve Plate}}{Fe_3O_4}$$

9.40. Electrical Characteristics

The e.m.f. of an Edison cell, when fully charged, is nearly 1.4 V which decreases rapidly to 1.3 V and then try slowly on 1.1 or 1.0 V on discharge. The average discharge voltage for a 5-hour discharge rate is 1.2 V. Hence, for the same average value of the voltage, an alkali accumulator will consist of 1.6 to 1.7 times as many cells as in a lead-acid battery. Internal resistance of an alkali cell is nearly five times that of the lead-acid cell, hence there is a relatively greater difference between its terminal voltage when charging and discharging.

The average charging voltage for an alkali cell is about 1.7 V. The general shapes of the charge and discharge curves for such cells are, however, similar to those for lead-acid cells. The rated capacity of nickel accumulators usually refers to either 5-hour or 8-hour discharge rate unless stated otherwise.

The plates of such cells have greater mechanical strength because of all-steel construction. They are comparatively lighter because ***(i)*** their plates are lighter and ***(ii)*** they require less quantity of electrolyte. They can withstand heavy charge and discharge currents and do not deteriorate even if left discharged for long periods.

Due to its relatively higher internal resistance, the efficiencies of an Edison cell are power than those of the lead acid cell. On the average, its Ah efficiency is about 80% and Wh efficiency 60 or 50%. It has an average density of 50 Wh/kg.

With increase in temperature, e.m.f. is increased slightly but capacity increases by an appreciable amount. With decrease in temperature, the capacity decreases becoming practically zero at 4°C even through the cell is fully charged. This is serious drawback in the back in the case of electrically driven vehicles in cold weather and previous have to be taken to heat up the battery before starting, though, in practice, the $I^2 R$ loss in the internal resistance of the battery is sufficient to keep the battery cells warm when running.

The principal disadvantage of the Edison battery on nickel-iron battery is its high initial cost (which will probably be sufficiently reduced when patents expire). At present, an Edison battery costs approximately twice as much as a lead-acid battery designed for similar service. But since the alkaline battery outlasts an indeterminate number of lead-acid batteries, it is cheaper in the end.

Because of their lightness, compact construction, increased mechanical strength, ability to withstand rapid charging and discharging without injury and freedom from corrosive liquids and fumes, alkaline batteries are ideally suited for traction work such as propulsion of electric factory trucks, mine locomotives, miner's lamps, lighting and starting of public service vehicles and other services involving rough usage etc.

9.41. Nickel-Cadmium Batteries

The reactive materials in a nickel-cadmium cell (Fig. 9.31) are :

(i) $Ni(OH)_4$ for the positive plate exactly as in the nickel-iron cell.

(ii) a mixture of cadmium or cadmium oxide and iron mass to which is added about 3 per cent of solar oil for stabilizing the electrode capacity. The use of cadmium results in reduced internal resistance of the cell.

(iii) the electrolyte is the same as in the nickel-iron cell.

The cell grouping and plate arrangement is identical with nickel-iron batteries except that the number of positive plates is more than the negative plates. Such batteries are more suitable than nickel-iron batteries for floating duties in conjunction with a charging dynamo because, in their case, the difference between charging and discharging e.m.f.s is not as great as in nickel-iron batteries.

Nickel-cadmium sintered plate batteries were first manufactured by Germans for military aircrafts and rockets. Presently, they are available in a variety of designs and sizes and have energy density going upto 55 Wh/kg. Their capacity is less affected by high discharge rates and low operating temperature than any other rechargeable batteries. Since such batteries have very low open-circuit losses, they are well-suited for pleasure yatches and launches which may be laid up for long periods. They are also used in commercial airliners, military aeroplanes and helicopters for starting main engines or auxiliary turbines and for emergency power supply.

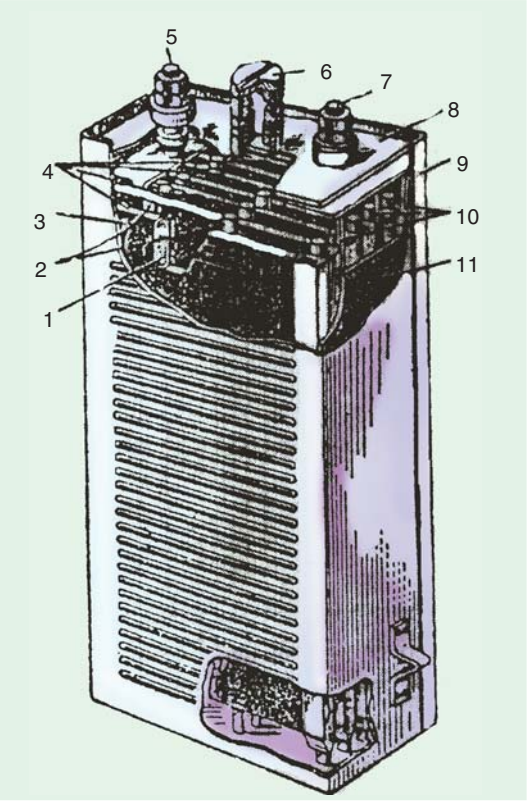


Fig. 9.31. Parts of Nickel-cadium alkaline cell.
1–active material 2–ebonite spacer sticks 3–pocket element 4–positive plates 5–positive terminal post 6–vent plug 7–negative terminal post 8–cover 9–container 10–negative plates 11–ebonite plate.

9.42. Chemical Changes

The chemical changes are more or less similar to those taking place in nickel-iron cell. As before, the electrolyte is split up into positive K ions and negative OH ions. The chemical reactions at the two plates are as under :

During discharge

Positive plate :	$Ni(OH)_4 + 2K = Ni(OH)_2 + 2\,KOH$
Negative plate :	$Cd + 2\,OH = Cd\,(OH)_2$

During Charging

Positive plate :	$Ni(OH)_2 + 2OH = Ni(OH)_4$
Negative plate :	$Cd(OH)_2 + 2\,K = Cd + 2\,KOH$

The above reaction can be represented by the following reversible equation :

$$Ni(OH)_4 + KOH + Cd \rightleftharpoons Ni(OH)_2 + KOH + Cd(OH)_2$$

9.43. Comparison : Lead-acid and Edison Cells

The relative strong and weak points of the cells have been summarised below :

Particulars	*Lead-acid cell*	*Edison cell*
1. Positive Plate	PbO_2 lead peroxide	Nickel hydroxide $Ni(OH)_4$ or NiO_2
2. Negative Plate	Sponge lead	Iron
3. Electrolyte	diluted H_2SO_4	KOH
4. Average e.m.f.	2.0 V/cell	1.2 V/cell
5. Internal resistance	Comparatively low	Comparatively higher
6. Efficiency : amp-hour watt-hour	90–95% 72–80%	nearly 80% about 60%
7. Cost	Comparatively less than alkaline line cell	almost twice that of Pb-acid cell
8. Life	gives nearly 1250 charges and discharges	Easy maintenance five years at least
9. Strength	Needs much care and maintenance. Sulphation occurs often due to incomplete charge of discharge.	robust, mechanically strong, can withstand vibration, is light, unlimited rates of charge and discharge. Can be left discharged, free from corrosive liquids and fumes.

Moreover, as copmpared to lead-acid, the alkaline cells operate much better at low temperature, do not emit obnoxious fumes, have very small self-discharge and their plates do not buckle or smell.

9.44. Silver-Zinc Batteries

The active material of the positive plates is silver oxide which is pressed into the plate and then subjected to a heat treatment. The active material of the negative plates is a mixture of zinc powder and zinc oxide.

The chemical changes taking place within the cell can be represented by the following single equation :

$$\underset{\text{+ ve plate}}{Ag_2O} + \underset{\text{–ve plate}}{Zn} \underset{\text{Charge}}{\overset{\text{discharge}}{\rightleftharpoons}} \underset{\text{+ ve plate}}{2\,Ag} + \underset{\text{–ve plate}}{ZnO}$$

A silver-zinc cell has a specific capacity (*i.e.* capacity per unit weight) 4 to 5 times greater than that of other type of cells. Their ground applications are mainly military *i.e.* communications equipment, portable radar sets and night-vision equipment. Moreover, comparatively speaking, their efficiency is high and self-discharge is small. Silver-zinc batteries can withstand much heavier discharge currents than are permissible for other types and can operate over a temperature range of –20° C to + 60°C. Hence, they are used in heavy – weight torpedoes and for submarine propulsion. It has energy density of 150 Wh/kg. Its life time in wet condition is 1-2 years and the dry storage life is upto 5 years. However, the only disadvantage of silver-zinc battery or cell is its higher cost.

9.45. High Temperature Batteries

It is a new group of source which requires operating temperatures above the embient. They possess the advantages of high specific energy and power coupled with low cost. They are particularly suitable for vehicle traction and load levelling purpose in the electric supply industry. We will beiefly describe the following cell from which high-temperature batteries are made.

1. *Lithium/Chlorine Cell*

It has an emf of 3.5 V, a theoretical specific energy of 2200 Wh/kg at 614°C and operating temperature of 650°C.

2. *Lithium/Sulphur Cell*

It has an emf of 2.25 V, specific energy of 2625 Wh/kg and an operating temperature of 365°C.

3. *Lithium-Aluminium/Iron-Sulphide Cells*

The emf of these cells is 1.3 V and a theoretical specific energy of 450 Wh/kg.

4. *Sodium/Sulphur Cells*

It utilises liquid sodium as negative electrode and sulphur as positive electrode and employs polycrystalline beta alumina as solid electrolyte. It was conceived in the 1960s by J.T. Kummer and N. Weber. The cell reaction can be written as $2\ Na + 3\ S = Na_2S_3$. The announcement of sodium/sulphur battery based on beta alumina was made by Ford Motor Company of USA in 1966. The open-circuit voltage of the cell is 2.1 V and it has a specific energy of 750 Wh/kg with an operating temperature of 350°C. The two unique features of this cell are (1) a Faradaic efficiency of 100% and an ampere-hour capacity which is invariant with discharge rate and (2) high self-life (which is critical for certain space applications).

9.46. Secondary Hybrid Cells

A hybrid cell may be defined as a galvanic electrotechnical generator in which one of the active reagents is in the gaseous state *i.e.* the oxygen of the air. Such cells take advantage of both battery and fuel cell technology. Examples of such cells are :

1. *Metal-air cells such as iron oxygen and zinc oxygen cells.*

The Zn/O_2 cell has an open-circuit voltage of 1.65 V and a theoretical energy density of 1090 Wh/kg. The Fe/O_2 cell has an OCV of 1.27 V and energy density of 970 Wh/kg.

2. *Metal-halogen cells such as zinc-chlorine and zinc-bromine cells.*

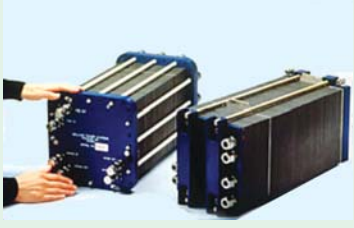
The zinc-chlorine cell has an OCV of 2.12 V at 25°C and a theoretical energy density of 100 Wh/kg. Such batteries are being developed for EV and load levelling applications. The zinc-bromine cell has an OCV of 1.83 V at 25°C and energy density of 400 Wh/kg.

3. *Metal-hydrogen cells such as nickel-hydrogen cell.*

Such cells have an OCV of 1.4 V and a specific energy of about 65 Wh/kg. Nickel-hydrogen batteries have captured large share of the space battery market in recent years and are rapidly replacing Nickel/cadmium batteries as the energy storage system of choice. They are acceptable for geosynchronous orbit applications where not many cycles are required over the life of the system (1000 cycles, 10 years).

The impetus for research and development of metal-air cells has arisen from possible EV applications where energy density is a critical parameter. An interesting application suggested for a secondary zinc-oxygen battery is for energy storage on-board space craft where the cell could be installed inside one of the oxygen tanks thereby eliminating need for gas supply pipes and valves etc. These cells could be reacharged using solar converters.

Some of the likely future developments for nickel-hydrogen batteries are **(1)** increase in cycle life for low earth orbit applications upto 40,000 cycles (7 years) **(2)** increase in the specific energy upto 100 W/kg for geosynchronous orbit applications and **(3)** development of a bipolar nickel-hydrogen battery for high pulse power applications.



Fuel Cells

9.47. Fuel Cells

As discussed earlier, a secondary battery produces electric current by oxidation-reduction chemical reaction. Similar chemical reactions take place in fuel cells but there is a basic difference between the two. Whereas in secondary batteries the

chemical energy is stored in the positive and negative electrodes, in fuel cells the oxidant and the fuel are stored outside the cells and must be fed to the electrodes continuously during the time the fuel cell supplies electric current. This gives an advantage to the fuel cells over the storage battery because fuels can be quickly replenished which is similar to filling up to the petrol tank of a car. Moreover, storage batteries when fully discharged take several hours to be recharged.

9.48. Hydrogen-Oxygen Fuel Cells

The first fuel battery was designed by F.T. Bacon in 1959. The construction of a simple fuel cell is shown in Fig. 9.32. The electrodes are made from sintered nickel plates having a coarse pore surface and a fine pore surface, the two surfaces being for gas and electrolyte respectively. The electrolyte used in KOH is of about 85 per cent concentration. The water vapour formed as a byproduct of the reaction is removed by condensation from the stream of hydrogen passing over the back of the fuel.

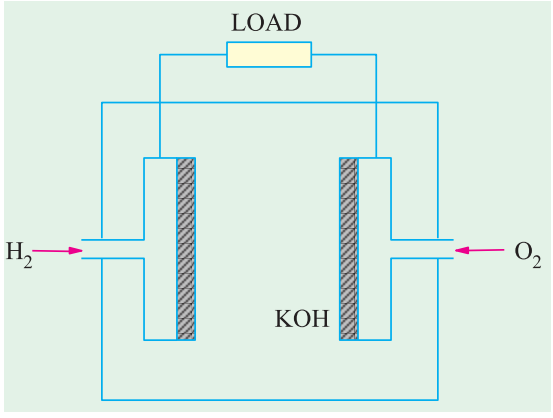


Fig. 9.32

The two electrodes of the fuel cell are fed with a continuous stream of hydrogen and oxygen (or air) as shown. The oxygen and hydrogen ions react with the potassium hydroxide electrolyte at the surface of the electrodes and produce water. The overall cell reaction is

$$2H_2 + O_2 = 2H_2O$$

The basic reaction taking place in the cells are shown in more details in the Fig. 9.32.

Fuel cell batteries have been used in the manned Apollo space mission for on-board power supply and also for power supply in unmanned satellites and space probes. These batteries have also been used for tractors, fork-lift trucks and golf carts etc. Research is being carried out to run these batteries with natural gas and alcohol. Fuel cell systems are particularly useful where electrical energy is required for long periods. Such applications include **(1)** road and rail traction **(2)** industrial trucks **(3)** naval craft and submarine **(4)** navigational aids and radio repeater stations etc.

9.49. Batteries for Aircraft

The on-board power requirements in aircraft have undergone many changes during the last three or four decades. The jet engines of the aircraft which require starting currents of about 1000 A, impose a heavy burden on the batteries. However, these days this load is provided by small turbogenerator sets and since batteries are needed only to start them, the power required is much less. These batteries possess good high-rate capabilities in order to supply emergency power for upto 1 h in the event of the generator failure. However, their main service is as a standby power for miscellaneous on-board equipment. Usually, batteries having 12 cells (of a nominal voltage of 24 V) with capacities of 18 and 34 Ah at the 10 h rate are used. In order to reduce weight, only light-weight high-impact polystyrene containers and covers are used and the cells are fitted with non-spill ventplugs to ensure complete unspillability in any aircraft position during aerobatics. Similarly, special plastic manifolds are moulded into the covers to provide outlet for gases evolved during cycling.

9.50. Batteries for Submarines

These batteries are the largest units in the traction service. In older types of submarines, the lead storage battery was the sole means of propulsion when the submarine was fully submerged and, additionally supplied the 'hotel load' power for lights, instruments and other electric equipment. When the introduction of the snorkel breathing tube made it possible to use diesel engines for propulsion, battery was kept in reserve for emergency use only. Even modern nuclear-powered submarines use storage batteries for this purpose. These lead-acid batteries may be flat, pasted plate or tubular

positive plate type with 5 h capacities ranging from 10,000 to 12,000 Ah. One critical requirement for this service is that the rate of evolution of hydrogen gas on open-circuit should not exceed the specified low limit.

Double plate separation with the help of felted glass fibre mats and microscoporous separators is used in order to ensure durability, high performance and low standing losses.

OBJECTIVE TESTS – 9

1. Active materials of a lead-acid cell are :
(*a*) lead peroxide (*b*) sponge lead
(*c*) dilute sulphuric acid (*d*) all the above

2. During the charging of a lead-acid cell :
(*a*) its cathode becomes dark chocolate brown in colour
(*b*) its voltage increases
(*c*) it gives out energy
(*d*) specific gravity of H_2SO_4 is decreased

3. The ratio of Ah efficiency to Wh efficiency of a lead-acid cell is :
(*a*) always less than one (*b*) just one
(*c*) always greater than one
(*d*) either (*a*) or (*b*)

4. The capacity of a cell is measured in :
(*a*) watt-hours (*b*) watts
(*c*) amperes (*d*) ampere-hours

5. The capacity of a lead-acid cell does NOT depend on its :
(*a*) rate of charge
(*b*) rate of discharge
(*c*) temperature
(*d*) quantity of active material

6. As compared to constant-current system, the constant-voltage system of charging a lead-acid cell has the advantage of :
(*a*) avoiding excessive gassing
(*b*) reducing time of charging
(*c*) increasing cell capacity
(*d*) both (*b*) and (*c*).

7. Sulphation in a lead-acid battery occurs due to:
(*a*) trickle charging
(*b*) incomplete charging
(*c*) heavy discharging (*d*) fast charging

8. The active materials of a nickel-iron battery are.
(*a*) nickel hydroxide
(*b*) powdered iron and its oxides
(*c*) 21% solution of caustic potash
(*d*) all of the above.

9. During the charging and discharging of a nickel iron cell :
(*a*) its e.m.f. remains constant
(*b*) water is neither formed nor absorbed
(*c*) corrosive fumes are produced
(*d*) nickel hydroxide remains unsplit

10. As compared to a lead-acid cell, the efficiency of a nickel-iron cell is less due to its :
(*a*) lower e.m.f.
(*b*) smaller quantity of electrolyte used
(*c*) higher internal resistance
(*d*) compactness.

11. Trickle charging of a storage battery helps to :
(*a*) prevent sulphation
(*b*) keep it fresh and fully charged
(*c*) maintain proper electrolyte level
(*d*) increase its reserve capacity

12. A dead storage battery can be revived by :
(*a*) a dose of H_2SO_4
(*b*) adding so-called battery restorer
(*c*) adding distilled water
(*d*) none of the above

13. The sediment which accumulates at the bottom of a lead-acid battery consists largely of :
(*a*) lead-peroxide (*b*) lead-sulphate
(*c*) antimony-lead alloy (*d*) graphite

14. The reduction of battery capacity at high rates of discharge is primarily due to :
(*a*) increase in its internal resistance
(*b*) decrease in its terminal voltage
(*c*) rapid formation of $PbSO_4$ on the plates
(*d*) non-diffusion of acid to the inside active materials.

15. Floating battery systems are widely used for :
(*a*) power stations
(*b*) emergency lighting
(*c*) telephone exchange installation
(*d*) all of the above

16. Any charge given to the battery when taken off the vehicle is called :
(*a*) bench charge (*b*) step charge
(*c*) float charge (*d*) trickle charge

ANSWERS

1. *d* **2.** *b* **3.** *c* **4.** *d* **5.** *a* **6.** *d* **7.** *b* **8.** *d* **9.** *b* **10.** *c* **11.** *b* **12.** *d* **13.** *c* **14.** *c* **15.** *d* **16.** *a*

Top

CHAPTER 10

ELECTRICAL INSTRUMENTS AND MEASURE-MENTS

Learning Objectives

- Absolute and Secondary Instruments
- Deflecting Torque
- Controlling Torque
- Damping Torque
- Moving-iron Ammeters and Voltmeters
- Moving-coil Instruments
- Permanent Magnet Type Instruments
- Voltmeter Sensitivity
- Electrodynamic or Dynamometer Type Instruments
- Hot-wire Instruments
- Megger
- Induction Voltmeter
- Wattmeters
- Energy Meters
- Electrolytic Meter
- Ampere-hour Mercury Motor Meter
- Friction Compensation
- Commutator Motor Meters
- Ballistic Galvanometer
- Vibration Galvanometer
- Vibrating-reed Frequency Meter
- Electrodynamic Frequency Meter
- Moving-iron Frequency Meter
- Electrodynamic Power Factor Meter
- Moving-iron Power Factor Meter
- Nalder-Lipman Moving-iron Power Factor Meter
- D.C. Potentiometer
- A.C. Potentiometers
- Instrument Transformers
- Potential Transformers.



Electrical instruments help us to measure the changes in variables such as voltage, current and resistance

10.1. Absolute and Secondary Instruments

The various electrical instruments may, in a very broad sense, be divided into ***(i) absolute*** instruments and ***(ii) secondary*** **instruments. Absolute instruments *are those which give the value of the quantity to be measured, in terms of the constants of the instrument and their deflection only.*** No previous calibration or comparision is necessary in their case. The example of such an instrument is tangent galvanometer, which gives the value of current, in terms of the tangent of deflection produced by the current, the radius and number of turns of wire used and the horizontal component of earth's field.


An absolute instrument

Secondary instruments are those, in which the value of electrical quantity to be measured can be determined from the deflection of the instruments, only when they have been pre-calibrated by comparison with an absolute instrument. Without calibration, the deflection of such instruments is meaningless.

It is the secondary instruments, which are most generally used in everyday work; the use of the absolute instruments being merely confined within laboratories, as standardizing instruments.

10.2. Electrical Principles of Operation

All electrical measuring instruments depend for their action on one of the many physical effects of an electric current or potential and are generally classified according to which of these effects is utilized in their operation. The effects generally utilized are :

1. Magnetic effect - for ammeters and voltmeters usually.
2. Electrodynamic effect - for ammeters and voltmeters usually.
3. Electromagnetic effect - for ammeters, voltmeters, wattmeters and watthour meters.
4. Thermal effect - for ammeters and voltmeters.
5. Chemical effect - for d.c. ampere-hour meters.
6. Electrostatic effect - for voltmeters only.

Another way to classify secondary instruments is to divide them into ***(i) indicating instruments (ii) recording instruments and (iii) integrating instruments.***

Indicating instruments are those which indicate the instantaneous value of the electrical quantity being measured *at the time* at which it is being measured. Their indications are given by pointers moving over calibrated dials. Ordinary ammeters, voltmeters and wattmeters belong to this class.

Recording instruments are those, which, instead of indicating by means of a pointer and a scale the instantaneous value of an electrical quantity, give a ***continuous record*** or the variations of such a quantity over a selected period of time. The moving system of the instrument carries an inked pen which rests lightly on a chart or graph, that is moved at a uniform and low speed, in a direction perpendicular to that of the deflection of the pen. The path traced out by the pen presents a continuous record of the variations in the deflection of the instrument.

Integrating instruments are those which measure and register by a set of dials and pointers either the ***total*** quantity of electricity (in amp-hours) or the ***total*** amount of electrical energy (in watt-hours or kWh) supplied to a circuit in a given time. This summation gives the product of time and the electrical quantity but gives no direct indication as to the ***rate*** at which the quantity or energy is being supplied because their registrations are independent of this rate provided the current flowing through the instrument is sufficient to operate it.

Ampere-hour and watt-hour meters fall in this class.

10.3. Essentials of Indicating Instruments

As defined above, indicating instruments are those which indicate the value of the quantity that is being measured at the time at which it is measured. Such instruments consist essentially of a pointer which moves over a calibrated scale and which is attached to a moving system pivoted in jewelled bearings. The moving system is subjected to the following three torques :

1. A deflecting (or operating) torque
2. A controlling (or restoring) torque
3. A damping torque.

10.4. Deflecting Torque

The deflecting or operating torque (T_d) is produced by utilizing one or other effects mentioned in Art. 10.2 *i.e.* magnetic, electrostatic, electrodynamic, thermal or inductive etc. The actual method of torque production depends on the type of instrument and will be discussed in the succeeding paragraphs. This deflecting torque causes the moving system (and hence the pointer attached to it) to move from its '***zero***' position *i.e.* its position when the instrument is disconnected from the supply.

10.5. Controlling Torque

The deflection of the moving system would be indefinite if there were no controlling or restoring torque. This torque oppose the deflecting torque and increases with the deflection of the moving system. The pointer is brought to rest at a position where the two opposing torques are equal. The deflecting torque ensures that currents of different magnitudes shall produce deflections of the moving system in proportion to their size. Without such at torque, the pointer would swing over to the maximum deflected position irrespective of the magnitude of the current to be measured. Moreover, in the absence of a restoring torque, the pointer once deflected, would not return to its zero position on removing the current. The controlling or restoring or balancing torque in indicating instruments is obtained either by a spring or by gravity as described below :

(*a*) Spring Control

A hair-spring, usually of phosphor-bronze, is attached to the moving system of the instrument as shown in Fig. 10.1 (*a*).

With the deflection of the pointer, the spring is twisted in the opposite direction. This twist in the spring produces restoring torque which is directly proportional to the angle of deflection of the moving system. The pointer comes to a position of rest (or equilibrium) when the deflecting torque (T_d) and controlling torque (T_c) are equal. For example, in permanent-magnet, moving-coil type of instruments, the deflecting torque is proportional to the current passing through them.

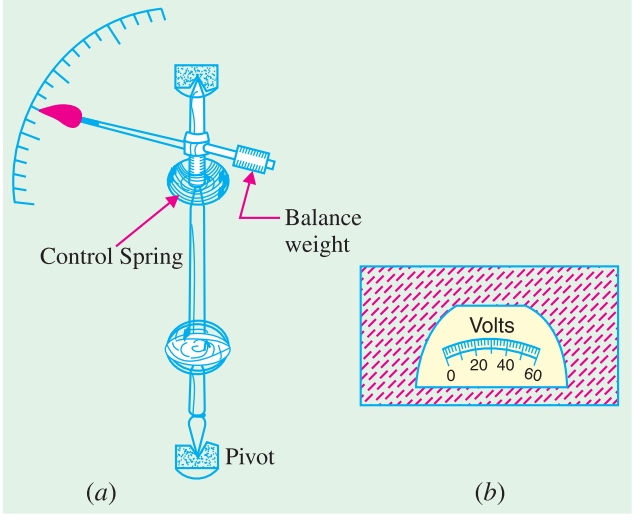


Fig. 10.1

$\therefore \quad T_d \propto I$

and for spring control $T_c \propto \theta$

As $T_c = T_d$

$\therefore \quad \theta \propto I$

Since deflection θ is directly proportional to current I, the spring-controlled instruments have a uniform or equally-spaced scales over the whole of their range as shown in Fig. 10.1 (*b*).

To ensure that controlling torque is proportional to the angle of deflection, the spring should have a fairly large number of turns so that angular deformation per unit length, on full-scale deflection, is small. Moreover, the stress in the spring should be restricted to such a value that it does not produce a permanent set in it.

Springs are made of such materials which

(*i*) are non-magnetic
(*ii*) are not subject to much fatigue
(*iii*) have low specific resistance-especially in cases where they are used for leading current in or out of the instrument
(*iv*) have low temperature-resistance coefficient.

The exact expression for controlling torque is $T_c = C\theta$ where C is *spring constant*. Its value is given by $C = \frac{Ebt^3}{L}$ N-m/rad. The angle θ is in radians.

(*b*) Gravity Control

Gravity control is obtained by attaching a small adjustable weight to some part of the moving system such that the two exert torques in the opposite directions. The usual arrangements is shown in Fig. 10. 2(*a*).

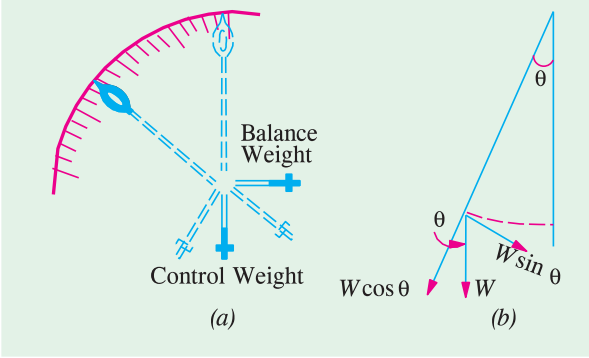


Fig. 10.2

It is seen from Fig. 10.2 (*b*) that the controlling or restoring torque is proportional to the sine of the angle of deflection *i.e.*

$$T_c \propto \sin\theta$$

The degree of control is adjusted by screwing the weight up or down the carrying system

It $T_d \propto I$

then for position of rest

$$T_d = T_c$$

or $I \propto \sin\theta$ (*not* θ)

It will be seen from Fig. 10.2 (*b*) that as θ approaches 90º, the distance AB increases by a relatively small amount for a given change in the angle than when θ is just increasing from its zero value. Hence, gravity-controlled instruments have scales which are not uniform but are cramped or crowded at their lower ends as shown in Fig. 10.3.

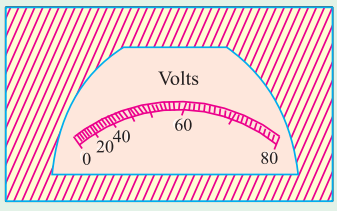


Fig. 10.3

As compared to spring control, the disadvantages of gravity control are :

1. it gives cramped scale
2. the instrument has to be kept vertical.

However, gravity control has the following advantages :

1. it is cheap
2. it is unaffected by temperature
3. it is not subjected to fatigue or deterioration with time.

Example 10.1 *The torque of an ammeter varies as the square of the current through it. If a current of 5 A produces a deflection of 90º, what deflection will occur for a current of 3 A when the instrument is* ***(i)*** *spring-controlled and* ***(ii)*** *gravity-controlled.*

(Elect. Meas. Inst and Meas. Jadavpur Univ.)

Solution. Since deflecting torque varies as (current)^2, we have $T_d \propto I^2$

For spring control, $T_c \propto \theta \quad \therefore \theta \propto I^2$

For gravity control, $T_c \propto \sin\theta \quad \therefore \sin\theta \propto I^2$

(*i*) For spring control $90^\circ \propto 5^2$ and $\theta \propto 3^2$; $\theta = 90^\circ \times 3^2/5^2 = \mathbf{32.4^\circ}$

(*ii*) For gravity control $\sin 90^\circ \propto 5^2$ and $\sin\theta \propto 3^2$

$\sin\theta = 9/25 = 0.36$; $\theta = \sin^{-1}(0.36) = \mathbf{21.1^\circ}$.

10.6. Damping Torque

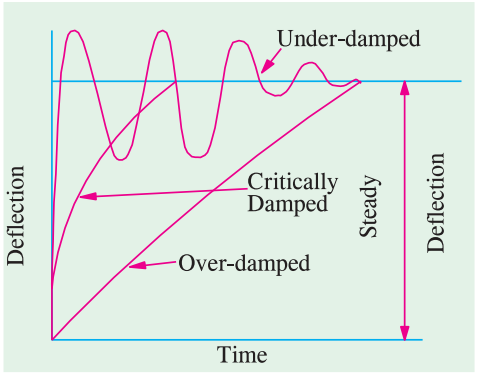


Fig. 10.4

A damping force is one which acts on the moving system of the instrument ***only when it is moving*** and always opposes its motion. Such stabilizing or demping force is necessary to bring the pointer to rest ***quickly,*** otherwise due to inertia of the moving system, the pointer will oscillate about its final deflected position for quite some time before coming to rest in the steady position. The degree of damping should be adjusted to a value which is sufficient to enable the pointer to rise quickly to its deflected position without overshooting. In that case, the instrument is said to be ***dead-beat.*** Any increase of damping above this limit *i.e.* overdamping will make the instruments slow and lethargic. In Fig. 10.4 is shown the effect of damping on the variation of position with time of the moving system of an instrument.

The damping force can be produced by **(*i*) *air frictions* (*ii*) *eddy currents*** and **(*iii*) *fluid friction*** (used occasionally).

Two methods of air-friction damping are shown in Fig. 10.5 (*a*) and 10.5 (*b*). In Fig.. 10.5 (*a*), the light aluminium piston attached to the moving system of the instrument is arranged to travel with

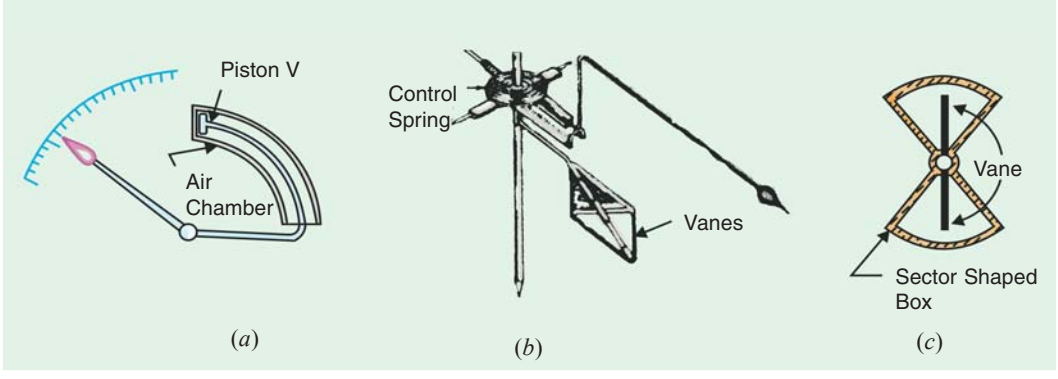


Fig. 10.5

a very small clearance in a fixed air chamber closed at one end. The cross-section of the chamber is either circular or rectangular. Damping of the oscillation is affected by the compression and suction actions of the piston on the air enclosed in the chamber. Such a system of damping is not much favoured these days, those shown in Fig. 10.5 (*b*) and (*c*) being preferred. In the latter method, one or two light aluminium vanes are mounted on the spindle of the moving system which move in a closed sector-shaped box as shown.

Fluid-friction is similar in action to the air friction. Due to greater viscosity of oil, the damping is more effective. However, oil damping is not much used because of several disadvantages such as objectionable creeping of oil, the necessity of using the instrument always in the vertical position and its obvious unsuitability for use in portable instruments.

The eddy-current form of damping is the most efficient of the three. The two forms of such a damping are shown in Fig. 10.6 and 10.7. In Fig. 10.6 (*a*) is shown a thin disc of a conducting but ***non-magnetic*** material like copper or aluminium mounted on the spindle which carries the moving system and the pointer of the instrument. The disc is so positioned that its edges, when in rotation, cut the magnetic flux between the poles of a permanent magnet. Hence, eddy currents are produced in the disc which flow and so produce a damping force in such a direction as to oppose the very cause producing them (Lenz's Law Art. 7.5). Since the cause producing them is the rotation of the disc, these eddy current retard the motion of the disc and the moving system as a whole.

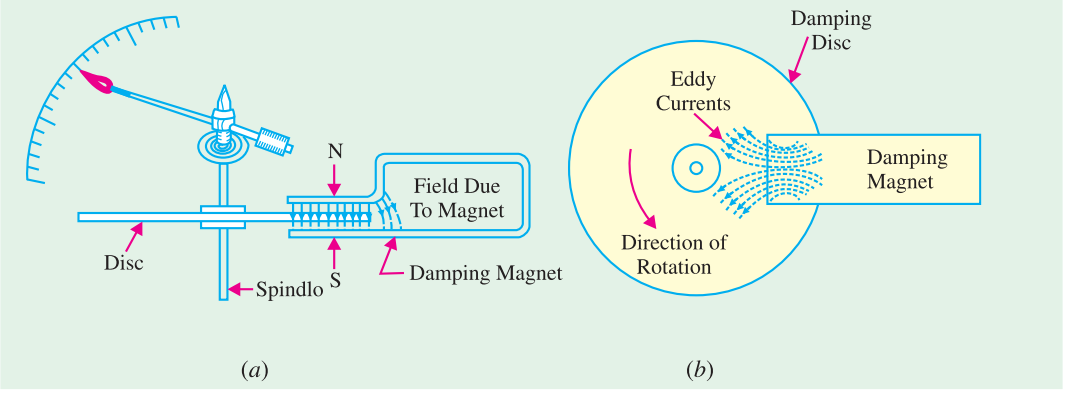


Fig. 10.6

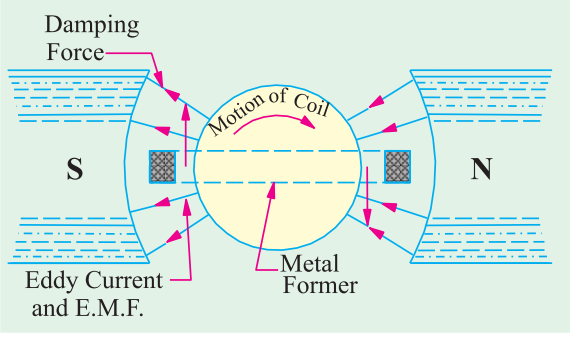


Fig. 10.7

In Fig. 10.7 is shown the second type of eddy-current damping generally employed in permanent-magnet moving coil instruments. The coil is wound on a thin light aluminium former in which eddy currents are produced when the coil moves in the field of the permanent magnet. The directions of the induced currents and of the damping force produced by them are shown in the figure.

The various types of instruments and the order in which they would be discussed in this chapter are given below.

Ammeters and voltmeters

1. Moving-iron type (both for D.C./A.C.)
   - (*a*) ***the attraction type***
   - (*b*) ***the repulsion type***
2. Moving-coil type
   - (*a*) ***permanent-magnet type*** (for D.C. only)
   - (*b*) ***electrodynamic or dynamometer type*** (*for D.C./A.C.*)
3. Hot-wire type (both for D.C./A.C.)
4. Induction type (for A.C. only)
   - (*a*) ***Split-phase type***
   - (*b*) ***Shaded-pole type***
5. Electrostatic type-for voltmeters only (for D.C./A.C.)

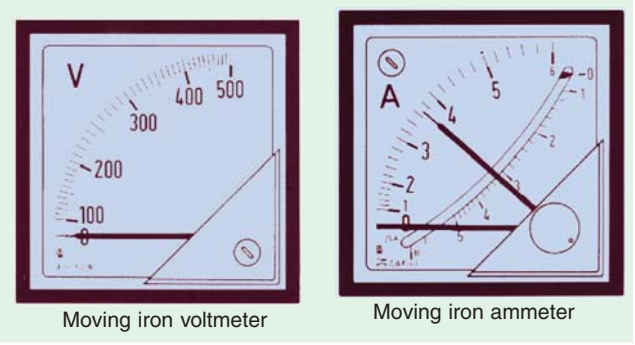
Wattmeter

6. ***Dynamometer type*** (both for D.C./A.C.),
7. ***Induction type*** (for A.C. only)
8. *Electrostatic type* (for D.C. only)

Energy Meters

9. ***Electrolytic type*** (for D.C. only)
10. **Motor Meters**
    - (*i*) ***Mercury Motor Meter.*** For d.c. work only. Can be used as amp-hour or watt-hour meter.
    - (*ii*) ***Commutator Motor Meter.*** Used on D.C./A.C. Can be used as Ah or Wh meter.
    - (*iii*) ***Induction type.*** For A.C. only.
11. Clock meters (as Wh-meters).

10.7. Moving-iron Ammeters and Voltmeters



Moving iron voltmeter Moving iron ammeter

There are two basic forms of these instruments *i.e.* the ***attraction*** type and the ***repulsion*** type. The operation of the attraction type depends on the attraction of a single piece of soft iron into a magnetic field and that of repulsion type depends on the repulsion of two adjacent pieces of iron magnetised by the same magnetic field. For both types of these instruments, the necessary magnetic field is produced by the ampere-turns of a current-carrying coil. In case the instrument is to be used as an ammeter, the coil has comparatively fewer turns of thick wire so that the ammeter has low resistance because it is connected in series with the circuit. In case it is to be used as a voltmeter, the coil has high impedance so as to draw as small a current as possible since it is connected in parallel with the circuit. As the current through the coil is small, it has large number of turns in order to produce sufficient ampere-turns.

10.8. Attraction Type M.I. Insturments

The basic working principle of an attraction-type moving-iron instrument is illustrated in Fig. 10.8. It is well-known that if a piece of an unmagnetised soft iron is brought up near either of the two ends of a current-carrying coil, it would be attracted into the coil in the same way as it would be attracted by the pole of a bar magnet. Hence, if we pivot an oval-shaped disc of soft iron on a spindle between bearings near the coil (Fig. 10.8), the iron disc will swing into the coil when the latter has an electric current passing through it. As the field strength would be strongest at the centre of

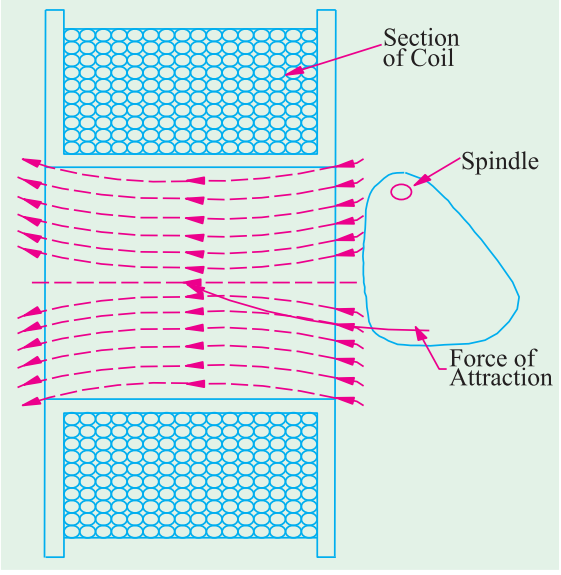


Fig. 10.8

the coil, the ovalshaped iron disc is pivoted in such a way that the greatest bulk of iron moves into the centre of the coil. If a pointer is fixed to the spindle carrying the disc, then the passage of current through the coil will cause the pointer to deflect. The amount of deflection produced would be greater when the current producing the magnetic field is greater. Another point worth noting is that ***whatever the direction of current through the coil, the iron disc would always be magnetised in such a way that it is pulled inwards.*** Hence, such instruments can be used both for direct as well as alternating currents.

A sectional view of the actual instrument is shown in Fig. 10.9. When the current to be measured is passed through the coil or solenoid, a magnetic field is produced, which attracts the eccentrically-mounted disc inwards, thereby deflecting the pointer, which moves over a calibrated scale.

Deflecting Torque

Let the axis of the iron disc, when in zero position, subtend an angle of ϕ with a direction perpendicular to the direction of the field H produced by the coil. Let the deflection produced be θ corresponding to a current I through the coil. The magnetisation of iron disc is proportional to the component of H acting along the axis of the disc *i.e.* proportional to $H \cos [90 - (\phi + \theta)]$ or $H \sin (\theta + \phi)$. The force F pulling the disc inwards is proportional to MH or $H^2 \sin (\theta + \phi)$. If the permeability of iron is assumed constant, then, $H \propto I$. Hence, $F \propto I^2 \sin (\theta + \phi)$. If this force acted at a distance of l from the pivot of the rotating disc, then deflecting torque $T_d = Fl \cos (\theta + \phi)$. Putting the value of F, we get

$T_d \propto I^2 \sin (\theta + \phi) \times l \cos (\theta + \phi) \propto I^2 \sin 2 (\theta + \phi) = KI^2 \sin 2 (\theta + \phi)$ ...sin l is constant

If spring-control is used, then controlling torque $T_c = K' \theta$

In the steady position of deflection, $T_d = T_c$

$\therefore \quad KI^2 \sin 2 (\theta + \phi) = K' \theta$; Hence $\theta \propto I^2$

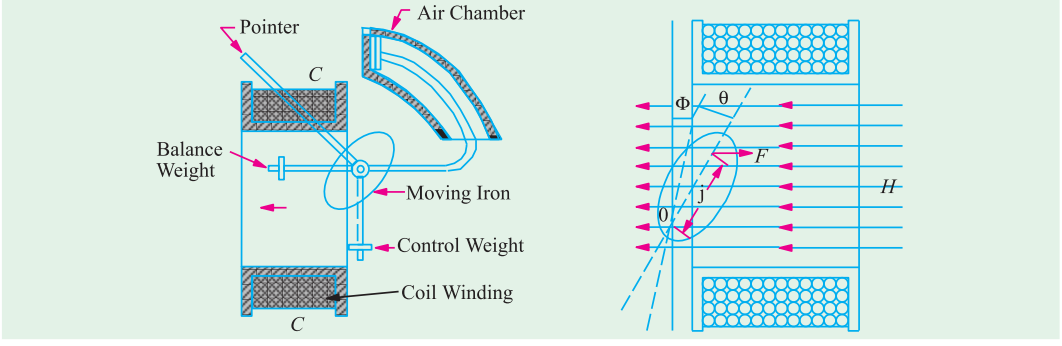


Fig. 10.9

Fig. 10.10

If A.C. is used, then $\theta \propto I^2_{\text{r.m.s.}}$

However, if gravity-control is used, then $T_c = K_1 \sin \theta$

$\therefore \quad KI^2 \sin 2 (\theta + \phi) = K_1 \sin \theta \quad \therefore \sin \theta \ \alpha I^2 \sin 2 (\theta + \phi)$

In both cases, the scales would be uneven.

Damping

As shown, air-friction damping is provided, the actual arrangement being a light piston moving in an air-chamber.

10.9. Repulsion Type M.I. Instruments

The sectional view and cut-away view of such an instrument are shown in Fig. 10.11 and 10.12. It consists of a fixed coil inside which are placed two soft-iron rods (or bars) A and B parallel to one another and along the axis of the coil. One of them *i.e.* A is fixed and the other B which is movable carries a pointer that moves over a calibrated scale. When the current to be measured is passed

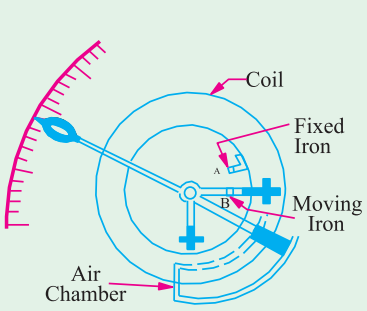


Fig. 10.11

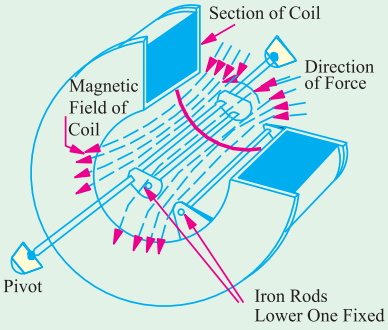


Fig. 10.12

through the fixed coil, it sets up its own magnetic field which magnetises the two rods similarly *i.e.* the adjacent points on the lengths of the rods will have the same magnetic polarity. Hence, they repel each other with the result that the pointer is deflected against the controlling torque of a spring or gravity. The force of repulsion is approximately proportional to the square of the current passing through the coil. Moreover, whatever may be the direction of the current through the coil, the two rods will be magnetised similaraly and hence will repel each other.

In order to achieve uniformity of scale, two tongue-shaped strips of iron are used instead of two rods. As shown in Fig. 10.13 (*a*), the fixed iron consists of a tongue-shaped sheet iron bent into a cylindrical form, the moving iron also consists of another sheet of iron and is so mounted as to move parallel to the fixed iron and towards its narrower end [Fig. 10.13 (*b*)].

Deflecting Torque

The deflecting torque is due to the repulsive force between the two similarly magnetised iron rods or sheets.

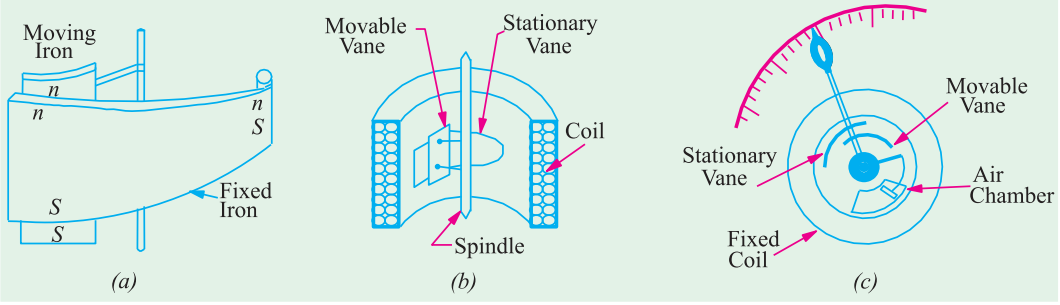


Fig. 10.13

Instantaneous torque $\propto$ repulsive force $\propto m_1 m_2$ ...product of pole strengths

Since pole strength are proportional to the magnetising force H of the coil,

$\therefore$ instantaneous torque $\propto H^2$

Since H itself is proportional to current (assuming ***constant*** permeability) passing through the coil, $\therefore$ instantaneous torque $\propto I^2$

Hence, the deflecting torque, which is proportional to the mean torque is, in effect, proportional to the mean value of I^2. Therefore, when used on a.c. circuits, the instrument reads the r.m.s. value of current.

Scales of such instruments are uneven if rods are used and uniform if suitable-shaped pieces of iron sheet are used.

The instrument is either gravity-controlled or as in modern makes, is spring-controlled.

Damping is pneumatic, eddy current damping cannot be employed because the presence of a permanent magnet required for such a purpose would affect the deflection and hence, the reading of the instrument.

Since the polarity of both iron rods reverses simultaneously, the instrument can be used both for a.c. and d.c. circuits *i.e.* instrument belongs to the unpolarised class.

10.10. Sources of Error

There are two types of possible errors in such instruments, firstly, those which occur both in a.c. and d.c. work and secondly, those which occur in a.c. work alone.

(*a*) Errors with both d.c. and a.c. work

(*i*) *Error due to hysteresis.* Because of hysteresis in the iron parts of the moving system, readings are higher for descending values but lower for ascending values.

The hysteresis error is almost completely eliminated by using Mumetal or Perm-alloy, which have negligible hysteresis loss.

(*ii*) *Error due to stray fields.* Unless shielded effectively from the effects of stray external fields, it will give wrong readings. Magnetic shielding of the working parts is obtained by using a covering case of cast-rion.

(*b*) Errors with a.c. work only

Changes of frequency produce **(*i*)** change in the impedance of the coil and **(*ii*)** change in the magnitude of the eddy currents. The increase in impedance of the coil with increase in the frequency of the alternating current is of importance in voltmeters (Ex. 10.2). For frequencies higher than the one used for calibration, the instrument gives lower values. However, this error can be removed by connecting a capacitor of suitable value in parallel with the swamp resistance R of the instrument. It can be shown that the impedance of the whole circuit of the instrument becomes independent of frequency if $C = L/R^2$ where C is the capacitance of the capacitor.

10.11. Advantages and Disadvantages

Such instruments are cheap and robust, give a reliable service and can be used both on a.c. and d.c. circuits, although they cannot be calibrated with a high degree of precision with d.c. on account of the effect of hysteresis in the iron rods or vanes. Hence, they are usually calibrated by comparison with an alternating current standard.

10.12. Deflecting Torque in terms of Change in Self-induction

The value of the deflecting torque of a moving-iron instrument can be found in terms of the variation of the self-inductance of its coil with deflection θ.

Suppose that when a direct current of I passes through the instrument, its deflection is θ and inductance L. Further suppose that when current changes from I to $(I + dI)$, deflection changes from θ to $(\theta + d\theta)$ and L changes to $(L + dL)$. Then, the increase in the energy stored in the magnetic field is

$$dE = d\left(\frac{1}{2}LI^2\right) = \frac{1}{2}L2I.dI + \frac{1}{2}I^2dL = LI.\,dI + \frac{1}{2}I^2.\,dL \text{ joule.}$$

If $T\frac{1}{2}$ N-m is the controlling torque for deflection θ, then extra energy stored in the control system is $T \times d\theta$ joules. Hence, the total increase in the stored energy of the system is

$$LI.dI + \frac{1}{2}I^2.\,dL + T \times d\,\theta \qquad \text{...(i)}$$

The e.m.f. induced in the coil of the instrument is $e = N.\frac{d\Phi}{dt}$ volt

where $d\phi$ = change in flux linked with the coil due to change in the position of the disc or the bars

dt = time taken for the above change ; N = No. of turns in the coil

Now $L = NF/I \quad \therefore \quad \Phi = LI/N \quad \therefore \quad \frac{d\Phi}{dt} = \frac{1}{N}.\frac{d}{dt}(LI)$

Induced e.m.f. $e = N.\frac{1}{N}.\frac{d}{dt}(LI) = \frac{d}{dt}(LI)$

The energy drawn from the supply to overcome this back e.m.f is

$$= e.Idt = \frac{d}{dt}(LI).Idt = I.d(LI) = I(L.dI + I.dL) = LI.dI + I^2.dL \qquad \text{...(ii)}$$

Equating **(*i*)** and **(*ii*)** above, we get $LI.dI + \frac{1}{2}I^2dL + T.d\theta = LI.dI + I^2.dL \quad \therefore \quad T = \frac{1}{2}I^2\frac{dL}{d\theta}$ N-m

where $dL/d\theta$ is henry/radian and I in amperes.

10.13. Extension of Range by Shunts and Multipliers

(*i*) As Ammeter. The range of the moving-iron instrument, when used as an ammeter, can be extended by using a suitable shunt across its terminals. So far as the operation with direct current is concerned, there is no trouble, but with alternating current, the division of current between the instrument and shunt changes with the change in the applied frequency. For a.c. work, both the inductance and resistance of the instrument and shunt have to be taken into account.

Obviously, $\frac{\text{current through instruments, } i}{\text{current through shunt, } I_s} = \frac{R_s + j\omega L_s}{R + j\omega L} = \frac{\mathbf{Z}_s}{\mathbf{Z}}$

where R, L = resistance and inductance of the instrument

R_s, L_s = resistance and inductance of the shunt

It can be shown that above ratio *i.e.* the division of current between the instrument and shunt would be independent of frequency if the time-constants of the instrument coil and shunt are the same *i.e.* if $L/R = L_s/R_s$. The multiplying power (N) of the shunt is given by

$$N = \frac{I}{i} = 1 + \frac{R}{R_s}$$

where I = line current ; i = full-scale deflection current of the instrument.

(*ii*) As Voltmeter. The range of this instrument, when used as a voltmeter, can be extended or multiplied by using a high non-inductive resistance R connected in series with it, as shown in Fig. 10.14. This series resistance is known as 'multiplier' when used on d.c. circuits. Suppose, the range of the instrument is to be extended from v to V. Then obviously, the excess voltage of $(V - v)$ is to be dropped across R. If i is the full-scale deflection current of the instrument, then

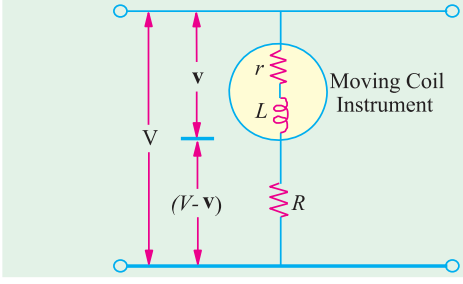


Fig. 10.14

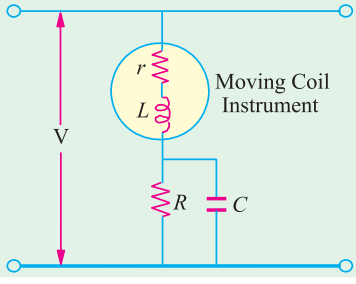


Fig. 10.15

$$iR = V - v; \quad R = \frac{V - v}{i} = \frac{V - ir}{i} = \frac{V}{i} - r$$

Voltage magnification = V/v. Since $iR = V - v$; $\therefore \frac{iR}{v} = \frac{V}{v} - 1$

or $$\frac{iR}{ir} = \frac{V}{v} - 1 \quad \therefore \quad \frac{V}{v} = \left(1 + \frac{R}{r}\right)$$

Hence, greater the value of R, greater is the extension in the voltage range of the instrument.

For d.c. work, the principal requirement of R is that its value should remain constant *i.e.* it should have low temperature-coefficient. But for a.c. work it is essential that total impedance of the voltmeter and the series resistance R should remain as nearly constant as possible at different frequencies. That is why R is made as non-inductive as possible in order to keep the inductance of the whole circuit to the minimum. The frequency error introduced by the inductance of the instrument coil can be compensated by shunting R by a capacitor C as shown in Fig. 10.15. In case $r \ll R$, the impedance of the voltmeter circuit will remain practically constant (for frequencies upto 1000 Hz) provided.

$$C = \frac{L}{(1+\sqrt{2})R^2} = 0.41\frac{L}{R^2}$$

Example 10.2. *A 250-volt moving-iron voltmeter takes a current of 0.05 A when connected to a 250-volt d.c. supply. The coil has an inductance of 1 henry. Determine the reading on the meter when connected to a 250-volt, 100-Hz a.c. supply.* **(Elect. Engg., Kerala Univ.)**

Solution. When used on d.c. supply, the instrument offers ohmic resistance only. Hence, resistance of the instrument = 250/0.05 = 5000 Ω

When used on a.c. supply, the instrument offers ***impedance*** instead of ohmic resistance.

$$\text{impedance at 100 Hz} = \sqrt{5000^2 + (2\pi \times 100 \times 1)^2} = 5039.3\ \Omega$$

$\therefore$ voltage of the instrument = 250 × 5000/5039.3 = **248 V**

Example 10.3. *A spring-controlled moving-iron voltmeter reads correctly on 250-V d.c. Calculate the scale reading when 250-V a.c. is applied at 50 Hz. The instrument coil has a resistance of 500 Ω and an inductance of 1 H and the series (non-reactive) resistance is 2000 Ω.*

(Elect. Instru. & Measure. Nagpur Univ. 1992)

Solution. Total circuit resistance of the voltmeter is

$$= (r + R) = 500 + 2{,}000 = 2{,}500\ \Omega$$

Since the voltmeter reads correctly on direct current supply, its full-scale deflection current is = 250/2500 = 0.1 A.

When used on a.c. supply, instrument offers an impedance

$$Z = \sqrt{2500^2 + (2\pi \times 50 \times 1)^2} = 2.520\ \Omega \quad \therefore\ I = 0.099\ \text{A}$$

$\therefore$ Voltmeter reading on a.c. supply = 250 × 0.099/0.1 = **248 V***

Note. Since swamp resistance R = 2,000 Ω, capacitor required for compensating the frequency error is

$$C = 0.41\ L/R^2 = 0.41 \times 1/2000^2 = 0.1\ \mu\text{F}.$$

Example 10.4. *A 150-V moving-iron voltmeter intended for 50 Hz has an inductance of 0.7 H and a resistance of 3 kΩ. Find the series resistance required to extend the range of the instrument to 300 V. If this 300-V, 50-Hz instrument is used to measure a d.c. voltage, find the d.c. voltage when the scale reading is 200 V.* **(Elect. Measur, A.M.I.E. Sec B, 1991)**

Solution. Voltmeter reactance = 2π × 50 × 0.7 = 220 Ω

Impedance of voltmeter = (3000 + j 220) = 3008 Ω

When the voltmeter range is doubled, its impedance has also to be doubled in order to have the same current for full-scale deflection. If R is the required series resistance, then $(3000 + R)^2 + 220^2 = (2 \times 3008)^2$ $\therefore$ R = 3012 Ω

When used on d.c. supply, if the voltmeter reads 200 V, the actual applied d.c. voltage would be = 200 × (Total A.C. Impedance)/total d.c. resistance) = 200 × (2 × 3008)/(3000 + 3012) = 200 × (6016 × 6012) = 200.134 V.

Example 10.5. *The coil of a moving-iron voltmeter has a resistance of 5,000 Ω at 15°C at which temperature it reads correctly when connected to a supply of 200 V. If the coil is wound with wire whose temperature coefficient at 15°C is 0.004, find the percentage error in the reading when the temperature is 50°C.*

In the above instrument, the coil is replaced by one of 2,000 Ω but having the same number of turns and the full 5,000 Ω resistance is obtained by connecting in series a 3,000 Ω resistor of negligible temperature-coefficient. If this instrument reads correctly at 15°C, what will be its percentage error at 50°C.

Solution. Current at 15°C = 200/5,000 = 0.04 A

Resistance at 50°C is $R_{50} = R_{15}(1 + \alpha_{15} \times 35)$

$\therefore$ $R_{50} = 5{,}000\ (1 + 35 \times 0.004) = 5{,}700\ \Omega$

* or reading = 250 × 2500/2520 = **248 V.**

$\therefore$ current at 50ºC = 200/5,700

$\therefore$ reading at 50ºC = $\frac{200 \times (200/5,700)}{0.04}$ = 175.4 V or = 200 × 5000/5700 = 175.4 V

$\therefore$ % error = $\frac{175.4 - 200}{200} \times 100$ = **–12.3%**

In the second case, swamp resistance is 3,000 Ω whereas the resistance of the instrument is only 2,000 Ω

Instrument resistance at 50ºC = 2,000 (1 + 35 × 0.004) = 2,280 Ω

$\therefore$ total resistance at 50ºC = 3,000 + 2,280 = 5,280 Ω

$\therefore$ current at 50ºC = 200/5,280 A

$\therefore$ instrument reading = $200 \times \frac{200/5,280}{0.04}$ = 189.3 V

$\therefore$ percentage error = $\frac{189.3 - 200}{200} \times 100$ = **–5.4 %**

Example 10.6. *The change of inductance for a moving-iron ammeter is 2μH/degree. The control spring constant is 5 × 10^{-7} N-m/degree.*

The maximum deflection of the pointer is 100º, what is the current corresponding to maximum deflection ? **(Measurement & Instrumentation Nagpur Univ. 1993)**

Solution. As seen from Art. 10.12 the deflecting torque is given by

$$T_d = \frac{1}{2} I^2 \frac{dL}{d\theta} \text{ N-m}$$

Control spring constant = 5 × 10^{-7} N-m/degree

Deflection torque for 100° deflection = 5 × 10^{-7} × 100 = 5 × 10^{-5} N-m ; $dL/d\theta$ = 2 μH/degree = 2 × 10^{-6} H/degree.

$\therefore$ 5 × 10^{-5} = $\frac{1}{2}$ I² × 2 × 10^{-6} $\therefore$ I^2 = **50** and I = 7.07 A

Example 10.7. *The inductance of attraction type instrument is given by L = (10 + 5θ – θ²)μH where θ is the deflection in radian from zero position. The spring constant is 12 × 10^{-6} N-m/rad. Find out the deflection for a current of 5 A.*

(Elect. and Electronics Measurements and Measuring Instruments Nagpur Univ. 1993)

Solution. $L = (10 + 5\theta - \theta^2) \times 10^{-6}$ H

$\therefore$ $\frac{dL}{d\theta} = (0 + 5 - 2 \times \theta) \times 10^{-6} = (5 - 2\theta) \times 10^{-6}$ H/rad

Let the deflection be θ radians for a current of 5A, then deflecting torque,

$$T_d = 12 \times 10^{-6} \times \theta \text{ N-m}$$

Also, $$T_d = \frac{1}{2} I^2 \frac{dL}{d\theta}$$ ...Art.

Equating the two torques, we get

$12 \times 10^{-6} \times \theta = \frac{1}{2} \times 5^2 \times (5 - 2\theta) \times 10^{-6}$ $\therefore$ θ = **1.689 radian**

Tutorial Problems No. 10.1

1. Derive an expression for the torque of a moving-iron ammeter. The inductance of a certain moving-iron ammeter is $(8 + 4\theta - \frac{1}{2}\theta^2)$ μH where θ is the deflection in radians from the zero position. The control-spring torque is 12 × 10^{-6} N-m/rad. Calaculate the scale position in radians for a current of 3A.

[1.09 rad] (*I.E.E. London*)

2. An a.c. voltmeter with a maximum scale reading of 50-V has a resistance of 500 Ω and an inductance of 0.09 henry, The magnetising coil is wound with 50 turns of copper wire and the remainder of the circuit is a non-inductive resistance in series with it. What additional apparatus is needed to make this instrument read correctly on both d.c. circuits or frequency 60 ? **[0.44 μF in parallel with series resistance]**

3. A 10-V moving-iron ammeter has a full-scale deflection of 40 mA on d.c. circuit. It reads 0.8% low on 50 Hz a.c. Hence, calculate the inductance of the ammeter. **[115.5 mH]**

4. It is proposed to use a non-inductive shunt to increase the range of a 10-A moving iron ammeter to 100 A. The resistance of the instrument, including the leads to the shunt, is 0.06 Ωand the inductance is 15 μ*H* at full scale. If the combination is correct on a.d.c circuit, find the error at full scale on a 50 Hz a.c. circuit. **[3.5 %](*London Univ.*)**

10.14. Moving-coil Instruments

There are two types of such instruments ***(i) permanent-magnet type*** which can be used for d.c. work only and ***(ii) the dynamometer type*** which can be used both for a.c. and d.c. work.

10.15. Permanent Magnet Type Instruments

The operation of a permanent-magnet moving-coil type instrument is be based upon the principle that when a current-carrying conductor is placed in a magnetic field, it is acted upon by a force which tends to move it to one side and out of the field.

Construction

As its name indicates, the instrument consists of a permanent magnet and a rectangular coil of many turns wound on a light aluminium or copper former inside which is an iron core as shown in

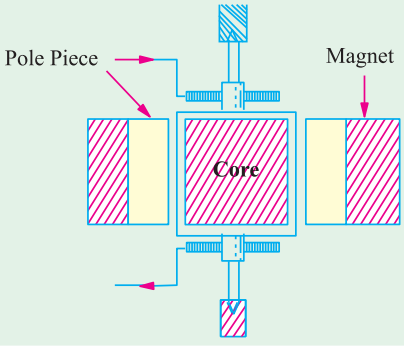


Fig. 10.16.

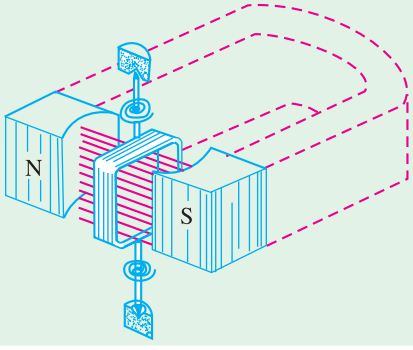


Fig. 10.17

Fig. 10.16. The powerfull U-shaped permanent magnet is made of Alnico and has soft-iron end-pole pieces which are bored out cylindrically. Between the magnetic poles is fixed a soft iron cylinder whose function is ***(i)*** to make the field radial and uniform and ***(ii)*** to decrease the reluctance of the air path between the poles and hence increase the magnetic flux. Surrounding the core is a rectangular coil of many turns wound on a light aluminium frame which is supported by delicate bearings and to which is attached a light pointer. The aluminium frame not only provides support for the coil but also provides damping by eddy currents induced in it. The sides of the coil are free to move in the two air-gaps between the poles and core as shown in Fig. 10.16 and Fig. 10.17. Control of the coil movement is affected by two phosphor-bronze hair springs, one above and one below, which additionally serve the purpose of lending the current in and out of the coil. The two springs are spiralled in opposite directions in order to neutralize the effects of temperature changes.

Deflecting Torque

When current is passed through the coil, force acts upon its both sides which produce a deflecting torque as shown in Fig. 10.18. Let

B = flux density in Wb/m^2

l = length or depth of the coil in metre

b = breadth of coil in metre

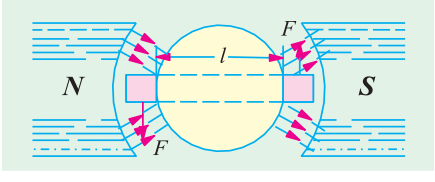


Fig. 10.18

N = number of turns in the coil

If I ampere is the current passing through the coil, then the magnitude of the force experienced by each of its sides is = BIl newton

For N turns, the force on each side of the coil is = $NBIl$ newton

$\therefore$ deflecting torque T_d = force × perpendicular distance

= $NBIl \times b = NBI(I \times b) = NBIA$ N-m

where A is the face area of the coil.

It is seen that if B is constant, then T_d is proportional to the current passing through the coil *i.e.* $T_d \propto I$.

Such instruments are invariable spring-controlled so that $T_c \propto$ deflection θ.

Since at the final deflected position, $T_d = T_c$ $\therefore \theta \propto I$

Hence, such instruments have uniform scales. Damping is electromagnetic *i.e.* by eddy currents induced in the metal frame over which the coil is wound. Since the frame moves in an intense magnetic field, the induced eddy currents are large and damping is very effective.

10.16. Advantage and Disadvantages

The permanent-magnet moving-coil (PMMC) type instruments have the following advantages and disadvantages :

Advantages

1. they have low power consumption.
2. their scales are uniform and can be designed to extend over an arc of 170° or so.
3. they possess high (torque/weight) ratio.
4. they can be modified with the help of shunts and resistances to cover a wide range of currents and voltages.
5. they have no hysteresis loss.
6. they have very effective and efficient eddy-current damping.
7. since the operating fields of such instruments are very strong, they are not much affected by stray magnetic fields.

Disadvantages

1. due to delicate construction and the necessary accurate machining and assembly of various parts, such instruments are somewhat costlier as compared to moving-iron instruments.
2. some errors are set in due to the aging of control springs and the parmanent magnets.

Such instruments are mainly used for d.c. work only, but they have been sometimes used in conjunction with rectifiers or thermo-junctions for a.c. measurements over a wide range or frequencies.

Permanent-magnet moving-coil instruments can be used as ammeters (with the help of a low resistance shunt) or as voltmeters (with the help of a high series resistance).

The principle of permanent-magnet moving-coil type instruments has been utilized in the construction of the following :

1. For a.c. galvanometer which can be used for detecting extremely small d.c. currents. A galvanometer may be used either as an ammeter (with the help of a low resistance) or as a voltmeter (with the help of a high series resistance). Such a galvanometer (of pivoted type) is shown in Fig. 10.19.
2. By eliminating the control springs, the instrument can be used for measuring the quantity of electricity passing through the coil. This method is used for *fluxmeters.*



Fig. 10.19

3. If the control springs of such an instrument are purposely made of large moment of inertia, then it can be used as ballistic galvanometer.

10.17. Extension of Range

(*i*) As Ammeter

When such an instrument is used as an ammeter, its range can be extended with the help of a low-resistance shunt as shown in Fig. 10.12 (*a*). This shunt provides a bypath for extra current because it is connected across (*i.e.* in parallel with) the instrument. These shunted instruments can be made to record currents many times greater than their normal full-scale deflection currents. The ratio of maximum current (with shunt) to the full-scale deflection current (without shunt) is known as the 'multiplying power' or 'multiplying factor' of the shunt.

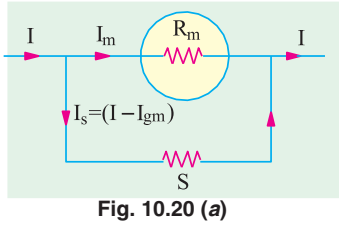


Fig. 10.20 (*a*)

Let R_m = instrument resistance
S = shunt resistance
I_m = full-scale deflection current of the instrument
I = line current to be measured

As seen from Fig. 10.20 (*a*), the voltage across the instrument coil and the shunt is the same since both are joined in parallel.

$$\therefore \quad I_m \times R_m = S\,I_s = S\,(I - I_m) \qquad \therefore \quad S = \frac{I_m R_m}{(I - I_m)}\,; \text{ Also } \frac{I}{I_m} = \left(1 + \frac{R_m}{S}\right)$$

$$\therefore \quad \text{multiplying power} = \left(1 + \frac{R_m}{S}\right)$$

Obviously, lower the value of shunt resistance, greater its multiplying power.

(*ii*) As voltmeter

The range of this instrument when used as a voltmeter can be increased by using a high resistance in series with it [Fig. 10.20 (*b*)].

Let I_m = full-scale deflection current
R_m = galvanometer resistance
$\nu = R_m I_m$ = full-scale p.d. across it
V = voltage to be measured
R = series resistance required

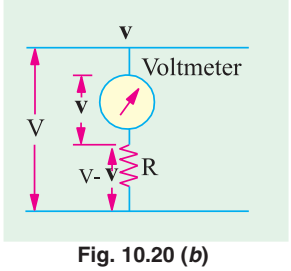


Fig. 10.20 (*b*)

Then it is seen that the voltage drop across R is $V - \nu$

$$\therefore \quad R = \frac{V - \nu}{I_m} \text{ or } \quad R \,.\, I_m = V - \nu$$

Dividing both sides by ν, we get

$$\frac{R I_m}{\nu} = \frac{V}{\nu} - 1 \quad \text{or} \quad \frac{R \,.\, I_m}{I_m R_m} = \frac{V}{\nu} - 1 \quad \therefore \quad \frac{V}{\nu} = \left(1 + \frac{R}{R_m}\right)$$

$$\therefore \quad \text{voltage multiplication} = \left(1 + \frac{R}{R_m}\right)$$

Obviously, larger the value of R, greater the voltage multiplication or range. Fig. 10.20 (*b*) shown a voltmeter with a single multiplier resistor for one range. A multi-range voltmeter requires on multiplier resistor for each additional range.

Example 10.8. *A moving coil ammeter has a fixed shunt of 0.02 Ω with a coil circuit rtesistance of R = 1 kΩ and need potential difference of 0.5 V across it for full-scale deflection.*

(1) *To what total current does this correspond ?*

(2) *Calculate the value of shunt to give full scale deflection when the total current is 10 A and 75 A.* **(Measurement & Instrumentation Nagpur Univ. 1993)**

Solution. It should be noted that the shunt and the meter coil are in parallel and have a common p.d. of 0.5 V applied across them.

(1) $\therefore I_m = 0.5/1000 = 0.0005$ A; $I_s = 0.5./0.02 = 25$ A

$\therefore$ line current = **25.0005 A**

(2) When total current is 10 A, $I_s = (10 - 0.0005) = 9.9995$ A

$$\therefore \quad S = \frac{I_m R_m}{I_s} = \frac{0.0005 \times 1000}{9.9995} = 0.05\ \Omega$$

When total current is 75 A, $I_s = (75 - 0.0005) = 74.9995$ A

$\therefore$ $S = 0.0005 \times 1000/74.9995 =$ **0.00667 Ω**

Example 10.9. *A moving-coil instrument has a resistance of 10 Ω and gives full-scale deflection when carrying a current of 50 mA. Show how it can be adopted to measure voltage up to 750 V and currents upto 1000 A.* **(Elements of Elect. Engg.I, Bangalore Univ.)**

Solution. (*a*) As Ammeter. As discussed above, current range of the meter can be extended by using a shunt across it [Fig. 10.21 (*a*)].

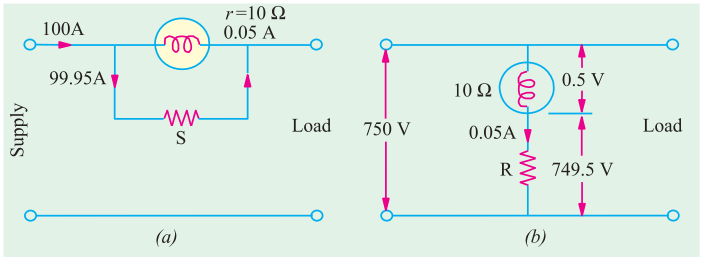


Fig. 10.21

Obviously,

$10 \times 0.05 = S \times 99.95$

$\therefore$ $S =$ **0.005 Ω**

(*b*) As Voltmeter. In this case, the range can be extended by using a high resistance R in series with it.

[Fig. 10.21 (*b*)]. Obviously, R must drop a voltage of (750-0.5) = 749.5 V while carrying 0.05 A.

$\therefore$ 0.05 R = 749.5 or $R =$ **14.990 Ω**

Example 10.10. *How will you use a P.M.M.C. instrument which gives full scale deflection at 50 mV p.d. and 10 mA current as*

(1) *Ammeter 0 - 10 A range*

(2) *Voltmeter 0-250 V range* **(Elect. Instruments & Measurements Nagpur Univ. 1993)**

Solution. Resistance of the instrument $R_m = 50$ mV/10 mA = 5

(*i*) As Ammeter

full-scale meter current, $I_m = 10$ mA = 0.01 A

shunt current $I_s = I - I_m = 10 - 0.01 = 9.99$ A

$$\text{Reqd. shunt resistance, } S = \frac{I_m . R_m}{(I - I_m)} = \frac{0.01 \times 5}{9.99} = 0.0005\ \Omega$$

(*ii*) As Voltmeter

Full-scale deflection voltage, v = 50 mV = 0.05 V; $V = 250$ V

$$\text{Reqd. series resistance, } R = \frac{V - v}{I_m} = \frac{250 - 0.05}{0.01} = 24{,}995\ \Omega$$

Example 10.11. *A current galvanometer has the following parameters :*

$B = 10 \times 10^{-3}$ *Wb/m*^{2} *; N = 200 turns, l = 16 mm;*

d = 16 mm; k = 12×10^{-9} *Nm/radian.*

Calculate the deflection of the galvanometer when a current of 1 μ*A flows through it.*

(Elect. Measurement Nagpur Univ. 1993)

Solution. Deflecting torque $T_d = NBIA$ N-m $= 200 \times (10 \times 10^{-3}) \times (1 \times 10^{-6}) \times (1 \times 10^{-3}) \times (16 \times 10^{-3})$ N-m $= 512 \times 10^{-12}$ N-m

Controlling torque T_c = controlling spring constant × deflection $= 12 \times 10^{-9} \times \theta$ N-m

Equating the deflecting and controlling torques, we have $12 \times 110^{-9} \times \theta = 512 \times 10^{-12}$

$\therefore \quad \theta = 0.0427$ radian = **2.45°**

Example 10.12. *The coil of a moving coil permanent magnet voltmeter is 40 mm long and 30 mm wide and has 100 turns on it. The control spring exerts a torque of 120×10^{-6} N-m when the deflection is 100 divisions on full scale. If the flux density of the magnetic field in the air gap is 0.5 Wb/m^2, estimate the resistance that must be put in series with the coil to give one volt per division. The resistance of the voltmeter coil may be neglected.* **(Elect. Mesur. AMIE Sec. B Summer 1991)**

Solution. Let I be the current for full-scale deflection. Deflection torque $T_d = NBIA$

$= 100 \times 0.5 \times I \times (1200 \times 10^{-6}) = 0.06\, I$ N-m

Controlling torque $T_c = 120 \times 10^{-6}$ N-m

In the equilibrium position, the two torques are equal *i.e.* $T_d = T_c$.

$\therefore \quad 0.06\, I = 120 \times 10^{-6} \quad \therefore \quad I = 2 \times 10^{-3}$ A.

Since the instrument is meant to read 1 volt per division, its full-scale reading is 100 V.

Total resistance $= 100/2 \times 10^{-3} = 50{,}000\ \Omega$

Since voltmeter coil resistance is negligible, it represents the additional required resistance.

Example 10.13. *Show that the torque produced in a permanent-magnet moving-coil instrument is proportional to the area of the moving coil.*

A moving-coil voltmeter gives full-scale deflection with a current of 5 mA. The coil has 100 turns, effective depth of 3 cm and width of 2.5 cm. The controlling torque of the spring is 0.5 cm for full-scale deflection. Estimate the flux density in the gap. **(Elect. Meas, Marathwads Univ.)**

Solution. The full-scale deflecting torque is $T_d = NBIA$ N-m

where I is the full-scale deflection current ; $I = 5$ mA $= 0.005$ A

$T_d = 100 \times B \times 0.005 \times (3 \times 2.5 \times 10^{-4}) = 3.75 \times 10^{-4}\, B$ N-m

The controlling torque is

$T_c = 0.5$ g-cm $= 0.5$ g. wt.cm $= 0.5 \times 10^{-3} \times 10^{-2}$ kg wt-m

$= 0.5 \times 10^{-5} \times 9.8 = 4.9 \times 10^{-5}$ N-m

For equilibrium, the two torques are equal and opposite.

$\therefore \quad 4.9 \times 10^{-5} = 3.75 \times 10^{-4}\, B \quad \therefore \quad B =$ **0.13 Wb/m^2**

Example 10.14. *A moving-coil milliammeter has a resistance of 5 Ω and a full-scale deflection of 20 mA. Determine the resistance of a shunt to be used so that the instrument could measure currents upto 500 mA at 20° C. What is the percentage error in the instrument operating at a temperature of 40°C ? Temperature co-efficient of copper = 0.0039 per °C.*

(Measu. & Instrumentation, Allahabad Univ. 1991)

Solution. Let R_{20} be the shunt resistance at 20°C. When the temperature is 20°C, line current is 500 mA and shunt current is = (500–20) = 480 mA.

$\therefore \quad 5 \times 20 = R_{20} \times 480, \quad R_{20} = 1/4.8\ \Omega$

If R_{40} is the shunt resistance at 40°C, then

$$R_{40}\, R_{20}\, (1 + 20\,\alpha) = \frac{1}{4.8}\,(1 + 0.0039 \times 20) = \frac{1.078}{4.8}\ \Omega$$

Shunt current at 40°C is $= \dfrac{5 \times 20}{1.078/4.8} = 445$ mA

Line current = 445 + 20 = 465 mA

Although, line current would be only 465 mA, the instrument will indicate 500 mA.

$\therefore \quad$ error $= 35/500 = 0.07$ or 7%

Example 10.15. *A moving-coil millivoltmeter has a resistance of 20Ω and full-scale deflection of 120° is reached when a potential difference of 100 mV is applied across its terminals. The moving coil has the effective dimensions of 3.1 cm × 2.6 cm and is wound with 120 turns. The flux density in the gap is 0.15 Wb/m^2. Determine the control constant of the spring and suitable diameter of copper wire for coil winding if 55% of total instrument resistance is due to coil winding. ρ for copper = 1.73 × 10^{-6} Ω cm.* **(Elect. Inst. and Meas. M.S. Univ. Baroda)**

Solution. Full-scale deflection current is = 100/20 = 5 mA

Deflecting torque for full-scale deflection of 120° is

$$T_d = NBIA = 120 \times 0.15 \times (5 \times 10^{-3}) \times (3.1 \times 2.6 \times 10^{-4}) = 72.5 \times 10^{-6} \text{ N-m}$$

Control constant is defined as the deflecting torque per radian (or degree) or deflection of moving coil. Since this deflecting torque is for 120° deflection.

Control constant = 72.5 × 10^{-6}/120 = **6.04 × 10–7 N-m/degree**

Now, resistance of copper wire = 55% of 20 Ω = 11 Ω

Total length of copper wire = 120 × 2 (3.1 + 2.6) = 1368 cm

Now $R = \rho l/A \quad \therefore A = 1.73 \times 10^{-6} \times 1368/11 = 215.2 \times 10^{-6} \text{ cm}^2$

$\therefore \quad \pi d^2/4 = 215.2 \times 10^{-6}$

$\therefore \quad d = \sqrt{215 \times 4 \times 10^{-6}/\pi} = 16.55 \times 10^{-3} \text{ cm} =$ **0.1655 mm**

10.18. Voltmeter Sensitivity

It is defined in terms of resistance per volt (Ω/V). Suppose a meter movement of 1 kΩ internal resistance has s full-scale deflection current of 50 μ A. Obviously, full-scale voltage drop of the meter movement is = 50 μA × 1000 Ω = 50 mV. When used as a voltmeter, its sensitivity would be 1000/50 × 10^{-3} = 20 kΩ/V. It should be clearly understood that a sensitivity of 20 kΩ/V means that the total resistance of the circuit in which the above movement is used should be 20 kΩ for a full-scale deflection of 1 V.

10.19. Multi-range Voltmeter

It is a voltmeter which measures a number of voltage ranges with the help of different series resistances. The resistance required for each range can be easily calculated provided we remember one basic fact that the sensitivity of a meter movement is always the same regardless of the range selected. Moreover, the full-scale deflection current is the same in every range. For any range, the total circuit resistance is found by multiplying the sensitivity by the full-scale voltage for that range.

For example, in the case of the above-mentioned 50μA, 1 kΩ meter movement, total resistance required for 1 V full-scale deflection is 20 kΩ. It means that an additional series resistance of 19 kΩ is required for the purpose as shown in Fig. 10.22 (*a*).

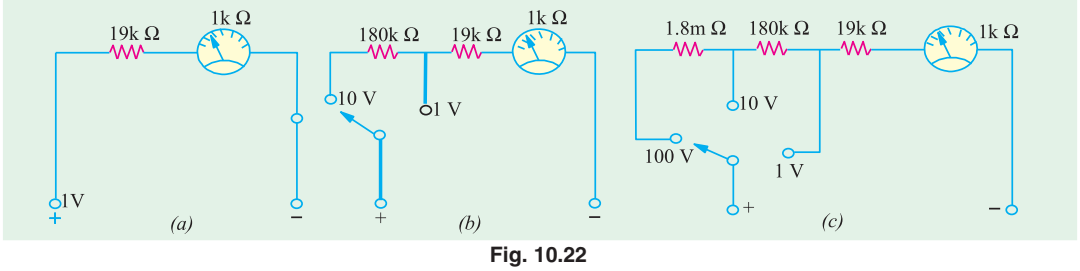


Fig. 10.22

For 10-V range, total circuit resistance must be (20 kΩ/V) (10 V) = 200 kΩ. Since total resistance for 1 V range is 20 kΩ, the series resistance *R* for 10-V range = 200 –20 = 180 kΩ as shown in Fig. 10.22 (*b*).

For the range of 100 V, total resistance required is (20 kΩ/V) (100 V) = 2 MΩ. The additional resistance required can be found by subtracting the existing two-range resistance from the total

resistance of 2 MΩ. Its value is

$$= 2\ M\ \Omega - 180\ k\Omega - 19\ k\Omega - 1\ k\Omega = 1.8\ M\Omega$$

It is shown in Fig. 10.22 (*c*).

Example 10.16. *A basic 1' Arsonval movement with internal resistance R_m = 100 Ω and full scale deflection current I_f = 1 mA is to be converted into a multirange d.c. voltmeter with voltage ranges of 0-10 V, 0-50 V, 0-250 V and 0-500 V. Draw the necessary circuit arrangement and find the values of suitable multipliers.* **(Instrumentation AMIE Sec. B Winter 1991)**

Solution. Full-scale voltage drop = (1 mA) (100 Ω) = 100 mV. Hence, sensitivity of this movement is $100/100 \times 10^{-3}$ = 1 kΩ/V.

(i) 0-10 V range

Total resistance required = (1 kΩ/V) (10 V) = 10 kΩ. Since meter resistance is 1 kΩ additional series resistance required for this range R_1 = 10 – 1 = **9 kΩ**

(ii) 0-50 V range

R_T = (1 kΩ/V) (50 V) = 50 kΩ; R_2 = 50-9-1 = **40 kΩ**

(iii) 0-250 V range

R_T = (1 kΩ/V) (250 V) = 250 kΩ; R_3 = 250 – 50 = **200 kΩ**

(iv) 0-500 V range

R_T = (1 kΩ/V) (500 V) = 500 kΩ ; R_4 = 500 – 250 = **250 kΩ**

The circuit arrangement is similar to the one shown in Fig. 10.22

Tutorial problem No. 10.2

1. The flux density in the gap of a 1-mA (full scale) moving '-coil' ammeter is 0.1 Wb/m^2. The rectangular moving-coil is 8 mm wide by 1 cm deep and is wound with 50 turns. Calculate the full-scale torque which must be provided by the springs. **[4 × 10-7 N-m] (*App. Elec. London Univ.*)**

2. A moving-coil instrument has 100 turns of wire with a resistance of 10 Ω, an active length in the gap of 3 cm and width of 2 cm. A p.d. of 45 mV produces full-scale deflection. The control spring exerts a torque of 490.5 × 10^{-7} N-m at full-scale deflection. Calculate the flux density in the gap.

[0.1817 Wb/m2] (*I.E.E. London*)

3. A moving-coil instrument, which gives full-scale deflection with 0.015 A has a copper coil having a resistance of 1.5 Ω at 15°C and a temperature coefficient of 1/234.5 at 0°C in series with a swamp resistance of 3.5 Ω having a negligible temperature coefficient.

Determine (*a*) the resistance of shunt required for a full-scale deflection of 20 A and (*b*) the resistance required for a full-scale deflection of 250 V.

If the instrument reads correctly at 15°C, determine the percentage error in each case when the temperature is 25°C. **[(a) 0.00376 Ω ; 1.3 % (b) 16,662 Ω, negligible] (*App. Elect. London Univ.*)**

4. A direct current ammeter and leads have a total resistance of 1.5 Ω. The instrument gives a full-scale deflection for a current of 50 mA. Calculate the resistance of the shunts necessary to give full-scale ranges of 2-5, 5.0 and 25.0 amperes **[0.0306 ; 1 0.01515 ; 0.00301 Ω] (*I.E.E. London*)**

5. The following data refer to a moving-coil voltmeter : resistance = 10,000 Ω, dimensions of coil = 3 cm × 3 cm ; number of turns on coil = 100, flux density in air-gap = 0.08 Wb/m^2, stiffness of springs = 3 × 10^{-6} N-m per degree. Find the deflection produced by 110 V. **[48.2°] (*London Univ.*)**

6. A moving-coil instrument has a resistance of 1.0 Ω and gives a full-scale deflection of 150 divisions with a p.d. of 0.15 V. Calculate the extra resistance required and show how it is connected to enable the instrument to be used as a voltmeter reading upto 15 volts. If the moving coil has a negligible temperature coefficient but the added resistance has a temperature coefficient of 0.004 Ω per degree *C*, what reading will a p.d. of 10 V give at 15ºC, assuming that the instrument reads correctly at 0°C. **[99 Ω, 9.45]**

10.20. Electrodynamic or Dynamometer Type Instruments

An electrodynamic instrument is a moving-coil instrument in which the operating field is produced, not by a permanent magnet but by another fixed coil. This instrument can be used either as an ammeter or a voltmeter but is generally used as a wattmeter.

As shown in Fig. 10.23, the fixed coil is usually arranged in two equal sections *F* and *F* placed close together and parallel to each other. The two fixed coils are air-cored to avoid hysteresis effects

when used on a.c. circuits. This has the effect of making the magnetic field in which moves the moving coil *M*, more uniform. The moving coil is spring-controlled and has a pointer attached to it as shown.

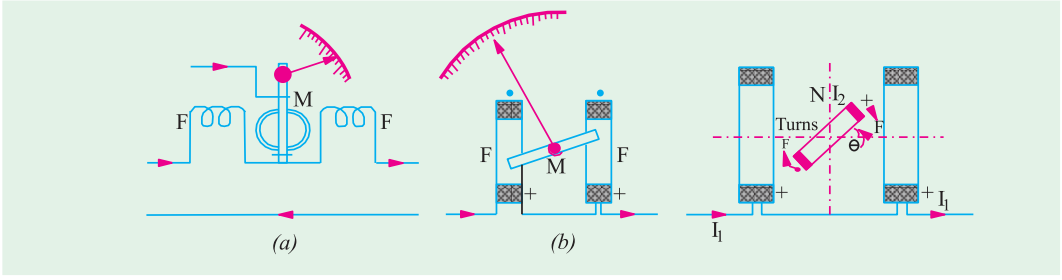


Fig. 10.23 Fig. 10.24

Deflecting Torque*

The production of the deflecting torque can be understood from Fig. 10.24. Let the current passing through the fixed coil be I_1 and that through the moving coil be I_2. Since there is no iron, the field strength and hence the flux density is proportional to I_1.

$\therefore \quad B = KI_1$ where K is a constant

Let us assume for simplicity that the moving coil is rectangular (it can be circular also) and of dimensions $l \times b$. Then, force on each side of the coil having N turns is (NBI_2l) newton.

The turning moment or deflecting torque on the coil is given by

$$T_d = NBI_2lb = NKI_1I_2lb \text{ N-m}$$

Now, putting $NKlb = K_1$, we have $T_d = K_1I_1I_2$ where K_1 is another constant.

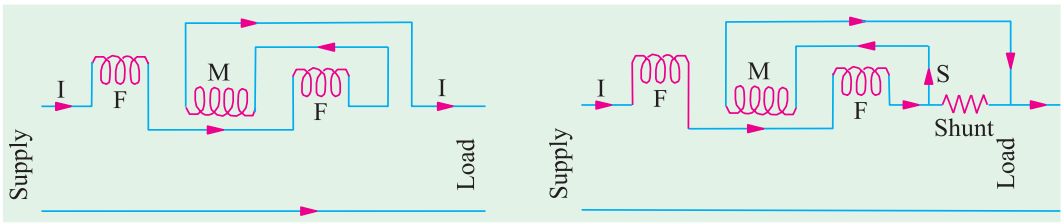


Fig. 10.25 Fig. 10.26

It shows that the deflecting torque is proportional to the product of the currents flowing in the fixed coils and the moving coil. Since the instrument tis spring-controlled, the restoring or control torque is proportional to the angular deflection θ.

i.e. $T_c \propto = K_2\,\theta \quad \therefore \quad K_1I_1I_2 = K_2\theta \quad \text{or } \theta \propto I_1I_2$

---

* As shown in Art. 10.12, the value of torque of a moving-coil instrument is

$$T_d = \frac{1}{2} I^2\, dL/d\theta \; N - m$$

The equivalent inductance of the fixed and moving coils of the electrodynamic instrument is

$$L = L_1 + L_2 + 2M$$

where M is the mutual inductance between the two coils and L_1 and L_2 are their individual self-inductances.

Since L_1 and L_2 are fixed and only M varies,

$\therefore \quad dL/d\theta = 2dM/d\theta \quad \therefore \quad T_d \frac{1}{2} I^2 \times 2.dM/d\theta = I^2.dM/d\theta$

If the currents in the fixed and moving coils are different, say I_1 and I_2 then

$$T_d = I_1 .I_2. dM/d\theta \; N\text{-}m$$

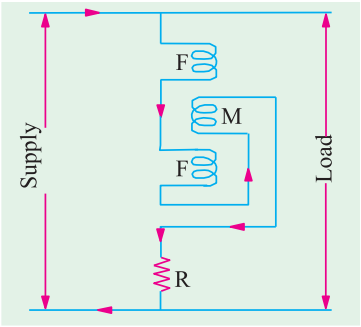


Fig. 10.27

When the instrument is used as an ammeter, the same current passes through both the fixed and the moving coils as shown in Fig. 10.25.

In that case $I_1 = I_2 = I$, hence $\theta \propto I^2$ or $I \propto \sqrt{\theta}$. The connections of Fig. 10.24 are used when small currents are to be measured. In the case of heavy currents, a shunt S is used to limit current through the moving coil as shown in Fig. 10.26.

When used as a voltmeter, the fixed and moving coils are joined in series along with a high resistance and connected as shown in Fig. 10.27. Here, again $I_1 = I_2 = I$, where $I = \frac{V}{R}$ in d.c. circuits and $I = V/Z$ in a.c. circuits.

$$\therefore \quad \theta \propto V \times V \quad \text{or} \quad \theta \propto V^2 \quad \text{or} \quad V \propto \sqrt{\theta}$$

Hence, it is found that whether the instrument is used as an ammeter or voltmeter, its scale is uneven throughout the whole of its range and is particularly cramped or crowded near the zero.

Damping is pneumatic, since owing to weak operating field, eddy current damping is inadmissible. Such instruments can be used for both a.c. and d.c. measurements. But it is more expensive and inferior to a moving-coil instrument for d.c. measurements.

As mentioned earlier, the most important application of electrodynamic principle is the wattmeter and is discussed in detail in Art. 10.34.

Errors

Since the coils are air-cored, the operating field produced is small. For producing an appreciable deflecting torque, a large number of turns is necessary for the moving coil. The magnitude of the current is also limited because two control springs are used both for leading in and for leading out the current. Both these factors lead to a heavy moving system resulting in frictional losses which are somewhat larger than in other types and so frictional errors tend to be relatively higher. The current in the field coils is limited for the fear of heating the coils which results in the increase of their resistance. A good amount of screening is necessary to avoid the influence of stray fields.

Advantages and Disadvantages

1. Such instruments are free from hysteresis and eddy-current errors.
2. Since (torque/weight) ratio is small, such instruments have low sensitivity.

Example 10.17. *The mutual inductance of a 25-A electrodynamic ammeter changes uniformly at a rate of 0.0035 μH/degree. The torsion constant of the controlling spring is 10^{-6} N-m per degree. Determine the angular deflection for full-scale.*

(Elect. Measurements, Poona Univ.)

Solution. By torsion constant is meant the deflecting torque per degree of deflection. If full-scale deflecting is θ degree, then deflecting torque on full-scale is $10^{-6} \times \theta$ N-m.

Now, $T_d = I^2 dM/d\theta$ Also, $I = 25$ A

$dM/d\theta = 0.0035 \times 10^{-6}$ H/degree $= 0.0035 \times 10^{-6} \times 180/\pi$ H/radian

$10^{-6} \times \theta = 25^2 \times 0.0035 \times 10^{-6} \times 180/\pi \quad \therefore \quad \theta =$ **125.4°**

Example 10.18. *The spring constant of a 10-A dynamometer wattmeter is 10.5×10^{-6} N-m per radian. The variation of inductance with angular position of moving system is practically linear over the operating range, the rate of change being 0.078 mH per radian. If the full-scale deflection of the instrument is 83 degrees, calculate the current required in the voltage coil at full scale on d.c. circuit.* **(Elect. Inst. and Means. Nagpur Univ. 1991)**

Solution. As seen from foot-note of Art. 10.20, $T_d = I_1 I_2 \, dM/d\theta$ N-m

Spring constant = 10.5×10^{-6} N/m/rad = $10.5 \times 10^{-6} \times \pi/180$ N-m/degree

T_d = spring constant × deflection = $(10.5 \times 10^{-6} \times \pi/180) \times 83 = 15.2 \times 10^{-6}$ N-m

$\therefore \quad 15.2 \times 10^{-6} = 10 \times I_2 \times 0.078$; I_2 = **19.5 μA.**

10.21. Hot-wire Instruments

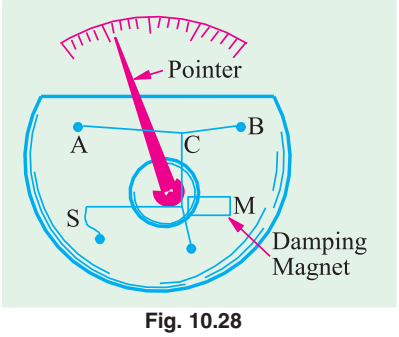


Fig. 10.28

The working parts of the instrument are shown in Fig. 10.28. It is based on the heating effect of current. It consists of platinum-iridium (It can withstand oxidation at high temperatures) wire *AB* stretched between a fixed end *B* and the tension-adjusting screw at *A*. When current is passed through *AB*, it expands according to I^2R formula. This sag in *AB* produces a slack in phosphor-bronze wire *CD* attached to the centre of *AB*. This slack in *CD* is taken up by the silk fibre which after passing round the pulley is attached to a spring *S*. As the silk thread is pulled by *S*, the pulley moves, thereby deflecting the pointer. It would be noted that even a small sag in *AB* is magnified (Art. 10.22) many times and is conveyed to the pointer. Expansion of *AB* is magnified by *CD* which is further magnified by the silk thread.

It will be seen that the deflection of the pointer is proportional to the extension of *AB* which is itself proportional to I^2. Hence, deflection is $\propto I^2$. If spring control is used, then $T_c \propto \theta$.

Hence $\theta \propto I^2$

So, these instruments have a 'squre law' type scale. They read the r.m.s. value of current and their readings are independent of its form and frequency.

Damping

A thin light aluminium disc is attached to the pulley such that its edge moves between the poles of a permanent magnet *M*. Eddy currents produced in this disc give the necessary damping.

These instruments are primarily meant for being used as ammeters but can be adopted as voltmeters by connecting a high resistance in series with them. These instruments are suited both for a.c. and d.c. work.

Advantages of Hot-wire Instruments :

1. As their deflection depends on the r.m.s. value of the alternating current, they can be used on direct current also.

2. Their readings are independent of waveform and frequency.

3. They are unaffected by stray fields.

Disadvantages

1. They are sluggish owing to the time taken by the wire to heat up.

2. They have a high power consumption as compared to moving-coil instruments. Current consumption is 200 mA at full load.

3. Their zero position needs frequent adjustment. **4.** They are fragile.

10.22. Magnification of the Expansion

As shown in Fig. 10.29 (*a*), let *L* be the length of the wire *AB* and *dL* its expansion after steady temperature is reached. The sag *S* produced in the wire as seen from Fig. 10.29 (*a*) is given by

$$S^2 = \left(\frac{L + dL}{2}\right)^2 - \left(\frac{L}{2}\right)^2 = \frac{2L.\,dL + (dL)^2}{4}$$

Neglecting $(dL)^2$, we have $S = \sqrt{L\,.\,dL/2}$

Magnification produced is $= \dfrac{S}{dL} = \dfrac{\sqrt{L\,.\,dL/2}}{dL} = \sqrt{\dfrac{L}{2.dL}}$

As shown in Fig. 10.29 (*b*), in the case of double-sag instruments, this sag is picked up by wire *CD* which is under the constant pull of the spring. Let L_1 be the length of wire *CD* and let it be pulled at its center, so as to take up the slack produced by the sag *S* of the wire *AB*.

$$S_1^2 = \left(\frac{L_1}{2}\right)^2 - \left(\frac{L_1 - S}{2}\right)^2 = \frac{2L_1 S - S^2}{4}$$

Neglecting S^2 as compared to $2L_1S$, we have $S_1 = \sqrt{L_1 S/2}$

Substituting the value of *S*, we get $S_1 = \sqrt{\frac{L_1}{2}\sqrt{\frac{L.dL}{2}}}$

$$\sqrt{\frac{L_1}{2\sqrt{2}}} \quad \sqrt{L.dL}$$

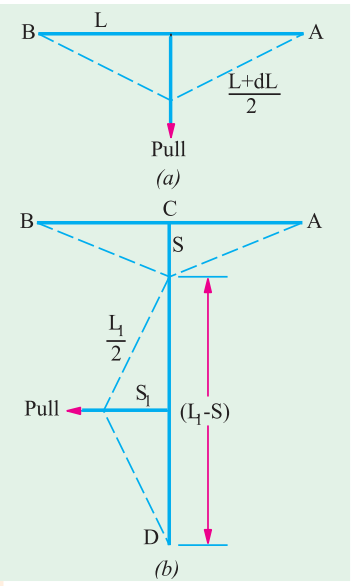


Fig. 10.29

Example 10.19. *The working wire of a single-sag hot wire instrument is 15 cm long and is made up of platinum-silver with a coefficient of linear expansion of 16 × 10^{-6}. The temperature rise of the wire is 85°C and the sag is taken up at the center. Find the magnification*

(i) *with no initial sag and* ***(ii)*** *with an initial sag of 1 mm.*

(Elect. Meas and Meas. Inst., Calcutta Univ.)

Solution. ***(i)*** Length of the wire at room temperature = 15 cm

Length when heated through 85°C is = $15\,(1 + 16 \times 10^{-6} \times 85) = 15.02$ cm

Increase in length, $dL = 15.02 - 15 = 0.02$ cm

Magnification $\sqrt{\frac{L}{2.dL}} \quad \sqrt{\frac{15}{0.04}}$ **19.36**

(ii) When there is an initial sag of 1 mm, the wire is in the position *ACB* (Fig. 10.30). With rise in temperature, the new position becomes *ADB*. From the right-angled Δ *ADE*,

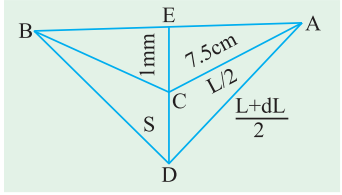


Fig. 10.30

we have $(S + 0.1)^2 = \left(\frac{L + dL}{2}\right)^2 - AE^2$

Now $AE^2 = AC^2 - EC^2 = \left(\frac{L}{2}\right)^2 - 0.1^2 = \frac{L^2}{4} - 0.1^2$

$$\therefore \quad (S + 0.1)^2 = \frac{L^2}{4} \quad \frac{(dL)^2}{4} \quad \frac{L.dL}{2} \quad \frac{L^2}{4} \quad 0.1^2 \quad \frac{L^2}{4} \quad \frac{(dL)^2}{4} \quad \frac{L.dL}{2} \quad \frac{L^2}{4} \quad 0.01$$

$$= \frac{L.dL}{2} + 0.01 \quad \text{...neglecting } (dL)^2/4$$

$$= \frac{15 \times 0.02}{2} + 0.01 = 0.16 \quad \therefore \quad (S + 0.1) = 0.4 \quad \therefore \quad S = 0.3 \text{ cm}$$

Magnification $= S/dL = 0.3/0.02 = \mathbf{15}$

10.23. Thermocouple Ammeter

The working principle of this ammeter is based on the Seebeck effect, which was discovered in 1821. A thermocouple, made of two dissimilar metals (usually bismuth and antimony) is used in the construction of this ammeter. The hot junction of the thermocouple is welded to a heater wire *AB*, both of which are kept in vacuum as shown in Fig. 10.31 (*a*). The cold junction of the thermocouple is connected to a moving-coil ammeter.

When the current to be measured is passed through the heater wire *AB*, heat is generated, which raises the temperature of the thermocouple junction *J*. As the junction temperature rises, the generated

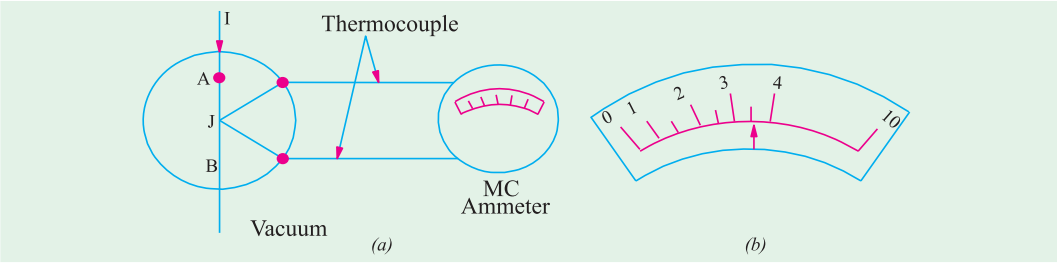


Fig. 10.31

thermoelectric EMF increases and derives a greater current through the moving-coil ammeter. The amount of deflection on the *MC* ammeter scale depend on the heating effect, since the amount of heat produced is directly proportional to the square of the current. The ammeter scale is non-linear so that, it is cramped at the low end and open at the high end as shown in Fig. 10.31 (*b*). This type of "current-squared: ammeter is suitable for reading both direct and alternating currents. It is particularly suitable for measuring radio-frequency currents such as those which occur in antenna system of broadcast transmitters. Once calibrated properly, the calibration of this ammeter remains accurate from dc upto very high frequency currents.



Megger

10.24. Megger

It is a portable instrument used for testing the insulation resistance of a circuit and for measuring resistances of the order of megaohms which are connected across the outside terminals XY in Fig. 10.32 (*b*).

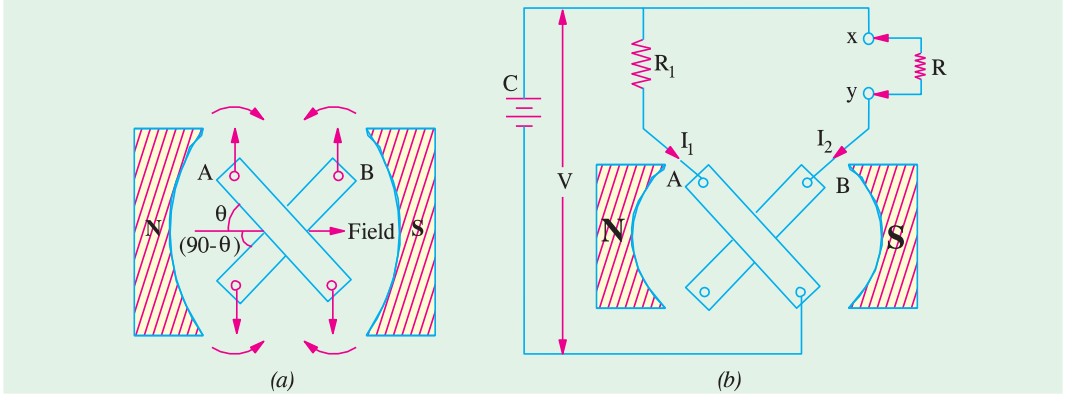


Fig. 10.32

1. Working Principle

The working principle of a 'corss-coil' type megger may be understood from Fig. 10.32 (*a*) which shows two coils *A* and *B* mounted rigidly at right angles to each other on a common axis and free to rotate in a magnetic field. When currents are passed through them, the two coils are acted upon by torques which are in opposite directions. The torque of coil *A* is proportional to $I_1 \cos \theta$ and that of *B* is proportional to $I_2 \cos (90 - \theta)$ or $I_2 \sin \theta$. The two coils come to a position of equilibrium where the two torques are equal and opposite *i.e.* where

$$I_1 \cos \theta = I_2 \sin \theta \quad \text{or} \quad \tan \theta = I_1/I_2$$

In practice, however, by modifying the shape of pole faces and the angle between the two coils, the ratio I_1/I_2 is made proportional to θ instead of tan θ in order to achieve a linear scale.

Suppose the two coils are connected across a common source of voltage *i.e.* battery *C*, as shown in Fig. 10.32 (*b*). Coil *A*, which is connected directly across *V*, is called the voltage (or control) coil. Its current $I_1 = V/R_1$. The coil *B* called current or deflecting coil, carries the current $I_2 = V/R$, where *R* is the external resistance to be measured. This resistance may vary from infinity (for good insulation or open circuit) to zero (for poor insulation or a short-circuit). The two coils are free to rotate in the field of a permanent magnet. The deflection θ of the instrument is proportional to I_1/I_2 which is equal to R/R_1. If R_1 is fixed, then the scale can be calibrated to read *R* directly (in practice, a current-limiting resistance is connected in the circuit of coil *B* but the presence of this resistance can be allowed for in scaling). The value of *V* is immaterial so long as it remains constant and is large enough to give suitable currents with the high resistance to be measured.

2. Construction

The essential parts of a megger are shown in Fig. 10.33. Instead of battery *C* of Fig. 10.32 (*b*), there is a hand-driven d.c. generator. The crank turns the generator armature through a clutch mechanism which is designed to slip at a pre-determined speed. In this way, the generator speed and voltage are kept constant and at their correct values when testing.

The generator voltage is applied across the voltage coil *A* through a fixed resistance R_1 and across deflecting coil *B* through a current-limiting resistance *R*′ and the external resistance is connected across testing terminal *XY*. The two coils, in fact, constitute a moving-coil voltmeter and an ammeter combined into one instrument.

(i) Suppose the terminals *XY* are open-circuited. Now, when crank is operated, the generator voltage so produced is applied across coil *A* and current I_1 flows through it but no current flows through coil *B*. The torque so produced rotates the moving element of the megger until the scale points to 'infinity', thus indicating that the resistance of the external circuit is too large for the instrument to measure.

(ii) When the testing terminals *XY* are closed through a low resistance or are short-circuited, then a large current (limited only by *R*′) passes through the deflecting coil *B*. The deflecting torque produced by coil *B* overcomes the small opposing torque of coil *A* and rotates the moving element until the needle points to 'zero', thus shown that the external resistance is too small for the instrument to measure.

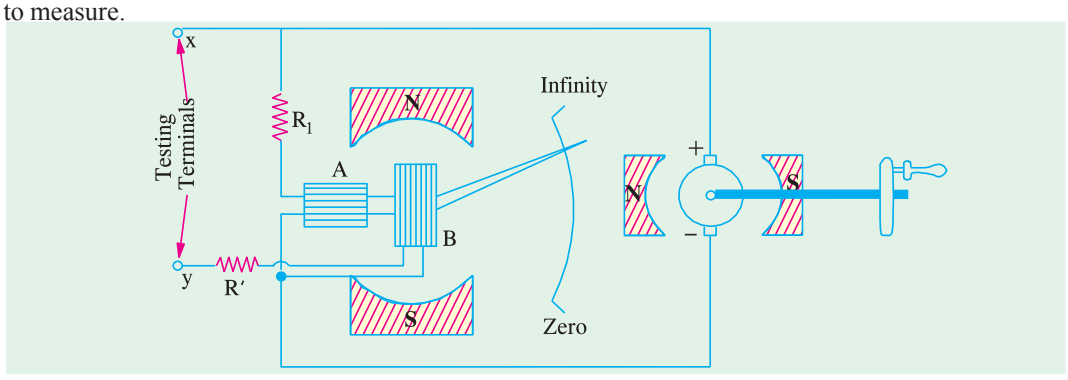


Fig. 10.33

Although, a megger can measure all resistance lying between zero and infinity, essentially it is a high-resistance measuring device. Usually, zero is the first mark and 10 kΩ is the second mark on its scale, so one can appreciate that it is impossible to accurately measure small resistances with the help of a megger.

The instrument described above is simple to operate, portable, very robust and independent of the external supplies.

10.25. Induction type Voltmeters and Ammeters

Induction type instruments are used only for a.c. measurements and can be used either as ammeter,

voltmeter or wattmeter. However, the induction principle finds its widest application as a watt-hour or energy meter. In such instruments, the deflecting torque is produced due to the reaction between the flux of an a.c. magnet and the eddy currents induced by this flux. Before discussing the two types of most commonly-used induction instruments, we will first discuss the underlying principle of their operation.

Principle

The operation of all induction instruments depends on the production of torque due to the reaction between a flux Φ_1 (whose magnitude depends on the current or voltage to be measured) and eddy currents induced in a metal disc or drum by another flux Φ_2 (whose magnitude also depends on the current or voltage to be measured). Since the magnitude of eddy currents also depend on the flux producing them, the *instantaneous* value of torque is proportional to the square of current or voltage under measurement and the value of *mean* torque is proportional to the mean square value of this current or voltage.

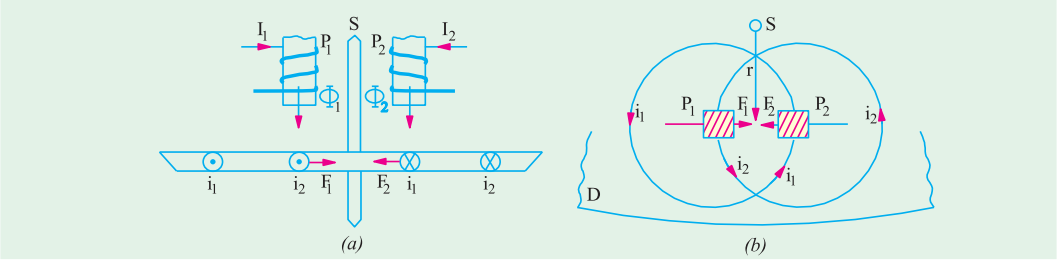


Fig. 10.34

Consider a thin aluminium or Cu disc *D* free to rotate about an axis passing through its centre as shown in Fig. 19.34. Two a.c. magnetic poles P_1 and P_2 produce alternating fluxes Φ_1 and Φ_2 respectively which cut this disc. Consider any annular portion of the disc around P_1 with center on the axis of P_1. This portion will be linked by flux Φ_1 and so an alternating e.m.f. e_1 be induced in it. This e.m.f. will circulate an eddy current i_1 which, as shown in Fig. 10.34, will pass under P_2. Similarly, Φ_2 will induce an e.m.f. e_2 which will further induce an eddy current i_2 in an annular portion of the disc around P_2. This eddy current i_2 flows under pole P_1.

Let us take the downward directions of fluxes as positive and further assume that at the instant under consideration, both Φ_1 and Φ_2 are increasing. By applying Lenz's law, the directions of the induced currents i_1 and i_2 can be found and are as indicated in Fig. 10.34.

The portion of the disc which is traversed by flux Φ_1 and carries eddy current i_2 experiences a force F_1 along the direction as indicated. As $F = Bil$, force $F_1 \propto \Phi_1 i_2$. Similarly, the portion of the disc lying in flux Φ_2 and carrying eddy current i_1 experiences a force $F_2 \propto \Phi_2 i_1$.

$\therefore \quad F_1 \propto \Phi_1 i_2 = K \Phi_1 i_2$ and $F_2 \propto \Phi_2 i_1 = K \Phi_2 i_1$.

It is assumed that the constant K is the same in both cases due to the symmetrically positions of P_1 and P_2 with respect to the disc.

If r is the effective radius at which these forces act, the net instantaneous torque T acting on the disc begin equal to the difference of the two torques, is given by

$$T = r(K\Phi_1 i_2 - K\Phi_2 i_1) = K_1(\Phi_1 i_2 - \Phi_2 i_1) \qquad ...(i)$$

Let the alternating flux Φ_1 be given by $\Phi_1 = \Phi_{1m} \sin \omega t$. The flux Φ_2 which is assumed to lag Φ_1 by an angle α radian is given by $\Phi_2 = \Phi_{2m} \sin(\omega t - \alpha)$

Induced e.m.f. $$e_1 = \frac{d\Phi_1}{dt} = \frac{d}{dt}(\Phi_{1m} \sin \omega t) = \omega \Phi_{1m} \cos \omega t$$

Assuming the eddy current path to be purely resistive and of value R*, the value of eddy current is

---

* If it has a reactance of X, then impedance Z should be taken, whose value is given by $Z = \sqrt{R^2 + X^2}$.

$i_1 \quad \frac{e_1}{R} \quad \frac{_{1m}}{R} \cos \quad t$ Similarly $e_2 \quad _{2m}(\quad t \quad)$ and $i_2 \quad \frac{_{2m}}{R} \cos(\quad t \quad)$ *

Substituting these values of i_1 and i_2 in Eq. **(*i*)** above, we get

$$T = \frac{K_1 \omega}{R}[\Phi_{1m} \sin \omega t . \Phi_{2m} \cos(\omega t - \alpha) - \Phi_{2m} \sin(\omega t - \alpha) \Phi_{1m} \cos \omega t]$$

$$= \frac{K_1 \omega}{R} . \Phi_{1m} \Phi_{2m} [\sin \omega t . \cos(\omega t - \alpha) - \cos \omega t . \sin(\omega t - \alpha)]$$

$$= \frac{K_1 \omega}{R} . \Phi_{1m} \Phi_{2m} \sin \alpha = k_2 \omega \Phi_{1m} \Phi_{2m} \sin \alpha \text{ (putting } K_1/R = K_2)$$

It is obvious that

(*i*) if $\alpha = 0$ *i.e.* if two fluxes are in phase, then *net* torque is zero. If on the other hand, $\alpha = 90°$, the net torque is maximum for given values of Φ_{1m} and Φ_{2m}.

(*ii*) the net torque is in such a direction as to rotate the disc from the pole with leading flux towards the pole with lagging flux.

(*iii*) since the expression for torque does not involve '*t*', it is independent of time *i.e.* it has a steady value at all time.

(*iv*) the torque *T* is inversely proportional to *R*-the resistance of the eddy current path. Hence, for large torques, the disc material should have low resistivity. Usually, it is made of Cu or, more often, of aluminium.

10.26. Induction Ammeters

It has been shown in Art.10.22 above that the net torque acting on the disc is

$$T = K_2 \omega \Phi_{1m} \Phi_{2m} \sin \alpha$$

Obviously, if both fluxes are produced by the same alternating current (of maximum value I_m) to be measured, then $T = K_3 \omega I_m^2 \sin \alpha$

Hence, for a given frequency ω and angle α, the torque is proportional to the square of the current. If the disc has spring control, it will take up a steady deflected position where controlling torque becomes equal to the deflecting torque. By attaching a suitable pointer to the disc, the apparatus can be used as an ammeter.

There are three different possible arrangements by which the operational requirements of induction ammeters can be met as discussed below.

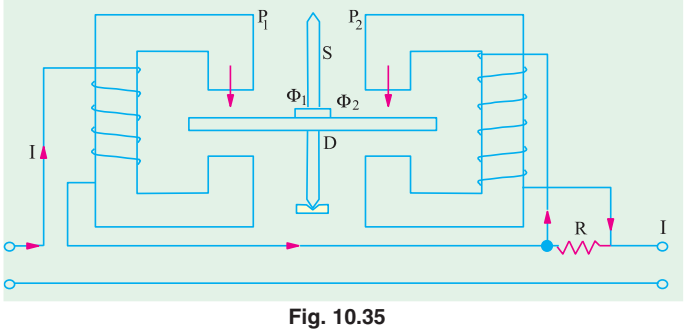


Fig. 10.35

(*i*) Disc Instrument with Split-phase Winding

In this arrangement, the windings on the two laminated a.c. magnets P_1 and P_2 are connected in series (Fig. 10.35). But, the winding of P_2 is shunted by a resistance *R* with the result that the current in this winding lags with respect to the total line current. In this way, the necessary phase angle α is produced between two fluxes Φ_1 and Φ_2 produced by P_1 and P_2 respectively. This angle is of the order of 60°. If the hysteresis effects etc. are neglected, then each flux would be proportional to the current to be measured *i.e.* line current *I*

$$T_d \propto \Phi_{1m} \Phi_{2m} \sin \alpha$$

or $T_d \propto I^2$ where *I* is the r.m.s. value.

If spring control is used, then $T_c \propto \theta$

* It being assumed that both paths have the same resistance.

In the final deflected position, $T_c = T_d \quad \therefore \ \theta \propto I^2$

Eddy current damping is employed in this instrument. When the disc rotates, it cuts the flux in the air-gap of the magnet and has eddy currents induced in it which provide efficient damping.

(*ii*) Cylindrical type with Split-phase Winding

The operating principle of this instrument is the same as that of the above instrument except that instead of a rotating disc, it employs a hollow aluminium drum as shown in Fig. 10.36. The poles P_1 produce the alternating flux Φ_1 which produces eddy current i_1 in those portions of the drum that lie under poles P_2. Similarly, flux F_2 due to poles P_2 produces eddy current i_2 in those parts of the drum that lie under poles P_1. The force F_1 which is $\propto \Phi_1 i_2$ and F_2 which is $\propto \Phi_2 i_1$ are tangential to the surface of the drum and the resulting torque tends to rotate the drum about its own axis. Again, the winding of P_2 is shunted by resistance R which helps to introduce the necessary phase difference α between F_1 and F_2.

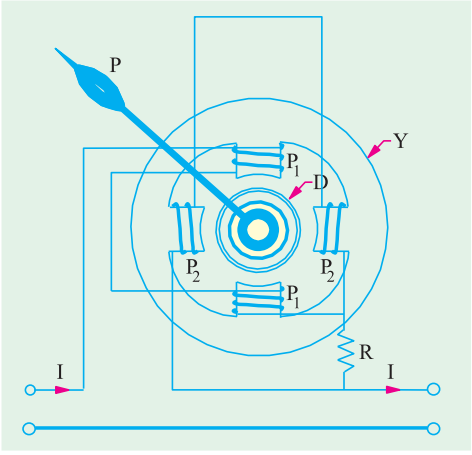


Fig. 10.36

The spiral control springs (not shown in the figure) prevent any continuous rotation of the drum and ultimately bring it to rest at a position where the deflecting torque becomes equal to the controlling torque of the springs. The drum has a pointer attached to it and is itself carried by a spindle whose two ends fit in jewelled bearings. There is a cylindrical laminated core inside the hollow drum whole function is to strengthen the flux cutting the drurm. The poles are laminated and magnetic circuits are completed by the yoke Y and the core.

Damping is by eddy currents induced in a separate aluminium disc (not shown in the figure) carried by the spindle when it moves in the air-gap flux of a horse-shoe magnet (also not shown in the figure).

(*iii*) Shaded-pole Induction Ammeter

In the shaded-pole disc type induction ammeter (Fig. 10.37) only single flux-producing winding is used. The flux F produced by this winding is split up into two fluxes Φ_1 and Φ_2 which are made to have the necessary phase difference of α by the device shown in Fig. 10.37. The portions of the upper and lower poles near the disc D are divided by a slot into two halves one of which carries a closed 'shading' winding or ring. This shading winding or ring acts as a short-circuited secondary and the main winding as a primary. The current induced in the ring by transformer action retards the phase of flux Φ_2 with respect to that of Φ_1 by about 50° or so. The two fluxes Φ_1 and Φ_2 passing through the unshaded and shaded parts respectively, react with eddy currents i_2 and i_1 respectively and so produce the net driving torque whose value is

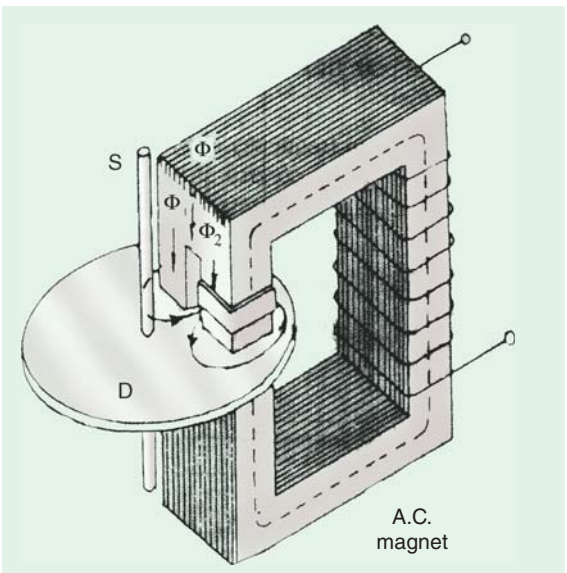


Fig. 10.37

$$T_d \propto \Phi_{1m}\ \Phi_{2m} \sin \alpha$$

Assuming that both Φ_1 and Φ_2 are proportional to the current I, we have

$$T_d \propto I^2$$

This torque is balanced by the controlling torque provided by the spiral springs.

The actual shaded-pole type induction instruments is shown in Fig. 10.38. It consists of a suitably-shaped aluminium or copper disc mounted on a spindly which is supported by jewelled bearings. The spindle carries a pointer and has a control spring attached to it. The edge or periphery of the disc moves in the air-gap of a laminated a.c. electromagnet which is energised either by the current to be measured (as ammeter) or by a current proportional to the voltage to be measured (as a voltmeter). Damping is by eddy currents induced by a permanent magnet embracing another portion of the ***same*** disc. As seen, the disc serves both for damping as well as operating purposes. The main flux is split into two components fluxes by shading one-half of each pole. These two fluxes have a phase difference of 40° to 50° between them and they induce two eddy currents in the disc. Each eddy current has a component in phase with the ***other*** flux, so that two torques are produced which are oppositely directed. The resultant torque is equal to the difference between the two. This torque ***deflects*** the disc-continuous rotation being prevented by the control spring and the deflection produced is proportional to the square of the current or voltage being measured.

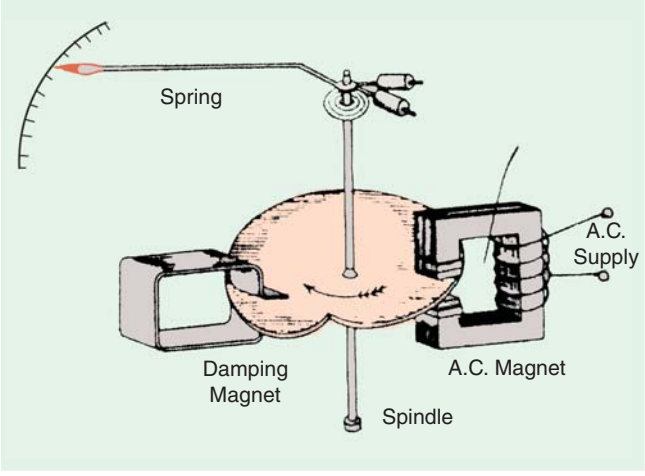


Fig. 10.38

As seen, for a given frequency, $T_d \propto I^2 \quad = KI^2$

For spring control $T_c \propto \theta$ or $T_c = K_1 \theta$

For steady deflection, we have $T_c = T_d$ or $\theta \propto I^2$

Hence, such instruments have uneven scales *i.e.* scales which are cramped at their lower ends. A more even scale can, however, be obtained by using a cam-shaped disc as shown in Fig. 10.38.

10.27. Induction Voltmeter

Its construction is similar to that of an induction ammeter except for the difference that its winding is wound with a large number of turns of fine wire. Since it is connected across the lines and carries very small current (5 –10mA), the number of turns of its wire has to be large in order to produce an adequate amount of m.m.f. Split phase windings are obtained by connecting a high resistance R in series with the winding of one magnet and an inductive coil in series with the winding of the other magnet as shown in Fig. 10.39.

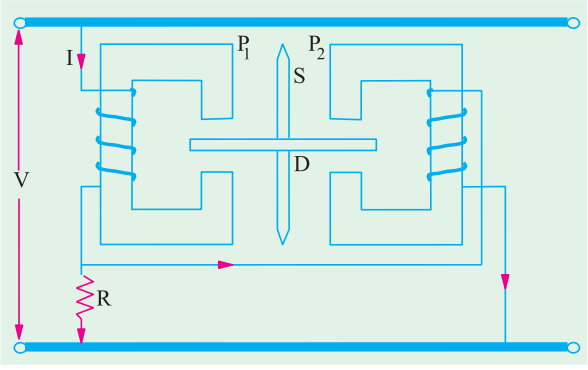


Fig. 10.39

10.28. Errors in Induction Instruments

There are two types of errors ***(i) frequency*** error and ***(ii) temperature*** error.

1. Since deflecting torque depends on frequency, hence unless the alternating current to be measured has same frequency with which the instrument was calibrated, there will be large

error in its readings. Frequency errors can be compensated for by the use of a non-inductive shunt in the case of ammeters. In voltmeters, such errors are not large and, to a great extent, are self-compensating.

2. Serious errors may occur due to the variation of temperature because the resistances of eddy current paths depends on the temperature. Such errors can, however, be compensated for by hunting in the case of ammeters and by combination of shunt and swamping resistances in the case of voltmeters.

10.29. Advantages and Disadvantages

1. A full-scale deflection of over 200° can be obtained with such instruments. Hence, they have long open scales.
2. Damping is very efficient.
3. They are not much affected by external stray fields.
4. Their power consumption is fairly large and cost relatively high.
5. They can be used for a.c. measurements only.
6. Unless compensated for frequency and temperature variations, serious errors may be introduced.

10.30. Electrostatic Voltmeters

Electrostatic instruments are almost always used as voltmeters and that too more as a laboratory rather than as industrial instruments. The underlying principle of their operation is the force of attraction between electric charges on neighboring plates between which a p.d. is maintained. This force gives rise to a deflecting torque. Unless the p.d. is sufficiently large, the force is small. Hence, such instruments are used for the measurement of very high voltages.

There are two general types of such instruments :

(i) the quadrant type-used upto 20 kV. ***(ii) the attracted disc type*** – used upto 500 kV.

10.31. Attracted-disc Type Voltmeter

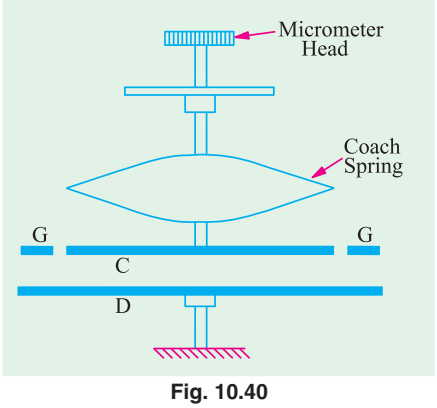


Fig. 10.40

As shown in Fig. 10.40, it consists of two-discs or plates C and D mounted parallel to each other. Plate D is fixed and is earthed while C suspended by a coach spring, the support for which carries a micrometer head for adjustment. Plate C is connected to the positive end of the supply voltage. When a p.d. (whether direct or alternating) is applied between the two plates, then C is attracted towards D but may be returned to its original position by the micrometer head. The movement of this head can be made to indicate the force F with which C is pulled downwards. For this purpose, the instrument can be calibrated by placing known weights in turn on C and observing the movement of micrometer head necessary to bring C back to its original position. Alternatively, this movement of plate C is balanced by a control device which actuates a pointer attached to it that sweeps over a calibrated scale.

There is a guard ring G surrounding the plate C and separated from it by a small air-gap. The ring is connected electrically to plate C and helps to make the field uniform between the two plates. The effective area of plate C, in that case, becomes equal to its actual area plus half the area of the air-gap.

Theory

In Fig. 10.41 are shown two parallel plates separated by a distance of x meters. Suppose the lower plate is fixed and carries a charge of $-Q$ coulomb whereas the upper plate is movable and

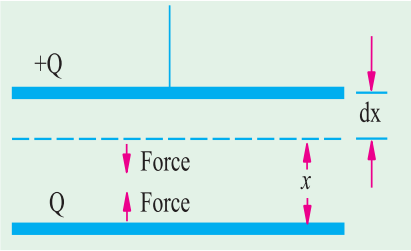


Fig. 10.41

carries a charge of + Q coulomb. Let the mutual force of attraction between the two plates be F newtons. Suppose the upper plates is moved apart by a distance dx. Then mechanical work done during this movement is $F \times dx$ joule. Since charge on the plate is constant, no electrical energy can move into the system from outside. This work is done at the case of the energy stored in the parallel-plate capacitor formed by the two plates.

Before movement, let the capacitance of the capacitor be C farad. Then,

Initial energy stored $= \frac{1}{2} \cdot \frac{Q^2}{C}$

If the capacitance changes to $(C + dC)$ because of the movement of plate, then

Final energy stored $= \frac{1}{2} \frac{Q^2}{(C + dC)} = \frac{1}{2} \frac{Q^2}{C} \cdot \frac{1}{\left(1 + \frac{dC}{C}\right)} = \frac{1}{2} \frac{Q^2}{C} \cdot \left(1 + \frac{dC}{C}\right)^{-1}$

$$= \frac{1}{2} \frac{Q^2}{C} \left(1 - \frac{dC}{C}\right) \text{ if } dC \leq C$$

Change in stored energy $= \frac{1}{2} \frac{Q^2}{C} - \frac{1}{2} \frac{Q^2}{C} \left(1 - \frac{dC}{C}\right) = \frac{1}{2} \frac{Q^2}{C} \cdot \frac{dC}{C}$

$$\therefore \quad F \times dx = \frac{1}{2} \frac{Q^2}{C} \cdot \frac{dC}{C} \text{ or } F \quad \frac{1}{2} \frac{Q^2}{C^2} \cdot \frac{dC}{dx} \quad \frac{1}{2} V^2 \frac{dC}{dx}$$

Now, $C = \frac{_0 A}{x} \quad \frac{dC}{dx} \quad \frac{_0 A}{x^2} \quad F \quad \frac{1}{2} V^2 \frac{_0 A}{x^2}$ N

Hence, we find that force is directly proportional to the square of the voltage to be measured. The negative sign merely shows that it is a force of attraction.

10.32. Quadrant Type Voltmeters

The working principle and basic construction of such instruments can be understood from Fig. 10.42. A light aluminium vane C is mounted on a spindle S and is situated partially within a hollow metal quadrant B. Alternatively, the vane be suspended in the quadrant. When the vane and the quadrant are oppositely charged by the voltage under measurement, the vane is further attracted inwards into the quadrant thereby causing the spindle and hence the pointer to rotate. The amount of rotation and hence the deflecting torque is found proportional to V^2. The deflecting torque in the case of arrangement shown in Fig. 10.42 is very small unless V is extremely large.

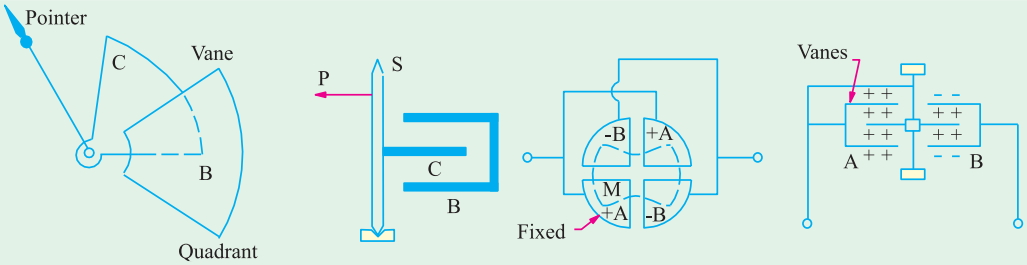


Fig. 10.42 Fig. 10.43

The force on the vane may be increased by using a larger number of quadrants and a double-ended vane. In Fig. 10.43 are shown four fixed metallic double quadrants arranged so as to form a circular box with short air-gaps between the quadrants in which is suspended or pivoted as aluminium vane. Opposite quadrants AA and BB are joined together and each pair is connected to on terminal of the

a.c. or d.c. supply and at the same time, one pair is connected to the moving vane *M*. Under these conditions [Fig. 10.43.] the moving vane is recalled by quadrants *AA* and attracted by quadrants *BB*. Hence, a deflecting torque is produced which is proportional to $(\text{p.d.})^2$. Therefore, such voltmeters have an uneven scale. Controlling torque is produced by torsion of the suspension spring or by the spring (used in pivoted type voltmeters). Damping is by a disc or vane immersed in oil in the case of suspended type or by air friction in the case of pivoted type instruments.

Theory

With reference to Fig 10.42, suppose the quadrant and vane are connected across a source of *V* volts and let the resulting deflection be θ. If *C* is a capacitance between the quadrant and vane in the deflected position, then the charge on the instrument will be *CV* coulomb. Suppose that the voltage is charged from *V* to $(V + dV)$, then as a result, let θ, *C* and *Q* charge to $(\theta + d\theta)$, $(C + dC)$ and $(Q + dQ)$ respectively. Then, the energy stored in the electrostatic field is increased by

$$dE = d\left(\frac{1}{2}CV^2\right) \quad \frac{1}{2}V^2.dC \quad CV\,dV \text{ joule}$$

If *T* is the value of controlling torque corresponding to a deflection of θ, then the additional energy stored in the control will be $T \times d\theta$ joule.

Total increase in stored energy $= T \times d\theta + \frac{1}{2}V^2\,dC + CVdV$ joule

It is seen that during this charge, the source supplies a change *dQ* at potential *V*. Hence, the value of energy supplied is

$$= V \times dQ = V \times d(CV) = V^2 \times dC + CV.dV$$

Since the energy supplied by the source must be equal to the extra energy stored in the field and the control

$$\therefore \quad T \times d\theta + \frac{1}{2}V^2\,dC + CV.dV = V^2.dC + CVdV$$

or

$$T \quad d \quad \frac{1}{2}V^2.dC \qquad T \quad \frac{1}{2}V^2\frac{dC}{d}\ \text{N-m}$$

The torque is found to be proportional to the square of the voltage to be measured whether that voltage is alternating or direct. However, in alternating circuits the scale will read r.m.s. values.

10.33. Kelvin's Multicellular Voltmeter

As shown in Fig. 10.44, it is essentially a quadrant type instrument, as described above, but with the difference that instead of four quadrants and one vane, it has a large number of fixed quadrants and vanes mounted on the same spindle. In this way, the deflecting torque for a given voltage is increased many times. Such voltmeters can be used to measure voltages as low as 30 V. As said above, this reduction in the minimum limit of voltage is due to the increasing operating force in proportion to the number of component units. Such an instrument has a torsion head for zero adjustment and a coach spring for protection against accidental fracture of suspension due to vibration etc. There is a pointer and scale of edgewise pattern and damping is by a vane immersed in an oil dashpot.

10.34. Advantages and Limitation of Electrostatic Voltmeters

Some of the main advantages and use of electrostatic voltmeters are as follows :

1. They can be manufactured with first grade accuracy.
2. They give correct reading both on d.c. and a.c. circuits. On a.c. circuits, the scale will, however, read r.m.s. values whatever the wave-form.
3. Since no iron is used in their construction, such instruments are free from hysteresis and eddy current losses and temperature errors.
4. They do not draw any continuous current on d.c. circuits and that drawn on a.c. circuits (due to the capacitance of the instrument) is extremely small. Hence, such voltmeters do not cause any disturbance to the circuits to which they are connected.

5. Their power loss is negligibly small.
6. They are unaffected by stray magnetic fields although they have to be guarded against any stray electrostatic field.
7. They can be used upto 1000 kHz without any serious loss of accuracy.

However, their main limitations are :

1. Low-voltage voltmeters (like Kelvin's Multicellular voltmeter) are liable to friction errors.
2. Since torque is proportional to the square of the voltage, their scales are not uniform although some uniformity can be obtained by suitably shaping the quadrants of the voltmeters.
3. They are expensive and cannot be made robust.

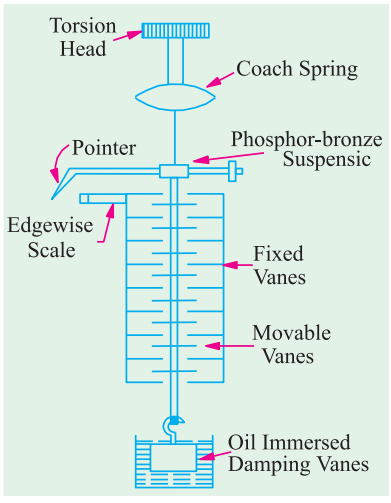


Fig. 10.44

10.35. Range Extension of Electrostatic Voltmeters

The range of such voltmeters can be extended by the use of multipliers which are in the form of a resistance potential divider or capacitance potential divider. The former method can be used both for direct and alternating voltages whereas the latter method is useful only for alternating voltages.

(*i*) Resistance Potential Divider

This divider consists of a high non-inductive resistance across a small portion which is attached to the electrostatic voltmeter as shown in Fig. 10.45. Let R be the resistance of the whole of the potential divider across which is applied the voltage V under measurement. Suppose V is the maximum value of the voltage which the voltmeter can measure without the multiplier. If r is the resistance of the portion of the divider across which voltmeter is connected, then the multiplying factor is given by

$$\frac{V}{v} = \frac{R}{r}$$

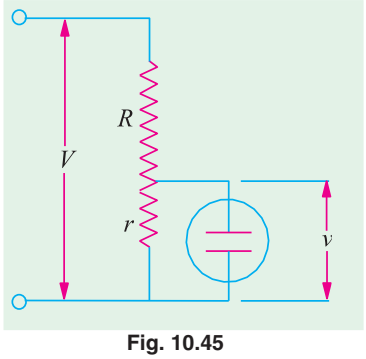


Fig. 10.45

The above expression is true for d.c. circuits but for a.c. circuits, the capacitance of the voltmeter (which is in parallel with r) has to be taken into account. Since this capacitance is variable, it is advisable to calibrate the voltmeter along with its multiplier.

(*ii*) Capacitance Potential Divider

In this method, the voltmeter may be connected in series with a single capacity C and put across the voltage V which is to be measured [Fig. 10.46 (*a*)] or a number of capacitors may be joined in series to form the potential divider and the voltmeter may be connected across one of the capacitors as shown in Fig. 10.46 (*b*).

Consider the connection shown in Fig. 10.46 (*a*). It is seen that the multiplying factor is given by

$$\frac{V}{v} = \frac{\text{reactance of total circuit}}{\text{reactance of voltmeter}}$$

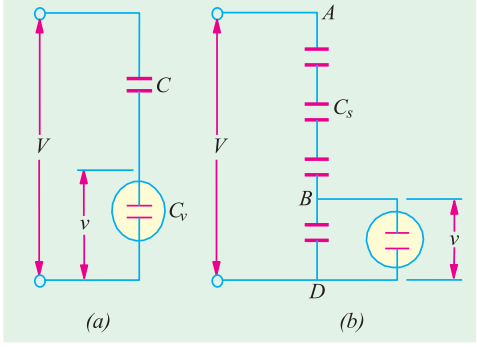


Fig. 10.46

Now, capacitance of the total circuit is $\frac{CC_v}{C + C_v}$ and its reactance is

$$= \frac{1}{\omega \times \text{capacitance}} = \frac{C + C_v}{\omega C C_v}$$

Reactance of the voltmeter $= \dfrac{1}{\omega C_v}$

$\therefore$ $\dfrac{V}{v}$ $\dfrac{(C \quad C_v)/ \quad C C_v}{1/ \quad C_v}$ $\dfrac{C \quad C_v}{C}$ Multiplying factor $\dfrac{C \quad C_v}{C}$ 1 $\dfrac{C_v}{C}$*.

Example 10.20. *The reading '100' of a 120-V electrosatatic voltmeter is to represent 10,000 volts when its range is extended by the use of a capacitor in series. If the capacitance of the voltmeter at the above reading is 70 μμF, find the capacitance of the capacitor multiplier required.*

Solution. Multiplying factor $= \dfrac{V}{v} = 1 + \dfrac{C_v}{C}$

Here, V = 10,000 volt, v = 100 volts ; C_v = capacitance of the voltmeter = 70μμ F

C = capacitance of the multiplier $\therefore \quad \dfrac{10,000}{100} = 1 + \dfrac{70}{C}$

or $70/C = 99 \quad \therefore \quad C = 70/99$ μμ $F =$ **0.707 μμ F (approx)**

Example 10.21 (*a*). *An electrostatic voltmeter is constructed with 6 parallel, semicircular fixed plates equal-spaced at 4 mm intervals and 5 interleaved semi-circular movable plates that move in planes midway between the fixed plates, in air. The movement of the movable plates is about an axis through the center of the circles of the plates system, perpendicular to the planes of the plates. The instrument is spring-controlled. If the radius of the movable plates is 4 cm, calculate the spring constant if 10 kV corresponds to a full-scale deflection of 100°. Neglect fringing, edge effects and plate thickness.* **(Elect. Measurements, Bombay Univ.)**

Solution. Total number of plates (both fixed and movable) is 11, hence there are 10 parallel plate capacitors.

Suppose, the movable plates are rotated into the fixed plates by an angle of θ radian. Then, overlap area between one fixed and one movable semi-circular plate is

$$A = \frac{1}{2} r^2 \theta = \frac{1}{2} \times 0.04^2 \times \theta = 8 \times 10^{-4}\ \theta\ \text{m}^2;\ d = 4/2 = 2\ \text{mm} = 2 \times 10^{-3}\ \text{m}$$

Capacitance of each of ten parallel-plate capacitors is

$$C = \frac{\varepsilon_0 A}{d} = \frac{8.854 \times 10^{-12} \times 8 \times 10^{-4}\ \theta}{2 \times 10^{-3}} = 3.54 \times 10^{-12}\ \theta\ \text{F}$$

Total capacitance $C = 10 \times 3.54 \times 10^{-12}\ \theta = 35.4 \times 10^{-12}\ \theta$ F $\therefore dC/d\theta = 35.4 \times 10^{-12}$ farad/radian

Deflecting torque $= \dfrac{1}{2} V^2 \dfrac{dC}{d\theta}$ N-m $= \dfrac{1}{2} \times (10,000)^2 \times 35.4 \times 10^{-12} = 17.7 \times 10^{-4}$ N-m

If S is spring constant *i.e.* torque per radian and θ is the plate deflection, then control torque is

$T_c = S\theta$

Here, $\theta = 100° = 100 \times \pi/180 = 5\pi/9$ radian

$\therefore \quad S \times 5\pi/9 = 17.7 \times 10^{-4} \quad \therefore \quad S =$ **10.1 × 10–4 N-m/rad.**

Example 10.21 (*b*). *A capacitance transducer of two parallel plates of overlapping area of $5 \times 10^{-4}\ m^2$ is immersed in water. The capacitance 'C' has been found to be 9.50 pF. Calculate the separation 'd' between the plates and the sensitivity, $S = \partial C/\partial d$, of this transducer, given : ε_r water = 81 ; $\varepsilon_0 = 8.854$ pF/m.* **(Elect. Measuer. A.M.I.E. Sec. B, 1992)**

Solution. Since $C = \varepsilon_0 \varepsilon_r A/d$, $d = 3\ \varepsilon_0 \varepsilon_r$A/C.

Substituting the given values we get, $d = 37.7 \times 10^{-3}$ m

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* It is helpful to compare it with a similar expression in Art. 10.17 for permanent magnet moving-coil instruments.

$$\text{Sensitivity } \frac{\partial C}{\partial d} = \frac{\partial}{d}\left(\frac{\varepsilon_0 \varepsilon_r A}{d}\right) = -\frac{8.854\times10^{-12}\times81\times5\times10^{-4}}{(37.7\times10^{-3})^2} = -0.025\times10^{-8} \text{ F/m}$$

10.36. Wattmeters

We will discuss the two main types of wattmeters in general use, that is, ***(i)*** the dynamometer or electrodynamic type and ***(ii)*** the induction type.

10.37. Dynamometer Wattmeter

The basic principle of dynamometer instrument has already been explained in detail in Art. 10.20. The connections of a dynamometer type wattmeter are shown in Fig. 10.47. The fixed

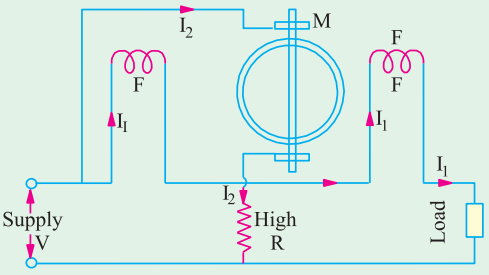


Fig. 10.47

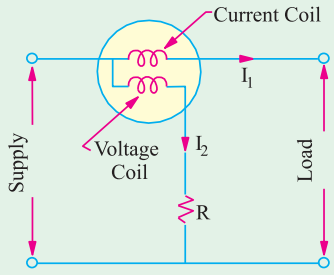


Fig. 10.48

circular coil which carries the main circuit current I_1 is wound in two halves positioned parallel to each other. The distance between the two halves can be adjusted to give a uniform magnetic field. The moving coil which is pivoted centrally carries a current I_2 which is proportional to the voltage V. Current I_2 is led into the moving coil by two springs which also supply the necessary controlling torque. The equivalent diagrammatic view is shown in Fig. 10.48.

Deflecting Torque

Since coils are air-cored, the flux density produced is directly proportional to the current I_1.

$\therefore$ $B \propto I_1$ or $B = K_1 I_1$; current $I_2 \propto V$ or $I_2 = K_2 V$

Now $T_d \propto BI_2 \propto I_1 V$ $\therefore$ $T_d = KV I_1 = K \times$ power

In d.c. circuits, power is given by the product of voltage and current in amperes, hence torque is directly proportional to the power.

Let us see how this instrument indicates true power on a.c. circuits.

For a.c. supply, the value of instantaneous torque is given by $T_{inst} \propto vi = K\, vi$

where v = instantaneous value of voltage across the moving coil

i = instantaneous value of current through the *fixed* coils.

However, owing to the large inertia of the moving system, the instrument indicates the mean or average power.

$\therefore$ Mean deflecting torque $T_m \propto$ average value of υi

Let $\upsilon = V_{max} \sin\theta$ and $i = I_{max} \sin(\theta - \phi)$ $\therefore$ $T_m \propto \frac{1}{2\pi}\int_0^{2\pi} V_{max} \sin\theta \times I_{max} \sin(\theta - \phi)\, d\theta$

$$\propto \frac{V_{max}\, I_{max}}{2}\int_0^{2} \sin \quad \sin(\quad)\, d \quad \frac{V_{max}\, I_{max}}{2}\int_0^{2} \frac{\cos \quad \cos(2 \quad)}{2}\, d$$

$$\propto \frac{V_{max}\, I_{max}}{4\pi}\left[\theta\cos\phi - \frac{\sin(2\theta - \phi)}{2}\right]_0^{2\pi} \propto \frac{V_{max}}{\sqrt{2}} \cdot \frac{I_{max}}{\sqrt{2}} \cdot \cos\phi \propto V I \cos\phi$$

where V and I are the r.m.s. values. $\therefore$ $T_m \propto VI \cos\phi \propto$ true power.

Hence, we find that in the case of a.c. supply also, the deflection is proportional to the true power in the circuit.

Scales of dynamometer wattmeters are more or less uniform because the deflection is proportional to the average power and for spring control, controlling torque is proportional to the deflection. Hence $\theta \propto$ power. Damping is pneumatic with the help of a piston moving in an air chamber as shown in Fig. 10.49.

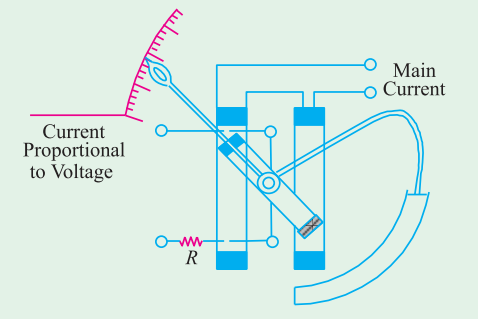


Fig. 10.49

Errors

The inductance of the moving or voltage coil is liable to cause error but the high non-inductive resistance connected in series with the coil swamps, to a great extent, the phasing effect of the voltage-coil inductance.

Another possible error in the indicated power may be due to **(*i*)** some voltage drop in the circuit or **(*ii*)** the current taken by the voltage coil. In standard wattmeters, this defect is overcome by having an additional compensating winding which is connected in series with the voltage coil but is so placed that is produces a field in opposite direction to that of the fixed or current coils.

Advantages and Disadvantages

By careful design, such instruments can be built to give a very high degree of accuracy. Hence they are used as a standard for calibration purposes. They are equally accurate on d.c. as well as a.c. circuits.

However, at low power factors, the inductance of the voltage coil causes serious error unless special precautions are taken to reduce this effect [Art. 10.38 (*ii*)].

10.38. Wattmeter Errors

(*i*) Error Due to Different Connections

Two possible ways of connecting a wattmeter in a single-phase a.c. circuit are shown in Fig. 10.50 along with their phasor diagrams. In Fig. 10.50 (*a*), the pressure or voltage-coil current does not pass through the current coil of the wattmeter whereas in the connection of Fig. 10.50 (*b*) is passes. A wattmeter is supposed to indicate the power consumed by the load but its actual reading is slightly higher due to power losses in the instrument circuits. The amount of error introduced depends on the connection.

(*a*) Consider the connection of Fig. 10.50 (*a*). If $\cos \phi$ is the power factor of the load, then power in the load is $VI \cos \theta$.

Now, voltage across the pressure-coil of the wattmeter is V_1 which is the phasor sum of the load voltage V and p.d. across current-coil of the instrument *i.e.* V' (= Ir where r is the resistance of the current coil).

Hence, power reading as indicated by the wattmeter is = $V_1 I \cos \theta$

where θ = phase difference between V_1 and I as shown in the phasor diagram of Fig. 10.50 (*a*).

As seen from the phasor diagram, $V_1 \cos \theta = (V \cos \phi + V')$

$\therefore$ wattmeter reading $= V_1 \cos \theta \, . \, I = (V \cos \phi + V') I$

$= VI \cos \phi + V' I = VI \cos \phi + I^2 r$ = power in load + power in current coil.

(*b*) Next, consider the connection of Fig. 10.50 (*b*). The current through the current-coil of the wattmeter is the phasor sum of load current I and voltage-coil current $I' = V/R$. The power reading indicated by the wattmeter is = $VI_1 \cos \theta$.

As seen from the phasor diagram of Fig. 10.50 (*b*), $I_1 \cos \theta = (I \cos \phi + I')$

$\therefore$ wattmeter reading $= V (I \cos \phi + I') = VI \cos \phi + VI' = VI \cos \phi + V^2/R$

= power in load + power in pressure-coil circuit

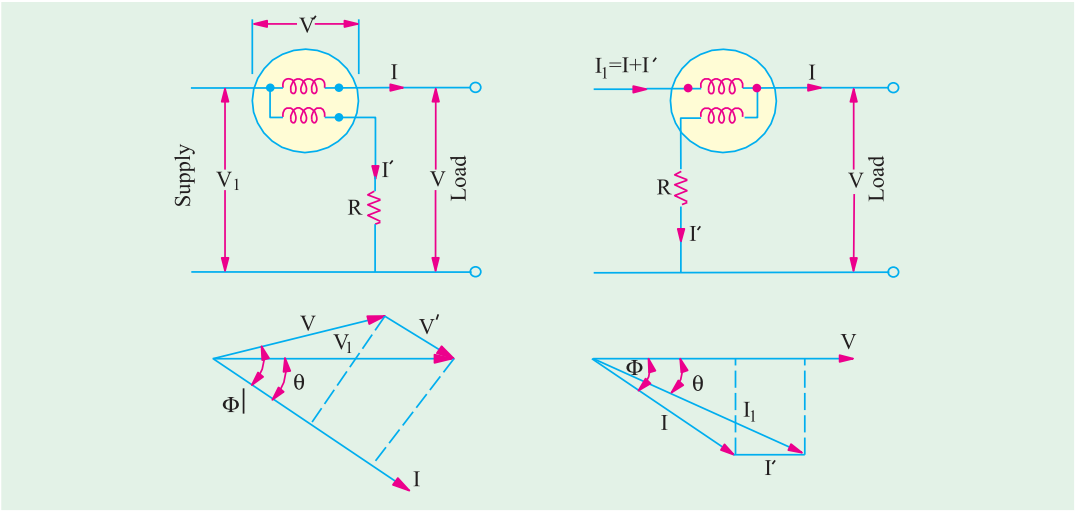


Fig. 10.50

(*ii*) Error Due to Voltage-coil Inductance

While developing the theory of electrodynamic instruments, it was assumed that pressure-coil does not posses any inductance (and hence reactance) so that current drawn by it was = V/R. The wattmeter reading is proportional to the mean deflecting torque, which is itself proportional to $I_1 I_2 \cos\theta$, where θ is the angle between two currents (Fig. 10.52).

In case the inductance of the voltage-coil is neglected.

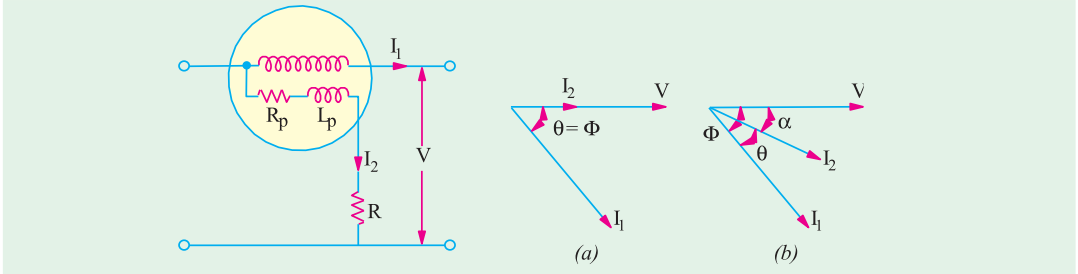


Fig. 10.51 Fig. 10.52

$$I_2 = V/(R + R_p) = V/R \text{ approximately}$$

and $\theta = \phi$ as shown in the phasor diagram of 10.52 (*a*)

$$\therefore \quad \text{wattmeter reading} \propto \frac{I_1 V}{R} \cos\phi \qquad \ldots(i)$$

In case, inductance of the voltage coil is taken into consideration, then

$$I_2 = \frac{V}{\sqrt{(R_p + R)^2 + X_L^2}} = \frac{V}{\sqrt{R^2 + X_L^2}} = \frac{V}{Z_p}$$

It lags behind V by an angle α [Fig. 10.52 (*b*)] such that

$$\tan\alpha = X_L/(R_p + R) = X_L/R \text{ (approx.)} = \omega L_p/R$$

$$\therefore \quad \text{wattmeter reading} \propto \frac{I_1 V}{Z_p} \cos\theta \propto \frac{I_1 V}{Z_p} \cos(\phi - \theta)$$

Now $$\cos\alpha = \frac{R_p + R}{Z_p} = \frac{R}{Z_p} \quad \therefore \; Z_p = \frac{R}{\cos\alpha}$$

$\therefore$ wattmeter reading in this case is $\propto I_1 \frac{V}{R} \cos\alpha \cos(\phi - \alpha)$ ...(*ii*)

Eq. (*i*) above, gives wattmeter reading when inductance of the voltage coil is neglected and Eq. (*ii*) gives the reading when it is taken into account.

The correction factor which is given by the ratio of the true reading (W_1) and the actual or indicated reading (W_a) of the wattmeter is

$$\frac{W_t}{W_a} = \frac{\frac{VI_1}{R_1}\cos}{\frac{VI_1}{R_1}\cos \quad \cos(\quad)} \quad \frac{\cos}{\cos \quad \cos(\quad)}$$

Since, in practice, α is very small, $\cos\alpha = 1$. Hence the correction factor becomes $= \frac{\cos\phi}{\cos(\phi - \alpha)}$

$\therefore$ True reading $= \frac{\cos\phi}{\cos\alpha\cos(\phi-\alpha)} \times$ actual reading $\cong \frac{\cos\phi}{\cos(\phi-\alpha)} \times$ actual reading

The error in terms of the actual wattmeter reading can be found as follows :

Actual reading–true reading

$= \text{actual reading} - \frac{\cos\phi}{\cos\alpha\cos(\phi-\alpha)} \times \text{actual reading}$

$= 1 \quad \frac{\cos}{\cos(\quad)}$ actual reading $\quad 1 \quad \frac{\cos}{\cos \quad \sin \quad \sin}$ actual reading

$= \frac{\sin \quad .\sin}{\cos \quad \sin \quad .\sin}$ actual reading $\quad \frac{\sin}{\cot \quad \sin}$ actual reading

The error, expressed as a fraction of the actual reading, is $= \frac{\sin}{\cot \quad \sin}$

Percentage error $= \frac{\sin}{\cot \quad \sin} \times 100$

(*iii*) Error Due to Capacitance in Voltage-coil Circuit

There is always present a small amount of capacitance in the voltage-coil circuit, particularly in the series resistor. Its effect is to reduce angle α and thus reduce error due to the inductance of the voltage coil circuit. In fact, in some wattmeters, a small capacitor is purposely connected in parallel with the series resistor for obtaining practically non-inductive voltage-coil circuit. Obviously, over-compensation will make resultant reactance capacitive thus making α negative in the above expressions.

(*iv*) Error Due to Stray Fields

Since operating field of such an instrument is small, it is very liable to stray field errors. Hence, it should be kept as far away as possible from stray fields. However, errors due to stray fields are, in general, negligible in a properly-constructed instrument.

(*v*) Error Due to Eddy Currents

The eddy current produced in the solid metallic parts of the instrument by the alternating field of the current coil changes the magnitude and strength of this operating field thus producing an error in the reading of the wattmeter. This error is not easily calculable although it can be serious if care is not taken to remove away solid masses of metal from the proximity of the current coil.

Example 10.22. *A dynamometer type wattmeter with its voltage coil connected across the load side of the instrument reads 250 W. If the load voltage be 200 V, what power is being taken by load? The voltage coil branch has a resistance of 2,000* Ω

(Elect. Engineering, Madras Univ.)

Solution. Since voltage coil is connected across the load side of the wattmeter (Fig. 10.53), the power consumed by it is also included in the meter reading.

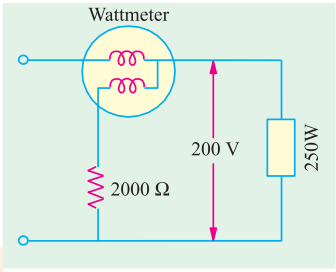


Fig. 10.53

Power consumed by voltage coil is

$= V^2/R = 200^2/2{,}000 = 20$ W

$\therefore$ Power being taken by load = 250 –20

= 230 W

Example 10.23. *A 250-V, 10-A dynamometer type wattmeter has resistance of current and potential coils of 0.5 and 12,500 ohms respectively. Find the percentage error due to each of the two methods of connection when unity p.f. loads at 250 volts are of* ***(a)*** *4A* ***(b)*** *12 A.*

Neglect the error due to the inductance of pressure coil.

(Elect. Measurements, Pune. Univ.)

Solution. (*a*) When I = 4 A

(i) Consider the type of connection shown in Fig. 10.50 (*a*)

Power loss in current coil of wattmeter = $I^2r = 4^2 \times 0.5 =$ **8 W**

Load power = $250 \times 4 \times 1 = 1000$ W ; Wattmeter reading = 1008 W

$\therefore$ percentage error = $(8/1008) \times 100 =$ **0.794%**

(ii) Power loss in pressure coil resistance = $V^2/R = 250^2/12{,}500 = 5$ W

$\therefore$ Percentage error = $5 \times 100/1005 =$ **0.497 %**

(b) **When I = 12 A**

(i) Power loss in current coil = $12^2 \times 0.5 = 72$ W

Load power = $250 \times 12 \times 1 = 3000$ W ; wattmeter reading = 3072 W

$\therefore$ percentage error = $72 \times 100/3072 =$ **2.34 %**

(ii) Power loss in the resistance of pressure coil is $250^2/12{,}500 = 5$ W

$\therefore$ percentage error = $5 \times 100/3005 =$ **0.166 %**

Example 10.24. *An electrodynamic wattmeter has a voltage circuit of resistance of 8000 Ω and inductance of 63.6 mH which is connected directly across a load carrying a current of 8A at a 50-Hz voltage of 240-V and p.f. of 0.1 lagging. Estimate the percentage error in the wattmeter reading caused by the loading and inductance of the voltage circuit.*

(Elect & Electronic Measu. & Instru. Nagpur, Univ. 1992)

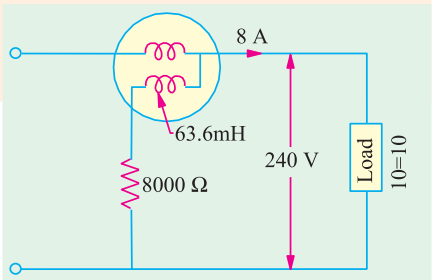


Fig. 10.54

Solution. The circuit connections are shown in Fig. 10.54.

Load power = $240 \times 8 \times 0.1 = 192$ W

$\cos \phi = 0.1$, $\phi = \cos^{-1}(0.1) = 84°\ 16'$

Power loss in voltage coil circuit is = V^2/R

$= 240^2/8000 = 7.2$ W

Neglecting the inductance of the voltage coil, the wattmeter reading would be

= 192 + 7.2 = **199.2 W**

Now, $X_p = 2\pi \times 50 \times 63.3 \times 10^{-3} = 20\ \Omega$

$\alpha = \tan^{-1}(20/8000) = \tan^{-1}(0.0025) = 0°\ 9'$

Error factor due to inductance of the voltage coil = $\dfrac{\cos(\phi - \alpha)}{\cos \phi} = \dfrac{\cos 84°\,7'}{\cos 84°\,16'} = 1.026$

Wattmeter reading = $1.026 \times 199.2 = 204.4$ W

Percentage error $= \left(\frac{204.4 - 199.2}{199.2}\right) \times 100 = 2.6\ \%$

Example 10.25. *The inductive reactance of the pressure-coil circuit of a dynamometer wattmeter is 0.4 % of its resistance at normal frequency and the capacitance is negligible.*

Calculate the percentage error and correction factor due to reactance for load at ***(i)*** *0.707 p.f. lagging and* ***(ii)*** *0.5 p.f. lagging.* **(Elect. Measurement. Bombay Univ.)**

Solution. It is given that $X_p/R = 0.4$ $R = 0.004$

$\tan \alpha = X_p/R = 0.004$ $\therefore$ $\alpha = 0°\ 14'$ and $\sin \alpha = 0.004$

(i) **When p.f. = 0.707** (*i.e.* $\phi = 45°$)

$$\text{Correction factor} = \frac{\cos}{\cos(\quad)} \quad \frac{\cos 45°}{\cos 44°\ 46} \quad 0.996$$

$$\text{Percentage error} = \frac{\sin \alpha}{\cot \phi + \sin \alpha} \times 100 = \frac{\sin 0°\ 14'}{\cot 45° + \sin 0°\ 14'} \times 100$$

$$= \frac{0.004 \times 100}{1 + 0.004} = \frac{0.4}{1.004} = 0.4 \text{ (approx)}$$

(ii) **When p.f. = 0.5** (*i.e.*, $\phi = 60°$)

$$\text{Correction factor} = \frac{\cos 60°}{\cos 59°\ 46} \quad 0.993$$

$$\text{Percentage error} = \frac{\sin 0°\ 14}{\cot 60° \quad \sin 0°\ 14} \quad 100 \quad \frac{0.004 \quad 100}{0.577 \quad 0.004} \quad \frac{0.4}{0.581} \quad 0.7$$

Example. 10.26. *The current coil of wattmeter is connected in series with an ammeter and an inductive load. A voltmeter and the voltage circuit of the wattmeter are connected across a 400-Hz supply. The ammeter reading is 4.5 A and voltmeter and wattmeter readings are respectively 240 V and 29 W. The inductance of the voltage circuit is 5 mH and its resistance is 4 kΩ If the voltage drops across the ammeter and current coil are negligible, what is the percentage error in wattmeter readings ?*

Solution. The reactance of the voltage-coil circuit is $X_p = 2\pi \times 400 \times 5 \times 10^{-3}\ \pi$ ohm

$$\tan \alpha = X_p/R = 4\pi/4000 = 0.00314^2$$

$\therefore$ $\alpha = 0.003142$ radian ($\because$angle is very small)

$= 0.18°$ or $0°11'$

Now, true reading $= \frac{\cos \phi}{\cos \alpha \cos(\phi - \alpha)} \times$ actual reading

or $VI \cos \phi = \frac{\cos \phi}{\cos \alpha \cos(\phi - \alpha)} \times$ actual reading

or $VI = \frac{\text{actual reading}}{\cos(\phi - \alpha)}$

taking $(\cos \alpha = 1)$

$\therefore$ $\cos(\phi - \alpha) = 29/240 \times 4.5 = 0.02685$

$\therefore$ $\phi - \alpha = 88°28'$ or $\phi = 88°\ 39'$

$$\therefore \text{Percentage error} = \frac{\sin \alpha}{\cot \phi + \sin \alpha} \times 100 = \frac{\sin 11'}{\cot 88°\ 39' + \sin 11'} \times 100$$

$$= \frac{0.0032}{0.235 \quad 0.0032} \quad \mathbf{100 = 12\ \%}$$

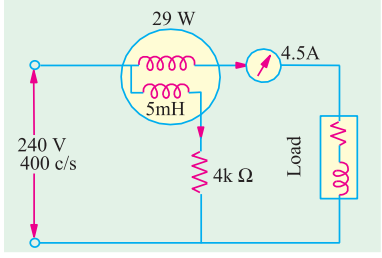


Fig. 10.55

10.39. Induction Wattmeters

Principle of induction wattmeters is the same as that of induction ammeters and voltmeters. They can be used on a.c. supply only in constant with dynamometer wattmeters, which can be used both on d.c. and a.c. supply. Induction wattmeters are useful only when the frequency and supply voltage are constant.

Since, both a current and a pressure element are required in such instrument, it is not essential to use the shaded-pole principle. Instead of this, two separate a.c. magnets are used, which produce two fluxes, which have the required phase difference.

Construction

The wattmeter has two laminated electromagnets, one of which is excited by the current in the main circuit-exciting winding being joined in series with the circuit, hence it is also called a ***series*** magnet. The other is excited by current which is proportional to the voltage of the circuit. Its exciting coil is joined in parallel with the circuit, hence this magnet is sometimes referred to as ***shunt*** magnet.

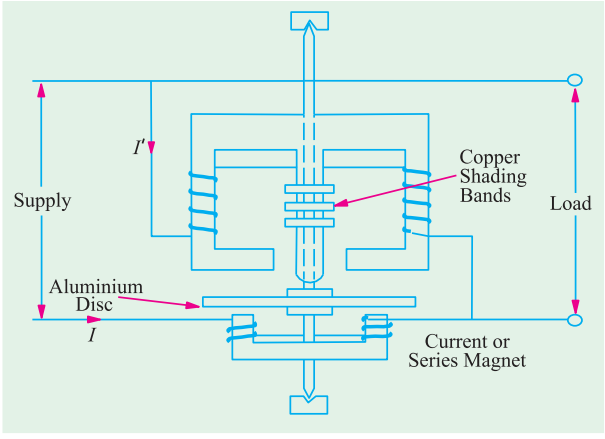


Fig. 10.56

A thin aluminium disc is so mounted that it cuts the fluxes of both magnets. Hence, two eddy currents are produced in the disc. The deflection torque is produced due to the interaction of these eddy current and the inducing fluxes. Two or three copper rings are fitted on the central limb of the shunt magnet and can be so adjusted as to make the resultant flux in the shunt magnet lag behind the applied voltage by 90°.

Two most common forms of the electromagnets are shown in Fig. 10.56 and 10.57. It is seen that in both cases, one magnet is placed above and the other below the disc. The magnets are so positioned and shaped that their fluxes are cut by the disc.

In Fig. 10.56, the two pressure coils are joined in series and are so wound that both send the flux through the central limb in the same direction. The series magnet carries two coils joined in series and so wound that they magnetise their respective cores in the same direction. Correct phase displacement between the shunt and series magnet fluxes can be obtained by adjusting the position of the copper shading bands as shown.

In the type of instrument shown in Fig. 10.57, there is only one pressure winding and one current winding. The two projecting poles of the shunt magnet are surrounded by a copper shading band whose position can be adjusted for correcting the phase of the flux of this magnet with respect to the voltage.

Both types of induction wattmeters shown above, are spring-controlled, the spring being fitted to the spindle of the moving system which also carries the pointer. The scale is uniformly even and extends over 300°.

Currents upto 100 A can be handled by such wattmeters directly but for currents greater than this value, they are used in conjunction with current transformers. The pressure coil is purposely made as much inductive as possible in order that the flux through it should lag behind the voltage by 90°.

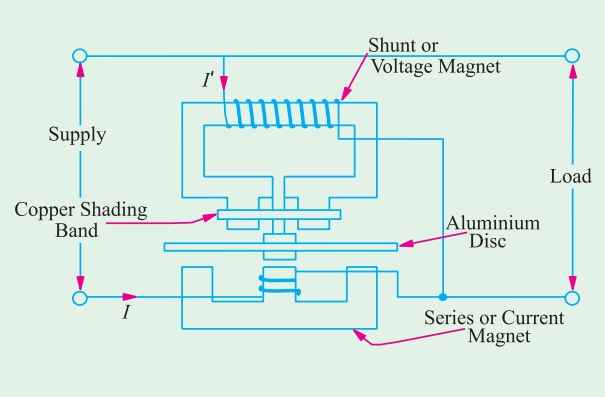


Fig. 10.57

Theory

The winding of one magnet carries line current I so that $\Phi_1 \propto I$ and is in phase with I (Fig. 10.58).

The other coil *i.e.*, pressure or voltage coil is made highly inductive having an inductance of L and and negligible resistance. This is connected across the supply voltage V. The current in the pressure coil is therefore, equal to $V/\omega L$. Hence, $\Phi_2 \propto V/\omega L$ and lags behind the voltage by 90°. Let the load current I lag behind V by ϕ *i.e.*, let the load power factor angle be ϕ. As shown in Fig. 10.56, the phase angle between Φ_1 and Φ_2 is $\alpha = (90 - \phi)$.

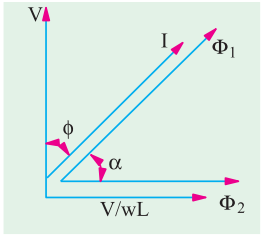


Fig. 10.58

The value of the torque acting on the disc is given by

$$T = k\omega \Phi_{1m} \Phi_{2m} \sin \alpha \qquad \text{– Art. 10.25}$$

or $$T \propto 2\,\omega I . \frac{V}{\omega L} . \sin (90 - \phi) \propto VI \cos \phi \propto \text{power}$$

Hence, the torque is proportional to the power in the load circuit. For spring control, the controlling torque $T_c \propto \theta$. $\therefore$ $\theta \propto$ power. Hence, the scale is even.

10.40. Advantage and Limitations of Induction Wattmeters

These wattmeters possess the advantages of fairly long scales (extending over 300°), are free from the effects of stray fields and have good damping. They are practically free from frequency errors. However, they are subject to (sometimes) serious temperature errors because the main effect of temperature is on the resistance of the eddy current paths.

10.41. Energy Meters

Energy meters are integrating instruments, used to measure quantity of electric energy supplied to a circuit in a given time. They give no direct indication of power *i.e.*, as to the rate at which energy is being supplied because their registrations are independent of the rate at which a given quantity of electric energy is being consumed. Supply or energy meters are generally of the following types :

(*i*) ***Electrolytic meters*** - their operation depends on electrolytic action.

(*ii*) ***Motor meters*** - they are really small electric motors.

(*iii*) ***Clock meters*** - they function as clock mechanisms.

10.42. Electrolytic Meter

It is used on d.c. circuits* only and is essentially an ampere-hour meter and not a true watt-hour meter. However, its registrations are converted into watt-hour by multiplying them by the voltage (assumed constant) of the circuits in which it is used. Such instruments are usually calibrated to read kWh directly at the declared voltage. Their readings would obviously be incorrect when used on any other voltage. ***Because of the question of power factor, such instrument cannot be used on a.c. circuits.***

The advantages of simplicity, cheapness and of low power consumption of ampere-hour meters are, to a large extent, discounted by the fact that variations in supply voltage are not taken into account by them. As an example suppose that the voltage of a supply whose nominal value is 220 V, has an average value of 216 volts in one hour during which a consumer draws a current of 100 A. Quantity of electricity as measured by the instrument which is calibrated on 220 V, is $220 \times 100/1000 = 22$ kWh. Actually, the energy consumed by the customer is only $216 \times 100/1000 = 21.6$ kWh. Obviously, the consumer is being overcharged to the extent of the cost of $22 - 21.6 = 0.4$ kWh of energy per hour. A true watthour-meter would have taken into account the decrease in the supply voltage and would have, therefore, resulted in a saving to the consumer. If the supply voltage would

* Recently such instruments have been marketed for measurement of kilovoltampere-hours on a.c. supply, using a small rectifier unit, which consists of a current transformer and full-wave copper oxide rectifier.

have been higher by that amount, then the supply company would have been the loser (Ex. 10.27).

In this instrument, the operating current is passed through a suitable electrolyte contained in a voltmeter. Due to electrolysis, a deposit of mercury is given or a gas is liberated (depending on the type of meter) in proportion to the quantity of electricity passed (Faraday's Laws of Electrolysis). The quantity of electricity passed is indicated by the level of mercury in a graduated tube. Hence, such instruments are calibrated in amp-hour or if constancy of supply voltage is assumed, are calibrated in watt-hour or kWh.

Such instruments are cheap, simple and are accurate even at very small loads. They are not affected by stray magnetic fields and due to the absence of any moving parts are free from friction errors.

10.43. Motor Meters

Most commonly-used instruments of this type are :

(*i*) ***Mercury motor*** meters **(*ii*)** ***Commutator motor*** meters and **(*ii*)** ***Induction motor*** meters.

Of these, mercury motor meter is normally used on d.c. circuits whereas the induction type instrument is used only on a.c. circuits. However, the commutator type meter can be used both for d.c. as well as a.c. work.

Instruments used for d.c. work can be either in the form of a amp-hour meters or watt-hour meters. In both cases, the moving system is allowed to revolve continuously instead of being merely allowed to deflect or rotate through a fraction of a revolution as in indicating instruments. The speed of rotation is directly proportional to the current in the case of amp-hour meter and to power in the case of watt-hour meter. Hence the number of revolutions made in a given time is proportional, in the case of amp-hour meter, to the quantity of electricity ($Q = i \times t$) and in the case of Wh meter, to the quantity of energy supplied to the circuit. The number of revolutions made are registered by a counting mechanism consisting of a train of gear wheels and dials.

The control of speed of the rotating system is brought about by a permanent magnet (known as braking magnet) which is so placed as to set up eddy currents in some parts of the rotating system. These eddy currents produce a retarding torque which is proportional to their magnitude-their magnitude itself depending on the speed of rotation of the rotating system. The rotating system attains a *steady speed* when the braking torque exactly balances the driving torque which is produced either by the current or power in the circuit.

The essential parts of motor meters are :

1. An operating system which produces an operating torque proportional to the current or power in the circuit and which causes the rotation of the rotating system.

2. A retarding or braking device, usually a permanent magnet, which produces a braking torque is proportional to the speed of rotation. Steady speed of rotation is achieved when braking torque becomes equal to the operating torque.

3. A registering mechanism for the revolutions of the rotating system. Usually, it consists of a train of wheels driven by the spindle of the rotating system. A worm which is cut on the spindle engages a pinion and so driven a wheel-train.

10.44. Errors in Motor Meters

The two main errors in such instruments are : **(*i*)** friction error and **(*ii*)** braking error. Friction error is of much more importance in their case than the corresponding error in indicating instruments because **(*a*)** it operates continuously and **(*b*)** it affects the speed of the rotor. The braking action in such meters corresponding to damping in indicating instruments. The braking torque directly affects the speed for a given driving torque and also the number of revolution made in a given time.

Friction torque can be compensated for providing a small constant driving torque which is applied to the moving system independent of the load.

As said earlier, steady speed of such instruments is reached when driving torque is euqal to the braking torque. The braking torque is proportional to the flux of the braking magnet and the eddy current induced in the moving system due to its rotation in the field of the braking magnet

$$\therefore \qquad T_B \propto \Phi_i$$

where Φ is the flux of the braking magnet and i the induced current. Now $i = e/R$ where e is the induced e.m.f. and R the resistance of the eddy current path. Also $e \propto \Phi n$ where n is the speed of the moving part of the instrument.

$$\therefore \qquad T_b \ \alpha \ \frac{n}{R} \ \frac{^2n}{R}$$

The torque T_B' at the steady speed of N is given $T_B' \propto \Phi^2 N/R$

Now $T_B' = T_D$ – the driving torque

$$\therefore \qquad T_D \propto \Phi^2 N/R \quad \text{or} \quad N \propto T_D R/\Phi^2$$

Hence for a given driving torque, the steady speed is direcly proportional to the resistance of the eddy current of path and inversely to the square of the flux.

Obviously, it is very important that the strength of the field of the brake magnet should be constant throughout the time the meter is in service. The constancy of field strengths can be assured by careful design treatment during the manufacturing of the brake magnet. Variations in temperature will affect the braking torque since the resistance of the eddy current path will change. This error is difficult to fully compensate for.

10.45. Quantity or ampere-hour Meters

The use of such meters is mostly confined to d.c. circuits. Their operation depends on the production of two torques **(*i*)** a driving torque which is proportional to the current I in the circuit and **(*ii*)** a braking torque which is proportional to the speed n of the spindle. This speed attains a steady value N when these two torques become numerically equal. In that case, speed becomes proportional to current *i.e.*, $N \propto I$. Over a certain period of time, the total number of revolutions $\int Ndt$ will be proportional to the quantity of electricity $\int Idt$ passing through the meter. A worm cut in the spindle as its top engages gear wheels of the recording mechanism which has suitably marked dials reading directly in ampere hours. Since electric supply charges are based on watt-hour rather than ampere-hours, the dials of ampere-hour meters are frequently marked in corresponding watt-hour at the normal supply voltage. Hence, their indications of watt-hours are correct only when the supply voltages remains constant, otherwise reading will be wrong.

10.46. Ampere-hour Mercury Motor Meter

It is one of the best and most popular forms of mercury Ah meter used for d.c. work.

Construction

It consists of a thin Cu disc D mounted at the base of a spindle, working in jewelled cup bearings and revolving between a pair of ***permanent*** magnets M_1 and M_2. One of the two magnets *i.e.*, M_2 is used for driving purposes whereas M_1 is used for braking. In between the poles of M_1 and M_2 is a hollow circular box B in which rotates the Cu disc and the rest of the space is filled up with mercury which exerts considerable upward thrust on the disc, thereby reducing the pressure on the bearings. The spindle is so weighted that it just sinks in the mercury bath. A worm cut in the spindle at its top engages the gear wheels of the recording mechanism as shown in Fig. 10.59.

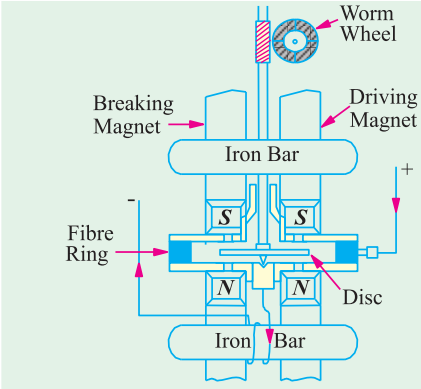


Fig. 10.59

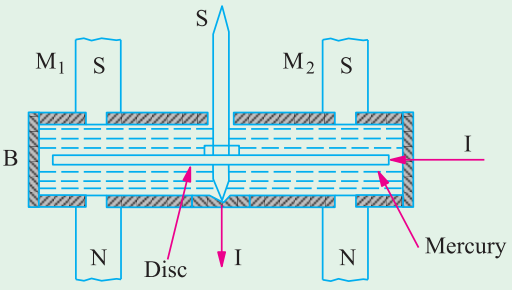


Fig. 10.60

Principle of Action

Its principle of action can be understood from Fig. 10.61 which shows a separate line drawing of the motor element.

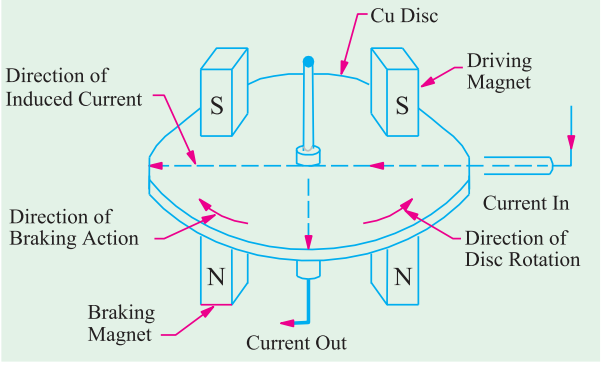


Fig. 10.61

The current to be measured is led into the disc through the mercury at a point at its circumference on the right-hand side. As shown by arrows, it flows radially to the centre of the disc where it passes out to the external circuit through the spindle and its bearings. It is worth noting ***that current flows takes place only under the right-hand side magnet M_2*** and not under the left-hand side magnet M_1. The field of M_2 will, therefore, exert a force on the right-side portion of the disc which carries the current (motor action). The direction of the force, as found by Fleming's Left-hand rule, is as shown by the arrow. The magnitude of the force depends on the flux density and current ($\because F = BIl$). The driving or motoring torque T_d so produced is given by the product of the force and the distance from the spindle at which this force acts. When the disc rotates under the influence of this torque, it cuts through the field of left-hand side magnet M_1 and hence eddy currents are produced in it which results in the production of braking torque. The magnitude of the retarding or braking torque is proportional to the speed of rotation of the disc.

Theory

Driving torque $T_d \propto$ force on the disc $\times$ $B\,I$

If the flux density of M_2 remains constant, then $T_d \propto I$.

The braking torque T_B is proportional to the flux Φ of braking magnet M_1 and eddy current i induced in the disc due to its rotation in the field of M_1.

$\therefore$ $T_B \propto \Phi_i$

Now $i = e/R$ where e is the induced e.m.f. and R the resistance of eddy current path.

Also $e \propto \Phi_n$ - where n is the speed of the disc $\therefore T_B \propto \Phi \times \frac{n}{R} \quad \frac{^2n}{R}$

The speed of the disc will attain a steady value N when the driving and braking torques becomes equal. In that case, $T_B \propto \Phi^2 N/R$.

If Φ and R are constant, then $I \propto N$

The total number of revolution in any given time t *i.e.*, $\int_0^t N.dt$ will become proportional to

$\int_0^t I.dt$ *i.e.*, to the total quantity of electricity passed through the meter.

10.47. Friction Compensation

There are two types of frictions in this ampere-hour meter.

(i) **Bearing Friction.** The effect of this friction is normally negligible because the disc and spindle float in mercury. Due to the upward thrust, the pressure on bearings is considerably reduced which results in freedom from wear as well as freat reduction in the bearing friction.

(ii) **Mercury Friction.** Since the disc revolves in mercury, there is friction between mercury and the disc, which gives rise to a torque, approximately proportional to the square of the speed of rotation. Hence, this friction causes the meter to run shown on heavy loads. It can be compensated for in the following two ways :

(a) a coil of few turns is wound on one of the poles of the driving magnet M_2 and the meter current is passed through it in a suitable direction so as to increase the strength of M_2. The additional driving torque so produced can be made just sufficient to compensate for the mercury friction.

(b) in the other method, two iron bars are placed across the permanent magnets, one above and other one below the mercury chamber as shown in Fig. 10.62. The lower bar carries a small compensating coil through which is passed the load current. The local magnetic field set up by this coil strengthens the field of driving magnet M_2 and weakens that of the braking magnet M_1, thereby compensating for mercury friction.

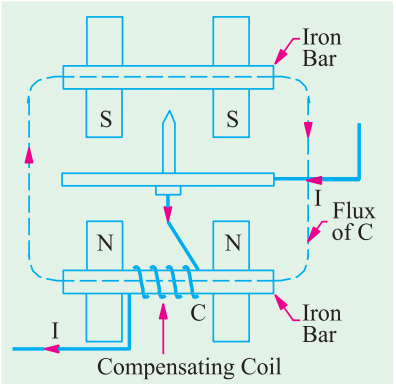


Fig. 10.62

10.48. Mercury Meter Modified as Watt-hour meter

If the permanent magnet M_2 of the amp-hour meter, used for producing the driving torque, is replaced by a wound electromagnet connected across the supply, the result is a watt-hour meter. The exciting current of this electromagnet is proportional to the voltage of the supply. The driving torque is exerted on the aluminium disc immersed in the mercury chamber below which is placed this electromagnet. The aluminimum disc has radial slots cut in it for ensuring the radial flow of current through it the current being led into and out of this disc through mercury contacts situated at diameterically opposite points. These radial slots, moreover, prevent the same disc being used for braking purposes. Braking is by a separate aluminium disc mounted on the same spindle and revolving in the air-gap of a separate braking magnet.

10.49. Commutator Motor Meters

These meters may be either ampere-hour or true watt-hour meters. In Fig. 10.63 is shown the principle of a common type of watt-hour meter known as Elihu-Thomson meter. It is based on the dynamometer principle (Art. 10.20) and is essentially an ironless motor with a wound armature having a commutator.

Construction

There are two fixed coils C_1 and C_2 each consisting of a few turns of heavy copper strip and joined in series with each other and with the supply circuit so that they carry the main current in

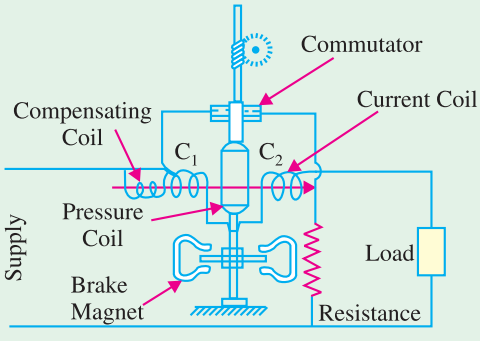


Fig. 10.63

the circuit (a shunt is used if the current is too heavy). The field produced by them is proportional to the current to be measured. In this field rotates an armature carrying a number of coils which are connected to the segments of a small commutator. The armature coils are wound on a former made of non-magnetic material and are connected through the brushes and in series with a large resistance across the supply lines. The commutator is made of silver and the brushes are silver tipped in order to reduce friction. Obviously, the current passing through the armature is proportional to the supply voltage.

The operating torque is produced due to the reaction of the field produced by the fixed coils and the armature coils. The magnitude of this torque is proportional to the product of the two currents *i.e.*,

$$T_d \propto \phi I_1 \quad \text{or} \quad T_d \propto I_1 \times I \qquad (\because \Phi \propto I)$$

where I = main circuit current

I_1 = current in armature coils.

since $I_1 \propto V \quad \therefore \; T_d \propto V I$ -power

Brake torque is due to the eddy currents induced in an aluminium disc mounted on the *same* spindle and running in the air-gaps of two permanent magnets. As shown in Art, 10.44, this braking torque is proportional to the speed of the disc if the flux of the braking magnet and the resistance of the eddy current paths are assumed constant.

When steady speed of rotation is reached, then

$$T_B = T_d \quad \therefore \; N \propto VI \propto \text{power } W$$

Hence, steady number of revolutions in a given time is proportional to $\int Wt$ = the energy in the circuit.

The friction effect is compensated for by means of a small compensating coil placed coaxially with the two currents coils and connected in series with the armature such that it strengthens the field of current coil. But its position is so adjusted that with zero line current the armature just fails to rotate. Such meters are now employed mainly for switchboard use, house service meters being invariably of the mercury ampere-hour type.

10.50. Induction Type Single-phase Watthour Meter

Induction type meters are, by far, the most common form of a.c. meters met with in every day domestic and industrial installations. These meters measure electric energy in kilo-watthours. The principle of these meters is practically the same as that of the induction wattmeters. Constructionally, the two are similar that the control spring and the pointer of the watt-meter are replaced, in the case of watthour meter, by a brake magnet and by a spindle of the meter.

The brake magnet induces eddy currents in the disc which revolves continuously instead of rotating through only a fraction of a revolution as in the case of wattmeters.

Construction

The meter consists of two a.c. electromagnets as shown in Fig. 10.64. (*a*), one of which *i.e.*, M_1 is excited by the line current and is known as ***series*** magnet. The alternating flux Φ_1 produced by it is proportional to and in phase with the line current (provided effects of

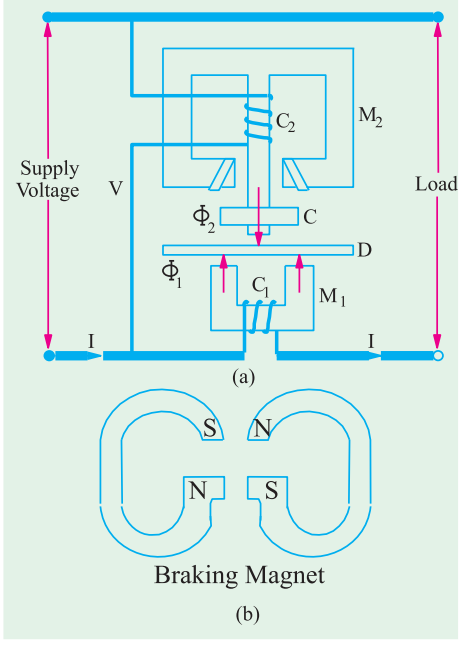


Fig. 10.64

hysteresis and iron saturation are neglected). The winding of the other magnet M_2 called ***shunt*** magnet, is connected across the supply line and carries current proportional to the supply voltage V. The flux Φ_2 produced by it is proportional to supply voltage V and lags behind it by 90°. This phase displacement of exact 90° is achieved by adjustment of the copper shading band C (also known as power factor compensator) on the shunt magnet M_2. Major portion of Φ_2 crosses the narrow gap between the centre and side limbs of M_2 but a small amount, which is the useful flux, passes through the disc D. The two fluxes Φ_1 and Φ_2 induce e.m.f.s in the disc which further produce the circulatory eddy currents. The reaction between these fluxes and eddy currents produces the driving torque on the disc in a manner similar to that explained in Art. 10.39. The braking torque is produced by a pair of magnets [Fig. 10.64 (*b*)] which are mounted diametrically opposite to the magnets M_1 and M_2. The arrangements minimizes the interaction between the fluxes of M_1 and M_2. This arrangements minimizes the interaction between the fluxes of M_1 and M_2 and that of the braking magnet. When the peripheral portion of the rotating disc passes through the air-gap of the braking magnet, the eddy currents are induced in it which give rise to the necessary torque. The braking torque $T_B \propto \Phi^2 N/R$ where Φ is the flux of braking magnet, N the speed of the rotating disc and R the resistance of the eddy current path. If Φ and R are constant, then $T_B \propto N$.

The register mechanism is either of pointer type of cyclometer type. In the former type, the pinion on the rotor shaft drives, with the help of a suitable train of reduction gears, a series of five or six pointers rotating on dials marked with ten equal divisions. The gearing between different pointers is such that each pointer advances by 1/10th of a revolution for a complete revolution of the adjacent pointer on the main rotor disc in the train of gearing as shown in Fig. 10.65.

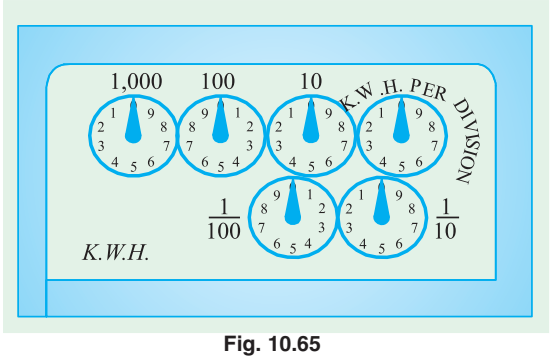


Fig. 10.65

Theory

As shown in Art. 10.39. and with reference to Fig. 10.58, the driving torque is given by $T_d \propto \omega\Phi_{1m}$ and Φ_{2m} where sin $\propto$, Φ_{1m} and Φ_{2m} are the maximum fluxes produced by magnets M_1 and α the angle between these fluxes. Assuming that fluxes are proportional to the current, we have

Current through the windings of $M_1 = I$ –the line current

Current through the winding of $M_2 = V/\omega L$

$\alpha = 90 - \phi$ where ϕ is the load p.f. angle

$T_d \propto \quad . \frac{V}{L} . I \cos(90 \quad) \quad VI \cos \quad$ power

Also, $T_b \propto N$

The disc achieves a steady speed N when the two torques are equal *i.e.*, when

$T_d = T_B \quad \therefore N \propto$ power W

Hence, in a given period of time, the total number of revolution $\int_0^t N.dt$ is proportional to $\int_0^t W.dt$ *i.e.*, the electric energy consumed.

10.51. Errors in Induction Watthour Meters

1. Phase and speed errors

Because ordinary the flux due to shunt magnet does not lag behind the supply voltage by exactly 90° owing to the fact that the coil has some resistance, the torque is not zero power factor. This is compensated for by means of an adjustable shading ring placed over the central limb of the sunt magnet. That is why this shading ring is known as ***power factor compensator.***

An error in the speed of the meter, when tested on a non-inductive load, can be eliminated by correctly adjusting the position of the brake magnet. Movement of the poles of the braking magnet towards the centre of the disc reduces the braking torque and ***vice-versa***.

The supply voltage, the full load current and the correct number of revolutions per kilowatthour are indicated on the name plate of the meter.

2. Friction compensation and creeping error

Frictional forces at the rotor bearings and in the register mechanism gives rise to unwanted braking torque on the disc rotor. This can be reduced to an unimportant level by making the ratio of the shunt magnet flux Φ_2 and series magnet flux Φ_1 large with the help of two shading bands. These bands embrace the flux contained in the two outer limbs of the shunt magnet and so eddy currents are induced in them which cause phase displacement between the enclosed flux and the main-gap flux. As a result of this, a small driving torque is exerted on the disc rotor solely by the pressure coil and independent of the main driving torque. The amount of this corrective torque is adjusted by the variation of the position of the two bands, so as to exactly compensate for firctional torque in the instrument. Correctness of friction compensation is achieved when the rotor does not run on no-load ***with only the supply voltage connected.***

By '***creeping***' is meant the slow but continuous rotation of the rotor when only the pressure coils are excited but with no current flowing in the circuit. It may be caused due to various factors like incorrect friction compensation, to vibration, to stray magnetic fields or due to the voltage supply being is excess of the normal. In order to prevent creeping on no-load, two holes are drilled in the disc on a diameter *i.e.*, on the opposite sides of the spindle.

This causes sufficient distortion of the field to prevent rotation when one of the holes comes under one of the poles of the sunt magnet.

3. Errors due to temperature variations

The errors due to temperature variations of the instruments are usually small, because the various effects produced tend to neutralise one another.

Example 10.27. *An ampere-hour meter, calibrated at 210 V, is used on 230 V circuit and indicates a consumption of 730 units in a certain period. What is the actual energy supplied ?*

If this period is reckoned as 200 hours, what is the average value of the current ?

(Elect. Technology, Utkal Univ.)

Solution. As explained in Art. 10.42, ampere-hour meters are calibrated to read directly in mWh at the *declared voltage*. Obviously, their readings would be incorrect when used on any other voltage.

Reading on 210 volt = 730 kWh

Reading on 230 volt = $730 \times 230/210 =$ **800 kWh (approx.)**

Average current = $800{,}000/230 \times 200 =$ **17.4 A**

Example 10.28. *In a test run of 30 min. duration with a constant current of 5 A, mercury-motor amp-hour meter, was found to register 0.51 kWh. If the meter is to be used in a 200-V circuit, find its error and state whether it is running fast or slow. How can the instrument be adjusted to read correctly ?* **(Elect. Mean Inst. and Mean., Jadavpur Univ.)**

Solution. Ah passed in 30 minutes = $5 \times 1/2 = 2.5$

Assumed voltage = $0.51 \times 1000/2.5 = 204$ V

When used on 200-V supply, it would obviously show higher values because actual voltage is less than the assumed voltage. It would be fast by $4 \times 100/200 =$ **2%**

Example 10.29. *An amp-hour meter is calibrated to read kWh on a 220-V supply. In one part of the gear train from the rotor to the first counting dial, there is a pinion driving a 75-tooth wheel. Calculate the number of teeth on a wheel which is required to replace 75-tooth wheel, in order to render the meter suitable for operation on 250-V supply.*

Solution. An amp-hour meter, which is calibrated on 220-V supply would run fast when operated on 250-V supply in the ratio 250/220 or 25/22. Hence, to neutralize the effect of increased

voltage, the number of teeth in the wheel should be reduced by the same ratio.

$\therefore$ Teeth on the new wheel = $75 \times 22/25 =$ **66**

Example 10.30. *A meter, whose constant is 600 revolutions per kWh, makes five revolution in 20 seconds. Calculate the load in kW.* **(Elect. Meas. and Meas. Inst. Gujarat Univ.)**

Solution. Time taken to make 600 revolution is = $600 \times 20/5 = 2{,}400$ second

During this time, the load consumes 1 kWh of energy. If W is load in kW, then

$$W \times 2400(60) \times 60 = 1 \quad \text{or} \quad W = \mathbf{1.5\ kW}$$

Example 10.31. *A current of 6 A flows for 20 minutes through a 220-V ampere hour meter. If during a test the initial and final readings on the meter are 3.53 and 4.00 kWh respectively, calculate the meter error as a percentage of the meter readings.*

If during the test, the spindle makes 480 revolutions, calculate the testing constant in coulomb/rev and rev/kWh.

Solution. Energy actually consumed = $6 \times \left(\frac{20}{60}\right) \times \frac{200}{1000} = 0.44$ kWh

Energy as registered by meter = $4.00 - 3.53 = 0.47$ kWh

Error = $0.47 - 0.44 = 0.03$ kWh ; % error = $0.03 \times 100/0.47 = 6.38$ %

No. of coulombs passed through in 20 minutes = $6 \times 20 \times 7{,}2000$ coulomb

Testing for 480 revolutions, only 0.44 kWh are consumed, hence testing constant = $480/0.44 =$ **1091 rev/kWh.**

Example 10.32. *A 230-V, single-phase domestic energy meter has a constant load of 4 A passing through it for 6 hours at unity power factor. If the meter disc makes 2208 revolutions during this period, what is the meter are 1472 when operating at 230 V and 5 V for 4 hours.*

(Elect, Measure, A.M.I.E. Sec B, 1991)

Solution. Energy consumption in 6 hr = $230 \times 4 \times 1 \times 6 = 5520$ W = 5.52 kW

Meter constant = $2208/5.52 = 400$ rev/kWh.

Now, 1472 revolution represents energy consumption of $1472/400 = 3.68$ kWh

$\therefore$ $VI \cos\phi \times$ hours $= 3.68 \times 10^{-10}$ or $230 \times 5 \times \cos\phi \times 4 = 3680$, $\therefore$ $\cos\phi =$ **0.8**

Example 10.33. *A 230 V, single-phase domestic energy meter has a constant load of 4 A passing through it for 6 hours at unity power factor. If the motor disc makes 2208 revolution during this period, what is the constant in rev kWh ? Calculate the power factor of the load if the No. of rev made by the meter are 1472 when operating at 230 V and 5 A for 4 hours.*

(Elect. Measuring. AMIE Sec. Winter 1991)

Solution. Energy supplied at unity p.f. = $230 \times 4 \times 6 \times 1/1000 = 5.52$

$\therefore$ $230 \times 5 \times 4 \times \cos\phi/2000 = 3.68$ $\therefore$ $\cos\phi =$ **0.8.**

Example 10.34. *The testing constant of a supply meter of the amp-hour type is given as 60 coulomb/revolution. It is found that with a steady current of 50 A, the spindle makes 153 revolutions in 3 minutes. Calculate the factor by which dial indications of the meter must be multiplied to give the consumption.* **(City and Guilds, London)**

Solution. Coulombs supplied in 3 min. = $50 \times 3 \times 60$

At the rate of 60 C/rev., the correct of revolution should have been

$$= 50 \times 3 \times 60/60 = 150$$

Registered No. of revolutions = 153

Obviously, the meter is fact. The registered readings should be multiplied by $150/153 =$ **0.9804** for correction.

Example 10.35. *A single phase kWhr meter makes 500 revolutions per kWh. It is found on testing as making 40 revolutions in 58.1 seconds at 5 kW full load. Find out the percentage error.*

(Elect. Measurement & Measuring Instrument Nagpur Univ. 1993)

Solution. The number of revolutions the meter will make in one hour on testing = 40 × 3600/58.1 = 2478.5

These revolutions correspond to an energy of 5 × 5 kWh

∴ No. of revolutions kWh = 2478.5/5 = 495.7

Percentage error = (500 –495.7) × 100/500 = **0.86 %**

Example 10.36. *An energy meter is designed to make 100 revolution of the disc for one unit of energy. Calculate the number of revolutions made by it when connected to a load carrying 40 A at 230-V and 0.4 power factor for an hour. If it actually makes 360 revolutions, find the percentage error.* **(Elect. Engg - I Nagpur Univ. 1993)**

Solution. Energy consumed in one hour = 230 × 40 × 0.4 × 1/1000 = 3.68 kWh

No. of revolutions the meter should make if it is correct = 3.68 × 100 = 368

No. of revolutions actually made = 360

∴ Percentage error = (369 –360) × 100/368 = **2.17 %**

Example 10.37. *The constant of a 25-ampere, 220-V meter is 500 rev/kWh. During a test at full load of 4.400 watt, the disc makes a 50 revolutions in 83 seconds. Calculate the meter error.*

Solution. In one hour, at full-load the meter should make (4400 × 1) × 500/1000 = 2200 revolutions. This corresponds to a speed of 2200/60 = 36.7 r.p.m.

Correct time for 50 rev. = (50 × 60)/36.7 = 81.7s

Hence, meter is slow by 83 –81.7 = 1.3 s

∴ Percentage error = 1.3 × 100/81.7 = **1.59 %**

Example 10.38. *A 16-A amp-hour meter with a dial marked in kWh, has an error of + 2.5 % when used on 250-V circuit. Find the percentage error in the registration of the meter if it is connected for an hour in series with a load taking 3.2 kW at 200.*

Solution. In one hour, the reading given by the meter is = 16 × 250/1000 = 4 kWh

Correct reading = 4 + 2.5 % of 4 = 4.1 kWh

Meter current on a 3.2 kW load at 200 V = 3200/200 = 16 A

Since on the given load, meter current is the same as the normal current of the meter, hence in one hour it would give a corrected reading of 4.1 kWh.

But actual load is 3.2 kWh. ∴ Error = 4.1 –3.2 = 0.9 kWh

% error = 0.9 × 100/3.2 = **28.12 %**

Example 10.39. *The disc of an energy meter makes 600 revolutions per unit of energy. When a 1000 watt load is connected, the disc rotates at 10.2 r.p.s if the load is on for 12 hours, how many units are recorded as error ?* **(Measurs, Instru. Allahabad Univ. 1992)**

Solution. Since load power is one kWh, energy actually consumed is

= 1 × 12 = 12 kWh

Total number of revolutions made by the disc during the period of 12 hours

= 10.2 × 60 × 12 = 7,344

since 600 revolutions record one kWh, energy recorded by the meter is

= 7,344/600 = 12.24 kWh

Hence, **0.24** unit is recorded extra.

Example 10.40. *A d.c. ampere-hour meter is rated at 5-A, 250-V. The declared constant is 5 A-s/rev. Express this constant in rev/kWh. Also calculate the full-load speed of the meter.*

(Elect. Meas. Inst. and Meas., Jadavpur Univ.)

Solution. Meter constant = 5 A-s/rev

Now, 1 kWh = 10^3 Wh = 10^3 × 3600 volt-second

= 36×10^5 volt × amp × second

Hence, on a 250-V circuit, this corresponds to $36 \times 10^5/250 = 14{,}400$ A-s

Since for every 5 A-s, there is one revolution, the number of revolution is one kWh is

$= 14{,}400/5 = 2{,}880$ revolutions

$\therefore$ Meter constant $=$ **2,800 rev/kWh**

Since full-load meter current is 5 A and its constant is 5 A-s/rev, it is obvious that it makes one revolution every second.

$\therefore$ full load speed $=$ 60 r.p.m.

Example 10.41. *The declared constant of a 5-A, 200-V amp-hour meter is 5 coulomb per revolution. Express the constant in rev/kWh and calculate the full-load speed of the meter.*

In a rest at half load, the meter disc completed 60 revolutions in 119-5 seconds. Calculate the meter error.

Solution. Meter constant $=$ 5 C/rev, or 5 A-s/rev.

1 kWh 1000 Wh $= 1000 \times 3600$ watt-second

$= 1000 \times 3600$ volt $\times$ amp $\times$ second

Hence, on a 200-V circuit, this corresponds to

$= 100 \times 3600/200 = 18{,}000$ A-s

Since for every 5 A-s, there is one revolution, hence number of revolution is one kWh = 18,000/5 = 3600 revolution $\therefore$ Meter constant = **3600 rev/kWh**

Since full-load meter current is 5 A and its constant 5 A-s/rev, it is obvious that it makes one revolution every one second.

$\therefore$ its full-load speed $=$ **60 r.p.m.**

At half-load

Quantity passed in 60 revolutions $= 119.5 \times 2.5$ A-s or 298.75 C

Correct No. of revolutions $= 298.75/5 = 69.75$

Obviously the meter is running fast because instead of making 59.75 revolutions, it is making 60 revolutions.

Error $= 60 - 59.75 = 0.25$ $\therefore$% error $= 0.25 \times 100/59.75 =$ **0.418 %**

Example 10.42 (*a*). *A single-phase energy meter of the induction type is rated 230-V; 10-A, 50-Hz and has a meter constant of 600 rev/kWh when correctly adjusted. If 'quadrature' adjustment is slightly disturbed so that the lag is 85°, calculate the percentage error at full-load 0.8 p.f. lag.*

(Measu & Instru. Nagpur Univ 1991)

Solution. As seen from Art. 10.50, the driving torque. T_d depends among other factors, on $\sin \alpha$ where α is the angle between the two alternating fluxes. $\therefore$ $T_d \propto \sin \alpha$

If the voltage flux-lagging adjustment is disturbed so that the phase angle between the voltage flux and the voltage is less than 90° (instead of being exactly 90°) the error is introduced.

Now $\cos \phi = 0.8$, $\phi = \cos^{-1}(0.8) = 36°52'$

$\therefore$ $\alpha = 85° - 36°52' = 48°8'$ where it should be $= 90 - 36°52' = 53°8'$

$\therefore$ error $= \dfrac{\sin 53°8 \quad \sin 48°8}{\sin 53°8} \quad 100$ **7%**

Example 10.42 (*b*). *A 50-A, 230-V energy meter is a full-load test makes 61 revolutions in 37 seconds if the normal speed of the disc is 520 revolutions/kWh, compute the percentage error.*

[Nagpur University November 1999]

Solution. Unity power-factor is assumed.

Energy consumed, in kWh, in 37 seconds

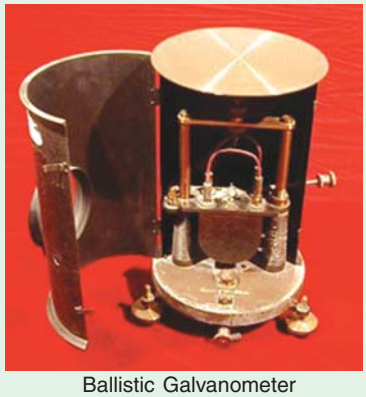
$$= \frac{50 \times 230}{1000} \times \frac{37}{3600} = 0.1182 \text{ kWh}$$

Number of revolutions corresponding to this energy = 520 × 0.1182 = 61.464

The meter makes 61 revolutions

$$\therefore \quad \% \text{ Error} = \frac{61 - 61.464}{61.464} \times 100\ \% = \mathbf{-0.755\ \%}$$

10.52. Ballistic Galvanometer


Ballistic Galvanometer

It is used principally for measuring small electric charges such as those obtained in magnetic flux measurements. Constructionally, it is similar to a moving-coil galvanometer except that **(*i*)** it has extremely small electromagnetic damping and **(*ii*)** has long period of undamped oscillation (several seconds). These conditions are necessary if the galvanometer is to measure electric charge. In fact, the moment of inertia of the coil is made so large that whole of the charge passes through the galvanometer before its coil has had time to move sufficiently. In that case, the first swing of the coil is proportional to the charge passing through the galvanometer. After this swing has been observed, the oscillating coil may be rapidly brought to rest by using eddy-current damping.

As explained above, the coil moves after the charge to be measured has passed through it. Obviously, during the movement of the coil, there is no current flowing through it. Hence, the equation of its motion is

$$J\frac{d^2\phi}{dt^2} + D\frac{d\theta}{dt} + C\theta = 0$$

where J is the moment of inertia, D is damping constant and C is restoring constant.

Since damping is extremely small, the approximate solution of the above equation is

$$0 = U_e^{-(D/2J)t} \sin(\omega_0 t + \phi)$$

At the start of motion, where $r = 0$, $\theta = 0$, hence $\phi = 0$ $\quad \therefore \ \theta = Ue^{-(D/2J)t} \sin \omega_0 t$ ...**(*i*)**

During the passage of charge, at any instant, there will be a deflecting torque of Gi acting on the coil. If t is the time taken by the whole charge to pass through, the torque impulse due to this charge is

$$\int_0^t Gi\,dt \text{ . Now } \int_0^t i\,dt = Q$$

Hence, torque impulse = GQ. This must be equal to the change of angular momentum produced *i.e.*, $j\alpha$ where α is the angular velocity of the coil at the end of the impulse period.

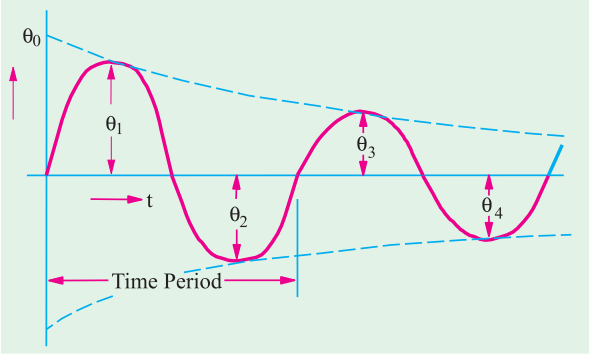


Fig. 10.66

$$\therefore \quad GQ = J\alpha$$

or $$\alpha = GQ/J$$

Differentiating Eq. **(*i*)** above, we get

$$\frac{d\theta}{dt} = U\left(e^{-(D/2J)t}\,\omega_0 t - \frac{D}{2}J\,e^{-(D/2J)t}\sin\omega_0 t\right)$$

Since duration of the passage of charge is very small, at the end of the passage, $t \cong 0$, so that from above, $dq/dt = U\omega_0$.

However, at this time, $\frac{d\theta}{dt} = \alpha = \frac{GQ}{J} \quad \therefore \quad \frac{GQ}{J} = U\,\omega_0 \quad \text{or} \quad Q = \frac{J\omega_0}{G}U$

Now, U being the amplitude which the oscillations would have if the damping were zero, it may be called undamped swing θ_0.

$$\therefore \qquad Q = \frac{J\omega_0}{G}\theta_0 \quad \text{or} \quad Q \propto \theta_0 \qquad \text{...(ii)}$$

However, in practice, due to the presence of small amount of damping, the successive oscillations diminish exponentially (Fig. 10.66). Even the first swing θ_1 is much less than θ_0. Hence, it becomes necessary to obtain the value of θ_0 from the observed value of first maximum swing θ_1.

As seen from Fig. 10.66, the successive peak value θ_1, θ_2, θ_3 etc. are ϕ radian apart or ϕ/ω_0 second apart. The ratio of the amplitude of any two successive peaks is

$$= \frac{\theta_0 e^{-(D/2J)t}\sin\omega_0 t}{\theta_0 e^{-(D/2J)t+(\pi/\omega_0)}\sin(\omega_0 t+\pi)} = \frac{e^{-(D/2J)t}\sin\omega_0 t}{e^{-(D/2J)t}\,e^{-(D/2J)(\pi/\omega_0)}(-\sin\omega_0 t)} = -e^{(D/2J)(\pi/\omega_0)}$$

$$\therefore \quad \frac{\theta_1}{\theta_2} = \frac{\theta_2}{\theta_3} = \frac{\theta_3}{\theta_4} = \ldots\ldots = e^{(D/2J)(\pi/\theta_0)}$$

Let $e^{(D/2J)(\pi/\omega_0)} = \Delta^2$ where Δ is called the damping factor*.

The time period of oscillation $T_0 = 2\pi/\omega_0$. If damping is very small $\theta_0 = \theta_1$, $t = T_0/4 = \pi/2\omega_0$ as a very close approximation.

Hence, from Eq. **(i)** above, putting $t = \pi/2\omega_0$, we have

$$\theta = \theta_0^{(D/2J)(\pi/2\omega_0)}\sin\pi/2 = \theta_0\Delta - 1 \quad \therefore \quad \theta_0 = \Delta\,\theta_1 \qquad \text{...(iii)}$$

or undamped swing = damping factor × 1st swing

Suppose, a steady current of I_s flowing through the galvanometer produces a steady deflection ϕ_s, then $C\theta_s = GI_s \quad \text{or} \quad G = C\theta_s/I_s$

Since damping is small, $\omega_0 = \sqrt{C/J} \quad \therefore C = J\omega_0^2 = J \times \left(\frac{2\pi}{T_0}\right)^2 = 4\pi^2\frac{J^2}{T_0^2} \quad \therefore \quad G = \frac{4\pi^2 J\theta s}{T_0^2 I_s}$

Substituting this value of G in Eq. **(ii)**, we get

$$Q = \frac{J\omega_0\theta_0}{4\pi^2 J\theta_s/T_0^2 I_s^2} \quad \text{or} \quad Q_0 = \frac{T_0}{2\pi}\cdot\frac{I_s}{\theta_s}\cdot\theta_0 = \frac{T_0}{2\pi}\cdot\frac{I_s}{2\pi}\cdot\Delta\cdot\theta_1 \ldots(iv)$$

Alternatively, let quantity $(D/2J)(\pi/\omega_0)$ be called the logarithmic decrement λ. Since, $\Delta^2 = e^{\lambda}$, we have

$$\Delta = e^{\lambda/2} = 1 + (\lambda/2) + \frac{(\lambda/2)^2}{2!} + \ldots \cong \left(1+\frac{\lambda}{2}\right) \text{ when } \lambda \text{ is small}^{**}$$

Hence, from Eq. **(iv)** above, we have $Q = \frac{T_0}{2\pi}\cdot\frac{I_s}{\theta_s}\left(1+\frac{\lambda}{2}\right)\theta_1$ ...(v)

In general, Eq. **(iv)** may be put as $Q = k\theta_1$

---

* Now, $\frac{\theta_1}{\theta_2} = \frac{\theta_2}{\theta_3} = \frac{\theta_{n-1}}{\theta_n} = \Delta^2 \quad \therefore \quad \frac{\theta_1}{\theta_2}\times\frac{\theta_2}{\theta_3}\times\frac{\theta_3}{\theta_4}\times\ldots\times\frac{\theta_{n-1}}{\theta_n} = (\Delta^2)^{n-1} \quad \therefore \frac{\theta_1}{\theta_2} = (\Delta^2)^{n-1} \quad \text{or } \Delta = \left(\frac{\theta_1}{\theta_2}\right)^{1/2(n-1)}$

Hence, Δ may be obtained by observing the first and nth swing.

** Since $\Delta^2 = e^{\lambda}$, taking logs, we have $2\log_c\Delta = \lambda\log_e e = \lambda$

$$\therefore \quad \frac{\lambda}{2} = \log_c\Delta = \log_e\left(\frac{\theta_1}{\theta_2}\right)^{1/2(n-1)} = \frac{1}{2(n-1)}\log_e\theta_1/\theta_n \quad \therefore \quad \lambda = \frac{1}{(n-1)}\log_e\frac{\theta_1}{\theta_n}$$

Example. 10.43. *A ballistic galvanometer has a free period of 10 seconds and gives a steady deflection of 200 divisions with a steady current of 0.1 μA. A charge of 121 μC is instantaneously discharged through the galvanometer giving rise to a first maximum deflection of 100 divisions . Calculate the 'decrement' of the resulting oscillations.*

(Electrical Measurements, Bombay Univ.)

Solution. From Eq. **(v)** of Art 10.52, we have $Q = \frac{T_0}{2} \cdot \frac{I_s}{s}\ 1\quad \overline{2}\quad 1$

Here, $Q = 121\ \mu C = 121 \times 10^{-6}\ C$; $T_0 = 10s$; $I_s = 0.1\ \text{mA} = 10^{-4}\ \text{A}$; $\theta_s = 200$; $\theta_1 = 100$

$$\therefore \quad 121 \times 10^{-6} = \frac{10}{2\pi} \times \frac{10^{-4}}{200}\left(1 + \frac{\lambda}{2}\right) \times 100; \quad \therefore\ \lambda = \mathbf{1.04}$$

10.53. Vibration Galvanometer

Such galvanometers are widely used as null-point detectors in a.c. bridges.

Construction

As shown in Fig. 10.67. (*a*), it consists of a moving coil suspended between poles of a strong permanent magnet. The natural frequency of oscillation of the coil is very high, this being achieved by the use of a large value of control constant and a moving system of very small inertia. The suspension (which provides control) is either a phosphor-bronze strip or is a bifilar suspension in which case the two suspension wires carry the coil and a small piece of mirror (or in some cases the two suspension wires themselves from the coil.)

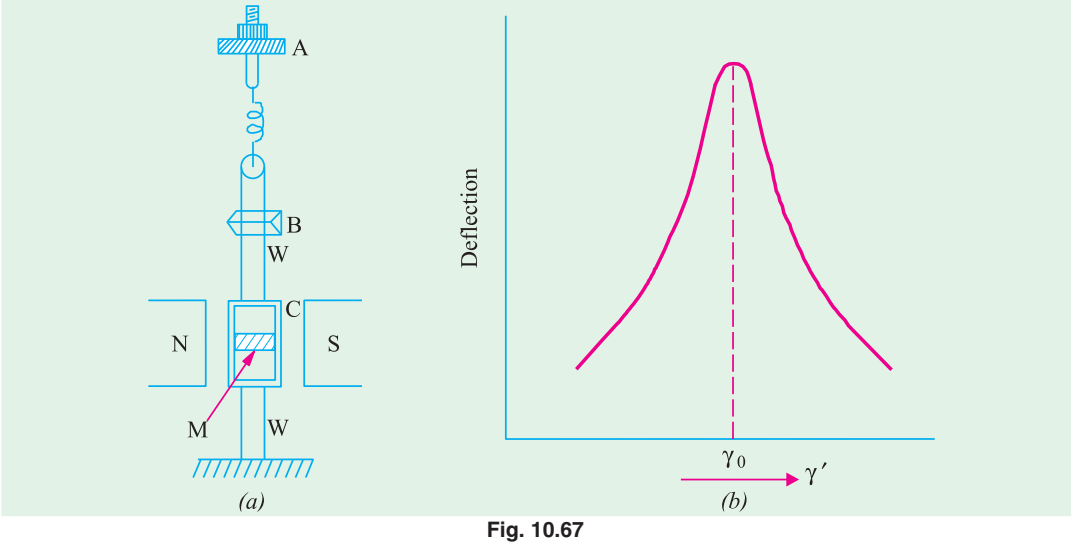


Fig. 10.67

As seen, *W* is the suspension, *C* is the moving coil and *M* the mirror on which as cast a beam of light. From mirror *M*, this beam is deflected on to a scale. When alternating current is passed through *C*, an alternating torque is applied to it so that the reflected spot of light on the scale is drawn out in the form of a band of light. The length of this band of light is maximum if the natural frequency of oscillation of *C* coincides with the the supply frequency due to resonance. The turning of *C* may be done in the following two ways :

(*i*) by changing the length of suspension *W*. This is achieved by raising or lowering bridge piece *B* against which the bifilar loop presses.

(*ii*) by adjusting tension in the suspension. This is achieved by turning the knurled knob *A*.

By making the damping very small, the resonance curve of the galvanometer can be made sharply-peaked [Fig. 10.67 (*b*)]. In that case, the instrument discriminates sharply against frequencies other than its own natural frequency. In other words, its deflection becomes very small even when the frequency of the applied current differs by a very small amount from its resonance frequency.

Theory

If the equation of the current passing through the galvanometer is $i = I_m \sin \omega t$, then the equation of motion of the coil is :

$$J\frac{d^2\theta}{dt^2} + D\frac{d\theta}{dt} + C\theta = Gi = GI_m \sin \omega t \qquad ...(i)$$

where J, D and C have the usual meaning and G is the deflection constant.

The complementary function of the solution represents the transient motion, which in the case of vibration galvanometers, is of no practical importance. The particular integral is of the form

$$\theta = A \sin (\omega t - \phi)$$

where A and ϕ are constant.

Now, $d\theta/dt = \omega A \cos (\omega t - \phi)$ and $d^2\theta/dt^2 = -\omega^2 A \sin (\omega t - \theta)$. Substituting these values in Eq. **(*i*)** above, we get

$$\omega^2 JA \sin (\omega t - \phi) + \omega DA \cos (\omega t - \phi) + CA \sin (\omega t - \phi) = GI_m \sin \omega t$$

It must be true for all values or i

When $\omega t = \phi$ $\quad DA\,\omega = G\,I_m \sin \phi$ ...**(*ii*)**

When $(wt - \phi) = \pi/2$ $\quad \omega^2 JA + CA = G\,I_m \cos \phi$ ...**(*iii*)**

Since the phase angle ϕ of oscillations is of no practical significance, it may be eliminated by squaring and adding Eq. **(*ii*)** and **(*iii*).**

Since the phase angle ϕ of oscillations is of no practical significance, it may be eliminated by squaring and adding Eq. **(*ii*)** and **(*iii*)**.

$$\therefore \quad \omega^2 D^2 A^2 + A^2 (C - \omega^2 J)^2 = G^2 I_m^2 \quad \text{or} \quad A = \frac{G\,I_m}{\sqrt{[D^2\omega^2 + (C - \omega^2 J)^2]}} \qquad ...(iv)$$

This represents the amplitude A of the resulting oscillation for a sinusoidally alternating current of peak value I_m flowing through the moving coil of the galvanometer.

10.54. The Vibrating-reed Frequency Meter

1. Working Principle

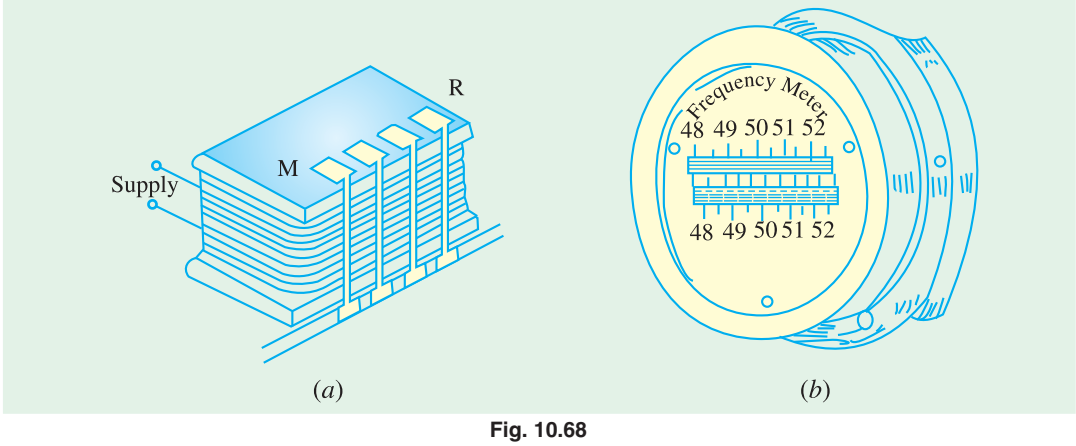


Fig. 10.68

The meter depends for its indication on the mechanical resonance of thin flat steel reeds arranged alongside and, close to, an electromagnet as shown in Fig. 10.68.

2. Construction

The electromagnet has a laminated armature and its winding, in series with a resistance, is connected across a.c. supply whose frequency is required. In that respect, the external connection of this meter is the same as that of a voltmeter.

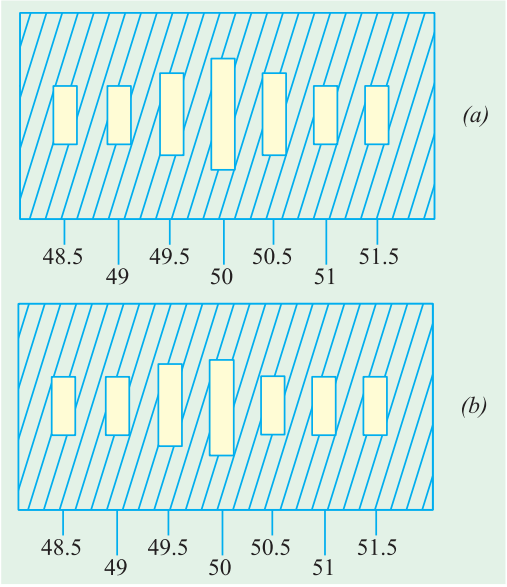


Fig. 10.69

The metallic reeds (about 4 mm wide and 0.5 mm thick) are arranged in a row and are mounted side by side on a common and slightly flexible base which also carries the armature of the electromagnet. The upper free ends of the reeds are bent over a right angles so as to serves as flags or targets and enamelled white for better visibility. The successive reeds are not exactly similar, their natural frequencies of vibration differing by $\frac{1}{2}$ cycle. The reeds are arranged in ascending order of natural frequency.

3. Working

When the electromagnet is connected across the supply whose frequency is to be measured, its magnetism alternates with the same frequency. Hence the electromagnet exerts attracting force on each reed once every half cycle. All reeds tend to vibrate but only that whose natural frequency is exactly double the supply frequency vibrates with maximum amplitude due to mechanical resonance [Fig. 10.69 (*a*)]. The supply frequency is read directly by noting the scale mark opposite the white painted flag which is vibrating the most (f = 50 Hz). The vibrations of other reeds would be so small as to be almost unobservable. For a frequency exactly midway between the natural frequencies of the two reeds (f = 49.75 Hz), both will vibrate with amplitudes which are equal but much less than when the supply frequency exactly coincides with that of the reeds.

4. Range

Such meters have a small range usually from 47 to 53 Hz or from 57 to 63 Hz etc.

The frequency range of a given set of reeds may be doubled by polarising the electromagnets as explained below. As seen from above description, each reeds is attracted twice per cycle of the supply *i.e.*, once every half-cycle and the reeds whose natural frequency is twice that of the current is of the one which responds most. Suppose the electromagnet carries an additional winding carrying direct current whose steady flux is equal in magnitude to the alternating flux of the a.c. winding. The resultant flux would be zero in one half-cycle and double in the other half-cycle when the two fluxes reinforce each other so that the reeds would receive one impulse per cycle. Obviously, a reed will indicate the frequency of the supply if the electromagnet is polarised and half the supply frequency if it is unpolarised. The polarisation may be achieved by using an extra d.c. winding on the electromagnet or by using a permanent tangent which is then wound with an a.c. winding.

5. Advantages

One great advantage of this reed-type meter is that its indications are independent of the waveform of the applied voltage and of the magnitude of the voltage, except that the voltage should be high enough to provide sufficient amplitude for reed vibration so as to make its readings reliable.

However, its limitations are :

(*a*) it cannot read closer than half the frequency difference between adjacent reeds.

(*b*) its error is dependent upon the accuracy with which reeds can be turned to a given frequency.

10.55. Electrodynamic Frequency Meter

It is also referred to as moving-coil frequency meter and is a ratiometer type of instrument.

1. Working Principle

The working principle may be understood from Fig. 10.70 which shows two moving coils rigidly fixed together with their planes at right angles to each other and mounted on the shaft or spindle situated in the field of a permanent magnet. There is no mechanical control torque acting on the two coils. If G_1 and G_2 are displacement constants of the two coils and I_1 and I_2 are the two currents, then their respective torques are $T_1 = G_1 I_1 \cos\theta$, $T_2 = G_2 I_2 \sin\theta$. These torques act in the opposite directions.

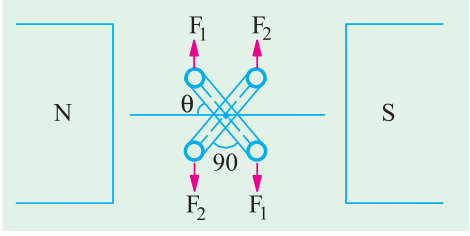


Fig. 10.70

Obviously, T_1 decreases with θ where as T_2 increases but an equilibrium position is possible for same angle θ for which

$$G_1 I_1 \cos\theta = G_2 I_2 \sin\theta \text{ or } \tan\theta = \frac{G_1}{G_2} \cdot \frac{I_1}{I_2}$$

By modifying the shape of pole faces and the angle between the planes of the two coils, the ratio I_1/I_2 is made proportional to angle θ instead of tan θ. In that case, for equilibrium $\theta \propto I_1/I_2$

2. Construction

The circuit connections are shown in Fig. 10.71. The two ratiometer coils X and Y are connected across the supply lines through their respective bridge rectifiers. The direct current I_1 through

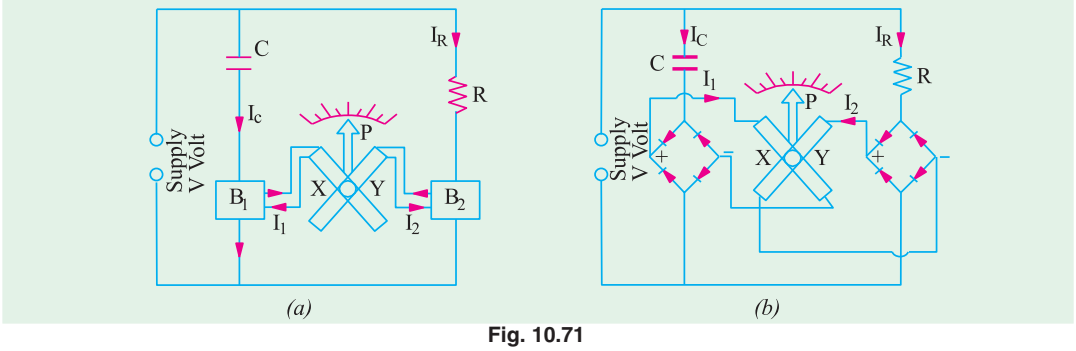


Fig. 10.71

coil X represents the R.M.S. value of capacitor current I_C as rectified by B_1. Similarly, direct current I_2 flowing through Y is the rectified current I_R passing through series resistance R.

3. Working

When the meter is connected across supply lines, rectified currents I_1 and I_2 pass through coils X and Y they come to rest at an angular position where their torques are equal but opposite. This angular position is dependent on the supply frequency which is read by a pointer attached to the coil.

As proved above, $\theta \propto I_1/I_2$

Assuming sinusoidal waveform, mean values of I_1 and I_2 are proportional to the R.M.S. values of I_C and I_R respectively.

$$\therefore \quad \theta \propto \frac{I_1}{I_2} \propto \frac{I_C}{I_R} \quad \text{Also } I_C \propto V_m \omega C \text{ and } I_R \propto V_m/R$$

where V_m is the maximum value of the supply voltage whose equation is assumed as $v = V_m \sin \omega t$.

$$\therefore \quad \theta \propto \frac{V_m \omega C}{V_m / R} \propto \omega CR \propto \omega \quad \therefore \ \theta \propto f \qquad (\because \omega = 2\pi f)$$

Obviously, such meters have linear frequency scales. Moreover, since their readings are independent of voltage, they can be used over a fairly wide range of voltage although at too low voltages, the distortions introduced by rectifier prevent an accurate indication of frequency.

It will be seen that the range of frequency covered by the meter depends on the value of R and C and these may be chosen to get ranges of 40–60 Hz, 1200–2000 Hz or 8000–12,000 Hz.

10.56. Moving-iron Frequency Meter

1. Working Principle

The action of this meter depends on the variation in current drawn by two parallel circuits – one inductive and the other non-inductive—when the frequency changes.

2. Construction

The construction and internal connections are shown in Fig. 10.72. The two coils A and B are so fixed that their magnetic axes are perpendicular to each other. At their centres is pivoted a long and thin soft-iron needle which aligns itself along the resultant magnetic field of the two coils. There is no control device used in the instrument.

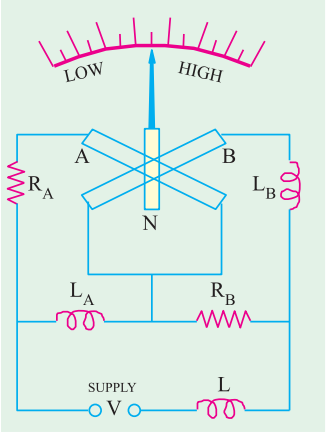


Fig. 10.72

It will be noted that the various circuit elements constitute a Wheatstone bridge which becomes balanced at the supply frequency. Coil A has a resistance R_A in series with it and a reactance L_A in parallel. Similarly R_B is in series with coil B and L_B is in parallel. The series inductance L helps to suppress higher harmonics in the current waveform and hence, tends to minimize the waveform errors in the indication of the instrument.

3. Working

On connecting the instrument across the supply, currents pass through coils A and B and produce opposing torques. When supply frequency is high, currents through coil A is more whereas that through coil B is less due to the increase in the reactance offered by L_B. Hence, magnetic field of coil A is stronger than that of coil B. Consequently, the iron needle lies more nearly to the magnetic axis of coil A than that of B. For low frequencies, coil B draws more current than coil A and, hence, the needle lies more nearly parallel to the magnetic axis of B than to that of coil A. The variations of frequency are followed by the needle as explained above.

The instrument can be designed to cover a broad or narrow range of frequencies determined by the parameters of the circuit.

10.57. Electrodynamic Power Factor Meter

1. Working Principle

The instrument is based on the dynamometer principle with spring control removed.

2. Construction

As shown in Fig. 10.73 and 10.74, the instrument has a stationary coil which is divided into two sections F_1 and F_2. Being connected in series with the supply line, it carries the load current. Obviously, the uniform field produced by F_1 and F_2 is proportional to the line current. In this field are situated two moving coils C_1 and C_2 rigidly attached to each other and mounted on the same shaft or spindle. The two moving coils are 'voltage' coils but C_1 has a series resistance R whereas C_2 has

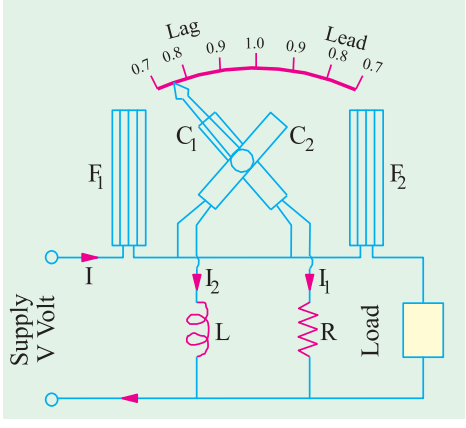


Fig. 10.73

a series inductance L. The values of R and L as well as turns on C_1 and C_2 are so adjusted that the ampere-turns of C_1 and C_2 are exactly equal. However, I_1 is in phase with the supply voltage V whereas I_2 lags behind V by nearly 90º. As mentioned earlier, there is no control torque acting on C_1 and C_2 – the currents being led into them by fine ligaments which exerts no control torque.

3. Working

Consider the case when load power factor is unity *i.e.* I is in phase with V. Then I_1 is in phase with I whereas I_2 lags behind by 90°. Consequently, a torque will act on C_1 which will set its plane perpendicular to the common magnetic axis of coils F_1 and F_2 *i.e.* corresponding to the pointer position of unity p.f. However, there will be no torque acting on coil C_2.

Now, consider the case when load power factor is zero *i.e.* I lags behind V by 90° (like current I_2). In that case, I_2 will be in phase with I where as I_1 will be 90° out of phase. As a result, there will be no toruqe on C_1 but that acting on C_2 will bring its plane perpendicular to the common magnetic axis of F_1 and F_2. For intermediate values of power factor, the deflection of the pointer corresponds to the load power factor angle ϕ or to $\cos\phi$, if the instrument has been calibrated to read to power factor direcly.

For reliable readings, the instrument has to be calibrated at the frequency of the supply on which it is to be used. At any other frequency (or when harmonics are present), the reactance of L will change so that the magnitude and phase of current through C_2 will be incorrect and that will lead to serious errors in the instrument readings.

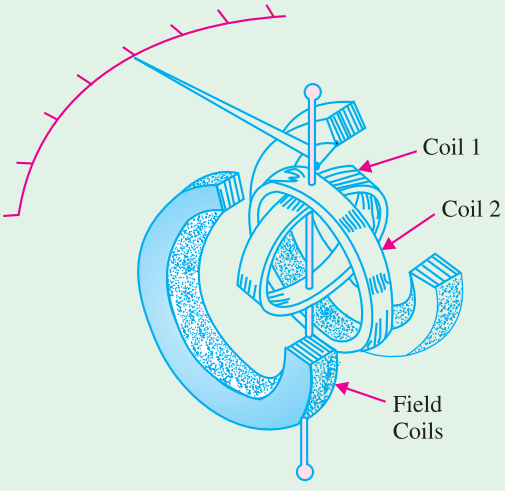


Fig. 10.74

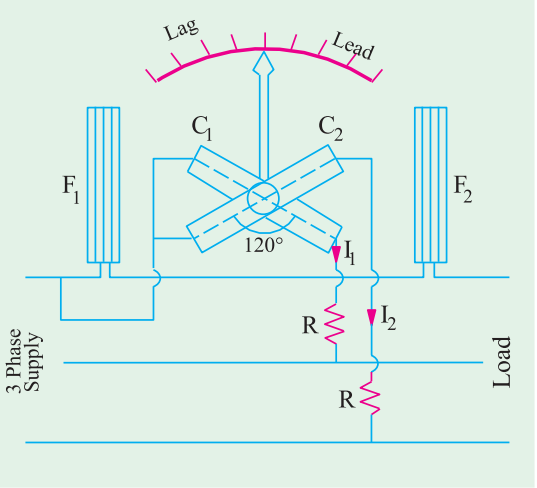


Fig. 10.75

For use on balanced 3-phase load, the instrument is modified, so as to have C_1 and C_2 at 120° to each other, instead of 90°, as in 1-phase supply. As shown in Fig. 10.75, C_1 and C_2 are connected across two different phase of the supply circuit, the stationary coils F_1 and F_2 being connected in series with the third phase (so that it carries the line current). Since there is no need of phase splitting between the currents of C_1 and C_2, I_1 and I_2 are not determined by the phase-splitting circuit and consequently, the instrument is not affected by variations in frequency or waveform.

10.58. Moving-iron Power Factor Meter

1. Construction

One type of power factor meter suitable for 3-phase balanced circuits is shown in Fig. 10.76. It consists of three fixed coils R, Y and B with axes mutually at 120° and intersecting on the centre line of the instrument. These coils are connected respectively in R, Y and B lines of the 3-phase supply through current transformers. When so energised, the three coils produce a synchronously rotating flux.

There is a fixed coil *B* at the centre of three fixed coils and is connected in series with a high resistance across one of the pair of lines, say, across *R* and *Y* lines as shown. Coil *B* is threaded by the instrument spindle which carries an iron cylinder *C* [Fig. 10.76 (*b*)] to which are fixed sector-shaped iron vanes V_1 and V_2. The same spindle also carries damping vanes and pointer (not shown in the figure) but ***there are no control springs.*** The moving system is shown separately in Fig. 10.76 (*b*).

2. Working

The alternating flux produced by coil *B* interacts with the fluxes produced by the three current coils and causes the moving system to take up a position determined by the power factor angle of the load. However, the instrument is calibrated to read the power factor cos ϕ directly instead of ϕ. In other words, the angular deflection ϕ of the iron vanes from the line *M N* in Fig. 10.76 (*a*) is equal to the phase angle ϕ.

Because of the rotating field produced by coils *R, Y* and *B*, there is a slight induction-motor action which tends to continuously turn the moving iron in the direction of the rotating flux. Hence, it becomes essential to design the moving iron as to make this torque negligibly small *i.e.* by using high-resistance metal for the moving iron in order to reduce eddy currents in it.

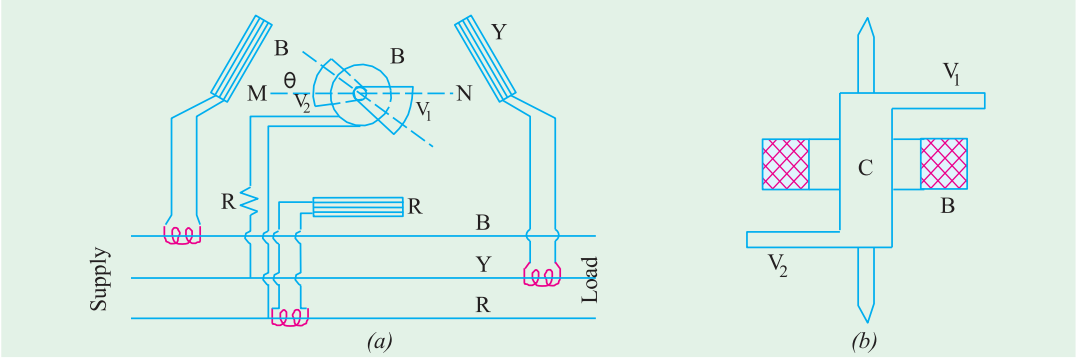


Fig. 10.76

3. Merits and Demerits

Moving iron p.f. meters are more commonly used as compared to the electrodynamic type because

(i) they are robust and comparatively cheap ***(ii)*** they have scales upto 360° and

(ii) in their case, all coils being fixed, there are no electrical connections to the moving parts.

On the other hand, they are not as accurate as the electrodynamic type of instruments and, moreover, suffer from erros introduced by the hysteresis and eddy-current losses in the iron parts–these losses varying with load and frequency.

10.59. Nalder-Lipman Moving-iron Power Factor Meter

1. Construction

The moving system of this instrument (Fig. 10.77) consists of three iron elements similar to the one shown in Fig. 10.76 (*b*). They are all mounted on a common shaft, one above the other, and are separated from one another by non-magnetic distance pieces D_1 and D_2. The three pairs of sectors are displaced in space by 120° relative to each other. Each iron vane is magnetised by one of the three voltage coils B_1, B_2 and B_3 which are connected (in series with a high resistance *R*) in star across the supply lines. The whole system is free to move in the space between two parallel halves F_1 and F_2 of a single current coil connected in one line of the supply. The common spindle also carries the damping vanes (not shown) and the pointer *P*.

2. Working

The angular position of the moving system is determined by the phase angle ϕ between the line current and the respective phase voltage. In other words, deflection θ is equal to ϕ, although, in practice, the instrument is calibrated to read the power factor directly.

3. Advantages

(i) Since no ***rotating*** magnetic field is produced, there is no tendency for the moving system to be dragged around continuously in one direction.

(ii) This instrument is not much affected by the type of variations of frequency, voltage and waveform as might be expected in an ordinary supply.

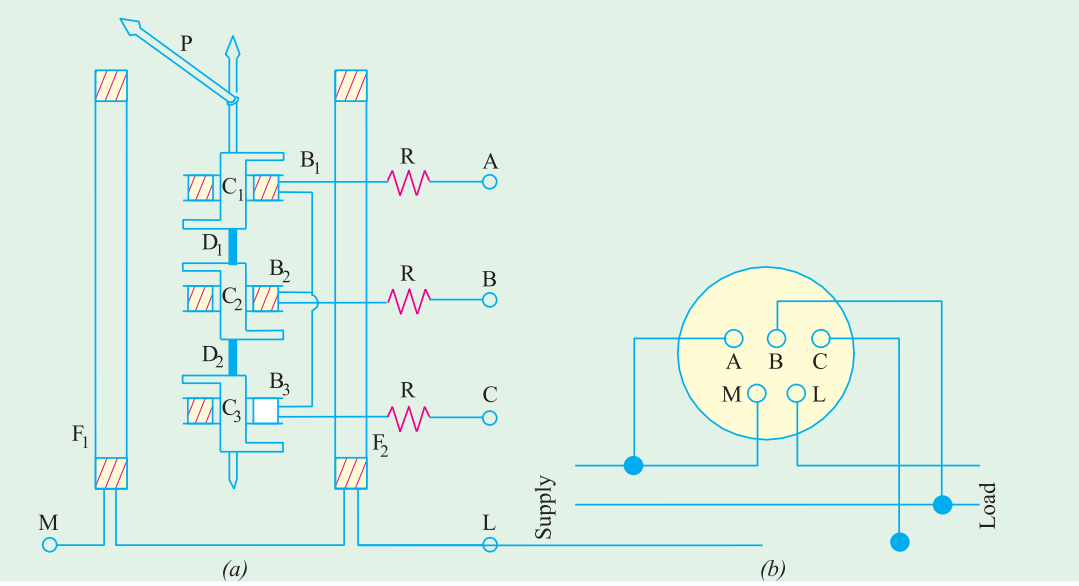


Fig. 10.77

10.60. D.C. Potentiometer



DC potentiometer

A potentiometer is used for measuring and comparing the e.m.fs. of different cells and for calibrating and standardizing voltmeters, ammeters etc. In its simplest from, it consists of a German silver or manganin wire usually one meter long and stretched between two terminals as shown in Fig. 10.78.

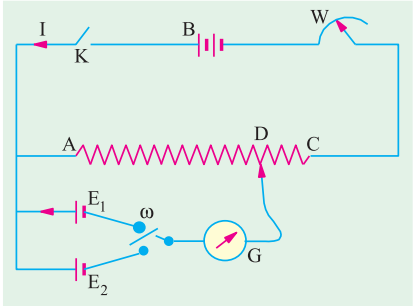


Fig. 10.78

This wire is connected in series with a suitable rheostat and battery B which sends a steady current through the resistance wire AC. As the wire is of uniform cross-section throughout, the fall in potential across it is uniform and the drop between any two points is proportional to the distance between them. As seen, the battery voltage is spread over the rheostat and the resistance wire AC. As we go along AC, there is a progressive fall of potential. If ρ is the resistance/cm of this wire, L its length, then for a current of I amperes, the fall of potential over the whole length of the wire is $\rho\ LI$ volts.

The two cells whose e.m.fs are to be compared are joined as shown in Fig. 10.78, always remembering that ***positive terminals of the cells and the battery must be joined together.*** The cells can be joined with the galvanometer in turn through a two-way key. The other end of the galvanometer is connected to a movable contact on AC. By this movable contact, a point like D is found when there is no current in and hence no deflection of G. Then, it means that the e.m.f. of the cell just balances the potential fall on AD due to the battery current passing through it.

Suppose that the balance or null point for first cell of e.m.f. E_1 occurs at a length L_1 as measured from point A. The $E_1 = \rho\ L_1 I$.

Similarly, if the balance point is at L_2 for th other cell, then $E_2 = \rho\ L_2 I$.

Dividing one equation by the other, we have $\dfrac{E_1}{E_2} = \dfrac{\rho L_1 I}{\rho L_2 I} = \dfrac{L_1}{L_2}$

If one of the cells is a standard cell, the e.m.f. of the other cell can be found.

10.61. Direct-reading Potentiometer

The simple potentiometer described above is used for educational purposes only. But in it commercial form, it is so calibrated that the readings of the potentiometer give the voltage directly, thereby eliminating tedious arithmetical calculations and so saving appreciable time.

Such a direct-reading potentiometer is shown in Fig. 10.79. The resistance R consists of 14 equal resistances joined in series, the resistance of each unit being equal to that of the whole slide were S (which is divided into 100 equal parts). The battery current is controlled by slide wire resistance W.

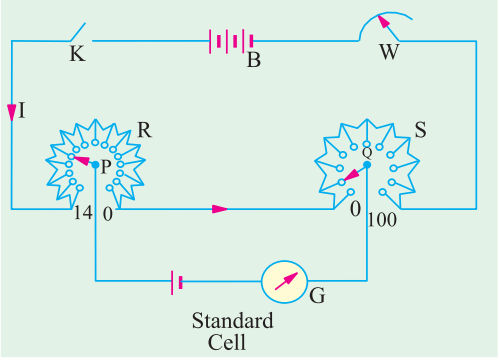


Fig. 10.79

10.62. Standardizing the Potentiometer

A standard cell *i.e.* Weston cadmium cell of e.m.f. 1.0183 V is connected to sliding contacts P and Q through a sensitive galvanometer G. First, P is put on stud No. 10 and Q on 18.3 division on S and then W is adjusted for zero deflection on G. In that case, potential difference between P and Q is equal to cell voltage *i.e.* 1.0183 V so that potential drop on each resistance of R is 1/10 = 0.1 V and every division of S represents 0.1/100 = 0.001 V. After standardizing this way, the ***position of W is not to be changed in any case*** otherwise the whole adjustment would go wrong. After this, the instrument becomes direct reading. Suppose in a subsequent experiment, for balance, P is moved to stud No. 7 and Q to 84 division, then voltage would be $= (7 \times\ 0.1) + (84 \times\ 0.001) = 0.784$ V.

It should be noted that since most potentiometers have fourteen steps on R, it is usually not possible to measure p.ds. exceeding 1.5 V. For measuring higher voltges, it is necessary to use a volt box.

10.63. Calibration of Ammeters

The ammeter to be calibrated is connected in series with a variable resistance and a standard resistance F, say, of 0.1 Ω across battery B_1 of ample current capacity as shown in Fig. 10.80. Obviously, the resistance of F should be such that with maximum current flowing through the ammeter A, the potential drop across F should not exceed 1.5 V. Some convenient current, say

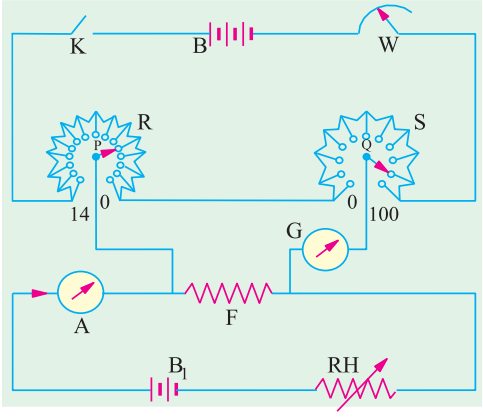


Fig. 10.80

6 amperes (as indicated by *A*) is passed through the circuit by adjusting the rheostat *RH*.

The potential drop across *F* is applied between *P* and *Q* as shown. Next the sliding contacts *P* and *Q* are adjusted for zero deflection on *G*. Suppose *P* reads 5 and *Q* reads 86.7. Then it means that p.d. across *F* is 0.5867 V and since *F* is of 0.1 Ω, hence ***true*** value of current through *F* is 0.5867/0.1 = 5.867 amperes. Hence, the ammeter reads high by (6 – 5.867) = 0.133 A. The test is repeated for various values of current over the entire ranges of the ammeter.

10.64. Calibration of Voltmeters

As pointed out in Art. 10.62, a voltage higher than 1.5 cannot be measured by the potentiometer directly, the limit being set by the standard cell and the type of the potentiometer (since it has only 14 resistances on *R* as in Fig. 10.79). However, with the help of a volt-box which is nothing else but a voltage reducer, measurements of voltage up to 150 V or 300 V can be made, the upper limit of voltage depending on the design of the volt-box.

The diagram of connections for calibration of voltmeters is shown in Fig. 10.81. By calibration is meant the determination of the extent of error in the reading of the voltmeter throughout its range. A high value resistor *AB* is connected across the supply terminals of high voltage battery B_1 so that it acts as a voltage divider. The volt-box consists of a high resistance *CD* with tapings at accurately determined points like *E* and *F* etc. The resistance *CD* is usually 15,000 to 300 Ω The two tappings *E* and *F* are such that the resistances of portions *CE* and *CF* are 1/100th and 1/10th the resistance *CD*. Obviously, whatever the potential drop across *CD*, the corresponding potential drop across *CE* is 1/100th and that across *CF*, 1/10th of that across *CD*.

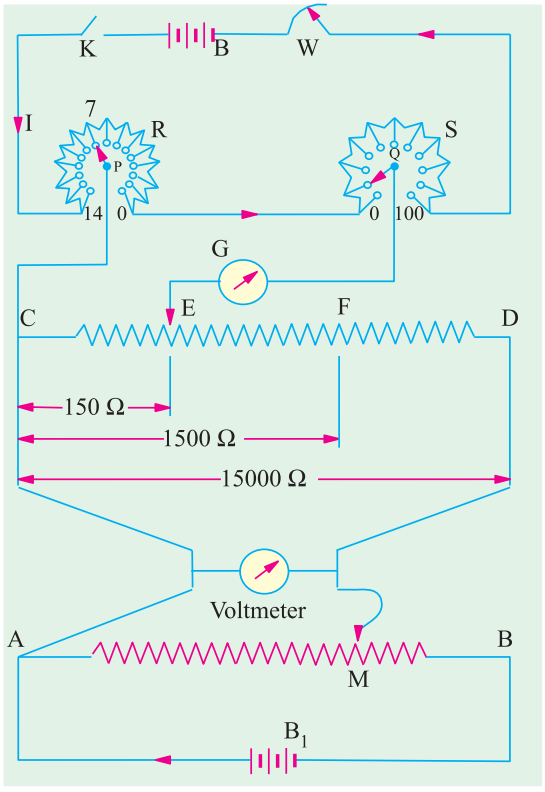


Fig. 10.81

If supply voltage is 150 V, then p.d. across *AB* is also 150 V and if *M* coincides with *B*, then p.d. across *CD* is also 150 V, so across *CF* is 15 volts and across *CE* is 1.5 V. Then p.d. across *CE* can be balanced over the potentiometer as shown in Fig. 10.81. Various voltages can be applied across the voltmeter by moving the contact point *M* on the resistance *AB*.

Suppose that *M* is so placed by voltmeter *V* reads 70 *V* and p.d. across *CE* is balanced by adjusting *P* and *Q*. If the readings on *P* and *Q* to give balance are 7 and 8.4 respectively, then p.d. across *CE* is 0.7084 V.

Hence, the true p.d. across *AM* or *CD* or voltmeter is 0.7048 × 100 = 70.84 V (because resistance of *CD* is 100 times greater than that of *CE*). In other words, the reading of the voltmeter is low by 0.84 V.

By shifting the position of *M* and then balancing the p.d. across *CE* on the potentiometer, the voltmeter can be calibrated throughout its range. By plotting the errors on a graph, a calibration curve of the instrument can also be drawn.

10.65. A.C. Potentiometer

An A.C. potentiometer basically works on the same principle as a d.c. potentiometer. However,

there is one very important difference between the two. In d.c. potentiometer, only the ***magnitudes*** of the unknown e.m.f. and slide-wire voltage drop are made equal for obtaining balance. But in an a.c. potentiometer, not only the magnitudes but *phases* as well have to be equal for obtaining balance. Moreover, to avoid frequency and waveform errors, the a.c. supply for slide-wire must be taken from same source as the voltage or current to be measured.

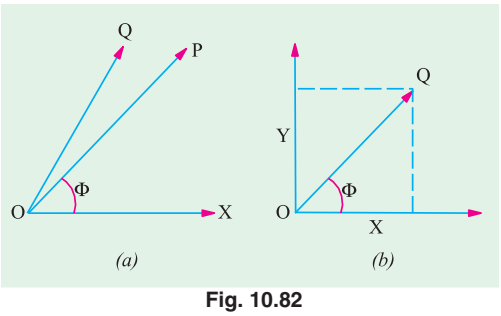


Fig. 10.82

A.C. potentiometers are of two general types differing in the manner in which the value of the unknown voltage is presented by the instrument dials or scales. The two types are :

(i) Polar potentiometers in which the unknown voltage is mesured in polar form *i.e.* in terms of magnitude and relative phase.

(ii) Co-ordinate potentiometers which measure the rectangular co-ordinates of the voltage under test.

The two products are illustrated in Fig. 10.82. In Fig. 10.82 (*a*), vector *OQ* denotes the test voltage whose magnitude and phase are to be imitated. In polar potentiometer, the length *r* of the vector *OP* can be varied with the help of a sliding contact on the slidewire while its phase ϕ is varied independently with the help of a phase-shifter. Drysdale potentiometer is of this type.

In co-ordinate type potentiometers, the unknown voltage vector *OQ* is copied by the adjustment of 'in phase' and 'quadrature' components *X* and *Y*. Their values are read from two scales of the potentiometer. The magnitude of the required vector is $= \sqrt{X^2 + Y^2}$ and its phase is given by $\phi = \tan^{-1}(X/Y)$. Examples of this type are ***(i)*** Gall potentiometer and ***(ii)*** Campbell-Larsen potentiometer.

10.66. Drysdale Polar Potentiometer

As shown in Fig. 10.83 for a.c. measurements, the slide-wire *MN* is supplied from a phase shifting circuit so arranged that magnitude of the voltage supplied by it remains constant while its

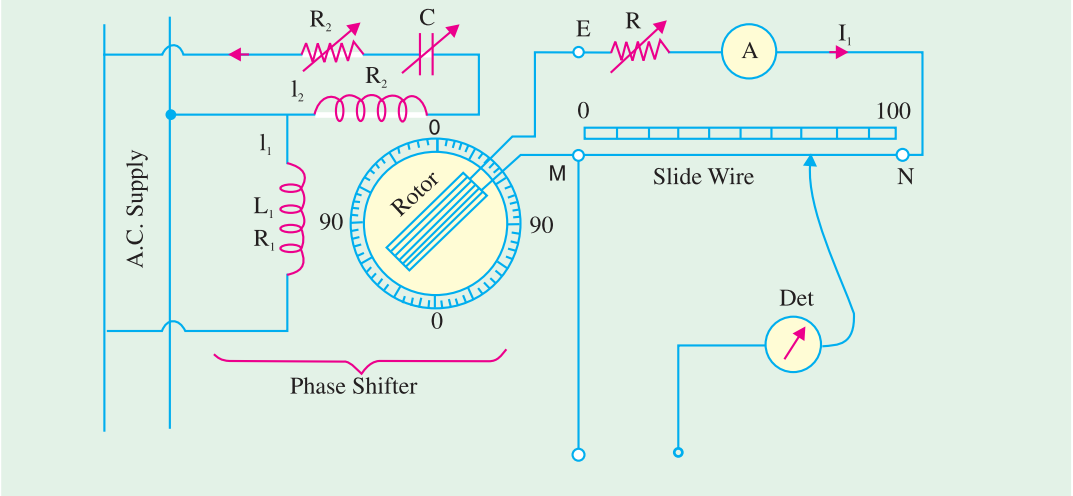


Fig. 10.83

phase can be varied through 360°. Consequently, slide-wire current *I* can be maintained constant in magnitude but varied in phase. The phase-shifting circuit consists of :

(i) Two stator coils supplied from the same source in parallel. Their currents I_1 and I_2 are made to differ by 90° by using well-known phase-splitting technique.

(ii) The two windings produce a rotating flux which induces a secondary e.m.f. in the rotor winding which is of constant magnitude but the phase of which can be varied by rotating the rotor in any position either manually or otherwise. The phase of the rotor e.m.f. is read from the circular graduated dial provided for the purpose.

The ammeter *A* in the slide-wire circuit is of electrodynamic of thermal type. Before using it for a.c. measurements, the potentiometer is first calibrated by using d.c. supply for slidwire and a standard cell for test terminals T_1 and T_2.

The unknown alternating voltage to be measured is applied across test terminals I_2 and T_2 balance is effected by the alternate adjustment of the slide-wire contact and the position of phase-shifting rotor. The slide-wire reading represents the magnitude of the test voltage phase-shifter reading gives its phase with reference to an arbitrary reference vector.

10.67. Gall Co-ordinate Potentiometer

This potentiometer uses two slide-wires *CD* and *MN* with their currents I_1 and I_2 (Fig. 10.84) having a mutual phase difference of 90°. The two currents are obtained from the single phase supply through isolating transformers, the circuit for 'quadrature' slidewire *MN* incorporating a phase shifting arrangement.

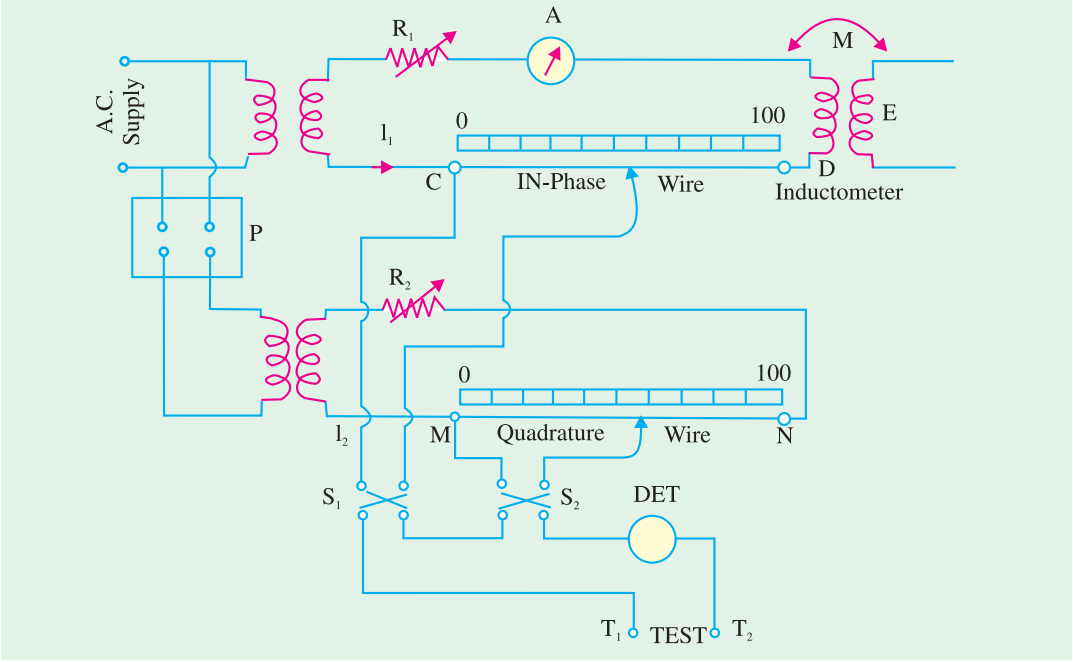


Fig. 10.84

Before use, then current I_1 is first standardised as described for Drysdale potentiometer (Art. 10.66). Next, current I_2 is standardised with the help of the mutually induced e.m.f. *E* in inductometer secondary. This e.m.f. $E = \omega M I_1$ and is in quadrature phase with I_1. Now, *E* is balanced against the voltage drop on slide-wire *MN*. This balance will be obtained only when I_2 is of correct magnitude and is in exact quadrature with I_1. Balance is achieved with the help of the phase-shifter and rheostat R_2.

The unknown voltage is applied across the test terminals T_1 and T_2. Slide-wire *MN* measures that component of the unknown voltage which is in phase with I_2. Similarly, slide-wire *CD* measures that component of the unknown voltage which is in phase with I_1. Since I_1 and I_2 are in quadrature, the

two measured values are quadrature components of the unknown voltage. If V_1 and V_2 are these values, then

$$V = \sqrt{V_1^2 + V_2^2} \quad \text{and} \quad \phi = \tan^{-1}(V_2/V_1) \quad \text{—with respect to } I_1$$

Reversing switches S_1 and S_2 are used for measuring both positive and negative in-phase and quadrature components of the unknown-voltage.

10.68. Instrument Transformers

The d.c. circuits when large currents are to be measured, it is usual to use low-range ammeters with suitable shunts. For measuring high voltages, low-range voltmeters are used with high resistances connected in series with them. But it is neither convenient nor practical to use this method with alternating current and voltage instruments. For this purpose, specially constructed accurate-ratio instrument transformers are employed in conjunction with standard low-range a.c. instruments. Their purpose is to reduce the line current or supply voltage to a value small enough to be easily measured with meters of moderates size and capacity. In other words, they are used for extending the range of a.c. ammeters and voltmeters. Instruments transformers are of two types :

(*i*) current transformers (*CT*) —for measuring large alternating currents.
(*ii*) potential transformers (*VT*) —for measuring high alternating voltages.

Advantages of using instrument transformers for range extension of a.c. meters are as follows :

(1) the instrument is insulated from the line voltage, hence it can be grounded. **(2)** the cost of the instrument (or meter) together with the instrument transformer is less than that of the instrument alone if it were to be insulated for high voltages. **(3)** it is possible to achieve standardisation of instruments and meters at secondary ratings of 100–120 volts and 5 or 1 amperes **(4)** if necessary, several instruments can be operated from a single transformer and 5 power consumed in the measuring circuits is low.

In using instrument transformers for current (or voltage) measurements, we must know the ratio of primary current (or voltage) to the secondary current (or voltage). These ratios give us the multiplying factor for finding the primary values from the instrument readings on the secondary side.

However, for energy or power measurements, it is essential to know not only the transformation ratio but also the phase angle between the primary and secondary currents (or voltages) because it necessitates further correction to the meter reading.

For range extension on a.c. circuits, instrument transformers are more desirable than shunts (for current) and multipliers (for voltage measurements) for the following reasons :

1. time constant of the shunt must closely match the time constant of the instrument. Hence, a different shunt is needed for each instrument.
2. range extension is limited by the current-carrying capacity of the shunt *i.e.* upto a few hundred amperes at the most.
3. if current is at high voltage, instrument insulation becomes a very difficult problem.
4. use of multipliers above 1000 becomes almost impracticable.
5. insulation of multipliers against leakage current and reduction of their distributed capacitance becomes not only more difficult but expensive above a few thousand volts.

10.69. Ratio and Phase-angle Errors

For satisfactory and accurate performance, it is necessary that the ratio of transformation of the instrument transformer should be constant within close limits. However, in practice, it is found that neither current transformation ratio I_1/I_2 (in the case of current transformers) nor voltage transformation ratio V_1/V_2 (in the case of potential transformers) remains constant. The transformation ratio is found to depend on the exciting current as well as the current and the power factor of the secondary circuit. This fact leads to an error called ratio error of the transformer which depends on the working component of primary.

It is seen from Fig. 1.85 (*a*) that the phase angle between the primary and secondary currents is not exactly 180° but slightly less than this value. This difference angle β may be found by reversing

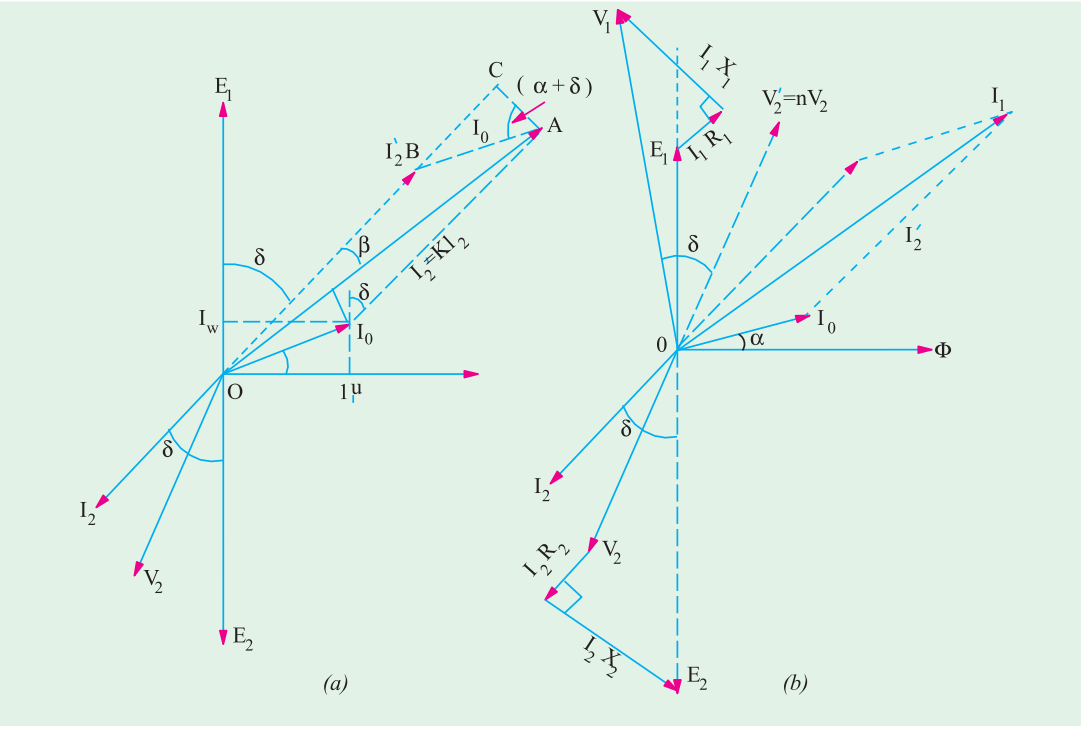


Fig. 10.85

vector I_2. The angular displacement between I_1 and I_2 reversed is called the phase angle error of the current transformer. This angle is reckoned positive if the reversed secondary current ***leads*** the primary current. However, on very low power factors, the phase angle may be negative. Similarly, there is an angle of γ between the primary voltage V_1 and secondary voltage reversed-this angle represents the phase angle error of a voltage transformer. In either case, the phase error depends on the magnetising component $I\mu$ of the primary current. It may be noted that ratio error is primarily due to the reason that the ***terminal*** voltage transformation ratio of a transformer is not exactly equal to its turn ratio. The divergence between the two depends on the resistance and reactance of the transformer windings as well as upon the value of the exciting current of the transformer. Accuracy of voltage ratio is of utmost importance in a voltage transformer although phase angle error does not matter if it is to be merely connected *to* ***a voltmeter.*** Phase-angle error becomes important only when voltage transformer supplies the voltage coil of a wattmeter *i.e.* in power measurement. In that case, phase angle error causes the wattmeter to indicate on a wrong power factor.

In the case of current transformers, constancy of current ratio is of paramount importance. Again, phase angle error is of no significance if the current transformer is merely feeding an ammeter but it assumes importance when feeding the current coil of a wattmeter. While discussing errors, it is worthwhile to define the following terms :

(i) Nominal transformation ratio (k_n). It is the ratio of the ***rated*** primary to the ***rated*** secondary current (or voltage).

$$k_n = \frac{\text{rated primary current } (I_1)}{\text{rated secondary current } (I_2)} \quad \text{—for } CT$$

$$= \frac{\text{rated primary voltage } (V_1)}{\text{rated secondary voltage } (V_2)} \quad \text{—for } VT$$

In the case of current transformers, it may be stated either as a fraction such as 500/5 or 100/1 or

simply as the number representing the numerator of the reduced fraction *i.e.*100. It is also known as ***marked*** ratio.

(ii) Actual transformation ratio (k). The actual transformation ratio or just ratio under any given *condition of loading is*

$$k = \frac{\text{primary current } (I_1)}{\text{corresponding secondary current } (I_2)}$$

In general, k differs from k_n except in the case of an ideal or perfect transformer when $k = k_n$ for all conditions of loading.

(iii) Ratio Error (σ). In most measurements it may be assumed that $I_1 = k_n I_2$ but for very accurate work, it is necessary to correct for the difference between k and k_n. It can be done with the help of ratio error which is defined as

$$\sigma = \frac{k_n - k}{k} = \frac{\text{nominal ratio-actual ratio}}{\text{actual ratio}}$$

Also,
$$\sigma = \frac{k_n \,.\, I_2 - kI_2}{k \,.\, I_2} = \frac{k_n \,.\, I_2 - I_1}{I_1}$$

Accordingly, ratio error may be defined as the difference ***between the primary current reading (assuming the nominal ratio) and the true primary current divided by the true primary current.***

(iv) Ratio Correction Factor (R.C.F.). It is given by

$$R.C.F. = \frac{\text{actual ratio}}{\text{nominal ratio}} \quad \frac{k}{k_n}$$

10.70. Current Transformer

A current transformer takes the place of shunt in d.c. measurements and enables heavy alternating current to be measured with the help of a standard 5-A range a.c. ammeter.

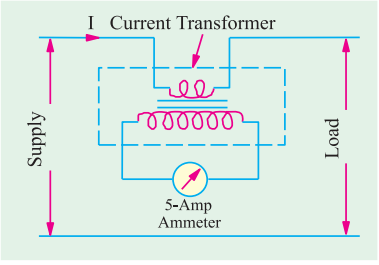


Fig. 10.86

As shown in Fig. 10.86, the current - or series-transformer has a primary winding of one or more turns of thick wire connected in series with the line carrying the current to be measured. The secondary consist of a large number of turns of fine wire and feeds a standard 5-A ammeter (Fig. 10.86) or the current coil of a watt-meter or watthour-meter (Fig. 10.87).

For example, a 1,000/5A current transformer with in single-turn primary will have 200 secondary turns. Obviously, it steps down the current in the 200 : 1 ratio whereas it steps up the voltage drop across the single-turn primary (an extremely small quantity) in the ratio 1 : 200. Hence if we know the current ratio of the transformer and the reading of the a.c. ammeter, the line current can be calculated.

It is worth noting that ammeter resistance being extremely low, a current transformer operates with its secondary under nearly short-circuit conditions. Should it be necessasry to remove the ammeter of the current coils of the wattmeter or a relay, the seondary winding must, first of all, be short-circuited *before* the instrument is disconnected.

If it is not done then due to the absence of counter ampere-turns of the secondary, the unopposed primary m.m.f. will set up an abnormally high flux in the core which will produce excessive core loss with subsequent heating of and damage of the transformer insulation and a high voltage across the secondary

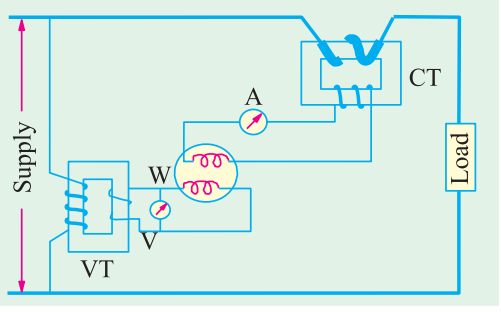


Fig. 10.87

terminals. This is not the case with the ordinary constant-potential transformers because their primary current is determined by the load on their secondary whereas in a current transformer, primary current is determined entirely by the load on the system and not by the load on its own secondary. Hence, the secondary of a current transformer should ***never be left open under any circumstances.***

10.71. Theory of Current Transformer

Fig. 10.85 (*b*) represents the general phase diagram for a current transformer. Current I_0 has been exaggerated for clarity.

(a) Ratio Error. For obtaining an expression for the ratio error, it will be assumed that the turn ratio n (= secondary turns, N_2/primary turns N_1) is made equal to the nominal current ratio *i.e.* $n = k_n$. In other words, it will be assumed that $I_1/I_2 = n$ although actually $n = I_1/I_2'$. As seen from Art. 10.63.

$$\sigma = \frac{n I_2 - I_1}{I_1} = \frac{I_2' - I_1}{I_1} = \frac{OB - OA}{OA} \qquad \text{— } [\because n = k_n]$$

$$\equiv \frac{OB - OC}{OA} \qquad (\because \beta \text{ is very small angle})$$

$$= -\frac{BC}{OA} = -\frac{AB \sin(\alpha + \delta)}{OA} = -\frac{I_0 \sin(\alpha + \delta)}{I_1} = -\frac{I_0 \sin(\alpha + \delta)}{nI_2}$$

For most instrument transformers, the power factor of the secondary burden is nearly unity so that δ is very small. Hence, very approximately.

$$\sigma = \frac{I_0 \sin \alpha}{I_1} - \frac{I_\omega}{I_1}$$

where I_ω is the iron-loss or working or wattful component of the exciting current I_0

Note. The transformation ratio R may be found from Fig. 10.85 (*a*) as under :

$I_1 = OA = OB + BC = nI_2 \cos\beta + I_0 \cos[90 - (\delta + \beta + \alpha)] = nI_2 \cos\beta + I_0 \sin(\delta + \beta + \alpha)$

Now $\beta = (\alpha + \delta)$ hence $I_1 = nI_2 + I_0 \sin(\alpha + \delta)$ where n is the turn ratio of the transformer.

$$\therefore \qquad \text{ratio } R = \frac{I_1}{I_2} = \frac{nI_2 + I_0 \sin(\alpha + \delta)}{I_2} \text{ or } R = n + \frac{I_0 \sin(\alpha + \delta)}{I_2} \qquad \text{...(i)}$$

If δ is negligible small, then $R = n + \dfrac{I_0 \sin \alpha}{I_2} = n + \dfrac{I_\omega}{I_2}$

It is obvious from **(*i*)** above that ratio error can be eliminated if secondary turn are reduced by a number

$$= I_0 \sin(\alpha + \delta)/I_2$$

(*b*) Phase angle (β)

Again from Fig. 10.85 (*a*), we fine that

$$\beta \equiv \sin\beta = \frac{AC}{OA} = \frac{AB \cos(\alpha + \delta)}{OA} = \frac{I_0 \cos(\alpha + \delta)}{I_1} = \frac{I_0 \cos(\alpha + \delta)}{nI_2}$$

Again, if the secondary power factor is nearly unity, then δ is very small, hence

$$\beta \equiv \frac{I_0 \cos \alpha}{I_1} = \frac{I_\mu}{I_1} \text{ or } \frac{I_\mu}{nI_2}$$

where I_μ is the magnetising component of the exciting current I_0.

$$\therefore \qquad \beta = \frac{I_\mu}{I_1} \text{ —in radian; } = \frac{180}{\pi} \times \frac{I_\mu}{I_1} \text{ —in degrees}$$

Note. As found above, $\beta = \dfrac{I_0 \cos(\quad)}{I_1}$ $\dfrac{I_0 (\cos \quad \cos \quad \sin \quad \sin \quad)}{I_1}$

$$= \frac{I_\mu \cos\delta - I_\omega \sin\delta}{I_1} = \frac{I_\mu \cos\delta - I_\omega \sin\delta}{nI_2} \text{ radian}$$

$$\therefore \quad \beta = \frac{180}{\pi} \times \frac{I_\mu \cos\delta - I_\omega \sin\delta}{nI_2} \text{ degrees.}$$

Dependence of ratio error on working component of I_0 and that of phase angle on the magnetising component is obvious. If R is to come closer to k and β is to become negligible small, then I_μ and I_ω and hence I_0 should be very small.

10.72. Clip-on Type Current Transformer

It has a laminated core which is so arranged that it can be opened out at a hinged section by merely pressing a trigger-like projection (Fig. 10.88). When the core is thus opened, it permits the admission of very heavy current-carrying bus-bars or feeders whereupon the trigger is released and the core is tightly closed by a spring. The current-carrying conductor of feeder acts as a single-turn primary whereas the secondary is connected across the standard ammeter conveniently mounted in the handle itself.

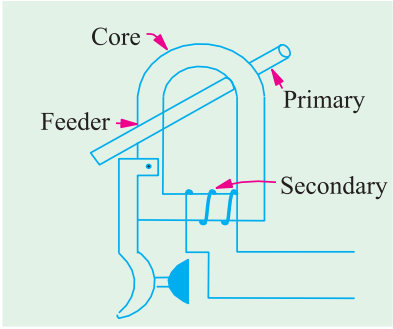


Fig. 10.88

10.73. Potential Transformers

These transformers are extremely accurate-ratio stepdown transformers and are used in conjunction with standard low-range voltmeters (100-120 V) whose deflection when divided by transformation ratio, gives the true voltage on the primary or high-voltage side. In general, they are of the shell type and do not differ much from the ordinary two-winding transformers except that their power rating is extremely small. Sine their secondary windings are required to operate instruments or relays or pilot lights, their ratings are usually of 40 to 100W. For safety, the secondary is completely insulated from the high voltage primary and is, in addition, grounded for affording protection to the operator. Fig. 10.89 shows the connection of such a transformer.



Potential transformer

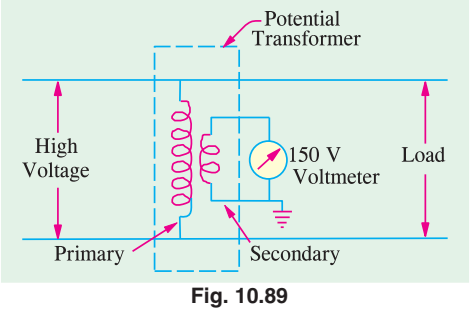


Fig. 10.89

10.74. Ratio and Phase-angle Errors

In the case of a potential transformer, we are interested in the ratio of the primary to the secondary terminal voltage and in the phase angle γ between the primary and reversed secondary terminal voltage V_2'.

The general theory of voltage transformer is the same as for the power transformers except that, as the current in the secondary burden is very small, the total primary current I_1 is not much greater than I_0.

In the phasor diagram of Fig. 10.90, vectors *AB, BC, CD* and *DE* represent small voltage drops due to resistances and reactances of the transformer winding (they have been exaggerated for the sake of clarity). Since the drops as well as the phase angle γ are small, the top portion of diagram 10.90 (*a*) can be drawn with negligible loss of accuracy as in Fig. 10.90 (*b*) where V_2' vector has been drawn parallel to the vector for V_1.

In these diagrams, V_2' is the secondary terminal voltage as referred to primary assuming transformation without voltage drops. All actual voltage drops have been referred to the primary. Vector AB represents total resistive drop as referred to primary *i.e.* $I_2'\ R_{01}$. Similarly, BC represents total reactive drop as referred to primary *i.e.* $I_2'\ X_{01}$.

In a voltage transformer, the relatively large no-load current produces appreciable resistive drops which have been represented by vectors CD and DE respectively. Their values are I_0R_1 and I_0X_1 respectively.

(*a*) Ratio Error

In the following theory, n would be taken to represent the ratio of ***primary turns to secondary turns*** (Art. 10.69). Further, it would be assumed, as before, that n equals the nominal transformation ratio *i.e.* $n = k_n$.

In other words, it would be assumed that $V_1/V_2 = n$, although, actually, $V_1/V_2' = n$.

Then
$$\sigma = \frac{k_n - k}{k} = \frac{k_n . V_2 - k V_2}{kV_2} = \frac{V_2' - V_1}{V_1} = -\frac{EN}{OE} \quad \text{...Fig. 10.90 } (a)$$
$$= -\frac{AG + FC + LD + EM}{OE} \quad \text{...Fig. 10.90 } (b)$$
$$= -\frac{I_2' R_{02} \cos\delta + I_2' X_{02} \sin\delta + I_0R_1 \sin\alpha + I_0X_1 \cos\alpha}{V_1}$$
$$= -\frac{I_2' R_{02} \cos\delta + I_2' X_{02} \sin\delta + I_\mu R_1 + I_\mu X_1}{V_1}$$

where I_ω and I_μ are the iron-loss and magnetising components of the no-load primary current I_0.

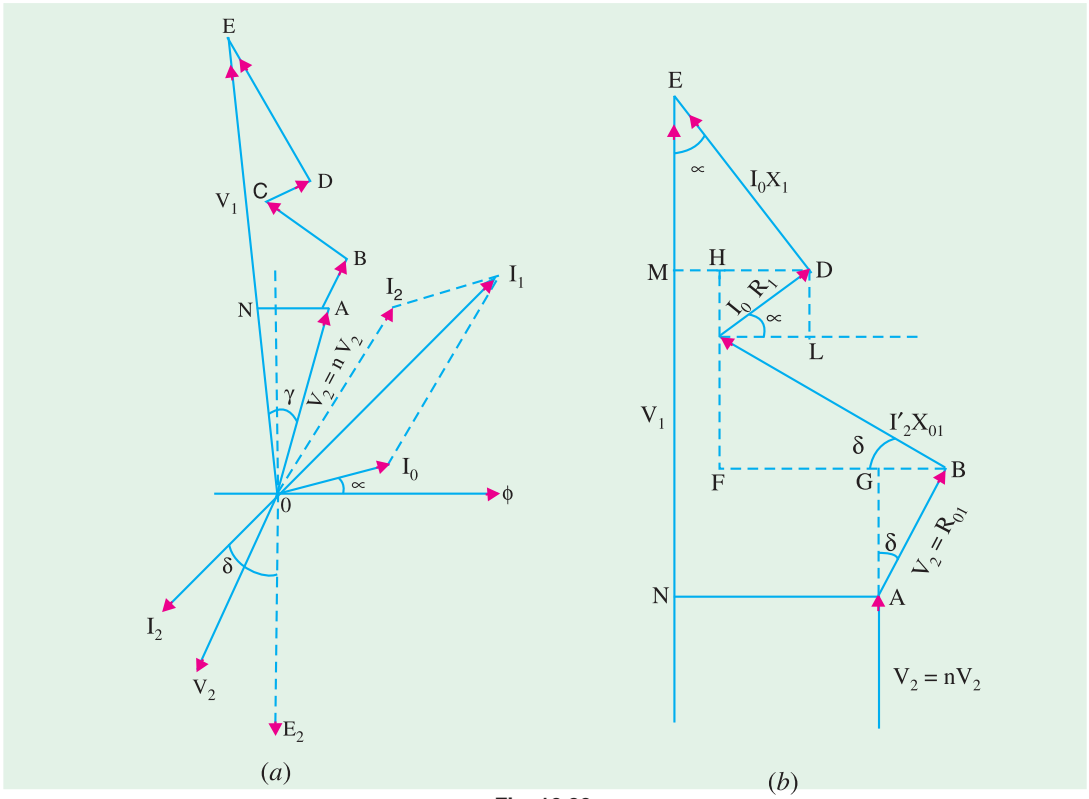


Fig. 10.90

(b) Phase Angle (γ)

To a very close approximation, value of γ is given by $\gamma = AN/OA$ —in radian

Now, $OA \cong OE$ provided ratio error is neglected. In that case,

$$\gamma = \frac{AN}{OE} = -\frac{GF + HM}{OE} \quad \text{...Fig. 10.90 } (b)$$

$$= -\frac{(BE - BG) + (DM - DH)}{OE}$$

$$= -\frac{I_2' X_{01} \cos\delta - I_2' R_{01} \sin\delta + I_0 X_1 \sin\alpha - I_0 R_1 \cos\alpha}{V_1}$$

$$= -\frac{I_2' X_{01} \cos\delta - I_2' R_{01} \sin\delta + I_\omega X_1 - I_\mu R_1}{V_1}$$

The negative sign has been given because reversed secondary voltage *i.e.* V_2' lags behind V_1.

Example 10.44. *A current transformer with 5 primary turns and a nominal ratio of 1000/5 is operating with a total secondary impedance of 0.4 + j 0.3 Ω. At rated load, the iron loss and magnetising components of no-load primary current are 1.5 A and 6 A respectively. Calculate the ratio error and phase angle at rated primary current if the secondary has* ***(a)*** *1000 turns and* ***(b)*** *990 turns.*

Solution. Phasor diagram of Fig. 10.87 may please be referred to.

(a) $\tan\delta = 0.3/0.4$ or $\delta = \tan^{-1}(0.3/0.4) = 36°52'$

$\alpha = \tan^{-1}(1.5/6) = 14°2'$ $\quad \therefore (\alpha + \beta) = 50°54'$

$I_0 = \sqrt{I_\omega^2 + I_\mu^2} = \sqrt{1.5^2 + 6^2} = 6.186$ A

$BC = I_0 \sin(\alpha + \beta) = 6.186 \times \sin 50°54' = 4.8$ A

Since β is amall, $OC \cong OA = 1000$ A $\quad \therefore I_2' = OC - BC = 1000 - 4.8 = 995.2$ A

$\therefore \quad \sigma = \frac{k_n I_2 - I_1}{I_1} = \frac{nI_2 - I_1}{I_1}$ because in this case $n\ (= N_2/N_1 = 1000/5)$ is equal to nominal ratio k_n (= 1000/5 A).

or $\quad \sigma = \frac{I_2 \quad I_1}{I_1} \quad \frac{995.2 \quad 1000}{1000} \quad 0.0048$ or **48%**

$\beta = \tan^{-1}(AC/OC)$

Now $AC = I_0 \cos(\alpha + \beta) = 6.186 \cos 50°54' = 3.9$ A

centre $\beta = \tan^{-1}(AC/OC) = \tan^{-1}(3.9/1000) =$ **0°13′**

(b) In this case, $I_2 = 995.2 \times \frac{5}{990}$; $k_n . I_2 = \frac{1000}{5} \times 995.2 \times \frac{5}{990} = 1005.3$ A

$\therefore \quad \sigma = \frac{k_n I_2 \quad I_1}{I_1} \quad \frac{1005.3 \quad 1000}{1000} \quad 0.0053$ or **0.53%**

The value of phase angle β would not be significantly different from the value obtained in (*a*) above.

Example 10.45. *A relay current-transformer has a bar primary and 200 secondary turns. The secondary burden is an ammeter of resistance 1.2 Ω and resistance of 0.5 Ω and the secondary winding has a resistance of 0.2 Ω and reactance of 0.3 Ω. The core requires the equivalent of 100 AT for magnetisation and 50 AT for core losses.*

(i) *Find the primary current and the ratio error when the secondary ammeter indicates 5.0 A.*

(ii) *By how many turns should the secondary winding be reduced to eliminate the ratio error for this condition ?* **(Electrical Measurements, Bombay Univ.)**

Solution. Total secondary impedance is

$$\mathbf{Z_2} = 1.4 + j\,0.8 = 1.612 \angle 29°45' \quad \therefore \ \delta = 29°45'$$

$$\mathbf{I0} = 100 + j50 = 111.8 \angle 26°34' \quad \therefore \ \alpha = 26°34'$$

Turn ratio, $n = 200/I = 200$; Transformation ratio $R = n + \dfrac{I_0 \sin(\alpha + \delta)}{I_2}$

$$\therefore \quad R = 200 + \frac{111.8 \sin 56°19'}{5} = 218.6$$

(*i*) Primary current $= 5 \times 218.6 = \mathbf{1093\ A}$

Ratio error $\sigma = \dfrac{I_0 \sin(\quad)}{nI_2} \quad \dfrac{111.8 \quad 0.8321}{200 \quad 5}$ **0.093 or 9.3%**

(*ii*) No. of secondary turns to be reduced $= I_0 \sin(\alpha + \delta)/I_2 = 93/5 =$ **19 (approx).**

Example 10.46. *A current transformer has 3 primary turns and 300 secondary turns. The total impedance of the secondary is (0.583 + j 0.25) ohm. The secondary current is 5 A. The ampere-turns required to supply excitation and iron losses are respectively 10 and 5 per volt induced in the secondary.*

Determine the primary current and phase angle of the transformer.

(Elect Meas; M.S. Univ. Baroda)

Solution. Z2 $= 0.583 = j\,0.25 = 0.6343 \angle 23°\ 10'$ $\therefore E_2 = I_2 Z_2 = 5 \times 0.6343 =$ **3.17 V**

Now, there are 10 magnetising *AT* per *secondary* volt induced in secondary.

$\therefore$ total magnetising $AT = 3.17 \times 10 = 31.7$; Similarly, iron-loss $AT = 3.17 \times 5 = 15.85$

Remembering that there are 3 primary turns, the magnetising and iron-loss components of primary current are as under :

Magnetising current, $I_\mu = 31.7/3 = 10.6$ A; iron-loss current $I_\omega = 15.85/3 = 5.28$ A

$$I_0 = \sqrt{10.6^2 + 5.28^2} = 11.84 \text{ A}$$

Now, $R = n \quad \dfrac{I_0 \sin(\quad)}{I_2}$

Here, $n = 300/3 = 100$; $\alpha = \tan^{-1}(I_\omega/I_\mu) = \tan^{-1}(5.28/10.6)$

$= \tan^{-1}(0.498) = 26°30'$

δ = secondary load angle $= 23°10'$ —found earlier

$\therefore \quad R = 100 + \dfrac{11.86}{5}(\sin 49°40') = 100 + 1.81 = 101.81$

$I_1 = R \times I_2 = 101.81 \times 5 =$ **509.05 A**

$$\beta = \frac{180}{\pi} \times \frac{I_\mu \cos\delta - I_\omega \sin\delta}{nI_2}$$

$$= \frac{180}{} \quad \frac{10.6 \cos 23\ 10 \quad 5.28 \sin 23\ 10}{100 \quad 5} \quad \mathbf{0.88°}$$

Example 10.47. *A current transformer with a bar primary has 300 turns in its secondary winding. The resistance and reactance of the secondary circuit are 1.5 Ω and 1.0 Ω respectively including the transformer winding. With 5 A flowing in the secondary circuit the magnetising ampere-turns required are 100 and iron loss is 1.2 W. Determine the ratio error at this condition.*

(Elect. Measure, A.M.I.E. Sec. B, 1992)

Solution. Turn ratio $n = 300/1 = 300$

Secondary impedance is $Z_2 = 1.5 + j\,1.0 = 1.8 \angle 33°42'$

Secondary induced e.m.f. $E_2 = I_2 Z_2 = 5 \times 1.8 = 9$ V

$E_1 = E_2/n = 9/300 = 0.03$ V

Let us now find the magnetising and working components of primary no-load current I_0

Magnetising $AT = 100$. Since threre is one primary turn, $\therefore I_\mu = 100/1 = 100$ A

Now, $E_1 I_\omega = 1.2 \quad \therefore I_\omega = 1.2/0.03 = 40$ A

$$I_0 = 100 + j\,40 = 107.7 \angle 21°48' \; ; \sigma = -\frac{I_0 \sin(\alpha + \delta)}{nI_2}$$

Now $\alpha = 21°48'$ and $\delta = 33°42'$

$$\therefore \quad \sigma = \frac{107.7 \sin 55\ 30}{300\ \ 5} \qquad \textbf{0.0592 or 5.92\%}$$

Phase angle $$\beta = \frac{I_0 \cos(\alpha + \delta)}{nI_2} = \frac{107.7 \times \cos 55°30'}{1500}$$

$$= \frac{180\ \ 107.7\ \ 0.5664}{1500} \qquad \textbf{2°20}$$

Tutorial Problems No. 10.4

1. A current transformer with 5 primary turns has a secondary burden consisting of a resistance of 0.16 Ω and an inductive reactance of 0.12Ω. When primary current is 200 A, the magnetising current is 1.5 A and the iron-loss component is 0.4 A. Determine the number of secondary turns needed to make the current ratio 100/1 and also the phase angle under these conditions. **[407 : 0.275°]**
2. A current transformer having a 1-turn primary is rated at 500/5 A, 50 Hz, with an output of 1.5 VA. At rated load with the non-inductive burden, the in-phase and quadrature components (referred to the flux) of the exciting ampere-turns are 8 and 10 respectively. The number of turns in the secondary is 98 and and the resistance and leakage reactance of the secondary winding are 0.35 Ω and 0.3 Ω respectively. Calculate the current ratio and the phase angle error. **[501.95/5; 0.533°]**
   (Elect. Inst. and Meas, M.S. Univ. Baroda)
3. A ring-core current transformer with a nominal ratio of 500/5 and a bar primary has a secondary resistance of 0.5 Ω and negligible secondary reactance. The resultant of the magnetising and iron-losss components of the primary current associated with a full-load secondary current of 5 A in a burden of 1.0 Ω (non-inductive) is 3 A at a power factor of 0.4. Calculate the true ratio and the phase-angle error of the transformer on full-load. Calculate also the total flux in the core, assuming that frequency is 50 Hz. **[501.2/5; 0.314°; 337 μWb]**
4. A current transformer has a single-turn primary and a 200-turn secondary winding. The secondary supplies a current of 5 A to a non-inductive burden of 1 Ω resistance, the requisite flux is set up in the core by 80 AT. The frequency is 50 Hz and the net cross-section of the core is 10 cm^2. Calculate the ratio and phase angle and the flux density in the core.
   [200.64; 4°35′ 0.079 Wb/m2] *(Electrical Measurements, Osmania Univ.)*
5. A potential transformer, ratio 1000/100-V, has the following constants :

   primary resistance = 94.5 Ω ; secondary resistance = 0.86 Ω
   primary reactance = 66.2 Ω ; equivalent reactance = 66.2 Ω
   magnetising current = 0.02 A at 0.4 p.f.

   Calculate (*i*) the phase angle at no-load between primary and secondary voltages (*ii*) the load in VA at u.p.f. at which the phase angle would be zero. **[(i) 0°4′ (ii) 18.1 VA]**
6. PMMC instrument has FSD current of 50 milliampere and 2 ohm resistance. How the instrument can be converted to
   (*i*) 0.5 A range Ammeter (*ii*) 0.100 V range Voltmeter? ***(Nagpur University, Summer 2002)***
7. What are the essential torques of an indicating instruments? Justify their necessity.
   (Nagpur University, Winter 2002)
8. Discuss the necessity of damping in Indicating instrument and explain eddy current damping.
   (U.P. Technical University 2002) (Nagpur University, Summer 2003)
9. PMMC instrument has FSD current of 50 milliampere and 2 ohm resistance. How the instrument can be converted to (*i*) 0-5 A range Ammeter (*ii*) 0-100 V range Voltmeter?
   (Nagpur University, Summer 2003)

10. What are the essential requirements of indicating type instruments? Explain each of them. ***(U.P. TechnicalUniversity 2002) (Nagpur University, Winter 2003)***

11. What are the different operating systems required in an instrument? Explain damping system in detail. ***(U.P. TechnicalUniversity 2002) (Nagpur University, Summer 2004)***

12. If a shunt for a moving coil instrument is a have a multiplicationfactor m, shown that its shunt resistance is given by

$$R_{sh} = \frac{R_m}{m - 1}$$

where R_m →resistance of the meter. ***(Nagpur University, Summer 2004)***

13. Find the value of a series resistance to be connected to a basic d' Arsonval movement with internal resistance R_m = 100 ohm and full scale deflection current is 1 mA, for conversion into 0–500 volt. ***(Nagpur University, Summer 2004)***

14. What is the basic principle of Induction type instruments? ***(Anna University, April 2002)***

15. What is damping torque in instruments? ***(Anna University, April 2002)***

16. Why does MI instruments have non-linear scale? ***(Anna University, April 2002)***

17. Explain how can you measure power by the Ammeters. ***(Anna University, April 2002)***

18. What is creeping in energy meter? ***(Anna University, April 2002)***

19. What is the advantage of Induction type meters? ***(Anna University, April 2002)***

20. What is a trivector meter? ***(Anna University, April 2002)***

21. What is the difference between PMMC instrument and ballistic galvanometer? ***(Anna University, April 2002)***

22. What is the basic principle of operation of reed type frequency meters? ***(Anna University, April 2002)***

23. Why can't you measure low resistance with meggar? ***(Anna University, April 2002)***

24. What are the advantages of electronic meters? ***(Anna University, April 2002)***

25. Explain the principle of operation, construction and the expression for deflection of a moving iron instrument. ***(Anna University, April 2002)***

26. Draw and explain the working of 2 element wattmeter. Explain how does the 2 element wattmeter reads total power in a 3 ϕ circuit. ***(Anna University, April 2002)***

27. Write the equations for wattmeter readings W_1 and W_2 in 3 phase power measurement and therefrom for power factor. ***(Anna University, Wimter 2002)***

28. Explain how power can be measured in a three phase circuit with the help of two wattmeters, for a balanced star connected load. Draw the phasor diagram. ***(Anna University Winter 2002)***

29. State the working principle of a dynamometer type wattmeter and show its connections. ***(V.T.U., Belgaum Karnataka University, February 2002)***

30. A.D.C. milliammeter having a resistance of 2Ω gives full scale deflection when the current is 50mA. How can it be used to measure (*i*) a current of 5A (*ii*) a voltage of 500 V. ***(V.T.U., Belgaum Karnataka University, February 2002)***

31. With the help of a neat diagram, explain the construction and principle of operation of single phase energy meter. ***(V.T.U., Belgaum Karnataka University, Wimter 2003)***

32. With a neat sketch explain the construction and working of a single phase induction type energymeter. ***(V.T.U., Belgaum Karnataka University, Summer 2003)***

OBJECTIVE TESTS—10

1. The kWh meter can be classified as a/an-instrument :

(*a*) deflecting (*b*) digital
(*c*) recording (*d*) indicating

2. The moving system of an indicating type of electrical instrument is subjected to :

(*a*) a deflecting torque
(*b*) a controlling torque
(*c*) a damping torque
(*d*) all of the above

3. The damping force acts on the moving system of an indicating instrument only when it is :

(*a*) moving (*b*) stationary
(*c*) near its full deflection
(*d*) just starting to move.

4. The most efficient form of damping employed in electrical instruments is :
   (*a*) air friction
   (*b*) fluid friction
   (*c*) eddy currents
   (*d*) none of the above.
5. Moving iron instruments can be used for measuring :
   (*a*) direct currents and voltages
   (*b*) alternating current and voltages
   (*c*) radio frequency currents
   (*d*) both (*a*) and (*b*).
6. Permanent-magnet moving-coil ammetere have uniform scales because :
   (*a*) of eddy current damping
   (*b*) they are spring-controlled
   (*c*) their deflecting torque varies directly as current
   (*d*) both (*b*) and (*c*).
7. The meter that is suitable for *only* direct current measurements is :
   (*a*) moving-iron type
   (*b*) permanent-magnet type
   (*c*) electrodynamic type
   (*d*) hot-wire type.
8. A moving coil voltmeters measures—
   (*a*) only a.c. voltages
   (*b*) only d.c. voltages
   (*c*) both a.c. and d.c. voltages
   (Principles of Elect. Engg. Delhi Univ.)
9. The reading of the voltmeter in Fig. 10.91 would be nearest to–volt :
   (*a*) 80
   (*b*) 120
   (*c*) 200
   (*d*) 0

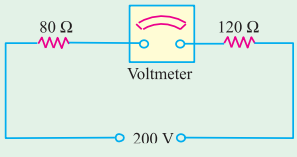


Fig. 10.91

10. The hot-wire ammeter :
   (*a*) is used only for d.c. circuits
   (*b*) is a high precision instrument
   (*c*) is used only for a.c. circuits
   (*d*) reads equally well on d.c. and/or a.c. circuits.

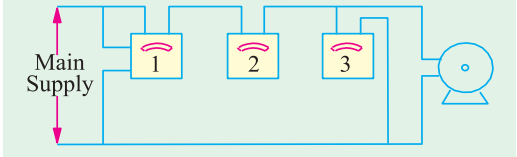


Fig. 10.92

11. If an energy meter disc makes 10 revolutions in 100 seconds when a load of 450 w is connected to it, the meter constant (in rev/k Wh) is
   (*a*) 1000 (*b*) 500
   (*c*) 1600 (*d*) 800
   (GATE 2001)
12. The inductance of a certain moving-iron ammeter is expressed as $L = 10 + 3\theta - \frac{\theta^2}{4}$ μH, where θ is the deflection in radians from the zero position. The control spring torque in 25×10^{-6} Nm/radian. The deflection of the pointer in radian when the meter carries a current of 5A, is
   (*a*) 2.4 (*b*) 2.0
   (*c*) 1.2 (*d*) 1.0
   (GATE 2003)
13. A dc potentiometer is designed to measure up to about 2 V with a slide wire of 800 mm. A standard cell of emf 1.18 V obtains balance at 600 mm. A test cell is seen to obtain balance at 680 mm. The emf of the test cell is
   (*a*) 1.00 V (*b*) 1.34 V
   (*c*) 1.50 V (*d*) 1.70 V
   (GATE 2004)
14. A galvanometer with a full scale current of 10 mA has a resistance of 1000 Ω The multiplying power (the ratio of measured current to galvanometer current) of a 100 Ω shunt with this galvanometer is
   (*a*) 110 (*b*) 100
   (*c*) 11 (*d*) 10
   (GATE 2004)
15. A moving coil of a meter has 100 turns, and a length and depth of 10 mm and 20 mm respectively. It is positioned in a uniform radial flux density of 200mT. The coil carries a current of 50 mA. The torque on the coil is
   (*a*) 200 μNm (*b*) 100 μNm
   (*c*) 2 μNm (*d*) 1 μNm
   (GATE 2004)
16. A dc A-h meter is rated for 15 A, 250 V. The meter constant is 14.4 A-sec/rev. The meter constant at rated voltage may be expressed as
   (*a*) 3750 rev/kWh (*b*) 3600 rev/kWh
   (*c*) 1000 rev/kWh (*d*) 960 rev/kWh
   (GATE 2004)

ANSWERS

1. *c* **2.** *d* **3.** *a* **4.** *c* **5.** *d* **6.** *d* **7.** *b* **8.** *b* **9.** *c* **10.** *d*

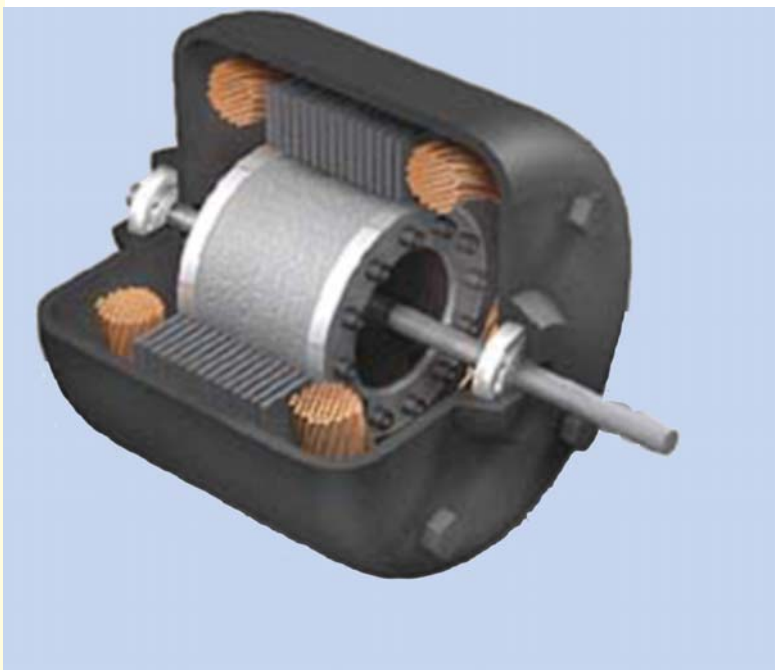
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CHAPTER 11

A.C. FUNDA-MENTALS

Learning Objectives

- Generation of Alternating Voltages and Currents
- Alternate Method for the Equations of Alternating Voltages and currents
- Simple Waveforms
- Cycle
- Different Forms of E.M.F. Equation
- Phase
- Phase Difference
- Root Mean Square (R.M.S.) Value
- Mid-ordinate Method
- Analytical Method
- R.M.S. Value of a Complex Wave
- Average Value
- Form Factor
- Crest or Peak Factor
- R.M.S. Value of H.W. Rectified A.C.
- Average Value
- Form Factor of H.W. Rectified
- Vector Diagrams Using R.M.S. Values
- Vector Diagrams of Sine Waves of Same Frequency
- Addition and Subtraction of Vectors
- A.C. Through Resistance, Inductance and Capacitance
- A.C. through Pure Ohmic Resistance alone
- A.C. through Pure Inductance alone
- Complex Voltage Applied to Pure Inductance
- A.C. through Capacitance alone



Alternating current circuits improves the versatility and usefulness of electrical power system. Alternating current plays a vital role in today's energy generation

11.1. Generation of Alternating Voltages and Currents

Alternating voltage may be generated by rotating a coil in a magnetic field, as shown in Fig. 11.1 (*a*) or by rotating a magnetic field within a stationary coil, as shown in Fig. 11.1 (*b*).

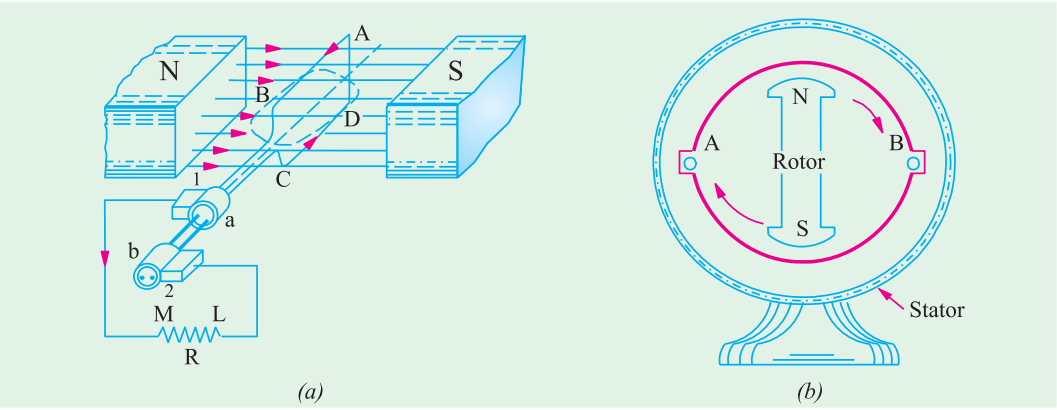


Fig. 11.1

The value of the voltage generated depends, in each case, upon the number of turns in the coil, strength of the field and the speed at which the coil or magnetic field rotates. Alternating voltage may be generated in either of the two ways shown above, but rotating-field method is the one which is mostly used in practice.

11.2. Equations of the Alternating Voltages and Currents

Consider a rectangular coil, having N turns and rotating in a uniform magnetic field, with an angular velocity of ω radian/second, as shown in Fig. 11.2. Let time be measured from the X-axis. Maximum flux Φ_m is linked with the coil, when its plane coincides with the X-axis. In time t seconds, this coil rotates through an angle $\theta = \omega t$. In this deflected position, the component of the flux which is perpendicular to the plane of the coil, is $\Phi = \Phi_m \cos \omega t$. Hence, ***flux linkages*** of the coil at any time are $N\Phi = N\Phi_m \cos \omega t$.

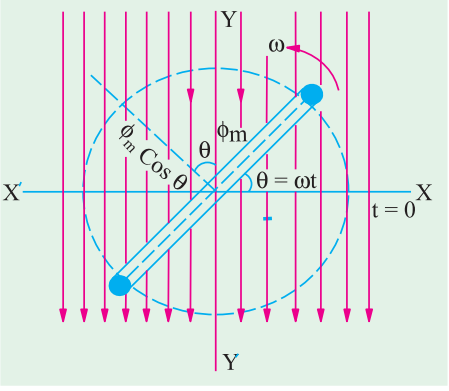


Fig. 11.2

According to Faraday's Laws of Electromagnetic Induction, the e.m.f. induced in the coil is given by the rate of change of flux-linkages of the coil. Hence, the value of the induced e.m.f. at this instant (*i.e.* when $\theta = \omega t$) or the instantaneous value of the induced e.m.f. is

$$e = -\frac{d}{dt}(N\Phi) \text{ volt} = -N.\frac{d}{dt}(\Phi_m \cos \omega t) \text{ volt} = -N\Phi_m\, \omega\, (-\sin \omega t) \text{ volt}$$

$$= \omega N \Phi_m \sin \omega t \text{ volt} = \omega N \Phi_m \sin \theta \text{ volt} \qquad \text{...(i)}$$

When the coil has turned through 90° *i.e.* when $\theta = 90°$, then $\sin \theta = 1$, hence e has maximum value, say E_m. Therefore, from Eq. **(*i*)** we get

$$E_m = \omega N \Phi_m = \omega N B_m A = 2\pi f N B_m A \text{ volt} \qquad \text{...(ii)}$$

where B_m = maximum flux density in Wb/m^2 ; A = area of the coil in m^2

f = frequency of rotation of the coil in rev/second

Substituting this value of E_m in Eq. **(*i*)**, we get $e = E_m \sin \theta = E_m \sin \omega t$ ...**(*iii*)**

Similarly, the equation of induced alternating current is $i = I_m \sin \omega t$ ...(iv)

provided the coil circuit has been closed through a resistive load.

Since $\omega = 2\pi f$, where f is the frequency of rotation of the coil, the above equations of the voltage and current can be written as

$$e = E_m \sin 2\pi f t = E_m \sin \left(\frac{2\pi}{T}\right) t \quad \text{and} \quad i = I_m \sin 2\pi f t = I_m \sin \left(\frac{2\pi}{T}\right) t$$

where T = time-period of the alternating voltage or current = $1/f$

It is seen that the induced e.m.f. varies as sine function of the time angle ωt and when e.m.f. is plotted against time, a curve similar to the one shown in Fig. 11.3 is obtained. This curve is known as sine curve and the e.m.f. which varies in this manner is known as ***sinusoidal*** e.m.f. Such a sine curve can be conveniently drawn, as shown in Fig. 11.4. A vector, equal in length to E_m is drawn. It rotates in the counter-clockwise direction with a velocity of ωradian/second, making one revolution while the generated e.m.f. makes two loops or one cycle. The projection of this vector on Y-axis gives the instantaneous value e of the induced e.m.f. *i.e.* $E_m \sin \omega t$.

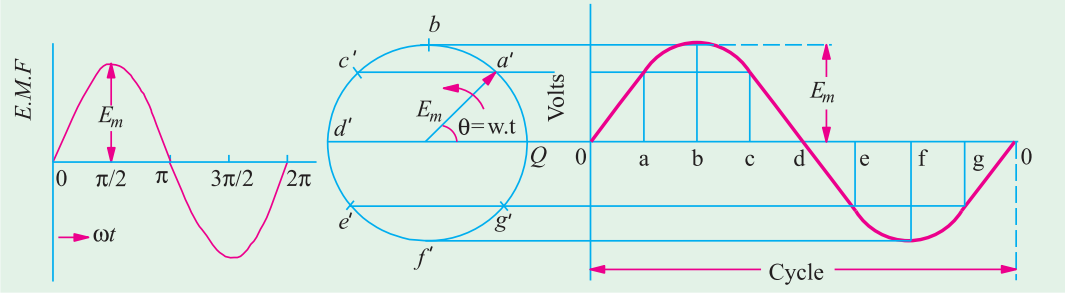


Fig. 11.3 Fig. 11.4

To construct the curve, lay off along X-axis equal angular distance oa, ab, bc, cd etc. corresponding to suitable angular displacement of the rotating vector. Now, erect coordinates at the points a, b, c and d etc. (Fig. 11.4) and then project the free ends of the vector E_m at the corresponding positions a', b', c', etc to meet these ordinates. Next draw a curve passing through these intersecting points. The curve so obtained is the graphic representation of equation ***(iii)*** above.

11.3. Alternate Method for the Equations of Alternating Voltages and Currents

In Fig. 11.5 is shown a rectangular coil AC having N turns and rotating in a magnetic field of flux density B Wb./m^2. Let the length of each of its sides A and C be l meters and their peripheral velocity v metre/second. Let angle be measured from the horizontal position *i.e.* from the X-axis. When in horizontal position, the two sides A and C move parallel to the lines of the magnetic flux. Hence, no flux is cut and so no e.m.f. is generated in the coil.

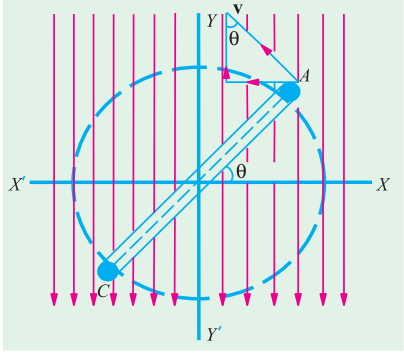


Fig. 11.5

When the coil has turned through angle θ, its velocity can be resolved into two mutually perpendicular components ***(i)*** $v \cos \theta$ component-parallel to the direction of the magnetic flux and ***(ii)*** $v \sin \theta$ component-perpendicular to the direction of the magnetic flux. The e.m.f. is generated due entirely to the perpendicular component *i.e.* $v \sin \theta$.

Hence, the e.m.f. generated in one side of the coil which contains N conductors, as seen from Art. 7.7, is given by, $e = N \times Blv \sin \theta$.

Total e.m.f. generated in both sides of the coil is

$$e = 2BNl\,v \sin\theta \text{ volt} \qquad ...(i)$$

Now, e has maximum value of E_m (say) when $\theta = 90°$. Hence, from Eq. (i) above, we get,

$E_m = 2\,B\,N\,l\,v$ volt. Therefore Eq. **(i)** can be rewritten as $e = E_m \sin\theta$ ...as before

If b = width of the coil in meters ; f = frequency of rotation of coil in Hz, then $v = \pi\,bf$

$$\therefore \quad E_m = 2\,B\,N\,l \times \pi\,b\,f = 2\,\pi\,f\,N\,B\,A \text{ volts} \qquad ...\text{as before}$$

Example 11.1. *A square coil of 10 cm side and 100 turns is rotated at a uniform speed of 1000 revolutions per minute, about an axis at right angles to a uniform magnetic field of 0.5 Wb/m^2. Calculate the instantaneous value of the induced electromotive force, when the plane of the coil is* ***(i)*** *at right angles to the field* ***(ii)*** *in the plane of the field.*

(Electromagnetic Theory, A.M.I.E. Sec B, 1992)



square coils

Solution. Let the magnetic field lie in the vertical plane and the coil in the horizontal plane. Also, let the angle θ be measured from X-axis.

Maximum value of the induced e.m.f., $E_m = 2\,\pi\,f\,N\,B_m\,A$ volt.

Instantaneous value of the induced e.m.f. $e = E_m \sin\theta$

Now $f = 100/60 = (50/3)$ rps, $N = 100$, $B_m = 0.5$ Wb/m^2, $A = 10^{-2}$ m^2

(i) In this case, $\theta = 0°$

$\therefore \quad e = 0$

(ii) Here $\theta = 90°$, $\quad \therefore \; e = E_m \sin 90° = E_m$

Substituting the given values, we get

$$e = 2\pi \times (50/3) \times 100 \times 0.5 \times 10^{-2} = \mathbf{52.3\ V}$$

11.4. Simple Waveforms

The shape of the curve obtained by plotting the instantaneous values of voltage or current as the ordinate against time as a abscissa is called its waveform or wave-shape.

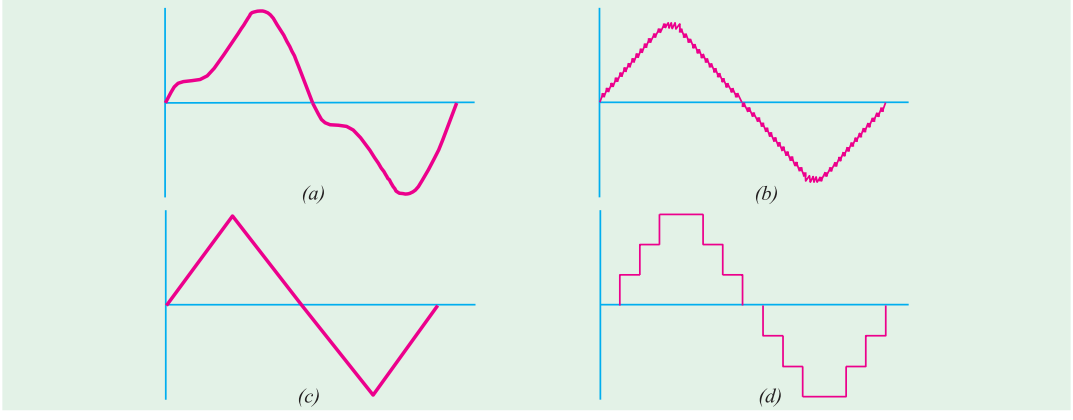


Fig. 11.6

An alternating voltage or current may not always take the form of a systematical or smooth wave such as that shown in Fig. 11.3. Thus, Fig. 11.6 also represents alternating waves. But while it is scarcely possible for the manufacturers to produce sine-wave generators or alternators, yet sine wave is the ideal form sought by the designers and is the accepted standard. The waves deviating from the standard sine wave are termed as distorted waves.

In general, however, ***an alternating current or voltage is one the circuit direction of which reverses at regularly recurring intervals.***

11.5. Complex Waveforms

Complex waves are those which depart from the ideal sinusoidal form of Fig. 11.4. All alternating complex waves, which are periodic and have equal positive and negative half cycles can be shown to be made up of a number of pure sine waves, having different frequencies but all these frequencies are integral multiples of that of the lowest alternating wave, called the ***fundamental*** (or first harmonic). These waves of higher frequencies are called ***harmonics.*** If the fundamental frequency is 50 Hz, then the frequency of the second ***harmonic*** is 100 Hz and of the third is 150 Hz and so on. The complex wave may be composed of the fundamental wave (or first harmonic) and any number of other harmonics.

In Fig. 11.7 is shown a complex wave which is made up of a fundamental sine wave of frequency of 50 Hz and third harmonic of frequency of 150 Hz. It is seen that

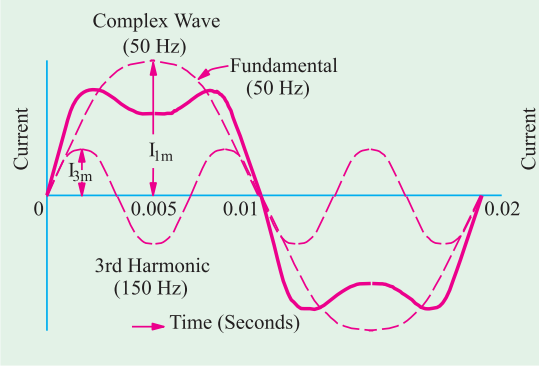


Fig. 11.7

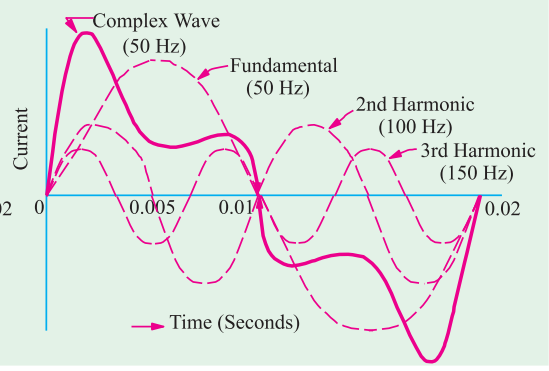


Fig. 11.8

(i) the two halves of the complex wave are identical in shape. In other words, there is no distortion. This is always the case when only ***odd*** harmonic (3rd, 5th, 7th, 9th etc.) are present.

(ii) frequency of the complex wave is 50 Hz *i.e.* the same as that of the fundamental sine wave.

In Fig. 11.8 is shown a complex wave which is a combination of fundamental sine wave of frequency 50 Hz and 2nd harmonic of frequency 100 Hz and 3rd harmonic of frequency 150 Hz. It is seen that although the frequency of the complex wave even now remains 50 Hz, yet :

(i) the two halves of the complex wave are not identical. It is always so when *even* harmonics (2nd, 4th, 6th etc.) are present.

(ii) there is distortion and greater departure of the wave shape from the purely sinusoidal shape.

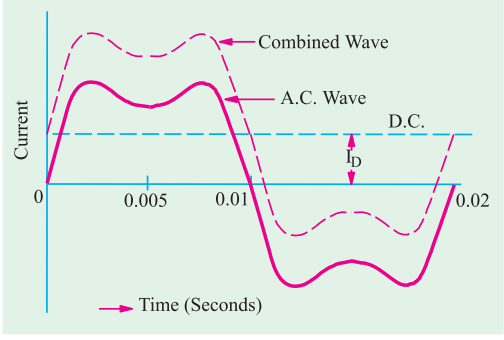


Fig. 11.9

Sometimes, a combination of an alternating and direct current flows simultaneously through a circuit In Fig. 11.9 is shown a complex wave (containing fundamental and third harmonic) combined with a direct current of value I_D. It is seen that the resultant wave remains undistorted in shape but is raised above the axis by an amount I_D. It is worth noting that with reference to the original axis, the two halves of the combined wave are not equal in area.

11.6. Cycle

One complete set of positive and negative values of alternating quantity is known as cycle. Hence, each diagram of Fig. 11.6 represents one complete cycle.

A cycle may also be sometimes specified in terms of angular measure. In that case, one complete cycle is said to spread over 360° or 2π radians.

11.7. Time Period

The time taken by an alternating quantity to complete one cycle is called its time period T. For example, a 50-Hz alternating current has a time period of 1/50 second.

11.8. Frequency

The number of cycles/second is called the frequency of the alternating quantity. Its unit is hertz (Hz).

In the simple 2-pole alternator of Fig. 24.1 (*b*), one cycle of alternating current is generated in one revolution of the rotating field. However, if there were 4 poles, then two cycles would have been produced in each revolution. In fact, the frequency of the alternating voltage produced is a function of the speed and the number of poles of the generator. The relation connecting the above three quantities is given as

$f = PN/120$ where N = revolutions in r.p.m. and P = number of poles

For example, an alternator having 20 poles and running at 300 r.p.m. will generate alternating voltage and current whose frequency is $20 \times 300/120 = 50$ hertz (Hz).

It may be noted that the frequency is given by the reciprocal of the time period of the alternating quantity.

$$\therefore \qquad f = 1/T \quad \text{or} \quad T = 1/f$$

11.9. Amplitude

The maximum value, positive or negative, of an alternating quantity is known as its amplitude.

11.10. Different Forms of E.M.F. Equation

The standard form of an alternating voltage, as already given in Art. 11.2, is

$$e = E_m \sin \theta = E_m \sin \omega t = E_m \sin 2\pi f t = E_m \sin \frac{2\pi}{T} t$$

By closely looking at the above equations, we find that

(i) the maximum value or peak value or amplitude of an ***alternating voltage is given by the coefficient of the sine of the time angle.***

(ii) the frequency f is given by the coefficient of time divided by 2π..

For example, if the equation of an alternating voltage is given by $e = 50 \sin 314t$ then its maximum value of 50 V and its frequency is $f = 314/2\pi = 50$ Hz.

Similarly, if the equation is of the form $e = I_m \sqrt{(R^2 + 4\omega^2 L^2)} \sin 2\omega t$, then its maximum value is $E_m = I_m \sqrt{(R^2 + 4\omega^2 L^2)}$ and the frequency is $2\omega/2\pi$ or ω/π Hz.

Example 11.2. *The maximum values of the alternating voltage and current are 400 V and 20 A respectively in a circuit connected to 50 Hz supply and these quantities are sinusoidal. The instantaneous values of the voltage and current are 283 V and 10 A respectively at t = 0 both increasing positively.*

(i) *Write down the expression for voltage and current at time t.*

(ii) *Determine the power consumed in the circuit.* **(Elect. Engg. Pune Univ.)**

Solution. ***(i)*** In general, the expression for an a.c. voltage is $v = V_m \sin (\omega t + \phi)$ where ϕ is the phase difference with respect to the point where $t = 0$.

Now, v = 283 V ; V_m = 400 V. Substituting $t = 0$ in the above equation, we get

283 = 400 (sin ω × 0 + ϕ) ∴sin ϕ = 283/400 = 0.707 ; ∴ ϕ = 45° or π/4 radian.

Hence, general expression for voltage is

v = 400 (sin 2π × 50 × t + π/4)

= 400 sin (100 π t + π/4)

Similarly, at t = 0, 10 = 20 sin (ω × 0 + ϕ) ∴ sin ϕ = 0.5 ∴ ϕ = 30° or π/6 radian

Hence, the general expression for the current is

i = 20 (sin 100 π t + 30°) = 20 sin (100 π t + π/6)

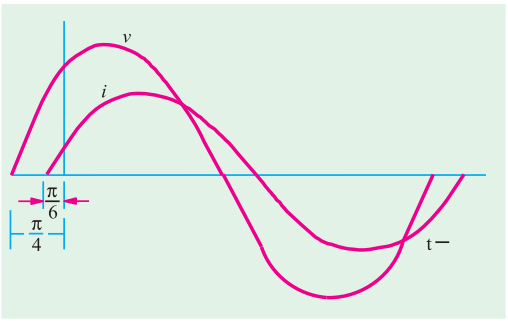


Fig. 11.10

(ii) $P = VI \cos \theta$ where V and I are rms values and θ is the phase difference between the voltage and current.

Now, $V = V_m/\sqrt{2} = 400/\sqrt{2}$; $I = 20/\sqrt{2}$; θ = 45° –30° = 15° (See Fig. 11.10)

∴ $P = (400/\sqrt{2}) \times (20/\sqrt{2}) \times \cos 15° =$ **3864 W**

Example 11.3. *An alternating current of frequency 60 Hz has a maximum value of 120 A. Write down the equation for its instantaneous value. Reckoning time from the instant the current is zero and is becoming positive, find* ***(a)*** *the instantaneous value after 1/360 second and* ***(b)*** *the time taken to reach 96 A for the first time.*

Solution. The instantaneous current equation is

$i = 120 \sin 2\pi ft = 120 \sin 120 \pi t$

Now when t = 1/360 second, then

(a) $i = 120 \sin (120 \times \pi \times 1/360)$ ...angle in radians

$= 120 \sin (120 \times 180 \times 1/360)$ ...angle in degree

$= 120 \sin 60° = 103.9$ A

(b) $96 = 120 \times \sin 2 \times 180 \times 60 \times t$ ...angle in degree

or $\sin (360 \times 60 \times t) = 96/120 = 0.8$ ∴ $360 \times 60 \times t = \sin^{-1} 0.8 = 53°$ (approx)

∴ $t = \theta/2\pi f = 53/360 \times 60 =$ **0.00245 second.**

11.11. Phase

By phase of an alternating current is meant the fraction of the time period of that alternating current which has elapsed since the current last passed through the zero position of reference. For example, the phase of current at point A is $T/4$ second, where T is time period or expressed in terms of angle, it is π/2 radians (Fig. 11.11). Similarly, the phase of the rotating coil at the instant shown in Fig. 11.1 is ωt which is, therefore, called its phase angle.

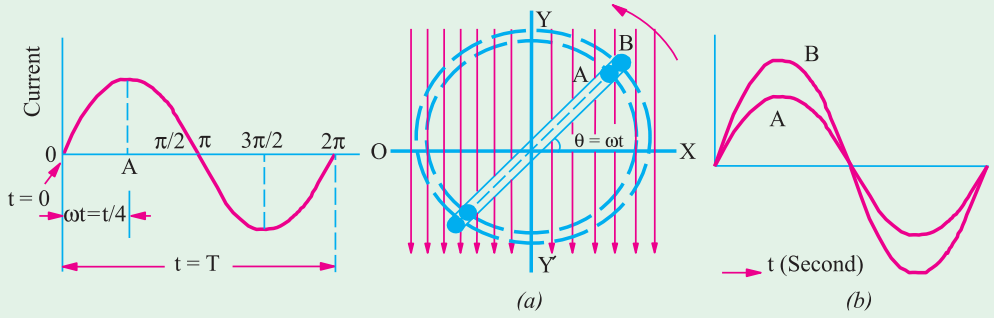


Fig. 11.11 Fig. 11.12

In electrical engineering, we are, however, more concerned with relative phases or phase differences between different alternating quantities, rather than with their absolute phases. Consider two single-turn coils of different sizes [Fig. 11.12 (*a*)] arranged radially in the same plane and rotating with the same angular velocity in a common magnetic field of uniform intensity. The e.m.fs. induced in both coils will be of the same frequency and of sinusoidal shape, although the values of instantaneous e.m.fs. induced would be different. However, the two alternating e.m.fs. would reach their maximum and zero values at the same time as shown in Fig. 11.12 (*b*). Such alternating voltages (or currents) are said to be in phase with each other. The two voltages will have the equations

$$e_1 = E_{m1} \sin \omega t \quad \text{and} \quad e_2 = E_{m2} \sin \omega t$$

11.12. Phase Difference

Now, consider three similar single-turn coils displaced from each other by angles α and β and rotating in a uniform magnetic field with the same angular velocity [Fig. 11.13 (*a*)].

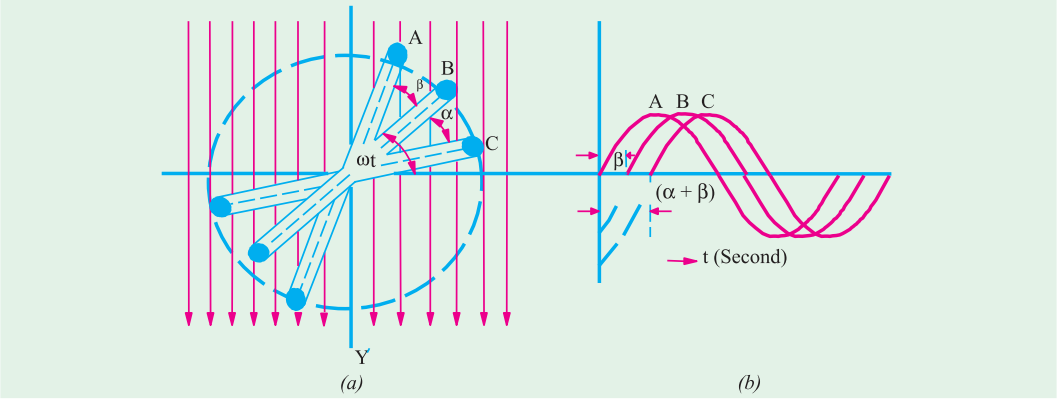


Fig. 11.13

In this case, the value of induced e.m.fs. in the three coils are the same, but there is one important difference. The e.m.fs. in these coils do not reach their maximum or zero values simultaneously but one after another. The three sinusoidal waves are shown in Fig. 11.13 (*b*). It is seen that curves *B* and *C* are displaced from curve *A* and angles β and $(\alpha + \beta)$ respectively. Hence, it means that phase difference between *A* and *B* is β and between *B* and *C* is α but between *A* and *C* is $(\alpha + \beta)$. The statement, however, does not give indication as to which e.m.f. reaches its maximum value first. This deficiency is supplied by using the terms ***'lag'*** or ***'lead'*.**

A leading alternating quantity is one which reaches its maximum (or zero) value earlier as compared to the other quantity.

Similarly, a lagging alternating quantity is one which reaches its maximum or zero value later than the other quantity. For example, in Fig. 11.13 (*b*), *B* lags behind *A* by β and *C* lags behind *A* by $(\alpha + \beta)$ because they reach their maximum values later.

The three equations for the instantaneous induced e.m.fs. are (Fig. 11.14)

$e_A = E_m \sin \omega t$ ...reference quantity

$e_B = E_m \sin (\omega t - \beta)$

$e_C = E_m \sin [\omega t - (\alpha + \beta)]$

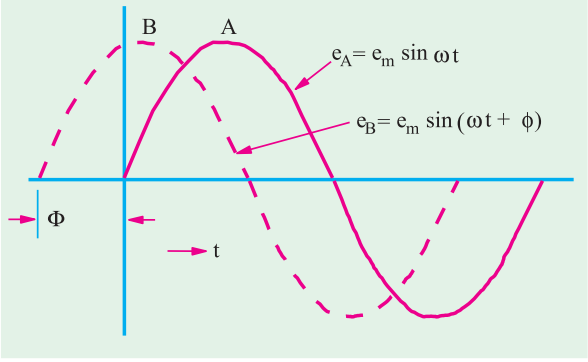


Fig. 11.14

In Fig. 11.14, quantity B leads A by an angle ϕ. Hence, their equations are

$e_A = E_m \sin \omega t$ ...reference quantity

$e_B = E_m \sin (\omega t - \phi)$

A plus (+) sign when used in connection with phase difference denotes 'lead' whereas a minus (–) sign denotes 'lag'.

11.13. Root-Mean-Square (R.M.S.) Value

The r.m.s. value of an alternating current ***is given by that steady (d.c.) current which when flowing through a given circuit for a given time produces the same*** **heat** ***as produced by the alternating current when flowing through the same circuit for the same time.***

It is also known as the ***effective*** or ***virtual*** value of the alternating current, the former term being used more extensively. For computing the r.m.s. value of symmetrical sinusoidal alternating currents, either mid-ordinate method or analytical method may be used, although for symmetrical but non-sinusoidal waves, the mid-ordinate method would be found more convenient.

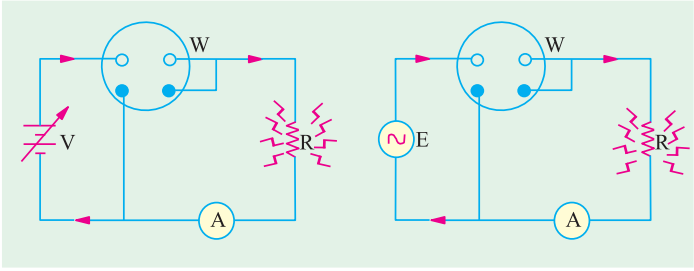


Fig. 11.15

A simple experimental arrangement for measuring the equivalent d.c. value of a sinusoidal current is shown in Fig. 11.15. The two circuits have identical resistances but one is connected to battery and the other to a sinusoidal generator. Wattmeters are used to measure heat power in each circuit. The voltage applied to each circuit is so adjusted that heat power production in each circuit is the same. In that case, the direct current will equal $I_m/\sqrt{2}$ which is called r.m.s. value of the sinusoidal current.

11.14. Mid-ordinate Method

In Fig. 11.16 are shown the positive half cycles for both symmetrical sinusoidal and non-sinusoidal alternating currents. Divide time base 't' into n equal intervals of time each of duration t/n seconds. Let the average values of instantaneous currents during these intervals be respectively i_1, i_2, i_3 .... i_n (*i.e.* mid-ordinates in Fig. 11.16). Suppose that this alternating current is passed through a circuit of resistance R ohms. Then,

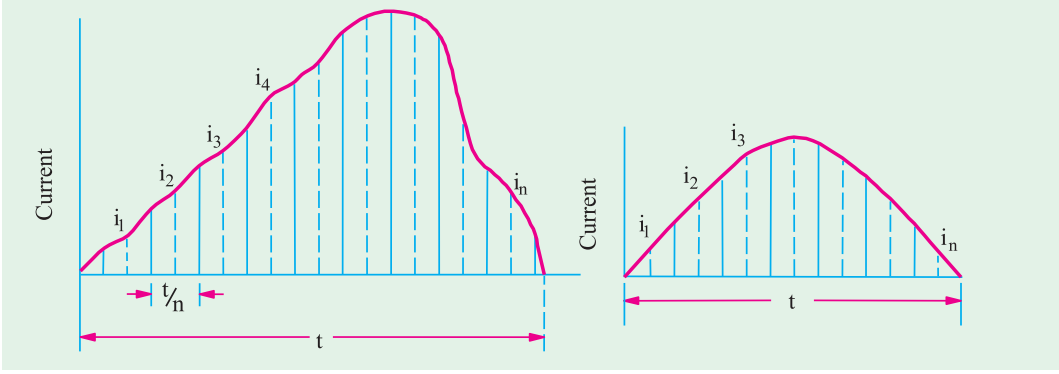


Fig. 11.16

Heat produced in 1st interval $= 0.24 \times 10^{-3}\, i_1^2\, Rt/n$ kcal $\quad (\because 1/J = 1/4200 = 0.24 \times 10^{-3})$

Heat produced in 2nd interval $= 0.24 \times 10^{-3}\, i_2^2\, Rt/n$ kcal

: : : : :

: : : : :

: : : : :

Heat produced in nth interval $= 0.24 \times 10^{-3}\, i_n^2\, Rt/n$ kcal

Total heat produced in t seconds is $= 0.24 \times 10^{-3}\, Rt \left(\dfrac{i_1^2 + i_2^2 + \ldots + i_n^2}{n} \right)$ kcal

Now, suppose that a direct current of value I produces the same heat through the same resistance during the same time t. Heat produced by it is $= 0.24 \times 10^{-3}\, I^2 Rt$ kcal. By definition, the two amounts of heat produced should be equal.

$$\therefore \quad 0.24 \times 10^{-3} I^2 Rt = 0.24 \times 10^{-3}\, Rt \left(\frac{i_1^2 + i_2^2 + \ldots + i_n^2}{n} \right)$$

$$\therefore \quad I^2 = \frac{i_1^2 + i_2^2 + \ldots + i_n^2}{n} \quad \therefore \; I = \sqrt{\left(\frac{i_1^2 + i_2^2 + \ldots + i_n^2}{n} \right)}$$

= square root of the mean of the squares of the instantaneous currents

Similarly, the r.m.s. value of alternating voltage is given by the expression

$$V = \sqrt{\left(\frac{v_1^2 + v_2^2 + \ldots + v_n^2}{n} \right)}$$

11.15. Analytical Method

The standard form of a sinusoidal alternating current is $i = I_m \sin \omega t = I_m \sin \theta$.

The mean of the squares of the instantaneous values of current over one complete cycle is (even the value over half a cycle will do).

$$= \int_0^{2\pi} \frac{i^2\, d\theta}{(2\pi - 0)}$$

The square root of this value is $= \sqrt{\left(\int_0^{2\pi} \frac{i^2\, d\theta}{2\pi} \right)}$

Hence, the r.m.s. value of the alternating current is

$$I = \sqrt{\left(\int_0^{2\pi} \frac{i^2 d\theta}{2\pi} \right)} = \sqrt{\left(\frac{I_m^2}{2\pi} \int_0^{2\pi} \sin^2 \theta\, d\theta \right)} \qquad (\text{put } i = I_m \sin \theta)$$

Now, $\cos 2\theta = 1 - 2 \sin^2 \theta \quad \therefore \; \sin^2 \theta = \dfrac{1 - \cos 2\theta}{2}$

$$\therefore \quad I = \sqrt{\left(\frac{I_m^2}{4\pi} \int_0^{2\pi} (1 - \cos 2\theta)\, d\theta \right)} = \sqrt{\left(\frac{I_m^2}{4\pi} \left| \theta - \frac{\sin 2\theta}{2} \right|_0^{2\pi} \right)}$$

$$= \sqrt{\frac{I_m^2}{4}\; 2} \qquad \sqrt{\frac{I_m^2}{2}} \quad \therefore \; \mathrm{I} = \frac{I_m}{\sqrt{2}} = 0.707\, I_m$$

Hence, we find that for a symmetrical sinusoidal current

r.m.s. value of current = 0.707 × max. value of current

The r.m.s. value of an alternating current is of considerable importance in practice, because the ammeters and voltmeters record the r.m.s. value of alternating current and voltage respectively. In

electrical engineering work, ***unless indicated otherwise, the values of the given current and voltage are always the r.m.s. values.***

It should be noted that the average heating effect produced during one cycle is

$$= I^2R = (I_m/\sqrt{2})^2 R = \frac{1}{2} I_m^2 R$$

11.16. R.M.S. Value of a Complex Wave

In their case also, either the mid-ordinate method (when equation of the wave is not known) or analytical method (when equation of the wave is known) may be used. Suppose a current having the equation $i = 12 \sin \omega t + 6 \sin (3\omega t - \pi/6) + 4 \sin (5\omega t + \pi/3)$ flows through a resistor of R ohm. Then, in the time period T second of the wave, the effect due to each component is as follows :

Fundamental ..... $(12/\sqrt{2})^2 \; RT$ watt

3rd harmonic ..... $(6/\sqrt{2})^2 \; RT$ watt

5th harmonic ..... $(4/\sqrt{2})^2 \; RT$ watt

$\therefore$ Total heating effect $= RT\,[(12/\sqrt{2})^2 + (6/\sqrt{2})^2 + (4/\sqrt{2})^2]$

If I is the r.m.s. value of the complex wave, then equivalent heating effect is I^2RT

$$\therefore \quad I^2RT = RT\,[(12/\sqrt{2})^2 + (6/\sqrt{2})^2 + (4/\sqrt{2})^2]$$

$$\therefore \quad I = \sqrt{[(12/\sqrt{2})^2 + (6/\sqrt{2})^2 + (4/\sqrt{2})^2]} = 9.74 \text{ A}$$

Had there been a direct current of (say) 5 amperes flowing in the circuit also*, then the r.m.s. value would have been

$$= \sqrt{(12/\sqrt{2})^2 \quad (6/\sqrt{2})^2 \quad (4/\sqrt{2})^2 \quad 5^2} = 10.93 \text{ A}$$

Hence, for complex waves the rule is as follows : ***The r.m.s. value of a complex current wave is equal to the square root of the sum of the squares of the r.m.s. values of its individual components.***

11.17. Average Value

The average value I_a of an alternating current is expressed ***by that steady current which transfers across any circuit the same charge as is transferred by that alternating current during the same time.***

In the case of a symmetrical alternating current (*i.e.* one whose two half-cycles are exactly similar, whether sinusoidal or non-sinusoidal), the average value over a complete cycle is zero. Hence, in their case, the average value is obtained by adding or integrating the instantaneous values of current over one half-cycle only. ***But in the case of an unsymmetrical alternating current (like half-wave rectified current) the average value must always be taken over the whole cycle.***

(*i*) Mid-ordinate Method

With reference to Fig. 11.16, $I_{av} = \dfrac{i_1 + i_2 + \ldots + i_n}{n}$

This method may be used both for sinusoidal and non-sinusoidal waves, although it is specially convenient for the latter.

(*ii*) Analytical Method

The standard equation of an alternating current is, $i = I_m \sin \theta$

$$I_{av} = \int_0^{\pi} \frac{i\,d\theta}{(\pi - 0)} = \frac{I_m}{\pi} \int_0^{\pi} \sin \theta \, d\theta \qquad \text{(putting value of } i\text{)}$$

---

* The equation of the complex wave, in that case, would be,
$i = 5 + 12 \sin \omega t + 6 \sin (3\omega t - \pi/6) + 4 \sin (5\omega t + \pi/3)$

$$= \frac{I_m}{\pi}\left|-\cos\theta\right|_0^{\pi} = \frac{I_m}{\pi}\left|+1-(-1)\right| = \frac{2I_m}{\pi} = \frac{I_m}{\pi/2} = \frac{\text{twice the maximum current}}{\pi}$$

$\therefore$ $I_{av} = I_m/\frac{1}{2}\pi = 0.637\, I_m$ $\therefore$ **average value of current = 0.637 × maximum value**

Note. R.M.S. value is always greater than average value except in the case of a rectangular wave when both are equal.

11.18. Form Factor

It is defined as the ratio, $K_f = \dfrac{\text{r.m.s. value}}{\text{average value}} = \dfrac{0.707\, I_m}{0.637\, I_m} = 1.1$. (for sinusoidal alternating currents only)

In the case of sinusoidal alternating voltage also, $K_f = \dfrac{0.707\, E_m}{0.637\, E_m} = 1.11$

As is clear, the knowledge of form factor will enable the r.m.s. value to be found from the arithmetic mean value and ***vice-versa.***

11.19. Crest or Peak or Amplitude Factor

It is defined as the ratio $K_a = \dfrac{\text{maximum value}}{\text{r.m.s. value}} = \dfrac{I_m}{I_m/\sqrt{2}} = \sqrt{2} = 1.414$ (for sinusoidal a.c. only)

For sinusoidal alternating voltage also, $K_a = \dfrac{E_m}{E_m/\sqrt{2}} = 1.414$

Knowledge of this factor is of importance in dielectric insulation testing, because the dielectric stress to which the insulation is subjected, is proportional to the maximum or peak value of the applied voltage. The knowledge is also necessary when measuring iron losses, because the iron loss depends on the value of maximum flux.

Example 11.4. *An alternating current varying sinusoidally with a frequency of 50 Hz has an RMS value of 20 A. Write down the equation for the instantaneous value and find this value* ***(a)*** *0.0025 second* ***(b)*** *0.0125 second after passing through a positive maximum value. At what time, measured from a positive maximum value, will the instantaneous current be 14.14 A ?*

(Elect. Science-I Allahabad Univ. 1992)

Solution. $I_m = 20\sqrt{2} = 28.2$ A, $\omega = 2\pi \times 50 = 100\,\pi$ rad/s.

The equation of the sinusoidal current wave with reference to point O (Fig. 11.17) as zero time point is $i = 28.2 \sin 100\,\pi t$ **ampere**

Since time values are given from point *A* where voltage has positive and maximum value, the equation may itself be referred to point *A*. In the case, the equation becomes :

$$i = 28.2 \cos 100\,\pi t$$

(*i*) When *t* = 0.0025 second

$i = 28.2 \cos 100\pi \times 0.0025$ ...angle in radian

$= 28.2 \cos 100 \times 180 \times 0.0025$ ...angle in degrees

$= 28.2 \cos 45° = 20$ A ...point *B*

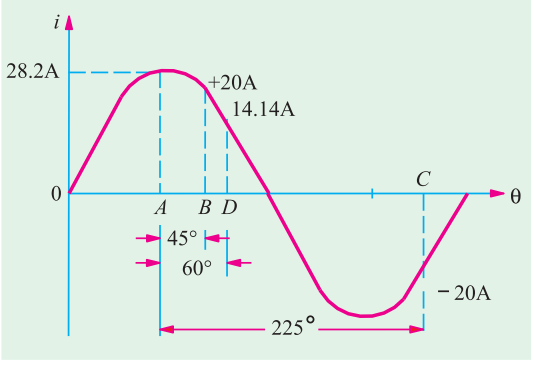


Fig. 11.17

(*ii*) When t = 0.0125 second

$i = 28.2 \cos 100 \times 180 \times 0.0125$

$= 28.2 \cos 225^\circ = 28.2 \times (-1/\sqrt{2})$

$= -20$ A ...point *C*

(*iii*) Here i = 14.14 A

$\therefore \quad 14.14 = 28.2 \cos 100 \times 180\, t \quad \therefore \quad \cos 100 \times 180\, t = \frac{1}{2}$

or $100 \times 180\, t = \cos^{-1}(0.5) = 60^\circ$, t = **1/300 second** ...point *D*

Example 11.5. *An alternating current of frequency 50 Hz has a maximum value of 100 A. Calculate* ***(a)*** *its value 1/600 second after the instant the current is zero and its value decreasing thereafter* ***(b)*** *how many seconds after the instant the current is zero (increasing thereafter wards) will the current attain the value of 86.6 A ?* **(Elect. Technology. Allahabad Univ. 1991)**

Solution. The equation of the alternating current (assumed sinusoidal) with respect to the origin *O* (Fig. 11.18) is

$$i = 100 \sin 2\pi \times 50t = 100 \sin 100\, \pi t$$

(a) It should be noted that, in this case, time is being measured from point *A* (where current is zero and decreasing thereafter) and not from point *O*.

If the above equation is to be utilized, then, this time must be referred to point *O*. For this purpose, half time-period *i.e.* 1/100 second has to be added to 1/600 second. The given time as referred to point *O* becomes

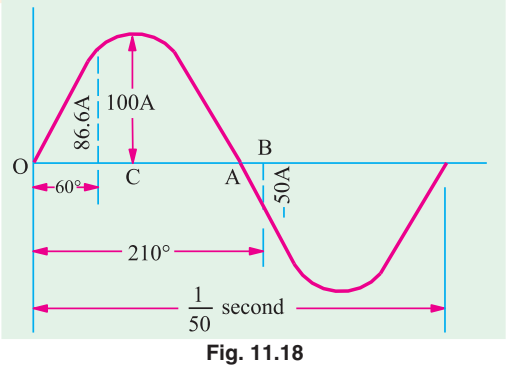


Fig. 11.18

$$= \frac{1}{100} + \frac{1}{600} = \frac{7}{600} \text{ second}$$

$\therefore \quad i = 100 \sin 100 \times 180 \times 7/600 = 100 \sin 210^\circ$

$= 100 \times -1/2 =$ **–50 A** ...point *B*

(b) In this case, the reference point is *O*.

$\therefore \quad 86.6 = 100 \sin 100 \times 180\, t$ or $\sin 18{,}000\, t = 0.866$

or $18{,}000\, t = \sin^{-1}(0.866) = 60^\circ \quad \therefore \quad t = 60/18{,}000 =$ **1/300 second**

Example 11.6. *Calculate the r.m.s. value, the form factor and peak factor of a periodic voltage having the following values for equal time intervals changing suddenly from one value to the next : 0, 5, 10, 20, 50, 60, 50, 20, 10, 5, 0, –5, –10 V etc. What would be the r.m.s value of sine wave having the same peak value ?*

Solution. The waveform of the alternating voltage is shown in Fig. 11.19. Obviously, it is not sinusoidal but it is symmetrical. Hence, though r.m.s value may be full one cycle, the average value has necessarily to be considered for half-cycle only, otherwise the symmetrical negative and positive half-cycles will cancel each other out.

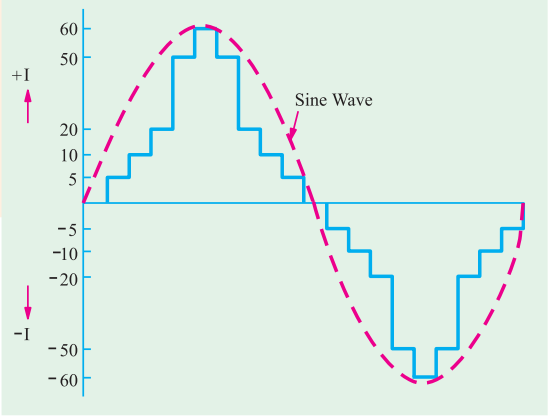


Fig. 11.19

Mean value of $v^2 = \dfrac{0^2 + 5^2 + 10^2 + 20^2 + 50^2 + 60^2 + 50^2 + 20^2 + 10^2 + 5^2}{10} = \mathbf{965\ V}$

$\therefore$ r.m.s. value = $\sqrt{965}$ = **31 V (approx.)**

Average value (half-cycle) = $\dfrac{0 + 5 + 10 + 20 + 50 + 60 + 50 + 20 + 10 + 5}{10} = \mathbf{23\ V}$

Form factor = $\dfrac{\text{r.m.s. value}}{\text{average value}} = \dfrac{31}{23} = \mathbf{1.35.}$ Peak factor = 60/31 = **2 (approx.)**

R.M.S. value of a sine wave of the same peak value = 0.707 × 60 = **42.2 V.**

Alternative Solution

If 't' be the regular time interval, then area of the half-cycle is

= (5t + 10t + 20t + 50t) 2 + 60t = 230t, Base = 10t $\therefore$ Mean value = 230t/10t = 23 V.

Area when ordinates are squared = (25t + 100t + 400t + 2500t) 2 + 3600t = 9650t, Base = 10t

$\therefore$ Mean height of the squared curve = 9650t/10 t = 965

$\therefore$ r.m.s. value = $\sqrt{965}$ = 31 V

Further solution is as before.

Example 11.7. *Calculate the reading which will be given by a hot-wire voltmeter if it is connected across the terminals of a generator whose voltage waveform is represented by*

$$v = 200 \sin \omega t + 100 \sin 3\omega t + 50 \sin 5\omega t$$

Solution. Since hot-wire voltmeter reads only r.m.s value, we will have to find the r.m.s. value of the given voltage. Considering one complete cycle,

R.M.S. value $V = \sqrt{\dfrac{1}{2\pi}\int_0^{2\pi} v^2 d\theta}$ where $\theta = \omega t$

or $V^2 = \dfrac{2}{2\pi}\int_0^{2\pi} (200 \sin \theta + 100 \sin 3\theta + 50 \sin 5\theta)^2\, d\theta$

$= \dfrac{1}{2\pi}\int_0^{2\pi} (200^2 \sin^2\theta + 100^2 \sin^2 3\theta + 50^2 \sin^2 5\theta$

$+ 2 \times 200.100 \sin\theta . \sin 3\theta + 2 \times 100.50 . \sin 3\theta . \sin 5\theta$

$+ 2 \times 50.200 . \sin 5\theta . \sin\theta)\, d\theta$

$= \dfrac{1}{2\pi}\left(\dfrac{200^2}{2} + \dfrac{100^2}{2} + \dfrac{50^2}{2}\right) 2\pi = 26{,}250$

$\therefore$ $V = \sqrt{26{,}250} = \mathbf{162\ V}$

Alternative Solution

The r.m.s. value of individual components are ($200/\sqrt{2}$), ($100/\sqrt{2}$) and ($50/\sqrt{2}$). Hence, as stated in Art. 11.16,

$$V = \sqrt{V_1^2 + V_2^2 + V_3^2} = \sqrt{(200/\sqrt{2})^2 + (100/\sqrt{2})^2 + (50/\sqrt{2})^2} = \mathbf{162\ V}$$

11.20. R.M.S. Value of H.W. Rectified Alternating Current

Half-wave (H.W.) rectified alternating current is one whose one half-cycle has been suppressed *i.e.* one which flows for half the time during one cycle. It is shown in Fig. 11.20 where suppressed half-cycle is shown dotted.

As said earlier, for finding r.m.s. value of such an alternating current, summation would be carried over the period for which current ***actually*** flows *i.e.* from 0 to π, though it would be averaged for the whole cycle *i.e.* from 0 to 2π.

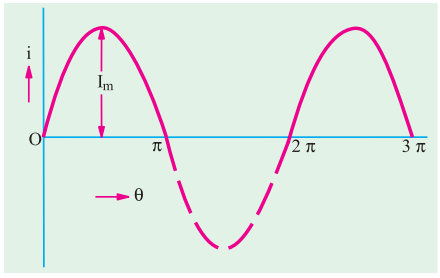


Fig. 11.20

∴ R.M.S. current

$$I = \sqrt{\left(\int_0^{\pi} \frac{i^2 d\theta}{2\pi}\right)} = \sqrt{\left(\frac{I_m^2}{2\pi}\int_0^{\pi} \sin^2\theta\, d\theta\right)}$$

$$= \sqrt{\frac{I_m^2}{4\pi}\int_0^{\pi}(1-\cos 2\theta)\,d\theta}$$

$$= \sqrt{\left(\frac{I_m^2}{4\pi}\left|\theta - \frac{\sin 2\theta}{2}\right|_0^{\pi}\right)} = \sqrt{\left(\frac{I_m^2}{4\pi}\times\pi\right)} = \sqrt{\left(\frac{I_m^2}{4}\right)} \quad \therefore\ I = \frac{I_m}{2} = \mathbf{0.5I_m}$$

11.21. Average Value of H.W. Rectified Alternating Current

For the same reasons as given in Art. 11.20, integration would be carried over from 0 –π

$$\therefore \quad I_{av} = \int_0^{\pi} \frac{i d\theta}{2\pi} = \frac{I_m}{2\pi}\int_0^{\pi} \sin\theta\, d\theta \qquad (\because\ i = I_m \sin\theta)$$

$$= \frac{I_m}{2\pi}\left|-\cos\theta\right|_0^{\pi} = \frac{I_m}{2\pi}\times 2 = \frac{I_m}{\pi}$$

11.22. Form Factor of H.W. Rectified Alternating Current

$$\text{Form factor} = \frac{\text{r.m.s. value}}{\text{average value}} = \frac{I_m/2}{I_m/\pi} = \frac{\pi}{2} = 1.57$$

Example 11.8. *An alternating voltage e = 200 sin 314t is applied to a device which offers an ohmic resistance of 20 Ω to the flow of current in one direction, while preventing the flow of current in opposite direction. Calculate RMS value, average value and form factor for the current over one cycle.* **(Elect. Engg. Nagpur Univ. 1992)**

Solution. Comparing the given voltage equation with the standard form of alternating voltage equation, we find that V_m = 200 V, R = 20 Ω, I_m = 200/20 = 10 A. For such a half-wave rectified current, RMS value = $I_m/2$ = 10/2 = **5A.**

Average current = I_m/π = 10/π = **3.18 A ;** Form factor = 5/3.18 = **1.57**

Example 11.9. *Compute the average and effective values of the square voltage wave shown in Fig. 11.21.*

Solution. As seen, for $0 < t < 0.1$ *i.e.* for the time interval 0 to 0.1 second, v = 20 V. Similarly, for $0.1 < t < 0.3$, v = 0. Also time-period of the voltage wave is 0.3 second.

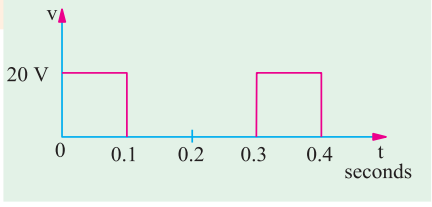


Fig. 11.21

$$\therefore \quad V_{av} = \frac{1}{T}\int_0^T v\ dt = \frac{1}{0.3}\int_0^{0.1} 20\ dt$$

$$= \frac{1}{0.3}(20\times 0.1) = \mathbf{6.67\ V}$$

$$V^2 = \frac{1}{T}\int_0^T v^2 dt = \frac{1}{0.3}\int_0^{0.1} 20^2\ dt = \frac{1}{0.3}(400\times 0.1) = 133.3;\ V = \mathbf{11.5\ V}$$

Example 11.10. *Calculate the RMS value of the function shown in Fig. 11.22 if it is given that for $0 < t < 0.1$, $y = 10(1 - e^{-100t})$ and $0.1 < t < 0.2$, $y = 10\, e^{-50(t-0.1)}$*

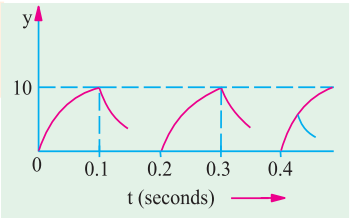


Fig. 11.22

Solution.

$$Y^2 = \frac{1}{0.2}\left\{\int_0^{0.1} y^2 dt + \int_{0.1}^{0.2} y^2 dt\right\}$$

$$= \frac{1}{0.2}\left\{\int_0^{0.1} 10^2 (1 - e^{-100t})^2 dt + \int_{0.1}^{0.2} (10e^{-50(t-0.1)})^2 dt\right\}$$

$$= \frac{1}{0.2}\left\{\int_0^{0.1} 100\,(1 + e^{-200t} - 2e^{-100t}) dt + \int_{0.1}^{0.2} 100\, e^{-100(t-0.1)} dt\right\}$$

$$= 500\left\{\left[t - 0.005e^{-200t} + 0.02\, e^{-100t}\right]_0^{0.1} + \left[-0.0\, e^{-100(t-0.1)}\right]_{0.1}^{0.2}\right\}$$

$$= 500\left\{\left[\left(0.1 - 0.005e^{-20} + 0.02e^{-10}\right) - (0 - 0.005 + 0.02)\right] + \left[\left(-0.01e^{-10}\right) - (-0.01)\right]\right\}$$

$$= 500 \times 0.095 = 47.5 \quad \therefore \quad Y = \sqrt{47.5} = \mathbf{6.9}$$

Example 11.11. *The half cycle of an alternating signal is as follows : It increases uniformly from zero at 0° to F_m at $\alpha°$, remains constant from $\alpha°$ $(180 - \alpha)°$, decreases uniformaly from F_m at $(180 - \alpha)°$ to zero at 180°. Calculate the average and effective values of the signal.*

(Elect. Science-I, Allahabad Univ. 1992)

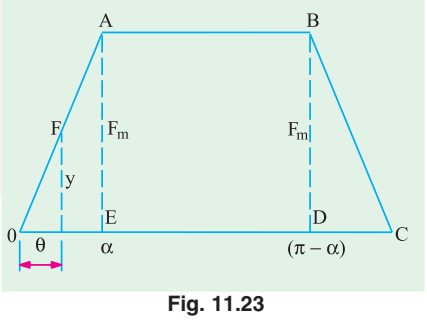


Fig. 11.23

Solution. For finding the average value, we would find the total area of the trapezium and divide it by π (Fig. 11.23).

Area $= 2 \times \Delta OAE + \text{rectangle } ABDE = 2 \times (1/2) \times F_m\alpha$
$+ (\pi - 2\alpha)\, F_m = (\pi - \alpha)\, F_m$

$\therefore$ average value $= (\pi - \alpha)\, F_m/\pi$

RMS Value From similar triangles, we get $\frac{y}{\theta} = \frac{F_m}{\alpha}$ or $y^2 \quad \frac{Fm^2}{2}$ 2

This gives the equation of the signal over the two triangles *OAE* and *DBC*. The signal remains constant over the angle α to $(\pi - \alpha)$ *i.e.* over an angular distance of $(\pi - \alpha) - \alpha = (\pi - 2\alpha)$

Sum of the squares $= \frac{2F_m^2}{\alpha^2}\int_\theta^\alpha \theta^2\, d\theta + F_m^2(\pi - 2\alpha) = F_m^2(\pi - 4\alpha/3)$.

The mean value of the squares is $= \frac{1}{\pi}F_m^2\left(\pi - \frac{4\alpha}{3}\right) = F_m^2\left(1 - \frac{4\alpha}{3\pi}\right)$

$$\text{r.m.s. value} = F_m\sqrt{\left(1 - \frac{4\,\alpha}{3\pi}\right)}$$

Example 11.12. *Find the average and r.m.s values of the a.c. voltage whose waveform is given in Fig. 11.24 (a)*

Solution. It is seen [(Fig. 11.24 (a)] that the time period of the waveform is 5s. For finding the average value of the waveform, we will calculate the net area of the waveform over one period and then find its average value for one cycle.

$$A_1 = 20 \times 1 = 20\ V\text{–}s,\ A_2 = -5 \times 2 = -10\ \text{V –s}$$

Net area over the full cycle $= A_1 + A_2 = 20 - 10 = 10$ V –s.

Average value $= 10$ V-s/5s $= 2$ V.

Fig. 11.24 (*b*) shows a graph of v^2 (*t*). Since the negative voltage is also squared, it becomes positive.

Average value of the area $= 400\ V^2 \times 1\ s + 25\ V^2 \times 2\ s = 450\ V^2 - s$. The average value of the sum of the square $= 450\ V^2$/- s/5s $= 90V^2$ rms value $= \sqrt{90\ V^2} = 9.49$ V.

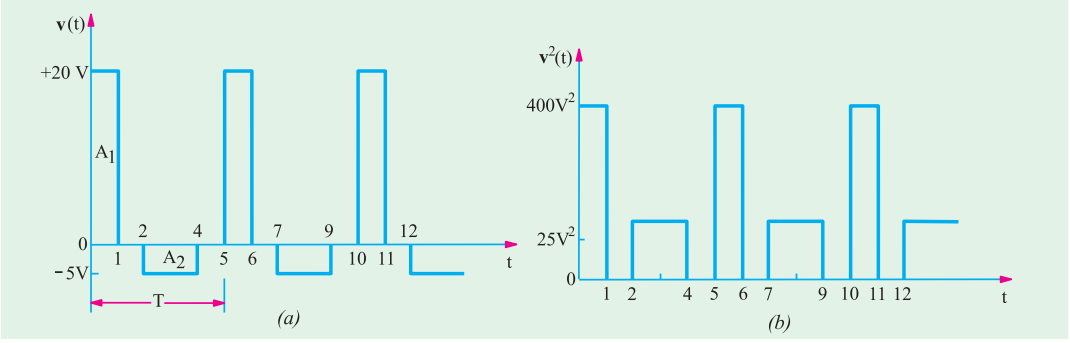


Fig. 11.24

Example 11.13. *What is the significance of the r.m.s and average values of a wave ? Determine the r.m.s. and average value of the waveform shown in Fig. 11.25*

(Elect. Technology, Indore Univ.)

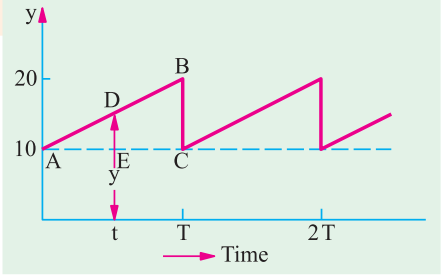


Fig. 11.25

Solution. The slope of the curve *AB* is *BC*/*AC* = 20/*T*. Next, consider the function *y* at any time *t*. It is seen that *DE*/*AE* = *BC*/*AC* = 10/*T*

or $(y - 10)/t = 10/T$

or $y = 10 + (10/T)t$

This gives us the equation for the function for one cycle.

$$Y_{av} = \frac{1}{T}\int_0^T y\,dt = \frac{1}{T}\int_0^T \left(10 + \frac{10}{T}t\right) dt$$

$$= \frac{1}{T}\int_0^T \left[10.dt + \frac{10}{T}.t.\,dt\right] = \frac{1}{T}\left|10t + \frac{5t^2}{T}\right|_0^T = \mathbf{15}$$

$$\text{Mean square value} = \frac{1}{T}\int_0^T y^2 dt = \int_0^T \left(10 + \frac{10}{T}t\right)^2 dt$$

$$= \frac{1}{T}\int_0^T \left(100 + \frac{100}{T^2}t^2 + \frac{200}{T}t\right)dt = \frac{1}{T}\left|100t + \frac{100t^3}{3T^2} + \frac{100t^2}{T}\right|_0^T = \frac{700}{3}$$

or RMS value $= 10\sqrt{7/3} = \mathbf{15.2}$

Example 11.14. *For the trapezoidal current wave-form of Fig. 11.26, determine the effective value.*

(Elect. Technology, Vikram Univ. Ujjain , Similar Example, Nagpur Univ. 1999)

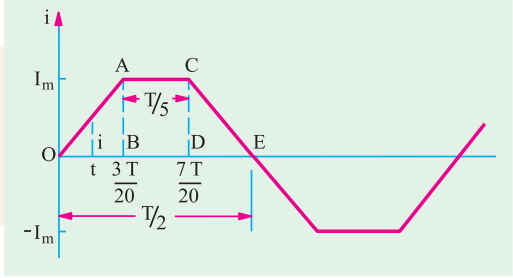


Fig. 11.26

Solution. For $0 < t < 3T/20$, equation of the current can be found from the relation

$$\frac{i}{t} = \frac{I_m}{3T/20} \quad \text{or } i = \frac{20I_m}{3T}.t$$

When $3T/20 < t < 7T/20$, equation of the current is given by $i = I_m$. Keeping in mind the fact that ΔOAB is identical with ΔCDE,

$$\text{RMS value of current} = \sqrt{\frac{1}{T/2}\left[2\int_0^{3T/20} i^2 dt + \int_{3T/20}^{7T/20} I_m^2 dt\right]}$$

$$= \sqrt{\frac{2}{T}\left[2\left(\frac{20I_m}{3T}\right)^2 \int_0^{3T/20} t^2 dt + I_m^2 \int_{3T/20}^{7T/20} dt\right]} = \frac{3}{5} I_m^2$$

$$\therefore \quad I = \sqrt{(3/5)}.I_m = \mathbf{0.775\ I_m}$$

Incidentally, the average value is given by

$$I_{ac} = \frac{2}{T}\left\{2\int_0^{3T/20} i dt + \int_{3T/20}^{7T/20} I_m dt\right\} = \frac{2}{T}\left\{2\int_0^{3T/20}\left(\frac{20I_m}{3T}\right)t dt + I_m \int_{3T/20}^{7T/20} dt\right\}$$

$$= \frac{2}{T}\left\{2\left(\frac{20I_m}{3T}\right)\left|\frac{t^2}{2}\right|_0^{3T/20} + I_m \left| t \right|_{3T/20}^{7T/20}\right\} = \frac{7}{10}.\mathbf{I_m}$$

Example 11.15. *A sinusoidal alternating voltage of 110 V is applied across a moving-coil ammeter, a hot-wire ammeter and a half-wave rectifier, all connected in series. The rectifier offers a resistance of 25 Ω in one direction and infinite resistance in opposite direction. Calculate* ***(i)*** *the readings on the ammeters* ***(ii)*** *the form factor and peak factor of the current wave.*

(Elect. Engg.-I Nagpur Univ. 1992)

Solution. For solving this question, it should be noted that

(a) Moving-coil ammeter, due to the inertia of its moving system, registers the average current for the *whole* cycle.

(b) The reading of hot-wire ammeter is proportional to the average heating effect over the ***whole*** cycle. It should further be noted that in a.c. circuits, the given voltage and current values, unless indicated otherwise, always refer to r.m.s values.

$$E_m = 110/0.707 = 155.5 \text{ V (approx.)} ; I_m/2 = 155.5/25 = 6.22 \text{ A}$$

Average value of current for positive half cycle = $0.637 \times 6.22 = 3.96$ A

Value of current in the negative half cycle is zero. But, as said earlier, due to inertia of the coil, M.C. ammeter reads the average value for the ***whole*** cycle.

(i) M.C. ammeter reading = $3.96/2 = \mathbf{1.98\ A}$

Let R be the resistance of hot-wire ammeter. Average heating effect over the positive half cycle is $\frac{1}{2} I_m^2 . R$ watts. But as there is no generation of heat in the negative half cycle, the average heating effect over the whole cycle is $\frac{1}{4} I_m^2 R$ watt.

Let I be the d.c. current which produces the same heating effect, then

$$I^2 R = \frac{1}{4} I_m^2 R \qquad \therefore \quad I = I_m/2 = 6.22/2 = 3.11\ A.$$

Hence, hot-wire ammeter will read **3.11 A**

(ii) Form factor $= \dfrac{\text{r.m.s value}}{\text{average value}} = \dfrac{3.11}{1.98} = \mathbf{1.57}$; Peak factor $= \dfrac{\text{max. value}}{\text{r.m.s. value}} = \dfrac{6.22}{3.11} = \mathbf{2}$

Example 11.16. *Find the form-factor of the wave form given in fig.*

[Nagpur University November 1991, Similar example, Sambalpur University]

Solution.

$$\text{Form-factor} = \frac{\text{RMS value}}{\text{Average value}}$$

Average value of the current

$$= 1/4 \times \int_0^4 (50/4) \times t \times dt = 25 \text{ amp}$$

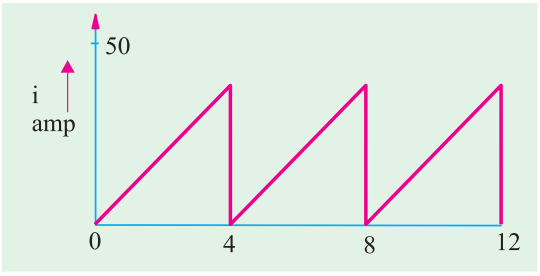


Fig. 11.27

Let RMS value of the current be I amp

$$I^2 \times 4 = \int_0^4 (12.5 \times t)^2 . dt$$

$$= \left[\frac{12.5 \times 12.5 \times t^2}{3}\right]_0^4 = (1/3) \times (12.5 \times 12.5 \times 4 \times 4 \times 4)$$

Thus $I = \frac{50}{\sqrt{3}} = 28.87$ amp, Hence, form factor $\frac{28.87}{25} = 1.1548$

Example 11.17. *A half-wave rectifier which prevents current flowing in one direction is connected in series with an a.c. ammeter and a permanent-magnet moving-coil ammeter. The supply is sinusoidal. The reading on the a.c. ammeter is 10 A. Find the reading given by the other ammeter. What should be the readings on the ammeters, if the other half-wave were rectified instead of being cut off ?*

Solution. It should be noted that an a.c. ammeter reads r.m.s. value whereas the d.c. ammeter reads the average value of the rectified current.

As shown in Art. 11.20 from H.W. rectified alternating current, $I = I_m/2$ and $I_{av} = I_m/\pi$

As a.c. ammeter reads 10 A, hence r.m.s. value of the current is 10 A.

$\therefore \quad 10 = I_m/2 \quad$ or $\quad I_m = 20$ A

$\therefore \quad I_{av} = 20/\pi =$ **6.365 A** –reading of d.c. ammeter.

The full-wave rectified current wave is shown in Fig. 11.28. In this case mean value of i^2 over a complete cycle is given as

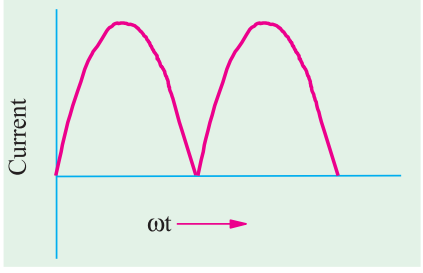


Fig. 11.28

$$= 2\int_0^{\pi} \frac{i^2 \, d\theta}{2\pi - 0} = \frac{1}{\pi}\int_0^{\pi} I_m^2 \; 2\sin^2\theta \, d\theta$$

$$= \frac{I_m^2}{2\pi}\int_0^{\pi} (1 - \cos 2\theta) d\theta = \frac{I_m^2}{2\pi}\left| \theta - \frac{\sin 2\theta}{2} \right|_0^{\pi} = \frac{I_m^2}{2}$$

$\therefore \quad I = I_m/\sqrt{2} = 20/\sqrt{2} = 14.14$ A $\quad \therefore$ a.c. ammeter will read **14.14 A**

Now, mean value of i over a complete cycle

$$= \frac{2}{2} \int_0 I_m \sin \, d \quad = \quad \frac{I_m}{} \int_0 \sin \, d \quad \frac{I_m}{} \left| \cos \right|_0 \quad \frac{2I_m}{} \quad \frac{2}{} \quad \frac{20}{} \quad \textbf{12.73 A}$$

This value, as might have been expected, is twice the value obtained in the previous case.

$\therefore$ d.c. ammeter will read **12.73 A.**

Example 11.18. *A full-wave rectified sinusoidal voltage is clipped at $1/\sqrt{2}$ of its maximum value. Calculate the average and RMS values of such a voltage.*

Solution. As seen from Fig. 11.29, the rectified voltage has a period of π and is represented by the following equations during the different intervals.

$$0 < \theta < \pi/4 \; ; v = V_m \sin\theta$$

$$\pi/4 < \theta < 3\pi/4 \; ; v = V_m/\sqrt{2} = 0.707\, V_m$$

$$3\pi/4 < \theta < \pi \; ; v = V_m \sin\theta$$

$$\therefore \quad V_{av} = \frac{1}{\pi}\left\{\int_0^{\pi/4} v\,d\theta + \int_{\pi/4}^{3\pi/4} v\,d\theta + \int_{3\pi/4}^{\pi} v\,d\theta\right\}$$

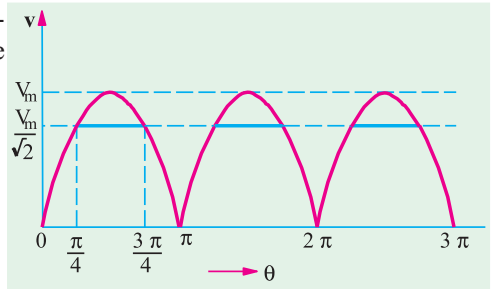


Fig. 11.29

$$= \frac{1}{\pi}\left\{\int_0^{\pi/4} V_m \sin\theta\, d\theta + \int_{\pi/4}^{3\pi/4} 0.707\, V_m d\theta + \int_{3\pi/4}^{\pi} V_m \sin\theta\, d\theta\right\}$$

$$= \frac{V_m}{\pi}\left\{|-\cos\theta|_0^{\pi/4} + 0.707\,|\theta|_{\pi/4}^{3\pi/4} + |-\cos\theta|_{3\pi/4}^{\pi}\right\} = \frac{V_m}{\pi}(0.293 + 1.111 + 0.293) = \mathbf{0.54\ V_m}$$

$$V^2 = \frac{1}{\pi}\left\{\int_0^{\pi/4} V_m^2 \sin^2\theta\, d\theta + \int_{\pi/4}^{3\pi/4} (0.707\, V_m)^2 d\,\theta + \int_{3\pi/4}^{\pi} V_m^2 \sin^2\theta\, d\theta\right\} = 0.341\, V_m^2$$

$$\therefore \quad V = \mathbf{0.584\ V_m}$$

Example 11.19. *A delayed full-wave rectified sinusoidal current has an average value equal to half its maximum value. Find the delay angle θ.* **(Basic Circuit Analysis, Nagpur 1992)**

Solution. The current waveform is shown in Fig. 11.30.

$$I_{av} = \frac{1}{\quad} \quad I_m \sin \quad d \quad \frac{I_m}{\quad} (-\cos\pi + \cos\theta)$$

Now, $I_{av} = I_m/2$

$$\therefore \quad \frac{I_m}{\pi}(-\cos\pi + \cos\theta) = \frac{I_m}{2}$$

$$\therefore \quad \cos\theta = 0.57,\ \theta = \cos^{-1}(0.57) = \mathbf{55.25^o}$$

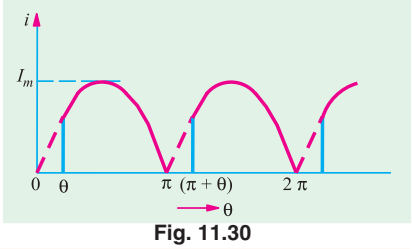


Fig. 11.30

Example 11.20. *The waveform of an output current is as shown in Fig. 11.31. It consists of a portion of the positive half cycle of a sine wave between the angle θ and 180°. Determine the effective value for θ = 30°.*

(Elect. Technology, Vikram Univ. 1984)

Solution. The equation of the given delayed half-wave rectified sine wave is $i = I_m \sin\omega t = I_m \sin\theta$. The effective value is given by

$$I = \sqrt{\frac{1}{2\pi}\int_{\pi/6}^{\pi} i^2 d\theta} \quad \text{or } I^2 = \frac{1}{2\pi}\int_{\pi/6}^{\pi} I_m^2 \sin^2\theta\, d\theta$$

$$= \frac{I_m^2}{4\pi}\int_{\pi/6}^{\pi}(1 - \cos 2\theta)\, d\theta = \frac{I_m^2}{4\pi}\left|\left(\theta - \frac{\sin 2\theta}{2}\right)_{\pi/6}^{\pi}\right|$$

$$= 0.242\, I_m^2$$

or $\quad I = \sqrt{0.242 I_m^2} = \mathbf{0.492\ I_m}$

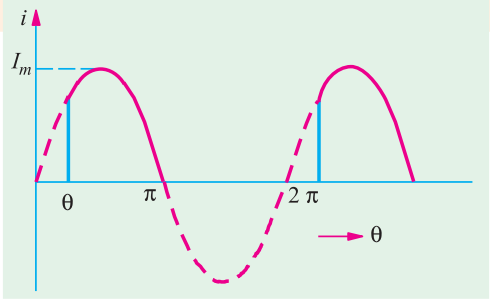


Fig. 11.31

Example 11.21. *Calculate the "form factor" and "peak factor" of the sine wave shown in Fig. 11.32.* **(Elect. Technology-I, Gwalior Univ.)**

Solution. For $0 < \theta < \pi$, $i = 100 \sin\theta$ and for $\pi < \theta < 2\pi$, $i = 0$. The period is 2π.

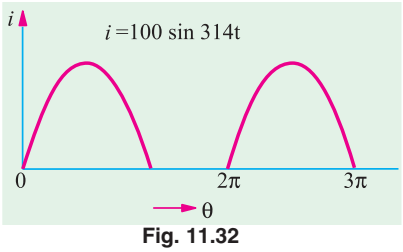


Fig. 11.32

$\therefore 0 \qquad I_{av} = \frac{1}{2\pi}\left\{\int_0^{\pi} i\,d\theta + \int_{\pi}^{2\pi} 0\,d\theta\right\}$

$= \frac{1}{2\pi}\left\{100\int_0^{\pi}\sin\theta\,d\theta\right\} = 31.8 \text{ A}$

$I^2 = \frac{1}{2\pi}\int_0^{\pi} i^2 d\theta = \frac{100^2}{2\pi}\int_0^{\pi}\sin^2\theta\,d\theta = \frac{100^2}{4} = 2500\,;\ I = 50\text{A}$

$\therefore$ form factor = 50/31.8 = **1.57 ;** peak factor = 100/50 = **2**

Example 11.22. *Find the average and effective values of voltage of sinusoidal waveform shown in Fig. 11.33.*

(Elect. Science-I Allahabad Univ. 1991)

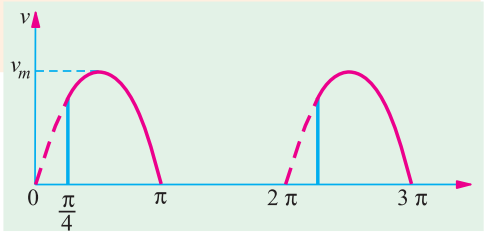


Fig. 11.33

Solution. Although, the given waveform would be integrated from π/4 to π, it would be averaged over the whole cycle because it is unsymmetrical. The equation of the given sinusoidal waveform is $v = 100 \sin\theta$.

$\therefore \qquad V_{av} = \frac{1}{2\pi}\int_{\pi/4}^{\pi} 100\sin\theta\,d\theta = \frac{100}{2\pi}\left|-\cos\theta\right|_{\pi/4}^{\pi} = 27.2 \text{ V}$

$V^2 = \frac{1}{2} \quad {}_{/4}\, 100^2\sin^2 \quad d \quad \frac{100^2}{4}\,\frac{}{4}(1 \quad \cos 2\ )\,d \quad \frac{100^2}{4}\left|\ \frac{\sin 2}{2}\ \right|_{/4} \quad \frac{100^2}{4} \quad \frac{}{4} \quad \frac{1}{2}$

$\therefore \qquad V = \mathbf{47.7\ V}$

Example 11.23. *Find the r.m.s. and average values of the saw tooth waveform shown in Fig. 11.34 (a).*

Solution. The required values can be found by using either graphical method or analytical method.

Graphical Method

The average value can be found by averaging the function from $t = 0$ to $t = 1$ in parts as given below :

$$\text{Average value of } (f) = \frac{1}{T}\int_0^T f(t)\,dt = \frac{1}{T} \times \text{(net area over one cycle)}$$

Now, area of a right-angled triangle = (1/2) × (base) × (altitude).

Hence, area of the triangle during $t = 0$ to $t = 0.5$ second is

$$A_1 = \frac{1}{2}\times(\Delta t)\times(-2) = \frac{1}{2}\times\frac{1}{2}\times -2 = -\frac{1}{2}$$

Similarly, area of the triangle from $t = 0.5$ to $t = 1$ second is

$$A_2 = \frac{1}{2}\times(\Delta t)\times(+2) = \frac{1}{2}\times\frac{1}{2}\times 2 = \frac{1}{2}$$

Net area from $t = 0$ to $t = 1.0$ second is $A_1 + A_2 = -\frac{1}{2} + \frac{1}{2} = 0$

Hence, average value of $f(t)$ over one cycle is zero.

For finding the r.m.s. value, we will first square the ordinates of the given function and draw a new plot for $f^2(t)$ as shown in Fig. 11.34 (*b*). It would be seen that the squared ordinates from a parabola.

Area under parabolic curve = $\frac{1}{3}\times$ base × altitude. The area under the curve from $t = 0$ to

$t = 0.5$ second is ; $A_1 = \frac{1}{3}(\Delta t) \times 2^2 = \frac{1}{3} \times \frac{1}{2} \times 4 = \frac{2}{3}$

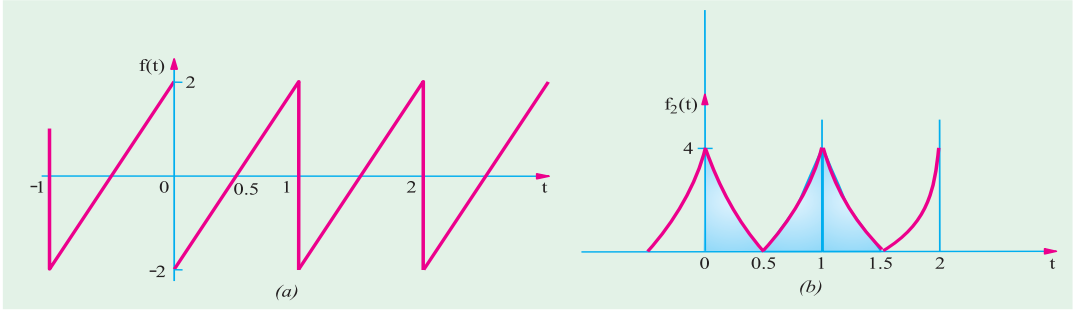


Fig. 11.34

Similarly, for $t = 0.5$ to $t = 1.0$ second $A_2 = \frac{1}{3}(\Delta t) \times 4 = \frac{1}{3} \times \frac{1}{2} \times 4 = \frac{2}{3}$

Total area $= A_1 + A_2 = \frac{2}{3} + \frac{2}{3} = \frac{4}{3}$, r.m.s. value $= \sqrt{\frac{1}{T}\int_0^T f^2(t)\,dt} = \sqrt{\text{average of } f^2(t)}$

$\therefore$ r.m.s. value $= \sqrt{4/3} = \mathbf{1.15}$

Analytical Method

The equation of the straight line from $t = 0$ to $t = 1$ in Fig. 11.34 (*a*) is

$f(t) = 4t - 2$; $f^2(t) = 16t^2 - 16t + 4$

$$\text{Average value} = \frac{1}{T}\int_0^T (4t - 2)\,dt = \frac{1}{T}\left|\frac{4t^2}{2} - 2t\right|_0^T = 0$$

$$\text{r.m.s value} = \sqrt{\frac{1}{T}\int_0^T (16t^2 - 16t + 4)dt} = \sqrt{\frac{1}{T}\left|\frac{16t^3}{3} - \frac{16t^2}{2} + 4t\right|_0^T} = \mathbf{1.15}$$

Example 11.24. *A circuit offers a resistance of 20 Ω in one direction and 100 Ω in the reverse direction. A sinusoidal voltage of maximum value 200 V is applied to the above circuit in series with*

(a) *a moving-iron ammeter* ***(b)*** *a moving-coil ammeter*

(c) *a moving-coil instrument with a full-wave rectifier* ***(d)*** *a moving-coil ammeter.*

Calculate the reading of each instrument.

Solution. *(a)* The deflecting torque of an *MI* instrument is proportional to (current)^2. Hence, its reading will be proportional to the average value of i^2 over the whole cycle. Therefore, the reading of such an instrument :

$$= \sqrt{\left[\frac{1}{2\pi}\left(\int_0^{\pi} 10^2 \sin^2\theta d\theta + \int_{\pi}^{2\pi} 2^2 \sin^2\theta d\theta\right)\right]}$$

$$= \sqrt{\frac{1}{2}\,\frac{100}{2}\left|\frac{\sin 2}{2}\right|_0\,\frac{4}{2}\left|\frac{\sin 2}{2}\right|^2} \qquad \sqrt{26} \qquad \mathbf{5.1\ A}$$

(b) An MC ammeter reads the average current over the whole cycle.

Average current over positive half-cycle is $= 10 \times 0.637 = 6.37$ A

Average current over positive half-cycle is $= -2 \times 0.637 = -1.27$ A

$\therefore$ Average value over the whole cycle is $= (6.37 - 1.27)/2 = \mathbf{2.55\ A}$

(c) In this case, due to the full-wave rectifier, the current passing through the operating coil of the instrument would flow in the positive direction during both the positive and negative half cycles.

$\therefore$ reading $= (6.37 + 1.27)/2 = \mathbf{3.82\ A}$

(d) Average heating effect over the positive half-cycle is $= \frac{1}{2} I_{m_1}^2 R$

Average heating effect over the negative half-cycle is = $\frac{1}{2} I_{m_2}^2 R$

where $I_{m1} = 200/20 = 10$ A; $I_{m2} = 200/100 = 2$ A

Average heating effect over the whole cycle is = $\left(\frac{1}{2}\times 10^2 R + \frac{1}{2}\times 2^2 \times R\right)/2 = 26\,R$

If I is the direct current which produces the same heating effect, then

$I^2 R = 26\,R \qquad \therefore \quad I = \sqrt{26} = 5.1$ **A**

Example 11.25. *A moving coil ammeter, a hot-wire ammeter and a resistance of 100 Ω are connected in series with a rectifying device across a sinusoidal alternating supply of 200 V. If the device has a resistance of 100 Ω to the current in one direction and 5.00 Ω to current in opposite direction, calculate the readings of the two ammeters.*

(Elect. Theory and Meas. Madras University)

Solution. R.M.S. current in one direction is = 200/(100 + 100) = 1 A
Average current in the first *i.e.* positive half cycle is = 1/1.11 = 0.9 A
Similarly, r.m.s. value in the negative half-cycle is = –200/(100 + 500) = –1/3 A
Average value = (–1/3)/1.11 = –0.3 A
Average value over the whole cycle is = (0.9 –0.3)/2 = 0.3 A
Hence, M/C ammeter reads **0.3 A**
Average heating effect during the +ve half cycle = $I_{rms}^2 \times R = I^2 \times R = R$
Similarly, average heating effect during the –ve half-cycle is = $(-1/3)^2 \times R = R/9$
Here, R is the resistance of the hot-wire ammeter.

Average heating effect over the whole cycle is = $\frac{1}{2}\left(R + \frac{R}{9}\right) = \frac{5R}{9}$

If I is the direct current which produces the same heating effect, then

$I^2 R = 5R/9 \qquad \therefore I = \sqrt{5}/3 = 0.745$ A

Hence, hot-wire ammeter indicates **0.745 A**

Example 11.26. *A resultant current wave is made up of two components : a 5A d.c. component and a 50-Hz a.c. component, which is of sinusoidal waveform and which has a maximum value of 5A.*

(i) *Draw a sketch of the resultant wave.*

(ii) *Write an analytical expression for the current wave, reckoning t = 0 at a point where the a.c. component is at zero value and when di/dt is positive.*

(iii) *What is the average value of the resultant current over a cycle ?*

(iv) *What is the effective or r.m.s. value of the resultant current ?*

[Similar Problem: Bombay Univ. 1996]

Solution. ***(i)*** The two current components and resultant current wave have been shown in Fig. 11.35.

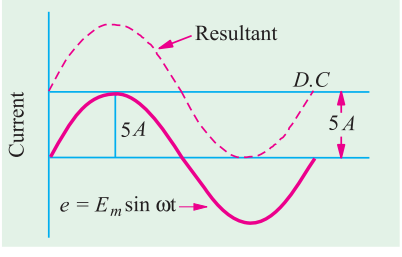


Fig. 11.35

(ii) Obviously, the instantaneous value of the resultant current is given by $i = (5 + 5 \sin \omega t) = (5 + 5 \sin \theta)$

(iii) Over one complete cycle, the average value of the alternating current is zero. Hence, the average value of the resultant current is equal to the value of d.c. component *i.e.* **5A**

(iv) Mean value of i^2 over complete cycle is

$$= \frac{1}{2\pi}\int_0^{2\pi} i^2\, d\theta = \frac{1}{2\pi}\int_0^{2\pi} (5 + 5\sin\theta)^2\, d\theta$$

$$= \frac{1}{2\pi}\int_0^{2\pi} (25 + 50\sin\theta + 25\sin^2\theta)\, d\theta$$

$$= \frac{1}{2\pi}\int_0^{2\pi}\left[25 + 50\sin\theta + 25\left(\frac{1-\cos 2\theta}{2}\right)\right]d\theta = \frac{1}{2\pi}\int_0^{2\pi}(37.5 + 50\sin\theta - 12.5\cos 2\theta)\,d\theta$$

$$= \frac{1}{2}\Big|\ 37.5 \quad 50\cos \quad \frac{12.5}{2}\sin 2 \ \Big|_0^{2} \quad \frac{75}{2} \quad 37.5 \quad \therefore \text{R.M.S. value } I = \sqrt{37.5} = \mathbf{6.12\ A}$$

Note. In general, let the combined current be given by $i = A + B\sqrt{2}\sin\omega t = A + B\sqrt{2}\sin\theta$ where A represents the value of direct current and B the r.m.s. value of alternating current.

The r.m.s. value of combined current is given by

$$I_{rms} = \sqrt{\left(\frac{1}{2\pi}\int_0^{2\pi} i^2\,d\theta\right)} \text{ or } I^2_{r.m.s.} = \frac{1}{2\pi}\int_0^{2\pi} i^2\,d\theta = \frac{1}{2\pi}\int_0^{2\pi}(A + B\sqrt{2}\sin\theta)^2\,d\theta$$

$$= \frac{1}{2\pi}\int_0^{2\pi}(A^2 + 2B^2\sin^2\theta + 2\sqrt{2}\,AB\sin\theta)\,d\theta = \frac{1}{2\pi}\int_0^{2\pi}(A^2 + B^2 - B^2\cos 2\theta + 2\sqrt{2}\,AB\sin\theta\,d\theta$$

$$= \frac{1}{2}\ A^2 \quad B^2 \quad \frac{B^2\sin 2}{2} \quad 2\sqrt{2}AB\cos \quad {}_0^{2} \quad \frac{1}{2}\ 2\ A^2 \quad 2\ B^2 \quad 2\sqrt{2}\,AB \quad 2\sqrt{2}AB$$

$$= A^2 + B^2 \quad \therefore\ I_{rms} = \sqrt{(A^2 + B^2)}$$

The above example could be easily solved by putting $A = 5$ and $B = 5/\sqrt{2}$ (because $B_{max} = 5$)

$$\therefore \quad I_{rms} = \sqrt{5^2 + (5/\sqrt{2})^2} = \mathbf{6.12\ A}$$

Example 11.27. *Determine the r.m.s. value of a semi-circular current wave which has a maximum value of a.*

Solution. The equation of a semi-circular wave (shown in Fig. 11.36) is

$$x^2 + y^2 = a^2 \quad \text{or} \quad y^2 = a^2 - x^2$$

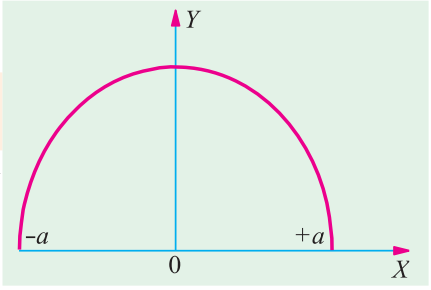


Fig. 11.36

$$\therefore \quad I_{rms} = \sqrt{\frac{1}{2a}\int_{-a}^{+a} y^2 dx} \quad \text{or} \quad I^2_{rms} = \frac{1}{2a}\int_{-a}^{+a}(a^2 - x^2)\,dx$$

$$= \frac{1}{2a}\int_{-a}^{+a}(a^2\,dx - x^2\,dx) \quad = \quad \frac{1}{2a}\left|a^2x - \frac{x^3}{3}\right|_{-a}^{+a} = \frac{1}{2a}\left(a^3 - \frac{a^3}{3} + a^3 - \frac{a^3}{3}\right) = \frac{2a^2}{3}$$

$$\therefore \quad I_{rms} = \sqrt{2a^2/3} = \mathbf{0.816\ a}$$

Example 11.28. *Calculate the r.m.s. and average value of the voltage wave shown in Fig. 11.37.*

Solution. In such cases, it is difficult to develop a single equation. Hence, it is usual to consider two equations, one applicable from 0 to 1 and an other form 1 to 2 millisecond.

For t lying between 0 and 1 ms, $v_1 = 4$, For t lying between 1 and 2 ms, $v_2 = -4t + 4$

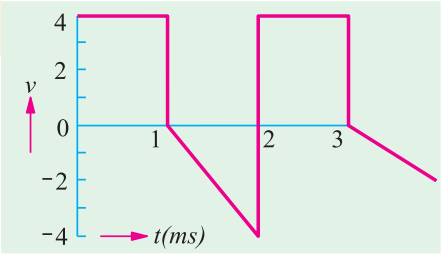


Fig. 11.37

$$\therefore v_{rms} = \sqrt{\frac{1}{2}\left(\int_0^1 v_1^2\,dt + \int_1^2 v_2^2\,dt\right)}$$

$$V^2_{rms} = \frac{1}{2}\left[\int_0^1 4^2\,dt + \int_1^2 (-4t+4)^2\,dt\right]$$

$$= \frac{1}{2}\left[\left|16\,t\right|_0^1 + \left|\frac{16\,t^3}{3}\right|_1^2 + \left|16\,t\right|_1^2 - \left|\frac{32\,t^2}{2}\right|_1^2\right]$$

$$= \frac{1}{2}\left[16 + \frac{16\times 8}{3} - \frac{16}{3} + 16\times 2 - 16\times 1 - \frac{32\times 4}{2} + \frac{32\times 1}{2}\right] = \frac{32}{3} \quad \therefore V_{rms} = \sqrt{32/3} = 3.265 \text{ volt}$$

$$V_{av} = \frac{1}{2}\left[\int_0^1 v_1 dt + \int_1^2 v_2\,dt\right] = \frac{1}{2}\left[\int_0^1 4dt + \int_1^2 (-4t+4)\,dt\right] = \frac{1}{2}\left[\left|4t\right|_0^1 + \left|-\frac{4t^2}{2} + 4t\right|_1^2\right] = 1 \text{ volt}$$

Tutorial Problems No. 11.1

1. Calculate the maximum value of the e.m.f. generated in a coil which is rotating at 50 rev/s in a uniform magnetic field of 0.8 Wb/m^3. The coil is wound on a square former having sides 5 cm in length and is wound with 300 turns. **[188.5 V]**

2. (*a*) What is the peak value of a sinusoidal alternating current of 4.78 r.m.s. amperes ?

(*b*) What is the r.m.s. value of a rectangular voltage wave with an amplitude of 9.87 V ?

(*c*) What is the average value of a sinusoidal alternating current of 31 A maximum value ?

(*d*) An alternating current has a periodic time of 0.03 second. What is its frequency ?

(*e*) An alternating current is represented by i = 70.7 sin 520 *t*. Determine (*i*) the frequency (*ii*) the current 0.0015 second after passing through zero, increasing positively.

[6.76 A ; 9.87 V ; 19.75 A ; 33.3 Hz ; 82.8 Hz ; 49.7 A]

3. A sinusoidal alternating voltage has an r.m.s. value of 200 V and a frequency of 50 Hz. It crosses the zero axis in a positive direction when $t = 0$. Determine (*i*) the time when voltage first reaches the instantaneous value of 200 V and (*ii*) the time when voltage after passing through its maximum positive value reaches the value of 141.4 V.

[(*i*) (0.0025 second (*ii*) 1/300 second)]

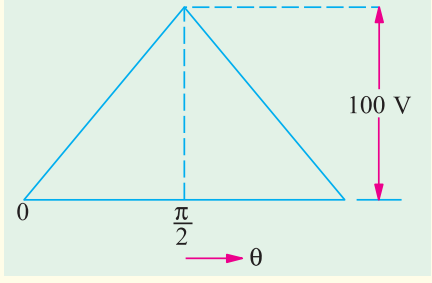


Fig. 11.38

4. Find the form factor and peak factor of the triangular wave shown in Fig. 11.38 **[1.155; 1.732]**

5. An alternating voltage of 200 sin 471 *t* is applied to a h.w. rectifier which is in series with a resistance of 40 Ω. If the resistance of the rectifier is infinite in one direction and zero in the other, find the r.m.s. value of the current drawn from the supply source. **[2.5 A]**

6. A sinusoidally varying alternating current has an average value of 127.4 A. When its value is zero, then its rate change is 62,800 A/s. Find an analytical expression for the sine wave. **[i = 200 sin 100 πt]**

7. A resistor carries two alternating currents having the same frequency and phase and having the same value of maximum current *i.e.* 10 A. One is sinusoidal and the other is rectangular in waveform. Find the r.m.s. value of the resultant current. **[12.24 A]**

8. A copper-oxide rectifier and a non-inductive resistance of 20 Ω are connected in series across a sinusoidal a.c. supply of 230 V (r.m.s.). The resistance of the rectifier is 2.5 Ω in forward direction and 3,000 Ω in the reverse direction. Calculate the r.m.s. and average values of the current.

[r.m.s. value = 5.1 A, average value = 3.22 A]

9. Find the average and effective values for the waveshape shown in Fig. 11.39 if the curves are parts of a sine wave. **[27.2 V, 47.7V] (*Elect. Technology, Indore Univ.*)**

10. Find the effective value of the resultant current in a wire which carries simultaneously a direct current of 10 A and a sinusoidal alternating current with a peak value of 15 A.

[14.58A] (*Elect. Technology, Vikram Univ. Ujjain*)

11. Determine the r.m.s. value of the voltage defined by $e = 5 + 5 \sin (314\, t + \pi/6)$

[6.12 V] (*Elect. Technology, Indore Univ.*)

12. Find the r.m.s. value of the resultant current in a wire which carries simultaneously a direct current of 10 A and a sinusoidal alternating current with a peak value of 10 A. **[12.25 A] (*Elect. Technology-I; Delhi Univ.*)**

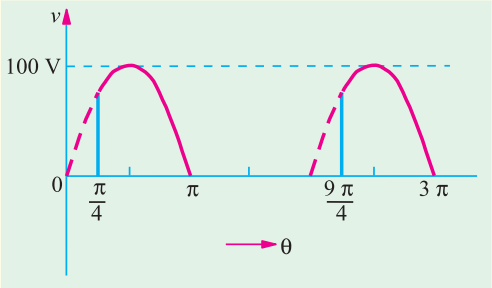


Fig. 11.39

13. An alternating voltage given by $e = 150 \sin 100\pi t$ is applied to a circuit which offers a resistance of 50 ohms to the current in one direction and completely prevents the flow of current in the opposite direction. Find the r.m.s. and average values of this current and its form factor.

[1.5 A, 0.95 A, 1.57] (*Elect. Technology, Indore Univ.*)

14. Find the relative heating effects of three current waves of equal maximum value, one rectangular, the second semi-circular and the third sinusoidal in waveform **[1: 2/2, 1/2] (*Sheffield Univ. U.K.*)**

15. Calculate the average and root mean-square value, the form factor and peak factor of a periodic current wave have the following values for equal time intervals over half-cycle, changing suddenly from one value of the next. **[0, 40, 60, 80, 100, 80, 60, 40, 0] (*A.M.I.E.*)**

16. A sinusoidal alternating voltage of amplitude 100 V is applied across a circuit containing a rectifying device which entirely prevents current flowing in one direction and offers a resistance of 10 ohm to the flow of current in the other direction. A hot wire ammeter is used for measuring the current. Find the reading of instrument. **(*Elect. Technology. Punjab Univ.*)**

11.23. Representation of Alternating Quantities

It has already been pointed out that an attempt is made to obtain alternating voltages and currents having sine waveform. In any case, a.c. computations are based on the assumption of sinusoidal voltages and currents. It is, however, cumbersome to continuously handle the instantaneous values in the form of equations of waves like $e = E_m \sin \omega t$ etc. A conventional method is to employ vector method of representing these sine waves. These vectors may then be manipulated instead of the sine functions to achieve the desired result. ***In fact, vectors are a shorthand for the represen-tation of alternating voltages and currents and their use greatly simplifies the problems in a.c. work.***

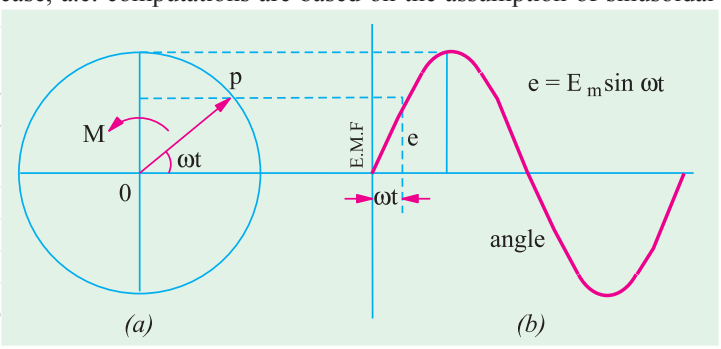


Fig. 11.40

A vector is a physical quantity which has magnitude as well as direction. Such vector quantities are completely known when particulars of their magnitude, direction and the sense in which they act, are given. They are graphically represented by straight lines called vectors. The length of the line represents the magnitude of the alternating quantity, the inclination of the line with respect to some axis of reference gives the direction of that quantity and an arrow-head placed at one end indicates the direction in which that quantity acts.

The alternating voltages and currents are represented by such vectors rotating counter-clockwise with the same frequency as that of the alternating quantity. In Fig. 11.40 (*a*), *OP* is such a vector which represents the maximum value of the alternating current and its angle with *X* axis gives its phase. Let the alternating current be represented by the equation $e = E_m \sin \omega t$. It will be seen that the projection of *OP* and *Y*-axis at any instant gives the instantaneous value of that alternating current.

$$\therefore \qquad OM = OP \sin \omega t \text{ or } e = OP \sin \omega t = E_m \sin \omega t$$

It should be noted that a line like *OP* can be made to represent an alternating voltage of current if it satisfies the following conditions :

(i) Its length should be equal to the peak or maximum value of the sinusoidal alternating current to a suitable scale. ***(ii)*** It should be in the horizontal position at the same instant as the alternating quantity is zero and increasing. ***(iii)*** Its angular velocity should be such that it completes one revolution in the same time as taken by the alternating quantity to complete one cycle.

11.24. Vector Diagram using R.M.S. Values

Instead of using maximum values as above, it is very common practice to draw vector diagrams using r.m.s. values of alternating quantities. But it should be understood that in that case, the projection of the rotating vector on the *Y*-axis does not give the instantaneous value of that alternating quantity.

11.25. Vector Diagrams of Sine Waves of Same Frequency

Two or more sine waves of the same frequency can be shown on the same vector diagram because the various vectors representing different waves all rotate counter-clockwise at the same frequency and maintain a fixed position relative to each other. This is illustrated in Fig. 11.41 where a voltage *e* and current *i* of the same frequency are shown. The current wave is supposed to pass upward through zero at the instant when $t = 0$ while at the same time the voltage wave has already advanced an angle α from its zero value. Hence, their equations can be written as

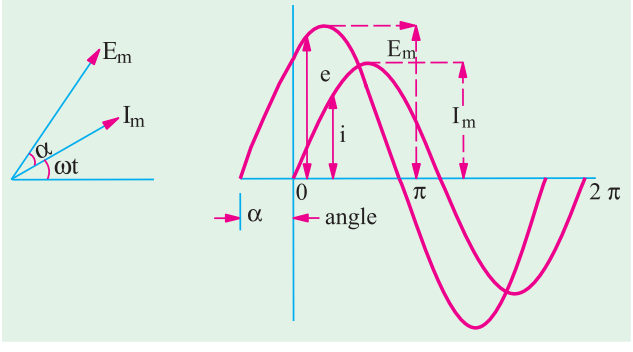


Fig. 11.41

$$i = I_m \sin \omega t$$

and $$e = E_m \sin (\omega t + \alpha)$$

Sine wave of different frequencies cannot be represented on the same vector diagram in a still picture because due to difference in speed of different vectors, the phase angles between them will be continuously changing.

11.26. Addition of Two Alternating Quantities

In Fig. 11.42 (*a*) are shown two rotating vectors representing the maximum values of two sinusoidal voltage waves represented by $e_1 = E_{m1} \sin \omega t$ and $e_2 = E_{m2} \sin (\omega t - \phi)$. It is seen that the sum of the two sine waves of the same frequency is another sine wave of the same frequency but of a different maximum value and phase. The value of

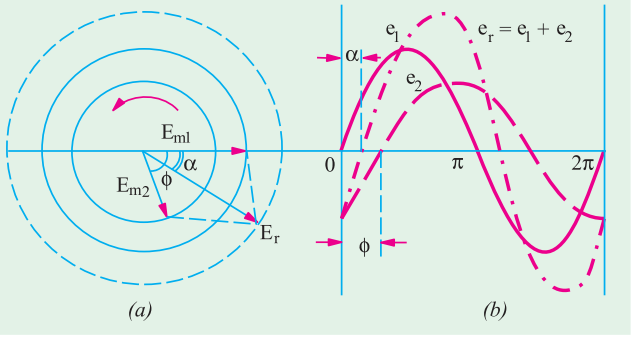


Fig. 11.42

the instantaneous resultant voltage e_r, at any instant is obtained by algebraically adding the projections of the two vectors on the *Y*-axis. If these projections are e_1 and e_2, then, $e_r = e_1 + e_2$ at that time. The resultant curve is drawn in this way by adding the ordinates. It is found that the resultant wave is a sine wave of the same frequency as the component waves but lagging behind E_{m1} by an angle α. The vector diagram of Fig. 11.42 (*a*) can be very easily drawn. Lay off E_{m2} lagging $\phi°$ behind E_{m1} and then complete the parallelogram to get E_r.

Example 11.29. *Add the following currents as waves and as vectors.*

$$i_1 = 7 \sin \omega t \text{ and } i_2 = 10 \sin (\omega t + \pi/3)$$

Solution. As Waves

$$i_r = i_1 + i_2 = 7 \sin \omega t + 10 \sin (\omega t + 60°)$$
$$= 7 \sin \omega t + 10 \sin \omega t \cos 60° + 10 \cos \omega t \sin 60°$$
$$= 12 \sin \omega t + 8.66 \cos \omega t$$

Dividing both sides by $\sqrt{(12^2 + 8.66^2)} = 14.8$, we get

$$i_r = 14.8 \left(\frac{12}{14.8} \sin \omega t + \frac{8.66}{14.8} \cos \omega t \right)$$
$$= 14.8 (\cos \alpha \sin \omega t + \sin \alpha \cos \omega t)$$

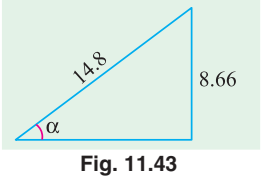


Fig. 11.43

where $\cos \alpha = 12/14.8$ and $\alpha = 8.66/14.8$ —as shown in Fig. 11.43

$\therefore$ $i_r = 14.8 \sin (\omega t + \alpha)$

where $\tan \alpha = 8.66/12$ or $\alpha = \tan^{-1}(8.66/12) = 35.8°$

$\therefore$ $i_r = 14.8 \sin (\omega t + 35.8°)$

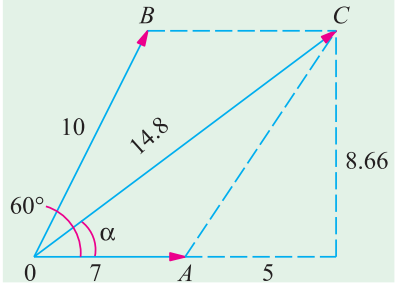


Fig. 11.44

As Vectors

Vector diagram is shown in Fig. 11.44. Resolving the vectors into their horizontal and vertical components, we have

$$X - \text{component} = 7 + 10 \cos 60° = 12$$
$$Y - \text{component} = 0 + 10 \sin 60° = 8.66$$
$$\text{Resultant} = \sqrt{(12^2 + 8.66^2)} = 14.8 \text{ A}$$
$$\text{and} \quad \alpha = \tan^{-1} (8.66/12) = 35.8°$$

Hence, the resultant equation can be written as

$$i_r = 14.8 \sin (\omega t + 35.8°)$$

11.27. Addition and Subtraction of Vectors

(*i*) Addition. In a.c. circuit problems we may be concerned with a number of alternating voltages or currents of the same frequency but of different phases and it may be required to obtain the resultant voltage or current. As explained earlier (Art. 11.23) if the quantities are sinusoidal, they may be represented by a number of rotating vectors having a common axis of rotation and displaced from one another by fixed angles which are equal to the phase differences between the respective alternating quantities. The instantaneous value of the resultant voltage is given by the algebraic sum of the projections of the different vectors on *Y*-axis. The maximum value (or r.m.s. value if the vectors represent that value) is obtained by compounding the several vectors by using the parallelogram and polygon laws of vector addition.

However, another easier method is to resolve the various vectors into their *X*-and *Y*-components and then to add them up as shown in Example 11.30 and 31.

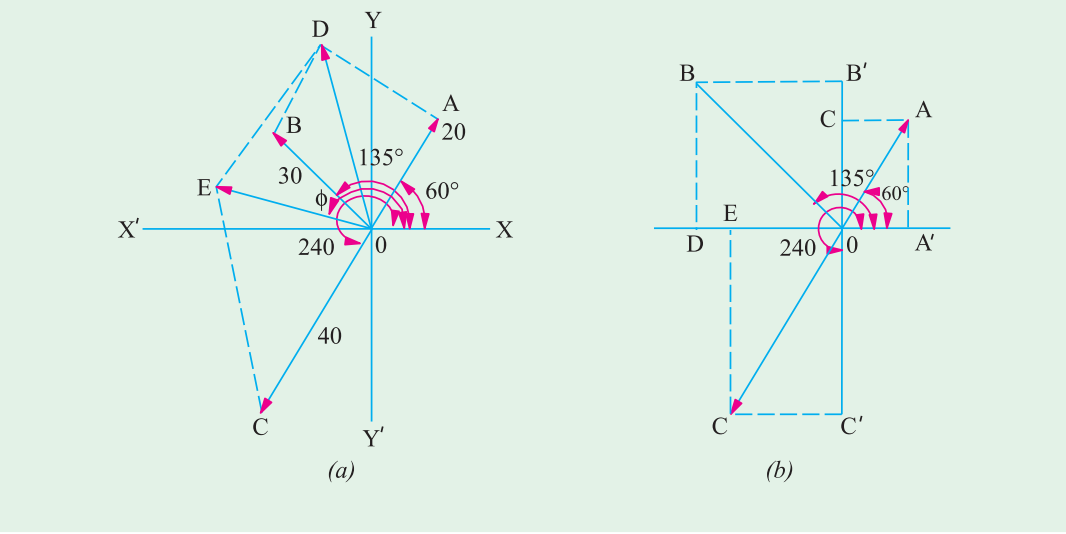


Fig. 11.45

Suppose we are given the following three alternating e.m.fs. and it is required to find the equation of the resultant e.m.f.

$$e_1 = 20 \sin (\omega t + \pi/3)$$
$$e_2 = 30 \sin (\omega t + 3\pi/4)$$
$$e_3 = 40 \sin (\omega t + 4\pi/3)$$

Then the vector diagram can be drawn as explained before and solved in any of the following three ways :

(i) By compounding according to parallelogram law as in Fig. 11.45 (*a*)

(ii) By resolving the various vectors into their *X*-and *Y*-components as in Fig. 11.45 (*b*).

(iii) By laying off various vectors end-on end at their proper phase angles and then measuring the closing vector as shown in Fig. 11.46.

Knowing the magnitude of the resultant vector and its inclination ϕ with *X* axis, the equation of the resultant e.m.f. can be written as $e = E_m \sin (\omega t + \phi)$.

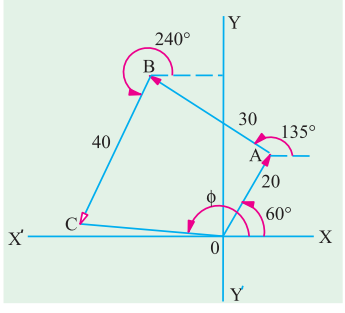


Fig. 11.46

Example 11.30. *Represent the following quantities by vectors :*
5 sin (2π ft –1) ; 3 cos (2π ft + 1) ; 2 sin (2π ft + 2.5) and 4 sin (2π ft –1)
Add the vectors and express the result in the form : A sin (2π ft ± Φ)

Solution. It should be noted that all quantities have the same frequency *f*, hence they can be represented vertically on the same vector diagram and added as outlined in Art. 11.27. But before doing this, it would be helpful to express all the quantities as sine functions. Therefore, the second expression 3 cos (2π *ft* + 1) can be written as

$$3 \sin \left(2\pi ft + 1 + \frac{\pi}{2}\right) = 3 \sin (2\pi ft + 1 + 1.57) = 3 \sin (2\pi ft + 2.57)$$

The maximum value of each quantity, its phase with respect to the quantity of reference *i.e.* *X* sin 2π*ft*, its horizontal and vertical components are given in the table on next page :

Quantity	Max. value	Phase		Horizontal component	Vertical component
		radians	angles		
(*i*) 5 sin ($2\pi ft - 1$)	5	–1	–57.3°	5 × cos (–57.3°) = 2.7	5 sin (–57.3°) = –4.21
(*ii*) 3 sin ($2\pi ft + 2.57$)	3	+2.57	147.2°	3 × cos 147.2° = –2.52	3 sin 147.2° = 1.63
(*iii*) 2 sin ($2\pi ft + 2.5$)	2	+ 2.5	143.2°	2 cos 143.2° = –1.6	2 sin 143.2° = 1.2
(*iv*) 4 sin ($2\pi ft - 1$)	4	– 1	– 57.3°	4 cos (–57.3°) = 2.16	4 sin (–57.3°) = –3.07
Total				0.74	–4.75

The vector diagram is shown in Fig. 11.47 in which *OA*, *OB*, *OC* and *OD* represent quantities **(*i*), (*ii*), (*iii*)** and **(*iv*)** given in the table.

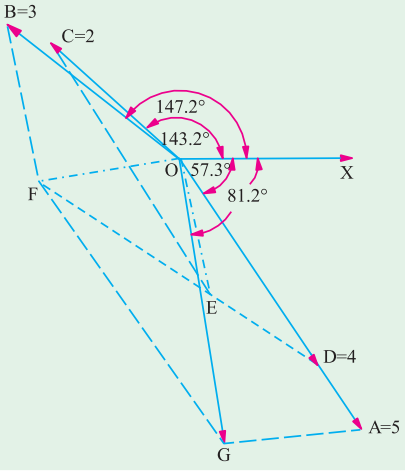


Fig. 11.47

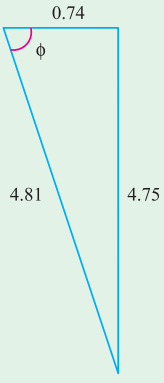


Fig. 11.48

Their resultant is given by *OG* and the net horizontal and vertical components are shown in Fig. 11.48. Resultant $= \sqrt{[0.74^2 + (-4.75)^2]} = 4.81$ and $\tan\theta = -4.75 / 0.74$

$\therefore \quad \theta = \tan^{-1}(-4.75/0.74) = -81.2° = -1.43$ radians

The equation of the resultant quantity is **$4.81 \sin (2\pi ft - 1.43)$**

Example 11.31. *Three voltages represented by*

$$e_1 = 20 \sin \omega t;\ e_2 = 30 \sin (\omega t - \pi/4) \text{ and } e_3 = 40 \cos (\omega t + \pi/6)$$

act together in a circuit. Find an expression for the resultant voltage. Represent them by appropriate vectors. **(Electro-technics Madras Univ.) (*Elec. Circuit Nagpur Unvi. 1991*)**

Solution. First, let us draw the three vectors representing the maximum values of the given alternating voltages.

$e_1 = 20 \sin \omega t$ –here phase angle with *X*-axis is zero, hence the vector will be drawn parallel to the *X*-axis

$e_2 = 30 \sin (\omega t - \pi/4)$ –its vector will be below *OX* by 45°

$e_3 = 40 \cos (\omega t + \pi/6) = 40 \sin (90° + \omega t + \pi/6)$*

$= 40 \sin (\omega t + 120°)$ –its vector will be at 120° with respect to *OX* in counter clockwise direction.

These vectors are shown in Fig. 11.49 (*a*). Resolving them into *X*-and *Y*-components, we get

X - component = 20 + 30 cos 45° – 40 cos 60°= 21.2 V

Y - component = 40 sin 60° –30 sin 45° = 13.4 V

* $\cos\theta = \sin(90° + \theta)$

As seen from Fig. 11.49 (*b*), the maximum value of the resultant voltage is

$$OD = \sqrt{21.2^2 + 13.4^2} = 25.1 \text{ V}$$

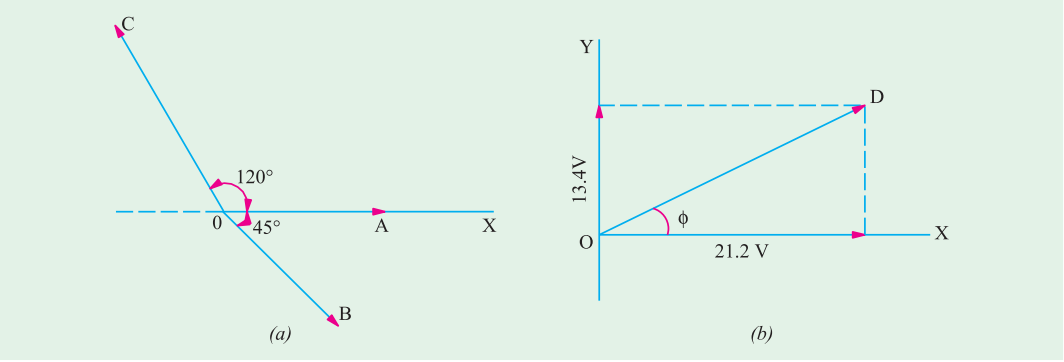


Fig. 11.49

The phase angle of the resultant voltage is given by $\tan \phi = \dfrac{\text{Y-component}}{\text{X-component}} = \dfrac{13.4}{27.2} = 0.632$

$\therefore \quad \phi = \tan^{-1} 0.632 = 32.3° = \mathbf{0.564 \text{ radian}}$

The equation of the resultant voltage wave is $e = 25.1 \sin (\omega t + 32.3°)$ or $e = 25.1 \sin (\omega t + 0.564)$

Example 11.32. *Four circuits A, B, C and D are connected in series across a 240 V, 50-Hz supply. The voltages across three of the circuits and their phase angles relative to the current through them are, V_A, 80 V at 50° leading, V_B, 120 V at 65° lagging : V_θ, 135 V at 80° leading. If the supply voltage leads the current by 15°, find from a vector diagram drawn to scale the voltage V_D across the circuit D and its phase angle.*

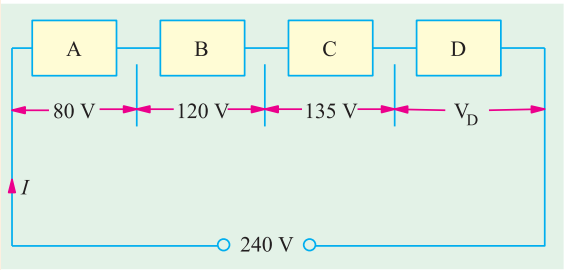


Fig. 11.50

Solution. The circuit is shown in Fig. 11.50.

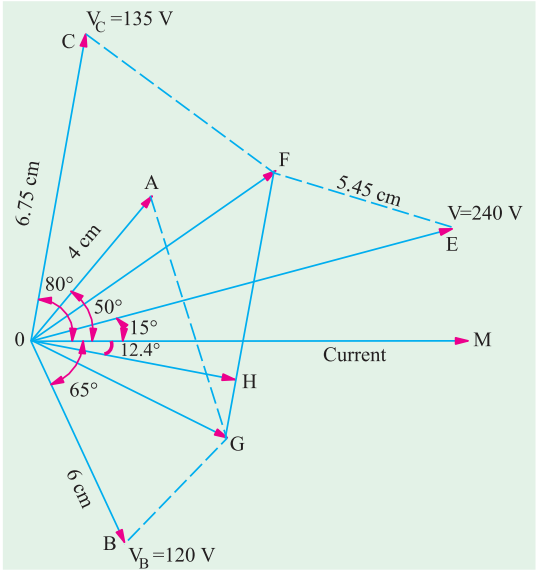


Fig. 11.51

(*a*) The vector diagram is shown in Fig. 11.50. The current vector *OM* is drawn horizontally and is taken as reference vector. Taking a scale of 1 cm = 20 V, vector *OA* is drawn 4 cm in length and leading *OM* by an angle of 50°. Vector *OB* represents 120 V and is drawn lagging behind *OM* by 65°. Their vector sum, as found by Parallelogram Law of Vectors, is given by vector *OG*.

Next, vector *OC* is drawn ahead of *OM* by 80° representing 135 V. Vector *OF* represents the vector sum of *OG* and *OC*. Vector *OE* represents the applied voltage of 240 V and is drawn 15° ahead of current vector *OM*. The vector difference of *OE* and *OF* gives the required voltage V_D. It is equal to *FE*. It measures 5.45 cm which means that it represents 20 × 5.45 = 109 V. This vector is transferred to position

OH by drawing *OH* parallel to *FE*. It is seen that *OH* lags behind the current vector *OM* by 12.4°.

Hence, V_D' = **109** volts lagging behind the current by **12.4°.**

(*ii*) Subtraction of Vectors

If difference of two vectors is required, then one of the vectors is reversed and this reversed vector is then compounded with the other vector as usual.

Suppose it is required to subtract vector *OB* from vector *OA*. Then *OB* is reversed as shown in Fig. 11.52 (*a*) and compounded with *OA* according to parallelogram law. The vector difference **(A-B)** is given by vector *OC*.

Similarly, the vector *OC* in Fig. 12.52 (*b*) represents (**B–A**) *i.e.* the subtraction of **OA** form **OB**.

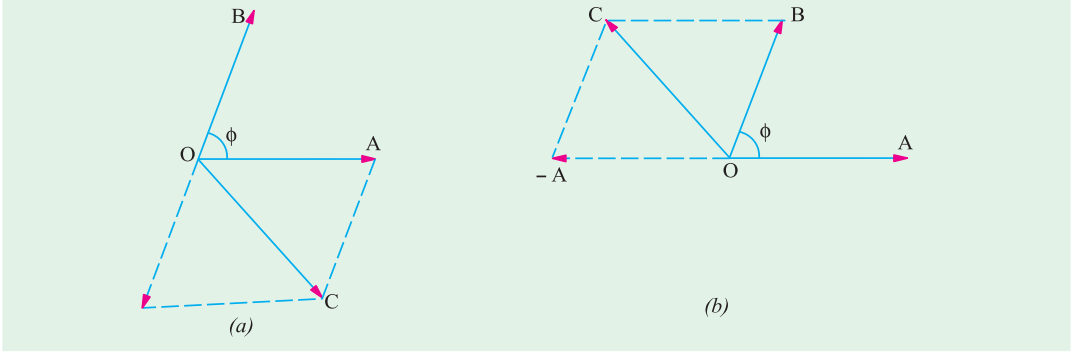


Fig 11.52

Example 11.33. *Two currents i_1 and i_2 are given by the expressions*

$$i_1 = 10 \sin (314t + \pi/4) \text{ amperes and } i_2 = 8 \sin (313\, t - \pi/3) \text{ amperes}$$

Find **(a)** $i_1 + i_2$ *and* **(b)** $i_1 - i_2$. *Express the answer in the form* $i = I_m \sin (314\, t \pm \phi)$

Solution. **(*a*)** The current vectors representing maximum values of the two currents are shown in Fig. 11.53 (*a*). Resolving the currents into their *X*-and *Y*-components, we get

X- component = 10 cos 45° + 8 cos 60° = $10/\sqrt{2} + 8/2 = 11.07$ A

Y- component = 10 sin 45° –8 sin 60 ° = 0.14 A

$\therefore \quad I_m = \sqrt{11.07^2 + 0.14^2} = 11.08$ A

tan ϕ = (0.14/11.07) = 0.01265 $\quad \therefore \ \phi = 44'$

Hence, the equation for the resultant current is $\boldsymbol{i = 11.08 \sin (314\, t + 44')}$ **amperes**

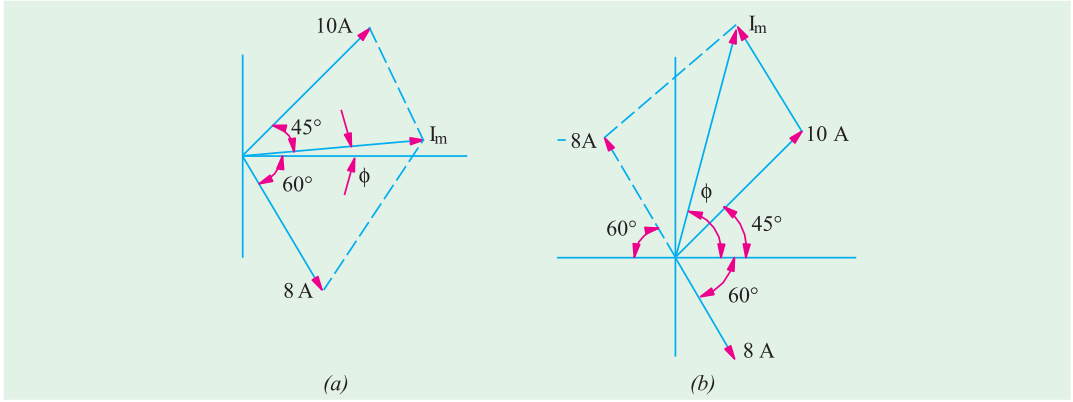


Fig. 11.53

(b) X –component $= 10 \cos 45° - 8 \cos 60° = 3.07$ A

Y –component $= 10 \sin 45° + 8 \sin 60° = 14$ A

$\therefore \quad I_m = \sqrt{3.07^2 + 14^2} = 14.33$ A ...Fig. 11.53 (*b*)

$\phi = \tan^{-1} (14/3.07) = 77° 38'$

Hence, the equation of the resultant current is

$$i = 14.33 \sin (314 + 77° 38') \text{ amperes}$$

Example 11.34. *The maximum values of the alternating voltage and current are 400 V and 20 A respectively in a circuit connected to a 50 Hz supply. The instantaneous values of voltage and current are 283 Vand 10 A respectively at time t = 0, both increasing positively.*

(i) *Write down the expression for voltage and current at time t.*

(ii) *Determine the power consumed in the circuit.*

Take the voltage and current to be sinusoidal **[Nagpur University]**

Solution. $V_m = 400$, $I_m = 20$, $\omega = 314$ rad./sec

(i) Let the expressions be as follows :

$v(t) = V_m \sin (\omega t + \theta_1) = 400 \sin (314\, t + \theta_1)$

$I(t) = I_m \sin (\omega t + \theta_2) = 20 \sin (314\, t + \theta_2)$

where θ_1 and θ_2 indicate the concerned phase-shifts with respect to some reference. Substituting the given instantaneous values at $t = 0$,

$\theta_1 = 45°$ and $\theta_2 = 30°$

The required expressions are :

$V(t) = 400 \sin (314\, t + 45°)$

$i(t) = 20 \sin (314\, t + 30°)$

Thus, the voltage leads the current by 15°.

V = RMS voltage = $400/1.414\ t = 283$ V

I = RMS voltage = $20/1.414 = 14.14$ A

Power-factor, $\cos \phi = \cos 15° = 0.966$ lagging, since current lags behind the voltage.

(ii) Power = V I $\cos \phi = 3865$ watts

Additional Hint : Draw these two wave forms

Example 11.35. *Voltage and current for a circuit with two elements in series are experssed as follows :*

$$v(t) = 170 \sin (6280\, t + \pi/3) \text{ Volts}$$
$$i(t) = 8.5 \sin (6280\, t + \pi/2) \text{ Amps}$$

(i) *Plot the two waveforms.* ***(ii)*** *Determine the frequency in Hz.* ***(iii)*** *Determine the power factor stating its nature.* ***(iv)*** *What are the values of the elements ?*

[Nagpur University, April 1996]

Solution. ***(ii)*** $\omega = 6280$ radiation/sec, $f = \omega/2\pi = 1000$ Hz

(i) Two sinusoidal waveforms with a phase-difference of 30° ($= \pi/2 - \pi/3$) are to be drawn. Each waveform completes a cycle in 1 milli-second, since $f = 1000$ Hz.

The waveform for current leads that for the voltage by 30°. At $\omega t = 0$, the current is at its positive peak, while the voltage will be at its positive peak for $\omega t = \pi/6 = 30°$. Peak value are 170 volts and 8.5 amp.

(iii) RMS value of voltage = $170/\sqrt{2} = 120$ volts

RMS value of current = $8.5/\sqrt{2} = 6$ amp.

Impedance = V/I = 120/6 = 20 ohms

Power factor = cos 30°, Leading = 0.866,

Since the current leads the voltage, the two elements must be *R* and *C*.

$R = Z \cos \phi = 20 \times 0.866 = 17.32$ ohms

$X_c = Z \sin \phi = 20 \times 0.50 = 10$ ohms

$$C = 1/(\omega X_c) = \frac{1000 \times 1000}{6280 \times 10} = 15.92 \text{ mF}$$

Example 11.36. *Three sinusoidally alternating currents of rms values 5,7.5, and 10 A are having same frequency of 50 Hz, with phase angles of 30°, –60° and 45°.*

(i) *Find their average values,* ***(ii)*** *Write equations for their instantaneous values,* ***(iii)*** *Draw waveforms and phasor diagrams taking first current as the reference,* ***(iv)*** *Find their instantaneous values at 100 mSec from the original reference.* **[Nagpur University, Nov. 1996]**

Solution. ***(i)*** Average value of alternating quantity in case of sinusoidal nature of variation = (RMS Value)/1.11

Average value of first current = 5/1.11 = 4.50 A

Average value of second current = 7.5/1.11 = 6.76 A

Average value of third current = 10/1.11 = 9.00 A

(ii) Instantaneous Values : $\omega = 2\pi \times 50 = 314$ rad/sec

$i_1(t) = 5\sqrt{2} \sin(314t \quad 30)$

$i_2(t) = 7.5\sqrt{2} \sin(314t \quad 60)$

$i_3(t) = 10\sqrt{2} \sin(314t \quad 45)$

(iii) First current is to be taken as a reference, now. From the expressions, second current lags behind the first current by 90°. Third current leads the first current by 15°. Waveforms with this description are drawn in Fig. 11.54 (*a*) and the phasor diagrams, in Fig. 11.54 (*b*).

(iv) A 50 Hz a.c. quantity completes a cycle in 20 m sec. In 100 m sec, it completes five cycles. Original reference is the starting point required for this purpose. Hence, at 100 m sec from the reference.

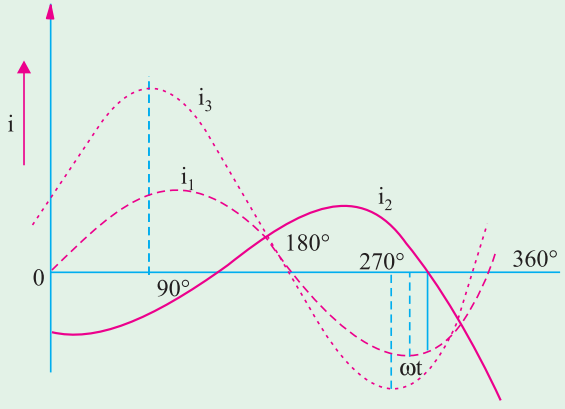


Fig. 11.54 (*a*)

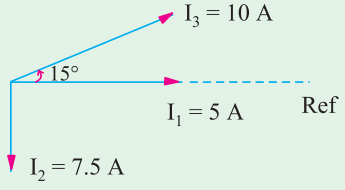


Fig. 11.54 (*b*)

(v) instantaneous value of $i_1(t) = 5\sqrt{2} \sin 30 \quad 3.53$ A

instantaneous value of $i_2(t) = 7.5\sqrt{2}\sin(\quad 60\ )$ 9.816 A

instantaneous value of $i_3(t) = 10\sqrt{2}\sin(45\ )$ 10 A

Example 11.37. *Determine the form factor and peak factor for the unshaded waveform,in Fig. 11.55.* **[Bombay University, 2000]**

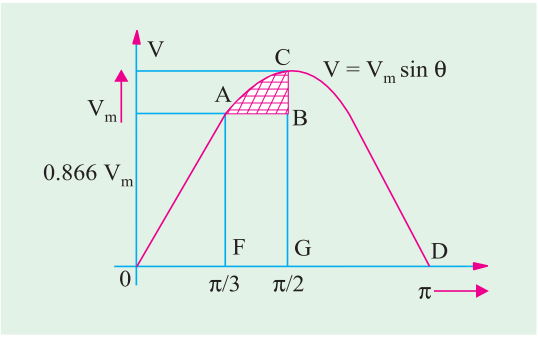


Fig. 11.55

Solution. $v(\theta) = V_m \sin\theta$, except for the region between

$\theta = 60°$ to $\theta = 90°$, wherein $v = 0.866\ V_m$.

Area under the curve will be worked out first, for calculating the average value.

$$\text{Area OAF} = V_m \int_0^{/3} \sin\ d \qquad 0.5\ V_m$$

Area FABG $= 0.866\ V_m\ (\pi/2 - \pi/3) = 0.4532\ V_m$

Area GCD $= V_m$. Total area under the curve $= V_m\ (1 + 0.432 + 0.5)$

Average value, $V_{av} = (1.9532\ V_m)/3.14$

$= 0.622\ V_m$

For evaluating rms value, the square of the function is to be taken, its mean value calculated and square-root of the mean value found out.

Area under the squared function :

For Portion OF : $V_m^2 \int_0^{\pi/2} \sin^2\theta d\theta = V_m^2/2 \int_0^{\pi/2} (1 - \cos 2\theta)\, d\theta = 0.307\ V_m^2$

For Portion FG : $(0.866\ V_m)^2 \times \pi \times (1/2 - 1/3) = 0.3925\ V_m^2$

For Portion GD : $V_m^2 \int_0^{\pi/2} \sin^2\theta\, d\theta = 0.785\ V_m^2$

Total area $= V_m^2\ [0.307 + 0.3925 + 0.785] = 1.4845\ V_m^2$

Let R.M.S. Value be V_c

$V_m^2\ \pi = 1.4845\ V_m^2$, or $V_c = 0.688 V_m$

Form factor = RMS Value/Average Value = 0.688/0.622 = 1.106

Peak factor = Peak Value/RMS Value = 1.0/0.688 = 1.4535

Tutorial Problems No. 11.2

1. The values of the instantaneous currents in the branches of a parallel circuit are as follows : $i_1 = 5 \sin 346\ t$; $i_2 = 10 \sin(346t + \pi/4)$; $i_3 = 7.5 \sin(346\ t + \pi/2)$; $i_4 = 8 \sin(346\ t - \pi/3)$ Express the resultant line current in the same form as the original expression and determine the r.m.s. value and the frequency of this current. **[12.5 A; 55 Hz]**
2. Four coils are connected in series. Each has induced in it a sinusoidal e.m.f. of 100 V, 50 Hz and there is a phase difference of 14 electrical degrees between one coil and the next. What is the total e.m.f. generated in the circuit ? **[384 V]**
3. The instantaneous voltage across each of the four coils connected in series is given by $v_1 = 100 \sin 471\ t$; $v_2 = 250 \cos 471\ t$; $v_3 = 150 \sin(471\ t + \pi/6)$; $v_4 = 200 \sin(471\ t - \pi/4)$ Determine the total p.d. expressed in similar form to those given. What will be the resultant p.d. If v_2 is reversed in sign ? **[v = 414 sin (471 t + 26.5°); v = 486 sin (471 t –40°)]**
4. An alternating voltage of $v = 100 \sin 376.8\ t$ is applied to a circuit consisting of a coil having a resistance of 6Ω and an inductance of 21.22 mH.

(*a*) Express the current flowing in the circuit in the form $i = I_n \sin(376.8\,t \pm \phi)$
(*b*) If a moving-iron voltmeter, a wattmeter and a frequency meter are connected in the circuit, what would be the respective readings on the instruments ?

[i = 10 sin (376.8 *t* –53.1°); 70.7 V; 300 W; 60 Hz]

5. Three circuits *A*, *B* and *C* are connected in series across a 200-V supply. The voltage across circuit *A* is 50 V lagging the supply voltage by 45° and the voltage across circuit *C* is 100 V leading the supply voltage by 30°. Determine graphically or by calculation, the voltage across circuit *B* and its phase displacement from the supply voltage. **[79.4 V ; 10° 38′ lagging]**

6. Three alternating currents are given by
$i_1 = 141 \sin(\omega t + \pi/4)$ $i_2 = 30 \sin(\omega t + \pi/2)$ $i_3 = 20 \sin(\omega t - \pi/6)$
and are fed into a common conductor. Find graphically or otherwise the equation of the resultant current and its r.m.s. value. **[i = 167.4 sin (ωt + 0.797), I_{rms} = 118.4 A]**

7. Four e.m.fs $e_1 = 100 \sin \omega t$, $e_2 = 80 \sin(\omega t - \pi/6)$, $e_3 = 120 \sin(\omega t + \pi/4)$ and $e_4 = 100 \sin(\omega t - 2\pi/3)$ are induced in four coils connected in series so that the vector sum of four e.m.fs. is obtained. Find graphically or by calculation the resultant e.m.f. and its phase difference with (*a*) e_1 and (*b*) e_2.If the connections to the coil in which the e.m.f. e_2 is induced are reversed, find the new resultant e.m.f . **[208 sin (ωt – 0.202) (*a*) 11°34′ lag (*b*) 18′ 26′ lead; 76 sin (ωt + 0.528)]**

8. Draw to scale a vector diagram showing the following voltages :
$v_1 = 100 \sin 500\,t$; $v_2 = 200 \sin(500\,t + \pi/3)$; $v_3 = -50 \cos 500\,t$; $v_4 = 150 \sin(500\,t - \pi/4)$
Obtain graphically or otherwise, their vector sum and express this in the form $V_m \sin(500\,t \pm \phi)$, using v_1 as the reference vector. Give the r.m.s. value and frequency of the resultant voltage.

[360.5 sin (500 *t* + 0.056); 217 V; 79.6 Hz]

11.28. A.C. Through Resistance, Inductance and Capacitance

We will now consider the phase angle introduced between an alternating voltage and current when the circuit contains resistance only, inductance only and capacitance only. ***In each case, we will assume that we are given the alternating voltage of equation $e = E_m \sin \omega t$*** and will proceed to find the equation and the phase of the alternating current produced in each case.

11.29. A.C. Through Pure Ohmic Resistance Alone

The circuit is shown in Fig. 11.56. Let the applied voltage be given by the equation.

$$v = V_m \sin\theta = V_m \sin \omega t \qquad \text{...(i)}$$

Let R = ohmic resistance ; i = instantaneous current

Obviously, the applied voltage has to supply ohmic voltage drop only. Hence for equilibrium

$$v = iR;$$

Putting the value of 'v' from above, we get $V_m \sin \omega t = iR$; $i = \dfrac{V_m}{R} \sin \omega t$ ...(ii)

Current 'i' is maximum when $\sin \omega t$ is unity $\therefore$ $I_m = V_m/R$ Hence, equation (*ii*) becomes,

$i = I_m \sin \omega t$ ...(iii)

Comparing **(*i*)** and **(*ii*)**, we find that the alternating voltage and current are in phase with each other as shown in Fig. 11.57. It is also shown vectorially by vectors V_R and I in Fig. 11.54.

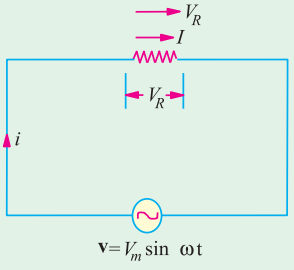


Fig. 11.56

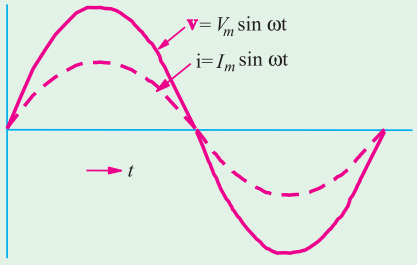


Fig. 11.57

Power. Instantaneous power, $p = vi = V_m I_m \sin^2 \omega t$ ...(Fig. 11.58)

$$= \frac{V_m I_m}{2}(1 - \cos 2\omega t) = \frac{V_m I_m}{2} - \frac{V_m I_m}{2} \cos 2\omega t$$

Power consists of a constant part $\frac{V_m I_m}{2}$ and a fluctuating part $\frac{V_m I_m}{2}\cos 2\omega t$ of frequency double that of voltage and current waves. For a complete cycle. the average value of $\frac{V_m I_m}{2}\cos 2\omega t$ is zero.

Hence, power for the whole cycle is

$$P = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}$$

or $P = V \times I$ watt

where V = r.m.s. value of applied voltage.

I = r.m.s. value of the current.

It is seen from Fig. 11.58 that no part of the power cycle becomes negative at any time. In other words, in a purely resistive circuit, power is never zero. This is so because the instantaneous values of voltage and current are always either both positive or negative and hence the product is always positive.

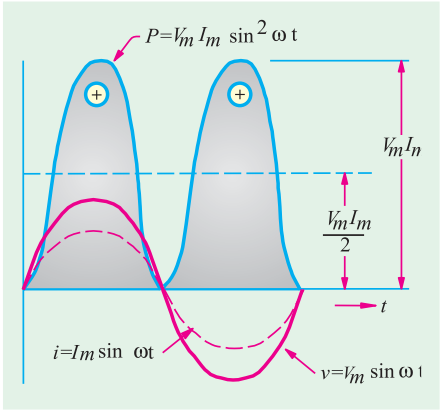


Fig. 11.58

Example 11.38. *A 60-Hz voltage of 115 V (r.m.s.) is impressed on a 100 ohm resistance :* ***(i)*** *Write the time equations for the voltage and the resulting current. Let the zero point of the voltage wave be at t = 0* ***(ii)*** *Show the voltage and current on a time diagram.* ***(iii)*** *Show the voltage and current on a phasor diagram.*

[Elect Technology. Hyderabad Univ. 1992, Similar Example, U.P. Technical Univ. 2001]

Solution. ***(i)*** $V_{max} = \sqrt{2}\ \text{V} = \sqrt{2} \times 115 = 163\ \text{V}$

$I_{max} = V_{max}/R = 163/100 = 1.63$ A; $\phi = 0$; $\omega = 2\pi f = 2\pi \times 60 = 377$ rad/s

The required equations are : $v\,(t) = 1.63 \sin 377\,t$ and $i\,(t) = 1.63 \sin 377\,t$

(ii) and ***(iii)*** These are similar to those shown in Fig. 11.56 and 11.57

11.30. A.C. Through Pure Inductance Alone

Whenever an alternating voltage is applied to a purely inductive coil*, a back e.m.f. is produced due to the self-inductance of the coil. The back e.m.f., at every step, opposes the rise or fall of current through the coil. As there is no ohmic voltage drop, the applied voltage has to overcome this self-induced e.m.f. only. So at every step

$$v = L\frac{di}{dt}$$

Now $v = V_m \sin \omega t$

$$\therefore \quad V_m \sin \omega t = L\frac{di}{dt} \quad \therefore \quad di = \frac{V_m}{L} \sin \omega t\, dt$$

Integrating both sides, we get $i = \frac{V_m}{L} \int \sin \omega t\, dt$

$$\frac{V_m}{L}(\ \cos\ \ t) \qquad \text{...(constant of integration = 0)}$$

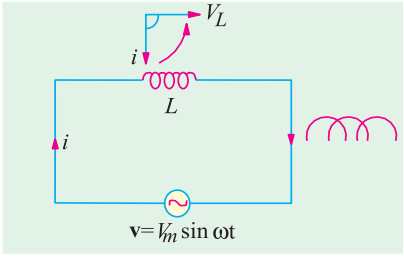


Fig. 11.59

* By purely inductive coil is meant one that has no ohmic resistance and hence no I^2R loss. Pure inductance is actually no attainable, though it is very nearly approached by a coil wound with such thick wire that its resistance is negligible. If it has some actual resistance, then it is represented by a separate equivalent resistance joined in series with it.

$$\therefore \quad = \frac{V_m}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right) = \frac{V_m}{X_L} \sin(\omega t - \pi/2) \qquad \text{...(ii)}$$

Max. value of i is $I_m = \frac{V_m}{\omega L}$ when $\sin\left(\omega t - \frac{\pi}{2}\right)$ is unity.

Hence, the equation of the current becomes $i = I_m \sin(\omega t - \pi/2)$.

So, we find that if applied voltage is represented by $v = V_m \sin \omega t$, then current flowing in a *purely* inductive circuit is given by $i = I_m \sin\left(\omega t - \frac{\pi}{2}\right)$

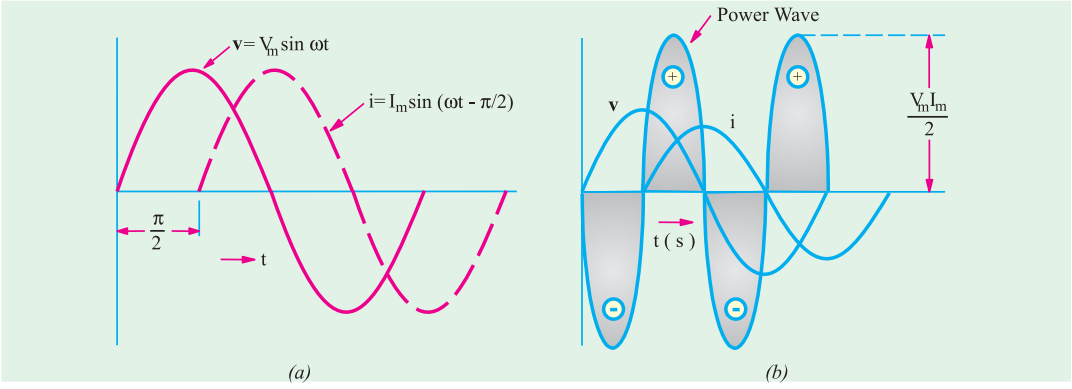


Fig. 11.60

Clearly, the current lags behind the applied voltage by a quarter cycle (Fig. 11.60) or the phase difference between the two is $\pi/2$ with voltage leading. Vectors are shown in Fig. 11.59 where voltage has been taken along the reference axis. We have seen that $I_m = V_m/\omega L = V_m/X_L$. Here '$\omega L$' plays the part of 'resistance'. It is called the (inductive) ***reactance*** X_L of the coil and is given in ohms if L is in henry and $\omega$is in radian/second.

Now, $X_L = \omega L = 2\pi f L$ ohm. It is seen that X_L depends directly on frequency of the voltage. Higher the value of f, greater the reactance offered and ***vice-versa***.

Power

Instantaneous power $= v_i = V_m I_m \sin t \sin t \frac{}{2} \quad V_m I_m \sin t.\cos t$* $\frac{V_m I_m}{2} \sin 2t$

Power for whole cycle is $P = -\frac{V_m I_m}{2} \int_0^{2\pi} \sin 2\omega t \, dt = 0$

It is also clear from Fig. 11.60 (*b*) that the average demand of power from the supply for a complete cycle is zero. Here again it is seen that power wave is a sine wave of frequency double that of the voltage and current waves. The maximum value of the instantaneous power is $V_m I_m/2$.

Example 11.39. *Through a coil of inductance 1 henry, a current of the wave-form shown in Fig. 11.61 (a) is flowing. Sketch the wave form of the voltage across the inductance and calculate the r.m.s. value of the voltage.* **(Elect. Technology, Indore Univ.)**

Solution. The instantaneous current i (t) is given by

(i) $0 < t < 1$ second, here slope of the curve is $1/1 = 1$.

$\therefore \quad i = 1 \times t = t$ ampere

(ii) $1 < t < 3$ second, here slope is $\frac{1-(-1)}{2} = 1$

* Or $p = \frac{1}{2} E_m I_m [\cos 90° - \cos(2\omega t - 90°)]$. The constant component $= \frac{1}{2} E_m I_m \cos 90° = 0$. The pulsating component is $-\frac{1}{2} E_m I_m \cos(2\omega t - 90°)$ whose average value over one complete cycle is zero.

$\therefore \quad t = 1 - (1)\,(t-1) = 1 - (t-1) = (2-t)$ ampere

(iii) $3 < t < 4$ second, here slope is $\dfrac{1-0}{1} = -1$

(b) $i = -1 - (-1)\,(t-3) = (t-4)$ ampere

The corresponding voltage are (*i*) $v_1 = Ldi/dt = 1 \times 1 = 1$ V (*ii*) $v_2 = Ldi/dt = 1 \times \dfrac{d}{dt}(2-t) = -1$ V

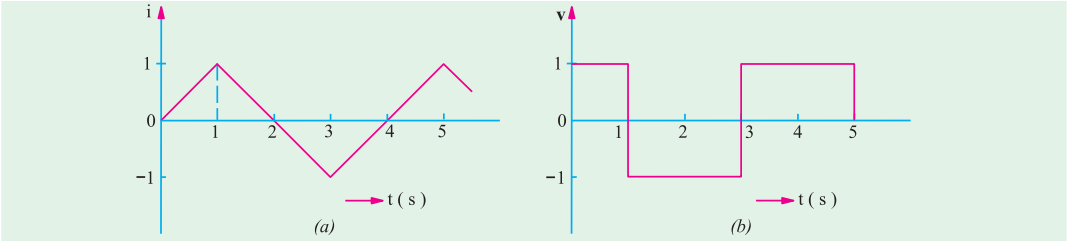


Fig. 11.61

(iii) $v_3 = Ldi/dt = 1 \times \dfrac{d}{dt}(t-4) = 1$ V

The voltage waveform is sketched in Fig. 11.61. Obviously, the r.m.s. value of the symmetrical square voltage waveform is **1 V.**

Example 11.40. *A 60-Hz voltage of 230-V effective value is impressed on an inductance of 0.265 H.*

(i) *Write the time equation for the voltage and the resulting current. Let the zero axis of the voltage wave be at t = 0.* ***(ii)*** *Show the voltage and current on a phasor diagram.* ***(iii)*** *Find the maximum energy stored in the inductance.* **(Elect. Engineering, Bhagalpur Univ.)**

Solution. $V_{max} \quad \sqrt{2}\,V \quad \sqrt{2} \quad 230\,V, f \quad 60$ *Hz.*

$\omega = 2\pi f = 2\pi \times 60 = 377$ rad/s, $X_L = \omega L = 377 \times 0.265 = 100\ \Omega$

(i) The time equation for voltage is $v\,(t) = 230\sqrt{2}\sin 377\,t$;

$I_{max} \quad V_{max}/X_L \quad 230\sqrt{2}/100 \quad 2.3\sqrt{2}, \quad 90$ (lag)

$\therefore$ Current equation is $i\,(t) = 2.3\sqrt{2}\sin(377\,t - \pi/2)$ or $= 2.3\sqrt{2}\cos 377\,t$.

(ii) It is shown in Fig. 11.56. ***(iii)*** $E_{max} = \dfrac{1}{2}LI_{max}^2 = \dfrac{1}{2} \times 0.265 \times (2.3\sqrt{2})^2 =$ **1.4 J**

11.31. Complex Voltage Applied to Pure Inductance

In Art. 11.30, the applied voltage was a pure sine wave (*i.e.* without harmonics) given by

$$v = V_m \sin \omega t.$$

The current was given by $i = I_m \sin(\omega t - \pi/2)$

Now, it is applied voltage has a complex form and is given by *

$$v = V_{1m} \sin \omega t + V_{3m} \sin 3\omega t + V_{5m} \sin 5\omega t$$

then the reactances offered to the fundamental voltage wave and the harmonics would be different.

For the fundamental wave, $X_1 = \omega L$. For 3rd harmonic ; $X_3 = 3\omega L$. For 5th harmonic ; $x_5 = 5\omega L$.

Hence, the current would be given by the equation.

$$i = \frac{V_{1m}}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right) + \frac{V_{3m}}{3\omega L}\sin\left(3\omega t - \frac{\pi}{2}\right) + \frac{V_{5m}}{5\omega L}\sin\left(5\omega t - \frac{\pi}{2}\right)$$

* It is assumed that the harmonics have no individual phase differences.

Obviously, the harmonics in the current wave are much smaller than in the voltage wave. For example, the 5th harmonic of the current wave is of only 1/5th of the harmonic in the voltage wave. It means that the self-inductance of a coil has the effect of 'smoothening' current waveform when the voltage waveform is complex *i.e.* contains harmonics.

Example 11.41. *The voltage applied to a purely inductive coil of self-inductance 15.9 mH is given by the equation, v = 100 sin 314 t + 75 sin 942 t + 50 sin 1570 t. Find the equation of the resulting current wave.*

Solution. Here $\omega = 314$ rad/s $\quad \therefore X_1 = \omega L = (15.9 \times 10^{-3}) \times 314 = 5\ \Omega$

$X_3 = 3\omega L = 3 \times 5 = 15\ \Omega$; $X_5 = 5\omega L = 5 \times 5 = 25\ \Omega$

Hence, the current equation is

$i = (100/5) \sin (314\ t - \pi/2) + (75/15) \sin (942t - \pi/2) + (50/25) \sin (1570t - \pi/2)$

or $i = \mathbf{20 \sin (314t - \pi/2) + 5 \sin (942t - \pi/2) + 2 \sin (1570\ t - \pi/2)}$

11.32. A.C. Through Pure Capacitance Alone

When an alternating voltage is applied to the plates of a capacitor, the capacitor is charged first in one direction and then in the opposite direction. When reference to Fig. 11.62, let

v = p.d. developed between plates at any instant

q = Charge on plates at that instant.

Then $q = Cv$ ...where C is the capacitance

$= C V_m \sin \omega t$ ...putting the value of v.

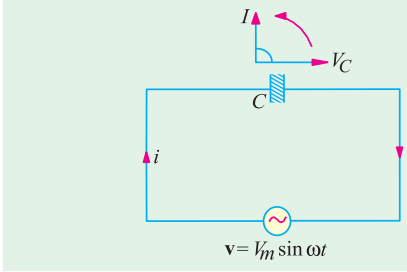


Fig. 11.62

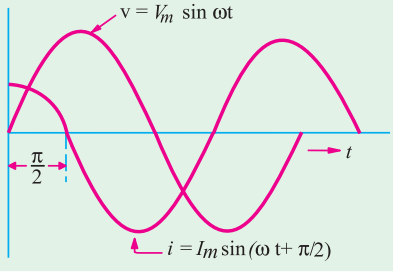


Fig. 11.63

Now, current i is given by the rate of flow of charge.

$$\therefore \quad i \quad \frac{dq}{dt} \quad \frac{d}{dt}(CV_m \sin \quad t) \quad CV_m \cos \quad t \text{ or } i \quad \frac{V_m}{1/\ C} \cos \quad t \quad \frac{V_m}{1/\ C} \sin \quad t \quad \frac{}{2}$$

Obviously, $I_m \quad \frac{V_m}{1/\ C} \quad \frac{V_m}{X_C}$ $\quad i \quad I_m \sin \quad t \quad \frac{}{2}$

The denominator $X_C = 1/\omega C$ is known as capacitive *reactance* and is in ohms if C is in farad and ω in radian/second. It is seen that if the applied voltage is given by $v = V_m \sin \omega t$, then the current is given by $i = I_m \sin (\omega t + \pi/2)$.

Hence, we find that the current in a pure capacitor leads its voltage by a quarter cycle as shown in Fig. 11.63 or phase difference between its voltage and current is $\pi/2$ with the current leading. Vector representation is given in Fig. 11.63. Note that V_c is taken along the reference axis.

Power. Instantaneous power

$p = vi = V_m \sin \omega t . I_m \sin (\omega t + 90°)$

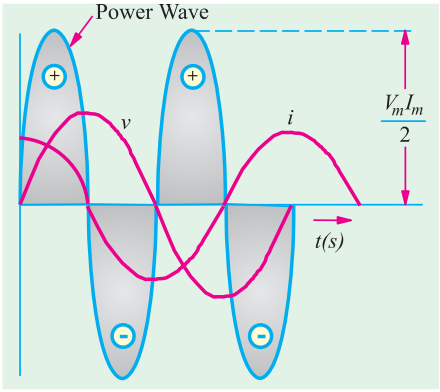


Fig. 11.64

$$= V_m I_m \sin t \cos t^{*} \quad \frac{1}{2} V_m I_m \sin 2t$$

Power for the whole cycle

$$= \frac{1}{2} V_m I_m \int_0^{2\pi} \sin 2\omega t \, dt = 0$$

This fact is graphically illustrated in Fig. 11.64. We find that in a purely capacitive circuit **, the average demand of power from supply is zero (as in a purely inductive circuit). Again, it is seen that power wave is a sine wave of frequency double that of the voltage and current waves. The maximum value of the instantaneous power is $V_m I_m/2$.

Example 11.42. *A 50-Hz voltage of 230 volts effective value is impressed on a capacitance of 26.5 μF.* **(a)** *Write the time equations for the voltage and the resulting current. Let the zero axis of the voltage wave be at t = 0.* **(b)** *Show the voltage and current on a time diagram.*

(c) *Show the voltage and current on a phasor diagram.* **(d)** *Find the maximum energy stored in the capacitance. Find the relative heating effects of two current waves of equal peak value, the one sinusoidal and the other rectangular in waveform.* **(Elect. Technology, Allahabad Univ. 1991)**

Solution. $V_{max} = 230\sqrt{2} \quad 325$ V

$\omega = 2\pi \times 50 = 314$ rad/s; $X_C = 1/\omega C = 10^6/314 \times 26.5 = 120\ \Omega$

$I_{max} = V_{max}/X_C = 325/120 = 2.71$ A, $\phi = 90°$ (lead)

(a) $v(t) = 325 \sin 314t$; $i(t) = 2.71 \sin (314t + \pi/2) = 2.71 \cos 314t$.

(b) and (c) These are shown in Fig. 11.59.

(d) $E_{max} = \frac{1}{2} C V^2_{max} = \frac{1}{2}(26.5 \times 10^{-6}) \times 325^2 =$ **1.4 J**

(e) Let I_m be the peak value of both waves.

For sinusoidal wave : $H \propto I^2 R \propto (I_m/\sqrt{2})^2 R \propto I_m^2 R/2$. For rectangular wave : $H \propto I_m^2 R$ - Art. 12.15.

$$\frac{H \text{ rectangular}}{H \text{ sinusoidal}} = \frac{I_m^2 R}{I_m^2 R/2} = \mathbf{2}$$

Example 11.43. *A 50-μF capacitor is connected across a 230-V, 50-Hz supply. Calculate* **(a)** *the reactance offered by the capacitor* **(b)** *the maximum current and* **(c)** *the r.m.s. value of the current drawn by the capacitor.*

Solution. **(a)** $X_C \quad \frac{1}{C} \quad \frac{1}{2 f_C} \quad \frac{1}{2 \quad 50 \quad 50 \quad 10^{-6}}$ **63.6 Ω**

(c) Since 230 V represents the r.m.s. value,

$\therefore \quad I_{r.m.s.} = 230/X_c = 230/63.6 =$ **3.62 A** **(b)** $I_m = I_{r.m.s.} \times \sqrt{2} = 3.62 \times \sqrt{2} =$ **5.11 A**

---

* Or power $p = \frac{1}{2} E_m I_m [\cos 90° - \cos (2\omega t - 90°)]$. The constant component is again zero. The pulsating component averaged over one complete cycle is zero.

** By pure capacitor is meant one that has neither resistance nor dielectric loss. If there is loss in a capacitor, then it may be represented by loss in (*a*) high resistance joined in parallel with the pure capacitor or (*b*) by a comparatively low resistance joined in series with the pure capacitor. But out of the two alternatives usually. (*a*) is chosen (Art. 13.8).

Example 11.44. *The voltage applied across 3-branched circuit of Fig. 11.65 is given by v = 100 sin (5000t + π/4). Calculate the branch currents and total current.*

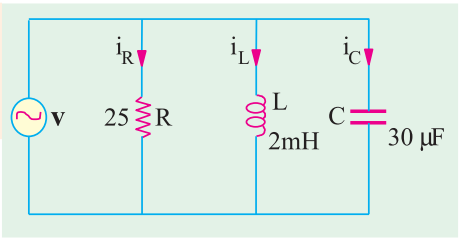


Fig. 11.65

Solution. The total instantaneous current is the vector sum of the three branch currents.

$i_r = i_R + i_L + i_C$.

Now $i_R = v/R = 100 \sin (5000\ t + \pi/4)/25$

$= 4 \sin (5000\ t + \pi/4)$

$$i_L = \frac{1}{L}\int v\,dt = \frac{10^3}{2}\int 100 \sin\left(5000t + \frac{\pi}{4}\right) dt$$

$$= \frac{10^3 \times 100}{2}\left[\frac{-\cos (5000\ t + \pi/4)}{5000}\right] = -10 \cos (5000\ t + \pi/4)$$

$$i_C = C\frac{dv}{dt} = C.\frac{d}{dt}[100 \sin (5000\ t + \pi/4)]$$

$= 30 \times 10^{-6} \times 100 \times 5000 \times \cos (5000\ t + \pi/4) = 15 \cos (5000\ t + \pi/4)$

$i_r = 4 \sin (5000\ t + \pi/4) - 10 \cos (5000\ t + \pi/4) + 15 \cos (5000t + \pi/4)$

$= \mathbf{4 \sin (5000\ t + \pi/4) + 5 \cos (5000\ t + \pi/4)}$

Tutorial Problems No. 11.2

1. Define r.m.s. and average value as applied to ac voltage, prove that in pure inductive circuit, current lags behind applied voltage at an angle 90^0. (Down waveform) *(Nagpur University, Summer 2002)*
2. Find the rms value, average value, form factor and peak factor for the waveform shown in figure. *(Nagpur University, Winter 2003)*

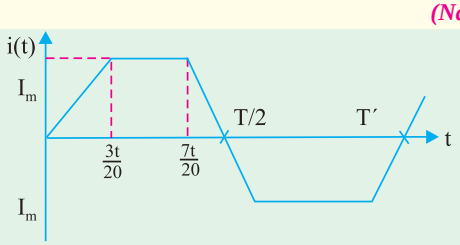


Fig. 11.66

3. Derive an expression for the instantaneous value of alternating sinusoidal e.m.f. in terms of its maximum value, angular freq. and time. *(Gujrat University, June/July 2003)*
4. Prove that average power consumption in pure inductor is zero when a.c. voltage is applied. *(Gujrat University, June/July 2003)*
5. Define and explain the following : (*i*) time period (*ii*) amplitude (*iii*) phase difference. *(Gujrat University, June/July 2003)*
6. What is the r.m.s value of an a.c. quantity? Obtain expression for the r.m.s. value of a sinusoidal current in terms of its maximum value. *(V.T.U., Belgaum Karnataka University, February 2002)*
7. Deduce an expression for the average power in a single phase series R.L. circuit and therefrom explain the term power factor. *(V.T.U., Belgaum Karnataka University, February 2002)*

8. Derive an expression for the RMS value of a sine wave.
(V.T.U., Belgaum Karnataka University, Summer 2002)
9. With a neat sketch briefly explain how an alternating voltage is produced when a coil is rotated in a magnetic field. *(V.T.U., Belgaum Karnataka University, Summer 2003)*
10. Derive expressions for average value and RMS value of a sinusoidally varying AC voltage.
(V.T.U., Belgaum Karnataka University, Summer 2003)
11. A circuit having a resistance of 12Ω, an inductance of 0.15 H and a capacitance of 100μ*f* in series is connected across a 100V, 50Hz supply. Calculate the impedance, current, the phase difference between the current and supply voltage.
(V.T.U., Belgaum Karnataka University, Summer 2003)
12. Two circuits with impedances of $Z_1 = 10 + j15\Omega$ and $Z_2 = 6 - j8\Omega$ are connected in parallel. If the supply current is 20A, what is the power dissipated in each branch?
(V.T.U., Belgaum Karnataka University, Summer 2003)
13. Show that the power consumed in a pure inductance is zero.
(U.P. TechnicalUniversity 2002) (RGPV Bhopal 2002)
14. What do you understand by the terms power factor, active power and reactive power?
(U.P. TechnicalUniversity 2002) (RGPV Bhopal 2002)
15. Current flowing through each line.
(RGPV Bhopal December 2002)
16. Distinguish between (*i*) apparent power (*ii*) active power and (*iii*) reactive power in A.C. circuits.
(U.P. TechnicalUniversity 2002) (RGPV Bhopal June 2003)

OBJECTIVE TESTS – 11

1. An a.c. current given by $i = 14.14 \sin(\omega t + \pi/6)$ has an r.m.s value of — amperes.
(*a*) 10
(*b*) 14.14
(*c*) 1.96
(*d*) 7.07
and a phase of — degrees.
(*e*) 180
(*f*) 30
(*g*) – 30
(*h*) 210
2. If $e_1 = A \sin \omega t$ and $e_2 = B \sin(\omega t - \phi)$, then
(*a*) e_1 lags e_2 by θ
(*b*) e_2 lags e_1 by θ
(*c*) e_2 leads e_2 by θ
(*d*) e_1 is in phase with e_2
3. From the two voltage equations $e_A = E_m \sin 100\pi t$ and $e_B = E_m \sin(100\pi t + \pi/6)$, it is obvious that
(*a*) *A* leads *B* by 30°
(*b*) *B* achieves its maximum value 1/600 second before *A* does.
(*c*) *B* lags behind *A*
(*d*) *A* achieves its zero value 1/600 second before *B*.
4. The r.m.s. value of a half-wave rectified current is 10A, its value for full-wave rectification would be — amperes.
(*a*) 20
(*b*) 14.14
(*c*) $20/\pi$
(*d*) $40/\pi$
5. A resultant current is made of two components : a 10 *A* d.c. component and a sinusoidal component of maximum value 14.14 A. The average value of the resultant current is — amperes.
(*a*) 0 (*b*) 24.14
(*c*) 10 (*d*) 4.14
and r.m.s. value is — amperes.
(*e*) 10 (*f*) 14.14
(*g*) 24.14 (*h*) 100
6. The r.m.s. value of sinusoidal a.c. current is equal to its value at an angle of — degree
(*a*) 60 (*b*) 45 (*c*) 30 (*d*) 90
7. Two sinusoidal currents are given by the equations : $i_2 = 10 \sin(\omega t + \pi/3)$ and $i_2 = 15 \sin(\omega t - \pi/4)$. The phase difference between them is — degrees.
(*a*) 105
(*b*) 75
(*c*) 15
(*d*) 60

8. As sine wave has a frequency of 50 Hz. Its angular frequency is — radian/second.

(*a*) $50/\pi$

(*b*) $50/2\ \pi$

(*c*) $50\ \pi$

(*d*) $100\ \pi$

9. An a.c. current is given by $i = 100 \sin 100$. It will achieve a value of 50 A after — second.

(*a*) 1/600

(*b*) 1/300

(*c*) 1/1800

(*d*) 1/900

10. The reactance offered by a capacitor to alternating current of frequency 50 Hz is 10Ω. If frequency is increased to 100 Hz reactance becomes—ohm.

(*a*) 20 (*b*) 5

(*c*) 2.5 (*d*) 40

11. A complex current wave is given by $i = 5 + 5 \sin 100\ \pi t$ ampere. Its average value is — ampere.

(*a*) 10 (*b*) 0

(*c*) $\sqrt{50}$ (*d*) 5

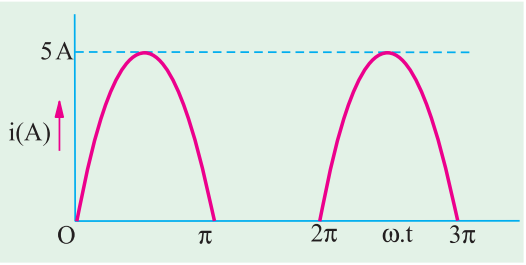


Fig. 11.67

12. The current through a resistor has a waveform as shown in Fig. 11.67. The reading shown by a moving coil ammeter will be—ampere.

(*a*) $5/\sqrt{2}$ (*b*) $2.5/\sqrt{2}$

(*c*) $5/\pi$ (*d*) 5

(Principles of Elect. Engg. Delhi Univ.)

ANSWERS

1. *a, f* **2.** *b* **3.** *b* **4.** *b* **5.** *c, f* **6.** *b* **7.** *a* **8.** *d* **9.** *a* **10.** *b*
11. *d* **12.** *c*

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Contents

CHAPTER 12

COMPLEX NUMBERS

Learning Objectives

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- Symbolic Notation
- Significance of Operator *j*
- Conjugate Complex Numbers
- Trigonometrical Form of Vector
- Exponential Form of Vector
- Polar Form of Vector Representation
- Addition and Subtraction of Vector Quantities
- Multiplication and Division of Vector Quantities
- Power and Root of Vectors
- The 120° Operator



In multi-phase generation of electricity, it is often convenient to express each phase as a complex number, Z, which has the form a + jb, where a and b are real numbers. This representation is known as Carterian form of Z.

12.1. Mathematical Representation of Vectors

There are various forms or methods of representing vector quantities, all of which enable those operations which are carried out graphically in a phasor diagram, to be performed analytically. The various methods are :

(i) Symbolic Notation. According to this method, a vector quantity is expressed algebraically in terms of its rectangular components. Hence, this form of representation is also known as Rectangular or Cartesian form of notation or representation.

(ii) Trigonometrical Form (iii) Exponential Form (iv) Polar Form.

12.2. Symbolic Notation

A vector can be specified in terms of its X-component and Y-component. For example, the vector OE_1 (Fig. 12.1) may be completely described by stating that its horizontal component is a_1 and vertical component is b_1. But instead of stating this verbally, we may express symbolically

$$\mathbf{E_1} = a_1 + jb_1$$

where symbol j, known as an operator, indicates that component b_1 is perpendicular to component a_1 and that the two terms are ***not*** to be treated like terms in any algebraic expression. The vector written in this way is said to be written in ***'complex form'***. In Mathematics, a_1 is known as real component and b_1 as imaginary component but in electrical engineering, these are known as ***in phase*** (or active) and ***quadrature*** (or reactive) components respectively.

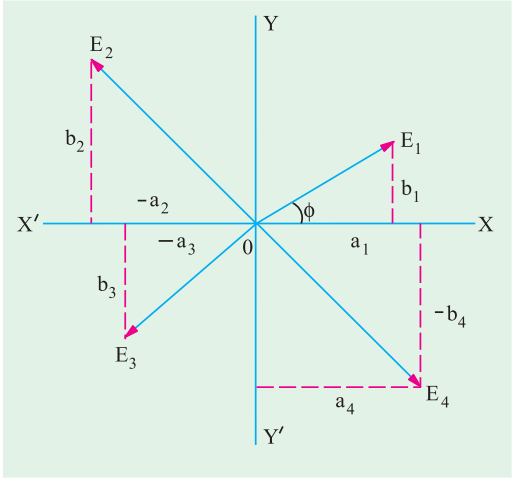


Fig. 12.1

The other vectors OE_2, OE_3 and OE_4 can similarly, be expressed in this form.

$$\mathbf{E_2} = -a_2 + jb_2 \; ; \; \mathbf{E_3} = -a_3 - jb_3 \; ; \; \mathbf{E_4} = +a_4 - jb_4$$

It should be noted that in this book, a vector quantity would be represented by letters in heavy type and its numerical or scalar value by the same letter in ordinary type.* Other method adopted for indicating a vector quantity is to put an arrow about the letter such as $\vec{E}$.

The numerical value of vector E_1 is $\sqrt{a_1^2 + b_1^2}$. Its angle with X-axis is given by $\phi = \tan^{-1}(b_1/a_1)$.

12.3. Significance of Operator *j*

The letter j used in the above expression is a symbol of an operation. Just as symbols $\times$, $+\sqrt{\ }$, $\int$ etc. are used with numbers for indicating certain operations to be performed on those numbers, similarly, symbol j is used to indicate the counter-clockwise rotation of a vector through 90°. It is assigned a value of $\sqrt{(-1)}$ **. The double operation of j on a vector rotates it counter-clockwise through 180° and hence reverses its sense because

---

* The magnitude of a vector is sometimes called 'modulus' and is represented by $|E|$ or E.

** In Mathematics, $\sqrt{(-1)}$ is denoted by i but in electrical engineering j is adopted because letter i is reserved for representing current. This helps to avoid confusion.

$$j\,j = j^2 = \sqrt{(-1)^2} = -1$$

When operator j is operated on vector **E**, we get the new vector j**E** which is displaced by 90° in counter-clockwise direction from **E** (Fig. 12.2). Further application of j will give j^2**E** = –**E** as shown.

If the operator j is applied to the vector j^2**E**, the result is j^3**E** = $-j$**E**. The vector j^3**E** is 270° counter-clockwise from the reference axis and is directly opposite to j**E**. If the vector j^3**E** is, turn, operated on by j, the result will be

$$j^4\mathbf{E} = \left[\sqrt{(-1)}\right]^4 \mathbf{E} = \mathbf{E}$$

Hence, it is seen that successive applications of the operator j to the vector **E** produce successive 90° steps of rotation of the vector in the counter-clockwise direction without in anyway affecting the magnitude of the vector.

It will also be seen from Fig. 12.2 that the application of $-j$ to **E** yields $-j$**E** which is a vector of identical magnitude but rotated 90° ***clockwise*** from **E**.

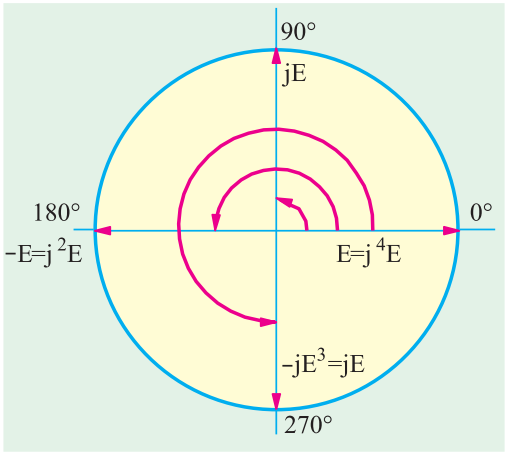


Fig. 12.2

Summarising the above, we have

j = 90° ccw rotation = $\sqrt{(-1)}$

j^2 = 180° ccw rotation = $[\sqrt{(-1)}]^2 = -1$;

j^3 = 270° ccw rotation = $[\sqrt{(-1)}]^3 = -\sqrt{(-1)} = -j$

j^4 = 360° ccw rotation = $[\sqrt{(-1)}]^4 = +1$;

j^5 = 450° ccw rotation = $[\sqrt{(-1)}]^5 = -\sqrt{(-1)} = j$

It should also be noted that $\dfrac{1}{j} = \dfrac{j}{j^2} = \dfrac{j}{-1} = -j$

12.4. Conjugate Complex Numbers

Two complex numbers are said to be conjugate if they differ only in the algebraic sign of their quadrature components. Accordingly, the numbers $(a + jb)$ and $(a - jb)$ are conjugate. The sum of two conjugate numbers gives in-phase (or active) component and their difference gives quadrature (or reactive) component.

12.5. Trigonometrical Form of Vector

From Fig. 12.3, it is seen that ***X-component of*** **E** is $E \cos\theta$ and Y-component is $E \sin\theta$. Hence, we can represent the vector **E** in the form : **E** = E $(\cos\theta + j \sin\theta)$

This is equivalent to the rectangular form **E** = $a + jb$ because $a = E \cos\theta$ and $b = E \sin\theta$. In general, **E** = E $(\cos\theta \pm j \sin\theta)$.

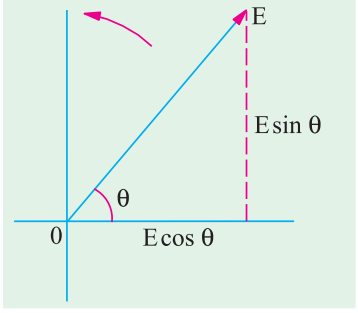


Fig. 12.3

12.6. Exponential Form of Vector

It can be proved that $e^{\pm j\theta} = (\cos\theta \pm j \sin\theta)$

This equation is known as Euler's equation after the famous mathematician of 18th century : Leonard Euler.

This equation follows directly from an inspection of Maclaurin* series expansions of sin θ, cos θ and $e^{j\theta}$.

When expanded into series form :

$$\cos\theta = 1 - \frac{\theta^2}{L\,2} + \frac{\theta^4}{L\,4} - \frac{\theta^6}{L\,6} + \ldots \text{ and } \sin\theta = \theta - \frac{\theta^3}{L\,3} + \frac{\theta^5}{L\,5} - \frac{\theta^7}{L\,7} + \ldots$$

$$e^{j\theta} = 1 + j\theta + \frac{(j\theta)^2}{L\,2} + \frac{(j\theta)^3}{L\,3} + \frac{(j\theta)^4}{L\,4} + \frac{(j\theta)^5}{L\,5} + \frac{(j\theta)^6}{L\,6} + \ldots$$

Keeping in mind that $j^2 = -1$, $j^3 = -j$, $j^4 = 1$, $j^5 = -j$, $j^6 = -1$, we get

$$e^{j\theta} = \left(1 - \frac{\theta^2}{L\,2} + \frac{\theta^4}{L\,4} - \frac{\theta^6}{L\,6} + \ldots\right) + j\left(\theta - \frac{\theta^3}{L\,3} + \frac{\theta^5}{L\,5} - \frac{\theta^7}{L\,7} + \ldots\right)$$

$$\therefore \quad e^{j\theta} = \cos\theta + j\sin\theta$$

Similarly, it can be shown that $e^{-j\theta} = \cos\theta - j\sin\theta$

Hence $\mathbf{E} = E(\cos\theta \pm y' \sin\theta)$ can be written as $\mathbf{E} = Ee^{\pm j\theta}$. This is known as exponential form of representing vector quantities. It represents a vector of numerical value E and having phase angle of $\pm\theta$ with the reference axis.

12.7. Polar Form of Vector Representation

The expression $E(\cos\theta + j\sin\theta)$ is written in the simplified form of $E \angle\theta$. In this expression, E represents the magnitude of the vector and θ its inclination (in ccw direction) with the X-axis. For angles in clockwise direction the expression becomes $E \angle -\theta$. In general, the expression is written as $E \angle \pm\theta$. ***It may be pointed out here that $E \angle \pm\theta$ is simply a short-hand or symbolic style of writing $Ee^{\pm j\theta}$.*** Also, the form is purely conventional and does not possess the mathematical elegance of the various other forms of vector representation given above.

Summarizing, we have the following alternate ways of representing vector quantities

(i) Rectangular form (or complex form) $\mathbf{E} = a + jb$

(ii) Trigonometrical form $\mathbf{E} = E(\cos\theta \pm j\sin\theta)$

(iii) Exponential form $\mathbf{E} = Ee^{\pm j\theta}$

(iv) Polar form (conventional) $\mathbf{E} = E \angle \pm\theta$.

Example 12.1. *Write the equivalent exponential and polar forms of vector 3 + j4. How will you illustrate the vector means of diagram ?*

Solution. With reference to Fig. 12.4., magnitude of the vector is = $\sqrt{3^2 + 4^2} = 5$. $\tan\theta = 4/3$.

$$\therefore \quad \theta = \tan^{-1}(4/3) = 53.1^\circ$$

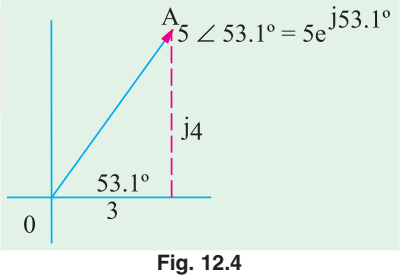


Fig. 12.4

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* Functions like cos θ, sin θ and $e^{j\theta}$ etc. can be expanded into series form with the help of Maclaurin's Theorem. The theorem states : $f(\theta) = f(0) + \frac{f'(0)\theta}{1} + \frac{f''(0)\theta^2}{\angle 2} + \frac{f'''(0)\theta^3}{\angle 3} + \ldots$ where $f(\theta)$ is function of θ which is to be expanded, $f(0)$ is the value of the function when $\theta = 0$, $f'(0)$ is the value of first derivative of $f(\theta)$ when $\theta = 0$, $f''(0)$ is the value of second derivative of function $f(\theta)$ when $\theta = 0$ etc.

$\therefore$ Exponential form = $5\, e^{j53.1^\circ}$

The angle may also be expressed in radians.

Polar form = $5 \angle 53.1^\circ$.

Example 12.2. *A vector is represented by 20 $e^{j2\pi/3}$. Write the various equivalent forms of the vector and illustrate by means of a vector diagram, the magnitude and position of the above vector.*

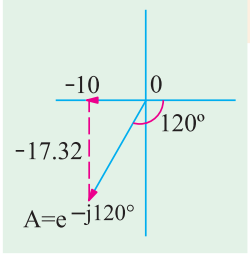


Fig. 12.5

Solution. The vector is drawn in a direction making an angle of $2\pi/3 = 120^\circ$ in the clockwise direction (Fig. 12.5). The clockwise direction is taken because the angle is negative.

(*i*) Rectangular Form $a = 20 \cos(-120^\circ) = -10$;

$b = 20 \sin(-120^\circ) = -17.32$

$\therefore$ Expression is = $(-10 - j17.32)$

(*ii*) Polar Form is $20 \angle -120^\circ$

12.8. Addition and Subtraction of Vector Quantities

Rectangular form is best suited for addition and subtraction of vector quantities. Suppose we are given two vector quantities $\mathbf{E_1} = a_1 + jb_1$ and $\mathbf{E_2} = a_2 + jb_2$ and it is required to find their sum and difference.

Addition. $\mathbf{E} = \mathbf{E_1} + \mathbf{E_2} = a_1 + jb_1 + a_2 + jb_2 = (a_1 + a_2) + j(b_1 + b_2)$

The magnitude of resultant vector **E** is $\sqrt{(a_1 + a_2)^2 + (b_1 + b_2)^2}$

The position of **E** with respect to *X*-axis is $\theta = \tan^{-1}\left(\dfrac{b_1 + b_2}{a_1 + a_2}\right)$

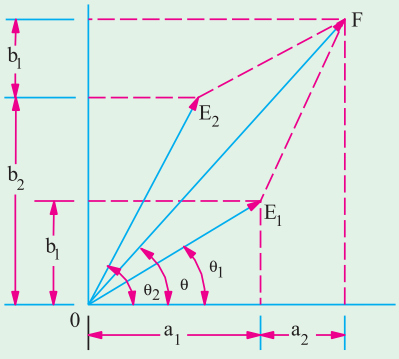


Fig. 12.6

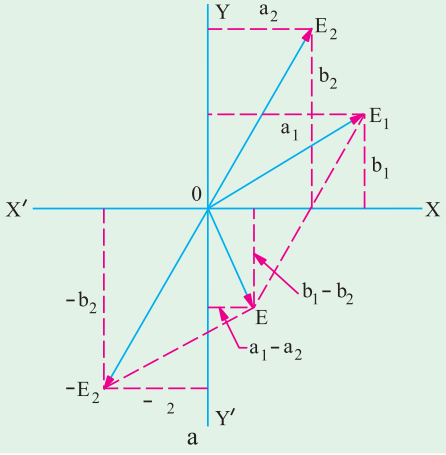


Fig. 12.7

A graphic representation of the addition process is shown in Fig. 12.6

Subtraction. $\mathbf{E} = \mathbf{E_1} - \mathbf{E_2} = (a_1 + jb_1) - (a_2 + jb_2) = (a_1 - a_2) + j(b_1 - b_2)$

Magnitude of $\mathbf{E} = \sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2}$

Its position with respect to *x*-axis is given by the angle $\theta = \tan^{-1}\left(\dfrac{b_1 - b_2}{a_1 - a_2}\right)$.

The graphic representation of the process of subtraction is shown in Fig. 12.7.

12.9. Multiplication and Division of Vector Quantities

Multiplication and division of vectors becomes very simple and easy ***if they are represented in the polar or exponential form***. As will be shown below, the rectangular form of representation is not well-suited for this process.

(*i*) Multiplication – *Rectangular form*

Let the two vectors be given by $\mathbf{A} = a_1 + jb_1$ and $\mathbf{B} = a_2 + jb_2$

$$\therefore \quad \mathbf{A} \times \mathbf{B} = \mathbf{C} = (a_1 + jb_1)(a_2 + jb_2) = a_1a_2 + j^2b_1b_2 + j(a_1b_2 + b_1a_2)$$
$$= (a_1a_2 - b_1b_2) + j(a_1b_2 + b_1a_2) \qquad (\because\ j^2 = -1)$$

The magnitude of $\mathbf{C} = \sqrt{[(a_1a_2 - b_1b_2)^2 + (a_1b_2 + b_1a_2)^2]}$

In angle with respect to X-axis is given by $\theta = \tan^{-1}\left(\dfrac{a_1b_2 + b_1a_2}{a_1a_2 - b_1b_2}\right)$

(ii) Division – *Rectangular Form :* $\dfrac{\mathbf{A}}{\mathbf{B}} = \dfrac{a_1 + jb_1}{a_2 + jb_2} = \dfrac{(a_1 + jb_1)(a_2 - jb_2)}{(a_2 + jb_2)(a_2 - jb_2)}$

Both the numerator and denominator have been multiplied by the conjugate of $(a_2 + jb_2)$ *i.e.* by $(a_2 - jb_2)$

$$\therefore \quad \frac{\mathbf{A}}{\mathbf{B}} = \frac{(a_1a_2 + b_1b_2) + j(b_1a_2 - a_1b_2)}{a_2^2 + b_2^2} = \frac{a_1a_2 + a_1b_2}{a_2^2 + b_2^2} + j\frac{b_1a_2 + a_1b_2}{a_2^2 + b_2^2}$$

The magnitude and the angle with respects X-axis can be found in the same way as given above.

As will be noted, both the results are somewhat awkward but unfortunately, there is no easier way to perform multiplication in rectangular form.

(*iii*) Multiplication – *Polar Form*

Let $\mathbf{A} = a_1 + jb_1 = A\angle\alpha = Ae^{j\alpha}$ where $\alpha = \tan^{-1}(b_1/a_1)$

$\mathbf{B} = a_2 + jb_2 = B\angle\beta = Be^{j\beta}$ where $\beta = \tan^{-1}(b_2/a_2)$

$\therefore$ $\mathbf{AB} = A\angle\alpha \times B\angle\beta = AB\angle(\alpha + \beta)$* or $AB = Ae^{j\alpha} \times Be^{j\beta} = ABe^{j(\alpha + \beta)}$

Hence, product of any two vector **A** and **B** is given by another vector equal in length to $\mathbf{A} \times \mathbf{B}$ and having a phase angle equal to the sum of the angles of **A** and **B**.

$$\frac{\mathbf{A}}{\mathbf{B}} = \frac{A\angle\alpha}{B\angle\beta} = \frac{A}{B}\angle(\alpha - \beta)$$

Hence, the quotient $\mathbf{A} \div \mathbf{B}$ is another vector having a magnitude of $\mathbf{A} \div \mathbf{B}$ and phase angle equal to angle of **A** minus the angle of **B**.

Also $$\frac{\mathbf{A}}{\mathbf{B}} = \frac{Ae^{j\alpha}}{Be^{j\beta}} = \frac{A}{B}e^{j(\alpha - \beta)}$$

As seen, the division and multiplication become extremely simple if vectors are represented in their polar or exponential form.

Example 12.3. *Add the following vectors given in rectangular form and illustrate the process graphically.*

$$A = 16 + j12, \quad B = -6 + j10.4$$

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* $\mathbf{A} = A(\cos\alpha + j\sin\alpha)$ and $\mathbf{B} = B(\cos\beta + j\sin\beta)$

$\therefore$ $\mathbf{AB} = AB(\cos\alpha\cos\beta + j\sin\alpha\cos\beta + j\cos\alpha\sin\beta + j^2\sin\alpha\sin\beta)$

$= AB[\cos\alpha\cos\beta - \sin\alpha\sin\beta) + j(\sin\alpha\cos\beta + \cos\alpha\sin\beta)]$

$= AB[\cos(\alpha + \beta) + j\sin(\alpha + \beta)] = AB\angle(\alpha + \beta)$

Solution. $\mathbf{A + B} = C = (16 + j\,12) + (-6 + j\,10.4) = 10 + j\,22.4$

$\therefore$ Magnitude of $\mathbf{C} = \sqrt{(10^2 + 22.4^2)} = 25.5$ units

Slope of $\mathbf{C} = \theta = \tan^{-1}\left(\frac{22.4}{10}\right) = 65.95°$

The vector addition is shown in Fig. 12.8.

$\alpha = \tan^{-1}(12/16) = 36.9°$

$\beta = \tan^{-1}(-10.4/6) = -240°$ or $120°$

The resultant vector is found by using parallelogram law of vectors (Fig. 12.8).

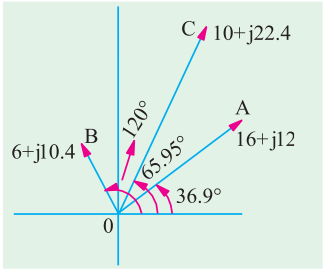


Fig. 12.8

Example 12.4. *Perform the following operation and express the final result in the polar form :* $5 \angle 30° + 8 \angle -30°$. **(Elect. Engg. & Electronics Bangalore Univ. 1989)**

Solution. $5 \angle 30° = 5(\cos 30° + j \sin 30°) = 4.33 + j\,2.5$

$8 \angle -30° = 8[\cos(-30°) + j \sin(-30°)] = 8(0.866 - 0.5) = 6.93 - j\,4$

$\therefore 5\angle 30° + 8\angle -30° = 4.33 + j2.5 + 6.93 - j4 = 11.26 - j1.5 = \sqrt{11.26^2 + 1.5^2}\ \angle \tan^{-1}(-1.5/11.26) = 11.35$

$\tan^{-1}(-0.1332) = 11.35 \angle 7.6°$

Example 12.5. *Subtract the following given vectors from one another :*

$A = 30 + j\,52$ *and* $B = -39.5 - j\,14.36$

Solution. $\mathbf{A - B} = \mathbf{C} = (30 + j\,52) - (-39.5 - j\,14.36) = 69.5 + j\,66.36$

$\therefore$ Magnitude of $C = \sqrt{(66.5^2 + 66.36^2)} = 96$

Slope of $C = \tan^{-1}(66.36/69.5) = 43.6°$ $\therefore$ $\mathbf{C} = 96 \angle 43.6°$.

Similarly $\mathbf{B - A} = -69.5 - j\,66.36 = 96 \angle 223.6°$* or $= 96 \angle -136.4°$

Example 12.6. *Given the following two vectors :*

$A = 20 \angle 60°$ *and* $B = 5 \angle 30°$

Perform the following indicated operations and illustrate graphically ***(i)*** $A \times B$ *and* ***(ii)*** A/B.

Solution. ***(i)*** $\mathbf{A \times B} = \mathbf{C} = 20 \angle 60° \times 5 \angle 30° = 100 \angle 90°$

Vectors are shown in Fig. 12.9.

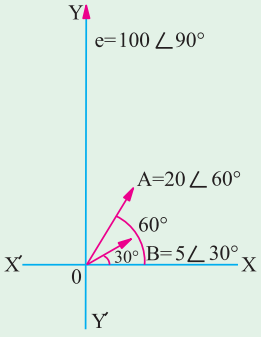


Fig. 12.9

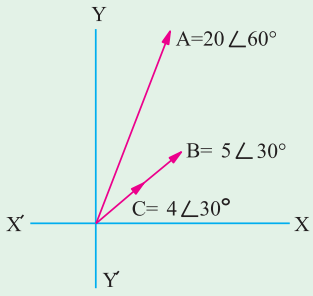


Fig. 12.10

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* $\tan\theta = 66.36/69.5$ or $\theta = \tan^{-1}(66.36/69.5) = 43.6°$. Since both components are negative, the vector lies in third quadrant. Hence, the angle measured from + ve direction of X-axis and in the CCW directions is $= (180 + 43.6) = 223.6°$.

(ii) $\dfrac{\mathbf{A}}{\mathbf{B}} = \dfrac{20 \angle 60°}{5 \angle 30°} = 4 \angle 30°$ —Fig. 12.10

Example 12.7. *Perform the following operation and the final result may be given in the polar form : (8 + j6) × (-10 –j 7.5)* **(Elect. Engg. & Electronics Bangalore Univ. 1990)**

Solution. We will use the following two methods to solve the above question.

Method No. 1

We know that multiplication of $(A + B)$ and $(C + D)$ can be found as under :

$$\begin{array}{l} A + B \\ \times\ C + D \\ \hline CA\ +\ CB \\ \quad +\ DA + DB \\ \hline CA + CD + DA + DB \end{array}$$

Similarly, the required multiplication can be carried out as follows :

$$\begin{array}{l} 8 + j\,6 \\ \times -10\ \ -j\ 7.5 \\ \hline -\ 80\ -\ j\ 60 \\ -j\ 60\ -\ j^2\ 45 \\ \hline -80\ -j120\ +\ 45 \end{array}$$

or $-35 - j120 = \sqrt{(-35)^2 + (-120)^2}\ \big|\tan^{-1}(120/35) = 125 \tan^{-1} 3.42 = 125\,\big|73.8°$

Since both the components of the vector are negative, it obviously lies in the third quadrant. As measured from the *X*-axis in the *CCW* direction, its angle is = 180° + 73.8° = 253.8°. Hence, the product vector can be written as 125 ∠ 53.8°.

Method No. 2 $8 + j\,6 = 10 \angle 36.9°$, $- 10 - j\,7.5 = 12.5 \tan^{-1} 0.75 = 12.5 \angle 36.9°$.

Again as explained in Method 1 above, the actual angle of the vector is 180° + 36.9° = 216.9°

$\therefore\ -10 - j\,7.5 = 12.5 \angle 216.9\ \ \therefore\ 10 \angle 36.9° \times 12.5 \angle 216.9° = 125 < 253.8°$

Example 12.8. *The following three vectors are given :*

$$A = 20 + j20,\ B = 30 \angle -120 \text{ and } C = 10 + j0$$

Perform the following indicated operations :

(i) $\dfrac{AB}{C}$ and **(ii)** $\dfrac{BC}{A}$.

Solution. Rearranging all three vectors in polar form, we get

$$\mathbf{A} = 28.3 \angle 45°\ ,\ \mathbf{B} = 30 \angle -120°,\ \mathbf{C} = 10 \angle 0°$$

(i) $\dfrac{\mathbf{AB}}{\mathbf{C}} = \dfrac{28.3 \angle 45° \times 30 \angle -120°}{10 \angle 0°} = 84.9 \angle -75°$

(ii) $\dfrac{\mathbf{BC}}{\mathbf{A}} = \dfrac{30 \angle -120 \times 10 \angle 0°}{28.3 \angle 45°} = 10.6 \angle -165°$

Example 12.9. *Given two current $i_1 = 10 \sin(\omega t + \pi/4)$ and $i_2 = 5 \cos(\omega t - \pi/2)$, find the r.m.s. value of $i_1 + i_2$ using the complex number representation.* **[Elect. Circuit Theory, Kerala Univ.]**

Solution. The maximum value of first current is 10 A and it leads the reference quantity by 45°. The second current can be written as

$$i_2 = 5 \cos(\omega t - \pi/2) = 5 \sin[90 + (\omega t - \pi/2)] = 5 \sin \omega t$$

Hence, its maximum value is 5 A and is in phase with the reference quantity.

$\therefore\ \ \mathbf{I}_{m1} = 10 (\cos 45° + j \sin 45°) = (7.07 + j\,7.07)$

$$\mathbf{I}_{m2} = 5(\cos 0° + j \sin 0°) = (5 + j\,0)$$

The maximum value of resultant current is

$$\mathbf{I}_m = (7.07 + j\,7.07) + (5 + j\,0) = 12.07 + j\,7.07 = 14 \angle 30.4°$$

$$\therefore \quad \text{R.M.S. value} = 14/\sqrt{2} = \mathbf{10\ A}$$

12.10. Power and Roots of Vectors

(a) Powers

Suppose it is required to find the cube of the vector $3 \angle 15°$. For this purpose, the vector has to be multiplied by itself three times.

$\therefore \quad (3 \angle 15)^3 = 3 \times 3 \times 3. \angle (15° + 15° + 15°) = 27 \angle 45°$. In general, $\mathbf{A^n = A^n \angle n\alpha}$

Hence, nth power of vector **A** is a vector whose magnitude is A^n and whose phase angle with respect to X-axis is $n\alpha$.

It is also clear that $\mathbf{A^n B^n} = A^n B^n \angle (n\alpha + n\beta)$

(b) Roots

It is clear that $\sqrt[3]{(8 \angle 45°)} = 2 \angle 15°$

In general, $\sqrt[n]{\mathbf{A}} = \sqrt[n]{A} = \angle \alpha/n$

Hence, nth root of a vector **A** is a vector whose magnitude is $\sqrt[n]{A}$ and whose phase angle with respect to X-axis is α/n.

12.11. The 120° Operator

In three-phase work where voltage vectors are displaced from one another by 120°, it is convenient to employ an operator which rotates a vector through 120° forward or backwards without changing its length. This operator is 'a'. Any operator which is multiplied by 'a' remains unchanged in magnitude but is rotated by 120° in the counter-clockwise (ccw) direction.

$$\therefore \quad \alpha = 1 \angle 120°$$

This, when expressed in the cartesian form, becomes

$$a = \cos 120° + j \sin 120° = -0.5 + j\,0.866$$

Similarly, $a^2 = 1 \angle 120° \times 1 \angle 120° = 1 \angle 240° = \cos 240° + j \sin 240° = -0.5 - j\,0.866$

Hence, operator 'a^2' will rotate the vector in ccw by 240°. This is the same as rotating the vector in *clockwise* direction by 120°.

$\therefore \quad a^2 = 1 \angle -120°$. Similarly, $a^3 = 1 \angle 360° = 1$*

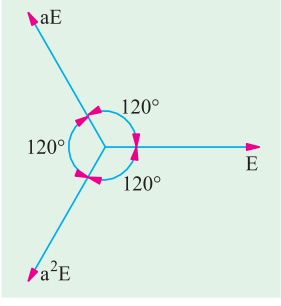


Fig. 12.11

As shown in Fig. 12.11, the 3-phase voltage vectors with standard phase sequence may be represented as E, a^2E and aE or as E, $E\,(-0.5 - j\,0.866)$ and $E\,(-0.5 + j\,0.866)$

It is easy to prove that

(i) $a^2 + a = -1$ **(ii)** $a^2 + a + 1 = 0$ **(iii)** $a^3 + a^2 + a = 0$

Note. We have seen in Art. 12.3 that operator $-j$ turns a vector through $-90°$ *i.e.* through 90° in *clockwise* direction. But it should be clearly noted that operator '$-a$' *does not* turn a vector through $-120°$. Rather '$-a$' turns a vector through $-60°$ as shown below.

Example 12.10. *Evaluate the following expressions in the polar form* **(i)** $a^2 - 1$, **(ii)** $1 - a - a^2$ **(iii)** $2a^2 + 3 + a$ **(iv)** ja. **[Elect. Meas and Meas. Inst., Madras Univ.]**

* Numerically, a is equivalent to the cube root of unity.

Solution. ***(i)*** $a^2 = a \times a = 1 \angle 120° = 1 \angle 240° = 1 \angle -120° = -\frac{1}{2} - j\frac{\sqrt{3}}{2}$

$\therefore \quad a^2 - 1 = \quad \frac{1}{2} \quad j\frac{\sqrt{3}}{2} \quad 1 \quad j\frac{\sqrt{3}}{2}\sqrt{3} \quad \frac{\sqrt{3}}{2} \quad j\frac{1}{2} \quad \sqrt{3} \qquad 180 \quad \sqrt{3} \quad \mathbf{210}$

(ii) $\qquad a = 1\angle 120° = -\frac{1}{2} + j\frac{\sqrt{3}}{2}\,;\, a^2 = 1\angle 240° = -\frac{1}{2} - j\frac{\sqrt{3}}{2}$

$\therefore \qquad 1 - a - a^2 = 1 \quad \frac{1}{2} \quad j\frac{\sqrt{3}}{2} \quad \frac{1}{2} \quad j\frac{\sqrt{3}}{2} \quad 2 \quad j0 \quad \mathbf{2} \quad \mathbf{0}$

(iii) $\qquad 2a^2 = 2\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) = -1 - j\sqrt{3}$

$\qquad 2a = 2 \quad \frac{1}{2} \quad j\frac{\sqrt{3}}{2} \qquad \mathbf{1} \quad \boldsymbol{j\sqrt{3}}$

$\therefore \qquad 2a^2 + 3 + 2a = 3 - 1 - j\sqrt{3} - 1 + j\sqrt{3} = 1\angle 0°$

(iv) $\qquad ja = j \times a = 1 \angle 90° \times 1 \angle 120° = 1 \angle 210°$

Tutorial Problem No. 12.1

1. Perform the following indicated operations :
(*a*) (60 + *j* 80) + (30 – *j* 40) (*b*) (12 – *j* 6) – (40 – *j* 20) (*c*) (6 + *j* 8) (3 – *j* 4) (*d*) 16 + *j*8) ÷ (3 – *j* 4)
[(*a*) (90 + *j* 40) (*b*) (–28 + *j* 14) (*c*) (50 + *j* 0) (*d*) (–0.56 + *j* 1.92)]

2. Two impedances $\mathbf{Z}_1 = 2 + j\,6\ \Omega$ and $\mathbf{Z}_2 = 6 - 12\ \Omega$ are connected in a circuit so that they are additive. Find the resultant impedance in the polar form. **[10 ∠ – 36.9°]**

3. Express in rectangular form and polar form a vector, the magnitude of which is 100 units and the phase of which with respect to reference axis is
(*a*) + 30° (*b*) + 180° (*c*) –60° (*d*) + 120° (*e*) –120° (*f*) –210°.
[(*a*) 86.6 + *j*50 ∠30° (*b*) (–100 + *j*0), 100 ∠180° (*c*) 50 –*j*86.6, 100 ∠–60° (*d*) (–50 + *j*86.6), 100 ∠–120° (*e*) (–50 –*j* 86.6), 100 ∠– 120° (*f*) (–50 + *j* 86.6), 100 ∠– 210°]

4. In the equation $\mathbf{V}_m = \mathbf{V} - \mathbf{ZI}$, $\mathbf{V} = 100 \angle 0°$ volts, $\mathbf{Z} = 10 \angle 60°\ \Omega$ and $\mathbf{I} = 8 \angle -30°$ amperes. Express $\mathbf{V}_m$ in polar form. **[50.5 ∠ – 52°]**

5. A voltage $\mathbf{V} = 150 + j\,180$ is applied across an impedance and the current flowing is found to be $\mathbf{I} = 5 - j\,4$. Determine (*i*) scalar impedance (*ii*) resistance (*iii*) reactance (*iv*) power consumed.
[(*i*) 3.73 Ω (*ii*) 0.75 Ω (*iii*) 36.6 Ω (*iv*) 30 W]

6. Calculate the following in polar form :
(*i*) Add (40 + j20) to (20 + j120) (*ii*) Subtract (10 + j30) from (20 – j20)
(*iii*) Multiply (15 + j20) with (20 + j30) (*iv*) Devide (6 + j7) by (5 + j3)
(Gujrat University, June/July 2003)

7. Why is impedance represented by a complex number? ***(RGPV Bhopal December 2002)***

OBJECTIVE TESTS – 12

1. The symbol *j* represents counterclockwise rotation of a vector through—degrees.
(*a*) 180 (*b*) 90
(*c*) 360 (*d*) 270

2. The operator *j* has a value of
(*a*) + 1 (*b*) – 1
(*c*) $\sqrt{-1}$ (*d*) $\sqrt{+1}$

3. The vector $j^5 E$ is the same as vector
(*a*) *jE* (*b*) $j^2 E$
(*c*) $j^3 E$ (*d*) $j^4 E$

4. The conjugate of (–*a* + *jb*) is
(*a*) (*a* – *jb*) (*b*) (– *a* – *jb*)
(*c*) (*a* + *jub*) (*d*) (*jb* – *a*)

ANSWERS

1. *b* **2.** *c* **3.** *a* **4.** *b*

Top

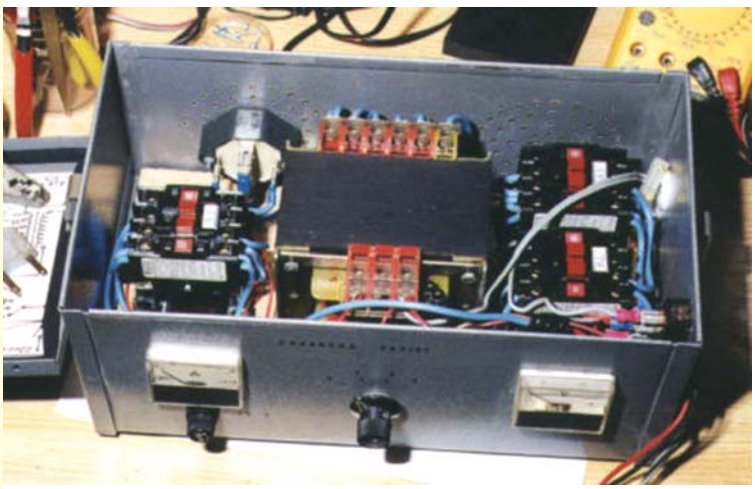
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CHAPTER 13

SERIES A.C. CIRCUITS

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- Active, Reactive and Apparent Power
- *Q*-factor of a Coil
- Power in an Iron-cored Chocking Coil
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- Relation Between Resonant Power P_0 and Off-resonant Power P



This chapter discusses series AC circuits, and how they function

13.1. A.C. Through Resistance and Inductance

A pure resistance R and a pure inductive coil of inductance L are shown connected in series in Fig. 13.1.

Let V = r.m.s. value of the applied voltage, I = r.m.s. value of the resultant current

$V_R = IR$ –voltage drop across R (in phase with I), $V_L = I \,.\, X_L$ –voltage drop across coil (ahead of I by 90°)

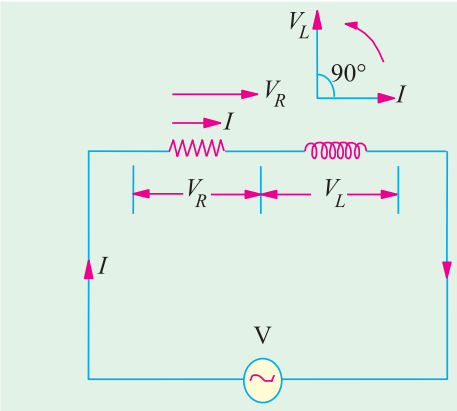


Fig. 13.1

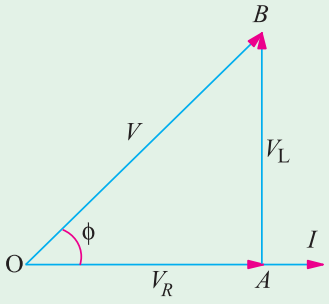


Fig. 13.2

These voltage drops are shown in voltage triangle OAB in Fig. 13.2. Vector OA represents ohmic drop V_R and AB represents inductive drop V_L. The applied voltage V is the vector sum of the two *i.e.* OB.

$$\therefore \quad V = \sqrt{(V_R^2 \quad V_L^2)} \quad \sqrt{[(IR)^2 \quad (I \,.\, X_L)^2]} \quad I\sqrt{R^2 \quad X_L^2}\,, \frac{V}{\sqrt{(R^2 \quad X_L^2)}} \quad I$$

The quantity $\sqrt{(R^2 + X_L^2)}$ is known as the ***impedance*** (Z) of the circuit. As seen from the impedance triangle ABC (Fig. 13.3) $Z^2 = R^2 + X_L{}^2$.

i.e. **(Impedance)^2 = (resistance)^2 + (reactance)^2**

From Fig. 13.2, it is clear that the applied voltage V leads the current I by an angle ϕ such that

$$\tan \phi = \frac{V_L}{V_R} = \frac{I \,.\, X_L}{I \,.\, R} = \frac{X_L}{R} = \frac{\omega L}{R} = \frac{\text{reactance}}{\text{reactance}} \quad \therefore \quad \phi = \tan^{-1} \frac{X_L}{R}$$

The same fact is illustrated graphically in Fig. 13.4.

In other words, current I lags behind the applied voltage V by an angle ϕ.

Hence, if applied voltage is given by $v = V_m \sin \omega t$, then current equation is

$$i = I_m \sin (\omega t - \phi) \text{ where } I_m = V_m / Z$$

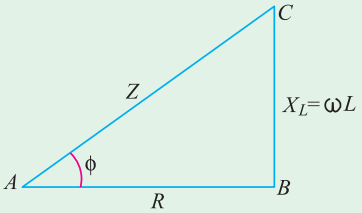


Fig. 13.3

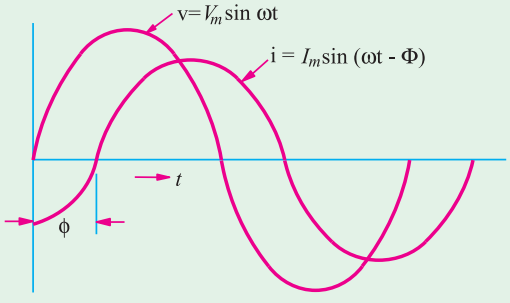


Fig. 13.4

In Fig. 13.5, I has been resolved into its two mutually perpendicular components, $I \cos \phi$ along the applied voltage V and $I \sin \phi$ in quadrature (*i.e.* perpendicular) with V.

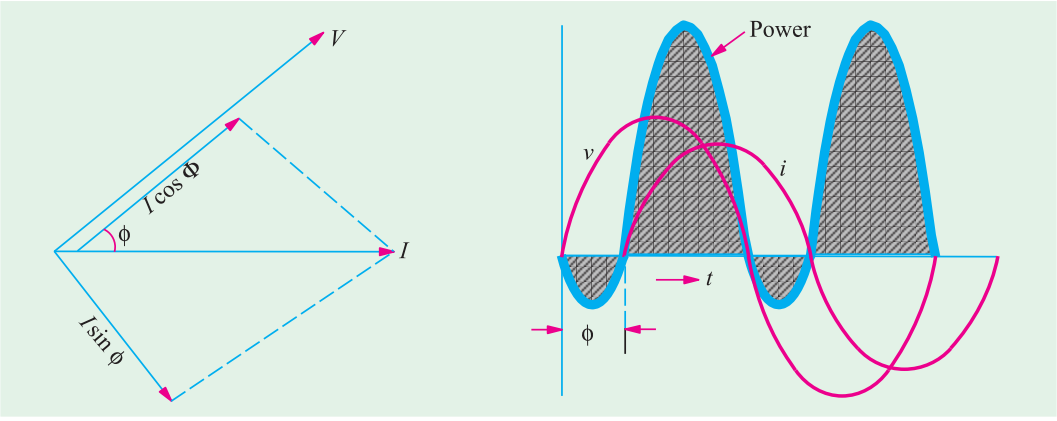


Fig. 13.5 Fig. 13.6

The mean power consumed by the circuit is given by the product of V and ***that component of the current I which is in phase with V.***

So $P = V \times I \cos \phi$ = r.m.s. voltage × r.m.s. current × $\cos \phi$

The term 'cos ϕ' is called the power factor of the circuit.

Remember that in an a.c. circuit, the product of r.m.s. volts and r.m.s. amperes gives volt-amperes (VA) and not true power in watts. True power (W) = volt-amperes (VA) × power factor.

or **Watts = VA × cos ϕ***

It should be noted that power consumed is due to ohmic resistance only because pure inductance does not consume any power.

Now $P = VI \cos \phi = VI \times (R/Z) = (V/Z) \times I.R = I^2 R$ ($\because \cos \phi = R/Z$) or $P = I^2 R$ watt

Graphical representation of the power consumed is shown in Fig. 14.6.

Let us calculate power in terms of instantaneous values.

Instantaneous power is $= v\ i = V_m \sin \omega t \times I_m \sin (\omega t - \phi) = V_m I_m \sin \omega t \sin (\omega t - \phi)$

$$= \frac{1}{2} V_m I_m [\cos \phi - \cos (2\omega t - \Phi)]$$

Obviously, this power consists of two parts (Fig. 13.7).

(i) a constant part $\frac{1}{2} V_m I_m \cos \phi$ which contributes to real power.

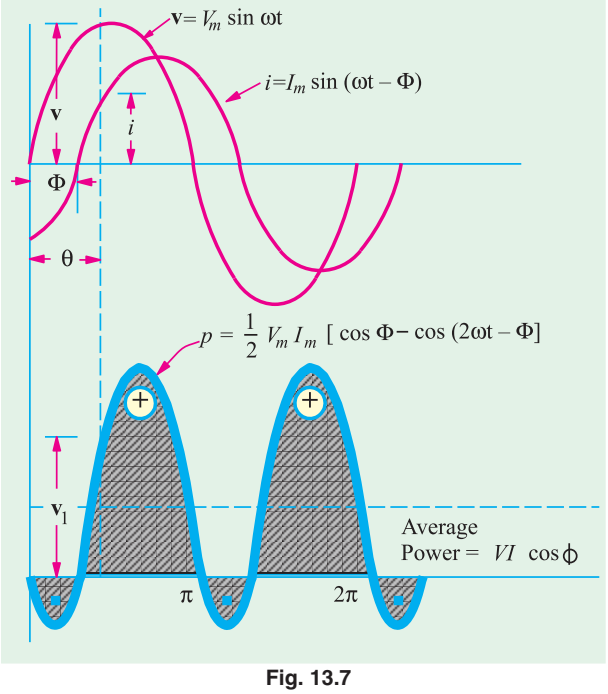


Fig. 13.7

* While dealing with large supplies of electric power, it is convenient to use kilowatt as the unit kW = kVA × cos ϕ

(ii) a pulsating component $\frac{1}{2} V_m I_m \cos(2\omega t - \phi)$ which has a frequency twice that of the voltage and current. It does not contribute to actual power since its average value over a complete cycle is zero.

Hence, average power consumed $= \frac{1}{2} V_m I_m \cos\phi = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos\phi = VI \cos\phi$, where V and I represent the r.m.s values.

Symbolic Notation. $Z = R + jX_L$

Impedance vector has numerical value of $\sqrt{(R^2 + X_L^2)}$.

Its phase angle with the reference axis is $\phi = \tan^{-1}(X_L/R)$

It may also be expressed in the polar form as $\mathbf{Z} = Z \angle \phi°$

(i) Assuming $\mathbf{V} = V \angle 0°$; $\mathbf{I} = \frac{V}{Z} = \frac{V \angle 0°}{Z \angle \phi°} = \frac{V}{Z} \angle -\phi°$ (Fig. 13.8)

It shows that current vector is lagging behind the voltage vector by $\phi°$. The numerical value of current is V/Z.

(ii) However, if we assumed that

$\mathbf{I} = I \angle 0$, then

$\mathbf{V} = \mathbf{IZ} = I \angle 0° \times Z \angle \phi°$

$= IZ \angle \phi°$

It shows that voltage vector is $\phi°$ ahead of current vector in ccw direction as shown in Fig. 13.9.

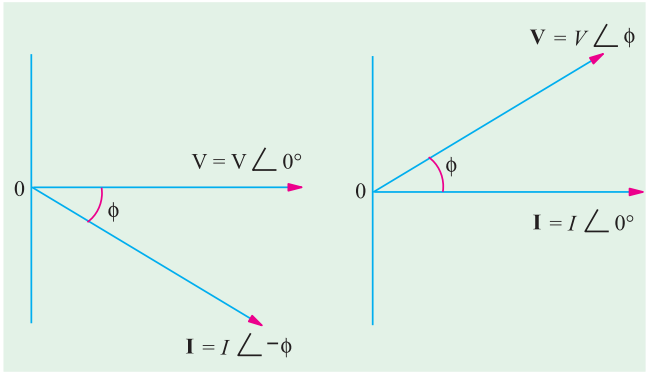


Fig. 13.8

Fig. 13.9

13.2. Power Factor

It may be defined as

(i) cosine of the angle of lead or lag

(ii) the ratio $\frac{R}{Z} = \frac{\text{resistance}}{\text{impedance}}$ (...Fig. 13.3) **(iii)** the ratio $\frac{\text{true power}}{\text{apparent power}} = \frac{\text{watts}}{\text{volt} - \text{amperes}} = \frac{W}{VA}$

13.3. Active and Reactive Components of Circuit Current I

Active component is that which is in phase with the applied voltage V *i.e.* $I \cos\phi$. It is also known as 'wattful' component.

Reactive component is that which in quadrature with V *i.e.* $I \sin\phi$. It is also known as 'wattless' or 'idle' component.

It should be noted that the product of volts and amperes in an a.c. circuit gives voltamperes (VA). Out of this, the actual power is $VA \cos\phi = W$ and reactive power is $VA \sin\phi$. Expressing the values in kVA, we find that it has two rectangular components :

(i) active component which is obtained by multiplying kVA by cos ϕ and this gives power in kW.

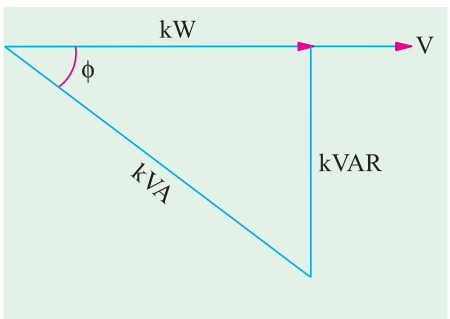


Fig. 13.10

(ii) the reactive component known as reactive kVA and is obtained by multiplying kVA by sin ϕ. It is written as kVAR (kilovar). The following relations can be easily deduced.

$$\text{kVA} = \sqrt{\text{kW}^2 + \text{kVAR}^2} \; ; \; \text{kW} = \text{kVA} \cos\phi \text{ and kVAR} = \text{kVA} \sin\phi$$

These relationships can be easily understood by referring to the kVA triangle of Fig. 13.10 where it should be noted that lagging kVAR has been taken as negative.

For example, suppose a circuit draws a current of 1000 A at a voltage of 20,000 V and has a power factor of 0.8. Then

input = 1,000 × 20,000/1000 = 20,000 kVA; $\cos\phi = 0.8$; $\sin\phi = 0.6$

Hence kW = 20,000 × 0.8 = 16,000 ; kVAR = 20,000 × 0.6 = 12,000

Obviously, $\sqrt{16000^2 + 12000^2} = 20{,}000$ *i.e.* $\text{kVA} = \sqrt{\text{kW}^2 + \text{kVAR}^2}$

13.4. Active, Reactive and Apparent Power

Let a series *R-L* circuit draw a current of *I* when an alternating voltage of r.m.s. value *V* is applied to it. Suppose that current lags behind the applied voltage by ϕ. The three powers drawn by the circuit are as under :

(i) apparent power (*S*)

It is given by the product of r.m.s. values of applied voltage and circuit current.

$\therefore \quad S = VI = (IZ) \, . \, I = I^2 Z$ volt-amperes (*VA*)

(ii) active power (*P* or *W*)

It is the power which is actually dissipated in the circuit resistance. $P = I^2R = VI \cos\phi$ watts

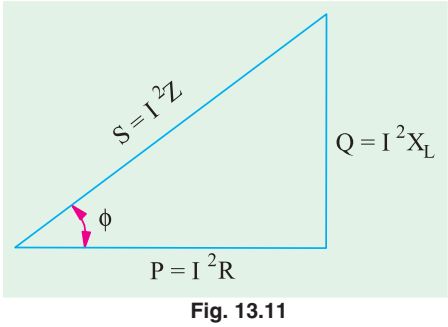


Fig. 13.11

(iii) reactive power (Q)

It is the power developed in the inductive reactance of the circuit.

$Q = I^2X_L = I^2 \, . \, Z \sin\phi = I \, . \, (IZ) . \sin\phi = VI \sin\phi$ volt-amperes-reactive (VAR)

These three powers are shown in the power triangle of Fig. 13.11 from where it can be seen that

$$S^2 = P^2 + Q^2 \text{ or } S = \sqrt{P^2 + Q^2}$$

13.5. Q-factor of a Coil

Reciprocal of power factor is called the *Q*-factor of a coil or its figure of merit. It is also known as quality factor of the coil.

$$Q \text{ factor} = \frac{1}{\text{power factor}} = \frac{1}{\cos\phi} = \frac{Z}{R}$$

If *R* is small as compared to reactance, then *Q*-factor = $Z/R = \omega L/R$

Also, $$Q = 2\pi \frac{\text{maximum energy stored}}{\text{energy dissipated per cycle}}$$ —in the coil

Example 13.1. *In a series circuit containing pure resistance and a pure inductance, the current and the voltage are expressed as :*

$$i\,(t) = 5 \sin(314\,t + 2\,\pi/3) \text{ and } v\,(t) = 15 \sin(314\,t + 5\,\pi/6)$$

(a) *What is the impedance of the circuit ?* **(b)** *What is the value of the resistance ?* **(c)** *What is the inductance in henrys ?* **(d)** *What is the average power drawn by the circuit ?* **(e)** *What is the power factor ?* **[Elect. Technology, Indore Univ.]**

Solution. Phase angle of current = $2\,\pi/3 = 2 \quad 180°/3 = 120°$ and phase angle of voltage = $5\,\pi/6 = 5 \times 180°/6 = 150°$. Also, $Z = V_m/I_m = 3\ \Omega$.

Hence, current *lags* behind voltage by 30°. It means that it is an *R-L* circuit. Also $314 = 2\pi f$ or $f = 50$ Hz. Now, $R/Z = \cos 30° = 0.866$; $R = 2.6\ \Omega$; $X_L/Z = \sin 30° = 0.5$

$\therefore \quad X_L = 1.5\ \Omega \qquad 314\,L = 1.5, \qquad L = 4.78$ mH

(a) $Z = \mathbf{3\ \Omega}$ (b) $R = \mathbf{2.6\ \Omega}$ **(c)** $L = \mathbf{4.78}$ mH

(d) $P = I^2 R = (5/\sqrt{2})^2 \times 2.6 = \mathbf{32.5\ W}$ **(e)** p.f. $= \cos 30° = \mathbf{0.866\ (lag).}$

Example 13.2. *In a circuit the equations for instantaneous voltage and current are givn by* $v = 141.4 \sin t \frac{2}{3}$ *volt and* $i = 7.07 \sin t \frac{}{2}$ *amp, where* $\omega = 314$ *rad/sec.*

(i) *Sketch a neat phasor diagram for the circuit.* **(ii)** *Use polar notation to calculate impedance with phase angle.* **(iii)** *Calculate average power & power factor.* **(iv)** *Calculate the instantaneous power at the instant* $t = 0$. **(F.Y. Engg. Pune Univ.)**

Solution. **(i)** From the voltage equation, it is seen that the voltage lags behind the reference quantity by $2\pi/3$ radian or $2 \times 180/3 = 120°$. Similarly, current lags behind the reference quantity by $\pi/2$ radian or $180/2 = 90°$. Between themselves, voltage lags behind the current by $(120 - 90) = 30°$ as shwon in Fig. 13.12 (*b*).

(ii) $V = V_m/\sqrt{2} = 141.4/\sqrt{2} = 100$ V; $I = I_m/\sqrt{2} = 7.07/\sqrt{2} = \mathbf{5\ A.}$

$\therefore \quad V = 100 \angle -120°$ and $I = 5 \angle -90° \ \therefore Z = \dfrac{100 \angle -120°}{5 \angle -90°} = 20 \angle -30°$

(iii) Average power $= VI \cos\phi = 100 \times 5 \times \cos 30° = \mathbf{433\ W}$

(iv) At $t = 0$; $v = 141.4 \sin (0 - 120°) = -122.45$ V; $i = 7.07 \sin (0 - 90°) = -7.07$ A.

$\therefore$ instantaneous power at $t = 0$ is given by $vi = (-122.45) \times (-7.07) = \mathbf{865.7\ W.}$

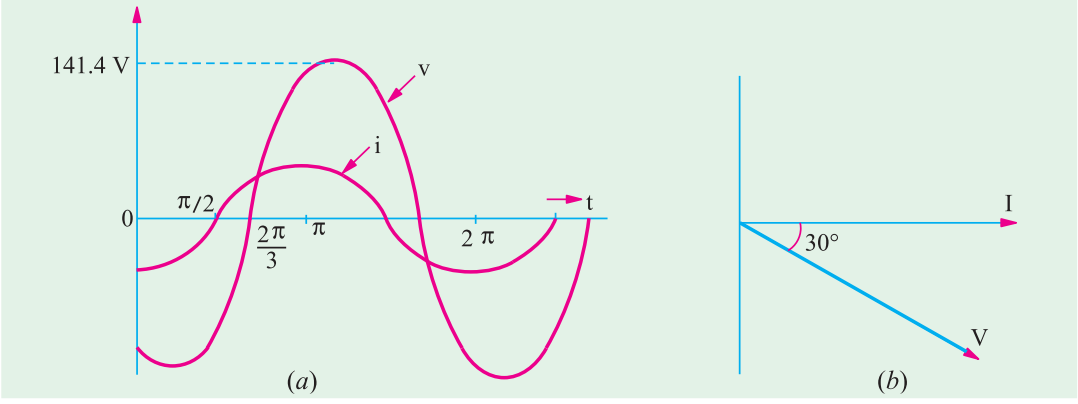


Fig. 13.12

Example 13.3. *The potential difference measured across a coil is 4.5 V, when it carries a direct current of 9 A. The same coil when carries an alternating current of 9 A at 25 Hz, the potential difference is 24 V. Find the current, the power and the power factor when it is supplied by 50 V, 50 Hz supply.* **(F.Y. Pune Univ.)**

Solution. Let R be the *d.c.* resistance and L be the inductance of the coil.

$\therefore \qquad R = V/I = 4.5/9 = 0.5\ \Omega$

With a.c. current of 25 Hz, $Z = V/I = 24/9 = 2.66\ \Omega$

$$X_L = \sqrt{Z^2 - R^2} = \sqrt{2.66^2 - 0.5^2} = 2.62\ \Omega. \quad \text{Now } X_L = 2\pi \times 25 \times L\ ; L = 0.0167\ \Omega$$

At 50 Hz

$$X_L = 2.62 \times 2 = 5.24\ \Omega\ ; Z = \sqrt{0.5^2 + 5.24^2} = 5.26\ \Omega$$

Current $I = 50/5.26 = \mathbf{9.5\ A}$; Power $= I^2 R = 9.5^2 \times 0.5 = \mathbf{45\ W.}$

Example 13.4. *In a particular R-L series circuit a voltage of 10 V at 50 Hz produces a current of 700 mA while the same voltage at 75 Hz produces 500 mA. What are the values of R and L in the circuit ?* **(Network Analysis A.M.I.E. Sec. B, S 1990)**

Solution. (*i*) $Z = \sqrt{R^2 + (2\pi \times 50\, L)^2} = \sqrt{R^2 + 98696\, L^2}$; $V = IZ$ or $10 = 700 \times 10^{-3} \sqrt{(R^2 + 98696\, L^2)}$

$$\sqrt{(R^2 + 98696\, L^2)} = 10/700 \times 10^{-3} = 100/7 \text{ or } R^2 + 98696\, L^2 = 10000/49 \quad \text{...(i)}$$

(*ii*) In the second case $Z = \sqrt{R^2 + (2\pi \times 75L)^2} = \sqrt{(R^2 + 222066\, L^2)}$

$\therefore 10 = 500 \times 10^{-3} \sqrt{(R^2 + 222066\, L^2)}$ *i.e.* $\sqrt{(R^2 + 222066\, L^2)} = 20$ or $R^2 + 222066 L^2 = 400$ **(*ii*)**

Subtracting Eq. **(*i*)** from **(*ii*)**, we get

$222066\, L^2 - 98696\, L^2 = 400 - (10000/49)$ or $123370\, L^2 = 196$ or $L = 0.0398$ H = **40 mH.**

Substituting this value of L in Eq. (*ii*), we get, $R^2 + 222066\, (0.398)^2 = 400$ $\therefore$ **$R = 6.9\ \Omega$**

Example 13.5. *A series circuit consists of a resistance of 6 Ω and an inductive reactance of 8 Ω. A potential difference of 141.4 V (r.m.s.) is applied to it. At a certain instant the applied voltage is + 100 V, and is increasing. Calculate at this current,* **(*i*)** *the current* **(*ii*)** *the voltage drop across the resistance and* **(*iii*)** *voltage drop across inductive reactance.* **(F.E. Pune Univ.)**

Solution. $Z = R + jX = 6 + j8 = 10 \angle 53.1°$

It shows that current lags behind the applied voltage by 53.1°. Let V be taken as the reference quantity. Then $v = (141.4 \times \sqrt{2})\ \sin \omega t = 200 \sin \omega t$; $i = (V_m/Z \sin \omega t) - 30° = 20 \sin (\omega t - 53.1°)$.

(*i*) When the voltage is + 100 Vand increasing; $100 = 200 \sin \omega t$; $\sin \omega t = 0.5$; $\omega t = 30°$

At this instant, the current is given by $i = 20 \sin (30° - 53.1°) = -20 \sin 23.1° =$ **-7.847 A.**

(*ii*) drop across resistor = $iR = -7.847 \times 6 = -47$ V.

(*iii*) Let us first find the equation of the voltage drop V_L across the inductive reactance. Maximum value of the voltage drop = $I_m X_L = 20 \times 8 = 160$ V. It leads the current by 90°. Since current itself lags the applied voltage by 53.1°, the reactive voltage drop across the applied voltage by (90° – 53.1°) = 36.9°. Hence, the equation of this voltage drop at the instant when $\omega t = 30°$ is

$$V_L = 160 \sin (30° + 36.9°) = 160 \sin 66.9° = \textbf{147.2 V.}$$

Example 13.6. *A 60 Hz sinusoidal voltage v = 141 sin ωt is applied to a series R-L circuit. The values of the resistance and the inductance are 3 Ω and 0.0106 H respectively.*

(*i*) *Compute the r.m.s. value of the current in the circuit and its phase angle with respect to the voltage.*

(*ii*) *Write the expression for the instantaneous current in the circuit.*

(*iii*) *Compute the r.m.s. value and the phase of the voltages appearing across the resistance and the inductance.*

(*iv*) *Find the average power dissipated by the circuit.*

(*v*) *Calculate the p.f. of the circuit.* **(F.E. Pune Univ.)**

Solution. $V_m = 141$ V; $V = 141/\sqrt{2} = 100\, V$ $\therefore$ $V = 100 + j0$

$$X_L = 2\pi \times 60 \times 0.0106 = 4\ \Omega.\ Z = 3 + j\,4 = 5 \angle 53.1°$$

(*i*) $I = V/Z = 100 \angle 0°/5 \angle 53.1° = 20 \angle -53.1°$

Since angle is minus, the current lags behind the voltage by 53.1

(*ii*) $I_m = \sqrt{2} \times 20 = 28.28$; $\therefore$ $i = 28.28 \sin (\omega t - 53.1°)$

(*iii*) $VR = \text{IR} = 20 \angle -53.1° \times 3 = 60 \angle -53.1°$ volt.

$V_L = jIX_L = 1 \angle 90° \times 20 \angle -53.1° \times 4 = 80 \angle 36.9°$

(*iv*) $P = VI \cos \phi = 100 \times 20 \times \cos 53.1° =$ **1200 W.**

(*v*) p.f. = $\cos \phi = \cos 53.1° =$ **0.6.**

Example 13.7. *In a given R-L circuit, R = 3.5 Ω and L = 0.1 H. Find* ***(i)*** *the current through the circuit and* ***(ii)*** *power factor if a 50-Hz voltage V = 220 ∠ 30° is applied across the circuit.*

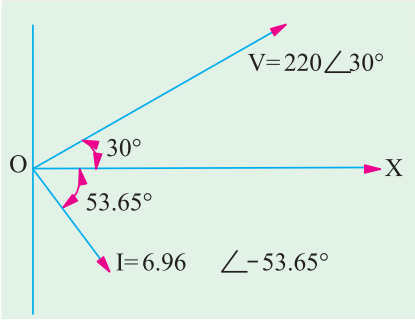


Fig. 13.13

Solution. The vector diagram is shown in Fig. 13.13.

$$X_L = 2\pi fL = 2\pi \times 50 \times 0.1 = 31.42\ \Omega$$

$$Z = \sqrt{(R^2 \quad X_L^2)} \quad \sqrt{3.5^2 \quad 31.42^2} \quad 31.6$$

$$\therefore \mathbf{Z} = 31.6 \angle \tan^{-1}(31.42/3.5) = 31.6 \angle 83.65°$$

$$\textbf{(i)}\ \mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} \quad \frac{220 \quad 30}{31.6 \quad 83.65} \quad 6.96 \quad 53.65$$

(ii) Phase angle between voltage and current is

= 53.65° + 30° = 83.65° with current lagging.

p.f. = cos 83.65° = **0.11 (lag).**

Example 13.8. *In an alternating circuit, the impressed voltage is given by V = (100 –j50) volts and the current in the circuit is I = (3 – j4) A. Determine the real and reactive power in the circuit.* **(Electrical Engg., Calcutta Univ. 1991)**

Solution. Power will be found by the conjugate method. Using current conjugate, we have

$$P_{VA} = (100 - j\,50)(3 + j\,4) = 300 + j\,400 - j\,150 + 200 = 500 + j\,250$$

Hence, real power is **500 W** and reactive power of VAR is **250.** Since the second term in the above expression is positive, the reactive volt-amperes of 250 are inductive.*

Example 13.9. *In the circuit of Fig. 14.14, applied voltage V is given by (0 + j10) and the current is (0.8 + j 0.6) A. Determine the values of R and X and also indicate if X is inductive or capacitive.* **(Elect. Technology, Nagpur Univ. 1991)**

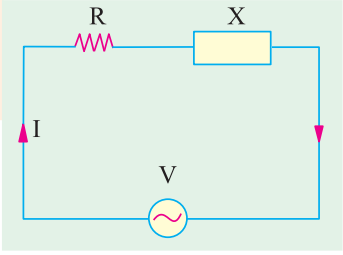


Fig. 13.14

Solution. $\mathbf{V} = 0 + j10 = \angle 90°$; $\mathbf{I} = 0.8 + j0.6 = 1\angle 36.9°$

As seen, *V* leads the reference quantity by 90° whereas *I* leads by 36.9°. In other words , *I lags* behind the applied voltage by (90° –36.9°) = 53.1°

Hence, the circuit of Fig. 13.14 is an *R-L* circuit.

Now, $\mathbf{Z = V/I} = 10\ \angle 90°/1\ \angle 36.9° = 10\ \angle 53.1° = 6 + j8$

Hence, $R =$ **6 Ω** and $X_L =$ **8 Ω**

Example 13.10. *A two-element series circuit is connected across an a.c. source* $e = 200\sqrt{2}\ \sin(\omega t + 20°)$ *V. The current in the circuit then is found to be* $i = 10\sqrt{2}\ \cos(314\,t - 25°)$ *A. Determine the parameters of the circuit.* **(Electromechanic Allahabad Univ. 1991)**

Solution. The current can be written as $i = 10\sqrt{2}\ \sin(314\,t - 25° + 90°) = 10\sqrt{2}\ \sin(314\,t + 65°)$. It is seen that applied voltage leads by 20° and current leads by 65° with regards to the reference quantity, their mutual phase difference is = 65° –(20°) = 45°. Hence, p.f. = cos 45° = **$1/\sqrt{2}$ (lead).**

Now, $V_m = 200\sqrt{2}$ and $I_m = 10\sqrt{2}$ $\quad \therefore\ Z = V_m/I_m = 200\sqrt{2}/10\sqrt{2} = 20\ \Omega$

$R = Z\cos\phi = 20/\sqrt{2}\ \Omega =$ **14.1 Ω** ; $X_c = Z\sin\phi = 20/\sqrt{2} =$ **14.1 Ω**

Now, $f = 314/2\pi = 50$ Hz. Also, $X_c = 1/2\pi fC$ $\therefore\ C = 1/2\pi \times 50 \times 14.1 =$ **226 μF**

Hence, the given circuit is an *R-C* circuit.

* If voltage conjugate is used, then capacitive VARs are positive and inductive VARs negative. If current conjugate is used, then capacitive VARs are negative and inductive VARs are positive.

Example 13.11. *Transform the following currents to the time domain :* **(i)** $6 - j\,8$ **(ii)** $-6 + j8$ **(iii)** $-j5$.

Solution. **(i)** Now, $(6 - j8)$ when expressed in the polar form is $\sqrt{6^2 + 8^2} \angle -\tan^{-1} 8/6 = 10 \angle 53.1°$. The time domain representation of this current is $i\,(t) = 10 \sin (\omega t - 53.1°)$

(ii) $-6 + j8 = \sqrt{6^2 + 8^2} \angle \tan^{-1} 8/-6 = 10 \angle 126.9°$

$\therefore \quad i\,(t) = 10 \sin (\omega t + 126.9°)$

(iii) $-j\,5 = 10 \angle -90° \quad \therefore i\,(t) = 10 \sin (\omega t - 90°)$

Example 13.12. *A choke coil takes a current of 2 A lagging 60° behind the applied voltage of 200 V at 50 Hz. Calculate the inductance, resistance and impedance of the coil. Also, determine the power consumed when it is connected across 100-V, 25-Hz supply.*

(Elect. Engg. & Electronics, Bangalore Univ.)



Choke coil

Solution. **(i)** $Z_{coil} = 200/2 = 100\ \Omega$; $R = Z \cos \phi = 100 \cos 60° = 50\ \Omega$

$X_L = Z \sin \phi = 100 \sin 60° = 86.6\ \Omega\ X_L = 2\pi fL = 86.6 \therefore L = 86.6/2\pi \times 50 = 0.275$ H

(ii) Now, the coil will have different impedance because the supply frequency is different but its resistance would remain the same *i.e.* 50 Ω. Since the frequency has been halved, the inductive reactance of the coil is also halved *i.e.* it becomes $86.6/2 = 43.3\ \Omega$.

$$Z_{\text{coil}} = \sqrt{50^2 + 43.3^2} = 66.1\ \Omega$$

$$I = 100/66.1 = 1.5 \text{ A, p.f.} = \cos \phi = 50/66.1 = 0.75$$

Power consumed by the coil $= VI \cos \phi = 100 \times 1.5 \times 0.75 =$ **112.5 W**

Example 13.13. *An inductive circuit draws 10 A and 1 kW from a 200-V, 50 Hz a.c. supply. Determine :*

(i) *the impedance in cartesian from (a + jb)* ***(ii)*** *the impedance in polar from* $Z \angle \theta$ ***(iii)*** *the power factor* ***(iv)*** *the reactive power* ***(v)*** *the apparent power.*

Solution. $Z = 200/10 = 20\ \Omega$; $P = I^2R$ or $1000 = 10^2 \times R$; $R = 10\ \Omega$; $X_L = \sqrt{20^2 - 10^2} = 17.32\Omega$.

(i) $\mathbf{Z} = 10 + j17.32$ **(ii)** $|Z| = \sqrt{(10^2 + 17.32^2)} = 20\ \Omega$; $\tan \phi = 17.32/10 = 1.732$; $\phi = \tan^{-1}(1.732)$ $= 60° \therefore$ **$Z = 20 \angle 60°$.** **(iii)** p.f. $= \cos \phi = \cos 60° =$ **0.5 lag** **(iv)** reactive power $= VI \sin \phi$ $= 200 \times 10 \times 0.866 =$ **1732 VAR** (v) apparent power $= VI = 200 \times 10 =$ **2000 VA.**

Example 13.14. *When a voltage of 100 V at 50 Hz is applied to a choking coil A, the current taken is 8 A and the power is 120 W. When applied to a coil B, the current is 10 A and the power is 500 W. What current and power will be taken when 100 V is applied to the two coils connected in series ?*

(Elements of Elect. Engg., Bangalore Univ.)

Solution.

$$Z_1 = 100/8 = 12.5\ \Omega\ ; P = I^2 . R_1 \text{ or } 120 = 8^2 \times R_1\ ; R_1 = 15/8\ \Omega$$

$$X_1 = \sqrt{Z_1^2 - R_1^2} = \sqrt{12.5^2 - (15/8)^2} = 12.36\ \Omega$$

$$Z_2 = 100/10 = 10\ \Omega\ ; 500 = 10^2 \times R_2 \quad \text{or} \quad R_2 = 5\ \Omega$$

$$X_2 = \sqrt{10^2 - 5^2} = 8.66\ \Omega$$

With Joined in Series

$R = R_1 + R_2 = (15/8) + 5 = 55/9\ \Omega$; $X = 12.36 + 8.66 = 21.02\ \Omega$

$Z = \sqrt{(55/8)^2 + (21.02)^2} = 22.1\ \Omega$, $I = 100/22.1 = 4.52$ A, $P = I^2R = 4.52^2 \times 55/8 =$ **140 W**

Example 13.15. *A coil takes a current of 6 A when connected to a 24-V d.c. supply. To obtain the same current with a 50-Hz a.c. supply, the voltage required was 30 V.*
Calculate ***(i)*** *the inductance of the coil* ***(ii)*** *the power factor of the coil.*
(F.Y. Engg. Pune Univ.)

Solution. It should be kept in mind the coil offers only resistance to direct voltage whereas it offers impedance to an alternating voltage.

$\therefore \quad R = 24/6 = 4\ \Omega$; $Z = 30/6 = 5\ \Omega$

(*i*) $\therefore \quad X_L = \sqrt{Z^2 - R^2} = \sqrt{5^2 - 4^2} = 3\ \Omega$ ***(ii)*** p.f. $= \cos\phi = R/Z = 4/5 =$ **0.8 (lag)**

Example 13.16. *A resistance of 20 ohm, inductance of 0.2 H and capacitance of 150 μF are connected in series and are fed by a 230 V, 50 Hz supply. Find X_L, X_C, Z, Y, p.f., active power and reactive power.* **(Elect. Science-I, Allahabad Univ. 1992)**

Solution. $X_L = 2\pi fL = 2\pi \times 50 \times 0.2 = 62.8\Omega$; $X_C = 1/2\pi fC$

$= 10^{-6}\, 2\pi \times 50 \times 150 = 21.2\ \Omega$; $X = (X_L - X_C) = 41.6\ \Omega$;

$Z = \sqrt{R^2 + X^2} = \sqrt{20^2 + 41.6^2} = 46.2\ \Omega$; $I = V/Z = 230/46.2 = 4.98\ A$

Also, $Z = R + jX = 20 + j\,41.6 = 46.2 \angle 64.3°$ ohm

$\therefore \quad Y = 1/Z = 1/46.2 \angle 64.3° = 0.0216 \angle -64.3°$ siemens

p.f. $= \cos 64.3° = 0.4336$ (lag)

Active power $= VI\cos\phi = 230 \times 4.98 \times 0.4336 = 497$ W

Reactive power $= VI\sin\phi = 230 \times 4.98 \times \sin 64.3° = 1031$ VAR

Example 13.17. *A 120-V, 60-W lamp is to be operated on 220-V, 50-Hz supply mains. Calculate what value of* ***(a)*** *non-inductive resistance* ***(b)*** *pure inductance would be required in order that lamp is run on correct voltage. Which method is preferable and why ?*

Solution. Rated current of the bulb = 60/120 = 0.5 A

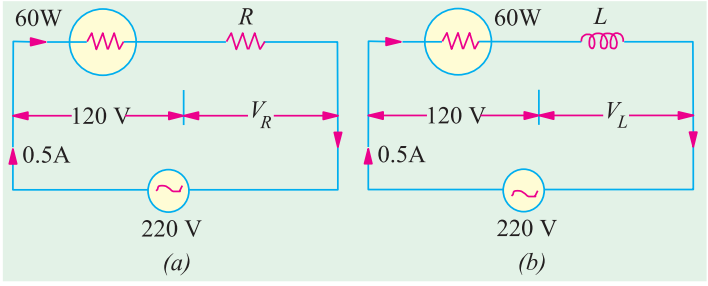


Fig. 13.15

(a) Resistor has been shown connected in series with the lamp in Fig. 13.15 (*a*).

P.D. across R is $V_R = 220 - 1200 = 100$ V

It is in phase with the applied voltage, $\therefore R = 100/0.5 =$ **200 Ω**

(b) P.D. across bulb = 120 V

P.D. across L is

$V_L = \sqrt{(220^2 - 120^2)} = 184.4$ V

(Remember that V_L is in quadrature with V_R –the voltage across the bulb).

Now, $V_L = 0.5 \times X_L$ or $184.4 = 0.5 \times L \times 2\pi \times 50 \quad \therefore \ L = 184.4/0.5 \times 3.14 =$ **1.17 H**

Method ***(b)*** is preferable to ***(a)*** because in method ***(b)***, there is no loss of power. Ohmic resistance of 200 Ω itself dissipates large power (*i.e.* $100 \times 0.5 =$ **50 W**).

Example 13.18. *A non-inductive resistor takes 8 A at 100 V. Calculate the inductance of a choke coil of negligible resistance to be connected in series in order that this load may be supplied from 220-V, 50-Hz mains. What will be the phase angle between the supply voltage and current ?*
(Elements of Elect. Engg.-I, Bangalore Univ.)

Solution. It is a case of pure resistance in series with pure inductance as shown in Fig. 13.16 (*a*).

Here $V_R = 100$ V,

$V_L = \sqrt{(220^2 - 100^2)} = 196$ V

Now, $V_L = I \,.\, X_L$

or $196 = 8 \times 2\pi \times 50 \times L = \mathbf{0.078\ H}$

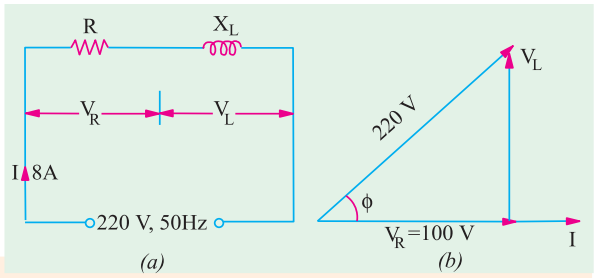


Fig. 13.16

Example 13.19. *A current of 5 A flows through a non-inductive resistance in series with a choking coil when supplied at 250-V, 50-Hz. If the voltage across the resistance is 125 V and across the coil 200 V, calculate*

***(a)** impedance, reactance and resistance of the coil **(b)** the power absorbed by the coil and **(c)** the total power. Draw the vector diagram.* **(Elect. Engg., Madras Univ.)**

Solution. As seen from the vector diagram of Fig. 13.17 (*b*).

$BC^2 + CD^2 = 200^2$ ...(*i*) $(125 + BC)^2 + CD^2 = 250^2$ ...(*ii*)

Subtracting Eq. (*i*) from (*ii*), we get, $(125 + BC)^2 - BC^2 = 250^2 - 200^2$

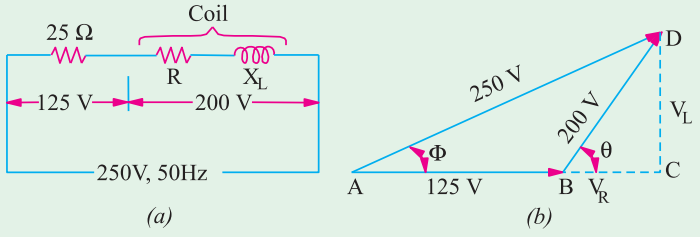


Fig. 13.17

$\therefore \quad BC = 27.5$ V; $CD = \sqrt{200^2 - 27.5^2} = 198.1$ V

(*i*) Coil impedance $= 200/5 = \mathbf{40\ \Omega}$

$V_R = IR = BC$ or $5R = 27.5 \quad \therefore \ P = 27.5/5 = \mathbf{5.5\ \Omega}$

Also $V_L = I \,.\, X_L = CD = 198.1 \quad \therefore \ X_L = 198.1/5 = \mathbf{39.62\ \Omega}$

or $X_L = \sqrt{40^2 - 5.5^2} = \mathbf{39.62\ \Omega}$

(*ii*) Power absorbed by the coil is $= I^2 R = 5^2 \times 5.5 = 137.5$ W

Also $P = 200 \times 5 \times 27.5/200 = \mathbf{137.5\ W}$

(*iii*) Total power $= VI \cos \phi = 250 \times 5 \times AC/AD = 250 \times 5 \times 152.5/250 = \mathbf{762.5\ W}$

The power may also be calculated by using $I^2 R$ formula.

Series resistance $= 125/5 = 25\ \Omega$

Total *circuit* resistance $= 25 + 5.5 = 30.5\ \Omega$

$\therefore$ Total power $= 5^2 \times 30.5 = \mathbf{762.5\ W}$

Example 13.20. *Two coils A and B are connected in series across a 240-V, 50-Hz supply. The resistance of A is 5 Ω and the inductance of B is 0.015 H. If the input from the supply is 3 kW and 2 kVAR, find the inductance of A and the resistance of B. Calculate the voltage across each coil.* **(Elect. Technology Hyderabad Univ. 1991)**

Solution. The kVA triangle is shown in Fig. 13.18 (*b*) and the circuit in Fig. 13.18(*a*). The circuit kVA is given by, kVA $= \sqrt{(3^2 + 2^2)} = 3.606$ or $VA = 3{,}606$ voltmeters

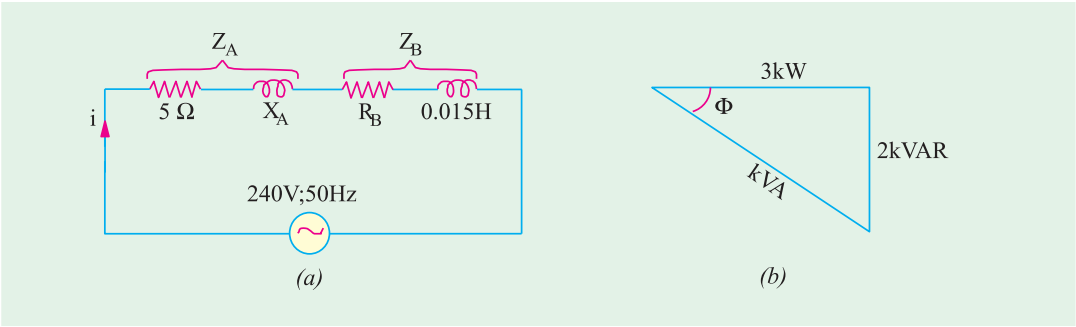


Fig. 13.18

Circuit current = 3,606/240 = 15.03 A $\quad\therefore\quad 15.03^2\,(R_A + R_B) = 3{,}000$

$\therefore\quad R_A + R_B = 3{,}000/15.03^2 = 13.3\ \Omega \quad\therefore\quad R_B = 13.3 - 5 = \mathbf{8.3\ \Omega}$

Now, impedance of the whole circuit is given by $Z = 240/15.03 = 15.97\ \Omega$

$$\therefore \quad X_A + X_B = \sqrt{(Z^2 - (R_A + R_B)^2} = \sqrt{15.97^2 - 13.3^3} = 8.84\ \Omega$$

Now $\quad X_B = 2\pi \times 50 \times 0.015 = 4.713\ \Omega \quad\therefore\quad X_A = 8.843 - 4.713 = 4.13\ \Omega$

or $\quad 2\pi \times 50 \times L_A = 4.13 \quad\therefore\quad L_A = \mathbf{0.0132\ H\ (approx)}$

Now $\quad Z_A = \sqrt{R_A^2 + X_A^2} = \sqrt{5^2 + 4.13^2} = 6.585\ \Omega$

P.D. across coil $A = I \,.\, Z_A = 15.03 \times 6.485 = \mathbf{97.5\ V;}\ Z_b = \sqrt{8.3^2 + 4.713^2} = 9.545\ \Omega$

$\therefore$ p.d. across coil $B = I \,.\, Z_B = 15.03 \times 9.545 = \mathbf{143.5\ V}$

Example 13.21. *An e.m.f. $e_0 = 141.4 \sin (377\ t + 30°)$ is impressed on the impedance coil having a resistance of 4 Ω and an inductive reactance of 1.25 Ω measured at 25 Hz. What is the equation of the current ? Sketch the waves for i, e_R, e_L and e_0.*

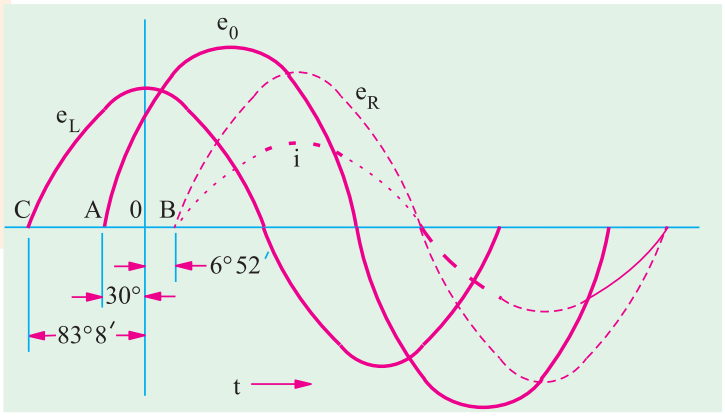


Fig. 13.19

Solution. The frequency of the applied voltage is $f = 377/2\pi = 60$ Hz

Since coil reactance is 1.25 Ω at 25 Hz, its value at 60 Hz = $1.25 \times 60/25 = 3\ \Omega$

Coil impedance, $Z = \sqrt{4^2 + 3^2}, = 5\ \Omega$; $\phi = \tan^{-1}(3/4) = 36°52'$

It means that circuit current lags behind the applied voltage by 36°52′ . Hence, equation of the circuit current is

$i = (141.4/5) \sin (377\ t + 30° - 36°52') = 28.3 \sin (377\ t - 6°52')$

Since, resistance drop is in phase with current, its equation is $e_R = iR = 113.2 \sin (377\ t - 6°52')$

The inductive voltage drop ***leads the current*** by 90°, hence its equation is

$$e_L = iX_L = 3 \times 28.3 \sin (377\ t - 6°52' + 90°) = 54.9 \sin (377\ t + 83°8')$$

The waves for i, e_R , e_L and e_0 have been drawn in Fig. 13.19.

Example 13.22. *A single phase, 7.46 kW motor is supplied from a 400-V, 50-Hz a.c. mains. If its efficiency is 85% and power factor 0.8 lagging, calculate* ***(a)*** *the kVA input* ***(b)*** *the reactive components of input current and* ***(c)*** *kVAR.*

Solution. Efficiency $= \frac{\text{output in watts}}{\text{input in watts}}$ $\therefore\ 0.85 = \frac{7.46 \quad 1000}{VI \quad \cos} \quad \frac{7,460}{VI \quad 0.8}$

$$\therefore \qquad VI = \frac{7460}{0.85 \times 0.8} = 10,970 \text{ voltamperes}$$

(a) $\therefore$ Input $= 10,970/1000 =$ **10.97 kVA**

(b) Input current $I = \frac{\text{voltamperes}}{\text{volts}} = \frac{10,970}{400} = 27.43$ A

Active component of current $= I \cos \phi =$ **27.43** $\times$ 0.8 = **21.94 A**

Reactive component of current $= I \sin \phi = 27.43 \times 0.6 =$ **16.46 A** ($\because \sin \phi = 0.6$)

(Reactive component $= \sqrt{27.43^2 \quad 21.94^2} \quad 16.46$ A

(*c*) kVAR = kVA $\sin \phi = 10.97 \times 0.6 =$ **6.58** (or kVAR $= VI \sin \phi \times 10^{-3} = 400 \times 16.46 \times 10^{-3}$ = 6.58)

Example 13.23. *Draw the phasor diagram for each of the following combinations :*

(i) *R and L in series and combination in parallel with C.*

(ii) *R, L and C in series with $X_C > X_L$ when ac voltage source is connected to it.*

[Nagpur University—Summer 2000]

Solution. ***(i)***

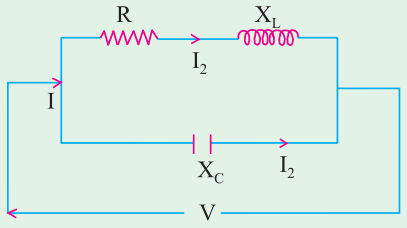


Fig. 13.20 (*a*) Circuit

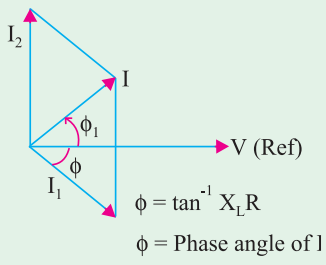


Fig. 13.20 (*b*) Phasor diagram

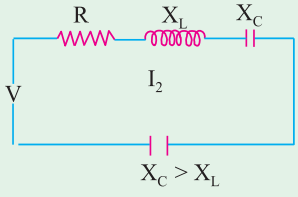


Fig. 13.20 (*c*) Circuit

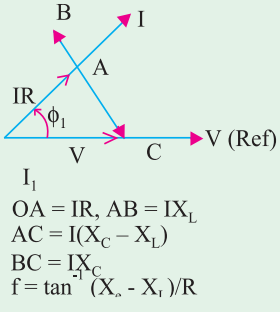


Fig . 13.20 (*d*) Phasor diagram

Example 13.24. *A voltage v (t) = 141.4 sin (314 t + 10°) is applied to a circuit and the steady current given by i(t) = 14.14 sin (314 t –20°) is found to flow through it.*

Determine

(i) *The p.f. of the circuit* ***(ii)*** *The power delivered to the circuit*

(iii) *Draw the phasor diagram.* **[Nagpur University Summer 2000]**

Solution. $v\,(t) = 141.4 \sin (314\,t + 10°)$

This expression indicates a sinusoidally varying alternating voltage at a frequency

$\omega = 314$ rad/sec, $f = 50$ Hz

V = RMS voltage (Peak voltage)/$\sqrt{2}$ = 100 volts

The expression for the current gives the following data :

I = RMS value = $14.14/\sqrt{2} = 10$ amp

frequency = 50 Hz, naturally.

Phase shift between I and $V = 30°$, I lags behind V.

(i) Power factor of the circuit = cos 30° = 0.866 lag

(ii) $P = VI \cos \phi = 100 \times 10 \times 0.866 = 866$ watts

(iii) Phasor diagram as drawn below, in Fig. 13.21 (*a*).

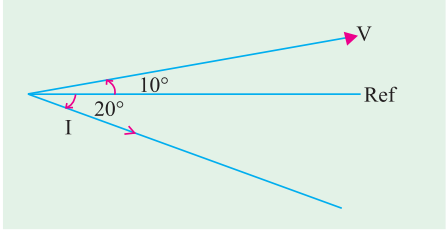


Fig. 13.21 (*a*) Phasor diagram

Example 13.25. *A coil of 0.8 p.f. is connected in series with 110 micro-farad capacitor. Supply frequency is 50 Hz. The potential difference across the coil is found to be equal to that across the capacitor. Calculate the resistance and the inductance of the coil. Calculate the net power factor.*

[Nagpur University, November 1997]

Solution. $X_C = 1/(3.14 \times C) = 28.952$ ohms

$\therefore$ Coil Impedance, $Z = 28.952\ \Omega$

Coil resistance = $28.952 \times 0.8 = 23.162\ \Omega$

Coil reactance = 17.37 ohms

Coil-inductance = 17.37/314 = 55.32 milli-henrys

Total impedance, $Z_T = 23.16 + j\,17.37 - j\,28.952 = 23.162 - j\,11.582 = 25.9$ ohms

Net power-factor = 23.162/25.9 = 0.8943 leading

Example 13.26. *For the circuit shown in Fig. 13.21 (c), find the values of R and C so that $V_b = 3V_a$, and V_b and V_a are in phase quadrature. Find also the phase relationships between V_s and V_b, and V_b and I.* **[Rajiv Gandhi Technical University, Bhopal, Summer 2001]**

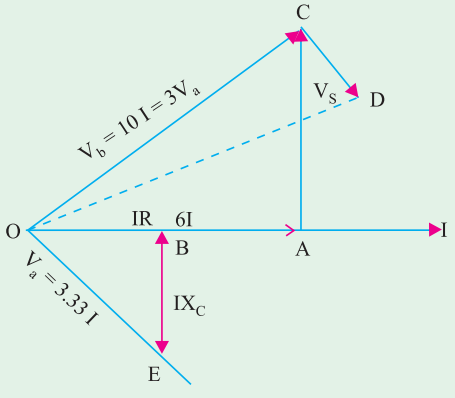


Fig. 13.21 (*b*)

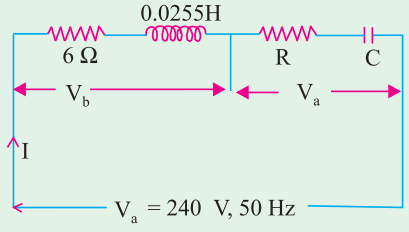


Fig . 13.21 (*c*)

Solution. $\angle COA = \phi = 53.13°$

$\angle BOE = 90° - 53.13° = 36.87°$

$\angle DOA = 34.7°$ Angle between V and I

Angle between V_s and $V_b = 18.43°$

$X_L = 314 \times 0.0255 = 8$ ohms

$Z_b = 6 + j\,8 = 10 \angle 53.13°$ ohms

$V_b = 10\,I = 3\,V_a$, and hence $V_a = 3.33\,I$

In phasor diagram, I has been taken as reference. V_b is in first quadrant. Hence V_a must be in the fourth quadrant, since Z_a consists of R and X_c. Angle between V_a and I is then 36.87°. Since Z_a and Z_b are in series, V is represented by the phasor OD which is at angle of 34.7°.

$$|V| = \sqrt{10}\,V_a = 10.53\,I$$

Thus, the circuit has a total effective impedance of 10.53 ohms.

In the phasor diagram, $OA = 6\,I$, $AC = 8\,I$, $OC = 10\,I = V_b = 3\,V_a$

Hence, $V_a = OE = 3.33\,I$,

Since $BOE = 36.87°$, $OB - RI = OE \times \cos 36.87° = 3.33 \times 0.8 \times I = 2.66\,I$.

Hence, $R = 2.66$

And $BE = OE \sin 36.87° = 3.33 \times 0.6 \times I = 2\,I$

Hence $X_c = 2$ ohms. For $X_c = 2$ ohms, $C = 1/(314 \times 2) = 1592\ \mu F$

Horizontal component of $OD = OB + OA = 8.66\,I$

Vertical component of $OD = AC - BE = 6\,I$

$OD = 10.54\,I = V_s$

Hence, the total impedance = 10.54 ohms = $8.66 + j\,6$ ohms

Angle between V_s and $I = \angle DOA = \tan^{-1}(6/866) = 34.7°$

Example 13.27. *A coil is connected in series with a pure capacitor. The combination is fed from a 10 V supply of 10,000 Hz. It was observed that the maximum current of 2 Amp flows in the circuit when the capacitor is of value 1 microfarad. Find the parameters (R and L) of the coil.*

[Nagpur University April 1996]

Solution. This is the situation of resonance in A.C. Series circuit, for which $X_L = X_C$

$Z = R = V/I = 10/2 = 5$ ohms

If ω_0 is the angular frequency, at resonance, L and C are related by $\omega_0^2 = 1/(LC)$,

which gives $L = 1/(\omega_0^2 C) = 2.5 \times 10^{-4}$ H = 0.25 mH

Example 13.28. *Two impedances consist of (resistance of 15 ohms and series-connected inductance of 0.04 H) and (resistance of 10 ohms, inductance of 0.1 H and a capacitance of 100 μF, all in series) are connectd in series and are connected to a 230 V, 50 Hz a.c. source. Find : **(i)** Current drawn, **(ii)** Voltage across each impedance, **(iii)** Individual and total power factor. Draw the phasor diagram.*

[Nagpur University, Nov. 1996]

Solution. Let suffix 1 be used for first impedance, and 2 for the second one. At 50 Hz,

$Z_1 = 15 + j\,(314 \times 0.04) = 15 + j\,12.56$ ohms

$Z_1 = \sqrt{15^2 + 12.56^2} = 19.56$ ohms,

Impedance-angle, $\theta_1 = \cos^{-1}(15/19.56) = +40°$,

$Z_2 = 10 + j\,(314 \times 0.1) - j\,\{1/(314 \times 100 \times 16^{-6})\}$

$= 10 + j\,31.4 - j\,31.85 = 10 - j\,0.45$

$= 10.01$ ohms, Impedance angle, $\theta_2 = -2.56°$,

Total Impedance, $Z = Z_1 = Z_2 = 15 + j\,12.56 + 10 - j\,0.85$

$= 25 + j\,12.11 = 27.78\ \underline{|\mathbf{25.85°}}$

For this, Phase-angle of + 25.85°, the power-factor of the total impedance
= cos 25.85° = 0.90, Lag.
Current drawn = 230/27.78 = 8.28 Amp, at 090 lagging p.f.
V_1 = 8.28 × 19.56 = 162 Volts
V_2 = 8.28 × 10.01 = 82.9 Volts

Individual Power-factor

$\cos \theta_1$ = cos 40° = 0.766 Lagging
$\cos \theta_2$ = cos 2.56° = 0.999 leading

Phasor diagram : In case of a series circuit, it is easier to treat the current as a reference. The phasor diagram is drawn as in Fig. 13.22.

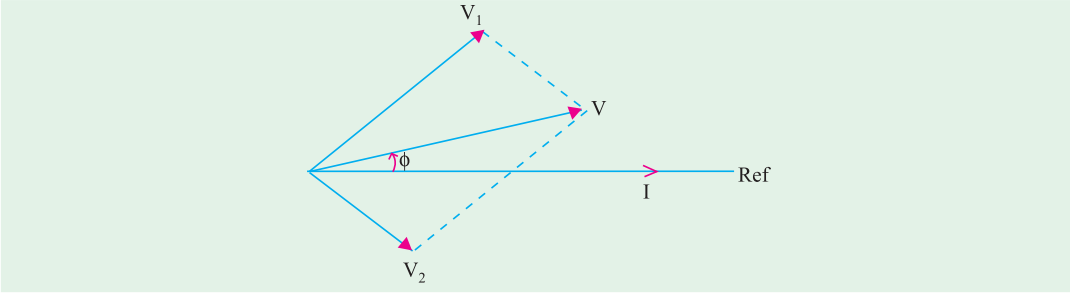


Fig. 13.22

Example 13.29. *Resistor (= R), choke-coil (r, L), and a capacitor of 25.2 μF are connected in series. When supplied from an A.C. source, in takes 0.4 A. If the voltage across the resistor is 20 V, voltage across the resistor and choke is 45 volts, voltage across the choke is 35 volts, and voltage across the capacitor is 50 V.*

Find : **(a)** *The values of r, L* **(b)** *Applied voltage and its frequency,* **(c)** *P.F. of the total circuit and active power consumed. Draw the phasor diagram.* **[Nagpur Univ. April 1998]**

Solution.

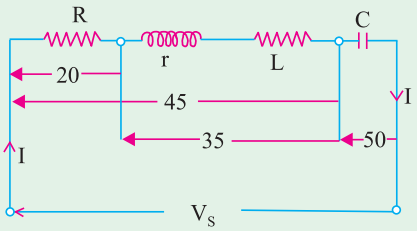


Fig. 13.23 (*a*)

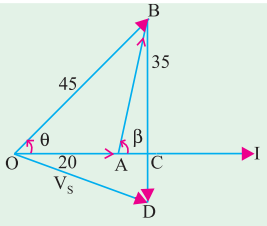


Fig. 13.23 (*b*)

(*b*) Since the current *I* is 0.4 amp, and voltage drop across *R* is 20 V,

R = 20/0.4 = 50 ohms

Similarly, Impedance of the coil, Z_1 = 35/0.4 = 87.5 ohms

Capacitive reactance X_c = 50/0.4 = 125 ohms

With a capacitor of 25.5 μF, and supply angular frequency of ω radians/sec

$$\frac{1}{\omega} = X_C \,.\, C = 125 \times 25.5 \times 10^{-6}, \text{ which gives } \omega = 314 \text{ ra./sec.}$$

The corresponding source frequency, f = 50 Hz

(*c*) The phasor diagram is drawn in Fig. 13.23 (*b*), taking *I* as the reference.

Solving triangle *OAB*,

$$\cos\phi = \frac{45^2 + 20^2 - 35^2}{2 \times 45 \times 20} = 0.667, \text{ and hence } \phi = 48.2°$$

Similarly, $\cos(180° - \beta) = \dfrac{400 + 1225 - 2025}{2 \times 20 \times 35} = 106.6°$

This gives $\beta = 73.4°$. From the phasor diagram is Fig. 13.23 (*b*),

$OC = OA + AC = 20 + 35 \cos\beta = 30$

$BC = 35 \sin\beta = 33.54$. The capacitive–reactance drop is BD.

Since $BD = 50$, $CD = 16.46$ volts.

$V_s = \sqrt{OC^2 + CD^2} = 34.22$ volts

$\angle COD = \phi = \cos^{-1}(OC/OD) = 28.75°$

The power-factor of the total circuit = $\cos\phi = 0.877$, Leading,

Since I Leads V_s in Fig. 13.23 (*b*).

(*a*) For the coil, ACB part of the phasor diagram is to be observed.

$r = AC/I = 10/0.4 = 25$ ohms

$X_L = BC/I = 33.54/0.4 = 83.85$ ohms

Hence, coil-inductance, $L = 83.85/314 = 267$ milli-henry.

P = Active Power Consumed = $V_s I \cos\phi = 12$ watts

or $P = (0.4)^2 \times (R + r) = 12$ watts

13.6. Power in an Iron-cored Choking Coil

Total power, P taken by an iron-cored choking coil is used to supply

(*i*) power loss in ohmic resistance *i.e.* $I^2 R$. **(*ii*)** iron-loss in core, P_i

$\therefore$ $P = I^2 R + P_i$ or $\dfrac{P}{I^2} = R + \dfrac{P_1}{I^2}$ is known as the **effective resistance*** of the choke.

$\therefore$ effective resistance = true resistance = equivalent resistance $\dfrac{P_i}{I^2}$ $\therefore R_{eff} = \dfrac{P}{I^2} = R + \dfrac{P_i}{I^2}$

Example 13.30. *An iron-cored choking coil takes 5 A when connected to a 20-V d.c. supply and takes 5 A at 100 V a.c. and consumes 250 W. Determine* ***(a)*** *impedance* ***(b)*** *the power factor* ***(c)*** *the iron loss* ***(d)*** *inductance of the coil.* **(Elect. Engg. & M.A.S.I. June, 1991)**

Solution. (*a*) $Z = 100/5 = \mathbf{20\ \Omega}$

(*b*) $P = VI \cos\phi$ or $250 = 100 \times 5 \times \cos\phi$ $\therefore$ $\cos\phi = 250/500 = \mathbf{0.5}$

(*c*) Total loss = loss in resistance + iron loss $\therefore 250 = 20 \times 5 + P_i$ $\therefore P_i = 250 - 100 = \mathbf{150\ W}$

(*d*) Effective resistance of the choke is $\dfrac{P}{I^2} = \dfrac{250}{25} = 10\ \Omega$

$\therefore$ $X_L = \sqrt{(Z^2 - R^2)} = \sqrt{(400 - 100)} = \mathbf{17.32\ \Omega}$

Example 13.31. *An iron-cored choking coil takes 5 A at a power factor of 0.6 when supplied at 100-V, 50 Hz. When the iron core is removed and the supply reduced to 15 V, the current rises to 6 A at power factor of 0.9.*

Determine ***(a)*** *the iron loss in the core* ***(b)*** *the copper loss at 5 A* ***(c)*** *the inductance of the choking coil with core when carrying a current of 5 A.*

* At higher frequencies like radio frequencies, there is skin-effect loss also.

Solution. When core is removed, then $Z = 15/6 = 2.5\ \Omega$

True resistance, $R = Z\cos\phi = 2.5 \times 0.9 = 2.25\ \Omega$

With Iron Core

Power input $= 100 \times 5 \times 0.6 = 300$ W

Power wasted in the true resistance of the choke when current is 5 A $= 5^2 \times 2.25 = 56.2$ W

(*a*) Iron loss $= 300 - 56.2 =$ **244 W (approx) (*b*)** Cu loss at 5 A = **56.2 W**

(*c*) $Z = 100/5 = 20\ \Omega$; $X_L = Z\sin\phi = 20 \times 0.8 = 16\Omega \therefore 2\pi \times 50 \times L = 16 \quad \therefore L =$ **0.0509 N**

Tutorial Problem No. 13.1

1. The voltage applied to a coil having $R = 200\ \Omega$, $L = 638$ mH is represented by $e = 20 \sin 100\pi t$. Find a corresponding expression for the current and calculate the average value of the power taken by the coil. **[$i = 0.707 \sin (100\pi t - \pi/4)$; 50 W] (*I.E.E. London*)**
2. The coil having a resistance of 10 Ω and an inductance of 0.2 *H* is connected to a 100-V, 50-Hz supply. Calculate (*a*) the impedance of the coil (*b*) the reactance of the coil (*c*) the current taken and (*d*) the phase difference between the current and the applied voltage.

   [(*a*) 63.5 Ω (*b*) 62.8 Ω (*c*) 1.575 A (*d*) 80°57′]
3. An inductive coil having a resistance of 15 Ω takes a current of 4 A when connected to a 100-V, 60 Hz supply. If the coil is connected to a 100 -V, 50 Hz supply, calculate (*a*) the current (*b*) the power (*c*) the power factor. Draw to scale the vector diagram for the 50-Hz conditions, showing the component voltages. **[(*a*) 4.46 A (*b*) 298 W (*c*) 0.669]**
4. When supplied with current at 240-V, single-phase at 50 Hz, a certain inductive coil takes 13.62 A. If the frequency of supply is changed to 40 Hz, the current increases to 16.12 A. Calculate the resistance and inductance of the coil. **[17.2 W, 0.05 H] (*London Univ.*)**
5. A voltage $v(t) = 141.4 \sin (314t + 10°)$ is applied to a circuit and a steady current given by $i(t) = 14.4 \sin (314t - 20°)$ is found to flow through it. Determine (*i*) the p.f. of the circuit and (*ii*) the power delivered to the circuit. **[0.866 (lag); 866 W]**
6. A circuit takes a current of 8 A at 100 V, the current lagging by 30° behind the applied voltage. Calculate the values of equivalent resistance and reactance of the circuit. **[10.81 Ω; 6.25 Ω]**
7. Two inductive impedance *A* and *B* are connected in series. *A* has $R = 5\ \Omega$, $L = 0.01$ *H;* B has $R = 3\ \Omega$, $L = 0.02$ H. If a sinusoidal voltage of 230 V at 50 Hz is applied to the whole circuit calculate (*a*) the current (*b*) the power factor (*c*) the voltage drops. Draw a complete vector diagram for the circuit.

   [(*a*) 18.6 (*b*) 0.648 (*c*) $V_A = 109.5$ V, $V_B = 129.5$ V] (*I.E.E. London*)
8. A coil has an inductance of 0.1 H and a resistance of 30 Ω at 20°C. Calculate (*i*) the current and (*ii*) the power taken from 100-V, 50-Hz mains when the temperature of the coil is 60° C, assuming the temperature coefficient of resistance to be 0.4% per°C from a basic temperature of 20°C.

   [(*i*) 2.13 A (*ii*) 158.5 W] (*London Univ.*)
9. An air-cored choking coil takes a current of 2 A and dissipates 200 W when connected to a 200-V, 50-Hz mains. In other coil, the current taken is 3 A and the power 270 W under the same conditions. Calculate the current taken and the total power consumed when the coils are in series and connected to the same supply. **[1.2, 115 W] (*City and Guilds, London*)**
10. A circuit consists of a pure resistance and a coil in series. The power dissipated in the resistance is 500 W and the drop across it is 100 V. The power dissipated in the coil is 100 W and the drop across it is 50 V. Find the reactance and resistance of the coil and the supply voltage.

    [9.168Ω; 4Ω; 128.5V]
11. A choking coil carries a current of 15 A when supplied from a 50-Hz, 230-V supply. The power in the circuit is measured by a wattmeter and is found to be 1300 watt. Estimate the phase difference between the current and p.d. in the circuit. **[0.3768] (*I.E.E. London*)**
12. An ohmic resistance is connected in series with a coil across 230-V, 50-Hz supply. The current is 1.8

A and p.ds. across the resistance and coil are 80 V and 170 V respectively. Calculate the resistance and inductance of the coil and the phase difference between the current and the supply voltage.

[61.1 Ω, 0.229 H, 34°20′] (*App. Elect. London Univ.*)

13. A coil takes a current of 4 A when 24 V d.c. are applied and for the same power on a 50-Hz a.c. supply, the applied voltage is 40. Explain the reason for the difference in the applied voltage. Determine (*a*) the reactance (*b*) the inductance (*c*) the angle between the applied p.d. and current (*d*) the power in watts. **[(*a*) 8 Ω (*b*) 0.0255 H (*c*) 53°7′ (*d*) 96 W]**

14. An inductive coil and a non-inductive resistance R ohms are connected in series across an a.c. supply. Derive expressions for the power taken by the coil and its power factor in terms of the voltage across the coil, the resistance and the supply respectively. If $R = 12\ \Omega$ and the three voltages are in order, 110 V, 180 V and 240 V, calculate the power and the power factor of the coil. **[546 W; 0.331]**

15. Two coils are connected in series. With 2 A d.c. through the circuit, the p.ds. across the coils are 20 and 30 V respectively. With 2 A a.c. at 40 Hz, the p.ds. across the coils are 140 and 100 V respectively. If the two coils in series are connected to a 230-V, 50-Hz supply, calculate (*a*) the current (*b*) the power (*c*) the power factor. **[(*a*) 1.55 A (*b*) 60 W (*c*) 0.1684]**

16. It is desired to run a bank of ten 100-W, 10-V lamps in parallel from a 230-V, 50-Hz supply by inserting a choke coil in series with the bank of lamps. If the choke coil has a power factor of 0.2, find its resistance, reactance and inductance. **[$R = 4.144\ \Omega$, $X = 20.35\ \Omega$, $L = 0.065$ H] (*London Univ.*)**

17. At a frequency for which $\omega = 796$, an e.m.f. of 6 V sends a current of 100 mA through a certain circuit. When the frequency is raised so that $\omega = 2866$, the same voltage sends only 50 mA through the same circuit. Of what does the circuit consist ? **[$R = 52\ \Omega$, L = 0.038 H in series] (*I.E.E. London*)**

18. An iron-cored electromagnet has a d.c. resistance of 7.5 Ω and when connected to a 400-V 50-Hz supply, takes 10 A and consumes 2 kW. Calculate for this value of current (*a*) power loss in iron core (*b*) the inductance of coil (*c*) the power factor (*d*) the value of series resistance which is equivalent to the effect of iron loss. **[1.25 kW, 0.11 H, 0.5; 12.5 Ω] (*I.E.E. London*)**

13.7. A.C. Through Resistance and Capacitance

The circuit is shown in Fig. 13.24 (*a*). Here $V_R = IR$ = drop across R in phase with I.

$V_C = IX_C$ = drop across capacitor –lagging I by $\pi/2$

As capacitive reactance X_C is taken negative, V_C is shown along negative direction of Y-axis in the voltage triangle [Fig. 13.24 (*b*)]

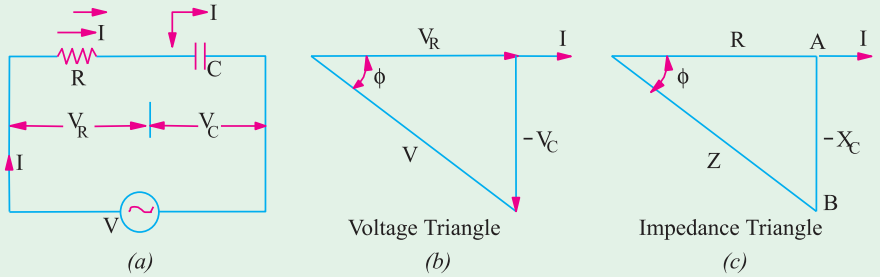


Fig. 13.24

Now $V = \sqrt{V_R^2 + (-V_C)^2} = \sqrt{(IR)^2 + (-IX_C)^2} = I\sqrt{R^2 + X_C^2}$ or $I = \dfrac{V}{\sqrt{R^2 + X_C^2}} = \dfrac{V}{Z}$

The denominator is called the ***impedance*** of the circuit. So, $Z = \sqrt{R^2 + X_C^2}$

Impedance triangle is shown in Fig. 13.24 (*c*)

From Fig. 13.24 (*b*) it is found that I leads V by angle ϕ such that $\tan \phi = -X_C/R$

Hence, it means that if the equation of the applied alternating voltage is $v = V_m \sin \omega t$, the equation of the resultant current in the *R-C* circuit is $i = I_m \sin (\omega t + \phi)$ so that current ***leads*** the applied voltage by an angle ϕ. This fact is shown graphically in Fig. 13.25.

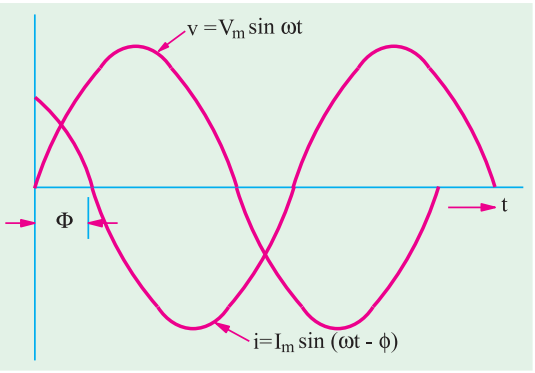


Fig. 13.25

Example 13.32. *An a.c. voltage (80 + j 60) volts is applied to a circuit and the current flowing is (–4 + j 10) amperes. Find* ***(i)*** *impedance of the circuit* ***(ii)*** *power consumed and* ***(iii)*** *phase angle.* **[Elect. Technology, Indore, Univ., Bombay Univ. 1999]**

Solution. $\mathbf{V} = (80 + j\,60) = 100 \angle 36.9°$;

$\mathbf{I} = -4 + j\,10 = 10.77 \angle \tan^{-1}(-2.5) = 10.77 \angle (180° - 68.2°) = 10.77 \angle 111.8°$

(i) $\mathbf{Z} = \mathbf{V/I} = 100 \angle 36.9°/10.77 \angle 111.8°$

$= 9.28 \angle -74.9°$

$= 9.28 (\cos 74.9° - j \sin 74.9°) = 2.42 - j\,8.96\ \Omega$

Hence $R = 2.42\ \Omega$ and $X_C = 8.96\ \Omega$ capacitive

(ii) $P = I^2 R = 10.77^2 \times 2.42 =$ **2.81 W**

(iii) Phase angle between voltage and current = 74.9° with current ***leading*** as shown in Fig. 13.26.

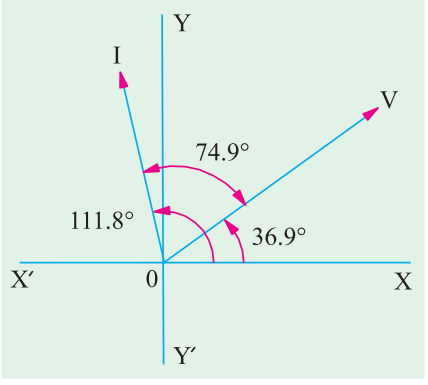


Fig. 13.26

Alternative Method for Power

The method of conjugates will be used to determine the real power and reactive volt-ampere. It is a convenient way of calculating these quantities when both voltage and current are expressed in cartesian form. If the conjugate of *current* is multiplied by the voltage in cartesian form, the result is a complex quantity, the real part of which gives the real power *j* part of which gives the reactive volt-amperes (*VAR*). It should, however, be noted that real power as obtained by this method of conjugates is the same regardless of whether *V* or *I* is reversed although sign of voltamperes will depend on the choice of *V* or *I*.*

Using current conjugate, we get $P_{VA} = (80 + j\,60)(-4 - j\,10) = 280 - j\,1040$

$\therefore$ Power consumed = **280 W**

Example 13.33. *In a circuit, the applied voltage is 100 V and is found to lag the current of 10 A by 30°.* ***(i)*** *Is the p.f. lagging or leading ?* ***(ii)*** *What is the value of p.f. ?* ***(iii)*** *Is the circuit inductive or capacitive ?* ***(iv)*** *What is the value of active and reactive power in the circuit ?* **(Basic Electricity, Bombay Univ.)**

Solution. The applied voltage lags behind the current which, in other words, means that current leads the voltage.

(i) $\therefore$p.f. is **leading** ***(ii)*** p.f. = $\cos \phi = \cos 30° =$ **0.866 (lead)** ***(iii)*** Circuit is **capacitive** ***(iv)*** Active power = $VI \cos \phi = 100 \times 10 \times 0.866 =$ **866 W**

Reactive power = $VI \sin \phi = 100 \times 10 \times 0.5 =$ **500 VAR (lead)**

or $VAR = \sqrt{(VA)^2 - W^2} = \sqrt{(100 \times 10)^2 - 866^2} =$ **500 (lead)**

* If voltage conjugate is used, then capacitive VARs are positive and inductive VARs negative. If current conjugate is used, then capacitive VARs are negative and inductive VARs are positive.

Example 13.34. *A tungsten filament bulb rated at 500-W, 100-V is to be connected to series with a capacitance across 200-V, 50-Hz supply. Calculate :*

(a) *the value of capacitor such that the voltage and power consumed by the bulb are according to the rating of the bulb.* ***(b)*** *the power factor of the current drawn from the supply.* ***(c)*** *draw the phasor diagram of the circuit.* **(Elect. Technology-1, Nagpur Univ. 1991)**

Solution. The rated values for bulb are : voltage = 100 V and current $I = W/V = 500/100 = 5$A. Obviously, the bulb has been treated as a pure resistance :

(a) $V_C = \sqrt{220^2 - 100^2} = 196\ V$

Now, $IX_C = 196$ or $5\,X_C = 196$, $X_C = 39.2\ \Omega$

$\therefore I/\omega C = 39.2$ or $C = 1/314 \times 39.2 =$ **81 μF**

(b) p.f. $= \cos\phi = V_R/V = 100/200 =$ **0.455 (lead)**

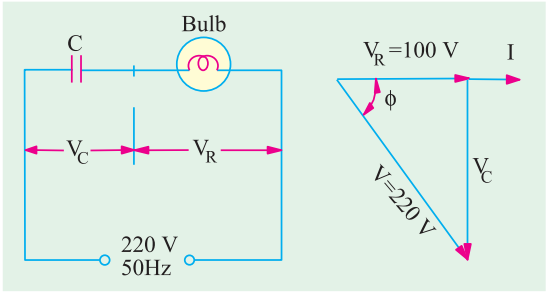


Fig 13.27

(c) The phasor diagram is shown in Fig 13.27.

Example 13.35. *A pure resistance of 50 ohms is in series with a pure capacitance of 100 microfarads. The series combination is connected across 100-V, 50-Hz supply. Find* ***(a)*** *the impedance* ***(b)*** *current* ***(c)*** *power factor* ***(d)*** *phase angle* ***(e)*** *voltage across resistor* ***(f)*** *voltage across capacitor. Draw the vector diagram.* **(Elect. Engg.-1, JNT Univ. Warrangel)**

Solution. $X_C = 10^6\pi/2\pi \times 50 \times 100 = 32\ \Omega$; $R = 50\ \Omega$

(a) $Z = \sqrt{50^2 + 32^2}$ **59.4 Ω** ***(b)*** $I = V/Z = 100/59.4 =$ **1.684 A**

(c) p.f. $= R/Z = 50 = 59.4 =$ **0.842 (lead)** ***(d)*** $\phi = \cos^{-1}(0.842) =$ **32°36′**

(e) $V_R = IR = 50 \times 1.684 =$ **84.2 V** ***(f)*** $V_C = IX_C = 32 \times 1.684 =$ **53.9 V**

Example 13.36. *A 240-V, 50-Hz series R-C circuit takes an r.m.s. current of 20 A. The maximum value of the current occurs 1/900 second before the maximum value of the voltage. Calculate* ***(i)*** *the power factor* ***(ii)*** *average power* ***(iii)*** *the parameters of the circuit.*

(Elect. Engg.-I, Calcutta Univ.)

Solution. Time-period of the alternating voltage is 1/50 second. Now a time interval of 1/50 second corresponds to a phase difference of 2 π radian or 360°. Hence, a time interval of 1/900 second corresponds to a phase difference of $360 \times 50/900 = 20°$.

Hence, current leads the voltage by 20°.

(i) power factor $= \cos 20° =$ **0.9397 (lead)**

(ii) averge power $= 240 \times 20 \times 0.9397 =$ **4,510 W**

(iii) $Z = 240/20 = 12\ \Omega$; $R = Z\cos\Phi = 12 \times 0.9397 =$ **11.28 Ω**

$X_C = Z\sin\Phi = 12 \times \sin 20° = 12 \times 0.342 = 4.1\ \Omega$

$C = 10^6/2\pi \times 50 \times 4.1 =$ **775 μF**

Example 13.37. *A voltage $v = 100 \sin 314\ t$ is applied to a circuit consisting of a 25 Ω resistor and an 80 μF capacitor in series. Determine :* ***(a)*** *an expression for the value of the current flowing at any instant* ***(b)*** *the power consumed* ***(c)*** *the p.d. across the capacitor at the instant when the current is one-half of its maximum value.*

Solution. $X_C = 1/(314 \times 80 \times 10^{-6}) = 39.8\ \Omega$, $Z = \sqrt{25^2 + 39.8^2} = 47\ \Omega$

$I_m = V_m/Z = 100/47 = 2.13$ A

$\phi = \tan^{-1}(39.8/25) = 57°52' = 1.01$ radian (lead)

(a) Hence, equation for the instantaneous current

i = **2.13 sin (314 t + 1.01) (b)** Power = $I^2 R = (2.13/\sqrt{2})^2 \times 25$ = **56.7 W (c)** The voltage across the capacitor lags the circuit current by π/2 radians. Hence, its equation is given by

$$v_c = V_{cm} \sin\left(314\,t + 1.01 - \frac{\pi}{2}\right) \text{ where } V_{cm} = I_m \times X_C = 2.13 \times 39.8 = 84.8\,V$$

Now, when i is equal to half the maximum current (say, in the positive direction) then

$$i = 0.5 \times 2.13 \text{ A}$$

$$\therefore \quad 0.5 \times 2.13 = 2.13 \sin (314\,t + 1.01) \text{ or } 314\,t + 1.01 = \sin^{-1}(0.5) = \frac{\pi}{6} \text{ or } \frac{5\pi}{6} \text{ radian}$$

$$\therefore \quad v_C = 84.8 \sin\left(\frac{\pi}{6} - \frac{\pi}{2}\right) = 84.8 \sin(-\pi/3) = -73.5 \text{ V}$$

$$\text{or} \quad v_c = 34.8 \sin\left(\frac{5\pi}{6} - \frac{\pi}{2}\right) = 84.4 \sin \pi/3 = \textbf{73.5 V}$$

Hence, p.d. across the capacitor is **73.5 V**

Example 13.38. *A capacitor and a non-inductive resistance are connected in series to a 200-V, single-phase supply. When a voltmeter having a non-inductive resistance of 13,500 Ω is connected across the resistor, it reads 132 V and the current then taken from the supply is 22.35 mA.*

Indicate on a vector diagram, the voltages across the two components and also the supply current ***(a)*** *when the voltmeter is connected and* ***(b)*** *when it is disconnected.*

Solution. The circuit and vector diagrams are shown in Fig. 13.28 (*a*) and (*b*) respectively.

(a) $$V_C = \sqrt{200^2 - 132^2} = \textbf{150 V}$$

It is seen that $\phi = \tan^{-1}(150/132) = 49°$ in Fig. 13.28 (*b*). Hence

(i) Supply voltage lags behind the current by 49°. ***(ii)*** V_R leads supply voltage by 49° (*iii*) V_C lags behind the supply voltage by (90° –49°) = 41°

The supply current is, as given equal to **22.35 mA.** The value of unknown resistance R can be found as follows :

Current through voltmeter = 132/13,500 = 9.78 mA

$\therefore$ Current through R = 22.35 –9.78 = 12.57 mA $\therefore R = 132/12.57 \times 10^{-3} = 10{,}500\ \Omega$;

$X_C = 150/22.35 \times 10^{-3} = 6.711\ \Omega$

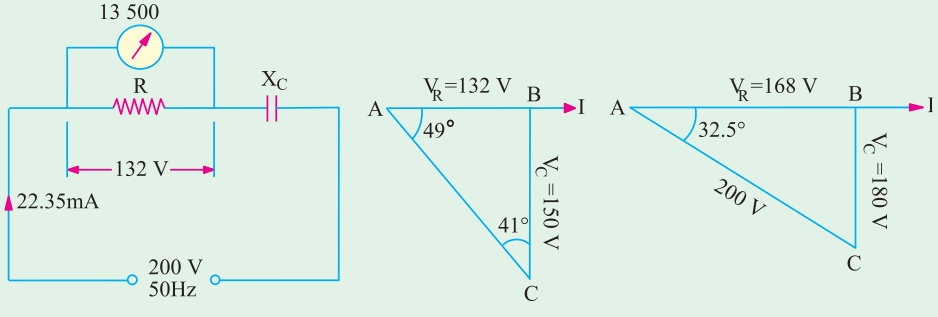


Fig. 13.28

(b) When voltmeter is disconnected, $Z = \sqrt{R^2 + X_L^2} = \sqrt{10{,}500^2 + 6{,}730^2} = 12{,}500\ \Omega$

Supply current = 200/12,461 = **16.0 mA**

In this case, $V_R = 16.0 \times 10^{-3} \times 10{,}500 =$ **168 V**

$V_C = 16.0 \times 10^{-3} \times 6711 =$ **107.4 V;** $\tan \phi = 107.4/168$

$\therefore \quad \phi = 32.5°$.

In this case, the supply voltage lags the circuit current by 32.5° as shown in Fig. 13.28 (*c*).

Example 13.39. *It is desired to operate a 100-W, 120-V electric lamp at its current rating from a 240-V, 50-Hz supply. Give details of the simplest manner in which this could be done using* ***(a)*** *a resistor* ***(b)*** *a capacitor and* ***(c)*** *an indicator having resistance of 10 Ω. What power factor would be presented to the supply in each case and which method is the most economical of power.*

(Principles of Elect. Engg.-I, Jadavpur Univ.)

Solution. Rated current of the bulb is = 100/120 = 5/6 A

The bulb can be run at its correct rating by any one of the three methods shown in Fig. 13.29. (*a*) With reference to Fig. 13.29 (*a*), we have

P.D. across R = 240 −120 = 120 V

$\therefore \quad R = 120/(5/6) =$ **144 Ω**

Power factor of the circuit is **unity.** Power consumed = 240 × 5/6 = **200 W**

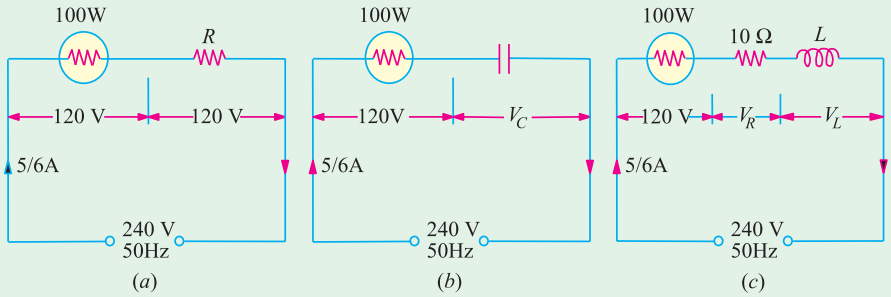


Fig. 13.29

(*b*) Referring to Fig. 13.29 (*b*), we have

$V_C = \sqrt{240^2 - 120^2} = 207.5\,V$; $X_C = 207.5\,(5/6) = 249\ \Omega$

$\therefore \quad 1/314\,C = 249$ or $C =$ **12.8 μF**; p.f. = cos ϕ = 120/240 = **0.5 (lead)**

Power consumed = 240 × (5/6) × 0.5 = **100 W**

(*c*) The circuit connections are shown in Fig 13.29 (*c*)

$$V_R = (5/6) \times 10 = 25/3 \text{ V} \quad \therefore \quad V_L = \sqrt{240^2 - \left(120 + \frac{25}{3}\right)^2} = 203\,V$$

$\therefore \quad 314\,L \times (5/6) = 203 \quad \therefore \quad L =$ **0.775 H**

Total resistive drop = 120 + (25/3) = 128.3 V; cos ϕ = 128.3/240 = **0.535 (lag)**

Power consumed = 240 × (5/6) × 0.535 = **107 W**

Method (*b*) is most economical because it involves least consumption of power.

Example 13.40. *A two-element series circuit consumes 700 W and has a p.f. = 0.707 leading. If applied voltage is v = 141.1 sin (314 t + 30°), find the circuit constants.*

Solution. The maximum value of voltage is 141.4 V and it leads the reference quantity by 30°. Hence, the given sinusoidal voltage can be expressed in the phase form as

V = $(141.4/\sqrt{2}) \angle 30° = 100 \angle 30°$ now, $P = VI \cos \phi \quad \therefore 700 = 100 \times I \times 0.707$; $I = 10$ A.

Since p.f. = 0.707 (lead); $\phi = \cos^{-1}(0.707) = 45°$ (lead).

It means that current leads the given voltage by 45° for it ***leads the common reference quantity***

by (30° + 45°) = 75°. Hence, it can be expressed as $\mathbf{I} = 10 \angle 75°$

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} \quad \frac{100 \quad 30}{10 \quad 75} \quad 10 \qquad j\,7.1 \quad R \quad \mathbf{7.1\,\Omega}$$

Since $X_C = 7.1 \;\therefore\; 1/314\,C = 7.1; \;\therefore\; C = \mathbf{450\ \mu F}$

13.8. Dielectric Loss and Power Factor of a Capacitor

An ideal capacitor is one in which there are no losses and whose current leads the voltage by 90° as shown in Fig. 13.30 (*a*). In practice, it is impossible to get such a capacitor although close approximation is achieved by proper design. In every capacitor, there is always some dielectric loss and hence it absorbs some power from the circuit. Due to this loss, the phase angle is somewhat less than 90° [Fig. 13.30 (*b*)]. In the case of a capacitor with a poor dielectric, the loss can be considerable and the phase angle much less than 90°. This dielectric loss appears as heat. By ***phase difference*** is meant the difference between the ideal and actual phase angles. As seen from Fig. 13.30 (*b*), the phase difference ψ is given by ψ = 90 − ϕ where ϕ is the actual phase angle, sin ψ = sin (90 − ϕ) = cos ϕ where cos ϕ is the power factor of the capacitor.

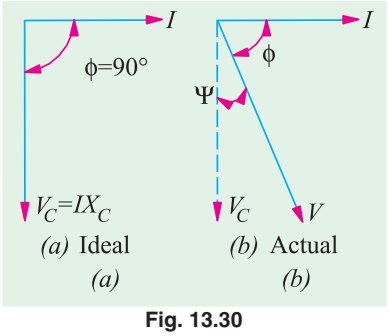


Fig. 13.30

Since ψ is generally small, sin ψ = ψ (in radians) ∴tan ψ = ψ = cos ϕ.

It should be noted that dielectric loss increases with the frequency of the applied voltage. Hence phase difference increases with the frequency *f*.

The dielectric loss of an actual capacitor is allowed for by imagining it to consist of a pure capacitor having an equivalent resistance either in series or in parallel with it as shown in Fig. 13.31. These resistances are such that $I^2 R$ loss in them is equal to the dielectric loss in the capacitor.

As seen from Fig. (13.27*b*), $\tan \psi = \frac{IR_{se}}{IX_C} \quad \frac{R_{se}}{1/C} \quad CR_{se} \quad \therefore\; R_{se} = \tan \psi/\omega C = \text{p.f.}/\omega C$

Similarly, as seen from Fig. 13.31 (*d*), $\tan \psi = \frac{I_1}{I_2} \quad \frac{V/R_{sh}}{V/X_C} \quad \frac{X_C}{R_{sh}} \quad \frac{1}{CR_{sh}}$

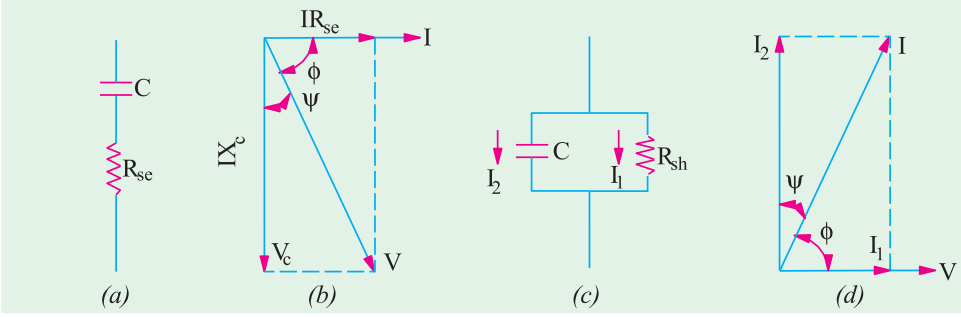


Fig. 13.31

$$R_{sh} = \frac{1}{\omega C \,.\, \tan \psi} = \frac{1}{\omega C \times \text{power factor}} = \frac{1}{\omega C \times \text{p.f.}}$$

The power loss in these resistances is $P = V^2/R_{sh} = \omega C V^2 \tan \psi = \omega C V^2 \times$ p.f.

or $= I^2 R_{se} = (I^2 \times \text{p.f.})/\omega C$

where p.f. stands for the power factor of the capacitor.

Note. (*i*) In case, ψ is not small, then as seen from Fig. 13.41 (*b*) $\tan \phi = \frac{X_C}{R_{se}}$ (Ex. 13.41) $R_{se} = X_C/\tan \phi$

From Fig. 13.31 (*d*), we get $\tan\phi = \frac{I_2}{I_1} = \frac{V/X_C}{V/R_{sh}} = \frac{R_{sh}}{X_C}$ $\therefore$ $R_{sh} = X_C \tan\phi = \tan\phi/\omega C$

(ii) It will be seen from above that both R_{se} and R_{sh} vary inversely as the frequency of the applied voltage. In other words, the resistance of a capacitor decreases in proportion to the increase in frequency.

$$\frac{R_{se1}}{R_{se2}} = \frac{f_2}{f_1}$$

Example 13.41. *A capacitor has a capacitance of 10 μF and a phase difference of 10°. It is inserted in series with a 100 Ω resistor across a 200-V, 50-Hz line, Find* ***(i)*** *the increase in resistance due to the insertion of this capacitor* ***(ii)*** *power dissipated in the capacitor and* ***(iii)*** *circuit power factor.*

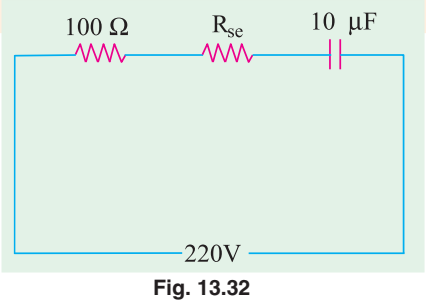


Fig. 13.32

Solution. $X_C = \frac{10^6}{2\pi \times 50 \times 10} = 318.3\ \Omega$

The equivalent series resistance of the capacitor in Fig. 13.32 is $R_{se} = X_C / \tan\phi$

Now $\phi = 90 - \psi = 90° - 10° = 80°$

$\tan\phi = \tan 80° = 5.671$

$\therefore$ $R_{se} = 318.3/5.671 = 56.1\ \Omega$

(i) Hence, resistance of the circuit increases by **56.1 Ω**

(ii) $Z = \sqrt{(R + R_{se})^2 + X_C^2} = \sqrt{156.1^2 + 318.3^2} = 354.4\ \Omega$; $I = 220/354 = 0.62$ A

Power dissipated in the capacitor $= I^2 R_{se} = 0.62^2 \times 56.1 =$ **21.6 W**

(iii) Circuit power factor is $= (R + R_{se})/Z = 156.1/354.4 =$ **0.44 (lead)**

Example 13.42. *Dielectric heating is to be employed to heat a slab of insulating material 2 cm thick and 150 sq. cm in area. The power required is 200 W and a frequency of 30 MHz is to be used. The material has a relative permittivity of 5 and a power factor of 0.03. Determine the voltage necessary and the current which will flow through the material. If the voltage were to be limited to 600-V, to what would the frequency have to be raised ?*

[Elect. Engg. AMIETE (New Scheme) June 1992]

Solution. The capacitance of the parallel-plate capacitor formed by the insulating slab is

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d} = \frac{8.854 \times 10^{-12} \times 5 \times 150 \times 10^{-4}}{2 \times 10^{-2}} = 33.2 \times 10^{-12}\ F$$

As shown in Art. 13.8 $R_{sh} = \frac{1}{\omega C \times p.f.} = \frac{1}{(2\pi \times 30 \times 10^6) \times 33.2 \times 10^{-12} \times 0.05} = 3196\ \Omega$

Now, $P = V^2/R_{sh}$ or $V = \sqrt{P \times R_{sh}} = \sqrt{200 \times 3196} = 800\ V$

Current $I = V/X_C = \omega C V = (2\pi \times 30 \times 10^6) \times 33.2 \times 10^{-12} \times 800 = 5\ A$

Now, as seen from above $P = \frac{V^2}{R_{sh}} = \frac{V^2}{1/\omega C \times p.f.} = V^2 \omega C \times \text{p.f.}$ or $P \propto V^2 f$

$\therefore$ $800^2 \times 30 = 600^2 \times f$ or $f = \left(\frac{800}{600}\right)^2 \times 30 =$ **53.3 MHz**

Tutorial Problem No. 13.2

1. A capacitor having a capacitance of 20 μF is connected in series with a non-inductive resistance of 120 Ω across a 100-V, 50-Hz supply, Calculate (*a*) voltage (*b*) the phase difference between the current and the supply voltage (*c*) the power. Also draw the vector diagram.

[(*a*) 0.501A (*b*) 52.9° (*c*) 30.2 W]

2. A capacitor and resistor are connected in series to an a.c. supply of 50 V and 50 Hz. The current is 2 A and the power dissipated in the circuit is 80 W. Calculate the resistance of the resistor and the capacitance of the capacitor. **[20 Ω ; 212 μF]**

3. A voltage of 125 V at 50 Hz is applied to a series combination of non-inductive resistor and a lossless capacitor of 50 μF. The current is 1.25 A. Find (*i*) the value of the resistor (*ii*) power drawn by the network (*iii*) the power factor of the network. Draw the phasor diagram for the network. **[(*i*) 77.3 Ω (*ii*) 121 W (*iii*) 0.773 (lead)] (*Electrical Technology-1, Osmania Univ.*)**

4. A black box contains a two-element series circuit. A voltage (40 $-j30$) drives a current of (40 $-j3$) A in the circuit. What are the values of the elements ? Supply frequency is 50 Hz. **[R = 1.05 ; C = 4750 μF] (*Elect. Engg. and Electronics Bangalore Univ.*)**

5. Following readings were obtained from a series circuit containing resistance and capacitance :
V = 150 V ; I = 2.5 A; P = 37.5 W, f = 60 Hz.
Calculate (*i*) Power factor (*ii*) effective resistance (*iii*) capacitive reactance and (*iv*) capacitance. **[(*i*) 0.1 (*ii*) 6 Ω (*iii*) 59.7 Ω (*iv*) 44.4 μF]**

6. An alternating voltage of 10 volt at a frequency of 159 kHz is applied across a capacitor of 0.01 μF. Calculate the current in the capacitor. If the power dissipated within the dielectric is 100 μW, calculate (*a*) loss angle (*b*) the equivalent series resistance (*c*) the equivalent parallel resistance. **[0.A (*a*) 10^{-4} radian (*b*) 0.01 Ω (*c*) 1 MΩ]**

13.9. Resistance, Inductance and Capacitance in Series

The three are shown in Fig. 13.33 (*a*) joined in series across an a.c. supply of r.m.s. voltage V.

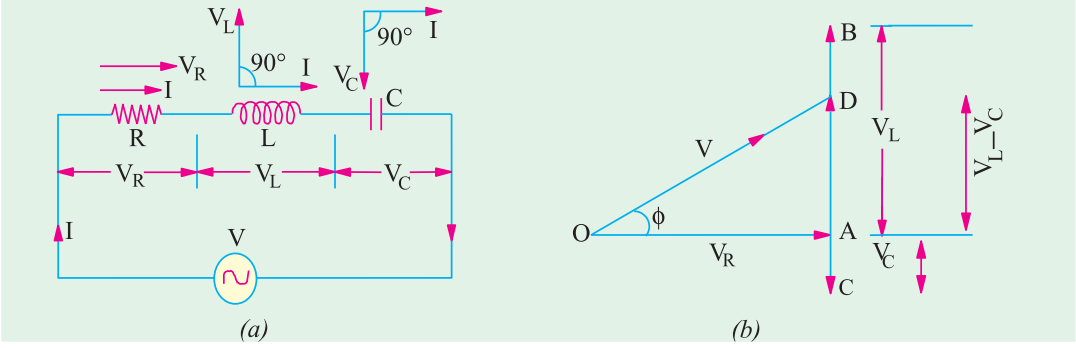


Fig. 13.33

Let $V_R = IR$ = voltage drop across R —in phase with I
$V_L = I.X_L$ = voltage drop across L —leading I by π/2
$V_C = I.X_C$ = voltage drop across C —lagging I by π/2

In voltage triangle of Fig. 13.33 (*b*), OA represents V_R, AB and AC represent the inductive and capacitive drops respectively. It will be seen that V_L and V_C are 180° out of phase with each other *i.e.* they are in direct opposition to each other.

Subtracting BD (= AC) from AB, we get the net reactive drop $AD = I(X_L - X_C)$
The applied voltage V is represented by OD and is the vector sum of OA and AD

$\therefore \quad OD = \sqrt{OA^2 \quad AD^2}$ or $V \quad \sqrt{(IR)^2 \quad (IX_L \quad IX_C)^2} \quad I\sqrt{R^2 \quad (X_L \quad X_C)^2}$

or $$I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{\sqrt{R^2 + X^2}} = \frac{V}{Z}$$

The term $\sqrt{R^2 + (X_L - X_C)^2}$ is known as the impedance of the circuit. Obviously,

(impedance)^2 = (resistance)^2 + (net reactance)^2

or $Z^2 = R^2 + (X_L - X_C)^2 = R^2 + X^2$

where X is the net reactance (Fig. 13.33 and 13.34).

Phase angle ϕ is given by $\tan\phi = (X_L - X_C)/R = X/R$ = net reactance/resistance

Power factor is $\cos\phi = \dfrac{R}{Z} = \dfrac{R}{\sqrt{R^2 + (X_L - X_C)^2}} = \dfrac{R}{\sqrt{R^2 + X^2}}$

Hence, it is seen that if the equation of the applied voltage is $v = V_m \sin \omega t$, then equation of the resulting current in an R-L-C circuit is given by $i = I_m \sin(\omega t \pm \phi)$

The + ve sign is to be used when current leads *i.e.* $X_C > X_L$.

The −ve sign is to be used when current lags *i.e.* when $X_L > X_C$.

In general, the current lags or leads the supply voltage by an angle ϕ such that $\tan\phi = X/R$

Using symbolic notation, we have (Fig. 13.35), $\mathbf{Z} = R + j(X_L - X_C)$

Numerical value of impedance $Z = \sqrt{R^2 + (X_L - X_C)^2}$

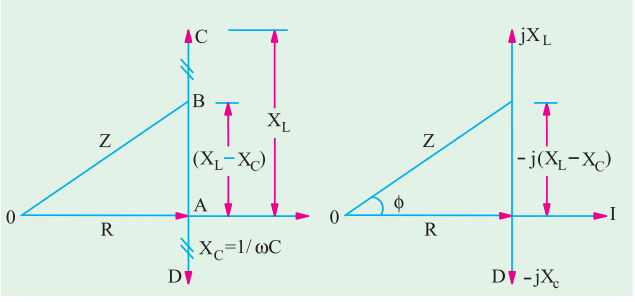


Fig. 13.34

Fig. 13.35

Its phase angle is $\Phi = \tan^{-1}[X_L - X_C/R]$

$Z = Z \angle \tan^{-1}[(X_L - X_C)/R] = Z \angle \tan^{-1}(X/R)$

If $\mathbf{V} = V \angle 0$, then, $\mathbf{I = V/Z}$

Summary of Results of Series AC Circuits

Type of Impedance	*Value of Impedance*	*Phase angle for current*	*Power factor*
Resistance only	R	0°	1
Inductance only	ωL	90° lag	0
Capacitance only	$1/\omega C$	90° lead	0
Resistance and Inductance	$\sqrt{[R^2 + (\omega L)^2]}$	$0 < \phi < 90°$ lag	1 > p.f. > 0 lag
Resistance and Capacitance	$\sqrt{[R^2 + (-1/\omega C)^2]}$	$0 < \phi < 90°$ lead	1 > p.f. > 0 lead
R-L-C	$\sqrt{[R^2 + (\omega L \sim 1/\omega C)^2]}$	between 0° and 90° lag or lead	between 0 and unity lag or lead

Example 13.43. *A resistance of 20 Ω, an inductance of 0.2 H and a capacitance of 100 μF are connected in series across 220-V, 50-Hz mains. Determine the following* ***(a)*** *impedance* ***(b)*** *current* ***(c)*** *voltage across R, L and C* ***(d)*** *power in watts and VA* ***(e)*** *p.f. and angle of lag.*

(Elect. Engg. A.M.Ae S.I. 1992)

Solution. $X_C = 0.2 \times 314 = 63\ \Omega$, $C = 10\ \mu F = 100 \times 10^{-6} = 10^{-4}$ farad

$X_C = \dfrac{1}{\omega C} = \dfrac{1}{314 \times 10^{-4}} = 32\ \Omega$, $X = 63 - 32 =$ **31 Ω (*inductive*)**

(a) $Z = \sqrt{(20^2 + 31)^2} =$ **37 Ω** ***(b)*** $I = 220/37 =$ **6 A (approx)**

(c) $V_R = I \times R = 6 \times 20 =$ **120 V;** $V_L = 6 \times 63 =$ **278 V,** $V_C = 6 \times 32 =$ **192 V**

(d) Power in $VA = 6 \times 220 =$ **1320**

Power in watts $= 6 \times 220 \times 0.54 =$ **713 W**

(e) p.f. $= \cos\phi = R/Z = 20/37 =$ **0.54;** $\phi = \cos^{-1}(0.54) =$ **57°18′**

Example 13.44. *A voltage e(t) = 100 sin 314 t is applied to series circuit consisting of 10 ohm resistance, 0.0318 henry inductance and a capacitor of 63.6 μF. Calculate* ***(i)*** *expression for i (t)* ***(ii)*** *phase angle between voltage and current* ***(iii)*** *power factor* ***(iv)*** *active power consumed* ***(v)*** *peak value of pulsating energy.* **(Elect. Technology, Indore Univ.)**

Solution. Obviously, $\omega = 314$ rad/s ; $X_l = \omega L = 314 \times 0.0318 = 10\ \Omega$

$X_C = 1/\omega C = 1/314 \times 63.6 \times 10^{-6} = 50\ \Omega$; $X = X_L - X_C = (10 - 50) = -40\ \Omega$ (capacitive)

$\mathbf{Z} = 10 - j\,40 = 41.2 \angle -76°$;

$\mathbf{I} = \dfrac{\mathbf{V}}{\mathbf{Z}} \quad \dfrac{(100/\sqrt{2})}{\quad} \quad 1.716$

$I_m = I \times \sqrt{2} = 1.716 \times \sqrt{2} = 2.43\ A$

(i) $i\,(t) = 2.43 \sin (314\,t + 76°)$

(ii) $\phi = 76°$ with current leading

(iii) p.f. $= \cos \phi = \cos 76° =$ **0.24 (lead)**

(iv) Active power, $P = VI \cos \phi$

$= (100/\sqrt{2})\,(2.43/\sqrt{2}) \times 0.24 =$ **29.16 W**

(v) As seen from Fig. 13.36, peak value of pulsating energy is $\dfrac{V_m I_m}{2} + \dfrac{V_m I_m}{2} \cos \phi$

$= \dfrac{V_m I_m}{2}(1 + \cos \phi) = \dfrac{100 \times 2.43}{2}(1 + 0.24) =$ **151 W**

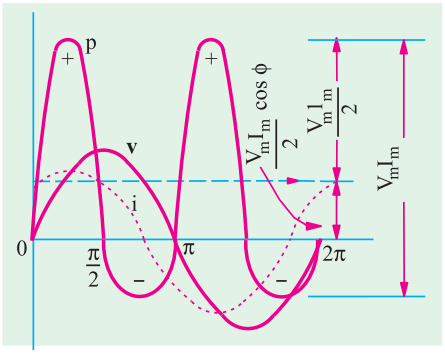


Fig. 13.36

Example 13.45. *Two impedances Z_1 and Z_2 when connected separately across a 230-V, 50-Hz supply consumed 100 W and 60 W at power factors of 0.5 lagging and 0.6 leading respectively. If these impedances are now connected in series across the same supply, find :*

(i) *total power absorbed and overall p.f.* ***(ii)*** *the value of the impedance to be added in series so as to raise the overall p.f. to unity.* **(Elect. Circuits-I, Bangalore Univ.)**

Solution. Inductive Impedance $V_1 I \cos \phi_1 =$ power; $230 \times I_1 \times 0.5 = 100$; $I_1 = 0.87$ A

Now, $I_1^2 R_1 =$ power or $0.87^2 R_1 = 100$; $R_1 = 132\ \Omega$; $Z_1 = 230/0.87 = 264\ \Omega$

$$X_L = \sqrt{Z_1^2 - R_1^2} = \sqrt{264^2 - 132^2} = 229\ \Omega$$

Capacitance Impedance $I_2 = 60/230 \times 0.6 = 0.434$ A ; $R_2 = 60/0.434^2 = 318\ \Omega$

$$Z_2 = 230/0.434 = 530\ \Omega\ ; X_C = \sqrt{530^2 - 318^2} = 424\ \Omega \text{ (capacitive)}$$

When Z_1 and Z_2 are connected in series

$$R = R_1 + R_2 = 132 + 318 = 450\ \Omega;\ X = 229 - 424 = -195\ \Omega \text{ (capacitive)}$$

$$Z = \sqrt{R^2 + X^2} = \sqrt{450^2 + (-195)^2} = 490\ \Omega,\ I = 230/490 = 0.47\ \text{A}$$

(i) Total power absorbed $= I^2 R = 0.47^2 \times 450 =$ **99 W,** $\cos \phi = R/Z = 450/490 =$ **0.92 (lead)**

(ii) Power factor will become unity when the net capacitive reactance is neutralised by an equal inductive reactance. The reactance of the required series pure inductive coil is **195 Ω**

Example 13.46. *A resistance R, an inductance L = 0.01 H and a capacitance C are connected in series. When a voltage v = 400 cos (300 t –10°) votls is applied to the series combination, the current flowing is $10\sqrt{2}$ cos (3000 t – 55°) amperes. Find R and C.*

(Elect. Circuits Nagpur Univ. 1992)

Solution. The phase difference between the applied voltage and circuit current is (55° –10°) = 45° with current lagging. The angular frequency is $\omega = 3000$ radian/second. Since current lags, $X_L > X_C$.

Net reactance $X = (X_L - X_C)$. Also $X_L = \omega L = 3000 \times 0.01 = 30\ \Omega$

$\tan \phi = X/R$ or $\tan 45° = X/R$ $\therefore X = R$ Now, $Z = \frac{V_m}{I_m} = \frac{400}{10\sqrt{2}} = 28.3\ \Omega$

$Z^2 = R^2 + X^2 = 2R^2$ $\therefore$ $R = Z/\sqrt{2} = 28.3/\sqrt{2} = 20\ \Omega$; $X = X_L - X_C = 30 - X_C = 20$

$X_C = 10\ \Omega$ or $\frac{1}{\omega C} = 10$ or $\frac{1}{3000\,C} =$ or $C =$ **33 μF**

Example 13.47. *A non-inductive resistor is connected in series with a coil and a capacitor. The circuit is connected to a single-phase a.c. supply. If the voltages are as indicated in Fig. 13.37 when current flowing through the circuit is 0.345 A, find the applied voltage and the power loss in coil.* **(Elect. Engg. Pune Univ.)**

Solution. It may be kept in mind that the coil has not only inductance L but also some resistance r which produces power loss. In the voltage vector diagram, AB represents drop across R = 25 V. Vector BC represents drop across coil which is due to L and r. Which value is 40 V and the vector BC is at any angle of ϕ with the current vector. AD represents 50 V which is the drop across R and coil combined. AE represents the drop across the capacitor and leads the current by 90°.

It will be seen that the total horizontal drop in the circuit is AC and the vertical drop is AG. Their vector sum AF represents the applied voltage V.

From triangle ABD, we get $50^2 = 40^2 + 25^2 + 2 \times 25 \times 40 \times \cos\phi$ $\therefore$ $\cos\phi = 0.1375$ and $\sin\phi = 0.99$. Considering the coil, $IZ_L = 40$ $\therefore$ $Z_L = 40/0.345 = 115.94\ \Omega$

Now $r = Z_L \cos\phi = 115.94 \times 0.1375 = 15.94\ \Omega$

Power loss in the coil = $I^2 r = 0.345^2 \times 15.94 =$ **1.9 W**

$BC = BD \cos\phi = 40 \times 0.1375 = 5.5$ V; $CD = BD \sin\phi = 40 \times 0.99 = 39.6$ V

$AC = 25 + 5.5 = 30.5$ V; $AG = AE - DC = 55 - 39.6 = 15.4$ V

$$AF = \sqrt{AC^2 + CF^2} = \sqrt{30.5^2 + 15.4^2} = \textbf{34.2 V}$$

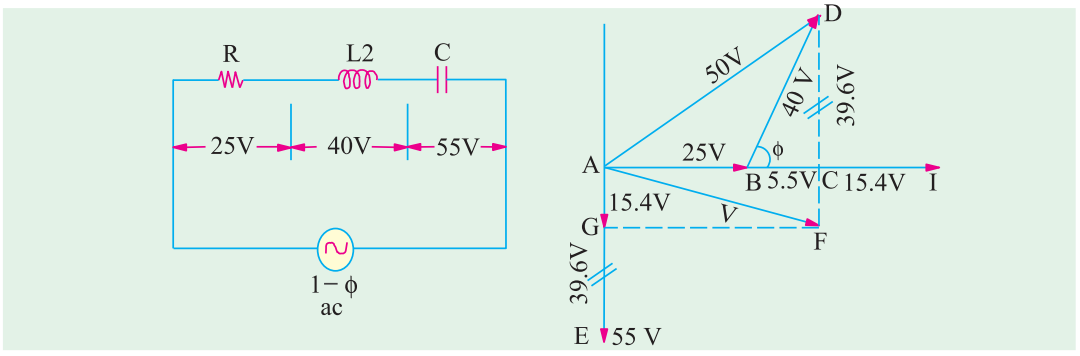


Fig. 13.37

Example 13.48. *A 4.7 H inductor which has a resistance of 20 Ω, a 4-μF capacitor and a 100-Ω non-inductive resistor are connected in series to a 100-V, 50-Hz supply. Calculate the time interval between the positive peak value of the supply voltage and the next peak value of power.*

Solution. Total resistance = 120 Ω; $X_L = 2\pi \times 50 \times 4.7 = 1477\ \Omega$

$X_C = 10^6/2\pi \times 50 \times 4 = 796\ \Omega$; $X = 1477 - 798 = 681\ \Omega$;

$Z = \sqrt{120^2 + 681^2} = 691.3\ \Omega$

$\cos\phi = R/Z = 120/691.3 = 0.1736$; $\phi = 80°$

Now, as seen from Fig. 13.38, the angular displace-

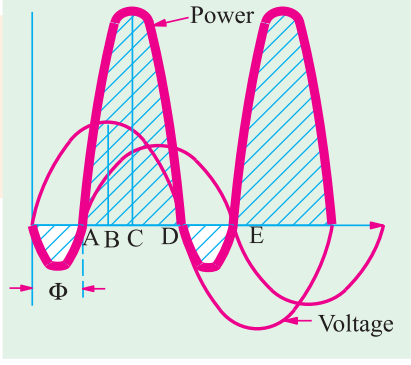


Fig. 13.38

ment between the peak values of supply voltage and power cycles is $BC = \phi/2$ because $AB = 90\text{-}\phi$ and $AD = 180 - \phi$.

Hence $AC = 90 - \phi/2$

$\therefore \quad BC = AC - AB = (90 - \phi/2) - (90 - \phi) = \phi/2$

Angle difference $= \phi/2 = 80°/2 = 40°$

Since a full cycle of 360° corresponds to a time interval of 1.50 second

$$\therefore \quad 40° \text{ angular interval} = \frac{40}{50 \times 360} = \mathbf{2.22 \text{ ms.}}$$

Example 13.49. *A coil is in series with a 20 μF capacitor across a 230-V, 50-Hz supply. The current taken by the circuit is 8 A and the power consumed is 200 W. Calculate the inductance of the coil if the power factor of the circuit is* ***(i)*** *leading* ***(ii)*** *lagging.*

Sketch a vector diagram for each condition and calculate the coil power factor in each case.

(Elect. Engg.-I Nagpur Univ. 1993)

Solution. ***(i)*** Since power factor is leading, net reactance $X = (X_C - X_L)$ as shown in Fig. 13.39 (*a*).

$$I^2R = 200 \text{ or } 8^2 \times R = 200; \quad \therefore \quad R = 200/64 = 25/8\ \Omega = 3.125\ \Omega$$

$$Z = V/I = 230/8 = 28.75\ \Omega; \ X_C = 10^6/2\pi \times 50 \times 20 = 159.15\ \Omega$$

$R_2 + X_2 = 28.75^2 \quad \therefore \ X = 28.58\ \Omega \quad \therefore \ (X_C - X_L) = 28.58 \quad \text{or} \quad 159.15 - X_L = 28.58$

$\therefore \quad X_L = 130.57\ \Omega$ or $2\pi \times 50 \times L = 130.57 \quad \therefore \ L = 0.416\ H$

If θ is the p.f. angle of the coil, then $\tan \theta = R/X_L = 3.125/130.57 = 0.024$; $\theta = 1.37°$, p.f. of the coil = 0.9997

(ii) When power factor is lagging, net reactance is $(X_L - X_C)$ as shown in Fig. 13.39 (*b*).

$\therefore \quad X_L - 159.15 = 28.58 \quad \text{or} \quad X_L = 187.73\ \Omega \quad \therefore \ 187.73 = 2\pi \times 50 \times L$ or $L = 0.597$ H.

In this case, $\tan \theta = 3.125/187.73 = 0.0167$; $\theta = 0.954° \ \therefore \cos \theta = 0.9998$.

The vector diagrams for the two conditions are shown in Fig. 13.35.

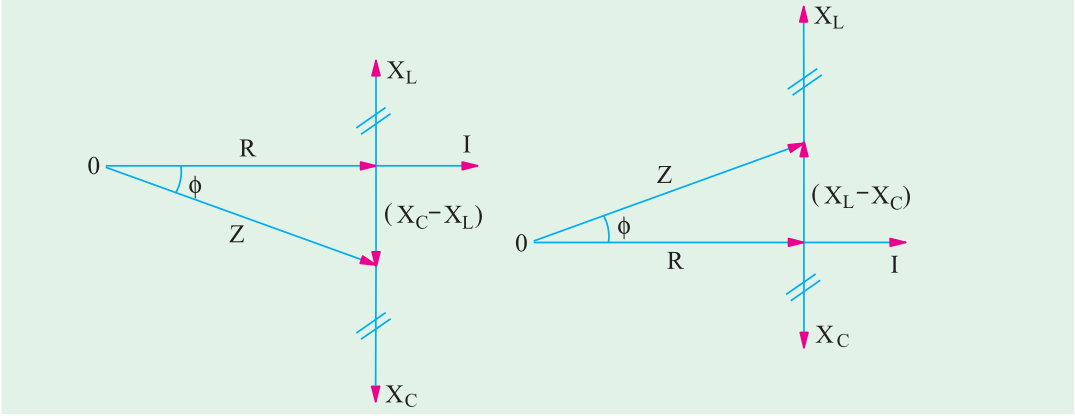


Fig. 13.39

Example 13.50. *In Fig. 13.40, calculate* ***(i)*** *current* ***(ii)*** *voltage drops V_1, V_2, and V_3 and* ***(iii)*** *power absorbed by each impedance and total power absorbed by the circuit. Take voltage vector along the reference axis.*

Solution. $\mathbf{Z_1} = (4 + j3)\ \Omega; \ \mathbf{Z_2} = (6 - j8)\ \Omega; \ \mathbf{Z_3} = (4 + j0)\Omega$

$\mathbf{Z = Z_1 + Z_2 + Z_3} = (4 + j3) + (6 - j8) + (4 + j0) = (14 - j5)\Omega$

Taking $\mathbf{V} = V \angle 0° = 100 \angle 0° = (100 + j0)$

$$\therefore \quad \mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} \quad \frac{100}{(14 \quad j5)} \quad \frac{100\,(14 \quad j5)}{(14 \quad j5)\,(14 \quad j5)} \quad 6.34 \quad j2.26$$

(i) Magnitude of the current

$= \sqrt{(6.34^2 + 2.26^2)} = \mathbf{6.73\ A}$

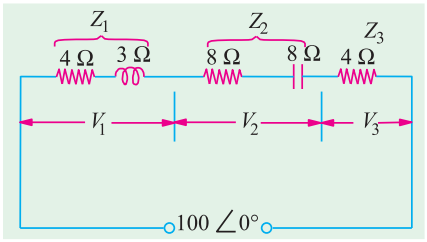


Fig. 13.40

(ii) $\mathbf{V_1 = IZ_1} = (6.34 + j\,2.26)\,(4 + j3) = 18.58 + j\,28.06$

$\mathbf{V_2 = IZ_2} = (6.34 + j\,2.26)\,(6 - j8) = 56.12 - j\,37.16$

$\mathbf{V_3 = IZ_3} = (6.34 + j\,2.26)\,(4 + j0) = 25.36 + j\,9.04$

$\mathbf{V} = 100 + j\,0$ (check)

(iii) $P_1 = 6.73^2 \times 4 = 181.13$ W.

$P_2 = 6.73^2 \times 6 = 271.74$ W, $P_3 = 6.73^2 \times 4 = 181.13$ W,

Total = 34 W

Otherwise $\mathbf{P_{VA}} = (100 + j\,0)\,(6.34 - j\,2.26)$ (using current conjugate)

$= 634 - j\,226$

real power = 634 W (as a check)

Example 13.51. *Draw a vector for the circuit shown in Fig. 13.41 indicating the resistance and reactance drops, the terminal voltages V_1 and V_2 and the current. Find the values of* **(i)** *the current I* **(ii)** *V_1 and V_2 and* **(iii)** *p.f.* **(Elements of Elect Engg-I, Bangalore Univ.)**

Solution. $L = 0.05 + 0.1 = 0.15$ H; $X_L = 314 \times 0.5 = 47.1\ \Omega$

$X_C = 10^6/314 \times 50 = 63.7\ \Omega$; $X = 47.1 - 63.7 = -16.6\ \Omega$, $R = 30\ \Omega$,

$Z = \sqrt{30^2 + (-16.6)^2} = 34.3\ \Omega$

(i) $I = V/Z = 200/34.3 = \mathbf{5.83\ A}$, from Fig. 13.41 (*a*)

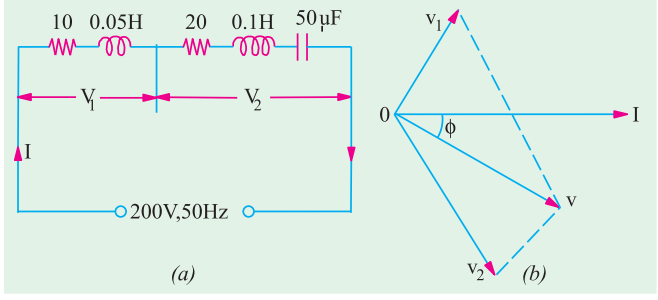


Fig. 13.41

(ii) $X_{L1} = 314 \times 0.05$

$= 15.7\ \Omega$

$Z_1 = \sqrt{10^2 + 15.7^2} = 18.6\ \Omega$

$V_1 = IZ_1 = 5.83 \times 18.6$

$= \mathbf{108.4\ V}$

$\phi_1 = \cos^{-1}(10/18.6) = 57.5°$ (lag)

$X_{L2} = 314 \times 0.1 = 31.4\ \Omega$, $X_c = -63.7\ \Omega$, $X = 31.4 - 63.7 = -32.2\ \Omega$, $Z_2 = \sqrt{20^2 + (-32.3)^2} = \mathbf{221\ V}$

$\phi_2 = \cos^{-1}(20/38) = 58.2$ (lead)

(iii) Combined p.f. = $\cos\phi = R/Z = 30/34.3 = \mathbf{0.875\ (lead)}$, from Fig. 13.41 (*b*).

Example 13.52. *In a circuit, the applied voltage is found to lag the current by 30°.*

(a) *Is the power factor lagging or leading ?* ***(b)*** *What is the value of the power factor ? (c) Is the circuit inductive or capacitive ?*

In the diagram of Fig 13.42, the voltage drop across Z_1 is (10 + j0) volts. Find out

(i) *the current in the circuit* ***(ii)*** *the voltage drops across Z_2 and Z_3* ***(iii)*** *the voltage of the generator.*

(Elect. Engg.-I, Bombay Univ. 1991)

Solution. (*a*) Power factor is ***leading*** because current leads the voltage.

(*b*) p.f. = cos 30° = **0.86 (lead)** **(*c*)** The circuit is **capacitive.**

(*i*) Circuit current can be found by dividing voltage drop $\mathbf{V_1}$ by $\mathbf{Z_1}$

$$\mathbf{I} = \frac{10 + j\,0}{3 + j\,4} = \frac{10 \angle 0°}{5 \angle 53.1°} = 2 \angle -53.1° = 2(\cos 53.1° - j \sin 53.1°)$$

$= 2\,(0.6 - j\,0.8) = 1.2 - j\,1.6$

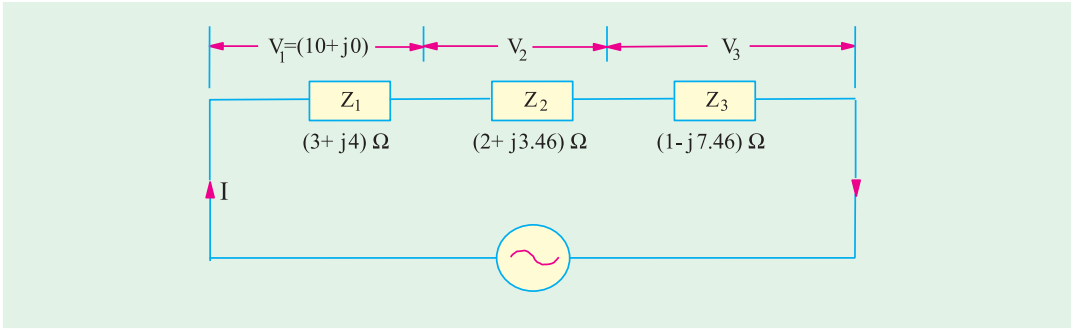


Fig. 13.42

$\mathbf{Z_2} = 2 + j\,3.46$; $\mathbf{V_2 = IZ_2} = (1.2 - j\,1.6)\,(2 + j3.46) = (7.936 + j\,0.952)$ volt

$\mathbf{V_3} = (1.2 - j\,1.6)\,(1 - j\,7.46) = (-10.74 - j\,10.55)$ volt

(ii) $\mathbf{V} = \mathbf{V_1} + \mathbf{V_2} + \mathbf{V_3} = (10 + j\,0) + (7.936 + j\,0.952) + (-10.74 - j\,10.55)$

$= (7.2 - j\,9.6) = 12 \angle - 53.1°$

Incidentally, it shows that current **I** and voltage **V** are in phase with each other.

Example 13.53. *A 230-V, 50-Hz alternating p.d. supplies a choking coil having an inductance of 0.06 henry in series with a capacitance of 6.8 μF, the effective resistance of the circuit being 2.5 Ω. Estimate the current and the angle of the phase difference between it and the applied p.d. If the p.d. has a 10% harmonic of 5 times the fundamental frequency, estimate* **(a)** *the current due to it and* **(b)** *the p.d. across the capacitance.* **(Electrical Network Analysis, Nagpur Univ. 1993)**

Solution. Fundamental Frequency : For the circuit in Fig. 13.43,

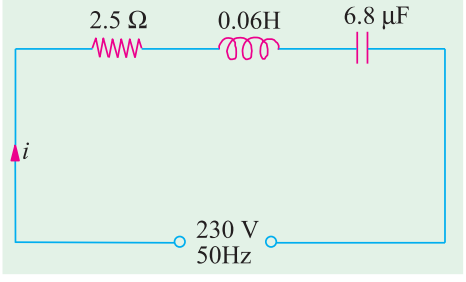


Fig. 13.43

$X_L = \omega L = 2\pi \times 50 \times 0.06 = 18.85\ \Omega$

$X_C = \dfrac{10^6}{2\pi \times 50 \times 6.8} = 648\ \Omega$

$\therefore \quad X = 18.85 - 468 = -449.15\ \Omega$

$Z = \sqrt{2.5^2 + (-449.15)^2} = 449.2\ \Omega$

Current I_f = 230/449.2 0.512 A, phase angle $\tan^{-1}\dfrac{449.2}{2.5}$ 89 .42

$\therefore$ current leads p.d. by 89° 42′ .

Fifth Harmonic Frequency $X_L = 18.85 \times 5 = 94.25\ \Omega$; $X_C = 468 + 5 = 93.6\ \Omega$

$X = 94.25 - 93.6 = 0.65\ \Omega$; **Z** $\sqrt{2.5^2 \quad 0.65}$ 2.585 , Harmonic p.d. = 230 × 10/100 = 23 V

$\therefore$ Harmonic current $I_h = 23/2.585 = 8.893$ A

P.D. across capacitor at harmonic frequency is, $V_h = 8.893 \times 93.6 = 832.6$ V

The total current flowing through the circuit, due to the complex voltage wave form, is found from the fundamental and harmonic components thus. Let,

I = the r.m.s. value of total circuit current,

I_f = r.m.s. value of fundamental current,

I_h = r.m.s. value of fifth harmonic current,

(a) $\therefore \quad I = \sqrt{I_f^2 + I_h^2} = \sqrt{0.512^2 + 8.893^2} = \mathbf{8.9\ A}$

(*b*) The r.m.s. value of p.d. across capacitor is found in a similar way.

$$V_f = 0.512 \times 468 = 239.6 \text{ V}$$

$$\therefore \quad V = \sqrt{(V_f^2 + V_h^2)} = \sqrt{239.6^2 + 832.6^2} = \mathbf{866.4\ V}$$

Tutorial Problem No. 13.3

1. An e.m.f. represented by $e = 100 \sin 100\pi t$ is impressed across a circuit consisting of 40-Ω resistor in series with a 40-μ*F* capacitor and a 0.25 H indicator.
Determine (*i*) the r.m.s. value of the current (*ii*) the power supplied (*iii*) the power factor.
[(*i*) 1.77 A (*ii*) 125 W (*iii*) 1.0] *(London Univ.)*

2. A series circuit with a resistor of 100 Ω capacitor of 25 μ*F* and inductance of 0.15 H is connected across 220-V, 60-Hz supply. Calculate (*i*) current (*ii*) power and (*iii*) power factor in the circuit.
[(*i*) 1.97 A; (*ii*) 390 W (*iii*) 0.9 (lead)] *(Elect. Engg. and Electronics Bangalore Univ.)*

3. A series circuit with $R = 10\ \Omega$, $L = 50\ mH$ and $C = 100\ \mu F$ is supplied with 200 V/50 Hz. Find (*i*) the impedance (*ii*) current (*iii*) power (*iv*) power factor.
[(*i*) 18.94 Ω (*ii*) 18.55 A (*iii*) 1966 W (*iv*) 0.53 (leading)]
(Elect. Engg. & Electronics Bangalore Univ.)

4. A coil of resistance 10 Ω and inductance 0.1 H is connected in series with a 150-μ*F* capacitor across a 200-V, 50-Hz supply. Calculate (*a*) the inductive reactance, (*b*) the capacitive reactance, (*c*) the impedance (*d*) the current, (*e*) the power factor (*f*) the voltage across the coil and the capacitor respectively. **[(*a*) 31.4 Ω (*b*) 21.2 Ω (*c*) 14.3 Ω (*d*) 14 A (*e*) 0.7 lag (*f*) 460 V, 297 V]**

5. A circuit is made up of 10 Ω resistance, 12 mH inductance and 281.5 μ*F* capacitance in series. The supply voltage is 100 V (constant). Calculate the value of the current when the supply frequency is (*a*) 50 Hz and (*b*) 150 Hz. **[8 A leading; 8 A lagging]**

6. A coil having a resistance of 10 Ω and an inductance of 0.2 H is connected in series with a capacitor of 59.7 μ*F*. The circuit is connected across a 100-V, 50-Hz a.c. supply. Calculate (*a*) the current flowing (*b*) the voltage across the capacitor (*c*) the voltage across the coil. Draw a vector diagram to scale. **[(*a*) 10 A (*b*) 628 V (*c*) 635 V]**

7. A coil is in series with a 20 μ*F* capacitor across a 230-V , 50-Hz supply. The current taken by the circuit is 8 A and the power consumed is 200 W. Calculate the inductance of the coil if the power factor of the circuit is (*a*) leading and (*b*) lagging.
Sketch a vector diagram for each condition and calculate the coil power factor in each case.
[0.415 H; 0.597 H; 0.0238 ; 0.0166]

8. A circuit takes a current of 3 A at a power factor of 0.6 lagging when connected to a 115-V, 50-Hz supply. Another circuit takes a current, of 5 A at a power factor of 0.707 leading when connected to the same supply. If the two circuits are connected in series across a 230-V, 50Hz supply, calculate (*a*) the current (*b*) the power consumed and (*c*) the power factor.
[(*a*) 5.5 A (*b*) 1.188 kW (*c*) 0.939 lag]

9. A coil of insulated wire of resistance 8 ohms and inductance 0.03 H is connected to an a.c. supply at 240 V, 50-Hz. Calculate (*a*) the current, the power and power factor (*b*) the value of a capacitance which, when connected in series with the above coil, causes no change in the values of current and power taken from the supply. **[(*a*) 19.4 A, 3012 W, 0.65 lag (*b*) 168.7 μF] *(London Univ.)***

10. A series circuit, having a resistance of 10 Ω, an inductance of 0.025 H and a variable capacitance is connected to a 100-V, 25-Hz single-phase supply. Calculate the capacitance when the value of the current is 8 A. At this value of capacitance, also calculate (*a*) the circuit impedance (*b*) the circuit power factor and (*c*) the power consumed. **[556 μF (*a*) 1.5 Ω (*b*) 0.8 leading (*c*) 640 W]**

11. An alternating voltage is applied to a series circuit consisting of a resistor and iron-cored inductor and a capacitor. The current in the circuit is 0.5 A and the voltages measured are 30 V across the resistor, 48 V across the inductor, 60 V across the resistor and inductor and 90 V across the capacitor. Find (*a*) the combined copper and iron losses in the inductor (*b*) the applied voltage.
[(*a*) 3.3 W (*b*) 56 V] *(City & Guilds, London)*

12. When an inductive coil is connected across a 250-V, 50-Hz supply, the current is found to be 10 A and the power absorbed 1.25 kW. Calculate the impedance, the resistance and the inductance of the coil. A capacitor which has a reactance twice that of the coil, is now connected in series with the coil across the same supply. Calculate the p.d. across the capacitor. **[25 Ω; 12.5 Ω; 68.7 mH; 433 V]**

13. A voltage of 200 V is applied to a series circuit consisting on a resistor, an inductor and a capacitor. The respective voltages across these components are 170, 150 and 100 V and the current is 4 A. Find the power factor of the inductor and of the circuit. **[0.16; 0.97]**

14. A pure resistance R, a choke coil and a pure capacitor of 50μ F are connected in series across a supply of V volts, and carry a current of 1.57 A. Voltage across R is 30 V, across choke coil 50 V and across capacitor 100 V. The voltage across the combination of R and choke coil is 60 volt. Find the supply voltage V, the power loss in the choke, frequency of the supply and power factor of the complete circuit. Draw the phasor diagram. **[60.7 V; 6.5 W; 0.562 lead] (*F.E. Pune Univ.*)**

13.10. Resonance in R-L-C Circuits

We have seen from Art. 13.9 that net reactance in an R-L-C circuit of Fig. 13.40 (a) is

$$X = X_L - X_C \text{ and } Z = \sqrt{[R^2 + (X_L - X_C)^2]} = \sqrt{R^2 + X^2}$$

Let such a circuit be connected across an a.c. source of constant voltage V but of frequency varying from zero to infinity. There would be a certain frequency of the applied voltage which would make X_L equal to X_C in magnitude. In that case, $X = 0$ an $Z = R$ as shown in Fig. 13.40 (c). Under this condition, the circuit is said to be in electrical resonance.

As shown in Fig. 13.40 (c), $V_L = I.\ X_L$ and $V_C = I.\ X_C$ and the two are equal in magnitude but opposite in phase. Hence, they cancel each other out. The two reactances taken together act as a short-circuit since no voltage develops across them. Whole of the applied voltage drops across R so that $V = V_R$. The circuit impedance $Z = R$. The phasor diagram for series resonance is shown in Fig. 13.40 (d).

Calculation of Resonant Frequency

The frequency at which the net reactance of the series circuit is zero is called the resonant frequency f_0. Its value can be found as under : $X_L - X_C = 0$ or $X_L = X_C$ or $\omega_0 L = 1/\omega_0 C$

or $$\omega_0^2 = \frac{1}{LC} \text{ or } (2\pi f_0)^2 = \frac{1}{LC} \text{ or } f_0 = \frac{1}{2\pi\sqrt{LC}}$$

If L is in henry and C in farad, then f_0 is given in Hz.

When a series R-L-C circuit is in resonance, it possesses minimum impedance $Z = R$. Hence, circuit current is maximum, it being limited by value of R alone. The current $I_0 = V/R$ and is in phase with V.

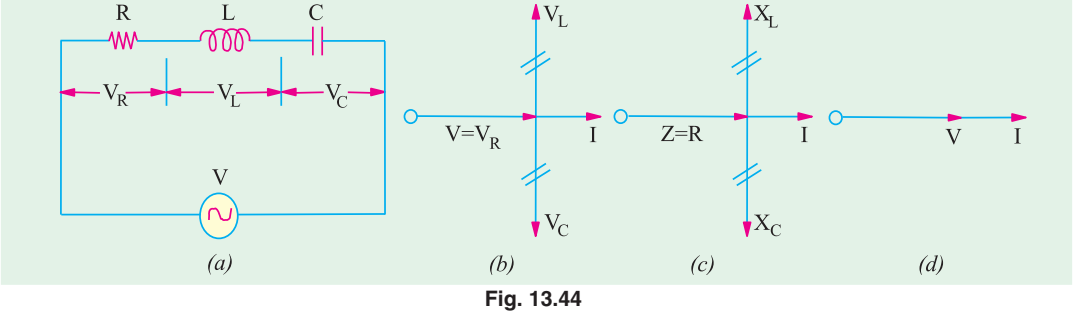


Fig. 13.44

Since circuit current is maximum, it produces large voltage drops across L and C. But these drops being equal and opposite, cancel each other out. Taken together, L an C from part of a circuit across which no voltage develops, however, large the current flowing. If it were not for the presence of R, such a resonant circuit would act like a short-circuit to currents of the frequency to which it resonates. Hence, a series resonant circuit is sometimes called ***acceptor*** circuit and the series resonance is often referred to as voltage resonance.

In fact, at resonance the series RLC circuit is reduced to a purely resistive circuit, as shown in Fig. 13.44.

Incidentally, it may be noted that if X_L and X_C are shown at any frequency f, that the value of the resonant frequency of such a circuit can be found by the relation $f_0 = f\sqrt{X_C/X_L}$.

Summary

When an *R-L-C* circuit is in resonance

1. net reactance of the circuit is zero *i.e.* $(X_L - X_C) = 0$. or $X = 0$.
2. circuit impedance is minimum *i.e.* $Z = R$. Consequently, circuit admittance is maximum.
3. circuit current is maximum and is given by $I_0 = V/Z_0 = V/R$.
4. power dissipated is maximum *i.e.* $P_0 = I_0^2 R = V^2/R$.
5. circuit power factor angle $\theta = 0$. Hence, power factor $\cos\theta = 1$.
6. although $V_L = V_C$ yet V_{coil} is greater than V_C because of its resistance.
7. at resonance, $\omega^2 LC = 1$
8. $Q = \tan\theta = \tan 0° = 0$*.

13.11. Graphical Representation of Resonance

Suppose an alternating voltage of constant magnitude, but of varying frequency is applied to an *R-L-C* circuit. The variations of resistance, inductive reactance X_L and capacitive reactance X_C with frequency are shown in Fig. 13.45 (*a*).

(i) Resistance : It is independent of *f*, hence, it is represented by a straight line.

(ii) Inductive Reactance : It is given by $X_L = \omega L = 2\pi fL$. As seen, X_L is directly proportional to *f i.e.* X_L increases linearly with *f*. Hence, its graph is a straight line passing through the origin.

(iii) Capacitive Reactance : It is given by $X_C = 1/\omega C = 1/2\pi fC$. Obviously, it is inversely proportional to *f*. Its graph is a rectangular hyperbola which is drawn in the fourth quadrant because X_C is regarded negative. It is asymptotic to the horizontal axis at high frequencies and to the vertical axis at low frequencies.

(iv) Net Reactance : It is given by $X = X_L \sim X_C$. Its graph is a hyperbola (not rectangular) and crosses the *X*-axis at point *A* which represents resonant frequency f_0.

(v) Circuit Impedance : It is given by $Z = \sqrt{[R^2 + (X_L \sim X_C)^2]} = \sqrt{R^2 + X^2}$

At low frequencies *Z* is large because X_C is large. Since $X_C > X_L$, the net circuit reactance *X* is capacitive and the p.f. is leading [Fig. 13.45 (*b*)]. At high frequencies, *Z* is again large (because X_L is large) but is inductive because $X_L > X_C$. Circuit impedance has minimum values at f_0 given by $Z = R$ because $X = 0$.

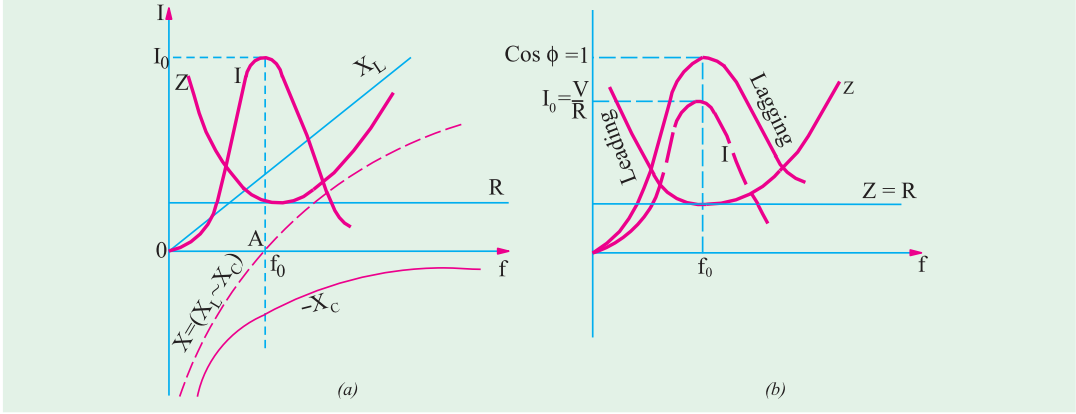


Fig. 13.45

(vi) Current I_0 : It is the reciprocal of the circuit impedance. When *Z* is low, I_0 is high and vice versa. As seen, I_0 has low value on both sides of f_0 (because *Z* is large there) but has maximum value of $I_0 = V/R$ at resonance. Hence, maximum power is dissipated by the series circuit under resonant conditions. At frequencies below and above resonance, current decreases as shown in Fig. 13.45

* However, value of Q_0 is as given in Art 13.5, 13.9 and 13.17.

(b). Now, $I_0 = V/R$ and $I = V/Z = V/\sqrt{(R^2 + X^2)}$. Hence $I/I_0 = R/Z = V/\sqrt{(R^2 + X^2)}$ where X is the net circuit reactance at any frequency f.

(vii) Power Factor

As pointed out earlier, X is capacitive below f_0. Hence, current leads the applied voltage. However, at frequencies above f_0, X is inductive. Hence, the current lags the applied voltage as shown in Fig. 13.45. The power factor has maximum value of unity at f_0.

13.12. Resonance Curve

The curve, between circuit current and the frequency of the applied voltage, is known as resonance curve. The shapes of such a curve, for different values of R are shown in Fig. 13.46. For smaller values of R, the resonance curve is sharply peaked and such a circuit is said to be sharply resonant or highly selective. However, for larger values of R, resonance curve is flat and is said to have poor selectivity. The ability of a resonant circuit to discriminate between one particular frequency and all others is called its selectivity. The selectivities of different resonant circuits are compared in terms of their half-power bandwidths (Art. 13.13).

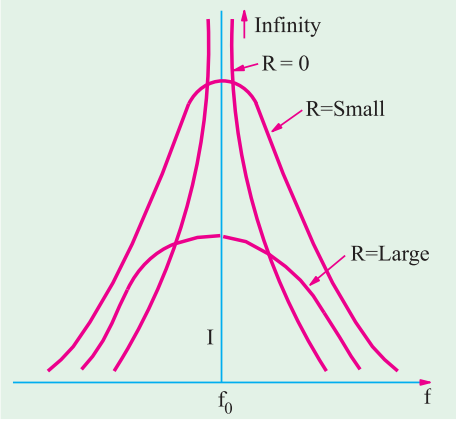


Fig. 13.46

13.13. Half-Power Bandwidth of a Resonant-Circuit

A discussed earlier, in an R-L-C circuit, the maximum current at resonance is solely determined by circuit resistance R ($\because X = 0$) but at off-resonance frequencies, the current amplitude depends on Z (where $X \neq 0$). The half-wave bandwidth of a circuit is given by the band of fequencies which lies between two points on either side of f_0 where current falls to $I_0/\sqrt{2}$. Narrower the bandwidth, higher the selectivity of the circuit and vice versa. As shown in Fig. 13.47 the half-power bandwidth AB is given by

$AB = \Delta f = f_2 - f_1$ or $AB = \Delta\omega = \omega_2 - \omega_1$ where f_1 and f_2 are the corner or edge frequencies.

As seen, $P_0 = I_0^2 R$. However, power at either of the two points A and B is

$$P_1 = P_2 = I^2R$$

$$= (I_0/\sqrt{2})^2 R = I_0^2R/2 = \frac{1}{2} I_0^2R = \frac{1}{2} \times \text{power at resonance}$$

That is why the two points A and B on the resonance curve are known as half-power points* and the corresponding value of the bandwidth is called half-power bandwidth B_{hp}. It is also called –3dB* bandwidth. The following points regarding half-power point A and B are worth noting. At these points,

1. current is $I_0/\sqrt{2}$
2. impedance is $\sqrt{2}.R$ or $\sqrt{2}.Z_0$
3. $P_1 = P_2 = P_0/2$
4. the circuit phase angle is $\theta = \pm 45°$
5. $Q = \tan\theta = \tan 45° = 1$

---

* The decibel power responses at these points, in terms of the maximum power at resonance, is

$$10 \log_{10} P/P_0 = 10 \log_{10} \frac{I_m^2 R/2}{I_m^2 R} = 10 \log_{10} \frac{1}{2} = -10 \log_{10} 2 = -3\ dB$$

Hence, the half-power points are also referred to as –3 dB points.

6. $B_{hp} = f_2 - f_1 = f_0/Q_0 = \sqrt{f_1 f_2}/Q_0 = R/2\pi L.$

It is interesting to note that B_{hp} is independent of the circuit capacitance.

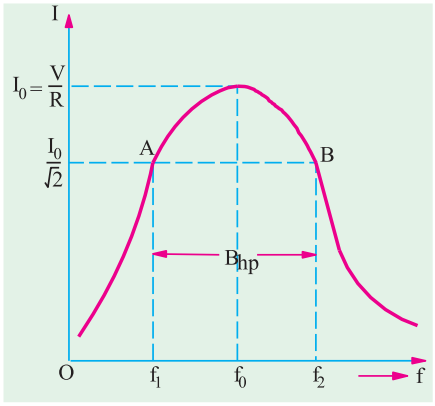


Fig. 13.47

13.14. Bandwidth B at any Off-resonance Frequency

It is found that the bandwidth of a given *R-L-C* circuit at any off-resonance frequencies f_1 and f_2 is given by

$B = f_0\, Q/Q_0 = \sqrt{f_1 f_2};\ .\ Q/Q_0 = f_2 - f_1$

where f_1 and f_2 are any frequencies (not necessarily half-power frequencies) below and above f_0.

Q = tangent of the circuit phase angle at the off-resonance frequencies f_1 and f_2.

Q_0 = quality factor at resonance $= \dfrac{\omega_0 L}{R} = \dfrac{1}{R}\sqrt{\dfrac{L}{C}}$

13.15. Determination of Upper and Lower Half-power Frequencies

As mentioned earlier, at lower half-power frequencies, $\omega_1 < \omega_0$ so that $\omega_1 L < 1/\omega_1 C$ and $\phi = -45°$

$$\therefore \quad \frac{1}{\omega_1 C} - \omega_1 L = R \quad \text{or} \quad \omega_1^2 + \frac{R}{L}\omega_1 - \frac{1}{LC} = 0$$

Putting $\dfrac{\omega_0}{Q_0} = \dfrac{R}{L}$ and $\omega_0^2 = \dfrac{1}{LC}$ in the above equation, we get $\omega_1^2 + \dfrac{\omega_0}{Q_0}\omega_1 - \omega_0^2 = 0$

The positive solution of the above equation is, $\omega_1 = \omega_0\left[\sqrt{\left(1 + \dfrac{1}{4Q_0^2}\right)} - \dfrac{1}{2Q_0}\right]$

Now at the upper half-power frequency, $\omega_2 > \omega_0$ so that $\omega_2 > 1/\omega_2 C$ and $\phi = +45°$

$$\therefore \quad \omega_2 L - \frac{1}{\omega_2 C} = R \quad \text{or} \quad \omega_2^2 - \frac{\omega_0}{Q_0}\omega_2 - \omega_0^2 = 0$$

The positive solution of the above equation is $\omega_2 = \omega_0\left[\sqrt{\left(1 + \dfrac{1}{4Q_0^2}\right)} + \dfrac{1}{2Q_0}\right]$

In case $Q_0 > 1$; then the term $1/4\,Q_0^2$ is negligible as compared to 1.

Hence, in that case $\omega_1 \cong \omega_0\left(1 - \dfrac{1}{2Q_0}\right)$ and $\omega_2 \cong \omega_0\left(1 + \dfrac{1}{2Q_0}\right)$

Incidentally, it may be noted from above that $\omega_2 - \omega_1 = \omega_0/Q_0$.

13.16. Values of Edge Frequencies

Let us find the values of ω_1 and ω_2, $I_0 = V/R$...at resonance

$$I = \frac{V}{[R^2 + (\omega L - 1/\omega C)^2]^{1/2}} \text{ ... at any frequency}$$

At points *A* and *B*, I $\dfrac{I_0}{\sqrt{2}}$ $\dfrac{1}{\sqrt{2}} \cdot \dfrac{V}{R}$

$$\therefore \quad \frac{1}{\sqrt{2}} \cdot \frac{V}{R} \quad \frac{V}{[(R^2 \quad (\ L \quad 1/\ C)^2]^{1/2}} \text{ or } R \quad (\ L \quad 1/\ C) \quad X$$

It shows that at half-power points, net reactance is equal to the resistance.

(Since resistance equals reactance, p.f. of the circuit at these points is = $1/\sqrt{2}$ *i.e.* 0.707, though leading at point *A* and lagging at point *B*).

Hence $R^2 = (\omega L - 1/\omega C)^2 \quad \therefore \quad \omega = \pm \frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}} = \pm \alpha \pm \sqrt{\alpha^2 + \omega_0^2}$

where $\alpha = \frac{R}{2L}$ and $\omega_0 = \frac{1}{\sqrt{LC}}$

Since $R^2/4L^2$ is much less then $1/\sqrt{LC} \quad \therefore \quad \omega = \pm \frac{R}{2L} \pm \frac{1}{\sqrt{LC}} = \pm \frac{R}{2L} \pm \omega_0$

Since only positive values of ω_0 are considered, $\omega = \omega_0 \pm R/2L = \omega_0 \pm \alpha$

$\therefore \qquad \omega_1 = \omega_0 - \frac{R}{2L}$ and $\omega_2 = \omega_0 + \frac{R}{2L}$

$\therefore \qquad \Delta\omega = \omega_2 - \omega_1 = \frac{R}{L}$ rad/s and $f_2 - f_1 = \frac{R}{2L}$ Hz $\quad \frac{f_0}{Q_0}$ Hz

Also $\qquad f_1 = f_0 - \frac{R}{4L}$ Hz and $f_2 = f_0 + \frac{R}{4L}$ Hz

It is obvious that f_0 is the *centre* frequency between f_1 and f_2.

Also, $\omega_1 = \omega_0 - \frac{1}{2}\Delta\omega$ and $\omega_2 = \omega_0 + \frac{1}{2}\Delta\omega$

As stated above, bandwidth is a measure of circuits selectivity. Narrower the bandwidth, higher the selectivity and vice versa.

13.17. Q-Factor of a Resonant Series Circuit

The *Q*-factor of an *R-L-C* series circuit can be defined in the following different ways.

(i) it is given by the voltage magnification produced in the circuit at resonance.

We have seen that at resonance, current has maximum value $I_0 = V/R$. Voltage across either coil or capacitor = $I_0 X_{L_0}$ or $I_0 X_{C_0}$, supply voltage $V = I_0 R$

$\therefore$ Voltage magnification $= \frac{V_{L_0}}{V} = \frac{I_0 X_{L_0}}{I_0 R} = \frac{\text{reactive power}}{\text{active power}} = \frac{X_{L_0}}{R} = \frac{\omega_0 L}{R} = \frac{\text{reactance}}{\text{resistance}}$

or $\qquad = \frac{V_{C0}}{V} = \frac{I_0 X_{C0}}{I_0 R} = \frac{\text{reactive power}}{\text{active power}} = \frac{X_{C0}}{R} = \frac{\text{reactance}}{\text{resistance}} = \frac{1}{\omega_0 CR}$

$\therefore \quad Q$ –factor, $\qquad Q_0 = \frac{\omega_0 L}{R} = \frac{2\pi f_0 L}{R} = \tan\phi$ ...***(i)***

where ϕ is power factor of the coil.

(ii) The *Q*-factor may also be defined as under.

Q-factor $= 2\pi \frac{\text{maximum stored energy}}{\text{energy dissipated per cycle}}$ ...in the circuit

$= 2\pi \frac{\frac{1}{2} L I_0^2}{I^2 R T_0} = 2\pi \frac{\frac{1}{2} L (\sqrt{2} I)^2}{I^2 R (1/f_0)} = \frac{I^2 2\pi f_0 L}{I^2 R} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}$ ...($T_0 = 1/f_0$)

In other words, $\qquad Q_0 = \frac{\text{energy stored}}{\text{energy lost}}$* ...**in the circuit**

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* The author often jokingly tells students in his class that these days the quality of a person is also measured in terms of a quality factor given by

$$Q = \frac{\text{money earned}}{\text{money spent}}$$

Obviously, a person should try to have a high quality factor as possible by minimising the denominator and/or maximizing the numerator.

(iii) We have seen above that resonant frequency, $f_0 = \dfrac{1}{2\pi\sqrt{(LC)}}$ or $2\pi f_0 = \dfrac{1}{\sqrt{(LC)}}$

Substituting this value in Eq. (*i*) above, we get the *Q*-factor, $Q_0 = \dfrac{1}{R}\sqrt{\left(\dfrac{L}{C}\right)}$

(iv) In the case of series resonance, higher *Q*-factor means not only higher voltage magnification but also higher selectivity of the tuning coil. In fact, *Q*-factor of a resonant series circuit may be written as

$$Q_0 = \frac{\omega_0}{\text{bandwidth}} = \frac{\omega_0}{\Delta\omega} = \frac{\omega_0}{R/L} = \frac{\omega_0 L}{R} = \frac{L}{R\sqrt{LC}} = \frac{1}{R}\sqrt{\frac{L}{C}}$$ ...as before

Obviously, *Q*-factor can be increased by having a coil of large inductance but of small ohmic resistance.

(v) In summary, we can say that

$$Q_0 = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R} = \frac{1}{R}\sqrt{\frac{L}{C}} = \sqrt{\frac{X_{L0}\, X_{C0}}{R}} = \frac{f_0}{B_{hp}} = \frac{\omega_0}{\omega_2 - \omega_1} = \frac{f_0}{f_2 - f_1}$$

13.18. Circuit Current at Frequencies Other Than Resonant Frequencies

At resonance, $I_0 = V/R$

At any other frequency above the resonant frequency, the current is given by I

$$\frac{V}{Z} = \frac{V}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

This current lags behind the applied voltage by a certain angle ϕ

$$\therefore \quad \frac{I}{I_0} = \frac{V}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \times \frac{R}{V} = \frac{1}{\sqrt{1 + \dfrac{1}{R^2}(\omega L - 1/\omega C)^2}} = \frac{1}{\left[1 + \left(\dfrac{\omega_0 L}{R}\right)^2 \left(\dfrac{\omega}{\omega_0} - \dfrac{\omega_0}{\omega}\right)^2\right]^{1/2}}$$

Now, $\omega_0 L/R = Q_0$ and $\omega/\omega_0 = f/f_0$ hence, $\dfrac{I}{I_0} = \dfrac{1}{\left[1 + Q_0^2\left(\dfrac{f}{f_0} - \dfrac{f_0}{f}\right)^2\right]^{1/2}}$

13.19. Relation Between Resonant Power P_0 and Off-resonant Power P

In a series *RLC* resonant circuit, current is maximum *i.e.* I_0 at the resonant frequency f_0. The maximum power P_0 is dissipated by the circuit at this frequency where X_L equals X_C. Hence, circuit impedance $Z_0 = R$.

$$\therefore \quad P_0 = I_0^2 R = (V/R)^2 \times R = V^2/R$$

At any other frequency either above or below f_0 the power is (Fig. 13.48).

$$P = I^2 R = \left(\frac{V}{Z}\right)^2 \times R = \frac{V^2 R}{Z^2} = \frac{V^2 R}{R^2 + X^2} = \frac{V^2 R}{R^2 + X^2 R^2/R^2}$$

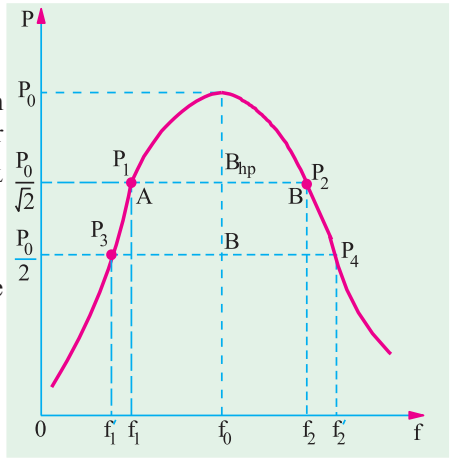


Fig. 13.48

$$= \frac{V^2R}{R^2 + R^2Q^2} = \frac{V^2R}{R^2(1+Q^2)} = \frac{V^2}{R(1+Q^2)} = \frac{P_0}{(1+Q^2)}$$

The above equation shows that any frequency other than f_0, the circuit power P is reduced by a factor of $(1 + Q^2)$ where Q is the tangent of the circuit phase angle (and not Q_0). At resonance, circuit phase angle $\theta = 0$, and $Q = \tan\theta = 0$. Hence, $P = P_0 = V^2/R$ (values of Q_0 are given in Art.)

Example 13.54. *For a series R.L.C circuit the inductor is variable. Source voltage is* $200\sqrt{2}$ *sin* $100\pi t$. *Maximum current obtainable by varying the inductance is 0.314 A and the voltage across the capacitor then is 300 V. Find the circuit element values.*

(Circuit and Field Theory, A.M.I.E. Sec B, 1993)

Solution. Under resonant conditions, $I_m = V/R$ and $V_L = V_C$.

$\therefore \quad R = V/I_m = 200/0.314 = 637\ \Omega,\ V_C = I_m \times X_{CD} = I_m/\omega_0 C$

$\therefore \quad C = I_m/\omega_0 V_C = 0.314/100\ \pi \times 300 = 3.33\ \mu F.$

$V_L = I_m \times X_L = I_m\ \omega_0 L;\ L = V_1/\omega_0 I_m = 300/100\ \pi \times 0.314 =$ **3.03 H**

Example 13.55. *A coil having an inductance of 50 mH and resistance 10* Ω *is connected in series with a 25* μF *capacitor across a 200 V ac supply. Calculate* **(a)** *resonance frequency of the circuit* **(b)** *current flowing at resonance and* **(c)** *value of* Q_0 *by using different data.*

(Elect. Engg. A.M.Ae. S.I, June 1991)

Solution. **(a)** $f_0 = \frac{1}{2\sqrt{LC}} \quad \frac{1}{2\sqrt{50\ \ 10^{\ 3}\ \ 25\ \ 10^{\ 6}}} \quad 142.3$ Hz

(b) $I_0 = V/R = 200/10 = 20$ A

(c) $Q_0 = \frac{\omega_0 L}{R} = \frac{2\pi \times 142.3 \times 50 \times 10^{-3}}{10} = 4.47$

$$Q_0 = \frac{1}{\omega_0 CR} = \frac{1}{2\pi \times 142.3 \times 25 \times 10^{-6} \times 10} = 4.47$$

$$Q_0 = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{50 \times 10^{-3}}{25 \times 10^{-6}}} = 4.47$$

Example 13.56. *A 20-*Ω *resistor is connected in series with an inductor, a capacitor and an ammeter across a 25-V variable frequency supply. When the frequency is 400-Hz, the current is at its maximum value of 0.5 A and the potential difference across the capacitor is 150 V. Calculate*

(a) *the capacitance of the capacitor*

(b) *the resistance and inductance of the inductor.*

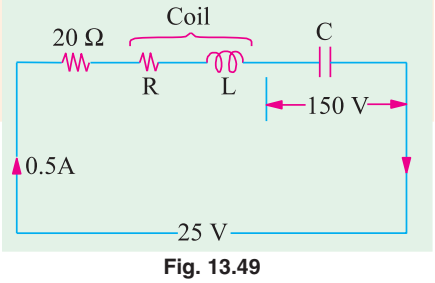


Fig. 13.49

Solution. Since current is maximum, the circuit is in resonance.

$X_C = V_C/I = 150/0.5 = 300\ \Omega$

(a) $X_C = 1/2\pi fC$ or $300 = 1/2\pi \times 400 \times C$

$\therefore \quad C = 1.325 \times 10^{-6}\ F =$ **1.325 μF**

(b) $X_L = X_C = 300\ \Omega$

$\therefore \quad 2\pi \times 400 \times L = 300 \quad \therefore\ L =$ **0.119 H**

(c) Now, at resonance,

circuit resistance = circuit impedance or $20 + R = V/I = 25/0.5 \quad \therefore\ R = 30\ \Omega$ ...Fig.13.49

Example 13.57. *An R-L-C series circuit consists of a resistance of 1000 Ω, an inductance of 100 mH an a capacitance of 10 μμ F. If a voltage of 100 V is applied across the combination, find* **(i)** *the resonance frequency* **(ii)** *Q-factor of the circuit and* **(iii)** *the half-power points.*

(Elect. Circuit Analysis, Bombay Univ.)

Solution. (*i*) $f_0 = \dfrac{1}{2\pi\sqrt{10^{-1}\times 10^{-11}}} = \dfrac{10^6}{2\pi} =$ **159 kHz**

(*ii*) $Q = \dfrac{1}{R}\sqrt{\dfrac{L}{C}} = \dfrac{1}{1000}\times\sqrt{\dfrac{10^{-1}}{10^{-11}}} =$ **100**

(*iii*) $f_1 = f_0 - \dfrac{R}{4\pi L} = 159\times 10^3 - \dfrac{1000}{4\pi\times 10^{-1}} =$ **158.2 kHz**

$f_2 = f_0 + \dfrac{R}{4\pi L} = 159\times 10^3 + \dfrac{1000}{4\pi\times 10^{-1}} =$ **159.8 kHz**

Example 13.58. *A series R-L-C circuit consists of R = 1000 Ω, L = 100 mH and C = 10 picofarads. The applied voltage across the circuit is 100 V.*

(*i*) *Find the resonant frequency of the circuit.*

(*ii*) *Find the quality factor of the circuit at the resonant frequency.*

(*iii*) *At what angular frequencies do the half power points occur ?*

(*iv*) *Calculate the bandwidth of the circuit.*

(Networks-I, Delhi Univ. & U.P. Tech. Univ. 2001)

Solution. (*i*) $f_0 = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{100\times 10^{-3}\times 10\times 10^{-12}}} = 159.15$ kHz

(*ii*) $Q_0 = \dfrac{1}{R}\sqrt{\dfrac{L}{C}}$ $\dfrac{1}{1000}$ $\sqrt{\dfrac{100\quad 10^{\ 3}}{10\quad 10^{\ 12}}}$ 100

(*iii*) $B_{hp} = \dfrac{R}{2\pi L} = \dfrac{1000}{2\pi\times 100\times 10^{-3}} = 1591.5$ Hz

Also. $B_{hp} = f_0/Q_0 = 159.15$ kHz$/100 = 1.5915$ kHz $=$ **1591.5 Hz** ...as above

(*iv*) $\omega_1 = {}_0\ 1\ \dfrac{1}{2Q_0}$ 2 159.15 1 $\dfrac{1}{2\quad 100}$ 994.969 radia/sec.

$\omega_2 = {}_0\ 1\ \dfrac{1}{2\,Q_0}$ 2 159.15 1 $\dfrac{1}{2\quad 100}$ 1004.969 rad/sec

(*v*) Band width $= (\omega_2 - \omega_1) = 1004.969 - 994.969 = 10.00$ rad/sec.

Example 13.59. *An R-L-C series resonant circuit has the following parameters :*
Resonance frequency = 5000/2π Hz; impedance at resonance = 56 Ω and Q-factor = 25.
Calculate the capacitance of the capacitor and the inductance of the inductor.
Assuming that these values are independent of the frequency, find the two frequencies at which the circuit impedance has a phase angle of π/4 radian.

Solution. Here $\omega_0 = 2\pi f_0 = 2\pi \times 5000/2\pi = 5000$ rad/s

Now, $Q = \dfrac{\omega_0 L}{R}$ or $25 = \dfrac{5000\,L}{56}$ or $L = 0.28$ H

Also at resonance $\omega_0 L = 1/\omega_0 C$ or $5000 \times 0.28 = 1/5000 \times C$ $\therefore$ $C = 0.143$ μF

The circuit impedance has a phase shift of 45° and the two half-power frequencies which can be found as follows :

$$BW = \frac{f_0}{Q} = \frac{5000/2\pi}{25} = \textbf{31.83 Hz}$$

Therefore lower half-power frequency = $(f_0 - 31.83/2) = 5000/2\pi - 15.9 =$ **779.8 Hz.**

Upper half-power frequency = $(f_0 + 31.83/2) = 5000/2\pi + 15.9 =$ **811.7 Hz.**

Example 13.60. *An R-L-C series circuit is connected to a 20-V variable frequency supply. If R = 20 Ω, L = 20 mH and C = 0.5 μF, calculate the following :*

***(a)** resonant frequency f_0 **(b)** resonant circuit Q_0 using L/C ratio **(c)** half-power bandwidth using f_0 and Q_0 **(d)** half-power bandwidth using the general formula for any bandwidth **(e)** half-power bandwidth using the given component values **(f)** maximum power dissipated at f_0.*

Solution. (a) $f_0 = 1/2\pi\sqrt{LC} = 1/2\pi\sqrt{(20\times10^{-3}\times 0.5\times10^{-6})} =$ **159 Hz**

(b) $Q_0 = \frac{1}{R}\cdot\sqrt{\frac{L}{C}} = \frac{1}{20}\cdot\sqrt{\frac{20\times10^{-3}}{0.5\times10^{-6}}} =$ **10**

(c) $B_{hp} = f_0/Q_0 = 1591/10 =$ **159.1 Hz**

(d) $B_{hp} = f_0\, Q/Q_0 = 1591 \times \tan 45°/10 =$ **159.1 Hz**

It is so because the power factor angle at half-power frequencies is ±45°.

(e) $B_{hp} = R/2\pi\, L = 20/2\pi \times 20 \times 10^{-3} =$ **159.1 Hz**

(f) $f_0 = V^2/R = 20^2/20 =$ **20 W**

Example 13.61. *An inductor having a resistance of 25 Ω and a Q_0 of 10 at a resonant frequency of 10 kHz is fed from a 100 ∠0° supply. Calculate*

***(a)** Value of series capacitance required to produce resonance with the coil*

***(b)** the inductance of the coil **(c)** Q_0 using the L/C ratio **(d)** voltage across the capacitor **(e)** voltage across the coil.*

Solution. (a) $X_{L0} = Q_0 R = 10 \times 25 = 250\ \Omega$

Now, $X_{C0} = X_{L0} = 250\ \Omega$

Hence, $C = 1/2\,\pi f_0 \times X_{C0} = 1/2\,\pi \times 10^4 \times 250 = 63.67 \times 10^{-9}\ F =$ **63.67 μF**

(b) $L = X_{L0}/2\pi f_0 = 250/2\pi \times 10^4 =$ **3.98 mH**

(c) $Q_0 = \frac{1}{R}\cdot\sqrt{\frac{L}{C}}$

Now, $\frac{L}{C} = \frac{3.98\times10^{-3}}{63.67\times10^{-9}} = 6.25\times10^4$

$\therefore$ $Q_0 = \frac{1}{25}\sqrt{6.25\quad 10^4}$ **10** (verification)

(d) $V_{C0} = -jQ_0V = -j\ 100\ \angle\ 0\ °\ \times\ 10 = -j\ 1000\text{V} =$ **−100 ∠− 90° V**

(e) Since $V_{L0} = V_{C0}$ in magnitude, hence, $V_{L0} = +j\ 1000$ V

$= 1000\ \angle 90°$V; Also, $V_R = V = 100\ \angle 0°$

Hence, $V_{coil} = V_R + V_{L0}$

$= 100 + 1000\ \angle 90° = 100 + j\ 1000 =$ **1005 ∠84.3°**

Example 13.62. *A series L.C circuit has L = 100 pH, C = 2500 μF and Q = 70. Find **(a)** resonant frequency f_0 **(b)** half-power points and **(c)** bandwidth.*

Solution. (a) $f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{10^9}{2\pi\sqrt{100\times2500}} =$ **318.3 kHz**

(b) $f_2 - f_1 = \Delta f = f_0/Q = 318.3/70 =$ **4.55 kHz**

(c) $f_1 f_2 = f_0^2 = 318.3^2; f_2 - f_1 = 4.55$ kHz

Solving for f_1 and f_2, we get, f_1 = **3104 kHz** and f_2 = **320.59 kHz**

Note. Since Q is very high, there would be negligible error in assuming that the half-power points are equidistant from the resonant frequency.

Example 13.63. *A resistor and a capacitor are connected in series across a 150 V ac supply. When the frequency is 40 Hz, the circuit draws 5 A. When the frequency is increased to 50 Hz, it draws 6 A. Find the values of resistance and capacitance. Also find the power drawn in the second case.* **[Bombay University, 1997]**

Solution. Suffix 1 for 40 Hz and 2 for 50 Hz will be given.

$Z_1 = 150/5 = 30$ ohms at 40 Hz

or $R^2 + X_{C2}^2 = 900$

Similarly, $R^2 + X_{C2}^2 = 625$ at 50 Hz, since $Z_2 = 25\ \Omega$

Further, capacitive reactance is inversely proportional to the frequency.

$X_{C1}/X_{C2} = 50/40$ or $X_{C1} = 1.25\ X_{C2}$

$X_{C1}^2 - X_{C2}^2 = 900 - 625 = 275$

$X_{C1}^2\ (1.25^2 - 1) = 275$ or $X_{C2}^2 = 488.9$, $X_{C2} = 22.11$

$X_{C1} = 1.25 \times 22.11 = 27.64$ ohms

$R^2 = 900 - X_{C1}^2 = 900 - 764 = 136$, $R = 11.662$ ohms

$$C \quad \frac{1}{2 \quad 40 \quad 27.64} \quad 144 \quad \text{F}$$

Power drawn in the second case $= 6^2 \times 11.662 = 420$ watts

Example 13.64. *A constant voltage at a frequency of 1 MHz is applied to an inductor in series with a variable capacitor. When the capacitor is set 500 pF, the current has its maximum value while it is reduced to one-half when the capacitance is 600 pF. Find*

(*i*) *the resistance,* (*ii*) *the inductance,* (*iii*) *the Q-factor of the inductor.*

[Bombay University, 1996]

Solution. Resonance takes palce at 1 MHz, for $C = 500$ pF.

$$LC = \frac{1}{\omega_0\ 2} = \frac{1}{(2\pi \times 10^6)^2} = \frac{10^{-12}}{4\pi^2}$$

$$L = 10^{-12}/(4 \times \pi^2 \times 500 \times 10^{-12}) = 1/(4 \times \pi^2 \times 500)$$

$$= 50.72 \text{ mH}$$

Z_1 = Impedance with 500 pF capacitor $= R + j\ \omega L - j\ 1/\omega C$

$$= R + j\ (2p \times 10^6 \times 50.72 \times 10^{-6}) - j\ \frac{1}{2\pi \times 10^6 \times 500 \times 10^{-12}}$$

$= R$, since resonance occurs.

Z_2 = Impedance with 600 pF capacitor

$$|Z|_2 = R + j\ \omega L - j\ \frac{1}{\omega \times 600 \times 10^{-12}} = 2\ R, \text{ since current is halved.}$$

$$\omega L - 1/\omega C = \sqrt{3}\ R$$

$$\sqrt{3}\ R = 2\pi \times 10^6 \times 50.7 \times 10^{-6} - \frac{1}{2\pi \times 10^6 \times 600 \times 10^{-12}}$$

$$= 2\pi \times 50.72\ \frac{10^6}{2\pi\ 600}$$

$$= 318.52 - 265.4 = 53.12$$

$$R = 30.67 \text{ ohms}$$

$$Q \text{ Factor of coil} = (\omega_0 L)/R$$

$$= 50.72 \times 10^{-6} \times 2\pi \times 10^6/30.67 = 50.72 \times 2\pi/30.67 = 10.38$$

Example 13.65. *A large coil of inductance 1.405 H and resistance 40 ohms is connected in series with a capacitor of 20 microfarads. Calculate the frequency at which the circuit resonates.*

If a voltage of 100 V at the corresponding frequency is applied to the circuit, calculate the current drawn from the supply and the voltages across the coil and across the capacitor.

[Nagpur University Nov. 1999]

Solution. $$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1.405 \times 20 \times 10^{-6}}} = 188.65 \text{ radians/sec.}$$

$$f_0 = \frac{188.65}{2\pi} = 30.04 \text{ Hz}$$

Reactance at 30.04 Hz have to be calculated for voltages across the coil and the capacitor

$$X_L = \omega_0 L = 188.65 \times 1.405 = 265\ \Omega$$

$$X_C = \frac{1}{\omega_0 C} = \frac{1}{188.65 \times 20 \times 10^{-6}} = 265\ \Omega$$

Coil Impedance $$= \sqrt{40^2 + 265^2} = 268\ \Omega$$

Impedance of the total circuit

$$= 40 + j\,265 - j\,265 = 40\ \Omega$$

Supply Current $$= \frac{100}{40} = 2.5 \text{ amp, at unity p.f.}$$

Voltage across the coil = 2.5 × 268 = 670 V

Voltage across the capacitor = 2.5 × 265 = 662.5 V

The phasor diagram is drawn below; in Fig. 13.50 (*a*) for the circuit in Fig. 13.50 (*b*)

$$OB = V, BC = V_C$$

$$AB = IX_L$$

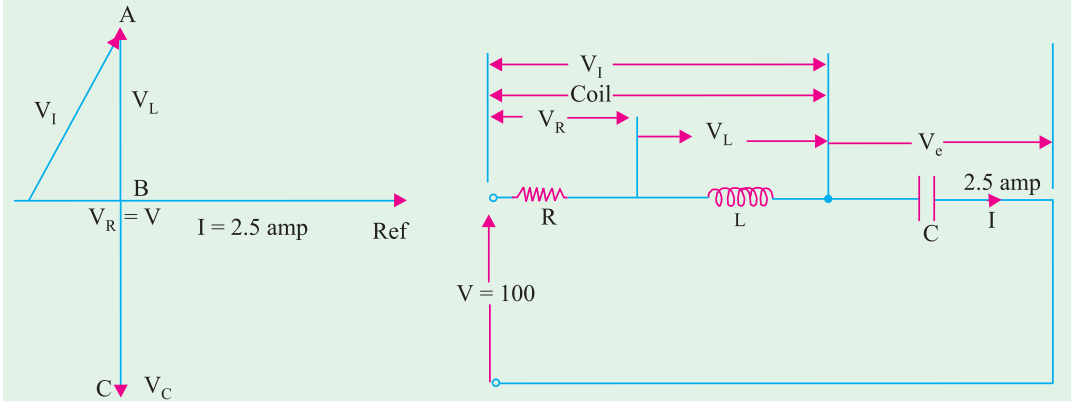


Fig. 13.50 (*a*) Phasor diagram

Fig. 13.50 (*b*) Series resonating circuit

Example 13.66. *A series R-L-C circuit is excited from a constant-voltage variable frequency source. The current in the circuit becomes maximum at a frequency of 600/2π Hz and falls to half the maximum value at 400/2π Hz. If the resistance in the circuit is 3 Ω, find L and C.*

(Grad. I.E.T.E. Summer 1991)

Solution. Current at resonance is $I_0 = V/R$

Actual current at any other frequency is $I = \dfrac{V}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}}$

$$\therefore \frac{I}{I_0} \quad \frac{V}{R^2 \quad L \quad \frac{1}{C}^{\,2}}^{1/2} \cdot \frac{R}{V} \quad \frac{1}{1 \quad \frac{1}{R^2} \quad L \quad \frac{1}{C}^{\,2}}^{1/2} \quad \frac{1}{1 \quad \frac{{}_0 L}{R}^{\,2} \quad \frac{}{{}_0} \quad \frac{{}_0}{}^{\,2}}^{1/2}$$

Now $\quad Q = \dfrac{\omega_0 L}{R}$ and $\dfrac{\omega}{\omega_0} = \dfrac{f}{f_0}$, hence $\dfrac{I}{I_0} = \dfrac{1}{\left[1 + Q^2\left(\dfrac{f}{f_0} - \dfrac{f_0}{f}\right)^2\right]^{1/2}}$

In the present case, $f_0 = 600/2\pi$ Hz, $f = 400/2\pi$ Hz and $I/I_0 = 1/2$

$$\therefore \quad \frac{1}{2} = \frac{1}{\left[1 + Q^2\left(\frac{400}{600} - \frac{600}{400}\right)^2\right]^{1/2}} = \frac{1}{\left[1 + Q^2\left(\frac{2}{2} - \frac{3}{2}\right)^2\right]^{1/2}}$$

or $\quad \dfrac{1}{4} = \dfrac{1}{1 + 25\,Q^2/36} \quad \therefore \; Q = 2.08$

Now, $\quad Q = \dfrac{1}{\omega_0 RC}$ or $2.08 = \dfrac{1}{600 \times 3 \times C} \quad \therefore \; C = 267 \times 10^{-6}\ F = \mathbf{267\ mF}$

Also $\quad Q = \omega_0\, L/R \;\therefore\; 2.08 = \dfrac{600L}{R} = \dfrac{600L}{3} \;\therefore\; L = \mathbf{10.4\ mH}$

Example 13.67. *Discuss briefly the phenomenon of electrical resonance in simple R-L-C circuits. A coil of inductance L and resistance R in series with a capacitor is supplied at constant voltage from a variable-frequency source. Call the resonance frequency ω_0 and find, in terms of L, R and ω_0, the values of that frequency at which the circuit current would be half as much as at resonance.*

(Basic Electricity, Bombay Univ.)

Solution. For discussion of resonance, please refer to Art. 13.10.

The current at resonance is maximum and is given by $I_0 = V/R$. Current at any other frequency is

$$I = \frac{V}{[R^2 + (\omega L - 1/\omega C)^2]^{1/2}}$$

$$\therefore \quad \frac{I_0}{I} = \frac{[R^2 + (\omega L - 1/\omega C)^2]^{1/2}}{R}$$

or $\quad N = \left[1 + \dfrac{1}{R^2}\left(\omega L - \dfrac{1}{\omega C}\right)^2\right]^{1/2}$

Now $\quad Q = \dfrac{\omega_0 L}{R} = \dfrac{1}{\omega_0 CR}$

$$\therefore \quad R = \frac{\omega_0 L}{Q} = \frac{1}{\omega_0 CQ}$$

Substituting this value in the above equation, we get

$$N = \left[1 + Q^2\left(\frac{f}{f_0} - \frac{f_0}{f}\right)^2\right]^{1/2}.$$

or $\quad \left(\dfrac{f}{f_0} - \dfrac{f_0}{f}\right)^2 = \dfrac{N^2 - 1}{Q^2} \;\therefore\; \dfrac{\sqrt{(N^2 - 1)}}{Q} = \pm\left(\dfrac{f}{f_0} - \dfrac{f_0}{f}\right)$

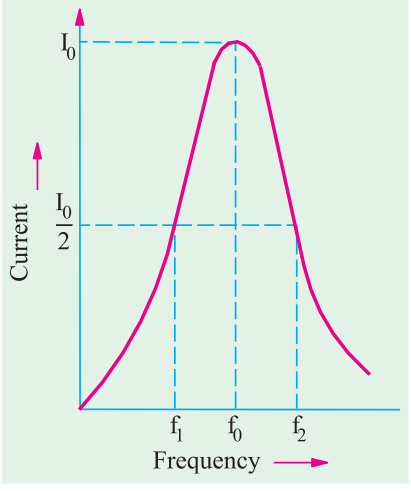


Fig. 13.51

or $$\frac{\sqrt{(N^2-1)}}{Q} = \frac{f_2}{f_0} - \frac{f_0}{f_2} = \frac{f_0}{f_1} - \frac{f_1}{f_0}$$

where $f_2 > f_0$ and $f_1 < f_0$ are the two frequencies at which the current has fallen to 1/N of the resonant value.

In the present case, $N = 2$ (Fig. 13.50) $\therefore \frac{f_2}{f_0} - \frac{f_0}{f_2} = \frac{\sqrt{3}}{Q}$ and $\frac{f_0}{f_1} - \frac{f_1}{f_0} = \frac{\sqrt{3}}{Q}$

From these equations, f_1 and f_2 may be calculated.

Example 13.68. *A coil of inductance 9 H and resistance 50 Ω in series with a capacitor is supplied at constant voltage from a variable frequency source. If the maximum current of 1A occurs at 75 Hz, find the frequency when the current is 0.5 A.*

(Principles of Elect. Engg. Delhi Univ.)

Solution. Here, $N = 1_0/I = I/0.5 = 2$; $Q = \omega_0 L/R = 2\pi \times 75 \times 9/50 = 84.8$

Let f_1 and f_2 be the frequencies at which current falls to half its maximum value at resonance frequency. Then, as seen from above

$$\frac{f_0}{f_1} - \frac{f_1}{f_0} = \frac{\sqrt{3}}{Q} \text{ or } \frac{75}{f_1} - \frac{f_1}{75} = \frac{\sqrt{3}}{84.4}$$

or $$(75^2 - f_1^2)/75 f_1 = 0.02 \text{ or } f_1^2 + 1.5 f_1 - 5625 = 0 \text{ or } f_1 = \mathbf{74.25\ Hz}$$

Also $$\frac{f_2}{75} - \frac{75}{f_2} = \frac{\sqrt{3}}{84.4} \text{ or } f_2^2 - 1.5 f_2 - 5625 = 0 \text{ or } f_2 = \mathbf{75.75\ Hz.}$$

Example 13.69. *Using the data given in Ex. 13.45 find the following when the power drops to 4 W on either side of the maximum power at resonance.*

(a) *circuit Q (b) circuit phase angle ϕ* **(c)** *4-W bandwidth B*

(d) *lower freqency f_1* **(e)** *upper frequency f_2.*

(f) *resonance frequency using the value of f_1 and f_2.*

Solution. (a) $$P = \frac{P_0}{(1+Q_0^2)} \therefore Q = \sqrt{(P_0/P_1) - 1} = \sqrt{(20/4) - 1} = \mathbf{2}$$

(b) $$\tan(\pm\theta) = 2\ j\ \pm\theta/\tan^{-1} 2 = \pm \mathbf{63.4°}$$

(c) $$B_{hp} = \frac{f_0 Q}{Q_0} = \frac{1591 \times 2}{10} = 318.2 \text{ Hz}$$

(d) $$f_1 = f_0 - B/2 = 1591 - (318.2/2) = 1431.9 \text{ Hz}$$

(e) $$f_2 = f_0 + B/2 = 1591 + (318.2/2) = 1750.1 \text{ Hz}$$

(f) $$f_0 = \sqrt{f_1 f_2} = \sqrt{1431.9 \times 1750.2} = \mathbf{1591\ Hz.}$$

It shows that regardless of the bandwidth magnitude, f_0 is always the geometric mean of f_1 and f_2.

Example 13.70. *A constant e.m.f. source of variable frequency is connected to a series R.L.C. circuit of Fig. 13.51.*

(a) *Shown in nature of the frequency $-V_R$ graph*

(b) *Calculate the following* **(i)** *frequency at which maximum power is consumed in the 2 Ω resistor*

(ii) *Q-factor of the circuit at the above frequency* **(iii)** *frequencies at which the power consumed in 2 Ω resistor is one-tenth of its maximum value.*

(Network Analysis A.M.I.E Sec. B.W.)

Solution. (a) The graph of angular frequency ω versus voltage drop across *R i.e.* V_R is shown in Fig. 13.52. It is seen that as frequency of the applied voltage increases, V_R increases till it reaches its

maximum value when the given *RLC* circuit becomes purely reactive *i.e.* when $X_L = X_C$ (Art. 13.10). (*b*) (*i*) maximum power will be consumed in the 2 Ω resistor when maximum current flows in the circuit under resonant condition.

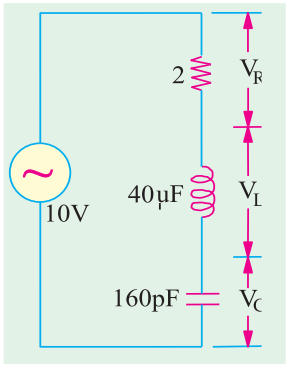


Fig. 13.52

For resonance $\omega_0 L = 1/\omega_0 X_C$ or

$$\omega_0 = 1/\sqrt{LC} = 1/\sqrt{40\times10^{-6}\times160\times10^{-12}} = 10^9/80 \text{ rad/s}$$

$\therefore$ $f_0 = \omega_0/2\pi = 10^9/2\pi \times 80 =$ **1.989 MHz**

(ii) *Q*-factor, $Q_0 = \dfrac{\omega_0 L}{R} = \dfrac{10^9\times40\times10^{-6}}{80\times2} =$ **250**

(iii) Maximum current $I_0 = V/R$ (Art. 13.10). Current at any other frequency is $I = \dfrac{V}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$

Power at any frequency $I^2R = \dfrac{V^2}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} . R$

Maximum power $I_0^2 R = \left(\dfrac{V}{R}\right)^2 . R$

Hence, the frequencies at which power consume would be one-tenth of the maximum power will be given by the relation. $\dfrac{1}{10} . \left(\dfrac{V}{r}\right)^2 R = \dfrac{V^2}{R^2 + (\omega L - 1/\omega C)^2} . R$

or cross multiplying, we get $R^2 + (\omega L - 1/\omega C)^2 = 10R^2$ or $(\omega L - 1/\omega C^2 = 9\ R^2$

$\therefore$ $(\omega L - 1/\omega C) = \pm 3\ R$ and $\omega_1 L - 1/\omega_1\ C = 3R$

and $\omega_2\ L - 1/\omega_2\ C = -3\ R$

Adding the above two equations, we get

$$(\ _1 \quad _2) L \quad \frac{1}{C} \quad \frac{2 \quad 1}{1 \cdot 2} \quad 0 \text{ or } \quad _1 \ 2 \quad \frac{1}{LC} \quad \frac{2}{0}$$

Subtracting the same two equations, we have $L(\ _1 \quad _2) \quad \frac{1}{C} \quad \frac{1}{2} \quad \frac{1}{1} \quad 6R$

$$(\ _1 \quad _2) \quad \frac{1}{LC} \quad \frac{1 \quad 2}{1 \cdot 2} \quad \frac{6R}{L}$$

Substituting the value of $1/LC = \omega^2 = \omega_1\omega_2$, we get

$$(\omega_1 - \omega_2) + \omega_1\omega_2\left(\frac{\omega_1 - \omega_2}{\omega_1\omega_2}\right) = \frac{6R}{L} \text{ or } (\omega_1 - \omega_2) = \Delta\omega = \frac{3R}{L}$$

Now, $\omega_2 = \omega_0 + \dfrac{\Delta\omega}{2} = \omega_0 + \dfrac{1.5R}{L}$ and $\omega_1 = \omega_0 - \dfrac{1.5R}{L}$

$\therefore f_2 = f_0 + 1.5\ R/2\pi L = 1.989 \times 10^6 + 1.5 \times 2.2\pi \times 40 \times 10^{-6} = 1.989 \times 10^6 + 0.0119 \times 10^6 =$ **2 MHz**

$f_1 = f_0 - 1.5\ R/2\pi L = 1.989 \times 10^6 - 0.0119 \times 10^6 =$ **1.977 MHz**

Example 13.71. *Show that in R-L-C circuit, the resonant frequency ω_0 is the geometric mean of the lower and upper half-power frequencies ω_1 and ω_2 respectively.*

Solution. As stated earlier, at lower half-power radiant frequency ω_1; $X_C > X_L$ and at frequencies higher than half-power frequencies $X_L > X_C$. However, the difference between the two equals *R*.

$\therefore$ at ω_1, $X_C - X_L = R$ or $1/\omega_1 C - \omega_1 L = R$ ...(*i*)

At ω_2, $X_L - X_C = R$ or $\omega_2 L - 1/\omega_2\ C = R$

Multiplying both sides of Eq. (*i*) by C and substituting $\omega_0^2 = 1/\sqrt{LC}$, we get

$$\frac{1}{\omega_1} - \frac{\omega_1}{\omega_0^2} = \frac{\omega_2}{\omega_0^2} - \frac{1}{\omega_2} \quad \text{or} \quad \frac{1}{\omega_1} + \frac{1}{\omega_2} = \frac{\omega_1 + \omega_2}{\omega_0^2} \quad \text{or} \quad \omega_0 = \sqrt{\omega_1 \omega_2}$$

Example 13.72. *Prove that in a sereis R-L-C circuit, $Q_0 = \omega_0 L/R = f_0/\text{bandwidth} = f_0/BW$.*

Solution. As has been proved in Art. 14.13, at half power frequencies, net reactance equals resistance. Moreover, at ω_1, capacitive reactance exceeds inductive reactance whereas at ω_2, inductive reactance exceeds capacitive reactance.

$$\therefore \qquad \frac{1}{2\pi f_1 C} - 2\pi f_1 L = R \quad \text{or} \quad f_1 = \frac{-R + \sqrt{R^2 + 4L/C}}{4\pi L}$$

$$\text{Also} \qquad 2\pi f_2 L - \frac{1}{2\pi f_2 C} = \quad \text{or} \quad f_2 = \frac{R + \sqrt{R^2 + 4L/C}}{4\pi L}$$

Now, $BW = f_2 - f_1 = R/2\pi L$. Hence, $Q_0 = f_0/BW = 2\pi f_0 L/R = \omega_0 L/R$.

Tutorial Problem No. 13.4

1. An a.c. series circuit has a resistance of 10 Ω, an inductance of 0.2 H and a capacitance of 60 μF. Calculate
   (*a*) the resonant frequency (*b*) the current and (*c*) the power at resonance.
   Give that the applied voltage is 200 V. **[46 Hz; 20 A; 4 kW]**
2. A circuit consists of an inductor which has a resistance of 10 Ω and an inductance of 0.3 H, in series with a capacitor of 30 μ*F* capacitance. Calculate
   (*a*) the impedance of the circuit to currents of 40 Hz (*b*) the resonant frequency (*c*) the peak value of stored energy in joules when the applied voltage is 200 V at the resonant frequency.
   [58.31 Ω; 53 Hz; 120 J]
3. A resistor and a capacitor are connected in series with a variable inductor. When the circuit is connected to a 240-V, 50-Hz supply, the maximum current given by varying the inductance is 0.5 A. At this current, the voltage across the capacitor is 250 V. Calculate the values of
   (*a*) the resistance (*b*) the capacitance (*c*) the inductance. **[480 Ω, 6.36 μF; 1.59 H]**
   Neglect the resistance of the inductor
4. A circuit consisting of a coil of resistance 12Ω and inductance 0.15 H in series with a capacitor of 12μF is connected to a variable frequency supply which has a constant voltage of 24 V.
   Calculate (*a*) the resonant frequency (*b*) the current in the circuit at resonance (*c*) the voltage across the capacitor and the coil at resonance. **[(*a*) 153 Hz (*b*) 2 A (*c*) 224 V]**
5. A resistance, a capacitor and a variable inductance are connected in series across a 200-V, 50-Hz supply. The maximum current which can be obtained by varying the inductance is 314 mA and the voltage across the capacitor is then 300 V. Calculate the capacitance of the capacitor and the values of the inductance and resistance. **[3.33 μF, 3.04 H, 637 Ω] (*I.E.E. London*)**
6. A 250-V circuit, consisting of a resistor, an inductor and a capacitor in series, resonates at 50 Hz. The current is then 1A and the p.d. across the capacitor is 500 V. Calculate (*i*) the resistance (*ii*) the inductance and (*iii*) the capacitance. Draw the vector diagram for this condition and sketch a graph showing how the current would vary in a circuit of this kind if the frequency were varied over a wide range, the applied voltage remaining constant.
   [(*i*) 250 Ω (*ii*) 0.798 H (*iii*) 12.72 μ F] (*City & Guilds, London*)
7. A resistance of 24 Ω, a capacitance of 150 μF and an inductance of 0.16 H are connected in series with each other. A supply at 240 V, 50 Hz is applied to the ends of the combination. Calculate (*a*) the current in the circuit (*b*) the potential differences across each element of the circuit (*c*) the frequency to which the supply would need to be changed so that the current would be at unity power-factor and find the current at this frequency.
   [(*a*) 6.37 A (*b*) V_R = 152.9 V, V_C = 320 V, V_C = 123.3 V (*c*) 32 Hz; 10 A] (*London Univ.*)
8. A series circuit consists of a resistance of 10 Ω, an inductance of 8 mH and a capacitance of 500 μμ*F*. A sinusoidal E.M.F. of constant amplitude 5 V is introduced into the circuit and its frequency varied

over a range including the resonant frequency.
At what frequencies will current be (*a*) a maximum (*b*) one-half the-maximum ?

[(*a*) 79.6 kHz (*b*) 79.872 kHz, 79.528 kHz] (*App. Elect. London Univ.*)

9. A circuit consists of a resistance of 12 ohms, a capacitance of 320 μF and an inductance of 0.08 H, all in series. A supply of 240 V, 50 Hz is applied to the ends of the circuit. Calculate :
(*a*) the current in the coil.
(*b*) the potential differences across each element of the circuit.
(*c*) the frequency at which the current would have unity power-factor.

[(*a*) 12.4 A (*b*) 149 V, 311 V (*c*) 32 Hz] (*London Univ.*)

10. A series circuit consists of a reactor of 0.1 henry inductance and 5 ohms resistance and a capacitor of 25.5 μ F capacitance.
Find the resonance frequency and the precentage change in the current for a divergence of 1 percent from the resonance frequency.

[100 Hz, 1.96% at 99 Hz; 4.2% at 101 Hz] (*City and Guilds, London*)

11. The equation of voltage and currents in two element series circuit are :
$v\ (t) = 325\ .\ 3 \sin (6.\ 28\ kt + \pi/3)$ volts
$i\ (t) = 14\ .\ 142 \sin (6.\ 28\ kt + \pi/2)$ Amp.
(*i*) Plot the power p (t) on wave diagram. (*ii*) Determine power factor and its nature. (*iii*) Determine the elements value. ***(Nagpur University, Winter 2002)***

12. A pure capacitor is connected in series with practical inductor (coil). The voltage source is of 10 volts, 10,000 Hz. It was observed that the maximum current of 2 Amp. flows in the circuit when the value of capacitor is 1 micro-farad. Find the parameter (R & L) of the coil. ***(Nagpur University, Winter 2002)***

13. Draw the phasor diagram for each of the following combinations :
(*i*) Rand L in series and combination in parallel with C.
(*ii*) Rand C in series and combination in parallel with L.
(*iii*) R, Land C in series, with $X_L > X_C$, when ac source is connected to it.

(Nagpur University, Summer 2003)

14. Two choke coils are connected in series as shown in Fig. 13.53. :

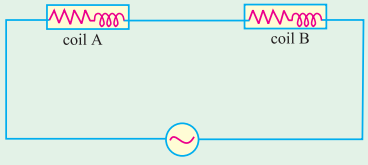


Fig. 13.53

Internal resistance and its inductive reactance of coil A is 4 Ω and 8 Ω resp. Supply voltage is 200 V. Total power consumed in the circuit is 2.2 kW and reactive power consumed is 1.5 kVAR. Find the internal resistance and inductive reactance of coil B. ***(Nagpur University, Summer 2004)***

15. A series circuit having a resistance of 10Ω, and inductance of (1/2π) H and a variable cap. is connected to 100 V, 50 Hz supply. Calculate the value of capacitor to form series resonance. Calculate resonant current, power and power factor. ***(Gujrat University, June/July 2003)***

16. Derive expressions for impedance, current and power factor for an R-L-C series circuit when applied with a.c. voltage. Draw also the vector diagram. ***(Gujrat University, June/July 2003)***

17. Explain the terms active power, reactive power and power factor. Also describe the series resonance of RLC circuit and list its important properties. ***(R.G.P.V. Bhopal University, June 2004)***

18. A 60 Hz sinusoidal voltage $v = 100 \sin \omega t$ is applied to a series R-L circuit. Assuming R = 10 Ω, L = 0.01 H, find the steady state current and relative phase angle. Also compute the effective magnitude and phase of voltage drops across each circuit elements.

(R.G.P.V. Bhopal University, June 2004)

19. With reference to Fig 13.54, find the values of R and X so that $V_1 = 3V_2$ and V_1 and V_2 are in quadrature. Applied voltage across AB is 240V.***(Belgaum Karnataka University, February 2002)***

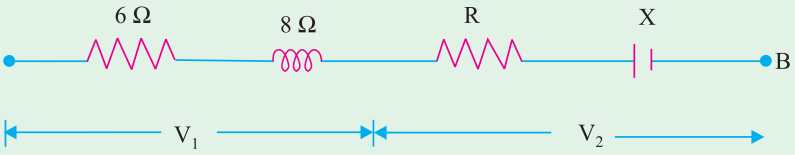


Fig. 13.54

20. A choke coil takes a current of 2 A lagging 60^0 behind the applied voltage of 220V at 50Hz. Calculate the inductance and resistance of the coil. ***(V.T.U., Belgaum Karnataka University Winter 2003)***

21. The instantaneous values ofthe voltage across a two element series circum and the current flowing through it are given by V = 100 sin $(314t - \pi/4)V$, $i = 20sin\ (314t - 90^0)A$. Find the frequency and the circuit elements. ***(V.T.U., Belgaum Karnataka University, Winter 2003)***

22. Show that the power consumed in a pure inductance is zero. ***(U.P. TechnicalUniversity 2003) (RGPV Bhopal 2002)***

23. What do you understand by the terms power factor, active power and reactive power? ***(Mumbai University 2003) (RGPV Bhopal 2002)***

24. Series R-L-C circuit ***(Mumbai University 2003) (RGPV Bhopal 2002)***

25. Describe the properties of (*i*) Resistance (*ii*) Inductance and (*iii*) capacitance used in A.C. Circuit. ***(RGPV Bhopal June 2003)***

26. Define Apparant Power and Power factor in a.c. circuit. Describe parallel resonance and list its important properties. ***(Mumbai University 2003) (RGPV Bhopal December 2003)***

OBJECTIVE TESTS -13

1. In a series *R-L* circuit, V_L—V_R by—degrees.
(*a*) lags, 45 (*b*) lags, 90
(*c*) leads, 90 (*d*) leads, 45

2. The voltage applied across an *R-L* circuit is equal to—of V_R and V_L.
(*a*) arithmetic sum (*b*) algebraic sum
(*c*) phasor sum (*d*) sum of the squares.

3. The power in an a.c. circuit is given by
(*a*) $VI \cos \phi$ (*b*) $VI \sin \phi$
(*c*) I^2Z (*d*) I^2X_L

4. The p.f. of an *R-C* circuit is
(*a*) often zero
(*b*) between zero and 1
(*c*) always unity
(*d*) between zero and –1.0

5. In a series RLC circuit at resonance, the magnitude of the voltage developed across the capacitor
(*a*) is always zero
(*b*) can never be greater than the input voltage.
(*c*) can be greater than the input voltage, however, it is 90^0 out of phase with the input voltage
(*d*) can be greater than the input voltage,and in in phase with the input voltage. ***(GATE 2001)***

6. The total impedance Z (jω) of the circuit shown above is

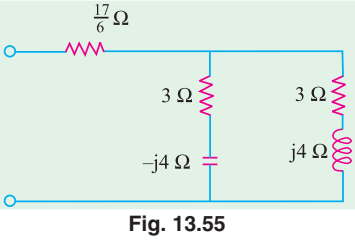


Fig. 13.55

(*a*) $(6 + j0)\ \Omega$ (*b*) $(7 + j0)\ \Omega$
(*c*) $(0 + j8)\ \Omega$ (*d*) $(6 + j8)\ \Omega$
(ESE 2003)

7. The impedance of a parallel RC network is $Z(s) = \dfrac{58}{s^2 + 0.5s + 100}$. Then the values of R, L and C are, respectively

(*a*) $10\ \Omega, \dfrac{1}{20}$ H, $\dfrac{1}{5}$ F (*b*) $1\ \Omega, \dfrac{1}{2}$ H, $\dfrac{1}{5}$ F

(*c*) $10\ \Omega, \dfrac{1}{20}$ H, $\dfrac{1}{5}$ F (*d*) $2\ \Omega, \dfrac{1}{20}$ H, $\dfrac{1}{5}$ F

(Engineering Services Exam. 2003)

ANSWERS

1. *c* **2.** *c* **3.** *a* **4.** *b*

Top

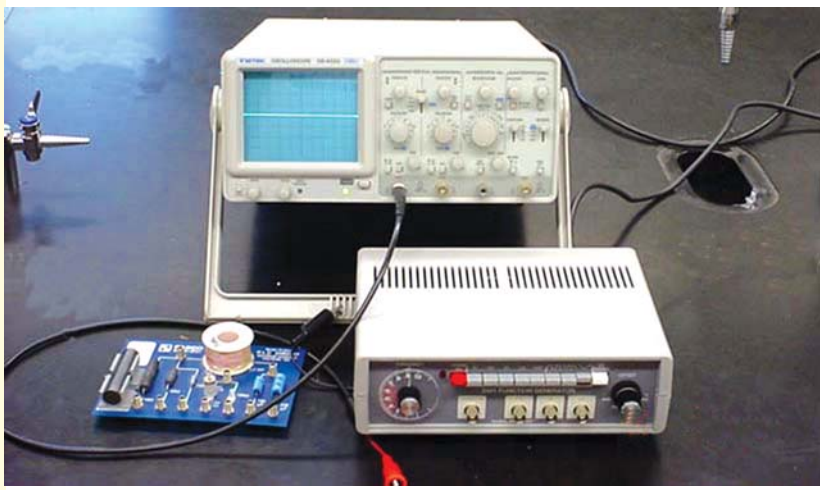
Contents

CHAPTER 14

PARALLEL A.C. CIRCUITS

Learning Objectives

- Solving Parallel Circuits
- Vector or Phasor Method
- Admittance Method
- Application of Admittance Method
- Complex or Phasor Algebra
- Series-Parallel Circuits
- Series Equivalent of a Parallel Circuit
- Parallel Equivalent of a Series Circuit
- Resonance in Parallel Circuits
- Graphic Representation of Parallel Resonance
- Points to Remember
- Bandwidth of a Parallel Resonant Circuit
- Q-factor of a Parallel Circuit



Parallel AC circuit combination is as important in power, radio and radar application as in series AC circuits

14.1. Solving Parallel Circuits

When impedances are joined in parallel, there are three methods available to solve such circuits:

(a) Vector or phasor Method (b) Admittance Method and (c) Vector Algebra

14.2. Vector or Phasor Method

Consider the circuits shown in Fig. 14.1. Here, two reactors *A* and *B* have been joined in parallel across an r.m.s. supply of *V* volts. The voltage across two parallel branches *A* and *B* is the same, but currents through them are different.

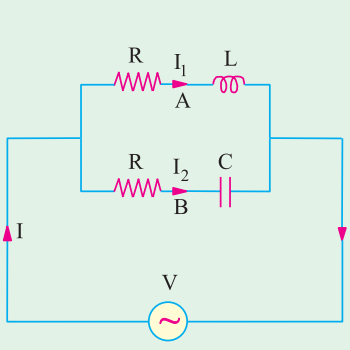


Fig. 14.1

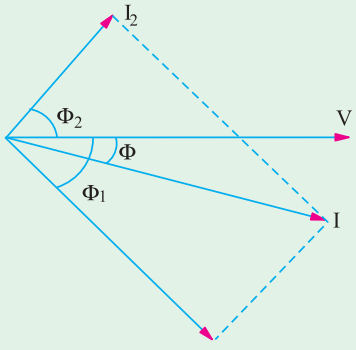


Fig. 14.2

For Branch *A*, $Z_1 = \sqrt{(R_1^2 + X_L^2)}$; $I_1 = V/Z_1$; $\cos \phi_1 = R_1/Z_1$ or $\phi_1 = \cos^{-1} (R_1/Z_1)$

Current I_1 lags behind the applied voltage by ϕ_1 (Fig. 14.2).

For Branch B, $Z_2 = \sqrt{(R_2^2 + X_c^2)}$; $I_2 = V/Z_2$; $\cos \phi_2 = R_2/Z_2$ or $\phi_2 = \cos^{-1} (R_2/Z_2)$

Current I_2 leads *V* by ϕ_2 (Fig. 14.2).

Resultant Current I

The resultant circuit current *I* is the vector sum of the branch currents I_1 and I_2 and can be found by ***(i)*** using parallelogram law of vectors, as shown in Fig. 14.2. or ***(ii)*** resolving I_2 into their *X*- and *Y*-components (or active and reactive components respectively) and then by combining these components, as shown in Fig. 14.3. Method ***(ii)*** is preferable, as it is quick and convenient.

With reference to Fig. 14.3. (*a*) we have

Sum of the active components of I_1 and I_2

$$= I_1 \cos \phi_1 + I_2 \cos \phi_2$$

Sum of the reactive components of I_1 and $I_2 = I_2 \sin \phi_2 - I_1 \sin \phi_1$

If *I* is the resultant current and ϕ its phase, then its active and reactive components must be equal to these *X*-and *Y*-components respectively [Fig. 14.3. (*b*)]

$I \cos \phi = I_1 \cos \phi_1 + I_2 \cos \phi_2$ and $I \sin \phi = I_2 \sin \phi_2 - I_1 \sin \phi_1$

$$\therefore \quad I = \sqrt{[(I_1 \cos \phi_1 + I_2 \cos \phi_2)^2 + (I_2 \sin \phi_2 - I_1 \sin \phi_1)^2}$$

and

$$\tan \phi = \frac{I_2 \sin \phi_2 - I_1 \sin \phi_1}{I_1 \cos \phi_1 + I_2 \cos \phi_2} = \frac{Y \text{ component}}{X \text{ component}}$$

If $\tan \phi$ is positive, then current leads and if $\tan \phi$ is negative, then current lags behind the applied

voltage V. Power factor for the whole circuit is given by

$$\cos\phi = \frac{I_1 \cos \Phi_1 \quad I_2 \cos \Phi_2}{I} \quad \frac{X \quad \text{comp.}}{I}$$

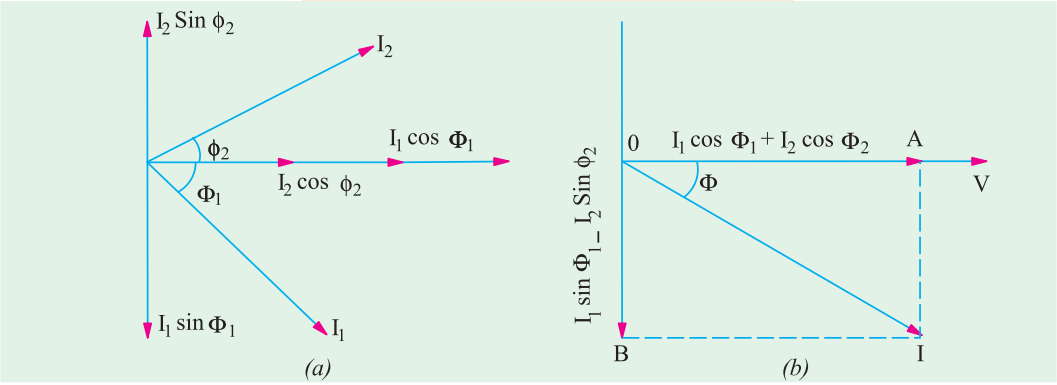


Fig. 14.3

14.3. Admittance Method

Admittance of a circuit is defined as ***the reciprocal of its impedance.*** Its symbol is Y.

$$\therefore \quad Y = \frac{1}{Z} = \frac{I}{V} \quad \text{or} \quad Y = \frac{\text{r.m.s. amperes}}{\text{r.m.s. volts}}$$

Its unit is Siemens (S). A circuit having an impedance of one ohm has an admittance of one Siemens. The old unit was mho (ohm spelled backwards).

As the impedance Z of a circuit has two components X and R (Fig. 14.4.), similarly, admittance Y also has two components as shown in Fig. 14.5. The X-component is known as ***conductance*** and Y-component as ***susceptance.***

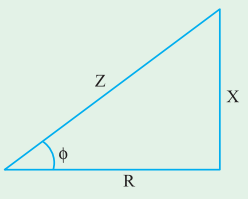


Fig. 14.4

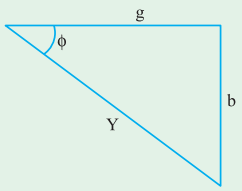


Fig. 14.5

Obviously, conductance $g = Y \cos\phi$

or $g = \frac{1}{Z} \cdot \frac{R}{Z}$ (from Fig. 14.4)

$$\therefore \quad g^* = \frac{R}{Z^2} = \frac{R}{R^2 + X^2}$$

Similarly, susceptance $b = Y \sin\phi = \frac{1}{2}, \frac{X}{Z}$ $\quad \therefore \; b^{**} = X/Z^2 = X/(R^2 + X^2)$ (from Fig. 14.5)

The admittance $Y = \sqrt{(g^2 + b^2)}$ just as $Z = \sqrt{(R^2 + X^2)}$

The unit of g, b and Y is Siemens. We will regard the ***capacitive susceptance as positive and inductive susceptance as negative.***

---

* In the special case when $X = 0$, then $g = 1/R$ *i.e.*, conductance becomes reciprocal of resistance, not otherwise.

** Similarly, in the special case when $R = 0$, $b = 1/X$ *i.e.*, susceptance becomes reciprocal of reactance, not otherwise.

14.4. Application of Admittance Method

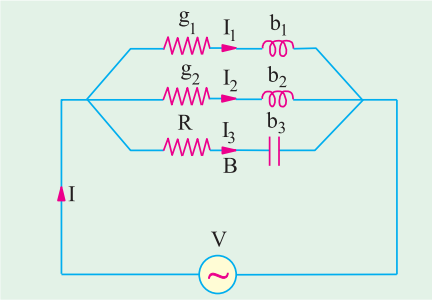


Fig. 14.6

Consider the 3-branched circuit of Fig. 14.6. Total conductance is found by merely adding the conductances of three branches. Similarly, total susceptance is found by ***algebraically*** adding the individual susceptances of different branches.

Total conductance $G = g_1 + g_2 + g_3$ .....

Total susceptance $B = (-b_1) + (-b_2) + b_3$ .....

(algebraic sum)

$\therefore$ Total admittance $Y = \sqrt{(G^2 + B^2)}$

Total current $I = VY$; Power factor $\cos\phi = G/Y$

14.5. Complex or Phasor Algebra

Consider the parallel circuit shown in Fig. 14.7. The two impedances, $\mathbf{Z_1}$ and $\mathbf{Z_2}$, being in parallel, have the same p.d. across them.

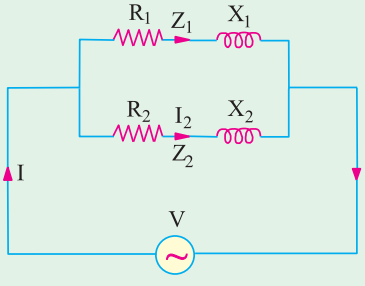


Fig. 14.7

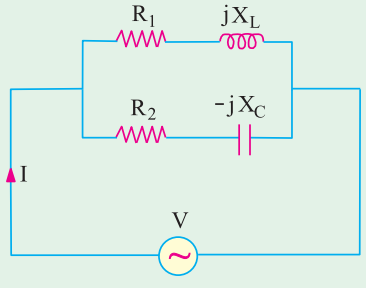


Fig. 14.8

Now $\mathbf{I_1 = \frac{V}{Z_1}}$ and $\mathbf{I_2 = \frac{V}{Z_2}}$

Total current $\mathbf{I = I_1 + I_2 = \frac{V}{Z_1} + \frac{V}{Z_2} = V\left(\frac{1}{Z_1} + \frac{1}{Z_2}\right) = V(Y_1 + Y_2) = VY}$

where $\mathbf{Y}$ = total admittance $= \mathbf{Y_1 + Y_2}$

It should be noted that ***admittances are added*** for parallel branches, whereas for branches in series, it is the *impedances* which are *added*. ***However, it is important to remember that since both admittances and impedances are complex quantities, all additions must be in complex form.*** Simple arithmetic additions must not be attempted.

Considering the two parallel branches of Fig. 14.8, we have

$$\mathbf{Y_1} = \frac{1}{\mathbf{Z_1}} = \frac{1}{R_1 + jX_L} = \frac{(R_1 - jX_L)}{(R_1 + jX_L)(R_1 - jX_L)}$$

$$= \frac{R_1}{R_1^2} \frac{jX_L}{X_L^2} \quad \frac{R_1}{R_1^2} \frac{}{X_L^2} \quad j\frac{X_L}{R_1^2} \frac{}{X_L^2} \quad g_1 \quad jb_1$$

where $g_1 = \frac{R_1}{R_1^2 + X_L^2}$ – conductance of upper branch,

$b_1 = -\frac{X_L}{R_1^2 + X_L^2}$ – susceptance of upper branch

Similarly, $\mathbf{Y_2} = \frac{1}{Z_2} = \frac{1}{R_2 - jX_C}$

$= \frac{R_2 + jX_C}{(R_2 - jX_C)(R_2 + jX_C)} = \frac{R_2 + jX_C}{R_2^2 + X_C^2} = \frac{R_2}{R_2^2 + X_C^2} + j\frac{X_C}{R_2^2 + X_C^2} = g_2 + jb_2$

Total admittance $\mathbf{Y} = \mathbf{Y_1} + \mathbf{Y_2} = (g_1 - jb_1) + (g_2 + jb_2) = (g_1 + g_2) - j(b_1 - b_2) = G - jB$

$\mathbf{Y} = \sqrt{[(g_1 + g_2)^2 + (b_1 - b_2)^2]}$; $\phi = \tan^{-1}\frac{b_1 \quad b_2}{g_1 \quad g_2}$

The polar form for admittance is $\mathbf{Y} = Y \angle \phi^\circ$ where ϕ is as given above.

$Y = \sqrt{G^2 + B^2} \angle \tan^{-1}(B/G)$

Total current $\mathbf{I} = \mathbf{VY}$; $\mathbf{I_1} = \mathbf{VY_1}$ and $\mathbf{I_2} = \mathbf{VY_2}$

If $\mathbf{V} = V \angle 0^\circ$ and $\mathbf{Y} = Y \angle \phi$ then $\mathbf{I} = \mathbf{VY} = V \angle 0^\circ \times Y \angle \phi = VY \angle \phi$

In general, if $\mathbf{V} = V \angle \alpha$ and $\mathbf{Y} = Y \angle \beta$, then $\mathbf{I} = \mathbf{VY} = V \angle \alpha \times Y \angle \beta = VY \angle \alpha + \beta$

Hence, it should be noted that when vector voltage is multiplied by admittance either in complex (rectangular) or polar form, the result is vector current in its proper phase relationship with respect to the voltage, ***regardless of the axis to which the voltage may have been referred to.***

Example 14.1. *Two circuits, the impedance of which are given by $Z_1 = 10 + j\,15$ and $Z_2 = 6 - j8$ ohm are connected in parallel. If the total current supplied is 15 A, what is the power taken by each branch ? Find also the p.f. of individual circuits and of combination. Draw vector diagram.*

(Elect. Technology, Vikram Univ, Ujjain)

Solution. Let $\mathbf{I} = 15 \angle 0^\circ$; $\mathbf{Z_1} = 10 + j15 = 18 \angle 57^\circ$

$\mathbf{Z_2} = 6 - j8 = 10 \angle -53.1^\circ$

Total impedance, $\mathbf{Z} = \frac{\mathbf{Z_1 Z_2}}{\mathbf{Z_1 + Z_2}} = \frac{(10 + j\,15)(6 - j\,8)}{16 + j\,7}$

$= 9.67 - j\,3.6 = 10.3 \angle -20.4^\circ$

Applied voltage is given by

$\mathbf{V} = \mathbf{IZ} = 15 \angle 0^\circ \times 10.3 \angle 20.4^\circ = 154.4 \angle 20.4^\circ$

$\mathbf{I_1} = \mathbf{V/Z_1} = 154.5 \angle 20.4^\circ / 18 \angle 57^\circ = 8.58 \angle 77.4^\circ$

$\mathbf{I_2} = \mathbf{V/Z_2} = 154.5 \angle 20.4^\circ / 10 \angle 53.1^\circ$

$= 15.45 \angle 32.7^\circ$

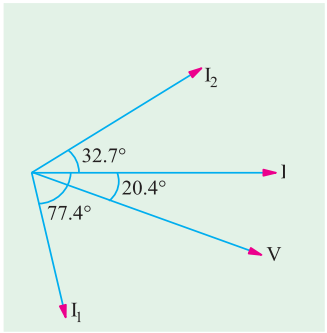


Fig. 14.9

We could also find branch currents as under :

$\mathbf{I_1} = \mathbf{I.Z_2/(Z_1 + Z_2)}$ and $\mathbf{I_2} = \mathbf{I.Z_1/(Z_1 + Z_2)}$

It is seen from phasor diagram of Fig. 14.9 that $\mathbf{I_1}$ lags behind $\mathbf{V}$ by $(77.4^\circ - 20.4^\circ) = 57^\circ$ and $\mathbf{I_2}$ leads it by $(32.7^\circ + 20.4^\circ) = 53.1^\circ$.

$\therefore \quad P_1 = I_1^2 R_1 = 8.58^2 \times 10 =$ **736 W; p.f.** $= \cos 57^\circ =$ **0.544 (lag)**

$P_2 = I_2^2 R_2 = 15.45^2 \times 6 =$ **1432 W** ; p.f. $= \cos 53.1^\circ =$ **0.6**

Combined p.f. $= \cos 20.4^\circ =$ **0.937 (lead)**

Example 14.2. *Two impedance $Z_1 = (8 + j6)$ and $Z_2 = (3 - j4)$ are in parallel. If the total current of the combination is 25 A, find the current taken and power consumed by each impedance.*

(F.Y. Engg. Pune Univ.)

Solution. $Z_1 = (8 + j6) = 10 \angle 36.87^\circ$; $Z_2 = (3 - j4) = 5 \angle -53.1^\circ$

$Z = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(10 \angle 36.87^\circ)(5 \angle -53.1^\circ)}{(8 + j6) + (3 - j4)} = \frac{50 \angle -16.23^\circ}{11 + j2} = \frac{50 \angle -16.23^\circ}{11.18 \angle 10.3^\circ} = 4.47 \angle 26.53^\circ$

Let $I = 25 \angle 0^\circ$; $V = I\,Z = 25 \angle 0^\circ \times 4.47 \angle 26.53^\circ = 111.75 \angle 26.53^\circ$

$I_1 = V/Z_1 = 111.75 \angle 26.53°/10\angle 36.87° = 11.175 \angle 63.4°$

$I_2 = 111.75 \angle 26.53/5 \angle 53.1° = 22.35 \angle 26.57°$

Now, the phase difference between V and I_1 is $63.4° - 26.53° = 36.87°$ with current lagging. Hence, $\cos \phi_1 = \cos 36.87° = 0.8$.

Power consumed in $Z_1 = VI_1 \cos \phi = 11.175 \times 111.75 \times 0.8 =$ **990 W**

Similarly, $\phi_2 = 26.57 - (-26.53) = 53.1°$; $\cos 53.1° = 0.6$

Power consumed in $Z_2 = VI_2 \cos \phi_2 = 111.75 \times 22.35 \times 0.6 =$ **1499 W**

Example 14.3. *Refer to the circuit of Fig. 14.10 (a) and determine the resistance and reactance of the lagging coil load and the power factor of the combination when the currents are as indicated.*

(Elect. Engg. A.M.Ae. S.I.)

Solution. As seen from the ΔABC of Fig. 14.10 (*b*).

$5.6^2 = 2^2 + 4.5^2 + 2 \times 2 \times 4.5 \times \cos \theta$, $\therefore \cos \theta = 0.395$, $\sin \theta = 0.919$. $Z = 300/4.5 = 66.67\ \Omega$

$R = Z \cos \theta = 66.67 \times 0.919 = 61.3\ \Omega$

p.f. $= \cos \phi = AC/AD = (2 + 4.5 \times 0.395)/5.6 =$ **0.67 (lag)**

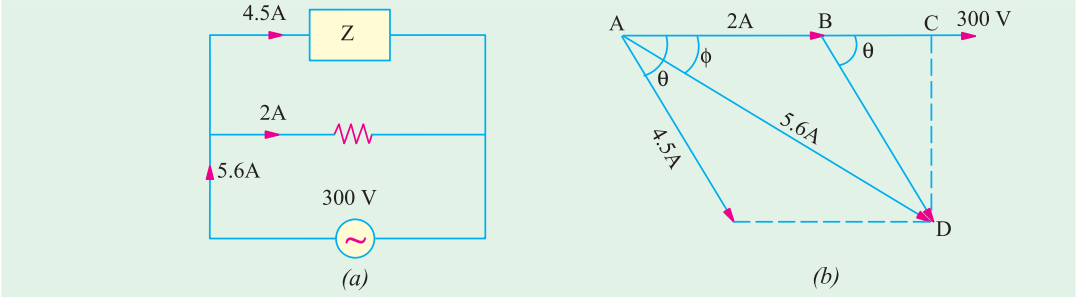


Fig. 14.10

Example 14.4. *A mercury vapour lamp unit consists of a 25μf condenser in parallel with a series circuit containing the resistive lamp and a reactor of negligible resistance. The whole unit takes 400 W at 240 V, 50 Hz at unity p.f. What is the voltage across the lamp ?*

(F.Y. Engg. Pune Univ.)

Solution. $X_C = \dfrac{1}{2\pi fC} = \dfrac{1}{2\pi \times 50 \times (25 \times 10^{-6})} = 127.3\ \Omega$, $\therefore I_C = \dfrac{240}{127.3} = 1.885$ A

$W = VI \cos \phi = VI \quad \therefore I = W/V = 400/240 = 1.667$ A

In the vector diagram of Fig. 14.10 (*b*) I_C leads V by 90° and current I_1 in the series circuit lags V by ϕ_1 where ϕ_1 is the power factor angle of the series circuit. The vector sum of I_C and ϕ_1 gives the total current I. As seen $\tan \phi_1 = I_C/I = 1.885/1.667 = 1.13077$. Hence, $\phi_1 = 48.5°$ lag. The applied voltage V is the vector sum of the drop across the resistive lamp which is in phase with I_1 and drop across the coil which leads I_1 by 90°.

Voltage across the lamp $= V \cos \Phi_1 = 340 \times \cos 48.5 = 240 \times 0.662 =$ **159 V.**

Example 14.5. *The currents in each branch of a two-branched parallel circuit are given by the expression $i_a = 7.07 \sin (314t - \pi/4)$ and $i_b = 21.2 \sin (314\ t + \pi/3)$*

The supply voltage is given by the expression $v = 354 \sin 314t$. Derive a similar expression for the supply current and calculate the ohmic value of the component, assuming two pure components in each branch. State whether the reactive components are inductive or capacitive.

(Elect. Engineering., Calcutta Univ.)

Solution. By inspection, we find that i_a lags the voltage by $\pi/4$ radian or 45° and i_b leads it by $\pi / 3$ radian or 60°. Hence, branch *A* consists of a resistance in series with a pure inductive reactance. Branch *B* consists of a resistance in series with pure capacitive reactance as shown in Fig. 14.11 (*a*).

Maximum value of current in branch A is 7.07 A and in branch *B* is 21.2 A. The resultant current can be found vectorially. As seen from vector diagram.

X-comp = 21.2 cos 60° + 7.07 cos 45° = 15.6 A

Y-comp = 21.2 sin 60° –7.07 sin 45° = 13.36 A

Maximum value of the resultant current is = $\sqrt{15.6^2 + 13.36^2}$ = 20.55 A

$\phi = \tan^{-1}(13.36/15.6) = \tan^{-1}(0.856) = 40.5°$ (lead)

Hence, the expression for the supply current is **$i = 20.55 \sin (314\,t + 40.5°)$**

$Z_A = 354/7.07 = 50\ \Omega$; $\cos \phi_A = \cos 4$:

$= 1/\sqrt{2}$. $\sin \phi_A = \sin 45° = 1/\sqrt{2}$

$R_A = Z_A \cos \phi_A = 50 \times 1/\sqrt{2} = 35.4\ \Omega$

$X_L = Z_A \sin \phi_A = 50 \times 1/\sqrt{2} = 35.4\ \Omega$

$Z_B = 354/20.2 = 17.5\ \Omega$

$R_B = 17.5 \times \cos 60° = 8.75\ \Omega$

$X_C = 17.5 \times \sin 60° = 15.16\ \Omega$

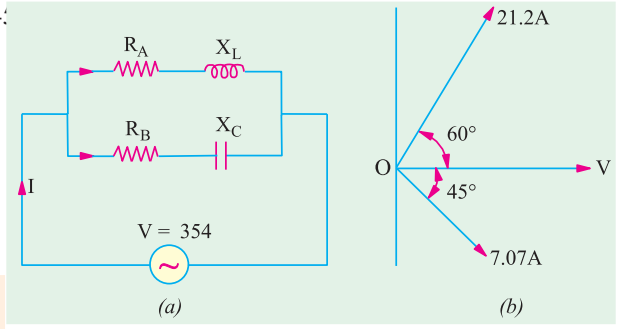


Fig. 14.11

Example 14.5 (*a*). *A total current of 10 A flows through the parallel combination of three impedance : (2 –j5) Ω, (6 + j3) Ω and (3 + j4) Ω. Calculate the current flowing through each branch. Find also the p.f. of the combination.*

(Elect. Engg., -I Delhi Univ.)

Solution. Let $\mathbf{Z_1} = (2 - j5)$, $\mathbf{Z_2} = (6 + j3)$, $\mathbf{Z_3} = (3 + j4)$

$\mathbf{Z_1 Z_2} = (2 - j5)(6 + j3) = 27 - j24$. $\mathbf{Z_2 Z_3} = (6 + j3)(3 + j4) = 6 + j33$

$\mathbf{Z_3 Z_1} = (3 + j4)(2 - j5) = 26 - j7$; $\mathbf{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1} = 59 + j2$

With reference to Art, 1.25

$$\mathbf{I_1} = \mathbf{I} \cdot \frac{\mathbf{Z_2 Z_3}}{\mathbf{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}} \quad (10 \quad j0) \quad \frac{6 \quad j33}{59 \quad j2} \quad 1.21 \quad j5.55$$

$$\mathbf{I_2} = \mathbf{I} \cdot \frac{\mathbf{Z_3 Z_1}}{\Sigma \mathbf{Z_1 Z_2}} \quad (10 \quad j0) \quad \frac{26 \quad j7}{59 \quad j2} \quad 4.36 \quad j1.33$$

$$\mathbf{I_3} = \mathbf{I} \cdot \frac{\mathbf{Z_1 Z_2}}{\Sigma \mathbf{Z_1 Z_2}} \quad (10 \quad j0) \quad \frac{27 \quad j24}{59 \quad j2} \quad 4.43 \quad j4.22$$

Now, $$\mathbf{Z} = \frac{\mathbf{Z_1 Z_2 Z_3}}{\mathbf{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}} \quad \frac{(2 \quad j5)(6 \quad j33)}{59 \quad j2} \quad 3.01 \quad j0.51$$

$\mathbf{V} = 10\ \angle 0° \times 3.05\ \angle 9.6° = 30.5\ \angle 9.6°$

Combination p.f. = cos 9.6° = **0.986 (lag)**

Example 14.6. *Two impedances given by Z_n = (10 + j 5) and Z_2 = (8 + j 6) are joined in parallel and connected across a voltage of V = 200 + j0. Calculate the circuit current, its phase and the branch currents. Draw the vector diagram.*

(Electrotechnics-I, M.S. Univ. Baroda)

Solution. The circuit is shown in Fig. 14.12

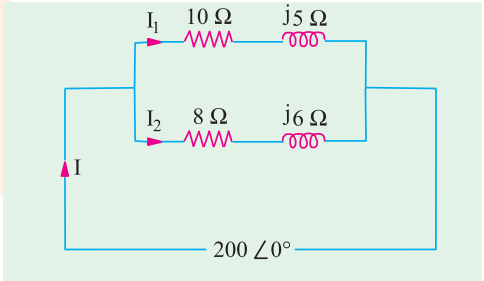


Fig. 14.12

Branch A, $\mathbf{Y_1} = \frac{\mathbf{1}}{\mathbf{Z_1}}$ $\quad \frac{1}{(10 \quad j5)}$

$$= \frac{10 - j5}{(10 + j5)(10 - j5)} = \frac{10 - j5}{100 + 25}$$

$$= 0.08 - j0.04 \text{ Siemens}$$

Branch B, Y_2 $= \frac{1}{Z_2} = \frac{1}{(8 + j6)}$

$$= \frac{8 - j6}{(8 + j6)(8 - j6)} = \frac{8 - j6}{64 + 36} = 0.08 - j0.06 \text{ Siemens}$$

$$\mathbf{Y} = (0.08 - j\,0.04) + (0.08 - j\,0.06) = 0.16 - j\,0.1 \text{ Siemens}$$

Direct Method

We could have found total impedance straightway like this : $\frac{1}{\mathbf{Z}} = \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} = \frac{\mathbf{Z}_1 + \mathbf{Z}_2}{\mathbf{Z}_1\mathbf{Z}_2}$

$$\therefore \quad \mathbf{Y} = \frac{\mathbf{Z}_1 + \mathbf{Z}_2}{\mathbf{Z}_1\mathbf{Z}_2} \quad \frac{(10 \quad j5)\ (8 \quad j6)}{(10 \quad j5)(8 \quad j6)} \quad \frac{18 \quad j11}{50 \quad j100}$$

Rationalizing the above, we get

$$\mathbf{Y} = \frac{(18 + j\,11)(50 - j\,100)}{(50 + j\,100)(50 - j\,100)} = \frac{200 - j\,1250}{12,500} = 0.16 - j\,0.1 \text{ (same as before)}$$

Now $\mathbf{V} = 200 \angle 0^\circ = 200 + j0$

$\therefore \quad \mathbf{I} = \mathbf{VY} = (200 + j0)(0.16 - j0.1)$

$= 32 - j20 = 37.74 \angle -32^\circ$....polar form

Power factor = $\cos 32^\circ = 0.848$

$\mathbf{I}_1 = \mathbf{VY}_1 = (200 + j0)(0.08 - j0.04)$

$= 16 - j8 = 17.88 \angle -26^\circ 32'$

It lags behind the applied voltage by $26^\circ 32'$.

$\mathbf{I}_2 = \mathbf{VY}_2 = (200 + j0)(0.08 - j0.06)$

$= 16 - j12 = 20 \angle -36^\circ 46'$

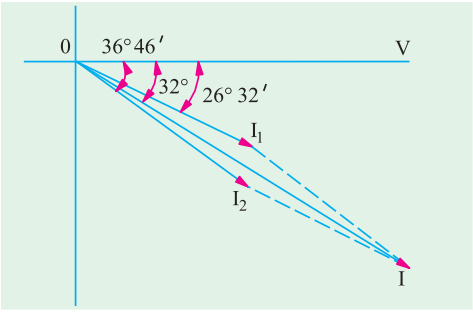


Fig. 14.13

It lags behind the applied voltage by $36^\circ 46'$. The vector diagram is shown in Fig. 14.13.

Example 14.7. *Explain the term admittance. Two impedance Z_1 = (6 –j 8) ohm and Z_2 = (16 + j12) ohm are connected in parallel. If the total current of the combination is (20 + j10) amperes, find the complexor for power taken by each impedance. Draw and explain the complete phasor diagram.* **(Basic Electricity, Bombay Univ.)**

Solution. Let us first find out the applied voltage, $\mathbf{Y} = \mathbf{Y}_1 + \mathbf{Y}_2 \quad \frac{1}{6 \quad j8} \quad \frac{1}{16 \quad j12}$

$$= (0.06 + j0.08) + (0.04 - j0.03) = 0.1 + j0.05 = 0.1118 \angle 26^\circ 34'$$

$$\mathbf{I} = 20 + j10 = 22.36 \angle 26^\circ 34'$$

Now $\mathbf{I} = \mathbf{VY} \quad \therefore \quad \mathbf{V} = \frac{I}{Y} = \frac{22.36 \angle 26^\circ 34'}{0.1118 \angle 26^\circ 34'} = 200 \angle 0^\circ$

$\mathbf{I}_1 = \mathbf{VY}_1 = (200 + j0)(0.06 + j0.008) = 12j + 16$ *A*, $\mathbf{I}_2 = 200\ (0.04 - j0.03) = 8 - j6$ A

Using the method of conjugates and taking voltage conjugate, the complexor power taken by each branch can be foudn as under :

$\mathbf{P}_1 = (200 - j0)(12 + j16) = 2400 + j3200$; $P_2 = (200 - j0)(8 - j6) = 100 - j1200$

Drawing of phasor diagram is left to the reader.

Note. Total voltamperes = 4000 + *j*2000

As a check, $\mathbf{P} = \mathbf{VI} = 200\ (20 + j10) = 4000 + j2000$

Example 14.8. *A 15-mH inductor is in series with a parallel combination of an 80 Ω resistor and 20 μF capacitor. If the angular frequency of the applied voltage is ω = 1000 rad/s, find the admittance of the network.* **(Basic Circuit Analysis Osmania Univ. Jan/Feb 1992)**

Solution. $X_L = \omega L = 1000 \times 15 \times 10^{-3} = 15\Omega$; $X_C = 1/\omega C = 10^6/1000 \times 20 = 50\ \Omega$

Impedance of the parallel combination is given by

$$Z_p = 80 \| -J\,50 = -j4000/(80 - j50) = 22.5 - j36,$$

Total impedance $= j15 + 22.5 - j36 = 22.5 - j21$

Admittance $Y = \dfrac{1}{Z} \quad \dfrac{1}{22.5 \quad j21} \quad 0.0238 \quad j0.022$ Siemens

Example 14.9. *An impedance (6 + j 8) is connected across 200-V, 50-Hz mains in parallel with another circuit having an impedance of (8 – j6) Ω. Calculate* ***(a)*** *the admittance, the conductance, the susceptance of the combined circuit* ***(b)*** *the total current taken from the mains and its p.f.*
(Elect. Engg-AMIE, S.I. 1992)

Solution. $\mathbf{Y_1} = \dfrac{1}{6 + j8} = \dfrac{6 - j8}{6^2 + 8^2} = 0.06 - j0.08$ Siemens, $\mathbf{Y} = \dfrac{1}{8 - j6} = \dfrac{8 + j6}{100} = 0.08 + j0.06$ Siemens

(*a*) Combined admittance is $\mathbf{Y = Y_1 + Y_2} = 0.14 - j0.02 = \mathbf{0.1414 \angle -88°'}$ **Siemens**

Conductance, G = **0.14 Siemens** ; Susceptance, B = **-0.02 Siemens** (inductive)

(*b*) Let $\mathbf{V} = 200 \angle 0°$; $\mathbf{I = VY} = 200 \times 0.1414 \angle 8°8'$ $V = 28.3 \angle 8°8'$

p.f. = cos 8°8′ = **0.99 (lag)**

Example 14.10. *If the voltmeter in Fig. 14.14 reads 60 V, find the reading of the ammeter.*

Solution. $I_2 = 60/4 = 15$ A. Taking it as reference quantity, we have $\mathbf{I}_2 = 15 \angle 0$.

Obviously, the applied voltage is

$\mathbf{V} = 15 \angle 0° \times (4 - j4) = 84.8 \angle 45°$

$\mathbf{I_1} = 84.8 \angle 45°/(6 + j\,3) = 84.8 \angle 45 + 6.7 \angle 26.6$

$= 12.6 \angle 71.6° = (4 - j\,12)$

$\mathbf{I} = \mathbf{I_1 + I_2} = (15 + j\,0) + (4 - j\,12)$

$= 19 - j\,12 = 22.47 \angle 32.3°$

Hence, ammeter **reads 22.47.**

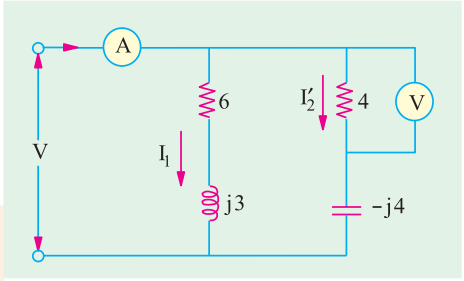


Fig. 14.14

Example 14.11. *Find the reading of the ammeter when the voltmeter across the 3 ohm resistor in the circuit of Fig. 14.15 reads 45 V.*
(Elect. Engg. & Electronics Bangalore Univ.)

Solution. Obviously $I_1 = 45/3 = 15$ A. If we take it as reference quantity, $I_1 = 3 \angle 0°$

Now, $Z_1 = 3 - j3 = 4.24 \angle 45°$.

Hence, $V = I_1 Z_1 = 15 \angle 0° \times 4.24 \angle 45° = 63.6 \angle 45°$

$$I_2 = \frac{V}{Z_2} = \frac{63.6 \angle -45°}{5 + j2} = \frac{63.6 \angle -45°}{5.4 \angle 21.8°}$$

$= 11.77 \angle -66.8° = 4.64 - j\ 10.8$

$I = I_1 + I_2 = 19.64 - j10.8 =$ **22.4 ∠28.8°**

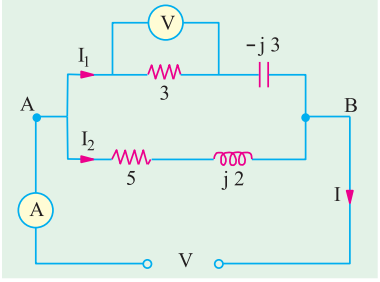


Fig. 14.15

Example 14.12. *A coil having a resistance of 5 Ω and an inductance of 0.02 H is arranged in parallel with another coil having a resistance of 1 Ω and an inductance of 0.08 H. Calculate the current through the combination and the power absorbed when a voltage of 100 V at 50 Hz is applied. Estimate the resistance of a single coil which will take the same current at the same power factor.*

Solution. The circuit and its phasor diagram are shown in Fig. 14.16.

Branch No. 1

$X_1 = 314 \times 0.02 = 6.28\ \Omega$

$Z_1 = \sqrt{5^2 + 6.28^2} = 8\ \Omega$

$I_1 = 100/8 = 12.5$ A

$\cos \phi_1 = R_1/Z_1 = 5/8$

$\sin \phi_1 = 6.28/8$

Branch No. 2

$X_2 = 314 \times 0.08 = 25.12\ \Omega$, $Z_2 = \sqrt{1^2 + 25.12^2} = 25.14\ \Omega$, $I_2 = 100/25.14 = 4$ A

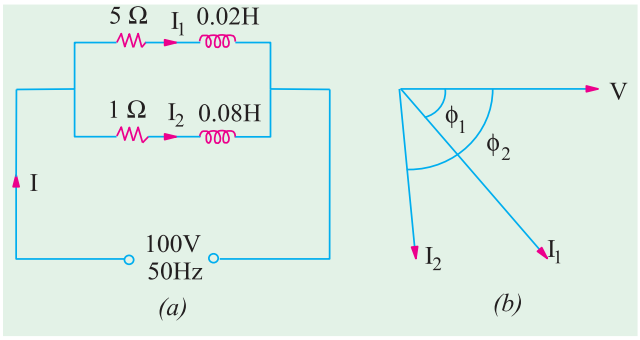


Fig. 14.16.

$\cos \phi_2 = 1/25.14$ and $\sin \phi_2 = 25.12/25.14$

X - components of I_1 and $I_2 = I_1 \cos \phi_1 + I_2 \cos \phi_2 = (12.5 \times 5/8) + (4 \times 1/25.14) = 7.97$ A

Y - components of I_1 and $I_2 = I_1 \sin \phi_1 + I_2 \sin \phi_2 = (12.5 \times 6.28/8) + (4 \times 25.12/25.14) = 13.8$ A

$I = \sqrt{7.97^2 + 13.8^2} = \mathbf{15.94\ A}$

$\cos \phi = 7.97/15.94 = 0.5$ (lag)

$\phi = \cos^{-1}(0.5) = 60°$

Power absorbed

$= 100 \times 15.94 \times 0.5 = \mathbf{797\ W}$

The equivalent series circuit is shown in Fig. 14.17 (*a*).

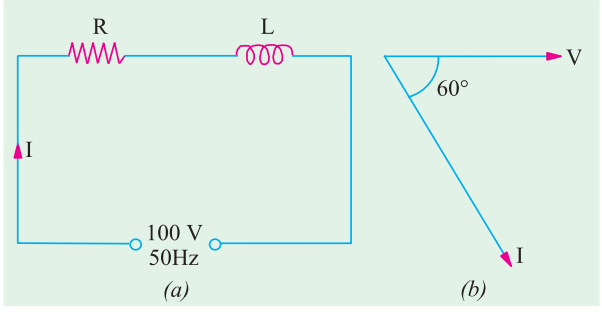


Fig. 14.17

$V = 100$ V ; $I = 15.94$ A ; $\phi = 60°$

$Z = 100/15.94 = 6.27\ \Omega$;

$R = Z \cos \phi = 6.27 \times \cos 60° = \mathbf{3.14\ \Omega}$

$X = Z \sin \phi = 6.27 \times \sin 60° = \mathbf{5.43\ \Omega}$

Admittance Method For Finding Equivalent Circuit

$$\mathbf{Y_1} = \frac{1}{5 + j6.28} = \frac{5 - j6.28}{5^2 + 6.28^2} = 0.078 - j\,0.098\ \text{S},$$

$$\mathbf{Y_2} = \frac{1}{1 + j25.12} = \frac{1 - j25.12}{1^2 + j25.12^2} = 0.00158 - j\,0.0397\ \text{S},$$

$$\mathbf{Y} = \mathbf{Y_1} + \mathbf{Y_2} = 0.0796 - j0.138 = 0.159 \angle -60°$$

Here $G = 0.0796$ S, $B = -0.138$ S, $Y = 0.159\ \Omega$

$\therefore$ $R_{eq} = G/Y^2 = 0.0796/0.159^2 = \mathbf{3.14\ \Omega}$, $X_{eq} = B/Y^2 = 0.138/0.159^2 = \mathbf{5.56\ \Omega}$

Example. 14.13. *A voltage of 200 ∠53°8′ is applied across two impedances in parallel. The values of impedances are (12 + j 16) and (10 –j 20). Determine the kVA, kVAR and kW in each branch and the power factor of the whole circuit.* **(Elect. Technology, Indore Univ.)**

Solution. The circuit is shown in Fig. 14.18.

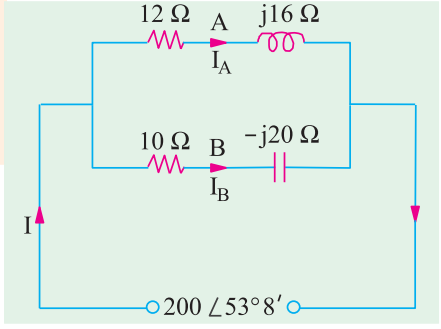


Fig. 14.18

$\mathbf{Y_A} = 1/(12 + j16) = (12 - j16)/[(12 + j16)(12 - j16)]$

$= (12 - j16)/400 = 0.03 - j0.04$ mho

$\mathbf{Y_B} = 1/(10 - j20) = 10 + j20/[(10 - j20)(10 + j20)]$

$$= \frac{10 \quad j20}{500} \quad 0.02 \quad j0.04 \text{ mho}$$

Now $V = 200 \angle 53°8' = 200 (\cos 53°8' + j \sin 53°8')$

$= 2000 (0.6 + j0.8) = 120 + j160$ volt

$\mathbf{I_A = VY_A} = (120 + j160) (0.03 - j0.04)$

$= (10 + j0)$ ampere (along the reference axis)

$\therefore \quad \mathbf{I_B = VY_B} = (120 + j160) (0.02 + j0.04)$

$= -4.0 + j8$ ampere (leading)

Example 14.14. *Two circuits, the impedances of which are given by $Z_1 = 15 + j12$ ohms and $Z_2 = 8 - j5$ ohms are connected in parallel. If the potential difference across one of the impedance is 250 + j0 V, calculate.*

(i) *total current and branch currents*

(ii) *total power and power consumed in each branch*

and ***(iii)*** *overall power-factor and power-factor of each branch.*

(Nagpur University, November 1998)

Solution. *(i)* $I_1 = (250 + j\ 0)/(15 + j\ 12) = 250 \angle 0°/19.21 \angle 38.6°$

$= 13 \angle -38.6°$ amp $= 13\ (0.78 - j\ 0.6247)$

$= 10.14 - j\ 8.12$ amp

$I_2 = (250 + j\ 0)/(8 - j\ 5) = 250 \angle 0°/9.434 \angle -32°$

$= 26.5 \angle + 32° = 26.5\ (0.848 + j\ 0.530)$

$= 22.47 + j14.05$ amp

$I = I_1 + I_2 = 32.61 + j\ 5.93 = 33.15 \angle + 10.36°$

(ii) Power in branch 1 $= 13^2 \times 15 = 2535$ watts

Power in branch 2 $= 26.5^2 \times 8 = 5618$ watts

Total power consumed $= 2535 + 5618 = 8153$ watts

(iii) Power factor of branch 1 $= \cos 38.60° = 0.78$ lag

Power factor of branch 2 $= \cos 32° = 0.848$ lead.

Overall power factor $= \cos 10.36° = 0.984$ lead.

Additional hint : Drawn phasor-diagram for these currents, in fig. 14.19, for the expressions written above,

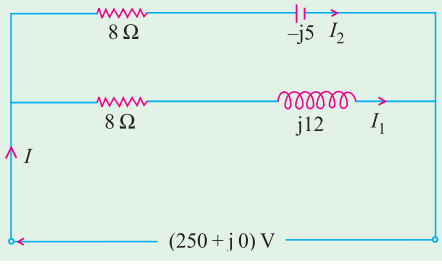


Fig. 14.19

Example 14.15. *An inductive circuit, in parallel with a resistive circuit of 20 ohms is connected across a 50 Hz supply. The inductive current is 4.3 A and the resistive current is 2.7 A. The total current is 5.8 A Find :* ***(a)*** *Power factor of the combined circuit. Also draw the phasor diagram.*

(Nagpur University, November 1997)

Solution. I_2 (= 2.7 A) is in phase with V which is 54 V in magnitude. The triangle for currents is drawn in the phasor diagram in fig. 14.20 (*b*)

Solving the triangle, $\phi_1 = 180^\circ - \cos^{-1}[(2.7^2 = 4.3^2 - 5.8^2)/(2 \times 4.3 \times 2.7)] = 70.2^\circ$

Further, $5.8 \sin \phi = 4.3 \sin \phi_1$, giving $\phi = 44.2^\circ$.

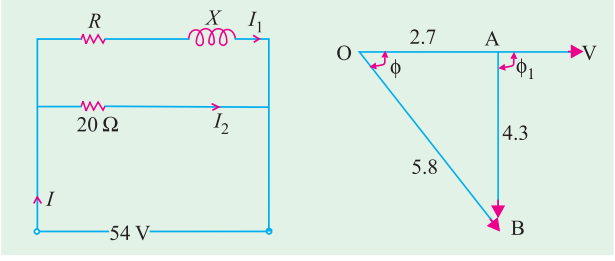


Fig. 14.20 (*a*) Fig. 14.20 (*b*)

OA = I_2 = 2.7 in phase with V
AB = I_1 = 4.3 lagging behind V by ϕ_1
OB = I = 5.8 lagging behind V by ϕ

$$|Z_1| = 54/4.3 = 12.56 \text{ ohms}$$
$$R = Z_1 \cos \phi_1 = 4.25 \text{ ohms}$$
$$X = Z_1 \sin \phi_1 = 11.82 \text{ ohms, since } \phi_1 \text{ is the lagging angle}$$

(*a*) Power absorbed by the Inductive branch
$$= 4.3^2 \times 4.25 = 78.6 \text{ watts}$$

(*b*) $L = 11.82/314 = 37.64$ mH

(*c*) P.f. of the combined circuit = $\cos \phi = 0.717$ lag

Check : Power consumed by 20 ohms resistor = $2.7^2 \times 20 = 145.8$ W

Total Power consumed in two branches = 78.6 + 145.8 = 224.4 W

This figure must be obtained by input power = VI cos ϕ

= $54 \times 5.8 \times \cos 44.2^\circ = 224.5$ W. Hence checked.

Example 14.16. *In a particular A.C. circuit, three impedances are connected in parallel, currents as shown in fig. 14.21 are flowing through its parallel branches.*

(i) *Write the equations for the currents in terms of sinusoidal variations and draw the waveforms.*

Find the total current supplied by the source. **[Nagpur University, April 1998]**

Solution. In Fig. 14.21, V is taken as reference, and is very convenient for phasor diagrams for parallel circuits.

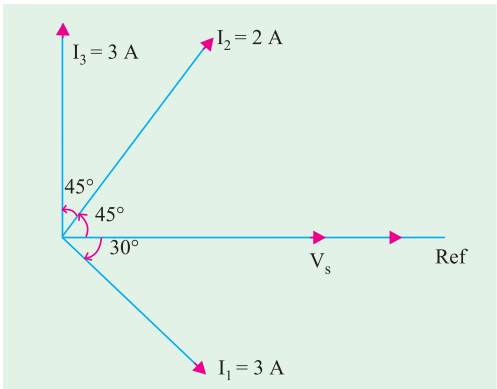


Fig. 14.21

(i) I_1 lags behind by 30°. Branch no. 1 must, therefore, have an R-L series combination. With 10-volt source, a current of 3 A in branch 1 means that its impedance Z_1 is given by

$Z_1 = 10/3 = 3.333$ ohms

The phase-angle for I_1 is 30° lagging

$R_1 = 3.333 \cos 30^\circ = 2.887$ ohms

$X_{L1} = 3.333 \sin 30^\circ = 1.6665$ ohms

(ii) I_2 is 2 amp and it leads the voltage by 45°. Branch 2 must, therefore, have R-C series combination.

$$Z_2 = 10/2 = 5 \text{ ohms}$$
$$R_2 = 5 \cos 45^\circ = 3.5355 \text{ ohms}$$

$$X_{c2} = 5 \sin 45^\circ = 3.5355 \text{ ohms}$$

(iii) Third branch draws a current of 3 amp which leads the voltage by 90°. Hence, it can only have a capacitive reactance.

$$|Z_3| = X_{c3} = 10/3 = 3.333 \text{ ohms}$$

Total current supplied by the source = I amp

$$I = I_1 + I_2 + I_3$$
$$= 3 [\cos 30^\circ - j \sin 30^\circ] + 2 [\cos 45^\circ + j \sin 45^\circ] + 3 [0 + j\ 1]$$
$$= 4.0123 + j\ 2.9142$$
$$|I| = 4.96 \text{ amp, leading } V_s, \text{ by } 36^\circ.$$

Expressions for currents : Frequency is assumed to be 50 Hz

$$v_s = 10\sqrt{2} \sin (314\ t)$$
$$i_1 = 3\sqrt{2} \sin (314\ t - 30^\circ)$$
$$i_2 = 2\sqrt{2} \sin (314\ t + 45^\circ)$$
$$i_3 = 3\sqrt{2} \sin (314\ t + 90^\circ)$$

Total current, $i\ (t) = 4.96\sqrt{2} \sin (314\ t + 36^\circ)$

Total power consumed = Voltage × active (or in phase-) component of current
$$= 10 \times 4.012 = 40.12 \text{ watts}$$

Example 14.17. *A resistor of 12 ohms and an inductance of 0.025 H are connected in series across a 50 Hz supply. What values of resistance and inductance when connected in parallel will have the same resultant impedance and p.f. Find the current in each case when the supply voltage is 230 V.* **(Nagpur University, Nov. 1996)**

Solution. At 50 Hz, the series R-L circuit has an impedance of Z_s given by

$$Z_s = 12 + j\ (314 \times 0.025) = 12 + j\ 7.85 = 14.34 + \angle 33.2^\circ$$
$$I_s = (230 + j0) / (12 + j\ 7.85) = 16.04 - \angle 33.2^\circ$$
$$= 13.42 - j\ 8.8 \text{ amp}$$

Out of these two components of I_s, the in-phase components is 13.42 amp and quadrature component (lagging) is 8.8 amp. Now let the *R-L* parallel combination be considered. In Fig. 14.22 (*b*), *R* carries the in-phase component, and *L* carries the quadrature-component (lagging). For the two systems to be equivalent,

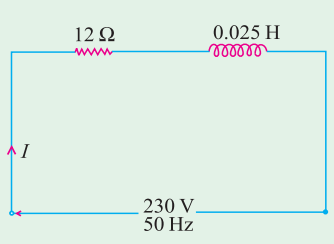


Fig. 14.22 (*a*)

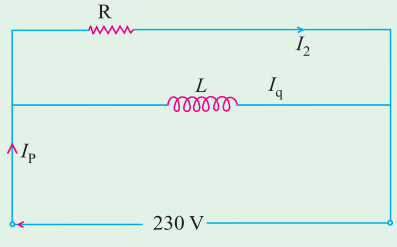


Fig. 14.22 (*b*)

$$I_s = I_p$$

It means
$$I_s = 13.42 \text{ amp}$$
$$I_q = 8.8 \text{ amp}$$

Thus,
$$R = 230 / 13.42 = 17.14 \text{ ohms}$$

$$X_L = 230 / 8.8 = 26.14 \text{ ohms}$$
$$L = 26.14/314 = 83.2 \text{ mH}$$

Example 14.18. *An inductive coil of resistance 15 ohms and inductive reactance 42 ohms is connected in parallel with a capacitor of capacitive reactance 47.6 ohms. The combination is energized from a 200 V, 33.5 Hz a.c. supply. Find the total current drawn by the circuit and its power factor. Draw to the scale the phasor diagram of the circuit.* **(Bombay University, 2000)**

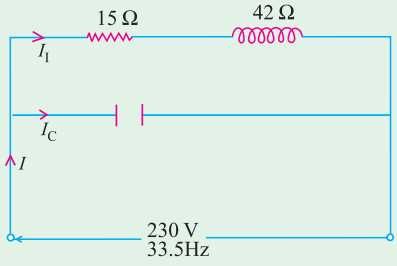


Fig. 14.23 (*a*)

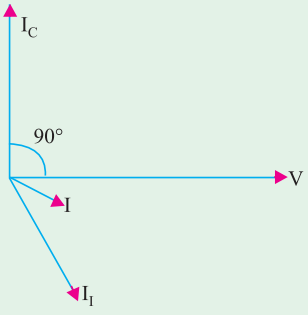


Fig. 14.23 (*b*)

Solution.

$$Z_1 = 15 + j42,\ Z_1 = 44.6 \text{ ohms},\ \cos \phi_1 = 15.44.6 = 0.3363$$
$$\phi_1 = 70.40 \text{ Lagging},\ I_1 = 200/44.6 = 4.484 \text{ amp}$$
$$I_c = 200/47.6 = 4.2 \text{ amp}$$
$$I = 4.484\ (0.3355 - j0.942) + j4.2 = 1.50 - j0.025 = 1.5002 \angle -1^\circ$$

For the circuit in Fig. 14.23 (*a*), the phasor diagram is drawn in Fig. 14.23 (*b*).

Power Calculation

Power etc. can be calculated by the method of conjugates as explained in Ex. 14.3

Branch A

The current conjugate of $(10 + j\ 0)$ is $(10 - j\ 0)$

$\therefore$ VIA $= (120 + j\ 160)\ (10 - j\ 0) = 1200 + j\ 1600$ $\therefore$ kW $= 1200/1000 = 1.2$

$\therefore$ kVAR $= 1600/1000 = 1.6$. The fact that it is positive merely shows the reactive volt-amperes are due to a lagging current* kVA $= \sqrt{(1.2^2 + 1.6^2)} = 2$

Branch B

The current conjugate of $(-4.0 + j8)$ is $(-4.0 - j8)$

$\therefore$ $\mathbf{VI_B} = (120 + j160)\ (-4 - j8) = 800 - j1600$

$\therefore$ kW $= 800/1000 =$ **0.8** $\therefore$ kVAR $= -1600/1000 =$ **-1.6**

The negative sign merely indicates that reactive volt-amperes are due to the leading current

$\therefore$ kVA $= \sqrt{[0.8^2 + (-1.6)^2]} =$ **1.788**

$$\mathbf{Y} = \mathbf{Y_A} + \mathbf{Y_B} = (0.03 - j0.04) + (0.02 + j0.04) = 0.05 + j0$$
$$\mathbf{I} = \mathbf{VY} = (120 + j160)\ (0.05 + j0) = 6 + j8 = 10 \angle 53°8'$$

or $\mathbf{I} = \mathbf{I_A} + \mathbf{I_B} = (10 + j0) + (-4 + j8) = 6 + j8$ (same as above)

Circuit p.f. $= \cos 0° = 1$ ($\because$ current is in phase with voltage)

* If voltage conjugate is used, then capacitive VARs are positive and inductive VARs negative.

Example 14.19. *An impedance $Z_1 = (8 - j5)\ \Omega$ is in parallel with an impedance $Z_2 = (3 + j7)\ \Omega$ If 100 V are impressed on the parallel combination, find the branch currents I_1, I_2 and the resultant current. Draw the corresponding phasor diagram showing each current and the voltage drop across each parameter. Calculate also the equivalent resistance, reactance and impedance of the whole circuit.* **(Elect. Techology-I, Gwalior Univ. 1998)**

Solution. Admittance Method

$\mathbf{Y_1} = 1/(8 - j5) = (0.0899 + j0.0562)\ S$

$\mathbf{Y_2} = 1/(3 + j7) = (0.0517 - j0.121)\ S$, $\mathbf{Y = Y_1 + Y_2} = (0.1416 - j0.065) S$

Let $\mathbf{V} = (100 + j0)$; $\mathbf{I_1 = VY_1} = 100\ (0.0899 + j0.0562) = 8.99 + j5.62$

$\mathbf{I_2 = VY_2} = 100\ (0.0517 - j0.121) = 5.17 - j12.1$; $\mathbf{I = VY} = 100\ (0.416 - j0.056) = 14.16 - j6.5$

Now, $G = 0.1416$S, $B = -0.065\ S$ (inductive;)

$$Y = \sqrt{G^2 + B^2} = \sqrt{0.1416^2 + 0.065^2} = 0.1558 \text{ S}$$

Equivalent series resistance, $R_{eq} = G/Y^2 = 0.1416/0.1558^2 = 5.38\ \Omega$

Equivalent series inductive reactance $X_{eq} = B/Y^2 = 0.065/0.1558^2 = 2.68\ \Omega$

Equivalent series impedance $\mathbf{Z} = 1/\mathbf{Y} = 1/0.1558 =$ **6.42 Ω**

Impedance Method

$$\mathbf{I_1} = \mathbf{V}/\mathbf{Z_1} = (100 + j0) / (8 - j5) = 8.99 + j5.62$$

$$\mathbf{I_2} = \mathbf{V}/\mathbf{Z_2} = 100/(3 + j7) = 5.17 - j12.1$$

$$\mathbf{Z} = \frac{\mathbf{Z_1}\,\mathbf{Z_2}}{\mathbf{Z_1}\ \ \mathbf{Z_2}} \quad \frac{(8\quad j5)\,(3\quad j7)}{(11\quad j2)} \quad \frac{59\quad j41}{(11\quad j2)} = 5.848 + j2.664 = 6.426 = \angle 24.5^\circ,$$

$$\mathbf{I} = 100/6.426\ \angle 24.5^\circ = 15.56\ \angle 24.5^\circ = 14.16 - j\ 6.54$$

As seen from the expression for Z, equivalent series resistance is **5.848** Ω and inductive reactance is **2.664 ohm.**

Example 14.20. *The impedances $Z_1 = 6 + j\ 8$, $Z_2 = 8 - j\ 6$ and $Z_3 = 10 + j\ 0$ ohms measured at 50 Hz, form three branches of a parallel circuit. This circuit is fed from a 100 volt. 50-Hz supply. A purely reactive (inductive or capacitive) circuit is added as the fourth parallel branch to the above three-branched parallel circuit so as to draw minimum current from the source. Determine the value of L or C to be used in the fourth branch and also find the minimum current.*

(Electrical Circuits, South Gujarat Univ.)

Solution. Total admittance of the 3-branched parallel circuit is

$$\mathbf{Y} = \frac{1}{6 + j8} + \frac{1}{8 - j6} + \frac{1}{10 + j0} = 0.06 - j0.08 + 0.08 + j0.06 + 0.1 = 0.24 - j0.02$$

Current taken would be minimum when net susceptance is zero. Since combined susceptance is inductive, it means that we must add capacitive susceptance to neutralize it. Hence, we must connect a pure capacitor in parallel with the above circuit such that its susceptance equals + j0.02 S

$\therefore \quad I/X_C = 0.02$ or $2\pi/C = 0.02$; $C = 0.2/314 =$ **63.7 μF**

Admittance of four parallel branches = $(0.24 - j0.02) + j0.02 = 0.24$ S

$\therefore$ Minimum current drawn by the circuit = $100 \times 0.24 =$ **24 A**

Example 14.21. *The total effective current drawn by parallel circuit of Fig. 14.24 (a) is 20 A. Calculate* ***(i)*** *VA* ***(ii)*** *VAR and* ***(iii)*** *watts drawn by the circuit.*

Solution. The combined impedance of the circuit is

$$\mathbf{Z} = \frac{\mathbf{Z_1 Z_2}}{\mathbf{Z_1 + Z_2}} \quad \frac{10\,(6 \quad j8)}{(16 \quad j\,8)} \quad (5 \quad j2.5)\ \text{ohm}$$

(iii) Power $= I^2R = 20^2 \times 5 =$ **2000 W** (ii) $Q = I^2X = 20^2 \times 2.5$ = **1000 VAR (leading)** **(i)** $S = P + j\,Q = 2000 + j1000 = 2236 \angle 27°$; $S =$ **2236 VA**

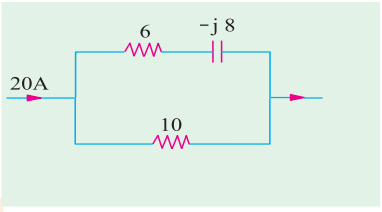


Fig. 14.24. (*a*)

Example 14.22. *Calculate (i) total current and (ii) equivalent impedance for the four-branched circuit of Fig. 14.24 (b).*

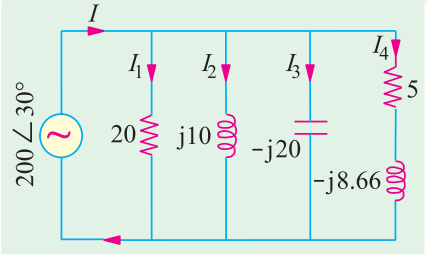


Fig. 14.24 (*b*)

Solution. $\mathbf{Y_1} = 1/20 = 0.05$ S, $\mathbf{Y_2} = 1/j10 = -j0.1$ S;

$\mathbf{Y_3} = 1/-j20 = j0.05$ S ; $\mathbf{Y_4} = 1/5 - j8.66 = 1/10 \angle 60°$

$= 0.1 \angle 60° = (0.05 - j0.0866)$ S

$\mathbf{Y} = \mathbf{Y_1} + \mathbf{Y_2} + \mathbf{Y_3} + \mathbf{Y_4} = (0.1 - j\ 0.1366)$ S

$= 0.169 \angle -53.8°$ S

(i) $\mathbf{I} = \mathbf{VY} = 200 \angle 30° \times 0.169 \angle -53.8° = 33.8 \angle -23.8°$ A

(ii) $\mathbf{Z} = \mathbf{1/Y} = 1/0.169 \angle -53.8° = 5.9 \angle 53.8°\ \Omega$

Example 14.23. *The power consumed by both branches of the circuit shown in Fig. 14.23 is 2200 W. Calculate power of each branch and the reading of the ammeter.*

Solution. $\mathbf{I_1} = \mathbf{V}/\mathbf{Z_1}$

$= \mathbf{V}/(6 + j\ 8) = \mathbf{V}/10 \angle 53.1°$, $\mathbf{I_2} = \mathbf{V}/\mathbf{Z_2} = \mathbf{V}/20$

$\therefore \quad I_1/I_2 = 20/10 = 2$, $P_1 = I_1^2\,R_1$ and $P_2 = I_2^2\,R_2$

$$\therefore \quad \frac{P_1}{P_2} = \frac{I_1^2 R_1}{I_2^2 R_2} = 2^2 \times \left(\frac{6}{20}\right) = \frac{6}{5}$$

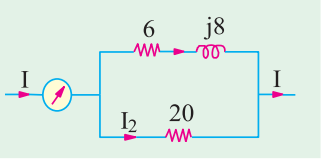


Fig. 14.25

Now, $P = P_1 + P_2$ or $\dfrac{P}{P_2} = \dfrac{P_1}{P_2} + 1 = \dfrac{6}{5} + 1 = \dfrac{11}{5}$

or $P_2 = 2200 \times \dfrac{5}{11} = 1000\,W \quad \therefore\ P_1 = 2200 - 1000 = 1200$ W

Since $P_1 = I_1^2\,R_1$ or $1200 = I_1^2 \times 6$; $I_1 = 14.14$ A

If $\mathbf{V} = V \angle 0°$, then $\mathbf{I_1} = 14.14 \angle -53.1° = 8.48 - j11.31$

Similarly, $P_2 = I_2^2\,R_2$ or $1000 = I_2^2 \times 20$; $\mathbf{I_2} = 7.07$ A or $\mathbf{I_2} = \mathbf{7.07} \angle 0°$

Total current $\mathbf{I} = \mathbf{I_1} + \mathbf{I_2} = (8.48 - j11.31) + 7.07 = 15.55 - j11.31 =$ **19.3** $\angle$**36°**

Hence, ammeter reads **19.3 A**

Example 14.24. *Consider an electric circuit shown in Figure 14.25 (a)*

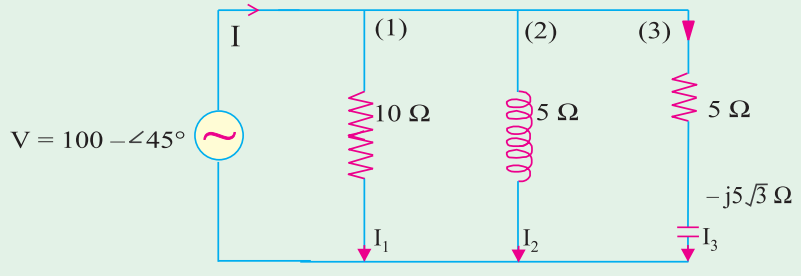


Fig. 14.25. (*a*)

Determine : ***(i)*** *the current and power consumed in each branch.*
(ii) *the supply current and power factor.* **(U.P. Technical University, 2001)**

Solution. Indicating branch numbers 1, 2, 3 as marked on the figure, and representing the source voltage by 100 ∠45°,

$$Z_1 = 10 + j\,0 = 10\angle 0°,\ I_1 = 100\angle 45°/10\angle 0° = 10\angle 45° \text{ amp}$$

$$Z_2 = 5 + j\,5\sqrt{3} = 10\angle 60°,\ I_2 = 100\angle 45°/10\angle 60° = 10\angle -15° \text{ amp}$$

$$Z_3 = 5 - j\,5\sqrt{3} = 10\angle -60°,\ I_3 = 100\angle 45°/10\angle -60° = 10\angle 105° \text{ amp}$$

Phasor addition of these three currents gives the supply current, I which comes out to be $I = 20\angle 45°$ amp.

This is in phase with the supply voltage.

(i) Power consumed by the branches :

Branch 1 : $10^2 \times 10 = 1000$ watts

Branch 2 : $10^2 \times 5 = 500$ watts

Branch 3 : $10^2 \times 5 = 500$ watts

Total power consumed = 2000 watts

(ii) Power factor = 1.0 since V and I are in phase

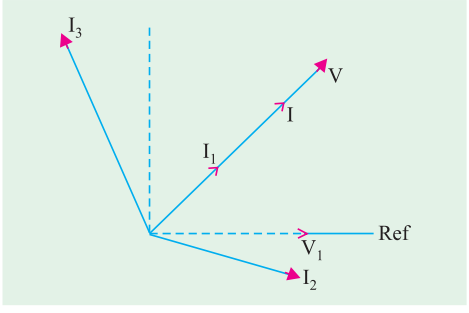


Fig. 14.25 (*b*)

14.6. Series-parallel Circuits

(i) By Admittance Method

In such circuits, the parallel circuit is first reduced to an equivalent series circuit and then, as usual, combined with the rest of the circuit. For a parallel circuit,

Equivalent series resistance $R_{eq} = Z\cos\phi = \dfrac{1}{Y}\cdot\dfrac{G}{Y} = \dfrac{G}{Y^2}$

- Sec Ex. 14.14

Equivalent series reactance $X_{eq} = Z\sin\phi = \dfrac{1}{Y}\cdot\dfrac{B}{Y} = \dfrac{B}{Y^2}$

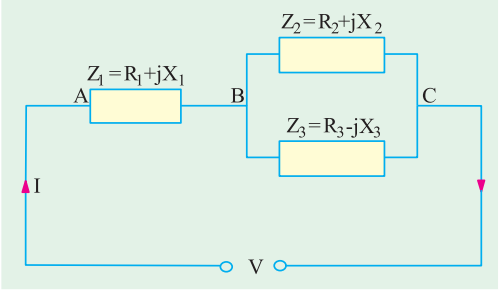


Fig. 14.26

(ii) By Symbolic Method

Consider the circuit of Fig. 14.26. First, equivalent impedance of parallel branches is calculated and it is then added to the series impedance to get the total circuit impedance. The circuit current can be easily found.

$$\mathbf{Y}_2 = \frac{1}{R_2 \quad jX_2};\ \mathbf{Y}_3 \quad \frac{1}{R_3 \quad jX_3}$$

$$\therefore \quad \mathbf{Y}_{23} = \frac{1}{R_2 + jX_2} + \frac{1}{R_3 - jX_3}$$

$$\therefore \quad \mathbf{Z}_{23} = \frac{1}{\mathbf{Y}_{23}};\ \mathbf{Z}_1 = R_1 + jX_1;\ \mathbf{Z} = \mathbf{Z}_{23} + \mathbf{Z}_1$$

$$\therefore \quad \mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}}$$ **(Sec Ex. 14.21)**

14.7. Series Equivalent of a Parallel Circuit

Consider the parallel circuit of Fig. 14.27 (*a*). As discussed in Art. 14.5

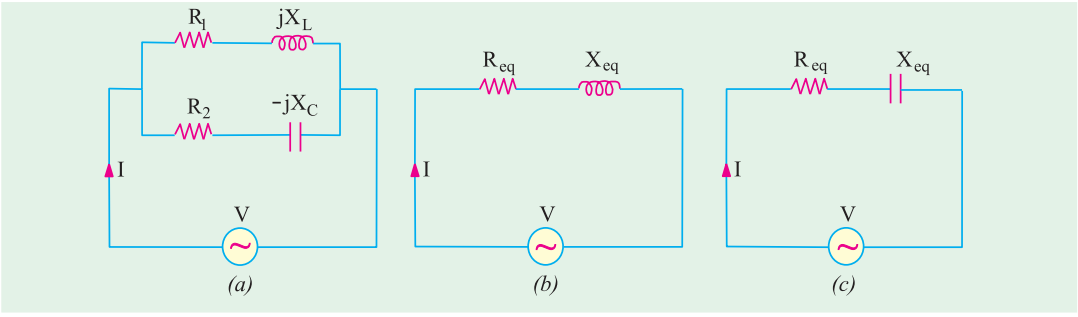


Fig. 14.27

$$\mathbf{Y_1} = \frac{R_1}{R_1^2 \quad X_L^2} \quad j\frac{X_L}{R_1^2 \quad X_L^2} \quad g_1 \quad jb_1 \; ; \; \mathbf{Y_2} \quad \frac{R_2}{R_2^2 \quad X_c^2} \quad j\frac{X_2}{R_2^2 \quad X_c^2} \quad g_2 \quad jb_2$$

$$\mathbf{Y = Y_1 + Y_2} = g_1 - jb_1 + g_2 + jb_2 = (g_1 + g_2) + j(b_2 - b_1) = G + jB = \sqrt{G^2 + B^2)} \; \angle \tan^{-1}(B/G)$$

As seen from Fig. 14.28.

$$R_{eq} = Z\cos\phi = \frac{1}{Y}\cdot\frac{G}{Y} = \frac{G}{Y^2}$$

$$X_{eq} = Z\sin\phi = \frac{1}{Y}\cdot\frac{B}{Y} = \frac{B}{Y^2}$$

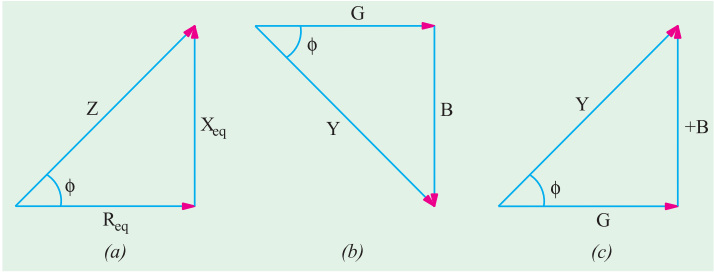


Fig. 14.28

Hence, equivalent series circuit is as shown in Fig. 14.27 (*b*) or (*c*) depending on whether net susceptance *B* is negative (inductive) or positive (capacitive). If *B* is negative, then it is an *R-L* circuit of Fig. 14.27 (*b*) and if *B* is positive, then it is an *R-C* circuit of Fig. 14.27 (*c*).

14.8. Parallel Equivalent of a Series Circuit

The two circuits will be equivalent if **Y** of Fig. 14.29 (*a*) is equal to the **Y** of the circuit of Fig. 14.29. (*b*).

Series Circuit

$$\mathbf{Y_s} = \frac{1}{R_s + jX_s}$$

$$= \frac{R_s - jX_s}{(R_s + jX_s)(R_s - jX_s)}$$

$$= \frac{R_s - jX_s}{R_2^2 + X_s^2} = \frac{R_s}{R_s^2 + X_s^2} - j\frac{X_s}{R_s^2 + X_s^2}$$

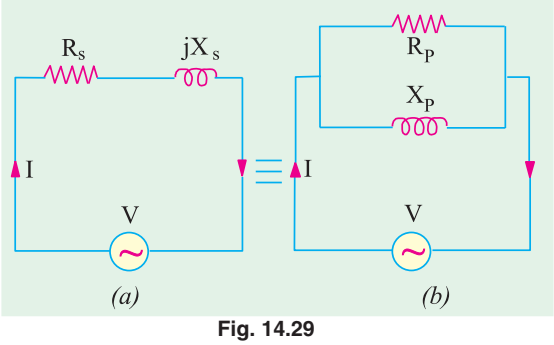


Fig. 14.29

Parallel Circuit

$$\mathbf{Y_p} = \frac{1}{R_p + j0} + \frac{1}{0 + jX_p} = \frac{1}{R_p} + \frac{1}{jX_p} = \frac{1}{R_p} - \frac{j}{X_p}$$

$$\therefore \quad \frac{R_s}{R_s^2+X_s^2} - j\frac{X_s}{R_s^2+X_s^2} = \frac{1}{R_P} - \frac{j}{X_p} \quad \therefore \frac{1}{R_p} = \frac{R_s}{R_s^2+X_s^2} \quad \text{or } R_p = R_s + \frac{X_s^2}{R_s} = R_s\left(1+\frac{X_s^2}{R_s^2}\right)$$

Similarly $X_p = X_s + \frac{R_s^2}{X_s} = X_s\left(1+\frac{R_s^2}{X_s^2}\right)$

Example 14.25. *The admittance of a circuit is (0.03 –j 0.04) Siemens. Find the values of the resistance and inductive reactance of the circuit if they are joined* ***(a)*** *in series and (b) in parallel.*

Solution. ***(a)*** $\mathbf{Y} = 0.03 - j0.04$

$$\therefore \quad \mathbf{Z} = \frac{1}{\mathbf{Y}} \quad \frac{1}{0.03 \quad j0.04} \quad \frac{0.03 \quad j0.04}{0.03^2 \quad 0.04^2} \quad \frac{0.03 \quad j0.04}{0.0025} \quad 12 \quad j16$$

Hence, if the circuit consists of a resistance and inductive reactance in series, then resistance is **12 Ω** and inductive reactance is 16 Ω as shown in Fig. 14.30.

(b) Conductance = 0.03 mho $\quad \therefore$ Resistance = 1/0.03 = 33.3 Ω

Susceptance (inductive) = 0.04 S $\quad \therefore$ Inductive reactance = 1/0.04 = 25 Ω

Hence, if the circuit consists of a resistance connected in parallel with an inductive reactance, then resistance is **33.3 Ω** and inductive reactance is **25 Ω** as shown in Fig. 14.31.

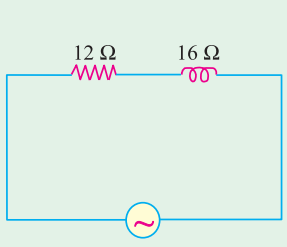


Fig. 14.30

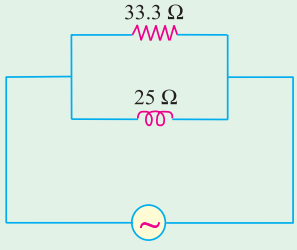


Fig. 14.31

Example 14.26. *A circuit connected to a 115-V, 50-Hz supply takes 0.8 A at a power factor of 0.3 lagging. Calculate the resistance and inductance of the circuit assuming* ***(a)*** *the circuit consists of a resistance and inductance in series and* ***(b)*** *the circuit consists of a resistance and inductance in paralllel.* **(Elect. Engg.-I, Sardar Patel Univ.)**

Solution. Series Combination

$$Z = 115/0.8 = 143.7\ \Omega;\ \cos\phi = R/Z = 0.3 \quad \therefore\ R = 0.3 \times 143.7 = \mathbf{43.1\ \Omega}$$

Now $\quad X_L = \sqrt{Z^2 - R^2} = \sqrt{143.7^2 - 43.1^2} = 137.1\ \Omega$

$\therefore \quad L = 137.1/2\pi \times 50 = \mathbf{0.436\ H}$

Parallel Combination

Active component of current (drawn by resistance)

$= 0.8 \cos\phi = 0.8 \times 0.3 = 0.24$ A; $\quad R = 115/0.24 = \mathbf{479\ \Omega}$

Quadrature component of current (drawn by inductance) $= 0.8 \sin\phi = 0.8\sqrt{1-0.3^2} = 0.763$ A

$\therefore \quad X_L = 115/0.763\ \Omega \quad \therefore\ L = 115/0.763 \times 2\pi \times 50 = \mathbf{0.48\ H}$

Example 14.27. *The active and lagging reactive components of the current taken by an a.c. circuit from a 250-V supply are 50 A and 25 A respectively. Calculate the conductance, susceptance, admittance and power factor of the circuit. What resistance and reactance would an inductive coil have if it took the same current from the same mains at the same factor ?*

(Elect. Technology, Sumbal Univ.)

Solution. The circuit is shown in Fig. 14.32.

Resistance $= 250/50 = 5\ \Omega$; Reactance $= 250/25 = 10\ \Omega$

$\therefore$ Conductance $g = 1/5 = \mathbf{0.2\ S}$, Susceptance $b = -1/10 = \mathbf{-0.1\ S}$

Admittance $Y = \sqrt{g^2 + b^2} = \sqrt{0.2^2 + (-0.1)^2} = \sqrt{0.05} = \mathbf{0.224\ S}$

$\mathbf{Y} = 0.2 - j\ 0.1 = 0.224 \angle -26°34'$. Obviously, the total current lags the supply voltage by $26°34'$, p.f. $= \cos 26°34' = \mathbf{0.894\ (lag)}$

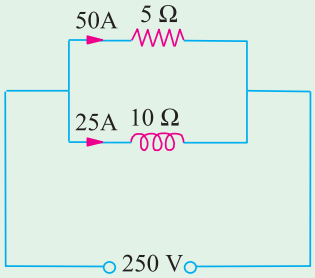


Fig. 14.32

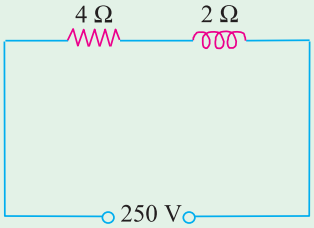


Fig. 14.33

Now $\mathbf{Z} = \frac{1}{\mathbf{Y}} \quad \frac{1}{0.2 \quad j0.1} \quad \frac{0.2 \quad j0.1}{0.05} \quad 4 \quad j2$

Hence, resistance of the coil = **4 Ω** Reactance of the coil = **2 Ω** (Fig. 14.33)

Example 14.28. *The series and parallel circuits shown in Fig. 14.34 have the same impedance and the same power factor. If $R = 3\ \Omega$ and $X = 4\ \Omega$, find the values of R_1 and X_1. Also, find the impedance and power factor.* **(Elect. Engg., Bombay Univ.)**

Solution. Series Circuit [Fig. 14.34 (*a*)]

$$\mathbf{Y}_S = \frac{1}{R + jX} = \frac{R - jX}{R^2 + X^2} = \frac{R}{R^2 + X^2} - j\frac{X}{R^2 + X^2}$$

Parallel Circuit [Fig. 14.34 (*b*)]

$$\mathbf{Y}_P = \frac{1}{R_1 + j0} = \frac{1}{0 + jX_1} = \frac{1}{R_1} + \frac{1}{jX_1} = \frac{1}{R_1} - \frac{j}{X_1}$$

$\therefore$ $$\frac{R}{R^2 + X^2} - j\frac{X}{R^2 + X^2} = \frac{1}{R_1} - \frac{j}{X_1}$$

$\therefore$ $R_1 = R + X^2/R$ and $X_1 = X + R^2/X$

$\therefore$ $R_1 = 3 + (16/3) = \mathbf{8.33\ \Omega}$; $X_1 = 4 + (9/4) = \mathbf{6.25\ \Omega}$

Impedance $= 3 + j\ 4 = 5 \angle 53.1°$; Power factor $= \cos 53.1° = \mathbf{0.6\ (lag)}$

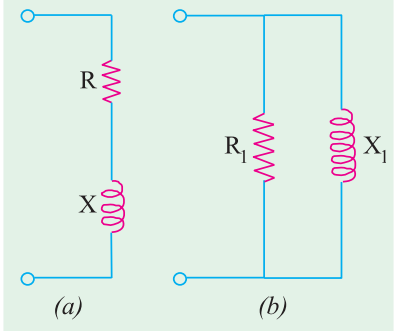


Fig. 14.34

Example 14.29. *Find the value of the resistance R and inductance L which when connected in parallel will take the same current at the same power factor from 400-V, 50-Hz mains as a coil of resistance $R_1 = 8\ \Omega$ and an induction $L_1 = 0.2$ H from the same source of supply.*

Show that when the resistance R_1 of the coil is small as compared to its inductance L_1, then R and L are respectively equal to $\omega^2 L_1^2/R_1$ and L_1.

(Elect. Technology, Utkal Univ.)

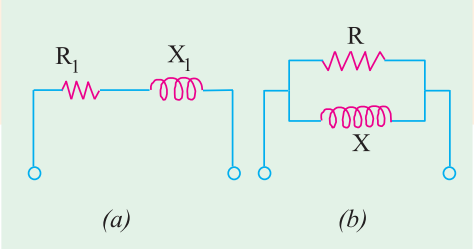


Fig. 14.35

Solution. As seen from Art. 14.8 in Fig. 14.35.

$R = R_1 + X_1^2/R_1$ ...(*i*)

$X = X_1 + R_1^2/X_1$ ...(*ii*)

Now

$R_1 = 8\,\Omega,\ X_1 = 2\pi \times 50 \times 0.2 = 62.8\,\Omega$

$\therefore \quad R = 8 + (62.8^2/8) = \mathbf{508\ \Omega}$

$X = 62.8 + (64/62.8) = \mathbf{63.82\ \Omega}$

From ***(i)***, it is seen that if R_1 is negligible, then $R = X_1^2/R_1 = \omega^2 L_1^2/R_1$

Similarly, from ***(ii)*** we find that the term R_1^2/X_1 is negligible as compared to X_1,

$\therefore \quad X = X_1$ or $L = L_1$

Example 14.30. *Determine the current drawn by the following circuit [Fig. 14.36 (a)[when a voltage of 200 V is applied across the same. Draw the phasor diagram.*

Solution. As seen from the figure

$$\mathbf{Z_2} = 10 - j12 = 15.6 \angle -50.2°;\ \mathbf{Z_3} = 6 + j10 = 11.7 \angle 58°$$

$$\mathbf{Z_1} = 4 + j6 = 7.2 \angle 56.3°\ ;\ \mathbf{Z_{BC}} = \frac{(10 - j12)(6 - j10)}{16 - j2} = 10.9 + j3.1 = 11.3 \angle 15.9°$$

$$\mathbf{Z} = \mathbf{Z_1} + Z_{BC} = (4 + j6) + (10.9 + j3.1) = 14.9 + j9.1 = 17.5 \angle 31.4°$$

Assuming $\mathbf{V} = 200 \angle 0°$; $\mathbf{I} = \dfrac{\mathbf{V}}{\mathbf{Z}}$ 200 / .5 31.4 11.4

For drawing the phasor diagram, let us find the following quantities :

(i) $\mathbf{V_{AB}} = \mathbf{IZ_1} = 11.4 \angle -31.4° \times 7.2 \angle 56.3° = 82.2 \angle 24.9°$

$\mathbf{V_{BC}} = \mathbf{I.}\ \mathbf{Z_{BC}} = 11.4 \angle -31.4° \times 11.3 \angle 15.9 = 128.8 \angle -15.5°$

$\mathbf{I_2} = \dfrac{\mathbf{V_{BC}}}{\mathbf{Z_2}} =$ 128. / 15.6 5.5 / 50.2° 8.25 34.7

$\mathbf{I_3} =$ 128.8 / 11.7 5.5 / 58° 15.1 74.5°

Various currents and voltages are shown in their phase relationship in Fig. 14.36 (*b*).

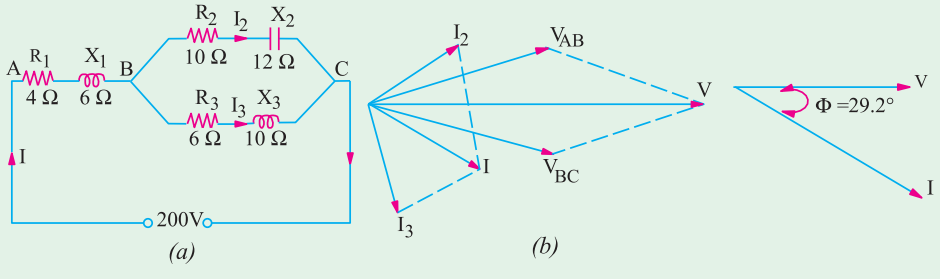


Fig. 14.36 Fig. 14.37 (*a*)

Example 14.31. *For the circuit shown in Fig. 14.37 (a), find **(i)** total impedance **(ii)** total current **(iii)** total power absorbed and power-factor. Draw a vector diagram.*

(Elect. Tech. Osmania Univ. Jan/Feb 1992)

Solution. $Z_{BC} = (4 + j8) \parallel (5 - j8) = 9.33 + j0.89$

(i) $Z_{AC} = 3 + j6 + 9.33 + j0.89 = 12.33 + j6.89$

$= 14.13 \angle 29.2°$

(ii) $I = 100/14.13 \angle 29.2°$, as drawn in Fig. 14.37 (*b*)

$= 7.08 \angle -29.2°$

(iii) $\phi = 29.2°$; $\cos\phi = 0.873$; $P = VI \cos\phi$

$= 100 \times 7.08 \times 0.873 = 618$ W

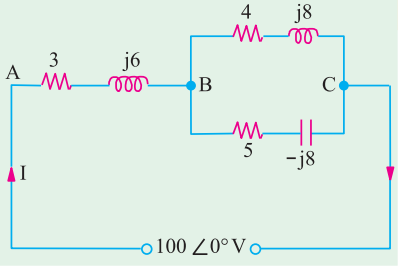


Fig. 14. 37 (*b*)

Example 14.32. *In a series-parallel circuit, the parallel branches A and B are in series with C. The* ***impedances*** *are : $Z_A = (4 + j\,3)$; $Z_B = (4 - j\,16/3)$; $Z_C = (2 + j8)$ ohm.*

If the current $I_C = (25 + j\,0)$, draw the complete phasor diagram determining the branch currents and voltages and the total voltage. Hence, calculate the complex power (the active and reactance powers) for each branch and the whole circuits. **(Basic Electricity, Bombay Univ.)**

Solution. The circuit is shown in Fig. 14.38 (*a*)

$\mathbf{Z_A} = (4 + j3) = 5 \angle 36°52'$; $\mathbf{Z_B} = (4 - j16/3) = 20/3 \angle -53°8'$; $\mathbf{Z_C} = (2 + j8) = 8.25 \angle 76°$

$\mathbf{I_C} = (25 + j0) = 25 \angle 0°$; $\mathbf{V_C} = \mathbf{I_C Z_C} = 206 \angle 76°$

$$\mathbf{Z_{AB}} = \frac{(4 + j3)(4 - j16/3)}{(8 - j7/3)} = \frac{(32 - j28/3)}{(8 - j7/3)} = 4 + j\,0 = 4 \angle 0°$$

$\mathbf{V_{AB}} = \mathbf{I_C Z_{AB}} = 25 \angle 0° \times 4 \angle 0° = 100 \angle 0°$

$\mathbf{Z} = \mathbf{Z_C} + \mathbf{Z_{AB}} = (2 + j8) + (4 + j0) = (6 + j8) = 10 \angle 53°8$; $\mathbf{V} = \mathbf{I_C Z} = 25 \angle 0° \times 10 \angle 53°8'$

$= 250 \angle 53°8'$

$\mathbf{I_A} = \dfrac{\mathbf{V_{AB}}}{\mathbf{Z_A}}$ 100 5 52 20 52 ; $\mathbf{I_B}$ $\dfrac{\mathbf{V_{AB}}}{\mathbf{Z_B}} =$ 100 (20/3) 53 8 15 53 8

Various voltages and currents are shown in Fig. 14.38 (*b*). Powers would be calculated by using voltage conjugates.

Power for whole circuit is $\mathbf{P} = \mathbf{VI_C} = 250 \angle 53°8' \times 25 \angle 0° = 6{,}250 \angle 53°8'$

$= 6250 (\cos 53°8' - j \sin 53°8') = 3750 - j5000$

$\mathbf{P_C} = 25 \times 206 \angle -76° = 5150 (\cos 76° - j \sin 76°) = 1250 - j5000$

$\mathbf{P_A} = 100 \times 20 \angle -36°52' = 2000 \angle -36°52' = 1600 - j1200$

$\mathbf{P_B} = 100 \times 15 \angle 53°8' = (900 + j1200)$; Total = 3,750 $-j5000°$ (as a check)

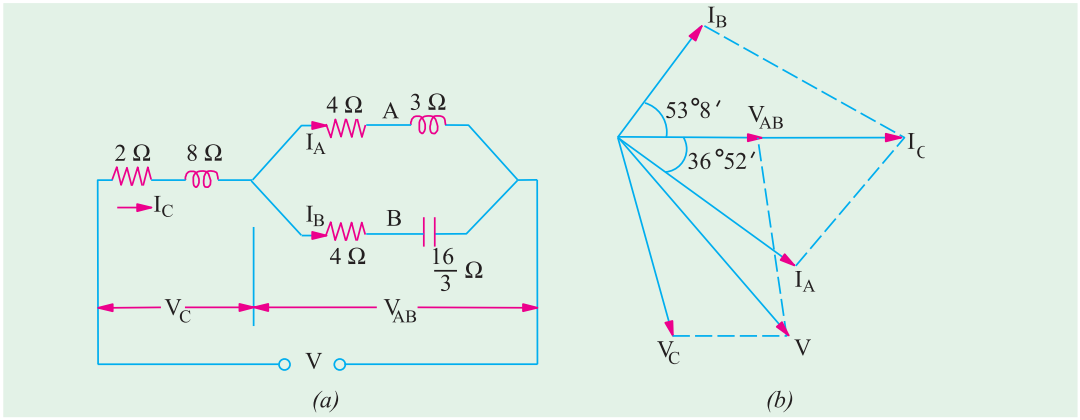


Fig. 14.38

Example 14.33. *Find the value of the power developed in each arm of the series-paralell circuit shown in Fig. 14.39.*

Solution. In order to find the circuit current, we must first find the equivalent impedance of the whole circuit.

$Z_{AB} = (5 + j12) \parallel (-j\,20)$

$$= \frac{(5 + j12)(-j20)}{5 + j12 - j20} = \frac{13 \angle 67.4° \times 20 \angle -90°}{9.43 \angle -58°}$$

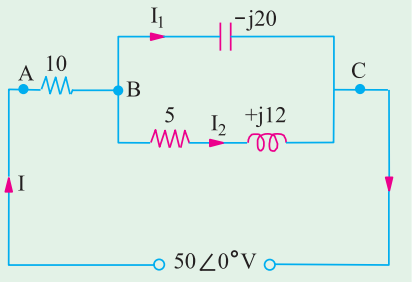


Fig. 14.39

$= 27.57 \angle 35.4° = (22.47 + j\ 15.97)$

$Z_{AC} = (10 + j0) + (22.47 + j15.97) = (32.47 + j14.97)$

$= 36.2 \angle 26.2°$

$$I = \frac{V}{Z} = \frac{50 \angle 0°}{36.2 \angle 26.2°} = 1.38 \angle -26.2° A$$

Power developed in 10 Ω resistor $= I^2 R = 1.38^2 \times 10 =$ **19 W.**

Potential difference across 10 Ω resistor is

$IR = 1.38 \angle -26.2° \times 10 = 13.8 \angle -26.2° = (12.38 - j\ 6.1)$

V_{BC} = supply voltage –drop across 10 Ω resistor

$= (50 + j0) - (12.38 - j6.1) = (37.62 + j6.1) = 38.1 \angle 9.21°$

$$I_2 = \frac{V_{BC}}{(5 + j12)} = \frac{38.1 \angle 9.21°}{13 \angle 67.4°} = 2.93 \angle -58.2°$$

Power developed $= I_2^2 \times 5 = 2.93^2 \times 5 =$ **43 W**

No power is developed in the capacitor branch because it has no resistance.

Example 14.34. *In the circuit shown in Fig. 14.40 determine the voltage at a frequency of 50 Hz to be applied across AB in order that the current in the circuit is 10 A. Draw the phasor diagram.* **(Elect. Engg. & Electronics Bangalore Univ.)**

Solution. $X_{L1} = 2\pi \times 50 \times 0.05 = 15.71\ \Omega$; $X_{L2} = -2\pi \times 50 \times 0.02 = 6.28\ \Omega$,

$X_C = 1/2\pi \times 50 \times 400 \times 10^{-6} = 7.95\ \Omega$

$\mathbf{Z_1} = R_1 + jX_{L1} = 10 + j15.71 = 18.6 \angle 57°33'$

$\mathbf{Z_2} = R_2 + jX_{L2} = 5 + j6.28 = 8 \angle 51°30'$

$\mathbf{Z_3} = R_3 - jX_C = 10 - j7.95 = 12.77 \angle -38°30'$

$\mathbf{Z_{BC}} = Z_2 \,||\, Z_3 = (5 + j6.28) \,||\, (10 - j7.95) = 6.42 + j2.25 = 6.8 \angle 19°18'$

$Z = Z_1 + Z_{BC} = (10 + j15.71) + (6.42 + j2.25) = 16.42 + j17.96 = 24.36 \angle 47°36'$

Let I = 10∠0°; ∴ $\mathbf{V} = IZ = 10\angle 0° \times 24.36\angle 47°36 = 243.6\angle 47°36'$

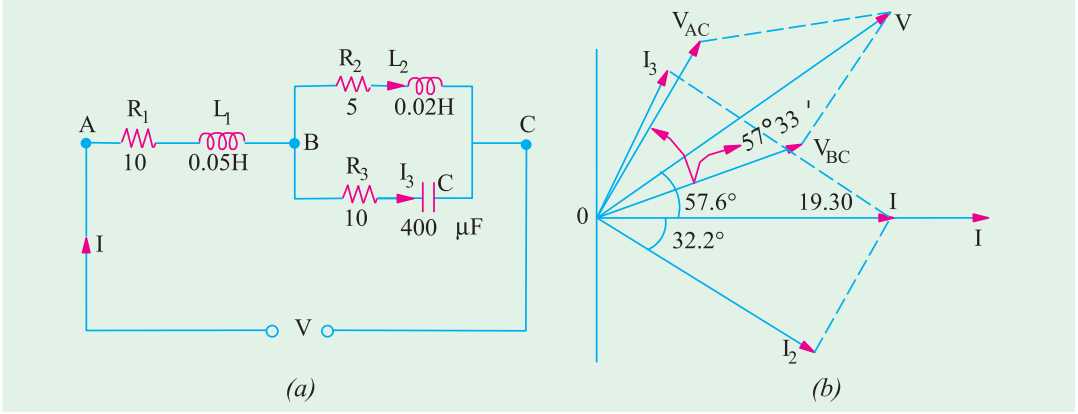


Fig. 14.40

$$V_{BC} = IZ_{BC} = 10 \angle 0° \times 6.8 \angle 19°18' = 68 \angle 19°18' \ ; I_2 = \frac{V_{BC}}{Z_2} = \frac{68 \angle 19°18'}{8 \angle 51°31'} = 8.5 \angle -32°12'$$

$$I_3 = \frac{V_{BC}}{Z_3} = \frac{68\angle 19°18'}{12.77\angle -38°30'} = 5.32\angle 57°48'\,;\; V_{AC} = IZ_1 = 10\angle 0° \times 18.6\angle 57°33' = 186\angle 57°33'$$

The phasor diagram is shown in Fig. 14.36 (*b*).

Example 14.35. *Determine the average power delivered to each of the three boxed networks in the circuit of Fig. 14.41.* **(Basic Circuit Analysis Osmania Univ. Jan/Feb 1992)**

Solution. $Z_1 = 6 - j8 = 10\angle -53°13°$; $Z_2 = 2 + j14 = 14.14\angle 81.87°$; $Z_3 = 6 - j8 = 10\angle -53.13°$

$$Z_{23} = \frac{Z_2 Z_3}{Z_2 + Z_3} = 14.14\angle -8.13° = 14 - j2$$

Drop across two parallel impedances is given by

$$V_{23} = 100\,\frac{14 - j2}{(6 - j8) + (14 - j2)} = 63.2\angle 18.43° = 60 + j20$$

$$V_1 = 100\,\frac{10\angle -53.13°}{6 - j8 + (14 - j2)} = 47.7\angle -26.57° = 40 - j20$$

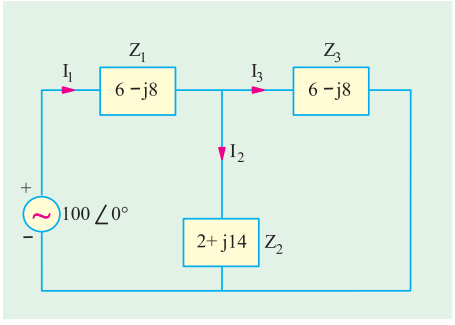


Fig. 14.41

$$I_1 = \frac{44.7\angle -26.57°}{10\angle -53.13°} = 4.47\angle 26.56°$$

$$I_2 = \frac{63.2\angle 18.43°}{14.14\angle 81.87°} = 4.47\angle -63.44°$$

$$I_3 = \frac{63.2\angle 18.43°}{10\angle -53.13°} = 6.32\angle 71.56°$$

$P_1 = V_1 I_1 \cos\phi_1 = 44.7 \times 4.47 \times \cos 53.13° = 120$ W

$P_2 = V_2 I_2 \cos\phi_2 = 63.2 \times 4.47 \times \cos 81.87° = 40$ W;

$P_3 = V_3 I_3 \cos\phi_3 = 63.2 \times 6.32 \times \cos 53.13° =$ **240 W,** Total = 400 W

As a check, power delivered by the 100-V source is,

$P = VI_1 \cos\phi = 100 \times 4.47 \times \cos 26.56° = 400$ W

Example 14.36. *In a series-parallel circuit of Fig. 14.42 (a), the parallel branches A and B are in series with C. The impedances are $Z_A = (4 + j3)$, $Z_B = (10 - j7)$ and $Z_C = (6 + j5)\ \Omega$.*

If the voltage applied to the circuit is 200 V at 50 Hz, calculate : (a) current I_A, I_B and I_C; (b) the total power factor for the whole circuit.

Draw and explain complete vector diagram.

Solution. $\mathbf{Z_A} = 4 + j3 = 5\angle 36.9°$; $\mathbf{Z_B} = 10 - j7 = 12.2\angle -35°$; $\mathbf{Z_C} = 6 + j5 = 7.8\angle 39.8°$

$$\mathbf{Z_{AB}} = \frac{\mathbf{Z_A Z_B}}{\mathbf{Z_A + Z_B}} \quad 5\ \ 36.9\ \ 12.2\ \ 5 \quad 14\ \ j4 \quad \frac{61\ \ 9}{14.56} \quad 4.19\ \ 9\ \ 4\ \ j1.3$$

$$\mathbf{Z} = \mathbf{Z_C} + \mathbf{Z_{AB}} = (6 + j5) + (4 + j1.3) = 10 + j6.3 = 11.8\angle 32.2°$$

Let $\mathbf{V} = 200\angle 0°$; $\mathbf{I_C} = (\mathbf{V}/\mathbf{Z}) = (200/11.8)\angle 32.2° = 16.35\angle -32.2°$

$$\mathbf{I_A} = \mathbf{I_C} \cdot \frac{\mathbf{Z_B}}{\mathbf{Z_A + Z_B}} \quad 16.35\ \ 32.2\ \ \frac{12.2\ \ 35°}{14.56\ \ 16°} \quad 13.7$$

$$\mathbf{I_B} = \mathbf{I_C} \cdot \frac{\mathbf{Z_A}}{\mathbf{Z_A + Z_B}} \quad 16.35\ \ 32.2\ \ \frac{36.9}{14.56} \quad 5.7\ \ 20\ 7$$

The phase angle between V and total circuit current I_C is 32.2°. Hence p.f. for the whole circuit is = cos 32.2° = **0.846 (lag)**

For drawing the phasor diagram of Fig. 14.42 (*b*) following quantities have to be calculated :

$$\mathbf{V_C = I_C Z_C} = 16.35 \angle -32.2° \times 7.8 \angle 39.8° = 127.53 \angle 7.6°$$

$$\mathbf{V_{AB} = I_C Z_{AB}} = 16.35 \angle -32.2° \times 4.19 \angle 17.9° = 18.5 \angle -14.3°$$

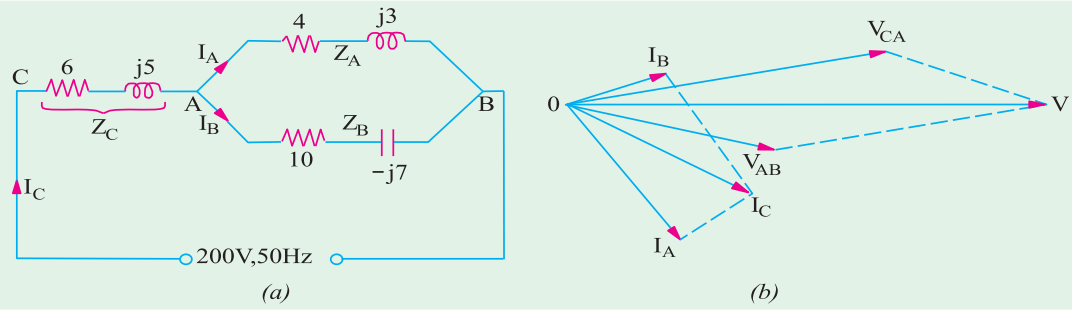


Fig. 14.42

The circuit and phasor diagrams are shown in Fig. 143.38.

Example 14.37. *A fluorescent lamp taking 80 W at 0.7 power factor lagging from a 230-V 50-Hz supply is to be corrected to unity power factor. Determine the value of the correcting apparatus required.*

Solution. Power taken by the 80-W lamp circuit can be found from the following equation,

$$230 \times I \times 0.7 = 80 \quad \therefore I = 80/230 \times 0.7 = 0.5 \text{ A}$$

Reactive component of the lamp current is $= I \sin \phi = 0.5\sqrt{1 - 0.7^2} = 0.357$ A

The power factor of the lamp circuit may be raised to unity by connecting a suitable capacitor across the lamp circuit. The leading reactive current drawn by it should be just equal to 0.357 A. In that case, the two will cancel out leaving only the in-phase component of the lamp current.

$$I_C = 0.357 \text{ A}, \quad X_C = 230/0.357 = \mathbf{645\ \Omega}$$

Now $X_C = I/\omega C \quad \therefore 645 = 1/2\pi \times 50 \times C, \quad C = \mathbf{4.95\ \mu F}$

Example 14.38. *For the circuit shown in Fig. 14.43, calculate I_1, I_2 and I_3. The values marked on the inductance and capacitance give their reactances.* **(Elect. Science-I Allahabad Univ. 1992)**

Solution. $$Z_{BC} = Z_2 \| Z_3 = \frac{(4+j2)(1-j5)}{(3+j2)+(1-j5)} = \frac{14-j18}{5-j3} = \frac{(14-j18)(5+j3)}{5^2+3^2} = 3.65 - j1.41 = 3.9 \angle 21.2°$$

$\mathbf{Z = Z_1 + Z_{BC}} = (2 + j3) + (3.65 - j1.41) = 5.65 + j1.59 = 5.82 \angle 74.3°$

Let $V = 10 \angle 0°$; $I_1 = V/Z = 10 \angle 0°/5.82 \angle 74.3° = \mathbf{1.72 \angle -74.3°}$

$V_{BC} = I_1 Z_{BC} = 1.72 \angle -74.3° \times 3.9 \angle 21.2° = 6.7 \angle -53.1$

Now, $Z_2 = 4 + j2 = 4.47 \angle 63.4°$;

$Z_3 = 1 - j5 = 5.1 \angle -11.3°$

$I_2 = V_{BC}/Z_2 = 6.7 \angle -53.1°/4.47 \angle 63.4° = \mathbf{1.5 \angle 10.3°}$

$I_3 = V_{BC}/Z_3 = 6.7 \angle -53.1°/5.1 \angle -11.3° = \mathbf{1.3 \angle -41.8°}$

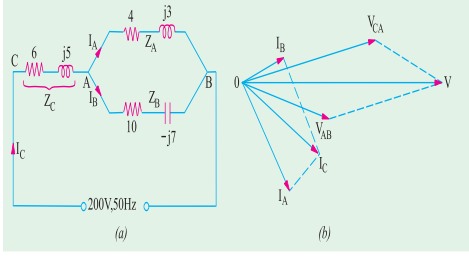


Fig. 14.43

Example 14.39. *A workshop has four 240-V, 50-Hz single-phase motors each developing 3.73 kW having 85% efficiency and operating at 0.8 power factor. Calculate the values 0.9 lagging and* ***(b)*** *0.9 leading. For each case, sketch a vector diagram and find the value of the supply current.*

Solution. Total motor power input = $4 \times 3730/0.85 = 17{,}550$ W

Motor current $I_m = 17{,}550/240 \times 0.8 = 91.3$A

Motor p.f. $= \cos \phi_m = 0.8 \quad \therefore \ \phi_m = \cos^{-1}(0.8) = 36°52'$

(a) Since capacitor does not consume any power, the power taken from the supply remains unchanged after connecting the capacitor. If I_s is current drawn from the supply, then $240 \times I_s \times 0.9 = 17{,}550$

$\therefore \quad I_s = 81.2$ A, $\cos \phi_s = 0.9$; $\phi_s = \cos^{-1}(0.9) = 25°50'$

As seen from vector diagram of Fig. 14.44 (*a*), I_s the vector sum of I_m and capacitor current I_C , $I_C = I_m \sin \phi_m - I_s \sin \phi_s = 91.3 \sin 36°52' - 81.2 \sin 25°50' = 54.8 - 35.4 = 19.4$ A

Now $I_C = \omega VC$

or $19.4 = 240 \times 2\pi \times 50 \times C$

$\therefore \quad C = 257 \times 10^{-6}$F = **257μF**

(b) In this case, I_s leads the supply voltage as shown in Fig. 14.44 (*b*)

$I_C = I_m \sin \phi_m + I_s \sin \phi_s$

$= 54.8 + 35.4 = 90.2$ A

Now $I_C = \omega VC$

$\therefore \quad 90.2 = 240 \times 2\pi \times 50 \times C$

$\therefore \quad C = 1196 \times 10^{-6}$ F = ***1196 μF***

The line or supply current is, as before, 81.2 A (leading)

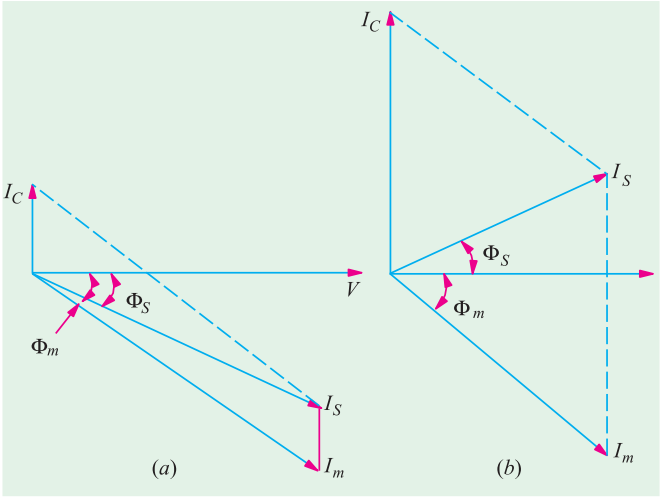


Fig. 14.44

Example 14.40. *The load taken from a supply consists of* ***(a)*** *lamp load 10 kW a unity power factor* ***(b)*** *motor load of 80 kVA at 0.8 power factor (lag) and* ***(c)*** *motor load of 40 kVA at 0.7 power factor leading. Calculate the total load taken from the supply in kW and in kVA and the power factor of the combined load.*

Solution. Since it is more convenient to adopt the tabular method for such questions, we will use the same as illustrated below. We will tabulate the kW, kVA and kVAR (whether leading or lagging) of each load. The lagging kVAR will be taken as negative and leading kVAR as positive.

Load	kVA	cos ϕ	sin ϕ	kW	kVAR
(*a*)	10	1	0	10	0
(*b*)	80	0.8	0.6	64	−48
(*c*)	40	0.7	0.714	28	_ 28/6
			Total	**102**	**−19.2**

Total kW = **102** ; Total kVAR = −19.4 (lagging) ; kVA taken = $\sqrt{102^2 + (-19.4)^2}$ = **103.9**

Power factor = kW/kVA = 102/103.9 = **0.9822 (lag)**

Example 14.41. *A 23-V, 50 Hz, 1-ph supply is feeding the following loads which are connected across it.*

(i) *A motor load of 4 kW, 0.8 lagging p.f.* ***(ii)*** *A rectifier of 3 kW at 0.6 leading p.f.*

(ii) *A lighter-load of 10 kVA at unity p.f.* ***(iv)*** *A pure capacitive load of 8 kVA*

Determine : Total kW, Total kVAR, Total kVA **(I BE Nagpur University Nov. 1999)**

Solution.

S. No.	Item	kW	P.f	kVA	kVAR	I	I_s	I_r
1	Motor	4	0.8 lag	5	3 –ve, Lag	21.74	17.4	13.04 Lag (-)
2	Rectifier	3	0.6 Lead	5	4 + ve, Lead	21.74	13.04	17.4 Lead (+)
3	Light-Load	10	1.0	10	zero	43.48	43.48	zero
4	Capacitive Load	Zero	0.0 Lead	8	8 + ve Lead	34.8	zero	34.8 Lead (+)
	Toal	17	-	Phasor Addition required	+ 9 + ve Lead	Phasor addition required	73.92	39.16 Lead (+)

Performing the calculations as per the tabular entries above, following answers are obtained

Total kW = 17

Total kVAR = + 9, leading

Total $kVA = \sqrt{17^2 + 9^2} = 19.2354$

Overall circuit p.f. $= \frac{kW}{kVA} \quad \frac{17}{19.2354} = 0.884$ leading

Overall Current $= \frac{19235.4}{230} = 83.63$ amp

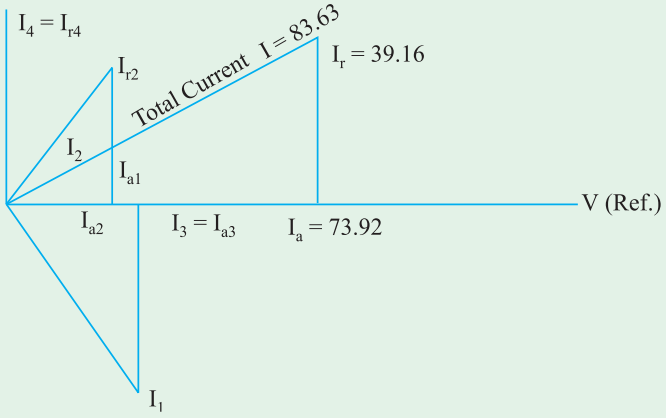


Fig. 14.45 Phasor diagram for currents corresponding to load

Example 14.42. *A three phase induction Motor delivers an output of 15 h.p. at 83 % efficiency. The motor is d (delta) connected and is supplied by 440 V, three phase, 50 Hz supply. Line current drawn by motor is 22.36 Amp. What is motor power factor ?*

It is now decided to improve the power factor to 0.95 lag by connecting three similar capacitors in delta across the supply terminals. Determine the value of the capacitance of each capacitor.

[Note : 1 h.p. = 745 watts] **(Bombay University, 2000)**

Solution. Power factor $= \frac{15 \times 745}{0.83 \times 1.732 \times 440 \times 22.36} = 0.79$, Lagging

$\phi = \cos^{-1} 0.79 = 37.8°$

$I_1 = I_{ph} = 22.36/1.732 = 12.91$ amp

Active Current $I_a = I_1 \cos \phi_1 = 12.91 \times 0.79 = 10.2$ amp

New Power-factor $\cos \phi_2 = 0.95, \phi_2 = 18.2°$

$I_2 = 10.2/0.95 = 10.74$ amp

Capacitive current per phase $= I_1 \sin \phi_1 - I_2 \sin \phi_2$

$= 4.563$

Capacitive reactance per phase $= 440/4.563 = 96.43$ ohms

Capacitance per phase $= 33\ \mu f$

These have to be delta-connected



Three phase induction motor

Example 14.43. *Draw admittance triangle between the terminals AB of Fig 14.46 (a) labelling its sides with appropriate values and units in case of :*

(i) $X_L = 4$ *and* $X_C = 8$ ***(ii)*** $X_L = 10$ *and* $X_C = 5$ **[Bombay University 1999]**

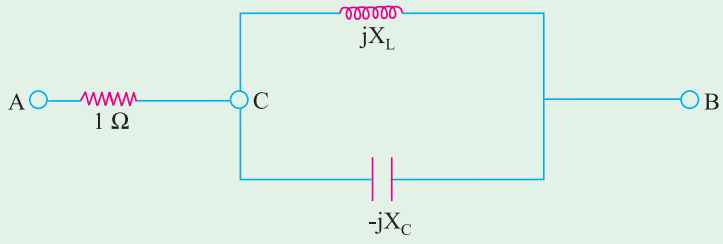


Fig. 14.46 (*a*)

Solution. *(i)* $X_L = 4\ \Omega,\ X_C = 8\ \Omega$

$$Z_{CB} = \frac{jX_L(-jX_C)}{j(X_L - X_C)} = j8$$

$Z_{AB} = 1 + j\,8$ ohms

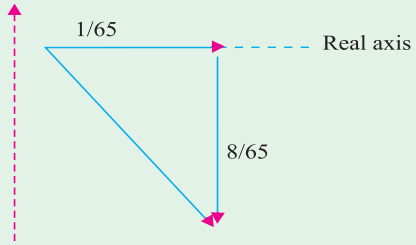
$Y_{AB} = 1/Z_{AB} = (1/65) - j\,(8/65)$ mho

(ii) $X_L = 10\ \Omega,\ X_C = 5\ \Omega$

$$Z_{CB} = \frac{j10 \times (-j5)}{j5} = -j10$$

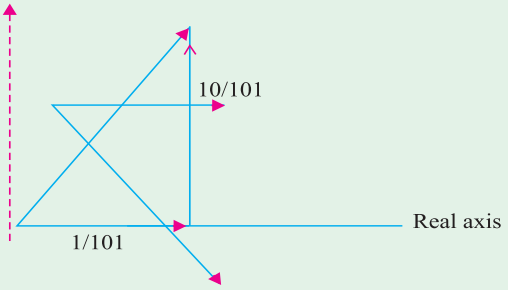
$Z_{AB} = 1 - j\ 10$ ohms

$Y_{AB} = (1/101) + j\,(10/101)$ mho



(*i*) Admittance triangle for first case

Fig. 14.46 (*b*)



(*ii*) Admittance triangle for second case

Fig. 14.46 (*c*)

Example 14.44. *For the circuit in Fig. 14.47 (a), given that L = 0.159 H*

$C = 0.3183\ mf$

$I_2 = 5 \angle 60°\ A$

$V_1 = 250 \angle 90°$ *volts.*

Find :-

(i) *Impedance Z_1 with its components.*

(ii) *Source voltage in the form of $V_m \cos(\omega t + \phi)$.*

(iii) *Impedance Z_2 with its components so that source p.f. is unity, without adding to the circuit power loss.*

(iv) *Power loss in the circuit*

(v) *Draw the phasor diagram.*

(Bombay University 1997)

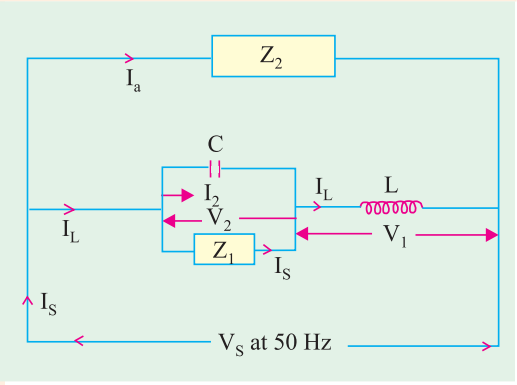


Fig. 14.47 (*a*)

Solution. $X_L = 314 \times 0.159 = 50$ ohms

$X_C = 1/(314 \times 0.3183 \times 10^{-3}) = 10$ ohms

$I_L = V_1/jX_L = (250 \angle 90°) / (50 \angle 90°) = 5 \angle 0°$ amps

$V_2 = -jI_2 X_C = (5 \angle 60°) \times (10 \angle -90°) = 50 \angle -30°$ volts $= 43.3 - j\ 25$ volts

$I_L = I_L - I_2 = 5 \angle 0° - 5 \angle 60° = 5 + j0 - 5\ (0.5 + j0.866)$

$= 2.5 - j4.33 = 5 \angle -60°$

(a) $Z_1 = V_2/I_1 = (50 \angle -30°) / 5 \angle -60° = 10 \angle +30°$

$= 10\ (\cos 30° + j \sin 30°) = 8.66 + j5$

(b) $V_s = V_1 + V_2 = 0\ j250 + 43.3 - j25 = 43.3 + j225$

$= 229.1 \angle 79.1°$ volts

V_s has a peak value of $(229.1 \times \sqrt{2} =)$ 324 volts

$V_s = 324 \cos(314\ t - 10.9°)$, taking V_1 as reference

or $V_s = 325 \cos(314\ t - 79.1°)$, taking I_L as reference.

(c) Source Current must be at unity P.f., with V_s

Component of I_L in phase with $V_s = 5 \cos 79.1° = 0.9455$ amp

Component of I_L in quadrature with V_S (and is lagging by 90°)

$= 5.00 \times \sin 79.1° = 4.91$ amp

Z_2 must carry I_a such that no power loss is there and I_S is at unity P.f. with V_s.

I_a has to be capacitive, to compensate, in magnitude, the quadrature component of I_L

$|I_a| = 4.91$ amp

$|Z_2| = V_s / |I_a| = 229.1/4.91 = 46.66$ ohms

Corresponding capacitance, $C_2 = 1 / (46.66 \times 314) = 68.34\ \mu F$

(d) Power-loss in the circuit $= I^2_1 \times 8.66 = 216.5$ watts or power $= V_s \times$ component of I_L in phase with $V_s = 299.1 \times 0.9455 = 216.5$ watts

(e) Phasor diagram is drawn in Fig. 14.47 (*b*)

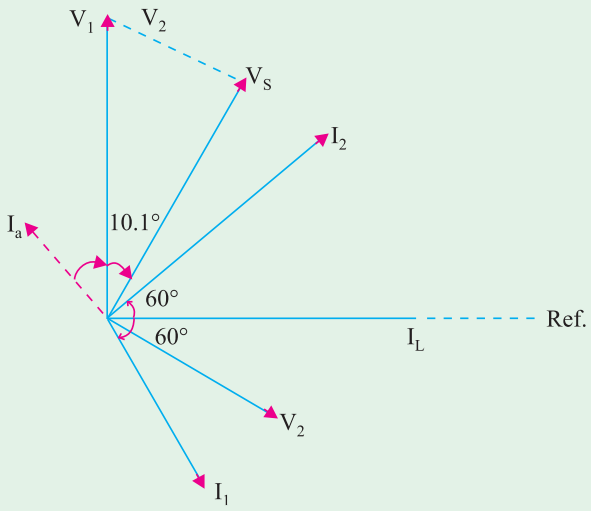


Fig. 14.47 (*b*) Phasor diagram

Tutorial Problem No. 14.1

1. A capacitor of 50 μF capacitance is connected in parallel with a reactor of 22 Ω resistance and 0.07 henry inductance across 200-V, 50-Hz mains. Calculate the total current taken. Draw the vector diagram in explanation. **[4.76 A lagging, 17°12′] (*City & Guilds, London*)**
2. A non-inductive resistor is connected in series with a capacitor of 100 μF capacitance across 200-V, 50-Hz mains. The p.d. measured across the resistor is 150 V. Find the value of resistance and the value of current taken from the mains if the resistor were connected in parallel-with the capacitor instead of in series. **[R = 36.1 Ω; 8.37 Ω] (*City & Guilds, London*)**
3. An impedance of $(10 + j15)$ Ω is connected in parallel with an impedance of $(6 - j8)$Ω. The total current is 15 A. Calculate the total power. **[2036 W] (*City & Guilds, London*)**
4. The load on a 250-V supply system is : 12 A at 0.8 power factor lagging ; 10 A at 0.5 power factor lagging ; 15 A at unity power factor ; 20 A at 0.6 power factor leading. Find (*i*) the total lead in kVA and (*ii*) its power factor. **[(*i*) 10.4 kVA (*ii*) 1.0] (*City & Guilds, London*)**
5. A voltage having frequency of 50 Hz and expressed by $\mathbf{V} = 200 + j100$ is applied to a circuit consisting of an impedance of 50 $\angle 30°$ Ω in parallel with a capacitance of 10 μF. Find (*a*) the reading on a ammeter connected in the supply circuit (*b*) the phase difference between the current and the voltage. **[(*a*) 4.52 (*b*) 26.6° lag] (*London University*)**
6. A voltage of 200° $\angle 30°$ V is applied to two circuits *A* and *B* connected in parallel. The current in *A* is 20 $\angle 60°$ A and that in *B* is 40 $\angle 30°$ A. Find the kVA and kW in each branch circuit and the main circuit. Express the current in the main circuit in the form $A + jB$. **(*City & Guilds, London*)**
   [$kVA_A = 4$, $kVA_B = 8$, $kV_A = 12$, $kW_A = 3.46$, $kW_B = 4$, kW = 7.46, I = 44.64 $-j$ 2.68] (*City & Guilds, London*)
7. A coil having an impedance of $(8 + j6)$ Ω is connected across a 200-V supply. Express the current in the coil in (*i*) polar and (*ii*) rectangular co-ordinate forms.
   If a capacitor having a susceptance of 0.1 S is placed in parallel with the coil, find (*iii*) the magnitude of the current taken from the supply. **[(*i*) 20 ∠ 36.8°A (*ii*) 16 –j12 A (*iii*) 17.9 A] (*City & Guilds, London*)**
8. A coil-A of inductance 80 mH and resistance 120 Ω, is connected to a 230-V, 50 Hz single-phase supply. In parallel with it in a 16 μF capacitor in series with a 40 Ω non-inductive resistor *B*. Determine (*i*) the power factor of the combined circuit and (*ii*) the total power taken from the supply. **[(*i*) 0.945 lead (*ii*) 473 W] (*London University*)**
9. A choking coil of inductance 0.08 H and resistance 12 ohm, is connected in parallel with a capacitor

of 120 μF. The combination is connected to a supply at 240 V, 50 Hz Determine the total current from the supply and its power factor. Illustrate your answers with a phasor diagram.

[3.94 A, 0.943 lag] (*London University*)

10. A choking coil having a resistance of 20 Ω and an inductance of 0.07 henry is connected with a capacitor of 60 μF capacitance which is in series with a resistor of 50 Ω. Calculate the total current and the phase angle when this arrangement is connected to 200-V, 50 Hz mains.

[7.15 A, 24°39′ lag] (*City & Guilds, London*)

11. A coil of resistance 15 Ω and inductance 0.05 H is connected in parallel with a non-inductive resistance of 20 Ω. Find (*a*) the current in each branch (*b*) the total current (*c*) the phase angle of whole arrangement for an applied voltage of 200 V at 50 Hz. **[9.22 A ; 10A ; 22.1°]**

12. A sinusoidal 50-Hz voltage of 200 V (r.m.s) supplies the following three circuits which are in parallel : (*a*) a coil of inductance 0.03 H and resistance 3 Ω (*b*) a capacitor of 400 μF in series with a resistance of 100 Ω (*c*) a coil of inductance 0.02 H and resistance 7 Ω in series with a 300 μF capacitor. Find the total current supplied and draw a complete vector diagram. **[29.4 A] (*Sheffield Univ. U.K.*)**

13. A 50-Hz, 250-V single-phase power line has the following loads placed across it in parallel : 4 kW at a p.f. of 0.8 lagging ; 6 kVA at a p.f. of 0.6 lagging; 5 kVA which includes 1.2 kVAR leading. Determine the overall p.f. of the system and the capacitance of the capacitor which, if connected across the mains would restore the power factor to unity. **[0.844 lag ; 336 μF]**

14. Define the terms admittance, conductance and susceptance with reference to alternating current circuits. Calculate their respective values for a circuit consisting of resistance of 20Ω, in series with an inductance of 0.07 H when the frequency is 50 Hz. **[0.336 S, 0.0226 S, 0.0248 S] (*City & Guilds, London*)**

15. Explain the terms admittance, conductance, susceptance as applied to a.c. circuits. One branch *A*, of a parallel circuit consists of a coil, the resistance and inductance of which are 30 Ω and 0.1 *H* respectively. The other branch *B*, consists of a 100 μF capacitor in series with a 20 Ω resistor. If the combination is connected 240-V, Hz mains, calculate (*i*) the line current and (*ii*) the power. Draw to scale a vector diagram of the supply current and the branch-circuit currents.

[(*i*) 7.38 A (*ii*) 1740 W] (*City & Guilds, London*)

16. Find the value of capacitance which when placed in parallel with a coil of resistance 22 Ω and inductance of 0.07 H, will make it resonate on a 50-Hz circuit. **[72.33 μF] (*City & Guilds, London*)**

17. A parallel circuit has two branches. Branch A consists of a coil of inductance 0.2 H and a resistance of 15 Ω ; branch B consists of a 30 mF capacitor in series with a 10 Ω resistor. The circuit so formed is connected to a 230-V, 50-Hz supply. Calculate (*a*) current in each branch (*b*) line current and its power factor (*c*) the constants of the simplest series circuit which will take the same current at the same power factor as taken by the two branches in parallel.

[3.57 A, 2.16 A ; 1.67 A, 0.616 lag, 8.48 Ω, 0.345 H]

18. A 3.73 kW, 1-phase, 200-V motor runs at an efficiency of 75% with a power factor of 0.7 lagging. Find (*a*) the real input power (*b*) the kVA taken (*c*) the reactive power and (*d*) the current. With the aid of a vector diagram, calculate the capacitance required in parallel with the motor to improve the power factor to 0.9 lagging. The frequency is 50 Hz.

[4.97 kW ; 7.1 kVA ; 5.07 kVAR ; 35.5 A ; 212μF]

19. The impedances of two parallel circuits can be represented by $(20 + j15)$ and $(1 - j60)$ Ω respectively. If the supply frequency is 50 Hz, find the resistance and the inductance or capacitance of each circuit. Also derive a symbolic expression for the admittance of the combined circuit and then find the phase angle between the applied voltage and the resultant current. State whether this current is leading or lagging relatively to the voltage. **[20 Ω; 0.0478 H; 10 Ω; 53 μF; (0.0347 –j 0.00778)S; 12°38′ lag]**

20. One branch *A* of a parallel circuit consists of a 60-μF capacitor. The other branch *B* consists of a 30 Ω resistor in series with a coil of inductance 0.2 *H* and negligible resistance. A 140 Ω resistor is connected in parallel with the coil. Sketch the circuit diagram and calculate (*i*) the current in the 30 Ω resistor and (*ii*) the line current if supply voltage is 230-V and the frequency 50 Hz.

[(*i*) 3.1 ∠ 44° (*ii*) 3.1 ∠ 45° A]

21. A coil having a resistance of 45 Ω and an inductance of 0.4 *H* is connected in parallel with a capacitor having a capacitance of 20 μ*F* across a 230-V, 50-Hz system. Calculate (*a*) the current taken from the

supply (*b*) the power factor of the combination and (*c*) the total energy absorbed in 3 hours.
[(*a*) 0.615 (*b*) 0.951 (*c*) 0.402 kWh] (*London University*)

22. A series circuit consists of a resistance of 10 Ω and reactance of 5 Ω. Find the equivalent value of conductance and susceptance in parallel. **[0.08 S, 0.04 S]**

23. An alternating current passes through a non-inductive resistance R and an inductance L in series. Find the value of the non-inductive resistance which can be shunted across the inductance without altering the value of the main current. **[$\omega^2L^2/2R$] (*Elec. Meas. London Univ.*)**

24. A p.d. of 200 V at 50 Hz is maintained across the terminals of a series-parallel circuit, of which the series branch consists of an inductor having an inductance of 0.15 H and a resistance of 30 Ω, one parallel branch consists of 100-μF capacitor and the other consists of a 40-Ω resistor.
Calculate (*a*) the current taken by the capacitor (*b*) the p.d. across the inductor and (*c*) the phase difference of each of these quantities relative to the supply voltage. Draw a vector diagram representing the various voltage and currents.
[(*a*) 29.5 A (*b*) 210 V (*c*) 7.25°, 26.25°] (*City & Guilds, London*)

25. A coil (*A*) having an inductance of 0.2 *H* and resistance of 3.5 Ω is connected in parallel with another coil (*B*) having an inductance of 0.01 H and a resistance of 5 Ω. Calculate (*i*) the current and (*ii*) the power which these coils would take from a 100-V supply system having a frequency of 50-Hz. Calculate also (*iii*) the resistance and (*iv*) the inductance of a single coil which would take the same current and power. **[(*i*) 29.9 A (*ii*) 2116 W (*ii*) 2.365 Ω (*iv*) 0.00752 H] (*London Univ.*)**

26. Two coils, one (*A*) having $R = 5$ Ω, $L = 0.031$ *H* and the other (*B*) having $R = 7$ Ω ; $L = 0.023$ *H*, are connected in parallel to an a.c. supply at 200 *V*, 50 Hz. Determine (*i*) the current taken by each coil and also (*ii*) the resistance and (*iii*) the inductance of a single coil which will take the same total current at the same power factor as the two coils in parallel.
[(*i*) $I_A = 18.28$ A, $I_B = 19.9$ A (*ii*) 3.12 Ω (*iii*) 0.0137 H] (*London Univ.*)

27. Two coils are connected in parallel across 200-V, 50-Hz mains. One coil takes 0.8 kW and 1.5 kVA and the other coil takes 1.0 kW and 0.6 kVAR. Calculate (*i*) the resistance and (*ii*) the reactance of a single coil which would take the same current and power as the original circuit.
[(*i*) 10.65 Ω (*ii*) 11.08 Ω] (*City & Guilds, London*)

28. An a.c. circuit consists of two parallel branches, one (*A*) consisting of a coil, for which $R = 20$ Ω and $L = 0.1$ H and the other (*B*) consisting of a 40-Ω non-inductive resistor in series with 60-μF capacitor. Calculate (*i*) the current in each branch (*ii*) the line current (*iii*) the power, when the circuit is connected to 230-V mains having a frequency of 50 Hz. Calculate also (*iv*) the resistance and (*b*) the inductance of a single coil which will take the same current and power from the supply.
[(*i*) 6.15 A, 3.46 A (*ii*) 5.89 (*iii*) 1235 W (*iv*) 35.7 Ω (*b*) 0.0509 H] (*London Univ.*)

29. One branch (*A*) of a parallel circuit, connected to 230-V, 50-Hz mains consists of an inductive coil ($L = 0.15$ H, $R = 40$ Ω) and the other branch (*B*) consists of a capacitor ($C = 50$ μF) in series with a 45 Ω resistor. Determine (*i*) the power taken (*ii*) the resistance and (*iii*) the reactance of the equivalent series circuit. **[(*i*) 946 W (*ii*) 55.4 Ω (*iii*) 4.6 Ω] (*London Univ.*)**

14.9. Resonance in Parallel Circuits

We will consider the practical case of a coil in parallel with a capacitor, as shown in Fig. 14.48. Such a circuit is said to be in electrical resonance when the reactive (or wattless) component of line current becomes zero. The frequency at which this happens is known as ***resonant*** frequency.

The vector diagram for this circuit is shown in Fig. 14.48 (*b*).

Net reactive or wattless component
$= I_C - I_L \sin \phi_L$

As at resonance, its value is zero, hence $I_C - I_L \sin \phi_L = 0$ or $I_L \sin \phi_L = I_C$

Now, $I_L = V/Z$; $\sin \phi_L = X_L$ and $I_C = V/X_C$

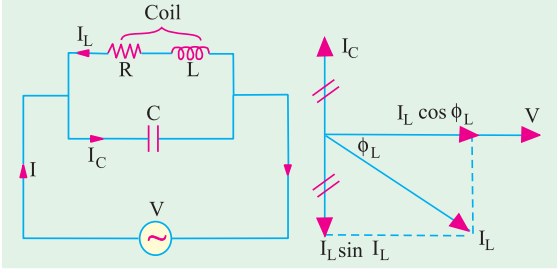


Fig. 14.48

Hence, condition for resonance becomes

$$\frac{V}{Z}\times\frac{X_L}{Z}=\frac{V}{X_C} \quad \text{or} \quad X_L\times X_C=Z^2$$

Now, $X_L=\omega L$, $X_C=\dfrac{1}{\omega C}$

$$\therefore \quad \frac{\omega L}{\omega C}=Z^2 \quad \text{or} \quad \frac{L}{C}=Z^2 \qquad \ldots (i)$$

$$\text{or} \quad \frac{L}{C}=R^2+X_L{}^2=R^2+(2\pi f_0 L)^2$$

$$\text{or} \quad (2\pi f_0 L)^2=\frac{L}{C}-R^2 \quad \text{or} \quad 2\pi f_0=\sqrt{\frac{1}{LC}-\frac{R^2}{L^2}} \quad \text{or} \quad f_0=\frac{1}{2}\sqrt{\frac{1}{LC}-\frac{R^2}{L^2}}$$

This is the resonant frequency and is given in Hz, R is in ohm, L is the henry and C is the farad.

If R is the negligible, then $f_0=\dfrac{1}{2\pi\sqrt{(LC)}}$

... same as for series resonance

Current at Resonance

As shown in Fig. 14.41 (*b*), since wattless component of the current is zero, the circuit current is

$$I=I_L\cos\phi_L=\frac{V}{Z}\cdot\frac{R}{Z} \quad \text{or} \quad I=\frac{VR}{Z^2}.$$

Putting the value of $Z^2=L/C$ from (*i*) above, we get $I=\dfrac{VR}{L/C}=\dfrac{V}{L/CR}$

The denominator L/CR is known as the ***equivalent*** or ***dynamic impedance*** of the parallel circuit at resonance. It should be noted that impedance is 'resistive' only. Since current is minimum at resonance, L/CR must, therefore, represent the maximum impedance of the circuit. In fact, parallel resonance is a condition of maximum impedance or minimum admittance.

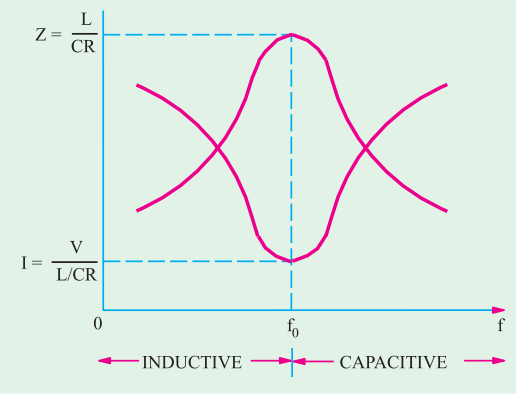


Fig. 14.49

Current at resonance is minimum, hence such a circuit (when used in radio work) is sometimes known as ***rejector*** circuit because it rejects (or takes minimum current of) that frequency to which it resonates. This resonance is often referred to as current resonance also because the current circulating ***between*** the two branches is many times greater than the line current taken from the supply.

The phenomenon of parallel resonance is of great practical importance because it forms the basis of tuned circuits in Electronics.

The variations of impedance and current with frequency are shown in Fig. 14.49. As seen, at resonant frequency, impedance is maximum and equals L/CR. Consequently, current at resonance is minimum and is $= V/(L/CR)$. At off-resonance frequencies, impedance decreases and, as a result, current increases as shown.

Alternative Treatment

$$\mathbf{Y_1}=\frac{1}{R+jX_L}=\frac{R}{R^2+X_L^2}-j\frac{X_L}{R^2+X_L^2}; \quad \mathbf{Y_2}=\frac{1}{-jX_C}=\frac{j}{X_C}$$

$$\mathbf{Y}=\frac{R}{R^2+X_L^2}+j\left(\frac{1}{X_C}-\frac{X_L}{R^2+X_L^2}\right)$$

Now, circuit would be in resonance when *j*-component of the complex admittance is zero *i.e.*

when $\frac{1}{X_C} - \frac{X_L}{R^2 + X_L^2} = 0$ or $\frac{X_L}{R^2 + X_L^2} = \frac{1}{X_C}$

or $X_L X_C = R^2 + X_L^2 = Z^2$ **–as before**

Talking in terms of susceptance, the above relations can be put as under :

Inductive susceptance $B_L = \frac{X_L}{R^2 \quad X_L^2}$; capacitive susceptance $B_C = \frac{1}{X_C}$

Net susceptance $B = (B_C - B_L)$ $\therefore$ $Y = G + j\,(B_C - B_L) = G + jB$.

The parallel circuit is said to be in resonance when $B = 0$.

$\therefore$ $B_C - B_L = 0$ or $\frac{1}{X_C} = \frac{X_L}{R^2 + X_L^2}$

The rest procedure is the same as above. It may be noted that at resonance, the admittance equals the conductance.

14.10. Graphic Representation of Parallel Resonance

We will now discuss the effect of variation of frequency on the susceptance of the two parallel branches. The variations are shown in Fig. 14.50.

(*i*) Inductive susceptance ;

$b = -1/X_L = -1/2\pi f\, L$

It is inversely proportional to the frequency of the applied voltage. Hence, it is represented by a rectangular hyperbola drawn in the fourth quadrant ($\therefore$ it is assumed negative).

(*ii*) Capacitive susceptance ;

$b = 1/X_C = \omega C = 2\pi f\, C$

It increases with increase in the frequency of the applied voltage. Hence, it is represented by a straight line drawn in the first quadrant (it is assumed positive).

(*iii*) Net Susceptance B

It is the difference of the two susceptances and is represented by the dotted hyperbola. At point *A*, net susceptance is zero, hence admittance is minimum (and equal to *G*). So at point *A*, line current is minimum.

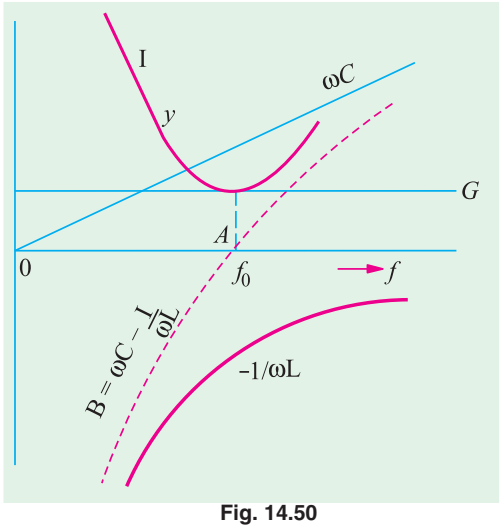


Fig. 14.50

Obviously, below resonant frequency (corresponding to point *A*), inductive susceptance predominates, hence line current lags behind the applied voltage. But for frequencies above the resonant frequency, capacitive susceptance predominates, hence line current leads.3

14.11. Points to Remember

Following points about parallel resonance should be noted and compared with those about series resonance. At resonance.

1. net susceptance is zero *i.e.* $1/X_C = X_L/Z^2$ or $X_L \times X_C = Z^2$ or $L/C = Z^2$
2. the admittance equals conductance
3. reactive or wattless component of line current is zero.
4. dynamic impedance = L/CR ohm.

5. line current at resonance is minimum and = $\frac{V}{L/CR}$ but is in phase with the applied voltage.
6. power factor of the circuit is unity.

14.12. Bandwidth of a Parallel Resonant Circuit

The bandwidth of a parallel circuit is defined in the same way as that for a series circuit. This circuit also has upper and lower half-power frequencies where power dissipated is half of that at resonant frequency.

At bandwidth frequencies, the net susceptance B equals the conductance. Hence, at f_2, $B = B_{C2} - B_{L2} = G$. At f_1, $B = B_{L1} - B_{C1} = G$. Hence, $Y = \sqrt{G^2 + B^2} = \sqrt{2}.G$ and $\phi = \tan^{-1}(B/G) = \tan^{-1}(1) = 45°$.

However, at off-resonance frequencies, $Y > G$ and $B_C \neq B_L$ and the phase angle is greater than zero.

Comparison of Series and Parallel Resonant Circuits

item	*series circuit (R–L–C)*	*parallel circuit (R–L and C)*
Impedance at resonance	Minimum	Maximum
Current at resonance	Maximum = V/R	Minimum = $V/(L/CR)$
Effective impedance	R	L/CR
Power factor at resonance	Unity	Unity
Resonant frequency	$1/2\pi\sqrt{(LC)}$	$\frac{1}{2\pi}\sqrt{\left(\frac{1}{LC} - \frac{R^2}{L^2}\right)}$
It magnifies	Voltage	Current
Magnification is	$\omega L/R$	$\omega L/R$

14.13. Q-factor of a Parallel Circuit

It is defined as ***the ratio of the current circulating between its two branches to the line current drawn from the supply or simply, as the current magnification.*** As seen from Fig. 14.51, the circulating current between capacitor and coil branches is I_C.

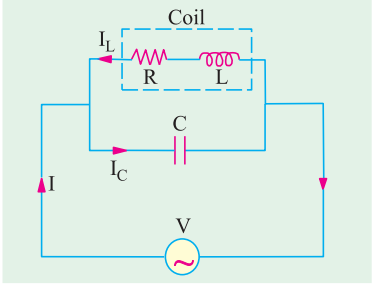


Fig. 14.51

Hence Q-factor = I_C/I

Now $I_C = V/X_C = V/(1/\omega C) = \omega CV$

and $I_C = V/(L/CR)$

$\therefore Q$ – factor = $CV \quad \frac{V}{L/CR} \quad \frac{L}{R} \quad \frac{2\,f_0 L}{R}$

= tan ϕ (same as for series circuit)

where ϕ is the power factor angle of the *coil*.

Now, resonant frequency when R is negligible is,

$$f_0 = \frac{1}{2\pi\sqrt{(LC)}}$$

Putting this value above, we get, Q-factor = $\frac{2\,f_0 L}{R} \quad \frac{1}{2\sqrt{(LC)}} \quad \frac{1}{R}\sqrt{\frac{L}{C}}$

It should be noted that in series circuits, Q-factor gives the voltage magnification, whereas in parallel circuits, it gives the current magnification.

Again, $$Q = 2\pi \frac{\text{maximum stored energy}}{\text{energy dissipated/cycle}}$$

Example 14.45. *A capacitor is connected in parallel with a coil having L = 5.52 mH and R = 10 Ω, to a 100-V, 50-Hz supply. Calculate the value of the capacitance for which the current taken from the supply is in phase with voltage.* **(Elect. Machines, A.M.I.E. Sec B, 1992)**

Solution. At resonance, $L/C = Z^2$ or $C = L/Z^2$

$X_L = 2\pi \times 50 \times 5.52 \times 10^{-3} = 1.734\ \Omega$, $Z^2 = 10^2 + 1.734^2$, $Z = 10.1\ \Omega$

$C = 5.52 \times 10^{-3}/10.1 =$ **54.6 μF**

Example 14.46. *Calculate the impedance of the parallel-turned circuit as shown in Fig. 14.52 at a frequency of 500 kHz and for bandwidth of operation equal to 20 kHz. The resistance of the coil is 5 Ω* **(Circuit and Field Theory, A.M.I.E. Sec. B, 1993)**

Solution. At resonance, circuit impedance is L/CR. We have been given the value of R but that of L and C has to be found from the given data.

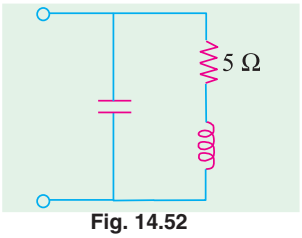


Fig. 14.52

$$BW = \frac{R}{2L}, \quad 20 \quad 10^3 \quad \frac{5}{2 \quad L} \quad \text{or} \quad L = 39\ \mu\text{H}$$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} = \frac{1}{2\pi}\sqrt{\frac{1}{39\times10^{-6}C} - \frac{5^2}{(39\times10^{-6})^2}}$$

$\therefore \quad C = 2.6 \times 10^{-9}$ F, $Z = L/CR = 39 \times 10^{-6}/2.6 \times 10^{-9} \times 5 =$ **$3 \times 10^3\ \Omega$**

Example 14.47. *An inductive circuit of resistance 2 ohm and inductance 0.01 H is connected to a 250-V, 50-Hz supply. What capacitance placed in parallel will produce resonance ?*

Find the total current taken from the supply and the current in the branch circuits.

(Elect. Engineering, Kerala Univ.)

Solution. As seen from Art. 14.9, at resonance $C = L/Z^2$

Now, $R = 2\ \Omega$, $X_L = 314 \times 0.01 = 3.14\ \Omega$; $Z = \sqrt{2^2 + 3.14^2} = 3.74\ \Omega$

$C = 0.01/3.74^2 = 714 \times 10^{-6}$ F = **714 μF** ; $I_{RL} = 250/3.74 =$ **66.83 A**

$\tan \phi_L = 3.14/2 = 1.57$; $\phi_L = \tan^{-1}(1.57) = 57.5°$

Hence, current in R-L branch lags the applied voltage by 57.5°

$$\therefore \quad I_C = \frac{V}{X_C} \quad \frac{V}{1/\ C} = \omega VC = 250 \times 314 \times 714 \times 10^{-6} = \textbf{56.1 A}$$

This current leads the applied voltage by 90°.

Total current taken from the supply under resonant condition is

$$I = I_{RL} \cos \phi_L = 66.83 \cos 57.5° = 66.83 \times 0.5373 = \textbf{35.9 A} \left(\text{or } \boldsymbol{I = \frac{V}{L/CR}}\right)$$

Example 14.48. *Find active and reactive components of the current taken by a series circuit consisting of a coil of inductance 0.1 henry and resistance 8 Ω and a capacitor of 120 μF connected to a 240-V, 50-Hz supply mains. Find the value of the capacitor that has to be connected in parallel with the above series circuit so that the p.f. of the entire circuit is unity.*

(Elect. Technology, Mysore Univ.)

Solution. $X_L = 2\pi \times 50 \times 0.1 = 31.4\ \Omega$, $X_C = 1/\omega C = 1/2\pi \times 50 \times 120 \times 10^{-6} = 26.5\ \Omega$

$X = X_L - X_C = 31.4 - 26.5 = 5\ \Omega$, $Z = \sqrt{(8^2 + 5^2)} = 9.43\ \Omega$; $I = V/Z = 240/9.43 = 25.45$ A

$\cos \phi = R/Z = 8/9.43 = 0.848$, $\sin \phi = X/Z = 5/9.43 = 0.53$

active component of current $= I \cos \phi = 25.45 \times 0.848 = 21.58$ A

reactive component of current $= I \sin \phi = 25.45 \times 0.53 = 13.49$ A

Let a capacitor of capacitance C be joined in parallel across the circuit.

$$\mathbf{Z_1} = R + jX = 8 + j5\ ; \ \mathbf{Z_2} = -jX_C\ ;$$

$$\mathbf{Y} = \mathbf{Y_1} + \mathbf{Y_2} = \frac{1}{\mathbf{Z}_1} \quad \frac{1}{\mathbf{Z}_2} \quad \frac{1}{8 \quad j5} \quad \frac{1}{jX_C}$$

$$= \frac{8 - j5}{89} + \frac{j}{X_C} = 0.0899 - j0.056 + \frac{j}{X_C} = 0.0899 + j\,(1/X_C - 0.056)$$

For p.f. to be unit , the j-component of **Y** must be zero.

$$\therefore \quad \frac{1}{X_C} - 0.056 = 0 \quad \text{or} \quad 1/X_C = 0.056 \quad \text{or} \quad \omega C = 0.056 \quad \text{or} \quad 2\pi \times 50C = 0.056$$

$$\therefore \quad C = 0.056/100\pi = 180 \times 10^{-6} \text{ F} = \mathbf{180\ \mu F}$$

Example 14.49. *A coil of resistance 20 Ω and inductance 200 μH is in parallel with a variable capacitor. This combination is in series with a resistor of 8000 Ω. The voltage of the supply is 200 V at a frequency of 10^6 H_z. Calculate*

(i) *the value of C to give resonance* ***(ii)*** *the Q of the coil*

(iii) *the current in each branch of the circuit at resonance.*

(Similar Question : Bombay Univ. 2000)

Solution. The circuit is shown in Fig. 14.53.

$X_L = 2\pi fL = 2\pi \times 10^6 \times 200 \times 10^{-6} = 1256\ \Omega$

Since coil resistance is negligible as compared to its reactance, the resonant frequency is given by

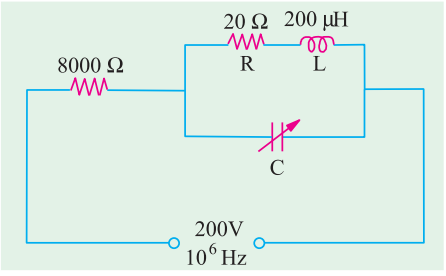


Fig. 14.53

$$f = \frac{1}{2\pi\sqrt{LC}}$$

$$\therefore \quad 10^6 = \frac{1}{2\pi\sqrt{200\times10^{-6}\times C}}$$

(i) $\therefore \quad C = \mathbf{125\ \mu F}$

(ii) $Q = \dfrac{2\pi f L}{R} = \dfrac{2\pi\times10^6\times200\times10^{-4}}{20} = \mathbf{62.8}$

(iii) Dynamic resistance of the circuit is $= \dfrac{L}{CR} = \dfrac{200\times10^{-6}}{125\times10^{-12}\times20} = 80{,}000\ \Omega$

Total equivalent resistance of the tuned circuit is 80,000 + 8,000 = 88,000 Ω

$\therefore$ Current $I = 200/88{,}000 = 2.27$ mA

p.d. across tuned circuit = current × dynamic resistance = $2.27 \times 10^{-3} \times 80{,}000 = 181.6$ V

Current through inductive branch $= \dfrac{181.6}{\sqrt{10^2 + 1256^2}} = 0.1445$ A $= \mathbf{144.5\ mA}$

Current through capacitor branch

$$= \frac{V}{1/\omega C} = \omega VC = 181.6 \times 2\pi \times 10^6 \times 125 \times 10^{-12} = \mathbf{142.7\ mA}$$

Note. It may be noted in passing that current in each branch is nearly 62.8 (*i.e.* Q-factor) times the resultant current taken from the supply.

Example 14.50. *Impedances Z_2 and Z_3 in parallel are in series with an impedance Z_1 across a 100-V, 50-Hz a.c. supply. Z_1 = (6.25 + j 1.25) ohm ; Z_2 = (5 + j0) ohm and Z_3 = (5 –j X_C) ohm. Determine the value of capacitance of X_C such that the total current of the circuit will be in phase with the total voltage. When is then the circuit current and power ?*

(Elect. Engg-I, Nagpur Univ, 1992)

Solution. $Z_{23} = \dfrac{5\,(5 - jX_c)}{(10 - jX_C)}$, for the circuit in Fig. 14.59.

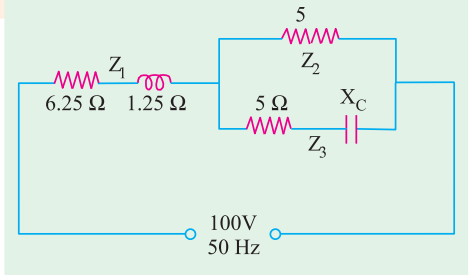


Fig. 14.54

$$= \frac{25 - j5X_C}{(10 - jX_C)} \times \frac{10 + jX_C}{10 + jX_C} = \frac{250 + 5X_C^2}{100 + X_C^2} - j\,\frac{25X_C}{100 + X_C^2}$$

$$\mathbf{Z} = 6.25 + j\,1.25 + \frac{250 + 5X_C^2}{100 + X_C^2} - j\,\frac{25X_C}{100 + X_C^2}$$

$$= 6.25 \quad \frac{250}{100} \quad \frac{5X_C^2}{X_C^2} \quad j \quad \frac{25X_C}{100} \quad \frac{5}{X_C^2} \quad \frac{5}{4}$$

Power factor will be unity or circuit current will be in phase with circuit voltage if the j term in the above equation is zero.

$$\therefore \quad \left(\frac{25X_C}{100 + X_C^2} - \frac{5}{4}\right) = 0 \quad \text{or} \quad X_C = 10 \quad \therefore \quad 1/\omega C = 10 \text{ or } C = 1/314 \times 10 = \mathbf{318\ \mu F}$$

Substituting the value of $X_C = 10\ \Omega$ above, we get

$Z = 10 - j0 = 10\angle 0°$ and $I = 100/10 = \mathbf{10\ A}$; Power $= I^2R = 10^2 \times 10 = \mathbf{1000\ W}$

Example 14.51. *In the circuit given below, if the value of $R = \sqrt{L/C}$, then prove that the impedance of the entire circuit is equal to R only and is independent of the frequency of supply. Find the value of impedance for L = 0.02 H and C = 100 μF.*

(Communication System, Hyderabad Univ. 1991)

Solution. The impedance of the circuit of Fig. 14.55 is

$$\mathbf{Z} = \frac{(R + j\omega L)(R - j/\omega C)}{2R + j(\omega L - 1/\omega C)} = \frac{R^2 + (L/C) + jR(\omega L - 1/\omega C)}{2R + j(\omega L - 1/\omega C)}$$

If $R^2 = L/C$ or $R = \sqrt{L/C}$, then

$$\mathbf{Z} = \frac{R^2 + R^2 + jR(\omega L - 1/\omega C)}{2R + j(\omega L - 1/\omega C)}$$

$$= R\left[\frac{2R + j(\omega L - 1/\omega C)}{2R + j(\omega L - 1/\omega C)}\right] \quad \text{or} \quad \mathbf{Z = R}$$

Now, $R = \sqrt{\frac{L}{C}} = \sqrt{\frac{0.02}{100 \times 10^{-6}}} = \mathbf{14.14\ \Omega}$

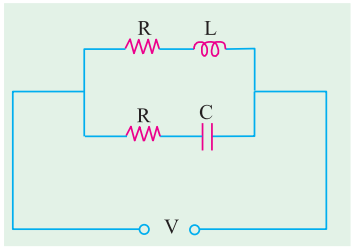


Fig. 14.55

Example 14.52. *Derive an expression for the resonant frequency of the parallel circuit shown in Fig. 14.46.* **(Electrical Circuit, Nagpur Univ, 1993)**

Solution. As stated in Art. 14.9 for resonance of a parallel circuit, total circuit susceptance should be zero. Susceptance of the R-L branch is

$$B_1 = -\frac{X_L}{R_1^2 + X_L^2}$$

Similarly, susceptance of the R-C branch is

$$B_2 = \frac{X_C}{R_2^2 + X_C^2}$$

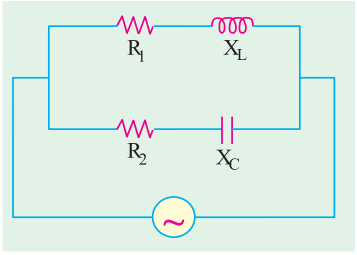


Fig. 14.56

Net susceptance is $B = -B_1 + B_2$

For resonance $B = 0$ or $0 = -B_1 + B_2 \quad \therefore B_1 = B_2$

$$\text{or} \quad \frac{X_L}{R_1^2 + X_L^2} = \frac{X_C}{R_2^2 + X_C^2} \quad \text{or} \quad X_L(R_2^2 + X_C^2) = X_C(R_1^2 + X_L^2)$$

$$2\ fL\ R_2^2 \quad \frac{1}{2\ fC}^2 \quad \frac{1}{2\ fC}[R_1^2 \quad (2\ fL)^2]; 4\ ^2f^2LC\ R_2^2 \quad \frac{1}{2\ fC}^2 \quad [R_1^2 \quad (2\ fL)^2]$$

$$\therefore \quad 4\pi^2f^2LCR_2^2 + \frac{L}{C} = R_1^2 + 4\pi^2f^2L^2 \ ; \ 4\pi^2f^2[L(L - CR_2^2)] = \frac{L}{C} - R_1^2$$

$$\therefore \quad f_0 = \frac{1}{2\pi}\sqrt{\left(\frac{L/C - R_1^2}{L(L - CR_2^2)}\right)} \quad \therefore \quad f_0 = \frac{1}{2\pi}\sqrt{\left(\frac{L - CR_1^2}{LC(L - CR_2^2)}\right)} \ ; \ \omega_0 = \frac{1}{\sqrt{LC}}\sqrt{\left(\frac{L - CR_1^2}{L - CR_2^2}\right)}$$

Note. If both R_1 and R_2 are negligible, then $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$ –as in Art. 14.9

Example 14.53. *Calculate the resonant frequency of the network shown in Fig. 14.57.*

Solution. Total impedance of the network between terminals A and B is

$$Z_{AB} = (R_1 \| jX_L) + [R_2 \| (-jX_C)] = \frac{jR_1X_L}{R_1 + jX_L} + \frac{R_2(-jX_C)}{R_2 - jX_C} = \frac{jR_1\omega L}{R_1 + j\omega L} - \frac{jR_2/\omega C}{R_2 - j/\omega C}$$

$$= \frac{R_1\omega^2L^2}{R_1^2 + \omega^2L^2} + \frac{R_2}{\omega C(R_2^2 + 1/\omega^2C^2)} + j\left[\frac{R_1^2\omega L}{R_1^2 + \omega^2L^2} - \frac{R_2^2}{\omega C(R_2^2 + 1/\omega^2C^2)}\right]$$

At resonance, $\omega = \omega_0$ and the j term of Z_{AB} is zero.

$$\therefore \frac{R_1^2\omega_0 L}{R_1^2 + \omega_0^2L^2} - \frac{R_2^2}{\omega_0 C(R_2^2 + 1/\omega_0^2C^2)} = 0$$

$$\text{or } \frac{R_1^2\omega_0 L}{R_1^2 + \omega_0^2L^2} = \frac{R_2^2\omega_0 C}{R_2^2\omega_0^2C^2 + 1}$$

Simplifying the above, we get

$$\omega_0^2 = \frac{G_2^2 - C/L}{LC(G_1^2 - C/L)} \text{ where } G_1 = \frac{1}{R_1} \text{ and } G_2 = \frac{1}{R_2}$$

The resonant frequency of the given network in Hz is

$$f_0 = \frac{0}{2} \quad \frac{1}{2}\sqrt{\frac{G_2^2 \quad C/L}{LC\,(G_1^2 \quad C/L)}}$$

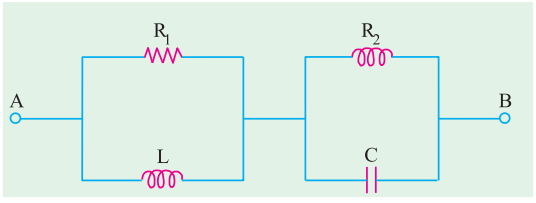


Fig. 14.57

Example 14.54. *Compute the value of C which results in resonance for the circuit shown in Fig. 14.58 when f = 2500/π Hz.*

Solution. $\mathbf{Y_1} = 1/(6 + j8)$

$\mathbf{Y_2} = 1/(4 - jX_C)$

$$\mathbf{Y} = \mathbf{Y_1} + \mathbf{Y_2} = \frac{1}{6 \quad j8} \quad \frac{1}{4 \quad jX_C}$$

$$= \left(0.06 + \frac{4}{16 + X_C^2}\right) + j\left(\frac{X_C}{16 + X_C^2} - 0.08\right)$$

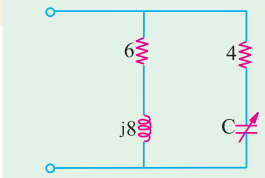


Fig. 14.58

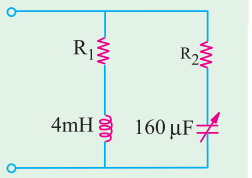


Fig. 14.59

For resonance, j part of admittance is zero, *i.e.* the complex admittance is real number.

$\therefore \quad X_C/(16 + X_C^2) - 0.08 = 0 \quad$ or $\quad 0.08\,X_C^2 - X_C + 1.28 = 0$

$\therefore \quad X_C = 11.05 \quad$ or $\quad 1.45 \quad \therefore \; 1/\omega C = 11.05 \quad$ or $\quad 1.45$

(i) $1/5000C = 11.05 \quad$ or $\quad C = \mathbf{18\ \mu F} \quad$ ***(ii)*** $1/5000\,C = 1.45 \quad$ or $\quad C = \mathbf{138\ \mu F}$

Example 14.55. *Find the values of R_1 and R_2 which will make the circuit of Fig. 14.59 resonate at all frequencies.*

Solution. As seen from Example 14.42, the resonant frequency of the given circuit is

$$\omega_0 = \frac{1}{\sqrt{LC}}\sqrt{\left(\frac{L - CR_1^2}{L - CR_2^2}\right)}$$

Now, ω_0 can assume any value provided $R_1^2 = R_2^2 = L/C$.

In the present case, $L/C = 4 \times 10^{-3}/60 \times 10^{-6} = 25$. Hence, $R_1 = R_2 = \sqrt{25} = \mathbf{5\ ohm.}$

Tutorial Problem No. 14.2

1. A resistance of 20 W and a coil of inductance 31.8 mH and negligible resistance are connected in parallel across 230 V, 50 Hz supply. Find (*i*) The line current (*ii*) power factor and (*iii*) The power consumed by the circuit. **[(*i*) 25.73 A (*ii*) 0.44 T lag (*iii*) 246 W] (*F. E. Pune Univ.*)**
2. Two impedances $Z_1 = (150 + j157)$ ohm and $Z_2 = (100 + j\ 110)$ ohm are connected in parallel across a 220-V, 50-Hz supply. Find the total current and its power factor.
[24 ∠– 47°A ; 0.68 (lag)] (*Elect. Engg. & Electronics Bangalore Univ.*)
3. Two impedances $(14 + j5)\Omega$ and $(18 + j10)\ \Omega$ are connected in parallel across a 200-V, 50-Hz supply. Determine (*a*) the admittance of each branch and of the entire circuit ; (*b*) the total current, power, and power factor and (*c*) the capacitance which when connected in parallel with the original circuit will make the resultant power factor unity. **[(*a*) (0.0634 –j0.0226), (0.0424 –j0.023) (0.1058 –j0.0462 S) (*b*) 23.1 A, 4.232 kW, 0.915 (*c*) 147 μF]**
4. A parallel circuit consists of two branches *A* and *B*. Branch A has a resistance of 10 Ω and an inductance of 0.1 H in series. Branch *B* has a resistance of 20 Ω and a capacitance of 100 μF in series. The circuit is connected to a single-phase supply of 250 V, 50 Hz. Calculate the magnitude and the phase angle of the current taken from the supply. Verify your answer by measurement from a phasor diagram drawn to scale. **[6.05 ∠– 15.2°] (*F. E. Pune Univ.*)**
5. Two circuits, the impedances of which are given by $Z_1 = (10 + j15)\ \Omega$ and $Z_2 = (6 - j8)\ \Omega$ are connected in parallel. If the total current supplied is 15 A, what is the power taken by each branch ?
[737 W ; 1430 W] (*Elect. Engg. A.M.A.E. S.I.*)
6. A voltage of 240 V is applied to a pure resistor, a pure capacitor, and an inductor in parallel. The resultant current is 2.3 A, while the component currents are 1.5, 2.0 and 1.1 A respectively. Find the resultant power factor and the power factor of the inductor. **[0.88 ; 0.5]**
7. Two parallel circuits comprise respectively (*i*) a coil of resistance 20 Ω and inductance 0.07 H and (*ii*) a capacitance of 60 μF in series with a resistance of 50 Ω. Calculate the current in the mains and the power factor of the arrangement when connected across a 200-V, 50-Hz supply.
[7.05 A ; 0.907 lag] (*Elect. Engg. & Electronics, Bangalore Univ.*)
8. Two circuits having the same numerical ohmic impedances are joined in parallel. The power factor of one circuit is 0.8 lag and that of other 0.6 lag. Find the power factor of the whole circuit.
[0.707] (*Elect. Engg. Pune Univ.*)
9. How is a current of 10 A shared by three circuits in parallel, the impedances of which are $(2 - j5)\ \Omega$, $(6 + j3)\Omega$ and $(3 + j4)\ \Omega$. **[5.68 A ; 4.57 A, 6.12 A]**
10. A piece of equipment consumes 2,000 W when supplied with 110 V and takes a lagging current of 25 A. Determine the equivalent series resistance and reactance of the equipment.
If a capacitor is connected in parallel with the equipment to make the power factor unity, find its capacitance. The supply frequency is 100 Hz. **[3.2 Ω, 3.02 Ω, 248 μF] (*Sheffield Univ. U.K.*)**
11. A capacitor is placed in parallel with two inductive loads, one of 20 A at 30° lag and one of 40° A at 60° lag. What must be current in the capacitor so that the current from the external circuit shall be at unity power factor ? **[44.5 A] (*City & Guilds, London*)**
12. An air-cored choking coil is subjected to an alternating voltage of 100 V. The current taken is 0.1 A and the power factor 0.2 when the frequency is 50 Hz. Find the capacitance which, if placed in parallel with the coil, will cause the main current to be a minimum. What will be the impedance of this parallel combination (*a*) for currents of frequency 50 (*b*) for currents of frequency 40 ?
[3.14 μF (*a*) 5000 Ω (*b*) 1940 Ω] (*London Univ.*)
13. A circuit, consisting of a capacitor in series with a resistance of 10 Ω, is connected in parallel with a coil having $L = 55.2$ mH and $R = 10\ \Omega$, to a 100-V, 50-Hz supply. Calculate the value of the capacitance for which the current take from the supply is in phase with the voltage. Show that for the particular values given, the supply current is independent of the frequency. **[153 μF] (*London Univ.*)**
14. In a series-parallel circuit, the two parallel branches *A* and *B* are in series with *C*. The impedances are $Z_A = (10 - j8)\ \Omega$, $Z_B = (9 - j6)\ \Omega$ and $Z_C = (100 + j0)$. Find the currents I_A and I_B and the phase difference between them. Draw the phasor diagram. **[$I_A = 12.71 \angle 30°58'$ $I_B = 15 \angle 35°56'$; 4°58′]**
(*Elect. Engg. & Electronics Bangalore Univ.*)
15. Find the equivalent series circuits of the 4-branch parallel circuit shown in Fig. 14.60.
[(4.41 + j2.87) Ω] [A resistor of 4.415 Ω in series with a 4.57 mH inductor]

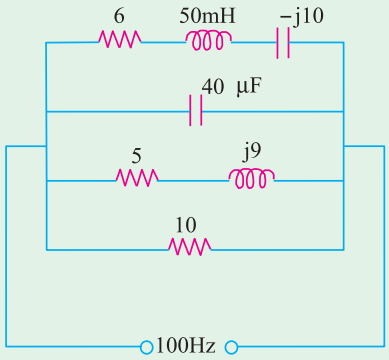


Fig. 14.60

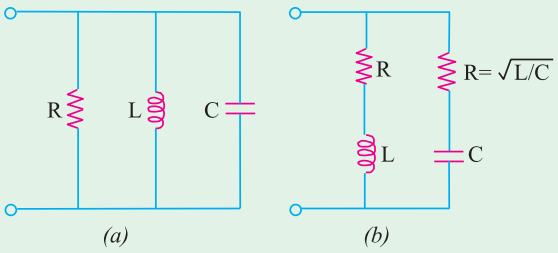


Fig. 14.61

16. A coil of 20 Ω resistance has an inductance of 0.2 H and is connected in parallel with a 100-μF capacitor. Calculate the frequency at which the circuit will act as a non-inductive resistance of *R* ohms. Find also the value of *R*. **[31.8 Hz; 100 Ω]**

17. Calculate the resonant frequency, the impedance at resonance and the Q-factor at resonance for the two circuits shown in Fig. 14.61.

(*a*) $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$; $Z_0 = R$; $Q_0 = \dfrac{R}{\sqrt{L/C}}$

(*b*) Circuit is resonant at all frequencies with a constant resistive impedance of $(\sqrt{L/C})$ ohm, Q = 0.]

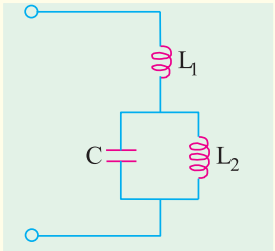


Fig. 14.62

18. Prove that the circuit shown in Fig. 14.62 exhibits both series and parallel resonances and calculate the frequencies at which two resonaces occur.

Parallel $f_0 = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{(L_2C_2)}}$; **series** $f_0 = \dfrac{1}{2\pi}\sqrt{\dfrac{(L_1+L_2)}{L_1L_2C_2}}$

19. Calculate the resonant frequency and the corresponding Q-factor for each of the networks shown in Fig. 14.63.

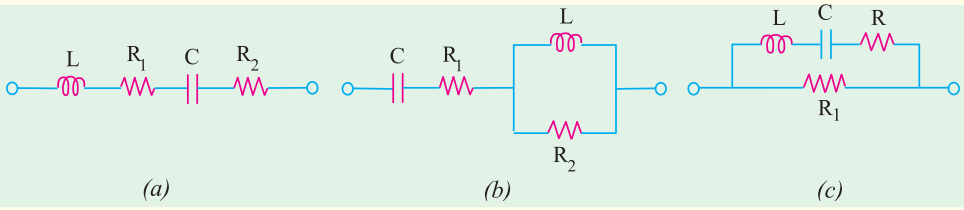


Fig. 14.63

$$\left[\ (a)\ f_0 = \frac{1}{2\pi\sqrt{LC}};\ Q = \frac{\omega_0 L}{R_1+R_2} = \frac{1}{\omega_0 C(R_1+R_2)} = \frac{1}{(R_1+R_2)}\cdot\sqrt{\left(\frac{L}{C}\right)}\right.$$

$$(b)\ f_0 = \frac{1}{2}\sqrt{\frac{1}{LC\ \ L^2/R_2^2}};\ Q = \frac{R_2}{\omega_0(L+R_1R_2C)}\quad (c)\ f_0 = \frac{1}{2\pi\sqrt{LC}};\ Q = \frac{\omega_0 L}{R}\times\frac{R_1}{R+R_1} = \frac{R_1}{R(R+R_1)}\cdot\sqrt{\frac{L}{C}}\ \Big]$$

20. A parallel *R-L-C* circuit is fed by a constant current source of variable frequency. The circuit resonates at 100 kHz and the *Q*-factor measured at this frequency is 5. Find the frequencies at which the amplitude of the voltage across the circuit falls to (*a*) 70.7% (*b*) 50% of the resonant frequency amplitude. **[(*a*) 90.5 kHz ; 110.5 kHz (*b*) 84.18 kHz ; 118.8 kHz]**

21. Two impedance Z_1 = (6 + j8) ohm and Z_2 = (8 - j6) ohm are connected in parallel across 100 V supply. Determine : (*i*) Current and power factor of each branch. (*ii*) Overall current and power factor (*iii*) Power consumed by each branch and total power. ***(Nagpur University, Winter 2003)***

22. The currents is each branch of a two branched parallel circuit is given as :

$$i_a = 8.07 \sin\left(314\,t - \frac{\pi}{4}\right)$$

$$i_b - 21.2 \sin\left(314\,t - \frac{\pi}{3}\right)$$

and supply voltage is v = 354 sin 314 t.
Calculate :
(*i*) Total current in the same form (*ii*) Calculate ohmic value of components in each branch.
(Nagpur University, Summer 2004)

23. Two coils are connected in parallel and a voltage of 200 V is applied between the terminals the total current taken by the circuit is 25 A and power dissipated in one of the coils is 1500 W. Calculate the resistance of each coil. ***(Gujrat University, June/July 2003)***

24. Compare the series and parallel resonance of R-L-C series and R-L-C parallel circuit.
(Gujrat University, June/July 2003)

25. Two circuits with impedances $Z_1 = (10 + r15)\ \Omega$ *and* $z_2 = (6 - r8_\Omega)$ are connected in parallel. If the supply current is 20A, what is the power dissipated in each branch.
(V.T.U., Belgaum Karnataka University, Winter 2003)

26. Three impedances $z_1 = 8 + j6\Omega$, $z_2 = 2 - j1.5\Omega$ and $z_3 = 2\Omega$ are connected in parallel across a 50Hz supply. If the current through z_1 is $3 + j4$amp, calculate the current through the other impedances and also power absorbed by this parallel circuit.
(V.T.U. Belgaum Karnataka University, Winter 2004)

27. Show that the power consumed in a pure inductance is zero.
(RGPV Bhopal 2002)

28. What do you understand by the terms power factor, active power and reactive power?
(RGPV Bhopal 2002)

29. Two circuits the impedances of which are given by $Z_1\ (10 + j\ 15)\Omega$ and $Z_2 = (6 - j\ 8)\Omega$ are connected in parallel. If the total current supplied is 15 A. What is the power taken by each branch?
(RGPV Bhopal 2002)

30. Does an inductance draw instantaneous power as well as average power?
(RGPV Bhopal December 2002)

31. Describe the properties of (*i*) Resistance (*ii*) Inductance and (*iii*) capacitance used in A.C. Circuit.
(RGPV Bhopal June 2003)

OBJECTIVE TYPES – 14

1. Fill in the blanks
(*a*) unit of admittance is .....
(*b*) unit of capacitive susceptance is .....
(*c*) admittance equals the reciprocal of .....
(*d*) admittance is given by the ..... sum of conductance and susceptance.

2. An *R-L* circuit has **Z** = (6 + *j*8) ohm. Its susceptance is -Siemens.
(*a*) 0.06 (*b*) 0.08 (*c*) 0.1 (*d*) -0.08

3. The impedances of two parallel branches of a circuit are (10 + *j*10) and (10 –*j*10) respectively. The impedance of the parallel combination is
(*a*) 20 + *j*0 (*b*) 10 + *j*0
(*c*) 5 –*j*5 (*d*) 0 –*j*20

4. The value of Z in Fig. 14.64 which is most appropriate to cause parallel resonance at 500 Hz is

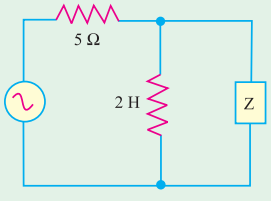


Fig. 14.64

(*a*) 125.00 mH (*b*) 304.20 μF
(*c*) 2.0 μF (*d*) 0.05 μF
(GATE 2004)

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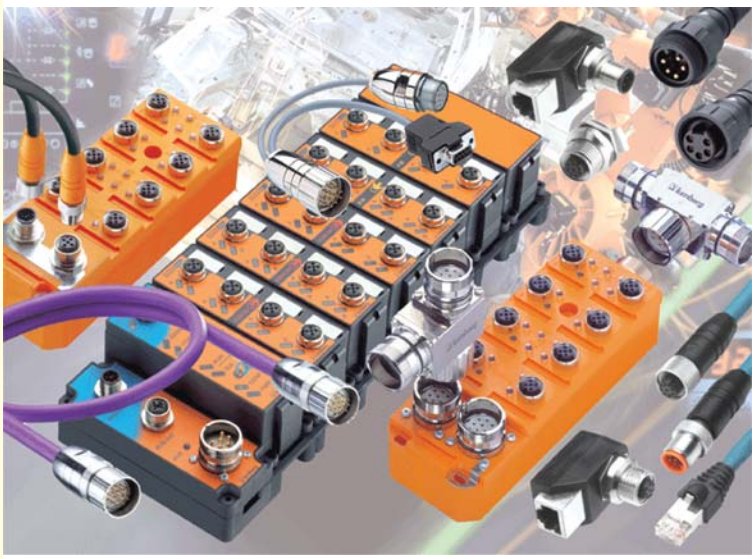
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CHAPTER 15

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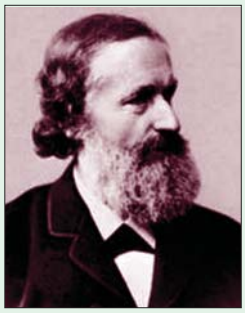
A.C. NETWORK ANAYLSIS



Engineers use AC network analysis in thousands of companies worldwide in the design, maintenance and operation of electrical power systems

15.1. Introduction

We have already discussed various d.c. network theorems in Chapter 2 of this book. The same laws are applicable to a.c. networks except that instead of resistances, we have impedances and instead of taking algebraic sum of voltages and currents we have to take the phasor sum.



Gustav Kirchhoff
(1824-1887)

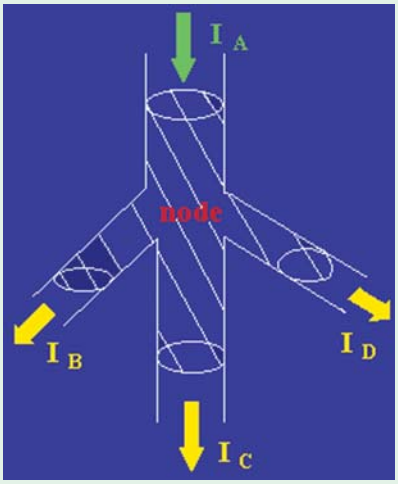
15.2. Kirchhoff's Laws

The statements of Kirchhoff's laws are similar to those given in Art. 2.2 for d.c. networks except that instead of algebraic sum of currents and voltages, we take phasor or vector sums for a.c. networks.

1. ***Kirchhoff's Current Law.*** According to this law, in any electrical network, the phasor sum of the currents meeting at a junction is zero.

In other words, $\sum I = 0$ ...at a junction

Put in another way, it simply means that in any electrical circuit the phasor sum of the currents flowing towards a junction is equal to the phasor sum of the currents going away from that junction.



2. ***Kirchhoff's Voltage Law.*** According to this law, the phasor sum of the voltage drops across each of the conductors in any closed path (or mesh) in a network plus the phasor sum of the e.m.fs. connected in that path is zero.

In other words, $\sum IR + \sum$e.m.f. $= 0$ ...round a mesh

Example 15.1. *Use Kirchhoff's laws to find the current flowing in each branch of the network shown in Fig. 15.1.*

Solution. Let the current distribution be as shown in Fig. 15.1 (*b*). Starting from point *A* and applying KVL to closed loop *ABEFA*, we get

$$-10(x + y) - 20\,x + 100 = 0 \quad \text{or} \quad 3x + y = 10 \qquad ...(i)$$

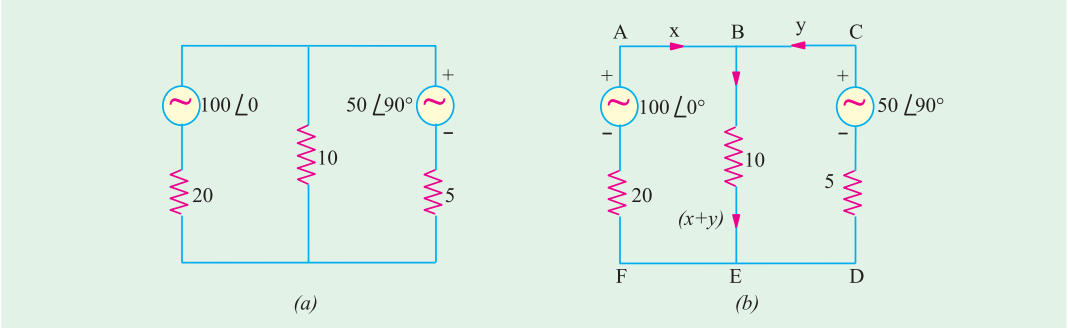


Fig. 15.1

Similarly, considering the closed loop *BCDEB* and starting from point *B*, we have

$$-50\ \angle 90^\circ + 5y + 10\,(x + y) = 0 \quad \text{or} \quad 2x + 3y = j10 \qquad ...(ii)$$

Multiplying Eq. **(*i*)** by 3 and subtracting it from Eq. **(*ii*)**, we get

$$7x = 30 - j10 \quad \text{or} \quad x = 4.3 - j1.4 = 4.52\ \angle -18^\circ$$

Substituting this value of x in Eq. ***(i)***, we have

$$y = 10 - 3x = 5.95 \angle 119.15^\circ = -2.9 + j5.2$$

$$\therefore \quad x + y = 4.3 - j1.4 - 2.9 + j5.2 = 1.4 + j3.8$$

Tutorial Problem No. 15.1

1. Using Kirchhoff's Laws, calculate the current flowing through each branch of the circuit shown in Fig. 15.2 **[I = 0.84 ∠47.15° A; I_1 = 0.7 ∠– 88.87° A; I_2 = 1.44 ∠ 67.12°A]**

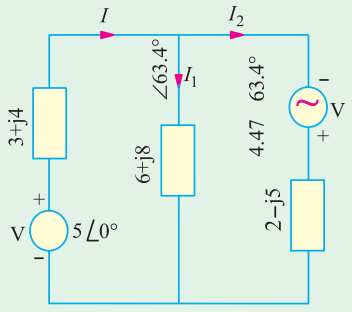


Fig. 15.2

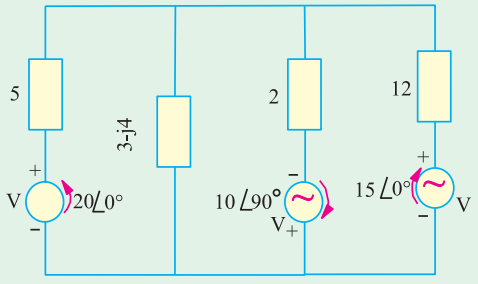


Fig. 15.3

2. Use Kirchhoff's laws to find the current flowing in the capacitive branch of Fig. 15.3 **[5.87 A]**

15.3. Mesh Analysis

It has already been discussed in Art. 2.3. Sign convention regarding the voltage drops across various impedances and the e.m.f.s is the same as explained in Art. 2.3. The circuits may be solved with the help of KVL or by use of determinants and Cramer's rule or with the help of impedance matrix $[Z_m]$.

Example 15.2. *Find the power output of the voltage source in the circuit of Fig. 15.4. Prove that this power equals the power in the circuit resistors.*

Solution. Starting from point A in the clockwise direction and applying KVL to the mesh *ABEFA*, we get.

$$-8\, I_1 - (-j6)\,(I_1 - I_2) + 100 \angle 0^\circ = 0$$

or $$I_1\,(8 - j6) + I_2.\,(j6) = 100 \angle 0^\circ \quad ...(i)$$

Similarly, starting from point *B* and applying *KVL* to mesh *BCDEB*, we get

$$-I_2(3 + j4) - (-j6)\,(I_2 - I_1) = 0$$

or $$I_1\,(j6) + I_2\,(3 - j2) = 0 \quad ...(ii)$$

The matrix form of the above equation is

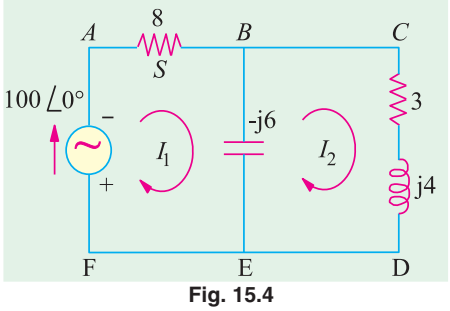


Fig. 15.4

$$\begin{bmatrix} (8 - j6) & j6 \\ j6 & (3 - j2) \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 100 \angle 0^\circ \\ -0 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} (8 - j6) & j6 \\ j6 & (3 - j2) \end{bmatrix} = (8 - j6)\,(3 - j2) - (j6)^2 = 62.5 \angle 39.8^\circ$$

$$\Delta_1 = \begin{vmatrix} 100\angle 0^\circ & j6 \\ 0 & (3 - j2) \end{vmatrix} = (300 - j\,200) = 360 \angle -26.6^\circ$$

$$\Delta_2 = \begin{vmatrix} (8 - j6) & 100\angle 0^\circ \\ j6 & 0 \end{vmatrix} = 600 \angle 90^\circ$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{360\angle -26.6^\circ}{62.5\angle -39.8^\circ} = 5.76\angle 13.2^\circ \; ; \; I_2 = \frac{\Delta_2}{\Delta} = \frac{600\angle 90^\circ}{62.5\angle -39.8^\circ} = 9.6\angle 129.8^\circ$$

Example 15.3. *Using Maxwell's loop current method, find the value of current in each branch of the network shown in Fig. 15.5 (a).*

Solution. Let the currents in the two loops be I_1 and I_2 flowing in the clockwise direction as shown in Fig. 15.5 (*b*). Applying KVL to the two loops, we get

Loop No. 1

$$25 - I_1 (40 + j50) - (-j\ 100)\ (I_1 - I_2) = 0$$

$$\therefore \quad 25 - I_1 (40 - j50) - j\ 100\ I_2 = 0 \qquad ...(i)$$

Loop No. 2

$$-60\ I_2 - (-j100)\ (I_2 - I_1) = 0$$

$$\therefore \quad -j100\ I_1 - I_2\ (60 - j100) = 0$$

$$\therefore \qquad I_2 = \frac{-j100 I_1}{(60 - j100)} = \frac{100\angle -90^\circ\, I_1}{116.62\angle -59^\circ} = 0.8575\angle 31^\circ I_1 \qquad ...(ii)$$

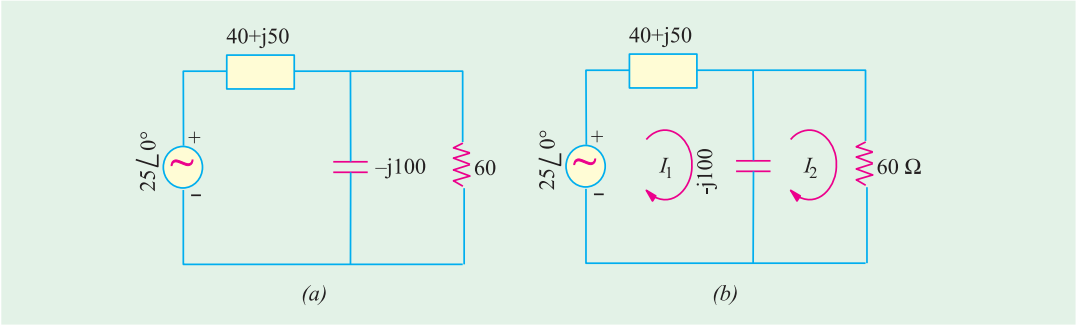


Fig. 15.5

Substituting this value of I_2 in (*i*) above, we get $25 - I_1 (40 - j50) - j100 \times 0.8575\ \angle 31^\circ\ I_1 = 0$

or $\quad 25 - 40\ I_1 + j50\ I_1 - 85.75\ \angle 59^\circ\ \ I_1 = 0$ ($j100 = 100\ \angle\ 90^\circ$)

or $\quad 25 - I_1 (84.16 + j\ 23.5) = 0.$

$$\therefore \qquad I_1 = \frac{25}{(84.16 + j23.5)} = \frac{25}{87.38\angle 15.6^\circ}\ 0.286\ \angle 15.6^\circ\ \text{A}$$

Also, $\qquad I_2 = 0.8575\ \angle\ 31^\circ\ I_1 \times 0.286\ \angle -15.6^\circ = 0.2452\ \angle\ 46.6^\circ \text{A}$

Current through the capacitor = $(I_1 - I_2) = 0.286\ \angle\ 15.6^\circ - 0.2452\ \angle 46.6^\circ = 0.107 + j0.1013 = 0.1473\ \angle\ 43.43^\circ$ A.

Example 15.4. *Write the three mesh current equations for network shown in Fig. 15.6.*

Solution. While moving along I_1, if we apply KVL, we get

$$-(-j10)\ I_1 - 10(I_1 - I_2) - 5\ (I_1 - I_3) = 0$$

or $\quad I_1\ (15 - j10) - 10\ I_2 - 5I_3 = 0 \qquad ...(i)$

In the second loop, current through the a.c. source is flowing upwards indicating that its upper end is positive and lower is negative. As we move along I_2, we go from the positive terminal of the voltage source to its negative terminal. Hence, we

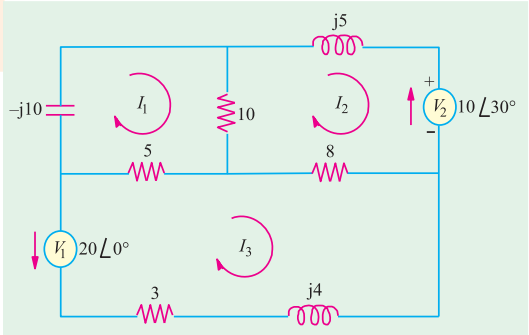


Fig. 15.6

experience a decrease in voltage which as per Art. would be taken as negative.

$$-j5\ I_2 - 10\ \angle 30^\circ - 8(I_2 - I_3) - 10\ (I_2 - I_1) = 0$$

or $10\ I_1 - I_2\ (18 + j5) + 8\ I_3 = 10\ \angle\ 30^\circ$ ...(*ii*)

Similarly, from third loop, we get

$$-20\ \angle 0^\circ - 5(I_3 - I_1) - 8\ (I_3 - I_2) - I_3\ (3 + j4) = 0$$

or $5\ I_1 + 8\ I_2 - I_3\ (16 + j4) = 20\ \angle 0^\circ$ ...(*iii*)

The values of the three currents may be calculated with the help of Cramer's rule. However, the same values may be found with the help of mesh impedance $[Z_m]$ whose different items are as under :

$$Z_{11} = -j\ 10 + 10 + 5 = (15 - j10);\ Z_{22} = (18 + j5)$$
$$Z_{33} = (16 + j5);\ Z_{12} = Z_{21} = -10;\ Z_{23} = Z_{32} = -8$$
$$Z_{13} = Z_{31} = -5;\ E_1 = 0;\ E_2 = -10\ \angle 30^\circ;\quad E_3 = -20\ \angle 0^\circ$$

Hence, the mesh equations for the three currents in the matrix form are as given below :

$$\begin{bmatrix} (15 - j10) & -10 & -5 \\ -10 & (18 + j5) & -8 \\ -5 & -8 & (16 + j5) \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -10\angle 30^\circ \\ -20\angle 0^\circ \end{bmatrix}$$

Example 15.5. *For the circuit shown in Fig. 15.7 determine the branch voltage and currents and power delivered by the source using mesh analysis.*

(Elect. Network Analysis Nagpur Univ. 1993)

Solution. Let the mesh currents be as shown in Fig. 15.7. The different items of the mesh resistance matrix $[E_m]$ are :

$$Z_{11} = (2 + j1 + j2 - j1) = (2 + j2)$$
$$Z_{22} = (-j2 + 1 - j1 + j2) = (1 - j1)$$
$$Z_{12} = Z_{21} = -(j2 - j1) = -j1$$

Hence, the mesh equations in the matrix form are

$$\begin{bmatrix} (2 + j2) & -j1 \\ -j1 & (1 - j1) \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 10\angle 0^\circ \\ -5\angle 30^\circ \end{bmatrix}$$

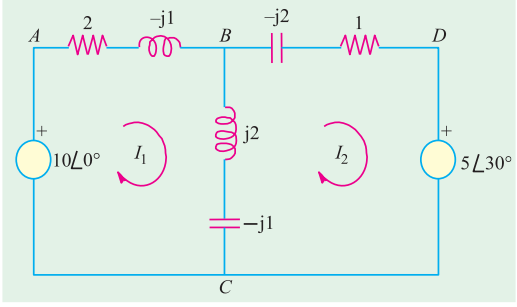


Fig. 15.7

$$\therefore\quad \Delta = (2 + j2)\ (1 - j1) + 1 = 5$$

$$\Delta_1 = \begin{bmatrix} 10 & -j1 \\ -(4.43 + j2.5 & (1 - j1) \end{bmatrix} = 10(1 - j1) - j1(4.43 + j2.5) = 12.5 - j\ 14.43 = 19.1\ \angle\ 49.1^\circ$$

$$\Delta_2 = \begin{bmatrix} (2 + j2) & 10 \\ -j1 & -(4.43 + j2.5) \end{bmatrix} = (2 + j2)\ (4.43 + j2.5) + j\ 10 = -3.86 - j\ 3.86$$

$$= 5.46\angle\ 135^\circ \text{ or } \angle 225^\circ$$

$$I_1 = \Delta_1/\Delta = 19.1\ \angle -49.1^\circ/5 = 3.82\ \angle -49.1^\circ = 2.5 - j2.89$$

$$I_2 = \Delta_2/\Delta = 5.46\ \angle\ 135^\circ/5 = 1.1\ \angle -135^\circ = -0.78 - j0.78$$

Current through branch $BC = I_1 - I_2 = 2.5 - j2.89 + 0.78 + j0.78 = 3.28 - j2.11 = 3.49\ \angle\ 32.75^\circ$

Drop over branch $AB = (2 + j1)(2.5 - j\ 2.89) = 7.89 - j\ 3.28$

Drop over branch $BD = (1 - j2)\ (-0.78 - j0.78) = 2.34 + j0.78$

Drop over branch $BC = j_1\ (I_1 - I_2) = j1\ (3.28 - j2.11) = 2.11 + j3.28$

Power delivered by the sources would be found by using conjugate method. Using current conjugate, we get

$VA_1 = 10(2.5 + j2.89) = 25 + j28.9$; $\therefore W_1 = 25$ W

$VA_2 = V_2 \times I_2$ — because $-I_2$ is the current coming out of the second voltage source. Again, using current conjugate, we have

$VA_2 = (4.43 + j2.5)(0.78 - j0.78)$ or $W_2 = 4.43 \times 0.78 + 2.5 \times 0.78 = 5.4$ W

$\therefore$ total power supplied by the two sources = 25 + 5.4 = 30.4 W

Incidentally, the above fact can be verified by adding up the powers dissipated in the three branches of the circuit. It may be noted that there is no power dissipation in the branch *BC*.

Power dissipated in branch $AB = 3.82^2 \times 2 = 29.2$ W

Power dissipated in branch $BD = 1.1^2 \times 1 = 1.21$ W

Total power dissipated = 29.2 + 1.21 = 30.41 W.

Tutorial Problems No. 15.2

1. Using mesh analysis, find current in the capacitor of Fig. 15.8. **[13.1 ∠70.12° A]**

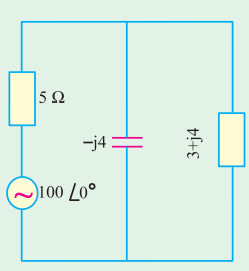


Fig. 15.8

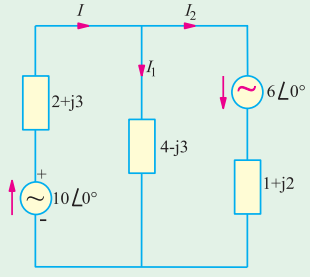


Fig. 15.9

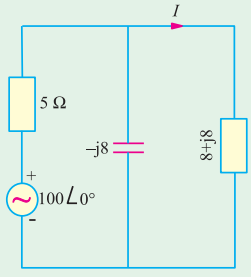


Fig. 15.10

2. Using mesh analysis or Kirchhoff's laws, determine the values of I, I_1 and I_2 (in Fig. 15.9)
[I = 2.7 ∠– 58.8° A ; I_1 = 0.1 ∠97° A; I_2 = 2.8 ∠59.6°A]

3. Using mesh current analysis, find the value of current I and active power output of the voltage source in Fig. 15.10. **[7 ∠– 50° A ; 645 W]**

4. Find the mesh currents I_1, I_2 and I_3 for the circuit shown in Fig. 15.11. All resistances and reactances are in ohms. **[I_1 = (1.168 + j1.281) ; I_2 = (0.527 –j0.135); I_3 = (0.718 + J0.412)]**

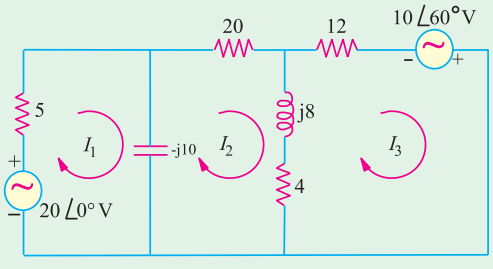


Fig.15.11

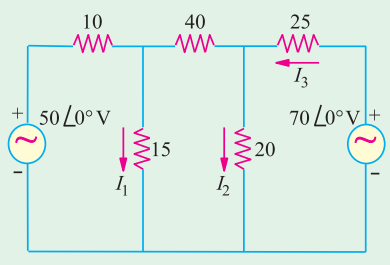


Fig. 15.12

5. Find the values of branch currents I_1.I_2 and I_3 in the circuit shown in Fig. 15.12 by using mesh analysis. All resistances are in ohms. **[I_1 = 2.008 ∠0° ; I_2 = 1.545 ∠0° ; I_3 = 1.564 ∠0°]**

6. Using mesh-current analysis, determine the current I_1, I_2 and I_3 flowing in the branches of the networks shown in Fig. 15.13. **[I_1 = 8.7 ∠1.37° A; I_2 = 3 ∠48.7° A ; I_3 = 7 ∠17.25° A]**

7. Apply mesh-current analysis to determine the values of current I_1 to I_5 in different branches of the circuit shown in Fig. 15.14.

[I_1 = 2.4 ∠52.5° A ; I_2 = 1.0 ∠ 46.18° A ; I_3 = 1.4 ∠ 57.17° A ; I_4 = 0.86 ∠166.3° A ; I_5 = 1.0 ∠83.7° A]

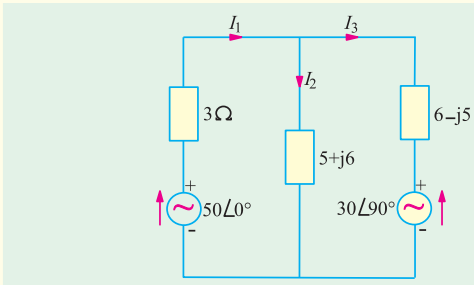


Fig. 15.13

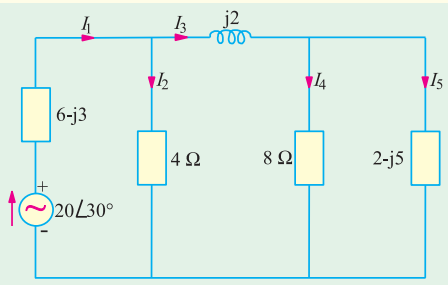


Fig. 15.14

15.4. Nodal Analysis

This method has already been discussed in details Chapter 2. This technique is the same although we have to deal with circuit impedances rather than resistances and take phasor sum of voltages and currents rather than algebraic sum.

Example 15.6. *Use Nodal analysis to calculate the current flowing in each branch of the network shown in Fig. 15.15.*

Solution. As seen, there are only two principal nodes out of which node No. 2 has been taken as the reference node. The nodal equations are :

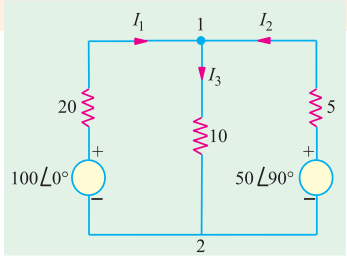


Fig. 15.15

$$V_1\left(\frac{1}{20}+\frac{1}{10}+\frac{1}{5}\right)-\frac{100\angle 0^\circ}{20}-\frac{50\angle 90^\circ}{5}=0$$

$$\therefore \quad 0.35\,V_1 = 5 + j10;\ V_1 = \frac{5+j10}{0.35}$$

$$= 14.3 + j28.6 = 32\angle 63.4^\circ$$

$$\therefore \quad I_1 = \frac{100\angle 0^\circ - V_1}{20} = \frac{100-14.3-j28.6}{20}$$

$$= 4.3 - j1.4 = 4.5\angle -18^\circ \text{ flowing towards node No. 1}$$

(or $4.5 \angle -18^\circ + 180^\circ = 45 \angle 162^\circ$ flowing away from node No. 1)

$$I_3 = \frac{V_1}{10} = \frac{32\angle 63.4^\circ}{10} = 3.2\angle 63.4^\circ = 1.4 + j2.9 \text{ flowing from node No. 1 to node No. 2}$$

$$I_2 = \frac{50\angle 90^\circ - V_1}{5} = \frac{j50-14.3-j28.6}{5} = \frac{-14.3+j21.4}{5} = -2.86 + j4.3 = 5.16\angle 123.6^\circ$$

flowing towards node No. 1

$$\angle 123.6^\circ - 180^\circ = 5.16 \angle 56.4^\circ \text{ flowing away from node No. 1).}$$

Example15.7. *Find the current I in the j10 Ω branch of the given circuit shown in Fig. 15.16 using the Nodal Method.* **(Principles of Elect. Engg. Delhi Univ.)**

Solution. There are two principal nodes out of which node No. 2 has been taken as the reference node. As per Art.

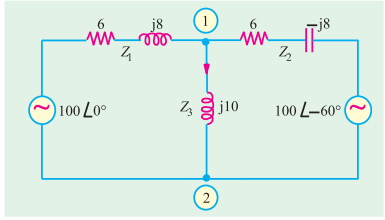


Fig. 15.16

$$V_1\left(\frac{1}{6+j8}+\frac{1}{6-j8}+\frac{1}{j10}\right)-\frac{100\angle 0^\circ}{6+j8}\,\frac{100\angle -60^\circ}{6-j8}=0$$

$V_1(0.06 - j0.08 + 0.06 + j0.08 - j0.1) = 6 - j8 + 9.93 - j1.2 = 18.4 \angle -30^\circ$

$$\therefore \quad V_1(0.12 - j0.1) = 18.4\angle -30^\circ \text{ or}$$

$$V_1 \times 0.156 \angle -85.6^\circ = 18.4 \angle -30^\circ$$

$$\therefore \quad V_1 = 18.4 \angle 30^\circ / 0.156 \angle 85.6^\circ = 118 \angle 55.6^\circ \text{V}$$

$$\therefore \quad V = V_1/j10 = 118 \angle 55.6^\circ / j10 = 11.8 \angle 34.4^\circ \text{A}$$

Example 15.8. *Find the voltage V_{AB} in the circuit of Fig. 15.17 (a). What would be the value of V_1 if the polarity of the second voltage source is reversed as shown in Fig. 15.17 (b).*

Solution. In the given circuit, there are no principle nodes. However, if we take point *B* as the reference node and point *A* as node 1, then using nodal mathod, we get

$$V_1\left(\frac{1}{10}+\frac{1}{8+j4}\right) - \frac{10\angle 0^\circ}{10} - \frac{10\angle 30^\circ}{8+j4} = 0$$

$$V_1 \times 0.2 \angle 14.1^\circ = 1 + 1.116 + j0.066 = 4.48 \angle 1.78^\circ$$

$$\therefore \quad V_1 = 4.48 \angle 1.78^\circ / 0.2 \angle 14.1^\circ = 22.4 \angle 15.88^\circ$$

When source polarity is Reversed

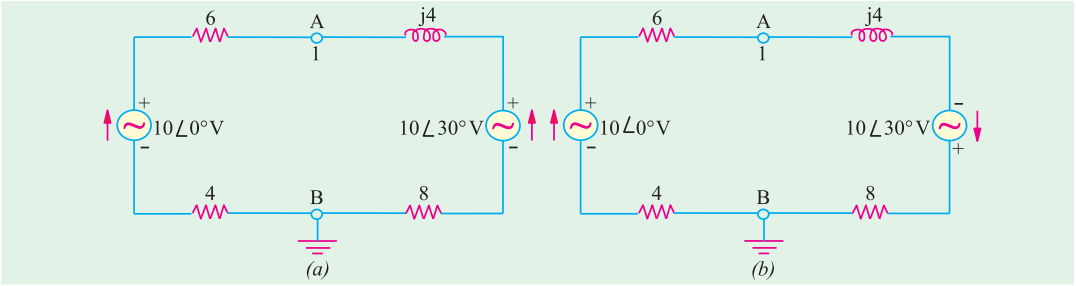


Fig. 15.17

$$V_1\left(\frac{1}{10}+\frac{1}{8+j4}\right) - \frac{10\angle 0^\circ}{10} + \frac{10\angle 30^\circ}{8+j4} = 0 \quad \text{or} \quad V_1 = 0.09 \angle 223.7^\circ$$

Example 15.9. *Write the nodal equations for the network shown in Fig. 15.18.*

Solution. Keeping in mind the guidance given in Art. 2.10, it would be obvious that since current of the second voltage source is flowing away from node 1, it would be taken as negative. Hence, the term containing this source will become positive because it has been reversed twice. As seen, node 3 has been taken as the reference node. Considering node 1, we have

$$V_1\left(\frac{1}{10}+\frac{1}{4+j4}+\frac{1}{j5}\right) - \frac{V_2}{4+j4} - \frac{10\angle 0^\circ}{10} + \frac{10\angle 30^\circ}{j5} = 0$$

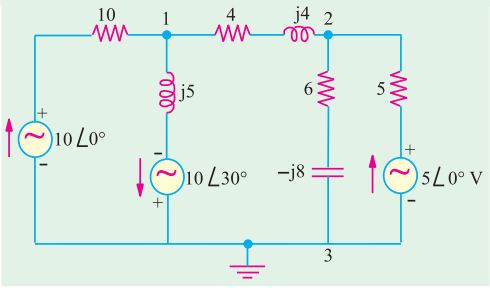


Fig. 15.18

Similarly, considering node 2, we have

$$V_2\left(\frac{1}{4+j4}+\frac{1}{5}+\frac{1}{6-j8}\right) - \frac{V_1}{4+j4} - \frac{5\angle 0^\circ}{5} = 0$$

Example 15.10. *In the network of Fig. 15.19 determine the current flowing through the branch of 4 Ω resistance using nodal analysis.* **(Network Analysis Nagpur Univ. 1993)**

Solution. We will find voltages V_A and V_B by using Nodal analysis and then find the current through 4 Ω resistor by dividing their difference by 4.

$$V_2\left(\frac{1}{5}+\frac{1}{4}+\frac{1}{j2}\right)-\frac{V_B}{4}-\frac{50\angle 30^\circ}{5}=0 \qquad \text{...for node A}$$

$\therefore \quad V_A(9-j10)-5V_B = 200\ \angle 30^\circ$ ...(*i*)

Similarly, from node *B*, we have

$$V_B\left(\frac{1}{4}+\frac{1}{2}+\frac{1}{-j2}\right)-\frac{V_A}{4}-\frac{50\angle 90^\circ}{2}=0 \quad \therefore V_B\,(3+j2)-V_A = 100\ \angle 90^\circ = j\,100 \qquad ...(ii)$$

V_A can be eliminated by multiplying. Eq. (*ii*) by (9+j10) and adding the result.

$$\therefore \quad V_B(42-j12) = 1173 + j1000 \quad \text{or} \quad V_B = \frac{1541.4\angle 40.40^\circ}{43.68\angle -15.9^\circ} = 35.29\angle 56.3^\circ = 19.58 + j29.36$$

Substituting this value of V_B in Eq. (*ii*), we get

$$V_A = V_B(3+j2)-j100 = (19.58 + j29.36)\,(3 + j2) - j100 = j27.26$$

$$\therefore \quad V_A - V_B = j\,27.26 - 19.58 - j29.36 = -19.58 - j2.1 = 19.69\angle 186.12^\circ$$

$$\therefore \quad I_2 = (V_A - V_B)/4 = 19.69\angle 186.12^\circ/4 = 4.92\angle 186.12^\circ$$

For academic interest only, we will solve the above question with the help of following two methods :

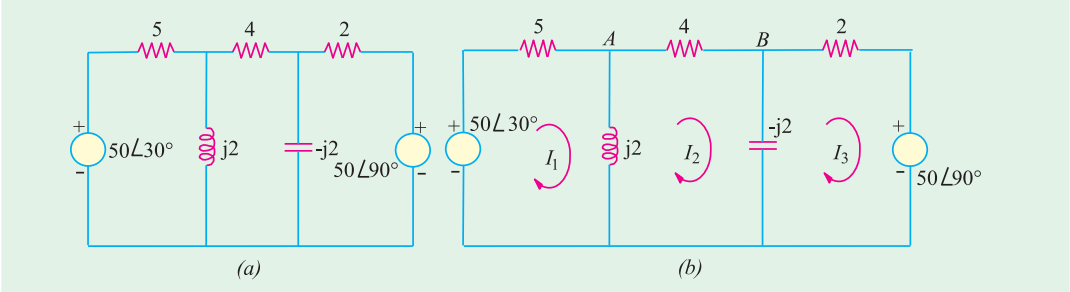


Fig. 15.19

Solution by Using Mesh Resistance Matrix

Let the mesh currents I_1, I_2 and I_3 be as shown in Fig. 15.19 (*b*). The different items of the mesh resistance matrix $[R_m]$ are as under :

$$R_{11} = (5 + j2)\ ;\ R_{22} = 4\ ;\ R_{33} = (2 - j2)\ ;\ R_{12} = R_{21} = -j2;$$

$$R_{23} = R_{32} = j2\ ;\ R_{31} = R_{13} = 0$$

The mesh equations in the matrix form are :

$$\begin{bmatrix}(5+j2) & -j2 & 0\\ -j2 & 4 & j2\\ 0 & j2 & (2-j2)\end{bmatrix}=\begin{bmatrix}I_1\\ I_2\\ I_3\end{bmatrix}=\begin{bmatrix}50\angle 30^\circ\\ 0\\ -j50\end{bmatrix}$$

$$\Delta = (5 + j2)\,[4(2 - j2) - (j2 \times j2)] - (-j2)\,[(-j2)\,(2-j2)] = 84 - j24 = 87.4\ \angle -15.9^\circ$$

$$\Delta_2 = \begin{bmatrix}(5+j2) & (43.3+j25) & -0\\ -j2 & 0 & j2\\ 0 & -j50 & (2-j2)\end{bmatrix} = (5 + j2)\,[-j2\,(-j50)] +$$

$$j2\,[43.3 + j25)\,(2 - j2)] = -427 + j73 = 433\angle 170.3^\circ$$

$$\therefore \qquad I_2\,\Delta_2/\Delta = 433\angle 170.3^\circ/87.4\angle -15.9^\circ = 4.95\angle 186.2^\circ$$

Solution by using Thevenin's Theorem

When the 4 Ω resistor is disconnected, the given figure becomes as shown in Fig. 15.20 (*a*). The voltage V_A is given by the drop across $j2$ reactance. Using the voltage-divider rule, we have

$$V_A = 50\angle 30^\circ \times \frac{j2}{5+j2} = 18.57\angle 98.2^\circ = -2.65 + j18.38$$

Similarly, $$V_B = 50\angle 90^\circ \frac{-j2}{2-j2} = 35.36\angle 45^\circ = 25 + j25$$

$$\therefore \qquad V_{th} = V_A - V_B = -2.65 + 18.38 - 25 - j25 = 28.43\angle 193.5^\circ$$

$$R_{th} = 5 \| j2 + 2 \| (-j2) = \frac{j10}{5+j2} + \frac{-j4}{2-j2} + 1.689 + j0.72$$

The Thevenin's equivalent circuit consists of a voltage source of 28.43 ∠ 193.5° V and an impedance of (1.689 + j0.72) Ω as shown in Fig. 15.20 (*c*). Total resistance is 4 + (1.689 + j0.72) = 5.689 + j 0.72 = 5.73∠7.2°. Hence, current through the 4 Ω resistor is 28.43 ∠193.5°/5.73∠7.20° = 4.96 ∠ 186.3°.

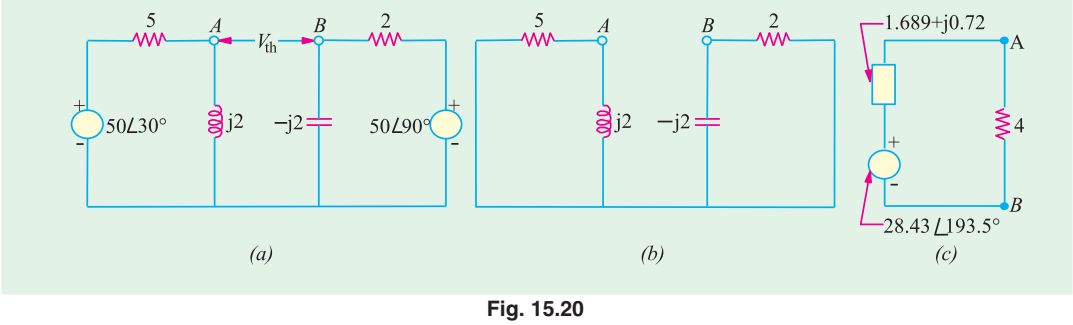


Fig. 15.20

Note. The slight variations in the answers are due to the approximations made during calculations.

Example 15.11. *Using any suitable method, calculate the current through 4 ohm resistance of the network shown in Fig. 15.21.* **(Network Analysis AMIE Sec. B Summer 1990)**

Solution. We will solve this question with the help of ***(i)*** Kirchhoff's laws ***(ii)*** Mesh analysis and ***(iii)*** Nodal analysis.

(*i*) Solution by using Kirchhoff's Laws

Let the current distribution be as shown in Fig. 15.21 (*b*). Using the same sign convention as given in Art. we have

First Loop $\quad -10(I_1 + I_2 + I_3) - (-j5)\, I_1 + 100 = 0$

or $\quad I_1\,(10 - j5) + 10\, I_3 + 10 I_2 = 100 \qquad ...(i)$

Second Loop $\quad -5(I_2 + I_3) - 4I_2 + (-j\,5)\, I_1 = 0$

or $\quad j5I_1 + 9\, I_2 + 5I_3 = 0 \qquad ...(ii)$

Third Loop $\quad -I_3\,(8 + j6) + 4I_2 = 0$

or $\quad 0I_1 + 4I_2 - I_3\,(8 + j6) = 0 \qquad ...(iii)$

The matrix form of the above three equations is

$$\begin{bmatrix} (10-j5) & -10 & 10 \\ j5 & 9 & 5 \\ 0 & 4 & -(8+j6) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix}$$

$$\Delta = (10 - j\,5)\,[-9\,(8 + j\,6) - 20] - j\,5\,[-10(8 + j\,6) - 40]$$
$$= -1490 + j\,520 = 1578\angle 160.8^\circ$$

Since we are interested in finding I_2 only, we will calculate the value of Δ_2.

$$\Delta_2 = \begin{bmatrix} (10-j5) & 100 & 10 \\ j5 & 0 & 5 \\ 0 & 0 & -(8+j6) \end{bmatrix}$$

$$= -j\,5\,(-800 - j\,600) = -3000 + j\,4000 = 5000\angle 126.9^\circ$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{500.0\angle 126.9^\circ}{1578\angle 160.8^\circ} = 3.17\angle -33.9^\circ \text{ A}$$

(*ii*) Solution by using Mesh Impedance Matrix

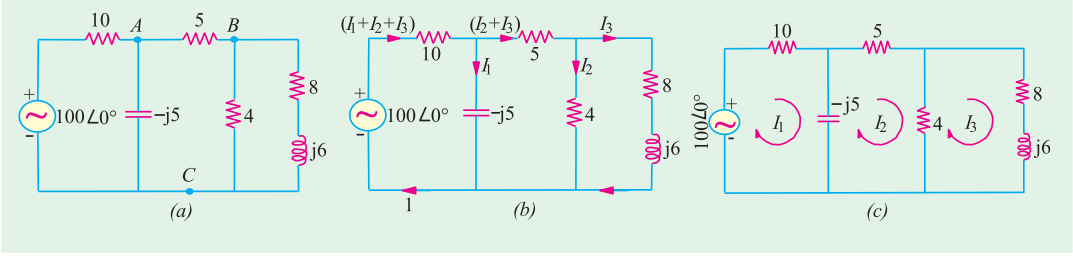


Fig. 15.21

Let the mesh currents I_1, I_2 and I_3 be as shown in Fig. 15.21 (*c*). From the inspection of Fig. 15.21 (*c*), the different items of the mesh impedance matrix $[Z_m]$ are as under :

$$Z_{11} = (10 - j\,5)\,;\, Z_{22} = (9 - j\,5);\, Z_{33} = (12 + j\,6)$$
$$Z_{21} = Z_{12} = -(-j\,5) = j\,5;\; Z_{23} = Z_{32} = -4;\, Z_{31} = Z_{13} = 0$$

Hence, the mesh equations in the matrix form are :

$$\begin{bmatrix} (10-j5) & j5 & 0 \\ j5 & (9-j5) & -4 \\ 0 & -4 & (12+j6) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore$ $$\Delta = (10 - j\,5)\,[(9 - j\,5)\,(12 + j\,6) - 16] - j5\,(j\,60 - 30)$$
$$= 1490 - j\,520 = 1578\angle 19.2^\circ$$

It should be noted that the current passing through 4 Ω resistance is the vector diference $(I_2 - I_3)$. Hence, we will find I_2 and I_3 only.

$$\Delta_2 = \begin{bmatrix} (10-j5) & 100 & 0 \\ j5 & 0 & -4 \\ 0 & 0 & (12+j6) \end{bmatrix} = j5\,(1200 + j600) = 3000 - j6000 = 6708\angle -63.4^\circ$$

$$\Delta_2 = \begin{bmatrix} (10-j5) & j5 & 100 \\ j5 & (9-j5) & 0 \\ 0 & -4 & 0 \end{bmatrix} = -j\,5\,(400) = -j\,2000 = 2000\angle -90^\circ$$

$\therefore$ $$I_2 = \frac{\Delta_2}{\Delta} = \frac{6708\angle -63.4^\circ}{1578\angle -19.2^\circ} = 4.25\angle -44.2^\circ = 3.05 - j\,2.96$$

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{2000\angle -90^\circ}{1578\angle -19.2^\circ} = 1.27\angle -70.8^\circ = 0.42 - j\,1.2$$

Current $(I_2 - I_3) = 2.63 - j\,1.76 = 3.17\angle 33.9^\circ$

(*iii*) Solution by Nodal Analysis

The current passing through 4 Ω resistance can be found by finding the voltage V_B of node B with the help of Nodal analysis. For this purpose point C in Fig. 15.21 (a) has been taken as the reference node. Using the Nodal technique as explained in Art. we have

$$V_A\left(\frac{1}{10}+\frac{1}{5}+\frac{1}{-j5}\right)-\frac{V_B}{5}-\frac{100\angle 0^\circ}{10}=0 \qquad \text{...for node A}$$

$$V_A(3 + j\,2) - 2\,V_B = 100 \qquad \text{...(}i\text{)}$$

Similarly, for node B, we have

$$V_B\left(\frac{1}{5}+\frac{1}{4}+\frac{1}{(8+j6)}\right)-\frac{V_A}{5}=0 \quad \text{or} \quad V_B(53 - j\,6) - 20\,V_A = 0 \qquad \text{...(}ii\text{)}$$

Estimating V_A from Eq. (i) and (ii), we have

$$V_B(131 + j88) = 2000 \quad \text{or} \quad V_B = 12.67\angle 33.9^\circ$$

Current through 4 Ω resistor $12.67\angle 33.9^\circ/4 = 3.17\angle 33.9^\circ$

Tutorial Problems No. 15.3

1. Apply nodal analysis to the network of Fig. 15.22 to determine the voltage at node A and the active power delivered by the voltage source. **[8∠3.7° V; 9.85 W]**

2. Using nodal analysis, determine the value of voltages at models 1 and 2 in Fig. 15.23.

[V_1 = 88.1 ∠33.88° A; V_2 = 58.7∠72.34°A]

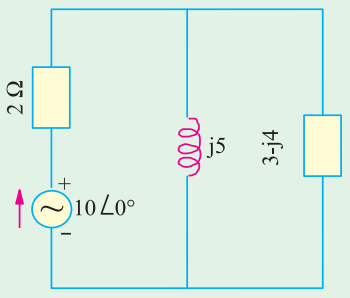


Fig. 15.22

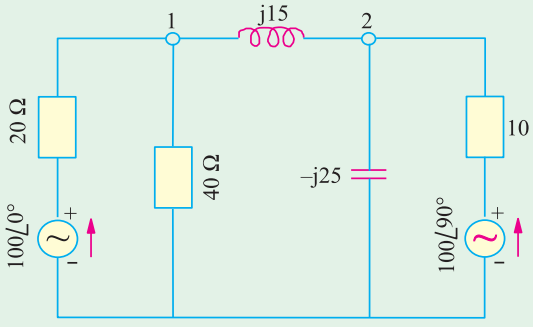


Fig. 15.23

3. Using Nodal analysis, find the nodal voltages V_1 and V_2 in the circuit shown in Fig. 15.24. All resistances are given in terms of siemens **[V_1 = 1.64 V : V_2 = 0.38 V]**

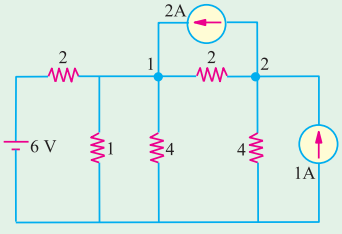


Fig. 15.24

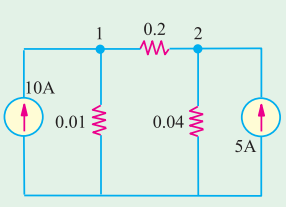


Fig. 15.25

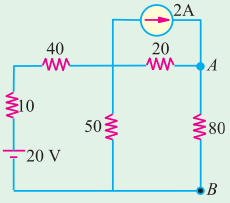


Fig. 15.26

4. Find the values of nodal voltages V_1 and V_2 in the circuit of Fig. 15.25. Hence, find the current going from node 1 to node 2. All resistances are given in siemens. **[V_1 = 327 V; V_2 = 293.35 V ; 6.73 A]**

5. Using Nodal analysis, find the voltage across points *A* and *B* in the circuit of Fig. 15.26: Check your answer by using mesh analysis. **[32 V]**

15.5. Superposition Theorem

As applicable to a.c. networks, it states as follows :

In any network made up of linear impedances and containing more than one source of e.m.f., the current flowing in any branch is the phasor sum of the currents that would flow in that branch if each source were considered separately, all other e.m.f. sources being replaced for the time being, by their respective internal impedances (if any).

Note. It may be noted that independent sources can be 'killed' *i.e.* removed leaving behind their internal impedances (if any) but dependent sources should not be killed.

Example 15.12. *Use Superposition theorem to find the voltage V in the network shown in Fig. 15.27.*

Solution. When the voltage source is killed, the circuit becomes as shown in the Fig. 15.27 (*b*) Using current-divider rule,

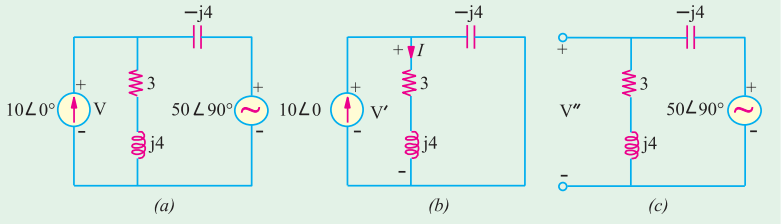


Fig. 15.27

$$I = 10\angle 0^\circ \times \frac{-j4}{(3+j4)-j4}, \text{ Now, } V' = I(3+j4)$$

$$\therefore \quad V' = 10\,\frac{-j4(3+j4)}{3} = 53.3 - j\,40$$

Now, when current source is killed, the circuit becomes as shown in Fig. 15.27 (*c*). Using the voltage-divider rule, we have

$$V'' = 50\angle 90^\circ \times \frac{(3+j4)}{(3+j4)-j4} = -66.7 + j\,50$$

$$\therefore \quad \text{drop V} = V' + V'' = 53.3 - j\,40\ (-66.7 + j\,50) = -13.4 + j\,10 = 16.7 \angle 143.3^\circ\ V$$

Tutorial Problems No. 15.4

1. Using Superposition theorem to find the magnitude of the current flowing in the branch *AB* of the circuit shown in Fig. 15.28.

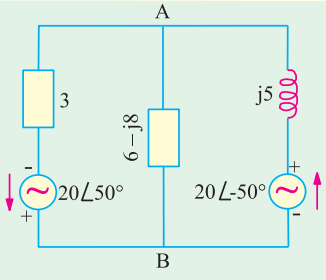


Fig. 15.28

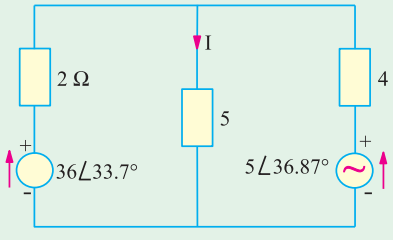


Fig. 15.29

2. Apply Superposition theorem to determine the circuit *I* in the circuit of Fig. 15.29. **[0.53 ∠ 5.7° A]**

15.6. Thevenin's Theorem


Leon-Charles Thevenin

As applicable to a.c. networks, this theorem may be stated as follows :

The current through a load impedance Z_L connected across any two terminals A and B of a linear network is given by $V_{th}/(Z_{th} + Z_L)$ wher V_{th} is the open-circuit voltage across A and B and Z_{th} is the internal impedance of the network as viewed from the open-circuited terminals A and B with all voltage sources replaced by their internal impedances (if any) and current sources by infinite impedance.

Example 15.13. *In the network shZown in Fig. 15.30.*

$Z_1 = (8 + j8)\ \Omega$; $Z_2 = (8 - j8)\ \Omega$; $Z_3 = (2 + j20)$; $V = 10 \angle 0^\circ$ and $Z_L = j\,10\ \Omega$

Find the current through the load Z_L using Thevenin's theorem.

Solution. When the load impedance Z_L is removed, the circuit becomes as shown in Fig. 15.30 (*b*). The open-circuit voltage which appears across terminals A and B represents the Thevenin voltage V_{th}. This voltage equals the drop across Z_2 because there is no current flow through Z_3.

Current flowing through Z_1 and Z_2 is

$$I = V(Z_1 + Z_2) = 10 \angle 0^\circ\ [(8 + j8) + (8 - j8)] = 10 \angle 0^\circ/16 = 0.625 \angle 0^\circ$$

$$V_{th} = IZ_2 = 0.625\ (8 - j\,8) = (5 - j\,5) = 7.07 \angle -45^\circ$$

The Thevenin impedance Z_{th} is equal to the impedance as viewed from open terminals A and B with voltage source shorted.

$$\therefore \quad Z_{th} = Z_3 + Z_1 \,||\, Z_2 = (2 + j\,20) + (8 + j\,8) \,||\, (8 - j\,8) = (10 + j\,20)$$

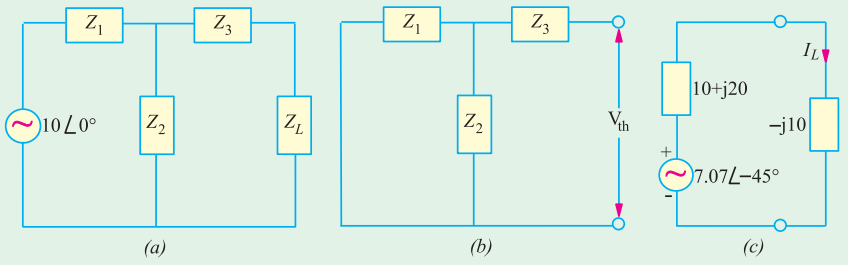


Fig. 15.30

The equivalent Thevenin circuit is shown in Fig. 15.30 (*c*) across which the load impedance has been reconnected. The load current is given by

$$\therefore \quad I_L = \frac{V_{th}}{Z_{th} + Z_L} = \frac{(5 - j5)}{(10 + j\,20) + (-j10)} = \frac{-j}{2}$$

Example 15.13 A. *Find the Thevenin equivalent circuit at terminals AB of the circuit given in Fig. 15.31 (a).*

Solution. For finding $V_{th} = V_{AB}$, we have to find the phasor sum of the voltages available on the way as we go from point B to point A because V_{AB} means voltage of point A with respect to that of point B (Art.). The value of current $I = 100 \angle 0^\circ/(6 - j\,8) = (6 + j\,8)A$.

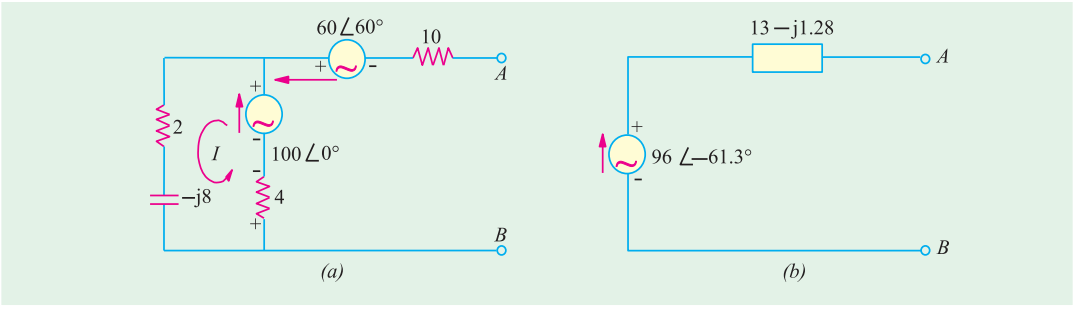


Fig. 15.31

Drop across 4 Ω resistor = 4 (6 + *j* 8) = (24 + *j* 32)

$$\therefore \quad V_{th} = V_{AB} = -(24 + j\,32) + (100 + j\,0) - 60(0.5 + j\,0.866)$$
$$= 46 - j\,84 = 96 \angle -61.3^{\circ}$$
$$Z_{AB} = Z_{th} = [10 + [4 \,||\, (2 - j\,8)] = (13 - j\,1.28)$$

The Thevenin equivalent circuit is shown in Fig. 15.31 (*b*).

Example 15.14. *Find the Thevenin's equivalent of the circuit shown in Fig. 15.32 and hence calculate the value of the current which will flow in an impedance of (6 + j 30) Ω connected across terminals A and B. Also calculate the power dissipated in this impedance.*

Solution. Let us first find the value of V_{th} *i.e.* the Thevenin voltage across open terminals *A* and *B*. With terminals *A* and *B* open, there is no potential drop across the capacitor. Hence, V_{th} is the drop across the pure inductor *j*3 ohm.

$$\text{Drop across the inductor} = \frac{10 + j0}{(4 + j3)} \times j3 = \frac{j30}{4 + j3} = \frac{j30(4 - j3)}{4^2 + 3^2} = (3.6 + j\,4.8)V$$

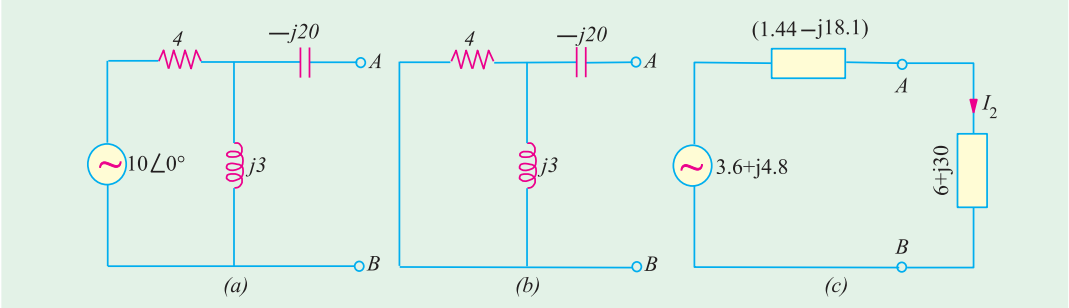


Fig. 15.32

Let us now find the impedance of the circuit as viewed from terminals *A* and *B* after replacing the voltage source by a short circuit as shown in Fig. 15.32 (*a*).

$$Z_{th} = -j\,20 + 4 \,||\, j3 = -j20 + 1.44 + j1.92 = 1.44 - j18.1$$

The equivalent Thevenin circuit along with the load impedance of (6 + *j*30) is shown in Fig. 15.32 (*c*).

$$\text{Load current} = \frac{(3.6 + j\,4.8)}{(1.44 - j18.1) + (6 + j30)} = \frac{(3.6 + j4.8)}{(7.44 + j11.9)} = \frac{6\angle 53.1^{\circ}}{14\angle 58^{\circ}} = 0.43 \angle 4.9^{\circ}$$

The current in the load is 0.43 A and lags the supply voltage by 4.9°

Power in the load impedance is $0.43^2 \times 6 =$ **1.1 W**

Example 15.15. *Using Thevenin's theorem, calculate the current flowing through the load connected across terminals A and B of the circuit shown in Fig. 15.33 (a). Also calculate the power delivered to the load.*

Solution. The first step is to remove the load from the terminals A and B. $V_{th} = V_{AB}$ = drop across $(10 + j10)$ ohm with A and B open.

$$\text{Circuit current } I = \frac{100}{-j10 + 10 + j10} = 10 \angle 0^\circ$$

$$\therefore \quad V_{th} = 10(10 + j10) = 141.4 \angle 45^\circ$$
$$Z_{th} = (-j10) \,||\, (10 + j10) = (10 - j10)$$

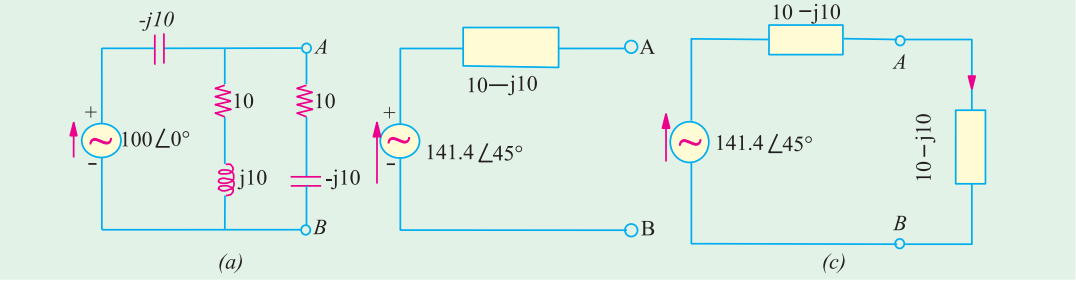


Fig. 15.33

The equivalent Thevenin's source is shown in Fig. 15.33 (*b*). Let the load be re-connected across A and B shown in Fig. 15.33 (*c*).

$$\therefore \quad I_L = \frac{141.4 \angle 45^\circ}{(10 - j10) + (10 - j10)} = \frac{141.4 \angle 45^\circ}{20 - j20} = \frac{141.4 \angle 45^\circ}{28.3^\circ \angle -45^\circ} = 5 \angle 90^\circ$$

Power delivered to the load = $I_L^2 R_L = 5^2 \times 10 = 250$ W

Example 15.16. *Find the Thevenin's equivalent across terminals A and B of the networks shown in Fig. 15.34. (a).*

Solution. The solution of this circuit involves the following steps :

(i) Let us find the equivalent Thevenin voltage V_{CD} and Thevenin impedance Z_{CD} as viewed from terminals C and D.

$$V_{CD} = V \frac{Z_2}{Z_1 + Z_2} = \frac{100 \angle 0^\circ \times 20 \angle -30^\circ}{10 \angle 30^\circ + 20 \angle -30^\circ} = 75.5 \angle 19.1^\circ V$$

$$Z_{CD} = Z_1 \,||\, Z_2 = \frac{10 \angle 30^\circ \times 20 \angle -30^\circ}{10 \angle 30^\circ + 20 \angle -30^\circ} = 7.55 \angle 10.9^\circ \text{ ohm}$$

(ii) Using the source conversion technique (Art) we will replace the $5 \angle 0^\circ$ current source by a voltage source as shown in Fig. 15.34 (*b*).

$$V_{EC} = 5 \angle 0^\circ \times 10 \angle 30^\circ = 50 \angle 30^\circ$$

Its series resistance is the same as $Z_3 = 10 \angle 30^\circ$ as shown in Fig. 15.34 (*b*).

The polarity of the voltage source is such that it sends current in the direction EC, as before.

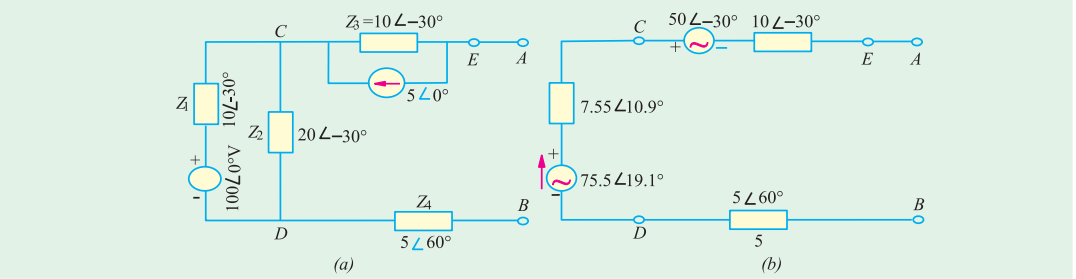


Fig. 15.34

(iii) From the above information, we can find V_{th} and Z_{th}

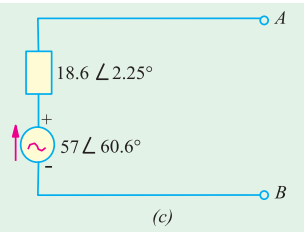


Fig. 15.34

$V_{th} = V_{CD} = 75.5\angle 19.1^\circ - 50 \angle 30^\circ = 57 \angle 60.6^\circ$

$Z_{th} = Z_{CD} = 10\angle -30^\circ + 7.55\angle 10.9^\circ + 5\angle 60^\circ = 18.6 \angle 2.25^\circ$

The Thevenin equivalent with respect to the terminals *A* and *B* is shown in Fig. 15.34 (*c*).

For finding V_{AB} *i.e.* voltage at point *A* with respect to point *B*, we start from point B in Fig. 15.34 (*b*) and go to point *A* and calculate the phasor sum of the voltages met on the way.

$\therefore V_{AB} = 75.1\angle 19.1^\circ - 50\angle 30^\circ = 57\angle 60.6^\circ$

$Z_{AB} = 10\angle 30^\circ + 7.55\angle 10.9^\circ + 5\angle 60^\circ = 18.6\angle 2.25^\circ$

Example 15.17. *For the network shown, determine using Thevenin's theorem, voltage across capacitor in. Fig. 15.35.* **(Elect. Network Analysis Nagpur Univ. 1993)**

$Z_{CD} = j5 \,||\, (10 + j5) = 1.25 + j3.75$. This impedance is in series with the 10Ω resistance. Using voltage divider rule, the drop over Z_{CD} is

Solution. When load of -j5 Ω is removed the circuit becomes as shown in Fig. 15.35 (*b*). Thevenin voltage is given by the voltage drop produced by 100-V source over (5 + *j*5)impedance. It can be calculated as under.

$$V_{CD} = 100\frac{(1.25 + j3.75)}{10 + (1.25 + j3.75)} = \frac{125 + j375}{11.25 + j3.75}$$

This V_{CD} is applied across j5 reactance as well as across the series combination of 5Ω and (5 + *j*5) Ω. Again, using voltage-divider rule for V_{CD}, we get

$$V_{AB} = V_{th} = V_{CD} \times \frac{5 + j5}{10 \times j5} = \frac{(125 + j375)}{11.25 + j3.75} \times \frac{5 + j5}{10 + j5} = 21.1\angle 71.57^\circ = 6.67 + j20$$

As looked into terminals *A* and *B*, the equivalent impedance is given by

$$R_{AB} = R_{th} = (5 + j5) \,||\, (5 + 10 \,||\, j5) = (5 + j5) \,||\, (7 + j4) = 3 + j\,2.33$$

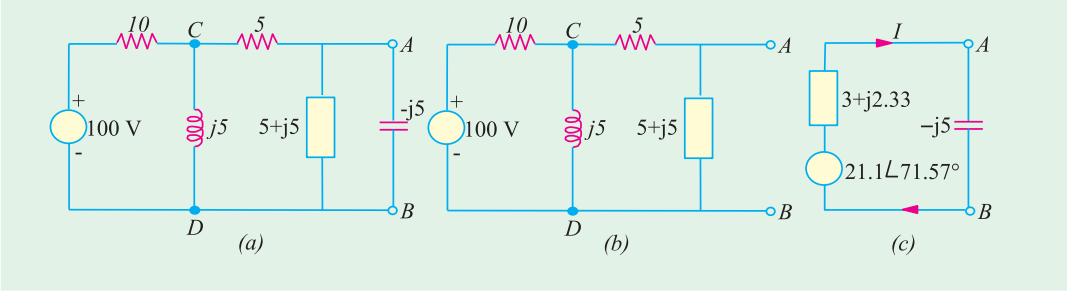


Fig. 15.35

The equivalent Thevenin's source along with the load is shown in Fig. 15.35 (*c*).

Total impedance $= 3 + j\,2.33 - j\,5 = 3 - j2.67 = 4.02 \angle 41.67^\circ$

$\therefore \quad I = 21.1\angle 71.57^\circ / 4.02\angle 41.67^\circ = 5.25\angle 113.24^\circ$

Solution by Mesh Resistance Matrix

The different items of the mesh resistance matrix $[R_m]$ are as under :

$R_{11} = 10 + j\,5$; $R_{22} = 10 + j\,10$; $R_{33} = 5$;

$R_{12} = R_{21} = -j5$;

$R_{23} = R_{32} = -(5 + j5)$; $R_{31} = R_{13} = 0$. Hence, the mesh equations in the matrix form are as given below :

$$\begin{bmatrix} (10+j5) & -j5 & 0 \\ -j5 & (10+j110) & -(5+j5) \\ 0 & -(5+j5) & 5 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore \quad \Delta = (10 + j5)\,[5(10 + j10) - (5 + j5)\,(5 + j5)] + j5\,(-j25) = 625 + j250 = 673\angle 21.8^\circ$

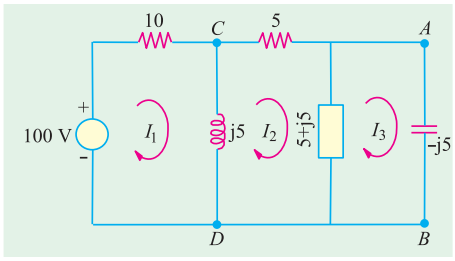


Fig. 15.36

$$\begin{bmatrix} (10+j5) & -j5 & 100 \\ -j5 & (10+j10) & 0 \\ 0 & -(5+j5) & 0 \end{bmatrix} = j5\,(500 + j500) = 3535 \angle 135^\circ$$

$$\therefore \qquad I_3 = \Delta_3/\Delta = 3535\angle 135^\circ / 673\angle 21.8^\circ = 5.25\angle 113.2^\circ$$

Tutorial Problems No. 15.5

1. Determine the Thevenin's equivalent circuit with respect to terminals *AB* of the circuit shown in Fig. 15.37. **[$V_{th} = 14.3\angle 6.38^\circ$, $Z_m = (4 + j0.55)\ \Omega$]**

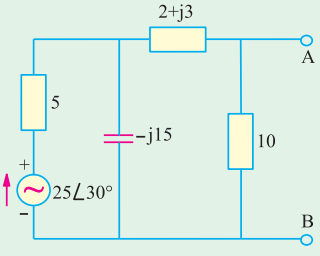


Fig. 15.37

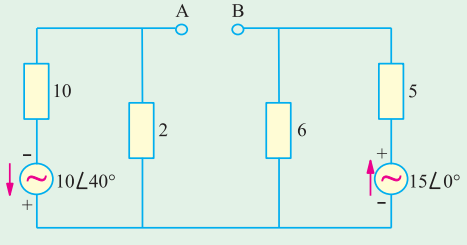


Fig. 15.38

2. Determine Thevenin's equivalent circuit with respect to terminals *AB* in Fig. 15.38.

[$V_{th} = 9.5\ \angle 6.46^\circ$; $Z_{th} = 4.4\ \angle 0^\circ$]

3. The e.m.fs. of two voltage source shown in Fig. 15.39 are in phase with each other. Using Thevenin's theorem, find the current which will flow in a 16 Ω resistor connected across terminals *A* and *B*.

[$V_{th} = 100$ V ; $Z_{th} = (48 + j32)$; $I = 1.44\ \angle -26.56^\circ$]

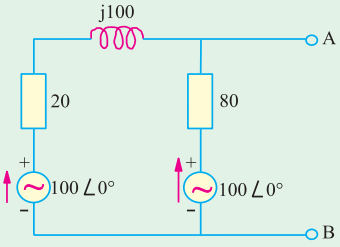


Fig. 15.39

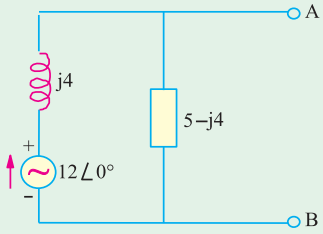


Fig. 15.40

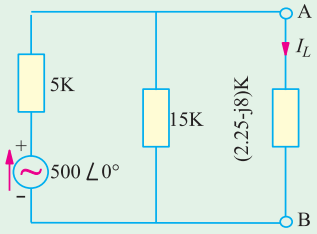


Fig. 15.41

4. Find the Thevenin's equivalent circuit for terminals *AB* for the circuit shown in Fig. 15.40.

[$V_{th} = 15.37\ \angle 38.66^\circ$; $Z_{th} = (3.2 + j4)\ \Omega$]

5. Using Thevenin's theorem, find the magnitude of the load current I_L passing through the load connected across terminals *AB* of the circuit shown in Fig. 15.41. **[37.5 mA]**

6. By using Thevenin's theorem, calculate the current flowing through the load connected across terminals *A* and *B* of circuit shown in Fig. 15.42. All resistances and reactances are in ohms.

[V_{th} = 56.9 ∠50.15° ; 3.11 ∠85.67°]

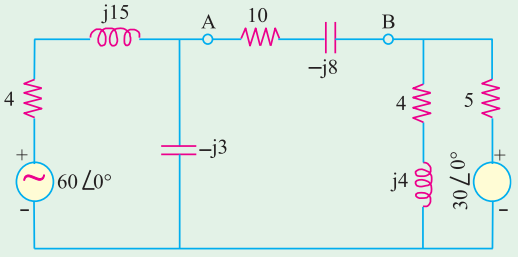


Fig. 15.42

7. Calculate the equivalent Thevenin's source with respect to the terminals *A* and *B* of the circuit shown in Fig. 15.43. **[V_{th} = (6.34 + j2.93) V ; Z_{th} = (3.17 –j5.07) Ω]**

8. What is the Thevenin's equivalent source with respect to the terminals *A* and *B* of the circuit shown in Fig. 15.44 ?

[V_{th} = (9.33 + j8) V ; Z_{th} = (8 –j11) Ω]

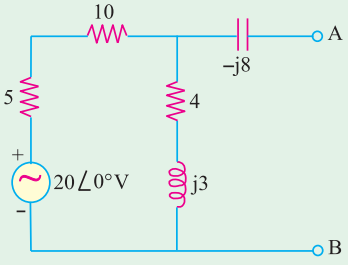


Fig. 15.43

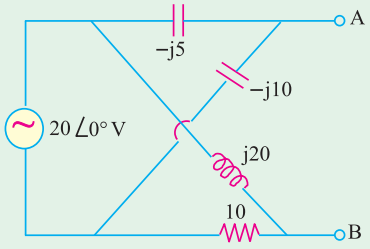


Fig. 15.44

9. What is the Thevenin's equivalent soure with respect to terminals *A* and *B* the circuit shown in Fig. 15.45 ? Also, calculate the value of impedance which should be connected across *AB* for *MPT*. All resistances and reactances are in ohms. **[V_{th} = (16.87 + j15.16) V; Z_{th} = (17.93 –j1.75) Ω ; (17.93 + j1.75 Ω]**

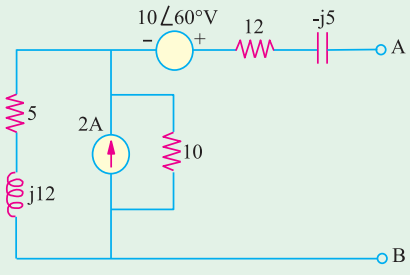


Fig. 15.45

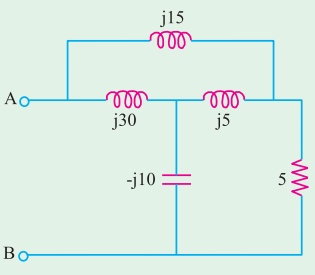


Fig.15.46

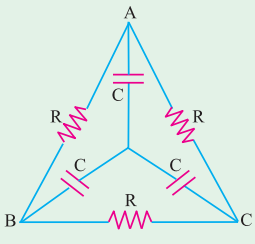


Fig. 15.47

10. Find the impedance of the network shown in Fig. 15.46, when viewed from the terminals *A* and *B*. All resistances and reactances are in ohms. **[(4.435 + j6.878)]**

11. Find the value of the impedance that would be measured across terminals *BC* of the circuit shown in Fig. 15.47.

$$\left[\frac{2R}{9+\omega^2C^2R^2}(3-j\omega CR)\right]$$

15.7. Reciprocity Theorem

This theorem applies to networks containing linear bilateral elements and a single voltage source or a single current source. This theorem may be stated as follows :

If a voltage source in branch *A* of a network causes a current of 1 branch *B*, then shifting the voltage source (but not its impedance) of branch *B* will cause the same current *I* in branch A.

It may be noted that currents in other branches will generally not remain the same. A simple way of stating the above theorem is that if an ideal voltage source and an ideal ammeter are inter-changed, the ammeter reading would remain the same. The ratio of the input voltage in branch *A* to the output current in branch *B* is called the transfer impedance.

Similarly, if a current source between nodes 1 and 2 causes a potential difference of *V* between nodes 3 and 4, shifting the current source (but not its admittance) to nodes 3 and 4 causes the same voltage *V* between nodes 1 and 2.

In other words, the interchange of an ideal current source and an ideal voltmeter in any linear bilateral network does not change the voltmeter reading.

However, the voltages between other nodes would generally not remain the same. The ratio of the input current between one set of nodes to output voltage between another set of nodes is called the transfer admittance.

Example 15.18. *Verify Reciprocity theorem for V & I in the circuit shown in Fig. 15.48.*

[Elect. Network Analysis, Nagpur Univ. 1993]

Solution. We will find the value of the current *I* as read by the ammeter first by applying series parallel circuits technique and then by using mesh resistance matrix (Art.)

1. Series Parallel Circuit Technique

The total impedance as seen by the voltage source is

$$= 1 + [j\,1 \,||\, (2 - j\,1)] \;=\; 1 + \frac{j1(2 - j1)}{2} \;=\; 15 + j\,1$$

$$\therefore \qquad \text{total circuit current } i \;=\; \frac{5\angle 0^{o}}{1.5 + j1}$$

This current gets divided into two parts at point *A*, one part going through the ammeter and the other going along *AB*. By using current-divider rule. (Art), we have

$$I \;=\; \frac{5}{1.5 + j1} \times \frac{J1}{(2 + j1 - j1)} = \frac{j5}{3 + j\,2}$$

2. Mesh Resistance Matrix

In Fig. 15.48 (*b*), $R_{11} = (1 + j1)$, $R_{22} = (2 + j1 - j1) = 2$; $R_{12} = R_{21} = -j1$

$$\therefore \qquad \begin{vmatrix} (1+j1) & -j1 \\ -j1 & 2 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \end{vmatrix} \;=\; \begin{vmatrix} 5 \\ I0 \end{vmatrix} \;;\; \Delta = 2(1+j\,1) - (-j1)\,(-j1) = 3 + j\,2$$

$$\Delta_2 = \begin{vmatrix} (1+j\,1) & 5 \\ -j1 & 0 \end{vmatrix} \;=\; 0 + j\,5 = j5 \;;\; I_2 = I = \frac{\Delta_2}{\Delta} = \frac{j5}{(3 + j\,2)}$$

As shown in Fig. 15.48 (*c*), the voltage source has been interchanged with the ammeter. The polarity of the voltage source should be noted in particular. It looks as if the voltage source has been pushed along the wire in the counterclockwise direction to its new position, thus giving the voltage polarity as shown in the figure. We will find the value of *I* in the new position of the ammeter by using the same two techniques as above.

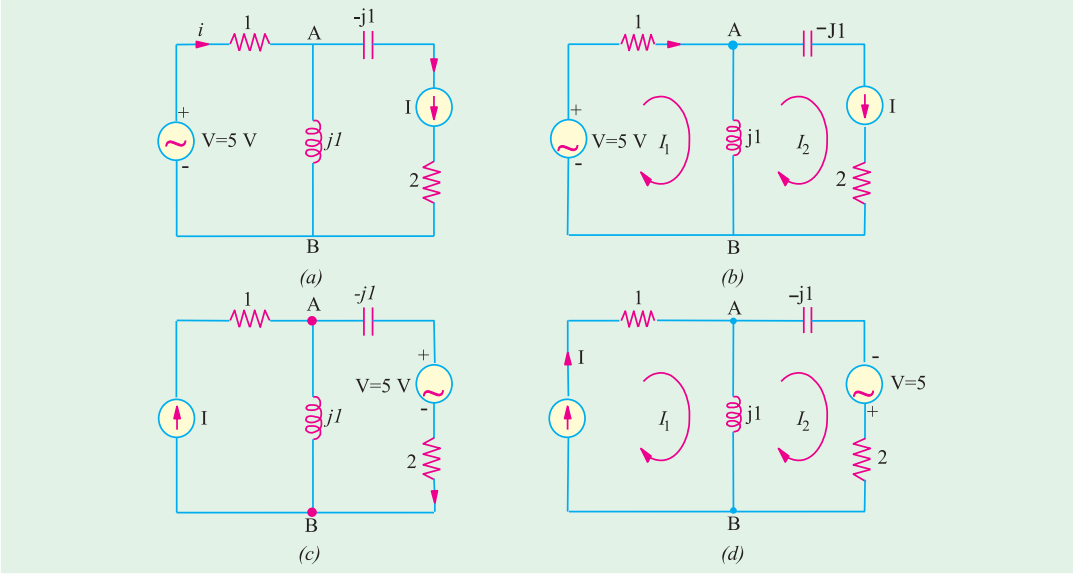


Fig. 15.48

1. Series Parallel Circuit Technique

As seen by the voltage source from its new position, the total circuit impedance is

$$= 2\,(2 - j\,1) + j\,1 \,||\, 1 = \frac{3 + j\,2}{1 + j1}$$

The total circuit current $i = 5 \times \dfrac{1 + j1}{3 + j\,2}$

This current i gets divided into two parts at point B as per the current-divider rule.

$$I = \frac{5(1 + j1)}{3 + j\,2} \times j\,\frac{1}{1 + j1} = \frac{j5}{3 + j\,2}$$

2. Mesh Resistance Matrix

As seen from Fig. 15.48 (*d*).

$$\begin{vmatrix} (1 + j1) & -j1 \\ -j1 & 2 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \end{vmatrix} = \begin{vmatrix} 0 \\ 5 \end{vmatrix}; \Delta = 2(1 + j\,1) + 1 = 3 + j\,2$$

$$\Delta = \begin{vmatrix} 0 & -j1 \\ 5 & 2 \end{vmatrix} = j5; I = I_1 = \frac{\Delta_1}{\Delta} = \frac{j\,5}{3 + j\,2}$$

The reciprocity theorem stands verified from the above results.

Tutorial problem No. 15.6

1. State reciprocity theorem. Verify for the circuit Fig. 15.49, with the help of any suitable current through any element.

(Elect. Network Analysis Nagpur Univ. 1993)

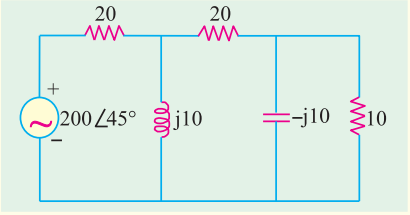


Fig. 15.49

15.8. Norton's Theorem

As applied to a.c. networks, this theorem can be stated as under :

Any two terminal active linear network containing voltage sources and impedances when viewed from its output terminals is equivalent to a constant current source and a parallel impedance. The constant current is equal to the current which would flow in a short-circuit placed across the terminals and the parallel impedance is the impedance of the network when viewed from open-circuited terminals after voltage sources have been replaced by their internal impedances (if any) and current sources by infinite impedance.

Example 15.19. *Find the Norton's equivalent of the circuit shown in Fig. 15.50. Also find the current which will flow through an impedance of (10 –j 20) Ω across the terminals A and B.*

Solution. As shown in Fig. 15.50 (*b*), the terminals *A* and *B* have been short-cicuited.

$\therefore \quad I_{SC} = I_N = 25/(10 + j\,20) = 25/22.36 \angle 63.4^\circ = 1.118 \angle -63.4^\circ$

When voltage source is replaced by a short, then the internal resistance of the circuit, as viewed from open terminals *A* and *B*, is $R_N = (10 + j20)\Omega$. Hence, Norton's equivalent circuit becomes as shown in Fig. 15.50(*c*).

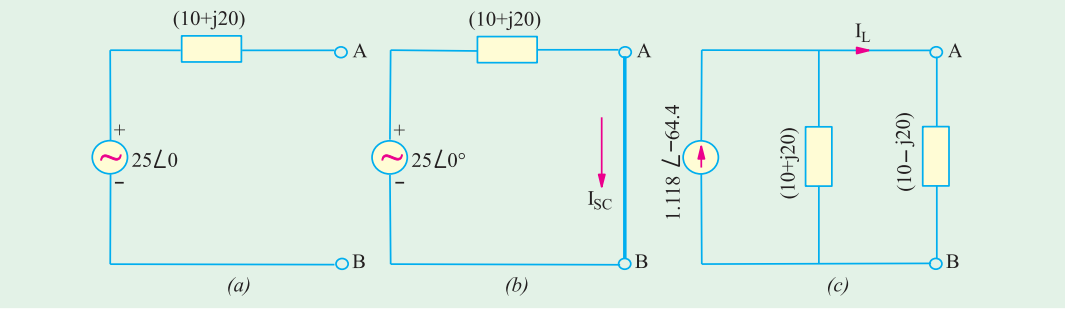


Fig. 15.50

When the load impedance of (10 –*j*20) is applied across the terminals *A* and *B*, current through it can be found with the help of current-divider rule.

$$\therefore \qquad I_L = 1.118 \angle -63.4^\circ \times \frac{(10 + j\,20)}{(10 + j\,20) + (10 - j\,20)} = 1.25 \text{ A}$$

Example 15.20. *Use Norton's theorem to find current in the load connected across terminals A and B of the circuit shown in Fig. 15.51 (a).*

Solution. The first step is to short-circuit terminals *A* and *B* as shown in Fig. 15.51 (*a*)*. The short across *A* and *B* not only short-circuits the load but the (10 + *j* 10) impedance as well.

$$I_N = 100 \angle 0^\circ / (-j\,10) = j\,10 = 10 \angle 90^\circ$$

Since the impedance of the Norton and Thevenin equivalent circuits is the same, $Z_N = 10 - j10$.

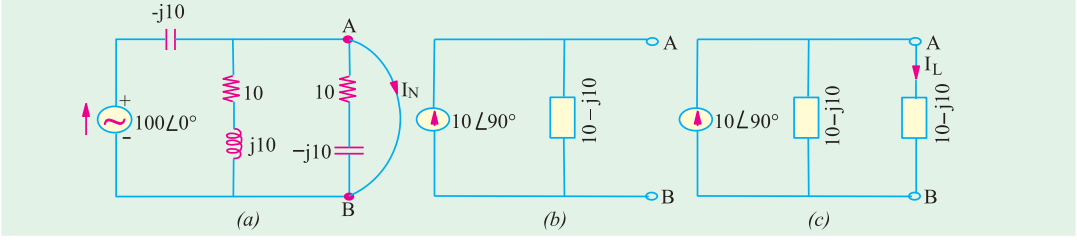


Fig. 15.51

* For finding I_N, we may or may not remove the load from the terminals (because, in either case, it would be short-circuited) but for finding R_N, it has to be removed as in the case of Thevenin's theorem.

The Norton's equivalent circuit is shown in Fig. 15.51 (*b*). In Fig. 15.51 (*c*), the load has been reconnected across the terminals *A* and *B*. Since the two impedances are equal, current through each is half of the total current *i.e.* $10\angle 90^\circ/2 = 5\angle 90^\circ$.

Tutorial Problems No. 15.7

1. Find the Norton's equivalent source with respect to terminals A and B of the networks shown in Fig. 15.51 (a) (*b*). All resistances and reactances are expressed in siemens in Fig. 15.51 (a) and in ohms in Fig. 15.52.

[(*a*) $I_N = -(2.1 - j3)$ A; $1/Z_N = (0.39 + j0.3)$S (b) $I_N = (6.87 + j0.5)$ A; $1/Z_N = (3.17 + j1.46$S]

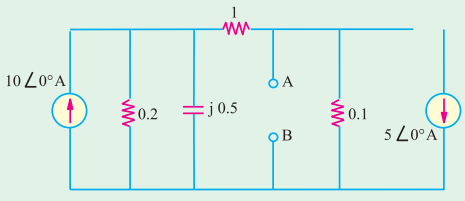


Fig. 15.52

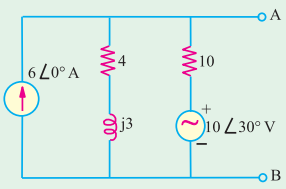


Fig. 15.53

2. Find the Nortons equivalent source with respect to terminals *A* and *B* for the circuit shown in Fig. 15.54. Hence, find the voltage V_L across the 100 Ω load and check its result by using Millman's theorem. All resistances are in ohms. **[$I_N = 9A$; $Y_N = 0.15\ S$; $V_L = 56.25\angle 0^\circ$]**

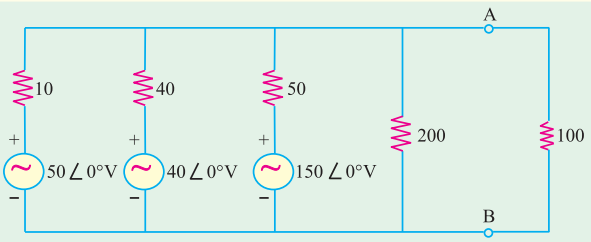


Fig. 15.54

3. Find the Norton's equivalent network at terminals *AB* of the circuit shown in Fig. 15.55.

[$I_{SC} = 2.17\angle -44^\circ$ A; $Z_N = (2.4 + j1.47)$ Ω]

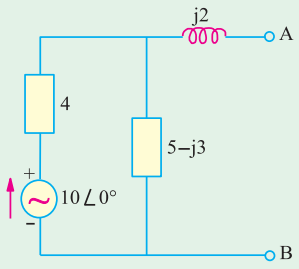


Fig. 15.55

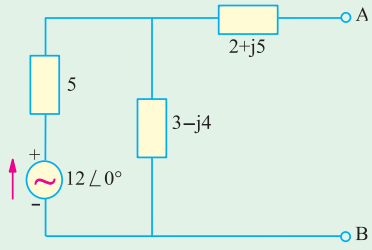


Fig. 15.56

4. What is the Norton equivalent circuit at terminals AB of the network shown in Fig. 15.56

[$I_{SC} = 1.15\angle -66.37^\circ$; $Z_N = (4.5 + j3.75)$ Ω]

15.9. Maximum Power Transfer Theorem

As explained earlier in Art. this theorem is particularly useful for analysing communication networks where the goals is transfer of maximum power between two circuits and not highest efficiency.

15.10. Maximum Power Transfer Theorems - General Case

We will consider the following maximum power transfer theorems when the source has a fixed complex impedance and delivers power to a load consisting of a variable resistance or a variable complex impedance.

Case 1. When load consists only for a variable resistance R_L [Fig. 15.57 (*a*)]. The circuit current is

$$I = \frac{V_g}{\sqrt{(R_g + R_L)^2 + X_g^2}}$$

Power delivered to R_L is $P_L = \dfrac{V_g^2 R_L}{(R_g + R_L)^2 + X_g^2}$

To determine the value of R_L for maximum transfer of power, we should set the first derivative dP_L/dR_L to zero.

$$\frac{dP_L}{dR_L} = \frac{d}{dR_L}\left[\frac{V_g^2 R_L}{(R_g + R_L)^2 + X_g^2}\right] = V_g^2\left\{\frac{[(R_g + R_L)^2 + X_g^2] - R_L(2)(Rg + R_L)}{[(R_g + R_L)^2 + X_g^2]^2}\right\} = 0$$

or $R_g^2 + 2R_gR_L + R_L^2 + X_g^2 - 2R_LR_g - 2R_L^2 = 0$ and $R_g^2 + X_g^2 = R_L^2$

$\therefore$ $R_L = \sqrt{R_g^2 + X_g^2} = |Z_g|$

It means that with a variable pure resistive load, maximum power is delivered across the terminals of an active network only when the load resistance is equal to the absolute value of the impedance of the active network. Such a match is called magnitude match.

Moreover, if X_g is zero, then for maximum power transfer $R_L = R_g$

Case 2. Load impedance having both variable resistance and variable reactance [Fig. 15.57 (*b*)].

The circuit current is $I = \dfrac{V_g}{\sqrt{(R_g + R_L)^2 + (X_g + X_L)^2}}$

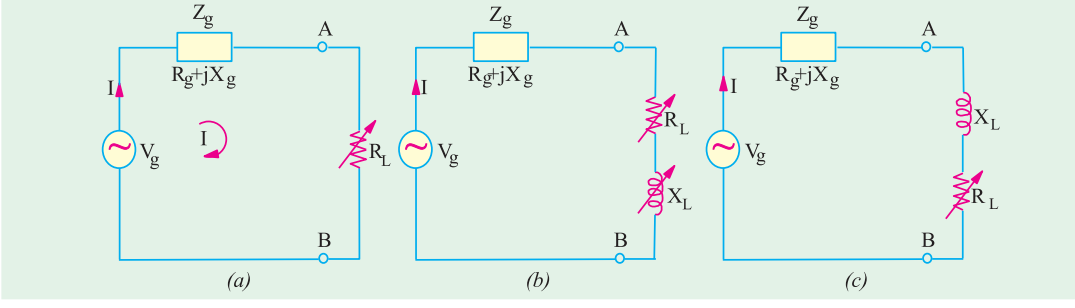


Fig. 15.57

The power delivered to the load is $= P_L = I^2R_L = \dfrac{V_g^2 R_L}{(R_g + R_L)^2 + (X_g + X_L)^2}$

Now, if R_L is held fixed, P_L is maximum when $X_g = -X_L$. In that case $P_{Lmax} = \dfrac{V_g^2 R_L}{(R_g + R_L)^2}$

If on the other hand, R_L is variable then, as in **Case 1** above, maximum power is delivered to the load when $R_L = R_g$. In that case if $R_L = R_g$ and $X_L = -X_g$, then $Z_L = Z_g$. Such a match is called conjugate match.

From the above, we come to the conclusion that in the case of a load impedance having both variable resistance and variable reactance, maximum power transfer across the terminals of the active network occurs when Z_L equals the complex conjugate of the network impedance Z_g *i.e.* the two impedances are conjugately matched.

Case 3. Z_L with variable resistance and fixed reactance [Fig. 15.57 (*c*)]. The equations for current I and power P_L are the same as in **Case** 2 above except that we will consider X_L to remain constant. When the first derivative of P_L with respect to R_L is set equal to zero, it is found that

$$R_L^{\,2} = R_g^{\,2} + (X_g + X_L)^2 \text{ and } R_L = |\, Z_g + jZ_L \,|$$

Since Z_g and X_L are both fixed quantities, these can be combined into a single impedance. Then with R_L variable, **Case** 3 is reduced to **Case** 1 and the maximum power transfer takes place when R_L equals the absolute value of the network impedance.

Summary

The above facts can be summarized as under :

1. When load is purely resistive and adjustable, MPT is achieved when $R_L = |\, Z_g \,| = \sqrt{R_g^2 + X_g^{\,2}}$.

2. When both load and source impedances are purely resistive (*i.e.* $X_L = X_g = 0$), MPT is achieved when $R_L = R_g$.

3. When R_L and X_L are both independently adjustable, MPT is achieved when $X_L = -X_g$ and $R_L = R_g$.

4. When X_L is fixed and R_L is adjustable, MPT is achieved when $R_L = \sqrt{[R_g^2 + (X_g + X_L)^2]}$

Example 15.21. *In the circuit of Fig. 15.58, which load impedance of p.f. = 0.8 lagging when connected across terminals A and B will draw the maximum power from the source. Also find the power developed in the load and the power loss in the source.*

Solution. For maximum power transfer $|Z_L| = |Z_1| \sqrt{(3^2 + 5^2)} = 5.83\ \Omega$

For p.f. = 0.8, $\cos\phi = 0.8$ and $\sin\phi = 0.6$.

$\therefore$ $R_L = Z_L \cos\phi = 5.83 \times 0.8 = 4.66\ \Omega$

$X_L = Z_L \sin\phi = 5.83 \times 0.6 = 3.5\ \Omega$

Total circuit impedance $Z = \sqrt{[(R_1 + R_L)^2 + (X_1 + X_L)^2]}$

$= \sqrt{[(3 + 4.66)^2 + (5 + 3.5)^2]} = 11.44\ \Omega$

$\therefore$ $I = V/Z = 20/11.44 = 1.75$ A.

Power in the load $= I^2 R_L = 1.75^2 \times 4.66 = 14.3$ W

Power loss in the source $= 1.75^2 \times 3 = 9.2$ W.

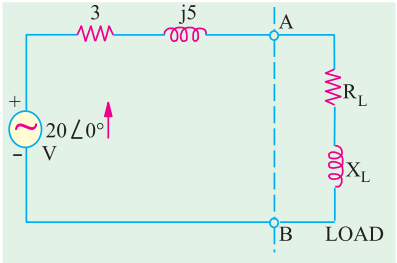


Fig. 15.58

Example 15.22. *In the network shown in Fig. 15.59 find the value of load to be connected across terminals AB consisting of variable resistance R_L and capacitive reactance X_C which would result in maximum power transfer.* **(Network Analysis, Nagpur Univ. 1993)**

Solution. We will first find the Thevenin's equivalent circuit between terminals *A* and *B*. When the load is removed , the circuit become as shown in Fig. 15.59 (*b*).

$$V_{th} = \text{drop across } (2 + j10) = 50\angle 45^\circ \times \frac{2 + j\,10}{7 + j\,10}$$

$$= 41.8 \times 68.7^\circ = 15.2 + j\,38.9$$

$$R_{th} = 5\,||\,(2 + j\,10) = 4.1\angle 23.7^\circ = 3.7 + j\,1.6$$

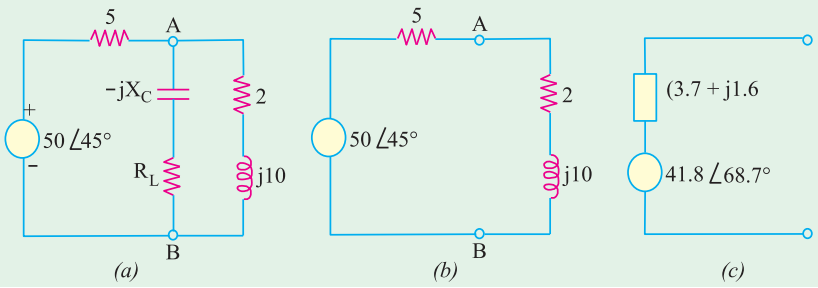


Fig. 15.59

The Thevenin's equivalent source is shown in Fig. 15.59 (*c*)

Since for MPT, conjugate match is required hence, $X_C = 1.6\ \Omega$ and $R_L = 3.7\ \Omega$

Tutorial Problem No. 15.8

1. In the circuit of Fig. 15.60 the load consists of a fixed inductance having a reactance of $j10\ \Omega$ and a variable load resistor R_L. Find the value of R_L for MPT and the value of this power.

[58.3 Ω ; 46.2 W]

2. In the circuit of Fig. 15.61, the source resistance R_g is variable between 5 Ω and 50 Ω but R_L has a fixed value of 25 Ω. Find the value of R_g for which maximum power is dissipated in the load and the value of this power.

[5 Ω ; 250 W]

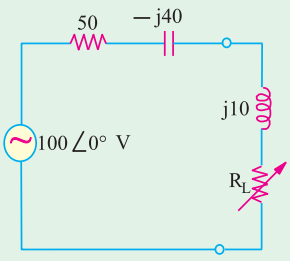


Fig. 15.60

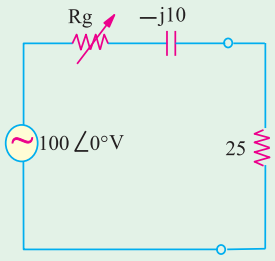


Fig. 15.61

15.11. Millman's Theorem

It permits any number of parallel branches consisting of voltage sources and impedances to be reduced to a single equivalent voltage source and equivalent impedance. Such multi-branch circuits are frequently encountered in both electronics and power applications.

Example 15.23. *By using Millman's theorem, calculate node voltage V and current in the j6 impedance of Fig. 15.62.*

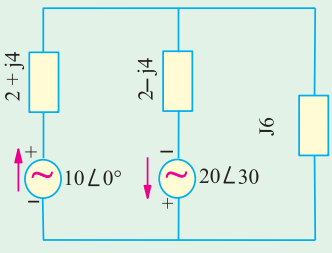


Fig. 15.62

Solution. According to Millman's theorem as applicable to voltage sources.

$$V = \frac{\pm V_1Y_1 \pm V_2Y_2 \pm V_3Y_3 \pm .... \pm V_nY_n}{Y_1 + Y_2 + Y_3 + ... + Y_n}$$

$$Y_1 = \frac{1}{2+j4} = \frac{2-j4}{20} = 0.01 - j0.02 = 0.022 \angle 63.4^\circ$$

$$Y_2 = \frac{1}{2-j4} = \frac{2+j4}{20} = 0.01 + j0.02 = 0.022 \angle 63.4^\circ$$

$$Y_3 = \frac{1}{j\,6} = 0.167 \angle 90^\circ = 0 - j\,0.167$$

$$Y_1 + Y_2 + Y_3 = 0.02 - j\,0.167$$

In the present case, $V_3 = 0$ and also V_2Y_2 would be taken as negative because current due to V_2 flows away from the node.

$$\therefore \quad V = \frac{V_1Y_1 - V_2Y_2}{Y_1 + Y_2 + Y_3} = \frac{10\angle 0^\circ \times 0.22\angle -63.4^\circ - 20\angle 30^\circ \times 0.022\angle 63.4^\circ}{0.02 - j\,0.167}$$

$$= 3.35\angle 177^\circ$$

Current through j 6 impedance $= 3.35\angle 177^\circ / 6\angle 90^\circ = 0.56\angle 87^\circ$

Tutorial Problems No. 15.9

1. With the help of Millman's theorem, calculate the voltage across the 1 K resistor in the circuit of Fig. 15.63.

[2.79 V]

2. Using Millman's theorem, calculate the voltage V_{ON} in the 3-phase circuit shown in Fig. 15.64. All load resistances and reactances are in milli-siemens.

[$V_{ON} = 69.73 \angle 113.53^\circ$]

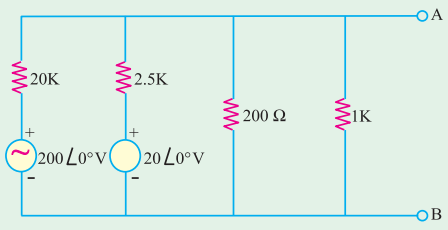


Fig. 15.63

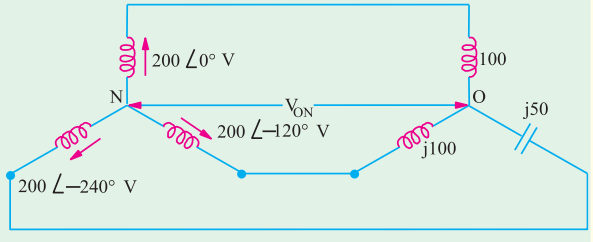


Fig. 15.64

3. Define mesh current and node voltage. ***(Madras University, April 2002)***

4. State superposition theorem. ***(Madras University, April 2002)***

5. State Millman's theorem. ***(Madras University, April 2002)***

6. Two circuits the impedances of which are given by Z_1 $(10 + j\ 15)\Omega$ and $Z_2 = (6 - j\ 8)\Omega$ are connected in parallel. If the total current supplied is 15 A. What is the power taken by each branch?

(RGPV Bhopal 2002)

OBJECTIVE TYPES – 15

1. The Thevenin's equivalent resistance R_{th} for the given network is

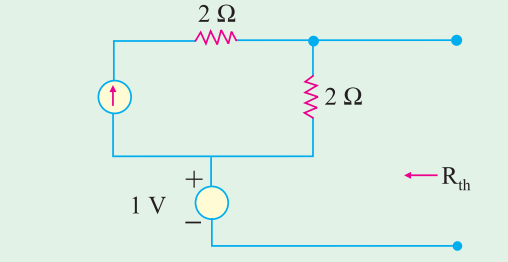


Fig. 15.65

(*a*) 1 Ω
(*b*) 2 Ω
(*c*) 4 Ω
(*d*) infinity

(ESE 2001)

2. The Norton's equivalent of circuit shown in Figure 15.66 is drawn in the circuit shown in Figure 15.67 The values of I_{SC} and R_{eq} in Figure II are respectively

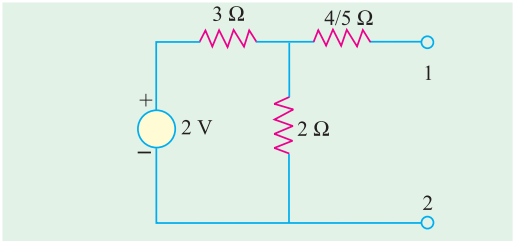


Fig. 15.66

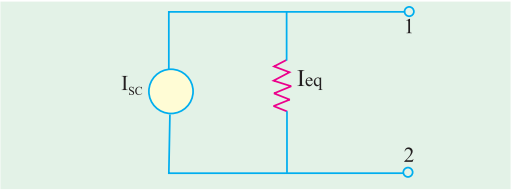


Fig. 15.67

(*a*) $\frac{5}{2}$ A and 2 Ω (*b*) $\frac{2}{5}$ A and 1 Ω

(*c*) $\frac{4}{5}$ A and $\frac{12}{5}$ Ω (*d*) $\frac{2}{5}$ A and 2 Ω

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CHAPTER 16

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- Schering Bridge
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A.C. BRIDGES



A wide variety of AC bridge circuits (such as wheatstone) may be used for the precision measurement of AC resistance, capacitance and inductance

16.1. A.C. Bridges

Resistances can be measured by direct-current Wheatstone bridge, shown in Fig. 16.1 (*a*) for which the condition of balance is that

$$\frac{R_1}{R_2} = \frac{R_4}{R_3} \quad \text{or} \quad R_1 R_3 \quad R_2 R_4 \ *$$

Inductances and capacitances can also be measured by a similar four-arm bridge, as shown in Fig. 16.1 (*b*); instead of using a source of direct current, alternating current is employed and galvanometer is replaced by a vibration galvanometer (for commercial frequencies or by telephone detector if frequencies are higher (500 to 2000 Hz)).

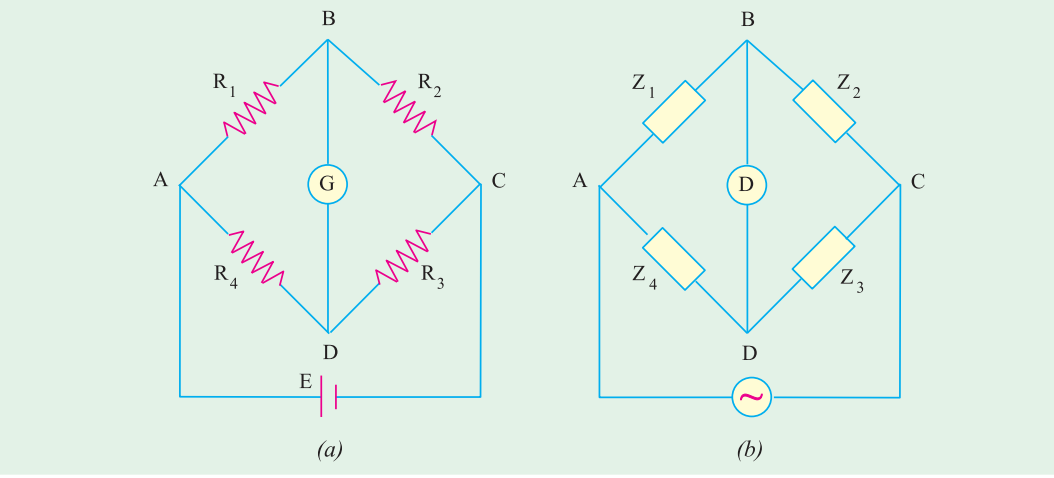


Fig. 16.1

The condition for balance is the same as before but instead of resistances, impedances are used *i.e.*

$$\mathbf{Z_1 / Z_2 = Z_4 / Z_3} \quad \text{or} \quad \mathbf{Z_1 Z_3 = Z_2 Z_4}$$

But there is one important difference *i.e.* not only should there be balance for the magnitudes of the impedances but also a phase balance. Writing the impedances in their polar form, the above condition becomes

$$Z_1 \angle \phi_1 . Z_3 \angle \phi_3 = Z_2 \angle \phi_2 . Z_4 \angle \phi_4 \quad \text{or} \quad Z_1 Z_3 \angle \phi_1 + \phi_3 = Z_2 Z_4 \angle \phi_2 + \phi_4$$

Hence, we see that, in fact, there are two balance conditions which must be satisfied simultaneously in a four-arm a.c. impedance bridge.

(i) $Z_1 Z_3 = Z_2 Z_4$ ... for magnitude balance

(ii) $\phi_1 + \phi_3 = \phi_2 + \phi_4$ ... for phase angle balance

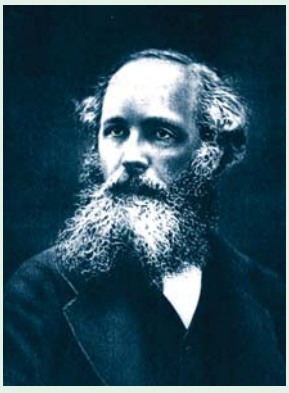
In this chapter, we will consider a few of the numerous bridge circuits used for the measurement of self-inductance, capacitance and mutual inductance, choosing as examples some bridges which are more common.

16.2. Maxwell's Inductance Bridge

The bridge circuit is used for medium inductances and can be arranged to yield results of considerable precision. As shown in Fig. 16.2, in the two arms, there are two pure resistances so

* Products of opposite arm resistances are equal.

that for balance relations, the phase balance depends on the remaining two arms. If a coil of an unknown impedance Z_1 is placed in one arm, then its positive phase angle ϕ_1 can be compensated for in either of the following two ways:



James Clark Maxwell

(*i*) A known impedance with an equal positive phase angle may be used in either of the *adjacent* arms (so that $\phi_1 =$ $_3$ or $_2$ $= \phi_4$), remaining two arms have zero phase angles (being pure resistances). Such a network is known as Maxwell's a.c. bridge or L_1/L_4 bridge.

(*ii*) Or an impedance with an equal *negative* phase angle (*i.e.* capacitance) may be used in *opposite* arm (so that $\phi_1 + \phi_3 = 0$). Such a network is known as Maxwell-Wien bridge (Fig. 16.5) or Maxwell's L/C bridge.

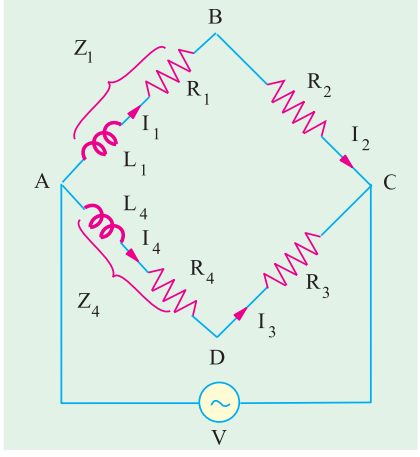


Fig. 16.2

Hence, we conclude that an inductive impedance may be measured in terms of another inductive impedance (of equal time constant) in either ***adjacent*** arm (Maxwell bridge) or the unknown inductive impedance may be measured in terms of a combination of resistance and capacitance (of equal time constant) in the ***opposite*** arm (Maxwell-Wien bridge). It is important, however, that in each case the time constants of the two impedances must be matched.

As shown in Fig. 16.2,

$Z_1 = R_1 + jX_1 = R_1 + j\omega L_1$ ... unknown; $\mathbf{Z_4} = R_4 + jX_4 = R_4 + j\omega L_4$ ...known

R_2, R_3 = known pure resistances; D = detector

The inductance L_4 is a variable self-inductance of constant resistance, its inductance being of the same order as L_1. The bridge is balanced by varying L_4 and one of the resistances R_2 or R_3.

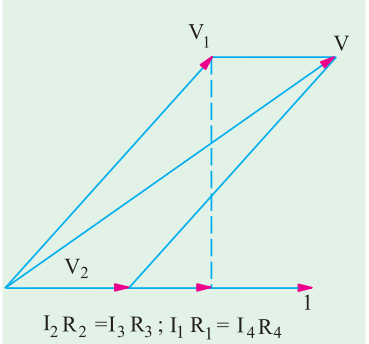


Fig. 16.3

Alternatively, R_2 and R_3 can be kept constant and the resistance of one of the other two arms can be varied by connecting an additional resistance in that arm (Ex. 16.1).

The balance condition is that $\mathbf{Z_1 Z_3 = Z_2 Z_4}$

$$\therefore \quad (R_1 + j\omega L_1)R_3 = (R_4 + j\omega L_4)R_2$$

Equating the real and imaginary parts on both sides, we have

$$R_1 R_3 = R_2 R_4 \text{ or } R_1 / R_4 = R_2 / R_3 \,*$$

(*i.e.* products of the resistances of opposite arms are equal).

and $$\omega L_1 R_3 = \omega L_4 R_2 \text{ or } L_1 = L_4 \frac{R_2}{R_3}$$

We can also write that L_1 $L_4 \cdot \frac{R_1}{R_4}$

---

* Or $\frac{L_1}{R_1} = \frac{L_4}{R_4}$ *i.e.*, the time constants of the two coils are matched.

Hence, the unknown self-inductance can be measured in terms of the known inductance L_4 and the two resistors. Resistive and reactive terms balance independently and the conditions are independent of frequency. This bridge is often used for measuring the iron losses of the transformers at audio frequency.

The balance condition is shown vectorially in Fig. 16.3. The currents I_4 and I_3 are in phase with I_1 and I_2. This is, obviously, brought about by adjusting the impedances of different branches, so that these currents lag behind the applied voltage V by the same amount. At balance, the voltage drop V_1 across branch 1 is equal to that across branch 4 and $I_3 = I_4$. Similarly, voltage drop V_2 across branch 2 is equal to that across branch 3 and $I_1 = I_2$.

Example 16.1. *The arms of an a.c. Maxwell bridge are arranged as follows: AB and BC are non-reactive resistors of 100 Ω each, DA is a standard variable reactor L_1 of resistance 32.7 Ω and CD comprises a standard variable resistor R in series with a coil of unknown impedance. Balance was obtained with L_1 = 47.8 mH and R = 1.36 Ω. Find the resistance and inductance of the coil.*

(Elect. Inst. & Meas. Nagpur Univ. 1993)

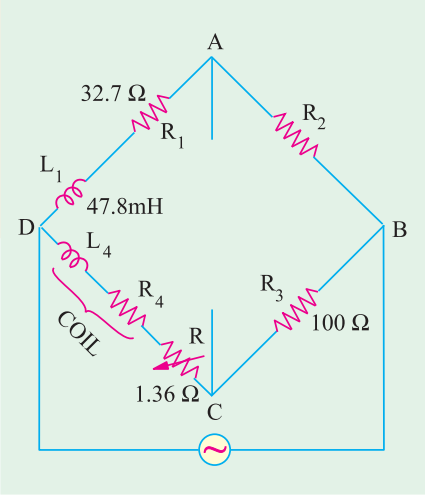


Fig. 16.4

Solution. The a.c. bridge is shown in Fig. 16.4.

Since the products of the resistances of opposite arms are equal

$$\therefore \quad 32.7 \times 100 = (1.36 + R_4)100$$

$$\therefore \quad 32.7 = 1.36 + R_4 \text{ or } R_4 = 32.7 - 1.36 = \mathbf{31.34\Omega}$$

Since $L_1 \times 100 = L_4 \times 100 \quad \therefore \quad L_4 = L_1 = \mathbf{47.8\ mH}$

or because time constants are the same, hence

$L_1/32.7 = L_4/(31.34 + 1.36) \quad \therefore \quad L_4 = 47.8$ mH

16.3. Maxwell-Wien Bridge or Maxwell's L/C Bridge

As referred to in Art. 16.2, the *positive* phase angle of an inductive impedance may be compensated by the *negative* phase angle of a capacitive impedance put in the *opposite* arm. The unknown inductance then becomes known in terms of this capacitance.

Let us first find the combined impedance of arm 1.

$$\frac{1}{Z_1} = \frac{1}{R_1} + \frac{1}{-jX_C} = \frac{1}{R_1} + \frac{j}{X_C} = \frac{1}{R_1} + j\omega C = \frac{1 + j\omega CR_1}{R_1}$$

$$\therefore \quad \mathbf{Z}_1 \quad \frac{R_1}{1 \quad j \quad CR_1}; \quad \mathbf{Z}_2 \quad R_2$$

$$\mathbf{Z_3} = R_3 + j\omega L_3 \text{ and } Z_4 = R_4$$

Balance condition is $\mathbf{Z_1 Z_3 = Z_2 Z_4}$

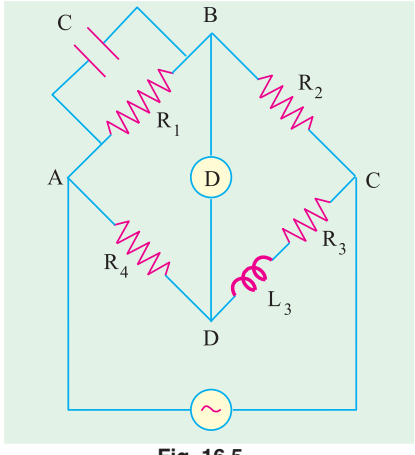


Fig. 16.5

or $$\frac{R_1(R_3 \quad j \quad L_3)}{1 \quad j \quad CR_1} \quad R_2R_4 \text{ or } R_1R_3 \quad j \quad L_3R_1 \quad R_2R_4 \quad j \quad CR_1R_2R_4$$

Separating the real and imaginaries, we get

$$R_1R_3 \quad R_2R_4 \text{ and } L_3R_1 \quad CR_1R_2R_4; R_3 \quad \frac{R_2R_4}{R_1} \text{ and } L_3 \quad CR_2R_4$$

Example 16.2. *The arms of an a.c. Maxwell bridge are arranged as follows: AB is a non-inductive resistance of 1,000 Ω in parallel with a capacitor of capacitance 0.5 μF, BC is a non-inductive resistance of 600 Ω CD is an inductive impedance (unknown) and DA is a non-inductive resistance of 400 Ω. If balance is obtained under these conditions, find the value of the resistance and the inductance of the branch CD.*

[Elect. & Electronic Meas, Madras Univ.]

Solution. The bridge is shown in Fig. 16.6. The conditions of balance have already been derived in Art. 16.3 above.

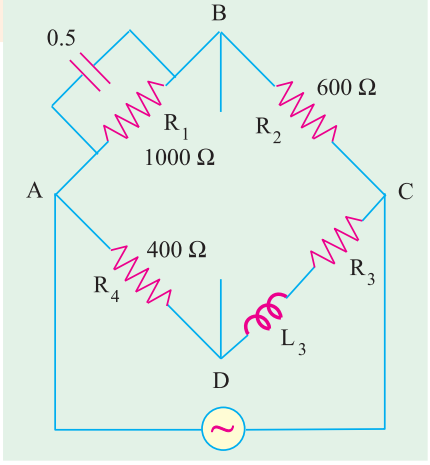


Fig. 16.6

Since $R_1R_3 = R_2R_4 \quad \therefore \quad R_3 = R_2R_4 / R_1$

$$\therefore \qquad R_3 = \frac{600 \times 400}{1000} = \mathbf{240\ \Omega}$$

Also $\quad L_3 = CR_2R_4$

$$= 0.5 \times 10^{-6} \times 400 \times 600$$

$$= 12 \times 10^{-2} = 0.12 \text{ H}$$

16.4. Anderson Bridge

It is a very important and useful modification of the Maxwell-Wien bridge described in Art. 16.3. In this method, the unknown inductance is measured in terms of a known capacitance and resistance, as shown in Fig. 16.7.

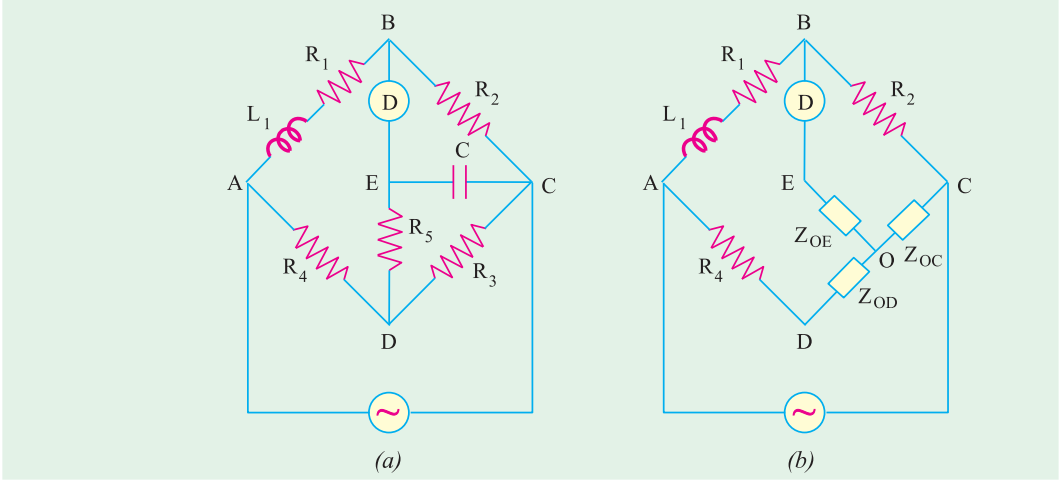


Fig. 16.7

The balance conditions for this bridge may be easily obtained by converting the mesh of impedances C, R_5 and R_3 to an equivalent star with star point O by Δ / Y transformation. As seen from Fig. 16.7 (b).

$$\mathbf{Z_{OD}} \quad \frac{R_3R_5}{(R_3 \quad R_5 \quad 1/j \quad C)}; \quad \mathbf{Z_{OC}} \quad \frac{R_3 / j \quad C}{(R_3 \quad R_5 \quad 1/j \quad C)} \quad \mathbf{Z_3}$$

With reference to Fig. 16.7 (b) it is seen that

$\mathbf{Z_1} = (R_1 + j\omega L_1)$ $\mathbf{Z_2} = R_2$; $\mathbf{Z_3} = \mathbf{Z_{OC}}$ and $\mathbf{Z_4} = R_4 + \mathbf{Z_{OD}}$

For balance $\mathbf{Z_1Z_3 = Z_2Z_4}$ $\therefore$ $(R_1 \quad j \quad L_1) \quad \mathbf{Z_{OC}} \quad R_2(R_4 \quad \mathbf{Z_{OD}})$

$\therefore$ $(R_1 \quad j \quad L_1)\dfrac{R_3 / j \quad C}{(R_3 \quad R_5 \quad 1/j \quad C)} \quad R_2 \quad R_4 \quad \dfrac{R_3R_5}{R_3 \quad R_5 \quad 1/j \quad C}$

Further simplification leads to $R_2R_3R_4 \quad R_2R_4R_5 \quad j\dfrac{R_2R_4}{C} \quad R_2R_3R_5 \quad j\dfrac{R_1R_3}{C} \quad \dfrac{R_3L_1}{C}$

$\therefore$ $\dfrac{jR_2R_4}{C} \quad \dfrac{jR_1R_3}{C}$ or $R_1 \quad R_2R_4 / R_3$

Also $\dfrac{R_3L_1}{C} \quad R_2R_3R_4 \quad R_2R_3R_5 \quad R_2R_4R_5 \quad \therefore \quad L_1 = CR_2 \quad R_4 + R_5 + \dfrac{R_4R_5}{R_3}$

This method is capable of precise measurements of inductances over a wide range of values from a few micro-henrys to several henrys and is one of the commonest and the best bridge methods.

Example 16.3. *An alternating current bridge is arranged as follows: The arms AB and BC consists of non-inductive resistances of 100-ohm each, the arms BE and CD of non-inductive variable resistances, the arm EC of a capacitor of 1 μF capacitance, the arm DA of an inductive resistance. The alternating current source is connected to A and C and the telephone receiver to E and D. A balance is obtained when resistances of arms CD and BE are 50 and 2,500 ohm respectively. Calculate the resistance and inductance of arm DA.*

Draw the vector diagram showing voltage at every point of the network.

(Elect. Measurements, Pune Univ.)

Solution. The circuit diagram and voltage vector diagram are shown in Fig. 16.8. As seen, I_2 is vector sum of I_C and I_3. Voltage $V_2 = I_2 R_2 = I_C X_C$. Also, vector sum of V_1 and V_2 is V as well as that of V_3 and V_4. I_C is at right angles to V_2.

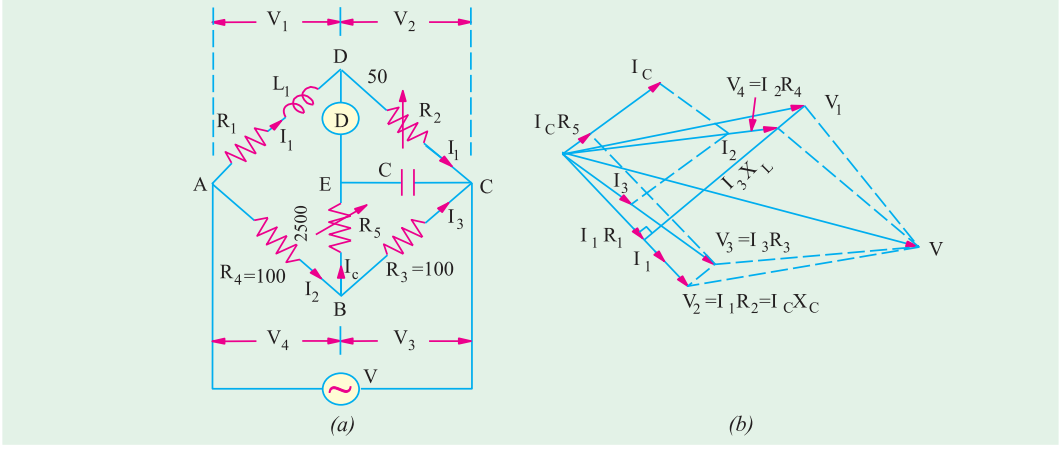


Fig. 16.8

Similarly, V_3 is the vector sum of V_2 and I_CR_5.

As shown in Fig. 16.8, $R_1 = R_2$. $R_4/R_3 = 50 \times 100/100 = 50\,\Omega$

The inductance is given by $L \quad CR_2(R_4 \quad R_5 \quad R_4R_5 / R_3)$

$\therefore$ $L \quad 1 \quad 10^{-6} \quad 50(100 \quad 2500 \quad 100 \quad 2500/100)$

$= \mathbf{0.2505\ H}$

Example 16.4. *Fig. 16.9 gives the connection of Anderson's bridge for measuring the inductance L_1 and resistance R_1 of a coil. Find R_1 and L_1 if balance is obtained when $R_3 = R_4 = 2000$ ohms, $R_2 = 1000$ ohms $R_5 = 200$ ohms and $C = 1\mu F$. Draw the vector diagram for the voltages and currents in the branches of the bridge at balance.*

(Elect. Measurements, AMIE Sec. B Summer 1990)

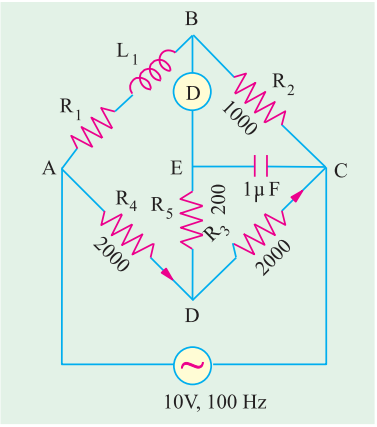


Fig. 16.9

Solution. $R_1 = R_2 R_4 / R_3 = 1000 \times 2000 / 2000 =$ **1000 Ω**

$$L_1 \quad CR_2 \quad R_4 \quad R_5 \quad \frac{R_4 R_5}{R_3}$$

$$= 1 \quad 10^{\ 6} \quad 1000 \quad 2000 \quad 200 \quad \frac{2000 \quad 200}{2000} \quad = \textbf{2.4 H}$$

16.5. Hay's Bridge

It is also a modification of the Maxwell-Wien bridge and is particularly useful if the phase angle of the inductive impedance $_{m} \quad \tan^{1}(\ L/R)$ is large. The network is shown in Fig. 16.10. It is seen that, in this case, a comparatively smaller series resistance R_1 is used instead of a parallel resistance (which has to be of a very large value).

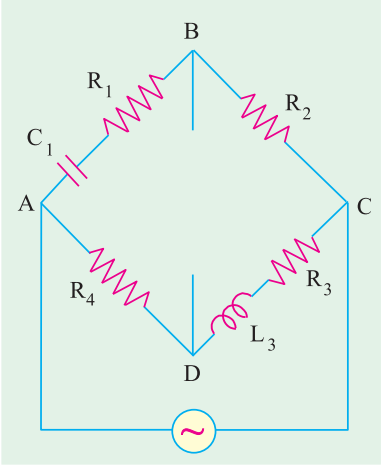


Fig. 16.10

Here $Z_1 \quad R_1 \quad \frac{j}{C_1}; Z_2 \quad R_2$

$Z_3 \quad R_3 \quad j \ L_3; Z_4 \quad R_4$

Balance condition is $\mathbf{Z_1 Z_3 = Z_2 Z_4}$

or $\quad R_1 \quad \frac{j}{C_1} \ (R_3 \quad j \ L_3) \quad R_2 R_4$

Separating the reals and the imaginaries, we obtain

$$R_1 R_3 \quad \frac{L_3}{C_1} \quad R_2 R_4 \text{ and } \quad L_3 R_1 \quad \frac{R_3}{C_1} \quad 0$$

Solving these simultaneous equations, we get

$$L_3 \quad \frac{C_1 R_2 R_4}{1 \quad {}^2 R_1^2 C_1^2} \text{ and } R_3 \quad \frac{{}^2 C_1^2 R_1 R_2 R_4}{1 \quad {}^2 R_1^2 C_1^2}$$

The symmetry of expressions should help the readers to remember the results even when branch elements are exchanged, as in Ex. 16.5.

Example 16.5. *The four arms of a Hay's a.c. bridge are arranged as follows: AB is a coil of unknown impedance; BC is a non-reactive resistor of 1000 Ω ; CD is a non-reactive resistor of 833 Ω in series with a standard capacitor of 0.38 μF ; DA is a non-reactive resistor of 16,800 Ω . If the supply frequency is 50 Hz, determine the inductance and the resistance at the balance condition.* **(Elect. Measu. A.M.I.E. Sec B, 1992)**

Solution. The bridge circuit is shown in Fig. 16.11.

277 50 314.22 rad/s; ^{2} 314.2^2 98,721

$$R_1 \quad \frac{98,721 \quad (0.38 \quad 10^{6})^2 \quad 833 \quad 16,800 \quad 1000}{1 \quad 98,721 \quad 833^2 \quad (0.38 \quad 10^{6})^2} = 210\ \Omega$$

$$L_1 \quad \frac{16,800 \quad 1000 \quad 0.38 \quad 10^{6}}{1 \quad 98,721 \quad 833^2 \quad (0.38 \quad 10^{6})^2} \quad 6.38\,\text{H}$$

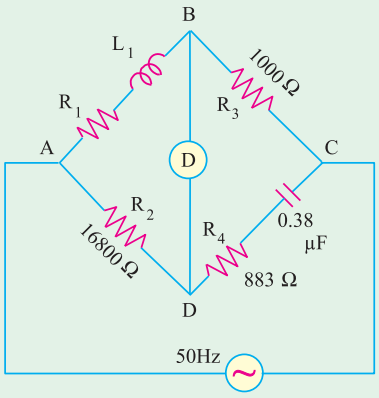


Fig. 16.11

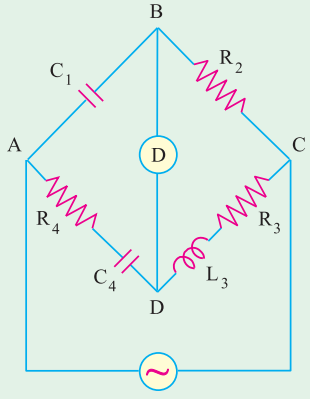


Fig. 16.12

16.6. The Owen Bridge

The arrangement of this bridge is shown in Fig. 16.12. In this method, also, the inductance is determined in terms of resistance and capacitance. This method has, however, the advantage of being useful over a very wide range of inductances with capacitors of reasonable dimensions.

Balance condition is $Z_1 Z_3 = Z_2 Z_4$

Here $Z_1 = -\frac{j}{\omega C_1}$; $\quad Z_2 \quad R_2; Z_3 \quad R_3 \quad j \quad L_3; \quad Z_4 \quad R_4 \quad \frac{j}{C_4}$

$\therefore \quad \frac{j}{C_1}(R_3 \quad j \quad L_3) \quad R_2 \quad R_4 \quad \frac{j}{C_4}$

Separating the reals and imaginaries, we get $R_3 = R_2 \frac{C_1}{C_4}$ and $L_3 = C_1 R_2 R_4$.

Since ω does not appear in the final balance equations, hence the bridge is unaffected by frequency variations and wave-form.

16.7. Heavisible-Campbell Equal Ratio Bridge

It is a mutual inductance bridge and is used for measuring self-inductance over a wide range in terms of mutual inductometer readings. The connections for Heaviside's bridge employing a standard variable mutual inductance are shown in Fig. 16.13. The primary of the mutual inductometer is inserted in the supply circuit and the secondary having self-inductance L_2 and resistance R_2 is put in arm 2 of the bridge. The unknown inductive impedance having self-inductance of L_1 and resistance R_1 is placed in arm 1. The other two arms have pure resistances of R_3 and R_4.

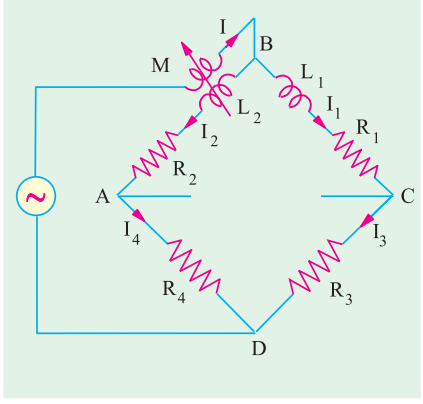


Fig. 16.13

Balance is obtained by varying mutual inductance M and resistances R_3 and R_4.

For balance, $I_1 R_3 \quad I_2 R_4$ ... ***(i)***

$$I_1(R_1 \quad j\ L_1) \quad I_2(R_2 \quad j\ L_2) \quad j\ MI$$ ... ***(ii)***

Since $I = I_1 + I_2$, hence putting the value of I in equation ***(ii)***, we get

$$I_1[R_1 \quad j \quad (L_1 \quad M)] \quad I_2[R_2 \quad j \quad (L_2 \quad M)]$$ ... ***(iii)***

Dividing equation ***(iii)*** by ***(i)***, we have $\dfrac{R_1 \quad j \quad (L_1 \quad M)}{R_3} \quad \dfrac{R_2 \quad j \quad (L_2 \quad M)}{R_4}$

$$\therefore \qquad R_3[R_2 \quad j \quad (L_2 \quad M)] \quad R_4[R_1 \quad j \quad (L_1 \quad M)]$$

Equating the real and imaginaries, we have $R_2 R_3 = R_1 R_4$ ... ***(iv)***

Also, $R_3 (L_2 + M) = R_4 (L_1 - M)$. If $R_3 = R_4$, then $L_2 + M = (L_1 - M) \ \therefore \ L_1 - L_2 = 2M$ ... ***(v)***

This bridge, as modified by Campbell, is shown in Fig. 16.14. Here $R_3 = R_4$. A balancing coil or a test coil of self-inductance equal to the self-inductance L_2 of the secondary of the inductometer and of resistance slightly greater than R_2 is connected in series with the unknown inductive impedance (R_1 and L_1) in arm 1. A non-inductive resistance box along with a constant-inductance rheostat are also introduced in arm 2 as shown.

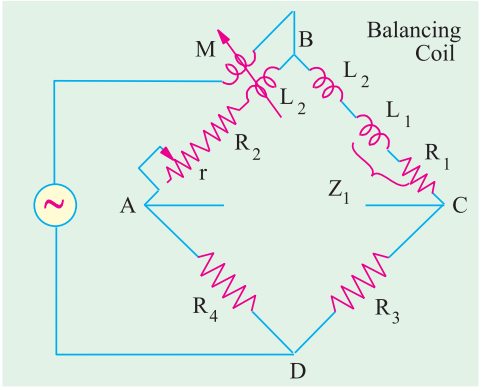


Fig. 16.14

Balance is obtained by varying M and r. Two readings are taken; one when Z_1 is in circuit and second when Z_1 is removed or short-circuited across its terminals.

With unknown impedance Z_1 still in circuit, suppose for balance the values of mutual inductance and r are M_1 and r_1. With Z_1 short-circuited, let these values be M_2 and r_2. Then

$$L_1 = 2(M_1 - M_2) \text{ and } R_1 = r_1 - r_2$$

By this method, the self-inductance and resistance of the leads are eliminated.

Example 16.6. *The inductance of a coil is measured by using the Heaviside-Campbell equal ratio bridge. With the test coil short-circuited, balance is obtained when adjustable non-reactive resistance is 12.63 Ω and mutual inductometer is set at 0.1 mH. When the test coil is in circuit, balance is obtained when the adjustable resistance is 25.9 Ω and mutual inductometer is set at 15.9 mH. What is the resistance and inductance of the coil?*

Solution. With reference to Art. 16.7 and Fig. 16.14, $r_1 = 25.9\ \Omega$, $M_1 = 15.9$ mH

With test coil short-circuited $r_2 = 12.63\ \Omega$; $M_2 = 0.1$ mH

$$L_1 = 2\ (M_1 - M_2) = 2\ (15.9 - 0.1) = \mathbf{31.6\ mH}$$

$$R_1 = -r_1 - r_2 = 25.9 - 12.63 = \mathbf{13.27\ \Omega}$$

16.8. Capacitance Bridges

We will consider only De Sauty bridge method of comparing two capacitances and Schering bridge used for the measurement of capacitance and dielectric loss.

16.9. De Sauty Bridge

With reference to Fig. 16.15, let

C_2 = capacitor whose capacitance is to be measured

C_3 = a standard capacitor

R_1, R_2 = non-inductive resistors

Balance is obtained by varying either R_1 or R_2. For balance, points B and D are at the same potential.

$$\therefore \quad I_1 R_1 \quad I_2 R_2 \text{ and } \frac{j}{C_2} . I_1 \quad \frac{j}{C_3} . I_2$$

Dividing one equation by the other, we get

$$\frac{R_1}{R_2} \quad \frac{C_2}{C_3}; \; C_2 \quad C_3 \frac{R_1}{R_2}$$

The bridge has maximum sensitivity when $C_2 = C_3$. The simplicity of this method is offset by the impossibility of obtaining a perfect balance if both the capacitors are not free from the dielectric loss. A perfect balance can only be obtained if air capacitors are used.

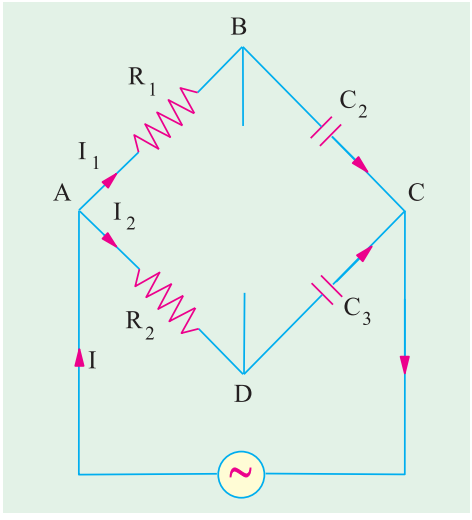


Fig. 16.15

16.10. Schering Bridge

It is one of the very important and useful methods of measuring the capacitance and dielectric loss of a capacitor. In fact, it is a device for comparing an imperfect capacitor C_2 in terms of a loss-free standard capacitor C_1 [Fig. 16.16 (*a*)]. The imperfect capacitor is represented by its equivalent loss-free capacitor C_2 in series with a resistance r [Fig. 16.16 (*b*)].

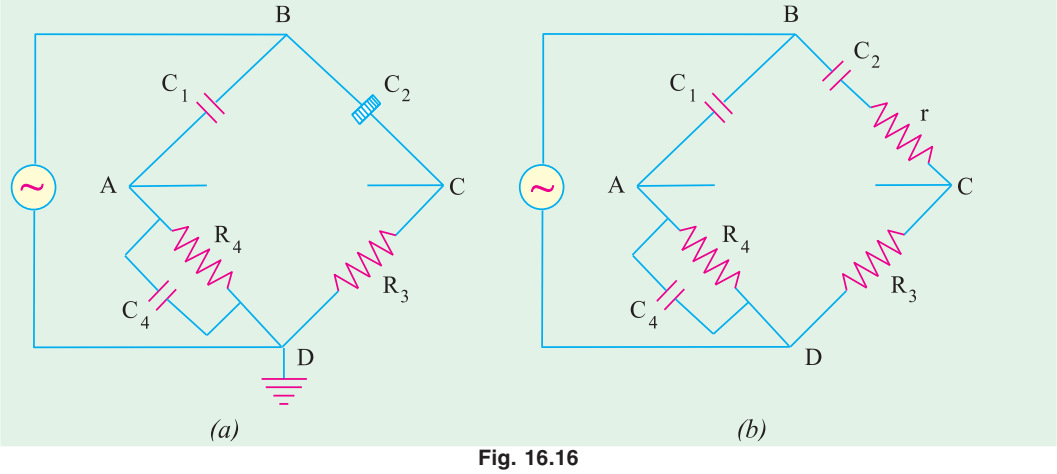


Fig. 16.16

For high voltage applications, the voltage is applied at the junctions shown in the figure. The junction between arms 3 and 4 is earthed. Since capacitor impedances at lower frequencies are much higher than resistances, most of the voltage will appear across capacitors. Grounding of the junction affords safety to the operator form the high-voltage hazards while making balancing adjustment in arms 3 and 4.

$$\text{Now} \quad Z_1 \quad \frac{j}{C_1}; Z_2 \quad r \quad \frac{j}{C_2}; Z_3 \quad R_3; Z_4 \quad \frac{1}{(1/R_4) \quad j \quad C_4} \quad \frac{R_4}{1 \quad j \quad C_4 R_4}$$

For balance, $\mathbf{Z_1 Z_3 = Z_2 Z_4}$

or $\frac{jR_3}{C_1} \quad r \quad \frac{j}{C_2} \quad \frac{R_4}{1 \quad j \quad C_4R_4}$ or $\frac{jR_3}{C_1}(1 \quad {}_aC_4R_4) \quad R_4 \quad r \quad \frac{j}{C_2}$

Separating the real and imaginaries, we have $C_2 = C_1(R_4 / R_3)$ and $r = R_3.(C_4 / C_1)$.

The quality of a capacitor is usually expressed in terms of its phase defect angle or dielectric loss angle which is defined as the angle by ***which current departs from exact quadrature from the applied voltage*** *i.e.* the complement of the phase angle. If ϕ is the actual phase angle and δ the defect angle, then 90 . For small values of δ, tan δ = sin δ = cos ϕ (approximately). Tan δ is usually called the *dissipation factor* of the *R–C* circuit. For low power factors, therefore, dissipation factor is approximately equal to the power factor.

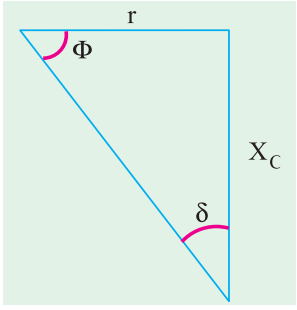


Fig. 16.17

As shown in Fig. 16.17,

Dissipation factor = power factor = tan δ

$$= \frac{r}{X_C} = \frac{r}{1/\omega C_2} = \omega r C_2$$

Putting the value of rC_2 from above,

Dissipation factor = $rC_2 \quad C_4R_4$ = power factor.

Example 16.7. *In a test on a bakelite sample at 20 kV, 50 Hz by a Schering bridge, having a standard capacitor of 106 pF, balance was obtained with a capacitance of 0.35 μF in parallel with a non-inductive resistance of 318 ohms, the non-inductive resistance in the remaining arm of the bridge being 130 ohms. Determine the capacitance, the p.f. and equivalent series resistance of the specimen. Derive any formula used. Indicate the precautions to be observed for avoiding errors.* **(Elect. Engg. Paper I, Indian Engg. Services 1991)**

Solution. Here C_1 = 106 pF, C_4 = 0.35 μF, R_4 = 318 Ω, R_3 = 130 Ω.

$C_2 \quad C_1.(R_4 \mid R_3) \quad 106 \quad 318/130$ = **259.3 pF**

$r \quad R_3.(C_4 / C_1) \quad 130 \quad 0.35 \quad 10^{-6}/106 \quad 10^{-12}$ = **0.429** MΩ

p.f. = $rC_2 \quad (2 \quad 50) \quad 0.429 \quad 10^{-6} \quad 259.3 \quad 10^{-12}$ = **0.035**

Example 16.8. *A losy capacitor is tested with a Schering bridge circuit. Balance obtained with the capacitor under test in one arm, the succeeding arms being, a non-inductive resistor of 100 Ω, a non-reactive resistor of 309 Ω in parallel with a pure capacitor of 0.5 μF and a standard capacitor of 109 μμF. The supply frequency is 50 Hz. Calculate from the equation at balance the equivalent series capacitance and power factor (at 50 Hz) of the capacitor under test.* **(Measu. & Instru., Nagpur Univ. 1992)**

Solution. Here, we are given

$C_1 \quad 109$ pF ; R_3 = 100 ; C_4 = 0.5 F ; R_4 = 309 Ω

Equivalent capacitance is $C_2 \quad 109 \quad 309/100$ = **336.8 pF**

p.f. = $C_4R_4 \quad 314 \quad 0.5 \quad 10^{-6} \quad 309$ = **0.0485**

16.11. Wien Series Bridge

It is a simple ratio bridge and is used for audio-frequency measurement of capacitors over a wide range. The bridge circuit is shown in Fig. 16.18.

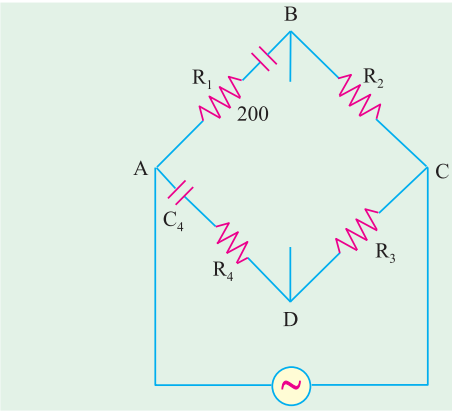


Fig. 16.18

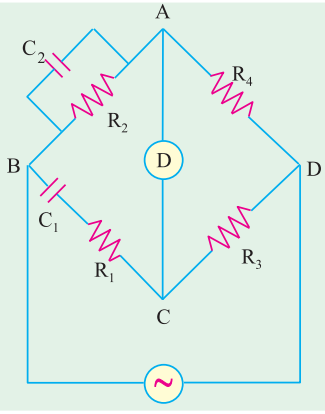


Fig. 16.19

The balance conditions may be obtained in the usual way. For balance

$$R_1 \quad R_2R_4 / R_3 \text{ and } C_1 = C_4(R_3 / R_2)$$

16.12. Wien Parallel Bridge

It is also a ratio bridge used mainly as the feedback network in the wide-range audio-frequency *R-C* oscillators. It may be used for measuring audio-frequencies although it is not as accurate as the modern digital frequency meters.

The bridge circuit is shown in Fig. 16.19. In the simple theory of this bridge, capacitors C_1 and C_2 are assumed to be loss-free and resistances R_1 and R_2 are separate resistors.

The usual relationship for balance gives

$$R_4 \; R_1 \; \frac{j}{C_1} \quad R_3 \; \frac{R_2}{1 \;\; j \;\; C_2R_2} \quad \text{or} \quad R_4 \;\; R_1 \;\; \frac{j}{C_1} \;\; (1 \;\; j \;\; C_2R_2) \quad R_2R_3$$

Separating the real and imaginary terms, we have

$$R_1R_4 \quad R_2R_4 \frac{C_2}{C_1} \quad R_2 \, R_3 \quad \text{or} \quad \frac{C_2}{C_1} \;\; \frac{R_3}{R_4} \;\; \frac{R_1}{R_2} \qquad \text{... } (i)$$

and

$$C_2R_2R_4 \;\; \frac{R_4}{C_1} \;\; 0 \quad \text{or} \quad {}^{2} \;\; \frac{1}{R_1R_2C_1C_2} \qquad \text{... } (ii)$$

or

$$f \;\; \frac{1}{2 \;\; \sqrt{R_1R_2C_1C_2}} \text{ Hz}$$

Note. Eq. (*ii*) may be used to find angular frequency ω of the source if terms are known. For such purposes, it is convenient to make $C_1 \quad 2C_2, R_3 \quad R_4$ and $R_2 \quad 2R_1$. In that case, the bridge has equal ratio arms so that Eq. (*i*) will always be satisfied. The bridge is balanced simultaneously by adjusting R_2 and R_1 (though maintaining $R_2 = 2R_1$). Then, as seen from Eq. (*ii*) above

$${}^{2} \;\; 1/(R_1.2R_1.2C_2.C_2) \text{ or } \quad 1/(2R_1C_2)$$

Example 16.9. *The arms of a four-arm bridge ABCD, supplied with a sinusoidal voltage, have the following values:*

AB : 200 ohm resistance in parallel with 1 μF capacitor; BC : 400 ohm resistance; CD : 1000 ohm resistance and DA : resistance R in series with a $2\mu F$ capacitor.

Determine ***(i)*** *the value of R and* ***(ii)*** *the supply frequency at which the bridge will be balanced.* **(Elect. Meas. A.M.I.E. Sec. 1991)**

Solution. The bridge circuit is shown in Fig. 16.20.

(*i*) As discussed in Art. 16.12, for balance we have

$$\frac{C_2}{C_1} \quad \frac{R_3}{R_4} \quad \frac{R_1}{R_2} \text{ or } \frac{2}{1} \quad \frac{1000}{4000} \quad \frac{R_1}{200}$$

$\therefore\ R_1 = 200 \times 0.5 =$ **100 Ω**

(*ii*) The frequency at which bridge is balanced is given by

$$f \quad \frac{1}{2\sqrt{R_1 R_2 C_1 C_2}} \text{ Hz}$$

$$= \frac{10^6}{2\sqrt{100 \quad 200 \quad 1 \quad 2}} = \textbf{796 Hz}$$

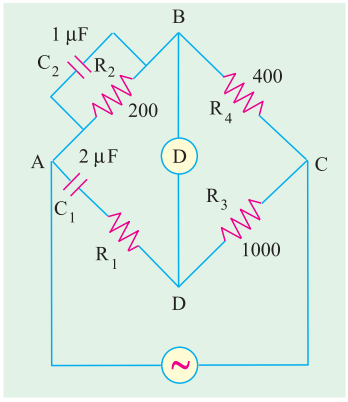


Fig. 16.20

Tutorial Problems No. 16.1

1. In Anderson a.c. bridge, an impedance of inductance L and resistance R is connected between A and B. For balance following data is obtained. An ohmic resistance of $1000\,\Omega$ each in arms AD and CD, a non-inductive resistance of $500\,\Omega$ in BC, a pure resistance of $200\,\Omega$ between points D and E and a capacitor of $2\mu F$ between C and E. The supply is 10 volt (A.C.) at a frequency of 100 Hz and is connected across points A and C. Find L and R. **[1.4 H; 500 Ω]**

2. A balanced bridge has the following components connected between its five nodes, A, B, C, D and E:

Between A and B : 1,000 ohm resistance; Between B and C : 1,000 ohm resistance

Between C and D : an inductor; Between D and A : 218 ohm resistance

Between A and E : 469 ohm resistance; Between E and B : 10 μ F capacitance

Between E and C : a detector; Between B and D : a power supply (a.c.)

Derive the equations of balance and hence deduce the resistance and inductance of the inductor.

[R = 218 Ω , L = 7.89 H] (*Elect. Theory and Meas. London Univ.*)

3. An a.c. bridge is arranged as follows: The arms AB and BC consist of non-inductive resistance of $100\,\Omega$, the arms, BE and CD of non-inductive variable resistances, the arm EC of a capacitor of $1\ \mu F$ capacitance, the arm DA of an inductive resistance. The a.c. source is connected to A and C and the telephone receiver to E and D. A balance is obtained when the resistances of the arms CD and BE are 50 Ω and 2500 Ω respectively.

Calculate the resistance and the inductance of the arm DA. What would be the effect of harmonics in the waveform of the alternating current source? **[50 Ω ; 0.25 H]**

4. For the Anderson's bridge of Fig. 16.21, the values are underbalance conditions. Determine the values of unknown resistance R and inductance L. **[R = 500 Ω ; L = 1.5 H]**
(*Elect. Meas & Inst. Madras Univ. Nov. 1978*)

5. An Anderson's bridge is arranged as under and balanced for the following values of the bridge components:

Branch AB – unknown coil of inductance L and resistance R

Branch BC – non-inductive resistance of 500 Ω

Branches AD & CD – non-inductive resistance of 100 Ω each

Branch DE – non-inductive resistance of 200 Ω

Branch EB – vibration galvanometer

Branch EC – 2.0 μF capacitance

Between A and C is 10 V, 100-Hz a.c. supply. Find the values of R and L of the unknown coil.

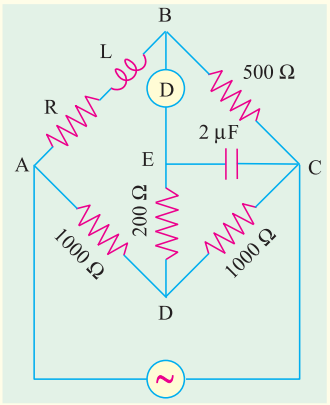


Fig. 16.21

[R = 500 Ω ; L = 0.5 H] (*Elect. Meas & Meas. Inst., Gujarat Univ.*)

6. An a.c. Anderson bridge is arranged as follows:

(*i*) branches BC and ED are variable non-reactive resistors

(*ii*) branches CD and DA are non-reactive resistors of 200 ohm each

(*iii*) branch CE is a loss-free capacitor of 1 μF capacitance.

The supply is connected across A and C and the detector across B and E. Balance is obtained when the resistance of BC is 400 ohm and that of ED is 500 ohm. Calculate the resistance and inductance of AB.
Derive the relation used and draw the vector diagram for balanced condition of the bridge.
[400 Ω ; 0.48 H] (*Elect. Measurements, Poona Univ.*)

7. In a balanced bridge network, AB is a resistance of 500 ohm in series with an inductance of 0.18 henry, the non-inductive resistances BC and DA have values of 1000 ohm and arm CD consists of a capacitance of C in series with a resistance R. A potential difference of 5 volts at a frequency $5000/2\pi$ is the supply between the points A and C. Find out the values of R and C and draw the vector diagram. **[472 Ω ; 0.235 μF] (*Elect. Measurements, Poona Univ.*)**

8. A sample of bakelite was tested by the Schering bridge method at 25 kV, 50-Hz. Balance was obtained with a standard capacitor of 106 pF capacitance, a capacitor of capacitance 0.4 μF in parallel with a non-reactive resistor of 318 Ω and a non-reactive resistor of 120 Ω. Determine the capacitance, the equivalent series resistance and the power factor of the specimen. Draw the phase or diagram for the balanced bridge. **[281 pF ; 0.452 MΩ ; 0.04] (*Elect. Measurements-II; Bangalore Univ.*)**

9. The conditions at balance of a Schering bridge set up to measure the capacitance and loss angle of a paper dielectric capacitor are as follows:

f = 500 Hz
Z_1 = a pure capacitance of 0.1 μF
Z_2 = a resistance of 500 Ω shunted by a capacitance of 0.0033 μF
Z_3 = pure resistance of 163 Ω
Z_4 = the capacitor under test

Calculate the approximate values of the loss resistance of the capacitor assuming–
(*a*) series loss resistance (*b*) shunt loss resistance. **[5.37 Ω , 197,000 Ω] (*London Univ.*)**

10. Name and draw the bridge used for measurements of Inductance. ***(Anna University, April 2002)***

11. A Wheat-stone bridge network has the following resistances :
AB = 10Ω, BC = 15Ω, CD = 25Ω, DA = 20Ω and BD = 10Ω
(V.T.U., Belgaum Karnataka University, February 2002)

OBJECTIVE TESTS – 16

1. Maxwell-Wien bridge is used for measuring
(*a*) capacitance (*b*) dielectric loss
(*c*) inductance (*d*) phase angle

2. Maxwell's L/C bridge is so called because
(*a*) it employs L and C in two arms
(*b*) ratio L/C remains constant
(*c*) for balance, it uses two opposite impedances in opposite arms
(*d*) balance is obtained when $L = C$

3. ..... bridge is used for measuring an unknown inductance in terms of a known capacitance and resistance.
(*a*) Maxwell's L/C (*b*) Hay's
(*c*) Owen (*d*) Anderson

4. Anderson bridge is a modification of ..... bridge.
(*a*) Owen (*b*) Hay's
(*c*) De Sauty (*d*) Maxwell-Wien

5. Hay's bridge is particularly useful for measuring
(*a*) inductive impedance with large phase angle
(*b*) mutual inductance
(*c*) self inductance
(*d*) capacitance and dielectric loss

6. The most useful ac bridge for comparing capacitances of two air capacitors is ..... bridge.
(*a*) Schering (*b*) De Sauty
(*c*) Wien series (*d*) Wien parallel

7. Heaviside-Campbell Equal Ratio bridge is used for measuring
(*a*) self-inductance in terms of mutual inductance
(*b*) capacitance in terms of inductance
(*c*) dielectric loss of an imperfect capacitor
(*d*) phase angle of a coil

ANSWERS

1. *c* **2.** *c* **3.** *d* **4.** *d* **5.** *a* **6.** *b* **7.** *a*

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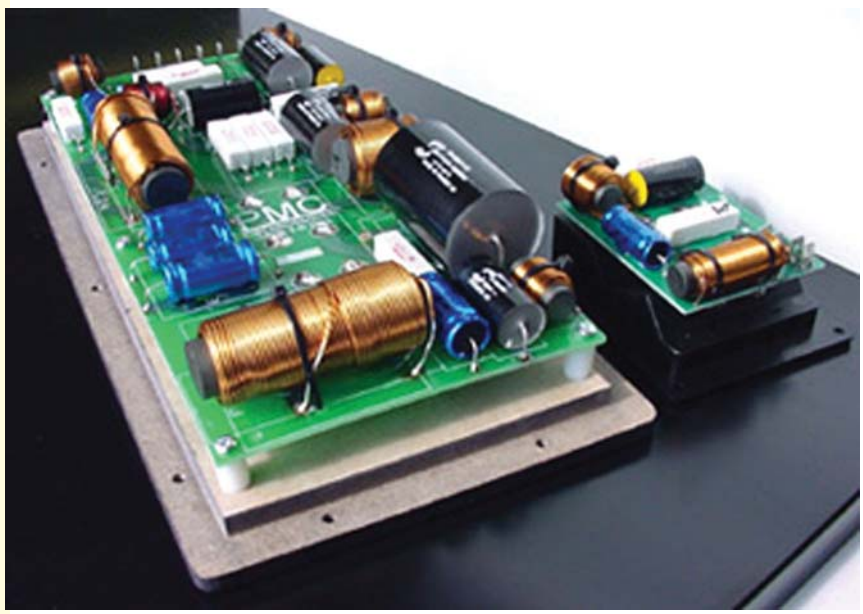
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CHAPTER 17

A.C. FILTER NETWORKS

Learning Objectives

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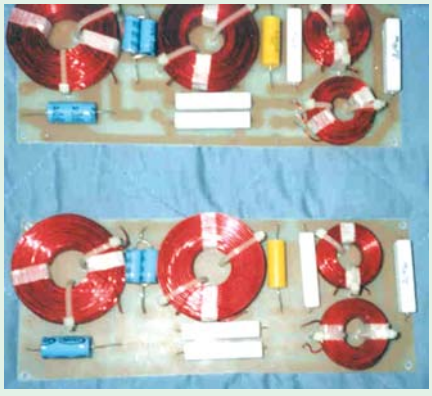


By using various combinations of resistors, inductors and capacitors, we can make circuits that have the property of passing or rejecting either low or high frequencies or bands of frequencies

17.1. Introduction

The reactances of inductors and capacitors depend on the frequency of the a.c. signal applied to them. That is why these devices are known as frequency-selective. By using various combinations of resistors, inductors and capacitors, we can make circuits that have the property of passing or rejecting either low or high frequencies or bands of frequencies. These frequency-selective networks, which alter the amplitude and phase characteristics of the input a.c. signal, are called filters. Their performance is usually expressed in terms of how much attenuation a band of frequencies experiences by passing through them. Attenuation is commonly expressed in terms of decibels (dB).

17.2. Applications



Closeup of a crossover network

A.C. filters find application in audio systems and television etc. Bandpass filters are used to select frequency ranges corresponding to desired radio or television station channels. Similarly, bandstop filters are used to reject undesirable signals that may contaminate the desirable signal. For example, low-pass filters are used to eliminate undesirable hum in d.c. power supplies.

No loudspeaker is equally efficient over the entire audible range of frequencies. That is why high-fidelity loudspeaker systems use a combination of low-pass, high-pass and bandpass filters (called crossover networks) to separate and then direct signals of appropriate frequency range to the different loudspeakers making up the system. Fig. 17.1 shows the output circuit of a high-fidelity audio amplifier, which uses three filters to separate, the low, mid-range and high frequencies, for feeding them to individual loudspeakers, best able to reproduce them.

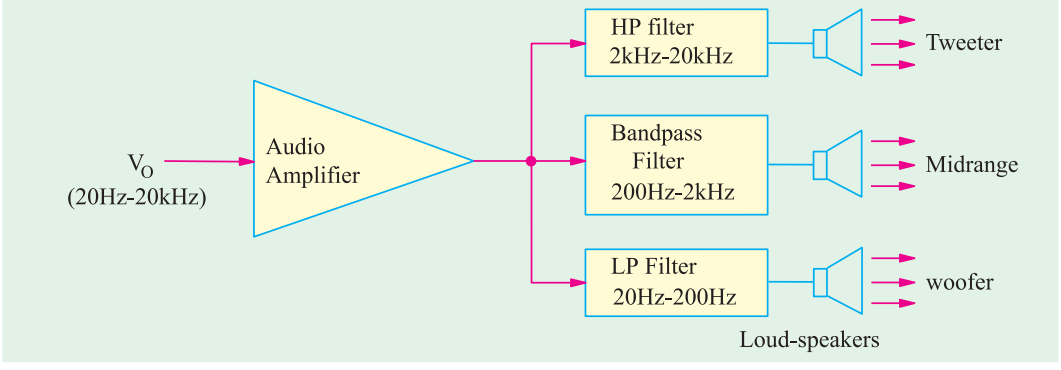


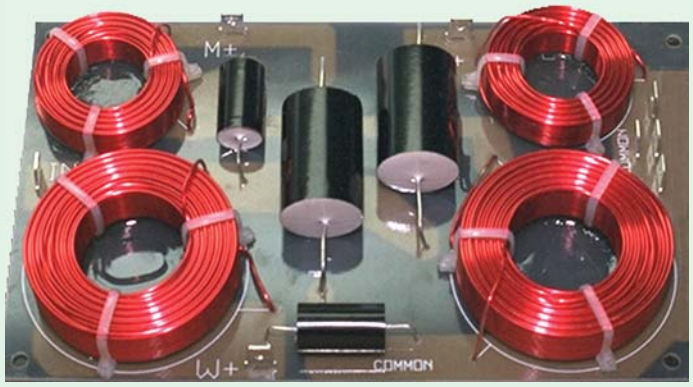
Fig. 17.1

17.3. Different Types of Filters

A.C. filter networks are divided into two major categories: ***(i)*** active networks and ***(ii)*** passive networks.

Active filter networks usually contain transistors and/or operational amplifiers in combination with *R*, *L* and *C* elements to obtain the desired filtering effect. These will not be discussed in this book. We will consider passive filter networks only which usually consist of series-parallel combinations of *R*, *L* and *C* elements. There are four types of such networks, as described below:

1. Low-Pass Filter. As the name shows, it allows only low frequencies to pass through, but attenuates (to a lesser or greater extent) all higher frequencies. The maximum frequency which it allows to pass through, is called cutoff frequency f_c (also called break frequency). There are R_L and R_C low-pass filters.



Three way passive crossover network

2. High-Pass Filter. It allows signals with higher frequencies to pass from input to output while rejecting lower frequencies. The minimum frequency it allows to pass is called cutoff frequency f_c. There are R_L and R_C high-pass filters.



High pass filter

3. Bandpass Filter. It is a resonant circuit which is tuned to pass a certain band or range of frequencies while rejecting all frequencies below and above this range (called passband).

4. Bandstop Filter. It is a resonant circuit that rejects a certain band or range of frequencies while passing all frequencies below and above the rejected band. Such filters are also called wavetraps, notch filters or band-elimination, band-separation or band-rejection filters.

17.4. Octaves and Decades of Frequency

A filter's performance is expressed in terms of the number of decibels the signal is increased or decreased per frequency octave or frequency decade. An octave means a doubling or halving of a frequency whereas a decade means tenfold increase or decrease in frequency.

17.5. The Decibel System

These system of logarithmic measurement is widely used in audio, radio, TV and instrument industry for comparing two voltages, currents or power levels. These levels are measured in a unit called bel (B) or decibel (dB) which is $1/10^{th}$ of a bel.

Suppose we want to compare the output power P_0 of a filter with its input power P_i. The power level change is

$$= 10 \log_{10}{}^{th} P_0/P_i \text{ dB}$$

It should be noted that dB is the unit of power change (*i.e.* increase or decrease) and not of power itself. Moreover, 20 dB is not twice as much power as 10 dB.

However, when voltage and current levels are required, then the expressions are:

$$\text{Current level} = 20 \log_{10} (I_0/I_i) \text{ dB}$$

$$\text{Similarly, voltage level} = 20 \log V_0/V_i \text{ dB}$$

Obviously, for power, we use a multiplying factor of 10 but for voltages and currents, we use a multiplying factor of 20.

17.6. Value of 1 dB

It can be proved that 1 dB represents the log of two powers, which have a ratio of 1.26.

$$1 \text{ dB} = 10\log_{10}(P_2/P_1) \text{ or } \log_{10}(P_2/P_1) \quad 0.1 \text{ or } \frac{P_2}{P_1} \quad 10^{0.1} \quad 1.26$$

Hence, it means that + 1 dB represents an increase in power of 26%.

Example 17.1. *The input and output voltages of a filter network are 16 mV and 8 mV respectively. Calculate the decibel level of the output voltage.*

Solution. Decibel level = $20\log_{10}(V_0/V_i)$dB = $-20\log_{10}(V_i/V_0)$ dB = $-20\log_{10}(16/8) = -6$ dB. Whenever voltage ratio is less than 1, its log is negative which is often difficult to handle. In such cases, it is best to invert the fraction and then make the result negative, as done above.

Example 17.2. *The output power of a filter is 100 mW when the signal frequency is 5 kHz. When the frequency is increased to 25 kHz, the output power falls to 50 mW. Calculate the dB change in power.*

Solution. The decibel change in power is

$= 10\log_{10}(50/100) = -10\log_{10}(100/50) = -10\log_{10} 2 = -10 \times 0.3 = -3$ dB

Example 17.3. *The output voltage of an amplifier is 10 V at 5 kHz and 7.07 V at 25 kHz. What is the decibel change in the output voltage?*

Solution. Decibel change = $20\log_{10}(V_0/V_i) = 20\log_{10}(7.07/10) = -20\log_{10}(10/7.07)$

$= -20\log_{10}(1.4/4)$

$= -20 \times 0.15 = -3$ dB

17.7. Low-Pass RC Filter

A simple low-pass *RC* filter is shown in Fig. 17.2 (*a*). As stated earlier, it permits signals of low frequencies upto f_c to pass through while attenuating frequencies above f_c. The range of frequencies upto f_c is called the passband of the filter. Fig. 17.2 (*b*) shows the frequency response curve of such a filter. It shows how the signal output voltage V_0 varies with the signal frequency. As seen at f_c, output signal voltage is reduced to 70.7% of the input voltage. The output is said to be – 3 dB at f_c. Signal outputs beyond f_c roll-off or attenuate at a fixed rate of – 6 dB/octave or – 20 dB/decade. As seen from the frequency-phase response curve of Fig. 17.2 (*c*), the phase angle between V_0 and V_i is 45° at cutoff frequency f_c.

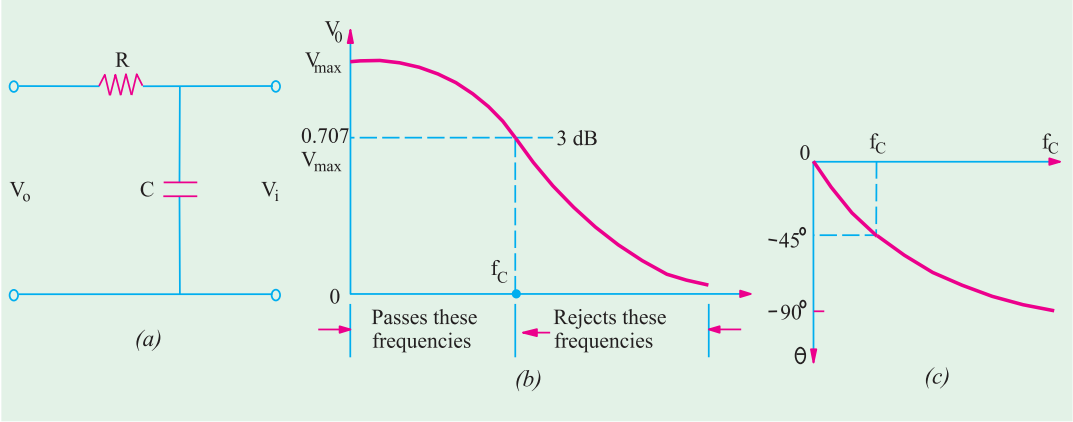


Fig. 17.2

By definition cutoff frequency f_c occurs where **(*a*)** $V_0 = 70.7\%\ V_i$ *i.e.* V_0 is – 3 dB down from V_i **(*b*)** $R = X_c$ and $V_R = V_C$ in magnitude. **(*c*)** The impedance phase angle $\theta = -45°$. The same is the angle between V_0 and V_i.

As seen, the output voltage is taken across the capacitor. Resistance R offers fixed opposition to frequencies but the reactance offered by capacitor C decreases with increase in frequency. Hence, low-frequency signal develops over C whereas high-frequency signals are grounded. Signal frequencies above f_c develop negligible voltage across C. Since R and C are in series, we can find the low-frequency output voltage V_0 developed across C by using the voltage-divider rule.

$$\therefore \qquad V_0 \quad V_i \frac{jX_C}{R \quad jX_C} \text{ and } f_c \quad \frac{1}{2 \ CR}$$

17.8. Other Types of Low-Pass Filters

There are many other types of low-pass filters in which instead of pure resistance, series chokes are commonly used alongwith capacitors.

(*i*) Inverted–L Type. It is shown in Fig. 17.3 (*a*). Here, inductive reactance of the choke blocks higher frequencies and C shorts them to ground. Hence, only low frequencies below f_c (for which X is very low) are passed without significant attenuation.

(*ii*) T-Type. It is shown in Fig. 17.3 (*b*). In this case, a second choke is connected on the output side which improves the filtering action.

(*iii*) π-Type. It is shown in Fig. 17.3 (*c*). The additional capacitor further improves the filtering action by grounding higher frequencies.

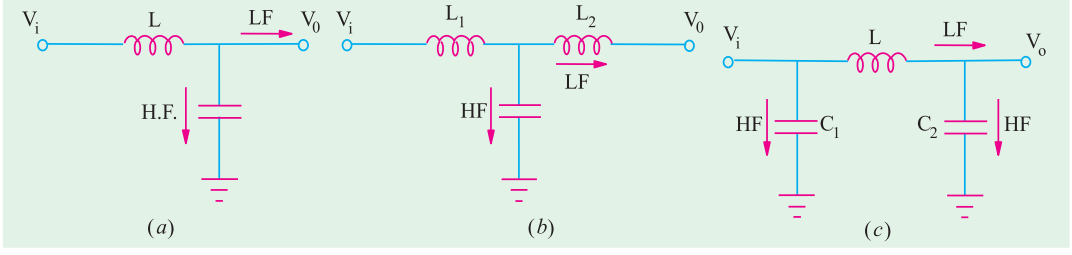


Fig. 17.3

It would be seen from the above figures that choke is always connected in series between the input and the output and capacitors are grounded in parallel. The output voltage is taken across the capacitor.

Example 17.4. *A simple low-pass RC filter having a cutoff frequency of 1 kHz is connected to a constant ac source of 10 V with variable frequency. Calculate the following :*

(*a*) value of C if $R = 10\,k\Omega$ **(*b*)** output voltage and its decibel level when

(*i*) $f = f_c$ **(*ii*)** $f = 2f_c$ and **(*iii*)** $f = 10f_c$.

Solution. (*a*) At f_c, $r = X_c = 1/2\pi f_c$ or $C = 1/2\pi \times 1 \times 10^3 \times 10 \times 10^3 = 15.9 \times 10^{-9} = 15.9$ nF

(*b*) (*i*) $f = f_c = 1$ kHz. Now, $-jX_c = R = -j10 = 10 \angle -90°\ \Omega$

$$\therefore \qquad V_0 = V_i \frac{-jX_c}{R - jX_c} = 10 \frac{-j10}{10 - j10} = 7.07 \angle -45°$$

Output decibel level = $20 \log_{10} (v_0/v_i) = -20 \log_{10} (V_i/V_0) = -20 \log_{10} (10/0.707) = -3$ dB

(*ii*) Here, $f = 2f_c = 2$ kHz *i.e.* octave of f_c. Since capacitive reactance is inversely proportional to frequency, $\therefore X_{c2} = C_{c1}(f_1 / f_2) = -j10(1/2) = -j5 = 5 \angle -90°\ k\Omega$

$$\therefore \quad V_0 = \frac{5\angle -90°}{10 - j5} = \frac{5°\angle -90°}{11.18\angle -26.6°} = 4.472\angle -63.4°$$

Decibel level = $-20 \log_{10} (V_i/V_0) = -20 \log_{10} (10/4.472) = -6.98$ dB

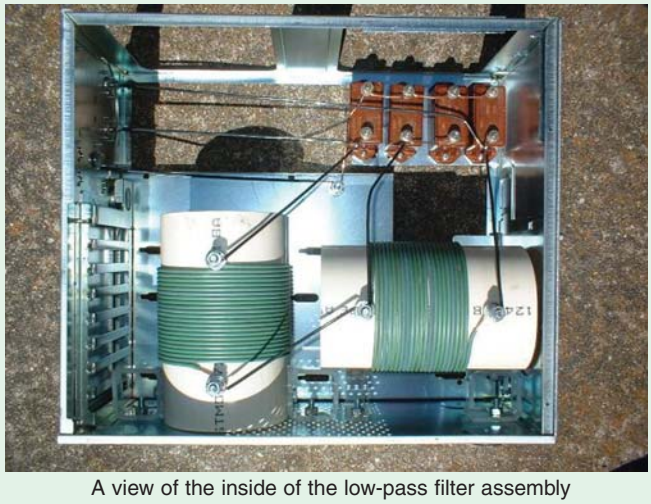
(iii) $X_{c3} = X_{c1}(f_1/f_3) = -j10\ (1/10) = j1 = 1\angle -90°$ $k\Omega$

$$\therefore \quad V_0 = 10\frac{10\angle -90°}{10 - j1} = 1\angle -84.3°$$

Decibel level = $-20 \log_{10}(10/1) = -20$ dB

17.9. Low-Pass RL Filter

It is shown in Fig. 17.4 (*a*). Here, coil offers high reactance to high frequencies and low reactance to low frequencies. Hence, low frequencies upto f_c can pass through the coil without much opposition. The output voltage is developed across *R*. Fig 17.4 (*b*) shows the frequency-output response curve of the filter. As seen at f_c, $V_0 = 0.707\ V_i$ and its attenuation level is –3 *dB* with respect to V_0 *i.e.* the voltage at $f = 0$.



A view of the inside of the low-pass filter assembly

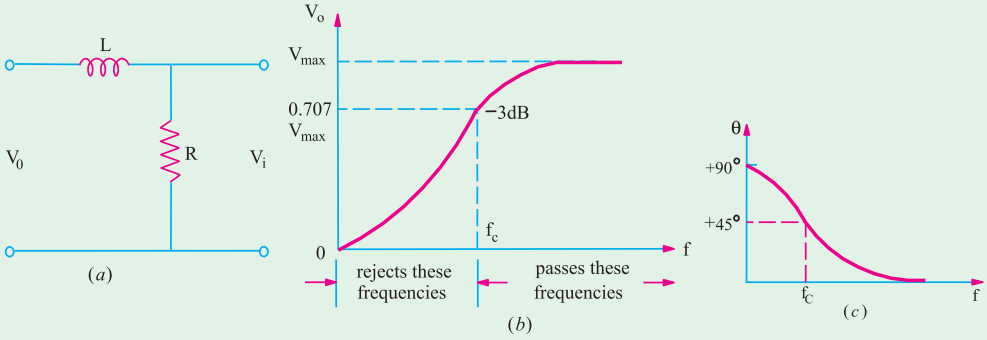


Fig. 17.4

However, it may be noted that being an *RL* circuit, the impedance phase angle is +45° (and not –45° as in low-pass *RC* filter). Again at f_c, $R = X_L$.

Using the voltage-divider rule, the output voltage developed across *R* is given by

$$V_0 = V_i \frac{R}{R + jX_L} \quad \text{and} \quad f_c = \frac{R}{2\pi L}$$

Example 17.5. *An ac signal having constant amplitude of 10 V but variable frequency is applied across a simple low-pass RL circuit with a cutoff frequency of 1 kHz. Calculate* ***(a)*** *value of L if* $R = 1\ k\Omega$ ***(b)*** *output voltage and its decibel level when* ***(i)*** $f = f_c$ *and* ***(iii)*** $f = 10\ f_c$.

Solution. ***(a)*** $L \quad R/2 \quad f_c \quad 1 \quad 10^3/2 \quad 3.14 \quad 10^3 \quad 159.2$ mH

(b) (i) $f = f_c = 1kHz\,;\, jX_L = R = j1; V_0 = 10\dfrac{1}{(1+j1)} = 7.07\angle -45°\,V$

Decibel decrease = –20 $\log_{10}$ (V_i / V_b) = –20 $\log_{10}$ 10/7.07 = –3 dB

(*ii*) $f = 2f_c$ = 2 kHz. Since X_L varies directly with

f, $X_{L2} = X_{L1}\,(f_2/f_1) = 1 \times 2/1 = 2\ k\Omega$

$$\therefore \qquad V_0 = 10\frac{1}{(1+j2)} = \frac{10}{2.236\angle 63.4°} = 4.472\angle -63.4°$$

Decibel decrease = –20 $\log_{10}$ (10/4.472) = – 6.98 dB

(*iii*) f = 10 f_c = 10 kHz ; $X_{L3} = 1 \times 10/1 = 10\,\Omega$, $V_0 = \dfrac{1}{(1+j10)} = 1\angle -84.3°$

Decibel decrease = – 20 $\log_{10}$(10/1) = –20 dB

17.10. High-Pass RC Filter

It is shown in Fig. 17.5 (*a*). Lower frequencies experience considerable reactance by the capacitor and are not easily passed. Higher frequencies encounter little reactance and are easily passed. The high frequencies passing through the filter develop output voltage V_0 across R. As seen from the frequency response of Fig. 13.5 (*b*), all frequencies above f_c are passed whereas those below it are attenuated. As before, f_c corresponds to –3 dB output voltage or half-power point. At f_c, $R = X_c$ and the phase angle between V_0 and V_i is +45° as shown in Fig. 17.5 (*c*). It may be noted that high-pass *RC* filter can be obtained merely by interchanging the positions of R and C in the low-pass *RC* filter of Fig. 17.5 (*a*).

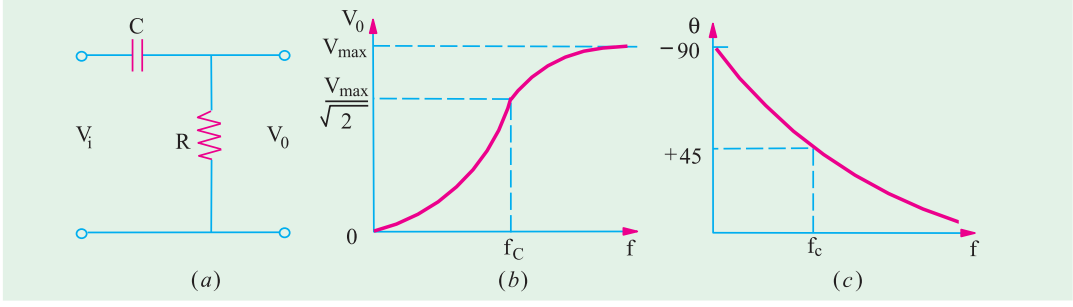


Fig. 17.5

Since R and C are in series across the input voltage, the voltage drop across R, as found by the voltage-divider rule, is

$$V_0 \quad V_i \frac{R}{R \quad jX_c} \text{ and } f_c \quad \frac{1}{2 \quad CR}$$

A very common application of the series capacitor high-pass filter is a coupling capacitor between two audio amplifier stages. It is used for passing the amplified audio-signal from one stage to the next and simultaneously block the constant d.c. voltage.

Other high-pass RC filter circuits exist besides the one shown in Fig. 17.5 (*a*). These are shown in Fig 17.6.



High-pass filter

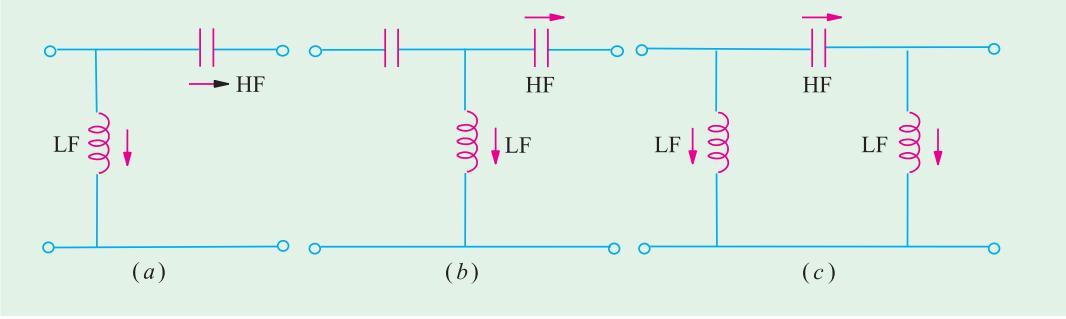


Fig. 17.6

***(i)* Inverted-L Type.** It is so called because the capacitor and inductor from an upside down *L*. It is shown in Fig. 17.6 (*a*). At lower frequencies, X_C is large but X_L is small. Hence, most of the input voltage drops across X_C and very little across X_L. However, when the frequency is increased, X_C becomes less but X_L is increased thereby causing the output voltage to increase. Consequently, high frequencies are passed while lower frequencies are attenuated.

***(ii)* T-Type.** It uses two capacitors and a choke as shown in Fig. 17.6 (*b*). The additional capacitor improves the filtering action.

***(iii)* π -Type.** It uses two inductors which shunt out the lower frequencies as shown in Fig. 17.6 (*c*).

It would be seen that in all high-pass filter circuits, capacitors are in series between the input and output and the coils are grounded. In fact, capacitors can be viewed as shorts to high frequencies but as open to low frequencies. Opposite is the case with chokes.

17.11. High-Pass RL Filter

It is shown in Fig. 17.7 and can be obtained by 'swapping' position of *R* and *L* in the low-pass *RL* circuit of Fig. 17.4 (*a*). Its response curves are the same as for high-pass *RC* circuit and are shown in Fig. 17.5 (*b*) and (*c*).

As usual, its output voltage equals the voltage which drops across X_L. It is given by

$$V_0 \quad V_i \frac{j X_L}{R \quad j X_L} \quad \text{and} \quad f_c \quad \frac{R}{2 \ L}$$

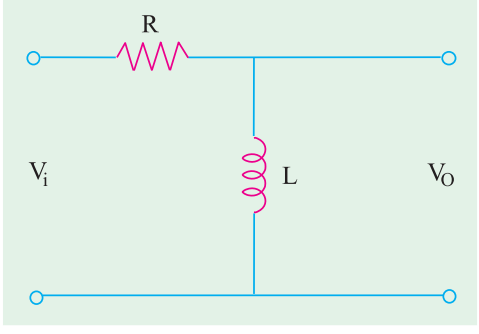


Fig. 17.7

Example 17.6. *Design a high-pass RL filter that has a cutoff frequency of 4 kHz when R = 3 kΩ . It is connected to a 10∠0° V variable frequency supply. Calculate the following :*

***(a)** Inductor of inductance L but of negligible resistance **(b)** output voltage V_0 and its decibel decrease at*

***(i)** f = 0 **(ii)** f = f_c **(iii)** 8 kHz and **(iv)** 40 kHz*

Solution. (*a*) $L = R/2\pi f_c = 3/2 \ \pi \ \times 4 = 119.4$ mH

(*b*) (*i*) At $f = 0$; $X_L = 0$ *i.e.* inductro acts as a short-circuit across which no voltage develops. Hence, $V_0 = 0$ V as shown in Fig. 17.74.

(*ii*) $f = f_c = 4$ kHz ; $X_L = R$. $\therefore \ jX_L = \text{j}3 = 3\angle 90°$ kΩ

$$\therefore \quad V_0 \quad V_i \frac{jX_L}{R \quad jX_L} \quad 10 \quad 0 \ \frac{3 \quad 90}{3 \quad j3} \quad \frac{30 \quad 90}{4.24 \quad 45} \quad 7.07 \quad 45 \text{ V}$$

Decibel decrease = – 20 $\log_{10}(10/7.07)$ = – 3 dB

(iii) $f = 2f_c$ = 8 kHz. $X_{L2} = 2 \times j3 = j6\ \text{k}\Omega$

$$\therefore \quad V_0 = 10\angle 0° \frac{6\angle 90°}{3 + j6} = \frac{60\angle 90°}{6.7\angle 63.4°} = 8.95\angle 26.6°\ \text{V}$$

Decibel decrease = – 20 $\log_{10}(10/8.95)$ = – 0.96 dB

(iv) $f = 10f_c$ = 40 kHz; $X_{L3} = 10 \times j3 = j30\ \text{k}\Omega$

$$\therefore \qquad B_0 = 10 \frac{j30}{3 + j30} = \frac{300\angle 90°}{30.15\angle 84.3°} = 19.95\angle 5.7°\ \text{V}$$

Decibel decrease = – 20 $\log_{10}(10/9.95)$ = 0.04 dB

As seen from Fig. 17.7, as frequency is increased, V_0 is also increased.

17.12. R-C bandpass Filter

It is a filter that allows a certain band of frequencies to pass through and attenuates all other frequencies below and above the passband. This passband is known as the bandwidth of the filter. As seen, it is obtained by cascading a high-pass *RC* filter to a low-pass *RC* filter. It is shown in Fig. 17.8 alongwith its response curve. The passband of this filter is given by the band of frequencies lying between f_{c1} and f_{c2}. Their values are given by

$$f_{c1} = 1/2\pi\, C_1R_1 \text{ and } \quad f_{c_2} \quad 1/2\ C_2R_2$$

The ratio of the output and input voltages is given by

$$\frac{V_0}{V_i} = \frac{R_1}{R_1 - jX_{C1}} \quad \text{... from } f_1 \text{ to } f_{C1};$$

$$= \frac{-jX_{C2}}{R_2 - jX_{C2}} \quad \text{... from } f_{c_2} \text{ to } f_2$$



Bandpass filters

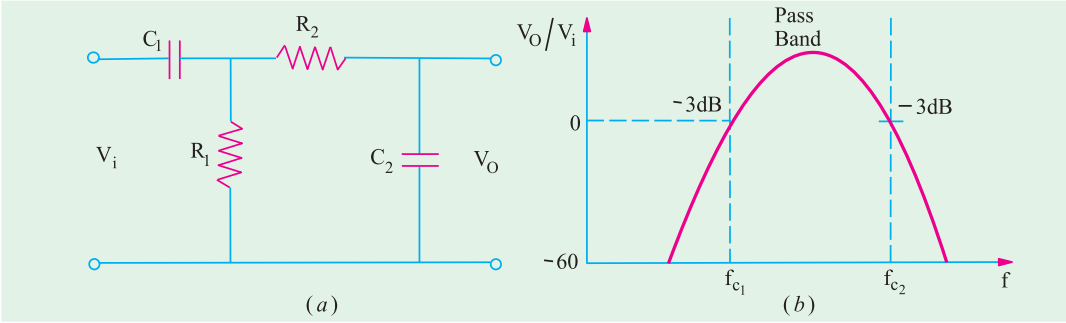


Fig. 17.8

17.13. R-C Bandstop Filter

It is a series combination of low-pass and high-pass RC filters as shown in Fig. 17.9 (*a*). In fact, it can be obtained by reversing the cascaded sequence of the RC bandpass filter. As stated earlier, this filter attenuates a single band of frequencies and allows those on either side to pass through. The stopband is represented by the group of frequencies that lie between f_1 and f_2 where response is below – 60 dB.

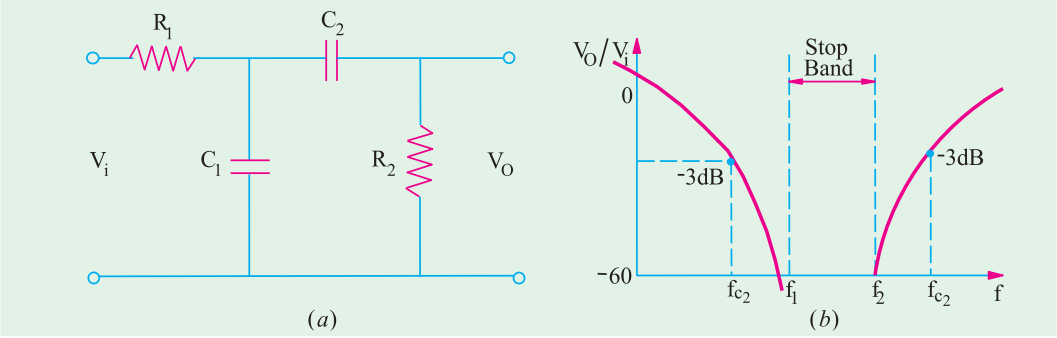


Fig. 17.9

For frequencies from f_{c1} to f_1, the following relationships hold good :

$$\frac{V_0}{V_i} \quad \frac{j\,X_{C1}}{(R_1 \quad j\,X_{C1})} \quad \text{and} \quad f_{c_1} \quad \frac{1}{2 \quad C_1 R_1}$$

For frequencies from f_2 to f_{c2}, the relationships are as under :

$$\frac{V_0}{V_i} \quad \frac{R_2}{(R_2 \quad j\,X_{C2})} \text{ and } f_{C2} \quad \frac{1}{2 \quad C_2 R_2}$$

In practices, several low-pass *RC* filter circuits cascaded with several high-pass *RC* filter circuits which provide almost vertical roll-offs and rises. Moreover, unlike *RL* filters, *RC* filters can be produced in the form of large-scale integrated circuits. Hence, cascading is rarely done with *RL* circuits.



A window filter contains one band pass and one low-pass or one high-pass and is used for filtering out unwanted channels, in CATV reception systems or for application cablenet or other communication systems

17.14. The – 3 dB Frequencies

The output of an a.c. filter is said to be down 3 dB or –3 dB at the cutoff frequencies. Actually at this frequency, the output voltage of the circuit is 70.7% of the maximum input voltage as shown in Fig. 17.10 (*a*) for low-pass filter and in Fig. 17.10 (*b*) and (*c*) for high-pass and bandpass filters respectively. Here, maximum voltage is taken as the 0 dB reference.

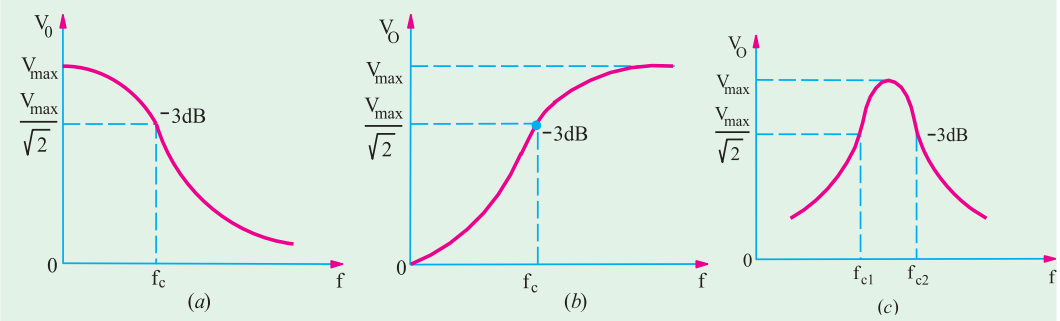


Fig. 17.10

It can also be shows that the power output at the cutoff frequency is 50% of that at zero frequency in the case of low-pass and high-pass filters and of that at f_0 in case of resonant-circuit filter.

17.15. Roll-off of the Response Curve

Gradual decreasing of the output of an a.c. filter is called roll-off. The dotted curve in Fig. 17.11 (*a*) shows an actual response curve of a low-pass *RC* filter. The maximum output is defined to be zero dB as a reference. In other words, 0 dB corresponds to the condition when $V_0 = v_i$ because $20 \log_{10} V_0/V_i = 20 \log 1 =$ dB. As seen, the output drops from 0 dB to – 3 dB at the cutoff frequency and then continues to decrease at a fixed rate. This pattern of decrease is known as the roll-off of the frequency response. The solid straight line in Fig. 17.11 (*a*) represents an ideal output response that is considered to be 'flat' and which cuts the frequency axis at f_c.

The roll-off for a basic IRC *or IRL* filter is 20 dB/decade or 6 dB/octave. Fig. 17.11 (*b*) shows the frequency response plot on a semi-log-scale where each interval on the horizontal axis represents a tenfold increase in frequency. This response curve is known as Bode plot. Fig. 17.11 (*c*) shown the Bode plot for a high-pass *RC* filter on a semi-log graph. The approximate actual response curve is shown by the dotted line. Here, the frequency is on the logarithmic scale and the filter output in decibel is alongwith the linear vertical scale. The filter output is flat beyond f_c. But as the frequency is reduced below f_c, the output drops at the rate of – 20 dB/decade.

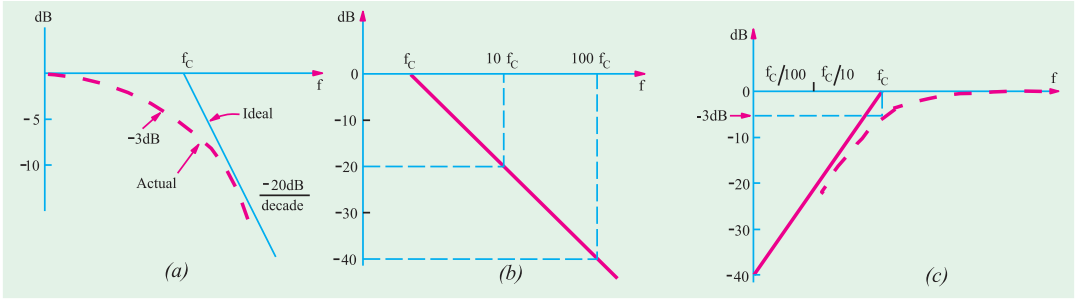


Fig. 17.11

17.16. Bandstop and Bandpass Resonant Filter Circuits

Frequency resonant circuits are used in electronic system to make either bandstop or bandpass filters because of their characteristic Q-rise to either current or voltage at the resonant frequency. Both series and parallel resonant circuits are used for the purpose. It has already been discussed in Chap. No. 7 that

(i) a series resonant circuit offers minimum impedance to input signal and provides maximum current. Minimum impedance equals *R* because $X_L = X_C$ and maximum current $I = V/R$.

(ii) a parallel circuit offers maximum impedance to the input signal and provides minimum current. Maximum impedance offered is = L/CR and minimum current $I = V/(L/CR)$.

17.17. Series-and Parallel-Resonant Bandstop Filters

The series resonant bandstop filter is shown in Fig. 17.12 (*a*) where the output is taken across the series resonant circuit. Hence, at resonant frequency f_0, the output circuit 'sees' a very low resistance *R* over which negligible output voltage V_0 is developed. That is why there is a shape resonant dip in the response curve of Fig. 17.12 (*b*). Such filters are commonly used to reject a particular frequency such as 50-cycle hum produced by transformers or inductors or turn table rumble in recording equipment.

For the series-resonant bandstop filter shown in Fig. 17.12 (*a*), the following relationships hold good :

$$\text{At } f_0, \frac{V_0}{V_i} = \frac{R_L}{(R_L + R_S)}; \; Q_0 = \frac{\omega_0 L}{(R + R_S)} \text{ and } B_{ph} = \frac{1/2\pi\sqrt{LC}}{Q_0}$$

At any other frequency f, $\dfrac{V_0}{V_i} = \dfrac{R_L + j(X_L - X_C)}{(R_L + R_S) + j(X_L - X_C)}$

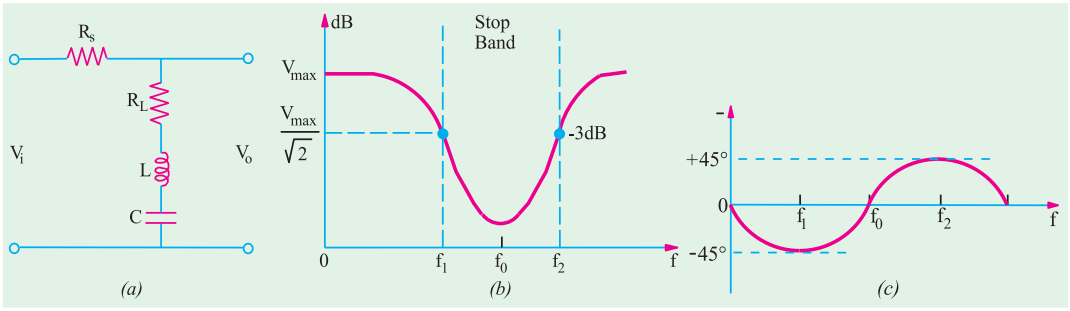


Fig. 17.12

17.18. Parallel-Resonant Bandstop Filter

In this filter, the parallel-resonant circuit is in series with the output resistor R as shown in Fig. 17.13. At resonance, the parallel circuit offers extremely high impedance to f_0 (and nearby frequencies) as compared to R. Hence the output voltage V_0 at f_0 developed across R is negligibly small as compared to that developed across the parallel-resonant circuit. Following relationships hold good for this filter :

At f_0; $\dfrac{V_0}{V_i} = \dfrac{R_0}{R_0 + Z_{p0}}$ where $Z_{p0} = Q_0^2 R_L$

At any frequency f, $\dfrac{V_0}{V_i} = \dfrac{R_0}{R_0 + Z_p}$

where $Z_p = \dfrac{Z_L Z_C}{R_L + j(X_L - X_C)}$

Also $Q_0 = \omega_0 L / R_L$ and $B_{hp} = (1/2\pi\sqrt{LC}) / Q_0$

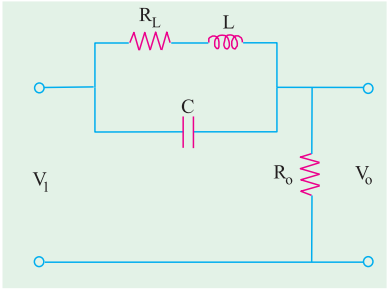


Fig. 17.13

It should be noted that the same amplitude phase response curves apply both to the series resonant and parallel-resonant bandstop filters. Since ***X_C predominates at lower frequencies, phase angle θ is negative below f_0, above f_0, X_L*** predominates and the phase current leads. At cutoff frequency f_1, $\theta = -45°$ and at other cutoff frequency f_2, $\theta = +45°$ as in the case of any resonant circuit.

Example 17.7. *A series-resonant bandstop filter consist of a series resistance of 2 k Ω across which is connected a series-resonant circuit consisting of a coil of resistance 10 Ω and inductance 350 mH and a capacitor of capacitance 181 pF. F if the applied signal voltage is $10\angle 0°$ of variable frequency, calculate*

***(a)** resonant frequency f_0 ; **(b)** half-power bandwidth B_{hp} ; **(c)** edge frequencies f_1 and f_2 ; **(d)** output voltage at frequencies f_0, f_1 and f_2.*

Solution. We are given that $R_S = 2$; $k\ \Omega$ $R = 10\ \Omega$; $L = 350$ mH; $C = 181$ pF.

(a) $f_0 = 1/2\pi\sqrt{LC} = 1/2\pi\sqrt{350\times10^{-3}\times181\times10^{-12}} = 20$ kHz

(b) $Q_0 = \omega_0 L/(R_S + R_L) = 2\pi\times20\times10^3\times350\times10^{-3}/2010 = 21.88$

$B_{hp} = f_0/Q_0 = 0.914$ kHz

(c) $f_1 = f_0 - B_{hp}/2 = 20 - 0.457 =$ **19.453 kHz**; $f_2 = 20 + 0.457 =$ **20.457 kHz**

(d) At f_0, $V_0 = V_i \dfrac{R_L}{(R_L + R_S)} = 10\angle0° \dfrac{10}{2110} =$ **$0.05\angle00°$V**

At f_1, $X_{L1} = 2\pi f_1 L = 2\pi \times 19.543 \times 10^3 \times 350 \times 10^{-3} = 42{,}977\,\Omega$

$$X_{C1} = 1/2\pi \times 19.543 \times 10^3 \times 181 \times 10^{-12} = 44{,}993\ \Omega$$

$\therefore\ (X_L - X_C) = (42{,}977 - 44{,}993) = -2016\ \Omega$

$$V_{01} = V_i \frac{R_L + j(X_L - X_C)}{(R_S + R_L) + j(X_L - X_C)} = 10\angle 0° \times \frac{f10 - j2016}{2010 - j2016}$$

$$= \frac{20160\angle -89.7°}{2847\angle -45°} = \mathbf{7.07\angle -44.7°V}$$

At f_2, $X_{L2} = X_{L1}\,(f_2/f_1) = 42977 \times 20.457/19.453$

$= 44{,}987\ \Omega$; $X_{C2} = X_{C1}\,(f_1/f_2) = 44{,}993 \times 19.543/20.457$

$= 42{,}983\ \Omega$; $(X_{L2} - X_{C2}) = 44{,}987 - 42{,}983 = 2004\ \Omega$

$$\therefore\ V_0 = 10\angle 0° \frac{10 + j2004}{2010 + j2004} = \frac{2004\angle 89.7°}{2837\angle 44.9°} = \mathbf{7.07\angle 44.8°V}$$

17.19. Series-Resonant Bandpass Filter

As shown in Fig. 17.14 (*a*), it consists of a series-resonant circuit shunted by an output resistance R_0. It would be seen that this filter circuit can be produced by 'swapping' as series resonant bandstop filter. At f_0, the series resonant impedance is very small and equal R_L which is negligible as compared to R_0. Hence, output voltage is maximum at f_0 and falls to 70.7% at cutoff frequency f_1 and f_2 and shown in the response curve of Fig. 17.14 (*b*). The phase angle is positive for frequencies above f_0 and negative for frequencies below f_0 as shown in Fig 17.14 (*c*) by the solid curve.

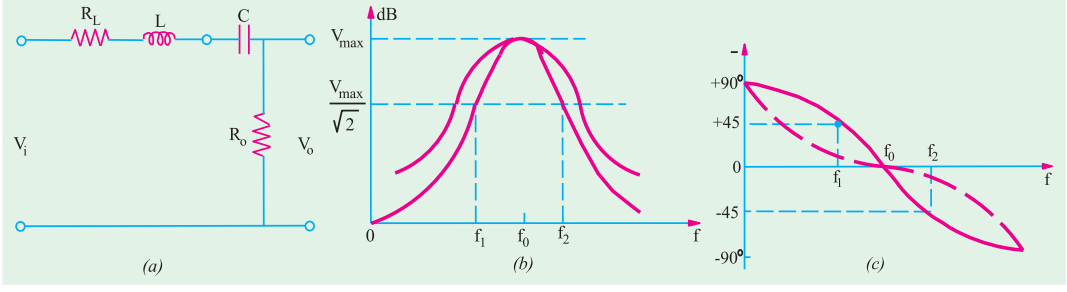


Fig. 17.14

Following relationships hold good for this filter circuit.

At f_0, $\dfrac{V_0}{V_i} = \dfrac{R_0}{(R_L + R_0)}$; $Q_0 = \dfrac{\omega_0 L}{(R_L + R_0)}$ and $B_{hp} = \dfrac{1/2\pi\sqrt{LC}}{Q_0}$

17.20. Parallel-Resonant Bandpass Filter

It can be obtained by transposing the circuit elements of a bandstop a parallel-resonant filter. As shown in Fig. 17.15, the output is taken across the two-branch parallel-resonant circuit. Since this circuit offers maximum impedance at resonance, this filter produces maximum output voltage V_0 at f_0. The amplitude-response curve of this filter is similar to that of the series-resonant bandpass filter discussed above [Fig. 17.14 (*b*)]. The dotted curve in Fig. 17.14 (*c*) represents the phase relationship between the input and output voltages of this filter. The following relationships apply to this filter :

At $f_0, \frac{V_0}{V_i} = \frac{R_0}{(R_0 + Z_{p0})}$ where $Z_{p0} = R_{p0} = Q_r^2 R_L$

and $Q_0 = \frac{R_{p0}}{X_{CO}}$ and $B_{hp} = \frac{1/2\pi\sqrt{LC}}{Q_0}$

At any frequency f,

$$\frac{V_0}{V_i} = \frac{Z_p}{R_0 + Z_p} \text{ where } Z_p = \frac{Z_L(-jX_C)}{R_L + j(X_L - X_C)}$$

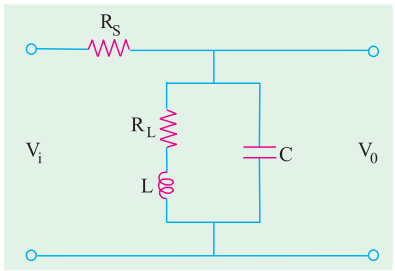


Fig. 17.15

OBJECTIVE TEST – 17

1. The decibel is a measure of
(*a*) power (*b*) voltage
(*c*) current (*d*) power level

2. When the output voltage level of a filter decreases by – 3 dB, its absolute value changes by a factor of
(*a*) $\sqrt{2}$ (*b*) $1/\sqrt{2}$
(*c*) 2 (*d*) 1/2

3. The frequency corresponding to half-power point on the response curve of a filter is known as —
(*a*) cutoff (*b*) upper
(*c*) lower (*d*) roll-off

4. In a low-pass filter, the cutoff frequency is represented by the point where the output voltage is reduced to — per cent of the input voltage.
(*a*) 50 (*b*) 70.7
(*c*) 63.2 (*d*) 33.3

5. In an *RL* low-pass filter, an attenuation of – 12 dB/octave corresponds to ..... dB/decade.
(*a*) – 6 (*b*) – 12
(*c*) – 20 (*d*) – 40

6. A network which attenuate a single band of frequencies and allows those on either side to pass through is called ..... filter.
(*a*) low-pass (*b*) high-pass
(*c*) bandstop (*d*) bandpass

7. In a simple high-pass RC filter, if the value of capacitance is doubled, the cutoff frequency is
(*a*) doubled (*b*) halved
(*c*) tripled (*d*) quadrupled

8. In a simple high-pass RL filter circuit, the phase difference between the output and input voltages at the cutoff frequency is .... degrees.
(*a*) – 90 (*b*) 45
(*c*) – 45 (*d*) 90

9. In a simple low-pass RC filter, attenuation is – 3 dB at f_c. At 2 f_c, attenuation is – 6 dB. At 10 f_c, the attenuation would be .... dB.
(*a*) – 30 (*b*) – 20
(*c*) – 18 (*d*) – 12

10. An a.c. signal of constant voltage 10 V and variable frequency is applied to a simple high-pass RC filter. The output voltage at ten times the cutoff frequency would be ..... volt.
(*a*) 1 (*b*) 5
(*c*) $10/\sqrt{2}$ (*d*) $10\sqrt{2}$

11. When two simple low-pass filters having same values of R and C are cascaded, the combined filter will have a roll-off of ..... dB/decade.
(*a*) – 20 (*b*) – 12
(*c*) – 40 (*d*) – 36

12. An a.c. signal of constant voltage but with frequency varying from dc to 25 kHz is applied to a high-pass filter. Which of the following frequency will develop the greatest voltage at the output load resistance?
(*a*) d. (*b*) 15 kHz
(*c*) 10 kHz (*d*) 25 kHz

13. A voltage signal source of constant amplitude with frequency varying from dc to 25 kHz is applied to a low-pass filter. Which frequency will develop greatest voltage across the output load resistance?
(*a*) d.c. (*b*) 10 kHz
(*c*) 15 kl (*d*) 25 kHz

14. The output of a filter drops from 10 to 5 V as the frequency is increased from 1 to 2 kHz. The dB change in the output voltage is
(*a*) – 3 dB/decade
(*b*) – 6 dB/octave
(*c*) 6 dB/octave
(*d*) – 3 dB/octave

ANSWERS

1. *d* **2.** *b* **3.** *a* **4.** *b* **5.** *d* **6.** *c* **7.** *b* **8.** *b* **9.** *b* **10.** *a* **11.** *c* **12.** *d* **13.** *a* **14.** *b*

Top

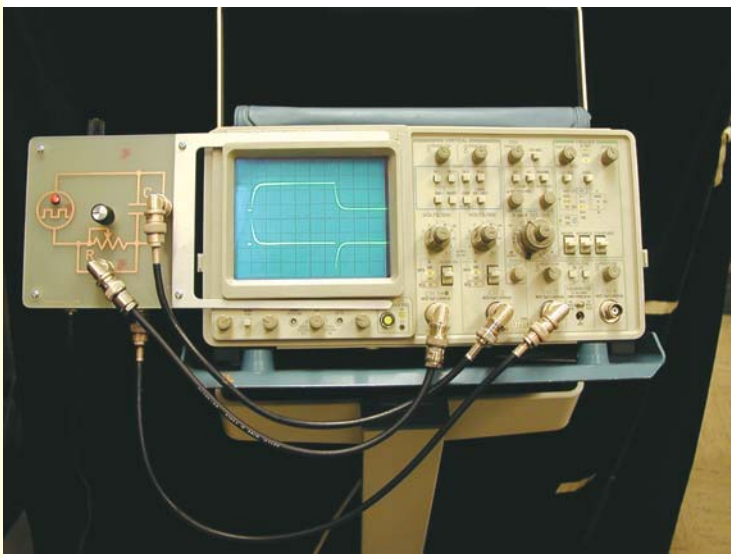
Contents

CHAPTER 18

CIRCLE DIAGRAMS

Learning Objectives

- Circle Diagram of a Series Circuit
- Rigorous Mathematical Treatment
- Constant Resistance but Variable Reactance
- Properties of Constant Reactance But Variable Resistance Circuit
- Simple Transmission Line Circuit



Combinations of R and C circuits

18.1. Circle Diagram of a Series Circuit

Circle diagrams are helpful in analysing the operating characteristics of circuits, which, under some conditions, are used in representing transmission lines and a.c. machinery (like induction motor etc.)

Consider a circuit having a constant reactance but variable resistance varying from zero to infinity and supplied with a voltage of constant magnitude and frequency (Fig. 18.1).

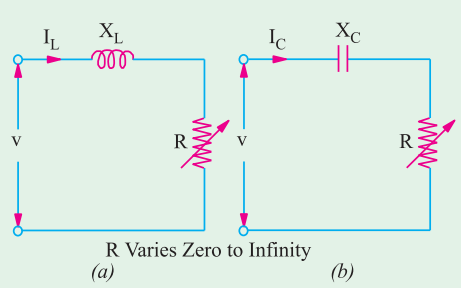


Fig. 18.1

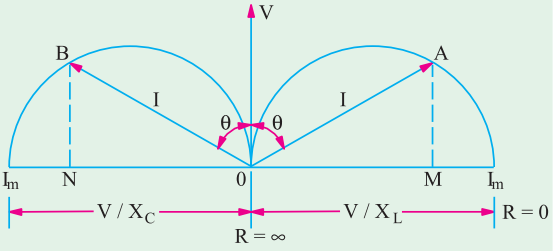


Fig. 18.2

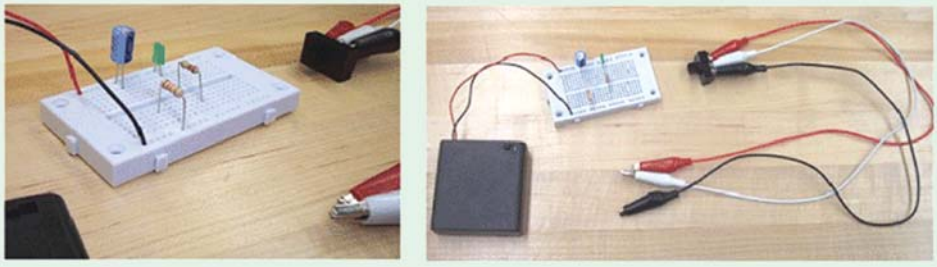
If $R = 0$, then $I = V/X_L$ or V/X_C and has maximum value . It will lag or lead the voltage by 90° depending on whether the reactance is inductive or capacitive. In Fig. 18.2, angle θ represents the phase angle. If R is now increased from its zero value, then I and θ will both decrease. In the limiting case, when $R = \infty$, then $I = 0$ and $\theta = 0°$. It is found that the locus of end point of current vector OA or OB represents a semi-circuit with diameter equal to V/X as shown in Fig. 18.2. It can be proved thus :

$I = V/Z$ and $\sin \theta = X/Z$ or $Z = X/\sin \theta \quad \therefore I = V \sin \theta / X$

For constant value of V and X, the above is the polar equation of a circle of diameter V/X. This equation is plotted in Fig. 18.2. Here, OV is taken as reference vector. It is also seen that for inductive circuit, the current semi-circle is on the right-hand side of reference vector OV so that current vector OA lags by $\theta°$. The current semi-circle for R-C circuit is drawn on the left hand side of OV so that current vector OB lends OV by $\theta°$. It is obvious that $AM = I \cos \theta$, hence AM represents, on a suitable scale, the power consumed by the R-L circuit, Similarly, BN represents the power consumed by the R-C circuit.

18.2. Rigorous Mathematical Treatment

We will again consider both R-L and R–C circuits. The voltage drops across R and X_L (or X_C) will be 90° out of phase with each other. Hence, for any given value of resistance, the vector diagram for the two voltage drops (*i.e.* IR and IX) is a rightangled triangle having applied voltage as the hypotenuse.



R.C. Circuits

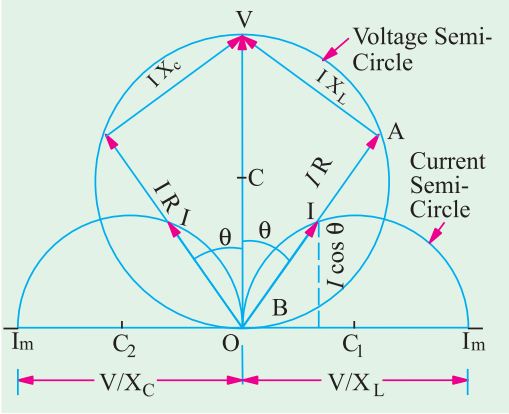


Fig. 18.3

For a constant applied voltage and reactance, the vector diagrams for different values of R are represented by a series of right-angled triangles having common hypotenuse as shown in Fig. 18.3. The locus of the apex of the right-angled voltage triangles is a semicircle described on the hypotenuse. The voltage semi–circle for R-L circuit (OAV) is on the right and for R-C circuit (OBV) on the left of the reference vector OV as shown in Fig. 18.3.

The foci of end points of current vectors are also semi-circles as shown but their centres lie on the opposite sides of and in an axis perpendicular to the reference vector OV.

(*ii*) R-L Circuit [Fig. 18.1 (*a*)]

The co-ordinates of point A with respect to the origin O are

$$y = I\cos\theta = \frac{V}{Z}\cdot\frac{R}{Z} = V\frac{R}{Z^2} = V\frac{R}{R^2+X_L^2}; x = I\sin\theta = \frac{V}{Z}\cdot\frac{X_L}{Z} = V\cdot\frac{X_L}{Z^2} = V\frac{X_L}{R^2+X_L^2} \quad \text{... (i)}$$

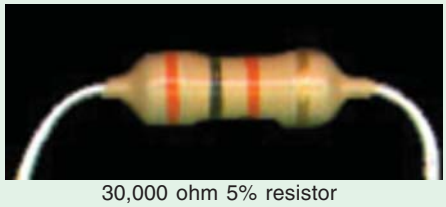
Squaring and adding, we get

$$x^2 \quad y^2 \quad \frac{VR}{R^2 \quad X_L^2}^2 \quad \frac{VX_L}{R^2 \quad X_L^2}^2 \quad \frac{V^2(R^2 \quad X_L^2)}{(R^2 \quad X_L^2)^2} \quad \frac{V^2}{R^2 \quad X_L^2}$$

From (*i*) above, $\frac{VX_L}{x} \quad R^2 \quad X_L^2 \qquad x^2 \quad y^2 \quad \frac{V^2}{VX_L/x} \quad \frac{xV}{X_L}$

$$\therefore \quad x^2 \quad y^2 \quad \frac{xV}{X_L} \quad \text{or} \quad y^2 \quad x \quad \frac{V}{2X_L}^2 \quad \frac{V^2}{4X_L^2}$$

This is the equation of a circle, the co-ordinates of the centre of which are $y = 0$, $x = V/2\ X_L$ and whose radius is $V/2X_L$.



30,000 ohm 5% resistor

(*ii*) R-C Circuit. In this case it can be similarly proved that the locus of the end point of current vector is a semi-circle. The equation of this circle is

$$y^2 \quad x \quad \frac{V}{2X_C}^2 \quad \frac{V^2}{4X_C^2}$$

The centre has co-ordinates of $y = 0$, $x = -V/2X_C$.

18.3. Constant Resistance But Variable Reactance

Fig. 18.4 shows two circuits having constant resistance but variable reactance X_L or X_C which vary from zero to infinity. When $X_L = 0$, current is maximum and equals V/R. For other values, $I = V/\sqrt{R^2+X_L^2}$. Current becomes zero when $X_L = \infty$. As seen from Fig. 18.5, the end point of the current vector describes a semi-circle with radius $OC = V/2R$ and centre lying in the reference sector *i.e.* voltage vector OV. For R-C circuit, the semi-circle lies to the left of OV. As before, it may be proved that the equation of the circle shown in Fig. 18.5 is

$$x^2 \quad y \quad \frac{V}{2R}^2 \quad \frac{V^2}{4R^2}$$

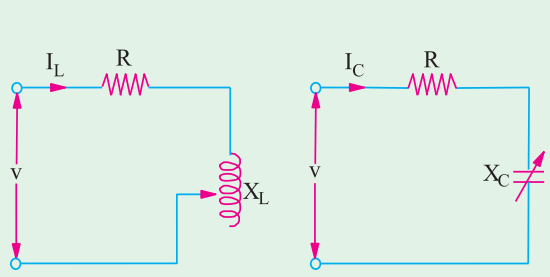


Fig. 18.4

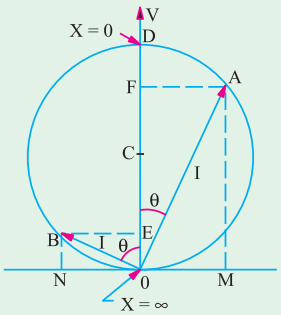


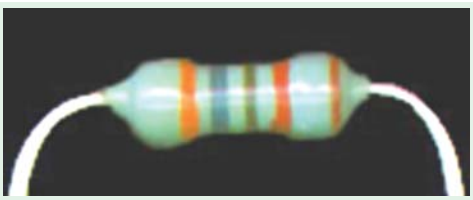
Fig. 18.5

The co-ordinates of the centre are $x = 0$, $y = V/2R$ and radius = $V/2R$.

As before, power developed would be maximum when current vector makes an angle of 45° with the voltage vector OV. In that case, current is $I_m / \sqrt{2}$ and $P_m = VI_m / 2$.

18.4. Properties of Constant Reactance But Variable Resistance Circuit

From the circle diagram of Fig. 18.3, it is seen that circuits having a constant reactance but variable resistance or ***vice-versa*** have the following properties :



36000 ohm 1% resistor

(i) the current has limiting value

(ii) the power supplied to the circuit has a limiting value also

(iii) the power factor corresponding to maximum power supply is 0.707 (= cos 45°)

Obviously, the maximum current in the circuit is obtained when $R = 0$.

$$\therefore \quad I_m = V / X_L = V / \omega L \quad \text{... for } R\text{-}L \text{ circuit}$$

$$= -V / X_C = -\omega VC \quad \text{... for } R\text{-}C \text{ circuit}$$

Now, power P taken by the circuit is $VI \cos \theta$ and if V is constant, then $P \propto I \cos\theta$. Hence, the ordinates of current semi-circles are proportional to $I \cos \theta$. The maximum ordinate possible in the semi-circle represents the maximum power taken by the circuit. The maximum ordinate passes through the centre of semi-circle so that current vector makes an angle of 45° with both the diameter and the voltage vector OV. Obviously, power factor corresponding to maximum power intake is cos 45° = 0.707.

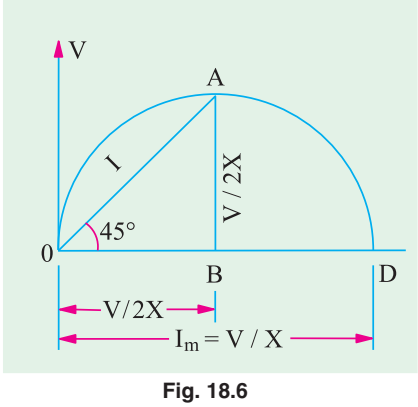


Fig. 18.6

Maximum power,

$$P_m = V \times AB = V \times \frac{I_m}{2} = \frac{1}{2} VI_m$$

Now, for R-L circuit, $I_m = V/X_L$

$\therefore \quad P_m \quad \frac{V^2}{2X_L} \quad \frac{V^2}{2\ L}$

For *R-C* circuit $P_m = \frac{V^2}{2X_C} = \frac{V^2 \omega C}{2}$

As said above, at maximum power, $\theta = 45°$, hence vector triangle for voltages is an isosceles triangle which means that voltage drops across resistance and reactance are each equal to 0.707 of supply voltage *i.e.* $V/\sqrt{2}$. As current is the same, for maximum power, resistance equals reactance *i.e.* $R = X_L$ (or X_C).

Hence, the expression representing maximum power may be written as $P_m = V^2/2R$.

18.5. Simple Transmission Line Circuit

In Fig. 18.7 (*a*) is shown a simple transmission circuit having negligible capacitance and reactance. R and X_L represent respectively the resistance and reactance of the line and R_L represents load resistance.

If R and X_L are constant, then as R_L is varied, the current AM follows the equation $I = (V/X) \sin \theta$ (Art. 18.1). The height AM in Fig. 18.7 (*b*) represents the power consumed by the circuit but, in the present case, this power is consumed both in R and R_L. The power absorbed by each resistance can be represented on the circle diagram.

In Fig. 18.7 (*b*), OB represents the line current when $R_L = 0$. The current $OB = V/\sqrt{(R^2 \quad X_L{}^2)}$ and power factor is cos θ_1. The ordinate BN then represents on a different scale the power dissipated in R only. OA represents current when R_L has some finite value *i.e.* $OA = V/\sqrt{(R+R_L)^2 + X_L^2}$. The ordinate AM represents total power dissipated, out of which ME is consumed in R and AE in R_L.

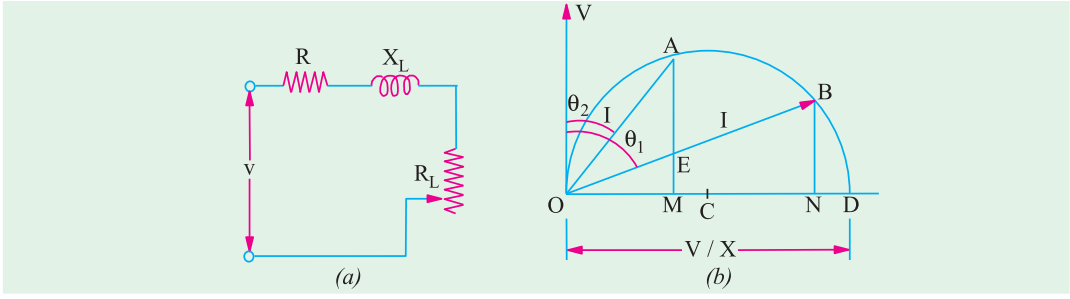


Fig. 18.7

In fact, if $OA^2 \times R_L$ ($= AE$) is considered to be the output of the circuit (the power transmitted by the line), then

$$\eta = \frac{AE}{AM}$$

With R and X_L constant, the maximum power that can be transmitted by such a circuit occurs when the extremity of current vector OA coincides with the point of tangency to the circle of a straight line drawn parallel to OB. Obviously, V times AE under these conditions represents the maximum power and the power factor at that time is cos θ_2.

Example 18.1. *A circuit consists of a reactance of 5 Ω in series with a variable resistance. A constant voltage of 100 V is applied to the circuit. Show that the current locus is circular. Determine* ***(a)*** *the maximum power input to the circuit* ***(b)*** *the corresponding current, p.f. and value of the resistance.* **(Electrical Science II, Allahabad Univ. 1992)**

Solution. For the first part, please refer to Art. 18.1

(a) $I_m = V/X = 100/5 = 20\ A$; $P_m = \frac{1}{2} VI_m = \frac{1}{2} \times 100 \times 20 =$ **1000 W**

(b) At maximum power input, current is = OA (Fig. 18.6)

$\therefore\ OA = I_m/\sqrt{2} = 20/\sqrt{2} = 14.14\ A$; *p.f.* = cos 45° = **0.707** ; $R = X =$ **5 Ω**

Example 18.2. *If a coil of unknown resistance and reactance is connected in series with a 100-V, 50-Hz supply, the current locus diagram is found to have a diameter of 5 A and when the value of series resistor is 15 Ω, the power dissipated is maximum. Calculate the reactance and resistance of the coil and the value of the maximum power in the circuit and the maximum current.*

Solution. Let the unknown resistance and reactance of the coil be R and X respectively

Diameter = V/X $\therefore$ $5 = 100/X$ or $X =$ **20 Ω**

Power is maximum when total resistance = reactance

or $15 + R = 20$ $\therefore$ $R = 5$ Ω

Maximum power $P_m = V^2/2X = 100^2/2 \times 20 =$ **250 W**

Maximum current $I_m = 100/\sqrt{(20^2 + 5^2)} =$ **4.85 A**

Example 18.3. *A constant alternating sinusoidal voltage at constant frequency is applied across a circuit consisting of an inductance and a variable resistance in series. Show that the locus diagram of the current vector is a semi-circle when the resistance is varied between zero and infinity.*

If the inductance has a value of 0.6 henry and the applied voltage is 100 V at 25 Hz, calculate ***(a)*** *the radius of the arc (in amperes) and* ***(b)*** *the value of variable resistance for which the power taken from the mains is maximum and the power factor of the circuit at the value of this resistance.*

Solution. $X_L = \omega L = 0.6 \times 2\pi \times 25 = 94.26\ \Omega$

(a) Radius = $V/2X_L = 100/2 \times 4.26 =$ **0.531 A**

Example 18.4. *A resistor of 10 Ω is connected in series with an inductive reactor which is variable between 2 Ω and 20 Ω. Obtain the locus of the current vector when the circuit is connected to a 250-V supply. Determine the value of the current and the power factor when the reactance is* ***(i)*** *5 Ω* ***(ii)*** *10 Ω* ***(iii)*** *15 Ω.* **(Basic Electricity, Bombay Univ.)**

Solution. As discussed in Art. 18.3, the end point of current vector describes a semi-circle whose diameter (Fig. 18.8) equals $V/R = 250/10 = 25$ A and whose centre lies to right side of the vertical voltage vector OV.

$I_{max} = 250/\sqrt{10^2 + 2^2} = 24$ A; $\theta = \tan^{-1}(2/10) = 11.3°$;

$I_{min} = 250/\sqrt{10^2 + 20^2} = 11.2$ A; $\theta = \tan^{-1}(20/10) = 63.5°$

(i) $\theta_1 = \tan^{-1}(5/10) = 26.7°$, p.f. = cos 26.7° = 0.89

$I = OA =$ **22.4 A**

(ii) $\theta_2 = \tan^{-1}(10/10) = 45°$, p.f. = cos 45° = **1**;

I = OB = **17.7 A**

(iii) $\theta_3 = \tan^{-1}(15/10) = 56.3$; p.f. = cos 56.3° = **0.55**;

$I = OC =$ **13.9 A**.

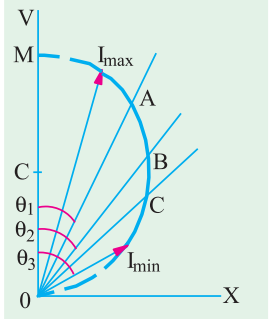


Fig. 18.8

Example 18.5. *A voltage of 100 sin 10,000 t is applied to a circuit consisting of a 1 μF capacitor in series with a resistance R. Determine the locus of the tip of the current phasor when R is varied from 0 to ∞. Take the applied voltage as the reference phasor.*

(Network Theory and Design, AMIETE 1990)

Solution. As seen from Art. 18.2 the locus of the tip of the current phasor is a circle whose equation is

$$y^2 \quad x \quad \frac{V}{2X_C}^{\,2} \quad \frac{V}{4X_C^2}$$

We are given that $V = V_m / \sqrt{2} = 100 / \sqrt{2} = 77.7$ V

$$\omega = 10{,}000 \text{ rad/s};\ X_C = 1/\omega \times C = 1/10{,}000 \times 1 \times 10^{-6} = 100\ \Omega\ \text{C},$$
$$(V/2\ X_C)^2 = (77.7/2 \times 100)^2 = 0.151, \quad \therefore\ y^2 + (x + 0.389)^2 = 0.151$$

Example 18.6. *Prove that polar locus of current drawn by a circuit of constant resistance and variable capacitive reactance is circular when the supply voltage and frequency are constant.*

If the constant resistance is 10 Ω and the voltage is 100 V, draw the current locus and find the values of the current and p.f. when the reactance is ***(i)*** *5.77 Ω* ***(ii)*** *10 Ω and* ***(iii)*** *17.32 Ω. Explain when the power will be maximum and find its value.*

(Electromechanics, Allahabad Univ. 1992)

Solution. For the first part, please refer to Art. 18.3. The current semicircle will be drawn on the vertical axis with a radius $OM = V/2\ R = 100/2 \times 10 = 5$ A as shown in Fig. 18.9 (*b*)

(i) $\theta_1 = \tan^{-1}(5.77/10) = 30°$; $\cos\theta_1 =$ **0.866 (lead)** ; current = $OA =$ **8.66 A**

(ii) $\theta_2 = \tan^{-1}(10/10) = 45°$; $\cos\theta_2 =$ **0.707 (lead)** current = $OB =$ **7.07 A**

(iii) $\theta_3 = \tan^{-1}(17.32/10) = 60°$; $\cos\theta_3 =$ **0.5 (lead)** current = $OC =$ **5 A**

Power would be maximum for point *B* when $\theta = 45°$; $I_m = V/R = 100/10 = 10$ A

$P_m = V \times OB \times \cos 45° = V \times I_m \cos 45° \times \cos 45° = \frac{1}{2} VI_m = \frac{1}{2} \times 100 \times 10 =$ **500 W**

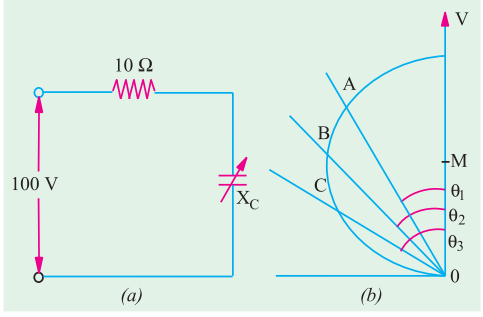


Fig. 18.9

Example 18.7. *Prove that the polar locus of the current drawn by a circuit of constant reactance and variable resistance is circular when the supply voltage and frequency are constant.*

If the reactance of such a circuit is 25 Ω and the voltage 250, draw the said locus and locate there on the point of maximum power and for this condition, find the power, current, power factor and resistance.

Locate also the point at which the power factor is 0.225 and for this condition, find the current, power and resistance. **(Basic Electricity, Bombay Univ.)**

Solution. For the first part, please refer to Art. 18.3.

Radius of the current semi-circle is = $V/2X = 250/2 \times 25 = 5$ A. As discussed in Art. 18.3, point A [Fig. 18.10 (*a*)] corresponds to maximum power.

Now, $I_m = V/X = 250/25 = 10$ A; $P_m = \frac{1}{2} VI_m = \frac{1}{2} \times 250 \times 10 =$ **1250 W**

Current $OA = I_m/\sqrt{2} = 10/\sqrt{2} =$ **7.07 A** ; *p.f.* = $\cos 45° =$ **0.707**.

Under condition of maximum power, $R = X =$ **25 Ω**.

Now, $\cos\theta = 0.225$;

$\theta = \cos^{-1}(0.225) = 77°$

In Fig. 18.10 (*b*), current vector *OA* has been drawn at an angle of 77° with the vertical voltage vector *OV*.

By measurement, current *OA* = 9.74 A

By calculate, *OA*, = I_m cos 13° = 10 × 0.974 = **9.74 A**

Power = *VI* cos θ = 250 × 9.74 × 0.225 = **548 W**

$P = I^2 R$; $R = P/I^2 = 548/9.74^2 = \mathbf{5.775}\ \Omega$.

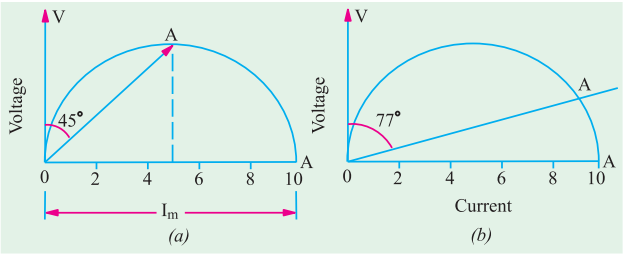


Fig. 18.10

Example 18.8. *A non-inductive resistance R, variable between 0 and 10* Ω*., is connected in series with a coil of resistance 3* Ω*. and reactance 4* Ω*. and the circuit supplied from a 240-V a.c. supply. By means of a locus diagram, determine the current supplied to the circuit when R is* ***(a)*** *zero* ***(b)*** *5* Ω*. and* ***(c)*** *10* Ω*.. By means of the symbolic method, calculate the value of the current when R = 5* Ω*..*

Solution. The locus of the current vector is a semi-circle whose centre is (0, *V*/2*X*) and whose radius is obviously equal to *V*/2*X*. Now, *V*/2 *X* = 240/2 × 4 = 30 A.

Hence, the semi-circle is drawn as shown in Fig. 18.11 (*b*).

(a) Total resistance = 3 Ω. and *X* = 4 Ω. ∴ tan θ_1 = 4/2 ∴ θ_1 = 53° 8′

Hence, current vector *OA* is drawn making an angle of 53°8′ with vector *OV*. Vector *OA* measures **49 A**.

(b) Total resistance = 3 + 5 = 8 Ω.

Reactance = 4 Ω.; tan θ_2 = 4/8 = 0.5 ∴ θ_2 = 26°34′

Current vector *OB* is drawn at an angle of 26°34′ with *OV*. It measure **27 A (approx.)**

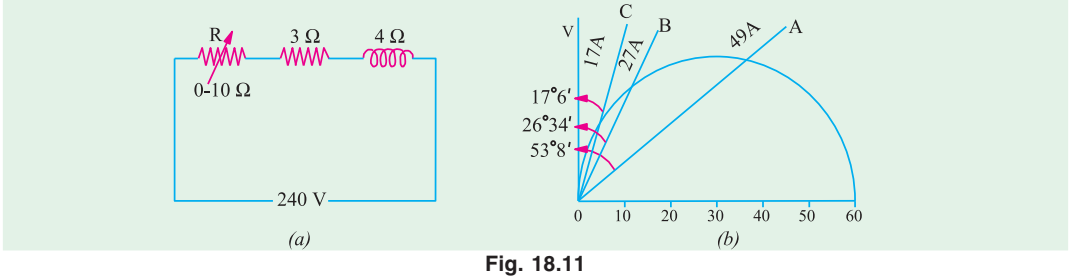


Fig. 18.11

(c) Total resistance = 3 + 10 = 13 Ω.

Reactance = 4 Ω.; tan θ_3 = 4/13 ∴ θ_3 = 17°6′

Current vector *OC* is drawn at an angle of 17°6′ with vector *OA*. It measures **17 A.**

Symbolic Method

$$\mathbf{I} \quad \frac{240 \quad j0}{(5 \quad 3) \quad j4} \quad \frac{240}{8 \quad j4} \quad \frac{240}{8.96 \quad 26.5} \quad 26.7 \quad 26.5$$

Note. There is difference in the magnitudes of the currents and the angles as found by the two different methods. It is so because one has been found exactly by mathematical calculations, whereas the other has been measured from the graph.

Example 18.9. *A circuit consisting of a 50-*Ω *resistor in series* with a variable reactor is shouted by a 100-Ω *resistor. Draw the locus of the extremitty of the total current vector to scale and determine the reactance and current corresponding to the minimum overall power factor, the supply voltage being 100 V.*

Solution. The parallel circuit is shown in Fig. 18.12 (*a*).

The resistive branch draws a fixed current I_2 =100/100 = 1 A. The current I_1 drawn by the reactive branch is maximum when X_L = 0 and its maximum value is = 100/50 = 2 A and is in phase with voltage.

In the locus diagram of Fig. 18.12 (*b*), the diameter *OA* of the reactive current semi-circle is = 2 A. *OB* is the value of I_1 for some finite value of X_L. $O'O$ represents I_2. Being in phase with voltage, it is drawn in phase with voltage vector *OV*. Obviously, $O'B$ represents total circuit current, being the vector sum of I_1 and I_2.

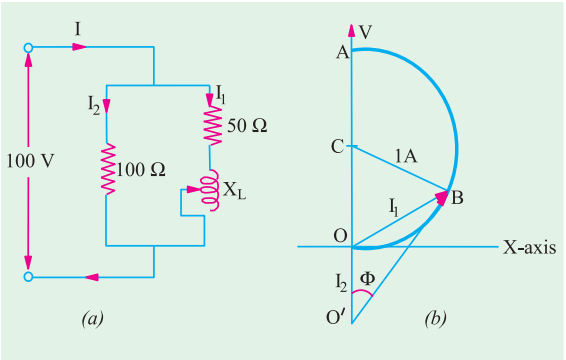


Fig. 18.12

The minimum power factor which corresponds to maximum phase difference between $O'B$ and $O'V$ occurs when $O'B$ is tangential to the semi-circle. In that case, $O'B$ is perpendicular to *BC*. It means that $O'BC$ is a right-angled triangle.

Now, $\sin \phi = BC / O'C = 1/(1+1) = 0.5;\ \phi = 30°$

$\therefore$ Minimum *p.f.* = cos 30° = 0.866 (lag)

Current corresponding to minimum *p.f.* is $O'B = O'C \cos \phi = 2 \times 0.866 =$ **1.732 A**.

Now, Δ *OBC* is an equilateral triangle, hence $I_1 = OB$ = 1 A. Considering reactive branch, Z = 100/1 = 100 Ω, $X_L = \sqrt{100^2 - 50^2}$ = **88.6** Ω

Example 18.10. *A coil of resistance 60 Ω and inductance 0.4 H is connected in series with a capacitor of 17.6 μF across a variable frequency source which is maintained at a fixed potential of 120 V. If the frequency is varied through a range of 40Hz to 80Hz, draw the complete current locus and calculate the following :*

(i) the resonance frequency, ***(ii)*** the current and power factor at 40 Hz and

(iii) *the current and power factor at 80 Hz.*

(Elect. Circuits, South Gujarat Univ.)

Solution. *(i)* $f_0 = 10^3 / 2\pi\sqrt{0.4 \times 17.6}$ = **60 Hz.**

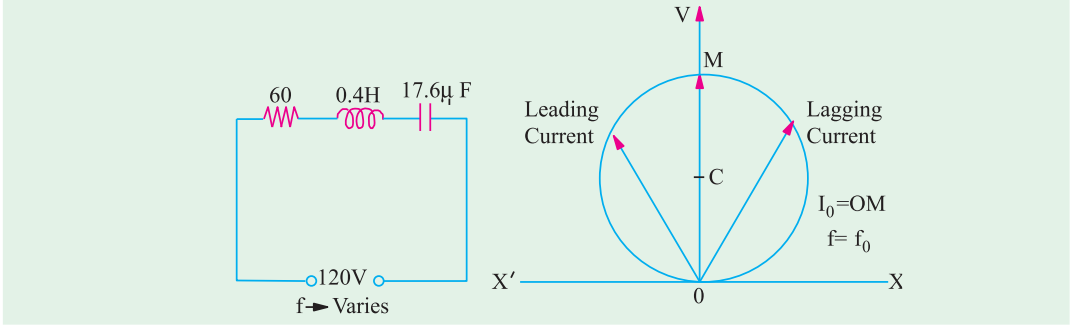


Fig. 18.13

(ii) ***f* = 40 Hz**

$X_L = 2\pi \times 40 \times 0.4 = 100\,\Omega;\ X_C = 10^6 / 2\pi \times 40 \times 17.6 = 226\ \Omega$

X = 100 – 226 = – 126 Ω (capacitive); $I = 120 / \sqrt{60^2 + (-126)^2}$ = **0.86 A**

p.f. = cos θ = *R*/*Z* = 60/139.5 = **0.43 (lead)**

(iii) f = 80 Hz

$X_L = 100 \times 2 = 200\ \Omega$; $X_C = 226/2 = 113\ \Omega$; $X = 200 - 113 = 87\ \Omega$ (inductive)

$$Z = \sqrt{R^2 + X^2} = \sqrt{60^2 + 87^2} = 105.3\Omega$$

$I = 120/105.3 =$ **1.14 A** ; p.f. cos $\theta = 60/105.3 =$ **0.57 (lag)**

Tutorial Problems No. 18.1

1. A circuit having a constant resistance of 60 Ω and a variable inductance of 0 to 0.4 H is connected across a 100-V, 50-Hz supply. Derive from first principles the locus of the extremity of the current vector. Find (*a*) the power and (*b*) the inductance of the circuit when the power factor is 0.8.

[(*a*) 107 W (*v*) 0.143 H] (*App. Elect. London Univ.*)

2. A constant reactance of 10 Ω is connected in series with a variable resistor and the applied voltage is 100 V. What is (*i*) the maximum power dissipated and (*ii*) at what value of resistance does it occur ?

[(*a*) 500 W (*ii*) 10 Ω] (*City & Guilds London*)

3. A variable capacitance and a resistance of 300 Ω are connected in series across a 240-V; 50-Hz supply. Draw the complex or locus of impedance and current as the capacitance changes from 5μ*F* to 30 μ*F*. From the diagram, find (*a*) the capacitance to give a current of 0.7 A and (*b*) the current when the capacitance is 10 μ*F*. **[19.2 μF, 0.55A] (*London Univ.*)**

4. An a.c. circuit consists of a variable resistor in series with a coil, for which R = 20 Ω and L = 0.1 H. Show that when this circuit is supplied at constant voltage and frequency and the resistance is varied between zero and infinity, t he locus diagram of the current vector is a circular arc. Calculate when the supply voltage is 100 V and the frequency 50 Hz (*i*) the radius (in amperes) of the arc (*ii*) the value of the variable resistor in order that the power taken from the mains may be a maximum.

[(*i*) 1.592 A (*ii*) 11.4 Ω] (*London Univ.*)

5. A circuit consists of an inductive coil (L = 0.2 H, R = 20 Ω) in series with a variable resistor (0 – 200 Ω). Draw to scale the locus of the current vector when the circuit is connected to 230-V, 50-Hz supply mains and the resistor is varied between 0 and 200 Ω . Determine (*i*) the value of the resistor which will give maximum power in the circuit, (*ii*) the power when the resistor is 150 Ω .

[(*i*) 42.8 Ω (*ii*) 275 W] (*London Univ.*)

6. A 15 μF capacitor, an inductive coil (L = 0.135 H, R = 50 Ω) and a variable resistor are in series and connected to a 230-V, 50-Hz supply.

Draw to scale the vector locus of the current when the variable resistor is varied between 0 and 500 Ω .

Calculate (*i*) the value of the variable resistor when the power is a maximum (*ii*) the power under these conditions. **[(*i*) 120 Ω (*ii*) 155.5 W] (*London Univ.*)**

7. As a.c. circuit supplied at 100 V, 50-Hz consists of a variable resistor in series with a fixed 100 μF capacitor.

Show that the extremity of the current vector moves on a circle. Determine the maximum power dissipated in the circuit the corresponding power factor and the value of the resistor.

[157 W ; 0.707 : 131.8 Ω]

8. A variable non-inductive resistor R of maximum value 10 Ω is placed in series with a coil which has a resistance of 3 Ω and reactance of 4 Ω . The arrangement is supplied from a 240-V a.c. supply. Show that the locus of the extremity of the current vector is a semi-circle. From the locus diagram, calculate the current supplied when R = 5 Ω . **[26.7 A]**

9. A 20- Ω reactor is connected in parallel with a series circuit consisting of a reactor of reactance 10 Ω and a variable resistance R. Prove that the extremity of the total current vector moves on a circle. If the supply voltage is constant at 100 V (r.m.s.), what is the maximum power factor ? Determine also the value of R when the p.f. has its maximum value. **[0.5 : 17.3 Ω]**

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CHAPTER 19

POLYPHASE CIRCUITS

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- Phase Sequence Indicators



This is a mercury arc rectifier 6-phase device, 150 A rating with grid control

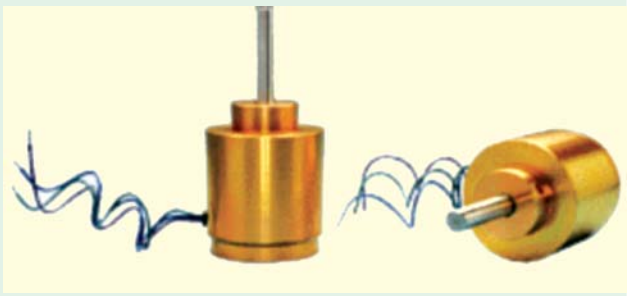
19.1. Generation of Polyphase Voltage

The kind of alternating currents and voltages discussed in chapter 12 to 15 are known as single-phase voltage and current, because they consist of a single alternating current and voltage wave. A single-phase alternator was diagrammatically depicted in Fig. 11.1 (*b*) and it was shown to have one armature winding only. But if the number of armature windings is increased, then it becomes polyphase alternator and it produces as many independent voltage waves as the number of windings or phases. These windings are displaced from one another by equal angles, the values of these angles being determined by the number of phases or windings. In fact, the word 'poly-phase' means poly (*i.e.* many or numerous) and phases (*i.e.* winding or circuit).

In a two-phase alternator, the armature windings are displaced 90 electrical degrees apart. A 3-phase alternator, as the name shows, has three independent armature windings which are 120 electrical degrees apart. Hence, the voltages induced in the three windings are 120° apart in time-phase. With the exception of two-phase windings, it can be stated that, in general, the electrical displacement between different phases is 360/*n* where *n* is the number of phases or windings.

Three-phase systems are the most common, although, for certain special jobs, greater number of phases is also used. For example, almost all mercury-arc rectifiers for power purposes are either six-phase or twelve-phase and most of the rotary converters in use are six-phase. All modern generators are practically three-phase. For transmitting large amounts of power, three-phase is invariably used. The reasons for the immense popularity of three-phase apparatus are that **(*i*)** it is more efficient **(*ii*)** it uses less material for a given capacity and **(*iii*)** it costs less than single-phase apparatus etc.

In Fig. 19.1 is shown a two-pole, stationary-armature, rotating-field type three-phase alternator. It has three armature coils aa', bb' and cc' displaced 120° apart from one another. With the position and clockwise rotation of the poles as indicated in Fig. 19.1, it is found that the e.m.f. induced in conductor '*a*' for coil aa' is maximum and its direction* is away from the reader. The e.m.f. in conductor '*b*' of coil bb' would be maximum and away from the reader when the N-pole has turned through 120° *i.e.* when *N-S* axis lies along bb'. It is clear that the induced e.m.f. in conductor '*b*' reaches its maximum value 120° later than the maximum value in conductor '*a*'. In the like manner, the maximum e.m.f. induced (in the direction away from the reader) in conductor '*c*' would occur 120° later than that in '*b*' or 240° later than that in '*a*'.



The rotary phase converter

Thus the three coils have three e.m.fs. induced in them which are similar in all respects except that they are 120° out of time phase with one another as pictured in Fig. 19.3. Each voltage wave is assumed to be sinusoidal and having maximum value of E_m.

In practice, the space on the armature is completely covered and there are many slots per phase per pole.

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* The direction is found with the help of Fleming's Right-hand rule. But while applying this rule, it should be remembered that the relative motion of the conductor with respect to the field is anticlockwise although the motion of the field with respect to the conductor is clockwise as shown. Hence, thumb should point to the left.

Fig. 19.2 illustrates the relative positions of the windings of a 3-phase, 4-pole alternator and Fig. 19.4 shows the developed diagram of its armature windings. Assuming full-pitched winding and the direction of rotation as shown, phase '*a*' occupies the position under the centres of *N* and *S*-poles. It starts at S_a and ends or finishes at F_a.

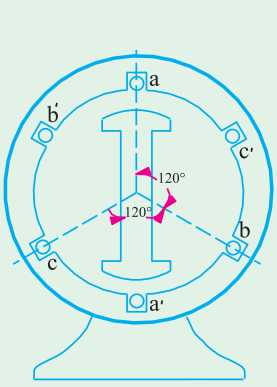


Fig. 19.1

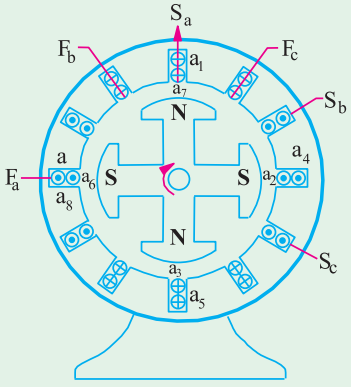


Fig. 19.2

The second phase '*b*' start at S_b which is 120 electrical degrees apart from the start of phase '*a*', progresses round the armature clockwise (as does '*a*') and finishes at F_b. Similarly, phase '*c*' starts at S_c, which is 120 electrical degrees away from S_b, progresses round the armature and finishes at F_c. As the three circuits are exactly similar but are 120 electrical degrees apart, the e.m.f. waves generated in them (when the field rotates) are displaced from each other by 120°. Assuming these waves to be sinusoidal and counting the time from the instant when the e.m.f. in phase '*a*' is zero, the instantaneous values of the three e.m.fs. will be given by curves of Fig. 193.

Their equations are :

$$e_a = E_m \sin \omega t \quad \text{... (i)}$$

$$e_b = E_m \sin(\omega t - 120°) \quad \text{... (ii)}$$

$$e_c = E_m \sin(\omega t - 240°) \quad \text{... (iii)}$$

As shown in Art. 11.23. alternating voltages may be represented by revolving vectors which indicate their maximum values (or r.m.s. values if desired). The actual values of these voltages vary from peak positive to zero and to peak negative values in one revolution of the vectors. In Fig. 19.5 are shown the three vectors representing the r.m.s. voltages of the three phases E_a, E_b and E_c (in the present case $E_a = E_b = E_c = E$, say).

It can be shown that the sum of the three phase e.m.fs. is zero in the following three ways :

(*i*) The sum of the above three equations **(*i*), (*ii*)** and **(*iii*)** is zero as shown below :

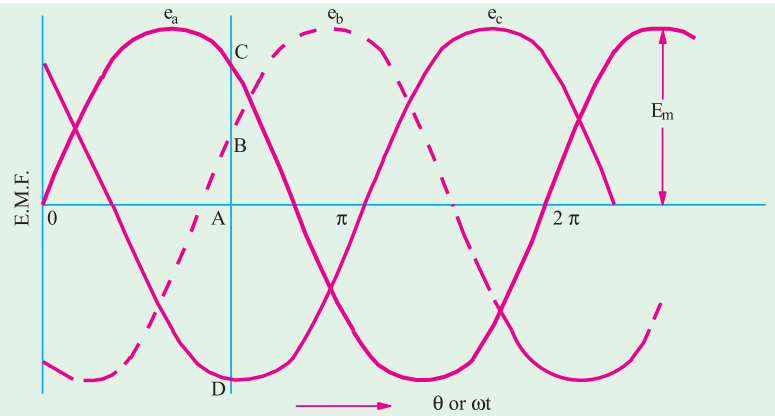


Fig. 19.3

Resultant instantaneous e.m.f. = $e_a + e_b + e_c$

$$= E_m \sin t \quad E_m \sin(t \quad 120) \quad E_m (t \quad 240)$$

$$= E_m[\sin t \quad 2\sin(t \quad 180)\cos 60]$$

$$= E_m[\sin t \quad 2\sin t \cos 60] \quad 0$$

(ii) The sum of ordinates of three e.m.f. curves of Fig. 19.3 is zero. For example, taking ordinates *AB* and *AC* as positive and *AD* as negative, it can be shown by actual measurement that

$$AB + AC + (- AD) = 0$$

(iii) If we add the three vectors of Fig. 19.5 either vectorially or by calculation, the result is zero.

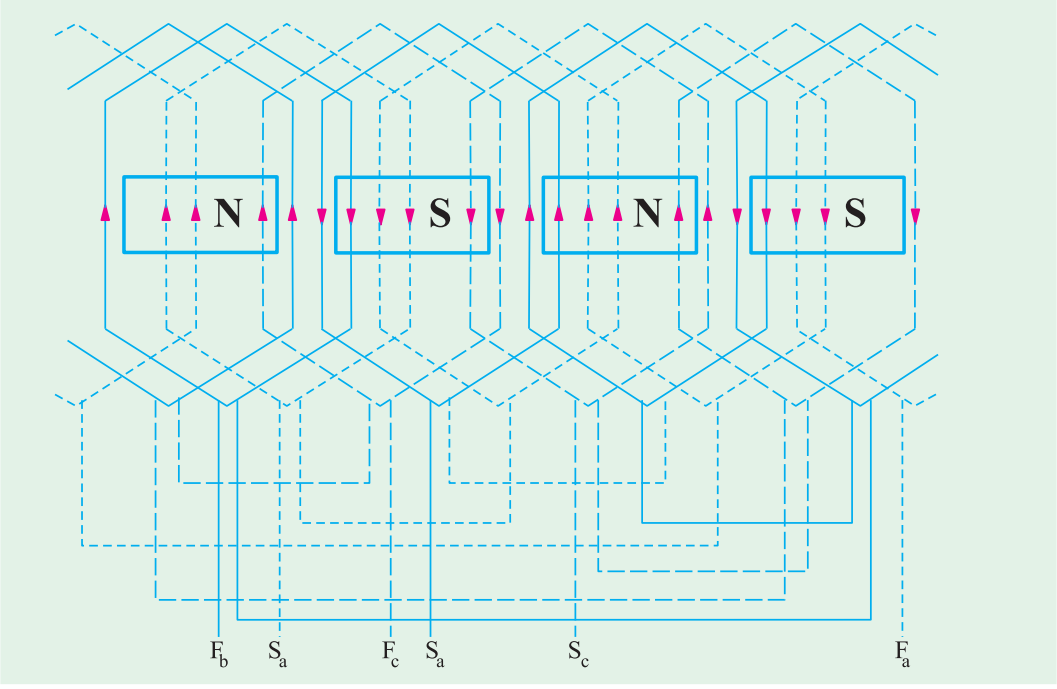


Fig. 19.4

Vector Addition

As shown in Fig. 19.6, the resultant of E_a and E_b is E_r and its magnitude is $2E \cos 60° = E$ where $E_a = E_b = E_c = E$.

This resultant E_r is equal and opposite to E_c. Hence, their resultant is zero.

By Calculation

Let us take E_a as reference voltage and assuming clockwise phase sequence

$$\boldsymbol{E_a} \quad E \quad 0 \quad E \quad j0$$

$$\boldsymbol{E_b} \quad E \quad 240 \quad E \quad 120 \quad E(\quad 0.05 \quad j0.866)$$

$$\boldsymbol{E_c} \quad E \quad 240 \quad E \quad 120 \quad E(\quad 0.05 \quad j0.866)$$

$$\therefore \boldsymbol{E_a} + \boldsymbol{E_b} + \boldsymbol{E_c} \quad (E \quad j0) \quad E(\quad 0.5 \quad 0.866) \quad E(\quad 0.05 \quad j0.866) \quad 0$$

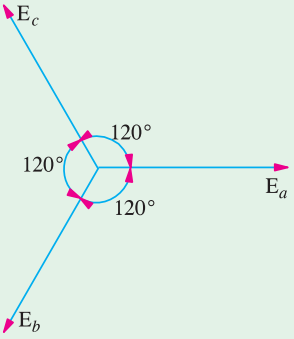


Fig. 19.5

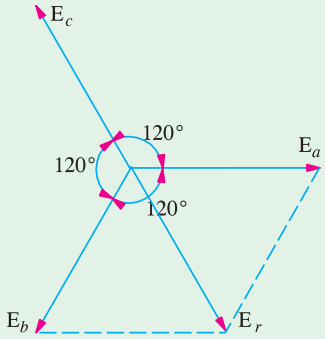


Fig. 19.6

19.2. Phase Sequence

By phase sequence is meant the order in which the three phases attain their peak or maximum values. In the development of the three-phase e.m.fs. in Fig. 19.7, clockwise rotation of the field system in Fig. 19.1 was assumed. This assumption made the e.m.fs. of phase '*b*' lag behind that

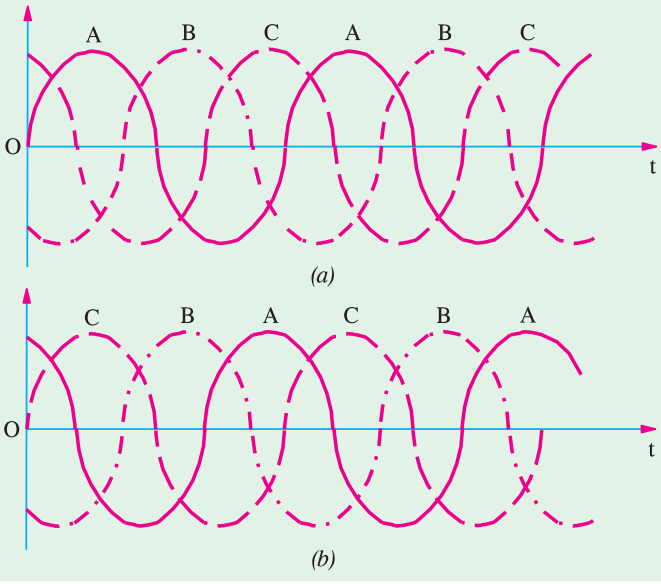


Fig. 19.7

of '*a*' by 120° and in a similar way, made that of '*c*' lag behind that of '*b*' by 120° (or that of '*a*' by 240°). Hence, the order in which the e.m.fs. of phases *a*, *b* and *c* attain their maximum values is *a b c*. ***It is called the phase order*** **or** ***phase sequence*** $a \rightarrow b \rightarrow c$ as illustrated in Fig. 19.7 (*a*).

If, now, the rotation of the field structure of Fig. 19.1 is reversed *i.e.* made anticlockwise, then the order in which the three phases would attain their corresponding maximum voltages would also be reversed. The phase sequence would become $a \rightarrow b \rightarrow c$. This means that e.m.f. of phase '*c*' would now lag behind that of phase '*a*' by 120° instead of 240° as in the previous case as shown in Fig. 19.7 (*b*). By repeating the letters, this phase sequence can be written as ***acbacba*** which is the same thing as ***cba***. Obviously, a three-phase system has only two possible sequences : ***abc*** and ***cba*** (*i.e.* ***abc*** read in the reverse direction).

19.3. Phase Sequence At Load

In general, the phase sequence of the voltages applied to load is determined by the order in which the 3-phase lines are connected. The phase sequence can be reversed by interchanging any pair of lines. In the case of an induction motor, reversal of sequence results in the reversed direction of motor rotation. In the case of 3-phase unbalanced loads, the effect of sequence reversal is, in general, to cause a completely different set of values of the currents. Hence, when working on such systems, it is essential that phase sequence be clearly specified otherwise unnecessary confusion will arise. Incidentally, reversing the phase sequence of a 3-phase generator which is to be paralleled with a similar generator can cause extensive damage to both the machines.

Fig. 19.8 illustrates the fact that by interchanging any two of the three

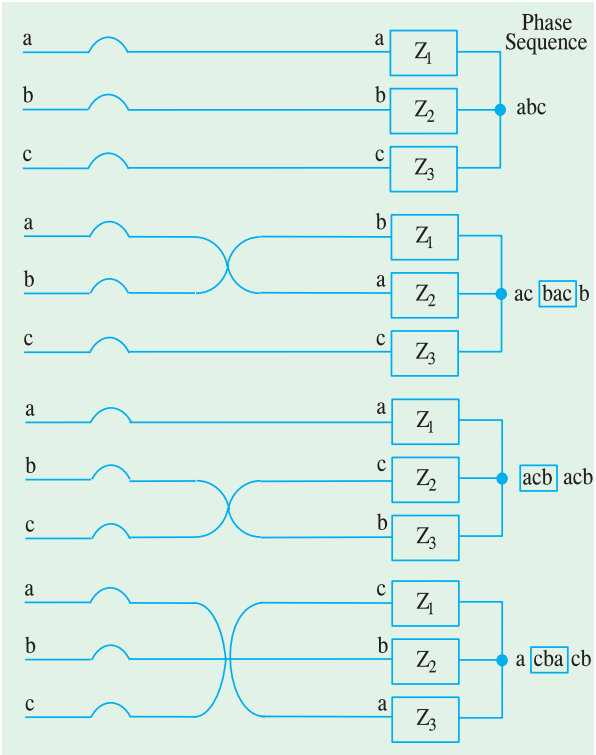


Fig. 19.8



Induction motor for drilling applications

cables the phase sequence ***at the load*** can be reversed though sequence of 3-phase supply remains the same *i.e.* ***abc***. It is customary to define phase sequence at the load by reading repetitively from top to bottom. For example, load phase sequence in Fig. 19.8 (*a*) would be read as ***abcabcabc***– or simply ***abc***. The changes are as tabulated below :

Cables Interchanged	*Phase Sequence*
a and *b*	*b a* [*c b a c b a*] *c* – or *c b a*
b and *c*	*a* [*c b a c b a*] *c b* – or *c b a*
c and *a*	[*c b a c b a*] *c b a* – or *c b a*

19.4. Numbering of Phases

The three phases may be numbered 1, 2, 3 or *a*, *b*, *c* or as is customary, they may be given three colours. The colours used commercially are red, yellow (or sometimes white) and blue. In this case, the sequence is *RYB*.

Obviously, in any three-phase system, there are two possible sequences in which the three coil or phase voltages may pass through their maximum values *i.e.* red → yellow → blue (*RYB*) or red → blue → yellow (*RBY*). By convention, sequence *RYB* is taken as positive and *RBY* as negative.

19.5. Interconnection of Three Phases

If the three armature coils of the 3-phase alternator (Fig. 19.8) are not interconnected but are kept separate, as shown in Fig. 19.9, then each phase or circuit would need two conductors, the total number of conductors, in that case, being six. It means that each transmission cable would contain six conductors which will make the whole system complicated and expensive. Hence, the three phases are generally inter-connected which results in substantial saving of copper. The general methods of interconnection are

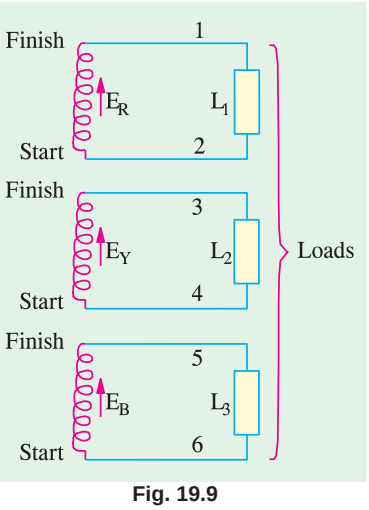


Fig. 19.9



3-phase alternator

(a) Star or Wye (Y) connection and

(b) Mesh or Delta (Δ) connection.

19.6. Star or Wye (Y) Connection

In this method of interconnection, the ***similar**** ends say, 'star' ends of three coils (it could be 'finishing' ends also) are joined together at point *N* as shown in Fig. 19.10 (*a*).

The point *N* is known as ***star point*** or ***neutral point***. The three conductors meeting at point *N* are replaced by a single conductor known as ***neutral conductor*** as shown in Fig. 19.10 (*b*). Such an interconnected system is known as four-wire, 3-phase system and is diagrammatically shown in Fig. 19.10 (*b*). If this three-phase voltage system is applied across a balanced symmetrical load, the neutral wire will be carrying three currents which are exactly equal in magnitude but are 120° out of phase with each other. Hence, their vector sum is zero.

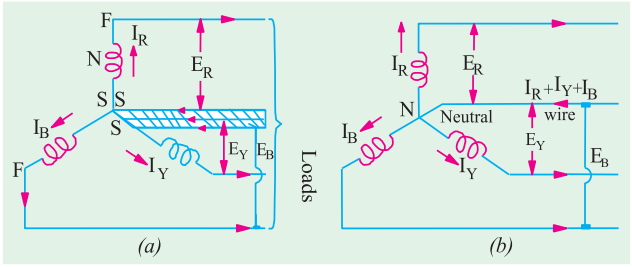


Fig. 19.10

i.e. $I_R + I_Y + I_B = 0$ ... vectorially

The neutral wire, in that case, may be omitted although its retention is useful for supplying lighting loads at low voltages (Ex. 19.22). The p.d. between any terminal (or line) and neutral (or star) point gives the ***phase*** or ***star*** voltage. But the p.d. between any two lines gives the line-to-line voltage or simply line voltage.

19.7. Values of Phase Currents

When considering the distribution of current in a 3-phase system, it is extremely important to bear in mind that :

---

* As an aid to memory, remember that first letter *S* of Similar is the same as that of Star.

(*i*) the arrow placed alongside the currents I_R I_Y and I_B flowing in the three phases [Fig. 19.10 (*b*)] indicate the directions of currents when they are assumed to be ***positive*** and not the directions at a particular instant. It should be clearly understood ***that at no instant will all the three currents flow in the same direction either outwards or inwards.*** The three arrows indicate that first the current flows outwards in phase *R*, then after a phase-time of 120°, it will flow outwards from phase *Y* and after a further 120°, outwards from phase *B*.

(*ii*) the current flowing outwards in one or two conductors is always equal to that flowing inwards in the remaining conductor or conductors. In other words, ***each conductor in turn, provides a return path for the currents of the other conductors.***

In Fig. 19.11 are shown the three phase currents, having the same peak value of 20 A but displaced from each other by 120°. At instant '*a*', the currents in phases *R* and *B* are each + 10 A (*i.e.* flowing outwards) whereas the current in phase *Y* is – 20A (*i.e.* flowing inwards). In other words, at the instant '*a*', phase *Y* is acting as return path for the currents in phases *R* and *B*. At instant *b*, I_R = +15 A and I_Y = +5 A but I_B = –20A which means that now phase *B* is providing the return path.

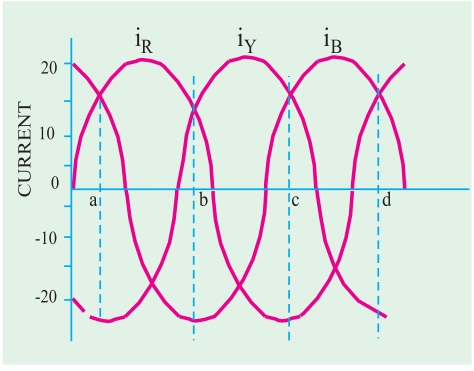


Fig. 19.11

At instant *c*, I_Y = +15 *A* and I_B = +5A and I_R = – 20A.

Hence, now phase *R* carries current inwards whereas *Y* and *B* carry current outwards. Similarly at point *d*, I_R = 0, I_B = 17.3 A and I_Y = – 17.3 A. In other words, current is flowing outwards from phase *B* and returning *via* phase *Y*.

In addition, it may be noted that although the distribution of currents between the three lines is continuously changing, yet at any instant the algebraic sum of the ***instantaneous*** values of the three currents is zero *i.e.* $i_R + i_Y + i_B = 0$ – algebraically.

19.8. Voltages and Currents in Y-Connection

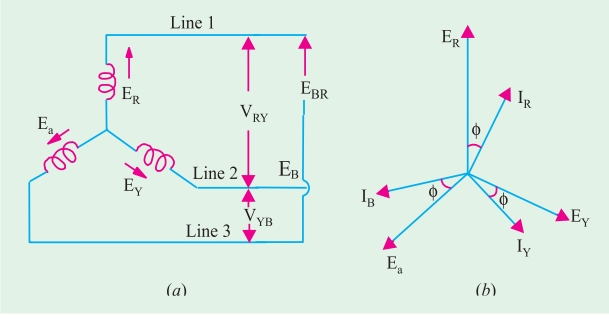


Fig. 19.12

The voltage induced in each winding is called the ***phase*** voltage and current in each winding is likewise known as ***phase*** current. However, the voltage available between any pair of terminals (or outers) is called ***line*** voltage (V_L) and the current flowing in each ***line*** is called ***line*** current (I_L).

As seen from Fig. 19.12 (*a*), in this form of interconnection, there are two phase windings between each pair of terminals but since their ***similar*** ends have been joined together, they are in opposition. Obviously, the ***instantaneous*** value of p.d. between any two terminals is the ***arithmetic difference*** of the two phase e.m.fs. concerned. However, the r.m.s. value of this p.d. is given by the ***vector difference*** of the two phase e.m.fs.

The vector diagram for phase voltages and currents in a star connection is shown in Fig. 19.12.

(*b*) where a balanced system has been assumed.* It means that $E_R = E_Y = E_{ph}$ (phase e.m.f.).

Line voltage V_{RY} between line 1 and line 2 is the vector difference of E_R and E_Y.

Line voltage V_{YB} between line 2 and line 3 is the vector difference of E_Y and E_B.

Line voltage V_{BR} between line 3 and line 1 is the vector difference of E_B and E_R.

(*a*) Line Voltages and Phase Voltages

The p.d. between line 1 and 2 is $V_{RY} = E_R - E_Y$ ... vector difference.

Hence, V_{RY} is found by compounding E_R and E_Y reversed and its value is given by the diagonal of the parallelogram of Fig. 19.13. Obviously, the angle between E_R and E_Y reversed is 60°. Hence if $E_R = E_Y = E_B$ = say, E_{ph} – the phase e.m.f., then

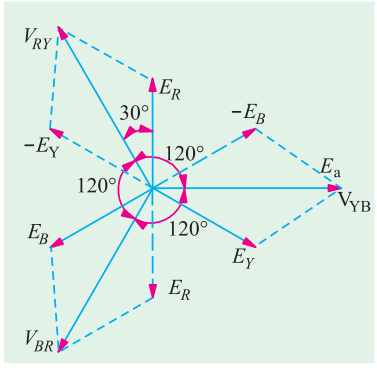


Fig. 19.13

$$V_{RY} = 2 \times E_{ph} \times \cos(60°/2)$$

$$= 2 \times E_{ph} \times \cos 30° = 2 \times E_{ph} \times \frac{\sqrt{3}}{2} = \sqrt{3}\, E_{ph}$$

Similarly, $V_{YB} = E_Y - E_B = \sqrt{3} \cdot E_{ph}$ ...vector difference

and $V_{BR} = E_B - E_R = \sqrt{3} \cdot E_{ph}$

Now $V_{RY} = V_{YB} = Y_{BR} =$ line voltage, say V_L. Hence, in star connection $V_L = \sqrt{3} \cdot E_{ph}$

It will be noted from Fig. 19.13 that

1. Line voltages are 120° apart.

2. Line voltages are 30° ahead of their respective ***phase*** voltages.

3. The angle between the line currents and the corresponding line voltages is (30 + ϕ) with current lagging.

(*b*) Line Currents and Phase Currents

It is seen from Fig. 19.12 (*a*) that each line is in series with its individual phase winding, hence the line current in each line is the same as the current in the phase winding to which the line is connected.

Current in line 1 = I_R ; Current in line 2 = I_Y; Current in line 3 = I_B

Since $I_R = I_Y = I_B$ = say, I_{ph} – the phase current

$\therefore$ line current $I_L = I_{ph}$

(*c*) Power

The total active or true power in the circuit is the sum of the three phase powers. Hence,

total active power = 3 × phase power or $P = 3 \times V_{ph}\, I_{ph} \cos \phi$

Now $V_{ph} = V_L / \sqrt{3}$ and $I_{ph} = I_L$

Hence, in terms of line values, the above expression becomes

$$P = 3 \times \frac{V_L}{\sqrt{3}} \times I_L \times \cos\phi \quad \text{or} \quad P = \sqrt{3}\, V_L\, I_L \cos\phi$$

---

* A balanced system is one in which (*i*) the voltages in all phases are equal in magnitude and offer in phase from one another by equal angles, in this case, the angle = 360/3 = 120°, (*ii*) the currents in the three phases are equal in magnitude and also differ in phase from one another by equal angles.

A 3-phase balanced load is that in which the loads connected across three phases are identical.

It should be particularly noted that ϕ is the angle between ***phase*** voltage and ***phase*** current and not between the line voltage and line current.

Similarly, total reactive power is given by **Q** = $\sqrt{3}V_L I_L \sin\phi$

By convention, reactive power of a coil is taken as positive and that of a capacitor as negative.

The total apparent power of the three phases is

$S = \sqrt{3}V_L I_L$ Obviously, $S = \sqrt{P^2 + Q^2}$ – Art. 13.4

Example 19.1. *A balanced star-connected load of (8 + j6) Ω per phase is connected to a balanced 3-phase 400-V supply. Find the line current, power factor, power and total volt-amperes.* **(Elect. Engg., Bhagalpur Univ.)**

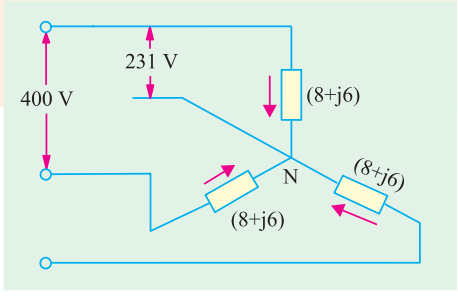


Fig. 19.14

Solution. $Z_{ph} = \sqrt{8^2 + 6^2} = 10\,\Omega$

$V_{ph} = 400/\sqrt{3} = 231$ V

$I_{ph} = V_{ph}/Z_{ph} = 231/10 = 23.1$ A

(i) $I_L = I_{ph}$ = **23.1 A**

(ii) p.f. = cos ϕ = R_{ph}/Z_{ph} = 8/10 = **0.8 (lag)**

(iii) Power P $\sqrt{3}V_L I_L \cos$

$= \sqrt{3} \times 400 \times 23.1 \times 0.8$ = **12,800 W** [Also, $P = 3I_{ph}^2 R_{ph} = 3(23.1)^2 \times 8 = 12{,}800$ *W*]

(iv) Total volt-amperes, $S = \sqrt{3}\,V_L I_L = \sqrt{3} \times 400 \times 23.1$ = **16,000 VA**

Example 19.2. *Phase voltages of a star connected alternator are* $E_R = 231 \angle 0°$ *V;* $E_Y = 231 \angle -120°$ *V ; and* $E_B = 231 \angle +120°$*V. What is the phase sequence of the system ? Compute the line voltages* E_{RY} *and* E_{YB}. **(Elect. Mechines AMIE Sec. B Winter 1990)**

Solution . The phase voltage $E_B = 231\angle -120°$ can be written as $E_B = 231\angle -240°$. Hence, the three voltages are: $E_R = 231\angle -0°$, $E_Y = 231\angle -120°$ and $E_B = 231\angle -240°$. It is seen that E_R is the reference voltage, E_Y lags behind it by 120° whereas E_B lags behind it by 240°. Hence, phase sequence is *RYB*. Moreover, it is a symmetrical 3-phase voltage system.

$\therefore$ $E_{RY} = E_{YB} = \sqrt{3} \times 231 =$ **400 V**

Example 19.3 *Three equal star-connected inductors take* 8 *kW at a power factor* 0.8 *when connected across a* 460 *V, 3-phase, 3-phase, 3-wire supply. Find the circuit constants of the load per phase.* **(Elect. Machines AMIE Sec. B 1992)**

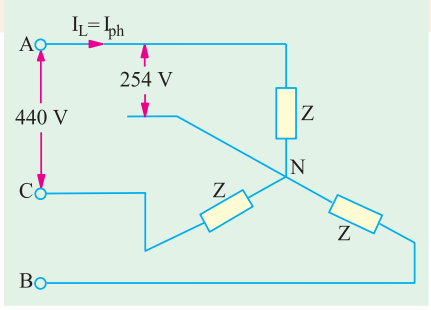


Fig. 19.15

Solution. $P = \sqrt{3}V_L I_L \cos\phi$ or

$8000 = \sqrt{3} \times 460 \times I_L \times 0.8$

$\therefore$ I_L = 12.55 A $\therefore$ I_{ph} = 12.55 *A*;

$V_{ph} = V_L/\sqrt{3}$ $460/\sqrt{3}$ 265*V*

$I_{ph} = V_{ph}/Z_{ph}$; $\therefore Z_{ph} = V_{ph}/I_{ph} = 265/12.55 = 21.1\Omega$

$R_{ph} = Z_{ph} \cos\phi = 21.1 \times 0.8 = 16.9\ \Omega$

$X_{ph} = Z_{ph} \sin\phi = 21.1 \times 0.6 = 12.66\,\Omega$;

The circuit is shown in Fig. 19.15.

Example 19.4. *Given a balanced* $3-\phi$, *3-wire system with Y-connected load for which line voltage is* 230 *V and impedance of each phase is* (6 + *J*8) *ohm. Find the line current and power absorbed by each phase.* **(Elect. Engg - II Pune Univ. 1991)**

Solution. $Z_{ph} = \sqrt{6^2 + 8^2} = 10\,\Omega$; $V_{ph} = V_L / \sqrt{3} = 230 / \sqrt{3} = 133\,\text{V}$

$\cos\phi = R / Z = 6 / 10 = 0.6; I_{ph} = V_{ph} / Z_{ph} = 133 / 10 = 13.3\,\text{A}$

$\therefore \quad I_L = I_{ph} =$ **13.3 A**

Power absorbed by each phase = $I_{ph}^2 R_{ph} = 13.3^2 \times 6 =$ **1061 W**

Solution by Symbolic Notation

In Fig. 19.16 (*b*), V_R, V_Y and V_B are the phase voltage whereas I_R, I_Y and I_B are phase currents. Taking V_R as the reference vector, we get

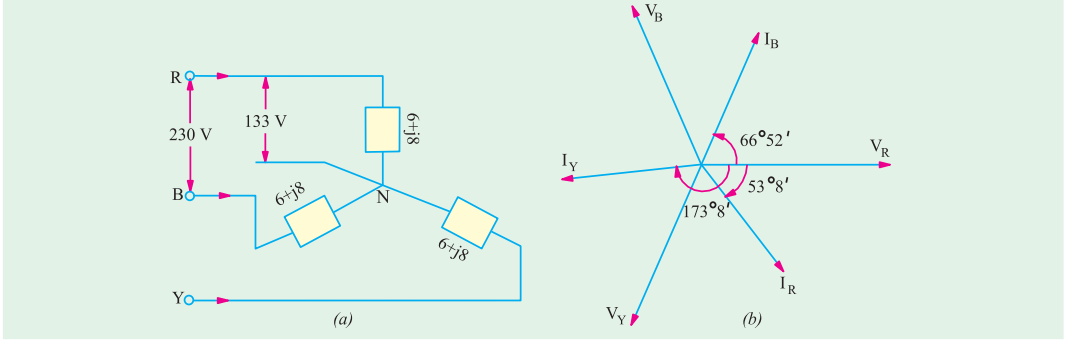


Fig 19.16

$\mathbf{V}_R \quad 133 \quad 0 \quad 133 \quad j0$ volt

$\mathbf{V}_Y \quad 133 \quad 120 \quad 133(\ 0.5 \quad j0.866) \quad (\ 66.5 \quad j115)$ volt

$\mathbf{V}_B = 133 \quad \angle 120° = 133\ (-0.5 + j\ 0.866) = (-66.5 + j\ 115)$ volt

$\mathbf{Z} = 6 + j8 = 10 \quad \angle 53°\,8'$; $\mathbf{I}_R = \dfrac{\mathbf{V}_R}{\mathbf{Z}} \quad \dfrac{133 \quad 0}{10 \quad 53\ 8} \quad 13.3 \quad 53°8$

This current lags behind the reference voltage by $53°8'$ [Fig. 19.16 (*b*)]

$\mathbf{I}_Y = \dfrac{\mathbf{V}_Y}{\mathbf{Z}} \quad \dfrac{133 \quad 120}{10 \quad 53\ 8} \quad 13.3 \quad 173\ 8$

It lags behind the reference vector *i.e.* V_R by $173°8'$ which amounts to lagging behind its phase voltage V_Y by $53°8'$.

$\mathbf{I}_B \quad \dfrac{\mathbf{V}_B}{\mathbf{Z}} \quad \dfrac{133 \quad 120}{10 \quad 53\ 8} \quad 13.3 \quad 66°52$

This current leads V_R By $66°52'$ which is the same thing as ***lagging*** behind its phase voltage by $53°8'$. For calculation of power, consider *R*-phase

$\mathbf{V}_R = (133 - j0)$; $I_R = 13.3\ (0.6 - j0.8) = (7.98 - j10.64)$

Using method of conjugates, we get

$\mathbf{P}_{VA} = (133 - j0)\ (7.98 - j10.64) = 1067 - j1415$

$\therefore$ Real power absorbed/phase = 1067 W – as before

Example 19.5. *When the three identical star-connected coils are supplied with 440 V, 50 Hz, 3-ϕ supply, the 1-ϕ wattmeter whose current coil is connected in line R and pressure coil across the phase R and neutral reads 6 kW and the ammeter connected in R-phase reads 30 Amp. Assuming RYB phase sequence find:*

(i) *resistance and reactance of the coil,* ***(ii)*** *the power factor, of the load*

(iii) *reactive power of 3-ϕ load.* **(Elect. Engg.-I, Nagpur Univ. 1993)**

Solution. $V_{ph} = 440/\sqrt{3} = 254$ V; $I_{ph} = 30$A (Fig. 19.17.)

Now, $V_{ph}\, I_{ph} \cos\phi = 6000$; $254 \times 30 \times \cos\phi = 6000$

$\therefore \cos\phi = 0.787$; $\phi = 38.06°$ and $\sin\phi = 0.616°$; $Z_{ph} = V_{ph} / I_{ph} = 254/30 = 8.47\,\Omega$

(i) Coil resistance $R = Z_{ph} \cos\phi = 8.47 \times 0.787 = 6.66\,\Omega$

$X_L = Z_{ph} \sin\phi = 8.47 \times 0.616 = \mathbf{5.22}\ \Omega$

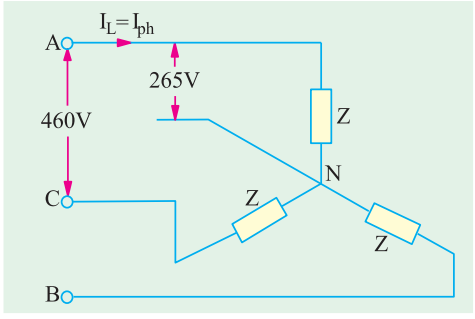


Fig. 19.17

(ii) p.f. = $\cos\phi$ = **0.787 (lag)**

(iii) Reactive power = $\sqrt{3}\, V_L I_L \sin$ $\sqrt{3}$ 440 30 0.616 14,083VA **14.083kVA**

Example 19.6 *Calculate the active and reactive components in each phase of Y-connected 10,000 V, 3-phase alternator supplying 5,000 kW at 0.8 p.f. If the total current remains the same when the load p.f is raised to 0.9, find the new output.*

(Elements of Elect. Engg.-I, Bangalore Univ.)

Solution. $5000 \times 10^3 = \sqrt{3}$ 10,000 I_L 0.8; I_L I_{ph} 361A

active component= $I_L \cos\phi = 361 \times 0.8 = \mathbf{288.8\ A}$

reactive component= $I_L \sin\phi = 361 \times 0.6 = \mathbf{216.6\ A}$

New power $P = \sqrt{3} V_L I_L \cos\phi = \sqrt{3} \times 10^4 \times 361 \times 0.9 = \mathbf{5,625\ kW}$

[or new power = 5000 × 0.9/0.8 = 5625 kW]

Example 19.7. *Deduce the relationship between the phase and line voltages of a three-phase star-connected alternator. If the phase voltage of a 3-phase star-connected alternator be 200 V, what will be the line voltages* ***(a)*** *when the phases are correctly connected and* ***(b)*** *when the connections to one of the phases are reversed.*

Solution. ***(a)*** When phases are correctly connected, the vector diagram is as shown in Fig. 19.12. (*b*). As proved in Art. 19.7

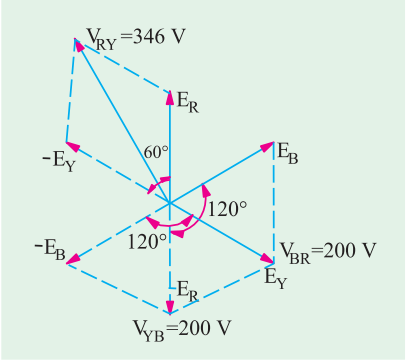


Fig. 19.18

$V_{RY} = V_{YB} V_{BR} = \sqrt{3} E_{Ph}$

Each line voltage = $\sqrt{3} \times 200 = \mathbf{346\ V}$

(b) Suppose connections to *B*-phase have been reversed. Then voltage vector diagram for such a case is shown in Fig. 19.18. It should be noted that E_B has been drawn in the reversed direction, so that angles between the three-phase voltages are 60° (instead of the usual 120°)

$V_{RY} = E_R - E_Y$ ... vector difference

$= 2 \times E_{ph} \times \cos 30° = \sqrt{3} \times 200 = \mathbf{346\ V}$

$V_{YB} = E_Y - E_B$ ... vector difference

$$= 2 \times E_{ph} \times \cos 60° = 2 \times 200 \times \frac{1}{2} = \mathbf{200V}$$

$V_{BR} = E_B - E_R$ ... vector difference $= 2 \times E_{ph} \times \cos 60° = 2 \times 200 \times \frac{1}{2} = \mathbf{200\ V}$

Example 19.8 *In a 4-wire, 3-phase system, two phases have currents of 10A and 6A at lagging power factors of 0.8 and 0.6 respectively while the third phase is open-circuited, Calculate the current in the neutral and sketch the vector diagram.*

Solution. The circuit is shown in Fig. 19.19 (*a*).

$\phi_1 = \cos^{-1}(0.8) = 36°54'; \phi_2 = \cos^{-1}(0.6) = 53°6'$

Let V_R be taken as the reference vector. Then

$\mathbf{I_R} = 10\angle -36°54' = (8 - j6)$ $\mathbf{I_y} = 6\angle -173°6' = (-6 - j0.72)$

The neutral current $\mathbf{I_N}$, as shown in Fig. 19.16 (*b*), is the sum of these two currents.

$\therefore$ $\mathbf{I_N} = (8 - j6) + (-6 - j0.72) = 2 - j6.72 = 7\angle -73°26'$

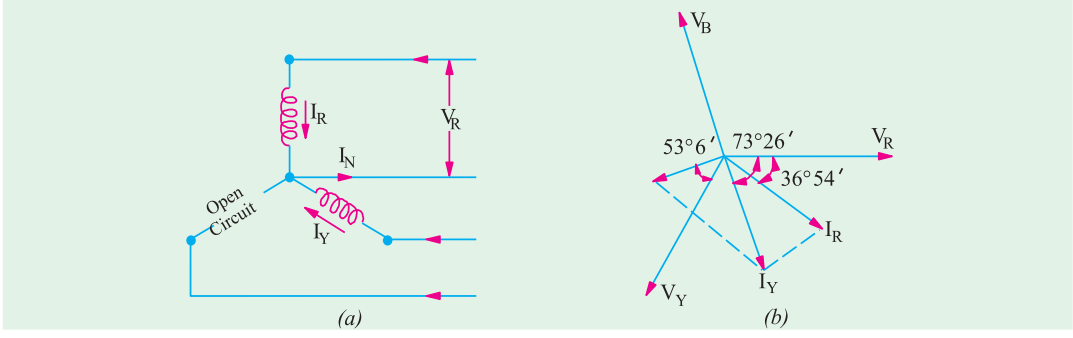


Fig. 19.19

Example 19.9 (*a*). *Three equal star-connected inductors take 8 kW at power factor 0.8 when connected a 460-V, 3-phase, 3-wire supply. Find the line currents if one inductor is short-circuited.*

Solution. Since the circuit is balanced, the three line voltages are represented by

$V_{ab} = 460\angle 0°; V_{bc} = 460\angle -120°$ and $V_{ca} = 460\angle 120°$

The phase impedance can be found from the given data :

$8000 = \sqrt{3} \times 460 \times I_L \times 0.8$ $\therefore$ $I_L = I_{ph} = 12.55$ A

$Z_{ph} = V_{ph} / I_{ph} = 460 / \sqrt{3} \times 12.55 = 21.2\ \Omega;$

$\therefore$ $Z_{ph} = 21.2\angle 36.9°$ because $\phi = \cos^{-1}(0.8) = 36.9°$

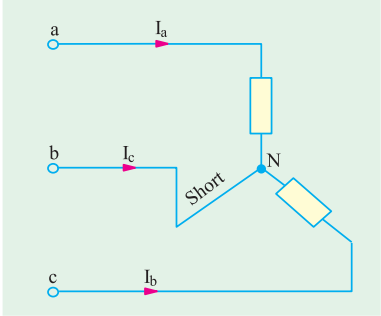


Fig. 19.20

As shown in the Fig. 19.20, the phase *c* has been short-circuited. The line current $I_a = V_{ac}/Z_{ph} = -V_{ca}/Z_{ph}$ because the current enters at point *a* and leaves from point *c*.

$\therefore$ $I_a = -460\angle 1200°/21.2\angle 36.9° = 21.7\angle 83.1°$

Similarly, $I_b = V_{bc}/Z_{ph} = 460\ \angle 120°/21.2\ \angle 36.9° = 21.7\ \angle -156.9°$. The current I_c can be found by applying KVL to the neutral point N.

$\therefore$ $I_a + I_b + I_c = 0$ or $I_c = -I_a - I_b$

$\therefore \quad I_C = 21.7\angle 83.1^\circ - 21.7\angle -156.9^\circ = 37.3\angle 53.6^\circ$

Hence, the magnitudes of the three currents are : 21.7 A; 21.7 Al 37.3 A.

Example 19.9 (*b*). *Each phase of a star-connected load consists of a non-reactive resistance of 100 Ω in parallel with a capacitance of 31.8 μF.*

Calculate the line current, the power absorbed, the total kVA and the power factor when connected to a 416-V, 3-phase, 50-Hz supply.

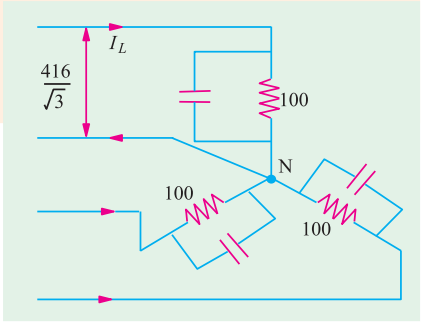


Fig. 19.21

Solution. The circuit is shown in Fig. 14.20.

$V_{ph} = (416/\sqrt{3})\angle 0^\circ = 240\angle 0^\circ = (240 + j0)$

Admittance of each phase is

$$\mathbf{Y}_{ph} \quad \frac{1}{R} \quad j \quad C \quad \frac{1}{100} \quad j314 \quad 31.8 \quad 10^{-6}$$

$$= 0.01 + j0.01$$

$\therefore \quad \mathbf{I_{ph} = V_{ph} \cdot Y_{ph}} = 240(0.01 + j0.01)$

$= 2.4 + j2.4 = 3.39 \angle 45^\circ$

Since $I_{ph} = I_L$ – for a star connection $\therefore \quad \mathbf{I_L = 3.39\ A}$

Power factor = cos 45° = **0.707 (leading)**

Now $\mathbf{V_{ph}} = (240 + j0)$; $I_{ph} = 2.4 + j2.4$

$\therefore \quad \mathbf{P}_{VA} = (240 + j0)\ (2.4 + j\ 2.4)$

$= 240 \times 2.4 - j2.4 \times 240 = 576 - j576 = 814.4 \angle -45^\circ$ ... per phase

Hence, total power = $3 \times 576 = 1728$ W = **1.728 kW**

Total voltampers = $814.4 \times 3 = 2{,}443$ VA ; kilovolt amperes = **2.433 kVA**

Example 19.10. *A three pahse 400-V, 50 Hz, a.c. supply is feeding a three phase delta-connected load with each phase having a resistance of 25 ohms, an inductance of 0.15 H, and a capacitor of 120 microfarads in series. Determine the line current, volt-amp, active power and reactive volt-amp.* **[Nagpur University, November 1999]**

Solution. Impedance per phase $r + jX_L - jX_C$

$X_L = 2\pi \times 50 \times 0.15 = 47.1\ \Omega$

$X_C = \dfrac{10^6}{32{\cdot}37} = 26.54\ \Omega$

$\cos\phi = \dfrac{25}{32.37}$ Lagging, since inductive reactance is dominating.

Phase Current = $\dfrac{400}{25 + j20.56} = 12.357$

Line Current = $\sqrt{3} \times 12.357 = 21.4$ amp

Since the power factor is 0.772 lagging,

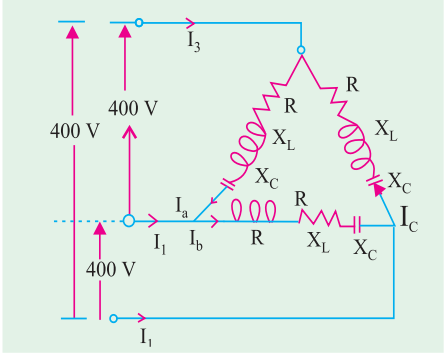


Fig. 19.22

P = total three phase power = $\sqrt{3}\,V_L\,I_L\cos\phi\times10^{-3}$ kW

= $\sqrt{3}\times400\times21.4\times0.772\times10^{-3}$ = 11.446 kW

S = total 3 ph kVA $\dfrac{11.446}{0.772}$ = 14.83 kVA 14.83 kVA

Q = total 3 ph "reactive kilo-volt-amp" $\sqrt{3}$ = $(S^2 - P^2)^{0.50}$ = 9.43 kVAR lagging

Example 19.11. *Three phase star-connected load when supplied from a 400 V, 50 Hz source takes a line current of 10 A at 66.86° w.r. to its line voltage. Calculate : **(i)** Impedance-Parameters, **(ii)** P.f. and active-power consumed. Draw the phasor diagram.*

[Nagpur University, April 1998]

Solution. Draw three phasors for phase-voltages.

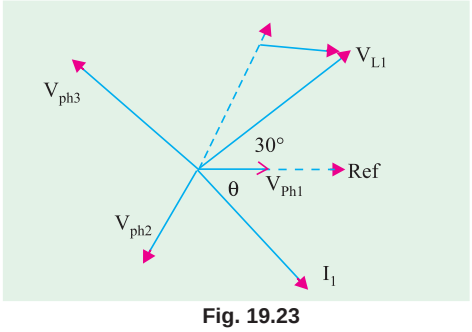


Fig. 19.23

These are V_{ph1}, V_{ph2}, V_{ph3} in Fig 19.23. As far as phase number 1 is concerned, its current is I_1 and the associated line voltage is V_{L1} . V_{L1} and V_{ph1} differ in phase by 30°. A current differing in phase with respect to line voltage by 66.86° and associated with V_{ph1} can only be lagging, as shown in Fig. 19.23. This means ϕ = 36.86°, and the corresponding load power factor is 0.80 lagging.

$Z = V_{ph}/I_{ph}$ = 231/10 = 23.1 ohms

$R = Z\cos\phi$ = 23.1 × 0.8 = 18.48 ohms

$X_L = Z\sin\phi$ = 23.1 × ?0.6 = 13.86 ohms

Total active power consumed = $3\,V_{ph}\,I_{ph}\cos\phi$

= $3\times231\times10\times0.8\times10^{-3}$ kW = 5.544 kW

or total active power = $3\times I^2R = 3\times10^2\times18.48$ = 5544 watts

For complete phasor diagram for three phases, the part of the diagram for Phase 1 in Fig 19.23 has to be suitably repeated for phase-numbers 2 and 3.

19.9. Delta (Δ)* or Mesh Connection

In this form, of interconnection the ***dissimilar*** ends of the three phase winding are joined together *i.e.* the 'starting' end of one phase is joined to the 'finishing' end of the other phase and so on as showing in Fig. 19.24 (*a*). In other words, the three windings are joined in series to form a closed mesh as shown in Fig. 19.24 (*b*).

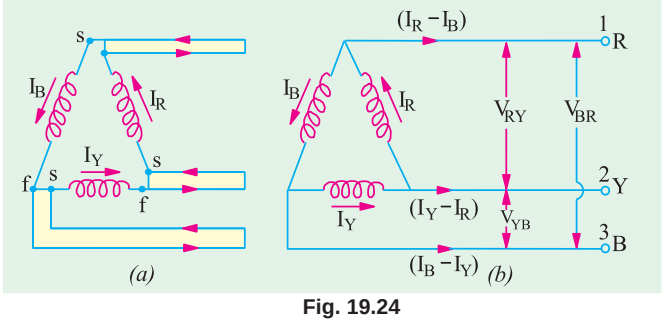


Fig. 19.24

Three leads are taken out from the three junctions as shown as outward directions are taken as positive.

It might look as if this sort of interconnection results in

* As an aid to memory, remember that first letter *D* of Dissimilar is the same as that of Delta.

shortcircuiting the three windings. However, if the system is balanced then sum of the three voltages round the closed mesh is zero, hence no current of fundamental frequency can flow around the mesh when the terminals are open. It should be clearly understood that at any instant, the e.m.f. in one phase is equal and opposite to the resultant of those in the other two phases.

This type of connection is also referred to as 3-phase, 3-wire system.

(*i*) Line Voltages and Phase Voltages

It is seen from Fig. 19.24 (*b*) that there is only one phase winding completely included between any pair of terminals. Hence, in Δ-connection, the voltage between any pair of lines is equal to the phase voltage of the phase winding connected between the two lines considered. Since phase sequence is *R Y B*, the voltage having its positive direction from *R* to *Y* leads by 120° on that having its positive direction from *Y* to *B*. Calling the voltage between lines 1 and 2 as V_{RY} and that between lines 2 and 3 as V_{YB}, we find that V_{RY} lead V_{YB} by 120°. Similarly, V_{YB} leads V_{BR} by 120° as shown in Fig. 19.23. Let $V_{RY} = V_{YB} = V_{BR}$ = line voltage V_L. Then, it is seen that $V_L = V_{ph}$.

(*ii*) Line Currents and Phase Currents

It will be seen from Fig. 19.24 (*b*) that current in each line is the ***vector difference*** of the two phase currents flowing through that line. For example

Current in line 1 is I_1 I_R I_B
Current in line 2 is I_2 I_Y I_R vector difference
Current in line 3 is I_3 I_B I_Y

Current in line No. 1 is found by compounding I_R and I_B reversed and its value is given by the diagonal of the parallelogram of Fig. 19.25. The angle between I_R and I_B reversed (*i.e.* $-I_B$) is 60°. If $I_R = I_Y$ = phase current I_{ph} (say), then

Current in line No. 1 is

$$I_1 = 2 \times I_{ph} \times \cos(60°/2) = 2 \times I_{ph} \sqrt{3}/2 = \sqrt{3} I_{ph}$$

Current in line No. 2 is

$I_2 = I_B - I_Y$ ... vector difference = $\sqrt{3} I_{ph}$ and current in line No. 3 is $I_3 = I_B - I_Y$ ∴ Vector difference = $\sqrt{3} \cdot I_{ph}$

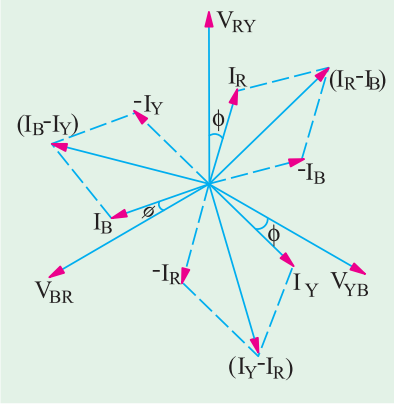


Fig. 19.25

Since all the line currents are equal in magnitude *i.e.*

$I_1 = I_2 - I_3 = I_L$

$$\therefore \quad I_L = \sqrt{3} I_{ph}$$

With reference to Fig. 19.25, it should be noted that

1. line currents are 120° apart ;

2. line currents are 30° behind the respective phase currents ;

3. the angle between the line currents and the corresponding line voltages is (30 + ϕ) with the current lagging.

(*iii*) Power

Power/phase = $V_{ph} I_{ph} \cos\phi$; Total power = $3 \times V_{ph} I_{ph} \cos\phi$. However, $V_{ph} = V_L$ and $I_{ph} = I_L/\sqrt{3}$

Hence, in terms of line values, the above expression for power becomes

$$P = 3 \times V_L \times \frac{I_L}{\sqrt{3}} \times \cos\phi = \sqrt{3} V_L I_L \cos\phi$$

where ϕ is the phase power factor angle.

19.10. Balanced Y/Δ and Δ/Y Conversion

In view of the above relationship between line and phase currents and voltages, any balanced *Y*-connected system may be completely replaced by an equivalent Δ-connected system. For example, a 3-phase, *Y*-connected system having the voltage of V_L and line current I_L may be replaced by a Δ-connected system in which phase voltage is V_L and phase current is $I_L / \sqrt{3}$.

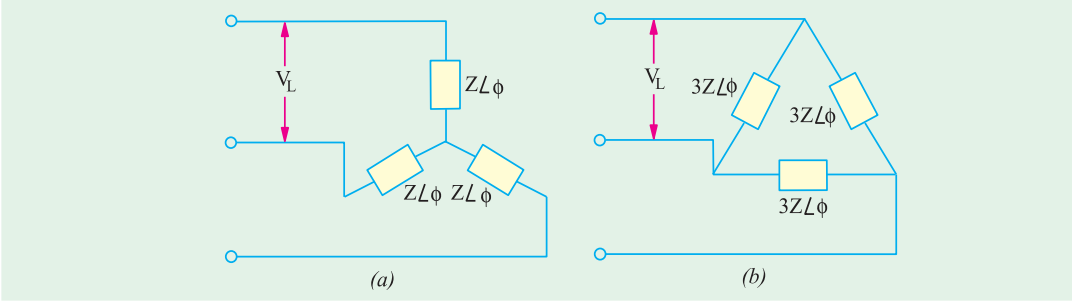


Fig. 19.26

Similarly, a balanced *Y*-connected load having equal branch impedances each of $Z \angle \phi$ may be replaced by an equivalent Δ-connected load whose each phase impedance is $3Z \angle \phi$. This equivalence is shown in Fig. 19.26.

For a balanced star-connected load, let

V_L = line voltage; I_L = line current ; Z_Y = impedance/phase

$\therefore \quad V_{ph} \quad V_L / \sqrt{3}. \quad I_{ph} \quad I_L; \quad Z_Y \quad V_L / (\sqrt{3} I_L)$

Now, in the equivalent Δ-connected system, the line voltages and currents must have the same values as in the *Y*-connected system, hence we must have

$$V_{ph} = V_L, \quad I_{ph} = I_L / \sqrt{3} \quad \therefore \quad Z_\Delta = V_L / (I_L / \sqrt{3}) = \sqrt{3} V_L / I_L = 3Z_Y$$

$$\therefore \qquad Z_\Delta \angle \phi = 3Z_Y \angle \phi \qquad (\quad V_L / I_L = \sqrt{3} Z_Y)$$

or $\quad \mathbf{Z}_\Delta = 3\mathbf{Z}_Y \quad$ or $\quad \mathbf{Z}_Y = \mathbf{Z}_\Delta / 3$

The case of unbalanced load conversion is considered later. (Art. 19.34)

Example 19.12. *A star-connected alternator supplies a delta connected load. The impedance of the load branch is (8 + j6) ohm/phase. The line voltage is 230 V. Determine* ***(a)*** *current in the load branch,* ***(b)*** *power consumed by the load,* ***(c)*** *power factor of load, (d) reactive power of the load.* **(Elect. Engg. A.M.Ae. S.I. June 1991)**

Solution. Considering the Δ-connected load, we have $Z_{ph} = \sqrt{8^2 + 6^2} = 10\,\Omega$; $V_{ph} = V_L =$ **230 V**

(a) $I_{ph} = V_{ph}/Z_{ph} = 230/10 = 23$ A

(b) $I_L = \sqrt{3} I_{ph} = \sqrt{3} \times 23 = 39.8$ *A*; $P \quad \sqrt{3} V_L I_L \cos \quad \sqrt{3} \quad 230 \quad 39.8 \quad 0.8$ **12,684 W**

(c) p.f. cos ϕ = *R* | *Z* = 8/10 = **0.8 (lag)**

(d) Reactive power *Q* = $\sqrt{3}$ V_L I_L sin $\quad \sqrt{3} \quad 230 \quad 39.8 \quad 0.6$ **9513 W**

Example 19.13. *A 220-V, 3-ϕ voltage is applied to a balanced delta-connected 3-ϕ load of phase impedance (15 + j20) Ω*

(a) *Find the phasor current in each line.* ***(b)*** *What is the power consumed per phase ?*

(c) *What is the phasor sum of the three line currents ? Why does it have this value ?*

(Elect. Circuits and Instruments, B.H.U.)

Solution. The circuit is shown in Fig. 19.27 (*a*).

$V_{ph} = V_L = 220\ V;\ Z_{ph} = \sqrt{15^2 + 20^2} = 25\,\Omega,\ I_{ph} = V_{ph} / Z_{ph} = 220/25 = 8.8\,\text{A}$

(*a*) $I_L = \sqrt{3}\ I_{ph} = \sqrt{3} \times 8.8 =$ **15.24 A** (*b*) $P = {I_{ph}}^2 R_{ph} = 8.8^2 \times 15 =$ **462 W**

(*c*) Phasor sum would be zero because the three currents are equal in magnitude and have a mutual phase difference of 120°.

Solution by Symbolic Notation

Taking V_{RY} as the reference vector, we have [Fig. 19.27 (*b*)]

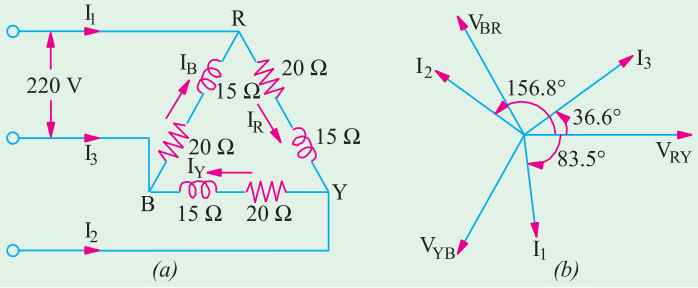


Fig. 19.27

$\mathbf{V_{RY}}$ 220 0 ; $\mathbf{V_{YB}}$ 220 120

V_{BR} 220 120 ; **Z** 15 *j*20 125 53°8

$$\mathbf{I_R} \quad \frac{V_{RY}}{Z} \quad \frac{220 \quad 0^\circ}{25 \quad 53^\circ 8} \quad 8.8 \quad 53^\circ 8 \quad (5.28 \quad j7.04)\ A$$

$$\mathbf{I_Y} \quad \frac{V_{YB}}{Z} \quad \frac{220 \quad 120^\circ}{25 \quad 53^\circ 8} \quad 8.8 \quad 173^\circ 8 \quad (\ 8.75 \quad j1.05)\ A$$

$$\mathbf{I_B} \quad \frac{V_{BR}}{Z} \quad \frac{220 \quad 120^\circ}{25 \quad 53^\circ 8} \quad 8.8 \quad 66^\circ 55 \quad (3.56 \quad j8.1)$$

(*a*) Current in line No. 1 is

$\mathbf{I_1 = I_R - I_B} = (5.28 - j\ 7.04) - (4.56 + j8.1) = (1.72 - j15.14) = 15.23 \angle -83.5^\circ$

$\mathbf{I_2 = I_Y - I_R} = (-8.75 - j\ 1.05) - (5.28 - j7.04) = (-14.03 + j\ 6.0) = 15.47 \angle -156.8^\circ$

$\mathbf{I_3 = I_B - I_Y} = (3.56 - j\ 8.1) - (-\ 8.75 - j1.05) = (12.31 + j\ 9.15) = 15.26 \angle 36.8^\circ$

(*b*) Using conjugate of voltage, we get for *R*-phase

$\mathbf{P_{VA} = V_{RY} \cdot I_R} = (220 - j0)\ (5.28 - 7.04) = (1162 - j\ 1550)$ voltampere

Real power per phase = **1162 W**

(*c*) Phasor sum of three line currents

$= \mathbf{I_1 + I_2 + I_3} = (1.72 - j\ 15.14) + (-\ 14.03 + j\ 6.0) + (12.31 + j\ 9.15) = 0$

As expected, phasor sum of 3 line currents drawn by a balanced load is zero because these are equal in magnitude and have a phase difference of 120° amount themselves.

Example 19.14 *A 3-ϕ, Δ-connected alternator drives a balanced 3-ϕ load whose each phase current is 10 A in magnitude. At the time when* $I_a = 10 \angle 30^\circ$*, determine the following, for a phase sequence of abc.*

***(i)** Polar expression for* I_b *and* I_c *and **(ii)** polar expressions for the three line current. Show the phase and line currents on a phasor diagram.*

Solution. (*i*) Since it is a balanced 3-phase system, I_b lags I_a by 120° and I_c lags I_a by 240° or leads it by 120°.

$$\therefore \quad I_b = I_a \angle -120° = 10\angle(30°-120°) = 10\angle -90°$$

$$I_c = I_a\angle 120° = 10\angle(30°+120°) = 10\angle 150°$$

The 3-phase currents have been represented on the phasor diagram of Fig. 19.28 (*b*).

As seen from Fig. 19.28 (*b*), the line currents lag behind their nearest phase currents by 30°.

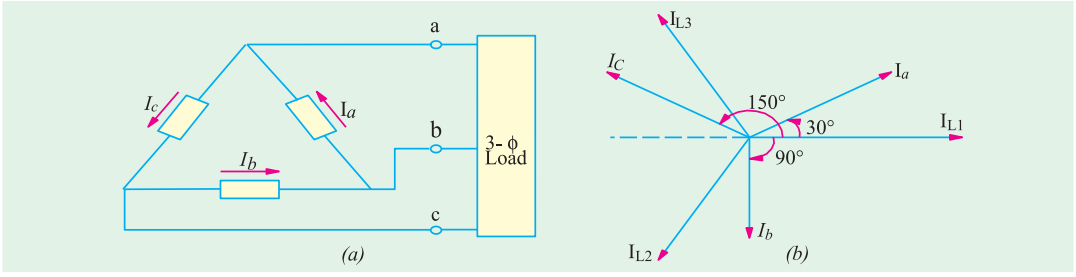


Fig. 19.28

$$\therefore \quad I_{L1} \quad \sqrt{3}\ I_a \quad (30°\ \ 30°) \quad \mathbf{17.3\ \ 0°}$$

$$I_{L2} \quad \sqrt{3}\ I_b \quad (\ 90°\ \ 30°) \quad \mathbf{17.3 \quad 120°}$$

$$I_{L3} \quad \sqrt{3}\ I_C \quad (150°\ \ 30°) \quad \mathbf{17.3\ \ 120°}$$

These line currents have also been shown in Fig. 19.28 (*b*).

Example 19.15. *Three similar coils, each having a resistance of 20 ohms and an inductance of 0.05 H are connected in* ***(i)*** *star* ***(ii)*** *mesh to a 3-phase, 50-Hz supply with 400-V between lines. Calculate the total power absorbed and the line current in each case. Draw the vector diagram of current and voltages in each case.* **(Elect. Technology, Punjab Univ. 1990)**

Solution. $X_L \quad 2 \quad 50 \quad 0.05 \quad 15 \quad , Z_{ph} \quad \sqrt{15^2 \quad 20^2} \quad \mathbf{25\,\Omega}$

(*i*) Star Connection. [Fig. 19.29 (*a*)]

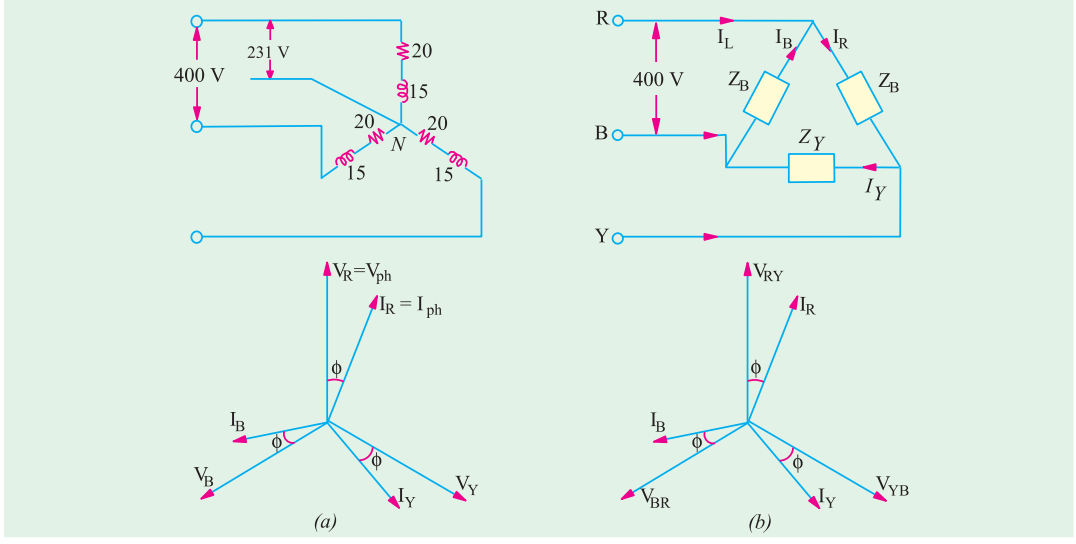


Fig. 19.29

$V_{ph} = 400/\sqrt{3} = 231$ V; $I_{ph} = V_{ph}/Z_{ph} = 231/25 =$ **9.24 Ω**

I_L I_{ph} **9.24 A;** P $\sqrt{3}$ 400 9.24 (20/25) **5120 W**

(*ii*) Delta Connection [Fig. 19.29 (*b*)]

V_{ph} V_L 400 *V*; I_{ph} 400/25 16 *A*; I_L $\sqrt{3}I_{ph}$ $\sqrt{3}$ 16 **27.7 A**

$P = \sqrt{3} \times 400 \times 27.7 \times (20/25) =$ **15,360 W**

Note. It may be noted that line current as well as power are three times the star values.

Example 19.16. *A Δ -connected balanced 3-phase load is supplied from a 3-phase, 400-V supply. The line current is 20 A and the power taken by the load is 10,000 W. Find (i) impedance in each branch (ii) the line current, power factor and power consumed if the same load is connected in star.* **(Electrical Machines, A.M.I.E. Sec. B. 1992)**

Solution. (*i*) Delta Connection.

$$V_{ph} = V_L = 400\ V;\ I_L = 20\ A;\ I_{ph} = 20/\sqrt{3}\ \text{A}$$

(*i*) $\therefore$ Z_{ph} $\dfrac{400}{20/\sqrt{3}}$ $20\sqrt{3}$ **34.64 Ω**

Now $P = \sqrt{3}\ V_L I_L \cos\phi$ $\therefore$ $\cos\phi = 10{,}000/\sqrt{3} \times 400 \times 20 =$ **0.7217**

(*ii*) Star Connection

$$V_{ph}\ \frac{400}{\sqrt{3}},\ I_{ph}\ \frac{400/\sqrt{3}}{\sqrt{3}}\ \frac{20}{3}A,\ I_L\ I_{ph}\ \frac{20}{3}\ \text{A}$$

Power factor remains the same since impedance is the same.

Power consumed = $\sqrt{3} \times 400 \times (20/3) + 0.7217 =$ **3,330 W**

Note. The power consumed is 1/3 of its value of Δ-connection.

Example 19.17. *Three similar resistors are connected in star across 400-V, 3-phase lines. The line current is 5 A. Calculate the value of each resistor. To what value should the line voltage be changed to obtain the same line current with the resistors delta-connected.*

Solution. Star Connection

$I_L = I_{ph} = 5\ A$; $V_{ph} = 400/\sqrt{3} = 231$V $\therefore$ $R_{ph} = 231/5 =$ **46.2 Ω**

Delta Connection

I_L = 5 A... (given); I_{ph} = $5/\sqrt{3}$ A; R_{ph} = **46.2 Ω**

... found above

V_{ph} I_{ph} R_{ph} 5 46.2/$\sqrt{3}$ **133.3 V**

Note. Voltage needed is 1/3rd the star value.

Example 19.18. *A balanced delta connected load, consisting of there coils, draws* $10\sqrt{3}$ *A at 0.5 power factor from 100 V, 3-phase supply. If the coils are re-connected in star across the same supply, find the line current and total power consumed.*

(Elect. Technology, Punjab Univ. Nov.)

Solution. Delta Connection

V_{ph} V_L 100*V*; I_L $10\sqrt{3}$ *A*; I_{ph} $10\sqrt{3}/\sqrt{3}$ **10 A**

$Z_{ph} = V_{ph} / I_{ph} = 100/10 = 10\,\Omega$; $\cos\phi = 0.5$ (given); $\sin\phi = 0.866$

$\therefore$ R_{ph} Z_{ph} cos 10 0.5 5 ; X_{ph} Z_{ph} sin 10 0.866 **8.66**

Incidentally, total power consumed = $\sqrt{3}\,V_L I_L \cos\phi = \sqrt{3}\times 100\times 10\sqrt{3}\times 0.5 = 1500$ W

Star Connection

V_{ph} $V_L/\sqrt{3}$ $100/\sqrt{3}$; Z_{ph} 10 ; I_{ph} V_{ph}/Z_{ph} $100/\sqrt{3}$ 10 $10\sqrt{3}$ A

Total power absorbed = $\sqrt{3}\times 100\times(10\sqrt{3})\times 0.5 =$ **500 W**

It would be noted that the line current as well as the power absorbed are one-third of that in the delta connection.

Example 19.19. *Three identical impedances are connected in delta to a 3 ϕ supply of 400 V. The line current is 35 A and the total power taken from the supply is 15 kW. Calculate the resistance and reactance values of each impedance.* **(Elect. Technology, Punjab Univ.,)**

Solution. $V_{ph} = V_L = 400$ V; $I_L = 35$ A $\therefore$ $I_{ph} = 35/\sqrt{3}$ A

$$Z_{ph} = V_{ph} / I_{ph} = 400\times\sqrt{3}/35 = 19.8 \text{ A}$$

Now,

Power $P = \sqrt{3}V_L I_L \cos\phi$ $\therefore \cos\phi = \dfrac{P}{\sqrt{3}V_L I_L} = \dfrac{15{,}000}{\sqrt{3}\times 400\times 35} = 0.619$; But $\sin\phi = 0.786$

$\therefore$ R_{ph} $Z_{ph}\cos$ 19.8 0.619 **12.25** ; X_{ph} $Z_{ph}\sin$ and X_{ph} 19.8 0.786 **15.5**

Example 19.20. *Three 100 Ω non-inductive resistances are connected in (a) star (b) delta across a 400-V, 50-Hz, 3-phase mains. Calculate the power taken from the supply system in each case. In the event of one of the three resistances getting open-circuited, what would be the value of total power taken from the mains in each of the two cases ?*

(Elect. Engg. A.M.Ae. S.I June, 1993)

Solution. (*i*) Star Connection [Fig. 19.30 (*a*)]

V_{ph} $400/\sqrt{3}$ V

$P = \sqrt{3}\,V_L I_L \cos\phi$

$= \sqrt{3}\times 400\times 4\times 1/\sqrt{3} =$ **1600 W**

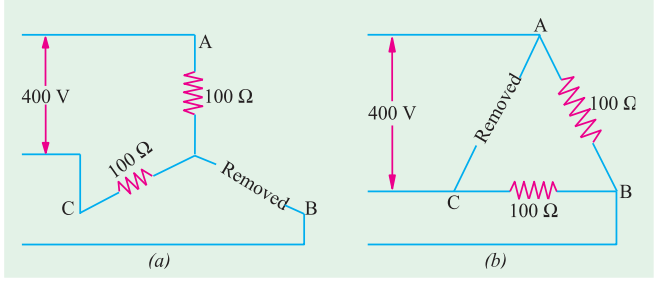


Fig. 19.30

(*ii*) Delta Connection Fig. 19.30 (*b*)

$V_{ph} = 400$ V; $R_{ph} = 100\,\Omega$

$I_{ph} = 400/100 = 4$ A

$I_L = 4\times\sqrt{3}$ A

$P = \sqrt{3}\times 400\times 4\times\sqrt{3}\times 1$ = **4800 W**

When one of the resistors is disconnected

(*i*) Star Connection [Fig. 19.28 (*a*)]

The circuit no longer remains a 3-phase circuit but consists of two 100 Ω resistors in series across a 400-V supply. Current in lines *A* and *C* is = 400/200 = 2 A

Power absorbed in both = 400 × 2 = 800 W

Hence, by disconnecting one resistor, the power consumption is reduced by half.

(*ii*) Delta Connection [Fig. 19.28 (*b*)]

In this case, currents in *A* and *C* remain as usual 120° out of phase with each other.

Current in each phase = 400/100 = 4 A

Power consumption in both = $2 \times 4^2 \times 100$ = **3200 W**

(or $P = 2 \times 4 \times 400 = 3200$ W)

In this case, when one resistor is disconnected, the power consumption is reduced by one-third.

Example 19.21. *A 200-V, 3-ϕ voltage is applied to a balanced Δ-connected load consisting of the groups of fifty 60-W, 200-V lamps. Calculate phase and line currents, phases voltages, power consumption of all lamps and of a single lamp included in each phase for the following cases :*

***(a)** under normal conditions of operation*

***(b)** after blowout in line $R'R$ **(c)** after blowout in phase YB*

Neglect impedances of the line and internal resistances of the sources of e.m.f.

Solution. The load circuit is shown in Fig. 19.31 where each lamp group is represented by two lamps only. It should be kept in mind that lamps remain at the line voltage of the supply irrespective of whether the Δ-connected load is balanced or not.

(*a*) Normal operating conditions [Fig. 19.31 (*a*)]

Since supply voltage equals the rated voltage of the bulbs, the power consumption of the lamps equals their rated wattage.

Power consumption/lamp = 60 W; Power consumption/phase = 50 × 60 = **3,000 W**

Phase current = 3000/200 = **15 A** ; Line current = $15 \times \sqrt{3}$ = **26 A**

(*b*) Line Blowout [Fig. 19.31 (*b*)]

When blowout occurs in line *R*, the lamp group of phase *Y-B* remains connected across line voltage $V_{YB} = V_{y'B'}$. However, the lamp groups of other two phases get connected in series across the same voltage V_{YB}. Assuming that lamp resistances remain constant, voltage drop across $YR = V_{YB}$ 200/2 = **100 V** and that across *RB* = **100 V**.

Hence, phase currents are as under :

I_{YB} = 3000/200 = **15 A**, $I_{YB} = I_{RB}$ = 15/2 = **7.5 A**

The line currents are :

I_{RR} 0, I_{YY} I_{BB} I_{YB} I_{YR} 15 7.5 **22.5 A**

Power in phase *YR* = 100 × 7.5 = 750 W; Power/lamp = 750/50 = **15 W**

Power in phase *YB* = 200 × 15 = 3000 W ; Power/lamp = 3000/50 = **60 W**

Power in phase *RB* = 100 × 7.5 = 750 W ; Power/lamp = 750/50 = **15 W**

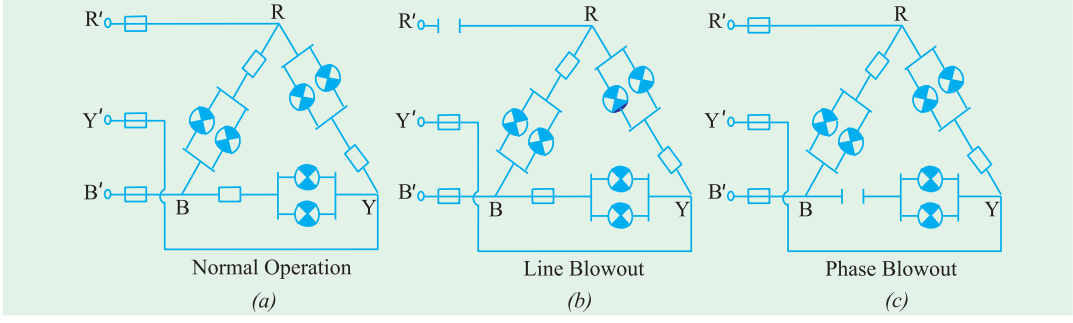


Fig. 19.31

(c) Phase Blowout [Fig. 19.31 (*c*)]

When fuse in phase *Y-B* blows out, the phase voltage becomes zero (though voltage across the open remains 200 V). However, the voltage across the other two phases remains the same as under normal operating conditions.

Hence, different phase currents are :

$$I_{RY} \quad \mathbf{15\,A,}\ I_{BR} \quad \mathbf{15\ A,}\ I_{YB} \quad \mathbf{0}$$

The line currents become

$$I_{RR} \quad 15\sqrt{3} \quad \mathbf{26\,A;}\ I_{YY} \quad \mathbf{15\,A,}\ I_{BB} \quad \mathbf{15\,A}$$

Power in phase *RY* = 200 × 15 = **3000 W**, Power/lamp = 3000/50 = **60 W**

Power in phase *RB* = 200 × 15 = **3000 W**, Power/lamp = 3000/50 = **60 W**

Power in phase *YB* = 0; power/lamp = 0.

Example 19.22. *The load connected to a 3-phase supply comprises three similar coils connected in star. The line currents are 25 A and the kVA and kW inputs are 20 and 11 respectively. Find the line and phase voltages, the kVAR input and the resistance and reactance of each coil.*

If the coils are now connected in delta to the same three-phase supply, calculate the line currents and the power taken.

Solution. Star Connection

$\cos\phi$ k = W/kVA = 11/20 I_L = 25 A P = 11 kW = 11,000 W

Now $P = \sqrt{3}\ V_L I_L \cos\phi$ $\therefore$ $11{,}000 = \sqrt{3} \times V_L \times 25 \times 11/20$

$\therefore$ V_L **462 V;** $V_{ph} = 462/\sqrt{3} = \mathbf{267\ V}$

kVAR $\sqrt{\text{kVA}^2 \quad \text{kW}^2}$ $\sqrt{20^2 \quad 11^2}$ **16.7;** Z_{ph} 267/2 10.68

$\therefore$ $R_{ph} = Z_{ph} \times \cos\phi = 10.68 \times 11/20 = \mathbf{5.87\ \Omega}$

$\therefore$ X_{ph} Z_{ph} sin 10.68 0.838 **897Ω**

Delta Connection

$V_{ph} = V_L = 462\ V$ and $Z_{ph} = 10.68\,\Omega$

$\therefore$ I_{ph} 462/10.68 A, I_L $\sqrt{3}$ 462/10.68 75 A

$P = \sqrt{3} \times 462 \times 75 \times 11/20 = \mathbf{33{,}000\ W}$

Example 19.23. *A 3-phase, star-connected system with 230 V between each phase and neutral has resistances of 4, 5 and 6 Ω respectively in the three phases. Estimate the current flowing in each phase and the neutral current. Find the total power absorbed.* **(I.E.E. London)**

Solution. Here, V_{ph} = 230 V [Fig. 19.32 (*a*)]

Current in 4-Ω resistor = 230/4 = 57.5 A

Current in 5-Ω resistor = 230/5 = 46 A

Current in 6-Ω resistor = 230/6 = 38.3 A

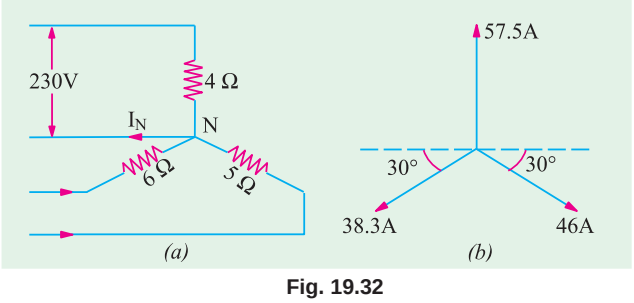


Fig. 19.32

These currents are mutually displaced by 120°. The neutral current I_N is the vector sum* of these three currents. I_N can be obtained by splitting up these three phase currents into their *X*-components and *Y*-components and then by combining them together, in diagram 19.32 (*b*).

X-component = 46 cos 30° – 38.3 cos 30° = **6.64 A**

Y-component = 57.5 – 46 sin 30° – 38.3 sin 30° = 15.3 A $I_N \sqrt{6.64^2 \quad 15.3^2}$ **16.71 A**

The power absorbed = 230 (57.5 + 46 + 38.3) = **32.610 W**

Example 19.24. *A 3-phase, 4-wire system supplies power at 400 V and lighting at 230 V. If the lamps in use require 70, 84 and 33 A in each of the three lines, what should be the current in the neutral wire ? If a 3-phase motor is now started, taking 200 A from the line at a power factor of 0.2, what would be the current in each line and the neutral current ? Find also the total power supplied to the lamps and the motor.* **(Elect. Technology, Aligarh Univ.)**

Solution. The lamp and motor connections are shown in Fig. 19.33.

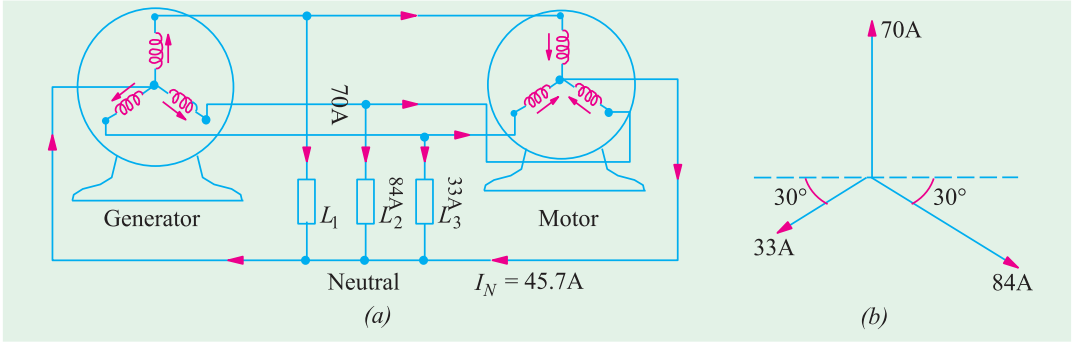


Fig. 19.33

When motor is not started

The neutral current is the vector sum of lamp currents. Again, splitting up the currents into their *X*- and *Y*-components, we get

X-component = 84 cos 30° – 33 cos 30° = 44.2 A

Y-component = 70 – 84 sin 30° – 33 sin 30° = 11.5 A

$$\therefore \quad I_N = \sqrt{44.2^2 + 11.5^2} = \mathbf{45.7\ A}$$

When motor is started

A 3-phase motor is a balanced load. Hence, when it is started, it will change the line currents but being a balanced load, it contributes nothing to the neutral current. Hence, ***the neutral current remains unchanged even after starting the motor.***

Now, the motor takes 200 A from the lines. It means that each line will carry motor current (which lags) as well as lamp current (which is in phase with the voltage). The current in each line would be the vector of sum of these two currents.

Motor p.f. = 0.2 ; sin ϕ = 0.9799 ... from tables

Active component motor current = 200 × 0.2 = 40 A

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* Some writers disagree with this statement on the ground that according to Kirchhoff's Current Law, at any junction, $I_N + I_R + I_Y + I_B = 0 \quad \therefore \quad I_N = -(I_R + I_Y + I_B)$

Hence, according to them, numerical value of I_N is the same but its phase is changed by 180°.

Reactive component of motor current = 200 × 0.9799 = 196 A

(i) Current in first line = $\sqrt{(40+70)^2+196^2}$ = **224.8 A**

(ii) Current in second line = $\sqrt{(40 \quad 84)^2 \quad 196^2}$ **232A**

(iii) Current in third line $= \sqrt{(40+33)^2+196^2} =$ **210.6 A**

Power supplied to lamps = 230 (33 + 84 + 70) = **43,000 W**

Power supplied to motor $= \sqrt{3} \times 200 \times 400 \times 0.2 =$ **27,700 W**

19.11. Star and Delta connected Lighting Loads

In Fig. 19.34 (*a*) is shown a *Y*-connected lighting network in a three storey house. For such a load, it is essential to have neutral wire in order to ensure uniform distribution of load among the three phases despite random switching on and off or burning of lamps. It is seen from Fig. 1934 (*a*),

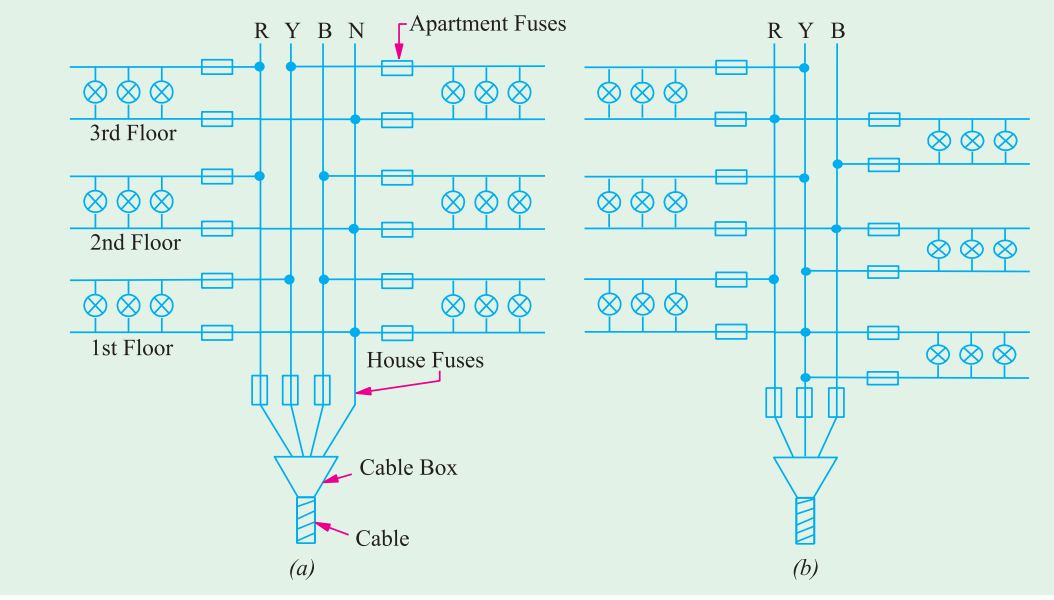


Fig. 19.34

that network supplies two flats on each floor of the three storey residence and there is balanced distribution of lamp load among the three phases. There are house fuses at the cable entry into the building which protect the two mains against short-circuits in the main cable. At the flat entry, there are apartment (or flat) fuses in the single-phase supply which protect the two mains and other flats in the same building from short-circuits in a given building. There is no fuse (or switch) on the neutral wire of the mains because blowing of such a fuse (or disconnection of such a switch) would mean a break in the neutral wire. This would result in unequal voltages across different groups of lamps in case they have different power ratings or number. Consequently, filaments in one group would burn dim whereas in other groups they would burn too bright resulting in their early burn-out.

The house-lighting wire circuit for Δ-connected lamps is shown in Fig. 19.34 (*b*).

19.12. Power Factor Improvement

The heating and lighting loads supplied from 3-phase supply have power factors, ranging

from 0.95 to unity. But motor loads have usually low lagging power factors, ranging from 0.5 to 0.9. Single-phase motors may have as low power factor as 0.4 and electric wedding units have even lower power factors of 0.2 or 0.3.

The power factor is given by $\cos \frac{\text{kW}}{\text{kVA}}$ or $\text{kVA} \frac{\text{kW}}{\cos}$

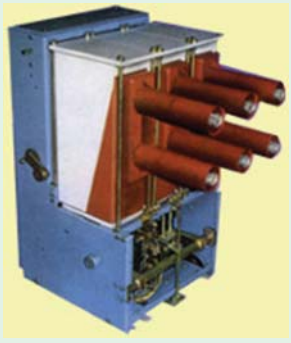
In the case of single-phase supply, $\text{kVA} = \frac{VI}{1000}$ or $I = \frac{1000\ k\text{VA}}{V}$ $\therefore \quad I \propto \text{kVA}$

In the case of 3-phase supply $\text{kVA} = \frac{\sqrt{3} V_L I_L}{1000}$ or $I_L = \frac{1000\ \text{kVA}}{\sqrt{3} \times V_L}$ $\therefore \quad I \propto \text{kVA}$

In each case, the kVA is directly proportional to current. The chief disadvantage of a low p.f. is that the current required for a given power, is very high. This fact leads to the following undesirable results.

(*i*) Large kVA for given amount of power

All electric machinery, like alternators, transformers, switchgears and cables are limited in their current-carrying capacity by the permissible temperature rise, which is proportional to I^2. Hence, they may all be fully loaded with respect to their rated kVA, without delivering their full power. Obviously, it is possible for an existing plant of a given kVA rating to increase its earning capacity (which is proportional to the power supplied in kW) if the overall power factor is improved *i.e.* raised.


Switchgear

(*ii*) Poor voltage regulation

When a load, having allow lagging power factor, is switched on, there is a large voltage drop in the supply voltage because of the increased voltage drop in the supply lines and transformers. This drop in voltage adversely affects the starting torques of motors and necessitates expensive voltage stabilizing equipment for keeping the consumer's voltage fluctuations within the statutory limits. Moreover, due to this excessive drop, heaters take longer time to provide the desired heat energy, fluorescent lights flicker and incandescent lamps are not as bright as they should be. Hence, all supply undertakings try to encourage consumers to have a high power factor.

Example 19.25. *A 50-MVA, 11-kV, 3-ϕ alternator supplies full load at a lagging power factor of 0.7. What would be the percentage increase in earning capacity if the power factor is increased to 0.95 ?*

Solution. The earning capacity is proportional to the power (in MW or kW) supplied by the alternator.

MW supplied at 0.7 lagging = 50 × 0.7 = 35

MW supplied at 0.95 lagging = 50 × 0.95 = 47.5

increase in MW = 12.5

The increase in earning capacity is proportional to 12.5

$\therefore$ Percentage increase in earning capacity = (12.5/35) × 100 = **35.7**

19.13. Power Correction Equipment

The following equipment is generally used for improving or correcting the power factor :

(*i*) Synchronous Motors (or capacitors)

These machines draw leading kVAR when they are over-excited and, especially, when they

are running idle. They are employed for correcting the power factor in bulk and have the special advantage that the amount of correction can be varied by changing their excitation.

(*ii*) Static Capacitors

They are installed to improve the power factor of a group of a.c. motors and are practically loss-free (*i.e.* they draw a current leading in phase by 90°). Since their capacitances are not variable, they tend to over-compensate on light loads, unless arrangements for automatic switching off the capacitor bank are made.

(*iii*) Phase Advancers

They are fitted with individual machines.

However, it may be noted that the economical degree of correction to be applied in each case, depends upon the tariff arrangement between the consumers and the supply authorities.

Example 19.26. *A 3-phase, 37.3 kW, 440-V, 50-Hz induction motor operates on full load with an efficiency of 89% and at a power factor of 0.85 lagging. Calculate the total kVA rating of capacitors required to raise the full-load power factor at 0.95 lagging. What will be the capacitance per phase if the capacitors are* ***(a)*** *delta-connected and* ***(b)*** *star-connected ?*

Solution. It is helpful to approach such problems from the 'power triangle' rather than from vector diagram viewpoint.

Motor power input P = 37.3/0.89 = 41.191 kW

Power Factor 0.85 (lag)

$$\cos\phi_1 = 0.85 : \phi_1 = \cos^{-1}(0.85) = 31.8^\circ ; \tan\phi_1 = \tan 31.8^\circ = 0.62$$

Motor $kVAR_1 = P \tan \phi_1 = 41.91 \times 0.62 = 25.98$

Power Factor 0.95 (lag)

Motor power input P = 41.91 kW ... as before

It is the same as before because capacitors are loss-free *i.e.* they do not absorb any power.

$$\cos\phi_2 = 0.95 \quad \therefore \phi_2 = 18.2^\circ ; \quad \tan 18.2^\circ = 0.3288$$

Motor $kVAR_2 = P \tan \phi_2 = 41.91 \times 0.3288 = 13.79$

The difference in the values of kVAR is due to the capacitors which supply ***leading*** kVAR to partially neutralize the ***lagging*** kVAR of the motor.

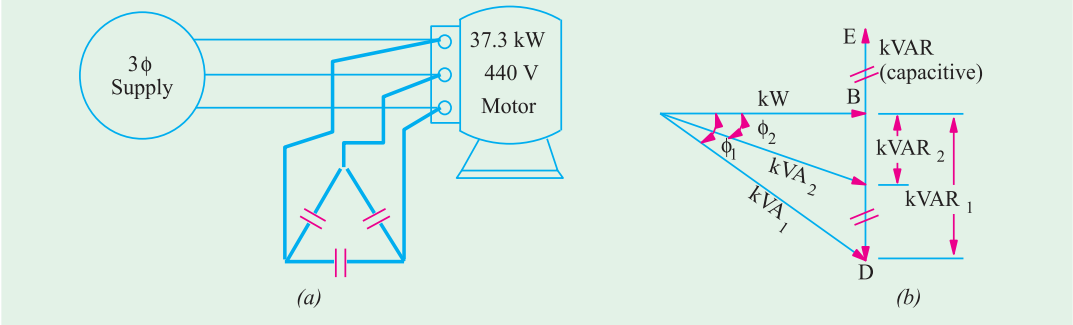


Fig. 19.35

$\therefore$ leading kVAR supplied by capacitors is

= $kVAR_1 - kVAR_2$ = 25.98 – 13.79 = **12.19** ... *CD* in Fing. 19.35 (*b*)

Since capacitors are loss-free, their kVAR is the same as kVA

$\therefore$ kVA/capacitor = 12.19/3 = 4.063 $\therefore$ VAR/capacitor = 4,063

(*a*) In Δ-connection, voltage across each capacitor is 440 V
Current drawn by each capacitor I_C = 4063/440 = 9.23 A

Now, $$I_c = \frac{V}{X_c} = \frac{V}{1/\omega C} = \omega VC$$

$\therefore$ $$C = I_c / \omega V = 9.23 / 2\pi \times 50 \times 440 = 66.8 \times 10^{-6}\ \text{F} = \mathbf{66.8\ \mu F}$$

(*b*) In star connection, voltage across each capacitor is = $440/\sqrt{3}$ volt

Current drawn by each capacitor, I_c $\frac{4063}{440/\sqrt{3}}$ 16.0 A

I_c $\frac{V}{X_c}$ VC or $16 = \frac{440}{\sqrt{3}} \times 2\pi \times 50 \times C$

$\therefore$ C = 200.4 × 10^{-6}F = **200.4 μF**

Note. Star value is three times the delta value.

Example 19.27. *If the motor of Example 19.24 is supplied through a cable of resistance 0.04 Ω per core, calculate*
(i) *the percentage reduction in cable Cu loss and*
(ii) *the additional balanced lighting load which the cable can supply when the capacitors are connected.*

Solution. Original motor $kVA_1 = P/\cos\phi_1 = 41.91/0.85 = 49.3$

Original line current, $$I_{L1} = \frac{kVA_1 \times 1000}{\sqrt{3} \times 440} = \frac{49.3 \times 1000}{\sqrt{3} \times 440} = 64.49\ \text{A}$$

$\therefore$ Original Cu loss/conductor = 64.69^2 × 0.04 = 167.4 W

From Fig . 19.34, it is seen that the new kVA *i.e.*kVA_2 when capacitors are connected is given by $kVA_2 = kW/\cos\phi_2 = 41.91/0.95 = 44.12$

New line current $$I_{L2} = \frac{44{,}120}{\sqrt{3} \times 440} = 57.89\ \text{A}$$

New Cu loss = $57.89^2 \times 0.04 = 134.1$ W

(i) $\therefore$ percentage reduction $= \frac{167.4 - 134.1}{167.4} \times 100 =$ **19.9**

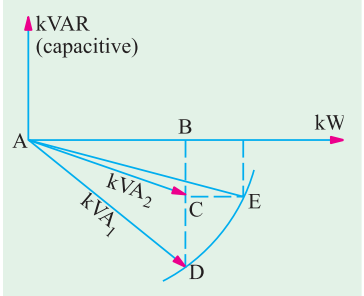


Fig. 19.36

The total kVA which the cable can supply is 49.3 kVA. When the capacitors are connected, the kVA supplied is 44.12 at a power factor of 0.94 lagging. The lighting load will be assumed at unity power factor. The kVA diagram is shown in Fig. 19.34. We will tabulate the different loads as follows. Let the additional lighting load be *x* kW.

Load	*kVA*	cos ϕ	*kW*	sin ϕ	*kVAR*
Motor	49.3	0.85 lag	41.91	0.527	–25.98
Capacitors	12.19	0 lead	0	1.0	+12.19
Lighting	–	1.0	*x*	0	0
			1.91 + *x*)		–13.79

From Fig. 19.36 it is seen that

$AF = 41.91 + x$ and $EF = 13.79$ AE = resultant kVA = 49.3

Also $AF^2 + EF^2 = AE^2$ or $(41.91 + x)^2 + 13.79^2 = 49.3^2$ $\therefore$ x = **5.42 kW**

Example 19.28. *Three impedance coils, each having a resistance of 20 Ω and a reactance of 15 Ω, are connected in star to a 400-V, 3-ϕ, 50-Hz supply. Calculate* ***(i)*** *the line current* ***(ii)*** *power supplied and* ***(iii)*** *the power factor.*

If three capacitors, each of the same capacitance, are connected in delta to the same supply so as to form parallel circuit with the above impedance coils, calculate the capacitance of each capacitor to obtain a resultant power factor of 0.95 lagging.

Solution. V_{ph} 100/√3V, Z_{ph} $\sqrt{20^2}$ 15^2 25

cos $_1$ R_{ph}/Z_{ph} 20/25 0.8lag; $_1$ 0.6 lag

where ϕ_1 is the power factor angle of the coils.

When capacitors are not connected

(i) $I_{ph} = 400/25 \times \sqrt{3} = 9.24$ A $\therefore I_L$ = **9.24 A**

(ii) $P = \sqrt{3}\, V_L I_L \cos\phi_1 = \sqrt{3} \times 400 \times 9.24 \times 0.8$ = **5.120 W**

(iii) Power factor = **0.8 (lag)**

$\therefore$ Motor VAR_1 = $\sqrt{3} V_L I_L \sin\phi_1 = \sqrt{3} \times 400 \times 9.24 \times 0.6 =$ **3,840**

When capacitors are connected

Power factor, $\cos\phi_2$ = 0.95, ϕ_2 = 18.2° ; tan 18.2° = 0.3288

Since capacitors themselves do not absorb any power, power remains the same *i.e.* 5,120 W even whin capacitors are connected. The only thing that changes is the VAR.

Now $VAR_2 = P \tan\phi_2$ = 5120 × 0.3288 = 1684

Leading VAR supplied by the three capacitors is

= $VAR_1 - VAR_2$ = 3840 – 1684 = 2156 BD or CE in Fig 19.37 (*b*)

VAR/ Capacitor = 2156/3 = 719

For delta connection, voltage across each capacitor is 400 V $\therefore$ I_c = 719/400 = 1.798 A

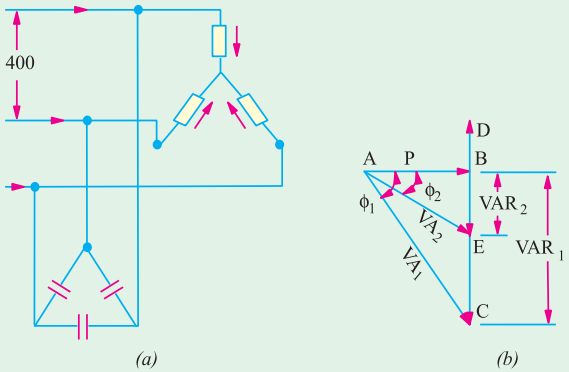


Fig. 19.37

Also $I_c = \dfrac{V}{I/\omega C} = \omega VC \therefore C = 1.798/\pi \times 50 \times 400 = 14.32 \times 10^{-6}$ F = **14.32 μF**

19.14. Parallel Loads

A combination of balanced 3-phase loads connected in parallel may be solved by any one of the following three methods :

1. All the given loads may be converted into equivalent Δ-loads and then combined together according to the law governing parallel circuits.

2. All the given loads may be converted into equivalent *Y*-loads and treated as in **(1)** above.

3. The third method, which requires less work, is to work in terms of volt-amperes. The special advantage of this approach is that voltameters can be added regardless of the kind of connection involved. The real power of various loads can be added arithmetically and VARs may be added algebraically so that total voltamperes are given by

$$VA = \sqrt{W^2 + VAR^2} \quad \text{or} \quad S = \sqrt{P^2 + Q^2}$$

where P is the power in water and Q represents reactive voltamperes.

Example 19.29. *For the power distribution system shown in Fig. 19.38, find*

(a) *total apparent power, power factor and magnitude of the total current I_T without the capacitor in the system*

(b) *the capacitive kVARs that must be supplied by C to raise the power factor of the system to unity ;*

(c) *the capacitance C necessary to achieve the power correction in part **(b)** above*

(d) *total apparent power and supply current I_T after the power factor correction.*

Solution. *(a)* We will take the inductive *i.e.* lagging kVARs as negative and capacitive *i.e.* leading kVARs as positive.

Total $Q = -16 + 6 - 12 = -22$ kVAR (lag); Total $P = 30 + 4 + 36 = 70$ kW

$\therefore$ apparent power $S = \sqrt{(-22)^2 + 70^2}$ = **73.4 kVA;** p.f. = cos ϕ = P/S = 70/73.4 = **0.95**

$$S = VI_T \text{ or } 73.4 \times 10^3 = 400 \times I_T \quad \therefore \; I_T = \mathbf{183.5\ A}$$

(b) Since total lagging kVARs are **– 22**, hence, for making the power factor unity, 22 leading kVARs must be supplied by the capacitor to neutralize them. In that case, total $Q = 0$ and $S = P$ and p.f. is unity.

(c) If I_C is the current drawn by the capacitor, then $22 \times 10^3 = 400 \times I_C$

Now, $I_C = V/X_C = V\omega C$

$= 400 \times 2\pi \times 50 \times C$

$\therefore 20 \quad 10^3 \quad 400 \quad (400 \quad 2 \quad 50 \quad C);$

$\therefore$ C = **483 μF**

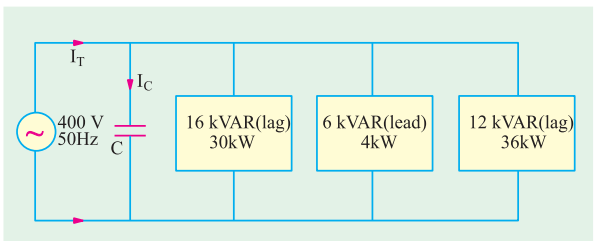


Fig. 19.38

(d) Since $Q = 0$,

hence, $S = \sqrt{10^2 + 70^2}$ = **70 kVA**

Now, $VI_T = 70 \times 10^3$;

$I_T = 70 \times 10^3 / 400$ = 175 A.

It would be seen that after the power correction, lesser amount of current is required to deliver the same amount of real power to the system.

Example 19.30. *A symmetrical 3-phase, 3-wire supply with a line voltage of 173 V supplies two balanced 3-phase loads; one Y-connected with each branch impedance equal to (6 + j8) ohm and the other Δ-connected with each branch impedance equal to (18 + j24) ohm. Calculate*

(i) *the magnitudes of branch currents taken by each 3-phase load*

(ii) *the magnitude of the total line current and*
(iii) *the power factor of the entire load circuit*
Draw the phasor diagram of the voltages and currents for the two loads.

(Elect. Engineering-I, Bombay Univ.)

Solution. The equivalent *Y*-load of the given Δ-load (Art.19.10) is = (18 + *j*24)/3 = (6 + *j*8) Ω.

With this, the problem now reduces to one of solving two equal *Y*-loads connected in parallel across the 3-phase supply as shown in Fig. 19.39 (*a*). Phasor diagram for the combined load for one phase only is given in Fig. 19.39 (*b*).

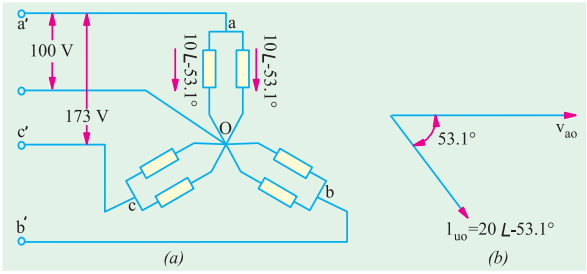


Fig. 19.39

Combined load impedance

$= (6 + j8)/2 = 3 + j4$

$= 5\angle 53.1°$ ohm

$V_{ph} = 173/\sqrt{3} = 100$ V

Let $V_{ph} = 100\angle 0°$

$\therefore \quad I_{ph} = \dfrac{100\angle 0°}{5\angle 53.1°} = 20\angle -53.1°$

Current in each load = $10\angle -53.1$ A

(*i*) branch current taken by each load is **10 A**; (*ii*) line current is **20 A**;

(*iii*) combined power factor = cos 53.1° = **0.6 (lag).**

Example 19.31. *Three identical impedances of* $30\angle 30°$ *ohms are connected in delta to a 3-phase, 3-wire, 208 V volt abc system by conductors which have impedances of (0.8 + j 0.63) ohm. Find the magnitude of the line voltage at the load end.*

(Elect. Engg. Punjab Univ. May 1990)

Solution. The equivalent Z_y, of the given Z_Δ is $30\angle 30/3 = 10\angle 30° = (8.86 + j5)$. Hence, the load connections become as shown in Fig. 19.40.

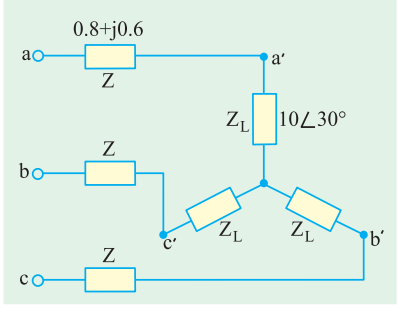


Fig. 19.40

$Z_{an} = (0.8 + j0.6) + (8.86 + j5)$

$= 9.66 + j5.6 = 11.16\angle 30.1°$

$V_{an} = V_{ph} = 208/\sqrt{3} = 120$ V

Let $V_{an} = 120\angle 0°$

$\therefore \quad I_{an} = 120\angle 0°/11.16\angle 30.1° = 10.75\angle -30.1°$

Now, $Z_{aa'} = 0.8 + j0.6 = 1\angle 36.9°$

Voltage drop on line conductors is

$V'_{aa} = I_{an} Z'_{aa} = 10.75\angle -30.1° \times 1\angle 36.9° = 10.75\angle 6.8° = 10.67 + j1.27$

$\therefore \quad V'_{an} = V_{an} - V'_{aa} = (120 + j0) - (10.67 + j1.27) = \mathbf{109.3 \quad 2.03°}$

Example 19.32. *A balanced delta-connected load having an impedance* Z_L *= (300 + j210) ohm in each phase is supplied from 400-V, 3-phase supply through a 3-phase line having an impedance of* Z_s *= (4 + j8) ohm in each phase. Find the total power supplied to the load as well as the current and voltage in each phase of the load.*

(Elect. Circuit Theory, Kerala Univ.)

Solution. The equivalent *Y*-load of the given Δ-load is

$= (300 + j210)/3 = (100 + j70)\Omega$

Hence, connections become as shown in Fig. 19.41

$\mathbf{Z}_{a0}$ (4 $j8$) (100 $j70$) 104 $j78$ 130 36.9

$\boldsymbol{V}_{a0}$ $400/\sqrt{3}$ $231V$,

$\boldsymbol{I}_{a0}$ 231 0 /130 36.9 1.78 36.9

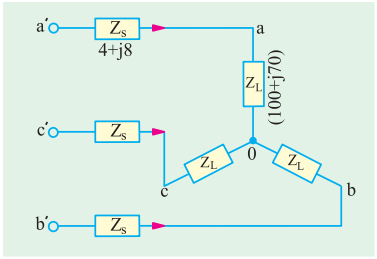


Fig. 19.41

Now, $Z'_{a0} = (4 + j8) = 8.94\angle 63.4°$

Line drop $\mathbf{V}_{aa}$ $\mathbf{I}_{aa}\mathbf{Z}_{aa}$ 1.78 36.9 8.94 63.4 15.9 26.5 14.2 $j7.1$

$\mathbf{V}_{a0}$ $\mathbf{V}_{a0}$ $\mathbf{V}_{aa}$ (231 $j0$) (14.2 $j7.1$)

$= (216.8 - j7.1) = 216.9\angle -1°52'$

Phase voltage at load end, V_{a0} = **216.9 V**

Phase current at load end, I_{a0} = **1.78 A**

Power supplied to load = 3 × 1.782 × 100 = **951 W**

Incidentally, line voltage at load end V_{ac} = 216.9 × $\sqrt{3}$ = **375.7 V***

Example 19.33. *A star connected load having R = 42.6 ohms/ph and X_L = 32 ohms/ph is connected across 400 V, 3 phase supply, calculate:*

***(i)** Line current, reactive power and power loss*

***(ii)** Line current when one of load becomes open circuited.*

[Nagpur University, Summer 2001]

Solution.

(*i*) Z = 42.6 + j32

|Z| = 53.28 ohms, Impedance angle, $\theta = \cos^{-1}\left(\frac{42.6}{53.28}\right) = \cos^{-1} 0.80$

$\theta = 36.9°$

Line Current = phase current, due to star-connection

$= \frac{\text{Voltage/phase}}{\text{Impedance/phase}} \quad \frac{400/\sqrt{3}}{53.28} = 4.336$ amp

Due to the phase angle of 36.9° lagging,

Reactive Power for the three-phases

$= 3\ V_{ph}\ I_{ph} \sin\phi = 3 \times 231 \times 4.336 \times 0.6 = 1803$ VAR

Total Power-loss = $3\ V_{ph}\ I_{ph} \cos\phi = 3 \times 231 \times 4.336 \times 0.8$ = 2404 watts

(*ii*) One of the Loads is open-circuited.

The circuit is shown in Fig. 19.42 (*b*).

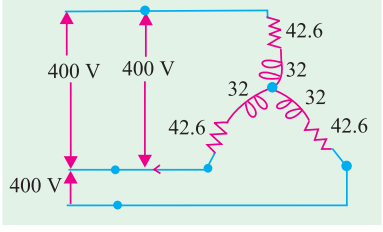


Fig. 19.42 (*a*)

* It should be noted that total line drop is not the numerical sum of the individual line drops because they are 120° out of phase with each other. By a laborious process $\mathbf{V}_{ac} = \mathbf{V}'_{ac} - \mathbf{V}'_{aa} - \mathbf{V}'_{ac}$.

Between A and B, the Line voltage of 400 V drives a current through two "phase-impedances" in series.

Total Impedance between A and B = (42.6 + j 32) × 2 ohms

Hence, the line current I for the two Lines A and B

$$= \frac{400}{2 \times 53.25} = 3.754 \text{ amp}$$

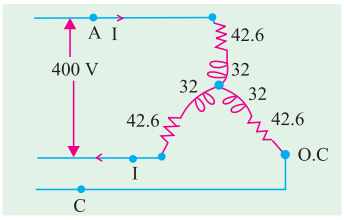


Fig. 19.42 (*b*) One phase open circuited

Note : Third Line 'C' does not carry any current.

Example 19.34. *Three non-inductive resistances, each of 100 ohms, are connected in star to a three-phase, 440-V supply. Three inductive coils, each of reactance 100 ohms connected in delta are also connected to the supply. Calculate: **(i)** Line-currents, and (ii) power factor of the system*

[Nagpur University, November 1998]

Solution. (*a*) Three resistances are connected in star. Each resistance is of 100 ohms and 254 – V appears across it. Hence, a current of 2.54 A flows through the resistors and the concerned power-factor is unity. Due to star-connection,

Line-current = Phase-current = 2.54A

(*b*) Three inductive reactance are delta connected.

Line-Voltage = Phase – Voltage = 440 V

Phase Current = 440/100 = 4.4 A

Line current = 1.732 × 4.4 = 7.62 A

The current has a zero lagging power-factor.

Total Line Current = 2.54 - j 7.62 A

= 8.032 A, in each of the lines.

Power factor = 2.54/8.032 = 0.32 Lag.

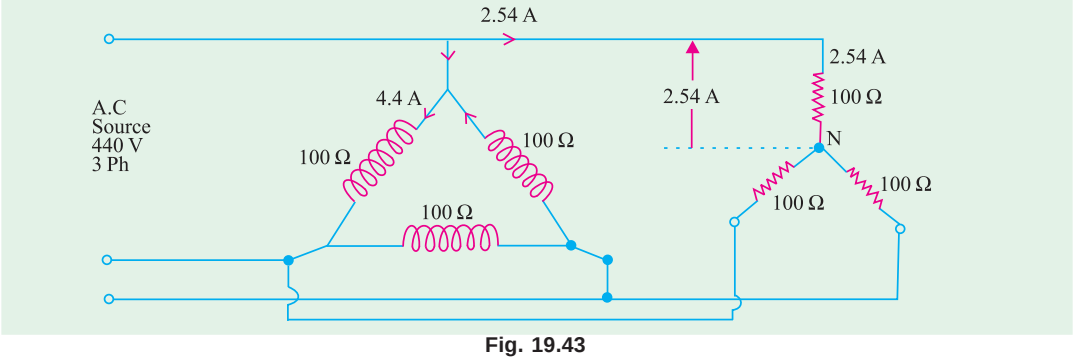


Fig. 19.43

Example 19.35. *The delta-connected generator of Fig 19.44 has the voltage; V_{RY} = 220 /0°, V_{YB}=220 /–120° and V_{BR} = 220 /–240° Volts.*

The load is balanced and delta-connected. Find:

***(a)** Impedance per phase, **(b)** Current per phase, **(c)** Other line – currents I_Y and I_B.*

[Nagpur University, November 1997]

Solution. Draw phasors for voltages as mentioned in the data. V_{RY} naturally becomes a reference-phasor, along which the phasor I_R also must lie, as shown in Fig. 19.44 (*b*) & (*c*). I_R is the line voltage which is related to the phase-currents I_{RY} and – I_{BR}. In terms of magnitudes,

$|I_{RY}| = |I_{BR}| = |I_R|/\sqrt{3} = 10/\sqrt{3} = 5.8$ Amp

Thus, I_{RY} leads V_{RY} by 30°. This can take place only with a series combination of a resistor and a capacitor, as the simplest impedance in each phase

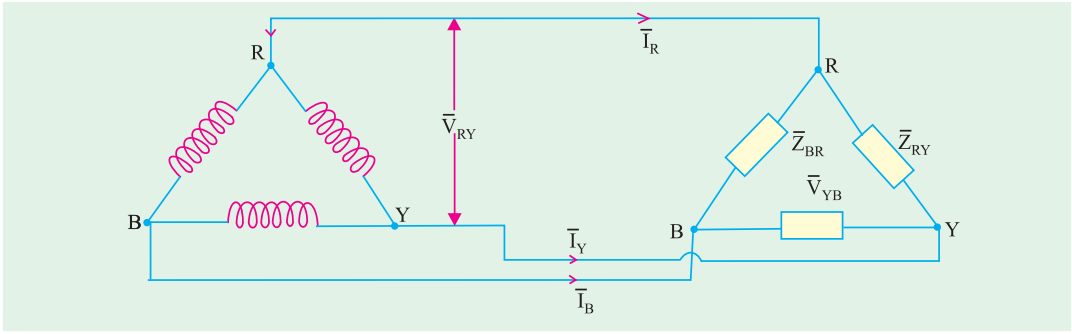


Fig 19.44 (*a*)

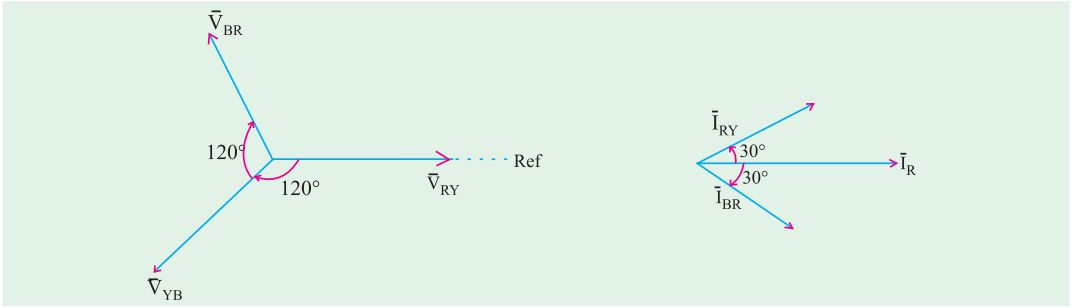


Fig. 19.44 (*b*) Fig. 19.44 (*c*)

(a) |Z| = 220/5.8 = 38.1 ohms

Resistance per phase = 38.1 × cos 30° = 33 ohms

Capacitive Reactance/phase = 38.1 × sin 30° = 19.05 ohms

(b) Current per phase = 5.8 amp, as calculated above.

(c) Otherline currents: Since a symmetrical three phase system is being dealt with, three currents have a mutual phase-difference of 120°. Hence

$I_R = 10 \angle 0°$ as given, $I_Y = 10 \angle -120°$ amp; $I_B = 10 \angle -240°$ amp.

Example 19.36. *A balanced 3-phase star-connected load of 8 + j 6 ohms per phase is connected to a three-phase 230 V supply. Find the line-current, power-factor, active power, reactive-power, and total volt-amperes.*

[Rajiv Gandhi Technical University, Bhopal, April 2001]

Solution. When a statement is made about three–phase voltage, when not mentioned otherwise, the voltage is the line-to-line voltage. Thus, 230 V is the line voltage, which means, in star-system, phase-voltage is 230/1.732, which comes to 132.8 V.

$|Z| = \sqrt{8^2 + 6^2} = 10$ ohms

Line current = Phase current

= 132.8/10 = 13.28 amp

Power – factor = R/Z = 0.8, Lagging

Total Active Power = P = 1.732 × Line Voltage × Line Current × P.f.

Or = 3. Phase Voltage × Phase-current × P.f

= 3 × 132.8 × 13.28 × 0.8 = 4232 watts

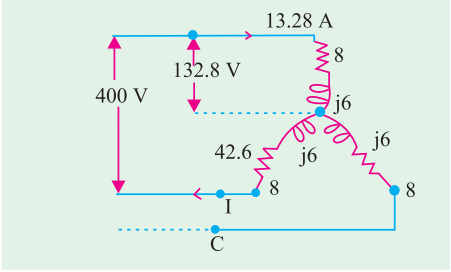


Fig. 19.45

Total Reactive Power = Q

= 3 × Phase-voltage × Phase-current × sin ϕ

= 3 × 132.8 × 13.28 × 0.60 = 3174 VAR

Total Volt-amps = S = $\sqrt{P^2+Q^2}$ = 5290 VA

Or S = $\sqrt{3}\times 230\times 13.28$ = 5290 VA

Example 19.37. *A balanced three-phase star connected load of 100 kW takes a leading current of 80 amp, when connected across a three-phase 1100 V, 50 Hz, supply. Find the circuit constants of the load per phase.* **[Nagpur University, April 1996]**

Solution. Voltage per phase = 1100/1.732 = 635 V

Impedance = 635/80 = 7.94 ohms.

Due to the leading current, a capacitor exists.

Resistance R can be evaluated from current and power consumed

3 I^2 R = 100 × 1000, giving R = 5.21 ohms

$X_c = (7.94^2 - 5.21^2)^{0.5}$ = 6 ohms

At 50 Hz, C = 1/(314 × 6) = 531 microfarads.

Tutorial Problem No. 19.1

1. Each phase of a delta-connected load comprises a resistor of 50 Ω and capacitor of 50 μF in series. Calculate (*a*) the line and phase currents (*b*) the total power and (*c*) the kilovoltamperes when the load is connected to a 440-V, 3-phase, 50-Hz supply. **[(*a*) 9.46 A; 5.46 A (*b*) 4480 W (*c*) 7.24 kVA]**

2. Three similar-coils, *A*, *B* and *C* are available. Each coil has 9 Ω resistance and 12 Ω reactance. They are connected in delta to a 3-phase, 440-V, 50-Hz supply. Calculate for this load:

(*a*) the line current (*b*) the power factor

(*c*) the total kilovolt-amperes (*d*) the total kilowatts

If the coils are reconnected in star, calculate for the new load the quantities named at (*a*), (*b*); (*c*) and (*d*) above. **[50.7 A; 0.6; 38.6 kVA; 23.16 kW; 16.9 A; 0.6; 12.867 kVA; 7.72 kW]**

3. Three similar choke coils are connected in star to a 3-phase supply. If the line currents are 15 A, the total power consumed is 11 kW and the volt-ampere input is 15 kVA, find the line and phase voltages, the VAR input and the reactance and resistance of each coil.

[577.3 V; 333.3 V; 10.2 kVAR; 15.1 Ω; 16.3 Ω]

4. The load in each branch of a delta-connected balanced 3-ϕ circuit consists of an inductance of 0.0318 H in series with a resistance of 10 Ω. The line voltage is 400 V at 50 Hz. Calculate (*i*) the line current and (*ii*) the total power in the circuit. **[(*i*) 49 A (*ii*) 24 kW] *(London Univ.)***

5. A 3-phase, delta-connected load, each phase of which has R = 10 Ω and X = 8 Ω, is supplied from a star-connected secondary winding of a 3-phase transformer each phase of which gives 230 V. Calculate

(*a*) the current in each phase of the load and in the secondary windings of the transformer

(*b*) the total power taken by the load

(*c*) the power factor of the load. **[(*a*) 31.1 A; 54 A (*b*) 29 kW (*c*) 0.78]**

6. A 3-phase load consists of three similar inductive coils, each of resistance 50 Ω and inductance 0.3 H. The supply is 415 V, 50 Hz, Calculate (*a*) the line current (*b*) the power factor and (*c*) the total power when the load is (*i*) star-connected and (*ii*) delta-connected.

[(*i*) 2.25 A, 0.47 lag, 762 W (*ii*) 6.75 A, 0.47 lag, 2280 W] (*London Univ.*)

7. Three 20 Ω non-inductive resistors are connected in star across a three phase supply the line voltage of which is 480 V. Three other equal non-inductive resistors are connected in delta across the same supply so as to take the same-line current. What are the resistance values of these other resistors and what is the current- flowing through each of them? **[60 Ω; 8A] (*Sheffield Univ. U.K.*)**

8. A 415-V, 3-phase, 4-wire system supplies power to three non-inductive loads. The loads are 25 kW between red and neutral, 30 kW between yellow and neutral and 12 kW between blue and neutral.

Calculate (*a*) the current in each-line wire and (*b*) the current in the neutral conductor.

[(*a*) 104.2 A, 125 A, 50 A (*b*) 67 A] (*London Univ.*)

9. Non-inductive loads of 10, 6 and 4 kW are connected between the neutral and the red, yellow and blue phases respectively of a three-phase, four-wire system. The line voltage is 400 V. Find the current in each line conductor and in the neutral. **[(*a*) 43.3 A, 26A, 173. A, 22.9] (*App. Elect. London Univ.*)**

10. A three-phase, star-connected alternator supplies a delta-connected load, each phase of which has a resistance of 20 Ω and a reactance of 10 Ω Calculate (*a*) the current supplied by the alternator (*b*) the output of the alternator in kW and kVA, neglecting the losses in the lines between the alternator and the load. The line voltage is 400 V. **[(*a*) 30.95 A (b) 19.2 kW, 21.45 kVA]**

11. Three non-inductive resistances, each of 100 Ω, are connected in star to 3-phase, 440-V supply. Three equal choking coils each of reactance 100 Ω are also connected in delta to the same supply. Calculate:

(*a*) line current (*b*) p.f. of the system. **[(*a*) 8.04 A (*b*) 0.3156] (*I.E.E. London*)**

12. In a 3-phase, 4-wire system, there is a balanced 3-phase motor load taking 200 kW at a power factor of 0.8 lagging, while lamps connected between phase conductors and the neutral take 50, 70 and 100 kW respectively. The voltage between phase conductors is 430 V. Calculate the current in each phase and in the neutral wire of the feeder supplying the load.

[512 A, 5.87 A, 699 A; 213.3 A] (*Elect. Power, London Univ.*)

13. A 440-V, 50-Hz induction motor takes a line current of 45 A at a power factor of 0.8 (lagging). Three Δ-connected capacitors are installed to improve the power factor to 0.95 (lagging). Calculate the kVA of the capacitor bank and the capacitance of each capacitor. **[11.45 kVA, 62.7 μF] (*I.E.E. London*)**

14. Three resistances, each of 500 Ω, are connected in star to a 400-V, 50-Hz, 3-phase supply. If three capacitors, when connected in delta to the same supply, take the same line currents, calculate the capacitance of each capacitor and the line current. **[2.123 μ F, 0.653 A] (*London Univ.*)**

15. A factory takes the following balanced loads from a 440-V, 3-phase, 50-Hz supply:

(*a*) a lighting load of 20 kW (*b*) a continuous motor load of 30 kVA at 0.5 p.f. lagging.

(*c*) an intermittent welding load of 30 kVA at 0.5 p.f. lagging.

Calculate the kVA rating of the capacitor bank required to improve the power factor of loads (*a*) and (*b*) together to unity. Give also the value of capacitor required in each phase if a star-connected bank is employed.

What is the new overall p.f. if, after correction has been applied, the welding load is switched on.

[30 kVAR; 490μ F; 0.945 kg]

16. A three-wire, three-phase system, with 400 V between the line wires, supplies a balanced delta-connected load taking a total power of 30 kW at 0.8 power factor lagging. Calculate (*i*) the resistance and (*ii*) the reactance of each branch of the load and sketch a vector diagram showing the line voltages and line currents. If the power factor of the system is to be raised to 0.95 lagging by means of three delta-connected capacitors, calculate (*iii*) the capacitance of each branch assuming the supply frequency to be 50 Hz.

[(*i*) 10.24 A (*ii*) 7.68 Ω (*iii*) 83.2 μF] (*London Univ.*)

19.15. Power Measurement in 3-phase Circuits

Following methods are available for measuring power in a 3-phase load.

(*a*) Three Wattmeter Method

In this method, three wattmeters are inserted one in each phase and the algebraic sum of their readings gives the total power consumed by the 3-phase load.

(*b*) Two Wattmeter Method

(*i*) This method gives true power in the 3-phase circuit without regard to balance or wave form provided in the case of *Y*-connected load. The neutral of the load is isolated from the neutral of the source of power. Or if there is a neutral connection, the neutral wire should not carry any current. This is possible only if the load is perfectly balanced and there are no harmonics present of triple frequency or any other multiples of that frequency.

(ii) This method can also be used for 3-phase, 4-wire system in which the neutral wire carries the neutral current. In this method, the current coils of the wattmeters are supplied from current transformers inserted in the principal line wires in order to get the correct magnitude and phase differences of the currents in the current coils of the wattmeter, because in the 3-phase, 4-wire system, the sum of the instantaneous currents in the principal line wires is not necessarily equal to zero as in 3-phase 3-wire system.

(*c*) One Wattmeter Method

In this method, a single wattmeter is used to obtain the two readings which are obtained by two wattmeters by the two-wattmeter method. This method can, however, be used only when the load is balanced.

19.16. Three Wattmeter Method

A wattmeter consists of ***(i)*** a low resistance current coil which is inserted in series with the line carrying the current and ***(ii)*** a high resistance pressure coil which is connected across the two points whose potential difference is to be measured.

A wattmeter shows a reading which is proportional to the product of the current through its current coil, the p.d. across its potential or pressure coil and cosine of the angle between this voltage and current.

As shown in Fig. 19.46 in this method three wattmeters are inserted in each of the three phases of the load whether Δ-connected or *Y*-connected. The current coil of each wattmeter carries the current of one phase only and the pressure coil measures the phase-voltage of this phase. Hence, each wattmeter measures the power in a single phase. The algebraic sum of the readings of three wattmeters must give the total power in the load.

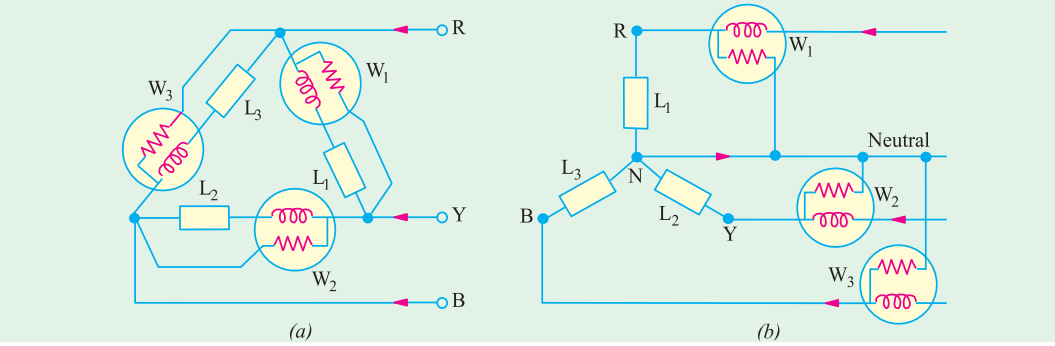


Fig. 19.46

The difficulty with this method is that under ordinary conditions it is not generally feasible to break into the phases of a delta-connected load nor is it always possible, in the case of a *Y*-connected load, to get at the neutral point which is required for connections as shown in Fig. 19.47 (*b*). However, it is not necessary to use three wattmeters to measure power, two wattmeters can be used for the purpose as shown below.

19.17. Two Wattmeter Method-Balanced or Unbalanced Load

As shown in Fig. 19.41, the current coils of the two wattmeters are inserted in ***any two*** lines and the potential coil of each joined to the third line. It can be proved that the sum of the instantaneous powers indicated by W_1 and W_2 gives the instantaneous power absorbed by the three loads L_1, L_2 and L_3. A star-connected load is considered in the following discussion although it can be equally applied to Δ-connected loads because a Δ-connected load can always be replaced by an equivalent *Y*-connected load.

Now, before we consider the currents through and p.d. across each wattmeter, it may be pointed out that ***it is important to take the direction of the voltage through the circuit the same as that taken for the current when establishing the readings of the two wattmeters.***

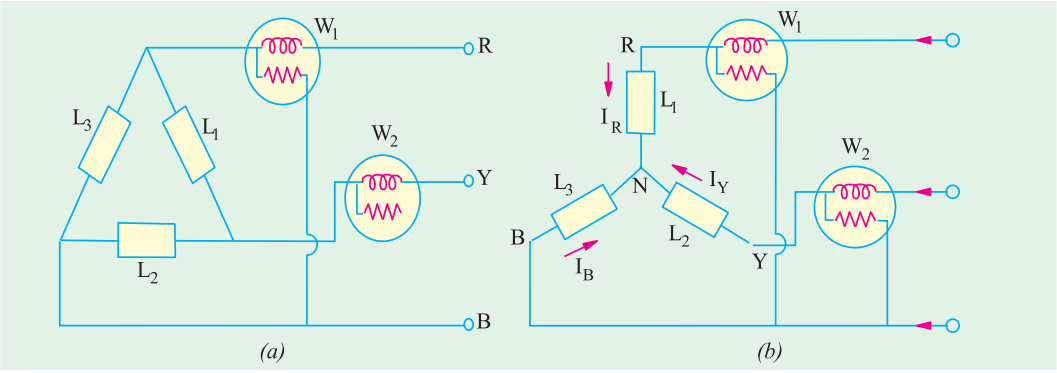


Fig. 19.47

Instantaneous current through $W_1 = i_R$

p.d. across $W_1 = e_{RB} = e_R - e_B$

p.d. across power read by $W_1 = i_R\,(e_R - e_B)$

Instantaneous current through $W_2 = i_Y$

Instantaneous p.d. across $W_2 = e_{YB} = (e_Y - e_B)$

Instantaneous power read by $W_2 = i_Y\,(e_Y - e_B)$

$$\therefore\ W_1 + W_2 = e_R(e_R - e_B) + i_Y(e_Y - e_B) = i_R e_R + i_Y e_Y - e_B(i_R + i_Y)$$

Now, $i_R + i_Y + i_B = 0$ ... Kirchhoff's Current Law

$$\therefore\ i_R + i_Y = -i_B$$

or $W_1 + W_2 = i_R.e_R + i_Y.e_y + i_B.e_B = p_1 + p_2 + p_3$

where p_1 is the power absorbed by load L_1, p_2 that absorbed by L_2 and p_3 that absorbed by L_3

$\therefore\ W_1 + W_2$ = total power absorbed

The proof is true whether the load is balanced or unbalanced. If the load is *Y*-connected, it should have no neutral connection (*i.e.* 3 – ϕ , 3-wire connected) and if it has a neutral connection (*i.e.* 3–ϕ , 4-wire connected) then it should be exactly balanced so that in each case there is no neutral current i_N otherwise Kirchoff's current law will give $i_N + i_R + i_y + i_B = 0$.

We have considered ***instantaneous*** readings, but in fact, the moving system of the wattmeter, due to its inertia, cannot quickly follow the variations taking place in a cycle, hence it indicates the ***average*** power.

$$\therefore\quad W_1 \quad W_2 \quad \frac{1}{T}\int_0^T i_R e_{RB}\,dt \quad \frac{1}{T}\int_0^T i_Y e_{YB}\,dt$$

19.18. Two Wattmeter Method– *Balanced Load*

If the load is balanced, then power factor of the load can also be found from the two wattmeter readings. The *Y*-connected load in Fig. 19.47 (*b*) will be assumed inductive. The vector diagram for such a balanced *Y*-connected load is shown in Fig. 19.48. We will now consider the problem in terms of r.m.s. values instead of instantaneous values.

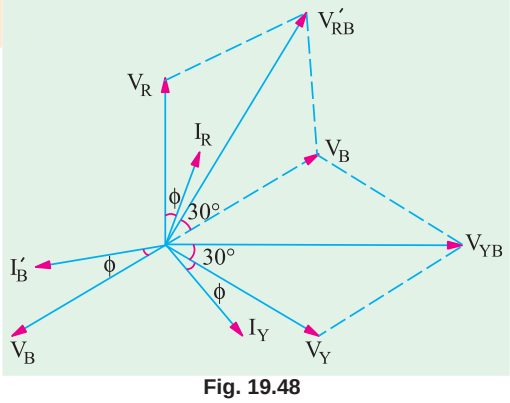


Fig. 19.48

Let V_R, V_Y and V_B be the r.m.s. values of the three phase voltages and I_R, I_Y and I_B the r.m.s. values of the currents. Since these voltages and currents are assumed sinusoidal, they can be represented by vectors, the currents lagging behind their respective phase voltages by ϕ.

Current through wattmeter W_1 [Fig. 19.47 (*b*)] is = I_R.

P.D. across voltage coil of W_1 is

$$V_{RB} = V_R - V_B \quad \text{... vectorially}$$

This V_{RB} is foundby compounding V_R and V_B reversed as shown in Fig. 19.42. It is seen that phase difference between V_{RB} and I_R = (30° – ϕ).

$\therefore$ Reading of $W_1 = I_R\, V_{RB} \cos (30° - \phi)$

Similarly, as seen from Fig. 19.47 (*b*). Current through $W_2 = I_Y$

P.D. across $W_2 = V_{YB} = V_Y - V_B$ ... vectorially

Again, V_{YB} is found by compounding V_Y and V_B reversed as shown in Fig. 19.48. The angle between I_Y and V_{YB} is (30° + ϕ). Reading of $W_2 = I_Y V_{YB} \cos (30° + \phi)$

Since load is balanced, $V_{RB} = V_{YB}$ = line voltage V_L; $I_Y = I_R$ = line current, I_L

$$\therefore \quad W_1 = V_L I_L \cos(30° - \Phi) \text{ and } W_2 = V_L I_L \cos(30° + \phi)$$

$$\therefore \quad W_1 + W_2 = V_L I_L \cos(30° - \phi) + V_L I_L (\cos(30° + \phi)$$

$$= V_L I_L [\cos 30° \cos\phi + \sin 30° \sin\phi + \cos 30° \cos\phi - \sin 30° \sin\phi]$$

$$= V_L I_L (2 \cos 30° \cos\phi) = \sqrt{3} V_L I_L \cos\phi$$ = total power in the 3-phase load

Hence, the sum of the two wattmeter readings gives the total power consumption in the 3-phase load.

It should be noted that phase sequence of *RYB* has been assumed in the above discussion. Reversal of phase sequence will interchange the readings of the two wattmeters.

19.19. Variations in Wattmeter Readings

It has been shown above that for a lagging power factor

$$W_1 = V_L I_L \cos(30° - \varphi) \text{ and } W_2 = V_L I_L \cos(30° + \phi)$$

From this it is clear that individual readings of the wattmeters not only depend ***on the load but upon its power factor also.*** We will consider the following cases:

(a) When ϕ = 0 *i.e.* power factor is unity (*i.e.* resistive load) then,

$$W_1 = W_2 = V_L I_L \cos 30°$$

Both wattmeters indicate equal and positive *i.e.* up-scale readings.

(b) When ϕ = 60° *i.e.* power factor = 0.5 (lagging)

Then $W_2 = V_L I_L \cos (30° + 60°) = 0$. Hence, the power is measured by W_1 alone.

(c) When 90° > ϕ > 60° *i.e.* 0.5 > p.f. > 0, then W_1 is still positive but reading of W_2 is reversed because the phase angle between the current and voltage is more than 90°. For getting the total power, the reading of W_2 is to be subtracted from that of W_1.

Under this condition, W_2 will read 'down scale' *i.e.* backwards. Hence, to obtain a reading on W_2 it is necessary

ϕ	0°	60°	90°
$\cos\phi$	1	0.5	0
W_1	+ve	+ve	+ve
W_2	+ve $W_1 = W_2$	0	–ve $W_1 = W_2$

to reverse either its pressure coil or current coil, usually the

All readings taken after reversal of pressure coil are to be taken as negative.

(d) When $\phi = 90°$ (*i.e.* pure inductive or capacitive load), then

$W_1 = V_L I_L \cos(30°-90°) = V_L I_L \sin 30°;$

$W_2 = V_L I_L \cos(30°+90°) = -V_L I_L \sin 30°$

As seen, the two readings are equal but of opposite sign.

$\therefore \quad W_1 + W_2 = 0$

The above facts have been summarised in the above table for a lagging power factor.

19.20. Leading Power Factor*

In the above discussion, lagging angles are taken positive. Now, we will see how wattmeter readings are changed if the power factor becomes leading. For ϕ = + 60° (lag), W_2 is zero. But for ϕ = – 60° (lead), W_1 is zero. So we find that for angles of lead, the reading of the two wattmeters are interchanged. Hence, for a ***leading*** power factor.

$W_1 = V_L I_L \cos(30°+\phi)$ and $W_2 = V_L I_L \cos(30°-\phi)$

19.21. Power Factor–*Balanced Load*

In case the load is balanced (and currents and voltages are sinusoidal) and for a *lagging* power factor:

$$W_1 + W_2 = V_L I_L \cos(30°-\phi) + V_L I_L \cos(30°+\phi) = \sqrt{3} V_L I_L \cos\phi \quad \text{... (i)}$$

Similarly $W_1 - W_2 = V_L I_L \cos(30°-\phi) - V_L I_L \cos(30°+\phi)$

$$= V_L I_L (2 \times \sin\phi \times 1/2) = V_L I_L \sin\phi \quad \text{... (ii)}$$

Dividing ***(ii)*** by ***(i)***, we have $\tan\phi = \dfrac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}$ ** ... ***(iii)***

Knowing $\tan\phi$ and hence ϕ, the value of power factor $\cos\phi$ can be found by consulting the trigonometrical tables. It should, however, be kept in mind that if W_2 reading has been taken after reversing the pressure coil *i.e.* if W_2 is negative, then the above relation becomes

$$\tan\phi = -\sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right) \quad \text{... Art 19.22}$$

$$\tan\phi = \sqrt{3}\frac{W_1 - (-W_2)}{W_1 + (-W_2)} = \sqrt{3}\frac{W_1 + W_2}{W_1 - W_2}$$

Obviously, in this expression, only *numerical* values of W_1 and W_2 should be substituted. We may express power factor in terms of the ratio of the two wattmeters as under:

Let $\dfrac{\text{smaller reading}}{\text{larger reading}} = \dfrac{W_2}{W_1} = r$

---

* For a leading p.f., conditions are just the opposite of this. In that case, W_1 reads negative (Art. 19.22).

** For a leading power factor, this expression becomes

$$\tan\phi = -\sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right) \quad \text{... Art 19.22}$$

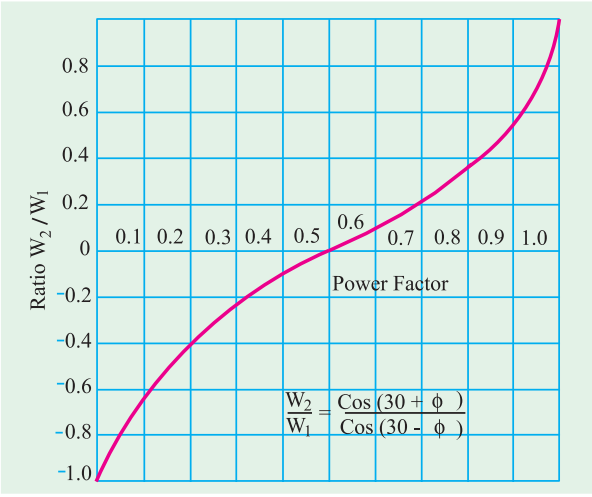


Fig. 19.49

Then from equation ***(iii)*** above,

$$\tan\phi = \frac{\sqrt{3}[1-(W_2/W_1)]}{1+(W_2/W_1)} = \frac{\sqrt{3}(1-r)}{1+r}$$

Now $\sec^2\phi = 1+\tan^2\phi$

or $\dfrac{1}{\cos^2\phi} = 1+\tan^2\phi$

$$\therefore \quad \cos\phi = \frac{1}{\sqrt{1+\tan^2\phi}}$$

$$= \frac{1}{\sqrt{1+3\left(\dfrac{1-r}{1+r}\right)^2}}$$

$$= \frac{1+r}{2\sqrt{1-r+r^2}}$$

If r is plotted against cos ϕ, then a curve called watt-ratio curve is obtained as shown in Fig. 19.49.

19.22. Balanced Load – *leading power factor*

In this case, as seen from Fig. 19.50

$$W_1 = V_L I_L \cos(30+\phi)$$

and $\quad W_2 = V_L I_L \cos(30-\phi)$

$$\therefore \quad W_1 + W_2 = \sqrt{3} V_L I_L \cos\phi \text{ – as found above}$$

$$W_1 - W_2 = -V_L I_L \sin\phi$$

$$\therefore \quad \tan\phi = -\frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}$$

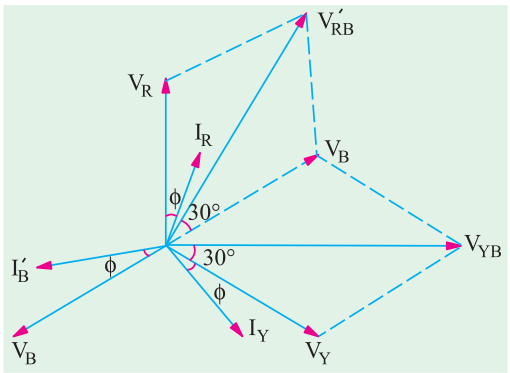


Fig. 19.50

Obviously, if ϕ > 60°, then phase angle between V_{RB} and I_R becomes more than 90°. Hence, W_1 reads 'down-scale' *i.e.* it indicates negative reading. However, W_2 gives positive reading even in the extreme case when ϕ = 90°.

19.23. Reactive Voltamperes with Two Wattmeters

We have seen that $\tan\phi = \dfrac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}$

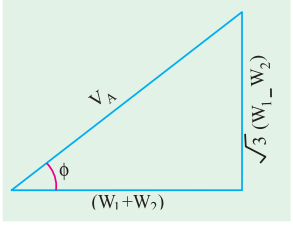


Fig. 19.51

Since the tangent of the angle of lag between phase current and phase voltage of a circuit is always equal to the ratio of the reactive power to the active power (in watts), it is clear that $\sqrt{3}(W_1 - W_2)$ represents the reactive power (Fig. 19.51). Hence, for a balanced load, the reactive power is given by $\sqrt{3}$ times the difference of the readings of the two wattmeters used to measure the power for a 3-phase circuit by the two wattmeter method. It may also be proved mathematically a follows:

$$= \sqrt{3}(W_1 - W_2) = \sqrt{3}[V_L I_L \cos(30° - \phi) - V_L I_L \cos(30° - \phi)]$$

$$= \sqrt{3} V_L I_L (\cos 30° \cos\phi + \sin 30° \sin\phi - \cos 30° \cos\phi + \sin 30° \sin\phi)$$

$$= \sqrt{3} V_L I_L \sin\phi$$

19.24. Reactive Voltamperes with One Wattmeter

For this purpose, the wattmeter is connected as shown in Fig. 19.52 (*a*) and (*b*). The pressure coil is connected across *Y* and *B* lines whereas the current coil is included in the *R* line. In Fig. 19.48 (*a*), the current coil is connected between terminals *A* and *B* whereas pressure coil is connected between terminals *C* and *D*. Obviously, current flowing through the wattmeter is I_R and p.d. is V_{YB}. The angle between the two, as seen from vector diagram of Fig. 19.48, is (30 + 30 + 30 – ϕ) = (90 – ϕ)

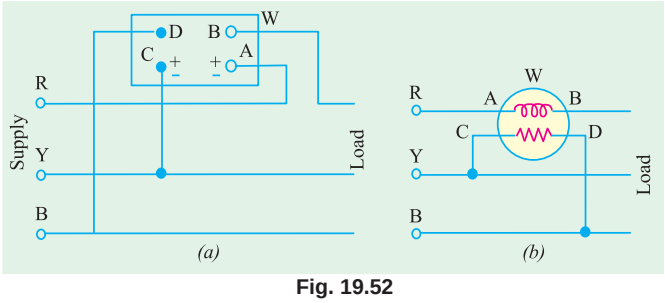


Fig. 19.52

Hence, reading of the wattmeter is $W = V_{YB} I_R \cos(90 - \phi) = V_{YB} I_R \sin\phi$

For a balanced load, V_{YB} equals the line voltage V_L and I_R equals the line current I_L, hence

$$W = V_L I_L \sin\phi$$

We know that the total reactive voltamperes of the load are $Q = \sqrt{3} V_L I_L \sin\phi$.

Hence, to obtain total *VARs*, the wattmeter reading must be multiplied by a factor of $\sqrt{3}$.

19.25. One Wattmeter Method

In this case, it is possible to apply two-wattmeter method by means of one wattmeter without breaking the circuit. The current coil is connected in any one line and the pressure coil is connected alternately between this and the other two lines (Fig. 19.53). The two readings so obtained, for a balanced load, correspond to those obtained by normal two wattmeter method. It should be kept in mind that this method is not of as much universal application as the two wattmeter method because it is restricted to fairly balanced loads only. However, it may be conveniently applied, for instance, when it is desired to find the power input to a factory motor in order to check the load upon the motor.

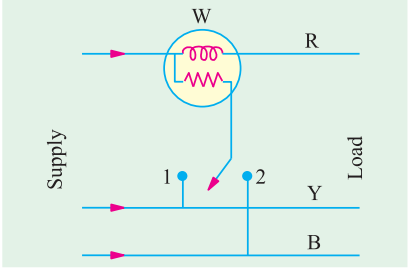


Fig. 19.53

It may be pointed out here that the two wattmeters used in the two-wattmeter method (Art. 19.17) are usually combined into a single instrument in the case of switchboard wattmeter which is then known as a polyphase wattmeter. The combination is affected by arranging the two sets of coils in such a way as to operate on a single moving system resulting in an indication of the total power on the scale.

19.26. Copper Required for Transmitting Power under Fixed Conditions

The comparison between 3-phase and single-phase systems will be done on the basis of a fixed amount of power transmitted to a fixed distance with the same amount of loss and at the same maximum voltage between conductors. In both cases, the weight of copper will be directly

proportional to the number of wires (since the distance is fixed) and inversely proportional to the resistance of each wire. We will assume the same power factor and same voltage.

$$P_1 = VI_1 \cos\phi \text{ and } P_3 = \sqrt{3}VI_3 \cos\phi$$

where I_1 = r.m.s. value of current in 1-phase system
I_3 = r.m.s. value of line current in 3-phase system

$$P_1 = P_2 \quad \therefore \quad VI_1 \cos\phi = \sqrt{3}VI_3 \cos\phi \quad \therefore \quad I_1 = \sqrt{3}I_3$$

also $I_1^2 R_1 \times 2 = I_2^3 R_3 \times 3$ or $\dfrac{R_1}{R_3} = \dfrac{3I_3^2}{2I_1^2}$

Substituting the value of I_1, we get $\dfrac{R_1}{R_3} = \dfrac{3I_3^2}{3I_3^2 \times 2} = \dfrac{1}{2}$

$$\therefore \quad \frac{\text{copper } 3-\text{phase}}{\text{copper } 1-\text{phase}} = \frac{\text{No. of wires } 3-\text{phase}}{\text{No. of wires } 1-\text{phase}} \times \frac{R_1}{R_3} = \frac{3}{2} \times \frac{1}{2} = \frac{3}{4}$$

Hence, we find that for transmitting the same amount of power over a fixed distance with a fixed line loss, we need only three-fourths of the amount of copper that would be required for a single phase or to put it in another way, one-third more copper is required for a 1-phase system than would be necessary for a three-phase system.

Example 19.38. *Phase voltage and current of a star-connected inductive load is 150 V and 25 A. Power factor of load is 0.707 (lag). Assuming that the system is 3-wire and power is measured using two wattmeters, find the readings of wattmeters.*

(Elect. Instrument & Measurements, Nagpur Univ. 1993)

Solution. V_{ph} 150V; V_L 150 $\sqrt{3}$V; I_{ph} I_L = 25 A

Total power = $\sqrt{3} V_L I_L \cos\phi = \sqrt{3} \times 150 \times \sqrt{3} \times 25 \times 0.707$ = 7954 W

$\therefore$ $W_1 + W_2$ = 7954 W ... ***(i)***

$\cos\phi = 0.707; \phi = \cos^{-1}(0.707) = 45°; \tan 45° = 1$

Now, for a lagging power factor, $\tan\phi = \sqrt{3}(W_1 - W_2)/(W_1 + W_2)$ or $1 = \sqrt{3}(W_1 - W_2)/7954$

$\therefore$ $(W_1 - W_2)$ = 4592 W ... ***(ii)***

From ***(i)*** and ***(ii)*** above, we get, W_1 = **6273 W; W_2 = 1681 W.**

Example 19.39. *In a balanced 3-phase 400-V circuit, the line current is 115.5 A. When power is measured by two wattmeter method, one meter reads 40 kW and the other zero. What is the power factor of the load? If the power factor were unity and the line current the same, what would be the reading of each wattmeter?*

Solution. Since W_2 = 0, the whole power is measured by W_1. As per Art. 19.18, in such a situation, p.f. = 0.5. However, it can be calculated as under.

Since total power is 40 kW, $\therefore$ 40,000 = $\sqrt{3}$ 400 115.5 cos ; cos 0.5

If the power factor is unity with line currents remaining the same, we have

$$\tan\phi = \frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)} = 0 \text{ or } W_1 = W_2$$

Also, $(W_1 + W_2) = \sqrt{3} \times 400 \times 115.5 \times 1$ = 80000 W = 80 kW

As per Art. 19.19, at unity p.f., $W_1 = W_2$. Hence, each wattmeter reads = 80/2 = **40 kW.**

Example 19.40. *The input power to a three-phase motor was measured by two wattmeter method. The readings were 10.4 KW and – 3.4 KW and the voltage was 400 V. Calculate **(a)** the power factor **(b)** the line current.* **(Elect. Engg. A.M.Ae, S.I. June 1991)**

Solution. As given in Art. 19.21, when W_2 reads negative, then we have

$\tan\phi = \sqrt{3}(W_1 + W_2)/(W_1 - W_2)$. Substituting numerical values of W_1 and W_2, we get

$\tan\phi = \sqrt{3}(10.4+3.4)/(10.4-3.4) = 1.97; \phi = \tan^{-1}(1.97) = 63.1°$

(a) p.f. = $\cos\phi = \cos 63.1°$ = **0.45 (lag)**

(b) W = 10.4 – 3.4 = 7 KW = 7,000 W

$7000 = \sqrt{3} I_L \times 400 \times 0.45; I_L$ = **22.4 A**

Example 19.41. *A three-phase, three-wire, 100-V, ABC system supplies a balanced delta connected load with impedance of* $20\angle 45°$ *ohm.*

***(a)** Determine the phase and line currents and draw the phase or diagram **(b)** Find the wattmeter readings when the two wattmeter method is applied to the system.*

(Elect. Machines, A.M.I.E. Sec B.)

Solution. (a) The phasor diagram is shown in Fig. 19.54 (*b*).

Let $V_{AB} = 100\angle 0°$. Since phase sequence is *ABC*, $V_{BC} = 100\angle -120°$ and $V_{CA} = 100°$ 120

Phase current $I_{AB} = \dfrac{V_{AB}}{Z_{AB}} = \dfrac{100\angle 0°}{20\angle 45°} = 5\angle -45°$

$$I_{BC} = \frac{V_{BC}}{Z_{BC}} = \frac{100\angle -120°}{20\angle 45°} = 5\angle -165°, I_{CA} = \frac{V_{CA}}{Z_{CA}} = \frac{100\angle 120°}{20\angle 45°} = 5\angle 75°$$

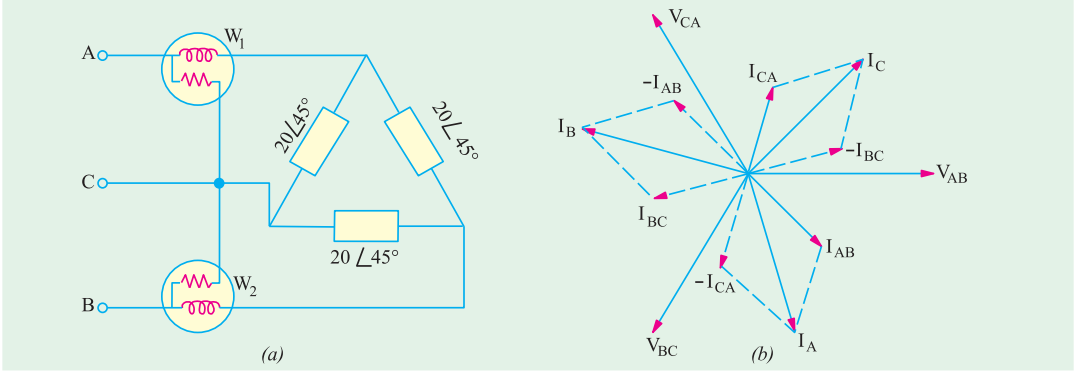


Fig. 19.54

Applying *KCL* to junction A, we have

$$I_A + I_{CA} - I_{AB} = 0 \text{ or } I_A = I_{AB} - I_{CA}$$

$\therefore$ Line current $I_A = 5\angle -45° - 5\angle 75° = 8.66\angle -75°$

Since the system is balanced, I_B will lag I_A by 120° and I_C will lag I_A by 240°.

$\because I_B = 8.66\angle(75° - 120°) = 8.66\angle -195°; I_C = 8.66\angle(-75° - 240°) = 8.66\angle -315° = 8.66\angle 45°$

(b) As shown in Fig. 19.54 (*b*), reading of wattmeter W_1 is $W_1 = V_{AC}\ I_C \cos\varphi$. Phasor V_{AC} is the reverse of phasor V_{CA}. Hence, V_{AC} is the reverse of phasor V_{CA}. Hence, V_{AC} lags the reference vector by 60° whereas I_A lags by 75°. Hence, phase difference between the two is (75° – 60°) = 15°

$\therefore$ W_1 = 100 × 8.66 × cos 15° = 836.5 W

Similarly $W_2 = V_{BC} I_B \cos\phi = 100 \times 8.66 \times \cos 75°$ = 224.1 W

$\therefore$ $W_1 + W_2$ = 836.5 + 224.1 = 1060.6 W

Resistance of each delta branch = 20 cos 45° = 14.14 Ω

Total power consumed = 3 I^2R = 3 × 5^2 × 14.14 = 1060.6 W

Hence, it proves that the sum of the two wattmeter readings gives the total power consumed.

Example 19.42. *A 3-phase, 500-V motor load has a power factor of 0.4 Two wattmeters connected to measure the power show the input to be 30 kW. Find the reading on each instrument.* **(Electrical Meas., Nagpur Univ. 1991)**

Solution. As seen from Art. 19.21

$$\tan\phi = \frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2} \quad \text{... (i)}$$

Now, $\cos\phi = 0.4; \phi = \cos^{-1}(0.4) = 66.6°$; tan 66.6° = 2.311

$$W_1 + W_2 = 30 \quad \text{... (ii)}$$

Substituting these values in equation (*i*) above, we get

$$2.311 = \frac{\sqrt{3}(W_1 - W_2)}{30} \quad \therefore \quad W_1 - W_2 = 40 \quad \text{... (iii)}$$

From Eq. **(*ii*)** and **(*iii*)**, we have W_1 = **45 kW** and W_2 = **– 5 kW**

Since W_2 comes out to be negative, second wattmeter reads 'down scale'. Even otherwise it is obvious that p.f. being less than 0.5, W_2 must be negative (Art. 19.19)

Example 19.43. *The power in a 3-phase circuit is measured by two wattmeters. If the total power is 100 kW and power factor is 0.66 leading, what will be the reading of each wattmeter? Give the connection diagram for the wattmeter circuit. For what p.f. will one of the wattmeter read zero?*

Solution. $\phi = \cos^{-1}(0.66) = 48.7°; \tan\phi = 1.1383$

Since p.f. is leading,

$$\therefore \quad \tan\phi = -\frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2} \quad \therefore \quad 1.1383 = \sqrt{3}(W_1 \quad W_2)/100$$

$\therefore$ $W_1 - W_2 = -65.7$ and $W_1 + W_2 = 100$ $\therefore$ W_1 = **17.14 kW;** W_2 = **82.85 kW**

Connection diagram is similar to that shown in Fig. 19.47 (*b*). One of the wattmeters will read zero when p.f. = **0.5**

Example 19.44. *Two wattmeters are used for measuring the power input and the power factor of an over-excited synchronous motor. If the readings of the meters are (– 2.0 kW) and (+ 7.0 kW) respectively, calculate the input and power factor of the motor.*

(Elect. Technology, Punjab Univ., June, 1991)

Solution. Since an over-excited synchronous motor runs with a leading p.f., we should use the relationship derived in Art. 19.22.

$$\tan\phi = \frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}$$

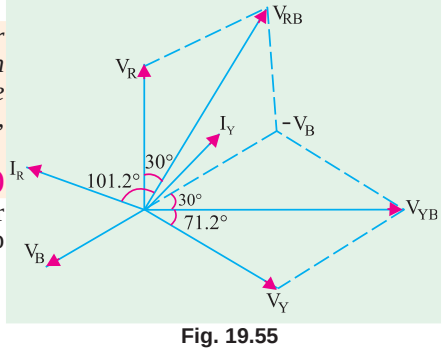


Fig. 19.55

Moreover, as explained in the same article, it is W_1 that gives negative reading and not W_2.

Hence, $W_1 = -2$ kW

$$\therefore \quad \tan\phi = -\frac{\sqrt{3}(-2-7)}{-2+7} = \sqrt{3} \times \frac{9}{5} = 3.1176$$

$$\therefore \quad \phi = \tan^{-1}(3.1176) = 71.2° \text{ (lead)}$$

$$\therefore \quad \cos\phi = \cos 71.2° = \textbf{0.3057 (lead)} \text{ and}$$

$$\text{Input} = W_1 + W_2 = -2 + 7 = \textbf{5 kW}$$

Example 19.45. *A 440-V, 3-phase, delta-connected induction motor has an output of 14.92 kW at a p.f. of 0.82 and efficiency 85%. Calculate the readings on each of the two wattmeters connected to measure the input. Prove any formula used.*

If another star-connected load of 10 kW at 0.85 p.f. lagging is added in parallel to the motor, what will be the current draw from the line and the power taken from the line?

(Elect. Technology-I, Bombay Univ.)

Solution. Motor input = 14,920/0.85 = 17,600 W $\therefore$ $W_1 + W_2$ = 17.6 kW ... ***(i)***

$\cos\phi = 0.82; \phi = 34.9°$, $\tan 34.9° = 0.6976$; $0.6976 = \sqrt{3}\dfrac{W_1 - W_2}{17.6}$

$\therefore$ $W_1 - W_2$ = 7.09 kW ... ***(ii)***

From ***(i)*** and ***(ii)*** above, we get W_1 = **12.35 kW** and W_2 = **5.26 kW**

Motor **kVA**, $S_m = \dfrac{\text{motor kW}}{\cos\phi_m} = \dfrac{17.6}{0.82} = 21.46$ $\therefore$ $\mathbf{S_m} = 21.46 \angle -34.9° = (17.6 - j\,12.28)$ kVA

Load p.f. = 0.85 $\therefore$ $\phi = \cos^{-1}(0.85) = 31.8°$; Load kVA, S_Y = 10/0.85 = 11.76

$\therefore$ $S_Y = 11.76 \angle -31.8° = (10 - j6.2)$ kVA

Combined kVA, $\mathbf{S = S_m + S_Y}$ = (27.6 – j 18.48) = 32.2 $\angle$ – 33.8° kVA

$$I = \frac{S}{\sqrt{3}.V} = \frac{33.2 \times 10^3}{\sqrt{3} \times 440} = \mathbf{43.56\ A}$$

Power taken = **27.6 kW**

Example 19.46. *The power input to a synchronous motor is measured by two wattmeters both of which indicate 50 kW. If the power factor of the motor be changed to 0.866 leading, determine the readings of the two wattmeters, the total input power remaining the same. Draw the vector diagram for the second condition of the load.* **(Elect. Technology, Nagpur Univ. 1992)**

Solution. In the first case both wattmeters read equal and positive. Hence motor must be running at unity power (Art. 19.22).

When p.f. is 0.866 leading

In this case; $W_1 = V_L I_L \cos(30° + \phi)$;

$W_2 = V_L I_L \cos(30° - \phi)$

$\therefore$ $W_1 + W_2 = \sqrt{3} V_L I_L \cos\phi$

$W_1 - W_2 = -V_L I_L \sin\phi$

$\therefore$ $\tan\phi = \dfrac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}$

$\phi = \cos^{-1}(0.866) = 30°$

$\tan\phi = 1/\sqrt{3}$

$\therefore$ $\dfrac{1}{\sqrt{3}} = \dfrac{-\sqrt{3}(W_1 - W_2)}{100}$

$\therefore$ $W_1 - W_2 = -100/3$

and $W_1 + W_2 = 100$

$\therefore$ $2W_1$ = 200/3; W_1 = **33.33 kW**; $\mathbf{W_2}$ = **66.67 kW**

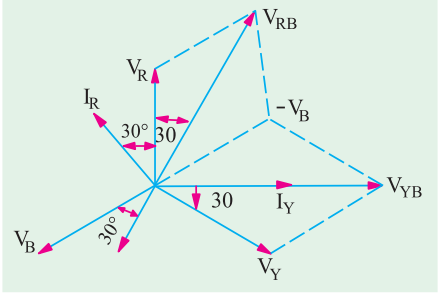


Fig. 19.56

For connection diagram, please refer to Fig. 19.47. The vector or phasor diagram is shown in Fig. 19.56.

Example 19.47 (*a*). *A star-connected balanced load is supplied from a* $3-\phi$ *balanced supply with a line voltage of 416 volts at a frequency of 50 Hz. Each phase of the load consists of a resistance and a capacitor joined in series and the reading on two wattmeters connected to measure the total power supplied are 782 W and 1980 W, both positive. Calculate*

(i) *power factor of circuit,* ***(ii)*** *the line current,* ***(iii)*** *the capacitance of each capacitor.*

(Elect. Engg. I, Nagpur Univ. 1993)

Solution. ***(i)*** As seen from Art. 19.21 $\tan\phi = -\dfrac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)} = -\dfrac{\sqrt{3}(782 - 1980)}{(782 + 1980)} = 0.75;$ $\phi = 36.9°, \cos\phi = 0.8$

(ii) $\sqrt{3} \times 416 \times I_L \times 0.8 = 2762, I_L = 4.8\,\text{A}$

(iii) $Z_{ph} = V_{ph} / I_{ph} = (416\sqrt{3}) / 4.8 = 50\Omega, X_C = Z_{ph} \sin\phi = 50 \times 0.6 = 30\,\Omega$

Now, $X_C = 1/2\pi fC = 1/2\phi \times 50 \times C = 106 \times 10^{-6}\,\text{F}$

Example 19.48. *Each phase of a 3-phase, Δ-connected load consists of an impedance $Z = 20\ \angle 60°$ ohm. The line voltage is 440 V at 50 Hz. Compute the power consumed by each phase impedance and the total power. What will be the readings of the two wattmeters connected?* **(Elect. and Mech. Technology, Osmania Univ.)**

Solution. $Z_{ph} = 20\Omega; V_{ph} = V_L = 440\text{V}; I_{ph} = V_{ph} / Z_{ph} = 440/20 = 22\ A$

Since $\phi = 60°; \cos\phi = \cos 60° = 0.5°;\ R_{ph} = Z_{ph} \times \cos 60° = 20 \times 0.5 = 10\,\Omega$

$\therefore$ Power/phase = $I_{ph}^2 R_{ph} = 22^2 \times 10$ = **4,840 W**

Total power = $3 \times 4{,}840$ = **14,520 W** [or $P = \sqrt{3} \times 440 \times (\sqrt{3} \times 22) \times 0.5$ = 14,520 W]

Now, $W_1 + W_2 = 14{,}520.$

Also $\tan\phi = \sqrt{3}.\dfrac{W_1 - W_2}{W_1 + W_2}$ $\therefore$ $\tan 60° = \sqrt{3} = \sqrt{3}.\dfrac{W_1 - W_2}{14{,}520}$

$\therefore$ $W_1 - W_2$ = 14,520. Obviously, W_2 = 0

Even otherwise it is obvious that W_2 should be zero because p.f. = cos 60° = 0.5 (Art. 19.19).

Example 19.49. *Three identical coils, each having a reactance of 20 Ω and resistance of 20 Ω are connected in **(a)** star **(b)** delta across a 440-V, 3-phase line. Calculate for each method of connection the line current and readings on each of the two wattmeters connected to measure the power.* **(Electro-mechanics, Allahabad Univ. 1992)**

Solution. (*a*) Star Connection

$Z_{ph} = \sqrt{20^2 + 20^2} = 20\sqrt{2} = 28.3\Omega; V_{ph} = 440/\sqrt{3} = 254\,\text{V}$

$I_{ph} = 254/28.3 = 8.97A; I_L$ = **8.97 A;** cos ϕ = R_{ph} / Z_{ph} = 20/28.3 = 0.707

Total power taken = $\sqrt{3} V_L I_L \cos\phi = \sqrt{3} \times 440 \times 8.97 \times 0.707$ = 4830 W

If W_1 and W_2 are wattmeter readings, then $W_1 + W_2$ = 4830 W ... ***(i)***

Now, $\tan\phi = 20/20 = \sqrt{3}(W_1 - W_2)/(W_1 + W_2); (W_1 - W_2); = 2790\,\text{W}$ ... ***(ii)***

From ***(i)*** and ***(ii)*** above, W_1 = **3810 W;** W_2 = **1020 W**

(*b*) Delta Connection

$Z_{ph} = 28.3\Omega, V_{ph} = 440\,V, I_{ph} = 440/28.3 = 15.5A; I_L = 15.5 \times \sqrt{3} = 28.8\,\text{A}$

$P = \sqrt{3} \times 440 \times 28.8 \times 0.707$ = 14,490 W (it is 3 times the *Y*-power)

$\therefore$ $W_1 + W_2$ = 14,490 W ... ***(iii)***

$\tan\phi = 20/20 = \sqrt{3}(W_1 - W_2)/14{,}490; W_1 - W_2$ = 8370 ... ***(iv)***

From Eq. ***(iii)*** and ***(iv)***, we get, W_1 = **11,430 W;** W_2 = **3060 W**

Note: These readings are 3-times the Y-readings.

Example 19.50. *Three identical coils are connected in star to a 200-V, three-phase supply and each takes 500 W. The power factor is 0.8 lagging. What will be the current and the total power if the same coils are connected in delta to the same supply? If the power is measured by two wattmeters, what will be their readings? Prove any formula used.* **(Elect. Engg. A.M. A. S.I. Dec. 1991)**

Solution. When connected in star as shown in Fig. 19.57 (*a*), $V_{ph} = 200/\sqrt{3}$ = 115.5 V

Now, $V_{ph} I_{ph} \cos\phi$ = power per phase or 115.5 × I_{ph} × 0.8 = 500

$\therefore\ I_{ph} = 5.41A; Z_{ph} = V_{ph}/I_{ph} = 115.5/5.41 = 21.34\,\Omega$

$R = Z_{ph}\cos\phi = 21.34 \times 0.8 = 17\Omega;\ X_L = Z_{ph}\sin\phi = 21.34 \times 0.6 = 12.8\,\Omega$

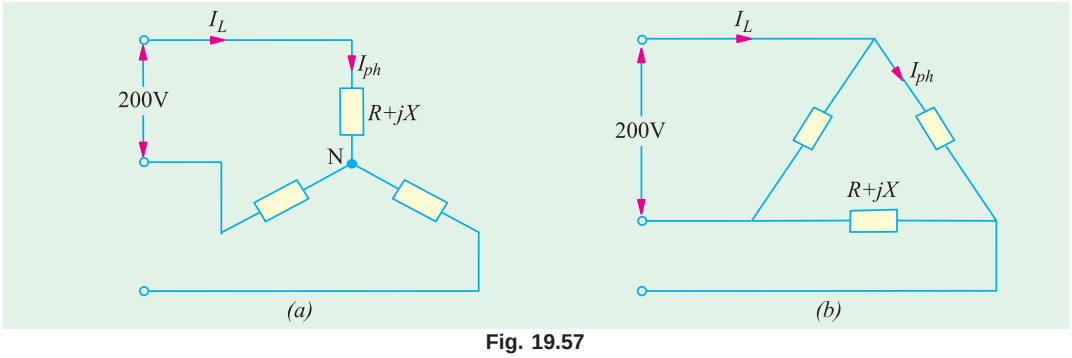


Fig. 19.57

The same three coils have been connected in delta in Fig. 19.57 (*b*). Here, $V_{ph} = V_L = 200$ V.

$$I_{ph} = 200/21.34 = 9.37A; I_L = \sqrt{3} I_{ph} = 9.37 \times 1.732 = \mathbf{16.23\ A}$$

Total power consumed = $\sqrt{3} \times 200 \times 16.23 \times 0.8$ = **4500 W**

It would be seen that when the same coils are connected in delta, they consume three times more power than when connected in star.

Wattmeter Readings

Now,
$$W_1 + W_2 = 4500; \tan\phi = \frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}$$

$$\phi = \cos^{-1}(0.8) = 36.87°; \tan\phi = 0.75$$

$$0.75 = \frac{\sqrt{3}(W_1 - W_2)}{4500} \quad \therefore\ (W_1 - W_2) = 1950\ \text{W}$$

$\therefore$ $W_1 = (4500 + 1950)/2 = 3225$ W; $\mathbf{W_2 = 1275\ W.}$

Example 19.51. *A 3-phase, 3-wire, 415-V system supplies a balanced load of 20 A at a power factor 0.8 lag. The current coil of wattmeter I is in phase R and of wattmeter 2 in phase B. Calculate* ***(i)*** *the reading on 1 when its voltage coil is across R and Y* ***(ii)*** *the reading on 2 when its voltage coil is across B and Y and* ***(iii)*** *the reading on 1 when its voltage coil is across Y and B. Justify your answer with relevant phasor diagram.* **(Elect. Machines, A.M.I.E. Sec. B, 1991)**

Solution. (*i*) As seen from phasor diagram of Fig. 19.57 (*a*)

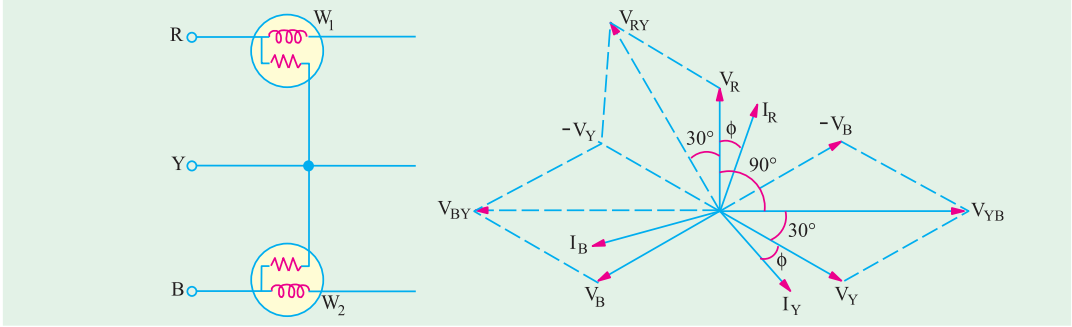


Fig. 19.57 (*a*)

$W_1 = V_{RY} I_A \cos(30+\phi) = \sqrt{3} \times 415 \times 20 \times \cos(36.87°+30°)$ = **5647 W**

(ii) Similarly, $W_2 = V_{BY} I_B \cos(30-\phi)$

It should be noted that voltage across W_2 is V_{BY} and not V_{YB}. Moreover, $\phi = \cos^{-1}(0.8) = 36.87°$,

$\therefore \quad W_2 = \sqrt{3} \times 415 \times 20 \times \cos(30°-36.87°)$ = **14,275 W**

(iii) Now, phase angle between I_R and V_{YB} is $(90°-\phi)$

$\therefore \quad W_2 = V_{YB} I_R \cos(90°-\phi) = \sqrt{3} \times 415 \times 20 \times \sin 36.87°$ = 8626 VAR

Example 19.52. *A wattmeter reads 5.54 kW when its current coil is connected in R phase and its voltage coil is connected between the neutral and the R phase of a symmetrical 3-phase system supplying a balanced load of 30 A at 400 V. What will be the reading on the instrument if the connections to the current coil remain unchanged and the voltage coil be connected between B and Y phases? Take phase sequence RYB. Draw the corresponding phasor diagram.* **(Elect. Machines, A.M.I.E., Sec. B, 1992)**

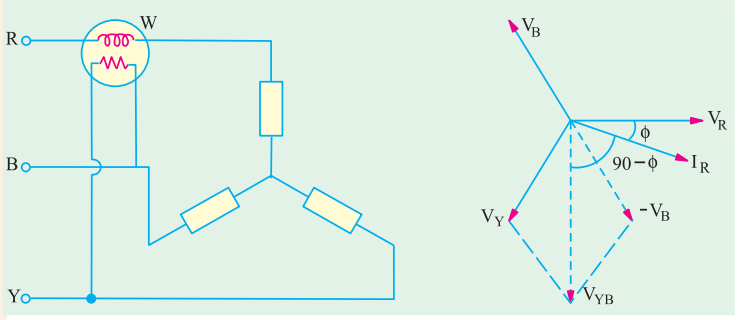


Fig. 19.57 (*b*)

Solution. As seen from Fig. 19.57 (*b*).

$W_1 = V_R I_R \cos\phi$ or $5.54 \times 10^3 = (400/\sqrt{3}) \times 30 \times \cos\phi$; $\therefore \quad \cos\phi = 0.8, \sin\phi = 0.6$

In the second case (Fig. 19.57 (*b*))

$W_2 = V_{YB} I_R \cos(90°-\phi) = 400 \times 30 \times \sin\phi = 400 \times 30 \times 0.6$ = **7.2 kW**

Example 19.53. *A 3-phase, 3-wire balanced load with a lagging power factor is supplied at 400 V (between lines). A I-phase wattmeter (scaled in kW) when connected with its current coil in the R-line and voltage coil between R and Y lines gives a reading of 6 kW. When the same terminals of the voltage coil are switched over to Y- and B-lines, the current-coil connections remaining the same, the reading of the wattmeter remains unchanged. Calculate the line current and power factor of the load. Phase sequence is* $R \to Y \to B$. **(Elect. Engg-1, Bombay Univ. 1985)**

Solution. The current through the wattmeter is I_R and p.d. across its pressure coil is V_{RY}. As seen from the phasor diagram of Fig. 19.58, the angle between the two is $(30°+\phi)$.

$\therefore \quad W_1 = V_{RY} I_R \cos(30°+\phi) = V_L I_L \cos(30°+\phi)$ ... ***(i)***

In the second case, current is I_R but voltage is V_{YB}. The angle between the two is $(90° - \phi)$

$\therefore \quad W_2 = V_{YB} I_R \cos(90°-\phi) = V_L I_L \cos(90°-\phi)$

Since $W_1 = W_2$ we have

$V_L I_L \cos(30°+\phi) = V_L I_L \cos(90°-\phi)$

$\therefore \quad 30°+\phi = 90°-\phi$

or $2\phi = 60° \quad \therefore \quad \phi = 30°$

$\therefore$ load power factor = cos 30° = **0.866 (lag)**

Now $W_1 = W_2$ = 6 kW.

Hence, from (*i*) above, we get

$6000 = 400 \times I_L \cos 60°; I_L$ = **30 A**

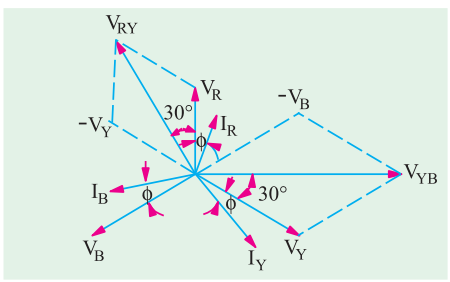


Fig. 19.58

Example 19.54. *A 3-phase, 400 V circuit supplies a Δ-connected load having phase impedances of* $Z_{AB} = 25\angle 0°; Z_{BC} = 25\angle 30°$ and $V_{CA} = 25\angle -30°$.

Two wattmeters are connected in the circuit to measure the load power. Determine the wattmeter readings if their current coils are in the lines ***(a)*** *A and B ;* ***(b)*** *B and C; and* ***(c)*** *C and A. The phase sequence is ABC. Draw the connections of the wattmeter for the above three cases and check the sum of the two wattmeter readings against total power consumed.*

Solution. Taking V_{AB} as the reference voltage, we have $Z_{AB} = 400\angle 0°; Z_{BC} = 400\angle -120°$ and $Z_{CA} = 400\angle 120°$.

The three phase currents can be found as follows:

$$I_{AB} = \frac{V_{AB}}{Z_{AB}} = \frac{400\angle 0°}{25\angle 0°} = 16\angle 0° = (16 + j0)$$

$$I_{BC} = \frac{V_{BC}}{Z_{BC}} = \frac{400\angle -120°}{25\angle 30°} = .16\angle -150° = (-13.8 - j8)$$

$$I_{CA} = \frac{V_{CA}}{Z_{CA}} = \frac{400\angle -120°}{25\angle 30°} = 16\angle -150° = (-13.8 - j8)$$

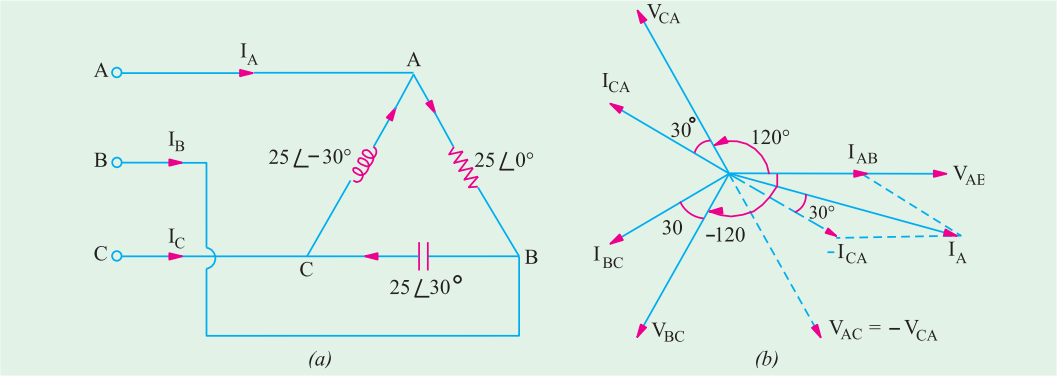


Fig. 19.59

The line currents I_A, I_B and I_C can be found by applying KCL at the three nodes *A*, *B* and *C* of the load.

$$I_A = I_{AB} + I_{AC} = I_{AB} - I_{CA} = (16 + j0) - (-13.8 + j8) = 29.8 - j8 = 30.8\angle -15°$$

$$I_B = I_{BC} - I_{AB} = (-13.8 - j8) - (16 + j0) = -29.8 - j8 = 30.8\angle -165°$$

$$I_C = I_{CA} - I_{BC} = (-13.8 + j8) - (-13.8 - j8) = 0 + j16 = 16\angle 90°$$

The phasor diagram for line and phase currents is shown in Fig. 19.59 (*a*) and (*b*).

(a) As shown in Fig. 19.60 (*a*), the current coils of the wattmeters are in the line *A* and *B* and the voltage coil of W_1 is across the lines *A* and *C* and that of W_2 is across the lines *B* and *C*. Hence, current through W_1 is I_A and voltage across it is V_{AC}. The power indicated by W_1 may be found in the following two ways:

(i) $P_1 = |V_{AC}|.|I_A|$ × (cosine of the angle between V_{AC} and I_A).

= 400 × 30.8 × cos (30° + 15°) = 8710 W

(ii) We may use current conjugate (Art.) for finding the power

$$P_{VA} = V_{AC}.I_A = -400\angle 120° \times 30.8\angle 15°$$

$\therefore$ P_1 = real part of P_{VA} = – 400 × 30.8 × cos 135° = 8710 W

P_2 = real part of $[V_{BC} Z_B] = 400\angle 120° \times 30.8\angle -165°$

$= 400 \times 30.8 \times \cos(-45°) = 8710$ W

$\therefore$ $P_1 + P_2 = 8710 + 8710 = 17{,}420$ W.

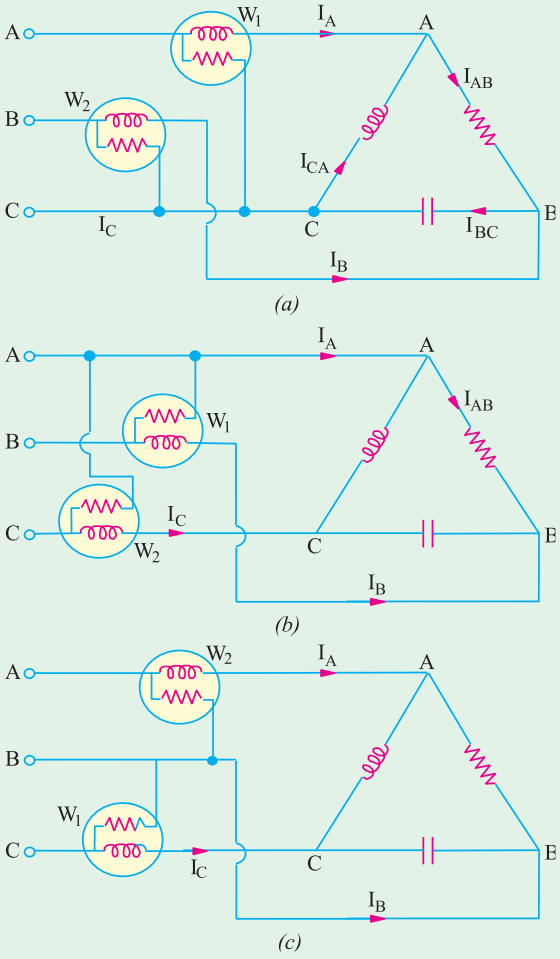


Fig. 19.60

(b) As shown in Fig. 19.60 (*b*), the current coils of the wattmeters are in the lines *B* and *C* whereas voltage coil of W_1 is across the lines *B* and *A* and that of W_2 is across lines *C* and *A*.

(i) $\therefore \quad P_1 = |V_{BA}| . |I_B|$ (cosine of the angle between V_{BA} and I_B)

$= 400 \times 30.8 \times \cos 15° = 11{,}900$ W

(ii) Using voltage conjugate (which is more convenient in this case), we have

$$P_{VA} = V_{BA}^{*} . I_B = -400\angle 0° \times 30.8\angle \times -165°$$

$\therefore \quad P_1$ = real part of $P_{VA} = -400 \times 30.8 \times \cos(-165°) = 11{,}900$ W

P_2 = real part of $[V_{CA}{}^{*} I_C] = [400 \ \angle -120° \times 16\angle 90°] = 400 \times 16 \times \cos(-30°) = 5{,}540$ W.

$\therefore \quad P_1 + P_2 = 11{,}900 + 5{,}540 = 17{,}440$ W.

(c) As shown in Fig. 19.60 (*c*), the current coils of the wattmeters are in the lines *C* and *A* whereas the voltage coil of W_1 is across the lines *C* and *B* and that of W_2 is across the lines *A* and *B*.

(i) P_1 = real part of $[V_{CB} * I_C]$ [(400 120) 16 90]

= 400 16 cos 210 5540W

P_2 = real part of $[V_{AB} * I_A] = [400\angle 0° \times 30.8\angle -15° = 400 \times 30.8 \times \cos\angle -15°$ = 11,900 W

$\therefore \quad P_1 + P_2 = 5{,}540 + 11{,}900 = 17{,}440$ W

Total power consumed by the phase load can be found directly as under :-

P_T = real part of $[V_{AB}I_{AB}^* + V_{BC}I_{BC}^* + V_{CA}I_{CA}^*]$

= real part of

$[(400\angle 0°)(16\angle -0°) + (400\angle -120°)(16\angle 150°) + (400\angle 120°)(16\angle -150°)]$

= 400 × 16 × real part of $(1\angle 0° + 1\angle 30° + 1\angle -30°)$

= 400 × 16 (cos 0° + cos° + cos (– 30°) = 17,485 W

Note. The slight variation in the different answers is due to the approximation made.

Example 19.55. *In a balanced 3-phase system load I draws 60 kW and 80 leading kVAR whereas load 2 draws 160 kW and 120 lagging kVAR. If line voltage of the supply is 1000 V, find the line current supplied by the generator. (Fig. 19.61)*

Solution. For load 1 which is a leading load, $\tan\phi_1 = Q_1/P_1 = 80/60 = -1.333$; $\phi_1 = 53.1°$, cos ϕ_1 = 0.6. Hence, line current of this load is

$$I_1 = 60{,}000/\sqrt{3}\times 1000\times 0.6 = 57.8 \text{ A}$$

For load 2, tan ϕ_2 = 120/160 = 0.75; ϕ_2 = 26.9°, cos ϕ_2 = 0.8. The line current drawn by this load is

$$I_2 = 160{,}000/\sqrt{3}\times 1000\times 0.8 = 115.5 \text{ A}$$

If we take the phase voltage as the reference voltage *i.e.* $V_{ph} = (1000/\sqrt{3})\angle 0° = 578\angle 0°$; then I_1 leads this voltage by 53.1° whereas I_2 lags it by 36.9°. Hence, I_1 = $57.8\angle 53.1°$ and I_2 = 115.5 36.9

$$\therefore\ I_{L1} = I_1 + I_2 = 57.8\ \angle 53.1° + 115.5\ \angle -36.9° = 171.7\angle 42.3° \text{ A}.$$

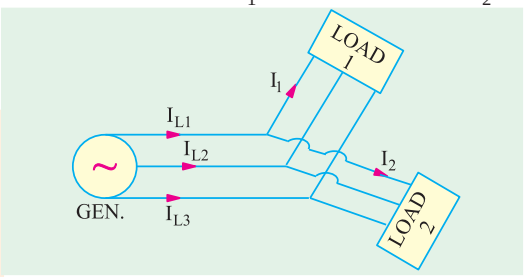


Fig. 19.61

Example 19.56. *A single-phase motor drawing 10A at 0.707 lagging power factor is connected across lines R and Y of a 3-phase supply line connected to a 3-phase motor drawing 15A at a lagging power factor of 0.8 as shown in Fig. 19.62(a). Assuming RYB sequence, calculate the three line currents.*

Solution. In the phasor diagram of Fig. 19.61 (*b*) are shown the three phase voltages and the one line voltage V_{RY} which is ahead of its phase voltage V_R. The current I_1 drawn by single-phase motor lags V_{RY} by $\cos^{-1}$ 0.707 or 45°. It lags behind the reference voltage V_R by 15° as shown. Hence, $I_1 = 10\angle -15° = 9.6 - j2.6$ A. The 3-phase motor currents lag behind their respective phase voltages by $\cos^{-1}$ 0.8 or 36.9°. Hence, $I_{R1} = 15\angle -36.9° = 12 - j9$.

$$I_{Y1} = 15\angle(-120° - 36.9°) = 15\angle -156.9° = -13.8 - j5.9$$

$$I_B = 15\angle(120° - 36.9°) = 15\angle 83.1°$$

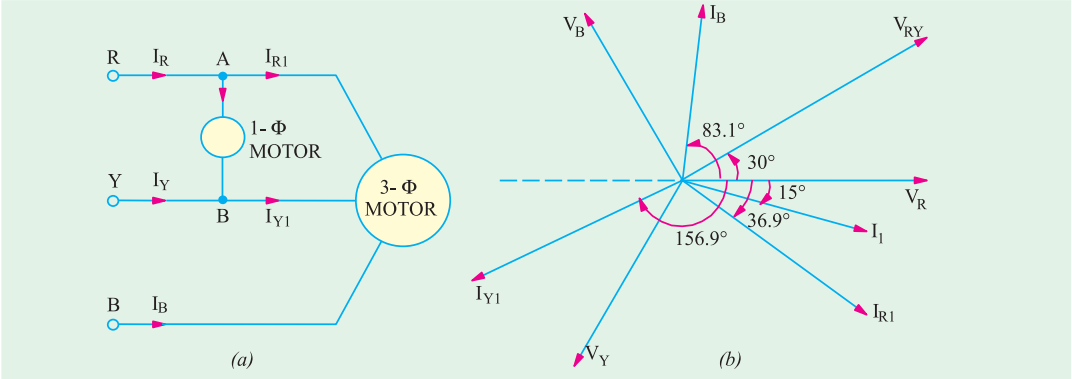


Fig. 19.62

Applying Kirchhoff's laws to point A of Fig. 19.62 (a), we get

$$I_R = I_1 + I_{R1} = 9.6 - j2.6 + 12 - j9 = 21.6 - j11.6 = 24.5\angle -28.2°$$

Similarly, applying KCL to point B, we get

$$I_Y + I_1 = I_{Y1} \text{ or } I_Y = I_{Y1} - I_1 = -13.8 - j5.9 - 9.6 + j2.6 = -23.4 - j3.3 = 23.6\angle -172°.$$

Example 19.57. *A 3-ϕ, 434-V, 50-Hz, supply is connected to a 3-ϕ, Y-connected induction motor and synchronous motor. Impedance of each phase of induction motor is (1.25 + j2.17) Ω. The 3-ϕ synchronous motor is over-excited and it draws a current of 120 A at 0.87 leading p.f. Two wattmeters are connected in usual manner to measure power drawn by the two motors. Calculate* ***(i)*** *reading on each wattmeter* ***(ii)*** *combined power factor.*

(Elect. Technology, Hyderabad Univ. 1992)

Solution. It will be assumed that the synchronous motor is *Y*-connected. Since it is over-excited it has a leading p.f. The wattmeter connections and phasor diagrams are as shown in Fig. 19.63.

$$\mathbf{Z_1} = 1.25 + j2.17 = 2.5\angle 60°$$

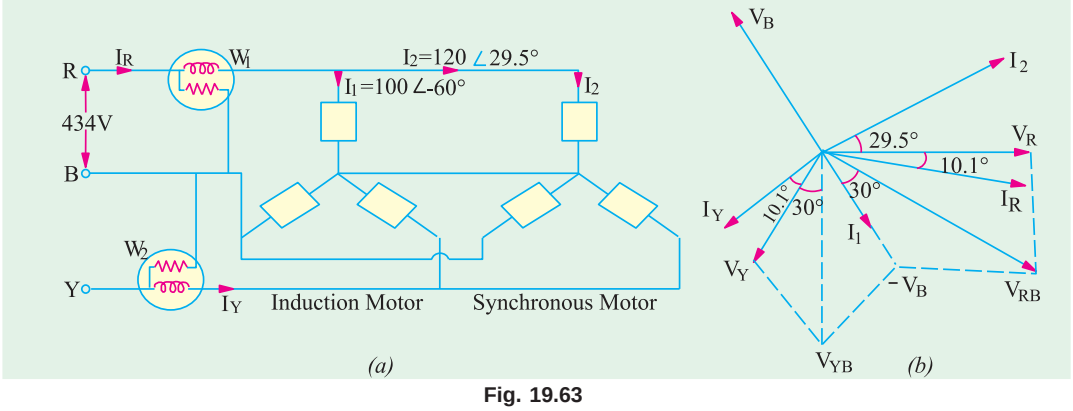


Fig. 19.63

Phase voltage in each case = $434/\sqrt{3}$ = 250 V

I_1 = 250/2.5 = 100 A lagging the reference vector V_R by 60°. Current I_2 = 120 A and leads V_R by an angle = $\cos^{-1}(0.87)$ = 29.5°

$$\therefore \quad \mathbf{I_1} = 100 \quad 60 \quad 50 \quad j86.6; \ \mathbf{I_2} \quad 120 \quad 29.5 \quad 104.6 \quad j59$$

$$\mathbf{I_R = I_1 + I_2} = 154.6 - j27.6 = 156.8\angle -10.1°$$

(*a*) As shown in Fig. 19.63 (*b*), I_R lags V_R by 10.1°. Similarly, I_Y lags V_Y by 10.1°.

As seen from Fig. 19.63 (*a*), current through W_1 is I_R and voltage across it is $V_{RB} = V_R - V_B$. As seen, V_{RB} = 434 V lagging by 30°. Phase difference between V_{RB} and I_R is = 30 – 10.1 = 19.9°.

$\therefore$ reading of W_1 = 434 × 156.8 × cos 19.9° = **63,970 W**

Current I_Y is also (like I_R) the vector sum of the line currents drawn by the two motors. It is equal to 156.8 A and lags behind its respective phase voltage V_Y by 10.1°. Current through W_2 is I_Y and voltage across it is $\mathbf{V_{YB} = V_Y - V_B}$. As seen, V_{YB} = 434 V. Phase difference between V_{YB} and I_Y = 30° + 10.1° = 40.1° (lag).

$\therefore$ reading of W_2 = 434 × 156.8 × cos 40.1° = **52,050 W**

(*b*) Combined p.f. = cos 10.1° = **0.9845 (lag)**

Example 19.58. *Power in a balanced 3-phase system is measured by the two-wattmeter method and it is found that the ratio of the two readings is 2 to 1. What is the power factor of the system?*

(Elect. Science-1, Allahabad Univ. 1991)

Solution. We are given that $W_1 : W_2 = 2 : 1$. Hence, $W_1/W_2 = r = 1/2 = 0.5$. As seen from Art. 19.21.

$$\cos\phi = \frac{1+r}{2\sqrt{1-r+r^2}} = \frac{1+0.5}{2\sqrt{1-0.5+0.5^2}} = \mathbf{0.866 \text{ lag}}$$

Example 19.59. *A synchronous motor absorbing 50 kW is connected in parallel with a factory load of 200 kW having a lagging power factor of 0.8. If the combination has a power factor of 0.9 lagging, find the kVAR supplied by the motor and its power factor.*

(Elect. Machines, A.M.I.E. Sec B)

Solution. Load kVA = 200/0.8 = 250

Load kVAR = 250 × 0.6 = 150 (lag) [cos ϕ = 0.8 sin ϕ = 0.6]

Total combined load = 50 + 200 = 250 kW

kVA of combined load = 250/0.9 = 277.8

Combined kVAR = 277.8 × 0.4356 = 121 (inductive) (combined cos ϕ = 0.9, sin ϕ = 0.4356)

Hence, leading kVAR supplied by synch, motor = 150 – 121 = 29 (capacitive)

kVA of motor alone = $\sqrt{(kW^2 + kVAR^2)} = \sqrt{50^2 + 29^2}$ = 57.8

p.f. of motor = kW/kVA = 50/57.8 = 0.865 (leading)

Example 19.60. *A star-connected balanced load is supplied from a 3-phase balanced supply with a line voltage of 416 V at a frequency of 50 Hz. Each phase of load consists of a resistance and a capacitor joined in series and the readings on two wattmeters connected to measure the total power supplied are 782 W and 1980 W, both positive. Calculate* ***(a)*** *the power factor of the circuit* ***(b)*** *the line current and* ***(c)*** *the capacitance of each capacitor.*

(Elect. Machinery-I, Bombay Univ.)

Solution. $W_1 = 728 \text{ and } W_2 = 1980$

For a leading p.f. $\tan\phi = -\sqrt{3}\dfrac{W_1 - W_2}{W_1 + W_2}$ $\therefore$ $\tan\phi = -\sqrt{3} \times \dfrac{(782 - 1980)}{782 + 1980}$ = 0.75

From tables, $\phi = 36°54'$

(*a*) $\therefore$ $\cos\phi = \cos 36°54'$ = **0.8 (leading)**

(*b*) power = $\sqrt{3} V_L I_L \cos\phi$ or $W_1 + W_2 = \sqrt{3} V_L I_L \cos\phi$

or $(782 + 1980) = \sqrt{3} \times 416 \times I_L \times 0.8$ $\therefore$ $I_L = I_{ph}$ = **4.8 A**

(*c*) Now $V_{ph} = 416/\sqrt{3}$ V $\therefore$ $Z_{ph} = 416/\sqrt{3} \times 4.8 = 50\,\Omega$

$\therefore$ In Fig. 19.64, $Z_{ph} = 50\angle -36°54' = 50(0.8 - j0.6) = 40 - j30$

Capacitive reactance X_C = 30; or $\dfrac{1}{2\pi \times 50 \times C} = 30$ $\therefore$ ***C* = 106** μF.

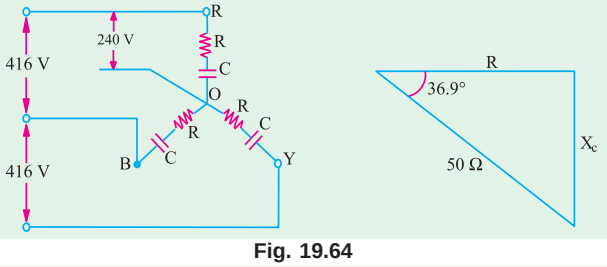


Fig. 19.64

Example 19.61. *The two wattmeters A and B, give readings as 5000 W and 1000 W respectively during the power measurement of 3-ϕ, 3-wire, balanced load system.* ***(a)*** *Calculate the power and power factor if* ***(i)*** *both meters read direct and* ***(ii)*** *one of them reads in reverse.* ***(b)*** *If the voltage of the circuit is 400 V, what is the value of capacitance which must be introduced in each phase to cause the whole of the power to appear on A. The frequency of supply is 50 Hz.*

(Elect. Engg-I, Nagpur Univ. 1992)

Solution. (*a*) (*i*) Both Meters Read Direct

$W_1 = 5000\text{ W}; W_2 = 1000\text{ W};$ $\therefore$ $W_1 + W_2 = 6000\text{ W}; W_1 - W_2 = 4000\text{ W}$

$$\tan\phi = \sqrt{3}(W_1 - W_2)/(W_1 + W_2) = \sqrt{3} \times 4000/6000 = 1.1547$$

$\therefore$ $\phi = \tan^{-1}(1.1547) = 49.1°$; p.f. = cos 49.1° = 0.655 (lag)

Total power = 5000 + 1000 = 6000 W

(*ii*) One Meter Reads in Reverse

In this case, $\tan\phi = \sqrt{3}(W_1 + W_2)/(W_1 - W_2) = \sqrt{3} \times 6000/4000 = 2.598$

$\therefore$ $\phi = \tan^{-1}(2.598) = 68.95°$; p.f. = cos 68.95° = 0.36 (lag)

Total power = $W_1 + W_2 = 5000 - 1000 = 4000$ W ...Art.

(*b*) The whole of power would be measured by wattmeter W_1 if the load power factor is 0.5 (lagging) or less. It means that in the present case p.f. of the load will have to be reduced from 0.655 to 0.5 In other words, capacitive reactance will have to be introduced in each phase of the load in order to partially neutralize the inductive-reactance.

Now, $\sqrt{3}V_L I_L \cos\phi = 6000$ or $\sqrt{3} \times 400\, I_L \times 0.655 = 6000$

$\therefore$ $I_L = 13.2$A; $\therefore$ $I_{ph} = 13.2/\sqrt{3} = 7.63$ A

$$Z_{ph} = V_{ph}/I_{ph} = 400/7.63 = 52.4\,\Omega$$

$$X_L = Z_{ph}\sin\phi = 52.4 \times \sin 49.1° = 39.6\,\Omega$$

When p.f. = 0.5

$$\sqrt{3} \times 400 \times I_L \times 0.5 = 6000;\ I_L = 17.32A;\ I_{ph} = 17.32/\sqrt{3} = 10A;\ Z_{ph} = 400/10 = 40\Omega$$

$$\cos\phi = 0.5;\ \phi = 60;\ \sin 60° = 0.886;\ X = Z_{ph}\sin\phi = 40 \times 0.886 = 35.4\,\Omega$$

$\therefore$ $X = X_L - X_C = 35.4$ or $39.6 - X_C = 35.4$; $\therefore$ $X_C = 4.2\,\Omega$.

If C is the required capacitance, then $4.2 = 1/2\pi \times 50 \times C$; $\therefore$ $C = 758\,\mu$F.

Tutorial Problems No. 19.2

1. Two wattmeters connected to measure the input to a balanced three-phase circuit indicate 2500 W and 500 W respectively. Find the power factor of the circuit (*a*) when both readings are positive and (*b*) when the latter reading is obtained after reversing the connections to the current coil of one instrument.

[(*a*) 0.655 (*b*) 0.3591] (*City & Guilds, London*)

2. A 400-V, 3-phase induction motor load takes 900 kVA at a power factor of 0.707. Calculate the kVA rating of the capacitor bank to raise the resultant power factor of the installation of 0.866 lagging.

Find also the resultant power factor when the capacitors are in circuit and the motor load has fallen to 300 kVA at 0.5 power factor. **[296 kVA, 0.998 leading] (*City & Guilds, London*)**

3. Two wattmeters measure the total power in three-phase circuits and are correctly connected. One reads 4,800 W while other reads backwards. On reversing the latter, it reads 400 W. What is the total power absorbed by the circuit and the power factor? **[4400 W; 0.49] (*Sheffield Univ. U.K.*)**

4. The power taken by a 3-phase, 400-V motor is measured by the two wattmeter method and the readings of the two wattmeters are 460 and 780 watts respectively. Estimate the power factor of the motor and the line current. **[0.913, 1.96 A] (*City & Guilds, London*)**

5. Two wattmeters, W_1 and W_2 connected to read the input to a three-phase induction motor running unloaded, indicate 3 kW and 1 kW respectively. On increasing the load, the reading on W_1 increases while that on W_2 decreases and eventually reverses.

Explain the above phenomenon and find the unloaded power and power factor of the motor.

[2 kW, 0.287 lag] (*London Univ.*)

6. The power flowing in a 3-ϕ, 3-wire, balanced-load system is measured by the two wattmeter method. The reading on wattmeter *A* is 5,000 W and on wattmeter *B* is – 1,000 W

(*a*) What is the power factor of the system?

(*b*) If the voltage of the circuit is 440, what is the value of capacitance which must be introduced into each phase to cause the whole of the power measured to appear on wattmeter A?

[0.359; 5.43 Ω] (*Meters and Meas. Insts. A.M.I.E.E. London*)

7. Two wattmeters are connected to measure the input to a 400 V; 3-phase, connected motor outputting 24.4 kW at a power factor of 0.4 (lag) and 80% efficiency. Calculate the

(*i*) resistance and reactance of motor per phase
(*ii*) reading of each wattmeters.

[(*i*) 2.55 Ω; 5.85 Ω; (*ii*) 34,915 W; – 4850 W] (*Elect. Machines, A.M.I.E. Sec. B, 1993*)

8. The readings of the two instruments connected to a balanced three-phase load are 128 W and 56 W. When a resistor of about 25 Ω is added to each phase, the reading of the second instrument is reduced to zero. State, giving reasons, the power in the circuit before the resistors were added. **[72 W] (*London Univ.*)**

9. A balanced star-connected load, each phase having a resistance of 10 Ω and inductive reactance of 30 Ω is connected to 400-*V*, 50-Hz supply. The phase rotation is red, yellow and blue. Wattmeters connected to read total power have their current coils in the red and blue lines respectively. Calculate the reading on each wattmeter and draw a vector diagram in explanation. **[2190 W, – 583 W] (*London Univ.*)**

10. A 7.46 kW induction motor runs from a 3-phase, 400-V supply. On no-load, the motor takes a line current of 4. A at a power factor of 0.208 lagging. On full load, it operates at a power factor of 0.88 lagging and an efficiency of 89 per cent. Determine the readings on each of the two wattmeters connected to read the total power on (*a*) no load and (*b*) full load. **[1070 W, – 494 W; 5500 W; 2890 W]**

11. A balanced inductive load, connected in star across 415-V, 50-Hz, three-phase mains, takes a line current of 25A. The phase sequence is *RYB*. A single-phase wattmeter has its current coil connected in the *R* line and its voltage coil across the line *YB*. With these connections, the reading is 8 kW. Draw the vector diagram and find (*i*) the kW (*ii*) the kVAR (*iii*) the kVA and (*iv*) the power factor of the load.

[(*i*) 11.45 kW (*ii*) 13.87 kVAR (*iii*) 18 kVA (iv) 0.637] (*City & Guilds, London*)

19.27. Double Subscript Notation

In symmetrically-arranged networks, it is comparatively easier and actually more advantageous, to use single-subscript notation. But for unbalanced 3-phase circuits, it is essential to use double subscript notation, in order to avoid unnecessary confusion which is likely to result in serious errors.

Suppose, we are given two coils are whose induced e.m.fs. are 60° out of phase with each other [Fig. 19.65 (*a*)]. Next, suppose that it is required to connect these coils in additive series *i.e.* in such

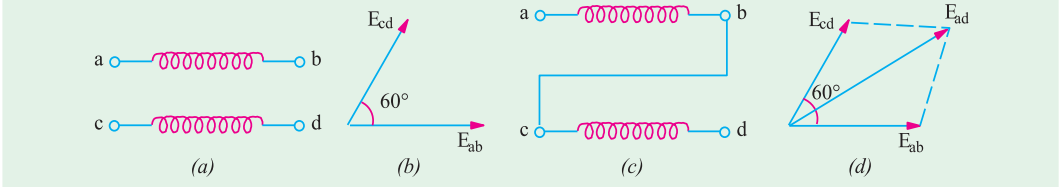


Fig. 19.65

a way that their e.m.fs. add at an angle of 60°. From the information given, it is impossible to know whether to connect terminal '*a*' to terminal '*c*' or to terminal '*d*'. But if additionally it were given that e.m.f. from terminal '*c*' to terminal '*d*' is 60° out of phase with that from terminal '*a*' to terminal '*b*', then the way to connect the coils is definitely fixed, as shown in Fig. 19.59 (*b*) and 19.60 (*a*). The double-subscript notation is obviously very convenient in such cases. The order in which these subscripts are written indicates the direction along which the voltage acts (or current flows). For example the e.m.f. '*a*' to '*b*' [Fig. 19.59 (*a*)], may be written as E_{ab} and that from '*c*' to '*d*' as E_{cd}..... The e.m.f. between '*a*' and '*d*' is E_{ad} where $\mathbf{E}_{ad} = \mathbf{E}_{ab} + \mathbf{E}_{ad}$ and is shown in Fig. 19.59 (*b*).

Example 19.62. *If in Fig. 19.60 (a), terminal 'b' is connected to 'd', find* E_{ac} *if E = 100 V.*

Solution. Vector diagram is shown in Fig. 19.60 (*b*)

Obviously, $\mathbf{E}_{ac} = \mathbf{E}_{ab} + \mathbf{E}_{dc} = \mathbf{E}_{ab} \times (-\mathbf{E}_{cd})$

Hence, $\mathbf{E}_{cd}$ is reversed and ***added*** to $\mathbf{E}_{ab}$ to get $\mathbf{E}_{ac}$ as shown in Fig. 19.60 (*b*). The magnitude of resultant vector is

$$E_{ac} = 2 \times 100 \cos 120°/2 = 100 \text{ V}; \quad \mathbf{E}_{ac} = 100 \angle -60°$$

Example 19.62(*a*). *In Fig. 19.66 (a) with terminal 'b' connected to 'd', find* E_{ca}.

Solution. $\mathbf{E}_{ca} = \mathbf{E}_{cd} + \mathbf{E}_{ba} = \mathbf{E}_{cd} + (-\mathbf{E}_{ab})$

As shown in Fig. 19.67, vector $\mathbf{E}_{ab}$ is reversed and then combined with $\mathbf{E}_{cd}$ to get $\mathbf{E}_{ca}$.

Magnitude of $\mathbf{E}_{ca}$ is given by $2 \times 100 \times \cos 60° = 100$ V but it leads $\mathbf{E}_{ab}$ by 120°.

$\therefore \quad \mathbf{E}_{ca} = 100\angle 120°$

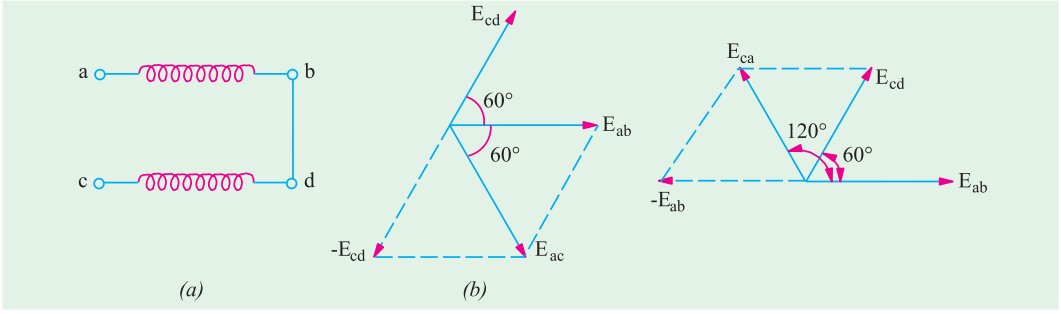


Fig. 19.66

Fig. 19.67

In Fig. 19.68 (*b*) is shown the vector diagram of the e.m.fs induced in the three phases 1, 2, 3 (or *R*, *Y*, *B*) of a 3-phase alternator [Fig.19.68 (*a*)]. According to double subscript notation, each phase e.m.f. may be written as $\mathbf{E}_{01}$, $\mathbf{E}_{02}$ and $\mathbf{E}_{03}$, the order of the subscripts indicating the direction in which the e.m.fs. act. It is seen that while passing from phase 1 to phase 2 through the external circuit, we are in opposition to $\mathbf{E}_{02}$.

$$\mathbf{E}_{12} = \mathbf{E}_{20} + \mathbf{E}_{01} = (-\mathbf{E}_{02}) + \mathbf{E}_{01} = \mathbf{E}_{01} - \mathbf{E}_{02}$$

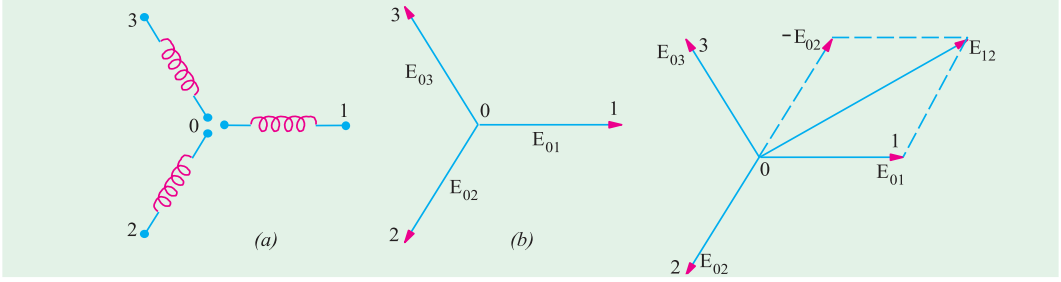


Fig. 19.68

Fig. 19.69

It means that for obtaining $\mathbf{E}_{12}$, $\mathbf{E}_{20}$ has to be reversed to obtain $-\mathbf{E}_{02}$ which is then combined with $\mathbf{E}_{01}$ to get $\mathbf{E}_{12}$ (Fig. 19.69). Similarly,

$$\mathbf{E}_{23} = \mathbf{E}_{30} + \mathbf{E}_{02} = (-\mathbf{E}_{03}) + \mathbf{E}_{02} = \mathbf{E}_{02} - \mathbf{E}_{03}$$
$$\mathbf{E}_{31} = \mathbf{E}_{10} + \mathbf{E}_{03} = (-\mathbf{E}_{01}) + \mathbf{E}_{03} = \mathbf{E}_{03} - \mathbf{E}_{01}$$

By now it should be clear that double-subscript notation is based on lettering every junction and terminal point of diagrams of connections and on the use of two subscripts with all vectors representing voltage or current. The subscripts on the vector diagram, taken from the diagram of connections, indicate that the positive direction of the current or voltage is from the first subscript to the second. For example, according to this notation $\mathbf{I}_{ab}$ represents a current whose + ve direction is from *a* to *b* in the branch *ab* of the circuit in the diagram of connections. In the like manner, $\mathbf{E}_{ab}$ represents the e.m.f. which produces this current. Further, $\mathbf{I}_{ba}$ will represent a current flowing from *b* to *a*, hence its vector will be drawn equal to but in a direction opposite to that of $\mathbf{I}_{ab}$ *i.e.* $\mathbf{I}_{ab}$ and $\mathbf{I}_{ba}$ differ in phase by 180° although they do not differ in magnitude.

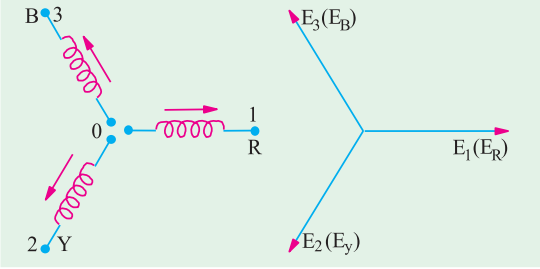


Fig. 19.69 (*a*)

In single subscript notation (*i.e.* the one in which single subscript is used) the + ve directions are fixed by putting arrows on the circuit diagrams as shown in Fig. 19.69 (*a*). According to this notation

$$\mathbf{E}_{12} = -\mathbf{E}_2 + \mathbf{E}_1 = \mathbf{E}_1 - \mathbf{E}_2; \ \mathbf{E}_{23} = -\mathbf{E}_3 + \mathbf{E}_2 = \mathbf{E}_2 - \mathbf{E}_3 \text{ and } \mathbf{E}_{31} = -\mathbf{E}_1 + \mathbf{E}_3 = \mathbf{E}_3 - \mathbf{E}_1$$

or $\mathbf{E}_{RY} = \mathbf{E}_R - \mathbf{E}_Y$; $\mathbf{E}_{YB} = \mathbf{E}_Y - \mathbf{E}_B$; $\mathbf{E}_{BR} = \mathbf{E}_B - \mathbf{E}_R$

Example 19.63. *Given the phasors* $V_{12} = 10\angle 30°; V_{23} = 5\angle 0°;\ V_{14} = 6\angle -60°;$ $V_{45} = 10\angle 90°$. *Find* ***(i)*** V_{13} ***(ii)*** V_{34} *and* ***(iii)*** V_{25}.

Solution. Different points and the voltage between them have been shown in Fig. 19.70.

(i) Using KVL, we have

$$V_{12} + V_{23} + V_{31} = 0 \text{ or } V_{12} + V_{23} - V_{13} = 0$$

or $V_{13} = V_{12} + V_{23} = 10\angle 30° + 5\angle 0° = 8.86 + j5 + 5$

$= 13.86 + j5 = 14.7\angle 70.2°$

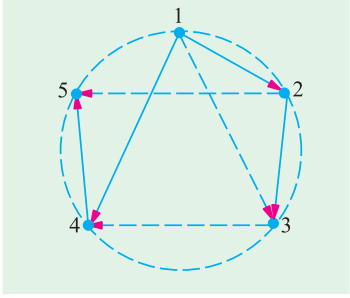


Fig. 19.70

(ii) Similarly, $V_{13} + V_{34} + V_{41} = 0$ or $V_{13} + V_{34} - V_{14} = 0$

or $V_{34} = V_{14} - V_{13} = 6\angle -60° - 14.7\angle 70.2°$

$= 3 - j5.3 - 13.86 - j5 = -10.86 - j10.3 = 15\angle 226.5°$

(iii) Similarly, $V_{23} + V_{34} + V_{45} + V_{52} = 0$

or $V_{23} + V_{34} + V_{45} - V_{52} = 0$

or $V_{25} = V_{23} + V_{34} + V_{45} = 5\angle 0° + 15\angle 226.5° + 10\angle 90°$

$= 5 - 10.86 - j10.3 + j10 = -5.86 - j0.3 = 5.86 - \angle 2.9°$.

Example 19.64. *In a balanced 3-phase Y-connected voltage source having phase sequence abc,* $V_{an} = 230\angle 30°$. *Calculate analytically* ***(i)*** V_{bn} ***(ii)*** V_{cn} ***(iii)*** V_{ab} ***(iv)*** V_{bc} *and* ***(v)*** V_{ca}. *Show the phase and line voltages on a phasor diagram.*

Solution. It should be noted that V_{an} stands for the voltage of terminal a with respect to the neutral point *n*. The usual positive direction of the phase voltages are as shown in Fig. 19.71 (*a*). Since the 3-phase system is balanced, the phase differences between the different phase voltages are 120°.

(i) $V_{bn} = \angle -120° = 230\ \angle(30° - 120°) = 230\angle -90°$

(ii) $V_{cn} = V_{an}\angle 120° = 230\ \angle(30° + 120°) = 230\angle 150°$ ... Fig. 19.71 (*b*)

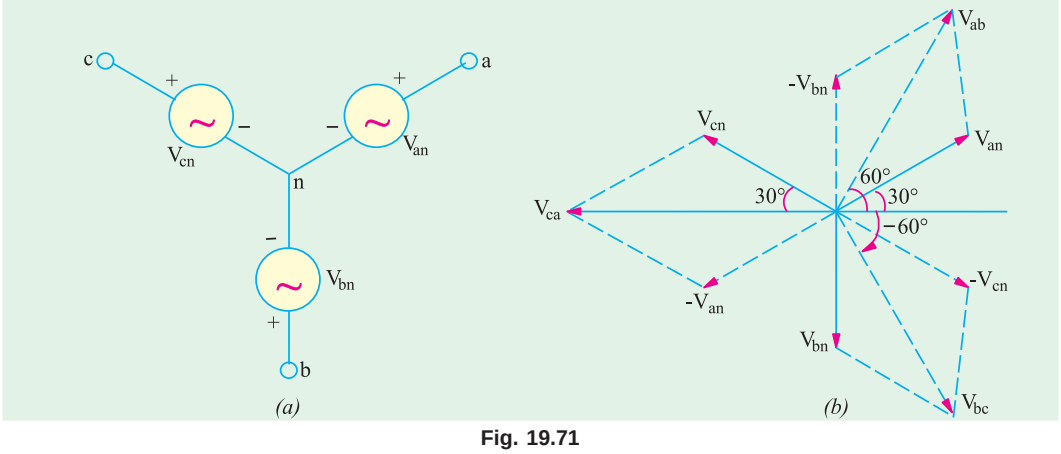


Fig. 19.71

(iii) It should be kept in mind that V_{ab} stands for the voltage of point a with respect to point *b*. For this purpose, we start from the reference point *b* in Fig.19.71 (*a*) and go to point *a* and find the sum of the voltages met on the way. As per sign convention given in Art, 19.27 as we go from *b* to *n*, there is a fall in voltage of by an amount equal to V_{bn}. Next as we go from *n* to *a*, there is increase of voltage given by V_{an}.

$\therefore\ V_{ab} = -V_{bn} + V_{an} = V_{an} - V_{bn} = 230\angle 30° - 230\angle -90°$

$= 230(\cos 30° + j\sin 30°) - 230(0 - j\sin 90°)$

$$= 230\ \frac{\sqrt{3}}{2}\ \ j\frac{1}{2}\quad j230\quad 230\ \frac{\sqrt{3}}{2}\ \ j\frac{3}{2}\ = 230\sqrt{3}\ \ \frac{1}{2}\ \ j\frac{\sqrt{3}}{2}\quad 400\quad 60$$

(*iv*) $V_{bc} = V_{bn} - V_{cn} = 230\angle -90° - 230\angle 150° = -j230 - 230$

$$\frac{\sqrt{3}}{2}\ \ j\frac{1}{2}\quad 230\sqrt{3}\ \ \frac{1}{2}\ \ j\frac{\sqrt{3}}{2}\quad 400\quad 60$$

(*v*) $V_{ca}\quad V_{cn}\quad V_{an}\quad 230\quad 150\quad 230\quad 30\quad \frac{\sqrt{3}}{2}\ \ j\frac{1}{2}\quad 230\ \frac{\sqrt{3}}{2}\ \ j\frac{1}{2}\quad 400\quad 400\quad 180$

These line voltages along with the phase voltages have been shown in the phasor diagram of Fig. 19.71 (*b*).

Example 19.65. *Three non-inductive resistances, each of 100* Ω *are connected in star to a 3-phase, 440-V supply. Three equal choking coils are also connected in delta to the same supply; the resistance of one coil being equal to 100* Ω*. Calculate* ***(a)*** *the line current and* ***(b)*** *the power factor of the system.* **(Elect. Technology-II, Sambal Univ.)**

Solution. The diagram of connections and the vector diagram of the *Y*-and Δ-connected impedances are shown in Fig. 19.72.

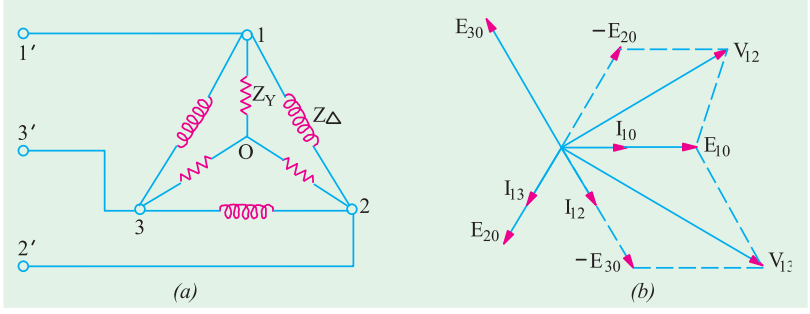


Fig. 19.72

The voltage $\mathbf{E}_{10}$ between line 1 and neutral is taken along the *X*-axis. Since the load is balanced, it will suffice to determine the current in one line only. Applying Kirchhoff's Law to junction 1, we have

$$\mathbf{I}'_{11} = \mathbf{I}_{10} + \mathbf{I}_{12} + \mathbf{I}_{13}$$

Let us first get the vector expressions for $\mathbf{E}_{10}$, $\mathbf{E}_{20}$ and $\mathbf{E}_{30}$

$$\mathbf{E}_{10} = \frac{440}{\sqrt{3}}(1 + j0) = 254 + j0,,\ \mathbf{E}_{20} = 254\ \ \frac{1}{2}\ \ j\frac{\sqrt{3}}{2}\ = -127 - j220$$

$$\mathbf{E}_{30} = 254\ \ \frac{1}{2}\ \ j\frac{\sqrt{3}}{2}\ = -127 + j220$$

Let us now derive vector expressions for $\mathbf{V}_{12}$ and $\mathbf{V}_{31}$.

$\mathbf{V}_{10} = \mathbf{E}_{10} + \mathbf{E}_{02} = \mathbf{E}_{10} - \mathbf{E}_{20} = (254 + j0) - (-127 - j220) = 381 + j220$

$\mathbf{V}_{13} = \mathbf{E}_{10} + \mathbf{E}_{03} = \mathbf{E}_{10} - \mathbf{E}_{30} = (254 - j0) - (-127 + j220) = 381 - j220$

$$\mathbf{I}_{10} = \frac{\mathbf{E}_{10}}{\mathbf{Z}_y}\quad \frac{254\quad j0}{100}\quad 2.54\quad j0,\ \mathbf{I}_{12} = \frac{\mathbf{V}_{13}}{\mathbf{Z}}\quad \frac{381\quad j220}{j100}\quad 2.2\quad j3.81\quad 4.4\quad 60$$

$$\mathbf{I}_{13} = \frac{V_{13}}{Z_\Delta} = \frac{381 - j220}{j100} = -2.2 - j3.81 = 4.4\angle -120°$$

(*a*) $\mathbf{I}'_{11} = (2.54 + j0) + (2.2 - j3.81) + (-2.2 - j3.81) = (2.54 - j7.62) = 8.03\ \angle\ -71.6°$

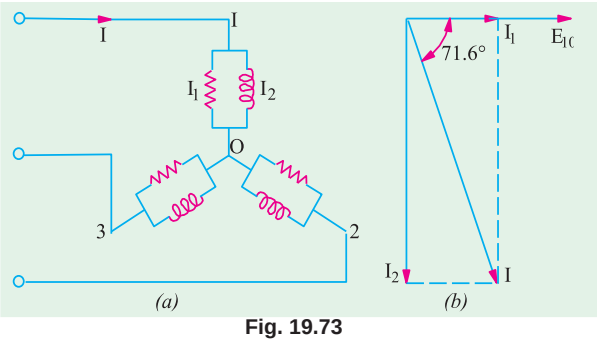


Fig. 19.73

(*b*) p.f. = cos 71.6° = **0.316 (lag)**

Alternative Method

This question may be easily solved by Δ/*Y* conversion. The star equivalent of the delta reactance is 100/3 Ω per phase.

As shown is Fig. 19.73, there are now two parallel circuits across each phase, one consisting of a resistance of 100 Ω and the other of a reactance of 100/3 Ω

Taking E_{10} as the reference vector, we have

$$\mathbf{E}_{10} = (254 + j0)$$

$$\mathbf{I}_1 \quad \frac{254 \quad j0}{100} \quad 2.54 \quad j0; \quad \mathbf{I}_2 \quad \frac{254 \quad j0}{j100/3} \quad j.7.62$$

Line current **I** = (2.54 + *j*0) + (–*j*7.62) = (2.54 – *j*7.62) = 8.03 ∠ – 71.6° ... Fig. 19.73 (*b*)

19.28. Unbalanced Loads

Any polyphase load in which the impedances in one or more phases differ from the impedances of other phases is said to be an unbalanced load. We will now consider different methods to handle unbalanced star-connected and delta-connected loads.

19.29. Unbalanced D-c onnected Load

Unlike unbalanced *Y*-connected load, the unbalanced Δ-connected load supplied from a balanced 3-phase supply does not present any new problems because the voltage across each load phase is fixed. It is independent of the nature of the load and is equal to line voltage. In fact, the problem resolves itself into three independent single-phase circuits supplied with voltages which are 120° apart in phase.

The different phase currents can be calculated in the usual manner and the three line currents are obtained by taking the vector difference of phase currents in pairs.

If the load consists of three different pure resistances, then trigonometrical method can be used with advantage, otherwise symbolic method may be used.

Example 19.66. *A 3-phase, 3-wire, 240 volt, CBA system supplies a delta-connected load in which* $Z_{AB} = 25 \angle 90°$, $Z_{BC} = 15 \angle 30°$, $Z_{CA} = 20 \angle 0°$ *ohms. Find the line currents and total power.* **(Advanced Elect. Machines A.M.I.E. Sec. B, Summer 1991)**

Solution. As explained in Art. 19.2, a 3-phase system has only two possible sequences : *ABC* and *CBA*. In the *ABC* sequence, the voltage of phase *B* lags behind voltage of phase *A* by 120° and that of phase *C* lags behind phase *A* voltage by 240°. In the *CBA* phase which can be written as A →*C* →*B*, voltage of *C* lags behind voltage *A* by 120° and that of *B* lags behind voltage *A* by 240°. Hence, the phase voltage which can be written as

$$E_{AB} = E \angle 0° \; ; \; E_{BC} = E \angle -120°$$

and $E_{CA} = E \angle -240°$ or $E_{CA} = \angle 120°$

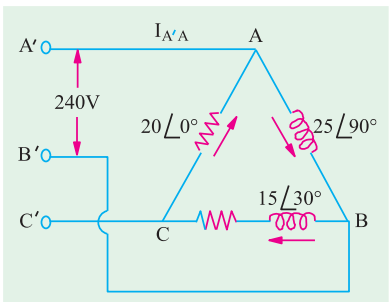


Fig. 19.74

$$\therefore \quad I_{AB} = \frac{E_{AB}}{Z_{AB}} = \frac{240\angle 0°}{25\angle 90°} = 9.6\angle -90° = -j9.6 \text{ A}$$

$$I_{BC} = \frac{E_{BC}}{Z_{BC}} = \frac{240\angle 120°}{15\angle 30°} = 16\angle 90° = j16 \text{ A}$$

$$I_{CA} = \frac{E_{CA}}{Z_{CA}} = \frac{240\angle -120°}{20\angle 0°} = 12\angle -120° = 12(0.5 - j0.866) = (-6 - j10.4) \text{ A}$$

The circuit is shown in Fig. 19.74.

Line current $I_{A'A} = I_{AB} + I_{AC} = I_{AB} - I_{CA} = -j9.6 - (-6 - j10.4) = 6 + j0.08$

Line current $I_{B'B} = I_{BC} - I_{AB} = j16 - (-j9.6) = j25.6$ A

$I_{C'C} = I_{CA} - I_{BC} = (-6 - j10.4) - j\ 16 = (-5 - j26.4)$ A

Now, $R_{AB} = 0$; $R_{BC} = 15 \cos 30 = 13\,\Omega$; $R_{CA} = 20\,\Omega$

Power

$W_{AB} = 0$; $W_{BC} = I_{BC}^{\ 2} R_{BC} = 16^2 \times 13 = 3328$ W; $W_{CA} = I_{CA}^{\ 2} \times R_{CA} = 27^2 \times 20 = 14{,}580$ W.

Total Power = 3328 + 14580 = 17,908 W.

Example 19.67. *In the network of Fig. 19.75, $E_{na} = 230\angle 0°$ and the phase sequence is abc. Find the line currents I_a, I_b and I_c as also the phase currents I_{AB}, I_{BC} and I_{CA}.*

E_{na}, E_{nb}, E_{nc} is a balanced three-phase voltage system with phase sequence abc.

(Network Theory, Nagpur Univ. 1993)

Solution. Since the phase sequence is *abc*, the generator phase voltages are:

$$E_{na} = 320\angle 0°; E_{nb} = 230\angle -120°; E_{nc} = 230\angle 120°$$

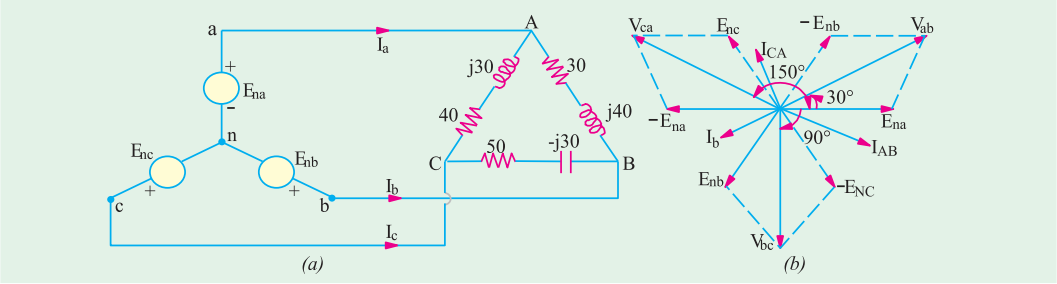


Fig. 19.75

As seen from the phasor diagram of Fig. 19.75 (*b*), the line voltages are as under :-

$V_{ab} = E_{na} - E_{nb}$; $V_{bc} = E_{nb} - E_{nc}$; $V_{ca} = E_{nc} - E_{na}$

$\therefore V_{ab} = \sqrt{3} \times 230\sqrt{30}° = 400\angle 30°$ *i.e* it is ahead of the reference generator phase voltage E_{na} by 30°.

$V_{bc} = \sqrt{3} \times 230\angle 90° = 400\angle -90°$. This voltage is 90° behind E_{na} but 120° behind V_{ab}.

$V_{ca} = \sqrt{3} \times 230\angle 150° = 400\angle 150°$ or $\angle -210°$. This voltage leads reference voltage E_{na} by 150° but leads V_{ab} by 120°.

These voltages are applied across the unbalanced Δ - connected lead as shown in Fig. 19.75 (*a*).

$Z_{AB} = 30 + j40 = 50\angle 53.1°; Z_{BC} = 50 - j30 = 58.3\angle -31°$,

$Z_{CA} = 40 + j30 = 50\angle 36.9°$

$$I_{AB} = \frac{V_{ab}}{Z_{AB}} = \frac{400\angle 30°}{50\angle 53.1°} = 8\angle -23.1° = 7.36 - j3.14$$

$$I_{BC} = \frac{V_{bc}}{Z_{BC}} = \frac{400\angle -90°}{58.3\angle -31°} = 6.86\angle -59° = 3.53 - j5.88$$

$$I_{CA} = \frac{V_{ca}}{Z_{CA}} = \frac{400\angle 150°}{50\angle 36.9°} = 8\angle 113.1° = 3.14 + j7.36$$

$$I_a = I_{AB} - I_{CA} = 7.36 - j3.14 + 3.14 - j7.36 = 10.5 - j10.5 = 14.85\angle -45°$$

$$I_b = I_{BC} - I_{AB} = 3.53 - j5.88 - 7.36 + j3.14 = -3.83 - j2.74 = 4.71\angle -215.6°$$

$$I_C = I_{CA} - I_{BC} = -3.14 + j7.36 - 3.53 + j5.88 = -6.67 + j13.24 = 14.8\angle 116.7°$$

Example 19.68. *For the unbalanced* Δ-connected *load of Fig. 19.76 (a), find, the phase currents, line currents and the total power consumed by the load when phase sequence is* **(a)** *abc and* **(b)** *acb.*

Solution. (*a*) Phase sequence abc (Fig. 19.76).

Let $\mathbf{V}_{ab} = 100\angle 0° = 100 + j0$

$\mathbf{V}_{bc}$ = 100 120 100 $\frac{1}{2}$ j$\frac{\sqrt{3}}{2}$ 50 j86.6

$\mathbf{V}_{ca}$ = 100 102 100 $\frac{1}{2}$ $j\frac{\sqrt{3}}{2}$ 50 j86.6

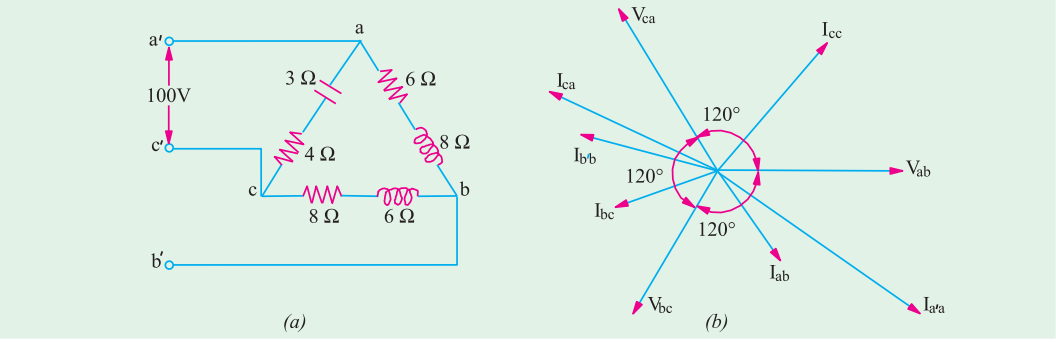


Fig. 19.76

(*i*) Phase currents

Phase current, $\mathbf{I}$ $\frac{\mathbf{V}_{ab}}{\mathbf{Z}_{ab}}$ $\frac{100 \quad j0}{6 \quad j8}$ 6 j8 10 53 8

Similarly, $\mathbf{I}_{bc}$ $\frac{\mathbf{V}_{bc}}{\mathbf{Z}_{bc}}$ $\frac{30 \quad j86.6}{8 \quad j6}$ 9.2 j3.93 10 156 52

$\mathbf{I}_{ca}$ $\frac{\mathbf{V}_{ca}}{\mathbf{Z}_{ca}}$ $\frac{50 \quad j86.6}{4 \quad j3}$ 18.39 j7.86 20 156 52

(*ii*) Line Currents

Line Current $\mathbf{I}_{a\ a}$ $\mathbf{I}_{ab}$ $\mathbf{I}_{ac}$ $\mathbf{I}_{ab}$ $\mathbf{I}_{ca}$ (6 j8) (18.39 j7.86)

= 24.39 – j15.86 = 29.1∠ – 33°2′

Similarly, $\mathbf{I}_{b\ b}$ $\mathbf{I}_{bc}$ $\mathbf{I}_{ba}$ $\mathbf{I}_{bc}$ $\mathbf{I}_{ab}$

(9.2 j3.93) (6 j8) 15.2 j4.07 15.73 165 30

$\mathbf{I}_{c\ c}$ $\mathbf{I}_{ca}$ $\mathbf{I}_{cb}$ $\mathbf{I}_{ca}$ $\mathbf{I}_{bc}$ (18.39 j7.86) (9.2 j3.93)

= 9.19 + j11.79 = 14.94∠52°3′

Check $\Sigma\mathbf{I} = 0 + j0$

(*iii*) Power

$$W_{ab} = I_{ab}^2 R_{ab} = 10^2 \times 6 = 600\,\text{W}$$

$$W_{bc} = I_{bc}^2 R_{bc} = 10^2 \times 8 = 800\,\text{W}$$

$$W_{ca} = I_{ca}^2 R_{ca} = 20^2 \times 4 = 1600\,\text{W}$$

Total = 3000 W

(*b*) Phase sequence acb (Fig. 19.77)

Here, $\mathbf{V}_{ab}$ 100 0 100 j9

$\mathbf{V}_{bc}$ 100 120 50 86.6

$\mathbf{V}_{ca}$ 100 120 50 *j*86.6

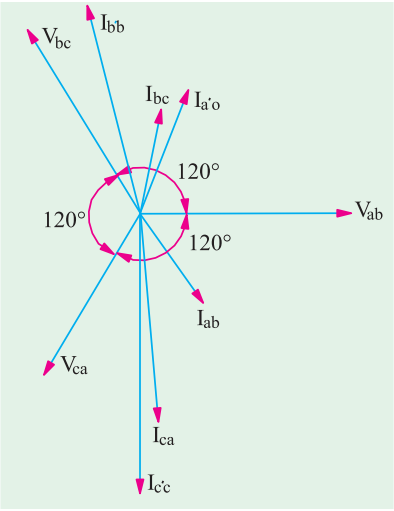


Fig. 19.77

(*i*) Phase Currents

$$\mathbf{I}_{ab} \quad \frac{100}{6 \quad j8} \quad 6 \quad j8 \quad 10 \quad 53\ 8$$

$$\mathbf{I}_{bc} \quad \frac{50 \quad j86.6}{8 \quad j6} = (1.2 + j9.93) = 10\angle 83°8'$$

$$\mathbf{I}_{ca} \quad \frac{50 \quad j86.6}{7 \quad j3} = (2.4 - j19.86) = 20 - 83°8'$$

(*ii*) Line Currents

$$\mathbf{I}_{a'a} = \mathbf{I}_{ab} + \mathbf{I}_{ac} = \mathbf{I}_{ab} - \mathbf{I}_{ca}$$

$$= (6 - j8) - (2.4 - j19.86) = (3.6 + j11.86) = 12.39\angle 73°6'$$

$\mathbf{I}_{bb}$ (1.2 *j*9.93) (6 *j*8) (4.8 *j*17.93) 18.56 105

$\mathbf{I}_{cc}$ (2.4 *j*19.86) (1.2 *j*9.93) (1.2 *j*29.79) 29.9 87 42

It is seen that $\Sigma\mathbf{I} = 0 + j0$

(*iii*) Power

$$W_{ab} = 10^2 \times 6 = 600\,\text{W}$$

$$W_{bc} = 10^2 \times 8 = 800\,\text{W}$$

$$W_{ca} = 20^2 \times 4 = 1600\,\text{W}$$

Total = 3000 W – as before

It will be seen that the effect of phase reversal on an unbalanced Δ-connected load is as under:

(*i*) phase currents change in angle only, their magnitudes remaining the same

(*ii*) consequently, phase powers remain unchanged

(*iii*) line currents change both in magnitude and angle.

The adjoining tabulation emphasizes the effect of phase sequence on the line currents drawn by an unbalanced 3-phase load.

Line	*Ampere Sequence a b c*	*Sequence c b a*
a	$29.1\angle -33°2'$	$12.39\angle 73.1°$
b	$15.73\angle 165°$	$18.56\angle 105°$
c	$14.94\angle 52°3'$	$29.9\angle -87.7°$

Example 19.69. *A balanced 3-phase supplies an unbalanced 3-phase delta-connected load made up of to resistors 100 Ω and a reactor having an inductance of 0.3 H with negligible resistance. V_L = 100 V at 50 Hz. Calculate (a) the total power in the system.*

(Elect. Engineering-I, Madras Univ.)

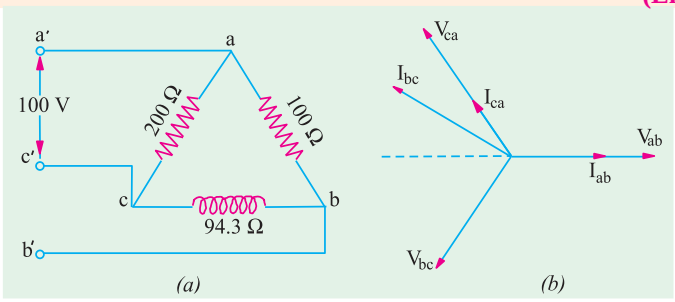


Fig. 19.78

Solution. The Δ-connected load and its phasor diagram are shown in Fig. 19.78 (*a*).

$$X_L = \omega L = 314.2 \times 0.2 = 94.3\Omega$$

Let $\mathbf{V}_{ab}$ 100 0 100 *j*0

$\mathbf{V}_{bc}$ 100 120

$= -50 - j86.6$

$\mathbf{V}_{ca}$ 100 120 50 *j*86.6

$\mathbf{I}_{ab}$ $\dfrac{V_{ab}}{Z_{ab}}$ $\dfrac{100\ 0}{100\ 0}$ 13 0 1 *j*0

$\mathbf{I}_{bc}$ $\dfrac{100\ 0}{100\ 0}$ 1.06 210 0.92 *j*0.53

$\mathbf{I}_{ca}$ $\dfrac{100\ 120}{200\ 0}$ 0.5 120 0.25 *j*0.43

Watts in branch $ab = V_{ab}^2 / R_{ab} = 100^2 / 100$ = 100 W; VARs = 0

Watts in branch $bc = 0$; VARs = 100 × 1.06 = 106 (lag)

Watts in branch $ca = V_{ca}^2 / R_{ca} = 100^2 / 200$ = 50 W; VRAs = 0

(a) Total power = 100 + 50 = 150 W; VARs = **106 (lag)**

19.30. Four-wire Star-connected Unbalanced Load

It is the simplest case of an unbalanced load and may be treated as three separate single-phase systems with a common return wire. It will be assumed that impedance of the line wires and source phase windings is zero. Should such an assumption be unacceptable, these impedances can be added to the load impedances. Under these conditions, source and load line terminals are at the same potential.

Consider the following two cases:

(*i*) Neutral wire of zero impedance

Because of the presence of neutral wire (assumed to behaving zero impedance), the star points of the generator and load are tied together and are at the same potential. Hence, the voltages across the three load impedances are ***equalized*** and each is equal to the voltage of the corresponding phase of the generator. In other words, due to the provision of the neutral, each phase voltage is a ***forced*** voltage so that the three phase voltages are balanced when line voltages are balanced even though phase impedances are unbalanced. However, it is worth noting that a break or open ($\mathbf{Z}_N = \infty$) in the neutral wire of a 3-phase, 2-wire system with ***unbalanced*** load always causes large (in most cases inadmissible) changes in currents and phase voltages. It is because of this reason that no fuses and circuit breakers are ever used in the neutral wire of such a 3-phase system.

The solution for currents follows a pattern similar to that for the unbalanced delta.

Obviously, the vector sum of the currents in the three lines is not zero but is equal to neutral current.

(*ii*) Neutral wire with impedance Z_N

Such a case can be easily solved with the help of Node-pair Voltage method as detailed below. Consider the general case of a *Y*- to -*Y* system with a neutral wire of impedance Z_n as shown in Fig. 19.79 (*a*). As before, the impedance of line wires and source phase windings would be assumed to be zero so that the line and load terminals, *R*,*Y*, *B* and R', Y', B' are the same respective potentials.

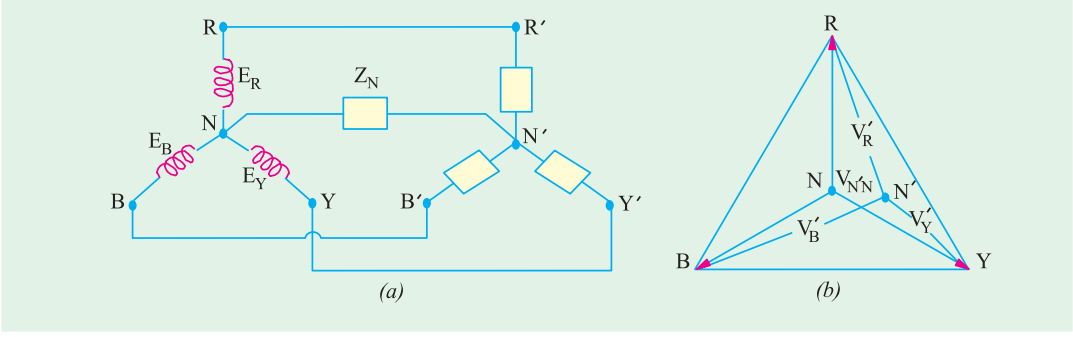


Fig. 19.79

According to Node-pair Voltage method, the above star-to-star system can be looked upon as multi-mesh network with a single pair of nodes *i.e.* neutral points *N* and *N*'. The node potential *i.e.* the potential difference between the supply and local neutrals is given by

$$V'_{NN} = \frac{E_R Y_R + E_Y Y_Y + E_B Y_B}{Y_R + Y_Y + Y_B + Y_N}$$

where $\mathbf{Y}_R$, $\mathbf{Y}_Y$ and $\mathbf{Y}_B$ represent the ***load phase*** admittances. Obviously, the load neutral N' does not coincide with source neutral *N*. Hence, load phase voltages are no longer equal to one another even when phase voltages are as seen from Fig. 19.79 (*b*).

The load phase voltage are given by

$$V'_R = E_R - V'_{NN;} V'_Y = E_Y - V_{NN} \text{ and } V_{B'} = E_B - V'_{NN}$$

The phase currents are

$$\mathbf{I}_R = \mathbf{V}'_R \mathbf{Y}_R,\ \mathbf{I}'_Y = \mathbf{V}'_Y \mathbf{Y}_Y \text{ and } \mathbf{I}_B = \mathbf{V}'_B \mathbf{Y}_B$$

The current in the neutral wire is $I_N = V_N Y_N$

Note. In the above calculations, $I_R \quad I_{R'} \quad I_{RR}$

Similarly, $I_Y = I'_Y = I'_{YY}$ and $I'_B = I_{BB}$.

Example 19.70. *A 3-phase, 4-wire system having a 254-V line-to-neutral has the following loads connected between the respective lines and neutral;* $Z_R = 10\angle 0°$ *ohm;* $Z_Y = 10\angle 37°$ *ohm and* $Z_B = 10\angle -53°$ *ohm. Calculate the current in the neutral wire and the power taken by each load when phase sequence is* ***(i)*** *RYB and* ***(ii)*** *RBY.*

Solution. (*i*) Phase sequence RYB (Fig. 19.80)

$\mathbf{V}_{RN} \quad 254 \quad 0\ ;\ \mathbf{V}_{YN} \quad 254 \quad 120\ ;\ \mathbf{V}_{BN} \quad 254 \quad 120$

$$I_R \quad I_{RN} \quad \frac{V_{RN}}{R_R} \quad \frac{254 \quad 0}{10 \quad 0} \quad 25.4 \quad 0 \ *$$

$$\mathbf{I}_r = \mathbf{I}_{YN} = \frac{254\angle -120°}{10\angle 37°} = 25.4\angle -157° = 25.4(-0.9205 - j0.3907) = -23.38 - j9.95$$

* This method is similar to Millman's Theorem of Art. 19.32.

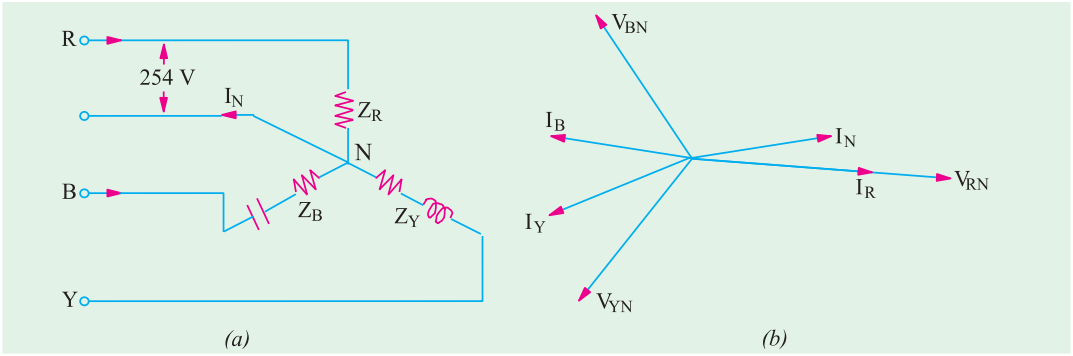


Fig. 19.80

$$I_B = I_{BN} = \frac{254\angle 120°}{10\angle -53°} = 25.4\angle 173° = 25.4(-0.9925 + j0.1219) = -25.2 + j3.1$$

$$I_N = -(I_R + I_Y + I_B) = -[25.4 + (-23.38 - j9.55) + (-25.21 + j3.1)] = 23.49 + j6.85$$
$$= \mathbf{24.46\angle 16°15'}$$

Now $R_R = 10\,\Omega; R_Y = 10\cos 37° = 8\Omega; R_B = 10\cos 53° = 6\Omega$

W_R 25.4^2 10 **6.452 W;** W_Y 25.4^2 8 **5,162 W**

W_B 25.4^2 6 **3,871 W**

(*ii*) Phase sequence RBY [Fig. 19.81]

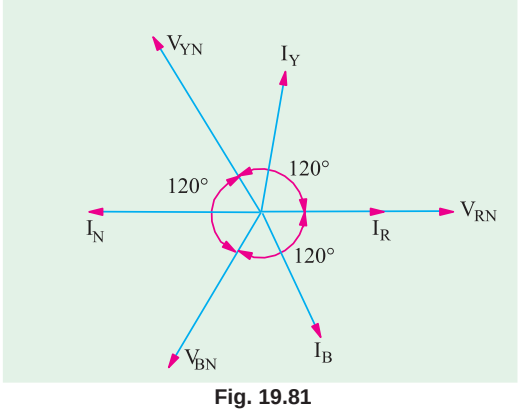


Fig. 19.81

V_{RN} 254 0 ; V_{YN} 254 120

V_{BN} 254 120

I_R $\frac{254\ \ 0}{10\ \ 0}$ 25.4 0

I_Y $\frac{254\ \ 120}{10\ \ 37}$ 25.4 83

$= (3.1 + j25.2)I_B$

I_B $\frac{254\ \ 120}{10\ \ 53}$ 25.4 67 (9.95 j23.4)

$\mathbf{I}_N$ ($\mathbf{I}_R$ $\mathbf{I}_Y$ $\mathbf{I}_B$) (38.45 j1.8)

$= -38.45 - j1.8 = 38.5\angle -177.3°$.

Obviously, power would remain the same because magnitude of branch currents is unaltered. From the above, we conclude that phase reversal in the case of a 4-wire unbalanced load supplied from a balanced voltage system leads to the following changes:

(*i*) it changes the angles of phase currents but not their magnitudes.

(*ii*) however, power remains unchanged.

(*iii*) it changes the magnitude as well as angle of the neutral current $\mathbf{I}_N$.

Example 19.71. *A* $3-\phi$, *4-wire, 380-V supply is connected to an unbalanced load having phase impedances of:* $Z_R = (8 + j6)\ \Omega$, $Z_y = (8 - j6)\ \Omega$ *and* $Z_B = 5\,\Omega$. *Impedance of the neutral wire is* $Z_N = (0.5 + j1)\,\Omega$.

Ignoring the impedances of line wires and internal impedances of the e.m.f. sources, find the phase currents and voltages of the load.

Solution. This question will be solved by using Node-pair Voltage method discussed in Art. 19.30. The admittances of the various branches connected between nodes *N* and *N*' in Fig. 19.82 (*a*).

$\mathbf{Y}_R = 1/\mathbf{Z}_R = 1/(8 + j6) = (0.08 - j0.06)$
$\mathbf{Y}_R = 1/\mathbf{Z}_R = 1/(8 - j6) = (0.08 + j0.06)$
$\mathbf{Y}_Y = 1/2/\mathbf{Z}_B = 1/(5 + j0) = 0.2$
$\mathbf{Y}_B = 1/\mathbf{Z}_N = 1/(0.5 + j1) = (0.4 - j0.8)$

Let $\mathbf{E}_R = (380/\sqrt{3})\angle 0° = 220\angle 0° = 220 + j0$

$\mathbf{E}_Y = 220\angle -120° = 220(-0.5 - j0.866) = -110 - j190$

$\mathbf{E}_B = 220\angle 120° = 220(-0.5 + j0.866) = -110 + j190$

The node voltage between N' and N is given by

$$\boldsymbol{V}'_{NN} = \frac{\boldsymbol{E}_R\boldsymbol{Y}_R + \boldsymbol{E}_Y\boldsymbol{Y}_Y + \boldsymbol{E}_B\boldsymbol{Y}_B}{\boldsymbol{Y}_R + \boldsymbol{Y}_Y + \boldsymbol{Y}_B + \boldsymbol{Y}_N}$$

$$= \frac{200(0.08 - j0.06) + (-110 - j190)(0.08 + j0.06) + (-110 + j190)\times 0.2}{(0.08 - j0.06) + (0.08 + j0.06) + 0.2 + (0.4 - j0.8)}$$

$$= \frac{-1.8 + j3}{0.76 - j0.8} = -3.41 + j0.76$$

The three load phase voltages are as under:

$\mathbf{V}_R \quad \mathbf{E}_R \quad \mathbf{V}'_{NN} \quad 220 \quad 3.41 \quad j0.76 \quad 223.41 \quad j0.76$

$\mathbf{V}_Y \quad \mathbf{E}_Y \quad \mathbf{V}'_{NN} \quad (\ 110 \quad j90) \quad 3.41 \quad j0.76 \quad 106.59 \quad j190.76$

$\mathbf{V}_B \quad \mathbf{E}_B \quad \mathbf{V}_{NN} \quad (\ 110 \quad j90) \quad 3.41 \quad j0.76 \quad 106.59 \quad j190.76$

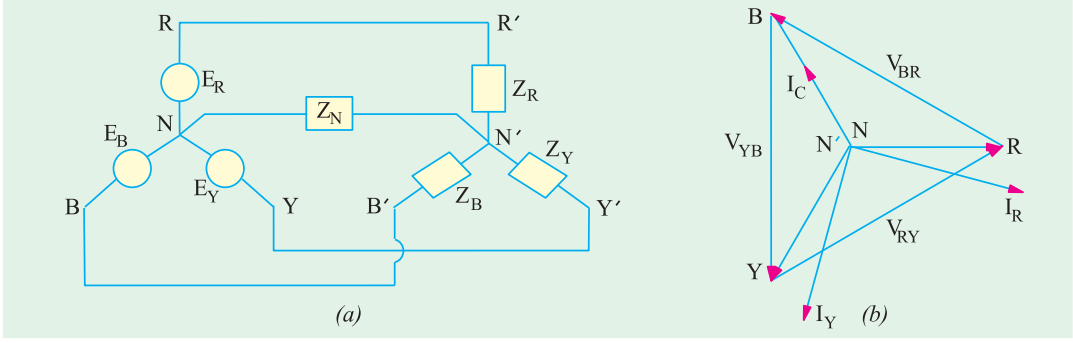


Fig 19.82

$\mathbf{I}_R \quad \mathbf{V}_R\mathbf{Y}_R \quad (223.41 \quad j0.76)(0.08 \quad j0.06) \quad 17.83 \quad j13.1 \quad 22.1 \quad 36.3 \text{ A}$

$\mathbf{I}_Y \quad \mathbf{V}_Y\mathbf{Y}_Y \quad (\ 106.59 \quad j190.76)(0.08 \quad j0.06) \quad 2.92 \quad j21.66 \quad 21.86 \quad 82.4 \text{ A}$

$\mathbf{I}_B \quad \mathbf{V}_B\mathbf{Y}_B \quad (\ 106.59 \quad j190.76) \quad 0.2 \quad 21.33 \quad j37.85 \quad 43.45 \quad 119.4 \text{ A}$

$\mathbf{I}_N \quad \mathbf{V}_N\mathbf{Y}_N \quad (\ 3.41 \quad j0.76)(0.4 \quad j0.8) \quad 0.76 \quad j3.03 \quad 3.12 \quad 104.1 \text{ A}$

These voltage and currents are shown in the phasor diagram of Fig. 19.82 (b) where displacement of the neutral point has not been shown due to the low value of $\boldsymbol{V}'_{NN}$.

Note. It can be shown that $\boldsymbol{I}_N = \boldsymbol{I}'_R + \boldsymbol{I}'_Y + \boldsymbol{I}'_B$

19.31. Unbalanced Y-connected Load Without Neutral

When a star-connected load is unbalanced and it has no neutral wire. Then its star point is isolated from the star-point of the generator. The potential of the load star-point is different from that of the generator star-point. The potential of the former is subject to variations according to the imbalance of the load and under certain conditions of loading, the potentials of the two star- point may differ considerably. Such an isolated load star-point or neutral point is called 'floating' neutral point because its potential is always changing and is not fixed.

All *Y*-connected unbalanced loads supplied from polyphase systems have floating neutral points without a neutral wire. Any unbalancing of the load causes variations not only of the potential of the star-point but also of the voltages across the different branches of the load. ***Hence, in that case, phase voltage of the load is not*** $1/\sqrt{3}$ ***of the line voltage.***

There are many methods to tackle such unbalanced Y-connected loads having isolated neutral points.

1. By converting the *Y*-connected load to an equivalent unbalanced Δ-connected load by using Y-Δ conversion theorem. The equivalent Δ-connection can be solved in Fig. 19.80. The line currents so calculated are equal in magnitude and phase to those taken by the original unbalanced *Y*-connected load.

2. By applying Kirchhoff's Laws.

3. By applying Millman's Theorem.

4. By using Maxwell's Mesh or Loop Current Method.

19.32. Millman's Theorem

Fig. 19.83 shows a number of linear bilateral admittances, Y_1, Y_2, ... connected to a common point or node O'. The voltages of the free ends of these admittances with respect to another common point O are V_{10}, V_{20} ... V_{no}. Then, according to this theorem, the voltage of O' with respect to O is given by $\mathbf{V}'_{00} = \dfrac{\mathbf{V}_{10}\mathbf{Y}_1 + \mathbf{V}_{20}\mathbf{Y}_2 + \mathbf{V}_{30}\mathbf{Y}_3 + .. \mathbf{V}_{no}\mathbf{Y}_n}{\mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3 + ... \mathbf{Y}_n}$

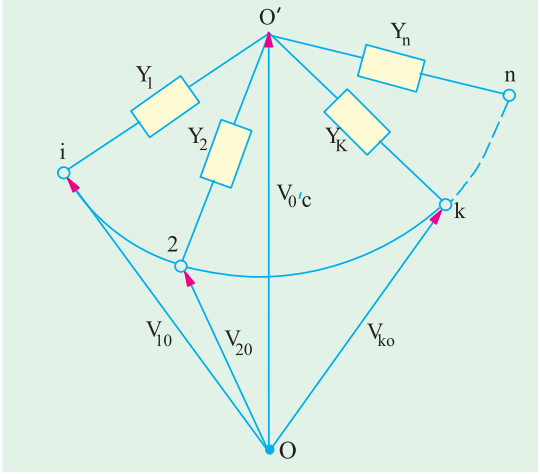


Fig. 19.83

or $\boldsymbol{V}'_{00} = \dfrac{\sum_{k=1}^{n} \boldsymbol{V}_{k0}\boldsymbol{Y}_k}{\sum_{k=1}^{n} \boldsymbol{Y}_k}$

Proof. Consider the closed loop 0' *Ok*. The sum of p.ds. around it is zero. Starting from O' and going anticlockwise, we have

$\boldsymbol{V}'_{00} + \boldsymbol{V}_{ok} + \boldsymbol{V}'_{ko} = 0$

$\therefore \quad \boldsymbol{V}'_{ko} = -\boldsymbol{V}_{ok} - \boldsymbol{V}'_{00} = \boldsymbol{V}_{ko} - \boldsymbol{V}'_{00}$

Current through $\boldsymbol{Y}_k$ is $\boldsymbol{I}'_{ko}$

$= \boldsymbol{V}'_{ko}\boldsymbol{Y}_k = (\boldsymbol{V}_{ko} - \boldsymbol{V}'_{00})\boldsymbol{Y}_k =$

By Kirchhoff's Current Law, sum of currents meeting at point O' is zero.

$\therefore \quad \boldsymbol{I}'_{10} + \boldsymbol{I}'_{20} + ... \boldsymbol{I}'_{ko} .... + \boldsymbol{I}'_{no} = 0$

$$(\mathbf{V}'_{10} - \mathbf{V}'_{00})\mathbf{Y}_1 + (\mathbf{V}_{20} - \mathbf{V}'_{00})\mathbf{Y}_2 + ...(\mathbf{V}_{ko} - \mathbf{V}'_{00})\mathbf{Y}_k + ... = 0$$

or $$\mathbf{V}'_{10}\mathbf{Y}_1 + \mathbf{V}_{20}\mathbf{Y}_2 + ...\mathbf{V}_{ko}\mathbf{Y}_k + ... = \mathbf{V}'_{00}(\mathbf{Y}_1 + \mathbf{Y}_2 + ... + \mathbf{Y}_k + ...)$$

$$\boldsymbol{V}'_{00} = \frac{\boldsymbol{V}_{10}\boldsymbol{Y}_1 + \boldsymbol{V}_{20}\boldsymbol{Y}_2 + ...}{\boldsymbol{Y}_1 + \boldsymbol{Y}_2 + ...}$$

19.33. Application of Kirchhoff's Laws

Consider the unbalanced *Y*-connected load of Fig. 19.84. Since the common point of the three load impedances is not at the potential of the neutral, it is marked 0' instead of *N**. Let us assume the

* For the sake of avoiding printing difficulties, we will take the load star point as 0 instead of 0' for this article.

phase sequence V_{ab}, V_{bc}, V_{ca} *i.e.* V_{ab} leads V_{bc} and V_{bc} leads V_{ca}. Let the three branch impedances be Z_{ca}, Z_{ab} and Z_{ac}, however, since double subscript notation is not necessary for a *Y*-connected impedances in order to indicate to which phase it belongs, single-subscript notation may be used with advantage. Therefore Z_{oa}, Z_{ab}, Z_{oc} can be written as Z_a, Z_b, Z_c respectively. It may be pointed out that double-subscript notation is essential for mesh-connected impedances in order to make them definite.

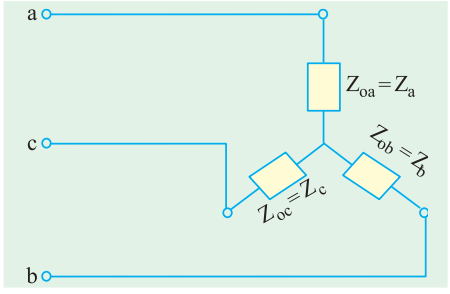


Fig. 19.84

From Kirchhoff's laws, we obtain

$$V_{ab} = I_{ao}Z_a + I_{ab}Z_b \quad \text{... (1)}$$

$$V_{bc} = I_{bo}Z_b + I_{oc}Z_c \quad \text{... (2)}$$

$$V_{ca} = I_{co}Z_c + I_{oa}Z_a \quad \text{... (3)}$$

and $I_{aa} + I_{ba} + I_{co} = 0$ – point law ... (4)

Equation (1), (3) and (4) can be used for finding I_{bo}.

Adding (1) and (3), we get

$$V_{ab} + V_{ca} = I_{ao}Z_a + I_{ao}Z_b + I_{co}Z_c + I_{oc}Z_a$$

$$= I_{ob}Z_b + I_{co}Z_c + I_{oa}Z_a - I_{oa}Z_a = I_{ob}Z_b + I_{co}Z_c \quad \text{... (5)}$$

Substituting $\mathbf{I}_{oa}$ from equation (4) in equation (3), we get

$$V_{ca} = I_{co}Z_c + (I_{bo} + I_{co})Z_a = I_{co}(Z_c + Z_a) + I_{bo}Z_a \quad \text{... (6)}$$

Putting the value of I_{co} from equation (5) in equation (6), we have

$$V_{ca} = (Z_c + Z_a)\frac{(V_{ab} + V_{ca}) - I_{ob}Z_b}{Z_c} + I_{bo}Z_a$$

$$V_{ca}Z_c = -I_{ob}Z_bZ_c - I_{ob}Z_bZ_a + I_{bo}Z_aZ_c + V_{ab}Z_a + V_{ab}Z_c + V_{ca}Z_c + V_{ca}Z_a$$

$$\therefore \quad I_{ob} = \frac{(V_{ab} + V_{ca})Z_a + V_{ab}Z_a}{Z_aZ_b + Z_bZ_c + Z_cZ_a}$$

Since $V_{ab} + V_{bc} + V_{ca} = 0$ $\quad \therefore \quad I_{ob} = \dfrac{V_{ab}Z_c - V_{bc}Z_a}{Z_aZ_b + Z_bZ_c + Z_cZ_a}$ ... (7)

From the symmetry of the above equation, the expressions for the other branch currents are,

$$I_{oc} = \frac{V_{bc}Z_a - V_{ca}Z_b}{Z_aZ_b + Z_bZ_c + Z_cZ_a} \quad \text{... (8)} \qquad I_{oa} = \frac{V_{ca}Z_b - V_{ab}Z_c}{Z_aZ_b + Z_bZ_c + Z_cZ_a} \quad \text{... (9)}$$

Note. Obviously, the three line currents can be written as

$$I_{ao} = -I_{oa} = \frac{V_{ab}Z_c - V_{ca}Z_b}{\sum Z_aZ_b} \quad \text{... (10)} \qquad I_{bo} = -I_{ob}\frac{V_{bc}Z_a - V_{ab}Z_c}{\sum Z_aZ_b} \quad \text{... (11)}$$

$$I_{co} = -I_{oc}\frac{V_{ca}Z_b - V_{bc}Z_a}{\sum Z_aZ_b} \quad \text{... (12)}$$

Example 19.72. *If in the unbalanced Y-connected load of Fig. 19.78, Z_a = (10 + j0), Z_b = (3 + j4) and Z_c = (0 – j10) and the load is put across a 3-phase, 200-V circuit with balanced voltages, find the three line currents and voltages across each branch impedance. Assume phase sequence of V_{ab}, V_{bc}, V_{ca}.*

Solution. Take V_{ab} along the axis of reference. The vector expressions for the three voltages are

$$V_{ab} = 200 + j0$$

$$V_{bc} \quad 200 \quad \frac{1}{2} \quad j\frac{\sqrt{3}}{2} \quad 100 \quad j173.2; \quad V_{ca} \quad 200 \quad \frac{1}{2} \quad j\frac{\sqrt{3}}{2} \quad 100 \quad j173.2$$

From equation **(9)** given above

$$\mathbf{I}_{oa} \quad \frac{(\ 100 \quad j173.2)(3 \quad j4) \quad (200 \quad j0)(0 \quad j10)}{(0 \quad j10)(10 \quad j0) \quad (10 \quad j0)(3 \quad j4) \quad (3 \quad j4)(0 \quad j10)}$$

$$= \frac{-992.8 + j2119.6}{70 - j90} = -20.02 + j4.54$$

$$\mathbf{I}_{ob} \quad \frac{(200 \quad j0)(0 \quad j10) \quad (\ 100 \quad j173.2)(10 \quad j0)}{(10 \quad j0)(3 \quad j4) \quad (3 \quad j4)(0 \quad j10) \quad (0 \quad j10)(10 \quad j0)}$$

$$= \frac{1000 - j268}{70 - j90} = 7.24 + j5.48$$

Now, $\mathbf{I}_{oc}$ may also be calculated in the same way or it can be found easily from equation (4) of Art. 19.33.

$$\mathbf{I}_{oc} = \mathbf{I}_{ao} + \mathbf{I}_{bo} = -\mathbf{I}_{oa} - \mathbf{I}_{ob} = 20.02 - j4.54 - 7.24 - j5.48 = 12.78 - j10.02$$

Now
$$\mathbf{V}_{oa} = \mathbf{I}_{oa}\mathbf{Z}_a = (-20.02 + j4.54)(10 + j0) = 200.2 + j45.4$$
$$\mathbf{V}_{ob} = \mathbf{I}_{ob}\mathbf{Z}_b = (7.24 + j5.48)(3 + j4) = -0.2 + j45.4$$
$$\mathbf{V}_{oc} = \mathbf{I}_{oc}\mathbf{Z}_c = (12.78 - j10.02)(0 - j10) = -100.2 - j127.8$$

As a check, we may combine $\mathbf{V}_{oa}$, $\mathbf{V}_{ob}$ and $\mathbf{V}_{oc}$ to get the line voltages which should be equal to the applied line voltages. In passing from a to b through the circuit internally, we find that we are in opposition to $\mathbf{V}_{oa}$ but in the same direction as the positive direction of $\mathbf{V}_{ob}$.

$$\mathbf{V}_{ab} \quad \mathbf{V}_{ao} \quad \mathbf{V}_{ob} \quad \mathbf{V}_{oa} \quad \mathbf{V}_{ob} \quad (\ 200.2 \quad j45.4) \quad (\ 0.2 \quad j45.4) \quad 200 \quad j0$$

$$\mathbf{V}_{bc} \quad \mathbf{V}_{bo} \quad \mathbf{V}_{oc} \quad \mathbf{V}_{ob} \quad \mathbf{V}_{oc} \quad (\ 0.2 \quad j45.4) \quad (\ 100.2 \quad j127.8) \quad 100 \quad j173.2$$

$$\mathbf{V}_{ca} \quad \mathbf{V}_{co} \quad \mathbf{V}_{oa} \quad \mathbf{V}_{oc} \quad \mathbf{V}_{oa} \quad (\ 100.2 \quad j127.8) \quad (\ 200.2 \quad j45.4) \quad 100 \quad j173.2$$

19.34. Delta/Star and Star/Delta Conversions

Let us consider the unbalanced Δ-connected load of Fig. 19.85 (*a*) and *Y*-connected load of Fig. 19.85 (*b*). If the two systems are to be equivalent, then the impedances between corresponding pairs of terminals must be the same.

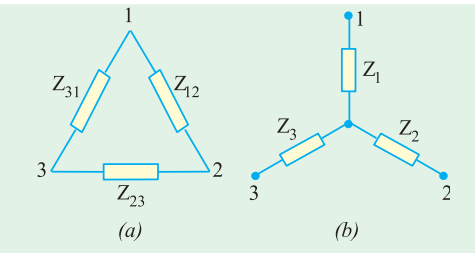


Fig. 19.85

(*i*) Delta/Star Conversion

For *Y*-load, total impedance between terminals 1 and 2 is = $\mathbf{Z}_1 + \mathbf{Z}_2$ (it should be noted that double subscript notation of $\mathbf{Z}_{01}$ and $\mathbf{Z}_{02}$ has been purposely avoided).

Considering terminals 1 and 2 of Δ-load, we find that there are two parallel paths having impedances of $\mathbf{Z}_{12}$ and $(\mathbf{Z}_{31} + \mathbf{Z}_{23})$. Hence, the equivalent impedance between terminals 1 and 2 is given by

$$\frac{1}{\mathbf{Z}} = \frac{1}{\mathbf{Z}_{12}} + \frac{1}{\mathbf{Z}_{23} + \mathbf{Z}_{31}} \quad \text{or} \quad \mathbf{Z} = \frac{\mathbf{Z}_{12}(\mathbf{Z}_{23} + \mathbf{Z}_{31})}{\mathbf{Z}_{12} + \mathbf{Z}_{23} + \mathbf{Z}_{31}}$$

Therefore, for equivalence between the two systems $\mathbf{Z}_1 + \mathbf{Z}_2 = \dfrac{\mathbf{Z}_{12}(\mathbf{Z}_{23} + \mathbf{Z}_{31})}{\mathbf{Z}_{12} + \mathbf{Z}_{23} + \mathbf{Z}_{31}}$ ... **(1)**

Similarly $\mathbf{Z}_2 + \mathbf{Z}_3 = \dfrac{\mathbf{Z}_{23}(\mathbf{Z}_{31} + \mathbf{Z}_{12})}{\mathbf{Z}_{12} + \mathbf{Z}_{23} + \mathbf{Z}_{31}}$ ... (2) $\mathbf{Z}_3 + \mathbf{Z}_1 = \dfrac{\mathbf{Z}_{31}(\mathbf{Z}_{12} + \mathbf{Z}_{23})}{\mathbf{Z}_{12} + \mathbf{Z}_{23} + \mathbf{Z}_{31}}$ ... (3)

Adding equation (3) to (1) and subtracting equation (2), we get

$$2\mathbf{Z}_1 = \frac{\mathbf{Z}_{12}(\mathbf{Z}_{23} + \mathbf{Z}_{31}) + \mathbf{Z}_{31}(\mathbf{Z}_{12} + \mathbf{Z}_{23}) - \mathbf{Z}_{23}(\mathbf{Z}_{31} + \mathbf{Z}_{12})}{\mathbf{Z}_{12} + \mathbf{Z}_{23} + \mathbf{Z}_{31}} = \frac{2\mathbf{Z}_{12}\mathbf{Z}_{31}}{\mathbf{Z}_{12} + \mathbf{Z}_{23} + \mathbf{Z}_{31}}$$

$$\therefore \quad \mathbf{Z}_1 = \frac{\mathbf{Z}_{12}\mathbf{Z}_{31}}{\mathbf{Z}_{12} + \mathbf{Z}_{23} + \mathbf{Z}_{31}} \quad \text{... (4)}$$

The other two results may be written down by changing the subscripts cyclically

$$\therefore \quad \mathbf{Z}_2 = \frac{\mathbf{Z}_{23}\mathbf{Z}_{12}}{\mathbf{Z}_{12} + \mathbf{Z}_{23} + \mathbf{Z}_{31}}; \quad \text{... (5)} \qquad \mathbf{Z}_3 = \frac{\mathbf{Z}_{31}\mathbf{Z}_{23}}{\mathbf{Z}_{12} + \mathbf{Z}_{23} + \mathbf{Z}_{31}} \quad \text{... (6)}$$

The above expression can be easily obtained by remembering that (Art. 2.19)

$$\text{Start } \mathbf{Z} = \frac{\text{Product of } \Delta\,\mathbf{Z}\text{'s connected to the same terminals}}{\text{Sum of } \Delta\,\mathbf{Z}\text{'s}}$$

In should be noted that all **Z'** are to be expressed in their complex form.

(*iii*) Star/Delta Conversion

The equations for this conversion can be obtained by rearranging equations **(4), (5)** and **(6)**, Rewriting these equations, we get

$$\mathbf{Z}_1(\mathbf{Z}_{12} + \mathbf{Z}_{23} + \mathbf{Z}_{31}) = \mathbf{Z}_{12}\mathbf{Z}_{31} \quad \text{... (7)}$$
$$\mathbf{Z}_2(\mathbf{Z}_{12} + \mathbf{Z}_{23} + \mathbf{Z}_{31}) = \mathbf{Z}_{23}\mathbf{Z}_{12} \quad \text{... (8)}$$
$$\mathbf{Z}_3(\mathbf{Z}_{12} + \mathbf{Z}_{23} + \mathbf{Z}_{31}) = \mathbf{Z}_{31}\mathbf{Z}_{23} \quad \text{... (9)}$$

Dividing equation (7) by (9), we get $\dfrac{\mathbf{Z}_1}{\mathbf{Z}_3} = \dfrac{\mathbf{Z}_{12}}{\mathbf{Z}_{23}}$ $\therefore \quad \mathbf{Z}_{23} = \mathbf{Z}_{12}\dfrac{\mathbf{Z}_3}{\mathbf{Z}_1}$

Dividing equation (8) by (9), we get $\dfrac{\mathbf{Z}_2}{\mathbf{Z}_3} = \dfrac{\mathbf{Z}_{12}}{\mathbf{Z}_{31}}$ $\therefore \quad \mathbf{Z}_{31} = \mathbf{Z}_{12}\dfrac{\mathbf{Z}_3}{\mathbf{Z}_2}$

Substituting these values in equation (7), we have $\mathbf{Z}_1 \quad \mathbf{Z}_{12} \quad \mathbf{Z}_{12}\dfrac{\mathbf{Z}_3}{\mathbf{Z}_1} \quad \mathbf{Z}_{31} \quad \mathbf{Z}_{12}.\mathbf{Z}_{12}\dfrac{\mathbf{Z}_3}{\mathbf{Z}_2}$

or $\mathbf{Z}_1\mathbf{Z}_{12} \quad 1 + \dfrac{\mathbf{Z}_3}{\mathbf{Z}_1} + \dfrac{\mathbf{Z}_{31}}{\mathbf{Z}_{12}} = \mathbf{Z}_{12}\mathbf{Z}_{12}\dfrac{\mathbf{Z}_3}{\mathbf{Z}_2}$ $\therefore$ $\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1 = \mathbf{Z}_{12} \times \mathbf{Z}_3$

$$\mathbf{Z}_{12} = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_3} \text{ or } \mathbf{Z}_{12} = \mathbf{Z}_1 + \mathbf{Z}_2 + \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_3}$$

Similarly, $\mathbf{Z}_{23} = \mathbf{Z}_2 + \mathbf{Z}_3 + \dfrac{\mathbf{Z}_2\mathbf{Z}_3}{\mathbf{Z}_1} = \dfrac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_1}$

$$\mathbf{Z}_{31} = \mathbf{Z}_3 + \mathbf{Z}_1 + \frac{\mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_2} = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_2}$$

As in the previous case, it is to be noted that all impedances must be expressed in their complex form.

Another point for noting is that *the line currents of this equivalent delta are the currents in the phases of the Y-connected load.*

Example 19.73. *An unbalanced star-connected load has branch impedances of* $Z_1 = 10 \angle 30°\ \Omega$, $Z_2 = 10 \angle -45°\ \Omega$, $Z_3 = 20 \angle 60°\ \Omega$ *and is connected across a balanced 3-phase, 3-wire supply of 200 V. Find the line currents and the voltage across each impedance using* Y/Δ *conversion method.*

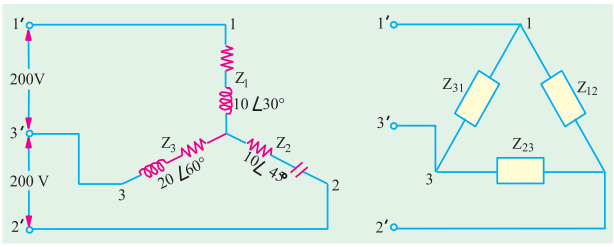


Fig. 19.86

Solution. The unbalanced *Y*-connected load and its equivalent Δ-connected load are shown in Fig. 19.86.

Now $\mathbf{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}$ = (10 ∠ 30°) (10 ∠ – 45°) + (10 ∠ – 45°) (20 ∠ 60°) + (20 ∠ 60°) (10 ∠ 30°) = 100 ∠ – 15° + 200 ∠ 15° + 200 ∠ 90°

Converting these into their cartesian form, we get

= 100 [cos (– 15°) – *j* sin 15°] + 200 (cos 15° + *j* sin 15°) + 200 (cos 90° + *j* sin 90°)

= 96.6 – *j*25.9 + 193.2 + *j*51.8 + 0 + *j*200 = 289.8 + *j*225.9 = 368 ∠ 38°

$\mathbf{Z}_{12}$ $\dfrac{\mathbf{Z_1Z_2}\ \ \mathbf{Z_2Z_3}\ \ \mathbf{Z_3Z_1}}{\mathbf{Z_3}}$ $\dfrac{368\ \ 38}{20\ \ 60}$ 18.4 22 17.0 *j*6.9

$\mathbf{Z}_{23}$ $\dfrac{368\ \ 38}{10\ \ 30}$ 36.8 8 36.4 *j*5.1

$\mathbf{Z}_{31}$ $\dfrac{368\ \ 38}{10\ \ 45}$ 36.8 83 4.49 *j*36.5

Assuming clockwise phase sequence of voltages $\mathbf{V}_{12}$, $\mathbf{Z}_{23}$ and $\mathbf{V}_{31}$, we have

$\mathbf{V}_{12}$ = 200 0 , $\mathbf{V}_{23}$ 200 120 , $\mathbf{V}_{31}$ 200 120

$\mathbf{I}_{12}$ $\dfrac{\mathbf{V}_{12}}{\mathbf{Z}_{12}}$ $\dfrac{200\ \ 0}{18.4\ \ 22}$ 10.86 22 10.07 *j*4.06

$\mathbf{I}_{23}$ $\dfrac{\mathbf{V}_{23}}{\mathbf{V}_{23}}$ $\dfrac{200\ \ 120}{36.8\ \ 8}$ 5.44 128 3.35 *j*4.29

$\mathbf{I}_{31}$ $\dfrac{\mathbf{V}_{31}}{\mathbf{Z}_{31}}$ $\dfrac{200\ \ 120}{36.8\ \ 83}$ 5.44 37 4.34 *j*3.2

Line current = $\mathbf{I}_{11}' = \mathbf{I}_{12} + \mathbf{I}_{13} = \mathbf{I}_{12} - \mathbf{I}_{31}$

= (10.07 + *j*4.06) – (4.34 + *j*3.2) = 5.73 + *j*0.86 = 5.76∠8°32′

$\mathbf{I}_{22}$ $\mathbf{I}_{23}$ $\mathbf{I}_{12}$ (3.35 *j*4.29) (10.07 *j*4.06) 13.42 *j*8.35 15.79 148 6

$\mathbf{I}_{33}$ $\mathbf{I}_{31}$ $\mathbf{I}_{23}$ (4.34 *j*3.2) (3.35 *j*4.29) 7.69 *j*7.49 10.73 44 16

These are currents in the *phases of the Y-connected unbalanced load*. Let us find voltage drop across each star-connected branch impedance.

Voltage drop across $\mathbf{Z}_1 = \mathbf{V}_{10} = \mathbf{I}_{11}\mathbf{Z}_1$ 5.76 8 32 .10 30 57.6 38 32

Voltage drop across $\mathbf{Z}_2 = \mathbf{V}_{20} = \mathbf{I}_{22}\ \mathbf{Z}_2$ 15.79 148 6.10 45 157.9 193 6

Voltage drop across $\mathbf{Z}_3 = \mathbf{V}_{30} = \mathbf{I}_{33}\ \mathbf{Z}_3$ 10.73 44 16 .20 60 214.6 *104 16

Example 19.74. *A 300-V (line) 3-phase supply feeds! star-connected load consisting of non-inductive resistors of 15, 6 and 10* Ω *connected to the R, Y and B lines respectively. The phase sequence is RYB. Calculate the voltage across each resistor.*

Solution. The *Y*-connected unbalanced load and its equivalent Δ-connected load are shown in Fig. 19.87. Using Y/Δ conversion method we have

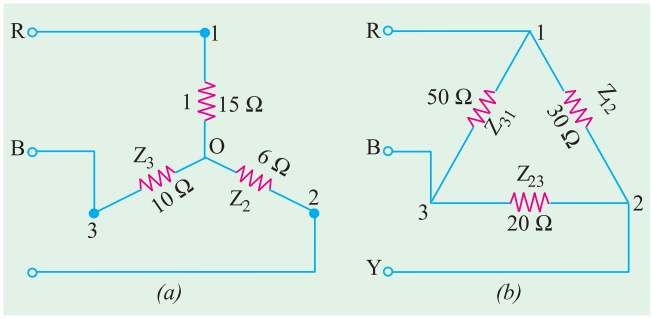


Fig. 19.87

$$\mathbf{Z}_{12} = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_3}$$

$$= \frac{90 + 60 + 150}{10} = 30\,\Omega$$

$\mathbf{Z}_{23}$ 300/15 20

$\mathbf{Z}_{31}$ 300/6 50

Phase current $I_{RY} = V_{RY} / Z_{12} = 300/30 = 10\,\text{A}$

Similarly $I_{YB} = V_{YB} / Z_{23} = 300/20 = 15\,\text{A}$

$I_{BR} = V_{BR} / Z_{31} = 300/50 = 6\,\text{A}$

Each current is in phase with its own voltage because the load is purely resistive.

The line currents for the delta connection are obtained by compounding these phase currents in pairs, either trigonometrically or by phasor algebra. Using phasor algebra and choosing $\mathbf{V}_{RY}$ as the reference axis, we get

$\mathbf{I}_{RY}$ 10 $j0$; $\mathbf{I}_{YB}$ 15($\frac{1}{2}$ $j\sqrt{3}/2$) 7.5 $j13.0$; $\mathbf{I}_{BR}$ 6($\frac{1}{2}$ $j\sqrt{3}/2$) 3.0 $j5.2$

Line currents for delta-connection [Fig. 19.66 (*b*)] are

$\mathbf{I}_R$ $\mathbf{I}_{RY}$ $\mathbf{I}_{RB}$ $\mathbf{I}_{RY}$ $\mathbf{I}_{BR}$ (10 $j0$) (3 $j5.2$) 13 $j5.2$ or 14 A in magnitude

$\mathbf{I}_Y$ $\mathbf{I}_{YR}$ $\mathbf{I}_{YB}$ $\mathbf{I}_{YB}$ $\mathbf{I}_{RY}$ (7.5 $j13.0$) (10 $j0$) 17.5 $j13$ or 21.8 A in magnitude

$\mathbf{I}_B$ $\mathbf{I}_{BR}$ $\mathbf{I}_{BY}$ $\mathbf{I}_{BR}$ $\mathbf{I}_{YB}$ (3.0 $j5.2$) (7.5 $j13.0$) 4.5 $j18.2$ 18.7 A in magnitude

These line currents for Δ-connection are the phase currents for *Y*-connection. Voltage drop across each limb of *Y*-connected load is

$\mathbf{V}_{RN}$ $\mathbf{I}_R\mathbf{Z}_1$ (13 $j5.2$)(15 $j0$) 195 78 volt or 210 V

$\mathbf{V}_{YN} = \mathbf{I}_Y\mathbf{Z}_2 = (-17.5 - j13.0)(6 + j0) = -105 - j78$ volt or 131 V

$\mathbf{V}_{BN} = \mathbf{I}_B\mathbf{Z}_3 = (4.5 + j18.2)(10 + j0) = 45 + j182$ volt or 187 V

As a check, it may be verified that the difference of phase voltages taken in pairs should give the three line voltages. Going through the circuit internally, we have

$\mathbf{V}_{RY} = \mathbf{V}_{RN} + \mathbf{V}_{NY} = \mathbf{V}_{RN} - \mathbf{V}_{YN} = (195 - j78) - (105 - j78) = 300 \angle 0°$

$\mathbf{V}_{YB} = \mathbf{V}_{YN} - \mathbf{V}_{BN} = (-105 - j78) - (45 + j182) = -150 - j260 = 300 \angle -120°$

$\mathbf{V}_{BR} = \mathbf{V}_{BN} - \mathbf{V}_{RN} = (45 + j182) - (195 - j78) = -150 + j260 = 300 \angle 120°$

This question could have been solved by direct geometrical methods as shown in Ex. 19.52.

Example 19.75 *A Y-connected load is supplied from a 400-V, 3-phase, 3-wire symmetrical system RYB. The branch circuit impedances are*

$$Z_R = 10\sqrt{3} + j10;\ Z_Y = 20 + j20\sqrt{3};\ Z_B = 0 - j10$$

Determine the current in each branch. Phase sequence is RYB.

(Network Analysis, Nagpur Univ. 1993)

Solution. The circuit is shown in Fig. 19.88. The problem will be solved by using all the four possible ways in which 3-wire unbalanced *Y* connected load can be handled.

Now, $\mathbf{Z}_R \quad 20 \quad 30 \quad (17.32 \quad j10)$

$\mathbf{Z}_Y \quad 40 \quad 60 \quad (20 \quad j34.64)$

$\mathbf{Z}_B = 10\angle -90° = -j10$

Also, let $\mathbf{V}_{RY} \quad 400 \quad 0 \quad 400 \quad j0$

$\mathbf{V}_{RB} = 400\angle -120° = -200 - j346$

$\mathbf{V}_R = 400\angle 120° = -200 + j346$

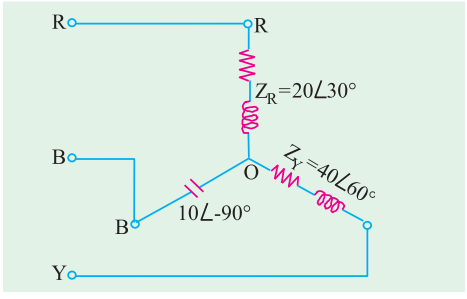


Fig. 19.88

(*a*) By applying Kirchhoff's Laws

With reference to Art. 19.33, it is seen that

$$\boldsymbol{I}_{RO} = \boldsymbol{I}_R = \frac{\boldsymbol{V}_{RY}\boldsymbol{Z}_B - \boldsymbol{V}_{BR}\boldsymbol{Z}_Y}{\boldsymbol{Z}_R\boldsymbol{Z}_Y + \boldsymbol{Z}_Y\boldsymbol{Z}_B + \boldsymbol{Z}_B\boldsymbol{Z}_R}; \boldsymbol{I}_{YO} = \mathrm{I}_Y = \frac{\boldsymbol{V}_{YB}\boldsymbol{Z}_R - \boldsymbol{V}_{RY}\boldsymbol{Z}_B}{\boldsymbol{Z}_R\boldsymbol{Z}_Y + \boldsymbol{Z}_Y\boldsymbol{Z}_B + \boldsymbol{Z}_B\boldsymbol{Z}_R};$$

$$\boldsymbol{I}_{BO} = \boldsymbol{I}_B = \frac{\boldsymbol{V}_{BR}\boldsymbol{Z}_Y - \boldsymbol{V}_{YB}\boldsymbol{Z}_R}{\boldsymbol{Z}_R\boldsymbol{Z}_Y + \boldsymbol{Z}_Y\boldsymbol{Z}_B + \boldsymbol{Z}_B\boldsymbol{Z}_R}$$

Now, $\mathbf{Z}_R\mathbf{Z}_Y + \mathbf{Z}_Y\mathbf{Z}_B + \mathbf{Z}_B\mathbf{Z}_R$

$= 20\angle 30° .40\angle 60° + 40\angle 60° .10\angle -90° + 10\angle -90° .20\angle 30°$

$= 800\angle 90° + 400\angle -30° + 200\angle -60° = 446 + j426 = 617\angle 43.7°$

$\mathbf{V}_{RY}\mathbf{Z}_B \quad \mathbf{V}_{BR}\mathbf{Z}_Y \quad 400 \quad 10 \quad 90 \quad 400 \quad 120 \quad .40 \quad 60$

$= 16{,}000 - j4000 = 16{,}490\angle -14°3'$

$\therefore \quad \mathbf{I}_R \quad \frac{16{,}490 \quad 14\ 3}{617 \quad 43.7} \quad 26.73 \quad 57\ 45$

$\mathbf{V}_{YB}\mathbf{Z}_R \quad \mathbf{V}_{RY}\mathbf{Z}_B \quad 400 \quad 120 \quad .20 \quad 30 \quad 400.10 \quad 90 \quad j4000 \quad 4000 \quad 90$

$\mathbf{I}_Y \quad \frac{4000 \quad 90}{617 \quad 43.7} \quad 6.48 \quad 133.7$

$\mathbf{V}_{BR}\mathbf{Z}_Y \quad \mathbf{V}_{YB}\mathbf{Z}_R \quad 400 \quad 120 \quad .40 \quad 60 \quad 400 \quad 120 \quad 20. \quad 30$

$= -16{,}000 + j8{,}000 = 17{,}890\angle 153°26'$

$\therefore \quad \mathbf{I}_B \quad \frac{17{,}890 \quad 153\ 26}{617 \quad 43.7} \quad 29 \quad 109 \quad 45$

(*b*) By Star/Delta Conversion (Fig. 19.89)

The given star may be converted into the equivalent delta with the help of equations given in Art. 19.34.

$\mathbf{Z}_{RY} \quad \frac{\mathbf{Z}_R\mathbf{Z}_Y \quad \mathbf{Z}_Y\mathbf{Z}_B \quad \mathbf{Z}_B\mathbf{Z}_R}{\mathbf{Z}_B} \quad \frac{617 \quad 43.7}{10 \quad 90} \quad 61.73 \quad 133.7$

$\mathbf{Z}_{YB} \quad \frac{\Sigma\mathbf{Z}_R\mathbf{Z}_Y}{\mathbf{Z}_R} \quad \frac{617 \quad 43.7}{20 \quad 30} \quad 30.87 \quad 13.7$

$\mathbf{Z}_{BR} \quad \frac{\Sigma\mathbf{Z}_R\mathbf{Z}_Y}{\mathbf{Z}_Y} \quad \frac{617 \quad 43.7}{40 \quad 60} \quad 15.43 \quad 16.3$

$\mathbf{I}_{RY} \quad \frac{\mathbf{V}_{RY}}{\mathbf{Z}_{RY}} \quad \frac{400}{61.73 \quad 133.7} \quad 6.48 \quad 133.7 \quad (\ 4.47 \quad j4.68)$

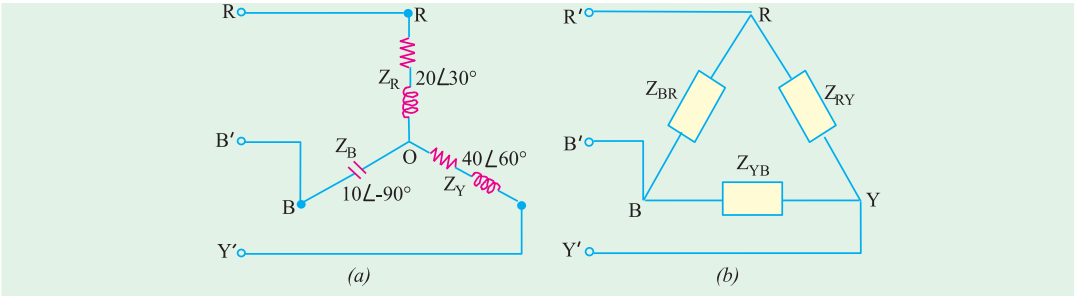


Fig. 19.89

$$\mathbf{I}_{YB} \quad \frac{\mathbf{V}_{YB}}{\mathbf{Z}_{YB}} \quad \frac{400 \quad 120}{30.87 \quad 13.7} \quad 12.95 \quad 133.7 \quad (\ 8.95 \quad j9.35)$$

$$\mathbf{I}_{BR} \quad \frac{\mathbf{V}_{BR}}{\mathbf{Z}_{BR}} \quad \frac{400 \quad 120}{15.43 \quad 16.3} \quad 25.9 \quad 136.3 \quad (\ 18.7 \quad j17.9)$$

$$\mathbf{I}_{RR} \quad \mathbf{I}_{RY} \quad \mathbf{I}_{BR} \quad 14.23 \quad j22.58 \quad 26.7 \quad 57 \quad 48$$

$$\mathbf{I}_{YY} \quad \mathbf{I}_{YB} \quad \mathbf{I}_{RY} \quad 4.48 \quad j4.67 \quad 6.47 \quad 134 \quad 6$$

$$\mathbf{I}_{YB} \quad \mathbf{I}_{BR} \quad \mathbf{I}_{YB} \quad 9.85 \quad j27.25 \quad 29 \quad 109 \quad 48$$

$$\mathbf{I} = (0 + j0) \qquad \text{–as a check}$$

As explained in Art. 19.34, these line currents of the equivalent delta represent the phase currents of the star-connected load of Fig. 19.89 (*a*).

Note. Minor differences are due to accumulated errors.

(*c*) By Using Maxwell's Loop Current Method

Let the loop or mesh currents be as shown in Fig. 19.90. It may be noted that

$$\mathbf{I}_R = \mathbf{I}_1; \mathbf{I}_Y = \mathbf{I}_2 - \mathbf{I}_1 \text{ and } \mathbf{I}_B = -\mathbf{I}_2$$

Considering the drops across *R* and *Y*-arms, we get

$$\mathbf{I}_1\mathbf{Z}_R + \mathbf{Z}_Y(\mathbf{I}_1 - \mathbf{I}_2) = \mathbf{V}_{RY}$$

or $$\mathbf{I}_1(\mathbf{Z}_R + \mathbf{Z}_Y) - \mathbf{I}_2\mathbf{Z}_Y = \mathbf{V}_{RY} \qquad \text{... (}i\text{)}$$

Similarly, considering the legs *Y* and *B*, we have

$$\mathbf{Z}_Y(\mathbf{I}_2 - \mathbf{I}_1) + \mathbf{Z}_B\mathbf{I}_2 = V_{YB}$$

or $$-\mathbf{I}_1\mathbf{Z}_Y + \mathbf{I}_2(\mathbf{Z}_B + \mathbf{Z}_Y) = \mathbf{V}_{YB} \qquad \text{... (}ii\text{)}$$

Solving for $\mathbf{I}_1$ and $\mathbf{I}_2$, we get

$$\mathbf{I}_1 = \frac{\mathbf{V}_{RY}(\mathbf{Z}_Y + \mathbf{Z}_B) + \mathbf{Z}_{YB}\mathbf{V}_Y}{(\mathbf{Z}_R + \mathbf{Z}_Y)(\mathbf{Z}_Y + \mathbf{Z}_B) - \mathbf{Z}_Y^2};$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_{YB}(\mathbf{Z}_R + \mathbf{Z}_Y) + \mathbf{V}_{RY}\mathbf{Z}_Y}{(\mathbf{Z}_R + \mathbf{Z}_Y)(\mathbf{Z}_Y + \mathbf{Z}_B) - \mathbf{Z}_Y^2}$$

$$\mathbf{I}_1 \quad \frac{400(20 \quad j24.64) \quad 400 \quad 120 \ .40 \quad 60}{(37.32 \quad j44.64)(20 \quad j24.64) \quad 1600 \quad 120}$$

$$= \frac{16{,}000 - j4{,}000}{448 + j427} = \frac{16{,}490\angle -14°3'}{617\angle 43.7°}$$

$$= 26\angle -57°45' = (13.9 - j22)$$

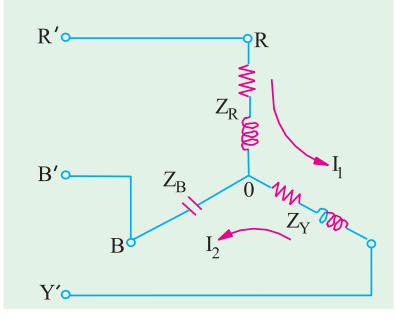


Fig. 19.90

$$\mathbf{I}_2 \quad \frac{(200 \quad j346)(37.32 \quad j44.64) \quad 400(20 \quad j34.64)}{484 \quad j427}$$

$$= \frac{16{,}000 - j8{,}000}{448 + j427} = \frac{17{,}890\angle -26°34'}{617\angle 43.7°} = 28.4\angle -70°16'$$

$$= 28.4\angle -70°16' = (9.55 - j26.7)$$

$\therefore$ $\mathbf{I}_R \quad \mathbf{I}_1 \quad 26 \quad 57\ 45$

$\mathbf{I}_Y \quad \mathbf{I}_2 \quad \mathbf{I}_1 \quad (9.55 \quad j26.7) \quad (13.9 \quad j22) \quad 4.35 \quad j4.7 \quad 6.5 \quad 134$

$\mathbf{I}_B \quad \mathbf{I}_2 \quad 28.4 \quad 70\ 16 \quad 28.4 \quad 109\ 44$

(*d*) By Using Millman's Theorem*

According to this theorem, the voltage of the load star point O' with respect to the star point or neutral O of the generator or supply (normally zero potential) is given by

$$\mathbf{V}_{OO'} = \frac{\mathbf{V}_{RO}\mathbf{Y}_R + \mathbf{V}_{YO}\mathbf{Y}_Y + \mathbf{V}_{BO}\mathbf{Y}_B}{\mathbf{Y}_R + \mathbf{Y}_Y + \mathbf{Y}_B}$$

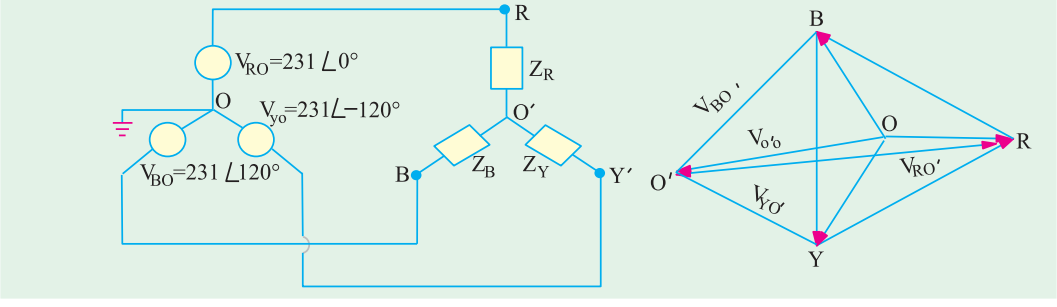


Fig. 19.91

where $\mathbf{V}_{RO}$, $\mathbf{V}_{YO}$ and $\mathbf{V}_{BO}$ are the phase voltages of the generator or 3-phase supply.

As seen from Fig. 19.91, voltage across each phase of the load is

$$V_{RO} \quad V_{RO} \quad V_{O\,O}V_{YO} \quad V_{YO} \quad V_{OO}\ V_{BO} \quad V_{BO} \quad V_{OO}$$

Obviously, $\mathbf{I}_{RO} \quad (\mathbf{V}_{RO} \quad \mathbf{V}_{OO})\mathbf{Y}_R ; \mathbf{I}_{YO} \quad (\mathbf{V}_{YO} \quad \mathbf{V}_{O\,O})\mathbf{Y}_Y$ and

$$\mathbf{I}_{BO}' = (\mathbf{V}_{BO} - \mathbf{V}_{OO})\mathbf{Y}_B$$

Here $\mathbf{Y}_R \quad \frac{1}{20 \quad 30} \quad 0.05 \quad 30 \quad (0.0433 \quad j0.025)$

$\mathbf{Y}_Y \quad \frac{1}{40 \quad 60} \quad 0.025 \quad 60 \quad (0.0125 \quad j0.0217)$

$\mathbf{Y}_B \quad \frac{1}{10 \quad 90} \quad 0.1 \quad 90 \quad 0 \quad j0.1$

---

* Incidentally, it may be noted that the p.d. between load neutral and supply neutral is given by

$$V_{OO'} = \frac{V_{RO}' + V_{YO}' + Y_{BO}'}{3}$$

$\therefore \quad \mathbf{Y}_R \quad \mathbf{Y}_Y \quad \mathbf{Y}_B \quad 0.0558 \quad j0.0533 \quad 0.077 \quad 43.7$

Let $\mathbf{V}_{RO} \quad \frac{400}{\sqrt{3}} \quad 0 \quad (231 \quad j0)$

$\mathbf{V}_{BO} \quad 231 \quad 120 \quad 115.5 \quad j200$

$\mathbf{V}_{BO} \quad 231 \quad 120 \quad 115.5 \quad j200$

$$\mathbf{V}_{OO} \quad \frac{231.0.05 \quad 30 \quad 231 \quad 120 \;.0.025 \quad 60 \quad 231 \quad 120 \;.01 \quad 90}{0.077 \quad 43.7}$$

$$= \frac{-15.8 - j17.32}{0.077\angle 43.7^\circ} = \frac{23.5\angle -132.4\angle}{0.077\angle 43.7} = 305\angle -176.1^\circ = (304.5 - j20.8)$$

$\mathbf{V}_{RO} \quad \mathbf{V}_{RO} \quad \mathbf{V}_{OO} \quad 231 \quad (\; 304.5 \quad j20.8) \quad 535.5 \quad j20.8 \quad 536 \quad 2.2$

$\mathbf{V}_{YO} \quad (\; 115.5 \quad j200) \quad (\; 304.5 \quad j20.8) \quad 189 \quad j179 \quad 260 \quad 43 \; 27$

$\mathbf{V}_{BO} \quad (\; 115.5 \quad j200) \quad (\; 304.5 \quad j20.8) \quad 189 \quad j221 \quad 291 \quad 49 \; 27$

$\therefore \quad \mathbf{I}_{RO} \quad 536 \quad 2.2 \quad 0.05 \quad 30 \quad 26.5 \quad 27.8$

$\mathbf{I}_{YO} \quad 260 \quad 43 \; 27 \quad 0.025 \quad 60 \quad 6.5 \quad 103 \; 27$

$\mathbf{I}_{BO} \quad 291 \quad 49 \; 27 \quad 0.1 \quad 90 \quad 29.1 \quad 139 \; 27$

Note. As seen from above, $\mathbf{V}'_{RO} = \mathbf{V}'_{RO} - \mathbf{V}'_{OO}$

Substituting the value of $\mathbf{V}'_{OO}$, we have

$$\mathbf{V_{RO}} = \mathbf{V_{RO}} - \frac{\mathbf{V_{RO}Y_R + V_{YO}Y_Y + V_{BO}Y_B}}{\mathbf{Y_R + Y_Y + Y_B}}$$

$$= \frac{(\mathbf{V}_{RO} - \mathbf{V}_{YO})\mathbf{Y}_Y + (\mathbf{Y}_{RO} - \mathbf{V}_{BO})\mathbf{Y}_B}{\mathbf{Y}_R + \mathbf{Y}_Y + \mathbf{Y}_B}$$

$$= \frac{\mathbf{V}_{RY}\mathbf{Y}_Y + \mathbf{V}_{RB}\mathbf{Y}_B}{\mathbf{Y}_R + \mathbf{Y}_Y + \mathbf{Y}_B}$$

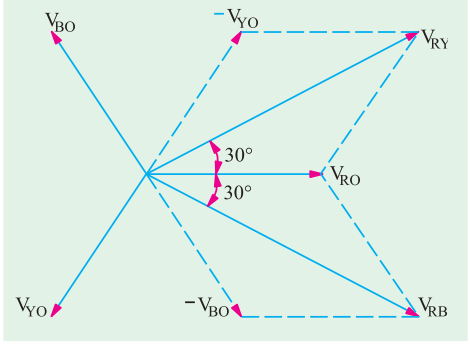


Fig. 19.92

Since $\mathbf{V}_{RO}$ is taken as the reference vector, then as seen from Fig. 19.92.

$\mathbf{V}_{RY} \quad 400 \quad 30$ and $\mathbf{V}_{RB} \quad 400 \quad 30$

$$\therefore \mathbf{V}_{RO} \quad \frac{400 \quad 30 \quad 0.025 \quad 60 \quad 400 \quad 30 \quad 0.1 \quad 90}{0.077 \quad 43.7}$$

$$= \frac{28.6 + j29.64}{0.077\angle 43.7^\circ} = \frac{41\angle 46^\circ}{0.077\angle 43.7^\circ} = 532.5\angle 2.3^\circ$$

$\mathbf{I}_{RO} \quad \mathbf{V}_{RO}\mathbf{Y}_R \quad 532.5 \quad 2.3 \quad 0.05 \quad 30 \quad 26.6 \quad 27.7$

Similarly, $\mathbf{V}_{YO}$ and $\mathbf{V}_{BO}$ may be found and $\mathbf{I}_Y$ and $\mathbf{I}_B$ calculated therefrom.

Example 19.76. *Three impedances, Z_R, Z_Y and Z_B are connected in star across a 440-V, 3-phase supply. If the voltage of star-point relative to the supply neutral is 200 $\angle$ 150° volt and Y and B line currents are 10 $\angle -90°$ A and 20 $\angle$ 90° A respectively, all with respect to the voltage between the supply neutral and the R line, calculate the values of Z_R, Z_Y and Z_B.*

(Elect Circuit; Nagpur Univ. 1991)

Solution. Let O and O' be the supply and load neutrals respectively. Also, let,

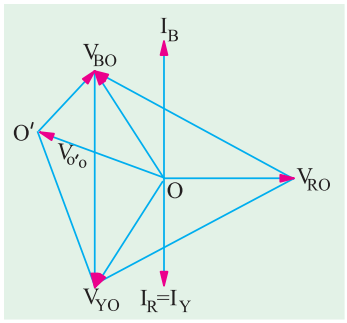


Fig. 19.93

$$\mathbf{V}_{RO} \quad \frac{440}{\sqrt{3}} \quad 0 \quad 254 \quad 0 \quad 254 \quad j0$$

$$\mathbf{Y}_{YO} \quad 254 \quad 120 \quad 127 \quad j220$$

$$\mathbf{V}_{BO} \quad 254 \quad 120 \quad 127 \quad j220$$

$$\mathbf{I}_Y = 10\angle -90° = -j10; \quad \mathbf{I}_B = 20\angle 90° = j20$$

$$\mathbf{I}_R = -(\mathbf{I}_Y + \mathbf{I}_B) = -j10$$

Also, $\mathbf{V}_{OO} \quad 200 \quad 150 \quad 173 \quad j100$

$$\mathbf{V}_{RO} \quad \mathbf{V}_{OO} \quad 254 \quad (173 \quad j100) \quad 427 \quad j100 \quad 438.5 \quad 13.2$$

$$\mathbf{V}_{YO} \quad \mathbf{V}_{OO} \quad (127 \quad j220) \quad (173 \quad j100) \quad 46 \quad j320 \quad 323 \quad 81.6$$

$$\mathbf{V}_{BO} \quad \mathbf{V}_{OO} \quad (127 \quad j220) \quad (173 \quad j100) \quad 46 \quad j120 \quad 128.6 \quad 69$$

As seen from Art. 19.32.

$$\mathbf{Z}_R \quad \frac{\mathbf{V}_{RO} \quad \mathbf{V}_{00}}{\mathbf{I}_R} \quad \frac{438.5 \quad 13.2}{10 \quad 90} \quad 43.85 \quad 76.8$$

$$\mathbf{Z}_Y \quad \frac{\mathbf{V}_{YO} \quad \mathbf{V}_{OO}}{\mathbf{I}_Y} \quad \frac{323}{10 \quad 90} \quad 32.3 \quad 8.4$$

$$\mathbf{Z}_B \quad \frac{\mathbf{V}_{BO} \quad \mathbf{V}_{OO}}{\mathbf{I}_B} \quad \frac{128.6 \quad 69}{20 \quad 90} \quad 6.43 \quad 21$$

19.35. Unbalanced Star-connected Non-inductive Load

Such a case can be easily solved by direct geometrical method. If the supply system is symmetrical, the line voltage vectors can be drawn in the form of an equilateral triangle *RYB* (Fig. 19.94). As the load is an unbalanced one, its neutral point will not, obviously, coincide with the centre of the gravity or centroid of the triangle. Let it lie at any other point like *N*. If point *N* represents the potential of the neutral point if the unbalanced load, then vectors drawn from *N* to points, *R*, *Y* and *B* represent the voltages across the branches of the load. These voltages can be represented in their rectangular co-ordinates with respect to the rectangular axis drawn through *N*. It is seen that taking co-ordinates of *N* as (0, 0), the co-ordinates of point *R* are $[(V/2 - x), -y]$

of point *Y* are $[-(V/2 + x), -y]$

and of point *B* are $[-x, (\sqrt{3}V/2 - y)]$

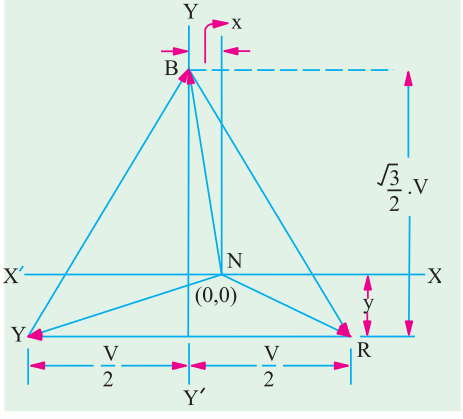


Fig. 19.94

$$\mathbf{V}_{RN} \quad \frac{V}{2} \quad x \quad jy; \quad \mathbf{V}_{YN} \quad \frac{V}{2} \quad x \quad iy$$

$$\mathbf{V}_{BN} \quad x \quad j \quad \frac{\sqrt{3}V}{2} \quad y$$

Let R_1, R_2 and R_3 be the respective branch impedances, Y_1, Y_2 and Y_3 the respective admittances and $\mathbf{I}_R$, $\mathbf{I}_Y$ and $\mathbf{I}_B$ the respective currents in them.

Then $\mathbf{I}_R = \mathbf{V}_{RN} / \mathbf{R}_1 = \mathbf{V}_{RN}\mathbf{Y}_1$

Similarly, $\mathbf{I}_Y = \mathbf{V}_{YN}\mathbf{Y}_2$ and $\mathbf{I}_B = \mathbf{V}_{BN}\mathbf{Y}_3$. Since $\mathbf{I}_R + \mathbf{I}_Y + \mathbf{I}_B = 0$

$\therefore \quad \mathbf{V}_{RN}\mathbf{Y}_1 + \mathbf{V}_{YN}\mathbf{Y}_2 + \mathbf{V}_{BN}\mathbf{Y}_3 = 0$

or $Y_1 \quad \frac{V}{2} \quad x \quad jy \quad Y_2 \quad \frac{V}{2} \quad x \quad jy \quad Y_3 \quad x \quad j \quad \frac{\sqrt{3}V}{2} \quad y \quad 0$

or $x(Y_1 \quad Y_2 \quad Y_3) \quad \frac{V}{2}(Y_1 \quad Y_2) \quad j \; Y_3 \frac{\sqrt{3}V}{2} \quad y(Y_1 \quad Y_2 \quad Y_3) \quad 0$

$\therefore \quad -x(Y_1 + Y_2 + Y_3) + \frac{V}{2}(Y_1 - Y_2) = 0 \quad \therefore \quad x = \frac{V(Y_1 - Y_2)}{2Y_1 + Y_2 + Y_3}$

Also $Y_3 \frac{\sqrt{3}V}{2} - y(Y_1 + Y_2 + Y_3) = 0 \quad \therefore \quad y = \frac{\sqrt{3}V}{2} \frac{Y_3}{(Y_1 + Y_2 + Y_3)}$

Knowing the values of x, the values of $\mathbf{V}_{RN}$, $\mathbf{V}_{YN}$ and $\mathbf{V}_{BN}$ and hence, of $\mathbf{I}_R$, $\mathbf{I}_Y$ and $\mathbf{I}_B$ can be found as illustrated by Ex. 19.68.

Example 19.77. *Three non-inductive resistances of 5, 10 and 15 Ω are connected in star and supplied from a 230-V symmetrical 3-phase system. Calculate the line currents (magnitudes).* **(Principles of Elect. Engg. Jadavpur Univ.)**

Solution.

(*a*) Star/Delta Conversion Method

The *Y*-connected unbalanced load and its equivalent Δ-connected load are shown in Fig. 19.95 (*a*) and (*b*) respectively. Using Y/Δ conversion, we have

$$\mathbf{Z}_{12} \quad \frac{\mathbf{Z}_1\mathbf{Z}_2 \quad \mathbf{Z}_2\mathbf{Z}_3 \quad \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_3} \quad \frac{50 \quad 150 \quad 75}{15} \quad \frac{55}{3}$$

$$Z_{23} \quad 275/5 \quad 55 \quad \text{and} \quad Z_{31} \quad 275/10 \quad 27.5$$

Phase current $\mathbf{I}_{RY} = \mathbf{V}_{RY}/\mathbf{Z}_{12} = 230/(55/3) = 12.56$ A

Similarly, $\mathbf{I}_{YB} = \mathbf{V}_{YB}/\mathbf{Z}_{23} = 230/55 = 4.18$ A; $\mathbf{I}_{BR} = \mathbf{V}_{BR}/\mathbf{Z}_{31} = 230/27.5 = 8.36$ A

The line currents for Δ-connection are obtained by compounding the above phase currents trigonometrically or vectorially. Choosing vector addition and taking $\mathbf{V}_{RY}$ as the reference vector, we get;

$\mathbf{I}_{RY} = (12.56 + j0)$

$\mathbf{I}_{YB} = 4.18 \quad \frac{1}{2} \; j\frac{\sqrt{3}}{2} \quad = -2.09 - j3.62$

$\mathbf{I}_{BR} = 8.36 \quad \frac{1}{2} \quad j\frac{\sqrt{3}}{2} \quad = -4.18 + j7.24$

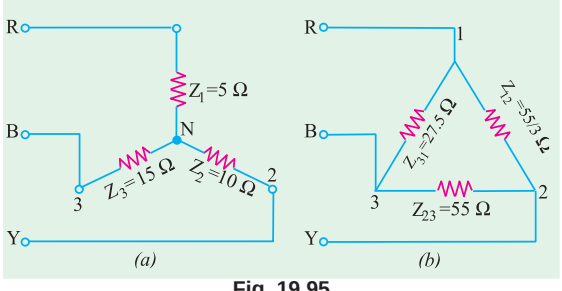


Fig. 19.95

Hence, line currents for Δ-connection of Fig. 19.95 (*b*) are

$\mathbf{I}_R = \mathbf{I}_{RY} + \mathbf{I}_{RB} = \mathbf{I}_{RY} - \mathbf{I}_{BR}$

$= (12.56 + j0) - (-4.18 + j7.24) = 16.74 - j7.24$ or 18.25 A – in magnitude

$\mathbf{I}_Y = \mathbf{I}_{YR} + \mathbf{I}_{YB} = \mathbf{I}_{YB} - \mathbf{I}_{RY}$

$= (-2.09 - j3.62) - (12.56 + j0) = -14.65 - j3.62$ or 15.08 A – in magnitude

$\mathbf{I}_B = \mathbf{I}_{BR} + \mathbf{I}_{BY} = \mathbf{I}_{BR} - \mathbf{I}_{YB}$

$= (-4.18 + j7.24) - (-2.09 - j3.62) = -2.09 + j10.86$ or 11.06 A –in magnitude

(*b*) Geometrical Method

Here, R_1 5 , R_2 10 ; and R_3 15 ; Y_1 1/5S; Y_2 1/10S; Y_3 1/15S

As found above in Art. 19.35 $x = \frac{V}{2}(Y_1 - Y_2)/(Y_1 + Y_2 + Y_3)$

$$= \frac{230}{2}\ \frac{1}{5}\ \frac{1}{10} \Big/ \frac{1}{5}\ \frac{1}{10}\ \frac{1}{15} \quad 31.4$$

$$y \quad \frac{\sqrt{3}V}{2}.Y_3 \Big/ (Y_1\ \ Y_2\ \ Y_3) \quad (\sqrt{3}\ \ 115\ \ 1/15)/(11/30) \quad 36.2$$

$$\mathrm{V}_{RN} \quad \frac{V}{2}\ \ x \quad jy \quad (115\ \ 31.4) \quad j36.2 \quad 83.6 \quad j36.2$$

$$\mathrm{V}_{YN} \quad \frac{V}{2}\ \ x \quad jy \quad 146.4 \quad j36.2$$

$$\mathrm{V}_{BN} \quad x \quad j\ \frac{\sqrt{3}V}{2}\ \ y \quad 31.4 \quad j163$$

$$\mathbf{I}_R \quad \mathbf{V}_{RN}\mathbf{Y}_1 \quad (83.6 \quad j36.2)\ \ 1/5 \quad 16.72 \quad j7.24$$

$$\mathbf{I}_Y \quad \mathbf{V}_{YN}\mathbf{Y}_2 \quad (\ 146.4 \quad j36.2)\ \ 1/10 \quad 14.64 \quad j3.62$$

$$\mathbf{I}_B \quad \mathbf{V}_{BN}\mathbf{Y}_3 \quad (\ 31.4 \quad j163)\ \ 1/15 \quad 2.1 \quad j10.9$$

These are the same currents as found before.

(*c*) Solution by Millman's Theorem

$\mathbf{Y}_R$ 1/5 0 ; $\mathbf{Y}_Y$ 1/10 0 ; $\mathbf{Y}_B$ 1/15 0 and $\mathbf{Y}_R$ $\mathbf{Y}_Y$ $\mathbf{Y}_B$ 11/30 0 Siemens

Let the supply voltages be represented (Fig. 19.96) by

$\mathbf{V}_{RO}$ $230/\sqrt{3}$ 0 133 0 ; $\mathbf{V}_{YO}$ 133 120 ; $\mathbf{V}_{BO}$ 133 120

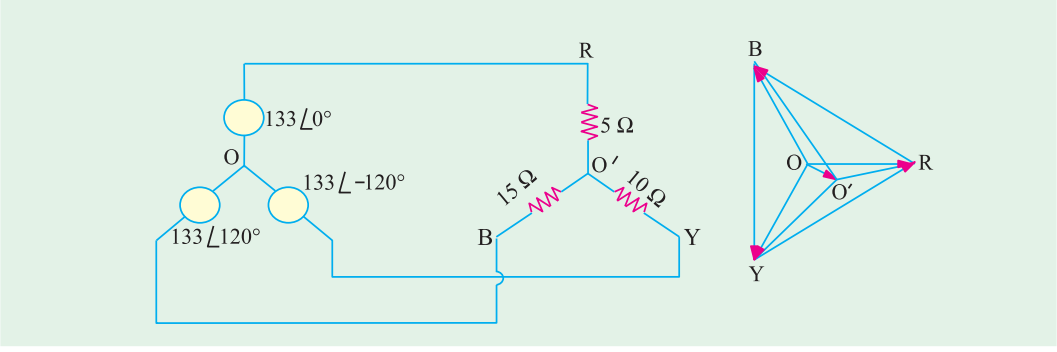


Fig. 19.96

The p.d. between load and supply neutral is

$$\mathbf{V}_{OO} \quad \frac{133/5 \quad (133/10) \quad 120 \quad (133/15) \quad 120}{30/11 \quad 0}$$

$$= 42.3 - j10.4 = 43.6\angle -13.8°$$

$\mathbf{V}_{RO}$ 133 (42.3 j10.4) 90.7 j10.4

$\mathbf{V}_{YO}$ 133 120 (42.3j 10.4)

$= (-66.5 - j115) - (42.3 - j10.4) = -108.8 - j104.6$

$\mathbf{V}_{BO}$ 133 120 V_{OO} (66.5 j115) (42.3 j10.4) 108.8 j125.4

$\mathbf{I}_R$ $\mathbf{V}_{RO}$ $\mathbf{Y}_R$ 1/5.(90.7 j10.4) 18.1 j2.1 or 18.22 A in magnitude

I_Y 10.88 j10.5 or 15.1 A in magnitude

$I_B = -7.5 + j8.4$ or 11.7 A in magnitude

Example 19.78. *The unbalanced circuit of Fig. 19.97 (a) is connected across a symmetrical 3-phase supply of 400-V. Calculate the currents and phase voltages. Phase sequence is RYB.*

Solution. The line voltages are represented by the sides of an equilateral triangle *ABC* in Fig. 19.97 (*b*). Since phase impedances are unequal, phase voltages are unequal and are represented by lengths, *NA*, *NB* and *NC* where *N* is the neutral point which is shifted from its usual position. *CM* and *ND* are drawn perpendicular to horizontal side *AB*. Let co-ordinates of point *N* be (0, 0). Obviously, $AM = BM = 200$ V, $CM = \sqrt{3} \times 200$ *V*, $CM = \sqrt{3} \times 200 = 346$ V. Let $DM = x$ volts and $ND = y$ volts.

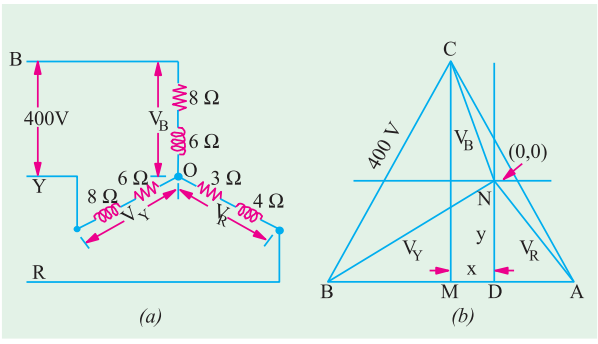


Fig. 19.97

Then, with reference to point *N*, the vector expressions for phase voltages are

$\mathbf{V}_R$ (200 x) jy, $\mathbf{V}_Y$ (200 x) jy; $\mathbf{V}_B$ x j(346 y)

$\mathbf{I}_R$ $\dfrac{\mathbf{V}_R}{\mathbf{Z}_R}$ $\dfrac{(200 \quad x) \quad jy}{3 \quad j4}$ $\dfrac{3 \quad j4}{3 \quad j4}$ (24 0.12x 0.16y) j(32 0.16x 0.12y)

$\mathbf{I}_Y$ $\dfrac{\mathbf{V}_Y}{\mathbf{Z}_Y}$ $\dfrac{(200 \quad x) \quad jy}{6 \quad j8}$ $\dfrac{6 \quad j8}{6 \quad j8}$

$= (-12 - 0.06x - 0.08y) + j(16 + 0.08x - 0.06y)$

$\mathbf{I}_B$ $\dfrac{\mathbf{V}_B}{\mathbf{Z}_B}$ $\dfrac{x \quad j(346 \quad y)}{8 \quad j6}$ $\dfrac{8 \quad j6}{8 \quad j6}$

$= (20.76 - 0.08x - 0.06y) + j(27.68 + 0.06x - 0.08y)$

Now, $\mathbf{I}_R + \mathbf{I}_Y + \mathbf{I}_B = 0$

$\therefore \quad (32.76 - 0.26x - 0.3y) + j(11.68 + 0.3x - 0.26y) = 0$

Obviously, the real component as well as the *j*-component must be zero.

$\therefore \quad 32.76 - 0.26 \times -0.3y = 0$ and $11.68 + 0.3x - 0.26y = 0$

Solving these equations for x and y, we have x = 31.9 V and y = 81.6 V

$\mathbf{V}_R$ (200 31.9) j81.6 168 j81.6 186.7 25.9

$\mathbf{V}_Y$ (200 31.9) j81.6 231.9 j81.6 245.8 199.4

$\mathbf{V}_B$ 31.9 j(346 81.6) 31.9 j264.4 266.3 83.1

Substituting these values of x and y in the expressions for currents, we get

$\mathbf{I}_R$ (24 0.12 31.9 0.16 81.6) j(32 0.16 31.9 0.12 81.6)
= 7.12 – j36.7

Similarly $\mathbf{I}_Y$ 20.44 j13.65; I_B 13.3 j23.06

$\Sigma I = (0.+j0)$ – as a check

Example 19.79. *A* $3-\phi$, *4-wire, 400-V symmetrical system supplies a Y-connected load having following branch impedances:*

$Z_R = 100\Omega, Z_Y = j10\,\Omega$ *and* $Z_B = -j10\Omega$

Compute the values of load phase voltages and currents and neutral current. Phase sequence is RYB.

How will these values change in the event of an open in the neutral wire?

Solution. (*a*) When Neutral Wire is Intact. [Fig 19.98 (*a*)]. As discussed in Art. 19.30, the *load* phase voltages would be the same as *supply* phase voltages despite imbalance in the load. The three load phase voltages are:

$\mathbf{V}_R$ 231 0 , $\mathbf{V}_Y$ 231 120 and $\mathbf{V}_B$ 231 120

$\mathbf{I}_R$ 231 0 /100 0 2.31 0 2.31 j0

$\mathbf{I}_Y$ 231 120 /10 90 2.31 210 20 j11.5

$\mathbf{I}_B$ 231 120 /10 90 23.1 210 20 j11.5

$\mathbf{I}_N$ ($\mathbf{I}_R$ $\mathbf{I}_Y$ $\mathbf{I}_B$) (2.31 20 j11.5 20 j11.5) 37.7 A

(*b*) When Neutral is Open [Fig.19.98 (*b*)]

In this case, the load phase voltages will be no longer equal. The node pair voltage method will be used to solve the question. Let the supply phase voltages be given by

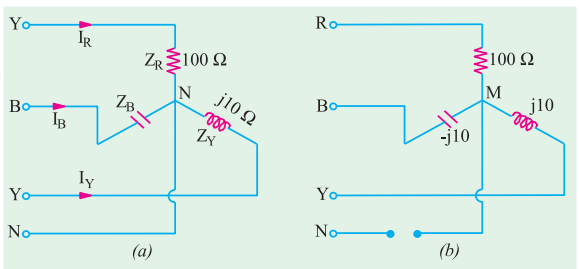


Fig. 19.98

$\mathbf{E}_R$ 231 0 , $\mathbf{E}_Y$ 231 120
= – 115.5 – j200

$\mathbf{E}_B = 231\angle 120° = -115\cdot 5 + j200$

$\mathbf{Y}_R$ 1/100 0.01; $\mathbf{Y}_Y$ 1/ j10 j0.1 and $\mathbf{Y}_B$ 1/ j10 j0.1

$$\mathbf{V}_{NN} = \frac{231\times 0.01 + (-j0.1)(-115.5 - j200) + j0.1(-115.5 + j200)}{0.01 + (-j0.1) + j0.1} = -3769 + j0$$

The load phase voltages are given by

$\mathbf{V}_R$ $\mathbf{E}_R$ $\mathbf{V}_{NN}$ (231 j0) (3769 j0) = 4000 V

$\mathbf{V}_Y$ $\mathbf{E}_Y$ $\mathbf{V}_{NN}$ 115.5 j200 (3769 j0) (3653.5 j200)

$\mathbf{V}_B$ $\mathbf{E}_B$ $\mathbf{V}_{NN}$ 115.5 j200 (3769 j0) (3653.5 j0)

$\mathbf{I}_R$ $\mathbf{V}_R$ $\mathbf{Y}_R$ 4000 0.01 40 A

$\mathbf{I}_Y$ (j0.1)(3653.5 j200) (20 j3653.5)

$\mathbf{I}_B$ (j0.1)(3653.5 j200) 20 j3653.5

Obviously, the neutral current will just not exist.

Note. As hinted in Art. 19.30 (*i*), the load phase voltages and currents become abnormally high.

Example 19.80. *For the circuit shown in Fig. 19.99 find the readings on the two wattmeters W_a and W_c.*

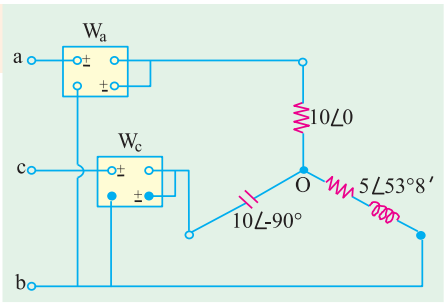


Fig. 19.99

Solution. The three line currents for this problem have already been determined in Example 19.43.

$$\mathbf{I}_{ao} = 20.02 - j4.54$$

$$\mathbf{I}_{bo} = -7.24 - j5.48$$

$$\mathbf{I}_{co} \quad 12.78 \quad j10.12$$

The line voltages are given by

$$\mathbf{V}_{ab} \quad 200 \quad j0$$

$$\mathbf{V}_{bc} = -100 - j\,173.2$$

$$\mathbf{V}_{ca} = -100 + j\,173.2$$

Wattmeter W_a carries a current of using $\mathbf{I}_{ao} = 20.02 - j\,4.54$ and has voltage $\mathbf{V}_{ab}$ impressed across its pressure coil. Power can be found by using current conjugate.

$$\mathbf{P}_{VA} = (200 + j\,0)\,(20.02 + j\,4.54) = (200)\,(20.02) + j\,(200)(4.54)$$

Actual power = $200 \times 20.02 = 4004$ W $\therefore$ W_a = **4004 W**

The other wattmeter W_c carries current of $\mathbf{I}_{co} = -12.78 + j\,10.02$ and has a voltage $\mathbf{V}_{cb} = -\mathbf{V}_{bc} = 100 + j\,173.2$ impressed across it. By the same method, wattmeter reading is

$$W_c = (100 \times -12.78) + (173.2 \times 10.02) = -1278 + 1735.5 = \mathbf{457.5\ W}$$

Example 19.81. *Three resistors 10, 20 and 20 Ω are connected in star to the terminals A, B and C of a $3-\phi$, 3 wire supply through two single-phase wattmeters for measurement of total power with current coils in lines A and C and pressure coils between A and B and C and B. Calculate **(i)** the line currents **(ii)** the readings of each wattmeter.*

The line voltage is 400-V. **(Electrical Engineering-I, Bombay Univ.)**

Solution. Let $\mathbf{V}_{AB}$ 400 0 ; $\mathbf{V}_{BC} = 400\angle -120°$ and $\mathbf{V}_{CA}$ 400 120

As shown in Fig. 19.100, current through wattmeter W_1 is $\mathbf{I}_{AO}$ or $\mathbf{I}_A$ and that through W_2 is $\mathbf{I}_{CO}$ or $\mathbf{I}_C$ and the voltages are $\mathbf{V}_{AB}$ and $\mathbf{V}_{CB}$ respectively. Obviously,

$$\mathbf{Z}_A = 10\angle 0°; \mathbf{Z}_B = 20\angle 0°, \mathbf{Z}_C = 20\angle 0°$$

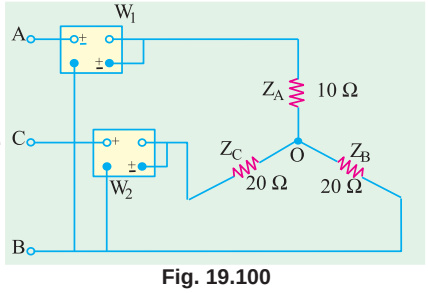


Fig. 19.100

The currents $\mathbf{I}_A$ and $\mathbf{I}_C$ may be found by applying either Kirchhoff's laws (Art. 19.33) or Maxwell's Mesh Method. Both methods will be used for illustration.

(a) From Eq. (10), (11) and (12) of Art. 19.33, we have

$$\mathbf{I}_A = \frac{400 \times 20 - 20(-200 + j346)}{(10 \times 20) + (20 \times 20) + (20 \times 10)}$$

$$= \frac{12{,}000 - j6{,}920}{800} = 15 - j8.65\,\text{A}$$

$$\mathbf{I}_C \quad \frac{20(\ 200 \quad j346) \quad 10(\ 200 \quad j346)}{800} \quad \frac{2000 \quad j10{,}380}{800} \quad 2.5 \quad j13$$

(b) From Eq. ***(i)*** and ***(ii)*** of solved example 17.48 (*c*) we get

$$\mathbf{I}_A \quad \mathbf{I}_1 \quad \frac{400 \quad 40 \quad 20(\ 200 \quad j346)}{30 \quad 40 \quad 20^2} \quad 15 \quad j8.65\,\text{A}$$

$$\mathbf{I}_C \quad \mathbf{I}_2 \quad \frac{30 \quad (\ 200 \quad j346) \quad 400 \quad 20}{800} \quad 25 \quad j13$$

As seen, wattmeter W_1 carries current $\mathbf{I}_A$ and has a voltage $\mathbf{V}_{AB}$ impressed across its pressure coil. Power may be found by using voltage conjugate.

$$\mathbf{P}_{VA} \quad (400 \quad j0)(15 \quad j8.65) \quad 6000 \quad j\,3{,}460$$

$\therefore$ reading of W_1 = 6000 W = **6 kW**

Similarly, W_2 carries $\mathbf{I}_C$ and has voltage $\mathbf{V}_{CB}$ impressed across its pressure coil.

Now, $\mathbf{V}_{CB} \quad \mathbf{V}_{BC} \quad (200 \quad j346)$. Using voltage conjugate, we get

$\mathbf{P}_{VA}$ (200 $j346$)(2.5 $j13$)

Real power = (200 × – 2.5) + (13 × 346) = 4000 W

∴ reading of W_2 = **4kW;** Total power = 10 kW

Example 19.82. *Three impedances Z_A, Z_B and Z_C are connected in delta to a 200-V, 3-phase three-wire symmetrical system RYB.*

Z_A = 10 ∠ 60° between lines R and Y ; Z_B = 10 ∠ 0° between lines Y and B

Z_C = 10 ∠ 60° between lines B and R

The total power in the circuit is measured by means of two wattmeters with their current coils in lines R and B and their corresponding pressure coils across R and Y and B and Y respectively. Calculate the reading on each wattmeter and the total power supplied. Phase sequence RYB.

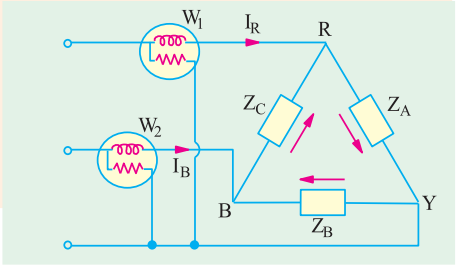


Fig. 19.101

Solution. The wattmeter connections are shown in Fig. 19.101.

$\mathbf{V}_{RY}$ 200 0 200 $j0$

$\mathbf{V}_{YB}$ 200 120 100 $j173.2$

$\mathbf{V}_{BR}$ 200 120 100 $j173.2$

$$\mathbf{I}_{BR} \quad \frac{200 \quad 0}{10 \quad 60} \quad 20 \quad 60 \quad 10 \quad j17.32$$

$$\mathbf{I}_{YB} \quad \frac{200 \quad 120}{10 \quad 0} \quad 20 \quad 120$$

$$= -10 - j17.32; \mathbf{I}_{BR} = \frac{200\angle 120°}{10\angle 60°} = 20\angle 60° = 10 + j17.32$$

As seen, current through W_1 is $\mathbf{I}_R$ and voltage across its pressure coil is $\mathbf{V}_{RY}$.

$\mathbf{I}_R$ $\mathbf{I}_{RY}$ $\mathbf{I}_{BR}$ $j34.64$ A

Using voltage conjugate, we have

$\mathbf{P}_{VA}$ (200 $j0$)($j34.64$) 0 $j6,928$

Hence, W_1 reads zero.

Current through W_2 is $\mathbf{I}_B$ and voltage across its pressure coil is $\mathbf{V}_{BY}$

$\mathbf{I}_B$ $\mathbf{I}_{BR}$ $\mathbf{I}_{YB}$ 20 $j34.64$; $\mathbf{V}_{BY}$ $\mathbf{V}_{YB}$ 100 $j173.2$

Again using voltage conjugate, we get

$\mathbf{P}_{YA}$ (100 $j173.2$)(20 $j34.64$)

= 8000 + $j0$

∴ reading of W_2 = **8000 W**

19.36. Phase Sequence Indicators

In unbalanced 3-wire star-connected loads, phase voltages change considerably if the phase sequence of the supply is reversed. One or the other load phase voltage becomes dangerously large which may result in damage to the equipment. Some phase voltage becomes too

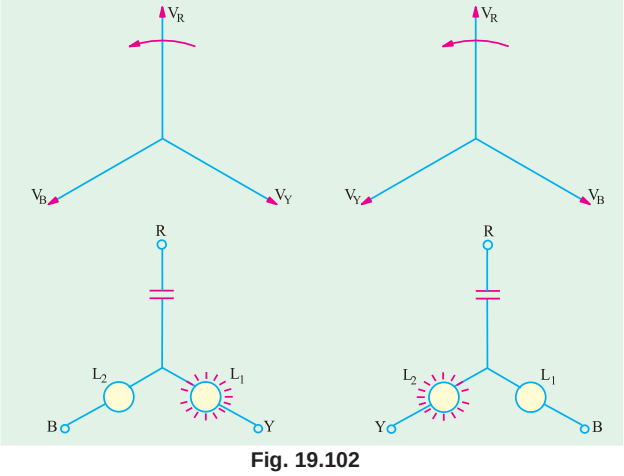


Fig. 19.102

small which is equally deterimental to some types of electrical equipment. Since phase voltage depends on phase sequence, this fact has been made the basis of several types of phase sequence indicators.* A simple phase sequence indicator may be made by connecting two suitable incandescent lamps and a capacitor in a *Y*-connection as shown in Fig. 19.102. It will be found that for phase sequence *RYB*, lamp L_1 will glow because its phase voltage will be large whereas L_2 will not glow because of low voltage across it.

When, phase sequence is *RBY*, opposite conditions develop so that this time L_2 glows but not L_1.

Another method of determining the phase sequence is by means of a small 3-phase motor. Once direction of rotation with a known sequence is found, the motor may be used thereafter for determining an unknown sequence.

Tutorial Problem No. 19.3

1. Three impedances $\mathbf{Z}_1$, $\mathbf{Z}_2$ and $\mathbf{Z}_3$ are mesh-connected to a symmetrical 3-phase, 400-V, 50-Hz supply of phase sequence $R \rightarrow Y \rightarrow B$.

$\mathbf{Z}_1$ = (10 + *j*0) ohm– between *R and* Y *lines*
$\mathbf{Z}_2$ = (5 + *j*6) ohm – between *Y* and *B* lines
$\mathbf{Z}_3$ = (5 – *j*5) ohm – between *B* and *R* lines

Calculate the phase and line currents and total power consumed.

[40 A, 40 A, 56.6 A ; 95.7 A, 78.4 A, 35.2 A; 44.8 kW]

2. A symmetrical $3\text{-}\phi$, 380-V supply feeds a mesh-connected load as follows :
Load *A* : 19 kVA at p.f. 0.5 lag ; Load *B* : 20 kVA at p.f. 0.8 lag : Load *C* : 10 kVA at p.f. 0.9 load
Determine the line currents and their phase angles for *RYB* sequence.

[74.6 $\angle$ –51° A, 98.6 $\angle$ 172.7° A ; 68.3 $\angle$ 41.8° A]

3. Determine the line currents in an unbalanced *Y* connected load supplied from a symmetrical $3\text{-}\phi$, 440-V, 3-wire system. The branch impedances of the load are : $\mathbf{Z}_1 = 5 \angle 30°$ ohm, $\mathbf{Z}_2 = 10 \angle 45°$ ohm and $\mathbf{Z}_3 = 10 \angle 45°$ ohm and $\mathbf{Z}_4 = 10 \angle 60°$ ohm. The sequence is *RYB*. **[35.7 A, 32.8 A; 27.7 A]**

4. A $3\text{-}\phi$, *Y*-connected alternator supplies an unbalanced load consisting of three impedances (10 + *j*20), (10 – *j*20) and 10 Ω respectively, connected in star. There is no neutral connection. Calculate the voltage between the star point of the alternator and that of the load. The phase voltage of the alternator is 230 V. **[–245.2 V]**

5. Non-reactive resistors of 10, 20 and 25 Ω are star-connected to the *R*, *Y* are *B* phases of a 400-V, symmetrical system. Determine the current and power in each resistor and the voltage between star point and neutral. Phase sequence, *RYB*. **[16.5 A, 2.72 kW ; 13.1 A, 3.43 kW; 11.2 A, 3.14 kW ; 68 V]**

6. Determine the line current in an unbalanced, star-connected load supplied from a symmetrical 3-phase, 440-V system. The branch impedance of the load are $\mathbf{Z}_R = 5 \angle 30°\ \Omega$, $Z_Y = 10 \angle 45°\ \Omega$ and $Z_B = 10 \angle 60°\ \Omega$. The phase sequence is *RYB*. **[35.7 A, 32.8 A, 27.7 A]**

7. Three non-reactive resistors of 3, 4 and 5-Ω respectively are star-connected to a 3-phase, 400-V symmetrical system, phase sequence *RYB*. Find (*a*) the current in each resistor (*b*) the power dissipated in each resistor (*c*) the phase angles between the currents and the corresponding line voltages (*d*) the star-point potential. Draw to scale the complete vector diagram.

[(*a*) 66.5 A, 59.5 A, 51.8 A (*b*) 13.2, 14.15, 13.4 kW (*c*) 26°24′, 38°10′, 25°20 (*d*) 34 V]

8. An unbalanced *Y*-connected load is supplied from a 400-V, $3\text{-}\phi$, 3-wire symmetrical system. The branch circuit impedances and their connection are (2 + *j*2) Ω, *R* to *N* ; (3 – *j*3) Ω, *Y* to *N* and (4 + *j*1) Ω, *B* to *N* of the load. Calculate (*i*) the value of the voltage between lines *Y* and *N* and (*ii*) the phase of this voltage relative to the voltage between line *R* and *Y*. Phase sequence *RYB*.

[(*i*) (–216–j 135.2) or 225.5 V (*ii*) 2° or –178°]

9. A star-connection of resistors R_a = 10 Ω ; R_b = 20 Ω is made to the terminals *A*, *B* and *C* respectively of a symmetrical 400-V, ϕ supply of phase sequence $A \rightarrow B \rightarrow C$. Find the branch voltages and currents and star-point voltage to neutral.

---

* It may, however, be noted that phase sequence of *currents* in an unbalanced load is not necessarily the same as the *voltage* phase sequence. Unless indicated otherwise, voltage phase sequence is implied.

[V_A = 148.5 + *j*28.6 ; I_A = 14.85 + j2.86 ; V_B = – 198 – j171.4 ; I_B = – 9.9 – j8.57
V_C = – 198 + j228.6 ; I_C = –4.95 + j5.71. V_N = 82.5 – j28.6 (to be subtracted from supply voltage)]

10. Three non-reactive resistance of 5, 10 and 5 ohm are star-connected across the three lines of a 230-V 3-phase, 3-wire supply. Calculate the line currents.

[(18.1 + J21.1) A ; (– 10.9–j 10.45) A ; (– 7.3 + j8.4) A]

11. A 3-ϕ , 400-V symmetrical supply feeds a star-connected load consisting of non-reactive resistors of 3, 4 and 5 Ω connected to the *R*, *Y* and *B* lines respectively. The phase sequence is *RYB*. Calculate (*i*) the load star point potential (*ii*) current in each resistor and power dissipated in each resistor.

[(*i*) 34.5 V (*ii*) 66.4 A, 59.7 A, 51.8 A (*iii*) 13.22 kW, 14.21 kW, 13.42 kW]

12. A 20-Ω resistor is connected between lines *R* and *Y*, a 50-Ω resistor between lines *Y* and *B* and a 10-Ω resistor between lines *B* and *R* of a 415-V, 3-phase supply. Calculate the current in each line and the reading on each of the two wattmeters connected to measure the total power, the respective current coils of which a connected in lines *R* and *Y*. **[(25.9 – j9); (– 24.9 – j7.2); (– 1.04 + j16.2); 8.6 kW; 7.75 kW]**

13. A three-phase supply, giving sinusoidal voltage of 400 V at 50 Hz is connected to three terminals marked *R*, *Y* and *B*. Between *R* and *Y* is connected a resistance of 100 Ω , between *Y* and *B* an inductance of 318 mH and negligible resistance and between *B* and *R* a capacitor of 31.8 μF . Determine (*i*) the current flowing in each line and (*ii*) the total power supplied. Determine (*iii*) the resistance of each phase of a balanced star-connected, non-reactive load, which will take the same total power when connected across the same supply. **[(*i*) 7.73 A, 7.73 A, (*ii*) 1,600 W (*iii*) 100 Ω (*London Univ.*)]**

14. An unbalanced, star-connected load is fed from a symmetrical 3-phase system. The phase voltages across two of the arms of the load are $V_B = 295 \angle 97° 30'$ and $V_R = 206 \angle -25°$. Calculate the voltage between the star-point of the load and the supply neutral. **[52.2 $\angle$ –49.54']**

15. A symmetrical 440-V, 3-phase system supplies a star-connected load with the following branch impedances: $Z_R = 100\ \Omega$, $Z_Y = j\,5\ \Omega$, $Z_B = -j5\ \Omega$. Calculate the voltage drop across each branch and the potential of the neutral point to earth. The phase sequence is *RYB*. Draw the vector diagram.

[$8800 \angle -30°,\ 8415 \angle -31.5°,\ 8420 - \angle -28.5°, 8545 \angle 150°$]

16. Three star-connected impedances, $Z_1 = (20 + j37.7)\ \Omega$ per phase are in parallel with three delta-connected impedances, $Z_2 = (30 - j159.3)\ \Omega$ per phase. The line voltage is 398 V. Find the line current, power factor, power and reactive volt-amperes taken by the combination.

[3.37 $\angle$10.4°; 0.984 lag; 2295 lag; 2295 W; 420 VAR.]

17. A 3-phase, 440-V, delta-connected system has the loads: branch *RY*, 20 KW at power factor. 1.0: branch *YB*, 30 kVA at power factor 0.8 lagging; branch *BR*, 20 kVA at power factor 0.6 leading. Find the line currents and readings on watt-meters whose current coils are in phases *R* and *B*.

[90.5 $\angle$ 176.5°; 111.4 $\angle$ 14°; 36.7 $\angle$ – 119; °39.8 kW; 16.1 kW]

18. A 415 V, 50 Hz, 3-phase supply of phase sequence *RYB* is connected to a delta connected load in which branch *RY* consists of $R_1 = 100\ \Omega$, branch Y_B consists of $R_2 = 20\ \Omega$ in series with $X_2 = 60\ \Omega$ and branch *BR* consists of a capacitor $C = 30\ \mu$F . Take V_{RY} as the reference and calculate the line currents. Draw the complete phasor diagrams. ***(Elect. Machines, A.M.I.E. Sec. B, 1989)***

[I_R = 7.78 $\angle$ 14.54°, I_Y = 10.66 $\angle$ 172.92°, I_B = 4.46 $\angle$ – 47°]

19. Three resistances of 5, 10 and 15 Ω are connected in delta across a 3-phase supply. Find the values of the three resistors, which if connected in star across the same supply, would take the same line currents.

If this star-connected load is supplied from a 4-wire, 3-phase system with 260 V between lines, calculate the current in the neutral. **[2.5 Ω, 1.67 Ω, 5 Ω; 52 A] (*London Univ.*)**

20. Show that the power consumed by three identical phase loads connected in delta is equal to three times the power consumed when the phase loads are connected in star.

(Nagpur University, Summer 2002)

21. Prove, that the power consumed in balanced three- phase Delta-connected load is three times the power consumed in starconnected load. ***(Nagpur University, Winter 2002)***

22. A three-phase 230 volts systems supplies a total load of 2000 watts at a line current of 6 Amp when three identical impedances are in star-connection across the line terminals of the systems. Determine

the resistive and reactive components of each impedance. *(Nagpur University, Winter 2002)*

23. Three simila coils each of impedance z = (8 + j10) ohms, are connected in star and supplied from 3-phase 400V, 50 Hz supply. Find the line current, power factor, power and total volt amperes.
(Nagpur University, Summer 2003)

24. Three similar each having a resistance of 20 ohm and an inductance of 0.05 H are connected in star to a 3-phase 50 Hz supply with 400 V between lines. Calculate power factor, total power absorb and line current. If the same coil are reconnected in delta across the same supply what will be the power factor, total power absorbed and line current?
(Pune University 2003) (Nagpur University, Winter 2003)

25. A 3 ϕ star connected load when supplied from 440 V, 50 Hz source takes a line current of 12 amp lagging w.r.t. line voltage by 70^0.Calculate : (*i*) limpedance parameters (*ii*) Power factor and its nature (*iii*) Draw phasor diagram indicating all voltages and currents. *(Nagpur University, Summer 2004)*

26. Derive the relationship between line current and phase current for Delta connected 3 phase load when supplied from 3 phase balanced supply. *(Nagpur University, Summer 2004)*

27. Derive the relationship between line voltage, phase voltage, line current and phase current in a 3 phase star connected and delta connected circuit. *(Gujrat University, June/July 2003)*

28. show that power input to a 3 phase circuit can be measured by two wattmeters connected properly in the circuit. Draw vector diagram. *(Gujrat University, June/July 2003)*

29. A balanced 3 phase star connected load of 100 kW takes a leading current of 100 A when connected across a 3 phase, 1100 V, 50 Hz supply. Calculate the circuit constants of the load per phase.
(Mumbai University 2003) (Gujrat University, June/July 2003)

30. Establish relationship between line and phase voltages and currents in a balanced 3-phase star connec tion. Draw complete phasor diagram for voltages and currents.
(R.G.P.V. Bhopal University, June 2004)

31. A delta connected load has the following impedances :
$Z_{RY} = j\ 10\ \Omega$, $Z_{YB} = 10 \angle 0^o\ \Omega$ and $Z_{BR} = -j\ 10\ \Omega$. If the load is connected across 100 volt balanced 3-phase supply, obtain the line currents. *(R.G.P.V. Bhopal University, June 2004)*

32. Two wattmeters ω_1 and ω_2 are used to measure power in a 3 phase balanced circuit. Mention the conditions under which (*i*) $\omega_1 = \omega_2$ (*ii*) $\omega_2 = 0$ (*iii*) $\omega_1 = 2\omega_2$.
(V.T.U. Belgaum Karnataka University, February 2002)

33. Three coils each of impedance $20 \mid 60^0\Omega$ are connected across a 400V, 3 phase supply. Find the reading of each of the two wattmeters connected to measure the power when the coils are connected in (*i*) star (*ii*) Delta. *(V.T.U. Belgaum Karnataka University, February 2002)*

34. The power input to a 3 phase circuit was measured by two wattmeter method and the readings were 3400 and - 1200 watts respectively. Calculate the total power and powerfactor.
(V.T.U. Belgaum Karnataka University, July/August 2002)

35. With the help of connection diagram and vector diagram, obtain expressions for the two wattmeter readings used to measure power in a 3 phase the DC generator is running.
(V.T.U. Belgaum Karnataka University, July/August 2002)

36. Obtain the relationship between line and phase values of current in a three phase, balanced, delta connected system. *(V.T.U. Belgaum Karnataka University, January/February 2003)*

37. Show that in a three phase, balanced circuit, two wattmeters are sufficient to measure the total three phase power and power factor of the circuit.
(V.T.U. Belgaum Karnataka University, January/February 2003)

38. Each of the two wattmeters connected to measure the input to a three phase circuit, reads 20kW. What does each instrument reads, when the load p.f. is 0.866 lagging with the total three phase power remaining unchanged in the altered condition? *(V.T.U. Belgaum Karnataka University, January/February 2003)*

39. Two wattmeters connected to measure power in a 3 phase circuit read 5KW and 1KW, the latter reading being obtained after reversing current coil connections. Calculate power factor of the load and the total power consumed. *(V.T.U. Belgaum Karnataka University, January/February 2003)*

39. Derive the relationship between phase and line values of voltages in a connected load.
(V.T.U. Belgaum Karnataka University, January/February 2003)

40. Three coils each of impedance $20 \angle 60^0\Omega$ are connected in delta across a 400, 3 phase, 50Hz, 50Hz Acsupply. Calculate line current and total power.
(V.T.U. Belgaum Karnataka University, January/February 2003)

41. What are the advantages of a three phase system over a single phase system?
(V.T.U. Belgaum Karnataka University, July/August 2003)

42. With a neat circuit diagram and a vector diagram prove that two wattmeters are sufficient to measure total power in a 3 phase system. *(V.T.U. Belgaum Karnataka University, July/August 2003)*

43. A balanced star connected load of $(8 + jb)\Omega$ is connected to a 3 phase, 230 V supply. Find the line current, power factor, power, reactive voltamperes and total voltamperes.
(V.T.U. Belgaum Karnataka University, July/August 2003)

44. Two watt meters are used to measure the power delivered to a balance 3 phase load of power factor 0.281. One watt meter reads 5.2kW. Determine the reading of the second watt meter. What is the line current if the line voltage is 415 wolt? *(V.T.U. Belgaum Karnataka University, January/February 2004)*

45. Write the equations for wattmeter reading W_1 and W_2 in 3 phase power measurement and therefrom for power factor. *(Anna University, October/November 2002)*

46. Show that the wattmeters will read equal in two wattmeter method under unity power factor loading condition. *(Anna University, November/December 2003)*

47. A star connected 3-phase load has a resistance of 6Ω and an inductive reactance of 8Ω in each brance. Line voltage is 220 volts. Write the phasor expressions for voltage across each branch, line voltages and line currents. Calculate the total power. *(Anna University, November/December 2003)*

48. Two wattmeters connected to measure the total power in a 3-phase balanced circuit. One measures 4,800 W, while the other reads backwards. On reversing the latter it is found to read 400 W. What is the total power and power and power factor? Draw the connection diagram and phasor diagram of the circuit. *(Mumbai University 2003) (RGPV Bhopal 2001)*

49. A star-network in which N is star point made up as follows :
AN = 70Ω, CN = 90 Ω. Find an equivalent delta network. If the above star-delta network are superimposed, what would be measured resistance between A and C?
(Pune University, 2003) (RGPV Bhopal 2001)

50. Explain with diagram the measurement of 3-phase power by two-wattmeter method.
(RGPV Bhopal 2002)

51. Show that the power taken by a 3-phase circuit can measured by two wattmeters connected properly in the circuit. *(RGPV Bhopal)*

52. With the aid of star-delta connection diagram, state the basic equation from which star-delta conversionequation canbe derived. *(Pune University, 2003) (RGPV Bhopal 2001)*

53. Star-delta connections in a 3-phase supply and their inter-relationship. *(RGPV Bhopal 2001)*

54. Measurement of power in three-phase circuit in a balanced condition. *(RGPV Bhopal 2001)*

55. Measurement of reactive power in three-phase circuit. *(RGPV Bhopal 2001)*

56. Differentiate between balanced and unbalancedthree-phase supply and balanced and unbalanced three-phase load. *(RGPV Bhopal June 2002)*

57. A 3-phase 3 wire supply feeds a load consisting of three equal resistors. By how much is the load reduced if one of the resistors be removed ? *(RGPV Bhopal June 2002)*

58. Establish relationship between line and phase voltages and currents in a balanced delta connection. Draw complete phasor diagram of voltages and currents. *(RGPV Bhopal December 2003)*

OBJECTIVE TESTS – 19

1. The minimum number of wattmeter (s) required to measure 3-phase, 3-wire balanced or unbalanced power is
(*a*) 1
(*b*) 2
(*c*) 3
(*d*) 4
(GATE 2001)

2. A wattmeter reads 400 W when its current coil is connected in the R phase and its pressure coil is connected between this phase and the neutral of a symmetrical 3-phase system supplying a balanced star connected 0.8 p.f. inductive load. The phase sequence is RYB. What will be the reading of this wattmeter if its pressure coil alone is reconnected between the B and Y phases, all other connections remaining as before?
(*a*) 400.0 (*b*) 519.6
(*c*) 300.0 (*d*) 692.8
(GATE 2003)

3. Total instantaneous power supplied by a 3-phase ac supply to a balanced R-L load is
(*a*) zero
(*b*) constant
(*c*) pulsating with zero average
(*d*) pulsating with non-zero average
(GATE 2004)

4. A balanced 3-phase, 3-wire supply feeds balanced star connected resistors. If one of the resistors is disconnected, then the percentage reduction in the load will be
(*a*) $33\frac{1}{3}$ (*b*) 50
(*c*) $66\frac{2}{3}$ (*d*) 75 *(GATE)*

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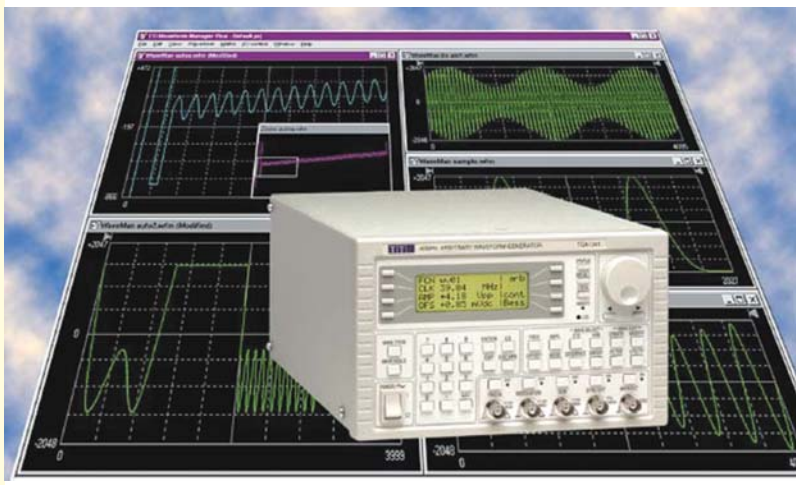
Contents

CHAPTER 20

HARMONICS

Learning Objectives

- Fundamental Wave and Harmonics
- Different Complex Waveforms
- General Equation of a Complex Wave
- R.M.S. Value of a Complex Wave
- Form Factor of a Copmplex Wave
- Power Supplied by a Complex Wave
- Harmonics in Single-phase A.C. Circuits
- Selective Resonance Due to Harmonics
- Effect of Harmonics on Measurement of Inductance and Capacitance
- Harmonics in Different Three-phase Systems
- Harmonics in Single and 3-Phase Transformers



Harmonics are the multiples of a sine wave (the fundamental frequency)

20.1. Fundamental Wave and Harmonics

Upto this stage, while dealing with alternating voltages and currents, it has been assumed that they have sinusoidal waveform or shape. Such a waveform is an ideal one and much sought after by the manufacturers and designers of alternators. But it is nearly impossible to realize such a waveform in practice. All the alternating waveforms deviate, to a greater or lesser degree, from this ideal sinusoidal shape. Such waveforms are referred to as ***non-sinusoidal*** or ***distorted*** or ***complex waveforms***.

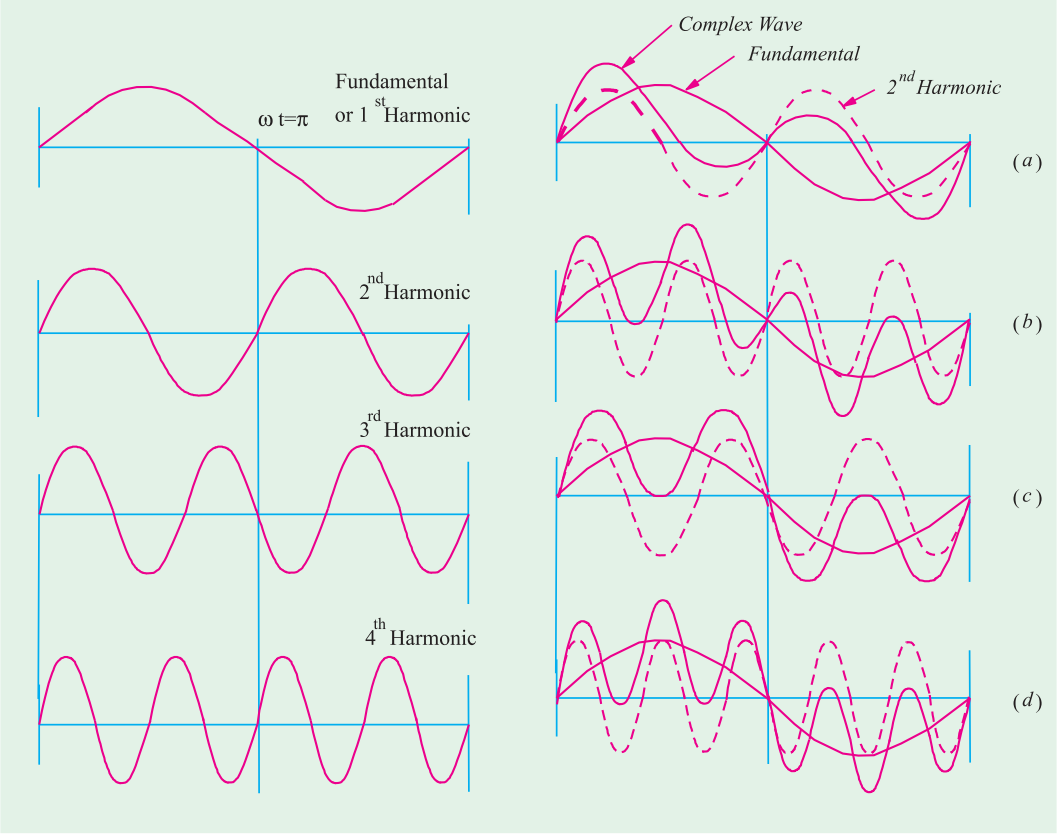


Fig. 20.1

Fig. 20.2

Complex waveforms are produced due to the superposition of sinusoidal waves of different frequencies. Such waves occur in speech, music, TV, rectifier outputs and many other applications of electronics. On analysis, it is found that a complex wave essentially consists of

(*a*) *a fundamental* wave – it has the lowest frequency, say '*f*'

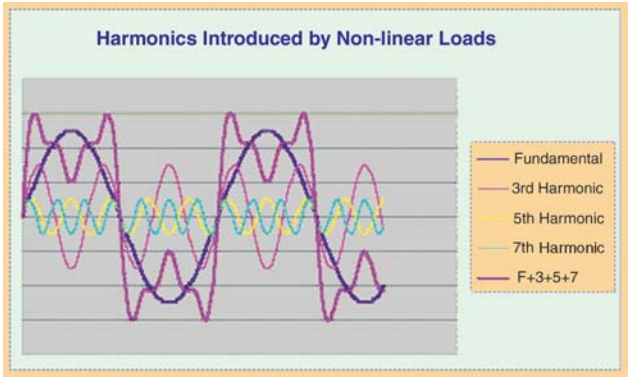
(*b*) a number of other sinusoidal waves whose frequencies are an integral multiple of the fundamental or basic frequency like 2*f*, 3*f* and 4*f* etc.

The fundamental and its higher multiples form a ***harmonic*** series.

As shown in Fig. 20.1, fundamental wave itself is called the ***first*** harmonic. The ***second*** harmonic has frequency ***twice*** that of the fundamental, the ***third*** harmonic has frequency ***thrice*** that of the fundamental and so on.

Waves having frequencies of 2*f*, 4*f* and 6*f* etc. are called ***even harmonics*** and those having frequencies of 3*f*, 5*f* and 7*f* etc. are called *odd* harmonics. Expressing the above is angular frequencies, we may say that successive odd harmonics have frequencies of 3ω, 5ω and 7ω etc. and even harmonics have frequencies of 2ω, 4ω and 6ω etc.

As mentioned earlier, harmonics are introduced in the output voltage of an alternator due to many reason such as the irregularities of the flux distribution in it. Considerations of waveform and form factor are very important in the transmission of a. c. power but they are of much greater importance in radio work where the intelligibility of a signal is critically dependent on the faithful transmission of the harmonic structure of sound waves. In fact, it is only the rich harmonic content of the consonants and lesser at still plentiful harmonic content of vowels which helps the ear to distinguish a well regulated speech from a more rhythmical succession of musical sounds.



20.2. Different Complex Waveforms

Let us now find out graphically what the resultant shape of a complex wave is when we combine the fundamental with one of its harmonics. Two cases would be considered ***(i)*** when the fundamental and harmonic are in phase with each other and have equal or unequal amplitudes and ***(ii)*** when there is some phase difference between the two.

In Fig. 20.2 (*a*), the fundamental and second harmonic, both having the same amplitude, have been shown by the firm and broken line respectively. The resultant complex waveform is plotted out by algebraically adding the individual ordinates and is shown by thick line.

It may be noted that since the maximum amplitude of the harmonic is equal to the maximum amplitude of the fundamental, the complex wave is said to contain 100% of second harmonic.

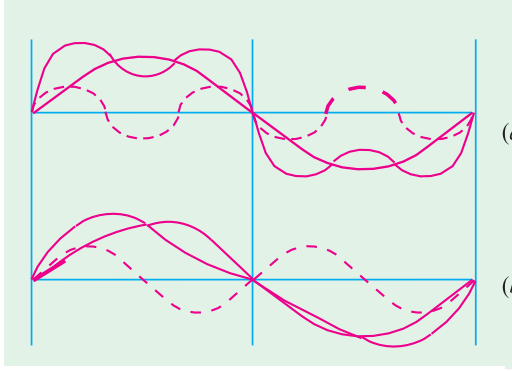


Fig. 20.3

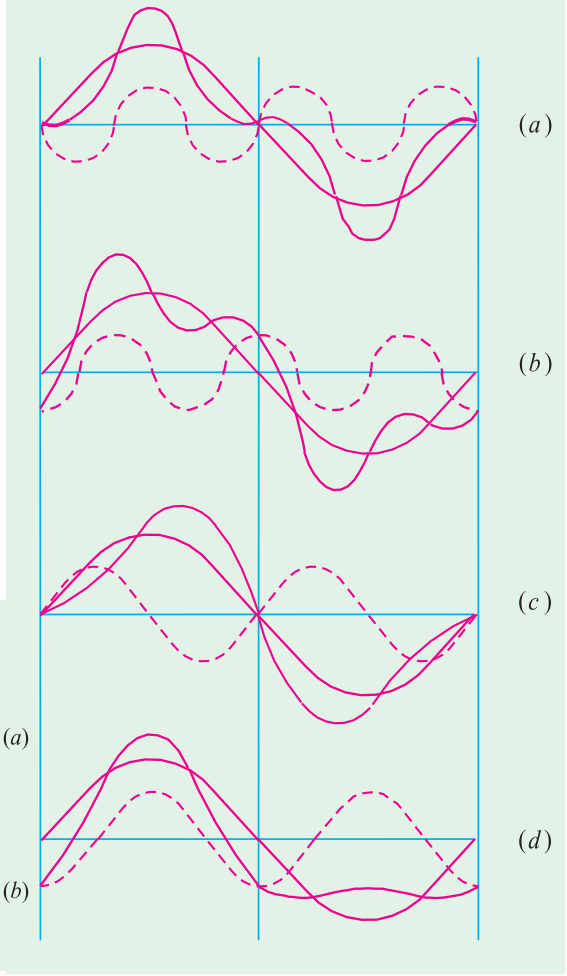


Fig. 20.4

The complex wave of Fig. 20.2 (*b*) is made up of the fundamental and 4th harmonic, that of Fig. 20.2 (*c*) consists of the fundamental and 3rd harmonic whereas that shown in Fig. 20.2 (*d*) is made up of the fundamental and 5th harmonic. Obviously, in all there cases, there is no phase difference between the fundamental and the harmonic.

Fig. 20.2 (*a*) and (*c*) have been reconstructed as Fig. 20.3 (*a*) and (*b*) respectively with the only difference that in this case, the amplitude of harmonic is half that of the fundamental *i.e.* the harmonic content is 50%.

The effect of phase difference between the fundamental and the harmonic on the shape of the resultant complex wave has been illustrated in Fig. 20.4.

Fig. 20.4 (*a*) shows the fundamental and second harmonic with phase difference of $\pi/2$ and Fig. 20.4 (*b*) shows the same with a phase difference of π. In Fig. 20.4 (*c*) and (*d*) are shown the fundamental and third harmonic with a phase difference of $\pi/2$ and π respectively. In all these figures, the amplitude of the harmonic has been taken equal to half that of the fundamental.

A careful examination of the above figures leads us to the following conclusions :

1. With ***odd*** harmonics, the positive and negative halves of the complex wave are symmetrical whatever the phase difference between the fundamental and the harmonic. In other words, the first and third quarters (*i.e.* ωt from 0 to $\pi/2$ and ωt from $3\pi/2$) and the second and fourth quarters (*i.e.* ωt from $\pi/2$ to π and ωt from $3\pi/2$ to 2π) are respectively similar.

2. (*i*) When ***even*** harmonics are present and their phase difference with the fundamental is 0 or π, then the first and fourth quarters of the complex wave are of the same phase but inverted and the same holds good for the second and third quarters.

(*ii*) When ***even*** harmonics are present and their phase difference with the fundamental is $\pi/2$ or $3\pi/2$, then there is no symmetry as shown in Fig. 20 (*a*).

3. It may also be noted that the resultant displacement of the complex wave (whether containing odd or even harmonics) is zero at $\omega t = 0$ only when the phase difference between the fundamental and the harmonics is either 0 to π.

The above conclusions are of great help in analysing a complex waveform into its harmonic constituents because a visual inspection of the complex wave enables us to rule out the presence of certain harmonics. For example, if the positive and negative half-cycles of a complex wave are symmetrical (*i.e.* the wave is symmetrical about $\omega t = 0$), then we need not look for even harmonics. In some cases, we may be able to forecast the types of harmonics to be expected from their mode of production. For example, in alternators which are symmetrically designed, we should expect *odd* harmonics only.

20.3. General Equation of a Complex Wave

Consider a complex wave which is built up of the fundamental and a few harmonics, each of which has its own peak value of phase angle. The fundamental may be represented by

$$e_1 = E_{1m} \sin(\omega t + \Psi_1)$$

the second harmonic by $e_2 = E_{2m} \sin(2\omega t + \Psi_2)$

the third harmonic by $e_3 = E_{3m} \sin(3\omega t + \Psi_3)$ and so on.

The equation for the instantaneous value of the complex wave is given by

$e = e_1 + e_2 + ... e_n = E_{1m} \sin(\omega t + \Psi_1) \times E_{2m} \sin(2\omega t + \Psi_2) + ... + I_{nm} \sin(n\omega t + \Psi_n)$

when E_{1m}, E_{2m} and E_{nm} etc. denote the maximum values or the amplitudes of the fundamental, second harmonic and *n*th harmonic etc. and Ψ_1, Ψ_2 and Ψ_n represent the phase differences with

respect to the complex wave* (*i.e.* angle between the zero value of complex wave and the corresponding zero value of the harmonic).

The number of terms in the series depends on the shape of the complex wave. In relatively simple waves, the number of terms in the series would be less, in others, more.

Similarly, the instantaneous value of the complex wave is given by

$$i = I_{1m}\sin(\omega t+\phi_1)+I_{2m}\sin(2\omega t+\phi_2)+.....+I_{nm}\sin(n\omega t+\phi_n)$$

Obviously $(\Psi_1-\phi_1)$ is the phase difference between the harmonic voltage and current for the fundamental, $(\Psi_2-\phi_2)$ for the second harmonic and $(\Psi_n-\Phi_2)$ for the nth harmonic.

20.4. R.M.S. Value of a Complex Wave

Let the equation of the given complex current wave be

$$i \quad I_{1m}\sin(\quad t \quad _1) \quad I_{2m}\sin(2 \quad t \quad _2) \quad ....I_{nm}\sin(n \quad t \quad _n)$$

Its r.m.s. value is given by $I=\sqrt{\text{average value of } i^2 \text{ over whole cycle}}$

Now $i^2=[I_{1m}\sin(\omega t+\phi_1)+I_{2m}\sin(2\omega t+\phi_1)+....I_{nm}\sin(n\omega t+\phi_n)]^2$

$$=I_{1m}{}^2\sin^2(\omega t+\phi_1)+I_{2m}{}^2\sin^2(2\omega t+\phi_2)+...I_{nm}{}^2\sin^2(n\omega t+\phi_n)]^2$$
$$+2I_{1m}I_{2m}\sin(\omega t+\phi_1)\sin(2\omega t+\phi_2)+2I_{1m}I_{3m}\sin(\omega t+\Phi_1)\sin(\omega t+\Phi_2)+....$$

The right-hand side of the above equation consists of two types of terms

(*i*) harmonic self-products, the general expression for which is $\mathbf{I}_{pm}{}^2\sin^2(p\omega t+\phi_p)$ for the pth harmonic and

(*ii*) the products of different harmonics of the general form $2I_{pm}I_{qm}\sin(p\omega t+\phi_n)\sin(q\omega t+\phi_q)$

The average value of i^2 is the sum of the average values of these individual terms in the above equation. Let us now find the average value of the general term $I_{pm}{}^2\sin^2(p\omega t+\phi_p)$ over a whole cycle.

$$\text{Average value} = \frac{1}{\quad}\ {}^{2}_{0}\ I_{pm}{}^2\sin^2(p \quad t \quad _p)d(\quad t) \quad \frac{I_{pm}{}^2}{2}\ {}^{2}_{0}\ \sin^2(p \quad \quad _p)d$$

$$=\frac{I_{pm}{}^2}{2}\ {}^{2}_{0}\ \frac{1\quad\cos 2(p \quad \quad _p)}{2}\ d \quad \frac{I_{pm}{}^2}{2}\Big|\quad \sin^2(p \quad \quad _p)\Big|^2_0$$

$$=\frac{I_{pm}{}^2}{4\pi}\times 2\pi=\frac{I_{pm}{}^2}{2}$$

From this result, we can generalize that

$$\text{Average value of } I_{1m}{}^2\sin^2(\omega t+\phi_1)=\frac{I_{1m}{}^2}{2}$$

$$\text{Average value of } I_{2m}{}^2\sin^2(2\omega t+\phi_2)=\frac{I_{2m}{}^2}{2}$$

---

* We could also express these phase angles with respect to the fundamentals wave instead of the complex wave.

Average value of $I_{nm}^2 \sin^2(n\omega t + \phi_n) = \frac{I_{nm}^2}{2}$ and so on.

Now, the average value of the product terms is

$$\frac{1}{2} \int_0^{2} I_{pm} I_{qm} \sin(p \quad t \quad _p) \sin(q \quad t \quad _q) d(\quad t)$$

$$\frac{I_{pm} I_{qm}}{5} \int_0^{2} \sin(p \quad _p) \sin(q \quad _q) d \quad 0$$

$\therefore$ Average value of $i^2 = \frac{I_{1m}^2}{2} + \frac{I_{2m}^2}{2} + ... + \frac{I_{nm}^2}{2}$

$\therefore$ r.m.s. value, $I = \sqrt{\text{average value of } i^2} \quad \sqrt{\frac{I_{1m}^2}{2} \quad \frac{I_{2m}^2}{2} \quad .....\frac{I_{nm}^2}{2}}$ ... ***(i)***

$$= 0.707\sqrt{I_{1m}^2 + I_{2m}^2 + ... + I_{nm}^2}$$

Equation ***(i)*** above may also be put in the form

$$I = \sqrt{\frac{I_{1m}}{\sqrt{2}}^2 \quad \frac{I_{2m}}{\sqrt{2}}^2 \quad ..... \quad \frac{I_{nm}}{\sqrt{2}}^2} \quad \sqrt{I_1^2 \quad I_2^2 \quad .....I_n^2}$$

where $I_1 = I_{1m}/\sqrt{2}$ – r.m.s. value of fundamental

$I_2 = I_{2m}/\sqrt{2}$ – r.m.s. value of 2nd harmonic

$I_n = I_{nm}/\sqrt{2}$ – r.m.s. value of nth harmonic

Similarly, the r.m.s. value of a complex voltage wave is

$$E = 0.707\sqrt{E_{1m}^2 + E_{2m}^2 + ...E_{nm}^2} = \sqrt{E_1^2 + E_2^2 + ...E_n^2}$$

Hence, the ***rule is that the r.m.s. value of the complex current (or voltage) wave is given by the square-root of the sum of the squares of the r.m.s. values of its individual components.***

Note. If complex current wave contains a d.c. component of constant value I_D then its equation is given by

$$i = I_D + I_{1m}\sin(\omega t + \phi_1) + I_{2m}\sin(2\omega t + \phi_2) + ...I_{nm}\sin(n\omega t + \phi_n)$$

r.m.s. value, $I \quad \sqrt{I_D^2 \quad (I_{1m}/\sqrt{2})^2 \quad (I_{2m}/\sqrt{2})^2 .....(I_{nm}/\sqrt{2})^2} \quad I_D^2 \quad I_1^2 \quad I_2^2 \quad ....I_n^2$

20.5. Form Factor of a Complex Wave

In general, it may be defined as $k_f = \frac{\text{R.M.S. value}}{\text{average value}}$

A general expression for form factor in some simple cases may be found as under :

***(i)* Sine Series.** Suppose the equation of a complex voltage wave is

$v = V_{1m}\sin\omega t \pm V_{3m}\sin 3\omega t \pm V_{5m}\sin 5\omega t$

$= V_{1m}\sin\theta \pm V_{3m}\sin 3\theta \pm V_{5m}\sin 5\theta$ where $\omega = 2\pi/T$.

Obviously, zeros occurs at $t = 0$ or at $\theta = 0°$ and $\theta = 180°$ or $t = T/2$.

Mean value over half-cycle is

$$V_{av} \quad \frac{1}{} \int_0 v d$$

$$\frac{1}{} V_{1m} \quad _0 \quad \sin \quad .d \quad V_{3m} \quad _0 \quad \sin 3 \quad .d \quad V_{5m} \quad _0 \quad \sin 5 \quad .d \quad \frac{2}{} \frac{V_{1m}}{1} \quad \frac{V_{3m}}{3} \quad \frac{V_{5m}}{5}$$

As found in Art. 20.4,

$$V = (V_1^2 + V_3^2 + V_5^2)^{1/2} = \frac{1}{\sqrt{2}} (V_{1m}^2 + V_{3m}^2 + V_{5m}^2)^{1/2}$$

$$\therefore \quad k_f \quad \frac{(1/\sqrt{2})(V_{1m}^2 \quad V_{3m}^2 \quad V_{5m}^2)^{1/2}}{\frac{2}{} \quad V_{1m} \quad \frac{V_{3m}}{3} \quad \frac{V_{5m}}{5}}$$

(*ii*) **Cosine Series.** Consider the following cosine series :

$$v = V_{1m} \cos \omega t \pm V_{3m} \cos 3\omega t \pm V_{5m} \cos 5\omega t$$

$$= V_{1m} \cos \theta \pm V_{3m} \cos 3\theta \pm V_{5m} \cos 5\theta$$

Obviously, in this case, zeros occur at $\theta = \pm \pi / 2$ or 90°. Moreover, positive and negative half-cycles are symmetrical.

$$\therefore \quad V_{av} \quad \frac{1}{} \quad _{/2}^{/2} (V_{1m} \cos \quad V_{3m} \cos 3 \quad V_{5m} \cos 5 \quad) d \quad \frac{2}{} \quad V_{1m} \quad \frac{V_{3m}}{3} \quad \frac{V_{5m}}{5}$$

$$\therefore \quad k_f \quad \frac{(1/\sqrt{2})(V_{1m}^2 \quad V_{3m}^2 \quad V_{5m}^2)^{1/2}}{\frac{2}{} \quad V_{1m} \quad \frac{V_{3m}}{3} \quad \frac{V_{5m}}{5}}$$

Example 20.1. *A voltage given by v = 50 + 24 sin ωt–20 sin 2ωt is applied across the circuit shown in Fig. 20.5. What would be the readings of the instruments if ω = 10,000 rad/s. A_1 is thermoelectric ammeter, A_2 a moving-coil ammeter and V an electrostatic voltmeter.*

Solution. It may be noted that the thermoelectric ammeter and the electrostatic voltmeter record the r.m.s. values of the current and voltage respectively. But the moving coil ammeter records the average values. Since the average values of the sinusoidal waves are zero, hence the moving coil ammeter reads the d.c. component of the current only. The d.c. will pass only through the inductive branch and not through the capacitive branch.

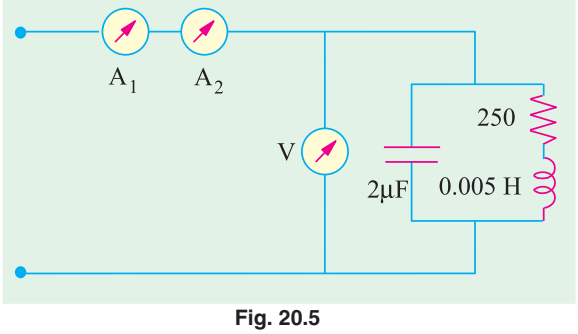


Fig. 20.5

$$\therefore \quad I_{DC} = \frac{V_{DC}}{R} = \frac{50}{250} = \mathbf{0.2\ A}$$

Equivalent impedance of the circuit at fundamental frequency is

$$\mathbf{Z_1} \quad \frac{(R \quad jX_{L1})(\quad jX_{C1})}{R \quad jX_{L1} \quad jX_{C1}} \quad \frac{(250 \quad j50)(\quad j50)}{250 \quad j(50 \quad 50)} \quad \frac{2{,}500 \quad j12{,}500}{250} \quad 10 \quad j50 \quad 51 \quad 78°42$$

$\therefore$ r.m.s. fundamental current $I_1 = I_{1m} / \sqrt{2} = 24 / 51 \times \sqrt{2} = 0.33$ A

Equivalent impedance of the circuit at the second harmonic is

$$\mathbf{Z_2} \quad \frac{(R \quad jX_{L2})(\quad jX_{C2})}{R \quad jX_{L2} \quad jX_{C2}} \quad \frac{(250 \quad 100)(\quad j25)}{200 \quad j75} \quad 31.5 \quad 88°43$$

$\therefore$ r.m.s. value of second harmonic current

$$I_2 = I_{2m} / \sqrt{2} = 20 / 31.5 \times \sqrt{2} = 0.449 \text{ A}$$

r.m.s. current in the circuit is

$$I = \sqrt{I_{DC}^2 + I_1^2 + I_2^2} = \sqrt{0.2^2 + 0.33^2 + 0.449^2} = 0.593 \text{ A}$$

Hence, the reading of the thermoelectric ammeter is **0.593 A**

The voltmeter reading $V = \sqrt{50^2 + (24/\sqrt{2})^2 + (20/\sqrt{2})^2} =$ **54.66**

Example 20.2. *Draw one complete cycle of the following wave*

$$i = 100 \sin \omega t + 40 \sin 5 \omega t$$

Determine the average value, the r.m.s. value and form factor of the wave.

(Elect. Engineering, Osmania Univ.)

Solution. $I_{av} \quad \frac{2}{} \quad \frac{I_{2m}}{1} \quad \frac{I_{5m}}{5} \quad \frac{2}{} \quad \frac{100}{1} \quad \frac{40}{5}$ **68.7 A**

$$I = \sqrt{\frac{I_{1m}^2}{2} + \frac{I_{5m}^2}{2}} = \frac{1}{\sqrt{2}}(100^2 + 40^2)^{1/2} = \textbf{76.2 A}$$

Form factor $= \frac{I}{I_{av}} = \frac{76.2}{68.7} =$ **1.109**

20.6. Power Supplied by a Complex Wave

Let the complex voltage be represented by the equation

$$e = E_{1m} \sin \omega t + E_{2m} \sin 2\omega t + \ldots . E_{nm} \sin n\omega t$$

be applied to a circuit. Let the equation of the resultant current wave be

$$i = I_{1m} \sin(\omega t + \phi_1) + I_{2m} \sin(2\omega t + \phi_2) + \ldots . . I_{nm} \sin(n\omega t + \phi_n)$$

The instantaneous value of the power in the circuit is $p = ei$ watt

For obtaining the value of this product, we will have to multiply every term of the voltage wave, in turn, by every term in the current wave. The average power supplied during a cycle would be equal to the sum of the average values over one cycle of each individual product term. However, as proved in Art. 20.4 earlier, the average value of all product terms involving harmonics of different frequencies will be zero over one cycle, so that we need consider only the products of current and voltage harmonics of the same frequency.

Let us consider a general term of this nature *i.e.* $E_{nm} \sin n\omega t \times I_{nm} \sin (n\omega t - \phi_n)$ and find its average value over one cycle of the fundamental.

Average value of power $\frac{1}{2} \int_0^{2} E_{nm} I_{nm} \sin n \; t \sin(n \; t \quad _n) d(\; t)$

$$\frac{E_{nm} I_{nm}}{2} \int_0^{2} \sin n \; \sin(n \quad _n) d$$

$$\frac{E_{nm} I_{nm}}{2} \int_0^{2} \frac{\cos \; _n \quad \cos(2n \quad _n)}{2} d$$

$$= \frac{E_{nm} I_{nm} \cos \phi}{2} = \frac{E_{nm}}{\sqrt{2}} \cdot \frac{I_{nm}}{\sqrt{2}} \cdot \cos \phi_n = E_n I_n \cos \phi_n$$

where E_n and I_n are the r.m.s. values of the voltage and current respectively. Hence, total average power supplied by a complex wave is ***the sum of the average power supplied by each harmonic component acting independently***.

$\therefore$ Total power is $P = E_1 I_1 \cos\phi_1 + E_2 I_2 \cos\phi_2 + ... E_n I_n \cos\phi_n$

The overall power factor is given by

$$\text{pf.}^* \quad \frac{\text{total watts}}{\text{total voltamperes}} \quad \frac{E_1 I_1 \cos_1 \quad E_2 I_2 \cos_2 \quad ....}{E \quad I}$$

when E = r.m.s. value of the complex voltage wave

I = r.m.s. value of the complex current wave

Example 20.3. *A single-phase voltage source 'e' is given by*

$$e = 141 \sin\omega t + 42.3 \sin 3\omega t + 28.8 \sin 5\omega t$$

The corresponding current in the load circuit is given by

$$i = 16.5 \sin(\omega t + 54.5°) + 8.43 \sin(3\omega t - 38°) + 4.65 \sin(5\omega t - 34.3°)$$

Find the power supplied by the source.

(Electrical Circuits, Nagpur Univ. 1991)

Solution. In problems of such type, it is best to deal with each harmonic separately

Power at fundamental

$$= E_1 I_1 \cos\phi_1 = \frac{E_{1m}}{\sqrt{2}} \cdot \frac{I_{1m}}{\sqrt{2}} \cos\phi_1 = \frac{E_{1m} I_{1m}}{2} \cos\phi_1 = \frac{141 \times 16.5}{2} \cos 54.5° = 675.5 \text{ W}$$

Power at 3rd harmonic $= \frac{E_{3m} I_{3m}}{2} \cos\phi_3 = \frac{42.3 \times 8.43}{2} \cos 38° = 140.5 \text{ W}$

Power at 5th harmonic $= \frac{28.8 \times 4.65}{2} \cos 34.3° = 55.5 \text{ W}$

Total power supplied = 675.5 + 140.5 + 55.5 = **871.5 W**

Example 20.4. *A complex voltage is given by e = 60 sin ωt + 24 sin (3 ωt + ω /6) + 12 sin (5 ω t + π /3) is applied across a certain circuit and the resulting current is given by*

$$i = 0.6 \sin(\omega t - 2\pi / 10) + 0.12 \sin(\omega t - 2\pi / 24) + 0.1 \sin(5\pi - 3\pi / 4)$$

Find ***(i)*** *r.m.s value of current and voltage* ***(ii)*** *total power supplied and* ***(iii)*** *the overall power factor.*

Solution. In such problems where harmonics are involved, it is best to deal with each harmonic separately.

Power at fundamental = $E_1 I_1 \cos\phi_1 = \frac{E_{1m} I_{1m}}{2} \cos\phi_1 = \frac{60 \times 0.6}{2} \times \cos 36° = 14.56 \text{ W}$

Power at 3rd harmonic = $\frac{E_{3m} I_{3m}}{2} \cos 45° = \frac{24 \times 0.12}{2} \times 0.707 = 1.02 \text{ W}$

Power at 5th harmonic = $\frac{E_{5m} I_{5m}}{2} \cos 75° = \frac{12 \times 0.1}{2} \times 0.2588 = 0.16 \text{ W}$

---

* When harmonics are present, it is obvious that the overall p.f. of the circuit cannot be stated lagging or leading. It is simply the ratio of power in watts of voltamperes.

(*i*) R.M.S. current $I = \sqrt{I_1^2 \quad I_3^2 \quad I_5^2} \quad \sqrt{\frac{I_{1m}}{\sqrt{2}}^2 \quad \frac{I_{3m}}{\sqrt{2}}^2 \quad \frac{I_{5m}}{\sqrt{2}}^2}$

$$= \sqrt{\frac{0.6^2}{2} + \frac{0.12^2}{2} + \frac{0.1^2}{2}} = \mathbf{0.438\ A}$$

R.M.S. volts, $E = \sqrt{\frac{60^2}{2} + \frac{24^2}{2} + \frac{12^2}{2}} = \mathbf{46.5\ V}$

(*ii*) Total power = 14.56 + 1.02 + 0.16 = **15.74 W**

(*iii*) Overall p.f. $= \frac{\text{watts}}{\text{voltamperes}} = \frac{15.74}{46.5 \times 0.438} = \mathbf{0.773}$

20.7. Harmonics in Single-phase A.C. Circuits

If an alternating voltage, containing various harmonics, is applied to a single-phase circuit containing linear circuit elements, then the current so produced also contains harmonics. Each harmonic voltage will produce its own current independent of others. By the principle of superposition, the combined current can be found. We will now consider some of the well-known elements like pure resistance, pure inductance and pure capacitance and then various combinations of these. In each case, we will assume that the applied complex voltage is represented by

$$e = E_{1m} \sin \omega t + E_{2m} \sin 2\omega t + ... + E_{nm} \sin n\omega t$$

(*a*) Pure Resistance

Let the circuit have a resistance of R which is independent of frequency.

The instantaneous current i_1 due to fundamental voltage is

$$i_1 = \frac{E_{1m} \sin \omega t}{R}$$

Similarly, $i_2 = \frac{E_{2m} \sin 2\omega t}{R}$ for 2nd harmonic

and $i_n \quad \frac{E_{nm} \sin n \quad t}{R}$ ... for nth harmonic

total current $i = i_1 + i_2 + ... + i_n$

$$= \frac{E_{1m} \sin \omega t}{R} + \frac{E_{2m} \sin 2\omega t}{R} + ... + \frac{E_{nm} \sin n\omega t}{R}$$

$$= I_{1m} \sin \omega t + I_{2m} \sin 2\omega t + ... + I_{nm} \sin n\omega t$$

It shows that

(*i*) the waveform of the resulting current is similar to that of the applied voltage *i.e.* the two waves are identical.

(*ii*) the percentage of harmonic content in the current wave is the same as in the applied voltage.

(*b*) Pure Inductance

Let the inductance of the circuit be L henry whose reactance varies directly as the frequency of the applied voltage. Its reactance for the fundamental would be $X_1 = \omega L$; for the second harmonic, $X_2 = 2\omega L$, for the third harmonic, $X_3 = 2\omega L$ and for the nth harmonic $X_n = n\omega L$.

However, for every harmonic term, the current will lag behind the voltage by 90°.

Current due to fundamental, $i_1 = \frac{E_{1m}}{\omega L}\sin(\omega t - \pi/2)$

Current due to 2nd harmonic, $i_2 = \frac{E_{2m}}{2\omega L}\sin(2\omega t - \pi/2)$

Current due to 3rd harmonic, $i_3 = \frac{E_{3m}}{3\omega L}\sin(3\omega t - \pi/2)$

Current due to nth harmonic, $i_n = \frac{E_{nm}}{n\omega L}\sin(n\omega t - \pi/2)$

$\therefore$ Total current $i = i_1 + i_2 + ...i_n$

$$\frac{E_{1m}}{L}\sin(\quad t \quad /2) \quad \frac{E_{2m}}{2\ L}\sin(2\ t \quad /2) \quad .... \quad \frac{E_{nm}}{n\ L}\sin(n\ t \quad /2)$$

It can be seen from the above equation that

(i) the waveform of the current differs from that of the applied voltage.

(ii) for the nth harmonic, the percentage harmonic content in the current-wave is $1/n$ of the corresponding harmonic content in the voltage wave. It means that in an inductive circuit, the current waveform shows less distortion that the voltage waveform. In this case, current more nearly approaches a sine wave than it does in a circuit containing resistance.

(c) Pure Capacitance

In this case,

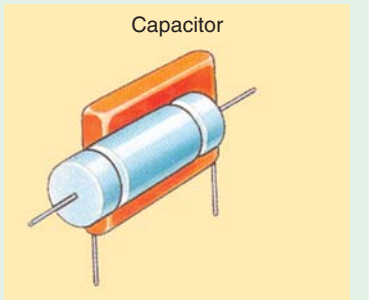
$X_1 = \frac{1}{\omega C}$ – for fundamental ; $X_2 = \frac{1}{2\omega C}$ – for 2nd harmonic

$X_3 = \frac{1}{3\omega C}$ – for 3rd harmonic ; $X_n = \frac{1}{n\omega C}$ – for nth harmonic

$$i_1 = \frac{E_{1m}}{1/\omega C}\sin(\omega t + \pi/2) = \omega C E_{1n}\sin(\omega t + \pi/2)$$

$$i_2 = \frac{E_{2m}}{1/2\omega C}\sin(2\omega t + \pi/2) = 2\omega C E_{2m}\sin(2\omega t + \pi/2)$$

$$i_n = \frac{E_{nm}}{1/n\omega C}\sin(n\omega t + \pi/2) = n\omega C E_{nm}\sin(n\omega t + \pi/2)$$



The amount of charge the device can store for a given voltage difference is called the capacitance

For every harmonic term, the current will lead the voltage by 90°.

Now $i = i_1 + i_2 + ... + i_n$

$$= \omega C E_{1m}\sin(\omega t + \pi/2) + 2\omega C E_{2m}\sin(2\omega t + \pi/2) + ... + n\omega C E_{nm}\sin(n\omega t + \pi/2)$$

This equation shows that

(i) the current and voltage waveforms are dissimilar.

(ii) percentage harmonic content of the current is larger than that of the applied voltage wave. For example, for nth harmonic, it would be n time larger.

(iii) as a result, the current wave is more distorted than the voltage wave.

(iv) effect of capacitor on distortion is just the reverse of that of inductance.

Example 20.5. *A complex wave of r.m.s. value 240 V has 20% 3rd harmonic content, 5% 5th harmonic content and 2% 7th harmonic content. Find the r.m.s. value of the fundamental and of each harmonic.* **(Elect. Circuits, Gujarat Univ.)**

Solution. Let V_1, V_3, V_5 and V_7 be the r.m.s. values of the fundamental and harmonic voltages. Then

$$V_3 = 0.2\ V_1\ ;\quad V_5 = 0.05\ V_1 \text{ and } V_7 = 0.02\ V_1$$

$$240 = (V_1^2 + V_3^2 + V_5^2 + V_7^2)^{1/2}$$

$$\therefore\ 240 = [V_1^2 + (0.2\ V_1)^2 + (0.05\ V_1)^2 + (0.02\ V_1)^2]^{1/2}$$

$$\therefore\ V_1 = \mathbf{235\ V}\ ;\ V_3 = 0.2 \times 235 = \mathbf{47\ V}$$

$$V_5 = 0.05 \times 235 = \mathbf{11.75\ V}\ ;\ V_7 = 0.02 \times 235 = \mathbf{4.7\ V}$$

Example 20.6. *Derive an expression for the power, power factor and r.m.s. value for a complex wave.*

A voltage $e = 250 \sin \omega t + 50 \sin (3\omega t + \pi/3) + 2 \sin (\omega t + 5\pi/6)$ *is applied to a series circuit of resistance* $20\ \Omega$ *and inductance 0.05 H. Derive* ***(a)*** *an expression for the current* ***(b)*** *the r.m.s. value of the current and for the voltage* ***(c)*** *the total power supplied and (d) the power factor. Take* $\omega = 314$ *rad/s.* **(Electrical Circuits, Nagpur Univ. 1991)**

Solution. For Fundamental

$$X_1 = \omega L = 314 \times 0.05 = 15.7\ \Omega;\ \mathbf{Z}_1 = 20 + j15.7 = 25.4 \angle 38.1^\circ\ \Omega$$

For Third Harmonic

$$X_3 = 3\omega L = 3 \times 15.7 = 47.1\ \Omega;\ \mathbf{Z}_3 = 20 + j47.1 = 51.2 \angle 67^\circ\ \Omega$$

For Fifth Harmonic

$$X_5 = 5\omega L = 5 \times 15.7 = 78.5\ \Omega;\ \mathbf{Z}_5 = 20 + j78.5 = 81 \angle 75.7^\circ\ \Omega$$

(a) Expression for the current is

$$i = \frac{250}{25.4}\sin(\omega t - 38.1^\circ) + \frac{50}{51.2}\sin(3\omega t + 60^\circ - 67^\circ) + \frac{20}{81}\sin(5\omega t + 150^\circ - 75.7^\circ)$$

$$\therefore\ i = 9.84\sin(\omega t - 38.1^\circ) + 0.9\sin(3\omega t - 7^\circ) + 0.25\sin(5\omega t + 74.3^\circ)$$

(b) *R.M.S. current* $I = \sqrt{\frac{I_{1m}^2}{2} + \frac{I_{3m}^2}{2} + \frac{I_{5m}^2}{2}}$

$$I^2 = \frac{9.84^2}{2} + \frac{0.9^2}{2} + \frac{0.25^2}{2} = 48.92$$

$$\therefore\quad I = \sqrt{48.92} = \mathbf{6.99\ A}$$

R.M.S. voltage $V = \sqrt{\frac{250^2}{2} + \frac{50^2}{2} + \frac{20^2}{2}} = \mathbf{180.8\ V}$

(c) Total power $= I^2R = 48.92 \times 20 = \mathbf{978\ W}$

(d) Power factor $\frac{\text{Watts}}{\text{VI}}$ $\frac{978}{180.8 \quad 6.99}$ **0.773**

Example 20.7. *An r.m.s. current of 5 A, which has a third harmonic content, is passed through a coil having a resistance of 1 Ω and an inductance of 10 mH. The r.m.s. voltage across the coil is 20 V. Calculate the magnitude of the fundamental and harmonic components of current if the fundamental frequency is 300/2π Hz. Also, find the power dissipated.*

Solution. (*i*) Fundamental Frequency

$\omega = 300$ rad/s ; $X_L = 300 \times 10^{-2} = 3\ \Omega$ $\therefore$ $\mathbf{Z}_1 = 1 + j\,3 = 3.16 \angle 71.6°$ ohm

If V_1 is the r.m.s. value of the fundamental voltage across the coil, then

$$V_1 = I_1 Z_1 = 3.16\, I_1$$

(*ii*) Third Harmonic

$X_3 = 3 \times 3 = 9\ \Omega$; $\mathbf{Z}_3 = 1 + j\,9 = 9.05 \angle 83.7°$ ohm ; $V_3 = I_3 Z_3 = 9.05\, I_3$

Since r.m.s. current of the complex wave is 5 A and r.m.s. voltage drop 20 V

$$5 = \sqrt{I_1^2 + I_3^2} \quad \text{and} \quad 20 = \sqrt{V_1^2 + V_3^2}$$

Substituting the values of V_1 and V_3, we get, $20 = [(3.16\, I_1)^2 + (9.05\, I_3)^2]^{1/2}$

Solving for I_1 and I_3, we have I_1 = **4.8 A** and I_3 = **1.44 A**

Power dissipated = $I^2R = 5^2 \times 1$ = **25 W**

Example 20.8. *An e.m.f. represented by the equation $e = 150 \sin 314\,t + 50 \sin 942\,t$ is applied to a capacitor having a capacitance 20 μF. What is the r.m.s. value of the charging current ?*

Solution. For Fundamental

X_{C1} 1/ C $10^6/20$ 314 159 ; I_{1m} E_{1m}/X_{C1} 150/159 0.943 A

For Third Harmonic

$X_{C1} = 1/3\ \omega C = 159/3 = 53\ \Omega$ $\therefore$ $I_{3m} = E_{3m}/X_{C1} = 50/53 = 0.943$ A

r.m.s. value of charging current,

$$I = \sqrt{\frac{I_{1m}^2}{2} + \frac{I_{3m}^2}{2}} = \sqrt{\frac{0.943^2}{2} + \frac{0.943^2}{2}}$$

or ***I* = 0.943 A**

Example 20.9. *The voltage given by $v = 100 \cos 314\,t + 50 \sin (1570t - 30°)$ is applied to a circuit consisting of a 10 Ω resistance, a 0.02 H inductance and a 50 μF capacitor. Determine the instantaneous current through the circuit. Also find the r.m.s. value of the voltage and current.*

Solution. For Fundamental

ω = 314 rad/s; $X_L = 314 \times 0.02 = 6.28\ \Omega$

$X_C = 10^6/314 \times 50 = 63.8\ \Omega$; $X = X_L - X_C = 6.28 - 63.8 = -57.32\ \Omega$

$Z = \sqrt{10^2 + (-57.32)^2}$ = 58.3 Ω ; $I_{1m} = 100/58.3 = 1.71$ A

$\phi_1 = \tan^{-1}(-57.32/10) = -80.2°$ (lead) ; $i_1 = 1.71 \cos (314t + 80.2°)$

For Fifth Harmonic

Inductive reactance = $5\,X_L = 5 \times 6.28 = 31.4\ \Omega$

Capacitive reactance = $X_C/5 = 63.8/5 = 12.76\ \Omega$

Net reactance = 31.4 – 12.76 = 18.64 Ω

$$Z = \sqrt{10^2 + 18.64^2} = 21.2\,\Omega$$

I_{5m} = 50/21.2 = 2.36 A; $\phi_5 = \tan^{-1}(18.64/10) = 61.8°$ (lag)

$i_5 = 2.36 \sin(1570\,t - 30° - 61.8°) = 2.36 \sin(1570t - 91.8°)$

Hence, total instantaneous current is

$i = i_1 + i_5 = 1.71 \cos(314\,t + 80.2°) + 2.36 \sin(1570t - 91.8°)$

$$\textit{R.M.S. voltage} = \sqrt{\frac{100^2}{2} + \frac{50^2}{5}} = \mathbf{79.2\ V}$$

$$\textit{R.M.S. current} = \sqrt{\frac{1.71^2}{2} + \frac{2.36^2}{2}} = \mathbf{2.06\ A}$$

Example 20.10. *A 6.36 μF capacitor is connected in parallel with a resistance of 500 Ω and the combination is connected in series with a 500-Ω resistor. The whole circuit is connected across an a.c. voltage given by e = 300 sin ωt + 100 sin (3ωt + π/6).*

If ω = 314 rad/s, find

(i) power dissipated in the circuit

(ii) an expression for the voltage across the series resistor

(iii) the percentage harmonic content in the resultant current.

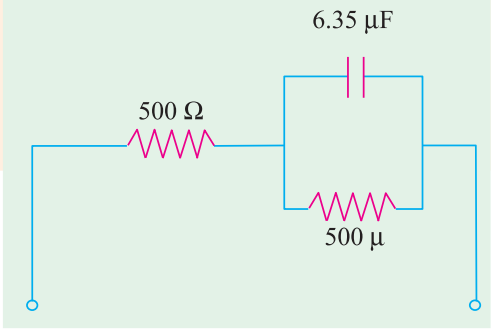


Fig. 20.6

Solution. For Fundamental

$$X_{C1} = \frac{1}{\omega C} = \frac{10^6}{314 \times 6.36} = 500\ \Omega$$

The impedance of the whole series-parallel circuit is given by

$$\mathbf{Z}_1 \quad 500 \quad \frac{500(\ j500)}{500 \quad j500} \quad 750 \quad j250 \quad 791 \quad 18.4°$$

For Third Harmonic

$$X_{C3} = 1/3\omega\,C = 500/3 = 167\ \Omega$$

$$\therefore \quad \mathbf{Z}_3 \quad 500 \quad \frac{500(\ j167)}{500 \quad j167} \quad 550 \quad j150 \quad 570 \quad 15.3°$$

$$\therefore \quad i = \frac{300}{791}\sin(\omega t + 18.4°) + \frac{100}{570}\sin(3\omega t + 45.3°)$$

$$= 0.397 \sin(\omega t + 18.4°) + 0.175 \sin(3\omega t + 45.3)$$

(*i*) Power dissipated = $\dfrac{E_{1m}I_{1m}}{2}\cos\phi_1 + \dfrac{E_{3m}I_{3m}}{2}\cos\phi_3$

$$= \frac{300 \times 0.379}{2} \times \cos 18.4° + \frac{100 \times 0.175}{2} \cos 15.3 = \mathbf{62.4\ W}$$

(ii) The voltage drop across the series resistor would be

$$E_R = iR = 500[0.379 \sin(\omega t + 18.4°) + 0.175 \sin(3\omega t + 45.3°)]$$

$$e_R = \mathbf{189.5 \sin (\omega t + 18.4°) + 87.5 \sin (3\omega t + 45.3°)}$$

(iii) The percentage harmonic content of the current is = 87.5/189.5 × 100 = **46.2%**

Example 20.11. *An alternating voltage of v = 1.0 sin 500t + 0.5 sin 1500t is applied across a capacitor which can be represented by a capacitance of 0.5 μF shunted by a resistance of 4,000 Ω. Determine*

(i) *the r.m.s. value of the current* ***(ii)*** *the r.m.s. value of the applied voltage*

(iii) *the p.f. of the circuit.* **(Circuit Theory and Components, Madras Univ.)**

Solution. For Fundamental [Fig. 20.7 (*a*)]

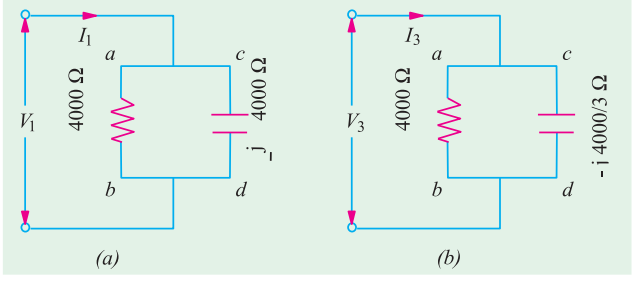


Fig. 20.7

$V_1 = 1.0/\sqrt{2} = 0.707$ V

Let, $V_1 = (0.707 + j0)$

Capacitive reactance

$= jX_{C1} = -j\ 10^6/500 \times 0.5$

$= j\ 4000\ \Omega$; $R = 4{,}000\ \Omega$

$\therefore\ I_{ab1} = 0.707/4{,}000 = 0.177$ mA

$I_{cd1} = 0.707/-j4{,}000 = j0.177$ m/A

$\therefore\ I_1 = 0.177 + j0.177$

$= 0.25\ \angle 45°$ mA

Hence, I_1 lead the fundamental voltage by 45°.

$P_{ab1} = 0.707 \times 0.177 = 0.125$ mW ; $P_{cd1} = 0$

For Third Harmonic [Fig. 20.7 (*b*)]

$\mathbf{V_3} = 0.5/\sqrt{2} = 0.3535\ \angle 0°$; $R = 4{,}000\ \Omega$: $\mathbf{X_{C3}} = -j4{,}000/3\ \Omega$

$\mathbf{I_{ab3}} = 0.3535/4000 = 0.0884\ \angle 0°$ mA ; $\mathbf{I_{cd3}} = 0.3535/-j(4{,}000/3) = j\ 0.265$ mA

$\mathbf{I_3} = 0.0884 + j0.265 = 0.28\ \angle 71.6°$ mA

$P_{ab3} = 0.3535 \times 0.0884 = 0.0313$ mW ; $P_{cd3} = 0$

(i) *R.M.S.* current $= \sqrt{I_1^2 + I_3^2} = \sqrt{0.25^2 + 0.28^2} = \mathbf{0.374\ mA}$

(ii) *R.M.S.* voltage $= \sqrt{(1/\sqrt{2})^2 + (0.5/\sqrt{2})^2} = \mathbf{0.79\ V}$

(iii) Power factor = watts/voltampere

Wattage = (0.125 + 0.0313) × 10^{-3} = 0.1563 × 10^{-3} W

Volt-amperes = 0.79 × 0.374 = 0.295 ; p.f. = 0.1563 × 10^{-3}/0.295 = **0.0005**

20.8. Selective Resonance due to Harmonics

When a complex voltage is applied across a circuit containing both inductance and capacitance, it may happen that the circuit resonates at one of the harmonic frequencies of the applied voltage. This phenomenon is known as ***selective resonance.***

If it is a series circuit, then large currents would be produced at resonance, even though the applied voltage due to this harmonic may be small. Consequently, it would result in large harmonic voltage appearing across both the capacitor and the inductance.

If it is a parallel circuit, then at resonant frequency, the resultant current drawn from the supply would be minimum.

It is because of the possibility of such selective resonance happening that every effort is made to eliminate harmonics in supply voltage.

However, the phenomenon of selective resonance has been usefully employed in some wave analyses for determining the harmonic content of alternating waveforms. For this purpose, a variable inductance, a variable capacitor, a variable non-inductive resistor and a fixed non-inductive resistance or shunt for an oscillograph are connected in series and connected to show the wave-form of the voltage across the fixed non-inductive resistance. The values of inductance and capacitance are adjusted successively to give resonance for the first, third, first and seventh harmonics and a record of the waveform is obtained by the oscillograph. A quick inspection of the shape of the waveform helps to detect the presence or absence of a particular harmonic.

Example 20.12. *An e.m.f. $e = 200 \sin \omega t + 40 \sin 3\omega t + 10 \sin 5\omega t$ is impressed on a circuit comprising of a resistance of 10 Ω, a variable inductor and a capacitance of 30 μF, all connected in series. Find the value of the inductance which will give resonance with triple frequency component of the pressure and estimate the effective p.f. of the circuit, $\omega = 300$ radian/second.* **(Elect. Engg. I, Bombay Univ.)**

Solution. For resonance at third harmonic

$$3\omega L = 1/3\omega C \quad \therefore \quad L = 1/9\,\omega^2 C = 10^6/9 \times 300^2 \times 30 = \mathbf{0.041\ H}$$

$$\mathbf{Z}_1 = 10 \quad j \quad 300 \quad 0.041 \quad \frac{10^6}{300 \quad 30} = 10 + j\,(12.3 - 111.1) = 10 - j98.8 = 99.3 \angle -84.2^\circ$$

$$\mathbf{Z}_3 \quad 10 \quad j \quad 3 \quad L \quad \frac{1}{3 \quad C} \quad 10 \quad j(36.9 \quad 37.0) \quad 10 \quad 0^\circ$$

$$\mathbf{Z}_5 \quad 10 \quad j \quad 5 \quad L \quad \frac{1}{5 \quad L} \quad 10 \quad j(61.5 \quad 22.2) \quad 10 \quad j39.3 \quad 40.56 \quad 75.7^\circ$$

$I_{1m} = 200/99.3 = 2.015$ A ; $I_{3m} = 40/10 = 4$A ; $I_{5m} = 10/40.56 = 0.246$ A

$$I = \sqrt{\frac{2.015^2}{2} + \frac{4^2}{2} + \frac{0.246^2}{2}} = \sqrt{10.06} = 3.172 \text{ A}$$

$$V = \sqrt{\frac{200^2}{2} + \frac{40^2}{2} + \frac{10^2}{2}} = 144.5 \text{ V ; Power} = I^2R = 10.06 \times 10 = 100.6 \text{ W}$$

Volt-amperes $VI = 144.5 \times 3.172 = 458$ VA ; Power factor $= 100.6/458 = \mathbf{0.22}$

Example 20.13. *A coil having $R = 100$ Ω and $L = 0.1$ H is connected in series with a capacitor across a supply, the voltage of which is given by $e = 200 \sin 314t + 5 \sin 3454t$. What capacitance would be required to produce resonance with the 11th harmonic. Find* ***(a)*** *the equation of the current and* ***(b)*** *the r.m.s. value of the current, if this capacitance is in circuit.*

Solution. For series resonance, $X_L = X_C$

Since resonance is required for 11th harmonic whose frequency is 3454 rad/s, hence

$$3454\,L = \frac{1}{3454\,C}; C = \frac{1}{3454^2 \times 0.1} \text{ farad} = \mathbf{0.838} \ \mu\mathbf{F}$$

(*a*) For Fundamental

Inductive reactance $= \omega L = 314 \times 0.1 = 31.4\ \Omega$

Capacitive reactance $= 1/\omega C = 10^6 / 0.836 \times 314 = 3796\,\Omega$

$\therefore$ Net reactance = 3796 – 31.4 = 3765 Ω ; Resistance = 100 Ω

$\therefore$ $Z_1 = \sqrt{100^2 + 3765^2} = 3767\ \Omega$; tan ϕ_1 = 3765/100 = 37.65

$\therefore$ $\phi_1 = 88°28'$ (leading) = 1.546 radian

Now E_{1m} = 200 V ; Z_1 = 3767 Ω $\therefore$ I_{1m} = 200/3767 = 0.0531 A

Eleventh Harmonic

New reactance = 0; Impedance Z_{11} = 100 Ω

$\therefore$ Current I_{11m} = 5/100 = 0.05 A ; ϕ_{11} = 0 ... at resonance

Hence, the equation of the current is

$$i = \frac{200}{3767}(\sin 314t + 1.546) + \frac{5}{100}\sin(3454t + 0)$$

$$i = 0.0531 \sin (314\ t + 1.546) + 0.05 \sin 3454\ t$$

(b) $I = \sqrt{(0.0531)^2/2 + (0.05)^2/2} =$ **0.052 A**

20.9. Effect of Harmonics on Measurement of Inductance and Capacitance

Generally, with the help of ammeter and voltmeter readings, the value of impedance, inductance and capacitance of a circuit can be calculated. But while dealing with complex voltages, the use of instrument readings does not, in general, give correct values of inductance and capacitance except in the case of a circuit containing only pure resistance. It is so because, in the case of resistance, the voltage and current waveforms are similar and hence the values of r.m.s. volts and r.m.s amperes (as read by the voltmeter and ammeter respectively) would be the same whether they ware sinusoidal or non-sinusoidal (*i.e.* complex).

(*i*) Effect on Inductances

Let L be the inductance of a circuit and E and I the r.m.s. values of the applied voltage and current as read by the instruments connected in the circuit. For a complex voltage

$$E = 0.707\sqrt{(E_{1m}^2 + E_{3n}^2 + E_{5m}^2 + ...)}$$

Hence $$I = 0.707\sqrt{\left(\frac{E_{1m}}{L}\right)^2 \left(\frac{E_{3m}}{3L}\right)^2 \left(\frac{E_{5m}}{5L}\right) .....}$$

$$\frac{0.707}{L}\sqrt{E_{1m}^2 \ \frac{1}{9}E_{3m}^2 \ \frac{1}{25}E_{5m}^2 \ ...}$$

$\therefore$ $$L \ \frac{0.707}{I}\sqrt{E_{1m}^2 \ \frac{1}{9}E_{3m}^2 \ \frac{1}{25}E_{5m}^2 \ .....}$$

For calculating the value of L from the above expression, it is necessary to known the absolute value of the amplitudes of several harmonic voltages. But, in practices, it is more convenient to deal with relative values than with absolute values. For this purpose, let us multiply and divide the right-hand side of the above expression by E but write the E ***in the denominator*** in its form $0.707\sqrt{(E_{1m}^2 + E_{3m}^2 + E_{5m}^2 + ...)}$

$\therefore$ $$L \ \frac{0.707}{I}\sqrt{E_{1m}^2 \ 1/9.E_{3m}^2 \ 1/25.E_{5m}^2 \ .....} \ \frac{E}{0.707\sqrt{(E_{1m}^2 \ E_{3m}^2 \ E_{5m}^2 \ .....)}}$$

or $$L \quad \frac{E}{I}\sqrt{\frac{E_{1m}^2 \quad 1/9\ E_{3m}^2 \quad 1/25\ E_{5m}^2 \quad .....)}{(E_{1m}^2 \quad E_{3m}^2 \quad E_{5m}^2 \quad ....)}} \quad \frac{E}{I}\sqrt{\frac{1 \quad 1/9\ (E_{3m}/E_{1m})^2 \quad 1/25\ (E_{5m}/E_{1m})^2 \quad .....}{1 \quad (E_{3m}/E_{1m})^2 \quad (E_{5m}/E_1)^2 \quad .....}}$$

If the effect of harmonics were to be neglected, then the value of the inductance would appear to be $E/\omega\ I$ but the true or actual value is less than this. The apparent value has to be multiplied by the quantity under the radical to get the true value of inductance when harmonics are present.

The quantity under the radical is called the correction factor *i.e.*

True inductance (L) = Apparent inductance (L') × correction factor

(*ii*) Effect on Capacitance

Let the capacitance of the circuit be C farads and E and I the instrument readings for voltage and current. Since the instruments read r.m.s. values, hence, as before,

$$E = 0.707\sqrt{(E_{1m}^2 + E_{3m}^2 + E_{5m}^2 + ...)}$$

Hence $$I \quad 0.707\sqrt{\frac{E_{1m}}{1/\ C}^2 \quad \frac{E_{3m}}{1/3\ \ C}^2 \quad \frac{E_{5m}}{1/5\ \ C} \quad .....}$$

$$= 0.707\sqrt{(\omega C E_{1m})^2 + (3\omega C E_{3m})^2 + (5\omega C\, E_{5m})^2 + ...}$$

$$= 0.707\ \omega C\sqrt{E_{1m}^2 + 9E_{3m}0^2 + 25E_{sm}^2 + ...}$$

$$\therefore \quad C = \frac{I}{0.707\omega\sqrt{(E_{1m}^2 + 9E_{3m}^2 + 25E_{5m}^2 + ...)}}$$

Again, we will multiply and divide the right-hand side E but in this case, we will write E in the *numerator* in its form $[0.707\sqrt{(E_{1m}^2 + E_{3m}^2 + E_{5m}^2 + ...)}]$

$$\therefore \quad C \quad \frac{1}{0.707 \quad E\sqrt{(E_{1m}^2 \quad 9E_{3m}^2 \quad 25E_{5m}^2 \quad .....)}} \quad 0.707\sqrt{(E_{1m}^2 \quad E_{3m}^2 \quad E_{5m}^2 \quad .....)}$$

$$\frac{1}{E}\sqrt{\frac{E_{1m}^2 \quad E_{3m}^2 \quad E_{5m}^2 .....}{E_{1m}^2 \quad 9E_{3m}^2 \quad 25E_{5m}^2 \quad .....}} \quad \frac{1}{E}\sqrt{\frac{1 \quad (E_{3m}/E_{1m})^2 \quad (E_{5m}/E_{1m})^2 .....}{1 \quad 9(E_{3m}/E_{1m})^2 \quad 25(E_{5m}/E_{1m})^2 \quad .....}}$$

Again, if the effects of harmonics were neglected, the value of capacitance would appear to be $I/\omega\ E$ but its true value is less than this. For getting the true value, this apparent value will have to be multiplied by the quantity under the radical (which, therefore, is referred to as correction factor).*

$\therefore$ True capacitance (C) = Apparent capacitance (C') × correction factor

Example 20.14. *A current of 50-Hz containing first, third and fifth harmonics of maximum values 100, 15 and 12 A respectively, is sent through an ammeter and an inductive coil of negligibly small resistance. A voltmeter connected to the terminals shows 75 V. What would be the current indicated by the ammeter and what is the exact value of the inductance of the coil in henrys ?*

---

* It may be noted that this correction factor is different from that in the case of pure inductance.

Solution. The r.m.s. current is

$$I = 0.707\sqrt{I_{1m}^{\ 2} + I_{3m}^{\ 2} + I_{5m}^{\ 2}} = 0.707\sqrt{(100^2 + 15^2 + 12^2)} = 72 \text{ A}$$

Hence, current indicated by the ammeter is **72 A**

Now $E = 0.707\sqrt{(E_{1m}^{\ 2} + E_{3m}^{\ 2} + E_{5m}^{\ 2})}$

Also $I_{1m} = \frac{E_{1m}}{\omega L}; I_{3m} = \frac{E_{3m}}{3\omega L}; I_{5m} = \frac{E_{5m}}{5\omega L}$

$\therefore \quad E_{1m} = I_{1m} \cdot \omega L; E_{3m} = I_{3m} 3\omega L; E_{5m} = I_{5m} \cdot 5\omega L$

$\therefore \quad E = 0.707\sqrt{(I_{1m}\omega L)^2 + (I_{3m} 3\omega L)^2 + (I_{5m} 5\omega L)^2} = 0.707\omega L\sqrt{I_{1m}^{\ 2} + 9I_{3m}^{\ 2} + 25I_{5m}^{\ 2}}$

$\therefore \quad 75 = 0.707\, L \times 2\pi \times 50\sqrt{100^2 + 9 \times 15^2 + 25 \times 12^2}$

$\therefore \quad L =$ **0.0027 H**

Note. Apparent inductance $L' = \frac{E}{\omega I} = \frac{75}{2\pi \times 50 \times 72} = 0.00331$ H

Example 20.15. *The capacitance of a 20 μF capacitor is checked by direct connection to an alternating voltage which is supposed to be sinusoidal, an electrostatic voltmeter and a dynamometer ammeter being used for measurement. If the voltage actually follows the law,*

$$e = 100 \sin 250\, t + 20 \sin (500 - \phi) + 10 \sin (750\, t - \phi)$$

Calculate the value of capacitance as obtained from the direct ratio of the instrument readings.

Solution. True value, $C = 20\ \mu F$

Apparent value C' = value read by the instruments

Now, $C = C' \times$ correction factor.

Let us find the value of correction factor.

Here $E_{1m} = 100$; $E_{2m} = 20$ and $E_{3m} = 10$

$\therefore$ Correction factor $\sqrt{\frac{E_{1m}^{\ 2}}{E_{1m}} \quad \frac{E_{2m}^{\ 2}}{4E_{2m}^{\ 2}} \quad \frac{E_{3m}^{\ 2}}{9E_{5m}^{\ 2}}}$ $\sqrt{\frac{100^2}{100^2} \quad \frac{20^2}{4} \quad \frac{10^2}{20^2} \quad \frac{}{9} \quad \frac{}{10^2}}$ 0.9166

$20 = C' \times 0.9166 \qquad \therefore \quad C' = 20 / 0.9166 =$ **21.82 μF.**

21.10. Harmonics in Different Three-phase Systems

In tree-phase systems, harmonics may be produced in the same way as in single-phase systems.

Hence, for all calculation they are treated in the same manner *i.e.* each harmonic is treated separately. Usually, even harmonics are absent in such systems. But care must be exercised when dealing with odd, especially, third harmonics and all multiples of 3rd harmonic (also called the triple-*n* harmonics).

(*a*) Expressions for Phase E.M.Fs.

Let us consider a 3-phase alternator having identical phase windings (*R*, *Y* and *B*) in which harmonics are produced. The three phase e.m.fs. would be represented in their proper phase sequence by the equation.

$$e_R = E_{1m}(\omega t + \Psi_1) + E_{3m}(3\omega t + \Psi_3) + E_{5m}\sin(5\omega t + \Psi_5) + \ldots\ldots$$

$$e_Y \quad E_{1m}\sin\ t\ \frac{2}{3}\ _1 \quad E_{3m}\sin\ 3\ t\ \frac{2}{3}\ _3 \quad E_{5m}\sin\ 5\ t\ \frac{2}{3}\ _5$$

$$e_B \quad E_{1m}\sin\ t\ \frac{4}{3}\ _1 \quad E_{3m}\sin\ 3\ t\ \frac{4}{3}\ _3 \quad E_{5m}\sin\ 5\ t\ \frac{4}{3}\ _5$$

On simplification, these become

$$e_R = E_{1m}\sin(\omega t + \Psi_1) + E_{3m}(3\omega t + \Psi_3) + E_{5m}\sin(5\omega t + \Psi_5) + \ldots\ldots \quad \text{– as before}$$

$$e_Y \quad E_{1m}\sin\ t\ \frac{2}{3}\ _2 \quad E_{3m}\sin(3\ t\ 2\ _3) \quad E_{5m}\sin\ 5\ t\ \frac{10}{3}\ _5 \quad \ldots\ldots$$

$$E_{1m}\sin\ t\ \frac{2}{3}\ _1 \quad E_{3m}\sin(3\ t\ _3) \quad E_{5m}\sin\ 5\ t\ \frac{4}{3}\ _5 \quad \ldots\ldots$$

$$e_B \quad E_{1m}\sin\ t\ \frac{4}{3}\ _1 \quad E_{3m}\sin(3\ t\ _3) \quad E_{5m}\sin\ 5\ t\ \frac{2}{3}\ _5 \quad \ldots\ldots$$

From these expressions, it is clear that

(*i*) All third harmonics are equal in all phases of the circuit *i.e.* they are in time phase.

(*ii*) Fifth harmonics in the three phases have a negative phase sequence of *R*, *B*, *Y* because the fifth harmonic of blue phase reaches its maximum value before that in the yellow phase.

(*iii*) All harmonics which are not multiples of three, have a phase displacement of 120° so that they can be dealt with in the usual manner.

(*iv*) At any instant, all the e.m.fs. have the same direction which means that in the case of a *Y*-connected system they are directed either away from or towards the neutral point and in the case of Δ-connected system, they flow in the same direction.

Main points can be summarized as below :

(*i*) all triple-*n* harmonics *i.e.* 3rd, 9th, 15th etc. are in phase,

(*ii*) the 7th, 13th and 19th harmonics have positive phase rotation of *R*, *Y*, *B*.

(*iii*) the 5th, 11th and 17th harmonics have a negative phase sequence of *R*, *B*, *Y*.

(*b*) Line Voltage for a Star-connected System

In this system, the line voltages will be the ***difference*** between successive phase voltages and hence will contain no third harmonic terms because they, being identical in each phase, will cancel out. The fundamental will have a line voltage $\sqrt{3}$ times the phase voltage. Also, fifth harmonic has line voltage $\sqrt{3}$ its phase voltage.

But it should be noted that in this case the r.m.s. value of the line voltage will be less than $\sqrt{3}$ times the r.m.s. value of the phase voltage due to the absence of third harmonic term from the line voltage. It can be proved that for any line voltage.

$$\text{Line value} = \sqrt{3}\sqrt{\frac{E_1^{\,2} + E_5^{\,2} + E_7^{\,2}}{E_1^{\,2} + E_3^{\,2} + E_5^{\,2} + E_7^{\,2}}}$$

where E_1, E_3 etc. are r.m.s. values of the phase e.m.fs.

(c) Line Voltage for a Δ-connected System

If the winding of the alternator are delta-connected, then the resultant e.m.f. acting round the closed mesh would be the sum of the phase e.m.fs. The sum of these e.m.fs. is zero for fundamental, 5th, 7th, 11th etc. harmonics. Since the third harmonics are in phase, there will be a resultant third harmonic e.m.f. of three times the phase value acting round the closed mesh. It will produce a circulating current whose value will depend on the impedance of the windings at the third harmonic frequency. It means that the third harmonic e.m.f. would be short-circuited by the windings with the result that there will be no third harmonic voltage across the lines. The same is applicable to all triple-*n* harmonic voltages. Obviously, the line voltage will be the phase voltage but without the triple-*n* terms.

Example 20.16. *A 3-ϕ generator has a generated e.m.f. of 230 V with 15 per cent third harmonic and 10 per cent fifth harmonic content. Calculate*

(i) *the r.m.s. value of line voltage for Y-connection.*

(ii) *the r.m.s. value of line voltage for Δ-connection.*

Solution. Let E_1, E_3, E_5 be the r.m.s. values of the phase e.m.fs. Then

$E_3 = 0.15\ E_1$ and $E_5 = 0.1\ E_1$

$$\therefore \qquad 230 = \sqrt{E_1^{\ 2} + (0.15E_1)^2 + (0.1E_1)^2}$$

$E_1 = 226$ V $\quad \therefore \quad E_3 = 0.15 \times 226 = 34$ V and $E_5 = 0.1 \times 226 = 22.6$ V

(i) r.m.s. value of the fundamental line voltage = $\sqrt{3} \times 226 = 392$ V

r.m.s. value of third harmonic line voltage = 0

r.m.s. value of 5th harmonic line voltage $\sqrt{3} \times 22.6 =$ **39.2 V**

$\therefore$ r.m.s. value of line voltage $V_L = \sqrt{392^2 + 39.2^2} =$ **394 V**

(ii) In Δ-connection, again the third harmonic would be absent from the line voltage

$\therefore$ r.m.s. value of line voltage $V_L = \sqrt{226^2 + 22.6^2} =$ **227.5 V**

(d) Circulating Current in Δ-connected Alternator

Let the three symmetrical phase e.m.fs. of the alternator be represented by the equations,

$$e_R = E_{1m} \sin(\omega t + \Psi_1) + E_{3m}(3\omega t + \Psi_3) + E_{5m} \sin(5\omega t + \Psi_5) + \ldots\ldots$$

$$e_Y = E_{1m} \sin(\omega t + \Psi_1 - 2\pi/3) + E_{3m} \sin(3\omega t + \Psi_3) + E_{5m} + \sin(5\omega t + \Psi_5 - 4\pi/3) \ldots\ldots$$

$$e_B = E_{1m} \sin(\omega t + \Psi_1 - 4\pi/3) + E_{3m} \sin(3\omega t + \Psi_3) + E_{5m} \sin(5\omega t + \Psi_5 - 2\pi/3) + \ldots.$$

The resultant e.m.f. acting round the Δ-connected windings of the armature is the sum of these e.m.fs. Hence it is given by $e = e_R + e_Y + e_B$

$$\therefore \quad e = 3E_{3m} \sin(3\omega t + \phi_3) + 3E_{9m} \sin(9\omega t + \Psi_9) + 3E_{15} \sin(15\omega t + \Psi_{15}) + \ldots\ldots$$

If R and L represent respectively the resistance and inductance per phase of the armature winding, then the circulating current due to the resultant e.m.f. is given by

$$i_i \quad \frac{3E_{3m} \sin(3\ \ t\ \ _3)}{3\sqrt{(R^2\ \ 9L^2\ \ ^2)}} \quad \frac{3E_{9m} \sin(9\ \ t\ \ _9)}{3\sqrt{(R^2\ \ 81L^2\ \ ^2)}} \quad \frac{3E_{15m} \sin(15\ \ t\ \ _{15})}{3\sqrt{(R^2\ \ 225L^2\ \ ^2)}} \quad \ldots.$$

$$= \frac{E_{3m}\sin(3\omega t+\Psi_3)}{\sqrt{(R^2+9L^2\omega^2)}} + \frac{E_{9m}\sin(9\omega t+\Psi_9)}{\sqrt{(R^2+81L^2\omega^2)}} + \frac{E_{15m}\sin(15\omega t+\Psi_{15})}{\sqrt{(R^2+225L^2\omega^2)}}$$

The r.m.s. value of the current is given by

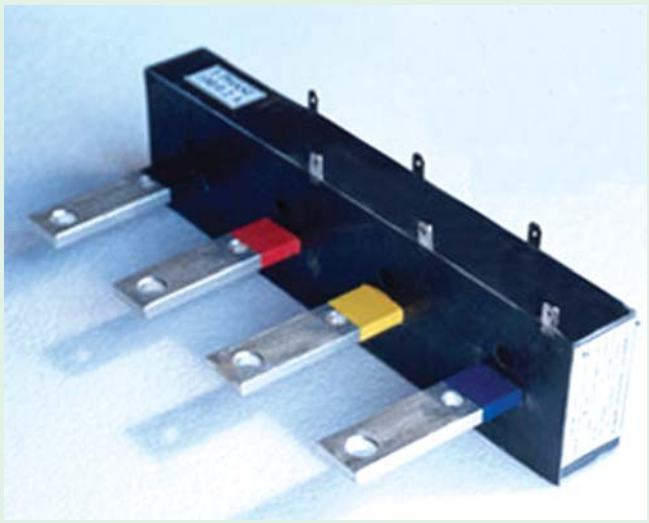
$$I_C = 0.707\sqrt{[{E_{3m}}^2/(R^2+9L^2\omega^2)+{E_{9m}}^2/(R^2+81L^2\omega^2)+({E_{15m}}^2/(R^2+225L^2\omega^2)+.....]}$$

(*e*) Three-phase four-wire System

In this case, there will be no third harmonic component in line voltage. For the 4-wire system, each phase voltage (*i.e.* line to neutral) may contain a third harmonic component. If it is actually present, then current will flow in the *Y*-connected load. In case load is balanced, the resulting third harmonic line currents will all be in phase so that neutral wire will have to carry three times the third harmonic line current. There will be no current in the neutral wire either at fundamental frequency, or any harmonic frequency other than the triple-*n* frequency.

20.11. Harmonics in Single and 3-phase Transformers

The flux density in transformer course is usually maintained at a fairly high value in order to keep the required volume of iron to the minimum. However, due to the non-linearity of magnetisation curve, some third harmonic distortion is always produced. Also, there is usually a small percentage of fifth harmonic. The magnetisation current drawn by the primary contains mainly third harmonic whose proportion depends on the size of the primary applied voltage. Hence, the flux is sinusoidal.



3-phase current transformer

In the case of three-phase transformers, the production of harmonics will be affected by the method of connection and the type of construction employed.

(*a*) Primary Windings Δ-CONNECTED

Each primary phase can be considered as separately connected across the sinusoidal supply.

(*i*) The core flux will be sinusoidal which means that magnetizing current will contain 3rd harmonic component in addition to relatively small amounts of other harmonics of higher order.

(*ii*) In each phase, these third harmonic currents will be in phase and so produce a circulating current round the mesh with the result that there will be no third harmonic component in the line current.

(*b*) Primary Winding Connected in 4-wire Star

Each phase of the primary can again be considered as separately connected across a sinusoidal supply.

(*i*) The flux in the transformer core would be sinusoidal and so would be the output voltage.

(*ii*) The magnetizing current will contain 3rd harmonic component. This component being in phase in each winding will, therefore, return through the neutral wire.

(*c*) Primary Windings Connected 3-wire Star

Since there is no neutral wire, there will be no return path for the 3rd harmonic component of the magnetizing current. Hence, there will exist a condition of forced magnetization so that core flux must contain third harmonic component which is in phase in each limb of the transformer core. Although there will be a magnetic path for these fluxes in the case of shell type 3-phase transformer, yet in the case of three-limb core type transformer, the third harmonic component of the flux must return *via* the air. Because of the high reluctance magnetic path in such transformers, the third harmonic flux is reduced to a very small value. However, if the secondary of the transformer is delta-connected, then a third harmonic circulating current would be produced. This current would be in accordance with Lenz's law tend to oppose the very cause producing it *i.e.* it would tend to minimize the third harmonic component of the flux.

Should the third and fifth secondary be *Y*-connected, then provision of an additional Δ-connected winding, in which this current can flow, becomes necessary. This tertiary winding additionally served the purpose of preserving magnetic equilibrium of the transformer in the case of unbalanced loads. In this way, the output voltage from the secondary can be kept reasonably sinusoidal.

Example 20.17. *Determine whether the following two waves are of the same shape*

$e = 10 \sin (\omega t + 30°) - 50 \sin (3\omega t - 60°) + 25 \sin (5\omega t + 40°)$

$i = 1.0 \sin (\omega t - 60°) + 5 \sin (3\omega t - 150°) + 2.5 \cos (5\omega t - 140°)$

(Principles of Elect. Engg-II Jadavpur Univ.)

Solution. Two waves possess the same waveshape

(*i*) if they contain the same harmonics

(*ii*) if the ratio of the corresponding harmonics to their respective fundamentals is the same

(*iii*) if the harmonics are similarly spaced with respect to their fundamentals.

In other words,

(*a*) the ratio of the magnitudes of corresponding harmonics must be constant and

(*b*) with fundamentals in phase, the corresponding harmonics of the two waves must be in phase.

The test is applied first by checking the ratio of the corresponding harmonics and then coinciding the fundamentals by shifting one wave. If the phase angles of the corresponding harmonics are the same, then the two waves have the same shape.

In the present case, condition **(*i*)** is fulfilled because the voltage and current waves contain the same harmonics, *i.e.* third and fifth.

Secondly, the ratio of the magnitude of corresponding current and voltage harmonics is the same *i.e.* 1/10.

Now, let the fundamental of the current wave be shifted ahead by 90° so that it is brought in phase with the fundamental of the voltage wave. It may be noted that the third and fifth harmonics of the current wave will be shifted by 3 × 90° = 270° and 5 × 90° = 450° respectively. Hence, the current wave becomes

$$i' = 1.0 \sin(\omega t - 60° + 90°) + 5 \sin(3\omega t - 150° + 270°) + 2.5 \cos(5\omega t - 140° + 450°)$$

$$= 1.0 \sin(\omega t + 30°) + 5 \sin(3\omega t + 120°) + 2.5 \cos(5\omega t + 310°)$$

$$= 1.0\sin(\omega t + 30°) - 5\sin(3\omega t - 60°) + 2.5\sin(5\omega t + 40°)$$

It is seen that now the corresponding harmonics of the voltage and current waves are in phase. Since all conditions are fulfilled, the two waves are of the same waveshape.

Tutorial Problem No. 20.1

1. A series circuit consist of a coil of inductance 0.1 H and resistance 25 Ω and a variable capacitor. Across this circuit is applied a voltage whose instantaneous value is given by

$v = 100\sin\omega t + 20\sin(3\omega t - 45) + 5\sin(5\omega t - 30)$ where $\omega = 314$ rad / s

Determine the value of C which will produce response at third harmonic frequency and with this value of C, find

(a) an expression for the current in the circuit (b) the r.m.s. value of this current (c) the total power absorbed.

[11.25μF, (a) i = 0.398 sin (ωt + 84.3) + 0.8 sin (3 ωt + 45) + 0.485 sin (5 ωt + 106) (b) 0.633 A (c) 10 W]

2. A voltage given by v = 200 sin 314 t + 520 sin (942 t + 45°) is applied to a circuit consisting of a resistance of 20 Ω, and inductance of 20 mH and a capacitance of 56.3 μF all connected in series.

Calculate the r.m.s. values of the applied voltage and current. Find also the total power absorbed by the circuit. **[146 V ; 3.16 A 200 W]**

3. A voltage given by v = 100 sin ωt + 8 sin 3 ωt is applied to a circuit which has a resistance of 1 Ω, an inductance of 0.02 H and a capacitance of 60μF. A hot-wire ammeter is connected in series with the circuit and a hot-wire voltmeter is connected to the terminals. Calculate the ammeter and voltameter readings and the power supplied to the circuit. **[71 V ; 5.18 A ; 26.8 W]**

4. A certain coil has a resistance of 20 Ω and an inductance of 0.04 H. If the instantaneous current flowing in it is represented by i = 5 sin 300 t + 0.8 sin 900 t amperes, derive an expression for the instantaneous value of the voltage applied across the ends of the coil and calculate the r.m.s. value of that voltage. **[V = 117 sin (300 t + 0.541) + 33 sin (900 t + 1.06) ; 0.86 V]**

5. A voltage given by the equation $v = \sqrt{2}\,100\sin 2\pi \times 50t + \sqrt{2.20}\sin 2\pi$. $150t$ is applied to the terminals of a circuit made up of a resistance of 5 Ω, an inductance of 0.0318 H and a capacitor of 12.5 μF all in series. Calculate the effective current and the power supplied to the circuit.

[0.547 A ; 1.5 W]

6. An alternating voltage given by the expression v = 1,000 sin 314t + 100 sin 942t is applied to a circuit having a resistance of 100 Ω and an inductance of 0.5 H. Calculate r.m.s. value of the current and p.f. **[3.81 A ; 0.535]**

7. The current in a series circuit consisting of a 159 μF capacitor, a reactor with a resistance of 10 Ω and an inductances of 0.0254 H is given by $i = \sqrt{2}(8\sin\omega t + 2\sin 3\omega t)$ amperes. Calculate the power input and the power factor. Given $\omega = 100\pi$ radian/second. **[680 W ; 0.63]**

8. If the terminal voltage of a circuit is 100 sin $\omega t + 50\sin(3\omega t + \pi/4)$ and the current is 10 sin$(\omega t + \pi 3)$ $+5\sin 3\omega t$, calculate the power consumption. **[522.6 W]**

9. A single-phase load takes a current of 4 sin $(\omega t + \pi/6) + 1.5\sin(3\omega t + \pi/3)$ A from source represented by 360 sin ωt volts. Calculate the power dissipated by the circuit and the circuit power factor.

[623.5 W ; 0.837]

10. An e.m.f. given by e = 100 sin t 40 sin(t /6) 10 sin(5 t /3) volts is applied to a series circuit having a resistance of 100 Ω, an inductance of 40.6 mH and a capacitor of 10 μF. Derive an expression for the current in the circuit. Also, find the r.m.s. value of the current and the power dissipated in the circuit. Take ω = 314 rad/second.

[0.329 A, 10.8 W]

11. A p.d. of the form $v = 400\sin\omega t + 30\sin 3\omega t$ is applied to a rectifier having a resistance of 50Ω in one direction and 200 in the reverse direction. Find the average and effective values of the current and the p.f. of the circuit.

[1.96 A, 4.1 A, 0.51]

12. A coil having R = 2 Ω and L = 0.01 H carries a current given by i = 50 + 20 sin 300 t

A moving-iron ammeter, a moving-coil voltmeter and a dynameter wattmeter, are used to indicate current, voltage and power respectively. Determine the readings of the instruments and equation for the p.d.

[121.1 V ; 52 A ; 5.4 kW, V = 100 + 72 sin (300 t + 0.982)]

13. Two circuits having impedances at 50 Hz of (10 + j6) Ω and (10 – j6) respectively are connected in parallel across the terminals of an a.c. system, the waveform of which is represented by v = 100 sin $\omega t + 35$ sin $3\omega t + 10\sin 5\omega t$, the fundamental frequency being 50 Hz. Determine the ratio of the readings of two ammeters, of negligible resistance, connected one in each circuit.

[6.35 ; 6.72]

14. Explain what is meant by harmonic resonance in a.c. circuits.

A current having an instantaneous value of 2 $(\sin\omega t + \sin 3\omega t)$ amperes is passed through a circuit which consists of a coil of resistance R and inductance L in series with a capacitor C. Derive an expression for the value of ω at which the r.m.s. circuit voltage is a minimum. Determine the voltage if the coil has inductance 0.1 H and resistance 150 Ω and the capacitance is 10 μ F. Determine also the circuit voltage at the fundamental resonant frequency.

[$\omega = 1/\sqrt{(LC)}$ **; 378 V ; 482 V]**

15. An r.m.s. current of 5 A which has a third-harmonic content, is passed through a coil having a resistance of 1Ω and an inductance of 10 mH. The r.m.s. voltage across the coil is 20 V. Calculate the magnitudes of the fundamental and harmonic components of current if the fundamental frequency is $300/2\pi$ Hz Also, find the power dissipated.

[4.8 A ; 1.44 A ; 25 W]

16. Derive a general expression for the form factor of a complex wave containing only odd-order harmonics. Hence, calculate the form factor of the alternating current represented by

i = 2.5 sin 157 t + 0.7 sin 471 t + 0.4 sin 785 t

[1.038]

OBJECTIVE TESTS – 20

1. Non-sinusoidal waveforms are made up of
(*a*) different sinusoidal waveforms
(*b*) fundamental and even harmonics
(*c*) fundamental and odd harmonics
(*d*) even and odd harmonics only.

2. The positive and negative halves of a complex wave are symmetrical when
(*a*) it contains even harmonics
(*b*) phase difference between even harmonics and fundamental is 0 or π
(*c*) it contains odd harmonics
(*d*) phase difference between even harmonics and fundamental is either π / 2 or 3π / 2.

3. The r.m.s. value of the complex voltage given by $v = 16\sqrt{2}\sin\omega t + 12\sqrt{2}\sin 3\omega t$ is

(*a*) $20\sqrt{2}$ (*b*) 20

(*c*) $28\sqrt{2}$ (*d*) 192

4. In a 3-phase system, ___th harmonic has negative phase sequence of RBY.

(a) 9 (*b*) 13

(*c*) 5 (*d*) 15

5. A complex current wave is given by the equation $i = 14 \sin \omega t + 2\sin 5\omega t$. The r.m.s. value of the current is ___ ampere.

(*a*) 16 (*b*) 12

(*c*) 10 (*d*) 8

6. When a pure inductive coil is fed by a complex voltage wave, its current wave

(*a*) has larger harmonic content

(*b*) is more distorted

(*c*) is identical with voltage wave

(*d*) shows less distortion.

7. A complex voltage wave is applied across a pure capacitor. As compared to the fundamental voltage, the reactance offered by the capacitor to the third harmonic voltage would be

(*a*) nine time (*b*) three times

(*b*) one-third (*d*) one-ninth

8. Which of the following harmonic voltage components in a 3-phase system would be in phase with each other ?

(*a*) 3rd, 9th, 15th etc.

(*b*) 7th, 13th, 19th etc.

(*c*) 5th, 11th, 17th etc.

(*d*) 2nd, 4th, 6th etc.

ANSWERS

1. *(a)* **2.** *(c)* **3.** *(b)* **4.** *(c)* **5.** *(c)* **6.** *(d)* **7.** *(c)* **8.** *(d)*

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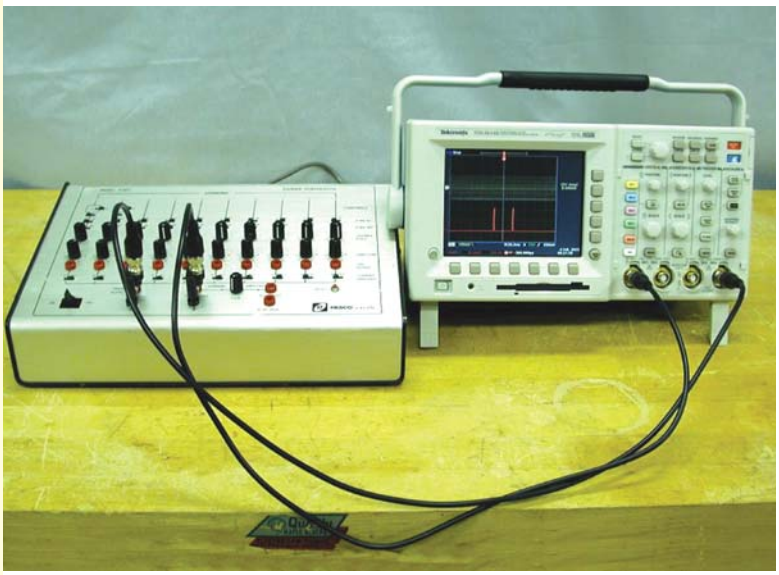
Contents

CHAPTER 21

FOURIER SERIES

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With the help of Fourier Theorem, it is possible to determine the magnitude, order and phase of the several hormonics present in a complex periodic wave

21.1. Harmonic Analysis

By harmonic analysis is meant the process of determining the magnitude, order and phase of the several harmonics present in a complex periodic wave.

For carrying out this analysis, the following methods are available which are all based on Fourier theorem:

(i) Analytical Method–the standard Fourier Analysis

(ii) Graphical Method– ***(a)*** by Superposition Method (Wedgemore' Method) ***(b)*** Twenty four Ordinate Method

(iii) Electronic Method–by using a special instrument called 'harmonic analyser'

We will consider the first and third methods only.

21.2. Periodic Functions

A function $f(t)$ is said to be periodic if $f(t+T) = f(t)$ for all values of t where T is some positive number. This T is the interval between two successive repetitions and is called the period of $f(t)$. A sine wave having a period of $T = 2\pi/\omega$ is a common example of periodic function.

21.3. Trigonometric Fourier Series

Suppose that a given function $f(t)$ satisfies the following conditions (known as Dirichlet conditions):

1. $f(t)$ is periodic having a period of T.

2. $f(t)$ is single-valued everywhere.

3. In case it is discontinuous, $f(t)$ has a finite number of discontinuities in any one period.

4. $f(t)$ has a finite number of maxima and minima in any one period.

The function $f(t)$ may represent either a voltage or current waveform. According to Fourier theorem, this function $f(t)$ may be represented in the trigonometric form by the infinite series.

$$f(t) = a_0 + a_1\cos\omega_0 t + a_2\cos 2\omega_0 t + a_3\cos 3\omega_0 t + \ldots + a_n\cos n\omega_0 t$$
$$+ b_1\sin\omega_0 t + b_2\sin 2\omega_0 t + b_3\sin 3\omega_0 t + \ldots + b_n\sin n\omega_0 t$$

$$= a_0 + \sum_{n=1}^{\infty}(a_n\cos n\omega_0 t + b_n\sin n\omega_0 t) \quad \ldots \textbf{(i)}$$

Putting $\omega_0 t = \theta$, we can write the above equation as under

$$f(\theta) = a_0 + a_1\cos\theta + a_2\cos 2\theta + a_3\cos 3\theta + \ldots + a_n\cos n\theta + b_1\sin\theta + b_2\sin 2\theta + b_3\sin 3\theta + \ldots + b_n\sin n\theta$$

$$= a_0 + \sum_{n=1}^{\infty}(a_n\cos n\theta + b_n\sin n\theta) \quad \ldots \textbf{(ii)}$$

Since $\omega_0 = 2\pi/T$, Eq. (*i*) above can be written as

$$f(t) = a_0 + \sum_{n=1}^{\infty}\left(a_n\cos\frac{2\pi n}{T}t + b_n\sin\frac{2\pi n}{T}t\right) \quad \ldots \textbf{(iii)}$$

where ω_0 is the fundamental angular frequency, T is the period and a_0, a_n and b_n are constants which depend on n and $f(t)$. The process of determining the values of the constants a_0, a_n and b_n is called Fourier Analysis. Also, $_0 \quad 2 /T \quad 2 f_0$ where f_0 is the fundamental frequency.

It is seen from the above Fourier Series that the periodic function consists of sinusoidal components of frequency 0, $\omega_0, 2\omega_0 \ldots . n\omega_0$. This representation of the function $f(t)$ is in the frequency domain. The first component a_0 with zero frequency is called the *dc* component. The sine and cosine terms represent the harmonics. The number n represents the order of the harmonics.

When n = 1, the component $(a_1 \cos \omega_0 t + b_1 \sin \omega_0 t)$ is called the first harmonic or the fundamental component of the waveform.

When n = 2, the component $(a_2 \cos 2\omega_0 t + b_2 \sin 2\omega_0 t)$ is called the second harmonic of the waveform.

The nth harmonic of the waveform is represented by $(a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$. It has a frequency of $n\omega_0$ *i.e.* n times the frequency of the fundamental component.

21.4. Alternate Forms of Trigonometric Fourier Series

Eq. (*i*) given above can be written as

$$f(t) = a_0 + (a_1 \cos \omega_0 t + b_1 \sin \omega_0 t) + (a_2 \cos 2\omega_0 t + b_2 \sin 2\omega_0 t) + \ldots + (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$$

Let, $a_n \cos n\omega_0 t + b_n \sin n\omega_0 t = A_n \cos(n\omega_0 t - \phi_n)$

$= A_n \cos n\omega_0 t \cos \phi_n + A_n \sin n\omega_0 t \sin \phi_n$

$\therefore \quad a_n = A_n \cos \phi_n$ and $b_n = A_n \sin \phi_n$

$\therefore \quad A_n \quad \sqrt{a_n^2 \quad b_n^2}$ and $_n \quad \tan^{-1} b_n / a_n$

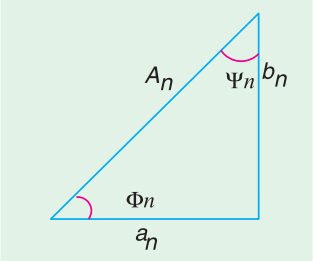


Fig. 21.1

Similarly, let $(a_n \cos n\omega_0 t + b_n \sin n\omega_0 t = A_n \sin(n\omega_0 t + \Psi_n))$

$= A_n \sin n\omega_0 t \cos \Psi_n + A_n \cos n\omega_0 t \sin \phi_n$

As seen from Fig. 21.1, $b_n = A_n \cos \psi_n$ and $a_n = A_n \sin \psi_n$

$\therefore \quad A_n = \sqrt{a_n^2 + b_n^2}$ and $\psi = \tan^{-1} a_n / b_n$

The two angles ϕ_n and ψ_n are complementary angles.

Hence, the Fourier series given in Art. 21.2 may be put in the following two alternate forms

$$f(t) = A_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t - \phi_n)$$

or $$f(t) = A_0 + \sum_{n=1}^{\infty} A_n \sin(n\omega_0 t + \psi_n)$$

21.5. Certain Useful Integral Calculus Theorems

The Fourier coefficients or constants $a_0, a_1, a_2 \ldots a_n$ and $b_1, b_2 \ldots . b_n$ can be evaluated by integration process for which purpose the following theorems will be used.

(i) $\int_0^{2\pi} \sin n\theta d\theta = \frac{1}{b}\left|\cos n\theta\right|_0^{2\pi} = \frac{1}{n}(1-1) = 0$

(ii) $\int_0^{2\pi} \sin n\theta d\theta = -\frac{1}{n}\left|\cos n\theta\right|_0^{2\pi} = -\frac{1}{n}(0-0) = 0$

(iii) $\int_0^{2} \sin^2 n \;\; d \quad \frac{1}{2}\int_0^{2} (1 \quad \cos 2n \;)d \quad \frac{1}{2}\left| \quad \frac{1}{2n}\sin 2n \;\right|_0^2$

(iv) $\int_0^{2\pi} \cos^2 n\theta d\theta = \frac{1}{2}\int_0^{2\pi} (\cos 2n\theta + 1)d\theta = \frac{1}{2}\left|\frac{1}{2n}\sin 2n\theta + \theta\right|_0^{2\pi} = \pi$

(v) $\int_0^{2} \sin m \;\; \cos n \;\; d \quad \frac{1}{2}\int_0^{2} \{\sin(m \quad n) \quad \sin(m \quad n)\}d$

$$= \frac{1}{2}\left|-\frac{1}{m+n}\cos(m+n)\theta - \frac{1}{m-n}\cos(m-n)\theta\right|_0^{2\pi} = 0$$

(vi) $\int_0^{2\pi} \cos m\theta \cos n\theta d\theta = \frac{1}{2}\int_0^{2\pi} \{\cos(m+n)\theta + \cos(m-n)\theta\}d\theta$

$$= \frac{1}{2}\left|\frac{1}{m+n}\sin(m+n)\theta + \frac{1}{m-n}\sin(m-n)\theta\right|_0^{2\pi} = 0 \ldots \text{for} \; n \neq m$$

(vii) $\int_0^{2} \sin m \;\; \sin n \;\; d \quad \frac{1}{2}\int_0^{2} \{\cos(m \quad n) \quad \cos(m \quad n) \;\}d$

$$= \frac{1}{2}\left|\frac{1}{m-n}\sin(m-n)\theta - \frac{1}{m+n}\sin(m+n)\theta\right|_0^{2\pi} = 0 \ldots \text{for} \; n \neq m$$

where m and n are any positive integers.

21.6. Evaluation of Fourier Constants

Let us now evaluate the constants a_0, a_n and b_n by using the above integral calculus theorems

(i) Value of a_0

For this purpose we will integrate both sides of the series given below over one period *i.e.* for $\theta = 0$ to $\theta = 2\pi$.

$$f(\theta) = a_0 + a_1\cos\theta + a_2\cos 2\theta + \ldots + a_n\cos n\theta \;+\; b_1\sin\theta + b_2\sin 2\theta + \ldots + b_n\sin n\theta$$

$$\therefore \int_0^{2\pi} f(\theta)d\theta = \int_0^{2\pi} a_0 d\theta + a_1\int_0^{2\pi}\cos\theta d\theta + a_2\int_0^{2\pi}\cos 2\theta d\theta + \ldots + a_n\int_0^{2\pi}\cos n\theta d\theta$$

$$+ \; b_1\int_0^{2\pi}\sin\theta d\theta + b_2\int_0^{2\pi}\sin 2\theta d\theta + \ldots + b_n\int_0^{2\pi}\sin n\theta d\theta$$

$= a_0|\theta|_0^{2\pi} + 0 + 0 + \ldots 0 + 0 + 0 + \ldots + 0 = 2\pi a_0$

$\therefore \quad a_0 = \frac{1}{2\pi}\int_0^{2\pi} f(\theta)d\theta$ or $= \frac{1}{2\pi}\int_{-\pi}^{\pi} f(\theta)d\theta$

= mean value of $f(\theta)$ between the limits 0 to 2π *i.e.* over one cycle or period.

Also, $a_0 \quad \frac{1}{2}[\text{net area}]_0^2$

If we take the periodic function as $f(t)$ and integrate over period T (which corresponds to 2π),

we get $a_0 = \frac{1}{T}\int_0^T f(t)dt = \frac{1}{T}\int_{-T/2}^{T/2} f(t)dt = \frac{1}{T}\int_{t_1}^{t_1+T} f(t)dt$

where t_1 can have any value.

(*ii*) Value of a_n

For finding the value of a_n, multiply both sides of the Fourier Series by cos $n\theta$ and integrate between the limits 0 to 2

$$\therefore \int_0^{2\pi} f(\theta)\cos n\theta d\theta = a_0\int_0^{2\pi}\cos n\theta d\theta + a_1\int_0^{2\pi}\cos\theta\cos n\theta d\theta + a_2\int_0^{2\pi}\cos 2\theta\cos n\theta d\theta$$

$$+a_n\int_0^{2\pi}\cos^2 n\theta d\theta + b_1\int_0^{2\pi}\cos\theta\cos n\theta d\theta + b_2\int_0^{2\pi}\sin 2\theta\cos n\theta d\theta + \ldots + b_n\int_0^{2\pi}\sin n\theta\cos n\theta d\theta$$

$$= 0 + 0 + 0 + \ldots + a_n\int_0^{2\pi}\cos^2 n\theta d\theta + 0 + 0 + \ldots + 0 = a_n\int_0^{2\pi}\cos^2 n\theta d\theta = \pi a_n$$

$$\therefore \quad a_n = \frac{1}{\pi}\int_0^{2\pi} f(\theta)\cos n\theta d\theta = 2\times\frac{1}{2\pi}\int_0^{2\pi} f(\theta)\cos n\theta\, d\theta$$

= 2 × average value of $f(\theta)$ cos $n\theta$ over one cycle of the fundamental.

Also, $a_n \quad \frac{1}{} \quad f(\,)\cos n \;\; d \quad 2 \;\; \frac{1}{2} \quad f(\,)\cos n \;\; d$

If we take periodic function as $f(t)$, then different expressions for a_n are as under.

$$a_n = \frac{2}{T}\int_0^T f(t)\cos\frac{2\pi n}{T}t\,dt = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos\frac{2\pi n}{T}t\,dt$$

Giving different numerical values to n, we get

a_1 = 2 × average of $f(\theta)$ cos θ over one cycle ....n = 1

a_2 = 2 × average value of $f(\theta)$ cos 2θ over one cycle etc. .... n = 2

(*iii*) Value of b_n

For finding its value, multiply both sides of the Fourier Series of Eq. (*i*) by sin $n\theta$ and integrate between limits $\theta = 0$ to $\theta = 2\pi$.

$$\therefore \int_0^{2\pi} f(\theta)\sin n\theta d\theta = a_0\int_0^{2\pi}\sin n\theta d\theta + a_1\int_0^{2\pi}\cos\theta\sin n\theta d\theta$$

$$+ a_2\int_0^{2\pi}\cos 2\theta\sin n\theta d\theta + \ldots + a_n\int_0^{2\pi}\cos n\theta\sin n\theta d\theta$$

$$+ b_1\int_0^{2\pi}\sin\theta\sin n\theta d\theta + b_2\int_0^{2\pi}\sin 2\theta\sin n\theta d\theta + \ldots + b_n\int_0^{2\pi}\sin^2 n\theta d\theta$$

$$= 0 + 0 + 0 + \ldots + 0 + 0 + \ldots + b_n\int_0^{2\pi}\sin^2 n\theta d\theta = b_n\int_0^{2\pi}\sin^2 n\theta\, d\theta = b_n\pi$$

$$\therefore \int_0^{2\pi} f(\theta)\sin\theta d\theta = b_n \times \pi$$

$$\therefore b_n = \frac{1}{\pi}\int_0^{2\pi} f(\theta)\sin n\theta d\theta = 2\times\frac{1}{2\pi}\int_0^{2\pi} f(\theta)\sin n\theta d\theta$$

= 2 × average value of $f(\theta)\sin n\theta$ over one cycle of the fundamental.

$\therefore b_1$ = 2 × average value of $f(\theta)\sin\theta$ over one cycle ... $n = 1$

b_2 = 2 × average value of $f(\theta)\sin 2\theta$ over one cycle ... $n = 2$

Also, $b_n = \frac{2}{t}\int_0^T f(t)\sin\frac{2\pi n}{T}t\,dt + \frac{2}{T}\int_{T/2}^{T/2} f(t)\sin\frac{2\pi n}{T}t\,dt$

$$= \frac{2}{T}\int_0^T f(t)\sin n\omega_0 t\,dt = \frac{2}{T}\int_{T/2}^{T/2} f(t)\sin n\omega_0 t dt$$

Hence, for Fourier analysis of a periodic function, the following procedure should be adopted:

(i) Find the term a_0 by integrating both sides of the equation representing the periodic function between limits 0 to 2π or 0 to T or $-T/2$ to $T/2$ or t_1 to $(t_1 + T)$.

$$\therefore a_0 = \frac{1}{2\pi}\int_0^{2\pi} f(\theta)d\theta = \frac{1}{T}\int_0^T f(t)dt = \frac{1}{T}\int_{-T/2}^{T/2} f(t)dt = \frac{1}{T}\int_{t_1}^{t_1+T} f(t)dt$$

= average value of the function over one cycle.

(ii) Find the value of a_n by multiplying both sides of the expression for Fourier series by $\cos n\theta$ and then integrating it between limits 0 to 2π or 0 to T or $-T/2$ to $T/2$ or t_1 to $(t_1 + T)$.

$$\therefore a_n = \frac{1}{\pi}\int_0^{2\pi} f(\theta)\cos n\theta d\theta = 2\times\frac{1}{2\pi}\int_0^{2\pi} f(\theta)\cos n\theta d\theta$$

Since $\pi = T/2$, we have

$$a_n = \frac{2}{T}\int_0^T f(t)\cos\frac{2\pi n}{T}t\,dt = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos\frac{2\pi n}{T}t\,dt = \frac{2}{T}\int_{t_1}^{t_1+T} f(t)\cos\frac{2\pi n}{T}t\,dt$$

$$= \frac{2}{T}\int_0^T f(t)\cos n\omega_0 t dt = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos n\omega_0 t dt = \frac{2}{T}\int_{t_1}^{t_1+T} f(t)\cos n\omega_0 t dt$$

= 2 × average value of $f(\theta)\cos n\theta$ over one cycle of the fundamental.

Values of a_1, a_2, a_3 etc. can be found from above by putting $n = 1, 2, 3$ etc.

(iii) Similarly, find the value of b_n by multiplying both sides of Fourier series by $\sin n\theta$ and integrating it between the limits 0 to 2π or 0 to T or $-T/2$ to $T/2$ or t_1 to $(t_1 + T)$.

$$\therefore \quad b_n = \frac{1}{\pi}\int_0^{2\pi} f(\theta)\sin n\theta = 2\times\frac{1}{2\pi}\int_0^{2\pi} f(\theta)\sin n\theta d\theta$$

$$= \frac{2}{T}\int_0^{T} f(t)\sin\frac{2\pi n}{T}t\,dt = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\sin\frac{2\pi n}{T}t\,dt = \frac{2}{T}\int_{t_1}^{t_1+T} f(t)\sin\frac{2\pi n}{t}t\,dt$$

$$= \frac{2}{T}\int_0^{T} f(t)\sin n\omega_0 t dt = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\sin n\omega_0 t\,dt = \frac{2}{T}\int_{t_1}^{t_1+T} f(t)\sin n\omega_0 t\,dt$$

= 2 × average value of $f(\theta)\ \sin n\theta$ of $f(t)\sin\frac{2\pi n}{T}t$ or $f(t)\ \sin n\omega_0 t$ over one cycle of the fundamental.

Values of b_1, b_2, b_3 etc. can be found from above by putting $n = 1, 2, 3$ etc.

21.7. Different Types of Functional Symmetries

A non-sinusoidal wave can have the following types of symmetry:

1. Even Symmetry

The function $f(t)$ is said to possess even symmetry if $f(t) = f(-t)$.

It means that as we travel equal amounts in time to the left and right of the origin (*i.e.* along the + X-axis and $-X$-axis), we find the function to have the same value. For example in Fig. 21.2 (*a*), points A and B are equidistant from point O. Here the two function values are equal and positive. At points C and D, the two values of the function are again equal, though negative. Such a function is symmetric with respect to the vertical axis. Examples of even function are: t^2,

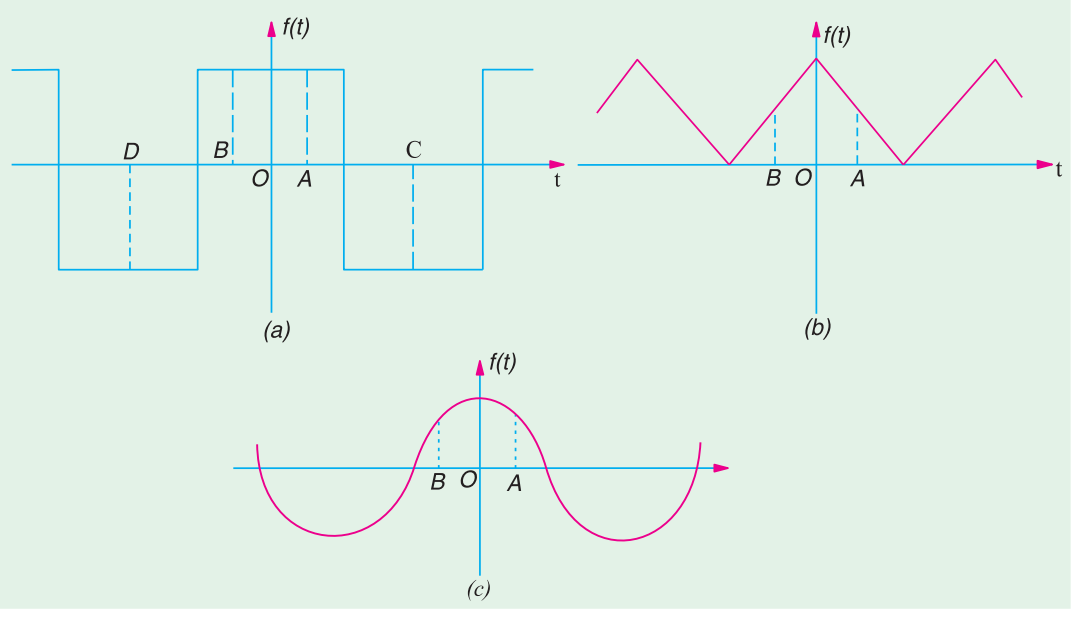


Fig. 21.2

cos 3t, sin^2 5t (2 + t^2 + t^4) and a constant A because the replacement of t by (– t) does not change the value of any of these functions. For example, $\cos \omega t = \cos(-\omega t)$.

This type of symmetry can be easily recognised graphically because mirror symmetry exists about the vertical or $f(t)$ axis. The function shown in Fig. 21.2 has even symmetry because when folded along vertical axis, the portions of the graph of the function for positive and negative time fit exactly, one on top of the other.

The effect of the even symmetry on Fourier series is that the constant $b_n = 0$ *i.e.* the wave has no sine terms. In general, b_1, b_2, b_3 ... $b_n = 0$. The Fourier series of an even function contains only a constant term and cosine terms *i.e.*

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t = a_0 + \sum_{n=1}^{\infty} a_n \cos \frac{2\pi n}{T} t$$

The value of a_n may be found by integrating over any half-period.

$$\therefore \quad a_n = \frac{2}{\pi}\int_0^{\pi} f(\theta)\cos n\theta d\theta = \frac{4}{T}\int_0^{T/2} f(t)\cos n\omega t\, dt$$

2. Odd Symmetry

A function $f(t)$ is said to possess odd symmetry if $f(-t) = -f(t)$.

It means that as we travel an equal amount in time to the left or right from the origin, we find the function to be the same except for a reversal in sign. For example, in Fig. 21.3 the two points A and B are equidistant from point O. The two function values at A and B are equal in magnitude but opposite in sign. In other words, if we replace t by (– t), we obtained the negative of the given function. The X-axis divides an odd function into two halves with equal areas above and below the X-axis. Hence, $a_0 = 0$.

Examples of odd functions are: t, sin t, t cos 50 $t(t + t^3 + t^5)$ and $t\sqrt{(1+t^2)}$

A sine function is an odd function because $\sin(-\omega t) = -\sin \omega t$.

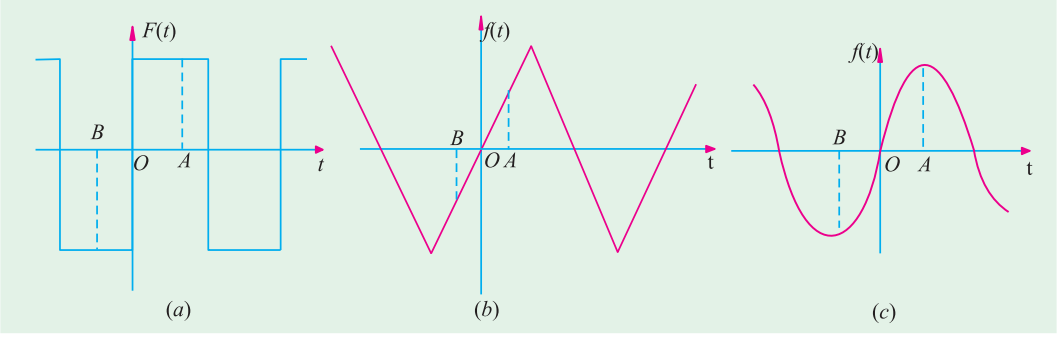


Fig. 21.3

An odd function has symmetry about the origin rather than about the $f(t)$ axis which was the case for an even function. The effect of odd symmetry on a Fourier series is that it contains no constant term or consine term. It means that $a_0 = 0$ and $a_n = 0$ *i.e.* a_1, a_2, a_3....$a_n = 0$. The Fourier series expansion contains only sine terms.

$$\therefore \quad f(t) = \sum_{n=1}^{\infty} b_n \sin n\omega_0 t$$

The value of b_n may be found by integrating over any half-period.

$$\therefore \quad b_n = \frac{2}{\pi}\int_0^{\pi} f(\theta)\sin n\theta d\theta = \frac{4}{T}\int_0^{T/2} f(t)\sin n\omega t\, dt$$

3. Half-wave Symmetry or Mirror-Symmetry or Rotational Symmetry

A function $f(t)$ is said to possess half-wave symmetry if $f(t) = -f(t \pm T/2)$ or $-f(t) = f(t \pm T/2)$.

It means that the function remains the same if it is shifted to the left or right by half a period and then flipped over (*i.e.* multiplied by – 1) in respect to the *t*-axis or horizontal axis. It is called mirror symmetry because the negative portion of the wave is the mirror image of the positive portion of the wave displaced horizontally a distance *T*/2.

In other words, a waveform possesses half symmetry only when we invert its negative half-cycle and get an exact duplicate of its positive half-cycle. For example, in Fig. 21.4 (*a*) if we invert the negative half-cycle, we get the dashed *ABC* half-cycle which is exact duplicate of the positive half-cycle. Same is the case with the waveforms of Fig. 21.4 (*b*) and Fig. 21.4 (*c*). In case

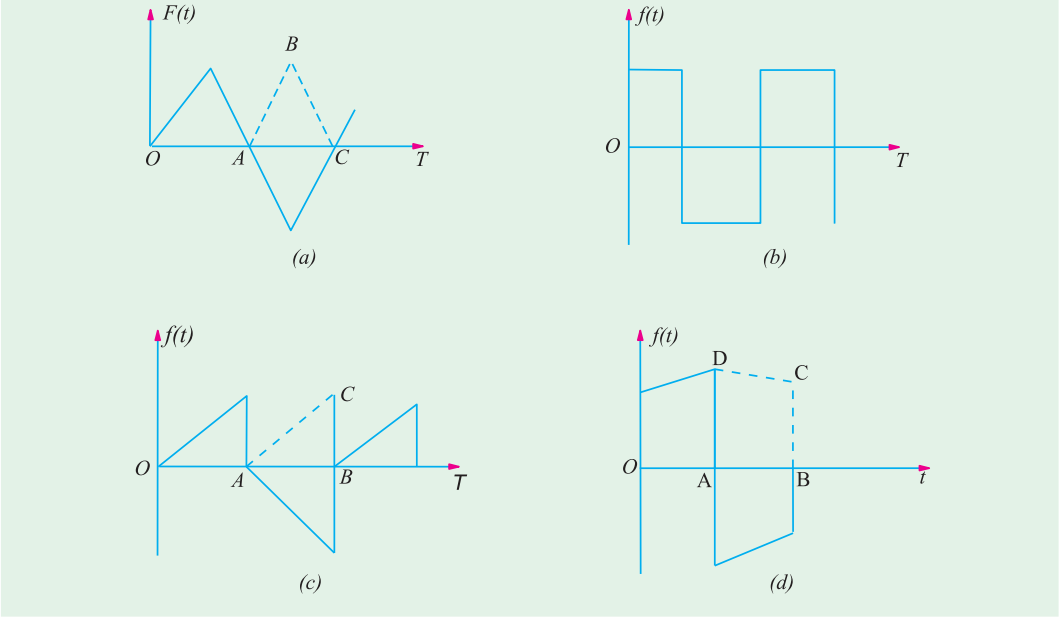


Fig. 21.4

of doubt, it is helpful to shift the inverted half-cycle by a half-period to the left and see if it superimposes the positive half-cycle. If it does so, there exists half-wave symmetry otherwise not. It is seen that the waveform of Fig. 21.4 (*d*) does not possess half-wave symmetry. It is so because when its negative half-cycle is inverted and shifted by half a period to the left it does not superimpose the positive half-cycle.

It may be noted that half-wave symmetry may be present in a waveform which also shows either odd symmetry or even symmetry:

For example, the square waveform shown in Fig. 21.4 (*a*) possesses even symmetry whereas the triangular waveform of Fig. 24.4 (*b*) has odd symmetry. All cosine and sine waves possess half-wave symmetry because

$$\cos\frac{2\pi}{T}\left(t \pm \frac{T}{2}\right) = \cos\left(\frac{2\pi}{T}t \pm \pi\right) = -\cos\frac{2\pi}{T}t; \sin\frac{2\pi}{T}\left(t \pm \frac{T}{2}\right) = \sin\left(\frac{2\pi}{T}t \pm \pi\right) = -\sin\frac{2\pi}{T}t$$

It is worth noting that the Fourier series of any function which possesses half-wave symmetry has zero average value and contains only odd harmonics and is given by

$$f(t)=\sum_{\substack{n=1\\odd}}^{\infty}\left(a_n\cos\frac{2\pi n}{T}t+b_n\sin\frac{2\pi n}{T}t\right)=\sum_{\substack{n=1\\odd}}^{\infty}(a_n\cos n\omega_0 t+b_n\sin n\omega_0 t)=\sum_{\substack{n=1\\odd}}^{\infty}(a_n\cos n\theta+b_n\sin n\theta)$$

where, $a_n=\frac{4}{T}\int_0^{T/2}f(t)\cos\frac{2\pi n}{T}dt=\frac{2}{\pi}\int_0^{\pi}f(\theta)\cos n\theta d\theta$ ... n odd

$$b_n=\frac{4}{T}\int_0^{T/2}f(t)\sin\frac{2\pi n}{T}dt=\frac{2}{\pi}\int_0^{\pi}f(\theta)\sin n\theta d\theta$$ ... n odd

4. Quarter-wave Symmetry

An odd or even function with rotational symmetry is said to possess quarter-wave symmetry.

Fig. 21.5 (*a*) possesses half-wave symmetry as well as odd symmetry. The wave shown in Fig. 21.5 (*b*) has both half-wave symmetry and even symmetry.

The mathematical test for quarter-wave symmetry is as under:

Odd quarter-wave $f(t)=-f(t+T/2)$ and $f(-t)=-f(t)$

Even quarter-wave $f(t)=-f(t+T/2)$ and $f(t)=f(-t)$

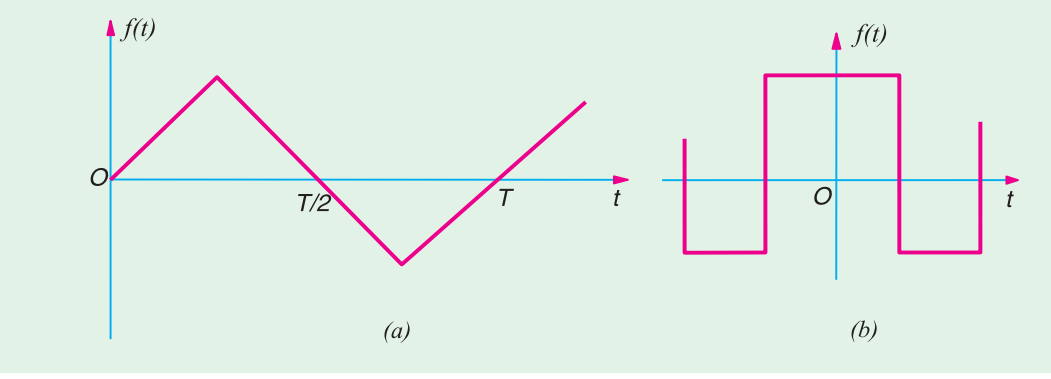


Fig. 21.5

Since each quarter cycle is the same in a way having quarter-wave symmetry, it is sufficient to integrate over one quarter period *i.e.* from 0 to $T/4$ and then multiply the result by 4.

(i) If $f(t)$ or $f(\theta)$ is odd and has quarter-wave symmetry, then a_0 is 0 and a_n is 0. Hence, the Fourier series will contain only odd sine terms.

$$\therefore\quad f(t)=\sum_{\substack{n=1\\odd}}^{\infty}b_n\sin\frac{2\pi nt}{T}\ \text{ or }\ f(\theta)=\sum_{\substack{n=1\\odd}}^{\infty}b_n\sin n\theta,\ \text{where}\ b_n=\frac{1}{\pi}\int_0^{2\pi}f(\theta)\sin n\theta d\theta$$

It may be noted that in the case of odd quarter-wave symmetry, the integration may be carried over a quarter cycle.

$$\therefore\quad a_n=\frac{4}{\pi}\int_0^{\pi/2}f(\theta)\cos n\theta d\theta$$ ... n odd

$$=\frac{8}{T}\int_0^{T/4}f(t)\sin n\omega t dt$$ ... n odd

(ii) If $f(t)$ or $f(\theta)$ is even and, additionally, has quarter-wave symmetry, then a_0 is 0 and b_n is 0. Hence, the Fourier series will contain only odd cosine terms.

$$\therefore \quad f(\theta) = \sum_{\substack{n=1 \\ odd}}^{\infty} a_n \cos n\theta d\theta = \sum_{\substack{n=1 \\ odd}}^{\infty} a_n \cos n\omega_0 t dt; \text{where } a_n = \frac{1}{\pi}\int_0^{2\pi} f(\theta)\cos n\theta d\theta$$

In this case a_n may be found by integrating over any quarter period.

$$a_n = \frac{4}{\pi}\int_0^{\pi/2} f(\theta)\sin n\theta d\theta \quad \text{... } n \text{ odd}$$

$$= \frac{8}{T}\int_0^{T/4} f(t)\cos n\omega t\, dt \quad \text{... } n \text{ odd}$$

21.8. Line or Frequency Spectrum

A plot which shows the amplitude of each frequency component in a complex waveform is called the line spectrum or frequency spectrum (Fig. 21.6). The amplitude of each frequency component is indicated by the length of the vertical line located at the corresponding frequency. Since the spectrum represents frequencies of the harmonics as discrete lines of appropriate height, it is also called a discrete spectrum. The lines decrease rapidly for waves having convergent series. Waves with discontinuities such as the sawtooth and square waves have spectra whose amplitudes decrease slowly because their series have strong high harmonics. On the other hand, the line spectra of waveforms without discontinuities and with a smooth appearance have lines which decrease in height very rapidly.

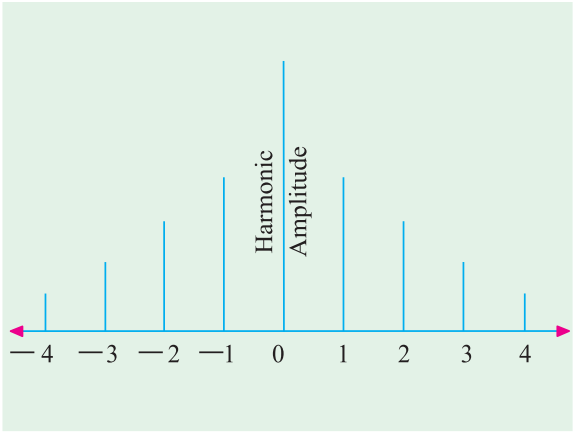


Fig. 21.6

The harmonic content and the line spectrum of a wave represent the basic nature of that wave and never change irrespective of the method of analysis. Shifting the zero axis changes the symmetry of a given wave and gives its trigonometric series a completely different appearance but the same harmonics always appear in the series and their amplitude remains constant.

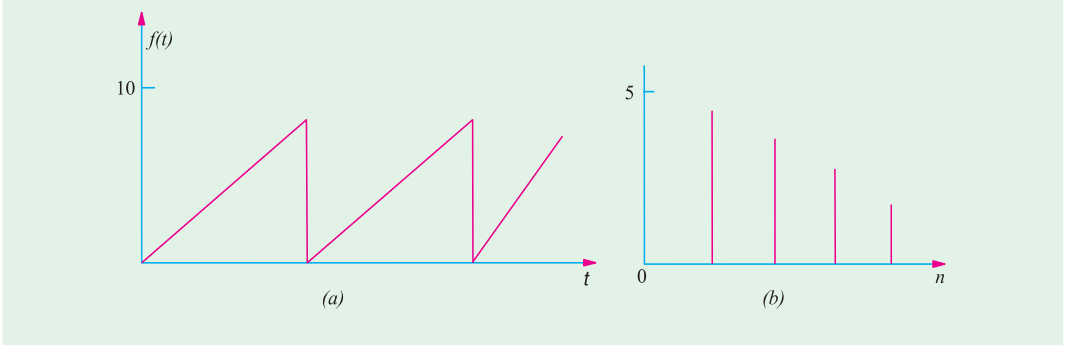


Fig. 21.7

Fig. 21.7 shows a smooth wave alongwith its line spectrum. Since there are only sine terms in its trigonometric series (apart from $a_0 = \pi$), the harmonic amplitudes are given by b_n.

21.9. Procedure for Finding the Fourier Series of a Given Function

It is advisable to follow the following steps:

1. Step No. 1

If the function is defined by a set of equations, sketch it approximately and examine for symmetry.

2. Step No. 2

Whatever be the period of the given function, take it as 2π (Ex. 20.6) and find the Fourier series in the form

$$f(\theta) = a_0 + a_1 \cos\theta + a_2 \cos 2\theta + \ldots + a_n \cos n\theta + b_1 \sin\theta + b_2 \sin 2\theta + \ldots + b_n \sin n\theta$$

3. Step No. 3

The value of the constant a_0 can be found in most cases by inspection. Otherwise it can be found as under:

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} f(\theta)d\theta = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(\theta)d\theta$$

4. Step No. 4

If there is no symmetry, then a_0 is found as above whereas the other two fourier constants can be found by the relation.

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(\theta)\cos n\theta d\theta = \frac{1}{\pi}\int_{-\pi}^{\pi} f(\theta)\cos n\theta d\theta$$

$$b_n = \int_0^{2\pi} f(\theta)\sin n\theta d\theta = \frac{1}{\pi}\int_{-\pi}^{\pi} f(\theta)\sin n\theta d\theta$$

5. Step No. 5

If the function has even symmetry *i.e.* $f(\theta) = f(-\theta)$, then b_n = 0 so that the Fourier series will have no sine terms. The series would be given by

$$f(\theta) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\theta d\theta \text{ where } a_n = \frac{1}{\pi}\int_0^{2\pi} f(\theta)\cos n\theta = \frac{2}{\pi}\int_0^{\pi} f(\theta)\cos n\theta d\theta$$

6. Step No. 6

If the given function has odd symmetry *i.e.* $f(-\theta) = -f(\theta)$ then a_0 = 0 and a_n = 0. Hence, there would be no cosine terms in the Fourier series which accordingly would be given by

$$f(\theta) = \sum_{n=1}^{\infty} b_n \sin\omega_0 t \text{ ; where } b_n = \frac{1}{\pi}\int_0^{2\pi} f(\theta)\sin n\theta d\theta = \frac{2}{\pi}\int_0^{\pi} f(\theta)\sin n\theta d\theta$$

7. Step No. 7

If the function possesses half-wave symmetry *i.e.* $f(\theta) = -f(\theta \pm \pi)$ or $f(t) = -f(t \pm T/2)$, then a_0 is 0 and the Fourier series contains only odd harmonics. The Fourier series is given by

$$f(\theta) = \sum_{\substack{n=1 \\ odd}}^{\infty} a_n(\cos n\theta + b_n \sin n\theta)$$

where $a_n = \frac{1}{\pi}\int_0^{2\pi} f(\theta)\cos n\theta d\theta \ldots n$ odd, $b_n = \frac{2}{\pi}\int_0^{\pi} f(\theta)\sin n\theta d\theta$ ... n odd

8. Step No. 8

If the function has even quarter-wave symmetry then $a_0 = 0$ and $b_n = 0$. It means the Fourier series will contain no sine terms but only odd cosine terms. It would be given by

$$f(\ ) \underset{n\ 1\ odd}{\ } a_n \cos n \ ; \text{where } a_n \ \frac{1}{\ } \ {}^{2}_{0} \ f(\ )\cos n \ d \ \ \frac{2}{\ } \ {}_{0} \ f(\ )\cos n \ d \ \ \frac{4}{\ } \ {}^{/2}_{0} f(\ )\cos n \ d$$

... n odd

9. Step No. 9

If the function has odd quarter-wave symmetry, then $a_0 = 0$ and $a_n = 0$. The Fourier series will contain only odd sine terms (but no cosine terms).

$$\therefore \ f(\ ) \underset{n\ 1\ odd}{\ } b_n \sin n \ ; \text{where } b_n \ \frac{1}{\ } \ {}^{2}_{0} \ f(\ )\sin n \ d \ \ \frac{2}{\ } \ {}_{0} \ f(\ )\sin n \ d \ \ \frac{4}{\ } \ {}^{/2}_{0} f(\ )\sin n \ d$$

... n odd

10. Step No. 10

Having found the coefficients, the Fourier series as given in step No. 2 can be written down.

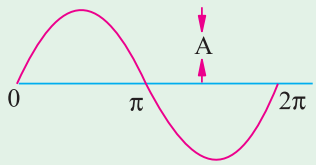
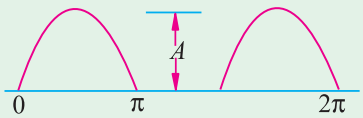
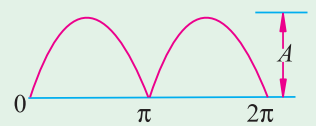
11. Step No. 11

The different harmonic amplitudes can be found by combining similar sine and cosine terms

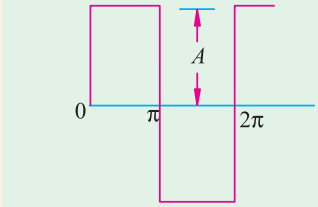
i.e. $A_n = \sqrt{a_n^2 + b_n^2}$

where A_n is the amplitude of the n_{th} harmonic.

Table No. 21.1

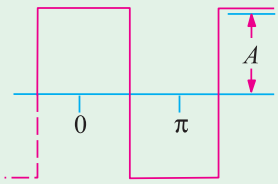
	Wave form	*Appearance*	*Equation*
A.	Sine wave		$f(t) = A = A \sin \omega t$
B.	Half-wave rectified sine wave		$f(t) = A\left(\frac{1}{\pi} + \frac{1}{2}\sin\omega t - \frac{2}{3\pi}\cos 2\omega t\right.$ $\left. - \frac{2}{15\pi}\cos 4\omega t - \frac{2}{35\pi}\cos 6\omega t \right.$ $\left. - \frac{2}{63\pi}\cos 8\omega t \ldots\ldots\right.$
C.	Full-wave rectified sine wave		$f(t) = \frac{2A}{\pi}(1 - \frac{2}{3}\cos 2\omega t$ $- \frac{2}{15}\cos 4\omega t - \frac{2}{35}\cos 6\omega t \ldots\ldots)$

D. Rectangular or square wave



$$f(t)=\frac{44}{\pi}(\sin\omega t+\frac{1}{3}\sin 3\omega t\ .....)$$

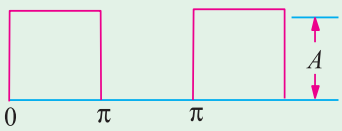
$$+\frac{1}{5}\sin 5\omega t+\frac{1}{7}\sin 7\omega t+....)$$



$$f(t)=\frac{4A}{\pi}(\cos\omega t-\frac{1}{3}\cos 3\omega t+$$

$$\frac{1}{5}\cos 5\omega t.....)$$

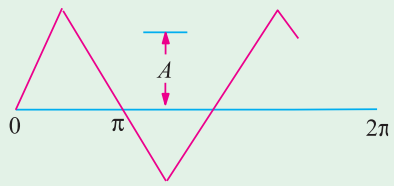
E. Rectangular or square wave pulse



$$f(t)=\frac{A}{2}+\frac{2A}{\pi}(\sin\omega t+\frac{1}{3}\sin 3\omega t+$$

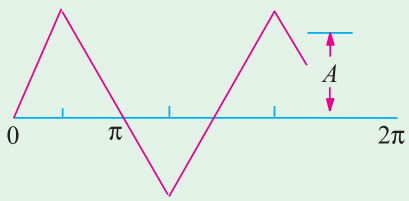
$$\frac{1}{5}\sin 5\omega t+\frac{1}{7}\sin 7\omega t+....)$$

F. Triangular wave



$$f(t)=\frac{8A}{\pi^2}(\sin\omega t-\frac{1}{9}\sin 3\omega t+\frac{1}{25}$$

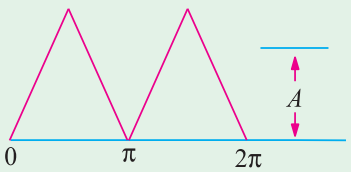
$$\sin 5\omega t-\frac{1}{49}\sin 7\omega t+....)$$



$$f(t)=\frac{8A}{\pi^2}(\cos\omega t+\frac{1}{9}\cos 3\omega t+$$

$$\frac{1}{25}\cos 5\omega t+\frac{1}{49}\cos 7\omega t+.....)$$

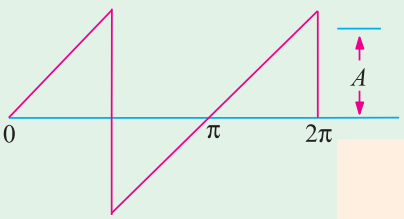
G. Triangular pulse



$$f(t)=\frac{A}{2}+\frac{4A}{\pi^2}(\sin\omega t-\frac{1}{9}\sin 3\omega t$$

$$+\frac{1}{25}\sin 5\omega t-\frac{1}{49}\sin 7\omega t+.....)$$

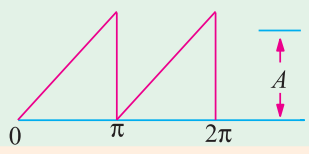
H. Sawtooth wave



$$f(t)=\frac{2A}{\pi}(\sin olegat-\frac{1}{2}\sin 2\omega t+$$

$$\frac{1}{3}\sin 3\omega t-\frac{1}{4}\sin 4\omega t+....)$$

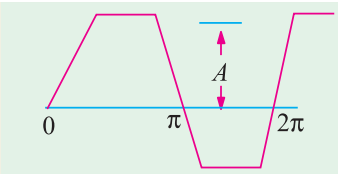
I. Sawtooth pulse



$$f(t)=\frac{A}{\pi}\left(\frac{\pi}{2}-\sin\omega_0 t-\frac{1}{2}\sin 2\omega t-\frac{1}{3}\sin 3\omega t\right.$$

$$\left.-\frac{1}{4}\sin 4\omega t\right)$$

J. Trapezoidal wave



$$f(t) = \frac{3\sqrt{3}A}{\pi^2}\left(\sin\omega t - \frac{1}{25}\sin 5\omega t + \frac{1}{49}\sin 7\omega t.....\right)$$

21.10. Wave Analyzer

A wave analyzer is an instrument designed to measure the individual amplitude of each harmonic

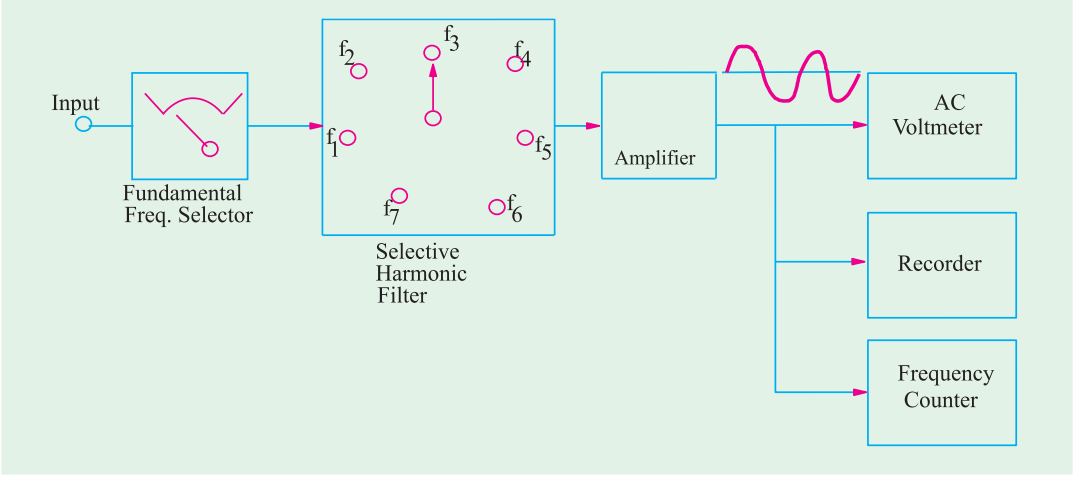


Fig. 21.8

component in a complex waveform. It is the simplest form of analysis in the frequency domain and can be performed with a set of tuned filters and a voltmeter. That is why such analyzers are also called frequency-selective voltmeters. Since such analyzers sample the frequency spectrum successively, *i.e.* one after the other, they are called non-real-time analyzers.



Wave analyzer

The block diagram of a simple wave analyzer is shown in Fig. 21.8. It consists of a tunable fundamental frequency selector that detects the fundamental frequency f_1 which is the lowest frequency contained in the input waveform.

Once tuned to this fundamental frequency, a selective harmonic filter enables switching to multiples of f_1. After amplification, the output is fed to an a.c. voltmeter, a recorder and a frequency counter. The voltmeter reads the r.m.s amplitude of the harmonic wave, the recorder traces its waveform and the frequency counter gives its frequency. The line spectrum of the harmonic component can be plotted from the above data.

For higher frequencies (MHz) heterodyne wave analyzers are generally used. Here, the input complex wave signal is heterodyned to a higher intermediate frequency (IF) by an internal local oscillator. The output of the IF amplifier is rectified and is applied to a dc voltmeter called heterodyned tuned voltmeter.

The block diagram of a wave analyzer using the heterodyning principle is shown in Fig. 21.9.

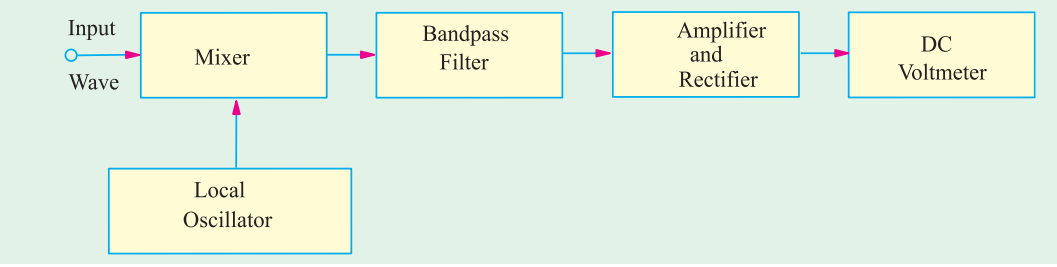


Fig. 21.9

The signal from the internal, variable-frequency oscillator heterodynes with the input signal in a mixer to produce output signal having frequencies equal to the sum and difference of the oscillator frequency f_0 and the input frequency f_1. Generally, the bandpass filter is tuned to the 'sum frequency' which is allowed to pass through. The signal coming out of the filter is amplified, rectified and then applied to a dc voltmeter having a decibel-calibrated scale. In this way, the peak amplitudes of the fundamental component and other harmonic components can be calculated.

21.11. Spectrum Analyzer

It is a real-time instrument *i.e.* it simultaneously displays on a CRT, the harmonic peak

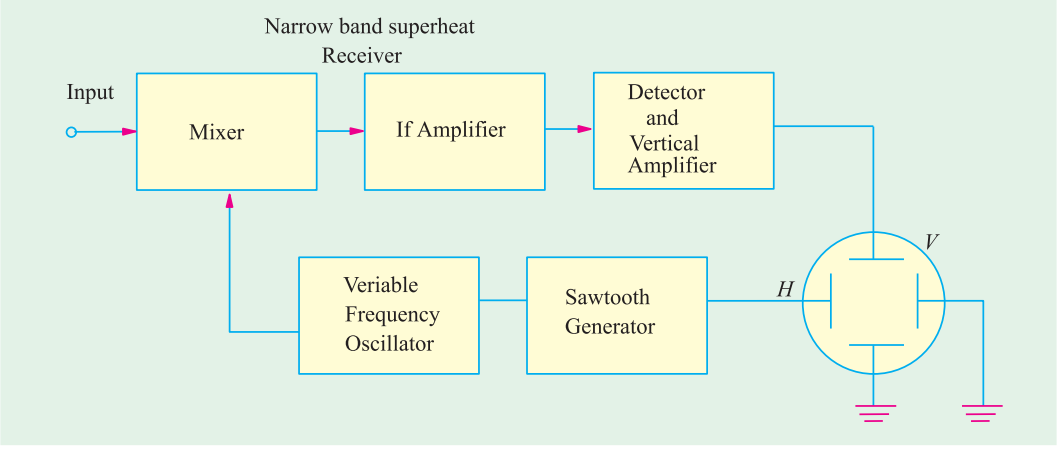
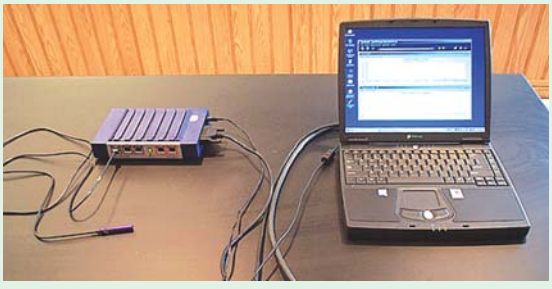


Fig. 21.10

values versus frequency of all wave components in the frequency range of the analyzer. The block diagram of such an analyzer is shown in Fig. 21.10.

As seen, the spectrum analyzer uses a CRT in combination with a narrow-band superheterodyne receiver. The receiver is tuned by varying the frequency of the voltage-tuned variable-frequency oscillator which also controls the sawtooth generator that sweeps the horizontal time base of the CRT deflection plates. As the oscillator is



Spectrum analyzer

swept through its frequency band by the sawtooth generator, the resultant signal mixes and beats with the input signal to produce an intermediate frequency (IF) signal in the mixer. The mixer output occurs only when there is a corresponding harmonic component in the input signal which matches with the sawtooth generator signal. The signals from the IF amplifier are detected and further amplified before applying them to the vertical deflection plates of the CRT. The resultant output displayed on the CRT represents the line spectrum of the input complex or nonsinusoidal waveform.

21.12. Fourier Analyzer

A Fourier analyzer uses digital signal processing technique and provides information regarding the contents of a complex wave which goes beyond the capabilities of a spectrum analyzer. These analyzers are based on the calculation of the discrete Fourier transform using an algorithm called the fast Fourier transformer. This algorithm calculates the amplitude and phase of each harmonic component from a set of time domain samples of the input complex wave signal.

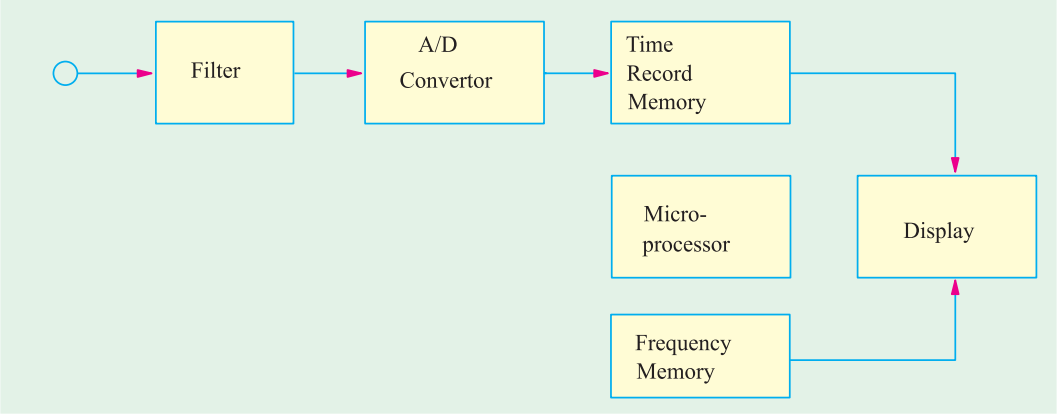


Fig. 21.11

A basic block diagram of a Fourier analyzer is shown in Fig. 21.11. The complex wave signal applied to the instrument is first filtered to remove out-of-band frequency components. Next, the signal is applied to an analog/digital (A/D) convertor which samples and digitizes it at regular time intervals until a full set of samples (called a time record) has been collected. The microprocessor then performs a series of computations on the time data to obtain the frequency-domain results *i.e.* amplitude versus frequency relationships. These results which are stored in memory can be displayed on a CRT or recorded permanently with a recorder or plotter.



Fourier Analyzer

Since Fourier analyzers are digital instruments, they can be easily interfaced with a computer or other digital systems. Moreover, as compared to spectrum analyzers, they provide a higher degree of accuracy, stability and repeatability.

21.13. Harmonic Synthesis

It is the process of building up the shape of a complex waveform by adding the instantaneous values of the fundamental and harmonics. It is a graphical procedure based on the knowledge of the different components of a complex waveform.

Example 21.1. *A complex voltage waveform contains a fundamental voltage of r.m.s. value 220 V and frequency 50 Hz alongwith a 20% third harmonic which has a phase angle lagging by* $3\pi/4$ *radian at t = 0. Find an expression representing the instantaneous complex voltage v. Using harmonic synthesis, also sketch the complex waveform over one cycle of the fundamental.*

Solution. The maximum value of the fundamental voltage is = $200\times\sqrt{2}$ = 310 V. Its angular velocity is $\omega = 2\pi\times 50$ = 100π rad/s. Hence, the fundamental voltage is represented by 310 sin 100π t.

The amplitude of the third harmonic = 20% of 310 = 62V. Its frequency is 3 × 50 = 150 Hz. Hence, its angular frequency is = $2\pi\times 150 = 300\pi$ rad/s. Accordingly, the third harmonic voltage can be represented by the equation 62 sin $(300\pi t - 3\pi/4)$. The equation of the complex voltage is given by υ = 310 sin $100\pi t + 62\sin(300\pi t - 3\pi/4)$.

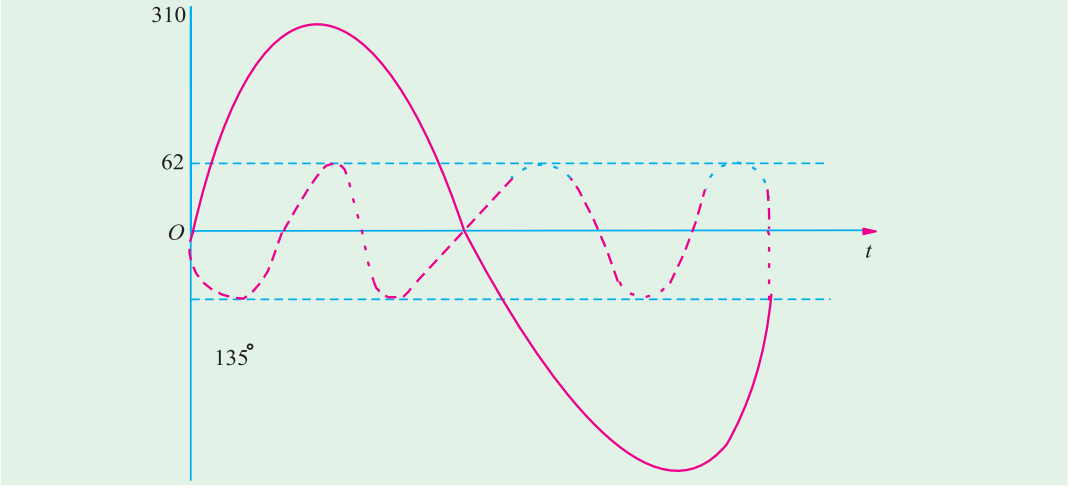


Fig 21.12

In Fig. 21.12 are shown one cycle of the fundamental and three cycles of the third harmonic component initially lagging by $3\pi/4$ radian or 135°. By adding ordinates at different intervals, the complex voltage waveform is built up as shown.

Incidentally, it would be seen that if the negative half-cycle is reversed, it is identical to the positive half-cycle. This is a feature of waveforms possessing half-wave symmetry which contains the fundamental and odd harmonics.

Example 21.2. *For the nonsinusoidal wave shown in Fig. 21.13, determine* ***(a)*** *Fourier coefficients* a_0, a_3 *and* b_4 *and* ***(b)*** *frequency of the fourth harmonic if the wave has a period of 0.02 second.*

Solution. The function $f(\theta)$ has a constant value from θ = 0 to θ = $4\pi/3$ radian and 0 value from $\theta = 4\pi/3$ radian to $\theta = 2\pi$ radian.

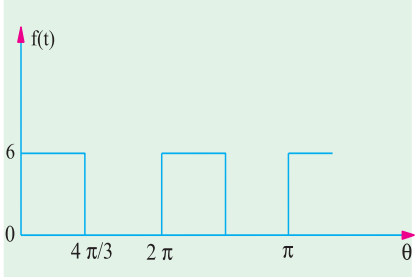


Fig. 21.13

(a) $a_0 = \dfrac{1}{2\pi}$ (net area per cycle)$_0^{2\pi} = \dfrac{1}{2\pi}\left(6\times\dfrac{4\pi}{3}\right) = 4$

$$a_3 = \frac{1}{\pi}\int_0^{2\pi} f(\theta)\cos 3\theta d\theta = \frac{1}{\pi}\int_0^{4\pi/3} 6\cos 3\theta d\theta$$

$$= \frac{6}{\pi}\left|\frac{\sin 3\theta}{3}\right|_0^{4\pi/3} = \frac{2}{\pi}(\sin 4\pi) = 0$$

$$b_4 = \frac{1}{\pi}\int_0^{2\pi} f(\theta)\sin 4\theta d\theta = \frac{1}{\pi}\int_0^{4\pi/3} 6\sin 4\theta d\theta$$

$$= \frac{6}{\pi}\left|\frac{-\cos 4\theta}{4}\right|_0^{4\pi/3} = -\frac{3}{2\pi}\left(\frac{\cos 16\pi}{3} - \cos 0\right) = \frac{-3}{2\pi}(-0.5-1) = \frac{9}{4\pi}$$

(b) Frequency of the fourth harmonic = $4f_0 = 4/T = 4/0.02$ = 200 Hz.

Example 21.3. *Find the Fourier series of the "half sinusoidal" voltage waveform which represents the output of a half-wave rectifier. Sketch its line spectrum.*

Solution. As seen from Fig. 21.14 (*a*), T = 0.2 second, $f_0 = 1/T = 1/0.2$ = 5 Hz and $\omega_0 = 2 \quad f_0 = 10\pi$ rad/s. Moreover, the function has even symmetry. Hence, the Fourier series will contain no sine terms because $b_n = 0$.

The limits of integration would not be taken from t = 0 to t = 0.2 second, but from $t = -0.5$ to t = 0.15 second in order to get fewer equations and hence fewer integrals. The function can be written as

$$f(t) = V_m \cos 10\pi t \qquad -0.05 < t < 0.05$$

$$= 0 \qquad 0.05 < t < 0.15$$

$$a_0 = \frac{1}{T}\int_{-0.05}^{0.15} f(t)dt = \frac{1}{0.2}\left[\int_{-0.05}^{0.05} V_m \cos 10\pi t dt + \int_{0.05}^{0.15} 0.dt\right]$$

$$= \frac{V_m}{0.2}\left|\frac{\sin 10\pi t}{10\pi}\right|_{-0.05}^{0.05} = \frac{V_m}{\pi} \quad a_n = \frac{2V_m}{0.2}\int_{-0.05}^{0.05} \cos 10\pi t.\cos 10\pi n t\, dt$$

The expression we obtain after integration cannot be solved when n = 1 although it can be solved when n is other than unity. For n = 1, we have

$$a_1 = 10V_m\int_{-0.05}^{0.05} \cos^2 10\pi t\, dt = \frac{V_m}{2}$$

When $n \neq 1$, then $a_n = 10V_m\int_{-0.05}^{0.05} \cos 10\pi t.\cos 10\pi n t dt$

$$= \frac{10V_m}{2}\int_{-0.05}^{0.05}[\cos 10\pi(1+n)t\cos 10\pi(1-n)t]dt = \frac{2V_m}{\pi}.\frac{\cos(\pi n/2)}{(1-n^2)} \quad \ldots n \neq 1$$

$$a_2 = \frac{2V_m}{\pi}.\frac{\cos\pi}{1-4} = \frac{2V_m}{\pi}.\frac{-1}{-3} = \frac{2V_m}{2\pi}; a_3 = \frac{2V_m}{\pi}.\frac{\cos 3\pi/2}{1-3^2} = 0; a_4 = \frac{2V_m}{\pi}.\frac{\cos 2\pi}{1-4^2} = -\frac{2V_m}{15\pi}$$

$$a_5 = 0; a_6 = \frac{2V_m}{\pi}.\frac{\cos 3\pi}{1-6^2} = \frac{2V_m}{35\pi} \quad \text{and so on}$$

Substituting the values of a_0, a_1, a_2, a_4 etc. in the standard Fourier series expression given in Art. 20.3. we have

$$f(t) \quad a_0 \quad a_1\cos 2 \;_0t \quad a_2\cos_2 \;_0t \quad a_4\cos 4 \;_0t \quad a_6\cos 6 \;_0t \quad \ldots.$$

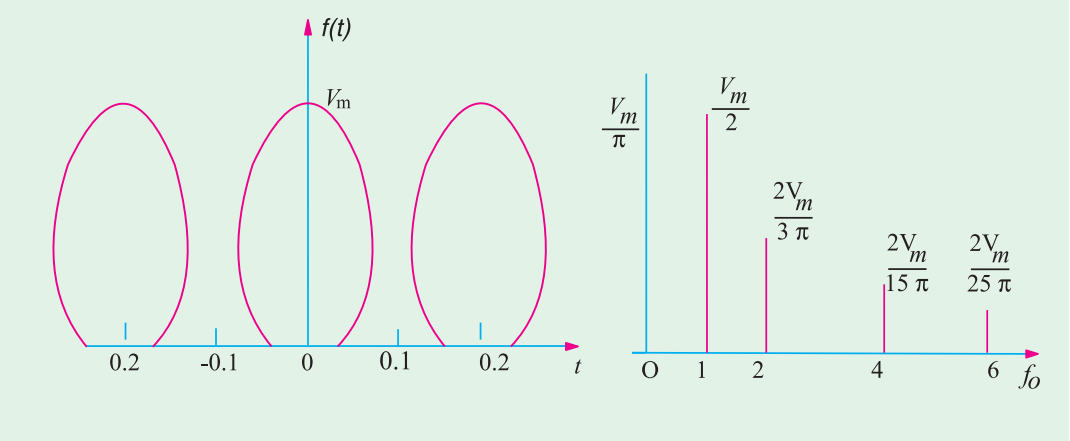


Fig. 21.14

$$= \frac{V_m}{\pi} + \frac{V_m}{2}\cos 10\pi t + \frac{2V_m}{3\pi}\cos 20\pi t - \frac{2V_m}{15\pi}\cos 40\pi t + \frac{2V_m}{35}\cos 60\pi t....$$

$$= V_m\left(\frac{1}{\pi} + \frac{1}{2}\cos\omega_0 t + \frac{2}{3\pi}\cos 2\omega_0 t\pi - \frac{2}{15\pi}\cos 4\omega_0 t + \frac{2}{35\pi}\cos 6\omega_0 t...\right)$$

The line spectrum which is a plot of the harmonic amplitudes versus frequency is given in Fig. 21.14(*b*).

Example 21.4. *Determine the Fourier series for the square voltage pulse shown in Fig. 21.15 (a) and plot its line spectrum.* **(Network Theory, Nagpur Univ. 1992)**

Solution. The wave represents a periodic function of θ or ωt or $\left(\frac{2\pi t}{T}\right)$ having a period extending over 2π radians or T seconds. The general expression for this wave can be written as

$$f(\theta) = a_0 + a_1\cos\theta + a_2\cos 2\theta + a_3\cos 3\theta + ...$$

$$... + b_1\sin\theta + b_2\sin 2\theta + b_3\sin 3\theta + ...$$

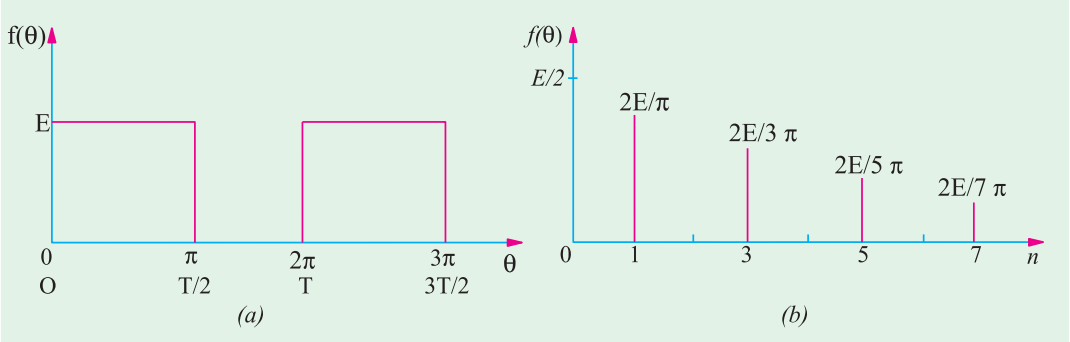


Fig. 21.15

(*i*) Now, $f(\theta) = V;\ \theta = 0 \text{ to } \theta = \pi;\ f(\theta) = 0,\ \text{from } \theta = \pi \text{ to } \theta = 2\pi$

$$\therefore \qquad a_0 = \frac{1}{2\pi}\int_0^{2\pi}(\theta)d\theta = \frac{1}{2\pi}\left\{\int_0^{\pi} f(\theta)d\theta + \int_{\pi}^{2\pi} f(\theta)d\theta\right\}$$

or $$a_0 = \frac{1}{2\pi}\left\{\int_0^{\pi} V d\theta + \int_{\pi}^{2\pi} 0 d\theta\right\} = \frac{V}{2\pi}|\theta|_0^{\pi} + 0 = V_{2\pi} \times \pi = \frac{V}{2}$$

(ii) $$a_n = \frac{1}{\pi}\int_0^{2\pi} f(\theta)\cos n\theta d\theta = \frac{1}{\pi}\left\{\int_0^{\pi} V\cos n\theta d\theta + \int_{\pi}^{2\pi} 0 \times \cos n\theta d\theta\right\}$$

$$= \frac{V}{\pi}\int_0^{\pi} \cos n\theta d\theta + 0 = \frac{V}{n\pi}|\sin n\theta|_0^{\pi} = 0 \quad \text{... whether } n \text{ is odd or even}$$

(iii) $$b_n = \frac{1}{\pi}\int_0^{2\pi} f(\theta)\sin n\theta d\theta = \frac{1}{\pi}\left\{\int_0^{\pi} V\sin n\theta d\theta + \int_{\pi}^{2\pi} 0 \times \sin n\theta d\theta\right\}$$

$$= \frac{V}{\pi}\int_0^{\pi} \sin n\theta d\theta + 0 = \frac{V}{\pi}\left|\frac{-\cos n\theta}{n}\right|_0^{\pi} = \frac{V}{n\pi}(-\cos n\pi + 1)$$

Now, when n is odd, $(1 - \cos n\pi) = 2$ but when n is even, $(1 - \cos n\pi) = 0$.

$$\therefore \quad b_1 = \frac{2V}{\pi} \ldots n = 1; \; b_2 = \frac{V}{2\pi} \times 0 = 0 \quad \ldots n = 2; \; b_3 = \frac{V}{3\pi} \times 2 = \frac{2V}{3\pi} \ldots n = 3 \text{ and so on.}$$

Hence, substituting the values of a_0, a_1, a_2 etc. and b_1, b_2 etc. in the above given Fourier series, we get

$$f(\theta) = \frac{V}{2} + \frac{2V}{\pi}\sin\theta + \frac{2V}{3\pi}\sin 3\theta + \frac{2V}{5\pi}\sin 5\theta + \ldots = \frac{E}{2} + \frac{2V}{\pi}\left(\sin\omega_0 t + \frac{1}{3}\sin 3\omega_0 t + \frac{1}{5}\sin 5\omega_0 t + \ldots\right)$$

It is seen that the Fourier series contains a constant term $V/2$ and odd harmonic components whose amplitudes are as under:

Amplitude of fundamental or first harmonic = $\frac{2V}{\pi}$

Amplitude of second harmonic = $\frac{2V}{3\pi}$

Amplitude of third harmonic = $\frac{2V}{5}$ and so on.

The plot of harmonic amplitude versus the harmonic frequencies (called line spectrum) is shown in Fig. 21.15 (*b*).

Example 21.5. *Obtain the Fourier series for the square wave pulse train indicated in Fig. 21.16.* **(Network Theory and Design, AMIETE June, 1990)**

Solution. Here $T = 2$ second, $\omega_0 = 2\pi/T = \pi$ rad/s. The given function is defined by

$f(t) = 1 \quad 0 < t < 1 = 0$ and

$= 0; \quad 1 < t < 2$

$$a_0 = \frac{1}{T}\int_0^T f(t)dt = \frac{1}{2}\int_0^2 1.dt = \frac{1}{2}\left[\int_0^1 1.dt + \int_1^2 0.dt\right] = \frac{1}{2}$$

Even otherwise by inspection $a_0 = (1 + 0)/2 = 1/2$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega_0 t dt = \frac{2}{2}\int_0^2 1.\cos n\pi t dt = \left[\int_0^1 1.\cos n\pi t dt + \int_1^2 (0).\cos n\pi t dt\right]$$

$$= \int_0^1 \cos n\pi t dt = \left| \frac{\sin n\pi t}{n\pi} \right|_0^1 = 0$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega_0 t dt = [\int_0^1 1.\sin n\pi t\, dt] + \int_1^2 (0)\sin n\pi t dt] = \int_0^1 \sin n\pi t\, dt$$

$$= \left| \frac{-\cos n\pi t}{n\pi} \right|_0^1 = \frac{1-\cos n\pi}{n\pi}$$

$\therefore$ $b_n = 2/n\pi$ ..... when n is odd; = 0

... when n is even

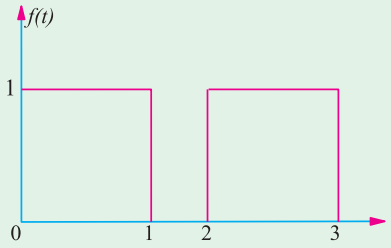


Fig. 21.16

$\therefore$ $f(t)$ a_0 $\frac{2}{}$ $\frac{1}{n}\sin 3n$ t $\frac{1}{2}$ $\frac{2}{}$ $\sin$ t $\frac{1}{3}\sin 3$ t $\frac{1}{5}\sin 5$ t, *etc.*

n 1
odd

Example 21.6. *Find the trigonometric Fourier series for the square voltage waveform shown in Fig. 21.17(a) and sketch the line spectrum.*

Solution. The function shown in Fig. 21.17 (*a*) is an odd function because at any time $f(-t) = -f(t)$. Hence, its Fourier series will contain only sine terms *i.e.* $a_n = 0$. The function also possesses half-wave symmetry, hence, it will contain only odd harmonics.

As seen from Art. 21.7 (2) the Fourier series for the above wave is given by

$$f(\theta) = \sum_{\substack{n=1 \\ odd}}^{\infty} b_n \sin n\theta t \quad \text{where} \quad b_n = \frac{1}{\pi}\int_0^{2\pi} f(\theta)\sin n\theta d\theta$$

$$= \frac{1}{\pi}\left\{\int_0^{\pi} V\sin n\theta d\theta + \int_{\pi}^{2\pi} -V\sin n\theta d\theta\right\} = \frac{V}{\pi n}\left|-\cos n\theta\right|_0^{\pi} + \frac{V}{\pi n}\left|\cos n\theta\right|_{\pi}^{2\pi}$$

$$= \frac{V}{\pi n}\{(-\cos n\pi + \cos 0) - \cos \pi n\}$$

$$= \frac{V}{\pi n}\{(1-\cos n\pi) + (1-\cos n\pi)\} = \frac{2V}{\pi n}(1-\cos n\pi)$$

Now, $1 - \cos n\pi = 2$ when n is odd

and $= 0$ when n is even

$\therefore \quad b_1 = \frac{2V}{\pi} \times 2 = \frac{4V}{\pi}$ ... putting $n = 1$; $b_2 = 0$ ... putting $n = 2$

$b_3 = \frac{2V}{\pi 3} \times 2 = \frac{4V}{3\pi}$ ... putting $n = 3$; $b_4 = 0$... putting $n = 4$

$b_5 = \frac{2V}{\pi.5} \times 2 = \frac{4V}{5\pi}$ .. putting $n = 5$ and so on.

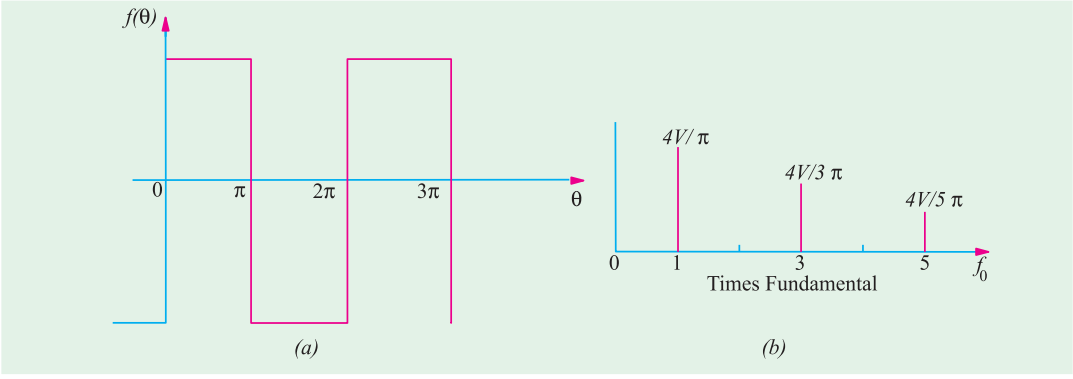


Fig. 21.17

Hence, the Fourier series for the given waveform is

$$f(\ ) \quad \frac{4V}{}\sin \quad \frac{4V}{3}\sin 3 \quad \frac{4V}{5}\sin 5 \quad ...$$

$$= \frac{4V}{\pi}\left(\sin\omega_0 t + \frac{1}{3}3\omega_0 t + \frac{1}{5}\sin 5\omega_0 t + ...\right) = \frac{4V}{\pi}\left(\sin\frac{2\pi}{T}t + \frac{1}{3}\sin\frac{6\pi}{T}t + \frac{1}{5}\sin\frac{10\pi}{T}t + ...\right)$$

The line spectrum of the function is shown in Fig. 21.17 (*b*). It would be seen that the harmonic amplitudes decrease as 1/*n*, that is, the third harmonic amplitude is 1/3 as large as the fundamental, the fifth harmonic is 1/5 as large and so on.

Example 21.7. *Determine the Fourier series for the square voltage waveform shown in Fig. 21.17 (a). Plot its line spectrum.*

Solution. This is the same question as given in Ex. 21.6 but has been repeated to illustrated a singhtly different technique. As seen from Fig. 21.17(*a*) $T = 2\pi$, hence, $\omega_0 = 2\pi f_0 = 2\pi / T = 2\pi / \pi = 1$. Over one period the function can be defined as

$$f(t) = V \quad 0 < t < 5$$

$$= -V, \ \pi < t < 2\pi$$

$$b_n \quad \frac{1}{} \int_0^{2} \quad f(t)\sin n \ _0 t dt \quad \frac{1}{} \int_0^{2} \quad f(t)\sin nt dt \quad \frac{1}{} \int_0 \quad f(t)\sin nt dt \quad \frac{1}{} \int \quad f(t)\sin nt dt$$

$$= \frac{1}{\pi}\int_0^{\pi} V\sin nt dt + \frac{1}{\pi}\int_{\pi}^{2\pi}(-V)\sin nt dt = \frac{V}{\pi}\left|\frac{-\cos nt}{n}\right|_0^{\pi} + \frac{V}{\pi}\left|\frac{\cos nt}{n}\right|_{\pi}^{2\pi}$$

$$= -\frac{V}{n\pi}(\cos n\pi - \cos 0) + \frac{V}{n\pi}(\cos 2n\pi - \cos n\pi)$$

Since cos 0 is 1 and cos $2n\pi = 1$ $\therefore$ $b_n = \frac{2V}{n\pi}(1 - \cos n\pi)$

When n is even, $\cos n\pi = 1$ $\therefore$ $b_n = 0$

When n is odd, cos $n\pi = -1$ $\therefore$ b_n $\frac{2V}{n}(1 \quad 1)$ $\frac{4V}{n}, n \quad 1,3,5...$

Substituting the value of b_n, the Fourier series become

$$f(t) = \sum_{\substack{n=1 \\ odd}}^{\infty} \frac{4V}{n\pi} \sin nt = \frac{4V}{\pi} \sum_{\substack{n=1 \\ odd}}^{\infty} \frac{1}{n} \sin nt = \frac{4V}{\pi}\left(\sin t + \frac{1}{3}\sin 3t + \frac{1}{5}\sin 5t + ...\right)$$

Since $\omega_0 = 1$, the above expression in general terms becomes

$$f(t) = \frac{4V}{\pi}\left(\sin \omega_0 t + \frac{1}{3}\sin 3\omega_0 t + \frac{1}{5}\sin 5\omega_0 t + ...\right)$$

The line spectrum is as shown in Fig. 21.17 (*b*).

Example 21.8. *Determine the Fourier series of the square voltage waveform shown in Fig. 21.18.*

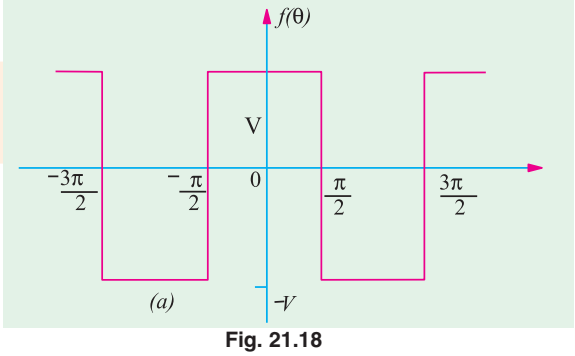


Fig. 21.18

Solution. As compared to Fig. 21.17 (*a*) given above, the vertical axis of figure has been shifted by $\pi/2$ radians. Replacing t by $(t + \pi/2)$ in the above equation, the Fourier series of the waveform shown in Fig. 21.18 becomes

$$f(t) = \frac{4V}{\pi}\left[\sin\left(t + \frac{\pi}{2}\right) + \frac{1}{3}\sin 3\left(t + \frac{\pi}{2}\right) + \frac{1}{5}\sin 5\left(t + \frac{\pi}{2}\right) + ...\right]$$

$$= \frac{4V}{\pi}\left(\cos t - \frac{1}{3}\cos 3t + \frac{1}{5}\cos 5t ...\right)$$

Example 21.9. *Determine the trigonometric Fourier series for the half-wave rectified sine wave form shown in Fig. 21.19 (a) and sketches line spectrum.*

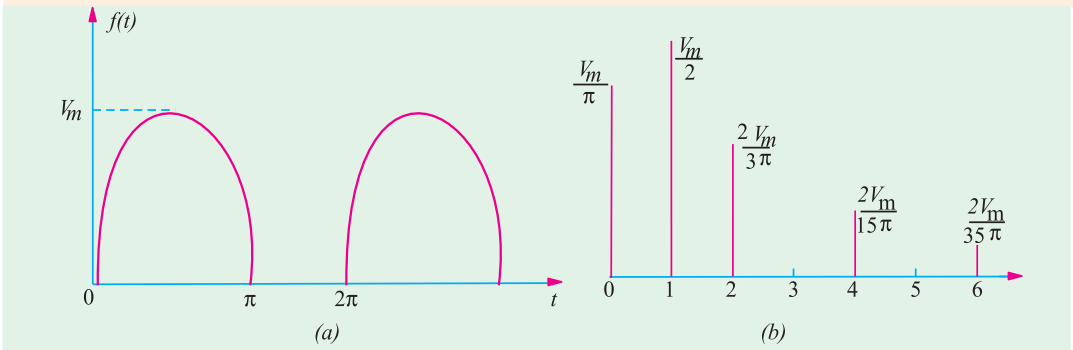


Fig. 21.19

Solution. The given waveform shows no symmetry, hence its series would contain both sine and cosine terms. Moreover, its average value is not obtainable by inpsection, hence it will have to be found by integration.

Here, $T \quad 2 \quad _0 \quad 2 \ / T \quad 1$. Hence, equation of the given waveform is $V = V_m \sin \omega t = V_m \sin t$.

The given waveform is defined by $f(t) = V_m \sin t, 0 < t < \pi = 0,\ \pi < t < 2\pi$

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} f(t)dt = \frac{1}{2\pi}\left[\int_0^{\pi} V_m \sin t dt + \int_{\pi}^{2\pi} dt = \frac{1}{2\pi}\int_0^{\pi} V_m \sin t dt\right]$$

$$= \frac{V_m}{2\pi}|-\cos t|_0^{\pi} = -\frac{V_m}{2\pi}(\cos\pi - \cos 0) = \frac{V_m}{\pi}$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(t)\cos nt dt = \frac{1}{\pi}\int_0^{\pi} V_m \sin t \cos nt dt$$

$$= \frac{V_m}{2\pi}\int_0^{\pi} [\sin(n+1)t - \sin(n-1)t]dt = \frac{V_m}{2\pi}\left|\frac{-\cos(n+1)t}{n+1} + \frac{\cos(n-1)t}{n-1}\right|_0^{\pi}$$

$$= \frac{V_m}{2\pi}\left[-\frac{\cos(n+1)\pi}{n+1} + \frac{\cos(n-1)\pi}{n-1} + \frac{1}{n+1} - \frac{1}{n-1}\right]$$

when n is even, $\cos\ (n+1)\pi = -1$ and $\cos(n-1)\pi = -1$

$$\therefore \quad a_n = \frac{V_m}{2\pi}\left[\frac{1}{n+1} - \frac{1}{n-1} + \frac{1}{n+1} - \frac{1}{n-1}\right] = -\frac{2V_m}{\pi(n^2-1)} \qquad \text{... } n = 2, 4, 6 \text{ etc.}$$

when n is odd and $\neq$ 1, $\cos\ (n+1)\pi\ = 1$ and $\cos\ (n-1)\pi\ = 1$

$$\therefore \quad a_n = \frac{V_m}{2\pi}\left(-\frac{1}{n+1} + \frac{1}{n-1} + \frac{1}{n+1} - \frac{1}{n-1}\right) = 0 \qquad \text{... } n = 3, 5, 7 \text{ etc.}$$

when $n \quad 1, a_1 \quad \frac{1}{\quad _0} \quad V_m \sin t.\cos t dt \quad \frac{V_m}{\quad _0} \quad \sin t \cos t dt \quad \frac{V_m}{2} \quad _0 \sin 2t dt \quad 0$

Hence, $a_n = 0$ ...n = 1, 3, 5...

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(t)\sin nt dt = \frac{1}{\pi}\left[\int_0^{\pi} V_m \sin t \sin nt dt + \int_{\pi}^{2\pi}(0)\sin nt dt\right]$$

$$= \frac{V_m}{\pi}\int_0^{\pi} \sin t \sin nt dt = 0 \text{ for } n = 2, 3, 4, 5 \text{ etc.}$$

However, the expression indeterminate for n = 1 so that b_1 has to be evaluated separately.

$$b_1 = \frac{1}{\pi}\int_0^{\pi} V_m \sin t.\sin t dt = \frac{V_m}{\pi}\int_0^{\pi} \sin^2 t dt = \frac{V_m}{2}$$

The required Fourier series for the half-wave rectified voltage waveform is

$$f(t) = \frac{V_m}{\pi} + \frac{V_m}{2}\sin t - \frac{2V_m}{\pi}\sum_{\substack{n=1 \\ even}}^{\infty}\left(\frac{\cos nt}{n^2-1}\right)$$

$$= \frac{V_m}{\pi}\left(1 + \frac{\pi}{2}\sin t - \frac{2}{3}\cos 2t - \frac{2}{15}\cos 4t - \frac{2}{35}\cos 6t...\right)$$

$$= \frac{V_m}{\pi}\left(1+\frac{\pi}{2}\sin\omega_0 t-\frac{2}{3}\cos 2\omega_0 t-\frac{2}{15}\cos\omega_0 t-\frac{2}{35}\cos 6\omega_0 t-\ldots\right)$$

$$= \frac{V_m}{T}\left(1+\frac{\pi}{2}\sin\theta-\frac{2}{3}\cos 2\theta-\frac{2}{15}\cos 4\theta-\frac{2}{35}\cos 6\theta-\ldots\right)$$

The line spectrum is shown in Fig. 21.19 (*b*) which has a strong fundamental term with rapidly decreasing amplitudes of the higher harmonics.

Example 21.10. *Find the trigonometrical Fourier series for the full wave rectified voltage sine wave shown in Fig. 21.20.*

Solution. Since the given function has even symmetry, $b_n = 0$ *i.e.* it will contain no sine terms in its series.

The equation of the sinusoidal sine wave given by $V = V_m \sin\theta$. In other words, $f(\theta) = V_m = \sin\theta$.

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} f(\theta)d\theta = \frac{1}{2\pi}\int_0^{2\pi} V_m \sin\theta d\theta = \frac{2V_m}{2\pi}\int_0^{\pi}\sin\theta d\theta$$

It is so because the two parts $0-\pi$ and $\pi-2$ are identical.

$$\therefore \quad a_0 = \frac{V_m}{\pi}|-\cos\theta|_0^{\pi} = \frac{2V_m}{\pi}$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(\theta)\cos n\theta d\theta = \frac{2V_m}{\pi}\int_0^{\pi}\sin\theta\cos n\theta d\theta$$

Now, $\sin A\ \cos B = \frac{1}{2}[\sin(A+B)+\sin(A-B)]$

$$\therefore \quad a_n = \frac{2V_m}{\pi}\int_0^{\pi}[\sin(1+n)\theta+\sin(1-n)\theta]d\theta$$

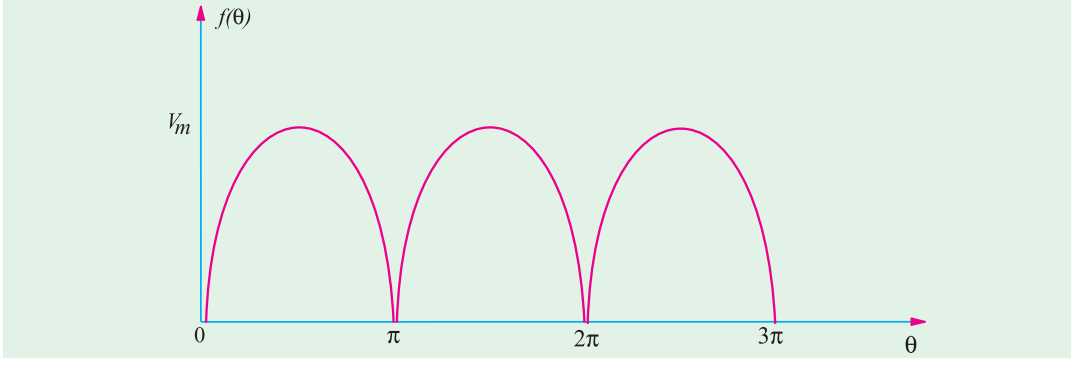


Fig. 21.20

$$= -\frac{V_m}{\pi}\frac{\cos(1+n)\theta}{(1+n)}+\frac{\cos(1-n)\theta}{(1-n)}$$

$$= 0 = \frac{-V_m}{\pi}\left[\frac{\cos(1+n)\pi}{1+n}+\frac{\cos(1-n)\pi}{1-n}-\frac{1}{1+n}-\frac{1}{1-n}\right] \text{ ... when } n \text{ is odd}$$

However, when n is even, then

$$a_n \quad \frac{V_m}{} \quad \frac{1}{1 \quad n} \quad \frac{1}{1 \quad n} \quad \frac{1}{1 \quad n} \quad \frac{1}{1 \quad n} \quad \frac{2V_m}{} \quad \frac{1}{1 \quad n} \quad \frac{1}{1 \quad n} \quad \frac{4V_m}{(n^2 \quad 1)}$$

$$\therefore \quad f(\theta) = a_0 - \frac{4V_m}{\pi} \sum_{\substack{n=2 \\ even}}^{\infty} \frac{\cos 2\theta}{(n^2 - 1)}$$

$$\therefore \quad f(\theta) = \frac{2V_m}{\pi} - \frac{4V_m}{\pi}\left(\frac{1}{3}\cos^2\theta - \frac{1}{15}\cos 4\theta + \frac{1}{35}\cos 6\theta + \ldots\right)$$

Example 21.11. *Determine the Fourier series for the waveform shown in Fig. 21.21 (a) and sketch its line spectrum.*

Solution. Its is seen from Fig. 21.21 (*a*) that the waveform equation is $f(\theta) = (V_m / \pi)\theta$. The given function $f(\theta)$ is defined by

$$f(\ ) \quad \frac{V_m}{} \quad , \quad 0$$

$$= 0, \qquad 2$$

Since the function possesses neither even nor odd symmetry, it will contain both sine and cosine terms.

Average value of the wave over one cycle is $V_m/4$ or $a_0 = V_m/4$. It is so because the average value over the first half cycle is $V_m/2$ and over the second half cycle is 0 hence, the average value for full cycle is $= \dfrac{(V_m / 2) + 0}{2} = \dfrac{V_m}{4}$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(\theta) n\theta d\theta = \frac{1}{\pi}\left[\int_0^{\pi} (V_m / \pi)\theta \cos n\theta d\theta + \int_{\pi}^{2\pi} (0)\cos n\theta d\theta\right]$$

$$= \frac{V_m}{\pi^2}\int_0^{\pi} \theta \cos n\theta d\theta = \frac{V_m}{\pi^2}\left|\frac{\cos n\theta}{n^2} + \frac{\theta}{n}\sin n\theta\right|_0^{\pi} = \frac{V_m}{\pi^2 n^2}(\cos n\pi - 1)$$

$\therefore$ $a_n = 0$ when n is odd because $\cos n\pi - 1 = 0$

$= -2V_m / \pi^2 n^2$ when n is even

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(\theta)\sin\theta d\theta = \frac{1}{\pi}\int_0^{\pi} (V_m / \pi)\theta \sin n\theta d\theta + \int_{\pi}^{2\pi} (0)\sin n\theta d\theta$$

$$= \frac{1}{\pi}\int_0^{\pi}\left(\frac{V_m}{\pi}\right)\theta \sin n\theta d\theta = \frac{V_m}{\pi^2}\left|\frac{\sin n\theta}{n^2} - \frac{\theta}{n}\cos n\theta\right|_0^{\pi} = \frac{-V_m}{\pi n}\cos n\pi$$

$\therefore$ $b_n = -V_m | \pi n when n is even b_n = +V_m | \pi_n$ when n is odd

Substituting the values of various constants in the general expression for Fourier series, we get

$$f(\theta) = \frac{V_m}{4} - \frac{2V_m}{\pi^2}\cos\theta - \frac{2V_m}{(3\pi)^2}\cos 3\theta - \frac{2V_m}{(5\pi)^2}\cos 5\theta \ldots$$

$$+ \frac{V_m}{\pi}\sin\theta - \frac{V_m}{2\pi}\sin 2\theta + \frac{V_m}{3\pi}\sin 3\theta \ldots.$$

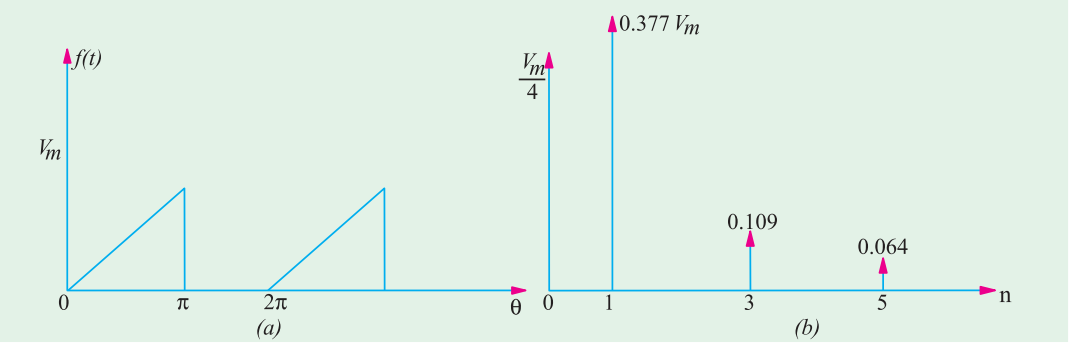


Fig. 21.21

The amplitudes of even harmonics are given directly by b_n but amplitudes of odd harmonics are given by $A_n = \sqrt{a_n^2 + b_n^2}$ (Art. 21.4)

For example,

$$A_1 = \sqrt{(2V_m/\pi^2)^2 + (V_m/\pi)^2} = 0.377\ V_m$$

$$A_3 = \sqrt{\left(\frac{2V_m}{(3\pi)^2}\right)^2 + \left(\frac{V_m}{2\pi}\right)^2} = 0.109\ V_m$$

$$A_5 = \left(\frac{2V_m}{(5\pi)^2}\right)^2 + \left(\frac{V_m}{5\pi}\right)^2 = 0.064\,V_m \text{ and so on.}$$

The line spectrum is as shown in Fig. 21.21 (*b*).

Example 21.12. *Find the Fourier series for the sawtooth waveform shown in Fig. 21.22 (a). Sketch its line spectrum.*

Solution. Using by the relation $y = mx$, the equation of the function becomes $f(t) = 1, t$ or $f(t) = t$.

$$T = 2, \omega_0 = 2\pi/t = 2\pi/2 = \pi$$

By inspection it is clear that $a_0 = 2/2 = 1$, $a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega_0 t = \int_0^2 t.\cos n\pi t.dt$

Since we have to find the integral of two functions, we use the technique of integration by parts *i.e.*

$$\int uvdx = u\int vdx - \int\left(\frac{du}{dx}\int vdx\right)dx$$

$$\therefore \qquad a_n = t.\int_0^2 \cos n\pi t dt - \int_0^2 \left(1.\int_0^2 \cos n\pi t dt\right) dt$$

$$= \left|\frac{t}{n\pi}\sin n\pi t\right|_0^2 + \left|\frac{\cos n\pi t}{(n\pi)^2}\right|_0^2 = 0 + \frac{1}{(n\pi)^2}(\cos 2n\pi - \cos 0)$$

Since $\cos 2n\pi = \cos 0$ for all values of n, hence $a_n = 0$

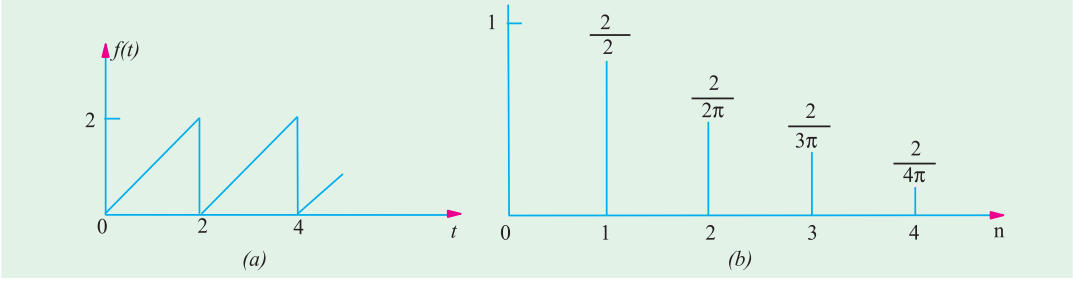


Fig. 21.22

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega_0 t dt = \int_0^T t\sin n\pi t dt$$

Employing integration by parts, we get,

$$b_n = t\int_0^2 \sin n\pi t\, dt - \int_0^2 \left(1.\int_0^2 \sin n\pi t dt\right) dt$$

$$= \left|t.\frac{-\cos n\pi t}{n\pi}\right|_0^2 dt - \int_0^2 \frac{-\cos n\pi t}{n\pi} = \left|\frac{-t}{n\pi}.\cos n\pi t\right|_0^2 + \left|\frac{\sin n\pi t}{(n\pi)^2}\right|_0^2 = \frac{\sin 2n\pi}{(n\pi)^2} - \frac{2}{n\pi}\cos 2n\pi$$

The sine term is 0 for all values of n because sign of any multiple of 2π is 0. Since value of cosine term is 1 for any multiple of 2π, we have, $b_n = -2/n\pi$.

$$\therefore \qquad f(t) = a_0 + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t = a_0 - \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{1}{n}\sin n\pi t$$

$$= 1 - \frac{2}{\pi}\left(\sin \pi t + \frac{1}{2}\sin 2\pi t + \frac{1}{3}\sin 3\pi t + \ldots\right)$$

The line spectrum showing the amplitudes of various harmonics is shown in Fig. 21.22 (*b*).

Example 21.13. *Determine the trigonometric Fourier series of the triangular waveform shown in Fig. 21.23.*

Solution. Since the waveform possesses odd symmetry, hence $a_0 = 0$ and $a_n = 0$ *i.e.* there would be n_o cosine terms in the series. Moreover, the waveform has half-wave symmetry. Hence, series will have only odd harmonics. In the present case, there would be only odd sine terms. Since the waveform possesses quarter-wave symmetry, it is necessary to integrate over only one quarter period of finding the Fourier coefficients.

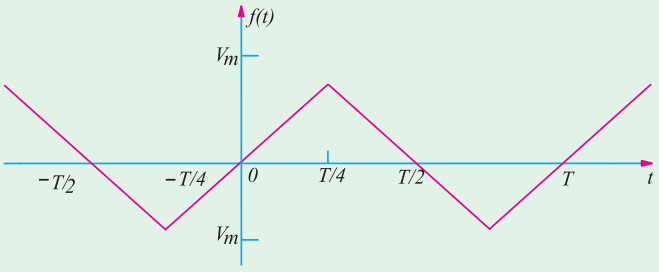


Fig. 21.23

$$f(t) = \sum_{\substack{n=1 \\ odd}}^{\infty} b_n \sin n\omega_0 t$$

where $b_n = \frac{8}{T}\int_0^{t/4} f(t)\sin n\omega_0 t$

The quarter-wave of the given waveform can be represented by equation of a straight line.

Slope of the straight line is $V_m/(T/40)$ $4V_m/T$.

Hence, using $Y = mx$, we have

$$f(t) = \left(\frac{4V_m}{T}\right)t 0 < t < T/4 \quad \therefore \quad b_n = \frac{8}{T}\int_0^{T/4}\left(\frac{4V_m}{T}\right)t.\sin n\omega_0 t = \frac{32V_m}{T^2}\int_0^{T/4} t.\sin n\omega_0 t dt$$

Using the theorem of integration by parts, we have

$$b_n = \frac{32V_m}{T^2}\left[t\int_0^{t/4}\sin n\omega_0 t\, dt - \int_0^{T/4}\left(1.\int_0^{T/4}\sin n\omega_0 t\, dt\right)dt\right]$$

$$\frac{32V_m}{T^2} \quad \frac{t}{4} \quad \frac{\cos n \quad {}_0 T/4}{n \quad {}_0} \quad \left|\frac{\sin n \quad {}_0 t}{(n \quad {}_0)^2}\right|^{T/4} \quad \frac{32V_m}{T^2} \quad \frac{T}{4}\frac{\cos n \quad {}_0 T/4}{n \quad {}_0} \quad \frac{\sin n \quad {}_0 T/4}{(n \quad {}_0)^2}$$

Now, $\omega_0 = 2\pi/T$ or $\omega_0 T = 2\pi$ $\therefore n\omega_0 T/4 = n\pi/2$

$\therefore$ $\cos n\omega_0 T/4 = \cos n\pi/2 = 0$ when n is odd

$$\therefore \quad b_n = \frac{32V_m}{n^2\omega_0^2 T^2}\sin n\omega_0 \frac{T}{4} = \frac{32V_m}{n^2(2\pi)^2}\sin\frac{n\pi}{2} = \frac{8V_m}{n^2\pi^2}\sin\frac{n\pi}{2}$$

$$\therefore \quad b_n = \frac{8V_m}{n^2\pi^2}....n = 1,5,9,13....b_n = \frac{-8V_m}{n^2\pi^2}....n = 3,7,11,15,...$$

Substituting this value of b_n, the Fourier series for the given waveform becomes

$$f(t) \quad \frac{8V_m}{2} \quad \sin \quad {}_0 t \quad \frac{1}{3^2}3 \quad {}_0 t \quad \frac{1}{5^2}\sin 5 \quad {}_0 t \quad \frac{1}{7^2}\sin 7 \quad {}_0 t \quad ...$$

Example 21.14. *Determine the Fourier series of the triangular waveform shown in Fig. 21.24.*

Solution. Since the function has even symmetry, $b_n = 0$. Moreover, it also has half-wave symmerty, hence, $a_0 = 0$. The Fourier series can be written as $f(t) = \sum_{n=1}^{\infty} a_n \cos n\omega_0 t$ where

$$a_n \quad \frac{2}{T} \quad {}_0^T \quad f(t)\cos n \quad {}_0 t\, dt.$$

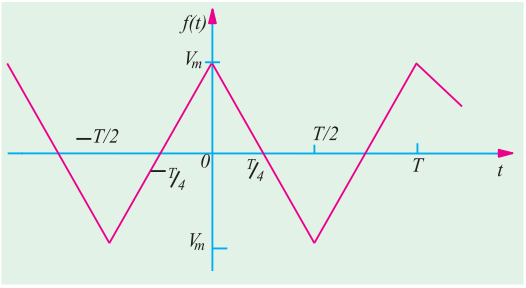


Fig. 21.24

The function is given by the relation $f(t) \quad \frac{4V_m}{T} \quad t \quad \frac{T}{4}$

It is so because for the interval $0 \le t \le T/2$, the slope of the line is $4V_m / T$.

$$\therefore \quad a_n \quad \frac{2}{T} \quad {}_0^T \quad f(t)\cos n \quad {}_0 t\, dt \quad \frac{4}{T} \quad {}_0^{T/4} \quad f(t)\cos n \quad {}_0 t\, dt$$

$$\frac{16V_m}{T^2} \quad {}_0^{T/2} \quad t \quad \frac{T}{4} \quad \cos n \quad {}_0 t\, dt \quad \frac{16V_m}{T^2} \quad {}_0^{T/2} \quad t\cos n \quad {}_0 t\, dt \quad \frac{4V_m}{T} \quad {}_0^{T/2} \quad \cos n \quad {}_0 t\, dt$$

$$= \frac{-16V_m}{T^2}\left|\frac{1}{4n^2\omega_0^2}\cdot\cos n\omega_0 t + \frac{t}{n\omega_0}\sin n\omega_0 t\right|_0^{T/2} + \frac{4V}{T}\left|\frac{\sin n\omega_0 t}{n\omega_0}\right|_0^{T/2}$$

Substituting $\omega = 2\pi / T$, we get

$$a_n \quad \frac{16V_m}{T^2} \quad \frac{T^2}{4 \quad {}^2 n^2}(\cos n \quad 1) \quad \frac{T^2}{4 \quad n}(\sin n \quad) \quad \frac{2V}{n} \quad \sin n$$

Now, $\sin n\pi = 0$ for all values of n, $\cos n\pi = 1$ when n is even add – 1 when n is odd.

$$\therefore \quad a_n = \frac{8V_m}{\pi^2 n^2}$$

$$\therefore \quad f(t) = \sum_{\substack{n=0 \\ odd}}^{\infty} \frac{8V_m}{\pi^2 n^2}\cos n\omega_0 t = \frac{8V_m}{\pi^2}\sum_{\substack{n=1 \\ odd}}^{\infty} \frac{\cos n\omega_0 t}{n^2}$$

$$= \frac{8V_m}{2} \quad \cos \quad {}_0 t \quad \frac{1}{9}\cos 3 \quad {}_0 t \quad \frac{1}{25}.\cos 5 \quad {}_0 t \quad \frac{1}{49}\cos 7 \quad {}_0 t \quad \ldots$$

Alternative Solution

We can deduce the Fourier series from Fig. of Ex. 21.11 by shifting the vertical axis by $\pi / 2$ radians to the right. Replacing t by $(t + \pi/2)$ in the Fourier series of Ex. 21.11, we get

$$f(t) \quad \frac{8V_m}{2} \quad \sin \quad {}_0 \quad t \quad \frac{}{2} \quad \frac{1}{3^2}\sin 3 \quad {}_0 \quad t \quad \frac{}{2} \quad \frac{1}{5^2}\sin 5 \quad {}_0 \quad t \quad \frac{}{2} \quad \frac{1}{7^2}\sin 7 \quad {}_0 \quad t \quad \frac{n}{2} \quad \ldots$$

$$\frac{8V_m}{2} \quad \cos \quad {}_0 t \quad \frac{1}{9}\cos 3 \quad {}_0 t \quad \frac{1}{25}\cos 5 \quad {}_0 t \quad \frac{1}{49}\cos \quad {}_0 t \quad \ldots$$

Example 21.15. *Obtain the Fourier series representation of the sawtooth waveform shown in Fig. 21.25 (a) and plot its spectrum.*

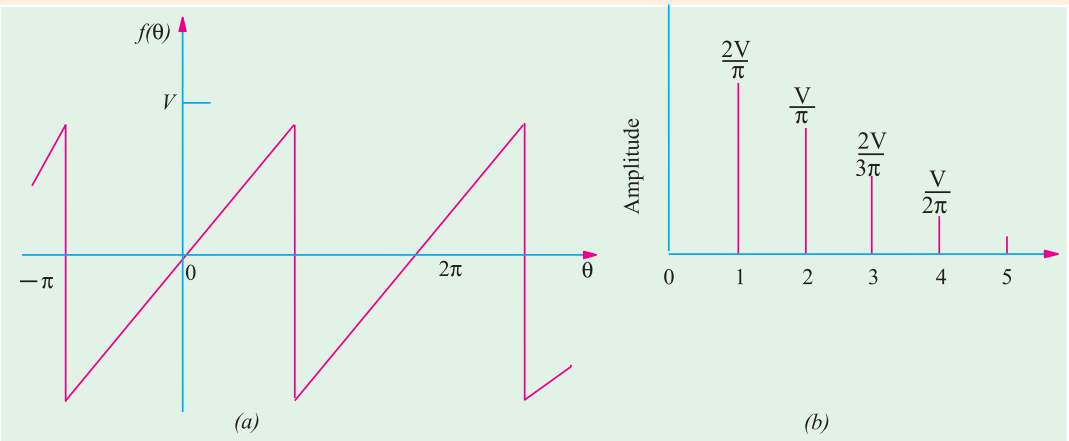


Fig. 21.25

Solution. By inspection, we know that the average value of the wave is zero over a cycle because the height of the curve below and above the X-axis is the same hence, $a_0 = 0$. Moreover, it has odd symmetry so that $a_n = 0$ *i.e.* there would be no cosine terms. The series will contain only sine terms.

$$\therefore \quad f(\quad) \qquad b_n \sin n \quad \text{where } b_n \quad \frac{1}{} \quad \int_0^{2} f(\quad)\sin n \quad d$$

$n \quad 1$

The slope of curve is $m = V/\pi$

$\therefore$ we get, $f(\theta) = (V/\pi)\,\theta.$

If we are the limit of integration form $-\pi$ to $+\pi$ then

$$b_n \frac{1}{} \qquad \frac{V}{} \quad \sin n \quad d \qquad \frac{V}{2}\left|\frac{1}{n^2}\sin n \quad \frac{}{n}\cos n\right| \qquad \frac{2V}{n}\cos n$$

The above result has been obtained by making use of integration by parts as explained earlier. $\cos n\pi$ is positive when n is even and is negative when n is odd and thus the signs of the coefficients alternate. The required Fourier series is

$$f(\quad) \quad \frac{2V}{} \quad \sin \qquad \frac{1}{2}\sin 2 \qquad \frac{1}{3}\sin 3 \qquad \frac{1}{4}\sin 4 \qquad \ldots$$

$$\text{or} \quad f(t) \quad \frac{2V}{} \quad \sin \quad {}_0t \quad \frac{1}{2}\sin 2 \quad {}_0t \quad \frac{1}{3}\sin 3 \quad {}_0t \quad \frac{1}{4}\sin 4 \quad {}_0t \quad \ldots$$

As seen, the coefficients decrease as $1/n$ so that the series converges slowly as shown by the line spectrum of Fig. 21.25 (*b*).

The amplitudes of the fundamental of first harmonic, second harmonic, third harmonic and fourth harmonic are $(2/\pi), (2V/2\pi), (2V/3\pi)$ and $(2V/4\pi)$ respectively.

Tutorial Problem. 21.1

1. Determine the Fourier series for the triangular waveform shown in Fig. 21.26 (*a*)

(Network Theory and Design, AMIETE June 1990)

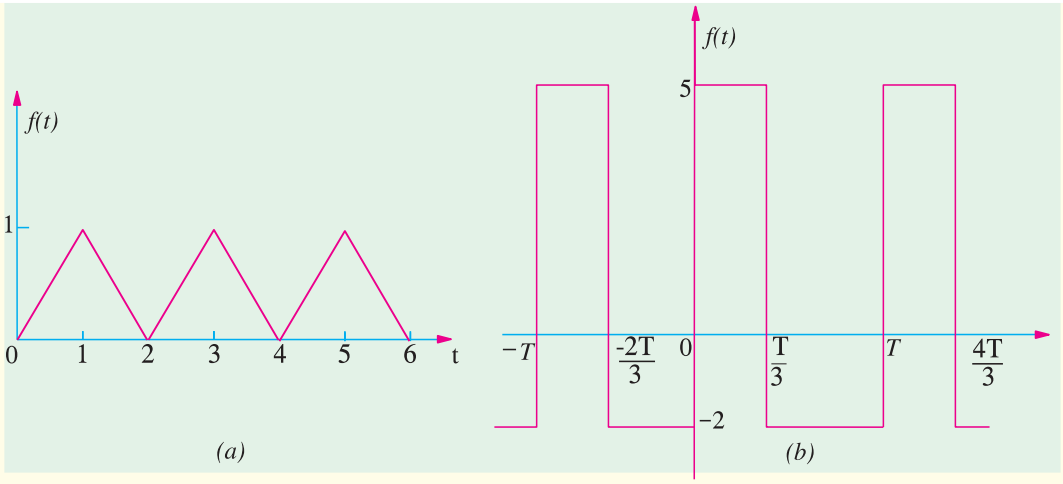


Fig. 21.26

$$\left[\frac{1}{2}-\frac{4}{\pi^2}\cos\omega_0 t+\frac{1}{3^2}\cos 3\omega_0 t+\frac{1}{5^2}\cos 5\omega_0 t+....\right]$$

2. Find the values of the Fourier coefficients a_0, a_n and b_n for the function given in fig. 21.26 (*b*).

$$\left[a_0=\frac{2}{3};a_n=\frac{7}{n\pi}\sin\frac{2\pi n}{3};b_n=\frac{7}{n\pi}\left(1-\cos\frac{2\pi n}{3}\right)\right]$$

3. Determine the trigonometric series of the triangular waveform shown in Fig. 21.27. Sketch its line spectrum.

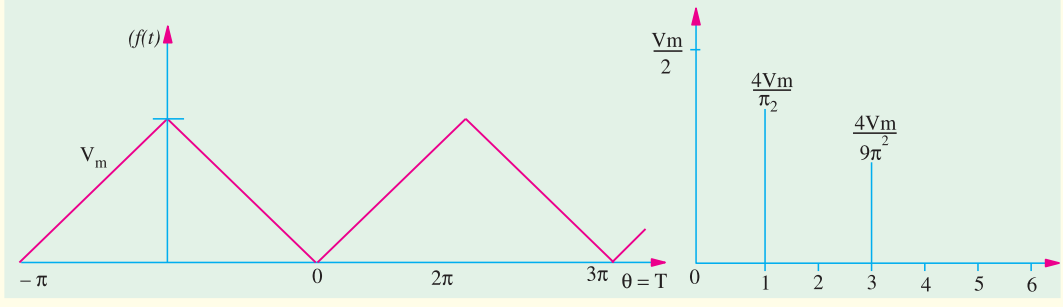


Fig. 21.27

$$\frac{V_m}{2}\quad\frac{4V_m}{\ 2}\ \cos\ {}_0t\quad\frac{1}{3^2}\cos 3\ {}_0t\quad\frac{1}{5^2}\cos 5\ {}_0t\ \ ...$$

4. Determine the Fourier series for the sawtooth waveform shown in Fig. 21.28.

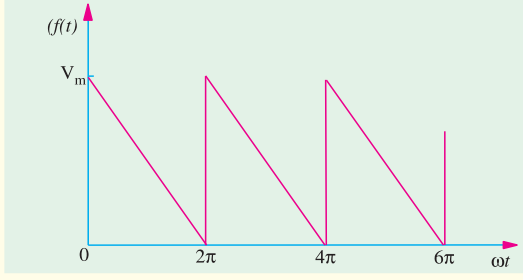


Fig. 21.28

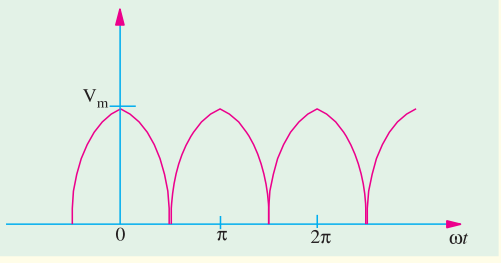


Fig. 21.29

$$\left[f(t)=\frac{V_m}{2}+\frac{V_m}{\pi}\left(\sin\omega_0 t+\frac{1}{2}\sin 2\omega_0 t+\frac{1}{3}\sin 3\omega_0 t+....\right)\right]$$

5. Represent the full-wave rectified voltage sine waveform shown in Fig. 21.29 by a Fourier series.

$$\left[f(t)=\frac{2V_m}{\pi}+\left(1+\frac{2}{3}\cos 2\omega t-\frac{2}{15}\cos 4\omega t+\frac{2}{35}\cos 6\omega t...\right)\right]$$

6. Obtain trigonometric Fourier series for the wave form shown in Fig. 21.30.

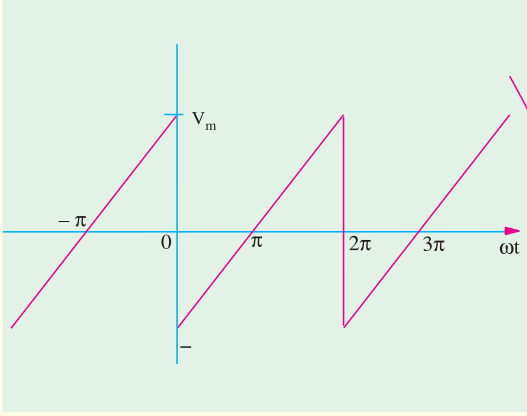


Fig. 21.30

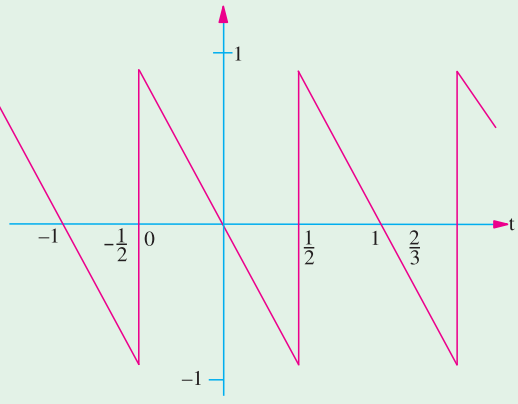


Fig. 21.31

$$\left[f(t)=-\frac{2V_m}{\pi}\left(\sin\omega_0 t+\frac{1}{2}\sin 2\omega_0 t+\frac{1}{3}\sin 3\omega_0 t+\frac{1}{4}\sin 4\omega_0 t...\right)\right]$$

7. Find the Fourier series for the sawtooth waveform shown in Fig. 21.31.

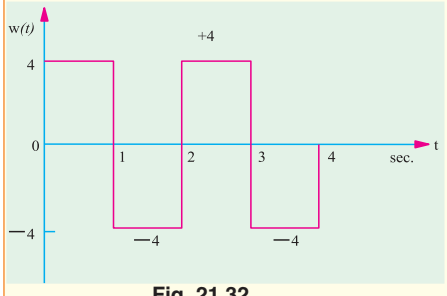


Fig. 21.32

$$\left[f(t)=\frac{2}{\pi}\left(-\sin 2\pi t+\frac{1}{2}\sin 4\pi t-\frac{1}{3}\sin 6\pi t+...\right)\right]$$

8. For the waveform of Fig. 21.32, find the Fourier series terms up to the 5th harmonic.

(Network Theory Nagpur Univ. 1993)

$$\left[V(t)=\frac{16}{\pi}\left(\sin t+\frac{2}{3}\sin 3t\right)+\frac{1}{5}\sin 5t+...\right]$$

9. Determine Fourier series of a repetitive triangular wave as shown in Fig. 21.33.

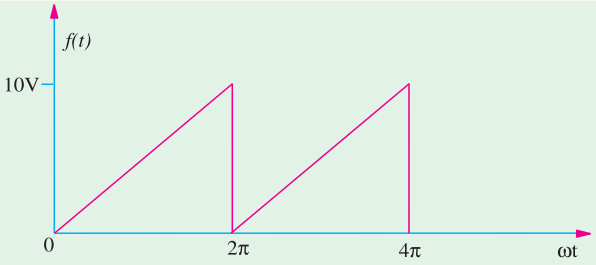


Fig. 21.33

(a) What is the magnitude of d.c. component?

(b) What is the fundamental frequency?

(c) What is the magnitude of the fundamental?

(d) Obtain its frequency spectrurm. ***(Network Theory Nagpur Univ.1993)***

[(a) 5 V (b) 1 Hz (c) 10 / π volt]

10. Determine the Fourier series of voltage responses obtained at the o/p of a half wave rectifier shown in Fig. 21.34. Plot the discrete spectrum of the waveform.

(Elect. Network Analysis Nagpur Univ. 1993)

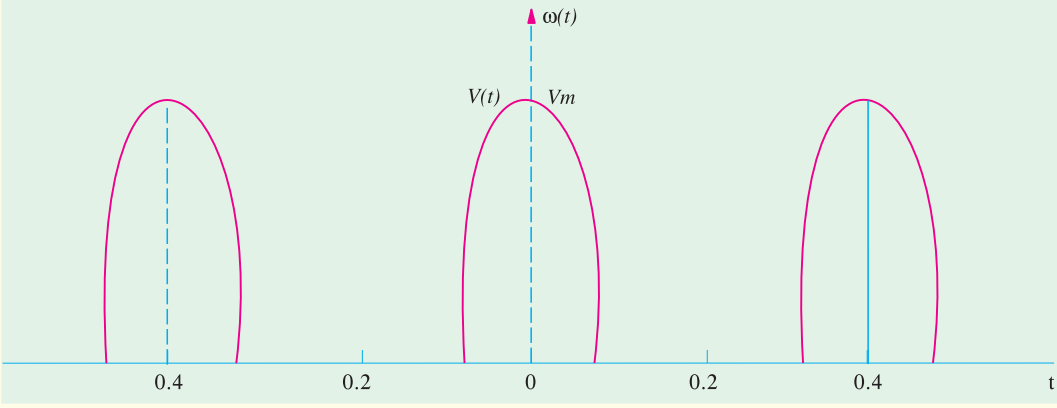


Fig. 21.34

$$\left[\mathbf{V(t)=\frac{V_m}{\pi}+\frac{V_m}{2}\cos\omega_0 t+\frac{2V_m}{3\pi}\cos 10\pi t-\frac{2V_m}{15}\cos 20\pi t+\frac{2V_m}{35\pi}\cos 30\pi t\ldots}\right]$$

11. Determine the Fourier coefficients and plot amplitude and phase spectral.

(Network Analysis Nagpur Univ. 1993)

$$\left[\mathbf{a_3=0, b_n=0, a_1=\frac{4V_m}{\pi}, a_2=0}\right.$$

$$\left.\mathbf{a_3=\frac{-4V_m}{3\pi}, a_4=0, a_5=\frac{4V_m}{\pi}}\right]$$

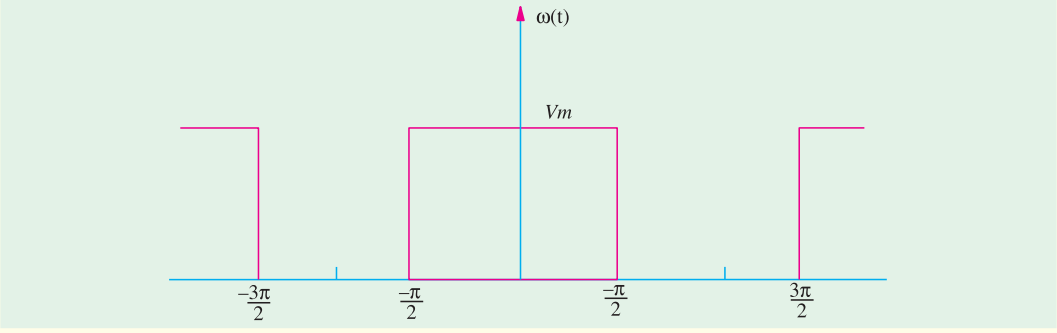


Fig. 21.35

OBJECTIVE TESTS – 21

1. A given function $f(t)$ can be represented by a Fourier series if it
(a) is periodic
(b) is single valued
(c) has a finite number of maxima and minima in any one period
(d) all of the above.

2. In a Fourier series expansion of a periodic function, the coefficient a_0 represents its
(a) net area per cycle
(b) d.c value

(*c*) average value over half cycle
(*d*) average a.c value per cycle

3. If in the Fourier series of a periodic function, the coefficient a_0 is zero, it means that the function has
(*a*) odd symmetry
(*b*) even quarter-wave symmetry
(*c*) odd quarter-wave symmetry
(*d*) any of the above.

4. A periodic function $f(t)$ is said to possess odd quarter-wave symmetry if
(*a*) $f(t) = f(-t)$
(*b*) $f(-t) = -f(t)$
(*c*) $f(t) = -f(t + T/2)$
(*a*) both (*b*) and (*c*).

5. If the average value of a periodic function over one period is zero and it consists of only odd harmonics then it must be possessing _____ symmetry.
(*a*) half-wave
(*b*) even quarter-wave
(*c*) odd quarter-wave
(*d*) odd.

6. If in the Fourier series of a periodic function, the coefficient $a_0 = 0$ and $a_n = 0$, then it must be having _____ symmetry.
(*a*) odd
(*b*) odd quarter-wave
(*c*) even
(*d*) either (*a*) or (*b*).

7. In the case of a periodic function possessing half-wave symmetry, which Fourier coefficient is zero ?
(*a*) a_n (*b*) b_n
(*c*) a_0 (*d*) none of above.

8. A periodic function has zero average value over a cycle and its Fourier series consists of only odd cosine terms. What is the symmetry possessed by this function.
(*a*) even
(*b*) odd
(*c*) even quarter-wave
(*d*) odd quarter-wave

9. Which of the following periodic function possesses even symmetry ?
(*a*) cos 3*t* (*b*) sin *t*
(*c*) *t* . cos 50 *t* (*d*) $(t + t^2 + t^5)$.

10. If the Fourier coefficient b_n of a periodic function is zero, then it must possess _____ symmetry.
(*a*) even
(*b*) even quarter-wave
(*c*) odd
(*d*) either (*a*) and (*b*).

11. A complex voltage waveform is given by $V = 120 \sin \omega t + 36(3\omega t + \pi/2) + 12 \sin (5\omega t + \pi)$. It has a time period of *T* seconds. The percentage fifth harmonic contents in the waveform is
(*a*) 12 (*b*) 10
(*c*) 36 (*d*) 5

12. In the waveform of *Q*. 11 above, the phase displacement of the third harmonic represents a time interval of —— seconds.
(*a*) T/12 (*b*) T/3
(*c*) 3T (*d*) T/36

13. When the negative half-cycle of a complex waveform is reversed, it becomes identical to its positive half-cycle. This feature indicates that the complex waveform is composed of
(*a*) fundamental
(*b*) odd harmonics
(*c*) even harmonics
(*d*) both (*a*) and (*b*)
(*e*) both (*a*) and (*c*)

14. A periodic waveform possessing half-wave symmetry has no
(*a*) even harmonics
(*b*) odd harmonics
(*c*) sine terms
(*d*) cosine terms

15. The Fourier series of a wave form possessing even quarter-wave symmetry has only
(*a*) even harmonics
(*b*) odd cosine terms
(*c*) odd sine terms
(*d*) both (*b*) and (*c*).

16. The Fourier series of a waveform possessing odd quarter-wave symmetry contains only
(*a*) even harmonics
(*b*) odd cosine terms
(*c*) odd cosine terms
(*d*) none of above

ANSWERS

1. (*d*) **2.** (*b*) **3.** (*d*) **4.** (*d*) **5.** (*a*) **6.** (*d*) **7.** (*c*) **8.** (*c*) **9.** (*a*) **10.** (*d*)
11. (*b*) **12.** (*a*) **13.** (*d*) **14.** (*a*) **15.** (*b*) **16.** (*c*)

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CHAPTER 22

TRANSIENTS

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- Transients in R-C Series Circuits (D.C.)
- Transients in R-C Series Circuits (A.C)
- Double Energy Transients



Transients are the surges or spikes in electric currents and voltages which are transmitted through power or data lines. The gadget shown in the above picture is claimed to protect the equipment against such surges or spikes

22.1. Introduction

It is quite an easy job to calculate the steady current, which flows in a circuit, when it is connected to a d.c. generator or a battery. Similarly, the alternating current which flows in a circuit when connected to an alternator can also be calculated by the various method discussed in Chapters 13 and 14. These currents are known as ***steady*** currents because in such cases, it is assumed that **(*i*)** the circuit components are constant and **(*ii*)** the circuit has been connected to the generator long enough for any disturbance produced on initial switching, to resolve itself.

In general, transients disturbances are produced whenever

(*a*) an apparatus or circuit is ***suddenly*** connected to or disconnected from the supply,

(*b*) a circuit is shorted and

(*c*) there is a ***sudden*** change in the applied voltage from one finite value to another.

We will now discuss the transients produced whenever different circuits are suddenly switched on or off from the supply voltage. In each case, we will assume that the resultant current consists of two parts **(*i*)** a final steady-stage or normal current and **(*ii*)** a transient current superimposed on the steady-stage current.

It is essential to remember that the transient currents are not driven by any part of the applied voltage but are entirely associated with the changes in the stored energy in inductors and capacitors. Since there is no stored energy in resistors, ***there are no transients in pure resistive circuits.***

22.2. Types of Transients

There are ***single-energy*** transients and ***double-energy*** transients. Single-energy transients are those in which only one form of energy, either electromagnetic or electrostatic is involved as in *R-L* and *R-C* circuits.

However, double-energy transients are those in which both electromagnetic or electrostatic is involved as in *R-L-C* circuits Transient disturbances may be further classified as follows :

(*a*) Initiation Transients : These are produced when a circuit, which is originally dead, is energised.

(*b*) Subsidence Transients : These are produced when an energised circuit is rapidly de-energised and reaches an eventual steady-stage of zero current or voltage, as in the case of short-circuiting an *R-L* or *R-C* circuit suddenly.

(*c*) Transition Transients : These are due to sudden but energetic changes from one steady state to another.

(*d*) Complex Transients : These are produced in a circuit which is simultaneously subjected to two transients due to two independent disturbances or when the disturbing force producing the transients is itself variable.

(*e*) Relaxation Transients : In these transients, the transition occurs cyclically towards states, which when reached, become unstable themselves.

A distinction may also be made between free and forced transients which are produced due to the applied voltage being itself transient.

22.3. Important Differential Equations

Some of the important differential equations, used in the treatment of single and double energy transients, are given below. We will consider both first-order and second-order differential equations.

1. First Order Equations

(*i*) Let $\frac{dy}{dx} + ay = 0$ where a is a constant.

Its solution is $y = k\,e^{-ax}$ where k is the constant of integration whose value can be found from the boundary conditions *i.e.* conditions prevalent at the instant when the voltage to a circuit is applied or excluded.

(*ii*) If $\frac{dy}{dx} + ay = b$ where a and b are constants, then solution is $y = \frac{b}{a} + k\,e^{-ax}$

The value of k can again be found from boundary conditions.

(*iii*) If $\frac{dy}{dx} + Ay = B$

where A and B are not constants but are functions of x, then the solution is given by

$$y \quad e^{\int A\,dx} \int e^{\int A\,dx} B dx \quad ke^{\int A\,dx}$$

If $A = a$ = constant, then the above equation simplifies to

$$y \quad e^{ax} \int e^{ax} B\,dx \quad ke^{ax}$$

2. Second Order Equations

(*i*) Suppose $\frac{d^2y}{dx^2} + a\frac{dy}{dx} + by = 0$ where a and b are constants, then the solution is

$$y \quad k_1 e^{\lambda_2 x} \quad k_2 e^{\lambda_2 x}$$

where λ_1 and λ_2 are constants of integration and whose values are,

$$\lambda_1 \quad \frac{a}{2} \sqrt{\frac{a^2}{4} \quad b} \quad \text{and} \quad \lambda_2 \quad \frac{a}{2} \sqrt{\frac{a^2}{4} \quad b}$$

(*a*) If $a^2/4 > b$, the roots are real and the above solution can be applied without any difficulty.

(*b*) If $a^2/4 < b$, the radicals contain a negative quantity. In that-case, the solution is given by

$$y = e^{-\frac{1}{2}ax} (k_3 \sin \lambda_0 x + k_4 \cos \lambda_0 x)$$

where k_3 and k_4 are the new constants of integration and

$$\lambda_0 = \sqrt{b - \frac{a^2}{4}}$$

(*c*) If $a^2/4 = b$, then both roots are equal and each is $= -a/2$.

Hence, in this case, the solution becomes $y = k_5\,e^{\lambda t} + k_6 t.e^{\lambda t}$

(*ii*) Let $\frac{d^2y}{dx^2} + a\frac{dy}{dx} + by = c$

where a, b and c are constant. In this case also, the solution will again depend on the root as discussed above.

$$y \quad k_1 e^{\quad 1^x} \quad k_2 e^{\quad \lambda_2 x} \quad c/b$$

(*iii*) (*a*) Let the differential equation be given by

$$\frac{d^2y}{dx^2} + a\frac{dy}{dx} + by = u$$

where a, b are c are constants but u is a particular function of the variable x. The solution of such an equation consists of a ***particular integral*** and a complementary function.

(*b*) Let y be a sinusoidal function of x, then

$$\frac{d^2y}{dx^2} + a\frac{dy}{dx} + by = c\sin\omega t$$

In this case, particular integral is

$$y_1 \quad \frac{c}{\sqrt{a^2 \;^2 \;(\;^2 \;b)^2}} \cos \quad x \quad \tan^{\;1} \frac{^2 \;b}{a}$$

The complementary function is given by

$$y_2 \quad k_1 e^{\;_1 x} \quad k_2 e^{\;_2 x} \text{ where } {}_1 \quad \frac{a}{2} \sqrt{\frac{a^2}{4} \quad b} \text{ and } {}_2 \quad \frac{a}{2} \sqrt{\frac{a^2}{4} \quad b}$$

The complete solution for the above equation is $y = y_1 + y_2$

Further treatment is the same as for case 2 (*i*) above.

22.4. Transients in R-L Circuits (D.C.)

If i, I_s and i_t be the resultant current, steady-state current and transient current respectively in R–L circuit of Fig. 22.1 (*a*), then by superimposition, the equation for the resultant current, for the duration of initiation transient, is

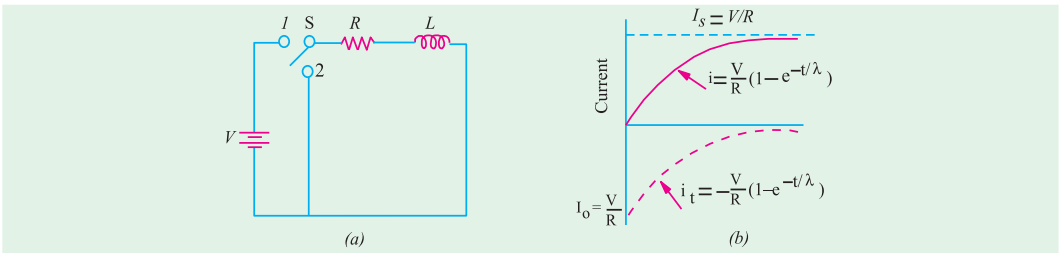


Fig. 22.1

$$i = I_s + i_t \qquad \text{... (i)}$$

Since the applied voltage V drives the steady-state current, hence

$$I_s = V/R$$

Since the transient current i_t is not associated with any voltage,

$$\therefore \qquad i_t R \quad L\frac{di_t}{dt} \quad 0 \qquad \text{... (ii)}$$

or $$\frac{di_t}{dt} \quad \frac{R}{L}dt \qquad \text{... (iii)}$$

or $$\frac{di_t}{i_t} \quad \frac{R}{L} \quad \text{dt} \;\therefore\; \text{logh } i_t \quad \frac{R}{L}t \quad K \;^* \qquad \text{... (iv)}$$

where K is the constant of integration whose value may be found from the initial conditions.

Now when $t = 0$, $i_t = I_0$ (say). Then from Eq. ***(iv)*** above we get, logh $I_0 = 0 + K$

---

* $\log_e x$ is written as logh x. Obviously, logh $x = 2.3 \log_{10} x$.

Putting this value of K in Eq. ***(iv)***, we have

$$\log_h i_t \quad \log_h I_0 \quad \frac{R}{L}t \quad \text{or} \quad \log_h i_t / I_0 \quad \frac{R}{L}t \quad \frac{t}{}$$

$$\therefore \quad i_t = I_0 e^{-t/\lambda} \qquad \text{... (v)}$$

where $\lambda = L/R$ is called the ***time-constant*** of the circuit. Its reciprocal R/L is called the ***damping coefficient*** of the circuit. The current decreases exponentially as shown in Fig. 22.1 (*b*). From Eq. ***(i)*** and ***(v)***, we have

$$i = I_s + I_0 e^{-t/\lambda} \qquad \text{... (vi)}$$

If the time is reckoned when the voltage V is applied, so that when $t = 0$, $i = 0$, then from equation ***(vi)***, we get

$$0 = I_s + i_0 e^{-0} = I_s + I_0 \quad \therefore \quad I_0 = -I_s = -\frac{V}{R}$$

In that case, Eq. ***(vi)*** becomes

$$i = \frac{V}{R} - \frac{V}{R} e^{-t/\lambda} \qquad \text{... (vii)}$$

$$= \frac{V}{R}(1 - e^{-t/\lambda}) \qquad \text{... (viii)}$$

Curves for I_s and i_t have been plotted in Fig. 22.1 (*b*). The curve for resultant current has been obtained by the superposition of steady-state current I_s (= V/R) and transient current

$$i_t = \frac{V}{R} e^{-t/\lambda}$$

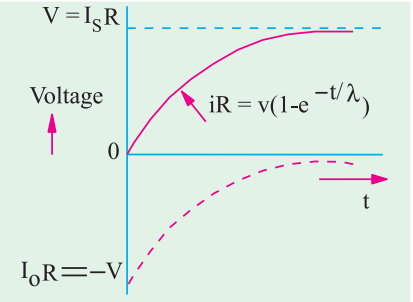


Fig. 22.2

Theoretically, the transient current i_t takes infinite time to die off but, in practice, it disappears in a very short time.

The values of resultant, steady-state and transient voltages across the resistor can be found by multiplying Eq. ***(vii)*** by R and are shown in Fig. 22.2. The e.m.f. of self-induction–Ldi_t/dt is only transient in nature and equals $i_t R$ as seen from Eq. ***(ii)*** above.

22.5. Short Circuit Current

After some time, the transient current would disappear and the only current flowing in the circuit would be the steady-state current $I_s = V/R$. Let the R-L circuit be closed upon itself *i.e.* be short-circuited by shifting the switch [Fig. 22.1 (*a*)] to position 2. Since the voltage V has been excluded from the circuit, the trapped current I_s will immediately cease to be a steady-state current, but on the other hand, will become the initial value I_0 of a new subsidence transient current i_t.



Large-capacity short-circuit current generator

If time is measured at the instant of short-circuit, so that when $t = 0$, the current is $I_s = V/R$, then Eq. **(v)** becomes

$$i_t = \frac{V}{R} e^{-t/\lambda} \qquad \text{... (ix)}$$

This equation has been plotted in Fig. 22.3. The only voltage acting in the circuit is that due to self-induction *i.e.* $-L\, di_t/dt$ which equals $i_t R$.

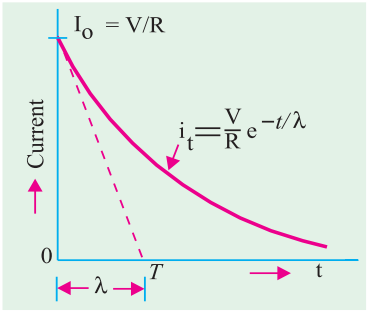


Fig. 22.3

22.6. Time Constant

The time constant of a circuit is defined as the time it would take for the transient current to decrease to zero, if the decrease were linear instead of being exponential.

In other words, it is the time during which the transient current would have decreased to zero, had it maintained its initial rate of decrease.

The initial rate of decrease can be found by differentiating Eq. (*vi*) and putting $t = 0$

$$\therefore \qquad \frac{di_t}{dt} = \frac{I_0}{\lambda} e^{-t/\lambda} \qquad \left(\frac{di_t}{dt}\right)_{t\,0} \qquad \frac{I_0}{}$$

If the rate of decrease were constant throughout and equal to $-I_0/\lambda$, then the straight line showing the relation between i_t and t would be given by

$$i_t = -\frac{I_0}{\lambda} t$$

The time-period would be equal to the sub-tangent OT drawn to the exponential curve of Fig. 22.3 at $i_t = I_0$ *i.e.* at the beginning of the curve.

If we put $t = \lambda$ in Eq. (*v*), then $i_t = I_0\, e^{-1} = I_0/e = I_0/2.718 = 0.37\, I_0$

Hence, time period of a circuit is the time during which the transient current decreases to 0.37 of its initial value.

Example 22.1. *A coil having a resistance of 30 ω and an inductance of 0.09 H is connected across a battery of 20 V. Plot the current and its two components. Assume that t = 0 when the circuit is completed.* **(Electromechanic Allahabad Univ. 1992)**

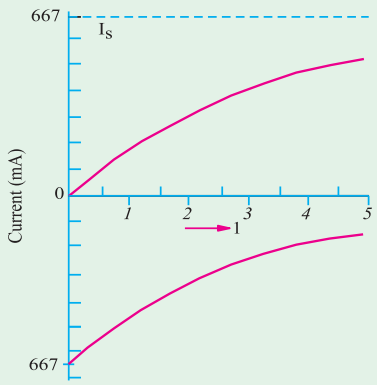


Fig. 22.4

Solution. The two components of the circuit current are **(i)** steady current $I_s = V/R = 20/30 = 2/3$ A = 667 mA and **(ii)** Transient current $i_t = -(V/R)e^{-t/\lambda}$.

Total current is $i = I_s + i_t$. Let us find the value of transient current after various time intervals. In the present case, $\lambda = L/R = 0.09/30 = 0.003$ second = 3 millisecond.

The values of i_t and i at various times are tabulated below. Value of $i = I_s + i_t$

t (ms)	$e^{-t/\lambda}$	i_t (mA)	i	t (ms)	$e^{-t/\lambda}$	i_t (mA)	i
0.0	1.000	–667	0	2.5	0.435	–290	377
0.5	0.847	–565	102	3.0	0.368	–244	423
1.0	0.716	–477	190	3.5	0.311	–208	459
1.5	0.606	–405	262	4.0	0.264	–176	491
2.0	0.514	-344	323	4.5	0.223	–148	519

It is seen that whereas transient current decreases exponentially, total circuit current increases exponentially as expected (Fig. 22.4).

Example 22.2. *A circuit of resistance 10 Ω and inductance 0.1 H in series has a direct voltage of 200 V suddenly applied to it. Find the voltage drop across the inductance at the instant of switching on and at 0.01 second. Find also the flux-linkages at these instants.*

(Basic Electricity, Bombay Univ.)

Solution. (*i*) Switching instant

At the instant of switching on, $i = 0$, so that $iR = 0$ hence all applied voltage must drop across the inductance only. Therefore, voltage drop across inductance = **200 V**.

Since at this instant $i = 0$, there are no flux-linkages of the coil.

(*ii*) When t = 0.01 second

As time passes, current grows so that the applied voltage is partly dropped across the resistance and partly across the coil. Let us first find iR drop for which purpose, we need the value of i at $t = 0.01$ second.

Now, time period of the circuit is $\lambda = L/R = 0.1/10 = 0.01$ second. Since the given time happens to be equal to time constant,

$\therefore \quad i = (200/10) \times 0.632 = 12.64$ A; $iR = 152.64 \times 10 =$ **126.4 V**

Drop across inductance = $\sqrt{200^2 - 126.4^2} =$ **155 V**

Now, $L = N\Phi/i$ or $N\Phi = Li$

$\therefore$ Flux-linkages $Li = 0.1 \times 12.64 =$ **1.264 Wb-turns.**

Example 22.3. *A coil of 10 H inductance and 5 Ω resistance is connected in parallel with a 20 Ω resistor across a 100-V d.c. supply which is suddenly disconnected. Find*

***(a)** the initial rate of change of current after switching.*

***(b)** the voltage across the 20 Ω resistor initially and after 0.3 s.*

***(c)** the voltage across the switch contacts at the instant of separation and*

***(d)** the rate at which the coil is losing stored energy 0.3 second after switching.*

Solution. (*a*) Since the steady-state current is zero, $i = I_0\, e^{-t/\lambda}$

Now, when $t = 0$, current is = 100/5 = 20 A. It means the current flowing through the coil immediately before opening the switch is 20 A.

$\therefore \quad I_0 = 20$ A

Hence, the above equation becomes $i = 20e^{-t/\lambda}$

Now $\lambda = L/R = 10/25 = 1/2.5 \quad \therefore \quad i = 20e^{-2.5t}$

$$\frac{di}{dt}_{t\ 0} \quad (\ 20 \quad 2.5e^{\ 2.5t})_{t\ 0} \quad \textbf{– 50 A/s}$$

The negative sign merely shows that the current is decreasing.

(b) After the supply has been disconnected, the current through the $20\text{-}\Omega$ resistor is i since it is in series with the coil.

Initial p.d. across the 20Ω resistor = (current at t = 0) × 20 = 20 × 20 = **400 V**

Current through the resistor after 0.3 second = $20e^{-2.5\times0.3} = 9.45$ A

$\therefore$ Voltage across the resistor after 0.3 second
= (current at t = 0.3 second) × 20 = 9.45 × 20 = **189 V**

(c) The e.m.f. induced in the coil at break tends to maintain the current through it in the original direction. Hence, the direction of the current through $2\,0\ \Omega$ resistor is upwards so that the p.d. across the switch contacts will be the *sum* of supply voltage and the voltage across $20\ \Omega$ resistor.

$\therefore$ Initial voltage across switch contacts = 400 + 100 = **500 V**

(d) The rate of loss of energy = power = induced e.m.f. in coil × current (after 0.3 s)

$$L\ \frac{di}{dt}_{t\ 0.3} \quad \text{(after 0.3 second)}$$

Now, after 0.3 second, i = 9.45 A

Value of di/dt after 0.3 second = $-20 \times 2.5 \times e^{-0.75} = -23.6$ A/second

$\therefore$Rate of loss of energy = $-10 \times 23.6 \times 9.45 = -2{,}230$ joule/second

22.7. Transients in R-L Circuits (A.C.)

Let a voltage given by $v = V_m \sin(\omega t + \Psi)$ be ***suddenly*** applied across an *R-L* circuit [Fig. 22.5 (*a*)] at a time when t = 0. It means that the voltage is applied when it is passing through the value $V_m \sin \Psi$. Since the contact may be closed at any point of the cycle, angle Ψ may have any value lying between zero and 2π radians. The resultant current, as before, is given by

$$i = i_s + i_t$$

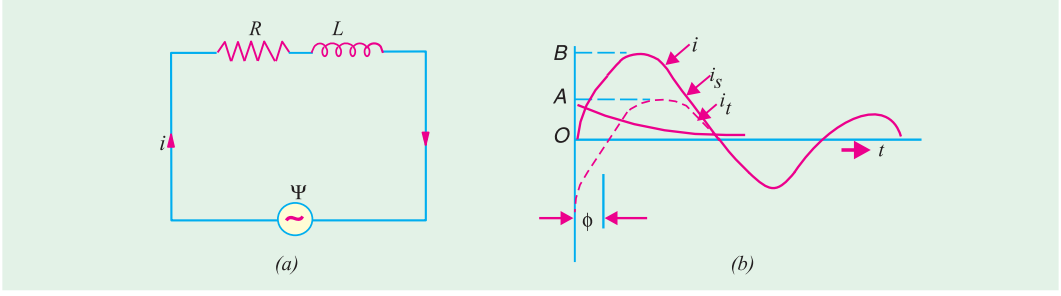


Fig. 22.5

The value of steady-state current is found by the normal circuit theory. The peak steady-state current is given by

$$I_m = \frac{V_m}{\sqrt{R^2 + X_L{}^2}} = \frac{V_m}{Z}$$

where $\sqrt{R^2 + X_L{}^2}$ is the impedance of the circuit. This current lags behind the applied voltage by an angle ϕ such that $\tan \phi = X_L/R$ or $\phi = \tan^{-1}(X_L/R)$

Hence, the equation for the instantaneous value of the steady-state current becomes

$$i_s = I_m \sin(\omega t + \Psi - \phi)$$

As before, the transient current is given by

$$i_t = I_0 e^{-t/\lambda} \qquad \therefore i = I_m \sin(\omega t + \Psi - \phi) + I_0 e^{-t/\lambda} \qquad \text{... (i)}$$

Now, when $t = 0$, $i = 0$, hence putting these values in Eq. (*i*) above, we get

$$0 = I_m \sin(\Psi - \phi) + I_0 \quad \therefore I_0 = -I_m \sin(\Psi - \phi)$$

Hence Eq. (*i*) can be written as

$$i = I_m \sin(\omega t + \Psi - \phi) - I_m \sin(\Psi - \phi) e^{-t/\lambda} \qquad \text{... (ii)}$$

From the above, it is seen that the value of I_0 and hence the size of the transient current depends on angle Ψ *i.e.* it depends on the instant in the cycle at which the circuit is closed. We will consider the following three cases :

Case 1

When $t = 0$, let the voltage pass through its zero value and become positive *i.e.* let $\Psi = 0$. In that case, putting this value of Ψ in Eq. (*ii*), we get

$$i = I_m \sin(\omega t - \phi) - I_m \sin(-\phi) e^{-t/\lambda} = I_m [\sin(\omega t - \phi) + \sin\phi e^{-t/\lambda}]$$

This is shown in Fig. 22.5 (*b*). It is seen that maximum instantaneous peak current *OB* is larger than the normal peak current *OA*.

Case 2

Let $t = 0$ when voltage is passing through its value $V_m \sin(\phi)$ so that $\Psi = \phi$ or $\Psi - \phi = 0$

In that, $I_0 = 0$, there is no transient current at the time of switching on (*i.e.* $i_t = 0$). It corresponds to the contacts closing at the instant when the steady state current itself is zero.

Case 3

When $t = 0$, let the voltage be passing through

$V_m \sin$ $\frac{}{2}$ *i.e.* $\frac{}{2}$ and / 2

In this case, the transient [as found from Eq. (*ii*)] would be given by

i_t $I_m \sin$ $\frac{}{2}$. $e^{\ t/}$ $I_m e^{\ t/}$

Under these conditions, the transient would have its maximum possible initial value.

Example 22.4. *A 1.0 H choke has a resistance of 50 Ω. This choke is supplied with an a.c. voltage given by e = 141 sin 314 t. Find the expression for the transient component of the current flowing through the choke after the voltage is suddenly switched on.*

(Principles of Elect. Engg-II, Jadavpur Univ.)

Solution. The equation of the transient component of the current is (Art. 22.7 Case 1)

$$i_t = I_m \sin\phi e^{-t/\lambda}$$

Here, $\lambda = L/R = 1/50 = 0.02$ second ; $\mathbf{Z} = 50 + j\,314 = 318 \angle 80.95^\circ$

$I_m = V_m/Z = 141/318 = 0.443$ A; $\sin 80.95^\circ = 0.9875$

$\therefore$ $i_t = 0.443 \times 0.9875\, e^{-t/0.02} = 0.4376 e^{-t/0.02}$

Example 22.5. *A 50-Hz sinusoidal voltage of maximum value of 400 V is applied to a series circuit of resistance 10 Ω and inductance 0.1 H. Find tan expression for the value of the current at any instant after the voltage is applied, assuming that voltage is zero at the instant of application. Calculate its value 0.02 second after switching on* **(Electric Circuit, Punjab Univ. 1990)**

Solution. In such cases, as seen from Art. 22.7 (Case 1), the current consists of a steady-state component and a transient component. The equation of the resultant current is

$$i = \underset{\substack{\text{steady-state}\\ \text{current}}}{I_m \sin(\omega t - \phi)} + \underset{\substack{\text{transient}\\ \text{current}}}{I_m \sin\phi\, e^{t/\lambda}}$$

where $I_m = V_m / Z$; $\phi = \tan^{-1}(X_L / R)$; $\lambda = L/R$ second

$R = 10\,\Omega; X_L = 314 \times 0.1 = 31.4\,\Omega; \mathbf{Z} = 10 + j\,31.4 = 33 \angle 72.3^\circ$

$I_m = 400/33 = 12.1$ A ; $\phi = 72.3^\circ = 1.26$ rad.

$\sin\phi = \sin 72.3^\circ = 0.9527$; $\lambda = 0.1/10 = 1/100$ second

$i = 121\ \{\sin(314\,t - 1.262) + 0.9527\, e^{100t}\}$

Substituting $t = 0.02$ second, we get

$i = 12.1\ (\sin(314 \times 0.02 - 1.262) + 0.9527\, e^{-2}\}$

$= 12.1\ (\sin 5.02 + 0.9527\, e^{-2}) = 12.1\ (\sin 288^\circ + 0.9527\, e^{-2})$

$= 12.1\ (-\sin 72^\circ + 0.9527 \times 0.1353) = 12.1\ (-0.9511 + 0.1289) =$ **–9.95 A**

Example 22.6. *An alternating voltage $v = 400 \sin(314\,t + \Psi)$ is suddenly applied across a coil of resistance 0.2 Ω and inductance 6.36 mH. Determine the first peak value of the resultant current when the transient current has maximum value.*

Solution. Obviously, $\omega = 314$ rad/s

$X_L = \omega L = 314 \times 6.36 \times 10^{-3} = 2\ \Omega$

Coil impedance $\mathbf{Z} = 0.2 + j2 \sim 2 \angle 84.3^\circ$

Max. value of steady-state current = 400/2 = 200 A

As seen from Art. 22.7, the maximum value of transient current will occur when

$\Psi = \phi \pm \pi/2$ where $\phi = 84.3^\circ$ *i.e.* the phase angle of the current w.r.t. voltage

$\therefore$ $\Psi = 84.3^\circ - 90^\circ = -5.7^\circ$

$\therefore$ resultant current, $i = 400 \sin(314\,t - 90^\circ) + I_0\, e^{-31.4t}$

Now, at $t = 0$, $i = 0$ $\therefore$ $0 = 400 \sin(-90^\circ) + I_0$

$\therefore I_0 = 400$ A

Hence, the above equation becomes

$i = 400 \sin(\omega t - 90^\circ) + 400\, e^{-31.4t}$

The procedure for determining an exact solution for the first peak of the resultant current is first to differentiate the above expression, next to equate the result to zero and then to solve the resulting expression graphically for t. However, sufficiently accurate result can be obtained by determining the instant at which steady-state current reaches its first positive peak value and then

to add to it the value of the transient current at this instant. The first peak value of steady-state current occurs when

$(314\,t - 90^\circ) = \pi/2$ rad ; *i.e.* when $t = \pi/314 = 0.01$ second

At this time, $i_t = 400\,e^{-0.314} = 292$ A

$\therefore$ resultant current i at this time = 200 + 292 = **492 A**

22.8. Transients in R-C Weeries Circuits (D.C.)

When a d.c. voltage V is ***suddenly*** applied to an R-C series circuit (Fig. 22.6), the voltage v_c across the capacitor rises from zero value to the steady-state value V. If v_c is the voltage across capacitor, V_{ct} the transient voltage, then

$$v_c = V + v_{ct} \qquad \text{... (i)}$$

The charging current is maximum at the beginning but then is reduced to zero so that there is no steady-state current but a transient one.

Since the transient current is not associated with any applied voltage, hence

$$i_t R + v_{ct} = 0 \qquad \text{.... (ii)}$$

Now, capacitor voltage $v_{ct} = q_1/C$

Hence, Eq. (*ii*) becomes

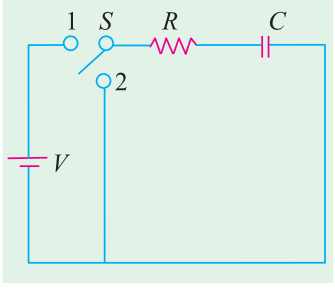


Fig. 22.6

$$i_t R + \frac{q_t}{C} = 0$$

or $$R \cdot \frac{di_t}{dt} + \frac{1}{C} \cdot \frac{dq_t}{dt} = 0 \quad \text{or} \quad \frac{di_t}{dt} = -\frac{1}{CR}\frac{dq_t}{dt} = -\frac{1}{CR} i_t \quad (\ dq_t / dt = i_t)$$

$$\therefore \quad \frac{di_t}{i_t} = -\frac{d_t}{CR}; \qquad \text{As before } i_t = I_0 e^{-t/CR} = I_0 e^{-t/\lambda}$$

where $CR = \lambda$ = time constant. The reciprocal I/CR is known as damping coefficient.

(*i*) Charging Current

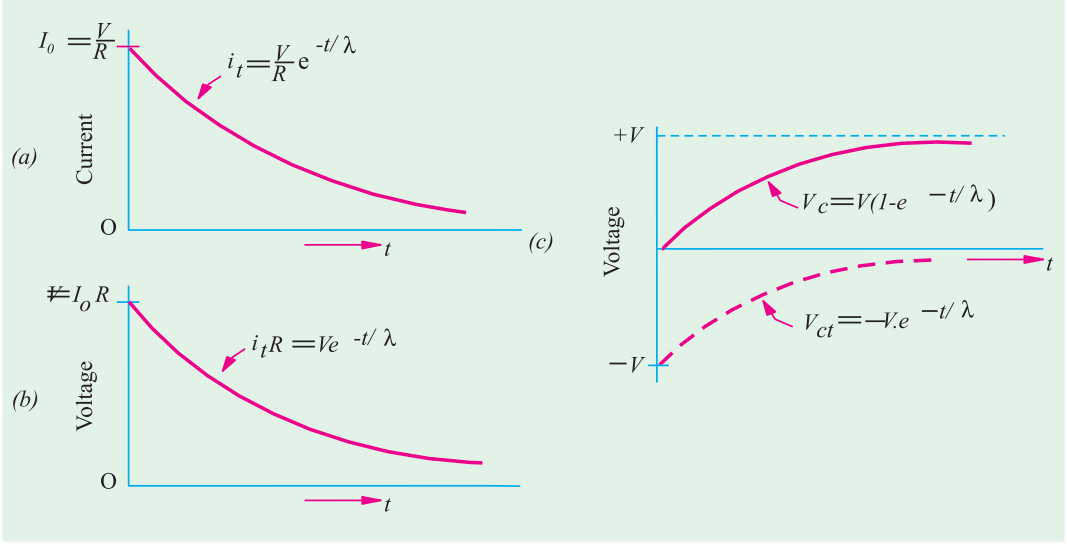


Fig. 22.7

When $t = 0$, transient current $i_t = I_0$, so that from Eq. (*ii*) $v_{ct} = -I_0R$. Moreover, when $t = 0$, $v_c = 0$, hence from Eq. **(*i*)**, $v_{ct} = -V$

Combining these results, we get

$$I_0 = V/R$$

$$\therefore \quad i_t = I_0 e^{-t/} = \frac{V}{R} e^{-t/}$$

This is plotted in Fig. 22.7 (*a*)

The transient voltage across the resistor R is given by

$$i_t R = \frac{V}{R} e^{-t/\lambda} \times R$$

$= Ve^{-t/\lambda}$ ... Fig. 19.7 (*b*)

From Eq. **(*ii*)** the value of transient voltage across the capacitor is $v_a = -i_t R$

Hence, Eq. **(*i*)** becomes

$$v_c = V - i_t R = V - Ve^{-t/\lambda}$$

or $$v_c = V(1 - e^{-t/\lambda})$$ ... **(*iii*)**

The voltage across the capacitor v_c which is the sum of the transient voltage v_{ct} and steady-state V has been plotted in Fig. 22.7 (*c*).

The charge across the capacitor is given by

$$q = v_c = CV(1 - e^{-t/\lambda}) \text{ or } q = Q(1 - e^{-t/\lambda}) \quad (\therefore Q = CV)$$

(*ii*) Discharge Current

When the capacitor has become fully charged so that charging current has ceased, then the *R-C* circuit is short-circuited by shifting the switch *S* from position 1 to position 2 (Fig. 22.6). On doing so, a transient discharge current will start flowing immediately. If time is reckoned from the instant of short-circuit, then when $t = 0$, $i_t = I_0$, hence from Eq. **(*ii*)** above $v_{ct} = -I_0 R$. Moreover, when $t = 0$, $v_c = V$. However, since there is no steady-state voltage across the capacitor, from Eq. **(*i*)**, we get $v_c = v_{ct}$. Combining these results, we get

$$I_0 = -V/R$$

$$\therefore \quad i_t = -\frac{V}{R} e^{-t/\lambda}$$

It is plotted in Fig. 22.8 (*a*). The negative sign shows that discharge current flows in a direction opposite to that in which the charging current flows. That is why the curve has been drawn below the *X*-axis. It may be noted that the only voltage in the circuit is v_{ct} which equals $-i_t R$.

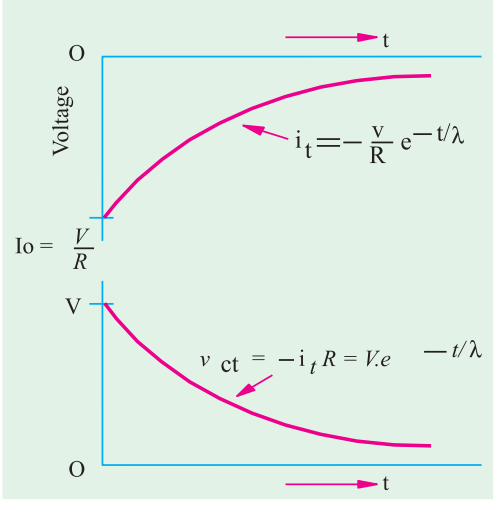


Fig. 22.8

Example 22.7. *In a simple saw-tooth generator circuit with the thyratron switches on at 150 V and switches off at 10 V. If this circuit is supplied with 250 V d.c. source; find the time period of saw-tooth wave. The resistance and capacitance have the values of 10 kΩ and 1 μF respectively.* **(Principles of Elect. Engg-II, Jadavpur Univ.)**

Solution. With reference to Fig. 22.9, let

V = applied voltage

v_{cl} = switching-off voltage of the thyratron = 10 V

v_{c2} = switching-on voltage of the thyratron = 150 V

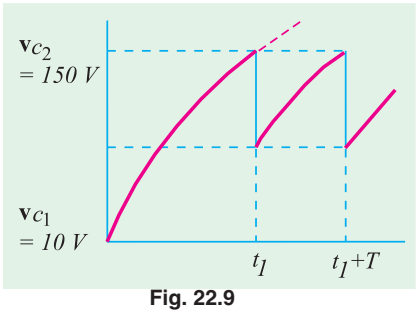


Fig. 22.9

Now, $v \quad V(1 \quad e^{\,t/}\,)$

$\therefore \qquad v_{c1} = V(1-e^{-t/\lambda}) \qquad$ ... ***(i)***

$V(1-e^{(-t_1+T)}/\lambda \qquad$ ... ***(ii)***

where T is the time-period of the saw-tooth wave. From Eq. ***(i)*** and ***(ii)***, we get

$$T = \lambda \log_e (V - v_{c1}) / (V - v_{c2})$$

Now $\lambda = CR = 10^4 \times 10^{-6} = 10^{-2}$ second

$$V - v_{c1} = 250 - 10 = 240\,\text{V}; V - v_{c2} = 250 - 150 = 100\,\text{V}$$

$$\therefore \quad \frac{V - v_{c1}}{V - v_{c2}} = \frac{240}{100} = 2.4 \quad \therefore \quad T = 10^{-2} \log_c{}^{2.4} = \mathbf{0.00875\ second}$$

Example 22.8. *A simple neon-tube time base for a cathode-ray oscillorgraphy employs a 300 $k\Omega$ and a 0.016 μF capacitor. The striking and extinction voltages of the neon-tube are 170 V and 140 V respectively. Calculate the frequency of the time base if the supply voltage is 200 V.*

Solution. The voltage across the capacitor increases according to the equation

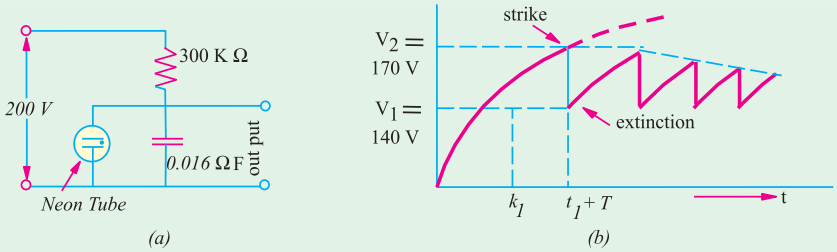


Fig. 22.10

$$v_c = V(1-e^{-t/CR})$$

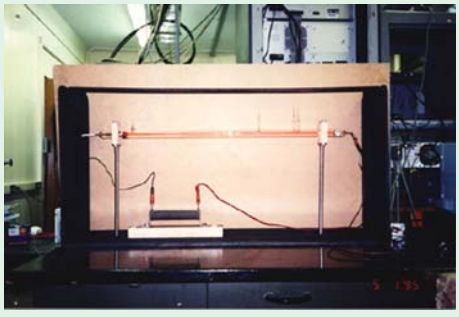
It is shown in Fig. 22.10 (*b*)

$$\therefore \qquad v_{c1} = V(1-e^{-t_1/\lambda}) \qquad \text{... ***(i)***}$$

$$v_{c2} = V(1-e^{(-t_1+T)/\lambda}) \qquad \text{... ***(ii)***}$$

From eq. ***(i)*** and ***(ii)***, we get

$$T = \lambda \log h\,(V - v_{C1})/(V - v_{C2})$$


A view of the neon tube experiment

Now

$$\lambda = CR = 0.016 \times 10^{-6} \times 300 \times 10^3 = 4.8 \times 10^{-3} \text{ second}$$

$$V - v_{c1} = 200 - 140 = 60\ \text{V and}\ V - v_{c2} = 200 - 170 = 30\ \text{V}$$

$\therefore \quad T = 4.8 \times 10^{-3} \log 60/30 = 1/300$ second

$\therefore$ Frequency of time base = $1/T$ = **300 Hz**

22.9. Transients in R-C Series Circuits (A.C.)

In this case, the resultant currents can be determined in the same way as for an *R-L* circuit (Art. 22.7). It is given by

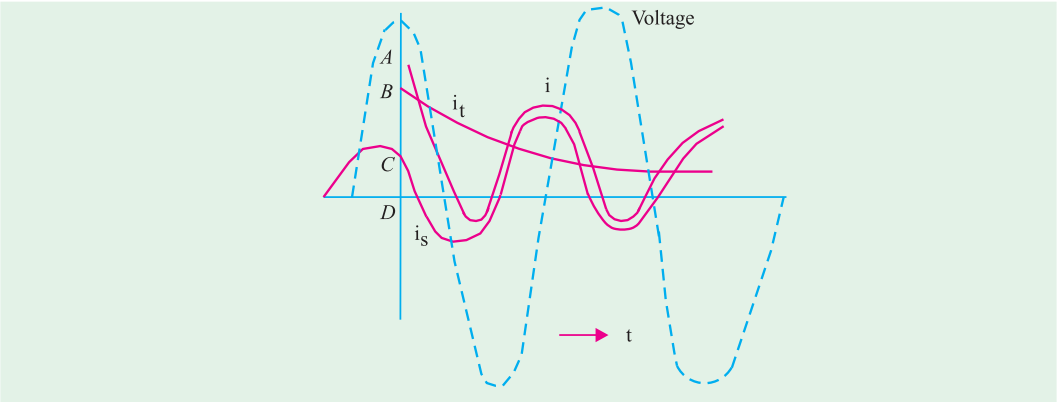


Fig. 22.11

$$i = i_s + i_t = I_m \sin(\omega t + \psi + \phi) + I_0 e^{-t/\lambda} \quad \text{where} \quad I_m = V_m / \sqrt{R^2 + X_C{}^2}$$

$$\text{and} \quad v = V_m \sin(\omega t + \psi)$$

The value of I_0 as found from initial known conditions ($t = 0$, $i = 0$) is given by $I_0 = -I_m \sin(\psi + \phi)$

Hence, the resultant current becomes

$$i = I_m \sin(\omega t + \psi + \phi) - I_m \sin(\psi + \phi) e^{-t/\lambda}$$

As shown in Fig. 22.11, the resultant current at the moment of switch closing is *OA* and is made up of steady-state current *OC* and transient current *OB*.

22.10. Double Energy Transients

In an *R-L-C* circuit, both electromagnetic and electrostatic energies are involved, hence any sudden change in the conditions of the circuit involves the redistribution of these two forms of energy. The transient currents produced due to this redistribution are known as double-energy transients. The transient current produced may be unidirectional or a decaying oscillatory current.

In an *R-L-C* circuit, the transient voltages across the three circuit parameters are $i_t, R, (L di_t/dt)$ and q_i/C. Hence, the equation of the transient voltage is

$$i_t R + L \frac{di_t}{dt} + \frac{q_t}{C} = 0 \qquad \text{... (i)}$$

Differentiating the above equation and putting i_t for dq_t/dt, we get

$$\frac{d^2 i_t}{dt^2} + \frac{R}{L}\frac{di_t}{dt} + \frac{1}{LC} i_t = 0 \qquad \text{... (ii)}$$

This is a linear differential equation of the second order with constant coefficient like 2 **(i)** given in Art. No. 22.3. Its solution is given by

$$i_t \quad k_1 e^{\ 1t} \quad k_2 e^{\ 1t} \qquad \text{... (iii)}$$

where k_1 and k_2 are constants whose values are found from the boundary conditions. The values of λ_1 and λ_2 are given by

$$\lambda_1 = -\frac{R}{2L} - \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}} \text{ and } {}_2 \quad \frac{R}{2L} \quad \sqrt{\frac{R^2}{4L^2} \quad \frac{1}{LC}}$$

Depending on the value of λ_1 and λ_2, four different conditions of the circuit are distinguishable. We will now examine these four conditions in the case of an *R-L-C* circuit.

Case 1. Loss-free Circuit, *R* = 0 *i.e.* Undamped

In this case, $\lambda_1 = \sqrt{-\frac{1}{LC}} = j\,1\sqrt{LC} = -j\omega$ and $\lambda_2 = -\sqrt{-\frac{1}{LC}} = -j\,1\sqrt{LC} = -j\omega$

Hence, Eq. ***(iii)*** given above becomes

$$i_t = k_1 e^{j\omega t} + k_2 e^{-j\omega t} = k_1(\cos\omega t + j\sin\omega t) + k_2(\cos\omega t - j\sin\omega t)$$

$$= (k_1 + k_2)\cos\omega t + j(k_1 - k_2)\sin\omega t$$

or $$i_t = A\cos\omega t + B\sin\omega t \qquad \text{... (iv)}$$

where $$A = k_1 + k_2 \text{ and } B = j(k_1 - k_2)$$

Eq. ***(iv)*** can be still further simplified to

$$i_t = I_m \sin(\omega t + \phi) \qquad \text{... (v)}$$

where $$I_m = \sqrt{A^2 + B^2} \text{ and } \phi = \tan^{-1}(A/B)$$

As seen from Eq. ***(v)***, the transient current in this case is sinusoidal wave of constant peak value and frequency $f = 1/2\pi\sqrt{LC}$ as shown in Fig. 22.12 (*a*). The values of two constant terms I_m and ϕ can be determined from any *two* known initial circuit conditions which are (*i*) the initial current in the inductance and (*ii*) the initial voltage across the capacitor.

Case 2. Low-loss Circuit: $\frac{R^2}{4L^2} < \frac{1}{LC}$ *i.e.* Under-damped

In this case, λ_1 and λ_2 would be conjugate complex numbers because the term under the square root sign in each case would be negative.

$$\therefore \quad {}_1 \quad \frac{R}{2L} \quad j\sqrt{\frac{1}{LC} \quad \frac{R^2}{4L^2}}$$

If $a = \frac{R}{2L}$ and $\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$ then $\lambda_1 = -a + j\omega$ and $\lambda_2 = -a - j\omega$

Putting these values in equation (*v*), we get

$$i_t = k_1 e^{(-a+j\omega)t} + k_2 e^{(-a-j\omega)t} = e^{-at}(k_1 e^{j\omega t} + k_2 e^{-j\omega t})$$

This equation can be reduced, as before, to the form

$$i_t = I_m e^{-at}\sin(\omega t + \phi) \qquad \text{... (iv)}$$

where I_m and ϕ are constants as before. Equation ***(vi)*** represents damped transient oscillatory current as shown in Fig. 22.12 (*b*).

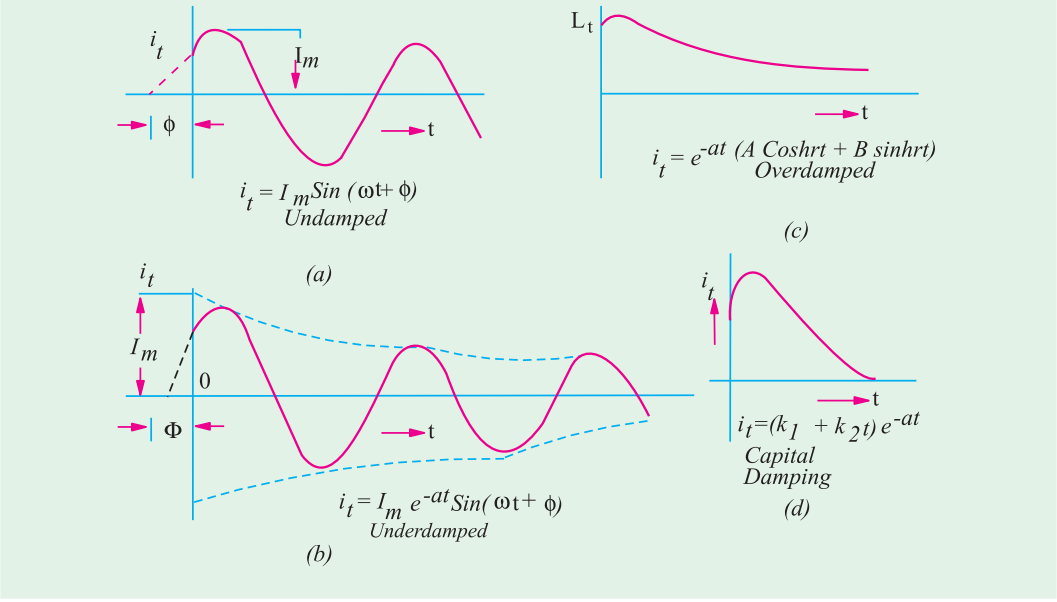


Fig. 22.12

The exponential term e^{-at} which accounts for the decay of oscillations, is called the decay or damping factor or merely ***decrement.*** It makes each current peak a definite fraction less than that preceding it. The logarithm to the Naperian base '*e*' of the ratio of peaks one cycle apart in time is $a/f = R/2fL$ and is referred to as ***logarithmic decrement.*** The frequency of damped oscillations is given by

$$f = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$ and is called the natural frequency of the circuit

If $\frac{R^2}{4L^2} < \frac{1}{LC}$, then $f = \frac{1}{2\pi\sqrt{LC}}$

Case 3. High-loss Circuit: $\frac{R^2}{4L^2} \quad \frac{1}{LC}$ *i.e.* **overdamped**

In this case, λ_1 and λ_2 will be pure numbers.

$$\lambda_1 - \frac{R}{2L} + \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}} = -a + \gamma \text{ and } \gamma_2 = -a - \gamma$$

$$\therefore \qquad i_t = k_1 e^{(-a+\gamma)t} + k_2 e^{(-a-\gamma)t} = e^{-at}(k_1 e^{\gamma t} + k_2 e^{-\gamma t})$$

Now $e^{rt} = \sinh \gamma t + \cosh \gamma t$

and $e^{-\gamma t} = \cosh \gamma t - \sinh \gamma t$

$$\therefore \quad i_t = e^{-at}\{(k_1 + k_2)\cosh \gamma t + (k_1 - k_2)\sinh \gamma t\}$$

or $$i_t = e^{-at}(A \cosh \gamma t + B \sinh \gamma t)$$

A typical curve of this equation is shown in Fig. 22.12(*c*)

Case 4. $\frac{R^2}{4L^2} = \frac{1}{LC}$ ***i.e.*** **Critical Damping**

In this case, $\lambda_1 = \lambda_2 = -\frac{R}{2L}$

Hence, equation ***(iii)*** is reduced to

$$i_t = (k_1 + k_2 t)e^{-\frac{R}{2L}t} \text{ or } i_t = (k_1 + k_2 t)e^{-at}$$

It is a case of critical damping because current is reduced to almost zero in the shortest possible time. The above equation has been plotted in Fig. 22.12 (*d*).

Hence, we can summarize as follows:

1. Transient current is an undamped sine wave if $R = 0$

2. Transient current is non-oscillatory if $R < 2\sqrt{L/C}$

3. Transient current is non-oscillatory if $R \geq 2\sqrt{L/C}$

4. Critical damping occurs if $R = 2\sqrt{L/C}$

Example 22.9. *A 5-μF capacitor is discharged suddenly through a coil having an inductance of 2H and a resistance of 200 Ω. The capacitor is initially charged to a voltage of 10 V. Find*

(a) *an expression for the current*

(b) *the additional resistance required to give critical damping.*

Solution. Since there is no battery or generator in the circuit (Fig. 22.13), the steady-state current must be zero. It means that resultant current is simply the transient current.

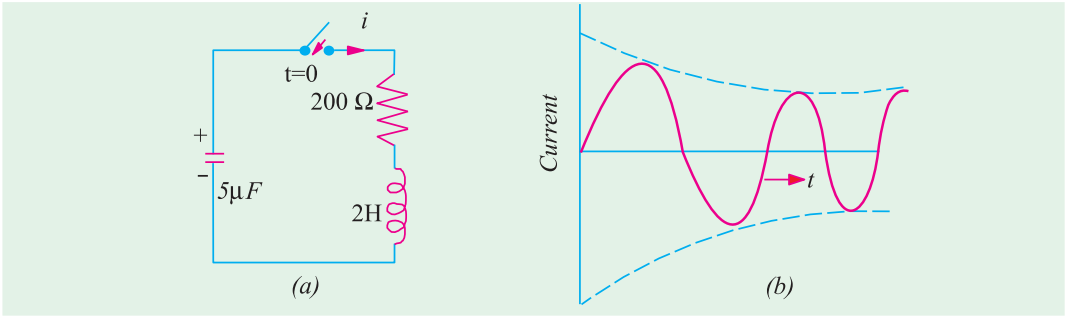


Fig. 22.13

Value of $2\sqrt{L/C} = 2\sqrt{2/5 \times 10^{-6}}$

$= 1265\ \Omega$

Since $R < 2\sqrt{L/C}$, the circuit is originally oscillatory.

(a) the expression for the transient current, therefore, is

$$i_t = I_m e^{-at} \sin(\omega t + \phi)$$

where $a = R/2L = 200/2 \times 2 = 50$

$$\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \quad \sqrt{100,000 \quad 2500} = 312.3 \text{ rad/s}$$

$\therefore \quad i_t = I_m e^{-50t} \sin(312.3t + \phi) = i$ ... **(*i*)**

Two initial conditions are known from which I_m and ϕ can be found (*a*) at $t = 0$; $i = 0$ and **(*b*)** at $t = 0$; $v_c = 10$ V. Applying condition **(*a*)** to Eq. (*i*), we get

$0 = I_m \sin\phi$, hence $\phi = 0 \quad \therefore i = I_m e^{-50t} \sin 312.3t$ ... **(*ii*)**

Now, at $t = 0$, the voltage across the inductance must be 10 V because the current in the resistance is zero.

i.e. $(Ldi/dt)_{t=0} = 10$ V $\therefore \ (di/dt)_{t=0} = 10/L = 5$ A/s ... **(*iii*)**

Now, from equation **(*ii*)**, we have

$$\frac{di}{dt} = -50 I_m e^{-50t} \sin 312.3t + 312.3 I_m e^{-50t} \cos 312.3t \quad \text{... (iv)}$$

Putting $t = 0$, it becomes $(di/dt)_{t=0} = 312.3 I_m$

From equation **(*iii*)**, we have

$312.3 I_m = 5 \qquad \therefore I_m = 5/312.3 = 0.016$ A

Hence, the general expression for the current becomes

$$\boldsymbol{i = 0.016\, e^{-50t} \sin 312.3t}$$

It is roughly plotted (the first few cycles only) in Fig. 22.13 (*b*).

(*b*) Critical damping is achieved when $R = 2\sqrt{L/C}$

$\therefore \quad R = 2\sqrt{2/5 \times 10^{-6}} = 1265\,\Omega$

$\therefore$ Additional resistance reqd. = 1265 – 200 = **1065** Ω

Example 22.10. *A damped oscillation has the equation* $i = 50e^{-10t}$ *sin 628t. Find the number of oscillations which occurs before the amplitude of the oscillations decays to 1/10th of its undamped value.*

Solution. Undamped amplitude = 50 A

1/10th amplitude = (1/10) × 50 = 5A

Let the time required for this decay be *t*. Now, the decay of the peak of the oscillations is given by the term $50e^{-10t}$

$\therefore \quad 5 = 50e_1^{-10t} \quad \therefore \ e_1^{10t_1} = 10$ or $10t_1 = \log h^{10} = 2.3 \log_{10}^{10} = 2.3$

$\therefore \qquad t_1 = 0.23$ second

Frequency of oscillations = $628/2\pi = 100$ Hz.

Hence, the number of oscillations which occur before the amplitude falls to 1/10th of its undamped value is = 0.23 × 100 = **23**

Example 22.11. *If, in Fig. 22.14, a break occurs at a point marked X, what would be the voltage across the break? It may be assumed that prior to the break, steady conditions existed in the circuit.*

Solution. Steady-state current through the inductance = 120/60 = 2A

Energy stored in the inductor prior to the break

$$= \frac{1}{2} LI^2 = \frac{1}{2} \times 12 \times 10^{-3} \times 4 = 24 \times 10^{-3}\ \text{J}$$

Energy initially stored in the capacitor

$$= \frac{1}{2} CV^2 = \frac{1}{2} \times 10^{-8} \times 120^2 = 72 \times 10^{-6}\ \text{J} = 0\text{-practically}$$

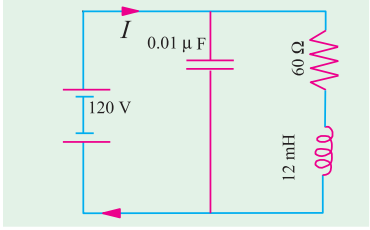


Fig. 22.14

When the break occurs, the energy stored in the inductor is transferred to the capacitor. If loss of energy during first transfer is neglected, then maximum energy stored in the capacitor is

$$= 20 \times 10^{-3}\ \text{J} \quad \therefore \quad \frac{1}{2}CV_m^2 = 24 \times 10^{-3}$$

$$\therefore \qquad V_m = \sqrt{2 \times 24 \times 10^{-3} \times 10^8} = 2{,}190\ \text{V}$$

Maximum voltage across the break is = 2190 + 120 = 2310 V

The voltage would be oscillatory because the energy alternates between the inductor and capacitor.

Frequency of voltage oscillation is

$$f = 1/2\pi\sqrt{LC} = 10^5/2\pi \times \sqrt{1.2} = 14{,}530\ \text{Hz}$$

Decay or damping factor = e^{-at} Art. 22.10, Case 2

Here, $a = R/2L = 60/2 \times 12 \times 10^{-3} = 2500$ $\therefore$ damping factor = e^{-2500t}

Hence, voltage across the break is

$$= 120 + 2190\,e^{-2500t} \sin 2\pi \times 14{,}500t = \mathbf{120 + 2190\,e^{2500t} \sin 91{,}290t}$$

Tutorial Problem No. 22.1

1. Deduce an expression for the growth of current in an inductive circuit.

A 15-H inductance coil of 10 Ω resistance is suddenly connected to a 20 V d.c. supply. Calculate:

(*a*) the initial rate of change of current

(*b*) the current after 2 second

(*c*) the rate of change of current after 2 second

(*d*) the energy stored in the magnetic field in this time

(*e*) the energy lost as heat in this time

(*f*) the time constant.

[(*a*) 1.33 A/s (*b*) 1.47 A (*c*) 0.352 A/s (*d*) 16.3 joules (*e*) 19.5 joules (*f*) 1.5 s]

2. A circuit consisting of a 20 Ω resistor in series with a 0.2 H inductor is supplied from 200 V (r.m.s.) 50 Hz a.c. mains. Deduce equations showing how the current varies with time if the supply is suddenly switched on (*a*) at the instant when the voltage is zero (*b*) at the instant when the voltage is a maximum.

[(*a*) 4.11 e^{-100} + 4.32 sin (314 t – 70°16') A (*b*) – 1.245 e^{-100t} + 4.32 cos (314t – 72°16') A]

3. A circuit consisting of a 20 Ω resistor, 20 mH inductor and a 100 μF capacitor in series is connected to a 200 V, d.c. supply. The capacitor is initially uncharged. Determine the equation relating the instantaneous current to the time and find the maximum instantaneous current.

[(20 e^{-500t} sin 500 t) A; 6.44 A]

4. Find an expression for the value of current at any instant after a sinusoidal voltage of amplitude 600 V at 50 Hz is applied to a series circuit of resistance 10 Ω and inductance 0.1 H, assuming that voltage is zero at the instant of switching. Also, find the value of transient current at t = 0.02 second.

[– 15.14 A; 2.17 A] (*Electric Circuits and Fields, Gujarat Univ.*)

5. A 40 Ω resistor and a 50 μF capacitor are connected in series and supplied with an alternating voltage v = 283 sin 314 t. The supply is switched on at the instant when the voltage is zero. Determine the expression for the instantaneous current at time t. **[– 3.18e^{-500t} + 3.76 sin (314 t + 57°50')]**

6. A d.c. voltage of 100 V is suddenly applied to a circuit consisting of a 100 Ω resistor, a 0.1 H inductor and a 100 μF capacitor in series. The capacitor is initially uncharged. Obtain the equation which shows how the capacitor voltage varies with time. **[100 – 115.3 e^{-500t} sin (866 t + π/3 V)]**

7. The voltage $v = 200 \sin 314t$ is suddenly applied at $t = 0$ to a circuit consisting of a 10 Ω resistor in series with a 0.1 H inductor. Deduce an equation showing how the current varies with time.

[$5.78\ e^{-100t} + 6.06 \sin (314\ t - 72°20')$]

8. A 20 Ω resistor, a 0.01 H inductor and a 100 μF capacitor are connected in series. A d.c. voltage of 100 V is suddenly applied to the circuit. Obtain the equation showing how the current through the circuit varies with time. Find the maximum current and the time at which it occurs.

[$10^4\ e^{-100t}$; 3.67 A; 0.001 second]

9. A $4 - \mu$ F capacitor is initially charged to 300 V. It is discharged through a 100 mH inductance and a resistor in series:

(*a*) find the frequency of the discharge if the resistance is zero.

(*b*) how many cycles at the above frequency will occur before the discharge oscillation decays to 1/10 of its initialy value if the resistance is 1 Ω.

(*c*) find the value of the resistance which would just prevent oscillations.

[(*a*) 796 Hz (*b*) 36.6 (*c*) 100 Ω]

OBJECTIVE TESTS – 22

1. Transient disturbance is produced in a circuit whenever

(*a*) it is suddenly connected or disconnected from the supply

(*b*) it is shorted

(*c*) its applied voltage is changed suddenly

(*d*) all of the above.

2. There are no transients in pure resistive circuits because they

(*a*) offer high resistance

(*b*) obey Ohm's law

(*c*) have no stored energy

(*d*) are linear circuits.

3. Transient currents in electrical circuit are associated with

(*a*) inductors

(*b*) capacitors

(*c*) resisters

(*d*) both (*a*) and (*b*).

4. The transients which are produced due to sudden but energetic changes from one steady state of a circuit to another are called ..... transients.

(*a*) initiation (*b*) transition

(*c*) relaxation (*d*) subsidence

5. In an *R-L* circuit connected to an alternating sinusoidal voltage, size of transient current primarily depends on

(*a*) the instant in the voltage cycle at which circuit is closed

(*b*) the peak value of steady-state current

(*c*) the circuit impedance

(*d*) the voltage frequency.

6. Double-energy transients are produced in circuits consisting of

(*a*) two or more resistors

(*b*) resistance and inductance

(*c*) resistance and capacitance

(*d*) resistance, inductance and capacitance.

7. The transient current in a loss-free *L-C* circuit when excited from an ac source is a/ an ..... sine wave.

(*a*) over damped (*b*) undamped

(*c*) under damped

(*d*) critically damped.

8. Transient current in an *R-L-C* circuit is oscillatory when

(*a*) $R = 0$ (*b*) $R > 2\sqrt{L/C}$

(*c*) $R < 2\sqrt{L/C}$ (*d*) $R = 2\sqrt{L/C}$

ANSWERS

1. (*d*) **2.** (*c*) **3.** (*d*) **4.** (*b*) **5.** (*a*) **6.** (*d*) **7.** (*b*) **8.** (*c*)

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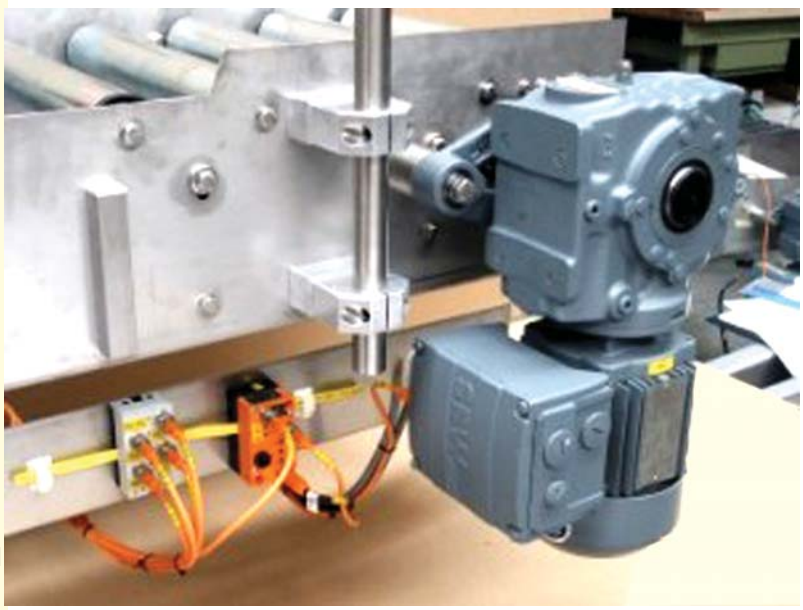
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CHAPTER 23

SYMMETRICAL COMPONENTS

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Any unbalanced 3-phase system of vectors (whether representing voltages or currents) can be resolved into three balanced systems of vectors which are called its 'symmetrical components'

23.1. Introduction

The method of symmetrical components was first proposed by C.L. Fortescue and has been found very useful in solving unbalanced polyphase circuits, for analytical determination of the

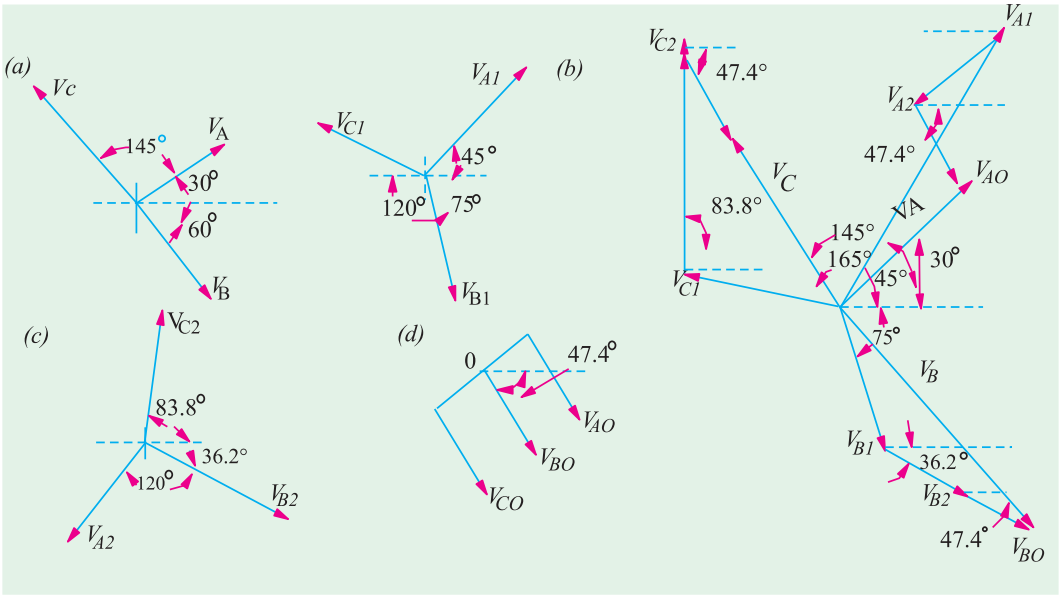


Fig. 23.1

Fig. 23.2

performance of polyphase electrical machinery when operated from a system of unbalanced voltages and for calculation of currents resulting from unbalanced faults. According to Fortescue's theorem, any unbalanced 3-phase system of vectors (whether representing voltages or currents) can be resolved into three ***balanced*** systems of vectors which are called its ***'symmetrical components'***. In Fig. 23.1 (*a*) is shown a set of three unbalanced voltage vectors V_A, V_B and V_C having phase sequence $A \rightarrow B \rightarrow C$. These can be regarded as made up of the following symmetrical components :-

(*i*) A balanced system of 3-phase vectors V_{A1}, V_{B1} and V_{C1} having the phase sequence $A \rightarrow B \rightarrow C$ as the original set of three unbalanced vectors. These vectors constitute the positive-sequence components [Fig. 23.1 (*b*)].

(*ii*) A balanced system of 3-phase vectors V_{A2}, V_{B2} and V_{C2} having phase sequence $A \rightarrow C \rightarrow B$ which is opposite to that of the original unbalanced vectors. These vectors constitute the negative-sequence components [Fig. 23.1 (*c*)].

(*iii*) A system of three vectors V_{A0}, V_{B0} and V_{C0} which are equal in magnitude and are in phase with each other *i.e.* $V_{A0} = V_{B0} = V_{C0}$. These three co-phasal vectors form a uniphase system and are known as zero-sequence components [Fig. 23.1(*d*).]

Hence, it means that an unbalanced 3-phase system of voltages or current can be regarded as due to the superposition of two symmetrical 3-phase systems having opposite phase sequences and a system of zero phase sequence *i.e.* ordinary single-phase current or voltage system. In Fig. 23.2, each of the original vectors has been



3-phase monitor continuously monitors 3-phase wave lines for abnormal conditions

reconstructed by the vector addition of its positive-sequence, negative - sequence and zero-sequence components. It is seen that

$$V_A = \overset{+ve}{V_{A1}} + \overset{-ve}{V_{A2}} + \overset{zero}{V_{A0}} \quad \text{... (i)}$$

$$V_B = V_{B1} + V_{B2} + V_{Bo} \quad \text{... (ii)}$$

$$V_C \quad V_{C1} \quad V_{C2} \quad V_{C0} \quad \text{... (iii)}$$

23.2. The Positive - sequence Components

As seen from above, the positive-sequence components have been designated as V_{A1}, V_{B1} and V_{C1}. The subscript 1 is meant to indicate that the vector belongs to the positive-sequence system. The letter refers to the original vector of which the positive-sequence vector is a component part.

These positive-sequence vectors are completely determined when the magnitude and phase of any one of these is known. Usually, these vectors are related to each other with the help of the operator a (for details, please refer to Art. 12.11). As seen from Fig. 23. 1 (*b*).

$$\mathbf{V}_{A1} = V_{A1};\ V_{B1} = a^2 V_{A1} = V_{A1} \angle -120°;\ \mathbf{V}_{C1} = a\mathbf{V}_{A1} == V_{A1} \angle 120°$$

23.3. The Negative - sequence Components

This system has a phase sequence of $A \to C \to B$. Since this system is also balanced, it is completely determined when the magnitude and phase of one of the vectors becomes known. The suffix 2 indicates that the vector belongs to the negative-sequence system. Obviously, as seen from Fig. From 23.1 (*c*).

$$\mathbf{V}_{A2} = V_{A2};\ \mathbf{V}_{B2} = a\,V_{A2} = \mathbf{V}_{A2} = \angle 120°\ ;\ \mathbf{V}_{C2} = a^2\mathbf{V}_{A2} = V_{A2} \angle -120°$$

23.4. The Zero - sequence Components

These three vectors are equal in magnitude and phase and hence form what is known as uniphase system. They are designated as V_{A0}, V_{B0} and V_{C0}. Since these are indentical in magnitude

$$\therefore \qquad V_{A0} = V_{B0} = V_{C0}$$

23.5. Graphical Composition of Sequence Vectors

Fig. 23.2 Shows how the original vector V_A has been obtained by the addition of V_{A1} , V_{A2} and V_{A0}. The same applies to other vectors V_B and V_C.

For simplicity, let us write V_{AI} as V_1, V_{A2} as V_2 and V_{A0} as V_0. Then

$$\mathbf{V}_A = V_1 + \mathbf{V}_2 + \mathbf{V}_0 \quad \text{... (iv)}$$

$$\mathbf{V}_B = a^2\mathbf{V}_1 + a\mathbf{V}_2 + \mathbf{V}_0 \quad \text{... (v)}$$

$$\mathbf{V}_C = a\mathbf{V}_1 + a^2\mathbf{V}_2 + \mathbf{V}_0 \quad \text{... (vi)}$$

23.6. Evaluation of V_{A1} or V_1

The procedure for evaluating $\mathbf{V}_1$ is as follows :

Multiplying **(*v*)** by *a* and **(*vi*)** by a^2, we get

$$a\mathbf{V}_B = a^3\mathbf{V}_1 + a^2\mathbf{V}_2 + a\mathbf{V}_0; \qquad a^2\mathbf{V}_C = a^3\mathbf{V}_1 + a^4\mathbf{V}_2 + a^2\mathbf{V}_0$$

Now $a^3 = 1$and $a^4 = a$, hence

$$a\mathbf{V}_B = \mathbf{V}_1 + a^2\mathbf{V}_2 + a\mathbf{V}_0 \quad \text{... (vii)}$$

$$a^2\mathbf{V}_C = \mathbf{V}_1 + a\mathbf{V}_2 + a^2\mathbf{V}_0 \quad \text{.... (viii)}$$

Adding **(iv)**, **(vii)** and **(viii)**, we get

$$\mathbf{V}_A + a\mathbf{V}_B + a^2\mathbf{V}_C = 3\mathbf{V}_1 + \mathbf{V}_2(1 + a + a^2) + \mathbf{V}_0(1 + a + a^2) = 3\mathbf{V}_1$$

$$\therefore \quad \mathbf{V}_1 = \frac{1}{3}(\mathbf{V}_A + a\mathbf{V}_B + a^2\mathbf{V}_C) = \frac{1}{3}(\mathbf{V}_A + \mathbf{V}_B\angle 120° + \mathbf{V}_C\angle -210°)$$

$$\frac{1}{3}\ V_A \quad V_B \quad \frac{1}{2} \quad j\frac{\sqrt{3}}{2} \quad V_C \quad \frac{1}{2} \quad j\frac{\sqrt{3}}{2}$$

This shows that, geometrically speaking, V_1 is a vector one-third as large as the vector obtained by the vector addition of the three original vectors V_A, $V_B \angle 120°$ and $V_C \angle -120°$.

23.7. Evaluation of V_{A2} or V_2

Multiplying **(vi)** by a and **(v)** by a^2 and adding them to **(iv)** we get

$$a\mathbf{V}_C = a^2\mathbf{V}_1 + a^3\mathbf{V}_2 + a\mathbf{V}_0; a^2\mathbf{V}_B = a^4\mathbf{V}_1 + a^3\mathbf{V}_2 + a^2\mathbf{V}_0$$

$\mathbf{V}_A + a^2\mathbf{V}_B + a\mathbf{V}_C = \mathbf{V}_1(1 + a + a^2) + 3\mathbf{V}_2 + \mathbf{V}_0(1 + a + a^2) = 3\mathbf{V}_2$ Now , $1 + a + a^2 = 0$

$$\therefore \quad \mathbf{V}_2 = \frac{1}{3}(\mathbf{V}_A + a^2\mathbf{V}_B + a\mathbf{V}_C) = \frac{1}{3}(\mathbf{V}_A + \mathbf{V}_B\angle -120° + \mathbf{V}_C\angle 120°)$$

$$= \frac{1}{3}\left[\mathbf{V}_A + \mathbf{V}_B\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) + \mathbf{V}_C\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)\right]$$

23.8. Evaluation of V_{A0} or V_0

Adding **(iv)**, **(v)** and **(vi)**, we get $\mathbf{V}_A + \mathbf{V}_B + \mathbf{V}_C = \mathbf{V}_1(1 + a + a^2) + \mathbf{V}_2(1 + a + a^2) + 3\mathbf{V}_0 = 3\mathbf{V}_0$

$$\therefore \quad \mathbf{V}_0 = \frac{1}{3}(\mathbf{V}_A + \mathbf{V}_B + \mathbf{V}_C)$$

It shows that V_0 is simply a vector one third as large as the vector obtained by adding the original vectors V_A, V_B and V_C.

To summarize the above results, we have

(i) $\mathbf{V}_1 = \frac{1}{3}(\mathbf{V}_A + a\mathbf{V}_B + a^2\mathbf{V}_C)$ **(ii)** $\mathbf{V}_2 \quad \frac{1}{3}(\mathbf{V}_A \quad a^2\mathbf{V}_B \quad a\mathbf{V}_C)$

(iii) $\mathbf{V}_0 = \frac{1}{3}(\mathbf{V}_A + \mathbf{V}_B + \mathbf{V}_C)$

Note. An unbalanced system of 3-phase currents can also be likewise resolved into its symmetrical components. Hence

$$\mathbf{I}_A = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_0; \mathbf{I}_B = a^2\mathbf{I}_1 + a\mathbf{I}_2 + \mathbf{I}_0; \mathbf{I}_C = a\mathbf{I}_1 + a^2\mathbf{I}_2 + \mathbf{I}_0$$

Also, as before $\mathbf{I}_1 = \frac{1}{3}(\mathbf{I}_A + a\mathbf{I}_B + a^2\mathbf{I}_C); \mathbf{I}_2 = \frac{1}{3}(\mathbf{I}_A + a^2\mathbf{I}_B + a\mathbf{I}_C); \mathbf{I}_0 = \frac{1}{3}(\mathbf{I}_A + \mathbf{I}_B + \mathbf{I}_C)$ ... **(ix)**

It shows that I_0 is one-third of the neutral or earth-return current and is zero for an unearthed 3-wire system. It is seen from **(ix)** above that I_0 is zero if the vector sum of the original current vectors is zero. This fact can be used with advantage in making numerical calculations because the original system of vectors can then be reduced to two balanced 3-phase systems having opposite phase sequences.

Example 23.1. *Find out the positive, negative and zero-phase sequence components of the following set of three unbalanced voltage vectors:*

$$V_A = 10\angle 30°; V_B = 30\angle -60°; V_C = 15\angle 145°$$

Indicate on an approximate diagram how the original vectors and their different sequence components are located. **(Principles of Elect. Engg. – I, Jadavpur Univ.)**

Solution. (*i*) Positive-sequence vectors

As seen from Art. 23.6

$$\mathbf{V}_1 \quad \frac{1}{3}(\mathbf{V}_A \quad a\mathbf{V}_B \quad a^2\mathbf{V}_C) \quad \frac{1}{3}(10 \quad 30 \quad a.30 \quad 60 \quad a^2.15 \quad 145\ )$$

$$= \frac{1}{3}(10\angle 30° + 30\angle 60° + 15\angle 25°) = 12.42 + j12.43 = 17.6\angle 45°$$

$$\therefore \quad \mathbf{V}_{A|} \quad 17.6 \quad 45\ ; \ \mathbf{V}_{B|} \quad 17.6 \quad 45 \quad 120 \quad 17.6 \quad 75$$

$$\mathbf{V}_{C1} \quad 17.6 \quad 45 \quad 120 \quad 17.6 \quad 165$$

These are shown in Fig. 23.1 (*b*)

(*ii*) Negative–sequence vectors

As seen from Art. 23.7,

$$\mathbf{V}_2 \quad \frac{1}{3}(\mathbf{V}_A \quad a^2\mathbf{V}_B \quad a\mathbf{V}_C) \quad \frac{1}{3}(10 \quad 30 \quad a^2.30 \quad 60 \quad a.15 \quad 145\ )$$

$$= \frac{1}{3}(10\angle 30° + 30\angle -180° + 15\angle 265°) = -7.55 - j3.32 = 8.24\angle -156.2°$$

$$\mathbf{V}_{A2} \quad 8.24 \quad 156.2\ ; \mathbf{V}_{B2} \quad 8.24 \quad 156.2 \quad 120 \quad 8.24 \quad 36.2$$

$$\mathbf{V}_{C2} \quad 8.24 \quad 156.2 \quad 120 \quad 8.24 \quad 276.2$$

These vectors are shown inFig. 23.1 (*c*)

(*iii*) Zero sequence vectors

$$\mathbf{V}_0 = \frac{1}{3}(\mathbf{V}_A + \mathbf{V}_B + \mathbf{V}_C)$$

$$= \frac{1}{3}(10\angle 30° + 30\angle -60° + 15\angle 145°) = 3.8 - j4.12 = 5.6\angle -47.4°$$

These vectors are shown in Fig. 23.1 (*d*).

Example 23.2. *Explain how an unsymmetrical system of 3-phase currents can be resolved into 3 symmetrical component systems.*

Determine the values of the symmetrical components of a system of currents

$I_R = 0 + j120A; I_Y = 50 - j100A; I_B = -100 - j50A$

Phase sequence is RYB. **(Elect. Engg.-I Bombay, Univ.)**

Solution. $\mathbf{I}_R \quad 0 \quad j120 \quad 120 \quad 90$

$$\mathbf{I}_Y \quad 50 \quad j100 \quad 111.8 \quad 63.5\ ; \mathbf{I}_B \quad 100 \quad j50 \quad 111.8 \quad 153.5$$

(*i*) Positive–sequence Components

$$\mathbf{I}_1 \quad \frac{1}{3}(\mathbf{I}_R \quad a\mathbf{I}_Y \quad a^2\mathbf{I}_B) \quad \frac{1}{3}(0 \quad j120) \quad \frac{1}{2} \quad j\frac{\sqrt{3}}{2}(50 \quad j100) \quad \frac{1}{2} \quad j\frac{\sqrt{3}}{2}(\ 100 \quad j5\ 0)$$

$= 22.8 + j108.3 = 110.7\angle 78.1°$ $\therefore \mathbf{I}_{R1}$ 110.7 78.1 ; $\mathbf{I}_{Y1}$ 110.7 41.9 ; $\mathbf{I}_{B1}$ 110.7 198.1

(*ii*) Negative–sequence components

$\mathbf{I}_2$ $\frac{1}{3}(\mathbf{I}_R$ $a^2\mathbf{I}_Y$ $a\mathbf{I}_B)$ $\frac{1}{3}($ 18.3 $j65.1)$ 6.1 $j21.7$ 22.5 105.7

$\therefore$ $\mathbf{I}_{R2}$ 22.5 105.7 ; $\mathbf{I}_{Y2}$ 22.5 225 ; $\mathbf{I}_{B2}$ 22.5 14.3

(*iii*) Zero-sequence component

$\mathbf{I}_0$ $\frac{1}{3}(\mathbf{I}_R$ $\mathbf{I}_Y$ $\mathbf{I}_B)$ $\frac{1}{3}[(0$ $j120)$ $(50$ $j100)$ $($ 100 $j50)]$ 16.7 $j10$

As a check, it may be found that

$\mathbf{I}_R = \mathbf{I}_{R1} + \mathbf{I}_{R2} + \mathbf{I}_0; \mathbf{I}_Y = \mathbf{I}_{Y1} + \mathbf{I}_{Y2} + \mathbf{I}_0; \mathbf{I}_{B1} + \mathbf{I}_{B2} + \mathbf{I}_0$

Example 23.3. *In a 3-phase, 4-wire system, the currents in the R, Y and B lines under abnormal conditions of loading were as follows:*

$$I_R = 100\angle 30°; I_Y = 50\angle 300°; I_B = 30\angle 180°$$

Calculate the positive, negative and zero-phase sequence currents in the R-line and the return current in the neutral conductor.

Solution. (*i*) The positive-sequence components of current in the *R*-line is

$$\mathbf{I}_1 = \frac{1}{3}(\mathbf{I}_R + a\mathbf{I}_Y + a^2\mathbf{I}_B)$$

Now $\mathbf{I}_R$ 100 30 $50(\sqrt{3}$ $j)$

$\mathbf{I}_Y$ 50 300 50 $\frac{1}{2}$ $j\frac{\sqrt{3}}{2}$ $25(1$ $j\sqrt{3})$

$\mathbf{I}_B = 30\angle 180° = (-30 + j0)$

$\mathbf{I}_1$ $\frac{1}{3}$ $50(\sqrt{3}$ $j)$ $25(1$ $j\sqrt{3}$ $)$ $\frac{1}{2}$ $j\frac{\sqrt{3}}{2}$ $($ 30) $\frac{1}{2}$ $j\frac{\sqrt{3}}{2}$ 58 48.4

(*ii*) The negative–sequence components of the current in the *R*-line is

$$\mathbf{I}_2 = \frac{1}{3}(\mathbf{I}_R + a^2\mathbf{I}_Y + a\mathbf{I}_B)$$

$= \frac{1}{3}$ $50(\sqrt{3}$ $j)$ $25(1$ $j\sqrt{3})$ $\frac{1}{2}$ $j\frac{\sqrt{3}}{2}$ $($ 30) $\frac{1}{2}$ $j\frac{\sqrt{3}}{2}$ 18.9 24.9

(*iii*) The zero–sequence component of current in the *R*-line is

$\mathbf{I}_0$ $\frac{1}{3}(\mathbf{I}_R$ $\mathbf{I}_Y$ $\mathbf{I}_B)$ $\frac{1}{3}[50(\sqrt{3}$ $j)$ $25(1$ $j\sqrt{3})$ 30] 27.2 4.7

The neutral current is

$\mathbf{I}_N$ $\mathbf{I}_R$ $\mathbf{I}_Y$ $\mathbf{I}_B$ 3 $\mathbf{I}_0$ 3 27.2 4.7 81.6 4.7

Example 23.4. *A 3-phase, 4-wire system supplies loads which are unequally distributed on the three phases. An analysis of the currents flowing in the direction of the loads in the R, Y and B lines shows that in the R-line, the positive phase sequence current is 200 $\angle 0°$ A and the*

negative phase sequence current is 100 $\angle 60°$. The total observed current flowing back to the supply in the neutral conductor is 300 $\angle 300°$ A. Calculate the currents in phase and magnitude in the three lines.

Assuming that the 3-phase supply voltages are symmetrical and that the power factor of the load on the R-phase is $\sqrt{3/2}$ leading, determine the power factor of the loads on the two other phases.

Solution. It is given that in *R*-phase [Fig. 23.3 (*a*)]

$$\mathbf{I}_{R1} = 200\angle 0° = (200 + j0)\,\text{A}; \mathbf{I}_{R2} = 100\angle 60° = (50 + j86.6)\,A$$

$$\mathbf{I}_{R0} = \frac{1}{3}\mathbf{I}_N = (300/3)\angle 300° = (50 - j86.6)\,\text{A}$$

$\mathbf{I}_R$ $\mathbf{I}_{R1}$ $\mathbf{I}_{R2}$ $\mathbf{I}_{R0}$ (200 $j0$) (50 $j86.6$) (50 $j86.6$) (300 $j0$) 300 0

Similarly, as seen from Fig. 23.3 (*b*) for the *Y*-phase

$$\mathbf{I}_Y = \mathbf{I}_{Y1} + \mathbf{I}_{Y2} + \mathbf{I}_{Y0} = a^2\mathbf{I}_{R1} + a\mathbf{I}_{R2} + \mathbf{I}_{R0}$$

$$= 200\angle 0° - 120° + 100\angle 60° + 120°$$

$$+ 100\angle 300° = -100 - j173.2 - 100 + 50 - j86.6 = -150 - j259.8 = 300\angle 240°\,\text{A}$$

Similarly, as seen from Fig. 23-3 (*c*) for the *B*-phase

$\mathbf{I}_B$ $\mathbf{I}_{B1}$ $\mathbf{I}_{B2}$ $\mathbf{I}_{B0}$ $a\mathbf{I}_{R1}$ $a^2\mathbf{I}_{R2}$ $\mathbf{I}_{R0}$ 200 0 120 100 60 120 100 300 0

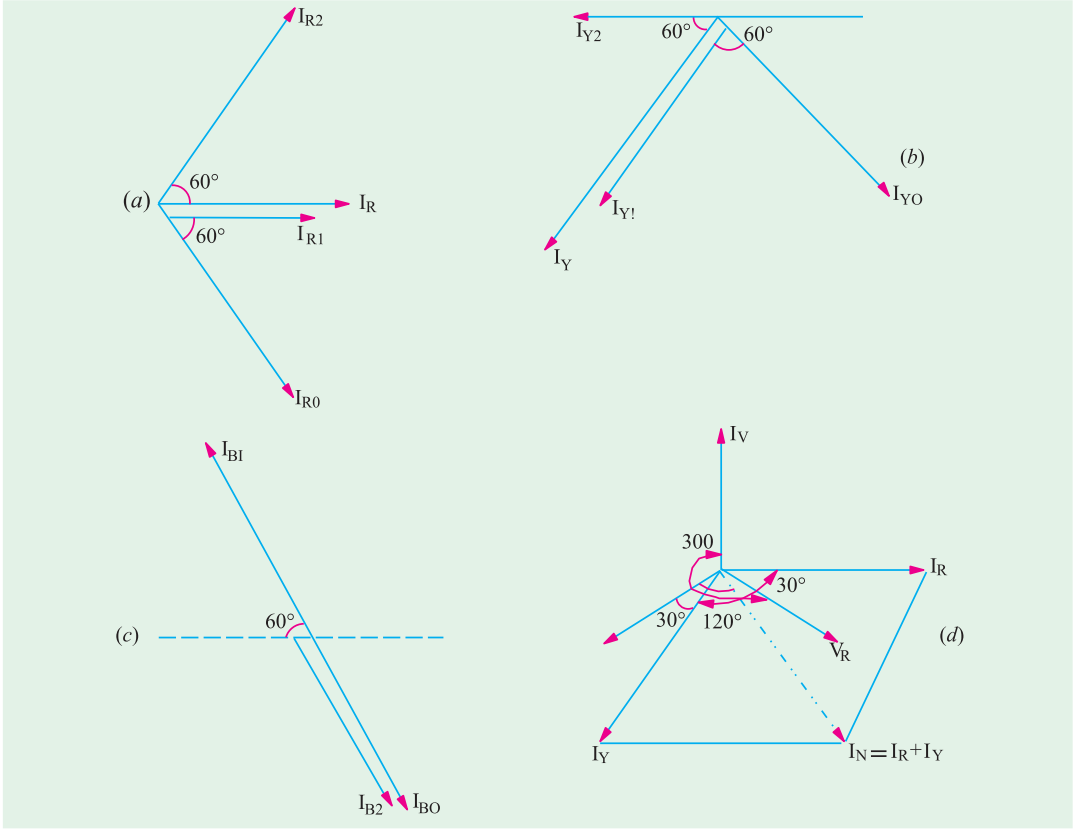


Fig. 23.3

Since the power factor of the *R*-phase is $\sqrt{3}/2$ leading, the current I_R leads the voltage V_R by 30° [Fig. 23.3(*d*)]

(*d*) Now, phase angle of I_Y is 240° relative to I_R so that I_Y leads its voltage V_Y by 30°. Hence, power factor of *Y* phase is also $\sqrt{3}/2$ leading. The power factor of *B* line is indeterminate because the current in this line is zero.

Example 23.5. *Prove that in a 3-phase system if V_1, V_2 and V_3 are the three balanced voltages whose phasor sum is zero, the positive and negative sequence components can be expressed as*

$$V_{1p} = \left\{\frac{1}{\sqrt{3}}(V_1 + V_2\angle 60°)\right\}\angle 30°; V_{1N} = \left\{\frac{1}{\sqrt{3}}(V_1 + V_2\angle -60°)\right\}\angle -30°$$

Phase sequence is 1-2-3.

A system of 3-phase currents is given as $I_1 = 10\ \angle 180°$, $I_2 = 14.14\ \angle -45°$ and $I_3 = 10\ \angle 90°$. Determine phasor expression for the sequence components of these currents. Phase sequence is 1-2-3. **(Elect. Engg-I, Bombay Univ.)**

Solution. As seen from Art. 23.6

$$\mathbf{V}_{1P} = \frac{1}{3}(\mathbf{V}_1 + a\mathbf{V}_2 + a^2\mathbf{V}_3); \text{Now, } \mathbf{V}_1 + \mathbf{V}_2 + \mathbf{V}_3 = 0 \ \therefore\ \mathbf{V}_3 = -(\mathbf{V}_1 + \mathbf{V}_2)$$

$$\mathbf{V}_{1P} = \frac{1}{3}[\mathbf{V}_1 + a\mathbf{V}_2 - a^2(\mathbf{V}_1 + \mathbf{V}_2)] = \frac{1}{3}[\mathbf{V}_1(1 - a^2) + \mathbf{V}_2(a - a^2)]$$

Now, $1 - a^2 = \frac{2}{3} + j\frac{\sqrt{3}}{2}$ and $a - a^2 = j\sqrt{3}$

$$\therefore\ \mathbf{V}_{1P} \quad \frac{1}{3}\ \mathbf{V}_1\ \frac{3}{2}\ j\frac{\sqrt{3}}{2} \quad j\sqrt{3}\mathbf{V}^2 \quad \frac{1}{\sqrt{3}}\ \mathbf{V}_1\ \frac{\sqrt{3}}{2}\ j\frac{1}{2} \quad j\mathbf{V}_2$$

$$= \frac{1}{\sqrt{3}}[\mathbf{V}_1\angle 30° + \mathbf{V}_2\angle 90°] = \left[\frac{1}{\sqrt{3}}(\mathbf{V}_1 + \mathbf{V}_2\angle 60°)\right]\angle 30°$$

Similarly, the negative-sequence component is given by

$$\mathbf{V}_{1N} = \frac{1}{3}(\mathbf{V}_1 + a^2\mathbf{V}_2 + a\mathbf{V}_3) = \frac{1}{3}[\mathbf{V}_1 + a^2\mathbf{V}_2 - a(\mathbf{V}_1 + \mathbf{V}_2)] = \frac{1}{3}[\mathbf{V}_1(1 - a) + \mathbf{V}_2(a^2 - a)]$$

Now, $a^2 - a = -j\sqrt{3}$

$$\therefore\ \mathbf{V}_{1N} = \frac{1}{3}\left[\mathbf{V}_1\left(\frac{3}{2} - j\frac{\sqrt{3}}{2}\right) - j\sqrt{3}\mathbf{V}_2\right] = \frac{1}{\sqrt{3}}\left[\mathbf{V}_1\left(\frac{\sqrt{3}}{2} - j\frac{1}{2}\right) - j\mathbf{V}_2\right]$$

$$= \frac{1}{\sqrt{3}}[\mathbf{V}_1\angle -30° + \mathbf{V}_2\angle -90°] = \left\{\frac{1}{\sqrt{3}}(\mathbf{V}_1 + \mathbf{V}_2\angle -60°)\right\}\angle -30°$$

Now $\mathbf{I}_1 = 10\angle 180° = -10 + j0; \mathbf{I}_2 = 14.14\angle -45° = 10 - j10$

$\mathbf{I}_3 = 10\angle 90° = j10$

$a\mathbf{I}_2$ 14.14 75 3.66 $j13.66$; $a^2\mathbf{I}_2$ 14.14 165 13.66 $j3.66$

$a\mathbf{I}_3$ 14.14 21 12.25 $j707$; $a^2\mathbf{I}_3$ 14.14 30 12.25 $j7.07$

$$\mathbf{I}_{1P} \quad \frac{1}{3}(\mathbf{I}_1 \quad a\mathbf{I}_2 \quad a^2\mathbf{I}_3) \quad \frac{1}{3}(5.91 \quad j6.59) \quad 1.97 \quad j2.2$$

$$\mathbf{I}_{1N} \quad \frac{1}{3}(\mathbf{I}_1 \quad a^2\mathbf{I}_2 \quad a\mathbf{I}_3) \quad \frac{1}{3}(\ 35.91 \quad j10.73) \quad 11.97 \quad j3.58$$

$$\mathbf{I}_{10} = \frac{1}{3}(\mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3) = 0$$

Tutorial Problems No. 23.1

1. The following currents were recorded in the *R*, *Y*, and *B* lines of a 3-phase system under abnormal conditions: **[$\mathbf{I_R} = 300\angle 300°$ A; $\mathbf{I_Y} = 500\angle 240°$ A; $\mathbf{I_B} = 1{,}000\angle 60°$ A]**

Calculate the values of the positive, negative and zero phase-sequence components.

[$\mathbf{I_1} = 536\angle -44°20'$ A; $\mathbf{I_2} = 372\angle 171°$ A; $\mathbf{I_0} = 145\angle 23°20'$]

2. Determine the symmetrical components of the three currents $I_0 = 10\angle 0°$; $\mathbf{I}_b = 100\angle 250°$ and $\mathbf{I}_C = 10\angle 110°$ A **[$\mathbf{I_1}$ = (39.45 + *j*522) A; $\mathbf{I_2}$ = (– 20.24 + *j*22.98) A; $\mathbf{I_0}$ = (– 9.21 – *j*28.19) A]**

(Elect. Meas & Measuring Instru., Madras Univ.)

3. The three current vectors of a 3-phase, four-wire system have the following values; $\mathbf{I_A} = 7 + j_0$, $\mathbf{I_B} = -12 - j13$ and $\mathbf{I_C} = -2 + j3$. Find the symmetrical components. The phase sequence is *A*, *B*, *C*

[$\mathbf{I_0}$ = – 2.33 – *j*3.33; $\mathbf{I_{A1}}$ = (27.75 – *j*3.67); $\mathbf{I_{B1}}$ = (– 17.05 – *j*22.22); $\mathbf{I_{C1}}$ = (– 10.7 + *j*25.9); $\mathbf{I_{A2}}$ = (0.25 + *j*13.67) A; $\mathbf{I_{B2}}$ = (– 11.97 – *j*6.62) A; $\mathbf{I_{C2}}$ = (11.73 – *j*7.05) A]

23.9. Zero-Sequence Components of Current and Voltage

Any circuit which allows the flow of positive-sequence currents will also allow the flow of negative-sequence currents because the two are similar. However, a fourth wire is necessary if zero-sequence components are to flow in the lines of the 3-phase system. It follows that the line currents of 3-phase 3-wire system can contain no zero sequence components whether it is delta - or star-connected. The zero sequence components of line-to-line voltages are non-existent regardless of the degree of imbalance in these voltages. It means that a set of unbalanced 3-phase, line-to-line voltages may be represented by a positive system and a negative system of balanced voltages. This fact is of considerable importance in the analysis of 3-phase rotating machinery. For example, the operation of an induction motor when supplied from an unbalanced system of 3-phase voltages, may be analysed on the basis of two balanced systems of voltages of opposite phase sequence.

Let us consider some typical 3-phase connections with reference to zero-sequence components of current and voltage.

(*a*) Four-wire Star Connection. Due to the presence of the fourth wire, the zero sequence currents may flow. The neutral wire carries only the zero-sequence current which is the sum of the zero-sequence currents in the three lines. Since the sum of line voltages is zero, there can be no zero sequence component of line voltages.

(*b*) Three-wire Star Connection. Since there is no fourth or return wire, zero-sequence components of current cannot flow. The absence of zero-sequence currents may be explained by considering that the impedance offered to these currents is infinite and that this impedance is situated between the star points of the generator and the load. If the two star points were joined by a neutral, only zero-sequence currents will flow through it so that only zero-sequence voltage can exist between the load and generator star points. Obviously, no zero-sequence component of voltage appears across the phase load.

(*c*) Three-wire Delta Connection. Due to the absence of fourth wire, zero-sequence components of currents cannot be fed into the delta-connected load. However, though line currents

have to sum up to zero (whereas phase currents need not do so) it is possible to have a zero-sequence current circulating in the delta-connected load.

Similarly, individual phase voltages will generally possess zero-sequence components though components are absent in the line-to line voltage.*

23.10. Unbalanced Star Load Supplied from Unbalanced Three-phase Three-wire System

In this case, line voltages and load currents will consist of only positive and negative-sequence components (but no zero-sequence component). But load voltages will consist of positive, negative and zero-sequence components.

Let the line voltages be denoted by $\mathbf{V}_{RY}$, $\mathbf{V}_{YB}$ and $\mathbf{V}_{BR}$, line (and load) currents by $\mathbf{I}_R$, $\mathbf{I}_Y$ and $\mathbf{I}_B$, the load voltages by $\mathbf{V}_{RN}$, $\mathbf{V}_{YN}$ and $\mathbf{V}_{BN}$ and load impedances by $\mathbf{Z}_R$, $\mathbf{Z}_Y$, and $\mathbf{Z}_B$ (their values being the same for currents of any sequence).

Obviously, $\mathbf{V}_{RN} = \mathbf{I}_R\mathbf{Z}_R$; $\mathbf{V}_{YN} = \mathbf{I}_Y\mathbf{Z}_Y$ and $\mathbf{V}_{BN} = \mathbf{I}_B\mathbf{Z}_B$

If $\mathbf{V}_{RN1}$, $\mathbf{V}_{RN2}$ and $\mathbf{V}_0$ are the symmetrical components of $\mathbf{V}_{RN}$, then we have

$$\mathbf{V}_0 = \frac{1}{3}(\mathbf{V}_{RN} + \mathbf{V}_{YN} + \mathbf{V}_{BN}) = \frac{1}{3}(\mathbf{I}_R\mathbf{Z}_R + \mathbf{I}_Y\mathbf{Z}_Y + \mathbf{I}_B\mathbf{Z}_B)$$

$$= \frac{1}{3}[\mathbf{Z}_R(\mathbf{I}_{R1} + \mathbf{I}_{R2}) + \mathbf{Z}_Y(\mathbf{I}_{Y1} + \mathbf{I}_{Y2}) + \mathbf{Z}_B(\mathbf{I}_{B1} + \mathbf{I}_{B2})]$$

$$= \frac{1}{3}[\mathbf{Z}_R(\mathbf{I}_{R1} + \mathbf{I}_{R2}) + \mathbf{Z}_Y(a^2\mathbf{I}_{R1} + a\mathbf{I}_{P2}) + \mathbf{Z}_B(a\mathbf{I}_{R1} + a^2\mathbf{I}_{R2})]$$

$$= \mathbf{I}_{R1}.\frac{1}{3}(\mathbf{Z}_R + a^2\mathbf{Z}_Y + a\mathbf{Z}_B) + \mathbf{I}_{R2}.\frac{1}{3}(\mathbf{Z}_R + a\mathbf{Z}_Y + a^2\mathbf{Z}_B) = \mathbf{I}_{R1}\mathbf{Z}_{R2} + \mathbf{I}_{R2}\mathbf{Z}_{R1}$$

... ***(i)***

$$\mathbf{V}_{RN1} = \frac{1}{3}(\mathbf{V}_{RN} + a\mathbf{V}_{YN} + a^2\mathbf{V}_{BN}) = \frac{1}{3}(\mathbf{I}_R\mathbf{Z}_R + a\mathbf{I}_Y\mathbf{Z}_Y + a^2\mathbf{I}_B\mathbf{Z}_B)$$

$$= \frac{1}{3}[\mathbf{Z}_R(\mathbf{I}_{R1} + \mathbf{I}_{R2})] + a\mathbf{Z}_Y(\mathbf{I}_{Y1} + \mathbf{I}_{Y2}) + a^2\mathbf{Z}_B(\mathbf{I}_{B1} + \mathbf{I}_{B2})$$

$$= \mathbf{I}_{R1}.\frac{1}{3}(\mathbf{Z}_R + \mathbf{Z}_Y + \mathbf{Z}_B) + \mathbf{I}_{R2}.\frac{1}{3}(\mathbf{Z}_R + a^2\mathbf{Z}_Y + \mathrm{aZ}_B) = \mathbf{I}_{R1}\mathbf{Z}_0 + \mathbf{I}_{R2}\mathbf{Z}_{R2} \quad \text{... } \textit{(ii)}$$

$$\mathbf{V}_{RN2} = \mathbf{I}_{R2}\mathbf{Z}_0 + \mathbf{I}_{R1}\mathbf{Z}_{R1}$$

Similarly, $\mathbf{V}_{YN1} = \mathbf{I}_{Y1}\mathbf{Z}_0 + \mathbf{I}_{Y2}\mathbf{Z}_{Y2} = a^2(\mathbf{I}_{R1}\mathbf{Z}_0 + \mathbf{I}_{R2}\mathbf{Z}_{R2}) = a^2\mathbf{V}_{RN1}$

$$\mathbf{V}_{BN1} = \mathbf{I}_{B1}\mathrm{Z}_0 + \mathbf{I}_{B2}\mathbf{Z}_{B2} = a(\mathbf{I}_{R1}\mathbf{Z}_0 + \mathbf{I}_{R2}\mathbf{Z}_{R2}) = a\mathbf{V}_{RN1}$$

$$\mathbf{V}_{YN2} = \mathbf{I}_{Y2}\mathbf{Z}_0 + \mathbf{I}_{Y1}\mathbf{Z}_{Y1} = a(\mathbf{I}_{R2}\mathbf{Z}_0 + \mathbf{I}_{R1}\mathbf{Z}_{R1}) = a\mathbf{V}_{RN2}$$

$$\mathbf{V}_{BN2} = \mathbf{I}_{B2}\mathbf{Z}_0 + \mathbf{I}_{B1}\mathbf{Z}_{B1} = a^2(\mathbf{I}_{R2} + \mathbf{Z}_0 + \mathbf{I}_{R1}\mathbf{Z}_{R1}) = a^2\mathbf{V}_{RN2}$$

Now, $\mathbf{V}_{RN1}$ and $\mathbf{V}_{RN2}$ may be determined from the relation between the line and phase voltages as given below:

$$\mathbf{V}_{RY} = \mathbf{V}_{RN} + \mathbf{V}_{NY} = \mathbf{V}_{RN} - \mathbf{V}_{YN} = \mathbf{V}_0 + \mathbf{V}_{RN1} + \mathbf{V}_{RN2} - (\mathbf{V}_0 + \mathbf{V}_{RN1} + \mathbf{V}_{YN2}]$$

---

* However, under balanced conditions, the phase voltages will possess no zero-sequence components.

$$= \mathbf{V}_{RN1}(1-a^2)+\mathbf{V}_{RN2}(1-a)=\frac{1}{2}\sqrt{3}[\mathbf{V}_{RN1}(\sqrt{3}+j1)+\mathbf{V}_{RN2}(\sqrt{3}-j1)]$$

... ***(iii)***

$$\mathbf{V}_{YB}=\mathbf{V}_{YN}+\mathbf{V}_{NB}=\mathbf{V}_{YN}-\mathbf{V}_{BN}=\mathbf{V}_0+\mathbf{V}_{YN1}+\mathbf{V}_{YN2}-(\mathbf{V}_0+\mathbf{V}_{BN1}+\mathbf{V}_{BN2})$$

$$= \mathbf{V}_{RN1}(a^2-a)-\mathbf{V}_{RN2}(a-a^2)=-j\sqrt{3}.\mathbf{V}_{RN1}+j\sqrt{3}\mathbf{V}_{RN2}$$

$$\therefore\ \mathbf{V}_{RN2}=\mathbf{V}_{RN1}-j\mathbf{V}_{YB}/\sqrt{3}$$

Substituting this value of $\mathbf{V}_{RN2}$ in Eq. ***(iii)*** above and simplifying, we have

$$\mathbf{V}_{RN1}=\frac{1}{3}[\mathbf{V}_{RY}+\frac{1}{2}\mathbf{V}_{YB}(1+j\sqrt{3})]$$

... ***(iv)***

$$\mathbf{V}_{RN2}=\frac{1}{3}[\mathbf{V}_{RY}+\frac{1}{2}\mathbf{V}_{YB}(1-j\sqrt{3})]$$

... ***(v)***

Having known $\mathbf{V}_{RN1}$ and $\mathbf{V}_{RN2}$, the currents $\mathbf{I}_{R1}$ and $\mathbf{I}_{R2}$ can be determined from the following equations:

$$\mathbf{I}_{R1}=\frac{(\mathbf{V}_{RN1}\mathbf{Z}_0-\mathbf{V}_{RN2}\mathbf{Z}_{R2})}{\mathbf{Z}_0^2-\mathbf{Z}_{R1}\mathbf{Z}_{R2}}$$

... ***(vi)***

$$\mathbf{I}_{R2}=\frac{(\mathbf{Z}_{RN2}\mathbf{Z}_0-\mathbf{V}_{RN1}\mathbf{Z}_{R1})}{\mathbf{Z}_0^2-\mathbf{Z}_{R1}\mathbf{Z}_{R2}}$$

... ***(vii)***

Alternatively, when $\mathbf{I}_{R1}$ becomes known, $\mathbf{I}_{R2}$ may be found from the relation

$$\mathbf{I}_{R2}=\frac{\mathbf{V}_{RN2}-\mathbf{I}_{R1}\mathbf{Z}_{R1}}{\mathbf{Z}_0}$$

... ***(viii)***

The symmetrical components of the currents in other phases can be calculated from $\mathbf{I}_{R1}$ and $\mathbf{I}_{R2}$ by using the relations given in Art. 23.8.

The phase voltages may be calculated by any one of the two methods given below:

(i) directly by calculating the products $\mathbf{I}_R\mathbf{Z}_R$, $\mathbf{I}_Y\mathbf{Z}_Y$ and $\mathbf{I}_B\mathbf{Z}_B$.

(ii) by first calculating the zero-sequence component $\mathbf{V}_0$ of the phase or load voltages and then adding this to the appropriate positive and negative sequence components.

For example, $\mathbf{V}_{RN}=\mathbf{V}_0+\mathbf{V}_{RN1}+\mathbf{V}_{RN2}$

23.11. Unbalanced Star Load Supplied from Balanced Three-phase, Three-wire System

It is a special case of the general case considered in Art. 23.10 above. In this case, the symmetrical components of the load voltages consist only of positive and zero-sequence components. This fact may be verified by substituting the value of **VYB** in Eq. (*v*) of Art. 23.10. Now, in a balanced or symmetrical system of positive-phase sequence

$$\mathbf{V}_{YB}\quad \mathbf{V}_{RY}\quad \frac{1}{2}\quad j\frac{\sqrt{3}}{2}$$

Substituting this value in Eq. ***(v)*** above, we have

$$\mathbf{V}_{RN2}=\frac{1}{3}[\mathbf{V}_{RY}+\frac{1}{2}\mathbf{V}_{YB}(1-j\sqrt{3})]$$

$$= \frac{1}{3}\left[\mathbf{V}_{RY}+\frac{1}{2}.\mathbf{V}_{RY}\left(-\frac{1}{2}-j\frac{\sqrt{3}}{2}\right)(1-j\sqrt{3})\right]=\frac{1}{3}-(\mathbf{V}_{RY}-\mathbf{V}_{RY})\ =0$$

Hence, substituting this value of $\mathbf{V}_{RN2}$ in Eq. (*iv*) and (*vi*) of Art. 23.10, we get

$$\mathbf{V}_{RN1} = j\mathbf{V}_{YB}\sqrt{3} = \frac{1}{2}\mathbf{V}_{RY} - j\frac{1}{2}\mathbf{V}_{RY}/\sqrt{3} \quad \text{... (i)}$$

$$\mathbf{I}_{R1} = \mathbf{V}_{RN1}\mathbf{Z}_0/(\mathbf{Z}_0^2 - \mathbf{Z}_{R1}\mathbf{Z}_{R2}) = \mathbf{V}_{RN1}\frac{\mathbf{Z}_R + \mathbf{Z}_Y + \mathbf{Z}_B}{\mathbf{Z}_R\mathbf{Z}_Y + \mathbf{Z}_Y\mathbf{Z}_B + \mathbf{Z}_B\mathbf{Z}_R} \quad \text{... (ii)}$$

$$\mathbf{I}_{R2} = -\mathbf{I}_{R1}.\mathbf{Z}_{R1}/\mathbf{Z}_0 = -\mathbf{R}\mathbf{I}_0 = -\mathbf{I}_{R1}\frac{\mathbf{Z}_R + a\mathbf{Z}_Y + a^2\mathbf{Z}_B}{\mathbf{Z}_R + \mathbf{Z}_Y + \mathbf{Z}_B} \quad \text{... (iii)}$$

Example 23.6 illustrates the procedure for calculating the current in an unbalanced star-connected load when supplied from a symmetrical three-wire system.

Example 23.6. *A symmetrical 3-phase, 3-wire, 440-V system supplies an unbalanced Y-connected load of impedances $Z_R = 5 \angle 30°\ \Omega$; $Z_Y = 10 \angle 45°\ \Omega$, $Z_B = 10 \angle 60°\ \Omega$. Phase sequence is $R \to Y \to B$. Calculate in the rectangular complex form the symmetrical components of the currents in the R-line.* **(Elect. Engg.-I, Bombay Univ.)**

Solution. As shown in Fig. 23.4, the branch impedances are

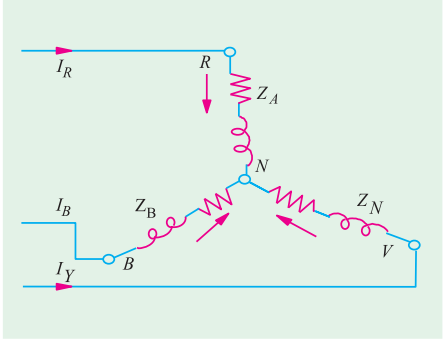


Fig. 23.4

$\mathbf{Z}_R$ 5 30 (4.33 j2.5)

$\mathbf{Z}_Y$ 10 45 (7.07 j7.07)

$\mathbf{Z}_B$ 10 60 (5 j8.66)

$a\mathbf{Z}_Y = 10\angle 165° = 10(-0.966 + j0.259) = -9.66 + j2.59\,\Omega$

$a^2\mathbf{Z}_Y$ 10 75 10(0.259 j0.966) 2.59 j9.66

$a\mathbf{Z}_B$ 10 180 (10 j0)

$a^2\mathbf{Z}_B$ 10 60 10(0.5 j0.866) (5 j8.66)

The symmetrical component impedances required for calculation purposes are as follows:

$$\mathbf{Z}_0 \quad \frac{1}{3}(\mathbf{Z}_R \quad \mathbf{Z}_Y \quad \mathbf{Z}_B) \quad \frac{1}{3}(16.4 \quad j18.23) \quad (5.47 \quad j6.08)$$

$$\mathbf{Z}_{R1} = \frac{1}{3}(\mathbf{Z}_R + \mathbf{Z}_Y + a^2\mathbf{Z}_B)$$

$$= \frac{1}{3}(4.33 + j2.5 - 9.66 + j2.59 + 5 - j8.66)$$

$$= \frac{1}{3}(-0.33 - j3.57) = -0.11 - j1.19$$

$$\mathbf{Z}_{R2} = \frac{1}{3}(\mathbf{Z}_R + a^2\mathbf{Z}_Y + a\mathbf{Z}_B)$$

$$= \frac{1}{3}(4.33 + j2.5 + 2.59 - j9.66 - 10 + j0) = \frac{1}{3}(-3.08 - j7.16) = -1.03 - j2.55$$

Now

$$\mathbf{Z}_0{}^2\mathbf{Z}_{R1}\mathbf{Z}_{R2} = \frac{1}{3}(\mathbf{Z}_R\mathbf{Z}_Y + \mathbf{Z}_Y\mathbf{Z}_B + \mathbf{Z}_B\mathbf{Z}_R)$$

$$= \frac{1}{3}[50 \quad 75 \quad 100 \quad 105 \quad 50 \quad 90\ ] \quad (\ 4.31 \quad j65)$$

Let $\mathbf{V}_{RY}$ be taken as the reference vectors so that $\mathbf{V}_{RY} = (440 + j0)$

Then, $\mathbf{V}_{RN1} = \frac{1}{2}\mathbf{V}_{RY} - j.\frac{1}{2}\mathbf{V}_{RY}/\sqrt{3} = \frac{1}{2}(440 + j0) - j\frac{1}{2}(440 + j0)/\sqrt{3} = (220 - j127)$

Now, $\mathbf{I}_{R_1} \quad \frac{\mathbf{V}_{RN1}.\mathbf{Z}_0}{(\mathbf{Z}_0{}^2 \quad \mathbf{Z}_{R_1}\mathbf{Z}_{R_2})} \quad \frac{(220 \quad j127)(5.47 \quad j6.08)}{(\ 4.31 \quad j65)} \quad \frac{1986 \quad j643}{(\ 4.31 \quad j65)}$

$$= \frac{2088\angle 18°}{66.4\angle 93.8°} = 31.6\angle -75.8° \neq (7.75 - j30.6)\text{A}$$

$$\mathbf{I}_{R_2} = -\mathbf{I}_{R_2}\mathbf{Z}_{R_1}/\mathbf{Z}_0$$

$$= \frac{-(7.75 - j30.6)(-0.11 - j1.19)}{(5.47 + j6.08)} = \frac{(37.25 + j5.88)}{(5.47 + j6.08)}$$

$$= \frac{37.75\angle 9°}{8.18\angle 48°} = 4.61\angle -39° = (3.28 - j2.66)\text{A}$$

Hence, the symmetrical components of $\mathbf{I}_R$ are:-

Positive-sequence component = (7.75 – *j*30.6) A

Negative-sequence component = (3.28 – *j*2.66) A

Note. (*i*) $\mathbf{I}_R = \mathbf{I}_{R1} + \mathbf{I}_{R2}$

(*ii*) Symmetrical components of other currents are

$$\mathbf{I}_Y = a^2\mathbf{I}_{B1} \text{ and } \mathbf{I}_{Y2} = a\mathbf{I}_{R2} \ ; \ \mathbf{I}_{B1} = a\mathbf{I}_{R1} \text{ and } \mathbf{I}_{B2} = a^2\mathbf{I}_{R2}$$

Example 23.7. *A balanced star-connected load takes 75 A from a balanced 3-phase, 4-wire supply. If the fuses in two of the supply lines are removed, find the symmetrical components of the line currents before and after the fuses are removed.*

Solution. The circuit is shown in Fig. 23.5.

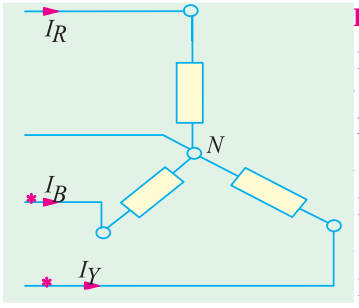


Fig. 23.5

Before uses are removed

$\mathbf{I}_R \quad 75 \quad 0\ ;\ \mathbf{I}_Y \quad 75 \quad 120\ ;\ \mathbf{I}_B \quad 75 \quad 120$

$\mathbf{I}_1 \quad \frac{1}{3}(\mathbf{I}_R \quad a\mathbf{I}_Y \quad a^2\mathbf{I}_B) \quad \frac{1}{3}(75 \quad 0 \quad 75 \quad 0 \quad 75 \quad 360\ ) \quad 75 \quad 0 \quad \text{A}$

$\mathbf{I}_2 \quad \frac{1}{3}(\mathbf{I}_R \quad a^2\mathbf{I}_Y \quad a\mathbf{I}_B) \quad \frac{1}{3}(75 \quad 0 \quad 75 \quad 120 \quad 75 \quad 240\ ) \quad 0$

$\mathbf{I}_0 \quad \frac{1}{3}(\mathbf{I}_R \quad \mathbf{I}_Y \quad \mathbf{I}_B) \quad \frac{1}{3}(75 \quad 0 \quad 75 \quad 120 \quad 75 \quad 120\ ) \quad 0.$

After fuses are removed

$$\mathbf{I}_R = 75\angle 0°; \mathbf{I}_Y = \mathbf{I}_B = 0$$

$\mathbf{I}_1 \quad \frac{1}{3}(75 \quad 0 \quad 0 \quad 0) \quad 25 \quad 0$

$\mathbf{I}_2 = 25\angle 0°, \mathbf{I}_0 = 25\angle 0°$

Example 23.8. *Explain the terms: positive-, negative- and zero-sequence components of a 3-phase voltage system. A star-connected load consists of three equal resistors, each of 1 Ω resistance. When the load is connected to an unsymmetrical 3-phase supply, the line voltages are 200 V, 346 V and 400 V.*

Find the magnitude of the current in any one phase by the method of symmetrical components. **(Power Systems-II, A.M.I.E.)**

Solution. This question could be solved by using the following two methods:

(a) As shown in Fig. 23.6(*a*), the line voltages form a closed right-angled triangle with an angle = $\tan^{-1}(346/200) = \tan^{-1}(1.732) = 60°$

Hence, if $\mathbf{V}_{AB} = 200 \angle 0°$, then $\mathbf{V}_{BC} = 346 \angle -90° = -j346$ and $\mathbf{V}_{CA} = 400 \angle 120°$.

As seen from Eq. (*iv*) and (*v*) of Art. 23.10

$$\mathbf{V}_{AN1} = \frac{1}{3}\left[\mathbf{V}_{AB} + \frac{1}{2}\mathbf{V}_{BC}(1 + j\sqrt{3})\right]$$

$$= \frac{1}{3}\left[200 + \left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)(-j346)\right] = 166.7 - j57.7$$

$$\mathbf{V}_{AN2} = \frac{1}{3}\left[\mathbf{V}_{AB} + \mathbf{V}_{BC}\left(\frac{1}{2} - \frac{\sqrt{3}}{2}\right)\right]$$

$$= \frac{1}{3}\left[200 + \left(\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)(-j346)\right] = -33.3 - j57.7$$

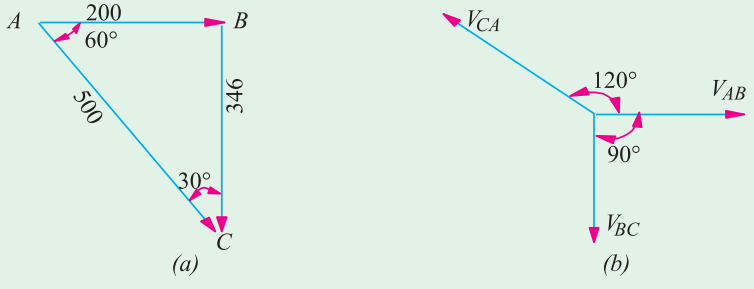


Fig. 23.6

Now, $\mathbf{Z}_A + \mathbf{Z}_B + \mathbf{Z}_C = 3$

$\mathbf{Z}_{A1} = \mathbf{Z}_A + a\mathbf{Z}_B + a^2\mathbf{Z}_C = 0$ (because $\mathbf{Z}_A$ $\mathbf{Z}_B$ $\mathbf{Z}_C$ and 1 a a^2 0)

Similarly, $\mathbf{Z}_{A2} = \mathbf{Z}_A + a^2\mathbf{Z}_B + a\mathbf{Z}_C = 0$

Hence, from Eq. ***(ii)*** of Art. 23.10 we have $\mathbf{V}_{AN1} = \mathbf{I}_{A1}\mathbf{Z}_0 \therefore \mathbf{I}_{A1} = \mathbf{A}_{AN1}/\mathbf{Z}_0$

Now, $\mathbf{Z}_0$ $\frac{1}{3}(\mathbf{Z}_A$ $\mathbf{Z}_B$ $\mathbf{Z}_C)$ 3/3 1 $\therefore$ $\mathbf{I}_{A1}$ = (166.7 – *j*57.7) A

$\mathbf{I}_{AS} = \mathbf{V}_{AN2}/\mathbf{Z}_0$ = – 33.3 – *j*57.7 $\therefore$ $\mathbf{I}_A = \mathbf{I}_{A1} + \mathbf{I}_{A2}$ ($\mathbf{I}_0$ 0)

$\therefore$ $\mathbf{I}_A$ (166.7 *j*57.7) (33.3 *j*57.7) 133.4 *j*115.4 176.4 40.7 A

(*b*) Using Millman's theorem and taking the line terminal *A* as reference point, the voltage between *A* and the neutral point *N* is

$$\mathbf{V}_{NA} \quad \frac{\mathbf{V}_{BA}\mathbf{Y}_B \quad \mathbf{V}_{CA}\mathbf{Y}_C}{\mathbf{Y}_A \quad \mathbf{Y}_B \quad \mathbf{Y}_C} \quad \frac{200 \quad 180 \quad 1 \quad 400 \quad 120 \quad 1}{3} \quad 133.3 \quad j115.3 \quad 176.4 \quad 139.3$$

$$\mathbf{V}_{AN} \quad 176.4 \quad 139.3 \quad 180 \quad 176.4 \quad 40.7 \quad \therefore \mathbf{I}_A \quad \mathbf{I}_{AN} / \mathbf{Z} \quad 176.4 \quad 40.7 \quad \text{... as before}$$

Example 23.9. *Three equal impedances of (8 + j6) are connected in star across a 3-phase, 3-wire supply. The phase voltages are V_A = (220 + j0), V_B = (– j220) and V_C = (– 100 + j220) V. If there is no connection between the load neutral and the supply neutral, calculate the symmetrical components of A-phase current and the three line currents.*

Solution. Since there is no fourth wire, there is no zero-component current. Moreover, $\mathbf{I}_A + \mathbf{I}_B + \mathbf{I}_C = 0$. The symmetrical components of the A-phase voltages are

$$\mathbf{V}_{0} \quad \frac{1}{3}(220 \quad j220 \quad 100 \quad j220) \quad 40\,\text{V} \ *$$

$$\mathbf{V}_{A1} \quad \frac{1}{3}[220 \quad (\ 0.5 \quad j0.866)(\ j220) \quad (\ 0.5 \quad j0.866)(\ 100 \quad j220)]$$

$$= \frac{1}{3}[(660 + j86.6)] = (220 + j28.9)\,\text{V}$$

$$\mathbf{V}_{A2} \quad \frac{1}{3}[220 \quad (\ 0.5 \quad j0.866)(\ j220) \quad (\ 0.5 \quad j0.866)(\ 100 \quad j220)]$$

$$= \frac{1}{3}(-120 - j86.6) = (-40 - j28.9)\,\mathbf{V}$$

The component currents in phase A are

$$\mathbf{I}_{A1} = \frac{\mathbf{V}_{A1}}{(8 + j6)} = \frac{220 + j28.9}{(8 + j6)} = (19.33 - j10.89)\,\text{A}$$

$$\mathbf{I}_{A2} \quad \frac{\mathrm{V}_{A2}}{(8 \quad j6)} \quad \frac{(\ 40 \quad j28.9)}{(8 \quad j6)} \quad (\ 4.93 \quad j0.09)\,\text{A}$$

$$\mathbf{I}_A = \mathbf{I}_{A1} + \mathbf{I}_{A2} = (14.4 - j10.8)\text{A}; \mathbf{I}_B = \mathbf{I}_{B1} + \mathbf{B}_{B2} = a^2\mathbf{I}_{A1} + a\mathbf{I}_{A2}$$

$$\mathbf{I}_B \quad \frac{1}{2} \quad j\frac{\sqrt{3}}{2} \ (19.93 \quad j10.89) \quad \frac{1}{2} \quad j\frac{\sqrt{3}}{2} \ (\ 4.93 \quad j0.09)$$

Example 23.10. *Two equal impedance arms AB and BC are connected to the terminals A, B, C of a 3-phase supply as shown in Fig. 23.7. Each capacitor has a reactance of $X = \sqrt{3}R$. A high impedance voltmeter V is connected to the circuit at points P and Q as shown. If the supply line voltages V_{AB}, V_{BC}, V_{CA} are balanced, determine the reading of the voltmeter (a) when the phase sequence of the supply voltages is $A \to B \to C$ and (b) when the phase sequence is reversed. Hence, explain how this network could be employed to measure, respectively, the positive and negative phase sequence voltage components of an unbalanced 3-phase supply.*

* However, it is not required in the problem.

Solution. The various currents and voltages are shown in Fig. 23.7.

Phase Sequence ABC

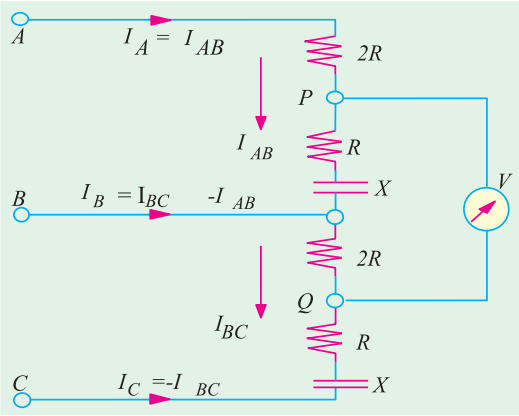


Fig. 23.7

Taking $\mathbf{V}_{AB}$ as the reference vector, we have

$\mathbf{V}_{AB} = V(1 + j0); \mathbf{V}_{BC} = a^2 V; \mathbf{V}_{CA} = aV$

As seen from the diagram,

$\mathbf{I}_{AB}$ $\frac{\mathbf{V}_{AB}}{3R \quad jX}$ $\frac{V}{3R \quad j\sqrt{3}R}$ $\frac{V}{\sqrt{3}R(\sqrt{3} \quad j)}$

$\mathbf{I}_{BC}$ $\frac{\mathbf{V}_{BC}}{3R \quad jX}$ $\frac{a^2 V}{\sqrt{3}R(\sqrt{3} \quad j)}$

Now, $\mathbf{V}_{PO} + \mathbf{V}_{OQ} = \mathbf{V}_{PQ}$

Also, $\mathbf{V}_{PQ} = \mathbf{I}_{AB}(R - jX) + \mathbf{I}_{BC}.2R$

$= \frac{V}{\sqrt{3}R(\sqrt{3} - j)}.R(1 - j\sqrt{3}) + \frac{a^2 V}{\sqrt{3}R(\sqrt{3} - j)}.2R$

$= \frac{V}{\sqrt{3}(\sqrt{3} - j)}.(1 - j\sqrt{3} + 2a^2)$ a^2 $\frac{1}{2}$ $j\frac{\sqrt{3}}{2}$

$= \frac{V}{\sqrt{3}(\sqrt{3} - j)}.(-j2\sqrt{3})$ $\therefore V_{PQ}$ $\frac{V}{\sqrt{3} \quad 2}$ $3\sqrt{3} \quad V$

Hence, the voltmeter which is not phase sensitive will read the line voltage when phase sequence is $A \rightarrow B \rightarrow C$.

Phase Sequence ACB

In this case, $\mathbf{V}_{AB} = V(1 + j0)$; $\mathbf{V}_{BC} = aV$; $\mathbf{V}_{CA} = a^2 V$

Also, $\mathbf{V}_{PQ} = \mathbf{V}_{PO} + \mathbf{V}_{OQ}$

$= \mathbf{I}_{AB}(R - jX) + \mathbf{I}_{BC}.2R. = \frac{V(R - jX)}{\sqrt{3}.R(\sqrt{3} - j)} + \frac{aV.2R}{\sqrt{3}.R(\sqrt{3} - j)}$

$= \frac{V}{\sqrt{3}(\sqrt{3} - j)}(1 - j\sqrt{3} + 2a) = \frac{V}{\sqrt{3}(\sqrt{3} - j)}(1 - j\sqrt{3} - 1 + j\sqrt{3}) = 0$

Hence, when the phase sequence is reversed, the voltmeter reads zero.

It can be proved that with sequence *ABC*, the voltmeter reads the positive sequence component ($\mathbf{V}_1$) and with phase sequence *ACB*, it reads the negative sequence component ($\mathbf{V}_2$) with phase sequence *ABC*

$\mathbf{V}_{PQ}$ $\frac{\mathbf{V}_{AB}}{3R \quad j\sqrt{3}.R}(R \quad j\sqrt{3}R)$ $\frac{\mathbf{V}_{BC}}{3R \quad j\sqrt{3}R}2R$ $\frac{1}{3 \quad j\sqrt{3}}[(\mathbf{V}_1 \quad \mathbf{V}_2)(1 \quad j\sqrt{3}) \quad 2(a^2\mathbf{V}_1 \quad a\mathbf{V}_2)]$

$= \frac{1}{3 - j\sqrt{3}}[\mathbf{V}_1 1 - j\sqrt{3} + 2a^2) + \mathbf{V}_2(1 - j\sqrt{3} + 2a)] = \frac{1}{3 - j\sqrt{3}}\mathbf{V}_1(-j2\sqrt{3})$ $\therefore \mathbf{V}_{PQ} = \mathbf{V}_1$

With phase sequence *ACB*

$$\mathbf{V}_{PQ} \quad \frac{1}{3 \quad j\sqrt{3}} [\ (\mathbf{V}_1 \quad \mathbf{V}_2)\ (1 \quad j\sqrt{3}) \quad 2(a\ \mathbf{V}_1 \quad a^2\ \mathbf{V}_2\ )]$$

$$= \frac{1}{3 - j\sqrt{3}} [\mathbf{V}_1(1 - j\sqrt{3} + 2a) + \mathbf{V}_2(1 - j\sqrt{3} + 2a^2)]$$

$$= \frac{1}{3 - j\sqrt{3}} [\mathbf{V}_2(-j2\sqrt{3})] \quad \therefore \quad V_{PQ} = V_2$$

23.12. Measurement of Symmetrical Components of Circuits

The apparatus consists of two identical current transformers, two impedances of the same ohmic value, one being more inductive than the other to the extent that its phase angle is 60° greater and two identical ammeters A_1 greater and two identical ammeters A_1 and A_2 as shown in Fig. 23.8 (*a*). It can be shown that A_1 reads positive-sequence current only while A_2 reads negative-sequence current only. If the turn ratio of the current transformer is *K*, then keeping in mind that zero-sequence component is zero, we have

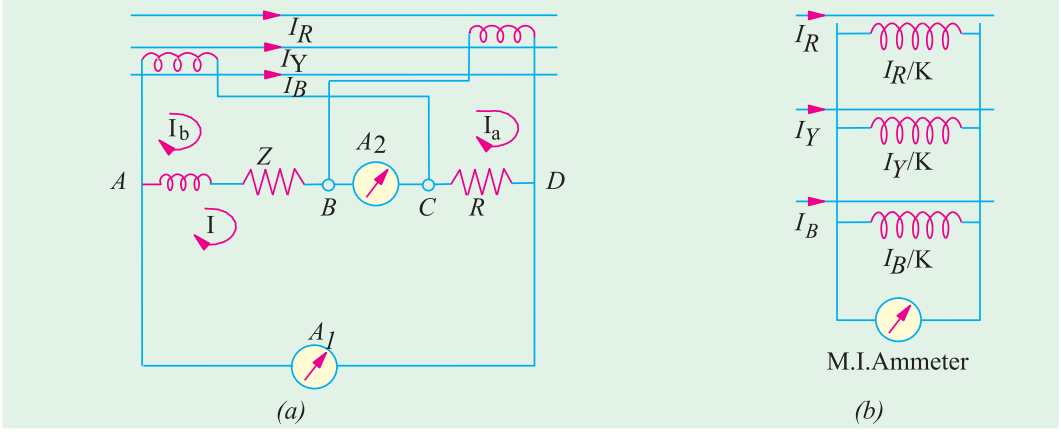


Fig. 23.8

$$\mathbf{I}_a = \mathbf{I}_R / K = (\mathbf{I}_1 + \mathbf{I}_2) / K$$

where $\mathbf{I}_1$ and $\mathbf{I}_2$ are the positive and negative-sequence components of the line current respectively.

Similarly,

$$\mathbf{I}_b = \mathbf{I}_Y / K = (a^2\mathbf{I}_1 + a\mathbf{I}_2) / K$$

If the impedance of each ammeter is $R_A + jK_A$, then impedance between points *B* and *D* is

$$\mathbf{Z}_{BD} \quad (R \quad R_A \quad jX_A)$$

The value of **Z** is so chosen that



Current transformers

$$\mathbf{Z}_{AC} \quad Z \quad R_A \quad jX_A \quad (R \quad R_A \quad jX_A) \quad 60 \quad Z_{BD} \quad 60$$

For finding the current read by A_1, imagine a break at point *X*. Thevenin voltage across the break *X* is

$$\mathbf{V}_{th} = \mathbf{I}_b\mathbf{Z}_{AC} + \mathbf{I}_a\mathbf{Z}_{BD} = \mathbf{I}_b\mathbf{Z}_{BD} = \mathbf{I}_b\mathbf{Z}_{BD}\angle 60° + \mathbf{I}a\mathbf{Z}_{BD} = (\mathbf{I}_b\angle 60° + \mathbf{I}_a)\mathbf{Z}_{BD}$$

Total impedance in series with this Thevenin voltage is

$$\mathbf{Z}_T \quad \mathbf{Z}_{AC} \quad \mathbf{Z}_{BD} \quad \mathbf{Z}_{BD} \quad 60 \quad \mathbf{Z}_{BD} \quad \frac{3}{2} \quad j\frac{\sqrt{3}}{2} \quad \mathbf{Z}_{BD}$$

The current flowing normally through the wire in which a break has been imagined is

$$\mathbf{I} \quad \frac{\mathbf{V}_{th}}{\mathbf{Z}_T} = \frac{\mathbf{I}_b\angle 60° + \mathbf{I}_a}{(3/2) + j(\sqrt{3}/2)} = \frac{1}{K}\frac{(a^2\mathbf{I}_1 + a\mathbf{I}_2)\angle 60° + \mathbf{I}_1 + \mathbf{I}_2}{\sqrt{3}\angle 30°}$$

$$= \frac{1}{K}\frac{I_1(1 \quad 300 \quad 1) \quad I_2(1 \quad 180 \quad 1)}{\sqrt{3} \quad 30} \quad \frac{I_1}{K} \quad 60$$

It means that A_1 reads positive-sequence current only. The ammeter A_2 reads current which is

$$= \mathbf{I}_a \quad \mathbf{I}_b \quad \mathbf{I} = \frac{1}{K}(I_1 + I_2 + I_1\angle 240° + I_2\angle 120° - I_2\angle -60°)$$

$$= \frac{I_1}{K}\left(1 - \frac{1}{2} - j\frac{\sqrt{3}}{2} - \frac{1}{2} + j\frac{\sqrt{3}}{2}\right) + \frac{I_2}{K}(\mathrm{I} + 1\angle 120°) = \frac{I_2}{K}\angle 60°$$

In other words, A_2 reads negative-sequence current only.

Now, it will be shown that the reading of the moving-iron ammeter of Fig. 23-8(*b*) is proportional to the zero-sequence component.

$$\mathbf{I}_0 = \frac{1}{3}(\mathbf{I}_R + \mathbf{I}_Y + \mathbf{I}_B) = \frac{K}{3}\left(\frac{\mathbf{I}_R}{K} + \frac{\mathbf{I}_Y}{K} + \frac{\mathbf{I}_B}{K}\right)$$

$$= \frac{K}{3} \times (\text{current through the ammeter}) = \frac{K}{3} \times \text{ammeter reading.}$$

It is obvious that for I_0 to be present, the system must be 3-phase, 4-wire. However, when the fourth wire is available, then I_0 may be found directly by finding the neutral current I_N. In that case $\mathbf{I}_0 \quad \frac{1}{3}I_N$.

23.13. Measurement of Positive and Negative-sequence *Voltages

With reference to Fig. 23.9 (*a*), it can be shown that the three voltmeters indicate only the positive-sequence component of the 3-phase system.

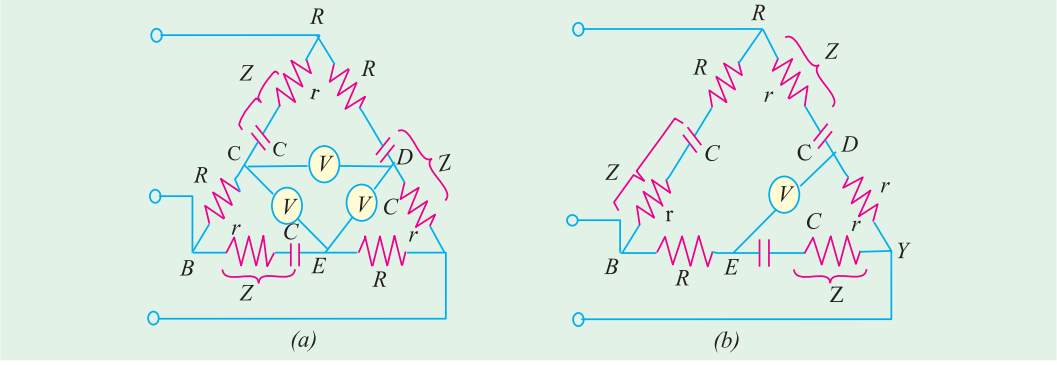


Fig. 23.9

* It is supposed to possess infinite impedance.

The line-to-neutral voltage can be written (with reference to the red phase) as

$$\mathbf{V}_{RN} = \mathbf{V}_1 + \mathbf{V}_2 + \mathbf{V}_0; \mathbf{V}_{YN} = a^2\mathbf{V}_1 + a\mathbf{V}_2 + \mathbf{V}_0; \mathbf{V}_{BN} = a\mathbf{V}_1 + a^2\mathbf{V}_2 + \mathbf{V}_0$$

$$\mathbf{V}_{RY} = \mathbf{V}_{RN} + \mathbf{V}_{NY} = \mathbf{V}_{RN} - \mathbf{V}_{YN} = (1-a^2)\mathbf{V}_1 + (1-a)\mathbf{V}_2$$

$$\mathbf{V}_{YB} = \mathbf{V}_{YN} + \mathbf{V}_{NB} = \mathbf{V}_{YN} - \mathbf{V}_{BN} = (a^2-a)\mathbf{V}_1 + (a-a^2)\mathbf{V}_2$$

$$\therefore \quad \mathbf{V}_{DY} \quad \frac{\mathbf{V}_{RY}(r \quad 1/j \quad C)}{(R \quad r \quad 1/j \quad C)} \qquad \mathbf{V}_{YE} \quad \frac{\mathbf{V}_{YB}.R}{(R \quad r \quad 1/j \quad C)}$$

$$\therefore \quad \mathbf{V}_{DE} \quad \frac{(r \quad 1/j \quad C)}{(R \quad r \quad 1/j \quad C)}\mathbf{V}_{RY} \quad \frac{R}{(r \quad 1/j \quad C)}\mathbf{V}_{YB}$$

The different equation of the bridge circuit are so chosen that

$$\frac{R}{r + j1/\omega C} = -a^2 = \frac{1}{2} + j\frac{\sqrt{3}}{2} \quad \text{or} \quad R = \frac{r}{2} + \frac{\sqrt{3}}{2\omega C} + \frac{1}{j2\omega C} + j\frac{\sqrt{3}r}{2}$$

Equating the *j*-terms or quadrature terms on both sides, we have

$$0 = \frac{1}{j2\omega C} + j\frac{\sqrt{3}}{2} \qquad \therefore \quad \frac{1}{\omega C} = \sqrt{3}r \qquad \text{... } \textit{(i)}$$

Similarly, equating the reference or real terms, we have

$$R = \frac{r}{2} + \frac{\sqrt{3}}{2\omega C} = \frac{r}{2} + \frac{3r}{2} = 2r \qquad \text{... } \textit{(ii)}$$

$$\therefore \quad \frac{r + 1/j\omega C}{R + r + 1/\omega C} = \frac{r - j\sqrt{3}r}{3r - j\sqrt{3}r} = \frac{r(1-j\sqrt{3})}{r(3-j\sqrt{3})} = \frac{1}{\sqrt{3}}\angle -30°$$

and $$\mathbf{V}_{DE} = \frac{1\angle -30°}{\sqrt{3}}[(1-a^2)\mathbf{V}_1 + (1-a)\mathbf{V}_2 - a^2(a^2-a)\mathbf{V}_1 - a^2(a-a^2)\mathbf{V}_2]$$

$$= \frac{1\angle -30°}{\sqrt{3}}(1 - a^2 - a^4 + a^3)\mathbf{V}_1$$

$$[\quad 1 - a = a^2(a - a^2)] = \sqrt{3}\mathbf{V}_1\angle -30°$$

Hence, the voltmeter connected between points *D* and *E* measures $\sqrt{3}$ times the positive sequence component of the phase voltage. So do the other two voltmeters.

In Fig. 23.9 (*b*), the elements have been reversed. It can be shown that provided the same relation is maintained between the elements, the high impedance voltmeter measures $\sqrt{3}$ times the negative-sequence component of phase voltage.

23.14. Measurement of Zero-sequence Component of Voltage

The zero-sequence voltage is given by

$$\mathbf{V}_0 = \frac{1}{3}(\mathbf{V}_{RN} + \mathbf{V}_{YN} + \mathbf{V}_{BN})$$

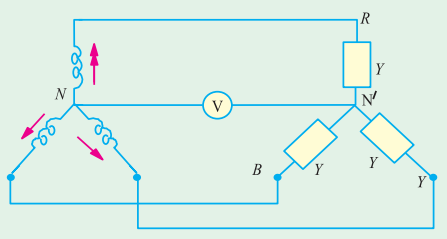


Fig. 23.10

Fig. 23.10 indicates one method of measuring $\mathbf{V}_0$. As seen

$$\mathbf{V}_{N'N} = \frac{\mathbf{V}_{RN}\mathbf{Y} + \mathbf{V}_{YN}\mathbf{Y} + \mathbf{V}_{BN}\mathbf{Y}}{3\mathbf{Y}}$$

$$= \frac{1}{3}(\mathbf{V}_{RN} + \mathbf{V}_{YN} + \mathbf{V}_{BN}) = \mathbf{V}_0$$

Hence, voltmeter connected between the neutral points measures the zero-sequence component of the voltage.

Tutorial Problem No. 23.2

1. Explain the essential features in the representation of an unsymmetrical three-phase system of voltages or currents by symmetrical components.

In a three-phase system, the three line currents are; $\mathbf{I}_R = (30 + j50)\,\text{A}, \mathbf{I}_Y = (15 - j45)\,\text{A}, \mathbf{I}_B = (-40 + j70)$ A. Determine the values of the positive, negative and zero sequence components.

[52.2 A, 19.3 A, 25.1 A] (*London Univ.*)

2. The phase voltages of a three-phase, four-wire system are; $\mathbf{V}_{RN} = (200 + j0)$ *V*, $\mathbf{V}_{YN} = (0 - j200)$ V, $\mathbf{V}_{BN} = (-100 + j\,100)$ *V*. Show that these voltages can be replaced by symmetrical components of positive, negative and zero sequence and calculate the magnitude of each component.

Sketch vector diagrams representing the positive, negative and zero-sequence components for each of the three phases. **[176 V', 38 V, 47.1 V] (*London Univ.*)**

3. Explain, with the aid of a diagram of connections, a method of measuring the symmetrical components of the currents in an unbalanced 3-phase, 3-wire system.

If in such a system, the line currents, in amperes, are

$$\mathbf{I}_R = (10 - j2), \mathbf{I}_Y = (-2 - j4), \mathbf{I}_B = (-8 + j6)$$

Calculate their symmetrical components. **[$\mathbf{I}_{RO} = \mathbf{I}_{YO} = \mathbf{I}_{BO} = 0$; $\mathbf{I}_{R1} = 7.89 + j0.732$; $\mathbf{I}_{Y1} = a^2\,\mathbf{I}_{R1}$; $\mathbf{I}_{B1} = a\mathbf{I}_{R1}$ $\mathbf{I}_{R2} = 2.113 - j2.732$; $\mathbf{I}_{Y2} = a\mathbf{I}_{R2}$; $\mathbf{I}_{B2} = a^2\mathbf{I}_{R2}$]**

OBJECTIVE TESTS – 23

1. The method of symmetrical components is very useful for
(*a*) solving unbalanced polyphase circuits
(*b*) analysing the performance of 3-phase electrical machinery
(*c*) calculating currents resulting from unbalanced faults
(*d*) all of the above

2. An unbalanced system of 3-phase voltages having *RYB* sequence actually consists of
(*a*) a positive-sequence component
(*b*) a negative-sequence component
(*c*) a zero-sequence component
(*d*) all of the above.

3. The zero-sequence component of the unbalanced 3-phase system of vectors V_A, V_B and V_C is .... of their vector sum.
(*a*) one-third (*b*) one-half
(*c*) two-third (*d*) one-fourth

4. In the case of an unbalanced star-connected load supplied from an unbalanced 3-ϕ, 3 wire system, load currents will consist of
(*a*) positive-sequence components
(*b*) negative-sequence components
(*c*) zero-sequence components
(*d*) only (*a*) and (*b*).

ANSWERS

1. (*d*) **2.** (*d*) **3.** (*a*) **4.** (*d*)

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CHAPTER 24

INTRODUCTION TO ELECTRICAL ENERGY GENERATION

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Electricity is generated at power plants and transmitted to different substrations from where it will be distributed to individual customers

24.1. Preference for Electricity

Energy is vital for all living-beings on earth. Modern life-style has further increased its importance, since a faster life means faster transport, faster communication, and faster manufacturing processes. All these lead to an increase in energy required for all those modern systems.

Arising out of comparison of status of nations, the progress is related in terms of per capita consumption of electrical energy (i.e. kWH consumed per person per year). At present, this parameter for India is about 300, for UK it is 12 to 15 times more, and for USA, it is about 30 times more.

It simply means that Electrical energy is the most popular form of energy, whether we require it in the usable thermal form (= heating applications), in mechanical form (= electrical motor-applications in Industries), for lighting purposes (= illumination systems), or for transportation systems.

Following are the main reasons for its popularity.

1. Cleaner environments for user
2. Higher efficiency
3. Better controllability
4. Easier bulk-power, long-distance transportation of power using overhead transmission or underground cables
5. Most versatile devices of energy conversions from Electrical to other forms are available for different purposes, such as thermal, illumination, mechanical, sound, chemical, etc.

24.2. Comparison of Sources of Power

While selecting a method of generating electricity, following factors are taken into account for purposes of comparison:

(*a*) Initial cost: For a given rating of a unit (in the minds of planners), investment must be known. Naturally, lower the initial cost, better it is.

(*b*) Running Cost:- To produce a given amount of electrical energy, the cost of conversion process (including proportional cost of maintenance/repairs of the system) has to be known.

(*c*) Limitations:- Whether a particular resource is available, whether a unit size of required rating is available from a single unit or from an array of large number of units, and whether a particular method of generation is techno-economically viable and is time-proven, are typical queries related to the limitations of the concerned method.

(*d*) **(1)** perpetuity, **(2)** efficiency, **(3)** reliability, **(4)** cleanliness and **(5)** simplicity.
It is naturally desirable that the source must have perpetuity (= be of endless duration), high conversion efficiency, and reliability (in terms of availability in appropriate quantity). The energy conversion must be through a cleaner process (specially from the view- points of toxicity, pollution or any other hazardous side effects). Further, a simpler overall system is always preferred with regards to maintenance/repairs problems and is supposed to be more reliable.

24.3. Sources for Generation of Electricity

Following types of resources are available for generating electrical energy (No doubt, this list can be extended to include some more up-coming resources. The following list, however, gives the popular and potential resources).

(*a*) Conventional methods

(*a*) **Thermal:** Thermal energy (from fossil fuels) or Nuclear Energy used for producing steam for turbines which drive the alternators (= rotating a.c. generators).

(*b*) **Hydro-electric:** Potential of water stored at higher altitudes is utilized as it is passed through water-turbines which drive the alternators.

(*b*) Non-conventional methods

(*c*) **Wind power:** High velocities of wind (in some areas) are utilized in driving wind turbines coupled to alternators. Wind power has a main advantage of having zero production cost. The cost of the equipment and the limit of generating-unit-rating is suitable for a particular

location (= geographically) are the important constraints. This method has exclusive advantages of being pollution free and renewable. It is available in plentiful quantity, at certain places. It suffers from the disadvantages of its availability being uncertain (since dependent on nature) and the control being complex (since wind-velocity has wide range of variation, as an input, and the output required is at constant voltage and constant frequency). Single large-power units cannot be planned due to techno-economic considerations.


Use of Wind Powder

(*d*) **Fuel cells:** These are devices which enable direct conversion of energy, chemically, into electrical form. This is an up-coming technology and has a special merit of being pollution-free and noise-free. It is yet to become popular for bulk-power generation.

(*e*) **Photo voltaic cells:** These directly convert solar energy into electrical energy through a chemical action taking place in solar cells. These operate based on the photo-voltaic effect, which develops an emf on absorption of ionizing radiation from Sun.

Power Scenario In India:-

Following approximate statistical data give an idea about some aspects in this regard.

Total installed capacity	:	150,000 MW
Hydropower	:	50,000 MW
Nuclear	:	10,000 MW
Thermal (fossil fuels)	:	80,000 MW
Other methods	:	10,000 MW

Other methods include partly exploited Potential such as Wind, Solar, Co-generation, Methods using Bio-fuels, etc.

24.4. Brief Aspects of Electrical Energy Systems

24.4.1. Utility and Consumers

At generating stations, power is generated at the best locations. Load-centres are generally away from these. Generation-units and Loads are connected by transmission systems. Thus, the energy system is divided into two main parts.

(*a*) Utility (including sources and transmission network) and,

(*b*) Consumers (who utilize the electrical energy)

24.4.2. Why is the three- phase a. c. system most popular?

(*a*) It is well known that a.c. generation is simpler (than d.c. generation through electrical machines because of absence of commutators in a.c. machines). Further, mechanical commutation system in d.c. machines sets an upper limit of their size, while the rating of the individual generators in ***modern power stations*** is too large, say about 1500 times the rating of a single largest feasible d.c. machine. A.C. further facilitates in stepping down or stepping-up of a voltage to suit a particular requirement, with the help of a simple device, the well-known transformer.

(*b*) Changing over from a.c. to d.c. is very easy these days due to the rectifiers of sufficiently high power ratings, so that a wide range of d.c. – applications can now be catered to.

(*c*) As the number of phases goes on increasing the power-output (from a device using a given quantity of active material, namely, that used for core and for winding) increases, but the number of circuits (*i.e.* connecting lines/ wires, switches, etc) also increases.

These two points are contradictory. A choice will be in favour of such a number (of phases) which will be high enough from power-output point of view but low enough from viewpoints

of complexity of connecting/controlling large number of phases. There is a Golden compromise. It has resulted into popularity of the three-phase a. c. systems over the entire time period. Now it has almost become a standard practice for all purposes.

24.4.3. Cost of Generation

Cost of generation for one unit of electrical energy depends on the method of generation, formulae worked to assess its running cost under the specified conditions, and the cost of transmission line loss to transport power upto the load. These days, a modern utility (= electricity board) has a large number of generators sharing the responsibility of supplying power to all the customers connected to the Grid (= common supply-network). Then, for an increase in load-demand, at a known location, the most economical generating unit is to be identified and that unit should be monitored to meet the increased demand.

24.4.4. Staggering of Loads during Peak-demand Hours

Incentives to consumers (by way of supplying at reduced rates during light load hours e.g. Late night 'hours, after noon hours) help in even demand throughout the day.

24.4.5. Classifications of Power Transmission

(*a*) Using underground cables or using overhead transmission lines.
(*b*) Extra High Voltage A.C.-versus-Extra High Voltage D.C. transmission systems.

24.4.6. Selecting A.C. Transmission Voltage for a Particular Case

In general, for transporting a given power of V I watts, either V can be high or V can be low. Accordingly, the current can be either low or high respectively. Higher voltage means higher cost of insulation, and larger clearances. Higher current means larger cross section of conductors. Considering these together, the most economical voltage has to be found out for a particular requirement.

Kelvin's Laws give a guideline for this. (These are discussed in Art. 47.21 in Vol. III, of this book).

24.5. Conventional Sources of Electrical Energy

Thermal (coal, gas, nuclear) and hydro-generations are the main conventional methods of generation of Electrical Energy. These enjoy the advantages of reaching perfections in technologies for these processes. Further, single units rated at large power-outputs can be manufactured along with main components, auxiliaries and switch- gear due to vast experiences during the past century. These are efficient and economical.

These suffer from the disadvantages listed below:

1. The fuels are likely to be depleted in near future, forcing us to conserve them and find alternative resources.
2. Toxic, hazardous fumes and residues pollute the environment.
3. Overall conversion efficiency is poor.
4. Generally, these are located at remote places with respect to main load centres, increasing the transmission costs and reducing the system efficiency.
5. Maintenance costs are high.

Out of these, only two such types will be dealt here, which have a steam turbine working as the prime mover. While remaining two use Internal Combustion Engines (I.C. engines) or Gas turbine as the prime mover, and these will *not* be dealt with, in this Introductory treatment.

The steam-turbine driven systems are briefly discussed below.

24.5.1. Steam Power Stations (Coal-fired)

India has rich stocks of coal as a natural resource. Chemical energy stored within the coal is finally transformed into Electrical energy through the process of these stations. Heat released by the combustion of coal produces steam in a boiler at elevated temperatures and pressures. It is then passed through steam turbines, which drive the alternator, the output of which is the electrical energy.

Figure 24.1 shows a simple schematic diagram of a modern coal-fired thermal station.

In India, coal is generally of low grade containing ash upto 40 %. It poses two problems. ***(i)*** Calorific value is low and hence system efficiency can be increased only by additional processes like pulverizing the coal and using oil firing to start with, ***(ii)*** large volumes of ash have to be handled after ensuring that ash is extracted to the maximum possible extent (upto 99 %) by using electrostatic precipitators, before flue gases from boiler are finally passed on to the atmosphere.

Coal is burnt in the boiler. This heat converts water into steam when passed through the boiler tubes. Modern plants have super heaters to raise the temperature and pressure of steam so that plant efficiency is increased. Condenser and cooling tower deal with steam coming out of turbine. Here, maximum heat is extracted from steam (which then takes the form of water) to pre-heat the incoming water and also to recycle the water for its best utilization.

Steam-turbine receives controlled steam from boiler and converts its energy into mechanical energy which drives the 3-ph a. c. generator (=alternator). The alternator delivers electrical energy, at its rated voltage (which may be between 11 to 30 kV). Through a circuit breaker, the step up transformer is supplied. Considering the bulk-power to be transmitted over long distances, the secondary rating

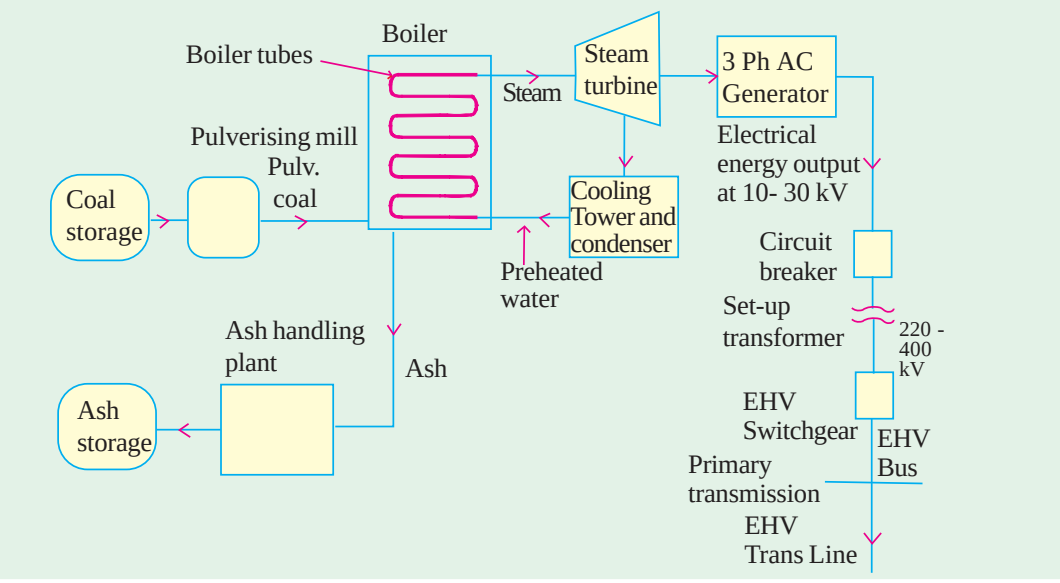


Fig. 24.1. Schematic Diagram of a Coal-fired Thermal Station

of this transformer may be 220 or 400 kV (as per figures for India). Through Extra-High Voltage Switch gear, the Bus is energized and the EHV primary transmission line can transport power to the Load centres connected to it.

A modern coal-fired thermal power station consumes about 10 % of its power for supplying to the Auxiliaries. These are mainly as follows.

(a) Main-exciter for alternator. ***(b)*** Water pumps.
(c) Fans: Forced draught and Induced draught fans for Pre-heaters and Chimney.
(d) Coal handling plant including pulverising mill.
(e) Ash handling plant including Electrolytic Precipitator.

Naturally, whenever such a station is to be brought into operation (either at commissioning or after repairs/maintenance schedule) the power required for the auxiliaries has to be supplied by the grid. Once the system is energized fully, it will look after supplying power to its own auxiliaries.

Merits of Coal-fired thermal stations

1. Fuel (=coal) is cheap. **2.** Less initial cost is required. **3.** It requires less space.
4. As a combination of all above points, the cost of generating unit of electrical energy is less.

Demerits

1. Atmospheric pollution is considerable.
2. Coal may have to be transported over long distances, in some cases, after some years, and then the energy cost may be quite high.

24.5.2. Nuclear Power Stations

Nuclear energy is available as a result of fission reaction. In a typical system, Uranium 235 is bombarded with neutrons and Heat energy is released. In chain-reaction, these release more neutrons, since more Uranium 235 atoms are fissioned. Speeds of Neutrons must be reduced to critical speeds for the chain reaction to take place. Moderators (= speed-reducing agents like graphite, heavy water, etc). are used for this purpose. Nuclear fuel rods (of Uranium 235) must be embedded in speed-reducing agents. Further, control rods (made of cadmium) are required since they are strong neutron-absorbers and help in finely regulating this reaction so that power control of the generator is precisely obtainable. When control rods are pulled out and are away from fuel rods, intensity of chain reaction increases, which increases the power output of the system. While if they are pushed in and closer to the fuel rods, the power-output decreases. Thus, the electrical load demand on the generator decides (automatically) the control-rod positions through a very sophisticated control system.

Fig. 24.2 shows a basic scheme of such a Nuclear power-station.

Advantages of Nuclear Generation

1. Quantity of fuel required is small for generating a given amount of electrical energy, compared to that with other fuels.
2. It is more reliable, cheaper for running cost, and is efficient when operated at rated capacity.

Disadvantages

1. Fuel is expensive and not abundantly available everywhere. **2.** It has high capital cost.
3. Maintenance charges are high. **4.** Nuclear waste disposal is a problem.

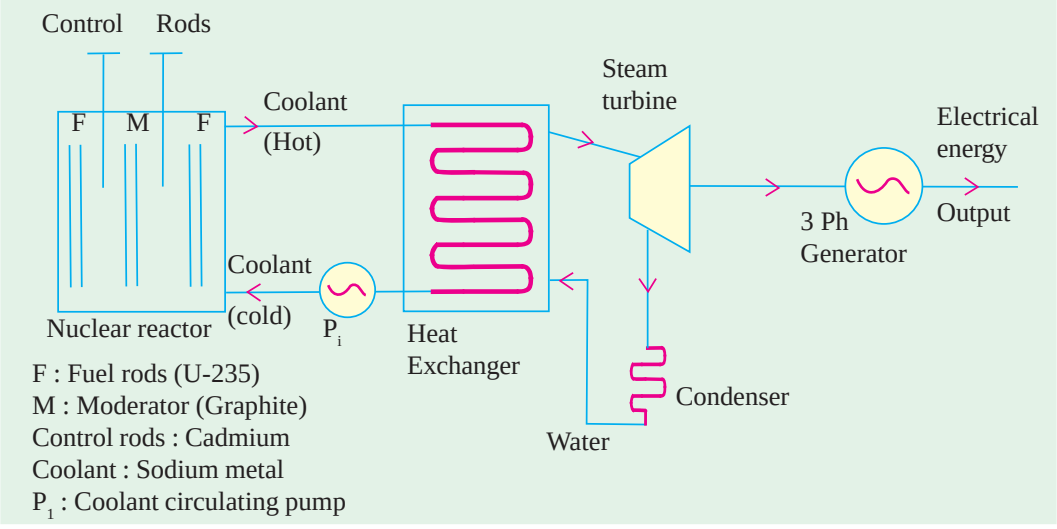


Fig. 24.2. Schematic Diagram of a Nuclear Power Station indicating Main Components

24.5.3. Hydroelectric Generation

Water-reservoir at higher altitudes is a pre-requisite for this purpose. Power-house is located at a lower level. The difference in these two levels is known as "Head."

Based on the "Heads", the Hydroelectric stations are categorized below:

1. Low head up to 60 metres. **2.** Medium head between 60 and 300 metres.
3. High heads above 300 metres.

In this method of generation, water from higher height is passed through penstock as controlled in the valve-house, into the water turbine. Thus, potential energy of water stored at higher altitudes is first converted into Kinetic energy. As the water reaches the turbine, it gains speed after losing the Potential energy. Kinetic energy of this speedy water drives the water turbine, which converts this into mechanical output. It drives the coupled generator, which gives Electrical energy output.

A schematic diagram of such a system is shown in fig.

The valve house has a controlling valve (=main sluice valve) and a protecting valve (= an automatic, isolating, "butterfly" type valve). As is obvious, power control is done by the main sluice valve, while "butterfly" valve comes into action if water flows in opposite direction as a result of a sudden drop in load on the generator. Otherwise, the penstock is subjected to extreme strains and it has a tendency to burst due to pressure of water as a result of sudden load reduction.

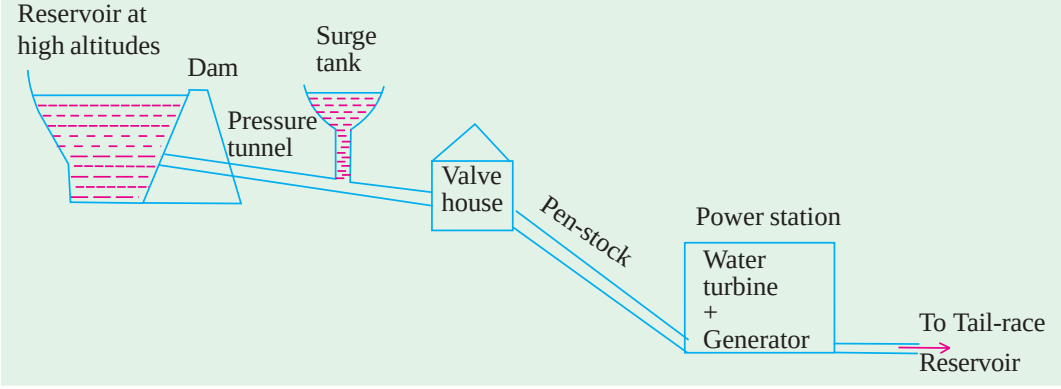


Fig. 24.3. Schematic of a typical Hydroelectric station

Note: Variation of Head (low-, medium-, or high-head) will affect the block-schematic

After doing the work (of imparting its energy to the water turbine), the water is allowed to pass into the tail-race reservoir.

The water turbines are essentially low-speed prime movers. In that, the best operating speed is dependent on the head. Alternators coupled to water turbines thus have large number of poles (since P= 120 f/N). Such alternators have the Salient-Pole type rotor.

There are different types of water turbines suitable for different cases (i.e. Heads, Power rating, Load-variation curve, etc). Since, this is only an elementary treatment, these aspects will not be discussed here.

24.6. Non-Conventional Energy Sources

Considering the previously discussed thermal methods of conventional energy-generations, it is necessary to understand the non-conventional energy sources, since they have two points in their favour.

(a) Non-polluting processes are used.

(b) Perpetuity and renewability of the main source (which is a natural atmospheric resource) generally exists.

The non-conventional energy sources are further advantageous due to virtually zero running cost, since wind energy or solar energy is the input-source of power.

However, they are disadvantageous due to high initial cost (per MW of installed capacity) and due to uncertainty resulting out of weather changes. For example, dense clouds (or night hours) lead to non-availability of solar energy. Similarly, "still-air" condition means no possibility of wind power generation, and during stormy weathers, wind turbines cannot be kept in operation (due to dangerously high speeds they would attain if kept in operation).

24.6.1. Photo Voltaic Cells (P.V. Cells or SOLAR Cells)

When ionized solar radiation is incident on a semi-conductor diode, energy conversion can take place with a voltage of 0.5 to 1 volt (d.c.) and a current density of 20-40 mA/cm^2, depending on the

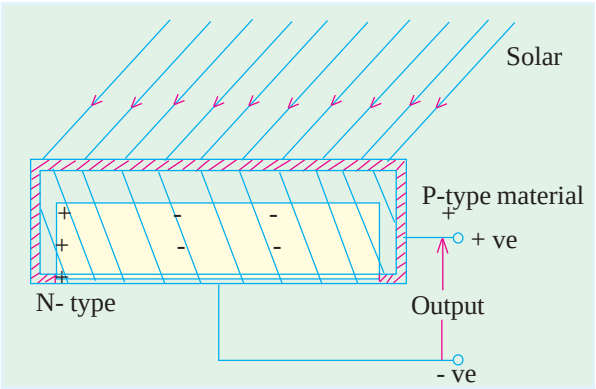


Fig. 24.4. Photo Voltaic (or Solar Cell)

materials used and the conditions of Sunlight. Area of these solar cells decides the current output. An array of large number of such diodes (i.e. Solar cells) results into higher d.c. output voltage.

Since, the final form of electrical energy required is generally an alternating current, it is realized from d.c. using inverters.

At quite a few locations in India, for realizing few hundred kilowatts of power-rating, huge arrays are accommodated in horizontal as well as vertical stacks, so that land area required is not too vast. Electrically, they are connected in series and in parallel combinations of cells so that rated voltage and current are realized.

Just to understand the principle of operation of solar cells, let a semi-conductor diode receive ionized radiation from Sun, as in fig. 24.4.

Typical materials used for these cells are: material doped with boron, cadmium sulphide, gallium-arsenide, etc. Their choice is mainly decided by conversion efficiency. Best material may lead to the efficiency being typically 15%. Since solar energy is available free of cost, this low-efficiency does not matter.

This method suffers from the disadvantages of having high initial cost and uncertainty (since dependent on weather conditions) including non operative night periods. Main advantages are: ***(i)*** no running cost (however, replacements of components may be a botheration), ***(ii)*** no pollution, ***(iii)*** location can be near the load (hence transportation of power is not required over long distances). ***(iv)*** since natural source is involved, it is perpetual.

Individual stations using solar cells are in operation with ratings of the order of 250-1000 kW. With manufacturing costs of semi-conductor devices going down and with the advent of better and better quality of cells which will be available in future, this method of generation has bright prospects.

24.6.2. Fuel Cells

24.6.2.1. Principle of Operation

In Fuel cells, negative porous electrode is fed by hydrogen and the positive porous electrode is fed by oxygen. Both the electrodes are immersed in an electrolyte. The porous electrodes are made of such a conducting material that both the fuel (oxygen and hydrogen) and the electrolyte can pass through them. Such a material for electrodes is nickel. The electrolyte is a solution of sulphuric acid or potassium hydroxide. The electrodes have a catalyst (= platinum or sintered nickel) which break the fuel compound into more reactive atoms.

24.6.2.2. Chemical Process (with Acidic Electrolyte)

At Negative Electrode : $2H_2 \longrightarrow 4H^+ + 4\bar{e}$

These hydrogen ions enter the solution (=electrolyte) leaving behind electrons which pass through external circuit to the positive electrode.

At Positive Electrode: $O_2 + 4H^+ + 4 \rightarrow 2H_2O$

Thus, the combination of Hydrogen and Oxygen results into water at the positive electrode. Water is the waste-product of the cell, which is harmless. The process is, thus, pollution free. There is no source of energy required, since the process is basically **"chemical"** in nature.

24.6.2.3. Schematic Diagram. It is given in fig 24.5

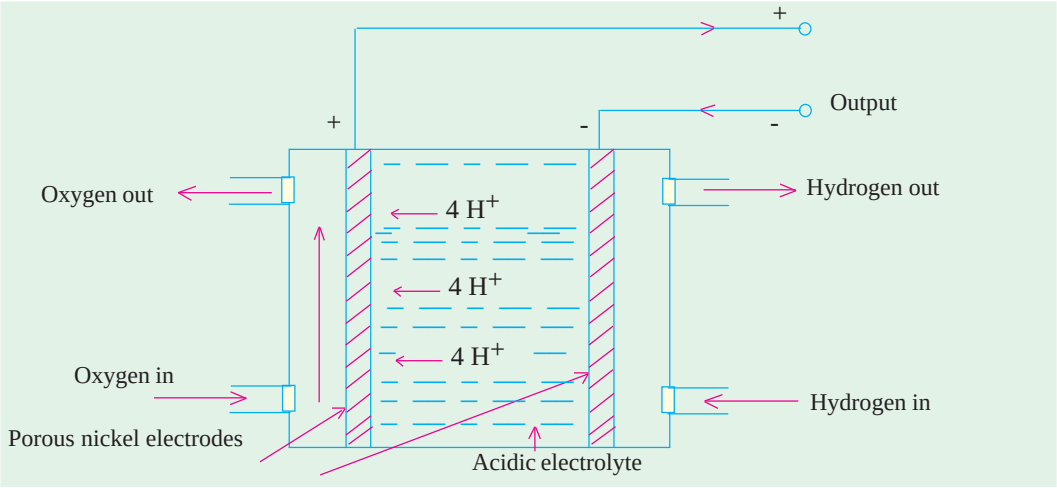


Fig. 24.5. Fuel Cell

24.6.2.4. Array for Large outputs

A fuel cell has a d. c. output voltage typically of 1.23 volts at normal atmospheric pressure and temperature. Raising pressure and temperature increases this voltage. To realize large output parameters (= voltage, current, and hence power), an array of a large number of fuel cells (connected in series as well as in parallel) is made. Voltage levels of 100 to 1000 V and power levels in kilowatts can be realized.

24.6.2.5. High Lights

1. Pollution-free, noiseless.
2. No outside source of energy is required.
3. Efficient.
4. No restriction on location

(*a*) High initial cost. (*b*) Working life is short.

Note: Solar energy can also be used for generating electrical energy through an intermediate stage of producing steam, which is used later for driving an alternator. However, this method is not discussed here.

24.6.3. Wind Power

24.6.3.1. Background

Wind power has been in use for serving the mankind, since centuries through what has been popularly known as "Wind-mills." There is no "electrical" stage of energy in old-styled uses where wind-velocity is directly used for performing the jobs such as wheat-grinding, pumping water for irrigation, sailing vessels, etc. It enjoys the advantages of being plentiful, inexhaustible, renewable and non-polluting, over and above being cheap for running costs. It suffers from disadvantages of being unreliable, and being economically un-viable for large power generation. In India, a large number of such units with small and medium power ratings (up to 100 kW) are already in operation mainly in coastal or hilly areas. With the modern system, it is now preferred to have suitable power-control circuits on the output side of wind-generators so that these can pump energy into low voltage lines of the grid over a wide range of variation of wind speeds.

24.6.3.2. Basic Scheme

A large variety of wind-turbines naturally exist arising out of large variation in wind-pattern

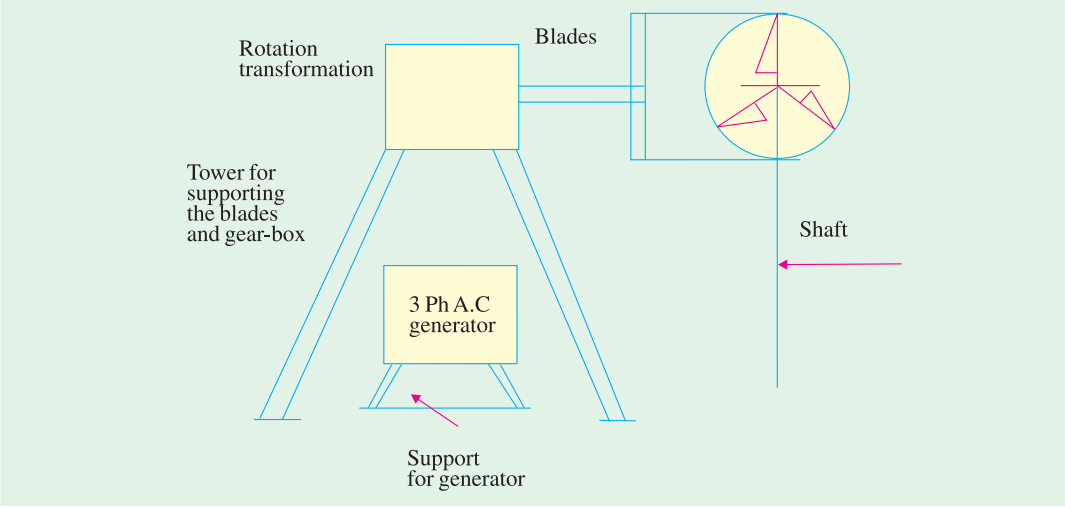


Fig. 24.6 (*a*) . Wind-generation, a Schematic view

Fig. 24.6 (*b*). Part-side-view to show a typical three bladed wind turbine

all over and out of different manufacturers producing systems with different designs. Since the aim here is to understand the basic system, only one type of system is presented here.

In Fig. 24.6 (*a*) an arrangement wherein, a horizontal three-bladed system is shown mounted on a tower. Through rotation-transformation using gears to step up the speed and to link the horizontal axis of turbine with vertical axis of generator. The speed of wind varies, as such turbine speed also varies so that output frequency and voltage of three-phase alternator vary over a wide range. Further, its waveform is also a distorted one. To increase its utility, it is necessary to modulate (through proper power-control) to derive line-frequency constant voltage output and hook-up to local grid for pumping the available wind-energy into it. This is schematically represented in fig 24.7.

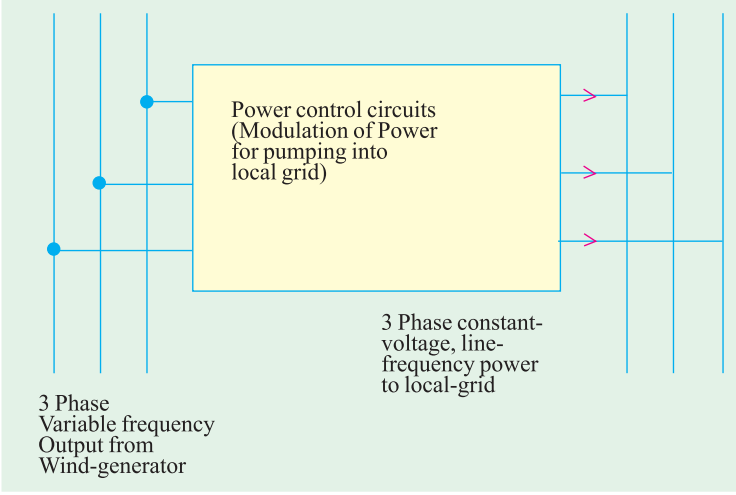


Fig. 24.7

24.6.3.3. Indian Scenario

Wind farms have been located where a large number of wind generators of ratings of few hundred kilowatts are in operation. For every unit, there is a safe wind - speed zone. If the wind-speed is below this, there is no appreciable power output, hence, it is better that the system is *not* brought into operation. If the wind speeds are too high, it is mechanically unsafe and hence it is *not* to be operated, even if the energy available is higher. This is decided by automated system. Such forms are located in coastal regions and in hilly areas. Because of the metering of energy received by the local grid, the investor can get a good return through payment from the grid-authorities. Hence, this has become a medium-level and attractive industrial investment.

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AC & DC MACHINES

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CHAPTER 25

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An electric motor is a machine which converts electrical energy into mechanical (rotational or kinetic) energy

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25.1. Introduction

"Energy can ***neither*** be created ***nor*** be destroyed". We can only change its forms, using appropriate energy-conversion processes.

An interesting aspect about the energy in "Electrical form" is that neither it is so available ***directly from nature*** nor it is required to be finally consumed in that form. Still, it is the most popular form of Energy, since it can be transported at remote Load-locations, for optimum utilization of resources. Further, technological progress has now made it possible to device Electrical-Power-modulation systems so flexible and controllable that modern systems tend to be energy-efficient, with increase in life-span of main equipment and the associated auxiliary components (like switches, connecting cables, contactors, etc.) since it is now possible to avoid overstrain (= over-currents or over-voltage) on the system. This means a lot for the total production process (for which electrical energy is being used) since the quality of the production improves, and plant-maintenance is minimal. Energy-conversion systems then assume still higher importance.



Generator

Energy conversion takes place between well known pairs of forms of Energy: Electrical ↔ Chemical, Electrical ↔ Thermal, Electrical ↔ Optical, Electrical ↔ Sound, and Electrical ↔ Mechanical are the common forms with numerous varieties of engineering - applications. Electrical ↔ mechanical conversion is the focus of discussion in this chapter.

The elements of electro-mechanical energy conversion shall deal with basic principles and systems dealing with this aspect. Purpose of the study is to have a general approach to understand to design, and later to modify the system with the help of modern technologies, for overall improvisation.

It is necessary to be aware about:

(a) basic conditions to be fulfilled by the conversion system.
(b) methods for innovating the conversion systems.

Electromechanical energy-conversion finds applications in following categories of systems:

(a) transducers: Devices for obtaining signals for measurement / control,
(b) force-producing devices: Solenoid-actuators, relays, electromagnets,
(c) devices for continuous-energy-conversion: Motors / Generators

These systems have different configurations. But the principles of their working are common. Understanding these principles enables us to analyze / design / improvise / innovate such systems. As a result of such development, newer types of motors and the associated modern power controllers have recently been manufactured and become popular. Controllers using power-

electronics switching devices offer energy-efficient, user-friendly, and high-performance drives. Their initial investment may be larger but two important parameters justify their use: ***(i)*** Considerable energy is saved, resulting into payback periods as short as 18-24 months, ***(ii)*** The controllers ensure to limit the currents to pre-set values under conditions of starting /overload/ unbalanced supply. Hence, the entire system enjoys longer life. Both these effects lead to better production-process and hence these are readily acceptable by industries.

25.2 Salient Aspects of Conversions

Purpose of electro-mechanical conversion device is to change the form of energy. Here, for simpler discussion, only rotary systems will be dealt with. When it is converting mechanical input to electrical output the device is "generating". With electrical input, when mechanical output is obtained, the device is motoring.

Some simple aspects of an electrical machine (motor / generator) have to be noted at this place:

(1) Electrical machine has a Stator, a Rotor, and an air-gap in between the two. For a flux path, the magnetic circuit has these three parts in series. In general, magnetic poles are established in Stator and in Rotor.

(2) Magnetic effects of following types can be categorized:

- **(*a*) Electromagnetic:** Due to currents passed through windings on Stator and / or Rotor, producing certain number of poles on these members.
- **(*b*) Permanent Magnets:** One side (Stator or Rotor) can have permanent magnets.
- **(*c*) Reluctance variation:** Surface of Rotor near the air-gap can be suitably shaped to have a particular pattern of Reluctance-variation so as to control the machine behaviour as per requirements.



Stator and Rotor

(3) Basic conditions which must be satisfied by such devices are:

- **(*a*)** Equal number of poles must be created on the two sides.
- **(*b*)** In some cases, reluctance-variation is primarily used for machine-action. The Stator side must accommodate a winding carrying current for the electromagnetic effect, when rotor surface is shaped so as to have the desired pattern of reluctance variation. Or, non-cylindrical rotor cannot have the current -carrying winding for machine action.

(4) Out of stator, rotor and air-gap, maximum energy-storage at any angular position takes place in the air-gap, since its reluctance is highest out of the three members.

(5) Stored energy must depend on rotor-position and the device tends to occupy that angular position which corresponds to maximum stored energy. If this position varies as a function of time, the device produces continuous torque.

(6) Ideal output of a motor is a constant unidirectional torque with given currents through its windings. In some cases, the output torque (as a compromise) is an average value of a cyclically varying torque.

(7) Where current-switching is done for motor-control, as in modern controllers, instantaneous effect has to be understood to conclude on any of the points mentioned above.

(8) A device can work either as a generator or as a motor, provided pertinent conditions are satisfied for the concerned mode of operation.

25.3. Energy – Balance

For an electro-mechanical system, following terms are important:

(*i*) Electrical port (= armature terminals): receiving / delivering electrical energy.

(*ii*) Mechanical port (= shaft): delivering / receiving mechanical energy.

(*iii*) Coupling field: Magnetic field *or* Electric field.



Conversion of electrical energy into mechanical energy

Even though, theoretically, both the types of fields mentioned above are able to convert the energy, the magnetic medium is most popular since the voltage levels required are not very high, and the devices of given power rating are smaller in size and are economical. Hence, only those will be dealt with.

It is obvious that an electrical motor receives energy at the electrical port and delivers it at the mechanical port. While an electric generator receives the energy at the mechanical port and delivers it at the electrical port. It is also known that the following losses take place in such systems and are dissipated away as heat: **(*i*)** i^2r losses in the windings of the machines, **(*ii*)** friction and windage losses, **(*iii*)** core-losses.

These can be either neglected or attached to electrical port, mechanical port and coupling magnetic field respectively, for simpler analysis. With this, the simple energy balance equation can be written as:

Change in Electrical Energy = Change in Mechanical Energy + Change in Field-Energy

$$dW_{elec} = dW_{mech} + dW_{fld} \quad \text{. . .(25.1)}$$

It is natural that this equation has +ve signs for electrical and mechanical-energy-terms when the device is motoring. For generating mode, however, both the terms assume –ve signs.

In case no mechanical work is done, eqn. (25.1) reduces to eqn. (25.2) below indicating that Electrical energy - input is stored in the magnetic field.

$$dW_{elec} = dW_{fld} \quad \text{. . . (25.2)}$$

25.4. Magnetic-field System: Energy and Co-energy

25.4.1 Linear System

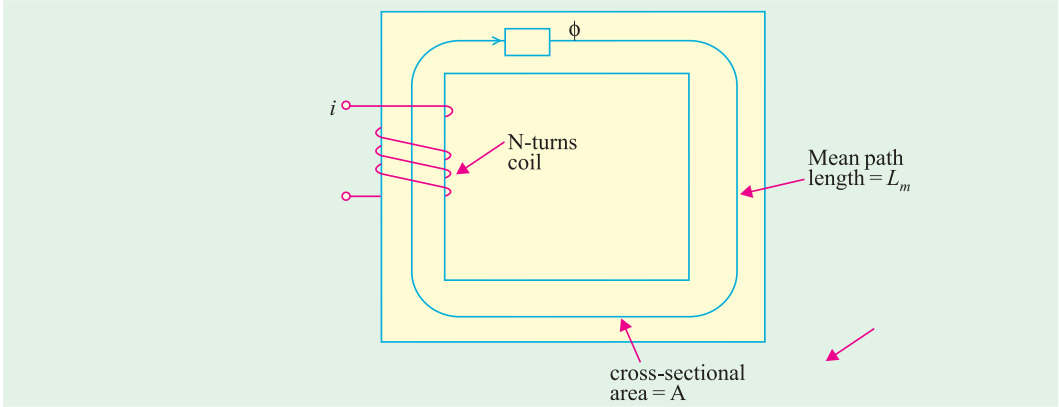


Fig. 25.1 (*a*) Magnetic circuit

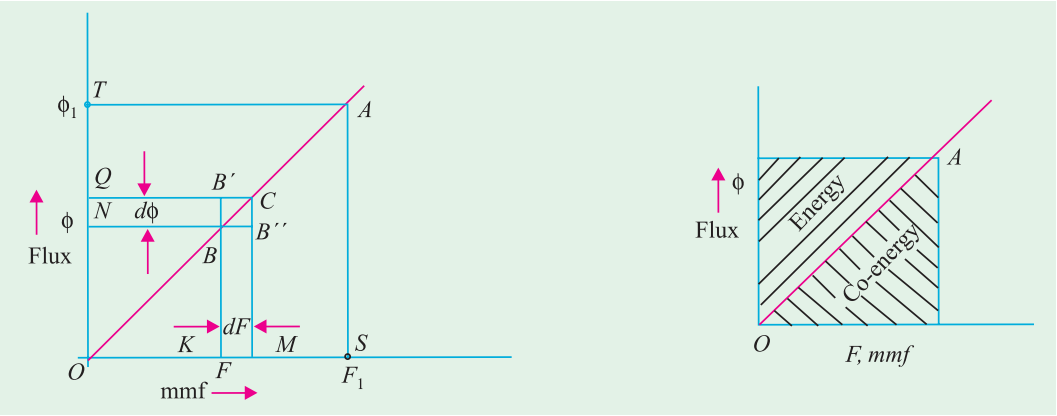


Fig. 25.1 (*b*) Characteristic of a magnetic circuit

Fig. 25.1 (*c*) Energy and co-energy

A simple magnetic current is shown in Fig 25.1 (*a*), with assumptions that air-gap length at the joints is negligible, and the magnetic medium is not saturated. With A as the cross-sectional area of the core and L_m as the mean length of the path, a coil with N turns carrying a current of i amp has an mmf of F, establishing a flux of ϕ, related by

$$\phi = F \times \mathcal{P}_m$$

where $\mathcal{P}_m$ = Permeance of the Magnetic circuit

$$= \mu_o \mu_r \, A/L_m$$

with μ_r = relative permeability of the magnetic medium,

This corresponds to the following relationships:

Coil Inductance, $L = N^2 \mathcal{P}_m = N\phi / i = \lambda / i$

where λ = flux-linkage of the coil, in weber-turns

W_{fld} = Energy stored in the coil $= \frac{1}{2} Li^2 = \frac{1}{2} N^2 \mathcal{P}_m i^2 = \frac{1}{2} F^2 \mathcal{P}_m$

$$= \tfrac{1}{2} F(F\mathcal{P}_m) = \tfrac{1}{2} F\phi \qquad \ldots \textbf{(25.3)}$$

In this eqn., $\mathcal{P}_m$ is the slope of the characteristic in Fig 25.1 (*b*). Hence, the inductance is proportional to the slope of F-ϕ plot. In Fig., 25.1 (*b*), for the operating point A, the mmf is F_1 and the flux ϕ_1. At the point A, the energy stored in the field is given by eqn. below:

$$W_{fld} = \tfrac{1}{2} F_1 \phi_1$$

F_1 is due to the current i_1. W_{fld} is given by area $OATO$ in Fig 25.1(*b*).

In Fig 25.1(*b*), the origin refers to the system without magnetization.

The system can reach the point A, starting from O as the current in the coil is increased from O to i_1.

Let us understand the intermediate events.

At point B, the flux is ϕ due to the mmf F.

An increment in coil current results into increase in mmf by dF. This increases the core-flux by $d\phi$. New operating point is C.

Eqn. (25.3) is to be suitably re-written in terms of these incremental values.

$$W_{fld} = \int dW_{fld} = \int_0^{\phi_1} Fd\phi = \int_0^{\phi_1} \text{(area of the strip } NBB'Q) = \text{area of triangle } OAT$$

$$= (1/\mathcal{P}_m) \int_0^{\phi_1} \phi . \, d\phi = \tfrac{1}{2}.\phi_1 . F_1 \qquad \ldots \textbf{(25.4)}$$

Alternatively, we have the area of elemental strip $kBB''M = \phi \, . \, dF$

Area of Δ OAS = $\int_0^{F_1}$ (area of the strip $KBB''M$)

$$= \int_0^{F_1} \phi \, . \, dF = \mathcal{P}_m \int_0^{F_1} F \, dF = \mathcal{P}_m \, F_1^2/2 = \tfrac{1}{2} F_1 \phi_1 \quad \ldots \textbf{(25.5)}$$

In order to distinguish with respect to the terms in eqn. (25.4), this area is called as the "Co-energy of the field" and is represented by W'_{fld}. For a Linear system, however, for a given operating point, say A, the two energy-terms are equal. Hence,

$$W_{fld} = W'_{fld} = \tfrac{1}{2} F_1 \phi_1 = \tfrac{1}{2} L i_1^2 \quad \ldots \textbf{(25.6)}$$

In order to have a simple and clear distinction between the two energy terms, it can be said that the differential variable ***"Current"*** (or mmf) is related to Co-energy and the differential variable "flux - linkage" (or flux) is related to Energy.

This energy stored in the magnetic field comes from the electrical source connected to the coil in Fig. 25.1(*a*).

25.4.2. A Simple Electromechanical System

A simple electro-mechanical system is shown in Fig. 25.2(*a*).

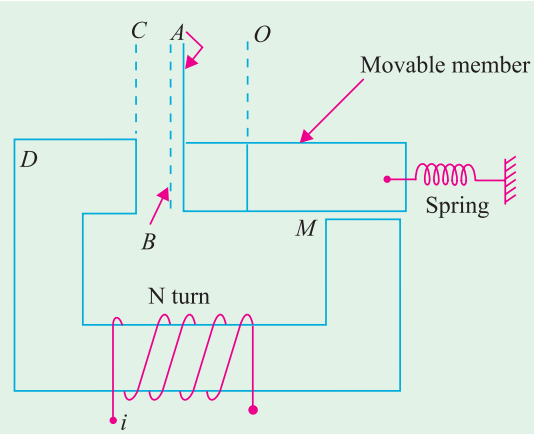


Fig. 25.2 (*a*)

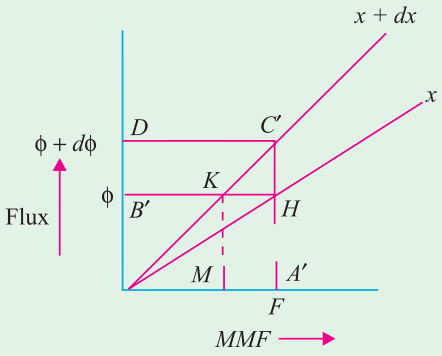
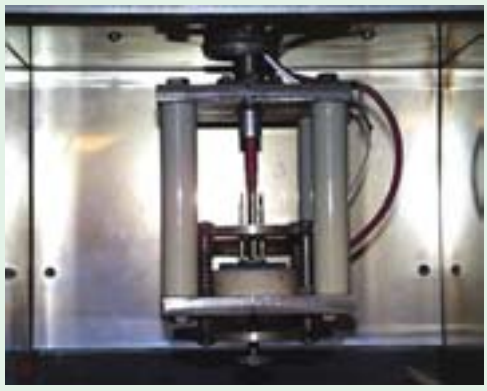


Fig. 25.2 (*b*)

Reference point *O* corresponds to the unstretched spring. Energy stored in the spring is then Zero. In position A of the movable member, the spring is elongated by *x*, and the corresponding energy stored in the spring is $\tfrac{1}{2} K_s x^2$, where K_s is "spring-constant" of the linear system in Nw / m. In Fig 25.2 (*a*), the distance *OA* is *x*. The elemental distance *AB* is *dx*, so that *OB* is $x + dx$. For simpler analysis, it is assumed that magnetic material is highly permeable and that the clearance at point M (for movement of the member) is negligible. So that the mmf of the coil is required to drive the flux in the region *OABC* only. The flux-mmf



A simple electromechanical system

relationships are plotted for these two positions in Fig 25.2 (*b*). In position A, the movable member has moved a distance of *x* from its unstretched position or reference point. Let the operating point be *H*, so that the coil-mmf *OA* (=*F*) establishes a flux *OB* (=ϕ). In this position, the movable member experiences a force in such a direction that the energy stored in the field tends to increase. It tends to reach *B*, so that an additional displacement of *dx* shifts the characteristic upwards and final operating point in position *B* is *C*′. From *H* to *C*′, the operating point can move in any one of the following ways:

(*a*) *HC*′ vertically, if the mechanical movement is too slow so that change of flux is slow and induced emf in the coil is negligible. This corresponds to the coil-mmf remaining constant at F during the transition. Constant mmf means vertical travel of the operating point from *H* to *C*′.

(*b*) *H* to *K* horizontally and then *K* to *C*′along the characteristic corresponding to (*x* + *dx*) as the displacement of the movable part. This is possible when the motion is very fast, resulting into flux remaining constant till the operating point traverses from *H* to *K*. Then, from *K* to *C*′, the flux increases, an emf in induced the coil and the mmf finally reaches its value of *F*, at the point *C*′.

(*c*) In reality, the transition from *H* to *C*′ will be somewhere in between these two extremes mentioned above.

However, for simplicity, one of these extreme conditions has to be accepted. In **(*a*)** above, the mmf remains constant. In **(*b*)** above, the flux (and hence the flux-linkage) remains constant. Let us take the case of constant-mmf. If the process has taken a time of *dt*,

Electrical-energy input during the process = dW_{elec}

= (voltage applied to the coil) × current × dt = $eidt$

= $d\lambda/dt \times i \times dt = id\lambda = i\,N\,d\phi = Fd\phi$ = area of rectangle $B'HC'D$

In this case, coil-resistance has been neglected.

25.4.2.1. In terms of Field Energy

At the previous operating point *H*, the energy stored in the magnetic field,

W_{fld1} = area of Δ *OHB*′

At the new position corresponding to the operating point *C*′, the field energy stored is given by W_{fld2} = area of the Δ *OC*′*D*

The difference of these two is the change in the energy stored in the magnetic field $= dW_{fld} = \frac{1}{2}[OA' \times A'C' - OA' \times A'H]$

$= \frac{1}{2}\,OA'\,[A'C' \times A'H] = \frac{1}{2}\,OA'\,HC'$

$= \frac{1}{2} \cdot F \cdot d\phi = \frac{1}{2}\,dW_{elec}$

Out of the energy delivered by the source, half is stored in the magnetic field. Where has the remaining half been utilized? Obviously, this must have been transformed into the mechanical work done. In this case, neglecting losses, it is finally stored in the stretched spring due to its elongation by *dx*.

Comparing this with the equation (25.1),

$$dW_{elec} = dW_{mech} + dW_{fld}$$
$$= dW_{mech} + \tfrac{1}{2}\,dW_{elec}$$

or $$dW_{mech} = dW_{fld} = \tfrac{1}{2}\,dW_{elec}$$

Consider that a force *F* is operative at the displacement of *x*. This force is in such a direction that *x* increases or the movable member is attracted towards *D*. In the same direction, a

displacement by dx results into the increase in the energy stored by the spring. Relating the concerned terms,

$$F = k_s.x$$

dW_{mech} = mechanical work done against the force of the stretched spring

$= -F\,dx = dW_{fld}$

or $F = -dW_{fld}/dx$, in this case

$= -\delta W_{fld}/\delta x$, in general

Alternatively, the difference in the energy stored in the spring also gives a very useful relationship.

In the position corresponding to $x + dx$, the energy stored in the spring $= \frac{1}{2}\,k_s\,(x + dx)^2$. Similarly, at x, the energy $= \frac{1}{2}\,k_s\,x^2$

Difference $= \frac{1}{2}\,k_s\,[(x + dx)^2 - (x)^2]$

$= \frac{1}{2}\,k_s\,[x^2 + 2.x.dx + (dx)^2 - x^2]$

$= \frac{1}{2}\,k_s\,[2.\,x.\,dx]$, neglecting $(dx)^2$

$= k_s.\,x.\,dx = F\,dx$

This difference is nothing but dW_{mech}, which is equal in magnitude to dW_{fld} and confirms the relationship obtained earlier.

25.4.2.2. In terms of field Co-energy

Proceeding along lines similar to those while dealing with field-energy above, following relationships exist. For simpler discussion, the transition is assumed to be along HKC'. Neglecting area of the small triangle HKC', we have

at x, W'_{fld1} = area of $\Delta\ OA'H$

at $x + dx$, W'_{fld2} = area of $\Delta\ OA'C'$ (neglecting $\Delta\ HKC'$)

$dW'_{fld} = W'_{fld2} - W'_{fld1} = \phi.\,dF$ where $dF = MA'$

= Co-energy in the field $F\,dx$

Hence, $F = +\,dW'_{fld}/dx$, in this case

$= +\,\delta W'_{fld}/\delta x$, in general

25.5. Energy in Terms of Electrical Parameters

In the preceding article, the energy and force were related in terms of magnetic-system parameters, namely flux and mmf, through the third parameter, the permeance.

It is at times convenient to relate these things in terms of electrical-system-parameters, namely, the inductances and currents. That is being dealt with here only for linear systems. Let $\mathcal{P}$ be the permeance of the magnetic circuit and L be the coil-inductance.

$$W_{fld} = \text{Field-energy} = \tfrac{1}{2}\,F\,\phi = \tfrac{1}{2}\,Ni\,(Ni\,\mathcal{P}) = \tfrac{1}{2}\,(N^2\mathcal{P})i^2$$

$$= \tfrac{1}{2}\,Li^2$$

$$F = -dW_{fld}/dx = -\tfrac{1}{2}\,.\,i^2.\,dL/dx$$

Thus, a force exists if the coil-inductance is dependent on x. Such analysis is more suitable when the system has more than one coils coupled through the magnetic circuit. If two such coils are considered, following data should be known for evaluation of the force, in case of linear displacement:

L_{11} = self inductance of coil - 1

L_{22} = self inductance of coil - 2

L_{12} = Mutual inductance between two coils, 1 and 2

i_1, i_2 = currents through the two coils.

W_{fld} = Total energy stored in the field $= \frac{1}{2} L_{11} i_1^2 + \frac{1}{2} L_{22} i^2_2 + L_{12} i_1 i_2$

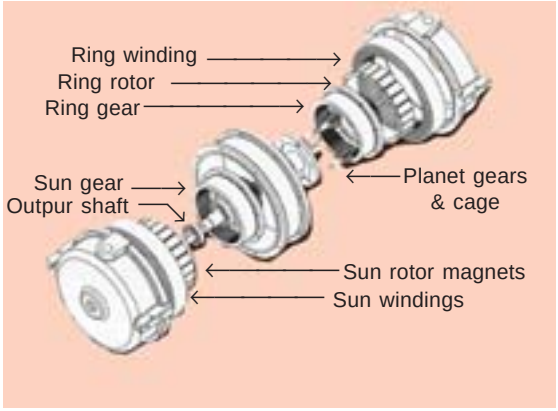
Magnitude of Force, $F = dW_{fld}/dx = \frac{1}{2} i^2_1 \, dL_{11}/dx + \frac{1}{2} i^2_2 \, dL_{22}/dx + i_1 i_2 \, dL_{12}/dx$

From the right-hand side of this equation, it is noted that the inductance-term which is dependent on x contributes to the force.

25.6. Rotary Motion

Most popular systems for electro-mechanical energy are Generators and Motors. The preceding discussion dealt with the Linear motions, wherein x represented the displacement parameter, and force was being calculated.

Now we shall deal with the rotary systems, wherein angular displacement parameters (such as θ) and corresponding torque developed by the system will be correlated, through a systematic procedure for a typical rotary machine.



Rotary motion

25.6.1. Description of Simple System

A simple rotary system has a 'stator' and a 'rotor'. Air-gap separates these two. Stator has two similar coils 'a' and 'b' located at 90^0 electrical, with respect to each other. Inner surface of the stator is cylindrical. Outer surface of the rotor is also cylindrical resulting into uniform air-gap length for the machine.

The diagram represents a two-pole machine. Axis of coil 'a' may be taken as reference, with respect to which the rotor-coil axis makes an angle of θ, at a particular instant of time. For a continuous rotation of the rotor at ω radians / sec, θ = ωt. Coil-'b'-axis is perpendicular to the reference, as shown. Due to the uniform air-gap length, and due to the perpendi-cularity between coils 'a' and 'b', inductance-parameters exhibit the following patterns:

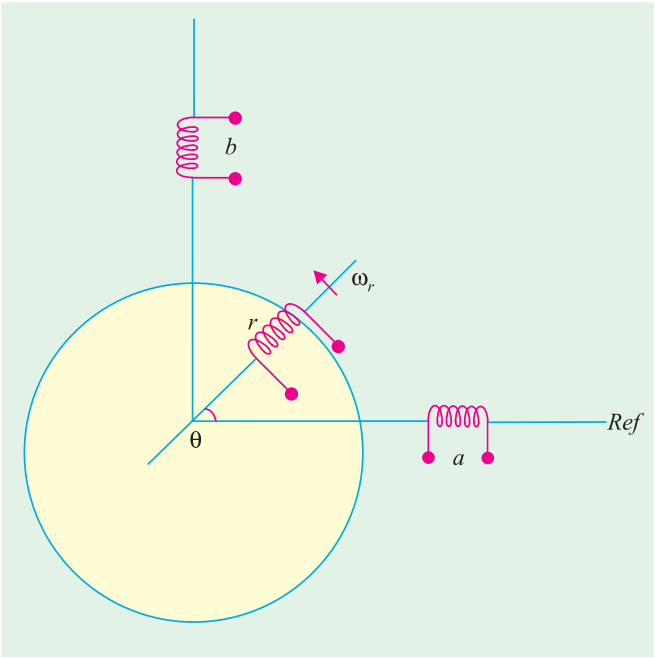


Fig. 25.3. Simple rotary system

Let x represents the inductance parameter as a function of θ. The subscripts indicate the particular parameter. x_{aa} = self-inductance of coil 'a' , x_{ab} = mutual inductance between coils a and b and so on. x represents value of the particular inductance parameter, which will help in knowing the variation of inductance with θ.

(a) Self inductances of coils '*a*' and '*b*' are not dependent on rotor position.

$x_{aa} = L_{aa}$ and $x_{bb} = L_{bb}$, at all values of θ.

(b) Mutual inductance between stator coils '*a*' and '*b*' is zero, due to perpendicularity.

x_{ab} = zero, for all values of θ.

(c) Self-inductance of rotor-coil is constant and not dependent on θ.

$x_{ra} = L_{rr}$ (= constant), at all values of θ.

(d) When '*r*' and '*a*' coils have their axes aligned, at θ = 0, the mutual inductance between them is maximum, which is denoted by x_{ar}. At $\theta = 90^0$, their axes are perpendicular resulting into no coupling or zero mutual inductance. When $\theta = 180^0$, *r* and a coils are aligned in anti-parallel way and hence maximum mutual inductance exists between them with negative sign.

Further, whatever happens to coupling between *r* and *a* at a value of θ happens to that between *r* and *b* with a delay of 90^0. All these are mathematically represented as :

$$x_{ra} = L_{ra} \cos\theta = L_{ra} \cos(\omega_r t)$$
$$x_{rb} = L_{ra} \sin\theta = L_{ra} \sin(\omega_r t) = L_{ra} \cos(\omega_r t - 90^0)$$

Since '*a*' and '*b*' coils are alike, the maximum mutual inductance is represented by the same term L_{ra}.

25.6.2. Energy stored in the coils

Energy stored in the magnetic field can *either* be expressed in terms of mmf and flux *or* be expressed in terms of inductance-terms and coil currents. If i_a, i_b and i_r are the coil-currents, stored-energy-terms are as given below:

(i) W_1 = Energy in Self-ind. of coil '*a*': $\frac{1}{2} L_{aa} i^2_a$

(ii) W_2 = Energy in Self-ind. of coil '*b*': $\frac{1}{2} L_{bb} i^2_b$

(iii) W_3 = Energy in Self-ind. of coil '*r*': $\frac{1}{2} L_{rr} i^2_r$

(iv) W_4 = Energy in mutual inductance between '*a*' and '*r*': $x_{ra} i_r i_a = x_{ra} i_r i_a = x_{ra} .\cos\theta . i_r . i_a$

(v) W_5 = Energy in mutual inductance between '*b*' and '*r*' : $x_{rb} i_r . i_b = x_{ra} \sin\theta . i_r . i_b$

W = Total energy stored in the system = Sum of all the energy-terms cited above

$= W_1 + W_2 + W_3 + W_4 + W_5$

T = Torque produced = $\delta W / \delta\theta$

If i_a, i_b, i_r are assumed to be constant currents, for simplicity, so that their derivatives with respect to θ (and hence with respect to time *t*) are zero, the energy-terms which include constant inductances do not contribute to torque. W_1, W_2, and W_3 thus cannot contribute to torque. W_4 and W_5 contribute to torque related by:

$$T = \delta W / \delta\theta = \delta / \delta\theta \; [W_4 + W_5] = \delta / \delta\theta \; [L_{ra} i_r i_a \cos\theta + L_{ra} i_r i_b \sin\theta]$$
$$= L_{ra} i_r \; [-i_a \sin\theta + i_b \cos\theta]$$

If $i_a = i_b = i_s$, $T = L_{ra} i_r i_s \; [-\sin\theta + \cos\theta]$

For such a system, the torque is zero at $\theta = +45^o$, and the torque is maximum at $\theta = -45^o$.

If one of the stator currents is reversed, the result differs. For this, let $i_a = -i_s$ and $i_b = +i_s$

$$T = L_{ra} i_r i_s [\sin\theta + \cos\theta]$$

And the maximum torque occurs at $\theta = 45^0$. This is a position for rotor, which is midway between the two stator coils.

25.6.2.1. Different Categories

From the torque expressions above, it is clear that the torque exists only when stator and rotor-coils carry currents. When only stator-coils (or only rotor coil) carry current, torque cannot be produced.

(*a*) One coil each on Stator and on Rotor

In the above mentioned case, let us excite only one stator-coil. Let $i_a = i_s$, $i_b = 0_1$ and i_r maintained as before.

$$T = -L_{ra}\ i_r i_s \sin\theta$$

Following observations are made for such a case:

(i) At $\theta = 0$, $T = 0$, Mutual inductance x_{ra} is maximum ($= L_{ra}$) and hence the stored energy in mutual inductance is maximum, but torque is zero.

(ii) At $\theta = 90^0$, T is maximum. x_{ra} is zero, hence the concerned stored energy is zero.

(iii) As seen earlier, $W_4 = L_{ra}\ i_r i_a \cos\theta = L_{ra}\ i_r i_s \cos\theta$

Torque $= dW_4 / d\theta = i_r i_s\ d(L_{ra} \cos\theta) / d\theta$

Contributed by W_4, the power is related as follows:

Power = rate of change of energy with time

$$= dW_4 / dt = dW_4 / d\theta .\ d\theta / dt = T.\ \omega_r = i_r i_s\ L_{ra}\ (-\sin\theta)\ \omega_r$$

Magnitudes of these terms are maximum for $\theta = 90^0$. If θ can be set at 90^0, at all instants of time, torque obtained is maximum. Such a situation does exist in a d. c. machine in which rotor carries an armature winding which is a lap- or wave-connected commutator winding. The brushes are so placed on the commutator that rotor-coil-axis satisfies the above-mentioned condition of $\theta = 90^0$, irrespective of the rotor-position or rotor speed. Such an equivalence of a **rotating** armature coil with such an **effectively stationary** coil is referred to as a **quasi-stationary** coil. It means that a rotating coil is being analyzed as a stationary coil due to its typical behaviour for electro-mechanical energy conversion purposes.



Permanent magnet synchronous motor for washing machine

(*b*) Two stator coils carrying two-phase currents and rotor-coil carrying d. c.: When two stator coils carry two-phase alternating currents, a synchronously rotating mmf is established. If the rotor-coil carries direct current, and the rotor is run at same synchronous speed, a unidirectional constant torque is developed. Mathematically, similar picture can be visualized, with a difference that the total system is imagined to rotate at synchronous speed. Such a machine is Synchronous machine, (to be discussed in Later chapters). It can be understood through the simple system described here.



Permanent magnet

(*c*) Machines with Permanent Magnets.

With suitable interpretation, the field side of the simple system can be imagined to be with permanent magnets in place of coil-excited electromagnets. All the interpretations made above are

valid, except for the difference that in this case there is no scope for controlling the rotor-coil-current-magnitude.

(*d*) Machines with no rotor coil, but with premeance variation.

Smooth cylindrical rotor surfaces do not exist in such cases. There are no rotor-coils. Due to geometry of the rotor surface, stator-coil- self inductances vary with rotor position. Thinking on lines of relating energy terms and their derivatives for torque-calculations, the working principles can be understood. With simple construction, Reluctance motors belong to this category.



Current in stator coils

(*e*) Switched currents in Stator Coils.

In yet another type, stator coils are distributed and properly grouped. One group carries currents during certain time interval. Then, this current is switched off. Another group carries current in the next time interval and so on. The rotor surface is so shaped that it responds to this current switching and torque is produced. Even though stator-coil-inductances are complicated functions of rotor position, the method of analysis for such machines is same. Prominent types of machines of this type are: switched reluctance motors, stepper motors, etc.

25.6.2.2. Vital Role of Air-gap

Magnetic circuit of an electrical machine has a flux established due to coil-mmfs. This flux is associated with stator core, rotor core and air-gap. An important point for understanding is to know which out of these three stores major portion of the field energy. Through an illustrative case, it will be clear below, in example 25.1.

Example 25.1. *Let a machine with following data be considered.*

Calculate the energy stored in the air-gap and compare the same with that stored in the cores.

Stator-core outer diameter = *15 cm*

Stator-core inner diameter = *10.05 cm*

Rotor-core outer diameter = *10.00 cm*

Rotor-core inner diameter = *5 cm*

Axial length of the machine = *8 cm*

Effect of slotting is neglected. The core volumes and air-gap volume for the machine shown in Fig 25.4 have to be calculated.

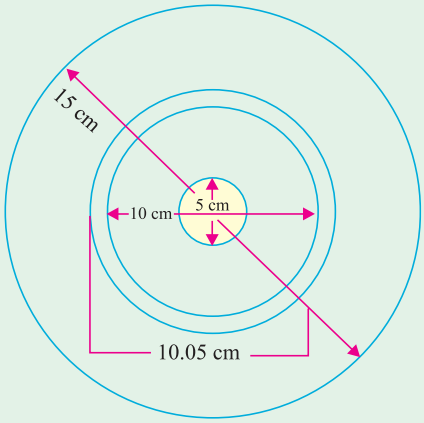


Fig. 25.4

Solution.

Volume of Stator-core $= (\pi / 4) \times (15^2 - 10.05^2) \times 8$ cm^3 $= 779$ cm^3

Volume of Rotor-core $= (\pi / 4) \times (10^2 - 5^2) \times 8$ cm^3 $= 471$ cm^3

Volume of air-gap in the machine $= (\pi / 4) \times (10.05^2 - 10^2) \times 8$ cm^3

$= 6.3$ cm^3

Let the relative permeability of the core material be 1000. If the flux density is B Wb / m^2, and μ is the permeability, the energy-density is ½ x B^2 / μ Joules / m^3. Let the flux density be 1.20 Wb / m^2. Energy density in air-gap = ½ × 1.20^2 / (4 π × 10^{-7}) = 572350 Joules / m^3

= 0.573 Joules / cm^3

Energy stored in air-gap = 0.573 × 6.3 = 3.6 Joules

Energy-density in Magnetic medium = ½ × 1.20^2 / (4π × 10^{-7} × 1000) = 573 J / m^3

It is assumed only for simplicity that the flux density is same for the entire core of stator and of rotor.

Energy stored in stator-core = 573 × 779 × 10^{-6} = 0.45 Joule

Energy stored in rotor-core = 573 × 471 × 10^{-6} = 0.27 Joule

It is worth noting that even though the ratio of volumes is 198, the ratio of energies is 0.2, since, for the present case,

$$K_v = \frac{\text{volume of (Stator - core + Rotor - core)}}{\text{Volume of air - gap}} = \frac{779+471}{6.3} = 198$$

k_E = Energy stored in cores / Energy stored in air-gap = (0.45 + 0.27) / 3.6 = 0.2

The ratio are like this due to μ_r being 1000, and k_v / k_E = 198 /0.2 = 1000

Alternatively, an air-gap of volume 6.3 cm^3, **[**surrounded by the magnetic medium of μ_r = 1000**]** is equivalent to the magnetic medium of volume 6.3 × 1000 cm^3.

$$\frac{\text{Converted equivalent volume of air-gap}}{\text{Volume of (Stator + Rotor)}} = \frac{6300}{779+471}$$

$$\frac{\text{Energy stored in air-gap}}{\text{Energy stored in (Stator + Rotor)}} = \frac{3.6}{(0.45+0.27)} = 5$$

This correlates the various parameters and confirms that the stored energy is maximum in the air-gap.

Or, one can now say that in the process of electro-mechanical energy-conversion, the air-gap plays a very vital role.

However, the stator-core and rotor-core help in completing the flux-path in a well defined manner for effective and efficient working of a rotary machine.

Example 25.2. *An electromagnetic relay has an exciting coil of 800 turns. The coil has a cross-sectional area of 5 cm × 5 cm. Neglect reluctance of the magnetic circuit and fringing.*

(a) (i) *Find the coil inductance if the air-gap length is 0.5 cm.*

(ii) *Find the field energy stored for a coil current of 1.25 amp.*

(b) *Coil-current remaining constant at 1.25 A, find the mechanical energy output based on field-energy changes when the armature moves to a position for which x = 0.25 cm. Assume slow movement of armature.*

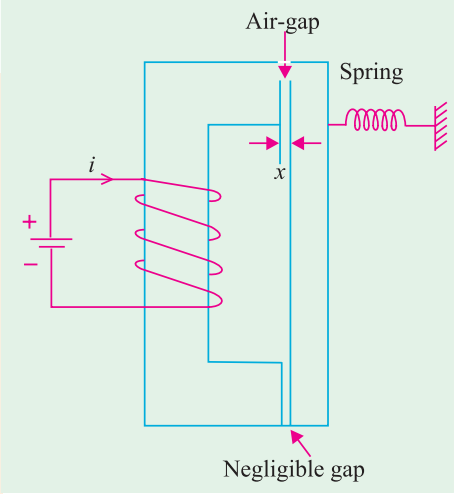


Fig. 25.5 Electro-magnetic relay

(c) Repeat (b) above based on force-calculations and mechanical displacement.

(d) What will be change in above results of mechanical work done, if the mechanical movement is fast, keeping the flux initially constant ?

Solution.

(a) (i) Permeance at air-gap = $\frac{\mu_0 5\times5\times10^{-4}}{0.5\times10^{-2}} = 4\pi \times 10^{-7} \times 10^{-2} = 6.28\times10^{-7}$

Coil Inductance = $N_2 \mathcal{P}_m = 800 \times 800 \times 6.28 \times 10^{-7} = 0.402$ H

(ii) Energy stored in magnetic field = $1/2\ Li^2 = 1/2 \times 0.402 \times 1.25^2$

$= 0.314$ joule

(iii) $W'_{fd} = 1/2\,L(x)i^2 = 1/2\left[\frac{H^2\mu_0 A}{l_x}\right] = \frac{1/2\times800\times800\times4\pi\times10^{-7}\times5\times5\times10^{-4}}{x}\times i^2$

$$= \frac{1.005\times10^{-3}}{x}\times i^2$$

$$F_f = \frac{\delta}{\delta_x}\frac{[i^2\times(1.005\times10^{-3})]}{x} = [1.005\times10^{-3}]\times i^2\times\frac{-1}{x^2}$$

This is to be evaluated at $x = 0.5 \times 10^{-2}$

$$= \frac{-1.005 \times 10^{-3} \times 1.25\times1.25}{(0.5\times10^{-2})^2} = -62.8\,\text{NW}$$

This force has to be balanced by the spring-tension.

(b) Energy-computations : Inductance for $x = 0.25$ cm is first calculated. $L(x_2) = N^2 \mathcal{P}_{m_2}$

$= 800 \times 800 \times 2 \times 6.28 \times 10^{-7} = 0.804$ Henry

If the mechanical movement is slow, net mmf remains unchanged and the operating point moves along *HC* vertically upwards and settles at *C*. Added Electrical Energy input during change-over of the operating point from *H* to *C*,

= area of rectangle $BDCH = (\phi_2 - \phi_1)\,F_1$

$$\left[\frac{L(x_2).i - L(x_1)i}{N}\right] Ni$$

$= i^2\ [L\,(x_2) - L\,(x_1)\,] = 1.25^2 \times [0.804 - 0.402]$

$= 0.628$ joule

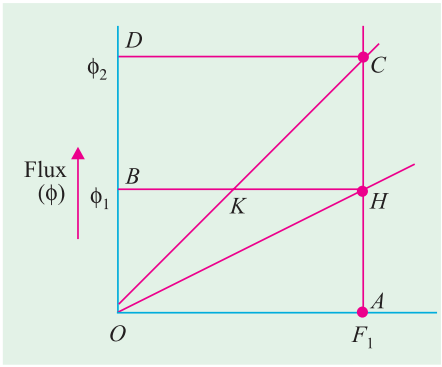


Fig. 25.6 Graphical correlation of energy-terms for the relay

Out of this, the additional stored energy in field, $dW_{fld} = \frac{1}{2}\,[L(x_2) - L(x_1)]$

$= 0.314$ joule

The remaining 0.314 joule is transformed into mechanical form and is related to the work-done. This is obtained when the force on moving member is multiplied by the displacement.

$$\therefore \int_{x_1}^{x_2} dW_{mech} = 0.314 \text{ joule}$$

(c) As in *a* (*iii*) above,

$F(x) = [1.005 \times 10^{-3} \times 1.25^2]\ [-1/x^2]$

$$dW_{mech} = F(x).dx$$

$$W_{\text{mech}} = \int_{x_1}^{x_2} F(x)dx = k.\ [\int_{x_1}^{x_2} -1/\ x^2 d_x] = -\ k.\ \int_{x_1}^{x_2} x^{-2}dx$$

$$=1.005\times10^{-3}\times1.25^2\times\left[\frac{1}{0.25}-\frac{1}{0.5}\right]\times10^2=\frac{1.005\times1.25^2}{10}\times2=0.314 \text{ joule}$$

This agrees with answer obtained in (*b*) above.

With fast movement of armature, the operating point will move from H to K first, then follow the path *KC*.

This means that the energy represented by the area of the triangle KHC corresponds to the reduced consumption of energy.

OC has a slope of $\mathcal{P}_{m_2} = 12.56 \times 10^{-7}$

OH has a slope of $\mathcal{P}_{m_1} = 6.28 \times 10^{-7}$

BK = mmf required for establishing a flux of ϕ_1 with an air-gap of 0.25 cm

BK = ½ *OA* = ½ × (800 × 1.25) = 500 amp-turns = *HK* in the present case.

Area of the triangle *KHC* = ½ × *KH* × *HC*

= ½ × 500 × $[\phi_2 - \phi_1]$

= ¼th of area of rectangle *BDCH*, in this case

= ¼ × 0.628 = 0.157 joule.

Hence, Electrical energy fed during this process = area *BKCD*

= area *BDCH* – area *KHC* = 0.628 – 0.157 = 0.471 joule

Increase in field energy stored $\Delta W'_{fld}$ = area *OKH*

= area OHC – area *KHC* = 0.314 – 0.157 = 0.157

Mechanical Energy output = 0.471 – 0.157 = 0.314 Joule

It indicates that with fast movement, the electrical energy-input and the field-stored energy have decreased by 0.157 J each but the mechanical-energy-term remains unaffected by fast or slow movements of armature.

25.7. Dynamic Equations and System-model of a Simple System

It is quite necessary to analyze electro-mechanical conversion system for predicting the performance and/or for monitoring the system. A simple system is being taken up here to deal with dynamic equations and a simple model with its components is being related to the system. The details will vary from system to system, and accordingly the equations will vary.

Fig. 25.7 shows different components of such a system meant for electrical to mechanical conversion. On one side, an electrical source feeds the device at the 'electrical port'. On the other side, a force f_e is developed at the 'mechanical port'. Mechanical load is connected to this port.

(*a*) *At Electrical Port* : A voltage source is shown to feed the device. r is its effective internal resistance. At the electrical port, the inputs are λ (= flux linkage with the coil) and *i*. From λ , the voltage induced in the coil can always be evaluated.

(*b*) *Role of the Conversion device*: With these inputs, the device converts the energy into mechanical form, and is available as a force f_e (in case of linear motions), and, displacement *x* measured from a suitable reference.

(*c*) *At the Mechanical Port*: The possible items are: spring, damper, mass and an applied mechanical force. Their natural and simple dependence on displacement x and its derivatives are indicated below:

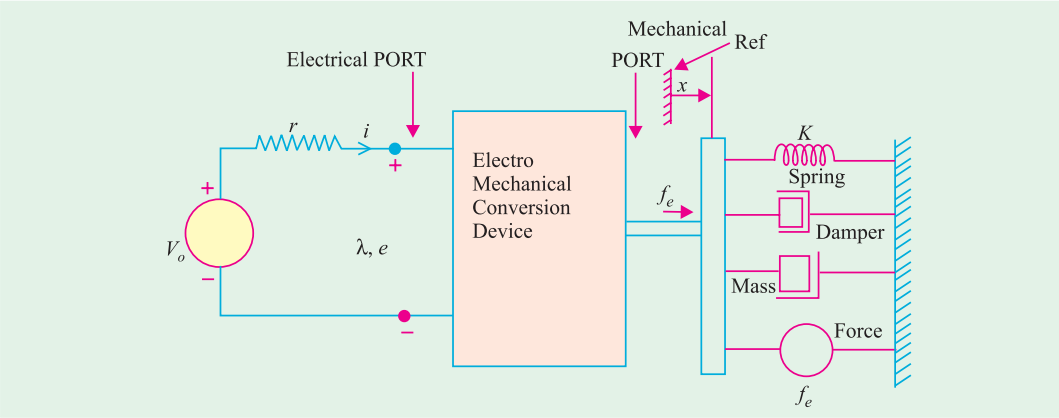


Fig. 25.7 Linear motion: MODEL

(*i*) Spring: Force required to overcome spring elongation is proportional to the displacement x.

(*ii*) Damper: Force required to overcome damping action in the system is proportional to derivative of x.

(*iii*) Mass: Force required to overcome acceleration of mass is proportional to second derivative of x.

(*iv*) Applied force, f_o: This has to be overcome by f_e . In terms of an equation, these terms are related as follows:

$$f_e = k_s (x - x_o) + B\dot{x} + M\ddot{x} + f_o$$

where

k_s = spring constant

x_o = value of x for unstretched spring

B = damping constant

M = Mass to be accelerated

f_o = External mechanical force applied to the system.

25.8. Statically induced emf and Dynamically Induced emf :

In Fig. 25.7 source voltage is v_o. Let $L(x)$ be the coil inductance as a function of displacement x. In a very general case,

$$\begin{aligned} v_o(t) &= ri + d\lambda/dt \\ &= ri + d/dt\ [L(x) \times i] \\ &= ri + L(x).\ di/dt + i.\ dL(x)/dx.\ dx/dt \end{aligned}$$

The second term on the right-hand side is statically induced emf (or transformer-emf), since change of current with time is responsible for it. This cannot produce any force (or torque) and hence cannot convert energy from electrical to mechanical form (or vice-versa).

The third term on the right hand side includes the speed (= dx/dt) and dependence of L (x) on x. Any of these, if non-existent, will mean that third term reduces to zero. This term relates dynamically induced emf (= speed emf) and is the main indicator of the process of electro-

mechanical energy conversion. So, for conversion, there must be an inductance which varies with the system position, and a motion must be there. In addition, coil must carry a current.

Having understood the linear-motion-system, it is easier to understand the system with rotary motion, with due modifications.

Example 25.3. *A doubly excited rotating machine has the following self and mutual inductances.*

$r_s = 40\ \Omega, \quad L_s = 0.16\ H$

$r_r = 2\ \Omega, \quad L_r = 0.04 + 0.02 \cos 2\theta$

$M_{sr} = 0.08 \cos \theta$

where θ is the space-angle between axes of rotor-coil and of stator-coil. The rotor is revolving at a speed of 100 radians/sec. For i_s = 10 Amp d. c., and i_r = 2 Amp d. c., obtain an expression for torque and corresponding electrical power.

[Rajiv Gandhi Technical University, Bhopal, Summer 2001]

Solution. W_{fld} = Total energy stored

$$= \tfrac{1}{2} L_s i^2_s + 1/2\, L_r i^2_r + M_{sr} i_s i_r$$

$$= \tfrac{1}{2} (0.16) i^2_s + \tfrac{1}{2} [0.04 + 0.02 \cos 2\theta] i^2_r + [0.08 \cos \theta]\, i_s i_r$$

since i_s and i_r are direct currents of constant magnitudes, there is no variation with ϕ or with t.

Relating torque with W_{fld} and substituting current-magnitudes,

$$\text{Torque, } T = \frac{-dW_{fld}}{d\theta} = -[0 + \tfrac{1}{2} \times 0.02 \times 2^2 (-2\sin 2\theta) + 0.08 (-\sin \theta)(10 \times 2)]$$

$$= 0.08 \sin 2\theta + 1.6 \sin \theta$$

$$= 1.6 \sin \theta + 0.08 \sin 2\theta \text{ Nw-m}$$

On the right hand side, the first term is electromagnetic Torque which is dependent on both the currents. Second term is dependent only on one current, and is of the type categorized as Reluctance-torque which depends on non-cylindrical shape,in this case, on the stator side, as shown in Fig. 25.8

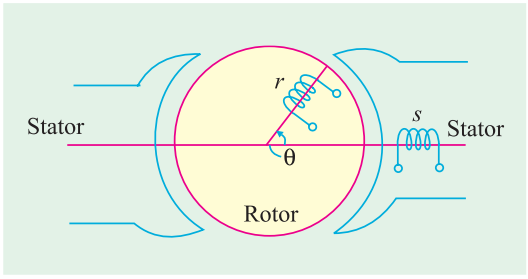


Fig. 25.8

Starting from W_{fld}, electrical power can be expressed, since it is well -known that

Power = time rate of change of energy

$$\text{Electrical power, } p = \frac{dW_{fld}}{dt}$$

$$= \frac{d}{dt}\left[\tfrac{1}{2} L_s i^2_s + \tfrac{1}{2} L_r i^2_r + M_{sr} i_s i_r\right]$$

$$= \tfrac{1}{2} i^2_s \frac{d}{dt}(L_s) + \tfrac{1}{2} i^2_r \frac{d}{dt}(L_r) + i_s i_r \frac{d}{dt}(M_{sr})$$

$$= \tfrac{1}{2} i^2{}_s \left[\frac{d}{d\theta}(L_s)\right]\frac{d\theta}{dt} + \tfrac{1}{2} i^2{}_r \left[\frac{d}{d\theta} + (L_r)\right]\frac{d\theta}{dt} + i_s i_r \left[\frac{d}{d\theta}(M_{st})\right]\frac{d\theta}{dt}$$

$$= \tfrac{1}{2} i^2{}_s [\text{zero}] \times 100 + \tfrac{1}{2} i_r{}^2 [-0.02 \times 2 \times \sin 2\theta] \times 100 + i_s i_r [-0.08 \sin\theta] \times 100$$

Substituting numerical values of currents,the electrical power is expressed as a function of θ, as below :

p = [zero + (– 8) sin 2θ + (–160) sin θ] watts

= [– 160 sin θ – 8 sin 2θ] watts

Proper interpretation of sign of power (as dependent on θ) is important. Positive power is received by the coils, while negative power is received by the source.

Example 25.4. *An inductor has an inductance which varies with displacement x as*

$$L = 2 L_o / [1 + (x / x_o)]$$

Where L_o *= 50 mh,* x_o *= 0.05cm, x = displacement in cm,*

The coil –resistance is 0.5 ohm.

(a) *The displacement x is held constant at 0.075 cm, and the current is increased from 0 to 3 amp. Find the resultant magnetic stored energy in the inductor.*

(b) *The current is then held constant at 3 amp and the displacement is increased to 0.15 cm. Find the corresponding change in the magnetic stored energy.*

Note. Assume that all electrical transients are negligible.

Solution. ***(a)*** Inductance at x = 0.075 cm is calculated first.

$$L_1 = \frac{2L_0}{1 + (0.075/0.05)} = 40\,\text{mH}$$

λ_1 = L_1 × current = 120 × 10^{-3}, corresponding to point A in Fig. 25.9

W_{fld} = ½ L_1 $(3)^2$ = ½ (40 × 10^{-3} × 9) = 0.18 joule

(b) Inductance for x = 0.15 cm is to be calculated now.

$$L_2 = \frac{2L_0}{1 + (0.075/0.05)} = 25\,\text{mH}$$

With current held constant at 3amp, the flux-Linkage is now λ_2= (25 × 10^{-3}) × 3 = 75 × 10^{-3}

Since the current is constant at 3amp, magnetic stored energy is reduced by the area of triangle *OAB*, in Fig. 25.9

Area of triangle *OAB* = ½ × 3 × (120 –75) 10^{-3} = 0.0675 joule

Check : Stored-energy at *B* in terms of L_2 and i, is given by

W_{fld} = ½ (25 x 10^{-3}) × 3^2 =0.1125 joule

Alternatively, $W_{fld2} = W_{fld1}$ – area of Δ OAB

= 0.18 – 0.0675 = 0.1125 Joule

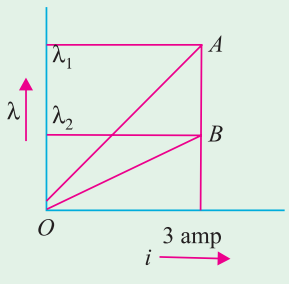


Fig. 25.9

Example 25.5. *If the inductor in the previous case is connected to a voltage source which increases from 0 to 3 V [part (a)] and then is held constant at 3 V [part (b)],repeat the problem, assuming that electrical transients are negligible.*

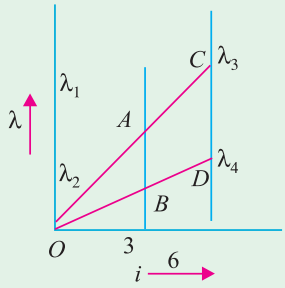


Fig. 25.10

Solution. Coil resistance is 0.5 ohm. When the voltage reaches 3 V, the coil current is 6 amp. In part (*a*), L_1 = 40 mH. Hence, W_{fld3} = energy stored = ½ $L_1 i^2_1$ = 0.72 joule, at point *C* in Fig. 25.10. In part (*b*), L_2 = 25 *mH*. The current is held constant at 6 amp. Working on similar lines,

ΔW_{fld} = change in the field energy stored = area of triangle *ODC* or $\Delta W_{fld} = W_{fld3} - W_{fld4}$

W_{fld} = ½ × 25 × 10^{-3} × 36 = 0.45 Joule, at point *D*

Change in energy stored in the field = $W_{fld3} - W_{fld4}$ = 0.72 – 0.45 = 0.27 joule

Or ΔW_{fld} = area of ΔODC = ½ × 6 × ($\lambda_3 - \lambda_4$)

Here λ_3 = 40 × 10^{-3} × 6, and λ_4 = 25 × 10^{-3} × 6

ΔW_{fld} = ½ × 6 × 6 × 10^{-3} (40 – 25) = 0.27 joule

Example 25.6. *A coil of an electromagnetic relay is associated with a magnetic circuit whose reluctance is given by*

$= a + bx$

where a and b are positive constants decided by the details of the magnetic circuit, in which x is the length of the air-gap between fixed and movable members. If the coil is connected to an A.C. source where voltage is described by

$v = V_m \sin \omega t$,

find the expression for the average force on armature, with air-gap held constant at x.

Solution. If ϕ = flux established, in Webers

N = number of turns on the coil, λ = flux-linkage in Weber-turns

W_{fld} = ½ $\mathcal{R}$ ϕ^2

And force $F = \dfrac{\delta W_{fld}}{\delta x} = \frac{1}{2}\ \phi^2 \dfrac{\delta \mathcal{R}}{\delta x} = -\frac{1}{2} b \phi^2$

The current in the coil is given by

$$v = R + L\frac{di}{dt}$$

for which, the steady-state solution for current with an a.c. voltage applied to the coil is given by

$$I = \frac{V}{\sqrt{R^2 + \omega^2 L^2}} \angle -\theta \text{ where } \theta = \tan^{-1}\frac{(\omega L)}{R}$$

RMS voltage, $V = (V_m) / \mathcal{R}\sqrt{2}$

Instantaneous current i is expressed as

$$i = \frac{\sqrt{2}V}{\sqrt{R^2 + \omega^2 L^2}} \sin(\omega t - \theta)$$

Further, L = $N^2/\mathcal{R}$

$$\phi = Ni/\mathcal{R} = \frac{\sqrt{2}\,NV}{(R\mathcal{R})^2 + (N^2\omega)^2} \sin(\omega t - \theta)$$

Force, $$F_f = \frac{\mathcal{R} - bN^2V^2}{(R\ )^2 + (N^2\omega)^2} \sin^2(\omega t - \theta)$$

The last term $\sin^2(\omega t - \theta)$ has a time average (over a cycle) of ½.

Hence, average force, $$F_f\,(\text{av}) = -\tfrac{1}{2}\frac{\mathcal{R}bN^2V^2}{(R\ )^2 + (N^2\omega)^2}$$

Force is in such a direction that x will be reduced, or that the energy stored tends to increase.

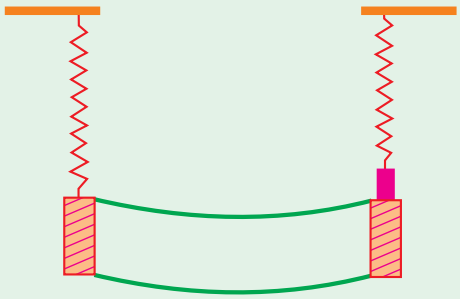
Example 25.7. *Two coupled coils have self- and mutual-inductances as expressed below:*

$$L_{11} = 1 + (1/x),$$
$$L_{22} = 0.5 + (1/x),$$
$$L_{12} = L_{21} = 1/x$$

These expressions are valid over a certain range of linear displacement x, in cms. The first coil is excited by a constant current of 20 A and the second one, by a constant current of – 10 A. Find

(a) *mechanical work done if x changes from 0.5 to 1.0 cm*

(b) *energy supplied by the two electrical sources in (a) above.*



Two coupled coils

Solution. With data given, substituting the values of currents,

$$W_{fld} = \tfrac{1}{2}L_{11}i_1^2 + L_{12}i_1i_2 + \tfrac{1}{2}L_{22}i_2^2$$
$$= 225 + (50/x)$$

$$F_f = -\delta W_{fld}/\delta x = 50/x^2$$

(a) $$\Delta W_{mech} = \int_{0.5}^{1.0} (50/x^2)\,dx$$

$$= 50[x^{-1}/-1]_{0.5}^{1.0} = +50 \text{ joules}$$

At $x = 0.5$, $W_{fld} = 325$ joules, and

at $x = 1.0$, $W_{fld} = 275$ joules

Thus, increase in x from 0.50 to 1.0 cm decreases the stored energy in the field from 325 to 275 joules. The field-system, thus, releases an energy of 50 joules.

(b) Calculations of Energy input from electrical sources –

$$\lambda_1 = L_{11} i_1 + L_{12} i_2$$

$$= 20 [1 + (1/x)] - 10 (1/x) = 20 + (10/x)$$

At $x = 0.5$, $\lambda_1 = 20 + 20 = 40$ Wb-turns

$x = 1.0$, $\lambda_1 = 20 + 10 = 30$ Wb-turns

$\Delta_{Welec1} = i_1$ [change in λ_1 due to displacement]

$= 20 \times (-10) = -200$ Joules

Similarly, $\lambda_2 = L_{12} i_1 + L_{22} i_2 = -5 + (10/x)$

At $x = 0.5$, $\lambda_2 = -5 + 20 = +15$ Wb-turns

$x = 1.0$, $\lambda_2 = -5 + 10 = +5$ Wb-turns

$\Delta W_{elec2} = (-10)(-10) = +100$ Joules.

Seeing the signs and numerical values, it can be seen that Source 1 receives an energy of 200 Joules, which comes from three constituents:

100 J from source 2,

50 J from field energy stored,

and 50 J from mechanical system.

Tutorial Problems 25.1

(a) A magnetic circuit has a coil with 1000 turns. Its reluctance is expressed as

$\mathcal{R} = [8.5 + 40\ g] \times 10^{+3}$ MKS units

where g = air- gap length in mm, between fixed and movable parts. For a coil current of 2.0 amp held constant and with slow movement, calculate the change in the field energy stored, if the length of the air-gap changes from 0.20 to 0.15 cm. Calculate the mechanical force experienced by the system.

Hint: $\Delta W_e = i(\lambda_2 - \lambda_1)$, $\Delta W_{fld} = \frac{1}{2} \Delta W_e$

Force, $F = -\Delta W_{fld} / \Delta x$

[**Ans.** $\Delta W_{fld} = 6.60$ J, Force $= 13200$ Nw]

(b) An electro-magnetic relay with an air-gap of x cm has the current and flux-linkage relationship as

$i = \lambda^2 + \lambda (0.5 - x)^2$ amp, for $x < 0.5$ cm

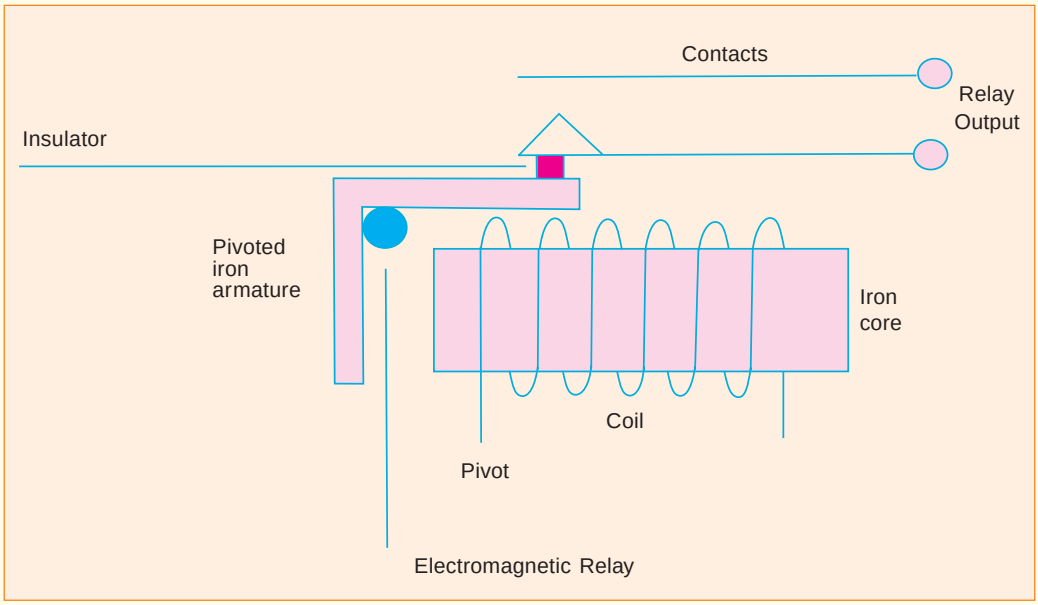
Find the force on armature as a function of λ and x.

Hint: $W_f(\lambda_1 x) = \int_{0.5}^{1.0} id\lambda$

And $F_f = -\delta W_f / \delta x$

[**Ans.** $W_f(\lambda, x) = (\lambda^3/3) + (\lambda^2/2)(0.5 - x)^2$

$F_f = \lambda^2 (0.5 - x)$



(c) For a rotary system, the stator-coil and the rotor-coil have self and mutual-inductances as described below, with suffix 1 for stator and 2 for rotor:

$L_{11} = L_{22} = 4 - (6\theta / \pi)$ for $0 < \theta < \pi / 2$

$= 1 + (6 / \pi)(\theta - 0.5\pi)$ for $\pi / 2 < \theta < \pi$

(Note: Self inductances cannot be negative.)

$L_{12} = L_{21} = 6 (1 - 2\theta/\pi)$ for $0 < \theta < \pi$

Evaluate the inductances and the torque for $\theta = \pi / 4$ and the two coil currents of 5 amp constant in magnitude.

Hint: $\delta L / \delta\theta$ contributes to torque.

[Ans. $L_{11} = L_{22} = 2.5\ H$

$L_{12} = +3\ H$

$T = 450 / \pi$ Nw**]**

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CHAPTER 26

D.C. GENERATORS

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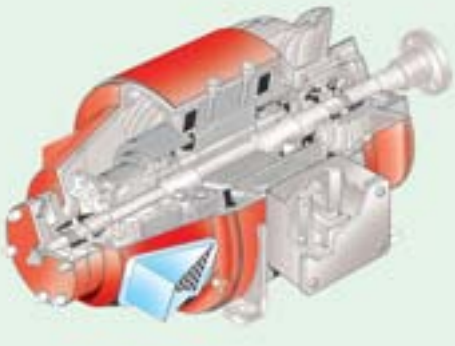
Generator converts mechanical energy into electrical energy using electromagnetic induction

26.1. Generator Principle

An electrical generator is a machine which converts mechanical energy (or power) into electrical energy (or power).

The energy conversion is based on the principle of the production of dynamically (or motionally) induced e.m.f. As seen from Fig. 26.1, whenever a conductor cuts magnetic flux, dynamically induced e.m.f. is produced in it according to Faraday's Laws of Electromagnetic Induction. This e.m.f. causes a current to flow if the conductor circuit is closed.

Hence, two basic essential parts of an electrical generator are ***(i)*** a magnetic field and ***(ii)*** a conductor or conductors which can so move as to cut the flux.



Cut-away view of dc generator

26.2. Simple Loop Generator

Construction

In Fig. 26.1 is shown a single-turn rectangular copper coil *ABCD* rotating about its own axis in a magnetic field provided by either permanent magnet is or electromagnets. The two ends of the coil

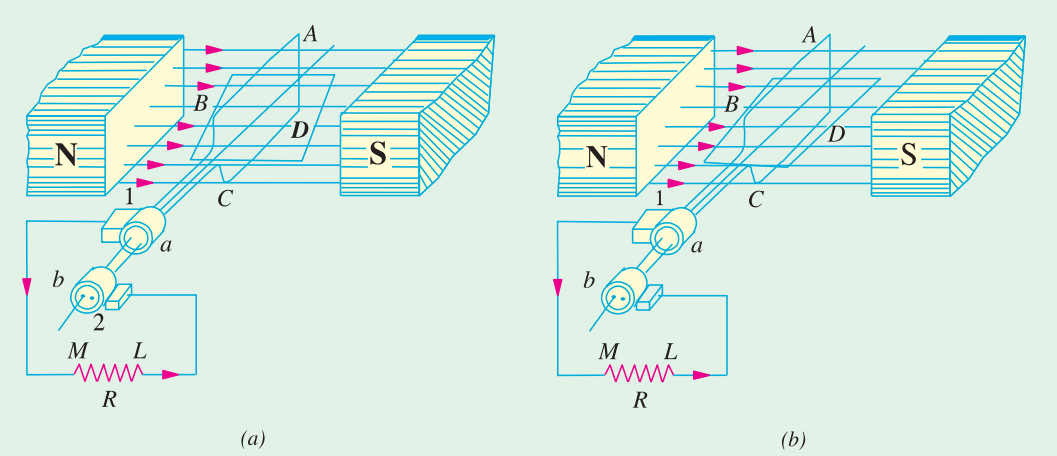


Fig. 26.1

are joined to two slip-rings '*a*' and '*b*' which are insulated from each other and from the central shaft. Two collecting brushes (of carbon or copper) press against the slip-rings. Their function is to collect the current induced in the coil and to convey it to the external load resistance *R*.

The rotating coil may be called 'armature' and the magnets as 'field magnets'.

Working

Imagine the coil to be rotating in clock-wise direction (Fig. 26.2). As the coil assumes successive positions in the field, the flux linked with it changes. Hence, an e.m.f. is induced in it which is

proportional to the rate of change of flux linkages ($e = Nd\Phi dt$). When the plane of the coil is at right angles to lines of flux *i.e.* when it is in position, 1, then flux linked with the coil is maximum but ***rate of change of flux linkages is minimum.***

It is so because in this position, the coil sides *AB* and *CD* do not cut or shear the flux, rather they slide along them *i.e.* they move parallel to them. Hence, there is no induced e.m.f. in the coil. Let us take this no-e.m.f. or vertical position of the coil as the starting position. The angle of rotation or time will be measured from this position.

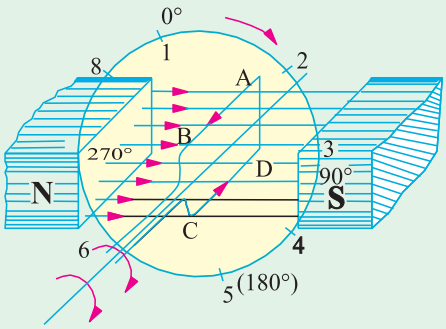


Fig. 26.2

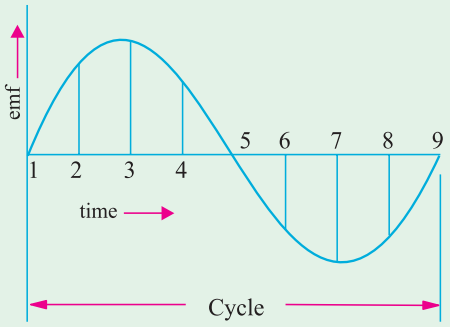


Fig. 26.3

As the coil continues rotating further, the rate of change of flux linkages (and hence induced e.m.f. in it) increases, till position 3 is reached where $\theta = 90^\circ$. Here, the coil plane is horizontal *i.e.* parallel to the lines of flux. As seen, the flux linked with the coil is minimum but ***rate of change of flux linkages is maximum***. Hence, maximum e.m.f. is induced in the coil when in this position (Fig. 26.3).

In the next quarter revolution *i.e.* from 90° to 180°, the flux linked with the coil gradually ***increases*** but the rate of change of flux linkages ***decreases*****.** Hence, the induced e.m.f. decreases gradually till in position 5 of the coil, it is reduced to zero value.

So, we find that in the first half revolution of the coil, no (or minimum) e.m.f. is induced in it when in position 1, maximum when in position 3 and no e.m.f. when in position 5. The direction of this induced e.m.f. can be found by applying Fleming's Right-hand rule which gives its direction from *A* to *B* and *C* to *D*. Hence, the direction of current flow is ***ABMLCD*** (Fig. 26.1). The current through the load resistance *R* flows from *M* to *L* during the first half revolution of the coil.

In the next half revolution *i.e.* from 180° to 360°, the variations in the magnitude of e.m.f. are similar to those in the first half revolution. Its value is maximum when coil is in position 7 and minimum when in position 1. But it will be found that the direction of the induced current is from *D* to *C* and *B* to *A* as shown in Fig. 26.1 (*b*). Hence, the path of current flow is along ***DCLMBA*** which is just the reverse of the previous direction of flow.

Therefore, we find that the current which we obtain from such a simple generator reverses its direction after every half revolution. Such a current undergoing periodic reversals is known as alternating current. It is, obviously, different from a direct current which continuously flows in one and the same direction. It should be noted that alternating current not only reverses its direction, it does not even keep its magnitude constant while flowing in any one direction. The two half-cycles may be called positive and negative half-cycles respectively (Fig. 26.3).

For making the flow of current unidirectional in the ***external*** circuit, the slip-rings are replaced by split-rings (Fig. 26.4). The split-rings are made out of a conducting cylinder which is cut into two halves or segments insulated from each other by a thin sheet of mica or some other insulating material (Fig. 26.5).

As before, the coil ends are joined to these segments on which rest the carbon or copper brushes.

It is seen [Fig. 26.6 (*a*)] that in the first half revolution current flows along ***(ABMNLCD)*** *i.e.* the brush No. 1 in contact with segment ***'a'*** acts as the positive end of the supply and ***'b'*** as the negative end. In the next half revolution [Fig. 26.6 (*b*)], the direction of the induced current in the coil has reversed. But at the same time, the positions of segments ***'a'*** and ***'b'*** have also reversed with the

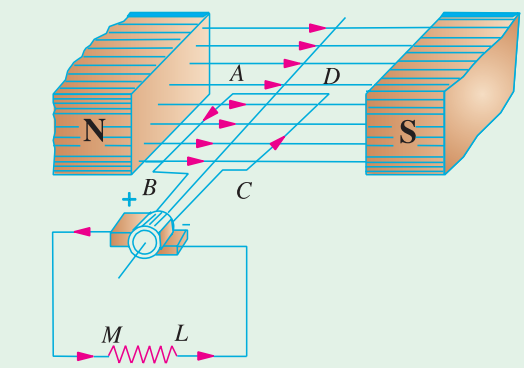


Fig. 26.4

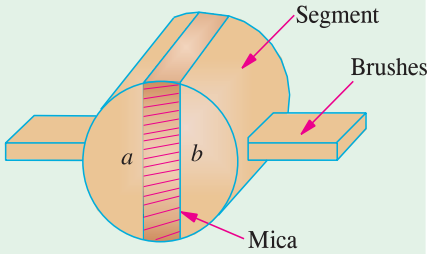


Fig. 26.5

result that brush No. 1 comes in touch with the segment which is positive *i.e.* segment ***'b'*** in this case. Hence, current in the load resistance again flows from ***M*** to ***L***. The waveform of the current through the external circuit is as shown in Fig. 26.7. ***This current is unidirectional but not continuous like pure direct current.***

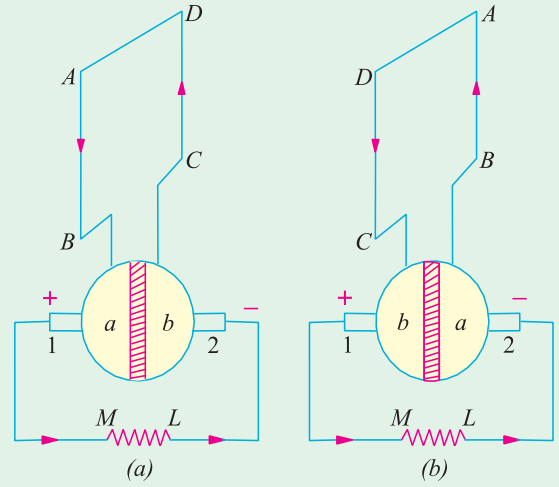


Fig. 26.6

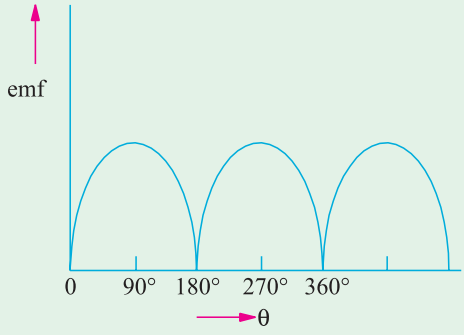


Fig. 26.7

It should be noted that the position of brushes is so arranged that the change over of segments ***'a'*** and ***'b'*** from one brush to the other takes place when the plane of the rotating coil is at right angles to the plane of the lines of flux. It is so because in that position, the induced e.m.f. in the coil is zero.

Another important point worth remembering is that even now the current induced in the coil sides is alternating as before. It is only due to the rectifying action of the split-rings (also called commutator) that it becomes unidirectional in the external circuit. Hence, it should be clearly understood that even in the armature of a d.c. generator, the induced voltage is alternating.

26.3. Practical Generator

The simple loop generator has been considered in detail merely to bring out the basic principle

underlying construction and working of an actual generator illustrated in Fig. 26.8 which consists of the following essential parts :

1. Magnetic Frame or Yoke
2. Pole-Cores and Pole-Shoes
3. Pole Coils or Field Coils
4. Armature Core
5. Armature Windings or Conductors
6. Commutator
7. Brushes and Bearings

Of these, the yoke, the pole cores, the armature core and air gaps between the poles and the armature core or the magnetic circuit whereas the rest form the electrical circuit.

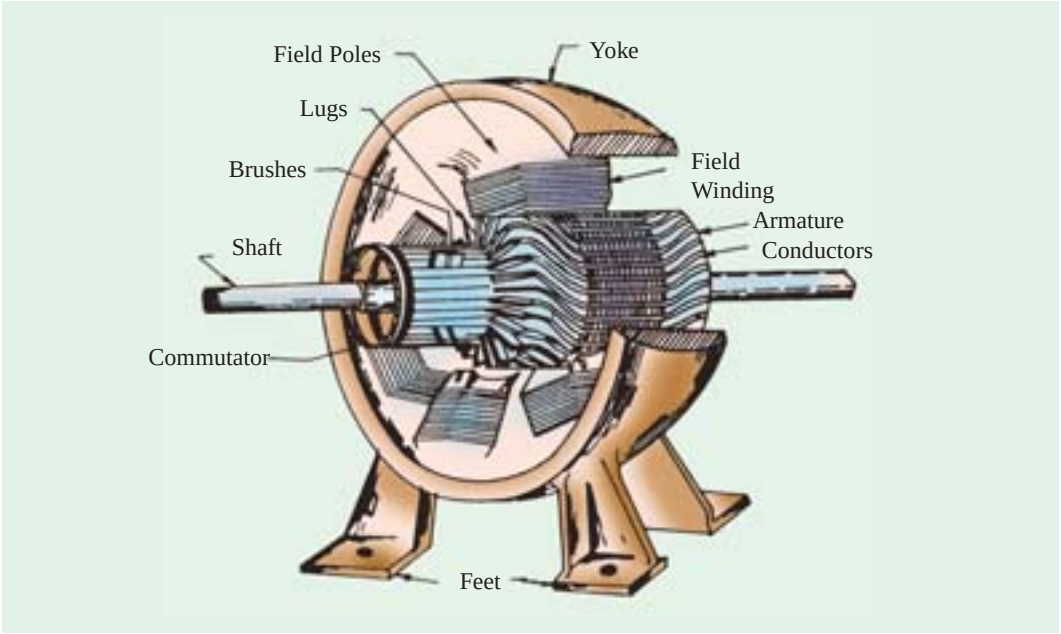


Fig. 26.8

26.4. Yoke

The outer frame or yoke serves double purpose :

(i) It provides mechanical support for the poles and acts as a protecting cover for the whole machine and

(ii) It carries the magnetic flux produced by the poles.



Yoke

In small generators where cheapness rather than weight is the main consideration, yokes are made of cast iron. But for large machines usually cast steel or rolled steel is employed. The modern process of forming the yoke consists of rolling a steel slab round a cylindrical mandrel and then welding it at the bottom. The feet and the terminal box etc. are welded to the frame afterwards. Such yokes possess sufficient mechanical strength and have high permeability.

26.5. Pole Cores and Pole Shoes

The field magnets consist of pole cores and pole shoes. The pole shoes serve two purposes

(*i*) they spread out the flux in the air gap and also, being of larger cross-section, reduce the reluctance of the magnetic path **(*ii*)** they support the exciting coils (or field coils) as shown in Fig. 26.14.

There are two main types of pole construction.

(*a*) The pole core itself may be a solid piece made out of either cast iron or cast steel but the pole shoe is laminated and is fastened to the pole face by means of counter sunk screws as shown in Fig. 24.10.

(*b*) In modern design, the complete pole cores and pole shoes are built of thin laminations of annealed steel which are rivetted together under hydraulic pressure (Fig. 26.11). The thickness of laminations varies from 1 mm to 0.25 mm. The laminated poles may be secured to the yoke in any of the following two ways :

(*i*) Either the pole is secured to the yoke by means of screws bolted through the yoke and into the pole body or

(*ii*) The holding screws are bolted into a steel bar which passes through the pole across the plane of laminations (Fig. 26.12).

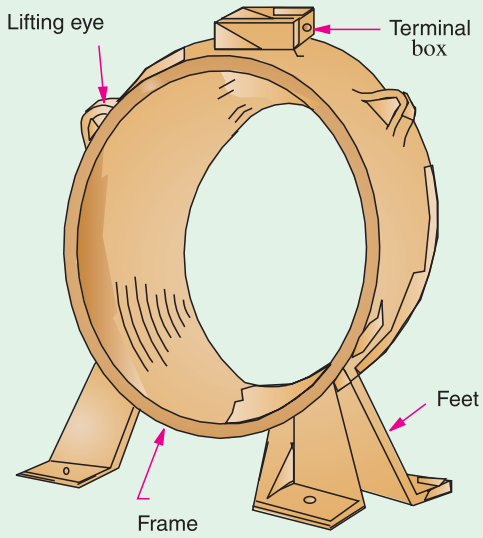


Fig. 26.9

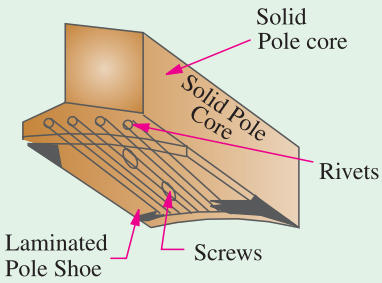


Fig. 26.10

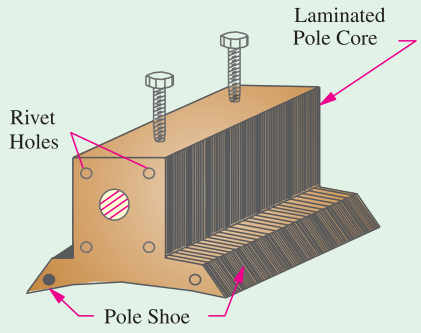


Fig. 26.11

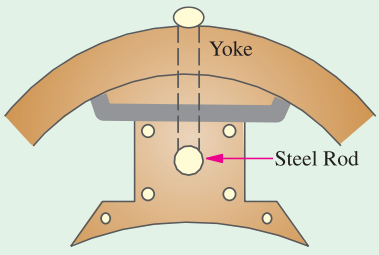


Fig. 26.12

26.6. Pole Coils

The field coils or pole coils, which consist of copper wire or strip, are former-wound for the correct dimension (Fig. 26.13). Then, the former is removed and wound coil is put into place over the core as shown in Fig. 26.14.

When current is passed through these coils, they electromagnetise the poles which produce the necessary flux that is cut by revolving armature conductors.

26.7. Armature Core

It houses the armature conductors or coils and causes them to rotate and hence cut the magnetic flux of the field magnets. In addition to this, its most important function is to provide a path of very low reluctance to the flux through the armature from a *N*-pole to a *S*-pole.

It is cylindrical or drum-shaped and is built up of usually circular sheet steel discs or laminations approximately 0.5 mm thick (Fig. 26.15). It is keyed to the shaft.

The slots are either die-cut or punched on the outer periphery of the disc and the keyway is located on the inner diameter as shown. In small machines, the armature stampings are keyed directly to the shaft. Usually, these laminations are perforated for air ducts which permits axial flow of air through the armature for cooling purposes. Such ventilating channels are clearly visible in the laminations shown in Fig. 26.16 and Fig. 26.17.

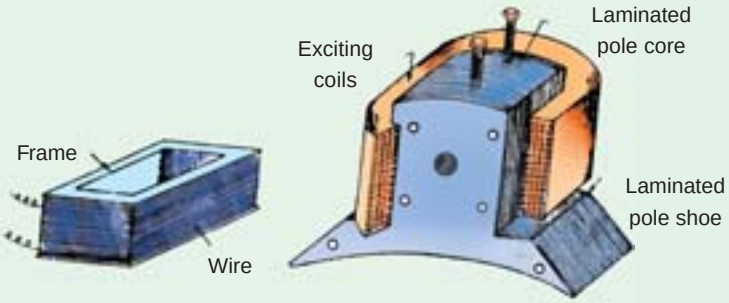


Fig. 26.13

Fig. 26.14

Up to armature diameters of about one metre, the circular stampings are cut out in one piece as shown in Fig. 26.16. But above this size, these circles, especially of such thin sections, are difficult to handle because they tend to distort and become wavy when assembled together. Hence, the circular laminations, instead of being cut out in one piece, are cut in a number of suitable sections or segments which form part of a complete ring (Fig. 26.17).

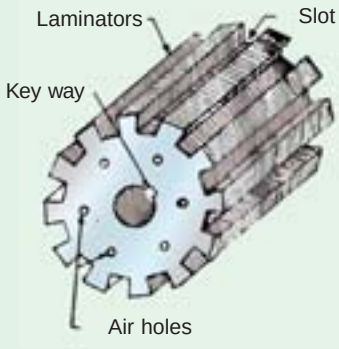


Fig. 26.15

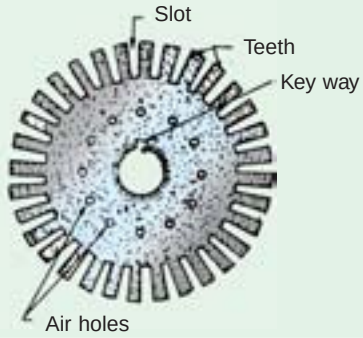


Fig. 26.16

A complete circular lamination is made up of four or six or even eight segmental laminations. Usually, two keyways are notched in each segment and are dove-tailed or wedge-shaped to make the laminations self-locking in position.

The purpose of using laminations is to reduce the loss due to eddy currents. Thinner the laminations, greater is the resistance offered to the induced e.m.f., smaller the current and hence lesser the $I^2 R$ loss in the core.

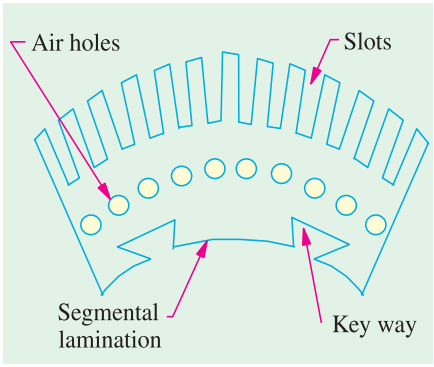


Fig. 26.17

26.8. Armature Windings

The armature windings are usually former-wound. These are first wound in the form of flat rectangular coils and are then pulled into their proper shape in a coil puller. Various conductors of the coils are insulated from each other. The conductors are placed in the armature slots which are lined with tough insulating material. This slot insulation is folded over above the armature conductors placed in the slot and is secured in place by special hard wooden or fibre wedges.

26.9. Commutator

The function of the commutator is to facilitate collection of current from the armature conductors. As shown in Art. 26.2, it rectified *i.e.* converts the alternating current induced in the armature conductors into unidirectional current in the external load circuit. It is of cylindrical structure and is built up of wedge-shaped segments of high-conductivity hard-drawn or drop forged copper. These

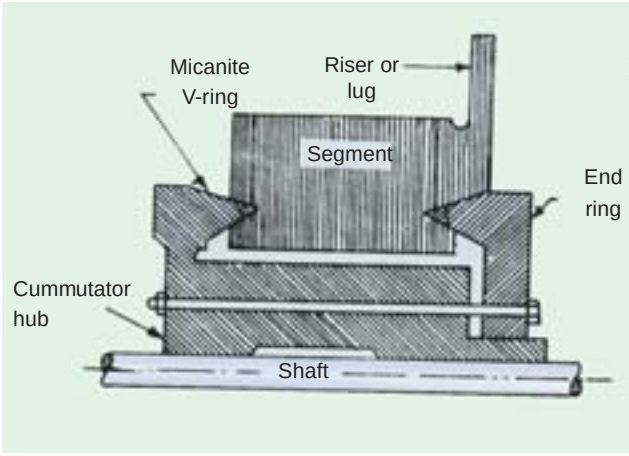


Fig. 26.18

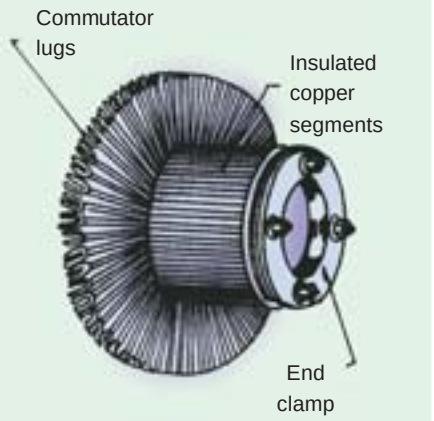


Fig. 26.19

segments are insulated from each other by thin layers of mica. The number of segments is equal to the number of armature coils. Each commutator segment is connected to the armature conductor by means of a copper lug or strip (or riser). To prevent them from flying out under the action of centrifugal forces, the segments have *V*-grooves, these grooves being insulated by conical micanite rings. A sectional view of commutator is shown in Fig. 26.18 whose general appearance when completed is shown in Fig. 26.19.

26.10. Brushes and Bearings

The brushes whose function is to collect current from commutator, are usually made of carbon or

graphite and are in the shape of a rectangular block. These brushes are housed in brush-holders usually of the box-type variety. As shown in Fig. 26.20, the brush-holder is mounted on a spindle and the brushes can slide in the rectangular box open at both ends. The brushes are made to bear down on the commutator by a spring whose tension can be adjusted by changing the position of lever in the notches. A flexible copper pigtail mounted at the top of the brush conveys current from the brushes to the holder. The number of brushes per spindle depends on the magnitude of the current to be collected from the commutator.


Bearing

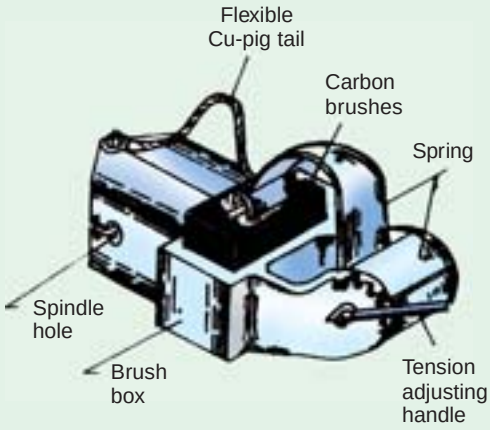


Fig. 26.20

Because of their reliability, ball-bearings are frequently employed, though for heavy duties, roller bearings are preferable. The ball and rollers are generally packed in hard oil for quieter operation and for reduced bearing wear, sleeve bearings are used which are lubricated by ring oilers fed from oil reservoir in the bearing bracket.

26.11. Armature Windings

Now, we will discuss the winding of an actual armature. But before doing this, the meaning of the following terms used in connection with armature winding should be clearly kept in mind.


Armature winding

26.12. Pole-pitch

It may be variously defined as :

(i) The periphery of the armature divided by the number of poles of the generator *i.e.* the distance between two adjacent poles.

(ii) It is equal to the number of armature conductors (or armature slots) per pole. If there are 48 conductors and 4 poles, the pole pitch is 48/4 = 12.

26.13. Conductor

The length of a wire lying in the magnetic field and in which an e.m.f. is induced, is called a conductor (or inductor) as, for example, length *AB* or *CD* in Fig. 26.21.

26.14. Coil and Winding Element

With reference to Fig. 26.21, the two conductors *AB* and *CD* along with their end connections constitute one coil of the armature winding. The coil may be single-turn coil (Fig. 26.21) or multi-turn coil (Fig. 26.22). A single-turn coil will have two conductors. But a multi-turn coil may have many conductors per coil side. In Fig. 26.22, for example, each coil side has 3 conductors. The

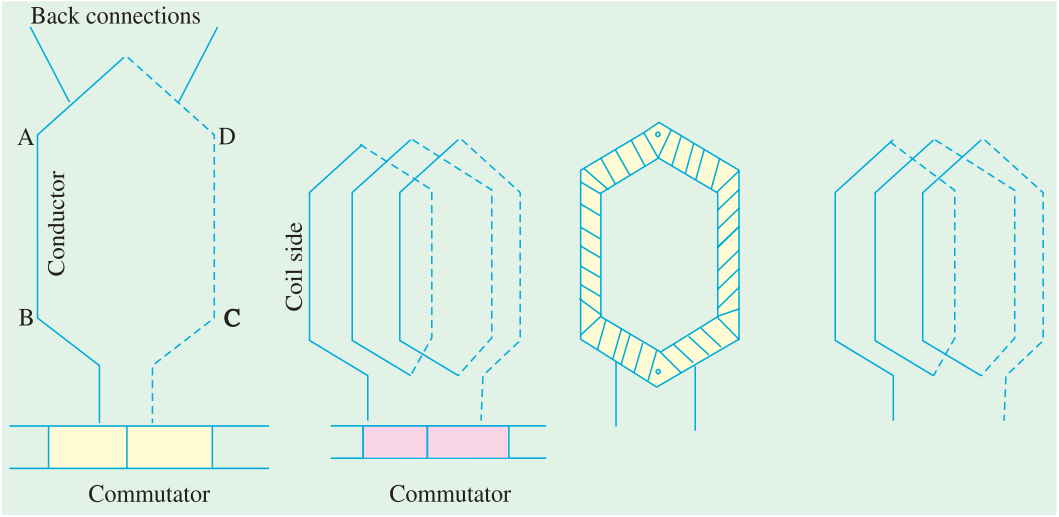


Fig. 26.21 Fig. 26.22 Fig. 26.23

group of wires or conductors constituting a coil side of a multi-turn coil is wrapped with a tape as a unit (Fig. 26.23) and is placed in the armature slot. It may be noted that since the beginning and the end of each coil must be connected to a commutator bar, there are as many commutator bars as coils for both the lap and wave windings (see Example 26.1).

The side of a coil (1-turn or multiturn) is called a winding element. Obviously, the number of winding elements is twice the number of coils.

26.15. Coil-span or Coil-pitch (Y_s)

It is the distance, measured in terms of armature slots (or armature conductors) between two sides of a coil. It is, in fact, the periphery of the armature spanned by the two sides of the coil.

If the pole span or coil pitch is equal to the pole pitch (as in the case of coil *A* in Fig. 26.24 where pole-pitch of 4 has been assumed), then winding is called ***full-pitched.*** It means that coil span is 180 electrical degrees. In this case, the coil sides lie under opposite poles, hence the induced e.m.fs. in them are additive. Therefore, maximum e.m.f. is induced in the coil as a whole, it being the sum of the e.m.f.s induced in the two coil sides. For example, if there are 36 slots and 4 poles, then coil span is 36/4 = 9 slots. If number of slots is 35, then Y_S = 35/4 = 8 because it is customary to drop fractions.

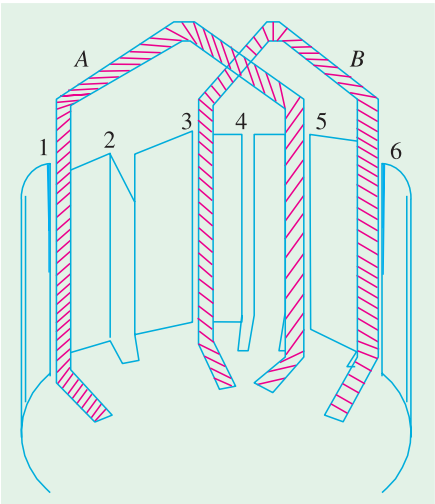


Fig. 26.24

If the coil span is less than the pole pitch (as in coil *B* where coil pitch is 3/4th of the pole pitch), then the

winding is fractional-pitched. In this case, there is a phase difference between the e.m.fs. in the two sides of the coil. Hence, the total e.m.f. round the coil which is the vector sum of e.m.fs. in the two coil sides, is less in this case as compared to that in the first case.

26.16. Pitch of a Winding (Y)

In general, it may be defined as the distance round the armature between two successive conductors which are directly connected together. Or, it is the distance between the beginnings of two consecutive turns.

$$Y = Y_B - Y_F \quad \text{.....for lap winding}$$
$$= Y_B + Y_F \quad \text{.....for wave winding}$$

In practice, coil-pitches as low as eight-tenths of a pole pitch are employed without much serious reduction in the e.m.f. Fractional-pitched windings are purposely used to effect substantial saving in the copper of the end connections and for improving commutation.

26.17. Back Pitch (Y_B)

The distance, measured in terms of the armature conductors, which a coil advances on the back of the armature is called back pitch and is denoted by Y_B.

As seen from Fig. 26.28, element 1 is connected on the back of the armature to element 8. Hence, $Y_B = (8 - 1) = 7$.

26.18. Front Pitch (Y_F)

The number of armature conductors or elements spanned by a coil on the front (or commutator end of an armature) is called the front pitch and is designated by Y_F. Again in Fig. 26.28, element 8 is connected to element 3 on the front of the armature, the connections being made at the commutator segment. Hence, $Y_F = 8 - 3 = 5$.

Alternatively, the front pitch may be defined as the distance (in terms of armature conductors) between the second conductor of one coil and the first conductor of the next coil which are connected together at the front *i.e.* commutator end of the armature. Both front and back pitches for lap and wave-winding are shown in Fig. 26.25 and 26.26.

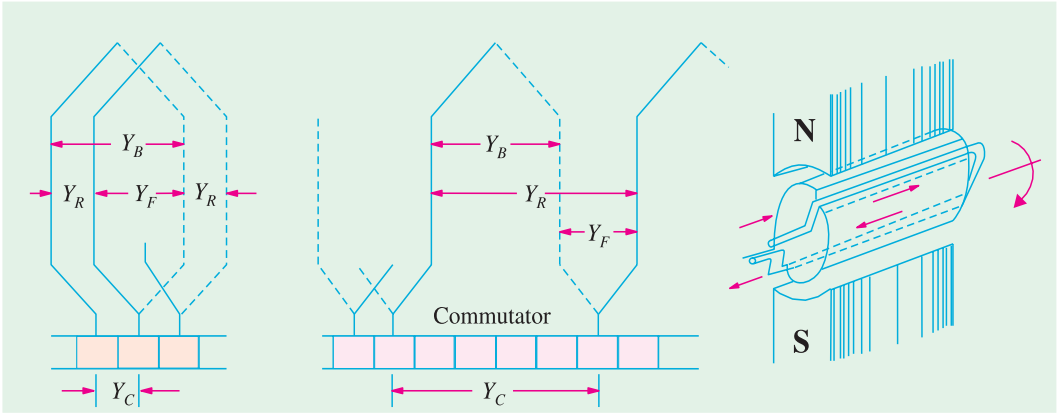


Fig. 26.25

Fig. 26.26

Fig. 26.27

26.19. Resultant Pitch (Y_R)

It is the distance between the beginning of one coil and the beginning of the next coil to which it is connected (Fig. 26.25 and 26.26).

As a matter of precaution, it should be kept in mind that all these pitches, though normally

stated in terms of armature conductors, are also sometimes given in terms of armature slots or commutator bars because commutator is, after all, an image of the winding.

26.20. Commutator Pitch (Y_G)

It is the distance (measured in commutator bars or segments) between the segments to which the two ends of a coil are connected. From Fig. 26.25 and 26.26 it is clear that for lap winding, Y_C is the ***difference*** of Y_B and Y_F whereas for wavewinding it is the *sum* of Y_B and Y_F. Obviously, commutator pitch is equal to the number of bars between coil leads. In general, Y_C equals the 'plex' of the lap-wound armature. Hence, it is equal to 1, 2, 3, 4 etc. for simplex-, duplex, triplex–and quadruplex etc. lap-windings.

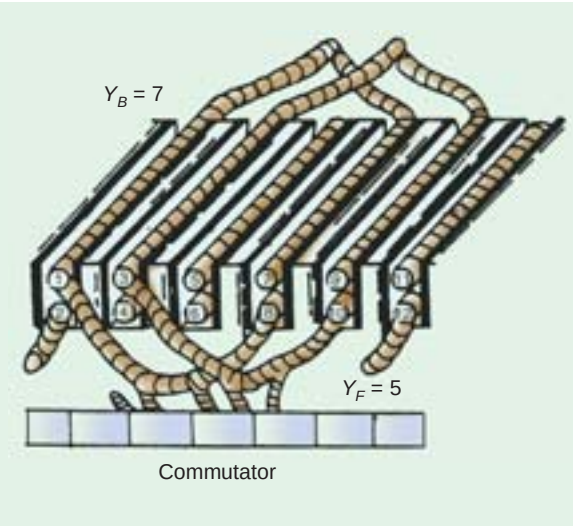


Fig. 26.28

26.21. Single-layer Winding

It is that winding in which one conductor or one coil side is placed in each armature slot as shown in Fig. 26.27. Such a winding is not much used.

26.22. Two-layer Winding

In this type of winding, there are two conductors or coil sides per slot arranged in two layers. Usually, one side of every coil lies in the upper half of one slot and other side lies in the lower half of some other slot at a distance of approximately one pitch away (Fig. 26.28). The transfer of the coil from one slot to another is usually made in a radial plane by means of a peculiar bend or twist at the back end as shown in Fig. 26.29. Such windings in which two coil sides occupy each slot are most commonly used for all medium-sized machines. Sometimes 4 or 6 or 8 coil sides are used in each slot in several layers because it is not practicable to have too many slots (Fig. 26.30). The coil sides lying at the upper half of the slots are numbered odd *i.e.* 1, 3, 5, 7 etc. while those at the lower half are numbered even *i.e.* 2, 4, 6, 8 etc.



Fig. 26.29

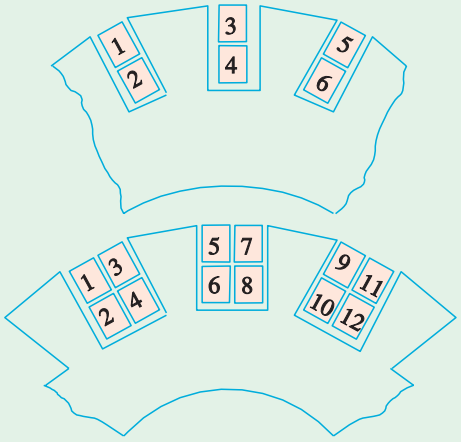


Fig. 26.30

26.23. Degree of Re-entrant of an Armature Winding

A winding is said to be single re-entrant if on tracing through it once, all armature conductors are included on returning to the starting point. It is double re-entrant if only half the conductors are included in tracing through the winding once and so on.



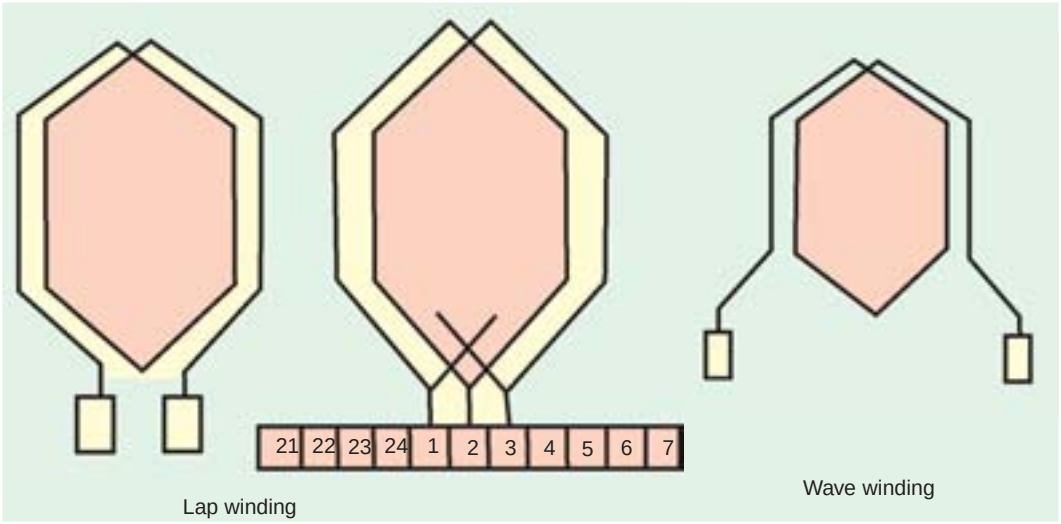
Multiplex Winding

26.24. Multiplex Winding

In such windings, there are several sets of completely closed and independent windings. If there is only one set of closed winding, it is called simplex wave winding. If there are two such windings on the same armature, it is called duplex winding and so on. The multiplicity affects a number of parallel paths in the armature. For a given number of armature slots and coils, as the multiplicity increases, the number of parallel paths in the armature increases thereby increasing the current rating but decreasing the voltage rating.

26.25. Lap and Wave Windings

Two types of windings mostly employed for drum-type armatures are known as Lap Winding and Wave Winding. The difference between the two is merely due to the different arrangement of the end connections at the front or commutator end of armature. Each winding can be arranged progressively or retrogressively and connected in simplex, duplex and triplex. The following rules, however, apply to both types of the windings :



Lap winding

Wave winding

(i) The front pitch and back pitch are each approximately equal to the pole-pitch *i.e.* windings should be full-pitched. This results in increased e.m.f. round the coils. For special purposes, fractional-pitched windings are deliberately used (Art. 26.15).

(ii) Both pitches should be odd, otherwise it would be difficult to place the coils (which are former-wound) properly on the armature. For exmaple, if Y_B and Y_F were both even, the *all* the coil sides and conductors would lie either in the upper half of the slots or in the lower

half. Hence, it would become impossible for one side of the coil to lie in the upper half. Hence, it would become impossible for one side of the coil to lie in the upper half of one slot and the other side of the same coil to lie in the lower half of some other slot.

(iii) The number of commutator segments is equal to the number of slots or coils (or half the number of conductors) because the front ends of conductors are joined to the segments in pairs.

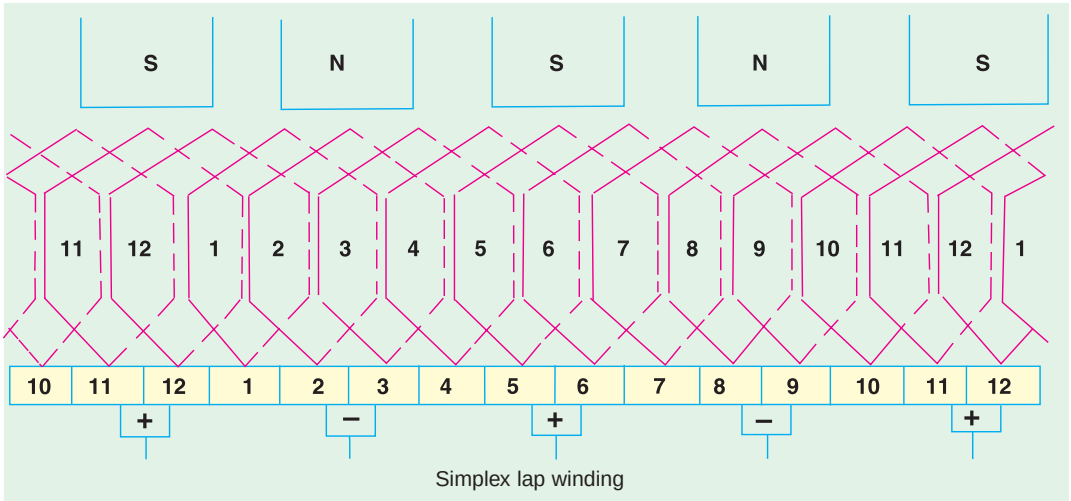
(iv) The winding must close upon itself *i.e.* if we start from a given point and move from one coil to another, then all conductors should be traversed and we should reach the same point again without a break or discontinuity in between.

26.26. Simplex Lap-winding*

It is shown in Fig. 26.25 which employs single-turn coils. In lap winding, the finishing end of one coil is connected to a commutator segment and to the starting end of the adjacent coil situated under the same pole and so on, till and the coils have been connected. This type of winding derives its name from the fact it doubles or laps back with its succeeding coils.

Following points regarding simplex lap winding should be carefully noted :

1. The back and front pitches are odd and of opposite sign. But they cannot be equal. They differ by 2 or some multiple thereof.
2. Both Y_B and Y_F should be nearly equal to a pole pitch.
3. The average pitch $Y_A = \frac{Y_B + Y_F}{2}$. It equals pole pitch = $\frac{Z}{P}$.
4. Commutator pitch $Y_C = \pm 1$. (In general, $Y_C = \pm m$)
5. Resultant pitch Y_R is even, being the arithmetical difference of two odd numbers, *i.e.*, $Y_R = Y_B - Y_F$.
6. The number of slots for a 2-layer winding is equal to the number of coils (*i.e.* half the number of coil sides). The number of commutator segments is also the same.



Simplex lap winding

* However, where heavy currents are necessary, duplex or triplex lap windings are used. The duplex lap winding is obtained by placing two similar windings on the same armature and connecting the even-numbered commutator bars to one winding and the odd-numbered ones to the second winding. Similarly, in triplex lap winding, there would be three windings, each connected to one third of the commutator bars.

7. The number of parallel paths in the armature = mP where m is the multiplicity of the winding and P the number of poles.

Taking the first condition, we have $Y_B = Y_F \pm 2$.**

(a) If $Y_B > Y_F$ *i.e.* $Y_B = Y_F + 2$, then we get a progressive or right-handed winding *i.e.* a winding which progresses in the clockwise direction as seen from the commutator end. In this case, obviously, $Y_C = +1$.

(b) If $Y_B < Y_F$ *i.e.* $Y_B = Y_F - 2$, then we get a retrogressive or left-handed winding *i.e.* one which advances in the anti-clockwise direction when seen from the commutator side. In this case, $Y_C = -1$.

(c) Hence, it is obvious that

$$\left.\begin{aligned} Y_F &= \frac{Z}{P} - 1 \\ Y_B &= \frac{Z}{P} + 1 \end{aligned}\right] \text{ for progressive winding and } \quad \left.\begin{aligned} Y_F &= \frac{Z}{P} + 1 \\ Y_B &= \frac{Z}{P} - 1 \end{aligned}\right] \text{ for retrogressive winding}$$

Obviously, Z/P must be even to make the winding possible.

26.27. Numbering of Coils and Commutator Segments

In the d.c. winding diagrams to follow, we will number the coils only (not individual turns). The upper side of the coil will be shown by a firm continuous line whereas the lower side will be shown by a broken line. The numbering of coil sides will be consecutive *i.e.* 1, 2, 3 ..... etc. and such that odd numbers are assigned to the top conductors and even numbers to the lower sides for a two-layer winding. The commutator segments will also be numbered consecutively, the number of the segments will be the same as that of the upper side connected to it.

Example 26.1. *Draw a developed diagram of a simple 2-layer lap-winding for a 4-pole generator with 16 coils. Hence, point out the characteristics of a lap-winding.*

(Elect. Engineering, Madras Univ. 1981)

Solution. The number of commutator segments = 16

Number of conductors or coil sides $16 \times 2 = 32$; pole pitch = 32/4 = 8

Now remembering that (*i*) Y_B and Y_F have to be odd and (*ii*) have to differ by 2, we get for a progressive winding $Y_B = 9$; $Y_F = -7$ (retrogressive winding will result if $Y_B = 7$ and $Y_F = -9$). Obviously, commutator pitch $Y_C = -1$.

[Otherwise, as shown in Art. 26.26, for progressive winding

$$Y_F = \frac{Z}{P} - 1 = \frac{32}{4} - 1 = 7 \text{ and } Y_B = \frac{Z}{P} - 1 = \frac{32}{4} + 1 = 9]$$

The simple winding table is given as under :

Back Connections		*Front Connections*
1 to (1 + 9) = 10	⟶	10 to (10 – 7) = 3
3 to (3 + 9) = 12	⟶	12 to (12 – 7) = 5
5 to (5 + 9) = 14	⟶	14 to (14 – 7) = 7
7 to (7 + 9) = 16	⟶	16 to (16 – 7) = 9
9 to (9 + 9) = 18	⟶	18 to (18 – 7) = 11
11 to (11 + 9) = 20	⟶	20 to (20 – 7) = 13
13 to (13 + 9) = 22	⟶	22 to (22 – 7) = 15
15 to (15 + 9) = 24	⟶	24 to (24 – 7) = 17
17 to (17 + 9) = 26	⟶	26 to (26 – 7) = 19
19 to (19 + 9) = 28	⟶	28 to (28 – 7) = 21

** In general, $Y_B = Y_F \pm 2m$ where $m = 1$ for simplex lap winding and $m = 2$ for duplex lap winding etc.

21 to (21 + 9) = 30	⟶	30 to (20 – 7) = 23
23 to (23 + 9) = 32	⟶	32 to (32 – 7) = 25
25 to (25 + 9) = 34 = (34 – 32) = 2	⟶	2 to (34 – 7) = 27
27 to (27 + 9) = 36 = (36 – 32) = 4	⟶	4 to (36 – 7) = 29
29 to (29 + 9) = 38 = (38 – 32) = 6	⟶	6 to (38 – 7) = 31
31 to (31 + 9) = 40 = (40 – 32) = 8	⟶	8 to (40 – 7) = 33 = (33 – 32) = 1

The winding ends here because we come back to the conductor from where we started.

We will now discuss the developed diagram which is one that is obtained by imagining the armature surface to be removed and then laid out flat so that the slots and conductors can be viewed without the necessity of turning round the armature in order to trace out the armature windings. Such a developed diagram is shown in Fig. 26.31.

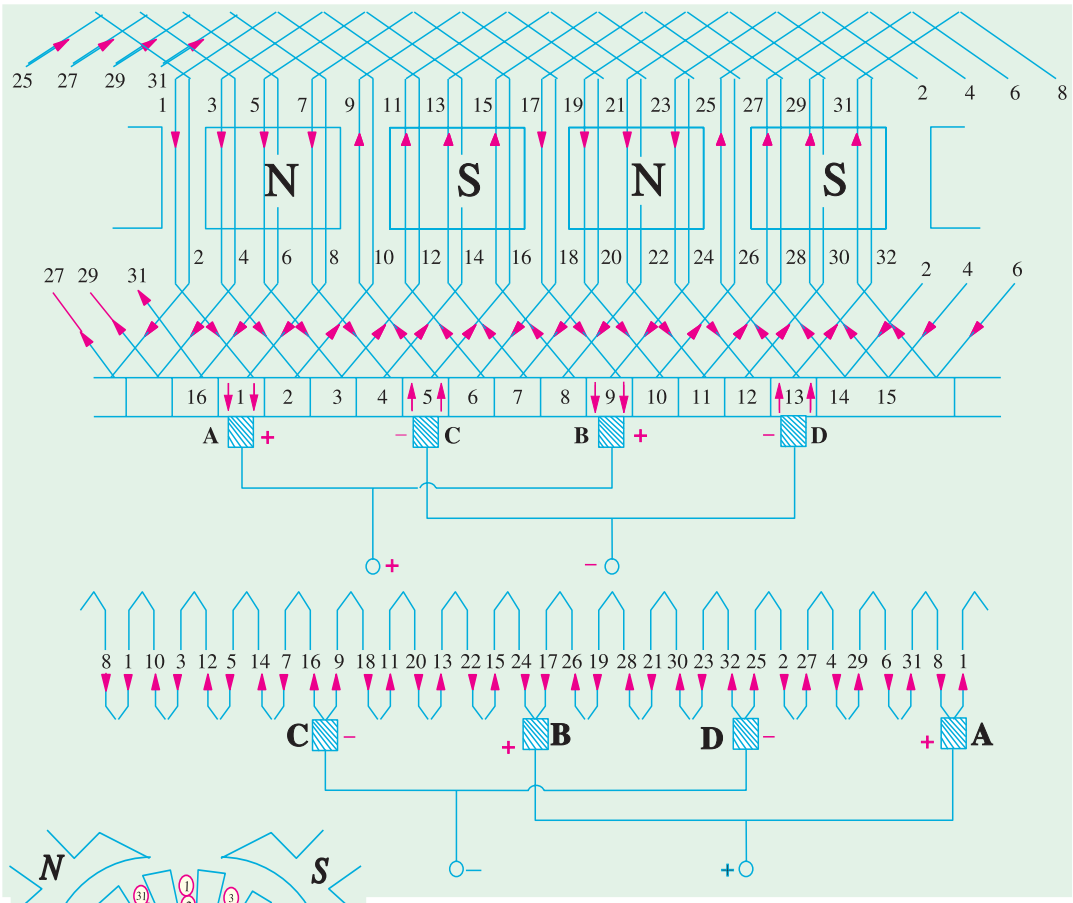


Fig. 26.31

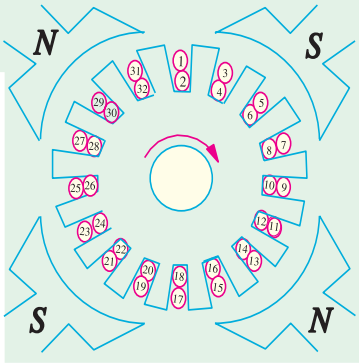


Fig. 26.32

The procedure of developing the winding is this :

Front end of the upper side of coil No. 1 is connected to a commutator segment (whose number is also 1). The back end is joined at the back to the 1 + 9 = 10th coil side in the *lower* half of 5th slot. The front end of coil side 10 is joined to commutator segment 2 to which is connected the front end of 10 – 7 = 3 *i.e.* 3rd coil side lying in the upper half of second armature slot. In this way, by travelling 9 coil sides to the right at the back and 7 to the left at the

front we complete the winding, thus including every coil side once till we reach the coil side 1 from where we started. Incidentally, it should be noted that all upper coil sides have been given odd numbers, whereas lower ones have been given even numbers as shown in the polar diagram (Fig. 26.32) of the winding of Fig. 26.31.

Brush positions can be located by finding the direction of currents flowing in the various conductors. If currents in the conductors under the influence of a *N*-pole are assumed to flow downwards (as shown), then these will flow upwards in conductors under the influence of *S*-pole. By putting proper arrows on the conductors (shown separately in the equivalent ring diagram), it is found that commutator bars No. 1 and 9 are the meeting points of e.m.fs. and hence currents are flowing out of these conductors. The positive brushes should, therefore, be placed at these commutator bars. Similarly, commutator bars No. 5 and 13 are the separating points of e.m.fs. hence negative brushes are placed there. In all, there are four brushes, two positive and two negative. If brushes of the same polarity are connected together, then all the armature conductors are divided into four parallel paths.

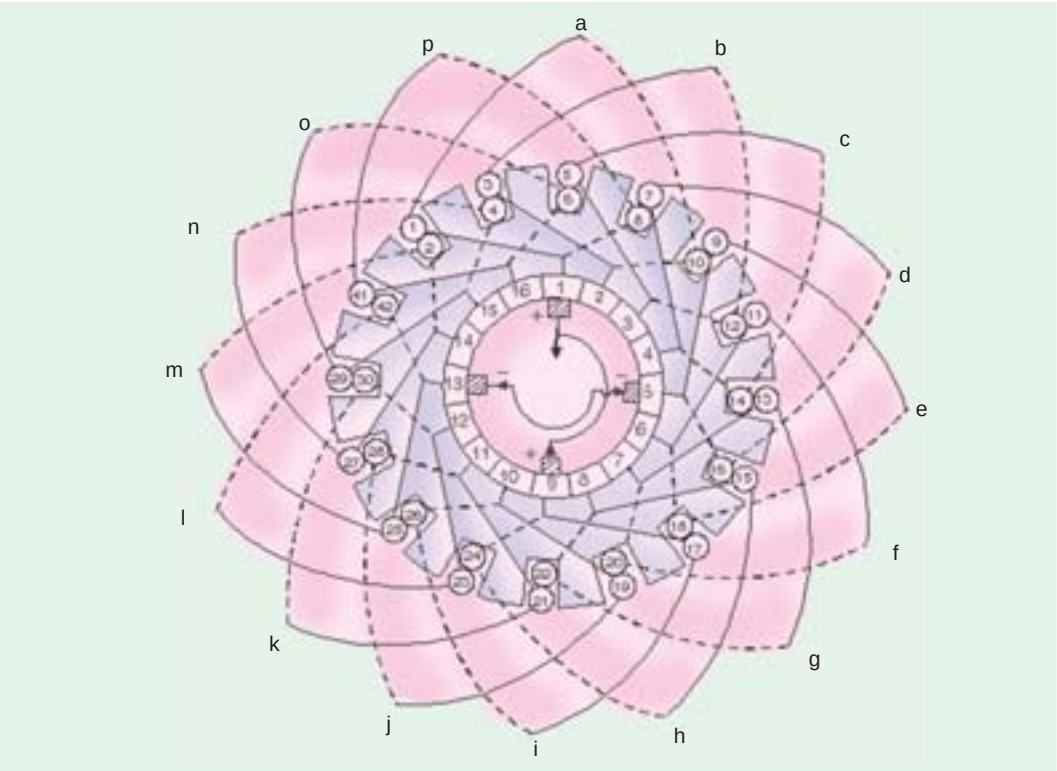


Fig. 26.33

Division of conductors into parallel paths is shown separately in the schematic diagram of Fig. 26.34. Obviously, if I_a is the total current supplied by the generator, then current carried by each parallel path is $I_a/4$.

Summarizing these conclusions, we have

1. The total number of brushes is equal to the number of poles.
2. There are as many parallel paths in the armature as the number of poles. That is why such a winding is sometimes known as 'multiple circuit' or 'parallel' winding. In general, number of parallel paths in armature = mP where m is the multiplicity (plex) of the lap winding. For example, a 6-pole duplex lap winding has $(6 \times 2) = 12$ parallel paths in its armature.
3. The e.m.f. between the +ve and –ve brushes is equal to the e.m.f. generated in any one of the parallel paths. If Z is the total number of armature conductors and P the number of poles, then the number of armature conductors (connected in series) in any parallel path is Z/P.

$\therefore$ Generated e.m.f. E_g = (Average e.m.f./conductor) $\times \frac{Z}{P} = e_{av} \times \frac{Z}{P}$

4. The total or equivalent armature resistance can be found as follows :

Let l = length of each armature conductor; S = its cross-section

A = No. of parallel paths in armature = P – for simplex lap winding

R = resistance of the whole winding then $R = \frac{\rho l}{S} \times Z$

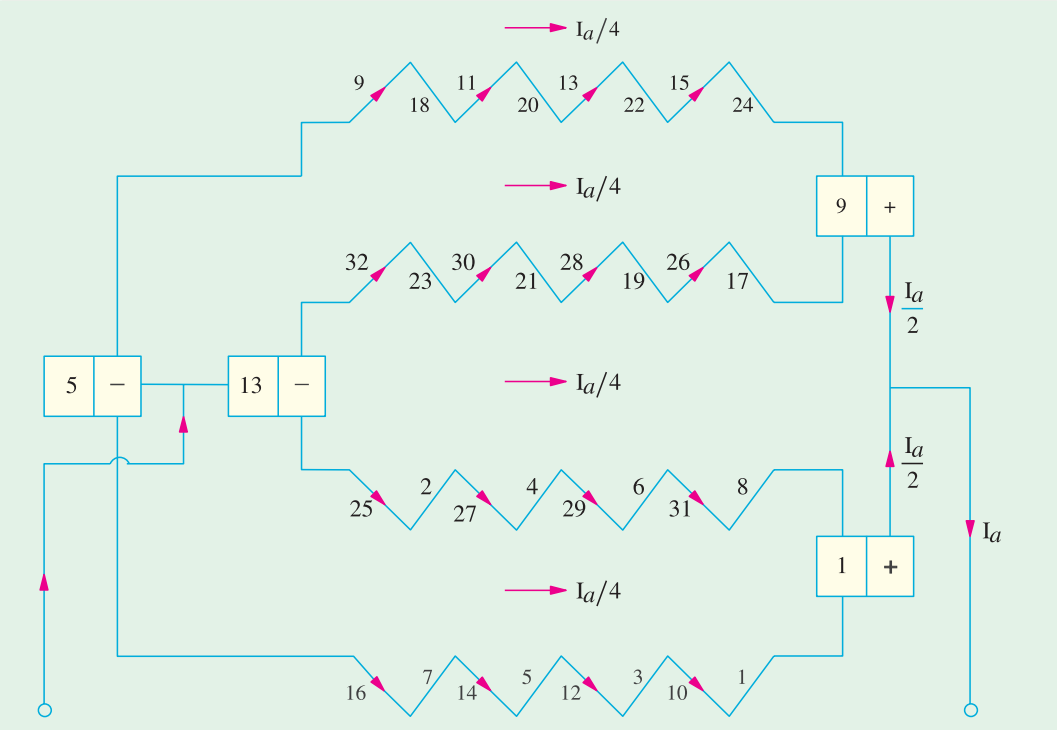


Fig. 26.34

Resistance of each path = $\frac{\rho l Z}{S \times A}$

There are P (or A) such paths in parallel, hence equivalent resistance

$$= \frac{1}{A} \times \frac{\rho l Z}{SA} = \frac{\rho l Z}{SA^2}$$

5. If I_a is the total armature current, then current per parallel path (or carried by each conductor) is I_a/P.

26.28. Simplex Wave Winding*

From Fig. 26.31, it is clear that in lap winding, a conductor (or coil side) under one pole is connected at the back to a conductor which occupies an almost corresponding position under the *next* pole of *opposite* polarity (as conductors 3 and 12). Conductor No. 12 is then connected to conductor No. 5 under the ***original*** pole but which is a little removed from the initial conductor No. 3. If, instead of returning to the same N-pole, the conductor No. 12 were taken *forward* to the next N-pole, it would make no difference so far as the direction and magnitude of the e.m.f. induced in the circuit are concerned.

* Like lap winding, a wave winding may be duplex, triplex or may have any degree of multiplicity. A simplex wave winding has two paths, a duplex wave winding four paths and a triplex one six paths etc.

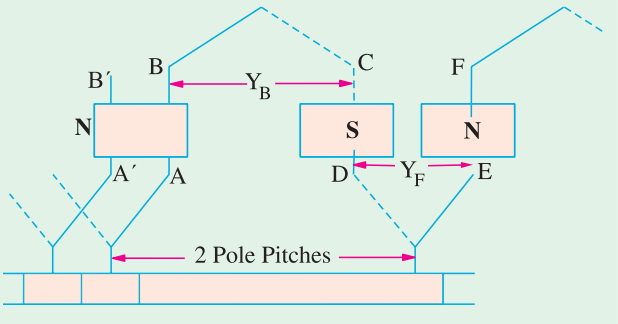


Fig. 26.35

As shown in Fig. 26.35, conductor *AB* is connected to *CD* lying under *S*-pole and then to *EF* under the next *N*-pole. In this way, the winding progresses, passing successively under every *N*-pole and *S*-pole till it returns to a conductor *A′B′* lying under the original pole. Because the winding progresses in one direction round the armature in a series of 'waves', it is known as wave winding.

If, after passing once round the armature, the winding falls in a slot to the left of its starting point (as *A′B′* in Fig. 26.35) then the winding is said to be retrogressive. If, however, it falls one slot to the right, then it is progressive.

Assuming a 2-layer winding and supposing that conductor *AB* lies in the upper half of the slot, then going once round the armature, the winding ends at *A′B′* which must be at the upper half of the slot at the left or right. Counting in terms of ***conductors,*** it means that *AB* and *A′B′* differ by two conductors (although they differ by one slot).

From the above, we can deduce the following relations. If *P* = No. of poles, then

$$\left.\begin{matrix} Y_B = \text{back pitch} \\ Y_F = \text{front pitch} \end{matrix}\right\} \text{nearly equal to pole pitch}$$

then $Y_A = \dfrac{Y_B + Y_F}{2}$ = average pitch ; Z = total No. of conductors or coil sides

Then, $Y_A \times P = Z \pm 2$ $\qquad Y_A = \dfrac{Z \pm 2}{P}$

Since *P* is always even and $Z = PY_A \pm 2$, hence Z must always be even. Put in another way, it means that $\dfrac{Z \pm 2}{P}$ must be an even integer.

The plus sign will give a progressive winding and the negative sign a retrogressive winding.

Points to Note :

1. Both pitches Y_B and Y_F are odd and of the same sign.
2. Back and front pitches are nearly equal to the pole pitch and may be equal or differ by 2, in which case, they are respectively one more or one less than the average pitch.
3. Resultant pitch $Y_R = Y_F + Y_B$.
4. Commutator pitch, $Y_C = Y_A$ (in lap winding $Y_C = \pm 1$).

   Also, $$Y_C = \frac{\text{No. of Commutator bars } \pm 1}{\text{No. of pair of poles}}$$
5. The average pitch which must be an integer is given by

$$Y_A = \frac{Z \pm 2}{P} = \frac{\frac{Z}{2} + 1}{P/2} = \frac{\text{No. of Commutator bars } \pm 1}{\text{No. of pair of poles}}$$

It is clear that for Y_A to be an integer, there is a restriction on the value of *Z*. With *Z* = 32, this winding is impossible for a 4-pole machine (though lap winding is possible). Values of *Z* = 30 or 34 would be perfectly alright.

6. The number of *coils i.e.* N_C can be found from the relation.

$$N_C = \frac{PY_A \pm 2}{2}$$

This relation has been found by rearranging the relation given in (5) above.

7. It is obvious from (5) that for a wave winding, the number of armature conductors with 2 either added or subtracted must be a multiple of the number of poles of the generator. This restriction eliminates many even numbers which are unsuitable for this winding.

8. The number of armature parallel paths = $2m$ where m is the multiplicity of the winding.

Example 26.2. *Draw a developed diagram of a simplex 2-layer wave-winding for a 4-pole d.c. generator with 30 armature conductors. Hence, point out the characteristics of a simple wave winding.* **(Elect. Engg-I, Nagpur Univ. 1991)**

Solution. Here, $Y_A = \frac{30 \pm 2}{4} = 8^*$ or 7. Taking $Y_A = 7$, we have $Y_B = Y_F = 7$

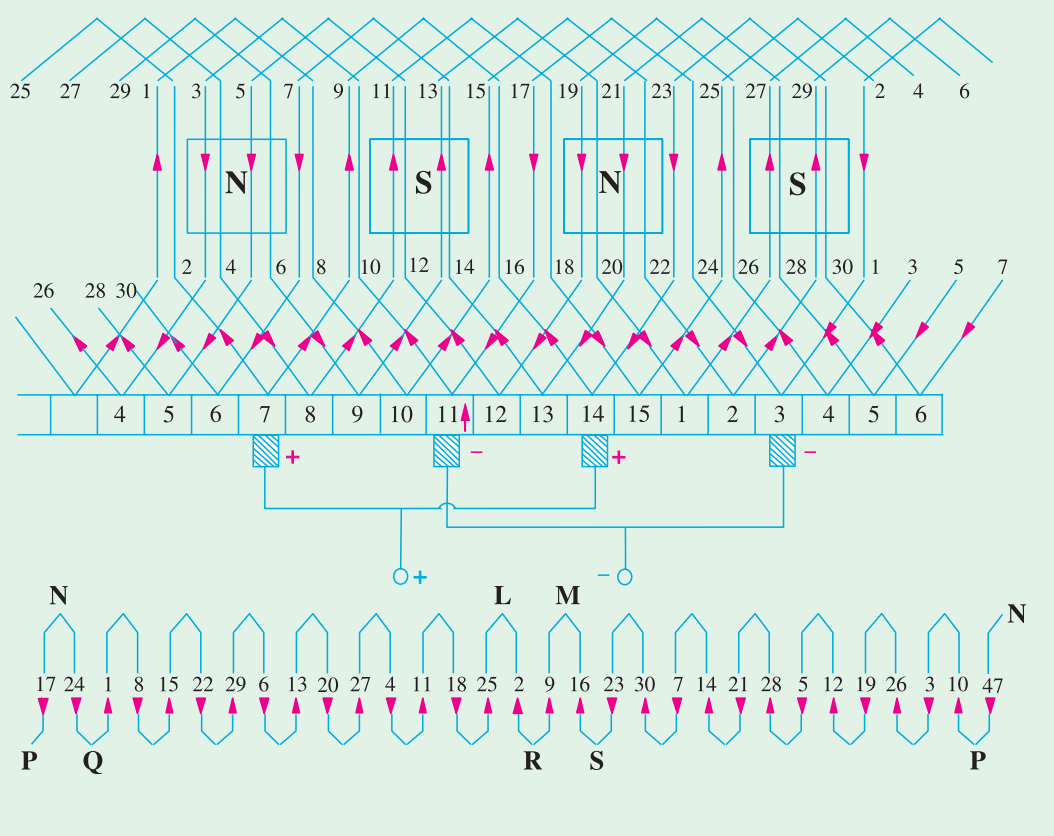


Fig. 26.36

As shown in Fig. 26.36 and 26.37, conductor No. 5 is taken to conductor No. 5 + 7 = 12 at the back and is joined to commutator segment 5 at the front. Next, the conductor No. 12 is joined to commutator segment 5 + 7 = 12 ($\because$ $Y_C = 7$) to which is joined conductor No. 12 + 7 = 19. Continuing this way, we come back to conductor No. 5 from where we started. Hence, the winding closes upon itself.

* If we take 8, then the pitches would be : $Y_B = 9$ and $Y_F = 7$ or $Y_B = 7$ and $Y_F = 9$. Incidentally, if $Y_A = Y_C$ is taken as 7, armature will rotate in one direction and if $Y_C = 8$, it will rotate in the opposite direction.

The simple winding table is as under :

Back Connections		*Front Connections*
1 to (1 + 7) = 8	⟶	8 to (8 + 7) = 15
15 to (15 + 7) = 22	⟶	22 to (22 + 7) = 29
29 to (29 + 7) = 36 = (36 – 30) = 6	⟶	6 to (6 + 7) = 13
13 to (13 + 7) = 20	⟶	20 to (20 + 7) = 27
27 to (27 + 7) = 34 = (34 – 30) = 4	⟶	4 to (4 + 7) = 11
11 to (11 + 7) = 18	⟶	18 to (18 + 7) = 25
25 to (25 + 7) = 32 = (32 – 30) = 2	⟶	2 to (2 + 7) = 9
9 to (9 + 7) = 16	⟶	16 to (16 + 7) = 23
23 to (23 + 7) = 30	⟶	30 to (30 + 7) = 37 = (37 – 30) = 7
7 to (7 + 7) = 14	⟶	14 to (14 + 7) = 21
21 to (21 + 7) = 28	⟶	28 to (28 + 7) = 35 = (35 – 30) = 5
5 to (5 + 7) = 12	⟶	12 to (12 + 7) = 19
19 to (19 + 7) = 26	⟶	26 to (26 + 7) = 33 = (33 – 30) = 3
3 to (3 + 7) = 10	⟶	10 to (10 + 7) = 17
17 to (17 + 7) = 24	⟶	24 to (24 + 7) = 31 = (31 – 30) = 1

Since we come back to the conductor No. 1 from where we started, the winding gets closed at this stage.

Brush Position

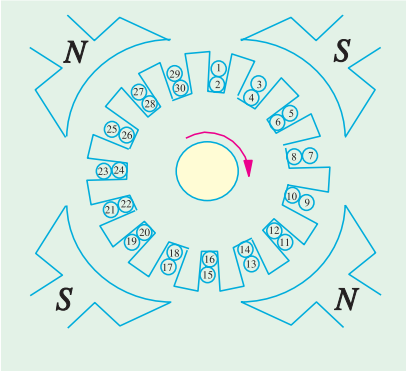


Fig. 26.37

Location of brush position in wave-winding is slightly difficult. In Fig. 26.36 conductors are supposed to be moving from left to right over the poles. By applying Fleming's Right-hand rule, the directions of the induced e.m.fs in various armature conductors can be found. The directions shown in the figure have been found in this manner. In the lower part of Fig. 26.36 is shown the equivalent ring or spiral diagram which is very helpful in understanding the formation of various parallel paths in the armature. It is seen that the winding is electrically divided into two portions. One portion consists of conductors lying between points *N* and *L* and the other of conductors lying between *N* and *M*. In the first portion, the general trend of the induced e.m.fs. is from left to right whereas in the second portion it is from right to left. Hence, in general, there are only two parallel paths through the winding, so that two brushes are required, one positive and one negative.

From the equivalent ring diagram, it is seen that point *N* is the separating point of the e.m.fs. induced in the two portions of the winding. Hence, this fixes the position of the negative brush. But as it is at the back and not at the commutator end of the armature, the negative brush has two alternative positions *i.e.* either at point *P* or *Q*. These points on the equivalent diagram correspond to commutator segments No. 3 and 11.

Now, we will find the position of the positive brush. It is found that there are two meeting points of the induced e.m.fs. *i.e.* points *L* and *M* but both these points are at the back or non-commutator end of the armature. These two points are separated by one loop only, namely, the loop composed of conductors 2 and 9, hence the middle point *R* of this loop fixes the position of the positive brush, which should be placed in touch with commutator segment No. 7. We find that for one position of the +ve brush, there are two alternative positions for the –ve brush.

Taking the +ve brush at point *R* and negative brush at point *P*, the winding is seen to be divided into the following two paths.

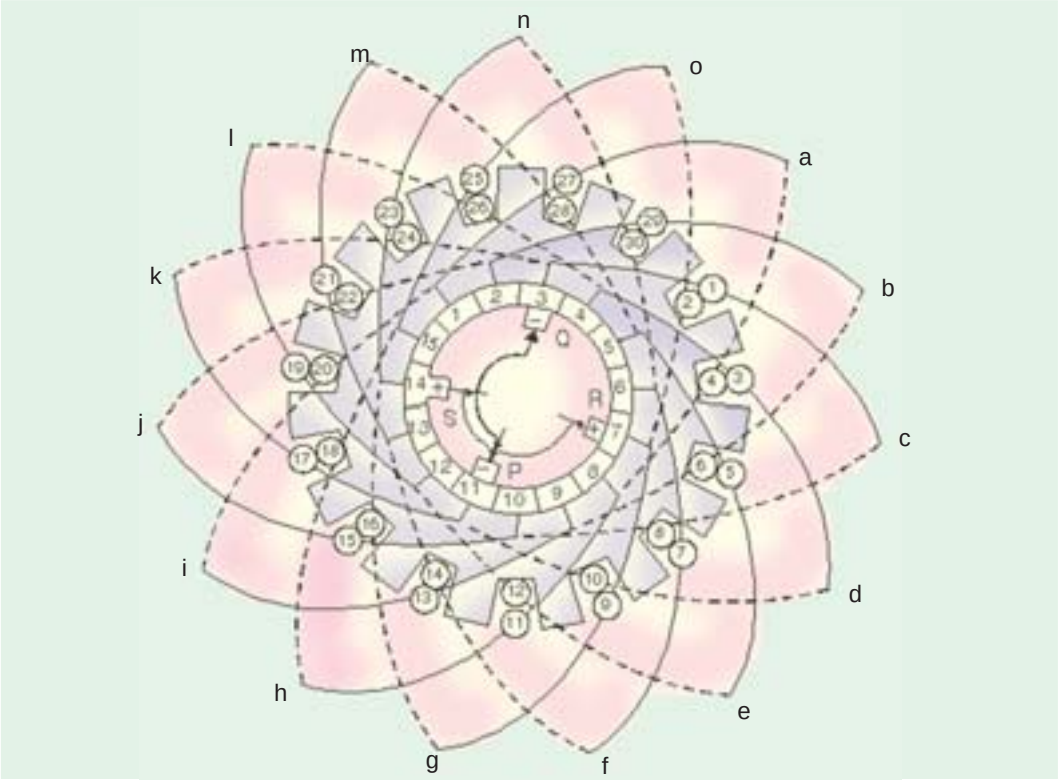


Fig. 26.38

In path 1 (Fig. 26.36) it is found that e.m.f. in conductor 9 is in opposition to the general trend of e.m.fs. in the other conductors comprising this path. Similarly, in path 2, the e.m.f. in conductor 2 is in position to the direction of e.m.fs. in the path as a whole. However, this will make no difference because these conductors lie almost in the interpolar gap and, therefore e.m.fs. in these conductors are negligible.

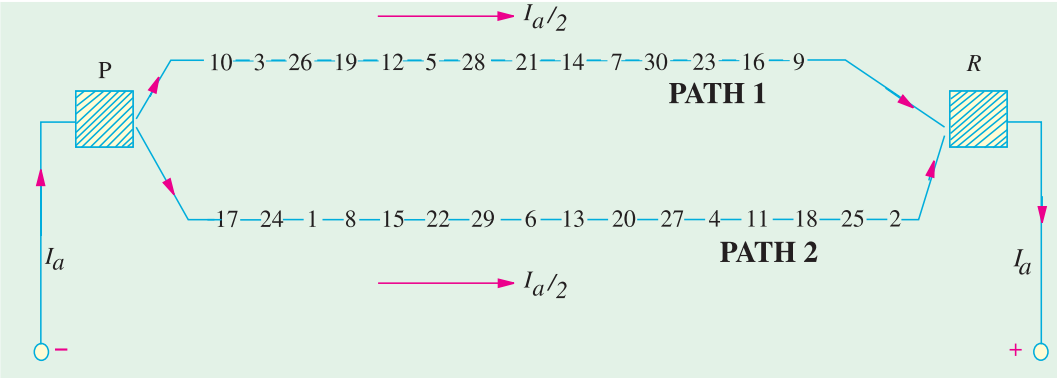


Fig. 26.39

Again, take the case of conductors 2 and 9 situated between points *L* and *M*. Since the armature conductors are in continuous motion over the pole faces, their positions as shown in the figure are only instantaneous. Keeping in this mind, it is obvious that conductor 2 is about to move from the influence of *S*-pole to that of the next *N*-pole. Hence, the e.m.f. in it is at the point of reversing. However, conductor 9 has already passed the position of reversal, hence its e.m.f. will not reverse,

rather it will increase in magnitude gradually. It means that in a very short interval, point *M* will

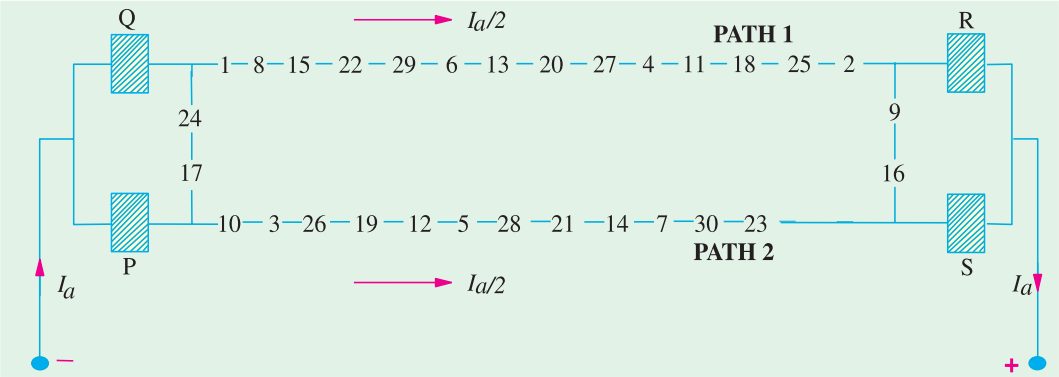


Fig. 26.40

become the meeting point of the e.m.fs. But as it lies at the back of the armature, there are two alternative positions for the +ve brush *i.e.* either point *R* which has already been considered or point *S* which corresponds to commutator segment 14. This is the second alternative position of the positive brush. Arguing in the same way, it can be shown that after another short interval of time, the alternative position of the positive brush will shift from segment 14 to segment 15. Therefore, if one positive brush is in the contact with segment 7, then the second positive brush if used, should be in touch with both segments 14 and 15.

It may be noted that if brushes are placed in both alternative positions for both positive and negative (*i.e.* if in all, 4 brushes are used, two +ve and two –ve), then the effect is merely to short-circuit the loop lying between brushes of the same polarity. This is shown in Fig. 26.40 where it will also be noted that irrespective of whether only two or four brushes are used, the number of parallel paths through the armature winding is still two.

Summarizing the above facts, we get

1. Only two brushes are necessary, though their number may be equal to the number of poles.
2. The number of parallel paths through the armature winding is two irrespective of the number of generator poles. That is why this winding is sometimes called 'two-circuit' or 'series' winding.
3. The generator e.m.f. is equal to the e.m.f. induced in any one of the two parallel paths. If e_{av} is the e.m.f. induced/conductor, then generator e.m.f. is $E_g = e_{av} \times Z/2$.
4. The equivalent armature resistance is nearly one-fourth of the total resistance of the armature winding.
5. If I_a is the total armature current, then current carried by each path or conductor is obviously $I_a/2$ whatever the number of poles.

26.29. Dummy or Idle Coils

These are used with wave-winding and are resorted to when the requirements of the winding are not met by the standard armature punchings available in armature-winding shops. These dummy coils do not influence the electrical characteristics of the winding because they are not connected to the commutator. They are exactly similar to the other coils except that their ends are cut short and taped. They are there simply to provide mechanical balance for the armature because an armature having some slots without windings would be out of balance mechanically. For example, suppose number of armature slots is 15, each containing 4 sides and the number of poles is 4. For a simplex wave-windings,



Dummy coils

$$Y_A = \frac{Z \pm 2}{P} = \frac{60 \pm 2}{4}$$

which does not come out to be an integer (Art. 26.28) as required by this winding. However, if we make one coil dummy so that we have 58 active conductors, then

$$Y_A = \frac{58 \pm 2}{4} = 14 \text{ or } 15$$

This makes the winding possible.

26.30. Uses of Lap and Wave Windings

The advantage of the wave winding is that, for a given number of poles and armature conductors, it gives more e.m.f. than the lap winding. Conversely, for the same e.m.f., lap winding would require large number of conductors which will result in higher winding cost and less efficient utilization of space in the armature slots. Hence, wave winding is suitable for small generators especially those meant for 500-600 V circuits.

Another advantage is that in wave winding, equalizing connections are not necessary whereas in a lap winding they definitely are. It is so because each of the two paths contains conductors lying under all the poles whereas in lap-wound armatures, each of the P parallel paths contains conductors which lie under one pair of poles. Any inequality of pole fluxes affects two paths equally, hence their induced e.m.fs. are equal. In lap-wound armatures, unequal voltages are produced which set up a circulating current that produces sparking at brushes.

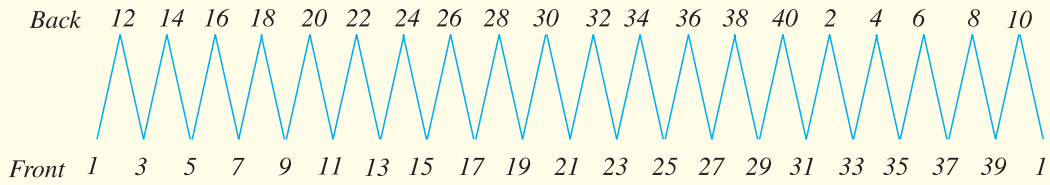
However, when large currents are required, it is necessary to use lap winding, because it gives more parallel paths.

Hence, lap winding is suitable for comparatively low-voltage but high-current generators whereas wave-winding is used for high-voltage, low-current machines.

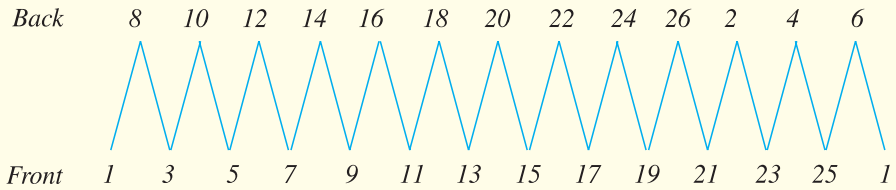
Tutorial Problem No. 26.1

1. Write down the winding table for a 2-layer simplex lap-winding for a 4-pole d.c. generator having **(*a*)** 20 slots and **(*b*)** 13 slots. What are the back and front pitches as measured in terms of armature conductors ?

[Hint : (*a*) No. of conductors = 40 ; $Y_B = 11$ and $Y_F = -9$] (Elect. Engineering, Madras Univ. 1978)

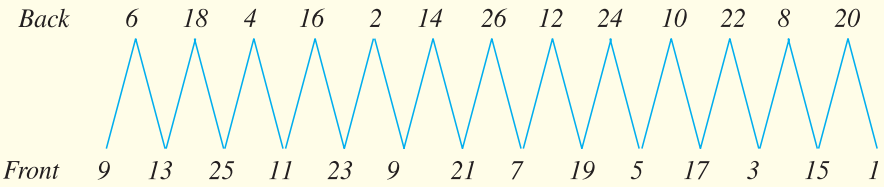


(*b*) No. of conductors = 26 ; $Y_B = 7$; $Y_F = -5$



2. With a simplex 2-layer wave winding having 26 conductors and 4-poles, write down the winding table. What will be the front and back pitches of the winding ?

[Hint : $Y_F = 7$ and $Y_B = 5$] (Electric Machinery-I, Madras Univ. Nov. 1979)



3. Is it possible to get simplex wave winding for a 4-pole d.c. machine with 28 conductors ? Explain the reason for your answer. **[No, it would contain only 4 conductors]**

4. State for what type of winding each of the following armatures could be used and whether the winding must be four or six-pole if no dummy coils are to be used (*a*) 33 slots, 165 commutator segments (*b*) 64 slots, 256 commutator segments (*c*) 65 slots, 260 commutator segments.

[(*a*) 4-pole lap with commutator pitch 82 or 83 or 6-pole lap.
(*b*) 4-pole lap or 6-pole wave with commutator pitch 85.
(*c*) 6-pole wave with commutator pitch 87.]

26.31. Types of Generators

Generators are usually classified according to the way in which their fields are excited. Generators may be divided into **(*a*)** separately-excited generators and **(*b*)** self-excited generators.

(*a*) Separately-excited generators are those whose field magnets are energised from an independent external source of d.c. current. It is shown diagramatically in Fig. 26.41.

(*b*) Self-excited generators are those whose field magnets are energised by the current produced by the generators themselves. Due to residual magnetism, there is always present some flux in the poles. When the armature is rotated, some e.m.f. and hence some induced current is produced which is partly or fully passed through the field coils thereby strengthening the residual pole flux.

There are three types of self-excited generators named according to the manner in which their field coils (or windings) are connected to the armature.

(*i*) Shunt wound

The field windings are connected across or in parallel with the armature conductors and have the full voltage of the generator applied across them (Fig. 26.42).

(*ii*) Series Wound

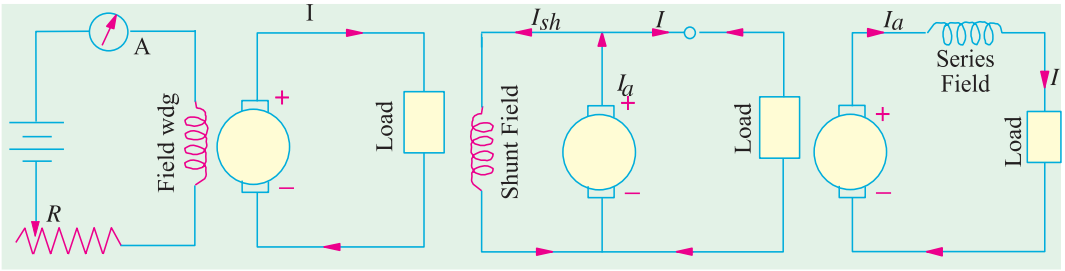


Fig. 26.41

Fig. 26.42

Fig. 26.43

In this case, the field windings are joined in series with the armature conductors (Fig. 26.43). As they carry full load current, they consist of relatively few turns of thick wire or strips. Such generators are rarely used except for special purposes *i.e.* as boosters etc.

(*iii*) Compound Wound

It is a combination of a few series and a few shunt windings and can be either short-shunt or long-shunt as shown in Fig. 26.44 (*a*) and

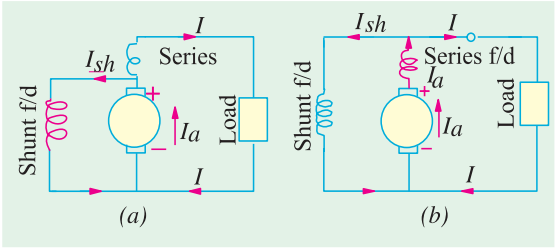


Fig. 26.44

(*b*) respectively. In a compound generator, the shunt field is stronger than the series field. When series field *aids* the shunt field, generator is said to be commutatively-compounded. On the other hand if series field ***opposes*** the shunt field, the generator is said to be differentially compounded. Various types of d.c. generators have been shown separately in Fig. 26.45.

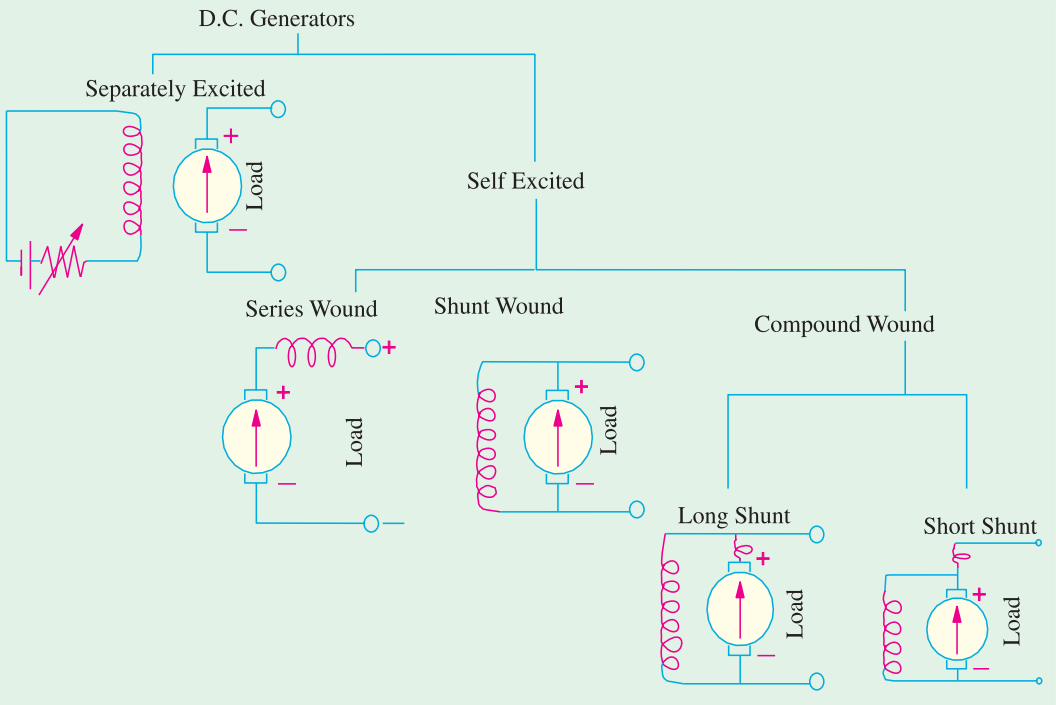


Fig. 26.45

26.32. Brush Contact Drop

It is the voltage drop over the brush contact resistance when current passes from commutator segments to brushes and finally to the external load. Its value depends on the amount of current and the value of contact resistance. This drop is usually small and includes brushes of both polarities. However, in practice, the brush contact drop is assumed to have following constant values for all loads.

0.5 V for metal-graphite brushes.

2.0 V for carbon brushes.

Example 26.3. *A shunt generator delivers 450 A at 230 V and the resistance of the shunt field and armature are 50 Ω and 0.03 Ω respectively. Calculate the generated e.m.f.*

Solution. Generator circuit is shown in Fig. 26.46.

Current through shunt field winding is

$$I_{sh} = 230/50 = 4.6 \text{ A}$$

Load current $I = 450$ A

$\therefore$ Armature current $I_a = I + I_{sh}$

$$= 450 + 4.6 = 454.6 \text{ A}$$

Armature voltage drop

$$I_a R_a = 454.6 \times 0.03 = \mathbf{13.6\ V}$$

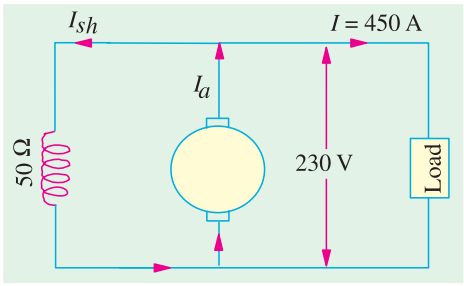


Fig. 26.46

Now E_g = terminal voltage + armature drop
$= V + I_a R_a$

∴ e.m.f. generated in the armature
$E_g = 230 + 13.6 = \mathbf{243.6\ V}$

Example 26.4. *A long-shunt compound generator delivers a load current of 50 A at 500 V and has armature, series field and shunt field resistances of 0.05 Ω, 0.03 Ω and 250 Ω respectively. Calculate the generated voltage and the armature current. Allow 1 V per brush for contact drop.*

(Elect. Science 1, Allahabad Univ. 1992)

Solution. Generator circuit is shown in Fig. 26.47.

$I_{sh} = 500/250 = 2$ A

Current through armature and series winding is
$= 50 + 2 = 52$ A

Voltage drop on series field winding
$= 52 \times 0.03 = 1.56$ V

Armature voltage drop
$I_a R_a = 52 \times 0.05 = 2.6$ V

Drop at brushes $= 2 \times 1 = 2$ V

Now, $E_g = V + I_a R_a$ + series drop + brush drop
$= 500 + 2.6 + 1.56 + 2 = \mathbf{506.16\ V}$

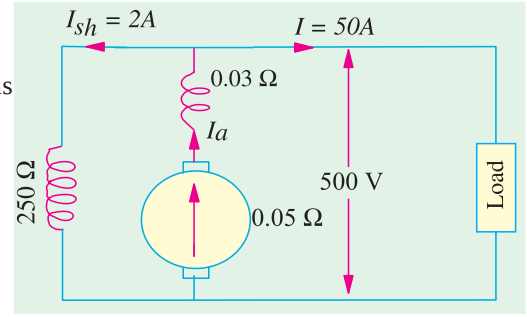


Fig. 26.47

Example 26.5. *A short-shunt compound generator delivers a load current of 30 A at 220 V, and has armature, series-field and shunt-field resistances of 0.05 Ω, 0.30 Ω and 200 Ω respectively. Calculate the induced e.m.f. and the armature current. Allow 1.0 V per brush for contact drop.*

(AMIE Sec. B. Elect. Machines 1991)

Solution. Generator circuit diagram is shown in Fig. 26.48.

Voltage drop in series winding $= 30 \times 0.3 = 9$ V

Voltage across shunt winding $= 220 + 9 = 229$ V

$I_{sh} = 229/200 = 1.145$ A

$I_a = 30 + 1.145 = 31.145$ A

$I_a R_a = 31.145 \times 0.05 = 1.56$ V

Brush drop $= 2 \times 1 = 2$ V

$E_g = V$ + series drop + brush drop + $I_a R_a$
$= 220 + 9 + 2 + 1.56 = 232.56$ V

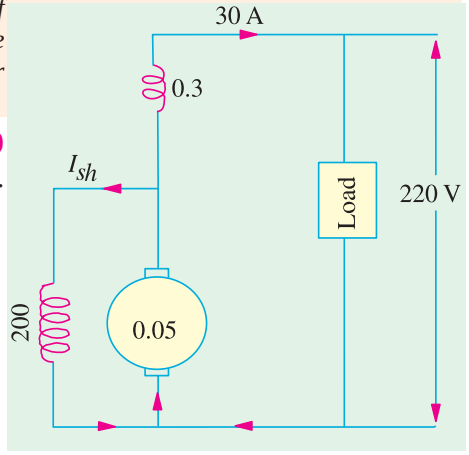


Fig. 26.48

Example 26.6. *In a long-shunt compound generator, the terminal voltage is 230 V when generator delivers 150 A. Determine **(i)** induced e.m.f. **(ii)** total power generated and **(iii)** distribution of this power. Given that shunt field, series field, divertor and armature resistance are 92 Ω, 0.015 Ω, 0.03 Ω and 0.032 Ω respectively.*

(Elect. Technology-II, Gwalior Univ. 1987)

Solution. $I_{sh} = 230/92 = 2.5$ A

$I_a = 150 + 2.5 = 152.5$ A

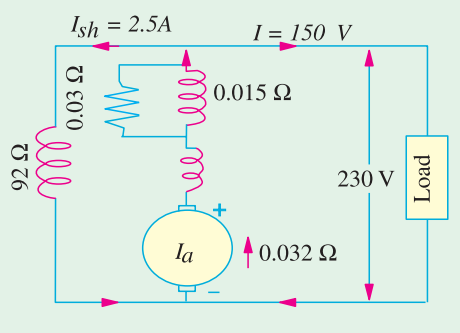


Fig. 26.49

Since series field resistance and divertor resistances are in parallel (Fig. 26.49) their combined resistance is

$= 0.03 \times 0.015/0.045 = 0.01\ \Omega$

Total armature circuit resistance is

$= 0.032 + 0.01 = 0.042\ \Omega$

Voltage drop = $152.5 \times 0.042 = 6.4$ V

(i) Voltage generated by armature

$E_g = 230 + 6.4 =$ **236.4 V**

(ii) Total power generated in armature

$E_g I_a = 236.4 \times 152.5 =$ **36,051 W**

(iii) Power lost in armature

$I_a R_a$	$= 152.5^2 \times 0.032$	=	744 W
Power lost in series field and divertor	$= 152.5^2 \times 0.01$	=	232 W
Power dissipated in shunt winding	$= VI_{sh} = 230 \times 0.01$	=	575 W
Power delivered to load	$= 230 \times 150$	=	34500 W
	Total	=	36,051 W.

Example 26.7. *The following information is given for a 300-kW, 600-V, long-shunt compound generator : Shunt field resistance = 75 Ω, armature resistance including brush resistance = 0.03 Ω, commutating field winding resistance = 0.011 Ω, series field resistance = 0.012 Ω, divertor resistance = 0.036 Ω. When the machine is delivering full load, calculate the voltage and power generated by the armature.*

(Elect. Engg-II, Pune Univ. Nov. 1989)

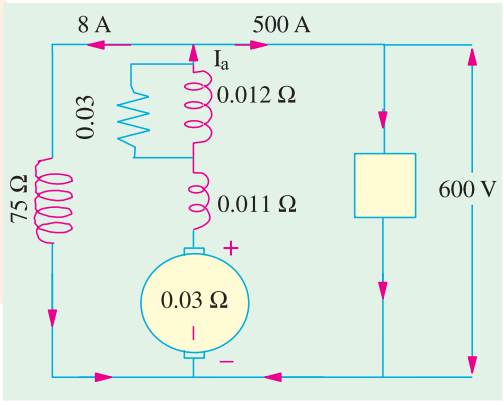


Fig. 26.50

Solution. Power output = 300,000 W

Output current = 300,000/600

= 500 A

I_{sh} = 600/75 = 8 A,

I_a = 500 + 8 = 508 A

Since the series field resistance and divertor resistance are in parallel (Fig. 26.50) their combined resistance is

$$= \frac{0.012 \times 0.036}{0.048} = 0.009\ \Omega$$

Total armature circuit resistance

$= 0.03 - 0.011 + 0.009 = 0.05\ \Omega$

Voltage drop = $508 \times 0.05 = 25.4$ V

Voltage generated by armature

$= 600 + 25.4 = 625.4$ V

Power generated = $625.4 \times 508 = 317{,}700$

$W =$ **317.7 kW**

26.33. Generated E.M.F. or E.M.F. Equation of a Generator

Let Φ = flux/pole in weber

Z = total number of armature conductors

= No. of slots × No. of conductors/slot
P = No. of generator poles
A = No. of parallel paths in armature
N = armature rotation in revolutions per minute (r.p.m.)
E = e.m.f. induced in any parallel path in armature

Generated e.m.f. E_g = e.m.f. generated in any one of the parallel paths *i.e.* E.

Average e.m.f. generated/conductor = $\frac{d\Phi}{dt}$ volt ($\because n = 1$)

Now, flux cut/conductor in one revolution $d\Phi = \Phi P$ Wb

No. of revolutions/second = $N/60$ $\therefore$ Time for one revolution, $dt = 60/N$ second

Hence, according to Faraday's Laws of Electromagnetic Induction,

E.M.F. generated/conductor = $\frac{d\Phi}{dt} = \frac{\Phi PN}{60}$ volt

For a simplex wave-wound generator

No. of parallel paths = 2

No. of conductors (in series) in one path = $Z/2$

$\therefore$ E.M.F. generated/path = $\frac{\Phi PN}{60} \times \frac{Z}{2} = \frac{\Phi ZPN}{120}$ volt

For a simplex lap-wound generator

No. of parallel paths = P

No. of conductors (in series) in one path = Z/P

$\therefore$ E.M.F. generated/path = $\frac{\Phi PN}{60} \times \frac{Z}{P} = \frac{\Phi ZN}{60}$ volt

In general generated e.m.f. $E_g = \frac{\Phi ZN}{60} \times \left(\frac{P}{A}\right)$ volt

where A = 2-for simplex wave-winding
= P-for simplex lap-winding

Also, $E_g = \frac{1}{2\pi} \cdot \left(\frac{2\pi N}{60}\right) \Phi Z \left(\frac{P}{A}\right) = \frac{\omega \Phi Z}{2\pi} \left(\frac{P}{A}\right)$ volt – ω in rad/s

For a given d.c. machine, Z, P and A are constant. Hence, putting $K_a = ZP/A$, we get

$E_g = K_a \Phi N$ volts—where N is in r.p.s.

Example 26.8. *A four-pole generator, having wave-wound armature winding has 51 slots, each slot containing 20 conductors. What will be the voltage generated in the machine when driven at 1500 rpm assuming the flux per pole to be 7.0 mWb ?* **(Elect. Machines-I, Allahabad Univ. 1993)**

Solution. $E_g = \frac{\Phi ZN}{60}\left(\frac{P}{A}\right)$ volts

Here, $\Phi = 7 \times 10^{-3}$ Wb, $Z = 51 \times 20 = 1020$, $A = P = 4$, $N = 1500$ r.p.m.

$\therefore$ $E_g = \frac{7 \times 10^{-3} \times 1020 \times 1500}{60}\left(\frac{4}{2}\right) =$ **178.5 V**

Example 26.9. *An 8-pole d.c. generator has 500 armature conductors, and a useful flux of 0.05 Wb per pole. What will be the e.m.f. generated if it is lap-connected and runs at 1200 rpm ? What must be the speed at which it is to be driven produce the same e.m.f. if it is wave-wound?*

(U.P. Technical Univ. 2001)

Solution. With lap-winding, $P = a = 8$

$$E = \phi\ (N/60)\ (P/a)$$
$$= 0.05 \times 500 \times 20 \times 1,$$
$$= 500 \text{ volts}$$

for lap-winding

If it is wave-wound, $P = 8, a = 2, P/a = 4$

and $E = 0.05 \times 500 \times (N/60) \times 4$

For $E = 500$ volts, $N = 300$ rpm

Hence, with wave-winding, it must be driven at 300 rpm to generate 500 volts.

Additional Explanation. Assume 1 amp as the current per conductor.

(*a*) Lap-wound, 1200 rpm : 500 V per coil-group, 8 groups in parallel

Net output current = 8 amp as in Fig. 26.51 (*a*).

Power output = 4 kW

(*b*) Wave-wound, 300 rpm : 2 groups in parallel, one group has four coils in series, as shown in Fig. 26.51 (*b*).

Total power-output is now

$$500 \times 2 = 1000 \text{ W.}$$

It is reduced to one fourth, being proportional to the speed.

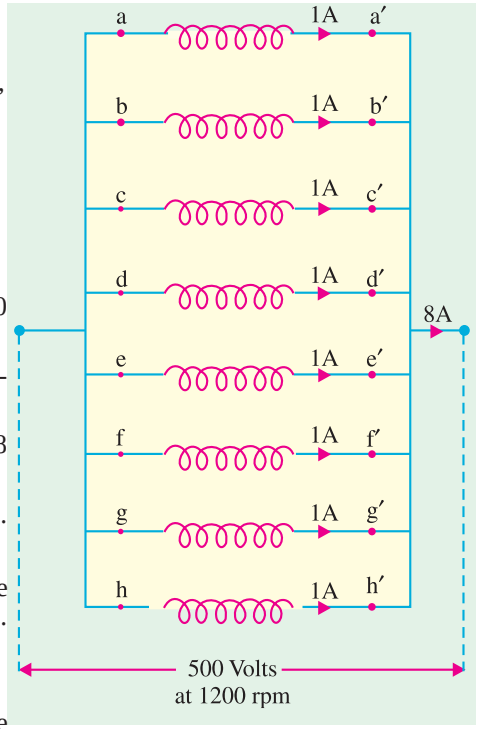


Fig. 26.51(*a*)

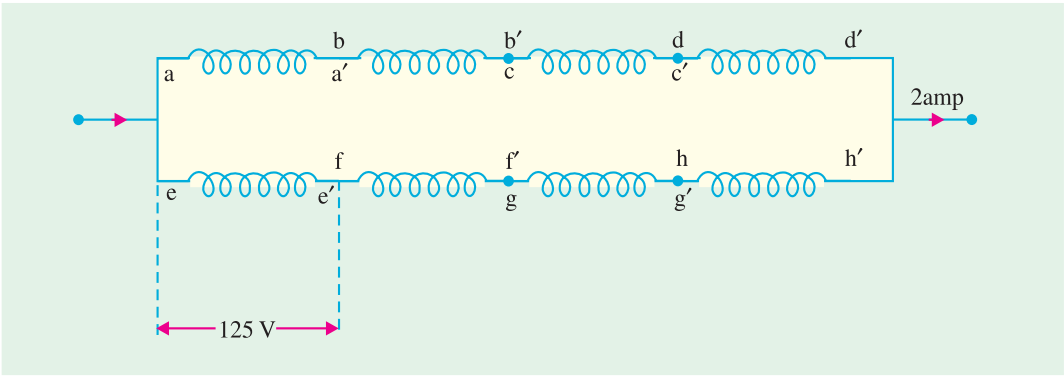


Fig. 26.51(*b*)

Example 26.10. *A d.c. shunt generator has an induced voltage on open-circuit of 127 volts. When the machine is on load, the terminal voltage is 120 volts. Find the load current if the field-circuit resistance is 15 ohms and the armature-resistance is 0.02 ohm. Ignore armature reaction.*

(Madras University April 1997, Bharathiar University Nov. 1997)

Solution.

Note : Even though the question does not specify some conditions, the solution given here is based on correct approach to deal with the case.

Generator on no load :

As shown in Fig. 26.52 (*a*), the machine is run at N_1 rpm.

$$E_g = 127 + 8.47 \times 0.02 = 127.17 \text{ volts}$$

As in Fig. 26.52 (*b*), $i_f = 8$ amp

E_g can be 127.17 volts, if the speed is increased to N_2 rpm, such that

$$8.47\ N_1 = 8N_2, \text{ or } N_2 = \frac{8.47}{8}\ N_1 = 1.05875\ N_1$$

Thus the effect due to 5.875% decrease in flux is compensated by 5.875% increase in speed.

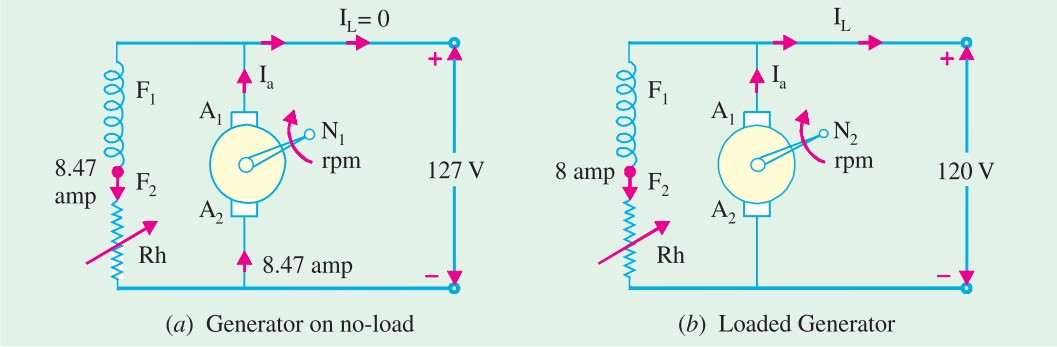


Fig. 26.52

If E_g is assumed to remain unaltered at 127.17 V,

$$I_a = \frac{127.17 - 120}{0.02} = 358.5 \text{ amp}$$

Hence, $I_L = 358.5 - 8 = 350.5$ amp.

Example 26.11(*a*). *An 8-pole d.c. shunt generator with 778 wave-connected armature conductors and running at 500 r.p.m. supplies a load of 12.5 Ω resistance at terminal voltage of 50 V. The armature resistance is 0.24 Ω and the field resistance is 250 Ω. Find the armature current, the induced e.m.f. and the flux per pole.*

(Electrical Engg-I, Bombay Univ. 1988)

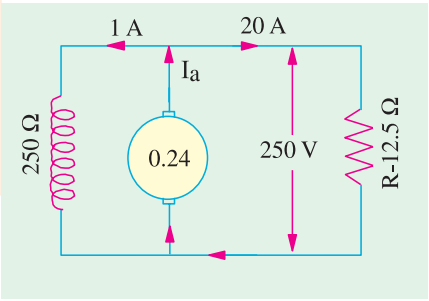


Fig. 26.53

Solution. The circuit is shown in Fig. 26.53

Load current = $V/R = 250/12.5 = 20$ A

Shunt current = $250/250 = 1$ A

Armature current = $20 + 1 =$ **21 A**

Induced e.m.f. = $250 + (21 \times 0.24) =$ **255.04 V**

Now, $$E_g = \frac{\Phi ZN}{60} \times \left(\frac{P}{A}\right)$$

$$\therefore \quad 255.04 = \frac{\Phi \times 778 \times 500}{60}\left(\frac{8}{2}\right)$$

$$\therefore \quad \Phi = \textbf{9.83 mWb}$$

Example 26.11(*b*). *A 4-pole lap-connected armature of a d.c. shunt generator is required to supply the loads connected in parallel :*

(1) 5 kW Geyser at 250 V, and

(2) 2.5 kW Lighting load also at 250 V.

The Generator has an armature resistance of 0.2 ohm and a field resistance of 250 ohms. The armature has 120 conductors in the slots and runs at 1000 rpm. Allowing 1 V per brush for contact drops and neglecting friction, find

(1) Flux per pole, (2) Armature-current per parallel path. **(Nagpur University Nov. 1998)**

Solution. Geyser current = 5000/250 = 20 A
Current for Lighting = 2500/250 = 10 A
Total current = 30 A
Field Current for Generator = 1 A
Hence, Armature Current = 31 A
Armature resistance drop = $31 \times 0.2 = 6.2$ volts
Generated e.m.f. = 250 + 6.2 + 2 = 258.2 V,
since $E = V_t + I_a r_a$ + Total brush contact drop
For a 4-pole lap-connected armature,
Number of parallel paths = number of poles = 4

(1) The flux per pole is obtained from the emf equation

$$258.2 = [\phi Z N/60] \times (p/a)$$
$$= [\phi \times 120 \times 1000/60] \times (4/4)$$
$$= 2000\,\phi$$
$$\phi = 129.1 \text{ mWb}$$

(2) Armature current per parallel path = 31/4 = 7.75 A.

Example 26.12. *A separately excited generator, when running at 1000 r.p.m. supplied 200 A at 125 V. What will be the load current when the speed drops to 800 r.p.m. if I_f is unchanged ? Given that the armature resistance = 0.04 ohm and brush drop = 2 V.*

(Elect. Machines Nagpur Univ. 1993)

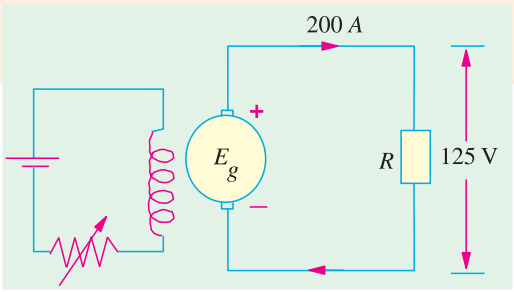


Fig. 26.54

Solution. The load resistance $R = 125/200 = 0.625\ \Omega$, in Fig. 26.54.

$E_{g1} = 125 + 200 \times 0.04 + 2 = 135$ V ; $N_1 = 1000$ r.p.m.

At 800 r.p.m. $E_{g2} = 135 \times 800/1000 = 108$ V

If I is the new load current, then terminal voltage V is given by

$$V = 108 - 0.04\,I - 2 = 106 - 0.04\,I$$

$$\therefore\ I = V/R = (106 - 0.04\,I)/0.635\ ;\ I = 159.4 \text{ A}$$

Example 26.13. *A 4-pole, 900 r.p.m. d.c. machine has a terminal voltage of 220 V and an induced voltage of 240 V at rated speed. The armature circuit resistance is 0.2 Ω. Is the machine operating as a generator or a motor ? Compute the armature current and the number of armature coils if the air-gap flux/pole is 10 mWb and the armature turns per coil are 8. The armature is wave-wound.* **(Elect. Machines AMIE Sec. B 1990)**

Solution. Since the induced voltage E is more than the terminal voltage v, the machine is working as a generator.

$$E - V = I_a R_a \quad \text{or} \quad 240 - 220 = I_a \times 0.2\ ;\ I_a = 100 \text{ A}$$

Now, $E_b = ZN\,(P/A)$ or $240 = 10 \times 10^{-3} \times z \times (900/600)\,(4/2)\ ;\ Z = 8000$

Since there are 8 turns in a coil, it means there are 16 active conductors/coil. Hence, the number of coils = 8000/16 = 500.

Example 26.14. *In a 120 V compound generator, the resistances of the armature, shunt and series windings are 0.06 Ω, 25 Ω and 0.04 Ω respectively. The load current is 100 A at 120 V. Find the induced e.m.f. and the armature current when the machine is connected as **(i)** long-shunt and as **(ii)** short-shunt. How will the ampere-turns of the series field be changed in **(i)** if a diverter of 0.1 ohm be connected in parallel with the series winding ? Neglect brush contact drop and ignore armature reaction.* **(Elect. Machines AMIE Sec. B, 1992)**

Solution. (*i*) Long Shunt [Fig. 26.55 (*a*)]

I_{sh} = 120/125 = 4.8 A ; I = 100 A ; I_a = 104.8 A

Voltage drop in series winding = $104.8 \times 0.04 = 4.19$ V

Armature voltage drop = $104.8 \times 0.06 = 6.29$ V

$\therefore$ E_g = 120 + 3.19 + 6.29 = 130.5 V

(*ii*) Short Shunt [Fig. 26.55 (*c*)]

Voltage drop in series winding = $100 \times 0.04 = 4$ V

Voltage across shunt winding = 120 + 4 = 124 V

$\therefore$ I_{sh} = 124/25 = 5 A ; $\therefore$ $I_a = 100 + 5 = 105$ A

Armature voltage drop = $105 \times 0.06 = 6.3$ V

E_g = 120 + 5 + 4 = 129 V

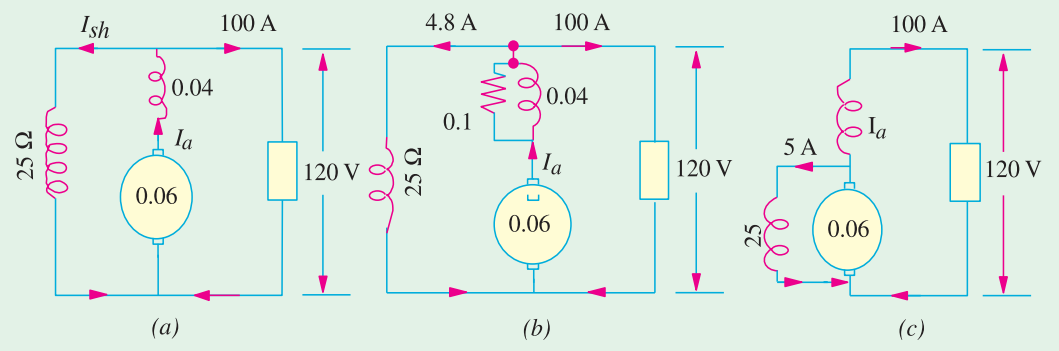


Fig. 26.55

When a diverter of 0.1 Ω is connected in parallel with the series winding, the diagram becomes as shown in Fig. 26.55 (*b*). As per current-divider rule, the current through the series winding is = $104.8 \times 0.1/(0.1 + 0.04) = 74.86$ A. It means that the series field current has decreased from an original value of 104.8 A to 74.86 A. Since No. of turns in the series winding remains the same, the change in series field ampere-turns would be the same as the change in the field current. Hence, the percentage decrease in the series field ampere-turns = $(74.86 - 104.8) \times 100/104.8 = -28.6\%$.

Example 26.15. *A 4-pole, long-shunt lap-wound generator supplies 25 kW at a termimal voltage of 500 V. The armature resistance is 0.03 ohm, series field resistance is 0.04 ohm and shunt field resistance is 200 ohm. The brush drop may be taken as 1.0 V. Determine the e.m.f. generated.*

Calculate also the No. of conductors if the speed is 1200 r.p.m. and flux per pole is 0.02 weber. Neglect armature reaction.

(Elec. Engineering-I, St. Patel Univ. 1986)

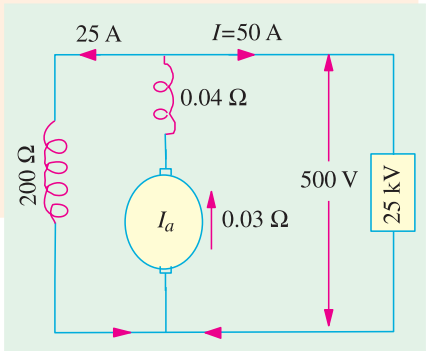


Fig. 26.56

Solution. I = 25,000/500 = 50 A, I_{sh} = 500/200 = 2.5 A (Fig. 26.56)

$I_a = I + I_{sh} = 50 + 2.5 = 52.5$ A

Series field drop = $52.5 \times 0.04 = 2.1$ V

Armature drop = $52.5 \times 0.03 = 1.575$ V

Brush drop = $2 \times 1 = 2$ V

Generated e.m.f., E_g = 500 + 2.1 + 1.575 + 2 = **505.67 V**

Now, $E_g = \frac{\Phi ZN}{60}\left(\frac{P}{A}\right)$

or $505.67 = \frac{0.02 \times Z \times 1200}{60}\left(\frac{4}{4}\right)$, Z = **1264**

Example 26.16. *A 4-pole d.c. generator runs at 750 r.p.m. and generates an e.m.f. of 240 V. The armature is wave-wound and has 792 conductors. If the total flux from each pole is 0.0145 Wb, what is the leakage coefficient ?*

Solution. Formula used :

$$E = \frac{\Phi ZN}{60}\left(\frac{P}{A}\right) \text{ volt} \quad \therefore \quad 240 = \frac{\Phi \times 750 \times 792}{60} \times \frac{4}{2}$$

$\therefore$ Working flux/pole, Φ = 0.0121 Wb ; Total flux/pole = 0.0145 Wb

$\therefore$ Leakage coefficient $\lambda = \frac{\text{total flux/pole}}{\text{working flux/pole}} = \frac{0.0145}{0.0121}$ = **1.2**

Example 26.17. *A 4-pole, lap-wound, d.c. shunt generator has a useful flux per pole of 0.07 Wb. The armature winding consists of 220 turns each of 0.004 Ω resistance. Calculate the terminal voltage when running at 900 r.p.m. if the armature current is 50 A.*

Solution. Since each turn has two sides,

$Z = 220 \times 2 = 400$; N = 900 r.p.m. ; Φ = 0.07 Wb ; $P = A = 4$

$\therefore \quad E_g = \frac{\Phi ZN}{60}.\left(\frac{P}{A}\right) = \frac{0.07 \times 440 \times 900}{60} \times \left(\frac{4}{4}\right)$ = 462 volt

Total resistance of 220 turns (or 440 conductors) = $220 \times 0.004 = 0.88\ \Omega$

Since there are 4 parallel paths in armature,

$\therefore$ Resistance of each path = $0.88/4 = 0.22\ \Omega$

Now, there are four such resistances in parallel each of value 0.22 Ω

$\therefore$ Armature resistance, $R_a = 0.22/4 = 0.055\ \Omega$

Armature drop $= I_a R_a = 50 \times 0.055 = 2.75\ \Omega$

Now, terminal voltage $V = E_g - I_a R_a = 462 - 2.75 =$ **459.25 volt.**

Example 26.18. *A 4-pole, lap-wound, long-shunt, d.c. compound generator has useful flux per pole of 0.07 Wb. The armature winding consists of 220 turns and the resistance per turn is 0.004 ohms. Calculate the terminal voltage if the resistance of shunt and series field are 100 ohms and 0.02 ohms respectively ; when the generator is running at 900 r.p.m. with armature current of 50 A. Also calculate the power output in kW for the generator.*

(Basic Elect. Machine Nagpur Univ. 1993)

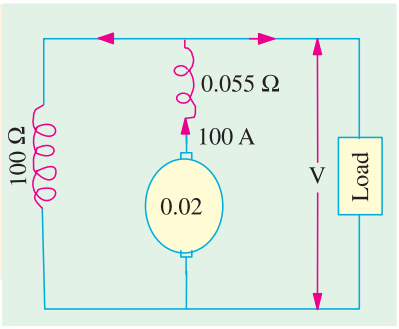


Fig. 26.57

Solution. $E_b = \frac{0.07 \times (220 \times 2) \times 900}{60} \times \left(\frac{4}{4}\right)$ = 462 V

As found in Ex. 26.17, R_a = 0.055 Ω

Arm. circuit resistance = $R_a + R_{se} = 0.055 + 0.02 = 0.075\ \Omega$

Arm. circuit drop = $50 \times 0.075 = 3.75$ V

$V = 462 - 3.75 = 458.25$ V, in Fig. 26.57.

$I_{sh} = 458.25/100 = 4.58$ A ; $I = 50 - 4.58 = 45.42$ A

Output = $VI = 458.25 \times 45.42 = 20{,}814$ W = 20.814 kW

Example 26.19. *A separately excited d.c. generator, when running at 1200 r.p.m. supplies 200 A at 125 V to a circuit of constant resistance. What will be the current when the speed is dropped to 1000 r.p.m. and the field current is reduced to 80% ? Armature resistance, 0.04 Ω and total drop at brushes, 2 V. Ignore saturation and armature reaction.*

(Elect. Machines AMIE Sec. B, 1991)

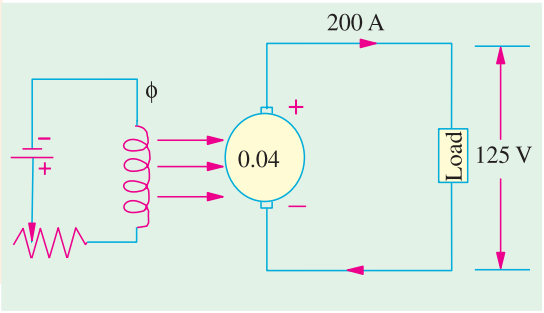


Fig. 26.58

Solution. We will find the generated e.m.f. when the load current is 200 A.

$E_{g1} = V$ + brush drop + $I_a R_a = 125 + 200 \times 0.04 = 135$ V, in Fig. 26.58.

Now, $E_{g1} \propto \Phi_1 N_1$ and $E_{g2} \propto \Phi_2 N_2$

$$\therefore \quad \frac{E_{g2}}{E_{g1}} = \frac{\Phi_2 N_2}{\Phi_1 N_1}$$

or $$\frac{E_{g2}}{135} = 0.8 \times \frac{1000}{1200} = 90 \text{ V}$$

Example 26.20(*a*). *A 4-pole, d.c. shunt generator with a shunt field resistance of 100 Ω and an armature resistance of 1 Ω has 378 wave-connected conductors in its armature. The flux per pole is 0.02 Wb. If a load resistance of 10 Ω is connected across the armature terminals and the generator is driven at 1000 r.p.m., calculate the power absorbed by the load.*

(Elect. Technology, Hyderabad Univ. 1991)

Solution. Induced e.m.f. in the generator is

$$E_s = \frac{\Phi Z N}{60}\left(\frac{P}{A}\right) \text{ volt}$$

$$= \frac{0.02 \times 378 \times 1000}{60}\left(\frac{4}{2}\right) = 252 \text{ volt}$$

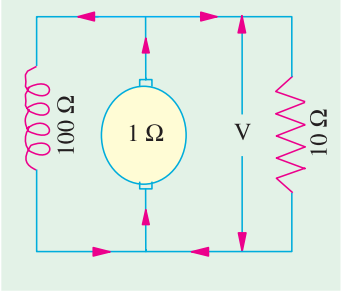


Fig. 26.59

Now, let V be the terminal voltage *i.e.* the voltage available across the load as well as the shunt resistance (Fig. 26.59).

Load current = V/10 A and Shunt current = V/100 A

Armature current = $\frac{V}{10} + \frac{V}{100} = \frac{11\ V}{100}$

Now, $V = E_g$ – armature drop

$$\therefore \quad V = 252 - 1 \times \frac{11\,V}{100} \quad \therefore \quad V = 227 \text{ volt}$$

Load current = 227/10 = 22.7 A, Power absorbed by the load is = 227 × 22.7 = **5,153 W**

Example 26.20(*b*). *A four-pole, lap-wound shunt generator has 300 armature-conductors and a flux/pole of 0.1 Wb. It runs at 1000 r.p.m. The armature and field-resistances are 0.2 ohm and 125 ohms respectively. Calculate the terminal voltage when it is loaded to take a load current of 90 A. Ignore armature reaction.* **(Nagpur University, April 1999)**

Solution. First, the e.m.f .should be calculated

$$E = 0.1 \times 300 \times (1000/60) \times (4/4) = 500 \text{ volts}$$

The field current 500/125 = 4 amp

For the load current of 90 amp, armature current = 94 amp

$$I_a r_a = 94 \times 0.20 = 18.8 \text{ volts}$$

Terminal voltage, $$V = 500 - 18.8 = 481.2 \text{ volts}$$

Note : Due to the reduction in terminal voltage (as an effect of loading), the shunt field current tends to decrease, which will further reduce V. To compensate for this, either increase the speed slightly or decrease the shunt-field-circuit resistance slightly.

Example 26.21(*a*). *A 6-pole dc generator runs at 1200 r.p.m. on no-load and has a generated e.m.f. of 250 V. Its armature diameter is 350 mm and the radial air-gap between the field poles and the armature is 3 mm. The axial length of the field poles is 260 mm and the field pole effective coverage is 80% including fringing. If the armature has 96 coils having 3 turns per coil and is wound duplex lap, calculate* ***(a)*** *flux per pole* ***(b)*** *effective pole arc length and* ***(c)*** *average air-gap flux density.*

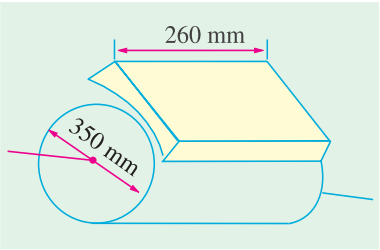


Fig. 26.60

Solution. (*a*) $Z = (96 \times 3) \times 2 = 576$, $P = 6$, $A = P \times$ plex $= 6 \times 2 = 12$, $N = 1200$ r.p.m.

$$\therefore \quad 250 = \frac{\Phi \times 576 \times 1200}{6}\left(\frac{6}{12}\right); \quad \therefore \quad \Phi = 0.0434 \text{ Wb}$$

(*b*) Inner diameter of the pole shoe circle is = 350 + 6 = 356 mm.

Since there are 6 poles, the net field pole flux coverage is 80% of one-sixth of the pole shoe circle. Hence, the effective pole arc length is

$$= \frac{1}{6} \times \pi d \times 0.8 = \frac{1}{6} \times \pi \times 356 \times 0.8 = 149 \text{ mm} = 0.149 \text{ m}.$$

(*c*) Pole surface area = pole shoe arc × axial length of the pole (Fig. 26.60).

$$= 0.149 \times 0.260 = 0.03874 \text{ mm}^2$$

$\therefore$ Flux density $B = 0.0434/0.03874 = 1.12$ T

Example 26.21(*b*). *A 4-pole d.c. Generator with 1200 conductors generates 250 volts on open circuit, when driven at 500 rpm. The pole-shoes have a bore of 35 cm and the ratio of pole-arc to pole pitch is 0.7, while, the length of the pole shoe is 20 cm. Find the mean flux density in the airgap.* **(Bharthiar Univ. Nov. 1972 & April 1998)**

Solution. For a diameter of 35, 4-pole machine has a pole-pitch of $(35\pi/4) = 27.5$ cm

Since pole-arc/pole pitch is 0.7, Pole-arc $= 0.7 \times 27.5 = 19.25$ cm

Pole area $= 19.25 \times 20 = 385$ sq. cm.

Substituting in the e.m.f. equation, $250 = (\phi ZN/60)(p/a)$

For Lap-winding, in the case, $p = a = 4$

Hence, flux/pole $= (250 \times 60)/(1200 \times 500) = 0.025$ Wb

This flux is uniformly distributed over the pole-area.

Mean flux density in the air-gap $= (0.025)/(385 \times 10^{-4}) = 0.65$ Wb/m^2

Example 26.21(*c*). *A four-pole lap-wound dc shunt generator having 80 slots with 10 conductors per slot generates at no-load an e.m.f. of 400 V, when run at 1000 rpm. How will you obtain a generated open-circuit voltage of 220 V ?* **(Nagpur University November 1996)**

Solution. (*i*) Keeping operating speed at 1000 rpm only, **change the flux per pole**

The O.C. e.m.f. is given by $E = (\phi ZN/60) \times (P/a)$

For the given operating conditions,

$$400 = \phi \times (80 \times 10) \times (1000/60) \times (4/4)$$

which gives $\phi = 30$ mWb

When speed is kept constant at 1000 rpm only,

$$E \propto \phi$$

Or to get 220 V on O.C., $\phi_2 = (220/400) \times 30 \text{ mWb} = 16.5 \text{ mWb}$

Thus, by increasing the shunt-field-circuit resistance with the help of adding external rheostatic, the current in the field-circuit is decreased so as to decrease the flux to 16.5 mWb.

(ii) Keep same flux per pole, change the speed.

If ϕ is held constant at 30 mWb, an O.C. e.m.f. of 220 V is obtained at a speed of N r.p.m., given by

$$220 = 30 \times 10^{-3} \times 800 \times N/60, \quad N = 550 \text{ rpm}$$

At 220 V, the flux can be maintained at 30 mWb provided the field **current is unchanged.**

$$400/R_{f1} = 200/R_{f2}$$

or $$R_{f2} = 0.55\, R_{f1}$$

Thus, the field circuit resistance must be reduced to the new value of 0.55 R_{f1} in order to obtain 30 mWb of flux per pole from a voltage of 220 V.

(iii) Any other combination of proper speed and flux/pole can be chosen and worked out on similar lines.

Example 26.21(*d*). *A short-shunt d.c. compound generator supplies 200 A at 100 V. The resistance of armature, series field and shunt field windings are 0.04, 0.03 and 60 ohms respectively. Find the emf generated. Also find the emf generated if same machine is connected as a long-shunt machine.* **(Nagpur University, April 1998)**

Solution. With short-shunt connection, shown in Fig. 26.61 (*a*).

V_a = armature terminal voltage = 100 (200 × 0.03) = 106 V

Shunt field current = 106/60 = 1.767 amp

Armature current = I_a = 200 + 1.767 = 201.767 amp

Armature induced e.m.f. = 106 + (201.767 × 0.04) = 114.07 volts

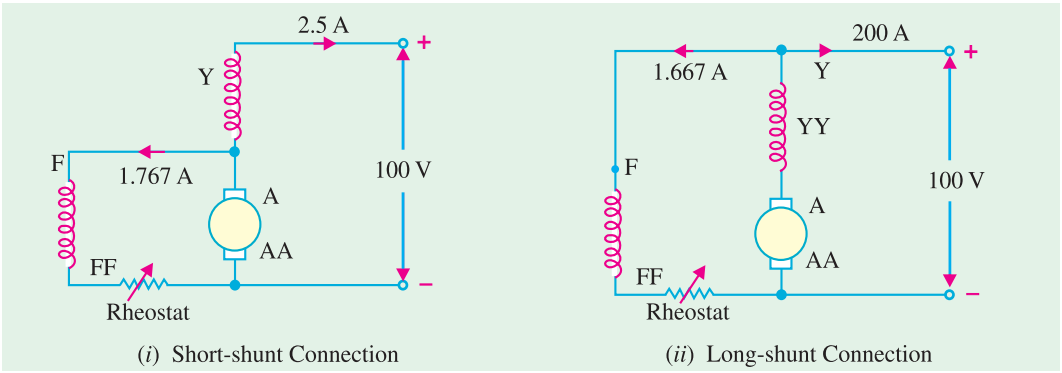


Fig. 26.61(*a*)

Now, with long-shunt connection shown in Fig. 26.61 (*b*),

Shunt field current = 100/60 = 1.667 amp

Armature current = 201.667 amp

Total voltage drop in armature and series field winding

= 201.667 (0.04 + 0.03) = 14.12 volts

Armature induced e.m.f. = 100 + 14.12 = 114.12 volts

Note : In case of long shunt connection, the generator has to develop the e.m.f. with shunt field current slightly reduced, compared to the case of short shunt connection. However, the series field winding carries a slightly higher current in latter case. Still, in practice, slight speed adjustment (or shunt field rheostatic variation) may be required to get this e.m.f., as per calculations done above.

Example 26.22. *A long shunt dynamo running at 1000 r.p.m. supplies 20 kW at a terminal voltage of 220 V. The resistance of armature, shunt field, and series field are 0.04, 110 and 0.05 ohm respectively. Overall efficiency at the above load is 85%. Find :*

(i) Copper loss,

(ii) Iron and friction loss,

(iii) Torque developed by the prime mover.

(Amravati University 1999)

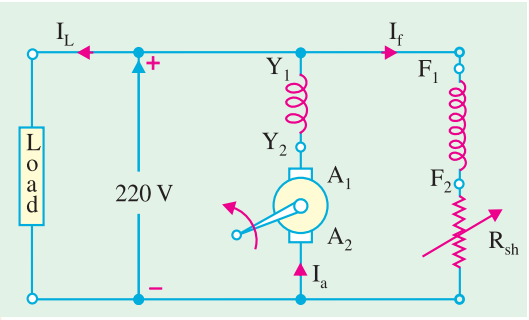


Fig. 26.61(*b*)

Solution. I_L, Load current $= \frac{20,000}{220} = 90.91$ amp

Shunt field current, $I_f = \frac{220}{110} = 2$ amp

Armature current, $I_a = 92.91$ amp

Input power = 20,000/0.85 = 23529 watts

Total losses in the machine = Input – Output = 23529 – 20,000 = 3529 watts

(*i*) *Copper losses* :

Power loss in series field-winding + armature winding = $92.91^2 \times 0.09$ watts = 777 watts

Power-loss in shunt field circuit : $2^2 \times 110 = 440$ watts

Total copper losses = 777 + 400 = 1217 watts

(*ii*) Iron and friction losses = Total losses – Copper losses

= 3529 – 1217 = 2312 watts

(*iii*) Let T = Torque developed by the prime-mover

At 1000 r.p.m., angular speed, $\omega = 2\pi \times 1000/60 = 104.67$ rad./sec

$T \times \omega$ = Input power

$\therefore$ T = 23529/104.67 = 224.8 Nw-m

26.34. Iron Loss in Armature

Due to the rotation of the iron core of the armature in the magnetic flux of the field poles, there are some losses taking place continuously in the core and are known as Iron Losses or Core Losses. Iron losses consist of **(*i*) Hysteresis** loss and **(*ii*) Eddy Current** loss.

(*i*) Hysteresis Loss (W_h)

This loss is due to the reversal of magnetisation of the armature core. Every portion of the rotating core passes under *N* and *S* pole alternately, thereby attaining *S* and *N* polarity respectively. The core undergoes one complete cycle of magnetic reversal after passing under one *pair* of poles. If *P* is the number of poles and *N*, the armature speed in r.p.m., then frequency of magnetic reversals is $f = PN/120$.

The loss depends upon the volume and grade of iron, maximum value of flux density B_{max} and frequency of magnetic reversals. For normal flux densities (*i.e.* upto 1.5 Wb/m^2), hysteresis loss is given by **Steinmetz** formula. According to this formula,

$$W_h = \eta B_{\max}^{1.6} f V \text{ watts}$$

where V = volume of the core in m^3

η = Steinmetz hysteresis coefficient.

Value of η for :

Good dynamo sheet steel = 502 J/m^3, Silicon steel = 191 J/m^2, Hard Cast steel = 7040 J/m^3, Cast steel = 750 – 3000 J/m^3 and Cast iron = 2700 – 4000 J/m^3.

(*ii*) Eddy Current Loss (W_e)

When the armature core rotates, it also cuts the magnetic flux. Hence, an e.m.f. is induced in the body of the core according to the laws of electromagnetic induction. This e.m.f. though small, sets up large current in the body of the core due to its small resistance. This current is known as eddy current. The power loss due to the flow of this current is known as eddy current loss. This loss would be considerable if solid iron core were used. In order to reduce this loss and the consequent heating of the core to a small value, the core is built up of thin laminations, which are stacked and then riveted at right angles to the path of the eddy currents. These core laminations are insulated from each other by a thin coating of varnish. The effect of laminations is shown in Fig. 26.62. Due to the core body being one continuous solid iron piece [Fig. 26.62 (*a*)], the magnitude of eddy currents is large. As armature cross-sectional area is large, its resistance is very small, hence eddy current loss is large. In Fig. 26.62 (*b*), the same core has been split up into thin circular discs insulated from each other. It is seen that now each current path, being of much less cross-section, has a very high resistance. Hence, magnitude of eddy currents is reduced considerably thereby drastically reducing eddy current loss.

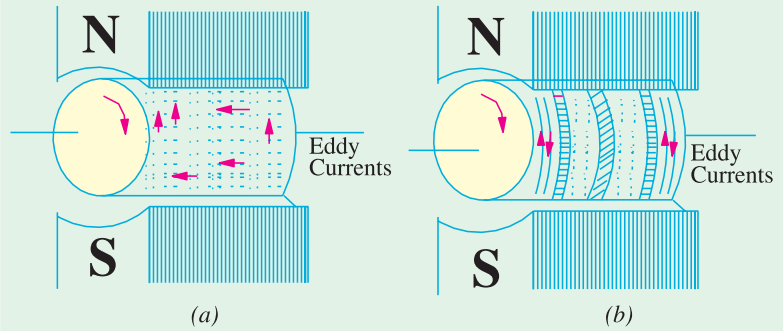


Fig. 26.62

It is found that eddy current loss W_e is given by the following relation :

$$W_e = KB_{max}^2 f^2 t^2 V^2 \text{ watt}$$

where B_{max} = maximum flux density f = frequency of magnetic reversals
t = thickness of each lamination V = volume of armature core.

It is seen from above that this loss varies directly as the square of the thickness of laminations, hence it should be kept as small as possible. Another point to note is that $W_h \propto f$ but $W_e \propto f^2$. This fact makes it possible to separate the two losses experimentally if so desired.

As said earlier, these iron losses if allowed to take place unchecked not only reduce the efficiency of the generator but also raise the temperature of the core. As the output of the machines is limited, in most cases, by the temperature rise, these losses have to be kept as small as is economically possible.

Eddy current loss is reduced by using laminated core but hysteresis loss cannot be reduced this way. For reducing the hysteresis loss, those metals are chosen for the armature core which have a low hysteresis coefficient. Generally, special silicon steels such as stalloys are used which not only have a low hysteresis coefficient but which also possess high electrical resistivity.

26.35. Total Loss in a D.C. Generator

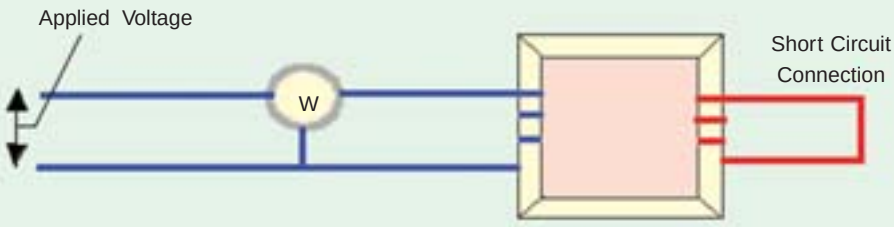
The various losses occurring in a generator can be sub-divided as follows :

(*a*) Copper Losses

(*i*) Armature copper loss = $I_a^2 R_a$ [**Note :** $E_g I_a$ is the power output from armature.]

where R_a = resistance of armature and interpoles and series field winding etc.

This loss is about 30 to 40% of full-load losses.



Short Circuit Connections for Copper Loss Test

(ii) Field copper loss. In the case of shunt generators, it is practically constant and $I_{sh}^2 R_{sh}$ (or VI_{sh}). In the case of series generator, it is = $I_{se}{}^2 R_{se}$ where R_{se} is resistance of the series field winding.

This loss is about 20 to 30% of F.L. losses.

(iii) The loss due to brush contact resistance. It is usually included in the armature copper loss.

(b) **Magnetic Losses** (also known as iron or core losses),

(i) hysteresis loss, $W_h \propto B_{max}^{1.6} f$ and ***(ii)*** eddy current loss, $W_e \propto B_{max}^2 f^2$

These losses are practically constant for shunt and compound-wound generators, because in their case, field current is approximately constant.

Both these losses total up to about 20 to 30% of F.L. losses.

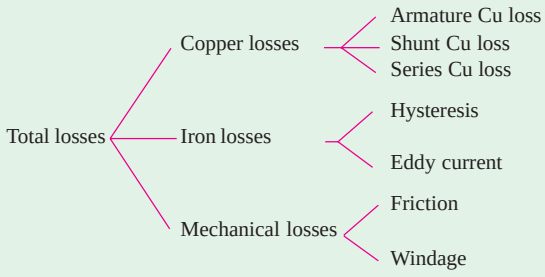
(c) **Mechanical Losses.** These consist of :

(i) friction loss at bearings and commutator.

(ii) air-friction or windage loss of rotating armature.

These are about 10 to 20% of F.L. Losses.

The total losses in a d.c. generator are summarized below :



26.36. Stray Losses

Usually, magnetic and mechanical losses are collectively known as ***Stray Losses*****.** These are also known as rotational losses for obvious reasons.

26.37. Constant or Standing Losses

As said above, field Cu loss is constant for shunt and compound generators. Hence, stray losses and shunt Cu loss are constant in their case. These losses are together known as standing or constant losses W_c.

Hence, for shunt and compound generators,

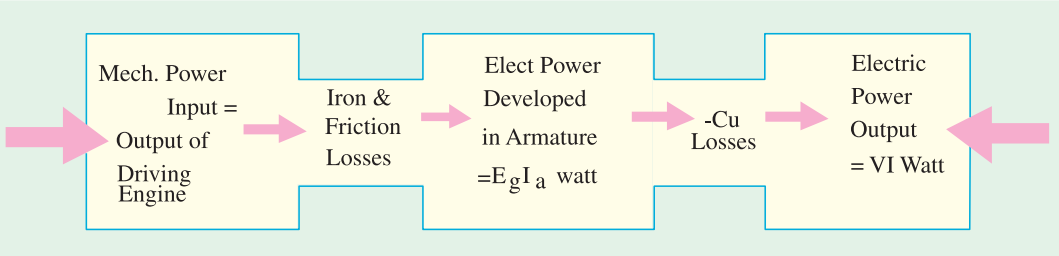
Total loss = armature copper loss + $W_c = I_a^2 R_a + W_c = (I + I_{sh})^2 R_a + W_c$.

Armature Cu loss $I_a^2 R_a$ is known as variable loss because it varies with the load current.

Total loss = variable loss + constant losses W_c

26.38. Power Stages

Various power stages in the case of a d.c. generator are shown below :



Following are the three generator efficiencies :

1. Mechanical Efficiency

$$\eta_m = \frac{B}{A} = \frac{\text{total watts generated in armature}}{\text{mechanical power supplied}} = \frac{E_g I_a}{\text{output of driving engine}}$$

2. Electrical Efficiency

$$\eta_e = \frac{C}{B} = \frac{\text{watts available in load circuit}}{\text{total watts generated}} = \frac{VI}{E_g I_a}$$

3. Overall or Commercial Efficiency

$$\eta_c = \frac{C}{A} = \frac{\text{watts available in load circuit}}{\text{mechanical power supplied}}$$

It is obvious that overall efficiency $\eta_c = \eta_m \times \eta_e$. For good generators, its value may be as high as 95%.

Note. Unless specified otherwise, commercial efficiency is always to be understood.

26.39. Condition for Maximum Efficiency

Generator output = VI

Generator input = output + losses

$$= VI + I_a^2 R_a + W_c = VI + (I + I_{sh})^2 R_a + W_c \qquad (\because I_a = I + I_{sh})$$

However, if I_{sh} is negligible as compared to load current, then $I_a = I$ (approx.)

$$\therefore \quad \eta = \frac{\text{output}}{\text{input}} = \frac{VI}{VI + I_a^2 R_a + W_c} = \frac{VI}{VI + I^2 R_a + W_c} \qquad (\because I_a = I)$$

$$= \frac{1}{1 + \left(\frac{IR_a}{V} + \frac{W_c}{VI}\right)}$$

Now, efficiency is maximum when denominator is minimum *i.e.* when

$$\frac{d}{dI}\left(\frac{IR_a}{V} + \frac{W_c}{VI}\right) = 0 \text{ or } \frac{R_a}{V} - \frac{W_c}{VI^2} = \quad \text{or} \quad I^2 R_a = W_c$$

Hence, generator efficiency is maximum when

Variable loss = constant loss.

The load current corresponding to maximum efficiency is given by the relation.

$$I^2R_a = W_c \quad \text{or} \quad I = \sqrt{\frac{W_c}{R_a}}.$$

Variation of η with load current is shown in Fig. 26.63.

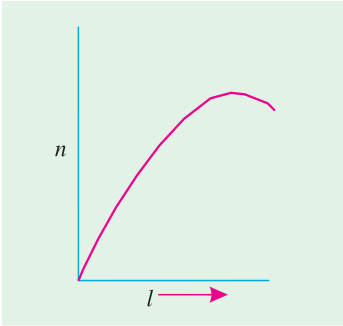


Fig. 26.63

Example 26.23. *A 10 kW, 250 V, d.c., 6-pole shunt generator runs at 1000 r.p.m. when delivering full-load. The armature has 534 lap-connected conductors. Full-load Cu loss is 0.64 kW. The total brush drop is 1 volt. Determine the flux per pole. Neglect shunt current.*

(Elect. Engg. & Electronics, M.S. Univ./Baroda 1987)

Solution. Since shunt current is negligible, there is no shunt Cu loss. The copper loss occurs in armature only.

$I = I_a = 10{,}000/250 = 40$ A ; $I_a^2R_a$ = Arm. Cu loss or $40^2 \times R_a = 0.64 \times 10^3$; $R_a = 0.4\ \Omega$

I_aR_a drop $= 0.4 \times 40 = 16$ V ; Brush drop $= 2 \times 1 = 2$ V

$\therefore$ Generated e.m.f. $E_g = 250 + 16 + 1 = 267$ V

Now, $E_g = \dfrac{\Phi ZN}{60}\left(\dfrac{P}{A}\right)$ volt $\therefore 267 = \dfrac{\Phi \times 534 \times 1000}{60}\left(\dfrac{6}{6}\right)$ $\therefore$ $\Phi = 30 \times 10^{-3}$ Wb = **30 mWb**

Example 26.24(*a*). *A shunt generator delivers 195 A at terminal p.d. of 250 V. The armature resistance and shunt field resistance are 0.02 Ω and 50 Ω respectively. The iron and friction losses equal 950 W. Find*

(a) E.M.F. generated (b) Cu losses (c) output of the prime motor

(d) commercial, mechanical and electrical efficiencies.

(Elect. Machines-I, Nagpur Univ. 1991)

Solution. (*a*) $I_{sh} = 250/50 = 5$ A ; $I_a = 195 + 5 = 200$ A

Armature voltage drop $= I_aR_a = 200 \times 0.02 = 4$ V

$\therefore$ Generated e.m.f. $= 250 + 4 =$ **254 V**

(*b*) Armature Cu loss $= I_a^2R_a = 200^2 \times 0.02 = 800$ W

Shunt Cu loss $= V.I_{sh} = 250 \times 5 = 1250$ W

$\therefore$ Total Cu loss $= 1250 + 800 =$ **2050 W**

(*c*) Stray losses $= 950$ W ; Total losses $= 2050 + 950 = 3000$ W

Output $= 250 \times 195 = 48{,}750$ W ; Input $= 48{,}750 + 3000 = 51750$ W

$\therefore$ Output of prime mover = **51,750 W**

(*d*) Generator input = 51,750 W ; Stray losses = 950 W

Electrical power produced in armature = 51,750 – 950 = 50,800

$\eta_m = (50{,}800/51{,}750) \times 100 =$ **98.2%**

Electrical or Cu losses = 2050 W

$\therefore$ $\eta_e = \dfrac{48,750}{48,750 + 2,050} \times 100 =$ **95.9%**

and $\eta_c = (48{,}750/51{,}750) \times 100 =$ **94.2%**

Example 26.24(*b*). *A 500 V, D.C. shunt motor draws a line current of 5 amps, on light load. If armature resistance is 0.15 ohm, and field resistance is 200 ohms, determine the efficiency of the machine running as a generator, delivering a load current of 40 Amp.*

(Bharathiar Univ. Nov. 1997)

Solution. ***(i)*** As a motor, on Light load, out of 5 Amps of line current, 2.5 Amps are required for field circuit and 2.5 Amps are required for field circuit and 2.5 Amps are required for armature. Neglecting copper-loss in armature at no load (since it works out to be just one watt), the armature-power goes towards armature-core-loss and no-load mechanical loss at the rated speed. This amounts to $(500 \times 2.5) = 1250$ watts.

(ii) As a generator, for a line current of 40 Amp, the total current for the armature is 42.5 amp.

Output of generator $= 500 \times 40 \times 10^{-3} = 20$ kW

Total losses as a generator = 1250 + field copper-loss + arm. copper-loss

$= (1250 + 1250 + 42.5^2 \times 0.15)$ watts $= 2.771$ kW

$$\text{Efficiency} = \frac{20}{20 + 2.771} \times 100 = 87.83\%$$

Example 26.25. *A shunt generator has a F.L. current of 196 A at 220 V. The stray losses are 720 W and the shunt field coil resistance is 55 Ω. If it has a F.L. efficiency of 88%, find the armature resistance. Also, find the load current corresponding to maximum efficiency.*

(Electrical Technology Punjab Univ. Nov. 1988)

Solution.

Output $= 220 \times 196 = 43{,}120$ W ; $\eta = 88\%$ (overall efficiency)

Electrical input $= 43{,}120/0.88 = 49{,}000$ W

Total losses $= 49{,}000 - 43{,}120 = 5{,}880$ W

Shunt field current $= 220/55 = 4$ A $\quad \therefore\ I_a = 196 + 4 = 200$ A

$\therefore$ Shunt Cu loss $= 220 \times 4 = 880$ W ; Stray losses $= 720$ W

Constant losses $= 880 + 720 = 1{,}600$

$\therefore$ Armature Cu loss $= 5{,}880 - 1{,}600 = 4{,}280$ W

$\therefore\ I_a^2 R_a = 4{,}280$ W

$200^2\ R_a = 4{,}280$ or $R_a = 4{,}280/200 \times 200 =$ **0.107 Ω**

For maximum efficiency,

$I^2 R_a =$ constant losses $= 1{,}600$ W ; $I = \sqrt{1{,}600 / 0.107} =$ **122.34 A**

Example 26.26. *A long-shunt dynamo running at 1000 r.p.m. supplies 22 kW at a terminal voltage of 220 V. The resistances of armature, shunt field and the series field are 0.05, 110 and 0.06 Ω respectively. The overall efficiency at the above load is 88%. Find* ***(a)*** *Cu losses* ***(b)*** *iron and friction losses* ***(c)*** *the torque exerted by the prime mover.*

(Elect. Machinery-I, Bangalore Univ. 1987)

Solution. The generator is shown in Fig. 26.64.

$I_{sh} = 220/110 = 2$ A

$I = 22{,}000/220 = 100$ A,

$I_a = 102$ A

Drop in series field winding $= 102 \times 0.06 = 6.12$ V

(a) $I_a^2 R_a = 102^2 \times 0.05 = 520.2$ W

Series field loss $= 102^2 \times 0.06 = 624.3$ W

Shunt field loss $= 4 \times 110 = 440$ W

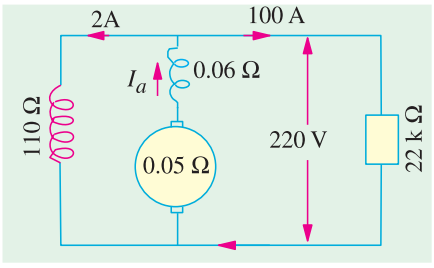


Fig. 26.64

Total Cu losses = 520.2 + 624.3 + 440 = **1584.5 W**

(*b*) Output = 22,000 W ; Input = 22,000/0.88 = 25,000 W

∴ Total losses = 25,000 – 22,000 = 3,000 W

∴ Iron and friction losses = 3,000 – 1,584.5 = **1,415.5 W**

Now, $T \times \frac{2\pi N}{60} = 25{,}000$; $T = \frac{25{,}000 \times 60}{1{,}000 \times 6.284} =$ **238.74 N-m**

Example 26.27. *A 4-pole d.c. generator is delivering 20 A to a load of 10 Ω. If the armature resistance is 0.5 Ω and the shunt field resistance is 50 Ω, calculate the induced e.m.f. and the efficiency of the machine. Allow a drop of 1 V per brush.*

(Electrical Technology-I, Osmania Univ., 1990)

Solution. Terminal voltage = 20 × 10 = 200 V

I_{sh} = 200/50 = 4 A ; I_a = 20 + 4 = 24 A

I_aR_a = 24 × 0.5 = 12 V ; Brush drop = 2 × 1 = 2 V

∴ E_g = 200 + 12 + 2 = **214 V**, as in Fig. 26.65.

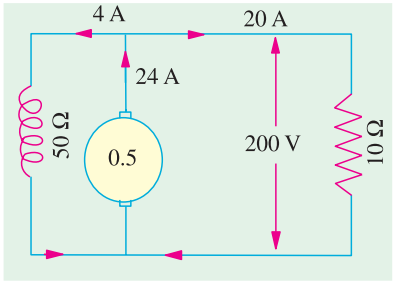


Fig. 26.65

Since iron and friction losses are not given, only electrical efficiency of the machine can be found out.

Total power generated in the armature = 214 × 24 = 5,136 W

Useful output = 200 × 20 = 4,000 W

∴ η_e = 4,000/5,136 = 0.779 or **77.9%**

Example 26.28. *A long-shunt compound-wound generator gives 240 volts at F.L. output of 100 A. The resistances of various windings of the machine are : armature (including brush contact) 0.1 Ω, series field 0.02 Ω, interpole field 0.025 Ω, shunt field (including regulating resistance) 100 Ω. The iron loss at F.L. is 1000 W ; windage and friction losses total 500 W. Calculate F.L. efficiency of the machine.*

(Electrical Machinery-I, Indore Univ. 1989)

Solution. Output = 240 × 100 = 24,000 W

Total armature circuit resistance = 0.1 + 0.02 + 0.025 = 0.145 Ω

I_{sh} = 240/100 = 2.4 A ∴ I_a = 100 + 2.4 = 102.4 A

∴ Armature circuit copper loss = $102.4^2 \times 0.145$ = 1,521 W

Shunt field copper loss = 2.4 × 240 = 576 W

Iron loss = 1000 W ; Friction loss = 500 W

Total loss = 1,521 + 1,500 + 576 = 3,597 W ; $\eta = \frac{24{,}000}{24{,}000 + 3{,}597} = 0.87 =$ **87%**

Example 26.29. *In a d.c. machine the total iron loss is 8 kW at its rated speed and excitation. If excitation remains the same, but speed is reduced by 25%, the total iron loss is found to be 5 kW. Calculate the hysteresis and eddy current losses at **(i)** full speed **(ii)** half the rated speed.*

(Similar Example, JNTU, Hyderabad, 2000)

Solution. We have seen in Art. 26.32 that

$$W_h \propto f \quad \text{and} \quad W_e \propto f^2$$

Since *f*, the frequency of reversal of magnetization, is directly proportional to the armature speed,

$$W_h \propto N \quad \text{and} \quad W_e \propto N^2$$

$\therefore$ $W_h = A \times N$ and $W_e = BN^2$, where A and B are constants.

Total loss $W = W_h + W_e = AN + BN^2$

Let the full rated speed be 1.

Then $8 = A \times 1 + B \times 1^2$ or $8 = A + B$ ...(*i*)

Now, when speed is 75% of full rated speed, then

$5 = A \times (0.75) + B(0.75)^2$ ...(*ii*)

Multiplying (*i*) by 0.75 and subtracting (*ii*) from it, we get

$0.1875\,B = 1$ $\therefore$ $B = 1/0.1875 =$ **5.33 kW**

Substituting this value in (*i*) above

$8 = 5.33 + A$ $\therefore$ $A =$ **2.67 kW**

(*i*) W_h at rated speed = 2.67 kW, W_e at rated speed = **5.33 kW**

(*ii*) W_h at half the rated speed $= 2.67 \times 0.5 =$ **1.335 kW**

W_e at half the rated speed $= 5.33 \times 0.5^2 =$ **1.3325 kW**

Example 26.30. *The hysteresis and eddy current losses in a d.c. machine running at 1000 r.p.m. are 250 W and 100 W respectively. If the flux remains constant, at what speed will be total iron losses be halved ?* **(Electrical Machines-I, Gujarat Univ. 1989)**

Solution. Total loss $W = W_h + W_e = AN + BN^2$

Now, $W_h = 250$ W $\therefore$ $A \times (1000/60) = 250$; $A = 15$

$W_e = 100$ W $\therefore$ $B \times (1000/60)^2 = 100$; $B = 9/25$

Let N be the new speed in r.p.s. at which total loss is one half of the loss at 1000 r.p.m. New loss $= (250 + 100)/2 = 175$ W

$\therefore$ $175 = 15\,N + (9/25)N^2$ or $9N^2 + 375\,N - 4{,}375 = 0$

$\therefore$ $$N = \frac{-375 \pm \sqrt{375^2 + 36 \times 4{,}375}}{2 \times 9} = \frac{-375 \pm 546}{18} = 9.5 \text{ r.p.s} =$$ **570 r.p.m.***

Note. It may be noted that at the new speed, $W_h = 250 \times (570/100) = 142.5$ W and $W_e = 100 \times (570/1000)^2 = 32.5$ W. Total loss = 142.5 + 32.5 = 175 W.

Example 26.31. *A d.c. shunt generator has a full load output of 10 kW at a terminal voltage of 240 V. The armature and the shunt field winding resistances are 0.6 and 160 ohms respectively. The sum of the mechanical and core-losses is 500 W. Calculate the power required, in kW, at the driving shaft at full load, and the corresponding efficiency.* **(Nagpur University November 99)**

Solution. Field current $= \frac{240}{160} = 1.5$ amp, Load current $= \frac{10{,}000}{240} = 41.67$ amp

Armature current $= 41.67 + 1.5 = 43.17$ amp

Field copper losses = 360 W, Armature copper losses $= 43.17^2 \times 0.6 = 1118$ W

Total losses in kW $= 0.36 + 1{,}118 + 0.50 = 1{,}978$ kW

Hence, Power input at the shaft = 11.978 kW

Efficiency $= \frac{10}{11.978} \times 100\% = 83.5\%$

* The negative value has been rejected–being mathematically absurd.

Example 26.32. *A long shunt d.c. compound generator delivers 110 kW at 220 V.*

If r_a = 0.01 ohm, r_{se} = 0.002 ohm, and shunt field has a resistance of 110 ohms, calculate the value of the induced e.m.f. **(Bharathithasan University Nov. 1997)**

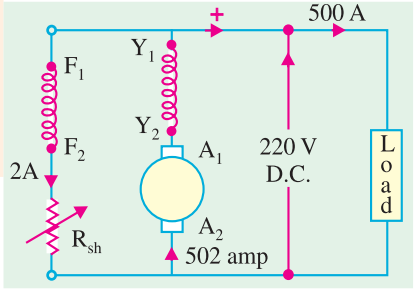


Fig. 26.66. Long-shunt d.c. compound generator

Solution.

Load current = 110 × 1000/220
= 500 A

Shunt field current = 220/110 = 2 A

Armature current = 502 A

$r_a + r_{se}$ = 0.012 ohm

E_a = 220 + [502 × (0.012)]
= 226.024 V

Example 26.33. *The armature of a four-pole d.c. shunt generator is lap-wound and generates 216 V when running at 600 r.p.m. Armature has 144 slots, with 6 conductors per slot. If this armature is rewound, wave-connected, find the e.m.f. generated with the same flux per pole but running at 500 r.p.m.* **(Bharathithasan University April 1997)**

Solution. Total number of armature conductors = $Z = 144 \times 6 = 864$

For a Lap winding, number of parallel paths in armature = number of poles

In the e.m.f. equation, $E = (\phi ZN/60)(P/a)$

Since $P = a$

$E = \phi ZN/60$

$216 = \phi \times 864 \times 600/60 = 8640\phi$

Hence ϕ = 25 milli-webers

If the armature is rewound with wave-connection, number of parallel paths = 2.

Hence, at 500 r.p.m., with 25 mWb as the flux per pole.

the armature emf $= (25 \times 10^{-3} \times 864 \times 500/60) \times 4/2$
$= 25 \times 864 \times 0.50 \times 2/60$
= 360 volts

Additional note :

Extension to Que : *Comment on the armature output power in the two cases.*

Solution. Assumption is that field side is suitably modified in the two cases.

Case (*i*) : Lap-wound Machine at 600 r.p.m.

Armature e.m.f.= 216 V

Let each armature-conductor be rated to carry a current of 10 amp.

In simple lap-wound machines, since a four-pole machine has four parallel paths in armature, the total armature output-current is 40 amp.

Hence, armature-output-power = $216 \times 40 \times 10^{-3}$ = 8.64 kW

Case (*ii*) : Wave-wound machine, at 500 r.p.m.

Armature e.m.f. = 360 V

Due to wave-winding, number of parallel paths in armature = 2

Hence, the total armature output current = 20 amp

Thus, Armature Electrical output-power = $360 \times 20 \times 10^{-3} = 7.2$ kW

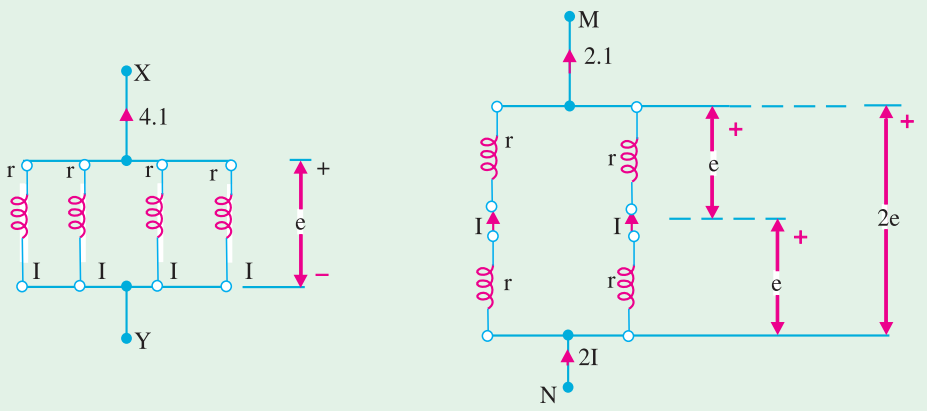
Observation. With same flux per pole, the armature power outputs will be in the proportion of the speeds, as (7.2/8.64) = 5/6).

Further Conclusion. In case of common speed for comparing Electrical Outputs with same machine once lap-wound and next wave-wound, there is no difference in the two cases. Lap-wound machine has lower voltage and higher current while the wave-wound machine has higher voltage and lower current.

Example 26.34. *A 4-pole, Lap-connected d.c. machine has an armature resistance of 0.15 ohm. Find the armature resistance of the machine is rewound for wave-connection.*

(Bharthiar Univ. Nov. 1997)

Solution. A 4-pole lap-winding has 4 parallel paths in armature. If it is rewound for wave-connection, the resistance across the terminal becomes $(4 \times 0.15) = 0.6$ ohm, as it obvious from Fig. 26.67.



4 parallel paths (Lap-winding)
Power rating = $eI \times 4$
Resistance Between X and $Y = r/4$
(*a*) Lap-winding

2 parallel paths (wave-winding)
Power rating = $(2e) \times (2I) = 4eI$
Resistance Between M and $N = r$
(*b*) Wave-winding

Fig. 26.67. Resistances for different methods

Tutorial Problem No. 26.2

1. A 4-pole, d.c. generator has a wave-wound armature with 792 conductors. The flux per pole is 0.0121 Wb. Determine the speed at which it should be run to generate 240 V on no-load. **[751.3 r.p.m.]**

2. A 20 kW compound generator works on full-load with a terminal voltage of 230 V. The armature, series and shunt field resistances are 0.1, 0.05 and 115 Ω respectively. Calculate the generated e.m.f. when the generator is connected short-shunt. **[243.25 V] (*Elect. Engg. Madras Univ. April, 1978*)**

3. A d.c. generator generates an e.m.f. of 520 V. It has 2,000 armature conductors, flux per pole of 0.013 Wb, speed of 1200 r.p.m. and the armature winding has four parallel paths. Find the number of poles.

[4] (*Elect. Technology, Aligarh Univ. 1978*)

4. When driven at 1000 r.p.m. with a flux per pole of 0.02 Wb, a d.c. generator has an e.m.f. of 200 V. If the speed is increased to 1100 r.p.m. and at the same time the flux per pole is reduced to 0.019 Wb per pole, what is then the induced e.m.f. ? **[209 V]**

5. Calculate the flux per pole required on full-load for a 50 kW, 400 V, 8-pole, 600 r.p.m. d.c. shunt generator with 256 conductors arranged in a lap-connected winding. The armature winding resistances is 0.1 Ω, the shunt field resistance is 200 Ω and there is a brush contact voltage drop of 1 V at each brush on full-load. **[0.162 Wb]**

6. Calculate the flux in a 4-pole dynamo with 722 armature conductors generating 500 V when running at 1000 r.p.m. when the armature is (*a*) lap connected (*b*) wave connected.

[(*a*) 41.56 mWb (*b*) 20.78 mWb] (*City & Guilds, London*)

7. A 4-pole machine running at 1500 r.p.m. has an armature with 90 slots and 6 conductors per slot. The flux per pole is 10 mWb. Determine the terminal e.m.f. as d.c. Generator if the coils are lap-connected. If the current per conductor is 100 A, determine the electrical power.

[810 V, 324 kW] (*London Univ.*)

8. An 8-pole lap-wound d.c. generator has 120 slots having 4 conductors per slot. If each conductor can carry 250 A and if flux/pole is 0.05 Wb, calculate the speed of the generator for giving 240 V on open circuit. If the voltage drops to 220 V on full load, find the rated output of the machine.

[600 V, 440 kW]

9. A 110-V shunt generator has a full-load current of 100 A, shunt field resistance of 55 Ω and constant losses of 500 W. If F.L. efficiency is 88%, find armature resistance. Assuming voltage to be constant at 110 V, calculate the efficiency at half F.L. And at 50% overload. Find the load current.

[0.078 Ω ; 85.8% ; 96.2 A]

10. A short-shunt compound d.c. Generator supplies a current of 100 A at a voltage of 220 V. If the resistance of the shunt field is 50 Ω, of the series field 0.025 Ω, of the armature 0.05 Ω, the total brush drop is 2 V and the iron and friction losses amount to 1 kW, find

(*a*) the generated e.m.f. **(*b*)** the copper losses **(*c*)** the output power of the prime-mover driving the generator and (*d*) the generator efficiency.

[(*a*) 229.7 V (*b*) 1.995 kW (*c*) 24.99 kW (*d*) 88%]

11. A 20 kW, 440-V, short-shunt, compound d.c. generator has a full-load efficiency of 87%. If the resistance of the armature and interpoles is 0.4 Ω and that of the series and shunt fields 0.25 Ω and 240 Ω respectively, calculate the combined bearing friction, windage and core-loss of the machine.

[725 W]

12. A long-shunt, compound generator delivers a load current of 50 A at 500 V and the resistances of armature, series field and shunt field are 0.05 ohm and 250 ohm respectively. Calculate the generated electromotive force and the armature current. Allow 1.0 V per brush for contact drop.

[506.2 V ; 52 A] (*Elect. Engg. Banaras Hindu Univ. 1977*)

13. In a 110-V compound generator, the resistances of the armature, shunt and the series windings are 0.06 Ω, 25 Ω and 0.04 Ω respectively. The load consists of 200 lamps each rated at 55 W, 110 V.

Find the total electromotive force and armature current when the machine is connected **(*i*)** long shunt **(*ii*)** short shunt. Ignore armature reaction and brush drop.

[(*a*) 1200.4, 104.4 A (*b*) 120.3 V, 104.6 A] (*Electrical Machines-I, Bombay Univ. 1979*)

14. Armature of a 2-pole, 200-V generator has 400 conductors and runs at 300 r.p.m. Calculate the useful flux per pole. If the number of turns in each field coil is 1200, what is the average value of e.m.f induced in each coil on breaking the field if the flux dies away completely in 0.1 sec ?

(*JNTU, Hyderabad, 2000*)

Hint: Calculate the flux per pole generating 200 V at 300 rpm. Calculate the e.m.f. induced in 1200-turn field coil due to this flux reducing to zero in 0.1 sec, from the rate of change of flux-linkage.

[ϕ = 0.1 Wb, e = 1200 V]

15. A 1500 kW, 550-V, 16 pole generator runs at 150 rev. per min. What must be the useful flux if there are 2500 conductors lap-connected and the full-load copper losses are 25 kW? Calculate the area of the pole shoe if the gap density has a uniform value of 0.9 wb/m^2 and find the no-load terminal voltage, neglecting armature reaction and change in speed.

(Rajiv Gandhi Techn. Univ., Bhopal, 2000) **[0.09944 m^2, 559.17 V]**

OBJECTIVE TESTS – 26

1. The basic requirement of a d.c. armature winding is that it must be
(*a*) a closed one
(*b*) a lap winding
(*c*) a wave winding
(*d*) either (*b*) or (*c*)

2. A wave winding must go at least ..... around the armature before it closes back where it started.
(*a*) once
(*b*) twice
(*c*) thrice
(*d*) four times

3. The d.c. armature winding in which coil sides are a pole pitch apart is called ..... winding.
(*a*) multiplex
(*b*) fractional-pitch
(*c*) full-pitch
(*d*) pole-pitch

4. For making coil span equal to a pole pitch in the armature winding of a d.c. generator, the back pitch of the winding must equal the number of
(*a*) commutator bars per pole
(*b*) winding elements
(*c*) armature conductors per path
(*d*) armature parallel paths.

5. The primary reason for making the coil span of a d.c. armature winding equal to a pole pitch is to
(*a*) obtain a coil span of 180° (electrical)
(*b*) ensure the addition of e.m.fs. of consecutive turns
(*c*) distribute the winding uniformly under different poles
(*d*) obtain a full-pitch winding.

6. In a 4-pole, 35 slot d.c. armature, 180 electrical-degree coil span will be obtained when coils occupy ..... slots.
(*a*) 1 and 10
(*b*) 1 and 9
(*c*) 2 and 11
(*d*) 3 and 12

7. The armature of a d.c. generator has a 2-layer lap-winding housed in 72 slots with six conductors/slot. What is the minimum number of commutator bars required for the armature?
(*a*) 72
(*b*) 432
(*c*) 216
(*d*) 36

8. The sole purpose of a commutator in a d.c. Generator is to
(*a*) increase output voltage
(*b*) reduce sparking at brushes
(*c*) provide smoother output
(*d*) convert the induced a.c. into d.c.

9. For a 4-pole, 2-layer, d.c., lap-winding with 20 slots and one conductor per layer, the number of commutator bars is
(*a*) 80
(*b*) 20
(*c*) 40
(*d*) 160

10. A 4-pole, 12-slot lap-wound d.c. armature has two coil-sides/slot. Assuming single turn coils and progressive winding, the back pitch would be
(*a*) 5
(*b*) 7
(*c*) 3
(*d*) 6

11. If in the case of a certain d.c. armature, the number of commutator segments is found either one less or more than the number of slots, the armature must be having a simplex ..... winding.
(*a*) wave
(*b*) lap
(*c*) frog leg
(*d*) multielement

12. Lap winding is suitable for ..... current, ... voltage d.c. generators.
(*a*) high, low
(*b*) low, high
(*c*) low, low
(*d*) high, high

13. The series field of a short-shunt d.c. generator is excited by ..... currents.
(*a*) shunt
(*b*) armature
(*c*) load
(*d*) external

14. In a d.c. generator, the generated e.m.f. is directly proportional to the
(*a*) field current
(*b*) pole flux
(*c*) number of armature parallel paths
(*d*) number of dummy coils

15. In a 12-pole triplex lap-wound d.c. armature, each conductor can carry a current of 100 A. The rated current of this armature is ..... ampere.
(*a*) 600
(*b*) 1200
(*c*) 2400
(*d*) 3600

16. The commercial efficiency of a shunt generator is maximum when its variable loss equals ..... loss.
(*a*) constant
(*b*) stray
(*c*) iron
(*d*) friction and windage

17. In small d.c. machines, armature slots are sometimes not made axial but are skewed. Though skewing makes winding a little more difficult, yet it results in
(*a*) quieter operation
(*b*) slight decrease in losses
(*c*) saving of copper
(*d*) both (*a*) and (*b*)

18. The critical resistance of the d.c. generator is the resistance of
(*a*) armature
(*b*) field
(*c*) load
(*d*) brushes **(Grad. I.E.T.E Dec. 1985)**

ANSWERS

1. (*a*) **2.** (*b*) **3.** (*c*) **4.** (*a*) **5.** (*b*) **6.** (*b*) **7.** (*c*) **8.** (*d*) **9.** (*b*) **10.** (*b*)
11. (*a*) **12.** (*a*) **13.** (*c*) **14.** (*b*) **15.** (*d*) **16.** (*a*) **17.** (*d*) **18.** (*b*)

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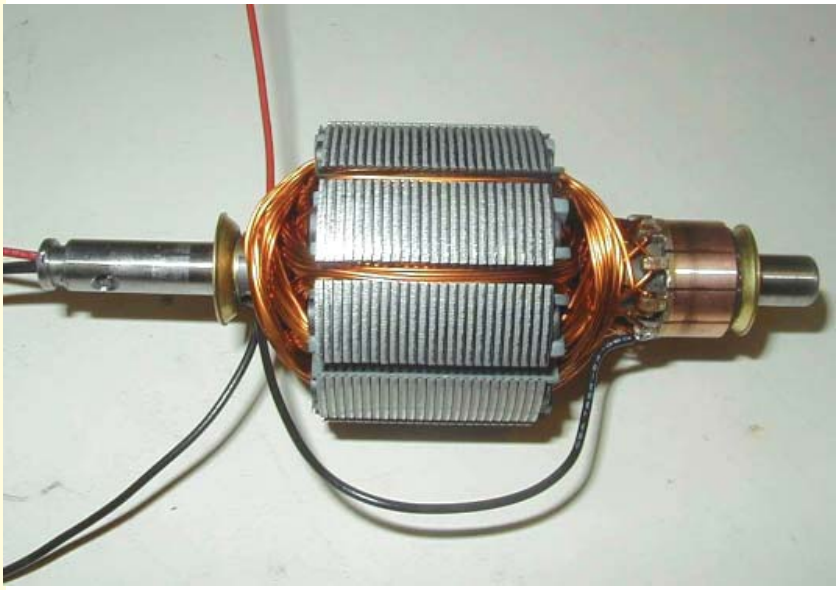
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CHAPTER 27

ARMATURE REACTION AND COMMUTATION

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- Demagnetising and Cross-magnetising Conductors
- Demagnetising AT per Pole
- Cross-magnetising AT per pole
- Compensating Windings
- No. of Compensating Windings
- Commutation
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- Equalising Connections
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Armature reaction is the change in the neutral plane and the reaction of the magnetic field

CONTENTS

27.1 Armature Reaction

By armature reaction is meant the effect of magnetic field set up by armature current on the distribution of flux under main poles of a generator. The armature magnetic field has two effects :

(*i*) ***It demagnetises* or *weakens the main flux*** and

(*ii*) ***It cross-magnetises* or *distorts it.***

The first effect leads to reduced generated voltage and the second to the sparking at the brushes.

These effects are well illustrated in Fig. 27.1 which shows the flux distribution of a bi-polar generator when there is no current in the armature conductors. For convenience, only two poles have been considered, though the following remarks apply to multipolar fields as well. Moreover, the brushes are shown touching the armature conductors directly, although in practice, they touch commutator segments, It is seen that

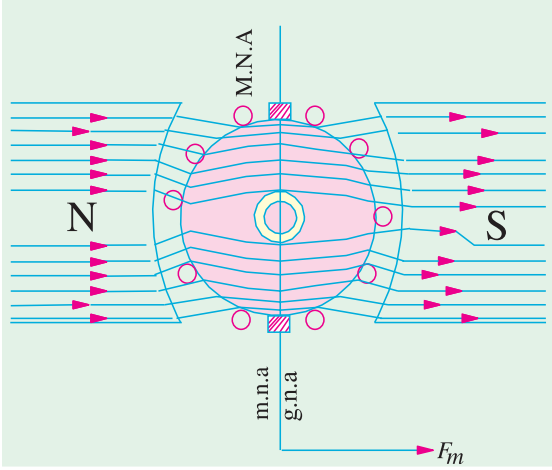


Fig. 27.1

(*a*) the flux is distributed symmetrically with respect to the polar axis, which is the line joining the centres of *NS* poles.

(*b*) The magnetic neutral axis or plane (*M.N.A.*) coincides with the geometrical neutral axis or plane (*G.N.A.*)

Magnetic neutral axis may be defined as the axis along which no e.m.f. is produced in the armature conductors because they then move parallel to the lines of flux.

Or *M.N.A.* is the axis which is perpendicular to the flux passing through the armature.

As hinted in Art. 27.2, brushes are always placed along *M.N.A.* Hence, *M.N.A.* is also called 'axis of commutation' because reversal of current in armature conductors takes place across this axis.

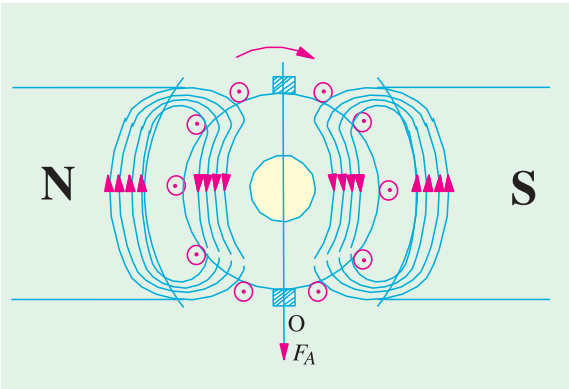


Fig. 27.2

In Fig. 27.1 is shown vector OF_m which represents, both in magnitude and direction, the m.m.f. producing the main flux and also *M.N.A.* which is perpendicular to OF_m.

In Fig. 27.2 is shown the field (or flux) set up by the armature conductors *alone* when carrying current, the field coils being unexcited. The direction of the armature current is the same as it would actually be when the generator is loaded. It may even be found by applying Fleming's Right-hand Rule. The current direction is downwards in conductors under *N*-pole and upwards in those under *S*-pole. The downward flow is represented by crosses and upward flow by dots.

As shown in Fig. 27.2, the m.m.fs. of the armature conductors combine to send flux downwards through the armature. The direction of the lines of force can be found by applying cork-screw rule. The armature m.m.f. (depending on the strength of the armature current) is shown separately both in magnitude and direction by the vector OF_A which is parallel to the brush axis.

So far, we considered the main m.m.f. and armature m.m.f. separately as if they existed independently, which is not the case in practice. Under actual load conditions, the two exist simultaneously in

the generator as shown in Fig. 27.3.

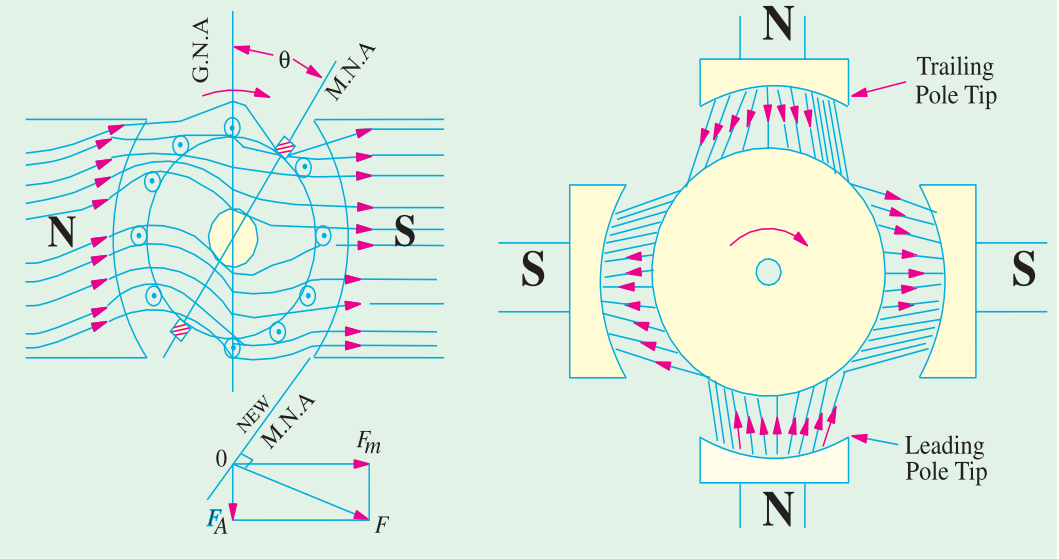
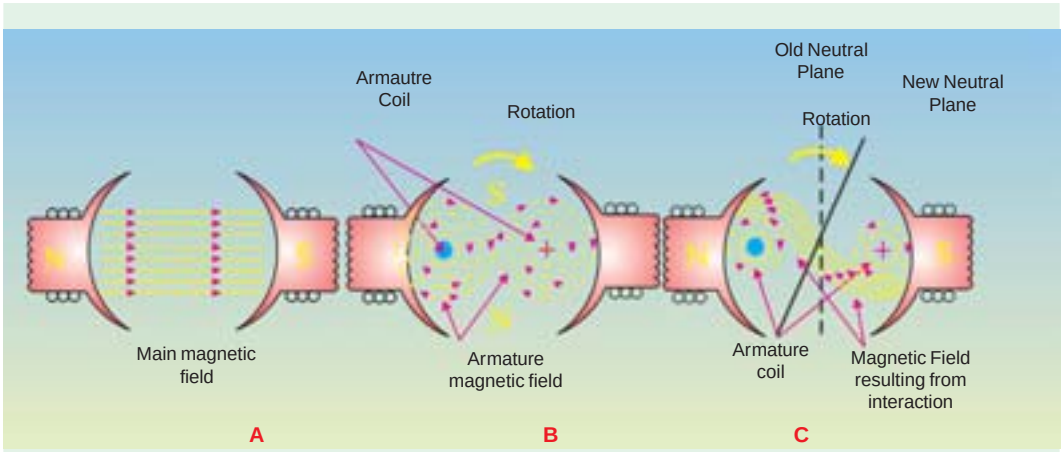


Fig. 27.3 Fig. 27.4

It is seen that the flux through the armature is no longer uniform and symmetrical about the pole axis, rather it has been distorted. The flux is seen to be crowded at the trailing pole tips but weakened or thinned out at the leading pole tips (the pole tip which is first met during rotation by armature conductors is known as the leading pole tip and the other as trailing pole tip). The strengthening and weakening of flux is separately shown for a four-pole machine in Fig. 27.4. As seen, air-gap flux density under one pole half is greater than that under the other pole half.

If Fig. 27.3 is shown the resultant m.m.f. *OF* which is found by vectorially combining OF_m and OF_A.

The new position of *M.N.A.*, which is always perpendicular to the resultant m.m.f. vector *OF*, is also shown in the figure. With the shift of *M.N.A.*, say through an angle θ, brushes are also shifted so as to lie along the new position of *M.N.A.* Due to this brush shift (or forward lead), the armature conductors and hence armature current is redistributed. Some armature conductors which were earlier



Armature reaction

under the influence of *N*-pole come under the influence of *S*-pole and ***vice-versa***. This regrouping is shown in Fig. 27.5, which also shows the flux due to armature conductors. Incidentally, brush position shifts in the *same* direction as the direction of armature rotation.

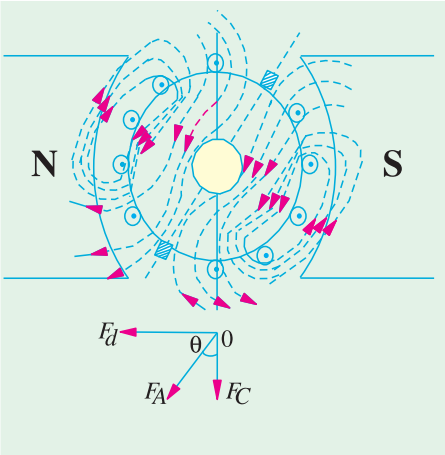


Fig. 27.5

All conductors to the left of new position of *M.N.A.* but between the two brushes, carry current downwards and those to the right carry current upwards. The armature m.m.f. is found to lie in the direction of the new position of *M.N.A.* (or brush axis). The armature m.m.f. is now represented by the vector OF_A which is not vertical (as in Fig 27.2) but is inclined by an angle θ to the left. It can now be resolved into two rectangular components, OF_d parallel to polar axis and OF_C perpendicular to this axis. We find that

(i) component OF_C is at right angles to the vector OF_m (of Fig. 27.1) representing the main m.m.f. It produces distortion in the main field and is hence called the cross-magnetising or distorting component of the armature reaction.

(ii) The component OF_d is in direct opposition of OF_m which represents the main m.m.f. It exerts a demagnetising influence on the main pole flux. Hence, it is called the demagnetising or weakening component of the armature reaction.

It should be noted that both distorting and demagnetising effects will increase with increase in the armature current.

27.2. Demagnetising and Cross-magnetising Conductors

The exact conductors which produce these distorting and demagnetising effects are shown in Fig. 27.6 where the brush axis has been given a forward lead of θ so as to lie along the new position of *M.N.A.* All conductors lying within angles *AOC* = *BOD* = 2θ at the top and bottom of the armature, are carrying current in such a direction as to send the flux *through* the armature from right to left. This fact may be checked by applying crockscrew rule. It is these conductors which act in direct opposition to the main field and are hence called the demagnetising armature conductors.

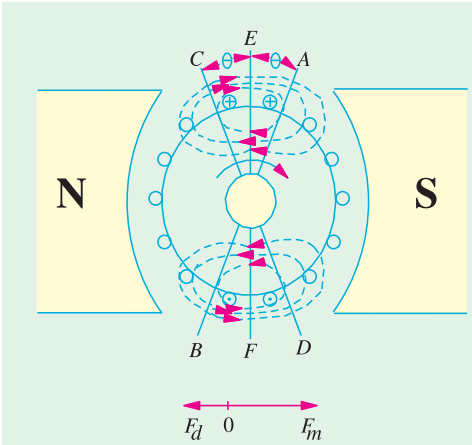


Fig. 27.6

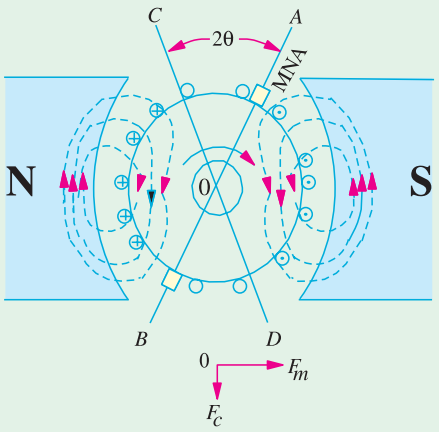


Fig. 27.7

Now consider the remaining armature conductors lying between angles *AOD* and *COB*. As shown in Fig. 27.7, these conductors carry current in such a direction as to produce a combined flux pointing vertically downwards *i.e.* at right angles to the main flux. This results in distortion of the main field. Hence, these conductors are known as cross-magnetising conductors and constitute distorting ampere-conductors.

27.3. Demagnetising AT per Pole

Since armature demagnetising ampere-turns are neutralized by adding extra ampere-turns to the main field winding, it is essential to calculate their number. But before proceeding further, it should be remembered that the number of turns is equal to half the number of conductors because two conductors-constitute one turn.

Let Z = total number of armature conductors

I = current in each armature conductor

$= I_a/2$ ... for simplex wave winding

$= I_a/P$ ... for simplex lap winding

θ_m = forward lead in mechanical or geometrical or angular degrees.

Total number of armature conductors in angles *AOC* and *BOD* is $\frac{4\theta_m}{360} \times Z$

As two conductors constitute one turn,

$\therefore$ Total number of turns in these angles $= \frac{2\theta_m}{360} \times ZI$

$\therefore$ Demagnetising amp-turns per pair of poles $= \frac{2\theta_m}{360} \times ZI$

$\therefore$ Demagnetising amp - turns/pole $= \frac{\theta_m}{360} \times ZI$ $\quad \therefore AT_d$ per pole $= ZI \times \frac{\theta_m}{360}$

27.4. Cross-magnetising AT per pole

The conductors lying between angles *AOD* and *BOC* constitute what are known as distorting or cross-magnetising conductors. Their number is found as under :

Total armature-*conductors*/pole both cross and demagnetising = Z / P

Demagnetising *conductors*/pole $= Z.\frac{2\theta_m}{360}$ (found above)

$\therefore$ Corss-magnetising *conductors*/pole $= \frac{Z}{P} - Z \times \frac{2\theta_m}{360} = Z\left(\frac{1}{P} - \frac{2\theta_m}{360}\right)$

$\therefore$ Cross-magnetising *amp-conductors*/pole $= ZI\left(\frac{1}{P} - \frac{2\theta_m}{360}\right)$

$\therefore$ Corss-magnetising *amp-turns*/pole $= ZI\left(\frac{1}{2P} - \frac{\theta_m}{360}\right)$

(Remembering that two conductors make one turn)

$\therefore$ AT_c/pole $= ZI\left(\frac{1}{2P} - \frac{\theta_m}{360}\right)$

Note. (*i*) For neutralizing the demagnetising effect of armature-reaction, an extra number of turns may be put on each pole.

No. of extra turns/pole $= \frac{AT_d}{I_{sh}}$ –for shunt generator

$= \frac{AT_d}{I_a}$ –for series generator

If the leakage coefficient λ is given, then multiply each of the above expressions by it.

(ii) If lead angle is given in electrical degrees, it should be converted into mechanical degrees by the following relation.

$$\theta \text{ (mechanical)} = \frac{\theta \text{ (electrical)}}{\text{pair of poles}} \text{ or } \theta_m = \frac{\theta_e}{P/2} = \frac{2\theta_e}{P}$$

27.5. Compensating Windings

These are used for large direct current machines which are subjected to large fluctuations in load *i.e.* rolling mill motors and turbo-generators etc. Their function is to neutralize the cross magnetizing effect of armature reaction. In the absence of compensating windings, the flux will be suddenly shifting backward and forward with every change in load. This shifting of flux will induce ***statically*** induced e.m.f. in the armature coils. The magnitude of this e.m.f. will depend upon the rapidity of changes in load and the amount of change. It may be so high as to strike an arc between the consecutive commutator segments across the top of the mica sheets separating them. This may further develop into a flash-over around the whole commutator thereby short-circuiting the whole armature.

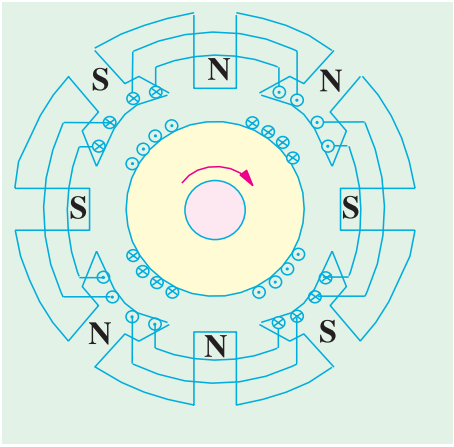
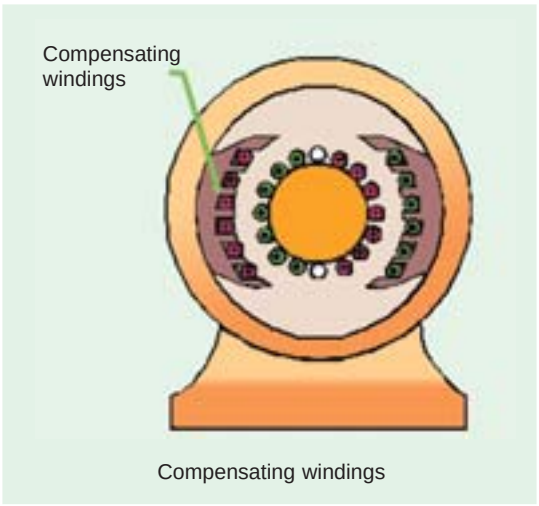


Fig. 27.8



Compensating windings

These windings are embedded in slots in the pole shoes and are connected in series with armature in such a way that the current in them flows in opposite direction to that flowing in armature conductors directly below the pole shoes. An elementary scheme of compensating winding is shown in Fig. 27.8.

It should be carefully noted that compensating winding must provide sufficient m.m.f so as to counterbalance the armature m.m.f. Let

Z_c = No. of compensating conductos/pole face

Z_a = No. of active armature conductors/pole,

I_a = Total armature current

I_a/A = current/armature conductor

$\therefore\ Z_c I_a = Z_a\,(I_a/A)$ or $Z_c = Z_a/A$

Owing to their cost and the room taken up by them, the compensating windings are used in the case of large machines which are subject to violent fluctuations in load and also for generators which have to deliver their full-load output at considerable low induced voltage as in the Ward-Leonard set.

27.6. No. of Compensating Windings

No. of armature *conductors*/pole $= \frac{Z}{P}$ No. of armature *turns*/pole $= \frac{Z}{2P}$

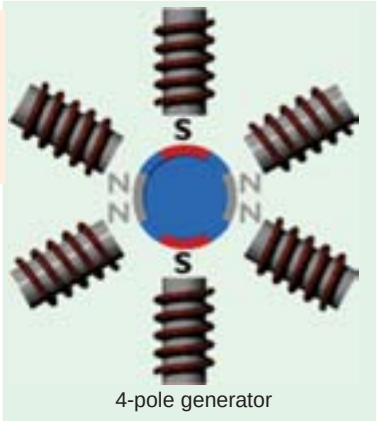
$\therefore$ No. of armature-turns immediately under one pole

$$= \frac{Z}{2P} \times \frac{\text{Pole arc}}{\text{Pole pitch}} = 0.7 \times \frac{Z}{2P} \text{ (approx.)}$$

$\therefore$ No. of armature amp-*turns*/pole for compensating winding

$$= 0.7 \times \frac{Z}{2P} = 0.7 \times \text{armature amp-turns/pole}$$

Example 27.1. *A 4-pole generator has a wave-wound armature with 722 conductors, and it delivers 100 A on full load. If the brush lead is 8°, calculate the armature demagnetising and cross-magnetising ampere turns per pole.* **(Advanced Elect. Machines AMIE Sec. B 1991)**



4-pole generator

Solution. $I = I_a/2 = 100/2 = 50\text{A}$; $Z = 722$; $\theta_m = 8°$

$$\text{AT}_d/\text{pole} = ZI.\frac{\theta_m}{360} = 722 \times 50 \times \frac{8}{360} = 802$$

$$\text{AT}_c/\text{pole} = ZI.\left(\frac{1}{2P} - \frac{\theta_m}{360}\right)$$

$$= 722 \times 50\left(\frac{1}{2 \times 4} - \frac{8}{360}\right) = 37/8$$

Example 27.2 *An 8-pole generator has an output of 200 A at 500 V, the lap-connected armature has 1280 conductors, 160 commutator segments. If the brushes are advanced 4-segments from the no-load neutral axis, estimate the armature demagnetizing and cross-magnetizing ampere-turns per pole.* **(Electrical Machines-I, South Gujarat Univ. 1986)**

Solution. $I = 200/8 = 25$ A, $Z = 1280$, $\theta_m = 4 \times 360/160 = 9°$; $P = 8$

$$\text{AT}_d/\text{pole} = ZI\theta_m/360 = 1280 \times 25 \times 9/360 = \mathbf{800}$$

$$\text{AT}_c/\text{pole} = ZI\left(\frac{1}{2p} - \frac{\theta_m}{360}\right) = 1280 \times 25\left(\frac{1}{2 \times 8} - \frac{9}{360}\right) = \mathbf{1200}$$

Note. No. of coils = 160, No. of conductors = 1280. Hence, each coil side contains 1280/160 = 8 conductors.

Example 27.3(*a*). *A 4-pole wave-wound motor armature has 880 conductors and delivers 120 A. The brushes have been displaced through 3 angular degrees from the geometrical axis. Calculate* ***(a)*** *demagnetising amp-turns/pole* ***(b)*** *cross- magnetising amp-turns/pole* ***(c)*** *the additional field current for neutralizing the demagnetisation of the field winding has 1100 turns/pole.*

Solution. $Z = 880$; $I = 120/2 = 60$ A ; $\theta = 3°$ angular

(a) $\therefore$ $\text{AT}_d = 880 \times 60 \times \frac{3}{360} = \mathbf{440\ AT}$

(b) $\therefore$ $\text{AT}_c = 880 \times 60\left(\frac{1}{8} - \frac{3}{360}\right) = 880 \times \frac{7}{60} \times 60 = \mathbf{6,160}$

or Total AT/pole = $440 \times 60/4 = 6600$

Hence, $\text{AT}_C/\text{pole} = \text{Total AT/pole} - \text{AT}_d/\text{pole} = 6600 - 440 = 6160$

(c) Additional field current = $440/1100 = \mathbf{0.4\ A.}$

Example 27.3(*b*). *A 4-pole lap-wound Generator having 480 armature conductors supplies a current of 150 Amps. If the brushes are given an actual lead of 10°, calculate the demagnetizing and cross-magnetizing amp-turns per pole.* **(Bharathiar Univ. April 1998)**

Solution. 10° mechanical (or actual) shift = 20° electrical shift for a 4-pole machine.

Armature current = 150 amp

For 4-pole lap-wound armature, number of parallel paths = 4. Hence, conductor-current = 150/4 = 37.5 amps.

$$\text{Total armature amp-turns/pole} = \frac{1}{2} \times \frac{(480 \times 37.5)}{14} = 2250$$

$$\text{Cross – magnetizing amp turns/pole} = 2250 \times \left(1 - \frac{2 \times 20°}{180°}\right) = 1750$$

$$\text{Demagnetizing amp turns/pole} = 2250 \times (2 \times 20°/180°) = 500$$

Example 27.4. *A 4-pole generator supplies a current of 143 A. It has 492 armature conductors* ***(a)*** *wave-wound* ***(b)*** *lap-wound. When delivering full load, the brushes are given an actual lead of 10°. Calculate the demagnetising amp-turns/pole. This field winding is shunt connected and takes 10 A. Find the number of extra shunt field turns necessary to neutralize this demagnetisation.*

(Elect. Machines, Nagpur Univ. 1993 & JNTU Hyderabad, 2000 & RGPU, Bhopal, 2000)

Solution. $Z = 492$; $\theta_m = 10°$; AT_d / pole $= Z I \times \frac{\theta_m}{360}$

$I_a = 143 + 10 = 153$ A ; $I = 153/2$ ... when wave-wound

$= 153/4$ ... when lap-wound

(a) $\therefore AT_d$ / pole $= 492 \times \frac{153}{2} \times \frac{10}{360} =$ **1046 AT**

Extra shunt field turns = 1046/10 = **105 (approx.)**

(b) AT_d / pole $= 492 \times \frac{153}{2} \times \frac{10}{360} =$ **523**

Extra shunt field turns = 523/10 = **52 (approx.)**

Example 27.5. *A 4-pole, 50-kW, 250-V wave-wound shunt generator has 400 armature conductors. Brushes are given a lead of 4 commutator segments. Calculate the demagnetisation amp-turns/pole if shunt field resistance is 50 Ω. Also, calculate extra shunt field turns/pole to neutralize the demagnetisation.*

Solution. Load current supplied = 50,000/250 = 200 A

$I_{sh} = 250/50 = 5$ A $\therefore I_a = 200 + 5 = 205$ A

Current in each conductor $I = 205/2$ A

No. of commutator segments $= N/A$ where $A = 2$ ... for wave-winding

$\therefore$ No. of segments $= \frac{400}{2} = 200$; $\theta = \frac{4}{200} \times 360 = \frac{36}{5}$ degrees

$\therefore AT_d$ / pole $= 400 \times \frac{205}{2} \times \frac{36}{5 \times 360} =$ **820 AT**

Extra shunt turns/poles $= \frac{AT_d}{I_{sh}} = \frac{820}{5} =$ **164**

Example 27.6. *Determine per pole the number* ***(i)*** *of cross-magnetising ampere-turns* ***(ii)*** *of back ampere-turns and* ***(iii)*** *of series turns to balance the back ampere-turns in the case of a d.c. generator having the following data.*

500 conductors, total current 200 A, 6 poles, 2-circuit wave winding, angle of lead = 10°, leakage coefficient = 1.3 **(Electrical Machines-I, Bombay University, 1986)**

Solution. Current/path, $I = 200/2 = 100$ A, $\theta = 10°$ (mech), $Z = 500$

(a) AT_c / pole $= ZI\left(\frac{1}{2P} - \frac{\theta_m}{360}\right) = 500 \times 100\left(\frac{1}{2 \times 6} - \frac{10}{360}\right) =$ **2,778**

(b) AT_d / pole $= 500 \times 100 \times 10/360 =$ **1390**

(c) Series turns required to balance the demagnetising ampere-turns are

$$= \lambda \times \frac{AT_d}{I_a} = 1.3 \times \frac{1390}{200} = \mathbf{9}$$

Example 27.7. *A 22.38 kW, 440-V, 4-pole wave-wound d.c. shunt motor has 840 armature conductors and 140 commutator segments. Its full-load efficiency is 88% and the shunt field current is 1.8 A. If brushes are shifted backward through 1.5 segments from the geometrical neutral axis, find the demagnetising and distorting amp-turns/pole.* **(Elect. Engg. Punjab Univ. 1991)**

Solution. The shunt motor is shown diagrammatically in Fig. 27.9.

Motor output = 22,380 W ; $\eta = 0.88$

$\therefore$ Motor input = 22,380/0.88 W

Motor input current $= \dfrac{22,380}{0.88 \times 440} = 57.8$ A

$I_{sh} = 1.8$ A ; $I_a = 57.8 - 1.8 = 56$ A

Current in each conductor = 56/2 = 28 A

$\theta = 1.5 \times 360/140$

$= 27/7$ degrees

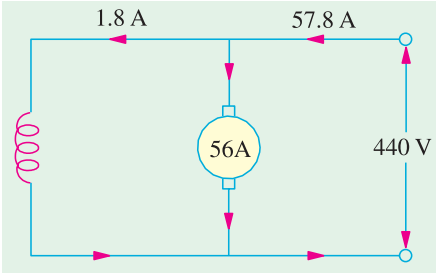


Fig. 27.9

$\therefore$ AT_d / pole $= 840 \times 28 \times \dfrac{27}{7 \times 360} = \mathbf{252}$

AT_c / pole $= ZI\left(\dfrac{1}{2P} - \dfrac{\theta_m}{360}\right) = 840 \times 28\left(\dfrac{1}{8} - \dfrac{27}{7 \times 360}\right) = \mathbf{2,688}$

Example 27.8. *A 400-V, 1000-A, lap-wound d.c. machine has 10 poles and 860 armature conductors. Calculate the number of conductors in the pole face to give full compensation if the pole face covers 70 % of pole span.*

Solution. AT/pole for compensating winding

$$= \text{armature amp-turn/pole} \times \frac{\text{pole arc}}{\text{pole pitch}} = 0.7 \times \frac{ZI}{2P}$$

Here I = current in each armature conductor = 1,000/10 = 100 A

Z = 860 ; P = 10

$\therefore$ AT/pole for compensating winding = $0.7 \times 860 \times 100/2 \times 10 = \mathbf{3,010}$

Tutorial Problem No. 27.1

1. Calculate the demagnetising amp-turns of a 4-pole, lap-wound generator with 720 turns, giving 50 A, if the brush lead is 10° (mechanical). **(250 AT/pole)**

2. A 250-V, 25-kW, 4-pole d.c. generator has 328 wave-connected armature conductors. When the machine is delivering full load, the brushes are given a lead of 7.2 electrical degrees. Calculate the cross-magnetising amp-turns/pole. **(1886, 164)**

3. An 8-pole lap-connected d.c. shunt generator delivers an output of 240 A at 500 V. The armature has 1408 conductors and 160 commutator segments. If the brushes are given a lead of 4 segments from the no-load neutral axis, estimate the demagnetising and cross-magnetising AT/pole.
(1056, 1584) (*Electrical Engineering, Bombay Univ. 1978*)

4. A 500-V, wave-wound, 750 r.p.m. shunt generator supplies a load of 195 A. The armature has 720 conductors and shunt field resistance is 100 Ω. Find the demagnetising amp-turns/pole if brushes are advanced through 3 commutator segments at this load. Also, calculate the extra shunt field turns required to neutralize this demagnetisation. **(600, 4800, 120)**

5. A 4-pole, wave-wound generator has 320 armature conductors and carries an armature current of 400 A. If the pole arc/pole pitch ratio is 0.68, calculate the AT/pole for a compensating winding to give uniform flux density in the air gap. **(5440)**

6. A 500-kW, 500-V, 10 pole d.c. generator has a lap-wound armature with 800 conductors. Calculate the number of pole-face conductors in each pole of a compensating winding if the pole face covers 75 percent of the pitch.

(6 conductors/pole)

7. Three shunt generators, each having an armature resistance of 0.1 ohm are connected across a common bus feeding a two ohms load. Their generated voltages are 127 V, 120 V, and 119 V. Neglecting field currents, calculate the bus voltage and modes of operations of the three machines. **(JNTU, Hyderabad, 200)**

Hint : Solve the circuit from the data given. Since the voltages differ considerably, first machine with 127 V as the generated voltage with supply the largest current.

(I_1 = 70 amp, Generating mode, I_2 = 0, Floating (= neither generating nor motoring). I_3 = –10 amp, motoring mode I_L = 60 amp.)

27.7. Commutation

It was shown in Art 26.2 that currents induced in armature conductors of a d.c. generator are alternating. To make their flow unidirectional in the external circuit, we need a commutator. Moreover, these currents flow in one direction when armature conductors are under *N*-pole and in the opposite direction when they are under *S*-pole. As conductors pass out of the influence of a *N*-pole and enter that of *S*-pole, the current in them is reversed. This reversal of current takes place along magnetic neutral axis or brush axis *i.e.* when the brush spans and hence short-circuits that particular coil undergoing reversal of current through it. This process by which current in the short-circuited coil is reversed while it crosses the *M.N.A.* is called commutation. The brief period during which coil remains short-circuited is known as commutation period T_c.


Commutation

If the current reversal *i.e.* the change from + *I* to zero and then to – *I* is completed by the end of short circuit or commutation period, then the commutation is ideal. If current reversal is not complete by that time, then sparking is produced between the brush and the commutator which results in progressive damage to both.

Let us discuss the process of commutation or current reversal in more detail with the help of Fig. 27.10 where ring winding has been used for simplicity. The brush width is equal to the width of one commutator segment and one mica insulation. In Fig. 27.10 (*a*) coil *B* is about to be short circuited because brush is about to come in touch with commutator segment '*a*'. It is assumed that each coil carries 20 A, so that brush current is 40 A. It is so because every coil meeting at the brush supplies half the brush current lap wound or wave wound. Prior to the beginning of short circuit, coil *B* belongs to the group of coils lying to the left of the brush and carries 20 A from left to right. In Fig. 27.10 (*b*) coil *B* has entered its period of short-circuit and is approximately at one-third of this period. The current through coil *B* has reduced down from 20 A to 10 A because the other 10 A flows *via* segment '*a*'. As area of contact of the brush is more with segment '*b*' than with segment '*a*', it receives 30 A from the former, the total again being 40 A.

Fig. 27.10 (*c*) shows the coil *B* in the middle of its short-circuit period. The current through it has decreased to zero. The two currents of value 20 A each, pass to the brush directly from coil *A* and *C* as shown. The brush contact areas with the two segments '*b*' and '*a*' are equal.

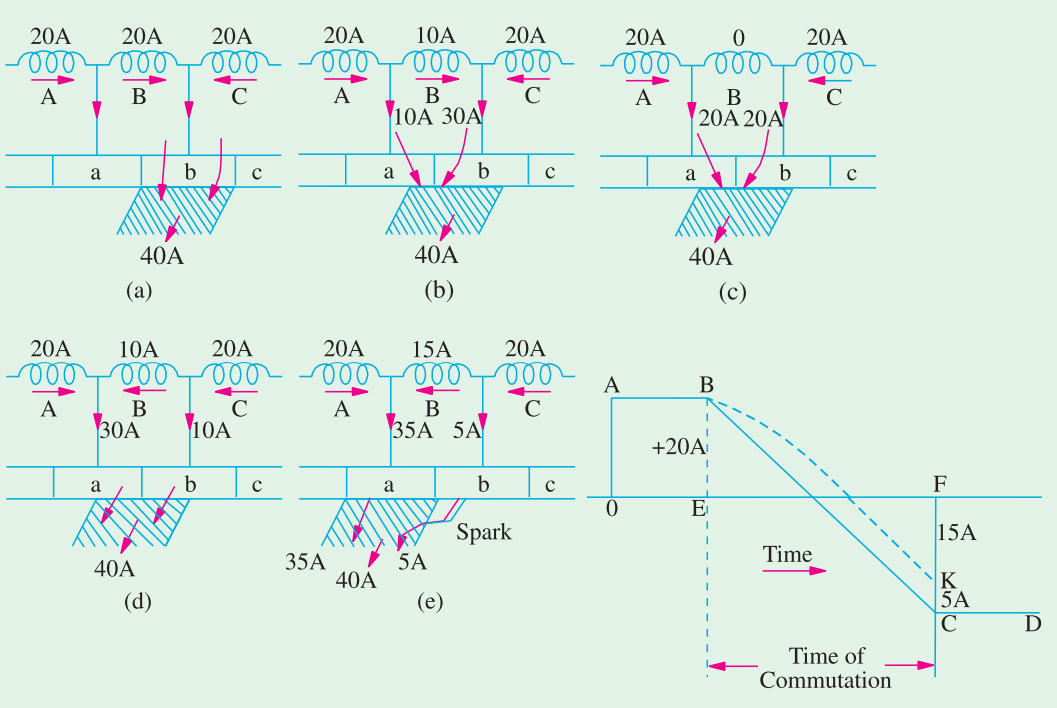


Fig. 27.10 **Fig. 27.11**

In Fig. 27.10 (*d*), coil *B* has become part of the group of coils lying to the right of the brush. It is seen that brush contact area with segment '*b*' is decreasing rapidly whereas that with segment '*a*' is increasing. Coil *B* now carries 10 A in the reverse direction which combines with 20 A supplied by coil *A* to make up 30 A that passes from segment '*a*' to the brush. The other 10 A is supplied by coil *C* and passes from segment '*b*' to the brush, again giving a total of 40 A at the brush.

Fig. 27.10 (*e*) depicts the moment when coil *B* is almost at the end of commutation or short-circuit period. For ideal commutation, current through it should have reversed by now but, as shown, it is carrying 15 A only (instead of 20 A). The difference of current between coils *C* and *B* *i.e.* 20.15 = 5 A in this case, jumps directly from segment *b* to the brush through air thus producing spark.

If the changes of current through coil *B* are plotted on a time base (as in Fig. 27.11) it will be represented by a horizontal line *AB* *i.e.* a constant current of 20 A up to the time of beginning of commutation. From the finish of commutation, the current will be represented by another horizontal line *CD*. Now, again the current value is *FC* = 20 A, although in the reversed direction. The way in which current changes from its positive value of 20 A (= *BE*) to zero and then to its negative value of 20 A (= *CF*) depends on the conditions under which the coil *B* undergoes commutation. If the current varies at a uniform rate *i.e.* if *BC* is a straight line, then it is referred to as linear commutation. However, due to the production of self-induced e.m.f. in the coil (discussed below) the variations follow the dotted curve. It is seen that, in that case, current in coil *B* has reached only a value of *KF* = 15 A in the reversed direction, hence the difference of 5 A (20 A – 15 A) passes as a spark.

So, we conclude that sparking at the brushes, which results in poor commutation is due to the inability of the current in the short-circuited coil to reverse completely by the end of short-circuit period (which is usually of the order of 1/500 second).

At this stage, the reader might ask for the reasons which make this current reversal impossibly in the specified period *i.e.* what factors stand in the way of our achieving ideal commutation. The main cause which retards or delays this quick reversal is the production of self-induced e.m.f. in the coil undergoing commutation. It may be pointed out that the coil possesses appreciable amount of self inductance because it lies embedded in the armature which is built up of a material of high magnetic permeability. This self-induced e.m.f. is known as ***reactance voltage*** whose value is found as given below. This voltage, even though of a small magnitude, produces a large current through the coil whose resistance is very low due to short circuit. It should be noted that if the brushes are set so that the coils undergoing short-circuit are in the magnetic neutral plane, where they are cutting no flux and hence have no e.m.f. induced in them due to armature rotation, there will still be the e.m.f. of self-induction which causes severe sparking at the brushes.

27.8. Value of Reactance Voltage

Reactance voltage = coefficient of self-inductance × rate of change of current.

It should be remembered that the time of short-circuit or commutation is the time required by the commutator to move a distance equal to the circumferential thickness of the brush minus the thickness of one insulating plate of strip of mica.

Let W_b = brush width in cm; W_m = width of mica insulation in cm

v = peripheral velocity of commutator segments in cm/second

Then T_c = time of commutation or short-circuit = $\frac{W_b - W_m}{v}$ second

Note. If brush width etc. are given in terms of commutator segments, then commutator velocity should also be converted in terms of commutator segments per second.

If I is the current through a conductor, then total change during commutation = $I - (-I) = 2I$.

$\therefore$ Self-induced or reactance voltage

$= L \times \frac{2I}{T_c}$ – if commutation is linear

$= 1.11\ L \times \frac{2I}{T_c}$ – if commutation is sinusodial

As said earlier, the reactance e.m.f. hinders the reversal of current. This means that there would be sparking at the brushes due to the failure of the current in short-circuited coil to reach its full value in the reversed direction by the end of short-circuit. This sparking will not only damage the brush and the commutator but this being a cumulative process, it may worsen and eventually lead to the short-circuit of the whole machine by the formation of an arc round the commutator from brush to brush.

Example 27.9. *The armature of a certain dynamo runs at 800 r.p.m. The commutator consists of 123 segments and the thickness of each brush is such that the brush spans three segments. Find the time during which the coil of an armature remains short-circuited.*

Solution. As W_m is not given, it is considered negligible.

W_b = 3 segments and $\nu = (800/60) \times 123$ segments/second

$\therefore$ commutation time = $\frac{3 \times 60}{800 \times 123}$ = 0.00183 second = **1.83 millisecond**

Example 27.10. *A 4-pole, wave-wound, d.c. machine running at 1500 r.p.m. has a commutator of 30 cm diameter. If armature current is 150 A, thickness of brush 1.25 cm and the self-inductance of each armature coil is 0.07 mH, calculate the average e.m.f. induced in each coil during commutation. Assume linear commutation.*

Solution. Formula : $E = L \dfrac{2I}{T_c}$

Here $L = 0.07 \times 10^{-3}$ H, $I = 150/2 = 75$ A (It is wave-wound)

$W_b = 1.25$ cm, $W_m = 0$ ...considered negligible

$\nu = \pi \times 30 \times (1500/60) = 2356$ cm/s ; $T_c = W_b/\nu = 1.25/2356 = 5.3 \times 10^{-4}$ second

$E = L \times 2I/T_c = 0.07 \times 10^{-3} \times 2 \times 75/5.3 \times 10^{-4} =$ **19.8 V**

Example 27.11. *Calculate the reactance voltage for a machine having the following particulars. Number of commutator segments = 55, Revolutions per minute = 900, Brush width in commutator segments = 1.74, Coefficient of self-induction = 153×10^{-6} henry, Current per coil = 27 A.*

(Advanced Elect. Machines. AMIE Sec. Winter 1991)

Solution. Current per coil, $I = 27$ A ; $L = 153 \times 10^{-6}$ H

$\nu = 55 \times (900/60) = 825$ segments/second; $T_c = W_b.\nu = 1.74/825 = 2.11 \times 10^{-3}$ second

Assuming linear commutation, $E = L \times 2I/T_C$

$\therefore \quad E = 153 \times 10^{-6} \times 2 \times 27/2.11 \times 10^{-3} =$ **3.91 V**

Example 27.12. *A 4-pole, lap-wound armature running at 1500 r.p.m. delivers a current of 150 A and has 64 commutator segments. The brush spans 1.2 segments and inductance of each armature coil is 0.05 mH. Calculate the value of reactance voltage assuming* ***(i)*** *linear commutation* ***(ii)*** *sinusoidal commutation. Neglect mica thickness.*

Solution. Formula : $E = L \cdot \dfrac{2I}{T_c}$ Now, $L = 0.05 \times 10^{-3}$ H; $W_b = 1.2$ segments

$$\nu = \frac{1500}{60} \times 64 = 1600 \text{ segments/second}$$

$$\therefore \quad T_c = \frac{1.2}{1600} = 7.5 \times 10^{-4} \text{ second} ; I = \frac{150}{4} \text{ A} = 37.5 \text{ A}$$

$$\therefore \quad \frac{2I}{T_c} = \frac{2 \times 37.5}{7.5 \times 10^{-4}} = 10^5 \text{ A/s}$$

For linear commutation, $E = 0.05 \times 10^{-3} \times 10^5 =$ **5 V**

For sinusoidal commutation, $E = 1.11 \times 5 =$ **5.55 V**

27.9. Methods of Improving Commutation

There are two practical ways of improving commutation *i.e.* of making current reversal in the short-circuited coil as sparkless as possible. These methods are known as ***(i)*** resistance commutation and ***(ii)*** e.m.f. commutation (which is done with the help of either brush lead or interpoles, usually the later).

27.10. Resistance Commutation

This method of improving commutation consists of replacing low-resistance Cu brushes by comparatively high-resistance carbon brushes.

From Fig. 27.12, it is seen that when current I from coil' C reaches the commutator segment b, it has two parallel paths open to it. The first part is straight from bar 'b' to the brush and the other parallel path is ***via*** the short-circuited coil B to bar 'a' and then to the brush. If the Cu brushes (which have low contact resistance) are used, then there is no inducement for the current to follow the sec-

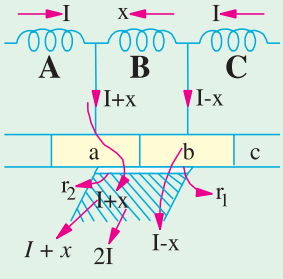


Fig. 27.12

ond longer path, it would preferably follow the first path. But when carbon brushes having high resistance are used, then current I coming from C will prefer to pass through the second path because **(*i*)** the resistance r_1 of the first path will increase due to the diminishing area of contact of bar 'b' with the brush and because **(*ii*)** resistance r_2 of second path will decrease due to rapidly increasing contact area of bar 'a' with the brush.

Hence, carbon brushes have, usually, replaced Cu brushes. However, it should be clearly understood that the main cause of sparking commutation is the self-induced e.m.f. (*i.e.* reactance voltage), so brushes alone do not give a sparkless commutation; though they do help in obtaining it.

The additional advantages of carbon brushes are that **(*i*)** they are to some degree self-lubricating and polish the commutator and **(*ii*)** should sparking occur, they would damage the commutator less than when Cu brushes are used.

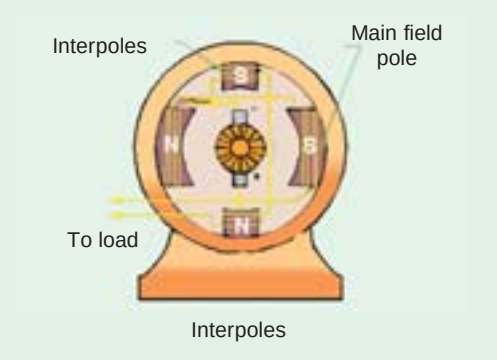
But some of their minor disadvantages are : **(*i*)** Due to their high contact resistance (which is beneficial to sparkless commutation) a loss of approximately 2 volt is caused. Hence, they are not much suitable for small machines where this voltage forms an appreciable percentage loss. **(*ii*)** Owing to this large loss, the commutator has to be made some what larger than with Cu brushes in order to dissipate heat efficiently without greater rise of temperature. **(*iii*)** because of their lower current density (about 7-8 A/cm^2 as compared to 25-30 A/cm^2 for Cu brushes) they need larger brush holders.

27.11. E.M.F. Commutation

In this method, arrangement is made to neutralize the reactance voltage by producing a reversing e.m.f. in the short-circuited coil under commutation. This reversing e.m.f., as the name shows, is an e.m.f. in opposition to the reactance voltage and if its value is made equal to the latter, it will completely wipe it off, thereby producing quick reversal of current in the short-circuited coil which will result in sparkless commutation. The reversing e.m.f. may be produced in two ways : **(*i*)** either by giving the brushes a forward lead sufficient enough to bring the short-circuited coil under the influence of next pole of opposite polarity or **(*ii*)** by using interpoles.

The first method was used in the early machines but has now been abandoned due to many other difficulties it brings along with.

27.12. Interpoles of Compoles



These are small poles fixed to the yoke and spaced in between the main poles. They are wound with comparatively few heavy gauge Cu wire turns and are connected in series with the armature so that they carry full armature current. Their polarity, in the case of a generator, is the same as that of the main pole ***ahead*** in the direction of rotation (Fig. 25.13).

The function of interpoles is two-fold :

(*i*) As their polarity is the same as that of the main pole ahead, they induce an e.m.f. in the coil (under commutation) which helps the reversal of current. The e.m.f. induced by the compoles is known as ***commutating* or *reversing e.m.f.*** The commutating e.m.f. neutralizes the reactance e.m.f. thereby making commutation sparkless. With interpoles, sparkless commutation can be obtained up to 20 to 30% overload with fixed brush position. In fact, interpoles raise sparking limit of a machine to *almost* the same value as heating limit. Hence, for a given output, an interpole machine can be made smaller and, therefore, cheaper than a non-interpolar machine.

As interpoles carry armature current, their commutating e.m.f. is proportional to the armature current. ***This ensures automatic neutralization of reactance voltage which is also due to armature current.*** Connections for a shunt generator with interpoles are shown in Fig. 27.14.

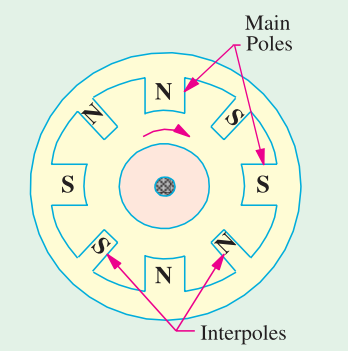


Fig. 27.13

(ii) Another function of the interpoles is to neutralize the cross-magnetising effect of armature reaction. Hence, brushes are not to be shifted from the original position. In Fig 27.15, *OF* as before, represents the m.m.f. due to main poles. *OA* represents the cross-magnetising m.m.f. due to armature. *BC* which represents m.m.f. due to interpoles, is obviously in opposition to *OA*, hence they cancel each other out. ***This cancellation of cross-magnetisation is automatic and for all loads because both are produced by the same armature current.***

The distinction between the interpoles and compensating windings should be clearly understood. Both are connected in series and thier m.m.fs. are such as to neutralize armature reaction. But compoles additionally supply m.m.f. for counteracting the reactance voltage induced in the coil undergoing commutation. Moreover, the action of the compoles is localized, they have negligible effect on the armature reaction occurring on the remainder of the armature periphery.

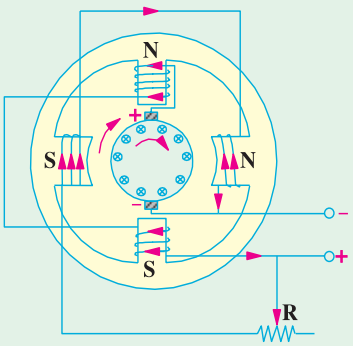


Fig. 27.14

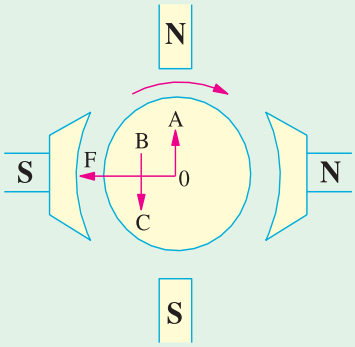


Fig. 27.15

Example 27.13. *Determine the number of turns on each commutating pole of a 6-pole machine, if the flux density in the air-gap of the commutating pole = 0.5 Wb/m^2 at full load and the effective length of the air-gap is 4 mm. The full-load current is 500 A and the armature is lap-wound with 540 conductors. Assume the ampere turns required for the remainder of the magnetic circuit to be one-tenth of that the air gap.* **(Advanced Elect. Machines AMIE Sec.B, 1991)**

Solution. It should be kept in mind that compole winding must be sufficient to oppose the armature m.m.f. (which is directed along compole axis) and to provide the m.m.f. for compole air-gap and its magnetic circuit.

$\therefore \quad N_{CP} I_a = ZI_C/2P + B_g l_g/\mu_0$

where N_{CP} = No. of turns on the compole ; I_a = Armature current

Z = No. of armature conductors ; I_c = Coil current

P = No. of poles ; l_g = Air-gap length under the compole

$Z = 540$; $I_a = 500$ A; $I_c = 500/6$ A; P = 6

$\therefore$ Arm. m.m.f. $= 540 \times (500/6)/2 \times 6 = 3750$

Compole air-gap m.m.f. = $B_g \times l_g/\mu_0 = 0.5 \times 4 \times 10^{-3}/4\pi \times 10^{-7} = 1591$

m.m.f. reqd. for the rest of the magnetic circuit = 10% of 1591 = 159

$\therefore$ Total compole air-gap m.m.f. = 1591 + 159 = 1750

Total m.m.f. reqd. = 3750 + 1750 = 5500

$\therefore$ $N_{cp}\, I_a = 5500$ *or* N_{cp} = 5500/500 = **11**

27.13. Equalizing Connections

It is characteristic of lap -winding that all conductors in any parallel path lie under one pair of poles. If fluxes from all poles are exactly the same, then e.m.f. induced in each parallel path is the same and each path carries the same current. But despite best efforts, some inequalities in flux inevitably occur due either to slight variations in air-gap length or in the magnetic properties of steel. Hence, there is always a slight imbalance of e.m.f. in the various parallel paths. The result is that conductors under stronger poles generate greater e.m.f. and hence carry larger current. The current distribution at the brushes becomes unequal. Some brushes are overloaded *i.e.* carry more than their normal current whereas others carry less. Overloaded brushes spark badly whatever their position may be. This results in poor commutation and may even limit the output of the machine.

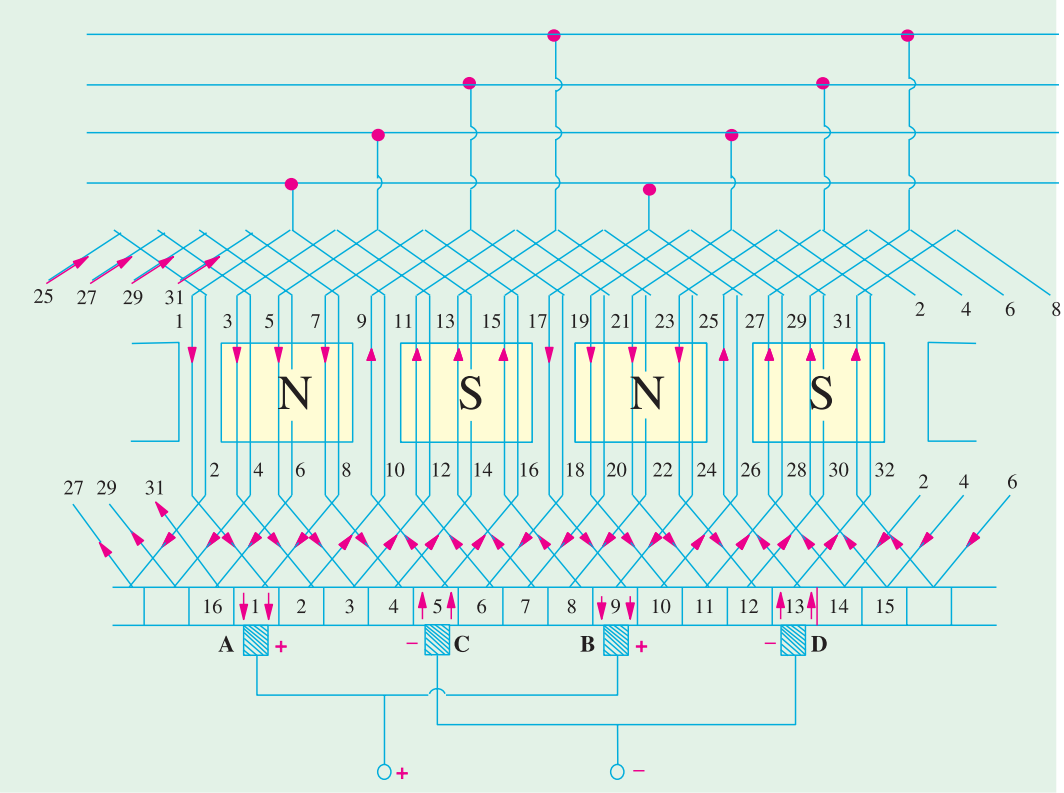


Fig. 27.16

By connecting together a number of symmetrical points on armature winding which would be at equal potential if the pole fluxes were equal, the difference in brush currents is diminished. This requires that there should be a whole number of slots per pair of poles so that, for example, if there is a slot under the centre of a *N*-pole, at some instant, then there would be one slot under the centre of every other *N*-pole. The equalizing conductors, which are in the form of Cu rings at the armature back and which connect such points are called *Equalizer Rings.* The circulating current due to the

slight difference in the e.m.fs. of various parallel paths, passes through these *equalizer* rings instead of passing through the brushes.

Hence, the function of equalizer rings is to avoid unequal distribution of current at the brushes thereby helping to get sparkless commutation.


Equalizer rings

One equalizer ring is connected to all conductors in the armature which are two poles apart (Fig. 27.17). For example, if the number of poles is 6, then the number of connections for each equalizer ring is 3 *i.e.* equal to the number of pair of poles. Maximum number of equalizer rings is equal to the number of conductors under one pair of poles. Hence, number of rings is

$$= \frac{\text{No. of conductors}}{\text{No. of pair of poles}}$$

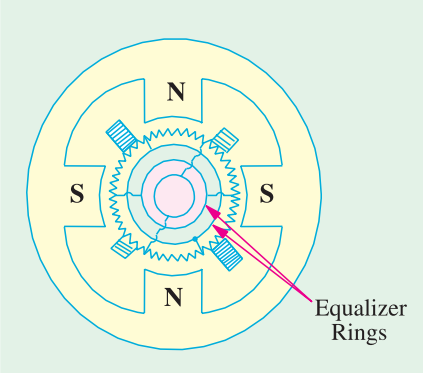


Fig. 27.17

In practice, however, the number of rings is limited to 20 on the largest machines and less on smaller machines. In Fig. 27.16 is shown a developed armature winding. Here, only 4 equalizing bars have been used. It will be seen that the number of equalizing connections to each bar is two *i.e.* half the number of poles. Each alternate coil has been connected to the bar. In this case, the winding is said to be 50% equalized. If all conductors were connected to the equalizer rings, then the winding would have been 100% equalized.

Equalizer rings are not used in wave-wound armatures, because there is no imbalance in the e.m.fs. of the two parallel paths. This is due to the fact that armature conductors in either parallel path are not confined under one pair of poles as in lap-winding but are distributed under all poles. Hence, even if there are inequalities in the pole flux, they will affect each path equally.

27.14. Parallel Operation of Shunt Generators

Power plants, whether in d.c. or a.c. stations, will be generally found to have several smaller generators running in parallel rather than large single units capable of supplying the maximum peak load. These smaller units can be run single or in various parallel combinations to suit the actual load demand. Such practice is considered extremely desirable for the following reasons :

(*i*) Continuity of Service

Continuity of service is one of the most important requirements of any electrical apparatus. This would be impossible if the power plant consisted only of a single unit, because in the event of breakdown of the prime mover or the generator itself, the entire station will be shut down. In recent years, the requirement of uninterrupted service has become so important especially in factories etc. that it is now recognized as an economic necessity.

(*ii*) Efficiency

Usually, the load on the electrical power plant fluctuates between its peak value sometimes during the day and its minimum value during the late night hours. Since generators operate most efficiently when delivering full load, it is economical to use a single small unit when the load is light. Then, as the load demand increases, a larger generator can be substituted for the smaller one or another smaller unit can be connected to run in parallel with the one already in operation.

(*iii*) Maintenance and Repair

It is considered a good practice to inspect generators carefully and periodically to forestall any possibility of failure or breakdown. This is possible only when the generator is at rest which means that there must be other generators to take care of the load. Moreover, when the generator does actually breakdown, it can be repaired with more care and not in a rush, provided there are other generators available to maintain service.

(*iv*) Additions to Plant

Additions to power plants are frequently made in order to deliver increasingly greater loads. Provision for future extension is, in fact, made by the design engineers fight from the beginning. It becomes easy to add other generators for parallel operation as the load demand increases.

27.15. Paralleling DC Generator

Whenever generators are in parallel, their +ve and –ve terminals are respectively connected to the +ve and –ve sides of the bus-bars. These bus-bars are heavy thick copper bars and they act as +ve and –ve terminals for the whole power station. If polarity of the incoming generator is not the same as the line polarity, as serious short-circuit will occur when S_1, is closed.

Moreover, paralleling a generator with reverse polarity effectively short-circuits it and results in damaged brushes, a damaged commutator and a blacked-out plant. Generators that have been tripped off the bus-because of a heavy fault current should always be checked for reversed polarity before paralleling.

In Fig. 27.18 is shown a shunt generator No. 1 connected across the bus-bars *BB* and supplying some of the load. For putting generator No. 2 in parallel with it the following procedure is adopted.

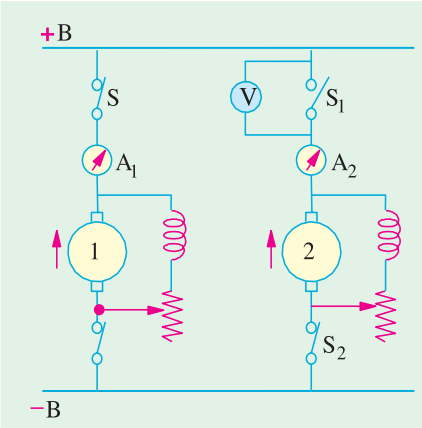


Fig. 27.18

The armature of generator No. 2 is speeded by the prime-mover up to its rated value and then switch S_2 is closed and circuit is completed by putting a voltmeter V across the open switch S_1. The excitation of the incoming generator No. 2 is changed till V reads zero. Then it means that its terminal voltage is the same as that of generator No. 1 or bus-bar voltage. After this, switch S_1 is closed and so the incoming machine is paralleled to the system. Under these conditions, however, generator No. 2 is not taking any load, because its ***induced*** e.m.f. is the same as bus-bar voltage and there can be no flow of current between two points at the same potential. The generator is said to be 'floating' on the bus-bar. If generator No. 2 is to deliver any current, then its induced e.m.f. E should be greater than the bus-bar voltage V. In that case, current supplied by it is $I = (E - V)/R_a$ where R_a is the resistance of the armature circuit. The induced e.m.f. of the incoming generator is increased by strengthening its field till it takes its proper share of load. At the same time, it may be found necessary to weaken the field of generator No. 1 to maintain the bus-bar voltage V constant.

27.16. Load Sharing

Because of their slightly drooping voltage characteristics, shunt generators are most suited for stable parallel operation. Their satisfactory operation is due to the fact that any tendency on the part of a generator to take more or less than its proper share of load results in certain changes of voltage in the system which immediately oppose this tendency thereby restoring the original division of load. Hence, once paralleled, they are automatically held in parallel.

Similarly, for taking a generator out of service, its field is weakened and that of the other generator is increased till the ammeter of the generator to be cleared reads zero. After that, its breaker and then the switch are opened thus removing the generator out of service. This method of connecting in and removing a generator from service helps in preventing any shock or sudden disturbance to the prime-mover or to the system itself.

It is obvious that if the field of one generator is weakened too much, then power will be delivered to it and it will run in its original direction as a motor, thus driving its prime-mover.

In Fig. 27.19. and 27.20 are shown the voltage characteristics of two shunt generators. It is seen that for a common terminal voltage V, the generator No. 1 delivers I_1 amperes and generator No. 2, I_2 amperes. It is seen that generator No. 1, having more drooping characteristic, delivers less current. It is found that two shunt generators will divide the load properly at all points if their characteristics are similar in form and each has the same voltage drop from no-load to full-load.

If it desired that two generators of different kW ratings automatically share a load in proportion to their ratings, then their external characteristics when plotted in terms of their percentage full-load currents (not actual currents) must be identical as shown in Fig. 27.21. If, for example, a 100-kW generator is working in parallel with a 200-kW generator to supply a total of 240-kW, then first generator will supply 80 kW and the other 160 kW.

When the individual characteristics of the generators are known, their combined characteristics can be drawn by adding the separate currents at a number of equal voltage (because generators are running in parallel). From this combined characteristic, the voltage for any combined load can be read off and from there, the current supplies by each generator can be found (Fig. 27.20).

If the generators have straight line characteristics, then the above result can be obtained by simple calculations instead of graphically.

Let us discuss the load sharing of two generators which have unequal no-load voltages.

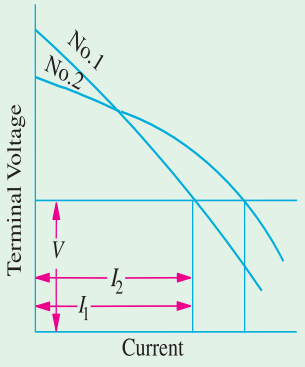


Fig. 27.19

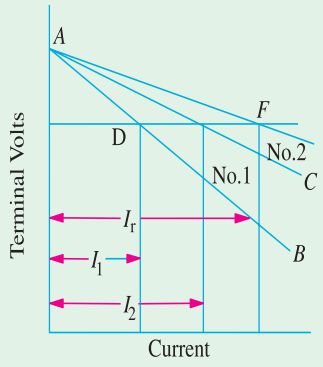


Fig. 27.20

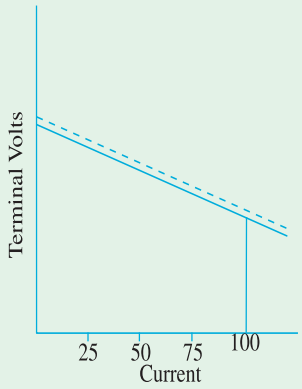


Fig. 27.21

Let E_1, E_2 = no-load voltages of the two generators
R_1, R_2 = their armature resistances
V = common terminal voltage

Then $$I_1 = \frac{E_1 - V}{R_1} \text{ and } I_2 = \frac{E_2 - V}{R_2}$$

$\therefore$ $$\frac{I_2}{I_1} = \frac{E_2 - V}{E_1 - V} \cdot \frac{R_1}{R_2} = \frac{K_2 N_2 \Phi_2 - V}{K_1 N_1 \Phi_1 - V} \cdot \frac{R_1}{R_2}$$

From the above equation, it is clear that bus-bar voltage can be kept constant (and load can be transferred from 1 to 2) by increasing Φ_2 or N_2 or by reducing N_1 and Φ_1.N_2 and N_1 are changed by changing the speed of driving engines and Φ_1 and Φ_2 are changed with the help of regulating shunt field resistances.

It should be kept in mind that

(i) Two parallel shunt generators having equal no-load voltages share the load in such a ratio that the load current of each machine produces the same drop in each generator.

(ii) In the case of two parallel generators having unequal no-load voltages, the load currents produce sufficient voltage drops in each so as to keep their terminal voltage the same.

(iii) The generator with the least drop assumes greater share of the change in bus load.

(iv) Paralleled generators with different power ratings but the same voltage regulation will divide any oncoming bus load in direct proportion to their respective power ratings (Ex. 27.14).

27.17. Procedure for Paralleling D.C. Generators

(i) Close the disconnect switch of the incoming generator

(ii) Start the prime-mover and adjust it to the rated speed of the machine

(iii) Adjust the voltage of the incoming machine a few volts higher than the bus voltage

(iv) Close the breaker of the incoming generator

(v) Turn the shunt field rheostat of the incoming machine in the raise-voltage direction and that of the other machine(s) already connected to the bus in the lower-voltage direction till the desired load distribution (as indicated by the ammeters) is achieved.

27.18. Compound Generators in Parallel

In Fig. 27.22 are shown two compound generators (designated as No. 1 and No. 2) running in parallel. Because of the rising characteristics of the usual compounded generators, it is obvious that in the absence of any corrective devices, the parallel operation of such generators is unstable. Let us suppose that, to begin with, each generator is taking its proper share of load. Let us now assume that for some reason, generator No. 1 takes a slightly increased load. In that case, the current passing through its series winding increases which further strengthens its field and so raises its generated e.m.f. thus causing it to take still more load. Since the system load is assumed to be constant, generator No. 2 will drop some of its load, thereby weakening its series field which will result in its further dropping off its load. Since this effect is cumulative. Generator No. 1 will, therefore, tend to take the entire load and finally drive generator No. 2 as a motor. The circuit breaker of at least one of the two generators will open, thus stopping their parallel operation.

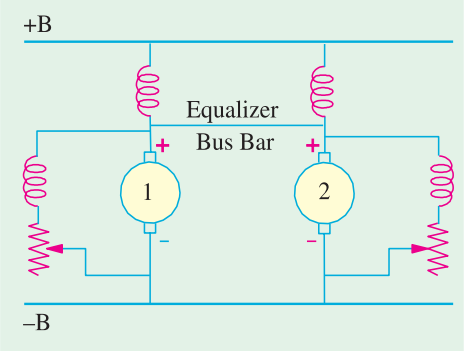


Fig. 27.22

For making the parallel operation of over-compound and level-compound generators stable,* they are always used with an equalizer bar (Fig. 27.22) connected to the armature ends of the series coils of the generators. The equalizer bar is a conductor of low resistance and its operation is as follows :

* Like shunt generators, the under-compound generators also do not need equalizers for satisfactory parallel operation.

Suppose that generator No. 1 starts taking more than its proper share of load. Its series field current is increased. But now this increased current passes partly through the series field coil of generator No. 1 and partly it flows via the equalizer bar through the series field winding of generator No. 2. Hence, the generators are affected in a similar manner with the result the generator No. 1 cannot take the entire load. For maintaining proper division of load from no-load to full-load, it is essential that

(i) the regulation of each generator is the same.

(ii) the series field resistances are inversely proportional to the generator rating.

27.19. Series Generators in Parallel

Fig 27.23 shows two identical series generators connected in parallel. Suppose E_1 and E_2 are initially equal, generators supply equal currents and have equal shunt resistances. Suppose E_1 increases slightly so that $E_1 > E_2$. In that case, I_1 becomes greater than I_2. Consequently, field of machine 1 is strengthened thus increasing E_1 further whilst the field of machine 2 is weakened thus decreasing E_2 further. A final stage is reached when machine 1 supplies not only the whole load but also supplies power to machine 2 which starts running as a motor. Obviously, the two machines will form a short-circuited loop and the current will rise indefinitely. This condition can be prevented by using equalizing bar because of which two similar machines pass approximately equal currents to the load, the slight difference between the two currents being confined to the loop made by the armatures and the equalizer bar.

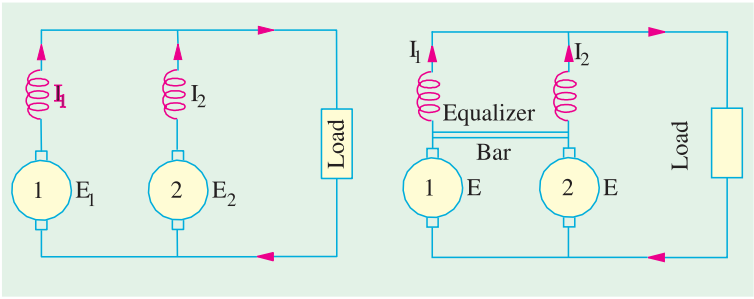


Fig. 27.23

Example 27.14. *A 100-kW, 250-V generator is paralleled with a 300 kW, 250-V generator. Both generators have the same voltage regulation. The first generator is supplying a current of 200 A and the other 500 A. What would be the current supplied by each generator if an additional load of 600 A is connected to the bus ?*

Solution. As explained in Art. 27.17, this additional load would be divided in direct proportion to the respective power ratings of the two generators.

$$\Delta I_1 = \left(\frac{100}{100+300}\right) \times 600 = 150 \text{ A};$$

$$\Delta I_2 = \left(\frac{300}{100+300}\right) \times 600 = 450 \text{ A}$$

Example 27.15. *Two 220–V, d.c. generators, each having linear external characteristics, operate in parallel. One machine has a terminal voltage of 270 V on no-load and 220 V at a load current of 35 A, while the other has a voltage of 280 V at no-load and 220 V at 50 A. Calculate the output current of each machine and the bus-bar voltage when the total load is 60 A. What is the kW output of each machine under this condition ?*

Solution. Generator No. 1.

Voltage drop for 35 A = 270–220 = 50 V

∴ Voltage drop/ampere = 50/35 = 10/7 V/A

Generator No. 2

Voltage drop/ampere = (280–220)/50 = 1.2 V/A

Let V = bus-bar voltage

I_1 = current output of generator No.1

I_2 = current output of generator No.2

then V = 270–(10/7) I_1 ...for generator No. 1

= 280–1.2 I_2 ...for generator No. 2

Since bus-bar voltage is the same.

$\therefore$ $270–10\ I_1/7 = 280–1.2\ I_2$ or $4.2\ I_2 – 5\ I_1 = 35$ ...(*i*)

Also $I_1 + I_2 = 60$ ...(*ii*)

Solving the two equations, we get I_1 = **23.6 A;** I_2 = **36.4 A**

Now $V = 280–1.2\ I_2 = 280–1.2 \times 36.4$

= **236.3 V**

Output of Ist machine

= 236.3 × 23.6/1000

= **5.577 kW**

Output of 2nd machine

= 236.3 × 36.4/1000

= **8.602 kW**

Graphical Solution.

In Fig. 27.24, total load current of 60 A has been plotted along *X*-axis and the terminal voltage along *Y*-axis. The linear characteristics of the two generators are drawn from the given data. The common bus-bar voltage is given by the point of intersection of the two graphs. From the graph, it is seen that V = 236.3 V ; I_1 = 23.6 A ; I_2 = 36.4 A.

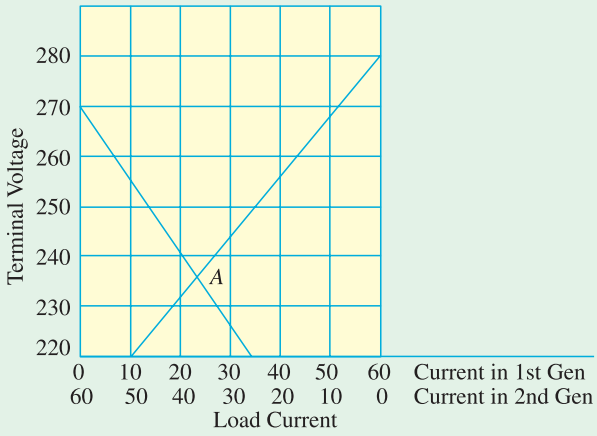


Fig. 27.24

Example 27.16. *Two shunt generators each with an armature resistance of 0.01 Ω and field resistance of 20 Ω run in parallel and supply a total load of 4000 A. The e.m.f.s are respectively 210 V and 220 V. Calculate the bus-bar voltage and output of each machine.* **(Electrical Machines-1, South Gujarat Univ. 1988)**

Solution. Generators are shown in Fig. 27.25.

Let V = bus-bar voltage

I_1 = output current of G_1

I_2 = output current of G_2

Now, $I_1 + I_2 = 4000$ A, $I_{sh} = V/20$.

$I_{a1} = (I_1 + V/20)$; $I_{a2} = (I_2 + V/20)$

In each machine,

V + armature drop = induced e.m.f.

$\therefore$ $V + I_{a1} R_a = E_1$

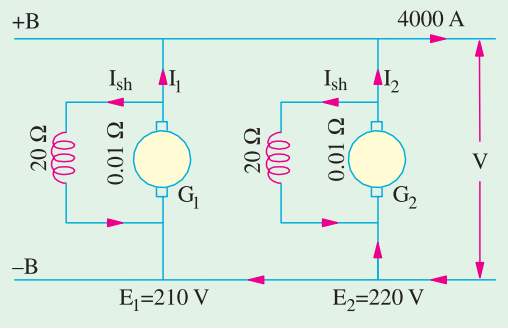


Fig. 27.25

or $V + (I_1 + V/20) \times 0.01$

$= 210$ ...1st machine

Also $V + I_{a2} R_a = E_2$

or $V + (I_2 + V/20) \times 0.01 = 220$ ...2nd machine

Subtracting, we have $0.01\ (I_1 - I_2) = 10$ or $I_1 - I_2 = 1000$

Also, $I_1 + I_2 = 4000$ A $\therefore$ I_1 = **2500 A ; I_2 = 1500 A**

Substituting the value of I_1 above, we get

$V + (2500 + V/20) \times 0.01 = 210$ $\therefore$ V = **184.9 V**

Output of Ist generator = $184.9 \times 2500/1000$ = **462.25 kW**

Output of 2nd generator = $184.49 \times 1500/1000$ = **277.35 kW**

Example 27.17. *Two shunt generators operating in parallel deliver a total current of 250 A. One of the generators is rated 50 kW and the other 100 kW. The voltage rating of both machine is 500 V and have regulations of 6 per cent (smaller one) and 4 percent. Assuming linear characteristics, determine* ***(a)*** *the current delivered by each machine* ***(b)*** *terminal voltage.*

(Elect. Machines, Nagpur Univ. 1991)

Solution. 50 kW generator

F.L. voltage drop = $500 \times 0.06 = 30$ V ; F.L. current = 50,000/500 = 100 A

Drop per ampere = 30/100 = 3/10 V/A

100 kW generator

F.L. drop = $500 \times 0.04 = 20$ V ; F.L. current = 100,000/5000 = 200 A

Drop per ampere = 20/200 = 1/10 V/A

If I_1 and I_2 are currents supplied by the two generators and V the terminal voltage, then

$V = 500 - (3I_1/10)$ –1st generator

$= 500 - (I_2/10)$ –2nd generator

$\therefore$ $3I_1/10 = I_2/10$ or $3I_1 = I_2$; Also $I_1 + I_2 = 250$ –given

(a) Solving the above two equations, we get I_1 = **62.5 A ; I_2 = 187.5 A**

(b) $V = 500 - (3 \times 62.5/10)$ = **481.25 V**

Example 27.18. *Two shunt generators, each with a no-load voltage of 125 V are running parallel. Their external characteristics can be taken as straight lines over this operating ranges. Generator No. 1 is rated at 25 kW and its full-load voltage is 119 V, Generator No. 2 is rated at 200 kW at 116 V. Calculate the bus-bar voltage when the total load is 3500 A. How is the load divided between the two ?*

(Elect. Machinery - I, Mysore Univ. 1988)

Solution. Let V = bus-bar voltage

x_1, x_2 = load carried by each generator in terms of percentage of rated load

P_1, P_2 = load carried by each generator in watts

$V = 125 - [(125 - 119)\ (x_1/100)]$ ...Generator No. 1

$V = 125 - [(125 - 116)\ (x_2/100)]$ ...Generator No. 2

$\therefore$ $125 - \dfrac{6x_1}{100} = 125 - \dfrac{9x_2}{100}$ $x_2 = \dfrac{6x_1}{9} = \dfrac{2x_1}{3}$

Since in d.c. circuits, power delivered is given by VI watt, the load on both generators is

$$\left(250\ x_1 \times \frac{1000}{100}\right) + \left(200\ x_2 \times \frac{1000}{100}\right) = V \times 3500$$

Now, replacing V and x_2 by terms involving x_1, we get as a result

$$\left(250\, x_1 \times \frac{1000}{100}\right) + \left(200 \times \frac{2x_1}{3} \times \frac{1000}{100}\right) = \left(125 - \frac{6x_1}{100}\right) \times 3500$$

x_1 = 108.2 per cent

$\therefore$ Bus-bar voltage $V = 125 - (6 \times 108.2/100) =$ **118.5 V**

The division of load between the two generators can be found thus :

$$x_1 = \frac{P_1 \times 1000}{250,000} \quad \text{and} \quad x_2 = \frac{P_2 \times 1000}{200,000}$$

$$\therefore \quad \frac{x_1}{x_2} = \frac{P_1 \times 200,000}{P_2 \times 250,000} = \frac{4P_1}{5P_2} = \frac{3}{2} \quad \therefore \quad \frac{3}{2} = \frac{4P_1}{5P_2} = \frac{4\,VI_1}{5\,VI_2} = \frac{4I_1}{5I_2} \qquad ...(i)$$

Since $I_1 + I_2 = 3500$ $\quad \therefore \; I_2 = 3500 - I_1$

Hence (i) above becomes, $\frac{3}{2} = \frac{4I_1}{5(3500 - I_1)}$

$\therefore$ I_1 = **2,283** A and I_2 = **1,217 A**

Example 27.19. *Two shunt generators A & B operate in parallel and their load-characteristics may be taken as straight lines. The voltage of generator A falls from 240 V at no load to 220 V at 200 A, while that of B falls from 245 V at no load to 220 V to 150 A. Determine the currents supplied by each machine to a common load of 300 A and the bus-bar voltage.*

(Bharathithasan Univ. April 1997)

Solution. Two graphs are plotted as shown in Fig. 27.26.

Their equations are :

$$240 - (20/200)\, I_A = 245 - (25/150)\, I_B$$

Futher, $I_A + I_B = 300$

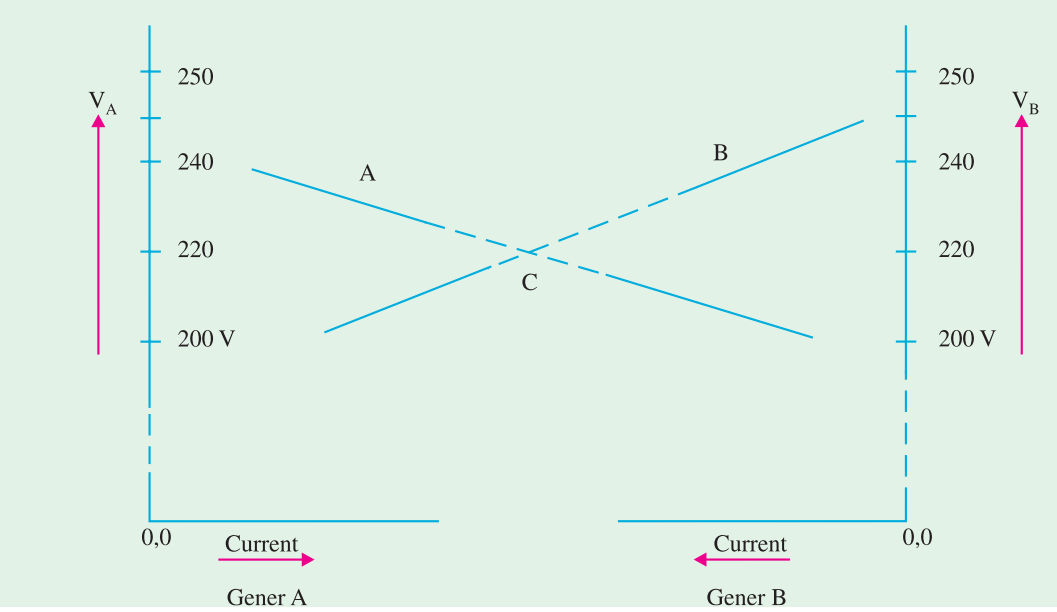


Fig. 27.26. Parallel operation of two D.C. Generators

This gives I_A = 168.75 A, I_B = 131.25 A

And common voltage of Bus-bar, V_{BUS}

$= 240 - (20/200) \times 168.75$, or

$V_{BUS} = 245 - (25/150) \times 131.25 = 223.125$ volts.

It is represented by the point C, in graph, as an intersection, satisfying the condition that two currents (I_A and I_B) add up to 300 amp.

Example 27.20. *In a certain sub-station, there are 5 d.c. shunt generators in parallel, each having an armature resistance of 0.1 Ω, running at the same speed and excited to give equal induced e.m.f.s. Each generator supplies an equal share of a total load of 250 kW at a terminal voltage of 500 V into a load of fixed resistance. If the field current of one generator is raised by 4%, the others remaining unchanged, calculate the power output of each machine and their terminal voltages under these conditions. Assume that the speeds remain constant and flux is proportional to field current.* **(Elect. Technology, Allahabad Univ. 1991)**

Solution. Generator connections are shown in Fig. 27.27.

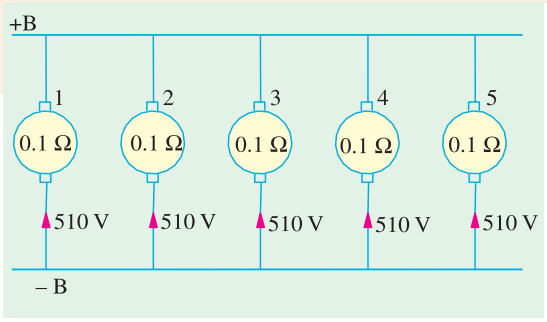


Fig. 27.27

Load supplied by each =250/5 = 50kW

∴ Output of each = 50,000/500 = 100 A

Terminal voltage of each = 500 V

Armature drop of each = 0.1 × 100 = 10 V

Hence, induced e.m.f. of each = 510 V

When field current of one is increased, its flux and hence its generated e.m.f. is increased by 4%. Now, 4% of 510 V = 20.4 V

∴ Induced e.m.f. of one =510 + 20.4 = 530.4 V

Let I_1 = current supplied by one generator after increased excitation

I_2 = current supplied by each of the other 4 generators

V = new terminal or bus-bar voltage

∴ $530.4 - 0.1\, I_1 = V$ ...(*i*)

$510 - 0.1\, I_2 = V$ ...(*ii*)

Now, fixed resistance of load = 500/500 = 1 Ω ; Total load current = $I_1 + 4I_2$

∴ $1 \times (I_1 + 4I_2) = V$ or $I_1 + 4I_2 = V$ ...(*iii*)

Subtracting (*ii*) from (*i*), we get, $I_1 - I_2 = 204$ ...(*iv*)

Subtracting (*iii*) from (*ii*), we have $I_1 + 4.1\, I_2 = 510$ ...(*v*)

From (*iv*) and (*v*), we get I_2 = 3060/51 = 59/99 = 60 A (approx.)

From (*iv*) I_1 = 204 + 60 = 264 A

From (*iii*) V = 264 + 240 = **504 Volt**

Output of Ist machine = 504 × 264 watt = **133 kW**

Output of *each* of other four generators = 504 × 60 W = **30.24 kW**

Example 27.21. *Two d.c. generators are connected in parallel to supply a load of 1500 A. One generator has an armature resistance of 0.5 Ω and an e.m.f. of 400 V while the other has an armature resistance of 0.04 Ω and an e.m.f. of 440 V. The resistances of shunt fields are 100 Ω and 80 Ω respectively. Calculate the currents I_1 and I_2 supplied by individual generator and terminal voltage V of the combination.* **(Power Apparatus-I, Delhi Univ. Dec. 1987)**

Solution. Generator connection diagram is shown in Fig. 27.28.

Let V = bust–bar voltage

I_1 = output current of one generator

I_2 = output current of other generator
= $(1500 - I_1)$

Now, I_{sh1} = V/100 A ; I_{sh2} = V/80 A

$I_{a1} = \left(I_1 + \frac{V}{100}\right)$ and $I_{a2} = \left(I_2 + \frac{V}{80}\right)$

or $I_{a2} = \left(1500 - I_1 + \frac{V}{80}\right)$

For each machine

E – armature drop = V

$\therefore \; 400 - \left(I_1 + \frac{V}{100}\right) \times 0.5 = V$

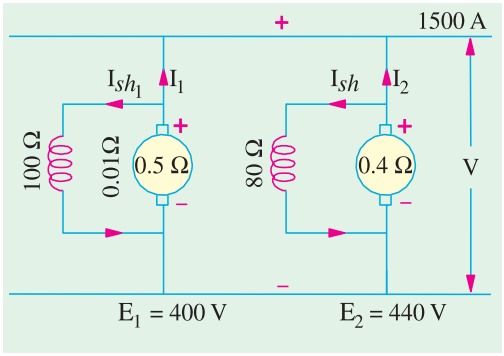


Fig. 27.28

or $400 - 0.5\, I_1 - 0.005\, V = V$ or $0.5\, I_1 = 400 - 1.0005 = V$ ...(*i*)

Also $440 - (1500 - I_1 + \frac{V}{80}) \times 0.04 = V$ or $0.04\, I_1 = 1.0005\, V - 380$ ...(*ii*)

Dividing Eq. (*i*) by (*ii*), we get

$$\frac{0.5\, I_1}{0.04\, I_2} = \frac{400 - 1.005\, V}{1.0005\, V - 380} \quad \therefore \; V = \mathbf{381.2\ V}$$

Substituting this value of V in Eq. (*i*), we get $0.5\, I_1 = 400 - 1.005 \times 381.2$

$\therefore \quad I_1 = \mathbf{33.8\ A}$; $I_2 = 1500 - 33.8 = \mathbf{1466.2\ A}$

Output of Ist generator = $381.2 \times 33.8 \times 10^{-3} = \mathbf{12.88\ kW}$

Output of 2nd generator = $381.2 \times 1466.2 \times 10^{-3} = \mathbf{558.9\ kW}$

Example 27.22. *Two shunt generators and a battery are working in parallel. The open circuit voltage, armature and field resistances of generators are 250 V, 0.24 Ω, 100 Ω are 248 V, 0.12 Ω and 100 Ω respectively. If the generators supply the same current when the load on the bus-bars is 40 A, calculate the e.m.f. of the battery if its internal resistance is 0.172 Ω.*

Solution. Parallel combination is shown in Fig. 27.29.

Values of currents and induced e.m.fs. are shown in the diagram.

$V + \left(I + \frac{V}{100}\right) \times 0.24 = 250$ ...(*i*)

$V + \left(I + \frac{V}{100}\right) \times 0.12 = 248$ ...(*ii*)

Also $I + I + I_b = 40$

$I_b + 2\, I = 40$ ...(*iii*)

Subtracting (*ii*) from (*i*), we get $\left(I + \frac{V}{100}\right) \times 0.12 = 2$ ...(*iv*)

Putting this value in (*ii*) above, V = 246 volt.

Putting this value of V in (*iv*), $\left(I + \frac{246}{100}\right) \times 0.12 = 2$ ...(*v*)

$\therefore \quad I = 50/3 - 2.46 = 14.2$ A

From (*iii*), we have $I_b = 40 - (2 \times 14.2) = 11.6$ A

Internal voltage drop in battery = $11.6 \times 0.172 = 2$ V $\quad \therefore \; E_b = 246 + 2 = \mathbf{248\ V}$

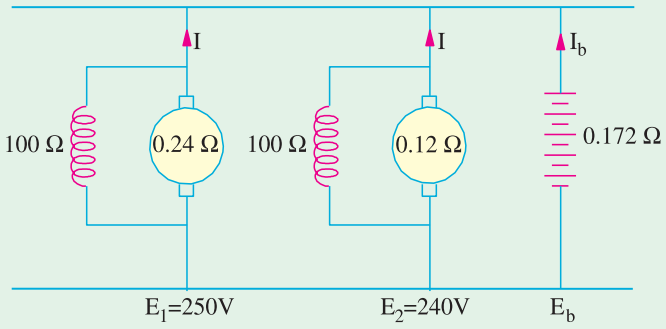


Fig. 27.29

Example 27.23. *Two d.c. generators A and B are connected to a common load. A had a constant e.m.f. of 400 V and internal resistance of 0.25 Ω while B has a constant e.m.f. of 410 V and an internal resistance of 0.4 Ω. Calculate the current and power output from each generator if the load voltage is 390 V. What would be the current and power from each and the terminal voltage if the load was open-circuited ?* **(Elect. Engg; I, Bangalore Univ. 1987)**

Solution. The generator connections are shown in Fig. 27.30 (*a*).

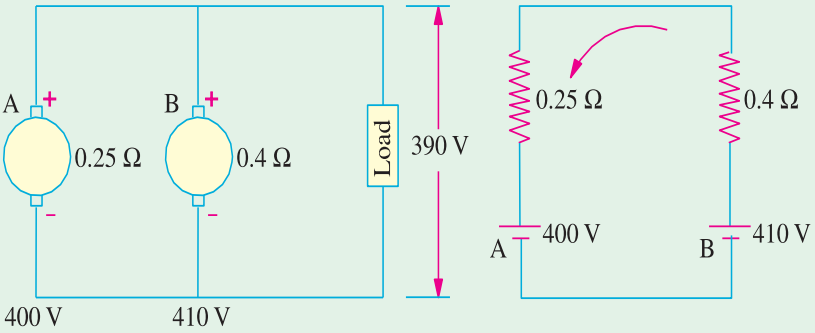


Fig. 27.30

Since the terminal or output voltage is 390 V, hence

Load supplied by A = (400 – 390)/0.25 = **40 A**

Load supplied by B = (410 – 390)/0.4 = **50A**

∴ Power output from A = 40 × 390 = **15.6 kW**

Power output from B = 50 × 390 = **19.5 kW**

If the load is open-circuited as shown in Fig. 27.30.(*b*), then the two generators are put in series with each other and a circulatory current is set up between them.

Net voltage in the circuit = 410 – 400 = 10 V

Total resistance = 0.4 + 0.25 = 0.65 Ω

∴ circulatory current = 10/0.65 = 15.4 A

The terminal voltage = 400 + (15.4 × 0.25) = **403.8 V**

Obviously, machine B with a higher e.m.f. acts as a generator and drives machine A as a motor.

Power taken by A from B = 403.8 × 15.4 = **6,219 W**

Part of this appears as mechanical output and the rest is dissipated as armature Cu loss.

Mechanical output = $400 \times 15.4 = 6.16$ kW ; Armature Cu loss = $3.8 \times 15.4 = 59$ W

Power supplied by B to A = 6,219 W ; Armature Cu loss = $6.16 \times 15.4 = 95$ W

Example 27.24. *Two compound generators A and B, fitted with an equalizing bar, supply a total load current of 500 A. The data regarding the machines are* :

	A	*B*
Armature resistance (ohm)	*0.01*	*0.02*
Series field winding (ohm)	*0.004*	*0.006*
Generated e.m.fs. (volt)	*240*	*244*

Calculate ***(a)*** *current in each armature* ***(b)*** *current in each series winding* ***(c)*** *the current flowing in the equalizer bar and* ***(d)*** *the bus-bar voltage. Shunt currents may be neglected.*

Solution. The two generators (with their shunt windings omitted) are shown in Fig. 27.31.

Let V = bus-bar voltage ; v = voltage between equalizer bus-bar and the negative

i_1, i_2 = armature currents of the two generators

Now, $i_1 + i_2 = 500$

or $$\frac{240-v}{0.01}+\frac{244-v}{0.01}=500$$

Multiplying both sides by $\frac{1}{100}$ we get

$$240 - v + 122 - (v/2) = 5$$

$\therefore \quad v = 238$ volts

(a) $\therefore \quad i_1 = \frac{240-238}{0.01} =$ **200 A**

$i_2 = \frac{244-238}{0.02} =$ **300 A**

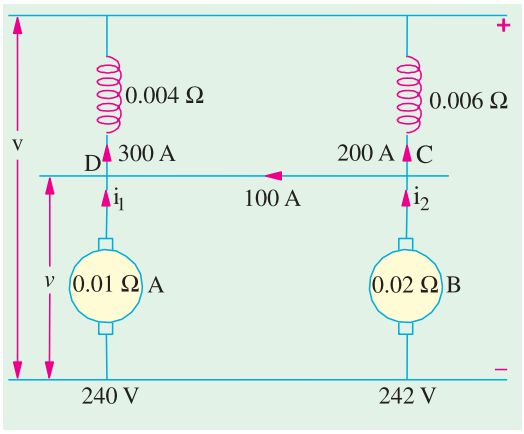


Fig. 27.31

(b) The total current of 500 A divides between the series windings in the inverse ratio of their resistance *i.e.* in the ratio of $\frac{1}{0.004} : \frac{1}{0.006}$ or in the ratio 3 : 2.

Hence, current in the series winding of generator $A = 500 \times 3/5 =$ **300 A**

Similarly, current in the series winding of generator $B = 500 \times 2/5 =$ **200 A**

(c) It is obvious that a current of 100 A flows in the equalizing bar from C to D. It is so because the armature current of generator A is 200 A only. It means that 100 A comes from the armature of generator, B, thus making 300 A for the series field winding of generator A.

(d) $V = v$ – voltage drop in one series winding = $238 - (300 \times 0.004) =$ **236.8 V**

Tutorial Problem No. 27.2

1. Two separately-excited d.c. generators are connected in parallel supply a load of 200 A. The machines have armature circuit resistances of 0.05 Ω and 0.1 Ω and induced e.m.fs. 425 V and 440 V respectively. Determine the terminal voltage, current and power output of each machine. The effect of armature reaction is to be neglected. **(423.3 V ; 33.3 A ; 14.1 kW ; 166.7 A ; 70.6 kW)**

2. Two shunt generators operating in parallel given a total output of 600 A. One machine has an armature resistance of 0.02 Ω and a generated voltage of 455 V and the other an armature resistance of 0.025 Ω and a generated voltage of 460 V. Calculate the terminal voltage and the kilowatt output of each machine. Neglect field currents.

(450.56 V ; 100 kW ; 170.2 kW)

3. The external characteristics of two d.c. shunt generators *A* and *B* are straight lines over the working range between no-load and full-load.

	Generator A		*Generator B*	
	No-load	*Full-load*	*No-load*	*Full-load*
Terminal P.D. (V)	400	360	420	370
Load current (A)	0	80	0	70

Determine the common terminal voltage and output current of each generator when sharing a total load of 100 A.

(57.7 A ; 42.3 A ; 378.8 V)

4. Two shunt generators operating in parallel have each an armature resistance of 0.02 Ω. The combined external load current is 2500 A. If the generated e.m.fs. of the machines are 560 V and 550 V respectively, calculate the bus-bar voltage and output in kW of each machine. **(530 V; 795 kW; 530 kW)**

5. Two shunt generators *A* and *B* operate in parallel and their load characteristics may be taken as straight lines. The voltage of *A* falls from 240 *V* at no-load to 220 V at 200 A, while that of *B* falls from 245 V at no-load to 220 V at 150 A. Determine the current which each machine supplies to a common load of 300 A and the bus-bar voltage at this load.

(169 A ; 131 A ; 223.1 V)

6. Two shunt-wound d.c. generators are connected in parallel to supply a load of 5,000 A. Each machine has an armature resistane of 0.03 Ω and a field resistance of 60 Ω, but the e.m.f. of one machine is 600 V and that of the other is 640 V. What power does each machine supply ?

(1,004 kW ; 1,730 kW including the fields)

7. Two shunt generators running in parallel share a load of 100 kW equally at a terminal voltage of 230 V. On no-load, their voltages rise to 240 V and 245 V respectively. Assuming that their volt-ampere characteristics are rectilinear, find how would they share the load when the total current is reduced to half its original value ? Also, find the new terminal voltage. **(20 kW ; 30 kW, 236 V)**

8. Two generators, each having no-load voltage of 500 V, are connected in parallel to a constant resistance load consuming 400 kW. The terminal p.d. of one machine falls linearly to 470 V as the load is increased to 850 A while that of the other falls linearly to 460 V when the load is 600 A. Find the load current and voltage of each generator.

If the induced e.m.f. of one machine is increased to share load equally, find the new current and voltage. **(I_1 = 626 A ; I_2 = 313 A ; V = 479 V ; I = 469.5 A ; V = 484.4 V)**

9. Estimate the number of turns needed on each interpole of a 6-pole generator delivering 200 kW at 200 V ; given : number of lap-connected armature conductors = 540 ; interpole air gap = 1.0 cm ; flux-density in interpole air-gap = 0.3 Wb/m^2. Ignore the effect of iron parts of the circuit and of leakage.

[10] (*Electrical Machines, B.H.U. 1980*)

OBJECTIVE TEST – 27

1. In d.c. generators, armature reaction is produced actually by

(*a*) its field current

(*b*) armature conductors

(*c*) field pole winding

(*d*) load current in armature

2. In a d.c. generator, the effect of armature reaction on the main pole flux is to

(*a*) reduce it (*b*) distort it

(*c*) reverse it (*d*) both (*a*) and (*b*)

3. In a clockwise-rotating loaded d.c. generator, brushes have to be shifted

(*a*) clockwise

(*b*) counterclockwise

(*c*) either (*a*) or (*b*)

(*d*) neither (*a*) nor (*b*).

4. The primary reason for providing compensating windings in a d.c. generator is to

(*a*) compensate for decrease in main flux

(*b*) neutralize armature mmf

(*c*) neutralize cross-magnetising flux

(*d*) maintain uniform flux distribution.

5. The main function of interpoles is to minimize ..... between the brushes and the commutator when the d.c. machine is loaded.

(*a*) friction (*b*) sparking

(*c*) current (*d*) wear and tear

6. In a 6-pole d.c. machine, 90 mechanical degrees correspond to ..... electrical degrees.
(*a*) 30 (*b*) 180
(*c*) 45 (*d*) 270

7. The most likely cause(s) of sparking at the brushes in a d.c. machine is /are
(*a*) open coil in the armature
(*b*) defective interpoles
(*c*) incorrect brush spring pressure
(*d*) all of the above

8. In a 10-pole, lap-wound d.c. generator, the number of active armature conductors per pole is 50. The number of compensating conductors per pole required is
(*a*) 5 (*b*) 50
(*c*) 500 (*d*) 10

9. The commutation process in a d.c. generator basically involves
(*a*) passage of current from moving armature to a stationary load
(*b*) reversal of current in an armature coil as it crosses MNA
(*c*) conversion of a.c. to d.c.
(*d*) suppression of reactance voltage

10. Point out the WRONG statement. In d.c. generators, commutation can be improved by
(*a*) using interpoles
(*b*) using carbon brushes in place of Cu brushes
(*c*) shifting brush axis in the direction of armature rotation
(*d*) none of the above

11. Each of the following statements regarding interpoles is true **except**
(*a*) they are small yoke-fixed poles spaced in between the main poles
(*b*) they are connected in parallel with the armature so that they carry part of the armature current
(*c*) their polarity, in the case of generators is the same as that of the main pole ahead
(*d*) they automatically neutralize not only reactance voltage but cross-magnetisation as well

12. Shunt generators are most suited for stable parallel operation because of their voltage characteristics.
(*a*) identical (*b*) dropping
(*c*) linear (*d*) rising

13. Two parallel shunt generators will divide the total load equally in proportion to their kilowatt output ratings only when they have the same
(*a*) rated voltage
(*b*) voltage regulation
(*c*) internal I_aR_a drops
(*d*) boths (*a*) and (*b*)

14. The main function of an equalizer bar is to make the parallel operation of two over-compounded d.c. generators
(*a*) stable (*b*) possible
(*c*) regular (*d*) smooth

15. The *essential* condition for stable parallel operation *A* Two d.c. generators having similar characteristics is that they should have
(*a*) same kilowatt ouput ratings
(*b*) droping voltage characterisitcs
(*c*) same percentage regulation
(*d*) same no-load and full-load speed

16. The main factor which loads to unstable parallel operation of flat-and over-compound d.c. generators is
(*a*) unequal number of turns in their series field windings
(*b*) unequal series field resistances
(*c*) their rising voltage characteristics
(*d*) unequal speed regulation of their prime movers

17. The simplest way to shift load from one d.c. shunt generator running in parallel with another is to
(*a*) adjust their field rheostats
(*b*) insert resistance in their armature circuits
(*c*) adjust speeds of their prime movers
(*d*) use equalizer connections

18. Which one of the following types of generators does NOT need equalizers for satisfactory parallel operation ?
(*a*) series (*b*) over-compound
(*c*) flat-compound (*d*) under-compound.

ANSWERS

1. (*d*) **2.** (*d*) **3.** (*a*) **4.** (*c*) **5.** (*b*) **6.** (*d*) **7.** (*d*) **8.** (*a*) **9.** (*b*) **10.** (*d*) **11.** (*b*) **12.** (*b*) **13.** (*d*) **14.** (*a*) **15.** (*b*) **16.** (*c*) **17.** (*a*) **18.** (*d*)

GO To FIRST

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CHAPTER 28

GENERATOR CHARACTERISTICS

Learning Objectives

- Characteristics of D.C. Generators
- Separately-excited Generator
- No-load Curve for Self-excited Generator
- How to find Critical Resistance R_c?
- How to draw O.C.C. at Different Speeds ?
- Critical Speed
- Voltage Build up of a Shunt Generator
- Conditions for Build-up of a Shunt Generator
- Other factors Affecting Voltage Building of a D.C. Generator
- External Characteristic
- Voltage Regulation
- Internal or Total Characteristic
- Series Generator
- Compound-wound Generator
- How to calculate Required Series Turns?
- Uses of D.C. Generators



Generator characteristics gives the relation between terminal voltage and load current. It is of great importance in judging the suitability of a generator for a particular purpose

CONTENTS

28.1. Characteristics of D.C. Generators

Following are the three ***most*** important characteristics or curves of a d.c. generator :

1. No-load saturation Characteristic (E_0/I_f)

It is also known as Magnetic Characteristic or Open-circuit Characteristic (*O.C.C.*). It shows the relation between the no-load generated e.m.f. in armature, E_0 and the field or exciting current I_f at a given fixed speed. It is just the magnetisation curve for the material of the electromagnets. Its shape is practically the same for all generators whether separately-excited or self-excited.

2. Internal or Total Characteristic (E/I_a)

It gives the relation between the e.m.f. E *actually* induces in the armature (after allowing for the demagnetising effect of armature reaction) and the armature current I_a. This characteristic is of interest mainly to the designer and can be obtained as explained in Art. 28.12.

3. External Characteristic (V/I)

It is also referred to as performance characteristic or sometimes voltage-regulating curve.

It gives relation between that terminal voltage V and the load current I. This curve lies below the internal characteristic because it takes into account the voltage drop over the armature circuit resistance. The values of V are obtained by subtracting I_aR_a from corresponding values of E. This characteristic is of great importance in judging the suitability of a generator for a particular purpose. It may be obtained in two ways **(*i*)** by making simultaneous measurements with a suitable voltmeter and an ammeter on a loaded generator (Art. 28.10) or **(*ii*)** graphically from the *O.C.C.* provided the armature and field resistances are known and also if the demagnetising effect (under rated load conditions) or the armature reaction (from the short-circuit test) is known.

28.2. Separately-excited Generator

(*a*) (*i*) No-load Saturation Characteristic (E_0/I_f)

The arrangement for obtaining the necessary data to plot this curve is shown in Fig. 28.1. The exciting or field current I_f is obtained from an external independent d.c. source. It can be varied from zero upwards by a potentiometer and its value read by an ammeter A connected in the field circuit as shown.

Now, the voltage equation of a d.c. generator is, $E_g = \dfrac{\Phi ZN}{60} \times \left(\dfrac{P}{A}\right)$ volt

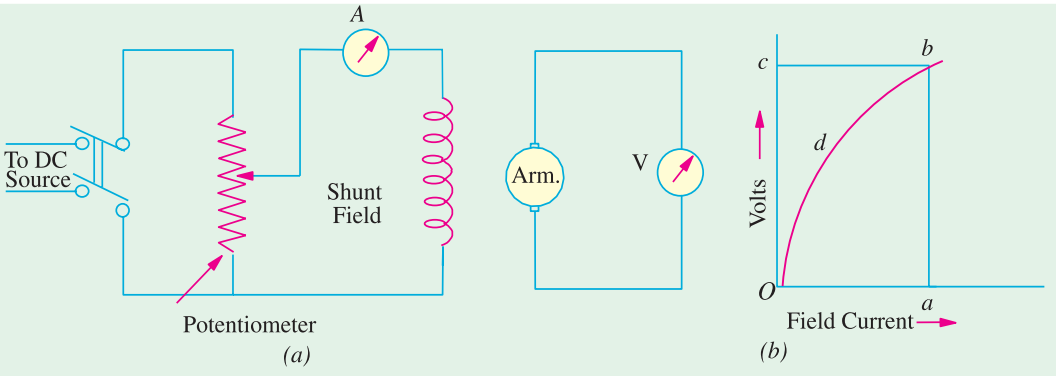


Fig. 28.1

Hence, if speed is constant, the above relation becomes $E = k\Phi$

It is obvious that when I_f is increased from its initial small value, the flux Φ and hence generated e.m.f. E_g increase directly as current so long as the poles are unsaturated. This is represented by the straight portion Od in Fig. 28.1 (*b*). But as the flux density increases, the poles become saturated, so a greater increase in I_f is required to produce a given increase in voltage than on the lower part of the curve. That is why the upper portion db of the curve Odb bends over as shown.

(*ii*) Load Saturation Curve (V/I_f)

The curve showing relation between the terminal voltage V and field current I_f when the generator is loaded, is known as Load Saturation Curve.

The curve can be deduced from the no-load saturation curve provided the values of armature reaction and armature resistance are known. While considering this curve, account is taken of the

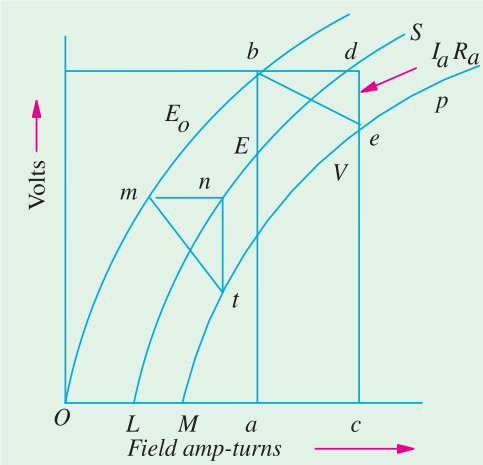


Fig. 28.2

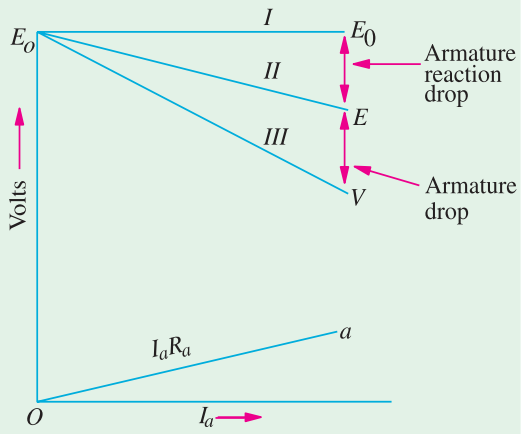


Fig. 28.3

demagnetising effect of armature reaction and the voltage drop in armature which are practically absent under no-load conditions. The no-load saturation curve of Fig. 28.1 has been repeated in Fig. 28.2 on a base of field amp-turns (and not current) and it is seen that at no-load, the field amp-turns required for rated no-load voltage are given by *Oa*. Under load conditions, the voltage will decrease due to demagnetising effect of armature reaction. This decrease can be made up by suitably increasing the field amp-turns. Let *ac* represent the equivalent demagnetising amp-turns per pole. Then, it means that in order to generate the same e.m.f. on load as at no-load, the field amp-turns/pole must be increased by an amount $ac = bd$. The point *d* lies on the curve *LS* which shows relation between the voltage *E* generated under ***load*** conditions and the field amp-turns. The curve *LS* is practically parallel to curve *Ob*. The terminal voltage *V* will be less than this generated voltage *E* by an amount $= I_a R_a$ where R_a is the resistance of the armature circuit. From point *d*, a vertical line $de = I_aR_a$ is drawn. The point *e* lies on the full-load saturation curve for the generator. Similarly, other points are obtained in the same manner and the full-load saturation curve *Mp* is drawn. The right-angled triangle *bde* is known as **drop reaction triangle**. Load saturation curve for half-load can be obtained by joining the mid-points of such lines as *mn* and *bd* etc. In the case of self-excited generators, load saturation curves are obtained in a similar way.

(*b*) Internal and External Characteristics

Let us consider a separately-excited generator giving its rated no-load voltage of E_0 for a certain constant field current. If there were no armature reaction and armature voltage drop, then this voltage would have remained constant as shown in Fig. 28.3. by the dotted horizontal line *I*. But when the generator is loaded, the voltage falls due to these two causes, thereby giving slightly dropping characteristics. If we subtract from E_0 the values of voltage drops due to armature reaction for different loads, then we get the value of *E*–the e.m.f. ***actually*** induced in the armature under load conditions. Curve II is plotted in this way and is known as the ***internal characteristic***. The straight line *Oa* represents the $I_aR_a$drops corresponding to different armature currents. If we subtract from *E* the armature drop I_aR_a, we get terminal voltage *V*. Curve *III* represents the external characteristic and is obtained by subtracting ordinates the line *Oa* from those of curve *II*.

28.3. No-load Curve for Self-excited Generator

The *O.C.C.* or no-load saturated curves for self-excited generators whether shunt or series-connected, are obtained in a similar way.

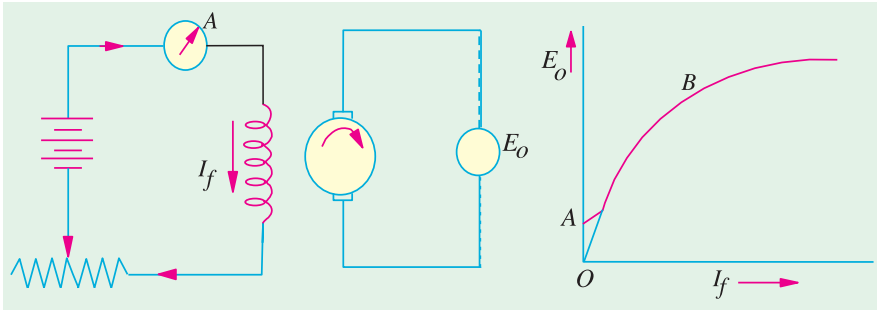


Fig. 28.4

Fig. 28.5

The field winding of the generator (whether ***shunt*** or ***series*** wound) is disconnected from the machine and connected to an external source of direct current as shown in Fig. 28.4. The field or exciting current I_f is varied rheostatically and its value read on the ammeter *A*. The machine is drived at constant speed by the prime mover and the generated e.m.f. on on-load is measured by the voltmeter connected across the armature. I_f is increased by suitable steps (starting from zero) and the corresponding values of E_0 are measured. On plotting the relation between I_f and E_0, a curve of this form shown in Fig. 28.5 is obtained.



Self Excited Generator

Due to residual magnetism in the poles, some e.m.f. (= *OA*) is generated even when $I_f = 0$. Hence, the curve starts a little way up. The slight curvature at the lower end is due to magnetic inertia. It is seen that the first part of the curve is practically straight. This is due to the fact that at low flux densities, reluctance of iron path being negligible (due to high permeability), total reluctance is given by the air-gap reluctance which is constant. Hence, the flux and consequently, the generated e.m.f. is directly proportional to the exciting current. However, at high flux densities, – where μ is small, iron path reluctance becomes appreciable and straight relation between *E* and I_f no longer holds good. In other words, after point *B*, saturation of poles starts. However, the initial slope of the curve is determined by air-gap width.

It should be noted that *O.C.C.* for a higher speed would lie above this curve and for a lower speed, would lie below it.

It should also be noted and the load-saturation curve for a shunt generator is similar to the one shown in Fig. 28.2.

Critical Resistance for Shunt Generator

Now, connect the field windings back to the armature and run the machine as a shunt generator. Due to residual magnetism in the poles, some e.m.f. and hence current, would be generated. This current while passing through the field coils will strengthen the magnetism of the poles (provided field coils are properly connected as regards polarity). This will increase the pole flux which will further increase the generated e.m.f. Increased e.m.f. means more current which further increases the flux and so on. This mutual reinforcement of e.m.f. and flux proceeds on till equilibrium is reached at some point like *P* (Fig. 28.6). The point lies on the resistance line *OA* of the field winding. Let *R* be the resistance of the field winding. Line *OA* is drawn such that its slope equals the field winding

resistance *i.e.* every point on this curve is such that volt/ampere = *R*.

The voltage *OL* corresponding to point *P* represents the maximum voltage to which the machine will build up with *R* as field resistance. *OB* represents smaller resistance and the corresponding voltage *OM* is slightly greater than *OL*. If field resistance is increased, then slope of the resistance line increased, and hence the maximum voltage to which the generator will build up at a given speed, decreases. If *R* is increased so much that the resistance line does not cut the *O.C.C.* at all (like *OT*), then obviously the machine will fail to excite *i.e.* there will be no 'build up' of the voltage. If the resistance line just lies along the slope, then with that value of field resistance, the machine will ***just*** excite. The value of the resistance represented by the tangent to the curve, is known as **critical resistance** R_c for a ***given speed***.

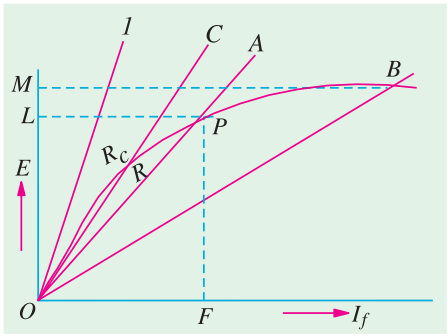


Fig. 28.6

28.4 How to Find Critical Resistance R_c ?

First, *O.C.C.* is plotted from the given data. Then, tangent is drawn to its initial portion. The slope of this curve gives the critical resistance for the speed at which the data was obtained.

28.5 How to Draw *O.C.C.* at Different Speeds ?

Suppose we are given the data for *O.C.C.* of a generator run at a fixed speed, say, N_1. It will be shown that *O.C.C.* at any other constant speed N_2 can be deduced from the *O.C.C.* for N_1. In Fig. 28.7 the *O.C.C.* for speed N_1 is shown.

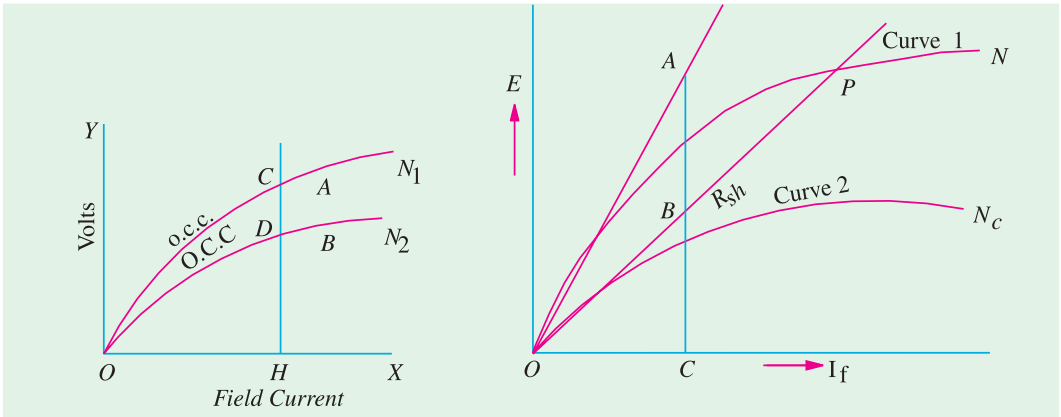


Fig. 28.7 **Fig. 28.8**

Since $E \propto N$ for any fixed excitation, hence $\frac{E_2}{E_1} = \frac{N_2}{N_1}$ or $E_2 = E_1 \times \frac{N_2}{N_1}$

As seen, for $I_f = OH$, $E_1 = HC$. The value of new voltage for the same I_f but at N_2

$$E_2 = HC \times \frac{N_2}{N_1} = HD$$

In this way, point *D* is located. In a similar way, other such points can be found and the new *O.C.C.* at N_2 drawn.

28.6. Critical Speed N_c

Critical speed of a shunt generator is that speed for which the given shunt field resistance represents critical resistance. In Fig. 28.8, curve 2 corresponds to critical speed because R_{sh} line is tangential to it. Obviously

$$\frac{BC}{AC} = \frac{N_c}{\text{Full Speed}} = \frac{N_c}{N} \quad \therefore \quad N_c = \frac{BC}{AC} \times \text{Full speed } N$$

Example 28.1. *The magnetization curve of a d.c. shunt generator at 1500 r.p.m. is :*

I_f (A) :	0	0.4	0.8	1.2	1.6	2.0	2.4	2.8	3.0
E_0 (V) :	6	60	120	172.5	202.5	221	231	237	240

For this generator find ***(i)*** *no load e.m.f. for a total shunt field resistance of 100 Ω* ***(ii)*** *the critical field resistance at 1500 r.p.m. and* ***(iii)*** *the magnetization curve at 1200 r.p.m. and therefrom the open-circuit voltage for a field resistance of 100 Ω.*

(b) *A long shunt, compound generator fitted with interpoles is cummutatively-compounded. With the supply terminals unchanged, the machine is now run as compound motor. Is the motor differentially or cumulatively compounded ? Explain.* **(Elect, Machines, A.M.I.E. Sec. B, 1990)**

Solution. The magnetisation curve at 1500 r.p.m. is plotted in Fig. 28.9 from the given data. The 100 Ω resistance line *OA* is obtained by joining the origin (0, 0) with the point (1A, 100 V). The voltage corresponding to point *A* is 227.5 V. Hence, no-load voltage to which the generator will build-up is 227.5 V.

The tangent *OT* represents the critical resistance at 1500 r.p.m. considering point *B*, R_c = 225/1.5 = 150 Ω.

For 1200 r.p.m., the induced voltages for different field currents would be (1200/1500) = 0.8 of those for 1500 r.p.m. The values of these voltages are tabulated below :

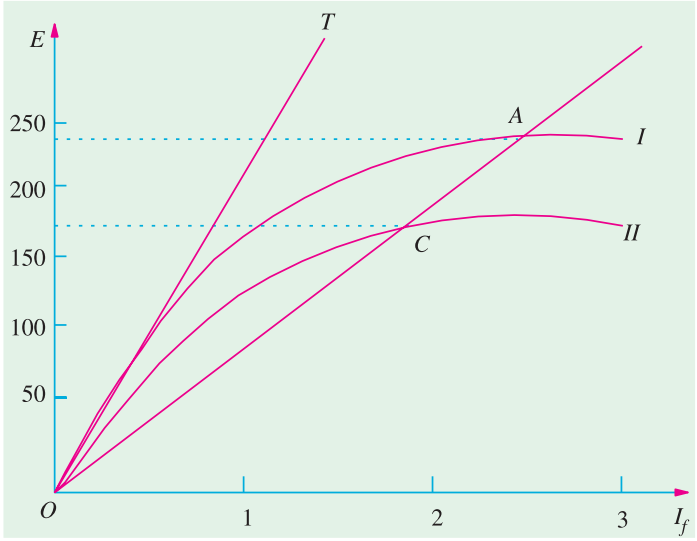


Fig. 28.9

I_f (A) :	0	0.4	0.8	1.2	1.6	2.0	2.4	2.8	3.0
E_0 (V) :	4.8	48	96	138	162	176.8	184.8	189.6	192

The new magnetisation curve is also plotted in Fig. 28.9. The 100 Ω line cuts the curve at point *C* which corresponds to an induced voltage of 166 V.

Example 28.2. *A shunt generator is to be converted into a level compounded generator by the addition of a series field winding. From a test on the machine with shunt excitation only, it is found that the shunt current is 3.1 A to give 400 V on no load and 4.8 A to give the same voltage when the machine is supplying its full load of 200 A. The shunt winding has 1200 turns per pole. Find the number of series turns required per pole.* **(Elect. Machines, A.M.I.E. Sec. B, 1989)**

Solution. At no-load the ampere turns required to produce 400 V = 3.1 × 1200 = 3720

On full-load ampere turns required to produce the same voltage = 4.8 × 1200 = 5760

Additional ampere turns required due to de-magnetising effect of load current = 5760 – 3720 = 2040.

If N is a number of series turns required when load current is 200 A, then

$$N \times 200 = 2040, \; N = 10.2$$

Example 28.3. *The open-circuit characteristic of a d.c. shunt generator driven at rated speed is as follows :*

Field Amperes :	*0.5*	*1.0*	*1.5*	*2.0*	*2.5*	*3.0*	*3.5 A*
Induced Voltage :	*60*	*120*	*138*	*145*	*149*	*151*	*152 V*

If resistance of field circuit is adjusted to 53 Ω, calculate the open circuit voltage and load current when the terminal voltage is 100 V. Neglect armature reaction and assume an armature resistance of 0.1 Ω. **(Electrical Technology Punjab Univ. Dec. 1989)**

Solution. Take I_f = 3 A, R_{sh} = 53 Ω,

Point *A* is (3A, 159V) point. Line *OA* is the 53 Ω line in Fig. 28.10. It cuts drop = 3 × 53 = 159 V. *O.C.C.* at *B*. Line *BM* is drawn parallel to the base. *OM* represents the *O.C.* voltage which equals **150 V.**

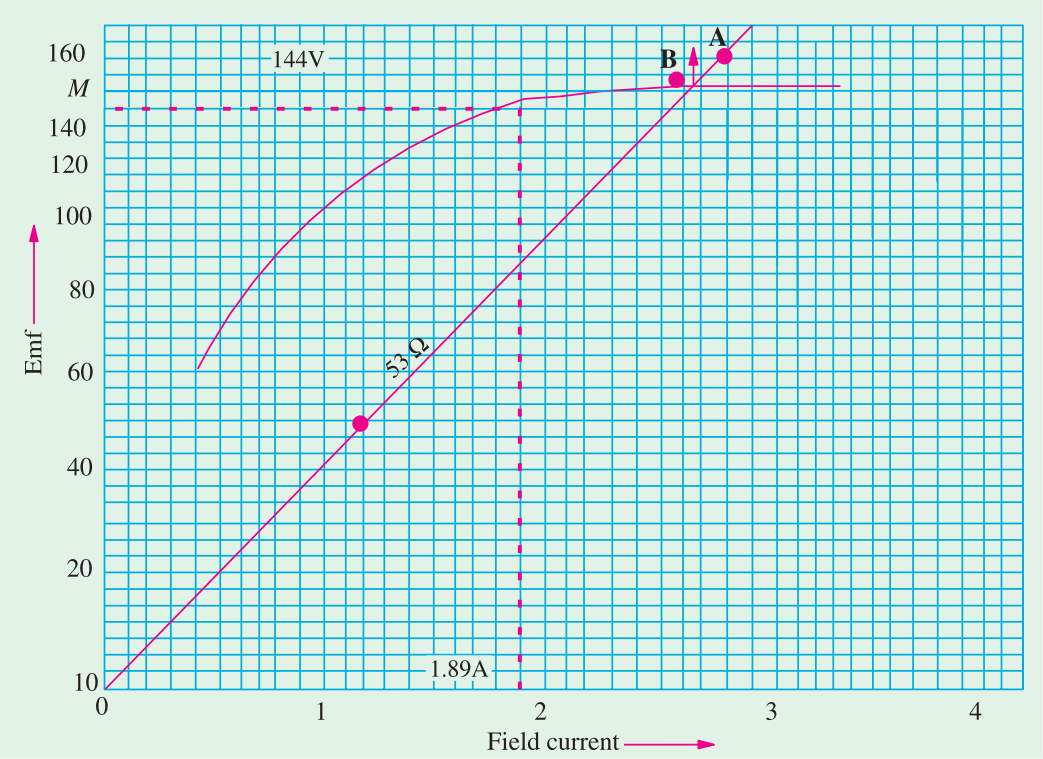


Fig. 28.10

Now, when $V = 100$ V, $I_{sh} = I_f = 100/53 = 1.89$ A

Generated or *O.C.* voltage corresponding to this exciting current as seen from graph of Fig. 26.10 is 144 V.

Now $E = V + I_a R_a$ or $I_a R_a = 144 - 100 = 44$ V

$\therefore$ $0.1\, I_a = 44$ or $I_a = 44/0.1 =$ **440 A**

Example 28.4. *The following figures give the O.C.C. of a d.c. shunt generator at 300 r.p.m.*

Field amperes :	0	2	3	4	5	6	7
Armature volt :	7.5	92	132	162	183	190	212

Plot the O.C.C. for 375 r.p.m. and determine the voltage to which the machine will excite if field circuit resistance is 40 Ω.

(a) *What additional resistance would have to be inserted in the field circuit to reduce the voltage to 200 volts at 375 r.p.m.?*

(b) *Without this additional resistance, determine the load current supplied by the generator, when its terminal voltage is 200 V. Ignore armature reaction and assume speed to be constant. Armature resistance is 0.4 Ω.* **(Elect. Machines - I, South Gujarat Univ. 1986)**

Solution. The e.m.f. induced at 375 r.p.m. would be increased in the ratio 375/300 corresponding to different shunt field current values. A new table is given with the voltages multiplied by the above ratio.

Field amperes :	0	2	3	4	5	6	7
Armature volt :	9.4	115	165	202.5	228.8	248.8	265

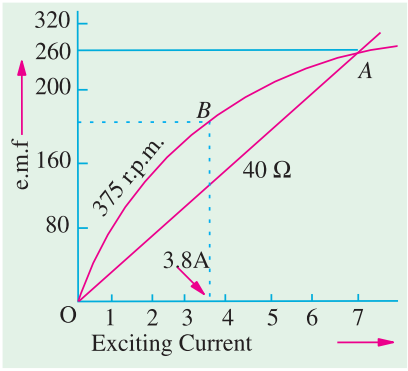


Fig. 28.11

The new *O.C.C.* at 375 r.p.m. is shown in Fig. 26.11. Line *OA* represents 40-Ω line.

The voltage corresponding to point *A* is 260 V. Hence machine will excite to 260 volt with 40 Ω shunt field resistance.

(a) From Fig. 28.11, it is clear that for exciting the generator to 200 V, exciting current should be 3.8 A.

∴ Field circuit resistance = 200/3.8 = 52.6 Ω

∴ Additional resistance required = 52.6 – 40

= **12.6 Ω**

(b) In this case, shunt field resistance = 40 Ω ...(as above)

Terminal voltage = 200 V ∴ Field current = 200/40 = 5 A

Generated e.m.f. for exciting current of 5 A = 228.8 V

For a generator $E = V + I_a R_a$ ∴ $I_a R_a = E - V$ or $0.4\, I_a = 228.8 - 20 = 28.8$

∴ $I_a = 28.8/0.4 = 72$ A ∴ Load current $I = 72 - 5 =$ **67 A**

Example 28.5. *The open-circuit characteristic of a separately-excited d.c. generator driven at 1000 r.p.m. is as follows :*

Field current :	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6
E.M.F. volts :	30.0	55.0	75.0	90.0	100.0	110.0	115.0	120.0

If the machine is connected as shunt generator and driven at 1,000 r.p.m. and has a field resistance of 100 Ω, find **(a)** *open-circuit voltage and exciting current* **(b)** *the critical resistance and* **(c)** *resistance to induce 115 volts on open circuit.*

(Elect. Machines, Nagpur, Univ. 1993)

Solution. The *O.C.C.* has been plotted in Fig. 28.12. The shunt resistance line *OA* is drawn as usual.

(a) *O.C.* voltage = **100 V**; Exciting current = **1 A**

(b) Line *OT* is tangent to the initial part of the *O.C.C.* It represents critical resistance. As seen from point *C*, value of critical resistance is 90/0.6 = **150 Ω.**

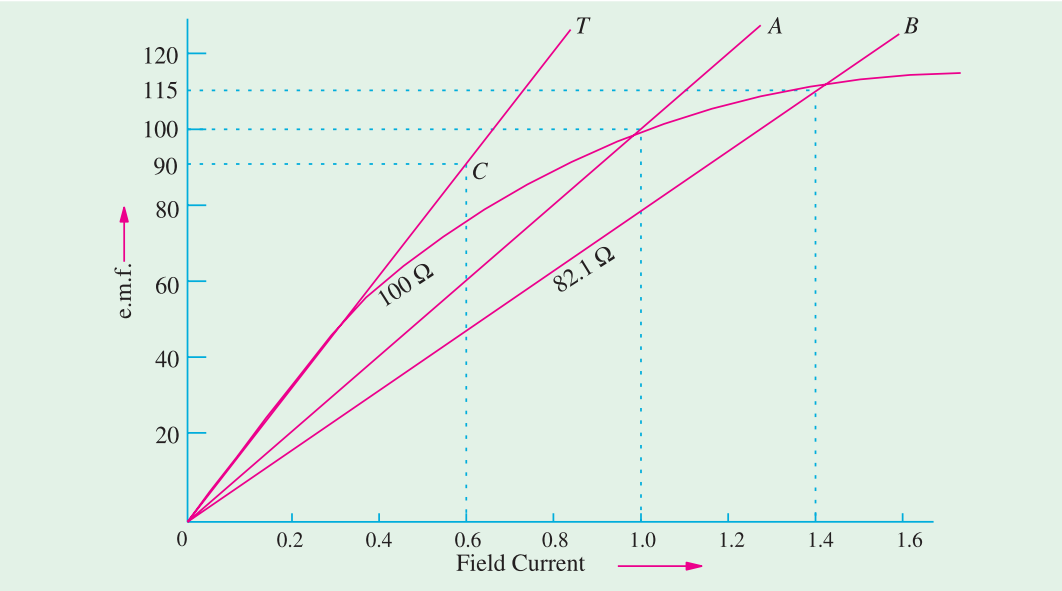


Fig. 28.12

(*c*) Line *OB* represents shunt resistance for getting 115 V on open-circuit. Its resistance = 115/1.4 = **82.1 Ω.**

Example 28.6. *A d.c. generator has the following magnetisation characteristics.*

Field current (A) :	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*
Generated e.m.f. (V) :	*23*	*45*	*67*	*85*	*100*	*112*	*121*	*126*

If the generator is shunt excited, determine the load current.

(*a*) *when terminal p.d. is 120 V, the field resistance is 15 Ω at a speed of 600 r.p.m. And*

(*b*) *when terminal p.d. is 144 V, the field resistance is 18 Ω at a speed of 700 r.p.m.*

Solution. (*a*) When terminal p.d. = 120 V, then field current $I_f = V/R_{sh} = 120/15 = 8$ Ω

From the given data, it is seen that the generated e.m.f. = 126 V*

Voltage drop = 126 – 120 = 6 V

Since drop due to armature reaction is neglected, this represents the armature drop.

$\therefore \quad I_a R_a = 6 \quad$ or $\quad I_a = 6/0.02 = 300$ A Load current = 300 – 8 = **292 A**

(*b*) The *O.C.* data at 700 r.p.m. can be obtained by multiplying the given values of generated e.m.f. by a factor of 700/600 = 7/6. Hence, the new data is :

Field current (*A*)	:	1	2	3	4	5	6	7	8
Generated e.m.f. (*V*)	:	26.8	52.5	78.2	99.2	116.6	131	141	146

When $V = 144$ V and $R_{sh} = 18$ Ω, field current = 144/18 = 8 A

From the given data, the corresponding generated e.m.f. is 146 V.

Voltage drop $I_a R_a = 146 - 144 = 2$V

$\therefore \quad I_a = 2/0.02 = 100$ A $\therefore$ Load current = 100 – 8 = **92 A**

Example 28.7. *The O.C.C. of a d.c. generator driven at 400 rev/min is as follows :*

Field current (A)	:	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
Terminal volts	:	*110*	*155*	*186*	*212*	*230*	*246*	*260*	*271*

* We need not plot the *O.C.C.* in this particular case.

Find :

(a) *voltage to which the machine will excite when run as a shunt generator at 400 rev/min with shunt field resistance equal to 34 Ω.*

(b) *resistance of shunt circuit to reduce the O.C. voltage to 220 V.*

(c) *critical value of the shunt field circuit resistance.*

(d) *the critical speed when the field circuit resistance is 34 Ω.*

(e) *lowest possible speed at which an O.C. voltage of 225 V can be obtained.*

(Electrical Technology, Bombay Univ. 1987)

Solution. The *O.C.C.* as plotted from the given data is shown in Fig. 28.13. The 34-Ω line *OA* is drawn as usual.

(a) The voltage to which machine will excite = *OM* = **255 V.**

(b) The horizontal line from *N* (220 V) is drawn which cuts the *O.C.C.* at point *B.* Resistance represented by line *OB* = 220/5.4 = **40.7 Ω.**

(c) Line *OC* has been drawn which is tangential at the origin to the *O.C.C.* This represents the value of critical resistance = 140/2.25 = **62.2 Ω.**

(d) Take any convenient point *D* and erect a perpendicular which cuts both *OA* and *OC*.

$$\frac{DE}{DF} = \frac{N_C}{400} \text{ or } \frac{110}{202} = \frac{N_C}{202}, N_C = \textbf{218 r.p.m.}$$

(e) From point *P* (225 V) drawn a horizontal line cutting *OA* at point *G*. From *G*, draw a perpendicular line *GK* cutting the *O.C.C.* at point *H*. If *N′* is the lowest speed possible for getting 225 volt with 34 Ω shunt circuit resistance, then

$$\frac{GK}{HK} = \frac{N'}{400} \text{ or } \frac{225}{241} \text{ or } N' = \textbf{375 r.p.m.}$$

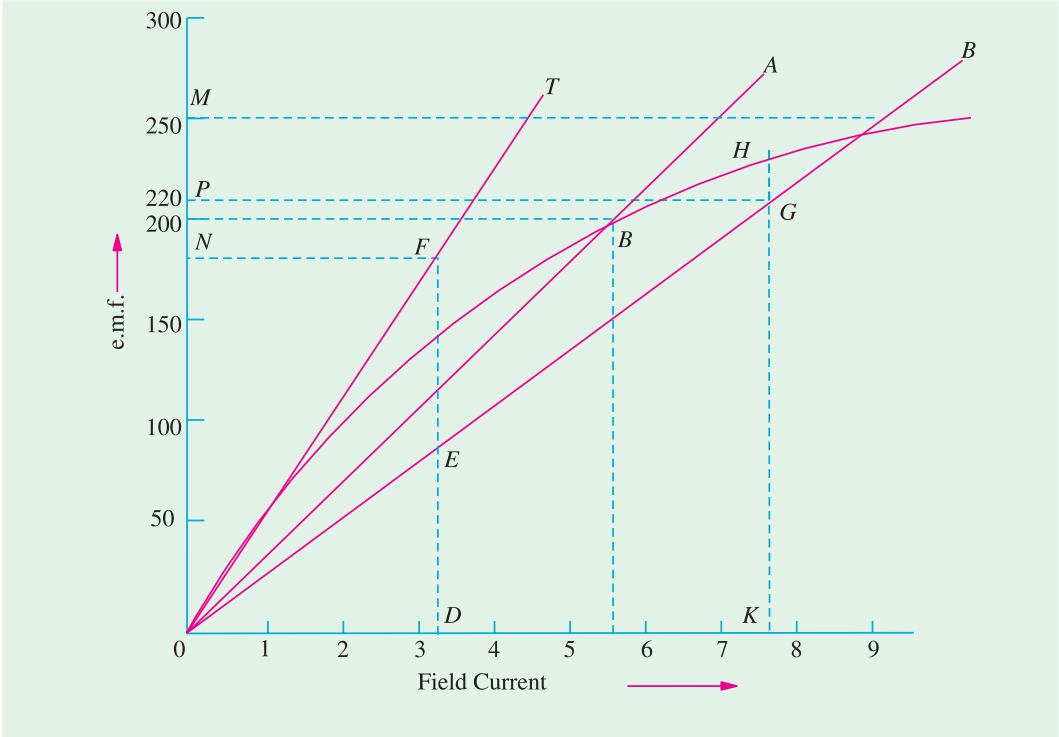


Fig. 28.13

Example 28.8. *The magnetization characteristic for a 4-pole, 110-V, 1000 r.p.m. shunt generator is as follows :*

Field current	*0*	*0.5*	*1*	*1.5*	*2*	*2.5*	*3 A*
O.C. voltage	*5*	*50*	*85*	*102*	*112*	*116*	*120 V*

Armature is lap-connected with 144 conductors. Field resistance is 45 ohms. Determine

(*i*) *voltage the machine will build up at no-load.*

(*ii*) *the critical resistance.*

(*iii*) *the speed at which the machine just fails to excite.*

(*iv*) *residual flux per pole.*

(Electrical Machinery- I, Mysore Unit. 1988)

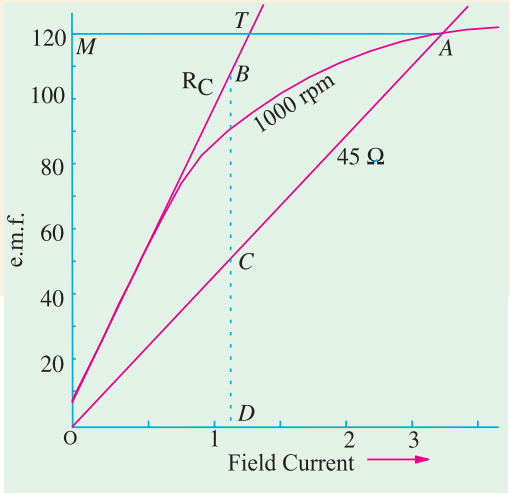


Fig. 28.14

Solution. In Fig. 28.14, *OA* represents the 45-Ω line which is drawn as usual.

(*i*) The voltage to which machine will build up = *OM* = **118 V.**

(*ii*) *OT* is tangent to the initial part of the *O.C.C.* It represents critical resistance. Take point *B* lying on this line. Voltage and exciting current corresponding to this point are 110 V and 1.1 A respectively.

$$\therefore \qquad R_c = 110/1.1 = \mathbf{100\ \Omega}$$

(*iii*) From any point on *OT*, say point *B*, drop the perpendicular *BD* on *X*-axis.

$$\frac{CD}{BD} = \frac{N_c}{1000} \quad \text{or} \quad \frac{49}{110} = \frac{N_c}{1000}$$

$$\therefore \qquad N_c = \mathbf{445\ r.p.m.}$$

(*iv*) As given in the table, induced e.m.f. due to residual flux (*i.e.* when there is no exciting current) is 5 V.

$$\therefore \qquad 5 = \frac{\Phi \times 144 \times 1000}{60}\left(\frac{4}{4}\right) \qquad \therefore\ \Phi = \mathbf{2.08\ mWb.}$$

Example 28.9. *A shunt generator gave the following results in the O.C.C. test at a speed of r.p.m.*

Field current (A) :	*1*	*2*	*3*	*4*	*6*	*8*	*10*
E.M.F. (volt) :	*90*	*185*	*251*	*290*	*324*	*345*	*360*

The field resistance is adjusted to 50 Ω and the terminal is 300 V on load. Armature resistance is 0.1 Ω and assuming that the flux is reduced by 5% due to armature reaction, find the load supplied by the generator. **(Electromechanic, Allahabad Univ.; 1992)**

Solution. When the terminal voltage is 300 V and R_{sh} = 50 Ω, then field current is

$$= 300/40 = 6 \text{ A}$$

With this shunt current, the induced e.m.f. as seen from the given table (we need not draw the *O.C.C.*) is 324 V.

Due to armature reaction, the flux and hence the induced e.m.f. is reduced to 0.95 of its no-load value.

Hence, induced e.m.f. when generator is no load = 324 × 0.95 = 307.8 V

Armature drop at the given load = 307.8 – 300 = 7.8 V

$I_a R_a$ = 7.8, I_a = 7.8/0.1 = 78 A

Load current = 78 – 6 = 72 A ; Generator output = 72 × 300/1000 = **21.6 kW**

Example 28.10. *A shunt generator gave the following open-circuit characteristic :*

Field current :	*0.5*	*1.0*	*1.5*	*2.0*	*2.5*	*3.0*	*3.5 A*
O.C. e.m.f. :	*54*	*107*	*152*	*185*	*210*	*230*	*245 V*

The armature and field resistances are 0.1 Ω and 80 Ω respectively. Calculate ***(a)*** *the voltage to which the machine will excite when run as a shunt generator at the same speed.*

(b) *The volts lost due to armature reaction when 100 A are passing in the armature at a terminal voltage of 175 V.*

(c) *The percentage reduction in speed for the machine to fail to excite on open circuit.*

(Electrical Machines-I, Bombay Univ. 1988)

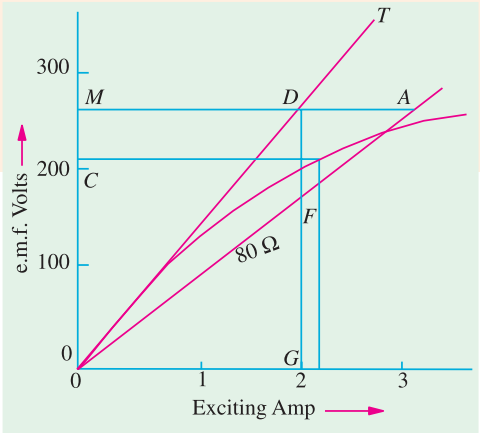


Fig. 28.15

Solution. ***(a)*** *O.C.C.* is shown in Fig. 28.15. *OA* represents 80 Ω line. The maximum voltage to which the generator will build up is given by *OM* = **222 V.**

(b) With 175 V terminal p.d. on load

I_{sh} = 175/80 = 2.2 A

Voltage corresponding to this field current is given by *OC* = 195 V.

Voltage lost due to armature reaction and armature drop = 195 – 175 = 20 V.

Now, armature drop = 0.1 × 100 = 10 V

Let '*x*' be the volts lost due to armature reaction.

Then 10 + *x* = 20 ∴ *x* = **10 V**

(c) Line *OT* is drawn tangential to the curve. *DFG* is perpendicular to the base line.

$$\frac{N_c}{N} = \frac{FG}{DG} = \frac{160}{220} \quad \text{or} \quad \frac{N_e - N}{N} = \frac{-60}{220}$$

Percentage reduction in speed = $\frac{-60}{220} \times 100 =$ **– 27.3 %**

Example 28.11. *The O.C.C. of a shunt generator running at 800 r.p.m. is as follows :*

Field current (amp.)	*:*	*1*	*2*	*3*	*4*	*5*	*6*
Induced e.m.f. (volt).	*:*	*82.5*	*180*	*225*	*252*	*273*	*282*

If the shunt field resistance is 60 Ω, find

(i) *the voltage to which the machine will build up running at the same speed* ***(ii)*** *the value of field regulating resistance if the machine is to build up to 120 V when its field coils are grouped in two parallel circuits and generator is runing at half the speed.*

(Electrical Technology, Hyderabad Univ. 1991)

Solution. *O.C.C.* is drawn from the given data and is shown in Fig. 28.16. *OA* represents 60 Ω. The voltage corresponding to point *A* is *OM* = 260 V.

(i) Generator will build up to **260 volts.**

(ii) Lower curve represents the induced e.m.f. for different exciting current values at 400 r.p.m. From point *B* which represents 120 V, draw *BC*. From *C* draw a perpendicular *CD* which gives the

exciting current of 3.6 A. It means that current through *each* parallel path of shunt field coils is 3.6 A. Total current which passes through the field regulator resistance is 3.6 × 2 = 7.2 A because it is in series with the field coils.

Hence, total shunt field resistance = 120/7.2 = 16.67 Ω

Now, resistance of each shunt parallel path = 60/2 = 30 Ω

Joint resistance of two parallel paths = 30/2 = 15 Ω

∴ Shunt field regulator resistance = 16.67 – 15 = **1.67 Ω**

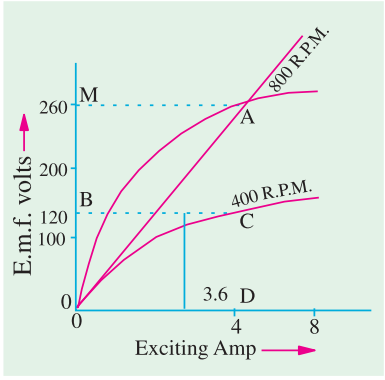


Fig. 28.16

28.7. Voltage Build up of a Shunt Generator

Before loading a shunt generator, it is allowed to build up its voltage. Usually, there is always present some residual magnetism in the poles, hence a small e.m.f. is produced initially. This e.m.f. circulates a small current in the field circuit which increases the pole flux (provided field circuit is properly connected to armature, otherwise this current may wipe off the residual magnetism). When flux is increased, generated e.m.f. is increased which further increases the flux and so on. As shown in Fig. 28.17, *Oa* is the induced e.m.f. due to residual magnetism which appears across the field circuit and causes a field current *Ob* to flow. This current aids residual flux and hence produces, a larger induced e.m.f. *Oc*. In turn, this increased e.m.f. *Oc* causes an even larger current *Od* which creates more flux for a still larger e.m.f. and so on.

Now, the generated e.m.f. in the armature has

(*a*) to supply the ohmic drop $I_f R_{sh}$ in the winding and (*b*) to overcome the opposing self-induced e.m.f. in the field coil *i.e. L.* ($d I_f / dt$)because field coils have appreciable self-inductance.

$$e_g = I_f R_{sh} + L \cdot d I_f/dt$$

If (and so long as), the generated e.m.f. is in excess of the ohmic drop $I_f R_{sh}$, energy would continue being stored in the pole fields. For example, as shown in Fig. 28.17, corresponding to field current *OA*, the generated e.m.f. is *AC*. Out of this, *AB* goes to supply ohmic drop $I_f R_{sh}$ and *BC* goes to overcome self-induced e.m.f. in the coil. Corresponding to $I_f = OF$, whole of the generated e.m.f. is used to overcome the ohmic drop. None is left to overcome $L.dI_f/dt$. Hence no energy is stored in the pole fields. Consequently, there is no further increase in pole flux and the generated e.m.f. With the given shunt field resistance represented by line *OP*, the maximum voltage to which the machine will build up is *OE*. If resistance is decreased, it will built up to a somewhat higher voltage. *OR* represents the resistance known as ***critical resistance***. If shunt field resistance is greater than this value, the generator will fail to excite.

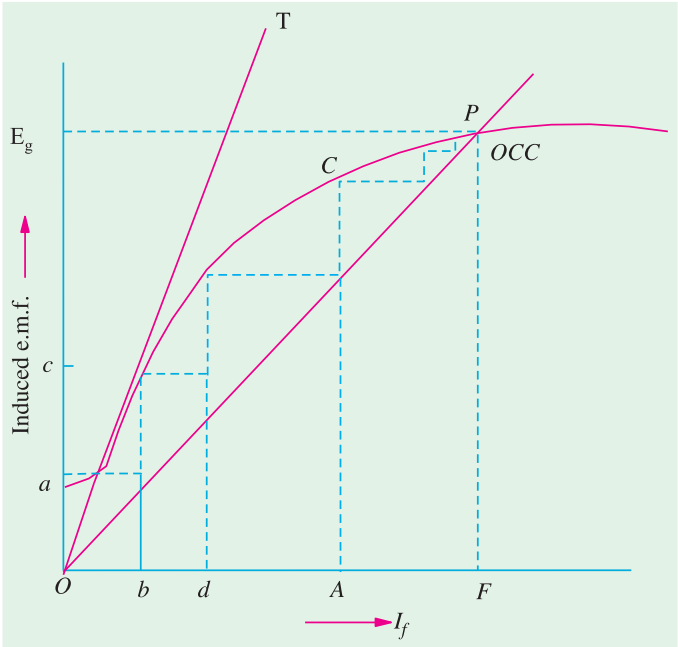


Fig. 28.17

28.8. Conditions for Build-up of a Shunt Generator

We may summarize the conditions necessary for the build-up of a (self-excited) short generator as follows :

1. There must be some residual magnetism in the generator poles.

2. For the given direction of rotation, the shunt field coils should be correctly connected to the armature *i.e.* they should be so connected that the induced current reinforces the e.m.f. produced initially due to residual magnetism.

3. If excited on open circuit, its shunt field resistance should be less than the critical resistance (which can be found from its *O.C.C.*)

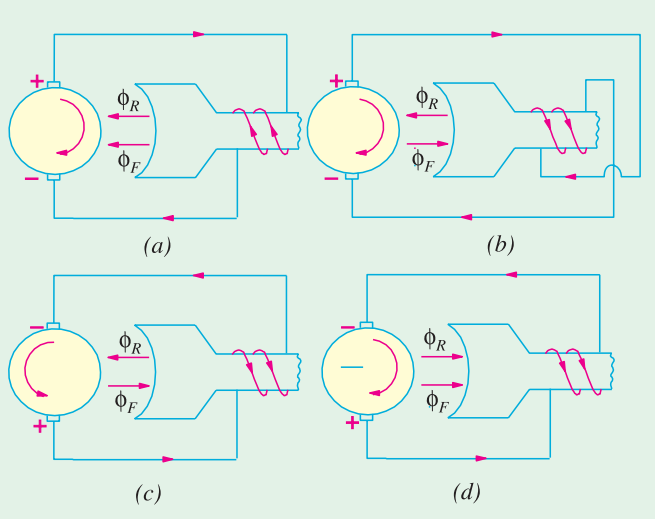


Fig. 28.18

4. If excited on load, then its shunt field resistance should be more than a certain minimum value of resistance which is given by internal characteristic (Art 28.11).

28.9. Other Factors Affecting Voltage Building of a DC Generator

In addition to the factors mentioned above, there are some other factors which affect the voltage building of a self-excited d.c. generator. These factors are ***(i)*** reversed shunt field connection ***(ii)*** reversed rotation and ***(iii)*** reversed residual magnetism. These adverse effects would be explained with the help of Fig. 28.18 and the right-hand rule for finding the direction of the coil flux. For the sake of simplicity, only one field pole has been shown in the Fig. 28.18.

Fig. 28.18 (*a*) represents the normal operation, the prime mover rotation is clockwise and both the residual flux Φ_R and the field flux Φ_F are directed to the left.

Fig. 28.18 (*b*) shows reversed connection of the field circuit which causes Φ_F to oppose Φ_R. Consequently, the generator voltage builds down from its original residual value.

In Fig. 28.18 (*c*), reversed armature rotation causes the reversal of the voltage produced by the residual magnetism. Even though the field coil connections are correct, the reversed field current flow causes Φ_F to oppose Φ_R so that the voltage builds down from its original residual value.

Fig. 28.18 (*d*) shows the case when due to some reason the residual magnetism gets reversed. Hence, the armature voltage is also reversed which further reverses the field current. Consequently, both Φ_F and Φ_R are reversed but are directed to the right as shown. Under this condition, the voltage buildup is in the reversed direction. Obviously, the generator will operate at rated voltage but with reversed polarity.

If desired, the reversed polarity can be corrected by using an external d.c. source to remagnetise the field poles in the correct direction. This procedure is known as field flashing.

Example 28.12. *The O.C.C. of a generator is given by the following :*

Field current :	*1.5*	*3.5*	*4.5*	*6*	*7.5*	*9*	*10.5*
E.M.F. :	*168*	*330*	*450*	*490*	*600*	*645*	*675*

The speed at which data is obtained is 1000 r.p.m. Find the value of the shunt field resistance that will give a p.d. of 600 V with an armature current of 300 A at the same speed. Due to armature

reaction, the shunt field current is given by I_{sh} (eff.) = I_{sh} – 0.003 I_a. Armature resistance, including brush contact resistance, is 0.1 Ω. What will be the p.d. on open circuit at the same speed ?

Solution. As shown in Fig. 28.19, the *O.C.C.* has been plotted from the given data.

Voltage drop due to armature resistance = 300 × 0.1 = 30 V.

Reduction of field current due to armature reaction = 0.003 × 300 = 0.9 A.

Any point *A* is taken on the *O.C.C.* A vertical distance *AB* = 30 V is taken and then the horizontal line *BC* = 0.9 A is drawn thus completing triangle *ABC* which is known as drop reaction triangle. Then, point *C* lies on the 300 ampere load saturation curve. This curve can be drawn by finding such similar points like *C′* etc.

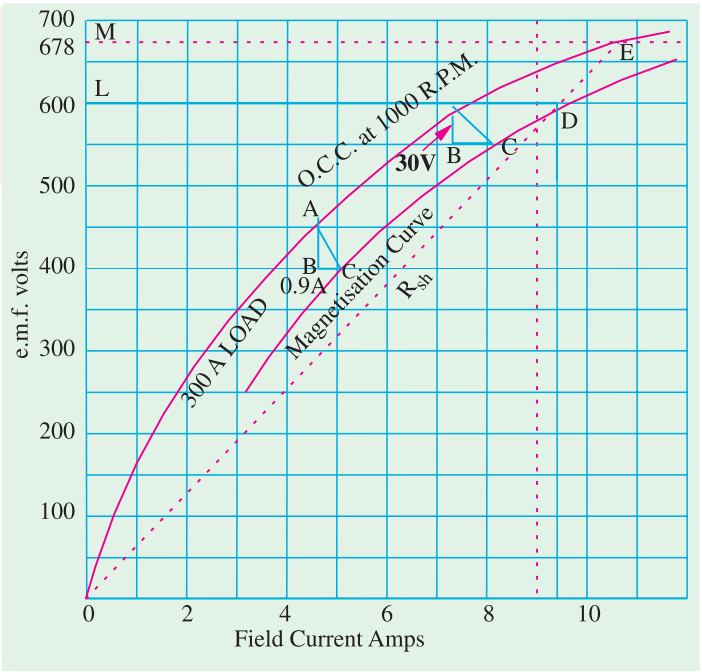


Fig. 28.19

From point *L* representing 600 V, a horizontal line is drawn cutting the load saturation curve at *D*. Join *OD*. Current corresponding to point *D* is 9.4 A. The slope of I_f the line *OD* gives the value of shunt resistance to give 600 V with 300 amperes of armature current.

$\therefore \quad R_{sh}$ = 600/9.4 = **63.8 Ω**

It is seen that e.m.f. on open circuit is **678 V.**

28.10. External Characteristic

After becoming familiar with the no-load characteristic of a shunt generator, we will now proceed to find its external characteristic (*V/I*) when loaded. It is found that if after building up, a shunt generator is loaded, then its terminal voltage *V* drops with increase in load current. Such a drop in voltage is undesirable especially when the generator is supplying current for light and power for which purpose it is desirable that *V* should remain practically constant and independent of the load. This condition of constant voltage is almost impossible to be fulfilled with a shunt generator unless the field current is being automatically adjusted by an automatic regulator. Without such regulation terminal voltage drops considerably as the load on the generator is increased. These are three main reasons for the drop in terminal voltage of a shunt generator when under load.

(*i*) Armature resistance drop :

As the load current increases, more and more voltage is consumed in the ohmic resistance of the armature circuit. Hence, the terminal voltage $V = E - I_a R_a$ is decreased where *E* is the induced e.m.f. in the armature under load condition.

(*ii*) Armature reaction drop

Due to the demagnetising effect of armature reaction, pole flux is weakened and so the induced e.m.f. in the armature is decreased.

(*iii*) The drop in terminal voltage *V* due to (*i*) and (*ii*) results in a decreased field current I_f which further reduces the induced e.m.f.

For obtaining the relation between the terminal voltage and load current, the generator is connected as shown in Fig. 28.20 (*a*).

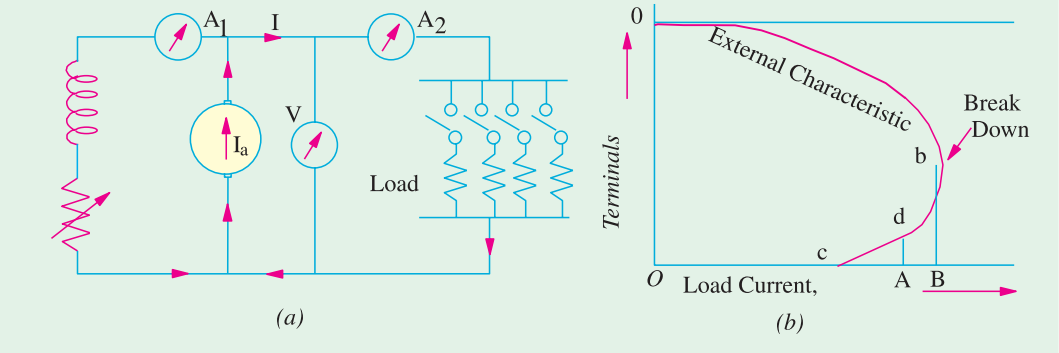


Fig. 28.20

The shunt generator is first excited on no-load so that it gives its full open circuit voltage = *Oa* [Fig. 28.20 (*b*)]. Then, the load is gradually applied and, at suitable intervals, the terminal voltage *V* (as read by the voltmeter) and the load current *I* (as read by the ammeter A_2) are noted. The field current as recorded by ammeter A_1 is kept constant by a rheostat (because during the test, due to heating, shunt field resistance is increased). By plotting these readings, the external characteristic of Fig. 28.20 (*b*) is obtained. The portion *ab* is the working part of this curve. Over this part, if the load resistance is decreased, load current is increased as usual, although this results in a comparatively small additional drop in voltage. These conditions hold good till point *b* is reached. This point is known as breakdown point. It is found that beyond this point (where load is maximum = *OB*) ***any effort to increase load current by further decreasing load resistance results in decreased load current (like OA) due to a very rapid decrease in terminal voltage.***

We will discuss the reason for this unusual behaviour of the generator in more details. Over the earlier portion *ab* [Fig. 28.20 (*b*)] where the load current is comparatively small, when external load resistance is decreased, it results in increased load current as might be expected keeping Ohm's law in mind. It should not, however, be forgotten that due to increase in load current, *V* is also decreased somewhat due to the cause (*iii*) given above. But over the portion *ab*, the effect of decrease in load resistance predominates the effect of decrease in *V* because load current is relatively small.

At point *b*, generator is delivering a very large current *i.e.* current which is many times greater than its normal current. If load resistance is decreased at this point so as to be able to draw a load current greater than *OB*, the current is ***increased*** momentarily. But due to the severe armature reaction for this heavy current and increased $I_a R_a$ drop, the terminal voltage *V* is drastically reduced. The effect of this drastic reduction in *V* results in less load current (= *OA*). In other words, over the portion *bdc* of the curve, the terminal voltage *V* decreases more rapidly than the load resistance. Hence, any, further decrease in load resistance actually causes a ***decrease*** in load current (it may seem to contravene Ohm's law but this law is not applicable, here since *V* is not constant). As load resistance is decreased beyond point *b*, the curve turns back till when the generator is actually short-circuited, it cuts the current axis at point *c*. Here, terminal voltage *V* is reduced to zero, though there would be some value of *E* due to residual magnetism (Fig. 28.22).

28.11. Voltage Regulation

By voltage regulation of a generator is meant the change in its terminal voltage with the change in load current when it is running at a constant speed. If the change in voltage between no-load and full load is small, then the generator is said to have good regulation but if the change in voltage is large, then it has poor regulation. The ***voltage regulation of a d.c. generator is the change in voltage when the load is reduced from rated value to zero, expressed as percentage of the rated load***

voltage.

If no-load voltage of a certain generator is 240 V and rated-load voltage is 220 V,

then, regn. = (240 – 220)/220 = 0.091 or 9.1 %

28.12. Internal or Total Characteristic

As defined before, internal characteristic gives the relation between E and I_a. Now in a shunt generator

$$I_a = I + I_f \quad \text{and} \quad E = V + I_a R_a$$

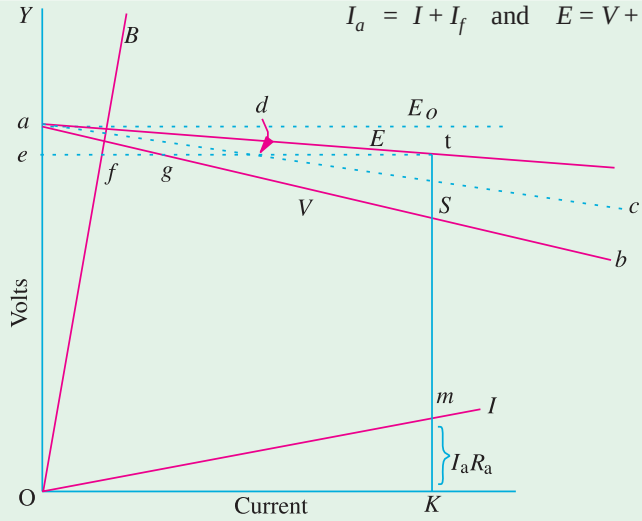


Fig. 28.21

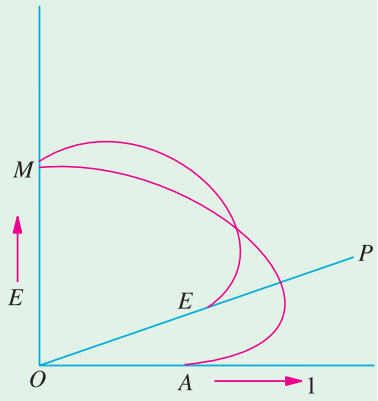


Fig. 28.22

Hence, E/I_a curve can be obtained from V/I curve as shown in Fig. 28.21. In this figure, *ab* represents the external characteristic as discussed above. The field resistance line *OB* is drawn as usual. The horizontal distances from *OY* line to the line *OB* give the values of field currents for different terminal voltages. If we add these distances horizontally to the external characteristic *ab*, then we get the curve for the total armature current *i.e.* dotted curve *ac*. For example, point *d* on *ac* is obtained by making *gd* = *ef*. The armature resistance drop line *Or* is then plotted as usual. If brush contact resistance is assumed constant, then armature voltage drop is proportional to the armature current. For any armature current = *OK*, armature voltage drop $I_a R_a = mK$. If we add these drops to the ordinates of curve *ac*, we get the internal characteristic. For example, $St = mK$. The point *t* lies on the internal characteristic. Other points like *t* can be found similarly at different armature currents as the total characteristic can be drawn.

It may be noted here, in passing, that product EI_a gives the total power developed within the armature. Some of this power goes to meet $I^2 R$ losses in armature and shunt field windings and the rest appears as output.

As explained in Art 28.10 if load resistance is decreased, the armature current increases up to a certain load current value. After that, any decrease in load resistance is not accompanied by increase in load current. Rather, it is decreased and the curve turns back as shown in Fig. 28.22. If the load resistance is too small, then the generator is short-circuited and there is no generated e.m.f. due to heavy demagnetisation of main poles.

Line *OP* is tangential to the internal characteristic *MB* and its slope gives the value of the ***minimum*** resistance with which the generator will excite if excited ***on load.***

28.13. Series Generator



Series generator

In this generator, because field windings are in series with the armature [Fig. 28.23 (*a*)], they carry full armature current I_a. As I_a is increased, flux and hence generated e.m.f. is also increased as shown by the curve. Curve *Oa* is the *O.C.C.* The extra exciting current necessary to neutralize the weakening effect of armature reaction at full load is given by the horizontal distance *ab*. Hence, point *b* is on the internal characteristic. If the ordinate *bc* = *gh* = armature voltage drop, then point *c* lies on the external characteristic [Fig. 28.23 (*b*)].

It will be noticed that a series generator has rising voltage characteristic *i.e.* with increase in load, its voltage is also increased. But it is seen that at high loads, the voltage starts decreasing due to excessive demagnetising effects of armature reaction. In fact, terminal voltage starts decreasing as load current is increased as shown by the dotted curve. For a load current *OC′*, the terminal voltage is reduced to zero as shown.

Example 28.13. *In a 220 V supply system, a series generator, working on a linear portion of its magnetisation characteristic is employed as a booster. The generator characteristic is such that induced e.m.f. increases by 1 volt for every increase of 6 amperes of load current through the generator. The total armature resistance of the generator is 0.02 Ω. If supply voltage remains constant, find the voltage supplied to the consumer at a load current of 96 A. Calculate also the power supplied by the booster itself.*

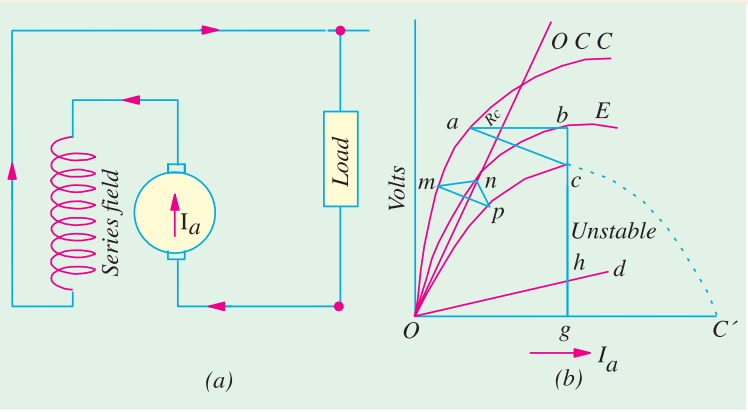


Fig. 28.23

Solution.

Voltage increase for 6 amperes = 1 V ∴ Voltage increase for 96 A = 96/6 = 16 V

Voltage drop in series coils = $96 \times 0.02 = 1.9$ V

Net Voltage rise due to booster = $16 - 1.9 = 14.1$ V

Voltage at consumer end = $220 + 14.1 =$ **234.1 V**

Power supplied by booster itself = $14.1 \times 96 = 1354$ W = **1.354 kW**

Example 28.14. *A d.c. series generator, having an external characteristic which is a straight line through zero to 50 V at 200 A is connected as a booster between a station bus bar and a feeder of 0.3 ohm resistance. Calculate the voltage difference between the station bus-bar and the far end of the feeder at a current of (i) 200 A and (ii) 50 A.*

(AIME Sec. B Elect. Machine Summer 1991)

Solution. (*i*) Voltage drop = $200 \times 0.3 = 60$ V

Booster voltage provided by series generator for 200 A current as given = 50 V.

$\therefore$ Net voltage decrease = 60 – 50 = 10 V

(ii) Feeder drop = 50 × 0.3 = 15 V

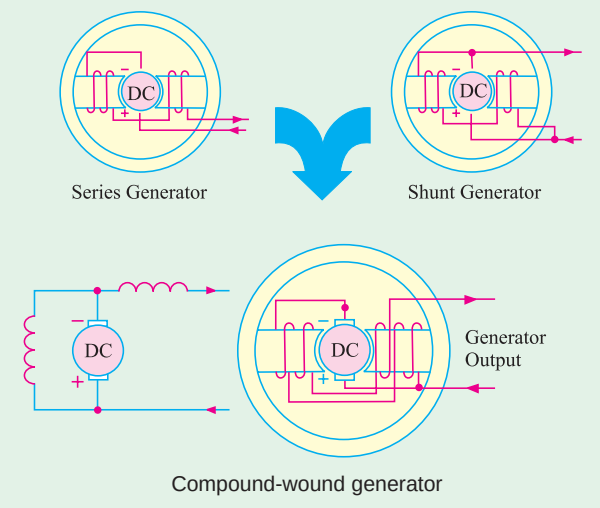
Booster voltage provided by series generator (by proportion) is

= 50 × 50/200 = 12.5 V

$\therefore$ Net decrease in voltage = 15 – 12.5 = 2.5 V

28.14. Compound-wound Generator

A shunt generator is unsuitable where constancy of terminal voltage is essential, because its terminal voltage decreases as the load on it increases. This decrease in *V* is particularly objectionable for lighting circuit where even slight change in the voltage makes an appreciable change in the candle power of the incandescent lamps. A shunt generator may be made to supply substantially constant voltage (or even a rise in voltage as the load increases) by adding to it a few turns joined in series with either the armature or the load (Fig. 28.24). These turns are so connected as to aid to shunt turns when the generator supplies load. As the load current increases, the current through the series windings also increase thereby increasing the flux. Due to the increase in flux, induced e.m.f. is also increased. By adjusting the number of series turns (or series amp-turns), this increase in e.m.f. can be made to balance the combined voltage drop in the generator due to armature reaction and the armature drop. Hence, *V* remains practically constant which means that field current is also almost unchanged. We have already discussed the three causes which decrease the terminal voltage of a shunt generator (Art 28.10). Out of these three, the first two are neutralized by the series field amp-turns and the third one, therefore, does not occur.



If the series field amp-turns are such as to produce the same voltage at rated load as at no-load, then the generator is ***flat-compounded***. It should be noted, however, that even in the case of a flat-compounded generator, the voltage is not constant from no-load to rated- load. At half the load, the voltage is actually greater than the rated voltage as seen from Fig. 28.24.

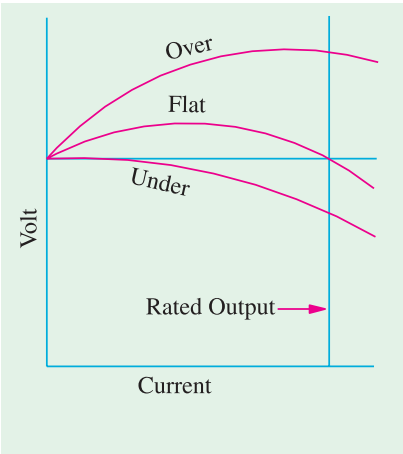


Fig. 28.24

If the series field amp-turns are such that the rated-load voltage is greater than the no-load voltage, then generator is ***over-compounded.*** If rated-load voltage is less than the no-load voltage, then the generator is ***under-compounded*** but such generators are seldom used.

For short distances such as in hotels and office buildings, flat-compound generators are used because the loss of voltage over small lengths of the feeder is negligible. But when it is necessary to maintain a constant voltage then an over-compounded generator, which combines the functions of a generator and a booster, is invariably used.

28.15. How to Calculate Required Series Turns ?

Consider a 110-V, 250-ampere generator. Suppose it gives its rated no-load voltage with a field current of 5.8 A. If, now, the series windings are disconnected and the shunt field rheostat is left unchanged then the machine will act as shunt generator, hence its voltage will fall with increase in load current. Further, supply that the field current has to be increased to 6.3 A in order to maintain the rated terminal voltage at full load. If the number of turns of the shunt field winding is 2000, then 2000 × (6.3 – 5.8) = 1000 amp-turns represent the additional excitation that has to be supplied by the series windings. As series turns will be carrying a full load current of 250 A, hence number of series turns = 1000/250 = 4.

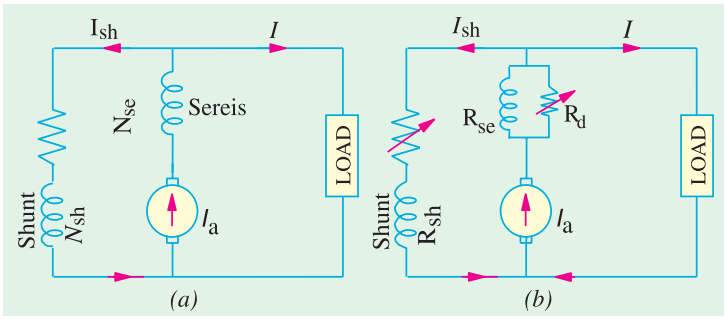


Fig. 28.25

In general, let

ΔI_{sh} = increase in shunt field current required to keep voltage constant from no-load to full-load

N_{sh} = No. of shunt field turns per pole (or the total number of turns)

N_{se} = No. of series turns per pole (or the total number of turns)

I_{se} = current through series winding

= armature current I_a —for long-shunt

= load current I —for short-shunt

It is seen that while running as a simple generator, the increase in shunt field ampere-turns necessary for keeping its voltage constant from no-load to full-load is $N_{sh} \cdot \Delta_{sh}$. This increase in field excitation can be alternatively achieved by adding a few series turns to the shunt generator [Fig. 28.25 (*a*)] thereby converting it into a compound generator.

$$\therefore \qquad N_{sh} \cdot \Delta I_{sh} = N_{se} I_{se}$$

If other things are known, N_{se} may be found from the above equation.

In practice, a few extra series amp-turns are taken in order to allow for the drop in armature. Any surplus amp-turns can be changed with the help of a divertor across the series winding as shown in Fig. 28.25 (*b*).

As said above, the degree of compounding can be adjusted with the help of a variable-resistance, divertor as shown in Fig. 28.25 (*b*). If I_d is the current through the divertor of resistance R_d, then remembering that series windings and divertor are in parallel,

$$\therefore \qquad I_{se} \cdot R_{se} = I_d R_d \quad \text{or} \quad R_d = I_{se} R_{se}/I_d$$

Example 28.15. *A shunt generator is converted into a compound generator by addition of a series field winding. From the test on the machine with shunt excitation only, it is found that a field current of 5 A gives 440 V on no-load and that 6 A gives 440 V at full load current of 200 A. The shunt winding has 1600 turns per pole. Find the number of series turns required.*

(Elect. Machines, A.M.I.E., Sec., B, 1991)

Solution. It would be assumed that shunt generator is converted into a short shunt compound generator. It is given that for keeping the voltage of shunt generator constant at 440 V both at no-load and full-load, shunt field ampere-turns per pole have to be increased from 1600 × 5 = 8000 to

$(1600 \times 6) = 9600$ *i.e.* an increase of $(9600 - 8000) = 1600$ *AT*. The same increase in field *AT* can be brought about by adding a few series turns.

Let *n* be the number of series turns required per pole. Since they carry 200 A,

$\therefore \quad n \times 200 = 1600; n =$ **8 turns/pole**

Example 28.16. *A long shunt compound generator has a shunt field winding of 1000 turns per pole and series field winding of 4 turns per pole and resistance 0.05 Ω. In order to obtain the rated voltage both at no-load and full-load for operation as shunt generator, it is necessary to increase field current by 0.2 A. The full-load armature current of the compound generator is 80 A. Calculate the divertor resistance connected in parallel with series field to obtain flat compound operation.*

(Elect. Machines A.M.I.E. Sec. B, 1993)

Solution.

Additional *AT* required to maintain rated voltage both at no-load and full-load (Fig. 28.26) $= 1000 \times 0.2 = 200$

No. of series turns/pole = 4

Current required to produce 200 *AT* by the series field = 200/4 = 50 A.

Since $I_a = 80$ A, the balance of 30 *A* must pass through the parallel divertor resistance.

$\therefore \quad 30\,R = 50 \times 0.05, R = 0.0833\ \Omega$

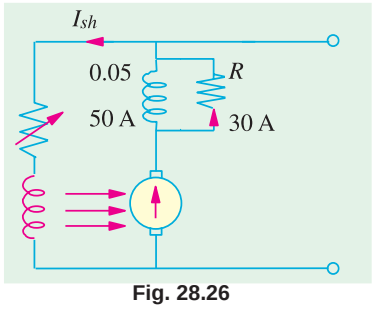


Fig. 28.26

Example 28.17. *A 220-V compound generator is supplying a load of 100 A at 220 V. The resistances of its armature, shunt and series windings are 0.1 Ω, 50 Ω and 0.06 Ω respectively. Find the induced e.m.f. and the armature current when the machine is connected* ***(a)*** *short shunt* ***(b)*** *long shunt* ***(c)*** *how will the series amp-turns be changed in* ***(b)*** *if a divertor of 0.14 Ω is connected in parallel with the series windings ? Neglect armature reaction and brush contact drop.*

Solution. (*a*) Short-shunt (Fig. 28.27).

Voltage drop in series $= 100 \times 0.06 = 6$ V; $I_{sh} = 220/50 = 4.4$ A.

$\therefore \quad I_a = 100 + 4.4 =$ **104.4**

Armature drop $= 104.4 \times 0.1 = 10.4$ V

$\therefore$ Induced e.m.f. $= 220 + 6 + 10.5 =$ **236.4 V**

(*b*) Long-shunt (Fig. 28.28)

$I_{sh} = 220/50 = 4.4$ A $\quad \therefore \quad I_a = 100 + 4.4 =$ **104.4 A**

Voltage drop over armature and series field winding $= 104.4 \times 0.16 = 16.7$ V

$\therefore$ Induced e.m.f. $= 200 + 16.7 =$ **216.7 V**

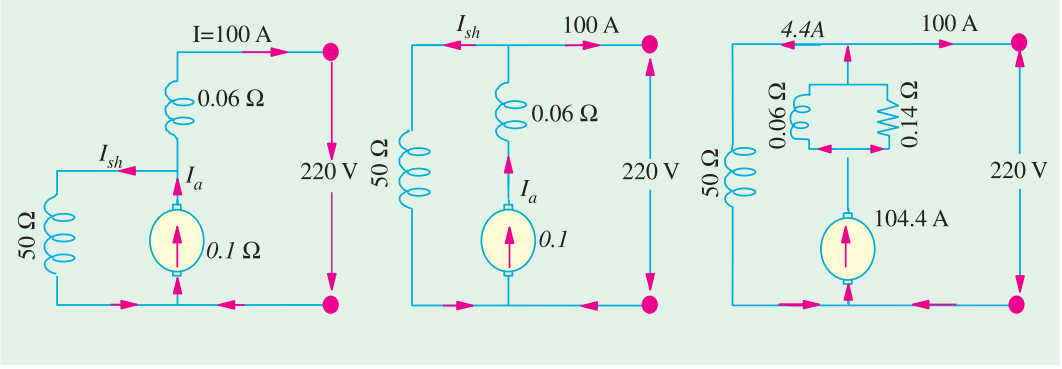


Fig. 28.27 Fig. 28.28 Fig. 28.29

(*c*) As shown in Fig. 28.29, a divertor of resistance 0.14 Ω is connected in parallel with the series field winding. Let n be the number of series turns.

Number of series amp-turns without divertor = $n \times 104.4 = 104.4\,n$

When divertor is applied, then current through series field is

$$= \frac{104.4 \times 0.14}{(0.14 + 0.06)} = 73.8\ A$$

$\therefore$ Series amp-turns = $73.8 \times n$ $\therefore$ Series amp-turns are reduced to $\frac{73.8\,n}{104.4\,n} \times 100 =$ **70 %**

Example 28.18. *A 250-kW, 240-V generator is to be compounded such that its voltage rises from 220 volts at no-load to 240 V at full load. When series field is cut out and shunt field is excited from an external source, then from the load test it is found that this rise in voltage can be obtained by increasing the exciting current from 7 A at no-load to 12 A at full-load. Given shunt turns/pole = 650, series turns/pole = 4 and resistance of series winding, 0.006 Ω. If the machine is connected long-shunt, find the resistance of the series amp-turns at no-load and drop in series winding resistance at full-load.*

Solution. Full-load current = $250 \times 10^3/240$

= 1042 A

Increase in shunt field ampere-turns to over-compound the shunt generator

= 650 (12 – 7) = 3,250. As seen from Fig. 28.30

$4 \times I_{se}$ = 3250

I_{se} = 3250/4 = 812.5 A

$\therefore$ I_d = 1042 – 812.5 = 229.5 A

It is so because no-load shunt current being negligible, $I_a = I$ = 1042 A.

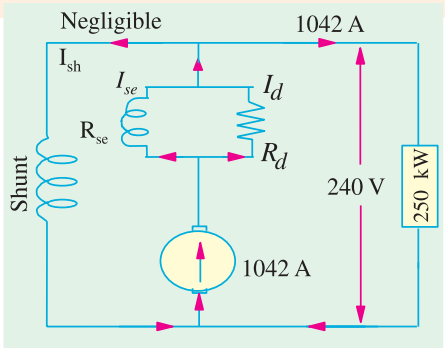


Fig. 28.30

Since series winding and divertor are in parallel, $I_d\,R_d = I_{se}\,R_{se}$ or $229.5\,R_d = 812.5 \times 0.006$

$\therefore$ R_d = **0.0212 Ω**

Example 28.19. *A 60-kW d.c. shunt generator has 1600 turns/pole in its shunt winding. A shunt field current of 1.25 A is required to generate 125 V at no-load and 1.75 A to generate 150 V at full load. Calculate*

(i) *the minimum number of series turns/pole needed to produce the required no-load and full-load voltages as a short-shunt compound generator.*

(ii) *if the generator is equipped with 3 series turns/pole having a resistance of 0.02 Ω, calculate divertor resistance required to produce the desired compounding.*

(iii) *voltage regulation of the compound generator.*

Solution. ***(i)*** Extra excitation ampere-turns required = 1600 (1.75 – 1.25) = 800

I_{se} = I = 60,000/150 = 400 A

$\therefore$ No. of series turns/pole required = 800/400 = **2**

Hence, minimum number of series turns/pole required for producing the desired compound generator terminal voltage is **2**.

(ii) Now, actual No. of series turns/pole is 3. Hence, current passing through it can be found from

$3 \times I_{se}$ = 800; I_{se} = 800/3 A

As shown in Fig. 28.31, I_d = 400 – (800/3) = 400/3 A

Also $(800/3) \times 0.02 = (400/3) \times R_d$; R_d = **0.04 Ω**

(iii) % regn. = $(125 - 150) \times 100/150$ = **– 16.7%**

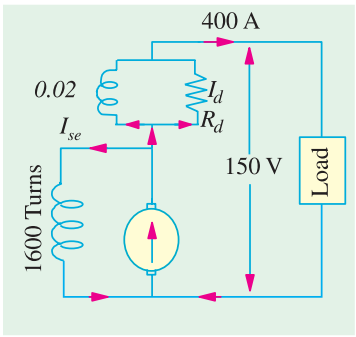


Fig. 28.31

Example 28.20. *A D.C. generator having an external characteristics which is a straight line through zero to 50 V at 200 Amp, is connected as a booster between a station bus-bar and a feeder of 0.3 ohm resistance. Calculate the voltage between the far end of the feeder and the bus-bar at a current of (i) 160 A, (ii) 50 A.* **(Manomaniam Surdaranar Univ. Nov. 1998)**

Solution. Due to the feeder resistance of 0.3 ohm there is a voltage drop of 1×0.3 volts in the direction of current, where I refers to the current flowing. Due to booster, there is a rise in voltage given by $I \times (50/200)$ according to the *V-I* characteristic given for the series generator. As a sum total of these two, the net feeder drop will be $I \times (0.3 - 0.25)$ or I volts, and that represents the voltage between far end of the feeder and the bus-bar.

(*i*) At 160 A, the net voltage drop in feeder = 160×0.05 or $48 - 40 = 9$ volts.

Since 48 volts drop is partially compensated by 40 volts boosted up.

(*ii*) At 50 A, the required answer = (15 volts drop 12.5 volts boosted up) = 2.5 volts drop.

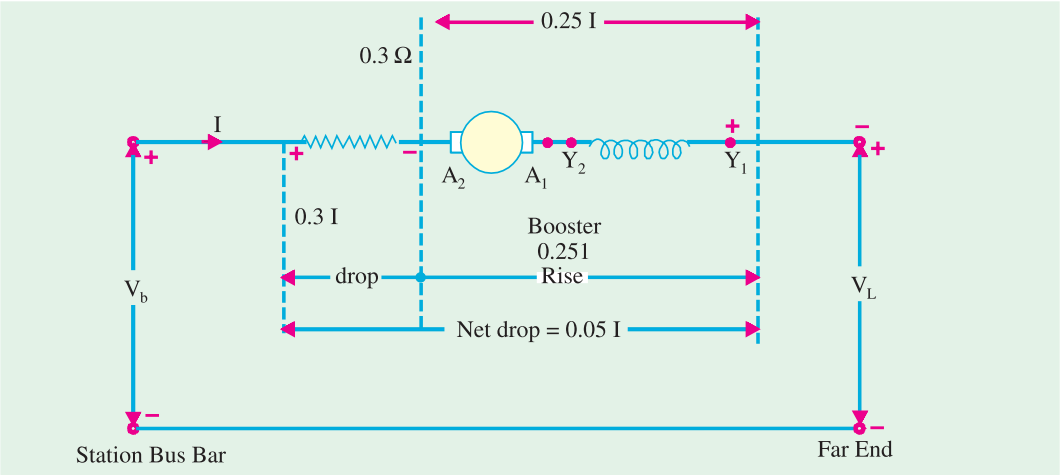


Fig. 28.32. Feeder and Booster

28.16. Uses of D.C. Generators

1. Shunt generators with field regulators are used for ordinary lighting and power supply purposes. They are also used for charging batteries because their terminal voltages are almost constant or can be kept constant.

2. Series generators are not used for power supply because of their rising characteristics. However, their rising characteristic makes them suitable for being used as boosters (Ex. 28.15) in certain types of distribution systems particularly in railway service.



Compound generators are used in electric railways

3. Compound generators

The cumulatively-compound generator is the most widely used d.c. generator because its external characteristic can be adjusted for compensating the voltage drop in the line resistance. Hence, such generators are used for motor driving which require d.c. supply at constant voltage, for lamp loads and for heavy power service such as electric railways.

The differential-compound generator has an external characteristic similar to that of a shunt generator but with large demagnetization armature reaction. Hence, it is widely used in arc welding where larger voltage drop is desirable with increase in current.

Tutorial Problem in 28.1

1. The *OC* curve of a d.c. shunt generator for a speed of 1000 r.p.m. is given by the following table.

Field current :	*2.0*	*3.0*	*4.0*	*5.0*	*6.0*	*7.0*
E.M.F. volts :	*102*	*150*	*188*	*215*	*232*	*245*

The shunt has a resistance of 37 Ω. Find the speed at which excitation may be expected to build up.

The aramture resistance of 0.04 Ω. Neglecting the effects of brush drop and armature reaction, estimate the p.d. when the speed is 1000 r.p.m. and the armature delivers a current of 100 A. **(725 r.p.m.; 231 V)**

2. A d.c. shunt generator running at 850 r.p.m. gave the followig *O.C.C.* data :

Field current (A) :	*0*	*0.5*	*1*	*2*	*3*	*4*	*5*
E.M.F. (V) :	*10*	*60*	*120*	*199*	*232*	*248*	*258*

If the resistance of the shunt field is 50 Ω, determine the additional resistance required in the shunt field circuit to give 240 V at a speed of 1000 r.p.m. **[64.3 Ω]**

3. Sketch the load characteristic of a d.c. generator with (*i*) shunt (*ii*) series excitation. Give reasons for the particular shape in each case.

The *O.C.C.* at 700 r.p.m. of a series generator with separately-excited field is as follows :

Field current (A) :	*20*	*40*	*50*	*60*	*75*
Armature e.m.f. (V) :	*190*	*360*	*410*	*450*	*480*

Determine the current and terminal voltage as a self-excited series machine when running at 600 r.p.m. with a load of 6 Ω connected to the terminal. Resistance of armature and series winding is 0.3Ω. Ignore effect of armature reaction. **[369 V; 61.5 A]**

4. The *O.C.C.* data for separately-excited generator when run at 130 r.p.m. on open circuit is

E.M.F. (V) :	*12*	*44*	*73*	*98*	*113*	*122*	*127*
Exciting current :	*0*	*0.2*	*0.4*	*0.6*	*0.8*	*1.0*	*1.2*

Deduce the curve of e.m.f. and excitation when the generator is running separately-excited at 1000 r.p.m. To what voltage will the generator build up on no-load when running at 1000 r.p.m.? To what voltage will the generator build up on no-load when running at 100 r.p.m. if the total field resistance is 100 Ω ? **[91 V]**

5. The following figures give the *O.C.C.* of d.c. Shunt generator driven at a constant speed of 700 r.p.m.

Terminal voltage (V):	*10*	*20*	*40*	*80*	*120*	*160*	*200*	*240*	*260*
Field Current (A) :	*0*	*0.1*	*0.24*	*0.5*	*0.77*	*1.2*	*1.92*	*3.43*	*5.2*

Determine the critical resistance at (*a*) 700 r.p.m. (*b*) 850 r.p.m. If resistance of field coils is 50 Ω, find the range of the field rheostat required to vary the voltage between the limits of 180 V and 250 V on open circuit at a speed of 700 r.p.m. **[160 Ω ; 194 Ω; 70 Ω to 10 Ω]**

6. The *O.C.C.* of a shunt generator when separately-excited and running at 1000 r.p.m. is given by :

O.C.C. volt :	*56*	*112*	*150*	*180*	*200*	*216*	*230*
Field amp. :	*0.5*	*1.0*	*1.5*	*2.0*	*2.5*	*3.0*	*3.5*

If the generator is shunt-connected and runs at 1100 r.p.m. with a total field resistance of 80 Ω, determine

(*a*) no-load e.m.f.

(*b*) the output when the terminal voltage is 200 V if the armature resistance is 0.1 Ω.

(*c*) the terminal voltage of the generator when giving the maximum output current.

Neglect the effect of armature reaction and of brush contact drop.

[236 V; 200 V; 460 V; 150 V (approx.)]

7. A long-shunt compound d.c. generator with armature, series field and shunt field resistance of 0.5, 0.4 and 250 Ω respectively gave the following readings when run at constant speed :

Load current (A) :	*0*	*10*	*20*	*30*	*40*
Terminal p.d. (V) :	*480*	*478*	*475*	*471*	*467*

Plot the curve of internal generated e.m.f. against load current. Explain fully the steps by which this curve is obtained and tabulate the values from which it is plotted. **[For 40 A; E = 504.7 V (approx.)]**

8. A shunt generator has the following open-circuit characteristic at 800 r.p.m.

Field amperes	:	0.5	1.0	1.5	2.0	2.5	3.0	3.5
E.M.F. Volt	:	54	107	152	185	210	230	245

Armature and shunt field resistances are respectively 0.1 Ω and 80 Ω. The terminal p.d. falls to 175 V when the armature current is 100 A. Find the O.C. volts and the volts lost due to (*i*) reduction in the field current (*ii*) armature resistance (*iii*) armature reaction. **[220 V (*i*) 27 V (*ii*) 120 V (*iii*) 8 V]**

9. The open-circuit characteristic of a shunt generator when driven at normal speed is as follows :

Field current	:	0.5	1.0	1.5	2.0	2.5	3.0	3.5 A
O.C. volts	:	54	107	152	185	210	230	240 V

The resistance of armature circuit is 0.1 Ω. Due to armature reaction the effective field current is given by the relation I_{sh} (eff.) = $I_{sh} - 0.003\, I_a$. Find the shunt field circuit resistance that will give a terminal voltage of 220 V with normal speed (*a*) on open circuit (*b*) at a load current of 100 A. Also find (*c*) number of series turns for level compounding at 220 V with 100 A armature current ; take number of shunt turns per pole as 1200 and (*d*) No. of series turns for over-compounding giving a terminal voltage of 220 V at no-load and 230 V with 100 A armature current. **[(*a*) 80 Ω (*b*) 66 Ω (*c*) 6.8 Ω (*d*) 9.1 turns]**

10. Find how many series turns per pole are needed on a 500-kW compound generator required to give 450 V on no-load and 500 V on full-load, the requisite number of ampere-turns per pole being 9,000 and 6,500 respectively. The shunt winding is designed to give 450 V at no-load when its temperature is 20°C. The final temperature is 60°C. Take α_0 = 1/234.5 per °C.

[2.76] (Electrical Technology, Allahabad Univ. 1977)

OBJECTIVE TESTS – 28

1. The external characteristic of a shunt generator can be obtained directly from its —— characteristic.
(*a*) internal (*b*) open-circuit
(*c*) load-saturation (*d*) performance

2. Load saturation characteristic of a d.c. generator gives relation between
(*a*) V and I_a (*b*) E and I_a
(*c*) E_0 and I_f (*d*) V and I_f

3. The slight curvature at the lower end of the *O.C.C.* of a self-excited d.c. generator is due to
(*a*) residual pole flux
(*b*) high armature speed
(*c*) magnetic inertia
(*d*) high field circuit resistance.

4. For the voltage built-up of a self-excited d.c. generator, which of the following is not an essential condition ?
(*a*) There must be some residual flux
(*b*) Field winding mmf must aid the residual flux
(*c*) Total field circuit resistance must be less than the critical value
(*d*) Armature speed must be very high.

5. The voltage build-up process of a d.c. generator is
(*a*) difficult (*b*) delayed
(*c*) cumulative (*d*) infinite

6. Which of the following d.c. generator cannot build up on open-circuit ?
(*a*) shunt (*b*) series
(*c*) short shunt (*d*) long shunt

7. If a self-excited d.c. generator after being installed, fails to build up on its first trial run, the first thing to do is to
(*a*) increase the field resistance
(*b*) check armature insulation
(*c*) reverse field connections
(*d*) increase the speed of prime mover.

8. If residual magnetism of a shunt generator is destroyed accidentally, it may be restored by connecting its shunt field
(*a*) to earth
(*b*) to an a.c. source
(*b*) in reverse
(*d*) to a d.c. source.

9. The three factors which cause decrease in the terminal voltage of a shunt generator are
(*a*) armature reactances
(*b*) armature resistance
(*c*) armature leakages
(*d*) armature reaction
(*e*) reduction in field current

10. If field resistance of a d.c. shunt generator is increased beyond its critical value, the generator
(*a*) output voltage will exceed its name-plate rating
(*b*) will not build up

(*c*) may burn out if loaded to its name-plate rating
(*d*) power output may exceed its name-plate rating

11. An ideal d.c. generator is one that has ..... voltage regulation.
(*a*) low (*b*) zero
(*c*) positive (*d*) negative

12. The ..... generator has poorest voltage regulation.
(*a*) series (*b*) shunt
(*c*) compound (*d*) high

13. The voltage regulation of an overcompound d.c. generator is always .....
(*a*) positive
(*b*) negative
(*c*) zero
(*d*) high

14. Most commercial compound d.c. generator are normally supplied by the manufacturers as over compound machines because
(*a*) they are ideally suited for transmission of d.c. energy to remotely-located loads
(*b*) degree of compounding can be adjusted by using a divertor across series field
(*c*) they are more cost effective than shunt generators
(*d*) they have zero percent regulation.

ANSWERS

1. (*b*) **2.** (*d*) **3.** (*c*) **4.** (*d*) **5.** (*c*) **6.** (*b*) **7.** (*c*) **8.** (*d*) **9.** (*b,d,e*) **10.** (*b*) **11.** (*b*) **12.** (*a*) **13.** (*b*) **14.** (*b*).

QUESTIONS AND ANSWERS ON D.C. GENERATORS

Q. 1. How may the number of parallel paths in an armature be increased ?
Ans. By increasing the number of magnetic poles.

Q. 2. How are brushes connected in a d.c. generator ?
Ans. Usually, all positive brushes are connected together and all the negative brushes together (Fig. 28.33).

Q. 3. What is meant by armature reaction ?
Ans. It is the effect of armature magnetic field on the distribution of flux under main poles of a generator. The armature magnetic field has two effects :
(i) It demagnetises or weakens the main flux and
(ii) It cross-magnetises or distorts it.

Q. 4. What is the effect of this distortion on the operation of the machine ?
Ans. It acts as a magnetic drag on the armature which consequently requires more power to turn it.

Q. 5. How can field distortion be remedied ?
Ans. By using compensating windings which are embedded in the slots in the pole-shoe and are connected in series with the armature.

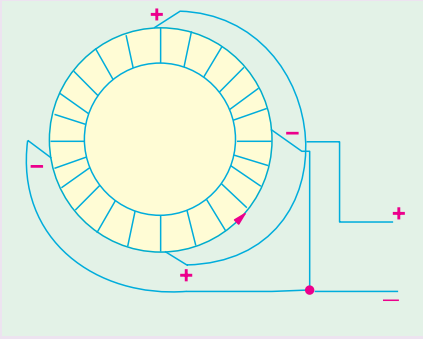


Fig. 28.33

Q. 6. What is meant by normal neutral plane ?
Ans. It is a plane which passes through the axis of the armature perpendicular to the magnetic field of the generator when there is no flow of current through the armature.

Q. 7. What is the importance of this plane in the working of the machine ?
Ans. It gives the position where brushes would be placed to prevent sparking during the operation of the generator where the main pole field not distorted by armature field and were there no self-induction in the coils.

Q. 8. How do you differentiate between normal neutral plane ?
Ans. The *NNP* is the position of zero induction and hence minimum sparking assuming no field distortion *i.e.* on no-load. It is perpendicular to magnetic axis. *NA* is the position of zero induction and hence minimum sparking with distorted field *i.e.* ***when generator is on load.***

Q. 9. How do you define 'commutating plane' ?
Ans. It is the plane which passes through the axis of the armature and through centre of contact of the brushes as shown in Fig. 28.34.

Q. 10. What is the angle of lead ?

Ans. It is the angle between the *NNP* and the commutating plane.

Q. 11. What affects this angle ?

Ans. For sparkless commutation, the angle of lead varies directly with load. Its value can be kept small by making main pole field considerably more powerful than the armature field.

Q. 12. What is the best way of minimizing eddy currents in an armature ?

Ans. Lamination.

Q. 13. How should the armature be laminated for the purpose ?

Ans. It should be laminated at right angle to its axis.

Q. 14. How does field distortion affect communication ?

Ans. The neutral plane no longer coincides with the normal neutral plane but is advanced by a certain angle in the direction of rotation of the armature.

Q. 15. Should the brushes of a loaded generator be placed in the neutral plane ?

Ans. No.

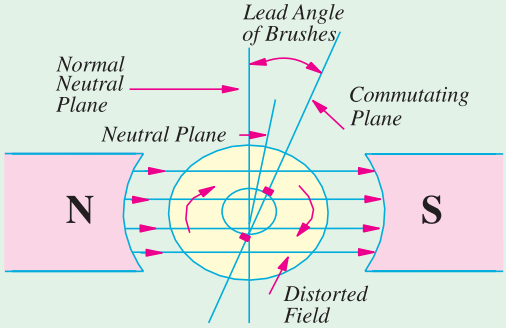


Fig. 28.34

Q. 16. Why not ?

Ans. The brushes must be advanced by a certain angle (called brush lead) beyond the neutral plane to prevent sparking.

Q. 17. What causes sparking at the brushes ?

Ans. It is due to the self-induction of the coil undergoing commutation.

Q. 18. What is the standard direction of rotation of the d.c. generators ?

Ans. Clockwise when viewed from the end opposite to the driven end.

Q. 19. What is meant by build-up of a generator ?

Ans. It means the gradual increase in the generator voltage to its maximum value after the generator is started from rest.

Q. 20. How should a generator be started ?

Ans. It is usually brought up to speed with the help of the driving engine called prime-mover.

Q. 21. How should a shunt or compound generator be started ?

Ans. Such machines excite best when all switches controlling the external circuit are open.

Q. 22. How about a series generator ?

Ans. In this case, the external circuit must be closed otherwise the generator will not build-up.

Q. 23. What is the procedure for shunting down a generator ?

Ans. First, the load should be gradually reduced, if possible, by easing down the driving engine, then when the generator is supplying little or no current, the main switch should be opened. When the voltmeter reads almost zero, then brushes should be raised from the commutator.

Q. 24. What are the indications and causes of an overloaded generator ?

Ans. A generator is said to be overloaded if a greater output is taken from it that it can safely carry. Overloading is indicated by **(*i*)** excessive sparking at brushes and **(*ii*)** overheating of the armature and other parts of the generator. Most likely causes of overloading are :

1. Excessive voltage-as indicated by the voltmeter or the increased brilliancy of the pilot lamp. This could be due to over-excitation of field magnets or too high speed of the engine.
2. Excessive current–which could be due to bad feeding of the load.
3. Reversal of polarity–this happens occasionally when the series or compound-wound generators are running in parallel. Polarity reversal occurs during stopping by the current from the machines at work.
4. Short-circuit or ground in the generator itself or in the external circuit.

Q. 25. Mention and explain the various causes for the failure of the generator to build up.

Ans. Principal causes due to which a generator may fail to excite are :

1. Brushes not properly adjusted–if brushes are not in their proper positions, then whole of the armature voltage will not be utilized and so would be insufficient to excite the machine.
2. Defective contacts-unclean contacts may interpose large resistance in the path of the exciting current and reduce it to such a small value that it fails to excite the machine.
3. Incorrect adjustment of regulators–in the case of shunt and compound generators, it is possibly that the resistance of field regulator may be too high to permit the passage of sufficient current through the field windings.
4. Speed too low–in the case of shunt–and compound-wound generators, there is certain critical armature speed below which they will not excite.
5. Open-circuit–in the case of series machines.
6. Short-circuit–in the generator of external circuit.
7. Reversed field polarity–usually caused by the reversed connections of the field coils.
8. Insufficient residual magnetism–The trouble normally occurs when the generator is new. It can be remedied by passing a strong direct current through the field coils.

Q. 26. How do we conclude that connections between field coils and armature are correct ?

Ans. If the generator builds up when brought to full speed. If it does not, then connections are reversed.

Q. 27. When a generator loses its residual magnetism either due to lighting or short circuit, how can it be made to build up ?

Ans. By temporarily magnetisng the main poles with the help of current from an external battery.

Q. 28. Can a generator be reversed by reversing the connections between the armature and field coils ?

Ans. No, because if these connections are reversed, the generator will not build up at all.

Q. 29. Will a generator build up if it becomes reversed ?

Ans. Yes.

Q. 30. Then, what is the objection to a reversed generator ?

Ans. Since the current of such a reversed generator is also reversed, serious trouble can occur if attempt is made to connect it in parallel with other machines which are not reversed.

Q. 31. What are the two kinds of sparking produced in a generator ?

Ans. One kind of sparking is due to bad adjustment of brushes and the other due to bad condition of the commutator. The sparking of the first are bluish whereas those of the other are reddish in colour.

Q. 32. What is the probable reason if sparking does not disappear in any position when brushes are rocked around the commutator ?

Ans. (*i*) The brushes may not be separated at correct distance.
(*ii*) The neutral plane may not be situated in the true theoretical position on the commutator due to faulty winding.

Q. 33. What is the permissible rise of temperature in a well-designed generator ?

Ans. 27°C above the surrounding air.

Q. 34. What are the causes of hot bearings ?

Ans. (*i*) lack of oil (*ii*) belt too tight (*iii*) armature not centered with respect of pole pieces (*iv*) bearing too tight or not in line.

Q. 35. What causes heating of armature ?

Ans.
1. Eddy currents.
2. Moisture which almost short-circuits the armature.
3. Unequal strength of magnetic poles.
4. Operation above rated voltage and below normal speed.

Q. 36. What is the commutator pitch of a 4-pole d.c. armature having 49 commutator bars?

Ans. $Y_c = (49 \pm 1)/2 = 24$ or 25.

Q. 37. Will it make any difference if lower figure of 24 is selected in preference to other.

Ans. Yes. Direction of armature rotation would be reversed.

GO To FIRST

CONTENTS

CHAPTER 29

D.C. MOTOR

Learning Objectives

- Motor Principle
- Comparison of Generator and Motor Action
- Significance of the Back e.m.f.—Voltage Equation of a Motor
- Conditions for Maximum Power
- Torque
- Armature Torque of a Motor
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- Speed of a D.C. Motor
- Speed RegulationTorque and Speed of a D.C. Motor
- Motor Characteristics
- Characteristics of Series Motors
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- Losses and Efficiency
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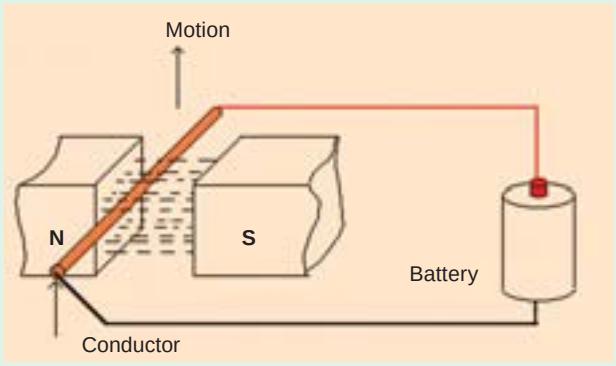


Ç Design for optimum performance and durability in demanding variable speed motor applications. D.C. motors have earned a reputation for dependability in severe operating conditions

CONTENTS

29.1. Motor Principle

An Electric motor is a machine which converts electric energy into mechanical energy. Its action is based on the principle that when a current-carrying conductor is placed in a magnetic field, it experiences a mechanical force whose direction is given by Fleming's Left-hand Rule and whose magnitude is given by $F = BIl$ Newton.



Principle of Motor

Constructionally, there is no basic difference between a d.c. generator and a d.c. motor. In fact, the same d.c. machine can be used interchangeably as a generator or as a motor. D.C. motors are also like generators, shunt-wound or series-wound or compound-wound.

In Fig. 29.1 a part of multipolar d.c. motor is shown. When its field magnets are excited and its armature conductors

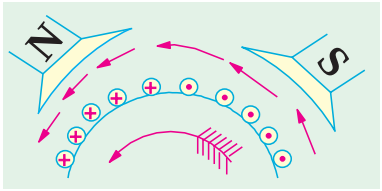


Fig. 29.1

are supplied with current from the supply mains, they experience a force tending to rotate the armature. Armature conductors under *N*-pole are assumed to carry current downwards (crosses) and those under *S*-poles, to carry current upwards (dots). By applying Fleming's Left-hand Rule, the direction of the force on each conductor can be found. It is shown by small arrows placed above each conductor. It will be seen that each conductor can be found. It will be seen that each conductor experiences a force F which tends to rotate the armature in anticlockwise direction. These forces collectively produce a driving torque which sets the armature rotating.

It should be noted that the function of a commutator in the motor is the same as in a generator. By reversing current in each conductor as it passes from one pole to another, it helps to develop a continuous and unidirectional torque.

29.2. Comparison of Generator and Motor Action

As said above, the same d.c. machine can be used, at least theoretically, interchangeably as a generator or as a motor. When operating as a generator, it is driven by a mechanical machine and it develops voltage which in turn produces a current flow in an electric circuit. When operating as a motor, it is supplied by electric current and it develops torque which in turn produces mechanical rotation.

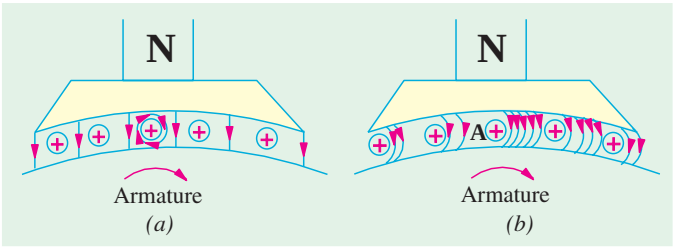


Fig. 29.2

Let us first consider its operation as a generator and see how exactly and through which agency, mechanical power is converted into electric power.

In Fig. 29.2 part of a generator whose armature is being driven clockwise by its prime mover is shown.

Fig. 29.2 (*a*) represents the fields set up independently by the main poles and the armature conductors like *A* in the figure. The resultant field or magnetic lines on flux are shown in Fig. 29.2 (*b*).

It is seen that there is a crowding of lines of flux on the right-hand side of *A*. These magnetic lines of flux may be likened to the rubber bands under tension. Hence, the bent lines of flux up a mechanical force on *A* much in the same way as the bent elastic rubber band of a catapult produces a mechanical force on the stone piece. It will be seen that this force is in a direction opposite to that of armature rotation. Hence, it is known as backward force or magnetic drag on the conductors. It is against this drag action on all armature conductor that the prime mover has to work. The work done in overcoming this opposition is converted into electric energy. Therefore, it should be clearly understood that it is only through the instrumentality of this magnetic drag that energy conversion is possible in a d.c. generator*.

Next, suppose that the above d.c. machine is uncoupled from its prime mover and that current is sent through the armature conductors under a *N*-pole in the downward direction as shown in Fig. 29.3 (*a*). The conductors will again experience a force in the anticlockwise direction (Fleming's Left hand Rule). Hence, the machine will start rotating anticlockwise, thereby developing a torque which can produce mechanical rotation. The machine is then said to be motoring.

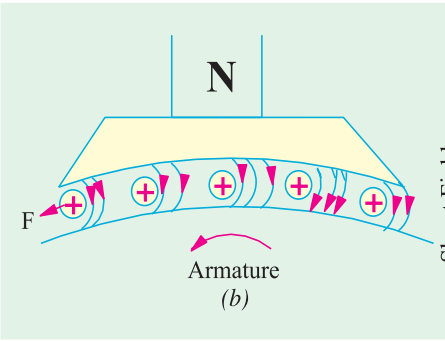


Fig. 29.3 (*a*)

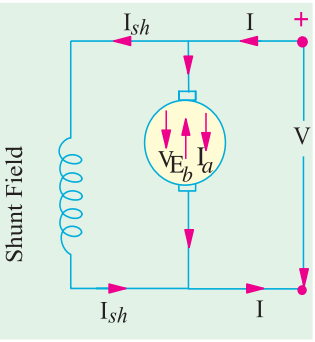


Fig. 29.3 (*b*)

As said above, energy conversion is not possible unless there is some opposition whose overcoming provides the necessary means for such conversion. In the case of a generator, it was the magnetic drag which provided the necessary opposition. But what is the equivalent of that drag in the case of a motor ? Well, it is the back e.m.f. It is explained in this manner :

As soon as the armature starts rotating, dynamically (or motionally) induced e.m.f. is produced in the armature conductors. The direction of this induced e.m.f. as found by Fleming's Right-hand Rule, is outwards *i.e.*, in direct opposition to the applied voltage (Fig. 29.3 (*b*)). This is why it is known as back e.m.f. E_b or counter e.m.f. Its value is the same as for the motionally induced e.m.f. in the generator *i.e.* $E_b = (\Phi ZN) \times (P/A)$ volts. The applied voltage V has to be force current through the armature conductors against this back e.m.f. E_b. The electric work done in overcoming this opposition is converted into mechanical energy developed in the armature. Therefore, it is obvious that but for the production of this opposing e.m.f. energy conversion would not have been possible.

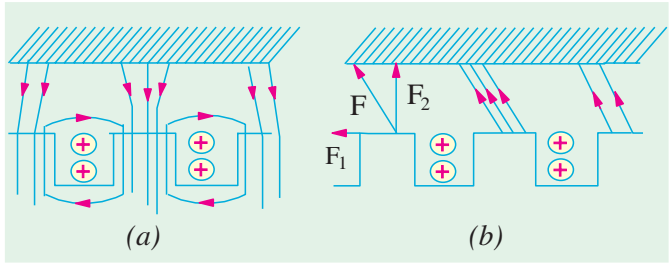


Fig. 29.4

Now, before leaving this topic, let it be pointed out that in an actual motor with slotted armature, the torque is not due to mechanical force on the conductors themselves, but due to tangential pull on the armature teeth as shown in Fig. 29.4.

It is seen from Fig. 29.4 (*a*) that the main flux is concentrated in the form of tufts at the armature teeth while the armature flux is shown by the dotted lines embracing the armature slots. The effect of

* In fact, it seems to be one of the fundamental laws of Nature that no energy conversion from one form to another is possible until there is some one to oppose the conversion. But for the presence of this opposition, there would simply be no energy conversion. In generators, opposition is provided by magnetic drag whereas in motors, back e.m.f. does this job. Moreover, it is only that part of the input energy which is used for overcoming this opposition that is converted into the other form.

armature flux on the main flux, as shown in Fig. 29.4 (*b*), is two-fold :

(i) It increases the flux on the left-hand side of the teeth and decreases it on the right-hand side, thus making the distribution of flux density across the tooth section unequal.

(ii) It inclines the direction of lines of force in the air-gap so that they are not radial but are disposed in a manner shown in Fig. 29.4 (*b*). The pull exerted by the poles on the teeth can now be resolved into two components. One is the tangential component F_1 and the other vertical component F_2. The vertical component F_2, when considered for all the teeth round the armature, adds up to zero. But the component F_1 is not cancelled and it is this tangential component which, acting on all the teeth, gives rise to the armature torque.

29.3. Significance of the Back e.m.f.

As explained in Art 29.2, when the motor armature rotates, the conductors also rotate and hence cut the flux. In accordance with the laws of electromagnetic induction, e.m.f. is induced in them whose direction, as found by Fleming's Right-hand Rule, is in opposition to the applied voltage (Fig. 29.5). Because of its opposing direction, it is referred to as counter e.m.f. or back e.m.f. E_b. The equivalent circuit of a motor is shown in Fig. 29.6. The rotating armature generating the back e.m.f. E_b is like a battery of e.m.f. E_b put across a supply mains of V volts. Obviously, V has to drive I_a against the opposition of E_b. The power required to overcome this opposition is $E_b I_a$.

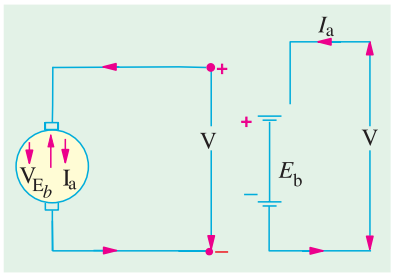


Fig. 29.5

In the case of a cell, this power over an interval of time is converted into chemical energy, but in the present case, it is converted into mechanical energy.

$$\text{It will be seen that } I_a = \frac{\text{Net voltage}}{\text{Resistance}} = \frac{V - V_b}{R_a}$$

where R_a is the resistance of the armature circuit. As pointed out above,

$E_b = \Phi ZN \times (P/A)$ volt where N is in r.p.s.

Back e.m.f. depends, among other factors, upon the armature speed. If speed is high, E_b is large, hence armature current I_a, seen from the above equation, is small. If the speed is less, then E_b is less, hence more current flows which develops motor torque (Art 29.7). So, we find that E_b acts like a governor *i.e.*, it makes a motor self-regulating so that it draws as much current as is just necessary.

29.4. Voltage Equation of a Motor

The voltage V applied across the motor armature has to

(i) overcome the back e.m.f. E_b and

(ii) supply the armature ohmic drop $I_a R_a$.

$$\therefore \quad V = E_b + I_a R_a$$

This is known as voltage equation of a motor.

Now, multiplying both sides by I_a, we get

$$V I_a = E_b I_a + I_a^2 R_a$$

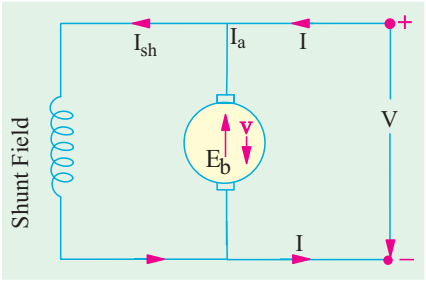


Fig.29.6

As shown in Fig. 29.6,

$V I_a$ = Eectrical input to the armature

$E_b I_a$ = Electrical equivalent of mechanical power developed in the armature

$I_a^2 R_a$ = Cu loss in the armature

Hence, out of the armature input, some is wasted in I^2R loss and the rest is converted into mechanical power ***within*** the armature.

It may also be noted that motor efficiency is given by the ratio of power developed by the arma-

ture to its input *i.e.*, $E_b I_a / V I_a = E_b / V$. Obviously, higher the value of E_b as compared to V, higher the motor efficiency.

29.5. Condition for Maximum Power

The gross mechanical power developed by a motor is $P_m = V I_a - I_a^2 R_a$.

Differentiating both sides with respect to I_a and equating the result to zero, we get

$$d P_m / d I_a = V - 2 I_a R_a = 0 \quad \therefore \quad I_a R_a = V/2$$

As $V = E_b + I_a R_a$ and $I_a R_a = V/2 \quad \therefore \quad E_b = V/2$

Thus gross mechanical power developed by a motor is maximum when back e.m.f. is equal to half the applied voltage. This condition is, however, not realized in practice, because in that case current would be much beyond the normal current of the motor. Moreover, half the input would be wasted in the form of heat and taking other losses (mechanical and magnetic) into consideration, the motor efficiency will be well below 50 percent.

Example 29.1. *A 220-V d.c. machine has an armature resistance of 0.5 Ω. If the full-load armature current is 20 A, find the induced e.m.f. when the machine acts as* ***(i)*** *generator* ***(ii)*** *motor.*

(Electrical Technology-I, Bombay Univ. 1987)

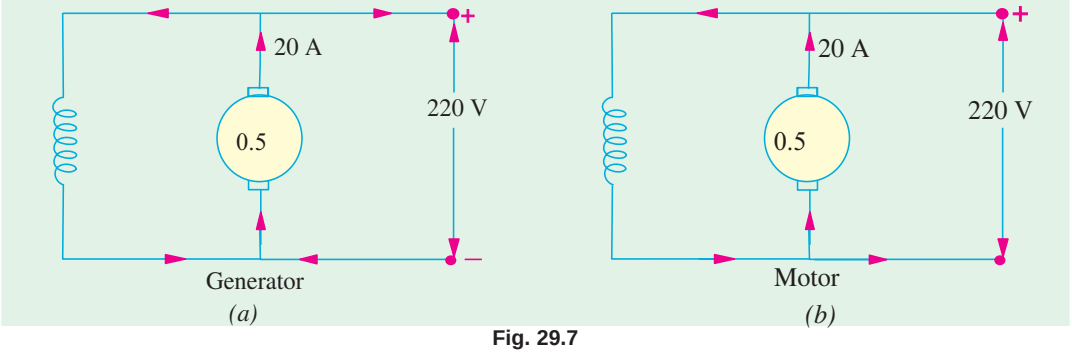


Fig. 29.7

Solution. As shown in Fig. 29.7, the d.c. machine is assumed to be shunt-connected. In each case, shunt current is considered negligible because its value is not given.

(*a*) As Generator [Fig. 29.7(*a*)] $E_g = V + I_a R_a = 220 + 0.5 \times 20 =$ **230 V**

(*b*) As Motor [Fig 29.7 (*b*)] $E_b = V - I_a R_a = 220 - 0.5 \times 20 =$ **210 V**

Example 29.2. *A separately excited D.C. generator has armature circuit resistance of 0.1 ohm and the total brush-drop is 2 V. When running at 1000 r.p.m., it delivers a current of 100 A at 250 V to a load of constant resistance. If the generator speed drop to 700 r.p.m., with field-current unaltered, find the current delivered to load.* **(AMIE, Electrical Machines, 2001)**

Solution. $R_L = 250/100 = 2.5$ ohms.

$E_{g1} = 250 + (100 \times 0.1) + 2 = 262$ V.

At 700 r.p.m., $E_{g2} = 262 \times 700/1000 = 183.4$ V

If I_a is the new current, $E_{g2} - 2 - (I_a \times 0.1) = 2.5\, I_a$

This gives $I_a = 96.77$ amp.

Extension to the Question : *With what load resistance will the current be 100 amp, at 700 r.p.m.?*

Solution. $E_{g2} - 2 - (I_a \times 0.1) = R_L \times I_a$

For $I_a = 100$ amp, and $E_{g2} = 183.4$ V, $R_L = 1.714$ ohms.

Example 29.3. *A 440-V, shunt motor has armature resistance of 0.8 Ω and field resistance of 200 Ω. Determine the back e.m.f. when giving an output of 7.46 kW at 85 percent efficiency.*

Solution. Motor input power $= 7.46 \times 10^3 / 0.85$ W

Motor input current = $7460/0.85 \times 440 = 19.95$ A ; $I_{sh} = 440/200 = 2.2$ A

$I_a = 19.95 - 2.2 = 17.75$ A ; Now, $E_b = V - I_a R_a$

$\therefore \quad E_b = 440 - (17.75 \times 0.8) =$ **425.8 V**

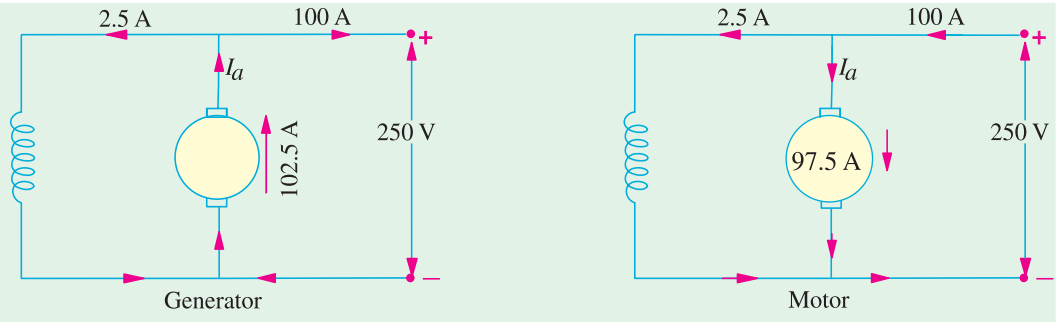


Fig. 29.8 (*a*) **Fig. 29.8 (*b*)**

Example 29.4. *A 25-kW, 250-V, d.c. shunt generator has armature and field resistances of 0.06 Ω and 100 Ω respectively. Determine the total armature power developed when working **(i)** as a generator delivering 25 kW output and **(ii)** as a motor taking 25 kW input.*

(Electrical Technology, Punjab Univ., June 1991)

Solution. As Generator [Fig. 29.8 (*a*)]

Output current = $25{,}000/250 = 100$ A ; $I_{sh} = 250/100 = 2.5$ A ; $I_a = 102.5$ A

Generated e.m.f. = $250 + I_a R_a = 250 + 102.5 \times 0.06 = 256.15$ V

Power developed in armature = $E_b I_a = \dfrac{256.15 \times 102.5}{1000} =$ **26.25 kW**

As Motor [Fig 29.8 (*b*)]

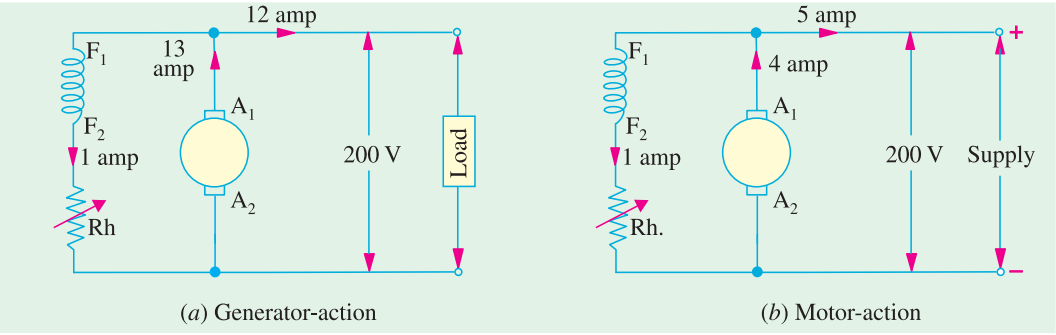
Motor input current = 100 A ; $I_{sh} = 2.5$ A, $I_a = 97.5$ A

$E_b = 250 - (97.5 \times 0.06) = 250 - 5.85 = 244.15$ V

Power developed in armature = $E_b I_a = 244.15 \times 97.5/1000 =$ **23.8 kW**

Example 29.5. *A 4 pole, 32 conductor, lap-wound d.c. shunt generator with terminal voltage of 200 volts delivering 12 amps to the load has $r_a = 2$ and field circuit resistance of 200 ohms. It is driven at 1000 r.p.m. Calculate the flux per pole in the machine. If the machine has to be run as a motor with the same terminal voltage and drawing 5 amps from the mains, maintaining the same magnetic field, find the speed of the machine.* **[Sambalpur University, 1998]**

Solution. Current distributions during two actions are indicated in Fig. 29.9 (*a*) and (*b*). As a generator, $I_a = 13$ amp



(*a*) Generator-action (*b*) Motor-action

Fig. 29.9

$E_g = 200 + 13 \times 2 = 226$ V

$$\phi \frac{Z N}{60} \times \frac{P}{a} = 226$$

For a Lap-wound armature,

$$P = a$$

$$\therefore \quad \phi = \frac{226 \times 60}{1000 \times 32} = 0.42375 \text{ wb}$$

As a motor, $I_a = 4$ amp

$$E_b = 200 - 4 \times 2 = 192 \text{ V}$$

$$= \phi ZN/60$$

$$\text{Giving } N = \frac{60 \times 192}{0.42375 \times 32}$$

$$= 850 \text{ r.p.m.}$$

Tutorial Problems 29.1

1. What do you understand by the term 'back e.m.f.' ? A d.c. motor connected to a 460-V supply has an armature resistance of 0.15 Ω. Calculate

(*a*) The value of back e.m.f. when the armature current is 120 A.

(*b*) The value of armature current when the back e.m.f. is 447.4 V. **[(*a*) 442 V (*b*) 84 A]**

2. A d.c. motor connected to a 460-V supply takes an armature current of 120 A on full load. If the armature circuit has a resistance of 0.25 Ω, calculate the value of the back e.m.f. at this load. **[430 V]**

3. A 4-pole d.c. motor takes an armature current of 150 A at 440 V. If its armature circuit has a resistance of 0.15 Ω, what will be the value of back e.m.f. at this load ? **[417.5 V]**

29.6. Torque

By the term torque is meant the turning or twisting moment of a force about an axis. It is measured by the product of the force and the radius at which this force acts.

Consider a pulley of radius r metre acted upon by a circumferential force of F Newton which causes it to rotate at N r.p.m. (Fig. 29.10).

Then torque $T = F \times r$ Newton-metre (N - m)

Work done by this force in one revolution

= Force × distance = $F \times 2\pi r$ Joule

Power developed = $F \times 2\pi r \times N$ Joule/second or Watt

= $(F \times r) \times 2\pi N$ Watt

Now $2\pi N$ = Angular velocity ω in radian/second and $F \times r$ = Torque T

$\therefore$ Power developed = $T \times \omega$ watt or $P = T\omega$ Watt

Moreover, if N is in r.p.m., then

$$\omega = 2\pi N/60 \text{ rad/s}$$

$$\therefore \quad P = \frac{2\pi N}{60} \times T \quad \text{or} \quad P = \frac{2\pi}{60} . NT = \frac{NT}{9.55}$$

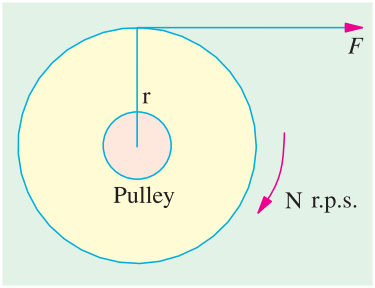


Fig. 29.10

29.7. Armature Torque of a Motor

Let T_a be the torque developed by the armature of a motor running at N r.p.s. If T_a is in N/M, then power developed = $T_a \times 2\pi N$ watt ...(*i*)

We also know that electrical power converted into mechanical power in the armature (Art 29.4)

$= E_b I_a$ watt ...(*ii*)

Equating (*i*) and (*ii*), we get $T_a \times 2\pi N = E_b I_a$ ...(*iii*)

Since $E_b = \Phi ZN \times (P/A)$ volt, we have

$$T_a \times 2\pi N = \Phi\, ZN \left(\frac{P}{A}\right) . I_a \text{ or } T_a = \frac{1}{2\pi} . \Phi Z I_0 \left(\frac{P}{A}\right) N\text{-}m$$

$= 0.159$ N newton metre

$\therefore \quad T_a = 0.159\, \Phi Z I_a \times (P/A)\ N\text{-}m$

Note. From the above equation for the torque, we find that $T_a \propto \Phi I_a$.

(*a*) In the case of a series motor, Φ is directly proportional to I_a (before saturation) because field windings carry full armature current $\therefore \quad T_a \propto I_a^2$

(*b*) For shunt motors, Φ is practically constant, hence $T_a \propto I_a$.

As seen from (*iii*) above

$$T_a = \frac{E_b\ I_a}{2\pi\ N} \text{ N - m - N in r.p.s.}$$

If N is in r.p.m., then

$$T_a = \frac{E_b\ I_a}{2\pi\ N/60} = 60\ \frac{E_b\ I_a}{2\pi N} = \frac{60}{2\pi}\ \frac{E_b\ I_a}{N} = 9.55\ \frac{E_b I_a}{N} \text{ N-m}$$

29.8. Shaft Torque (T_{sh})

The whole of the armature torque, as calculated above, is not available for doing useful work, because a certain percentage of it is required for supplying iron and friction losses in the motor.

The torque which is available for doing useful work is known as shaft torque T_{sh}. It is so called because it is available at the shaft. The motor output is given by

Output = $T_{sh} \times 2\pi N$ Watt provided T_{sh} is in N-m and N in r.p.s.

$$\therefore \quad T_{sh} = \frac{\text{Output in watts}}{2\pi N} \text{ N- m } -N \text{ in r.p.s}$$

$$= \frac{\text{Output in watts}}{2\pi N/60} \text{ N-m } -N \text{ in r.p.m.}$$

$$= \frac{60}{2\pi}\ \frac{\text{output}}{N} = 9.55\ \frac{\text{Output}}{N} \text{ N-m.}$$

The difference $(T_a - T_{sh})$ is known as lost torque and is due to iron and friction losses of the motor.

Note. The value of back e.m.f. E_b can be found from

(*i*) the equation, $E_b = V - I_a R_a$

(*ii*) the formula $E_b = \Phi ZN \times (P/A)$ volt

Example 29.6. *A d.c. motor takes an armature current of 110 A at 480 V. The armature circuit resistance is 0.2 Ω. The machine has 6-poles and the armature is lap-connected with 864 conductors. The flux per pole is 0.05 Wb. Calculate (i), the speed and (ii) the gross torque developed by the armature.* **(Elect. Machines, A.M.I.E. Sec B, 1989)**

Solution. $E_b = 480 - 110 \times 0.2 = 458$ V, $\Phi = 0.05$ W, $Z = 864$

Now, $E_b = \frac{\Phi\ ZN}{60} \left(\frac{P}{A}\right)$ or $458 = \frac{0.05 \times 864 \times N}{60} \times \left(\frac{6}{6}\right)$

$\therefore \quad N = 636$ r.p.m.

$T_a = 0.159 \times 0.05 \times 864 \times 110\ (6/6) = 756.3$ N-m

Example 29.7. *A 250-V, 4-pole, wave-wound d.c. series motor has 782 conductors on its armature. It has armature and series field resistance of 0.75 ohm. The motor takes a current of 40 A. Estimate its speed and gross torque developed if it has a flux per pole of 25 mWb.*

(Elect. Engg.-II, Pune Univ. 1991)

Solution. $E_b = \Phi ZN\,(P/A)$

Now, $E_b = V - I_a R_a = 50 - 40 \times 0.75 = 220$ V

$\therefore \quad 220 = 25 \times 10^{-3} \times 782 \times N \times 0.75 = 220$ V

$\therefore \quad 220 = 0.159\ \Phi\ ZI_a\ (P/A)$

$= 0.159 \times 25 \times 10^{-3} \times 782 \times 40 \times (4/2) =$ **249 N-m**

Example 29.8. *A d.c. shunt machine develops an a.c. e.m.f. of 250 V at 1500 r.p.m. Find its torque and mechanical power developed for an armature current of 50 A. State the simplifying assumptions.* **(Basic Elect. Machine Nagpur Univ., 1993)**

Solution. A given d.c. machine develops the same e.m.f. in its armature conductors whether running as a generator or as a motor. Only difference is that this armature e.m.f. is known as back e.m.f. when the machine is running as a motor.

Mechanical power developed in the arm = $E_b I_a = 250 \times 50 = 12{,}500$ W

$T_a = 9.55\ E_b\ I_a/N = 9.55 \times 250 \times 50/1500 =$ **79.6 N-m.**

Example 29.9. *Determine developed torque and shaft torque of 220-V, 4-pole series motor with 800 conductors wave-connected supplying a load of 8.2 kW by taking 45 A from the mains. The flux per pole is 25 mWb and its armature circuit resistance is 0.6 Ω.*

(Elect. Machine AMIE Sec. B Winter 1991)

Solution. Developed torque or gross torque is the same thing as armature torque.

$\therefore \quad T_a = 0.159\ \Phi\ ZA\ (P/A)$

$= 0.159 \times 25 \times 10^{-3} \times 800 \times 45\ (4/2) = 286.2$ N-m

$E_b = V - I_a R_a = 220 - 45 \times 0.6 = 193$ V

Now, $E_b = \Phi\ ZN\ (P/A)$ or $193 = 25 \times 10^{-3} \times 800 \times N\ \pi \times (4/2)$

$\therefore \quad N = 4.825$ r.p.s.

Also, $2\pi N\ T_{sh}$ = output or $2\pi \times 4.825\ T_{sh} = 8200 \qquad \therefore\ T_{sh} = 270.5$ N-m

Example 29.10. *A 220-V d.c. shunt motor runs at 500 r.p.m. when the armature current is 50 A. Calculate the speed if the torque is doubled. Given that $R_a = 0.2\ \Omega$.*

(Electrical Technology-II, Gwalior Univ. 1985)

Solution. As seen from Art 27.7, $T_a \propto \Phi I_a$. Since Φ is constant, $T_a \propto I_a$

$\therefore \quad T_{a1} \propto I_{a1}$ and $T_{a2} \propto I_{a2} \quad \therefore \quad T_{a2}/T_{a1} = I_{a2}/I_{a1}$

$\therefore \quad 2 = I_{a2}/50$ or $I_{a2} = 100$ A

Now, $N_2/N_1 = E_{b2}/E_{b1}$ – since Φ remains constant.

$F_{b1} = 220 - (50 \times 0.2) = 210$ V $\qquad E_{b2} = 220 - (100 \times 0.2) = 200$ V

$\therefore \quad N_2/500 = 200/210 \qquad \therefore\ N_2 =$ **476 r.p.m.**

Example 29.11. *A 500-V, 37.3 kW, 1000 r.p.m. d.c. shunt motor has on full-load an efficiency of 90 percent. The armature circuit resistance is 0.24 Ω and there is total voltage drop of 2 V at the brushes. The field current is 1.8 A. Determine **(i)** full-load line current **(ii)** full load shaft torque in N-m and **(iii)** total resistance in motor starter to limit the starting current to 1.5 times the full-load current.* **(Elect. Engg. I; M.S. Univ. Baroda 1987)**

Solution. *(i)* Motor input = $37{,}300/0.9 = 41{,}444$ W

F.L. line current = $41{,}444/500 =$ **82.9 A**

(ii) $T_{sh} = 9.55 \dfrac{\text{output}}{N} = 9.55 \times \dfrac{37,300}{1000} = 356$ N-m

(iii) Starting line current $= 1.5 \times 82.9 = 124.3$ A

Arm. current at starting $= 124.3 - 1.8 = 122.5$ A

If R is the starter resistance (which is in series with armature), then

$122.5\ (R + 0.24) + 2 = 500 \quad \therefore \quad R =$ **3.825 Ω**

Example 29.12. *A 4-pole, 220-V shunt motor has 540 lap-wound conductor. It takes 32 A from the supply mains and develops output power of 5.595 kW. The field winding takes 1 A. The armature resistance is 0.09 Ω and the flux per pole is 30 mWb. Calculate* ***(i)*** *the speed and* ***(ii)*** *the torque developed in newton-metre.* **(Electrical Technology, Nagpur Univ. 1992)**

Solution. $I_a = 32 - 1 = 31$ A ; $E_b = V - I_a R_a = 220 - (0.09 \times 31) = 217.2$ V

Now, $E_b = \dfrac{\Phi ZN}{60}\left(\dfrac{P}{A}\right) \quad \therefore \quad 217.2 = \dfrac{30 \times 10^{-3} \times 540 \times N}{60}\left(\dfrac{4}{4}\right)$

(i) $\therefore \quad N =$ **804.4 r.p.m.**

(ii) $T_{sh} = 9.55 \times \dfrac{\text{output in watts}}{N} = 9.55 \times \dfrac{5,595}{804.4} =$ **66.5 N-m**

Example 29.13 (a). *Find the load and full-load speeds for a four-pole, 220-V, and 20-kW, shunt motor having the following data :*

Field–current = 5 amp, armature resistance = 0.04 ohm,

Flux per pole = 0.04 Wb, number of armature-conductors = 160, Two-circuit wave-connection, full load current = 95 amp, No load current =9 A. Neglect armature reaction.

(Bharathithasan Univ. April 1997)

Solution. The machine draws a supply current of 9 amp at no load. Out of this, 5 amps are required for the field circuit, hence the armature carries a no-load current of 4 amp.

At load, armature-current is 90 amp. The armature-resistance-drop increases and the back e.m.f. decreases, resulting into decrease in speed under load compared to that at No-Load.

At No Load : $E_{ao} = 220 - 4 \times 0.04 = 219.84$ volts

Substituting this,

$0.04 \times 160 \times (N/60) \times (4/2) = 219.84$

No-Load speed, $N_0 = 1030.5$ r.p.m.

At Full Load : Armature current = 90 A, $E_a = 200 - 90 \times 0.04 = 216.4$ V

$N = (216.4/219.84) \times 1030.5 = 1014.4$ r.p.m.

Example 29.13 (b). *Armature of a 6-pole, 6-circuit D.C. shunt motor takes 400 A at a speed of 350 r.p.m. The flux per pole is 80 milli-webers, the number of armature turns is 600, and 3% of the torque is lost in windage, friction and iron-loss. Calculate the brake-horse-power.*

(Manonmaniam Sundaranar Univ. Nov. 1998)

Solution. Number of armature turns = 600

Therefore, Z = Number of armature conductors = 1200

If electromagnetic torque developed is T Nw – m,

Armature power $= T\omega = T \times 2\pi\, 350/60$

$= 36.67\ T$ watts

To calculate armature power in terms of Electrical parameters, E must be known.

$E = \phi Z (N/60)(P/A)$

$$= 80 \times 10^{-3} \times 1200 \times (350/60) \times (6/6)$$
$$= 560 \text{ volts}$$

With the armature current of 400 *A*, Armature power = 560×400 watts

Equating the two,

$T = 560 \times 400/36.67 = 6108.5$ Nw – m. Since 3 % of this torque is required for overcoming different loss-terms,

$$\text{Net torque} = 0.97 \times 6180.5 = 5925 \text{ Nw - m}$$

For Brake-Horse-Power, net output in kW should be computed first. Then "kW" is to be converted to "BHP", with 1 HP = 0.746 kW.

Net output in kW = $5925 \times 36.67 \times 10^{-3} = 217.27$ kW

Converting this to BHP, the output = 291.25 HP

Example 29.13 (*c*). *Determine the torque established by the armature of a four-pole D.C. motor having 774 conductors, two paths in parallel, 24 milli-webers of pole-flux and the armature current is 50 Amps.* **(Bharathiar Univ. April 1998)**

Solution. Expression for torque in terms of the parameters concerned in this problem is as follows :

$$T = 0.159 \, \phi \, Z \, I_a \, p/a \text{ Nw - m}$$

Two paths in parallel for a 4-pole case means a wave winding.

$$T = 0.159 \times (24 \times 10^{-3}) \times 774 \times 50 \times 4/2$$
$$= 295.36 \text{ Nw-m}$$

Example 29.13 (*d*). *A 500-V D.C. shunt motor draws a line-current of 5 A on light-load. If armature resistance is 0.15 ohm and field resistance is 200 ohms, determine the efficiency of the machine running as a generator delivering a load current of 40 Amps.*

(Bharathiar Univ. April 1998)

Solution. **(*i*)** *No Load, running as a motor* :

$$\text{Input Power} = 500 \times 5 = 2500 \text{ watts}$$
$$\text{Field copper-loss} = 500 \times 2.5 = 1250 \text{ watts}$$

Neglecting armature copper-loss at no load (since it comes out to be $2.5^2 \times 0.15 = 1$ watt), the balance of 1250 watts of power goes towards no load losses of the machine running at rated speed. These losses are mainly the no load mechanical losses and the core-loss.

(*ii*) *As a Generator, delivering 40 A to load* :

$$\text{Output delivered} = 500 \times 40 \times 10^{-3} = 20 \text{ kW}$$

Losses : **(*a*)** Field copper-loss = 1250 watts

(*b*) Armature copper-loss = $42.5^2 \times 0.15 = 271$ watts

(*c*) No load losses = 1250 watts

Total losses = 2.771 kW

Generator Efficiency = $(20/22.771) \times 100$ % = 87.83 %

Extension to the Question : *At what speed should the Generator be run, if the shunt-field is not changed, in the above case ? Assume that the motor was running at 600 r.p.m. Neglect armature reaction.*

Solution. As a motor on no-load,

$$E_{b0} = 500 - I_a \, r_a = 500 - 0.15 \times 2.5 = 499.625 \text{ V}$$

As a Generator with an armature current of 42.5 A,

$$E_{b0} = 500 + 42.5 \times 0.15 = 506.375 \text{ V}$$

Since, the terminal voltage is same in both the cases, shunt field current remains as 2.5 amp. With armature reaction is ignored, the flux/pole remains same. The e.m.f. then becomes proportional to the speed. If the generator must be driven at N r.p.m.

$$N = (506.375/449.625) \times 600 = 608.1 \text{ r.p.m.}$$

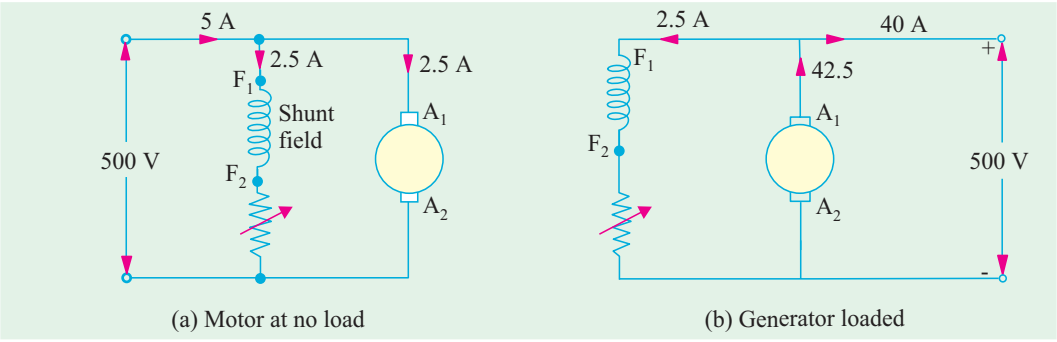


Fig. 29.11

Note. Alternative to this slight increase in the speed is to increase the field current with the help of decreasing the rheostatic resistance in the field-circuit.

Example 29.13 (e). *A d.c. series motor takes 40 A at 220 V and runs at 800 r.p.m. If the armature and field resistance are 0.2 Ω and 0.1 Ω respectively and the iron and friction losses are 0.5 kW, find the torque developed in the armature. What will be the output of the motor ?*

Solution. Armature torque is given by $T_a = 9.55 \dfrac{E_b I_a}{N}$ N-m

Now $E_b = V - I_a (R_a + R_{se}) = 220 - 40\ (0.2 + 0.1) = 208$ V

$\therefore$ $T_a = 9.55 \times 208 \times 40/800 =$ **99.3 N-m**

Cu loss in armature and series-field resistance = $40^2 \times 0.3 = 480$ W

Iron and friction losses = 500 W ; Total losses = 480 + 500 = 980 W

Motor power input = $220 \times 40 = 8{,}800$ W

Motor output = 8,800 – 980 = 7,820 W = **7.82 kW**

Example 29.14. *A cutting tool exerts a tangential force of 400 N on a steel bar of diameter 10 cm which is being turned in a simple lathe. The lathe is driven by a chain at 840 r.p.m. from a 220 V d.c. Motor which runs at 1800 r.p.m. Calculate the current taken by the motor if its efficiency is 80 %. What size is the motor pulley if the lathe pulley has a diameter of 24 cm ?*

(Elect. Technology-II, Gwalior Univ. 1985)

Solution. Torque T_{sh} = Tangential force × radius = $400 \times 0.05 = 20$ N-m

Output power = $T_{sh} \times 2\pi N$ watt = $20 \times 2\pi \times (840/60)$ watt = 1,760 W

Motor $\eta = 0.8$ $\therefore$ Motor input = 1,760/0.8 = 2,200 W

Current drawn by motor = 2200/220 = **10 A**

Let N_1 and D_1 be the speed and diameter of the driver pulley respectively and N_2 and D_2 the respective speed and diameter of the lathe pulley.

Then $N_1 \times D_1 = N_2 \times D_2$ or $1{,}800 \times D_1 = 840 \times 0.24$

$\therefore$ $D_1 = 840 \times 0.24/1{,}800 = 0.112$ m = **11.2 cm**

Example 29.15. *The armature winding of a 200-V, 4-pole, series motor is lap-connected. There are 280 slots and each slot has 4 conductors. The current is 45 A and the flux per pole is 18 mWb. The field resistance is 0.3 Ω; the armature resistance 0.5 Ω and the iron and friction losses total 800 W. The pulley diameter is 0.41 m. Find the pull in newton at the rim of the pulley.*

(Elect. Engg. AMIETE Sec. A. 1991)

Solution. $E_b = V - I_a R_a = 200 - 45\,(0.5 + 0.3) = 164$ V

Now $E_b = \dfrac{\Phi ZN}{60} \cdot \left(\dfrac{P}{A}\right)$ volt

$\therefore \quad 164 = \dfrac{18 \times 10^{-3} \times 280 \times 4 \times N}{60} \times \dfrac{4}{4} \quad \therefore \quad N = 488$ r.p.m.

Total input $= 200 \times 45 = 9{,}000$ W ; Cu loss $= I_a^2 R_a = 45^2 \times 0.8 = 1{,}620$ W

Iron + Friction losses $= 800$ W ; Total losses $= 1{,}620 + 800 = 2{,}420$ W

Output $= 9{,}000 - 2{,}420 = 6{,}580$ W

$\therefore \quad T_{sh} = 9 \times 55 \times \dfrac{6580}{488} = 128$ N-m

Let F be the pull in newtons at the rim of the pulley.

Then $F \times 0.205 = 128.8 \quad \therefore \quad F = 128.8/0.205$ N = **634 N**

Example 29.16. *A 4-pole, 240 V, wave connected shunt motor gives 1119 kW when running at 1000 r.p.m. and drawing armature and field currents of 50 A and 1.0 A respectively. It has 540 conductors. Its resistance is 0.1 Ω. Assuming a drop of 1 volt per brush, find* ***(a)*** *total torque* ***(b)*** *useful torque* ***(c)*** *useful flux / pole* ***(d)*** *rotational losses and* ***(e)*** *efficiency.*

Solution. $E_b = V - I_a R_a$ – brush drop $= 240 - (50 \times 0.1) - 2 = 233$ V

Also $I_a = 50$ A

(a) Armature torque $T_a = 9.55 \dfrac{E_b I_a}{N}$ N-m $= 9.55 \times \dfrac{233 \times 50}{1000}$ = **111 N-m**

(b) $T_{sh} = 9.55 \dfrac{\text{output}}{N} = 9.55 \times \dfrac{11{,}190}{1000}$ = **106.9 N-m**

(c) $E_b = \dfrac{\Phi ZN}{60} \times \left(\dfrac{P}{A}\right)$ volt

$\therefore \quad 233 = \dfrac{\Phi \times 540 \times 1000}{60} \times \left(\dfrac{4}{2}\right) \quad \therefore$ **Φ = 12.9 mWb**

(d) Armature input $= V I_a = 240 \times 50 =$ **12,000 W**

Armature Cu loss $= I_a^2 R_a = 50^2 \times 0.1 = 250$ W ; Brush contact loss $= 50 \times 2 = 100$ W

$\therefore$ Power developed $= 12{,}000 - 350 = 11{,}650$ W ; Output $= 11.19$ kW $= 11{,}190$ W

$\therefore$ Rotational losses $= 11{,}650 - 11{,}190 =$ **460 W**

(e) Total motor input $= VI = 240 \times 51 = 12{,}340$ W ; Motor output $= 11{,}190$ W

$\therefore$ Efficiency $= \dfrac{11{,}190}{12{,}240} \times 100$ = **91.4 %**

Example 29.17. *A 460-V series motor runs at 500 r.p.m. taking a current of 40 A. Calculate the speed and percentage change in torque if the load is reduced so that the motor is taking 30 A. Total resistance of the armature and field circuits is 0.8 Ω. Assume flux is proportional to the field current.*

(Elect. Engg.-II, Kerala Univ. 1988)

Solution. Since $\Phi \propto I_a$, hence $T \propto I_a^2$

$\therefore \quad T_1 \propto 40^2$ and $T_2 \propto 30^2 \quad \therefore \quad \dfrac{T_2}{T_1} = \dfrac{9}{16}$

$\therefore$ Percentage change in torque is

$$= \frac{T_1 - T_2}{T_1} \times 100 = \frac{7}{16} \times 100 = \textbf{43.75 \%}$$

Now $E_{b1} = 460 - (40 \times 0.8) = 428$ V ; $E_{b2} = 460 - (30 \times 0.8) = 436$ V

$\dfrac{N_2}{N_1} = \dfrac{E_{b2}}{E_{b1}} \times \dfrac{I_{a1}}{I_{a2}} \quad \therefore \quad \dfrac{N_2}{500} = \dfrac{436}{428} \times \dfrac{40}{30} \quad \therefore \quad N_2 =$ **679 r.p.m.**

Example 29.18. *A 460-V, 55.95 kW, 750 r.p.m. shunt motor drives a load having a moment of inertia of 252.8 kg-m^2. Find approximate time to attain full speed when starting from rest against full-load torque if starting current varies between 1.4 and 1.8 times full-load current.*

Solution. Let us suppose that the starting current has a steady value of (1.4 + 1.8)/2 = 1.6 times full-load value.

Full-load output = 55.95 kW = 55,950 W ; Speed = 750 r.p.m. = 12.5 r.p.s.

F.L. shaft torque T = power/ω = power/$2\pi N$ = 55,950 $\pi \times (750/60)$ = 712.4 N-m

During starting period, average available torque

$$= 1.6\,T - T = 0.6\,T = 0.6 \times 712.4 = 427.34 \text{ N-m}$$

This torque acts on the moment of inertial I = 252.8 km-m^2.

$$\therefore \quad 427.4 = 252.8 \times \frac{d\omega}{dt} = 252.8 \times \frac{2\pi \times 12.5}{dt}, \quad \therefore \quad dt = \mathbf{46.4\ s}$$

Example 29.19. *A 14.92 kW, 400 V, 400 -r.p.m. d.c. shunt motor draws a current of 40 A when running at full-load. The moment of inertia of the rotating system is 7.5 kg-m^2. If the starting current is 1.2 times full-load current, calculate*

(a) *full-load torque*

(b) *the time required for the motor to attain the rated speed against full-load.*

(Electrical Technology, Gujarat Univ. 1988)

Solution. ***(a)*** F.L. output 14.92 kW = 14,920 W ; Speed = 400 r.p.m. = 20/3 r.p.s

Now, $T\omega$ = output $\therefore$ $T = 14{,}920/2\pi \times (20/3) =$ **356 N-m**

(b) During the starting period, the torque available for accelerating the motor armature is

$$= 1.2\,T - T = 0.2\,T = 0.2 \times 356 = 71.2 \text{ N-m}$$

Now, torque $= I\,\dfrac{d\omega}{dt}$ $\quad \therefore \quad 71.2 = 7.5 \times \dfrac{2\pi \times (20/3)}{dt} \quad \therefore \quad dt =$ **4.41 second**

29.9. Speed of a D.C. Motor

From the voltage equation of a motor (Art. 27.4), we get

$$E_b = V - I_a R_a \quad \text{or} \quad \frac{\Phi Z N}{60}\left(\frac{P}{A}\right) = V - I_a R_a$$

$$\therefore \quad N = \frac{V - I_a R_a}{\Phi} \times \left(\frac{60A}{ZP}\right) \text{ r.p.m.}$$

Now $\quad V - I_a R_a = E_b \quad \therefore \quad N = \dfrac{E_b}{\Phi} \times \left(\dfrac{60A}{ZP}\right)$ r.p.m. or $N = K\,\dfrac{E_b}{\Phi}$

It shows that speed is directly proportional to back e.m.f. E_b and inversely to the flux Φ on $N \propto E_b/\Phi$.

For Series Motor

Let N_1 = Speed in the 1st case ; I_{a1} = armature current in the 1st case

Φ_1 = flux/pole in the first case

N_2, I_{a2}, Φ_2 = corresponding quantities in the 2nd case.

Then, using the above relation, we get

$$N_1 \propto \frac{E_{b1}}{\Phi_1} \text{ where } E_{b1} = V - I_{a1} R_a \,;\, N_2 \propto \frac{E_{b2}}{\Phi_2} \text{ where } E_{b2} = V - I_{a2} R_a$$

$$\therefore \quad \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$$

Prior to saturation of magnetic poles ; $\Phi \propto I_a$ $\quad \therefore \quad \dfrac{N_2}{N_1} = \dfrac{E_{b2}}{E_{b1}} \times \dfrac{I_{a1}}{I_{a2}}$

For Shunt Motor

In this case the same equation applies,

i.e., $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$ If $\Phi_2 = \Phi_1$, then $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}}$.

29.10. Speed Regulation

The term speed regulation refers to the change in speed of a motor with change in applied load torque, other conditions remaining constant. By change in speed here is meant the change which occurs under these conditions due to inherent properties of the motor itself and not those changes which are affected through manipulation of rheostats or other speed-controlling devices.

The speed regulation is defined ***as the change in speed when the load on the motor is reduced from rated value to zero, expressed as percent of the rated load speed***.

$$\therefore \quad \% \text{ speed regulation} = \frac{\text{N.L. speed} - \text{F.L. speed}}{\text{F.L. speed}} \times 100 = \frac{dN}{N} \times 100$$

29.11. Torque and Speed of a D.C. Motor

It will be proved that though torque of a motor is admittedly a function of flux and armature current, ***yet it is independent of speed***. In fact, it is the speed which depends on torque and not ***vice-versa***. It has been proved earlier that

$$N = K \frac{V - I_a R_a}{\phi} = \frac{KE_b}{\Phi} \quad \text{...Art. 27.9}$$

Also, $T_a \propto \Phi I_a$ ...Art. 27.7

It is seen from above that increase in flux would decrease the speed but increase the armature torque. It cannot be so because torque always tends to produce rotation. If torque increases, motor speed must ***increase*** rather than ***decrease***. The apparent inconsistency between the above two equations can be reconciled in the following way :

Suppose that the flux of a motor is decreased by decreasing the field current. Then, following sequence of events take place :

1. Back e.m.f. E_b (= $N\Phi/K$) drops instantly (the speed remains constant because of inertia of the heavy armature).

2. Due to decrease in E_b, I_a is increased because $I_a = (V - E_b)/R_a$. Moreover, a small reduction in flux produces a proportionately large increase in armature current.

3. Hence, the equation $T_a \propto \Phi I_a$, a small decrease in ϕ is more than counterbalanced by a *large* increase in I_a with the result that there is a net *increase* in T_a.

4. This increase in T_a produces an increase in motor speed.

It is seen from above that with the applied voltage V held constant, motor speed varies inversely as the flux. However, it is possible to increase flux and, at the same time, increase the speed provided I_a is held constant as is actually done in a d.c. servomotor.

Example 29.20. *A 4-pole series motor has 944 wave-connected armature conductors. At a certain load, the flux per pole is 34.6 mWb and the total mechanical torque developed is 209 N-m. Calculate the line current taken by the motor and the speed at which it will run with an applied voltage of 500 V. Total motor resistance is 3 ohm.*

(Elect. Engg. AMIETE Sec. A Part II June 1991)

Solution.

$$T_a = 0.159\ \phi\ Z I_a\ (P/A) \text{ N-m}$$

$$\therefore \quad 209 = 0.159 \times 34.6 \times 10^{-3} \times 944 \times I_a\ (4/2);\ I_a = 20.1 \text{ A}$$

$$E_a = V - I_a R_a = 500 - 20.1 \times 3 = 439.7 \text{ V}$$

Now, speed may be found either by using the relation for E_b or T_a as given in Art.

$$E_b = \Phi ZN \times (P/A) \text{ or } 439.7 = 34.6 \times 10^{-3} \times 944 \times N \times 2$$

$$\therefore \quad N = 6.73 \text{ r.p.s. or } \mathbf{382.2 \text{ r.p.m.}}$$

Example 29.21. *A 250-V shunt motor runs at 1000 r.p.m. at no-load and takes 8A. The total armature and shunt field resistances are respectively 0.2 Ω and 250 Ω. Calculate the speed when loaded and taking 50 A. Assume the flux to be constant.* **(Elect. Engg. A.M.Ae. S.I. June 1991)**

Solution. Formula used : $\frac{N}{N_0} = \frac{E_b}{E_{b0}} \times \frac{\Phi_0}{\Phi}$; Since $\Phi_0 = \Phi$ (given); $\frac{N}{N_0} = \frac{E_b}{E_{b0}}$

$$I_{sh} = 250/250 = 1 \text{ A}$$

$$E_{b0} = V - I_{a0} R_a = 250 - (7 \times 0.2) = 248.6 \text{ V}; \; E_b = V - I_a R_a = 250 - (49 \times 0.2) = 240.2 \text{ V}$$

$$\therefore \quad \frac{N}{1000} = \frac{240.2}{248.6}; \; N = \mathbf{9666.1 \text{ r.p.m.}}$$

Example 29.22. *A d.c. series motor operates at 800 r.p.m. with a line current of 100 A from 230-V mains. Its armature circuit resistance is 0.15 Ω and its field resistance 0.1 Ω. Find the speed at which the motor runs at a line current of 25 A, assuming that the flux at this current is 45 per cent of the flux at 100 A.* **(Electrical Machinery - I, Banglore Univ. 1986)**

Solution.

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}; \; \Phi_2 = 0.45 \, \Phi_1 \text{ or } \frac{\Phi_1}{\Phi_2} = \frac{1}{0.45}$$

$$E_{b1} = 230 - (0.15 + 0.1) \times 100 = 205 \text{ V}; \; E_{b2} = 230 - 25 \times 0.25 = 223.75 \text{ V}$$

$$\frac{N_2}{800} = \frac{223.75}{205} \times \frac{1}{0.45}; \; N_2 = \mathbf{1940 \text{ r.p.m.}}$$

Example 29.23. *A 230-V d.c. shunt motor has an armature resistance of 0.5 Ω and field resistance of 115 Ω. At no load, the speed is 1,200 r.p.m. and the armature current 2.5 A. On application of rated load, the speed drops to 1,120 r.p.m. Determine the line current and power input when the motor delivers rated load.* **(Elect. Technology, Kerala Univ. 1988)**

Solution.

$$N_1 = 1200 \text{ r.p.m.}, \; E_{b1} = 230 - (0.5 \times 2.5) = 228.75 \text{ V}$$

$$N_2 = 1120 \text{ r.p.m.}, \; E_{b2} = 230 - 0.5 \, I_{a2}$$

Now, $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \quad \therefore \quad \frac{1120}{1200} = \frac{230 - 0.5 I_{a2}}{228.75}; \; I_{a2} = \mathbf{33 \text{ A}}$

Line current drawn by motor $= I_{a2} + I_{sh} = 33 + (230/115) = \mathbf{35 \text{ A}}$

Power input at rated load $= 230 \times 35 = \mathbf{8{,}050 \text{ W}}$

Example 29.24. *A belt-driven 100-kW, shunt generator running at 300 r.p.m. on 220-V bus-bars continues to run as a motor when the belt breaks, then taking 10 kW. What will be its speed ? Given armature resistance = 0.025 Ω, field resistance = 60 Ω and contact drop under each brush = 1 V, Ignore armature reaction.* **(Elect. Machines (E-3) AMIE Sec.C Winter 1991)**

Solution. As Generator [Fig. 29.12 (*a*)]

Load current, $I = 100{,}000/220 = 454.55 \; A; \; I_{sh} = 220/60 = 3.67 \text{ A}$

$$I_a = I + I_{sh} = 458.2 \; A \; ; \; I_a R_a = 458.2 \times 0.025 = 11.45$$

$$E_b = 220 + 11.45 + 2 \times 1 = 233.45 \; V; \; N_1 = 300 \text{ r.p.m.}$$

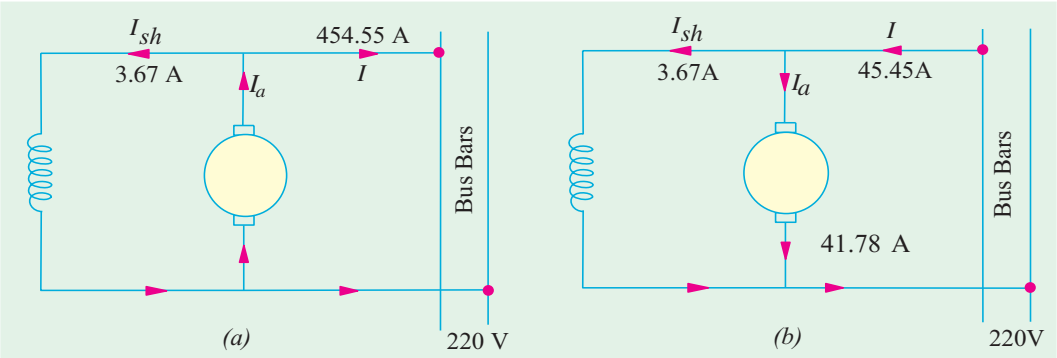


Fig. 29.12

As Motor [Fig. 29.12 (*b*)]

Input line current = 100,000/220 = 45.45 A; I_{sh} = 220/60 = 3.67 A

I_a = 45.45 – 3.67 = 41.78 A; $I_a R_a$ = 41.78 × 0.025 = 1.04 V; E_{b2} = 220 – 1.04 – 2 × 1 = 216.96 V

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}; \text{ since } \Phi_1 = \Phi_2 \text{ because } I_{sh} \text{ is constant}$$

$$\therefore \quad \frac{N_2}{300} = \frac{216.96}{233.45}; \; N_2 = 279 \text{ r.p.m.}$$

Example 29.25. *A d.c. shunt machine generates 250-V on open circuit at 1000 r.p.m. Effective armature resistance is 0.5 Ω, field resistance is 250 Ω, input to machine running as a motor on no-load is 4 A at 250 V. Calculate speed of machine as a motor taking 40 A at 250 V. Armature reaction weakens field by 4%.* **(Electrical Machines-I, Gujarat Univ. 1987)**

Solution. Consider the case when the machine runs as a motor on no-load.

Now, I_{sh} = 250/250 = 1 A; Hence, I_{a0} = 4 – 1 = 3A; E_{b0} = 250 – 0.5 × 3 = 248.5 V

It is given that when armature runs at 1000 r.p.m., it generates 250 V. When it generates 248.5 V, it must be running at a speed = 1000 × 248.5/250 = 994 r.p.m.

Hence, N_0 = 994 r.p.m.

When Loaded

I_a = 40 – 1 = 39 *A*; E_b = 250 – 39 × 0.5 = 230.5 *V* Also, Φ_0/Φ = 1/0.96

$$\frac{N}{E} = \frac{E_b}{E_{b0}} \quad \therefore \quad \frac{N}{994} = \frac{230.5}{248.5} \times \frac{1}{0.96} \qquad N = \mathbf{960 \text{ r.p.m.}}$$

Example 29.26. *A 250-V shunt motor giving 14.92 kW at 1000 r.p.m. takes an armature current of 75 A. The armature resistance is 0.25 ohm and the load torque remains constant. If the flux is reduced by 20 percent of its normal value before the speed changes, find the instantaneous value of the armature current and the torque. Determine the final value of the armature current and speed.*
(Elect. Engg. AMIETE (New Scheme) 1990)

Solution. E_{b1} = 250 – 75 × 0.25 = 231.25 V, as in Fig. 29.13. When flux is reduced by 20%, the back e.m.f. is also reduced instantly by 20% because speed remains constant due to inertia of the heavy armature (Art. 29.11).

$\therefore$ Instantaneous value of back e.m.f. $(E_b)_{inst}$ = 231.25 × 0.8 = 185 V

$(I_a)_{inst} = [V - (E_b)_{inst}]/R_a$ = (250 – 185)/0.25 = 260 A

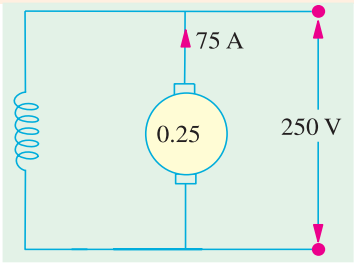


Fig. 29.13

Instantaneous value of the torque $= 9.55 \times \dfrac{(E_b)_{inst} \times (I_a)_{inst}}{N \ (in \ r.p.m.)}$

or $(T_a)_{inst} = 9.55 \times 185 \times 260/1000 = 459$ N-m

Steady Conditions

Since torque remains constant, $\Phi_1 I_{a1} = \Phi_2 I_{a2}$

$I_{a2} = \Phi_1 I_{a1}/\Phi_2 = 75 \times \Phi_1/0.8\,\Phi_1 =$ **93.7 A**

$\therefore \quad E_{b2} = 250 - 93.7 \times 0.25 = 226.6$ V

Now, $\dfrac{N_2}{N_1} = \dfrac{E_{b2}}{E_{b1}} \times \dfrac{\Phi_1}{\Phi_2} = \dfrac{226.6}{231.25} \times \dfrac{1}{0.8}$; $N_2 = 1225$ r.p.m.

Example 29.27. *A 220-V, d.c. shunt motor takes 4 A at no-load when running at 700 r.p.m. The field resistance is 100 Ω. The resistance of armature at standstill gives a drop of 6 volts across armature terminals when 10 A were passed through it. Calculate* ***(a)*** *speed on load* ***(b)*** *torque in N-m and* ***(c)*** *efficiency. The normal input of the motor is 8 kW.*

(Electrotechnics-II; M.S. Univ. Baroda 1988)

Solution. ***(a)*** $I_{sh} = 200/100 = 2$ A

F.L. Power input $= 8{,}000$ W; F.L. line current $= 8{,}000/200 = 40$ A

$I_a = 40 - 2 = 38$ A; $R_a = 6/10 = 0.6\ \Omega$

$E_{b0} = 200 - 2 \times 0.6 = 198.8$ V; $E_b = 200 - 38 \times 0.6 = 177.2$ V

Now, $\dfrac{N}{N_0} = \dfrac{E_b}{E_{b0}}$ or $\dfrac{N}{700} = \dfrac{177.2}{198.8}$; $N =$ **623.9 r.p.m.**

(b) $T_a = 9.55\, E_b I_a/N = 9.55 \times 177.2 \times 38/623.9 =$ **103 N-m**

(c) N.L. power input $= 200 \times 4 = 800$ W; Arm. Cu loss $= I_a^2 R_a = 2^2 \times 0.6 = 2.4$ W

Constant losses $= 800 - 2.4 = 797.6$ W; F.L. arm. Cu loss $= 38^2 \times 0.6 = 866.4$ W

Total F.L. losses $= 797.6 + 866.4 = 1664$ W; F.L. output $= 8{,}000 - 1664 = 6336$ W

F.L. Motor efficiency $= 6336/8{,}000 = 0.792$ or **79.2 %**

Example 29.28. *The input to 230-V, d.c. shunt motor is 11kW. Calculate* ***(a)*** *the torque developed* ***(b)*** *the efficiency* ***(c)*** *the speed at this load. The particulars of the motor are as follows :*

No-load current = 5 A; No-load speed = 1150 r.p.m.

Arm. resistance = 0.5 Ω; shunt field resistance = 110 Ω.

(Elect. Technology ; Bombay University 1988)

Solution. No-load input $= 220 \times 5 = 1{,}100$ W; $I_{sh} = 220/110 = 2$A; $I_{ao} = 5 - 2 = 3$A

No-load armature Cu loss $= 3^2 \times 0.5 = 4.5$ W

$\therefore$ Constant losses $= 1{,}100 - 4.5 = 1{,}095.5$ W

When input is 11 kW.

Input current $= 11{,}000/220 = 50$A ; Armature current $= 50 - 2 =$ **48 A**

Arm. Cu loss $= 48^2 \times 0.5 = 1{,}152$ W ;

Total loss $=$ Arm. Cu loss + Constant losses $= 1152 + 1095.5 = 2248$ W

Output $= 11{,}000 - 2{,}248 = 8{,}752$ W

(b) Efficiency $= 8{,}752 \times 100/11{,}000 =$ **79.6%**

(c) Back e.m.f. at no-load $= 220 - (3 \times 0.5) = 218.5$ V

Back e.m.f. at given load $= 220 - (48 \times 0.5) = 196$ V

$\therefore$ Speed $N = 1{,}150 \times 196/218.5 =$ **1,031 r.p.m.**

(a) $T_a = 9.55 \times \frac{196 \times 48}{1031} = \mathbf{87.1\ N\text{-}m}$

Example 29.29. *The armature circuit resistance of a 18.65 kW 250-V series motor is 0.1 Ω, the brush voltage drop is 3V, and the series field resistance is 0.05. When the motor takes 80 A, speed is 600 r.p.m. Calculate the speed when the current is 100 A.*

(Elect. Machines, A.M.I.E. Sec. B, 1993)

Solution. $E_{b1} = 250 - 80\,(0.1 + 0.05) - 3 = 235$ V.

$E_{b2} = 250 - 100\,(0.1 + 0.05) - 3 = 232$ V

Since $\Phi \propto I_a$, hence, $\Phi_1 \propto 80$, $\Phi_2 \propto 100$, $\Phi_1/\Phi_2 = 80/100$

Now $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$ or $\frac{N_2}{600} = \frac{232}{235} \times \frac{80}{100}$; $N_2 = 474$ r.p.m.

Example 29.30. *A 220-volt d.c. series motor is running at a speed of 800 r.p.m. and draws 100 A. Calculate at what speed the motor will run when developing half the torque. Total resistance of the armature and field is 0.1 ohm. Assume that the magnetic circuit is unsaturated.*

(Elect. Machines ; A.M.I.E. Sec. B, 1991)

Solution. $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{a1}}{I_{a2}}$ $(\because\ \Phi \propto I_a)$

Since field is unsaturated, $T_a \propto \Phi I_a \propto I_a^2$. $(\because\ T_1 \propto I_{a1}^2$ and $T_2 \propto I_{a2}^2)$

or $T_2/T_1 = (I_{a2}/I_{a1})^2$ or $1/2 = (I_{a2}/I_{a1})^2$; $I_{a1} = I_{a1}/\sqrt{2} = 70.7$ A

$E_{b1} = 220 - 100 \times 0.1 = 210$ V ; $E_{b2} = 220 - 0.1 \times 70.7 = 212.9$ V

$\therefore \quad \frac{N_2}{800} = \frac{212.9}{210} \times \frac{100}{70.7}$; $N_2 = \mathbf{1147\ r.p.m.}$

Example 29.31. *A 4-pole d.c. motor runs at 600 r.p.m. on full load taking 25 A at 450 V. The armature is lap-wound with 500 conductors and flux per pole is expressed by the relation.*

$$\Phi = (1.7 \times 10^{-2} \times I^{0.5})\ \text{weber}$$

where 1 is the motor current. If supply voltage and torque are both halved, calculate the speed at which the motor will run. Ignore stray losses. **(Elect. Machines, Nagpur Univ. 1993)**

Solution. Let us first find R_a.

Now $N = \frac{E_b}{Z\Phi}\left(\frac{60\,A}{P}\right)$ r.p.m.

$\therefore \quad 600 = \frac{E_b}{1.7 \times 10^{-2} \times 25^{0.5}} \times \frac{60 \times 4}{500 \times 4}$

$\therefore \quad E_b = 10 \times 1.7 \times 10^{-2} \times 5 \times 500 = 425$ V

$I_a R_a = 450 - 425 = 25$ V ; $R_a = 25/25 = 1.0\ \Omega$

Now in the Ist Case

$T_1 \propto \Phi_1 I_1 \quad \therefore \quad T_1 \propto 1.7 \times 10^{-2} \times \sqrt{25} \times 25$

Similarly $T_2 \propto 1.7 \times 10^{-2} \times \sqrt{1 \times I}$; Now $T_1 = 2T_2$

$\therefore \quad 1.7 \times 10^{-2} \times 125 = 1.7 \times 10^{-2} \times I^{3/2} \times 2 \quad \therefore \quad I = (125/2)^{2/3} = 15.75$ A

$E_{b1} = 425$ V ; $E_{b2} = 225 - (15.75 \times 1) = 209.3$ V

Using the relation $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$; we have

$$\frac{N_2}{600} = \frac{209.3}{425} \times \frac{1.7 \times 10^{-2} \times 5}{1.7 \times 10^{-2} \times \sqrt{15.75}}; \quad N_2 = \textbf{372 r.p.m.}$$

Tutorial Problems 29.2

1. Calculate the torque in newton-metre developed by a 440-V d.c. motor having an armature resistance of 0.25 Ω and running at 750 r.p.m. when taking a current of 60 A. **[325 N-m]**

2. A 4-pole, lap-connected d.c. motor has 576 conductors and draws an armature current of 10 A. If the flux per pole is 0.02 Wb, calculate the armature torque developed. **[18.3 N-m]**

3. **(*a*)** A d.c. shunt machine has armature and field resistances of 0.025 Ω and 80 Ω respectively. When connected to constant 400-V bus-bars and driven as a generator at 450 r.p.m., it delivers 120 kW. Calculate its speed when running as a motor and absorbing 120 kW from the same bus-bars.

(*b*) Deduce the direction of rotation of this machine when it is working as a motor assuming a clockwise rotation as a generator. **[(*a*) 435 r.p.m. (*b*) Clockwise]**

4. The armature current of a series motor is 60 A when on full-load. If the load is adjusted to that this current decreases to 40-A, find the new torque expressed as a percentage of the full-load torque. The flux for a current of 40 A is 70% of that when current is 60 A. **[46%]**

5. A 4-pole, d.c. shunt motor has a flux per pole of 0.04 Wb and the armature is lap-wound with 720 conductors. The shunt field resistance is 240 Ω and the armature resistance is 0.2 Ω. Brush contact drop is 1V per brush. Determine the speed of the machine when running (*a*) as a motor taking 60 A and (*b*) as a generator supplying 120 A. The terminal voltage in each case is 480 V. **[972 r.p.m. ; 1055 r.p.m.]**

6. A 25-kW shunt generator is delivering full output to 400-V bus-bars and is driven at 950 r.p.m. by belt drive. The belt breaks suddenly but the machine continues to run as a motor taking 25 kW from the bus-bars. At what speed does it run ? Take armature resistance including brush contact resistance as 0.5 Ω and field resistance as 160 Ω. **[812.7 r.p.m.] (*Elect. Technology, Andhra Univ. Apr. 1977*)**

7. A 4-pole, d.c. shunt motor has a wave-wound armature with 65 slots each containing 6 conductors. The *flux* per pole is 20 mWb and the armature has a resistance of 0.15 Ω. Calculate the motor speed when the machine is operating from a 250-V supply and taking a current of 60 A. **[927 r.p.m.]**

8. A 500-V, d.c. shunt motor has armature and field resistances of 0.5 Ω and 200 Ω respectively. When loaded and taking a total input of 25 kW, it runs at 400 r.p.m. Find the speed at which it must be driven as a shunt generator to supply a power output of 25 kW at a terminal voltage of 500 V. **[442 r.p.m.]**

9. A d.c. shunt motor runs at 900 r.p.m. from a 400 V supply when taking an armature current of 25 A. Calculate the speed at which it will run from a 230 V supply when taking an armature current of 15 A. The resistance of the armature circuit is 0.8 Ω. Assume the flux per pole at 230 V to have decreased to 75% of its value at 400 V. **[595 r.p.m.]**

10. A shunt machine connected to 250-A mains has an armature resistance of 0.12 Ω and field resistance of 100 Ω. Find the ratio of the speed of the machine as a generator to the speed as a motor, if line current is 80 A in both cases. **[1.08] (*Electrical Engineering-II, Bombay Univ. April. 1977, Madras Univ. Nov. 1978*)**

11. A 20-kW d.c. shunt generator delivering rated output at 1000 r.p.m. has a terminal voltage of 500 V. The armature resistance is 0.1 Ω, voltage drop per brush is 1 volt and the field resistance is 500 Ω.

Calculate the speed at which the machine will run as a motor taking an input of 20 kW from a 500 V d.c. supply. **[976.1 r.p.m.] (*Elect. Engg-I Bombay Univ. 1975*)**

12. A 4-pole, 250-V, d.c. shunt motor has a lap-connected armature with 960 conductors. The flux per pole is 2×10^{-2} Wb. Calculate the torque developed by the armature and the useful torque in newton-metre when the current taken by the motor is 30A. The armature resistance is 0.12 ohm and the field resistance is 125 Ω. The rotational losses amount to 825 W.

[85.5 N-m ; 75.3 N-m] (*Electric Machinery-I, Madras Univ. Nov. 1979*)

29.12. Motor Characteristics

The characteristic curves of a motor are those curves which show relationships between the following quantities.

1. Torque and armature current *i.e.* T_a/I_a characteristic. It is known as ***electrical characteristic***.

2. Speed and armature current *i.e.* N/I_a characteristic.

3. Speed and torque *i.e.* N/T_a characteristic. It is also known as ***mechanical characteristic***. It can be found from (1) and (2) above.

While discussing motor characteristics, the following two relations should always be kept in mind :

$$T_a \propto \Phi I_a \quad \text{and} \quad N \propto \frac{E_b}{\Phi}$$

29.13. Characteristics of Series Motors

1. T_a/I_a Characteristic. We have seen that $T_a \propto \Phi I_a$. In this case, as field windings also carry the armature current, $\Phi \propto I_a$ up to the point of magnetic saturation. Hence, before saturation,

$$T_a \propto \Phi I_a \quad \text{and} \quad \therefore \quad T_a \propto I_a^2$$

At light loads, I_a and hence Φ is small. But as I_a increases, T_a increases as the square of the current. Hence, T_a/I_a curve is a parabola as shown in Fig. 29.14. After saturation, Φ is almost independent of I_a hence $T_a \propto I_a$ only. So the characteristic becomes a straight line. The shaft torque T_{sh} is less than armature torque due to stray losses. It is shown dotted in the figure. So we conclude that (prior to magnetic saturation) on heavy loads, a series motor exerts a torque proportional to the square of armature current. Hence, in cases where huge starting torque is required for accelerating heavy masses quickly as in hoists and electric trains etc., series motors are used.

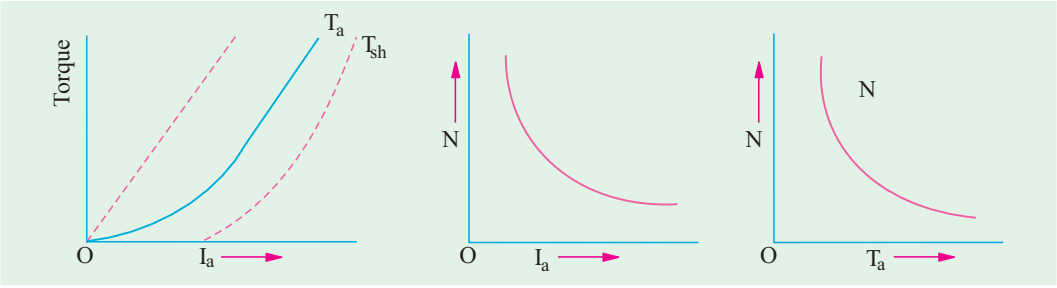


Fig. 29.14 Fig. 29.15 Fig. 29.16

2. N/I_a Characteristics. Variations of speed can be deduced from the formula :

$$N \propto \frac{E_b}{\Phi}$$

Change in E_b, for various load currents is small and hence may be neglected for the time being. With increased I_a, Φ also increases. Hence, speed varies inversely as armature current as shown in Fig. 29.15.

When load is heavy, I_a is large. Hence, speed is low (this decreases E_b and allows more armature current to flow). But when load current and hence I_a falls to a small value, speed becomes dangerously high. Hence, a series motor should never be started without some mechanical (not belt-driven) load on it otherwise it may develop excessive speed and get damaged due to heavy centrifugal forces so produced. It should be noted that series motor is a variable speed motor.

3. N/T_a or mechanical characteristic. It is found from above that when speed is high, torque is low and ***vice-versa***. The relation between the two is as shown in Fig. 29.16.

29.14. Characteristics of Shunt Motors

1. T_a/I_a Characteristic

Assuming Φ to be practically constant (though at heavy loads, ϕ decreases somewhat due to increased armature reaction) we find that $T_a \propto I_a$.

Hence, the electrical characteristic as shown in Fig. 29.17, is practically a straight line through the origin. Shaft torque is shown dotted. Since a heavy starting load will need a heavy starting current, shunt motor should never be started on (heavy) load.

2. N/I_a Characteristic

If Φ is assumed constant, then $N \propto E_b$. As E_b is also practically constant, speed is, for most purposes, constant (Fig. 29.18).

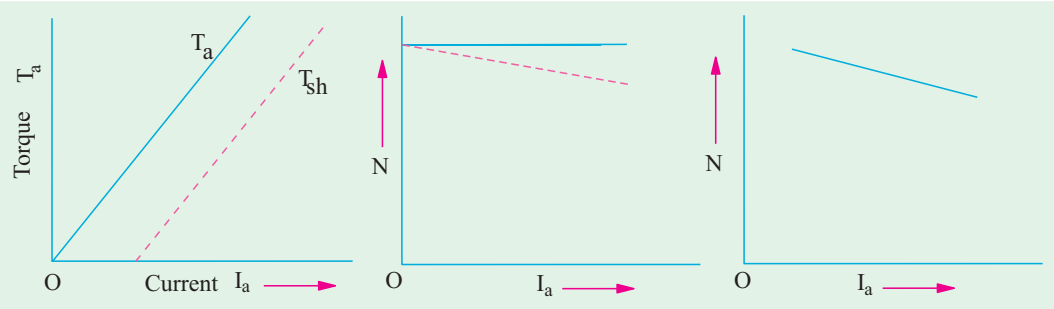


Fig. 29.17 Fig. 29.18 Fig. 29.19

But strictly speaking, both E_b and Φ decrease with increasing load. However, E_b decreases slightly more than ϕ so that on the whole, there is some decrease in speed. The drop varies from 5 to 15% of full-load speed, being dependent on saturation, armature reaction and brush position. Hence, the actual speed curve is slightly drooping as shown by the dotted line in Fig. 29.18. But, for all practical purposes, shunt motor is taken as a constant-speed motor.

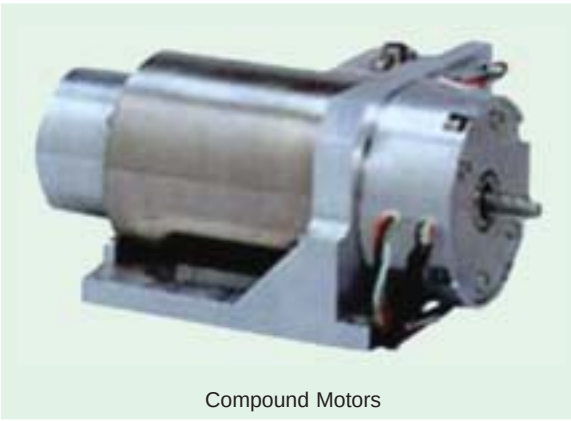
Because there is no appreciable change in the speed of a shunt motor from no-load to full-load, it may be connected to loads which are totally and suddenly thrown off without any fear of excessive speed resulting. Due to the constancy of their speed, shunt motors are suitable for driving shafting, machine tools, lathes, wood-working machines and for all other purposes where an approximately constant speed is required.

3. N/T_a Characteristic can be deduced from **(1)** and **(2)** above and is shown in Fig. 29.19.

29.15. Compound Motors

These motors have both series and shunt windings. If series excitation helps the shunt excitation *i.e.* series flux is in the ***same*** direction (Fig. 29.20); then the motor is said to be cummulatively compounded. If on the other hand, series field opposes the shunt field, then the motor is said to be differentially compounded.

The characteristics of such motors lie in between those of shunt and series motors as shown in Fig. 29.21.



Compound Motors

(*a*) Cumulative-compound Motors

Such machines are used where series characteristics are required and where, in addition,

the load is likely to be removed totally such as in some types of coal cutting machines or for driving heavy machine tools which have to take sudden cuts quite often. Due to shunt windings, speed will not become excessively high but due to series windings, it will be able to take heavy loads. In conjunction with fly-wheel (functioning as load equalizer), it is employed where there are sudden temporary loads as in rolling mills. The fly-wheel supplies its stored kinetic energy when motor slows down due to sudden heavy load. And when due to the removal of load motor speeds up, it gathers up its kinetic energy.

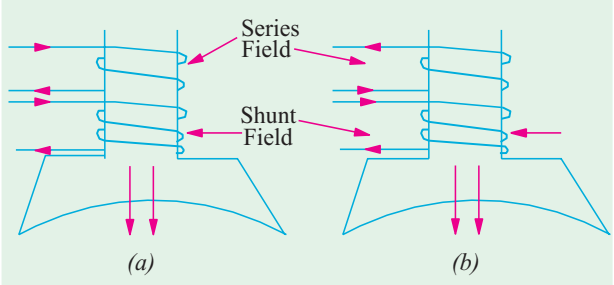


Fig. 29.20

Compound-wound motors have greatest application with loads that require high starting torques or pulsating loads (because such motors smooth out the energy demand required of a pulsating load). They are used to drive electric shovels, metal-stamping machines, reciprocating pumps, hoists and compressors etc.

(*b*) Differential-compound Motors

Since series field opposes the shunt field, the flux is decreased as load is applied to the motor. This results in the motor speed remaining almost constant or even increasing with increase in load (because, $N \propto E_b/(\Phi)$. Due to this reason, there is a decrease in the rate at which the motor torque increases with load. Such motors are not in common use. But because they can be designed to give an accurately constant speed under all conditions, they find limited application for experimental and research work.

One of the biggest drawback of such a motor is that due to weakening of flux with increases in load, there is a tendency towards speed instability and motor running away unless designed properly.

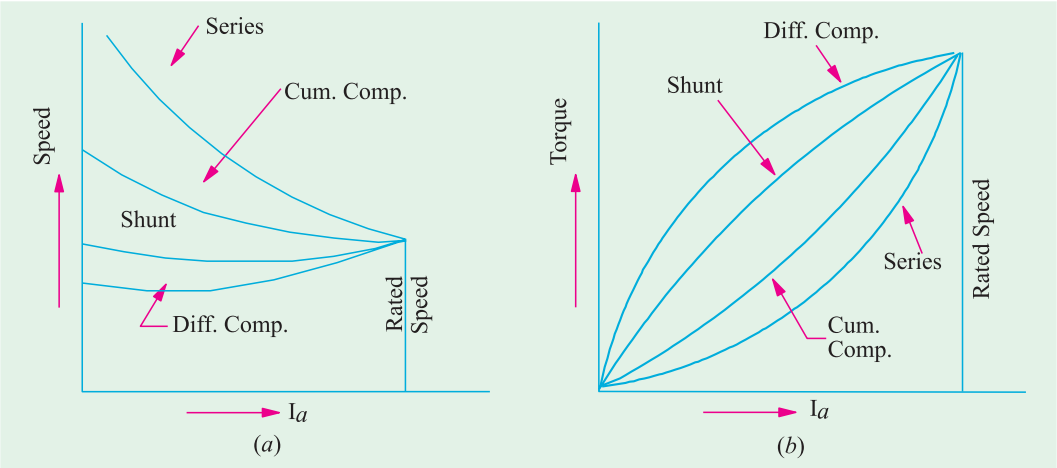


Fig. 29.21

Example 29.32. *The following results were obtained from a static torque test on a series motor :*

Current (A)	:	*20*	*30*	*40*	*50*
Torque (N - m)	:	*128.8*	*230.5*	*349.8*	*46.2*

Deduce the speed/torque curve for the machine when supplied at a constant voltage of 460 V. Resistance of armature and field winding is 0.5 Ω. Ignore iron and friction losses.

Solution. Taking the case when input current is 20 A, we have

Motor input = $460 \times 20 = 9{,}200$ W

Field and armature Cu loss

$= 20^2 \times 0.5 = 200$ W

Ignoring iron and friction losses,

output = $9{,}200 - 200 = 9{,}000$ W

Now, $T_{sh} \times 2\pi N$ = Output in watts.

$\therefore \quad 128.8 \times 2\pi \times N = 9{,}000$

$\therefore \quad N = 9{,}000/2\pi \times 128.8$

$= 11.12$ r.p.s. $= 667$ r.p.m.

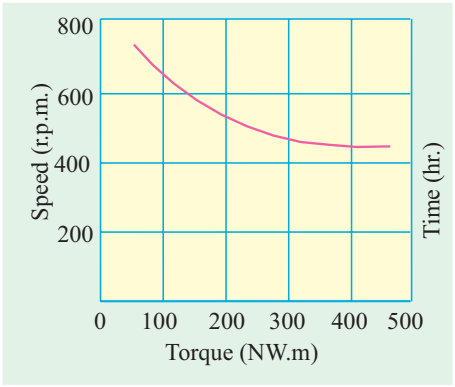


Fig. 29.22

Similar calculations for other values of current are tabulated below :

Current (A)	20	30	40	50
Input (W)	9,200	13,800	18,400	23,000
I^2R loss (W)	200	450	800	1,250
Output (W)	9,200	13,350	17,600	21,850
Speed (r.p.m.)	667	551	480	445
Torque (N-m)	128.8	230.5	349.8	469.2

From these values, the speed/torque curve can be drawn as shown in Fig. 29.22.

Example 29.33. *A fan which requires 8 h.p. (5.968 kW) at 700 r.p.m. is coupled directly to a d.c. series motor. Calculate the input to the motor when the supply voltage is 500 V, assuming that power required for fan varies as the cube of the speed. For the purpose of obtaining the magnetisation characteristics, the motor was running as a self-excited generator at 600 r.p.m. and the relationship between the terminal voltage and the load current was found to be as follows :*

Load current (A)	:	*7*	*10.5*	*14*	*27.5*
Terminal voltage (V)	:	*347*	*393*	*434*	*458*

The resistance of both the armature and field windings of the motor is 3.5 Ω and the core, friction and other losses may be assumed to be constant at 450 W for the speeds corresponding to the above range of currents at normal voltage. **(I.E.E. London)**

Solution. Let us, by way of illustration, calculate the speed and output when motor is running off a 500-V supply and taking a current of 7A.

Series voltage drop = $7 \times 3.5 = 24.5$ V

Generated or back e.m.f. E_b = $500 - 24.5 = 475.5$ V

The motor speed is proportional to E_b for a given current. For a speed of 600 r.p.m. and a current of 7A, the generated e.m.f is 347 V. Hence,

$N = 600 \times 475.5/347 = 823$ r.p.m.

Power to armature = $E_b I_a = 475.5 \times 7 = 3{,}329$ W

Output = Armature power – 450 = 3,329 – 450 = 2.879 W = **2.879 kW**

Power required by the fan at 823 r.p.m. is = $5.968 \times 823^2/700^2 = 9.498$ kW

These calculations are repeated for the other values of current in the following table.

Input currrent (A)	7	10.5	14	27.5
Series drop (V)	24.5	36.7	49	96.4
Back e.m.f. (V)	475.5	463.3	451	403.6

E.M.F. at 600 r.p.m. (V)	347	393	434	458
Speed N (r.p.m.)	823	707	623	528
Armature power (W)	3329	4870	6310	11,100
Motor output (kW)	2.879	4.420	5.860	10.65
Power required by fan (kW)	9.698	6.146	4.222	2.566

In Fig. 29.23 ***(i)*** the motor output in kW and ***(ii)*** power required by fan in kW against input currentis plotted. Since motor output equals the input to fan, hence the intersection point of these curves gives the value of motor input current under the given conditions.

Input current corresponding to intersection point = 12 A

$\therefore$ Motor input = 500×12 = **6,000 W**

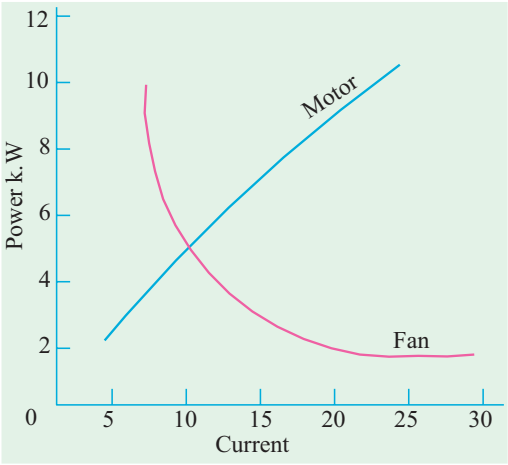


Fig. 29.23

29.16. Performance Curves

(*a*) Shunt Motor

In Fig. 29.24 the four essential characteristics of a shunt motor are shown *i.e.* torque, current speed and efficiency, each plotted as a function of motor output power. These are known as the ***performance curves*** of a motor.

It is seen that shunt motor has a definite no-load speed. Hence, it does not 'run away' when load is suddenly thrown off provided the field circuit remains closed. The drop in speed from no-load to full-load is small, hence this motor is usually referred to as ***constant speed*** motor. The speed for any load within the operating range of the motor can be readily obtained by varying the field current by means of a field rheostat.

The efficiency curve is usually of the same shape for all electric motors and generators. The shape of efficiency curve and the point of maximum efficiency can be varied considerably by the designer, though it is advantageous to have an efficiency curve which is farily flat, so that there is little change in efficiency between load and 25% overload and to have the maximum efficiency as near to the full load as possible.

It will be seen from the curves, that a certain value of current is required even when output is zero. The motor input under no-load conditions goes to meet the various losses occuring within the machine.

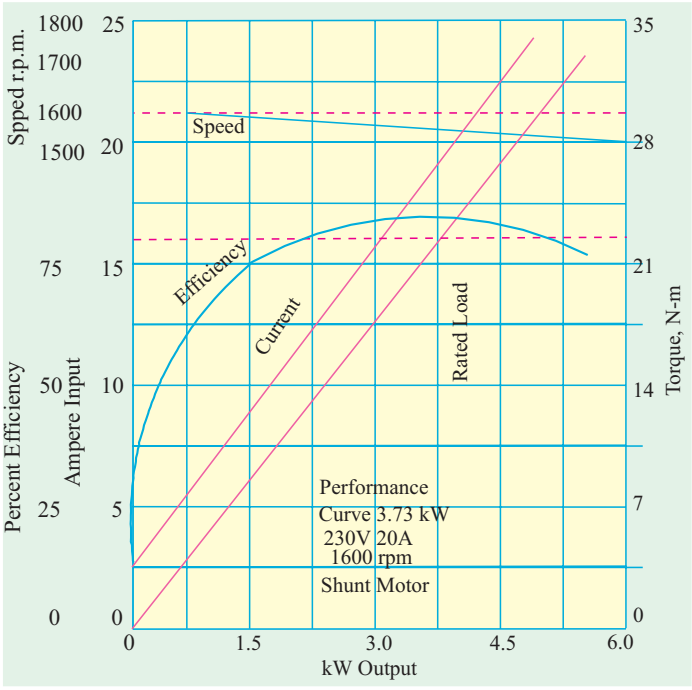


Fig. 29.24

As compared to other motors, a shunt motor is said to have a lower starting torque. But this should not be taken of mean that a shunt motor is incapable of starting a heavy load. Actually, it means that series and compound motors are capable of starting heavy loads with less excess of current inputs over normal values than the shunt motors and that consequently the depreciation on the motor will be relatively less. For example, if twice full load torque is required at start, then shunt motor draws twice the full-load current ($T_a \propto I_a$ or $I_a \propto \sqrt{T_a}$) whereas series motor draws only approximately one and a half times the full load current ($T_a \propto I_a^2$ or $I_a \propto \sqrt{T_a}$).

The shunt motor is widely used with loads that require essentially constant speed but where high starting torques are not needed. Such loads include centrifugal pumps, fans, winding reels conveyors and machine tools etc.

(*b*) Series Motor

The typical performance curves for a series motor are shown in Fig. 29.25.

It will be seen that drop in speed with increased load is much more prominent in series motor than in a shunt motor. Hence, a series motor is not suitable for applications requiring a substantially constant speed.

For a given current input, the starting torque developed by a series motor is greater than that developed by a shunt motor. Hence, series motors are used where huge starting torques are necessary *i.e.* for street cars, cranes, hoists and for electric-railway operation. In addition to the huge starting torque, there is another unique characteristic of series motors which makes them especially desirable for traction work *i.e.* when a load comes on a series motor, it responds by decreasing its speed (and hence, E_b) and supplies the increased torque with a small increase in current. On the other hand a shunt motor under the same conditions would hold its speed nearly constant and would supply the required increased torque with a large increase of input current. Suppose that instead of a series motor, a shunt motor is used to drive a street car. When the car ascends a grade, the shunt motor maintains the speed for the car at approximately the same value it had on the level ground, but the motor tends to take an excessive current. A series motor, however, automatically slows down on such a grade because of increased current demand, and so it develops more torque at reduced speed. The drop in speed permits the motor to develop a large torque with but a moderate increase of power. Hence, under the same load conditions, rating of the series motor would be less than for a shunt motor.

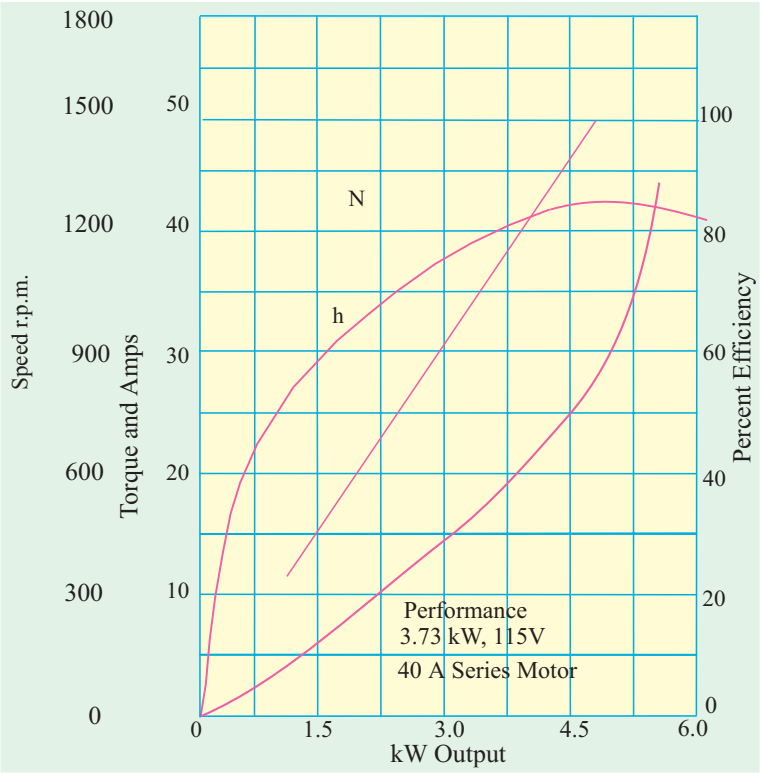


Fig. 29.25

29.17. Comparison of Shunt and Series Motors



Shunt Motors

(*a*) Shunt Motors

The different characteristics have been discussed in Art. 29.14. It is clear that

(*a*) speed of a shunt motor is sufficiently constant.

(*b*) for the same current input, its starting torque is not a high as that of series motor. Hence, it is used.

(*i*) When the speed has to be maintained approximately constant from N.L. to F.L. *i.e.* for driving a line of shafting etc.

(*ii*) When it is required to drive the load at various speeds, any one speed being kept constant for a relatively long period *i.e.* for individual driving of such machines as lathes. The shunt regulator enables the required speed control to be obtained easily and economically.

Summary of Applications

Type of motor	*Characteristics*	*Applications*
Shunt	Approximately constant speed Adjustable speed Medium starting torque (Up to 1.5 F.L. torque)	For driving constant speed line shafting Lathes Centrifugal pumps Machine tools Blowers and fans Reciprocating pumps
Series	Variable speed Adjustable variying speed High Starting torque	For traction work *i.e.* Electric locomotives Rapid transit systems Trolley, cars etc. Cranes and hoists Conveyors
Comulative Compound	Variable speed Adjustable varying speed High starting torque	For intermittent high torque loads For shears and punches Elevators Conveyors Heavy planers Heavy planers Rolling mills; Ice machines; Printing presses; Air compressors

(*b*) Series Motors

The operating characteristics have been discussed in Art 29.13. These motors

1. have a relatively huge starting torques.

2. have good accelerating torque.

3. have low speed at high loads and dangerously high speed at low loads.

Hence, such motors are used

1. when a large starting torque is required *i.e.* for driving hoists, cranes, trams etc.

2. when the motor can be directly coupled to a load such as a fan whose torque increases with speed.

3. if constancy of speed is not essential, then, in fact, the decrease of speed with increase of load has the advantage that the power absorbed by the motor does not increase as rapidly as the torque. For instance, when torque is doubled, the power approximately increases by about 50 to 60% only. ($\therefore I_a \propto \sqrt{T_a}$).

4. a series motor should not be used where there is a possibility of the load decreasing to a very small value. Thus, it should not be used for driving centrifugal pumps or for a belt-drive of any kind.



Series Motors

29.18. Losses and Efficiency

The losses taking place in the motor are the same as in generators. These are **(*i*)** Copper losses **(*ii*)** Magnetic losses and **(*iii*)** Mechanical losses.

The condition for maximum *power* developed by the motor is

$$I_a R_a = V/2 = E_b.$$

The condition for maximum ***efficiency*** is that armature Cu losses are equal to constant losses. (Art. 26.39).

29.19. Power Stages

The various stages of energy transformation in a motor and also the various losses occurring in it are shown in the flow diagram of Fig. 29.26.

Overall or commercial efficiency $\eta_c = \frac{C}{A}$, Electrical efficiency $\eta_e = \frac{B}{A}$, Mechanical efficiency $\eta_m = \frac{C}{B}$.

The efficiency curve for a motor is similar in shape to that for a generator (Art. 24.35).

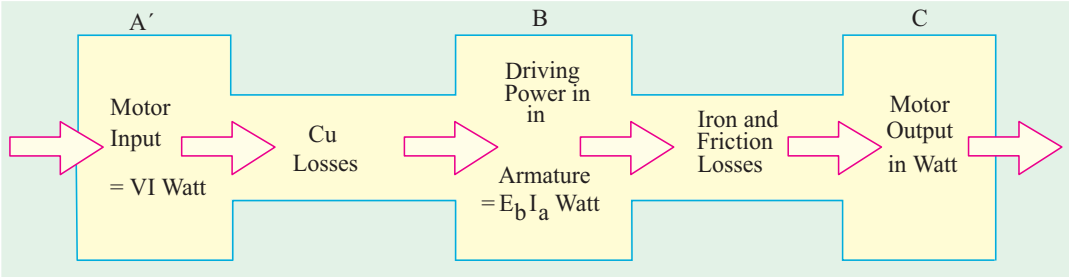


Fig. 29.26

It is seen that $A - B$ = copper losses and $B - C$ = iron and friction losses.

Example 29.34. *One of the two similar 500-V shunt machines A and B running light takes 3 A. When A is mechanically coupled to B, the input to A is 3.5 A with B unexcited and 4.5 A when B is separately-excited to generate 500 V. Calculate the friction and windage loss and core loss of each machine.* **(Electric Machinery-I, Madras Univ. 1985)**

Solution. When running light, machine input is used to meet the following losses **(*i*)** armature Cu loss **(*ii*)** shunt Cu loss **(*iii*)** iron loss and **(*iv*)** mechanical losses *i.e.* friction and windage losses. Obviously, these no-load losses for each machine equal $500 \times 3 = 1500$ W.

(*a*) With B unexcited

In this case, only mechanical losses take place in *B*, there being neither Cu loss nor iron-loss because *B* is unexcited. Since machine *A* draws 0.5 A more current.

Friction and windage loss of $B = 500 \times 0.5 =$ **250 W**

(*b*) With B excited

In this case, both iron losses as well as mechanical losses take place in machine *B*. Now, machine *A* draws, 4.5 – 3 = 1.5 A more current.

Iron and mechanical losses of $B = 1.5 \times 500 = 750$ W

Iron losses of $B = 750 - 250 =$ **500 W**

Example 29.35. *A 220 V shunt motor has an armature resistance of 0.2 ohm and field resistance of 110 ohm. The motor draws 5 A at 1500 r.p.m. at no load. Calculate the speed and shaft torque if the motor draws 52 A at rated voltage.* **(Elect. Machines Nagpur Univ. 1993)**

Solution.

$$I_{sh} = 220/110 = 2 \text{ A}; I_{a1} = 5 - 2 = 3 \text{ A}; I_{a2} = 52 - 2 = 50 \text{ A}$$

$$E_{b1} = 220 - 3 \times 0.2 = 219.4 \text{ V}; E_{b2} = 220 - 50 \times 0.2 = 210 \text{ V}$$

$$\frac{N_2}{1500} = \frac{210}{219.4}; N_2 = 1436 \text{ r.p.m.} \quad (\because \Phi_1 = \Phi_2)$$

For finding the shaft torque, we will find the motor output when it draws a current of 52 A. First we will use the no-load data for finding the constant losses of the motor.

No load motor input $= 220 \times 5 = 1100$ W; Arm. Cu loss $= 3^2 \times 0.2 = 2$ W

$\therefore$ Constant or standing losses of the motor $= 1100 - 2 = 1098$

When loaded, arm. Cu loss $= 50^2 \times 0.2 = 500$ W

Hence, total motor losses $= 1098 + 500 = 1598$ W

Motor input on load $= 220 \times 52 = 11{,}440$ W; output $= 11{,}440 - 1598 = 9842$ W

$\therefore$ $T_{sh} = 9.55 \times \text{output}/N = 9.55 \times 9842/1436 =$ **65.5 N-m**

Example 29.36. *250 V shunt motor on no load runs at 1000 r.p.m. and takes 5 amperes. Armature and shunt field resistances are 0.2 and 250 ohms respectively. Calculate the speed when loaded taking a current of 50 A. The armature reaction weakens the field by 3%.*

(Elect. Engg.-I Nagpur Univ. 1993)

Solution.

$$I_{sh} = 250/250 = 1 \text{ A}; I_{a1} = 5 - 1 = 4 \text{ A}; I_{a2} = 50 - 1 = 49 \text{ A}$$

$$E_{b1} = 250 - 4 \times 0.2 = 249.2 \text{ V}; E_{b2} = 250 - 49 \times 0.2 = 240.2 \text{ V}$$

$$\frac{N_2}{1000} = \frac{240.2}{249.2} \times \frac{\Phi_1}{0.97\,\Phi_1}; N_2 = 944 \text{ r.p.m.}$$

Example 29.37. *A 500 V d.c. shunt motor takes a current of 5 A on no-load. The resistances of the armature and field circuit are 0.22 ohm and 250 ohm respectively. Find* ***(a)*** *the efficiency when loaded and taking a current of 100 A* ***(b)*** *the percentage change of speed. State precisely the assumptions made.* **(Elect. Engg-I, M.S. Univ. Baroda 1987)**

Solution. No-Load condition

$I_{sh} = 500/250 = 2$ A; $I_{a0} = 5 - 2 = 3$ A; $E_{b0} = 500 - (3 \times 0.22) = 499.34$ V

Arm. Cu loss $= 3^2 \times 0.22 = 2$ W; Motor input $= 500 \times 5 = 2500$ W

Constant losses $= 2500 - 2 = 2498$ W

It is assumed that these losses remain constant under all load conditions.

Load condition

(*a*) Motor current $= 100$ A; $I_a = 100 - 2 = 98$ A; $E_b = 500 - (98 \times 0.22) = 478.44$ V

Arm. Cu loss = $98^2 \times 0.22 = 2110$ W, Total losses = 2110 + 2498 = 4608 W

Motor input = $500 \times 100 = 50{,}000$ W, Motor output = 50,000 – 4,608 = 45,392 W

Motor η = 45,392/50,000 = 0.908 or **90.8%**

$$\frac{N}{N_0} = \frac{E_b}{E_{b0}} = \frac{478.44}{499.34} \quad \text{or} \quad \frac{N - N_0}{N_0} = \frac{-20.9}{499.34} = -0.0418 \text{ or } \textbf{-4.18\%}$$

Example 29.38. *A 250 V d.c. shunt motor runs at 1000 r.p.m. while taking a current of 25 A. Calculate the speed when the load current is 50 A if armature reaction weakens the field by 3%. Determine torques in both cases.*

R_a = *0.2 ohm ; R_f = 250 ohms*

Voltage drop per brush is 1 V. **(Elect. Machines Nagpur Univ. 1993)**

Solution.

$I_{sh} = 250/250 = 1$ A; $I_{a1} = 25 - 1 = 24$ A

E_{bh} = 250 – arm. drop – brush drop

= $250 - 24 \times 0.2 - 2 = 243.2$ V

$I_{a2} = 50 - 1 = 49$ A; $E_{b2} = 250 - 49 \times 0.2 - 2 = 238.2$ V

$$\frac{N_2}{1000} = \frac{238.2}{243.2} \times \frac{\Phi_1}{0.97\,\Phi_1}; \quad N_2 = 1010 \text{ r.p.m.}$$

$T_{a1} = 9.55\, E_{b1}\, I_{a1}/N_1 = 9.55 \times 243.2 \times 24/1000 = 55.7$ N-m

$T_{a2} = 9.55 \times 238.2 \times 49/1010 = 110.4$ r.p.m.

Example 29.39. *A d.c. shunt machine while running as generator develops a voltage of 250 V at 1000 r.p.m. on no-load. It has armature resistance of 0.5 Ω and field resistance of 250 Ω. When the machine runs as motor, input to it at no-load is 4 A at 250 V. Calculate the speed and efficiency of the machine when it runs as a motor taking 40 A at 250 V. Armature reaction weakens the field by 4 %.* **(Electrical Technology, Aligarh Muslim Univ. 1989)**

Solution.

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$$

Now, when running as a generator, the machine gives 250 V at 1000 r.p.m. If this machine was running as motor at 1000 r.p.m., it will, obviously, have a back e.m.f. of 250 V produced in its armature. Hence N_1 = 1000 r.p.m. and E_{b1} = 250 V.

When it runs as a motor, drawing 40 A, the back e.m.f. induced in its armature is

$E_{b2} = 250 - (40 - 1) \times 0.5 = 230.5$ V; Also $\Phi_2 = 0.96\, \Phi_1$, N_2 = ?

Using the above equation we have

$$\frac{N_2}{1000} = \frac{230.5}{250} \times \frac{\Phi_1}{0.96\,\Phi_1}; \quad N_2 = \textbf{960 r.p.m.}$$

Efficiency

No-load input represents motor losses which consists of

(a) armature Cu loss = $I_a^2 R_a$ which is variable.

(b) constant losses W_c which consists of ***(i)*** shunt Cu loss ***(ii)*** magnetic losses and ***(iii)*** mechanical losses.

No-load input or total losses = $250 \times 4 = 1000$ W

Arm. Cu loss = $I_a^2 R_a = 3^2 \times 0.5 = 4.5$ W, $\therefore$ $W_c = 1000 - 4.5 = 995.5$ W

When motor draws a line current of 40 A, its armature current is (40 – 1) = 39 A

Arm. Cu loss = $39^2 \times 0.5 = 760.5$ W; Total losses = 760.5 + 955.5 = 1756 W

Input = $250 \times 40 = 10{,}000$ W; output = 10,000 – 1756 = 8,244 W

$\therefore$ $\eta = 8{,}244 \times 100/10{,}000 =$ **82.44%**

Example 29.40. *The armature winding of a 4-pole, 250 V d.c. shunt motor is lap connected. There are 120 slots, each slot containing 8 conductors. The flux per pole is 20 mWb and current taken by the motor is 25 A. The resistance of armature and field circuit are 0.1 and 125 Ω respectively. If the rotational losses amount to be 810 W find,*

(i) *gross torque* ***(ii)*** *useful torque and* ***(iii)*** *efficiency.* **(Elect. Machines Nagpur Univ. 1993)**

Solution. $I_{sh} = 250/125 = 2$ A; $I_a = 25 - 2 = 23$ A; $E_b = 250 - (23 \times 0.1) = 247.7$ V

Now, $E_b = \dfrac{\Phi\, ZN}{60}\left(\dfrac{P}{A}\right)$ $\quad \therefore 247.7 = \dfrac{20 \times 10^{-3} \times 960 \times N}{60}\left(\dfrac{4}{4}\right)$; $N = 773$ r.p.m.

(i) Gross torque or armature torque $T_a = 9.55\,\dfrac{E_b I_a}{N} = 9.55 \times \dfrac{247.7 \times 23}{773} =$ **70.4 N - m**

(ii) Arm Cu loss $= 23^2 \times 0.1 = 53$ W; Shunt Cu loss $= 250 \times 2 = 500$ W

Rotational losses = 810 W; Total motor losses = 810 + 500 + 53 = 1363 W

Motor input = $250 \times 25 = 6250$ W; Motor output = 6250 – 1363 = 4887 W

$T_{sh} = 9.55 \times$ output/$N = 9.55 \times 4887/773 = 60.4$ N-m

(iii) Efficiency = 4887/6250 = 0.782 = 78.2%

Example 29.41. *A 20-hp (14.92 kW); 230-V, 1150-r.p.m. 4-pole, d.c. shunt motor has a total of 620 conductors arranged in two parallel paths and yielding an armature circuit resistance of 0.2 Ω. When it delivers rated power at rated speed, it draws a line current of 74.8 A and a field current of 3 A. Calculate* ***(i)*** *the flux per pole* ***(ii)*** *the torque developed* ***(iii)*** *the rotational losses* ***(iv)*** *total losses expressed as a percentage of power.* **(Electrical Machinery-I, Banglore Univ. 1987)**

Solution. $I_a = 74.8 - 3 = 71.8$ A; $E_b = 230 - 71.8 \times 0.2 = 215.64$ V

(i) Now, $E_b = \dfrac{\Phi\, ZN}{60}\left(\dfrac{P}{A}\right)$; $215.64 = \dfrac{\Phi \times 620 \times 1150}{60}\left(\dfrac{4}{2}\right)$; $\Phi =$ **9 mWb**

(ii) Armature Torque, $T_a = 9.55 \times 215.64 \times 71.8/1150 =$ **128.8 N-m**

(iii) Driving power in armature $= E_b I_a = 215.64 \times 71.8 = 15{,}483$ W

Output = 14,920 W; Rotational losses = 15,483 – 14,920 = **563 W**

(iv) Motor input = $VI = 230 \times 74.8 = 17{,}204$ W; Total loss = 17,204 – 14,920 = 2,284 W

Losses expressed as percentage of power input = 2284/17,204 = 0.133 or **13.3%**

Example 29.42. *A 7.46 kW, 250-V shunt motor takes a line current of 5 A when running light. Calculate the efficiency as a motor when delivering full load output, if the armature and field resistance are 0.5 Ω and 250 Ω respectively. At what output power will the efficiency be maximum ? Is it possible to obtain this output from the machine ?* **(Electrotechnics-II, M.S. Univ. Baroda 1985)**

Solution. When loaded lightly

Total motor input (or total no-load losses) $= 250 \times 5 = 1{,}250$ W

$I_{sh} = 250/250 = IA \quad \therefore I_a = 5 - 1 = 4$ A

Field Cu loss $= 250 \times 1 = 250$ W; Armature Cu loss $= 4^2 \times 0.5 = 8$ W

$\therefore$ Iron losses and friction losses $= 1250 - 250 - 8 = 992$ W

These losses would be assumed constant.

Let I_a be the full-load armature current, then armature input is $= (250 \times I_a)$ W

F.L. output $= 7.46 \times 1000 = 7{,}460$ W

The losses in the armature are :

(i) Iron and friction losses = 992 W

(ii) Armature Cu loss $= I_a^2 \times 0.05$ W $\quad \therefore \quad 250\, I_a = 7{,}460 + 992 + I_a^2 \times 0.5$

or $0.5\,I_a^2 - 250\,I_a + 8{,}452 = 0$ $\therefore$ $I_a = 36.5$ A

$\therefore$ F.L. input current = 36.5 + 1 = 37.5 A ; Motor input = 250×37.5 W

F.L. output = 7,460 W

$\therefore$ F.L. efficiency = $7460 \times 100/250 \times 37.5 = 79.6\%$

Now, efficiency is maximum when armature Cu loss equals constant loss.

i.e. $I_a^2 R_d = I_a^2 \times 0.5 = (1{,}250 - 8) = 1{,}242$ W or $I_a = 49.84$ A

$\therefore$ Armature input = $250 \times 49.84 = 12{,}460$ W

Armature Cu loss = $49.84^2 \times 0.5 = 1242$ W; Iron and friction losses = 992 W

$\therefore$ Armature output = 12,460 – (1,242 + 992) = 10,226 W

$\therefore$ Output power = 10,226 W = **10.226 kW**

As the input current for maximum efficiency is beyond the full-load motor current, it is never realised in practice.

Example 29.43. *A d.c. series motor drives a load, the torque of which varies as the square of the speed. Assuming the magnetic circuit to be remain unsaturated and the motor resistance to be negligible, estimate the percentage reduction in the motor terminal voltage which will reduce the motor speed to half the value it has on full voltage. What is then the percentage fall in the motor current and efficiency ? Stray losses of the motor may be ignored.*

(Electrical Engineering-III, Pune Univ. 1987)

Solution. $T_a \propto \Phi I_a \propto I_a^2$. Also, $T_a \propto N^2$. Hence $N^2 \propto I_a^2$ or $N \propto I_a$

$\therefore$ $N_1 \propto I_{a1}$ and $N_2 \propto I_{a2}$ or $N_2/N_1 = I_{a2}/I_{a1}$

Since, $N_2/N_1 = 1/2$ $\therefore$ $I_{a2}/I_{a1} = 1/2$ or $I_{a2} = I_{a1}/2$

Let V_1 and V_2 be the voltages across the motor in the two cases. Since motor resistance is negligible, $E_{b1} = V_1$ and $E_{b2} = V_2$. Also $\Phi_1 \propto I_{a1}$ and $\Phi_2 \propto I_{a2}$ or $\Phi_1/\Phi_2 = I_{a1}/I_{a2} = I_{a1} \times 2/I_{a1} = 2$

Now, $\dfrac{N_2}{N_1} = \dfrac{E_{b2}}{E_{b1}} \times \dfrac{\Phi_1}{\Phi_2}$ or $\dfrac{1}{2} = \dfrac{V_2}{V_1} \times 2$ or $\dfrac{V_2}{V_1} = \dfrac{1}{4}$

$\therefore$ $\dfrac{V_1 - V_2}{V_1} = \dfrac{4-1}{4} = 0.75$

$\therefore$ Percentage reduction in voltage = $\dfrac{V_1 - V_2}{V_1} \times 100 = 0.75 \times 100 =$ **75%**

Percentage change in motor current = $\dfrac{I_{a1} - I_{a2}}{I_{a1}} \times 100 = \dfrac{I_{a1} - I_{a1}/2}{I_{a1}} \times 100 =$ **50%**

Example 29.44. *A 6-pole, 500-V wave-connected shunt motor has 1200 armature conductors and useful flux/pole of 20 mWb. The armature and field resistance are 0.5 Ω and 250 Ω respectively. What will be the speed and torque developed by the motor when it draws 20 A from the supply mains ? Neglect armature reaction. If magnetic and mechanical losses amount to 900 W, find **(i)** useful torque **(ii)** output in kW and **(iii)** efficiency at this load.*

Solution. (*i*) $I_{sh} = 500/250 = 2$ A $\therefore$ $I_a = 20 - 2 = 18$ A

$\therefore$ $E_b = 500 - (18 \times 0.5) = 491$ V; Now, $E_b = \dfrac{\Phi ZN}{60} \times \left(\dfrac{P}{A}\right)$ volt

$\therefore$ $491 = \dfrac{20 \times 10^{-3} \times 1200 \times N}{60} \times \left(\dfrac{6}{2}\right)$; $N =$ **410 r.p.m. (approx.)**

Now $T_a = 9.55 \dfrac{E_b I_a}{N} = 9.55 \dfrac{491 \times 18}{410} =$ **206 N - m**

Armature Cu loss = $18^2 \times 0.5 = 162$ W; Field Cu loss = $500 \times 2 = 1000$ W

Iron and friction loss = 900 W ; Total loss = 162 + 1000 + 900 = 2,062 W

Motor input = $500 \times 20 = 10{,}000$ W

(i) $T_{sh} = 9.55 \times \frac{7938}{410}$ = **184.8 N - m**

(ii) Output = 10,000 – 2062 = **7,938 kW**

(iii) $\% \eta = \frac{\text{Output}}{\text{Input}} \times 100 = \frac{7,938 \times 100}{10,000} = 0.794 =$ **79.4%**

Example 29.45. *A 50-h.p. (37.3 kW), 460-V d.c. shunt motor running light takes a current of 4 A and runs at a speed of 660 r.p.m. The resistance of the armature circuit (including brushes) is 0.3 Ω and that of the shunt field circuit 270 Ω.*

Determine when the motor is running at full load

(i) *the current input* ***(ii)*** *the speed. Determine the armature current at which efficiency is maximum. Ignore the effect of armature reaction.* **(Elect. Technology Punjab, Univ. 1991)**

Solution. I_{sh} = 460/270 = 1.7 A; Field Cu loss = $460 \times 1.7 = 783$ W

When running light

$I_a = 4 - 1.7 = 2.3$ A; Armature Cu loss = $2.3^2 \times 0.3 = 1.5$ W (negligible)

No-load armature input = $460 \times 2.3 = 1{,}058$ W

As armature Cu loss is negligible, hence 1,058 W represents iron, friction and windage losses which will be assumed to be constant.

Let full-load armature input current be I_a. Then

Armature input = $460\ I_a$ W; Armature Cu loss = $I_a^2 \times 0.3$ W

Output = 37.3 kW = 37,300 W

$\therefore$ $460\ I_a = 37{,}300 + 1{,}058 + 0.3\ I_a^2$ or $0.3\ I_a^2 - 460\ I_a + 38{,}358 = 0$

$\therefore$ $I_a = 88.5$ A

(i) Current input = 88.5 + 1.7 = **90.2 A**

(ii) $E_{b1} = 460 - (2.3 \times 0.3) = 459.3$V; $E_{b2} = 460 - (88.5 \times 0.3) = 433.5$ V

$\therefore$ $N_2 = 660 \times 433.5/459.3 =$ **624 r.p.m.**

For maximum efficiency, $I_a^2 R_a$ = constant losses (Art. 24.37)

$\therefore$ $I_a^2 \times 0.3 = 1058 + 783 = 1{,}841$ $\therefore$ $I_a = (1841/0.3)^{1/2} =$ **78.33 A**

Tutorial Problems 29.3

1. A 4-pole 250-V, d.c. series motor has a wave-wound armature with 496 conductors. Calculate

(*a*) the gross torque (*b*) the speed

(*b*) the output torque and (*d*) the efficiency, if the motor current is 50 A

The value of flux per pole under these conditions is 22 mWb and the corresponding iron, friction and windage losses total 810 W. Armature resistance = 0.19 Ω, field resistance = 0.14 Ω.

[(*a*) **173.5 N-m** (*b*) **642 r.p.m.** (*c*) **161.4 N-m** (*d*) **86.9%**]

2. On no-load, a shunt motor takes 5 A at 250 V, the resistances of the field and armature circuits are 250 Ω and 0.1 Ω respectively. Calculate the output power and efficiency of the motor when the total supply current is 81 A at the same supply voltage. State any assumptions made.

[18.5 kW; 91%. It is assumed that windage, friction and eddy current losses are independent of the current and speed]

3. A 230 V series motor is taking 50 A. Resistance of armature and series field windings is 0.2 Ω and 0.1 Ω respectively. Calculate :

(*a*) brush voltage (*b*) back e.m.f.

(*c*) power wasted in armature (*d*) mechanical power developed

[(*a*) **215 V** (*b*) **205 V** (*c*) **500 W** (*d*) **13.74 h.p.**] **(10.25 kW)**

4. Calculate the shaft power of a series motor having the following data; overall efficiency 83.5%, speed 550 r.p.m. when taking 65 A; motor resistance 0.2 Ω, flux per pole 25 mWb, armature winding lap with 1200 conductor.

(15.66 kW)

5. A shunt motor running on no-load takes 5 A at 200 V. The resistance of the field circuit is 150 Ω and of the armature 0.1 Ω. Determine the output and efficiency of motor when the input current is 120 A at 200 V. State any conditions assumed. **(89.8%)**

6. A d.c. shunt motor with interpoles has the following particulars :

Output power ; 8,952 kW, 440-V, armature resistance 1.1 Ω, brush contact drop 2 V, interpole winding resistance 0.4 Ω shunt resistance 650 Ω, resistance in the shunt regulator 50 Ω. Iron and friction losses on full-load 450 W. Calculate the efficiency when taking the full rated current of 24 A.

(85%)

7. A d.c. series motor on full-load takes 50 A from 230 V d.c. mains. The total resistance of the motor is 0.22 Ω. If the iron and friction losses together amount to 5% of the input, calculate the power delivered by the motor shaft. Total voltage drop due to the brush contact is 2 A. **(10.275 kW)**

8. A 2-pole d.c shunt motor operating from a 200 V supply takes a full-load current of 35 A, the no-load current being 2 A. The field resistance is 500 Ω and the armature has a resistance of 0.6 Ω. Calculate the efficiency of the motor on full-load. Take the brush drop as being equal to 1.5 V per brush arm. Neglect temperature rise.

[Rajiv Gandhi Tech. Univ. Bhopal,2000] **(82.63%)**

OBJECTIVE TESTS – 29

1. In a d.c. motor, undirectional torque is produced with the help of

(*a*) brushes (*b*) commutator

(*c*) end-plates (*d*) both (*a*) and (*b*)

2. The counter e.m.f. of a d.c. motor

(*a*) often exceeds the supply voltage

(*b*) aids the applied voltage

(*c*) helps in energy conversion

(*d*) regulates its armature voltage

3. The normal value of the armature resistance of a d.c. motor is

(*a*) 0.005 (*b*) 0.5

(*c*) 10 (*d*) 100

(Grad. I.E.T.E. June 1987)

4. The E_b/V ratio of a d.c. motor is an indication of its

(*a*) efficiency (*b*) speed regulation

(*c*) starting torque (*d*) Running Torque

(Grad. I.E.T.E. June 1987)

5. The mechanical power developed by the armature of a d.c. motor is equal to

(*a*) armature current multiplied by back e.m.f.

(*b*) power input minus losses

(*c*) power output multiplied by efficiency

(*d*) power output plus iron losses

6. The induced e.m.f. in the armature conductors of a d.c. motor is

(*a*) sinusoidal (*b*) trapezoidal

(*c*) rectangular (*d*) alternating

7. A d.c. motor can be looked upon as d.c. generator with the power flow

(*a*) reduced (*b*) reversed

(*c*) increased (*d*) modified

8. In a d.c. motor, the mechanical output power actually comes from

(*a*) field system

(*b*) air-gap flux

(*c*) back e.m.f.

(*d*) electrical input power

9. The maximum torque of d.c. motors is limited by

(*a*) commutation (*b*) heating

(*c*) speed (*d*) armature current

10. Which of the following quantity maintains the same direction whether a d.c. machine runs as a generator or as a motor ?

(*a*) induced e.m.f. (*b*) armature current

(*c*) field current (*d*) supply current

11. Under constant load conditions, the speed of a d.c. motor is affected by

(*a*) field flux (*b*) armature current

(*c*) back e.m.f. (*d*) both (*b*) and (*c*)

12. It is possible to increase the field flux and, at the same time, increase the speed of a d.c. motor provided its ..... is held constant.

(*a*) applied voltage

(*b*) torque

(*c*) Armature circuit resistance

(*d*) armature current

13. The current drawn by a 120 - V d.c. motor of armature resistance 0.5 Ω and back e.m.f. 110 V is ..... ampere.

(*a*) 20 (*b*) 240

(*c*) 220 (*d*) 5

14. The shaft torque of a d.c. motor is less than its armature torque because of ..... losses.

(*a*) copper (*b*) mechanical

(*c*) iron (*d*) rotational

15. A d.c. motor develops a torque of 200 N-m at 25 rps. At 20 rps it will develop a torque of ..... N-m.

(*a*) 200 (*b*) 160

(*c*) 250 (*d*) 128

16. Neglecting saturation, if current taken by a series motor is increased from 10 A to 12 A, the percentage increase in its torque is ..... percent

(*a*) 20 (*b*) 44

(*c*) 30.5 (*d*) 16.6

17. If load on a d.c. shunt motor is increased, its speed is decreased due primarily to

(*a*) increase in its flux

(*b*) decrease in back e.m.f.

(*c*) increase in armature current

(*d*) increase in brush drop

18. If the load current and flux of a d.c. motor are held constant and voltage applied across its armature is increased by 10 per cent, its speed will

(*a*) decrease by about 10 per cent

(*b*) remain unchanged

(*c*) increase by about 10 per cent

(*d*) increase by 20 per cent.

19. If the pole flux of a d.c. motor approaches zero, its speed will

(*a*) approach zero

(*b*) approach infinity

(*c*) no change due to corresponding change in back e.m.f.

(*d*) approach a stable value somewhere between zero and infinity.

20. If the field circuit of a loaded shunt motor is suddenly opened

(*a*) it would race to almost infinite speed

(*b*) it would draw abnormally high armature current

(*c*) circuit breaker or fuse will open the circuit before too much damage is done to the motor

(*d*) torque developed by the motor would be reduced to zero.

21. Which of the following d.c. motor would be suitable for drives requiring high starting torque but only fairly constant speed such as crushers ?

(*a*) shunt (*b*) series

(*c*) compound (*d*) permanent magnet

22. A d.c. shunt motor is found suitable to drive fans because they require

(*a*) small torque at start up

(*b*) large torque at high speeds

(*c*) practically constant voltage

(*d*) both (*a*) and (*b*)

23. Which of the following load would be best driven by a d.c. compound motor ?

(*a*) reciprocating pump

(*b*) centrifugal pump

(*c*) electric locomotive

(*d*) fan

24. As the load is increased, the speed of a d.c. shunt motor

(*a*) increases proportionately

(*b*) remains constant

(*c*) increases slightly

(*d*) reduces slightly

25. Between no-load and full-load, ... motor develops the *least* torque

(*a*) series

(*b*) shunt

(*c*) cumulative compound

(*d*) differential compound

26. The T_a/I_a graph of a d.c. series motor is a
(*a*) parabola from no-load to overload
(*b*) straight line throughout
(*c*) parabola throughout
(*d*) parabola upto full-load and a straight line at overloads.

27. As compared to shunt and compound motors, series motor has the highest torque because of its comparatively ..... at the start.
(*a*) lower armature resistance
(*b*) stronger series field
(*c*) fewer series turns
(*d*) larger armature current

28. Unlike a shunt motor, it is difficult for a series motor to stall under heavy loading because
(*a*) it develops high overload torque
(*b*) its flux remains constant
(*c*) it slows down considerably
(*d*) its back e.m.f. is reduced to almost zero.

29. When load is removed, ... motor will run at the ***highest*** speed.
(*a*) shunt
(*b*) cumulative-compound
(*c*) differential compound
(*d*) series

30. A series motor is best suited for driving
(*a*) lathes
(*b*) cranes and hoists
(*c*) shears and punches
(*d*) machine tools

31. A 220 V shunt motor develops a torque of 54 N-m at armature current of 10 A. The torque produced when the armature current is 20 A, is
(*a*) 54 N-m (*b*) 81 N-m
(*c*) 108 N-m (*d*) None of the above
(Elect. Machines, A.M.I.E. Sec. B, 1993)

32. The d.c. series motor should never be switched on at no load because
(*a*) the field current is zero
(*b*) The machine does not pick up
(*c*) The speed becomes dangerously high
(*d*) It will take too long to accelerate.
(Grad. I.E.T.E. June 1988)

33. A shunt d.c. motor works on a.c. mains
(*a*) unsatisfactorily (*b*) satisfactorily
(*c*) not at all (*d*) none of the above
(Elect. Machines, A.M.I.E. Sec. B, 1993)

34. A 200 V, 10 A motor could be rewound for 100 V, 20 A by using ..... as many turns per coil of wire, having ..... the cross-sectional area.
(*a*) twice, half
(*b*) thrice, one third
(*c*) half, twice
(*d*) four times, one-fourth

ANSWERS

1. (*d*) **2.** (*c*) **3.**(*b*) **4.** (*a*) **5.** (*a*) **6.** (*a*) **7.** (*b*) **8.** (*d*) **9.** (*a*) **10.** (*a*) **11.** (*a*)
12. (*d*) **13.** (*a*) **14.** (*d*) **15.** (*a*) **16.** (*b*) **17.** (*b*) **18.** (*c*) **19.** (*b*) **20.** (*c*) **21.** (*c*) **22.** (*d*)
23. (*a*) **24.** (*d*) **25.** (*a*) **26.** (*d*) **27.** (*b*) **28.** (*a*) **29.** (*d*) **30.** (*b*) **31.** (*c*) **32.** (*c*) **33.** (*a*)
34. (*c*).

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CHAPTER 30

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SPEED CONTROL OF D.C. MOTORS



DC motor speed controller control the speed of any common dc motor rated upto 100 V. It operates on 5V to 15 V.

30.1. Factors Controlling Motor Speed

It has been shown earlier that the speed of a motor is given by the relation

$$N = \frac{V - I_a R_a}{Z\Phi} \cdot \left(\frac{A}{P}\right) = K \frac{V - I_a R_a}{\Phi} \text{ r.p.s.}$$

where R_a = armature circuit resistance.

It is obvious that the speed can be controlled by varying ***(i)*** flux/pole, Φ (Flux Control) ***(ii)*** resistance R_a of armature circuit (Rheostatic Control) and ***(iii)*** applied voltage V (Voltage Control). These methods as applied to shunt, compound and series motors will be discussed below.

30.2. Speed Control of Shunt motors

(i) Variation of Flux or Flux Control Method

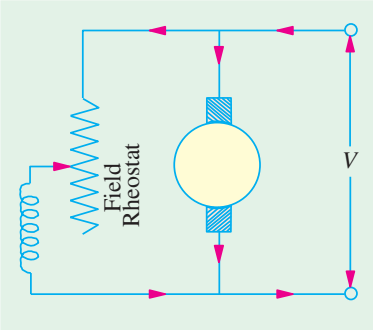


Fig. 30.1

It is seen from above that $N \propto 1/\Phi$. By decreasing the flux, the speed can be increased and ***vice versa***. Hence, the name ***flux*** or ***field control*** method. The flux of a d.c. motor can be changed by changing I_{sh} with help of a shunt field rheostat (Fig. 30.1). Since I_{sh} is relatively small, shunt field rheostat has to carry only a small current, which means I^2R loss is small, so that rheostat is small in size. This method is, therefore, very efficient. In non-interpolar machine, the speed can be increased by this method in the ratio 2 : 1. Any further weakening of flux Φ adversely affects the communication and hence puts a limit to the maximum speed obtainable with the method. In machines fitted with interpoles, a ratio of maximum to minimum speed of 6 : 1 is fairly common.

Example 30.1. *A 500 V shunt motor runs at its normal speed of 250 r.p.m. when the armature current is 200 A. The resistance of armature is 0.12 ohm. Calculate the speed when a resistance is inserted in the field reducing the shunt field to 80% of normal value and the armature current is 100 ampere.* **(Elect. Engg. A.M.A.E. S.I. June 1992)**

Solution. $E_{b1} = 500 - 200 \times 0.12 = 476$ V; $E_{b2} = 500 - 100 \times 0.12 = 488$ V

$$\Phi_2 = 0.8\,\Phi_1 \; ; \; N_1 = 250 \text{ rpm} \; ; \; N_2 = ?$$

Now, $$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} \text{ or } \frac{N_2}{250} = \frac{488}{476} \times \frac{\Phi_1}{0.8\Phi_1}, \; N_2 = \mathbf{320 \text{ r.p.m.}}$$

Example 30.2. *A 250 volt d.c. shunt motor has armature resistance of 0.25 ohm, on load it takes an armature current of 50 A and runs at 750 r.p.m. If the flux of motor is reduced by 10% without changing the load torque, find the new speed of the motor.* **(Elect. Eng-II, Pune Univ. 1987)**

Solution. $$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$$

Now, $T_a \propto \Phi I_a$. Hence $T_{a1} \propto \Phi_1 I_{a1}$ and $T_{a2} \propto \Phi_2 I_{a2}$.

Since $T_{a1} = T_{a2} \quad \therefore \quad \Phi_1 I_{a1} = \Phi_2 I_{a2}$

Now, $\Phi_2 = 0.9\,\Phi_1 \quad \therefore \quad 50\,\Phi_1 = 0.9\,\Phi, \; I_{a2} = 55.6$ A

$\therefore \quad E_{b1} = 250 - (50 \times 0.25) = 237.5$ V ; $E_{b2} = 250 - (55.6 \times 0.25) = 231.1$ V

$\therefore \quad \frac{N_2}{750} = \frac{231.1}{237.5} \times \frac{\Phi_1}{0.9\Phi_1}$; $N_2 = \mathbf{811 \text{ r.p.m.}}$

Example 30.3. *Describe briefly the method of speed control available for dc motors.*

A 230 V d.c. shunt motor runs at 800 r.p.m. and takes armature current of 50 A. Find resistance

to be added to the field circuit to increase speed to 1000 r.p.m. at an armature current of 80 A. Assume flux proportional to field current. Armature resistance = 0.15 Ω and field winding resistance = 250 Ω. **(Elect. Technology, Hyderabad Univ. 1991)**

Solution. $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{sh1}}{I_{sh2}}$ since flux ∝ field current

$E_{b1} = 230 - (50 \times 0.15) = 222.5$ V ; $E_{b2} = 230 - (80 \times 0.15) = 218$V

Let R_t = total shunt resistance = $(250 + R)$ where R is the additional resistance

$I_{sh1} = 230/250 = 0.92$ A, $I_{sh2} = 230/R_t$; $N_1 = 800$ r.p.m. ; $N = 1000$ r.p.m.

$\therefore \quad \frac{1000}{800} = \frac{218}{222.5} \times \frac{0.92}{230/R_t}$; $R_t = 319\ \Omega \quad \therefore \quad R = 319 - 250 = \mathbf{69\ \Omega}$,

$I_{sh2} = \frac{230}{319} = 0.721$

Ratio of torque in two cases $= \frac{T_2}{T_1} = \frac{I_{sh2}\, I_{a2}}{I_{sh1}\, I_{a1}} = \frac{0.721 \times 80}{0.92 \times 50} =$ **1.254**

Example 30.4. *A 250 V, d.c. shunt motor has shunt field resistance of 250 Ω and an armature resistance of 0.25 Ω. For a given load torque and no additional resistance included in the shunt field circuit, the motor runs at 1500 r.p.m. drawing an armature current of 20 A. If a resistance of 250 Ω is inserted in series with the field, the load torque remaining the same, find out the new speed and armature current. Assume the magnetisation curve to be linear.*

(Electrical Engineering-I, Bombay Univ. 1987)

Solution. In this case, the motor speed is changed by changing the flux.

Now, $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$

Since it is given that magnetisation curve is linear, it means that flux is directly proportional to shunt current. Hence $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{sh1}}{I_{sh2}}$ where $E_{b2} = V - I_{a2} R_a$ and $E_{b1} = V - I_{a1} R_a$.

Since load torque remains the same $\therefore \quad T_a \propto \Phi_1 I_{a1} \propto \Phi_2 I_{a2}$ or $\Phi_1 I_{a1} = \Phi_2 Ia_2$

$\therefore \quad I_{a2} = I_{a1} \times \frac{\Phi_1}{\Phi_2} = I_{a1} \times \frac{I_{sh1}}{I_{sh2}}$

Now, $I_{sh1} = 250/250 = 1$ A; $I_{sh2} = 250/(250 + 250) = 1/2$ A

$\therefore \quad I_{a2} = 20 \times \frac{1}{1/2} =$ **40 A** $\quad \therefore \quad E_{b2} = 250 - (40 \times 0.25) = 240$ V and

$E_{b1} = 250 - (20 \times 0.25) = 245$ V $\quad \therefore \quad \frac{N_2}{1500} = \frac{240}{245} \times \frac{1}{1/2}$

$\therefore \quad N_2 =$ **2, 930 r.p.m.**

Example 30.5. *A 250 V, d.c. shunt motor has an armature resistance of 0.5 Ω and a field resistance of 250 Ω. When driving a load of constant torque at 600 r.p.m., the armature current is 20 A. If it is desired to raise the speed from 600 to 800 r.p.m., what resistance should be inserted in the shunt field circuit ? Assume that the magnetic circuit is unsaturated.*

(Elect. Engg. AMIETE, June 1992)

Solution. $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$

Since the magnetic circuit is unsaturated, it means that flux is directly proportional to the shunt current.

$$\therefore \quad \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{sh1}}{I_{sh2}} \quad \text{where } E_{b2} = V - I_{a2} R_a \text{ and } E_{b1} = V - I_{a1} R_a$$

Since motor is driving at load of constant torque,

$$T_a \propto \Phi_1 I_{a1} \propto \Phi_2 I_{a2} \quad \therefore \quad \Phi_2 Ia_2 = \Phi_1 I_{a1} \quad \text{or} \quad I_{a2} = I_{a1} \times \frac{\Phi_1}{\Phi_2} = I_{a1} \times \frac{I_{sh1}}{I_{sh2}}$$

Now, $I_{sh1} = 250/250 = 1$ A ; $I_{sh2} = 250/R_t$

where R_t is the total resistance of the shunt field circuit.

$$\therefore \quad I_{a2} = 20 \times \frac{1}{250/R_t} = \frac{2R_t}{25} \; ; E_{b1} = 250 - (20 \times 0.5) = 240 \text{ V}$$

$$E_{b2} = 250 - \left(\frac{2R_t}{25} \times 0.5\right) = 250 - (R_t/25) \quad \therefore \quad \frac{800}{600} = \frac{250 - (R_t/25)}{240} \times \frac{1}{250/R_t}$$

$$0.04 R_t^2 - 250 R_t + 80{,}000 = 0$$

or $$R_t = \frac{250 \pm \sqrt{62{,}500 - 12{,}800}}{0.08} = \frac{27}{0.08} = 337.5\ \Omega$$

Additional resistance required in the shunt field circuit = 337.5 – 250 = **87.5 Ω**.

Example 30.6. *A 220 V shunt motor has an armature resistance of 0.5 Ω and takes a current of 40 A on full-load. By how much must the main flux be reduced to raise the speed by 50% if the developed torque is constant ?* **(Elect. Machines, AMIE, Sec B, 1991)**

Solution. Formula used is $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$. Since torque remains constant,

Hence $$\Phi_1 I_{a1} = \Phi_2 I_{a2} \quad \therefore \quad I_{a2} = I_{a1} \cdot \frac{\Phi_1}{\Phi_2} = 40x \text{ where } x = \frac{\Phi_1}{\Phi_2}$$

$$E_{b1} = 220 - (40 \times 0.5) = 200 \text{ V} \; ; E_{b2} = 220 - (40x \times 0.5) = (220 - 20x) \text{ V}$$

$$\frac{N_2}{N_1} = \frac{3}{2} \quad \text{... given} \quad \therefore \quad \frac{3}{2} = \frac{(220 - 20x)}{200} \times x \quad \therefore \quad x^2 - 11x + 15 = 0$$

or $$x = \frac{11 \pm \sqrt{121 - 60}}{2} = \frac{11 \pm 7.81}{2} = 9.4^* \quad \text{or} \quad 1.6$$

$$\therefore \quad \frac{\Phi_1}{\Phi_2} = 1.6 \quad \text{or} \quad \frac{\Phi_2}{\Phi_1} = \frac{1}{1.6}$$

$$\therefore \quad \frac{\Phi_1 - \Phi_2}{\Phi_1} = \frac{1.6 - 1}{1.6} = \frac{3}{8} \quad \therefore \quad \text{percentage change in flux} = \frac{3}{8} \times 100 = \mathbf{37.5\%}$$

Example 30.7. *A 220-V, 10-kW, 2500 r.p.m. shunt motor draws 41 A when operating at rated conditions. The resistances of the armature, compensating winding, interpole winding and shunt field winding are respectively 0.2 Ω, 0.05 Ω, 0.1 Ω and 110 Ω. Calculate the steady-state values of armature current and motor speed if pole flux is reduced by 25%, a 1 Ω resistance is placed in series with the armature and the load torque is reduced by 50%.*

Solution. $I_{sh} = 220/110 = 2$ A; $I_{a1} = 41 - 2 = 39$ A (Fig. 30.2)

$$T_1 \propto \Phi_1 I_{a1} \text{ and } T_2 \propto \Phi_2 I_{a2}$$

$$\therefore \quad \frac{T_2}{T_1} = \frac{\Phi_1}{\Phi_2} \times \frac{I_{a1}}{I_{a2}}$$

---

* This figure is rejected as it does not give the necessay increase in speed.

or $\frac{1}{2} = \frac{3}{4} \times \frac{I_{a2}}{39} \quad \therefore \quad I_{a2} = 26$ A

$E_{b1} = 220 - 39\,(0.2 + 0.1 + 0.05) = 206.35$ V

$E_{b2} = 220 - 26\,(1 + 0.35) = 184.9$ V

Now, $\frac{N_2}{2500} = \frac{184.9}{206.35} \times \frac{4}{3}$; $N_2 =$ **2987 r.p.m.**

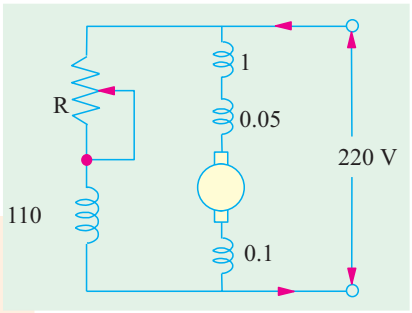


Fig. 30.2

Example 30.8. *A 220 V, 15 kW, 850 r.p.m. shunt motor draws 72.2 A when operating at rated condition. The resistances of the armature and shunt field are 0.25 Ω and 100 Ω respectively. Determine the percentage reduction in field flux in order to obtain a speed of 1650 r.p.m. when armature current drawn is 40 A.*

Solution. $I_{sh} = 220/100 = 2.2$ A; $I_{a1} = 72.2 - 2.2 = 70$ A

$E_{b1} = 220 - 70 \times 0.25 = 202.5$ V, $E_{b2} = 220 - 40 \times 0.25 = 210$ V.

Now, $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$ or $\frac{1650}{850} = \frac{210}{202.5} \times \frac{\Phi_1}{\Phi_2}$

$\therefore \quad \Phi_2 = 0.534\,\Phi_1$;

$\therefore$ Reduction in field flux $= \frac{\Phi_1 - 0.534\,\Phi_1}{\Phi_1} \times 100 =$ **46.6%.**

Example 30.9. *A 220 V shunt motor has an armature resistance of 0.5 ohm and takes an armature current of 40 A on a certain load. By how much must the main flux be reduced to raise the speed by 50% if the developed torque is constant ? Neglect saturation and armature reaction.*

(Elect. Machines, AMIE, Sec B, 1991)

Solution. $T_1 \propto \Phi_1 \times \Phi_1, \rightarrow \Phi_1 I_{a1}$, and $T_2 \propto \Phi_2 I_{a2}$ Since, $T_1 = T_2$

$\therefore \quad \Phi_2 I_{a2} = \Phi_1 I_{a1}$ or $\Phi_2/\Phi_1 = I_{a1}/I_{a2} = 40/I_{a2}$

$E_{b1} = 220 - 40 \times 0.5 = 200$ V ; $E_{b2} = 220 - 0.5\, I_{a2}$

Now, $\frac{E_{b2}}{E_{b1}} = \frac{N_2 \Phi_2}{N_1 \Phi_1}$ or $\frac{220 - 0.5\, I_{a2}}{200} = \frac{1.5\, N_1}{N_1} \times \frac{40}{I_{a2}}$

$\therefore \quad I_{a2} - 40\, I_{a2} + 24{,}000 = 0$ or $I_{a2} = 63.8$ A

$\therefore \quad \frac{\Phi_2}{\Phi_1} = \frac{40}{63.8} = 0.627$ or $\Phi_2 = 0.627\,\Phi_1 =$ **62.7% of Φ_1**

Example 30.10. *A d.c. shunt motor takes an armature current of 20 A from a 220 V supply. Armature circuit resistance is 0.5 ohm. For reducing the speed by 50%, calculate the resistance required in the series, with the armature, if*

(a) *the load torque is constant*

(b) *the load torque is proportional to the square of the speed.* **(Sambalpur Univ., 1998)**

Solution. $E_{b1} = V - I_a r_a = 220 - 20 \times 0.5 = 210$ V

$210 \propto N_1$

(*a*) Constant Load torque

In a shunt motor, flux remains constant unless there is a change in terminal voltage or there is a change in the field-circuit resistance.

If torque is constant, armature-current then must remain constant. $I_a = 20$ amp

With an external armature-circuit resistor of R ohms, $20 \times (R + 0.5) = 220 - E_{b2}$

The speed required now is $0.5\, N_1$.

With constant flux, $E_b \propto$ speed. Hence, $210 \propto N_1$

$$E_{b2} \propto 0.5\,N_1,\ E_{b2} = 105$$

$$R + 0.5 = (220 - 105)/20 = 5.75, \text{ giving } R = \mathbf{5.25\ ohms}$$

(*b*) Load torque is proportional to the square of speed.

With constant flux, Developed Torque at N_1 r.p.m. $\propto I_{a1}$

$$T_{m1} \propto 20$$

From the Load Side, $T_{L1} \propto N_1^2$

Since $T_{m1} = T_{L1}$

$$20 \propto N_1^2 \qquad \text{...(a)}$$

At 50% speed, Load Torque, $T_{L2} \propto (0.5\,N_1)^2$

For motor torque, $T_{m2} \propto I_{a2}$

Since $T_{L2} = T_{m2},\ I_{a2} \propto (0.5\,N_1)^2$ ...(*b*)

From eqn. **(*a*)** and **(*b*)** above

$$\frac{I_{a2}}{I_{a1}} = 0.25,\ I_{a2} = 5 \text{ amp}$$

$$\frac{E_{b2}}{E_{b1}} = \frac{220 - I_{a2}\,(R + 0.5)}{210} = 0.5\,N_1$$

$$220 - I_{a2}\,(R + 0.5) = 0.5 \times 210$$

$$R + 0.5 = \frac{220 - 105}{5} = 23,\ R = \mathbf{22.5\ ohms}$$

Check : With the concept of armature power output : (applied here for part **(*b*)** only as an illustration).

Armature power–output $= E_b \times I_a = T \times \omega$

When $T \propto (\text{speed})^2$, $E_b\,I_a = k_1\,T_\omega = K_2\,\omega^3$

At N_1 r.p.m. $210 \times 20 = K_3\,N_1^3$ ...(*c*)

With constant flux, at half speed

$$E_{b2} = 105$$

$$105 \times I_{a2} = K_3\,(0.5\,N_1)^3 \qquad \text{...(d)}$$

From eqs. **(*c*)** and **(*d*)**,

$$\frac{105 \times I_{a2}}{210 \times 20} = \frac{0.125 \times N_1^3}{N_1^3}, \text{ giving } I_{a2} = 5 \text{ amp}$$

This gives $R = \mathbf{22.5\ ohms.}$

Example 30.11. *A 250 V shunt motor runs at 1000 r.p.m. at no-load and takes 8 A. The total armature and shunt field resistances are respectively 0.2 ohm, 250 ohm. Calculate the speed when loaded and taking 50 A. Assume the flux to be constant.* **(Nagpur Univ. Summer 2000)**

Solution. The current distribution is shown in Fig. 30.3.

At no load, $I_L = 8$ amp, $I_f = 1$ amp,

Hence, $I_a = 7$ amp

$$E_{b0} = 250 - 7 \times 0.2 = 248.6 \text{ volts},$$

$$= K\phi \times 1000$$

$\therefore$ $K\phi = 0.2486$

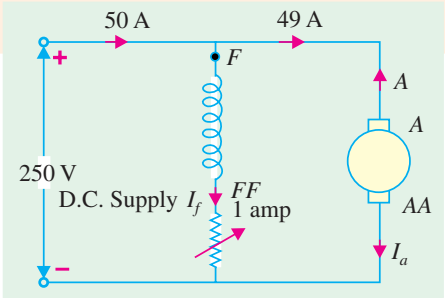


Fig. 30.3

At load, $I_a = 49$ amp

$$E_{b1} = 250 - 49 \times 0.2 = 240.2$$

$$N_1 = \frac{240.2}{0.2486} = 966.2 \text{ r.p.m.}$$

Notes : **(*i*)** The assumption of constant flux has simplified the issue. Generally, armature reaction tends to weaken the flux and then the speed tends to increase slightly.

(*ii*) The no load armature current of 7 amp is required to overcome the mechanical losses of motor as well as driven load, at about 1000 r.p.m.

Example 30.12. *A 240 V d.c. shunt motor has an armature-resistance of 0.25 ohm, and runs at 1000 r.p.m., taking an armature current 40 A. It is desired to reduce the speed to 800 r.p.m.*

(*i*) *If the armature current remains the same, find the additional resistance to be connected in series with the armature-circuit.*

(*ii*) *If, with the above additional resistance in the circuit, armature current decreases to 20 A, find the speed of the motor.* **(Bhartiar Univ., November 1997)**

Solution. $E_b = 240 - 0.25 \times 40 = 230$ V, $230 \propto 1000$ ...**(*a*)**

(*i*) For 800 r.p.m., $E_{b2} \propto 800$ ...**(*b*)**

From **(*a*)** and **(*b*)**, $E_{b2} = \frac{800}{1000} \times 230 = 184$

$$240 - (R + 0.25) \times 40 = 184, \quad R = \mathbf{1.15 \text{ ohm}}$$

(*ii*) $E_{b3} = 240 - 20\,(1.40) = 212$

$$E_{b3} = \frac{N_3}{1000} \times 230 = 212, \quad N_3 = \frac{212}{230} \times 1000 = \mathbf{922 \text{ r.p.m.}}$$

Example 30.13. *A 7.48 kW, 220 V, 990 r.p.m. shunt motor has a full load efficiency of 88%, the armature resistance is 0.08 ohm and shunt field current is 2 A. If the speed of this motor is reduced to 450 r.p.m. by inserting a resistance in the armature circuit, find the motor output, the armature current, external resistance to be inserted in the armature circuit and overall efficiency. Assume the load torque to remain constant.* **(Nagpur Univ., November 1998)**

Solution. With an output of 7.48 kW and an efficiency of 88%, the input power is 8.50 kW. Losses are 1.02 kW.

Input Current = 85000/220 = 38.64 A

Armature-Current = 38.64 – 2.00 = **36.64 A**

Power Loss in the shunt field circuit = $220 \times 2 = 440$ W

Copper-Loss in armature-circuit = $36.64^2 \times 0.08 = 107.4$ W

No-Load-Loss at 900 r.p.m. = 1020 – 107.4 – 440 = 473 W

At 900 r.p.m. Back-emf = $E_{b1} = 200 - (36.64 \times 0.08) = 217.3$ V

Motor will run at 450 r.p.m. with flux per pole kept constant, provided the back-emf = E_{b2}

$= (459/900) \times 217.1\text{V} = 108.5$ V

There are two simplifying assumptions in this case, which must be stated before further calculations :

1. Load-torque is constant,
2. No Load Losses are constant.

(These statements can be different which leads to variations in the next steps of calculations.)

For constant load-torque, the condition of constant flux per pole results into constant armature current, which is 36.64 A.

With an armature current of 36.64 A, let the external resistance required for this purpose be R.

$$36.64\,R = 217.1 - 108.5 = 108.6 \text{ V}, \quad R = 2.964 \text{ ohms}$$

Total Losses

Total i^2r - loss in armature = $36.64^2 \times (2.964 + 0.08) = 4086$ W

Field-copper-loss + No load loss = 440 + 473 = 913

Total Loss = 4999 W

Total Output = 8500 – 4999 = 3501 W

Hence, Efficiency = $(3501/8500) \times 100 =$ **41.2%**

(Note : Because of missing data and clarification while making the statements in the question, there can be variations in the assumption and hence in the final solutions.**)**

Example 30.14. *A d.c. shunt motor supplied at 230 V runs at 990 r.p.m. Calculate the resistance required in series with the armature circuit to reduce the speed to 500 r.p.m. assuming that armature current is 25 amp.* **(Nagpur Univ., November 1997)**

Solution. It is assumed that armature resistance is to

(*a*) At 900 r.p.m. : $E_{b1} = 230 = K \times 900$

(*b*) At 500 r.p.m. : $E_{b2} = K \times 500$

Therefore, $E_{b2} = E_{b1} \times 500/900 = 127.8$ volts

The difference between E_{b1} and E_{b2} must be the drop in the external resistance to be added to the armature circuit for the purpose of reducing the speed to 500 r.p.m.

$E_{b1} - E_{b2} = 25 \times R$

$R = (230 - 127.8)/25 =$ **4.088 ohms.**

Example 30.15. *A 220 V d.c. shunt motor has an armature resistance of 0.4 ohm and a field circuit resistance of 200 ohms. When the motor is driving a constant-torque load, the armature-current is 20 A, the speed being 600 r.p.m. It is desired to run the motor at 900 r.p.m. by inserting a resistance in the field circuit. Find its value, assuming that the magnetic circuit is not saturated.* **(Nagpur Univ., November 1996)**

Solution. (*i*) At 600 r.p.m. $i_{f1} = 220/200 = 1.1$ amp

$E_{b1} = 220 - (20 \times 0.4) = 212$ volts

$T_L = K_1 \times 1.1 \times 20$

The back e.m.f. $= 212 = K_2 \times 1.1 \times 600$

or $K_2 = 212/660 = 0.3212$

(ii) At 900 r.p.m. : $T_L = K_1 \times i_{f2} \times I_{a2}$

Due to constant load torque,

$i_{f2} \times I_{a2} = 1.1 \times 20 = 22$

$E_{b2} = 220 - (0.4\, I_{a2}) = K_2 \times i_{f2} \times 900 = 289\, i_{f2}$

$220 - (0.4 \times 22/i_{f2}) = 289\, i_{f2}$

Guess for approximate value of i_{f2} : Neglecting armature -resistance drop and saturation, 50% rise in speed is obtained with proportional decrease in i_{f2} related by

$$\frac{600}{900} \cong \frac{i_{f2}}{i_{f1}} \text{ giving } i_{f2} \cong 0.73 \text{ amp}$$

[In place of i_{f2}, I_{a2} can be evaluated first. Its guess-work will give $I_{a2} \cong 1.5 \times 20 \cong 30$ amp]

Continuing with the solution of the equation to evaluate a value of i_{f2}, accepting that value which is near 0.73 amp, we have $i_{f2} = 0.71865$. [Note that other value of i_{f2}, which is 0.04235, is not acceptable.]. Corresponding $I_{a2} = 30.6$ amp. Previous shunt field current, $i_{f1} = 1.1$, $R_{f1} = 200\ \Omega$. New shunt field current, $i_{f2} = 0.71865$, $R_{f2} = 220/0.71865 = 306\ \Omega$. Final answer is that a resistor of 106 ohms is to be added to the field circuit to run the motor at 900 r.p.m. at constant torque.

Example 30.16. *A 220 V d.c. shunt motor has an armature resistance of 0.40 ohm and field-resistance of 200 ohms. It takes an armature current of 22 A and runs at 600 r.p.m. It drives a load whose torque is constant. Suggest a suitable method to raise the speed to 900 r.p.m. Calculate the value of the controllable parameter.* **(Nagpur Univ., April 1998)**

Solution. At 600 r.p.m. I_{a1} = 22 amp, N_1 = 600 r.p.m., i_{f1} = 220/200 = 1.1 amp.

$$E_{b1} = 220 - (22 \times 0.40) = 211.2 \text{ volts.}$$

Let the Load-torque be denoted by T_2, k_1 and k_2 in the equations below represent machine constants appearing in the usual emf-equation and torque-equation for the d.c. shunt motor.

$$E_{b1} = 211.2 = k_1 \times i_{f1} \times N_1 = k_1 \times 1.10 \times 600 \quad \text{or} \quad k_1 = 211.2/660 = 0.32$$

$$T_L = k_2 \times i_{f1} \times I_{a1} = k_2 \times 1.10 \times 22$$

Since the load torque will remain constant at 900 r.p.m. also, the corresponding field current (= i_{f2}) and armature current (= i_{a2}) must satisfy the following relationships :

$$T_L = k_2\, i_{f2}\, I_{a2} = k_2 \times 1.10 \times 22$$

or $$i_{f2} \times I_{a2} = 24.2$$

And $$E_{b2} = 220 - (I_{a2} \times 0.40) = k_1 \times i_{f2} \times 900$$

$$220 - (I_{a2} \times 0.40) = 0.32 \times (24.2 \times I_{a2}) \times 900$$

(Alternatively, the above equation can also lead to a quadratic in i_{f2}.)

This leads to a quadratic equation in I_{a2}.

Guess for i_{f2} : Approximately, speed of a d.c. shunt motor is inversely proportional to the field current. Comparing the two speeds of 600 and 900 r.p.m., the value of i_{f2} should be approximately given by

$$i_{f2} \cong i_{f1} \times (600/900) = 0.733 \text{ amp}$$

Guess for I_{a2} : For approximate conclusions, armature-resistance drop can be ignored. With constant load-torque, armature-power must be proportional to the speed.

$$\frac{\text{Armature-power at 900 r.p.m.}}{\text{Armature-power at 600 r.p.m.}} = \frac{900}{600} = 1.5$$

$$E_{b2}\, I_{a2} = 1.5 \times E_{b1}\, I_{a1}$$

Neglecting armature-resistance drops, $E_{b1} - V$ and $E_{b2} - V$.

This gives $$I_{a2} = 1.5 \times 22 = 33 \text{ amps}$$

Thus, out of the two roots for i_{f2}, that which is close to 0.733 is acceptable. If quadratic equation for I_{a2} is being handled, that root which is near 33 amp is acceptable.

Continuing with the solution to quadratic equation for I_{a2}, we have

$$220 - 0.40\, I_{a2} = 0.32 \times (24.2 \times I_{a2}) \times 900$$

$$220 - 0.40\, I_{a2} = 6969/I_{a2}$$

$$I_{a2}^2 - 550\, I_{a2} + 17425 = 0$$

This gives I_{a2} as ***either*** 33.75 amp *or* 791.25 amp.

From the reasoning given above, acceptable root corresponds to I_{a2} = 33.75 amp.

Corresponding field current, i_{f2} = 24.2/33.75 = 0.717 amp

Previous field circuit resistance = 200 ohms

New field circuit resistance = 220/0.717 = 307 ohms

Hence, additional resistance of 107 ohms must be added to the shunt field circuit to run the motor at 900 r.p.m. under the stated condition of constant Load torque.

Additional Check : Exact calculations for proportions of armature-power in two cases will give the necessary check.

$$E_{b2} = 220 - (33.75 \times 0.40) = 206.5$$

As mentioned above, while guessing the value of I_{a2}, the proportion of armature-power should be 1.5.

$$\frac{E_{b2}\, I_{a2}}{E_{b1}\, I_{a1}} = \frac{206.56 \times 33.75}{211.2 \times 22} = 1.50$$

Thus, the results obtained are confirmed.

Example 30.17. *A 250 V, 25 kW d.c. shunt motor has an efficiency of 85% when running at 1000 r.p.m. on full load. The armature resistance is 0.1 ohm and field resistance is 125 ohms. Find the starting resistance required to limit the starting current to 150% of the rated current.*

(Amravati Univ., 1999)

Solution.

Output power = 25 kW, at full-load.

Input power $= \frac{25,000}{0.85} = 29412$ watts

At Full load, Input Current = 29412/250 = 117.65 amp

Field Current = 250/125 = 2 amp

F.L. Armature Current = 117.65 – 2 = 115.65 amp

Limit of starting current = 1.50 × 115.65 = 173.5 amp

Total resistance in armature circuit at starting

$$= \frac{250}{173.5} = 1.441 \text{ ohms}$$

External resistance to be added to armature circuit

= 1.441 – 0.1 = 1.341 ohm.

Tutorial Problems 30.1

1. A d.c. shunt motor runs at 900 r.p.m. from a 460 V supply when taking an armature current of 25 A. Calculate the speed at which it will run from a 230-V supply when taking an armature current of 15 A. The resistance of the armature circuit is 0.8 Ω. Assume the flux per pole at 230 V to have decreased to 75% of its value at 460 V. **[595 r.p.m.]**
2. A 250 V shunt motor has an armature resistance of 0.5 Ω and runs at 1200 r.p.m. when the armature current is 80 A. If the torque remains unchanged, find the speed and armature current when the field is strengthened by 25%. **[998 r.p.m. ; 64 A]**
3. When on normal full-load, a 500 V, d.c. shunt motor runs at 800 r.p.m. and takes an armature current 42 A. The flux per pole is reduced to 75% of its normal value by suitably increasing the field circuit resistance. Calculate the speed of the motor if the total torque exerted on the armature is (*a*) unchanged (*b*) reduced by 20%.

   The armature resistance is 0.6 Ω and the total voltage loss at the brushes is 2 V.

   [(*a*) 1,042 r.p.m. (*b*) 1,061 r.p.m.]
4. The following data apply to d.c. shunt motor.

   Supply voltage = 460 V ; armature current = 28 A ; speed = 1000 r.p.m. ; armature resistance = 0.72 Ω. Calculate (*i*) the armature current and (*ii*) the speed when the flux per pole is increased to 120% of the initial value, given that the total torque developed by the armature is unchanged. **[(*i*) 23.33 A (*ii*) 840 r.p.m.]**
5. A 100-V shunt motor, with a field resistance of 50 Ω and armature resistance of 0.5 Ω runs at a speed of 1,000 r.p.m. and takes a current of 10 A from the supply. If the total resistance of the field circuit is reduced to three quarters of its original value, find the new speed and the current taken from the supply. Assume that flux is directly proportional to field current. **[1,089 r.p.m. ; 8.33 A]**

6. A 250 V d.c. shunt motor has armature circuit resistance of 0.5 Ω and a field circuit resistance of 125 Ω. It drives a load at 1000 r.p.m. and takes 30 A. The field circuit resistance is then slowly increased to 150 Ω. If the flux and field current can be assumed to be proportional and if the load torque remains constant, calculate the final speed and armature current. **[1186 r.p.m. 33.6 A]**

7. A 250 V, shunt motor with an armature resistance of 0.5 Ω and a shunt field resistance of 250 Ω drives a load the torque of which remains constant. The motor draws from the supply a line current of 21 A when the speed is 600 r.p.m. If the speed is to be raised to 800 r.p.m., what change must be affected in the shunt field resistance ? Assume that the magnetization curve of the motor is a straight line. **[88 Ω]**

8. A 240 V, d.c. shunt motor runs at 800 r.p.m. with no extra resistance in the field or armature circuit, on no-load. Determine the resistance to be placed in series with the field so that the motor may run at 950 r.p.m. when taking an armature current of 20 A. Field resistance = 160 Ω. Armature resistance = 0.4 Ω. It may be assumed that flux per pole is proportional to field current. **[33.6 Ω]**

9. A shunt-wound motor has a field resistance of 400 Ω and an armature resistance of 0.1 Ω and runs off 240 V supply. The armature current is 60 A and the motor speed is 900 r.p.m.; Assuming a straight line magnetization curve, calculate (*a*) the additional resistance in the field to increase the speed to 1000 r.p.m. for the same armature current and (*b*) the speed with the original field current of 200 A. **[(*a*) 44.4 Ω (*b*) 842.5 r.p.m.]**

10. A 230 V d.c. shunt motor has an armature resistance of 0.5 Ω and a field resistance of $76\,^{2}/_{3}$ Ω. The motor draws a line current of 13 A while running light at 1000 r.p.m. At a certain load, the field circuit resistance is increased by $38\,^{1}/_{3}$ Ω. What is the new speed of the motor if the line current at this load is 42 A ? **[1400 r.p.m.] (*Electrical Engg. ; Grad I.E.T.E. Dec. 1986*)**

11. A 250 V d.c. shunt motor runs at 1000 r.p.m. and takes an armature current of 25 amp. Its armature resistance is 0.40 ohm. Calculate the speed with increased load with the armature current of 50 amp. Assume that the increased load results into flux-weakening by 3%, with respect to the flux in previous loading condition. **(*Nagpur Univ., April 1996*)**

Hint : (*i*) First Loading condition :

$$E_{b1} = 250 - 25 \times 0.40 = K_1 \times 1000$$

(*ii*) Second Loading condition :

$E_{b2} = 250 - 50 \times 0.40 = 230\ K_1 \times (0.97\ \phi) \times N_2$. This gives N_2. **[988 r.p.m.]**

(*ii*) Armature or Rheostatic Control Method

This method is used when speeds below the no-load speed are required. As the supply voltage is normally constant, the voltage across the armature is varied by inserting a variable rheostat or resistance (called controller resistance) in series with the armature circuit as shown in Fig. 30.4 (*a*). As controller resistance is increased, p.d. across the armature is decreased, thereby decreasing the armature speed. For a load constant torque, speed is approximately proportional to the p.d. across the armature. From the speed/armature current characteristic [Fig. 30.4 (*b*)], it is seen that greater the resistance in the armature circuit, greater is the fall in the speed.

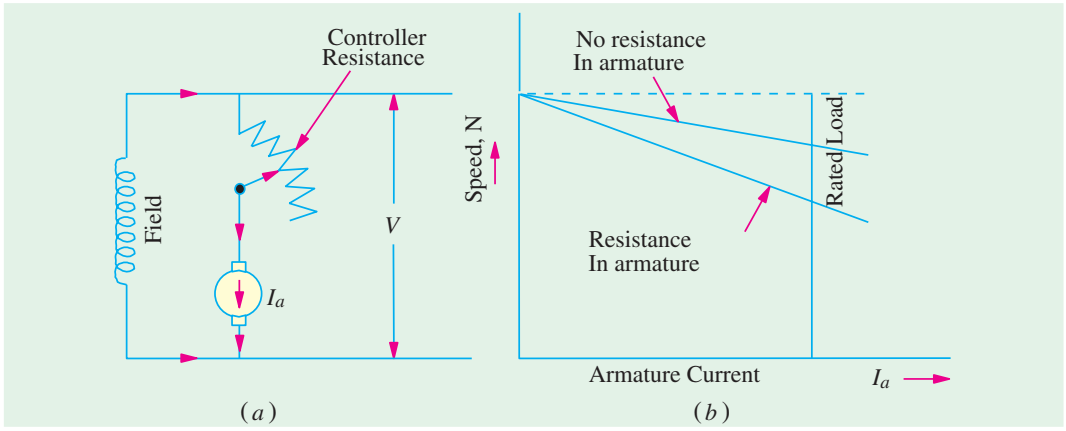


Fig. 30.4

Let I_{a1} = armature current in the first case
I_{a2} = armature current in the second case
(If $I_{a1} = I_{a2}$, then the load is of constant torque.)
N_1, N_2 = corresponding speeds, V = supply voltage

Then $N_1 \propto V - I_{a1} R_a \propto E_{b1}$

Let some controller resistance of value R be added to the armature circuit resistance so that its value becomes $(R + R_a) = R_t$.

Then $N_2 \propto V - I_{a2} R_t \propto E_{b2} \quad \therefore \quad \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}}$

(In fact, it is a simplified form of relation given in Art. 27.9 because here $\Phi_1 = \Phi_2$.)

Considering no-load speed, we have $\frac{N}{N_0} = \frac{V - I_a R_t}{V - I_{a0} R_a}$

Neglecting $I_{a0} R_a$ with respect to V, we get

$$N = N_0 \left(1 - \frac{I_a R_t}{V}\right)$$

It is seen that for a given resistance R_t the speed is a linear function of armature current I_a as shown in Fig. 30.5 (*a*).

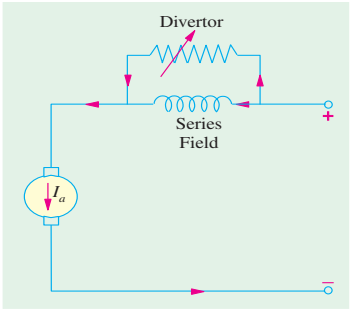


Fig. 30.5 (*a*)

The load current for which the speed would be zero is found by putting $N = 0$ in the above relation.

$$\therefore \quad 0 = N_0 \left(1 - \frac{I_a R_t}{V}\right) \quad \text{or} \quad I_a = \frac{V}{R_t}$$

This is the maximum current and is known as ***stalling*** current.

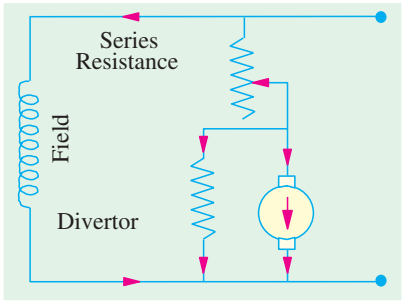


Fig. 30.5 (*b*)

As will be shown in Art. 30.5 (*a*), this method is very wasteful, expensive and unsuitable for rapidly changing loads because for a given value of R_t, speed will change with load. A more stable operation can be obtained by using a divertor across the armature in addition to armature control resistance (Fig. 30.5 (*b*)). Now, the changes in armature current (due to changes in the load torque) will not be so effective in changing the p.d. across the armature (and hence the armature speed).

Example 30.18. *A 200 V d.c. shunt motor running at 1000 r.p.m. takes an armature current of 17.5 A. It is required to reduce the speed to 600 r.p.m. What must be the value of resistance to be inserted in the armature circuit if the original armature resistance is 0.4 Ω ? Take armature current to be constant during this process.* **(Elect. Engg. I Nagpur Univ. 1993)**

Solution. $N_1 = 1000$ r.p.m. ; $E_{b1} = 200 - 17.5 \times 0.4 = 193$ V

R_t = total arm. circuit resistance ; $N_2 = 600$ r.p.m. ; $E_{b2} = (200 - 17.5\, R_t)$

Since I_{sh} remains constant ; $\Phi_1 = \Phi_2$

$$\therefore \quad \frac{600}{1000} = \frac{(200 - 17.5\, R_t)}{193} ; \quad R_t = 4.8\ \Omega$$

$\therefore$ Additional resistance reqd. $R = R_t - R_a = 4.8 - 0.4 =$ **4.4 Ω.**

It may be noted that brush voltage drop has not been considered.

Example 30.19. *A 500 V d.c. shunt motor has armature and field resistances of 1.2 Ω and 500 Ω respectively. When running on no-load, the current taken is 4 A and the speed is 1000 r.p.m. Calculate the speed when motor is fully loaded and the total current drawn from the supply is 26 A. Estimate the speed at this load if* ***(a)*** *a resistance of 2.3 Ω is connected in series with the armature and* ***(b)*** *the shunt field current is reduced by 15%.* **(Electrical Engg. I, Sd. Patel Univ. 1985)**

Solution. $I_{sh} = 500/500 = 1$ A ; $I_{a1} = 4 - 1 = 3$ A

$E_{b1} = 500 - (3 \times 1.2) = 496.4$ V $I_{a2} = 26 - 1 = 25$ A

$E_{b2} = 500 - (25 \times 1.2) = 470$ V $\therefore \dfrac{N_2}{1000} = \dfrac{470}{496.4}$; $N_2 =$ **947 r.p.m.**

(*a*) In this case, total armature circuit resistance = 1.2 + 2.3 = 3.5 Ω

$\therefore \quad E_{b2} = 500 - (25 \times 3.5) = 412.5$ V $\therefore \quad \dfrac{N_2}{1000} = \dfrac{412.5}{496.4}$; $N_2 =$ **831 r.p.m.**

(*b*) When shunt field is reduced by 15%, $\Phi_2 = 0.85\ \Phi_1$ assuming straight magnetisation curve.

$$\frac{N_2}{1000} = \frac{412.5}{496.4} \times \frac{1}{0.85} \text{ ; } N_2 = \textbf{977.6 r.p.m.}$$

Example 30.20. *A 250-V shunt motor (Fig. 30.6) has an armature current of 20 A when running at 1000 r.p.m. against full load torque. The armature resistance is 0.5 Ω. What resistance must be inserted in series with the armature to reduce the speed to 500 r.p.m. at the same torque and what will be the speed if the load torque is halved with this resistance in the circuit ? Assume the flux to remain constant throughout and neglect brush contact drop.*

(Elect. Machines AMIE Sec. B Summer 1991)

Solution. $E_{b1} = V - I_{a1} R_a = 250 - 20 \times 0.5 = 240$ V

Let R_t to be total resistance in the armature circuit *i.e.* $R_t = R_a + R$, where R is the additional resistance.

$\therefore \quad E_{b2} = V - I_{a2} R_t = 250 - 20\ R_t$

It should be noted that $I_{a1} = I_{a2} = 20$ A because torque remains the same and $\Phi_1 = \Phi_2$ in both cases.

$$\therefore \quad \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} = \frac{E_{b2}}{E_{b1}} \text{ or } \frac{500}{1000} = \frac{250 - 20\ R_t}{240}$$

$\therefore \quad R_t = 6.5\ \Omega$; hence, $R = 6.5 - 0.5 =$ **6 Ω**

Since the load is halved, armature current is also halved because flux remains constant. Hence, $I_{a3} = 10$ A.

$$\therefore \frac{N_3}{1000} = \frac{250 - 10 \times 6.5}{240} \quad \text{or} \quad N_3 = \textbf{771 r.p.m.}$$

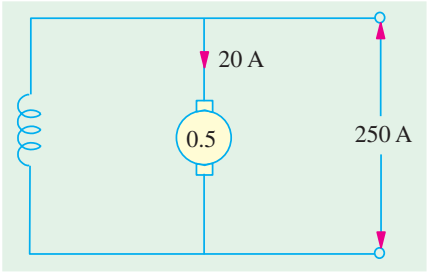


Fig. 30.6

Example 30.21. *A 250-V shunt motor with armature resistance of 0.5 ohm runs at 600 r.p.m. on full-load and takes an armature current of 20 A. If resistance of 1.0 ohm is placed in the armature circuit, find the speed at (i) full-load torque (ii) half full-load torque.*

(Electrical Machines-II, Punjab Univ. May 1991)

Solution. Since flux remains constant, the speed formula becomes $\dfrac{N_2}{N_1} = \dfrac{E_{b2}}{E_{b1}}$.

(*i*) In the first case, full-load torque is developed.

$N_1 = 600$ r.p.m.; $E_{b1} = V - I_{a1} R_{a1} = 250 - 20 \times 0.5 = 240$ V

Now, $T \propto \Phi I_a \propto I_a$ ($\because$ Φ is constant)

$\therefore \quad \dfrac{T_2}{T_1} = \dfrac{I_{a2}}{I_{a1}}$ Since $T_2 = T_1$; $I_{a2} = I_{a1} = 20$ A

$E_{b2} = V - I_{a2} R_{a2} = 250 - 20 \times 1.5 = 220$ V,

$$N_2 = \frac{N_2}{600} = \frac{220}{240} \text{ ; } N_2 = \frac{600 \times 220}{240} = \textbf{550 r.p.m.}$$

(*ii*) In this case, the torque developed is half the full-load torque.

$$\frac{T_2}{T_1} = \frac{I_{a2}}{I_{a1}} \quad \text{or} \quad \frac{T_1/2}{T_1} = \frac{I_{a2}}{20} \; ; I_{a2} = 10 \text{ A} \; ; E_{b2} = 250 - 10 \times 1.5 = 235 \text{ V}$$

$$\frac{N_2}{600} = \frac{235}{240} \; ; \; N_2 = 600 \times \frac{235}{240} = \textbf{587.5 r.p.m.}$$

Example 30.22. *A 220 V shunt motor with an armature resistance of 0.5 ohm is excited to give constant main field. At full load the motor runs of 500 rev. per minute and takes an armature current of 30 A. If a resistance of 1.0 ohm is placed in the armature circuit, find the speed at* ***(a)*** *full-load torque* ***(b)*** *double full-load torque.* **(Elect. Machines-I, Nagpur Univ. 1993)**

Solution. Since flux remains constant, the speed formula becomes $N_2/N_1 = E_{b2}/E_{b1}$.

(a) Full-load torque

With no additional resistance in the armature circuit,

$$N_1 = 500 \text{ r.p.m.} \; ; I_{a1} = 30 \text{ A} \; ; E_{b1} = 220 - 30 \times 0.5 = 205 \text{ V}$$

Now, $T \propto I_a$ (since Φ is constant.) $\therefore \quad \frac{T_2}{T_1} = \frac{I_{a2}}{I_{a1}}$ Since $T_2 = T_1 \; ; I_{a2} = I_{a1} = 30$ A

When additional resistance of 1 Ω is introduced in the armature circuit,

$$E_{b2} = 220 - 30\,(1 + 0.5) = 175 \text{ V} \; ; N_2 = ? \quad \frac{N_2}{500} = \frac{175}{205} \; ; N_2 = \textbf{427 r.p.m.}$$

(b) Double Full-load Torque

$$\frac{T_2}{T_1} = \frac{I_{a2}}{I_{a1}} \quad \text{or} \quad \frac{2T_1}{T_1} = \frac{I_{a2}}{30} \; ; I_{a2} = 60 \text{ A}$$

$$\therefore \quad E_{b2} = 220 - 60\,(1 + 0.5) = 130 \text{ V}$$

$$\therefore \quad \frac{N_2}{500} = \frac{130}{205} \; ; \; N_2 = \textbf{317 r.p.m.}$$

Example 30.23. *The speed of a 50 h.p (37.3 kW) series motor working on 500 V supply is 750 r.p.m. at full-load and 90 per cent efficiency. If the load torque is made 350 N-m and a 5 ohm resistance is connected in series with the machine, calculate the speed at which the machine will run. Assume the magnetic circuit to be unsaturated and the armature and field resistance to be 0.5 ohm.* **(Electrical Machinery I, Madras Univ. 1986)**

Solution. Load torque in the first case is given by

$$T_1 = 37{,}300/2\pi\,(750/60) = 474.6 \text{ N-m}$$

Input current, $I_{a1} = 37{,}300/0.9 \times 500 = 82.9$ A

Now, $T_2 = 250$ N-m ; $I_{a2} = ?$

In a series motor, before magnetic saturation,

$$T \propto \Phi I_a \propto I_{a2} \quad \therefore \quad T_1 \propto I_{a1}^2 \text{ and } T_2 \propto I_{a2}^2$$

$$\therefore \quad \left(\frac{I_{a2}}{I_{a1}}\right)^2 = \frac{T_2}{T_1} \quad \therefore \quad I_{a2} = 82.9 \times \sqrt{250/474.6} = 60.2 \text{ A}$$

Now, $E_{b1} = 500 - (82.9 \times 0.5) = 458.5$ V ;

$$E_{b2} = 500 - 60.2\,(5 + 0.5) = 168.9 \text{ V}$$

Using $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{a2}}{I_{a1}}$, we get $\frac{N_2}{750} = \frac{168.9}{458.5} \times \frac{82.9}{60.2} \quad \therefore \; N_2 = \textbf{381 r.p.m.}$

Example 30.24. *A 7.46 kW, 220 V, 900 r.p.m. shunt motor has a full-load efficiency of 88 per cent, an armature resistance of 0.08 Ω and shunt field current of 2 A. If the speed of this motor is*

reduced to 450 r.p.m. by inserting a resistance in the armature circuit, the load torque remaining constant, find the motor output, the armature current, the external resistance and the overall efficiency.

(Elect. Machines, Nagpur Univ. 1993)

Solution. Full-load motor input current $I = 7460/220 \times 0.88 = 38.5$ A

$\therefore \quad I_{a1} = 38.5 - 2 = 36.5$ A

Now, $T \propto \Phi I_a$. Since flux remains constant.

$\therefore \quad T \propto I_a \quad \therefore \quad T_{a1} \propto I_{a1}$ and $T_{a2} \propto I_{a2}$ or $\dfrac{T_{a2}}{T_{a1}} = \dfrac{I_{a2}}{I_{a1}}$

It is given that $T_{a1} = T_{a2}$; hence $I_{a1} = I_{a2} = \mathbf{36.5\ A}$

$E_{b1} = 220 - (36.5 \times 0.08) = 217.1\ \Omega$

$E_{b2} = 220 - 36.5\, R_t$; $N_1 = 900$ r.p.m.; $N_2 = 450$ r.p.m.

Now, $\dfrac{N_2}{N_1} = \dfrac{E_{b2}}{E_{b1}} \times \dfrac{\Phi_1}{\Phi_2} = \dfrac{E_{b2}}{E_{b1}}$ $(\because \quad \Phi_1 = \Phi_2)$

$\therefore \quad \dfrac{450}{900} = \dfrac{200 - 36.5\, R_t}{217.1}$; $R_t = 3.05\ \Omega$

$\therefore$ External resistance $R = 3.05 - 0.08 = \mathbf{2.97\ \Omega}$

For calculating the motor output, it will be assumed that all losses except copper losses vary directly with speed.

Since motor speed is halved, stray losses are also halved in the second case. Let us find their value.

In the first case, motor input = $200 \times 38.5 = 8{,}470$ W; Motor output = 7,460 W

Total Cu losses + stray losses = $8470 - 7460 = 1010$ W

Arm. Cu loss = $I_{a1}^2 R_a = 36.5^2 \times 0.08 = 107$ W ; Field Cu loss = $220 \times 2 = 440$ W

Total Cu loss = $107 + 440 = 547$ W $\quad \therefore$ Stray losses in first case = $1010 - 547 = 463$ W

Stray losses in the second case = $463 \times 450/900 = 231$ W

Field Cu loss = 440 W, as before ; Arm. Cu loss = $36.5^2 \times 3.05 = 4{,}064$ W

Total losses in the 2nd case = $231 + 440 + 4{,}064 = 4{,}735$ W

Input = 8,470 W – as before

Output in the second case = $8{,}470 - 4{,}735 = \mathbf{3{,}735\ W}$

$\therefore \quad$ Overall $\eta = 3{,}735/8{,}470 = 0.441$ or **44.1 per cent***

Example 30.25. *A 240 V shunt motor has an armature current of 15 A when running at 800 r.p.m. against F.L. torque. The arm. resistance is 0.6 ohms. What resistance must be inserted in series with the armature to reduce the speed to 400 r.p.m., at the same torque ?*

What will be the speed if the load torque is halved with this resistance in the circuit ? Assume the flux to remain constant throughout. **(Elect. Machines-I Nagpur Univ. 1993)**

Solution. Here, $N_1 = 800$ r.p.m., $E_{b1} = 240 - 15 \times 0.6 = 231$ V

Flux remaining constant, $T \propto I_a$. Since torque is the same in both cases, $I_{a2} = I_{a1} = 15$ A. Let R be the additional resistance inserted in series with the armature. $E_{b2} = 240 - 15\,(R + 0.6)$; $N_2 = 400$ r.p.m.

$\therefore \quad \dfrac{400}{800} = \dfrac{240 - 15\,(R + 0.6)}{231}$; $R = 7.7\ \Omega$

* It may be noted that efficiency is reduced almost in the ratio of the two speeds.

When load torque is halved :

With constant flux when load torque is halved, I_a is also halved. Hence, $I_{a3} = I_{a1}/2 = 15/2 = 7.5$ A

$$\therefore \quad E_{b3} = 240 - 7.5\,(7.7 + 0.6) = 177.75 \text{ V} ; N_3 = ?$$

$$\frac{N_3}{N_1} = \frac{E_{b3}}{E_{b1}} \text{ or } \frac{N_3}{800} = \frac{177.75}{231} ; N_3 = \mathbf{614.7 \text{ r.p.m.}}$$

Example 30.26. ***(a)*** *A 400 V shunt connected d.c. motor takes a total current of 3.5 A on no load and 59.5 A at full load. The field circuit resistance is 267 ohms and the armature circuit resistance is 0.2 ohms (excluding brushes where the drop may be taken as 2 V). If the armature reaction effect at 'full-load' weakens the flux per pole by 2 percentage change in speed from no-load to full-load.*

(b) *What resistant must be placed in series with the armature in the machine of (a) if the full-load speed is to be reduced by 50 per cent with the gross torque remaining constant ? Assume no change in the flux.* **(Electrical Machines, AMIE Sec. B, 1989)**

Solution. ***(a)*** Shunt current $I_{sh} = 400/267 = 1.5$ A. At no load, $I_{a1} = 3.5 - 1.5 = 2$ A, $E_{b1} = V - I_{a1} R_{ai}$ – brush drop $= 400 - 2 \times 0.2 - 2 = 397.6$ V. On full-load, $I_{a2} = 59.5 - 1.5 = 58$ A, $E_{b2} = 400 - 58 \times 0.2 - 2 = 386.4$ V.

$$\therefore \quad \frac{E_{b1}}{E_{b2}} = \frac{\Phi_1 N_1}{\Phi_2 N_2} \text{ or } \frac{397.6}{386.4} = \frac{\Phi_1 N_1}{0.98\,\Phi_1 N_2} ; \frac{N_1}{N_2} = 1.0084$$

$$\% \text{ change in speed} = \frac{N_1 - N_2}{N_1} \times 100$$

$$= \left(1 - \frac{1}{1.0084}\right) \times 100 = 0.833$$

(b) Since torque remains the same, I_a remains the same, hence $I_{a3} = I_{a2}$. Let R be the resistance connected in series with the armature.

$$E_{b3} = V - I_{a2}\,(R_a + R) - \text{brush drop}$$

$$= 400 - 58\,(0.2 + R) - 2 = 386.4 - 58\,R$$

$$\therefore \quad \frac{E_{b2}}{E_{b3}} = \frac{\Phi_2 N_2}{\Phi_3 N_3} = \frac{N_2}{N_3} \qquad (\because \quad \Phi_2 = \Phi_3)$$

$$\frac{386.4}{386.4 - 58\,R} = \frac{1}{0.5} ; \quad R = \mathbf{3.338\ \Omega}$$

Example 30.27. *A d.c. shunt drives a centrifugal pump whose torque varies as the square of the speed. The motor is fed from a 200 V supply and takes 50 A when running at 1000 r.p.m. What resistance must be inserted in the armature circuit in order to reduce the speed to 800 r.p.m. ? The armature and field resistance of the motor are 0.1 Ω and 100 Ω respectively.*

(Elect. Machines, Allahabad Univ. 1992)

Solution. In general, $T \propto \Phi I_a$

For shunt motors whose excitation is constant,

$$T \propto I_a \propto N^2, \text{ as given.}$$

$$\therefore \quad I_a \propto N^2. \text{ Now } I_{sh} = 200/100 = 2 \text{ A} \quad \therefore \quad I_{a1} = 50 - 2 = 48 \text{ A}$$

Let I_{a2} = new armature current at 800 r.p.m.

then $48 \propto N_1^2 \propto 1000^2$ and $I_{a2} \propto N_2^2 \propto 800^2$

$$\therefore \quad \frac{I_{a2}}{48} = \left(\frac{800}{1000}\right)^2 = 0.8^2 \quad \therefore \quad I_{a2} = 48 \times 0.64 = 30.72 \text{ A}$$

$\therefore \quad E_{b1} = 200 - (48 \times 0.1) = 195.2$ V ; $E_{b2} = (200 - 30.72\ R_t)$ V

Now, $\dfrac{N_2}{N_1} = \dfrac{E_{b2}}{E_{b1}} \quad \therefore \quad \dfrac{800}{1000} = \dfrac{200 - 30.72\ R_t}{195.2}$, $R_t = 1.42\ \Omega$

Additional resistance = $1.42 - 0.1 =$ **1.32 Ω**

Example 30.28. *A 250 V, 50 h.p. (373 kW) d.c. shunt motor has an efficiency of 90% when running at 1,000 r.p.m. on full-load. The armature and field resistances are 0.1 Ω and 115 Ω respectively. Find*

(a) *the net and developed torque on full-load.*

(b) *the starting resistance to have the line start current equal to 1.5 times the full-load current.*

(c) *the torque developed at starting.* **(Elect. Machinery-I, Kerala Univ. 1987)**

Solution. (a) $T_{sh} = 9.55 \times 37{,}300/1000 =$ **356.2 N-m**

Input current = $\dfrac{37{,}300}{250 \times 0.9} = 165.8$ A ; $I_{sh} = \dfrac{250}{125} = 2$ A

$\therefore \quad I_a = 165.8 - 2 = 163.8$ A ; $E_b = 250 - (163.8 \times 0.1) = 233.6$ V

$\therefore \quad T_a = 9.55\ \dfrac{233.6 \times 163.8}{1000} =$ **365.4 N-m**

(b) F.L. input line I = 165.8 A ; Permissible input $I = 165.8 \times 1.5 = 248.7$ A

Permissible armature current = $248.7 - 2 = 246.7$ A

Total armature resistance = $250/246.7 = 1.014\ \Omega$

$\therefore$ Starting resistance required = $1.014 - 0.1 =$ **0.914 Ω**

(c) Torque developed with 1.5 times the F.L. current would be practically 1.5 times the F.L. torque.

i.e. $1.5 \times 365.4 =$ **548.1 N-m.**

Example 30.29. *A 200 V shunt motor with a shunt resistance of 40 Ω and armature resistance of 0.02 Ω takes a current of 55 A and runs at 595 r.p.m. when there is a resistance of 0.58 Ω in series with armature. Torque remaining the same, what change should be made in the armature circuit resistance to raise the speed to 630 r.p.m. ? Also find*

(i) *At what speed will the motor run if the load torque is reduced such that armature current is 15 A.*

(ii) *Now, suppose that a divertor of resistance 5 Ω is connected across the armature and series resistance is so adjusted that motor speed is again 595 r.p.m., when armature current is 50 A. What is the value of this series resistance ? Also, find the speed when motor current falls of 15 A again.*

Solution. The circuit is shown in Fig. 30.7.

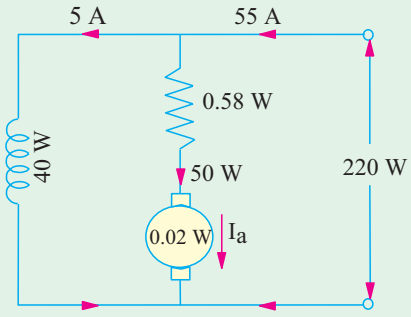


Fig. 30.7

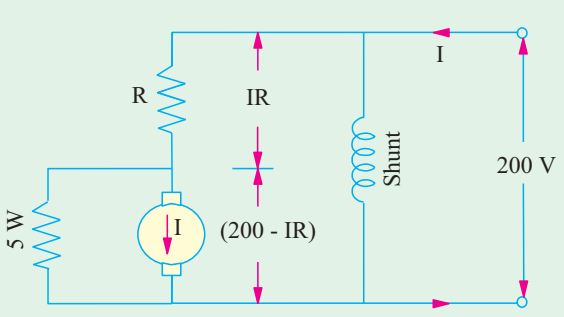


Fig. 30.8

I_{sh} = 200/40 = 5 A ∴ I_{a1} = 55 – 5 = 50 A

Armature circuit resistance = 0.58 + 0.02 = 0.6 Ω

∴ E_{b1} = 200 – (50 × 0.6) = 170 V

Since torque is the same in both cases, $I_{a1}\,\Phi_1 = I_{a2}\,\Phi_2$

Moreover, $\Phi_1 = \Phi_2$ ∴ $I_{a1} = I_{a2}$ ∴ I_{a2} = 50 A

Now E_{b1} = 170 V, N_1 = 595 r.p.m., N_2 = 630 r.p.m., E_{b2} = ?

Using $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}}$ ($\because$ $\Phi_1 = \Phi_2$)

we get E_{b2} = 170 × (630/595) = 180 V

Let R_2 be the new value of armature circuit resistance for raising the speed from 595 r.p.m. to 630 r.p.m.

∴ 180 = 200 – 50 R_2 ∴ R = 0.4 Ω

Hence, armature circuit resistance should be reduced by 0.6 – 0.4 = **0.2 Ω.**

(i) We have seen above that

I_{a1} = 50 A, E_{b1} = 170 V, N_1 = 595 r.p.m.

If I_{a2} = 15 A, E_{b2} = 200 – (15 × 0.6) = 191 V

∴ $\frac{N_2}{595} = \frac{191}{170}$ ∴ N_2 = **668.5 r.p.m.**

(ii) When armature divertor is used (Fig. 30.8).

Let R be the new value of series resistance

∴ E_{b3} = 200 – IR – (50 × 0.02) = 199 – IR

Since speed is 595 r.p.m., E_{b3} must be equal to 170 V

∴ 170 = 199 – IR ∴ IR = 29 V; P.D. across divertor = 200 – 29 = 171 V

Current through divertor I_d = 171/5 = 34.2 A ∴ I = 50 + 34.2 = 84.2 A

As IR = 29 V ∴ R = 29/84.2 = **0.344 W**

When I_a = 15 A, then I_d = (I – 15) A

P.D. across divertor = 5 (I – 15) = 200 – 0.344 I ∴ I = 51.46 A

E_{b4} = 200 – 0.344 I – (15 × 0.02)

= 200 – (0.344 × 51.46) – 0.3 = 182 V

∴ $\frac{N_4}{N_1} = \frac{E_{b4}}{E_{b1}}$ or $\frac{N_4}{595} = \frac{182}{170}$ ∴ N_4 = **637 r.p.m.**

The effect of armature divertor is obvious. The speed without divertor is 668.5 r.p.m. and with armature divertor, it is 637 r.p.m.

(iii) Voltage Control Method

(*a*) Multiple Voltage Control

In this method, the shunt field of the motor is connected permanently to a fixed exciting voltage, but the armature is supplied with different voltages by connecting it across one of the several different voltages by means of suitable switchgear. The armature speed will be approximately proportional to these different voltages. The intermediate speeds can be obtained by adjusting the shunt field regulator. The method is not much used, however.

(*b*) Ward-Leonard System

This system is used where an unusually wide (upto 10 : 1) and very sensitive speed control is required as for colliery winders, electric excavators, elevators and the main drives in steel mills and blooming and paper mills. The arrangement is illustrated in Fig. 30.9.

M_1 is the main motor whose speed control is required. The field of this motor is permanently connected across the d.c. supply lines. By applying a variable voltage across its armature, any desired

speed can be obtained. This variable voltage is supplied by a motor-generator set which consists of either a d.c. or an a.c. motor M_2 directly coupled to generator G.

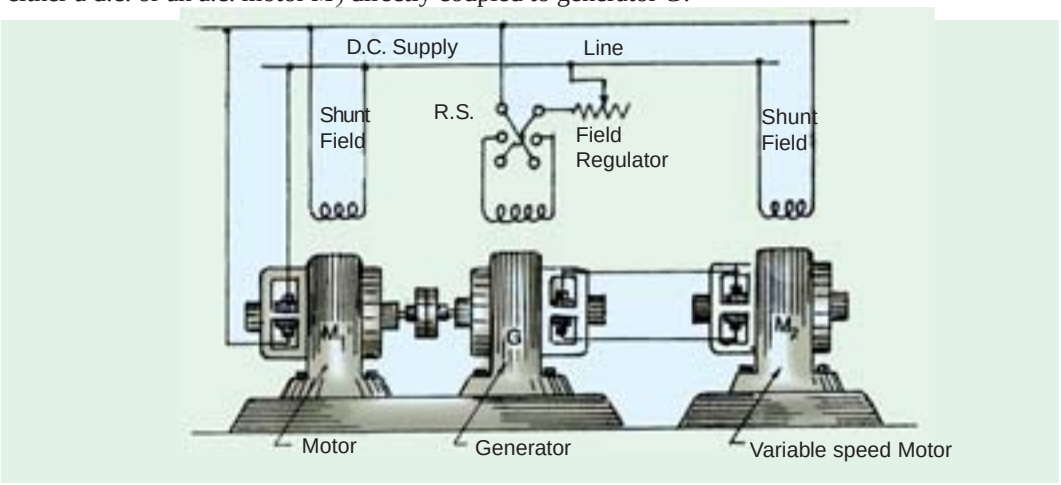


Fig. 30.9

The motor M_2 runs at an approximately constant speed. The output voltage of G is directly fed to the main motor M_1. The voltage of the generator can be varied from zero up to its maximum value by means of its field regulator. By reversing the direction of the field current of G by means of the reversing switch RS, generated voltage can be reversed and hence the direction of rotation of M_1. It should be remembered that motor generator set always runs in the same direction.

Despite the fact that capital outlay involved in this system is high because **(*i*)** a large output machine must be used for the motor generator set and **(*ii*)** that two extra machines are employed, still it is used extensively for elevators, hoist control and for main drive in steel mills where motor of ratings 750 kW to 3750 kW are required. The reason for this is that the almost unlimited speed control in either direction of rotation can be achieved entirely by field control of the generator and the resultant economies in steel production outwiegh the extra expenditure on the motor generator set.

A modification of the Ward-Leonard system is known as Ward-Leonard-Ilgner system which uses a smaller motor-generator set with the addition of a flywheel whose function is to reduce fluctuations in the power demand from the supply circuit. When main motor M_1 becomes suddenly overloaded, the driving motor M_2 of the motor generator set slows down, thus allowing the inertia of the flywheel to supply a part of the overload. However, when the load is suddenly thrown off the main motor M_1, then M_2 speeds up, thereby again storing energy in the flywheel.

When the Ilgner system is driven by means of an a.c. motor (whether induction or synchronous) another refinement in the form of a 'slip regulator' can be usefully employed, thus giving an additional control.

The chief disadvantage of this system is its low overall efficiency especially at light loads. But as said earlier, it has the outstanding merit of giving wide speed control from maximum in one direction through zero to the maximum in the opposite direction and of giving a smooth acceleration.

Example 30.30. *The O.C.C. of the generator of a Ward-Leonard set is*

Field amps :	*1.4*	*2.2*	*3*	*4*	*5*	*6*	*7*	*8*
Armature volts :	*212*	*320*	*397*	*472*	*522*	*560*	*586*	*609*

The generator is connected to a shunt motor, the field of which is separately-excited at 550 V. If the speed of motor is 300 r.p.m. at 550 V, when giving 485 kW at 95.5% efficiency, determine the excitation of the generator to give a speed of 180 r.p.m. at the same torque. Resistance of the motor

armature circuit = 0.01 Ω, resistance of the motor field = 60 Ω, resistance of generator armature circuit = 0.01 Ω. Ignore the effect of armature reaction and variation of the core factor and the windage losses of the motor.

Solution. Motor input = $485 \times 10^3/0.955 = 509{,}300$ W

Motor to motor field = $550/60 = 55/6$ A

Input to motor field = $550 \times 55/6 = 5{,}040$ W

$\therefore$ Motor armature input = $509{,}300 - 5{,}040 = 504{,}260$ W

$\therefore$ Armature current = $504{,}260/550 = 917$ A

Back e.m.f. E_{b1} at 300 r.p.m. = $550 - (917 \times 0.01) = 540.83$ V

Back e.m.f. E_{b2} at 180 r.p.m. = $540.83 \times 180/300 = 324.5$ V

Since torque is the same, the armature current of the main motor is also the same *i.e.* 917 A because its excitation is independent of its speed.

$\therefore$ $V = 324.5 + (917 \times 0.01) = 333.67$ V

Generated e.m.f. = $V + I_a R_a$ ...for generator

$333.67 + (917 \times 0.011) = 343.77$ V.

If O.C.C. is plotted from the above given data, then it would be found that the excitation required to give 343.77 V is 2.42 A.

$\therefore$ Generator exciting current = **2.42 A**

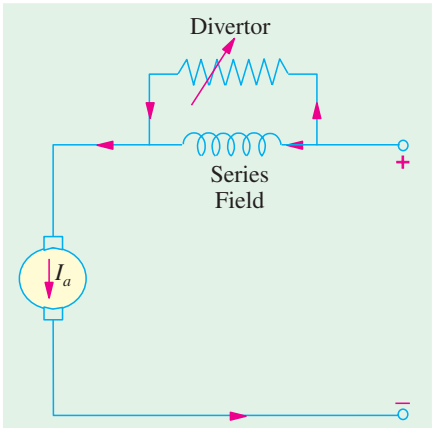


Fig. 30.10

30.3. Speed Control of Series Motors

1. Flux Control Method

Variations in the flux of a series motor can be brought about in any one of the following ways :

(*a*) Field Divertors

The series winding are shunted by a variable resistance known as field divertor (Fig. 30.10). Any desired amount of current can be passed through the divertor by adjusting its resistance. Hence the flux can be decreased and consequently, the speed of the motor increased.

(*b*) Armature Divertor

A divertor across the armature can be used for giving speeds lower than the normal speed (Fig. 30.11). For a ***given constant load torque***, if I_a is reduced due to armature divertor, the Φ must increase.

($\because$ $T_a \propto \Phi I_a$). This results in an increase in current taken from the supply (which increases the flux and a fall in speed ($N \propto I/\Phi$)). The variation in speed can be controlled by varying the divertor resistance.

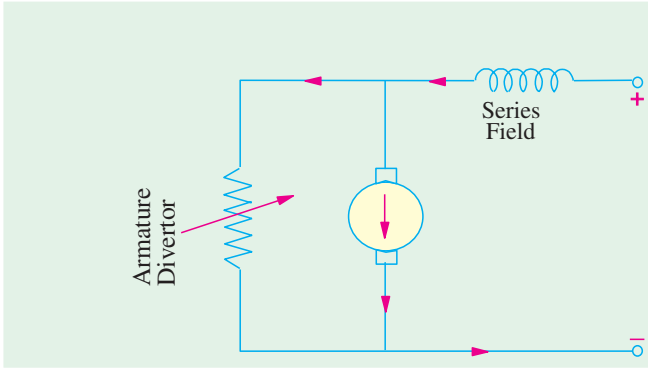


Fig. 30.11

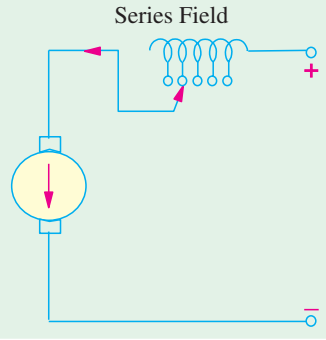


Fig. 30.12

(c) Trapped Field Control Field

This method is often used in electric traction and is shown in Fig. 30.12.

The number of series filed turns in the circuit can be changed at will as shown. With full field, the motor runs at its minimum speed which can be raised in steps by cutting out some of the series turns.

(d) Paralleling Field coils

In this method, used for fan motors, several speeds can be obtained by regrouping the field coils as shown in Fig. 30.13. It is seen that for a 4-pole motor, three speeds can be obtained easily. (Ex.30.35)

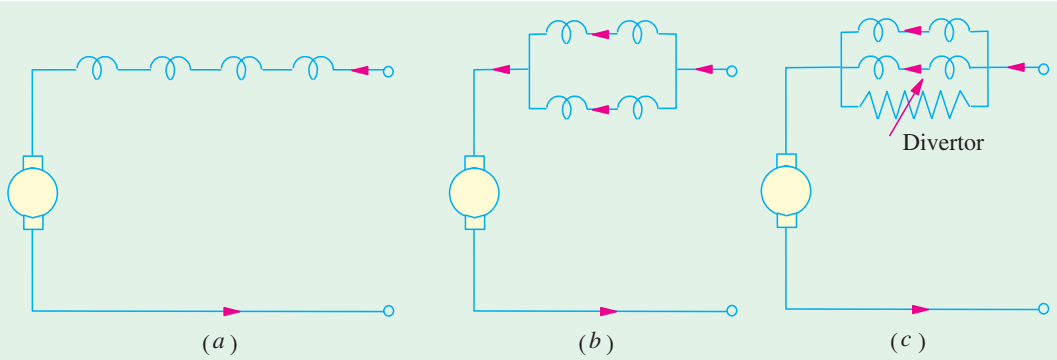


Fig. 30.13

2. Variable Resistance in Series with Motor

By increasing the resistance in series with the armature (Fig. 30.14) the voltage applied across the armature terminals can be decreased.

With reduced voltage across the armature, the speed is reduced. However, it will be noted that since full motor current passes through this resistance, there is a considerable loss of power in it.

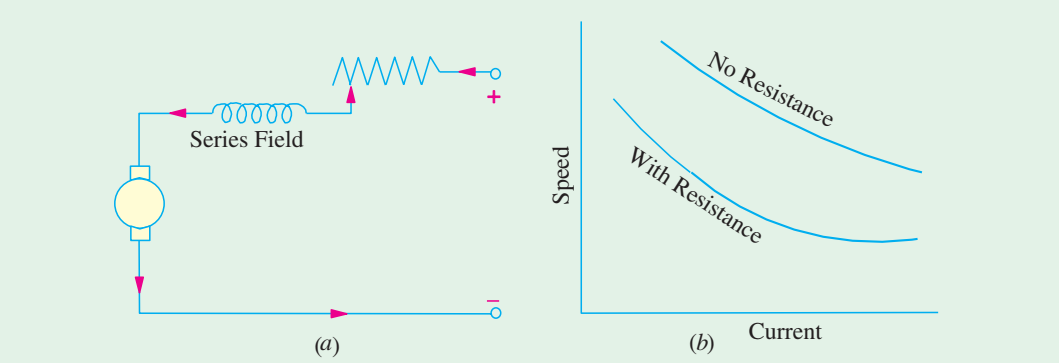


Fig. 30.14

Example 30.31. *A d.c. series motor drives a load the torque of which varies as the square of the speed. The motor takes a current of 15 A when the speed is 600 r.p.m. Calculate the speed and the current when the motor field winding is shunted by a divertor of the same resistance as that of the field winding. Mention the assumptions made, if any.* **(Elect. Machines, AMIE Sec B, 1993)**

Solution.

$$T_{a1} \propto N_1^2, T_{a2} \propto N_2^2 \quad \therefore \quad T_{a2}/T_{a1} = N_2^2/N_1^2$$

Also, $$T_{a1} \propto \Phi_1 I_{a1} \propto I_{a1}^2, T_{a2} \propto \Phi_2 I_{a2} \propto (I_{a2}/2) I_{a2} \propto I_{a2}^2/2$$

It is so because in the second case, field current is half the armature current.

$$\therefore \quad \frac{N_2^2}{N_1^2} = \frac{I_{a2}^2/2}{I_{a1}^2} \quad \text{or} \quad \frac{N_2}{N_1} = \frac{I_{a2}}{\sqrt{2}\, I_{a1}} \qquad \text{... (i)}$$

Now, $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$

If we neglect the armature and series winding drops as well as brush drop, then $E_{b1} = E_{b2} = V$

$$\frac{N_2}{N_1} = \frac{\Phi_1}{\Phi_2} = \frac{\tau_1}{\tau_2 \varepsilon 2} = \frac{2\tau_1}{\tau_2} \quad \text{...(ii)}$$

From **(i)** and **(ii)**, $\frac{I_{a2}}{\sqrt{2}\, I_{a1}} = \frac{2 I_{a1}}{I_{a2}}$ or $I_{a2}^2 = 2\sqrt{2}\, I_{a1}^2 = 2\sqrt{2} \times 15^2$ or $I_{a2} = 25.2$ A

From **(ii)**, we get, $N_2 = 600 \times 2 \times 15/252 =$ **714 r.p.m.**

Example 30.32. *A 2-pole series motor runs at 707 r.p.m. when taking 100 A at 85 V and with the field coils in series. The resistance of each field coil is 0.03 Ω and that of the armature 0.04 Ω. If the field coils are connected in parallel and load torque remains constant, find* **(a)** *speed* **(b)** *the additional resistance to be inserted in series with the motor to restore the speed to 707 r.p.m.*

Solution. Total armature circuit resistance = 0.04 + (2 × 0.03) = 0.1 Ω

$I_{a1} = 100$ A ; $E_{b1} = 85 - (100 \times 0.1) = 75$ V

When series field windings are placed in parallel, the current through each is half the armature current.

If I_{a2} = new armature current ; then $\Phi_2 \propto I_{a2}/2$.

As torque is the same in the two cases,

$\therefore \quad \Phi_1 I_{a1} = \Phi_2 I_{a2}$ or $I_{a1}^2 = \frac{I_{a2}}{2} \times I_{a2} = \frac{I_{a2}^2}{2}$

$\therefore \quad 100^2 = I_{a2}^2 \quad \therefore \quad I_{a2} = 100\sqrt{2} = 141.4$ A

In this case, series field resistance = 0.03/2 = 0.015 Ω

$\therefore \quad E_{b2} = 85 - 141.4\,(0.04 + 0.015) = 77.22$ V

$$\frac{N_2}{707} = \frac{77.22}{75} \times \frac{100}{141.4/2} \qquad (\because \ \Phi_2 \propto I_{a2}/2)$$

(a) $\therefore \quad N_2 = 707 \times \frac{77.22}{75} \times \frac{200}{141.4} =$ **1029 r.p.m.**

(b) Let the total resistance of series circuit be R_t.

Now, $E_{b1} = 77.22$ V, $N_1 = 1029$ r.p.m. ; $E_{b2} = 85 - 141.4\, R_t$, $N_2 = 707$ r.p.m.

$$\frac{707}{1029} = \frac{85 - 141.4\, R_t}{77.22} \quad \therefore \quad R_t = 0.226\ \Omega$$

$\therefore$ Additional resistance = 0.226 – 0.04 – 0.015 = **0.171 Ω**

Example 30.33. *A 240 V series motor takes 40 amperes when giving its rated output at 1500 r.p.m. Its resistance is 0.3 ohm. Find what resistance must be added to obtain rated torque (i) at starting (ii) at 1000 r.p.m.* **(Elect. Engg., Madras Univ. 1987)**

Solution. Since torque remains the same in both cases, it is obvious that current drawn by the motor remains constant at 40 A. $(\because \ T_a \propto I_a^2)$

(i) If R is the series resistance added, then $40 = 240/(R + 0.3) \quad \therefore \quad R = 5.7\ \Omega$

(ii) Current remaining constant, $T_a \propto E_b/N$ – Art. 29.7

$$\frac{E_{b1}}{N_1} = \frac{E_{b2}}{N_2}$$

Now, $E_{b1} = 240 - 40 \times 0.3 = 228$ V ; $N_1 = 1500$ r.p.m.

$E_{b2} = 240 - 40\,(R + 0.3)$ V ; $N_2 = 1000$ r.p.m.

$$\therefore \quad \frac{228}{1500} = \frac{240 - 40\,(R + 0.3)}{1000} \; ; \; R = \mathbf{1.9\ \Omega}$$

Example 30.34. *A 4-pole, series-wound fan motor runs normally at 600 r.p.m. on a 250 V d.c. supply taking 20 A. The field coils are connected at in series. Estimate the speed and current taken by the motor if the coils are reconnected in two parallel groups of two in series. The load torque increases as the square of the speed. Assume that the flux is directly proportional to the current and ignore losses.* **(Elect. Machines, AMIE, Sec B. 1990)**

Solution. When coils are connected in two parallel groups, current through each becomes $I_{a2}/2$ where I_{a2} is the new armature current.

Hence, $\Phi_2 \propto I_{a2}/2$

Now $T_a \propto \Phi I_a$

$\propto N^2$ – given

$$\therefore \quad \Phi_1 I_{a1} \propto N_1^{\,2} \text{ and } \Phi_2 I_{a2} \propto N_2^{\,2} \quad \therefore \quad \left(\frac{N_2}{N_1}\right)^2 = \frac{\Phi_2 I_{a2}}{\Phi_1 I_{a1}} \qquad \textbf{... (i)}$$

Since losses are negligible, field coil resistance as well as armature resistance are negligible. It means that armature and series field voltage drops are negligible. Hence, back e.m.f. in each case equals the supply voltage.

$$\therefore \quad \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} \text{ becomes } \frac{N_2}{N_1} = \frac{\Phi_1}{\Phi_2} \qquad \textbf{... (ii)}$$

Putting this value in **(i)** above, we get

$$\left(\frac{\Phi_1}{\Phi_2}\right)^2 = \frac{\Phi_2 I_{a2}}{\Phi_1 I_{a1}} \quad \text{or} \quad \frac{I_{a2}}{I_{a1}} = \left(\frac{\Phi_1}{\Phi_2}\right)^3$$

Now, $\Phi_1 \propto 20$ and $\Phi_2 \propto I_{a2}/2 \quad \therefore \quad \frac{I_{a2}}{20} = \left(\frac{20}{I_{a2}/2}\right)^3$ or $I_{a2} = 20 \times 2^{3/4} = \mathbf{33.64\ A}$

From **(ii)** above, we get $\frac{N_2}{N_1} = \frac{\Phi_1}{\Phi_2} = \frac{I_{a1}}{I_{a2}/2} = \frac{2\,I_{a1}}{I_{a2}}$; $N_2 = 600 \times 2 \times 20/33.64 = \mathbf{714\ r.p.m.}$

Example 30.35. *A d.c. series motor having a resistance of 1 Ω drives a fan for which the torque varies as the square of the speed. At 220 V, the set runs at 350 r.p.m. and takes 25 A. The speed is to be raised to 500 r.p.m. by increasing the voltage. Determine the necessary voltage and the corresponding current assuming the field to be unsaturated.*

(Electrical Engg., Banaras Hindu Univ. 1998)

Solution. Since $\Phi \propto I_a$, hence $T_a \propto \Phi I_a \propto I_a^{\,2}$. Also $T_a \propto N^2$ ...(given)

$$\therefore \quad I_a^{\,2} \propto N^2 \quad \text{or} \quad I_a \propto N \quad \text{or} \quad I_{a1} \propto N_1 \text{ and } I_{a2} \propto N_2$$

$$\therefore \quad \frac{I_{a2}}{I_{a1}} = \frac{N_2}{N_1} = \frac{500}{350} \; ; \; I_{a2} = 25 \times \frac{500}{350} = \frac{250}{7}\ \mathbf{A}$$

$$E_{b1} = 220 - 25 \times 1 = 195\ \text{V} \; ; \; E_{b1} = \text{V} - (250/7) \times 1, \; \frac{\Phi_1}{\Phi_2} = \frac{25}{250/7} = \frac{7}{10}$$

Now $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} \; ; \; \frac{500}{350} = \frac{V - (250/7)}{195} \times \frac{7}{10} \; ; \; V = \mathbf{433.7\ V}$

Example 30.36. *A d.c. series motor runs at 1000 r.p.m. when taking 20 A at 200 V. Armature resistance is 0.5 Ω. Series field resistance is 0.2 Ω. Find the speed for a total current of 20 A when a divertor of 0.2 Ω resistance is used across the series field. Flux for a field current of 10 A is 70 per cent of that for 20 A.*

Solution. $E_{b1} = 200 - (0.5 + 0.2) \times 20 = 186$ V ; $N_1 = 1000$ r.p.m.

Since divertor resistance equals series field resistance, the motor current of 20 A is divided equally between the two. Hence, a current of 10 A flows through series field and produces flux which is 70% of that corresponding to 20 A. In other words, $\Phi_2 = 0.7$ or $\Phi_1/\Phi_2 = 1/0.7$

Moreover, their combined resistance = 0.2/2 = 0.1 Ω

Total motor resistance becomes = 0.5 + 0.1 = 0.6 Ω

$$\therefore \quad E_{b2} = 200 - 0.6 \times 20 = 188 \text{ V} ; N_2 = ?$$

$$\therefore \quad \frac{N_2}{1000} = \frac{188}{186} \times \frac{1}{0.7} ; \quad N_2 = \textbf{1444 r.p.m.}$$

Example 30.37. *A 200 V, d.c. series motor takes 40 A when running at 700 r.p.m. Calculate the speed at which the motor will run and the current taken from the supply if the field is shunted by a resistance equal to the field resistance and the load torque is increased by 50%.*

Armature resistance = 0.15 Ω, field resistance = 0.1 Ω

It may be assumed that flux per pole is proportional to the field.

Solution. In a series motor, prior to magnetic saturation

$$T \propto \Phi I_a \propto I_a^2 \quad \therefore \quad T_1 \propto I_{a1}^2 \propto 40^2 \qquad \text{...(i)}$$

If I_{a2} is the armature current (or motor current) in the second case when divertor is used, then only $I_{a2}/2$ passes through the series field winding.

$$\therefore \quad \Phi_2 \propto I_{a2}/2 \text{ and } T_2 \propto \Phi_2 I_{a2} \propto (I_{a2}/2) \times I_{a2} \propto I_{a2}^2/2 \qquad \text{...(ii)}$$

From **(i)** and **(ii)**, we get $\frac{T_2}{T_1} = \frac{I_{a2}^2}{2 \times 40^2}$

Also $T_2/T_1 = 1.5 \quad \therefore \quad 1.5 = I_{a2}/2 \times 40^2$

$$\therefore \quad I_{a2} = \sqrt{1.5 \times 2 \times 40^2} = 69.3 \text{ A} ;$$

Now $E_{b1} = 220 - (40 \times 0.25) = 210$ V

$E_{b2} = 220 - (69.3 \times 0.2^*) = 206.14$ V ; $N_1 = 700$ r.p.m. ; $N_2 = ?$

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} \quad \therefore \quad \frac{N_2}{700} = \frac{206.14}{210} \times \frac{40}{69.3/2} \quad \therefore \quad N_2 = \textbf{794 r.p.m.}$$

Example 30.38. *A 4-pole, 250 V d.c. series motor takes 20 A and runs at 900 r.p.m. Each field coil has resistance of 0.025 ohm and the resistance of the armature is 0.1 ohm. At what speed will the motor run developing the same torque if :*

(i) *a divertor of 0.2 ohm is connected in parallel with the series field*

(ii) *rearranging the field coils in two series and parallel groups*

Assume unsaturated magnetic operation.

(Electric Drives and their Control, Nagpur Univ. 1993)

Solution. The motor with its field coils all connected in series is shown in Fig. 30.15 (*a*). Here, $N_1 = 900$ r.p.m., $E_{b1} = 250 - 20 \times (0.1 + 4 \times 0.025) = 246$ V.

In Fig. 30.15 (*b*), a divertor of resistance 0.2 Ω has been connected in parallel with the series field coils. Let I_{a2} be the current drawn by the motor under this condition. As per current-divider rule, part of the current passing through the series fields is $I_{a2} \times 0.2/(0.1 + 0.2) = 2\,I_{a2}/3$. Obviously, $\Phi_2 \propto 2\,I_{a2}/3$.

---

* The combined resistance of series field winding and the divertor is 0.1/2 = 0.05 Ω. Hence, the total resistance = 0.15 + 0.05 = 0.2 Ω, in example 30.37.

Now, $T_1 \propto \Phi_1 I_{a1} \propto I_{a1}^2$; $T_2 \propto \Phi_2 I_{a2} \propto (2 I_{a2}/3) I_{a2} \propto 2 I_{a2}^2/3$

Since $T_1 = T_2$; $\therefore\ I_{a1}^2 = 2 I_{a2}^2/3$ or $20^2 = 2 I_{a2}^2/3$; $\therefore\ I_{a2} = 24.5$ A.

Combined resistance of the field and divertor $= 0.2 \times 0.1/0.3 = 0.667\ \Omega$; Arm. circuit resistance $= 0.1 + 0.0667 = 0.1667\ \Omega$; $E_{b2} = 250 - 24.5 \times 0.1667 = 250 - 4.1 = 245.9$ V

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} ; \frac{N_2}{900} = \frac{245.9}{246} \times \frac{20}{(2/3)\,24.5} ; N_2 = 1102 \text{ r.p.m.} \quad ...(\because\ \Phi_2 \propto 2 I_{a2}/3)$$

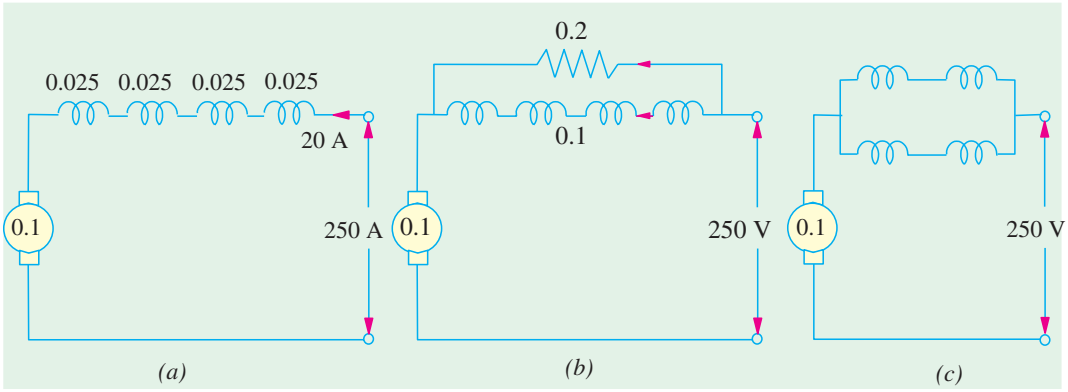


Fig. 30.15

(ii) In Fig. 30.15 (*c*), the series field coils have been arranged in two parallel groups. If the motor current is I_{a2}, then it is divided equally between the two parallel paths. Hence, $\Phi_2 \propto I_{a2}/2$.

Since torque remains the same,

$$T_1 \propto \Phi_1 I_{a1} \propto I_{a1}^2 \propto 20^2 ; T_2 \propto \Phi_2 I_{a2} \propto (I_{a2}/2) I_{a2} \propto I_{a2}^2/2$$

Since $T_1 = T_2$; $\therefore\ 20^2 = I_{a2}^2/2$; $I_{a2} = 28.28$ A

Combined resistance of the two parallel paths $= 0.05/2 = 0.025\ \Omega$

Total arm. circuit resistance $= 0.1 + 0.025 = 0.125\ \Omega$

$\therefore\ E_{b2} = 250 - 28.28 \times 0.125 = 246.5$ V

$$\frac{N_2}{900} = \frac{246.5}{246} \times \frac{20}{28.28/2} ; N_2 = \mathbf{1275\ r.p.m.}$$

Example 30.39. *A 4-pole, 230 V series motor runs at 1000 r.p.m., when the load current is 12 A. The series field resistance is 0.8 Ω and the armature resistance is 1.0 Ω. The series field coils are now regrouped from all in series to two in series with two parallel paths. The line current is now 20 A. If the corresponding weakening of field is 15%, calculate the speed of the motor.*

(Electrotechnology-I, Guwahati Univ. 1987)

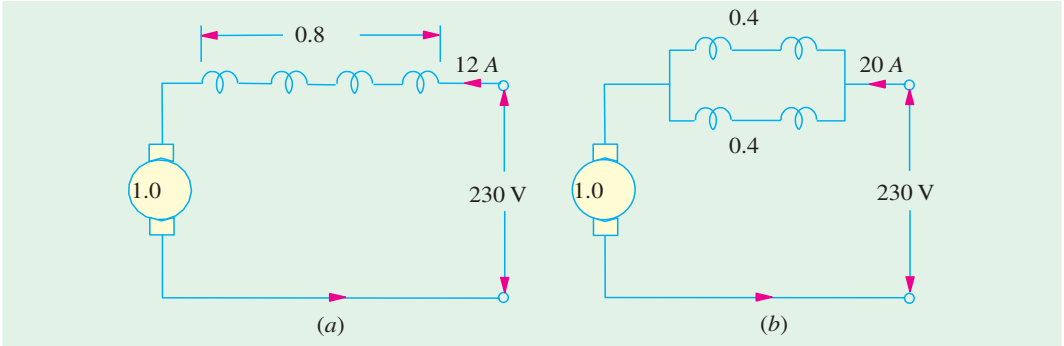


Fig. 30.16

Solution. $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$, $E_{b1} = 230 - 12 \times 1.8 = 208.4$ V, as in Fig. 30.16(a)

For circuit in Fig. 30.16 (b),

$E_{b2} = 230 - 20\,(1 + 0.4/2) = 206$ V ;

$\Phi_2 = 0.85\,\Phi_1$ or $\Phi_1/\Phi_2 = 1/0.85$

$\therefore \quad \frac{N_2}{1000} = \frac{206}{208.4} \times \frac{1}{0.85} \quad \therefore \quad N_2 =$ **1163 r.p.m.**

Example 30.40. *A 200 V, d.c. series motor runs at 500 r.p.m. when taking a line current of 25 A. The resistance of the armature is 0.2 Ω and that of the series field 0.6 Ω. At what speed will it run when developing the same torque when armature divertor of 10 Ω is used ? Assume a straight line magnetisation curve.* **(D.C. Machines, Jadavpur Univ. 1988)**

Solution. Resistance of motor = 0.2 + 0.6 = 0.8 Ω

$\therefore \quad E_{b1} = 200 - (25 \times 0.8) = 180$ V

Let the motor input current be I_2, when armature divertor is used, as shown in Fig. 30.17.

Series field voltage drop $= 0.6\,I_2$

$\therefore$ P.D. at brushes $= 200 - 0.6\,I_2$

$\therefore$ Arm. divertor current $= \left(\frac{200 - 0.6\,I_2}{10}\right)$ A

$\therefore$ Armature current $= I_2 \left(\frac{200 - 0.6\,I_2}{10}\right)$

$\therefore \quad I_{a2} = \frac{10.6\,I_2 - 200}{10}$

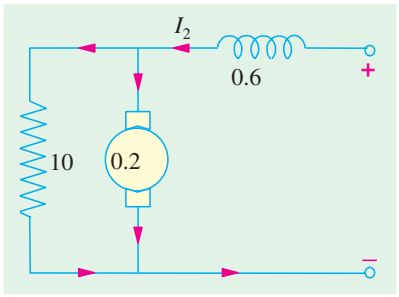


Fig. 30.17

As torque in both cases is the same, $\therefore \quad \Phi_1 I_{a1} = \Phi_2 I_{a2}$

$\therefore \quad 25 \times 25 = I_2 \left(\frac{10.6\,I_2 - 200}{10}\right)$ or $6{,}250 = 10.6\,I_2^2 - 200\,I_2$

or $10.6\,I_2^2 - 200\,I_2 - 6250 = 0$ or $I_2 = 35.6$ A

P.D. at brushes in this case $= 200 - (35.6 \times 0.6) = 178.6$ V

$\therefore \quad I_{a2} = \frac{10.6 \times 35.6 - 200}{10} = 17.74$ A;

$E_{b2} = 178.6 - (17.74 \times 0.2) = 175$ V

Now $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$ or $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{I_1}{I_2}$ $(\because \quad \Phi \propto I)$

$\therefore \quad \frac{N_2}{500} = \frac{175}{180} \times \frac{25}{35.6} \quad \therefore \quad N_2 =$ **314 r.p.m.**

Example 30.41. *A series motor is running on a 440 V circuit with a regulating resistance of R Ω for speed adjustment. The armature and field coils have a total resistance of 0.3 Ω. On a certain load with R = zero, the current is 20 A and speed is 1200 r.p.m. With another load and R = 3 Ω, the current is 15 A. Find the new speed and also the ratio of the two values of the power outputs of the motor. Assume the field strength at 15 A to be 80% of that at 20 A.*

Solution. $I_{a1} = 20$ A, $R_a = 0.3$ Ω ; $E_{b1} = 440 - (20 \times 0.3) = 434$ V

$I_{a2} = 15$ A, $R_a = 3 + 0.3 = 3.3$ Ω $\therefore E_{b2} = 440 - (3.3 \times 15) = 390.5$ V

$\Phi_2 = 0.8\,\Phi_1$, $N_1 = 1200$ r.p.m.

Using $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$, we get $N_2 = 1200 \times \frac{390.5}{434} \times \frac{1}{0.8}$ = **1350 r.p.m.**

Now, in a series motor,

$$T \propto \Phi I_a \text{ and power } P \propto T \times N \quad \text{or} \quad P \propto \Phi N I_a$$

$$\therefore \quad P_1 \propto \Phi_1 \times 1200 \times 20 \quad \text{and} \quad P_2 \propto 0.8\,\Phi_1 \times 1350 \times 15$$

$$\therefore \quad \frac{P_1}{P_2} = \frac{1200 \times 20\,\Phi_1}{1350 \times 15 \times 0.8\,\Phi_1} = \mathbf{1.48}$$

Hence, power in the first case is 1.48 times the power in the second case.

Example 30.42. *A d.c. series with an unsaturated field and negligible resistance, when running at a certain speed on a given load takes 50 A at 460 V. If the load-torque varies as the cube of the speed, calculate the resistance required to reduce the speed by 25%.*

(Nagpur, Univ. November 1999, Madras Univ. 1987)

Solution. Let the speed be ω radians/sec and the torque, T_{em} Nw-m developed by the motor. Hence power handled = T. ω watts

Let load torque be T_L, $T_L \propto \omega^3$

$$T_L = T_{em}, \; T_{em} \propto (50)^2$$

Hence Load power = $T_L\,\omega$

Since no losses have to be taken into account, $50^2 \propto \omega^3$

Armature power, $460 \times 50 \propto \omega^4$

Based on back e.m.f. relationship, $E_b \propto \omega I_a$

$$460 \propto \omega \times 50$$

To reduce the speed by 25%, operating speed = 0.75 ω rad/sec

Let the new current be I.

From Load side torque $\propto (0.75\,\omega)^3$

From electro-mech side, $T \propto I^2$

$$I^2 \propto (0.75\,\omega)^3$$

Comparing similar relationship in previous case,

$$\frac{I^2}{50^2} = \frac{(0.75\,\omega)^3}{\omega^3} = 0.75^3$$

$$I^2 = 50^2 \times 0.422 = 1055$$

$$I = 32.48 \text{ amp}$$

$$E_{b2} \propto I \times \text{speed}$$

$$\propto I \times 0.75\,\omega$$

$$\propto 32.48 \times 0.75\,\omega$$

$$\frac{E_{b2}}{460} = \frac{32.48 \times 0.75}{50}$$

$$E_{b2} = 224 \text{ volts}$$

If R is the resistance externally connected in series with the motor,

$$E_{b2} = 460 - 32.48 \times R = 224$$

$$\therefore \quad R = 7.266 \text{ ohms}$$

Previous armature power = $460 \times 50 \times 10^{-3} = 23$ kW

New armature power should be

$$23 \times (0.75)^4 = 7.28 \text{ kW}$$

With E_b as 224 V and current as 32.48 amp

$$\text{Armature power} = 224 \times 32.48 \text{ watts}$$
$$= 7.27 \text{ kW}$$

Thus, the final answer is checked by this step, since the results agree.

Example 30.43. *A d.c. series motor drives a load, the torque of which varies as the square of speed. The motor takes a current of 15 A when the speed is 600 r.p.m. Calculate the speed and current when the motor-field-winding is shunted by a divertor of equal resistance as that of the field winding. Neglect all motor losses and assume the magnetic circuit as unsaturated.*

(Bharathithasan Univ. April 1997)

Solution. Let the equations governing the characteristics of series motor be expressed as follows, with no losses and with magnetic circuit unsaturated.

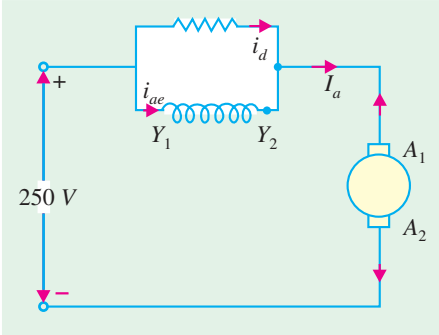


Fig. 30.18 (Divertor for speed control

$$\text{Torque developed by motor} = \text{Load torque}$$
$$k_m \times i_{se} \times I_a = k_L \times (600)^2$$

where k_m, k_L are constants

I_{se} = series field current

I_a = Armature current

In the first case $i_{se} = I_a$

With divertor, $i_{se} = i_d = 0.5\, I_a$,

Since, the resistances of divertor and series field are equal.

In the first case, $k_m \times 15 \times 15 = k_L \times (600)^2$ ...(*a*)

Let the supply voltage be V_L volts.

Since Losses are to be neglected, armature receives a power of $(V_L \times 15)$ watts.

Case ***(ii)*** Let the new speed = N_2 r.p.m. and the new armature current = I_{a2} amp

So that new series-field current = $0.5\, I_{a2}$

$$\text{Torque developed by motor} = \text{Load torque}$$
$$k_m \times (0.5\, I_{a2}) \times I_{a2} = k_L \times (N_2)^2 \quad \text{...(}b\text{)}$$

From equations (*a*) and (*b*) above,

$$\frac{0.5\, I_{a2}^2}{225} = \frac{N_2^2}{600^2}$$

or $$\left(\frac{N_2^2}{I_{a2}^2}\right) = \frac{600^2}{450} = 800 \quad \text{...(}c\text{)}$$

or $$N_2/I_{a2} = 28.28 \quad \text{...(}d\text{)}$$

Now armature receives a power of $I_{a2}\, V_L$ watts. Mechanical outputs in the two cases have to be related with these electrical-power-terms.

$$k_L = (600)^2 \times 2600/60 = 15\, V_L \quad \text{...(}e\text{)}$$
$$k_L = N^2 \times 2\, N/60 = I_{a2}\, V_L \quad \text{...(}f\text{)}$$

From these two equations,

$$N^3/600^3 = I_{a2}/15 \quad \text{...(}g\text{)}$$

From ***(c)*** and ***(g)***, $N_2\, I_{a2} = 18{,}000$ ...(*h*)

From **(*d*)** and **(*h*)**, $N_2 = 713.5$ r.p.m.

And $I_{a2} = 25.23$ amp

Additional Correlation : Since the load-torque is proportional to the square of the speed, the mechanical output power is proportional to the cube of the speed. Since losses are ignored, electrical power (input) must satisfy this proportion.

$$(15\ V_L) / (25.23\ V_L) = (600/713.5)^3$$

L.H.S. = 0.5945, R.H.S. = 0.5947

Hence, correlated and checked.

30.4. Merits and Demerits of Rheostatic Control Method

1. Speed changes with every change in load, because speed variations depend not only on controlling resistance but on load current also. This double dependence makes it impossible to keep the speed sensibly constant on rapidly changing loads.

2. A large amount of power is wasted in the controller resistance. Loss of power is directly proportional to the reduction in speed. Hence, efficiency is decreased.

3. Maximum power developed is diminished in the same ratio as speed.

4. It needs expensive arrangement for dissipation of heat produced in the controller resistance.

5. It gives speeds ***below*** the normal, not above it because armature voltage can be decreased (not increased) by the controller resistance.

This method is, therefore, employed when low speeds are required for a short period only and that too occasionally as in printing machines and for cranes and hoists where motor is continually started and stopped.

Advantages of Field Control Method

This method is economical, more efficient and convenient though it can give speeds *above* (not below) the normal speed. The only limitation of this method is that commutation becomes unsatisfactory, because the effect of armature reaction is greater on a weaker field.

It should, however, be noted that by combining the two methods, speeds above and below the normal may be obtained.

30.5. Series-parallel Control

In this system of speed control, which is widely used in electric traction, two or more similar mechanically-coupled series motors are employed. At low speeds, the motors are joined in series Fig. 30.19 (*a*) and for high speeds, are joined in parallel Fig. 30.19 (*b*).

When in series, the two motors have the same current passing through them, although the voltage across each motor is $V/2$ *i.e.*, half the supply voltage. When joined in parallel, voltage across each machines is V, though current drawn by each motor is $I/2$.

When in Parallel

Now speed $\propto E_b / \phi \propto E_b /$ current (being series motors)

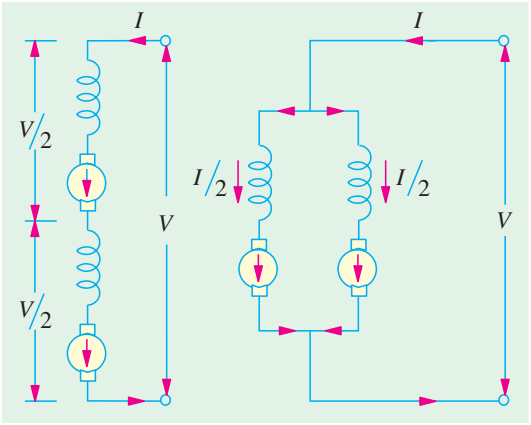


Fig. 30.19 (*a*) Fig. 30.19 (*b*)

Since E_b is approximately equal to the applied voltage V :

$$\therefore \quad \text{speed} \propto \frac{V}{I/2} \propto \frac{2V}{I}$$

Also, $\text{torque} \propto \Phi I \propto I^2 \quad (\because \Phi \propto I)$ ...(*i*)

$\therefore \quad T \propto (I/2)^2 \propto I^2/4$ ...(*ii*)

When in Series

Here $\text{speed} \propto \frac{E_b}{\Phi} \propto \frac{V/2}{I} \propto \frac{V}{2I}$ ...(*iii*)

This speed is one-fourth of the speed of the motors when in parallel.

Similarly $T \propto \Phi I \propto I^2$

The torque is four times that produced by motors when in parallel.

This system of speed control is usually combined with the variable resistance method of control described in Art. 30.3 (2).

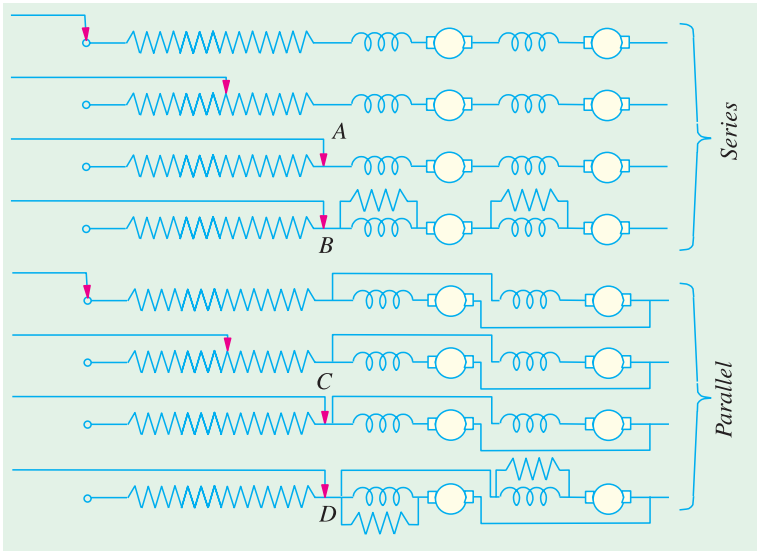


Fig. 30.20

The two motors are started up in series with each other and with variable resistance which is cut out in sections to increase the speed. When all the variable series resistance is cut out, the motors are connected in parallel and at the same time, the series resistance is reinserted. The resistance is again reduced gradually till full speed is attained by the motors. The switching sequence is shown in Fig.30.20. As the variable series controller resistance is not continuously rated, it has to be cut out of the circuit fairly quickly although in the four running positions *A*, *B*,*C* and *D*, it may be left in circuit for any length of time.

Example 30.44. *Two series motors run at a speed of 500 r.p.m. and 550 r.p.m. respectively when taking 50 A at 500 V. The terminal resistance of each motor is 0.5 Ω. Calculate the speed of the combination when connected in series and coupled mechanically. The combination is taking 50 A on 500 V supply.* **(Electrical Machinery-I, Mysore Univ. 1985)**

Solution. First Motor

$$E_{b1} = 500 - (50 \times 0.5) = 475 \text{ V} \; ; \; I = 50 \text{ A}$$

Now, $N_1 \propto E_{b1}/\Phi_1$ or $E_{b1} \propto N_1 \Phi_1$ or $E_{b1} = k N_1 \Phi_1$

$\therefore \quad 475 = k \times 500 \times \Phi_1 \quad \therefore \quad k\Phi_1 = 475/500$

Second Motor $E_{b2} = 500 - (50 \times 0.5) = 475$ V. Similarly, $k\Phi_2 = 475/550$

When both motors are in series

$$E_b' = 500 - (50 \times 2 \times 0.5) = 450 \text{ V}$$

Now, $E_b' = E_{b1} + E_{b2} = k\,\Phi_1\,N + k\,\Phi_2\,N$

where N is the common speed when joined in series.

$\therefore \quad 450 = \frac{475}{500}N + \frac{475}{550}N \quad \therefore \quad N = \mathbf{248\ r.p.m.}$

Example 30.45. *Two similar 20 h.p. (14.92 kW), 250 V, 1000 r.p.m. series motors are connected in series with each other across a 250 V supply. The two motors drive the same shaft through a reduction gearing 5 : 1 and 4 : 1 respectively. If the total load torque on the shaft is 882 N-m, calculate (i) the current taken from the supply main (ii) the speed of the shaft and (iii) the voltage across each motor. Neglect all losses and assume the magnetic circuits to be unsaturated.*

(Elect. Machines, Punjab Univ., 1991)

Solution. (*i*) Rated current of each motor = 14,920/250 = 59.68 A

Back e.m.f. $E_b = 250$ V (neglecting $I_a R_a$ drop)

Now, $E_b \propto N\Phi$ As $\Phi \propto I \therefore E_b \propto NI$ or $E_b = kNI$

$250 = k \times (1000/60) \times 59.68 \quad \therefore \quad k = 0.25$

Let N_{sh} be the speed of the shaft.

Speed of the first motor $N_1 = 5\,N_{sh}$; Speed of the second motor $N_2 = 4\,N_{sh}$

Let I be the new current drawn by the series set, then

$E_b' = E_{b1} + E_{b2} = kI \times 5\,N_1 + kI \times N_2 = kI \times 5\,N_{sh} + kI \times 4\,N_{sh}$

$250 = 9 \times kI\,N_{sh}$ ...(*i*)

Now, torque $T = 0.159\frac{E_b\,I}{N} = 0.159 \times \frac{kI\,N_{sh} \times I}{N_{sh}} = 0.159\,k\,I^2$

Shaft torque due to gears of 1st motor = $5 \times 0.159\,kI^2$

Shaft torque due to gears of 2nd motor = $4 \times 0.159\,kI^2$

$\therefore \quad 882 = kI^2\,(5 \times 0.159 + 4 \times 0.159) = 1.431\,kI^2$

$\therefore \quad I^2 = 882/1.431 \times 0.25 = 2{,}449$ A $\quad \therefore \quad I = \mathbf{49.5\ A}$

(ii) From equation (*i*), we get

$250 = 9 \times 0.25 \times 49.5 \times N_{sh} \quad \therefore \quad N_{sh} = 2.233$ r.p.s. $= \mathbf{134\ r.p.m.}$

(iii) Voltage across the armature of Ist motor is

$E'_{b1} = 5\,kI\,N_{sh} = 5 \times 0.25 \times 49.5 \times 2.233 = \mathbf{139\ V}$

Voltage across the armature of 2nd motor

$E_{b2} = 4\,kI\,N_{sh} = 4 \times 0.25 \times 49.5 \times 2.233 = \mathbf{111\ V}$

Note that E_{b1} and E_{b2} are respectively equal to the applied voltage across each motor because $I_a R_a$ drops are negligible.

30.6. Electric Braking

A motor and its load may be brought to rest quickly by using either ***(i)*** Friction Braking or ***(ii)*** Electric Braking. The commonly-used mechanical brake has one drawback : it is difficult to achieve a smooth stop because it depends on the condition of the braking surface as well as on the skill of the operator.



Strong electric brake for airwheels

The excellent electric braking methods are available which eliminate the need of brake lining levers and other mechanical gadgets. Electric braking, both for shunt and series motors, is of the following three types: ***(i)*** rheostatic or dynamic braking

(*ii*) plugging *i.e.*, reversal of torque so that armature tends to rotate in the opposite direction and **(*iii*)** regenerative braking.

Obviously, friction brake is necessary for holding the motor even after it has been brought to rest.

30.7. Electric Braking of Shunt Motors

(*a*) Rheostatic or Dynamic Braking

In this method, the armature of the shunt motor is disconnected from the supply and is connected across a variable resistance R as shown in Fig. 30.21 (*b*). The field winding is, however, left connected across the supply undisturbed. The braking effect is controlled by varying the series resistance R. Obviously, this method makes use of generator action in a motor to bring it to rest.* As seen from Fig. 30.21 (*b*), armature current is given by

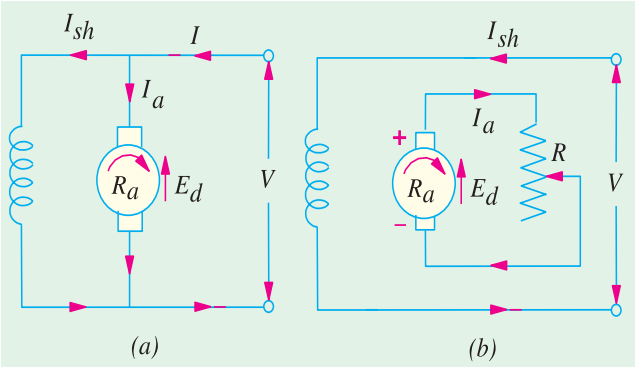


Fig. 30.21

$$I_a = \frac{E_b}{R + R_a} = \frac{\Phi ZN(P/A)}{R + R_a}$$

$$= \frac{k_1 \Phi N}{R + R_a}$$

Braking torque is given by

$$T_B = \frac{1}{2\pi} \Phi Z I_a \left(\frac{P}{A}\right) \text{N-m}$$

$$= \frac{1}{2\pi} \Phi Z \left(\frac{P}{A}\right) . \frac{\Phi ZN(P/A)}{R + R_a} = \frac{1}{2\pi} \left(\frac{ZP}{A}\right)^2 \frac{\Phi^2 N}{R + R_a} = k_2 \Phi^2 N \quad \therefore \; T_B \propto N$$

Obviously, T_B decreases as motor slows down and disappear altogether when it comes to a stop.

(*b*) Plugging or Reverse Current Braking

This method is commonly used in controlling elevators, rolling mills, printing presses and machine tools etc.

In this method, connections to the armature terminals are reversed so that motor tends to run in the opposite direction (Fig. 30.22). Due to the reversal of armature connections, applied voltage V and E_b start acting ***in the same direction*** around the circuit. In order to limit the armature current to a reasonable value, it is necessary to insert a resistor in the circuit while reversing armature connections.

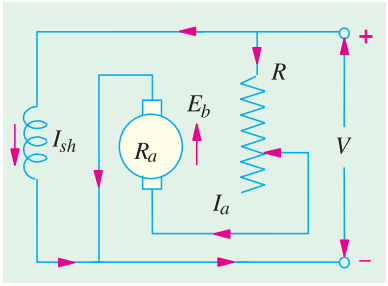


Fig. 30.22

$$I_a = \frac{V + E_b}{R + R_a} = \frac{V}{R + R_a} + \frac{E_b}{R + R_a}$$

$$= \frac{V}{R + R_a} + \frac{\Phi ZN(P/A)}{R + R_a} = \frac{V}{R + R_a} + \frac{k_1 \Phi Z}{R + R_a}$$

$$T_B = \frac{1}{2\pi} . \Phi Z I_a \left(\frac{P}{A}\right) = \frac{1}{2\pi} . \left(\frac{\Phi ZP}{A}\right) . I_a = \frac{1}{2\pi} \left(\frac{\Phi ZP}{A}\right) \left[\frac{V}{R + R_a} + \frac{\Phi ZN(P/A)}{R + R_a}\right]$$

* The motor while acting as a generator feeds current to the resistor dissipating heat at the rate of I^2R. The current I_a produced by dynamic braking flow in the opposite direction, thereby producing a counter torque that slows down the machine.

$$= \frac{1}{2\pi}\left(\frac{ZP}{A}\right)\left(\frac{V}{R+R_a}\right).\Phi + \frac{1}{2\pi}.\left(\frac{ZP}{A}\right)^2.\frac{\Phi^2 N}{R+R_a} = k_2\Phi + k_3\Phi^2 N$$

or $T_b = k_4 + k_5 N$, where $k_4 = \frac{1}{2\pi}\left(\frac{\Phi ZP}{A}\right)\left(\frac{V}{R+R_a}\right)$

and $k_5 = \frac{1}{2\pi}\left(\frac{\Phi ZP}{A}\right)^2 \times \frac{1}{(R+R_a)}$.

Plugging gives greater braking torque than rheostatic braking. Obviously, during plugging, power is drawn from the supply and is dissipated by R in the form of heat. It may be noted that even when motor is reaching zero speed, there is some braking torque $T_B = k_4$ (see Ex. 30.47).

(c) Regenerative Braking

This method is used when the load on the motor has overhauling characteristic as in the lowering of the cage of a hoist or the downgrade motion of an electric train. Regeneration takes place when E_b becomes grater than V. This happens when the overhauling load acts as a prime mover and so drives the machines as a generator. Consequently, direction of I_a and hence of armature torque is reversed and speed falls until E_b becomes lower than V. It is obvious that during the slowing down of the motor, power is returned to the line which may be used for supplying another train on an upgrade, thereby relieving the powerhouse of part of its load (Fig. 30.23).

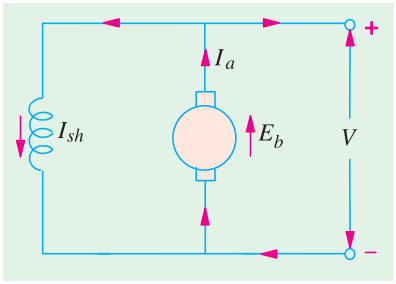


Fig. 30.23


Regenerative braking demonstrations

For protective purposes, it is necessary to have some type of mechanical brake in order to hold the load in the event of a power failure.

Example 30.46. *A 220 V compensated shunt motor drives a 700 N-m torque load when running at 1200 r.p.m. The combined armature compensating winding and interpole resistance is 0.008 Ω and shunt field resistance is 55 Ω. The motor efficiency is 90%. Calculate the value of the dynamic braking resistor that will be capable of 375 N-m torque at 1050 r.p.m. The windage and friction losses may be assumed to remain constant at both speeds.*

Solution. Motor output $= \omega T = 2\pi NT = 2\pi\,(1200/60) \times 700 = 87{,}965$ W

Power drawn by the motor $= 87{,}965/0.9 = 97{,}740$ W

Current drawn by the motor $= 97{,}740/220 = 444$ A.

$I_{sh} = 220/55 = 4$ A ; $I_{a1} = 444 - 4 = 440$ A

$E_{b1} = 220 - 440 \times 0.008 = 216.5$ V

Since field flux remains constant, T_1 is proportional to I_{a1} and T_2 to I_{a2}.

$$\frac{T_2}{T_1} = \frac{I_{a2}}{I_{a1}} \quad \text{or} \quad I_{a2} = 440 \times \frac{375}{100} = 2650 \text{ A}$$

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \quad \text{or} \quad \frac{1050}{1200} = \frac{E_{b2}}{216.5} \; ; \; E_{b2} = 189.4 \text{ V}$$

With reference to Fig. 30.23, we have

$189.4 = 2650\,(0.008 + R)$; $R = 0.794\ \Omega$

30.8. Electric Braking of Series Motor

The above-discussed three methods as applied to series motors are as follows :

(*a*) Rheostatic (or Dynamic) Braking

The motor is disconnected from the supply, the field connections ***are reversed*** and the motor is connected in series with a variable resistance R as shown in Fig. 30.24. Obviously, now, the machine is running as a generator. The field connections are reversed to make sure that current through field winding flows in the *same* direction as before (*i.e.*, from M to N) in order to assist residual magnetism. In practice, the variable resistance employed for starting purpose is itself used for braking purposes. As in the case of shunt motors,

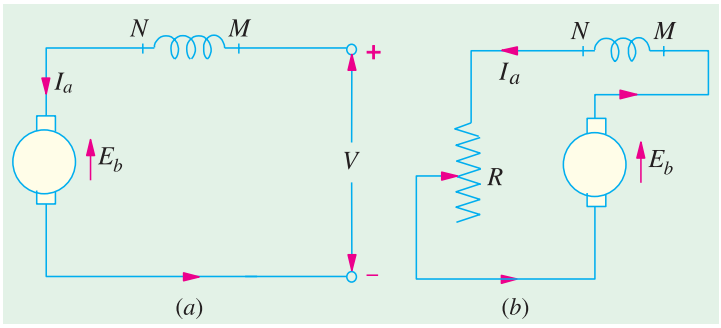


Fig. 30.24

$$T_B = k_2 \Phi^2 N = k_3 I_{a2} N \quad \text{... prior to saturation}$$

(*b*) Plugging or Reverse Current Braking

As in the case of shunt motors, in this case also the connections of the armature are reversed and a variable resistance R is put in series with the armature as shown in Fig. 30.25. As found in Art. 30.7 (*b*),

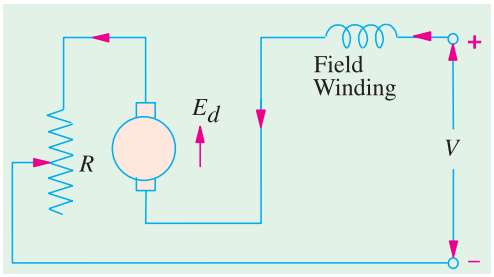


Fig. 30.25

$$T_B = k_2 \Phi + k_3 \Phi^2 N$$

(*c*) Regenerative Braking

This type of braking of a sereis motor is not possible without modification because reversal of I_a would also mean reversal of the field and hence of E_b. However, this method is sometimes used with traction motors, special arrangements being necessary for the purpose.

Example 30.47. *A 400 V, 25 h.p. (18.65 kW), 45 r.p.m., d.c. shunt motor is braked by plugging when running on full load. Determine the braking resistance necessary if the maximum braking current is not to exceed twice the full-load current. Determine also the maximum braking torque and the braking torque when the motor is just reaching zero speed. The efficiency of the motor is 74.6% and the armature resistance is 0.2* Ω. **(Electrical Technology, M.S. Univ. Baroda, 1988)**

Solution.

F.L. Motor input current $I = 18{,}650/0.746 \times 400 = 62.5$ A

$I_a = 62.5$ A (neglecting I_{sh}); $E_b = 400 - 62.5 \times 0.2 = 387.5$ V

Total voltage around the circuit is $= 400 + 387.5 = 787.5$ V

Max. braking current $= 2 \times 62.5 = 125$ A

Total resistance required in the circuit $= 787.5/125 = 6.3\ \Omega$

Braking resistance $R = 6.3 - 0.2 =$ **6.1 Ω**

Maximum braking torque will be produced initially when the motor speed is maximum *i.e.*, 450 r.p.m. or 7.5 r.p.s.

Maximum value of $T_B = k_4 + k_5 N$ – Art. 25.7(*b*)

Now, $k_4 = \frac{1}{2\pi}\left(\frac{\Phi ZP}{A}\right)\left(\frac{V}{R + R_a}\right)$ and $k_5 = \frac{1}{2\pi}\left(\frac{\Phi ZP}{A}\right)^2 \cdot \frac{1}{(R + R_a)}$

Now, $E_b = \Phi ZN\,(P/A)$; also $N = 450/60 = 7.5$ r.p.s.

$\therefore \quad 387.5 = 7.5\,(\Phi ZP/A)$ or $(\Phi ZP/A) = 51.66$

$\therefore \quad k_4 = \frac{1}{2\pi} \times 51.66 \times \frac{400}{6.3} = 522$ and $k_5 = \frac{1}{2\pi} \times (51.66)^2 \times \frac{1}{6.3} = 67.4$

$\therefore \quad$ Maximum $T_B = 522 + 67.4 \times 7.5 =$ **1028 N-m**

When speed is also zero *i.e.*, $N = 0$, the value of torque is $T_B = K_4 =$ **522 N-m.**

30.9. Electronic Speed Control Method for DC Motors

Of late, solid-state circuits using semiconductor diodes and thyristors have become very popular for controlling the speed of a.c. and d.c. motors and are progressively replacing the traditional electric power control circuits based on thyratrons, ignitrons, mercury arc rectifiers, magnetic amplifiers and motor-generator sets etc. As compared to the electric and electromechanical systems of speed control, the electronic methods have higher accuracy, greater reliability, quick response and also higher efficiency as there are no I^2R losses and moving parts. Moreover, full 4-quadrant speed control is possible to meet precise high-speed standards.

All electronic circuits control the motor speed by adjusting either ***(i)*** the voltage applied to the motor armature or ***(ii)*** the field current or ***(iii)*** both.

DC motors can be run from d.c. supply if available or from a.c. supply after it has been converted into d.c. supply with the help of rectifiers which can be either half-wave or full-wave and either controlled (by varying the conduction angle of the thyristors used) or uncontrolled.

AC motors can be run on the a.c. supply or from d.c. supply after it has been converted into a.c. supply with the help of inverters (opposite of rectifiers).

As stated above, the average output voltage of a thyristor-controlled rectifier can be changed by changing its conduction angle and hence the armature voltage of the d.c. motor can be adjusted to control its speed.

When run on a d.c. supply, the armature d.c. voltage can be changed with the help of a thyristor chopper circuit which can be made to interrupt d.c. supply at different rates to give different average values of the d.c. voltage. If d.c. supply is not available, it can be obtained from the available a.c. supply with the help of uncontrolled rectifiers (using only diodes and not thyristors). The d.c. voltages so obtained can be then chopped with the help of a thyristor chopper circuit.

A brief description of rectifiers, inverters* and d.c. choppers would now be given before discussing the motor speed control circuits.

30.10. Uncontrolled Rectifiers

As stated earlier, rectifiers are used for a.c. to d.c. conversion *i.e.*, when the supply is alternating but the motor to be controlled is a d.c. machine.

Fig. 30.26 (*a*) shows a half-wave uncontrolled rectifier. The diode *D* conducts only during positive half-cycles of the single-phase a.c. input *i.e.*, when its anode *A* is positive with respect to its cathode *K*. As shown, the average voltage available across the load (or motor) is 0.45 V where *V* is the r.m.s. value of the a.c. voltage (in fact, $V = V_m/\sqrt{2}$). As seen it is a pulsating d.c. voltage.

In Fig. 30.26 (*b*) a single-phase, full-wave bridge rectifier which uses four semiconductor diodes and provides double the voltage *i.e.*, 0.9 V is shown. During positive input half-cycles when end *A* is positive with respect to end *B*, diodes D_1 and D_4 conduct (*i.e.* opposite diodes) whereas during negative input half-cycles, D_2 and D_3 conduct. Hence, current flows through the load during both half-cycles in the *same* direction. As seen, the d.c. voltage supplied by a bridge rectifier is much less pulsating than the one supplied by the half-wave rectifier.

* Rectifiers convert a.c. power into d.c power, whereas inverters convert d.c. power into a.c. power. However, converter is a general term embracing both rectifiers and inverters.

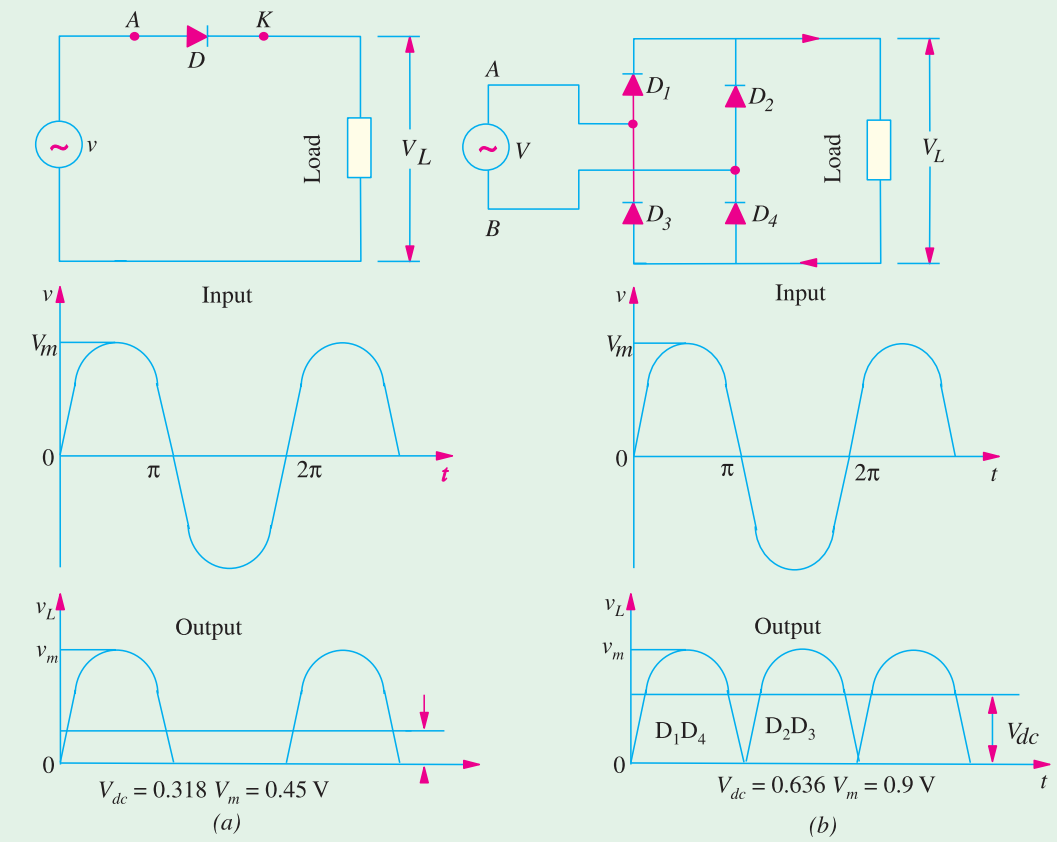


Fig. 30.26

30.11. Controlled Rectifiers

In these rectifiers, output load current (or voltage) can be varied by controlling the point in the input a.c. cycle at which the thyristor is turned ON with the application of a suitable low-power gate pulse. Once triggered (or fired) into conduction, the thyristor remains in the conducting state for the rest of the half-cycle *i.e.*, upto 180°. The firing angle α can be adjusted with the help of a control circuit. When conducting, it offers no resistance *i.e.*, it acts like a short-circuit.

Fig. 30.27 (*a*) shows an elementary half-wave rectifier in which thyristor triggering is delayed by angle α with the help of a phase-control circuit. As shown, the thyristor starts conducting at point *A* and not at point *O* because its gate pulse is applied after a delay of α. Obviously, the conduction angle is reduced from 180° to (180° – α) with a consequent decrease in output voltage whose value is given by

$$V_L = \frac{V_m}{2\pi}(1 + \cos\alpha) = 0.16\, V_m (1 + \cos\alpha) = 0.32\, V_m \cos^2\frac{\alpha}{2}$$

where V_m is the peak value of a.c. input voltage. Obviously, V_L is maximum when $\alpha = 0$ and is zero when $\alpha = 180°$.

Fig. 30.27 (*b*) shows the arrangement where a thyristor is used to control current through a load connected in series with the a.c. supply line.

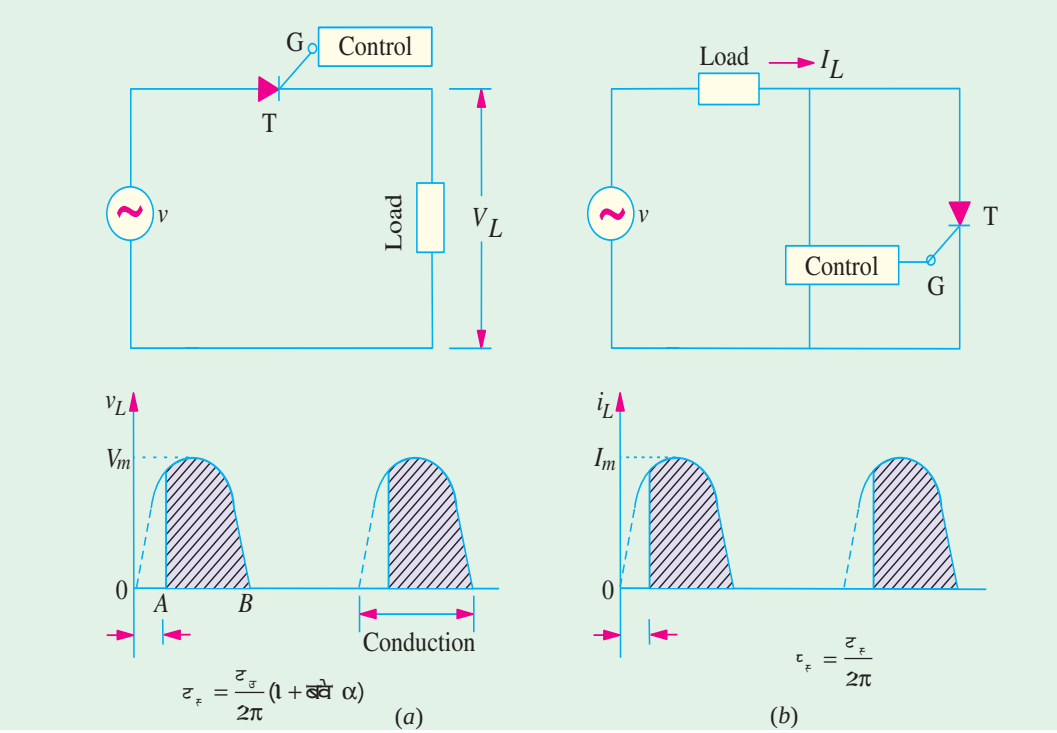


Fig. 30.27

The load current is given by

$$I_L = \frac{V_L}{R_L} = \frac{V_m}{2\pi R_L}(1 + \cos \alpha) = \frac{V_m}{\pi R_L} \cos^2 \frac{\alpha}{2}$$

Fig. 30.28 (*a*) shows a single-phase, full-wave half-controlled rectifier. It is called half-controlled because it uses two thyristors and two diodes instead of four thyristors. During positive input half-cycle when *A* is positive, conduction takes place via T_1, load and D_1. During the negative half-cycle when *B* becomes positive, conduction route is via T_2, load and D_2.

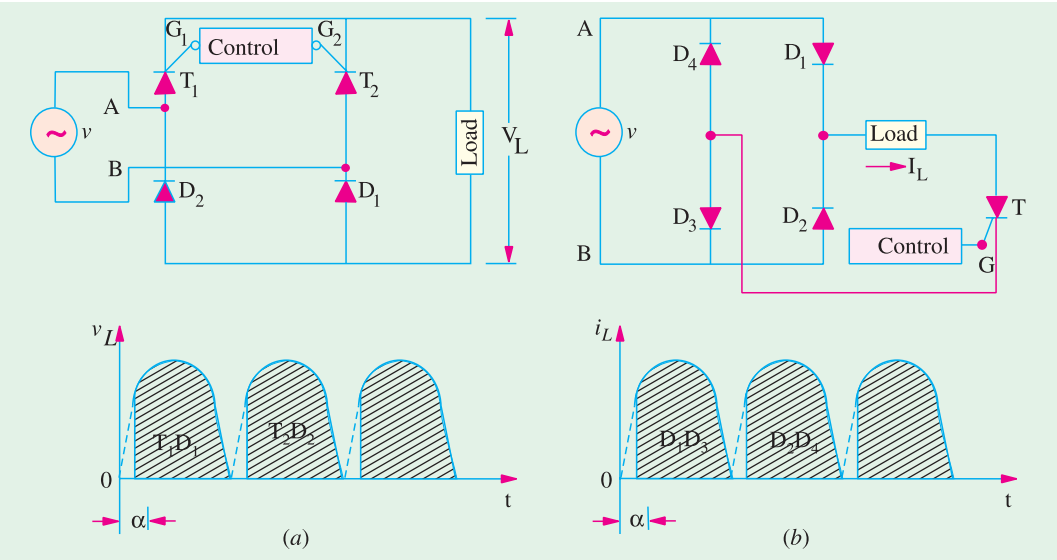


Fig. 30.28

The average output voltage V_L or V_{dc} is given by $V_L = 2 \times$ half-wave rectifier output

$$\therefore \quad V_L = 2 \times \frac{\text{ट उ}}{2\pi} (\text{ा} + \text{बवे } \alpha) = \frac{\text{ट उ}}{\pi} (\text{ा} + \text{बवे } \alpha)' = \frac{2\text{ट उ}}{\pi} \text{बवे}^2 \frac{\alpha}{2}$$

Similarly, Fig. 30.28 (*b*) shows a 4-diode bridge rectifier controlled by a single thyristor. The average load current through the series load is given by

$$I_L = \frac{V_m}{\pi R_L} (1 + \cos \alpha) = \frac{2V_m}{\pi R_L} \cos^2 \frac{\alpha}{2}$$

As seen from the figure, when *A* is positive, D_1 and D_3 conduct provided *T* has been fired. In the negative half-cycle, D_2 and D_4 conduct via the load.

30.12. Thyristor Choppers

Since thyristors can be switched ON and OFF very rapidly, they are used to interrupt a d.c. supply at a regular frequency in order to produce a lower (mean) d.c. voltage supply. In simple words, they can produce low-level d.c. voltage from a high-voltage d.c. supply as shown in Fig. 30.29.

The mean value of the output voltage is given by

$$V_{dc} = V_L = V \frac{T_{ON}}{T_{ON} + T_{OFF}} = V \frac{T_{ON}}{T}$$

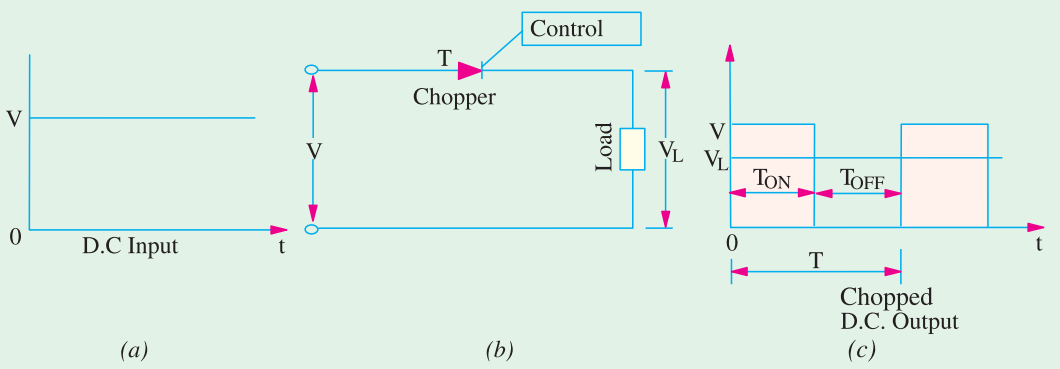


Fig. 30.29

Fig. 30.30 (*a*) shows a simple thyristor chopper circuit alongwith extra commutating circuitry for switching T_1 OFF. As seen, T_1 is used for d.c. chopping, whereas *R*, T_2 and *C* are used for commutation purposes as explained below.

When T_1 is fired into conduction by its control circuit (not shown), current is set up through the load and commutation capacitor *C* gets charged via *R* with the polarity shown in the figure during this ON period.

For switching T_1 OFF, second thyristor T_2 is triggered into conduction allowing *C* to discharge through it (since it acts as a short-circuit while conducting) which reverse-biases T_1 thus turning it OFF. The discharge from *C* leaves T_2 with reverse polarity so that it is turned OFF, whereas T_1 is triggered into conduction again.

Depending upon the frequency of switching ON and OFF, the input d.c. voltage is cut into d.c. pulses as shown in Fig. 30.30 (*c*).

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* For a fully-controlled bridge rectifier, its value is

$$V_L = \frac{2V_m}{\pi} \cos \alpha \, .$$

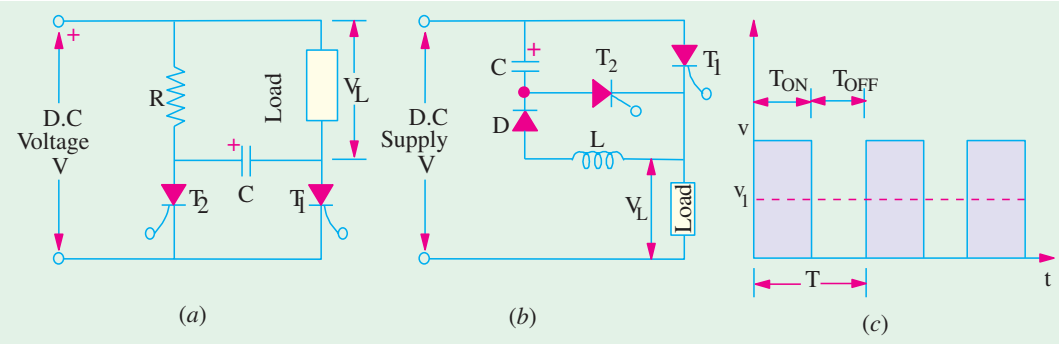


Fig. 30.30

In Fig. 30.30 (*b*), T_1 is the chopping thyristor, whereas *C*, *D*, T_2 and *L* constitute the commutation circuitry for switching T_1 OFF and ON at regular intervals.

When T_2 is fired, *C* becomes charged via the load with the polarity as shown. Next, when T_1 is fired, *C* reverse-biases T_2 to OFF by discharging via T_1, *L* and *D* and then recharges with reverse polarity. T_2 is again fired and the charge on *C* reverse-biases T_1 to non-conducting state.

It is seen that output (or load) voltage is present only when T_1 is ON and is absent during the interval it is OFF. The mean value of output d.c. voltage depends on the relative values of T_{ON} and T_{OFF}. In fact, output d.c. voltage is given by

$$V_{dc} = V_L \frac{T_{ON}}{T_{ON} + T_{OFF}} = V \frac{T_{ON}}{T}$$

Obviously, by varying thyristor ON/OFF ratio, V_L can be made any percentage of the input d.c. voltage *V*.

Example 30.48. *The speed of a separately excited d.c. motor is controlled by a chopper. The supply voltage is 120 V, armature circuit resistance = 0.5 ohm, armature circuit inductance = 20 mH and motor constant = 0.05 V/r.p.m. The motor drives a constant load torque requiring an average current of 20 A. Assume motor current is continuous. Calculate* ***(a)*** *the range of speed control* ***(b)*** *the range of duty cycle.* **(Power Electronics-I, Punjab Univ. Nov. 1990)**

Solution. The minimum speed is zero when $E_b = 0$

$$V_t = E_b + I_a R_a = I_a \times R_a = 200 \times 0.5 = 10 \text{ V}$$

Now, $V_t = \frac{\bar{v}_{s\,g}}{\bar{v}} \tau = \alpha \tau \Omega \cdot 10 = 120 \Omega \alpha = \frac{1}{12}$

Maximum speed corresponds to $\alpha = 1$ when $V_t = V = 120$ V

$\therefore$ $E_b = 120 - 20 \times 0.5 = 110$ V

Now, $N = E_b / K_a \Phi = 110/0.05 = 2200$ r.p.m.

(a) Hence, speed range is from 0 to 2200 r.p.m.

(b) Range of duty cycle is from $\frac{1}{12}$ to 1.

30.13. Thyristor Inverters

Such inverters provide a very efficient and economical way of converting direct current (or voltage) into alternating current (or voltage). In this application, a thyristor serves as a controlled switch alternately opening and closing a d.c. circuit. Fig. 30.31 (*a*), shows a basic inverter circuit where an a.c. output is obtained by alternately opening and closing switches S_1 and S_2. When we

replace the mechanical switches by two thyristors (with their gate triggering circuits), we get the thyristor inverter in Fig. 30.31 (*b*).

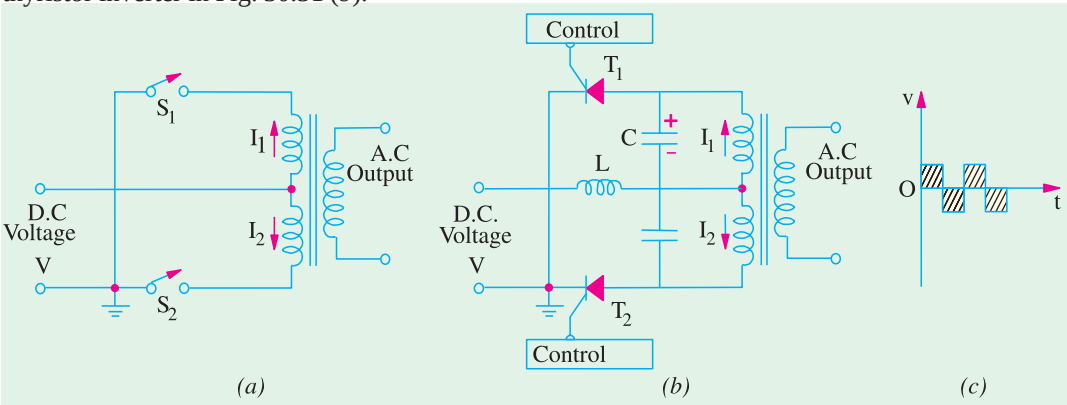


Fig. 30.31

Before discussing the actual circuit, it is worthwhile to recall that thyristor is a latching device which means that once it starts conducting, gate loses control over it and cannot switch it OFF whatever the gate signal. A separate ***commutating*** circuitry is used to switch the thyristor OFF and thus enable it to perform ON-OFF switching function.



Second generation thyristor inverter

Suppose T_1 is fired while T_2 is still OFF. Immediately I_1 is set up which flows through L, one half of transformer primary and T_1. At the same time, C is charged with the polarity as shown.

Next when T_2 is fired into condition, I_2 is set up and C starts discharging through T_1 thereby reverse-biasing it to CUT-OFF.

When T_1 is again pulsed into condition, I_1 is set up and C starts discharging thereby reverse-biasing T_2 to *OFF* and the process just described repeats. As shown in Fig. 30.31 (*c*), the output is an alternating voltage whose frequency depends on the switching frequency to thyristors T_1 and T_2.

30.14. Thyristor Speed Control of Separately-excited D.C. Motor

In Fig. 30.32, the bridge rectifier converts a voltage into d.c. voltage which is then applied to the armature of the separately-excited d.c. motor *M*.

As we know, speed of a motor is given by

$$N = \frac{V - I_a R_a}{\Phi}\left(\frac{A}{ZP}\right)$$

If Φ is kept constant and also if $I_a R_a$ is neglected, then, $N \propto V \propto$ voltage across the armature. The value of this voltage furnished by the rectifier can be changed by varying the firing angle α of the thyristor T with the help of its contol circuit. As α is increased *i.e.*, thyristor firing is delayed

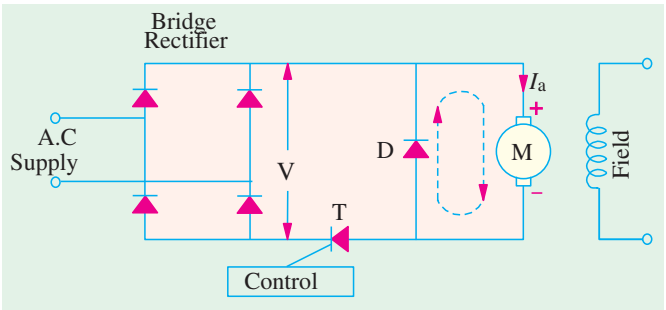


Fig. 30.32

more, its conduction period is reduced and, hence, armature voltage is decreased which, in turn, decreases the motor speed. When α is decreased *i.e.*, thyristor is fired earlier, conduction period is increased which increases the mean value of the voltage applied across the motor armature. Consequently, motor speed is increased. In short, as α increases, V decreases and hence N decreases. Conversely, as α decreases, V increases and so, N increases. The free-wheeling diode D connected across the motor provides a circulating current path (shown dotted) for the energy stored in the inductance of the armature winding at the time T turns OFF. Without D, current will flow through T and bridge rectifier, prohibiting T from turning OFF.

30.15. Thyristor Speed Control of a D.C. Series Motor

In the speed control circuit of Fig. 30.33, an RC network is used to control the diac voltage that triggers the gate of a thyristor. As the a.c. supply is switched ON, thyristor T remains OFF but the capacitor C is charged through motor armature and R towards the peak value of the applied a.c. voltage. The time it takes for the capacitor voltage V_C to reach the breakover voltage of the diac* depends on the setting of the variable resistor T. When V_C becomes equal to the breakover voltage of diac, it conducts and a triggering pulse is applied to the thyristor gate G. Hence, T is turned ON and allows current to pass through the motor. Increasing R delays the rise of V_C and hence the breakover of diac so that thyristor is fired later in each positive half cycle of the a.c. supply. It reduces the conduction angle of the thyristor which, consequently, delivers less power to the motor. Hence, motor speed is reduced.

If R is reduced, time-constant of the RC network is decreased which allows V_C to rise to the breakover voltage of diac more quickly. Hence, it makes the thyristor fire early in each positive input half-cycle of the supply. Due to increase in the conduction angle of the thyristor, power delivered to the motor is increased with a subsequent increase in its speed. As before D is the free-wheeling diode which provides circulating current path for the energy stored in the inductance of the armature winding.

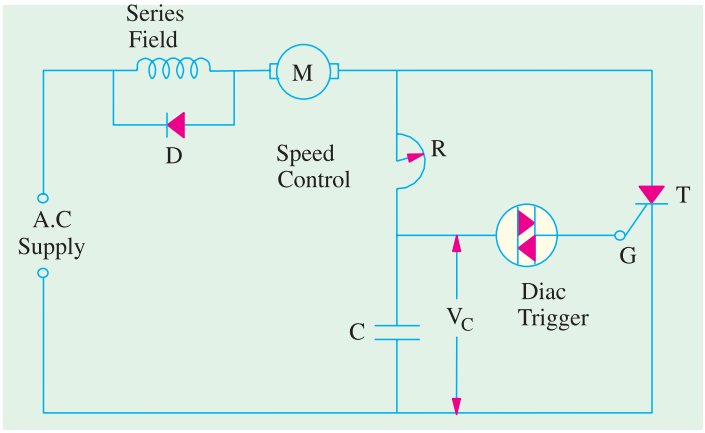


Fig. 30.33

30.16. Full-wave Speed Control of a Shunt Motor

Fig. 30.34 shows a circuit which provides a wide range of speed control for a fractional kW shunt d.c. motor. The circuit uses a bridge circuit for full-wave rectification of the a.c. supply. The shunt field winding is permanently connected across the d.c. output of the bridge circuit. The armature voltage is supplied through thyristor T. The magnitude of this

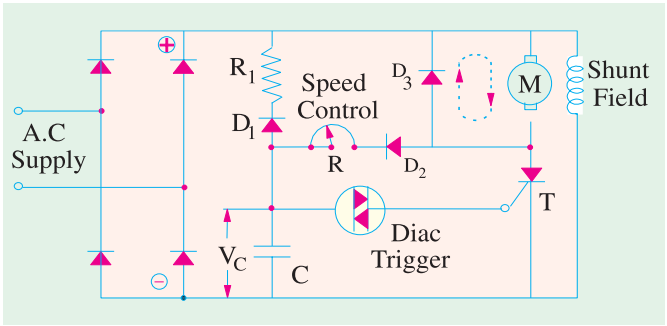


Fig. 30.34

* Like a triac, it has directional switching characteristics.

voltage (and hence, the motor speed) can be changed by turning T_{ON} at different points in each half-cycle with the help of R. The thyristor turns OFF only at the end of each half-cycle. Free-wheeling diode D_3 provides a circulating current path (shows dotted) for the energy stored in the armature winding at the time T turns OFF. Without D_3. This current would circulate through T and the bridge rectifier thereby prohibiting T from turning OFF.

At the beginning of each half-cycle, T is the OFF state and C starts charging up via motor armature, diode D_2 and speed-control variable resistor R (it cannot charge through R_1 because of reverse-biased diode D_1). When voltage across C *i.e.*, V_C builds up to the breakover voltage of diac, diac conducts and applies a sudden pulse to T thereby turning it ON. Hence, power is supplied to the motor armature for the remainder of that half-cycle. At the end of each half-cycle, C is discharged through D_1, R_1 and shunt field winding. The delay angle α depends on the time it takes V_C to become equal to the breakover voltage of the diac. This time, in turn, depends on the time-constant of the R-C circuit and the voltage available at point A. By changing R, V_C can be made to build-up either slowly or quickly and thus change the angle α at will. In this way, the average value of the d.c. voltage across the motor armature can be controlled. It further helps to control the motor speed because it is directly proportional to the armature voltage.

Now, when load is increased, motor tends to slow down. Hence, E_b is reduced. The voltage of point A is increased because it is equal to the d.c. output voltage of the bridge rectifier ***minus*** back e.m.f. E_b. Since V_A increases *i.e.*, voltage across the R-C charging circuit increases, it builds up V_C more quickly thereby decreasing which leads to early switching ON of T in each half-cycle. As a result, power supplied to the armature is increased which increases motor speed thereby compensating for the motor loading.

30.17. Thyristor Speed Control of a Shunt Motor

The speed of a shunt d.c.motor (upto 5 kW) may be regulated over a wide range with the help of the full-wave rectifier using only one main thyristor (or *SCR*) T as shown in Fig. 30.35. The firing angle α of T is adjusted by R_1 thereby controlling the motor speed. The thyristor and *SUS* (silicon unilateral switch) are reset (*i.e.*, stop conduction) when each half-wave of voltage drops to zero. Before switching on the supply, R_1 is increased by turning it in the counter-clockwise direction. Next, when supply is switched ON, C gets charged via motor armature and diode D_1 (being forward biased). It means that it takes much longer for V_C to reach the breakdown voltage of *SUS** due to large time constant of R_1– C network. Once V_C reaches that value, *SUS* conducts suddenly and triggers T into conduction. Since thyristor starts conducting late (*i.e.*, its α is large), it furnishes low voltage to start the motor. As speed selector R_1 is turned clockwise (for less resistance), C charges up more rapidly (since time constant is decreased) to the breakover voltage of *SUS* thereby firing T into conduction earlier. Hence, average value of the d.c. voltage across the motor armature increases thereby increasing its speed.

While the motor is running at the speed set by R_1, suppose that load on the motor is increased. In that case, motor will tend to slow down thereby decreasing armature back e.m.f. Hence, potential of point 3 will rise which will charge C faster to the breakover voltage of *SUS*. Hence, thyristor will be fired earlier thereby applying greater armature voltage which will return the motor speed to its desired value. As seen, the speed is automatically regulated to offset changes in load.

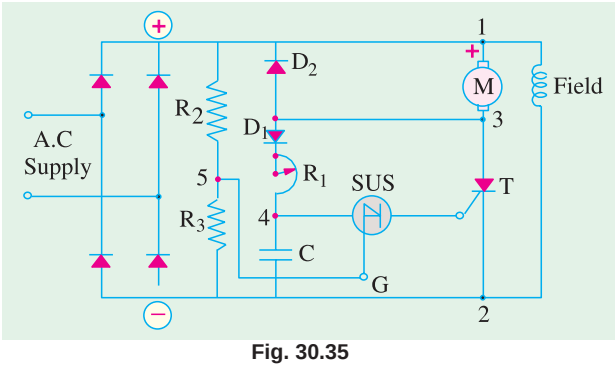


Fig. 30.35

The function of free-wheeling diode D_2 is to allow dissipation of energy stored in motor

* It is a four-layer semiconductor diode with a gate terminal. Unlike diac, it conducts in one direction only.

armature during the time the full-wave rectified voltage drops to zero between half-cycles. If D_2 is not there, then decreasing armature current during those intervals would be forced to flow through T thereby preventing its being reset. In that case, T would not be ready to be fired in the next half-cycle.

Similarly, towards the end of each half cycle as points 1 and 5 decrease towards zero potential, the negative going gate G turns SUS on thereby allowing C to discharge completely through SUS and thyristor gate-cathode circuit so that it can get ready to be charged again in the next half-cycle.

30.18. Thyristor Speed Control of a Series D.C. Motor

Fig. 30.36 shows a simple circuit for regulating the speed of a d.c. motor by changing the average value of the voltage applied across the motor armature by changing the thyristor firing angle α. The trigger circuit $R_1 - R_2$ can give a firing range of almost 180°. As the supply is switched on, full d.c. voltage is applied across $R_1 - R_2$. By changing the variable resistance R_2, drop across it can be made large enough to fire the SCR at any desired angle from 0° – 180°. In this way, output voltage of the bridge rectifier can be changed considerably, thus enabling a wide-range control of the motor speed. The speed control can be made somewhat smoother by joining a capacitor C across R_2 as shown in the figure.

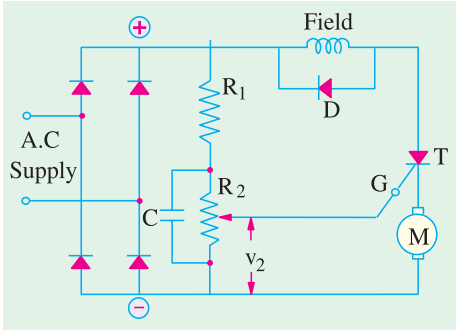


Fig. 30.36

30.19. Necessity of a Starter

It has been shown in Art 29.3 that the current drawn by a motor armature is given by the relation

$$I_a = (V - E_b)/R_a$$

where V is the supply voltage, E_b the back e.m.f. and R_a the armature resistance.

When the motor is at rest, there is, as yet, obviously no back e.m.f. developed in the armature. If, now, full supply voltage is applied across the stationary armature, it will draw a very large current because armature resitance is relatively small. Consider the case of a 440-V, 5 H.P. (3.73 kW) motor having a cold armature resistance of 0.25 Ω and a full-load current of 50 A. If this motor is started from the line directly, it will draw a starting current of 440/0.25 = 1760 A which is 1760/50 = 35.2 times its full-load current. This excessive current will blow out the fuses and, prior to that, it will damage the commutator and brushes etc. To avoid this happening, a resistance is introduced in series with the armature (for the duration of starting period only, say 5 to 10 seconds) which limits the starting current to a safe value. The starting resistance is gradually cut out as the motor gains speed and develops the back e.m.f. which then regulates its speed.

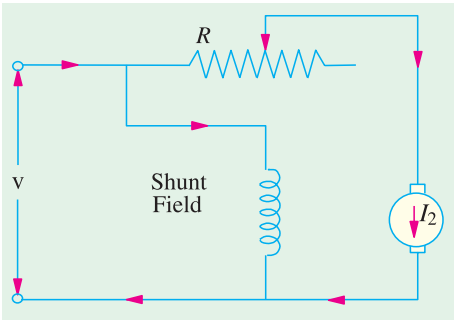


Fig. 30.37

Very small motors may, however, be started from rest by connecting them directly to the supply lines. It does not result in any harm to the motor for the following reasons :

1. Such motors have a relatively higher armature resistance than large motors, hence their starting current is not so high.
2. Being small, they have low moment of inertia, hence they speed up quickly.
3. The momentary large starting current taken by them is not sufficient to produce a large disturbance in the voltage regulation of the supply lines.

In Fig. 30.37 the resistance R used for starting a shunt motor is shown. It will be seen that the starting resistance R is in series with the ***armature*** and not with the ***motor*** as a whole. The field winding is connected directly across the lines, hence shunt field current is independent of the

resistance R. If R wes introduced in the motor circuit, then I_{sh} will be small at the start, hence starting torque T_{st} would be small ($\because\ T_a \propto \Phi I_a$) and there would be experienced some difficulty in starting the motor. Such a simple starter is shown diagramatically in Fig. 30.38.

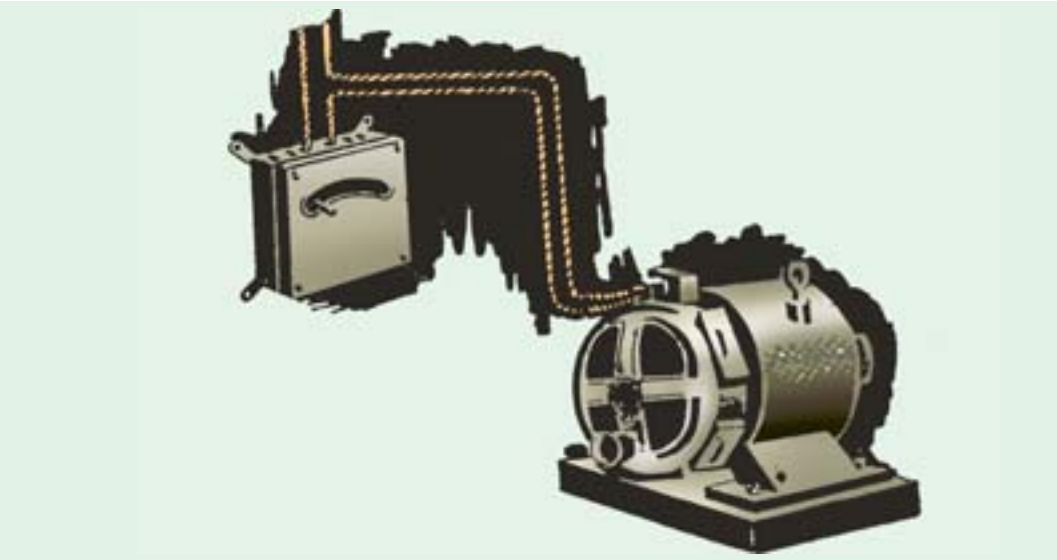


Fig. 30.38

30.20. Shunt Motor Starter

The face-plate box type starters used for starting shunt and compound motors of ordinary industrial capacity are of two kinds known as three-point and four-point starters respectively.

30.21. Three-point Starter

The internal wiring for such a starter is shown in Fig. 30.39 and it is seen that basically the connections are the same as in Fig. 30.37 except for the additional protective devices used here. The three terminals of the starting box are marked A, B and C. One line is directly connected to one armature terminal and one field terminal which are tied together. The other line is connected to point A which is further connected to the starting arm L, through the overcurrent (or overload) release M.

To start the motor, the main switch is first closed and then the starting arm is slowly moved to the right. As soon as the arm makes contact with stud No. 1, the field circuit is directly connected across the line and at the same time full starting resistance R, is placed in series with the armature. The starting current drawn by the armature = $V/(R_a + R_s)$ where R_s is the starting

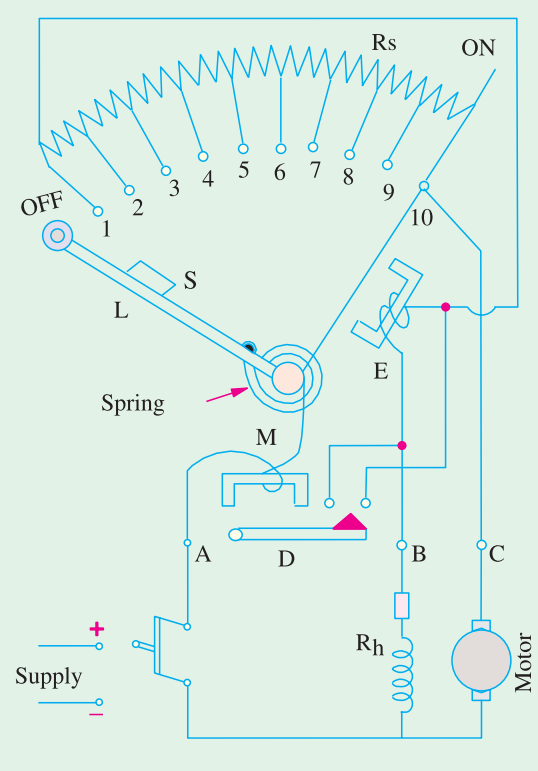


Fig. 30.39

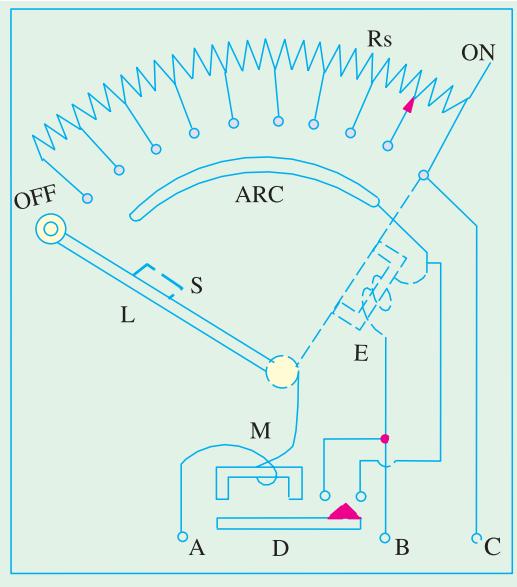


Fig. 30.40

resistance. As the arm is further moved, the starting resistance is gradually cut out till, when the arm reaches the running position, the resistance is all cut out. The arm moves over the various studs against a strong spring which tends to restore it to OFF position. There is a soft iron piece *S* attached to the arm which in the full 'ON' or running position is attracted and held by an electromagnet *E* energised by the shunt current. It is variously known as 'HOLD-ON' coil, LOW VOLTAGE (or NO-VOLTAGE) release.

It will be seen that as the arm is moved from stud NO. 1 to the last stud, the field current has to travel back through that portion of the starting resistance that has been cut out of the armature circuit. This results is slight decrease of shunt current. But as the value of starting resistance is very small as compared to shunt field resistance, this slight decreases in I_{sh} is negligible. This defect can, however, be remedied by using a brass arc which is connected to stud No. 1 (Fig. 30.40). The field circuit is completed through the starting resistance as it did in Fig. 30.39.

Now, we will discuss the action of the two protective devices shown in Fig. 30.39. The normal function of the HOLD-ON coil is to hold on the arm in the full running position when the motor is in normal operation. But, in the case of failure or disconnection of the supply or a break in the field circuit, it is de-energised, thereby releasing the arm which is pulled back by the spring to the OFF position. This prevents the stationary armature from being put across the lines again when the supply is restored after temporary shunt down. This would have happened if the arm were left in the full ON position. One great advantage of connecting the HOLD-ON coil in series with the shunt field is that, should the field circuit become open, the starting arm immediately springs back to the OFF position thereby preventing the motor from running away.

The overcurrent release consists of an electromagnet connected in the supply line. If the motor becomes overloaded beyond a certain predetermined value, then *D* is lifted and short circuits the electromagnet. Hence, the arm is released and returns to OFF position.

The form of overload protection described above is becoming obsolete, because it cannot be made either as accurate or as reliable as a separate well-designed circuit breaker with a suitable time element attachment. Many a times a separated magnetic contactor with an overload relay is also used.

Often the motors are protected by thermal overload relays in which a bimetallic strip is heated by the motor current at approximately the same rate at which the motor is itself heating up. Above a certain temperature, this relay trips and opens the line contactor, thereby isolating the motor from the supply.

If it is desired to control the speed of the motor in addition, then a field rheostat is connected in the filed circuit as indicated in Fig.30.39.The motor speed can be increased by weakening the flux ($\because N \propto I/\Phi$). Obviously, there is a limit to the speed increase obtained in this way, although speed ranges of three to four are possible. The connections of a starter and speed regulator with the motor are shown diagrammatically in Fig. 30.41. But there is one difficulty with such an arrangement for speed control. If too much resistance is 'cut in' by the field rheostat, then field current is reduced very much so that it is unable to create enough electromagnetic pull to overcome the spring tension. Hence,

the arm is pulled back to OFF position. It is this undesirable feature of a three-point starter which makes it unsuitable for use with variable-speed motors. This has resulted in widespread application of four-point starter discussed below.

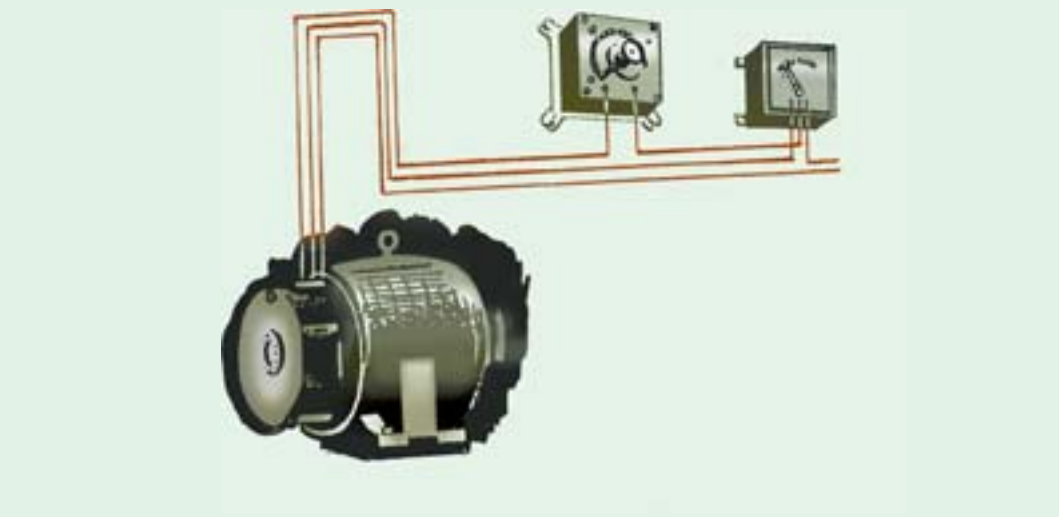


Fig. 30.41

30.22. Four-point Starter

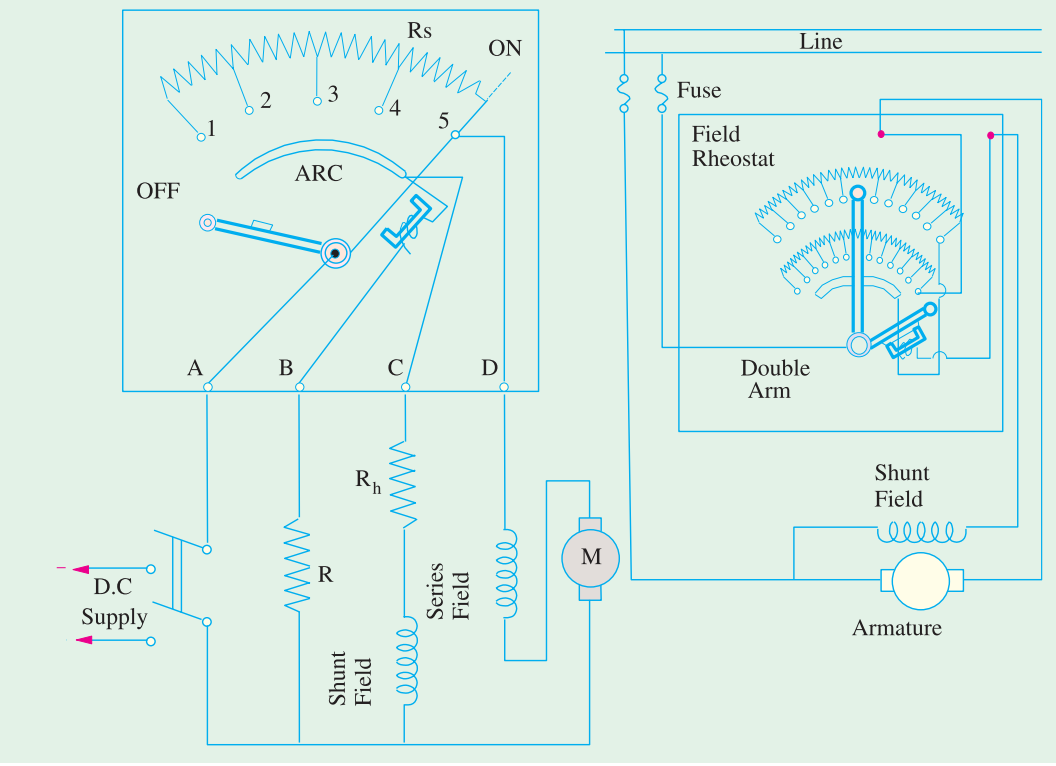


Fig. 30.42 Fig. 30.43

Such a starter with its internal wiring is shown, connected to a long-shunt compound motor in Fig. 30.42. When compared to the three-point starter, it will be noticed that one important change has been made *i.e.*, the HOLD-ON coil has been taken out of the shunt field circuit and has been connected directly across the line through a protecting resistance as shown. When the arm touches stud No. 1, then the line current divides into three parts (*i*) one part passes through starting resistance R_s, series field and motor armature (*ii*) the second part passes through the shunt field and its field rheostat R_h and (*iii*) the third part passes through the HOLD-ON coil and current-protecting resistance R. It should be particularly noted that with this arrangement any change of current in the shunt field circuit does not at all affect the current passing through the HOLD-ON coil because the two circuits are independent of each other. It means that the electromagnetic pull exerted by the HOLD-ON coil will always be sufficient and will prevent the spring from restoring the starting arm to OFF position no matter how the field rheostat or regulator is adjusted.

30.23. Starter and Speed-control Rheostats

Sometimes, for convenience, the field rheostat is also contained within the starting box as shown in Fig. 30.43. In this case, two arms are used. There are two rows of studs, the lower ones being connected to the armature. The inside starting arm moves over the lower studs on the starting resistor, whereas the outside field lever moves over the upper ones on the field rheostat. Only the outside field arm is provided with an operating handle. While starting the motor, the two arms are moved together, but field lever is electrically inoperative because the field current flows directly from the starting arm through the brass arc to HOLD-ON coil and finally to the shunt field winding. At the end of the starting period, the starting arm is attracted and held in FULL-ON position by the HOLD-ON coil, and the contact between the starting arm and brass arc is broken thus forcing field current to pass through the field rheostat. The field lever can be moved back to increase the motor speed. It will be seen that now the upper row of contacts is operative because starting arm no longer touches the brass arc.



Speed-control Rheostats

When motor is stopped by opening the main switch, the starting arm is released and on its way back it strikes the field lever so that both arms are returned simultaneously to OFF position.

30.24. Starting and Speed Control of Series Motor

For starting and speed control of series motor either a face-plate type or drum-type controller is used which usually has the reversing feature also. A face-plate type of reversing controller is shown in Fig. 30.44.

Except for a separate overload circuit, no inter-locking or automatic features are required because the operator watches the performance continuously.

As shown, the regulating lever consists of three pieces separated by strips of insulation. The outside parts form the electrical connections and the middle one is insulated from them. By moving the regulating lever, resistance can be cut in and out of the motor circuit. Reversing is obtained by moving the lever in the opposite direction as shown, because in that case, connections to the armature are reversed. Such an arrangement is employed where series motors are used as in the case of cranes, hoists and streetcars etc.

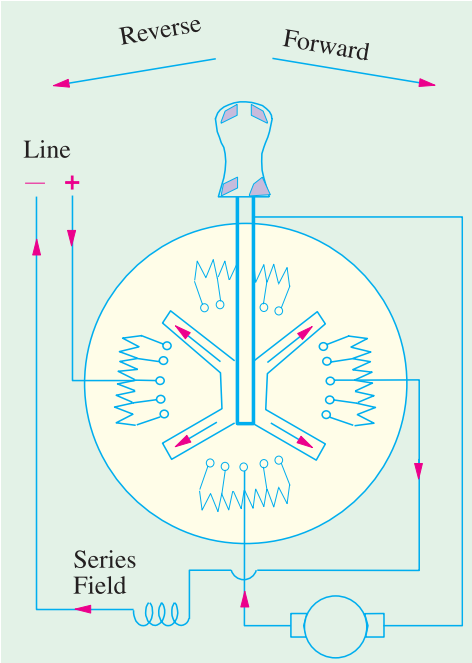


Fig. 30.44

cuit at this stage, then when resistance in the armature circuit is completely cut out, further rotation of the handle inserts resistance into the field circuit. Turning of the handle in the opposite direction starts and speeds up the motor in the reverse direction.

30.25. Grading of Starting Resistance for Shunt Motors

T_{st} would be small in designing shunt motor starters, it is usual to allow an overload of 50% for starting and to advance the starter a step when armature current has fallen to definite lower value. Either this lower current limit may be fixed or the number of starter steps may be fixed. In the former case, the number of steps are so chosen as to suit the upper and lower current limits whereas in the latter case, the lower current limit will depend on the number of steps specified. It can be shown that the resistances in the circuit on successive studs from geometrical progression, having a common ratio equal to lower current limit/upper current limit *i.e.*, I_2/I_1.

In Fig. 30.45 the starter connected to a shunt motor is shown. For the sake of simplicity, four live studs have been taken. When arm *A* makes contact with stud No. 1, full shunt field is

However, for adjustable speed service in connection with the operation of machine tools, a drum controller is preferred. It is called 'controller' because in addition to accelerating the motor to its normal speed, it provides the means for reversing the direction of the motor. Other desirable features such as safety protection against an open field or the temporary failure of power supply and overloads are frequently provided in this type of controller.

The controller consists of armature resistance grids of cross-section sufficient to carry the full-load operating current continuously and are used for adjusting the motor speed to values lower than the base speed obtained with no external resistance in the armature of field circuit. As the operating handle is gradually turned, the resistance is cut out of the armature circuit–there being as yet no resistance in the field cir-

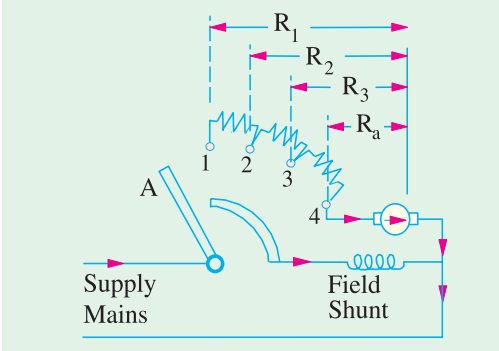


Fig. 30.45

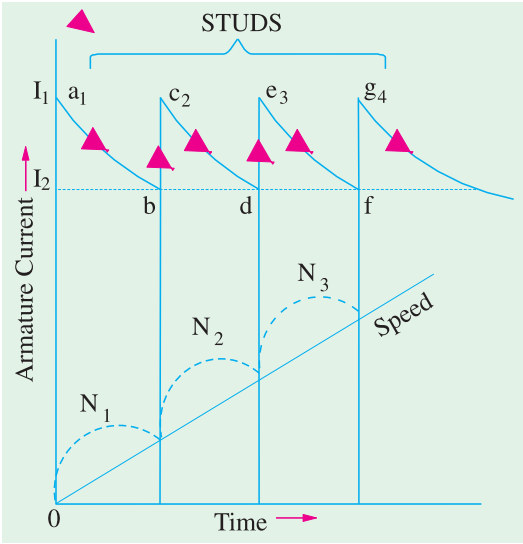


Fig. 30.46

established and at the same time the armature current immediately jumps to a maximum value I_1 given by $I_1 = V/R_1$ where R_1 = armature and starter resistance (Fig. 30.45).

I_1 the maximum permissible armature current at the start (I_{max}) and is, as said above, usually limited to 1.5 times the full-load current of the motor. Hence, the motor develops 1.5 times its full-load torque and accelerates very rapidly. As the motor speeds up, its back e.m.f. grows and hence decreases the armature current as shown by curve *ab* in Fig. 30.46.

When the armature current has fallen to some predetermined value I_2 (also called I_{min}) arm *A* is moved to stud No. 2. Let the value of back e.m.f. be E_{b1} at the time of leaving stud No. 1. Then

$$I_2 = \frac{V - E_{b1}}{R_1} \qquad ...(i)$$

It should be carefully noted that I_1 and I_2 [(I_{max}) and (I_{min})] are respectively the maximum and minimum currents of the motor. When arm *A* touches stud No. 2, then due to diminution of circuit resistance, the current again jumps up to its previous value I_1. Since speed had no time to change, the back e.m.f. remains the same as initially.

$$\therefore \quad I_1 = \frac{V - E_{b1}}{R_2} \qquad ...(ii)$$

From **(*i*)** and **(*ii*)**, we get $\frac{I_1}{I_2} = \frac{R_1}{R_2}$ ... **(*iii*)**

When arm *A* is held on stud No. 2 for some time, then speed and hence the back e.m.f. increases to a value E_{b2}, thereby decreasing the current to previous value I_2, so that

$$I_2 = \frac{V - E_{b2}}{R_2} \qquad ...(iv)$$

Similarly, on first making contact with stud No. 3, the current is

$$I_1 = \frac{V - E_{b2}}{R_3} \qquad ...(v)$$

From **(*iv*)** and **(*v*)**, we again get $\frac{I_1}{I_2} = \frac{R_2}{R_3}$ ...**(*vi*)**

When arm *A* is held on stud No. 3 for some time, the speed and hence back e.m.f. increases to a new value E_{b3}, thereby decreasing the armature current to a value I_2 such that

$$I_2 = \frac{V - E_{b3}}{R_3} \qquad ...(vii)$$

On making contact with stud No. 4, current jumps to I_1 given by

$$I_1 = \frac{V - E_{b3}}{R_a} \qquad ...(viii)$$

From **(*vii*)** and **(*viii*)**, we get $\frac{I_1}{I_2} = \frac{R_3}{R_a}$ ...**(*ix*)**

From **(*iii*)**, **(*vi*)** and **(*ix*)**, it is seen that

$$\frac{I_1}{I_2} = \frac{R_1}{R_2} = \frac{R_2}{R_3} = \frac{R_3}{R_a} = K \text{ (say)} \qquad ...(x)$$

Obviously, $R_3 = KR_a$; $R_2 = KR_3 = K^2R_a$

$R_1 = KR_2 = K.K^2R_a = K^3R_a$

In general, if *n* is the number of live studs and therefore $(n-1)$ the number of sections in the starter resistance, then

$$R_1 = K^{n-1}.R_a \text{ or } \frac{R_1}{R_a} = K^{n-1} \text{ or } \left(\frac{I_1}{I_2}\right)^{n-1} = \frac{R_1}{R_a} \quad ...\text{from } (x)$$

Other variations of the above formula are

(a) $$K^{n-1} = \frac{R_1}{R_a} = \frac{V}{I_1 R_a} = \frac{V}{I_{max}\, R_a}$$

(b) $$K^{n} = \frac{V}{I_1 R_a} \cdot \frac{I_1}{I_2} = \frac{V}{I_2 R_a} = \frac{V}{I_{min.} R_a} \text{ and}$$

(c) $$n = 1 + \frac{\log (V / R_a . I_{max})}{\log K} \text{ from } (a) \text{ above.}$$

Since $R_1 = V/I_1$ and R_a are usually known and K is known from the given values of maximum and minimum currents (determined by the load against which motor has to start), the value of n can be found and hence the value of different starter sections.

When Number of Sections is Specified.

Since I_1 would be given, R_1 can be found from $R_1 = V/I_1$.

Since n is known, K can be found from $R_1/R_a = K^{n-1}$ and the lower current limit I_2 from $I_1/I_2 = K$.

Example 30.49. *A 10 b.h.p. (7.46 kW) 200-V shunt motor has full-load efficiency of 85%. The armature has a resistance of 0.25 Ω. Calculate the value of the starting resistance necessary to limit the starting current to 1.5 times the full-load current at the moment of first switching on. The shunt current may be neglected. Find also the back e.m.f. of the motor, when the current has fallen to its full-load value, assuming that the whole of the starting resistance is still in circuit.*

Solution. Full-load motor current $= 7{,}460/200 \times 0.85 = 43.88$ A

Starting current, $I_1 = 1.5 \times 43.88 = 65.83$ A

$R_1 = V/I_1 = 200/65.83 = 3.038\ \Omega$; $R_a = 0.25\ \Omega$

$\therefore$ Starting resistance $= R_1 - R_a = 3.038 - 0.25 = 2.788\ \Omega$

Now, full-load current $I_2 = 43.88$ A

Now, $$I_2 = \frac{V - E_{b1}}{R_1}$$

$\therefore$ $E_{b1} = V - I_2 R_1 = 200 - (43.88 \times 3.038) =$ **67 V**

Example 30.50. *A 220-V shunt motor has an armature resistance of 0.5 Ω. The armature current at starting must not exceed 40 A. If the number of sections is 6, calculate the values of the resistor steps to be used in this starter.* **(Elect. Machines, AMIE Sec. B, 1992)**

Solution. Since the number of starter sections is specified, we will use the relation.

$R_1/R_a = K^{n-1}$ or $R_1 = R_a K^{n-1}$

Now, $R_1 = 220/40 = 5.5\Omega$, $R_a = 0.4\ \Omega$; $n-1 = 6$, $n = 7$

$\therefore$ $5.5 = 0.4\, K^6$ or $K^6 = 5.5/0.4 = 13.75$

$6 \log_{10} K = \log_{10} 13.75 = 1.1383$; $K = 1.548$

Now, $R_2 = R_1/K = 5.5/1.548 = 3.553\ \Omega$

$R_3 = R_2/K = 3.553/1.548 = 2.295\ \Omega$

$R_4 = 2.295/1.548 = 1.482\ \Omega$

$R_5 = 1.482/1.548 = 0.958\ \Omega$

$R_6 = 0.958/1.548 = 0.619\ \Omega$

Resistance of Ist section $= R_1 - R_2 = 5.5 - 3.553 =$ **1.947 Ω**

,, 2nd ” $= R_2 - R_3 = 3.553 - 2.295 =$ **1.258 Ω**

,, 3rd ” $= R_3 - R_4 = 2.295 - 1.482 =$ **0.813 Ω**

,, 4th ” $= R_4 - R_5 = 1.482 - 0.958 =$ **0.524 Ω**

,, 5th ” = $R_5 - R_6$ = 0.958 – 0.619 = **0.339 Ω**

,, 6th ” = $R_6 - R_a$ = 0.619 – 0.4 = **0.219 Ω**

Example 30.51. *Find the value of the step resistance in a 6-stud starter for a 5 h.p. (3.73 kW), 200-V shunt motor. The maximum current in the line is limited to twice the full-load value. The total Cu loss is 50% of the total loss. The normal field current is 0.6 A and the full-load efficiency is found to be 88%.* **(D.C. Machines, Jadavpur Univ. 1988)**

Solution. Output = 3,730 W

Total loss = 4,238 – 3,730 = 508 W

Armature Cu loss alone = 508/2 = 254 W

Input current = 4,238/200 = 21.19 A

Armature current = 21.19 – 0.6 = 20.59 A

$\therefore \quad 20.59^2 R_a = 254 \quad \therefore \quad R_a = 254/20.59^2 = 0.5989\ \Omega$

Permissible input current = 21.19 × 2 = 42.38 A

Permissible armature current = 42.38 – 0.6 = 41.78 A

$\therefore \quad R_1 = 200/41.78 = 4.787\ \Omega; \quad n = 6\ ;\ n - 1 = 5$

$\therefore \quad 4.787 = K^5 \times 0.5989 \quad \therefore \quad K^5 = 4.787/0.5989 = 7.993$

or $5 \log K = \log 7.993 = 0.9027$

$\therefore \quad \log K = 0.1805\ ; \quad K = 1.516$

Now $R_2 = R_1/K = 4.789/1.516 = 3.159\ \Omega$

$R_3 = 2.084\ \Omega\ ;\ R_4 = 1.376\ \Omega\ ;\ R_5 = 0.908\ \Omega$

Resistance in 1st step = $R_1 - R_2$ = 4.787 – 3.159 = **1.628 Ω**

Resistance in 2nd step = $R_2 - R_3$ = 3.159 – 2.084 = **1.075 Ω**

Resistance in 3rd step = $R_3 - R_4$ = 2.084 – 1.376 = **0.708 Ω**

Resistance in 4th step = $R_4 - R_5$ = 1.376 – 0.908 = **0.468 Ω**

Resistance in 5th step = $R_5 - R_a$ = 0.908 – 0.5989 = **0.309 Ω**

The various sections are shown in Fig. 28.47.

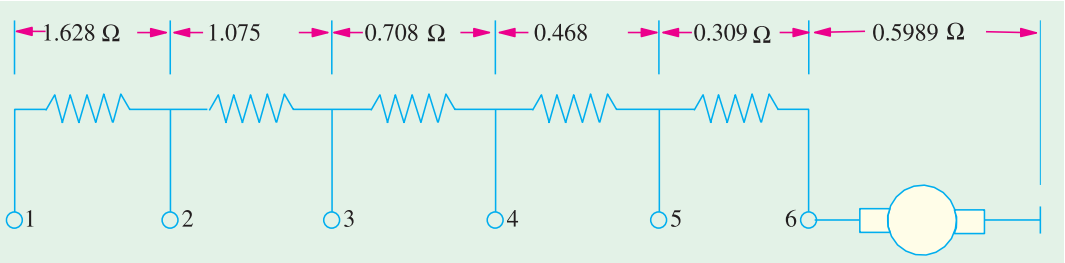


Fig. 30.47

Example 30.52. *Design the resistance sections of a seven-stud starter for 36.775 kW, 400V, d.c. shunt motor. Full-load efficiency is 92%, total Cu losses are 5% of the input. Shunt field resistance is 200 Ω. The lower limit of the current through the armature is to be full-load value.* **(Elec. Machines, Gujarat Univ. 1987)**

Solution. Output = 36,775 W ; Input = 36,775/0.92 = 39,980 W

Total Cu loss = 0.05 × 39,980 = 1,999W

Shunt Cu loss = V^2/R_{sh} = $400^2/200$ = 800 W

Armature Cu loss = 1,999 – 800 = 1199 W

F.L. input current = 39,980/400 = 99.95 A

I_{sh} = 400/200 = 2A ; I_a = 99.95 – 2 = 97.95 A

$\therefore$ $97.95^2 R_a$ = 1199 W or R_a = 0.125 Ω

Now, minimum armature current equals full-load current *i.e.* I_a = 97.95 A. As seen from Art. 30.25 in its formula given in (*b*), we have

$$K^n = \frac{\text{ट}}{\text{ट. ट.}}$$

or K^7 = 400/97.95 × 0.125 = 32.68

$\therefore$ $K = 32.68^{1/7} = 1.645$

I_1 = maximum permissible armature current

= KI_2 = 1.645 × 97.94 = 161 A

$\therefore$ $R_1 = V/I_1$ = 400/161 = 2.483 Ω

$R_2 = R_1/K$ = 2.483/1.645 = 1.51 Ω

R_3 = 1.51/1.645 = 0.917 Ω

R_4 = 0.917/1.645 = 0.557 Ω

R_5 = 0.557/1.645 = 0.339 Ω

R_6 = 0.339/1.645 = 0.206 Ω

R_7 = 0.206/1.645 = 0.125 Ω

Resistance in 1st step = $R_1 - R_2$ = **0.973 Ω**

Resistance in 2nd step = $R_2 - R_3$ = **0.593 Ω**

Resistance in 3rd step = $R_3 - R_4$ = **0.36 Ω**

Resistance in 4th step = $R_4 - R_5$ = **0.218 Ω**

Resistance in 5th step = $R_5 - R_6$ = **0.133 Ω**

Resistance in 6th step = $R_6 - R_a$ = **0.081 Ω**

The various starter sections are shown in Fig 30.48.

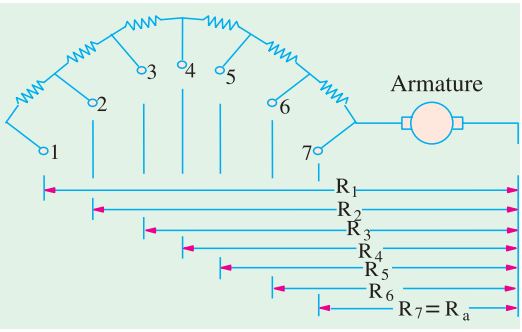


Fig. 30.48

Example 30.53. *Calculate the resistance steps for the starter of a 250-V, d.c. shunt motor having an armature resistance of 0.125 Ω and a full-load current of 150 A. The motor is to start against full-load and maximum current is not to exceed 200 A*

. **(Elect.Engineering-I, Bombay Univ. 1989)**

Solution. As the motor is to start against its full-load, the minimum current is its F.L. current *i.e.* 150A. We will use the formula given in Art. 30.25.

(I_1/I_2^{n-1}) = R_1 / R_a

Here I_1 = 200 A ; I_2 = 150 A ; R_1 = 250/200 = **1.25 Ω**

R_a = 0.125 Ω ; n = No. of live studs

$\therefore$ $(200/150)^{n-1}$ = 1.25/0.125 = 10 or $(4/3)^{n-1}$ = 10

$(n-1)\log 4/3 = \log 10$ or $(n-1) \times 0.1249 = 1$

$\therefore$ $(n-1)$ = 1/0.1249 = 8

Hence, there are 9 studs and 8 steps.

Now $R_2 = R_1 \times I_2/I_1$ = 1.25 × 3/4 = 0.938 Ω

R_3 = 0.938 × 3/4 = 0.703 Ω

R_4 = 0.703 × 3/4 = 0.527 Ω

$R_5 = 0.527 \times 3/4 = 0.395\ \Omega$
$R_6 = 0.395 \times 3/4 = 0.296\ \Omega$
$R_7 = 0.296 \times 3/4 = 0.222\ \Omega$
$R_8 = 0.222 \times 3/4 = 0.167\ \Omega$
$R_a = 0.167 \times 3/4 = 0.125\ \Omega$

$\therefore$ Resistance of 1st element $= 1.25 - 0.938 =$ **0.312 Ω**
,, 2nd ,, $= 0.938 - 0.703 =$ **0.235 Ω**
,, 3rd ,, $= 0.703 - 0.527 =$ **0.176 Ω**
,, 4th ,, $= 0.527 - 0.395 =$ **0.132 Ω**
,, 5th ,, $= 0.395 - 0.296 =$ **0.099 Ω**
,, 6th ,, $= 0.296 - 0.222 =$ **0.074 Ω**
,, 7th ,, $= 0.222 - 0.167 =$ **0.055 Ω**
,, 8th ,, $= 0.167 - 0.125 =$ **0.042 Ω**

Example 30.54. *The 4-pole, lap-wound armature winding of a 500-V, d.c. shunt motor is housed in a total number of 60 slots each slot containing 20 conductors. The armature resistance is 1.31 Ω. If during the period of starting, the minimum torque is required to be 218 N-m and the maximum torque 1.5 times the minimum torque, find out how many sections the starter should have and calculate the resistances of these sections. Take the useful flux per pole to be 23 mWb.*

(Elect. Machinery-II, Bangalore Univ. 1991)

Solution. From the given minimum torque, we will be able to find the minimum current required during starting. Now

$$T_a = 0.159\ \Phi\ ZI_a\ (P/A)$$

$\therefore$ $218 = 0.159 \times 23 \times 10^{-3} \times (60 \times 20)\ I_a \times (4/4)$ $\therefore$ $I_a = 50$ A (approx.)

Maximum current $= 50 \times 1.5 = 75$ A

$\therefore$ $I_1 = 75$ A ; $I_2 = 50$ A $\therefore$ $I_1/I_2 = 75/50 = 1.5$

$R_1 = 500/75 = 6.667\ \Omega$

If n is the number of stater studs, then

$$(I_1/I_2)^{n-1} = R_1/R_a \quad \text{or} \quad 1.5^{n-1} = 6.667/1.31 = 5.09$$

$\therefore$ $(n-1)\log_{10} 1.5 = \log_{10} 5.09$ $\therefore$ $(n-1) \times 0.1761 = 0.7067$ $\therefore$ $(n-1) = 4$ or $n = 5$

Hence, there are five studs and four sections.

$R_2 = R_1 \times I_2/I_1 = 6.667 \times 2/3 = 4.44\ \Omega$
$R_3 = 4.44 \times 2/3 = 2.96\ \Omega$; $R_4 = 2.96 \times 2/3 = 1.98\ \Omega$

Resistance of 1st section $= R_1 - R_2 = 6.67 - 4.44 =$ **2.23 Ω**
,, 2nd ,, $= R_2 - R_3 = 4.44 - 2.96 =$ **1.48 Ω**
,, 3rd ,, $= R_3 - R_4 = 2.96 - 1.98 =$ **0.98 Ω**
,, 4th ,, $= R_4 - R_a = 1.98 - 1.31 =$ **0.67 Ω**

30.26. Series Motor Starters

The basic principle employed in the design of a starter for series motor is the same as for a shunt motor *i.e.*, the motor current is not allowed to exceed a certain upper limit as the starter arm moves from one stud to another. However, there is one significant difference. In the case of a series motor, the flux does not remain constant but varies with the current because armature current is also the exciting current. The determination of the number of steps is rather complicated as illustrated in Example 30.55. It may however, be noted that the section resistances form a geometrical progression.

The face-plate type of starter formerly used for d.c. series motor has been almost entirely replaced by automatic starter in which the resistance steps are cut out automatically by means of a contactor operated by electromagnets. Such starters are well-suited for remote control.

However, for winch and crane motors where frequent starting, stopping, reversing and speed variations are necessary, drum type controllers are used. They are called controllers because they can be left in the circuit for any length of time. In addition to serving their normal function of starters, they also used as speed controllers.

Example 30.55. **(*a*)** *Show that, in general, individual resistances between the studs for a rheostat starter for a series d.c. motor with constant ratio of maximum to minimum current at starting, are in geometrical progression, stating any assumptions made.*

(*b*) *Assuming that for a certain d.c. series motor the flux per pole is proportional to the starting current, calculate the resistance of the each rheostat section in the case of a 50 b.h.p. (37.3 kW) 440-V motor with six sections.*

The total armature and field voltage drop at full-load is 2% of the applied voltage, the full-load efficiency is 95% and the maximum starting current is 130% of full-load current.

Solution. **(*a*)** Let I_1 = maximum current, I_2 = minimum current

Φ_1 = flux/pole for I_1 ; Φ_2 = flux/pole for I_2

$\frac{I_1}{I_2}$ = K and $\frac{\Phi_1}{\Phi_2} = \alpha$.

Let us now consider the conditions when the starter arm is on the nth and $(n + 1)$th stud. When the current is I_2, then $E_b = V - I_2 R_n$.

If, now, the starter is moved up to the $(n + 1)$th stud, then

$$E_b' = \frac{\Phi_1}{\Phi_2} . E_b = \alpha E_b$$

$$\therefore \quad R_{n+1} = \frac{V - E_b'}{I_1} = \frac{V - \alpha E_b}{I_1} = \frac{V - \alpha (V - I_2 R_n)}{I_1} = \frac{V}{I_1}(1 - \alpha) + \alpha \frac{I_2}{I_1} \cdot R_n$$

Now, $V/I_1 = R_1$–the total resistance in the circuit when the starter arm is on the first stud.

$$\therefore \quad R_{n+1} = R_1 (1 - \alpha) + \frac{\alpha}{K} R_n$$

Similarly, by substituting $(n - 1)$ for n, we get $R_n = R_1 (1 - \alpha) + \frac{\alpha}{K} R_{n-1}$

Therefore, the resistance between the nth and $(n + 1)$th studs is

$$r_n = R_n - R_{n+1} = \frac{\alpha}{K} R_{n-1} - \frac{\alpha}{K} R_n = \frac{\alpha}{K} (R_{n-1} - R_n) = \frac{\alpha}{K} r_{n-1}$$

$$\therefore \quad \frac{r_n}{r_{n-1}} = \frac{\alpha}{K} = \frac{\Phi_1}{\Phi_2} \times \frac{I_2}{I_1} = b \qquad \text{– constant}$$

Obviously, the resistance elements form a geometrical progression series.

(*b*) Full-load input current $= \frac{37,300}{440 \times 0.95} = 89.2$ A

Max. starting current $I_1 = 1.3 \times 89.2 = 116$ A

Arm. and field voltage drop on full-load = 2% of 440 $= 0.02 \times 440 = 8.8$ V

Resistance of motor $= 8.8/89.2 = 0.0896\ \Omega$

Total circuit resistance on starting, $R_1 = V/I_1 = 440/116 = 3.79\ \Omega$

Assuming straight line magnetisation, we have $I_1 \propto \Phi_1$ and $I_2 \propto \Phi_2$

$\therefore \quad I_1/I_2 = \Phi_1/\Phi_2 \qquad \therefore \quad \alpha = K$ and $b = \alpha/K = 1 \qquad \therefore \quad r_n = b \times r_{n-1}$

In other words, all sections have the same resistance.

$$\therefore \qquad r = \frac{R_1 - R_{motor}}{\text{No. of sections}} = \frac{3.79 - 0.0896}{6} = \mathbf{0.6176\ \Omega}$$

Example 30.56. *A 75 h.p. (55.95 kW) 650-V, d.c. series tractions motor has a total resistance of 0.51 Ω. The starting current is to be allowed to fluctuate between 140 A and 100 A the flux at 140 A being 20 % greater than at 100 A. Determine the number of steps required in the controller and the resistance of each step.*

Solution. Let R_1 = total resistance on the first stud = 650/140 = 4.65 Ω

When motor speeds up, then back e.m.f. is produced and current falls to I_2.

$$V = E_{b1} + I_2 R_1 \qquad \text{...(i)}$$

When the starter moves to the next stud, the speed is still the same, but since current rises to I_1 for which flux is 1.2 times greater than for I_2, hence back e.m.f. becomes 1.2 E_{b1}. Since resistance in the circuit is now R_2.

$$\therefore \qquad V = I_1 R_2 + 1.2\ E_{b1} \qquad \text{...(ii)}$$

From **(i)** and **(ii)** we get, $0.2\ V = 1.2\ I_2 R_1 - I_1 R_2$

$$\therefore \qquad R_2 = (1.2 I_2/I_1)\ R_1 - 0.2\ V/I_1 = (1.2 I_2/I_1)\ R_1 - 0.2\ R_1 = (1.2 I_2/I_1 - 0.2)\ R_1$$

$$\text{Similarly} \qquad R_3 = (1.2 I_2/I_1)\ R_2 - 0.2\ R_1$$

In this way, we continue till we reach the value of resistance equal to the armature resistance. Hence, we obtain R_1, R_2 etc. and also the number of steps.

In the present case, $I_1 = 140$ A, $I_2 = 100$A, $V = 650$ V and $I_1/I_2 = 100/140 = 1.14$

$$\therefore \qquad R_1 = 4.65\ \Omega;\ R_2 = (1.2/1.4 - 0.2)\ 4.65 = 3.07\ \Omega$$

$$R_3 = (1.2/1.4) \times 3.07 - 0.2 \times 4.65 = 1.70\ \Omega$$

$$R_4 = (1.2/1.4) \times 1.7 - 0.2 \times 4.65 = 0.53\ \Omega$$

We will stop here because R_4 is very near the value of the motor resistance. Hence, there are 4 studs and 3 sections or steps.

$$R_1 - R_2 = 4.65 - 3.07 = 1.58\ \Omega,\ R_2 - R_3 = 3.07 - 1.7 = 1.37\ \Omega$$

$$R_3 - R_4 = 1.70 - 0.53 = 1.17\ \Omega$$

Note. It will be seen that

$$R_2 - R_3 = (1.2\ I_2/I_1)\ (R_1 - R_2) \text{ and } R_3 - R_4 = (1.2\ I_2/I_1)\ (R_2 - R_3) \text{ and so on.}$$

or
$$\frac{R_2 - R_3}{R_1 - R_2} = \frac{R_3 - R_4}{R_2 - R_3} = 1.2\ \frac{I_2}{I_1} = \frac{\Phi_1}{\Phi_2} \times \frac{I_2}{I_1} = \frac{\alpha}{K} = b$$

It is seen that individual resistances of various sections decrease in the ratio of $\alpha/K = b$.

30.27. Thyristor Controller Starters

The moving parts and metal contacts etc., of the resistance starters discussed in Art. 30.21 can be eliminated by using thyristors which can short circuit the resistance sections one after another. A thyristor can be switched on to the conducting state by applying a suitable signal to its gate terminal. While conducting, it offers zero resistance in the forward (*i.e.*, anode-to-cathode) direction and thus acts as a short-circuit for the starter resistance section across which it is connected. It can be switched off (*i.e.*, brought back to the non-conducting state) by reversing the polarity of its anode-cathode voltage. A typical thyristor-controlled starter for d.c. motors is shown in Fig. 30.49.

After switching on the main supply, when switch S_1 is pressed, positive signal is applied to gate G of thyristor T_1 which is, therefore, turned ON. At the same time, shunt field gets established since it is directly connected across the d.c. supply. Consequently, motor armature current I_a flows *via* T_1,

R_2, R_3 and R_4 because T_2, T_3 and T_4 are, as yet in the non-conducting state. From now onwards, the starting procedure is automatic as detailed below :

1. As S_1 is closed, capacitor C starts charging up with the polarity as shown when I_a starts flowing.
2. The armature current and field flux together produce torque which accelerates the motor and load.

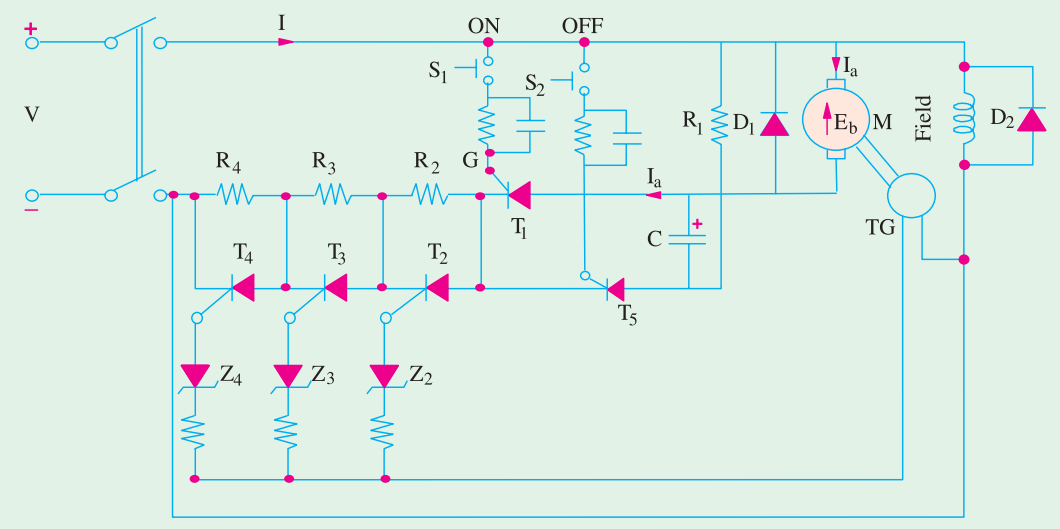


Fig. 30.49

3. As motor speeds up, voltage provided by tachogenerator (*TG*) is proportionately increased because it is coupled to the motor.
4. At some motor speed, the voltage provided by *TG* becomes large enough to breakdown Zener diode Z_2 and hence trigger T_2 into conduction. Consequently, R_2 is shorted out and now I_a flows via motor armature, T_1, T_2, R_3 and R_4 and back to the negative supply terminal.
5. As R_2 is cut out, I_a increases, armature torque increases, motor speed increases which further increases the voltage output of the tachogenerator. At some speed, Z_3 breaks down, thereby triggering T_3 into conduction which cuts out R_3.
6. After sometime, R_4 is cut out as Z_4 breaks down and triggers T_4 into conduction. In fact, Zener diodes Z_2, Z_3 and Z_4 can be rated for 1/3, 1/2 and 3/4 full speed respectively.

For stopping the motor, switch S_2 is closed which triggers T_5 into conduction, thereby establishing current flow via R_1. Consequently, capacitor C starts discharging thereby reverse-biasing T_1 which stops conducting. Hence I_a ceases and, at the same time, T_2, T_3 and T_4 also revert back to their non-conducting state.

Incidentally, it may be noted that the function of C is to switch T_1, ON and OFF. Hence, it is usually called ***commutating capacitor.***

The function of the diodes D_1 and D_2 is to allow the decay of inductive energy stored in the motor armature and field when supply is disconnected. Supply failure will cause the thyristors to block because of this current decay, thereby providing protection usually given by no-voltage release coil.

Recently, thyristor starting circuits have been introduced which use no starting resistance at all, thereby making the entire system quite efficient and optimized as regards starting time. These are based on the principle of 'voltage chopping' (Art. 30.12). By varying the chopping frequency, the ratio of the time the voltage is ON to the time it is OFF can be varied. By varying this ratio, the average voltage applied to the motor can be changed. A low average voltage is needed to limit the

armature current while the motor is being started and gradually the ratio is increased to reach the maximum at the rated speed of the motor.

Tutorial Problems 32.2

1. A shunt-wound motor runs at 600 r.p.m. from a 230-V supply when taking a line current of 50 A. Its armature and field resistances are 0.4 Ω and 104.5 Ω respectively. Neglecting the effects of armature reaction and allowing 2 V brush drop, calculate (*a*) the no-load speed if the no-load line current is 5A (*b*) the resistance to be placed in armature circuit in order to reduce the speed to 500 r.p.m. when motor is taking a line current of 50 A (*c*) the percentage reduction in the flux per pole in order that the speed may be 750 r.p.m. when the armature current is 30 A with no added resistance in the armature circuit. **[(*a*) 652 r.p.m. (*b*) 0.73 Ω (*c*) 1.73 %]**

2. The resistance of the armature of a 250-V shunt motor is 0.3 Ω and its full-load speed is 1000 r.p.m. Calculate the resistance to be inserted in series with the armature to reduce the speed with full-load torque to 800 r.p.m., the full-load armature current being 5A. If the load torque is then halved, at what speed will the motor run ? Neglect armature reaction. **[0.94 Ω; 932 r.p.m.]**

3. A 230-V d.c. shunt motor takes an armature current of 20 A on a certain load. Resistance of the armature is 0.5 Ω. Find the resistance required in series with the armature to half the speed if (*a*) the load torque is constant (*b*) the load torque is proportional to the square of the speed.

[(*a*) 5.5 Ω (*b*) 23.5 Ω]

4. A 230-V series motor runs at 1200 r.p.m. at a quarter full-load torque, taking a current of 16 A. Calculate its speed at half and full-load torques. The resistance of the armature brushes, and field coils is 0.25 Ω. Assume the flux per pole to be proportional to the current. Plot torque/speed graph between full and quarter-load. **[842 r.p.m. ; 589 r.p.m.]**

5. A d.c. series motor drives a load the torque of which is proportional to the square of the speed. The motor current is 20 A when speed is 500 r.p.m. Calculate the speed and current when the motor field winding is shunted by a resistance of the same value as the field winding. Neglect all motor losses and assume that the magnetic field is unsaturated. **[595 r.p.m. ; 33.64 A]**

(Electrical Machines-I, Aligarh Muslim Univ. 1979)

6. A d.c. series motor, with unsaturated magnetic circuit and with negligible resistance, when running at a certain speed on a given load takes 50 A at 500 V. If the load torque varies as the cube of the speed, find the resistance which should be connected in series with machine to reduce the speed by 25 per cent. **[7.89 Ω]**

(Electrical Engg-I, M.S. Univ. Baroda 1980)

7. A series motor runs at 500 r.p.m. on a certain load. Calculate the resistance of a divertor required to raise the speed to 650 r.p.m. with the same load current, given that the series field resistance is 0.05 Ω and the field is unsaturated. Assume the ohmic drop in the field and armature to be negligible.

[0.1665 Ω]

8. A 230-V d.c. series motor has armature and field resistances of 0.5 Ω and 0.3 Ω respectively. The motor draws a line current of 40 A while running at 400 r.p.m. If a divertor of resistance 0.15 W is used, find the new speed of the motor for the same armature current.

It may be assumed that flux per pole is directly proportional to the field current. **[1204 r.p.m.]**

(Electrical Engineering Grad. I.E.T.E. June 1986)

9. A 250-V, d.c. shunt motor runs at 700 r.p.m. on no-load with no extra resistance in the field and armature circuit. Determine :

(*i*) the resistance to be placed in series with the armature for a speed of 400 r.p.m. when taking a total current of 16 A.

(*ii*) the resistance to be placed in series with the field to produce a speed of 1,000 r.p.m. when taking an armature current of 18 A.

Assume that the useful flux is proportional to the field. Armature resistance = 0.35 Ω, field resistance = 125 Ω. **[(*i*) 7.3 Ω (*ii*) 113 Ω] (Elect. Engg. Grad. I.E.T.E., June 1984)**

10. A d.c. series motor is operating from a 220-V supply. It takes 50 A and runs at 1000 r.p.m. The resistance of the motor is 0.1 Ω. If a resistance of 2 Ω is placed in series with the motor, calculate the resultant speed if the load torque is constant. **[534 r.p.m.]**

11. A d.c. shunt motor takes 25 A when running at 1000 r.p.m. from a 220-V supply.

Calculate the current taken form the supply and the speed if the load torque is halved, a resistance of 5 Ω is placed in the armature circuit and a resistance of 50 Ω is placed in the field circuit.

Armature resistance = 0.1 Ω ; field resistance = 100 Ω

Assume that the field flux per pole is directly proportional to the field current. **[17.1 A; 915 r.p.m.]**

(Elect. Technology, Gwalior Univ. Nov. 1977)

12. A 440-V shunt motor takes an armature current of 50 A and has a flux/pole of 50 mWb. If the flux is suddenly decreased to 45 mWb, calculate (*a*) instantaneous increase in armature current (*b*) percentage increase in the motor torque due to increase in current (*c*) value of steady current which motor will take eventually (*d*) the final percentage increase in motor speed. Neglect brush contact drop and armature reaction and assume an armature resistance of 0.6 Ω.

[(*a*) 118 A (*b*) 112 % (*c*) 5.55 A (*d*) 10%]

13. A 440-V shunt motor while running at 1500 r.p.m. takes an armature current of 30 A and delivers a mechanical output of 15 h.p. (11.19 kW). The load torque varies as the square of the speed. Calculate the value of resistance to be connected in series with the armature for reducing the motor speed to 1300 r.p.m. and the armature current at that speed. **[2.97 Ω, 22.5 A]**

14. A 460-V series motor has a resistance of 0.4 Ω and takes a current of 25 A when there is no additional controller resistance in the armature circuit. Its speed is 1000 r.p.m. The control resistance is so adjusted as to reduce the field flux by 5%. Calculate the new current drawn by the motor and its speed. Assume that the load torque varies as the square of the speed and the same motor efficiency under the two conditions of operation.

[22.6 A; 926 r.p.m.] (Elect. Machines, South Gujarat Univ. Oct. 1977)

15. A 460-V, series motor runs at 500 r.p.m. taking a current of 40 A. Calculate the speed and percentage change and torque if the load is reduced so that the motor is taking 30 A. Total resistance of armature and field circuit is 0.8 Ω. Assume flux proportional to the field current.

[680 r.p.m. 43.75%]

16. A 440-V, 25 h.p (18.65 kW) motor has an armature resistance of 1.2 Ω and full-load efficiency of 85%. Calculate the number and value of resistance elements of a starter for the motor if maximum permissible current is 1.5 times the full-load current. **[1.92 Ω, 1.30 Ω, 0.86 Ω; 0.59 Ω]**

(Similar example in JNTU, Hyderabad, 2000)

17. A 230-V, d.c. shunt motor has an armature resistance of 0.3 Ω. Calculate (*a*) the resistance to be connected in series with the armature to limit the armature current to 75 A at starting and (*b*) value of the generated e.m.f. when the armature current has fallen to 50 A with this value of resistance still in circuit. **[(*a*) 2.767 Ω (*b*) 76.7 A]**

18. A 200-V, d.c. shunt motor takes full-load current of 12 A. The armature circuit resistance is 0.3 Ω and the field circuit resistance is 100 Ω. Calculate the value of 5 steps in the 6-stud starter for the motor. The maximum starting current is not to exceed 1.5 times the full-load current.

[6.57 Ω, 3.12 Ω, 1.48 Ω, 0.7 Ω, 0.33 Ω]

19. The resistance of a starter for a 200-V, shunt motor is such that maximum starting current is 30 A. When the current has decreased to 24 A, the starter arm is moved from the first to the second stud.

Calculate the resistance between these two studs if the maximum current in the second stud is 34 A. The armature resistance of the motor is 0.4 Ω. **[1.334 Ω]**

20. A totally-enclosed motor has thermal time constant of 2 hr. and final temperature rise at no-load and 40° on full load.

Determine the limits between which the temperature fluctuates when the motor operates on a load cycle consisting of alternate period of 1 hr. on full-load and 1 hr. on no-load, steady state conditions having been established. **[28.7° C, 21.3°C]**

21. A motor with a thermal time constant of 45 min. has a final temperature rise of 75°C on continuous rating (*a*) What is the temperature rise after one hour at this load ? (*b*) If the temperature rise on one-hour rating is 75°C, find the maximum steady temperature at this rating (*c*) When working at its one-hour rating, how long does it take the temperature to increase from 60°C to 75°C ? **[(*a*) 55 °C (*b*) 102 °C (*c*) 20 min]**

(Electrical Technology, M.S. Univ. Baroda. 1976)

OBJECTIVE TESTS – 30

1. The speed of a d.c. motor can be controlled by varying
(*a*) its flux per pole
(*b*) resistance of armature circuit
(*c*) applied voltage
(*d*) all of the above

2. The most efficient method of increasing the speed of a 3.75 kW d.c. shunt motor would be the .....method.
(*a*) armature control
(*b*) flux control
(*c*) Ward-Leonard
(*d*) tapped-field control

3. Regarding Ward-Leonard system of speed control which statement is **false ?**
(*a*) It is usually used where wide and very sensitive speed control is required.
(*b*) It is used for motors having ratings from 750 kW to 4000 kW
(*c*) Capital outlay involved in the system is right since it uses two extra machines.
(*d*) It gives a speed range of 10 : 1 but in one direction only.
(*e*) It has low overall efficiency especially at light loads.

4. In the rheostatic method of speed control for a d.c. shunt motor, use of armature divertor makes the method
(*a*) less wasteful
(*b*) less expensive
(*c*) unsuitable for changing loads
(*d*) suitable for rapidly changing loads

5. The chief advantage of Ward-Leonard system of d.c. motor speed control is that it
(*a*) can be used even for small motors
(*b*) has high overall efficiency at all speeds
(*c*) gives smooth, sensitive and wide speed control
(*d*) uses a flywheel to reduce fluctuations in power demand

6. The flux control method using paralleling of field coils when applied to a 4-pole series d.c. motor can give ..... speeds.
(*a*) 2 (*b*) 3
(*c*) 4 (*d*) 6

7. The series-parallel system of speed control of series motors widely used in traction work gives a speed range of about
(*a*) 1 : 2 (*b*) 1 : 3
(*c*) 1 : 4 (*d*) 1 : 6

8. In practice, regenerative braking is used when
(*a*) quick motor reversal is desired
(*b*) load has overhauling characteristics
(*c*) controlling elevators, rolling mills and printing presses etc.
(*d*) other methods can not be used.

9. **Statement 1.** A direct-on-line (**DOL**) starter is used to start a small d.c. motor because
Statement 2. it limits initial current drawn by the armature circuit.
(*a*) both statement 1 and 2 are incorrect
(*b*) both statement 1 and 2 are correct
(*c*) statement 1 is correct but 2 is wrong
(*d*) statement 2 is correct but 1 is wrong

10. Ward-Leonard system of speed control is **NOT** recommended for
(*a*) wide speed range
(*b*) constant-speed operation
(*c*) frequent motor reversals
(*d*) very low speeds

11. Thyristor chopper circuits are employed for
(*a*) lowering the level of a d.c. voltage
(*b*) rectifying the a.c. voltage
(*c*) frequency conversion
(*d*) providing commutation circuitry

12. An invertor circuit is employed to convert
(*a*) a.c. voltage into d.c. voltage
(*b*) d.c.voltage into a.c. voltage
(*c*) high frequency into low frequency
(*d*) low frequency into high frequency

13. The phase-control rectifiers used for speed of d.c. motors convert fixed a.c. supply voltage into
(*a*) variable d.c. supply voltage
(*b*) variable a.c. supply voltage
(*c*) full-rectified a.c. voltage
(*d*) half-rectified a.c. voltage

14. If some of the switching devices in a convertor are controlled devices and some are diodes, the convertor is called
(*a*) full convertor (*b*) semiconvertor
(*c*) solid-state chopper
(*d*) d.c. convertor

15. A solid-state chopper converts a fixed-voltage d.c. supply into a
(*a*) variable-voltage a.c. supply
(*b*) variable-voltage d.c. supply
(*c*) higher-voltage d.c. supply
(*d*) lower-voltage a.c. supply

16. The d.c. motor terminal voltage supplied by a solid-state chopper for speed control purposes varies.....with the duty ratio of the chopper
(*a*) inversely (*b*) indirectly
(*c*) linearly (*d*) parabolically

ANSWERS

1. *d* **2.** *b* **3.** *d* **4.** *d* **5.** *c* **6.** *b* **7.** *c* **8.** *b* **9.** *c* **10.** *b* **11.** *a*
12. *b* **13.** *a* **14.** *b* **15.** *b* **16.** *c*

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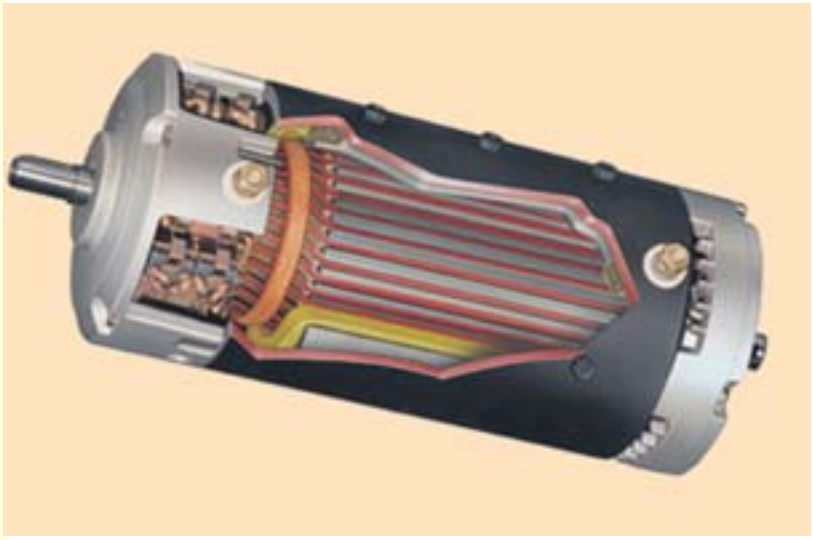
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CHAPTER 31

TESTING OF D.C. MACHINES

Learning Objectives

- Brake Test
- Swinburne's Test
- Advantages of Swinburne's Test
- Main Disadvantages
- Regenerative or Hopkinson's Test
- Alternative Connections for Hopkinson's Test
- Merits of Hopkinson's Test
- Retardation or Running Down Test
- Field's Test for Series Motors
- Objective Test
- Questions and Answers on D.C. Motors.



Testing is performed on D.C. machines to determine efficiency and power losses

CONTENTS

31.1 Brake Test

It is a direct method and consists of applying a brake to a water-cooled pulley mounted on the motor shaft as shown in Fig. 31.1. The brake band is fixed with the help of wooden blocks gripping the pulley. One end of the band is fixed to earth via a spring balance S and the other is connected to a suspended weight W_1. The motor is running and the load on the motor is adjusted till it carries its full load current.


Motor shaft

Let W_1 = suspended weight in kg

W_2 = reading on spring balance in kg-wt

The net pull on the band due to friction at the pulley is $(W_1 - W_2)$ kg. wt. or 9.81 $(W_1 - W_2)$ newton.

If R = radius of the pulley in metre

and N = motor or pulley speed in r.p.s.

Then , shaft torque T_{sh} developed by the motor

$= (W_1 - W_2)\, R$ kg-m $= 9.81\, (W_1 - W_2)\, R$ N-m

Motor output power $= T_{sh} \times 2\pi N$ watt

$= 2\pi \times 9.81\, N\, (W_1 - W_2)\, R$ watt

$= 61.68\, N\, (W_1 - W_2)\, R$ watt

Let V = supply voltage ; I = full-load current taken by the motor.

Then, input power = VI watt

$$\therefore \quad \eta = \frac{\text{Output}}{\text{Input}} = \frac{61.68\ N(W_1 - W_2)R}{VI}$$

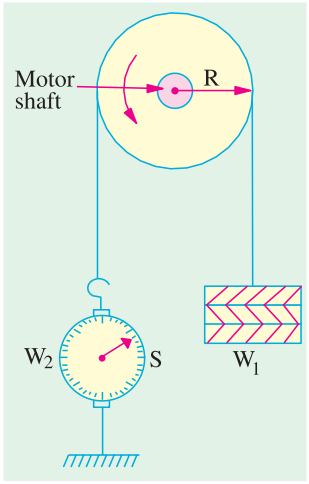


Fig. 31.1

The simple brake test described above can be used for small motors only, because in the case of large motors, it is difficult to dissipate the large amount of heat generated at the brake.

Another simple method of measuring motor output is by the use of poney brake one form of which is shown in Fig. 31.2 (*a*). A rope is wound round the pulley and its two ends are attached to two spring balances S_1 and S_2. The tension of the rope can be adjusted with the help of swivels. Obviously, the force acting tangentially on the pulley is equal to the difference between the readings of the two spring balances. If R is the pulley radius, the torque at the pulley is $T_{sh} = (S_1 - S_2)R$. If $\omega\,(= 2\pi N)$ is the angular velocity of the pulley, then

motor output $= T_{sh} \times \omega = 2\,\pi\, N\, (S_1 - S_2)R$ m-kg. wt. $= 9.81 \times 2\pi N\, (S_1 - S_2)\, R$ watt.

The motor input may be measured as shown in Fig. 31.2 (*b*). Efficiency may, as usual, be found by using the relation η = output/input.

Example 31.1. *In a brake test the effective load on the branch pulley was 38.1 kg, the effective diameter of the pulley 63.5 cm and speed 12 r.p.s. The motor took 49 A at 220 V. Calculate the output power and the efficiency at this load.*

Solution. Effective load $(W_1 - W_2) = 38.1$ kg. wt ; radius $= 0.635/2 = 0.3175$ m

Shaft torque $= 38.1 \times 0.3175$ kg-m $= 9.81 \times 38.1 \times 0.3175 = 118.6$ N-m

Power output = torque × angular velocity in rad/s $= 118.6 \times 2\pi \times 12 =$ **8,945 W**

Now, motor input $= 49 \times 220$ W $\quad \therefore \quad$ Motor $\eta = \dfrac{8,945}{49 \times 220} =$ **0.83** or **83%**

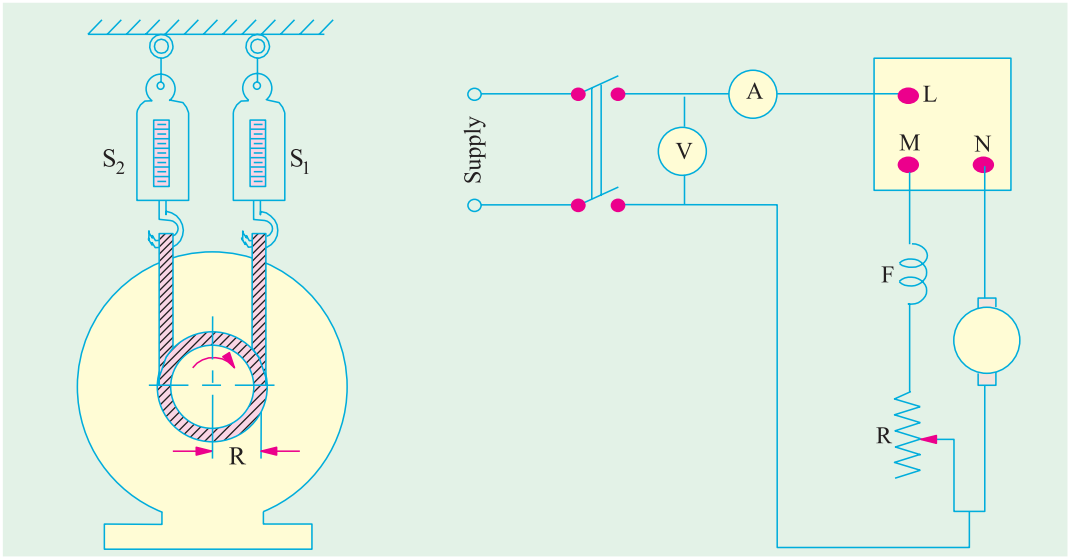


Fig. 31.2 (*a*) Fig. 31.2 (*b*)

Example 31.2(*a*). *The following readings are obtained when doing a load test on a d.c. shunt motor using a brake drum :*

Spring balance reading	*10 kg and 35 kg*	*Diameter of the drum*	*40 cm*
Speed of the motor	*950 r.p.m.*	*Applied voltage*	*200 V*
Line current	*30 A*		

Calculate the output power and the efficiency. **(Electrical Engineering, Madras Univ. 1986)**

Solution. Force on the drum surface $F = (35 - 10) = 25$ kg wt $= 25 \times 9.8$ N

Drum radius $R = 20$ cm $= 0.2$ m ; Torque $T_{sh} = F \times R = 25 \times 9.8 \times 0.2 = 49$ N

$N = 950/60 = 95/6$ r.p.s. ; $\omega = 2\pi\,(95/6) = 99.5$ rad/s

Motor output $= T_{sh} \times \omega$ watt $= 49 \times 99.5 = 4{,}876$ W

Motor input $= 200 \times 30 = 6000$ W ; $\eta = 4876/6000 = 0.813$ or **81.3%**

Example 31.2(*b*). *In a brake-test, on a d.c. shunt motor, the tensions on the two sides of the brake were 2.9 kg and 0.17 kg. Radius of the pulley was 7 cm. Input current was 2 amp at 230 volts. The motor speed was 1500 rpm. Find the torque, power-output and efficiency.*

(Bharathiar Univ. April 1998)

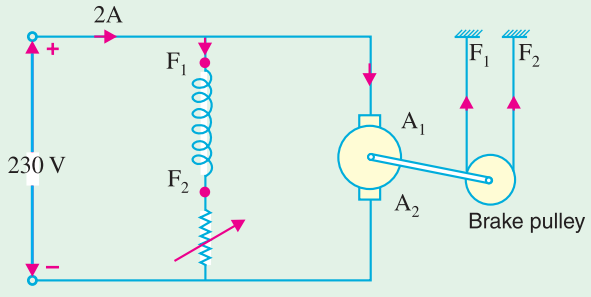


Fig. 31.3. D.C. Shunt Motor Brake Test

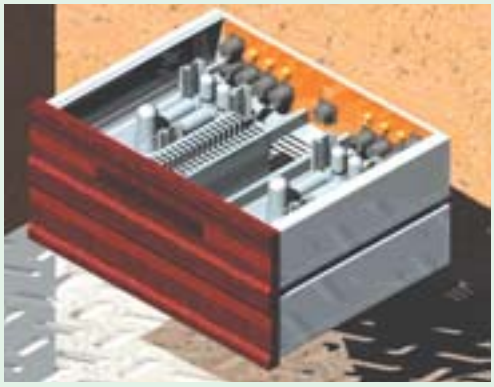
Solution. Net force on pulley $= 2.90 - 0.17 = 2.73$ kg

$= 2.73 \times 9.81 = 26.78$ Nw

Net torque = Force × Radius = 26.78 × 7/100
= 1.8746 Nw-m
Power output = Torque × Radians/sec.
= 1.8746 × 2π 1500/60
= 294 watts
Efficiency = 294/(230 × 2) = 0.639
% efficiency = 63.9%

31.2. Swinburne's* Test (or No-load Test or Losses Method)

It is a simple method in which losses are measured separately and from their knowledge, efficiency at any desired load can be predetermined in advance. The only running test needed is no-load test. *However,* ***this test is applicable to those machines in which flux is practically constant i.e. shunt and compound-wound machines.***


Shunt regulator

The machine is running as a motor on no-load at its rated voltage *i.e.* voltage stamped on the name-plate. The speed is adjusted to the rated speed with the help of shunt regulator as shown in Fig. 31.4.

The no-load current I_0 is measured by the ammeter A_1 whereas shunt field current I_{sh} is given by ammeter A_2. The no-load armature current is $(I_0 - I_{sh})$ or I_{a0}.

Let, supply voltage = V no-load input = VI_0 watt
∴ Power input to armature = $V(I_0 - I_{sh})$; Power input to shunt = VI_{sh}

No-load power input to armature supplies the following :

(i) Iron losses in core ***(ii)*** friction loss ***(iii)*** windage loss and
(iv) armature Cu loss, $(I_0 - I_{sh})^2 R_a$ or $I_{a0}^2 R_a$

In calculating armature Cu loss, 'hot' resistance of armature should be used. A stationary measurement of armature circuit resistance at the room-temperature of, say, 15°C is made by passing current through the armature from a low voltage d.c. supply [Fig. 31.5 (*a*)].

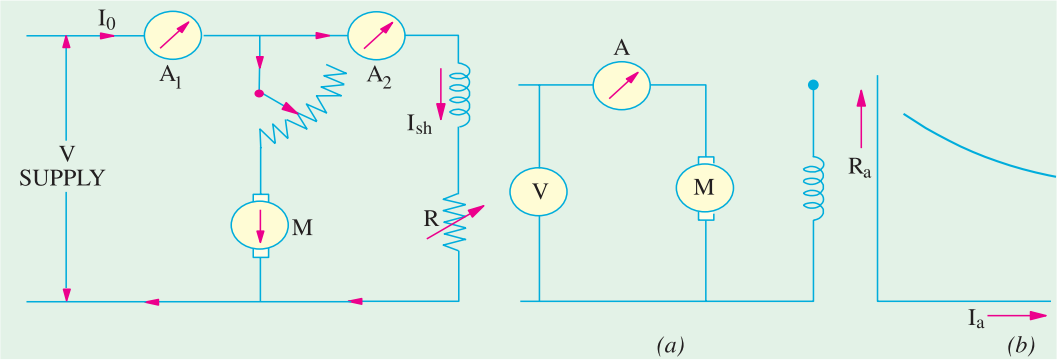


Fig. 31.4 Fig. 31.5

* Sir James Swinburne (1858-1958) made outstanding contributions to the development of electric lamps, electric machines and synthetic resins.

Then, the 'hot' resistance, allowing a temperature rise of 50°C is found thus :

$R_{15} = R_0(1 + 15\alpha_0)$; $R_{65} = (1 + 65\alpha_0)$, $R_{65} = R_{15} \times \dfrac{1 + 65\alpha_0}{1 + 15\alpha_0}$

Taking $\alpha_0 = 1/234.5$, we have $R_{65} = R_{15} \times \dfrac{234.5 + 65}{234.5 + 15} = 1.2\ R_{15}$ (approx.*)

If we subtract from the total input the no-load armature Cu loss, then we get constant losses.

$\therefore$ Constant losses $W_c = VI_0 - (I_0 - I_{sh})^2 R_a$

Knowing the constant losses of the machine, its efficiency at any other load can be determined as given below. Let I = load current at which efficiency is required.

Then, armature current is $I_a = I - I_{sh}$ ...if machine is motoring

$= I + I_{sh}$ ...if machine is generating

Efficiency when running as a motor

Input = VI, Armature Cu loss = $I_a^2 R_a = (I - I_{sh})^2 R_a$

Constant losses $= W_c$ ...found above

$\therefore$ Total losses = $(I - I_{sh})^2 R_a + W_c$; $\eta_m = \dfrac{\text{input} - \text{losses}}{\text{input}} = \dfrac{VI - (I - I_{sh})^2 R_a - W_c}{VI}$

Efficiency when running as a generator

Output = VI ; Armature, Cu loss = $(I + I_{sh})^2 R_a$; Constant loss = W_c ...found above

$\therefore$ Total losses = $(I + I_{sh})^2 R + W_c$; $\eta_g = \dfrac{\text{output}}{\text{output} + \text{losses}} = \dfrac{VI}{VI + (I + I_{sh})^2 R_a + W_c}$

31.3. Advantages of Swinburne's Test

1. It is convenient and economical because power required to test a large machine is small *i.e.* only no-load input power.

2. The efficiency can be predetermined at any load because constant-losses are known.

31.4. Main Disadvantages

1. No account is taken of the change in iron losses from no-load to full-load. At full-load, due to armature reaction, flux is distorted which increases the iron losses in some cases by as much as 50%.

2. As the test is on no-load, it is impossible to know whether commutation would be satisfactory at full-load and whether the temperature rise would be within the specified limits.

Example 31.3. *A 220 V, d.c. shunt motor at no load takes a current of 2.5 A. The resistances of the armature and shunt field are 0.8 Ω and 200 Ω respectively. Estimate the efficiency of the motor when the input current is 20 A. State precisely the assumptions made.*

(Electrical Technology, Kerala Univ. 1986)

Solution. No-load input = $220 \times 2.5 = 550$ W

This input meets all kinds of no-load losses *i.e.* armature Cu loss and constant losses.

$I_{sh} = 220/200 = 1.1$ A. No-load arm current, $I_{a0} = 2.5 - 1.1 = 1.4$ A

* The armature resistance is found to decrease slightly with increasing armature current as shown in Fig. 31.5 (*b*). This is due to the fact that brush contact resistance is inversely proportional to the armature current.

No-load armature Cu loss = $I_{a0}^2 R_a = 1.4^2 \times 0.8 = 1.6$ W

Constant losses = 550 – 1.6 = 548.4 W

When input current is 20 A

I_a = 32 – 1.1 = 30.9 A ; Armature Cu loss = $30.9^2 \times 0.8$ = 764 W

Total loss = 764 + 548.4 = 1312 W (approx.) ; Input = 220 × 20 = 4,400 W

Output = 4,400 – 1,312 = 3,088 W ; Efficiency = (3088/4400) × 100 = **70.2%**

In the above calculations, it has been assumed that :

1. mechanical losses remain constant even through motor speed changes from no-load to the given load.

2. effect of armature reaction on main pole flux with a consequent change in iron losses has been neglected.

3. decrease in flux due to increase in shunt resistance by heating has been neglected.

Example 31.4. *When running on no-load, a 400-V shunt motor takes 5 A. Armature resistance is 0.5 Ω and field resistance 200 Ω. Find the output of the motor and efficiency when running on full-load and taking a current of 50 A. Also, find the percentage change in speed from no-load to full-load.* **(Electro Mechanics, Allahabad Univ. 1991)**

Solution. No-load input = 400 × 5 = 2,000 W

This input goes to meet all kinds of no-load losses *i.e.* armature Cu loss and constant losses.

I_{sh} = 400/200 = 2 A ; No-load I_a = 5 – 2 = 3 A

No-load arm. Cu loss = $3^2 \times 0.5$ = 4.5 W ; Constant losses = 2,000 – 4.5 = **1,995.5 W**

When line current is 50 A

I_a = 50 – 2 = 48 A ; Arm. Cu loss = $48^2 \times 0.5$ = 1,152 W

Total loss on F.L. = 1,152 + 1,995.5 = 3,147.5 W ; Input = 50 × 400 = 20,000 W

Output = 20,000 – 3,147.5 = **16,852.5 W = 16.8 kW**

F.L. efficiency = 16,852.5/20,000 = 0.8426 or **84.26%**

Now, $E_{b1} = 400 - (3 \times 0.5) = 398.5$ V ; $E_{b2} = 400 - (48 \times 0.5) = 376$ V

$$\frac{N_1}{N_2} = \frac{E_{b1}}{E_{b2}} = \frac{398.5}{376} \quad \therefore \quad \frac{N_1 - N_2}{N_2} = \frac{22.5}{376} = 0.0598$$

∴ percentage change in speed = **5.98**

Example 31.5. *The no-load test of a 44.76 kW, 220-V, d.c. shunt motor gave the following figures:*

Input current = 13.25 A ; field current = 2.55 A ; resistance of armature at 75°C = 0.032 Ω and brush drop = 2 V. Estimate the full-load current and efficiency.

(Electrical Engineering, Madras Univ. 1987)

Solution. No-load Condition

No-load input = 220 × 13.25 = 2915 W ; Armature current = 13.25 – 2.55 = 10.7 A

Armature Cu loss = $10.7^2 \times 0.032$ = 3.6 W

Loss due to brush drop = 2 × 10.7 = 21.4 W

Variable loss = 21.4 + 3.6 = 25 W_e, Constant losses W_c = 2915 – 25 = 2890 W

Full-load Condition

If I_a is the full-load armature current, then full-load motor input current is (I_a + 2.55) A.

F.L. motor power input = 220 (I_a + 2.55) W

This input must be equal to the sum of

(i) output = 44.76 kW = 44,760 W ***(ii)*** W_c = 2,890 W

(iii) brush loss = $2I_a$ watt ***(iv)*** Arm. Cu loss = $0.032\, I_a^2$

$\therefore \quad 220\,(I_a + 2.55) = 44{,}750 + 2{,}890 + 2I_a + 0.032\,I_a^2$

or $0.032I_a^2 - 218\,I_a + 47{,}090 = 0$

or $$I_a = \frac{218 \pm \sqrt{218^2 - 4 \times 0.032 \times 47{,}090}}{2 \times 0.032} = 223.5 \text{ A}$$

Line input current $I = I_a + I_{sh} = 223.5 + 2.55 =$ **226 A**

F.L. power input $= 226 \times 220 = 49{,}720$ W

$\therefore$ F.L. efficiency $= 44{,}760/49{,}720 = 0.9$ or **90%.**

Example 31.6. *A 200-V, shunt motor develops an output of 17.158 kW when taking 20.2 kW. The field resistance is 50 Ω and armature resistance 0.06 Ω. What is the efficiency and power input when the output is 7.46 kW ?* **(Elect. Machines-I, Aligarh Muslim Univ. 1989)**

Solution. In the first case :

Output = 17,158 W Input = 20,200 W

Total losses = 20,200 – 17,158 = 3,042 W ; Input current = 20,200/200 = 101 A

$I_{sh} = 200/50 = 4$ A ; $I_a = 101 - 4 = 97$ A

$\therefore$ Armature Cu loss $= 97^2 \times 0.06 = 564.5$ W

$\therefore$ Constant losses $= 3{,}042 - 564.5 = 2{,}477.5 = 2478$ W (approx.)

In the second case :

Let, I_a = armature current Input current = $(I_a + 4)$ A

Now, input power = output + $I_a^2 R_a$ + constant losses

$\therefore \quad 200(I_a + 4) = 7{,}460 + 0.06\,I_a^2 + 2{,}478$

or $0.06I_a^2 - 200I_a + 9{,}138 = 0$

$\therefore$ $$I_a = \frac{200 \pm \sqrt{200^2 - 4 \times 0.06 \times 9{,}138}}{2 \times 0.06} = \frac{200 \pm 194}{0.12} = 3{,}283.3 \text{ A or } 46 \text{ A}$$

We will reject the larger value because it corresponds to unstable operation of the motor. Hence, take $I_a = 46$ A.

$\therefore$ Input current $I = I_a + I_{sh} = 46 + 4 = 50$ A

Power input $= \frac{50 \times 200}{1000} =$ **10 kW** $\therefore \quad \eta = \frac{7{,}460 \times 100}{10{,}000} =$ **74.6%**

Example 31.7. *A 200-V, 14.92 kW dc shunt motor when tested by the Swinburne method gave the following results :*

Running light : armature current was 6.5 A and field current 2.2 A. With the armature locked, the current was 70 A when a potential difference of 3 V was applied to the brushes. Estimate the efficiency of the motor when working under full-load conditions.

(Electrical Engg.-I, Bombay Univ. 1985)

Solution. No-load input current = 6.5 + 2.2 = 8.7 A

No-load power input $= 200 \times 8.7 = 1{,}740$ W

No-load input equals Cu losses and stray losses.

Field Cu loss $= 200 \times 2.2 = 440$ W

Armature Cu loss $= 6.5^2 \times 0.04286 = 1.8$ W ($\because R_a = 3/70 = 0.04286\ \Omega$)

$\therefore$ Constant losses $= 1{,}740 - 1.8 = 1738$ W

We will assume that constant losses are the same at full-load also.

Let, I_a = full-load armature current

F.L. armature Cu loss = $0.04286\, I_a^2$ W ; Constant losses = 1,738 W

F.L. total loss = $1{,}738 + 0.04286\, I_a^2$

F.L. output = 14,920 W ; F.L. input = $200\,(I_a + 2.2)$ W

We know, input = output + losses

or $200\, I_a + 440 = 14{,}920 + 1{,}738 + 0.04286\, I_a^2$

or $0.04286\, I_a^2 - 200\, I_a + 16{,}218 = 0 \quad \therefore \quad I_a = 82.5$ A

$\therefore$ Input current = 82.5 + 2.2 = 84.7 A

F.L. power input = 200×84.7 A = **16,940 W**

$\therefore$ $\eta = 14{,}920 \times 100/16{,}940 =$ **88%**

Example 31.8. *In a test on a d.c. shunt generator whose full-load output is 200 kW at 250 V, the following figures were obtained :*

(a) *When running light as a motor at full speed, the line current was 36 A, the field current 12 A, and the supply voltage 250.*

(b) *With the machine at rest, a p.d. of 6 V produced a current of 400 A through the armature circuit. Explain how these results may be utilised to obtain the efficiency of generator at full-load and half-load. Neglect brush voltage drop.*

Solution. At no-load :

$I_a = 36 - 12 = 24$ A ; $R_a = 6/400 = 0.015\ \Omega$

$\therefore$ Armature Cu loss = $24^2 \times 0.015 = 8.64$ watt

No-load input = total losses in machine = $250 \times 36 = 9{,}000$ W

Constant losses = $9{,}000 - 8.64 =$ **8,991.4 W**

At full-load :

Output = 200,000 W ; Output current = 200,000/250 = 800 A ; $I_{sh} = 12$ A

$\therefore$ F.L. armature current = 800 + 12 = 812 A

$\therefore$ F.L. armature Cu losses = $812^2 \times 0.015 = 9890$ W

$\therefore$ F.L. total losses = 9890 + 8,991.4 = 18881 W $\quad \therefore \quad \eta = \dfrac{200,000 \times 100}{200,000 + 18,881} =$ **91.4%**

At half-load :

Output = 100,000 W ; Output current = 100,000/250 = 400 A

$I_a = 400 + 12 = 412$ A $\quad \therefore \quad I_a^2 R_a = 412^2 \times 0.015 = 2{,}546$ W

Total losses = $8{,}991.4 + 2{,}546 = 11{,}537$ W $\quad \therefore \quad \eta = \dfrac{100,000 \times 100}{111,537} =$ **89.6%**

Example 31.9. *A 250-V, 14.92 kW shunt motor has a maximum efficiency of 88% and a speed of 700 r.p.m. when delivering 80% of its rated output. The resistance of its shunt field is 100 Ω. Determine the efficiency and speed when the motor draws a current of 78 A from the mains.*

Solution. Full-load output = 14,920 W

80% of F.L. output = $0.8 \times 14.920 = 11{,}936$ W ; $\eta = 0.88$

Input = 11,936/0.88 = 13,564 W

Total losses = 13,564 – 11,936 = 1,628 W

As efficiency is maximum at this load, the variable loss is equal to constant losses.

$\therefore$ $W_c = I_a^2 R_a = 1{,}628/2 \quad \therefore \quad I_a^2 R_a = 814$ W

Now, input current = 13,564/250 = 54.25 A

$\therefore$ $I_{sh} = 250/100 = 2.5$ A $\quad \therefore \quad I_a = 54.25 - 2.5 = 51.75$ A

$\therefore \quad 51.75^2 R_a = 814 \quad \therefore \quad R_a = 814/51.75^2 = 0.3045\ \Omega$

When input current is 78 A

$I_a = 78 - 2.5 = 75.5$ A $\quad \therefore \quad I_a^2 R_a = 75.5^2 \times 0.3045 = 1{,}736$ W

Total losses $= 1{,}736 + 814 = 2{,}550$ W ; Input $= 250 \times 78 = 19{,}500$ W

$$\eta = \frac{19{,}500 - 2{,}550}{19{,}550} \times 100 = \mathbf{86.9\%}$$

Speed :

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \quad \text{or} \quad \frac{N_2}{700} = \frac{250 - (75.5 \times 0.3045)}{250 - (51.75 \times 0.3045)} = \frac{227}{234.25} \; ; N_2 = \mathbf{680\ r.p.m.}$$

31.5. Regenerative or Hopkinson's Test (Back-to-Back Test)

By this method, ***full-load test*** can be carried out on two shunt machines, preferably identical ones, without wasting their outputs. The two machines are mechanically coupled and are so adjusted electrically that one of them runs as a motor and the other as a generator. The mechanical output of the motor drives the generator and the electrical output of generator is used in supplying the greater part of input to the motor. If there were no losses in the machines, they would have run without any external power supply. But due to these losses, generator output is not sufficient to drive the motor and ***vice-versa***. The losses are supplied either by an extra motor which is belt-connected to the motor-generator set or as suggested by Kapp, electrically from the supply mains.

Essential connections for the test are shown in Fig. 31.6. The two shunt machines are connected in parallel. They are, to begin with, started as unloaded motors. Then, the field of one is weakened and that of the other is strengthened so that the former runs as a motor and the latter as a generator. The usual method of procedure is as follows :

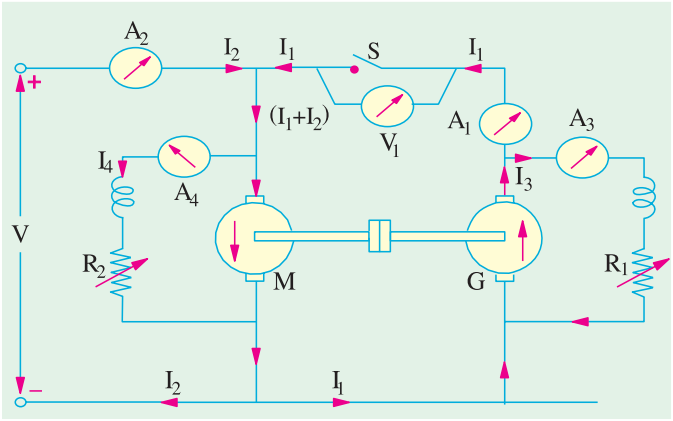


Fig. 31.6

Machine *M* is started up from the supply mains with the help of a starter (not shown) whereas main switch *S* of the other machine is kept open. Its speed is adjusted to normal value by means of its shield regulator. Machine *M* drives machine *G* as a generator and its voltage is read on voltmeter V_1. The voltage of *G* is adjusted by its field regulator until voltmeter V_1 reads zero, thereby showing that its voltage is the same, both in polarity and magnitude as that of the main supply. Thereafter, *S* is closed to parallel the machines. By adjusting the respective field regulators, any load can now be thrown on to the machines. Generator current I_1 can be adjusted to any desired value by increasing the excitation of *G* or by reducing the excitation of *M* and the corresponding values of different ammeters are read.



Motor-cum generator set

The electrical output of the generator ***plus*** the small power taken from the supply, is taken by the motor and is given out as a mechanical power ***after*** supplying the motor losses.

If supply voltage is V, then

Motor input = $V(I_1 + I_2)$, where I_2 is the current taken from the supply.

Generator output = VI_1 ...(*i*)

Assuming that both machines have the same efficiency η,

Output of motor = $\eta \times$ input = $\eta\, V(I_1 + I_2)$ = generator input

Output of generator = $\eta \times$ input = $\eta \times \eta\, V(I_1 + I_2) = \eta^2\, V(I_1 + I_2)$ ...(*ii*)

Hence, from (*i*) and (*ii*), we get

$$\eta^2 V(I_1 + I_2) = VI_1 \quad \text{or} \quad \eta = \sqrt{\frac{I_1}{I_1 + I_2}}$$

However, it is not quite correct to assume equal efficiencies for two machines because their armature currents as well as excitations are different. We will not find the efficiencies separately.

Let R_a = armature resistance of each machine

I_3 = exciting current of the generator

I_4 = exciting current of the motor

Armature Cu loss in generator = $(I_1 + I_3)^2 R_a$; Armature Cu loss in motor = $(I_1 + I_2 - I_4)^2 R_a$

Shunt Cu loss in generator = VI_3 ; Shunt Cu loss in motor = VI_4

But total motor and generator losses are equal to the power supplied by the mains.

Power drawn from supply = VI_2

If we subtract the armature and shunt Cu losses from this, we get the stray losses of *both* machines.

∴ Total stray losses for the set

$$= VI_2 - [(I_1 + I_3)^2 R_a + (I_1 + I_2 - I_4)^2 R_a + VI_3 + VI_4] = \text{W (say)}$$

Making one assumption that stray losses are equally divided between the two machines, we have

Stray loss per machine = W/2

For Generator

Total losses = $(I_1 + I_3)^2\, R_a + VI_3 + \text{W}/2 = W_g$ (say)

Output = $VI_1 \quad \therefore \quad \eta_g = \dfrac{VI_1}{VI_1 + W_g}$

Total losses = $(I_1 + I_2 - I_4)^2 R_a + VI_d + \text{W}/2 = W_m$ (say)

Input = $V(I_1 + I_2) \quad \therefore \quad \eta_m = \dfrac{V(I_1 + I_2) - W_m}{V(I_1 + I_2)}$

31.6. Alternative Connections for Hopkinson's Test

In Fig. 31.7 is shown in slightly different method of connecting the two machines to the supply. Here, the main difference is that the shunt windings are directly connected across the lines. Hence, the line input current is I_1 ***excluding the field currents.*** The efficiencies can be calculated as detailed below :

Motor armature Cu loss = $(I_1 + I_2)^2 R_a$; Generator armature Cu loss = $I_2^2 R_a$

Power drawn from the supply = VI_1

∴ Total stray losses *i.e.* iron, friction and windage losses for the two machines are

$$= VI_1 - [(I_1 + I_2)^2\, R_a - I_2^2\, R_a] = \text{W (say)}$$

$\therefore$ stray loss for each machine = W/2

Motor Efficiency

Motor input = armature input + shunt field input = $V(I_1 + I_2) + VI_3 + W_{input}$

Motor losses = armature Cu loss + shunt Cu loss + stray losses

$= (I_1 + I_2)^2 R_a + VI_3 + W/2 = W_m$ (say)

Motor $\eta = \dfrac{W_{input} - W_m}{W_{input}} \times 100$

Generator Efficiency

Generator output = VI_2 ; Generator losses = $I_2^2 R_a + V I_4 + W/2 = W_g$ (say)

$\therefore$ Generator $\eta = \dfrac{VI_2}{VI_2 + W_g}$

31.7. Merits of Hopkinson's Test

1. Power required for the test is small as compared to the full-load powers of the two machines.

2. As machines are being tested under full-load conditions, the temperature rise and the commutation qualities of the machines can be observed.

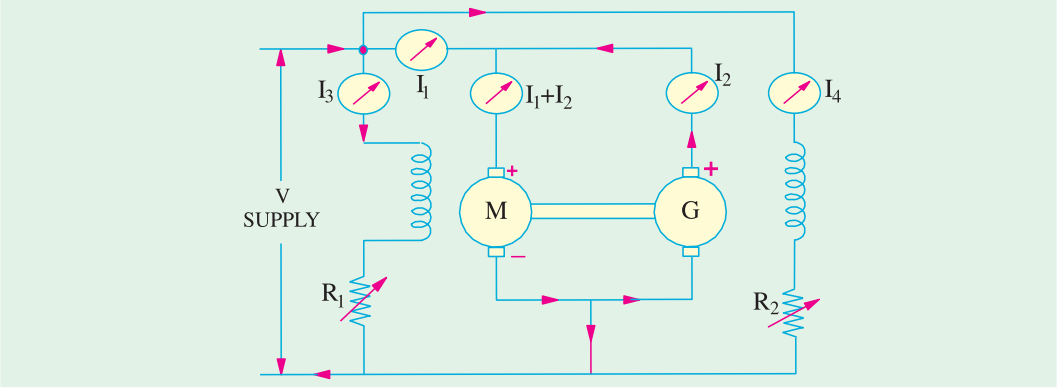


Fig. 31.7

3. Because of full-load conditions, any change in iron loss due to flux distortion at full-load, is being taken into account.

The only disadvantage is with regard to the availability of two identical machines.

Example 31.10 (*a*). *In a Hopkinson's test on two 220-V, 100-kW generators, the circulating current is equal to the full-load current and, in addition, 90 A are taken from the supply. Obtain the efficiency of each machine.*

Solution. Output current of the generator

$$I_1 = \frac{100,000}{220} = \frac{5,000}{11} = 454.4 \text{ A}, I_2 = 90 \text{ A}$$

Assuming equal efficiencies, from Art. 29.5, we have

$$\eta = \sqrt{\frac{I_1}{I_1 + I_2}} = \sqrt{\frac{454.5}{454.5 + 90}} = 0.914 \text{ or } \mathbf{91.4\%}$$

Example 31.10 (*b*). *In the Hopkinson's test on two d.c. machines, machine A has a field current of 1.4 A and machine B has a field current of 1.3 A. Which machine acts as a generator ?*

(Bharathithasan University April 1997)

Solution. In Hopkinson's test (on two identical d.c. shunt machines), since the two machines are coupled, the speed is common and is decided by the field current of the motor. The field windings of both the machines are in parallel with a separate D.C. source.

Since the machines are identical and are running at the ***same speed***, their e.m.fs are in proportion to their field currents.

$$\frac{\text{E.M.F. induced in the armature of machine A}}{\text{E.M.F. induced in the armature of machine B}} = \frac{1.4}{1.3}$$

$$E_A = (1.4/1.3) \times E_B = 1.077\, E_B$$

Since E_A is larger than E_B, Machine A supplies power to Machine B. It means, A is working as a generator, and B is motoring.

Example 31.11. *Two shunt machines loaded for the Hopkinson's test take 15 A at 200 V from the supply. The motor current is 100 A and the shunt currents are 3 A and 2.5 A. If the armature resistance of each machine is 0.05 ohm, calculate the efficiency of each machine for this particular load-condition.* **(Bharathithasan Univ. April 1997)**

Solution. Line current into armature circuits = 15 A, Motor armature copper-loss = 500 W

Motor-armature-current = 100 A, Generator armature copper loss = 361 W

Hence generator-armature-current = 85 A

For each machine, No load Mechanical losses + Core-loss + Stray losses

$$= \tfrac{1}{2}(VI_a - I_{am}^2\, r_{am} - I_{ag}^2\, r_{ag})$$

$$= \tfrac{1}{2}\,(200 \times 15 - 100^2 \times 0.05 - 85^2 \times 0.05)$$

$$= \tfrac{1}{2}\,(3000 - 500 - 361) = 1069.5 \text{ W}$$

Motor field copper-loss $= 200 \times 3 = 600 \text{ W} \cong 1.07 \text{ kW}$

Generator field copper-loss $= 200 \times 2.5 = 500 \text{ W}$

Total Losses in motor $= 600 + 1069.5 + 500 = 2169.5 \text{ W}$

Total Losses in Generator $= 500 + 1069.5 + 361 = 1931 \text{ W}$

$$\text{Efficiency of motor} = \frac{\text{Motor output}}{\text{Motor input}} \times 100\%$$

Motor Input : ***(a)*** $200 \times 100 = 20$ kW to armature

(b) 0.6 kW to field winding

Total Input to motor = 20.6 kW

From armature side, losses to be catered are :

(i) Stray losses + No Load Mech. Losses + Core Losses = 1.07 kW

(ii) Armature copper-loss = 0.5 kW

Motor Output from armature = 20 – 0.5 – 1.07 = 18.43 kW

$$\text{Motor efficiency} = \frac{18.43}{20.6} \times 100\% = 89.47\%$$

Generator armature output $= 200 \times 85 \times 10^{-3} = 17$ kW

Generator losses : ***(a)*** Field wdg : 0.5 kW

(b) Total no-load-losses : 1.07 kW

(c) armature copper-loss = 0.36 kW

Total losses in Generator = 1.93 kW

$$\text{Generator efficiency} = \frac{17}{17 + 1.93} \times 100\% = \mathbf{89.80\%}$$

Special Note: 15 A current for d.c. supply is related here to armature-input for two machines which are under back-to-back regenerative tests. There are different variations in handling and giving the test data. It is always desirable to draw the circuit diagram according to which the calculations are being related.

Example 31.12. *The Hopkinson's test on two similar shunt machines gave the following full-load data :*

Line voltage = *110 V* — *Field currents are 3 A and 3.5 A*
Line current = *48 A* — *Arm. resistance of each is 0.035 Ω*
Motor arm. current = *230 A*

Calculate the efficiency of each machine assuming a brush contact drop of 1 volt per brush.

(Electrical Machines, Nagpur Univ. 1992)

Solution. The motor-generator set is shown in Fig. 31.8. It should also be noted that the machine with lesser excitation is motoring. We will find the total armature Cu losses and brush contact loss for both machines.

Motor

Arm. Cu loss = $230^2 \times 0.035 = 1{,}851.5$ W
Brush contact loss = $230 \times 2 = 460$ W
Total arm. Cu loss = 1851.5 + 460 = 2,312 W
Shunt Cu loss = $110 \times 3 = 330$ W
Total Cu loss = 2,312 + 330 = 2,642 W

Generator

Generator arm. current = 233 – 48 + 3.5
= 188.5 W
Arm. Cu loss = $188.5^2 \times 0.035 = 1{,}244$ W
Brush contact Cu loss = $188.5 \times 2 = 377$ W
Total arm. Cu loss = 1,244 + 377 = 1,621 W
Shunt Cu loss = $110 \times 3.5 = 385$ W ;
Total Cu loss = 1,621 + 385 = 2,006 W

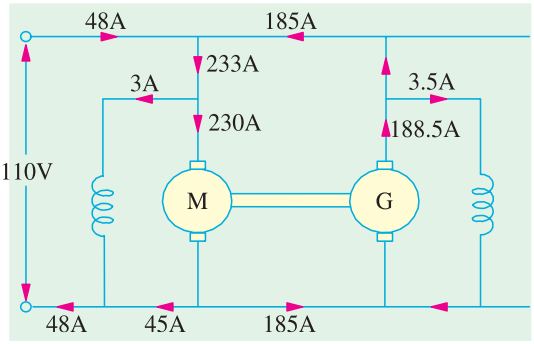


Fig. 31.8

For the Set

Total arm. and shunt Cu loss for the set
= 2,642 + 2,006 = 4,648 W
Total input = $110 \times 48 = 5{,}280$ W ; Stray losses for the set = 5,280 – 4,648 = 632 W
Stray losses per machine = 632/2 = 316 W

Motor Efficiency

Arm. Cu + brush drop loss = 2,312 W — Shunt Cu loss = 330 W
Stray losses = 316 W — Total loss = 2,312 + 330 + 316 = 2,958 W
Motor input = $110 \times 233 = 25{,}630$ W ; Motor output = 25,630 – 2,958 = 22,672
$\therefore$ η = $22{,}672 \times 100/25{,}630$ = **88.8%**

Generator Efficiency

Total losses = 2,006 + 316 = 2,322 W ; Output = $110 \times 185 = 20{,}350$ W
Generator input = 20,350 + 2,322 = 22,672 W = motor input
η = 20,350/22,672 = 0.894 or **89.4%**

Example 31.13. *In a Hopkinson's test on a pair of 500-V, 100-kW shunt generators, the following data was obtained :*

Auxiliary supply, 30 A at 500 V : Generator output current, 200 A
Field currents, 3.5 A and 1.8 A
Armature circuit resistances, 0.075 Ω each machine. Voltage drop at brushes, 2 V (each machine).
Calculate the efficiency of the machine acting as a generator.

(Elect. Technology-1, Gwalior Univ. 1986)

Solution. Motor arm. current = 200 + 30 = 230 A, as shown in Fig. 31.9.

Motor arm. Cu loss = $230^2 \times 0.075 + 230 \times 2 = 4{,}428$ W

Motor field Cu loss = $500 \times 1.8 = 900$ W

Generator arm. Cu loss = $200^2 \times 0.075 + 200 \times 2 = 3{,}400$ W

Geneator field Cu loss = $500 \times 3.5 = 1{,}750$ W

Total Cu loss for two machines

= 4,428 + 900 + 3400 + 1750

= 10,478 W

Power taken from auxiliary supply

= 500 × 30 = 15,000 W

Stray losses for the two machines

= 15,000 – 10,478 = 4,522 W

Stray loss per machine = 4,522/2 = 2,261 W

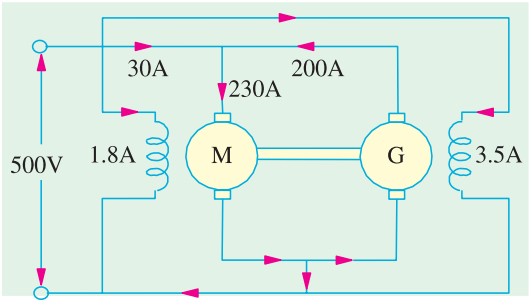


Fig. 31.9

Total losses in generator = 3400 + 1750 + 2261 = 7,411 W

Generator output = 500 × 200 = 100,000 W

$$\therefore \quad \eta_g = \frac{\text{output}}{\text{output + losses}} = \frac{100,000}{107,411} \times 100 = \mathbf{93.09\%}$$

Example 31.14. *Explain the Hopkinson's test on a pair of shunt motors.*

In such a test on 250-V machines, the line current was 50 A and the motor current 400 A not including the field currents of 6 A and 5 A. The armature resistance of each machine was 0.015 Ω. Calculate the efficiency of each machine. **(Adv. Elect. Machines, A.M.I.E. Sec. B, 1991)**

Solution. The connections are shown in Fig. 31.10.

Motor armature Cu loss

= $400^2 \times 0.015 = 2{,}400$ W

Generator armature Cu loss

= $350^2 \times 0.015 = 1{,}838$ W

Power drawn from supply

= 250 × 50 = 12,500 W

∴ Iron, friction and windage losses for the two machines

= 12,500 – (2,400 + 1,838)

= 8,262 W

Iron, friction and windage loss per machine

= 8.262/2 = 4,130 W* (approx.)

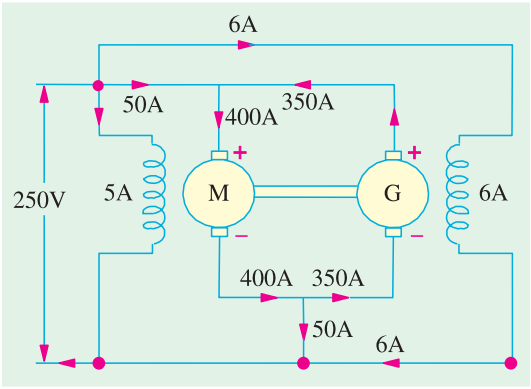


Fig. 31.10

Motor Losses and Efficiency

Motor arm. Cu loss = 2,400 W ; Motor field Cu loss = 250 × 5 = 1,250 W

Iron, friction and windage losses = 4,130 W

Total motor losses = 2,400 + 1,250 + 4,130 = 7,780 W

Motor input = 250 × 400 + 250 × 5 = 101,250 W

∴ Motor efficiency = (101,250 – 7,780)/101,250 = 0.923 or **92.3%**

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* We could also get this value as follows :

Total supply input = 250 × 61 = 15,250 W ; Gen. and motor field Cu loss = 250 × 6 + 250 × 5 = 2,750 W

Iron, friction and windage losses for both machines

= 15,250 – (2,400 + 1,838 + 2,750) = **8,262 W** – as before

Generator Losses and Efficiency

Generator arm. Cu loss = 1,838 W ; Generator field Cu loss = 250 × 6 = 1,500 W
Iron, friction and windage loss = 4,130 W
Total losses = 1,838 + 1,500 + 4,130 = 7.468 W
Generator output = 250 × 350 = 87,500 W
∴ Generator efficiency = (87,500 – 7.468)/87,500 = 0.915 or **91.5%**

Example 31.15. *The Hopkinson's test on two shunt machines gave the following results for full-load :*

Line voltage = 250 V ; current taken from supply system excluding field currents = 50 A ; motor armature current = 380 A ; field currents 5 A and 4.2 A. Calculate the efficiency of the machine working as a generator. Armature resistance of each machine is 0.2 Ω.

(Electrical Machinery-I Mysore Univ. 1988)

Solution. The connections are shown in Fig. 31.11.

Motor arm. Cu loss = $380^2 \times 0.02$ = 2,888 W
Generator arm. Cu loss = $330^2 \times 0.02$ = 2,178 W
Power drawn from supply = 250 × 50 = 12,500 W
Stray losses for the two machines
= 12,500 – (2,888 + 2,178) = 7.434 W
Stray losses per machine = 7,434/2 = 3,717 W

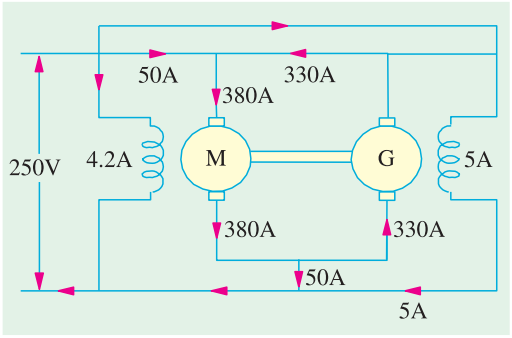


Fig. 31.11

Motor Efficiency

Arm. Cu loss = 2,888 W
Field Cu loss = 250 × 4.2 = 1050 W
Stray losses = 3,717 W
Total loss = 2,888 + 1050 + 3,717
= 7,655 W
Motor input = 250 × 380 + 250 × 4.2
= 96,050 W
Motor output = 96,050 – 7,655
= 88,395 W
∴ η = 88,395/96,050 = 0.9203 or **92.03%**

Generator Efficiency

Arm. Cu loss = 2,178 W ; Field Cu loss = 250 × 5 = 1250 W
Stray losses = 3,717 W ; Total losses = 7,145 W
Generator output = 250 × 330 = 82,500 W
Generator input = 82,500 + 7,145 = 89,645 W
η = 82,500/89,645 = 0.9202 or **92.02%**

31.8. Retardation or Running Down Test

This method is applicable to shunt motors and generators and is ***used for finding stray losses.*** Then, knowing the armature and shunt Cu losses at a given load current, efficiency can be calculated.

The machine under test is speeded up slightly beyond its normal speed and then supply is cut off from the armature while keeping the field excited. Consequently, the armature slows down and its kinetic energy is used to meet the ***rotational losses i.e.*** friction, windage and iron losses.*

Kinetic energy of the armature is $K.E. = \frac{1}{2} I\omega^2$

where I = moment of inertia of the armature and ω = angular velocity
∴ Rotational losses, W = Rate of loss of $K.E.$

* If armature slows down with no excitation, then energy of the armature is used to overcome mechanical losses only, there being no iron losses (see Ex. 31.19).

$$\therefore \quad W = \frac{d}{dt}\left(\frac{1}{2} I\omega^2\right) = I\omega . \frac{d\omega}{dt}$$

Two quantities need be known (i) moment of inertia (I) of the armature and (ii) $\frac{d\omega}{dt}$ or $\frac{dN}{dt}$ because $\omega \propto N$. These are found as follows :

(a) Finding $\frac{d\omega}{dt}$

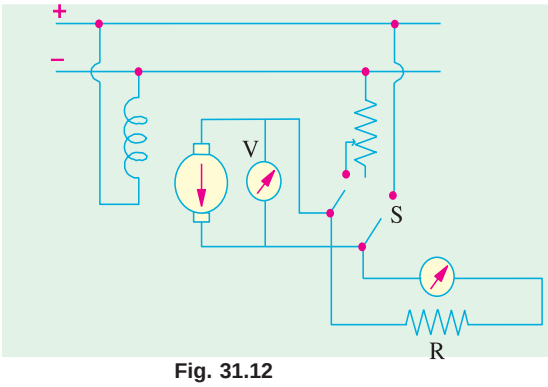


Fig. 31.12

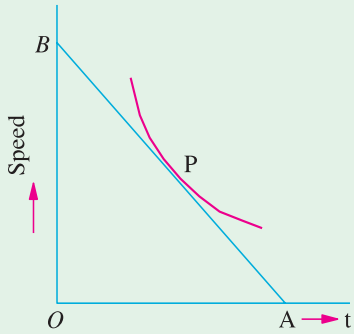


Fig. 31.13

As shown in Fig. 31.12, a voltmeter V is connected across the armature. This voltmeter is used as a speed indicator by suitably graduating it, because $E \propto N$. When supply is cut off, the armature speed and hence voltmeter reading falls. By noting different amounts of voltage fall in different amounts of time, a curve is drawn between time and the speed (obtained from voltage values) as shown in Fig. 31.13.

From any point P which corresponds to normal speed, a tangent AB is drawn.

Then $$\frac{dN}{dt} = \frac{OB \text{ (in r.p.m.)}}{OA \text{ (in seconds)}}$$

From (i), above $$W = I\,\omega\,\frac{d\omega}{dt}$$

Now $$\omega = \frac{2\pi N}{60} \quad \text{...}(N \text{ in r.p.m.})$$

$$W = I\left(\frac{2\pi N}{60}\right) \times \frac{d}{dt}\left(\frac{2\pi N}{60}\right); \; W = \left(\frac{2\pi}{60}\right)^2 I.N.\frac{dN}{dt} = 0.011\, I.N.\frac{dN}{dt} \quad \text{...}(ii)$$



Shunt wound generator

(ii) Finding Moment of Inertia (I)

(a) First Method–*where I is calculated.*

First, slowing down curve is drawn with armature alone. Next, a fly-wheel of known moment of inertia I_1 is keyed onto the shaft and slowing down curve is drawn again. Obviously, slowing down time will be longer due to combined increased moment of inertia of the two. For any given speed, (dN/dt_1) and (dN/dt_2) are determined as before. It should be noted that the losses in both cases would be almost the same, because addition of a fly-wheel will not make much difference to the losses.

Hence, from equation **(ii)** above

In the first case, $$W = \left(\frac{2\pi}{60}\right)^2 IN\left(\frac{dN}{dt_1}\right)$$

In the second case, $W = \left(\frac{2\pi}{60}\right)^2 (I + I_1)N \times \left(\frac{dN}{dt_2}\right)$

$\therefore \quad (I + I_1)\left(\frac{dN}{dt_2}\right) = I\left(\frac{dN}{dt_1}\right)$ or $\left(\frac{I + I_1}{I}\right) = \left(\frac{dN}{dt_1}\right) \Big/ \left(\frac{dN}{dt_2}\right)$

$\therefore \quad I = I_1 \times \frac{(dN/dt_2)}{(dN/dt_1) - (dN/dt_2)} = I_1 \frac{dt_1}{dt_2 - dt_1}$; $I = I_1 \frac{t_1}{t_2 - t_1}$

(*b*) Second Method–*where I is eliminated.*

In this method, first, time taken to slow down, say by 5%, is noted with armature alone. Next, a retarding torque–mechanical or preferably electrical, is applied to the armature and again time is noted. The method using electrical torque is shown in Fig. 31.12. The double-throw switch *S* while cutting off the armature from supply, automatically joins it to a non-inductive resistance *R* as shown. The power drawn by this resistance acts as a retarding torque on the armature, thereby making it slow down comparatively quickly. The additional loss is $I_a^2 (R_a + R)$ or VI_a,

where I_a = *average* current through *R* ; *V* = *average* voltage across *R*.

Let *W′* be this power. Then from (*i*) above

$$\left.\begin{aligned} W &= \left(\frac{2\pi}{60}\right)^2 I.N.\frac{dN}{dt_1} \\ W + W' &= \left(\frac{2\pi}{60}\right)^2 I.N.\frac{dN}{dt_2} \end{aligned}\right\} \text{If } dN \text{ is the same}$$

$\therefore \quad \frac{W + W'}{W} = \frac{dt_1}{dt_2} \quad \therefore \quad W = W' \times \frac{dt_2}{dt_1 - dt_2}$ or $W = W' \times \frac{t_2}{t_1 - t_2}$

where $\frac{dN}{dt_1}$ = rate of change of speed ***without*** extra load

$\frac{dN}{dt_2}$ = rate of change of speed ***with*** extra electrical load.

Example 31.16. *In a retardation test on a separately-excited motor, the induced e.m.f. in the armature falls from 220 V to 190 V in 30 seconds on disconnecting the armature from the supply. The same fall takes place in 20 seconds if, immediately after disconnection, armature is connected to a resistance which takes 10 A (average) during this fall. Find stray losses of the motor.*

(Adv. Elect. Machines, A.M.I.E. Sec. B, 1992)

Solution. Let W = stray losses (mechanical and magnetic losses)

Average voltage across resistance = (200 + 190)/2 = 195 V, Average current = 10 A

$\therefore$ Power absorbed W' = 1950 W

Using the relation $\frac{W}{W'} = \frac{t_2}{t_1 - t_2}$; we get $W = 1950 \times \frac{20}{30 - 20}$ = **3,900 watt**

Example 31.17. *In a retardation test on a d.c. motor, with its field normally excited, the speed fell from 1525 to 1475 r.p.m. in 25 seconds. With an average load of 1.0 kW supplied by the armature, the same speed drop occurred in 20 seconds. Find out the moment of inertia of the rotating parts in kg.m^2.*

(Electrical Machines-III, Gujarat Univ. 1984)

Solution. As seen from Art. 31.8 (*ii*) (*b*).

$W' = \left(\frac{2\pi}{60}\right)^2 I.N.\frac{dN}{dt}$. Also $W = W' \times \frac{t_2}{t_1 - t_2}$

Here, W' = 1 kW = 1000 W, t_1 = 25 second, t_2 = 20 second

$\therefore \quad W = 1000 \times 20/(25 - 20) = 4000$ W

Now, $N = 1500$ r.p.m. (average speed) ; $dN = 1525 - 1475 = 50$ r.p.m. ; $dt = 25$

$\therefore \quad 4000 = (2\pi/60)^2 I.1500 \times 50/25 \quad \therefore \quad I =$ **121.8 kg.m^2.**

Example 31.18. *A retardation test is made on a separately-excited d.c. machine as a motor. The induced voltage falls from 240 V to 225 V in 25 seconds on opening the armature circuit and 6 seconds on suddenly changing the armature connection from supply to a load resistance taking 10 A (average). Find the efficiency of the machines when running as a motor and taking a current of 25 A on a supply of 250 V. The resistance of its armature is 0.4 Ω and that of its field winding is 250 Ω.*

(Elect. Technology, Allahabad Univ. 1991)

Solution. Average voltage across load

$= (240 + 225)/2 = 232.5$ V ; $I_{av} = 10$ A

$\therefore$ Power absorbed $W' = 232.5 \times 10 = 2{,}325$ W

and $t_1 = 30$ second, $t_2 = 6$ second ; W = stray loss

Using $\dfrac{W}{W'} = \dfrac{t_2}{t_1 - t_2} =$ **734.1 W**, we get

Stray losses $W = 2325 \times \dfrac{6}{25 - 6} =$ **734.1 W**

Input current $= 25$ A ; $I_{sh} = 250/250 = 1$ A ; $I_a = 25 - 1 = 24$ A

Armature Cu loss $= 24^2 \times 0.4 = 230.4$ W ; Shunt Cu loss $= 250 \times 1 = 250$ W

$\therefore$ Total losses $= 734.1 + 230.4 + 250 = 1{,}215$ W (approx.)

Input $= 250 \times 25 = 6{,}250$ W ; Output $= 6{,}250 - 1{,}215 = 5{,}035$ W

$\therefore \quad \eta = 5{,}035/6{,}250 = 0.806$ or **80.6%**

Example 31.19. *A retardation test is carried out on a 1000 r.p.m. d.c. machine. The time taken for the speed to fall from 1030 r.p.m. to 970 r.p.m. is :*

(a) *36 seconds with no excitation*

(b) *15 seconds with full excitation and*

(c) *9 seconds with full excitation and the armature supplying an extra load of 10 A at 219 V.*

Calculate (i) the moment of inertia of the armature in kg. m^2 (ii) iron losses and (iii) the mechanical losses at the mean speed of 1000 r.p.m.

Solution. It should be noted that

(i) when armature slows down with no excitation, its kinetic energy is used to overcome mechanical losses only ; because due to the absence of flux, there is no iron loss.

(ii) with excitation, kinetic energy is used to supply mechanical and iron losses collectively known as stray losses.

(iii) If I is taken in kg-m^2 unit, then rate of loss of energy is in watts.

Mechanical loss $W_m = \left(\dfrac{2\pi}{60}\right)^2 I.N.\dfrac{dN}{dt}$ ...Art. 31.8

Here $dN = 1030 - 970 = 60$ r.p.m., $dt = 36$ seconds, $N = 1000$ r.p.m.

$W_m = \left(\dfrac{2\pi}{60}\right)^2 I.N.\dfrac{60}{36}$ ...***(i)***

Similarly $W_s = \left(\dfrac{2\pi}{60}\right)^2 I.N.\dfrac{60}{15}$ ...***(ii)***

Also $W_s = W' \dfrac{t_2}{t_1 - t_2} = 219 \times 10 \times \dfrac{9}{15 - 9} = 3{,}285$ W

Using equation (*ii*), we get

$$3{,}285 = \left(\frac{2\pi}{60}\right)^2 \times I \times 1000 \times \frac{60}{15}$$

(*i*) $\therefore \quad I = 75$ **kg.m^2**

Dividing (*i*) by (*ii*), we get $\frac{W_m}{W_s} = \frac{15}{36}$

(*ii*) $\therefore \quad W_m = 3{,}285 \times 15/36 =$ **1,369 W**

(*iii*) $\therefore$ Iron losses $= W_s - W_m = 3{,}285 - 1{,}369 =$ **1,916 W**

31.9. Field's Test for Series Motor

This test is applicable to two similar ***series motors***. Series motors which are mainly used for traction work are easily available in pairs. The two machines are coupled mechanically.

One machine runs normally as a motor and drives generator whose output is wasted in a variable load R (Fig. 31.14). Iron and friction losses of two machines are made equal (*i*) by joining the series field winding of the generator in the motor armature circuit so that both

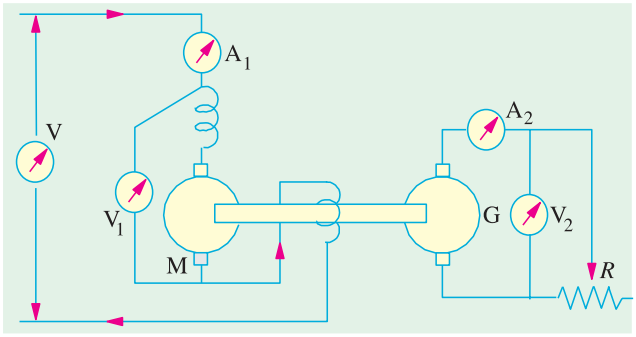


Fig. 31.14



Micro series motors

machines are equally excited and (*ii*) by running them at equal speed. Load resistance R is varied till the motor current reaches its full-load value indicated by ammeter A_1. After this adjustment for full-load current, different ammeter and voltmeter readings are noted.

Let V = supply voltage ; I_1 = motor current ; V_2 = terminal p.d. of generator ; I_2 = load current.

$\therefore$ Intake of the whole set $= VI_1$; output $= V_2I_2$.

Total losses in the set, $W_t = VI_1 - V_2I_2$

Armature and field Cu losses $W_{cu} = (R_a + 2R_{se})I_1^2 + I_2^2R_a$

where R_a = hot armature resistance of each machine

R_{se} = hot series field resistance of each machine

$\therefore$ Stray losses for the set $= W_t - W_{cu}$

Stray losses per machine $W_s = \frac{W_t - W_{cu}}{2}$

Stray losses are equally divided between the machines because of their equal excitation and speed.

Motor Efficiency

Motor input $= V_1I_1$

Motor losses = armature + field Cu losses + stray losses

$= (R_a + R_{se})I_1^2 + W_s = W_m$ (say)

$$\eta_m = \frac{V_1I_1 - W_m}{V_1I_1}$$

Generator Efficiency

The generator efficiency will be of little use because it is running under abnormal conditions of *separate* excitation. However, the efficiency under these unusual conditions can be found if desired.

Generator output = $V_2 I_2$

Field Cu loss = $I_1^2 R_{se}$ ($\because$ Motor current is passing through it.)

Armature Cu loss = $I_2^2 R_a$; Stray losses = W_s

Total losses = $I_1^2 R_{se} + I_2^2 R_a + W_s = W_g$ (say)

$$\eta_g = \frac{V_2 I_2}{V_2 I_2 + W_g}$$

It should be noted that although the two machines are mechanically coupled yet it is not a regenerative method, because the generator output is wasted instead of being fed back into the motor as in Hopkinson's (back-to-back) test.

Example 31.20. *A test on two coupled similar tramway motors, with their fields connected in series, gave the following results when one machine acted as a motor and the other as a generator.*

Motor : Armature current = 56 A ; Armature voltage = 590 V

Voltage drop across field winding = 40 V

Generator :

Armature current = 44 A ; Armature voltage = 400 V

Field voltage drop = 40 V ; Resistance of each armature = 0.3 Ω

Calculate the efficiency of the motor and gearing at this load.

(Elect. Machinery-II, Nagpur Univ. 1992 & JNTU, Hyderabad, 2000)

Solution. The connection for the two machines are shown in Fig. 31.15.

Total input = $630 \times 56 = 35{,}280$ W

Output = $400 \times 44 = 17{,}600$ W

Total losses in the two machines are

= $35{,}280 - 17{,}600 = 17{,}680$ W

Series field resistance

R_{se} = 40/56

= 0.714 Ω

Total Cu loss = $(0.3 + 2 \times 0.714) \times 56^2 + 44^2 \times 0.3 = 5{,}425 + 581 = 6{,}006$ W

Stray losses of the set= 17,680 – 6,006

= 11,674 W

$\therefore$ Stray losses/machine= 11,674/2 = 5,837 W

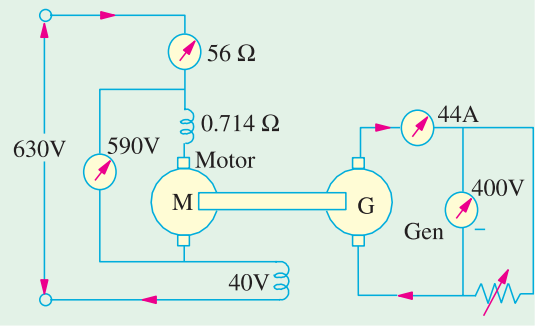


Fig. 31.15

Motor Efficiency

Motor armature input = arm. voltage × motor current = $590 \times 56 = 33{,}040$ W

Armature circuit Cu loss

= $(0.3 + 0.714) \times 56^2 = 3{,}180$ W

Stray loss = 5,837 W – found above

Total losses = 3,180 + 5,837 = 9,017 W, Output = 33,040 – 9,017 = 24,023 W

$\therefore$ η_m = 24,023/33,040 = 0.727 or **72.7%**

Generator Efficiency

Armature Cu loss = $44^2 \times 0.3 = 581$ W, Series field Cu loss = $40 \times 56 = 2{,}240$ W

Stray losses = 5,837 W ; Total losses = 581 + 2,240 + 5,837 = 8,658 W

Output = $400 \times 44 = 17{,}600$ W

$\therefore \quad \eta_g = 17{,}600/(17{,}600 + 8{,}658) = 0.67$ or 67%

Tutorial Problem No. 31.1

1. A 500-V, shunt motor takes a total current of 5 A when running unloaded. The resistance of armature circuit is 0.25 Ω and the field resistance is 125 Ω. Calculate the efficiency and output when the motor is loaded and taking a current of 100 A. **[90.4% ; 45.2 kW]**

2. A d.c. shunt motor rated at 12.5 kW output runs at no-load at 1000 r.p.m. from a 250-V supply consuming an input current of 4 A. The armature resistance is 0.5 Ω and shunt field resistance is 250 Ω. Calculate the efficiency of the machine when delivering full-load output of 12.5 kW while operating at 250 V. **[81.57%]** (*Elect. Technology-I Madras Univ. 1979*)

3. The following results were obtained during Hopkinson's test on two similar 230-V machines; armature currents 37 A and 30 A; field currents 0.85 A and 0.8 A. Calculate the efficiencies of machines if each has an armature resistance of 0.33 Ω. **[Generator 87.9%, Motor 87.7%]**

4. In a Field's test on two 230-V, 1.492 kW mechanically-coupled similar series motors, the following figures were obtained. Each has armature and compole winding resistance of 2.4 Ω, series field resistance of 1.45 Ω and total brush drop of 2 V. The p.d. across armature and field was 230 V with a motor current of 10.1 A. The generator supplied a current of 8.9 A at a terminal p.d. of 161 V. Calculate the efficiency and output of the motor for this load. **[76.45%, 1.775 kW]**

5. Describe the Hopkinson's test for obtaining the efficiency of two similar shunt motors. The readings obtained in such a test were as follows ; line voltage 100 V ; motor current 30 A ; generator current 25 A ; armature resistance of each machine 0.25 Ω. Calculate the efficiency of each machine from these results, ignoring the field currents and assuming that their iron and mechanical losses are the same.

[Motor 90.05%, Generator 92.5%]

6. The Hopkinson's test on two similar d.c. shunt machines gave the following results :

Line voltage = 220 V ; line current excluding field currents = 40 A ; the armature current of motoring machine = 200 A ; field currents 6 A and 7 A. Calculate the efficiency of each of the machines at the given load conditions. The armature resistance of each of the machines is 0.05 Ω.

[η_m = 86.58% ; η_g = 86.3%] (*Electrical Engg-I, M.S. Univ. Baroda 1980*)

OBJECTIVE TEST – 31

1. One of the main advantages of Swinburne's test is that it
- (*a*) is applicable both to shunt and compound motors
- (*b*) needs one running test
- (*c*) is very economical and convenient
- (*d*) ignores any charge in iron loss

2. The main disadvantage of Hopkinson's test for finding efficiency of shunt d.c. motors is that it
- (*a*) requires full-load power
- (*b*) ignores any change in iron loss
- (*c*) needs one motor and one generator
- (*d*) requires two identical shunt machines

3. The most economical method of finding no-load losses of a large d.c. shunt motor is—test.
- (*a*) Hopkinson's
- (*b*) Swinburne's
- (*c*) retardation
- (*d*) Field's

4. Retardation test on a d.c. shunt motor is used for finding—losses.
- (*a*) stray
- (*b*) copper
- (*c*) friction
- (*d*) iron

5. The main thing common between Hopkinson's test and Field's test is that both
   (*a*) require two electrically-coupled series motors
   (*b*) need two similar mechanically-coupled motors
   (*c*) use negligible power
   (*d*) are regenerative tests

6. The usual test for determining the efficiency of a traction motor is the ..... test.
   (*a*) Field's
   (*b*) retardation
   (*c*) Hopkinson's
   (*d*) Swinburne's

ANSWERS

1. *c* **2.** *d* **3.** *b* **4.** *a* **5.** *b* **6.** *a*

QUESTIONS AND ANSWERS ON D.C. MOTORS

Q. 1. How may the direction of rotation of a d.c. motor be reversed ?

Ans. By reversing either the field current or current through the armature. Usually, reversal of current through the armature is adopted.

Q. 2. What will happen if both currents are reversed ?

Ans. The motor will run in the original direction.

Q. 3. What will happen if the field of a d.c. shunt motor is opened ?

Ans. The motor will achieve dangerously high speed and may destroy itself.

Q. 4. What happens if the direction of current at the terminals of a series motor is reversed ?

Ans. It does not reverse the direction of rotation of motor because current flows through the armature in the same direction as through the field.

Q. 5. Explain what happens when a d.c. motor is connected across an a.c. supply ?

Ans. **1.** Since on a.c. supply, reactance will come into the picture, the a.c. supply will be offered impedance (not resistance) by the armature winding. Consequently, with a.c. supply, current will be much less. The motor will run but it would not carry the same load as it would on d.c. supply.

2. There would be more sparking at the brushes.

3. Though motor armature is laminated as a rule, the field poles are not. Consequently, eddy currents will cause the motor to heat up and eventually burn on a.c. supply.

Q. 6. What will happen if a shunt motor is directly connected to the supply line ?

Ans. Small motors up to 1 kW rating may be line-started without any adverse results being produced. High rating motors must be started through a suitable starter in order to avoid the huge starting current which will

(i) damage the motor itself and ***(ii)*** badly affect the voltage regulation of the supply line.

Q. 7. What is the function of interpoles and how are interpole windings connected ?

Ans. Interpoles are small poles placed in between the main poles. Their function is to assist commutation by producing the auxiliary or commutating flux. Consequently, brush sparking is practically eliminated. Interpole windings are connected in series with the armature windings.

Q. 8. In rewinding the armature of a d.c. motor, progressive connections are changed to retrogressive ones. Will it affect the operation in any way ?

Ans. Yes. Now, the armature will rotate in the opposite direction.

Q. 9. A d.c. motor fails to start when switched on. What could be the possible reasons and remedies ?

Ans. Any one of the following reasons could be responsible :

1. Open-circuit in controller–should be checked for open starting resistance or open switch or open fuse.
2. Low terminal voltage–should be adjusted to name-plate value.
3. Overload–should be reduced if possible otherwise larger motor should be installed.
4. Excessive friction–bearing lubrication should be checked.

Q. 10. A d.c. motor is found to stop running after a short period of time. What do you think could be the reasons ? How would you remedy each ?

Ans. Possible causes are as under :

1. Motor not getting enough power–check voltage at motor terminals as well as fuses, clups and overload relay.
2. Weak or no field–in the case of adjustable-speed motors, check if rheostat is correctly set. Also, check field winding for any 'open'. Additionally, look for any loose winding or broken connection.
3. Motor torque insufficient for driving the given load–check line voltage with name-plate voltage. If necessary, use larger motor to match the load.

Q. 11. What are the likely causes if a d.c. motor is found to run too slow under load ? And the remedy ?

Ans.

1. Supply line voltage too low–remove any excessive resistance in supply line, connections or controller.
2. Brushes ahead of neutral–set them on neutral.
3. Overload–reduce it to allowable value or use larger motor.

Q. 12. Why does a d.c. motor sometime run too fast when under load ? Give different possible causes and their remedies.

Ans. Different possible causes are as under :

1. Weak field–remove any extra resistance in shunt field circuit. Also, check for 'grounds'.
2. Line voltage too high–reduce it to name-plate value.
3. Brushes back of neutral–set them on neutral.

Q. 13. Under what conditions is sparking produced at the brushes of a d.c. motor ? How would you remedy it ?

Ans.

1. Commutator in bad condition–clean and reset brushes.
2. commutator either eccentric or rough–grind and true the commutator. Also, undercut mica.
3. Excessive vibration–balance armature. Make sure that brushes ride freely in holders.
4. Brush-holding spring broken or sluggish–replace spring and adjust pressure to recommended value.
5. Motor overloaded–reduce load or install motor of proper rating.
6. Short-circuit in armature circuit–remove any metallic particles between commutator segments and check for short between adjacent commutator risers. Locate and repair internal armature short if any.

Q. 14. Sometimes a hissing noise (or brush chatter) is heard to emanate from the commutator end of a running d.c. motor. What could it be due to and how could it be removed ?

Ans. Any one of the following causes could produce brush chatter :

1. Excessive clearance of brush holders–adjust properly
2. Incorrect angle of brushes–adjust to correct value
3. Unsuitable brushes–replace them
4. High mica–undercut it
5. Wrong brush spring pressure–adjust to correct value.

Q. 15. What are the possible causes of excessive sparking at brushes in a d.c. motor ?

Ans.

1. Poor brush fit on commutator–sand-in the brushes and polish commutator.
2. Brushes binding in the brush holders–clean holders and brushes and remove any irregularities on surfaces of brush holders or rough spots on brushes.
3. Excessive or insufficient pressure on brushes–adjust pressure.
4. Brushes off neutral–set them on neutral.

Q. 16. Why does a d.c. motor sometime spark on light load ?

Ans. Due to the presence of paint spray, chemical, oil or grease etc. on commutator.

Q. 17. When is the armature of a d.c. motor likely to get over-heated ?

Ans.

1. When motor is over-loaded.
2. When it is installed at a place having restricted ventilation.
3. When armature winding is shorted.

Q. 18. What causes are responsible for over-heating of commutator in a d.c. motor ?

Ans. It could be due either to the brushes being off neutral or being under excessive spring pressure. Accordingly, brushes should be adjusted properly and the spring pressure should be reduced but not to the point where sparking is introduced.

GO To FIRST

CONTENTS

CHAPTER 32

TRANSFORMER

Learning Objectives

- Working Principle of Transformer
- Transformer Construction
- Core-type Transformers
- Shell-type Transformers
- E.M.F. Equation of Transformer
- Voltage Transformation Ratio
- Transformer with losses
- Equivalent Resistance
- Magnetic Leakage
- Transformer with Resistance and Leakage Reactance
- Total Approximate Voltage Drop in Transformer
- Exact Voltage Drop
- Separation of Core Losses
- Short-Circuit or Impedance Test
- Why Transformer Rating in KVA?
- Regulation of a Transformer
- Percentage Resistance, Reactance and Impedance
- Kapp Regulation Diagram
- Sumpner or Back-to-back-Test
- Efficiency of a Transformer
- Auto-transformer



To overcome losses, the electricity from a generator is passed through a step-up transformer, which increases the voltage. Throughout the distribution system, the voltages are changed using step-down transformers to voltages suitable to the applications at industry and homes.

CONTENTS

32.1. Working Principle of a Transformer

A transformer is a static (or stationary) piece of apparatus by means of which electric power in one circuit is transformed into electric power of the same frequency in another circuit. It can raise or lower the voltage in a circuit but with a corresponding decrease or increase in current. The physical basis of a transformer is ***mutual induction*** between two circuits linked by a common magnetic flux. In its simplest form, it consists of two inductive coils which are electrically separated but magnetically linked through a path of low reluctance as shown in Fig. 32.1. The two coils possess high mutual inductance. If one coil is connected to a source of alternating voltage, an alternating flux is set up in the laminated core, most of which is linked with the other coil in which it produces mutually-induced e.m.f. (according to Faraday's Laws of Electromagnetic Induction $e = MdI/dt$). If the second coil circuit is closed, a current flows in it and so electric energy is transferred (entirely magnetically) from the first coil to the second coil. The first coil, in which electric energy is fed from the a.c. supply mains, is called ***primary*** winding and the other from which energy is drawn out, is called ***secondary*** winding. In brief, a transformer is a device that

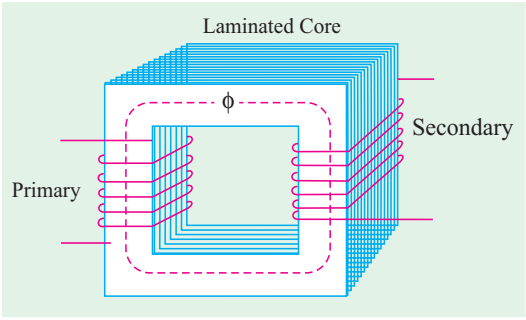
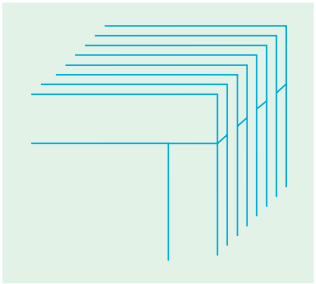


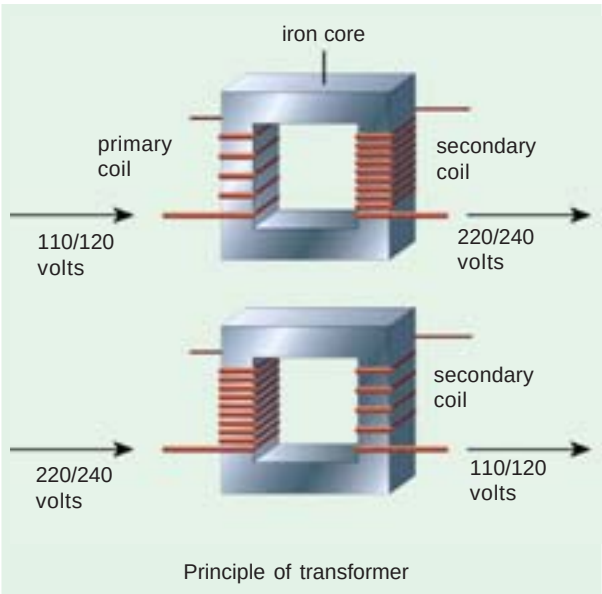
Fig. 32.1

1. transfers electric power from one circuit to another
2. it does so without a change of frequency
3. it accomplishes this by electromagnetic induction and
4. where the two electric circuits are in mutual inductive influence of each other.

32.2. Transformer Construction

The simple elements of a transformer consist of two coils having mutual inductance and a laminated steel core. The two coils are insulated from each other and the steel core. Other necessary parts are : some suitable container for assembled core and windings ; a suitable medium for insulating the core and its windings from its container ; suitable bushings (either of porcelain, oil-filled or capacitor-type) for insulating and bringing out the terminals of windings from the tank.

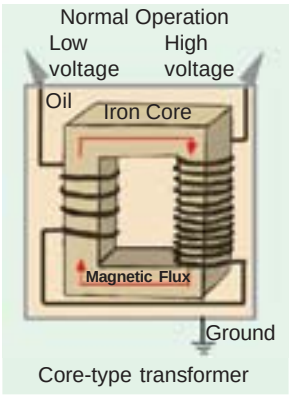

Fig. 32.2



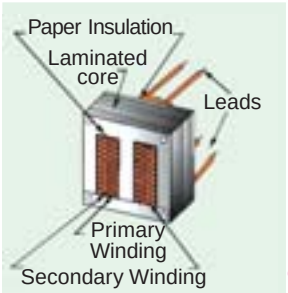
Principle of transformer

In all types of transformers, the core is constructed of transformer sheet steel laminations assembled to provide a continuous magnetic path with a minimum of air-gap included. The steel used is of high silicon content, sometimes heat treated to produce a high permeability and a low hysteresis loss at the

usual operating flux densities. The eddy current loss is minimised by laminating the core, the laminations being insulated from each other by a light coat of core-plate varnish or by an oxide layer on the surface. The thickness of laminations varies from 0.35 mm for a frequency of 50 Hz to 0.5 mm for a frequency of 25 Hz. The core laminations (in the form of strips) are joined as shown in Fig. 32.2. It is seen that the joints in the alternate layers are staggered in order to avoid the presence of narrow gaps right through the cross-section of the core. Such staggered joints are said to be 'imbricated'.



Core-type transformer



Shell-Type transformer

Constructionally, the transformers are of two general types, distinguished from each other merely by the manner in which the primary and secondary coils are placed around the laminated core. The two types are known as **(*i*) core-type** and **(*ii*) shell-type.** Another recent development is ***spiral-core* or *wound-core type***, the trade name being *spirakore* transformer.

In the so-called core type transformers, the ***windings surround a considerable part of the core*** whereas in shell-type transformers, the *core* ***surrounds a considerable portion of the windings*** as shown schematically in Fig. 32.3 (*a*) and (*b*) respectively.

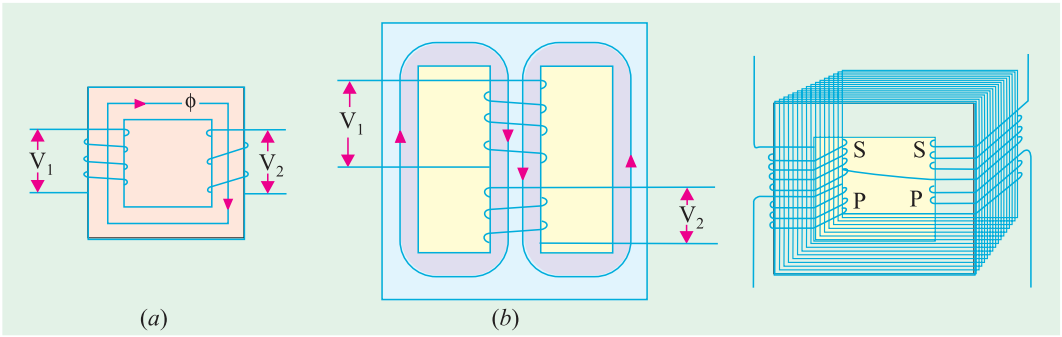


Fig. 32.3

Fig. 32.4

In the simplified diagram for the core type transformers [Fig. 32.3 (*a*)], the primary and secondary winding are shown located on the opposite legs (or limbs) of the core, but in actual construction, these are always interleaved to reduce leakage flux. As shown in Fig. 32.4, half the primary and half the secondary winding have been placed side by side or concentrically on each limb, not primary on one limb (or leg) and the secondary on the other.

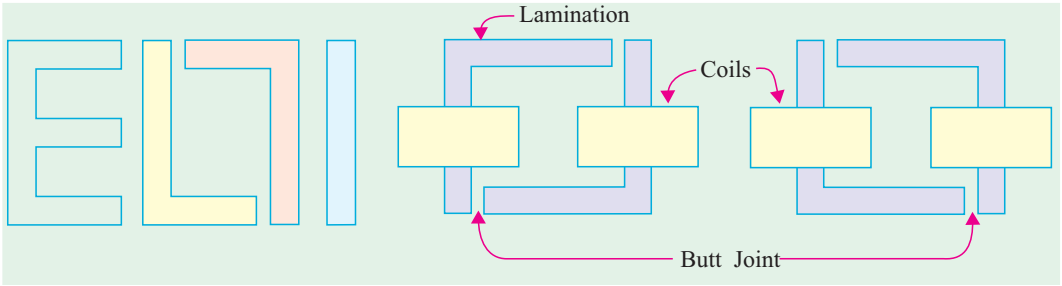


Fig. 32.5 Fig. 32.6

In both core and shell-type transformers, the individual laminations are cut in the form of long strips of *L*'s, *E*'s and *I*'s as shown in Fig. 32.5. The assembly of the complete core for the two types of transformers is shown in Fig. 32.6 and Fig. 32.7.

As said above, in order to avoid high reluctance at the joints where the laminations are butted against each other, the alternate layers are stacked differently to eliminate these joints as shown in Fig. 32.6 and Fig. 32.7.

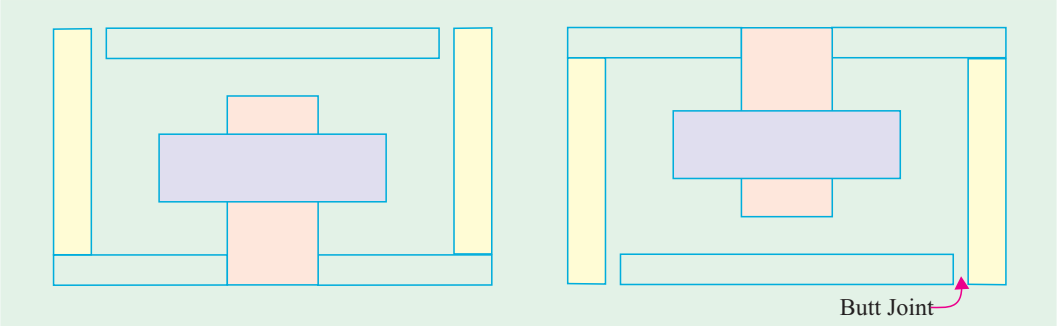
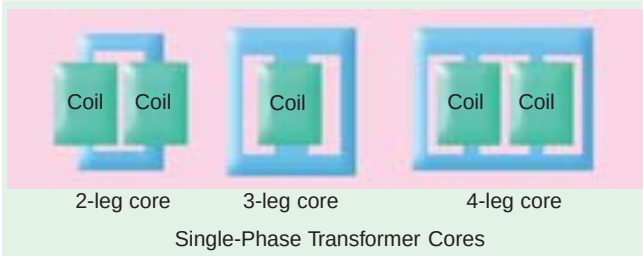


Fig. 32.7

32.3. Core-type Transformers

The coils used are form-wound and are of the cylindrical type. The general form of these coils may be circular or oval or rectangular. In small size core-type transformers, a simple rectangular core is used with cylindrical coils which are either circular or rectangular in form. But for large-size core-type transformers, round or circular cylindrical coils are used which are so wound as to fit over a cruciform core section as shown in Fig. 32.8(*a*). The circular cylindrical coils are used in most of the core-type transformers because of their mechanical strength. Such cylindrical coils are wound in helical layers with the different layers insulated from each other by paper, cloth, micarta board or cooling ducts. Fig. 32.8(*c*) shows the general arrangement of these coils with respect to the core. Insulating cylinders of fuller board are used to separate the cylindrical windings from the core and from each other. Since the low-voltage (LV) winding is easiest to insulate, it is placed nearest to the core (Fig. 32.8).



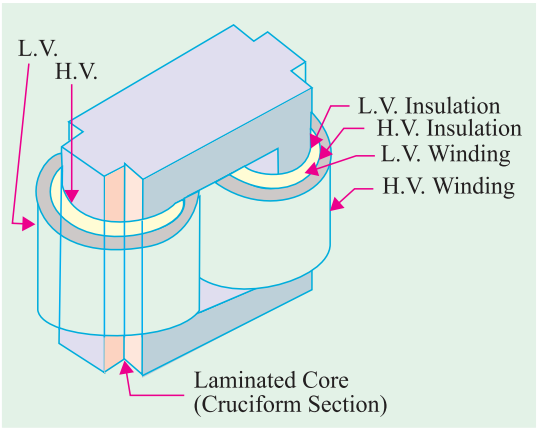


Fig. 32.8 (a)

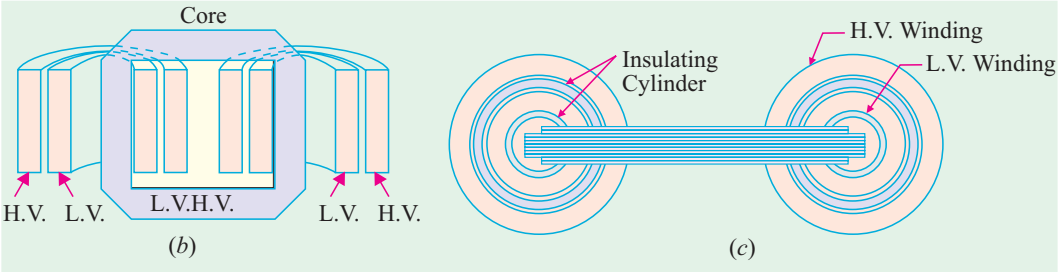


Fig. 32.8

Because of laminations and insulation, the net or effective core area is reduced, due allowance for which has to be made (Ex. 32.6). It is found that, in general, the reduction in core sectional area due to the presence of paper, surface oxide etc. is of the order of 10% approximately.

As pointed out above, rectangular cores with rectangular cylindrical coils can be used for small-size core-type transformers as shown in Fig. 32.9 (*a*) but for large-sized transformers, it becomes wasteful to use rectangular cylindrical coils and so circular cylindrical coils are preferred. For such purposes, square cores may be used as shown in Fig. 32.9 (*b*) where circles represent the tubular former carrying the coils. Obviously, a considerable amount of useful space is still wasted. A common improvement on square core is to employ cruciform core as in Fig. 32.9 (*c*) which demands, at least, two sizes of core strips. For very large transformers, further core-stepping is done as in Fig. 32.9 (*d*) where at least three sizes of core plates are necessary. Core-stepping not only gives high space factor but also results in reduced length of the mean turn and the consequent I^2R loss. Three stepped core is the one most commonly used although more steps may be used for very large transformers as in Fig. 32.9 (*e*). From the geometry of Fig. 32.9, it can be shown that maximum gross core section for Fig. 32.9 (*b*) is 0.5 d^2 and for Fig. 32.9 (*c*) it is 0.616 d^2 where d is the diameter of the cylindrical coil.

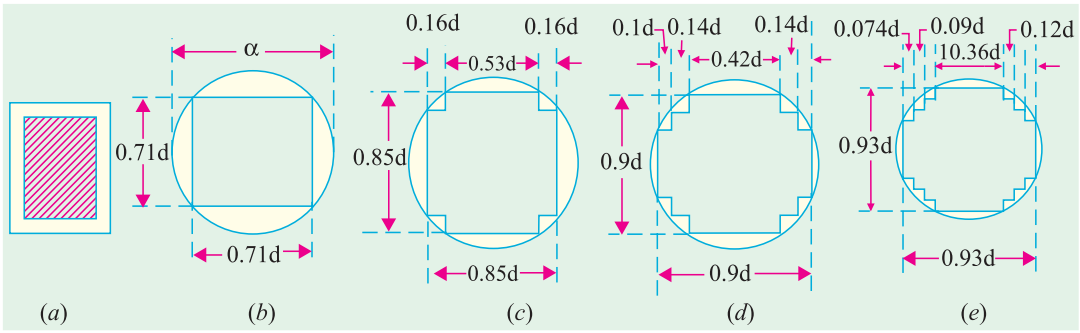


Fig. 32.9

32.4. Shell-type Transformers

In these case also, the coils are form-would but are multi-layer disc type usually wound in the form of pancakes. The different layers of such multi-layer discs are insulated from each other by paper. The complete winding consists of stacked discs with insulation space between the coils–the spaces forming horizontal cooling and insulating ducts. A shell-type transformer may have a simple rectangular form as shown in Fig. 32.10 or it may have distributed form as shown in Fig. 32.11.

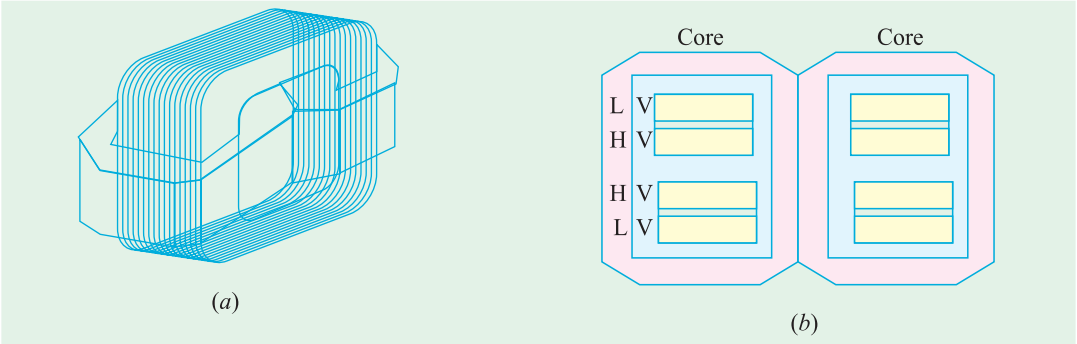


Fig. 32.10

A very commonly-used shell-type transformer is the one known as Berry Transformer–so called after the name of its designer and is cylindrical in form. The transformer core consists of laminations arranged in groups which radiate out from the centre as shown in section in Fig. 32.12.

It may be pointed out that cores and coils of transformers must be provided with rigid mechanical bracing in order to prevent movement and possible insulation damage. Good bracing reduces vibration and the objectionable noise–a humming sound–during operation.

The spiral-core transformer employs the newest development in core construction. The core is assembled of a continuous strip or ribbon of transformer steel wound in the form of a circular or elliptical cylinder. Such construction allows the core flux to follow the grain of the iron. Cold-rolled steel of high silicon content enables the designer to use considerably higher operating flux densities with lower loss per kg. The use of higher flux density reduces the weight per kVA. Hence, the advantages of such construction are ***(i)*** a relatively more rigid core ***(ii)*** lesser weight and size per kVA rating ***(iii)*** lower iron losses at higher operating flux densities and ***(iv)*** lower cost of manufacture.

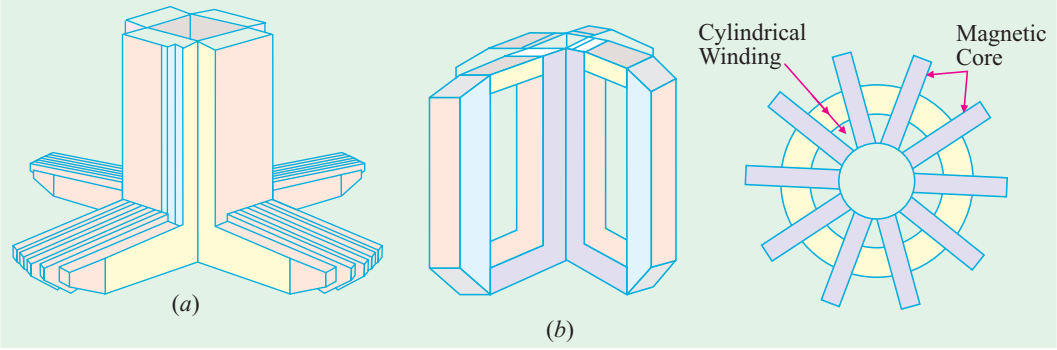


Fig. 32.11 **Fig. 32.12**

Transformers are generally housed in tightly-fitted sheet-metal ; tanks filled with special insulating oil*. This oil has been highly developed and its function is two-fold. By circulation, it not only keeps the coils reasonably cool, but also provides the transformer with additional insulation not obtainable when the transformer is left in the air.

In cases where a smooth tank surface does not provide sufficient cooling area, the sides of the tank are corrugated or provided with radiators mounted on the sides. Good transformer oil should be absolutely free from alkalies, sulphur and particularly from moisture. The presence of even an extremely small percentage of moisture in the oil is highly detrimental from the insulation viewpoint because it lowers the dielectric strength of the oil considerably. The importance of avoiding moisture in the transformer oil is clear from the fact that even an addition of 8 parts of water in 1,000,000 reduces the insulating quality of the oil to a value generally recognized as below standard. Hence, the tanks are sealed air-tight in smaller units. In the case of large-sized transformers where complete air-tight construction is impossible, chambers known as ***breathers*** are provided to permit the oil inside the tank to expand and contract as its temperature increases or decreases. The atmospheric moisture is entrapped in these breathers and is not allowed to pass on to the oil. Another thing to avoid in the oil is sledging which is simply the decomposition of oil with long and continued use. Sledging is caused principally by exposure to oxygen during heating and results in the formation of large deposits of dark and heavy matter that eventually clogs the cooling ducts in the transformer.

No other feature in the construction of a transformer is given more attention and care than the insulating materials, because the life on the unit almost solely depends on the quality, durability and handling of these materials. All the insulating materials are selected on the basis of their high quality and ability to preserve high quality even after many years of normal use.

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* Instead of natural mineral oil, now-a-days synthetic insulating fluids known as ASKARELS (trade name) are used. They are non-inflammable and, under the influence of an electric arc, do not decompose to produce inflammable gases. One such fluid commercially known as PYROCLOR is being extensively used because it possesses remarkable stability as a dielectric and even after long service shows no deterioration through sledging, oxidation, acid or moisture formation. Unlike mineral oil, it shows no rapid burning.

All the transformer leads are brought out of their cases through suitable bushings. There are many designs of these, their size and construction depending on the voltage of the leads. For moderate voltages, porcelain bushings are used to insulate the leads as they come out through the tank. In general, they look almost like the insulators used on the transmission lines. In high voltage installations, oil-filled or capacitor-type bushings are employed.

The choice of core or shell-type construction is usually determined by cost, because similar characteristics can be obtained with both types. For very high-voltage transformers or for multiwinding design, shell-type construction is preferred by many manufacturers. In this type, usually the mean length of coil turn is longer than in a comparable core-type design. Both core and shell forms are used and the selection is decided by many factors such as voltage rating, kVA rating, weight, insulation stress, heat distribution etc.

Another means of classifying the transformers is according to the type of cooling employed. The following types are in common use :

(a) oil-filled self-cooled ***(b) oil-filled water-cooled*** ***(c) air-blast type***

Small and medium size distribution transformers–so called because of their use on distribution systems as distinguished from line transmission–are of type (*a*). The assembled windings and cores of such transformers are mounted in a welded, oil-tight steel tank provided with steel cover. After putting the core at its proper place, the tank is filled with purified, high quality insulating oil. The oil serves to convey the heat from the core and the windings to the case from where it is radiated out to the surroundings. For small size, the tanks are usually smooth-surfaced, but for larger sizes, the cases are frequently corrugated or fluted to get greater heat radiation area without increasing the cubical capacity of the tank. Still larger sizes are provided with radiators or pipes.

Construction of very large self-cooled transformers is expensive, a more economical form of construction for such large transformers is provided in the oil-immersed, water-cooled type. As before, the windings and the core are immersed in the oil, but there is mounted near the surface of oil, a cooling coil through which cold water is kept circulating. The heat is carried away by this water. The largest transformers such as those used with high-voltage transmission lines, are constructed in this manner.

Oil-filled transformers are built for outdoor duty and as these require no housing other than their own, a great saving is thereby effected. These transformers require only periodic inspection.

For voltages below 25,000 V, transformers can be built for cooling by means of an air-blast. The transformer is not immersed in oil, but is housed in a thin sheet-metal box open at both ends through which air is blown from the bottom to the top by means of a fan or blower.

32.5. Elementary Theory of an Ideal Transformer

An ideal transformer is one which has no losses *i.e.* its windings have no ohmic resistance, there is no magnetic leakage and hence which has no I^2R and core losses. In other words, an ideal transformer consists of two purely inductive coils wound on a loss-free core. It may, however, be noted ***that it is impossible to realize such a transformer in practice, yet for convenience, we will start with such a transformer and step by step approach an actual transformer***.

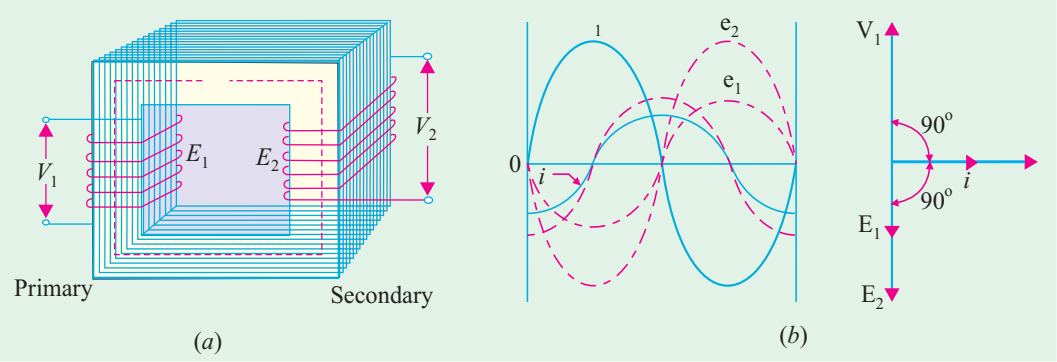
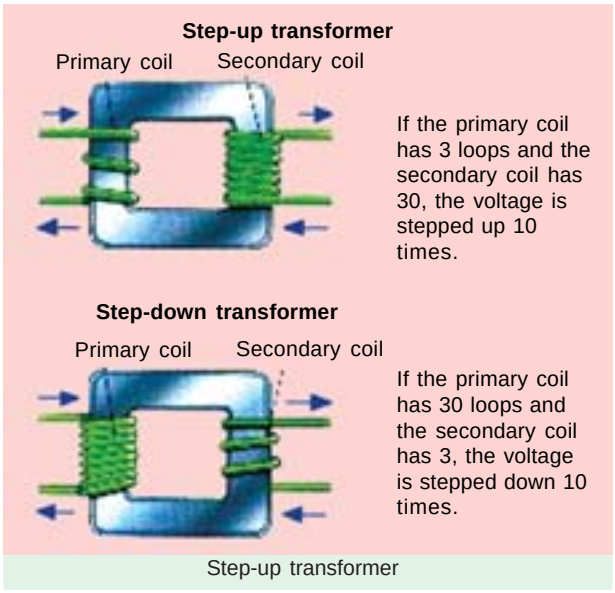


Fig. 32.13

Consider an ideal transformer [Fig. 32.13 (*a*)] whose secondary is open and whose primary is connected to sinusoidal alternating voltage V_1. This potential difference causes an alternating current to flow in the primary. Since the primary coil is purely inductive and there is no output (secondary being open) the primary draws the magnetising current I_μ only. The function of this current is merely to magnetise the core, it is small in magnitude and lags V_1 by 90°. This alternating current I_μ produces an alternating flux ϕ which is, at all times, proportional to the current (assuming permeability of the magnetic circuit to be constant) and, hence, is in phase with it. This changing flux is linked both with the primary and the secondary windings. Therefore, it produces self-induced e.m.f. in the primary. This *self-induced* e.m.f. E_1 is, at every instant, equal to and in opposition to V_1. It is also known as counter e.m.f. or back e.m.f. of the primary.

Similarly, there is produced in the secondary an induced e.m.f. E_2 which is known as *mutually* induced e.m.f. This e.m.f. is antiphase with V_1 and its magnitude is proportional to the rate of change of flux and the number of secondary turns.

The instantaneous values of applied voltage, induced e.m.fs, flux and magnetising current are shown by sinusoidal waves in Fig. 32.13 (*b*). Fig. 32.13 (*c*) shows the vectorial representation of the effective values of the above quantities.



Step-up transformer

32.6. E.M.F. Equation of a Transformer

Let N_1 = No. of turns in primary
N_2 = No. of turns in secondary
Φ_m = Maximum flux in core in webers
= $B_m \times A$
f = Frequency of a.c. input in Hz

As shown in Fig. 32.14, flux increases from its zero value to maximum value Φ_m in one quarter of the cycle *i.e.* in 1/4 f second.

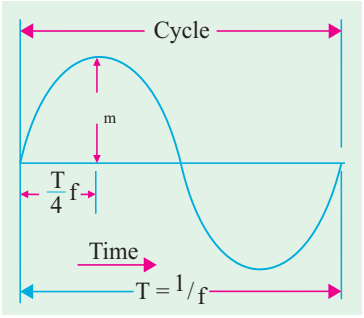


Fig. 32.14

$\therefore$ Average rate of change of flux = $\dfrac{\Phi_m}{1/4f}$

$= 4f\Phi_m$ Wb/s or volt

Now, rate of change of flux per turn means induced e.m.f. in volts.

$\therefore$ Average e.m.f./turn = $4f\Phi_m$ volt

If flux Φ varies *sinusoidally*, then r.m.s. value of induced e.m.f. is obtained by multiplying the average value with form factor.

$$\text{Form factor} = \frac{\text{r.m.s. value}}{\text{average value}} = 1.11$$

$\therefore$ r.m.s. value of e.m.f./turn = $1.11 \times 4f\Phi_m = 4.44f\Phi_m$ volt

Now, r.m.s. value of the induced e.m.f. in the whole of primary winding

= (induced e.m.f/turn) × No. of primary turns

$E_1 = 4.44f N_1 \Phi_m = 4.44f N_1 B_m A$ ...(*i*)

Similarly, r.m.s. value of the e.m.f. induced in secondary is,

$$E_2 = 4.44\, f\, N_2\, \Phi_m = 4.44\, f\, N_2\, B_m A \quad \text{...(ii)}$$

It is seen from (*i*) and (*ii*) that $E_1/N_1 = E_2/N_2 = 4.44\, f\, \Phi_m$. It means that e.m.f./turn is the *same* in both the primary and secondary windings.

In an ideal transformer on no-load, $V_1 = E_1$ and $E_2 = V_2$ where V_2 is the terminal voltage (Fig. 32.15).

32.7 Voltage Transformation Ratio (K)

From equations (*i*) and (*ii*), we get

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = K$$

This constant K is known as voltage transformation ratio.

(*i*) If $N_2 > N_1$ *i.e.* $K > 1$, then transformer is called ***step-up*** transformer.

(*ii*) If $N_2 < N_1$ *i.e.* $K < 1$, then transformer is known as ***step-down*** transformer.

Again, for an *ideal* transformer, input *VA* = output *VA*.

$$V_1 I_1 = V_2 I_2 \text{ or } \frac{I_2}{I_1} = \frac{V_1}{V_2} = \frac{1}{K}$$

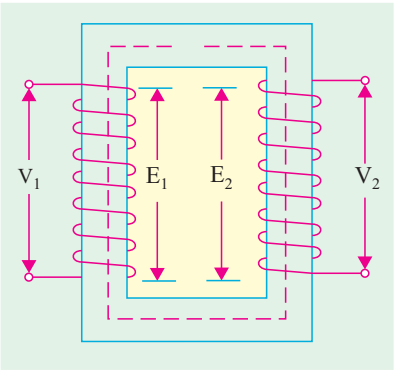


Fig. 32.15

Hence, currents are in the inverse ratio of the (voltage) transformation ratio.

Example 32.1. *The maximum flux density in the core of a 250/3000-volts, 50-Hz single-phase transformer is 1.2 Wb/m^2. If the e.m.f. per turn is 8 volt, determine*

(*i*) *primary and secondary turns* **(*ii*)** *area of the core.*

(Electrical Engg.-I, Nagpur Univ. 1991)

Solution. (*i*) $E_1 = N_1 \times$ e.m.f. induced/turn

$N_1 = 250/8 = \mathbf{32}$; $N_2 = 3000/8 = \mathbf{375}$

(*ii*) We may use $E_2 = -4.44\, f\, N_2\, B_m A$

$\therefore \quad 3000 = 4.44 \times 50 \times 375 \times 1.2 \times A$; $\mathbf{A = 0.03 m^2}$.

Example 32.2. *The core of a 100-kVA, 11000/550 V, 50-Hz, 1-ph, core type transformer has a cross-section of 20 cm × 20 cm. Find* **(*i*)** *the number of H.V. and L.V. turns per phase and* **(*ii*)** *the e.m.f. per turn if the maximum core density is not to exceed 1.3 Tesla. Assume a stacking factor of 0.9.*

What will happen if its primary voltage is increased by 10% on no-load ?

(Elect. Machines, A.M.I.E. Sec. B, 1991)

Solution. (*i*) $B_m = 1.3\ T$, $A = (0.2 \times 0.2) \times 0.9 = 0.036\ m^2$

$\therefore \quad 11{,}000 = 4.44 \times 50 \times N_1 \times 1.3 \times 0.036$, $N_1 = 1060$

$550 = 4.44 \times 50 \times N_2 \times 1.3 \times 0.036$; $N_2 = 53$

or, $N_2 = KN_1 = (550/11{,}000) \times 1060 = 53$

(*ii*) e.m.f./turn = 11,000/1060 = 10.4 V or 550/53 = 10.4 V

Keeping supply frequency constant, if primary voltage is increased by 10%, magnetising current will increase by much more than 10%. However, due to saturation, flux density will increase only marginally and so will the eddy current and hysteresis losses.

Example 32.3. *A single-phase transformer has 400 primary and 1000 secondary turns. The net cross-sectional area of the core is 60 cm^2. If the primary winding be connected to a 50-Hz supply at 520 V, calculate* **(*i*)** *the peak value of flux density in the core* **(*ii*)** *the voltage induced in the secondary winding.*

(Elect. Engg-I, Pune Univ. 1989)

Solution. $K = N_2/N_1 = 1000/400 = 2.5$

(i) $E_2/E_1 = K \quad \therefore \quad E_2 = KE_1 = 2.5 \times 520 =$ **1300 V**

(ii) $E_1 = 4.44\, f\, N_1\, B_m A$

or $520 = 4.44 \times 50 \times 400 \times B_m \times (60 \times 10^{-4}) \therefore B_m =$ **0.976 Wb/m^2**

Example 32.4. *A 25-kVA transformer has 500 turns on the primary and 50 turns on the secondary winding. The primary is connected to 3000-V, 50-Hz supply. Find the full-load primary and secondary currents, the secondary e.m.f. and the maximum flux in the core. Neglect leakage drops and no-load primary current.* **(Elect. & Electronic Engg., Madras Univ. 1985)**

Solution. $K = N_2/N_1 = 50/500 = 1/10$

Now, full-load $I_1 = 25{,}000/3000 =$ **8.33 A.** F.L. $I_2 = I_1/K = 10 \times 8.33 =$ **83.3 A**

e.m.f. per turn on primary side $= 3000/500 = 6$ V

$\therefore$ secondary e.m.f. $= 6 \times 50 =$ **300 V** (or $E_2 = KE_1 = 3000 \times 1/10 = 300$ *V*)

Also, $E_1 = 4.44\, f\, N_1 \Phi_m$; $3000 = 4.44 \times 50 \times 500 \times \Phi_m \therefore \Phi_m =$ **27 mWb**

Example 32.5. *The core of a three phase, 50 Hz, 11000/550 V delta/star, 300 kVA, core-type transformer operates with a flux of 0.05 Wb. Find*

(i) *number of H.V. and L.V. turns per phase.* **(ii)** *e.m.f. per turn*

(iii) *full load H.V. and L.V. phase-currents.* **(Bharathithasan Univ. April 1997)**

Solution. Maximum value of flux has been given as 0.05 Wb.

(ii) e.m.f. per turn $= 4.44\, f\, \phi_m$

$= 4.44 \times 50 \times 0.05 = 11.1$ volts

(i) Calculations for number of turns on two sides :

Voltage per phase on delta-connected primary winding = 11000 volts

Voltage per phase on star-connected secondary winding = 550/1.732 = 317.5 volts

T_1 = number of turns on primary, per phase

= voltage per phase/e.m.f. per turn

= 11000/11.1 = 991

T_2 = number of turns on secondary, per phase

= voltage per phase/e.m.f. per turn

= 317.5/11.1 = 28.6

Note : (i) Generally, Low-voltage-turns are calculated first, the figure is rounded off to next higher even integer. In this case, it will be 30. Then, number of turns on primary side is calculated by turns-ratio.

In this case, $T_1 = T_2 (V_1/V_2) = 30 \times 11000/317.5 = 1040$

This, however, reduces the flux and results into less saturation. This, in fact, is an elementary aspect in Design-calculations for transformers. (Explanation is added here only to overcome a doubt whether a fraction is acceptable as a number of L.V. turns).

(ii) Full load H.V. and L.V. phase currents :

Output per phase $= (300/3) = 100$ kVA

H.V. phase-current $= \dfrac{100 \times 1000}{11{,}000} = 9.1$ Amp

L.V. phase-current $= (100 \times 1000/317.5) = 315$ Amp

Example 32.6. *A single phase transformer has 500 turns in the primary and 1200 turns in the secondary. The cross-sectional area of the core is 80 sq. cm. If the primary winding is connected to a 50 Hz supply at 500 V, calculate* ***(i)*** *Peak flux-density, and* ***(ii)*** *Voltage induced in the secondary.*

(Bharathiar University November 1997)

Solution. From the e.m.f. equation for transformer,

$$500 = 4.44 \times 50 \times \phi_m \times 500$$

$$\phi_m = 1/222 \text{ Wb}$$

(*i*) Peak flux density, $B_m = \phi_m / (80 \times 10^{-4}) = 0.563 \text{ wb/m}^2$

(*ii*) Voltage induced in secondary is obtained from transformation ratio or turns ratio

$$\frac{V_2}{V_1} = \frac{N_2}{N_1}$$

or $V_2 = 500 \times 1200/500 = 1200$ volts

Example 32.7. *A 25 kVA, single-phase transformer has 250 turns on the primary and 40 turns on the secondary winding. The primary is connected to 1500-volt, 50 Hz mains. Calculate* ***(i)*** *Primary and Secondary currents on full-load,* ***(ii)*** *Secondary e.m.f.,* ***(iii)*** *maximum flux in the core.* **(Bharathiar Univ. April 1998)**

Solution. (*i*) If V_2 = Secondary voltage rating, = secondary e.m.f.,

$$\frac{V_2}{1500} = \frac{40}{250}, \text{ giving } V_2 = 240 \text{ volts}$$

(*ii*) Primary current = 25000/1500 = 16.67 amp

Secondary current = 25000/240 = 104.2 amp

(*iii*) If ϕ_m is the maximum core-flux in *Wb*,

$$1500 = 4.44 \times 50 \times \phi_m \times 250, \text{ giving } \phi_m = 0.027 \text{ Wb or } 27 \text{ mWb}$$

Example 32.8. *A single-phase, 50 Hz, core-type transformer has square cores of 20 cm side. Permissible maximum flux-density is 1 Wb/m^2. Calculate the number of turns per Limb on the High and Low-voltage sides for a 3000/220 V ratio.* **(Manonmaniam Sundaranar Univ. April 1998)**

Solution. E.M.F. equation gives the number of turns required on the two sides. We shall first calculate the L.V.-turns, round the figure off to the next higher even number, so that given maximum flux density is not exceeded. With the corrected number of L.V. turns, calculate H.V.-turns by transformation ratio. Further, there are two Limbs. Each Limb accommodates half-L.V. and half H.V. winding from the view-point of reducing leakage reactance.

Starting with calculation for L.V. turns, T_2,

$4.44 \times 50 \times [(20 \times 20 \times 10^{-4}) \times 1] \times T_2 = 220$

$$T_2 = 220/8.88 = 24.77$$

Select $T_2 = 26$

$$T_1/T_2 = V_1/V_2$$

$T_1 = 26 \times 3000/220 = 354$, selecting the nearest even integer.

Number of H.V. turns on each Limb = 177

Number of L.V. turns on each Limb = 13

32.8. Transformer with Losses but no Magnetic Leakage

We will consider two cases **(*i*)** when such a transformer is on no load and **(*ii*)** when it is loaded.

32.9. Transformer on No-load

In the above discussion, we assumed an ideal transformer *i.e.* one in which there were no core losses and copper losses. But practical conditions require that certain modifications be made in the foregoing

theory. When an ***actual*** transformer is put on load, there is iron loss in the core and copper loss in the windings (both primary and secondary) and these losses are not entirely negligible.

Even when the transformer is on no-load, the primary input current is not wholly reactive. The primary input current under no-load conditions has to supply **(*i*)** iron losses in the core *i.e.* hysteresis loss and eddy current loss and **(*ii*)** a very small amount of copper loss in primary (there being no Cu loss in secondary as it is open). Hence, the no-load primary input current I_0 is not at 90° behind V_1 but lags it by an angle $\phi_0 <$ 90°. No-load input power

$$W_0 = V_1 I_0 \cos \phi_0$$

where $\cos \phi_0$ is primary power factor under no-load conditions. No-load condition of an actual transformer is shown vectorially in Fig. 32.16.

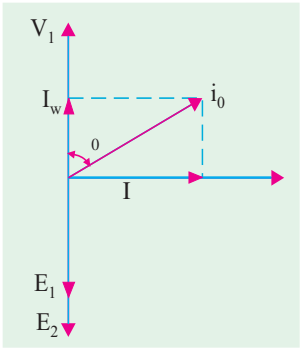


Fig. 32.16

As seen from Fig. 32.16, primary current I_0 has two components :

(*i*) One in phase with V_1. This is known as ***active*** or ***working*** or ***iron*** loss component I_w because it mainly supplies the iron loss plus small quantity of primary Cu loss.

$$I_w = I_0 \cos \phi_0$$

(*ii*) The other component is in quadrature with V_1 and is known as ***magnetising*** component I_μ because its function is to sustain the alternating flux in the core. It is wattless.

$$I_\mu = I_0 \sin \phi_0$$

Obviously, I_0 is the vector sum of I_w and I_μ, hence $I_0 = (I_\mu^2 + I_\omega^2)$.

The following points should be noted carefully :

1. The no-load primary current I_0 is very small as compared to the full-load primary current. It is about 1 per cent of the full-load current.

2. Owing to the fact that the permeability of the core varies with the instantaneous value of the exciting current, the wave of the exciting or magnetising current is not truly sinusoidal. As such it should not be represented by a vector because only sinusoidally varying quantities are represented by rotating vectors. But, in practice, it makes no appreciable difference.

3. As I_0 is very small, the no-load primary Cu loss is negligibly small which means ***that no-load primary input is practically equal to the iron loss in the transformer***.

4. As it is principally the core-loss which is responsible for shift in the current vector, angle ϕ_0 is known as ***hysteresis angle of advance***.

Example 32.9. ***(a)*** *A 2,200/200-V transformer draws a no-load primary current of 0.6 A and absorbs 400 watts. Find the magnetising and iron loss currents.*

(b) *A 2,200/250-V transformer takes 0.5 A at a p.f. of 0.3 on open circuit. Find magnetising and working components of no-load primary current.*

Solution. **(*a*)** Iron-loss current

$$= \frac{\text{no-load input in watts}}{\text{primary voltage}} = \frac{400}{2,200} = \mathbf{0.182\ A}$$

Now $I_0^2 = I_w^2 + I_\mu^2$

Magnetising component $I_\mu = \sqrt{(0.6^2 - 0.182)^2} = \mathbf{0.572\ A}$

The two components are shown in Fig. 29.15.

(b) $I_0 = 0.5$ A, $\cos \phi_0 = 0.3 \therefore I_w = I_0 \cos \phi_0 = 0.5 \times 0.3 = 0.15$ A

$$I_\mu = \sqrt{0.5^2 - 0.15^2} = 0.476 \text{ A}$$

Example 32.10. *A single-phase transformer has 500 turns on the primary and 40 turns on the secondary winding. The mean length of the magnetic path in the iron core is 150 cm and the joints are equivalent to an air-gap of 0.1 mm. When a p.d. of 3,000 V is applied to the primary, maximum flux density is 1.2 Wb/m^2. Calculate (a) the cross-sectional area of the core (b) no-load secondary voltage (c) the no-load current drawn by the primary (d) power factor on no-load. Given that AT/cm for a flux density of 1.2 Wb/m^2 in iron to be 5, the corresponding iron loss to be 2 watt/kg at 50 Hz and the density of iron as 7.8 gram/cm^3.*

Solution. (a) $3{,}000 = 4.44 \times 50 \times 500 \times 1.2 \times A \therefore A = 0.0225 \text{ m}^2 = \mathbf{225 \text{ cm}^2}$

This is the net cross-sectional area. However, the gross area would be about 10% more to allow for the insulation between laminations.

(b) $K = N_2/N_1 = 40/500 = 4/50$

$\therefore$ N.L. secondary voltage $= KE_1 = (4/50) \times 3000 = \mathbf{240 \text{ V}}$

(c) AT per cm $= 5 \therefore AT$ for iron core $= 150 \times 5 = \mathbf{750}$

$$AT \text{ for air-gap} = Hl = \frac{B}{\mu_0} \times l = \frac{1.2}{4\pi \times 10^{-7}} \times 0.0001 = 95.5$$

Total AT for given $B_{max} = 750 + 95.5 = 845.5$

Max. value of magnetising current drawn by primary $= 845.5/500 = 1.691$ A

Assuming this current to be sinusoidal, its r.m.s. value is $I_\mu = 1.691/\sqrt{2} = 1.196$ A

Volume of iron = length $\times$ area $= 150 \times 225 = 33{,}750 \text{ cm}^3$

Density $= 7.8 \text{ gram/cm}^3 \therefore$ Mass of iron $= 33{,}750 \times 7.8/1000 = 263.25$ kg

Total iron loss $= 263.25 \times 2 = 526.5$ W

Iron loss component of no-load primary current I_0 is $I_w = 526.5/3000 = 0.176$ A

$$I_0 = \sqrt{I_u^2 + I_w^2} = \sqrt{1.196^2 + 0.176^2} = \mathbf{0.208 \text{ A}}$$

(d) Power factor, $\cos \phi_0 = I_w/I_0 = 0.176/1.208 = \mathbf{0.1457}$

Example 32.11. *A single-phase transformer has 1000 turns on the primary and 200 turns on the secondary. The no load current is 3 amp. at a p.f. of 0.2 lagging. Calculate the primary current and power-factor when the secondary current is 280 Amp at a p.f. of 0.80 lagging.*

(Nagpur University, November 1997)

Solution. V_2 is taken as reference. $\cos^{-1} 0.80 = 36.87°$

$$I_2 = 280 \angle -36.87° \text{ amp}$$

$$I'_2 = (280/5) \angle -36.87° \text{ amp}$$

$$\phi = \cos^{-1} 0.20 = 78.5°, \sin \phi = 0.98$$

$$I_1 = I_0 + I'_2 = 3(0.20 - j\,0.98) + 56\,(0.80 - j\,0.60)$$

$$= 0.6 - j\,2.94 + 44.8 - j\,33.6$$

$$= 45.4 - j\,2.94 + 44.8 - j\,33.6$$

$$= 45.4 - j\,36.54 = 58.3 \angle 38.86°$$

Thus I lags behind the supply voltage by an angle of 38.86°.

Tutorial Problems 32.1

1. The number of turns on the primary and secondary windings of a 1–ϕ transformer are 350 and 35 respectively. If the primary is connected to a 2.2 kV, 50-Hz supply, determine the secondary voltage on no-load. **[220 V] (*Elect. Engg.-II, Kerala Univ. 1980*)**
2. A 3000/200-V, 50-Hz, 1-phase transformer is built on a core having an effective cross-sectional area of 150 cm^2 and has 80 turns in the low-voltage winding. Calculate
   (*a*) the value of the maximum flux density in the core
   (*b*) the number of turns in the high-voltage winding. **[(*a*) 0.75 Wb/m^2 (*b*) 1200]**
3. A 3,300/230-V, 50-Hz, 1-phase transformer is to be worked at a maximum flux density of 1.2 Wb/m^2 in the core. The effective cross-sectional area of the transformer core is 150 cm^2. Calculate suitable values of primary and secondary turns. **[830; 58]**
4. A 40-kVA, 3,300/240-V, 50 Hz, 1-phase transformer has 660 turns on the primary. Determine
   (*a*) the number of turns on the secondary
   (*b*) the maximum value of flux in the core
   (*c*) the approximate value of primary and secondary full-load currents.
   Internal drops in the windings are to be ignored. **[(*a*) 48 (*b*) 22.5 mWb (*c*) 12.1 A; 166.7 A]**
5. A double-wound, 1-phase transformer is required to step down from 1900 V to 240 V, 50-Hz. It is to have 1.5 V per turn. Calculate the required number of turns on the primary and secondary windings respectively.
   The peak value of flux density is required to be not more than 1.2 Wb/m^2. Calculate the required cross-sectional area of the steel core. If the output is 10 kVA, calculate the secondary current.
   [1,267; 160; 56.4 cm^2; 41.75 A]
6. The no-load voltage ratio in a 1-phase, 50-Hz, core-type transformer is 1,200/440. Find the number of turns in each winding if the maximum flux is to be 0.075 Wb. **[24 and 74 turns]**
7. A 1-phase transformer has 500 primary and 1200 secondary turns. The net cross-sectional area of the core is 75 cm^2. If the primary winding be connected to a 400-V, 50 Hz supply, calculate.
   (*i*) the peak value of flux density in the core and (*ii*) voltage induced in the secondary winding.
   [0.48 Wb/m^2; 60 V]
8. A 10-kVA, 1-phase transformer has a turn ratio of 300/23. The primary is connected to a 1500-V, 60 Hz supply. Find the secondary volts on open-circuit and the approximate values of the currents in the two windings on full-load. Find also the maximum value of the flux. **[115 V; 6.67 A; 87 A; 11.75 mWb]**
9. A 100-kVA, 3300/400-V, 50 Hz, 1 phase transformer has 110 turns on the secondary. Calculate the approximate values of the primary and secondary full-load currents, the maximum value of flux in the core and the number of primary turns.
   How does the core flux vary with load ? **[30.3 A; 250 A; 16.4 mWb; 907]**
10. The no-load current of a transformer is 5.0 A at 0.3 power factor when supplied at 230-V, 50-Hz. The number of turns on the primary winding is 200. Calculate (*i*) the maximum value of flux in the core (*ii*) the core loss (*iii*) the magnetising current. **[5.18 mWb; 345 W; 4.77 A]**
11. The no-load current of a transformer is 15 at a power factor of 0.2 when connected to a 460-V, 50-Hz supply. If the primary winding has 550 turns, calculate
    (*a*) the magnetising component of no-load current
    (*b*) the iron loss
    (*c*) the maximum value of the flux in the core. **[(*a*) 14.7 A (*b*) 1,380 W (*c*) 3.77 mWb]**
12. The no-load current of a transformer is 4.0 A at 0.25 p.f. when supplied at 250-V, 50 Hz. The number of turns on the primary winding is 200. Calculate
    (*i*) the r.m.s. value of the flux in the core (assume sinusoidal flux)
    (*ii*) the core loss
    (*iii*) the magnetising current. **[(*i*) 3.96 mWb (*ii*) 250 W (*iii*) 3.87 A]**

13. The following data apply to a single- phase transformer:
output : 100 kVA, secondary voltage; 400 V; Primary turns: 200; secondary turns: 40; Neglecting the losses, calculate: (*i*) the primary applied voltage (*ii*) the normal primary and secondary currents (*iii*) the secondary current, when the load is 25 kW at 0.8 power factor.
***(Rajiv Gandhi Technical University, Bhopal 2000)* [(*i*) 2000 V, (*ii*) 50 amp, (*iii*) 78.125 amp]**

32.10. Transformer on Load

When the secondary is loaded, the secondary current I_2 is set up. The magnitude and phase of I_2 with respect to V_2 is determined by the characteristics of the load. Current I_2 is in phase with V_2 if load is non-inductive, it lags if load is inductive and it leads if load is capacitive.

The secondary current sets up its own m.m.f. ($=N_2I_2$) and hence its own flux Φ_2 which is in opposition to the main primary flux Φ which is due to I_0. The secondary ampere-turns $N_2\ I_2$ are known as ***demagnetising*** amp-turns. The opposing secondary flux Φ_2 weakens the primary flux Φ momentarily, hence primary back e.m.f. E_1 tends to be reduced. For a moment V_1 gains the upper hand over E_1 and hence causes more current to flow in primary.

Let the additional primary current be I_2'. It is known as ***load component of primary current***. This current is antiphase with I_2'. The additional primary m.m.f. $N_1\ I_2$ sets up its own flux Φ_2' which is in opposition to Φ_2 (but is in the same direction as Φ) and is equal to it in magnitude. Hence, the two cancel each other out. So, we find that the magnetic effects of secondary current I_2 are immediately neutralized by the additional primary current I_2' which is brought into existence exactly at the same instant as I_2. The whole process is illustrated in Fig. 32.17.

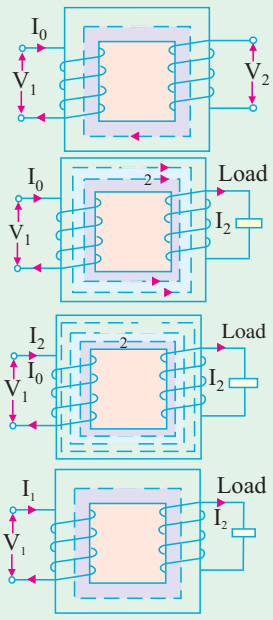


Fig. 32.17

Hence, whatever the load conditions, ***the net flux passing through the core is approximately the same as at no-load***. An important deduction is that due to the constancy of core flux at all loads, ***the core loss is also practically the same under all load conditions***.

As $\Phi_2 = \Phi_2' \quad \therefore \quad N_2I_2 = N_1I_2' \quad \therefore \quad I_2' = \dfrac{N_2}{N_1} \times I_2 = KI_2$

Hence, when transformer is on load, the primary winding has two currents in it; one is I_0 and the other is I_2' which is anti-phase with I_2 and K times in magnitude. ***The total primary current is the vector sum of I_0 and I_2'.***

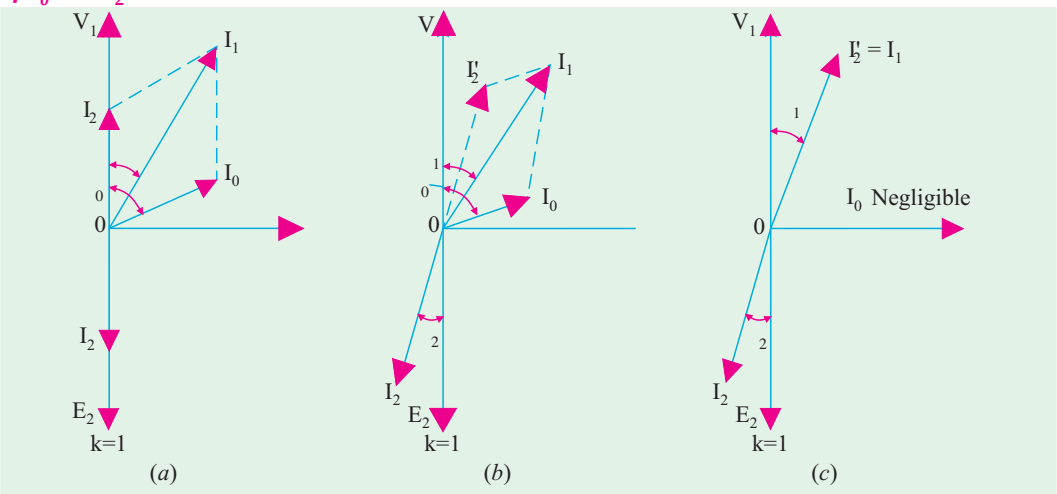


Fig. 32.18

In Fig. 32.18 are shown the vector diagrams for a load transformer when load is non-inductive and when it is inductive (a similar diagram could be drawn for capacitive load). Voltage transformation ratio of unity is assumed so that primary vectors are equal to the secondary vectors. With reference to Fig. 32.18 (*a*), I_2 is secondary current in phase with E_2 (strictly speaking it should be V_2). It causes primary current I_2' which is anti-phase with it and equal to it in magnitude ($\because K = 1$). Total primary current I_1 is the vector sum of I_0 and I_2' and lags behind V_1 by an angle ϕ_1.

In Fig. 32.18 (*b*) vectors are drawn for an inductive load. Here I_2 lags E_2 (actually V_2) by ϕ_2. Current I_2' is again antiphase with I_2 and equal to it in magnitude. As before, I_1 is the vector sum of I_2' and I_0 and lags behind V_1 by ϕ_1.

It will be observed that ϕ_1 is slightly greater than ϕ_2. But if we neglect I_0 as compared to I_2' as in Fig. 32.18 (*c*), then $\phi_1 = \phi_2$. Moreover, under this assumption

$$N_1 I_2' = N_2 I_1 = N_1 I_2 \quad \therefore \quad \frac{I_2'}{I_2} = \frac{I_1}{I_2} = \frac{N_2}{N_1} = K$$

It shows that under full-load conditions, the ratio of primary and secondary currents is constant. This important relationship is made the basis of current transformer–a transformer which is used with a low-range ammeter for measuring currents in circuits where the direct connection of the ammeter is impracticable.

Example 32.12. *A single-phase transformer with a ratio of 440/110-V takes a no-load current of 5A at 0.2 power factor lagging. If the secondary supplies a current of 120 A at a p.f. of 0.8 lagging, estimate the current taken by the primary.* **(Elect. Engg. Punjab Univ. 1991)**

Solution. $\cos\phi_2 = 0.8, \phi_2 = \cos^{-1}(0.8) = 36°54'$

$\cos\phi_0 = 0.2 \quad \therefore \quad \phi_0 = \cos^{-1}(0.2) = 78°30'$

Now $K = V_2/V_1 = 110/440 = 1/4$

$\therefore \quad I_2' = KI_2 = 120 \times 1/4 = 30$ A

$I_0 = 5$ A.

Angle between I_0 and I_2'

$= 78°30' - 36°54' = 41°36'$

Using parallelogram law of vectors (Fig. 32.19) we get

$$I_1 = \sqrt{(5^2 + 30^2 + 2 \times 5 \times 30 \times \cos 41°36')}$$

$$= \mathbf{34.45\ A}$$

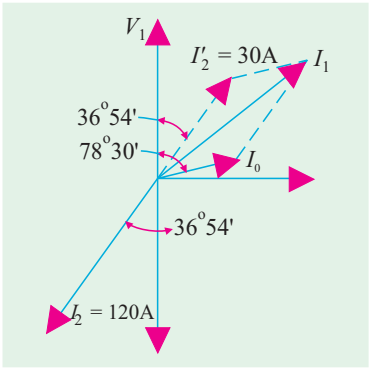


Fig. 32.19

The resultant current could also have been found by resolving I_2' and I_0 into their X and Y-components.

Example 32.13. *A transformer has a primary winding of 800 turns and a secondary winding of 200 turns. When the load current on the secondary is 80 A at 0.8 power factor lagging, the primary current is 25 A at 0.707 power factor lagging. Determine graphically or otherwise the no-load current of the transformer and its phase with respect to the voltage.*

Solution. Here $K = 200/800 = 1/4$; $I_2' = 80 \times 1/4 = 20$ A

$\phi_2 = \cos^{-1}(0.8) = 36.9°$; $\phi_1 = \cos^{-1}(0.707) = 45°$

As seen from Fig. 32.20, I_1 is the vector sum of I_0 and I_2'. Let I_0 lag behind V_1 by an angle ϕ_0.

$$I_0 \cos\phi_0 + 20 \cos 36.9° = 25 \cos 45°$$

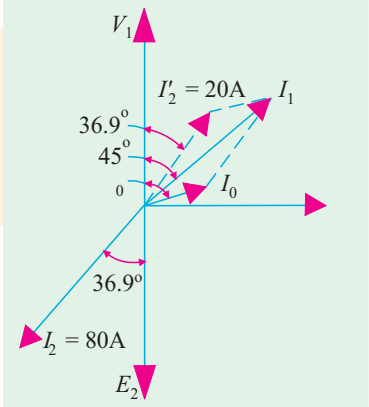


Fig. 32.20

$$\therefore \quad I_0 \cos \phi_2 = 25 \times 0.707 - 20 \times 0.8$$
$$= 1.675 \text{ A}$$
$$I_0 \sin \phi_0 + 20 \sin 36.9° = 25 \sin 45°$$
$$\therefore \quad I_0 \sin \phi_0 = 25 \times 0.707 - 20 \times 0.6$$
$$= 5.675 \text{ A}$$
$$\therefore \quad \tan \phi_0 = 5.675/1.675 = 3.388$$
$$\therefore \quad \phi_0 = \mathbf{73.3°}$$
$$\text{Now,} \quad I_0 \sin \phi_0 = 5.675$$
$$\therefore \quad I_0 = 5.675/\sin 73.3° = \mathbf{5.93 \text{ A}}$$

Example 32.14. *A single phase transformer takes 10 A on no load at p.f. of 0.2 lagging. The turns ratio is 4 : 1 (step down). If the load on the secondary is 200 A at a p.f. of 0.85 lagging. Find the primary current and power factor.*

Neglect the voltage-drop in the winding. **(Nagpur University November 1999)**

Solution. Secondary load of 200 A, 0.85 lag is reflected as 50 A, 0.85 lag in terms of the primary equivalent current.

$$I_0 = 10 \angle -\phi_0, \text{ where } \phi_0 = \cos^{-1} 0.20 = 78.5° \text{ lagging}$$
$$= 2 - j\,9.8 \text{ amp}$$
$$I_2' = 50 \angle -\phi_L \text{ where } \phi_L = \cos^{-1} 0.85 = 31.8°, \text{ lagging}$$
$$I_2' = 42.5 - j\,26.35$$

Hence primary current I_1

$$= I_0 + I_2'$$
$$= 2 - j\,9.8 + 42.5 - j\,26.35$$
$$= 44.5 - j\,36.15$$
$$|I_1| = 57.333 \text{ amp}, \phi = 0.776 \text{ Lag.}$$
$$\phi = \cos^{-1} \frac{44.5}{57.333} = 39.10° \text{ lagging}$$

The phasor diagram is shown in Fig. 32.21.

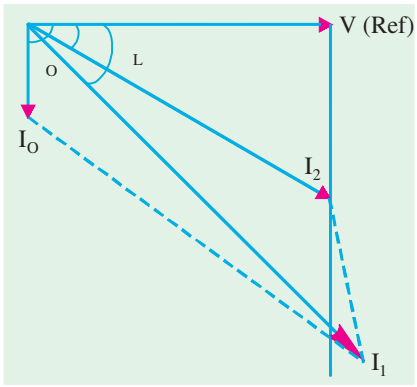


Fig. 32.21

Tutorial Problems 32.2

1. The primary of a certain transformer takes 1 A at a power factor of 0.4 when it is connected across a 200-V, 50-Hz supply and the secondary is on open circuit. The number of turns on the primary is twice that on the secondary. A load taking 50 A at a lagging power factor of 0.8 is now connected across the secondary. What is now the value of primary current ? **[25.9 A]**
2. The number of turns on the primary and secondary windings of a single-phase transformer are 350 and 38 respectively. If the primary winding is connected to a 2.2 kV, 50-Hz supply, determine
   (*a*) the secondary voltage on no-load,
   (*b*) the primary current when the secondary current is 200 A at 0.8 p.f. lagging, if the no-load current is 5 A at 0.2 p.f. lagging,
   (*c*) the power factor of the primary current. **[239 V; 25-65 A; 0.715 lag]**
3. A 400/200-V, 1-phase transformer is supplying a load of 25 A at a p.f. of 0.866 lagging. On no-load the current and power factor are 2 A and 0.208 respectively. Calculate the current taken from the supply. **[13.9 A lagging V1 by 36.1°]**
4. A transformer takes 10 A on no-load at a power factor of 0.1. The turn ratio is 4 : 1 (step down). If

a load is supplied by the secondary at 200 A and p.f. of 0.8, find the primary current and power factor (internal voltage drops in transformer are to be ignored). **[57.2 A; 0.717 lagging]**

5. A 1-phase transformer is supplied at 1,600 V on the h.v. side and has a turn ratio of 8 : 1. The transformer supplies a load of 20 kW at a power factor of 0.8 lag and takes a magnetising current of 2.0 A at a power factor of 0.2. Calculate the magnitude and phase of the current taken from the h.v. supply. **[17.15 A ; 0.753 lag]** ***(Elect. Engg. Calcutta Univ. 1980)***

6. A 2,200/200-V, transformer takes 1 A at the H.T. side on no-load at a p.f. of 0.385 lagging. Calculate the iron losses.

   If a load of 50 A at a power of 0.8 lagging is taken from the secondary of the transformer, calculate the actual primary current and its power factor. **[847 W; 5.44 A; 0.74 lag]**

7. A 400/200-V, I-phase transformer is supplying a load of 50 A at a power factor of 0.866 lagging. The no-load current is 2 A at 0.208 p.f. lagging. Calculate the primary current and primary power factor. **[26.4 A; 0.838 lag]** ***(Elect. Machines-I, Indore Univ. 1980)***

32.11. Transformer with Winding Resistance but No Magnetic Leakage

An ideal transformer was supposed to possess no resistance, but in an actual transformer, there is always present some resistance of the primary and secondary windings. Due to this resistance, there is some voltage drop in the two windings. The result is that :

(i) The secondary terminal voltage V_2 is vectorially less than the secondary induced e.m.f. E_2 by an amount $I_2 R_2$ where R_2 is the resistance of the secondary winding. Hence, V_2 is equal to the vector difference of E_2 and resistive voltage drop $I_2 R_2$.

$$\therefore \quad V_2 = E_2 - I_2 R_2 \quad \text{...vector difference}$$

(ii) Similarly, primary induced e.m.f. E_1 is equal to the vector difference of V_1 and $I_1 R_1$ where R_1 is the resistance of the primary winding.

$$E_1 = V_1 - I_1 R_1 \quad \text{...vector difference}$$

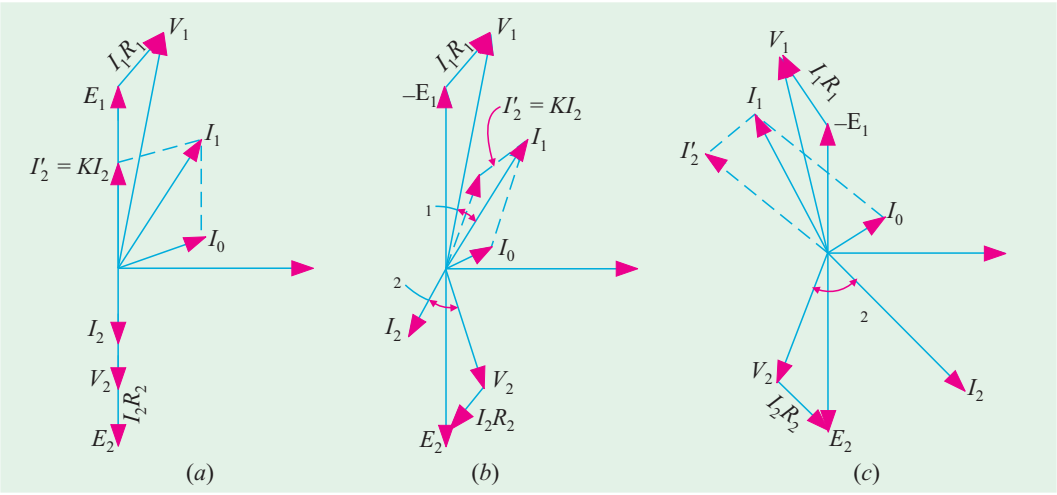


Fig. 32.22

The vector diagrams for non-inductive, inductive and capacitive loads are shown in Fig. 32.22 (*a*), (*b*) and (*c*) respectively.

32.12. Equivalent Resistance

In Fig. 32.23 a transformer is shown whose primary and secondary windings have resistances of R_1 and R_2 respectively. The resistances have been shown external to the windings.

It would now be shown that the resistances of the two windings can be transferred to any one of the two windings. The advantage of concentrating both the resistances in one winding is that it makes calculations very simple and easy because one has then to work in one winding only. It will be proved that a resistance of R_2 in secondary is equivalent to R_2/K^2 in primary. The value R_2/K^2 will be denoted by R_2'– ***the equivalent secondary resistance as referred to primary***.

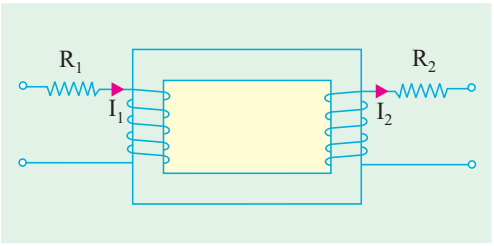


Fig. 32.23

The copper loss in secondary is $I_2^2 R_2$. This loss is supplied by primary which takes a current of I_1. Hence if R_2' is the ***equivalent resistance in primary which would have caused the same loss*** as R_2 in secondary, then

$$I_1^2 R_2' = I_2^2 R_2 \text{ or } R_2' = (I_2/I_1)^2 R_2$$

Now, if ***we neglect no-load current*** I_0, then $I_2/I_1 = I/K$*. Hence, $R_2' = R_2/K^2$

Similarly, equivalent primary resistance as referred to secondary is $R_1' = K^2 R_1$

In Fig. 32.24, secondary resistance has been transferred to primary side leaving secondary circuit resistanceless. The resistance $R_1 + R_2' = R_1 + R_2/K^2$ is known as the ***equivalent or effective resistance of the transformer as referred to primary*** and may be designated as R_{01}.

$$\therefore \quad R_{01} = R_1 + R_2' = R_1 + R_2/K^2$$

Similarly, the ***equivalent resistance of the transformer as referred to secondary is***

$$R_{02} = R_2 + R_1' = R_2 + K^2 R_1.$$

This fact is shown in Fig. 32.25 where all the resistances of the transformer has been concentrated in the secondary winding.

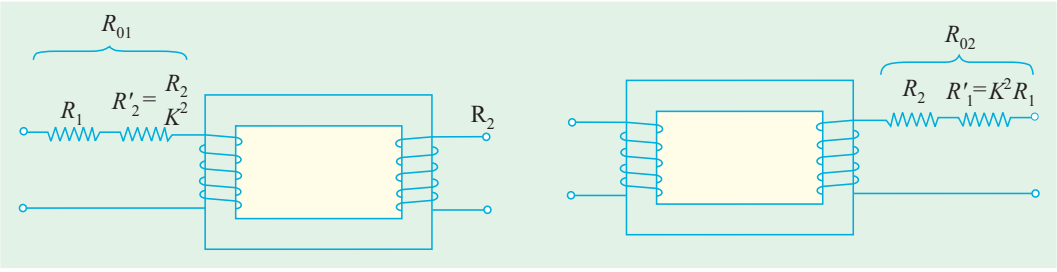


Fig. 32.24 Fig. 32.25

It is to be noted that

1. a resistance of R_1 in primary is equivalent to $K^2 R_1$ in secondary. Hence, it is called ***equivalent resistance as referred to secondary*** *i.e.* R_1.

2. a resistance of R_2 in secondary is equivalent to R_2/K^2 in primary. Hence, it is called the ***equivalent secondary resistance as referred to primary*** *i.e.* R_2'.

3. Total or effective resistance of the transformer as referred to primary is

R_{01} = primary resistance + equivalent secondary resistance as referred to primary

$= R_1 + R_2' = R_1 + R_2/K^2$

4. Similarly, total transformer resistance as referred to secondary is,

R_{02} = secondary resistance + equivalent primary resistance as referred to secondary

$= R_2 + R_1' = R_2 + K^2 R_1$

---

* Actually $I_2 \# 2/I_2' = I/K$ and not $I_2 \# 2/I_1$. However, if I_0 is neglected, then $I_2' = I_1$.

Note : It is important to remember that

(*a*) when shifting any primary resistance to the secondary, ***multiply*** it by K^2 *i.e.* (transformation ratio)^2.

(*b*) when shifting secondary resistance to the primary, ***divide*** it by K^2.

(*c*) however, when shifting any voltage from one winding to another only K is used.

32.13. Magnetic Leakage

In the preceding discussion, it has been assumed that all the flux linked with primary winding also links the secondary winding. But, in practice, it is impossible to realize this condition. It is found, however, that all the flux linked with primary does not link the secondary but part of it *i.e.* Φ_{L_1} completes its magnetic circuit by passing through air rather than around the core, as shown in Fig. 32.26. This leakage flux is produced when the m.m.f. due to primary ampere-turns existing between points *a* and *b*, acts along the leakage paths. Hence, this flux is known as ***primary leakage flux*** and is proportional to the primary ampere-turns alone because the secondary turns do not link the magnetic circuit of Φ_{L_1}. The flux Φ_{L_1} is in time phase with I_1. It induces an e.m.f. e_{L_1} in primary but not in secondary.

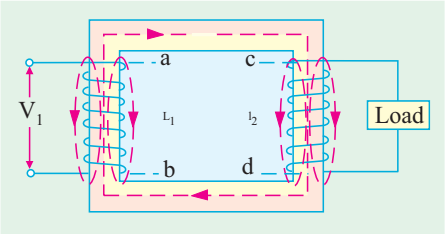


Fig. 32.26

Similarly, secondary ampere-turns (or m.m.f.) acting across points *c* and *d* set up leakage flux Φ_{L_2} which is linked with secondary winding alone (and not with primary turns). This flux Φ_{L_2} is in time phase with I_2 and produces a self-induced e.m.f. e_{L_2} in secondary (but not in primary).

At no load and light loads, the primary and secondary ampere-turns are small, hence leakage fluxes are negligible. But when load is increased, both primary and secondary windings carry huge currents. Hence, large m.m.f.s are set up which, while acting on leakage paths, increase the leakage flux.

As said earlier, the leakage flux linking with each winding, produces a self-induced e.m.f. in that winding. Hence, in effect, it is equivalent to a small choker or inductive coil in series with each winding such that voltage drop in each series coil is equal to that produced by leakage flux. In other words, ***a transformer with magnetic leakage is equivalent to an ideal transformer with inductive coils connected in both primary and secondary circuits*** as shown in Fig. 32.27 such that the internal e.m.f. in each inductive coil is equal to that due to the corresponding leakage flux in the actual transformer.

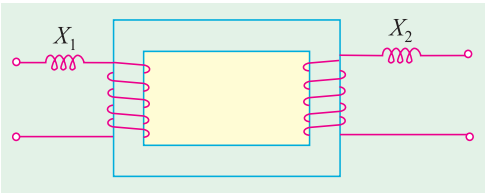


Fig. 32.27

$X_1 = e_{L1}/I_1$ and $X_2 = e_{L2}/I_2$

The terms X_1 and X_2 are known as primary and secondary *leakage* reactances respectively.

Following few points should be kept in mind :

1. The leakage flux links one or the other winding but ***not both***, hence it in no way contributes to the transfer of energy from the primary to the secondary winding.

2. The primary voltage V_1 will have to supply reactive drop I_1X_1 in addition to I_1R_1. Similarly E_2 will have to supply I_2R_2 and I_2X_2.

3. In an actual transformer, the primary and secondary windings are not placed on separate legs or limbs as shown in Fig. 32.27 because due to their being widely separated, large primary and secondary leakage fluxes would result. These leakage fluxes are minimised by sectionalizing and interleaving the primary and secondary windings as in Fig. 32.6 or Fig. 32.8.

32.14. Transformer with Resistance and Leakage Reactance

In Fig. 32.28 the primary and secondary windings of a transformer with reactances taken out of the windings are shown. The primary impedance is given by

$$Z_1 = \sqrt{(R_1^2 + X_1^2)}$$

Similarly, secondary impedance is given by

$$Z_2 = \sqrt{(R_2^2 + X_2^2)}$$

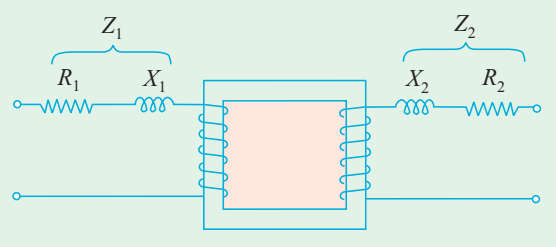


Fig. 32.28

The resistance and leakage reactance of each winding is responsible for some voltage drop in each winding. In primary, the leakage reactance drop is I_1X_1 (usually 1 or 2% of V_1). Hence

$$\mathbf{V_1} = \mathbf{E_1} + \mathbf{I_1}(R_1 + jX_1) = \mathbf{E_1} + \mathbf{I_1}\mathbf{Z_1}$$

Similarly, there are I_2R_2 and I_2X_2 drops in secondary which combine with V_2 to give E_2.

$$\mathbf{E_2} = \mathbf{V_2} + \mathbf{I_2}(R_2 + jX_2) = \mathbf{V_2} + \mathbf{I_2}\mathbf{Z_2}$$

The vector diagram for such a transformer for different kinds of loads is shown in Fig. 32.29. In these diagrams, vectors for resistive drops are drawn parallel to current vectors whereas reactive drops are perpendicular to the current vectors. The angle ϕ_1 between V_1 and I_1 gives the power factor angle of the transformer.

It may be noted that leakage reactances can also be transferred from one winding to the other in the same way as resistance.

$\therefore$ $\quad X_2' = X_2/K^2$ and $X_1' = K^2X_1$

and $\quad X_{01} = X_1 + X_2' = X_1 + X_2/K^2$ and $X_{02} = X_2 + X_1' = X_2 + K^2X_1$

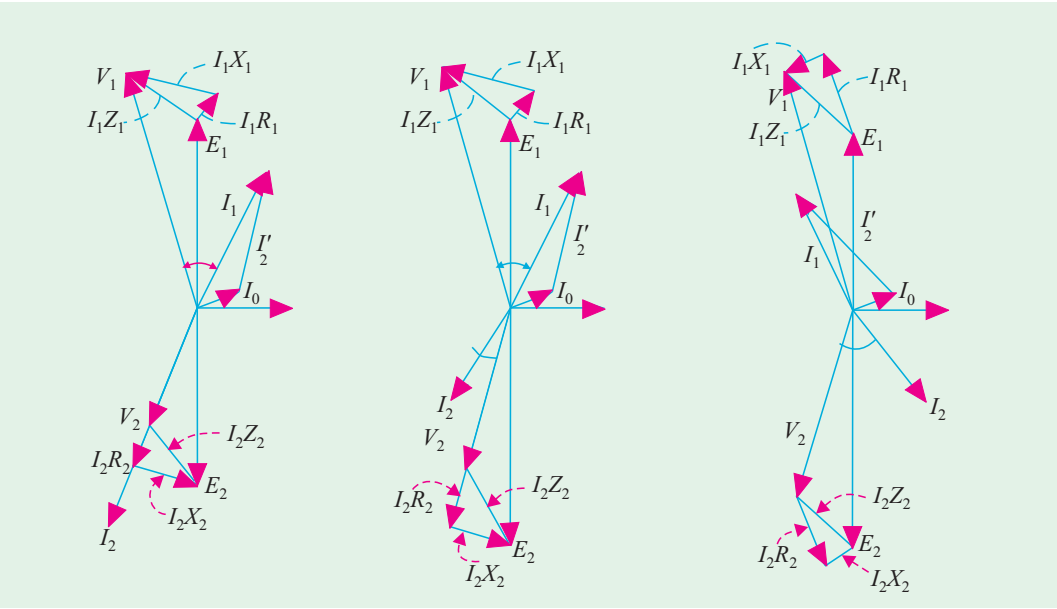


Fig. 32.29

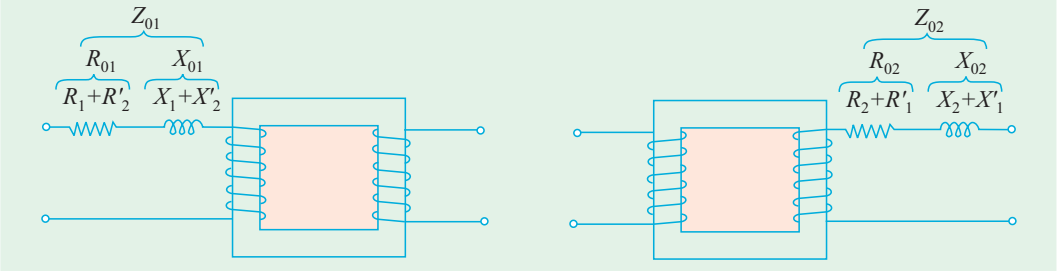


Fig. 32.30 (a) Fig. 32.30 (b)

It is obvious that total impedance of the transformer as referred to primary is given by

$$Z_{01} = \sqrt{(R_{01}^2 + X_{01}^2)} \quad \text{...Fig. 32.30 } (a)$$

and $$Z_{02} = \sqrt{(R_{02}^2 + X_{02}^2)} \quad \text{...Fig. 32.30 } (b)$$

Example 32.15. *A 30 kVA, 2400/120-V, 50-Hz transformer has a high voltage winding resistance of 0.1 Ω and a leakage reactance of 0.22Ω. The low voltage winding resistance is 0.035 Ω and the leakage reactance is 0.012 Ω. Find the equivalent winding resistance, reactance and impedance referred to the* ***(i)*** *high voltage side and* ***(ii)*** *the low-voltage side.*

(Electrical Machines-I, Bangalore Univ. 1987)

Solution. $K = 120/2400 = 1/20$; $R_1 = 0.1\ \Omega$, $X_1 = 0.22\ \Omega$

$R_2 = 0.035\ \Omega$ and $X_2 = 0.012\ \Omega$

(i) Here, high-voltage side is, obviously, the primary side. Hence, values as referred to primary side are

$$R_{01} = R_1 + R_2' = R_1 + R_2/K^2 = 0.1 + 0.035/(1/20)^2 = \mathbf{14.1\ \Omega}$$
$$X_{01} = X_1 + X_2' = X_1 + X_2/K^2 = 0.22 + 0.12/(1/20)^2 = \mathbf{5.02\ \Omega}$$
$$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2} = \sqrt{14.1^2 + 5.02^2} = \mathbf{15\ \Omega}$$

(ii)
$$R_{02} = R_2 + R_1' = R_2 + K^2R_1 = 0.035 + (1/20)^2 \times 0.1 = \mathbf{0.03525\ \Omega}$$
$$X_{02} = X_2 + X_1' = X_2 + K^2X_1 = 0.012 + (1/20)^2 \times 0.22 = \mathbf{0.01255\ \Omega}$$
$$Z_{02} = \sqrt{R_{02}^2 + X_{02}^2} = \sqrt{0.0325^2 + 0.01255^2} = \mathbf{0.0374\ \Omega}$$
$$(\text{or } Z_{02} = K^2Z_{01} = (1/20)^2 \times 15 = 0.0375\ \Omega)$$

Example 32.16. *A 50-kVA, 4,400/220-V transformer has $R_1 = 3.45\ \Omega$, $R_2 = 0.009\ \Omega$. The values of reactances are $X_1 = 5.2\ \Omega$ and $X_2 = 0.015\ \Omega$. Calculate for the transformer* ***(i)*** *equivalent resistance as referred to primary* ***(ii)*** *equivalent resistance as referred to secondary* ***(iii)*** *equivalent reactance as referred to both primary and secondary* ***(iv)*** *equivalent impedance as referred to both primary and secondary* ***(v)*** *total Cu loss, first using individual resistances of the two windings and secondly, using equivalent resistances as referred to each side.*

(Elect. Engg.-I, Nagpur Univ. 1993)

Solution. Full-load $I_1 = 50{,}000/4{,}400 = 11.36$ A (assuming 100% efficiency)

Full-load $I_2 = 50{,}000/2220 = 227$ A; $K = 220/4{,}400 = 1/20$

(i) $$R_{01} = R_1 + \frac{R_2}{K^2} = 3.45 + \frac{0.009}{(1/20)^2} = 3.45 + 3.6 = \mathbf{7.05\ \Omega}$$

(ii) $$R_{02} = R_2 + K^2R_1 = 0.009 + (1/20)^2 \times 3.45 = 0.009 + 0.0086 = \mathbf{0.0176\ \Omega}$$

Also, $$R_{02} = K^2 R_{01} = (1/20)^2 \times 7.05 = 0.0176\ \Omega \text{ (check)}$$

(iii) $X_{01} = X_1 + X_2' = X_1 + X_2/K^2 = 5.2 + 0.015/(1/20)^2 = \mathbf{11.2\ \Omega}$

$X_{02} = X_2 + X_1' = X_2 + K^2 X_1 = 0.015 + 5.2/20^2 = \mathbf{0.028\ \Omega}$

Also $X_{02} = K^2 X_{01} = 11.2/400 = 0.028\ \Omega$ (check)

(iv) $Z_{01} = \sqrt{(R_{01}^2 + X_{01}^2)} = \sqrt{(7.05^2 + 11.2)^2} = \mathbf{13.23\ \Omega}$

$Z_{02} = \sqrt{(R_{02}^2 + X_{02}^2)} = \sqrt{(0.0176^2 + 0.028)^2} = \mathbf{0.03311\ \Omega}$

Also $Z_{02} = K^2 Z_{01} = 13.23/400 = 0.0331\ \Omega$ (check)

(v) Cu loss $= I_1^2 R_2 + I_2^2 R_2 = 11.36^2 \times 3.45 + 227^2 \times 0.009 = \mathbf{910\ W}$

Also Cu loss $= I_1^2 R_{01} = 11.36^2 \times 7.05 = \mathbf{910\ W}$

$= I_2^2 R_{02} = 227^2 \times 0.0176 = \mathbf{910\ W}$

Example 32.17. *A transformer with a 10 : 1 ratio and rated at 50-kVA, 2400/240-V, 50-Hz is used to step down the voltage of a distribution system. The low tension voltage is to be kept constant at 240 V.*

(a) *What load impedance connected to low-tension size will be loading the transformer fully at 0.8 power factor (lag) ?*

(b) *What is the value of this impedance referred to high tension side ?*

(c) *What is the value of the current referred to the high tension side ?*

(Elect. Engineering-I, Bombay Univ. 1987)

Solution. (a) F. L. $I_2 = 50{,}000/240 = 625/3$ A; $Z_2 = \dfrac{240}{(625/3)} = \mathbf{1.142\ \Omega}$

(b) $K = 240/2400 = 1/10$

The secondary impedance referred to primary side is

$Z_2' = Z_2/K^2 = 1.142/(1/10)^2 = \mathbf{114.2\ \Omega}$

(c) Secondary current referred to primary side is $I_2' = KI_2 = (1/10) \times 625/3 = \mathbf{20.83\ A}$

Example 32.18. *The full-load copper loss on the h.v. side of a 100-kVA, 11000/317-V, 1-phase transformer is 0.62 kW and on the L.V. side is 0.48 kW.*

(i) *Calculate R_1, R_2 and R_3 in ohms* **(ii)** *the total reactance is 4 per cent, find X_1, X_2 and X_3 in ohms if the reactance is divided in the same proportion as resistance.*

(Elect. Machines A.M.I.E,. Sec. B, 1991)

Solution. (i) F.L. $I_1 = 100 \times 10^3/11000 = 9.1$ A. F.L. $I_2 = 100 \times 10^3/317 = 315.5$ A

Now, $I_1^2 R_1 = 0.62$ kW or $R_1 = 620/9.1^2 = 7.5\ \Omega$

$I_2^2 R_2 = 0.48$ kW, $R_2 = 480/315.5^2 = 0.00482\ \Omega$

$R_2' = R_2/K^2 = 0.00482 \times (11{,}000/317)^2 = 5.8\ \Omega$

% reactance $= \dfrac{I_1 \times X_{01}}{V_1} \times 100$ or $4 = \dfrac{9.1 \times X_{01}}{11000} \times 100$, $X_{01} = 48.4\ \Omega$

$X_1 + X_2' = 48.4\ \Omega$. Given $R_1/R_2' = X_1/X_2'$

or $(R_1 + R_2')/R_2' = (X_1 + X_2')/X_2'$ $(7.5 + 5.8)/5.8 = 48.4/X_2'$ $\therefore$ $X_2' = 21.1\ \Omega$

$\therefore$ $X_1 = 48.4 - 21.1 = 27.3\ \Omega$, $X_2 = 21.1 \times (317/11000)^2 = \mathbf{0.175\ \Omega}$

Example 32.19. *The following data refer to a I-phase transformer :*

Turn ratio 19.5 : 1 ; $R_1 = 25\ \Omega$; $X_1 = 100\ \Omega$; $R_2 = 0.06\ \Omega$; $X_2 = 0.25\ \Omega$. No-load current = 1.25 A leading the flux by 30°.

The secondary delivers 200 A at a terminal voltage of 500 V and p.f. of 0.8 lagging. Determine

by the aid of a vector diagram, the primary applied voltage, the primary p.f. and the efficiency.
(Elect. Machinery-I, Madras Univ. 1989)

Solution. The vector diagram is similar to Fig. 30.28 which has been redrawn as Fig. 32.31. Let us take V_2 as the reference vector.

$$\therefore \quad \mathbf{V_2} = 500\angle 0° = 500 + j0$$
$$\mathbf{I_2} = 200\,(0.8 - j\,0.6) = 160 - j\,120$$
$$\mathbf{Z_2} = (0.06 + j\,0.25)$$
$$\mathbf{E_2} = V_2 + \mathbf{I_2 Z_2}$$
$$= (500 + j\,0) + (160 - j\,120)\,(0.06 + j\,0.25)$$
$$= 500 + (39.6 + j\,32.8) = 539.6 + j\,32.8 = 541\angle 3.5°$$

Obviously, $\beta = 3.5°$

$$\mathbf{E_1} = \mathbf{E_2}/K = 19.5\,\mathbf{E_2} = 19.5\,(539.6 + j\,32.8)$$
$$= 10{,}520 + j\,640$$
$$\therefore -\mathbf{E_1} = -10{,}520 - j\,640 = 10{,}540\angle 183.5°$$
$$\mathbf{I_2'} = -\mathbf{I_2}K = (-160 + j\,120)/19.5$$
$$= -8.21 + j\,6.16$$

As seen from Fig. 32.31, $\mathbf{I_0}$ leads $\mathbf{V_2}$ by an angle

$$= 3.5° + 90° + 30° = 123.5°$$
$$\therefore \quad \mathbf{I_0} = 1.25\angle 123.5°$$
$$= 1.25\,(\cos 123.5° + j \sin 123.5°)$$
$$= 1.25\,(-\cos 56.5° + j \sin 56.5°)$$
$$= -0.69 + j\,1.04$$
$$\mathbf{I_1} = \mathbf{I_2'} + \mathbf{I_0} = (-8.21 + j\,6.16) + (-0.69 + j\,1.04)$$
$$= -8.9 + j\,7.2 = 11.45\angle 141°$$
$$\mathbf{V_2} = -\mathbf{E_1} + \mathbf{I_1 Z_1}$$
$$= -10{,}520 - j\,640 + (-8.9 + j\,7.5)\,(25 + j\,100)$$
$$= -10{,}520 - j\,640 - 942 - j\,710$$
$$= -11{,}462 - j\,1350$$
$$= 11{,}540\angle 186.7°$$

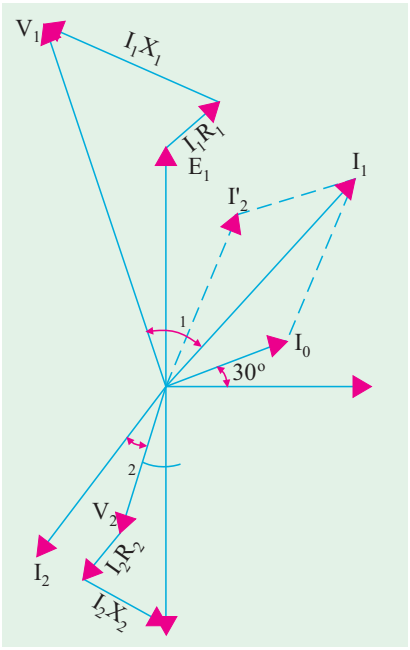


Fig. 32.31

Phase angle between $\mathbf{V_1}$ and $\mathbf{I_1}$ is $= 186.7° - 141° = 45.7°$

$\therefore$ primary p.f. $= \cos 45.7° =$ **0.698 (lag)**

No-load primary input power $= V_1 I_0 \sin \phi_0$
$= 11{,}540 \times 1.25 \times \cos 60° = 7{,}210$ **W**

$$R_{02} = R_2 + K^2 R_1 = 0.06 + 25/19.5^2 = 0.1257\ \Omega$$

Total Cu loss as referred to secondary $= I_2^2 R_{02} = 200^2 \times 0.1257 = 5{,}030$ W

Output $= V_2 I_2 \cos \phi_2 = 500 \times 200 \times 0.8 = 80{,}000$ W

Total losses $= 5030 + 7210 = 12{,}240$ W

Input $= 80{,}000 + 12{,}240 = 92{,}240$ W

$\eta = 80{,}000/92{,}240 = 0.8674$ or **86.74%**

Example 32.20. *A 100 kVA, 1100/220 V, 50 Hz, single-phase transformer has a leakage impedance of (0.1 + 0/40) ohm for the H.V. winding and (0.006 + 0.015) ohm for the L.V. winding. Find the equivalent winding resistance, reactance and impedance referred to the H.V. and L.V. sides.*
(Bharathiar Univ. Nov. 1997)

Solution. Turns ratio = $(N_1/N_2) = (V_1/V_2) = 1100/220 = 5$

(i) Referred to H.V. side :

Resistance = $r_1 + r_2 = 0.1 + (25 \times 0.006) = 0.25$ ohm

Reactance = $x_1 + x_2' = 0.4 + (25 \times 0.015) = 0.775$ ohm

Impedance = $(0.25^2 + 0.775^2)^{0.5} = 0.8143$ ohm

(ii) Referred to L.V. side :

Resistance = $0.25/25 = 0.01$

(or resistance = $0.006 + (0.1/25) = 0.01$ ohm)

Reactance = $0.775/25 = 0.031$ ohm

Impedance = $0.8143/25 = 0.0326$ ohm

32.15. Simplified Diagram

The vector diagram of Fig. 32.29 may be considerably simplified if the no-load current I_0 is neglected. Since I_0 is 1 to 3 per cent of full-load primary current I_1, it may be neglected without serious error. Fig. 32.32 shows the diagram of Fig. 32.29 with I_0 omitted altogether.

In Fig. 32.32, V_2, V_1, ϕ_2 are known, hence E_2 can be found by adding vectorially $I_2 R_2$ and $I_2 X_2$ to V_2. Similarly, V_1 is given by the vector addition of $I_1 R_1$ and $I_1 X_1$ to E_1. All the voltages on the primary side can be transferred to the secondary side as shown in figure, where the upper part of the diagram has been rotated through 180°. However, it should be noted that each voltage or voltage drop should be multiplied by transformation ratio K.

The lower side of the diagram has been shown separately in Fig. 32.34 laid horizontally where vector for V_2 has been taken along X-axis.

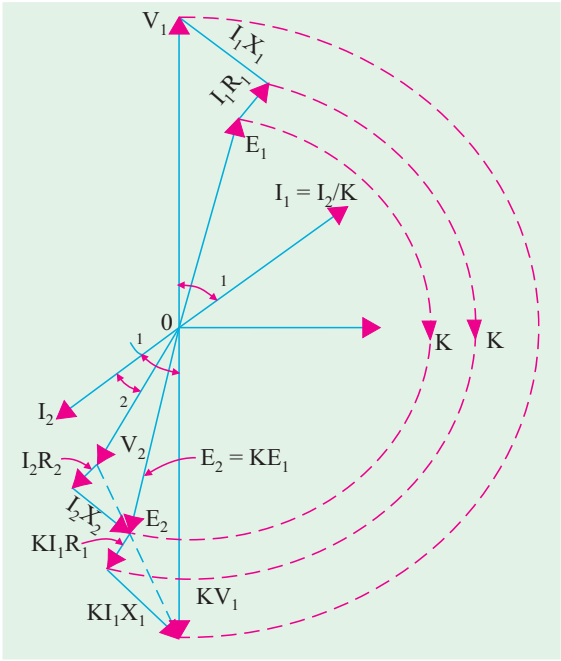


Fig. 32.32

It is a simple matter to find transformer regulation as shown in Fig. 32.34 or Fig. 32.35.

It may be noted that $\mathbf{V_2} = K\mathbf{V_1} - \mathbf{I_2}(R_{02} + jX_{02}) = K\mathbf{V_1} - \mathbf{I_2 Z_{02}}$.

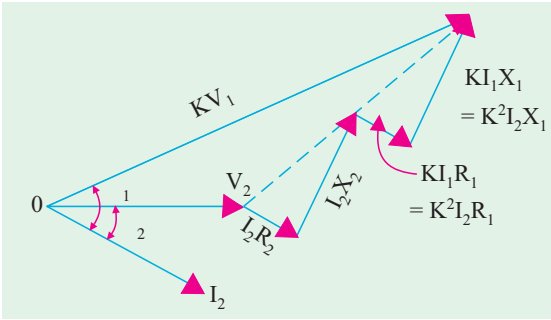


Fig. 32.33

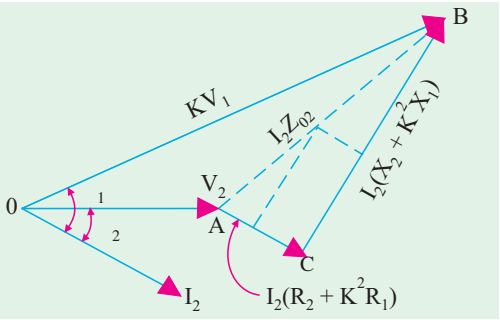


Fig. 32.34

* Also, $\mathbf{V_1} = (\mathbf{V_2} + \mathbf{I_2 Z_{02}})\ \#3/K$

32.16. Total Approximate Voltage Drop in a Transformer

When the transformer is on *no-load*, then V_1 is approximately equal to E_1. Hence $E_2 = KE_1 = KV_1$. Also, $E_2 = {}_0V_2$ where ${}_0V_2$ is secondary terminal voltage on ***no-load***, hence no-load secondary terminal voltage is KV_1. The secondary voltage on load is V_2. The difference between the two is $I_2 Z_{02}$ as shown in Fig. 32.35. The approximate voltage drop of the transformer ***as referred to secondary*** is found thus :

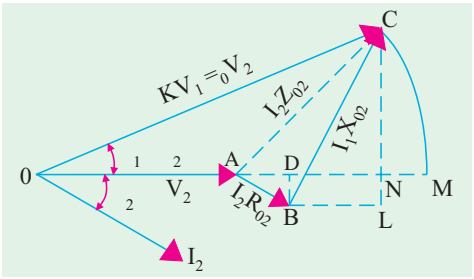


Fig. 32.35

With O as the centre and radius OC draw an arc cutting OA produced at M. The total voltage drop $I_2 Z_{02} = AC = AM$ which is approximately equal to AN. From B draw BD perpendicular on OA produced. Draw CN perpendicular to OM and draw BL parallel to OM.

Approximate voltage drop

$$= AN = AD + DN$$

$$= I_2R_{02}\cos\phi + I_2X_{02}\sin\phi$$

where $\phi_1 = \phi_2 = \phi$ (approx).

This is the value of approximate voltage drop for a ***lagging*** power factor.

The different figures for unity and leading power factors are shown in Fig. 32.36 (*a*) and (*b*) respectively.

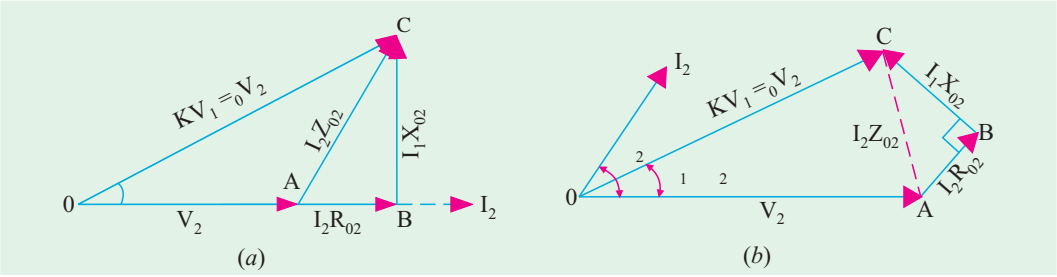


Fig. 32.36

The approximate voltage drop for ***leading*** power factor becomes

$$(I_2R_{02}\cos\phi \pm I_2X_{02}\sin\phi)$$

In general, approximate voltage drop is $(I_2R_{02}\cos\phi \pm I_2X_{02}\sin\phi)$

It may be noted that approximate voltage drop as referred to primary is

$$(I_1R_{01}\cos\phi \pm I_1X_{01}\sin\phi)$$

$$\% \text{ voltage drop in secondary is} = \frac{I_2 R_{02}\cos\phi \pm I_2 X_{02}\sin\phi}{{}_0V_2} \times 100$$

$$= \frac{100 \times I_2 R_{02}}{{}_0V_2}\cos\phi \pm \frac{100\, I_2 X_{02}}{{}_0V_2}\sin\phi$$

$$= v_r\cos\phi \pm v_x\sin\phi$$

where

$$v_r = \frac{100\, I_2R_{02}}{{}_0V_2} = \text{percentage resistive drop} = \frac{100\, I_1 R_{01}}{V_1}$$

$$v_x = \frac{100\, I_2X_{02}}{{}_0V_2} = \text{percentage reactive drop} = \frac{100\, I_1 X_{01}}{V_1}$$

32.17. Exact Voltage Drop

With reference to Fig. 32.35, it is to be noted that exact voltage drop is AM and not AN. If we add the quantity NM to AN, we will get the exact value of the voltage drop.

Considering the right-angled triangle OCN, we get

$$NC^2 = OC^2 - ON^2 = (OC + ON)(OC - ON) = (OC + ON)(OM - ON) = 2\,OC \times NM$$

$$\therefore \quad NM = NC^2/2.OC \quad \text{Now, } NC = LC - LN = LC - BD$$

$$\therefore \quad NC = I_2 X_{02} \cos\phi - I_2 R_{02} \sin\phi \qquad \therefore NM = \frac{(I_2 X_{02} \cos\phi - I_2 R_{02} \sin\phi)^2}{2_0 V_2}$$

$\therefore$ For a ***lagging*** power factor, exact voltage drop is

$$= AN + NM = (I_2 R_{02} \cos\phi + I_2 X_{02} \sin\phi) + \frac{(I_2 X_{02} \cos\phi - I_2 R_{02} \sin\phi)^2}{2_0 V_2}$$

For a ***leading*** power factor, the expression becomes

$$= (I_2 R_{02} \cos\phi - I_2 X_{02} \sin\phi) + \frac{(I_2 X_{02} \cos\phi + I_2 R_{02} \sin\phi)^2}{2_0 V_2}$$

In general, the voltage drop is

$$= (I_2 R_{02} \cos\phi \pm I_2 R_{02} \sin\phi) + \frac{(I_2 X_{02} \cos\phi \pm I_2 R_{02} \sin\phi)^2}{2_0 V_2}$$

Percentage drop is

$$= \frac{(I_2 R_{02} \cos\phi \pm I_2 X_{02} \sin\phi) \times 100}{{}_0 V_2} + \frac{(I_2 X_{02} \cos\phi \mp I_2 R_{02} \sin\phi)^2 \times 100}{2_0 V_2^2}$$

$$= (v_r \cos\phi \pm v_x \sin\phi) + (1/200)(v_x \cos\phi \mp v_r \sin\phi)^2$$

The upper signs are to be used for a ***lagging*** power factor and the lower ones for a ***leading*** power factor.

Example 32.21. *A 230/460-V transformer has a primary resistance of 0.2 Ω and reactance of 0.5 Ω and the corresponding values for the secondary are 0.75 Ω and 1.8 Ω respectively. Find the secondary terminal voltage when supplying 10 A at 0.8 p.f. lagging.*

(Electric. Machines-II, Bangalore Univ. 1991)

Solution.

$$K = 460/230 = 2;\ R_{02} = R_2 + K^2 R_1 = 0.75 + 2^2 \times 0.2 = 1.55\ \Omega$$

$$X_{02} = X_2 + K^2 X_1 = 1.8 + 2^2 \times 0.5 = 3.8\ \Omega$$

$$\text{Voltage drop} = I_2 (R_{02} \cos\phi + X_{02} \sin\phi) = 10\,(1.55 \times 0.8 + 3.8 \times 0.6) = 35.2\text{V}$$

$\therefore$ Secondary terminal voltage $= 460 - 35.2 =$ **424.8 V**

Example 32.22. *Calculate the regulation of a transformer in which the percentage resistance drops is 1.0% and percentage reactance drop is 5.0% when the power factor is* ***(a)*** *0.8 lagging* ***(b)*** *unity and* ***(c)*** *0.8 leading.* **(Electrical Engineering, Banaras Hindu Univ. 1988)**

Soluion. We will use the approximate expression of Art 30.16.

(a) **p.f. = cos ϕ = 0.8 lag** $\quad \mu = v_r \cos\phi + v_x \sin\phi = 1 \times 0.8 + 5 \times 0.6 =$ **3.8%**

(b) **p.f. = cos ϕ = 1** $\quad \mu = 1 \times 1 + 5 \times 0 =$ **1%**

(c) **p.f. = cos ϕ = 0.8 lead** $\quad \mu = 1 \times 0.8 - 5 \times 0.6 =$ **– 2.2%**

Example 32.23. *A transformer has a reactance drop of 5% and a resistance drop of 2.5%. Find the lagging power factor at which the voltage regulation is maximum and the value of this regulation.* **(Elect. Engg. Punjab Univ. 1991)**

Solution. The percentage voltage regulation (μ) is given by

$$\mu = v_r \cos\phi + v_x \sin\phi$$

where v_r is the percentage resistive drop and v_x is the percentage reactive drop.

Differentiating the above equation, we get $\frac{d\mu}{d\phi} = -v_r \sin\phi + v_x \cos\phi$

For regulation to be maximum, $d\mu/d\phi = 0 \quad \therefore \quad -v_r \sin\phi + v_x \cos\phi = 0$

or $\tan\phi = v_x/v_r = 5/2.5 = 2 \therefore \phi = \tan^{-1}(2) = 63.5°$ Now, $\cos\phi = 0.45$ and $\sin\phi = 0.892$

Maximum percentage regulation $= (2.5 \times 0.45) + (5 \times 0.892) = 5.585$

Maximum percentage regulation is **5.585** and occurs at a power factor of **0.45** (lag).

Example 32.24. *Calculate the percentage voltage drop for a transformer with a percentage resistance of 2.5% and a percentage reactance of 5% of rating 500 kVA when it is delivering 400 kVA at 0.8 p.f. lagging.* **(Elect. Machinery-I, Indore Univ. 1987)**

Solution. $$\% \text{ drop} = \frac{(\%R)\ I \cos\phi}{I_f} + \frac{(\%X)\ I \sin\phi}{I_f}$$

where I_f is the full-load current and I the actual current.

$$\therefore \quad \% \text{ drop} = \frac{(\%R)kW}{kVA \text{ rating}} + \frac{(\%X)\ kVAR}{kVA \text{ rating}}$$

In the present case, $kW = 400 \times 0.8 = 320$ and $kVAR = 400 \times 0.6 = 240$

$$\therefore \quad \% \text{ drop} = \frac{2.5 \times 320}{500} + \frac{5 \times 240}{500} = 4\%$$

32.18. Equivalent Circuit

The transformer shown diagrammatically in Fig. 32.37 (*a*) can be resolved into an equivalent circuit in which the resistance and leakage reactance of the transformer are imagined to be external to the winding whose only function then is to transform the voltage (Fig. 32.37 (*b*)). The no-load

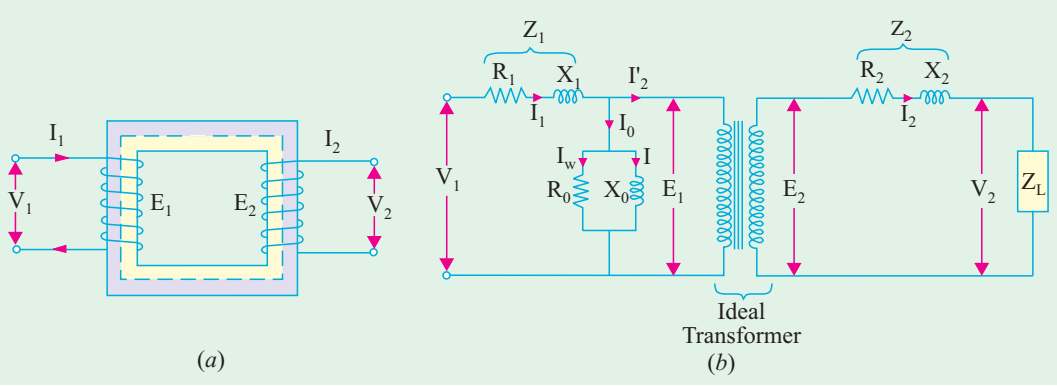


Fig. 32.37

current I_0 is simulated by pure inductance X_0 taking the magnetising component I_μ and a non-inductive resistance R_0 taking the working component I_w connected in parallel across the primary circuit. The value of E_1 is obtained by subtracting vectorially $I_1 Z_1$ from V_1. The value of $X_0 = E_1/I_0$ and of $R_0 = E_1/I_w$. It is clear that E_1 and E_2 are related to each other by expression

$$E_2/E_1 = N_2/N_1 = K.$$

To make transformer calculations simpler, it is preferable to transfer voltage, current and impedance

either to the primary or to the secondary. In that case, we would have to work in one winding only which is more convenient.

The primary equivalent of the secondary induced voltage is $E_2' = E_2/K = E_1$.

Similarly, primary equivalent of secondary terminal or output voltage is $V_2' = V_2/K$.

Primary equivalent of the secondary current is $I_2' = KI_2$.

For transferring secondary impedance to primary K^2 is used.

$$R_2' = R_2/K^2, X_2' = X_2/K^2, Z_2' = Z_2/K^2$$

The same relationship is used for shifting an external load impedance to the primary.

The secondary circuit is shown in Fig. 32.38 (*a*) and its equivalent primary values are shown in Fig. 32.38 (*b*).

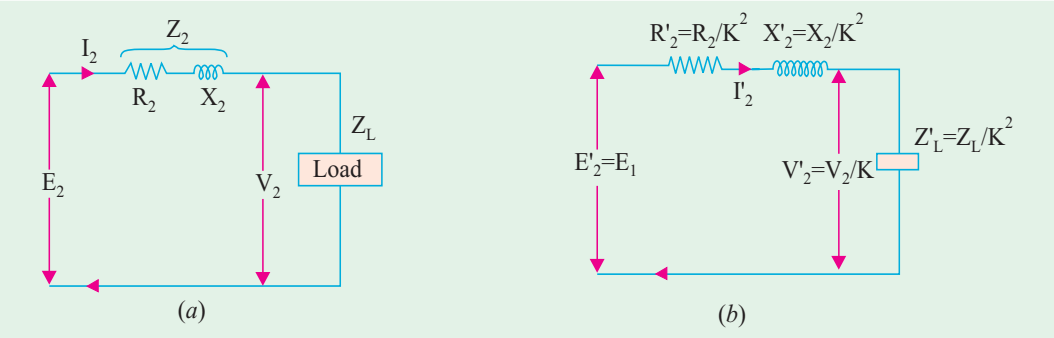


Fig. 32.38

The total equivalent circuit of the transformer is obtained by adding in the primary impedance as shown in Fig. 32.39. This is known as the exact equivalent circuit but it presents a somewhat harder circuit problem to solve. A simplification can be made by transferring the exciting circuit across the terminals as in Fig. 32.40 or in Fig. 32.41 (*a*). It should be noted that in this case $X_0 = V_1/I_\mu$.

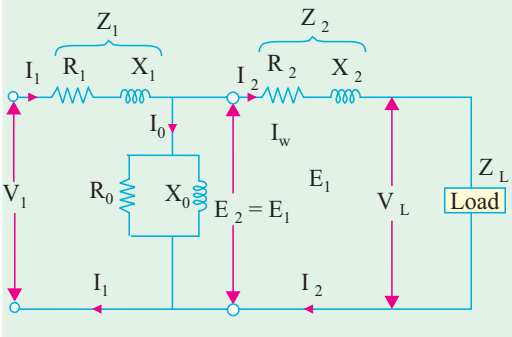


Fig. 32.39

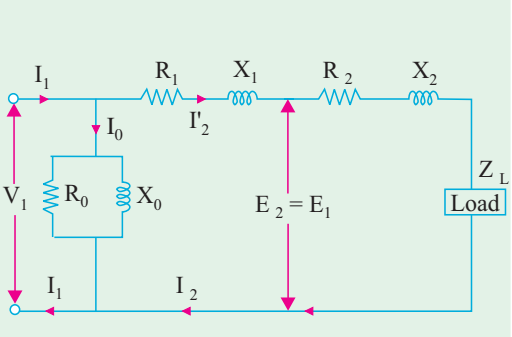


Fig. 32.40

Further simplification may be achieved by omitting I_0 altogether as shown in Fig. 32.41(*b*).

From Fig. 32.39 it is found that total impedance between the input terminal is

$$\mathbf{Z} = \mathbf{Z}_1 + \mathbf{Z}_m \,||\, (\mathbf{Z}_2' + \mathbf{Z}_L') = \left(\mathbf{Z}_1 + \frac{\mathbf{Z}_m\,(\mathbf{Z}_2' + \mathbf{Z}_L')}{\mathbf{Z}_m + (\mathbf{Z}_2' + \mathbf{Z}_L')} \right)$$

where $\mathbf{Z}_2' = R_2' + jX_2'$ and $\mathbf{Z}_m$ = impedance of the exciting circuit.

This is so because there are two parallel circuits, one having an impedance of $\mathbf{Z}_m$ and the other having $\mathbf{Z}_2'$ and $\mathbf{Z}_L'$ in series with each other.

$$\therefore \quad \mathbf{V}_1 = \mathbf{I}_1 \left[\mathbf{Z}_1 + \frac{\mathbf{Z}_m\,(\mathbf{Z}_2' + \mathbf{Z}_L')}{\mathbf{Z}_m + (\mathbf{Z}_2' + \mathbf{Z}_L')} \right]$$

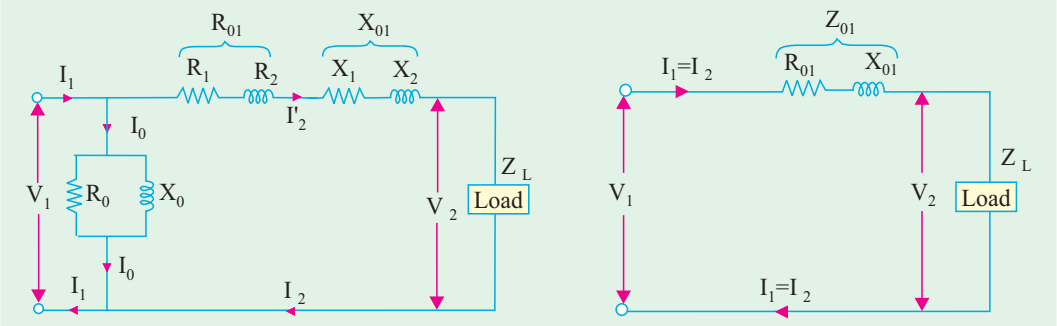


Fig. 32.41 (a) Fig. 32.41 (b)

Example 32.25. *The parametres of a 2300/230 V, 50-Hz transfomer are given below :*

$R_1 = 0.286\ \Omega$ $R_2' = 0.319\ \Omega$ $R_0 = 250\ \Omega$

$X_1 = 0.73\ \Omega$ $X_2' = 0.73\ \Omega$ $X_0 = 1250\ \Omega$

The secondary load impedance $Z_L = 0.387 + j\ 0.29$. Solve the exact equivalent circuit with normal voltage across the primary.

Solution.

$$K = 230/2300 = 1/10; \quad Z_L = 0.387 + j\,0.29$$

$$\mathbf{Z}_L' = \mathbf{Z}_L/K^2 = 100\,(0.387 + j\,0.29) = 38.7 + j\,29 = 48.4\angle 36.8°$$

$$\therefore \quad \mathbf{Z}_2' + \mathbf{Z}_L' = (38.7 + 0.319) + j(29 + 0.73) = 39.02 + j29.73 = 49.0\angle 37.3°$$

$$Y_m = (0.004 - j\,0.0008);\ \mathbf{Z}_m = 1/\mathbf{Y}_m = 240 + j48 = 245\angle 11.3°$$

$$\mathbf{Z}_m + (\mathbf{Z}_2' + \mathbf{Z}_L') = (240 + j48) + (39 + j29.7) = 290\angle 15.6°$$

$$\therefore \quad \mathbf{I}_1 = \frac{\mathbf{V}_1}{\mathbf{Z}_1 + \dfrac{\mathbf{Z}_m(\mathbf{Z}_2' + \mathbf{Z}_L')}{\mathbf{Z}_m(\mathbf{Z}_2' + \mathbf{Z}_L')}} = \left[\frac{2300\ \angle 0°}{0.286 + j0.73 + 41.4\ \angle 33°}\right]$$

$$= \frac{2300\ \angle 0°}{42\angle 33.7°} = 54.8\ \angle -33.7°$$

Now $$\mathbf{I}_2' = \mathbf{I}_1 \times \frac{\mathbf{Z}_m}{(\mathbf{Z}_2' + \mathbf{Z}'_L) + \mathbf{Z}_m} = 54.8\ \angle -33.7° \times \frac{245\ \angle 11.3°}{290\angle 15.6°}$$

$$= 54.8\ \angle -33.7° \times 0.845\ \angle -4.3° = 46.2\ \angle -38°$$

$$\mathbf{I}_0 = \mathbf{I}_1 \times \frac{(\mathbf{Z}_2' + \mathbf{Z}'_L)}{\mathbf{Z}_m + (\mathbf{Z}_2' + \mathbf{Z}_L')} = 54.8\ \angle -33.7° \times \frac{49\ \angle 37.3°}{290\angle 15.6°}$$

$$= 54.8\ \angle -33.7° \times 0.169\ \angle 21.7° = 9.26\ \angle -12°$$

Input power factor $= \cos 33.7° = 0.832$ lagging

Power input $= V_1 I_1 \cos\phi_1 = 2300 \times 54.8 \times 0.832 = 105$ kW

Power output $= 46.2^2 \times 38.7 = 82.7$ kW

Primary Cu loss $= 54.8^2 \times 0.286 = 860$ W

Secondary Cu loss $= 46.2^2 \times 0.319 = 680$ W; Core loss $= 9.26^2 \times 240 = 20.6$ kW

$\eta = (82.7/105) \times 100 = 78.8\%$; $V_2' = I_2' Z_L' = 46.2 \times 48.4 = 2{,}240$V

$$\therefore \quad \text{Regulation} = \frac{2300 - 2240}{2240} \times 100 = \mathbf{2.7\%}$$

Example 32.26. *A transformer has a primary winding with a voltage-rating of 600 V. Its secondary-voltage rating is 1080 V with an additional tap at 720 V. An 8 kW resistive load is connected across 1080-V output terminals. A purely inductive load of 10kVA is connected across the tapping point and common secondary terminal so as to get 720 V. Calculate the primary current and its power-factor. Correlate it with the existing secondary loads. Neglect losses and magnetizing current.* **(Nagpur University, Winter 1999)**

Solution. Loads are connected as shown in Fig. 32.42.

$$I_{r_2} = \frac{8000}{1080} = 7.41 \text{ at unity p.f.}$$

$$I_{L_2} = 10000/720 = 13.89 \text{ at zero lagging p.f.}$$

These are reflected on to the primary sides with appropriate ratios of turns, with corresponding power-factors. If the corresponding transformed currents are represented by the above symbols modified by dashed superscripts,

$$I'_{r_2} = 7.41 \times 1080/600 = 13.34 \text{ A at unity p.f.}$$

$$I'_{L_2} = 13.89 \times 720/600 = 16.67 \text{ A at zero lag. p.f.}$$

Hence, $$I_{r_2} = [I'^{2}_{r_2} + I'^{2}_{L_2}]^{0.5} = 21.35 \text{ A, at } 0.625 \text{ lag p.f.}$$

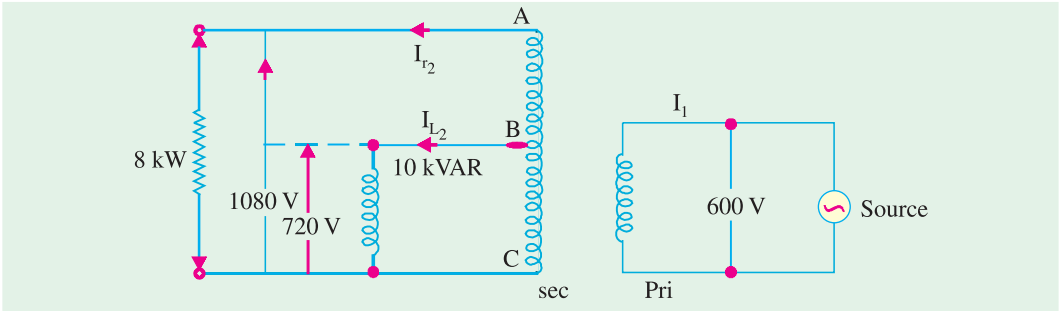


Fig. 32.42

Correlation : Since losses and magnetizing current are ignored, the calculations for primary current and its power-factor can also be made with data pertaining to the two Loads (in kW/kVAR), as supplied by the 600 V source.

S = Load to be supplied : 8 kW at unity p.f. and 10 kVAR lagging

Thus, $$S = P + jQ = 8 - j\ 10 \text{ kVA}$$

$$S = (8^2 + 10^2)^{0.5} = 12.8 \text{ kVA}$$

$$\text{Power – factor} = \cos\phi = 8/12.8 = 0.625 \text{ lag}$$

$$\text{Primary current} = 12.8 \times 1000/600 = 21.33 \text{ A}$$

32.19. Transformer Tests

As shown in Ex 32.25, the performance of a transformer can be calculated on the basis of its equivalent circuit which contains (Fig. 32.41) four main parameters, the equivalent resistance R_{01} as referred to primary (or secondary R_{02}), the equivalent leakage reactance X_{01} as referred to primary (or secondary X_{02}), the core-loss conductance G_0 (or resistance R_0) and the magnetising susceptance B_0 (or reactance X_0). These constants or parameters can be easily determined by two tests **(*i*) open-circuit test** and **(*ii*) short-circuit test.** These tests are very economical and convenient, because they furnish the required information without actually loading the transformer. In fact, the testing of very large a.c. machinery consists of running two tests similar to the open and short-circuit tests of a transformer.



Small transformer

32.20. Open-circuit or No-load Test

The purpose of this test is to determine no-load loss or core loss and no-load I_0 which is helpful in finding X_0 and R_0.

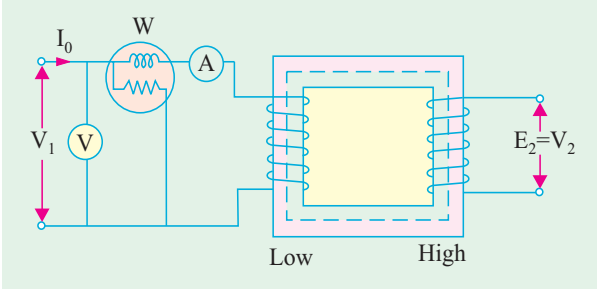


Fig. 32.43

One winding of the transformer – whichever is convenient but usually high voltage winding – is left open and the other is connected to its supply of normal voltage and frequency. A wattmeter W, voltmeter V and an ammeter A are connected in the low-voltage winding *i.e.* primary winding in the present case. With normal voltage applied to the primary, normal flux will be set up in the core, hence normal iron losses will occur which are recorded by the wattmeter. As the primary no-load current I_0 (as measured by ammeter) is small (usually 2 to 10% of rated load current), Cu loss is negligibly small in primary and nil in secondary (it being open). Hence, the wattmeter reading represents practically the core loss under no-load condition (and which is the same for all loads as pointed out in Art. 32.9).

It should be noted that since I_0 is itself very small, the pressure coils of the wattmeter and the voltmeter are connected such that the current in them does not pass through the current coil of the wattmeter.

Sometimes, a high-resistance voltmeter is connected across the secondary. The reading of the voltmeter gives the induced e.m.f. in the secondary winding. This helps to find transformation ratio K.

The no-load vector diagram is shown in Fig. 32.16. If W is the wattmeter reading (in Fig. 32.43), then

$$W = V_1 I_0 \cos\phi_0 \quad \therefore \quad \cos\phi_0 = W/V_1 I_0$$

$$\therefore \quad I_\mu = I_0 \sin\phi_0, \; I_w = I_0 \cos\phi_0 \; \therefore \; X_0 = V_1/I_\mu \text{ and } R_0 = V_1/I_w$$

Or since the current is practically all-exciting current when a transformer is on no-load (*i.e.* $I_0 \cong I_\mu$) and as the voltage drop in primary leakage impedance is small*, hence the exciting admittance Y_0 of the transformer is given by $I_0 = V_1 Y_0$ or $Y_0 = I_0/V_1$.

The exciting conductance G_0 is given by $W = V_1^2 G_0$ or $G_0 = W/V_1^2$.

The exciting susceptance $B_0 = \sqrt{(Y_0^2 - G_0^2)}$

Example. 32.27. *In no-load test of single-phase transformer, the following test data were obtained :*

Primary voltage : 220 V ; Secondary voltage : 110 V ;

Primary current : 0.5 A ; Power input : 30 W.

Find the following :

(i) *The turns ratio* ***(ii)*** *the magnetising component of no-load current* ***(iii)*** *its working (or loss) component* ***(iv)*** *the iron loss.*

Resistance of the primary winding = 0.6 ohm.

Draw the no-load phasor diagram to scale. **(Elect. Machine A.M.I.E. 1990)**

Solution. *(i)* Turn ratio, $N_1/N_2 = 220/110 = 2$

(ii) $W = V_1 I_0 \cos\phi_0$; $\cos\phi_0 = 30/220 \times 0.5 = 0.273$; $\sin\phi_0 = 0.962$

$$I_\mu = I_0 \sin\phi_0 = 0.5 \times 0.962 = 0.48 \text{ A}$$

---

* If it is not negligibly small, then $I_0 = E_1 Y_0$ *i.e.* instead of V_1 we will have to use E_1.

(iii) $I_w = I_0 \cos \phi_0 = 0.5 \times 0.273 = 0.1365$ A

(iv) Primary Cu loss $= I_0^2 R_1 = 0.5^2 \times 0.6 = 0.15$ W

$\therefore$ Iron loss $= 30 - 0.15 = 29.85$ W

Example 32.28. *A 5 kVA 200/1000 V, 50 Hz, single-phase transformer gave the following test results :*

O.C. Test (L.V. Side) : 2000 V, 1.2 A, 90 W

S.C. Test (H.V. Side) : 50 V, 5A, 110 W

(i) *Calculate the parameters of the equivalent circuit referred to the L.V. side.*

(ii) *Calculate the output secondary voltage when delivering 3 kW at 0.8 p.f. lagging, the input primary voltage being 200 V. Find the percentage regulation also.*

(Nagpur University, November 1998)

Solution. (i) Shunt branch parameters from O.C. test (L.V. side) :

$R_0 = V^2/P_i = 200^2/90 = 444$ ohms, $I_{ao} = 200/444 = 0.45$ amp

$I_\mu = (1.2^2 - 0.45^2)^{0.5} = 1.11$ amp, $X_m = 200/1.11 = 180.2$ ohms

All these are referred to L.V. side.

(ii) Series-branch Parameters from S.C test (H.V side) :

Since the S.C. test has been conducted from H.V. side, the parameters will refer to H.V. side.

They should be converted to the parameters referred to L.V. side by transforming them suitably.

From S.C. Test readings, $Z = 50/5 = 10$ ohms

$R = 110/25 = 4.40$ ohms, $X = (10^2 - 4.4^2)^{0.5} = 8.9$ ohms

These are referred to H.V. side.

For referring these to L.V. side, transform these using the ratio of turns, as follows :

$r_1 = 4.40 \times (200/1000)^2 = 0.176$ ohm

$x_1 = 8.98 \times (200/1000)^2 = 0.36$ ohm

Equivalent circuit can be drawn with R_0 and X_m calculated above and r_1 and x_1 as above.

L.V. Current at rated load= $5000/200 = 25$ A

L.V. Current at 3 kW at 0.8 lagging p.f. = $(3000/0.80)/200 = 18.75$ A

Regulation at this load $= 18.75 (r_1 \cos \phi + x_1 \sin \phi)$

$= 18.75 (0.176 \times 0.80 + 0.36 \times 0.6)$

$= +6.69$ Volts $= +(6.69/200) \times 100\% = +3.345\%$

This is referred to L.V. side, and positive sign means voltage drop.

Regulation in volts ref. to H.V. side $= 6.69 \times 1000/200 = 33.45$ V

With 200 V across primary (*i.e.* L.V. side), the secondary (*i.e.* H.V. side)

terminal voltage $= 1000 - 33.45 = 966.55$ V

Note : Since approximate formula for voltage regulation has been used, the procedure is simpler, and faster.

32.21. Separation of Core Losses

The core loss of a transformer depends upon the frequency and the maximum flux density when the volume and the thickness of the core laminations are given. The core loss is made up of two parts (*i*) hysteresis loss $W_h = PB_{max}^{1.6} f$ as given by Steinmetz's empirical relation and (*ii*) eddy current loss $W_e = QB_{max}^2 f^2$ where Q is a constant. The total core-loss is given by

$$W_i = W_h + W_e = PB_{max}^{1.6} f^2 + QB_{max}^2 f^2$$

If we carry out two experiments using two different frequencies but the same maximum flux density, we should be able to find the constants P and Q and hence calculate hysteresis and eddy current losses separately.

Example 32.29. *In a transformer, the core loss is found to be 52 W at 40 Hz and 90 W at 60 Hz measured at same peak flux density. Compute the hysteresis and eddy current losses at 50 Hz.*

(Elect. Machines, Nagpur Univ. 1993)

Solution. Since the flux density is the same in both cases, we can use the relation

$$\text{Total core loss } W_i = Af + Bf^2 \quad \text{or} \quad W_i/f = A + Bf$$

$\therefore$ $52/40 = A + 40B$ and $90/60 = A + 60B$; $\therefore$ $A = 0.9$ and $B = 0.01$

At 50 Hz, the two losses are

$$W_h = A_f = 0.9 \times 50 = 45 \text{ W}; \; W_e = Bf^2 = 0.01 \times 50^2 = 25 \text{ W}$$

Example 32.30. *In a power loss test on a 10 kg specimen of sheet steel laminations, the maximum flux density and waveform factor are maintained constant and the following results were obtained:*

Frequency (Hz)	*25*	*40*	*50*	*60*	*80*
Total loss (watt)	*18.5*	*36*	*50*	*66*	*104*

Calculate the eddy current loss per kg at a frequency of 50 Hz.

(Elect. Measur. A.M.I.E. Sec B, 1991)

Solution. When flux density and wave form factor remain constant, the expression for iron loss can be written as

$$W_i = Af + Bf^2 \quad \text{or} \quad W_i/f = A + Bf$$

The values of W_i/f for different frequencies are as under :

f	25	40	50	60	80
W_i/f	0.74	0.9	1.0	1.1	1.3

The graph between f and W_i/f has been plotted in Fig. 32.44. As seen from it, $A = 0.5$ and $B = 0.01$

$\therefore$ Eddy current loss at 50 Hz $= Bf^2 = 0.01 \times 50^2 = 25$ W

$\therefore$ Eddy current loss/kg $= 25/10 = 2.5$ W

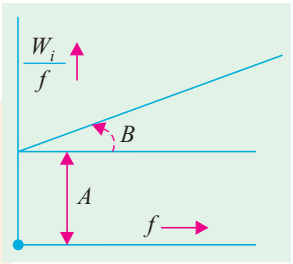


Fig. 32.44

Example 32.31. *In a test for the determination of the losses of a 440-V, 50-Hz transformer, the total iron losses were found to be 2500 W at normal voltage and frequency. When the applied voltage and frequency were 220 V and 25 Hz, the iron losses were found to be 850 W. Calculate the eddy-current loss at normal voltage and frequency.*

(Elect. Inst. and Meas. Punjab Univ. 1991)

Solution. The flux density in both cases is the same because in second case voltage as well as frequency are halved. Flux density remaining the same, the eddy current loss is proportional to f^2 and hysteresis loss $\propto f$.

Hysteresis loss $\propto f = Af$ and eddy current loss $\propto f^2 = Bf^2$

where A and B are constants.

$$\text{Total iron loss} \quad W_i = Af + Bf^2 \quad \therefore \quad \frac{W_i}{f} = A + Bf \qquad \ldots(i)$$

Now, when $f = 50$ Hz : $W_i = 2500$ W

and when $f = 25$ Hz ; $W_i = 850$ W

Using these values in (i) above, we get, from Fig. 32.44

$$2{,}500/50 = A + 50B \text{ and } 850/25 = A + 25B \quad \therefore \quad B = 16/25 = 0.64$$

Hence, at normal p.d. and frequency

$$\text{eddy current loss} = Bf^2 = 0.64 \times 50^2 = \mathbf{1600 \text{ W}}$$

$$\text{Hysteresis loss} = 2500 - 1600 = 900 \text{ W}$$

Example 32.32. *When a transformer is connected to a 1000-V, 50-Hz supply the core loss is 1000 W, of which 650 is hysteresis and 350 is eddy current loss. If the applied voltage is raised to 2,000 V and the frequency to 100 Hz, find the new core losses.*

Solution. Hysteresis loss $W_h \propto B_{max}^{1.6} f = PB_{max}^{1.6} f$

Eddy current loss $W_e \propto B^2_{max} f^2 = QB^2_{max} f^2$

From the relation $E = 4.44 fNB_{max}$ A volt, we get $B_{max} \propto E/f$

Putting this value of B_{max} in the above equations, we have

$$W_h = P\left(\frac{E}{f}\right)^2 f = PE^{1.6} f^{-0.6} \text{ and } W_e = Q\left(\frac{E}{f}\right)^2 f^2 = QE^2$$

In the first case, $E = 1000$ V, $f = 50$ Hz, $W_h = 650$ W, $W_e = 350$ W

$\therefore \quad 650 = P \times 1000^{1.6} \times 50^{-0.6} \quad \therefore \quad P = 650 \times 1000^{-1.6} \times 50^{0.6}$

Similarly, $350 = Q \times 1000^2 \quad \therefore \quad Q = 350 \times 1000^{-2}$

Hence, constants P and Q are known.

Using them in the second case, we get

$$W_h = (650 \times 1000^{-1.6} \times 50^{0.6}) \times 2000^{1.6} \times 100^{-0.6} = 650 \times 2 = 1{,}300 \text{ W}$$

$$W_e = (350 \times 1000^{-2}) \times 2{,}000^2 = 350 \times 4 = 1{,}400 \text{ W}$$

$\therefore$ Core loss under new condition is = 1,300 + 1,400 = **2700 W**

Alternative Solution

Here, both voltage and frequency are doubled, leaving the flux density unchanged.

With 1000 V at 50 Hz

$$W_h = Af \text{ or } 650 = 50\,A;\ A = 13$$

$$W_e = Bf^2 \text{ or } 350 = B \times 50^2;\ B = 7/50$$

With 2000 V at 100 Hz

$$W_h = Af = 13 \times 100 = 1300 \text{ W and}$$

$$W_e = Bf^2 = (7/50) \times 100^2 = 1400 \text{ W}$$

$\therefore$ New core loss = 1300 + 1400 = **2700 W**

Example 32.33. *A transformer with normal voltage impressed has a flux density of 1.4 Wb/m^2 and a core loss comprising of 1000 W eddy current loss and 3000 W hysteresis loss. What do these losses become under the following conditions ?*

(*a*) *increasing the applied voltage by 10% at rated frequency.*

(*b*) *reducing the frequency by 10% with normal voltage impressed.*

(*c*) *increasing both impressed voltage and frequency by 10 per cent.*

(Electrical Machinery-I, Madras Univ. 1985)

Solution. As seen from Ex. 32.32

$$W_h = PE^{1.6} f^{-0.6} \text{ and } W_e = QE^2$$

From the given data, we have $3000 = PE^{1.6} f^{-0.6}$ ...(*i*)

and $1000 = QE^2$ ...(*ii*)

where E and f are the normal values of primary voltage and frequency.

(*a*) Here voltage becomes $= E + 10\%\,E = 1.1\,E$

The new hysteresis loss is $W_h = P(1.1\,E)^{1.6} f^{-0.6}$ ...(*iii*)

Dividing Eq. (*iii*) by (*i*), we get $\frac{W_h}{3000} = 1.1^{1.6};\ W_h = 3000 \times 1.165 =$ **3495 W**

The new eddy-current loss is

$$W_e = Q\,(1.1.\,E)^2 \quad \therefore \quad \frac{W_e}{1000} = 1.1^2$$

$$\therefore \quad W_e = 1000 \times 1.21 = \mathbf{1210\ W}$$

(*b*) As seen from Eq. **(*i*)** above eddy-current loss would not be effected. The new hysteresis loss is

$$W_h = PE^{1.6}\,(0.9\,f)^{-0.6} \quad \text{...(iv)}$$

From **(*i*)** and **(*iv*)**, we get $\frac{W_h}{3000} = 0.9^{-0.6}$, $W_h = 3000 \times 1.065 = \mathbf{3,196\ W}$

(*c*) In this case, both E and f are increased by 10%. The new losses are as under :

$$W_h = P\,(1.1\,E)^{1.6}\,(1.1\,f)^{-0.6}$$

$$\therefore \quad \frac{W_h}{3000} = 1.1^{1.6} \times 1.1^{-0.6} = 1.165 \times 0.944$$

$$\therefore \quad W_h = 3000 \times 1.165 \times 0.944 = \mathbf{3,299\ W}$$

As W_e is unaffected by changes in f, its value is the same as found in (*a*) above *i.e.* **1210 W**

Example 32.34. *A transformer is connected to 2200 V, 40 Hz supply. The core-loss is 800 watts out of which 600 watts are due to hysteresis and the remaining, eddy current losses. Determine the core-loss if the supply voltage and frequency are 3300 V and 60 Hz respectively.*

(Bharathiar Univ. Nov. 1997)

Solution. For constant flux density (*i.e.* constant V/f ratio), which is fulfilled by 2200/40 or 3300/60 figures in two cases,

$$\text{Core-loss} = A\,f + B\,f^2$$

First term on the right-hand side represents hysteresis-loss and the second term represents the eddy-current loss.

At 40 Hz, 800 = 600 + eddy current loss.

Thus, $Af = 600$, or $A = 15$

$Bf^2 = 200$, or $B = 200/1600 = 0.125$

At 60 Hz, core-loss $= 15 \times 60 + 0.125 \times 60^2$

$= 900 + 450$

$= 1350$ watts

32.22 Short-Circuit or Impedance Test

This is an economical method for determining the following :

(*i*) Equivalent impedance (Z_{01} or Z_{02}), leakage reactance (X_{01} or X_{02}) and total resistance (R_{01} or R_{02}) of the transformer as referred to the winding in which the measuring instruments are placed.

(*ii*) Cu loss at full load (and at any desired load). This loss is used in calculating the efficiency of the transformer.

(*iii*) Knowing Z_{01} or Z_{02}, the total voltage drop in the transformer as referred to primary or secondary can be calculated and hence regulation of the transformer determined.

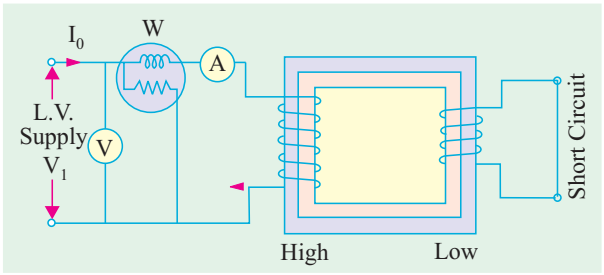


Fig. 32.45

In this test, one winding, usually the low-voltage winding, is solidly short-circuited by a thick conductor (or through an ammeter which may serve the additional purpose of indicating rated load current) as shown in Fig. 32.45.

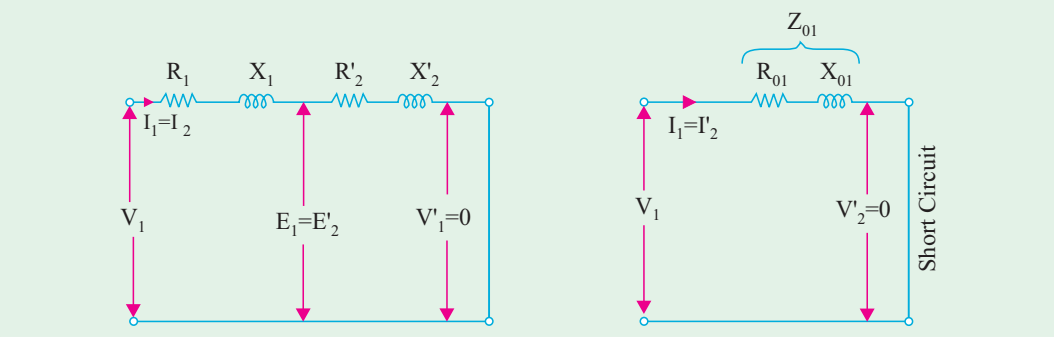


Fig. 32.46

A low voltage (usually 5 to 10% of normal primary voltage) at correct frequency (though for Cu losses it is not essential) is applied to the primary and is cautiously increased till full-load currents are flowing both in primary and secondary (as indicated by the respective ammeters).

Since, in this test, the applied voltage is a small percentage of the normal voltage, the mutual flux Φ produced is also a small percentage of its normal value (Art. 32.6). Hence, core losses are very small with the result that the wattmeter reading represent the full-load Cu loss or I^2R loss for the whole transformer *i.e.* both primary Cu loss and secondary Cu loss. The equivalent circuit of the transformer under short-circuit condition is shown in Fig. 32.46. If V_{sc} is the voltage required to circulate rated load currents, then $Z_{01} = V_{sc}/I_1$

Also $W = I_1^2 R_{01}$

$\therefore \quad R_{01} = W/I_1$

$\therefore \quad X_{01} = \sqrt{\left(Z_{01}^2 - R_{01}^2\right)}$

In Fig. 32.47 (*a*) the equivalent circuit vector diagram for the short-circuit test is shown. This diagram is the same as shown in Fig. 32.34 except that all the quantities are referred to the primary side. It is obvious that the entire voltage V_{SC} is consumed in the impedance drop of the two windings.

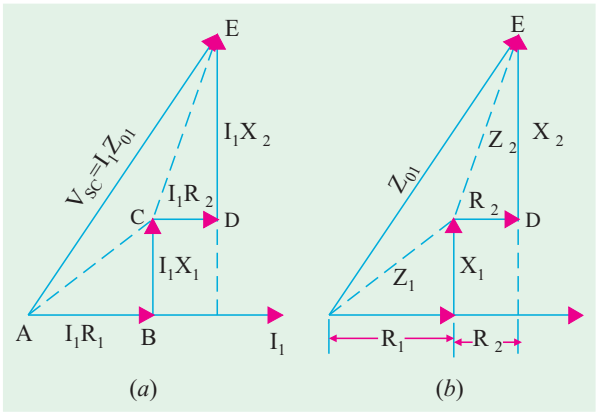


Fig. 32.47

If R_1 can be measured, then knowing R_{01}, we can find $R_2' = R_{01} - R_1$. The impedance triangle can then be divided into the appropriate equivalent triangles for primary and secondary as shown in Fig. 32.47 (*b*).

32.23. Why Transformer Rating in kVA ?

As seen, Cu loss of a transformer depends on current and iron loss on voltage. Hence, total transformer loss depends on volt-ampere (VA) and not on phase angle between voltage and current *i.e.* it is independent of load power factor. That is why rating of transformers is in kVA and not in kW.

Example 32.35. *The primary and secondary windings of a 30 kVA 76000/230, V, 1-phase transformer have resistance of 10 ohm and 0.016 ohm respectively. The reactance of the transformer referred to the primary is 34 ohm. Calculate the primary voltage required to circulate full-load current when the secondary is short-circuited. What is the power factor on short circuit ?*

(Elect. Machines AMIE Sec. B 1991)

Solution. $K = 230/6000 = 23/600$, $X_{01} = 34\,\Omega$,

$R_{01} = R_1 + R_2/K^2 = 10 + 0.016\ (600/23)^2 = 20.9\,\Omega$

$$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2} = \sqrt{20.9^2 + 34^2} = 40\ \Omega$$

F.L., $I_1 = 30{,}000/6000 = 5$ A ; $V_{SC} = I_1 Z_{01} = 5 \times 40 = 200$ V

Short circuit p.f. $= R_{01}/Z_{01} = 20.9/40 = 0.52$

Example 32.36. *Obtain the equivalent circuit of a 200/400-V, 50-Hz, 1-phase transformer from the following test data :*

O.C test : 200 V, 0.7 A, 70 W – on L.V. side

S.C. test : 15 V, 10 A, 85 W – on H.V. side

Calculate the secondary voltage when delivering 5 kW at 0.8 p.f. lagging, the primary voltage being 200V. **(Electrical Machinery-I, Madras Univ. 1987)**

Solution. From O.C. Test

$$V_1 I_0 \cos\phi_0 = W_0$$

$$\therefore \quad 200 \times 0.7 \times \cos\phi_0 = 70$$

$$\cos\phi_0 = 0.5 \text{ and } \sin\phi_0 = 0.866$$

$$I_w = I_0 \cos\phi_0 = 0.7 \times 0.5 = 0.35 \text{ A}$$

$$I_\mu = I_0 \sin\phi_0 = 0.7 \times 0.866 = 0.606 \text{ A}$$

$$R_0 = V_1/I_w = 200/0.35 = 571.4\ \Omega$$

$$X_0 = V_1/I_\mu = 200/0.606 = 330\ \Omega$$

As shown in Fig. 32.48, these values refer to primary *i.e.* low-voltage side.

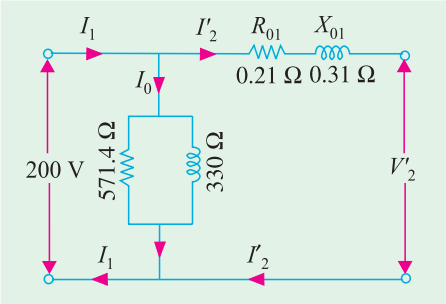


Fig. 32.48

From S.C. Test

It may be noted that in this test, instruments have been placed in the secondary *i.e.* high-voltage winding whereas the low-voltage winding *i.e.* primary has been short-circuited.

Now, as shown in Art. 32.32

$$Z_{02} = V_{sc}/I_2 = 15/10 = 1.5\ \Omega ; \ K = 400/200 = 2$$

$$Z_{01} = Z_{02}/K^2 = 1.5/4 = 0.375\ \Omega$$

Also $I_2^2 R_{02} = W ; \ R_{02} = 85/100 = 0.85\ \Omega$

$$R_{01} = R_{02}/K^2 = 0.85/4 = 0.21\ \Omega$$

$$X_{01} = \sqrt{Z_{01}^2 - R_{01}^2} = \sqrt{0.375^2 - 0.21^2} = 0.31\ \Omega$$

Output kVA $= 5/0.8$; Output current $I_2 = 5000/0.8 \times 400 = 15.6$ A

This value of I_2 is approximate because V_2 (which is to be calculated as yet) has been taken equal to 400 V (which, in fact, is equal to E_2 or ${}_0V_2$).

Now, $Z_{02} = 1.5\ \Omega, \ R_{02} = 0.85\ \Omega \quad \therefore \quad X_{02} = \sqrt{1.5^2 - 0.85^2} = 1.24\ \Omega$

Total transformer drop as referred to secondary

$$= I_2 (R_{02} \cos\phi_2 + X_{02} \sin\phi_2) = 15.6\,(0.85 \times 0.8 + 1.24 \times 0.6) = 22.2 \text{ V}$$

$$\therefore \quad V_2 = 400 - 22.2 = \mathbf{377.8\ V}$$

Example 32.37. *Starting from the ideal transformer, obtain the approximate equivalent circuit of a commercial transformer in which all the constants are lumped and represented on one side.*

A 1-phase transformer has a turn ratio of 6. The resistance and reactance of primary winding are 0.9 Ω and 5 Ω respecitvely and those of the secondary are 0.03 Ω and 0.13 Ω respectively. If 330 -V at 50-Hz be applied to the high voltage winding with the low-voltage winding short-circuited, find the current in the low-voltage winding and its power factor. Neglect magnetising current.

Solution. Here $K = 1/6$; $R_{01} = R_1 + R'_2 = 0.9 + (0.03 \times 36) = 1.98\ \Omega$

$X_{01} = X_1 + X_2' = 5 + (0.13 \times 36) = 9.68\ \Omega$

$\therefore \quad Z_{01} = \sqrt{(9.68^2 + 1.98^2)} = 9.9\ \Omega;\ V_{SC} = 330$ V

$\therefore$ Full-load primary current $I_1 = V_{sc}/Z_{01} = 330/0.9 = 100/3$ A

As I_0 is negligible, hence $I_1 = I'_2 = 100/3$ A. Now, $I'_2 = KI_2$

F.L. secondary current $I_2 = I'_2 K = (100/3) \times 6 = 200$ A

Now, Power input on short-circuit $= V_{SC} I_1 \cos \phi_{SC}$ = Cu loss $= I_1^2 R_{01}$

$\therefore \quad (100/3)^2 \times 1.98 = 330 \times (100/3) \times \cos \phi_{SC}$; $\cos \phi_{SC} =$ **0.2**

Example 32.38. *A 1-phase, 10-kVA, 500/250-V, 50-Hz transformer has the following constants:*

Reactance : primary 0.2 Ω; secondary 0.5 Ω

Resistance : primary 0.4 Ω; secondary 0.1 Ω

Resistance of equivalent exciting circuit referred to primary, $R_0 = 1500\ \Omega$

Reactance of equivalent exciting circuit referred to primary, $X_0 = 750\ \Omega$

What would be the reading of the instruments when the transformer is connected for the open-circuit and short-circuit tests ?

Solution. While solving this question, reference may please be made to Art. 30.20 and 30.22.

O.C. Test

$I_\mu = V_1/X_0 = 500/750 = 2/3$ A ; $I_w = V_1/R_0 = 500/1500 = 1/3$ A

$\therefore \quad I_0 = \sqrt{[1/3)^2 + (2/3)^2]} = 0.745$ A

No-load primary input $= V_1 I_w = 500 \times 1/3 = 167$ W

Instruments used in primary circuit are : voltmeter, ammeter and wattmeter, their readings being 500 V, 0.745 A and 167 W respectively.

S.C. Test

Suppose S.C. test is performed by short-circuiting the *l.v.* winding *i.e.* the secondary so that all instruments are in primary.

$R_{01} = R_1 + R'_2 = R_1 + R_2/K^2$; Here $K = 1/2 \quad \therefore \quad R_{01} = 0.2 + (4 \times 0.5) = 2.2\ \Omega$

Similarly, $X_{01} = X_1 + X'_2 = 0.4 + (4 \times 0.1) = 0.8\ \Omega$

$Z_{01} = \sqrt{(2.2^2 + 0.8^2)} = 2.341\ \Omega$

Full-load primary current

$I_1 = 10{,}000/500 = 20$ A $\therefore V_{SC} = I_1 Z_{01} = 20 \times 2.341 = 46.8$ V

Power absorbed $= I_1^2 R_{01} = 20^2 \times 2.2 = 880$ W

Primary instruments will read : **46.8 V, 20 A, 880 W.**

Example 32.39. *The efficiency of a 1000-kVA, 110/220 V, 50-Hz, single-phase transformer, is 98.5 % at half full-load at 0.8 p.f. leading and 98.8 % at full-load unity p.f. Determine (i) iron loss (ii) full-load copper loss and (iii) maximum efficiency at unity p.f.*

(Elect. Engg. AMIETE Sec. A Dec. 1991)

Solution. Output at F.L. unity p.f. = $1000 \times 1 = 1000$ kW

F.L. input = 1000/0.988 = 1012.146 kW

F.L. lossses = 1012.146 – 1000 = 12.146 kW

If F.L. Cu and iron losses are x and y respectively then

$x + y = 12.146$ kW ...**(*i*)**

Input at half F.L. 0.8 p.f. = $500 \times 0.8/0.985 = 406.091$ kW

Total losses at half F.L. = $406.091 - 400 = 6.091$ kW

Cu loss at half-load = $x\,(1/2)^2 = x/4$

$\therefore \quad x/4 + y = 6.091$ ...(*ii*)

From Eqn. (*i*) and (*ii*), we get (*i*) $x = 8.073$ kW and (*ii*) $y = 4.073$ kW

(*iii*) kVA for $\eta_{max} = 1000 \times \sqrt{4.073/8.073} = 710.3$ kVA

Output at u.p.f. $= 710.3 \times 1 = 710.3$ kW

Cu loss = iron loss = 4.037 kW ; Total loss = $2 \times 4.037 = 8.074$ kW

$\therefore \quad \eta_{max} = 710.3/(710.3 + 8.074) = 0.989$ or 98.9 %

Example 32.40. *The equivalent circuit for a 200/400-V step-up transformer has the following parameters referred to the low-voltage side.*

Equivalent resistance = 0.15 Ω ; Equivalent reactance = 0.37 Ω

Core-loss component resistance = 600 Ω ; Magnetising reactance = 300 Ω

When the transformer is supplying a load at 10 A at a power factor of 0.8 lag, calculate (i) the primary current (ii) secondary terminal voltage. **(Electrical Machinery-I, Bangalore Univ. 1989)**

Solution. We are given the following :

$R_{01} = 0.15\ \Omega, X_{01} = 0.37\ \Omega ; R_0 = 600\ \Omega, X_0 = 300\ \Omega$

Using the approximate equivalent circuit of Fig. 32.41, we have,

$$I_\mu = V_1/X_0 = 200/300 = (2/3)\text{ A}$$
$$I_w = V_1/R_0 = 200/600 = (1/3)\text{ A}$$
$$I_0 = \sqrt{I_\mu^2 + I_w^2} = \sqrt{(2/3)^2 + (1/3)^2} = 0.745\text{ A}$$

As seen from Fig. 32.49

$$\tan\theta = \frac{I_w}{I_\mu} = \frac{1/3}{2/3} = \frac{1}{2} ; \theta = 26.6^\circ$$

$\therefore \quad \phi_0 = 90^\circ - 26.6^\circ = 63.4^\circ$; Angle between I_0 and

$I_2' = 63.4^\circ - 36.9^\circ = 26.5^\circ$; $K = 400/200 = 2$

$I_2' = KI_2 = 2 \times 10 = 20$ A

(*i*) $I_1 = (0.745^2 + 20^2 + 2 \times 0.745 \times 20 \times \cos 26.5^\circ)^{1/2}$

$= \mathbf{20.67\ A}$

(*ii*) $R_{02} = K^2 R_{01} = 2^2 \times 0.15 = 0.6\ \Omega$

$X_{02} = 22 \times 0.37 = 1.48\ \Omega$

Approximate voltage drop

$= I_2 (R_{02} \cos\phi + X_{02} \sin\phi)$

$= 10\,(0.6 \times 0.8 + 1.48 \times 0.6) = 13.7$ V

$\therefore$ Secondary terminal voltage = $400 - 13.7$

$= \mathbf{386.3\ V}$

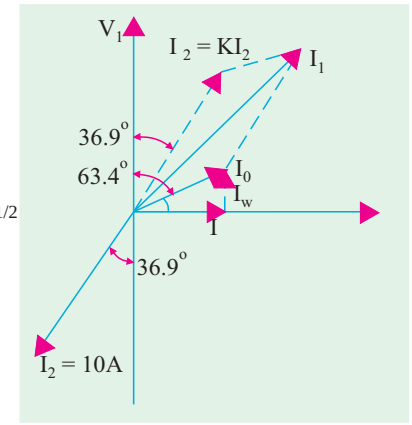


Fig. 32.49

Example 32.41. *The low voltage winding of a 300-kVA, 11,000/2500-V, 50-Hz transformer has 190 turns and a resistance of 0.06. The high-voltage winding has 910 turns and a resistance of 1.6 Ω. When the l.v. winding is short-circuited, the full-load current is obtained with 550-V applied to the h.v. winding. Calculate (i) the equivalent resistance and leakage reactance as referred to h.v. side and (ii) the leakage reactance of each winding.*

Solution. Assuming a full-load efficiency of 0.985, the full-load primary current is

$= 300{,}000/0.985 \times 11{,}000 = 27.7\text{A}$

$\therefore \quad Z_{01} = 550/27.7 = 19.8\ \Omega\ ;\ R'_2 = R^2/K^2 = 0.06\ (910/190)^2 = 1.38\ \Omega$

$\therefore \quad R_{01} = R_1 + R'_2 = 1.6 + 1.38 = 2.98\ \Omega$

$X_{01} = \sqrt{\left(Z_{01}^2 - R_{01}^2\right)} = \sqrt{(19.8^2 - 2.98^2)} = 19.5\ \Omega$

Let us make another assumption that for each winding the ratio (reactance/resistance) is the same, then

$X_1 = 19.5 \times 1.6/2.98 = 10.5\ \Omega$

$X_2' = 19.5 \times 1.38/2.98 = 9.0\ \Omega\ ;\ X_2 = 9(190/910)^2 = 0.39\ \Omega$

(a) $R_{01} =$ **2.98 Ω** ; $X_{01} =$ **19.5 Ω** *(b)* $X_1 =$ **10.5 Ω** : $X_2 =$ **0.39 Ω**

Example 32.42. *A 230/115 volts, single phase transformer is supplying a load of 5 Amps, at power factor 0.866 lagging. The no-load current is 0.2 Amps at power factor 0.208 lagging. Calculate the primary current and primary power factor.*

(Nagpur University Summer 2000)

Solution. L.V. current of 5 amp is referred to as a 2.5 amp current on the primary (= H.V.) side, at 0.866 lagging p.f. To this, the no load current should be added, as per the phasor diagram in Fig. 32.50. The phase angle of the load-current is 30° lagging. The no load current has a phase angle of 80° lagging. Resultant of these two currents has to be worked out. Along the reference, active components are added.

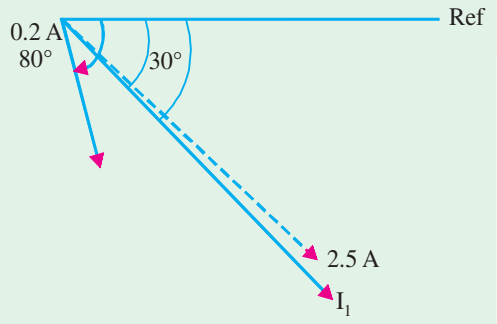


Fig. 32.50. Phasor diagram for Currents

Active components of currents $= 2.5 \times 0.866 + 0.2 \times 0.208$

$= 2.165 + 0.0416$

$= 2.2066$ amp

Along the perpendicular direction, the reactive components get added up.

Reactive component $= 2.5 \times\ 0.5 + 0.2 \times 0.9848$

$= 1.25 + 0.197 = 1.447$ amp.

$I_1 = 2.2066 - j\ 1.447$

$\phi = \tan^{-1} \dfrac{1.447}{2.2066} =$ **33.25° ; as shown**

Tutorial Problems 32.3

1. The S.C. test on a 1-phase transformer, with the primary winding short-circuited and 30 V applied to the secondary gave a wattmeter reading of 60 W and secondary current of 10 A. If the normal applied primary voltage is 200, the transformation ratio 1 :2 and the full-load secondary current 10 A, calculate the secondary terminal p.d. at full-load current for (*a*) unity power factor (*b*) power factor 0.8 lagging. If any approximations are made, they must be explained. **[394 V, 377.6 V]**
2. A single-phase transformer has a turn ratio of 6, the resistances of the primary and secondary windings are 0.9 Ω and 0.025 Ω respectively and the leakage reactances of these windings are 5.4 Ω and 0.15 Ω respectively. Determine the voltage to be applied to the low-voltage winding to obtain a current of 100 A in the short-circuited high voltage winding. Ignore the magnetising current. **[82 V]**
3. Draw the equivalent circuit for a 3000/400-V, I-phase transformer on which the following test results were obtained. Input to high voltage winding when 1.v. winding is open-circuited : 3000 V, 0.5 A, 500

W. Input to 1.v. winding when h.v. winding is short-circuited : 11 V, 100 A, 500 W. Insert the appropriate values of resistance and reactance.

[R_0 = 18,000 Ω, X_0 = 6,360 Ω, R_{01} = 2.81 Ω, X_{01} = 5.51 Ω] (*I.E.E. London*)

4. The iron loss in a transformer core at normal flux density was measured at frequencies of 30 and 50 Hz, the results being 30 W and 54 W respectively. Calculate (*a*) the hysteresis loss and (*b*) the eddy current loss at 50 Hz. **[44 W, 10 W]**

5. An iron core was magnetised by passing an alternating current through a winding on it. The power required for a certain value of maximum flux density was measured at a number of different frequencies. Neglecting the effect of resistance of the winding, the power required per kg of iron was 0.8 W at 25 Hz and 2.04 W at 60 Hz. Estimate the power needed per kg when the iron is subject to the same maximum flux density but the frequency is 100 Hz. **[3.63 W]**

6. The ratio of turns of a 1-phase transformer is 8, the resistances of the primary and secondary windings are 0.85 Ω and 0.012 Ω respectively and leakage reactances of these windings are 4.8 Ω and 0.07 Ω respectively. Determine the voltage to be applied to the primary to obtain a current of 150 A in the secondary circuit when the secondary terminals are short-circuited. Ignore the magnetising current. **[176.4 W]**

7. A transformer has no-load losses of 55 W with a primary voltage of 250 V at 50 Hz and 41 W with a primary voltage of 200 V at 40 Hz. Compute the hysteresis and eddy current losses at a primary voltage of 300 volts at 60 Hz of the above transformer. Neglect small amount of copper loss at no-load. **[43.5 W ; 27 W] (*Elect. Machines AMIE Sec. B. (E-3) Summer 1992*)**

8. A 20 kVA, 2500/250 V, 50 Hz, 1-phase transformer has the following test results :

O.C. Test (1.v. side) : 250 V, 1.4 A, 105 W

S.C. Test (h.v. side) : 104 V, 8 A, 320 W

Compute the parameters of the approximate equivalent circuit referred to the low voltage side and draw the circuit. **(R_0 = 592.5 Ω ; X_0 = 187.2 Ω ; R_{02} = 1.25 Ω ; X_{12} = 3 Ω)**

(*Elect. Machines A.M.I.E. Sec. B Summer 1990*)

9. A 10-kVA, 2000/400-V, single-phase transformer has resistances and leakage reactances as follows :

R_1 = 5.2 Ω, X_1 = 12.5 Ω, R_2 = 0.2 Ω, X_2 = 0.5 Ω

Determine the value of secondary terminal voltage when the transformer is operating with rated primary voltage with the secondary current at its rated value with power factor 0.8 lag. The no-load current can be neglected. Draw the phasor diagram. **[376.8 V] (*Elect. Machines, A.M.I.E. Sec B, 1989*)**

10. A 1000-V, 50-Hz supply to a transformer results in 650 W hysteresis loss and 400 W eddy current loss. If both the applied voltage and frequency are doubled, find the new core losses.

[W_h = 1300 W ; W_e = 1600 W] (*Elect. Machine, A.M.I.E. Sec. B, 1993*)

11. A 50 kVA, 2200/110 V transformer when tested gave the following results :

O.C. test (L.V. side) : 400 W, 10 A, 110 V.

S.C. test (H.V. side) : 808 W, 20.5 A, 90 V.

Compute all the parameters of the equivalent ckt. referred to the H.V. side and draw the resultant ckt.

(Rajiv Gandhi Technical University, Bhopal 2000)

[Shunt branch : R_0 = 12.1 k-ohms, X_m = 4.724 k-ohms Series branch : r = 1.923 ohms, x = 4.39 ohms]

32.24. Regulation of a Transformer

1. When a transformer is loaded with a ***constant primary voltage***, the secondary voltage decreases* because of its internal resistance and leakage reactance.

Let $_0V_2$ = secondary terminal voltage at *no-load*.

* Assuming lagging power factor. It will increase if power factor is leading.

$= E_2 = EK_1 = K V_1$ because at no-load the impedance drop is negligible.

V_2 = secondary terminal voltage on *full-load*.

The change in secondary terminal voltage from no-load to full-load is $= {}_0V_2 - V_2$. This change divided by ${}_0V_2$ is known as regulation 'down'. If this change is divided by V_2, *i.e.*, full-load secondary terminal voltage, then it is called regulation 'up'.

$$\therefore \quad \%\text{ regn 'down'} = \frac{{}_0V_2 - V_2}{{}_0V_2} \times 100 \text{ and } \%\text{ regn 'up'} = \frac{{}_0V_2 - V_2}{V_2} \times 100$$

In further treatment, unless stated otherwise, regulation is to be taken as regulation 'down'.

We have already seen in Art. 32.16 (Fig. 32.35) that the change in secondary terminal voltage from no-load to full-load, expressed as a percentage of no-load secondary voltage is,

$$= \nu_r \cos\phi \pm \nu_x \sin\phi \qquad \text{(approximately)}$$

Or more accurately

$$= (\nu_r \cos\phi \pm \nu_x \sin\phi) + \frac{1}{200}(\nu_x \cos\phi \mp \nu_r \sin\phi)^2$$

$$\therefore \quad \%\text{ regn} = \nu_r \cos\phi \pm \nu_x \sin\phi \qquad \text{...approximately.}$$

The lesser this value, the better the transformer, because a good transformer should keep its secondary terminal voltage as constant as possible under all conditions of load.

(2) The regulation may also be explained in terms of primary values.

In Fig. 32.51 (*a*) the approximate equivalent circuit of a transformer is shown and in Fig. 32.51 (*b*), (*c*) and (*d*) the vector diagrams corresponding to different power factors are shown.

The secondary ***no-load*** terminal voltage as referred to primary is $E_2' = E_2/K = E_1 = V_1$ and if the secondary full-load voltage as referred to primary is V_2' $(= V_2/K)$ then

$$\%\text{ regn} = \frac{V_1 - V_2'}{V_1} \times 100$$

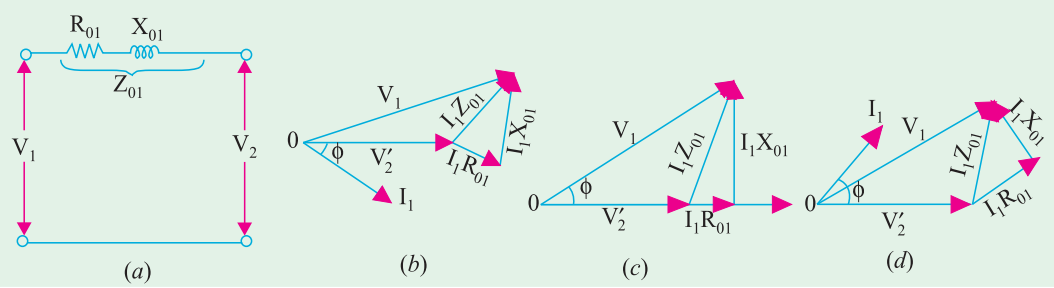


Fig. 32.51

From the vector diagram, it is clear that if angle between V_1 and V_2' is neglected, then the value of numerical difference $V_1 - V_2'$ is given by $(I_1 R_{01} \cos\phi + I_1 X_{01} \sin\phi)$ for lagging p.f.

$$\therefore \quad \%\text{ regn} = \frac{I_1 R_{01} \cos\phi + I_1 X_{01} \sin\phi}{V_1} \times 100 = \nu_r \cos\phi + \nu_x \sin\phi$$

where $\dfrac{I_1 R_{01} \times 100}{V_1} = \nu_r$ and $\dfrac{I_1 X_{01} \times 100}{V_1} = \nu_x$ ...Art. 32.16

As before, if angle between V_1 and V_2' is not negligible, then

$$\%\text{ regn} = (\nu_r \cos\phi \pm \nu_x \sin\phi) + \frac{1}{200}(\nu_x \cos\phi \mp \nu_r \sin\phi)^2$$

(3) In the above definitions of regulation, ***primary voltage was supposed to be kept constant*** and the changes in secondary terminal voltage were considered.

As the transformer is loaded, the secondary terminal voltage falls (for a lagging p.f.). Hence, to keep the output voltage constant, the primary voltage must be increased. The rise in primary voltage required to maintain rated output voltage from no-load to full-load at a given power factor expressed as percentage of rated primary voltage gives the regulation of the transformer.

Suppose primary voltage has to be raised from its rated value V_1 to V_1', then

$$\% \text{ regn.} = \frac{V_1' - V_1}{V_1} \times 100$$

Example 32.43. *A-100 kVA transformer has 400 turns on the primary and 80 turns on the secondary. The primary and secondary resistances are 0.3 Ω and 0.01 Ω respectively and the corresponding leakage reactances are 1.1 and 0.035 Ω respectively. The supply voltage is 2200 V. Calculate* ***(i)*** *equivalent impedance referred to primary and* ***(ii)*** *the voltage regulation and the secondary terminal voltage for full load having a power factor of 0.8 leading.*

(Elect. Machines, A.M.I.E. Sec. B, 1989)

Solution. $K = 80/400 = 1/5$, $R_1 = 0.3\ \Omega$, $R_{01} = R_1 + R_2/K^2 = 0.3 + 0.01/(1/5)^2 = 0.55\ \Omega$

$$X_{01} = X_1 + X_2/K^2 = 1.1 + 0.035/(1/5)^2 = 1.975\ \Omega$$

(i) $$Z_{01} = 0.55 + j\,1.975 = 2.05 \angle 74.44^\circ$$

(ii) $$Z_{02} = K^2 Z_{01} = (1/5)^2\,(0.55 + j\,1.975) = (0.022 + j\,0.079)$$

No-load secondary voltage $= K V_1 = (1/5) \times 2200 = 440$ V, $I_2 = 10 \times 10^3/440 = 227.3$ A

Full-load voltage drop as referred to secondary

$$= I_2 (R_{02} \cos\phi - X_{02} \sin\phi)$$

$$= 227.3\,(0.022 \times 0.8 - 0.079 \times 0.6) = -6.77 \text{ V}$$

$$\% \text{ regn.} = -6.77 \times 100/440 = -1.54$$

Secondary terminal voltage on load $= 440 - (-6.77) =$ **446.77 V**

Example 32.44. *The corrected instrument readings obtained from open and short-circuit tests on 10-kVA, 450/120-V, 50-Hz transformer are :*

O.C. test : $V_1 = 120$ V; $I_1 = 4.2$ A; $W_1 = 80$ W; V_1, W_1 and I_1 were read on the low-voltage side.

S.C. test : $V_1 = 9.65$ V; $I_1 = 22.2$ A ; $W_1 = 120$ W – with low-voltage winding short-circuited

Compute :

(*i*) *the equivalent circuit (approximate) constants,*
(*ii*) *efficiency and voltage regulation for an 80% lagging p.f. load,*
(*iii*) *the efficiency at half full-load and 80% lagging p.f. load.*

(Electrical Engineering-I, Bombay Univ. 1988)

Soluion. It is seen from the O.C. test, that with primary open, the secondary draws a no-load current of 4.2 A. Since $K = 120/450 = 4/15$, the corresponding no-load primary current $I_0 = 4.2 \times 4/15 = 1.12$ A.

(i) Now, $V_1 I_0 \cos\phi_0 = 80 \quad \therefore \quad \cos\phi_0 = 80/450 \times 1.12 = 0.159$

$\therefore \quad \phi_0 = \cos^{-1}(0.159) = 80.9°$; $\sin\phi_0 = 0.987$

$I_w = I_0 \cos\phi_0 = 1.12 \times 0.159 = 0.178$A and $I_\mu = 1.12 \times 0.987 =$ **1.1 A**

$\therefore \quad R_0 = 450/0.178 =$ **2530 Ω** and $X_0 = 450/1.1 =$ **409 Ω**

During S.C. test, instruments have been placed in primary.

$\therefore \quad Z_{01} = 9.65/22.2 = 0.435\ \Omega$

$$R_{01} = 120/22.2^2 = 0.243\ \Omega$$

$$X_{01} = \sqrt{0.435^2 - 0.243^2} = 0.361\ \Omega$$

The equivalent circuit is shown in Fig. 32.52.

(ii) Total approximate voltage drop as referred to primary is $I_1\,(R_{01} \cos\phi + X_{01} \sin\phi)$.

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* *Assuming* $\phi_1 = \phi_2 = \cos^{-1}(0.8)$.

Now, full-load I_1 = 10,000/450 = 22.2 A

$\therefore$ Drop = 22.2 (0.243 × 0.8 + 0.361 × 0.6) = 9.2 V

Regulation = 9.2 × 100/450 = 2.04%

F.L. losses = 80 + 120 = 200 W;

F.L. output = 10,000 × 0.8 = 8000 W

η = 8000/8200 = 0.9757 or **97.57%**

(iii) Half-load

Iron loss = 80 W; Cu loss = $(1/2)^2 \times 120$ = 30 W

Total losses = 110 W; Output = 5000 × 0.8 = 4000 W

$\therefore$ η = 4000/4110 = 0.9734 or **97.34%**

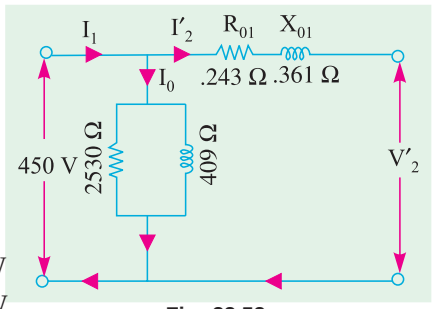


Fig. 32.52

Example 32.45. *Consider a 20 kVA, 2200/220 V, 50 Hz transformer. The O.C./S.C. test results are as follows :*

O.C. test : 220 V, 4.2 A, 148 W (l.v. side)

S.C. test : 86 V, 10.5 A, 360 W (h.v. side)

Determine the regulation at 0.8 p.f. lagging and at full load. What is the p.f. on short-circuit ?

(Elect. Machines Nagpur Univ. 1993)

Solution. It may be noted that O.C. data is not required in this question for finding the regulation. Since during S.C. test instruments have been placed on the h.v. side *i.e.* primary side.

$\therefore$ Z_{01} = 86/10.5 = 8.19 Ω; $R_{01} = 360/10.5^2 = 3.26\ \Omega$

$X_{01} = \sqrt{8.19^2 - 3.26^2} = 7.5\ \Omega$

F.L. primary current, I_1 = 20,000/2200 = 9.09 A

Total voltage drop as referred to primary = $I_1 (R_{01} \cos\phi + X_{01} \sin\phi)$

Drop = 9.09 (3.26 × 0.8 + 7.5 × 0.6) = 64.6 V

% age regn. = 64.6 × 100/2200 = **2.9%**, p.f. on short-circuit = R_{01}/Z_{01} = 3.26/8.19 = **0.4 lag**

Example 32.46. A *short-circuit test when performed on the h.v. side of a 10 kVA, 2000/400 V single phase transformer, gave the following data ; 60 V, 4 A, 100 W.*

If the 1.v. side is delivering full load current at 0.8 p.f. lag and at 400 V, find the voltage applied to h.v. side. **(Elect. Machines-I, Nagpur Univ. 1993)**

Solution. Here, the test has been performed on the h.v. side *i.e.* primary side.

Z_{01} = 60/4 = 15 Ω ; $R_{01} = 100/4^2 = 6.25\ \Omega$; $X_{01} = \sqrt{15^2 - 6.25^2} = 13.63\ \Omega$

F.L. I_1 = 10,000/2000 = 5A

Total transformer voltage drop as referred to primary is

$I_1 (R_{01} \cos\phi + X_{01} \sin\phi)$ = 5 (6.25 × 0.8 + 13.63 × 0.6) = 67 V

Hence, primary voltage has to be raised from 2000 V to 2067 V in order to compensate for the total voltage drop in the transformer. In that case secondary voltage on load would remain the same as on no-load.

Example 32.47. *A 250/500-V transformer gave the following test results :*

Short-circuit test : with low-voltage winding short-circuited :

20 V; 12 A, 100 W

Open-circuit test : 250 V, 1 A, 80 W on low-voltage side.

Determine the circuit constants, insert these on the equivalent circuit diagram and calculate applied voltage and efficiency when the output is 10 A at 500 volt and 0.8 power factor lagging.

(Elect. Machines, Nagpur Univ. 1993)

Solution. Open-circuit Test :

$$V_1 I_0 \cos \phi_0 = 80 \quad \therefore \quad \cos \phi_0 = 80/250 \times I = 0.32$$

$$I_w = I_0 \cos \phi_0 = I \times 0.32 = 0.32\ A,\ I_\mu = \sqrt{(1^2 - 0.32^2)} = 0.95\ A$$

$$R_0 = V_1/I_w = 250/0.32 = 781.3\ \Omega,\ X_0 = V_1/I_\mu = 250/0.95 = 263.8\ \Omega$$

The circuit is shown in Fig. 32.53 (*a*).

Short-circuit Test :

As the primary is short-circuited, all values refer to secondary winding.

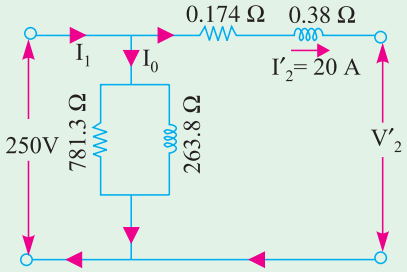


Fig. 32.53 (a)

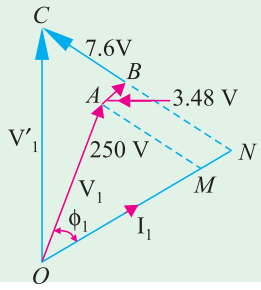


Fig. 32.53 (b)

$$\therefore \quad R_{02} = \frac{\text{short-circuit power}}{\text{F.L. secondary current}} = \frac{100}{12^2} = 0.694\ \Omega$$

$$Z_{02} = 20/12 = 1.667\ \Omega\ ;\ X_{02} = \sqrt{(1.667^2 - 0.694^2)} = 1.518\ \Omega$$

As R_0 and X_0 refer to primary, hence we will transfer these values to primary with the help of transformation ratio.

$$K = 500/250 = 2 \quad \therefore \quad R_{01} = R_{02}/K^2 = 0.694/4 = 0.174\ \Omega$$

$$X_{01} = X_{02}/K^2 = 1.518/4 = 0.38\ \Omega\ ;\ Z_{01} = Z_{02}/K^2 = 1.667/4 = 0.417\ \Omega$$

The equivalent circuit is shown in Fig. 32.53 (*a*).

Efficiency

Total Cu loss = $I_2^2 R_{02} = 100 \times 0.694 = 69.4$ W ; Iron loss = 80 W

Total loss = 69.4 + 80 = 149.4 W $\quad \therefore \quad \eta = \dfrac{5000 \times 0.8 \times 100}{4000 + 149.4} = \mathbf{96.42\%}$

The applied voltage V_1' is the vector sum of V_1 and $I_1 Z_{01}$ as shown in Fig. 32.53 (*b*).

$$I_1 = 20\ \text{A}\ ;\ I_1 R_{01} = 20 \times 0.174 = 3.84\ \text{V}\ ;\ I_1 X_{01} = 20 \times 0.38 = 7.6\ \text{V}$$

Neglecting the angle between V_1 and V_1', we have

$$V_1'^2 = OC^2 = ON^2 + NC^2 = (OM + MN)^2 + (NB + BC)^2$$

$$= (250 \times 0.8 + 3.48)^2 + (250 \times 0.6 + 7.6)^2$$

$$V_1'^2 = 203.5^2 + 157.6^2 \quad \therefore \quad V_1' = \mathbf{257.4\ V}$$

Example 32.48. *A 230/230 V, 3 kVA transformer gave the following results :*

O.C. Test : 230 V, 2 amp, 100 W

S.C. Test : 15 V, 13 amp, 120 W

Determine the regulation and efficiency at full load 0.80 p.f. lagging.

(Sambalpur University, 1998)

Solution. This is the case of a transformer with turns ratio as 1 : 1. Such a transformer is mainly required for isolation.

Rated Current = $\frac{3000}{230}$ = 13 amp

Cu-losses at rated load = 120 watts, from S.C. test

Core losses = 100 watts, from O.C. test

At full load, VA output = **3000**

At 0.8 lag p.f., Power output = $3000 \times 0.8 = 2400$ watts

Required efficiency = $\frac{2400}{2400 + 220} \times 100\% = 91.6\%$

From S.C. test, $Z = \frac{15}{13} = 1.154$ ohms

$R = \frac{120}{15 \times 15} = 0.53$ ohm, $X = \sqrt{1.154^2 - 0.53^2} = 1.0251$ ohm

Approximate voltage regulation

$= IR \cos\phi + IX \sin\phi = 13[0.53 \times 0.8 + 1.0251 \times 0.6]$

$= 13[0.424 + 0.615] = 13.51$ volts

In terms of %, the voltage regulation = $\frac{13.51}{230} \times 100\% =$ **5.874%**

Example 32.49. *A 10 kVA, 500/250 V, single-phase transformer has its maximum efficiency of 94% when delivering 90% of its rated output at unity p.f. Estimate its efficiency when delivering its full-load output at p.f. of 0.8 lagging.* **(Nagpur University, November 1998)**

Solution. Rated output at unity p.f. = 10000 W. Hence, 90% of rated output = 9,000 W

Input with 94% efficiency = 9000/0.94 W

Losses = $9000((1/0.94) - 1) = 574$ W

At maximum efficiency, variable copper-loss = constant = Core loss = 574/2 = 287 W

At rated current, Let the copper-loss = P_c watts

At 90% load with unity p.f., the copper-loss is expressed as $0.90^2 \times P_c$.

Hence, $P_c = 287/0.81 = 354$ W

(b) Output at full-load, 0.8 lag p.f. = $10{,}000 \times 0.80 = 8000$ W

At the corresponding load, Full Load copper-loss = 354 W

Hence, efficiency = $8000/(8000 + 354 + 287) = 0.926 =$ **92.6%**

Example 32.50. *Resistances and Leakage reactance of 10 kVA, 50 Hz, 2300/230 V single phase distribution transformer are $r_1 = 3.96$ ohms, $r_2 = 0.0396$ ohms, $x_1 = 15.8$ ohms, $x_2 = 0.158$ ohm. Subscript 1 refers to HV and 2 to LV winding (a) transformer delivers rated kVA at 0.8 p.f. Lagging to a load on the L.V. side. Find the H.V. side voltage necessary to maintain 230 V across Load-terminals. Also find percentage voltage regulation. (b) Find the power-factor of the rated load-current at which the voltage regulation will be zero, hence find the H.V. side voltage.*

(Nagpur University, November 1997)

Solution. ***(a)*** Rated current on L.V. side = 10,000/230 = 43.5 A. Let the total resistance and total leakage reactance be referred to L.V. side. Finally, the required H.V. side voltage can be worked out after transformation.

Total resistance, $r = r_1' + r_2 = 3.96 \times (230/2300)^2 + 0.0396$

$= 0.0792$ ohms

Total leakage-reactance, $x = x_1' + x_2 = 15.8 \times (230/2300)^2 + 0.158$

$= 0.316$ ohm

For purpose of calculation of voltage-magnitudes, approximate formula for voltage regulation can be used. For the present case of 0.8 lagging p.f.

$$V_1' = V_2 + I[r\cos\phi + x\sin\phi]$$
$$= 230 + 43.5[(0.0792 \times 0.8) + (0.316 \times 0.6)]$$
$$= 230 + 43.5[0.0634 + 0.1896] = 230 + 11 = 241 \text{ volts}$$

Hence, $V_1 = 241 \times (2300/230) = 2410$ volts.

It means that H.V. side terminal voltage must be 2410 for keeping 230 V at the specified load.

(b) Approximate formula for voltage regulation is : $V_1' - V_2 = I[r\cos\phi \pm x\sin\phi]$

With Lagging p.f., +ve sign is retained. With leading power-factor, the –ve sign is applicable. For the voltage-regulation to be zero, only leading P.f. condition can prevail.

Thus, $r\cos\phi - x\sin\phi = 0$

or $\tan\phi = r/x = 0.0792/0.316 = 0.25$

or $\phi = 14^\circ$, $\cos\phi = 0.97$ leading

Corresponding $\sin\phi = \sin 14^\circ =$ **0.243**

H.V. terminal voltage required is 2300 V to maintain 230 V at Load, since Zero regulation condition is under discussion.

Example 32.51. *A 5 kVA, 2200/220 V, single-phase transformer has the following parameters.*

H.V. side : $r_1 = 3.4$ ohms, $x_1 = 7.2$ ohms

L.V. side : $r_2 = 0.028$ ohms, $x_2 = 0.060$ ohms

Transformer is made to deliver rated current at 0.8 lagging P.f. to a load connected on the L.V. side. If the load voltage is 220 V, calculate the terminal voltage on H.V. side (Neglect the exciting current). **(Rajiv Gandhi Technical University, Bhopal, Summer 2001)**

Solution. Calculations may be done referring all the parameters the L.V. side first. Finally, the voltage required on H.V. side can be obtained after transformation.

Rated current ref. to L.V. side = 5000/220 = 22.73 A

Total winding resistance ref. to L.V. side = $r_1' + r_2 = (220/2200)^2 \times 3.4 + 0.028$

Total winding-leakage-reactance ref. to L.V. side = $x_1' + x_2$

$= (220/2200)^2 \times 7.2 + 0.060 = 0.132$ ohm

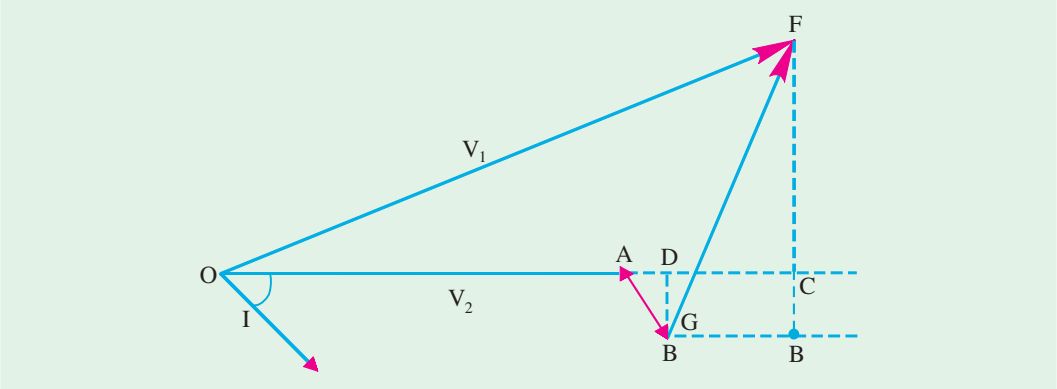


Fig. 32.53(c)

In the phasor diagram of Fig. 32.53 (*c*).

$QA = V_2 = 220$ volts, $I = 22.73$ A at lagging phase angle of 36.87°

$AB = Ir$, $AD = Ir\cos\phi = 22.73 \times 0.062 \times 0.80 = 1.127$ V

$DC = Ix\sin\phi = 22.73 \times 0.132 \times 0.60 = 1.80$ V

OC = 220 + 1.127 + 1.80 = 222.93 volts

BD = $Ir \sin \phi$ = 0.85 V

$B'F$ = $x \cos \phi$ = 2.40 V

CF = 2.40 – 0.85 = 1.55 V

V_1' = $OF = (222.93^2 + 1.55^2)^{0.50}$ = 222.935 volts

Required terminal voltage of H.V. side = $V_1 = 222.935 \times (2200/220) = 2229.35$ volts

[**Note.** In approximate and fast calculations, CF is often ignored for calculation of magnitude of V_1'. The concerned expression is : $V_1' = V_2 + Ir \cos \phi + Ix \sin \phi$, for lagging P.f.]

Example 32.52. *A 4-kVA, 200/400 V, single-phase transformer takes 0.7 amp and 65 W on Open-circuit. When the low-voltage winding is short-circuited and 15 V is applied to the high-voltage terminals, the current and power are 10 A and 75 W respectively. Calculate the full-load efficiency at unity power factor and full-load regulation at 0.80 power-factor lagging.*

(Nagpur University April 1999)

Solution. At a load of 4 kVA, the rated currents are :

L.V. side : 4000/200 = 20 amp

And H.V. side : 4000/400 = 10 amp

From the test data, full-load copper-loss = 75 W

And constant core-loss = 65 W

From S.C. test, Z = 15/10 = 1.5 ohms

R = 75/100 = 0.75 ohm

Hence $x = \sqrt{1.5^2 - 0.75^2}$ = 1.30 ohms

All these series-parameters are referred to the H.V. side, since the S.C. test has been conducted from H.V. side.

Full-load efficiency at unity p.f. = 4000 / (4000 + 65 + 75)

= 0.966 = 96.6%

Full load voltage regulation at 0.80 lagging p.f.

= $Ir \cos \phi + I x \sin \phi$

= 10 (0.75 × 0.80 + 1.30 × 0.60) = 16.14 Volts

Thus, due to loading, H.V. side voltage will drop by 16.14 volts (*i.e.* terminal voltage for the load will be 383.86 volts), when L.V. side is energized by 200-V source.

32.25. Percentage Resistance, Reactance and Impedance

These quantities are usually measured by the voltage drop at full-load current expressed as a percentage of the normal voltage of the winding on which calculations are made.

(i) Percentage resistance at full-load

$$\%R = \frac{I_1 R_{01}}{V_1} \times 100 = \frac{I_1^2 R_{01}}{V_1 I_1} \times 100$$

$$= \frac{I_2^2 R_{02}}{V_2 I_2} \times 100 = \%\text{ Cu loss at full-load}$$

$\%R$ = % Cu loss = v_r ...Art.32.16

(ii) Percentage reactance at full-load

$$\%X = \frac{I_1 X_{01}}{V_1} \times 100 = \frac{I_2 X_{02}}{V_2} \times 100 = v_x$$

(iii) Percentage impedance at full-load

$$\%\,Z = \frac{I_1 Z_{01}}{V_1} \times 100 = \frac{I_2 Z_{02}}{V_2} \times 100$$

(iv) $$\%Z = \sqrt{(\%R^2 + \%X^2)}$$

It should be noted from above that the reactances and resistances in ohm can be obtained thus :

$$R_{01} = \frac{\%R \times V_1}{100 \times I_1} = \frac{\%\,\text{Cu loss} \times V_1}{100 \times I_1}\ ;\ \text{Similarly}\ R_{02} = \frac{\%R \times V_2}{100 \times I_2} = \frac{\%\,\text{Cu loss} \times V_2}{100 \times I_2}$$

$$X_{01} = \frac{\%X \times V_1}{100 \times I_1} = \frac{\nu_z \times V_1}{100 \times I_1}\ ;\ \text{Similarly}\ X_{02} = \frac{\%X \times V_2}{100 \times I_2} = \frac{\nu_x \times V_2}{100 \times I_2}$$

It may be noted that percentage resistance, reactance and impedance have the same value whether referred to primary or secondary.

Example 32.53. *A 3300/230 V, 50-kVA, transformer is found to have impedance of 4% and a Cu loss of 1.8% at full-load. Find its percentage reactance and also the ohmic values of resistance, reactance and impedance as referred to primary. What would be the value of primary short-circuit current if primary voltage is assumed constant ?*

Solution. $$\%\,X = \sqrt{(\%Z^2 - \%R^2)} = \sqrt{(4^2 - 1.8^2)} = 3.57\%\ (\because\ \text{Cu loss} = \%\,R)$$

Full load $I_1 = 50{,}000/3300 = 15.2$ A (assuming 100% efficiency). Considering primary winding, we have

$$\%\,R = \frac{R_{01} I_1 \times 100}{V_L} = 1.8 \qquad \therefore R_{01} = \frac{1.8 \times 3300}{100 \times 15.2} = \mathbf{3.91\ \Omega}$$

Similarly $$\%\,X = \frac{X_{01} I_1 \times 100}{V_1} = 3.57 \qquad \therefore X_{01} = \frac{3.57 \times 3300}{100 \times 15.2} = \mathbf{7.76\ \Omega}$$

Similarly $$Z_{01} = \frac{4 \times 3300}{100 \times 15.2} = \mathbf{8.7\ \Omega}$$

Now $$\frac{\text{Short-circuit current}^*}{\text{Full load current}} = \frac{100}{4} \qquad \therefore \text{S.C. current} = 15.2 \times 25 = \mathbf{380\ A}$$

Example 32.54. *A 20-kVA, 2200/220-V, 50-Hz distribution transformer is tested for efficiency and regulation as follows :*

O.C. test : 220 V 4.2 A, 148 W –l.v side

S.C. test : 86 V 10.5 A, 360 W – l.v. side

Determine ***(a)*** *core loss* ***(b)*** *equivalent resistance referred to primary* ***(c)*** *equivalent resistance referred to secondary* ***(d)*** *equivalent reactance referred to primary* ***(e)*** *equivalent reactance referred to secondary* ***(f)*** *regulation of transformer at 0.8 p.f. lagging current* ***(g)*** *efficiency at full-load and half the full-load at 0.8 p.f. lagging current.*

Solution. **(a)** As shown in Art 32.9, no-load primary input is practically equal to the core loss. Hence, core loss as found from no-load test, is **148 W.**

(b) From S.C. test, $R_{01} = 360/10.5^2 = \mathbf{3.26\ \Omega}$

(c) $R_{02} = K^2 R_{01} = (220/2200)^2 \times 3.26 = \mathbf{0.0326\ \Omega}$

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* Short circuit $I_{SC} = \frac{V_1}{Z_{01}}$ Now, $Z_{01} = \frac{V_1 \times \%Z}{100 \times I_1}$

$$I_{SC} = \frac{V_1 \times 100 \times I_1}{V_1 \times \%Z} = \frac{100 \times I_1}{\%Z} \qquad \therefore \frac{I_{SC}}{I_1} = \frac{100}{\%Z}$$

(d) $Z_{10} = \frac{V_{SC}}{I_{SC}} = \frac{86}{10.5} = 8.19\ \Omega$

$X_{01} = \sqrt{(8.19^2 - 3.26^2)} = \mathbf{7.51\ \Omega}$

(e) $X_{02} = K^2 X_{01} = (220/2200)^2 \times 7.51 = \mathbf{0.0751\ \Omega}$

(f) We will use the definition of regulation as given in Art. 32.24 (3).

We will find the rise in primary voltage necessary to maintain the output terminal voltage constant from no-load to full-load.

Rated primary current = 20,000/2200 = 9.1 A

$V_1' = \sqrt{[2200 \times 0.8 + 9.1 \times 3.26)^2 + (2200 \times 0.6 + 9.1 \times 7.51)^2]} = \mathbf{2265\ V}$

$\therefore$ % regn $= \frac{2265 - 2200}{2200} \times 100 = \mathbf{2.95\%}$

We would get the same result by working in the secondary. Rated secondary current = **91 A.**

${}_0V_2 = \sqrt{(220 \times 0.8 + 91 \times 0.0326)^2 + (220 \times 0.6 + 91 \times 0.0751)^2]} = 226.5\ V$

$\therefore$ % regns. $= \frac{226.5 - 220}{220} \times 100 = \mathbf{2.95\%}$

(g) Core loss = 1.48 W. It will be the same for all loads.

Cu loss at full load = $I_1^2 R_{01} = 9.1^2 \times 3.26 = 270$ W

Cu loss at half full-load = $4.55^2 \times 3.26 = 67.5$ W (or F.L. Cu loss/4)

$\therefore$ η at full-load $= \frac{22,000 \times 0.8 \times 100}{20,000 \times 0.8 + 148 + 270} = \mathbf{97.4\%}$

$\therefore$ η at half-load $= \frac{10,000 \times 0.8 \times 100}{10,000 \times 0.8 + 148 + 67.5} = \mathbf{97.3\%}$

Example 32.55. *Calculate the regulation of a transformer in which the ohmic loss is 1% of the output and the reactance drop is 5% of the voltage, when the power factor is (i) 0.80 Lag (ii) unity (iii) 0.80 Leading.* **(Madras University, 1997)**

Solution. When 1% of output is the ohmic loss, p.u. resistance of the transformer, $\varepsilon_r = 0.01$

When 5% is the reactance drop, p.u. reactance of the transformer $\varepsilon_x = 0.05$

(i) Per Unit regulation of the transformer at full-load, 0.8 Lagging p.f.

$= 0.01 \times \cos\phi + 0.05 \times \sin\phi = 0.01 \times 0.8 + 0.05 \times 0.06 = 0.038$ or 3.8%

(ii) Per Unit regulation at unity p.f. $= 0.01 \times 1 = 0.01$ or 1%

(iii) Per Unit regulation at 0.08 Leading p.f. $= 0.01 \times 0.8 - 0.05 \times 0.6 = -0.022$ or -2.2%

Example 32.56. *The maximum efficiency of a 500 kVA, 3300/500 V, 50 Hz, single phase transformer is 97% and occurs at 3/4th full-load u.p.f. If the impedance is 10% calculate the regulation at fullload, 0.8 p.f. Lag.* **(Madurai Kamraj University, November 1997)**

Solution. At unity p.f. with 3/4th full load, the output of the transformer

$= 500 \times 0.75 \times 1$ kW $= 375$ kW

$0.97 = \frac{375}{375 + 2P_i}$

where P_i = core loss in kW, at rated voltage.

At maximum efficiency, $x^2 P_c = P_i$

$(0.75)^2 P_c = P_i$

where $x = 0.75$, *i.e.* 3/4th which is the fractional loading of the transformer

P_c = copper losses in kW, at rated current

$$P_i = \frac{1}{2}\left\{(375)\times\left(\frac{1}{0.97}-1\right)\right\} = ½ \times 375 \times \frac{3}{97} = 5.8 \text{ kW}$$

$$P_c = 5.8/(0.75)^2 = 10.3 \text{ kW}$$

Full load current in primary (H.V.) winding = $\frac{500\times1000}{3300}$ = **151.5 amp**

Total winding resistance ref. to primary

$$= \frac{10.3\times1000}{(151.5)^2} = 0.44876 \text{ ohm}$$

$$\varepsilon_r = \%\text{ resistance} = \frac{151.5\times0.44876}{3300}\times100\ \% = \textbf{2.06\%}$$

$$\varepsilon_z = \%\text{ Impedance} = 10\%$$

$$\varepsilon_x = \%\text{ reactance} = \sqrt{100-4.244} = \textbf{9.7855\%}$$

By Approximate formula at 0.8 p.f. lag

$$\%\text{ regulation} = \varepsilon_r\cos\phi + \varepsilon_x\sin\phi$$
$$= 2.06\times0.8 + 9.7855\times0.6$$
$$= 1.648 + 5.87 = \textbf{7.52 \%}$$

Example 32.57. *A transformer has copper-loss of 1.5% and reactance-drop of 3.5% when tested at full-load. Calculate its full-load regulation at (i) u.p.f. (ii) 0.8 p.f. Lagging and (iii) 0.8 p.f. Leading.* **(Bharathithasan Univ. April 1997)**

Solution. The test-data at full-load gives following parameters :

p.u. resistance = 0.015, p.u. reactance = 0.035

(i) Approximate Voltage – Regulation at unity p.f. full load

$$= 0.015\cos\phi + 0.035\sin\phi$$
$$= 0.015 \text{ per unit} = \textbf{1.5\%}$$

(ii) Approximate Voltage – Regulation at 0.80 Lagging p.f.

$$= (0.015\times0.8) + (0.035\times0.6) = 0.033 \text{ per unit} = \textbf{3.3\%}$$

(iii) Approximate Voltage Regulation at 0.8 leading p.f.

$$= I_r\cos\phi - I_x\sin\phi$$
$$= (0.015\times0.8) - (0.035\times0.6) = -0.009 \text{ per unit} = \textbf{– 0.9\%}$$

32.26. Kapp Regulation Diagram

It has been shown that secondary terminal voltage falls as the load on the transformer is increased when p.f. is lagging and it increases when the power factor is leading. In other words, secondary terminal voltage not only depends on the load but on power factor also (Art. 32.16). For finding the voltage drop (or rise) which is further used in determining the regulation of the transformer, a graphical construction is employed which was proposed by late Dr. Kapp.

For drawing Kapp regulation diagram, it is necessary to know the equivalent resistance and reactance as referred to secondary *i.e.* R_{02} and X_{02}. If I_2 is the secondary load current, then secondary terminal voltage on load V_2, is obtained by subtracting I_2R_{02} and I_2X_{02} voltage drops vectorially from secondary no-load voltage ${}_0V_2$.

Now, ${}_0V_2$ is constant, hence it can be represented by a circle of constant radius OA as in Fig. 32.54. This circle is known as no-load or open-circuit e.m.f. circle. For a given load, OI_2 represents the load current and is taken as the reference vector, CB represents I_2R_{02} and is parallel to OI_2, AB represents I_2X_{02} and is drawn at right angles to CB. Vector OC obviously represents I_2X_{02} and is drawn at right

angles to CB. Vector OC obviously represents secondary terminal voltage V_2. Since I_2 is constant, the drop triangle ABC remains constant in size. It is seen that end point C of V_2 lies on another circle whose centre is O'. This point O' lies at a distance of $I_2 X_{02}$ vertically below the point O and a distance of $I_2 R_{02}$ to its left as shown in Fig. 32.54.

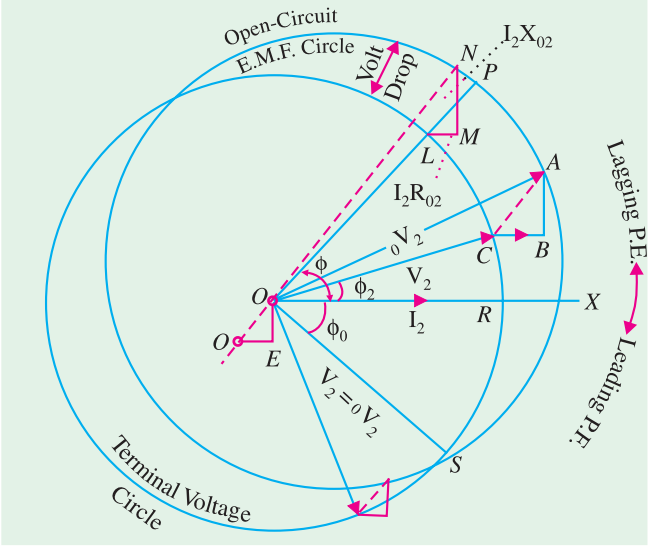


Fig. 32.54

Suppose it is required to find the voltage drop on full-load at a lagging power factor of cos ϕ, then a radius OLP is drawn inclined at an angle of ϕ with OX. $LM = I_2 R_{02}$ and is drawn horizontal $MN = I_2 X_{02}$ and is drawn perpendicular to LM. Obviously, ON is no-load voltage ${}_0V_2$. Now, $ON = OP = {}_0V_2$. Similarly, OL is V_2. The voltage drop = $OP - OL = LP$.

Hence, percentage regulation 'down' is $= \dfrac{OP - OL}{OP} \times 100 = \dfrac{LP}{OP} \times 100$

It is seen that for finding voltage drop, triangle LMN need not be drawn, but simply the radius OLP.

The diagram shows clearly how the secondary terminal voltage falls as the angle of lag increases. Conversely, for a leading power factor, the fall in secondary terminal voltage decreases till for an angle of ϕ_0 leading, the fall becomes zero; hence $V_2 = {}_0V_2$. For angles greater than ϕ_0, the secondary terminal voltage V_2 becomes greater than ${}_0V_2$.

The Kapp diagram is very helpful in determining the variation of regulation with power factor but it has the disadvantage that since the lengths of the sides of the impedance triangle are very small as compared to the radii of the circles, the diagram has to be drawn on a very large scale, if sufficiently accurate results are desired.

32.27. Sumpner of Back-to-Back Test

This test provides data for finding the regulation, efficiency and heating under load conditions and is employed only when two similar transformers are available. One transformer is loaded on the other and both are connected to supply. The power taken from the supply is that necessary for supplying the losses of both transformers and the negligibly small loss in the control circuit.

As shown in Fig. 32.55, primaries of the two transformers are connected in parallel across the same a.c. supply. With switch S open, the wattmeter W_1 reads the core loss for the two transformers.

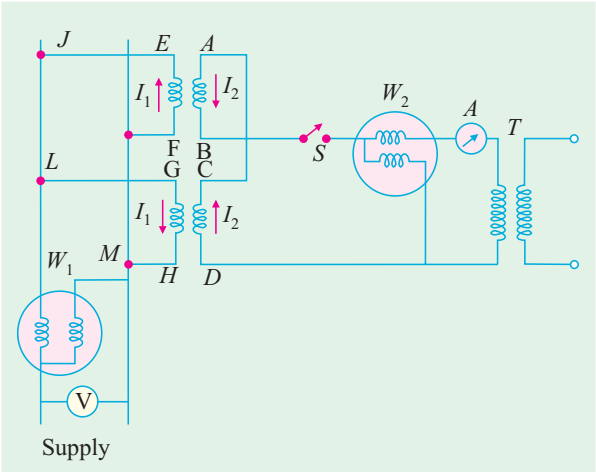


Fig. 32.55

The secondaries are so connected that their potentials are in opposition to each other. This would so if $V_{AB} = V_{CD}$ and A is joined to C whilst B is joined to D. In that case, there would be no secondary current flowing around the loop formed by the two secondaries. T is an auxiliary low-voltage transformer which can be adjusted to give a variable voltage and hence current in the secondary loop circuit. By proper adjustment of T, full-load secondary current I_2 can be made to flow as shown. It is seen, that I_2 flows from D to C and then from A to B. Flow of I_1 is confined to the loop $FEJLGHMF$ and it does not pass through W_1. Hence, W_1 continues to read the core loss and W_2 measures full-load Cu loss (or at any other load current value I_2). Obviously, the power taken in is twice the losses of a single transformer.

Example 32.58. *Two similar 250-kVA, single-phase transformers gave the following results when tested by back-to-back method :*

Mains wattmeter, $W_1 = 5.0$ *kW*

Primary series circuit wattmeter, $W_2 = 7.5$ *kW (at full-load current).*

Find out the individual transformer efficiencies at 75% full-load and 0.8 p.f. lead.

(Electrical Machines-III, Gujarat Univ. 1986)

Solution. Total losses for both transformers = 5 + 7.5 = 12.5 kW

F.L. loss for each transformer = 12.5/2 = 6.25 kW

$$\text{Copper-loss at 75\% load} = \left(\frac{3}{4}\right)^2 \times \frac{7.5}{2}\ \text{kW} = 2.11\ \text{kW}$$

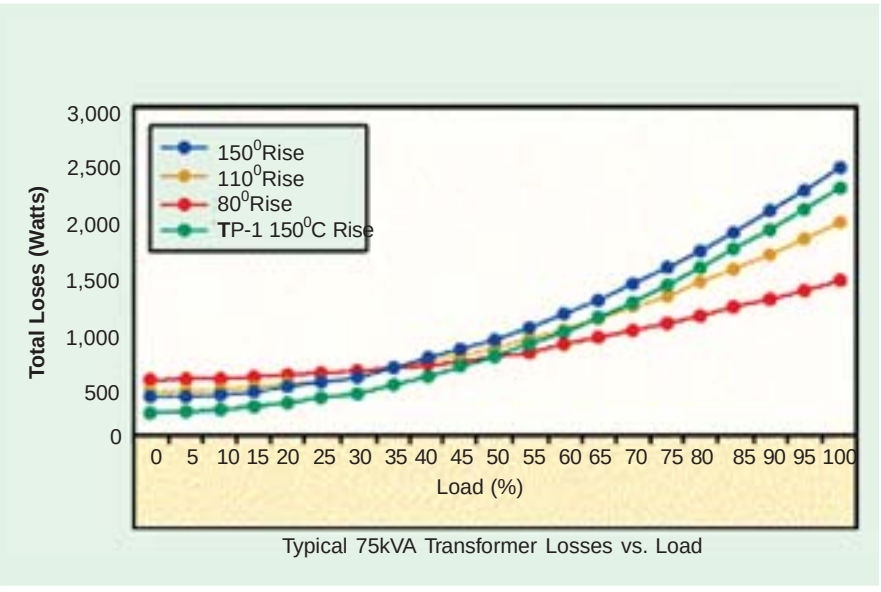
Output of each transformer at 75% F.L. and 0.8 p.f. = $(250 \times 0.75) \times 0.8 = 150$ kW

$$\eta = \frac{150}{150 + 2.5 + 2.11} = \mathbf{97\%}$$

32.28. Losses in a Transformer

In a static transformer, there are no friction or windage losses. Hence, the only losses occuring are :

(*i*) Core or Iron Loss : It includes both hysteresis loss and eddy current loss. Because the core flux in a transformer remains practically constant for all loads (its variation being 1 to 3% from no-load to full-load). The core loss is practically the same at all loads.



Typical 75kVA Transformer Losses vs. Load

Hysteresis loss $W_h = \eta B^{1.6}_{max} f V$ watt; eddy current loss $W_e = PB^2_{max} f^2 t^2$ watt

These losses are minimized by using steel of high silicon content for the core and by using very thin

laminations. Iron or core loss is found from the *O.C.* test. ***The input of the transformer when on no-load measures the core loss***.

***(ii)* Copper loss.** This loss is due to the ohmic resistance of the transformer windings. Total Cu loss $= I_1^2R_1 + I_2^2R_2 = I_1^2R_{01} + I_2^2R_{02}$. It is clear that Cu loss is proportional to (current)^2 or kVA^2. In other words, Cu loss at half the full-load is one-fourth of that at full-load.

The value of Cu loss is found from the short-circuit test (Art. 32.22).

32.29. Efficiency of a Transformer

As is the case with other types of electrical machines, the efficiency of a transformer at a particular load and power factor is defined as the output divided by the input–the two being measured in the same units (either watts or kilowatts).

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}}$$

But a transformer being a highly efficient piece of equipment, has very small loss, hence it is impractical to try to measure transformer, efficiency by measuring input and output. These quantities are nearly of the same size. A better method is to determine the losses and then to calculate the efficiency from ;

$$\text{Efficiency} = \frac{\text{Output}}{\text{Output + losses}} = \frac{\text{Output}}{\text{Output + Cu loss + iron loss}}$$

or
$$\eta = \frac{\text{Input} - \text{Losses}}{\text{Input}} = 1 - \frac{\text{losses}}{\text{Input}}$$

It may be noted here that efficiency is based on power output in watts and not in volt-amperes, although losses are proportional to VA. Hence, at any volt-ampere load, the efficiency depends on power factor, being maximum at a power factor of unity.

Efficiency can be computed by determining core loss from no-load or open-circuit test and Cu loss from the short-circuit test.

32.30. Condition for Maximum Efficiency

$$\text{Cu loss} = I_1^2R_{01} \quad \text{or} \quad I_2^2R_{02} = W_{cu}$$
$$\text{Iron loss} = \text{Hysteresis loss + Eddy current loss} = W_h + W_e = W_i$$

Considering primary side,

$$\text{Primary input} = V_1I_1\cos\phi_1$$
$$\eta = \frac{V_1I_1\cos\phi_1 - \text{losses}}{V_1I_1\cos\phi_1} = \frac{V_1I_1\cos\phi_1 - I_1^2R_{01} - W_i}{V_1I_1\cos\phi_1}$$
$$= 1 - \frac{I_1R_{01}}{V_1\cos\phi_1} - \frac{W_i}{V_1I_1\cos\phi_1}$$

Differentiating both sides with respect to I_1, we get

$$\frac{d\eta}{dI_1} = 0 - \frac{R_{01}}{V_1\cos\phi_1} + \frac{W_i}{V_1I_1^2\cos\phi_1}$$

For η to be maximum, $\frac{d\eta}{dI_1} = 0$. Hence, the above equation becomes

$$\frac{R_{01}}{V_1\cos\phi_1} = \frac{W_i}{V_1I_1^2\cos\phi_1} \quad \text{or} \quad W_i = I_1^2R_{01} \quad \text{or} \quad I_2^2R_{02}$$

or **Cu loss = Iron loss**

The output current corresponding to maximum efficiency is $I_2 = \sqrt{(W_i/R_{02})}$.

It is this value of the output current which will make the Cu loss equal to the iron loss. By proper design, it is possible to make the maximum efficiency occur at any desired load.

Note. (*i*) If we are given iron loss and full-load Cu loss, then the load at which two losses would be equal (*i.e.* corresponding to maximum efficiency) is given by

$$= \text{Full load} \times \sqrt{\left(\frac{\text{Iron loss}}{\text{F.L. Cu loss}}\right)}$$

In Fig. 32.56, Cu losses are plotted as a percentage of power input and the efficiency curve as deduced from these is also shown. It is obvious that the point of intersection of the Cu and iron loss curves gives the point of maximum efficiency. It would be seen that the efficiency is high and is practically constant from 15% full-load to 25% overload.

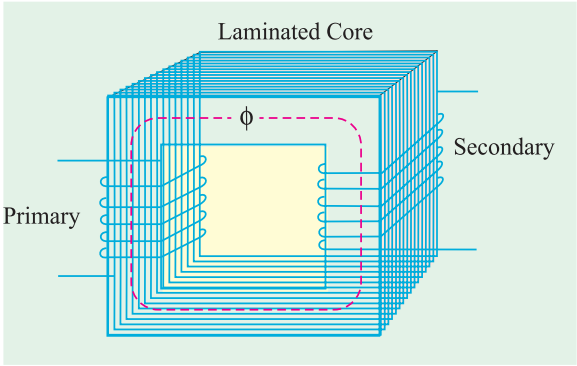


Fig. 32.56

(ii) The efficiency at any load is given by

$$\eta = \frac{x \times \text{full-load kVA} \times \text{p.f.}}{(x \times \text{full-load kVA} \times \text{p.f.}) + W_{cu} + W_i} \times 100$$

where x = ratio of actual to full-load kVA

W_i = iron loss in kW ; W_{cu} = Cu loss in kW.

Example 32.59. *In a 25-kVA, 2000/200 V, single-phase transformer, the iron and full-load copper losses are 350 and 400 W respectively. Calculate the efficiency at unity power factor on* ***(i)*** *full load* ***(ii)*** *half full-load.* **(Elect. Engg. & Electronic, Bangalore Univ. 1990 and Similar example in U.P. Technical University 2001)**

Solution. (*i*) Full-load Unity p.f.

Total loss = 350 + 400 = 750 W

F.L. output at u.p.f. = 25 × 1 = 25 kW ; η = 25/25.75 = 0.97 or **97%**

(*ii*) Half F.L. Unity p.f.

Cu loss = 400 × $(1/2)^2$ = 100 W. Iron loss remains constant at 350 W, Total loss = 100 + 350 = 450 W.

Half-load output at u.p.f. = 12.5 kW

$\therefore \quad \eta = 12.5/(12.5 + 0.45) =$ **96.52%**

Example 32.60. *If P_1 and P_2 be the iron and copper losses of a transformer on full-load, find the ratio of P_1 and P_2 such that maximum efficiency occurs at 75% full-load.* **(Elect. Machines AMIE Sec. B, Summer 1992)**

Solution. If P_2 is the Cu loss at full-load, its value at 75% of full-load is = $P_2 \times (0.75)^2 = 9P_2/16$. At maximum efficiency, it equals the iron loss P_1 which remains constant throughout. Hence, at maximum efficiency.

$$P_1 = 9P_2/16 \quad \text{or} \quad P_1/P_2 = 9/16.$$

Example 32.61. *A 11000/230 V, 150-kVA, 1-phase, 50-Hz transformer has core loss of 1.4 kW and F.L. Cu loss of 1.6 kW. Determine*

(i) *the kVA load for max. efficiency and value of max. efficiency at unity p.f.*

(ii) *the efficiency at half F.L. 0.8 p.f. leading* **(Basic Elect. Machine, Nagpur Univ. 1993)**

Solution. (*i*) Load kVA corresponding to maximum efficiency is

$$= \text{F.L. kVA} \times \sqrt{\frac{\text{Iron loss}}{\text{F.L. Cu loss}}} = 250 \times \sqrt{\frac{1.6}{1.4}} = 160 \text{ kVA}$$

Since Cu loss equals iron loss at maximum efficiency, total loss = 1.4 + 1.4 = 2.8 kW ; output = 160 × 1 = 160 kW

$$\eta_{max} = 160/162.8 = 0.982 \text{ or } \mathbf{98.2\%}$$

(*ii*) Cu loss at half full-load = $1.6 \times (1/2)^2 = 0.4$ kW ; Total loss = 1.4 + 0.4 = 1.8 kW

Half F.L. output at 0.8 p.f.= (150/2) × 0.8 = 60 kW

$\therefore$ Efficiency = 60/(60 + 1.8) = 0.97 or **97%**

Example 32.62. *A 5-kVA, 2,300/230-V, 50-Hz transformer was tested for the iron losses with normal excitation and Cu losses at full-load and these were found to be 40 W and 112 W respectively. Calculate the efficiencies of the transformer at 0.8 power factor for the following kVA outputs :*

1.25 2.5 3.75 5.0 6.25 7.5

Plot efficiency vs kVA output curve. **(Elect. Engg. -I, Bombay Univ. 1987)**

Solution. F.L. Cu loss = 112 W ; Iron loss = 40 W

(*i*) Cu loss at 1.25 kVA = $112 \times (1.25/5)^2 = 7$ W

Total loss = 40 + 7 = 47 W Output = 1.25 × 0.8 = 1 kW = 1,000 W

η = 100 × 1,000/1,047 = **95.51 %**

(*ii*) Cu loss at 2.5 kVA = $112 \times (2.5/5)^2 = 28$ W

Total loss = 40 + 28 = 68 W

Output = 2.5 × 0.8 = 2 kW

η = 2,000 × 100/2,068 = **96.71%**

(*iii*) Cu loss at 3.75 kVA

= $112 \times (3.75/5)^2 = 63$ W

Total loss = 40 + 63 = 103 W

η = 3,000 × 100/3,103 = **96.68 %**

(*iv*) Cu loss at 5 kVA

= 112 W

Total loss = 152 W = 0.152 kW

Output = 5 × 0.8 = 4 kW

η = 4 × 100/4.142 = **96.34 %**

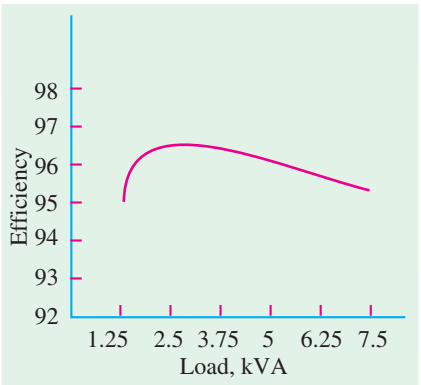


Fig. 32.57

(*v*) Cu loss at 6.25 kVA

= $112 \times (6.25/5)^2 = 175$ W

Total loss = 125 W = 0.125 kW ; Output = 6.25 × 0.8 = 5 kW

η = 5 × 100/5.215 = **95.88 %**

(*vi*) Cu loss at 7.5 kVA = $112 \times (7.5/5)^2 = 252$ W

Total loss = 292 W = 0.292 kW ; Output = 7.5 × 0.8 = 6 kW

η = 6 × 100/6.292 = **95.36 %**

The curve is shown in Fig. 32.57.

Example 32.63. *A 200-kVA transformer has an efficiency of 98% at full load. If the max. efficiency occurs at three quarters of full-load, calculate the efficiency at half load. Assume negligible magnetizing current and p.f. 0.8 at all loads.* **(Elect. Technology Punjab Univ. Jan. 1991)**

Solution. As given, the transformer has a F.L. efficiency of 98 % at 0.8 p.f.

F.L. output = $200 \times 0.8 = 160$ kW ; F.L. input = 160/0.98 = 163.265 kW

F.L. losses = 163.265 – 160 = 3.265 kW

This loss consists of F.L. Cu loss x and iron loss y.

$\therefore \quad x + y = 3.265$ kW ...(i)

It is also given that η_{max} occurs at three quarters of full-load when Cu loss becomes equal to iron loss.

$\therefore$ Cu loss at 75 % of F.L. = $x\,(3/4)^2 = 9x/16$

Since y remains constant, hence $9x/16 = y$ ...(ii)

Substituting the value of y in Eqn. (i), we get $x + 9x/16 = 3265$ or $x = 2090$ W; $y = 1175$ W

Half-load Unity p.f.

Cu loss = $2090 \times (1/2)^2 = 522$ W ; total loss = 522 + 1175 = 1697 W

Output = $100 \times 0.8 = 80$ kW ; $\eta = 80/81.697 = 0.979$ or **97.9 %**

Example 32.64. *A 25-kVA, 1-phase transformer, 2,200 volts to 220 volts, has a primary resistance of 1.0 Ω and a secondary resistance of 0.01 Ω. Find the equivalent secondary resistance and the full-load efficiency at 0.8 p.f. if the iron loss of the transformer is 80% of the full-load Cu loss.*

(Elect. Technology, Utkal Univ. 1998)

Solution. $K = 220/2{,}200 = 1/10$; $R_{02} = R_2 + K_2 R_1 = 0.01 + 1/100 =$ **0.02 Ω**

Full-load I_2 = 25,000/220 = 113.6 A ; F.L. Cu loss = $I_2^2 R_{02} = 113.6^2 \times 0.02 = 258$ W.

Iron loss = 80% of 258 = 206.4 W ; Total loss = 258 + 206.4 = 464.4 W

F.L. output = $25 \times 0.8 = 20$ kW = 20,000 W

Full-load η = 20,000 × 100/(20,000 + 464.4) = **97.7 %**

Example 32.65. *A 4-kVA, 200/400-V, 1-phase transformer has equivalent resistance and reactance referred to low-voltage side equal to 0.5 Ω and 1.5 Ω respectively. Find the terminal voltage on the high-voltage side when it supplies 3/4th full-load at power factor of 0.8, the supply voltage being 220 V. Hence, find the output of the transformer and its efficiency if the core losses are 100 W.*

(Electrical Engineering ; Bombay Univ. 1985)

Solution. Obviously, primary is the low-voltage side and the secondary, the high voltage side.

Here, $R_{01} = 0.5\,\Omega$ and $X_{01} = 1.5\,\Omega$. These can be transferred to the secondary side with the help of the transformation ratio.

$K = 400/200 = 2$; $R_{02} = K^2 R_{01} = 2^2 \times 0.5 = 2\,\Omega$; $X_{02} = K^2 X_{01} = 4 \times 1.5 = 6\,\Omega$

Secondary current when load is 3/4 the, full-load is = $(1{,}000 \times 4 \times 3/4)/400 = 7.5$ A

Total drop as referred to transformer secondary is

$= I_2 (R_{02} \cos\phi + X_{02} \sin\phi)$* $= 7.5\,(2 \times 0.8 + 6 \times 0.6) = 39$ V

$\therefore$ Terminal voltage on high-voltage side under given load condition is

= 400 – 39 = **361 V**

Cu loss = $I_2^2 R_{02} = 7.5^2 \times 2 = 112.5$ W Iron loss = 100 W

Total loss = 212.5 W output = $(4 \times 3/4) \times 0.8 =$ **2.4 kW**

Input = 2,400 + 212.5 = 2,612.5 W $\eta = 2{,}400 \times 100/2{,}612.5 =$ **91.87 %**

Example 32.66. *A 20-kVA, 440/220 V, I-ϕ, 50 Hz transformer has iron loss of 324 W. The Cu loss is found to be 100 W when delivering half full-load current. Determine (i) efficiency when*

* Assuming a lagging power factor

delivering full-load current at 0.8 lagging p.f. and (ii) the percent of full-load when the efficiency will be maximum. **(Electrotechnique-II, M.S. Univ., Baroda 1987)**

Solution. F.L. Cu loss $= 2^2 \times 100 = 400$ W ; Iron loss = 324 W

(i) F.L. efficiency at 0.8 p.f. $= \dfrac{20 \times 0.8}{(20 \times 0.8) + 0.724} \times 100 =$ **95.67 %**

(ii) $\dfrac{\text{kVA for maximum}}{\text{F.L. kVA}} = \sqrt{\dfrac{\text{Iron loss}}{\text{F.L. Cu loss}}} = \sqrt{\dfrac{324}{400}} = 0.9$

Hence, efficiency would be maximum at **90** % of F.L.

Example 32.67. *Consider a 4-kVA, 200/400 V single-phase transformer supplying full-load current at 0.8 lagging power factor. The O.C./S.C. test results are as follows :*

O.C. test : 200 V, 0.8 A, 70 W (I.V. side)
S.C. test : 20 V, 10 A, 60 (H.V. side)

Calculate efficiency, secondary voltage and current into primary at the above load.
Calculate the load at unity power factor corresponding to maximum efficiency.

(Elect. Machines Nagpur Univ. 1993)

Solution. Full-load, $I_2 = 4000/400 = 10$ A

It means that S.C. test has been carried out with full secondary flowing. Hence, 60 W represents full-load Cu loss of the transformer.

Total F.L. losses = 60 + 70 = 130 W ; F.L. output $= 4 \times 0.8 = 3.2$ kW

F.L. $\eta = 3.2/3.33 = 0.96$ or **96 %**

S.C. Test

$\mathbf{Z}_{02} = 20/10 = 2\ \Omega$; $I_2^2 R_{02} = 60$ or $R_{02} = 60/10^2 = 0.6\ \Omega$; $X_{02} = \sqrt{2^2 - 0.6^2} = 1.9\ \Omega$

Transformer voltage drop as referred to secondary

$= I_2 (R_{02} \cos\phi + X_{02} \sin\phi) = 10\,(0.6 \times 0.8 + 1.9 \times 0.6) = 16.2$ V

$\therefore \quad V_2 = 400 - 16.2 = 383.8$ V

Primary current = 4000/200 = 20 A

kVA corresponding to $\eta_{max} = 4 \times \sqrt{70/60} = 4.32$ kVA

$\therefore$ Load at u.p.f. corresponding to $\eta_{max} = 4.32 \times 1 =$ **4.32 kW**

Example 32.68. *A 600 kVA, 1-phase transformer has an efficiency of 92 % both at full-load and half-load at unity power factor. Determine its efficiency at 60 % of full-load at 0.8 power factor lag.*

(Elect. Machines, A.M.I.E. Sec. B, 1992)

Solution.

$$\eta = \frac{x \times kVA \times \cos\phi}{(x \times kVA) \times \cos\phi + W_i + x^2 W_{Cu}} \times 100$$

where x represents percentage of full-load

W_i is iron loss and W_{Cu} is full-load Cu loss.

At F.L. u.p.f. Here $x = 1$

$\therefore \quad 92 = \dfrac{1 \times 600 \times 1}{1 \times 600 \times 1 + W_1 + I^2 W_{Cu}} \times 100,\ W_i + W_{Cu} = 52.174$ kW ...***(i)***

At half F.L. UPF. Here $x = 1/2$

$$92 = \frac{1/2 \times 600 \times 1}{(1/2) \times 600 \times 1 + W_i + (1/2)^2 W_{Cu}} \times 100\ ;$$

$\therefore \quad W_i + 0.25 \qquad W_{Cu} = 26.087$ kW ...(*ii*)

From (*i*) and (*ii*), we get, $W_i = 17.39$ kW, $W_{Cu} = 34.78$ kW

60 % F.L. 0.8 p.f. (lag) Here, $x = 0.6$

$$\eta = \frac{0.6 \times 600 \times 0.8 \times 100}{(0.6 \times 600 \times 0.8) + 17.39 + (0.6)^2\ 34.78} = \mathbf{85.9\ \%}$$

Example 32.69. *A 600-kVA, 1-ph transformer when working at u.p.f. has an efficiency of 92 % at full-load and also at half-load. Determine its efficiency when it operates at unity p.f. and 60 % of full-load.* **(Electric. Machines, Kerala Univ. 1987)**

Solution. The fact that efficiency is the same *i.e.* 92 % at both full-load and half-load will help us to find the iron and copper losses.

At full-load

Output = 600 kW ; Input = 600/0.92 = 652.2 kW ; Total loss = 652.2 – 600 = 52.2 kW

Let x = Iron loss – It remains constant at all loads.

y = F.L. Cu loss – It is $\propto$ (kVA)^2. $\therefore \quad x + y = 52.2$ ...(*i*)

At half-load

Output = 300 kW ; Input = 300/0.92 $\therefore$ Losses = (300/0.92 – 300) = 26.1 kW

Since Cu loss becomes one-fourth of its F.L. value, hence

$$x + y/4 = 26.1$$

Solving for x and y, we get x = **17.4 kW** ; y = 34.8 kW ...(*ii*)

At 60 % full-load

Cu loss = $0.62 \times 34.8 = 12.53$ kW ; Total loss = 17.4 + 12.53 = 29.93 kW

Output = $600 \times 0.6 = 360$ kW $\therefore$ $\eta = 360/389.93 = 0.965$ or **96.5 %**

Example 32.70. *The maximum efficiency of a 100-kVA, single phase transformer is 98% and occurs at 80% of full load at 8 p.f. If the leakage impedance of the transformer is 5 %, find the voltage regulation at rated load of 0.8 power factor lagging.*

(Elect. Machines-I, Nagpur Univ. 1993)

Solution. Since maximum efficiency occurs at 80 percent of full-load at 0.8 p.f.,

Output at $\eta_{max} = (100 \times 0.8) \times 0.8 = 64$ kW ; Input = 64/0.98 = 65.3 kW

$\therefore$ Total loss = 65.3 – 64 = 1.3 kW. This loss is divided equally between Cu and iron.

$\therefore$ Cu loss at 80 % of full-load = 1.3/2 = 0.65 kW

Cu loss at full-load = $0.65/0.8^2 = 1$ kW

$$\%\ R = \frac{\text{Cu loss}}{V_2 I_2} \times 100 = 1 \times \frac{100}{100} = 1\ \% = v_r\,;\ v_x = 5\ \%$$

$$\therefore \quad \%\ \text{age regn.} = (1 \times 0.8 + 5 \times 0.6) + \frac{1}{200}(5 \times 0.8 - 1 \times 0.6)^2 = \mathbf{0.166\ \%}$$

Example 32.71. *A 10 kVA, 5000/440-V, 25-Hz single phase transformer has copper, eddy current and hysteresis losses of 1.5, 0.5 and 0.6 per cent of output on full load. What will be the percentage losses if the transformer is used on a 10-kV, 50-Hz system keeping the full-load current constant ? Assume unity power factor operation. Compare the full load efficiencies for the two cases.*

(Elect. Machines, A.M.I.E., Sec. B, 1991)

Solution. We know that $E_1 = 4{,}44\ f\ N_1 B_1 A$.. When both excitation voltage and frequency are doubled, flux remains unchanged.

F.L. output at upf = 10 kVA $\times$ 1 = 10 kW

F.L. Cu loss = $1.5 \times 10/100 = 0.15$ kW ; Eddy current loss

= $0.5 \times 10/100 = 0.05$ kW ; Hysteresis loss = $0.6 \times 10/100 = 0.06$ kW

Now, full-load current is kept constant but voltage is increased from 5000 V to 10,000 V. Hence, output will be doubled to 20 kW. Due to constant current, Cu loss would also remain constant.

New Cu loss = 0.15 kW, % Cu loss = $(0.15/20) \times 100 = 0.75$ %

Now, eddy current loss $\propto f^2$ and hysteresis loss $\propto f$.

New eddy current loss = $0.05\ (50/25)^2 = 0.2$ kW, % eddy current loss = $(0.2/20) \times 100 = 1$ %

Now, $W_h = 0.06 \times (50/25) = 0.12$ kW, % $W_h = (0.12/20) \times 100 = 0.6$ %

$$\eta_1 = \frac{10}{10 + 0.15 + 0.05 + 0.06} \times 100 = \mathbf{87.4\ \%}$$

$$\eta_2 = \frac{20}{20 + 0.15 + 0.2 + 0.12} \times 100 = \mathbf{97.7\ \%}$$

Example 32.72. *A 300-kVA, single-phase transformer is designed to have a resistance of 1.5 % and maximum efficiency occurs at a load of 173.2 kVA. Find its efficiency when supplying full-load at 0.8 p.f. lagging at normal voltage and frequency.* **(Electrical Machines-I, Gujarat Univ. 1985)**

Solution. $\%R = \frac{\text{F.L. Cu loss}}{\text{Full-load } V_2 I_2} \times 100$; $1.5 = \frac{\text{F.L. Cu loss}}{300 \times 1000} \times 100$

$\therefore$ F.L. Cu loss = $1.5 \times 300 \times 1000/100 = 4500$ W

Also, $173.2 = 300 \sqrt{\frac{\text{Iron Loss}}{4500}}$; Iron loss = 1500 W

Total F.L. loss = $4500 + 1500 = 6$ kW

F.L. η at 0.8 p.f. = $\frac{300 \times 0.8}{(300 \times 0.8) + 6} \times 100 = \mathbf{97.6\ \%}$

Example 32.73. *A single phase transformer is rated at 100-kVA, 2300/230-V, 50 Hz. The maximum flux density in the core is 1.2 Wb/m^2 and the net cross-sectional area of the core is 0.04 m^2. Determine*

(a) *The number of primary and secondary turns needed.*

(b) *If the mean length of the magnetic circuit is 2.5 m and the relative permeability is 1200, determine the magnetising current. Neglect the current drawn for the core loss.*

(c) *On short-circuit with full-load current flowing, the power input is 1200 W and an open-circuit with rated voltage, the power input was 400 W. Determine the efficiency of the transformer at 75 % of full-load with 0.8 p.f. lag.*

(d) *If the same transformer is connected to a supply of similar voltage but double the frequency (i.e., 100 Hz). What is the effect on its efficiency ?* **(Elect. Engg., Bombay Univ. 1988)**

Solution. ***(a)*** Applying e.m.f. equation of the transformer to the primary, we have

$2300 = 4.44 \times 50 \times N_1 \times (1.2 \times 0.0.4)$ $\therefore$ $N_1 = \mathbf{216}$

$K = 230/2300 = 1/10$ $N_2 = KN_1 = 216/10 = 21.6$ or **22**

(b) $AT = H \times l = \frac{B}{\mu_0 \mu_r} \times l = \frac{1.2 \times 2.5}{4\pi \times 10^{-7} \times 1200} = 1989$ $\therefore I = \frac{1989}{216} = \mathbf{9.21\ A}$

(c) F.L. Cu loss = 1200 W – S.C. test ; Iron loss = 400 W – O.C. test

Cu loss at 75 % of F.L. = $(0.75)^2 \times 1200 = 675$ W

Total loss = $400 + 675 = 1075$ kW

Output = $100 \times (3/4) \times 0.8 = 60$ kW ; $\eta = (60/61.075) \times 100 = \mathbf{98.26\ \%}$

(*d*) When frequency is doubled, iron loss is increased because

(*i*) hysteresis loss is doubled – $W_h \propto f$

(*ii*) eddy current loss is quadrupled – $W_e \propto f^2$

Hence, efficiency will be decreased.

Example 32.74. *A transformer has a resistance of 1.8 % and a reactance of 5.4 %. (a) At full load, what is the power-factor at which the regulation will be : (i) Zero, (ii) positive-maximum ? (b) If its maximum efficiency occurs at full-load (at unity p.f.), what will be the efficiency under these conditions ?*

Solution : Approximate percentage regulation is given, in this case, by the relationship

$1.8 \cos\phi \pm 5.4 \sin\phi$.

(*a*) Regulation :

(*i*) If regulation is zero, negative sign must be applicable. This happens at leadings p.f.

$$\text{Corresponding p.f.} = \tan\phi = 1.8/5.4 = 0.333 \text{ leading}$$

$$\phi = \mathbf{18.44^0 \text{ leading}}$$

(*ii*) For maximum positive regulation, lagging p.f. is a must. From phasor diagram, the result can be obtained.

$$\text{Corresponding } \tan\phi = 5.4/1.8 = 3, \phi = 71.56 \text{ lagging}$$

$$\text{\% Voltage regulation} = 1.8 \cos\phi + 5.4 \sin\phi = \mathbf{5.7\ \%}$$

(*b*) Efficiency : Maximum efficiency occurs at such a load when

$$\text{Iron losses} = \text{Copper losses}$$

This means Iron-losses are 1.8 %.

$$\text{Efficiency} = 100/(100 + 1.8 + 1.8) = \mathbf{96.52\ \%}$$

Example 32.75. *A 10 kVA, 1 phase, 50 Hz, 500/250 V transformer gave following test results :*

OC test (LV) side : 250 V, 3.0 A, 200 W

SC test (LV) side : 15 V, 30 A, 300 W.

Calculate efficiency and regulation at full load, 0.8 p.f. lagging.

(Nagpur University, Summer 2000)

Solution. For efficiency calculations, full load current should be calculated, on the L.V. side in this case,

$$\text{F.L. Current} = \frac{10,000}{250} = 40 \text{ amp}$$

Short-circuit test data have been given at 30 A current on the L.V. side.

$$i^2r \text{ losses at 40 A L.V. side} = \left(\frac{40}{30}\right)^2 \times 300 \text{ Watts} = 533.3 \text{ Watts}$$

At rated voltage, iron losses (from O.C. test) = 200 Watts

F.L. Output at 0.8 P.F. = $10,000 \times 0.8 = 8000$ Watts

$$\text{Hence, } \eta = \frac{8000}{8000 + 4733.3} \times 100\ \% = \mathbf{91.6\ \%}$$

For regulation, series resistance and reactance parameters of the equivalent circuit have to be evaluated, from the S.C. test.

$$\text{Series Impedance, } Z = \frac{15}{30} = 0.5 \text{ ohm}$$

$$\text{Series resistance, } r = \frac{300}{30 \times 30} = 0.333 \text{ ohm}$$

Series reactance, $x = \sqrt{0.5^2 - 0.333^2} = \mathbf{0.373\ ohm}$

By Approximate formula,

p.u. regulation at full load, 0.8 p.f. lagging

$$= \frac{40}{250}\,[0.333 \times 0.8 - 0.373 \times 0.6] = 6.82 \times 10^{-3}\ \text{p.u.}$$

When converted into volts, this is $6.82 \times 10^{-3} \times 250 = \mathbf{1.70\ volt}$

Example 32.76. *A 40 kVA, 1-ph, transformer has an iron loss of 400 W, and full copper loss of 800 W. Find the load at which maximum efficiency is achieved at unity power factor.*

(Amravati University, Winter 1999)

Solution. If x = fraction of rated load at which the efficiency is maximum.

$$P_i = \text{Iron–loss} = 400\ \text{W}$$

$$P_c = \text{F.L. copper–loss} = 800\ \text{W}$$

Then $x^2 P_c = P_i$

On substitution of numerical values of P_i and P_c, we get

$$x = 0.707$$

Hence, the efficiency is maximum, at unity p.f. and at 70.7 % of the rated Load. At this load, copper–loss = Iron–loss = 0.40 kW

$$\text{Corresponding output} = 40 \times 0.707 \times 1$$

$$= 28.28\ \text{kW}$$

$$\text{Corresponding efficiency} = \frac{28.28}{28.28 + 0.4 + 0.4} = \mathbf{97.25\ \%}$$

Extension to Question : (*a*) *At what load (s) at unity p.f. the efficiency will be 96.8 % ?*

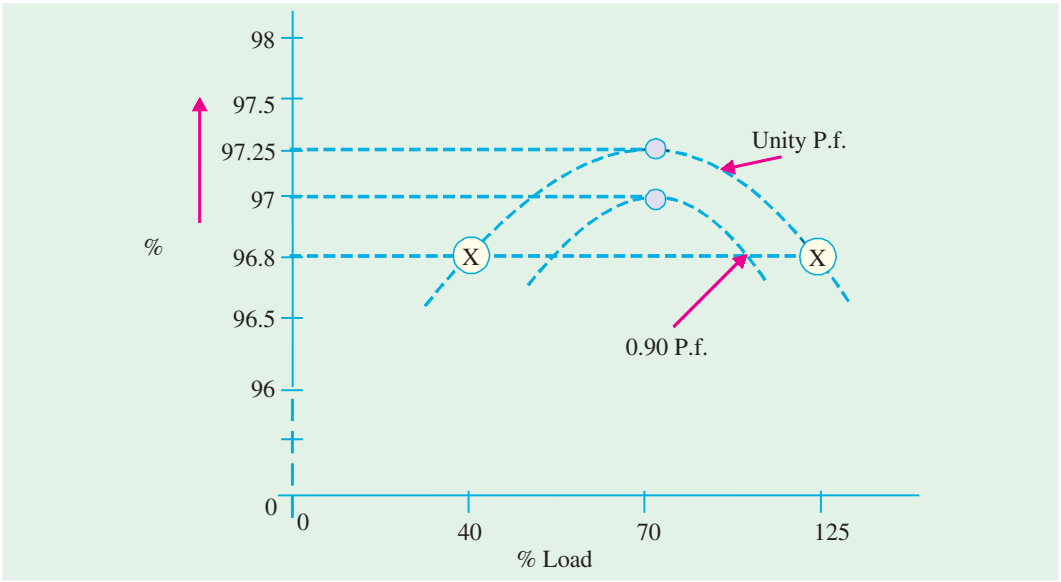


Fig. 32.58. Efficiency variation with load

Solution. Let x = Fractional load at which the concerned efficiency occurs, at unity p.f.

$$\frac{40\,x}{40\,x + 0.8\,x^2 + 0.40} = 0.968$$

This gives the following values of x : $x_1 = 1.25$ $x_2 = 0.40$

Thus, at 40 % and at 125 % of the rated load, the efficiency will be **96.8 %** as marked on the graph, in Fig. 32.58.

(b) How will maximum-efficiency condition be affected if the power factor is 0.90 lagging ?

Solution.

The condition for efficiency-variation-statement is that the power factor remains constant. Thus, for 0.90 lagging p.f., another curve (Lower curve in Fig. 32.58) will be drawn for which the maximum efficiency will occur at the same value of x (= 0.707), but

$$\text{Maximum efficiency} = \frac{40\ x \cos\phi}{40\cos\phi + 0.80\ x^2 + 0.40}$$

$$= \frac{28.28 \times 0.90}{(28.28 \times 0.90) + 0.80} = \mathbf{97\ \%}$$

Example 32.77. *A 10 kVA, 500/250 V, single-phase transformer gave the following test results:*

S.C. Test (H.V. side) : 60 V, 20 A, 150 W

The maximum efficiency occurs at unity power factor and at 1.20 times full-load current. Determine full-load efficiency at 0.80 p.f. Also calculate the maximum efficiency.

(Rajiv Gandhi Technical University, Bhopal, Summer 2001)

Solution. Full-load current on H.V. side = 10,000/500 = 20 Amp

S.C. test has been conducted from H.V. side only. Hence, full-load copper-loss, at unity p.f. = 150 watts

(a) Maximum efficiency occurs at 1.2 times full-load current, at unity p.f. corresponding copper-loss = $(1.2)^2 \times 150 = 216$ watts

At maximum efficiency, copper-loss = core-loss = 216 watts

Corresponding Power-output = $1.2 \times 10{,}000 \times 1.0 = 12$ kW

Hence, maximum efficiency at unity P.f. = (12)/(12 + 0.216 + 0.2160) = 0.9653 = 96.53 %

(b) Full-load efficiency at 0.80 P.f.

Output Power at full-load, 0.80 P.f. = $10{,}000 \times 0.8 = 8000$ W, constant core-loss = 216 W

Corresponding copper-loss = 150 W

Total losses = 366 W

Hence, efficiency = $(8000/8366) \times 100\ \% = 95.63\ \%$.

32.31. Variation of Efficiency with Power Factor

The efficiency of a transformer is given by

$$\eta = \frac{\text{Output}}{\text{Input}} = \frac{\text{Input - Losses}}{\text{Input}}$$

$$= 1 - \frac{\text{Losses}}{\text{Input}} = 1 - \frac{\text{Losses}}{(V_2\ I_2 \cos\phi + \text{losses})}$$

Let, losses/$V_2\ I_2 = x$

$$\therefore \quad \eta = 1 - \frac{\text{losses}/V_2 I_2}{\cos\phi + (\text{losses}/V_2 I_2)}$$

$$= 1 - \frac{x}{(\cos\phi + x)} = 1 - \frac{x/\cos\phi}{1 + (x/\cos\phi)}$$

The variations of efficiency with power factor at different loadings on a typical transformer are shown in Fig. 32.59.

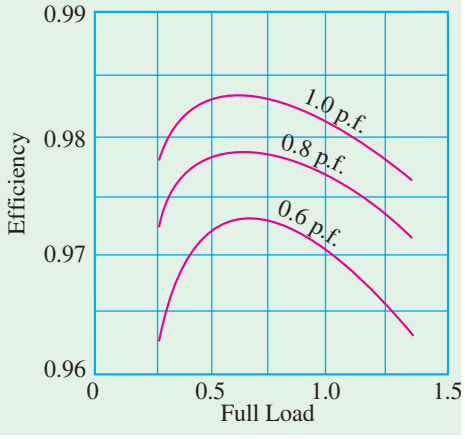


Fig. 32.59

Tutorial Problems 32.4

1. A 200-kVA transformer has an efficiency of 98 % at full-load. If the maximum efficiency occurs at three-quarters of full-load, calculate (*a*) iron loss at F.L. (*b*) Cu loss at F.L. (*c*) efficiency at half-load. Ignore magnetising current and assume a p.f. of 0.8 at all loads.

[(*a*) 1.777 kW (*b*) 2.09 kW (*c*) 97.92%]

2. A 600 kVA, 1-ph transformer has an efficiency of 92 % both at full-load and half-load at unity power factor. Determine its efficiency at 60 % of full load at 0.8 power factor lag.

[90.59%] (*Elect. Machines, A.M.I.E. Sec. B, 1992*)

3. Find the efficiency of a 150 kVA transformer at 25 % full load at 0.8 p.f. lag if the copper loss at full load is 1600 W and the iron loss is 1400 W. Ignore the effects of temperature rise and magnetising current. **[96.15%] (*Elect. Machines, A.M.I.E. Sec. B, 1991*)**

4. The F.L. Cu loss and iron loss of a transformer are 920 W and 430 W respectively. (*i*) Calculate the loading of the transformer at which efficiency is maximum (*ii*) what would be the losses for giving maximum efficiency at 0.85 of full-load if total full-load losses are to remain unchanged ?

[(*a*) 68.4% of F.L. (*ii*) W_i = 565 W ; W_{cu} = 785 W]

5. At full-load, the Cu and iron losses in a 100-kVA transformer are each equal to 2.5 kW. Find the efficiency at a load of 65 kVA, power factor 0.8. **[93.58%] (City & Guilds London)**

6. A transformer, when tested on full-load, is found to have Cu loss 1.8% and reactance drop 3.8%. Calculate its full-load regulation (*i*) at unity p.f. (*ii*) 0.8 p.f. lagging (*iii*) 0.8 p.f. leading.

[(*i*) 1.80% (*ii*) 3.7 % (*iii*) −0.88%]

7. With the help of a vector diagram, explain the significance of the following quantities in the open-circuit and short-circuit tests of a transformer (*a*) power consumed (*b*) input voltage (*c*) input current.

When a 100-kVA single-phase transformer was tested in this way, the following data were obtained : On open circuit, the power consumed was 1300 W and on short-circuit the power consumed was 1200 W. Calculate the efficiency of the transformer on (*a*) full-load (*b*) half-load when working at unity power factor. **[(*a*) 97.6% (*b*) 96.9%] (*London Univ.*)**

8. An 11,000/230-V, 150-kVA, 50-Hz, 1-phase transformer has a core loss of 1.4 kW and full-load Cu loss of 1.6 kW. Determine (*a*) the kVA load for maximum efficiency and the minmum efficiency (*b*) the efficiency at half full-load at 0.8 power factor lagging. **[140.33 kVA, 97.6% ; 97%]**

9. A single-phase transformer, working at unity power factor has an efficiency of 90 % at both half-load and a full-load of 500 kW. Determine the efficiency at 75 % of full-load. **[90.5%] (*I.E.E. London*)**

10. A 10-kVA, 500/250-V, single-phase transformer has its maximum efficiency of 94 % when delivering 90 % of its rated output at unity power factor. Estimate its efficiency when delivering its full-load output at p.f. of 0.8 lagging. **[92.6%] (*Elect. Machinery, Mysore Univ, 1979*)**

11. A single-phase transformer has a voltage ratio on open-circuit of 3300/660-V. The primary and secondary resistances are 0.8 Ω and 0.03 Ω respectively, the corresponding leakage reactance being 4 Ω and 0.12 Ω. The load is equivalent to a coil of resistance 4.8 Ω and inductive reactance 3.6 Ω. Determine the terminal voltage of the transformer and the output in kW. **[636 V, 54 kW]**

12. A 100-kVA, single-phase transformer has an iron loss of 600 W and a copper loss of 1.5 kW at full-load current. Calculate the efficiency at (*a*) 100 kVA output at 0.8 p.f. lagging (*b*) 50 kVA output at unity power factor. **[(*a*) 97.44% (*b*) 98.09%]**

13. A 10-kVA, 440/3300-V, 1-phase transformer, when tested on open circuit, gave the following figures on the primary side : 440V ; 1.3 A ; 115 W.

When tested on short-circuit with full-load current flowing, the power input was 140 W. Calculate the efficiency of the transformer at (*a*) full-load unity p.f. (*b*) one quarter full-load 0.8 p.f.

[(*a*) 97.51% (*b*) 94.18%] (*Elect. Engg-I, Sd. Patel Univ. June 1977*)

14. A 150-kVA single-phase transformer has a core loss of 1.5 kW and a full-load Cu loss of 2 kW. Calculate the efficiency of the transformer (*a*) at full-load, 0.8 p.f. lagging (*b*) at one-half full-load

unity p.f. Determine also the secondary current at which the efficiency is maximum if the secondary voltage is maintained at its rated value of 240 V. **[(*a*) 97.17% (*b*) 97.4% ; 541 A]**

15. A 200-kVA, 1-phase, 3300/400-V transformer gave the following results in the short-circuit test. With 200 V applied to the primary and the secondary short-circuited, the primary current was the full-load value and the input power 1650 W. Calculate the secondary p.d. and percentage regulation when the secondary load is passing 300 A at 0.707 p.f. lagging with normal primary voltage.

[380 V ; 480%]

16. The primary and secondary windings of a 40-kVA, 6600/250-V, single-phase transformer have resistances of 10 Ω and 0.02 Ω respectively. The leakage reactance of the transformer referred to the primary is 35 Ω. Calculate

(*a*) the primary voltage required to circulate full-load current when the secondary is short-circuited.

(*b*) the full-load regulations at (*i*) unity (*ii*) 0.8 lagging p.f. Neglect the no-load current.

[(*a*) 256 V (*b*) (*i*) 2.2% (*ii*) 3.7%] (*Elect. Technology, Kerala Univ. 1979*)

17. Calculate :

(*a*) F.L. efficiency at unity p.f.

(*b*) The secondary terminal voltage when supplying full-load secondary current at p.f. (*i*) 0.8 lag (*ii*) 0.8 lead for the 4-kVA, 200/400 V, 50 Hz, 1-phase transformer of which the following are the test figures :

Open circuit with 200 V supplied to the primary winding-power 60 W. Short-circuit with 16 V applied to the h.v. winding-current 8 A, power 40 W. **[0.97 ; 383 V ; 406 V]**

18. A 100-kVA, 6600/250-V, 50-Hz transformer gave the following results :

O.C. test : 900 W, normal voltage.

S.C. test (data on h.v. side) : 12 A, 290 V, 860 W

Calculate

(*a*) the efficiency and percentage regulation at full-load at 0.8 p.f. lagging.

(*b*) the load at which maximum efficiency occurs and the value of this efficiency at p.f. of unity, 0.8 lag and 0.8 lead. **[(*a*) 97.3%, 4.32% (*b*) 81 kVA, 97.8%, 97.3% ; 97.3%]**

19. The primary resistance of a 440/110-V transformer is 0.5 Ω and the secondary resistance is 0.04 Ω. When 440 V is applied to the primary and secondary is left open-circuited, 200 W is drawn from the supply. Find the secondary current which will give maximum efficiency and calculate this efficiency for a load having unity power factor.

[53 A ; 93.58%] (*Basic Electricity & Electronics. Bombay Univ. 1981*)

20. Two tests were performed on a 40-kVA transformer to predetermine its efficiency. The results were:

Open circuit : 250 V at 500 W

Short circuit : 40 V at F.L. current, 750 W both tests from primary side.

Calculate the efficiency at rated kVA and 1/2 rated kVA at (*i*) unity p.f. (*ii*) 0.8 p.f.

[96.97% ; 96.68% ; 96.24% ; 95.87%]

21. The following figures were obtained from tests on a 30-kVA, 3000/110-V transformer :

O.C. test : 3000 V 0.5 A 350 W ; S.C. test ; 150 V 10 A 500 W

Calculate the efficiency of the transformer at

(*a*) full-load, 0.8 p.f. (*b*) half-load, unity p.f.

Also, calculate the kVA output at which the efficiency is maximum. **[96.56% ; 97% ; 25.1 kVA]**

22. The efficiency of a 400 kVA, 1-phase transformer is 98.77 % when delivering full load at 0.8 power factor, and 99.13 % at half load and unity power factor. Calculate (*a*) the iron loss, (*b*) the full load copper loss. **[(*a*) 1012 W (*b*) 2973 W] (*Rajiv Gandhi Technical University, 2000*)**

32.32. All-day Efficiency

The ordinary or commercial efficiency of a transformer is given by the ratio

$$\frac{\text{Output in watts}}{\text{Input in watts}}.$$



The world first 5,000 KVA amorphous transformer commissioned in August 2001 in Japan

But there are certain types of transformers whose performance cannot be judged by this efficiency. Transformers used for supplying lighting and general network *i.e.*, distribution transformers have their primaries energised all the twenty-four hours, although their secondaries supply little or no-load much of the time during the day except during the house lighting period. It means that whereas core loss occurs throughout the day, the Cu loss occurs only when the transformers are loaded. Hence, it is considered a good practice to design such transformers so that core losses are very low. The Cu losses are relatively less important, because they depend on the load. The performance of such is compared on the basis of *energy* consumed during a certain time period, usually a day of 24 hours.

$$\therefore \quad \eta_{\text{all-day}} = \frac{\text{Output in kWh}}{\text{Input in kWh}} \text{ (For 24 hours)}$$

This efficiency is always less than the commercial efficiency of a transformer.

To find this all-day efficiency or (as it is also called) energy efficiency, we have to know the load cycle on the transformer *i.e.*, how much and how long the transformer is loaded during 24 hours. Practical calculations are facilitated by making use of a ***load factor***.

Example 32.78. *Find the all-day efficiency of 500-kVA distribution transformer whose copper loss and iron loss at full load are 4.5 kW and 3.5 kW respectively. During a day of 24 hours, it is loaded as under :*

No. of hours	*Loading in kW*	*Power factor*
6	*400*	*0.8*
10	*300*	*0.75*
4	*100*	*0.8*
4	*0*	–

(Elect. Machines, Nagpur Univ. 1993)

Solution. It should be noted that a load of 400 kW at 0.8 p.f. is equal to 400/0.8 = 500 kVA. Similarly, 300 kW at 0.75 p.f. means 300/0.75 = 400 kVA and 100 kW at 0.8 p.f. means 100/0.8 = 125 kVA *i.e.*, one-fourth of the full-load.

$$\text{Cu loss at F.L. of 500 kVA} = 4.5 \text{ kW}$$

$$\text{Cu loss at 400 kVA} = 4.5 \times (400/500)^2 = 2.88 \text{ kW}$$

$$\text{Cu loss at 125 kVA} = 4.5 \times (125/500)^2 = 0.281 \text{ kW}$$

$$\text{Total Cu loss in 24 hrs} = (6 \times 4.5) + (10 \times 2.88) + (4 \times 0.281) + (4 \times 0)$$

$$= 56.924 \text{ kWh}$$

The iron loss takes place throughout the day irrespective of the load on the transformer because its primary is energized all the 24 hours.

$\therefore$ Iron loss in 24 hours $= 24 \times 3.5 = 84$ kWh

Total transformer loss $= 56.924 + 84 = 140.924$ kWh

Transformer output is 24 hrs $= (6 \times 400) + (10 \times 300) + (4 \times 100) = 5800$ kWh

$\therefore \quad \eta_{\text{all-day}} = \dfrac{\text{output}}{\text{output + losses}} = \dfrac{5800}{5800 + 140.924} = 0.976$ or **97.6 %**

Example 32.79. *A 100-kVA lighting transformer has a full-load loss of 3 kW, the losses being equally divided between iron and copper. During a day, the transformer operates on full-load for 3 hours, one half-load for 4 hours, the output being negligible for the remainder of the day. Calculate the all-day efficiency.* **(Elect. Engg. Punjab Univ. 1990)**

Solution. It should be noted that lighting transformers are taken to have a load p.f. of unity.

Iron loss for 24 hour $= 1.5 \times 24 = 36$ kWh ; F.L. Cu loss $= 1.5$ kW

$\therefore$ Cu loss for 3 hours on F.L. $= 1.5 \times 3 = 4.5$ kWh

Cu loss at half full-load $= 1.5/4$ kW

Cu loss for 4 hours at half the load $= (1.5/4) \times 4 = 1.5$ kWh

Total losses $= 36 + 4.5 + 1.5 = 42$ kWh

Total output $= (100 \times 3) + (50 \times 4) = 500$ kWh

$\therefore \quad \eta_{\text{all-day}} = 500 \times 100/542 =$ **92.26 %**

Incidentally, ordinary or commercial efficiency of the transformer is

$= 100/(100 + 3) = 0.971$ or **97.1 %**

Example 32.80. *Two 100-kW transformers each has a maximum efficiency of 98 % but in one the maximum efficiency occurs at full-load while in the other, it occurs at half-load. Each transformer is on full-load for 4 hours, on half-load for 6 hours and on one-tenth load for 14 hours per day. Determine the all-day efficiency of each transformer.* **(Elect. Machines-I, Vikram Univ. 1988)**

Solution. Let x be the iron loss and y the full-load Cu loss. If the ordinary efficiency is a maximum at $1/m$ of full-load, then $x = y/m^2$.

Now, output $= 100$ kW ; Input $= 100/0.98$

$\therefore$ Total losses $= 100/0.98 - 100 = 2.04$ kW

$\therefore \quad y + x/m^2 = 2.04$

Ist Transformer

Here $m = 1$; $\quad y + y = 2.04$; $y = 1.02$ kW and $x = 1.02$ kW

Iron loss for 24 hours $= 1.02 \times 24 = 24.48$ kWh

Cu loss for 24 hours $= 4 \times 1.02 + 6 \times (1.02/4) + 14\,(1.02/10^2) = 5.73$ kWh

Total loss $= 24.48 + 5.73 = 30.21$ kWh

$= 4 \times 100 + 6 \times 50 + 14 \times 10 = 840$ kWh

$\therefore \quad \eta_{\text{all-day}} = 840/870.21 = 0.965$ or **96.5 %**

2nd Transformer

Here $1/m = 1/2$ or $m = 2$ $\therefore \; y + y/4 = 2.04$

or $y = 1.63$ kW ; $x = 0.14$ kW

Output $= 840$ kWh ...as above

Iron loss for 24 hours $= 0.41 \times 24 = 9.84$ kWh

Cu loss for 24 hours $= 4 \times 1.63 + 6\,(1.63/4) + 14\,(1.63/10^2) = 9.19$ kWh

Total loss $= 9.84 + 9.19 = 19.03$ kWh

$\therefore \quad \eta_{\text{all-day}} = 840/859.03 = 0.978$ or **97.8 %**

Example 32.81. *A 5-kVA distribution transformer has a full-load efficiency at unity p.f. of 95 %, the copper and iron losses then being equal. Calculate its all-day efficiency if it is loaded throughout the 24 hours as follows :*

No load for 10 hours Quarter load for 7 hours
Half load for 5 hours Full load for 2 hours
Assume load p.f. of unity. **(Power Apparatus-I, Delhi Univ. 1987)**

Solution. Let us first find out the losses from the given commercial efficiency of the transformer.

Output = $5 \times 1 = 5$ kW ; Input = $5/0.95 = 5.264$ kW

Losses = $(5.264 - 5.000) = 0.264$ kW = 264W

Since efficiency is maximum, the losses are divided equally between Cu and iron.

$\therefore$ Cu loss at F.L. of 5 kVA = $264/2 = 132$ W ; Iron loss = 132 W

Cu loss at one-fourth F.L. = $(1/4)^2 \times 132 = 8.2$ W

Cu loss at one-half F.L. = $(1/2)^2 \times 132 = 33$ W

Quarter load Cu loss for 7 hours = $7 \times 8.2 = 57.4$ Wh

Half-load Cu loss for 5 hours = $5 \times 33 = 165$ Wh

F.L. Cu loss for 2 hours = $2 \times 132 = 264$ Wh

Total Cu loss during one day = $57.4 + 165 + 264 = 486.4$ Wh = 0.486 kWh

Iron loss in 24 hours = $24 \times 132 = 3168$ Wh = 3.168 kWh

Total losses in 24 hours = $3.168 + 0.486 = 3.654$ kWh

Since load p.f. is to be assumed as unity.

F.L. output = $5 \times 1 = 5$ kW ; Half F.L. output = $(5/2) \times 1 = 2.5$ kW

Quarter load output = $(5/4) \times 1 = 1.25$ kW

Transformer output in a day of 24 hours = $(7 \times 1.25) + (5 \times 2.5) + (2 \times 5) = 31.25$ kWh

$$\eta_{\text{all-day}} = \frac{31.25}{(31.25 + 3.654)} \times 100 = \mathbf{89.53\ \%}$$

Example 32.82. *Find "all day" efficiency of a transformer having maximum efficiency of 98 % at 15 kVA at unity power factor and loaded as follows :*

12 hours – 2 kW at 0.5 p.f. lag
6 hours – 12 kW at 0.8 p.f. lag
6 hours – at no load. **(Elect. Machines-I, Nagpur Univ. 1993)**

Solution. Output = $15 \times 1 = 15$ kW, input = $15/0.98$

Losses = $(15/0.98 - 15) = 0.306$ kW = 306 W

Since efficiency is maximum, the losses are divided equally between Cu and iron.

$\therefore$ Cu loss at 15 kVA = 306/2 = 153 W, Iron loss = 153 W

2 kW at 0.5 p.f. = 2/0.5 = 4 kVA, 12 kW at 0.8 p.f. = 12/0.8 = 15 kVA

Cu loss at 4 kVA = $153\ (4/15)^2 = 10.9$ W ; Cu loss at 15 kVA = 153 W.

Cu loss in 12 hrs = $12 \times 10.9 = 131$ Wh ; Cu loss in 6 hr = $6 \times 153 = 918$ Wh.

Total Cu loss for 24 hr = $131 + 918 = 1050$ Wh = 1.05 kWh

Iron loss for 24 hrs = $24 \times 153 = 3{,}672$ Wh = 3.672 kWh

Output in 24 hrs = $(2 \times 12) + (6 \times 12) = 96$ kWh

Input in 24 hrs = $96 + 1.05 + 3.672 = 100.72$ kWh

$\therefore$ $\eta_{\text{all-day}} = 96 \times 100/100.72 = \mathbf{95.3\ \%}$

Example 32.83. *A 150-kVA transformer is loaded as follows :*

Load increases from zero to 100 kVA in 3 hours from 7 a.m. to 10.00 a.m., stays at 100 kVA from 10 a.m. to 6 p.m. and then the transformer is disconnected till next day. Assuming the load to be resistive and core-loss equal to full-load copper loss of 1 kW, determine the all-day efficiency and the ordinary efficiency of the transformer. **(Electrical Machines-II, Indore Univ. 1990)**

Solution. Since load is resistive, its p.f. is unity.

Average load from 7 a.m. to 10 a.m. = (0 + 100)/2 = 50 kVA *i.e.*, one-third F.L.

Load from 10 a.m. to 6 p.m. = 100 kVA *i.e.*, 2/3 of F.L.

Ordinary Efficiency

In this case, load variations are not relevant.

Output = 150 × 1 = 150 kW ; Iron loss = Cu loss = 1 kW ; Total loss = 2 kW.

∴ Ordinary η = 150/(150 + 2) = 0.9868 or **98.68 %**

All-day Efficiency

Cu loss from 7-10 a.m. = $3 \times (1/3)^2 \times 1 = 0.333$ kWh

Cu loss from 10 a.m. to 6.p.m. = $8 \times (2/3)^2 \times 1 = 3.555$ kWh

Total Cu loss for 24 hrs = 0.333 + 3.555 = 3.888 kWh

Total iron loss for 24 hrs = 24 × 1 = 24 kWh

Losses for a day of 24 hrs = 27.888 kWh

Output for 24 hrs = 3 × (50 × 1) + 8 (100 × 1) = 950 kWh

$$\therefore \quad \eta_{\text{all-day}} = \frac{950 \times 100}{(950 + 27.888)} = \mathbf{97.15\ \%}$$

Example 32.84. *Find the all-day efficiency of a 50 kVA distribution transformer having full load efficiency of 94 % and full-load copper losses are equal to the constant iron losses. The loading of the transformer is as follows, the power factor being 1.0.*

(i) No load for 10 hours *(ii) Half load for 5 hours*

(iii) 25 % load for 6 hours *(iv) Full load for 3 hours.* **(Sambalpur University, 1998)**

Solution. At full load unity p.f.

$$\text{efficiency} = 94\,\% = \frac{50,000}{50,000 + 2P_i}$$

$$2P_i = \left[\frac{50,000}{0.94} - 50,000\right], \quad \text{or} \quad P_i = \frac{1}{2} \times 50,000 \left[\frac{1 - 0.94}{0.94}\right],$$

$$P_i = 25,000 \times \frac{0.06}{0.94} = 1596 \text{ Watts}$$

Hence, full load Cu-losses = 1596 Watts

(*a*) Energy required in overcoming Cu-losses, during 24 hours

(*i*) No load for 10 hours : zero

(*ii*) At half load, Cu-losses = $(0.5)^2 \times 1596$ Watts = 399

Energy in 5 hours = $\frac{399 \times 5}{1000}$ kWh = 1.995 kWh

(*iii*) At 25 % load, Cu-loss = $(0.25)^2 \times 1596 = 99.75$ Watts

Energy in 6 hours = $\frac{6 \times 99.75}{1000} = 0.5985$ kWh

(*iv*) Energy lost during 3 hours of full load = $\frac{1596 \times 3}{1000} = 4.788$ kWh

(*b*) Energy lost in constant core-losses for 2 hours = $\frac{1596}{1000} \times 24 = 38.304$ kWh

(c) Energy required by the load = $25 \times 5 + 12.5 \times 6 + 50 \times 3 = 125 + 75 + 150 = 350$ kWh

$$\text{All-day efficiency} = \frac{350}{350 + 38.304 + 7.3815} \times 100 = \mathbf{88.454\ \%}$$

Example 32.85. *A 10 kVA, 1-ph transformer has a core-loss of 40 W and full load ohmic loss of 100 W. The daily variation of load on the transformer is as follows :*

6 a.m. to 1 p.m. — 3 kW at 0.60 p.f.
1 p.m. to 5 p.m. — −8kW at 0.8 p.f.
5 p.m. to 1 a.m. — full load at u.p.f.
1 a.m. to 6 a.m. — no load

Determine all day efficiency of the transformer **(Amravati University, 1999)**

Solution. Fractional loading (= x) and the output kWh corresponding to load variations can be worked out in tabular form, as below :

S.N.	Number of hours	$x = \frac{\text{load kVA}}{\text{Xmer Rating}}$	$x^2 P_c$ in kW	Output in kWh	Copper Loss in kWh
1	7	$\frac{3/0.6}{10} = 0.5$	$0.50^2 \times 0.10 = 0.025$	$3 \times 7 = 21$	$0.025 \times 7 = 0.175$
2	4	$\frac{8/0.8}{10} = 1.0$	0.10	$8 \times 4 = 32$	$0.1 \times 4 = 0.40$
3	8	$\frac{10/1}{10} = 1.0$	0.10	$10 \times 8 = 80$	$0.1 \times 8 = 0.8$
4	5	Zero	Zero	Zero	Zero
			Output in kWh	21 + 32 + 80 = 133	
			Ohmic Loss, in kWh		0.175 + 0.40 + 0.80 = 1.375
		Core loss during 24 Hrs = $\frac{400}{1000} \times 24 = 0.96$ kWh			

Hence, Energy efficiency (= All day Efficiency) = $\frac{133}{133 + 1.375 + 0.96} \times 100 = \mathbf{98.3\ \%}$

Example 32.86. *A transformer has its maximum efficiency of 0.98 at 15 kVA at unity p.f. During a day, it is loaded as follows :*

12 hours : 2 kW at 0.8 p.f.
6 hours : 12 kW at 0.8 p.f.
6 hours : 18 kW at 0.9 p.f.

Find the all day efficiency. **(Manomaniam Sundaranar Univ. April 1998)**

Solution. Let 15 kVA be treated as full load.

Output at maximum efficiency = 15000×1 watts

Input = 15000/0.98 watts

Losses = Input – Output = $15000\ (1/0.98 - 1) = 15000 \times 2/98 = 306$ Watts

At maximum efficiency, since the variable copper-loss and constant core-loss are equal.

Full load copper-loss = Constant core-loss = 306/2 = 153 Watts

Let the term x represents the ratio of required Load/Full load.

Output = $15 \times \cos\phi$

Following tabular entries simplify the calculations for all-day efficiency.

S.N.	x	Hrs	x^2P_c in kW	Energy in Copper -loss in kWh	Output during the period in kWh
1	$\frac{2/0.5}{15} = \frac{4}{15}$	12	0.01088	0.131	24
2	$\frac{12/0.8}{15} = 1.0$	6	0.153	0.918	72
3	$\frac{18/0.9}{15} = 4/3$	6	0.272	1.632	108

Total output during the day = 204 kWh

Total copper-loss during the day = 2.681 kWh

Total core-loss during the day = $0.153 \times 24 = 3.672$

All day efficiency = $(204/210.353) \times 100 = 96.98$ %

Tutorial Problems 32.5

1. A 100-kVA distribution transformer has a maximum efficiency of 98 % at 50 % full-load and unity power factor. Determine its iron losses and full-load copper losses.

The transformer undergoes a daily load cycle as follows :

Load	*Power factor*	*Load duration*
100 kVA	1.0	8 hrs
50 kVA	0.8	6 hrs
No load		10 hrs

Determine its all-day efficiency. ***(Electrical Engineering, MS Univ. Baroda 1979)***

2. What is meant by energy efficiency of a transformer ?

A 20-kVA transformer has a maximum efficiency of 98 percent when delivering three-fourth full-load at u.p.f. If during the day, the transformer is loaded as follows :

12 hours	No load
6 hours	12 kWh, 0.8 p.f.
6 hours	20 kW, u.p.f.

Calculate the energy efficiency of the transformer.

(Electrical Technology-III, Gwalior Univ., 1980)

32.33. Auto-transformer

It is a transformer with one winding only, part of this being common to both primary and secondary. Obviously, in this transformer the primary and secondary are not electrically *isolated* from each other as is the case with a 2-winding transformer. But its theory and operation are similar to those of a two-winding transformer. Because of one winding, it uses less copper and hence is cheaper. It is used where transformation ratio differs little from unity. Fig. 32.60 shows both step down and step-up auto-transformers.

As shown in Fig. 32.60 (*a*), *AB*, is primary winding having N_1 turns and *BC* is secondary winding having N_2 turns. Neglecting iron losses and no-load current.

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2} = K$$

The current in section CB is vector difference* of I_2 and I_1. But as the two currents are practically in phase opposition, the resultant current is $(I_2 - I_1)$ where I_2 is greater than I_1.

As compared to an ordinary 2-winding transformer of same output, an auto-transformer has higher efficiency but smaller size. Moreover, its voltage regulation is also superior.

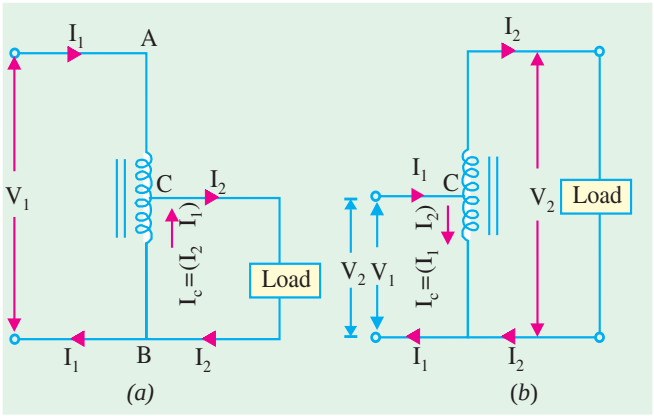


Fig. 32.60

Saving of Cu

Volume and hence weight of Cu, is proportional to the length and area of the cross-section of the conductors. Now, length of conductors is proportional to the number of turns and cross-section depends on current. Hence, weight is proportional to the product of the current and number of turns.

With reference to Fig. 32.60,

Wt. of Cu in section AC is $\propto (N_1 - N_2)\, I_1$; Wt. of Cu in section BC *is* $\propto N_2 (I_2 - I_1)$.

$\therefore$ Total Wt. of Cu in auto-transformer $\propto (N_1 - N_2)\, I_1 + N_2 (I_2 - I_1)$

If a two-winding transformer were to perform the same duty, then

Wt. of Cu on its primary $\propto N_1 I_1$; Wt. of Cu on secondary $\propto N_2 I_2$

Total Wt. of Cu $\propto N_1 I_1 + N_2 I_2$

$$\therefore \quad \frac{\text{Wt. of Cu in auto-transformer}}{\text{Wt. of Cu in ordinary transformer}} = \frac{(N_1 - N_2)\, I_1 + N_2 (I_2 - I_1)}{N_1 I_1 + N_2 I_2}$$

$$= I - \frac{2\dfrac{N_2}{N_1}}{1 + \dfrac{N_2}{N_1} \times \dfrac{I_2}{I_1}} = 1 - \frac{2K}{2} = 1 - K \left(\because \frac{N_1}{N_1} = K \; ; \frac{I_2}{I_1} = \frac{1}{K} \right)$$

Wt. of Cu in auto-transformer $(W_a) = (1 - K) \times$ (Wt. of Cu in ordinary transformer W_0)

$\therefore$ Saving $= W_0 - W_a$

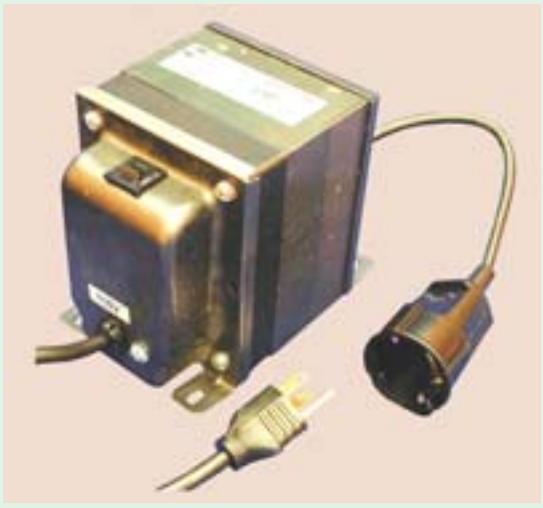
$= W_0 - (1 - K)\, W_0 = K W_0$

$\therefore$ Saving $= K \times$ (Wt. of Cu in ordinary transformer)

Hence, saving will increase as K approaches unity.

It can be proved that power transformed ***inductively*** is input $(1 - K)$.

The rest of the power $= (K \times \text{input})$ is ***conducted*** directly from the source to the load *i.e.*, it is transferred **conductively** to the load.



Step up auto-transformer

* In fact, current flowing in the common winding of the auto-transformer is always equal to the difference between the primary and secondary currents of an ordinary transformer.

Uses

As said earlier, auto-transformers are used when K is nearly equal to unity and where there is no objection to electrical connection between primary and secondary. Hence, such transformers are used :

1. to give small boost to a distribution cable to correct the voltage drop.
2. as auto-starter transformers to give upto 50 to 60 % of full voltage to an induction motor during starting.
3. as furnace transformers for getting a convenient supply to suit the furnace winding from a 230-V supply
4. as interconnecting transformers in 132 kV/330 kV system.
5. in control equipment for 1-phase and 3-phase electrical locomotives.

Example 32.87. *An auto-transformer supplies a load of 3 kW at 115 volts at a unity power factor. If the applied primary voltage is 230 volts, calculate the power transferred to the load (a) inductively and (b) conductively.* **(Basic Elect. Machines, Nagpur Univ, 1991)**

Solution. As seen from Art 32.33

Power transferred inductively = Input $(1 - K)$

Power transferred conductively = Input $\times K$

Now, $K = 115/230 = 1/2$, input $\cong$ output = 3 kW

$\therefore$ Inductively transferred power = $3\,(1 - 1/2) =$ **1.5 kW**

Conductivley transferred power = $(1/2) \times 3 =$ **1.5 kW**

Example 32.88. *The primary and secondary voltages of an auto-transformer are 500 V and 400 V respectively. Show with the aid of diagram, the current distribution in the winding when the secondary current is 100 A and calculate the economy of Cu in this particular case.*

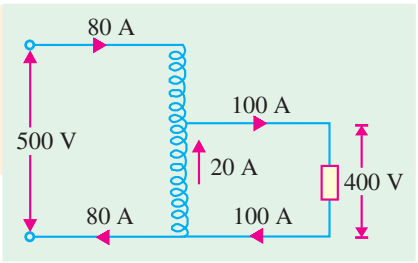


Fig. 32.61

Solution. The circuit is shown in Fig, 30.61.

$K = V_2/V_1 = 400/500 = 0.8$

$\therefore \quad I_1 = KI_2 = 0.8 \times 100 = 80$ A

The current distribution is shown in Fig. 32.61.

Saving = $K\,W_0 = 0.8\,W_0$ - Art 32.33

$\therefore$ Percentage saving = $0.8 \times 100 =$ **80**

Example 32.89. *Determine the core area, the number of turns and the position of the tapping point for a 500-kVA, 50-Hz, single-phase, 6,600/5,000-V auto-transformer, assuming the following approximate values : e.m.f. per turn 8 V. Maximum flux density 1.3 Wb/m^2.*

Solution.

$$E = 4.44\,f\,\Phi_m\,N \text{ volt}$$

$$\Phi_m = \frac{E/N}{4.44\,f} = \frac{8}{4.44 \times 50} = 0.03604 \text{ Wb}$$

Core area = $0.03604/1.3 = 0.0277$ m^2 = **277 cm^2**

Turns of h.v. side = $6600/8 =$ **825** ; Turns of $L.V.$ side = $5000/8 =$ **625**

Hence, tapping should be 200 turns from high voltage end or 625 turns from the common end.

32.34. Conversion of 2-Winding Transformer into Auto-transformer

Any two-winding transformer can be converted into an auto-transformer either step-down or step-up. Fig. 32.62 (*a*) shows such a transformer with its polarity markings. Suppose it is a 20-kVA,

2400/240 V transformer. If we employ ***additive*** polarity between the high-voltage and low-voltage sides, we get a step-up auto-transformer. If, however, we use the ***subtractive*** polarity, we get a step-down auto-transformer.

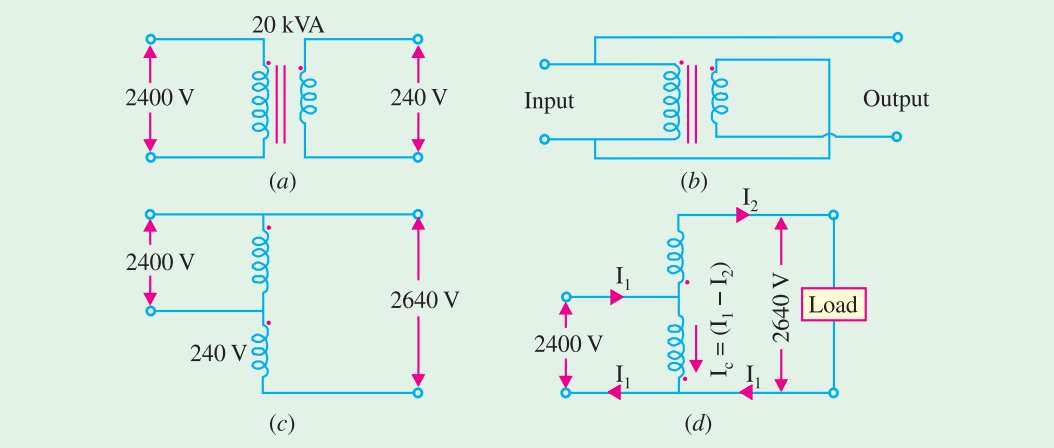


Fig. 32.62

(*a*) Additive Polarity

Connections for such a polarity are shown in Fig. 32.62 (*b*). The circuit is re-drawn in Fig. 32.62 (*c*) showing common terminal of the transformer at the top whereas Fig. 32.62 (*d*) shows the same circuit with common terminal at the bottom. Because of additive polarity, $V_2 = 2400 + 240 = 2640$ V and V_1 is 2400 V. There is a marked increase in the kVA of the auto-transformer (Ex. 32.90). As shown in Fig. 32.62 (*d*), common current flows ***towards*** the common terminal. The transformer acts as a step-up transformer.

(*b*) Subtractive Polarity

Such a connection is shown in Fig. 32.63 (*a*). The circuit has been re-drawn with common polarity at top in Fig. 32.63 (*b*) and at bottom in Fig. 32.63 (*c*). In this case, the transformer acts as a step-down auto-transformer.

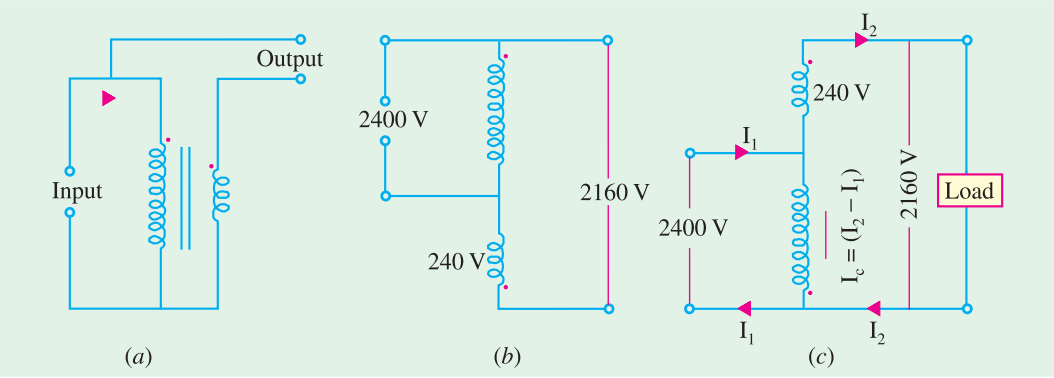


Fig. 32.63

The common current flows ***away*** from the common terminal. As will be shown in Example 32.91, in this case also, there is a very large increase in kVA rating of the auto-transformer though not as marked as in the previous case. Here, $V_2 = 2400 - 240 = 2160$ V.

Example 32.90. *For the 20-kVA, 2400/240-V two-winding step-down transformer shown in Fig. 32.63 (a) connected as an auto-transformer with additive polarity as shown in Fig. 30.61 (d), compute*

(*i*) *original current capacity of HV-windings.*

(*ii*) *original current capacity of LV-windings.*

(*iii*) *k VA rating of auto-transformer using current capacity of current LV winding as calculated in (ii) above.*

(*iv*) *per cent increase in kVA capacity of auto-transformer as compared to original two-winding transformer.*

(*v*) *values of I_1 and I_c in Fig. 30.61 (d) from value of I_2 used in (iii) above.*

(*vi*) *per cent overload of 2400-V winding when used as an auto-transformer.*

(*vii*) *comment on the results obtained.*

Solution. (*i*) $I_1 = 20 \times 10^3/2400 =$ **8.33 A** **(*ii*)** $I_2 = I_1/K = 8.33 \times 10 =$ **83.3 A**

(*iii*) kVA rating of auto-transformer $V_2 I_2 = 2640 \times 83.3 \times 10^{-3} =$ **220 kVA**

(*iv*) Per cent increase in kVA rating $= \frac{220}{20} \times 100 =$ **1100%**

(*v*) $I_1 = 220 \times 10^3/2400 =$ **91.7A**, $I_c = I_1 - I_2 = 91.7 - 83.3 =$ **8.4 A**

(*vi*) Per cent overload of 2400 V winding $= 8.4 \times 100/8.33 =$ **100.8%**

(*vii*) As an auto-transformer, the kVA has increased tremendously to 1100% of its original value with *LV* coil at its rated current capacity and *HV* coil at negligible overload *i.e.* 1.008 × rated load.

Example 32.91. *Repeat Example 30.64 for subtractive polarity as shown in Fig. 32.62 (c).*

Solution. (*i*) $I_1 =$ **8.33 A** (*ii*) $I_2 =$ **83.3 A**

(*iii*) New kVA rating of auto-transformer is $2160 \times 83.3 \times 10^{-3} =$ **180 kVA**

(*iv*) Per cent increase in kVA rating $= \frac{180}{20} \times 100 =$ **900%**

(*v*) $I_1 = 180 \times 10^3/2400 =$ **75 A**, $I_c = I_2 - I_1 = 83.3 - 75 =$ **8.3 A**

(*vi*) Per cent overload of 2400V winding $= 8.3 \times 100/8.33 =$ **100%**

(*vii*) In this case, kVA has increased to 900% of its original value as a two-winding transformer with both low-voltage and high-voltage windings carrying their rated currents.

The above phenomenal increase in kVA capacity is due to the fact that in an auto-transformer energy transfer from primary to secondary is by both conduction as well as induction whereas in a 2-winding transformer it is by induction only. This extra conductive link is mainly responsible for the increase in kVA capacity.

Example 32.92. *A 5-kVA, 110/110-V, single-phase, 50-Hz transformer has full-load efficiency of 95% and an iron loss of 50 W. The transformer is now connected as an auto-transformer to a 220-V supply. If it delivers a 5-kW load at unity power factor to a 110-V circuit, calculate the efficiency of the operation and the current drawn by the high-voltage side.*

(Electric Machinery-II, Banglore Univ. 1991)

Solution. Fig. 32.64 (*a*) shows the normal connection for a 2-winding transformer. In Fig. 32.64 (*b*) the same unit has been connected as an auto transformer. Since the two windings are connected in series, voltage across each is 110 V.

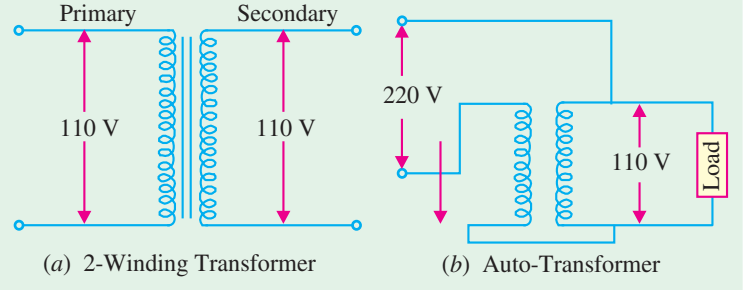


Fig. 32.64

The iron loss would

remain the same in both connections. Since the auto-transformer windings will each carry but half the current as compared to the conventional two-winding transformer, the copper loss will be one-fourth of the previous value.

Two-winding Transformer

$$\eta = 0.95 \qquad \therefore\ 0.95 = \frac{\text{output}}{\text{output + losses}} = \frac{5,000}{5,000 + 50 + \text{Cu loss}}$$

$\therefore$ Cu loss = 212 W

Auto-transformer

Cu loss = 212/4 = 53 W; Iron loss = 50 W $\therefore\ \eta = \dfrac{5,000}{5,000 + 53 + 50} = 0.9797$ or **97.97**%

Current of the h.v. side = 5103/220 = **23.2 A**

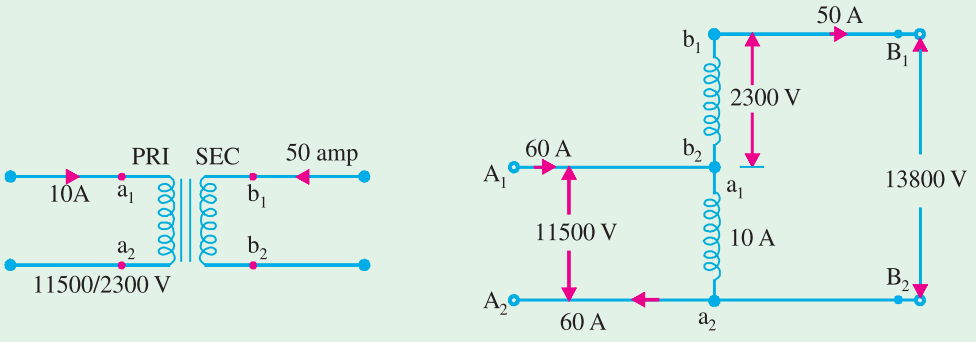
Example 32.93. *A transformer has a primary voltage rating of 11500 volts and secondary voltage rating of 2300 volts. Two windings are connected in series and the primary is connected to a supply of 11500 volts, to act as a step-up auto transformer. Determine the voltage output of the transformer.*

Question extended : If the two winding transformer is rated at 115 kVA, what will be the kVA raitng of the auto-transformer ? **(Madras University, 1997)**

Solution. As in Fig. 32.65 (*a*), 115 kVA, 11500/2300 V, transformer has the current ratings of 10 A and 50 A.

Referring to Fig. 32.65 (*b*), the step-up connections have been shown. Winding currents have to be at the same rated values. As in Fig. 32.65 (*b*), the voltage obtainable at $B_1 - B_2$ is 13800 *V*, and from b_1 a load-current of 50 A can be supplied.

kVA rating = $13800 \times 50 \times 10^{-3} = 690$



(*b*) 2 winding transformer Rating : 115kVA (*b*) Auto transformer

Fig. 32.65

Example 32.94. *An 11500/2300 V transformer is rated at 100 kVA as a 2 winding transformer. If the two windings are connected in series to form an auto-transformer, what will be the possible voltage ratios ?* **(Manonmaniam Sundaranar Univ. April 1998)**

Solution. Fig. 32.66 (*a*) shows this 2-winding transformer with rated winding currents marked .

Rated current of 11.5 kV winding = 100 × 100/11500 = 8.7 Amp

Rated current of 2300 V winding = 43.5 Amp

Fig. 32.66 (*b*) and Fig. 32.66 (*c*) show autotransformer

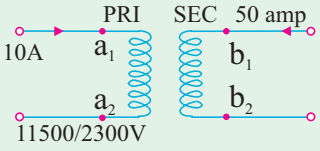


Fig. 32.66 (*a*). 2 winding transformer

connections. On H.V. side, they have a rating of 13.8 kV. On L.V. side, with connection as in Fig. 32.66 (*b*), the rating is 2300 V. On L.V. side of Fig. 32.66 (*c*), the output is at 11.5 kV.

Thus, possible voltage ratios are : 13800/2300 V and 13800/11500 V.

With both the connections, step-up or step-down versions are possible.

Extension of Question : Calculate kVA ratings in the two cases.

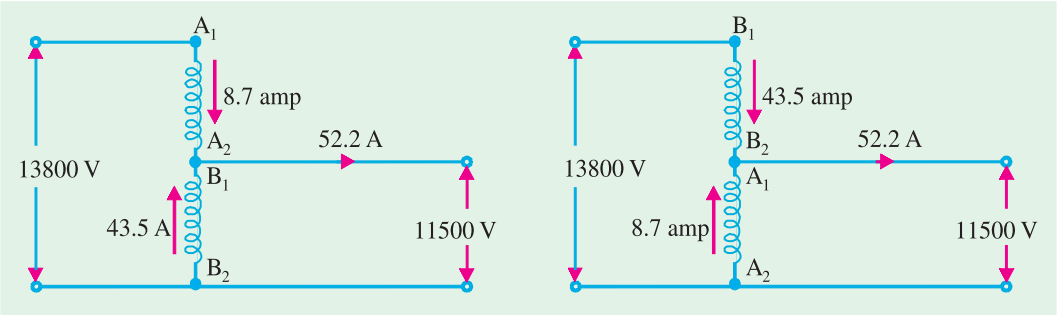


Fig. 32.66

Windings will carry the rated currents, while working out kVA outputs.

In Fig. 32.66 (*b*), Input current (into terminal A_1 of windings A_1-A_2) can be 8.7 Amp with H.V.-side-voltage ratings as 13.8 kV. Transformation ratio = 13800/2300 = 6

Hence, kVA rating = $13.8 \times 8.7 = 120$

Output current = $120 \times 1000/2300 = 52.2$ Amp

Current in the winding B_1-B_2

= Difference of Output current and Input current

$= 52.2 - 8.7 = 43.5$ A, which is the rated current of the winding B_1-B_2.

In Fig. 32.66 (*c*). Similarly, transformation ratio = 13800/11500 = 1.2

kVA rating = $13800 \times 43.5 \times 10^{-3} = 600$

Output current = $600 \times 1000/11500 = 52.2$ Amp

Current carried by common winding = 52.2 – 43.5 = 8.7 A, which is rated current for the winding A_1-A_2. Thus, with the same two windings give, a transformation ratio closer to unity gives higher kVA rating as an auto transformer.

Thus, a 100 kVA two winding transformer is reconnected as an autotransformer of 120 kVA with transformation ratio as 6, and becomes a 600 kVA autotransformer with transformation ratio as 1.2.

Example 32.95. *A two-winding transformer is rated at 2400/240 V, 50-kVA. It is re-connected as a step-up auto-transformer, with 2400 V input. Calculate the rating of the auto-transformer and the inductively and conductively transferred powers while delivering the rated output at unity power-factor.*

(Nagpur University, Winter 1999)

Solution. With 50 kVA as the rating, the rated currents on the two sides are 20.8 A (2400-V side) and 208 A (240 – V side). With the required re-connection, the 2400-V winding will work as a common winding. As shown in Fig. 32.67, the winding common to input and output can carry 20.8 A, the output current can be 208 A with a voltage of 2640 V, which means that the output of this auto

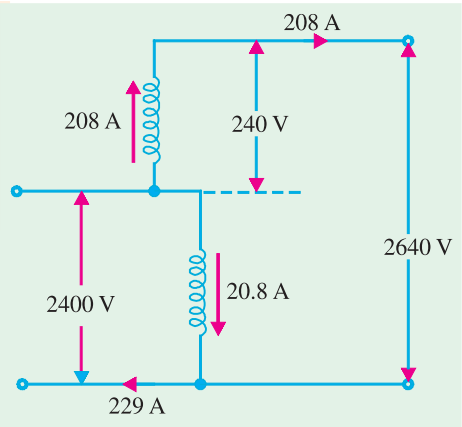


Fig. 32.67

transformer is (2640 × 208) = 550 kVA.

The corresponding input-current is

(208 × 2640/2400) = 229 A.

The ratio of turns in this case is given by

k = 2640/2400 = 1.1

With the step-up job, and $k = 1.1$

$$\frac{\text{Rating of Auto-transformer}}{\text{Rating as two-winding transformer}} = \frac{k}{k \sim 1} = \frac{1.1}{0.1}$$

This gives the rating as auto-transformer of 550 kVA.

At unity power-factor, the rated load = 550 kW.

Out of this, the "***inductively***" transferred power

= Power handled by the common winding

= (2400 V) × (20.8 A) × 10^{-3} = 50 kW.

= Rated output as a two-winding transformer.

Remaining Power = 550 kW – 50 kW = 500 kW.

This power of 500 kW is "***conductively***" transferred as is clear from the division of currents at the input node. Out of the total current of 229 A from the source, 208 A goes straight to the output. The remaining current of 20.8 A is through the *"**common**"* and *"**inductive**"* path, as marked in the Fig. 32.67.

32.35. Parallel Operation of Single-phase Transformers

For supplying a load in excess of the rating of an existing transformer, a second transformer may be connected in parallel with it as shown in Fig. 32.68. It is seen that primary windings are connected to the supply bus bars and secondary windings are connected to the load bus-bars. In connecting two or more than two transformers in parallel, it is essential that their terminals of similar polarities are joined to the same bus-bars as in Fig. 32.68. If this is not done, the two e.m.fs. induced in the secondaries which are paralleled with incorrect polarities, will act together in the local secondary circuit even when supplying no load and will hence produce the equivalent of a dead short-circuit as shown in Fig. 32.69.

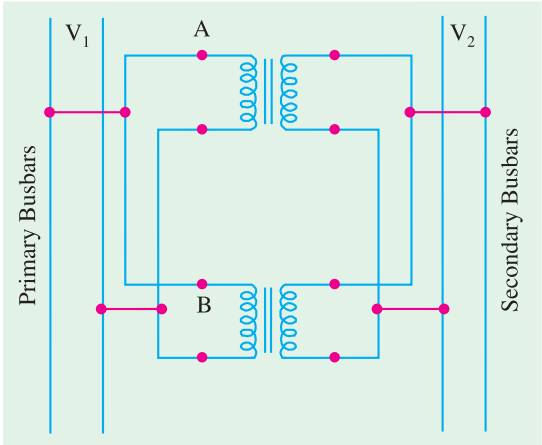


Fig. 32.68

There are certain definite conditions which must be satisfied in order to avoid any local circulating currents and to ensure that the transformers share the common load in proportion to their kVA ratings. The conditions are :

1. Primary windings of the transformers should be suitable for the supply system voltage and frequency.
2. The transformers should be properly connected with regard to polarity.
3. The voltage ratings of both primaries and secondaries should be identical. In other words, the transformers should have the same turn ratio *i.e.* transformation ratio.
4. The percentage impedances should be equal in magnitude and have the same X/R ratio in order to avoid circulating currents and operation at different power factors.

5. With transformers having different kVA ratings, the equivalent impedances should be inversely proportional to the individual kVA rating if circulating currents are to be avoided.

Of these conditions, **(1)** is easily comprehended ; condition **(2)** is absolutely essential (otherwise paralleling with incorrect polarities will result in dead short-circuit). There is some lattitude possible with conditions **(3)** and **(4)**. If condition **(3)** is not exactly satisfied *i.e.* the two transformers have slightly different transformation or voltage ratios, even then parallel operation is possible. But due to inequality of induced e.m.fs. in secondaries, there will be even on no-load, some circulating current between them (and therefore between the primary windings also) when secondary terminals are connected in parallel. When secondaries are loaded, this localized circulating current will tend to produce unequal loading condition. Hence, it may be impossible to take full kVA output from the parallel connected group without one of the transformers becoming over-heated.

If condition **(4)** is not exactly satisfied *i.e.* impedance triangles are not identical in shape and size, parallel operation will still be possible, but the power factors at which the two transformers operate will be different from the power factor of the common load. Therefore, in this case, the two transformers will not share the load in proportion to their kVA ratings.

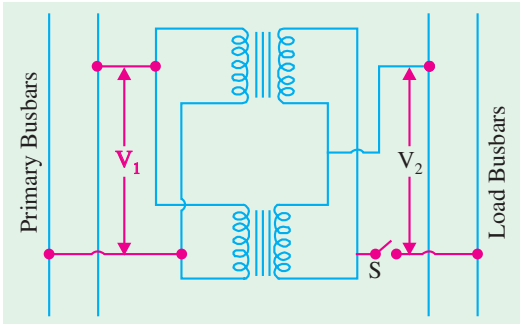


Fig. 32.69

It should be noted that the impedances of two transformers may differ in magnitude and in quality (*i.e.* ratio of equivalent resistance to reactance). It is worthwhile to distinguish between the ***percentage*** and ***numerical*** value of an impedance. For example, consider two transformers having ratings in the ratio 1 : 2. It is obvious that to carry double the current, the latter must have half the impedance of the former for the same regulation. For parallel operation, the regulation must be the same, this condition being enforced by the very fact of their being connected in parallel. It means that the currents carried by the two transformers are proportional to their ratings provided their ***numerical*** impedances are inversely proportional to these ratings and their ***percentage*** impedances are identical.

If the ***quality*** of the two percentage impedances is different (*i.e.* ratio of percentage resistance to reactance is different), then this will result in divergence of phase angle of the two currents, with the result that one transformer will be operating with a higher and the other with a lower power factor than that of the combined load.

(*a*) Case 1. Ideal Case

We will first consider the ideal case of two transformers having the same voltage ratio and having impedance voltage triangles identical in size and shape.

Let E be the no-load secondary voltage of each transformer and V_2 the terminal voltage ; I_A and I_B the currents supplied by them and I-the total current, lagging behind V_2 by an angle ϕ (Fig. 32.70(*a*)

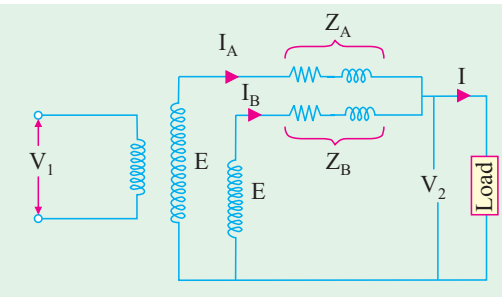


Fig. 32.70 (a)

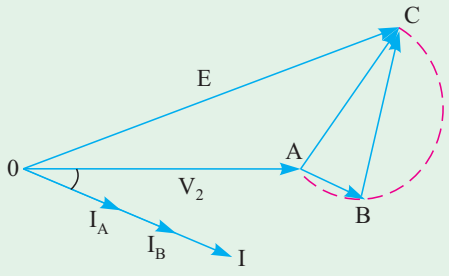


Fig. 32.70 (b)

In Fig. 32.70 (*b*) a single triangle *ABC* represents the identical impedance voltage triangles of both the transformers. The currents I_A and I_B of the individual transformers are in phase with the load current *I* and are inversely proportional to the respective impedances. Following relations are obvious.

$$\mathbf{I} = \mathbf{I}_A + \mathbf{I}_B;\ \mathbf{V}_2 = \mathbf{E} - \mathbf{I}_A\mathbf{Z}_A = \mathbf{E} - \mathbf{I}_B\mathbf{Z}_B = \mathbf{E} - \mathbf{I}\mathbf{Z}_{AB}$$

Also $$\mathbf{I}_A\mathbf{Z}_A = \mathbf{I}_B\mathbf{Z}_B \text{ or } \mathbf{I}_A/\mathbf{I}_B = \mathbf{Z}_B/\mathbf{Z}_A$$

$\therefore$ $$\mathbf{I}_A = \mathbf{I}\mathbf{Z}_B/(\mathbf{Z}_A + \mathbf{Z}_B) \text{ and } \mathbf{I}_B = \mathbf{I}\mathbf{Z}_A/(\mathbf{Z}_A + \mathbf{Z}_B)$$

(*b*) Case 2. Equal Voltage Ratios

Let us assume that no-load voltages of both secondaries is the same *i.e.* $E_A = E_B = E$, and that the two voltages are coincident *i.e.* there is no phase difference between E_A *and* E_B, which would be true if the magnetising currents of the two transformers are not much different from each other. Under these conditions, both primaries and secondaries of the two transformers can be connected in parallel and there will circulate no current between them on ***on-load***.

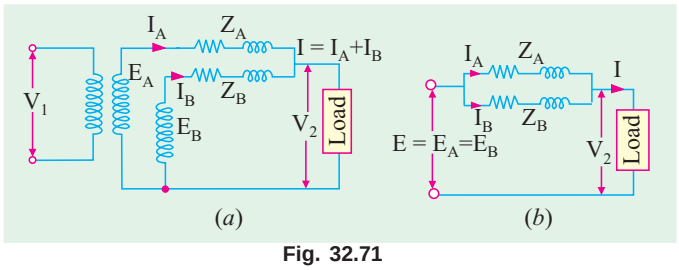


Fig. 32.71

However, if we neglect the magnetising admittances, the two transformers can be connected as shown by their equivalent circuits in Fig. 32.71. The vector diagram is shown in Fig. 32.72.

From Fig. 32.71 (*a*) or (*b*) it is seen that it represents two impedances in parallel. Considering all values ***consistently with reference to secondaries***, let

$\mathbf{Z}_A, \mathbf{Z}_B$ = impedances of the transformers

$\mathbf{I}_A, \mathbf{I}_B$ = their respective currents

$\mathbf{V}_2$ = common terminal voltage

$\mathbf{I}$ = combined current

It is seen that $\mathbf{I}_A\mathbf{Z}_A = \mathbf{I}_B\mathbf{Z}_B = \mathbf{I}\mathbf{Z}_{AB}$ ...(*i*)

where $\mathbf{Z}_{AB}$ is the combined impedance of $\mathbf{Z}_A$ and $\mathbf{Z}_B$ in parallel.

$$1/\mathbf{Z}_{AB} = 1/\mathbf{Z}_A + 1/\mathbf{Z}_B$$

Hence $$\mathbf{Z}_{AB} = \mathbf{Z}_A\mathbf{Z}_B/(\mathbf{Z}_A + \mathbf{Z}_B) \quad ...(ii)$$

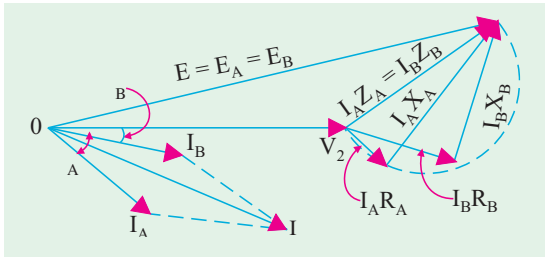


Fig. 32.72

From equation (*i*), we get

$$\mathbf{Z}_A = \mathbf{I}\mathbf{Z}_{AB}\mathbf{Z}_A = \mathbf{I}\mathbf{Z}_B/(\mathbf{Z}_A + \mathbf{Z}_B) \text{ and } \mathbf{I}_B = \mathbf{I}\mathbf{Z}_{AB}/\mathbf{Z}_B = \mathbf{I}\mathbf{Z}_A/(\mathbf{Z}_A + \mathbf{Z}_B)$$

Multiplying both sides by common terminal voltage $\mathbf{V}_2$, we have

$$\mathbf{V}_2\mathbf{I}_A = \mathbf{V}_2\mathbf{I}\frac{\mathbf{Z}_B}{\mathbf{Z}_A + \mathbf{Z}_B}\ ;\ \text{Similarly } \mathbf{V}_2\mathbf{I}_B = \mathbf{V}_2\mathbf{I}\frac{\mathbf{Z}_A}{\mathbf{Z}_A + \mathbf{Z}_B}$$

Let $\mathbf{V}_2\mathbf{I} \times 10^{-3} = \mathbf{S}$-the combined load kVA. Then, the kVA carried by each transformer is

$$\mathbf{S}_A = \mathbf{S}\frac{\mathbf{Z}_B}{\mathbf{Z}_A + \mathbf{Z}_B} = \mathbf{S}\frac{1}{1 + \mathbf{Z}_A/\mathbf{Z}_B} \quad \text{and} \quad \mathbf{S}_B = \mathbf{S}\frac{\mathbf{Z}_A}{\mathbf{Z}_A + \mathbf{Z}_B} = \mathbf{S}\frac{1}{1 + \mathbf{Z}_B/\mathbf{Z}_A} \quad ...(iii)$$

Hence, $\mathbf{S}_A$ and $\mathbf{S}_B$ are obtained in magnitude as well as in phase from the above vectorial expressions.

The above problem may be solved graphically, although somewhat more laboriously. As shown in Fig. 32.72, drawn I_A and I_B with an angular difference of $(\phi_A - \phi_B)$ and magnitude (according to some suitable scale) inversely proportional to the respective impedances. Vector sum of I_A and I_B gives total combined current *I*. The phase angle and magnitude of *I* will be known from the conditions of loading, so that angle ϕ

between V_2 and I will be known. Inserting this, the transformer currents I_A and I_B become known in magnitude and phase with respect to V_2.

Note. **(*a*)** In equation (*iii*) above, it is not necessary to use the ohmic values of resistances and reactances, because only impedance ratios are required.

(*b*) The two percentage impedances must be adjusted to the same kVA in the case of transformers of different rating as in Ex. 32.101.

(*c*) From equation (*iii*) above, it is seen that if two transformers having the same rating and the same transformation ratio are to share the load equally, then their impedances should be equal *i.e.* equal resistances and reactances and not numerical equality of impedances. In general, for transformers of different ratings but same transformation ratio, their equivalent impedances must be inversely proportional to their ratings if each transformer is to assume a load in proportion to its rating. For example, as said earlier, a transformer operating in parallel with another of twice the rating, must have an impedance twice that of the large transformer in order that the load may be properly shared between them.

Example 32.96. *Two 1-phase transformers with equal turns have impedances of (0.5 + j3) ohm and (0.6 + j10) ohm with respect to the secondary. If they operate in parallel, determine how they will share a total load of 100 kW at p.f. 0.8 lagging ?* **(Electrical Technology, Madras Univ. 1987)**

Solution. $\mathbf{Z}_A = 0.5 + j\,3 = 3.04 \angle 80.6°$ $\quad \mathbf{Z}_B = 0.6 + j\,10 = 10.02 \angle 86.6°$

$$\mathbf{Z}_A + \mathbf{Z}_B = 1.1 + j13 = 13.05 \angle 85.2°$$

Now, a load of 100 kW at 0.8 p.f. means a kVA of 100/0.8 = 125. Hence,

$$\mathbf{S} = 125 \angle -36.9°$$

$$\mathbf{S}_A = \mathbf{S}\frac{\mathbf{Z}_A}{\mathbf{Z}_A + \mathbf{Z}_B} = \frac{125 \angle -36.9° \times 10.02 \angle 86.6°}{13.05 \angle 85.2°} = 96 \angle -35.5°$$

$$= \text{a load of } 96 \times \cos 35.5° = 78.2 \text{ kW}$$

$$\mathbf{S}_B = \mathbf{S}\frac{\mathbf{Z}_A}{\mathbf{Z}_A + \mathbf{Z}_B} = \frac{125 \angle -36.9° \times 3.04 \angle 80.6°}{13.05 \angle 85.2°} = 29.1 \angle -41.5°$$

$$= \text{a load of } 29.1 \times \cos 41.5° = \mathbf{21.8\ kW}$$

Note. Obviously, transformer *A* is carrying more than its due share of the common load.

Example 32.97. *Two single-phase transformers A and B are connected in parallel. They have same kVA ratings but their resistances are respectively 0.005 and 0.01 per unit and their leakage reactances 0.05 and 0.04 per unit. If A is operated on full-load at a p.f. of 0.8 lagging, what will be the load and p.f. of B ?* **(A.C. Machines-I, Jadavpur Univ. 1985)**

Solution. In general, $\mathbf{S}_A = \mathbf{S}\frac{\mathbf{Z}_B}{\mathbf{Z}_A + \mathbf{Z}_B}$ and $\mathbf{S}_B = \mathbf{S}\frac{\mathbf{Z}_A}{\mathbf{Z}_A + \mathbf{Z}_B}$

where S is the total kVA supplied and $\mathbf{Z}_A$ and $\mathbf{Z}_B$ are the percentage impedances of these transformers.

$$\frac{\mathbf{S}_B}{\mathbf{S}_A} = \frac{\mathbf{Z}_A}{\mathbf{Z}_B}$$

Now, $\quad \mathbf{Z}_A = 0.005 + j\,0.05$ per unit ; % $\mathbf{Z}_A = 0.5 + j\,5$ and % $\mathbf{Z}_B = 1 + j\,4$

Let $\quad \mathbf{S}_A = S_A \angle -36.87°$

where S_A represents the rating of transformer *A* (and also of *B*).

$$\mathbf{S}_B = \mathbf{S}_A \angle -36.87° \times \frac{(0.5 + j5)}{(1 + j4)}$$

$$= S_A \angle -36.87° \times \frac{20.7}{17} \angle 8.3° = 1.22\, S_A \angle -28.57°$$

It is obvious that transformer *B* is working **22%** over-load and its power factor is

$$\cos 28.57° = \textbf{0.878 (lag)}$$

Example 32.98. *Two 1-phase transformers A and B rated at 250 kVA each are operated in parallel on both sides. Percentage impedances for A and B are (1 + j 6) and (1.2 + j 4.8) respectively. Compute the load shared by each when the total load is 500 kVA at 0.8 p.f. lagging.*

(Electrical Machines-II, Indore Univ. 1989)

Solution.

$$\frac{\mathbf{Z}_A}{\mathbf{Z}_A+\mathbf{Z}_B} = \frac{1+j\,6}{2.2+j\,10.8} = 0.55\angle 2.1° \; ; \; \frac{\mathbf{Z}_B}{\mathbf{Z}_A+\mathbf{Z}_B} = \frac{1.2+j\,4.8}{2.2+j\,10.8} = 0.45\angle -2.5°$$

$$\mathbf{S}_A = \mathbf{S}\frac{\mathbf{Z}_B}{\mathbf{Z}_A+\mathbf{Z}_B} = 500\angle -36.9° \times 0.45\angle -2.5° = \textbf{225}\angle \textbf{-39.4°}$$

$$\mathbf{S}_B = \mathbf{S}\frac{\mathbf{Z}_A}{\mathbf{Z}_A+\mathbf{Z}_B} = 500\angle -36.9° \times 0.55\angle 2.1° = \textbf{275}\angle \textbf{-34.8°}$$

Obviously, transformer *B* is overloaded to the extent of $(275 - 250) \times 100/250 = 10\%$. It carries $(275/500) \times 100 = 55\%$ of the total load.

Example 32.99. *Two 100-kW, single-phase transformers are connected in parallel both on the primary and secondary. One transformer has an ohmic drop of 0.5% at full-load and an inductive drop of 8% at full-load current. The other has an ohmic drop of 0.75% and inductive drop of 2%. Show how will they share a load of 180 kW at 0.9 power factor.*

(Elect. Machines-I, Calcutta Univ. 1988)

Solution. A load of 180 kW at 0.9 p.f. means a kVA of 180/0.9 = 200

$\therefore$ Load $\mathbf{S} = 200 - 25.8°$

$$\frac{\mathbf{Z}_1}{\mathbf{Z}_1+\mathbf{Z}_2} = \frac{(0.5+j\,8)}{(1.25+j\,12)} = \frac{(0.5+j\,8)(1.25-j\,12)}{1.25^2+12^2}$$

$$= \frac{96.63+j\,4}{145.6} = \frac{96.65\angle 2.4°}{145.6} = 0.664\angle 2.4°$$

$$\frac{\mathbf{Z}_2}{\mathbf{Z}_1+\mathbf{Z}_2} = \frac{(0.75+j\,4)(1.25-j\,12)}{145.6}$$

$$= \frac{48.94-j\,4}{145.6} = \frac{49.1\angle -5°}{145.6}$$

$$= 0.337\angle -5°$$

$$\mathbf{S}_1 = \mathbf{S}\frac{\mathbf{Z}_2}{\mathbf{Z}_1+\mathbf{Z}_2} = 200\angle -25.8° \times 0.337\angle -5° = 67.4\angle -30.8°$$

$\therefore$ $kW_1 = 67.4 \times \cos 30.8° = 67.4 \times 0.859 = 57.9$ kW

$\mathbf{S}_2 = 200\angle -25.8° \times 0.664\angle 2.4° = 132.8\angle -23.4$

$kW_2 = 132.8 \times \cos 23.4° = 132.8 \times 0.915 = 121.5$ kW

Note. Second transformer is working 21.5% over-load. Also, it shares 65.7% of the total load.

Example 32.100. *A load of 200 kW at 0.85 power factor lagging is to be shared by two transformers A and B having the same ratings and the same transformation ratio. For transformer A, the full-load resistive drop is 1% and reactance drop 5% of the normal terminal voltage. For transformer B the corresponding values are : 2% and 6%. Calculate the load kVA supplied by each transformer.*

Solution. $\mathbf{Z}_A = 1 + j5; Z_B = Z + j\,6$

$$\frac{\mathbf{Z_A}}{\mathbf{Z_B}} = \frac{1 + j\,5}{2 + j\,6} = 0.8 + j\,0.1\ ;\quad \frac{\mathbf{Z_B}}{\mathbf{Z_A}} = \frac{2 + j\,6}{1 + j\,5} = 1.23 - j\,0.514$$

Load kVA = kW/p.f. = 200/0.85 = 235

kVA = S = 235(0.85 – *j* 0.527)

= 200 – *j* 123.8 ($\because \cos\phi = 0.85; \sin\phi = 0.527$)

$$\mathbf{S_A} = \mathbf{S}\frac{\mathbf{Z_B}}{\mathbf{Z_A + Z_B}} = \mathbf{S}\frac{\mathbf{1}}{\mathbf{1 + (Z_A/Z_B)}}$$

$$= \frac{200 - j\,123.8}{1 + (0.8 + j0.1)} = \frac{200 - j\,123.8}{1.8 + j0.1} = 107.3 - j\,74.9$$

$$\mathbf{S_A} = \sqrt{(107.3^2 + 74.9^2)} = 131;\ \cos\phi_A = 107.3/131 = 0.82\ (\text{lag})$$

Similarly $$\mathbf{S_B} = \mathbf{S}\frac{\mathbf{Z_A}}{\mathbf{Z_A + Z_B}} = \mathbf{S}\frac{\mathbf{1}}{\mathbf{1 + (Z_B/Z_A)}}$$

$$= \frac{200 - j\,123.8}{1 + (1.23 - j0.514)} = \frac{200 - j\,123.8}{2.23 - j\,0.154} = 93 - j\,49$$

$\therefore$ $$\mathbf{S_B} = \sqrt{(93^2 + 49^2)} = 105;\ \cos\phi_B = 90/105 = 0.888\ (\text{lag})$$

(As a check, $S = S_A + S_B = 131 + 105 = 236$. The small error is due to approximations made in calculations.)

Example 32.101. *Two 2,200/110-V, transformers are operated in parallel to share a load of 125 kVA at 0.8 power factor lagging. Transformers are rated as below :*

A : 100 kVA ; 0.9% resistance and 10% reactance

B : 50 kVA ; 1.0% resistance and 5% reactance

Find the load carried by each transformer. **(Elect. Technology, Utkal Univ. 1989)**

Solution. It should be noted that the percentages given above refer to different ratings. As pointed out in Art. 32.35, these should be adjusted to the same basic kVA, say, 100 kVA.

$$\%\,\mathbf{Z_A} = 0.9 + j\,10\ ;\quad \%\,\mathbf{Z_B} = (100/50)(1 + j\,5) = (2 + j\,10)$$

$$\frac{\mathbf{Z_A}}{\mathbf{Z_A + Z_B}} = \frac{0.9 + j\,10}{(2.9 + j20)} = \frac{(0.9 + j\,10)\,(2.9 - j\,20)}{2.9^2 + 20^2}$$

$$= \frac{(202.6 + j\,11)}{408.4} = \frac{202.9\angle 3.1°}{408.4} = 0.4968\angle 3.1°$$

$$\frac{\mathbf{Z_B}}{\mathbf{Z_A + Z_B}} = \frac{(2 + j\,10)(2.9 - j\,20)}{408.4} = \frac{206 - j\,11}{408.4}$$

$$= \frac{206.1\angle -3.1°}{408.4} = 0.504\angle -3.1°$$

Also $\cos\phi = 0.8, \phi = \cos^{-1}(0.8) = 36.9°$

$$\mathbf{S_A} = \mathbf{S}\frac{\mathbf{Z_B}}{\mathbf{Z_A + Z_B}} = 125\angle -36.9° \times 0.504\angle -3.1° = 63\angle -40°$$

$$\mathbf{S_B} = \mathbf{S}\frac{\mathbf{Z_A}}{\mathbf{Z_A + Z_B}} = 125\angle -36.9° \times 0.4968\angle 3.1° = 62.1\angle -33.8°$$

Example 32.102. *A 500-kVA transformer with 1% resistance and 5% reactance is connected in parallel with a 250-kVA transformer with 1.5% resistance and 4% reactance. The secondary voltage of each transformer is 400 V on no-load. Find how they share a load of 750-kVA at a p.f. of 0.8 lagging.* **(Electrical Machinery-I, Madras Univ. 1987)**

Solution. It may be noted that percentage drops given above refer to different ratings. These should be adjusted to the same basic kVA *i.e.* 500 kVA.

$$\%\,\mathbf{Z_A} = 1 + j\,5 = 5.1 \angle 78.7°\,;\ \%\,\mathbf{Z_B} = \left(\frac{500}{250}\right)(1.5 + j\,4) = 3 + j\,8 = 8.55 \angle 69.4°$$

$$\%(\mathbf{Z_A} + \mathbf{Z_B}) = 4 + j\,13 = 13.6 \angle 72.9°\,;\ \mathbf{S} = 750 \angle -36.9°$$

$$\mathbf{S_A} = \mathbf{S}\frac{\mathbf{Z_B}}{\mathbf{Z_A} + \mathbf{Z_B}} = \frac{750 \angle -36.9° \times 8.55 \angle 69.4°}{13.6 \angle 72.9°} = 470 \angle -40.4°$$

$$= 470 \text{ kVA at p.f. of } 0.762 \text{ lagging}$$

$$\mathbf{S_B} = \mathbf{S}\frac{\mathbf{Z_A}}{\mathbf{Z_A} + \mathbf{Z_B}} = \frac{750 \angle -36.9° \times 5.1 \angle 78.7°}{13.6 \angle 72.9°} = 280 \angle -31.1°$$

$$= 280 \text{ kVA at p.f. } 0.856 \text{ lagging}$$

Note. The above solution has been attempted vectorially, but in practice, the angle between I_A and I_B is so small that if instead of using vectorial expressions, arithmetic expressions were used, the answer would not be much different. In most cases, calculations by both vectorial and arithmetical methods generally yield results that do not differ sufficiently to warrant the more involved procedure by the vector solution. The above example will now be attempted arithmetically :

$$Z_A = 5.1\,\Omega, Z_B = 8.55\,\Omega\,;\ I_A/I_B = Z_B/Z_A = 8.55/5.1 = 1.677 \therefore I_A = 1.677\,I_B$$

$$\text{Total current} = 750{,}000/400 = 1875 \text{ A}$$

$$I = I_A + I_B\,;\ 1875 = 1.677 I_B + I_B = 2.677\,I_B \therefore I_B = 1875/2.677$$

$$\therefore \quad S_B = 400 \times 1875/2.677 \times 100 = 280 \text{ kVA}$$

$$I_A = 1.677 \times 1875/2.677$$

$$S_A = 400 \times 1.677 \times 1875/2.677 \times 1000 = 470 \text{ kVA}$$

Example 32.103. *Two single-phase transformers A and B of equal voltage ratio are running in parallel and supply a load of 1000 A at 0.8 p.f. lag. The equivalent impedances of the two transformers are (2 + j3) and (2.5 + j5) ohms respectively. Calculate the current supplied by each transformer and the ratio of the kW output of the two transformers.*

(Electrical Machines-I, Bombay Univ. 1986)

Solution. $\mathbf{Z_A} = (2 + j\,3),\ \mathbf{Z_B} = (2.5 + j\,5)$

Now, $\dfrac{\mathbf{I_A}}{\mathbf{I_B}} = \dfrac{\mathbf{Z_B}}{\mathbf{Z_A}} = \dfrac{2.5 + j\,5}{2 + j\,3} = (1.54 + j\,0.2)\,;\ \mathbf{I_A} = \mathbf{I_B}(1.54 + j\,0.2)$

Taking secondary terminal voltage as reference vector, we get

$$\mathbf{I} = 1000(0.8 - j\,0.6) = 800 - j\,600 = 200\,(4 - j\,3)$$

Also, $\mathbf{I} = \mathbf{I_A} + \mathbf{I_B} = \mathbf{I_B}(1.54 + j\,0.2) + \mathbf{I_B} = \mathbf{I_B}(2.54 + j\,0.2)$

$\therefore$ $200(4 - j3) = \mathbf{I_B}(2.54 + j\,0.2)\,;\ \mathbf{I_B} = 294.6 - j\,259.5 = \mathbf{392.6 \angle -41.37°}$

$$\mathbf{I_A} = \mathbf{I_B}(1.54 + j\,0.2) = (294.6 - j\,259.5)(1.54 + j\,0.2) = 505.6 - j\,340.7$$

$$= \mathbf{609.7 \angle -33.95°}$$

The ratio of the kW output is given by the ratio of the in-phase components of the two currents.

$$\frac{\text{output of } A}{\text{output of } B} = \frac{505.6}{294.6} = \frac{1.7}{1}$$

Note. Arithmetic solution mentioned above could also be attempted.

Example 32.104. *Two transformers A and B, both of no-load ratio 1,000/500-V are connected in parallel and supplied at 1,000 V. A is rated at 100 kVA, its total resistance and reactance being 1% and 5% respectively, B is rated at 250 kVA, with 2% resistance and 2% reactance. Determine the load on each transformer and the secondary voltage when a total load of 300 kVA at 0.8 power factor lagging is supplied.*

Solution. Let the percentage impedances be adjusted to the common basic kVA of 100.

Then $\%\mathbf{Z_A} = (1 + j\,5)\ ;\ \%\mathbf{Z_B} = (100/250)(2 + j\,2) = (0.8 + j\,0.8)$

$$\frac{\mathbf{Z_A}}{\mathbf{Z_A} + \mathbf{Z_B}} = \frac{1 + j5}{(1.8 + j5.8)} = 0.839 \angle 5.9$$

$$\frac{\mathbf{Z_B}}{\mathbf{Z_A} + \mathbf{Z_B}} = \frac{0.8 + j0.8}{(1.8 + j5.8)} = 0.1865 \angle -27.6°$$

Now, $\mathbf{S} = 300 \angle -36.9° = 240 - j\,180$

$$\mathbf{S_A} = \mathbf{S}.\frac{\mathbf{Z_B}}{\mathbf{Z_A} + \mathbf{Z_B}} = 300 \angle -36.9° \times 0.1865 \angle -27.6° = 55.95 \angle -64.5°$$

$$\mathbf{S_B} = \mathbf{S}\frac{\mathbf{Z_A}}{\mathbf{Z_A} + \mathbf{Z_B}} = 300 \angle -36.9° \times 0.839 \angle 5.9° = 251.7 \angle -31°$$

Since $\mathbf{Z_A}$ and $\mathbf{Z_B}$ are in parallel, their combined impedance on 100 kVA basis is

$$\mathbf{Z_{AB}} = \frac{\mathbf{Z_A}\,\mathbf{Z_B}}{\mathbf{Z_A} + \mathbf{Z_B}} = \frac{(1 + j5)(0.8 + j0.8)}{(1.8 + j5.8)} = 0.6 + j\,0.738$$

Percentage drop over $\mathbf{Z_{AB}}$ is $= (0.6 \times 240/100) + (0.738 \times 180/100) = 1.44 + 1.328 = 2.768\%$

$$\therefore \quad V_2 = \left(500 - \frac{500 \times 2.768}{100}\right) = \mathbf{486.12\ V}$$

Example 32.105. *Two 1-ϕ transformers are connected in parallel at no-load. One has a turn ratio of 5,000/440 and a rating of 200 kVA, the other has a ratio of 5,000/480 and a rating of 350 kVA. The leakage reactance of each is 3.5%.*

What is the no-load circulation current expressed as a percentage of the nominal current of the 200 kVA transformer.

Solution. The normal currents are

$$200 \times 10^3/440 = 455\ \text{A and}\ 350 \times 10^3/480 = 730\ \text{A}$$

Reactances seen from the secondary side are

$$\frac{3.5}{100} \times \frac{440}{455} = 0.034\,\Omega, \quad \frac{3.5}{100} \times \frac{480}{730} = 0.023\,\Omega$$

The difference of induced voltage is 40 V. The circulating current is

$I_C = 40/0.057 = 704$ A = **1.55 times** the normal current of 200 kVA unit.

Tutorial Problems 32.6

1. Two single-phase transformers *A* and *B* of equal voltage ratio are running in parallel and supplying a load requiring 500 A at 0.8 power factor lagging at a terminal voltage of 400 V. The equivalent impedances of the transformers, as referred to secondary windings, are $(2 + j\,3)$ and $(2.5 + j\,5)$ ohm. Calculate the current supplied by each transformer.

   (Note. The student is advised to try by arithmetic method also). **[I_A = 304 A ; I_B = 197 A]**

2. Two single-phase transformers *A* and *B* are operating in parallel and supplying a common load of 1000 kVA at 0.8 p.f. lagging. The data regarding the transformers is as follows :

Transformer	Rating	%Resistance	% Reactance
A	750 kVA	3	5
B	500 kVA	2	4

Determine the loading of each transformer. **[S_A = 535 ∠– 34.7° ; S_B = 465 ∠– 39.3°]**

3. Two transformers *A* and *B* give the following test results. With the low-tension side short-circuited, *A* takes a current of 10 A at 200 V, the power input being 1000 W. Similarly, *B* takes 30 A at 200 V ; the power input being 1,500 W. On open circuit, both transformers give a secondary voltage of 2200 when 11,000 volts are applied to the primary terminals. These transformers are connected in parallel on both high tension and low tension sides Calculate the current and power in each transformer when supplying a load of 200 A at 0.8 power factor lagging. The no-load currents may be neglected.

(Hint : Calculate Z_A and Z_B from S.C. test data**)**

[I_A = 50.5 A, P_A = 100 kW ; I_B = 151 A, P_B = 252 kW] (*London University*)

4. Two 6600/250-V transformers have the following short-circuit characteristics : Applied voltage 200 V, current 30 A, power input 1,200 W for one of the transformers ; the corresponding data for the other transformer being 120 V, 20 A and 1,500 W. All values are measured on the H.V. side with the L.V. terminals short circuited. Find the approximate current and the power factor of each transformer when working in parallel with each other on the high and low voltage sides and taking a total load of 150 kW at a p.f. of 0.8 lagging from the high voltage bus-bars.

[I_A = 13.8 A, cos ϕA = 0.63 ; I_B = 15.35 A , cos ϕB = 0.91] (*Electrical Engg-IV, Baroda Univ. 1978*)

5. Two 11,000/2,200-V, 1-phase transformers are connected in parallel to supply a total load of 200 at 0.8 p.f. lagging at 2,200 V. One transformer has an equivalent resistance of 0.4 Ω and equivalent reactance of 0.8 Ω referred to the low-voltage side. The other has equivalent resistance of 0.1 Ω and a reactance of 0.3 Ω. Determine the current and power supplied by each transformer.

[52 A ; 148 A ; 99 A ; 252 kW]

6. A 2,000-kVA transformer (*A*) is connected in parallel with a 4,000-kVA transformer (B) to supply a 3-phase load of 5,000 kVA at 0.8 p.f. lagging. Determine the kVA supplied by each transformer assuming equal no-load voltages. The percentage volt drops in the windings at the rated loads are as follows :

Transformer *A* : resistance 2% ; reactance 8%

Transformer *B* : resistance 1.6% ; reactance 3%

[A : 860 kVA, 0.661 lag ; B : 4170 kVA, 0.824 lag] (*A.C. Machines-I, Jadavpur Univ. 1979*)

7. Two single-phase transformers work in parallel on a load of 750 A at 0.8 p.f. lagging. Determine secondary voltage and the output and power factor of each transformer. Test data are :

Open circuit : 11,00/13,300 V for each transformer

Short circuit : with h.v. winding short-circuit

Transformer *A* : secondary input 200 V, 400 A, 15 kW

Transformer *B* : secondary input 100 V, 400 A, 20 kW

[3,190 VA : 80 kVA, 0.65 lag ; B : 1,615 kVA ; 0.86 lag]

(*c*) Case 3. Unequal Voltage Ratios

In this case, the voltage ratios (or transformation ratios) of the two transformers are different. It means that their no-load secondary voltages are unequal. Such cases can be more easily handled by phasor algebra than graphically.

Let E_A, E_B = no-load secondary e.m.f.s of the two transformers.

Z_L = load impedance across the secondary.

The equivalent circuit and vector diagram are also shown in Fig. 32.73 and 32.74.

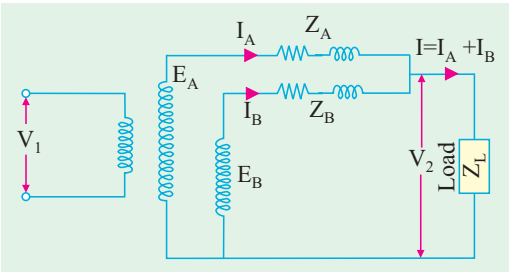


Fig. 32.73

It is seen that even when secondaries are on no-load, there will be some cross-current in them because of inequality in their induced e.m.fs. This circulating current I_C is given by

$$I_C = (E_A - E_B)/(Z_A + Z_B) \quad ...(i)$$

As the induced e.m.fs. of the two transformers are equal to the total drops in their respective circuits.

$$\therefore \quad Z_A = I_A Z_A + V_2 \; ; \; E_B = I_B Z_B + V_2$$

Now, $V_2 = IZ_L = (I_A + I_B) Z_L$

where Z_L = load impedance

$$E_A = I_A Z_A + (I_A + I_B) Z_L \quad ...(ii)$$

$$E_B = I_B Z_B + (I_A + I_B) Z_L \quad ...(iii)$$

$$\therefore \quad E_A - E_B = I_A Z_A - I_B Z_B \quad ...(iv)$$

$$I_A = [(E_A - E_B) + I_B Z_B]/Z_A$$

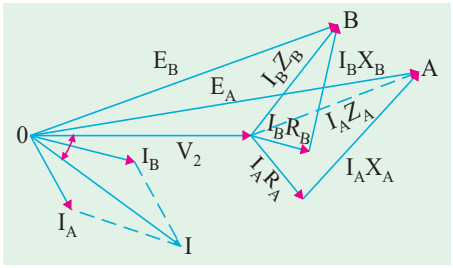


Fig. 32.74

Substituting this value of I_A in equation (*iii*), we get

$$E_B = I_B Z_B + [\{(E_A - E_B) + I_B Z_B\}/Z_A + I_B]/Z_L$$

$$I_B = [E_B Z_A - (E_A - E_B) Z_L]/[Z_A Z_B + Z_L.(Z_A + Z_B)] \quad ...(v)$$

From the symmetry of the expression, we get

$$I_A = [E_A Z_B + (E_A - E_B) Z_L [Z_A Z_B + Z_L (Z_A + Z_B)] \quad ...(vi)$$

Also, $$I = I_A + I_B = \frac{E_A Z_B + E_B Z_A}{Z_A Z_B + Z_L (Z_A + Z_B)}$$

By multiplying the numerator and denominator of this equation by $1/Z_A Z_B$ and the result by Z_L we get

$$V_2 = IZ_L = \frac{E_A/Z_A + E_B Z_B}{1/Z_A + 1/Z_B + 1/Z_L}$$

The two equations (*v*) and (*vi*) then give the values of secondary currents. The primary currents may be obtained by the division of transformation ratio *i.e. K* and by addition (if not negligible) of the no-load current. Usually, E_A and E_B have the same phase (as assumed above) but there may be some phase difference between the two due to some difference of internal connection in parallel of a star/star and a star/delta 3-phase transformers.

If Z_A and Z_B are small as compared to Z_L *i.e.* when the transformers are not operated near short-circuit conditions, then equations for I_A and I_B can be put in a simpler and more easily under-standable form. Neglecting $Z_A Z_B$ in comparison with the expression $Z_L(Z_A + Z_B)$, we have

$$I_A = \frac{E_A Z_B}{Z_L (Z_A + Z_B)} + \frac{E_A - E_B}{Z_A + Z_B} \quad ...(vii)$$

$$I_B = \frac{E_B Z_A}{Z_L (Z_A + Z_B)} - \frac{E_A - E_B}{Z_A + Z_B} \quad ...(viii)$$

The physical interpretation of the second term in equations (*vii*) and (*viii*) is that it represents the cross-current between the secondaries. The first term shows how the actual load current divides between the loads. The value of current circulating in transformer secondaries (even when there is no-load) is given by* $I_C = (E_A - E_B)/(Z_A + Z_B)$ assuming that $E_A > E_B$. It lags behind E_A by an angle α given by $\tan \alpha = (X_A + X_B)/(R_A + R_B)$. If $E_A + E_B$ the ratios of the currents are inversely as the impedances (numerical values).

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* Under load conditions, the circulating current is

$$I_C = \frac{E_A - E_B}{Z_A + Z_B + Z_A Z_B / Z_L}$$

If $Z_L = \infty$ *i.e.* on open-circuit, the expression reduces to that given above.

If in Eq. (*iv*) we substitute $I_B = I - I_A$ and simplify, we get

$$I_A = \frac{IZ_B}{Z_A + Z_B} + \frac{E_A - E_B}{Z_A + Z_B} \quad ...(ix)$$

Similarly, if we substitute $I_A = I - I_B$ and simplify, then

$$I_B = \frac{IZ_A}{Z_A + Z_B} + \frac{E_A - E_B}{Z_A + Z_B} \quad ...(x)$$

In a similar manner, value of terminal voltage V_2 is given by

$$V_2 = IZ_L = \frac{E_A Z_B + E_B Z_A - IZ_A Z_B}{Z_A + Z_B} \quad ...(xi)$$

These expressions give the values of transformer currents and terminal voltage in terms of the load current. The value of V_2 may also be found as under :

As seen from Fig. 30.70.

$$I_A = (E_A - V_2)/Z_A = (E_A - V_2)Y_A; I_B = (E_B - V_2)Y_B$$

$$I = V_2 Y_L = I_A + I_B \text{ or } V_2 Y_L = (E_A - V_2)Y_A + (E_B - V_2)Y_B$$

$$\therefore \quad V_2(Y_L + Y_A + Y_B) = E_A Y_A + E_B Y_B \text{ or } V_2 = \frac{E_A Y_A + E_B Y_B}{Y_L + Y_A + Y_B} \quad ...(xii)$$

Eq. (*xi*) gives V_2 in terms of load current. But if only load kVA is given, the problem becomes more complicated and involves the solution of a quadratic equation in V_2.

Now, $S = V_2 I$. When we substitute this value of **I** in Eq. (*xi*), we get

$$V_2 = \frac{E_A Z_B + E_B Z_A - SZ_A Z_B / V_2}{(Z_A + Z_B)}$$

or $$V_2^2(Z_A + Z_B) - V(E_A Z_B E_B Z_A) + SZ_A Z_B = 0$$

When V_2 becomes known, then I_A and I_B may be directly found from

$$V_2 = E_A - I_A Z_A \text{ and } V_2 = E_B - I_B Z_B.$$

Note. In the case considered above, it is found more convenient to work with *numerical* values of impedances instead of % values.

Example 32.106. *Two transformers A and B are joined in parallel to the same load. Determine the current delivered by each transformer having given : open-circuit e.m.f. 6600 V for A and 6,400 V for B. Equivalent leakage impedance in terms of the secondary = 0.3 + j3 for A and 0.2 + j1 for B. The load impedance is 8 + j6.* **(Elect. Machines-I, Indore Univ. 1987)**

Solution.

$$I_A = \frac{E_A Z_B + (E_A - E_B)Z_L}{Z_A Z_B + Z_L(Z_A + Z_B)}$$

Here $E_A = 6{,}600$ V ; $E_B = 6{,}400$ V ; $Z_L = 8 + j\,6$; $Z_A = 0.3 + j\,3$; $Z_B = 0.2 + j\,1$

$$I_A = \frac{6600(0.2 + j1) + (6600 - 6400)(8 + j\,6)}{(0.3 + j\,3)(0.2 + j1) + (8 + j\,6)(0.3 + j\,3 + 0.2 + j1)}$$

$$117 - j\,156 = 195\text{A in magnitude}$$

Similarly,

$$I_B = \frac{E_B Z_A - (E_A - E_B)Z_L}{Z_A Z_B + Z_L(Z_A + Z_B)}$$

$$= \frac{6400(0.3 + j\,3) - (6600 - 6400)(8 + j\,6)}{(0.3 + j\,3) + (0.2 + j1)(8 + j\,6) + (0.5 + j\,4)}$$

$$= 349 - j\,231 = 421 \text{ A (in magnitude)}$$

Example 32.107. *Two 1-ϕ transformers, one of 100 kVA and the other of 50 kVA are connected in parallel to the same bus-bars on the primary side, their no-load secondary voltages being 1000 V and 950 V respectively. Their resistances are 2.0 and 2.5 per cent respectively and their reactances 8 and 6 percent respectively. Calculate no-load circulating current in the secondaries.*

(Adv. Elect Machines, A.M.I.E. Sec. B, 1991)

Solution. The circuit connections are shown in Fig. 32.75.

Ist transformer

Normal secondary current = 100,000/1000 = 100 A

$$R_A = \frac{1000 \times 2.0}{100 \times 100} = 0.2\ \Omega\ ;\ X_A = \frac{1000 \times 8}{100 \times 100} = 0.8\ \Omega$$

2nd Transformer

Normal secondary current = 50,000/950 = 52.63 A

$$R_B = 950 \times 2.5/100 \times 52.63 = 0.45\ \Omega$$

$$X_B = 850 \times 6/100 \times 52.63 = 1.08\ \Omega$$

Combined impedance of the two secondaries is

$$Z = \sqrt{(R_A + R_B)^2 + (X_A + X_B)^2}$$

$$= \sqrt{0.65^2 + 1.88^2} = 1.99\ \Omega$$

$$\therefore\ I_c = (1000 - 950)/1.99 = \mathbf{25.1\ A}\ ;\ \alpha = \tan^{-1}(1.88/0.65) = \mathbf{71°}$$

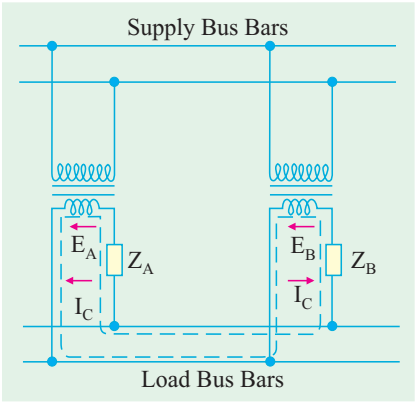


Fig. 32.75

Example 32.108. *Two single-phase transformers, one of 1000-kVA and the other of 500-kVA are connected in parallel to the same bus-bars on the primary side ; their no-load secondary voltages being 500 V and 510 V respectively. The impedance voltage of the first transformer is 3% and that of the second 5%. Assuming that ratio of resistance to reactance is the same and equal to 0.4 in each. What will be the cross current when the secondaries are connected in parallel ?*

(Electrical Machines-I, Madras Univ. 1985)

Solution. Let us first determine the ohmic value of the two impedances. Also, let the secondary voltage be 480 V*.

Full-load $I_A = 1000 \times 1000/480 = 2083$ A ; F.L. $I_B = 500 \times 1000/480 = 1042$ A

$$Z_A = \frac{\%Z_A \times E_A}{100 \times I_A} = \frac{9 \times 500}{100 \times 2083} = 0.0072\ \Omega$$

$$Z_B = \frac{5 \times 510}{100 \times 1042} = 0.0245\ \Omega$$

$$I_C = \frac{E_B - E_A}{Z_A + Z_B} = \frac{510 - 500}{(0.0072 + 0.0245)} = \mathbf{315.4\ A}$$

Note. Since the value of X/R is the same for the two transformers, there is no phase difference between E_A and E_B.

Example 32.109. *Two transformers A and B of ratings 500 kVA and 250 kVA are supplying a load kVA of 750 at 0.8 power factor lagging. Their open-circuit voltages are 405V and 415 V respectively. Transformer A has 1% resistance and 5% reactance and transformer B has 1.5% resistance and 4% reactance. Find (a) cross-current in the secondaries on no-load and (b) the load shared by each transformer.*

* Though it is chosen arbitrarily, its value must be less than either of the two no-load e.m.fs.

Solution. As said earlier, it is more convenient to work with ohmic impedances and for that purpose, we will convert percentage value into numerical values by assuming 400 volt as the terminal voltage (this value is arbitrary but this assumption will not introduce appreciable error).

Now $I_A R_A = 1\%$ of $400 \quad \therefore \quad R_A = \frac{1}{100} \times \frac{400}{1250} = 0.0032\ \Omega$

where $I_A = 500{,}000/400 = 1250$ A

$I_A X_A = 5\%$ of 400 ; $X_A = \frac{5}{100} \times \frac{400}{1250} = 0.016\ \Omega$ (*i.e.* $X_A = 5R_A$)

In a similar way, we can find R_B and X_B ; $R_B = 0.0096\ \Omega$; $X_B = 0.0256\ \Omega$

$\therefore \quad \mathbf{Z_A} = 0.0032 + j\,0.016 = 0.0163 \angle 78.5°$; $\mathbf{Z_B} = 0.0096 + j\,0.0256 = 0.0275 \angle 69.4°$

$Z_A + Z_B = 0.0128 + j\,0.0416 = 0.0436 \angle 72.9°$

Next step is to calculate load impedance. Let $\mathbf{Z_L}$ be the load impedance and V_2 the terminal voltage which has been assumed as 400 *V*.

$\therefore \quad (V_2^2/Z_L) = 750 \angle -36.9°$

$\therefore \quad \mathbf{Z_L} = 400^2 \times 10^{-3}/750 \angle -36.9° = 0.214 \angle 36.9° = (0.171 + j\,0.128)\ \Omega$

(a) $$\mathbf{I_C} = \frac{\mathbf{E_A - E_B}}{\mathbf{Z_A + Z_B}} = \frac{(405 - 415)}{0.0436 \angle 72.9°} = -230 \angle -72.9°$$

(b) $$\mathbf{I_A} = \frac{405 \times 0.0275 \angle 69.4° + (405 - 415) \times 0.214 \angle 36.9°}{0.0163 \angle 78.5° \times 0.0275 \angle 69.4° + 0.214 \angle 36.9° \times 0.0436 \angle 72.9°}$$

$$= 970 \angle -35°$$

Similarly, $$I_B = \frac{415 \times 0.0163 \angle 78.5° - (405 - 415) \times 0.214 \angle 36.9°}{0.0163 \angle 78.5° \times 0.0275 \angle 69.4° + 214 \angle 369° \times 0.0436 \angle 72.9°}$$

$\therefore \quad \mathbf{S_A} = 400 \times 970 \times 10^{-3} \angle -35° = 388 \angle -35°$ kVA ; $\cos \phi_A = \cos 35° = 0.82$ (lag)

$\mathbf{S_B} = 400 \times 875 \times 10^{-3} \angle 42.6° = 350 \angle -42.6°$ kVA

$\cos \phi_B = \cos 42.6° = 0.736$ (lag)

Example 32.110. *Two transformers A and B are connected in parallel to a load of (2 + j 1.5) ohms. Their impedances in secondary terms are Z_A = (0.15 + j 0.5) ohm and Z_B = (0.1 + j 0.6) ohm. Their no-load terminal voltages are E_A = 207 ∠ 0° volt and E_B =205 ∠0° volt. Find the power output and power factor of each transformer.* **(Elect. Machines-I, Punjab Univ. 1991)**

Solution. Using the equations derived in Art. 30.34 (*c*), we have

$$\mathbf{I_A} = \frac{\mathbf{E_A Z_B} + (\mathbf{E_A - E_B})\mathbf{Z_L}}{\mathbf{Z_A Z_B} + \mathbf{Z_L}(\mathbf{Z_A + Z_B})}$$

$\mathbf{Z_A} = (0.15 + j\,0.5)\ \Omega$; $Z_B = (0.1 + j\,0.6)\ \Omega$; $\mathbf{Z_L} = (2 + j\,1.5) = 2.5 \angle 36.9°$

$\therefore \quad \mathbf{I_A} = \frac{207(0.1 + j\,0.6) + (207 - 205)(2 + j1.5)}{(0.15 + j\,0.5)(0.1 + j\,0.6) + (2 + j1.5)(0.25 + j1.1)}$

$= \frac{24.7 + j\,127.2}{-1.435 + j\,2.715} = \frac{129.7 \angle 79°}{3.07 \angle 117.9°} = 42.26 \angle -38.9° A = (32.89 - j\,26.55) A$

$$\mathbf{I_B} = \frac{\mathbf{E_B Z_A} - (\mathbf{E_A - E_B})\mathbf{Z_L}}{\mathbf{Z_A Z_B} + \mathbf{Z_L}(\mathbf{Z_A + Z_B})} = \frac{205(0.15 + j\,0.5) - 2(2 + j\,1.5)}{-1.435 + j\,2.715} = \frac{103 \angle 75°}{3.07 \angle 117.9°}$$

$= 33.56 \angle -42.9° = (24.58 - j\,22.84)\ A$

Now $\mathbf{V_2'} = \mathbf{IZ_L} = \mathbf{(I_A + I_B)\,Z_L}$

$= (57.47 - j\,49.39)\,(2 + j\,1.5) = 189 - j\,12.58 = 189.4 \angle -3.9°$

p.f. angle of transformer $A = -3.9° - (-38.9°) = 35°$

$\therefore$ p.f. of $A = \cos 35° = 0.818$ (lag) ; p.f. of $B = \cos [-3.9° - (-42.9°)] = 0.776$ (lag)

Power output of transformer A is $P_A = 189.4 \times 42.26 \times 0.818 =$ **6,548 W**

Similarly, $P_B = 189.4 \times 33.56 \times 0.776 =$ **4,900 W**

Example 32.111. *Two transformers have the following particulars :*

	Transformer A	*Transformer B*
Rated current	*200 A*	*600 A*
Per unit resistance 0.02	*0.025*	
Per unit reactance	*0.05*	*0.06*
No-load e.m.f.	*245 V*	*240 V*

Calculate the terminal voltage when they are connected in parallel and supply a load impedance of (0.25 + j 0.1) Ω. **(Elect. Machines-I, Sd. Patel Univ. 1981)**

Solution. Impedance in ohms = $Z_{pu} \times$ N.L. e.m.f./full-load current

$$Z_A = (245/200)(0.02 + j\,0.05) = 0.0245 + j\,0.0613\ \Omega = 0.066 \angle 68.2°$$

$$Z_B = (240/600)(0.025 + j\,0.06) = 0.01 + j\,0.024\ \Omega = 0.026 \angle 67.3°$$

$$Z = (0.25 + j\,0.1) = 0.269 \angle 21.8° \ ; \ Z_A + Z_B = 0.0345 + j\,0.0853 = 0.092 \angle 68°$$

$$Z_L(Z_A + Z_B) = 0.269 \times 0.092 \angle 89.8° = 0.0247 \angle 89.8° = (0 + j\,0.0247)$$

$$Z_A Z_B = 0.066 \times 0.026 \angle 135.5° = (-0.001225 + j\,0.001201)$$

$\therefore$ $Z_A Z_B + Z_L(Z_A + Z_B) = (-0.00125 + j\,0.259) = 0.0259 \angle 92.7°$

Let us take E_A as reference quantity.

Also E_b is in phase with E_A because transformers are in parallel on both sides.

$$E_A Z_B = 245(0.01 + j\,0.0245) = 2.45 + j\,5.87$$

$$E_B Z_A = 240(0.0245 + j\,0.0613) = 5.88 + j\,14.7$$

$$E_A Z_B + E_B Z_A = 8.33 + j\,20.57 = 22.15 \angle 67.9°$$

Now, $$I = \frac{E_A Z_B + E_B Z_A}{Z_A Z_B + Z_L(Z_A + Z_B)} = \frac{22.15 \angle 67.9°}{0.0259 \angle 92.7°} = 855 \angle -24.8°$$

$\therefore$ $V_2 = IZ_L = 885 \angle -24.8° \times 0.269 \angle 21.8° = 230 - 3°$

Tutorial Problems 29.1

1. A 1000-kVA and a 500-kVA, 1-phase transformers are connected to the same bus-bars on the primary side. The secondary e.m.fs. at no-load are 500 and 510 V respectively. The impedance voltage of the first transformer is 3.4% and of the second 5%. What cross-current will pass between them when the secondaries are connected together in parallel ? Assuming that the ratio of resistance to reactance is the same in each, what currents will flow in the windings of the two transformers when supplying a total load of 1200 kVA.

[(*i*) 290 A (*ii*) 1577 and 900 A] (*City & Guilds, London*)

2. Two transformers A and B are connected in parallel to supply a load having an impedance of $(2 + j\,1.5\Omega)$. The equivalent impedances referred to the secondary windings are $0.15 + j\,0.5\ \Omega$ and $0.1 + j\,0.6\ \Omega$ respectively. The open-circuit e.m.f. of A is 207 V and of B is 205 V. Calculate (*i*) the voltage at the load (*ii*) the power supplied to the load (*iii*) the power output of each transformer and (*iv*) the kVA input to each transformer.

[(*i*) 189 $\angle -3.8°$V (*ii*) 11.5 kW (*iii*) 6.5 kW, 4.95 kW (iv) 8.7 kVA, 6.87 kVA]

QUESTIONS AND ANSWERS ON TRANSFORMERS

Q.1. How is magnetic leakage reduced to a minimum in commerical transformers ?

Ans. By interleaving the primary and secondary windings.

Q.2. Mention the factors on which hysteresis loss depends ?

Ans. (*i*) Quality and amount of iron in the core (*ii*) Flux density and (*iii*) Frequency.

Q.3. How can eddy current loss be minimised ?

Ans. By laminating the core.

Q.4. In practice, what determines the thickness of the laminae or stampings ?

Ans. Frequency.

Q.5. Does the transformer draw any current when its secondary is open ?

Ans. Yes, no-load primary current.

Q.6. Why ?

Ans. For supplying no-load iron and copper losses in primary.

Q.7. Is Cu loss affected by power factor ?

Ans. Yes, Cu loss varies inversely with power factor.

Q.8. Why ?

Ans. Cu loss depends on current in the primary and secondary windings. It is well-known that current required is higher when power factor is lower.

Q.9. What effects are produced by change in voltage ?

Ans.

1. Iron loss.....varies approximately as V^2.
2. Cu loss.....it also varies as V^2 but decreases with an increase in voltage ***if constant kVA output is assumed***.
3. Efficiency.....for distribution transformers, efficiency at fractional loads decreases with increase in voltage while at full load or overload it increases with increase in voltage and *vice-versa*.
4. Regulation.....it varies as V^2 but decreases with increase in voltage ***if constant kVA output is assumed.***
5. Heating.....for constant kVA output, iron temperatures increase whereas Cu temperatures decrease with increase in voltages and ***vice-versa***.

Q. 10. How does change in frequency affect the operation of a given transformer ?

Ans.

1. Iron loss .....increases with a decrease in frequency. A 60-Hz transformer will have nearly 11% higher losses when worked on 50Hz instead of 60 Hz. However, when a 25-Hz transformer is worked on 60 Hz, iron losses are reduced by 25%.
2. Cu loss.....in distribution transformers, it is independent of frequecy.
3. Efficiency.....since Cu loss is unaffected by change in frequency, a given transformer efficiency is less at a lower frequency than at a higher one.
4. Regulation.....regulation at unity power factor is not affected because *IR* drop is independent of frequency. Since reactive drop is affected, regulation at low power factors decreases with a decrease in frequency and *vice-versa*. For example, the regulation of a 25-Hz transformer when operated at 50-Hz and low power factor is much poorer.
5. Heating.....since total loss is greater at a lower frequency, the temperature is increased with decrease in frequency.

OBJECTIVE TESTS – 32

1. A transformer transforms
(*a*) frequency
(*b*) voltage
(*c*) current
(*d*) voltage and current.

2. Which of the following is not a basic element of a transformer ?
(*a*) core
(*b*) primary winding
(*c*) secondary winding
(*d*) mutual flux.

3. In an ideal transformer,
(*a*) windings have no resistance
(*b*) core has no losses
(*c*) core has infinite permeability
(*d*) all of the above.

4. The main purpose of using core in a transformer is to
(*a*) decrease iron losses
(*b*) prevent eddy current loss
(*c*) eliminate magnetic hysteresis
(*d*) decrease reluctance of the common magnetic circuit.

5. Transformer cores are laminated in order to
(*a*) simplify its construction
(*b*) minimise eddy current loss
(*c*) reduce cost
(*d*) reduce hysteresis loss.

6. A transformer having 1000 primary turns is connected to a 250-V a.c. supply. For a secondary voltage of 400 V, the number of secondary turns should be
(*a*) 1600 (*b*) 250
(*c*) 400 (*d*) 1250

7. The primary and secondary induced e.m.fs. E_1 and E_2 in a two-winding transformer are **always**
(*a*) equal in magnitude
(*b*) antiphase with each other
(*c*) in-phase with each other
(*d*) determined by load on transformer secondary.

8. A step-up transformer increases
(*a*) voltage (*b*) current
(*c*) power (*d*) frequency.

9. The primary and secondary windings of an ordinary 2-winding transformer **always** have
(*a*) different number of turns
(*b*) same size of copper wire
(*c*) a common magnetic circuit
(*d*) separate magnetic circuits.

10. In a transformer, the leakage flux of each winding is proportional to the current in that winding because
(*a*) Ohm's law applies to magnetic circuits
(*b*) leakage paths do not saturate
(*c*) the two windings are electrically isolated
(*d*) mutual flux is confined to the core.

11. In a two-winding transformer, the e.m.f. per turn in secondary winding is **always**.....the induced e.m.f. power turn in primary.
(*a*) equal to K times
(*b*) equal to $1/K$ times
(*c*) equal to
(*d*) greater than.

12. In relation to a transformer, the ratio 20 : 1 indicates that
(*a*) there are 20 turns on primary one turn on secondary
(*b*) secondary voltage is 1/20th of primary voltage
(*c*) primary current is 20 times greater than the secondary current.
(*d*) for every 20 turns on primary, there is one turn on secondary.

13. In performing the short circuit test of a transformer
(*a*) high voltage side is usually short circuited
(*b*) low voltage side is usually short circuited
(*c*) any side is short circuited with preference
(*d*) none of the above.
(Elect. Machines, A.M.I.E. Sec. B, 1993)

14. The equivalent resistance of the primary of a transformer having $K = 5$ and $R_1 = 0.1$ ohm when referred to secondary becomes.....ohm.
(*a*) 0.5
(*b*) 0.02
(*c*) 0.004
(*d*) 2.5

15. A transformer has negative voltage regulation when its load power factor is
(*a*) zero
(*b*) unity
(*c*) leading
(*d*) lagging.

16. The primary reason why open-circuit test is performed on the low-voltage winding of the transformer is that it
(*a*) draws sufficiently large on-load current for convenient reading
(*b*) requires least voltage to perform the test
(*c*) needs minimum power input
(*d*) involves less core loss.

17. No-load test on a transformer is carried out to determine
(*a*) copper loss
(*b*) magnetising current
(*c*) magnetising current and no-load loss
(*d*) efficiency of the transformer.

18. The main purpose of performing open-circuit test on a transformer is to measure its
(*a*) Cu loss
(*b*) core loss
(*c*) total loss
(*d*) insulation resistance.

19. During short-circuit test, the iron loss of a transformer is negligible because
(*a*) the entire input is just sufficient to meet Cu losses only
(*b*) flux produced is a small fraction of the normal flux
(*c*) iron core becomes fully saturated
(*d*) supply frequency is held constant.

20. The iron loss of a transformer at 400 Hz is 10 W. Assuming that eddy current and hysteresis losses vary as the square of flux density, the iron loss of the transformer at rated voltage but at 50 Hz would be..... watt.
(*a*) 80 (*b*) 640
(*c*) 1.25 (*d*) 100

21. In operating a 400 Hz transformer at 50 Hz
(*a*) only voltage is reduced in the same proportion as the frequency
(*b*) only kVA rating is reduced in the same proportion as the frequency
(*c*) both voltage and kVA rating are reduced in the same proportion as the frequency
(*d*) none of the above.

22. The voltage applied to the h.v. side of a transformer during short-circuit test is 2% of its rated voltage. The core loss will be.....percent of the rated core loss.
(*a*) 4 (*b*) 0.4
(*c*) 0.25 (*d*) 0.04

23. Transformers are rated in kVA instead of kW because
(*a*) load power factor is often not known
(*b*) kVA is fixed whereas kW depends on load p.f.
(*c*) total transformer loss depends on volt-ampere
(*d*) it has become customary.

24. When a 400-Hz transformer is operated at 50 Hz its kVA rating is
(*a*) raduced to 1/8
(*b*) increased 8 times
(*c*) unaffected
(*d*) increased 64 times.

25. At relatively light loads, transformer efficiency is low because
(*a*) secondary output is low
(*b*) transformer losses are high
(*c*) fixed loss is high in proportion to the output
(*d*) Cu loss is small.

26. A 200 kVA transformer has an iron loss of 1 kW and full-load Cu loss of 2kW. Its load kVA corresponding to maximum efficiency is ..... kVA.
(*a*) 100 (*b*) 141.4
(*c*) 50 (*d*) 200

27. If Cu loss of a transformer at 7/8th full load is 4900 W, then its full-load Cu loss would be .....watt.
(*a*) 5600 (*b*) 6400
(*c*) 375 (*d*) 429

28. The ordinary efficiency of a given transformer is maximum when
(*a*) it runs at half full-load
(*b*) it runs at full-load
(*c*) its Cu loss equals iron loss
(*d*) it runs slightly overload.

29. The output current corresponding to maximum efficiency for a transformer having core loss of 100 W and equivalent resistance referred to secondary of 0.25 Ω is ..... ampere.
(*a*) 20
(*b*) 25
(*c*) 5
(*d*) 400

30. The maximum efficiency of a 100-kVA transformer having iron loss of 900 kW and F.L. Cu loss of 1600 W occurs at ..... kVA.
(*a*) 56.3 (*b*) 133.3
(*c*) 75 (*d*) 177.7

31. The all-day efficiency of a transformer depends primarily on
(*a*) its copper loss
(*b*) the amount of load
(*c*) the duration of load
(*d*) both (*b*) and (*c*).

32. The marked increase in kVA capacity produced by connecting a 2 winding transformer as an autotransfomer is due to
(*a*) increase in turn ratio
(*b*) increase in secondary voltage
(*c*) increase in transformer efficiency
(*d*) establishment of conductive link between primary and secondary.

33. The kVA rating of an ordinary 2-winding transformer is increased when connected as an autotransformer because
(*a*) transformation ratio is increased
(*b*) secondary voltage is increased
(*c*) energy is transferred both inductively and conductivity
(*d*) secondary current is increased.

34. The saving in Cu achieved by converting a 2-winding transformer into an autotransformer is determined by
(*a*) voltage transformation ratio
(*b*) load on the secondary
(*c*) magnetic quality of core material
(*d*) size of the transformer core.

35. An autotransformer having a transformation ratio of 0.8 supplies a load of 3 kW. The power transferred conductively from primary to secondary is.....kW.
(*a*) 0.6 (*b*) 2.4
(*c*) 1.5 (*d*) 0.27

36. The essential condition for parallel opearation of two 1-ϕ transformers is that they should have the same
(*a*) polarity
(*b*) kVA rating
(*c*) voltage ratio
(*d*) percentage impedance.

37. If the impedance triangles of two transformers operating in parallel are not identical in shape and size, the two transformers will
(*a*) share the load unequally
(*b*) get heated unequally
(*c*) have a circulatory secondary current even when unloaded
(*d*) run with different power factors.

38. Two transformers *A* and *B* having equal outputs and voltage ratios but unequal percentage impedances of 4 and 2 are operating in parallel. Transformer *A* will be running over-load by ..... percent.
(*a*) 50 (*b*) 66
(*c*) 33 (*d*) 25

ANSWERS

1. *d* **2.** *d* **3.** *d* **4.** *d* **5.** *b* **6.** *a* **7.** *c* **8.** *a* **9.** *c* **10.** *b* **11.** *c* **12.** *d* **13.** *b* **14.** *d* **15.** *c* **16.** *a* **17.** *c* **18.** *b* **19.** *b* **20.** *b* **21.** *b* **22.** *d* **23.** *c* **24.** *a* **25.** *c* **26.** *b* **27.** *b* **28.** *c* **29.** *a* **30.** *c* **31.** *d* **32.** *d* **33.** *c* **34.** *a* **35.** *b* **36.** *a* **37.** *d* **38.** *c*

GO To FIRST

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CHAPTER 33

TRANSFORMER: THREE PHASE

Learning Objectives

- Three-phase Transformers
- Three-phase Transformer Connections
- Star/Star or Y/Y Connection
- Delta-Delta or Connection
- Wye/Delta or Y/ Connection
- Delta/Wye or /Y Connection
- Open-Delta or V-V Connection
- Power Supplied by V-V Bank
- Scott Connection or T-T Connection
- Three-phase to Two-Phase Conversion and vice-versa
- Parallel Operation of 3-phase Transformers
- Instrument Transformers
- Current Transformers
- Potential Transformers

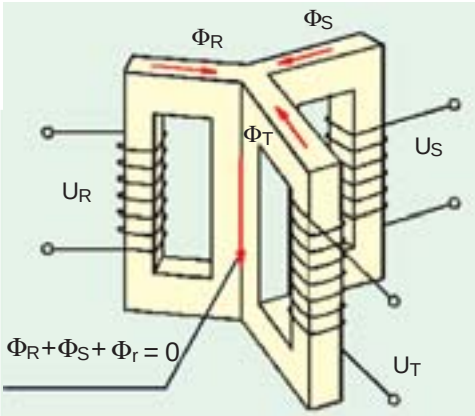


Three phase transformers are used throughout industry to change values of three phase voltage and current. Three phase power is the most common way in which power is produced.

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33.1. Three-Phase Transformer

Large scale generation of electric power is usually 3-phase at generated voltages of 13.2 kV or somewhat higher. Transmission is generally accomplished at higher voltages of 110, 132, 275, 400 and 750 kV for which purpose 3-phase transformers are necessary to step up the generated voltage to that of the transmission line. Next, at load centres, the transmission voltages are reduced to distribution voltages of 6,600, 4,600 and 2,300 volts. Further, at most of the consumers, the distribution voltages are still reduced to utilization voltages of 440, 220 or 110 volts. Years ago, it was a common practice to use suitably interconnected three single-phase transformers instead of a single 3-phase transformer. But these days, the latter is gaining popularity because of improvement in design and manufacture but principally because of better acquaintance of operating men with the three-phase type. As compared to a bank of single-phase transformers, the main advantages of a 3 phase transformer are that it occupies less floor space for equal rating, weighs less, costs about 15% less and further, that only one unit is to be handled and connected.



Three phase transformer inner circuits

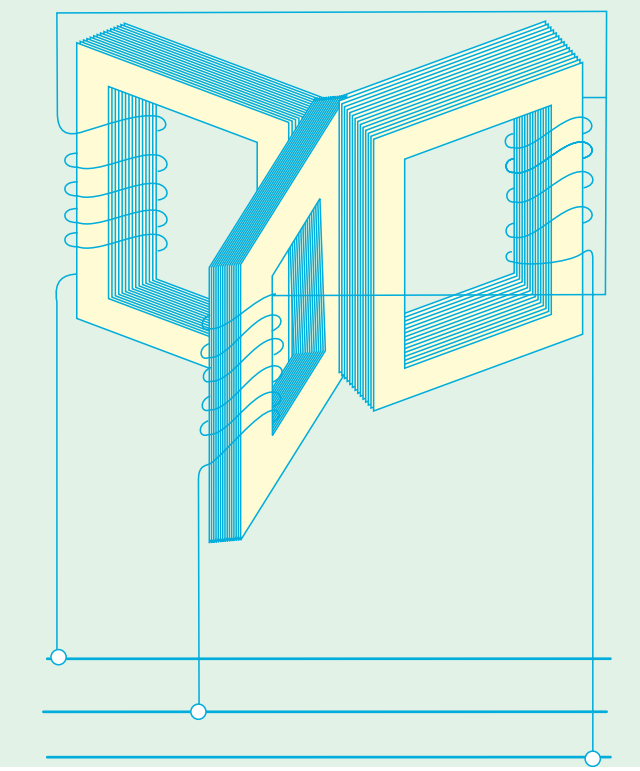


Fig. 33.1

Like single-phase transformers, the three-phase transformers are also of the core type or shell type. The basic principle of a 3-phase transformer is illustrated in Fig. 33.1 in which only primary windings have been shown interconnected in star and put across 3-phase supply. The three cores are 120° apart and their empty legs are shown in contact with each other. The centre leg, formed by these three, carries the flux produced by the three-phase currents I_R, I_Y and I_B. As at any instant $I_R + I_Y + I_B = 0$, hence the sum of three fluxes is also zero. Therefore, it will make no difference if the common leg is removed. In that case any two legs will act as the return for the third just as in a 3-phase system any two conductors act as the return for the current

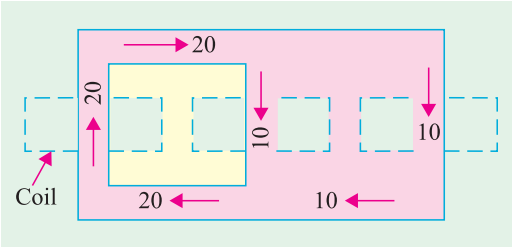


Fig. 33.2 (*a*)

in the third conductor. This improved design is shown in Fig. 33.2 (*a*) where dotted rectangles indicate the three windings and numbers in the cores and yokes represent the directions and magnitudes of fluxes at a particular instant. It will be seen that at any instant, the amount of 'up' flux in any leg is equal to the sum of 'down' fluxes in the other two legs. The core type transformers are usually wound with circular cylindrical coils.

In a similar way, three single-phase shell type transformers can be combined together to form a 3-phase shell type unit as shown in Fig. 33.2(*b*). But some saving in iron can be achieved in

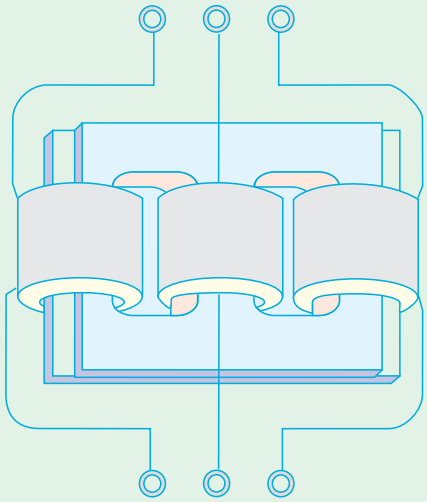


Fig. 33.2 (*b*)

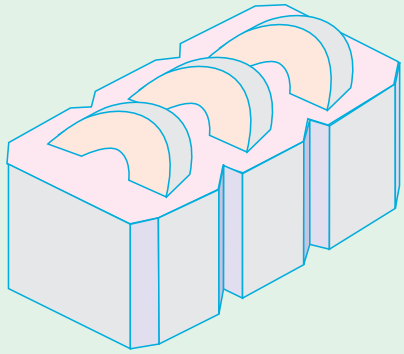
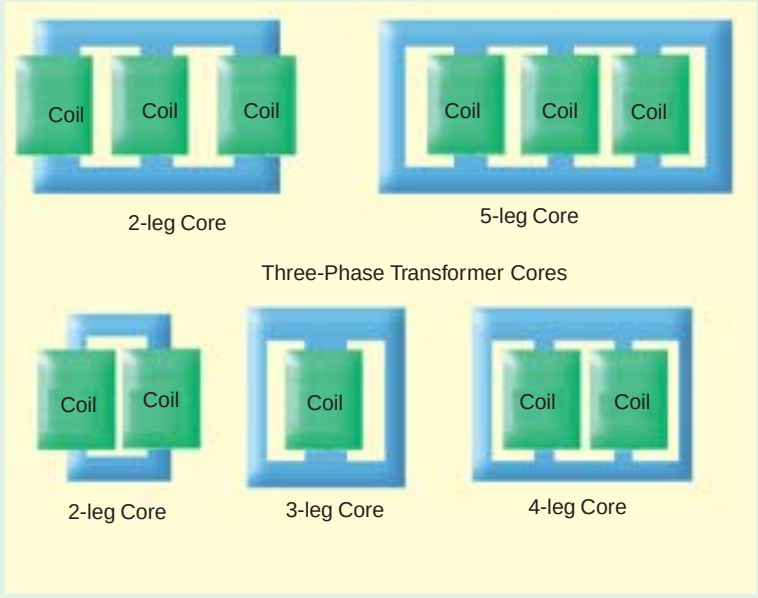


Fig. 33.3



constructing a single 3-phase transformer as shown in Fig. 33.3. It does not differ from three single-phase transformers put side by side. Saving in iron is due to the joint use of the magnetic paths between the coils. The three phases, in this case, are more independent than they are in the core type transformers, because each phase has a magnetic circuit independent of the other.

One main drawback in a 3-phase transformer is that if any one phase becomes disabled, then the whole transformer has to be ordinarily removed from service for repairs (the shell type may be operated open

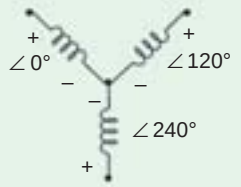
Δ or Vee but this is not always feasible). However, in the case of a 3-phase bank of single-phase transformers, if one transformer goes out of order, the system can still be run open-Δ at reduced capacity or the faulty transformer can be readily replaced by a single spare.

33.2. Three-phase Transformer Connections

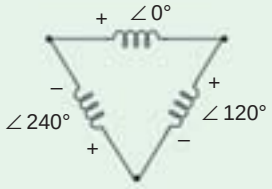
There are various methods available for transforming 3-phase voltages to higher or lower 3-phase voltages *i.e.* for handling a considerable amount of power. The most common connections are ***(i)*** $Y-Y$ ***(ii)*** $\Delta-\Delta$ ***(iii)*** $Y-\Delta$ ***(iv)*** $\Delta-Y$ ***(v)*** open-delta or $V-V$ ***(vi)*** Scott connection or $T-T$ connection.

33.3. Star/Star or *Y/Y* Connection

This connection is most economical for small, high-voltage transformers because the number of turns/phase and the amount of insulation required is minimum (as phase voltage is only $1/\sqrt{3}$ of line voltage). In Fig. 33.4 a bank of 3 transformers connected in Y on both the primary and the secondary sides is shown. The ratio of line voltages on the primary and secondary sides is the same as the transformation ratio of each transformer. However, there is a phase shift of 30° between the phase voltages and line voltages both on the primary and secondary sides. Of course, line voltages on both sides as well as primary voltages are respectively in phase with each other. ***This connection works satisfactorily only if the load is balanced.*** With the unbalanced load to the neutral, the neutral point shifts thereby making the three line-to-neutral (*i.e.* phase) voltages unequal. The effect of unbalanced loads can be illustrated by placing a single load between phase (or coil) *a* and the neutral on the secondary side. The power to the load has to be supplied by primary phase (or coil) *A*. This primary coil *A* cannot supply the required power because it is in series with primaries *B* and *C* whose secondaries are open. Under these conditions, the primary coils *B* and *C* act as very high impedances so that primary coil *A* can obtain but very little current through them from the line. Hence, secondary coil *a* cannot supply any appreciable power. In fact, a very low resistance approaching a short-circuit may be connected between point *a* and the neutral and only a very small amount of current will flow. This, as said above, is due to the reduction of voltage E_{an} because of neutral shift. In other words, under short-circuit conditions, the neutral is pulled too much towards coil *a*. This reduces E_{an} but increases E_{bn} and E_{cn} (however line voltage E_{AB}, E_{BC} and E_{CA} are unaffected). On the primary side, E_{AN} will be



With these phase angles, the center points of the Y must tie either all "–" or all "+" winding ends together.



With these phase angles, the winding polarities must stack together in a complementary manner (+ to –).

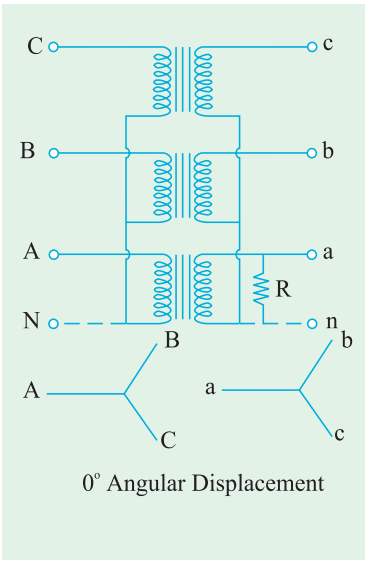


Fig. 33.4

practically reduced to zero whereas E_{BN} and E_{CN} will rise to nearly full primary line voltage. This difficulty of shifting (or floating) neutral can be obviated by connecting the primary neutral (shown dotted in the figure) back to the generator so that primary coil *A* can take its required power from between its line and the neutral. It should be noted that if a single phase load is connected between the lines *a* and *b*, there will be a similar but less pronounced neutral shift which results in an overvoltage on one or more transformers.

Another advantage of stabilizing the primary neutral by connecting it to neutral of the generator is that it eliminates distortion in the secondary phase voltages. This is explained as follows. For delivering a sine wave of voltage, it is necessary to have a sine wave of flux in the core, but on account of the characteristics of iron, a sine wave of flux requires a third harmonic component in the exciting current. As the frequency of this component is thrice the frequency of the circuit, at any given instant, it tends to flow either towards or away from the neutral point in all the three transformers. If the primary neutral is isolated, the triple frequency current cannot flow. Hence, the flux in the core cannot be a sine wave and so the voltages are distorted. But if the primary neutral is earthed *i.e.* joined to the generator neutral, then this provides a path for the triple-frequency currents and e.m.fs. and the difficulty is overcome. Another way of avoiding this trouble of oscillating neutral is to provide each of the transformers with a third or tertiary winding of relatively low kVA rating. This tertiary winding is connected in Δ and provides a circuit in which the triple-frequency component of the magnetising current can flow (with an isolated neutral, it could not). In that case, a sine wave of voltage applied to the primary will result in a sine wave of phase voltage in the secondary. As said above, the advantage of this connection is that insulation is stressed only to the extent of line to neutral voltage *i.e.* 58% of the line voltage.

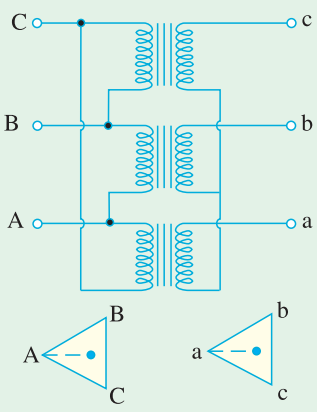
33.4. Delta-Delta or Δ – Δ Connection

This connection is economical for large, low-voltage transformers in which insulation problem is not so urgent, because it increases the number of turns/phase. The transformer connections and voltage triangles are shown in Fig. 33.5. The ratio of transformation between primary and secondary line voltage is exactly the same as that of each transformer. Further, the secondary voltage triangle *abc* occupies the same relative position as the primary voltage triangle *ABC i.e.* there is no angular displacement between the two. Moreover, there is no internal phase shift between phase and line voltages on either side as was the case in *Y* – *Y* connection. This connection has the following advantages :

1. As explained above, in order that the output voltage be sinusoidal, it is necessary that the magnetising current of the transformer must contain a third harmonic component. In this case, the third harmonic component of the magnetising current can flow in the Δ-connected transformer primaries without flowing in the line wires. The three phases are 120° apart which is $3 \times 120 = 360°$ with respect to the third harmonic, hence it merely circulates in the Δ. Therefore, the flux is sinusoidal which results in sinusoidal voltages.

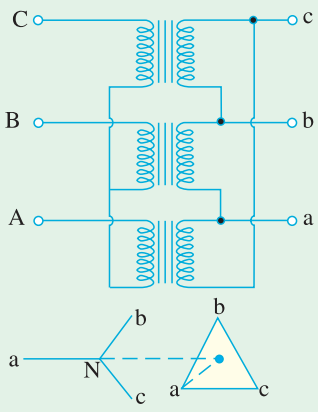
2. No difficulty is experienced from unbalanced loading as was the case in *Y* – *Y* connection. The three-phase voltages remain practically constant regardless of load imbalance.

3. An added advantage of this connection is that if one transformer becomes disabled, the system can continue to operate in open-delta or in *V* – *V* although with reduced available capacity. The reduced capacity is 58% and not 66.7% of the normal value, as explained in Art. 33.7.



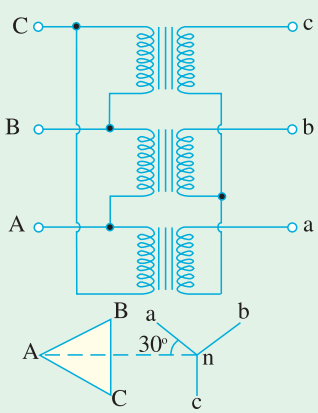
0° Angular Displacement

Fig. 33.5



30° Angular Displacement

Fig. 33.6



30° Angular Displacement

Fig. 33.7

33.5. Wye/Delta or *Y*/Δ Connection

The main use of this connection is at the substation end of the transmission line where the voltage is to be stepped down. The primary winding is *Y*-connected with grounded neutral as shown in Fig. 33.6. The ratio between the secondary and primary line voltage is $1/\sqrt{3}$ times the transformation ratio of each transformer. There is a 30° shift between the primary and secondary line voltages which means that a $Y-\Delta$ transformer bank cannot be paralleled with either a $Y-Y$ *or* a $\Delta-\Delta$ bank. Also, third harmonic currents flows in the Δ to provide a sinusoidal flux.

33.6. Delta/Wye or Δ/*Y* Connection

This connection is generally employed where it is necessary to step up the voltage as for example, at the beginning of high tension transmission system. The connection is shown in Fig. 33.7. The neutral of the secondary is grounded for providing 3-phase 4-wire service. In recent years, this connection has gained considerable popularity because it can be used to serve both the 3-phase power equipment and single-phase lighting circuits.

This connection is not open to the objection of a floating neutral and voltage distortion because the existence of a Δ-connection allows a path for the third-harmonic currents. It would be observed that the primary and secondary line voltages and line currents are out of phase with each other by 30°. Because of this 30° shift, it is impossible to parallel such a bank with a $\Delta-\Delta$ or $Y-Y$ bank of transformers even though the voltage ratios are correctly adjusted. The ratio of secondary to primary voltage is $\sqrt{3}$ times the transformation ratio of each transformer.

Example 33.1. *A 3-phase, 50-Hz transformer has a delta-connected primary and star-connected secondary, the line voltages being 22,000 V and 400 V respectively. The secondary has a star-connected balanced load at 0.8 power factor lagging. The line current on the primary side is 5 A. Determine the current in each coil of the primary and in each secondary line. What is the output of the transformer in kW ?*

Solution. It should be noted that in three-phase transformers, the *phase* transformation ratio is equal to the turn ratio but the terminal or line voltages depend upon the method of connection employed. The Δ/*Y* connection is shown in Fig. 33.8.

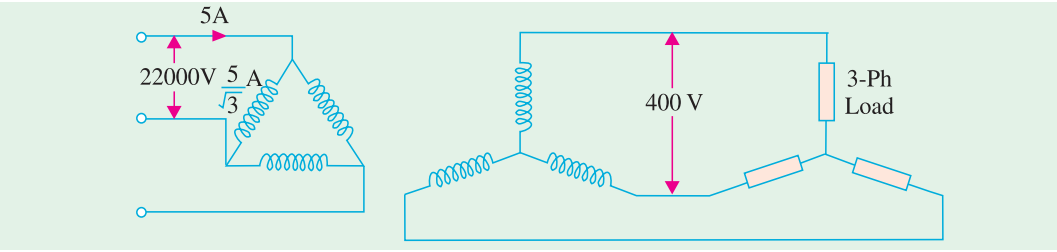


Fig. 33.8

Phase voltage on primary side = 22,000 V

Phase voltage on secondary side = $400/\sqrt{3}$

$\therefore$ $K = 400/22{,}000 \times \sqrt{3} = 1/55\sqrt{3}$

Primary phase current = $5/\sqrt{3}$ A

Secondary phase current = $\frac{5}{\sqrt{3}} \div \frac{1}{55\sqrt{3}} = 275$ A

Secondary line current = **275 A**

$\therefore$ Output = $\sqrt{3} V_L I_L \cos\phi = \sqrt{3} \times 400 \times 275 \times 0.8 =$ **15.24 kW.**

Example 33.2. *A 500-kVA, 3-phase, 50-Hz transformer has a voltage ratio (line voltages) of 33/11-kV and is delta/star connected. The resistances per phase are : high voltage 35 Ω, low voltage 0.876 Ω and the iron loss is 3050 W. Calculate the value of efficiency at full-load and one-half of full-load respectively (a) at unity p.f. and (b) 0.8 p.f.* **(Electrical Machinery, Madras Univ. 1985)**

Solution. Transformation ratio $K = \frac{11{,}000}{\sqrt{3} \times 33{,}000} = \frac{1}{3\sqrt{3}}$

Per phase $R_{02} = 0.876 + (1/3\sqrt{3})^2 \times 35 = 2.172\ \Omega$

Secondary phase current = $\frac{500{,}000}{\sqrt{3} \times 11{,}000} = \frac{500}{11\sqrt{3}}$ A

Full-load condition

Full load total Cu loss = $3 \times (500/11\sqrt{3})^2 \times 2.172 = 4{,}490$ W; Iron loss = 3,050 W

Total full-load losses

= 4,490 + 3,050 = 7,540 W; Output at unity p.f. = 500 kW

$\therefore$ F.L. efficiency = 500,000/507,540 = 0.9854 or **98.54%**; Output at 0.8 p.f. = 400 kW

$\therefore$ Efficiency = 400,000/407.540

= 0.982 or **98.2%**

Half-load condition

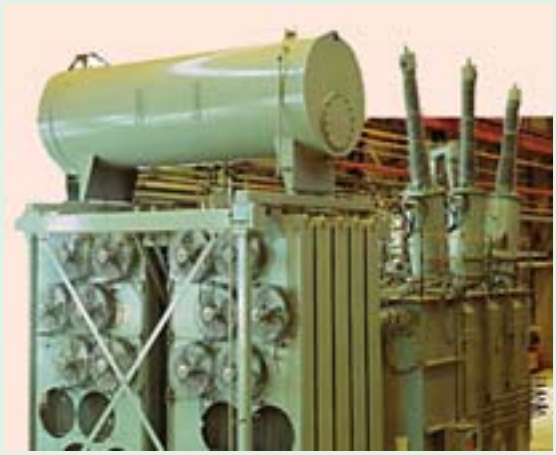
Output at unity p.f.= 250 kW

Cu losses = $(1/2)^2 \times 4{,}490$

= **1,222 W**

Total losses

= 3,050 + 1,122 = 4,172 W



Three-phase transformer

$\therefore$ η = 250,000/254,172 = 0.9835 = **98.35 %**

Output at 0.8 p.f. = 200 kW $\therefore$ η = **200,000/204,172 = 0.98 or 98%**

Example 33.3. *A 3-phase, 6,600/415-V, 2,000-kVA transformer has a per unit resistance of 0.02 and a per unit leakage reactance of 0.1. Calculate the Cu loss and regulation at full-load 0.8 p.f. lag.* **(Electrical Machines-I, Bombay Univ. 1987)**

Solution. As seen from Art. 27 – 16, $\%R = \%$ Cu loss $= \dfrac{\text{Cu loss}}{VA} \times 100$

Now, $\%R = 0.02 \times 100 = 2\%$ $\therefore$ $2 = \dfrac{\text{Cu loss}}{2{,}000} \times 100$ $\therefore$ Cu loss = **40 kW**

Now, percentage leakage reactance = $0.1 \times 100 = 10\%$

Regn. = $v_r \cos\phi + v_x \sin\phi = 2 \times 0.8 + 10 \times 0.6 =$ **7.6%**

Example 33.4. *A 120-kVA, 6,000/400-V, Y/Y 3-ph, 50-Hz transformer has an iron loss of 1,600 W. The maximum efficiency occurs at 3/4 full load.*

Find the efficiencies of the transformer at

(*i*) *full-load and 0.8 power factor* (*ii*) *half-load and unity power factor*

(*iii*) *the maximum efficiency.* **(Elect. Technology Utkal Univ. 1987)**

Solution. Since maximum efficiency occurs at 3/4 full-load, Cu loss at 3/4 full-load equals iron loss of 1,600 W.

Cu loss at 3/4 F.L. = 1,600 W; Cu loss at F.L. = $1{,}600 \times (4/3)^2 = 2{,}845$ W

(i) F.L. output at 0.8 p.f. = $120 \times 0.8 = 96$ kW = 96,000 W

Total loss = 1,600 + 2,845 = 4,445 W

$\therefore$ $\eta = \dfrac{96{,}000}{100{,}445} \times 100 =$ **95.57 %**

(ii) Cu loss at 1/2 full-load = $(1/2)^2 \times 2{,}845 = 710$ W

Total loss = 710 + 1,600 = 2310 W

Output at 1/2 F.L. and u.p.f. is= 60 kW = 60,000 W; $\eta = \dfrac{60{,}000}{62{,}310} \times 100 =$ **96.57%**

(iii) Maximum efficiency occurs at 3/4 full-load when iron loss equals Cu loss.

Total loss = $2 \times 1{,}600 = 3{,}200$ W

Output at u.p.f. = $(3/4) \times 120 = 90$ kW = 90,000 W

Input = 90,000 + 3,200 = 93,200 W $\therefore$ $\eta = \dfrac{90{,}000}{93{,}200} \times 100 =$ **96.57%**

Example 33.5. *A 3-phase transformer, ratio 33/6.6-kV, Δ/Y, 2-MVA has a primary resistance of 8 Ω per phase and a secondary resistance of 0.08 ohm per phase. The percentage impedance is 7%. Calculate the secondary voltage with rated primary voltage and hence the regulation for full-load 0.75 p.f. lagging conditions.* **(Elect. Machine-I, Nagpur, Univ. 1993)**

Solution. F.L. secondary current $= \dfrac{2 \times 10^6}{\sqrt{3} \times 6.6 \times 10^3} = 175$ A

$K = 6.6/\sqrt{3} \times 33 = 1/8.65$; $R_{02} = 0.08 + 8/8.65^2 = 0.1867\ \Omega$ per phase

Now, secondary impedance drop per phase $= \dfrac{7}{100} \times \dfrac{6{,}600}{\sqrt{3}} = 266.7$ V

$\therefore$ Z_{02} = 266.7/175 = 1.523 Ω per phase

$$X_{02} = \sqrt{Z_{02}^2 - R_{02}^2} = \sqrt{1.523^2 - 0.1867^2} = 1.51\,\Omega/\text{phase}$$

Drop per phase $= I_2 (R_{02}\cos\phi + X_{02}\sin\phi) = 175\,(0.1867 \times 0.75 + 1.51 \times 0.66) = 200$ V

Secondary voltage/phase $= 6,600/\sqrt{3} = 3,810$ V $\quad\therefore\quad V_2 = 3,810 - 200 = 3,610$ V

$\therefore$ Secondary *line* voltage $= 3,610 \times \sqrt{3} =$ **6,250 V**

%regn. $= 200 \times 100/3,810 =$ **5.23 %**

Example 33.6. *A 100-kVA, 3-phase, 50-Hz 3,300/400 V transformer is Δ-connected on the h.v. side and Y-connected on the l.v. side. The resistance of the h.v. winding is 3.5 Ω per phase and that of the l.v. winding 0.02 Ω per phase. Calculate the iron losses of the transformer at normal voltage and frequency if its full-load efficiency be 95.8% at 0.8 p.f. (lag).*

(A.C. Machines-I, Jadavpur Univ. 1989)

Solution. F.L. output $= 100 \times 0.8 = 80$ kW; Input $= 80/0.958 = 83.5$ kW

Total loss $=$ Input – Output $= 83.5 - 80 = 3.5$ kW

Let us find full-load Cu losses for which purpose, we would first calculate R_{02}.

$$K = \frac{\text{secondary voltage/phase}}{\text{primary voltage/phase}} = \frac{400/\sqrt{3}}{3,300} = \frac{4}{33\sqrt{3}}$$

$$R_{02} = R_2 + K^2 R_1 = 0.02 + (4/\sqrt{3} \times 33)^2 \times 3.5 = 0.037\,\Omega$$

Full-load secondary phase current is $I_2 = 100,000/\sqrt{3} \times 400 = 144.1$ A

Total Cu loss $= 3 I_2^2 R_{02} = 3 \times 144.1^2 \times 0.037 = 2,305$ W

Iron loss $=$ Total loss – F.L. Cu loss $= 3,500 - 2,305 =$ **1,195 W**

Example 33.7. *A 5,000-kVA, 3-phase transformer, 6.6/33-kV, Δ/Y, has a no-load loss of 15 kW and a full-load loss of 50 kW. The impedance drop at full-load is 7%. Calculate the primary voltage when a load of 3,200 kW at 0.8 p.f. is delivered at 33 kV.*

Solution. Full-load $I_2 = 5 \times 10^6/\sqrt{3} \times 33,000 = 87.5$ A

Impedance drop/phase $= 7\%$ of $(33/\sqrt{3}) = 7\%$ of 19 kV $= 1,330$ V

$\therefore\quad Z_{02} = 1,330/87.5 = 15.3\,\Omega$/phase; F.L. Cu loss $= 50 - 15 = 35$ kW

$\therefore\quad 3 I_2 R_{02} = 35,000; \quad R_{02} = 35,000/3 \times 8.75^2 = 1.53\,\Omega$/phase

$\therefore\quad X_{02} = \sqrt{15.3^2 - 1.53^2} =$ **15.23 Ω**

When load is 3,200 kW at 0.8 p.f.

$I_2 = 3,200/\sqrt{3} \times 33 \times 0.8 = 70$ A; drop $= 70\,(1.53 \times 0.8 + 15.23 \times 0.6) = 725$ V/phase

$\therefore$ % regn. $= \dfrac{725 \times 100}{19,000} = 3.8\,\%$

Primary voltage will have to be increased by 3.8%.

$\therefore$ Primary voltage $= 6.6 + 3.8\%$ of $6.6 = 6.85$ kV $=$ **6,850 V**

Example 33.8. *A 3-phase transformer has its primary connected in Δ and its secondary in Y. It has an equivalent resistance of 1% and an equivalent reactance of 6%. The primary applied voltage is 6,600 V. What must be the ratio of transformation in order that it will deliver 4,800 V at full-load current and 0.8 power factor (lag) ?* **(Elect. Technology-II, Magadh Univ. 1991)**

Solution. Percentage regulation

$$= v_r \cos\phi + v_x \sin\phi$$

$= 1 \times 0.8 + 6 \times 0.6 = 4.4\%$

Induced secondary e.m.f. (line value)

= 4,800 + 4.4% of 4,800 = 5,010 V, as in Fig. 33.9.

Secondary phase voltage

$= 5,010/\sqrt{3} = 2,890$ V

Transformation ratio

K = 2,890/6,600 = **0.437.**

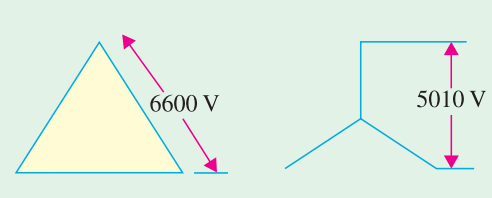


Fig. 33.9

Example 33.9. *A 2000-kVA, 6,600/400-V, 3-phase transformer is delta-connected on the high voltage side and star-connected on the low-voltage side. Determine its % resistance and % reactance drops, % efficiency and % regulation on full load 0.8 p.f. leading given the following data :*

S.C. test ; H.V. data : 400 V, 175 A and 17 kW

O.C. test; L.V. data : 400 V, 150 A and 15 kW **(Basic Elect., Machines Nagpur Univ. 1993)**

Solution. From S.C. test data, we have

Primary voltage/phase = 400 V; Primary current/phase = $175/\sqrt{3} = 100$ A

$$\therefore \quad Z_{01} = \frac{400}{101} = 3.96\ \Omega$$

$$I_1^2 R_{01} = \frac{17000}{3} \text{ or } R_{01} = 0.555\ \Omega;\ X_{01} = \sqrt{3.96^2 - 0.555^2} = 3.92\ \Omega$$

$$\%\, R = \frac{I_1 R_{01}}{V_1} \times 100 = \frac{101 \times 0.555}{6,600} \times 100 = \mathbf{0.849}$$

$$\%\, X = \frac{I_1 X_{01}}{V_1} \times 100 = \frac{101 \times 3.92}{6,600} \times 100 = \mathbf{6}$$

$$\%\ \text{regn} = v_r \cos\phi - v_x \sin\phi = 0.49 \times 0.8 - 6 \times 0.6 = -2.92\%$$

Full-load primary line current can be found from

$$\sqrt{3} \times 6,600 \times I_1 = 2000 \times 1,000;\ I_1 = 175\ \text{A}$$

It shows that S.C. test has been carried out under full-load conditions.

Total losses = 17 + 15 = 32 kW; F.L. output = $2,000 \times 0.8$ = 1600 kW

η = 1,600/1,632 = 0.98 or **98%**

Example 33.10. *A 3-ph, delta/star connected 11,000/440 V, 50 Hz transformer takes a line current of 5 amp, when secondary Load of 0.8 Lagging p.f. is connected. Determine each coil-current and output of transformer.* **(Amravati Univ. 1999)**

Solution. Due to delta/star connections the voltage ratings of the two sides on per phase basis are :

Primary coil rating = 11,000 V, Secondary coil rating = $\frac{440}{\sqrt{3}} = 254$ volts

Primary coil-current = $5/\sqrt{3} = 2.887$ amp

Each coil is delivering equal volt. amps.

Since three phase volt – amps = $3 \times 11,000 \times 5/\sqrt{3}$

= 95266

Volts amps/phase = 31755

This corresponds to the secondary coil-current of I_2, given by

$I_2 = \frac{31755}{254} = 125$ amp. This is shown in Fig. 33.10.

Total Output of transformer, in kVA = 31,755

Since, the p.f. given is 0.8 lagging.

The total output power in kW = $31{,}755 \times 0.80$ = **25.4 kW**

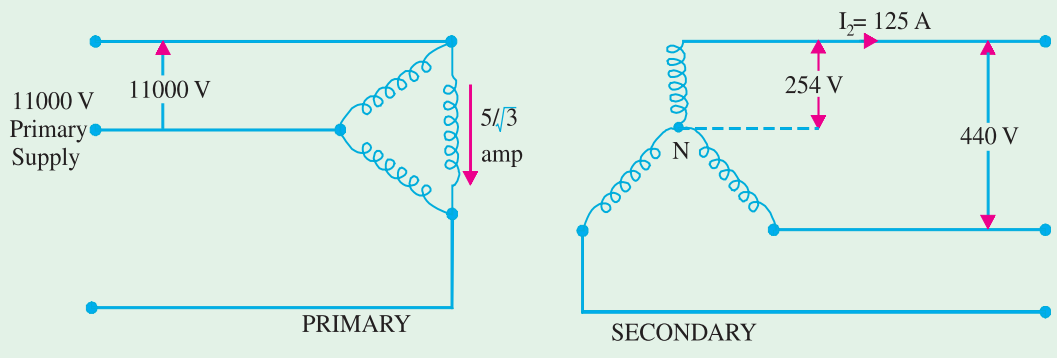


Fig. 33.10. Transformer coil currents

Example 33.11. *A load of 1000 kVA at 0.866 p.f. lagging is supplied by two 3 phase transformers of 800 kVA capacity operating in parallel. Ratio of transformation is same : 6600/400 V, delta/star. If the equivalent impedances referred to secondary are (0.005 + j.015) ohm and (0.012 + j 0.030) ohm per phase respectively. Calculate load and power factor of each transformer.*

(Amravati Univ. 1999)

Solution. Total load = 1000 kVA

$\cos \phi$ = 0.866 Lag, $\phi = 30°$ lag

Total power output = 866 kW

Secondary current with star connection,

$$I_2 = \frac{1000}{\sqrt{3} \times 440} \times 1000 = \textbf{1312.2 amp}$$

If the two transformers are identified as *A* and *B*, with their parameters with subscripts of *a* and *b*, we have :

$\dot{Z}_a = 0.005 + j\,0.015 = 0.0158 \angle 71.56°$ ohm

$\dot{Z}_b = 0.012 + j\,0.030 = 0.0323 \angle 68.2°$ ohm

$\dot{Z}_a + \dot{Z}_b = 0.017 + j\,0.045 = 0.0481 \angle 69.3°$ ohm

$\dot{I}_{2a}$ = secondary current of transformer *A*

$\dot{I}_{2b}$ = secondary current of transformer *B*

$$\dot{I}_{2a} = \frac{\dot{Z}_b}{\dot{Z}_b + \dot{Z}_a} \times \dot{I}_2 = \frac{0.0323 \angle 68.2°}{0.0481 \angle 69.3°} \times 1312 \angle -30°$$

$$= 88.1 \angle -31.1°$$

$$\dot{I}_{2b} = \frac{\dot{Z}_a}{\dot{Z}_a + \dot{Z}_b} \times \dot{I}_2 = \frac{0.0158 \angle 71.56°}{0.0481 \angle 69.3°} \times 1312 \angle -30°$$

$$= 43.1 \angle -27.74°$$

For Transformer *A*

Load = $3 \times 254 \times 881 \times 10^{-3}$ = 671.3 kVA

Power factor = cos 31.1° lag = 0.856 lag

For Transformer *B*

Load = $3 \times 254 \times 431 \times 10^{-3}$ = 328.4 kVA

Power factor = cos 23.74° lag = 0.885 lag

Check : Total kW gives a check.

1000 kVA at 0.866 lag means 866 kW.

Output, in kW, of transformer *A* = 671.3×0.856 = **574.6 kW**

Output in kW of transformer *B* = 328.4×0.885 = **290.6 kW**

Sum of these two outputs = 574.6 + 290.6 = **865.2 kW**

Note. Total kVAR also gives a check.

Depending on leading or lagging p.f., appropriate sign (+ *ve* or – *ve*) must be assigned to the kVAR-term.

Tutorial Problem No. 33.1

1. A 3-phase, star-connected alternator generates 6,360 V per phase and supplies 500 kW at a p.f. 0.9 lagging to a load through a step-down transformer of turns 40 : 1. The transformer is delta connected on the primary side and star-connected on the secondary side. Calculate the value of the line volts at the load. Calculate also the currents in (*a*) alternator windings (*b*) transformer primary windings (*c*) transformen secondary windings. **[476 V (*a*) 29.1 A (*b*) 16.8 A (*c*) 672 A]**

2. A 11,000/6,600 V, 3-ϕ, transformer has a star-connected primary and a delta-connected secondary. It supplies a 6.6 kV motor having a star-connected stator, developing 969.8 kW at a power factor of 0.9 lagging and an efficiency of 92 per cent. Calculate (*i*) motor line and phase currents (*ii*) transformer secondary current and (*iii*) transformer primary current.

[(*a*) Motor ; $I_L = I_{ph}$ = 126.3 A (*b*) phase current 73 A (*c*) 75.8 A]

33.7. Open-Delta or *V* – *V* connection

If one of the transformers of a Δ – Δ is removed and 3-phase supply is connected to the primaries as shown in Fig. 33.11, then three equal 3-phase voltages will be available at the secondary terminals on no-load. This method of transforming 3-phase power by means of only two transformers is called the open – Δ or *V* – *V* connection.

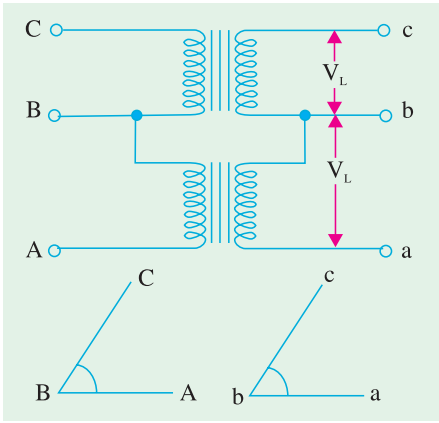


Fig. 33.11

It is employed :

1. when the three-phase load is too small to warrant the installation of full three-phase transformer bank.
2. when one of the transformers in a Δ – Δ bank is disabled, so that service is continued although at reduced capacity, till the faulty transformer is repaired or a new one is substituted.
3. when it is anticipated that in future the load will increase necessitating the closing of open delta.

One important point to note is that the total load that can be carried by a *V* – *V* bank is *not* two-third of the capacity of a Δ – Δ bank but it is only 57.7% of it. That is a reduction of 15% (strictly, 15.5%) from its normal rating. Suppose there is Δ – Δ bank of three 10-kVA transformers. When one transformer is removed, then it runs in *V* – *V*. The total rating of the two transformers is 20 kVA. But the capacity of the *V* – *V* bank is not the sum of the transformer kVA ratings but only 0.866 of it *i.e.* $20 \times 0.866 = 17.32$ (or $30 \times 0.57 = 17.3$ kVA). The fact that the ratio of V-capacity to Δ-capacity is $1/\sqrt{3}$ = 57.7% (or nearly 58%) instead of $66\frac{2}{3}$ per cent can be proved as follows :

As seen from Fig. 33.12 (*a*)

$\Delta - \Delta$ capacity $= \sqrt{3} \,.\, V_L \,.\, I_L = \sqrt{3} \,.V_L \,(\sqrt{3} \,.I_S) = 3V_L I_S$

In Fig. 33.12 (*b*), it is obvious that when $\Delta - \Delta$ bank becomes $V - V$ bank, the secondary line current I_L becomes equal to the secondary phase current I_S.

$$\therefore \qquad V - V \text{ capacity} = \sqrt{3} \,.V_L I_L = \sqrt{3}\, V_L \,.\, I_S$$

$$\therefore \qquad \frac{V - V \text{ capacity}}{\Delta - \Delta \text{ capacity}} = \frac{\sqrt{3} \,.V_L I_S}{3 V_L \, I_S} = \frac{1}{\sqrt{3}} = 0.577 \text{ or } 58 \text{ per cent}$$

It means that the 3-phase load which can be carried ***without exceeding the ratings*** of the transformers is 57.7 per cent of the original load rather than the expected 66.7%.

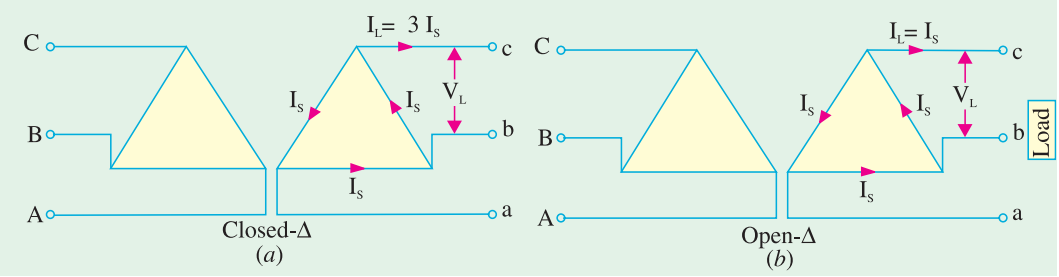


Fig. 33.12

It is obvious from above that when one transformer is removed from a $\Delta - \Delta$ bank.

1. the bank capacity is reduced from 30 kVA to $30 \times 0.577 = 17.3$ kVA and not to 20 kVA as might be thought off-hand.
2. only 86.6% of the rated capacity of the two remaining transformers is available (*i.e.* $20 \times 0.866 = 17.3$ kVA). In other words, ratio of *operating* capacity to *available* capacity of an open-Δ is 0.866. This factor of 0.866 is sometimes called the ***utility factor***.
3. each transformer will supply 57.7% of load and not 50% when operating in $V - V$ (Ex. 33.13).

However, it is worth noting that if three transformers in a $\Delta - \Delta$ bank are delivering their *rated load** and one transformer is removed, the overload on *each* of the two remaining transformers is 73.2% because

$$\frac{\text{total load in } V - V}{VA/\text{transformer}} = \frac{\sqrt{3} \,.V_L I_S}{V_L I_S} = \sqrt{3} = 1.732$$

This over-load may be carried temporarily but some provision must be made to reduce the load if overheating and consequent breakdown of the remaining two transformers is to be avoided.

The disadvantages of this connection are :

1. The average power factor at which the *V*-bank operates is less than that of the load. ***This power factor is actually 86.6% of the balanced load power factor.*** Another significant point to note is that, except for a balanced unity power factor load, the two transformers in the $V - V$ bank operate at different power factors (Art. 33.8).
2. Secondary terminal voltages tend to become unbalanced to a great extent when the load is increased, this happens even when the load is perfectly balanced.

It may, however, be noted that if two transformers are operating in $V - V$ and loaded to rated capacity (in the above example, to 17.3 kVA), the addition of a third transformer increases the total capacity by $\sqrt{3}$ or 173.2% (*i.e.* to 30 kVA). It means that for an increase in cost of 50% for the third transformer, the increase in capacity is 73.2% when converting from a $V - V$ system to a $\Delta - \Delta$ system.

* In Ex. 33.13, the three transformers are not supplying their rated load of $20 \times 3 = 60$ kVA but only 40 kVA.

33.8. Power Supplied by $V - V$ Bank

When a $V-V$ bank of two transformers supplies a balanced 3-phase load of power factor cos ϕ, then one transformer operates at a p.f. of cos $(30° - \phi)$ and the other at cos $(30° + \phi)$. Consequently, the two transformers will not have the same voltage regulation.

$\therefore$ P_1 = kVA cos $(30° - \phi)$ and P_2 = kVA cos $(30° + \phi)$

(*i*) When $\phi = 0$ *i.e.* load p.f. = 1

Each transformer will have a p.f. = cos 30° = 0.866

(*ii*) When $\phi = 30°$ *i.e.* load p.f. = 0.866.

In this case, one transformer has a p.f. of cos $(30° - 30°) = 1$ and the other of cos $(30° + 30°) = 0.866$.

(*iii*) When $\phi = 60°$ *i.e.* load p.f. = 0.5

In this case, one transformer will have a p.f. = cos $(30 - 60°)$ = cos $(-30°) = 0.866$ and the other of cos $(30° + 60°) = 0$. It means that one of the transformers will not supply any load whereas the other having a p.f. = 0.866 will supply the entire load.

Example 33.12. *What should be the kVA rating of each transformer in a $V - V$ bank when the 3-phase balanced load is 40 kVA ? If a third similar transformer is connected for operation, what is the rated capacity ? What percentage increase in rating is affected in this way ?*

Solution. As pointed out earlier, the kVA rating of each transformer has to be 15% greater.

$\therefore$ kVA/trasformer = $(40/2) \times 1.15 =$ **23**

$\Delta - \Delta$ bank rating = $23 \times 3 = 69$; Increase = $[(69 - 40)/40] \times 100 =$ **72.5%**

Example 33.13. *A $\Delta - \Delta$ bank consisting of three 20-kVA, 2300/230-V transformers supplies a load of 40 kVA. If one transformer is removed, find for the resulting $V - V$ connection*

(*i*) *kVA load carried by each transformer*

(*ii*) *per cent of rated load carried by each transformer*

(*iii*) *total kVA rating of the V-V bank*

(*iv*) *ratio of the V-V bank to $\Delta - \Delta$ bank transformer ratings.*

(*v*) *per cent increase in load on each transformer when bank is converted into V-V bank.*

Solution. (*i*) As explained earlier in Art. 33.7, $\dfrac{\text{total kVA load in } V-V \text{ bank}}{VA/\text{transformer}} = \sqrt{3}$

$\therefore$ kVA load supplied by *each* of the two transformers = $40/\sqrt{3}$ = **23.1 kVA**

Obviously, each transformer in V - V bank does not carry 50% of the original load but 57.7%.

(*ii*) per cent of rated load = $\dfrac{\text{kVA load/transformer}}{\text{kVA rating/transformer}} = \dfrac{23.1}{20}$ = **115.5 %**

carried by each transformer.

Obviously, in this case, each transformer is overloaded to the extent of 15.5 per cent.*

(*iii*) kVA *rating* of the V - V bank = $(2 \times 20) \times 0.866 =$ **34.64 kVA**

(*iv*) $\dfrac{V-V \text{ rating}}{\Delta - \Delta \text{ rating}} = \dfrac{34.64}{60} = 0.577$ or **57.7%**

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* Overloading becomes 73.2% only when full rated load is supplied by the $\Delta - \Delta$ bank (*i.e.* $3 \times 20 = 60$ kVA in this case) before it becomes $V - V$ bank.

As seen, the rating is reduced to 57.7% of the original rating.

(*v*) Load supplied by each transformer in $\Delta - \Delta$ bank = 40/3 = 13.33 kVA

$\therefore$ Percentage increase in load supplied by each transformer

$$= \frac{\text{kVA load/transformer in } V-V \text{ bank}}{\text{kVA load/transformer in } \Delta-\Delta \text{ bank}} = \frac{23.1}{13.3} = 1.732 = \mathbf{173.2\ \%}$$

It is obvious that each transformer in the $\Delta - \Delta$ bank supplying 40 kVA was running underloaded (13.33 *vs* 20 kVA) but runs overloaded (23.1 *vs* 20 kVA) in $V - V$ connection.

Example 33.14. *A balanced 3-phase load of 150 kW at 1000 V, 0.866 lagging power factor is supplied from 2000 V, 3-phase mains through single-phase transformers (assumed to be ideal) connected in (i) delta-delta (ii) Vee-Vee. Find the current in the windings of each transformer and the power factor at which they operate in each case. Explain your calculations with circuit and vector diagrams.*

Solution. (*i*) Delta-Delta Connection

$$\sqrt{3}\ V_L I_L \cos\phi = 150{,}000$$

$$\sqrt{3} \times 1000 \times I_L \times 0.866 = 150{,}000 \quad \therefore I_L = 100 \text{ A}$$

$\therefore$ Secondary line current = 100 A; secondary phase current = $100/\sqrt{3}$ = **57.7 A**

Transformation ratio = 1000/2000 = 1/2

$\therefore$ Primary phase current = 57.7/2 = **28.85 A**

(*ii*) Vee-Vee Connection

Let I be the secondary line current which is also the phase current in $V - V$ connection. Then

$$\sqrt{3} \times 1000 \times I \times 0.866 = 150{,}000 \quad \therefore \quad I = 100 \text{ A}$$

$\therefore$ Secondary phase current = **100 A**; primary phase current = $100 \times 1/2$ = **50 A**

Transformer power factor = 86.6 per cent of 0.866 = **0.75 (lag).**

Example 33.15. (*a*) *Two identical 1-phase transformers are connected in open-delta across 3-phase mains and deliver a balanced load of 3000 kW at 11 kV and 0.8 p.f. lagging. Calculate the line and phase currents and the power factors at which the two transformers are working.*

(*b*) *If one more identical unit is added and the open delta is converted to closed delta, calculate the additional load of the same power factor that can now be supplied for the same temperature rise. Also calculate the phase and line currents.* **(Elect. Machinery-I, Madras Univ. 1987)**

Solution. (*a*) If I is the line current, then

$$\sqrt{3} \times 11{,}000 \times I \times 0.8 = 3{,}000{,}000 \qquad \mathbf{I = 197\ A}$$

Since, this also represents the phase current,

$\therefore$ Secondary phase current = **197 A**; Transformer p.f. = 86.6 per cent of 0.8 = **0.693**

(*b*) Additional load = 72.5 per cent of 3000 = **2175 kW**

Total load = 3000 + 2175 = 5175 kW

Now, $\sqrt{3} \times V_L I_L \cos\phi = 5{,}175{,}000$ or $\sqrt{3} \times 11{,}000 \times I_L \times 0.8 = 5{,}175{,}000$

$\therefore$ I_L = **340 A;** phase current = $340/\sqrt{3}$ = **196 A**

Example 33.16. *Two transformers connected in open delta supply a 400-kVA balanced load operating at 0.866 p.f. (lag). The load voltage is 440 V. What is the (a) kVA supplied by each transformer ? (b) kW supplied by each transformer ?* **(Elect. Machines-I, Gwalior Univ. 1991)**

Solution. As stated in Art 33.7, the ratio of operating capacity to available capacity in an open-Δ is 0.866. Hence, kVA of each transformer is one-half of the total kVA load divided by 0.866.

(a) kVA of each transformer $= \dfrac{(400/2)}{0.866} =$ **231 kVA**

(b) As stated in Art 33.8, the two transformers have power factors of cos (30° – ϕ) and cos (30° + ϕ).

$\therefore \quad P_1 = \text{kVA} \cos(30° - \phi)$ and $P_2 = \text{kVA} \cos(30 + \phi)$

Now, load p.f. $= \cos\phi = 0.866; \quad \phi = \cos^{-1}(0.866) = 30°$

$\therefore \quad P_1 = 231 \times \cos 0° =$ **231 kW**; $P_2 = 231 \times \cos 60° =$ **115.5 kW**

Obviously, $P_1 + P_2$ must equal $400 \times 0.86 = 346.5$ kW

Tutorial Problem No. 33.2

1. Three 1100/110-V transformers connected delta-delta supply a lighting load of 100 kW. One of the transformers is damaged and removed for repairs. Find

(*a*) What currents were flowing in each transformer when the three transformers were in service ?

(*b*) What current flows in each transformer when the third is removed ? and

(*c*) The output kVA of each transformer if the transformers connected in open Δ supply the full-load with normal heating ?

[(*a*) primary = 30.3 A; secondary 303 A (*b*) primary = 30.3 $\sqrt{3}$A ; secondary = 303$\sqrt{3}$A (*c*) 33.33 kVA] ***(Elect. Machines-I, Gwalior Univ. Apr. 1977)***

33.9. Scott Connection or *T* - *T* Connection

This is a connection by which 3-phase to 3-phase transformation is accomplished with the help of two transformers as shown in Fig. 33.13. Since it was first proposed by Charles *F*. Scott, it is frequently referred to as Scott connection. This connection can also be used for 3-phase to 2-phase transformation as explained in Art. 33.10.

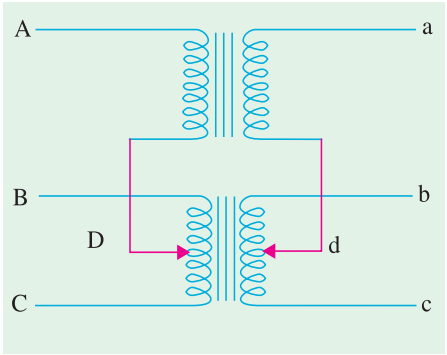


Fig. 33.13

One of the transformers has centre taps both on the primary and secondary windings (Fig. 33.13) and is known as the ***main*** transformer. It forms the horizontal member of the connection (Fig. 33.14).

The other transformer has a 0.866 tap and is known as ***teaser*** transformer. One end of both the primary and secondary of the teaser transformer is joined to the centre taps on both primary and secondary of the main transformer respectively as shown in Fig. 33.14 (*a*). The other end *A* of the teaser primary and the two ends *B* and *C* of the main transformer primary are connected to the 3-phase supply.

The voltage diagram is shown in Fig. 33.14 (*a*) where the 3-phase supply line voltage is assumed to be 100 V and a transformation ratio of unity. For understanding as to how 3-phase transformation results from this arrangement, it is desirable to think of the primary and secondary vector voltages as forming geometrical T_S' (from which this connection gets its name).

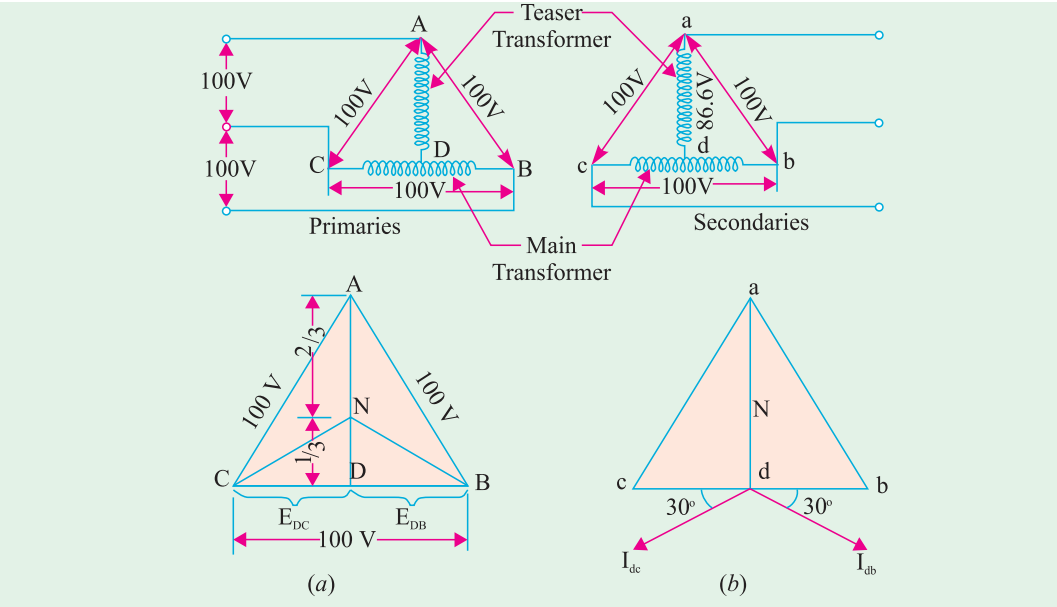


Fig. 33.14

In the primary voltage *T* of Fig. 33.14 (*a*), E_{DC} and E_{DB} are each 50 *V* and differ in phase by 180°, because both coils *DB* and *DC* are on the same magnetic circuit and are connected in opposition. Each side of the equilateral triangle represents 100 V. The voltage E_{DA} being the altitude of the equilateral triangle is equal to $(\sqrt{3}/2) \times 100 = 86.6$ V and lags behind the voltage across the main by 90°. The same relation holds good in the secondary winding so that *abc* is a symmetrical 3-phase system.

With reference to the secondary voltage triangle of Fig. 33.14 (*b*), it should be noted that for a load of unity power factor, current I_{db} lags behind voltage E_{db} by 30° and I_{dc} leads E_{dc} by 30°. In other words, the teaser transformer and each half of the main transformer, all operate at different power factors.

Obviously, the full rating of the transformers is not being utilized. The teaser transformer operates at only 0.866 of its rated voltage and the main transformer coils operate at cos 30° = 0.866 power factor, which is equivalent to the main transformer's coils working at 86.6 per cent of their kVA rating. Hence the capacity to rating ratio in a *T–T.* connection is 86.6% — the same as in *V – V* connection if two identical units are used, although heating in the two cases is not the same.

If, however, both the teaser primary and secondary windings are designed for 86.6 volts only, then they will be operating at full rating, hence the combined rating of the arrangement would become (86.6 + 86.6)/(100 + 86.6) = 0.928 of its total rating.* In other words, ratio of kVA utilized to that available would be 0.928 which makes this connection more economical than open-Δ with its ratio of 0.866.

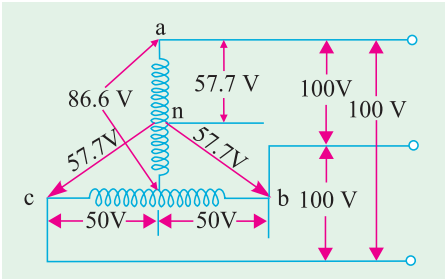


Fig. 33.15

* Alternatively, *VA* capacity available is = $V_L I_L + (0.866\ V_L) I_L = 1.866\ V_L I_L$ where I_L is the primary line current. Since 3-phase power is supplied, volt-amperes actually utilized = $1.732\ V_L I_L$. Hence, ratio of kVA actually utilized to those available is = $1.732\ V_L I_L / 1.866\ V_L I_L = 0.928$.

Fig. 33.15 shows the secondary of the $T-T$ connection with its different voltages based on a nominal voltage of 100 V. As seen, the neutral point n is one third way up from point d. If secondary voltage and current vector diagram is drawn for load power factor of unity, it will be found that

1. current in teaser transformer is in phase with the voltage.
2. in the main transformer, current leads the voltage by 30° across one half but lags the voltage by 30° across the other half as shown in Fig. 33.14 (*b*).

Hence, when a balanced load of p.f. = cos ϕ, is applied, the teaser current will lag or lead the voltage by Φ while in the two halves of the main transformer, the angle between current and voltage will be (30° – Φ) and (30° + Φ). The situation is similar to that existing in a $V-V$ connection.

Example 33.17. *Two T-connected transformers are used to supply a 440-V, 33-kVA balanced load from a balanced 3-phase supply of 3300 V. Calculate (a) voltage and current rating of each coil (b) kVA rating of the main and teaser transformer.*

Solution. **(*a*)** Voltage across main primary is 3300 V whereas that across teaser primary is $= 0.866 \times 3300 = 2858$ V.

The current is the same in the teaser and the main and equals the line current.

$\therefore \quad I_{LP} = 33,000/\sqrt{3} \times 3300 = 5.77$ A —Fig. 33.16

The secondary main voltage equals the line voltage of 440 V whereas teaser secondary voltage $= 0.866 \times 440 = 381$ V.

The secondary line current, $I_{LS} = I_{LP}/k = 5.77/(440/3300) = 43.3$ A as shown in Fig. 33.16.

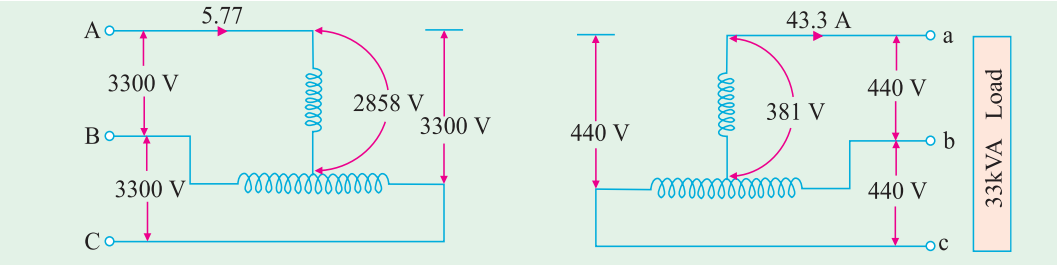


Fig. 33.16

(*b*) Main kVA $= 3300 \times 5.77 \times 10^{-3} =$ **19 kVA**

Teaser kVA $= 0.866 \times$ main kVA $= 0.866 \times 19 =$ **16.4 kVA**

33.10. Three-phase to Two-phase Conversion and *vice-versa*

This conversion is required to supply two-phase furnaces, to link two-phase circuit with 3-phase system and also to supply a 3-phase apparatus from a 2-phase supply source. For this purpose, Scott connection as shown in Fig. 33.17 is employed. This connection requires two transformers of different ratings although for interchangeability and provision of spares, both transformers may be identical but having suitable tappings.

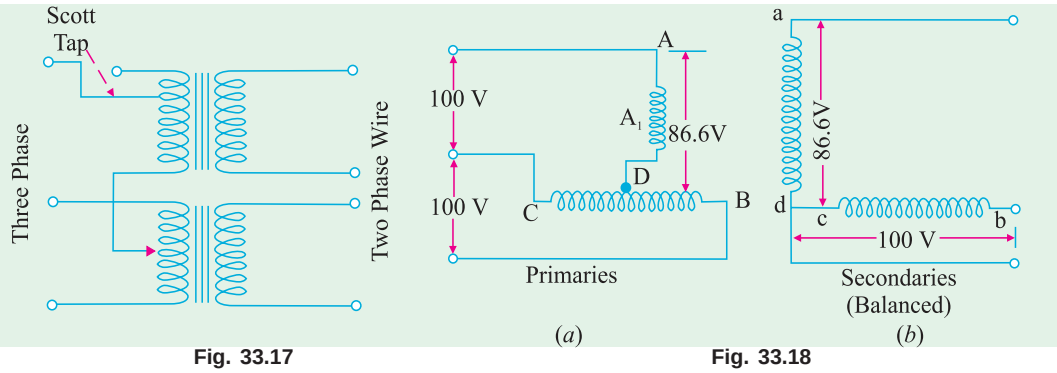


Fig. 33.17 **Fig. 33.18**

If, in the secondaries of Fig. 33.14 (*b*), points *c* and *d* are connected as shown in Fig. 33.18 (*b*), then a 2-phase, 3-wire system is obtained. The voltage E_{dc} *is 86.6 V but* E_{Cb} = 100 V, hence the

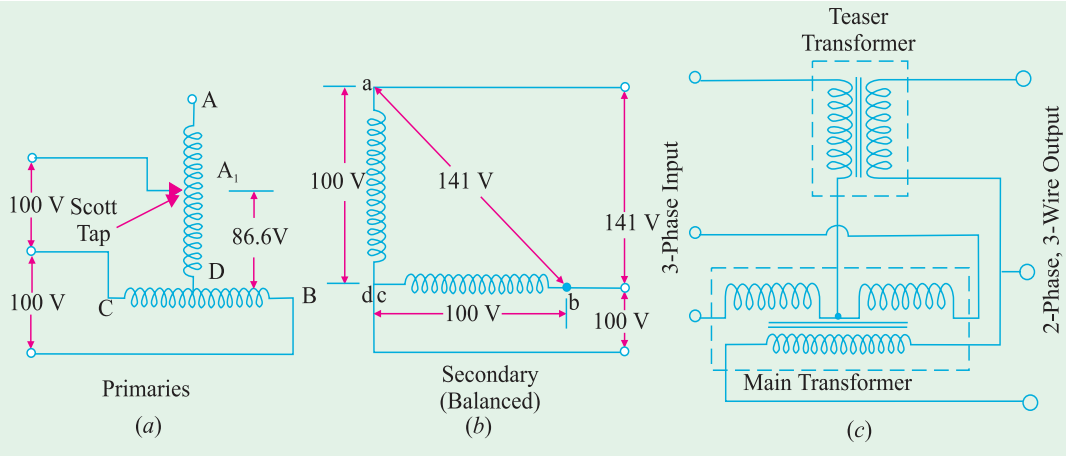


Fig. 33.19

resulting 2-phase voltages will be unequal. However, as shown in Fig. 33.19 (*a*) if the 3-phase line is connected to point A_1, such that DA_1 represents 86.6% of the teaser primary turns (which are the same as that of main primary), then this will increase the volts/turn in the ratio of 100 : 86.6, because now 86.6 volts are applied across 86.6 per cent of turns and not 100% turns. In other words, this will make volts/turn the same both in primary of the teaser and that of the main transformer. If the secondaries of both the transformers have the same number of turns, then secondary voltage will be equal in magnitude as shown, thus resulting in a symmetrical 2-phase, 3-wire system.

Consider the same connection drawn slightly differently as in Fig. 33.20. The primary of the main transformer having N_1 turns is connected between terminals *CB* of a 3-phase supply. If supply line voltage is *V*, then obviously $V_{AB} = V_{BC} = V_{CA} = V$ but voltage between *A* and *D* is $V \times \sqrt{3}/2$. As said

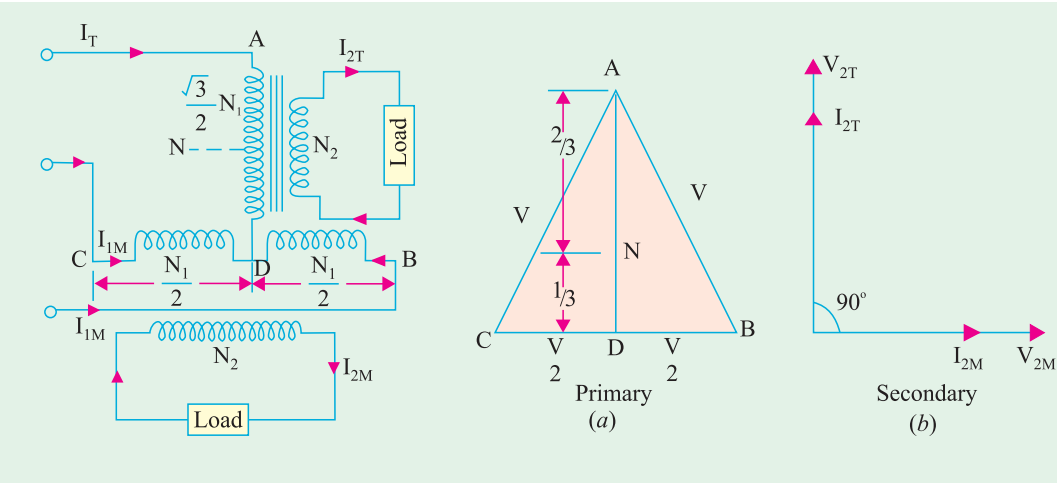


Fig. 33.20 **Fig. 33.21**

above, the number of turns between *A* and *D* should be also $(\sqrt{3}/2)\ N_1$ for making volt/turn the same in both primaries. If so, then for secondaries having equal turns, the secondary terminal voltages will be equal in magnitude although in phase quadrature.

It is to be noted that point D is not the neutral point of the primary supply because its voltage with respect to any line is not $V/\sqrt{3}$. Let N be the neutral point. Its position can be determined as follows. Voltage of N with respect to A must be $V/\sqrt{3}$ and since D to A voltage is $V \times \sqrt{3}/2$, hence N will be $(\sqrt{3}V/2 - V/\sqrt{3})$ = 0.288 V or 0.29 V from D. Hence, N is above D by a number of turns equal to 29% of N_1. Since 0.288 is one-third of 0.866, hence N divides the teaser winding AD in the ratio 2 : 1.

Let the teaser secondary supply a current I_{2T} at unity power factor. If we neglect the magnetizing current I_0, then teaser primary current is $I_{1T} = I_{2T} \times$ transformation ratio.

$\therefore \quad I_{1T} = I_{2T} \times N_2/(\sqrt{3}N_1/2) = (2/\sqrt{3}) \times (N_2/N_1) \times I_{2T} = 1.15\,(N_2/N_1)\,I_{2T} = 1.15\,KI_{2T}$ where $K = N_2/N_1$ = transformation ratio of ***main*** transformer. The current is in phase with star voltage of the primary supply (Fig. 33.21).

The total current I_{1M} in each half of the primary of the ***main*** transformer consists of two parts :

(i) One part is that which is necessary to balance the main secondary current I_{2M}. Its value is

$$= I_{2M} \times \frac{N_2}{N_1} = KI_{2M}$$

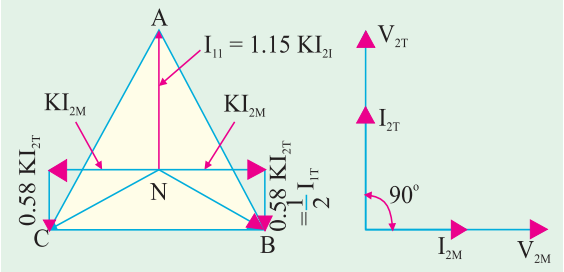


Fig. 33.22

(ii) The second part is equal to one-half of the teaser primary current *i.e.* $\frac{1}{2} I_{1T}$. This is so because the main transformer primary forms a return path for the teaser primary current which divides itself into two halves at mid-point D in either direction. The value of each half is = $I_{1T}/2 = 1.15\,KI_{2T}/2 = 0.58\,KI_{2T}$.

Hence, the currents in the lines B and C are obtained vectorially as shown in Fig. 33.22. It should be noted that as the two halves of the teaser primary current flow in opposite directions from point D, they have no magnetic effect on the core and play no part at all in balancing the secondary ampere-turns of the main transformer.

The line currents thus have rectangular components of KI_{2M} and 0.58, KI_{2T} and, as shown in Fig. 33.22, are in phase with the primary star voltages V_{NB} and V_{NC} and are equal to the teaser primary current. Hence, the three-phase side is balanced when the two-phase load of unity power factor is balanced.

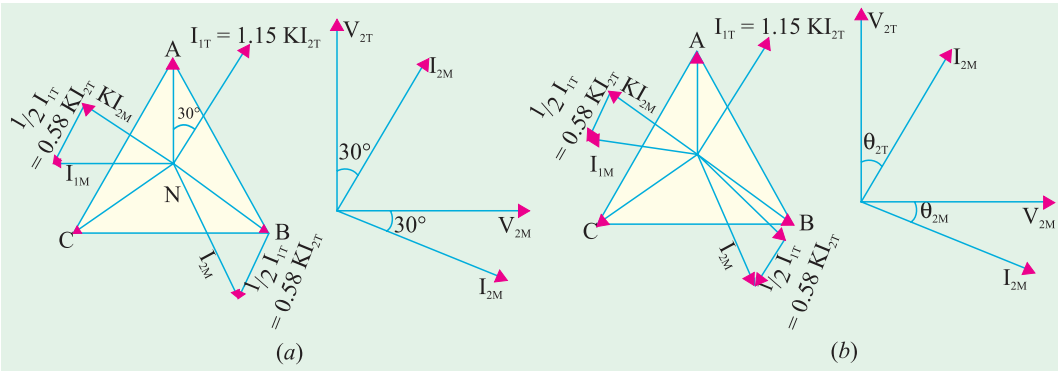


Fig. 33.23

Fig. 33.23 (*a*) illustrates the condition corresponding to a balanced two-phase load at a lagging power factor of 0.866. The construction is the same as in Fig. 33.22. It will be seen that the 3-phase side is again balanced. But under these conditions, the main transformer rating is 15% greater than that of the teaser, because its voltage is 15% greater although its current is the same.

Hence, **we conclude *that if the load is balanced on one side, it would always be balanced on the other side.***

The conditions corresponding to an unbalanced two-phase load having different currents and power factors are shown in Fig. 33.23 (*b*). The geometrical construction is similar to those explained in Fig. 33.22 and 33.23 (*a*).

Summarizing the above we have :

1. Teaser transformer primary has $\sqrt{3}/2$ times the turns of main primary. But volt/turn is the same. Their secondaries have the same turns which results in equal secondary terminal voltages.

2. If main primary has N_1 turns and main secondary has N_2 turns, then main transformation ratio is N_2/N_1. However, the transformation ratio of teaser is

$$N_2/(\sqrt{3}N_1/2) = 1.15\ N_2/N_1 = 1.15\ K$$

3. If the load is balanced on one side, it is balanced on the other side as well.

4. Under balanced load conditions, main transformer rating is 15% greater than that of the teaser.

5. The currents in either of the two halves of main primary are the vector sum of KI_{2M} and $0.58\ KI_{2T}$ (or $\frac{1}{2} I_{1T}$).

Example 33.18. *Two transformers are required for a Scott connection operating from a 440-V, 3-phase supply for supplying two single-phase furnaces at 200 V on the two-phase side. If the total output is 150 kVA, calculate the secondary to primary turn ratio and the winding currents of each transformer.*

Solution. Main Transformer

Primary volts = 440 V ; secondary volts = 200 V $\therefore \frac{N_2}{N_1} = \frac{200}{440} = \frac{1}{2.2}$

Secondary current = $150{,}000/2 \times 200 =$ **375 A**

$\therefore$ Primary current = $375 \times 1/_{2.2} =$ **197 A**

Teaser Transformer

Primary volts = $(\sqrt{3}/2 \times 440) = 381$ V : Secondary volts = 200 V

$$\frac{\text{secondary turns}}{\text{primary turns}} = \frac{200}{381} = \frac{1}{1.905}$$ (also teaser ratio = $1.15 \times 1/2.2 = 1/1.905$)

Example 33.19. *Two single-phase furnaces working at 100 V are connected to 3300-V, 3-phase mains through Scott-connected transformers. Calculate the current in each line of the 3-phase mains when the power taken by each furnace is 400-kW at a power factor of 0.8 lagging. Neglect losses in the transformers.* **(Elect. Machines-III, South Gujarat Univ. 1988)**

Solution. Here $K = 100/3{,}300 = 1/33$ (main transformer)

$$I_2 = \frac{400{,}000}{0.8 \times 100} = 5{,}000\text{A}$$ (Fig. 33.24); Here $I_{2T} = I_{2M} = I_2 = 5{,}000$ A

As the two-phase load is balanced, the 3-phase side is also balanced.

Primary phase currents are = $1.15\ KI_2 = 1.15 \times (1/33) \times 5{,}000 = 174.3$ A

Since for a star-connection, phase current is equal to line current,

$\therefore$ Line current = **174.3 A**

Note. We have made use of the fact that since secondary load is balanced, primary load is also balanced. If necessary, I_{1M} can also be found.

I_{1M} is the vector sum of (*i*) KI_{2M} and (*ii*) $\frac{1}{2}\ I_{1T}$ or $0.58\ KI_{2T}$.

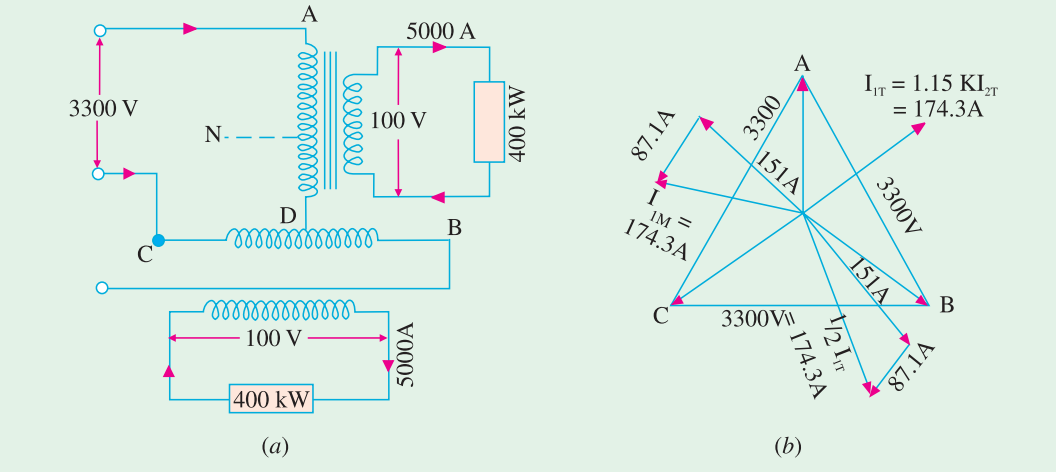


Fig. 33.24

Now, $KI_{2M} = (1/33) \times 5{,}000 = 151$ A and $0.58\ KI_2 = \frac{1}{2}\ I_{1T} = 174.3/2 = 87.1$ A

$\therefore \quad I_{1M} = \sqrt{151^2 + 87.1^2} = \mathbf{174.3\ A}$

Example 33.20. *In a Scott-connection, calculate the values of line currents on the 3-phase side if the loads on the 2-phase side are 300 kW and 450 kW both at 100 V and 0.707 p.f. (lag) and the 3-phase line voltage is 3,300 V. The 300-kW load is on the leading phase on the 2-phase side. Neglect transformer losses.* **(Elect. Technology, Allahabad Univ. 1991)**

Solution. Connections are shown in Fig. 33.25 (*a*) and phasor diagram in Fig. 33.25 (*b*).

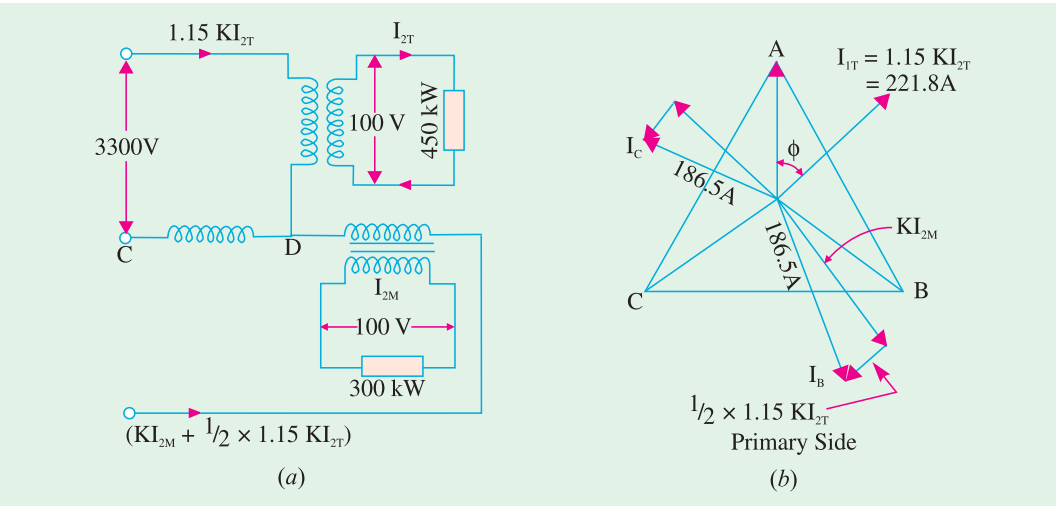


Fig. 33.25

Here, $K = 100/3{,}300 = 1/33$

Teaser secondary current is $I_{2T} = 450{,}000/100 \times 0.707 = 6360$ A

Teaser primary current is $I_{1T} = 1.15\ KI_{2T} = 1.5 \times (1/33) \times 6360 = 221.8$ A

As shown in Fig. 33.25 (*b*), main primary current I_{1M} has two rectangular components.

(*i*) KI_{2M} where I_{2M} is the secondary current of the main transformer and

(*ii*) Half of the teaser primary current $\frac{1}{2} I_{1T} = \frac{1}{2} \times 1.15\, KI_{2T} = 0.577\, KI_{2T}$

Now $KI_{2M} = \frac{1}{33} \times \frac{300,000}{100 \times 0.707} = 128.58$ A ; Also $\frac{1}{2} I_{1T} = \frac{1}{2} \times 221.8 = 110.9$A

Main Primary current $= \sqrt{128.58^2 + 110.9^2} = 169.79$A

Hence, the 3-phase line currents are **221.8 A** in one line and **169.79 A** in each of the other two.

Example 33.21. *Two electric furnaces are supplied with 1-phase current at 80 V from a 3-phase, 11,000 V system by means of two single-phase Scott-connected transformers with similar secondary windings. When the load on one furnace is 500 kW (teaser secondary) and on the other 800 kW (secondary of main transformer) what current will flow in each of the 3-phase lines (a) at unity power factor and (b) at 0.5 power factor ? Neglect phase displacement in and efficiency of, the transformers.* **(Electrical Engineering, Madras Univ. 1987)**

Solution. The connections are shown in Fig. 33.26 and the phasor diagrams for unity and 0.5 p.f. are shown in Fig. 33.27 (*a*) and (*b*) respectively.

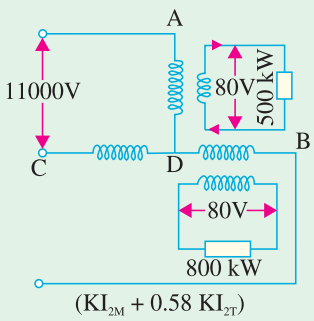


Fig. 33.26

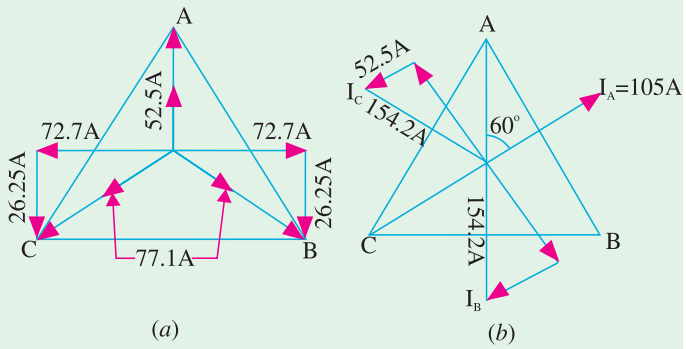


Fig. 33.27

Here, $K = 80/11,000 = 2/275$

(*a*) Unity p.f.

With reference to Fig. 33.27 (*a*), we have $I_{2T} = 500,000/80 \times 1 = 6,250$ A

Teaser primary current $I_{1T} = 1.15\, KI_{2T} = 1.15 \times (2/275) \times 6,250 = 52.5$ A

For the main transformer primary

(*i*) $KI_{2M} = \frac{2}{275} \times \frac{800,000}{80 \times 1} = 72.7$ A (*ii*) $\frac{1}{2} \times I_{1T} = 52.5/2 = 26.25$ A

Current in the primary of the main transformer is $= \sqrt{72.7^2 + 26.25^2} = 77.1$ A

Hence, one 3-phase line carries **52.5 A** whereas the other 2 carry **77.1 A** each [Fig. 33.27 (*a*)].

(*b*) 0.5 p.f.

With reference to Fig. 33.27 (*b*) we have $I_{2T} = 500,000/80 \times 0.5 = 12,500$ A

Teaser primary current $I_{1T} = 1.15 \times (2/275) \times 12,500 = 105$ A

For the main transformer primary

(*i*) $KI_{2M} = \frac{2}{275} \times \frac{800,000}{80 \times 0.5} = 145.4$ A (*ii*) $\frac{1}{2} I_{1T} = 105/2 = 52.5$ A.

Current in the primary of the main transformer is $= \sqrt{145.4^2 + 52.5^2} = 154.2$ A.

Hence, one 3-phase line carries **105 A** and the other two carry **154.2 A** each.

Note : Part (*b*) need not be worked out in full because at 0.5 p.f., each component current and hence the resultant are doubled. Hence, in the second case, answers can be found by multiplying by a factor of 2 the line currents found in (*a*).

Example 33.22. *Two furnaces are supplied with 1-phase current at 50 V from a 3-phase, 4.6 kV system by means of two 1-phase, Scott-connected transformers with similar secondary windings. When the load on the main transformer is 350 kW and that on the other transformer is 200 kW at 0.8 p.f. lagging, what will be the current in each 3-phase line ? Neglect phase displacement and losses in transformers.* **(Electrical Machinery-II, Bangalore Univ. 1991)**

Solution. Connections and vector diagrams are shown in Fig. 33.28.

$$K = 50/4,600 = 1/92; I_{2T} = 200,000/50 \times 0.8 = 5,000 \text{ A}$$

$$I_{1T} = 1.15\, KI_{2T} = 1.1 \times (1/92) \times 5,000 = 62.5 \text{ A}$$

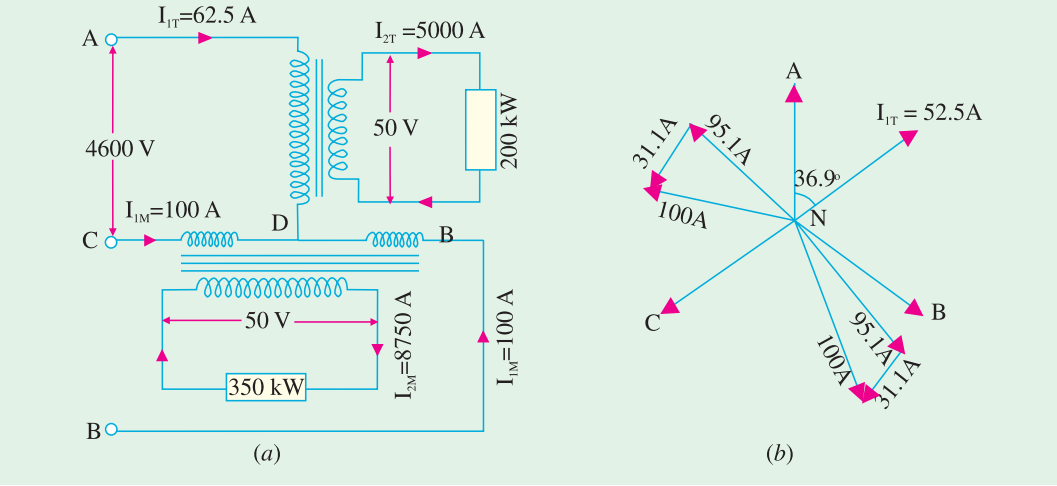


Fig. 33.28

As shown in Fig. 33.28 (*b*), main primary current I_{1M} has two rectangular components.

(*i*) KI_{2M} where $I_{2M} = 350,000/50 \times 0.8 = 8,750$ A $\therefore KI_{2M} = 8,750/92 = 95.1$ A

(*ii*) $(1/2)\, I_{1T} = 62.5/2 = 31.3$ A $\quad \therefore \quad I_{1M} = \sqrt{95.1^2 + 31.3^2} = 100$ A

$\therefore$ Current in line *A* = **62.5 A;** Current in line *B* = **100 A**; Current in line *C* = **100 A.**

Example 33.23. *Two single-phase Scott-connected transformers supply a 3-phase four-wire distribution system with 231 volts between lines and the neutral. The h.v. windings are connected to a two-phase system with a phase voltage of 6,600 V. Determine the number of turns in each section of the h.v. and l.v. winding and the position of the neutral point if the induced voltage per turn is 8 volts.*

Solution. As the volt/turn is 8 and the h.v. side voltage is 6,600 V, the h.v. side turns are = 6,600/8 = **825** on both transformers.

Now, voltage across points *B* and *C* of main winding = line voltage = $231 \times \sqrt{3} = 400$ V

No. of turns on the l.v. side of the main transformer = 400/8 = **50**

No. of turns on the l.v. side of teaser transformer = $\sqrt{(3/2)} \times$ mains turns

$= \sqrt{3} \times 50/2 =$ **43** (whole number)

The neutral point on the 3-phase side divides teaser turns in the ratio 1 : 2.

$\therefore$ Number of turns between *A* and *N* = (2/3) × *AD* = (2/3) × 43 = **29**

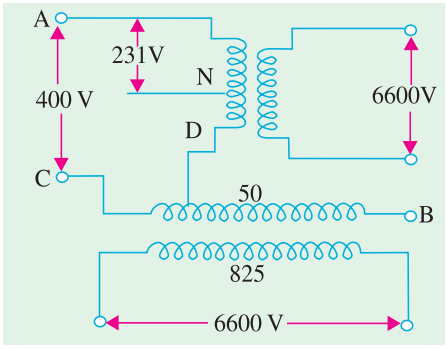


Fig. 33.29

Hence, neutral point is located on the 29th turn from *A* downwards (Fig. 33.29).

Example 33.24. *A Scott-connected (2 to 3-phase) transformer links a 6,000 V, 2-phase system with a 440 V; 3-phase system. The frequency is 50 Hz, the gross core area is 300 cm^2, while the maximum flux density is to be about 1.2 Wb/m^2. Find the number of turns on each winding and the point to be tapped for the neutral wire on the 3-phase side. If the load is balanced on the one side of such a transformer, find whether it will also be balanced on the other side.* **(London Univ.)**

Solution. Use the transformer voltage equation 1,

$E = 4.44 f N \Phi_x$ volt

Gross core area = 300 cm^2

Assuming net iron = 0.9 of gross area, and considering the h.v. side, we have

$6000 = 4.44 \times 50 \times N_1 \times 1.2 (300 \times 0.9 \times 10^{-4})$

$N_1 =$ **834**

Hence, h.v. sides of both transformers have 834 turns each.

Now $K = 440/6000 = 11/150$

$\therefore$ Turns on the *l*.v. side of main transformer

$N_2 = 834 \times 11/150 =$ **61**

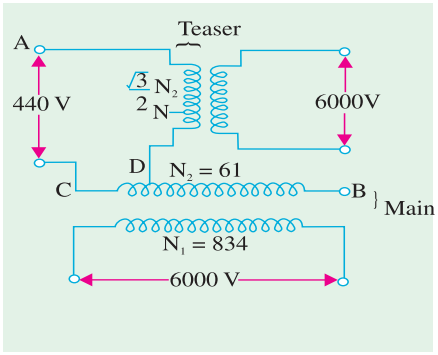


Fig. 33.30

Turns on the *l*.v. side of teaser = $(\sqrt{3}/2) \times 61 =$ **53**

With reference to Fig. 33.30, number of turns in *AN* = 53 × 2/3 = **35**

Example 33.25. *A 2-phase, 4-wire, 250 V system is supplied to a plant which has a 3-phase motor load of 30 kVA. Two Scott-connected transformers supply the 250 V motors. Calculate (a) voltage (b) kVA rating of each transformer. Draw the wiring connection diagram.*

Solution. ***(a)*** Both the main and the teaser have the same voltage rating as the supply voltage *i.e.* 250 V. The current in the main and the teaser coils is the same as the supply current and is

$$= \frac{\text{Total kVA}}{2 \times \text{Line voltage}} = \frac{30,000}{2 \times 250} = 60 \text{ A} \arcsin \theta$$

On the three-phase side, current is the same in all coils and is equal to the load line current $= 30,000/\sqrt{3} \times 250 = 69.3$ A

Load voltage on main secondary = line voltage = 250 V

Load voltage on teaser secondary = $0.866 \times 250 = 216.5$ V

Hence, voltage rating of main transformer is **250/250** whereas that of teaser transformer is **250/216.5.**

The current rating of main transformer is **60/69.3** and it is the same for the teaser transformer.

(*b*) The volt-amp rating of the teaser primary as well as secondary is the same *i.e.* $60 \times 250 \times 10^{-3} = 69.3 \times 216.5 \times 10^{-3} =$ **15 kVA**

The main volt-ampere rating of secondary is = $250 \times 69.3 \times 10^{-3} =$ **17.3 kVA**

Incidentally, if two identical transformers are used for providing inter-changeability, then both must be rated at 17.3 kVA. In that case, a total capacity of 34.6 kVA would be required to provide a 30 kVA load.

The wiring connections are shown in Fig. 33.31.

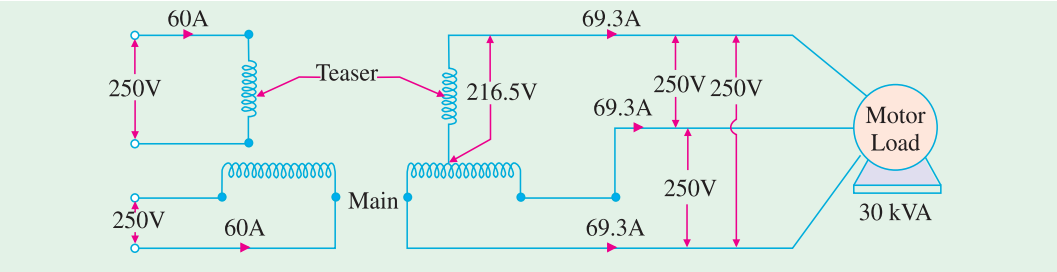


Fig. 33.31

Tutorial Problem No. 33.3

1. A Scott-connected transformer is fed from a 6,600-V, 3-phase network and supplies two single-phase furnaces at 100 V. Calculate the line currents on the 3-phase side when the furnaces take 400 kW and 700 kW respectively at 0.8 power factor lagging. **(Elect. Machines II, Indore Univ. 1977)**

[With 400 kW on teaser, line currents are 87. 2 A; 139 A; 139 A]

2. Two 220-V, 1-phase electrical furnaces take loads of 350 kW and 500 kW respectively at a power factor of 0.8 lagging. The main supply is at 11-kV, 3-phase, 50 Hz. Calculate current in the 3-phase lines which energise a Scott-connected transformer combination. **(Elect. Machines, Madras Univ. 1978)**

[With 350 kW on teaser line currents are : 45.7 A ; 61.2 A; 61.2 A]

3. Two electric furnaces are supplied with 1-phase current at 80 V from 3-phase, 11,000-V supply mains by means of two Scott-connected transformers with similar secondary windings. Calculate the current flowing kW respectively in each of the 3-phase lines at U.P.P. when the loads on the two transformers are 550 kW of 800 kW.

[With 550 kW on teaser, line currents are : 57.5 A; 78.2; 78.2 A] (Electrical Machines-I, Madras University, 1977)

33.11. Parallel Operation of 3-phase Transformers

All the conditions which apply to the parallel operation of single-phase transformers also apply to the parallel running of 3-phase transformers but with the following additions :

1. The voltage ratio must refer to the terminal ***voltage of primary and secondary***. It is obvious that this ratio may not be equal to the ratio of the number of turns per phase. For example, if V_1, V_2 are the primary and secondary terminal voltages, then for Y/Δ connection, the turn ratio is $V_2/(V_1/\sqrt{3})$ $= \sqrt{3}V_2/V_1$.

2. The phase displacement between primary and secondary voltages must be the same for all transformers which are to be connected for parallel operation.

3. The phase sequence must be the same.

4. All three transformers in the 3-phase transformer bank will be of the same construction either core or shell.

Note. (*i*) In dealing with 3-phase transformers, calculations are made for one phase only. The value of equivalent impedance used is the equivalent impedance per phase referred to secondary.

(*ii*) In case the impedances of primary and secondary windings are given separately, then primary impedance must be referred to secondary by multiplying it with (transformation ratio)^2.

(*iii*) For Y/Δ or Δ/Y transformers, it should be remembered that the voltage ratios as given in the questions, refer to terminal voltages and are quite different from turn ratio.

Example 33.26. *A load of 500 kVA at 0.8 power factor lagging is to be shared by two three-phase transformers A and B of equal ratings. If the equivalent delta impedances as referred to secondary are (2 + j6) Ω for A and (2 + j5) Ω for B, calculate the load supplied by each transformer.*

Solution. $$S_A = S\frac{Z_B}{Z_A + Z_B} = S\frac{1}{1 + (Z_A/Z_B)}$$

Now $S = 500\,(0.8 - j0.6) = (400 - j300)$

$Z_A/Z_B = (2 + j6)/(2 + j5) = 1.17 + j\,0.07$; $Z_B/Z_A = (2 + j5)/(2 + j6) = 0.85 - j\,0.05$

$S_A = (400 - j300)/(2.17 + j0.07) = 180 - j\,144.2 = 230.7 \angle -38.7°$

$\cos \phi_A = 0.78$ lagging

$S_B = (400 - j\,300)/(1.85 - j0.05) = 220.1 - j\,156 = 270 \angle -40°28'$ $\therefore \cos \Phi_B =$ **0.76 lagging.**

Example 33.27. *State (i) the essential and (ii) the desirable conditions to be satisfied so that two 3-phase transformers may operate successfully in parallel.*

A 2,000-kVA transformer (A) is connected in parallel with a 4,000 kVA transformer (B) to supply a 3-phase load of 5,000 kVA at 0.8 p.f. lagging. Determine the kVA supplied by each transformer assuming equal no-load voltages. The percentage voltage drops in the windings at their rated loads are as follows :

Transformer A	*resistance 2% ;*	*reactance 8 %*
Transformer B	*resistance 1.6 % ;*	*reactance 3 %*

(Elect. Engineering-II, Bombay Univ. 1987)

Solution. On the basis of 4,000 kVA

$\%\ Z_A = (4{,}000/2{,}000)\,(2 + j8) = (4 + j16) = 16.5 \angle 76°$

$\%\ Z_B = (1.6 + j3)$; $\%\ Z_A + \%\ Z_B = (5.6 + j16) = 19.8 \angle 73.6°$

$S = 5{,}000 \angle -36.9° = (4{,}000 - j\,3{,}000)$

Now $$S_B = S.\frac{Z_A}{Z_A + Z_B} = 5{,}000 \angle -36.9° \times \frac{16.5 \angle 76°}{19.8 \angle 73.6°}$$

$= 5{,}000 \angle -36.9° \times 0.832 \angle 2.4° = 4{,}160 \angle -34.5° = (3{,}425 - j\,2{,}355)$

$S_A = S - S_B = (4{,}000 - j\,3{,}000) - (3{,}425 - j\,2355)$

$= (575 - j\,645) = 864 \angle -48.3°$

$\cos \phi_B = \cos 34.5° = 0.824$ (lag); $\cos \phi_A = \cos 48.3° =$ **0.665 (lag).**

Example 33.28. *A load of 1,400 kVA at 0.866 p.f. lagging is supplied by two 3-phase transformers of 1,000 kVA and 500 kVA capacity operating in parallel. The ratio of transformation is the same in both : 6,600/400 delta-star. If the equivalent secondary impedances are (0.001 + j 0.003) ohm and (0.0028 + j0.005) ohm per phase respectively, calculate the load and power factor of each transformer.* **(Elect. Engg-I, Nagpur Univ. 1993)**

Solution. On the basis of 1000 kVA, $\mathbf{Z_A} = (0.001 + j\,0.003)\ \Omega$

$$\mathbf{Z_B} = (1000/500)\,(0.0028 + j\,0.005) = (0.0056 + j\,0.01)\ \Omega$$

$$\frac{\mathbf{Z_A}}{\mathbf{Z_A} + \mathbf{Z_B}} = \frac{(0.001 + j\,0.003)}{(0.0066 + j\,0.013)} = \frac{3.162 \times 10^{-3}\ \angle 71.6°}{14.57 \times 10^{-3}\ \angle 63.1°} = 0.2032\ \angle 8.5°$$

$$\mathbf{S} = 1400\ \angle \cos^{-1}(0.866) = 1400\ \angle -30° = (1212 - j\,700)$$

$$\mathbf{S_B} = \mathbf{S} \cdot \frac{\mathbf{Z_A}}{\mathbf{Z_A} + \mathbf{Z_B}} = 1400\ \angle -30° \times 0.2032\ \angle 8.5°$$

$$= 284.5\ \angle -21.5° = 265 - j\,104$$

$$\mathbf{S_A} = \mathbf{S} - \mathbf{S_B} = (1212 - j\,700) - (265 - j\,104) = (947 - j\,596) = 1145\ \angle -32.2°$$

$\cos \phi_A = \cos 32.3° = 0.846$ (lag); $\cos \phi_B = \cos 21.5° =$ **0.93 (lag).**

Example 33.29. *Two 3-phase transformers A and B having the same no-load line voltage ratio 3,300/400-V supply a load of 750 kVA at 0.707 lagging when operating in parallel. The rating of A is 500 kVA, its resistance is 2% and reactance 3%. The corresponding values for B are 250 kVA; 1.5% and 4% respectively. Assuming that both transformers have star-connected secondary windings, calculate*

(*a*) *the load supplied by each transformer,*

(*b*) *the power factor at which each transformer is working,*

(*c*) *the secondary line voltage of the parallel circuit.*

Solution. On the basis of 500 kVA,

$$\%\ \mathbf{Z_A} = 2 + j\,3,\ \%\ \mathbf{Z_B} = (500/200)\,(1.5 + j\,4) = (3 + j8)$$

$$\frac{\mathbf{Z_A}}{\mathbf{Z_A} + \mathbf{Z_B}} = \frac{2 + j\,3}{5 + j1} = 0.3\ \angle -9.3°;\ \frac{\mathbf{Z_B}}{\mathbf{Z_A} + \mathbf{Z_B}} = \frac{3 + j8}{5 + j11} = 0.711\ \angle 3.8°$$

Now, $\mathbf{S} = 750\ \angle -45°$

(*a*) $$\mathbf{S_A} = \mathbf{S}\,\frac{\mathbf{Z_B}}{\mathbf{Z_A} + \mathbf{Z_B}} = 750\ \angle -45° \times 0.711\ \angle 3.8°$$

$$= 533\ \angle -41.2° = (400 - j\,351)$$

$$\mathbf{S_B} = 750\ \angle -45° \times 0.3\ \angle 09.3° = 225\ \angle -54.3°$$

(*b*) $\cos \phi_A = \cos 41.2° = 0.752$ (lag); $\cos \phi_B = \cos 54.3° =$ **0.5835** (lag)

(*c*) Since voltage drop of each transformer is the same, its value in the case of transformer *A* would only be calculated. Now, for transformer *A*, kW = 400 for the active component of the current and kVAR = 351 for the reactive component.

$\therefore$ % resistive drop $= 2 \times 400/500$

$= 1.6\%$; % reactive drop $= 3 \times 351/500 = 2.1\%$

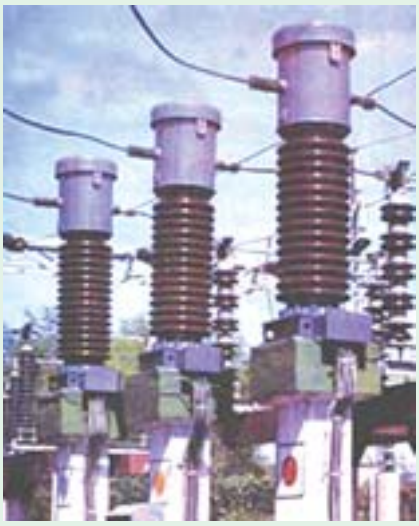
Total percentage drop = 1.6 + 2.1 = 3.7

Secondary line voltage = $400 - (3.7 \times 400/100) =$ **385.2 V.**

33.12. Instrument Transformers

In d.c. circuit when large currents are to be measured, it is usual to use low-range ammeters with suitable shunts. For measuring high voltages, low-range voltmeters are used with a high resistance connected

in series with them. But it is not convenient to use this methods with alternating current and voltage instruments. For this purpose, specially constructed accurate ratio instrument transformers are employed in conjunction with standard low-range a.c. instruments. These instrument transformers are of two kinds : (*i*) current transformers for measuring large alternating currents and (*ii*) potential transformers for measuring high alternating voltages.


Current transformers

33.13. Current Transformers

These transformers are used with low-range ammeters to measure currents in high-voltage alternating-current circuits where it is not practicable to connect instruments and meters directly to the lines. In addition to insulating the instrument from the high voltage line, they step down the current in a known ratio. The current (or series) transformer has a primary coil of one or more turns of thick wire connected in ***series*** with the line whose current is to be measured as shown in Fig. 33.32. The secondary consists of a large number of turns of fine wire and is connected across the ammeter terminals (usually of 5-ampere bracket should be removed or 1-ampere range).

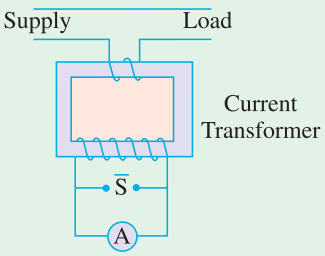


Fig. 33.32

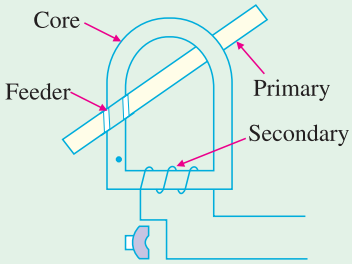


Fig. 33.33

As regards voltage, the transformers is of step-up variety but it is obvious that current will be stepped down. Thus, if the current transformer has ***primary to secondary*** current ratio of 100 : 5, then it steps up the voltage 20 times whereas it steps down the current to 1/20th of its actual value. Hence, if we know current ratio (I_1/I_2) of the transformer and the reading of the a.c. ammeter, the line current can be calculated. In fact, line current is given by the current transformation ratio times the reading on the ammeter. One of the most commonly used current transformer is the one known as ***clamp-on or clip-on type.*** It has a laminated core which is so arranged that it can be opened out at hinged section by merely pressing a triggr-like projection (Fig. 33.33). When the core is thus opened, it permits the admission of very heavy current-carrying bus bars or feeders whereupon the trigger is released and the core is tightly closed by a spring. The current carrying conductor or feeder acts as a single-turn primary whereas the secondary is connected across the standard ammeter conveniently mounted in the handle.


Small current transformer

It should be noted that, since the ammeter resistance is very low, the current transformer normally works short circuited. If for any reason, the ammeter is taken out of the secondary winding, then this winding

must be short-circuited with the help of short-circulating switch *S*. If this is not done, then due to the absence of counter amp-turns of the secondary, the unopposed primary m.m.f. will set up an abnormally high flux in the core which will produce excessive core loss with subsequent heating and a high voltage across the secondary terminals. This is not the case with ordinary constant-potential transformers, because their primary current is determined by the load in their secondary whereas in a current transformer, the primay current is determined entirely by the load on the system and not by the load on its own secondary.

Hence, ***the secondary of a current transformer should never be left open under any circumstances.***

Example 33.30. *A 100 : 5 transformer is used in conjunction with a 5-amp ammeter. If the latter reads 3.5 A, find the line current.*

Solution. Here, the ratio 100 : 5 stands for the ratio of primary-to-secondary currents *i.e.* $I_1/I_2 = 100/5$

$\therefore$ Primary (or line) current = $3.5 \times (100/5) =$ **70 A**

Example 33.31. *It is desired to measure a line current of the order of 2,000 A to 2,500 A. If a standard 5-amp ammeter is to be used along with a current transformer, what should be the turn ratio of the latter ? By what factor should the ammeter reading be multiplied to get the line current in each case ?*

Solution. $I_1/I_2 = 2000/5 = 400$ or $2500/5 = 500$. Since $I_1/I_2 = N_2/N_1$ hence $N_2/N_1 = 400$ in the first case and 500 in the second case. It means that $N_1 : N_2 :: 1 : 400$ or $1 : 500$.

Ratio or multiplication factor in the first case is 400 and in the second in the second case 500.

33.14. Potential Transformers

These transformers are extremely accurate-ratio step-down transformers and are used in conjunction with standard low-range voltmeters (usually 150-V) whose deflection when divided by voltage transformation ratio, gives the true voltage on the high voltage side. In general, they are of the shell-type and do not differ much from the ordinary two-winding transformers discussed so far, except that their power rating is extremely small. Upto voltages of 5,000, potential transformers are usually of the dry type, between 5,000 and 13,800 volts, they may be either dry type or oil immersed type, although for voltages above 13,800 they are always oil immersed type. Since their secondary windings are required to operate instruments or relays or pilot lights, their ratings are usually of 40 to 100 W. For safety, the secondary should be completely insulated from the high-voltage primary and should be, in addition, grounded for affording protection to the operator. Fig. 33.34 shows the connections of such a transformer.



Small Potential transformer

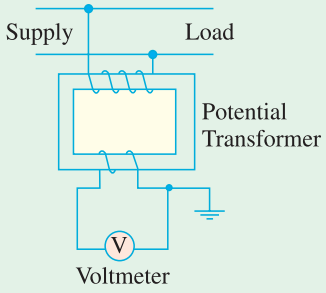


Fig. 33.34

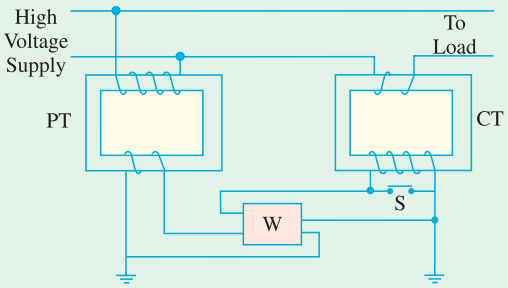


Fig. 33.35

Fig. 33.35 shows the connections of instrument transformers to a wattmeter. While connecting the wattmeter, the relative polarities of the secondary terminals of the transformers with respect to their primary terminals must be known for connections of the instruments.

OBJECTIVE TEST – 33

1. Which of the following connections is best suited for 3-phase, 4-wire service ?
(*a*) Δ – Δ (*b*) Y – Y
(*c*) Δ – Y (*d*) Y – Δ

2. In a three-phase Y-Y transformer connection, neutral is fundamental to the
(*a*) suppression of harmonics
(*b*) passage of unbalanced currents due to unbalanced loads
(*c*) provision of dual electric service
(*d*) balancing of phase voltages with respect to line voltages.

3. As compared to Δ – Δ bank, the capacity of the *V* – *V* bank of transformers is ..... percent.
(*a*) 57.7 (*b*) 66.7
(*c*) 50 (*d*) 86.6

4. If three transformers in a Δ – Δ are delivering their rated load and one transformer is removed, then overload on *each* of the remaining transformers is ..... percent.
(*a*) 66.7 (*b*) 173.2
(*c*) 73.2 (*d*) 58

5. When a V – V system is converted into a Δ – Δ system, increase in capacity of the system is ..... percent.
(*a*) 86.6 (*b*) 66.7
(*c*) 73.2 (*d*) 50

6. For supplying a balanced 3 – ϕ load of 40-kVA, rating of each transformer in V – V bank should be nearly ..... kVA.
(*a*) 20 (*b*) 23
(*c*) 34.6 (*d*) 25

7. When a closed – Δ bank is converted into an open – Δ bank, each of the two remaining transformers supplies ..... percent of the original load.
(*a*) 66.7 (*b*) 57.7
(*c*) 50 (*d*) 73.2

8. If the load p.f. is 0.866, then the average p.f. of the V – V bank is
(*a*) 0.886 (*b*) 0.75
(*c*) 0.51 (*d*) 0.65

9. A T – T connection has higher ratio of utilization that a V – V connection only when
(*a*) identical transformers are used
(*b*) load power factor is leading
(*c*) load power factor is unity
(*d*) non-identical transformers are used.

10. The biggest advantage of T - T connection over the V - V connection for 3-phase power transformation is that it provides
(*a*) a set of balanced voltages under load
(*b*) a true 3-phase, 4-wire system
(*c*) a higher ratio of utilization
(*d*) more voltages.

11. Of the following statements concerning parallel operation of transformers, the one which is **not** correct is
(*a*) transformers must have equal voltage ratings
(*b*) transformers must have same ratio of transformation
(*c*) transformers must be operated at the same frequency
(*d*) transformers must have equal kVA ratings.

12. Statement

An auto-transformer is more efficient in transferring energy from primary to secondary circuit.

Reason

Because it does so both inductively and conductively.

Key

(*a*) statement is false, reason is correct and relevant

(*b*) statement is correct, reason is correct but irrelevant

(*c*) both statement and reason are correct and are connected to each other as cause and effect

(*d*) both statement and reason are false.

13. Out of the following given choices for poly phase transformer connections which one will you select for three-to-two phase conversion ?

(*a*) Scott (*b*) star/star

(*c*) double Scott (*d*) star/double-delta

14. A T – T transformer cannot be paralleled with ..... transformer.

(*a*) V – V (*b*) Y – Δ

(*c*) Y – Y (*d*) Δ – Δ

15. Instrument transformers are used on a.c. circuits for extending the range of

(*a*) ammeters (*b*) voltmeters

(*c*) wattmeters (*d*) all of the above.

16. Before removing the ammeter from a current transformer, its secondary must be short-circuited in order to avoid

(*a*) excessive heating of the core

(*b*) high secondary e.m.f.

(*c*) increase in iron losses

(*d*) all of the above.

ANSWERS

1. *c* **2.** *a* **3.** *a* **4.** *c* **5.** *c* **6.** *b* **7.** *b* **8.** *b* **9.** *d* **10.** *b* **11.** *d* **12.** *c* **13.** *a* **14.** *b* **15.** *d* **16.** *d*

GO To FIRST

CONTENTS

CHAPTER 34

INDUCTION MOTOR

Learning Objectives

- Classification of AC Motors
- Induction Motor: General Principal
- Construction
- Phase-wound Rotor
- Mathematical Proof
- Relation between Torque and Rotor Power Factor
- Condition for Maximum Starting Torque
- Rotor E.M.F and Reactance under Running Conditions
- Condition for Maximum Torque Under Running Conditions
- Rotor Torque and Breakdown Torque
- Relation between Torque and Slip
- Full-load Torque and Maximum Torque
- Starting Torque and Maximum Torque
- Torque/Speed Characteristic Under Load
- Complete Torque/Speed Curve of a Three-phase Machine
- Power Stages in an Induction Motor
- Torque Developed by an Induction Motor
- Induction Motor Torque Equation
- Variation in Rotor Current
- Sector Induction Motor
- Magnetic Levitation
- Induction Motor as a Generalized Transformer
- Power Balance Equation
- Maximum Power Output



The high-speed magnetic levitation trains employ the principle of linear induction motor

CONTENTS

34.1. Classification of A.C. Motors

With the almost universal adoption of a.c. system of distribution of electric energy for light and power, the field of application of a.c. motors has widened considerably during recent years. As a result, motor manufactures have tried, over the last few decades, to perfect various types of a.c. motors suitable for all classes of industrial drives and for both single and three-phase a.c. supply. This has given rise to bewildering multiplicity of types whose proper classification often offers considerable difficulty. Different a.c. motors may, however, be classified and divided into various groups from the following different points of view :

1. AS REGARDS THEIR PRINCIPLE OF OPERATION

(A) Synchronous motors

(*i*) plain and (*ii*) super—

(B) Asynchronous motors

(*a*) **Induction motors**

(*i*) Squirrel cage { single / double

(*ii*) Slip-ring (external resistance)

(*b*) **Commutator motors**

(*i*) Series { single phase / universal

(*ii*) Compensated { conductively / inductively

(*iii*) shunt { simple / compensated

(*iv*) repulsion { straight / compensated

(*v*) repulsion-start induction **(*vi*)** repulsion induction



Three phase high voltage asynchronous motors

2. AS REGARDS THE TYPE OF CURRENT

(*i*) single phase **(*ii*)** three phase

3. AS REGARDS THEIR SPEED

(*i*) constant speed **(*ii*)** variable speed **(*iii*)** adjustable speed

4. AS REGARDS THEIR STRUCTURAL FEATURES

(*i*) open **(*ii*)** enclosed **(*iii*)** semi-enclosed

(*iv*) ventilated **(*v*)** pipe-ventilated **(*vi*)** riverted frame eye etc.



Fig. 34.1 Squirrel cage AC induction motor opened to show the stator and rotor construction, the shaft with bearings, and the cooling fan.

34.2. Induction Motor : General Principle

As a general rule, conversion of electrical power into mechanical power takes place in the ***rotating*** part of an electric motor. In d.c. motors, the electric power is ***conducted*** directly to the armature (*i.e.* rotating part) through brushes and commutator (Art. 29.1). Hence, in this sense, a d.c. motor can be called a ***conduction*** motor. However, in a.c. motors, the rotor does not receive electric power by conduction but by ***induction*** in exactly the same way as the secondary of a 2-winding transformer receives its power from

the primary. That is why such motors are known as ***induction*** motors. In fact, an induction motor can be treated as a ***rotating transformer*** *i.e.* one in which primary winding is stationary but the secondary is free to rotate (Art. 34.47).

Of all the a.c. motors, the polyphase induction motor is the one which is extensively used for various kinds of industrial drives. It has the following main advantages and also some dis-advantages:

Advantages:

1. It has very simple and extremely rugged, almost unbreakable construction (especially squirrel-cage type).
2. Its cost is low and it is very reliable.
3. It has sufficiently high efficiency. In normal running condition, no brushes are needed, hence frictional losses are reduced. It has a reasonably good power factor.
4. It requires minimum of maintenance.
5. It starts up from rest and needs no extra starting motor and has not to be synchronised. Its starting arrangement is simple especially for squirrel-cage type motor.

Disadvantages:

1. Its speed cannot be varied without sacrificing some of its efficiency.
2. Just like a d.c. shunt motor, its speed decreases with increase in load.
3. Its starting torque is somewhat inferior to that of a d.c. shunt motor.

34.3. Construction

An induction motor consists essentially of two main parts :

(*a*) a stator and **(*b*)** a rotor.

(*a*) Stator

The stator of an induction motor is, in principle, the same as that of a synchronous motor or generator. It is made up of a number of stampings, which are slotted to receive the windings [Fig.34.2 (*a*)]. The stator carries a 3-phase winding [Fig.34.2 (*b*)] and is fed from a 3-phase supply. It is wound for a definite number of poles*, the exact number of poles being determined by the requirements of speed. Greater the number of poles, lesser the speed and ***vice versa***. It will be shown in Art. 34.6 that the stator windings, when supplied with 3-phase currents, produce a magnetic flux, which is of constant magnitude but which revolves (or rotates) at synchronous speed (given by $N_s = 120\,f/P$). This revolving magnetic flux induces an e.m.f. in the rotor by mutual induction.

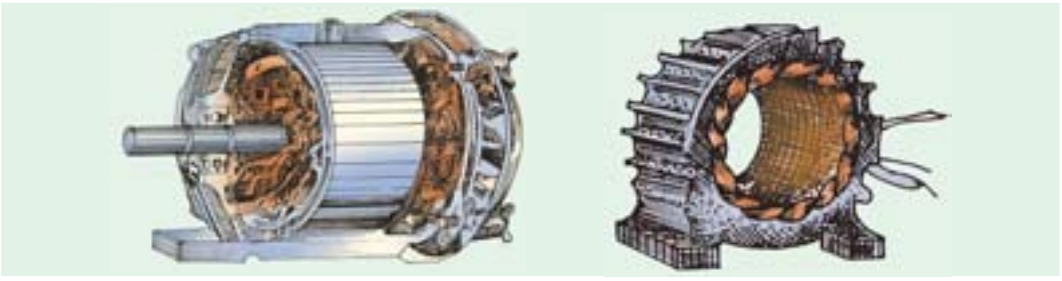


Fig. 34.2 (a) Unwound stator with semi-closed slots. Laminations are of high-quality low-loss silicon steel. (*Courtesy : Gautam Electric Motors*)

Fig. 34.2 (*b*) Completely wound stator for an induction motor. (*Courtesy : Gautam Electric Motors*)

* The number of poles P, produced in the rotating field is $P = 2n$ where n is the number of stator slots/pole/phase.

(*b*) Rotor

(*i*) *Squirrel-cage rotor* : Motors employing this type of rotor are known as squirrel-cage induction motors.

(*ii*) *Phase-wound or wound rotor* : Motors employing this type of rotor are variously known as 'phase-wound' motors or 'wound' motors or as 'slip-ring' motors.

34.4. Squirrel-cage Rotor

Almost 90 per cent of induction motors are squirrel-cage type, because this type of rotor has the simplest and most rugged construction imaginable and is almost indestructible. The rotor consists of a cylindrical laminated core with parallel slots for carrying the rotor conductors which, it should be noted clearly, are not wires but consist of heavy bars of copper, aluminium or alloys. One bar is placed in each slot, rather the bars are inserted from the end when semi-closed slots are used. The rotor bars are brazed or electrically welded or bolted to two heavy and stout short-circuiting end-rings, thus giving us, what is so picturesquely called, a squirrel-case construction (Fig. 34.3).



Fig. 34.3 (*a*) Squirrel-cage rotor with copper bars and alloy brazed end-rings (*Courtesy : Gautam Electric Motors*)



Fig. 34.3 (*b*) Rotor with shaft and brings (*Courtesy : Gautam Electric Motors*)

It should be noted that the ***rotor bars are permanently short-circuited on themselves*,** hence it is not possible to add any external resistance in series with the rotor circuit for starting purposes.

The rotor slots are usually not quite parallel to the shaft but are purposely given a slight skew (Fig. 34.4). This is useful in two ways :

(*i*) it helps to make the motor run quietly by reducing the magnetic hum and

(*ii*) it helps in reducing the locking tendency of the rotor *i.e.* the tendency of the rotor teeth to remain under the stator teeth due to direct magnetic attraction between the two.*

In small motors, another method of construction is used. It consists of placing the entire rotor core in a mould and casting all the bars and end-rings in one piece. The metal commonly used is an aluminium alloy.

Another form of rotor consists of a solid cylinder of steel without any conductors or slots at all. The motor operation depends upon the production of eddy currents in the steel rotor.

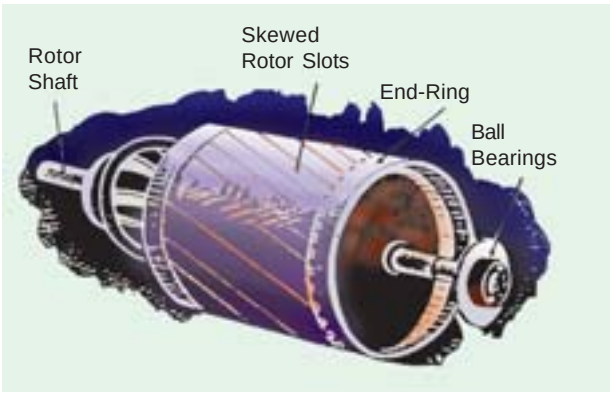


Fig. 34.4

* Other results of skew which may or may not be desirable are (*i*) increase in the effective ratio of transformation between stator and rotor (*ii*) increased rotor resistance due to increased length of rotor bars (*iii*) increased impedance of the machine at a given slip and (*iv*) increased slip for a given torque.

34.5. Phase-wound Rotor

This type of rotor is provided with 3-phase, double-layer, distributed winding consisting of coils as used in alternators. The rotor is wound for as many poles as the number of stator poles and is always wound 3-phase even ***when the stator is wound two-phase***.

The three phases are starred internally. The other three winding terminals are brought out and connected to three insulated slip-rings mounted on the shaft with brushes resting on them [Fig. 34.5 (*b*)]. These three brushes are further externally connected to a 3-phase star-connected rheostat [Fig. 34.5 (*c*)]. This makes possible the introduction of additional resistance in the rotor circuit during the starting period for increasing the starting torque of the motor, as shown in Fig. 34.6 (*a*) (Ex.34.7 and 34.10) and for changing its speed-torque/current characteristics. When running under normal conditions, the ***slip-rings are automatically short-circuited*** by means of a metal collar, which is pushed along the shaft and connects all the rings together. Next, the brushes are automatically lifted from the slip-rings to reduce the frictional losses and the wear and tear. Hence, it is seen that under normal running conditions, the wound rotor is short-circuited on itself just like the squirrel-case rotor.



Fig. 34.5 (*a*)

Fig. 34.6 (*b*) shows the longitudinal section of a slip-ring motor, whose structural details are as under :



Fig. 34.5 (*b*) Slip-ring motor with slip-rings brushes and short-circuiting devices
(*Courtesy : Kirloskar Electric Company*)



Fig. 34.5 (*c*)

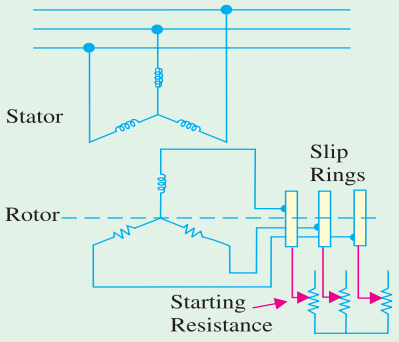


Fig. 34.6 (a)

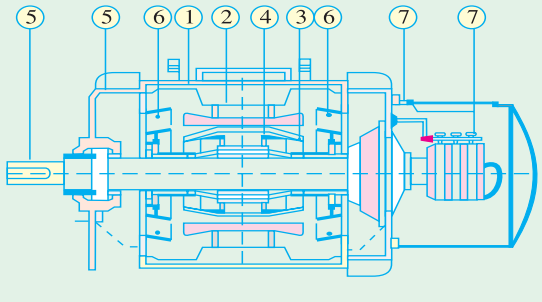


Fig. 34.6 (*b*) Longitudinal section of a Jyoti splash-proof slip-ring motor
(*Courtesy : Jyoti Colour-Emag Ltd.*)

1. **Frame.** Made of close-grained alloy cast iron.
2. **Stator and Rotor Core.** Built from high-quality low-loss silicon steel laminations and flash-enamelled on both sides.
3. **Stator and Rotor Windings.** Have moisture proof tropical insulation embodying mica and high quality varnishes. Are carefully spaced for most effective air circulation and are rigidly braced to withstand centrifugal forces and any short-circuit stresses.
4. **Air-gap.** The stator rabbets and bore are machined carefully to ensure uniformity of air-gap.
5. **Shafts and Bearings.** Ball and roller bearings are used to suit heavy duty, toruble-free running and for enhanced service life.
6. **Fans.** Light aluminium fans are used for adequate circulation of cooling air and are securely keyed onto the rotor shaft.
7. **Slip-rings and Slip-ring Enclosures.** Slip-rings are made of high quality phosphor-bronze and are of moulded construction.

Fig. 34.6 (*c*) shows the disassembled view of an induction motor with squirrel-cage rotor. According to the labelled notation **(*a*)** represents stator **(*b*)** rotor **(*c*)** bearing shields **(*d*)** fan **(*e*)** ventilation grill and **(*f*)** terminal box.

Similarly, Fig. 34.6 **(*d*)** shows the disassembled view of a slip-ring motor where (*a*) represents stator **(*b*)** rotor **(*c*)** bearing shields **(*d*)** fan **(*e*)** ventilation grill **(*f*)** terminal box **(*g*)** slip-rings **(*h*)** brushes and brush holders.

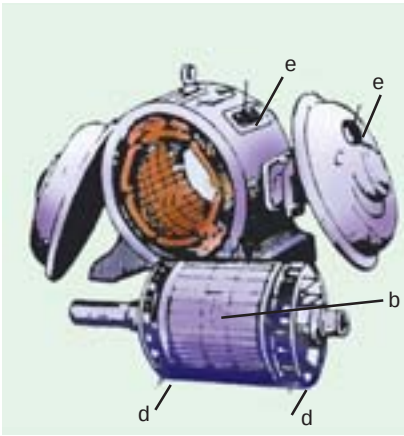


Fig. 34.6 (*c*)

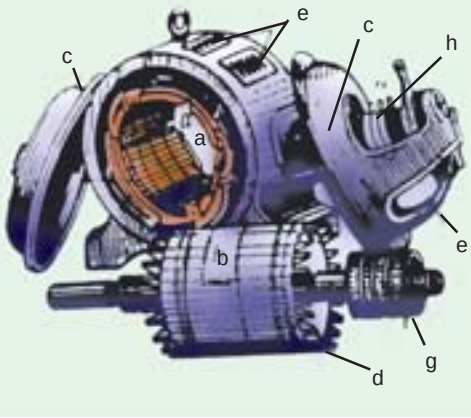


Fig. 34.6 (*d*)

34.6. Production of Rotating Field

It will now be shown that when stationary coils, wound for two or three phases, are supplied by two or three-phase supply respectively, a uniformly-rotating (or revolving) magnetic flux of constant value is produced.

Two-phase Supply

The principle of a 2-ϕ, 2-pole stator having two identical windings, 90 space degrees apart, is illustrated in Fig. 34.7.



Induction motor

The flux due to the current flowing in each phase winding is assumed sinusoidal and is represented in Fig. 34.9. The assumed positive directions of fluxes are those shown in Fig. 34.8.

Let Φ_1 and Φ_2 be the instantaneous values of the fluxes set up by the two windings. The resultant flux Φ_r at any time is the vector sum of these two fluxes (Φ_1 and Φ_2) at that time. We will consider conditions at intervals of 1/8th of a time period *i.e.* at intervals corresponding to angles of 0°, 45°, 90°, 135° and 180°. It will be shown that resultant flux Φ_r is constant in magnitude *i.e.* equal to Φ_m-the maximum flux due to either phase and is making one revolution/cycle. In other words, it means that the resultant flux rotates synchronously.

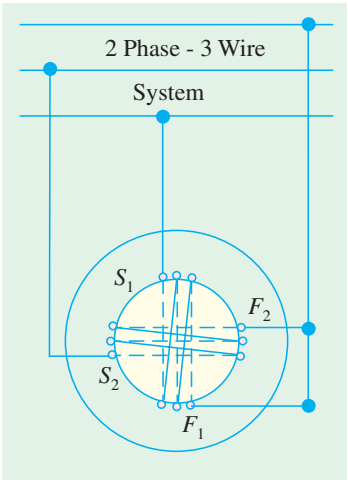


Fig. 34.7

(*a*) When θ = 0° *i.e.* corresponding to point 0 in Fig. 34.9, $\Phi_1 = 0$, but Φ_2 is maximum *i.e.* equal to Φ_m and negative. Hence, resultant flux $\Phi_r = \Phi_m$ and, being negative, is shown by a vector pointing downwards [Fig. 34.10 (*i*)].

(*b*) When θ = 45° *i.e.* corresponding to point 1 in Fig. 34.9. At this instant, $\Phi_1 = \Phi_m / \sqrt{2}$ and is positive; $\Phi_2 = \Phi_m / \sqrt{2}$ but is still negative. Their resultant, as shown in Fig. 34.10 (*ii*), is $\Phi_r = \sqrt{[(\phi_m / \sqrt{2})^2 + (\phi_m / \sqrt{2})^2]} = \Phi_m$ although this resultant has shifted 45° clockwise.

(*c*) When θ = 90° *i.e.* corresponding to point 2 in Fig. 34.9. Here $\Phi_2 = 0$, but $\Phi_1 = \Phi_m$ and is positive. Hence, $\Phi_r = \Phi_m$ and has further shifted by an angle of 45° from its position in (*b*) or by 90° from its original position in (*a*).

(*d*) When θ = 135° *i.e.* corresponding to point 3 in Fig. 34.9. Here, $\Phi_1 = \Phi_m / \sqrt{2}$ and is positive, $\Phi_2 = \Phi_m / \sqrt{2}$ and is also positive. The resultant $\Phi_r = \Phi_m$ and has further shifted clockwise by another 45°, as shown in Fig. 34.10 (*iv*).

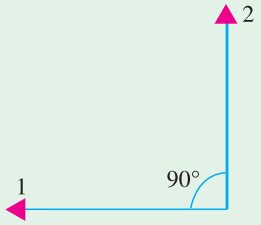


Fig. 34.8

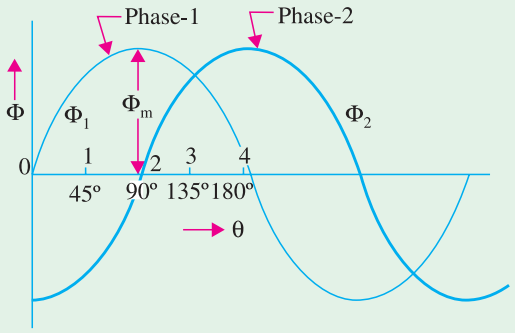


Fig. 34.9

(*e*) When θ = 180° *i.e.* corresponding to point 4 in Fig. 34.9. Here, $\Phi_1 = 0$, $\Phi_2 = \Phi_m$ and is positive. Hence, $\Phi_r = \Phi_m$ and has shifted clockwise by another 45° or has rotated through an angle of 180° from its position at the beginning. This is shown in Fig. 34.10(*v*).

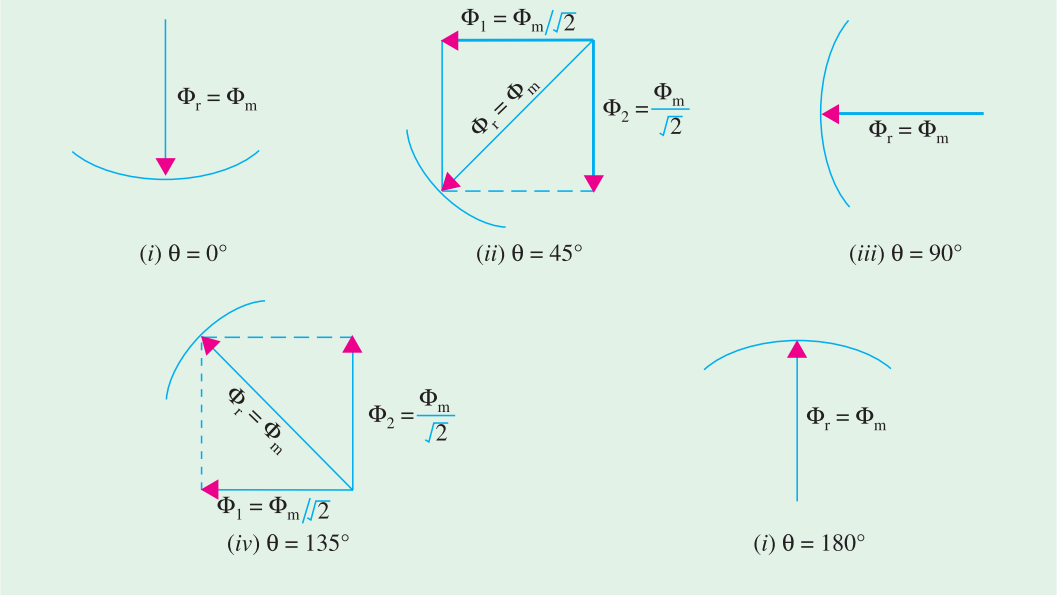


Fig. 34.10

Hence, we conclude

1. ***that the magnitude of the resultant flux is constant and is equal to Φ_m — the maximum flux due to either phase.***
2. ***that the resultant flux rotates at synchronous speed given by $N_s = 120\ f/P$ rpm.***

However, it should be clearly understood that in this revolving field, there is no actual revolution of the flux. The flux due to each phase changes periodically, according to the changes in the phase current, but the magnetic flux itself does not move around the stator. It is only the *seat* of the resultant flux which keeps on shifting synchronously around the stator.

Mathematical Proof

Let $\Phi_1 = \Phi_m \sin \omega t$ and $\Phi_2 = \Phi_m \sin (\omega t - 90^\circ)$

$\therefore \quad \Phi_r^2 = \Phi_1^2 + \Phi_2^2$

$\Phi_r^2 = (\Phi_m \sin \omega t)^2 + [\Phi_m \sin (\omega t - 90^\circ)]^2 = \Phi_m^2 (\sin^2 \omega t + \cos^2 \omega t) = \Phi_m^2$

$\therefore \quad \Phi_r = \Phi_m$

It shows that the flux is of constant value and does not change with time.

34.7. Three-phase Supply

It will now be shown that when three-phase windings displaced in space by 120°, are fed by three-phase currents, displaced in time by 120°, they produce a resultant magnetic flux, which rotates in space as if actual magnetic poles were being rotated mechanically.

The principle of a 3-phase, two-pole stator having three identical windings placed 120 space degrees apart is shown in Fig. 34.11. The flux (assumed sinusoidal) due to three-phase windings is shown in Fig 34.12.

The assumed positive directions of the fluxes are shown in Fig 34.13. Let the maximum value of flux due to any one of the three phases be Φ_m. The resultant flux Φ_r, at any instant, is given by the vector sum of the individual fluxes, Φ_1, Φ_2 and Φ_3 due to three phases. We will consider values of Φ_r at four instants 1/6th time-period apart corresponding to points marked 0, 1, 2 and 3 in Fig. 34.12.

(i) When $\theta = 0^\circ$ *i.e.* corresponding to point 0 in Fig. 34.12.

Here $\Phi_1 = 0$, $\Phi_2 = -\frac{\sqrt{3}}{2}\Phi_m$, $\Phi_3 = \frac{\sqrt{3}}{2}\Phi_m$. The vector for Φ_2 in Fig. 34.14 (*i*) is drawn in a direction opposite to the direction assumed positive in Fig. 34.13.

$$\therefore \quad \Phi_r = 2 \times \frac{\sqrt{3}}{2}\Phi_m \cos\frac{60^\circ}{2} = \sqrt{3} \times \frac{\sqrt{3}}{2}\Phi_m = \frac{3}{2}\Phi_m$$

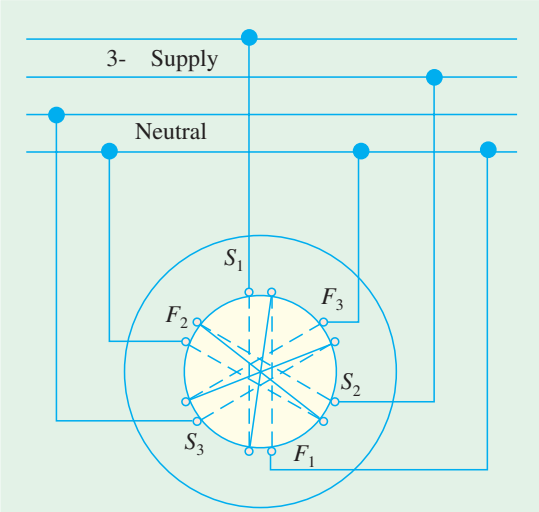


Fig. 34.11

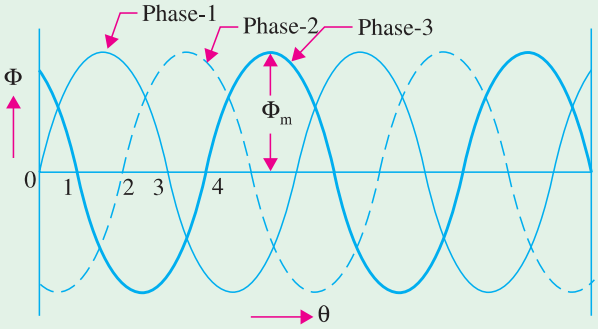


Fig. 34.12

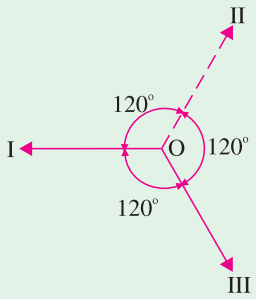


Fig. 34.13

(ii) when $\theta = 60^\circ$ *i.e.* corresponding to point 1 in Fig. 34.12.

Here $\Phi_1 = \frac{\sqrt{3}}{2}\Phi_m$ ...drawn parallel to *OI* of Fig. 34.13 as shown in Fig. 34.14 (*ii*)

$\Phi_2 = -\frac{\sqrt{3}}{2}\Phi_m$ ...drawn in opposition to *OII* of Fig. 34.13.

$\Phi_3 = 0$

$$\therefore \quad \Phi_r = 2 \times \frac{\sqrt{3}}{2}\Phi_m \times \cos 30^\circ = \frac{3}{2}\Phi_m$$ [Fig. 34.14 (*ii*)]

It is found that the resultant flux is again $\frac{3}{2}\Phi_m$ but has rotated clockwise through an angle of 60°.

(iii) When $\theta = 120^\circ$ *i.e.* corresponding to point 2 in Fig. 34.12.

Here, $\Phi_1 = \frac{\sqrt{3}}{2}\Phi_m$, $\Phi_2 = 0$, $\Phi_3 = -\frac{\sqrt{3}}{2}\Phi_m$

It can be again proved that $\Phi_r = \frac{3}{2}\Phi_m$.

So, the resultant is again of the same value, but has further rotated clockwise through an angle of 60° [Fig. 34.14 (*iii*)].

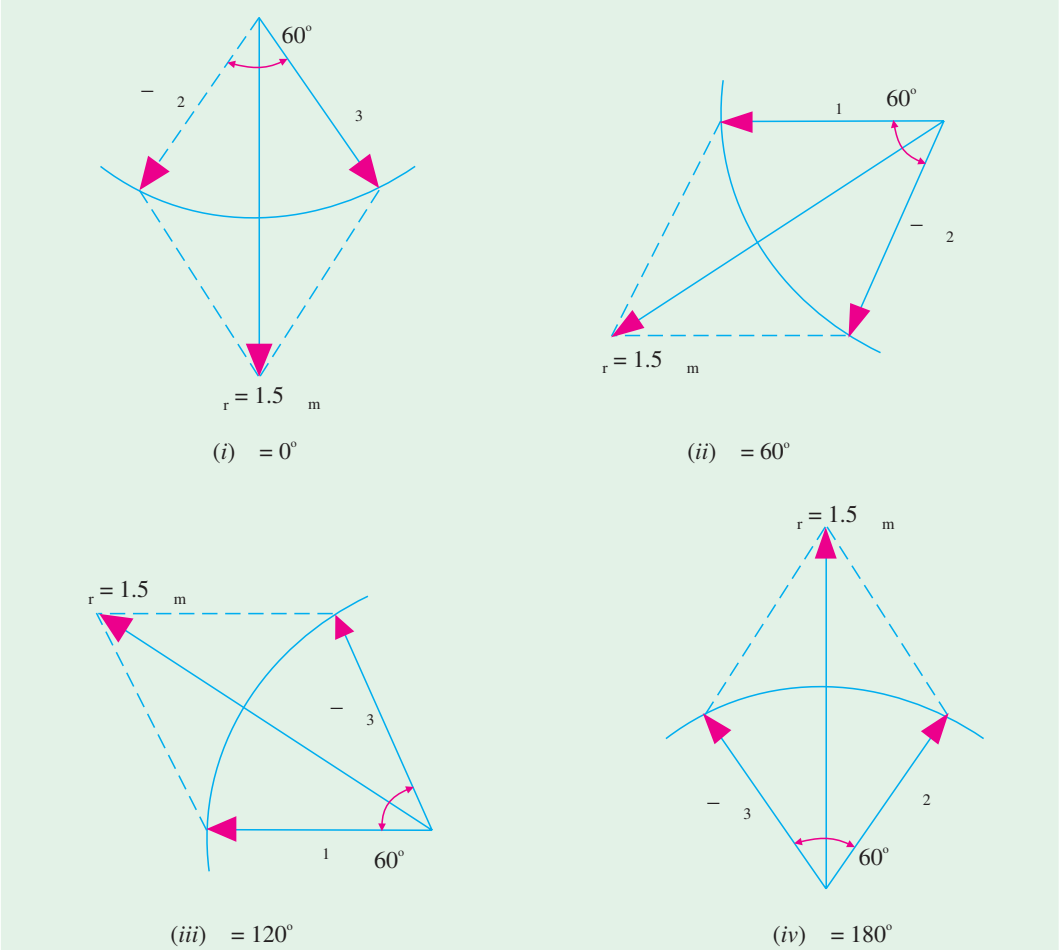


Fig. 34.14

(*iv*) When $\theta = 180°$ *i.e.* corresponding to point 3 in Fig. 34.12.

$$\Phi_1 = 0, \Phi_2 = \frac{\sqrt{3}}{2}\Phi_m, \Phi_3 = -\frac{\sqrt{3}}{2}\Phi_m$$

The resultant is $\frac{3}{2}\Phi_m$ and has rotated clockwise through an additional angle 60° or through an angle of 180° from the start.

Hence, we conclude that

1. the resultant flux is of constant value = $\frac{3}{2}\Phi_m$ *i.e. 1.5* ***times the maximum value of the flux due to any phase.***
2. ***the resultant flux rotates around the stator at synchronous speed given by $N_s = 120\ f/P$.***

Fig. 34.15 (*a*) shows the graph of the rotating flux in a simple way. As before, the positive directions of the flux phasors have been shown separately in Fig. 34.15 (*b*). Arrows on these flux phasors are reversed when each phase passes through zero and becomes negative.

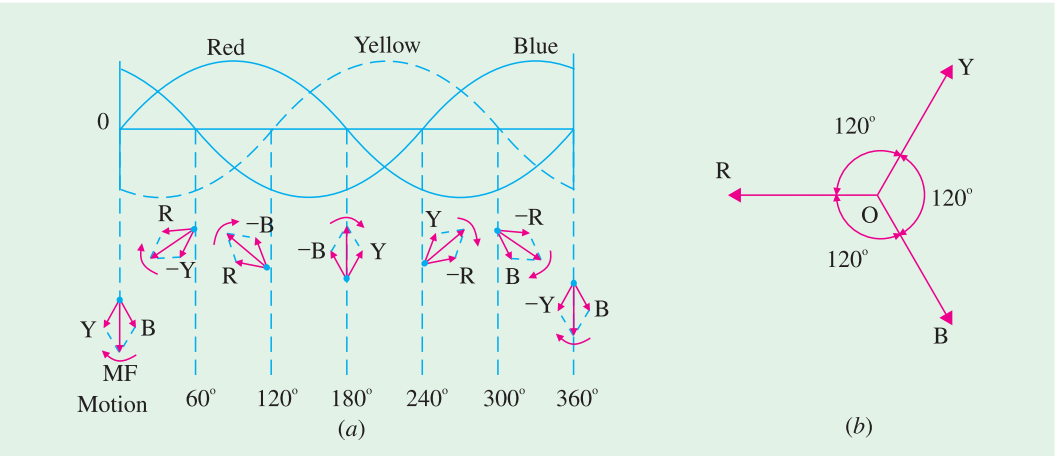


Fig. 34.15

As seen, positions of the resultant flux phasor have been shown at intervals of 60° only. The resultant flux produces a field rotating in the clockwise direction.

34.8. Mathematical Proof

Taking the direction of flux due to phase 1 as reference direction, we have

$$\Phi_1 = \Phi_m (\cos 0° + j \sin 0°) \sin \omega t$$
$$\Phi_2 = \Phi_m (\cos 240° + j \sin 240°) \sin (\omega t - 120°)$$
$$\Phi_3 = \Phi_m (\cos 120° + j \sin 120°) \sin (\omega t - 240°)$$

Expanding and adding the above equations, we get

$$\Phi_r = \frac{3}{2}\Phi_m (\sin \omega t + j \cos \omega t) = \frac{3}{2} \Phi_m \angle 90° - \omega t$$

The resultant flux is of constant magnitude and does not change with time 't'.

34.9. Why Does the Rotor Rotate ?

The reason why the rotor of an induction motor is set into rotation is as follow:

When the 3-phase stator windings, are fed by a 3-phase supply then, as seen from above, a magnetic flux of constant magnitude, but rotating at synchronous speed, is set up. The flux passes through the air-gap, sweeps past the rotor surface and so cuts the rotor conductors which, as yet, are stationary. Due to the relative speed between the rotating flux and the stationary conductors, an e.m.f. is induced in the latter, according to Faraday's laws of electro-magnetic induction. ***The frequency of the induced e.m.f. is the same as the supply frequency.*** Its magnitude is proportional to the relative velocity between the flux and the conductors and its direction is given by Fleming's Right-hand rule. Since the rotor bars or conductors form a closed circuit, rotor current is produced whose direction, as given by Lenz's law, is such as to oppose the very cause producing it. In this case, the cause which produces the rotor current is the relative velocity between the rotating flux of the stator and the stationary rotor conductors. Hence, to reduce the relative speed, the rotor starts running in the ***same*** direction as that of the flux and tries to catch up with the rotating flux.


Windings of induction electric motor

The setting up of the torque for rotating the rotor is explained below :

In Fig 34.16 (*a*) is shown the stator field which is assumed to be rotating clockwise. The relative motion of the rotor with respect to the stator is ***anticlockwise.*** By applying Right-hand rule, the direction of the induced e.m.f. in the rotor is found to be outwards. Hence, the direction of the flux due to rotor current *alone*, is as shown in Fig. 34.16 (*b*). Now, by applying the Left-hand rule, or by the effect of combined field [Fig. 34.16(*c*)] it is clear that the rotor conductors experience a force tending to rotate them in clockwise direction. Hence, the rotor is set into rotation in the same direction as that of the stator flux (or field).

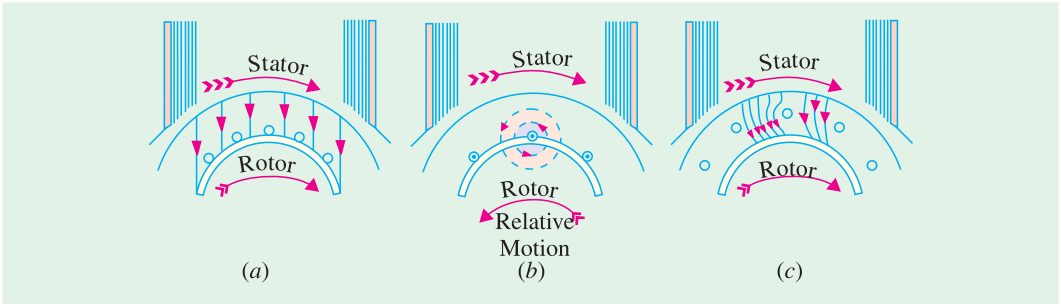


Fig. 34.16

34.10. Slip

In practice, the rotor never succeeds in 'catching up' with the stator field. If it really did so, then there would be no relative speed between the two, hence no rotor e.m.f., no rotor current and so no torque to maintain rotation. That is why the rotor runs at a speed which is always less than the speed of the stator field. The difference in speeds depends upon the load on the motor.*

The difference between the synchronous speed N_s and the actual speed N of the rotor is known as ***slip***. Though it may be expressed in so many revolutions/second, yet it is usual to express it as a percentage of the synchronous speed. Actually, the term '***slip***' is descriptive of the way in which the rotor 'slips back' from synchronism.

$$\% \text{ slip} \quad s = \frac{N_s - N}{N_s} \times 100$$

Sometimes, $N_s - N$ is called the ***slip speed.***

Obviously, rotor (or motor) speed is $N = N_s (1 - s)$.

It may be kept in mind that revolving flux is rotating synchronously, relative to the stator (*i.e.* stationary space) but at slip speed relative to the rotor.

34.11. Frequency of Rotor Current

When the rotor is stationary, the frequency of rotor current is *the same as the supply frequency.* But when the rotor starts revolving, then the frequency depends upon the relative speed or on slip-speed. Let at any slip-speed, the frequency of the rotor current be f'. Then

$$N_s - N = \frac{120\ f'}{P} \quad \text{Also,} \quad N_s = \frac{120\ f}{P}$$

Dividing one by the other, we get, $\frac{f'}{f} = \frac{N_s - N}{N_s} = s$ $\quad \therefore \quad f' = sf$

As seen, rotor currents have a frequency of $f' = sf$ and when flowing through the individual

* It may be noted that as the load is applied, the natural effect of the load or braking torque is to slow down the motor. Hence, slip increases and with it increases the current and torque, till the driving torque of the motor balances the retarding torque of the load. This fact determines the speed at which the motor runs on load.

phases of rotor winding, give rise to rotor magnetic fields. These individual rotor magnetic fields produce a combined rotating magnetic field, whose speed relative to rotor is

$$= \frac{120\ f'}{P} = \frac{120\ sf}{P} = sN_S$$

However, the rotor itself is running at speed N with respect to space. Hence,

speed of rotor field in ***space*** = speed of rotor magnetic field relative to rotor + speed of rotor relative to space

$$= sN_s + N = sN_s + N_s\ (1 - s) = N_s$$

It means that no matter what the value of slip, rotor currents and stator currents each produce a sinusoidally distributed magnetic field of constant magnitude and constant space speed of N_s. In other words, both the rotor and stator fields rotate synchronously, which means that they are stationary with respect to each other. These two synchronously rotating magnetic fields, in fact, superimpose on each other and give rise to the actually existing rotating field, which corresponds to the magnetising current of the stator winding.

Example 34.1. *A slip-ring induction motor runs at 290 r.p.m. at full load, when connected to 50-Hz supply. Determine the number of poles and slip.*

(Utilisation of Electric Power AMIE Sec. B 1991)

Solution. Since N is 290 rpm; N_s has to be somewhere near it, say 300 rpm. If N_s is assumed as 300 rpm, then $300 = 120 \times 50/P$. Hence, $P = 20$. $\therefore\ s = (300 - 290)/300 = 3.33\%$

Example 34.2. *The stator of a 3-ϕ induction motor has 3 slots per pole per phase. If supply frequency is 50 Hz, calculate*

(i) *number of stator poles produced and total number of slots on the stator*

(ii) *speed of the rotating stator flux (or magnetic field).*

Solution. *(i)* $P = 2n = 2 \times 3 =$ **6 poles**

Total No. of slots = 3 slots/pole/phase × 6 poles × 3 phases = **54**

(ii) $N_s = 120\ f/P = 120 \times 50/6 =$ **1000 r.p.m.**

Example 34.3. *A 4-pole, 3-phase induction motor operates from a supply whose frequency is 50 Hz. Calculate :*

(i) *the speed at which the magnetic field of the stator is rotating.*

(ii) *the speed of the rotor when the slip is 0.04.*

(iii) *the frequency of the rotor currents when the slip is 0.03.*

(iv) *the frequency of the rotor currents at standstill.*

(Electrical Machinery II, Banglore Univ. 1991)

Solution. *(i)* Stator field revolves at synchronous speed, given by

$N_s = 120\ f/P = 120 \times 50/4 =$ **1500 r.p.m.**

(ii) rotor (or motor) speed, $N = N_s\ (1 - s) = 1500(1 - 0.04) =$ **1440 r.p.m.**

(iii) frequency of rotor current, $f' = sf = 0.03 \times 50 = 1.5$ r.p.s = **90 r.p.m**

(iv) Since at standstill, $s = 1$, $f' = sf = 1 \times f = f =$ **50Hz**

Example 34.4. *A 3-ϕ induction motor is wound for 4 poles and is supplied from 50-Hz system. Calculate **(i)** the synchronous speed **(ii)** the rotor speed, when slip is 4% and **(iii)** rotor frequency when rotor runs at 600 rpm.* **(Electrical Engineering-I, Pune Univ. 1991)**

Solution. *(i)* $N_s = 120\ f/P = 120 \times 50/4 = 1500$ rpm

(ii) rotor speed, $N = N_s\ (1 - s) = 1500\ (1 - 0.04) =$ **1440 rpm**

(iii) when rotor speed is 600 rpm, slip is

$$s = (Ns - N)/N_s = (1500 - 600)/1500 = 0.6$$

rotor current frequency, $f' = sf = 0.6 \times 50 =$ **30 Hz**

Example 34.5. *A 12-pole, 3-phase alternator driven at a speed of 500 r.p.m. supplies power to an 8-pole, 3-phase induction motor. If the slip of the motor, at full-load is 3%, calculate the full-load speed of the motor.*

Solution. Let N = actual motor speed; Supply frequency, $f = 12 \times 500/120 = 50$ Hz. Synchronous speed $N_s = 120 \times 50/8 = 750$ r.p.m.

$$\% \text{ slip } s = \frac{N_S - N}{N} \times 100\,; \quad 3 = \frac{750 - N}{750} \times 100 \qquad \therefore N = \textbf{727.5 r.p.m.}$$

Note. Since slip is 3%, actual speed N is less than N_s by 3% of N_s *i.e.* by $3 \times 750/100 = 22.5$ r.p.m.

34.12. Relation Between Torque and Rotor Power Factor

In Art. 29.7, it has been shown that in the case of a d.c. motor, the torque T_a is proportional to the product of armature current and flux per pole *i.e.* $T_a \propto \phi I_a$. Similarly, in the case of an induction motor, the torque is also proportional to the product of flux per stator pole and the rotor current. However, there is one more factor that has to be taken into account *i.e.* the power factor of the rotor.

$\therefore \quad T \propto \phi I_2 \cos\phi_2 \quad$ or $\quad T = k \phi I_2 \cos\phi_2$

where I_2 = rotor current at standstill

ϕ_2 = angle between rotor e.m.f. and rotor current

k = a constant

Denoting rotor e.m.f. at ***standstill*** by E_2, we have that $E_2 \propto \phi$

$\therefore \quad T \propto E_2 I_2 \cos\phi_2$

or $\quad T = k_1 E_2 I_2 \cos\phi_2$

where k_1 is another constant.

The effect of rotor power factor on rotor torque is illustrated in Fig. 34.17 and Fig. 34.18 for various values of ϕ_2. From the above expression for torque, it is clear that as ϕ_2 increases (and hence, cos ϕ_2 decreases) the torque decreases and ***vice versa.***

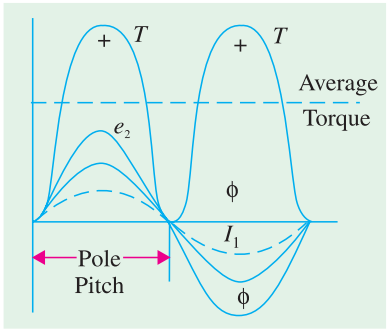


Fig. 34.17

In the discussion to follow, the stator flux distribution is assumed sinusoidal. This revolving flux induces in each rotor conductor or bar an *e.m.f.* whose value depends on the flux density, in which the conductor is lying at the instant considered ($\because e = Blv$ volt). Hence, the induced e.m.f. in the rotor is also sinusoidal.

(i) Rotor Assumed Non-inductive (or $\phi_2 = 0$)

In this case, the rotor current I_2 is in phase with the e.m.f. E_2 induced in the rotor (Fig. 34.17). The instantaneous value of the torque acting on each rotor conductor is given by the product of instantaneous value of the flux and the rotor current ($\because F \propto BI_2 l$). Hence, torque curve is obtained by plotting the products of flux ϕ (or flux density B) and I_2. It is seen that the torque is always positive *i.e.* unidirectional.

(ii) Rotor Assumed Inductive

This case is shown in Fig. 34.18. Here, I_2 lags behind E_2 by an angle $\phi_2 = \tan^{-1} X_2/R_2$ where R_2 = rotor resistance/phase; X_2 = rotor reactance/phase at ***standstill.***

It is seen that for a portion '*ab*' of the pole pitch, the torque is negative *i.e.* reversed. Hence, the total torque which is the difference of the forward and the backward torques, is considerably reduced. If $\phi_2 = 90°$, then the total torque is zero because in that case the backward and the forward torques become equal and opposite.

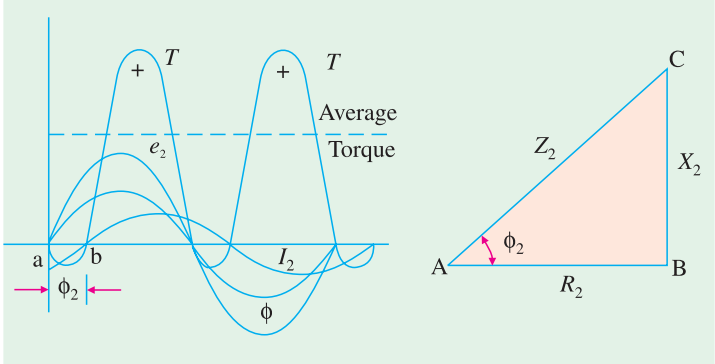


Fig. 34.18

Fig. 34.19

34.13. Starting Torque

The torque developed by the motor at the instant of starting is called starting torque. In some cases, it is greater than the normal running torque, whereas in some other cases it is somewhat less.

Let E_2 = rotor *e.m.f.* per phase at ***standstill***;
R_2 = rotor resistance/phase
X_2 = rotor reactance/phase at ***standstill***

$\therefore$ $Z_2 = \sqrt{(R_2^2 + X_2^2)}$ = rotor impedance/phase at ***standstill*** ...Fig. 34.19

Then, $I_2 = \dfrac{E_2}{Z_2} = \dfrac{E_2}{\sqrt{(R_2^2 + X_2^2)}}$; $\cos\phi_2 = \dfrac{R_2}{Z_2} = \dfrac{R_2}{\sqrt{(R_2^2 + X_2^2)}}$

Standstill or starting torque $T_{st} = k_1 E_2 I_2 \cos\phi_2$ ...Art. 34.12

or $T_{st} = k_1 E_2 \cdot \dfrac{E_2}{\sqrt{(R_2^2 + X_2^2)}} \times \dfrac{R_2}{\sqrt{(R_2^2 + X_2^2)}} = \dfrac{k_1 E_2^2 R_2}{R_2^2 + X_2^2}$ ...(*i*)

If supply voltage V is constant, then the flux Φ and hence, E_2 both are constant.

$\therefore$ $T_{st} = k_2 \dfrac{R_2}{R_2^2 + X_2^2} = k_2 \dfrac{R_2}{Z_2^2}$ where k_2 is some other constant.

Now, $k_1 = \dfrac{3}{2\pi N_s}$, $\therefore$ $T_{st} = \dfrac{3}{2\pi N_s} \cdot \dfrac{E_2^2 R_2}{R_2^2 + X_2^2}$

Where Ns $\rightarrow$ synchronous speed in rps.

34.14. Starting Torque of a Squirrel-cage Motor

The resistance of a squirrel-cage motor is fixed and small as compared to its reactance which is very large especially at the start because at standstill, the frequency of the rotor currents equals the supply frequency. Hence, the starting current I_2 of the rotor, though very large in magnitude, lags by a very large angle behind E_2, with the result that the starting torque per ampere is very poor. It is roughly 1.5 times the full-load torque, although the starting current is 5 to 7 times the full-load current. Hence, such motors are not useful where the motor has to start against heavy loads.

34.15. Starting Torque of a Slip-ring Motor

The starting torque of such a motor is increased by improving its power factor by adding external resistance in the rotor circuit from the star-connected rheostat, the rheostat resistance being progres-

sively cut out as the motor gathers speed. Addition of external resistance, however, increases the rotor impedance and so reduces the rotor current. At first, the effect of improved power factor predominates the current-decreasing effect of impedance. Hence, starting torque is increased. But after a certain point, the effect of increased impedance predominates the effect of improved power factor and so the torque starts decreasing.

34.16. Condition for Maximum Starting Torque

It can be proved that starting torque is maximum when rotor resistance equals rotor reactance.

Now $T_{st} = \dfrac{k_2 R_2}{R_2^2 + X_2^2}$ $\quad\therefore\quad \dfrac{dT_{st}}{dR_2} = k_2\left[\dfrac{1}{R_2^2 + X_2^2} - \dfrac{R_2\,(2R_2)}{(R_2^2 + X_2^2)^2}\right] = 0$

or $R_2^{\,2} + X_2^{\,2} = 2R_2^{\,2}$ $\quad\therefore\quad R_2 = X_2.$

34.17. Effect of Change in Supply Voltage on Starting Torque

We have seen in Art. 34.13 that $T_{st} = \dfrac{k_1 E_2^2 R_2}{R_2^2 + X_2^2}$. Now $E_2 \propto$ supply voltage V

$\therefore$ $T_{st} = \dfrac{k_3 V^2 R_2}{R_2^2 + X_2^2} = \dfrac{k_3 V^2 R_2}{Z_2^2}$ where k_3 is yet another constant. Hence $T_{st} \propto V^2$.

Clearly, the torque is very sensitive to any changes in the supply voltage. A change of 5 per cent in supply voltage, for example, will produce a change of approximately 10% in the rotor torque. This fact is of importance in star-delta and auto transformer starters (Art. 33-11).

Example 34.6. *A 3-6 induction motor having a star-connected rotor has an induced e.m.f. of 80 volts between slip-rings at standstill on open-circuit. The rotor has a resistance and reactance per phase of 1 Ω and 4 Ω respectively. Calculate current/phase and power factor when* ***(a)*** *slip-rings are short-circuited* ***(b)*** *slip-rings are connected to a star-connected rheostat of 3 Ω per phase.*

(Electrical Technology, Bombay Univ. 1987, and similar example: Rajiv Gandhi Techn. Univ. Bhopal, Dec. 2000)

Solution. Standstill e.m.f./rotor phase = $80/\sqrt{3} = 46.2$ V

(a) Rotor impedance/phase = $\sqrt{(1^2 + 4^2)} = 4.12\ \Omega$

Rotor current/phase = $46.2/4.12 =$ **11.2 A**

Power factor = $\cos\phi = 1/4.12 =$ **0.243**

As p.f. is low, the starting torque is also low.

(b) Rotor resistance/phase = $3 + 1 = 4\ \Omega$

Rotor impedance/phase = $\sqrt{(4^2 + 4^2)} = 5.66\ \Omega$

$\therefore$ Rotor current/phase = $46.2/5.66 =$ **8.16 A**; $\cos\phi = 4/5.66 =$ **0.707**.

Hence, the starting torque is increased due to the improvement in the power factor. It will also be noted that improvement in p.f. is much more than the decrease in current due to increased impedance.

Example 34.7. *A 3-phase, 400-V, star-connected induction motor has a star-connected rotor with a stator to rotor turn ratio of 6.5. The rotor resistance and standstill reactance per phase are 0.05 Ω and 0.25 Ω respectively. What should be the value of external resistance per phase to be inserted in the rotor circuit to obtain maximum torque at starting and what will be rotor starting current with this resistance?*

Solution. Here $K = \dfrac{1}{6.5}$ because transformation ratio K is defined as

$$= \frac{\text{rotor turns/phase}}{\text{stator turns/phase}}$$

Standstill rotor e.m.f./phase, $E_2 = \frac{400}{\sqrt{3}} \times \frac{1}{6.5} = 35.5$ volt

It has been shown in Art. 34.16 that starting torque is maximum when $R_2 = X_2$ *i.e.* when $R_2 = 0.25\ \Omega$ in the present case

$\therefore$ External resistance/phase required $= 0.25 - 0.05 =$ **0.2 Ω**

Rotor impedance/phase $= \sqrt{(0.25^2 + 0.25^2)} = 0.3535\ \Omega$

Rotor current/phase, $I_2 = 35.5/0.3535 =$ **100 A (approx)**

Example 34.8. *A 1100-V, 50-Hz delta-connected induction motor has a star-connected slip-ring rotor with a phase transformation ratio of 3.8. The rotor resistance and standstill leakage reactance are 0.012 ohm and 0.25 ohm per phase respectively. Neglecting stator impedance and magnetising current determine.*

(i) *the rotor current at start with slip-rings shorted*

(ii) *the rotor power factor at start with slip-rings shorted*

(iii) *the rotor current at 4% slip with slip-rings shorted*

(iv) *the rotor power factor at 4% slip with slip-rings shorted*

(v) *the external rotor resistance per phase required to obtain a starting current of 100 A in the stator supply lines.* **(Elect. Machines AMIE Sec. B 1992)**

Solution. It should be noted that in a Δ/Y connection, primary phase voltage is the same as the line voltage. The rotor phase voltage can be found by using the phase transformation ratio of 3.8 *i.e.* $K = 1/3.8$.

Rotor phase voltage at standstill $= 1100 \times 1/3.8 = 289.5$V

(i) Rotor impedence/phase $= \sqrt{0.012^2 + 0.25^2} = 0.2503\ \Omega$

Rotor phase current at start $= 289.5/0.2503 =$ **1157 A**

(ii) $p.f. = R_2/Z_2 = 0.012/0.2503 =$ **0.048 lag**

(iii) at 4% slip, $X_r = sX_2 = 0.04 \times 0.25 = 0.01\ \Omega$

$\therefore$ $Z_r = \sqrt{0.012^2 + 0.01^2} =$ **0.0156 Ω**

$E_r = sE_2 = 0.04 \times 289.5 = 11.58$ V; $I_2 = 11.58/0.0156 =$ **742.3 A**

(iv) $p.f. = 0.012/0.0156 =$ **0.77**

(v) $I_2 = I_1/K = 100 \times 3.8 = 380$ A; E_2 at standstill $= 289.5$ V

$Z_2 = 289.5/380 = 0.7618\ \Omega$; $R_2 = \sqrt{Z_2^2 - X_2^2} = \sqrt{0.7618^2 - 0.25^2} = 0.7196\ \Omega$

$\therefore$ External resistance reqd./phase $= 0.7196 - 0.012 =$ **0.707 Ω**

Example 34.9. *A 150-kw, 3000-V, 50-Hz, 6-pole star-connected induction motor has a star-connected slip-ring rotor with a transformation ratio of 3.6 (stator/rotor). The rotor resistance is 0.1Ω/phase and its per phase leakage reactance is 3.61 mH. The stator impedance may be neglected. Find the starting current and starting torque on rated voltage with short-circuited slip rings.* **(Elect. Machines, A.M.I.E. Sec. B, 1989)**

Solution. $X_2 = 2\pi \times 50 \times 5.61 \times 10^{-3} = 1.13\ \Omega$

$K = 1/3.6$, $R_2' = R_2/K^2 = (3.6)^2 \times 0.1 = 1.3\ \Omega$

$X_2 = 2\pi \times 50 \times 3.61 \times 10^{-3} = 1.13\ \Omega$; $X_2' = (3.6)^2 \times 1.13 = 14.7\Omega$

$$I_{st} = \frac{V}{(R_2')^2 + (X_2')^2} \cdot \frac{3000/\sqrt{3}}{\sqrt{(1.3)^2 + (14.7)^2}} = \mathbf{117.4\ A}$$

Now, $N_s = 120 \times 50/6 = 1000 \text{ rpm} = (50/3) \text{ rps}$

$$T_{st} = \frac{3}{2\pi N_s} \cdot \frac{V^2 R_2'}{(R_2')^2 + (X_2')^2} = \frac{3}{2\pi\,(50/3)} \times \frac{(3000/\sqrt{3})^2 \times 1.3}{(1.3^2 + 14.7^2)} = \mathbf{513\ N\text{-}m}$$

Tutorial Problem No. 34.1

1. In the case of an 8-pole induction motor, the supply frequency was 50-Hz and the shaft speed was 735 r.p.m. What were the magnitudes of the following: ***(Nagpur Univ., Summer 2000)***

(*i*) synchronous speed (*ii*) speed of slip

(*iii*) per unit slip (*iv*) percentage slip

[750 r.p.m. ; 15 r.p.m.; 0.02 ; 2%]

2. A 6-pole, 50-Hz squirrel-cage induction motor runs on load at a shaft speed of 970 r.p.m. Calculate:-

(*i*) the percentage slip

(*ii*) the frequency of induced current in the rotor. **[3% ; 1.5 Hz]**

3. An 8-pole alternator runs at 750. r.p.m. and supplies power to a 6-pole induction motor which has at full-load a slip of 3%. Find the full-load speed of the induction motor and the frequency of its rotor e.m.f. **[970 r.p.m. ; 1.5 Hz]**

4. A 3-phase, 50-Hz induction motor with its rotor star-connected gives 500 V (r.m.s.) at standstill between the slip-rings on open-circuit. Calculate the current and power factor at standstill when the rotor winding is joined to a star-connected external circuit, each phase of which has a resistance of 10 Ω and an inductance of 0.04 H. The resistance per phase of the rotor winding is 0.2 Ω and its inductance is 0.04 H.

Also, calculate the current and power factor when the slip-rings are short-circuited and the motor is running with a slip of 5 per cent. Assume the flux to remain constant.

[10.67 A; 0.376; 21.95 A; 0.303]

5. Obtain an expression for the condition of maximum torque of an induction motor. Sketch the torque-slip curves for several values of rotor circuit resistance and indicate the condition for maximum torque to be obtained at starting.

If the motor has a rotor resistance of 0.02 Ω and a standstill reactance of 0.1 Ω, what must be the value of the total resistance of a starter for the rotor circuit for maximum torque to be exerted at starting ? **[0.08 Ω] (*City and Guilds, London*)**

6. The rotor of a 6-pole, 50-Hz induction motor is rotated by some means at 1000 r.p.m. Compute (*i*) rotor voltage (*ii*) rotor frequency (*iii*) rotor slip and (iv) torque developed. Can the rotor rotate at this speed by itself ? **[(*i*) 0 (*ii*) 0 (*iii*) 0 (*iv*) 0; No] (*Elect. Engg. Grad I.E.T.E. June 1985*)**

7. The rotor resistances per phase of a 4-pole, 50-Hz, 3-phase induction motor are 0.024 ohm and 0.12 ohm respectively. Find the speed at maximum torque. Also find the value of the additional rotor resistance per phase required to develop 80% of maximum torque at starting.

[1200 r.p.m. 0.036 Ω] (*Elect. Machines, A.M.I.E. Sec. B, 1990*)

8. The resistance and reactance per phase of the rotor of a 3-phase induction motor are 0.6 ohm and 5 ohms respectively. The induction motor has a star-connected rotor and when the stator is connected to a supply of normal voltage, the induced e.m.f. between the slip rings at standstill is 80 V. Calculate the current in each phase and the power factor at starting when (*i*) the slip-rings are shorted, (*ii*) slip-rings are connected to a star-connected resistance of 4 ohm per phase.

[(*i*) 9.17 amp, 0.1194 lag (*ii*) 6.8 amp, 0.6765 lag][*Rajiv Gandhi Technical University, Bhopal, 2000*]

34.18. Rotor E.M.F. and Reactance Under Running Conditions

Let E_2 = ***standstill*** rotor induced e.m.f./phase

X_2 = ***standstill*** rotor reactance/phase, f_2 = rotor current frequency at ***standstill***

When rotor is stationary *i.e.* $s = 1$, the frequency of rotor e.m.f. is the same as that of the stator supply frequency. The value of e.m.f. induced in the rotor at standstill is maximum because the relative speed between the rotor and the revolving stator flux is maximum. In fact, the motor is equivalent to a 3-phase transformer with a short-circuited rotating secondary.

When rotor starts running, the relative speed between it and the rotating stator flux is decreased. Hence, the rotor induced e.m.f. which is directly proportional to this relative speed, is also decreased (and may disappear altogether if rotor speed were to become equal to the speed of stator flux). Hence, for a slip s, the rotor induced e.m.f. will be s times the induced e.m.f. at standstill.

Therefore, under ***running conditions*** $E_r = sE_2$

The frequency of the induced e.m.f. will likewise become $f_r = sf_2$

Due to decrease in frequency of the rotor e.m.f., the rotor reactance will also decrease.

$$\therefore \quad X_r = sX_2$$

where E_r and X_r are rotor e.m.f. and reactance under ***running*** conditions.

34.19. Torque Under Running Conditions

$$T \propto E_r I_r \cos \phi_2 \text{ or } T \propto \phi I_r \cos \phi_2 \qquad (\because E_r \propto \phi)$$

where E_r = rotor e.m.f./phase under ***running conditions***

I_r = rotor current/phase under ***running conditions***

Now $E_r = sE_2$

$$\therefore \quad I_r = \frac{E_r}{Z_r} = \frac{sE_2}{\sqrt{[R_2^2 + (sX_2)^2]}}$$

$$\cos \phi_2 = \frac{R_2}{\sqrt{[R_2^2 + (sX_2)^2]}} \quad \text{—Fig. 34.20}$$

$$\therefore \quad T \propto \frac{s\,\Phi\, E_2 R_2}{R_2^2 + (sX_2)^2} = \frac{k\Phi \,.\, s \,.\, E_2 R_2}{R_2^2 + (sX_2)^2}$$

$$\text{Also } T = \frac{k_1 \,.\, sE_2^2\, R_2}{R_2^2 + (sX_2)^2} \qquad (\because E_2 \propto \phi)$$

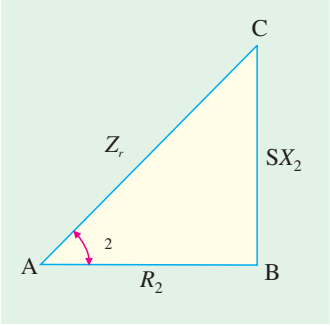


Fig. 34.20

where k_1 is another constant. Its value can be proved to be equal to $3/2\,\pi N_s$ (Art. 34.38). Hence, in that case, expression for torque becomes

$$T = \frac{3}{2\pi N_S} \cdot \frac{sE_2^2\, R_2}{R_2^2 + (sX_2)^2} = \frac{3}{2\pi N_S} \cdot \frac{sE_2^2\, R_2}{Zr^2}$$

At standstill when $s = 1$, obviously

$$T_{st} = \frac{k_1 E_2^2\, R_2}{R_2^2 + X_2^2} \left(\text{or} = \frac{3}{2\pi N_S} \cdot \frac{E_2^2 R_2}{R_2^2 + X_2^2} \right)$$ the same as in Art. 34.13.

Example 34.10. *The star connected rotor of an induction motor has a standstill impedance of (0.4 + j4) ohm per phase and the rheostat impedance per phase is (6 + J2) ohm.*

The motor has an induced emf of 80 V between slip-rings at standstill when connected to its normal supply voltage. Find

(*i*) *rotor current at standstill with the rheostat is in the circuit.*

(*ii*) *when the slip-rings are short-circuited and motor is running with a slip of 3%.*

(Elect.Engg. I, Nagpur Univ. 1993)

Solution. (1) Standstill Conditions

Voltage/rotor phase = $80/\sqrt{3}$ = 46.2. V; rotor and
starter impedance/phase = $(6.4 + j6) = 8.77 \angle 43.15°$
Rotor current/phase = 46.2/8.77 = **5.27 A** (p.f. = cos 43.15° = 0.729)

(2) Running Conditions. Here, starter impedance is cut out.

Rotor voltage/phase, $E_r = sE_2 = 0.03 \times 46.2 = 1.386$ V
Rotor reactance/phase, $X_r = 0.03 \times 4 = 0.12\ \Omega$
Rotor impedance/phase, $Z_r = 0.4 + j0.12 = 0.4176 \angle 16.7°$
Rotor current/phase = 1.386/0.4176 = **3.32 A** (p.f. = cos 16.7° = 0.96)

Note. It has been assumed that flux across the air -gap remains constant

34.20. Condition for Maximum Torque Under Running Conditions

The torque of a rotor under ***running*** conditions is

$$T = \frac{k\Phi sE_2R_2}{R_2^2 + (sX_2)^2} = k_1 \frac{sE_2^2\, R_2}{R_2^2 + (sX_2)^2} \qquad ...(i)$$

The condition for maximum torque may be obtained by differentiating the above expression with respect to slip s and then putting it equal to zero. However, it is simpler to put $Y = \frac{1}{T}$ and then differentiate it.

$$\therefore \quad Y = \frac{R_2^2 + (sX_2)^2}{k\ \Phi\ sE_2R_2} = \frac{R_2}{k\ \Phi\ sE_2} + \frac{sX_2^2}{k\Phi\ E_2R_2};\ \frac{dY}{ds} = \frac{-\ R_2}{k\Phi\ s^2E_2} + \frac{X_2^2}{k\Phi E_2R_2} = 0$$

$$\therefore \quad \frac{R_2}{k\ \Phi\ s^2E_2} = \frac{X_2^2}{k\ \Phi\ E_2R_2} \quad \text{or} \quad R_2^{\,2} = s^2\, X_2^{\,2} \text{ or } R_2 = sX_2$$

Hence, torque under ***running condition*** is maximum at that value of the slip s which makes rotor reactance per phase equal to rotor resistance per phase. This slip is sometimes written as s_b and the maximum torque as T_b.

Slip corresponding to maximum torque is $s = R_2/X_2$

Putting $R_2 = sX_2$ in the above equation for the torque, we get

$$T_{max} = \frac{k\ \Phi\ s^2\ E_2X_2}{2\ s^2\ X_2^2}\left(\text{or } \frac{k\ \Phi\ sE_2R_2}{2R_2^2}\right) \text{or } T_{max} = \frac{k\ \Phi\ E_2}{2\ X_2}\left(\text{or } \frac{k\ \Phi\ sE_2}{2\ R_2}\right) \qquad ...(ii)$$

Substituting value of $s = R_2/X_2$ in the other equation given in (*i*) above, we get

$$T_{max} = k_1 \frac{(R_2/X_2)\,.\,E_2^2\,.\,R_2}{R_2^2 + (R_2/X_2)^2\,.\,X_2^2} = k_1 \frac{E_2^2}{2\ X_2}$$

Since, $k_1 = 3/2\pi N_s$, we have $T_{max} = \frac{3}{2\pi N_s} \cdot \frac{E_2^2}{2X_2}$ N-m

From the above, it is found

1. *that the maximum torque is independent of rotor resistance as such.*
2. *however, the speed or slip at which maximum torque occurs is determined by the rotor*

resistance. As seen from above, torque becomes maximum when rotor reactance equals its resistance. Hence, by varying rotor resistance (possible only with slip-ring motors) maximum torque can be made to occur at any desired slip (or motor speed).

3. *maximum torque varies inversely as standstill reactance. Hence, it should be kept as small as possible.*
4. *maximum torque varies directly as the square of the applied voltage.*
5. *for obtaining maximum torque at starting (s =1), rotor resistance must be equal to rotor reactance.*

Example 34.11. *A 3-phase, slip-ring, induction motor with star-connected rotor has an induced e.m.f. of 120 volts between slip-rings at standstill with normal voltage applied to the stator. The rotor winding has a resistance per phase of 0.3 ohm and standstill leakage reactance per phase of 1.5 ohm.*

*Calculate **(i)** rotor current/phase when running short-circuited with 4 percent slip and **(ii)** the slip and rotor current per phase when the rotor is developing maximum torque.*

(Elect. Engg.-II, Pune Univ. 1989)

Solution. (*i*) induced e.m.f./rotor phase, $E_r = sE_2 = 0.04 \times (120/\sqrt{3}) =$ **2.77V**

rotor reactance/phase, $X_r = sX_2 = 0.04 \times 1.5 = 0.06\ \Omega$

rotor impedance/phase $= \sqrt{0.3^2 + 0.06^2} = 0.306\ \Omega$

rotor current/phase $= 2.77/0.306 =$ **9A**

(ii) For developing maximum torque,

$R_2 = sX_2$ or $s = R_2/X_2 = 0.3/1.5 =$ **0.2**

$X_r = 0.2 \times 1.5 = 0.3\ \Omega, Z_r = \sqrt{0.3^2 + 0.3^2} = 0.42\ \Omega$

$E_r = sE_2 = 0.2 \times (120/\sqrt{3}) = 13.86$ V

$\therefore$ Rotor current/phase $= 13.86/0.42 =$ **33 A**

34.21. Rotor Torque and Breakdown Torque

The rotor torque at any slip s can be expressed in terms of the maximum (or breakdown) torque T_b by the following equation

$$T = T_b \left[\frac{2}{(s_b / s) + (s / s_b)} \right]$$ where s_b is the breakdown or pull-out slip.

Example 34.12. *Calculate the torque exerted by an 8-pole, 50-Hz, 3-phase induction motor operating with a 4 per cent slip which develops a maximum torque of 150 kg-m at a speed of 660 r.p.m. The resistance per phase of the rotor is 0.5 Ω.* **(Elect. Machines, A.M.I.E. Sec. B, 1989)**

Solution. $N_s = 120 \times 50/8 = 750$ r.p.m.

Speed at maximum torque $= 660$ r.p.m. Corresponding slip $s_b = \dfrac{750 - 660}{750} = 0.12$

For maximum torque, $R_2 = s_b X_2$

$\therefore$ $X_2 = R_2/s_b = 0.5/0.12 =$ **4.167 Ω**

As seen from Eq. (*ii*) of Art. 34.20,

$$T_{max} = k\,\Phi\,E_2 \cdot \frac{s_b}{2R_2} = k\,\Phi\,E_2 \cdot \frac{0.12}{2 \times 0.5} = 0.12\,k\,\Phi\,E_2 \quad \text{...(i)}$$

When slip is 4 per cent

As seen from Eq. (*i*) of Art. 34.20

$$T = k\,\Phi\,E_2\,\frac{sR_2}{R_2^2 + (sX_2)^2} = k\Phi E_2\,\frac{0.04\times 0.5}{0.5^2 + (0.04\times 4.167)^2} = \frac{0.02\,k\,\Phi\,E_2}{0.2778}$$

$$\therefore \quad \frac{T}{T_{max}} = \frac{T}{150} = \frac{0.02}{0.2778\times 0.12} \quad \therefore \quad T = \mathbf{90\ kg\text{-}m}$$

Alternative Solution

$$T_b = 150 \text{ kg.m}; s_b = 0.12, s = 4\% = 0.04, T = ?$$

$$T = T_b\left(\frac{2}{(s_b/s) + (s/s_b)}\right) \qquad \text{...Art 34.21}$$

$$= 150\left(\frac{2}{(0.12/0.04) + (0.04/0.12)}\right) = \mathbf{90\ kg\text{-}m}$$

34.22. Relation Between Torque and Slip

A family of torque/slip curves is shown in Fig. 34.21 for a range of $s = 0$ to $s = 1$ with R_2 as the parameter. We have seen above in Art. 34.19 that

$$T = \frac{k\,\Phi\,sE_2R_2}{R_2^2 + (sX_2)^2}$$

It is clear that when $s = 0$, $T = 0$, hence the curve starts from point O.

At normal speeds, close to synchronism, the term $(s\,X_2)$ is small and hence negligible *w.r.t.* R_2.

$$\therefore \quad T \propto \frac{s}{R_2}$$

or $T \propto s$ if R_2 is constant.

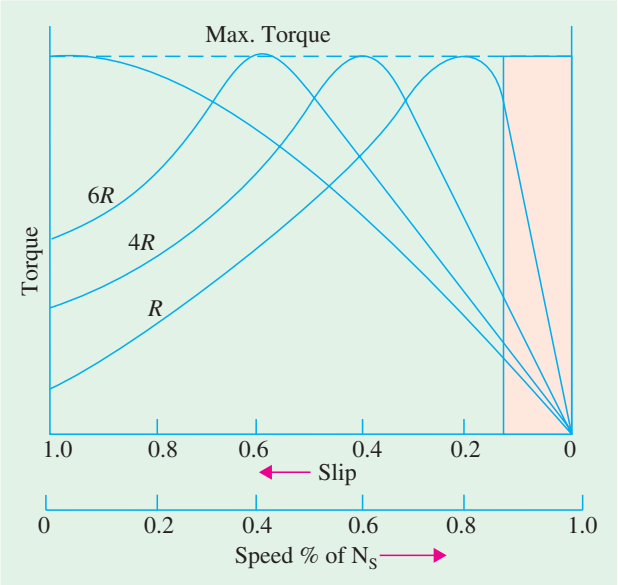


Fig. 34.21

Hence, for low values of slip, the torque/slip curve is approximately a straight line. As slip increases (for increasing load on the motor), the torque also increases and becomes maximum when $s = R_2/X_2$. This torque is known as '***pull-out***' or '***breakdown***' torque T_b or stalling torque. As the slip further increases (*i.e.* motor speed falls) with further increase in motor load, then R_2 becomes negligible as compared to (sX_2). Therefore, for large values of slip

$$T \propto \frac{s}{(sX_2)^2} \propto \frac{1}{s}$$

Hence, the torque/slip curve is a rectangular hyperbola. So, we see that beyond the point of maximum torque, any further increase in motor load results in decrease of torque developed by the motor. The result is that the motor slows down and eventually stops. The circuit-breakers will be tripped open if the circuit has been so protected. In fact, the stable operation of the motor lies between the values of $s = 0$ and that corresponding to maximum torque. The operating range is shown shaded in Fig. 34.21.

It is seen that although maximum torque does not depend on R_2, yet the *exact location* of T_{max} is dependent on it. Greater the R_2, greater is the value of slip at which the maximum torque occurs.

34.23. Effect of Change in Supply Voltage on Torque and Speed

As seen from Art. 34.19, $T = \dfrac{k\Phi\ sE_2R_2}{R_2^2 + (sX_2)^2}$

As $E_2 \propto \phi \propto V$ where V is supply voltage $\therefore T \propto sV^2$

Obviously, torque at any speed is proportional to the square of the applied voltage. If stator voltage decreases by 10%, the torque decreases by 20%. Changes in supply voltage not only affect the starting torque T_{st} but torque under running conditions also. If V decreases, then T also decreases. Hence, for maintaining the same torque, slip increases *i.e.* speed falls.

Let V change to V', s to s' and T to T'; then $\dfrac{T}{T'} = \dfrac{sV^2}{s'V'^2}$

34.24. Effect of Changes in supply Frequency on Torque and Speed

Hardly any important changes in frequency take place on a large distribution system except during a major disturbance. However, large frequency changes often take place on isolated, low-power systems in which electric energy is generated by means of diesel engines or gas turbines. Examples of such systems are : emergency supply in a hospital and the electrical system on a ship etc.

The major effect of change in supply frequency is on motor ***speed***. If frequency drops by 10%, then motor speed also drops by 10%. Machine tools and other motor-driven equipment meant for 50 Hz causes problem when connected to 60-Hz supply. Everything runs $(60 - 50) \times 100/50 = 20\%$ faster than normal and this may not be acceptable in all applications. In that case, we have to use either gears to reduce motor speed or an expensive 50-Hz source.

A 50-Hz motor operates well on a 60-Hz line provided its terminal voltage is raised to 60/50 = 6/5 (*i.e.* 120%) of the name-plate rating. In that case, the new breakdown torque becomes equal to the original breakdown torque and the starting torque is only slightly reduced. However, power factor, efficiency and temperature rise remain satisfactory.

Similarly, a 60-Hz motor can operate satisfactorily on 50-Hz supply provided its terminal voltage is reduced to 5/6 (*i.e.* 80%) of its name-plate rating.

34.25. Full-load Torque and Maximum Torque

Let s_f be the slip corresponding to full-load torque, then

$$T_f \propto \frac{s_f\ R_2}{R_2^2 + (s_f\ X_2)^2} \quad \text{and} \quad T_{\max} \propto \frac{1}{2 \times X_2} \qquad \text{...Art 34.20}$$

$$\therefore \quad \frac{T_f}{T_{\max}} = \frac{2s_f\ R_2\ X_2}{R_2^2 + (s_f\ X_2)^2}$$

Dividing both the numerator and the denominator by X_2^2, we get

$$\frac{T_f}{T_{\max}} = \frac{2s_f\ .\ R_2 / X_2}{(R_2 / X_2)^2 + s_f^2} = \frac{2as_f}{a^2 + s_f^2}$$

where $a = R_2/X_2$ = resistance/standstill reactance*

---

* In fact $a = s_m$ —slip corresponding to maximum torque. In that case, the relation becomes

$\dfrac{T_f}{T_{\max}} = \dfrac{2s_m s_f^2}{s_m^2 + s_f^2}$ — where s_f = full-load slip.

In general, $\dfrac{\text{operating torque at any slip } s}{\text{maximum torque}} = \dfrac{2\,as}{a^2 + s_f^{\,2}}$

34.26. Starting Torque and Maximum Torque

$$T_{st} \propto \frac{R_2}{R_2^2 + X_2^2} \qquad \text{...Art 34.13}$$

$$T_{\max} \propto \frac{1}{2\,X_2} \qquad \text{...Art. 34.20}$$

$$\therefore \quad \frac{T_{st}}{T_{\max}} = \frac{2R_2X_2}{R_2^2 + X_2^2} = \frac{2R_2 / X_2}{1 + (R_2 / X_2)^2} = \frac{2a}{1 + a^2}$$

where $a = \dfrac{R_2}{X_2} = \dfrac{\text{rotor resistance}}{\text{stand still reactance}}$ per phase*

Example. 34.13(*a*). *A 3-ϕ induction motor is driving full-load torque which is independent of speed. If line voltage drops to 90% of the rated value, find the increase in motor copper losses.*

Solution. As seen from Art. 34.23, when T remains constant, $s_1 V_1^{\,2} = s_2 V_2^{\,2}$

$$\therefore \quad \frac{s_2}{s_1} = \left(\frac{V_1}{V_2}\right)^2 = \left(\frac{1}{0.9}\right)^2 = 1.23$$

Again from Art. 34-19, $I_2 \propto sV$ $\therefore$ $\dfrac{I_2'}{I_2} = \dfrac{s_2V_2}{s_1V_1} = 1.2 \times 0.9 = 1.107$

Now, Cu losses are nearly proportional to $I_2^{\,2}$

$$\therefore \quad \frac{\text{Cu loss in the 2nd case}}{\text{Cu loss in the 1st case}} = \frac{(I_2')^2}{I_2^2} = 1.107^2 = 1.23$$

Thus a reduction of 10% in line voltage causes about 23% increase in Cu losses.

Example. 34.13 (*b*). *A 230-V, 6-pole, 3-ϕ, 50-Hz, 15-kW induction motor drives a constant torque load at rated frequency, rated voltage and rated kW output and has a speed of 980 rpm and an efficiency of 93%. Calculate **(i)** the new operating speed if there is a 10% drop in voltage and 5% drop in frequency and **(ii)** the new output power. Assume all losses to remain constant.*

Solution. (*i*) $V_2 = 0.9 \times 230 = 207$ V; $f_2 = 0.95 \times 50 = 47.5$ Hz; $N_{s1} = 120 \times 50/6 = 1000$ rpm; $N_{s2} = 120 \times 47.5/6 = 950$ rpm; $s_1 = (1000 - 980)/1000 = 0.02$

Since the load torque remains constant, the product (sV^2/f) remains constant.

$$\therefore \quad s_1V_1^{\,2}/f_1 = \frac{s_2V_2^{\,2}}{f_2} \text{ or } s_2 = s_1\left(\frac{V_1}{V_2}\right)^2 . f_2/f_1 = 0.02\,(230/230 \times 0.9)^2 \times 47.5/50 = 0.2234$$

$$\therefore \quad N_2 = Ns_2\,(1 - s_2) = 950\,(1 - 0.0234) = 928 \text{ rpm}$$

(ii) $P \propto TN$. Since torque remains constant, $P \propto N$

$\therefore$ $P_1 \propto N_1$; $P_2 \propto N_2$; or $P_2 = P_1 \times N_2/N_1 = 15 \times 928/980 =$ **14.2 kW**

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* Similarly, the relation becomes $\dfrac{T_f}{T_{\max}} = \dfrac{2s_m}{1 + s_m^2}$

Example 34.14 (*a*). *A 3-phase, 400/200-V, Y-Y connected wound-rotor induction motor has 0.06Ω rotor resistance and 0.3 Ω standstill reactance per phase. Find the additional resistance required in the rotor circuit to make the starting torque equal to the maximum torque of the motor.*
(Electrical Technology, Bombay Univ. 1990)

Solution. $\frac{T_{st}}{T_{\max}} = \frac{2a}{1+a^2}$; Since $T_{st} = T_{\max}$

$\therefore \quad 1 = \frac{2a}{1+a^2}$ or $a = 1$ Now, $a = \frac{R_2 + r}{X_2}$

where r = external resistance per phase added to the rotor circuit

$\therefore \quad 1 = \frac{0.06 + r}{0.3} \quad \therefore \quad r = 0.3 - 0.06 = \mathbf{0.24\ \Omega}$

Example 34.14 (*b*). *3-phase, 50-Hz, 8-pole, induction motor has full-load slip of 2%. The rotor-resistance and stand still rotor-reactance per phase are 0.001 ohm and 0.005 ohm respectively. Find the ratio of the maximum to full-load torque and the speed at which the maximum torque occurs.*
(Amravati University, 1999)

Solution. Synchronous speed, $N_s = 120 \times 50/8 = 750$ rpm

Slip at maximum torque, $s_{mT} = r_2/x_2$

Thus, let a $= \frac{r_2}{x_2} = \frac{0.001}{0.005} = 0.2$

Corresponding speed $= (1 - 0.2) \times 750 = 600$ rpm

$$\frac{\text{Full} - \text{load torque}}{\text{Maximum torque}} = \frac{2\, s_{mT}\, s_{fL}^2}{s_{mT}^2 + s_{fL}^2} \quad \therefore \quad \frac{T_{fL}}{T_{\max}} = \frac{2 \times 0.2 \times 0.02^2}{0.20^2 + 0.02^2} = \frac{1.6 \times 10^{-4}}{0.0404}$$

$\therefore \quad \frac{T_{\max}}{T_f} = 252.5$

$= \mathbf{3.96 \times 10^{-3}}$

Example 34.14 (*c*). *A 12-pole, 3-phase, 600-V, 50-Hz, star-connected, induction motor has rotor-resistance and stand-still reactance of 0.03 and 0.5 ohm per phase respectively. Calculate:*

***(a)** Speed of maximum torque. **(b)** ratio of full-load torque to maximum torque, if the full-load speed is 495 rpm.*
(Nagpur University, April 1999)

Solution. For a 12-pole, 50 Hz motor,

Synchronous speed = $120 \times 50 / 12 = 500$ rpm

For $r = 0.03$ and $x = 0.5$ ohm, the slip for maximum torque is related as :

$S_{mT} = a = r/x = 0.03/0.5 = 0.06$

(*a*) Corresponding speed $= 500\,(1 - s_{mT}) = 470$ rpm

(*b*) Full-load speed $=$ 495 rpm, slip $s = 0.01$, at full load.

$$\frac{\text{Full-load torque}}{\text{Maximum torque}} = \frac{2\, as}{a^2 + s^2} = \frac{2 \times 0.06 \times 0.01}{0.06^2 + 0.01^2} = \mathbf{0.324}$$

Example 34.15. *A 746-kW, 3-phase, 50-Hz, 16-pole induction motor has a rotor impedance of (0.02 + j 0.15) W at standstill. Full-load torque is obtained at 360 rpm. Calculate **(i)** the ratio of maximum to full-load torque **(ii)** the speed of maximum torque and **(iii)** the rotor resistance to be added to get maximum starting torque.*
(Elect. Machines, Nagpur Univ. 1993)

Solution. Let us first find out the value of full-load slip s_f

$N_s = 120 \times 50/16 = 375$ rpm.; F.L. Speed = 360 rpm.

$s_f = (375 - 360)/375 = 0.04$; $a = R_2/X_2 = 0.02/0.15 = 2/15$

(i) $\dfrac{T_f}{T_{max}} = \dfrac{2as_f}{a^2+s_f^2} = \dfrac{2\times(2/15)\times0.04}{(2/15)^2+(0.04)^2} = 0.55$ or $\dfrac{T_{max}}{T_f} = \dfrac{1}{0.55} = \mathbf{1.818}$

(ii) At maximum torque, $a = s_m = R_2/X_2 = 0.02/0.15 = 2/15$

$N = N_s(1-s) = 375\,(1-2/15) = \mathbf{325\ r.p.m.}$

(iii) For maximum starting torque, $R_2 = X_2$. Hence, total rotor resistance per phase = **0.15Ω**

$\therefore$ external resistance required/phase = 0.15 – 0.02 = **0.13 Ω**

Example 34.16. *The rotor resistance and reactance per phase of a 4-pole, 50-Hz, 3-phase induction motor are 0.025 ohm and 0.12 ohm respectively. Make simplifying assumptions, state them and :*

(i) *find speed at maximum torque*

(ii) *find value of additional rotor resistance per phase required to give three-fourth of maximum torque at starting. Draw the equivalent circuit of a single-phase induction motor.*

(Elect. Machines, Nagpur Univ. 1993)

Solution. **(i)** At maximum torque, $s = R_2/X_2 = 0.025/0.12 = 0.208$.

$N_s = 120\times50/4 = 1500$ rpm $\quad \therefore N = 1500\,(1-0.208) = \mathbf{1188\ rpm}$

(ii) It is given that $T_{st} = 0.75\,T_{max}$ Now, $\dfrac{T_{st}}{T_{max}} = \dfrac{2a}{1+a^2} = \dfrac{3}{4}$

$\therefore \quad 3a^2 - 8a + 3 = 0\,;$ $\quad a = \dfrac{8 \pm \sqrt{64-36}}{6} = 0.45\,\Omega$ *

Let, r = additional rotor resistance reqd., then

$a = \dfrac{R_2+r}{R_2}$ or $0.45 = \dfrac{0.025+r}{0.12}$ $\quad \therefore r = \mathbf{0.029\Omega}$

Example. 34.17. *A 50-Hz, 8-pole induction motor has F.L. slip of 4%. The rotor resistance/phase = 0.01 ohm and standstill reactance/phase = 0.1 ohm. Find the ratio of maximum to full-load torque and the speed at which the maximum torque occurs.*

Solution. $\dfrac{T_f}{T_{max}} = \dfrac{2as_f}{a^2+s_f^2}$

Now, $a = R_2/X_2 = 0.01/0.1 = 0.1,\ s_f = 0.04$

$\therefore \quad \dfrac{T_f}{T_{max}} = \dfrac{2\times0.1\times0.04}{0.1^2+0.04^2} = \dfrac{0.008}{0.0116} = 0.69 \quad \therefore \quad \dfrac{T_{max}}{T_f} = \dfrac{1}{0.69} = \mathbf{1.45}$

$N_s = 120\times50/8 = 750$ rpm, $s_m = 0.1$

$N = (1-0.1)\times750 = 675$ rpm

Example 34.18. *For a 3-phase slip-ring induction motor, the maximum torque is 2.5 times the full-load torque and the starting torque is 1.5 times the full-load torque. Determine the percentage reduction in rotor circuit resistance to get a full-load slip of 3%. Neglect stator impedance.*

(Elect. Machines, A.M.I.E. Sec. B, 1992)

Solution. Given, $T_{max} = 2.5\,T_f$; $T_{st} = 1.5\,T_f$; $T_{st}/T_{max} = 1.5/2.5 = 3/5$.

* The larger value of 2.214 Ω has been rejected.

Now, $\frac{T_{st}}{T_f} = \frac{3}{5} = \frac{2a}{1+a^2}$ or $3a^2 - 10a + 3 = 0$ or $a = 1/3$

Now, $a = R_2/X_2$ or $R_2 = X_2/3$

When F.L. slip is 0.03

$$\frac{T_f}{T_{st}} = \frac{2as}{a^2+s^2} \quad \text{or} \quad \frac{2}{2.5} = \frac{2a \times 0.03}{a^2 + 0.03^2}$$

$$a^2 - 0.15\,a + 0.009 = 0 \quad \text{or} \quad a = 0.1437$$

If R'_2 is the new rotor circuit resistance, then $0.1437 = R'_2/X_2$ or $R'_2 = 0.1437\,X_2$

% reduction in rotor resistance is

$$= \frac{(X_2/3) - 0.1437 \times X_2}{(X_2/3)} \times 100 = \mathbf{56.8\%}$$

Example 34.19. *An 8-pole, 50-Hz, 3-phase slip-ring induction motor has effective rotor resistance of 0.08 Ω/phase. Stalling speed is 650 r.p.m. How much resistance must be inserted in the rotor phase to obtain the maximum torque at starting? Ignore the magnetising current and stator leakage impedance.* **(Elect. Machines-I, Punjab Univ. 1991)**

Solution. It should be noted that stalling speed corresponds to maximum torque (also called stalling torque) and to maximum slip ***under running conditions***.

$N_s = 120 \times 50/8 = 750$ r.p.m.; stalling speed is = 650 r.p.m.

$s_b = (750 - 650)/750 = 2/15 = 0.1333$ or 13.33%

Now, $s_b = R_2/X_2 \quad \therefore \quad X_2 = 0.08 \times 15/2 = 0.6\,\Omega$

$$\frac{T_{st}}{T_{max}} = \frac{2a}{1+a^2}. \text{ Since } T_{st} = T_{max} \quad \therefore \quad 1 = \frac{2a}{1+a^2} \text{ or } a = 1$$

Let r be the external resistance per phase added to the rotor circuit. Then

$$a = \frac{R_2 + r}{X_2} \quad \text{or} \quad 1 = \frac{0.08 + r}{0.6} \quad \therefore \quad \mathbf{r = 0.52\,\Omega \text{ per phase.}}$$

Example 34.20. *A 4-pole, 50-Hz, 3-ϕ induction motor develops a maximum torque of 162.8 N-m at 1365 r.p.m. The resistance of the star-connected rotor is 0.2 Ω/phase. Calculate the value of the resistance that must be inserted in series with each rotor phase to produce a starting torque equal to half the maximum torque.*

Solution. $N_s = 120 \times 50/4 = 1500$ r.p.m. $N = 1365$ r.p.m.

$\therefore$ Slip corresponding to maximum torque is

$s_b = (1500 - 1365)/1500 = 0.09$ But $s_b = R_2/X_2 \quad \therefore X_2 = 0.2/0.09 = 2.22\,\Omega$

Now, $T_{max} = \frac{k\,\Phi\,E_2}{2\,X_2} = \frac{K}{2X_2}$ (where $K = k\Phi E_2$) ...Art 34.20

$$= \frac{K}{2 \times 2.22} = 0.225\,K$$

Let 'r' be the external resistance introduced per phase in the rotor circuit, then

Starting torque $T_{st} = \frac{k\,\Phi\,E_2\,(R_2 + r)}{(R_2 + r)^2 + (X_2)^2} = \frac{K\,(0.2 + r)}{(0.2 + r)^2 + (0.2/0.09)^2}$

$\because \quad T_{st} = \frac{1}{2}.T_{max} \quad \therefore \quad \frac{K\,(0.2 + r)}{(0.2 + r)^2 + (2.22)^2} = \frac{0.225\,K}{2}$

Solving the quadratic equation for 'r', we get $r = \mathbf{0.4\,\Omega}$

Example 34.21. *A 4-pole, 50-Hz, 7.46.kW motor has, at rated voltage and frequency, a starting torque of 160 per cent and a maximum torque of 200 per cent of full-load torque. Determine (i) full-load speed (ii) speed at maximum torque.* **(Electrical Technology-I, Osmania Univ. 1990)**

Solution. $\frac{T_{st}}{T_f} = 1.6$ and $\frac{T_{max}}{T_f} = 2$ $\quad \therefore \quad \frac{T_{st}}{T_{max}} = \frac{1.6}{2} = 0.8$

Now, $\frac{T_{st}}{T_{max}} = \frac{2a}{1+a^2}$ $\quad \therefore \quad \frac{2a}{1+a^2} = 0.8$

or $0.8a^2 - 2a + 0.8 = 0 \quad a = 0.04 \quad \therefore \quad a = R_2/X_2 = 0.04$ or $R_2 = 0.04X_2$

Also, $\frac{T_f}{T_{max}} = \frac{2as_f}{a^2 + s_f^2} = \frac{1}{2}$ or $\frac{2 \times 0.04\, s_f}{0.0016 + s_f^2} = \frac{1}{2}$ or $s_f = 0.01$

(*i*) full-load speed occurs at a slip of 0.01 or 1 per cent. Now,

$N_s = 120 \times 50/4 = 1500$ r.p.m.; $N = 1500 - 15 =$ **1485 r.p.m.**

(*ii*) Maximum torque occurs at a slip given by $s_b = R_2/X_2$. As seen from above slip corresponding to maximum torque is 0.04.

$\therefore \quad N = 1500 - 1500 \times 0.04 =$ **1440 r.p.m.**

Example 34.22. *A 3-phase induction motor having a 6-pole, star-connected stator winding runs on 240-V, 50-Hz supply. The rotor resistance and standstill reactance are 0.12 ohm and 0.85 ohm per phase. The ratio of stator to rotor turns is 1.8. Full load slip is 4%.*

Calculate the developed torque at full load, maximum torque and speed at maximum torque.

(Elect. Machines, Nagpur Univ. 1993)

Solution. Here, $K = \frac{\text{rotor turns/phase}}{\text{stator turns/phase}} = \frac{1}{1.8}$

$E_2 = KE_1 = \frac{1}{1.8} \times \frac{240}{\sqrt{3}} = 77$ V; $\quad s = 0.04;$

$N_s = 120 \times 50/6 = 1000$ rpm $= 50/3$ rps

$T_f = \frac{3}{2\pi N_s} \cdot \frac{sE_2^2 R_2}{R_2^2 + (sX_2)^2}$ ...Art. 34.19

$= \frac{3}{2\pi (50/3)} \cdot \frac{0.04 \times 77^2 \times 0.12}{0.12^2 + (0.14 \times 0.85)^2} = 52.4$ N-m

For maximum torque, $s = R_2/X_2 = 0.12/0.85 = 0.14$

$\therefore \quad T_{max} = \frac{3}{2\pi (50/3)} \cdot \frac{0.14 \times 77^2 \times 0.12}{0.12^2 + (0.14 \times 0.85)^2} = 99.9$ N-m

Alternatively, as seen from Art 34.20.

$T_{max} = \frac{3}{2\pi N_s} \cdot \frac{E_2^2}{2\, X_2}$

$\therefore \quad T_{max} = \frac{3}{2\pi (50/3)} \cdot \frac{77^2}{2 \times 0.85} = 99.9$ N-m

Speed corresponding to maximum torque, $N = 1000(1 - 0.14) =$ **860 rpm**

Example 34.23. *The rotor resistance and standstill reactance of a 3-phase induction motor are respectively 0.015 Ω and 0.09 Ω per phase. At normal voltage, the full-load slip is 3%. Estimate the percentage reduction in stator voltage to develop full-load torque at half full-load speed. Also, calculate the power factor.* **(Adv. Elect. Machines, A.M.I.E. 1989)**

Solution. Let $N_s = 100$ r.p.m. F.L. speed $= (1 - 0.03)100 = 97$ r.p.m.

Let the normal voltage be V_1 volts.

Speed in second case = 97/2 = 48.5 r.p.m.

$\therefore$ slip = (100 – 48.5)/100 = 0.515 or 51.5%

Now, $$T = \frac{k\,\Phi\, sE_2R_2}{R_2^2+(sX_2)^2} = \frac{ksV^2R_2}{R_2^2+(sX_2)^2} \qquad (\because\ E_2 \propto \Phi \propto V)$$

Since torque is the same in both cases,

$$\frac{kV_1^2\, s_1R_2}{R_2^2+(s_1X_2)^2} = \frac{kV_2^2\, s_2R_2}{R_2^2+(s_2X_2)^2}$$ where V_2 = stator voltage in second case

$$\therefore \quad \left(\frac{V_1}{V_2}\right)^2 = \frac{s_2}{s_1}\cdot\frac{R_2^2+(s_1X_2)^2}{R_2^2+(s_2X_2)^2}$$

$$= \frac{51.3}{3}\cdot\frac{0.015^2+(0.03\times0.09)^2}{0.015^2+(0.515\times0.09)^2} = 1.68$$

$$\therefore \quad \frac{V_1}{V_2} = \sqrt{1.68} = 1.296 \quad \text{or} \quad \frac{V_1-V_2}{V_1} = \frac{0.296}{1.296}$$

Hence, percentage, reduction in stator (or supply voltage) is

$$= \frac{V_1-V_2}{V_1}\times100 = \frac{0.296\times100}{1.296} = \mathbf{22.84\%}$$

In the second case, $\tan\phi = s_2X_2/R_2 = 0.515\times0.09/0.015 = 3.09$

$\therefore$ $\phi = \tan^{-1}(3.09) = 72°4'$ and p.f. = $\cos\phi = \cos 72°4' = \mathbf{0.31}$

34.27. Torque/Speed Curve

The torque developed by a conventional 3-phase motor depends on its speed but the relation between the two cannot be represented by a simple equation. It is easier to show the relationship in the form of a curve (Fig. 34.22). In this diagram, T represents the nominal full-load torque of the motor. As seen, the starting torque (at $N = 0$) is 1.5 T and the maximum torque (also called breakdown torque) is 2.5 T.

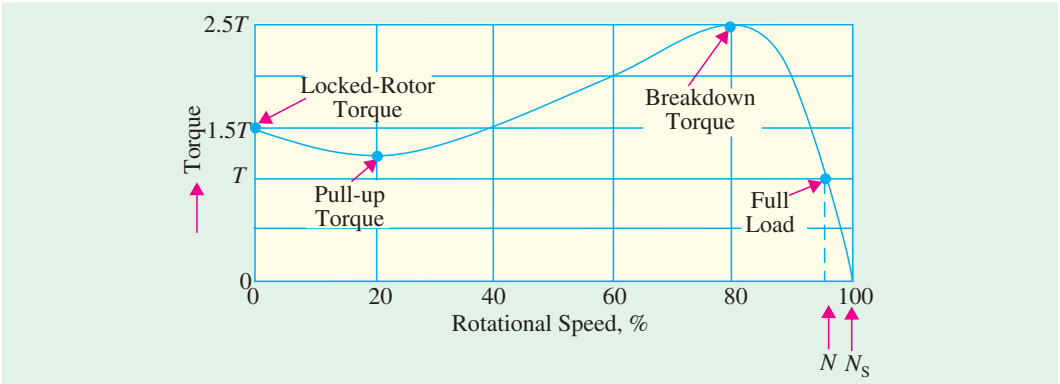


Fig. 34.22

At full-load, the motor runs at a speed of N. When mechanical load increases, motor speed decreases till the motor torque again becomes equal to the load torque. As long as the two torques are in balance, the motor will run at constant (but lower) speed. However, if the load torque exceeds 2.5 T, the motor will suddenly stop.

34.28. Shape of Torque/Speed Curve

For a squirrel-cage induction motor (SCIM), shape of its torque/speed curve depends on the voltage and frequency applied to its stator. If f is fixed, $T \propto V^2$ (Art 34.22). Also, synchronous speed

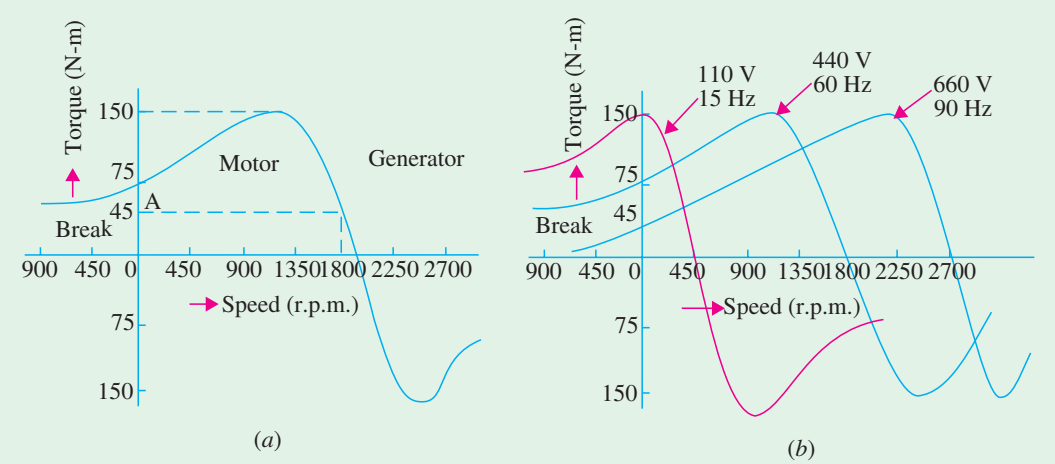


Fig. 34.23

depends on the supply frequency. Now, let us see what happens when *both* stator voltage and frequency are changed. In practice, supply voltage and frequency are varied in the ***same*** *proportion* in order to maintain a constant flux in the air-gap. For example, if voltage is doubled, then frequency is also doubled. Under these conditions, shape of the torque/speed curve remains the same but its position along the X-axis (*i.e.*speed axis) shifts with frequency.

Fig. 34.23 (*a*) shows the torque/speed curve of an 11.kW, 440-V, 60-Hz 3-ϕ SCIM. As seen, full-load speed is 1728 rpm and full-load torque is 45 N-m (point-A) whereas breakdown torque is 150 N-m and locked-rotor torque is 75 N–m.

Suppose, we now reduce both the voltage and fequency to ***one-fourth*** their original values *i.e.* to 110 V and 15 Hz respectively. As seen in Fig. 34.23 (*b*), the torque/speed curve shifts to the left. Now, the curve crosses the X-axis at the synchronous speed of $120 \times 15/4 = 450$ rpm (*i.e.* 1800/4 = 450 rpm). Similarly, if the voltage and frequency are increased by 50% (660 V 90 Hz), the curve shifts to the right and cuts the X-axis at the synchronous speed of 2700 rpm.

Since the *shape* of the torque/speed curve remains the same at all ***frequencies*****,** it follows that torque developed by a SCIM is the same ***whenever slip-speed is the same*****.**

Exampel 34.26. *A 440-V, 50-Hz, 4-pole, 3-phase SCIM develops a torque of 100 N-m at a speed of 1200 rpm. If the stator supply frequency is reduced by half, calculate*

(a) *the stator supply voltage required for maintaining the same flux in the machine.*

(b) *the new speed at a torque of 100 N-m.*

Solution. ***(a)*** The stator voltage must be reduced in proportion to the frequency. Hence, it should also be reduced by half to 440/2 = **220 V.**

(b) Synchronous speed at 50 Hz frequency = $120 \times 50/4 = 1500$ rpm. Hence, slip speed for a torque of 100 N-m = 1500 – 1200 = 300 rpm.

Now, synchronous speed at 25 Hz = 1500/2 = 750 rpm.

Since slip-speed has to be the same for the same torque irrespective of the frequency, the new speed at 100 N-m is = 750 + 300 = **1050 rpm.**

34.29. Current/Speed Curve of an Induction Motor

It is a V-shaped curve having a minimum value at synchronous speed. This minimum is equal to

the magnetising current which is needed to create flux in the machine. Since flux is purposely kept constant, it means that magnetising current is the same at all synchronous speeds.

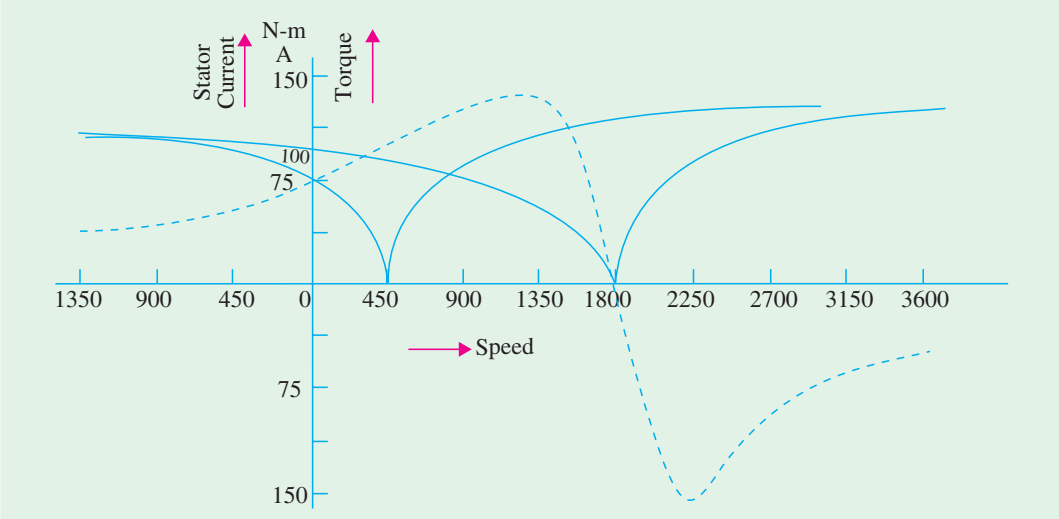


Fig. 34.24

Fig. 34.24. shows the current/speed curve of the SCIM discussed in Art. 34.28 above. Refer Fig. 34.23(*b*) and Fig. 34.24, As seen, locked rotor current is 100 A and the corresponding torque is 75 N-m. If stator voltage and frequency are varied in the same proportion, current/speed curve has the same shape, but shifts along the speed axis. Suppose that voltage and frequency are reduced to one-fourth of their previous values *i.e.* to 110 V, 15 Hz respectively. Then, locked rotor current decreases to 75 A but corresponding torque ***increases*** to 150 N-m which is equal to full breakdown torque (Fig. 34.25). It means that by reducing frequency, we can obtain ***a larger torque with a reduced current***. This is one of the big advantages of frequency control method. By progressively increasing the voltage and current during the start-up period, a SCIM can be made to develop close to its breakdown torque all the way from zero to rated speed.

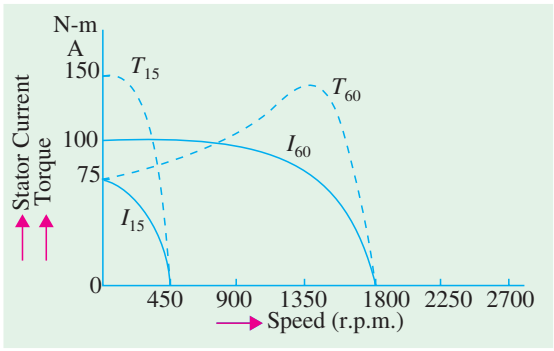


Fig. 34.25

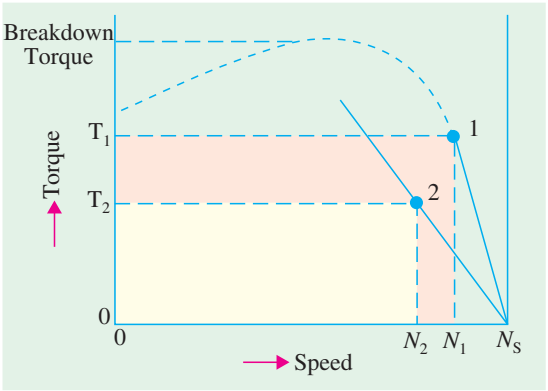


Fig. 34.26

Another advantage of frequency control is that it permits regenerative braking of the motor. In fact, the main reason for the popularity of frequency-controlled induction motor drives is their ability to develop high torque from zero to full speed together with the economy of regenerative braking.

34.30. Torque/Speed Characteristic Under Load

As stated earlier, stable operation of an induction motor lies over the linear portion of its torque/speed curve. The slope of this straight line depends

mainly on the rotor resistance. Higher the resistance, sharper the slope. This linear relationship between torque and speed (Fig. 34.26) enables us to establish a very simple equation between different parameters of an induction motor. The parameters under two different load conditions are related by the equation

$$s_2 = s_1 \cdot \frac{T_2}{T_1} \cdot \frac{R_2}{R_1}\left(\frac{V_1}{V_2}\right)^2 \qquad ...(i)$$

The only restriction in applying the above equation is that the new torque T_2 must not be greater than $T_1 (V_2/V_1)^2$. In that case, the above equation yields an accuracy of better than 5% which is sufficient for all practical purposes.

Example 34.24. *A 400-V, 60-Hz, 8-pole, 3-ϕ induction motor runs at a speed of 1140 rpm when connected to a 440-V line. Calculate the speed if voltage increases to 550V.*

Solution. Here, $s_1 = (1200 - 1140)/1200 = 0.05$. Since everything else remains the same in Eq. (*i*) of Art. 34.30 except the slip and voltage, hence

$$S_2 = s_1(V_1/V_2)^2 = 0.05 \times (440/550)^2 = 0.032 \qquad \text{...Art. 34.23}$$

$$\therefore \quad N_2 = 1200(1 - 0.032) = \mathbf{1161.6\ rpm.}$$

Example 34.25. *A 450.V, 60.Hz, 8-Pole, 3-phase induction motor runs at 873 rpm when driving a fan. The initial rotor temperature is 23°C. The speed drops to 864 rpm when the motor reaches its final temperature. Calculate **(i)** increase in rotor resistance and **(ii)** approximate temperature of the hot rotor if temperature coefficient of resistance is 1/234 per °C.*

Solution. $s_1 = (900 - 873)/900 = 0.03$ and $s_2 = (900 - 864)/900 = 0.04$

Since voltage and frequency etc. are fixed, the change in speed is entirely due to change in rotor resistance.

(*i*) $s_2 = s_1(R_2/R_1)$ or $0.04 = 0.03\,(R_2/R_1)$; $R_2 = 1.33\,R_1$

Obviously, the rotor resistance has increased by **33 percent.**

(*ii*) Let t_2 be temperature of the rotor. Then, as seen from Art.1-11,

$$R_2 = R_1[1 + \alpha(t_2 - 23)] \text{ or } 1.33\,R_1 = R_1\left[1 + \frac{1}{234}(t_2 - 23)\right] \qquad \therefore\ t_2 = \mathbf{100.2°C}$$

34.31. Plugging of an Induction Motor

An induction motor can be quickly stopped by simply inter-changing any of its two stator leads. It reverses the direction of the revolving flux which produces a torque in the reverse direction, thus applying brake on the motor. Obviously, during this so-called plugging period, ***the motor acts as a brake.*** It absorbs kinetic energy from the still

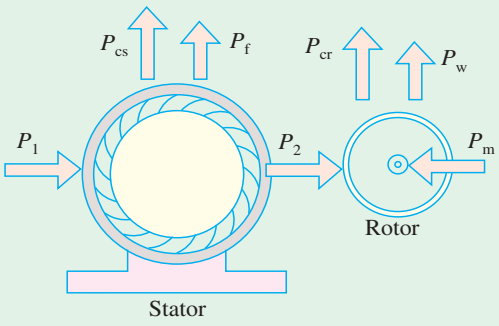


Fig. 34.27



Induction asynchronous motor

revolving load causing its speed to fall. The associated Power P_m is dissipated as heat in the rotor. At the same time, the rotor also continues to receive power P_2 from the stator (Fig. 34.27) which is also dissipated as heat. Consequently, plugging produces rotor I^2R losses which even exceed those when the rotor is locked.

34.32. Induction Motor Operating as a Generator

When run *faster than* its synchronous speed, an induction motor runs as a generator called a ***Induction generator***. It converts the mechanical energy it receives into electrical energy and this energy is released by the stator (Fig. 34.29). Fig. 34.28 shows an ordinary squirrel-cage motor which is driven by a petrol engine and is connected to a 3-phase line. As soon as motor speed exceeds its synchronous speed, it starts delivering ***active*** power P to the 3-phase line. However, for creating its own magnetic field, it absorbs *reactive* power Q from the line to which it is connected. As seen, Q flows in the ***opposite*** direction to P.

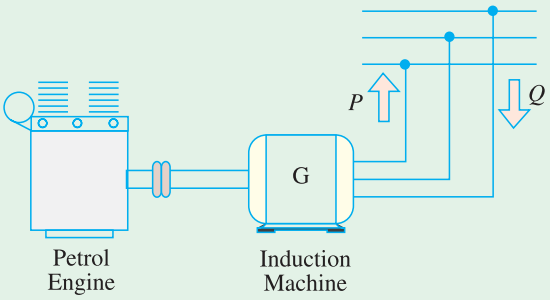


Fig. 34.28

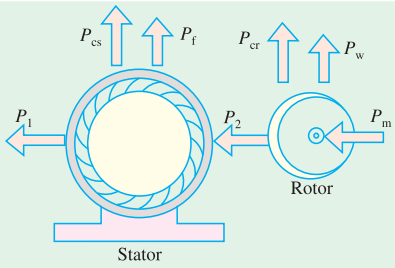


Fig. 34.29

The active power ***is directly proportional to the slip*** above the synchronous speed. The reactive power required by the machine can also be supplied by a group of capacitors connected across its terminals (Fig. 34.30). This arrangement can be used to supply a 3-phase load without using an external source. The frequency generated is slightly less than that corresponding to the speed of rotation.

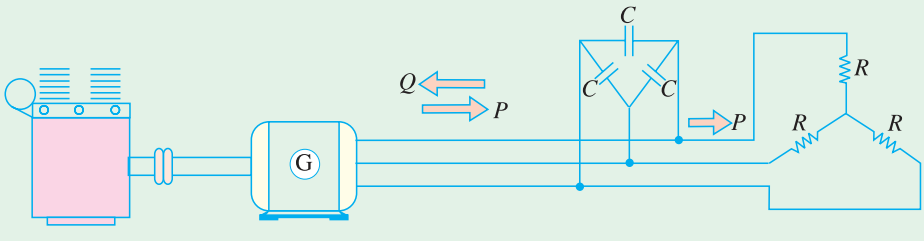


Fig. 34.30

The terminal voltage increases with capacitance. If capacitance is insufficient, the generator voltage will not build up. Hence, capacitor bank must be large enough to supply the reactive power normally drawn by the motor.

Example 34.26. *A 440-V, 4-pole, 1470 rpm. 30-kW, 3-phase induction motor is to be used as an asynchronous generator. The rated current of the motor is 40 A and full-load power factor is 85%. Calculate*

(a) *capacitance required per phase if capacitors are connected in delta.*

(b) *speed of the driving engine for generating a frequency of 50 Hz.*

Solution. *(i)* $S = \sqrt{3}.VI = 1.73 \times 440 \times 40 = 30.4$ kVA

$$P = S\cos\phi = 30.4 \times 0.85 = 25.8 \text{ kW}$$

$$Q = \sqrt{S^2 - P^2} = \sqrt{30.4^2 - 25.8^2} = 16 \text{ kVAR}$$

Hence, the Δ-connected capacitor bank (Fig. 32.31) must provide 16/3 = 5.333 kVAR per phase.

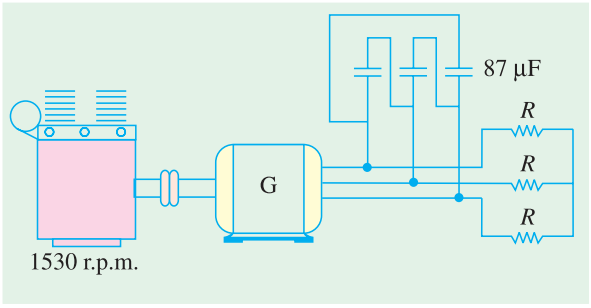


Fig. 34.31

Capacitor current per phase is = 5,333/440 = 12 A. Hence $X_c = 440/12 = 36.6\ \Omega$. Now,

$$C = \frac{1}{2\pi f X_C} = 1/2\pi \times 50 \times 36.6 = \mathbf{87\mu F}$$

(*ii*) The driving engine must run at slightly more than synchronous speed. The slip speed is usually the same as that when the machine runs as a motor *i.e.* 30 rpm.

Hence, engine speed is = 1500 + 30 = **1530 rpm.**

34.33. Complete Torque/Speed Curve of a Three-Phase Machine

We have already seen that a 3-phase machine can be run as a ***motor***, when it takes electric power and supplies mechanical power. The directions of torque and rotor rotation are in the ***same*** direction. The same machine can be used as an ***asynchronous generator*** when driven at a speed ***greater*** than the synchronous speed. In this case, it receives mechanical energy in the rotor and supplies electrical energy from the stator. The torque and speed are ***oppositely-directed***.

The same machine can also be used as a ***brake*** during the plugging period (Art. 34.31). The three modes of operation are depicted in the torque/speed curve shown in Fig. 34.32.

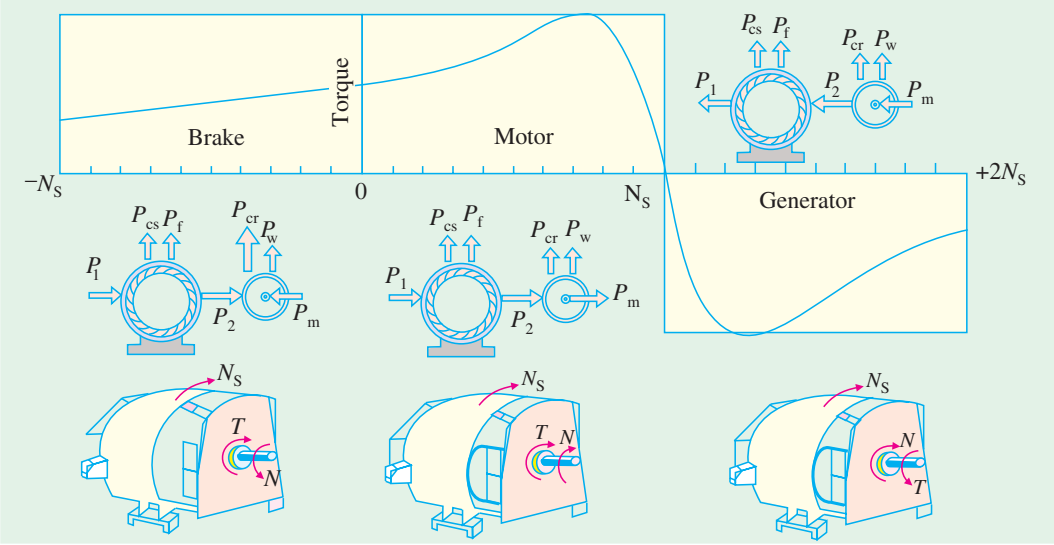


Fig. 34.32

Tutorial Problem No. 34.2

1. In a 3-phase, slip-ring induction motor, the open-circuit voltage across slip-rings is measured to be 110 V with normal voltage applied to the stator. The rotor is star-connected and has a resistance of 1 Ω and reactance of 4 Ω at standstill condition. Find the rotor current when the machine is (*a*) at standstill with slip-rings joined to a star-connected starter with a resistance of 2Ω per phase and negligible reactance (*b*) running normally with 5% slip. State any assumptions made.

[12.7 A ; 3.11 A] (*Electrical Technology-I, Bombay Univ. 1978*)

2. The star-connected rotor of an induction motor has a standstill impedance of $(0.4 + j4)$ ohm per phase and the rheostat impedance per phase is $(6+j2)$ ohm. The motor has an induced e.m.f. of 80 V between slip-rings at standstill when connected to its normal supply voltage. Find (*a*) rotor current at standstill with the rheostat in the circuit (*b*) when the slip-rings are short-circuited and the motor is running with a slip of 3%. **[5.27 A ; 3.3 A]**

3. A 4-pole, 50-Hz induction motor has a full-load slip of 5%. Each rotor phase has a resistance of 0.3 Ω and a standstill reactance of 1.2 Ω. Find the ratio of maximum torque to full-load torque and the speed at which maximum torque occurs. **[2.6 ; 1125 r.p.m.]**

4. A 3-phase, 4-pole, 50-Hz induction motor has a star-connected rotor. The voltage of each rotor phase at standstill and on open-circuit is 121 V. The rotor resistance per phase is 0.3 Ω and the reactance at standstill is 0.8 Ω. If the rotor current is 15 A, calculate the speed at which the motor is running. Also, calculate the speed at which the torque is a maximum and the corresponding value of the input power to the motor, assuming the flux to remain constant. **[1444 r.p.m.; 937.5 r.p.m.]**

5. A 4-pole, 3-phase, 50 Hz induction motor has a voltage between slip-rings on open-circuit of 520 V. The star-connected rotor has a standstill reactance and resistance of 2.0 and 0.4 Ω per phase respectively. Determine :

(*a*) the full-load torque if full-load speed is 1,425 r.p.m.

(*b*) the ratio of starting torque to full-load torque

(*c*) the additional rotor resistance required to give maximum torque at standstill

[(*a*) 200 N-m (*b*) 0.82 (*c*) 1.6 Ω] (*Elect. Machines-II, Vikram Univ. Ujjain 1977*)

6. A 50-Hz, 8-pole induction motor has a full-load slip of 4 per cent. The rotor resistance is 0.001 Ω per phase and standstill reactance is 0.005 Ω per phase. Find the ratio of the maximum to the full-load torque and the speed at which the maximum torque occurs.

[2.6; 600 r.p.m.] (*City & Guilds, London*)

7. A 3-ϕ, 50-Hz induction motor with its rotor star-connected gives 500 V (r.m.s.) at standstill between slip-rings on open circuit. Calculate the current and power factor in each phase of the rotor windings at standstill when joined to a star-connected circuit, each limb of which has a resistance of 10 Ω and an inductance of 0.03 H. The resistance per phase of the rotor windings is 0.2 Ω and inductance 0.03 H. Calculate also the current and power factor in each rotor phase when the rings are short-circuited and the motor is running with a slip of 4 per cent.

[13.6 A, 0.48; 27.0 A, 0.47] (*London University*)

8. A 4-pole, 50-Hz, 3-phase induction motor has a slip-ring rotor with a resistance and standstill reactance of 0.04 Ω and 0.2 Ω per phase respectively. Find the amount of resistance to be inserted in each rotor phase to obtain full-load torque at starting. What will be the approximate power factor in the rotor at this instant ? The slip at full-load is 3 per cent. **[0.084 Ω, 0.516 p.f.] (*London University*)**

9. A 3-ϕ induction motor has a synchronous speed of 250 r.p.m. and 4 per cent slip at full-load. The rotor has a resistance of 0.02 Ω/phase and a standstill leakage reactance of 0.15 Ω/phase. Calculate (*a*) the ratio of maximum and full-load torque (*b*) the speed at which the maximum torque is developed. Neglect resistance and leakage of the stator winding.

[(*a*) 1.82 (*b*) 217 r.p.m.] (*London University*)

10. The rotor of an 8-pole, 50-Hz, 3-phase induction motor has a resistance of 0.2 Ω/phase and runs at 720 r.p.m. If the load torque remains unchanged. Calculate the additional rotor resistance that will reduce this speed by 10% **[0.8 Ω] (*City & Guilds, London*)**

11. A 3-phase induction motor has a rotor for which the resistance per phase is 0.1 Ω and the reactance per phase when stationary is 0.4 Ω. The rotor induced e.m.f. per phase is 100 V when stationary. Calculate the rotor current and rotor power factor (*a*) when stationary (*b*) when running with a slip of 5 per cent. **[(*a*) 242.5 A; 0.243 (*b*) 49 A; 0.98]**

12. An induction motor with 3-phase star-connected rotor has a rotor resistance and standstill reactance of 0.1 Ω and 0.5 Ω respectively. The slip-rings are connected to a star-connected resistance of 0.2 Ω per phase. If the standstill voltage between slip-rings is 200 volts, calculate the rotor current per phase when the slip is 5%, the resistance being still in circuit. **[19.1 A]**

13. A 3-phase, 50-Hz induction motor has its rotor windings connected in star. At the moment of starting

the rotor, induced e.m.f. between each pair of slip-rings is 350 V. The rotor resistance per phase is 0.2 Ω and the standstill reactance per phase is 1 Ω. Calculate the rotor starting current if the external starting resistance per phase is 8 Ω and also the rotor current when running with slip-rings short-circuited, the slip being 3 per cent. **[24.5 A ; 30.0 A]**

14. In a certain 8-pole, 50-Hz machine, the rotor resistance per phase is 0.04 Ω and the maximum torque occurs at a speed of 645 r.p.m. Assuming that the air-gap flux is constant at all loads, determine the percentage of maximum torque (a) at starting (b) when the slip is 3%.
[(*a*) 0.273 (*b*) 0.41] (*London University*)

15. A 6-pole, 3-phase, 50-Hz induction motor has rotor resistance and reactance of 0.02 Ω and 0.1 Ω respectively per phase. At what speed would it develop maximum torque ? Find out the value of resistance necessary to give half of maximum torque at starting.
[800 rpm; 0.007 Ω] (*Elect.Engg. Grad I.E.T.E. June 1988*)

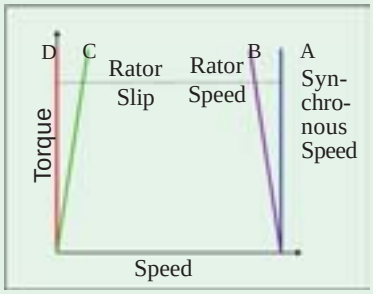
34.34. Measurement of Slip

Following are some of the methods used for finding the slip of an induction motor whether squirrel-cage or slip-ring type.

(*i*) By actual measurement of motor speed

This method requires measurement of actual motor speed N and calculation of synchronous speed N_s. N is measured with the help of a speedometer and N_s calculated from the knowledge of supply frequency and the number of poles of the motor.* Then slip can be calculated by using the equation.

$$s = (N_s - N) \times 100/N_s$$



The speed curve of an induction motor. The slip is the difference in rotor speed relative to that of the synchronous speed. CD = AD - BD = AB.

(*ii*) By comparing rotor and stator supply frequencies

This method is based on the fact that $s = f_r/f$

Since f is generally known, s can be found if frequency of rotor current can be measured by some method. In the usual case, where f is 50 Hz, f_r is so low that individual cycles can be easily counted. For this purpose, a d.c. moving-coil millivoltmeter, preferably of centre-zero, is employed as described below :

(*a*) In the case of a slip-ring motor, the leads of the millivoltmeter are lightly pressed against the adjacent slip-rings as they revolve (Fig. 34.33). Usually, there is sufficient voltage drop in the brushes and their short-circuiting strap to provide an indication on the millivoltmeter. The current in the millivoltmeter follows the variations of the rotor current and hence the pointer oscillates about its mean zero position. The number of complete cycles made by the pointer per second can be easily counted (it is worth remembering that one cycle consists of a movement from zero to a maximum to the right, back to zero and on to a maximum to the left and then back to zero).

As an example, consider the case of a 4-pole motor fed from a 50-Hz supply and running at 1,425 r.p.m. Since

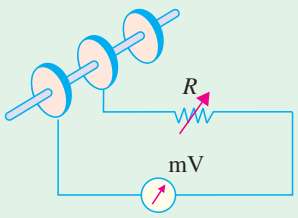


Fig. 34.33

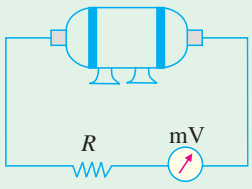


Fig. 34.34

* Since an induction motor does not have salient poles, the number of poles is usually inferred from the no-load speed or from the rated speed of the motor.

N_s = 1,500 r.p.m., its slip is 5% or 0.05. The frequency of the rotor current would be $f_r = s_f = 0.05 \times 50 = 2.5$ Hz which (being slow enough) can be easily counted.

(b) For squirrel-cage motors (which do not have slip-rings) it is not possible to employ the millivoltmeter so ***directly***, although it is sometimes possible to pick up some voltage by connecting the millivoltmeter across the ends of the motor shaft (Fig. 34.34)

Another method, sometime employed, is as follows :

A large flat search coil of many turns is placed centrally against the end plate on the non-driving end of the motor. Quite often, it is possible to pick up sufficient voltage by induction from the leakage fluxes to obtain a reading on the millivoltmeter. Obviously, a large 50-Hz voltage will also be induced in the search coil although it is too rapid to affect the millivoltmeter. Commercial slip-indicators use such a search coil and, in addition, contain a low-pass filter amplifier for eliminating fundamental frequency and a bridge circuit for comparing stator and rotor current frequencies.

(*iii*) Stroboscopic Method

In this method, a circular metallic disc is taken and painted with alternately black and white segments. The number of segments (both black and white) is equal to the number of poles of the motor. For a 6-pole motor, there will be six segments, three black and three white, as shown in Fig. 34.35(*a*). The painted disc is mounted on the end of the shaft and illuminated by means of a neon-filled stroboscopic lamp, which may be supplied preferably with a combined d.c. and a.c. supply although only a.c. supply will do*. The connections for combined supply are shown in Fig. 34.36 whereas Fig. 34.35 (*b*) shows the connection for a.c. supply only. It must be noted that with combined d.c. and a.c. supply, the lamp will flash once per cycle**. But with a.c. supply, it will flash twice per cycle.

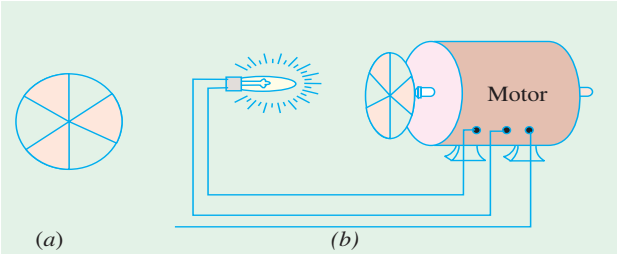


Fig. 34.35

Consider the case when the revolving disc is seen in the flash light of the bulb which is fed by the combined d.c. and a.c. supply.

If the disc were to rotate at synchronous speed, it would appear to be stationary. Since, in actual practice, its speed is slightly less than the synchronous speed, it appears to rotate slowly backwards. The reason for this apparent backward movement is as follows :

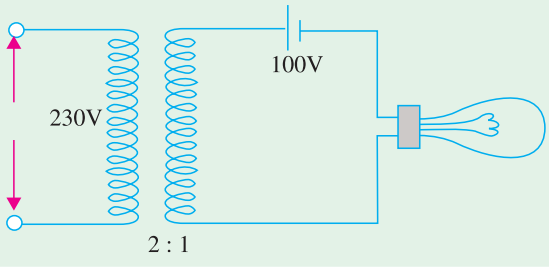


Fig. 34.36

* When combined d.c. and a.c. supply is used, the lamp should be tried both ways in its socket to see which way it gives better light.

** It will flash only when the two voltages add and remain extinguished when they oppose.

Let Fig. 34.37(*a*) represent the position of the white lines when they are illuminated by the first flash. When the next flash comes, they have ***nearly*** reached positions 120° ahead (but not quite), as shown in Fig. 34.37(*b*). Hence, line No. 1 has ***almost*** reached the position previously occupied by line No. 2 and one flash still later [Fig. 34.37 (*c*)] it has ***nearly*** reached the position previously occupied by line No. 3 in Fig. 34.37(*a*).

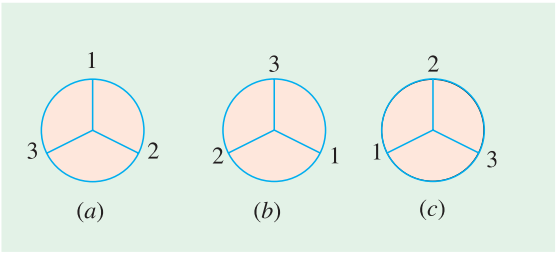


Fig. 34.37

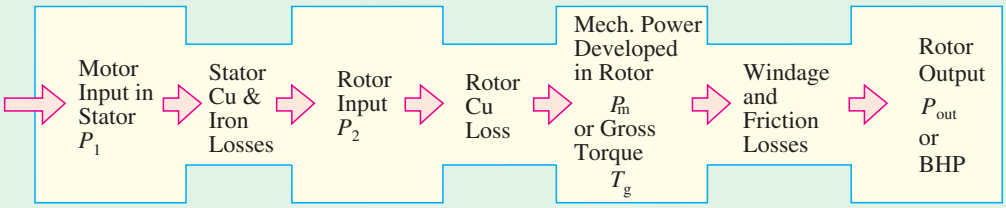
By counting the number of lines passing a fixed point in, say, a minute and dividing by the number of lines seen (*i.e.* three in the case of a 6-pole motor and so on) the apparent backward speed in r.p.m. can be found. This gives slip-speed in r.p.m. *i.e.* $N_s - N$. The slip may be found from the relation $s = \dfrac{N_S - N}{N_S} \times 100$

Note. If the lamp is fed with a.c. supply alone, then it will flash twice per cycle and twice as many lines will be seen rotating as before.

34.35. Power Stages in an Induction Motor

Stator iron loss (consisting of eddy and hysteresis losses) depends on the supply frequency and the flux density in the iron core. It is practically constant. The iron loss of the rotor is, however, negligible because frequency of rotor currents under normal running conditions is always small. Total rotor Cu loss = $3\ I_2^2 R_2$.

Different stages of power development in an induction motor are as under :



A better visual for power flow, within an induction motor, is given in Fig. 34.38.

34.36. Torque Developed by an Induction Motor

An induction motor develops gross torque T_g due to gross rotor output P_m (Fig 34.38). Its value can be expressed either in terms of rotor input P_2 or rotor gross output P_m as given below.

$$T_g = \frac{P_2}{\omega_s} = \frac{P_2}{2\pi N_s}$$ ...in terms of rotor input

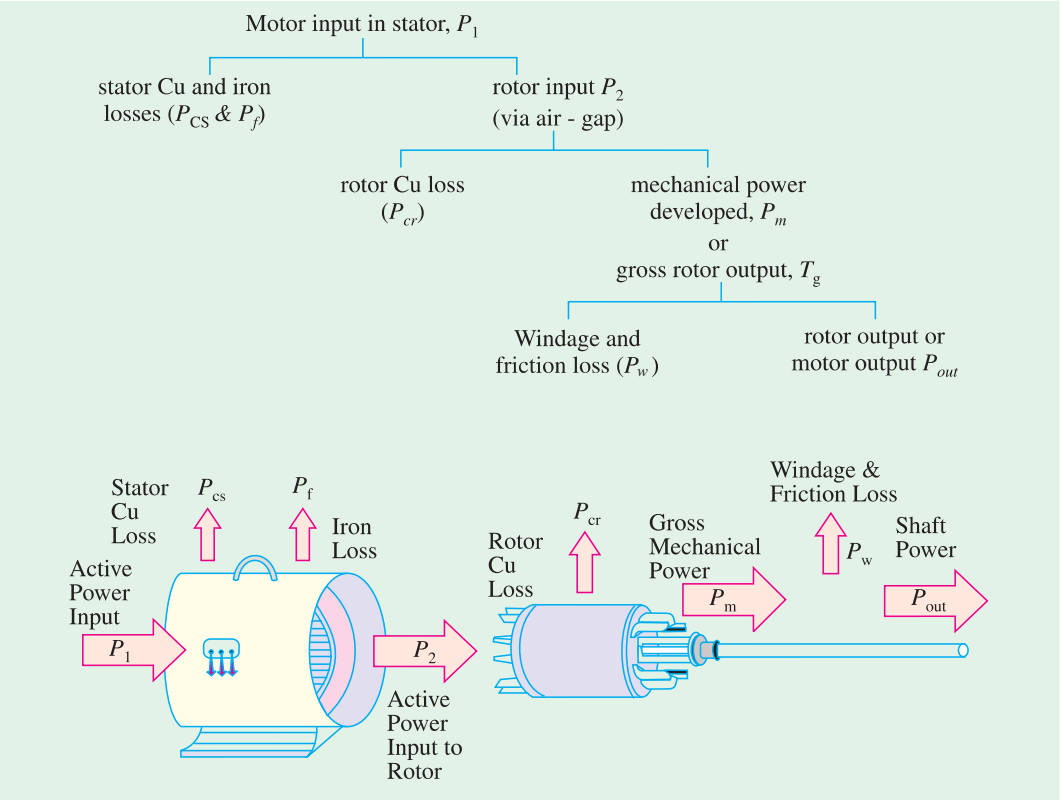


Fig. 34.38

$$T_g = \frac{P_m}{\omega} = \frac{P_m}{2\pi N}$$ ... in terms of rotor output

The shaft torque T_{sh} is due to output power P_{out} which is less than P_m because of rotor friction and windage losses.

$$\therefore \quad T_{sh} = P_{out}/\omega = P_{out}/2\pi N$$

The difference between T_g and T_{sh} equals the torque lost due to friction and windage loss in the motor.

In the above expressions, N and N_s are in r.p.s. However, if they are in r.p.m., the above expressions for motor torque become

$$T_g = \frac{P_2}{2\pi N_s/60} = \frac{60}{2\pi}\cdot\frac{P_2}{N_s} = 9.55\frac{P_2}{N_s} \text{ N-m}$$

$$= \frac{P_m}{2\pi N/60} = \frac{60}{2\pi}\cdot\frac{P_m}{N} = 9.55\frac{P_m}{N} \text{ N-m}$$

$$T_{sh} = \frac{P_{out}}{2\pi N/60} = \frac{60}{2\pi}\cdot\frac{P_{out}}{N} = 9.55\frac{P_{out}}{N} \text{ N-m}$$

34.37. Torque, Mechanical Power and Rotor Output

Stator input P_1 = stator output + stator losses

The stator output is transferred entirely inductively to the rotor circuit.

Obviously, rotor input P_2 = stator output

Rotor gross output, P_m = rotor input P_2 — rotor Cu losses



The type DF200 diesel electric locomotive is the first motor-driven train equipped with diesel generators since 1958. It adopted induction motors to realize high acceleration, high speed, and large torque, which resulted in a quick-response generator brake system.

This rotor output is converted into mechanical energy and gives rise to gross torque T_g. Out of this gross torque developed, some is lost due to windage and friction losses in the rotor and the rest appears as the useful or shaft torque T_{sh}.

Let N r.p.s. be the actual speed of the rotor and if T_g is in N-m, then

$T_g \times 2\pi N$ = rotor gross output in watts, P_m

$$\therefore\ T_g = \frac{\text{rotor gross output in watts, } P_m}{2\pi N}\ \text{N-m}^*$$ ...(1)

If there were no Cu losses in the rotor, then rotor output will equal rotor input and the rotor will run at synchronous speed.

$$\therefore\quad T_g = \frac{\text{rotor input } P_2}{2\pi N_S}$$ ...(2)

From (1) and (2), we get,

Rotor gross output $P_m = T_g\,\omega = T_g \times 2\pi N$

Rotor input $P_2 = T_g\,\omega_s = T_g \times 2\pi N_s$ ...(3)

The difference of two equals rotor Cu loss.

$\therefore$ rotor Cu loss $= P_2 - P_m = T_g \times 2\pi (N_s - N)$ ...(4)

From (3) and (4), $\dfrac{\text{rotor Cu loss}}{\text{rotor input}} = \dfrac{N_s - N}{N_s} = s$

$\therefore$ rotor Cu loss = $s \times$ rotor input = $s \times$ power across air-gap = $s\,P_2$ ...(5)

Also, rotor input = rotor Cu loss/s

Rotor gross output, P_m = input P_2 – rotor Cu loss = input – $s \times$ rotor input

= $(1 - s)$ input P_2 ...(6)

$\therefore$ rotor gross output $P_m = (1 - s)$ rotor input P_2

or $\dfrac{\text{rotor gross output, } P_m}{\text{rotor input, } P_2} = 1 - s = \dfrac{N}{N_s}$; $\dfrac{P_m}{P_2} = \dfrac{N}{N_s}$

$\therefore$ rotor efficiency $= \dfrac{N}{N_s}$ Also, $\dfrac{\text{rotor Cu loss}}{\text{rotor gross output}} = \dfrac{s}{1 - s}$

Important Conclusion

If some power P_2 is delivered to a rotor, then a part sP_2 is lost in the rotor itself as copper loss (and appears as heat) and the remaining $(1 - s)P_2$ appears as gross mechanical power P_m (including friction and windage losses).

$\therefore$ $P_2 : P_m : I^2R :: 1 : (1 - s) : s$ or $P_2 : P_m : P_{cr} :: 1 : (1 - s) : s$

---

* The value of gross torque in kg-m is given by

$$T_g = \frac{\text{rotor gross output in watts}}{9.81 \times 2\pi N}\ \text{kg-m.} = \frac{P_m}{9.81 \times 2\pi N}\ \text{kg-m}$$

The rotor input power will always divide itself in this ratio, hence it is advantageous to run the motor with as small a slip as possible.

Example 34.27. *The power input to the rotor of 440 V, 50 Hz, 6-pole, 3-phase, induction motor is 80 kW. The rotor electromotive force is observed to make 100 complete alterations per minute. Calculate* ***(i)*** *the slip,* ***(ii)*** *the rotor speed,* ***(iii)*** *rotor copper losses per phase.* **[Madras University, 1997]**

Solution. 100 alterations/minute $= \frac{100}{60}$ cycles/sec

$1.6667\text{ Hz} = sf$

Hence, the slip, $s = \frac{1.6667}{50} =$ **0.3333 P.u. or 3.333%**

(ii) rotor speed, $N = (1-s)N_S = (1-0.03333)\times 1000$

Since $N_s = \frac{120\times 50}{6} = 1000$ rpm, $N =$ **966.67 rpm**

(iii) rotor copper losses/phase $= \frac{1}{3}\times(s\times\text{rotor input})$

total rotor power input = 80 kW

rotor power input per phase = 80/3 kW

rotor copper losses per phase

$= \frac{0.0333\times 80}{3}$ kW = **0.8888 kW**

Example 34.28. *A 440-V, 3-ϕ, 50-Hz, 4-pole, Y-connected induction motor has a full-load speed of 1425 rpm. The rotor has an impedance of (0.4 + J 4) ohm and rotor/stator turn ratio of 0.8. Calculate* ***(i)*** *full-load torque* ***(ii)*** *rotor current and full-load rotor Cu loss* ***(iii)*** *power output if windage and friction losses amount to 500 W* ***(iv)*** *maximum torque and the speed at which it occurs* ***(v)*** *starting current and* ***(vi)*** *starting torque.*

Solution. $N_s = 120\times 50/4 = 1500$ rpm $= 25$ rps, $s = 75/1500 = 0.05$

$E_1 = 440/1.73 = 254$ V/phase

(i) Full-load $T_f = \frac{3}{2\pi\times 25}\times\frac{0.05\,(0.8\times 254)^2\times 0.4}{(0.4)^2+(0.05\times 4)^2}$

$=$ **78.87 N-m**

(ii) $I_r = \frac{sE_2}{\sqrt{R_2^2+(sX_2)^2}} = \frac{sKE_1}{\sqrt{R_2^2+(sX_2)^2}} = \frac{0.05\times(0.8\times 254)}{\sqrt{(0.4)^2+(0.05\times 4)^2}}$

$=$ **22.73 A**

Total Cu loss $= 3I_r^2 R = 3\times 22.73^2\times 0.4 =$ **620 W**

(iii) Now, $P_m = 2\pi NT = 2\,\pi\times(1425/60)\times 78.87 = 11{,}745\ \Omega$

$\therefore$ $P_{out} = P_m -$ windage and friction loss $= 11{,}745 - 500 =$ **11,245 W**

(iv) For maximum torque, $s = R_2/X_2 = 0.4/4 = 0.1$

$\therefore$ $T_{max} = \frac{3}{2\pi\times 25}\times\frac{0.1\times(0.8\times 254)^2\times 0.4}{(0.4)^2+(0.1\times 4)^2} =$ **98.5 N-m**

Since $s = 0.1$, slip speed $= s\,N_s = 0.1\times 1500 = 150$ rpm.

$\therefore$ Speed for maximum torque $= 1500 - 150 =$ **1350 rpm.**

(v) starting current $= \dfrac{E_2}{\sqrt{R_2^2 + X_2^2}} = \dfrac{KE_1}{\sqrt{R_2^2 + X_2^2}} = \dfrac{0.8 \times 254}{\sqrt{0.4^2 + 4^2}} =$ **50.5 A**

(vi) At start, $s = 1$, hence

$$T_{st} = \frac{3}{2\pi \times 25} \times \frac{(0.8 \times 254)^2 \times 0.4}{(0.4)^2 + 4^2} =$$ **19.5 N-m**

It is seen that as compared to full-load torque, the starting torque is much less-almost 25 per cent.

34.38. Induction Motor Torque Equation

The gross torque T_g developed by an induction motor is given by

$$T_g = P_2 / 2\pi N_s \quad \text{—} N_s \text{ in r.p.s.}$$
$$= 60\, P_2 / 2\pi N_s = 9.55\, P_2 / N_s \quad \text{—} N_s \text{ in r.p.m.}$$

Now, $P_2 =$ rotor Cu loss$/s = 3I_2^2 R_2 / s$

As seen from Art. 34.19, $I_2 = \dfrac{sE_2}{\sqrt{R_2^2 + (s\, X_2)^2}} = \dfrac{sKE_1}{\sqrt{R_2^2 + (s\, X_2)^2}}$

where K is rotor/stator turn ratio per phase.

$$\therefore \quad P_2 = 3 \times \frac{s^2 E_2^2 R_2}{R_2^2 + (s\, X_2)^2} \times \frac{1}{s} = \frac{3\, s\, E_2^2 R_2}{R_2^2 + (s\, X_2)^2}$$

Also, $$P_2 = 3 \times \frac{s^2 K^2 E_1^2 R_2}{R_2^2 + (s\, X_2)^2} \times \frac{1}{s} = \frac{3\, s\, K^2 E_1^2 R_2}{R_2^2 + (s\, X_2)^2}$$

$$\therefore \quad T_g = \frac{P_2}{2\pi N_s} = \frac{3}{2\pi N_S} \times \frac{s\, E_2^2 R_2}{R_2^2 + (s\, X_2)^2} \quad \text{— in terms of } E_2$$

or $$= \frac{3}{2\pi N_S} \times \frac{s\, K^2 E_1^2 R_2}{R_2^2 + (sX_2)^2} \quad \text{— in terms of } E_1$$

Here, E_1, E_2, R_2 and X_2 represent phase values.

In fact, $3K^2/2\pi N_s = k$ is called the constant of the given machine. Hence, the above torque equation may be simplified to

$$T_g = k \frac{s\, E_1^2 R_2}{R_2^2 + (sX_2)^2} \quad \text{— in terms of } E_1$$

34.39. Synchronous Watt

It is clear from the above relations that torque is proportional to rotor input. By defining a new unit of torque (instead of the force-at-radius unit), we can say that the rotor torque ***equals*** rotor input. The new unit is synchronous watt. When we say that a motor is developing a torque of 1,000 synchronous watts, we mean that the rotor input is 1,000 watts and that the torque is such that power developed would be 1,000 watts provided the rotor were running synchronously and developing the same torque.

Or

Synchronous watt is that torque which, at the synchronous speed of the machine under consideration, would develop a power of 1 watt.

$$\text{rotor input} = T_{sw} \times 2\pi N_S \qquad \therefore \quad T_{sw} = \frac{\text{rotor input, } P_2}{2\pi \times \text{synch. speed}}$$

$$= \frac{1}{\omega_s}.\frac{N_S}{N}.P_g = \frac{1}{\omega_s}.\frac{N_S}{N}.P_m$$

Synchronous wattage of an induction motor equals the power transferred across the air-gap to the rotor.

$\therefore$ torque in synchronous watt

$$= \text{rotor input} = \frac{\text{rotor Cu loss}}{s} = \frac{\text{gross output power, } P_m}{(1-s)}$$

Obviously, at $s = 1$, torque in synchronous watt equals the total rotor Cu loss because at standstill, entire rotor input is lost as Cu loss.

Suppose a 23-kW, 4-pole induction motor has an efficiency of 92% and a speed of 1440 r.p.m. at rated load. If mechanical losses are assumed to be about 25 per cent of the total losses, then

motor input = 23/0.92 = 25 kW, total loss = 25 – 23 = 2 kW.

Friction and windage loss = 2/4 = 0.5 kW

$\therefore \quad P_m = 23 + 0.5 = 23.5$ kW

Power in synchronous watts $P_{sw} = P_2 = 23.5 \times 1500/1440 = 24.5$ kW

synchronous speed $w_S = 2\pi(1500/60) = 157$ rad / s

$\therefore$ synchronous torque, $T_{sw} = 24.5 \times 10^3/157 = 156$ N-m

or $$T_{sw} = P_m \frac{N_S}{N}.\frac{1}{\omega_S} = 23.5 \times \frac{1500}{1440} \times \frac{1}{2\pi(1500/60)}$$

$= 156$ N–m —Art 34.39

34.40. Variations in Rotor Current

The magnitude of the rotor current varies with load carried by the motor.

As seen from Art. 34.37

$$\frac{\text{rotor output}}{\text{rotor input}} = \frac{N}{N_S} \text{ or rotor output} = \text{rotor input} \times \frac{N}{N_S}$$

$\therefore$ rotor input = rotor output $\times N_s / N$

Also, rotor output $\propto 2\pi N T = k N T$

$\therefore$ rotor input $= k N T \times N_s / N = k N_s T$

Now, $$\frac{\text{rotor Cu loss}}{\text{rotor input}} = s \text{ or } \frac{3 I_2^2 R_2}{s} = \text{rotor input}$$

$\therefore \quad 3 I_2^2 R_2/s = k N_s T \text{ or } T \propto I_2^2 R_2/s$

$T_{st} \propto I_{2st}^2 R_2$ — since $s = 1$

$T_f \propto I_{2f}^2 R_2 / s_f$ — s_f = full-load slip

$\therefore$ $$\frac{T_{st}}{T_f} = s_f \left(\frac{I_{2st}}{I_{2f}}\right)^2$$

where I_{2st} and I_{2f} are the rotor currents for starting and full-load running conditions.

34.41. Analogy with a Mechanical Clutch

We have seen above that, rotor Cu loss = slip × rotor input

This fact can be further clarified by considering the working of a mechanical clutch (though it is

not meant to be a proof for the above) similar to the one used in automobiles. A plate clutch is shown in Fig. 34.39. It is obvious that the torque on the driving shaft must exactly equal the torque on the driven shaft. In fact, these two torques are actually one and the same torque, because the torque is caused by friction between the discs and it is true whether the clutch is slipping or not. Let ω_1 and ω_2 be the angular velocities of the shaft when the clutch is slipping.

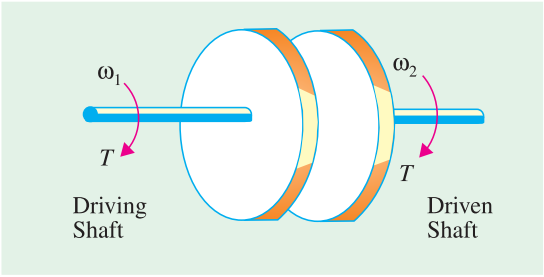


Fig. 34.39

Then, input = $T\omega_1$ and output = $T\omega_2 = T\omega_1(1-s)$: [$\because \omega_2 = \omega_1(1-s)$]

loss = $T\omega_1 - T\omega_2 = T\omega_1 - T\omega_1(1-s) = sT\omega_1$ = slip × input

34.42. Analogy with a D.C. Motor

The above relations could also be derived by comparing an induction motor with a d.c. motor. As shown in Art 29.3, in a d.c. shunt motor, the applied voltage is always opposed by a back e.m.f. E_b. The power developed in the motor armature is $E_b I_a$ where I_a is armature current. This power, as we know, is converted into mechanical power in the armature of the motor.

Now, in an induction motor, it is seen that the induced e.m.f in the rotor decreases from its standstill value of E_2 to sE_2 when in rotation. Obviously, the difference $(1-s)E_2$ is the e.m.f. called forth by the rotation of the rotor similar to the back e.m.f. in a d.c. motor. Hence, gross power P_m developed in the rotor is given by the product of the back e.m.f., armature current and rotor power factor.

$$P_m = (1-s)E_2 \times I_2 \cos\phi_2 \; ; \qquad \text{Now } I_2 = \frac{sE_2}{\sqrt{[R_2^2 + (sX_2)^2]}} \text{ and } \cos\phi_2 = \frac{R_2}{\sqrt{[R_2^2 + (sX_2)^2]}}$$

$$\therefore \quad P_m = (1-s)E_2 \times \frac{sE_2}{\sqrt{[R_2^2 + (sX_2)^2]}} \times \frac{R_2}{\sqrt{[R_2^2 + (sX_2)^2]}} = \frac{s(1-s)E_2^2 R_2}{R_2^2 + (sX_2)^2}$$

Multiplying the numerator and the denominator by s, we get

$$P_m = \left(\frac{1-s}{s}\right) R_2 \times \frac{s^2 E_2^2}{[R_2^2 + (sX_2)^2]} = \left(\frac{1-s}{s}\right) I_2^2 R_2 \qquad \left(\because \; I_2 = \frac{sE_2}{\sqrt{[R_2^2 + (sX_2)^2]}}\right)$$

Now, $I_2^2 R_2$ = rotor Cu loss/phase $\quad \therefore \quad \dfrac{\text{Cu loss}}{\text{rotor output}} = \dfrac{s}{1-s}$

This is the same relationship as derived in Art. 34-37.

Example 34.30. *The power input to a 3-phase induction motor is 60 kW. The stator losses total 1 kW. Find the mechanical power developed and the rotor copper loss per phase if the motor is running with a slip of 3%.*

(Elect. Machines AMIE Sec. E Summer 1991)

Solution. Rotor input, P_2 = stator input – stator losses = 60 – 1 = 59 kW

$P_m = (1-s)P_2 = (1-0.03) \times 59 = 57.23$ kW

Total rotor Cu loss = $sP_2 = 0.03 \times 59 = 1.77$ kW = 1770 W

Rotor Cu loss/phase = 1770/3 = 590 W

Example 34.31. *The power input to the rotor of a 400 V, 50.Hz, 6-pole, 3-phase induction motor is 20 kW. The slip is 3%. Calculate* ***(i)*** *the frequency of rotor currents* ***(ii)*** *rotor speed* ***(iii)*** *rotor copper losses and* ***(iv)*** *rotor resistance per phase if rotor current is 60 A.*
(Elect. Engg. Punjab Univ. 1991)

Solution. ***(i)*** Frequency of rotor current = $sf = 0.03 \times 50 = 1.5$ Hz

(ii) $N_s = 120 \times 50/6 = 1000$ rpm; $N = 1000(1 - 0.03) = 700$ rpm

(iii) rotor Cu loss = $s \times$ rotor input = $0.03 \times 20 = 0.6$ kW = 600 W

(iv) rotor Cu loss/phase = 200 W; $\therefore\ 60^2 R_2 = 200$; $R_2 = 0.055\ \Omega$

Example 34.32. *A 3-phase, 6-pole, 50-Hz induction motor develops 3.73 kW at 960 rpm. What will be the stator input if the stator loss is 280 W?* **(Madurai Kamraj Univ. 1999)**

Solution. As seen from Art. 34.37, $\dfrac{\text{power developed in rotor}}{\text{rotor input}} = \dfrac{N}{N_s}$

Now, mechanical power developed in rotor = 3.73 kW., $N_s = 120 \times 50/6 = 1000$ r.p.m.

$\therefore$ 3,730/rotor input = 960/1000 $\therefore$ rotor input = 3,885 W

Stator input = rotor input + stator losses = 3885 + 280 = **4,156 W**

Example 34.33. *The power input to the rotor of a 400 V, 50-Hz, 6-pole, 3-ϕ induction motor is 75 kW. The rotor electromotive force is observed to make 100 complete alteration per minute. Calculate:* ***(i)*** *slip* ***(ii)*** *rotor speed* ***(iii)*** *rotor copper losses per phase* ***(iv)*** *mechanical power developed.*
(Elect. Engg. I, Nagpur Univ. 1993)

Solution. Frequency of rotor emf, $f' = 100/60 = 5/3$ Hz

(i) Now, $f' = sf$ or $5/3 = s \times 50$; $s = 1/30 =$ **0.033 or 3.33%**

(ii) $N_s = 120 \times 50/6 = 1000$ rpm ; $N = N_s(1 - s) = 1000\,(1 - 1/30) = 966.7$ rpm

(iii) $P_2 = 75\ kW$; total rotor Cu loss = $sP_2 = (1/30) \times 75 = 2.5$ kW

rotor Cu loss/phase = 2.5/3 = **0.833 kW**

(iv) $P_m = (1 - s)P_2 = (1 - 1/30) \times 75 =$ **72.5 kW**

Example 34.34. *The power input to a 500 V, 50-Hz, 6-pole, 3-phase induction motor running at 975 rpm is 40 kW. The stator losses are 1 kW and the friction and windage losses total 2 kW. Calculate :* ***(i)*** *the slip* ***(ii)*** *the rotor copper loss* ***(iii)*** *shaft power and* ***(iv)*** *the efficiency.*
(Elect. Engg. - II, Pune Univ. 1989)

Solution. ***(i)*** $N_s = 120 \times 50/6 = 1000$ rpm ; $s = (100 - 975)/1000 = 0.025$ or 2.5%

(ii) Motor input P_1=40 kW ; stator loss = 1 kW; rotor input $P_2 = 40 - 1 = 39$ kW

$\therefore$ rotor Cu loss = $s \times$ rotor input = $0.025 \times 39 =$ **0.975 kW**

(iii) $P_m = P_2$ – rotor Cu loss = 39 – 0.975 = 38.025 kW

$P_{out} = P_m$ – friction and windage loss = 38.025 – 2 = **36.025 kW**

(iv) $\eta = P_{out}/P_1 = 36.025/40 = 0.9$ or **90%**

Example 34.35. *A 100-kW (output), 3300-V, 50-Hz, 3-phase, star-connected induction motor has a synchronous speed of 500 r.p.m. The full-load slip is 1.8% and F.L. power factor 0.85. Stator copper loss = 2440 W. Iron loss = 3500 W. Rotational losses = 1200 W. Calculate* ***(i)*** *the rotor copper loss* ***(ii)*** *the line current* ***(iii)*** *the full-load efficiency.* **(Elect. Machines, Nagpur Univ. 1993)**

Solution. P_m = output + rotational loss = 100 + 1.2 = 101.2 kW

(i) rotor Cu loss $= \frac{s}{1-s} \times P_m = \frac{0.018}{1-0.018} \times 101.2 =$ **1.855 kW**

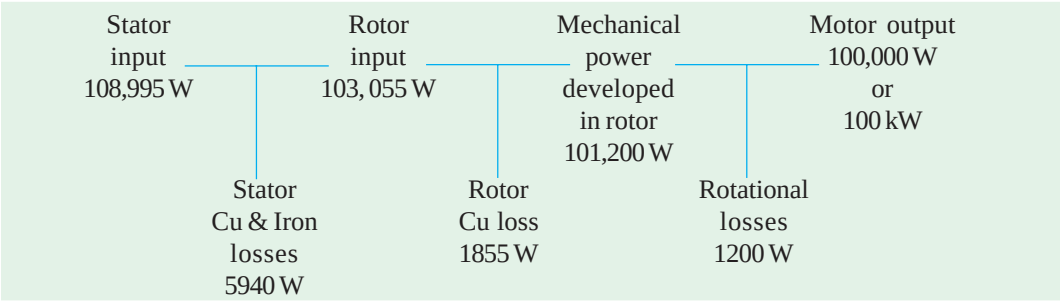
(ii) rotor input, P_2 = P_m + rotor Cu loss = 101.2 + 1.855 = 103.055 kW

Stator input = P_2 + stator Cu and iron losses

= 103.055 + 2.44 + 3.5 = 108.995 kW

$\therefore$ $108{,}995 = \sqrt{3} \times 3300 \times I_L \times 0.85$; $I_L =$ **22.4 A**

The entire power flow in the motor is given below.



(iii) F.L. efficiency = 100,000/108,995 = 0.917 or **91.7%**

Example 34.36. *The power input to the rotor of a 440 V, 50-Hz, 6-pole, 3-phase induction motor is 100 kW. The rotor electromotive force is observed to make 120 cycles per minute, Calculate* ***(i)*** *the slip* ***(ii)*** *the rotor speed* ***(iii)*** *mechanical power developed* ***(iv)*** *the rotor copper loss per phase and* ***(v)*** *speed of stator field with respect to rotor.* **(Elect. Engg. AMIETE Sec. A June 1991)**

Solution. ***(i)*** $f' = sf$ or $(120/60) = s \times 50$; $s =$ **0.01**

(ii) $N_s = 120 \times 50/6 = 1000$ rpm; $N = 1000(1 - 0.01) =$ **990 rpm**

(iii) $P_m\ (1 - s)P_2 = (1 - 0.01) \times 100 =$ **99 kW**

(iv) total rotor Cu loss = $sP_2 = 0.01 \times 100 = 1$ kW ; Cu loss/phase = **1/3 kW**

(v) $N_s = 1000$ rpm; $N = 990$ rpm. Hence, speed of stator field with respect to rotor is

$= 1000 - 990 =$ **10 rpm.**

Example 34.37. *An induction motor has an efficiency of 0.9 when delivering an output of 37 kW. At this load, the stator Cu loss and rotor Cu loss each equals the stator iron loss. The mechanical losses are one-third of the no-load loss. Calculate the slip.*

(Adv. Elect. Machines, A.M.I.E. Sec. B Winter 1993)

Solution. Motor input = 37,000/0.9 = 41,111 W

$\therefore$ total loss = 41,111 – 37,000 = 4,111 W

This includes ***(i)*** stator Cu and iron losses ***(ii)*** rotor Cu loss (its iron loss being negligibly small) and ***(iii)*** rotor mechanical losses.

Now, no-load loss of an induction motor consists of ***(i)*** stator iron loss and ***(ii)*** mechanical losses provided we neglect the small amount of stator Cu loss under no-load condition. Moreover, these two losses are independent of the load on the motor.

no-load loss $= W_i + W_m = 3W_m$

$\therefore$ $W_m = W_i / 2$

where W_i is the stator iron loss and W_m is the rotor mechanical losses.

Let, stator iron loss = x; then stator Cu loss = x; rotor Cu loss = x; mechanical loss = $x/2$

$\therefore \quad 3x + x/2 = 4{,}111$ or $x = 1175$ W

Now, rotor input = gross output + mechanical losses + rotor Cu loss

$= 37{,}000 + (1175/2) + 1175 = 38{,}752$ W

$$s = \frac{\text{rotor Cu loss}}{\text{rotor input}} = \frac{1175}{38{,}752} = 0.03 \text{ or } \mathbf{3\%}.$$

Example 34.38. *A 400 V, 50-Hz, 6-pole, Δ-connected, 3-ϕ induction motor consumes 45 kW with a line current of 75 A and runs at a slip of 3%. If stator iron loss is 1200 kW, windage and fricition loss is 900 W and resistance between two stator terminals is 0.12 Ω, calculate* ***(i)*** *power supplied to the rotor* P_2 ***(ii)*** *rotor Cu loss* P_{cr} ***(iii)*** *power supplied to load* P_{out} ***(iv)*** *efficiency and* ***(v)*** *shaft torque developed.*

Solution. $\text{Cos}\,\phi = \dfrac{45 \times 1000}{\sqrt{3} \times 400 \times 75} = 0.866$ lag

A line current of 75 amp means a phase-current of $75/\sqrt{3}$ *i.e.* 43.3 amp

Next, winding resistance has to be worked out

Refer to Fig. 34.40.

r and 2*r* in parallel have an equivalent resistance measured at *a* and *b* in delta connected motor as $r \times 2r/3r = 2r/3$ ohms

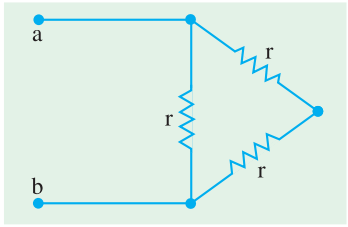


Fig. 34.40

From the data given $\dfrac{2r}{3} = 0.12, \; r = 0.18$

Total stator copper loss $= 3 \times 43.3^2 \times 0.18 = 1012$ Watts

Total input to stator = 45,000 Watts

Stator copper loss = 1012 Watts, stator core loss = 1200 Watts

Stator output = Rotor input = 42,788 Watts

Rotor copper loss = Slip × Rotor input = 0.03 × 42,788 = 1284 Watts

Rotor mech. output power = 42,788 – 1284 = 41,504 Watts

Shaft output = Mech. output of rotor – Mech losses

$= 41504 - 900 = 40604$ Watts

$$\text{Efficiency} = \frac{40604}{45000} \times 100\% = 92.23\%$$

$$\text{Shaft output torque, } T = \frac{40604 \times 60}{2\pi \times 970} = \mathbf{400 \text{ Nw-m}}$$

Example 34.39(*a*). *A 3-phase induction motor has a 4-pole, star-connected stator winding and runs on a 220-V, 50-Hz supply. The rotor resistance per phase is 0.1 Ω and reactance 0.9 Ω. The ratio of stator to rotor turns is 1.75. The full-load slip is 5%. Calculate for this load:*

(a) *the load torque in kg-m*

(b) *speed at maximum torque*

(c) *rotor e.m.f. at maximum torque.* **(Electrical Machines-I, South Gujarat Univ. 1985)**

Solution. ***(a)*** K = rotor turns/stator turns = 1/1.75

stator voltage/phase, $E_1 = 220/\sqrt{3}$ V

$\therefore$ standstill rotor e.m.f./phase, $E_2 = K E_1 = \dfrac{220}{\sqrt{3}} \times \dfrac{1}{1.75} = \mathbf{72.6 \text{ V}}$

$$Z_r = \sqrt{R_2^2 + (sX_2^2)} = \sqrt{0.1^2 + (0.05 \times 0.9)^2} = 0.11\,\Omega$$

$I_2 = sE_2 / Z_r = 0.05 \times 72.6/0.11 = 33$ A

Rotor Cu loss $P_{cr} = 3I_2^2 R_2 = 3 \times 33^2 \times 0.1 = 327$ W

$$\frac{\text{rotor Cu loss}}{\text{mech. power developed}} = \frac{s}{1-s}; \quad \frac{327}{P_m} = \frac{0.05}{1-0.05}; \quad P_m = 6213 \text{ W}$$

$T_g = 9.55\, P_m / N$; $N = N_s(1 - s) = 1500\,(1 - 0.05) = 1425$ rpm

$\therefore$ $T_g = 9.55 \times 6213/1425 = 41.6$ N-m $= 41.6/9.81 =$ **4.24 kg.m**

(b) For maximum torque, $s_m = R_2 / X_2 = 0.1/0.81 =$ **1/9**

$\therefore$ $N = N_s(1 - s) = 1500(1 - 1/9) =$ **1333 r.p.m.**

(c) rotor e.m.f./phase at maximum torque $= (1/9) \times 72.6 =$ **8.07 V**

Example 34.39 (*b*). *A 400 V, 3-phase, 50 Hz, 4-pole, star-connected induction-motor takes a line current of 10 A with 0.86 p. f. lagging. Its total stator losses are 5 % of the input. Rotor copper losses are 4 % of the input to the rotor, and mechanical losses are 3 % of the input of the rotor. Calculate **(i)** slip and rotor speed, **(ii)** torque developed in the rotor, and **(iii)** shaft-torque.*

[Nagpur University, April 1998]

Solution. Input to motor $= \sqrt{3} \times 400 \times 10 \times 0.86 = 5958$ Watts

Total stator losses = 5% of 5958 = 298 Watts

Rotor input = Stator Output = 5958 – 298 = 5660 Watts

Rotor Copper-loss = 4 % of 5660 = 226.4 Watts

Mechanical losses = 3% of 5660 = 169.8 Watts

Shaft output = 5660 – 226.4 – 169.8 = 5264 Watts

(i) slip, s = Rotor-copper-loss / rotor-input = 4 %, as given

Synchronous speed, $N_s = 120 \times f/P = 1500$ rpm

Rotor speed, $N = N_s(1 - s) = 1500\ (1 - 0.04) = 1440$ rpm

(ii) Let the torque developed in the rotor $= T_r$

ω_r, Angular speed of rotor $= 2\pi$ N/60 = 150.72 radians/sec

Rotor output = Rotor input – Rotor-copper-loss

= 5660 – 226.4 = 5433.6 watts

$T_r \omega_r$ = 5433.6, giving $T_r = 5433.6 / 150.72 = 36.05$ Nw-m

(Alternatively, synchronous angular speed $= \omega_s = 2\pi \times 1500 / 60 = 157$ rad / sec

$T_r \omega_s$ = rotor input in Watts = 5660

$T_r = 5660 / 157 = 36.05$ Nw – m

(iii) Shaft-torque = T_m = Shaft output in Watts $/\omega_r$

= 5264 / 150.72 = 34.93 Nw – m.

Example 34.40. *A 3-phase, 440-V, 50-Hz, 40-pole, Y-connected induction motor has rotor resistance of 0.1 Ω and reactance 0.9 Ω per phase. The ratio of stator to rotor turns is 3.5. Calculate:*

(a) *gross output at a slip of 5%*

(b) *the maximum torque in synchronous watts and the corresponding slip.*

Solution. (*a*) Phase voltage $= \frac{440}{\sqrt{3}}$ V ; $K = \frac{\text{rotor turns}}{\text{stator turns}} = \frac{1}{3.5}$

Standstill e.m.f. per rotor phase is $E_2 = KE_1 = \frac{440}{\sqrt{3}} \times \frac{1}{3.5} = 72.6$ V

$E_r = s.E_2 = 0.05 \times 72.6 = 3.63$ V ; $Z_r = \sqrt{0.1^2 + (0.05 \times 0.9)^2} = 0.1096\ \Omega$

$I_2 = \dfrac{E_r}{Z_r} = \dfrac{3.63}{0.1096} = 33.1$ A ; Total Cu loss $= 3\, I_2^2 R = 3 \times (33.1)^2 \times 0.1 = 330$ W

$\dfrac{\text{rotor Cu loss}}{\text{rotor gross output}} = \dfrac{s}{1-s}$ $\quad \therefore$ rotor gross output $= \dfrac{330 \times 0.95}{0.05} =$ **7,250 W**

(*b*) For Maximum Torque

$R_2 = s_m \,.\, X_2 \quad \therefore\ s_m = R_2 / X_2 = 0.1/0.9 =$ **1/9**; $E_r = (1/9) \times 72.6 = 8.07$ V

$Z_r = \sqrt{0.1^2 + (0.9/9)^2} = 0.1414\ \Omega$; $I_2 = 8.07/0.1414 = 57.1$ A

Total rotor Cu loss $= 3 \times 57.1^2 \times 0.1 = 978$ W

rotor input $= \dfrac{\text{rotor Cu loss}}{S} = \dfrac{978}{1/9} = 8{,}802$ W

$\therefore$ Maximum torque in synchronous watts = rotor input = **8,802 W**

Example 34.41. *An 18.65-kW, 4-pole, 50-Hz, 3-phase induction motor has friction and windage losses of 2.5 per cent of the output. The full-load slip is 4%. Compute for full load* ***(a)*** *the rotor Cu loss* ***(b)*** *the rotor input* ***(c)*** *the shaft torque* ***(d)*** *the gross electromagnetic torque.*

(Elect. Machines-II, Indore Univ. 1987)

Solution. Motor output $P_{out} = 18{,}650$ W

Friction and windage loss $P_w = 2.5\%$ of $18{,}650 = 466$ W

Rotor gross output $P_m = 18{,}650 + 466 = 19{,}116$ W

(a) $\dfrac{\text{rotor Cu loss}}{\text{rotor gross output}} = \dfrac{s}{1-s}$, rotor Cu loss $= \dfrac{0.04}{(1-0.04)} \times 19{,}116 =$ **796.6 W**

(b) Rotor input $P_2 = \dfrac{\text{rotor Cu loss}}{s} = \dfrac{796.5}{0.04} =$ **19,912.5 W**

(or rotor input = 19,116 + 796.6 = 19,913 W)

(c) $T_{sh} = 9.55\, P_{out} / N$: $N_s = 120 \times 50/4 = 1500$ r.p.m.

$N = (1 - 0.04) \times 1500 = 1440$ r.p.m

$\therefore$ $T_{sh} = 9.55 \times 18{,}650/1440 =$ **123.7 N-m**

(d) Gross torque $T_g = 9.55\, P_m / N = 9.55 \times 19{,}116/1440 =$ **126.8 N-m**

(or $T_g = P_2 / 2\pi N_s = 19{,}913/2\pi \times 25 = 127$ N-m)

Example 34.42. *An 8-pole, 3-phase, 50 Hz, induction motor is running at a speed of 710 rpm with an input power of 35 kW. The stator losses at this operating condition are known to be 1200 W while the rotational losses are 600 W. Find* ***(i)*** *the rotor copper loss,* ***(ii)*** *the gross torque developed,* ***(iii)*** *the gross mechanical power developed,* ***(iv)*** *the net torque and* ***(v)*** *the mechanical power output.*

(Elect. Engg. AMIETE Sec. A 1991 & Rajiv Gandhi Techn. Univ., Bhopal, 2000)

Solution. ***(i)*** $P_2 = 35 - 1.2 = 33.8$ kW ; $N_s = 120 \times 50/8 = 750$ rpm ; $N = 710$ rpm; $s = (750 - 710)/750 = 0.0533$

$\therefore$ Rotor Cu loss $= sP_2 =$ **1.8 kW**

(ii) $P_m = P_2 -$ rotor Cu loss $= 33.8 - 1.8 =$ **32 kW**

(iii) $T_g = 9.55\, P_m / N = 9.55 \times 32000/710 = 430.42$ N-m

(iv) $P_{out} = P_m -$ rotational losses $= 32000 - 600 =$ **31400 W**

(v) $T_{sh} = 9.55\, P_{out}/N = 9.55 \times 31400/710 =$ **422.35 N-m**

Example 34.43. *A 6-pole, 50-Hz, 3-phase, induction motor running on full-load with 4% slip develops a torque of 149.3 N-m at its pulley rim. The friction and windage losses are 200 W and the stator Cu and iron losses equal 1,620 W. Calculate* ***(a)*** *output power* ***(b)*** *the rotor Cu loss and* ***(c)*** *the efficiency at full-load.* **(Elect. Technology, Mysore Univ. 1989)**

Solution. $N_s = 120 \times 50/6 = 1{,}000$ r.p.m; $N = (1 - 0.04) \times 1{,}000 = 960$ r.p.m.

Out put power $= T_{sh} \times 2\pi N = 2\pi \times (960/60) \times 149.3 =$ **15 kW**

Now, output = 15,000 W

Friction and windage losses = 200 W ; Rotor gross output = 15,200 W

$\frac{P_m}{P_2} = \frac{N}{N_S}$ $\therefore$ rotor input $P_2 = 15{,}200 \times 1{,}000/960 = 15{,}833$ W

(b) $\therefore$ rotor Cu loss $= 15{,}833 - 15{,}200 =$ **633 W**

(rotor Cu loss is given by : $\frac{\text{rotor Cu loss}}{\text{rotor output}} = \frac{s}{1-s}$)

(c) stator output = rotor input = 15,833 W

Stator Cu and iron losses = 1,620 W

$\therefore$ Stator input $P_1 = 15{,}833 + 1{,}620 = 17{,}453$ W

overall efficiency, $\eta = 15{,}000 \times 100/17{,}453 =$ **86%**

Example 34.44. *An 18.65-kW, 6-pole, 50-Hz, 3-ϕ slip-ring induction motor runs at 960 r.p.m. on full-load with a rotor current per phase of 35 A. Allowing 1 kW for mechanical losses, find the resistance per phase of 3-phase rotor winding.* **(Elect. Engg-I, Nagpur Univ. 1992)**

Solution. Motor output = 18.65 kW; Mechanical losses = 1 kW

$\therefore$ Mechanical power developed by rotor, $P_m = 18.65 + 1 = 19.65$ kW

Now, $N_s = 120 \times 50/6 = 1000$ r.p.m. ; $s = (1000 - 960)/1000 = 0.04$

rotor Cu loss $= \frac{s}{1-s} \times P_m = \frac{0.04}{(1-0.04)} \times 19.65 = 0.819$ kW $= 819$ W

$\therefore$ $3I_2^2 R_2 = 819$ or $3 \times 35^2 \times R_2 = 819$ $R_2 =$ **0.023 Ω/phase**

Example 34.45. *A 400 V, 4-pole, 3-phase, 50-Hz induction motor has a rotor resistance and reactance per phase of 0.01 Ω and 0.1 Ω respectively. Determine* ***(a)*** *maximum torque in N-m and the corresponding slip* ***(b)*** *the full-load slip and power output in watts, if maximum torque is twice the full-load torque. The ratio of stator to rotor turns is 4.*

Solution. Applied voltage/phase $E_1 = 400/\sqrt{3} = 231$ V

Standstill e.m.f. induced in rotor, $E_2 = KE_1 = 231/4 = 57.75$ V

(a) Slip for maximum torque, $s_m = R_2/X_2 = 0.01/0.1 = 0.1$ or **10%**

$T_{max} = \frac{3}{2\pi N_S} \times \frac{E_2^2}{2X_2}$ ... Art. 34.20

$N_s = 120 \times 50/4 = 1500$ r.p.m. $= 25$ r.p.s.

$\therefore$ $T_{max} = \frac{3}{2\pi \times 25} \times \frac{57.75^2}{2 \times 0.1} =$ **320 N-m**

(b) $\frac{T_f}{T} = \frac{2as_f}{a^2 + s^2} = \frac{1}{2}$; Now, $a = R_2/X_2 = 0.01/0.1 = 0.1$

$\therefore \quad 2 \times 0.1 \times s_f/(0.1^2 + s_f^2) = 1/2 \quad \therefore \; s_f = 0.027$ or 0.373

Since $s_f = 0.373$ is not in the operating region of the motor, we select $s_f = 0.027$.

Hence, $s_f = \mathbf{0.027.}$ $N = 1500\,(1 - 0.027) = 1459.5$ r.p.m.

Full-load torque $T_f = 320/2 = 160$ N-m

F.L. Motor output $= 2\pi N T_f/60 = 2\pi \times 1459.5 \times 160/60 = \mathbf{24{,}454\ W}$

Example 34.46. *A 3-phase induction motor has a 4-pole, Y-connected stator winding. The motor runs on 50-Hz supply with 200V between lines. The motor resistance and standstill reactance per phase are 0.1 Ω and 0.9 Ω respectively. Calculate*

(a) *the total torque at 4% slip* ***(b)*** *the maximum torque*

(c) *the speed at maximum torque if the ratio of the rotor to stator turns is 0.67. Nelgect stator impedance.* **(Elect. Machinery, Mysore Univ. 1987)**

Solution. *(a)* Voltage/phase $= \dfrac{200}{\sqrt{3}}$ V; $K = \dfrac{\text{rotor turns}}{\text{stator turns}} = 0.67$

Standstill rotor e.m.f. per phase is $E_2 = \dfrac{200}{\sqrt{3}} \times 0.67 = 77.4$ V and $s = 0.04$

$$Z_r = \sqrt{R_2^2 + \left(sX_2^2\right)} = \sqrt{0.1^2 + (0.04 \times 0.9)^2} = 0.106\,\Omega$$

$$I_2 = \frac{sE_2}{Z_r} = \frac{77.4 \times 0.04}{0.106} = 29.1\text{ A}$$

Total Cu loss in rotor $= 3 I_2^2 R_2 = 3 \times 29.1^2 \times 0.1 = 255$ W

Now, $\dfrac{\text{rotor Cu loss}}{\text{rotor gross output}} = \dfrac{s}{1-s} \quad \therefore \quad P_m = 255 \times 0.96/0.04 = 6{,}120$ W

$N_s = 120 \times 50/4 = 1{,}500$ r.p.m; $N = (1 - 0.04) \times 1{,}500 = 1440$ r.p.m.

Gross torque developed, $T_g = 9.55\, P_m/N = 9.55 \times 6120/1440 = \mathbf{40.6\ N\text{-}m}$

(b) For maximum torque, $s_m = R_2 / X_2 = 0.1/0.9 = 1/9$ and $E_r = sE_2 = 77.4 \times 1/9 = 8.6$ V

$Z_r = \sqrt{0.1^2 + (0.9 \times 1/9)^2} = 0.1414\,\Omega$; $I_2 = 8.6/0.1414 = 60.8$ A

Total rotor Cu loss $= 3 \times 60.8^2 \times 0.1 = 1{,}110$ W

Rotor gross output $= \dfrac{1{,}100 \times (1 - 1/9)}{1/9} = 8{,}800$ W

$N = (1 - 1/9) \times 1{,}500 = 1{,}333$ rpm

$\therefore \quad T_{max} = 9.55 \times 8{,}800/1333 = \mathbf{63\ N\text{-}m}$

(c) Speed at maximum torque, as found above, is **1333.3 r.p.m.**

Example 34.47. *The rotor resistance and standstill reactance of a 3-phase induction motor are respectively 0.015 Ω and 0.09 Ω per phase.*

(i) *What is the p.f. of the motor at start?*

(ii) *What is the p.f. at a slip of 4 percent ?*

(iii) *If the number of poles is 4, the supply frequency is 50-Hz and the standstill e.m.f. per rotor phase is 110 V, find out the full-load torque. Take full-load slip as 4 per cent.*

(Electrical Technology-I, Osmania Univ. 1990)

Solution. *(i)* rotor impedance/phase $= \sqrt{0.015^2 + 0.09^2} = 0.0912\,\Omega$

p.f. $= 0.015/0.0912 = \mathbf{0.164}$

(ii) reactance/phase $= sX_2 = 0.04 \times 0.09 = 0.0036\,\Omega$

rotor impedance/phase $= \sqrt{0.015^2 + 0.0036^2} = 0.0154\ \Omega$

p.f. $= 0.015/0.0154 = \mathbf{0.974}$

(iii) $N_s = 120 \times 50/4 = 1{,}500$ r.p.m.; $N = 1{,}500 - (0.04 \times 1{,}500) = 1{,}440$ r.p.m.

$E_r = sE_2 = 0.04 \times 110 = 4.4$ V ; $Z_r = 0.0154\ \Omega$ ...found above

$I_2 = 4.4/0.0154 = 286$ A

Total rotor Cu loss $= 3I_2^2 R_2 = 3 \times 286^2 \times 0.015 = 3{,}650$ W

Now, $\dfrac{\text{rotor Cu loss}}{\text{rotor gross output}} = \dfrac{s}{1-s}$

$\therefore$ Rotor gross output $P_m = 3{,}650 \times 0.96/0.04 = 87{,}600$ W

If T_g is the gross torque developed by the rotor, then

$T_g = 9.55\, P_m/N = 9.55 \times 87{,}600/1440 = \mathbf{581\ N\text{-}m}$

Example 34.48. *The useful full load torque of 3-phase, 6-pole, 50-Hz induction motor is 162.84 N-m. The rotor e.m. f. is observed to make 90 cycles per minute. Calculate* ***(a)*** *motor output* ***(b)*** *Cu loss in rotor* ***(c)*** *motor input and* ***(d)*** *efficiency if mechanical torque lost in windage and friction is 20.36 N-m and stator losses are 830 W.* **(Elect. Machines-II, Indore Univ. 1988)**

Solution. $N_s = 120 \times 50/6 = 1{,}000$ r.p.m.

Frequency of rotor e.m.f. $= 90/60 = 1.5$ Hz; $s = f_r/f = 1.5/50 = 0.03$

Rotor speed $= 1{,}000(1 - 0.03) = 970$ r.p.m. ; Useful F.L. torque $= 162.84$ N-m

(a) motor output $= T_{sh}\dfrac{2\pi N}{60} = \dfrac{2\pi \times 970 \times 162.84}{60} = \mathbf{16{,}540\ W}$

(b) *gross torque* $T_g = 162.84 + 20.36 = 183.2$ N-m

Now, $T_g = 9.55\dfrac{P_2}{N_s}$ $\therefore$ $183.2 = 9.55 \times \dfrac{P_2}{1000}$

$\therefore$ rotor input, $P_2 = 183.2 \times 1000/9.55 = 19{,}170$ W

$\therefore$ rotor Cu loss $= s \times$ rotor input $= 0.03 \times 19{,}170 = \mathbf{575.1\ W}$

(c) Motor input, $P_1 = 19{,}170 + 830 = \mathbf{20{,}000\ W}$

(d) $\eta = (16{,}540/20{,}000) \times 100 = \mathbf{82.27\%}$

Example 34.49. *Estimate in kg-m the starting torque exerted by an 18.65-kW, 420-V, 6-pole, 50-Hz, 3-phase induction motor when an external resistance of 1 Ω is inserted in each rotor phase.*

stator impedance : (0.25 + j 0.75) Ω *rotor impedance : (0.173 + J 0.52) Ω*

stato/rotor voltage ratio: 420/350 *connection: Star-Star*

Solution. $K = E_2/E_1 = 350/420 = 5/6$

Equivalent resistance of the *motor* as referred to rotor is

$R_{02} = R_2 + K^2 R_1 = 0.173 + (5/6)^2 \times 0.25 = 0.346\ \Omega$ / phase

Similarly, $X_{02} = 0.52 + (5/6)^2 \times 0.75 = 1.04\ \Omega$ / phase

When an external resistance of 1 ohm/phase is added to the rotor circuit, the equivalent motor impedance as referred to rotor circuit is

$Z_{02} = \sqrt{(1 + 0.346)^2 + 1.04^2} = 1.7\ \Omega$

Short-circuit rotor current is $I_2 = \dfrac{350/\sqrt{3}}{1.7} = 119$ A

Rotor Cu loss per phase on short-circuit $= 119^2 \times 1.173 = 16{,}610$ W

Now, rotor power input $= \dfrac{\text{rotor Cu loss}}{s}$; On short-circuit, $s = 1$

$\therefore$ rotor power input = rotor Cu loss on short-circuit

= 16,610 W/phase = 49,830 W for 3 phases

$N_s = 120 \times 50/6 = 1000$ r.p.m.

If T_{st} is the starting torque in newton-metres, then

$T_{st} = 9.55 \quad P_2 / N_s = 9.55 \times 49{,}830/1000 = 476$ N-m $= 476/9.81 =$ **46.7 kg-m.**

Example 34.50. *An 8-pole, 37.3-kW, 3-phase induction motor has both stator and rotor windings connected in star. The supply voltage is 280 V per phase at a frequency of 50 Hz. The short-circuit current is 200 A per phase at a short-circuit power factor of 0.25. The stator resistance per phase is 0.15 Ω. If transformation ratio between the stator and rotor windings is 3, find*

(i) *the resistance per phase of the rotor winding*

(ii) *the starting torque of the motor.*

Solution. Under short circuit, all the power supplied to the motor is dissipated in the stator and rotor winding resistances. Short-circuit power supplied to the motor is

$W_{sc} = 3V_1 I_{sc} \cos \phi_{sc} = 3(280 \times 200 \times 0.25)$ W

Power supplied/phase $= 280 \times 200 \times 0.25 = 14{,}000$ W

Let r_2' be the rotor resistance per phase as referred to stator. If r_1 is the stator resistance per phase, then

$I_{sc}^2 (r_1 + r_2') = 14{,}000 \quad \therefore \quad r_1 + r_2' = 14{,}000 / 200^2 = 0.35\ \Omega$

$\therefore \quad r_2' = 0.35 - 0.15 = 0.2\ \Omega$; Now, $r_2' = r_2 / K^2$ where $K = 1/3$

(i) $\therefore \quad r_2 = K^2 r_2' = 0.2/9 =$ **0.022 Ω** per phase

(ii) Power supplied to the rotor circuit is

$= 3I_{sc}^2 r_2' = 3 \times 200^2 \times 0.2 = 24{,}000$ W

$N_s = 120\, f/P = 120 \times 50/8 = 750$ r.p.m. $= 12.5$ r.p.s.

$\therefore$ Starting torque $= 9.55\, P_2 / N_s = 9.55 \times 24{,}000/750 =$ **305.6 N-m**

Example 34.51. *A 3-phase induction motor, at rated voltage and frequency has a starting torque of 160 per cent and a maximum torque of 200 per cent of full-load torque. Neglecting stator resistance and rotational losses and assuming constant rotor resistance, determine*

(a) *the slip at full-load*

(b) *the slip at maximum torque*

(c) *the rotor current at starting in terms of F.L. rotor current.*

(Electrical Machine - II, Bombay Univ. 1987)

Solution. As seen from Example 34.22 above,

(a) $s_f = 0.01$ or **1** %

(b) From the same example it is seen that at maximum torque, $a = s_b = 0.04$ or **4** %

(c) As seen from Art. 34.40.

$$\frac{I_{2st}}{I_{2f}} = \sqrt{\frac{T_{st}}{s_f . T_f}} = \sqrt{\frac{1.6}{0.01}} = 12.65$$

$\therefore$ Starting rotor current = 12.65 × F.L. rotor current

34.43. Sector Induction Motor

Consider a standard 3-phase, 4-pole, 50-Hz, Y-connected induction motor. Obviously, its N_s = 1500 rpm. Suppose we cut the stator in half *i.e.* we remove half the stator winding with the result that only two complete *N* and *S* poles are left behind. Next, let us star the three phases without making any other changes in the existing connections. Finally, let us mount the original rotor above this ***sector stator*** leaving a small air-gap between the two. When this stator is energised from a 3-phase 50-Hz source, the rotor is found to run at almost 1500 rpm. In order to prevent saturation, the stator voltage should be reduced to half its original value because the sector stator winding has only half the original number of turns. It is found that under these conditions, this half-truncated sector motor still develops about 30% of its original rated power.

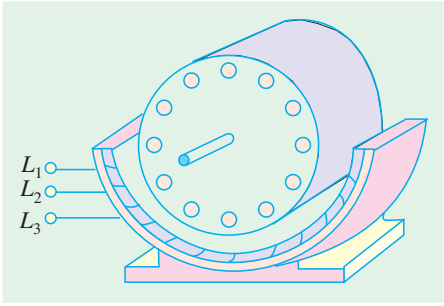


Fig. 34.41

The stator flux of the sector motor revolves at the same peripheral speed as the flux in the original motor. But instead of making a complete round, the flux in the sector motor simply travels ***continuously from one end of the stator to the other***.

34.44. Linear Induction Motor

If, in a sector motor, the sector is laid out flat and a flat squirrel-cage winding is brought near to it, we get a linear induction motor (Fig. 34.42). In practice, instead of a flat squirrel-cage winding, an aluminium or copper or iron plate is used as a 'rotor'. The flat stator produces a flux that moves in a straight line from its one end to the other at a linear synchronous speed given by

$$\upsilon_s = 2\,w.\,f.$$

where υ_s = linear synchronous speed (m/s)
w = width of one pole-pitch (m)
f = supply frequency (Hz)

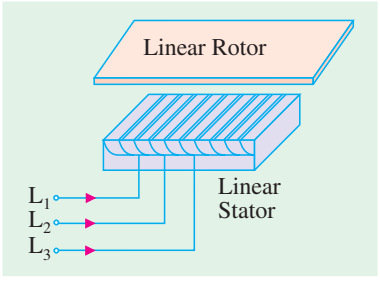


Fig. 34.42

It is worth noting that speed ***does not depend on the number of poles***, but only on the pole-pitch and stator supply frequency. As the flux moves linearly, it drags the rotor plate along with it in the ***same direction***. However, in many practical applications, the 'rotor' is stationary, while the stator moves. For example, in high-speed trains, which utilize magnetic levitation (Art. 34.46), the rotor is composed of thick aluminium plate that is fixed to the ground and extends over the full length of the track. The linear stator is bolted to the undercarriage of the train.

34.45. Properties of a Linear Induction Motor

These properties are almost identical to those of a standard rotating machine.

1. ***Synchronous speed***. It is given by $\upsilon_s = 2wf$
2. ***Slip***. It is given by $s = (v_s - v) / v_s$

where v is the actual speed.

3. ***Thrust or Force***. It is given by $F = P_2/v_s$

where P_2 is the active power supplied to the rotor.

4. ***Active Power Flow***. It is similar to that in a rotating motor.

(*i*) $P_{cr} = sP_2$ and (*ii*) $P_m = (1 - s)\,P_2$

Example 34.52. *An electric train, driven by a linear motor, moves with 200 km/h, when stator frequency is 100 Hz. Assuming negligible slip, calculate the pole-pitch of the linear motor.*

Solution. $\upsilon_s = 2\,\omega f \qquad \omega = \dfrac{(200 \times 5/18)}{2 \times 100} =$ **277.8 mm**

Example 34.53. *An overhead crane in a factory is driven horizontally by means of two similar linear induction motors, whose 'rotors' are the two steel I-beams, on which the crane rolls. The 3-phase, 4-pole linear stators, which are mounted on opposite sides of the crane, have a pole-pitch of 6 mm and are energised by a variable-frequency electronic source. When one of the motors was tested, it yielded the following results:*

Stator frequency = 25 Hz; Power to stator = 6 kW
Stator Cu and iron loss = 1.2 kW; crane speed = 2.4 m/s

Calculate ***(i)*** *synchronous speed and slip* ***(ii)*** *power input to rotor* ***(iii)*** *Cu losses in the rotor* ***(iv)*** *gross mechanical power developed and* ***(v)*** *thrust.*

Solution. (*i*) $v_s = 2\,\omega f = 2 \times 0.06 \times 25 =$ **3 m/s**

$s = (\upsilon_s - \upsilon)/\upsilon_s = (3 - 2.4)/3 = 0.2$ or **20%**

(*ii*) $P_2 = 6 - 1.2 =$ **4.8 kW**

(*iii*) $P_{cr} = sP_2 = 0.2 \times 4.8 =$ **0.96 kW**

(*iv*) $P_m = P_2 - P_{cr} = 4.8 - 0.96 =$ **3.84 kW**

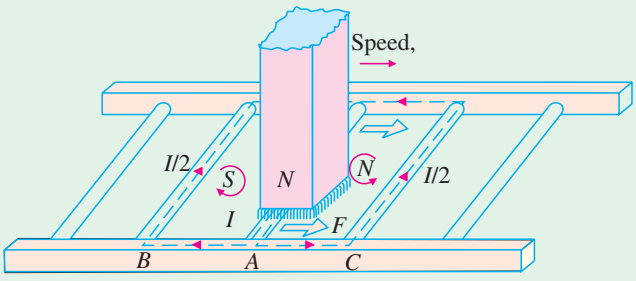
(*v*) $F = P_2 / \upsilon_s = 4.8 \times 10^3/3 = 1600$ N = **1.60 kN**

34.46. Magnetic Levitation

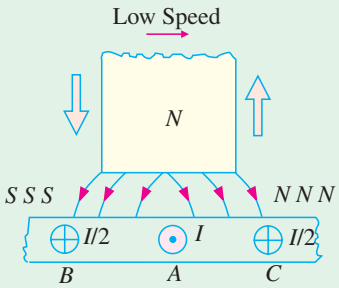
As shown in Fig. 34.43 (*a*), when a moving permanent magnet sweeps across a conducting ladder, it tends to drag the ladder along with, because it applies a horizontal tractive force $F = BIl$. It will now be shown that this horizontal force is also accompanied by a ***vertical*** force (particularly, at high magnet speeds), which tends to push the magnet away from the ladder in the upward direction.



Magnetic levitation



(*a*)



(*b*) Gateway

Fig. 34.43

A portion of the conducting ladder of Fig. 34.43 (*a*) has been shown in Fig. 34.43 (*b*). The voltage induced in conductor (or bar) *A* is maximum because flux is greatest at the centre of the *N* pole. If the magnet speed is very low, the induced current reaches its maximum value in *A* at virtually the ***same time*** (because delay due to conductor inductance is negligible). As this current flows via conductors *B* and *C*, it produces induced *SSS* and *NNN* poles, as shown. Consequently, the front half of

the magnet is pushed upwards while the rear half is pulled downwards. Since distribution of *SSS* and *NNN* pole is symmetrical with respect to the centre of the magnet, the vertical forces of attraction and repulsion, being equal and opposite, cancel each other out, leaving behind only horizontal tractive force.

Now, consider the case when the magnet sweeps over the conductor *A* with a *very high speed*, as shown in Fig. 34.44. Due to conductor inductance, current in A reaches its maximum value a fraction of a second (Δt) after voltage reaches its maximum value. Hence, by the time *I* in conductor *A* reaches its maximum value, the centre of the magnet is already ahead of *A* by a distance $= \upsilon . \Delta t$ where υ is the magnet velocity. The induced poles *SSS* and *NNN* are produced, as before, by the currents returning via conductors *B* and *C* respectively. But, by now, the *N* pole of the permanent magnet lies over the induced *NNN* pole, which pushes it upwards with a strong vertical force. * This forms the basis of ***magnetic levitation*** which literally means 'floating in air'.

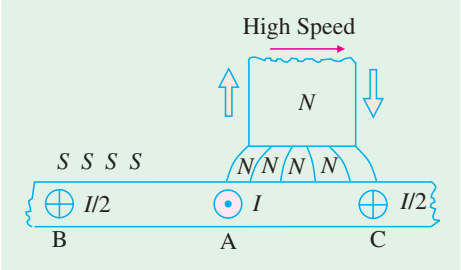


Fig. 34.44

Magnetic levitation is being used in ultrahigh speed trains (upto 300 km/h) which float in the air about 100 mm to 300 mm above the metallic track. They do not have any wheels and do not require the traditional steel rail. A powerful electromagnet (whose coils are cooled to about 4 °K by liquid helium) fixed underneath the train moves across the conducting rail, thereby inducing current in the rail. This gives rise to vertical force (called force of levitation) which keeps the train pushed up in the air above the track. Linear motors are used to propel the train.

A similar magnetic levitation system of transit is being considered for connecting Vivek Vihar in East Delhi to Vikaspuri in West Delhi. The system popularly known as Magneto-Bahn (*M*-Bahn) completely eliminates the centuries-old 'steel-wheel-over steel rail' traction. The *M*-Bahn train floats in the air through the principle of magnetic levitation and propulsion is by linear induction motors. There is 50% decrease in the train weight and 60% reduction in energy consumption for propulsion purposes. The system is extraordinarily safe (even during an earthquake) and the operation is fully automatic and computer-based.

Tutorial Problem No. 34.3

1. A 500-V, 50-Hz, 3-phase induction motor develops 14.92 kW inclusive of mechanical losses when running at 995 r.p.m., the power factor being 0.87. Calculate (*a*) the slip (*b*) the rotor Cu losses (*c*) total input if the stator losses are 1,500 W (*d*) line current (*e*) number of cycles per minute of the rotor e.m.f. **[(*a*) 0.005 (*b*) 75 W (*c*) 16.5 kW (*d*) 22 A (*e*) 15] (*City & Guilds, London*)**
2. The power input to a 3-phase induction motor is 40 kW. The stator losses total 1 kW and the friction and winding losses total 2 kW. If the slip of the motor is 4%, find (*a*) the mechanical power output (*b*) the rotor Cu loss per phase and (*c*) the efficiency. **[(*a*) 37.74 kW (*b*) 0.42 kW (*c*) 89.4%]**
3. The rotor e.m.f. of a 3-phase, 440-V, 4-pole, 50-Hz induction motor makes 84 complete cycles per minute when the shaft torque is 203.5 newton-metres. Calculate the h.p. of the motor. **[41.6 h.p. (31.03 kW)] (*City & Guilds, London*)**
4. The input to a 3-phase induction motor, is 65 kW and the stator loss is 1 kW. Find the total mechanical power developed and the rotor copper loss per phase if the slip is 3%. Calculate also in

* The induced current is always delayed (even at low magnet speeds) by an interval of time Δt which depends on the L/R time-constant of the conductor circuit. This delay is so brief at slow speed that voltage and the current reach their maximum value virtually at the same *time* and *place*. But at high speed, the same delay Δt is sufficient to produce large shift *in space between* the points where the voltage and current achieve their maximum values.

terms of the mechanical power developed the input to the rotor when the motor yields full-load torque at half speed. **[83.2 h.p. (62.067 kW) : 640 W, Double the output] (*City & Guilds. London*)**

5. A 6-pole, 50-Hz, 8-phase induction motor, running on full-load, develops a useful torque of 162 N-m and it is observed that the rotor electromotive force makes 90 complete cycles per min. Calculate the shaft output. If the mechanical torque lost in friction be 13.5 Nm, find the copper loss in the rotor windings, the input to the motor and the efficiency. Stator losses total 750 W.

[16.49 kW; 550 W; 19.2 kW; 86%]

6. The power input to a 500-V, 50-Hz, 6-pole, 3-phase induction motor running at 975 rpm is 40 kW. The stator losses are 1 kW and the friction and windage losses total 2 kW. Calculate (*a*) the slip (*b*) the rotor copper loss (*c*) shaft output (*d*) the efficiency. **[(*a*) 0.025 (*b*) 975 W (*c*) 36.1 kW (*d*) 90%]**

7. A 6-pole, 3-phase induction motor develops a power of 22.38kW, including mechanical losses which total 1.492 kW at a speed of 950 rpm on 550-V, 50-Hz mains. The power factor is 0.88. Calculate for this load (*a*) the slip (*b*) the rotor copper loss (*c*) the total input if the stator losses are 2000 W (*d*) the efficiency (*e*) the line current (*f*) the number of complete cycles of the rotor electromotive force per minute. **[(*a*) 0.05 (*b*) 1175 (*c*) 25.6 kW (*d*) 82% (*e*) 30.4 A (*f*) 150]**

8. A 3-phase induction motor has a 4-pole, star-connected stator winding. The motor runs on a 50-Hz supply with 200 V between lines. The rotor resistance and standstill reactance per phase are 0.1 Ω and 0.9 Ω respectively. The ratio of rotor to stator turns is 0.67. Calculate (*a*) total torque at 4% slip (*b*) total mechanical power at 4% slip (*c*) maximum torque (*d*) speed at maximum torque (*e*) maximum mechanical power. Prove the formulae employed, neglecting stator impedance.

[(*a*) 40 Nm (*b*) 6 kW (*c*) 63.7 Nm (*d*) 1335 rpm (*e*) 8.952 kW.]

9. A 3-phase induction motor has a 4-pole, star-connected, stator winding and runs on a 220-V, 50-Hz supply. The rotor resistance is 0.1 Ω and reactance 0.9. The ratio of stator to rotor turns is 1.75. The full load slip is 5%. Calculate for this load (*a*) the total torque (*b*) the shaft output. Find also (*c*) the maximum torque (*d*) the speed at maximum torque.

[(*a*) 42 Nm (*b*) 6.266 kW (*c*) 56 Nm (*d*) 1330 rpm]

10. A 3000-V, 24-pole, 50-Hz 3-phase, star-connected induction motor has a slip-ring rotor of resistance 0.016 Ω and standstill reactance 0.265 Ω per phase. Full-load torque is obtained at a speed of 247 rpm.

Calculate (*a*) the ratio of maximum to full-load torque (*b*) the speed at maximum torque. Neglect stator impedance. **[(*a*) 2.61 (*b*) 235 rpm]**

11. The rotor resistance and standstill reactance of a 3-phase induction motor are respectively 0.015 Ω and 0.09 Ω per phase. At normal voltage, the full-load slip is 3%. Estimate the percentage reduction in stator voltage to develop full-load torque at one-half of full-load speed. What is then the power factor ? **[22.5%; 0.31]**

12. The power input to a 3-phase, 50-Hz induction motor is 60 kW. The total stator loss is 1000 W. Find the total mechanical power developed and rotor copper loss if it is observed that the rotor e.m.f. makes 120 complete cycles per minute.

[56.64 kW; 2.36 kW] (*AMIE Sec. B Elect. Machine (E-3) Summer 1990*)

13. A balanced three phase induction motor has an efficiency of 0.85 when its output is 44.76 kW. At this load both the stator copper loss and the rotor copper loss are equal to the core losses. The mechanical losses are one-fourth of the no-load loss. Calculate the slip.

[4.94%] (*AMIE Sec. B Elect. Machines (E-3) Winter 1991*)

14. An induction motor is running at 20% slip, the output is 36.775 kW and the total mechanical losses are 1500 W. Estimate Cu losses in the rotor circuit. If the stator losses are 3 kW, estimate efficiency of the motor. **[9,569 W, 72.35%] (*Electrical Engineering-II, Bombay Univ. 1978*)**

15. A 3- ϕ, 50-Hz, 500-V, 6-pole induction motor gives an output of 37.3 kW at 955 r.p.m. The power factor is 0.86, frictional and windage losses total 1.492 kW; stator losses amount to 1.5 kW. Determine (*i*) line current (*ii*) the rotor Cu loss for this load.

[(*i*) 56.54 A (*ii*) 88.6% (*iii*) 1.828 kW] (*Electrical Technology, Kerala Univ. 1977*)

16. Determine the efficiency and the output horse-power of a 3-phase, 400-V induction motor running

on load with a slip of 4 per cent and taking a current of 50 A at a power factor of 0.86. When running light at 400 V, the motor has an input current of 15 A and the power taken is 2,000 W, of which 650 W represent the friction, windage and rotor core loss. The resistance per phase of the stator winding (delta-connected) is 0.5 Ω.

[85.8 per cent; 34.2 h.p. (25.51 kW)] (*Electrical Engineering-II, M.S. Univ. Baroda 1977*)

17. The power input to the rotor of a 440-V, 50-Hz, 3-phase, 6-pole induction motor is 60 kW. It is observed that the rotor e.m.f. makes 90 complete cycles per minute. Calculate (*a*) the slip (*b*) rotor speed (*c*) rotor Cu loss per phase (*c*) the mechanical power developed and (*e*) the rotor resistance per phase if rotor current is 60 A. **[(*a*) 0.03 (*b*) 970 r.p.m. (*c*) 600 W (*d*) 58.2 kW (*e*) 0.167 Ω]**

18. An induction motor is running at 50% of the synchronous speed with a useful output of 41.03 kW and the mechanical losses total 1.492 kW. Estimate the Cu loss in the rotor circuit of the motor. If the stator losses total 3.5 kW, at what efficiency is the motor working ?

[42.52 kW; 46.34%] (*Electrical Engineering-II, Bombay Univ. 1975*)

19. Plot the torque/speed curve of a 6-pole, 50-Hz, 3-phase induction motor. The rotor resistance and reactance per phase are 0.02 Ω and 0.1 Ω respectively. At what speed is the torque a maximum? What must be the value of the external rotor resistance per phase to give two-third of maximum torque at starting ? **[(*a*) 800 rpm (*b*) 0.242 Ω or 0.018 Ω]**

34.47. Induction Motor as a Generalized Transformer

The transfer of energy from stator to the rotor of an induction motor takes place entirely *inductively*, with the help of a flux mutually linking the two. Hence, an induction motor is essentially a transformer with stator forming the primary and rotor forming (the short-circuited) rotating secondary (Fig. 34.45). The vector diagram is similar to that of a transformer (Art. 32.15).

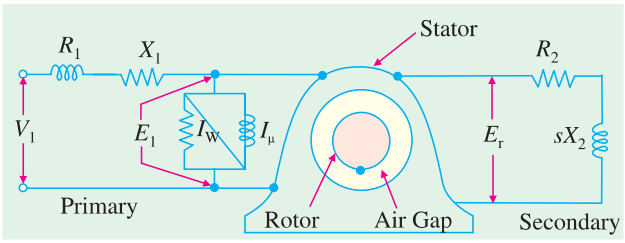


Fig. 34.45

In the vector diagram of Fig. 34.46, V_1 is the applied voltage per stator phase, R_1 and X_1 are stator resistance and leakage reactance per phase respectively, shown external to the stator winding in Fig. 34.45. The applied voltage V_1 produces a magnetic flux which links both primary and secondary thereby producing a counter e.m.f of self-induction E_1 in primary (*i.e.* stator) and a mutually-induced e.m.f. E_r (= sE_2) in secondary (*i.e.* rotor). There is no secondary terminal voltage V_2 in secondary because whole of the induced e.m.f. E_r is used up in circulating the rotor current as the rotor is closed upon itself (which is equivalent to its being short-circuited).

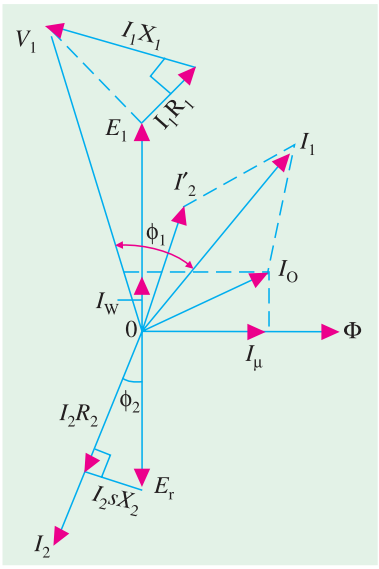


Fig. 34.46

Obviously $\mathbf{V_1} = \mathbf{E_1} + I_1R_1 + j\,I_1X_1$

The magnitude of $\mathbf{E_r}$ depends on voltage transformation ratio K between stator and rotor and the slip. As it is wholly absorbed in the rotor impedance.

$\therefore \quad \mathbf{E_r} = \mathbf{I_2Z_2} = I_2\,(R_2 + jsX_2)$

In the vector diagram, I_0 is the no-load primary current. It has two components ***(i)*** the working or iron loss components I_w which supplies the no-load motor losses and ***(ii)*** the magnetising component I_μ which sets up magnetic flux in the core and the air-gap.

Obviously $I_0 = \sqrt{\left(I_{\omega^2} + I_{\mu^2}\right)}\, I_n$ Fig. 34.45, I_ω and I_μ are taken care of by an exciting circuit containing $R_0 = E_1/I_\omega$ and $X_0 = E_1/I_\mu$ respectively.

It should be noted here, in passing that in the usual two-winding transformer, I_0 is quite small (about **1 %** of the full-load current). The reason is that the magnetic flux path lies almost completely in the steel core of low reluctance, hence I_μ is small, with the result that I_0 is small But in an induction motor, the presence of an air-gap (of high reluctance) necessitates a large I_μ hence I_0 is very large (approximately 40 to 50% of the full-load current).

In the vector diagram, I_2' is the equivalent load current in primary and is equal to KI_2. Total primary current is the ***vector*** sum of I_0 and I_2'.

At this place, a few words may be said to justify the representation of the stator and rotor quantities on the same vector diagram, even though the frequency of rotor current and e.m.f. is only a fraction of that of the stator. We will now show that even though the frequencies of stator and rotor currents are different, yet magnetic fields due to them are synchronous with each other, when seen by an observer stationed in space—both fields rotate at synchronous speed N_s (Art. 34.11).

The current flowing in the short-circuited rotor produces a magnetic field, which revolves round the rotor in the same direction as the stator field. The speed of rotation of the rotor field is

$$= \frac{120\, f_r}{P} = \frac{120\, sf}{P} = sN_s = N_s \times \frac{N_S - N}{N_s} = (N_S - N)$$

Rotor speed $N = (1 - s)\, N_s$

Hence, speed of the rotating field of the rotor with respect to the stationary stator or space is

$$= sN_S + N = (N_S - N) + N = N_s$$

34.48. Rotor Output

Primary current I_1 consists of two parts, I_0 and I_2'. It is the latter which is transferred to the rotor, because I_0 is used in meeting the Cu and iron losses in the stator itself. Out of the applied primary voltage V_1, some is absorbed in the primary itself (= I_1Z_1) and the remaining E_1 is transferred to the rotor. If the angle between E_1 and I_2' is ϕ, then

Rotor input / phase $= E_1 I_2' \cos\phi$; Total rotor input $= 3E_1 I_2' \cos\phi$

The electrical input to the rotor which is wasted in the form of heat is

$$= 3I_2\, E_r \cos\phi \qquad (\text{or} = 3\, I_2^{\,2}\, R_2)$$

Now $I_2' = KI_2$ or $I_2 = I_2'/K$

$E_r = s\, E_2$ or $E_2 = KE_1$

$\therefore \quad E_r = sKE_1$

$\therefore$ electrical input wasted as heat

$$= 3 \times (I_2'/K) \times sK\, E_1 \times \cos\phi = 3E_1 I'_2 \cos\phi \times s = \text{rotor input} \times s$$

Now, rotor output = rotor input – losses $= 3E_1\, I_2' \cos\phi - 3E_1 I_2' \cos\phi \times s$

$= (1 - s)\, 3E_1 I_2' \cos\phi = (1 - s) \times$ rotor input

$$\therefore \quad \frac{\text{rotor output}}{\text{rotor input}} = 1 - s \quad \therefore \quad \text{rotor Cu loss} = s \times \text{rotor input}$$

$$\text{rotor efficiency} = 1 - s = \frac{N}{N_S} = \frac{\text{actual speed}}{\text{synchronous speed}}$$

In the same way, other relation similar to those derived in Art. 34.37 can be found.

34.49. Equivalent Circuit of the Rotor

When motor is loaded, the rotor current I_2 is given by

$$I_2 = s\frac{E_2}{\sqrt{\left[R_2^2 + (sX_2)^2\right]}} = \frac{E_2}{\sqrt{\left[(R_2/s)^2 + X_2^2\right]}}$$

From the above relation it appears that the rotor circuit which actually consists of a fixed resistance R_2 and a variable reactance sX_2 (proportional to slip) connected across $E_r = sE_2$ [Fig. 34.47 (*a*)] can be looked upon as equivalent to a rotor circuit having a fixed reactance X_2 connected in series with a variable resistance R_2/s (inversely proportional to slip) and supplied with constant voltage E_2, as shown in Fig. 34.47 (*b*).

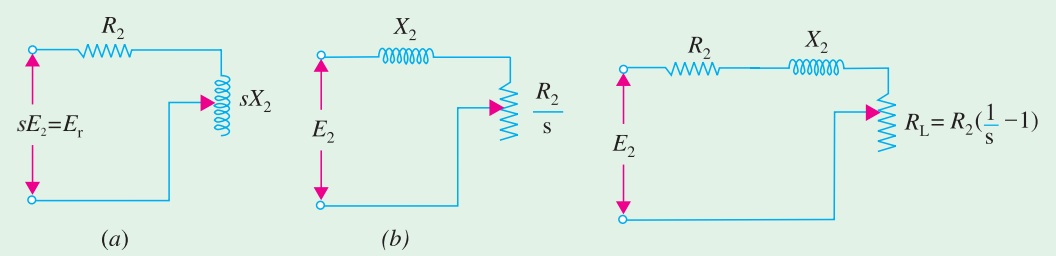


Fig. 34.47

Fig. 34.48

Now, the resistance $\frac{R_2}{s} = R_2 + R_2\left(\frac{1}{s} - 1\right)$. It consists of two parts :

(i) the first part R_2 is the rotor resistance itself and represents the rotor Cu loss.

(ii) the second part is $R_2\left(\frac{1}{s} - 1\right)$

This is known as the load resistance R_L and is the electrical equivalent of the mechanical load on the motor. In other words, the mechanical load on an induction motor can be represented by a non-inductive resistance of the value $R_2\left(\frac{1}{s} - 1\right)$. The equivalent rotor circuit along with the load resistance R_L may be drawn as in Fig. 34.48.

34.50. Equivalent Circuit of an Induction Motor

As in the case of a transformer (Fig. 32.14), in this case also, the secondary values may be transferred to the primary and *vice versa*. As before, it should be remembered that when shifting impedance or resistance from secondary to primary, it should be ***divided*** by K^2 whereas current should be ***multiplied*** by K. The equivalent circuit of an induction motor where all values have been referred to primary *i.e.* stator is shown in Fig. 34.49.

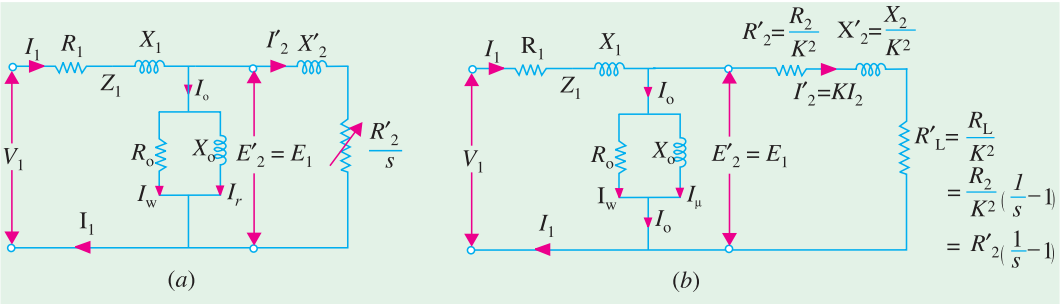


Fig. 34.49

As shown in Fig. 34.50, the exciting circuit may be transferred to the left, because inaccuracy involved is negligible but the circuit and hence the calculations are very much simplified. This is

known as the ***approximate equivalent*** circuit of the induction motor.

If transformation ratio is assumed unity *i.e.* $E_2/E_1 = 1$, then the equivalent circuit is as shown in Fig. 34.51 instead of that in Fig. 34.49.

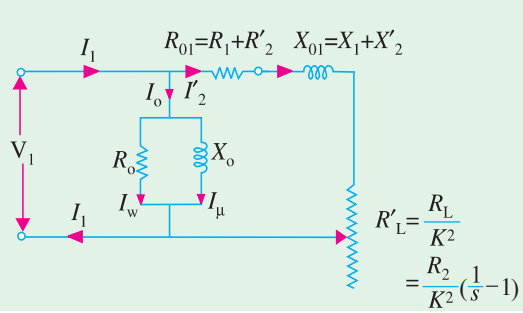


Fig. 34.50

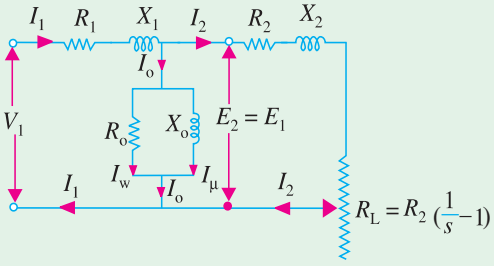


Fig. 34.51

34.51. Power Balance Equations

With reference to Fig. 34.49 (*a*), following power relations in an induction motor can be deduced:

Input power = $3V_1 I_1 \cos\phi_1$; stator core loss = $I_\omega^{\,2} R_0$; stator Cu loss = $3 I_1^{\,2} R_1$

Power transferred to rotor = $3 I_2'^2 R_2'/s$; Rotor Cu loss = $3 I_2'^2 R_2'$

Mechanical power developed by rotor (P_m) or gross power developed by rotor (P_g)

= rotor input – rotor Cu losses

$$= 3 I_2'^2 R_2' / s - 3 I_2'^2 R_2' = 3 I_2'^2 R_2' \left(\frac{1-s}{s}\right) \text{ Watt}$$

If T_g is the gross torque* developed by the rotor, then

$$T_g \times \omega = T_g \times 2\pi \frac{N}{60} = 3 I_2'^2 R_2' \left(\frac{1-s}{s}\right)$$

$$\therefore \quad T_g = \frac{3 I_2'^2 R_2' \left(\frac{1-s}{s}\right)}{2\pi N/60} \text{ N-m}$$

Now, $N = N_S(1-s)$. Hence gross torque becomes

$$T_g = \frac{3 I_2'^2 R_2'/s}{2\pi N_s/60} \text{ N-m} = 9.55 \times \frac{3 I_2'^2 R_2'/s}{N_s} \text{ N-m}$$

Since gross torque in ***synchronous watts*** is equal to the power transferred to the rotor across the air-gap.

$$\therefore \quad T_g = 3 I_2'^2 R_2'/s \text{ synch. watt.}$$

It is seen from the approximate circuit of Fig. 34.50 that

$$I_2' = \frac{V_1}{(R_1 + R_2'/s) + j\,(X_1 + X_2')}$$

$$T_g = \frac{3}{2\pi N_s/60} \times \frac{V_1^2}{(R_1 + R_2'/s)^2 + (X_1 + X_2')^2} \times \frac{R_2'}{s} \text{ N – m}$$

* It is different from shaft torque, which is less than T_g by the torque required to meet windage and frictional losses.

34.52. Maximum Power Output

Fig. 34.52 shows the approximate equivalent circuit of an induction motor with the simplification that :

(i) exciting circuit is omitted *i.e.* I_0 is neglected and

(ii) K is assumed unity.

As seen, gross power output for 3-phase induction motor is

$$P_g = 3 I_1^2 R_L$$

Now, $$I_1 = \frac{V_1}{\sqrt{\left[\left(R_{01} + R_L\right)^2 + X_{01}^2\right]}} \qquad \therefore \quad P_g = \frac{3V_1^2 R_L}{\left(R_{01} + R_L\right)^2 + X_{01}^2}$$

The condition for maximum power output can be found by differentiating the above equation and by equating the first derivative to zero. If it is done, it will be found that

$$R_L^2 = R_{01}^2 + X_{01}^2 = Z_{01}^2$$

where Z_{01} = leakage impedance of the motor as referred to primary

$$\therefore \qquad R_L = Z_{01}$$

Hence, the power output is maximum ***when the equivalent load resistance is equal to the stand-still leakage impedance of the motor.***

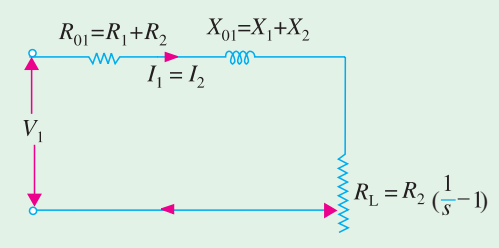


Fig. 34.52

34.53. Corresponding Slip

Now $$R_L = R_2[(1/s) - 1] \qquad \therefore \quad Z_{01} = R_L = R_2\,[(1/s) - 1] \text{ or } s = \frac{R_2}{R_2 + Z_{01}}$$

This is the slip corresponding to maximum gross power output. The vlaue of P_{gmax} is obtained by substituting R_L by Z_{01} in the above equation.

$$\therefore \qquad P_{gmax} = \frac{3V_1^2 Z_{01}}{\left(R_{01} + Z_{01}\right)^2 + X_{01}^2} = \frac{3V_1^2 Z_{01}}{R_{01}^2 + Z_{01}^2 + 2R_{01} Z_{01} + X_{01}^2} = \frac{3 V_1^2}{2\,(R_{01} + Z_{01})}$$

It should be noted that V_1 is voltage/phase of the motor and K has been taken as unity.

Example 34.54. *The maximum torque of a 3-phase induction motor occurs at a slip of 12%. The motor has an equivalent secondary resistance of 0.08 Ω/phase. Calculate the equivalent load resistance R_L, the equivalent load voltage V_L and the current at this slip if the gross power output is 9,000 watts.*

Solution. $R_L = R_2[\,(1/s) - 1] = 0.08\,[\,(1/0.12) - 1\,] =$ **0.587 Ω /phase.**

As shown in the equivalent circuit of the rotor in Fig. 34.53, V is a fictitious voltage drop equivalent to that consumed in the load connected to the secondary *i.e.* rotor. The value of $V = I_2 R_L$

Now, gross power $P_g = 3I_2^2 R_L = 3V^2/R_L$

$$V = \sqrt{(P_g \times R_L / 3)}$$

$$= \sqrt{(0.587 \times 9000 / 3)} = \textbf{42 V}$$

Equivalent load current $= V/R_L = 42/0.587 =$ **71.6A**

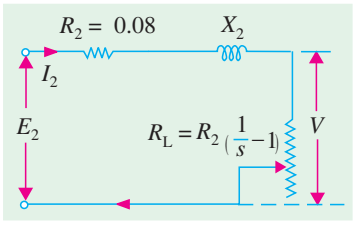


Fig. 34.53

Example 34.55. *A 3-phase, star-connected 400 V, 50-Hz, 4-pole induction motor has the following per phase parameters in ohms, referred to the stators.*

$$R_1 = 0.15,\ X_1 = 0.45,\ R_2' = 0.12,\ X_2' = 0.45,\ X_m = 28.5$$

Compute the stator current and power factor when the motor is operated at rated voltage and frequency with s = 0.04. **(Elect. Machines, A.M.I.E. Sec. B, 1990)**

Solution. The equivalent circuit with all values referred to stator is shown in Fig. 34.54.

$$R_L' = R_2'\left(\frac{1}{s} - 1\right)$$

$$= 0.12\left(\frac{1}{0.04} - 1\right) = 2.88\ \Omega$$

$$I_2' = \frac{V}{(R_{01} + R_L') + j\,X_{01}}$$

$$= \frac{400/\sqrt{3}}{(0.15 + 0.12 + 2.88) + j(0.45 + 0.45)}$$

$$= 67.78 - j\,19.36$$

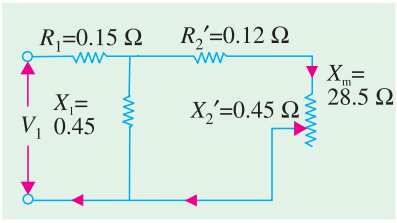


Fig. 34.54

$$I_0 = \frac{400/\sqrt{3}}{X_m} = \frac{400}{\sqrt{3} \times j\ 28.5} = -j\ 8.1$$

Stator current, $I_1 = I_0 + I_2' = 67.78 - j\ 19.36 - j\ 8.1 =$ **$73.13 \angle -22°$**

p.f. $= \cos\phi = \cos 22° = 0.927$ (lag)

Example 34.56. *A 220-V, 3-ϕ, 4-pole, 50-Hz, Y-connected induction motor is rated 3.73 kW. The equivalent circuit parameters are:*

$$R_1 = 0.45\ \Omega,\ X_1 = 0.8\ \Omega;\ R_2' = 0.4\ \Omega,\ X_2' = 0.8\ \Omega,\ B_0 = -1/30\ \text{mho}$$

The stator core loss is 50 W and rotational loss is 150 W. For a slip of 0.04, find ***(i)*** *input current* ***(ii)*** *p.f.* ***(iii)*** *air-gap power* ***(iv)*** *mechanical power* ***(v)*** *electro-magnetic torque* ***(vi)*** *output power and* ***(vii)*** *efficiency.*

Solution. The exact equivalent circuit is shown in Fig. 34.55. Since R_0 (or G_0) is negligible in determining I_1, we will consider B_0 (or X_0) only

$$Z_{AB} = \frac{j\,X_m\,[(R_2'/s) + j\,X_2']}{(R_2'/s) + j\,(X_2' + X_m)} = \frac{j\,30\,(10 + j\,0.8)}{10 + j\,30.8} = 8.58 + j\,3.56 = 9.29 \angle 22.5°$$

$$\mathbf{Z}_{01} = \mathbf{Z}_1 + \mathbf{Z}_{AB} = (0.45 + j\,0.8) + (8.58 + j\,3.56) = 9.03 + j\,4.36 = 10 \angle 25.8°$$

$$V_{ph} = \frac{220}{\sqrt{3}} \angle 0° = 127 \angle 0°$$

(i) $\therefore\ \mathbf{I}_1 = V_1 / Z_{01} = 127 \angle 0°/10 \angle 25.8°$

$= \mathbf{12.7 \angle -25.8°\ A}$

(ii) p.f. $= \cos 25.8° =$ **0.9**

(iii) air-gap power, $P_2 = 3\,I_2'^2\,(R_2'/s) = 3I_1^2 R_{AB}$

$= 3 \times 12.7^2 \times 8.58 =$ **4152 W**

(iv) $P_m = (1 - s)\,P_2 = 0.96 \times 4152 =$ **3,986 W**

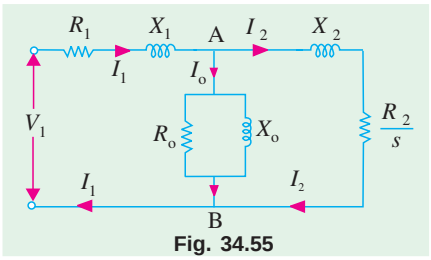


Fig. 34.55

(v) Electromagnetic torque (*i.e.* gross torque)

$$T_g = \frac{P_m}{2\pi N/60} = 9.55\,\frac{P_m}{N}\ \text{N–m}$$

Now, $N_s = 1500$ r.p.m., $N = 1500\ (1 - 0.04) = 1440$ r.p.m.

$$T_g = 9.55 \times 3986/1440 = \mathbf{26.4\ N\text{-}m}$$

$$(\text{or } T_g = 9.55\frac{P_2}{N_s} = 9.55 \times \frac{4152}{1500} = \mathbf{26.4\ N\text{-}m}$$

(vi) output power = $3986 - 150 = \mathbf{3836\ W}$

(vii) stator core loss = 50 W; stator Cu loss = $3 I_1^2 R_1 = 3 \times 12.7^2 \times 0.45 = 218$ W

Rotor Cu loss = $3 I_2'^2 R_2' = sP_2 = 0.04 \times 4152 = 166$ W; Rotational losses = 150 W

Total loss = 50 + 218 + 166 + 150 = 584 W; $\eta = 3836/(3836 + 584) = 0.868$ or **86.8%**

Example 34.57. *A 440-V, 3-ϕ 50-Hz, 37.3 kW, Y-connected induction motor has the following parameters:*

$$R_1 = 0.1\ \Omega, X_1 = 0.4\ \Omega, R_2' = 0.15\ \Omega, X_2' = 0.44\ \Omega$$

*Motor has stator core loss of 1250 W and rotational loss of 1000 W. It draws a no-load line current of 20 A at a p.f. of 0.09 (lag). When motor operates at a slip of 3%, calculate **(i)** input line current and p.f. **(ii)** electromagnetic torque developed in N-m **(iii)** output and **(iv)** efficiency of the motor.* **(Elect. Machines -II, Nagpur Univ. 1992)**

Solution. The equivalent circuit of the motor is shown in Fig. 34.49 (*a*). Applied voltage per phase = $440/\sqrt{3} = 254$ V.

$$I_2' = \frac{V_1}{(R_1 + R_2'/s) + j(X_1 + X_2')} = \frac{254 \angle 0°}{(0.1 + 0.15/0.03) + j(0.4 + 0.44)}$$

$$= \frac{254 \angle 0°}{5.1 + j\,0.84} = \frac{254 \angle 0°}{5.17 \angle 9.3°} = 49.1 \angle -9.3° = 48.4 - j\,7.9$$

For all practical purposes, no-load motor current may be taken as equal to magnetising current I_0. Hence, $I_0 = 20 \angle -84.9° = 1.78 - j\,19.9$.

(i) $\mathbf{I_1 = I_0 + I_2'} = (48.4 - j\,7.9) + (1.78 - j\,19.9) = 50.2 - j\,27.8 = \mathbf{57.4 \angle -29°}$

$\therefore$ p.f. = $\cos 29° = \mathbf{0.875\ (lag).}$

(ii) $P_2 = 3 I_2'^2 (R_2'/s) = 3 \times 49.1^2 \times (0.15/0.03) = 36{,}160$ W

$N_s = 1500$ r.p.m.

$\therefore$ $T_g = 9.55 \times 36{,}160/1500 = \mathbf{230\ N\text{-}m}$

(iii) $P_m = (1 - s) P_2 = 0.97 \times 36{,}160 = 35{,}075$ W

Output power = $35{,}075 - 1000 = \mathbf{34{,}075\ W}$

Obviously, motor is delivering less than its rated output at this slip.

(iv) Let us total up the losses.

Core loss = 1250 W, stator Cu loss = $3 I_1^2 R_1 = 3 \times 57.4^2 \times 0.1 = 988$ W

Rotor Cu loss = $3 I_2'^2 R_2' = sP_2 = 0.03 \times 36{,}160 = 1085$ W

rotational *i.e.* friction and windage losses = 1000 W

Total losses = 1250 + 988 + 1085 + 1000 = 4323 W

$\eta = 34{,}075/(34{,}075 + 4323) = 0.887$ or **88.7%**

or input = $\sqrt{3} \times 440 \times 57.4 \times 0.875 = 38{,}275$ W

$\therefore$ $\eta = 1 - (4323/38{,}275) = 0.887$ or **88.7%**

Example 34.58. *A 400 V, 3-ϕ, star-connected induction motor has a stator exciting impedance of (0.06 + j 0.2) Ω and an equivalent rotor impedance of (0.06 + j 0.22) Ω. Neglecting exciting current, find the maximum gross power and the slip at which it occurs.* **(Elect. Engg.-II, Bombay Univ. 1987)**

Solution. The equivalent circuit is shown in Fig. 34.56.

$R_{01} = R_1 + R_2' = 0.06 + 0.06 = 0.12\ \Omega$

$X_{01} = X_1 + X_2' = 0.2 + 0.22 = 0.42\ \Omega$

$\therefore \quad Z_{01} = \sqrt{(0.12^3 + 0.42^2)} = 0.44\ \Omega$

As shown in Art. 34.53, slip corresponding to maximum gross power output is given by

$$s = \frac{R_2}{R_2 + Z_{01}} = \frac{0.06}{0.06 + 0.44}$$

$= 0.12$ or **12%**

Voltage/phase, $V_1 = 400/\sqrt{3}V$

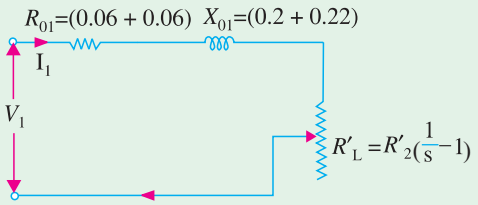


Fig. 34.56

$$P_{g\ max} = \frac{3V_1^2}{2(R_{01} + Z_{01})} = \frac{3(400/\sqrt{3})^2}{2(0.12 + 0.44)} = \mathbf{142{,}900\ W.}$$

Example 34.59. *A 115V, 60-Hz, 3-phase, Y-Connected, 6-pole induction motor has an equivalent T-circuit consisting of stator impedance of (0.07 + j 0.3) Ω and an equivalent rotor impedance at standstill of (0.08 + j 0.3) Ω. Magnetising branch has G_o = 0.022 mho, B_o = 0.158 mho. Find* ***(a)*** *secondary current* ***(b)*** *primary current* ***(c)*** *primary p.f.* ***(d)*** *gross power output* ***(e)*** *gross torque* ***(f)*** *input* ***(g)*** *gross efficiency by using approximate equivalent circuit. Assume a slip of 2% .*

Solution. The equivalent circuit is shown in Fig. 34.57 $R_L' = R_2'\ [(1/s) - 1]$

$$= 0.88\left(\frac{1}{0.02} - 1\right) = 3.92\ \Omega/\text{phase}$$

The impedance to the right of terminals *c* and *d* is

$Z_{cd} = R_{01} + R_L' + j\,X_{01}$

$= (0.07 + 0.08) + 3.92 + j\,0.6$

$= 4.07 + j\,0.6$

$= 4.11 \angle 8.4°\ \Omega$ / phase

$V = 115/\sqrt{3} = 66.5$ V

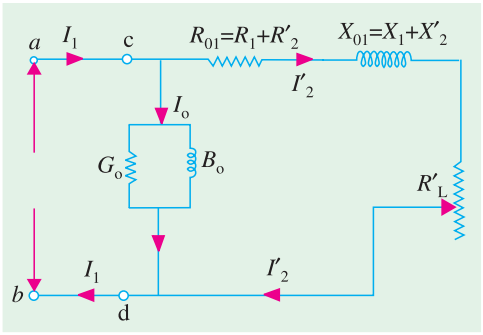


Fig. 34.57

(a) Secondary current $\mathbf{I_2' = I_2}$

$$= \frac{66.5}{4.11 \angle 8.4°} = 16.17 \angle -j\,8.4°$$

$= 16 - j\,2.36$ A

The exciting current $\mathbf{I_0} = V(G_0 - j\,B_0) = 66.5\,(0.022 - j\,0.158) = 1.46 - j\,10.5$ A

(b) $\mathbf{I_1 = I_0 + I_2' = I_0 + I_2} = (1.46 - j\,10.5) + (16 - j\,2.36)$

$= 17.46 - j\,12.86 = 21.7 \angle -36.5°$

(c) Primary p.f. $= \cos 36.5° = \mathbf{0.804}$

(d) $P_g = 3\,I_2^2\,R_L' = 3 \times 16.17^2 \times 3.92 = \mathbf{3{,}075\ W}$

(e) Synchronous speed $N_s = 120 \times 60/6 = 1{,}200$ r.p.m.

Actual rotor speed $N = (1 - s)\,N_s = (1 - 0.02) \times 1200 = 1{,}176$ r.p.m.

$\therefore \quad T_g = 9.55.\frac{P_m}{N} = 9.55 \times \frac{3075}{1176} = \mathbf{24.97\ N - m}$

(f) Primary power input $= \sqrt{3}\,V_1\,I_1 \cos\phi = \sqrt{3} \times 115 \times 21.7 \times 0.804 = \mathbf{3{,}450\ W}$

(g) Gross efficiency $= 3{,}075 \times 100/3{,}450 = \mathbf{89.5\ \%}$

Alternative Solution

Instead of using the equivalent circuit of Fig. 34.57, we could use that shown in Fig. 34.49 which is reproduced in Fig. 34.58.

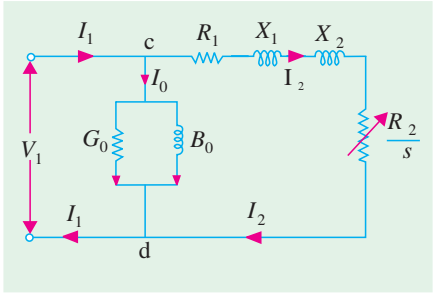


Fig. 34.58

(a) $I_2' = \dfrac{V_1}{(R_1 + R_2'/s) + j(X_1 + X_2')}$

$= \dfrac{66.5 \angle 0°}{(0.07 + 0.08/0.02) + j(0.3 + 0.3)}$

$= \dfrac{66.5}{4.07 + j\,0.6}$

$= 16 - j\,2.36 = 16.17 \angle -8.4°$

(b) $\mathbf{I_1 = I_0 + I_2} = 21.7 \angle -36.5°$ ...as before

(c) primary p.f. = **0.804** ...as before

(d) gross power developed, $P_g = 3\,I_2'^2\,R_2' \left(\dfrac{1-s}{s}\right)$

$= 3 \times 16.17^2 \times 0.08 \left(\dfrac{1-0.02}{0.02}\right) =$ **3075 W**

The rest of the solution is the same as above.

Example 34.60. *The equivalent circuit of a 400 V, 3-phase induction motor with a star-connected winding has the following impedances per phase referred to the stator at standstill:*

Stator : (0.4 + j 1) ohm; Rotor : (0.6 + j 1) ohm; Magnetising branch : (10 + j 50) ohm.

Find (i) maximum torque developed (ii) slip at maximum torque and (iii) p.f. at a slip of 5%. Use approximate equivalent circuit. **(Elect. Machinery-III, Bangalore Univ. 1987)**

Solution. (ii) Gap power transferred and hence the mechanical torque developed by rotor would be maximum when there is maximum transfer of power to the resistor R_2'/s shown in the approximate equivalent circuit of the motor in Fig. 34.59. It will happen when R_2'/s equals the impedance looking back into the supply source. Hence,

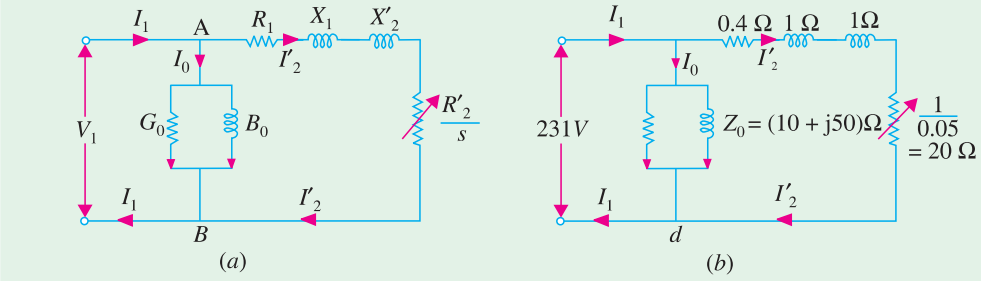


Fig. 34.59

$$\frac{R_2'}{s_m} = \sqrt{R_1^2 + (X_1 + X_2')^2}$$

or $$s_m = \frac{R_2'}{\sqrt{R_1^2 + (X_1 + X_2')^2}} = \frac{0.6}{\sqrt{0.4^2 + 2^2}} = 0.29 \text{ or } \mathbf{29\%}$$

(i) Maximum value of gross torque developed by rotor

$$T_{g\,max} = \frac{P_{g\,max}}{2\pi N_s/60} = \frac{3\,I_2'^2\,R_2'/s_m}{2\pi N_s/60}\ \text{N-m}$$

Now, $I_2' = \dfrac{V_1}{\sqrt{(R_1 + R_2')^2 + (X_1 + X_2')^2}} = \dfrac{400/\sqrt{3}}{\sqrt{(0.4+0.6)^2 + (1+1)^2}} = 103.3$ A

$\therefore \quad T_{g\text{-}max} = \dfrac{3 \times 103.3^2 \times 1/0.29}{2\pi \times 1500/60} = \mathbf{351\ N-m}$ ... assuming $N_S = 1500$ r.pm.

(iii) The equivalent circuit for one phase for a slip of 0.05 is shown in Fig. 32.59 (*b*).

$I_2' = 231/[(20 + 0.4) + j\,2] = 11.2 - j\,1.1$

$I_0 = 231/(10 + j\,50) = 0.89 - j\,4.4$

$I_1 = I_0 + I_2' = 12.09 - j\,5.5 = 13.28 \angle -24.4°$; p.f. = cos 24.4° = **0.91 (lag)**

Tutorial Problem No. 34.4.

1. A 3-phase, 115-volt induction motor has the following constants : $R_2 = 0.07\ \Omega$; $R_2' = 0.08\ \Omega$, $X_1 = 0.4\ \Omega$ and $X_2' = 0.2\ \Omega$. All the values are for one phase only. At which slip the gross power output will be maximum and the value of the gross power output ? **[11.4% ; 8.6 kW]**

2. A 3-phase, 400-V, Y-connected induction motor has an equivalent *T*-circuit consisting of $R_1 = 1\ \Omega$, $X_1 = 2\ \Omega$, equivalent rotor values are $R_2' = 1.2\ \Omega$, $X_2' = 1.5\ \Omega$. The exciting branch has an impedance of $(4 + j\,40)\ \Omega$. If slip is 5% find (*i*) current (*ii*) efficiency (*iii*) power factor (*iv*) output. Assume friction loss to be 250 W. ***[(i) 10.8 A (ii) 81% (iii) 0.82 (iv) 5 kW]***

3. A 50 HP, 440 Volt, 3-phase, 50 Hz Induction motor with star-connected stator winding gave the following test results:

(*i*)No load test: Applied line voltage 440 V, line current 24 A, wattmeter reading 5150 and 3350 watts.

(*ii*)Blocked rotor test: applied line voltage 33.6 volt, line current 65 A, wattmeter reading 2150 and 766 watts.

Calculate the parameters of the equivalent circuit.

[Rajiv Gandhi Technical University, Bhopal, 2000]

[(*i*) Shunt branch : $R_o = 107.6$ ohms, $X_m = 10.60$ ohms (*ii*) Series branch : $r = 0.23$ ohm, $x = 0.19$ ohm]

OBJECTIVE TESTS – 34

1. Regarding skewing of motor bars in a squirrel-cage induction motor, (SCIM) which statement is false ?

(*a*) it prevents cogging

(*b*) it increases starting torque

(*c*) it produces more uniform torque

(*d*) it reduces motor 'hum' during its operation.

2. The principle of operation of a 3-phase. Induction motor is most similar to that of a

(*a*) synchronous motor

(*b*) repulsion-start induction motor

(*c*) transformer with a shorted secondary

(*d*) capacitor-start, induction-run motor.

3. The magnetising current drawn by transformers and induction motors is the cause of their .....power factor.

(*a*) zero (*b*) unity

(c) lagging (*d*) leading.

4. The effect of increasing the length of air-gap in an induction motor will be to increase the

(*a*) power factor

(*b*) speed

(*c*) magnetising current

(*d*) air-gap flux.

(Power App-II, Delhi Univ. Jan. 1987)

5. In a 3-phase induction motor, the relative speed of stator flux with respect to .....is zero.

(*a*) stator winding (*b*) rotor

(c) rotor flux (*d*) space.

6. An eight-pole wound rotor induction motor operating on 60 Hz supply is driven at 1800 r.p.m. by a prime mover in the opposite direction of revolving magnetic field. The frequency of rotor current is

(*a*) 60 Hz (*b*) 120 Hz

(c) 180 Hz (*d*) none of the above.

(Elect. Machines, A.M.I.E. Sec. B, 1993)

7. A 3-phase, 4-pole, 50-Hz induction motor runs at a speed of 1440 r.p.m. The rotating field produced by the rotor rotates at a speed of .....r.p.m. with respect to the rotor.

(*a*) 1500 (*b*) 1440

(c) 60 (*d*) 0.

8. In a 3-ϕ induction motor, the rotor field rotates at synchronous speed with respect to

(*a*) stator (*b*) rotor

(*c*) stator flux (*d*) none of the above.

9. Irrespective of the supply frequency, the torque developed by a SCIM is the same whenever ..... is the same.

(*a*) supply voltage (*b*) external load

(*c*) rotor resistance (*d*) slip speed.

10. In the case of a 3-ϕ induction motor having N_s = 1500 rpm and running with s = 0.04

(*a*) revolving speed of the stator flux is space is .....rpm

(*b*) rotor speed is .....rpm

(*c*) speed of rotor flux relative to the rotor is .....rpm

(*d*) speed of the rotor flux with respect to the stator is .....rpm.

11. The number of stator poles produced in the rotating magnetic field of a 3-ϕ induction motor having 3 slots per pole per phase is

(*a*) 3 (*b*) 6

(c) 2 (*d*) 12

12. The power factor of a squirrel-cage induction motor is

(*a*) low at light loads only

(*b*) low at heavy loads only

(*c*) low at light and heavy loads both

(*d*) low at rated load only.

(Elect. Machines, A.M.I.E. Sec.B, 1993)

13. Which of the following rotor quantity in a SCIM does NOT depend on its slip ?

(*a*) reactance (*b*) speed

(*c*) induced emf (*d*) frequency.

14. A 6-pole, 50-Hz, 3-ϕ induction motor is running at 950 rpm and has rotor Cu loss of 5 kW. Its rotor input is .....kW.

(*a*) 100 (b) 10

(c) 95 (*d*) 5.3.

15. The efficiency of a 3-phase induction motor is approximately proportional to

(*a*) (1– s) (*b*) s

(*c*) N (*d*) N_s.

16. A 6-pole, 50-Hz, 3-ϕ induction motor has a full-load speed of 950 rpm. At half-load, its speed would be .....rpm.

(*a*) 475 (*b*) 500

(*c*) 975 (*d*) 1000

17. If rotor input of a SCIM running with a slip of 10% is 100 kW, gross power developed by its rotor is ..... kW.

(*a*) 10 (*b*) 90

(*c*) 99 (*d*) 80

18. Pull-out torque of a SCIM occurs at that value of slip where rotor power factor equals

(*a*) unity (*b*) 0.707

(*c*) 0.866 (*d*) 0.5

19. Fill in the blanks.

When load is placed on a 3-phase induction motor, its

(*i*) speed .....

(*ii*) slip .....

(*iii*) rotor induced emf .....

(*iv*) rotor current .....

(*v*) rotor torque .....

(*vi*) rotor continues to rotate at that value of slip at which developed torque equals ..... torque.

20. When applied rated voltage per phase is reduced by one-half, the starting torque of a SCIM becomes ..... of the starting torque with full voltage.

(*a*) 1/2 (*b*) 1/4

(*c*) $1/\sqrt{2}$ (*d*) $\sqrt{3}/2$

21. If maximum torque of an induction motor is 200 kg-m at a slip of 12%, the torque at 6% slip would be ..... kg-m.

(*a*) 100 (*b*) 160

(*c*) 50 (*d*) 40

22. The fractional slip of an induction motor is the ratio

(*a*) rotor Cu loss/rotor input

(*b*) stator Cu loss/stator input

(*c*) rotor Cu loss/rotor output

(*d*) rotor Cu loss/stator Cu loss

23. The torque developed by a 3-phase induction motor depends on the following three factors:

(*a*) speed, frequency, number of poles

(*b*) voltage, current and stator impedance

(*c*) synchronous speed, rotor speed and frequency

(*d*) rotor emf, rotor current and rotor p.f.

24. If the stator voltage and frequency of an induction motor are reduced proportionately, its
(*a*) locked rotor current is reduced
(*b*) torque developed is increased
(*c*) magnetising current is decreased
(*d*) both (*a*) and (*b*)

25. The efficiency and p.f. of a SCIM increases in proportion to its
(*a*) speed (*b*) mechanical load
(*c*) voltage (*d*) rotor torque

26. A SCIM runs at ***constant*** speed only so long as
(*a*) torque developed by it remains constant
(*b*) its supply voltage remains constant
(*c*) its torque exactly equals the mechanical load
(*d*) stator flux remains constant

27. The synchronous speed of a linear induction motor does NOT depend on
(*a*) width of pole pitch
(*b*) number of poles
(*c*) supply frequency
(*d*) any of the above

28. Thrust developed by a linear induction motor depends on
(*a*) synchronous speed
(*b*) rotor input
(*c*) number of poles
(*d*) both (*a*) and (*b*)

ANSWERS

1. *b* **2.** *c* **3.** *c* **4.** *c* **5.** *c* **6.** *c* **7.** *c* **8.** *a* **9.** *d* **10.** (*i*) 1500 (*ii*) 1440 (*iii*) 60 (*iv*) 1500 **11.** *b* **12.** *a* **13.** *b* **14.** *a* **15.** *a* **16.** *c* **17.** *b* **18.** *b* **19.** (*i*) decreases (*ii*) increases (*iii*) increases (*iv*) increases (*v*) increases (*vi*) applied **20.** *b* **21.** *b* **22.** *a* **23.** *d* **24.** *d* **25.** *b* **26.** *c* **27.** *b* **28.** *d*

ROUGH WORK

GO To FIRST

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CHAPTER 35

COMPUTATIONS AND CIRCLE DIAGRAMS

Learning Objectives

- General
- Circle Diagram for a Series Circuit
- Circle Diagram of the Approximate Equivalent Circle
- Determination of G0 and B0
- No-load Test
- Blocked Rotor Test
- Construction of the Circle Diagram
- Maximum Quantities
- Starting of Induction Motors
- Direct-Switching or Line Starting of Induction Motors
- Squirrel-cage Motors
- Starting of Slip-ring Motors
- Starter Steps
- Crawling
- Cogging or Magnetic Locking
- Double Sqiurrel-cage Motor
- Equivalent circuit
- Speed Control of Induction Motor
- Three-Phase A.C. Commutator Motors
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- Motor Enclosures
- Standard type of Squirrel-cage Motors
- Class A Motors
- Class B Motors
- Class C Motors
- Class D Motors
- Class E Motors
- Class F Motors
- Questions and Answer on Induction Motors

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This chapter explains you how to derive performance characteristics of induction motors using circular diagrams

35.1. General

In this chapter, it will be shown that the performance characteristics of an induction motor are derivable from a circular locus. The data necessary to draw the circle diagram may be found from no-load and blocked-rotor tests, corresponding to the open-circuit and short-circuit tests of a transformer. The stator and rotor Cu losses can be separated by drawing a torque line. The parameters of the motor, in the equivalent circuit, can be found from the above tests, as shown below.

35.2. Circle Diagram for a Series Circuit

It will be shown that the end of the current vector for a series circuit with constant reactance and voltage, but with a variable resistance is a circle. With reference to Fig. 35.1, it is clear that

$$I = \frac{V}{Z} = \frac{V}{\sqrt{(R^2 + X^2)}}$$

$$= \frac{V}{X} \times \frac{X}{\sqrt{(R^2 + X^2)}} = \frac{V}{X} \sin \phi$$

$$\because \sin \phi = \frac{X}{\sqrt{(R^2 + X^2)}} \quad \text{– Fig. 35.2}$$

$$\therefore \quad I = (V/X) \sin \phi$$

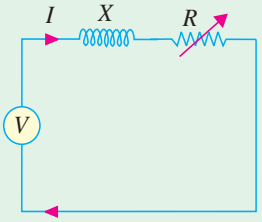


Fig. 35.1

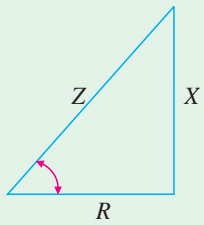


Fig. 35.2

It is the equation of a circle in polar co-ordinates, with diameter equal to V/X. Such a circle is drawn in Fig. 35.3, using the magnitude of the current and power factor angle ϕ as polar co-ordinates of the point A. In other words, as resistance R is varied (which means, in fact, ϕ is changed), the end of the current vector lies on a circle with diameter equal to V/X. For a lagging current, it is usual to orientate the circle of Fig. 35.3 (*a*) such that its diameter is horizontal and the voltage vector takes a vertical position, as shown in Fig. 35.3 (*b*). There is no difference between the two so far as the magnitude and phase relationships are concerned.

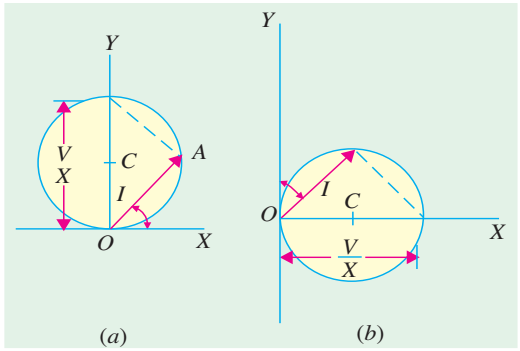


Fig. 35.3

35.3. Circle Diagram for the Approximate Equivalent Circuit

The approximate equivalent diagram is redrawn in Fig. 35.4. It is clear that the circuit to the right of points *ab* is similar to a series circuit, having a constant voltage V_1 and reactance X_{01} but variable resistance (corresponding to different values of slip s).

Hence, the end of current vector for I_2' will lie on a circle with a diameter of V/X_{01}. In Fig. 35.5, I_2' is the rotor current referred to stator, I_0 is no-load current (or exciting current) and I_1 is the total stator current and is the vector sum of the first two. When I_2' is lagging and $\phi_2 = 90°$, then the position of vector for I_2' will be along OC *i.e.* at right angles to the voltage vector OE. For any other value of ϕ_2, point A will move along the circle shown dotted. The exciting current I_0 is drawn lagging V by an angle ϕ_0. If conductance G_0 and susceptance

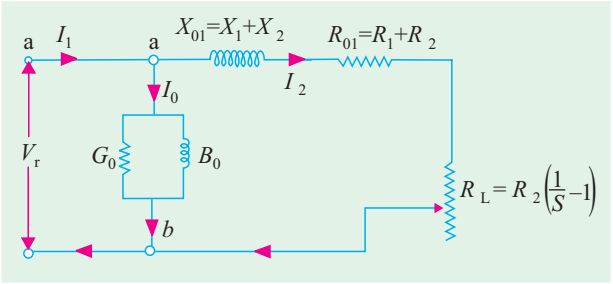


Fig. 35.4

B_0 of the exciting circuit are assumed constant, then I_0 and ϕ_0 are also constant. The end of current vector for I_1 is also seen to lie on another circle which is displaced from the dotted circle by an amount I_0. Its diameter is still V/X_{01} and is parallel to the horizontal axis OC. Hence, we find that if an induction motor is tested at various loads, the locus of the end of the vector for the current (drawn by it) is a circle.

35.4. Determination of G_0 and B_0

If the total leakage reactance X_{01} of the motor, exciting conductance G_0 and exciting susceptance B_0 are found, then the position of the circle $O'BC'$ is determined uniquely. One method of finding G_0 and B_0 consists in running the motor synchronously so that slip $s = 0$. In practice, it is impossible for an induction motor to run at synchronous speed, due to the inevitable presence of friction and windage losses. However, the induction motor may be run at synchronous speed by another machine which supplies the friction and windage losses. In that case, the circuit to the right of points *ab* behaves like an open circuit, because with $s = 0$, $R_L = \infty$ (Fig. 35.6). Hence, the current drawn by the motor is I_0 only. Let

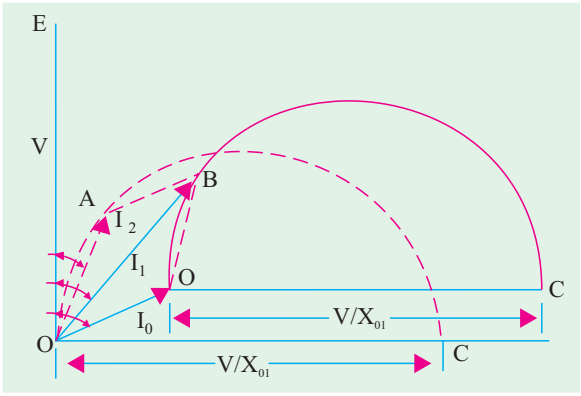


Fig. 35.5

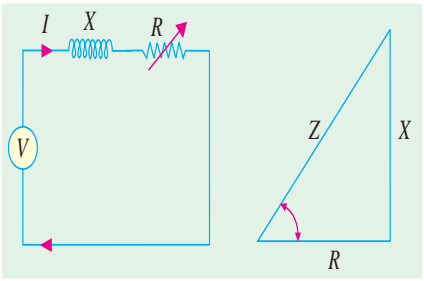


Fig. 35.6

V = applied voltage/phase; I_0 = motor current / phase

W = wattmeter reading *i.e.* input in watt ; Y_0 = exciting admittance of the motor. Then, for a 3-phase induction motor

$$W = 3G_0V^2 \quad \text{or} \quad G_0 = \frac{W}{3V^2} \quad \text{Also, } I_0 = V\,Y_0 \quad \text{or} \quad Y_0 = I_0/V$$

$$B_0 = \sqrt{(Y_0^2 - G_0^2)} = \sqrt{[(I_0/V)^2 - G_0^2]}$$

Hence, G_0 and B_0 can be found.

35.5. No-load Test

In practice, it is neither necessary nor feasible to run the induction motor synchronously for getting G_0 and B_0. Instead, the motor is run without any external mechanical load on it. The speed of the rotor would not be synchronous, but very much near to it ; so that, for all practical purposes, the speed may be assumed synchronous. The no load test is carried out with different values of applied voltage, below and above the value of normal voltage. The power input is measured by two wattmeters,

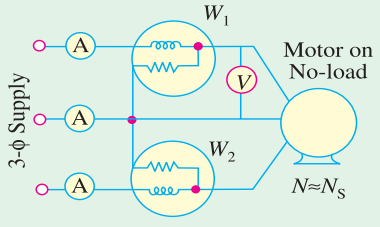


Fig. 35.7

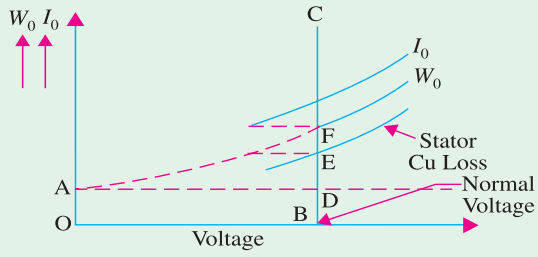


Fig. 35.8

I_0 by an ammeter and V by a voltmeter, which are included in the circuit of Fig. 35.7. As motor is running on light load, the p.f. would be low *i.e.* less than 0.5, hence total power input will be the difference of the two wattmeter readings W_1 and W_2. The readings of the total power input W_0, I_0 and voltage V are plotted as in Fig. 35.8. If we extend the curve for W_0, it cuts the vertical axis at point A. OA represents losses due to friction and windage. If we subtract loss corresponding to OA from W_0, then we get the no-load electrical and magnetic losses in the machine, because the no-load input W_0 to the motor consists of

(*i*) small stator Cu loss $3 I_0^2 R_1$ **(*ii*)** stator core loss $W_{CL} = 3G_0 V^2$

(*iii*) loss due to friction and windage.

The losses (*ii*) and (*iii*) are collectively known as fixed losses, because they are independent of load. OB represents normal voltage. Hence, losses at normal voltage can be found by drawing a vertical line from B.

BD = loss due to friction and windage DE = stator Cu loss EF = core loss

Hence, knowing the core loss W_{CL}, G_0 and B_0 can be found, as discussed in Art. 35.4.

Additionally, ϕ_0 can also be found from the relation $W_0 = \sqrt{3}\, V_L\, I_0 \cos\phi_0$

$$\therefore \quad \cos\phi_0 = \frac{W_0}{\sqrt{3}\, V_L\, I_0}$$

where V_L = line voltage and W_0 is no-load stator input.

Example 35.1. *In a no-load test, an induction motor took 10 A and 450 watts with a line voltage of 110 V. If stator resistance/phase is 0.05 Ω and friction and windage losses amount to 135 watts, calculate the exciting conductance and susceptance/phase.*

Solution. stator Cu loss $= 3 I_0^2 R_1 = 3 \times 10^2 \times 0.05 = 15$ W

$\therefore$ stator core loss $= 450 - 135 - 15 = 300$ W

Voltage/phase $V = 110/\sqrt{3}$ V ; Core loss $= 3 G_0 V^2$

$$300 = 3 G_0 \times (110/\sqrt{3})^2 \; ; \; G_0 = \frac{300}{3 \times (110/\sqrt{3})^2}$$

$= \mathbf{0.025}$ **siemens/phase**

$Y_0 = I_0 / V = (10 \times \sqrt{3})/110 = 0.158$ siemens/phase

$$B_0 = \sqrt{(Y_0^2 - G_0^2)} = \sqrt{(0.158^2 - 0.025^2)}$$

$= \mathbf{0.156}$ **siemens/phase.**

35.6. Blocked Rotor Test

It is also known as locked-rotor or short-circuit test. This test is used to find–

1. short-circuit current with ***normal*** voltage applied to stator

2. power factor on short-circuit

Both the values are used in the construction of circle diagram

3. total leakage reactance X_{01} of the motor as referred to primary (*i.e.* stator)

4. total resistance of the motor R_{01} as referred to primary.

In this test, the rotor is locked (or allowed very slow rotation)



This vertical test stand is capable of absorbing up to 10,000 N-m of torque at continuous load rating (max 150.0 hp at 1800 rpm). It helps to develop speed torque curves and performs locked rotor testing

and the rotor windings are short-circuited at slip-rings, if the motor has a wound rotor. Just as in the case of a short-circuit test on a transformer, a reduced voltage (up to 15 or 20 per cent of normal value) is applied to the stator terminals and is so adjusted that full-load current flows in the stator. As in this case $s = 1$, the equivalent circuit of the motor is exactly like a transformer, having a short-circuited secondary. The values of current, voltage and power input on short-circuit are measured by the ammeter, voltmeter and wattmeter connected in the circuits as before. Curves connecting the above quantities may also be drawn by taking two or three additional sets of readings at progressively reduced voltages of the stator.

(*a*) It is found that relation between the short-circuit current and voltage is approximately a straight line. Hence, if V is normal stator voltage, V_s the short-circuit voltage (a fraction of V), then short-circuit or standstill rotor current, if normal voltage were applied to stator, is found from the relation

$$I_{SN} = I_s \times V/V_s$$

where I_{SN} = short-circuit current obtainable with normal voltage

I_s = short-circuit current with voltage V_S

(*b*) Power factor on short-circuit is found from

$$W_S = \sqrt{3}\, V_{SL}\, I_{SL} \cos\phi_S ; \qquad \therefore \quad \cos\phi_S = W_S / (\sqrt{3}\, V_{SL}\, I_{SL})$$

where W_S = total power input on short-circuit

V_{SL} = line voltage on short-circuit

I_{SL} = line current on short-circuit.

(*c*) Now, the motor input on short-circuit consists of

(*i*) mainly stator and rotor Cu losses

(*ii*) core-loss, which is small due to the fact that applied voltage is only a small percentage of the normal voltage. This core-loss (if found appreciable) can be calculated from the curves of Fig. 35.8.

$\therefore$ Total Cu loss $= W_S - W_{CL}$

$$3\, I_s^2\, R_{01} = W_s - W_{CL} : \quad R_{01} = (W_s - W_{CL}) / 3\, I_s^2$$

(*d*) With reference to the approximate equivalent circuit of an induction motor (Fig. 35.4), motor leakage reactance per phase X_{01} as referred to the stator may be calculated as follows :

$$Z_{01} = V_S / I_S \qquad X_{01} = \sqrt{(Z_{01}^2 - R_{01}^2)}$$

Usually, X_1 is assumed equal to X_2' where X_1 and X_2 are stator and rotor reactances per phase respectively as referred to stator. $X_1 = X_2' = X_{01} / 2$

If the motor has a wound rotor, then stator and rotor resistances are separated by dividing R_{01} in the ratio of the d.c. resistances of stator and rotor windings.

In the case of squirrel-cage rotor, R_1 is determined as usual and after allowing for 'skin effect' is subtracted from R_{01} to give R_2' – the effective rotor resistance as referred to stator.

$\therefore$ $$R_2' = R_{01} - R_1$$

Example 35.2. *A 110-V, 3-ϕ, star-connected induction motor takes 25 A at a line voltage of 30 V with rotor locked. With this line voltage, power input to motor is 440 W and core loss is 40 W. The d.c. resistance between a pair of stator terminals is 0.1 Ω. If the ratio of a.c. to d.c. resistance is 1.6, find the equivalent leakage reactance/phase of the motor and the stator and rotor resistance per phase.* **(Electrical Technology, Madras Univ. 1987)**

Solution. S.C. voltage/phase, $V_s = 30 / \sqrt{3} = 17.3$ V : $I_s = 25$ A per phase

$Z_{01} = 17.3/25 = 0.7\ \Omega$ (approx.) per phase

Stator and rotor Cu losses = input – core loss = 440 – 40 = 400 W

$\therefore \quad 3 \times 25^2 \times R_{01} = 400 \quad \therefore R_{01} = 400/3 \times 625 =$ **0.21 Ω**

where R_{01} is equivalent resistance/phase of motor as referred to stator.

Leakage reactance/phase $X_{01} = \sqrt{(0.7^2 - 0.21^2)} = 0.668\ \Omega$

d.c. resistance/phase of stator = 0.1/2 = 0.05 Ω

a.c. resistance/phase $R_1 = 0.05 \times 1.6 = 0.08\ \Omega$

Hence, effective resistance/phase of rotor as referred to stator

$R_2' =$ 0.21 – 0.08 = **0.13 Ω**

35.7. Construction of the Circle Diagram



Windings inside a motor

Circle diagram of an induction motor can be drawn by using the data obtained from **(1) no-load (2) short-circuit test** and **(3) stator resistance test**, as shown below.

Step No. 1

From no-load test, I_0 and ϕ_0 can be calculated. Hence, as shown in Fig. 35.9, vector for I_0 can be laid off lagging ϕ_0 behind the applied voltage V.

Step No. 2

Next, from blocked rotor test or short-circuit test, short-circuit current I_{SN} ***corresponding to normal voltage*** and ϕ_S are found. The vector ***OA*** represents $I_{SN} = (I_S V/V_S)$ in magnitude and phase. Vector $O'A$ represents rotor current I_2' as referred to stator.

Clearly, the two points O' and A lie on the required circle. For finding the centre C of this circle, chord $O'A$ is bisected at right angles–its bisector giving point C. The diameter $O'D$ is drawn perpendicular to the voltage vector.

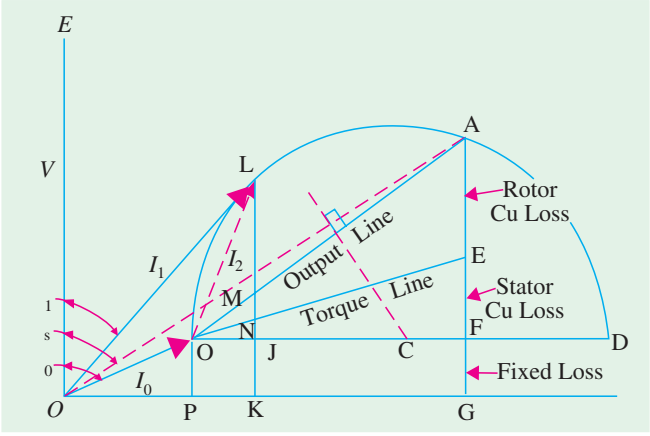


Fig. 35.9

As a matter of practical contingency, it is recommended that the scale of current vectors should be so chosen that the diameter is more than 25 cm, in order that the performance data of the motor may be read with reasonable accuracy from the circle diagram. With centre C and radius = CO', the circle can be drawn. The line $O'A$ is known as **out-put line.**

It should be noted that as the voltage vector is drawn vertically, all vertical distances represent the active or power or energy components of the currents. For example, the vertical component $O'P$ of no-load current OO' represents the no-load input, which supplies core loss, friction and windage loss and a negligibly small amount of stator I^2R loss. Similarly, the vertical component AG of short-circuit current OA is proportional to the motor input on short-circuit or if measured to a proper scale, may be said to equal power input.

Step No. 3

Torque line. ***This is the line which separates the stator and the rotor copper losses***. When the

rotor is locked, then all the power supplied to the motor goes to meet core losses and Cu losses in the stator and rotor windings. The power input is proportional to AG. Out of this, FG (= $O'P$) represents fixed losses *i.e.* stator core loss and friction and windage losses. AF is proportional to the sum of the stator and rotor Cu losses. The point E is such that

$$\frac{AE}{EF} = \frac{\text{rotor Cu loss}}{\text{stator Cu loss}}$$

As said earlier, line $O'E$ is known as torque line.

How to locate point E ?

(i) Squirrel-cage Rotor. Stator resistance/phase *i.e.* R_1 is found from stator-resistance test. Now, the short-circuit motor input W_s is approximately equal to motor Cu losses (neglecting iron losses).

Stator Cu loss = $3I_S^2 R_1$ $\quad\therefore$ rotor Cu loss = $W_S - 3I_S^2 R_1$ $\quad\therefore$ $\dfrac{AE}{EF} = \dfrac{W_S - 3I_S^2 R_1}{3I_S^2 R_1}$

(ii) Wound Rotor. In this case, rotor and stator resistances per phase r_2 and r_1 can be easily computed. For any values of stator and rotor currents I_1 and I_2 respectively, we can write

$$\frac{AE}{EF} = \frac{I_2^2 r_2}{I_1^2 r_1} = \frac{r_2}{r_1}\left(\frac{I_2}{I_1}\right)^2; \qquad \text{Now,} \quad \frac{I_1}{I_2} = K = \text{transformation ratio}$$

$$\frac{AE}{EF} = \frac{r_2}{r_1} \times \frac{1}{K^2} = \frac{r_2 / K^2}{r_1} = \frac{r_2'}{r_1} = \frac{\text{equivalent rotor resistance per phase}}{\text{stator resistance per phase}}$$

Value of K may be found from short-circuit test itself by using two ammeters, both in stator and rotor circuits.

Let us assume that the motor is running and taking a current OL (Fig. 35.9). Then, the perpendicular JK represents fixed losses, JN is stator Cu loss, NL is the rotor input, NM is rotor Cu loss, ML is rotor output and LK is the total motor input.

From our knowledge of the relations between the above-given various quantities, we can write :

$\sqrt{3} \cdot V_L \cdot LK$ = motor input $\qquad$ $\sqrt{3} \cdot V_L \cdot JK$ = fixed losses

$\sqrt{3} \cdot V_L \cdot JN$ = stator copper loss $\qquad$ $\sqrt{3} \cdot V_L \cdot MN$ = rotor copper loss

$\sqrt{3} \cdot V_L \cdot MK$ = total loss $\qquad$ $\sqrt{3} \cdot V_L \cdot ML$ = mechanical output

$\sqrt{3} \cdot V_L \cdot NL$ = rotor input $\propto$ torque

1. ML / LK = output/input = efficiency

2. MN / NL = (rotor Cu loss)/(rotor input) = slip, s.

3. $\dfrac{ML}{NL} = \dfrac{\text{rotor output}}{\text{rotor input}} = 1 - s = \dfrac{N}{N_S} = \dfrac{\text{actual speed}}{\text{synchronous speed}}$

4. $\dfrac{LK}{OL}$ = power factor

Hence, it is seen that, at least, theoretically, it is possible to obtain all the characteristics of an induction motor from its circle diagram. As said earlier, for drawing the circle diagram, we need ***(a)*** stator-resistance test for separating stator and rotor Cu losses and ***(b)*** the data obtained from ***(i)*** no-load test and ***(ii)*** short-circuit test.

35.8. Maximum Quantities

It will now be shown from the circle diagram (Fig. 35.10) that the maximum values occur at the positions stated below :

(i) Maximum Output

It occurs at point M where the tangent is parallel to output line $O'A$. Point M may be located by

drawing a line *CM* from point *C* such that it is perpendicular to the output line *O′A*. Maximum output is represented by the vertical MP.

(*ii*) Maximum Torque or Rotor Input

It occurs at point *N* where the tangent is parallel to torque line *O′E*. Again, point *N* may be found by drawing *CN* perpendicular to the torque line. Its value is represented by *NQ* . Maximum torque is also known as stalling or pull-out torque.

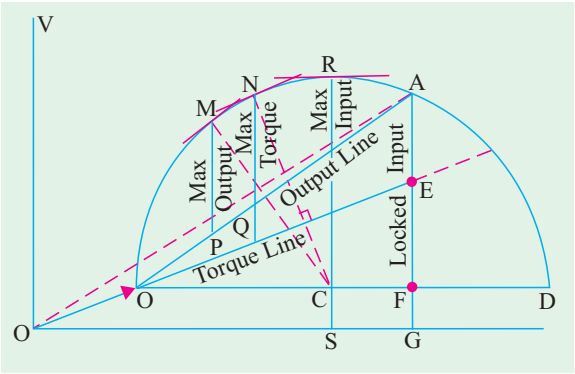


Fig. 35.10

(*iii*) Maximum Input Power

It occurs at the highest point of the circle *i.e.* at point R where the tangent to the circle is horizontal. It is proportional to RS. As the point R is beyond the point of maximum torque, the induction motor will be unstable here. However, the maximum input is a measure of the size of the circle and is an indication of the ability of the motor to carry short-time over-loads. Generally, RS is twice or thrice the motor input at rated load.

Example 35.3. *A 3-ph, 400-V induction motor gave the following test readings;*
No-load : 400 V, 1250 W, 9 A, Short-circuit : 150 V, 4 kW, 38 A
Draw the circle diagram.
If the normal rating is 14.9 kW, find from the circle diagram, the full-load value of current, p.f. and slip. **(Electrical Machines-I, Gujarat Univ. 1985)**

Solution. $\cos \phi_0 = \dfrac{1250}{\sqrt{3} \times 400 \times 9} = 0.2004 ; \quad \phi_0 = 78.5^\circ$

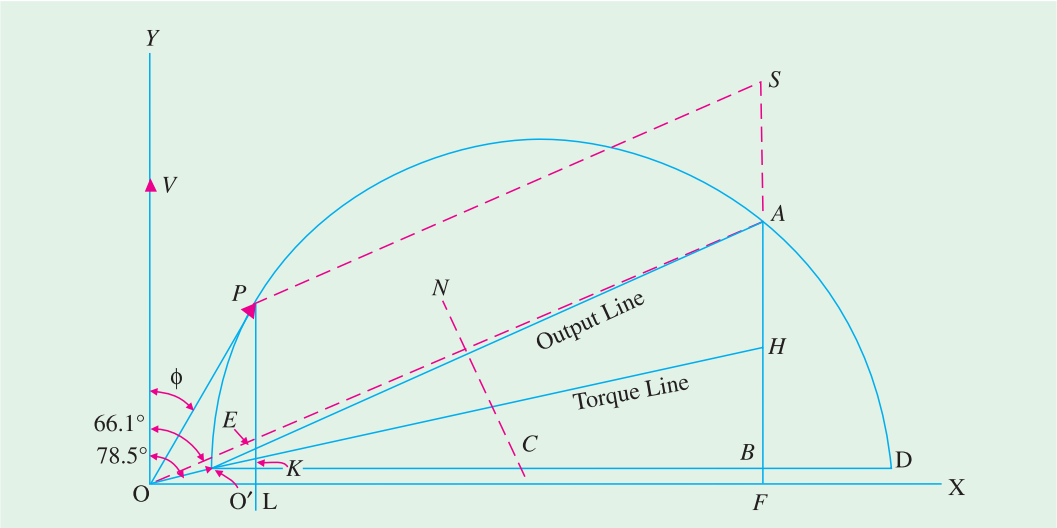


Fig. 35.11

$$\cos \phi_S = \frac{4000}{\sqrt{3} \times 150 \times 38} = 0.405 ; \quad \phi_S = 66.1^\circ$$

Short-circuit current with normal voltage is $I_{SN} = 38\ (400/150) = 101.3$ A. Power taken would be $= 4000\ (400/150)^2 = 28{,}440$ W. In Fig. 33.11, OO' represents I_0 of 9 A. If current scale is 1 cm = 5 A,

then vector OO' = 9/5 = 1.8 cm* and is drawn at an angle of ϕ_0 = 78.5° with the vertical OV (which represents voltage). Similarly, OA represents I_{SN} (S.C. current with normal voltage applied) equal to 101.3 A. It measures 101.3/5 = 20.26* cm and is drawn at an angle of 66.1°, with the vertical OV.

Line $O'D$ is drawn parallel to OX. NC is the right-angle bisector of $O'A$. The semi-circle $O'AD$ is drawn with C as the centre. This semi-circle is the locus of the current vector for all load conditions from no-load to short-circuit. Now, AF represents 28,440 W and measures 8.1 cm. Hence, power scale becomes : 1 cm = 28,440/8.1 = 3,510 W. Now, full-load motor output = 14.9×10^3 = 14,900 W. According to the above calculated power scale, the intercept between the semi-circle and output line $O'A$ should measure = 14,900/3510 = 4.25 cm. For locating full-load point P, BA is extended. AS is made equal to 4.25 cm and SP is drawn parallel to output line $O'A$. PL is perpendicular to OX.

Line current = OP = 6 cm = 6 × 5 = **30 A**; ϕ = 30° (by measurement)

p.f. = cos 30° = **0.886** (or cos $\phi = PL/OP$ = 5.2/6 = 0.865)

Now, $$\text{slip} = \frac{\text{rotor Cu loss}}{\text{rotor input}}$$

In Fig. 35.11, EK represents rotor Cu loss and PK represents rotor input.

$$\therefore \quad \text{slip} = \frac{EK}{PK} = \frac{0.3}{4.5} = 0.067 \text{ or } \mathbf{6.7\%}$$

Example 35.4. *Draw the circle diagram for a 3.73 kW, 200-V, 50-Hz, 4-pole, 3-ϕ star-connected induction motor from the following test data :*

No-load : Line voltage 200 V, line current 5 A; total input 350 W

Blocked rotor : Line voltage 100 V, line current 26 A; total input 1700 W

Estimate from the diagram for full-load condition, the line current, power factor and also the maximum torque in terms of the full-load torque. The rotor Cu loss at standstill is half the total Cu loss. **(Electrical Engineering, Bombay Univ. 1987)**

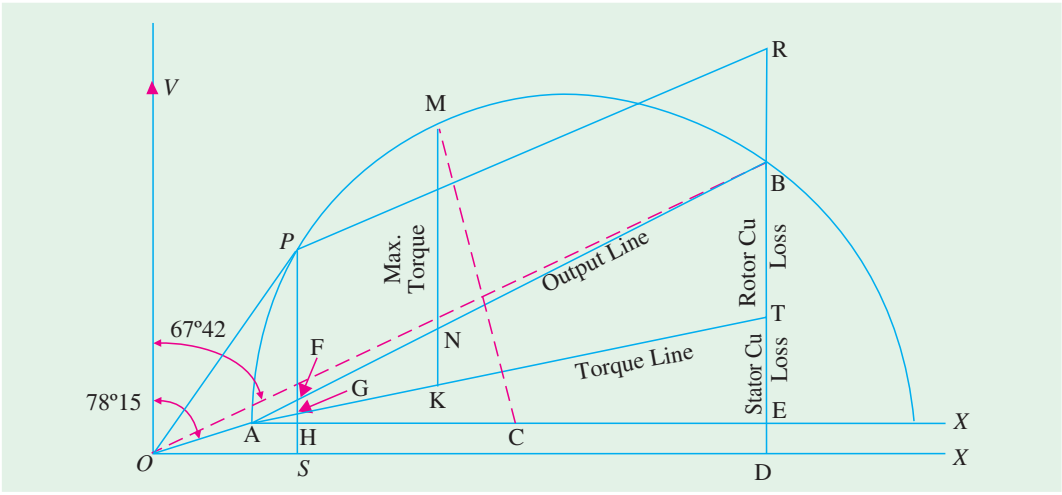


Fig. 35.12

Solution. No-load test

$$I_0 = 5 \text{ A}, \cos\phi_0 = \frac{350}{\sqrt{3} \times 200 \times 5} = 0.202 \; ; \; \phi_0 = 78°15'$$

* The actual lengths are different from these values, due to reduction in block making.

Blocked-rotor test :

$$\cos \phi_s = \frac{1700}{\sqrt{3} \times 100 \times 26} = 0.378 \; ; \; \phi_s = 67°42'$$

Short-circuit current with normal voltage, $I_{SN} = 26 \times 200/100 = 52$ A

Short-circuit/blocked rotor input with normal voltage = $1700(52/26)^2 = 6{,}800$ W

In the circle diagram of Fig. 35.12, voltage is represented along OV which is drawn perpendicular to OX. Current scale is 1 cm = 2 A

Line OA is drawn at an angle of $\phi_0 = 78°15'$ with OV and 2.5 cm in length. Line AX' is drawn parallel to OX. Line OB represents short-circuit current with normal voltage *i.e.* 52 A and measures 52/2 = 26 cm. AB represents output line. Perpendicular bisector of AB is drawn to locate the centre C of the circle. With C as centre and radius = CA, a circle is drawn which passes through points A and B. From point B, a perpendicular is drawn to the base. BD represents total input of 6,800 W for blocked rotor test. Out of this, ED represents no-load loss of 350 W and BE represents 6,800 – 350 = 6,450 W. Now BD = 9.8 cm and represents 6,800 W

$\therefore$ power scale = 6,800/9.8 = 700 watt/cm or 1 cm = **700 W**

BE which represents total copper loss in rotor and stator, is bisected at point T to separate the two losses. AT represents torque line.

Now, motor output = 3,730 watt. It will be represented by a line = 3,730/700 = 5.33 cm

The output point P on the circle is located thus :

DB is extended and BR is cut = 5.33 cm. Line RP is drawn parallel to output line AB and cuts the circle at point P. Perpendicular PS is drawn and P is joined to origin O.

Point M corresponding to maximum torque is obtained thus :

From centre C, a line CM is drawn such that it is perpendicular to torque line AT. It cuts the circle at M which is the required point. Point M could also have been located by drawing a line parallel to the torque line. MK is drawn vertical and it represents maximum torque.

Now, in the circle diagram, OP = line current on full-load = 7.6 cm. Hence, OP represents $7.6 \times 2 = 15.2$ A

$$\text{Power factor on full-load} = \frac{SP}{OP} = \frac{6.45}{7.6} = 0.86$$

$$\frac{\text{Max. torque}}{\text{F.L. torque}} = \frac{MK}{PG} = \frac{10}{5.6} = 1.8$$

$\therefore$ Max. torque = 180% of full-load torque.

Example. 35.5. *Draw the circle diagram from no-load and short-circuit test of a 3-phase. 14.92 kW, 400-V, 6-pole induction motor from the following test results (line values).*

No-load : 400-V, 11 A, p.f. = 0.2

Short-circuit : 100-V, 25 A, p.f. = 0.4

Rotor Cu loss at standstill is half the total Cu loss.

From the diagram, find (a) line current, slip, efficiency and p.f. at full-load (b) the maximum torque. **(Electrical Machines-I, Gujarat Univ. 1985)**

Solution. No-load p.f. = 0.2; $\phi_0 = \cos^{-1}(0.2) = 78.5°$

Short-circuit p.f. = 0.4: $\phi_s = \cos^{-1}(0.4) = 66.4°$

S.C. current I_{SN} if normal voltage were applied = 25 (400/100) = 100 A

S.C. power input with this current = $\sqrt{3} \times 400 \times 100 \times 0.4 = 27{,}710$ W

Assume a current scale of 1 cm = 5 A.* The circle diagram of Fig. 35.13 is constructed as follows :

(*i*) No-load current vector OO' represents 11 A. Hence, it measures 11/5 = 2.2 cm and is drawn at an angle of 78.5° with OY.

(*ii*) Vector OA represents 100 A and measures 100/5 = 20 cm. It is drawn at an angle of 66.4° with OY.

(*iii*) $O'D$ is drawn parallel to OX. NC is the right angle bisector of $O'A$.

(*iv*) With C as the centre and CO' as radius, a semicircle is drawn as shown.

(*v*) AF represents power input on short-circuit with normal voltage applied. It measures 8 cm and (as calculated above) represents 27,710 W. Hence, power scale becomes

1 cm = 27,710/8 = 3,465 W

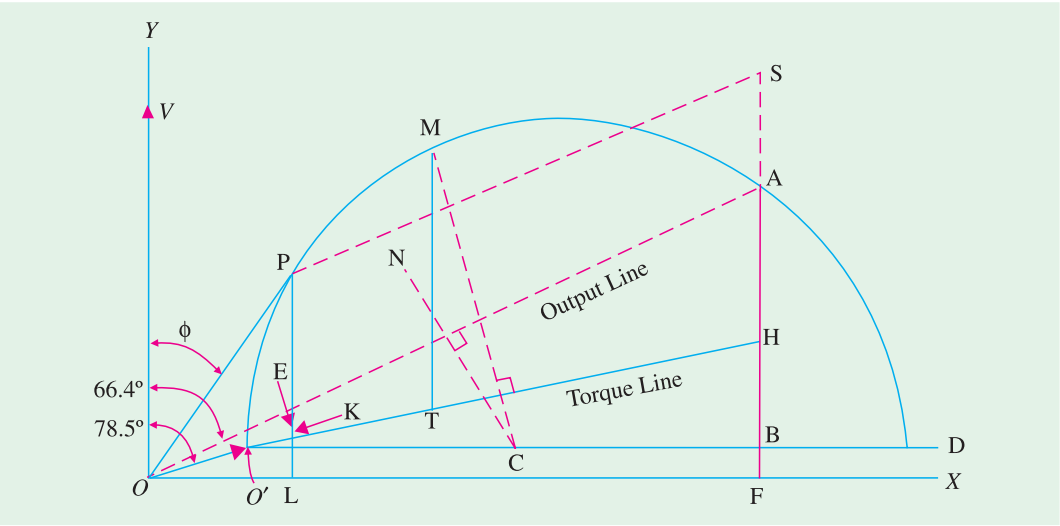


Fig. 35.13

(*a*) F.L motor output = 14,920 W. According to the above power scale, the intercept between the semicircle and the output line $O'A$ should measure = 14,920/3,465 = 4.31 cm. Hence, vertical line PL is found which measures 4.31 cm. Point P represents the full-load operating point.**

(*a*) Line current = OP = 6.5 cm which means that full-load line current

= 6.5×5 = **32.5 A.** $\phi = 32.9°$ (by measurement)

$\therefore$ $\cos 32.9° =$ **0.84** (or $\cos\phi = PL/OP = 5.4/6.5 = 0.84$)

slip $= \frac{EK}{PK} = \frac{0.3}{5.35} = 0.056$ or **5.6%** ; $\eta = \frac{PE}{PL} = \frac{4.3}{5.4} = 0.8$ or **80%**

(*b*) For finding maximum torque, line CM is drawn $\perp$ to torque line $O'H$. MT is the vertical intercept between the semicircle and the torque line and represents the maximum torque of the motor in synchronous watts

Now, MT = 7.8 cm (by measurement) $\therefore$ $T_{max} = 7.8 \times 3465 =$ **27,030 synch. watt**

Example 35.6. *A 415-V, 29.84 kW, 50-Hz, delta-connected motor gave the following test data :*

No-load test : 415 V, 21 A, 1,250 W

Locked rotor test : 100 V, 45 A, 2,730 W

Construct the circle diagram and determine

* The actual scale of the book diagram is different because it has been reduced during block making.

** The operating point may also be found by making AS = 4.31 cm and drawing SP parallel to $O'A$.

(a) *the line current and power factor for rated output* ***(b)*** *the maximum torque. Assume stator and rotor Cu losses equal at standstill.* **(A.C. Machines-I, Jadavpur Univ. 1990)**

Solution. Power factor on no-load is $= \dfrac{1250}{\sqrt{3}\times 415\times 21} = 0.0918$

$\therefore \quad \phi_0 = \cos^{-1}(0.0918) = 84°44'$

Power factor with locked rotor is $= \dfrac{2,730}{\sqrt{3}\times 100\times 45} = 0.3503$

$\therefore \quad \phi_S = \cos^{-1}(0.3503) = 69°30'$

The input current I_{SN} on short-circuit if normal voltage were applied = 45 (415/100) = 186.75 A and power taken would be = 2,730 $(415/100)^2$ = 47,000 W.

Let the current scale be 1 cm = 10 A. The circle diagram of Fig. 35.14 is constructed as follows :

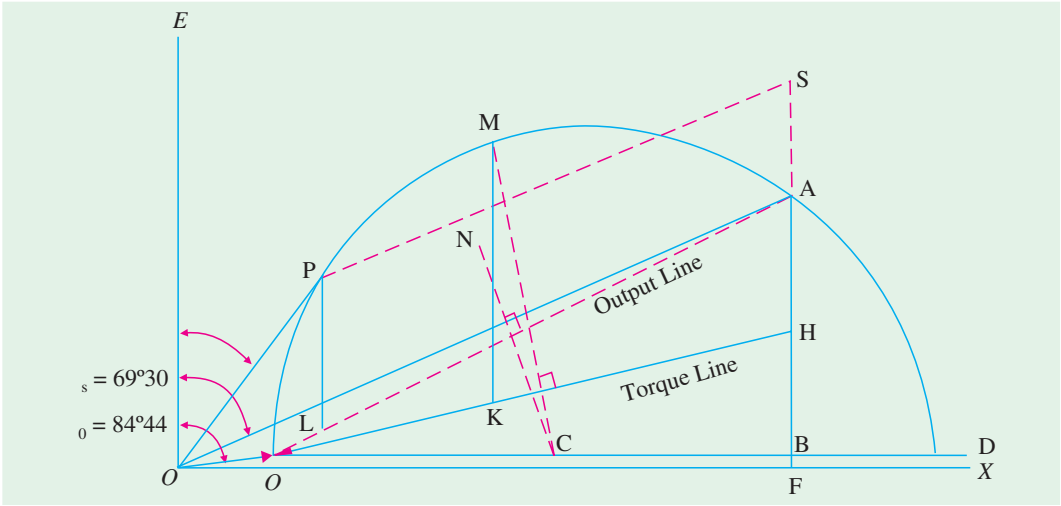


Fig. 35.14

(i) Vector OO' represents 21 A so that it measures 2.1 cm and is laid at an angle of 84°44′ with OE (which is vertical *i.e.* along Y-axis).

(ii) Vector OA measures 186.75/10 = 18.675 cm and is drawn at an angle of 69°30′ with OE.

(iii) $O'D$ is drawn parallel to OX. NC is the right-angle bisector of $O'A$

(iv) With C as the centre and CO' as radius, a semi-circle is drawn as shown. This semi-circle is the locus of the current vector for all load conditions from no-load to short-circuit.

(v) The vertical AF represents power input on short-circuit with normal voltage applied. AF measures 6.6 cm and (as calculated above) represents 47,000 W. Hence, power scale becomes, 1 cm = 47,000/6.6 = 7,120 W

(a) Full-load output = 29,840 W. According to the above power scale, the intercept between the semicircle and output line $O'A$ should measure 29,840/7,120 = 4.19 cm. Hence, line PL is found which measures 4.19 cm. Point P represents the full-load operating point.*

Phase current $= OP$ = 6 cm = 6 × 10 = 60 A; Line current = $\sqrt{3}\times 60$ = **104 A**

Power factor = cos ∠ POE = cos 35° = **0.819**

(b) For finding the maximum torque, line CM is drawn ⊥ to the torque line $O'H$. Point H is such that

$$\frac{AH}{BH} = \frac{\text{rotor Cu loss}}{\text{stator Cu loss}}$$

* The operating point may also by found be making AS = 4.19 cm and drawing SP parallel to $O'A$.

Since the two Cu losses are equal, point H is the mid-point of AB.

Line MK represents the maximum torque of the motor in synchronous watts

MK = 7.3 cm (by measurement) = 7.3 × 7,120 = **51,980 synch. watt.**

Example 35.7. *Draw the circle diagram for a 5.6 kW, 400-V, 3-ϕ, 4-pole, 50-Hz, slip-ring induction motor from the following data :*

No-load readings : 400 V, 6 A, cos ϕ_0 = 0.087 : Short-circuit test : 100 V, 12 A, 720 W.

The ratio of primary to secondary turns = 2.62, stator resistance per phase is 0.67 Ω and of the rotor is 0.185 Ω. Calculate

(*i*) *full-load current* **(*ii*)** *full-load slip*

(*iii*) *full-load power factor*

(*iv*) $\frac{\text{maximum torque}}{\text{full - load torque}}$ **(*v*)** *maximum power.*

Solution. No-load condition

$$\phi_0 = \cos^{-1}(0.087) = 85^\circ$$

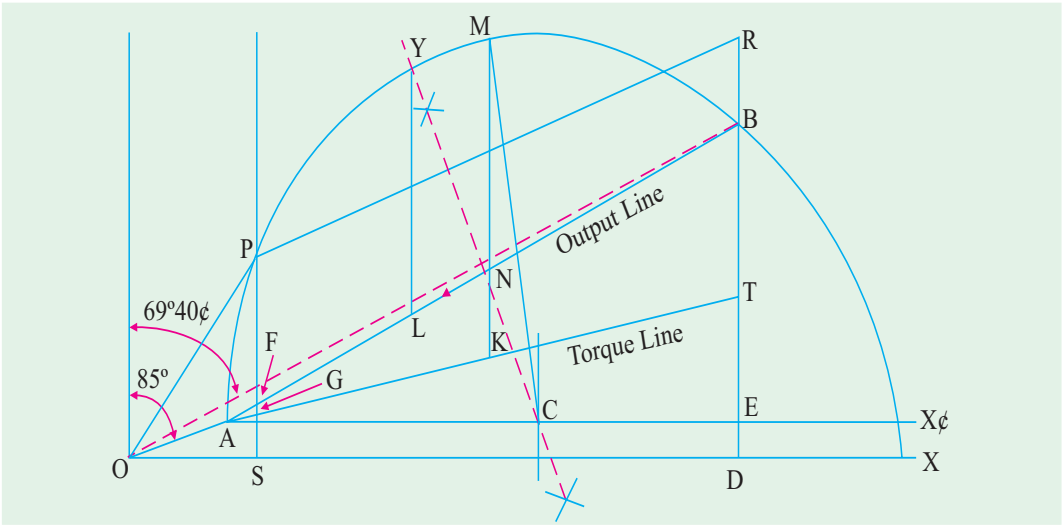


Fig. 35.15

Short-circuit condition

Short-circuit current with normal voltage = 12 × 400/100 = 48 A

$$\text{Total input} = 720 \times (48/12)^2 = 11.52 \text{ kW}$$

$$\cos\phi_s = \frac{720}{\sqrt{3} \times 100 \times 12} = 0.347 \quad \text{or} \quad \phi_s = 69^\circ\,40'$$

Current scale is, 1 cm = 2 A

In the circle diagram of Fig. 35.15, OA = 3 cm and inclined at 85° with OV. Line OB represents short-circuit current with normal voltage. It measures 48/2 = 24 cm and represent 48 A. BD is perpendicular to OX.

For Drawing Torque Line

$K = 2.62$ $R_1 = 0.67\ \Omega$ $R_2 = 0.185\ \Omega$

(in practice, an allowance of 10% is made for skin effect)

$$\therefore \quad \frac{\text{rotor Cu loss}}{\text{stator Cu loss}} = 2.62^2 \times \frac{0.185}{0.67} = 1.9 \qquad \therefore \quad \frac{\text{rotor Cu loss}}{\text{total Cu loss}} = \frac{1.9}{2.9} = 0.655$$

Now BD = 8.25 cm and represents 11.52 kW

power scale = 11.52/8.25 = 1.4 kW/cm

$\therefore$ 1 cm = 1.4 kW

BE represents total Cu loss and is divided at point *T* in the ratio 1.9 : 1.

$BT = BE \times 1.9/2.9 = 0.655 \times 8 = 5.24$ cm

AT is the torque line

Full-load output = 5.6 kW

It is represented by a line = 5.6/1.4 =4 cm

DB is produced to *R* such that *BR* = 4 cm. Line *RP* is parallel to output line and cuts the circle at *P*. OP represents full-load current.

PS is drawn vertically. Points *M* and *Y* represent points of maximum torque and maximum output respectively.

(*i*) F.L. current = $OP = 5.75$ cm $= 5.75 \times 2 =$ **11.5 A**

(*ii*) F.L. slip = $\frac{FG}{PG} = \frac{0.2}{4.25} = 0.047$ or **4.7%**

(*iii*) p.f. = $\frac{SP}{OP} = \frac{4.6}{5.75} =$ **0.8**

(*iv*) $\frac{\text{max. torque}}{\text{full-load torque}} = \frac{MK}{PG} = \frac{10.05}{4.25} =$ **2.37**

(*v*) Maximum output is represented by $YL = 7.75$ cm.

$\therefore$ Max. output = $7.75 \times 1.4 =$ **10.8 kW**

Example 35.8. *A 440-V, 3-ϕ, 4-pole, 50-Hz slip-ring motor gave the following test results :*

No-load reading : 440 V, 9 A, p.f. = 0.2

Blocked rotor test : 110 V, 22 A, p.f. = 0.3

The ratio of stator to rotor turns per phase is 3.5/1. The stator and rotor Cu losses are divided equally in the blocked rotor test. The full-load current is 20 A. Draw the circle diagram and obtain the following :

(*a*) *power factor, output power, efficiency and slip at full-load*

(*b*) *standstill torque or starting torque.*

(*c*) *resistance to be inserted in the rotor circuit for giving a starting torque 200 % of the full-load torque. Also, find the current and power factor under these conditions.*

Solution. No-load p.f. = 0.2 $\therefore$ $\phi_0 = \cos^{-1}(0.2) = 78.5^\circ$

Short-circuit p.f. = 0.3 $\therefore$ $\phi_S = 72.5^\circ$

Short-circuit current at normal voltage = $22 \times 440/110 = 88$ A

S.C. input = $\sqrt{3} \times 440 \times 88 \times 0.3 = 20{,}120$ W = 20.12 kW

Take a current scale of 1 cm = 4 A

In the circle diagram of Fig. 35.16, *OA* = 2.25 cm drawn at an angle of 78.5° behind *OV*. Similarly, *OB* = 88/4 = 22 cm and is drawn at an angle of 72.50 behind OV. The semi-circle is drawn as usual. Point *T* is such that *BT* = *TD*. Hence, torque line *AT* can be drawn. *BC* represents 20.12 kW. By measurement *BC* = 6.6 cm.

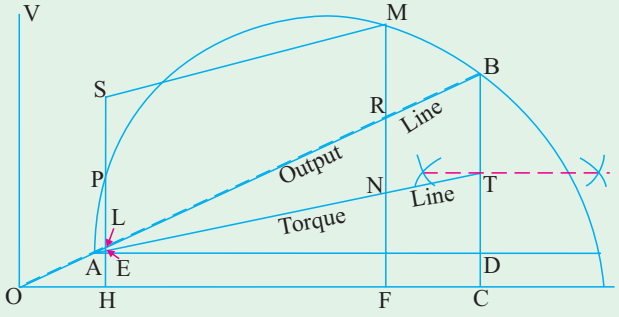


Fig. 35.16

$\therefore$ power scale = 20.12/6.6 = 3.05

$\therefore$ 1 cm = 3.05 kW

Full-load current = 20 A. Hence, it is represented by a length of 20/4 = 5 cm. With *O* as centre and 5 cm as radius, an arc is drawn which cuts the semi-circle at point *P*. This point represents full-load condition. *PH* is drawn perpendicular to the base *OC*.

(*a*) (*i*) p.f. = cos ϕ = *PH*/*OP* = 4.05/5 = **0.81**

(*ii*) Torque can be found by measuring the input.

Rotor input = *PE* = 3.5 cm = 3.5 × 3.05 = 10.67 kW

Now N_s = 120 × 50/4 = 1500 r.p.m.

$\therefore$ $T_g = 9.55\, P_2/N_s = 9.55 \times 10{,}670/1500 =$ **61 N-m**

(*iii*) output = *PL* = 3.35 × 3.05 = **10.21 kW**

(*iv*) $\text{efficiency} = \dfrac{\text{output}}{\text{input}} = \dfrac{PL}{PH} = \dfrac{3.35}{4.05} = 0.83$ or **83%**

(*v*) $\text{slip } s = \dfrac{\text{rotor Cu loss}}{\text{rotor input}} = \dfrac{LE}{PE} = \dfrac{0.1025}{3.5} = 0.03$ or **3%**

(*b*) Standstill torque is represented by *BT*.

BT = 3.1 cm = 3.1 × 3.05 = 9.45 kW $\quad\therefore\quad T_{st} = 9.55 \times \dfrac{9.45 \times 10^3}{1500} =$ **60.25 N-m**

(*c*) We will now locate point *M* on the semi-circle which corresponds to a starting torque twice the full-load torque *i.e.* 200% of F.L. torque.

Full-load torque = *PE*. Produce *EP* to point *S* such that *PS* = *PE*. From point *S* draw a line parallel to torque line *AT* cutting the semi-circle at *M*. Draw *MN* perpendicular to the base.

At starting when rotor is stationary, *MN* represents total rotor copper losses.

NR = Cu loss in rotor itself as before ; *RM* = Cu loss in external resistance

RM = 4.5 cm = 4.5 × 3.05 = 13.716 kW = 13,716 watt.

Cu loss/phase = 13,716/3 =4,572 watt

Rotor current *AM* = 17.5 cm = 17.5 × 4 = 70 A

Let r_2' be the additional external resistance in the rotor circuit (as referred to stator) then

$r_2' \times 70^2 = 4{,}572$ or r_2' = 4,572/4,900 = 0.93 Ω

Now K = 1/3.5

$\therefore$ rotor resistance/phase, $r_2 = r_2' \times K^2 = 0.93/3.5^2 =$ **0.076 Ω**

Stator current = *OM* = 19.6 × 4 = **78.4 A** ; power factor = $\dfrac{MF}{OM} = \dfrac{9.75}{18.7} =$ **0.498**

Example 35.9. *Draw the circle diagram of a 7.46 kW, 200-V, 50-Hz, 3-phase slip-ring induction motor with a star-connected stator and rotor, a winding ratio of unity, a stator resistance of 0.38 ohm/phase and a rotor resistance of 0.24 ohm/phase. The following are the test readings ;*

No-load : 200 V, 7.7 A, $\cos\phi_0$ = *0.195*

Short-circuit : 100 V, 47.6 A, $\cos\phi_s$ = *0.454*

Find **(*a*)** *starting torque and* **(*b*)** *maximum torque, both in synchronous watts*

(*c*) *the maximum power factor* **(*d*)** *the slip for maximum torque*

(*e*) *the maximum output* **(Elect. Tech.-II, Madras Univ. 1989)**

Solution. $\phi_0 = \cos^{-1}(0.195) = 78°45'$; $\phi_S = \cos^{-1}(0.454) = 63°$

The short-circuit I_{SN} with normal voltage applied is = 47.6 × (200/100) = 95.2 A

The circle diagram is drawn as usual and is shown in fig. 35.17.

With a current scale of 1 cm = 5 A, vector *OO'* measures 7.7/5 =1.54 cm and represents the no-load current of 7.7 A.

Similarly, vector OA represents I_{SN} *i.e.* short-circuit current with normal voltage and measures 95.2/5 = 19.04 cm

Both vectors are drawn at their respective angles with OE.

The vertical line AF measures the power input on short-circuit with normal voltage and is = $\sqrt{3} \times 200 \times 95.2 \times 0.454 = 14{,}970$ W.

Since AF measures 8.6 cm, the power scale is 1 cm = 14,970/8.6 = 1740 W

The point H is such that

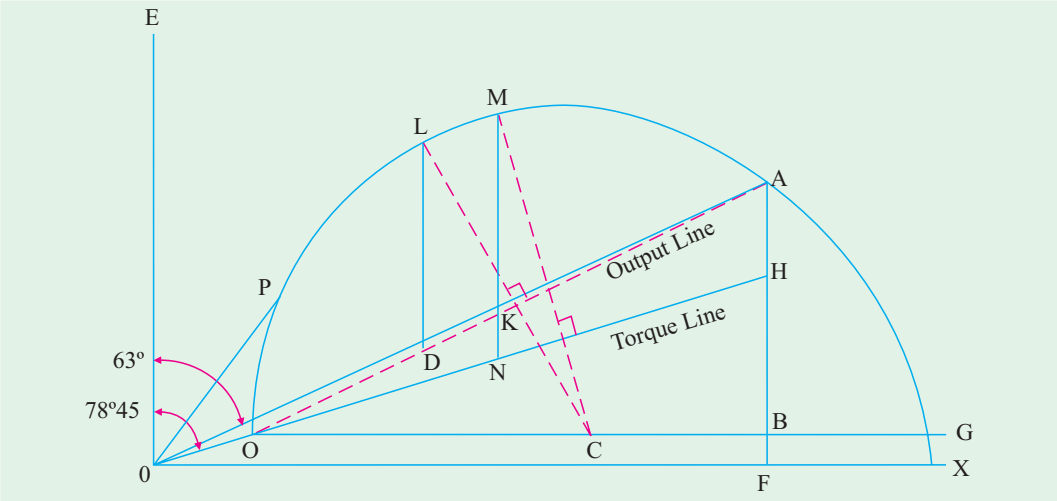


Fig. 35.17

$$\frac{AH}{AB} = \frac{\text{rotor Cu loss}}{\text{total Cu loss}} = \frac{\text{rotor resistance}^*}{\text{rotor + stator resistance}} = \frac{0.24}{0.62}$$

Now AB = 8.2 cm (by measurement) $\therefore AH = 8.2 \times 0.24/0.62 = 3.2$ cm

(*a*) Starting torque = AH = 3.2 cm = 3.2×1740 = **5,570 synch. watt.**

(*b*) Line CM is drawn perpendicular to the torque line $O'H$. The intercept MN represents the maximum torque in synchronous watts.

Maximum torque = MN = 7.15 cm = 7.15×1740 = **12,440 synch. watts.**

(*c*) For finding the maximum power, line OP is drawn tangential to the semi-circle.

$\angle POE = 28.5°$

$\therefore$ maximum p.f. = $\cos 28.5°$ = **0.879**

(*d*) The slip for maximum torque is = KN/MN= 1.4/7.15 = **0.195**

(*e*) Line CL is drawn perpendicular to the output line $O'A$. From L is drawn the vertical line LD. It measures 5.9 cm and represents the maximum output.

$\therefore$ maximum output = 5.9×1740 = **10,270 W**

Tutorial Problems 35.1

1. A 300 h.p. (223.8 kW), 3000-V, 3-ϕ, induction motor has a magnetising current of 20 A at 0.10 p.f. and a short-circuit (or locked) current of 240 A at 0.25 p.f. Draw the circuit diagram, determine the p.f. at full-load and the maximum horse-power. **[0.85 p.f. 621 h.p. (463.27 kW)] (*I.E.E. London*)**
2. The following are test results for a 18.65 kW, 3-ϕ, 440.V slip-ring induction motor :
   Light load : 440-V, 7.5 A, 1350 W (including 650 W friction loss).

* Because K = 1, otherwise it should be $R_2' = R_2/K^2$.

S.C. test : 100 V, 32 A, 1800 W

Draw the locus diagram of the stator current and hence obtain the current, p.f. and slip on full-load. On short-circuit, the rotor and stator copper losses are equal. **[30 A, 0.915, 0.035] (*London Univ*)**

3. Draw the circle diagram for 20 h.p. (14.92 kW), 440-V, 50-Hz, 3-ϕ induction motor from the following test figures (line values) :

No-load : 440 V, 10A, p.f. 0.2 Short-circuit : 200 V, 50 A, p.f. 0.4

From the diagram, estimate (*a*) the line current and p.f. at full-load (*b*) the maximum power developed (*c*) the starting torque. Assume the rotor and stator I^2R losses on short-circuit to be equal.

[(*a*) 28.1 A at 0.844 p.f. (*b*) 27.75 kW (*c*) 11.6 synchronous kW/phase] (*London Univ.*)

4. A 40 h.p. (29.84.kW), 440-V, 50-Hz, 3-phase induction motor gave the following test results

No. load : 440 V, 16 A, p.f. = 0.15 S.C. test : 100 V, 55 A, p.f. = 0.225

Ratio of rotor to stator losses on short-circuit = 0.9. Find the full-load current and p.f., the pull-out torque and the maximum output power developed.

[49 A at 0.88 p.f. ; 78.5 synch. kW or 2.575 times F.L. torque ; 701.2 kW] (*I.E.E. London*)

5. A 40 h.p. (29.84 kW), 50-Hz, 6-pole, 420-V, 3-ϕ, slip-ring induction motor furnished the following test figures :

No-load : 420 V, 18 A, p.f. = 0.15 S.C. test : 210 V, 140 A, p.f. = 0.25

The ratio of stator to rotor Cu losses on short-circuit was 7 : 6. Draw the circle diagram and find from it (*a*) the full-load current and power factor (*b*) the maximum torque and power developed.

[(*a*) 70 A at 0.885 p.f. (*b*) 89.7 kg.m ; 76.09 kW] (*I.E.E. London*)

6. A 500 h.p. (373 kW), 8-pole, 3-ϕ, 6,000-V, 50-Hz induction motor gives on test the following figures :

Running light at 6000 V, 14 A/phase, 20,000 W ; Short-circuit at 2000 V, 70 A/phase, 30,500 W

The resistance/phase of the star-connected stator winding is 1.1 Ω, ratio of transformation is 4 :1. Draw the circle diagram of this motor and calculate how much resistance must be connected in each phase of the rotor to make it yield full-load torque at starting. **[0.138 Ω] (*London Univ.*)**

7. A 3-phase induction motor has full-load output of 18.65 kW at 220 V, 720 r.p.m. The full-load p.f. is 0.83 and efficiency is 85%. When running light, the motor takes 5 A at 0.2 p.f. Draw the circle diagram and use it to determine the maximum torque which the motor can exert (*a*) in N-m (*b*) in terms of full-load torque and (*c*) in terms of the starting torque.

[(*a*) 268.7 N-m (*b*) 1.08 (*c*) 7.2 approx.] (*London Univ.*)

8. A 415-V, 40 h.p. (29.84 kW), 50.Hz, Δ-connected motor gave the following test data :

No-load test : 415 V, 21 A, 1250 W ; Locked rotor test : 100 V, 45 A, 2,730 W

Construct the circle diagram and determine

(*a*)the line current and power factor for rated output (*b*) the maximum torque. Assume stator and rotor Cu losses equal at standstill.

[(*a*) 104 A : 0.819 (*b*) 51,980 synch watt] (*A.C. Machines-I, Jadavpur Univ. 1978*)

9. Draw the no-load and short circuit diagram for a 14.92 kW., 400-V, 50-Hz, 3-phase star-connected induction motor from the following data (line values) :

No load test : 400 V, 9 A, cos ϕ = 0.2

Short circuit test : 200 V, 50 A, cos ϕ = 0.4

From the diagram find (*a*) the line current and power factor at full load, and (*b*) the maximum output power. **[(*a*) 32.0 A, 0.85 (*b*) 21.634 kW]**

35.9. Starting of Induction Motors

It has been shown earlier that a plain induction motor is similar in action to a polyphase transformer with a short-circuited rotating secondary. Therefore, if normal supply voltage is applied to the stationary motor, then, as in the case of a transformer, a very large initial current is taken by the primary, at least, for a short while. It would be remembered that exactly similar conditions exist in the case of a d.c. motor, if it is thrown directly across the supply lines, because at the time of starting it, there is no back e.m.f. to oppose the initial inrush of current.

Induction motors, when direct-switched, take five to seven times their full-load current and develop only 1.5 to 2.5 times their full-load torque. This initial excessive current is objectionable because it will produce large line-voltage drop that, in turn, will affect the operation of other electrical equipment connected to the same lines. Hence, it is not advisable to line-start motors of rating above 25 kW to 40 kW.

It was seen in Art. 34.15 that the starting torque of an induction motor can be improved by increasing the resistance of the rotor circuit. This is easily feasible in the case of slip-ring motors but not in the case of squirrel-cage motors. However, in their case, the initial in-rush of current is controlled by applying a reduced voltage to the stator during the starting period, full normal voltage being applied when the motor has run up to speed.

35.10. Direct-switching or Line starting of Induction Motors

It has been shown earlier that

Rotor input $= 2\pi N_s T = k\,T$ –Art. 34.36

Also, rotor Cu loss $= s \times$ rotor input

$\therefore \quad 3 I_2^2 R_2 = s \times kT \quad \therefore \quad T \propto I_2^2/s$ (if R_2 is the same)

Now $I_2 \propto I_1 \quad \therefore \quad T \propto I_1^2/s$ or $T = K I_1^2/s$

At starting moment $s = 1 \quad \therefore \quad T_{st} = K I_{st}^2$ where I_{st} = starting current

If I_f = normal full-load current and s_f = full-load slip

then $T_f = K I_f^2 / s_f \quad \therefore \quad \dfrac{T_{st}}{T_f} = \left(\dfrac{I_{st}}{I_f}\right)^2 . s_f$

When motor is direct-switched onto normal voltage, then starting current is the short-circuit current I_{sc}.

$$\therefore \quad \frac{T_{st}}{T_f} = \left(\frac{I_{sc}}{I_f}\right)^2 \cdot s_f = a^2 . s_f \quad \text{where } a = I_{sc}/I_f$$

Suppose in a case, $I_{sc} = 7 I_f$, $s_f = 4\% = 0.04$, the $T_{st}/T_f = 7^2 \times 0.04 = 1.96$

$\therefore$ starting torque $= 1.96 \times$ full-load torque

Hence, we find that with a current as great as seven times the full-load current, the motor develops a starting torque which is only 1.96 times the full-load value.

Some of the methods for starting induction motors are discussed below :

Squirrel-cage Motors

(*a*) Primary resistors (or rheostat) or reactors
(*b*) Auto-transformer (or autostarter)
(*c*) Star-delta switches

In all these methods, terminal voltage of the squirrel-cage motor is reduced during starting.

Slip-ring Motors

(*a*) Rotor rheostat

35.11. Squirrel-cage Motors

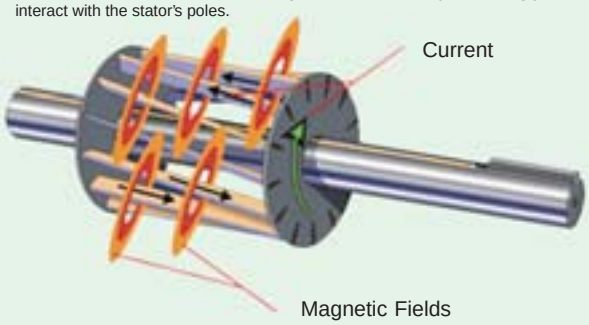
(*a*) Primary resistors

Their purpose is to drop some voltage and hence reduce the voltage applied across the motor terminals. In this way, the initial current drawn by the motor is reduced. However, it should be noted that whereas current varies directly as the voltage, the torque varies as square of applied voltage*

* When applied voltage is reduced, the rotating flux Φ is reduced which, in turn, decreases rotor e.m.f. and hence rotor current I_2. Starting torque, which depends both on Φ and I_2 suffers on two counts when impressed voltage is reduced.

Squirrel Cage Rotor

When the stator's moving magnetic field cuts across the rotor's conductor bars, it induces voltage in them.
This voltage produces current, which circulates through the bars and around the rotor end ring. This current in turn produces magnetic fields around each rotor bar. The continuously changing stator magnetic field results in a continuously changing rotor field. The rotor becomes an electromagnet with continuously alternating poles, which interact with the stator's poles.



Squirrel cage rotor

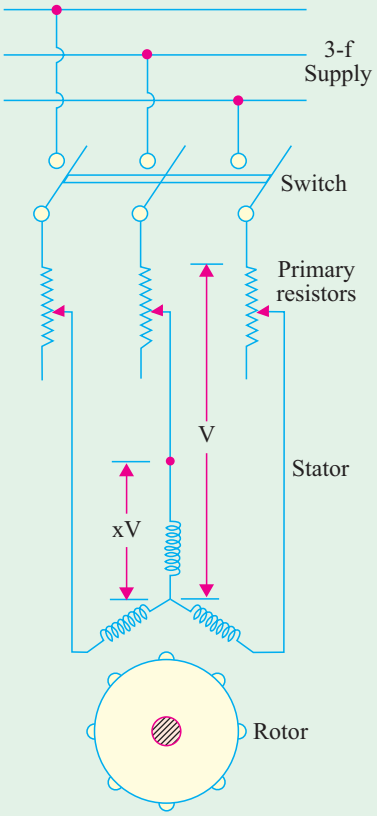


Fig. 35.18

(Art 34.17). If the voltage applied across the motor terminals is reduced by 50%, starting current is reduced by 50%, but torque is reduced to 25% of the full-voltage value.

By using primary resistors (Fig. 35.18), the applied voltage/phase can be reduced by a fraction 'x' (and it additionally improves the power factor of the line slightly).

$$I_{st} = x\, I_{sc} \quad \text{and} \quad T_{st} = x^2\, T_{sc}$$

As seen from Art 35.10, above,

$$\frac{T_{st}}{T_f} = \left(\frac{I_{st}}{I_f}\right)^2 . s_f = \left(\frac{x\, I_{sc}}{I_f}\right)^2 s_f$$

$$= x^2 \left(\frac{I_{sc}}{I_f}\right)^2 s_f = x^2 . a^2 . s_f$$

It is obvious that the ratio of the starting torque to full-load torque is x^2 of that obtained with direct switching or across-the-line starting. This method is useful for the smooth starting of small machines only.



Variable auto-transformer

(*b*) Auto-transformers

Such starters, known variously as ***auto-starters or compensators***, consist of an auto-transformer, with necessary switches. We may use either two auto-transformers connected as usual [Fig. 35.19 (*b*)] or 3 auto-transformers connected in open delta [Fig. 35.19 (*a*)]. ***This method can be used both for star-and delta-connected motors***. As shown in Fig. 35.20 with starting connections, a reduced voltage is applied across the motor terminals. When the motor has ran up to say, 80% of its normal speed, connections are so changed that auto-transformers are cut out and full supply voltage is applied across the motor. The switch making these changes from 'start' to 'run' may be airbreak (for small motors) or may be oil-immersed (for large motors) to reduce sparking. There is also provision for no-voltage and over-load protection, along with a time-delay device, so that momentary interruption of voltage or momentary over-load do not disconnect the motor from supply line. Most of the auto-starters are provided with 3 sets of taps, so as to reduce voltage to 80, 65 or 50 per cent of the line voltage, to suit the local conditions of supply. The

V-connected auto-transformer is commonly used, because it is cheaper, although the currents are unbalanced during starting period. This is, however, not much objectionable firstly, because the current imbalance is about 15 per cent and secondly, because balance is restored as soon as running conditions are attained.

The quantitative relationships between the motor current, line current, and torque developed can be understood from Fig.35.20.

In Fig 35.20 (*a*) is shown the case when the motor is direct-switched to lines. The motor current is, say, 5 times the full-load current. If V is the line voltage, then voltage/phase across motor is $V/\sqrt{3}$.

$$\therefore \qquad I_{sc} = 5\, I_f = \frac{V}{\sqrt{3}Z}$$ where Z is stator impedance/phase.

In the case of auto-transformer, if a tapping of transformation ratio K is used, then phase voltage across motor is $KV/\sqrt{3}$, as marked in Fig. 35.20 (*b*).

$$\therefore \quad \text{motor current at starting } I_2 = \frac{KV}{\sqrt{3}Z} = K \,.\, \frac{V}{\sqrt{3}Z} = K \,.\, I_{sc} = K \,.\, 5\, I_f$$

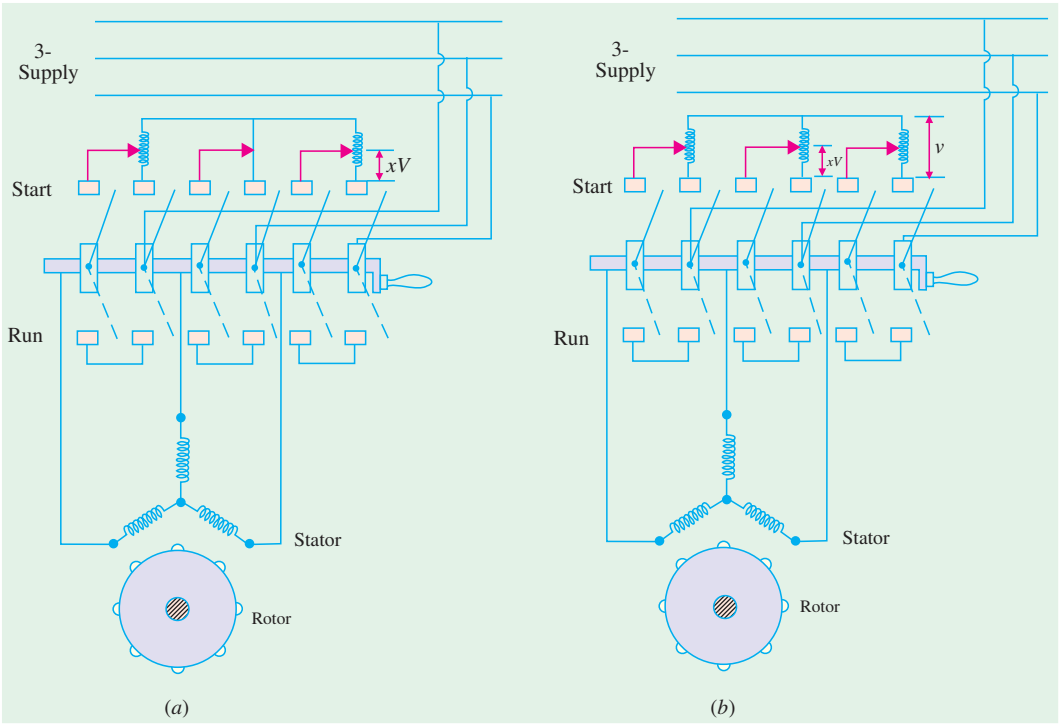


Fig. 35.19

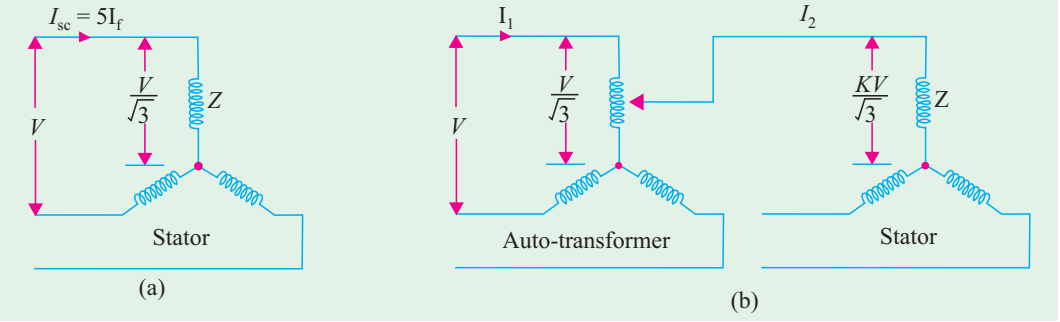


Fig. 35.20

The current taken from supply or by auto-transformer is $I_1 = K I_2 = K^2 \times 5 I_f = K^2 I_{sc}$ if magnetising current of the transformer is ignored. Hence, we find that although motor current per phase is reduced only K times the direct-switching current ($\because$ $K < 1$), the current taken by the line is reduced K^2 times.

Now, remembering that torque is proportional to the square of the voltage, we get

With direct-switching, $T_1 \propto (V/\sqrt{3})^2$; With auto-transformer, $T_2 \propto (KV/\sqrt{3})^2$

$\therefore$ $T_2/T_1 = (KV/\sqrt{3})^2/(V/\sqrt{3})^2$ or $T_2 = K^2 T_1$ or $T_{st} = K^2 . T_{sc}$

$\therefore$ torque with auto-starter $= K^2 \times$ torque with direct-switching.

Relation Between Starting and F.L. Torque

It is seen that voltage across motor phase on direct-switching is $V/\sqrt{3}$ and starting current is $I_{st} = I_{sc}$. With auto-starter, voltage across motor phase is $KV/\sqrt{3}$ and $I_{st} = K I_{sc}$

Now, $T_{st} \propto I_{st}^2 \, (s = 1)$ and $T_f \propto \dfrac{I_f^2}{s_f^2}$

$\therefore$ $\dfrac{T_{st}}{T_f} = \left(\dfrac{I_{st}}{I_f}\right)^2 s_f$ or $\dfrac{T_{st}}{T_f} = K^2 \left(\dfrac{I_{sc}}{I_f}\right)^2 s_f = K^2 . a^2 . s_f$ ($\because$ $I_{st} = K I_{sc}$)

Note that this expression is similar to the one derived in Art. 34.11. (*a*) except that x has been replaced by transformation ratio K.

Example 35.10. *Find the percentage tapping required on an auto-transformer required for a squirrel-cage motor to start the motor against 1/4 of full-load torque. The short-circuit current on normal voltage is 4 times the full-load current and the full-load slip is 3%.*

Solution. $\dfrac{T_{st}}{T_f} = \dfrac{1}{4}$, $\dfrac{I_{sc}}{I_f} = 4$, $s_f = 0.03$

$\therefore$ Using $\dfrac{T_{st}}{T_f} = K^2 \left(\dfrac{I_{sc}}{I_f}\right)^2 s_f$, we get $\dfrac{1}{4} = K^2 \times 4^2 \times 0.03$

$\therefore$ $K^2 = \dfrac{1}{64 \times 0.03}$ $\therefore$ $K = 0.722$ or $K = \mathbf{72.2\%}$

Example 35.11. *A 20 h.p. (14.92 kW), 400-V, 950 r.p.m., 3-ϕ, 50-Hz, 6-pole cage motor with 400 V applied takes 6 times full-load current at standstill and develops 1.8 times full-load running torque. The full-load current is 30 A.*

(a) *what voltage must be applied to produce full-load torque at starting ?*

(b) *what current will this voltage produce ?*

(c) *if the voltage is obtained by an auto-transformer, what will be the line current ?*

(d) *if starting current is limited to full-load current by an auto-transformer, what will be the starting torque as a percentage of full-load torque ?*

Ignore the magnetising current and stator impedance drops.

Solution. *(a)* Remembering that $T \propto V^2$, we have

In the first case, $1.8 \, T_f \propto 400^2$, ; In the second case, $T_f \propto V^2$

$\therefore$ $\left(\dfrac{V}{400}\right)^2 = \dfrac{1}{1.8}$ or $V = \dfrac{400}{\sqrt{1.8}} = \mathbf{298.1\ V}$

(b) Currents are proportional to the applied voltage.

$\therefore$ $6 I_f \propto 400$; $I \propto 298.1$ $\therefore$ $I = 6 \times \dfrac{298.1}{400}$, $I_f = \dfrac{6 \times 298.1 \times 30}{400} = \mathbf{134.2\ A}$

(c) Here $K = 298.1/400$

Line current $= K^2 I_{sc} = (298.1/400)^2 \times 6 \times 30 =$ **100 A**

(d) We have seen in Art. 33.11 (*b*) that line current $= K^2 I_{sc}$

Now, line current = full-load current I_f (given)

$\therefore \quad 30 = K^2 \times 6 \times 30 \quad \therefore \quad K^2 = 1/6$

Now, using $\frac{T_{st}}{T_f} = K^2 \left(\frac{I_{sc}}{I_f}\right)^2 \times s_f$ we get $\frac{T_{st}}{T_f} = \frac{1}{6} \times \left(\frac{6 I_f}{I_f}\right)^2 \times 0.05 = 0.3$

Here $N_s = 120 \times 50/6 = 1000$ r.p.m. $N = 950$ r.p.m.; $s_f = 50/1000 = 0.05$

$\therefore \quad T_{st} = 0.3\, T_f$ or **30%** F.L. torque

Example 35.12. *Determine the suitable auto-transformation ratio for starting a 3-phase induction motor with line current not exceeding three times the full-load current. The short-circuit current is 5 times the full-load current and full-load slip is 5%.*

Estimate also the starting torque in terms of the full-load torque.

(Elect. Engg.II, Bombay Univ. 1987)

Solution. Supply line current $= K^2 I_{sc}$

It is given that supply line current at start equals $3 I_f$ and short-circuit current $I_{sc} = 5 I_f$ where I_f is the full-load current

$\therefore \quad 3 I_f = K^2 \times 5 I_f$ or $K^2 = 0.6 \quad \therefore \quad K = 0.775$ or **77.5%**

In the case of an auto starter,

$$\frac{T_{st}}{T_f} = K^2 \left(\frac{I_{sc}}{I_f}\right)^2 \times s_f \quad \therefore \quad \frac{T_{st}}{T_f} = 0.6 \times \left(\frac{5 I_f}{I_f}\right)^2 \times 0.05 = 0.75$$

$\therefore \quad T_{st} = 0.75\, T_f =$ **75%** of full-load torque.

Example 35.13. *The full-load slip of a 400-V, 3-phase cage induction motor is 3.5% and with locked rotor, full-load current is circulated when 92 volt is applied between lines. Find necessary tapping on an auto-transformer to limit the starting current to twice the full-load current of the motor. Determine also the starting torque in terms of the full-load torque.*

(Elect. Machines, Banglore Univ. 1991)

Solution. Short-circuit current with full normal voltage applied is

$I_{sc} = (400/92)\, I_f = (100/23)\, I_f$

Supply *line* current $= I_{st} = 2 I_f$

Now, line current $I_{st} = K^2 I_{sc}$

$\therefore \quad 2 I_f = K^2 \times (100/23)\, I_f \quad \therefore K^2 = 0.46$; $K = 0.678$ or **67.8%**

Also, $\frac{T_{st}}{T_f} = K^2 \left(\frac{I_{sc}}{I_f}\right)^2 \times s_f = 0.46 \times (100/23)^2 \times 0.035 = 0.304$

$\therefore \quad T_{st} =$ **30.4%** of full-load torque

Tutorial Problems 35.2

1. A 3-ϕ motor is designed to run at 5% slip on full-load. If motor draws 6 times the full-load current at starting at the rated voltage, estimate the ratio of starting torque to the full-load torque.

   [1.8] (*Electrical Engineering Grad, I.E.T.E. Dec. 1986*)

2. A squirrel-cage induction motor has a short-circuit current of 4 times the full-load value and has a full-load slip of 5%. Determine a suitable auto-transformer ratio if the supply line current is not to

exceed twice the full-load current. Also, express the starting torque in terms of the full-load torque. Neglect magnetising current. **[70.7%, 0.4]**

3. A 3-ϕ, 400-V, 50-Hz induction motor takes 4 times the full-load current and develops twice the full-load torque when direct-switched to 400-V supply. Calculate in terms of full-load values (*a*) the line current, the motor current and starting torque when started by an auto-starter with 50% tap and (*b*) the voltage that has to be applied and the motor current, if it is desired to obtain full-load torque on starting. **[(*a*) 100%, 200%, 50% (*b*) 228 V, 282%]**

(*c*) Star-delta Starter

This method is used in the case of motors which are built to run normally with a delta-connected stator winding. It consists of a two-way switch which connects the motor in star for starting and then in delta for normal running. The usual connections are shown in Fig. 35.21. When star-connected, the applied voltage over each motor phase is reduced by a factor of $1/\sqrt{3}$ and hence the torque developed becomes 1/3 of that which would have been developed if motor were directly connected in delta. The line current is reduced to 1/3. Hence, during starting period when motor is *Y*-connected, it takes 1/3rd as much starting current and develops 1/3rd as much torque as would have been developed were it directly connected in delta.

Relation Between Starting and F.L. Torque

$$I_{st} \text{ per phase } = \frac{1}{\sqrt{3}} I_{sc} \text{ per phase}$$

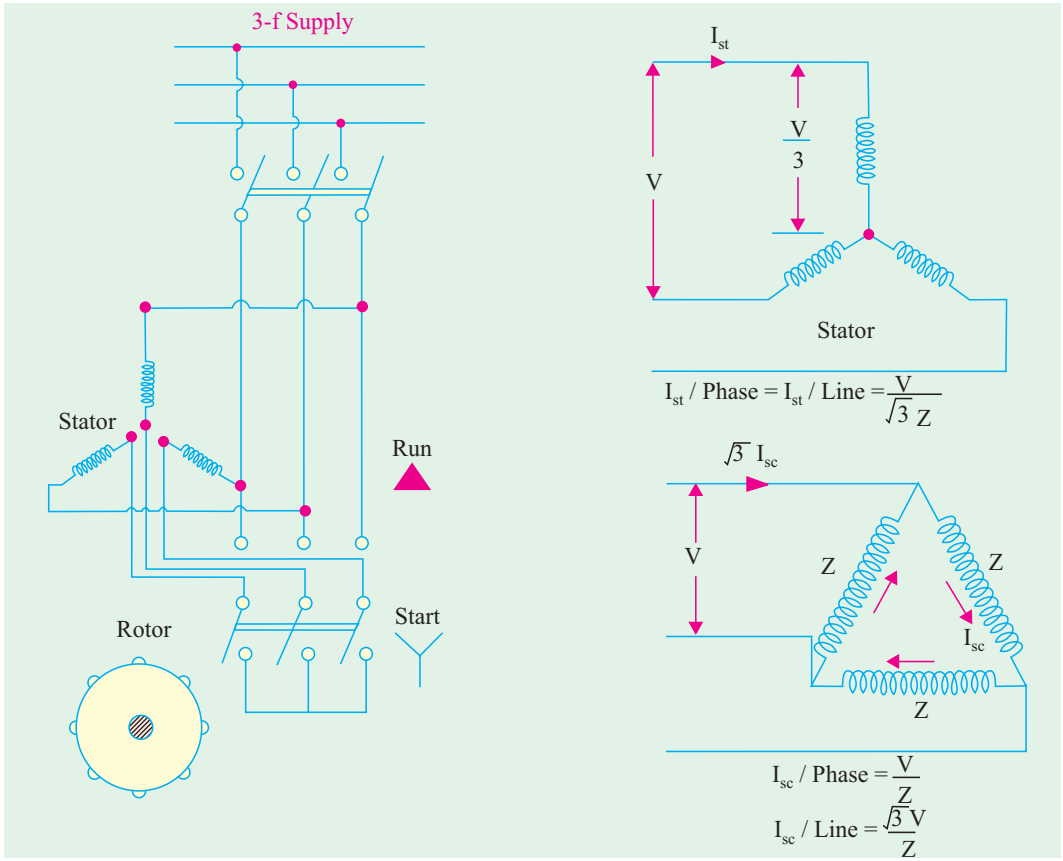


Fig. 35.21

where I_{sc} is the current/phase which Δ-connected motor would have taken if switched on to the supply directly (however, line current at start = 1/3 of line I_{sc})

Now $T_{st} \propto I_{st}^2 \quad (s = 1)$ – Art. 35.10

$T_f \propto I_f^2 / s_f$

$$\therefore \quad \frac{T_{st}}{T_f} = \left(\frac{I_{st}}{I_f}\right)^2 s_f = \left(\frac{I_{sc}}{\sqrt{3} I_f}\right)^2 s_f = \frac{1}{3}\left(\frac{I_{sc}}{I_f}\right)^2 s_f = \frac{1}{3} a^2 s_f$$

Here, I_{st} and I_{sc} represent phase values.

It is clear that the star-delta swith is equivalent* to an auto-transformer of ratio 1/$\sqrt{3}$ or 58% approximately.

This method is cheap and effective provided the starting torque is required not to be more than 1.5 times the full-load torque. Hence, it is used for machine tools, pumps and motor-generators etc.

Example 35.14. *The full-load efficiency and power factor of a 12-kW, 440-V, 3-phase induction motor are 85% and 0.8 lag respectively. The blocked rotor line current is 45 A at 220 V. Calculate the ratio of starting to full-load current, if the motor is provided with a star-delta starter. Neglect magnetising current.* **(Elect. Machines, A.M.I.E. Sec. B, 1991)**

Solution. Blocked rotor current with full voltage applied

$$I_{sc} = 45 \times 440/220 = 90 \text{ A}$$

Now, $\sqrt{3} \times 440 \times I_f \times 0.8 = 12{,}000/0.85, \quad \therefore \quad I_f = 23.1$ A

In star-delta starter, $I_{st} = I_{sc}/\sqrt{3} = 90/\sqrt{3} = 52$ A

$\therefore \quad I_{st}/I_f = 52/23.1 = \mathbf{2.256}$

Example 35.15. *A 3-phase, 6-pole, 50-Hz induction motor takes 60 A at full-load speed of 940 r.p.m. and develops a torque of 150 N-m. The starting current at rated voltage is 300 A. What is the starting torque? If a star/delta starter is used, determine the starting torque and starting current.*
(Electrical Machinery-II, Mysore Univ. 1988)

Solution. As seen from Art. 33.10, for direct-switching of induction motors

$$\frac{T_{st}}{T_f} = \left(\frac{I_{sc}}{I_f}\right)^2 s_f. \quad \text{Here,} \quad I_{st} = I_{sc} = 300 \text{ A (line value)}; \; I_f = 60 \text{ A (line value)},$$

$s_f = (1000 - 940)/1000 = 0.06$; $T_f = 150$ N-m

$\therefore \quad T_{st} = 150(300/60)^2 \times 0.06 = \mathbf{225 \text{ N-m}}$

When star/delta starter is used

Starting current = 1/3 × starting current with direct starting = 300/3 = **100 A**

Starting torque = 225/3 = **75 N-m** – Art 35-11 (*c*)

Example 35.16. *Determine approximately the starting torque of an induction motor in terms of full-load torque when started by means of* **(*a*)** *a star-delta switch* **(*b*)** *an auto-transformer with 70.7 % tapping. The short-circuit current of the motor at normal voltage is 6 times the full-load current and the full-load slip is 4%. Neglect the magnetising current.*
(Electrotechnics, M.S. Univ. Baroda 1986)

Solution. (*a*) $$\frac{T_{st}}{T_f} = \frac{1}{3}\left(\frac{I_{sc}}{I_f}\right)^2 s_f = \frac{1}{3} \times 6^2 \times 0.04 = 0.48$$

$\therefore \quad T_{st} = 0.48 \, T_f$ or **48% of F.L. value**

(*b*) Here $K = 0.707 = 1/\sqrt{2}$; $K^2 = 1/2$

* By comparing it with the expression given in Art. 35.11 (*b*)

Now, $\frac{T_{st}}{T_f} = K^2 \left(\frac{I_{sc}}{I_f}\right)^2 s_f = \frac{1}{2} \times 6^2 \times 0.04 = 0.72$

$\therefore \quad T_{st} = 0.72\, T_f$ or **72% of T_f**

Example 35.17. *A 15 h.p. (11.2 kW), 3-ϕ, 6-pole, 50-HZ, 400-V, Δ-connected induction motor runs at 960 r.p.m. on full-load. If it takes 86.4 A on direct starting, find the ratio of starting torque to full-load torque with a star-delta starter. Full-load efficiency and power factor are 88% and 0.85 respectively.*

Solution. Here, I_{sc}/phase $= 86.4/\sqrt{3}$ A

$$I_{st} \text{ per phase} = \frac{1}{\sqrt{3}} . I_{sc} \text{ per phase} = \frac{86.4}{\sqrt{3} \times \sqrt{3}} = 28.8 \text{ A}$$

Full-load input line current may be found from

$\sqrt{3} \times 400 \times I_L \times 0.85 = 11.2 \times 10^3/0.88$ $\quad \therefore$ Full-load $I_L = 21.59$ A

F.L. $I_{ph} = 21.59/\sqrt{3}$ A ; $\quad I_f = 21.59/\sqrt{3}$ A per phase

$N_s = 120 \times 50/6 = 1000$ r.p.m., $\quad N = 950$; $s_f = 0.05$

$$\frac{T_{st}}{T_f} = \left(\frac{I_{st}}{I_f}\right)^2 s_f = \left(\frac{28.8 \times \sqrt{3}}{21.59}\right)^2 \times 0.05 \quad \therefore \quad T_{st} = 0.267\, T_f \text{ or } \textbf{26.7\% F.L. torque}$$

Example 35.18. *Find the ratio of starting to full-load current in a 10 kW (output), 400-V, 3-phase induction motor with star/delta starter, given that full-load p.f. is 0.85, the full-load efficiency is 0.88 and the blocked rotor current at 200 V is 40 A. Ignore magnetising current.*

(Electrical Engineering, Madras Univ. 1985)

Solution. F.L. line current drawn by the Δ-connected motor may be found from

$\sqrt{3} \times 400 \times I_L \times 0.85 = 10 \times 1000/0.88$ $\quad \therefore \quad I_L = 19.3$ A

Now, with 200 V, the line value of S.C. current of the Δ-connected motor is 40 A. If full normal voltage were applied, the line value of S.C. current would be $= 40 \times (400/200) = 80$ A.

$\therefore \quad I_{sc}$ (line value) $= 80$ A ; $\quad I_{sc}$ (phase value) $= 80/\sqrt{3}$ A

When connected in star across 400 V, the starting current per phase drawn by the motor stator during starting is

$$I_{st} \text{ per phase} = \frac{1}{\sqrt{3}} \times I_{sc} \text{ per phase} = \frac{1}{\sqrt{3}} \times \frac{80}{\sqrt{3}} = \frac{80}{3} \text{ A} \qquad \text{– Art. 35.10}$$

Since during starting, motor is star-connected, I_{st} per phase = line value of $I_{sc} = 80/3$A

$$\therefore \frac{\text{line value of starting current}}{\text{line value of F.L. current}} = \frac{80/3}{19.3} = \textbf{1.38}$$

Example 35.19. *A 5 h.p. (3.73 kW), 400-V, 3-ϕ, 50-Hz cage motor has a full-load slip of 4.5%. The motor develops 250% of the rated torque and draws 650% of the rated current when thrown directly on the line. What would be the line current, motor current and the starting torque if the motor were started (i) be means of a star/delta starter and (ii) by connecting across 60% taps of a starting compensator.*

(Elect. Machines-II, Indore Univ. 1989)

Solution. (*i*) Line current $= (1/3) \times 650 =$ **216.7%**

Motor being star-connected, line current is equal to phase current.

$\therefore \quad$ motor current $= 650/3 =$ **216.7%**

As shown earlier, starting torque developed for star-connection is one-third of that developed on direct switching with delta-connection $\quad \therefore \quad T_{st} = 250/3 =$ **83.3%**

(ii) Line current $= K^2 \times I_{sc} = (60/100)^2 \times 650 =$ **234%**

Motor current $= K \times I_{sc} = (60/100) \times 650 =$ **390%**

$T_{st} = K^2 \times T_{sc} = (60/100)^2 \times 250 =$ **90%**

Example. 35.20. *A squirrel-cage type induction motor when started by means of a star/delta starter takes 180% of full-load line current and develops 35% of full-load torque at starting. Calculated the starting torque and current in terms of full-load values, if an auto-transformer with 75% tapping were employed.* **(Utilization of Elect. Power, A.M.I.E. 1987)**

Solution. With star-delta starter, $\frac{T_{st}}{T_f} = \frac{1}{3}\left(\frac{I_{sc}}{I_f}\right)^2 s_f$

Line current on line-start $I_{sc} = 3 \times 180\%$ of $I_f = 3 \times 1.8\, I_f = 5.4\, I_f$

Now, $T_{st}/T_f = 0.35$ (given) ; $I_{sc}/I_f = 5.4$

$\therefore$ $0.35 = (1/3) \times 5.4^2\, s_f$ or $5.4^2\, s_f = 1.05$

Autostarter : Here, $K = 0.75$

Line starting current $= K^2\, I_{sc} = (0.75)^2 \times 5.4\, I_f =$ **$3.04\, I_f$ = 304% of F.L. current**

$\frac{T_{st}}{T_f} = K^2\left(\frac{I_{sc}}{I_f}\right)^2 s_f$; $\frac{T_{st}}{T_f} = (0.75)^2 \times 5.4^2\, s_f$

$= (0.75)^2 \times 1.05 = 0.59$

$T_{st} = 0.59\, T_f =$ **59% F.L. torque**

Example. 35.21. *A 10 h.p. (7.46 kW) motor when started at normal voltage with a star-delta switch in the star position is found to take an initial current of 1.7 × full-load current and gave an initial starting torque of 35% of full-load torque. Explain what happens when the motor is started under the following conditions : (a) an auto-transformer giving 60% of normal voltage (b) a resistance in series with the stator reducing the voltage to 60% of the normal and calculate in each case the value of starting current and torque in terms of the corresponding quantities at full-load.*

(Elect. Machinery-III, Kerala Univ. 1987)

Solution. If the motor were connected in delta and direct-switched to the line, then it would take a line current three times that which it takes when star-connected.

$\therefore$ line current on line start or $I_{sc} = 3 \times 1.7\, I_f = 5.1\, I_f$

We know $\frac{T_{st}}{T_f} = \frac{1}{3}\left(\frac{I_{sc}}{I_f}\right)^2 s_f$

Now $\frac{T_{st}}{T_f} = 0.35$ ...given ; $\frac{I_{sc}}{I_f} = 5.1$ ...calculated

$\therefore$ $0.35 = (1/3) \times 5.1^2 \times s_f$

We can find s_f from $5.1^2 \times s_f = 1.05$

(a) When it is started with an auto-starter, then $K = 0.6$

Line starting current $= K^2 \times I_{sc} = 0.6^2 \times 5.1\, I_f =$ **$0.836\, I_f$**

$T_{st}/T_f = 0.6^2 \times 5.1^2 \times s_f = 0.6^2 \times 1.05 = 0.378$ $\therefore$ $T_{st} =$ **37.8%** of F.L. torque

(b) Here, voltage across motor is reduced to 60% of normal value. In this case *motor* current is the same as line current but it decreases in proportion to the decrease in voltage.

As voltage across motor = 0.6 of normal voltage

$\therefore$ line starting current $= 0.6 \times 5.1\, I_f =$ **$3.06\, I_f$**

Torque at starting would be the same as before.

$T_{st}/T_f = 0.6^2 \times 5.1^2 \times s_f = 0.378$ $\therefore$ $T_{st} =$ **37.8%** of F.L. torque.

Tutorial Problems 35.3

1. A 3-phase induction motor whose full-load slip is 4 per cent, takes six times full-load current when switched directly on to the supply. Calculate the approximate starting torque in terms of the full-load torque when started by means of an auto-transformer starter, having a 70 percent voltage tap.
[0.7 T_f]
2. A 3-phase, cage induction motor takes a starting current at normal voltage of 5 times the full-load value and its full-load slip is 4 per cent. What auto-transformer ratio would enable the motor to be started with not more than twice full-load current drawn from the supply ?
What would be the starting torque under these conditions and how would it compare with that obtained by using a stator resistance starter under the same limitations of line current ?
[63.3% tap ; 0.4 T_f ; 0.16 T_f]
3. A 3-phase, 4-pole, 50-Hz induction motor takes 40 A at a full-load speed of 1440 r.p.m. and develops a torque of 100 N-m at full-load. The starting current at rated voltage is 200 A. What is the starting torque ? If a star-delta starter is used, what is the starting torque and starting current ? Neglect magnetising current.
[100 N-m; 33.3 N-m; 66.7 A] (*Electrical Machines-IV, Bangalore Univ. Aug. 1978*)
4. Determine approximately the starting torque of an induction motor in terms of full-load torque when started by means of (*a*) a star-delta switch (*b*) an auto-transformer with 50% tapping. Ignore magnetising current. The short-circuit current of the motor at normal voltage is 5 times the full-load current and the full-load slip is 4 per cent. **[(*a*) 0.33 (*b*) 0.25] (*A.C. Machines, Madras Univ. 1976*)**
5. Find the ratio of starting to full-load current for a 7.46 kW, 400-V, 3-phase induction motor with star/delta starter, given that the full-load efficiency is 0.87, the full-load p.f. is 0.85 and the short-circuit current is 15 A at 100 V. **[1.37] (*Electric Machinery-II, Madras Univ. April 1978*)**
6. A four-pole, 3-phase, 50-Hz, induction motor has a starting current which is 5 times its full-load current when directly switched on. What will be the percentage reduction in starting torque if (*a*) star-delta switch is used for starting (*b*) auto-transformer with a 60 per cent tapping is used for starting ? **(*Electrical Technology-III, Gwalior Univ. Nov. 1917*)**
7. Explain how the performance of induction motor can be predicted by circle diagram. Draw the circle diagram for a 3-phase, mesh-connected, 22.38 kW, 500-V, 4-pole, 50-Hz induction motor. The data below give the measurements of line current, voltage and reading of two wattmeters connected to measure the input :

No load	500 V	8.3 A	2.85 kW	– 1.35 kW
Short circuit	100 V	32 A	– 0.75 kW	2.35 kW

From the diagram, find the line current, power factor, efficiency and the maximum output.
[83 A, 0.9, 88%, 50.73 kW] (*Electrical Machines-II, Vikram Univ. Ujjan 1977*)

35.12. Starting of Slip-ring Motors

These motors are practically always started with full line voltage applied across the stator terminals. The value of starting current is adjusted by introducing a variable resistance in the rotor circuit. The controlling resistance is in the form of a rheostat, connected in star (Fig. 35.22), the resistance being gradually cut-out of the rotor circuit, as the motor gathers speed. It has been already shown that by increasing the rotor resistance, not only is the rotor (and hence stator) current reduced at starting, but at the same time, the starting torque is also increased due to improvement in power factor.

The controlling rheostat is either of stud or contactor type and


Slip-ring electric motor



Rheostat

may be hand-operated or automatic. The starter unit usually includes a line switching contactor for the stator along with no-voltage (or low- voltage) and over-current protective devices. There is some form of interlocking to ensure proper sequential operation of the line contactor and the starter. This interlocking prevents the closing of stator contactor unless the starter is 'all in'.

As said earlier, the introduction of additional external resistance in the rotor circuit enables a slip-ring motor to develop a high starting torque with reasonably moderate starting current. Hence, such motors can be started under load. This additional resistance is for starting purpose only. It is gradually cut out as the motor comes up to speed.

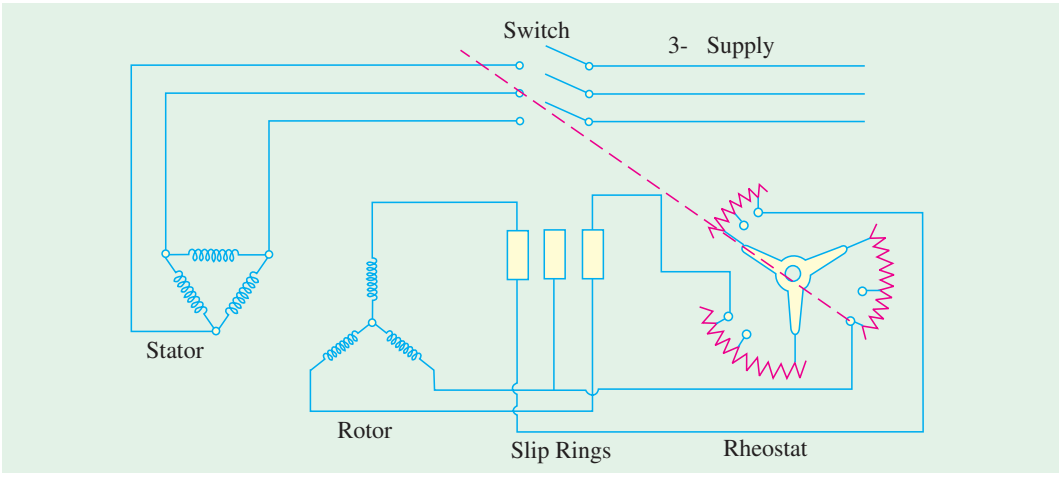


Fig. 35.22

The rings are, later on, short-circuited and brushes lifted from them when motor runs under normal conditions.

35.13. Starter Steps

Let it be assumed, as usually it is in the case of starters, that (*i*) the motor starts against a constant torque and (*ii*) that the rotor current fluctuates between fixed maximum and minimum values of I_{2max} and I_{2min} respectively.

In Fig. 35.23 is shown one phase of the 3-phase rheostat AB having n steps and the rotor circuit. Let R_1, R_2 ....etc. be the *total* resistances of the rotor circuit on the first, second step...etc. respectively. The resistances R_1, R_2 ..., etc. consist of rotor resistance per phase r_2 and the external resistances ρ_1, ρ_2 .... etc. Let the corresponding values of slips be s_1, s_2 ...etc. at stud No.1, 2...etc. At the commencement of each step, the current is I_{2max} and at the instant of leaving it, the current is I_{2min}. Let E_2 be the standstill e.m.f. induced in each phase of the rotor. When the handle touches first stud, the current rises to a maximum value I_{2max}, so that

$$I_{2max} = \frac{s_1 E_2}{\sqrt{[R_1^2 + (s_1 X_2)^2]}} = \frac{E_2}{\sqrt{[(R_1/s_1)^2 + X_2^2]}}$$

where s_1 = slip at starting *i.e.* unity and X_2 = rotor reactance/phase

Then, *before* moving to stud No. 2, the current is reduced to I_{2min} and slip changes to s_2 such that

$$I_{2min} = \frac{E_2}{\sqrt{[(R_1/s_2)^2 + X_2^2]}}$$

As we now move to stud No. 2, the speed momentarily remains the same, but current rises to I_{2max} because some resistance is cut out.

Hence, $I_{2max} = \dfrac{E_2}{\sqrt{[(R_2/s_2)^2 + X_2^2]}}$

After some time, the current is again reduced to I_{2min} and the slip changes to s_3 such that

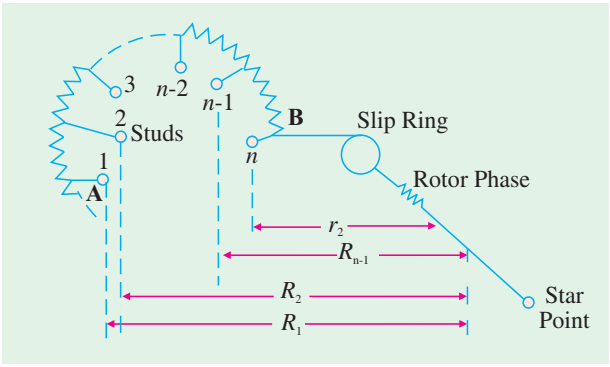


Fig. 35.23

$$I_{2min} = \frac{E_2}{\sqrt{[(R_2/s_3)^2 + X_2^2]}}$$

As we next move over to stud No.3, again current rises to I_{2max} although speed remains momentarily the same.

$$\therefore \quad I_{2max} = \frac{E_2}{\sqrt{[(R_2/s_3)^2 + X_2^2]}} \qquad \text{Similarly} \quad I_{2min} = \frac{E_2}{\sqrt{[(R_3/s_4)^2 + X_2^2]}}$$

At the last stud *i.e.* nth stud, $I_{2max} = \dfrac{E_2}{\sqrt{[(r_2/s_{max})^2 + X_2^2]}}$ where s_{max} = slip under normal running conditions, when external resistance is completely cut out.

It is found from above that

$$I_{2max} = \frac{E_2}{\sqrt{[(R_1/s_1)^2 + X_2^2]}} = \frac{E_2}{\sqrt{[(R_2/s_2)^2 + X_2^2]}} = \ldots\ldots = \frac{E_2}{\sqrt{[(r_2/s_{max})^2 + X_2^2]}}$$

or $$\frac{R_1}{s_1} = \frac{R_2}{s_2} = \frac{R_3}{s_3} = \ldots\ldots = \frac{R_{n-1}}{s_{n-1}} = \frac{R_n}{s_n} = \frac{r_2}{s_{max}} \qquad \ldots(i)$$

Similarly,

$$I_{2min} = \frac{E_2}{\sqrt{[(R_1/s_2)^2 + X_2^2]}} = \frac{E_2}{\sqrt{[(R_2/s_3)^2 + X_2^2]}} = \ldots\ldots = \frac{E_2}{\sqrt{[(R_{n-1}/s_{max})^2 + X_2^2]}}$$

or $$\frac{R_1}{s_2} = \frac{R_2}{s_3} = \frac{R_3}{s_4} = \ldots\ldots = \frac{R_{n-1}}{s_{max}} \qquad \ldots(ii)$$

From **(*i*)** and **(*ii*)**, we get

$$\frac{s_2}{s_1} = \frac{s_3}{s_2} = \frac{s_4}{s_3} = \ldots\ldots = \frac{R_2}{R_1} = \frac{R_3}{R_2} = \frac{R_4}{R_3} = \ldots\ldots = \frac{r_2}{R_{n-1}} = K \text{ (say)} \qquad \ldots(iii)$$

Now, from (*i*) it is seen that $R_1 = \dfrac{s_1 \times r_2}{s_{max}}$.

Now, $s_1 = 1$ at starting, when rotor is stationary.

$\therefore \quad R_1 = r_2/s_{max}$. Hence, R_1 becomes known in terms of rotor resistance/phase and normal slip. From **(*iii*)**, we obtain

$$R_2 = KR_1 \,;\, R_3 = KR_2 = K^2R_1 \,;\, R_4 = KR_3 = K^3R_1 \text{ and } r_2 = KR_{n-1} = K^{n-1}.R_1$$

or $$r_2 = K^{n-1}.\frac{r_2}{s_{max}} \qquad \text{(putting the value of } R_1\text{)}$$

$K = (s_{max})^{1/n-1}$ where n is the number of starter studs.

The resistances of various sections can be found as given below :

$\rho_1 = R_1 - R_2 = R_1 - KR_1 = (1-K)R_1$; $\rho_2 = R_2 - R_3 = KR_1 - K^2R_1 = K\rho_1$

$\rho_3 = R_3 - R_4 = K^2 \rho_1$ etc.

Hence, it is seen from above that if s_{max} is known for the assumed value I_{2max} of the starting current, then n can be calculated.

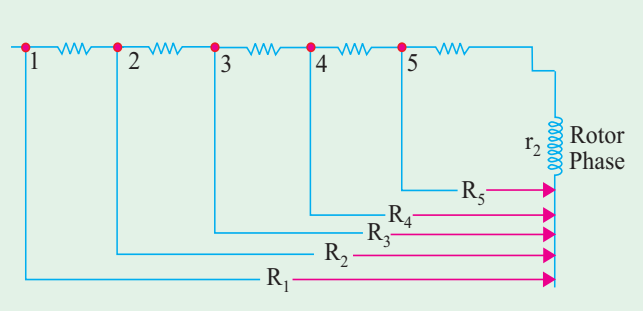


Fig. 35.24

Example 35.22. *Calculate the steps in a 5-step rotor resistance starter for a 3 phase induction motor. The slip at the maximum starting current is 2% with slip-ring short-circuited and the resistance per rotor phase is 0.02 Ω.*

Solution. Here, $s_{max} = 2\% = 0.02$; $r_2 = 0.02\ \Omega$, $n = 6$

R_1 = total resistance in rotor circuit/phase on first stud

$= r_2/s_{max} = 0.02/0.02 = 1\ \Omega$

Now, $K = (s_{max})^{1/n-1} = (0.02)^{1/5} = 0.4573$

$R_1 = 1\ \Omega$; $R_2 = KR_1 = 0.4573 \times 1 = 0.4573\ \Omega$

$R_3 = KR_2 = 0.4573 \times 0.4573 = 0.2091\ \Omega$; $R_4 = KR_3 = 0.4573 \times 0.2091 = 0.0956\ \Omega$

$R_5 = KR_4 = 0.4573 \times 0.0956 = 0.0437\ \Omega$; $r_2 = KR_5 = 0.4573 \times 0.0437 = 0.02\ \Omega$ (as given)

The resistances of various starter sections are as found below :

$\rho_1 = R_1 - R_2 = 1 - 0.4573 =$ **0.5427 Ω**; $\rho_2 = R_2 - R_3 = 0.4573 - 0.2091 =$ **0.2482 Ω**

$\rho_3 = R_3 - R_4 = 0.2091 - 0.0956 =$ **0.1135 Ω**; $\rho_4 = R_4 - R_5 = 0.0956 - 0.0437 =$ **0.0519 Ω**

$\rho_5 = R_5 - r_2 = 0.0437 - 0.02 =$ **0.0237 Ω**

The resistances of various sections are shown in Fig. 35.24.

35.14. Crawling

It has been found that induction motors, particularly the squirrel-cage type, sometimes exhibit a tendency to run stably at speeds as low as one-seventh of their synchronous speed N_s. This phenomenon is known as crawling of an induction motor.

This action is due to the fact that the a.c. winding of the stator produces a flux wave, which is not a pure sine wave. It is a complex wave consisting of a fundamental wave, which revolves synchronously and odd harmonics like 3rd, 5th, and 7th etc. which rotate either in the forward or backward direction at $N_s / 3$, $N_s / 5$ and $N_s / 7$ speeds respectively. As a result, in addition to the fundamental torque, harmonic torques are also developed, whose synchronous speeds are 1/nth of the speed for the fundamental torque *i.e.* N_s / n, where n is the order of the harmonic torque. Since 3rd harmonic currents are absent in a balanced 3-phase system, they produce no rotating field and, therefore, no torque. Hence, total motor torque has three components : (*i*) the fundamental torque, rotating with the synchronous speed N_s (*ii*) 5th harmonic torque* rotating at $N_s / 5$ speed and (*iii*) 7th harmonic torque, having a speed of $N_s / 7$.

* The magnitude of the harmonic torques is $1/n^2$ of the fundamental torque.

Now, the 5th harmonic currents have a phase difference of $5 \times 120^o = 600^o = -120^o$ in three stator windings. The revolving field, set up by them, rotates in the *reverse* direction at $N_s / 5$. The forward speed of the rotor corresponds to a slip greater than 100%. The small amount of 5th harmonic reverse torque produces a braking action and may be neglected.

The 7th harmonic currents in the three stator windings have a phase difference of $7 \times 120^o = 2 \times 360^o + 120^o = 120^o$. They set up a forward rotating field, with a synchronous speed equal to 1/7th of the synchronous speed of the fundamental torque.

If we neglect all higher harmonics, the resultant torque can be taken as equal to the sum of the fundamental torque and the 7th harmonic torque, as shown in Fig. 35.25. It is seen that the 7th harmonic torque reaches its maximum positive value just before 1/7th synchronous speed N_s, beyond which it becomes negative in value. Consequently, the resultant torque characteristic shows a dip which may become very pronounced with certain slot combinations. If the mechanical load on the shaft involves a constant load torque, it is possible that the torque developed by the motor may fall below this load torque. When this happens, the motor will not accelerate upto its normal speed but will remain running at a speed, which is nearly 1/7th of its full-speed. This is referred to as crawling of the motor.

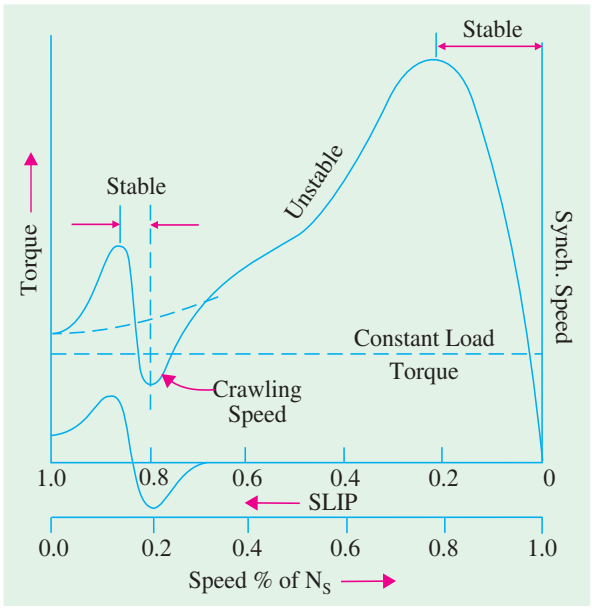


Fig. 35.25

35.15. Cogging or Magnetic Locking

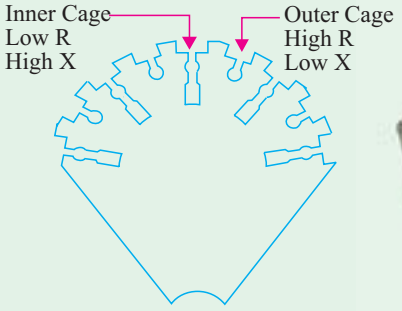


Fig. 35.26

Double-cage, 30-kW, 400/440-V, 3-f, 960 r.p.m.

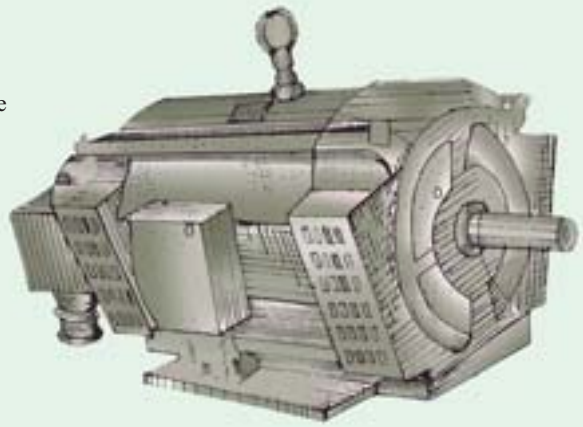


Fig. 35.27

squirrel-cage motor. (*Courtesy : Jyoti Ltd., Baroda*)

The rotor of a squirrel-cage motor sometimes refuses to start at all, particularly when the voltage is low. This happens when the number of stator teeth S_1 is equal to the number of rotor teeth S_2 and is due to the magnetic locking between the stator and rotor teeth. That is why this phenomenon is sometimes referred to as teeth-locking.

It is found that the reluctance of the magnetic path is minimum when the stator and rotor teeth face each other rather than when the teeth of one element are opposite to the slots on the other. It is in such positions of minimum reluctance, that the rotor tends to remain fixed and thus cause serious trouble during starting. Cogging of squirrel cage motors can be easily overcome by making the number of rotor slots prime to the number of stator slots.

35.16. Double Squirrel Cage Motor

The main disadvantage of a squirrel-cage motor is its poor starting torque, because of its low rotor resistance. The starting torque could be increased by having a cage of high resistance, but then the motor will have poor efficiency under normal running conditions (because there will be more rotor Cu losses). The difficulty with a cage motor is that its cage is permanently short-circuited, so no external resistance can be introduced temporarily in its rotor circuit during starting period. Many efforts have been made to build a squirrel-cage motor which should have a high starting torque without sacrificing its electrical efficiency, under normal running conditions. The result is a motor, due to Boucheort, which has two independent cages on the same rotor, one inside the other. A punching for such a double cage rotor is shown in Fig. 35.26.

The ***outer*** cage consists of bars of a ***high-resistance*** metal, whereas the inner cage has low-resistance copper bars.

Hence, outer cage has high resistance and low ratio of reactance-to-resistance, whereas the inner cage has low resistance but, being situated deep in the rotor, has a large ratio of reactance-to-resistance. Hence, the outer cage develops maximum torque at starting, while the inner cage does so at about 15% slip.

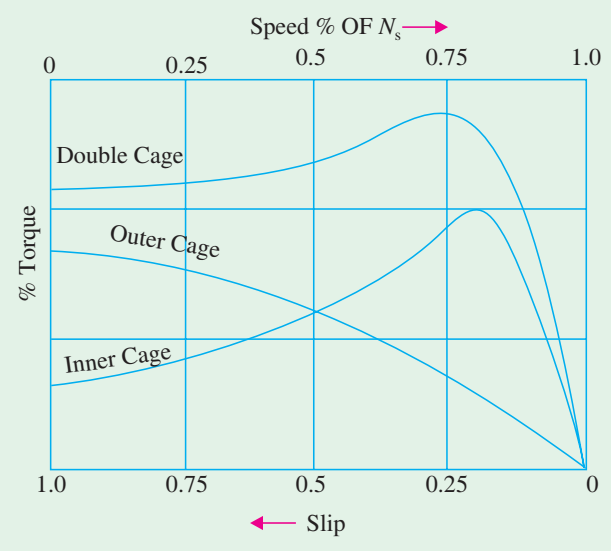


Fig. 35.28

As said earlier, at starting and at large slip values, frequency of induced e.m.f in the rotor is high. So the reactance of the inner cage ($= 2\pi f L$) and therefore, its impedance are both high. Hence, very little current flows in it. Most of the starting current is confined to outer cage, despite its high resistance. Hence, the motor develops a high starting torque due to high-resistance outer cage. Double squirrel-cage motor is shown in Fig. 35.27.

As the speed increases, the frequency of the rotor e.m.f. decreases, so that the reactance and hence the impedance of inner cage decreases and becomes very small, under normal running conditions. Most of the current then flows through it and hence it develops the greater part of the motor torque.

In fact, when speed is normal, frequency of rotor e.m.f. is so small that the reactance of both cages is practically negligible. The current is carried by two cages in parallel, giving a low combined resistance.

Hence, it has been made possible to construct a single machine, which has a good starting torque

with reasonable starting current and which maintains high efficiency and good speed regulation, under normal operating conditions.

The torque-speed characteristic of a double cage motor may be approximately taken to be the sum of two motors, one having a high-resistance rotor and the other a low-resistance one (Fig. 35.28).

Such motors are particularly useful where frequent starting under heavy loads is required.

35.17. Equivalent Circuit

The two rotor cages can be considered in parallel, if it is assumed that both cages completely link the main flux. The equivalent circuit for one phase of the rotor, as referred to stator, is shown in Fig. 35.29. If the magnetising current is neglected, then the figure is simplified to that shown in Fig. 35.30. Hence, R_0'/s and R_i'/s are resistances of outer and inner rotors as referred to stator respectively and X_0' and X_i' their reactances

Total impedance as referred to primary is given by

$$Z_{01} = R_1 + j X_1 + \frac{1}{1/Z_1' + 1/Z_0'} = R_1 + j X_1 + \frac{Z_i' Z_0'}{Z_0' + Z_i'}$$

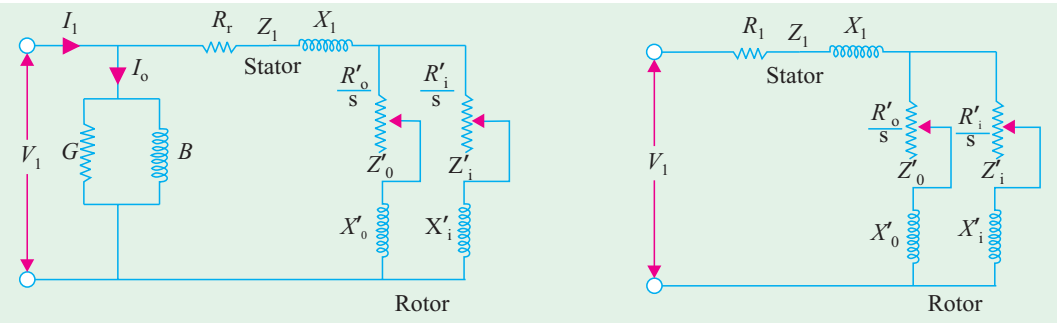


Fig. 35.29

Fig. 35.30

Example 35.23. *A double-cage induction motor has the following equivalent circuit parameters, all of which are phase values referred to the primary :*

Primary	$R_1 = 1\ \Omega$	$X_1 = 3\ \Omega$
Outer cage	$R_0' = 3\ \Omega$	$X_0' = 1\ \Omega$
Inner cage	$R_i' = 0.6\ \Omega$	$X_i' = 5\ \Omega$

The primary is delta-connected and supplied from 440 V. Calculate the starting torque and the torque when running at a slip of 4%. The magnetising current may be neglected.

Solution.

Refer to Fig. 35.31

(*i*) At start, $s = 1$

$$Z_{01} = 1 + j3 + \frac{1}{1/(3 + j1) + 1/(0.6 + j5)}$$

$$= 2.68 + j\,4.538\ \Omega$$

$$\text{Current / phase} = 440 / (2.68^2 + 4.538^2)^{1/2} = 83.43\ \text{A}$$

$$\text{Torque} = 3 \times 83.43^2 \times (2.68 - 1)$$

$$= \mathbf{35{,}000\ synch\ watt.}$$

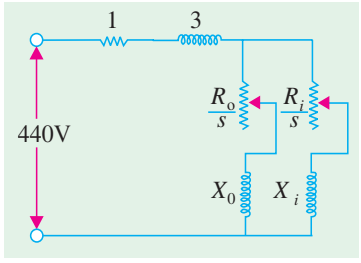


Fig. 35.31

(*ii*) when s = 4 per cent

In this case, approximate torque may be found by neglecting the outer cage impedance altogether. However, it does carry some current, which is almost entirely determined by its resistance.

$$Z_{01} = 1 + j\,3 + \frac{1}{1/(3/0.04) + 1/(0.6/0.04 + j\,5)} = 13.65 + j\,6.45$$

Current/phase $= 440/(13.65^2 + 6.45^2)^{1/2} = 29.14$ A

Torque $= 3 \times 29.14^2 \times (13.65 - 1) =$ **32,000 synch watt.**

Example 35.24. *At standstill, the equivalent impedance of inner and outer cages of a double-cage rotor are (0.4 + j2) Ω and (2+ j0.4) Ω respectively. Calculate the ratio of torques produced by the two cages (i) at standstill (ii) at 5% slip.* **(Elect. Machines-II, Punjab Univ. 1989)**

Solution. The equivalent circuit for one phase is shown in Fig. 35.32.

(*i*) At standstill, $s = 1$

Impedance of inner cage $= Z_i = \sqrt{0.4^2 + 2^2} = 2.04\ \Omega$

Impedance of outer cage $= Z_0 = \sqrt{2^2 + 0.4^2} = 2.04\ \Omega$

If I_0 and I_i are the current inputs of the two cages, then

power input of inner cage, $P_i = I_i^2 R_i = 0.4\, I_i^2$ watt

power input of outer cage, $P_0 = I_0^2 R_0 = 2\, I_0^2$

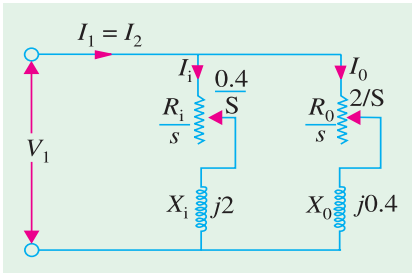


Fig. 35.32

$$\therefore \quad \frac{\text{torque of outer cage, } T_0}{\text{torque of inner cage, } T_i} = \frac{P_0}{P_i} = \frac{2\,I_0^2}{0.4\,I_i^2} = 5\left(\frac{I_0}{I_i}\right)^2$$

$$= 5\left(\frac{Z_i}{Z_0}\right)^2 = 5\left(\frac{2.04}{2.04}\right)^2 = 5 \qquad \therefore \quad T_0 : T_i :: 5 : 1$$

(*ii*) When $s = 0.05$

$$Z_0 = \sqrt{[(R_0/s)^2 + X_0^2]} = \sqrt{(2/0.05)^2 + 0.4^2} = 40\ \Omega$$

$$Z_i = \sqrt{[(R_i/s)^2 + X_i^2]} = \sqrt{(0.4/0.05)^2 + 2^2} = 8.25\ \Omega$$

$$\therefore \quad \frac{I_o}{I_i} = \frac{Z_i}{Z_o} = \frac{8.25}{40} = 0.206$$

$$P_0 = I_0^2 R_0 / s = 40\, I_0^2;\ P_i = I_i^2 R_i / s = 8\, I_i^2$$

$$\therefore \quad \frac{T_o}{T_i} = \frac{P_o}{P_i} = \frac{40\,I_o^2}{8\,I_i^2} = 5\left(\frac{I_o}{I_i}\right)^2 = 5\,(0.206)^2 = 0.21$$

$\therefore$ **$T_0 : T_i :: 0.21 : 1$**

It is seen from above, that the outer cage provides maximum torque at starting, whereas inner cage does so later.

Example 35.25. *A double-cage rotor has two independent cages. Ignoring mutual coupling between cages, estimate the torque in synchronous watts per phase (i) at standstill and at 5 per cent slip, given that the equivalent standstill impedance of the inner cage is (0.05 + j 0.4) ohm per phase and of the outer cage (0.5 + j 0.1) ohm per phase and that the rotor equivalent induced e.m.f. per phase is 100 V at standstill.*

Solution. The equivalent circuit of the double-cage rotor is shown in Fig. 35.33.

(*i*) At standstill, $s = 1$

The combined impedance of the two cages is

$$Z = \frac{Z_0 Z_i}{Z_0 + Z_i}$$

where Z_0 = impedance of the outer cage

Z_i = impedance of the inner cage

$$Z = \frac{(0.5 + j\,0.1)(0.05 + j\,0.4)}{(0.55 + j\,0.5)}$$

$$= 0.1705 + j\,0.191 \text{ ohm}$$

$$\therefore \quad Z = \sqrt{0.1705^2 + 0.191^2} = 0.256\ \Omega$$

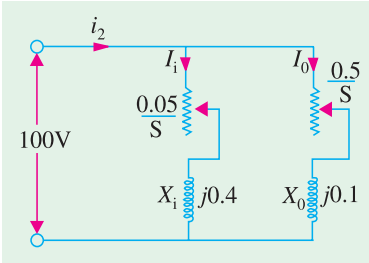


Fig. 35.33

Rotor current $I_2 = 100/0.256$ A; Combined resistance $R_2 = 0.1705\ \Omega$

Torque at standstill in synchronous watts per phase is

$$= I_2^2 R_2 = \left(\frac{100}{0.256}\right)^2 \times 0.1705 = \textbf{26,000 synch watts}$$

(*ii*) Here s = 0.05

$$Z = \frac{\left(\frac{0.5}{0.05} + j\,0.1\right)\left(\frac{0.05}{0.05} + j\,0.4\right)}{\left(\frac{0.05}{0.5} + \frac{0.5}{0.05} + j\,0.5\right)} = 1.01 + j\,0.326 \text{ ohm}$$

$$Z = \sqrt{(1.01)^2 + 0.326^2} = 1.06\ \Omega$$

Combined resistance $R_2 = 1.01\ \Omega$; rotor current = 100/1.06 A

Torque in synchronous watts per phase is

$$= I_2^2 R_2 = (100/1.06)^2 \times 1.1 = \textbf{9,000 synch.watt.}$$

Example 35.26. *In a double-cage induction motor, if the outer cage has an impedance at standstill of (2 + j 1.2) ohm, determine the slip at which the two cages develop equal torques if the inner cage has an impedance of (0.5 + j 3.5) ohm at standstill.*

(Electric Machines, Osmania Univ. 1991)

Solution. Let s be the slip at which two cage develop equal torques.

Then $Z_1 = \sqrt{(2/s)^2 + 1.2^2}$ and $Z_2 = \sqrt{(0.5/s)^2 + 3.5^2}$

$$\therefore \quad \left(\frac{I_1}{I_2}\right)^2 = \left(\frac{Z_2}{Z_1}\right)^2 = \frac{(0.25/s^2) + 3.5^2}{(4/s^2) + 1.44}$$

Power input to outer cage $P_1 = I_1^2 R_1/s$

$$\therefore \quad P_1 = I_1^2 \times \frac{2}{s}\ ; \qquad P_2 = I_2^2 \times \frac{0.5}{s}$$

$$\therefore \quad \frac{T_1}{T_2} = \frac{P_1}{P_2} = \left(\frac{I_1}{I_2}\right)^2 \times 4 = \frac{(0.25/s^2) + 3.5^2}{(4/s^2) + 1.44} \times 4$$

As $T_1 = T_2$

$$\therefore \quad \frac{4}{s^2} + 1.44 = \left(\frac{0.25}{s^2} + 12.25\right) \times 4 \quad \therefore \quad s = 0.251 = \textbf{25.1\%}$$

Example 35.27. *The resistance and reactance (equivalent) values of a double-cage induction motor for stator, outer and inner cage are 0.25, 1.0 and 0.15 ohm resistance and 3.5, zero and 3.0 ohm reactance respectively. Find the starting torque if the phase voltage is 250 V and the synchronous speed is 1000 r.p.m.* **(I.E.E. London)**

Solution. The equivalent circuit is shown in Fig. 35.34 where magnetising current has been neglected. At starting, $s = 1$

Impedance of outer cage $Z_0' = (1 + j\,0)$

Impedance of inner cage $Z_i' = (0.15 + j\,3)$

The two impedances are in parallel. Hence, their equivalent impedance

$$Z_2' = \frac{Z_0' Z_i'}{Z_0' + Z_i'} = \frac{(1 + j\,0)(0.15 + j\,3)}{(1 + j\,0) + (0.15 + j\,3)}$$

$$= 0.889 + j\,0.29$$

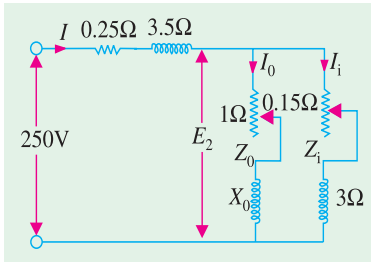


Fig. 35.34

Stator impedance = $(0.25 + j\,3.5)$

$\therefore$ total impedance $Z_{01} = (0.889 + j\,0.29) + (0.25 + j\,3.5)$

$= (1.14 + j\,3.79)\ \Omega$ (approx.)

$$\text{Current } I = \frac{\text{phase voltage}}{\text{total phase impedance}} = \frac{250 + j\,0}{1.14 + j\,3.79} = 18.2 - j\,60.5 = 66.15 \text{ A}$$

Rotor Cu loss/phase = (current)^2 × total resistance/phase of two rotors

$$= 66.15^2 \times 0.889 = 3{,}890 \text{ W}$$

Total Cu loss in 3-phases = $3 \times 3890 = 11{,}670$ W

Now, rotor input = $\frac{\text{rotor Cu loss}}{s}$; At starting, $s = 1$ $\therefore$ rotor input = 11,670 W

$\therefore$ T_{start} = 11,670 synchronous watts

Also $T_{start} \times 2\pi N_S = 11{,}670$ or $T_{start} = \frac{11{,}670}{2\pi \times (1000/60)}$ = **111.6 N-m (approx.)**

Note. If torques developed by the two rotors separately are required, then find E_2 (Fig. 35.34), then I_1 and I_2. Knowing these values, T_1 and T_2 can be found as given in previous example.

Example 35.28. *A double-cage induction motor has the following equivalent circuit parameters all of which are phase values referred to the primary :*

Primary : $R_1 = 1$ *ohm* $X_1 = 2.8$ *ohm*

Outer cage : $R_0' = 3$ *ohm* $X_0' = 1.0$ *ohm*

Inner cage : $R_i' = 0.5$ *ohm* $X_i' = 5$ *ohm*

The primary is delta-connected and supplied from 440 V. Calculate the starting torque and the torque when running at a slip of 4 per cent. The magnetizing branch can be assumed connected across the primary terminals.

(Electrical Machines-II, South Gujarat Univ. 1987)

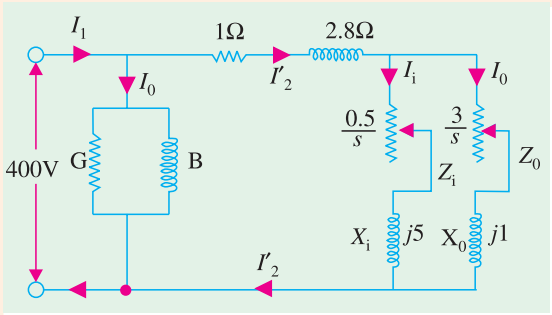


Fig. 35.35

Solution. The equivalent circuit for one phase is shown in Fig. 35.35. It should be noted that magnetising impedance Z_0 has no bearing on the torque and speed and hence, can be neglected so far as these two quantities are concerned

(i) At standstill s = 1

$$Z_2' = \frac{Z_0' Z_i'}{Z_0' + Z_i'} = \frac{(3 + j1.0)(0.5 + j\,5)}{(3.5 + j\,6)}$$

$$= 1.67 + j\,1.56$$

$$Z_{01} = Z_1 + Z_2' = (1 + j\,2.8) + (1.67 + j\,1.56)$$

$$= (2.67 + j\,4.36) = 5.1 \angle 58.5^\circ$$

Voltage per phase, V_1 = 440 V. Since stator is delta-connected.

$\mathbf{I_2'} = \mathbf{V_1/Z_{01}} = 440/5.1 \angle 58.5^\circ = 86.27 \angle -58.5^\circ$

Combined resistance R_2 = 1.6 Ω

∴ starting torque per phase = $I_2'^2 R_2 = 86.27^2 \times 1.67 =$ **12,430 synch. watt.**

***(ii)* when** $\mathbf{s = 0.04}$

$\mathbf{Z_0'} = (3/0.04) + j\,1.0 = 75 + j\,1.0$; $\mathbf{Z_i'} = (0.5/0.04) + j\,5 = 12.5 + j\,5$

$$\mathbf{Z_2'} = \frac{Z_0' Z_i'}{Z_0' + Z_i'} = \frac{(75 + j1.0)(12.5 + j5)}{(87.5 + j6)} = 10.3 + j\,3.67$$

$\mathbf{Z_{01}} = Z_1 + Z_2' = (1 + j\,2.8) + (10.3 + j\,6.47) = 11.3 + j\,6.47 = 13.03 \angle 29.8^\circ$

$\mathbf{I_2'} = \mathbf{V_1/Z_{01}} = 440/13.03 \angle 29.8^\circ = 33.76 \angle -29.8^\circ$

combined resistance $\mathbf{R_2}$ = 10.3 Ω

∴ full-load torque per phase = $I_2' R_2 = 33.76^2 \times 10.3 =$ **11.740 synch. watts**

Obviously, starting torque is higher than full-load torque.

Tutorial Problems 35.4

1. Calculate the steps in a 5-section rotor starter of a 3-phase induction motor for which the starting current should not exceed the full-load current, the full-load slip is 0.018 and the rotor resistance is 0.015 Ω per phase

$\rho_1 = 0.46\ \Omega$; $\rho_2 = 0.206\ \Omega$; $\rho_3 = 0.092\ \Omega$; $\rho_4 = 0.042\ \Omega$; $\rho_5 = 0.0185\ \Omega$

(Electrical Machinery-III, Kerala Univ. Apr. 1976)

2. The full-load slip of a 3-phase double-cage induction motor is 6% and the two cages have impedances of (3.5 + *j* 1.5) Ω and (0.6 + *j* 7.0) Ω respectively. Neglecting stator impedances and magnetising current, calculate the starting torque in terms of full-load torque. **[79%]**

3. In a double-cage induction motor, if the outer cage has an impedance at standstill of (2 + *j* 2) ohm and the inner cage an impedance of (0.5 + *j* 5) Ω, determine the slip at which the two cages develop equal torques. **[17.7%]**

4. The two independent cages of a rotor have the respective standstill impedance of (3 + *j* 1) ohm and (1 + *j* 4) ohm. What proportion of the total torque is due to the outer cage (*a*) at starting and (*b*) at a fractional slip of 0.05 ? **[(*a*) 83.6% (*b*) 25.8%] (*Principle of Elect. Engg.I, Jadavpur Univ. 1975*)**

5. An induction motor has a double cage rotor with equivalent impedance at standstill of (1.0 + *j* 1.0) and (0.2 + *j* 4.0) ohm. Find the relative value of torque given by each cage at a slip of 5%.

[(*a*) 40.1 (*b*) 0.4 : 1] ***(Electrical Machines-I, Gwalior Univ. Nov. 1977)***

35.18. Speed Control of Induction Motors*

A 3-phase induction motor is practically a constant-speed machine, more or less like a d.c. shunt motor. The speed regulation of an induction motor (having low resistance) is usually less than 5% at full-load. However, there is one difference of practical importance between the two. Whereas d.c. shunt motors can be made to run at any speed within wide limits, with good efficiency and speed regulation, merely by manipulating a simple field rheostat, the same is not possible with induction motors. In their case, speed reduction is accompanied by a corresponding loss of efficiency and good speed regulation. That is why it is much easier to build a good adjustable-speed d.c. shunt motor than an adjustable speed induction motor.

Different methods by which speed control of induction motors is achieved, may be grouped under two main headings :

* For Electronic Control of AC Motors, please consult the relevant chapter of this book, in vol. III.

1. Control from stator side

(*a*) by changing the applied voltage **(*b*)** by changing the applied frequency

(*c*) by changing the number of stator poles

2. Control from rotor side

(*d*) rotor rheostat control

(*e*) by operating two motors in concatenation or cascade

(*f*) by injecting an e.m.f. in the rotor circuit.

A brief description of these methods would be given below :

(*a*) Changing Applied Voltage

This method, though the cheapest and the easiest, is rarely used because

(*i*) a large change in voltage is required for a relatively small change in speed

(*ii*) this large change in voltage will result in a large change in the flux density thereby seriously disturbing the magnetic conditions of the motor.

(*b*) Changing the Applied Frequency

This method is also used very rarely. We have seen that the synchronous speed of an induction motor is given by $N_s = 120\ f/P$. Clearly, the synchronous speed (and hence the running speed) of an induction motor can be changed by changing the supply frequency f. However, this method could only be used in cases where the induction motor happens to be the only load on the generators, in which case, the supply frequency could be controlled by controlling the speed of the prime movers of the generators. But, here again the range over which the motor speed may be varied is limited by the economical speeds of the prime movers. This method has been used to some extent on electrically-driven ships.

(*c*) Changing the Number of Stator Poles

This method is ***easily applicable to squirrel-cage motors*** because the squirrel-cage rotor adopts itself to any reasonable number of stator poles.

From the above equation it is also clear that the synchronous (and hence the running)speed of an induction motor could also be changed by changing the number of stator poles. This change of number of poles is achieved by having two or more entirely independent stator windings in the same slots. Each winding gives a different number of poles and hence different synchronous speed. For example, a 36-slot stator may have two 3-ϕ windings, one with 4 poles and the other with 6-poles. With a supply frequency of 50-Hz, 4-pole winding will give $N_s = 120 \times 50/4 = 1500$ r.p.m. and the 6-pole winding will give $N_s = 120 \times 50/6 = 1000$ r.p.m. Motors with four independent stator winding are also in use and they give four different synchronous (and hence running) speeds. Of course, one winding is used at a time, the others being entirely disconnected.

This method has been used for elevator motors, traction motors and also for small motors driving machine tools.

Speeds in the ratio of 2:1 can be produced by a single winding if wound on the consequent-pole principle. In that case, each of the two stator windings can be connected by a simple switch to give two speeds, each, which means four speeds in all. For example, one stator winding may give 4 or 8-poles and the other 6 or 12-poles. For a supply frequency of 50-Hz, the four speeds will be 1500, 750, 1000 and 500 r.p.m. Another combination, commonly used, is to group 2- and 4-pole winding with a 6- and 12-pole winding, which gives four synchronous speeds of 3000, 1500, 1000 and 500 r.p.m.

(*d*) Rotor Rheostat Control

In this method (Fig. 35.36), which is applicable to slip-ring motors alone, the motor speed is reduced by introducing an external resistance in the rotor circuit. For this purpose, the rotor starter

may be used, provided it is continuously rated. ***This method is, in fact, similar to the armature rheostat control method of d.c. shunt motors.***

It has been shown in Art 34.22 that near synchronous speed (*i.e.* for very small slip value), $T \propto s/R_2$.

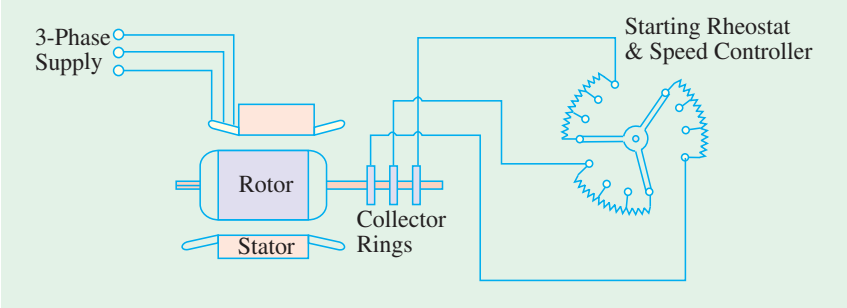


Fig. 35. 36

It is obvious that for a given torque, slip can be increased *i.e.* speed can be decreased by increasing the rotor resistance R_2.

One serious disadvantage of this method is that with increase in rotor resistance, I^2R losses also increase which decrease the operating efficiency of the motor. In fact, the loss is directly proportional to the reduction in the speed.

The second disadvantage is the double dependence of speed, not only on R_2 but on load as well.

Because of the wastefulness of this method, it is used where speed changes are needed for short periods only.

Example 35.29. *The rotor of a 4-pole, 50-Hz slip-ring induction motor has a resistance of 0.30 Ω per phase and runs at 1440 rpm. at full load. Calculate the external resistance per phase which must be added to lower the speed to 1320 rpm, the torque being the same as before.*

(Advanced Elect. Machines AMIE Sec.E1992)

Solution. The motor torque is given by $T = \dfrac{K\,s\,R_2}{R_2^2 + (s\,X_2)^2}$

Since, X_2 is not given, $T = \dfrac{K\,s\,R_2}{R_2^2} = \dfrac{K\,s}{R_2}$

In the first case, $T_1 = Ks_1/R_2$; in the second case, $T_2 = Ks_2 / (R_2 + r)$

where r is the external resistance per phase, added to the rotor circuit

Since $T_1 = T_2$ $\quad\therefore\quad K\,s_1/R_2 = K\,s_2 / (R_2 + r)$ or $(R_2 + r) / R_2 = s_2 / s_1$

Now, $N_s = 120 \times 50/4 = 1500$ rpm; $N_1 = 1440$ rpm; $N_2 = 1320$ rpm

$\therefore s_1 = (1500 - 1440)/1500 = 0.04$; $s_2 = (1500 - 1320)/1500 = 0.12$

$\therefore \quad \dfrac{0.3 + r}{0.3} = \dfrac{0.12}{0.04} \quad \therefore \quad r = \mathbf{0.6\ \Omega}$

Example 35.30. *A certain 3-phase, 6-pole, 50-Hz induction motor when fully-loaded, runs with a slip of 3%. Find the value of the resistance necessary in series per phase of the rotor to reduce the speed by 10%. Assume that the resistance of the rotor per phase is 0.2 ohm.*

(Electrical Engineering-II (M), Bangalore Univ. 1989)

Solution. $T = \dfrac{K\,s\,R_2}{R_2^2 + (s\,X_2)^2} = \dfrac{K_s\,R_2}{R_2^2} = \dfrac{K\,s}{R_2}$ - neglecting $(s\,X_2)$

$\therefore \quad T_1 = Ks_1 / R_2$ and $T_2 = K\,s_2 / (R_2 + r)$ where r is the external resistance per phase added to the rotor circuit.

Since $\quad T_1 = T_2, K\,s_1 / R_2 = K\,s_2 / (R_2 + r)$ or $(R_2 + r)/R_2 = s_2 / s_1$

Now, $\quad N_s = 120 \times 50/6 = 1000$ rpm., $s_1 = 0.03$, $N_1 = 1000\ (1 - 0.030) = 970$ rpm.

N_2 = 970 – 10% of 970 = 873 rpm., s_2 = (1000 – 873)/1000 = 0.127

$$\therefore \quad \frac{0.2 + r}{0.2} = \frac{0.127}{0.03}; \quad r = \mathbf{0.65\ \Omega.}$$

(e) Cascade or Concatenation or Tandem Operation

In this method, two motors are used (Fig. 35.37) and are ordinarily mounted on the same shaft, ***so that both run at the same speed*** (or else they may be geared together).

The stator winding of the main motor A is connected to the mains in the usual way, while that of the auxiliary motor B is fed from the rotor circuit of motor A. For satisfactory operation, the main motor A should be phase-wound *i.e.* of slip-ring type with stator to rotor winding ratio of 1 : 1, so that, in addition to concatenation, each motor may be run from the supply mains separately.

There are at least three ways (and sometimes four ways) in which the combination may be run.

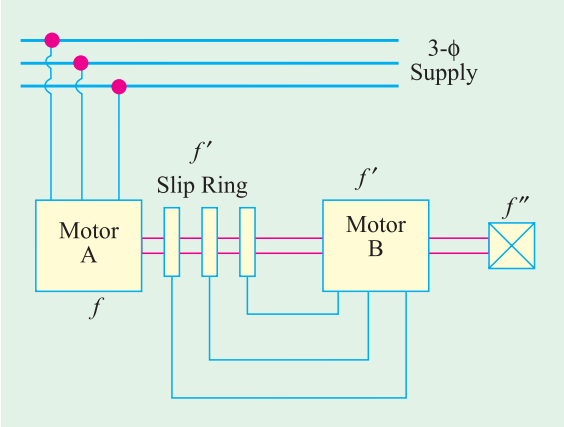


Fig. 35. 37

1. Main motor A may be run separately from the supply. In that case, the synchronous speed is $N_{sa} = 120\ f / P_a$ where P_a = Number of stator poles of motor A.

2. Auxiliary motor B may be run separately from the mains (with motor A being disconnected). In that case, synchronous speed is $N_{sb} = 120 \times f / P_b$ where P_b = Number of stator poles of motor B.

3. The combination may be connected in cumulative cascade *i.e.* in such a way that the phase rotation of the stator fields of both motors is in the same direction. The synchronous speed of the cascaded set, in this case, is $N_{sc} = 120\ f/(P_a + P_b)$.

Proof

Let N = actual speed of concatenated set ;

N_{sa} = synchronous speed of motor A, it being independent of N.

Clearly, the relative speed of rotor A with respect to its stator field is $(N_{sa} - N)$. Hence, the frequency f' of the induced e.m.f. in rotor A is given by

$$f' = \frac{N_{sa} - N}{N_{sa}} \times f$$

This is also the frequency of the e.m.f. applied to the stator of motor B. Hence, the synchronous speed of motor B with this input frequency is

$$N' = 120\ \frac{f'}{P_b} = \frac{120\ (N_{sa} - N)\ f}{P_b \times N_{sa}} \qquad \text{...(i)}$$

(Note that N' is not equal to N_{sb} which is the synchronous speed of motor B with supply frequency f).

This will induce an e.m.f. of frequency, say, f'' in the rotor B. Its value is found from the fact that the stator and rotor frequencies are proportional to the speeds of stator field and the rotor

$$\therefore \quad f'' = \frac{N' - N}{N'}\ f$$

Now, on no-load, the speed of rotor B is almost equal to its synchronous speed, so that the frequency of induced e.m.f. is, to a first approximation, zero.

$$f'' = 0, \quad \text{or} \quad \frac{N' - N}{N'} f = 0 \quad \text{or} \quad N' = N \qquad \text{...(ii)}$$

From (*i*) above $$N' = \frac{120\ f\ (N_{sa} - N)}{P_b \times N_{sa}} = \frac{120\ f}{P_b}\left(1 - \frac{N}{N_{sa}}\right)$$

Hence, from (*ii*) above,

$$\frac{120\ f}{P_b}\left(1 - \frac{N}{N_{sa}}\right) = N \quad \text{or} \quad \frac{120\ f}{P_b} = N\left(1 + \frac{1}{N_{sa}} \times \frac{120\ f}{P_b}\right)$$

Putting $N_{sa} = 120\ f/P_a$, we get

$$\frac{120\,f}{P_b} = \mathrm{N}\left(1 + \frac{P_a}{120\,f} \times \frac{120\ f}{P_b}\right) = N\left(1 + \frac{P_a}{P_b}\right) \therefore N = \frac{120\ f}{(P_a + P_b)}$$

Concatenated speed of the set = $120\ f\ /\ (P_a + P_b)$

How the Set Starts ?

When the cascaded set is started, the voltage at frequency f is applied to the stator winding of machine A. An induced e.m.f. of the same frequency is produced in rotor A which is supplied to auxiliary motor B. Both the motors develop a forward torque. As the shaft speed rises, the rotor frequency of motor A falls and so does the synchronous speed of motor B. The set settles down to a stable speed when the shaft speed becomes equal to the speed of rotating field of motor B.

Considering load conditions, we find that the electrical power taken in by stator A is partly used to meet its I^2R and core losses and the rest is given to its rotor. The power given to rotor is further divided into two parts : one part, proportional to the speed of set *i.e.* N is converted into mechanical power and the other part proportional to $(N_{sa} - N)$ is developed as electrical power at the slip frequency, and is passed on to the auxliary motor B, which uses it for producing mechanical power and losses. Hence, approximately, the mechanical outputs of the two motors are in the ratio $N : (N_{sa} - N)$. ***In fact, it comes to that the mechanical outputs are in the ratio of the number of poles of the motors***.

It may be of interest to the reader to know that it can be proved that

(*i*) $s = f'' / f$ where s = slip of the set referred to its synchronous speed N_{sc}.

$= (N_{sc} - N) / N_{sc}$

(*ii*) $s = s_a\, s_b$

where s_a and s_b are slips of two motors, referred to their respective stators *i.e*

$$s_a = \frac{N_{sa} - N}{N_{sa}} \quad \text{and} \quad s_b = \frac{N' - N}{N}$$

Conclusion

We can briefly note the main conclusions drawn from the above discussion :

(*a*) the mechanical outputs of the two motors are in the ratio of their number of poles.

(*b*) $s = f''/f$ **(*c*)** $s = s_a.s_b$

4. The fourth possible connection is the ***differential cascade***. In this method, the phase rotation of stator field of the motor B is opposite to that of the stator of motor A. This reversal of phase rotation of stator of motor B is obtained by interchanging any of its two leads. It can be proved in the same way as above, that for this method of connection, the synchronous speed of the set is

$$N_{sc} = 120\ f\ /\ (P_a - P_b)$$

As the differentially-cascaded set has a very small or zero starting torque, this method is rarely used. Moreover, the above expression for synchronous speed becomes meaningless for $P_a = P_b$.

Example 35.31. *Two 50-Hz, 3-ϕ induction motors having six and four poles respectively are cumulatively cascaded, the 6-pole motor being connected to the main supply. Determine the*

frequencies of the rotor currents and the slips referred to each stator field if the set has a slip of 2 per cent. **(Elect. Machinery-II Madras Univ. 1987)**

Solution. Synchronous speed of set $N_{sc} = 120 \times 50/10 = 600$ r.p.m.

Actual rotor speed $N = (1 - s)\, N_{sc} = (1 - 0.02)\, 600 = 588$ r.p.m.

Synchronous speed of the stator field of 6-pole motor, $N_{sa} = 120 \times 50/6 = 1000$ r.p.m.

Slip referred to this stator field is

$$s_a = \frac{N_{sa} - N}{N_{sa}} = \frac{1000 - 588}{1000} = 0.412 \quad \text{or} \quad \mathbf{41.2\%}$$

Frequency of the rotor currents of 6-pole motor $f' = s_a f = 0.412 \times 50 = \mathbf{20.6\ Hz}$

This is also the frequency of stator currents of the four pole motor. The synchronous speed of the stator of 4-pole motor is

$$N' = 120 \times 20.6/4 = 618 \text{ r.p.m.}$$

This slip, as referred to the 4-pole motor, is $s_b = \frac{N' - N}{N'} = \frac{618 - 588}{618}$

$= 0.0485$ or $\mathbf{4.85\ \%}$

The frequency of rotor current of 4-pole motor is

$$f'' = s_b f' = 0.0485 \times 20.6 = \mathbf{1.0\ Hz\ (approx)}$$

As a check, $f'' = sf = 0.02 \times 50 = 1.0$ Hz

Example 35.32. *A 4-pole induction motor and a 6-pole induction motor are connected in cumulative cascade. The frequency in the secondary circuit of the 6-pole motor is observed to be 1.0 Hz. Determine the slip in each machine and the combined speed of the set. Take supply frequency as 50 Hz.* **(Electrical Machinery-II, Madras Univ. 1986)**

Solution. With reference to Art. 35.18 (*e*) and Fig. 35.38

$N_{sc} = 120 \times 50/(4 + 6) = 600$ r.p.m.

$s = f''/f = 1/50 = 0.02$

N = actual speed of the concatenated set

$\therefore \quad 0.02 = \frac{600 - N}{600}$ or $N = \mathbf{588\ r.p.m.}$

$N_{sa} = 120 \times 50/4 = 1500$ r.p.m.

$s_a = (1500 - 588)/1500$

$= 0.608$ or $\mathbf{60.8\%}$

$f' = s_a f = 0.608 \times 50 = 30.4$ Hz

N' = synchronous speed of 6-pole motor with frequency f'

$= 120 \times 30.4/6 = 608$ r.p.m.

$s_b = \frac{N' - N}{N'} = \frac{608 - 588}{608} = 0.033$ or $\mathbf{3.3\%}$

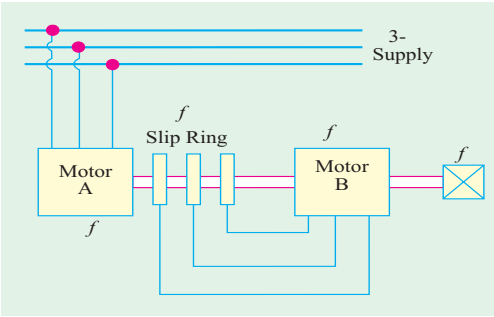


Fig. 35.38

Example 35.33. *The stator of a 6-pole motor is joined to a 50-Hz supply and the machine is mechanically coupled and joined in cascade with a 4-pole motor, Neglecting all losses, determine the speed and output of the 4-pole motor when the total load on the combination is 74.6 kW.*

Solution. As all losses are neglected, the actual speed of the rotor is assumed to be equal to the synchronous speed of the set.

Now, $N_{sc} = 120 \times 50/10 = \mathbf{600\ r.p.m.}$

As said earlier, mechanical outputs are in the ratio of the number of poles of the motors.

$\therefore$ output of 4-pole motor = $74.6 \times 4/10$ = **29.84 kW**

Example 35.34. *A cascaded set consists of two motors A and B with 4 poles and 6 poles respectively. The motor A is connected to a 50-Hz supply. Find*

(*i*) the speed of the set

(*ii*) *the electric power transferred to motor B when the input to motor A is 25 kW. Neglect losses.* **(Electric Machines-I, Utkal Univ. 1990)**

Solution. Synchronous speed of the set is*

$N_{sc} = 120\ f/(P_a + P_b) = 120 \times 50/(6 + 4)$ = **600 r.p.m.**

(ii) The outputs of the two motors are proportional to the number of their poles.

$\therefore$ output of 4-pole motor $B = 25 \times 4/10$ = **10 kW**

(f) Injecting an e.m.f. in the Rotor Circuit

In this method, the speed of an induction motor is controlled by injecting a voltage in the rotor circuit, it being of course, necessary for the injected voltage to have the same frequency as the slip frequency. There is, however, no restriction as to the phase of the injected e.m.f.

When we insert a voltage which is in ***phase opposition*** to the induced rotor e.m.f., it amounts to increasing the rotor resistance, whereas inserting a voltage which is in phase with the induced rotor e.m.f., is equivalent to ***decreasing*** its resistance. Hence, by changing the phase of the injected e.m.f. and hence the rotor resistance, the speed can be controlled.

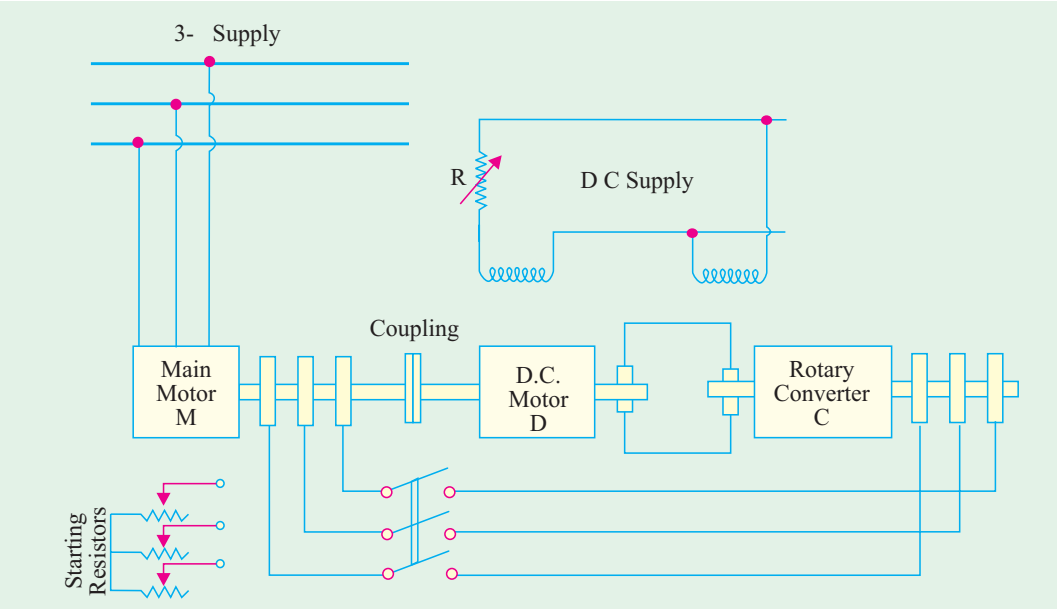


Fig. 35.39

One such practical method of this type of speed control is Kramer system, as shown in Fig. 35.39, which is used in the case of large motors of 4000 kW or above. It consists of a rotary converter *C* which converts the low-slip frequency a.c. power into d.c. power, which is used to drive a d.c. shunt motor *D*, mechanically coupled to the main motor *M*.

The main motor is coupled to the shaft of the d.c. shunt motor *D*. The slip-rings of *M* are connected to those of the rotary converter *C*. The d.c. output of *C* is used to drive *D*. Both *C* and *D*

* It is assumed that the two motors are connected in cumulative cascade.

are excited from the d.c. bus-bars or from an exciter. There is a field regulator which governs the back e.m.f. E_b of D and hence the d.c. potential at the commutator of C which further controls the slip-ring voltage and therefore, the speed of M.

One big advantage of this method is that any speed, within the working range, can be obtained instead of only two or three, as with other methods of speed control.

Yet another advantage is that if the rotary converter is over-excited, it will take a leading current which compensates for the lagging current drawn by main motor M and hence improves the power factor of the system.

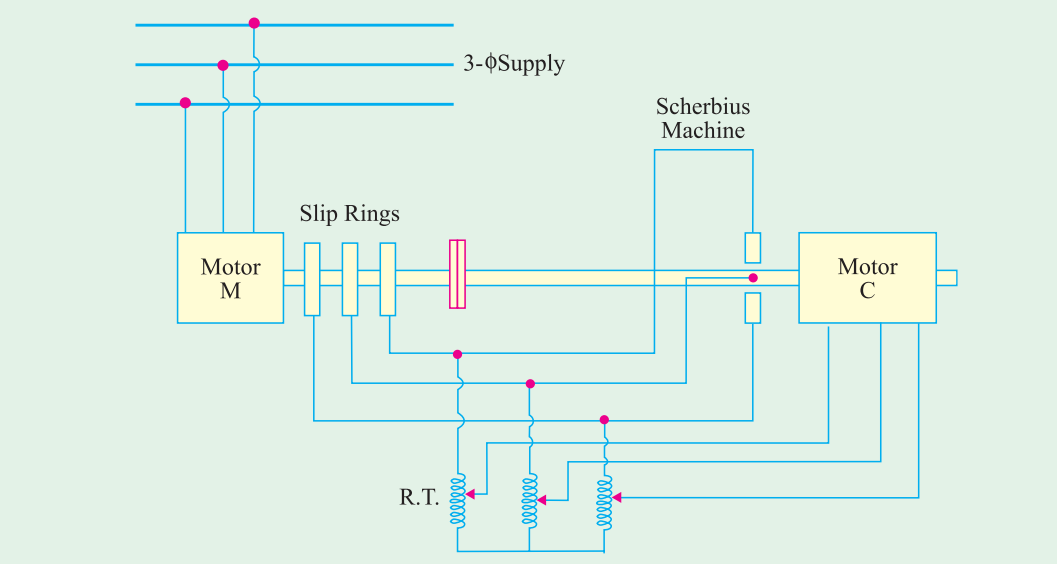


Fig. 35.40

In Fig. 35.40 is shown another method, known as Scherbius system, for controlling the speed of large induction motors. The slip energy is not converted into d.c. and then fed to a d.c. motor, rather it is fed directly to a special 3-phase (or 6-phase) a.c. commutator motor-called a, Scherbius machine.

The polyphase winding of machine C is supplied with the low-frequency output of machine M through a regulating transformer RT. The commutator motor C is a variable-speed motor and its speed (and hence that of M) is controlled by either varying the tappings on RT or by adjusting the position of brushes on C.

Tutorial Problems 35.5

1. An induction motor has a double-cage rotor with equivalent impedances at standstill of $(1.0 + j\,1.0)$ and $(0.2 + j\,4.0)\ \Omega$. Find the relative values of torque given by each cage (a) at starting and (b) at 5 % slip **[(a) 40:1 (b) 0.4:1]** ***(Adv. Elect. Machines AMIE Sec. B 1991)***
2. The cages of a double-cage induction motor have standstill impedances of $(3.5 + j\,1.5)\ \Omega$ and $(0.6 + j\,7.0)\ \Omega$ respectively. The full-load slip is 6%. Find the starting torque at normal voltage in terms of full-load torque. Neglect stator impedance and magnetizing current.
   [300%] ***(Elect. Machines-I, Nagpur Univ. 1993)***
3. The rotor of a 4 pole, 50 Hz, slip ring induction motor has a resistance of 0.25 ohm per phase and runs at 1440 rpm at full-load. Calculate the external resistance per phase, which must be added to lower the speed to 1200 rpm, the torque being same as before.
   [1 Ω] ***(Utilisation of Electric Power (E-8) AMIE Sec. B Summer 1992)***

35.19. Three-phase A.C. Commutator Motors

Such motors have shunt speed characteristics *i.e.* change in their speed is only moderate, as compared to the change in the load. They are ideally suited for drives, requiring a uniform accelerating torque and continuously variable speed characteristics over a wide range. Hence, they find wide use in high-speed lifts, fans and pumps and in the drives for cement kilns, printing presses, pulverised fuel plants, stokers and many textile machines. Being more complicated, they are also more expensive than single-speed motors. Their efficiency is high over the whole speed range and their power factor varies from low value at synchronous speed to unity at maximum (supersynchronous) speed.

The speed control is obtained by injecting a variable voltage at ***correct*** frequency into the secondary winding of the motor via its commutator. If injected voltage assists the voltage induced in the secondary winding, the speed is increased but if it is in the opposing direction, then motor speed is reduced. The commutator acts as a frequency changer because it converts the supply frequency of the regulating voltage to the slip frequency corresponding to the speed required.

Following are the two principal types of such motors :

(i) Schrage or rotor-fed or brush shift motor and ***(ii)*** stator-fed or induction-regulator type motor.

35.20. Schrage Motor*

It is a rotor-fed, shunt-type, brush-shifting, 3-phase commutator induction motor which has built-in arrangement both for ***speed control and power factor improvement***. In fact, it is an induction motor with a built-in slip-regulator. It has **three** windings:two in rotor and one in stator as shown in Fig. 35.41 and 35.42 (*a*) The three windings are as under:

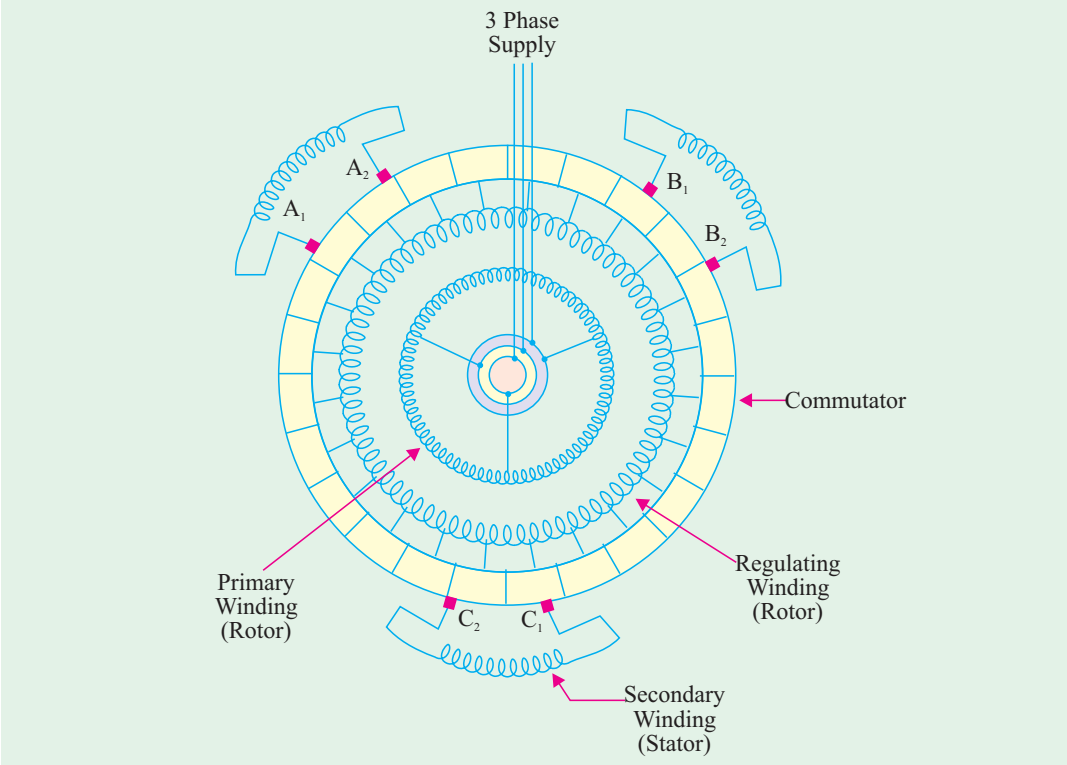


Fig. 35.41

(i) **Primary winding.** It is housed in the lower part of the rotor slots (not stator) and is supplied through slip-rings and brushes at ***line*** frequency. It generates the working flux in the machine.

* After the name of its inventor K.H. Schrage of Sweden.

(*ii*) Regulating winding. It is variously known as ***compensating*** winding or ***tertiary*** winding. It is also housed in ***rotor slots*** (in the upper part) and is connected to the commutator in a manner similar to the armature of a d.c. motor.

(*iii*) Secondary winding. It is contained in the ***stator slots***, but end of each phase winding is connected to one of the pair of brushes arranged on the commutator. These brushes are mounted on two separate brush rockers, which are designed to move in opposite directions relative to the centre line of the corresponding stator phase (usually by a rack and pinion mechanism). Brushes A_1, B_1, and C_1 move together and are 120 electrical degrees apart. Similarly, brushes A_2, B_2 and C_2 move together and are also 120 electrical degrees apart. A sectional drawing of the motor is shown in Fig. 35.44.

(*a*) Working

When primary is supplied at line frequency, there is transformer action between primary and regulating winding and normal induction motor action between primary and secondary winding. Hence, voltage at line frequency is induced in the regulating winding by transformer action. The commutator, acting as a frequency changer, converts this line-frequency voltage of the regulating winding to the slip frequency for feeding it into the secondary winding on the stator. The voltage across brush pairs A_1A_2, B_1B_2 and C_1C_2 increases as brushes are separated. In fact, magnitude of the voltage injected into the secondary winding depends on the angle of separation of the brushes A_1 and A_2, B_1 and B_2 and C_1 and C_2. How slip-frequency e.m.f. is induced in secondary winding is detailed below:

When 3-ϕ power is connected to slip-rings, synchronously rotating field is set up in the rotor core. Let us suppose that this field revolves in the clockwise direction. Let us further suppose that brush pairs are on ***one commutator segment***, which means that secondary is short-circuited. With rotor still at rest, this field cuts the secondary winding, thereby inducing voltage and so producing currents in it which react with the field to produce ***clockwise (CW)*** torque in the ***stator***. Since stator cannot rotate, as a reaction, it makes the rotor rotate in the ***counterclockwise (CCW)*** direction.

Suppose that the rotor speed is N rpm. Then

1. rotor flux is still revolving with synchronous speed relative to the primary and regulating winding.

2. however, this rotor flux will rotate at slip speed (N_s - N) relative to the stator. It means that the revolving rotor flux will rotate at slip speed in space.

3. if rotor could rotate at synchronous speed *i.e.* if $N = N_s$, then flux would be stationary in space (*i.e.* relative to stator) so that there would be no cutting of the secondary winding by the flux and, consequently, no torque would be developed in it.

As seen from above, in a Schrage motor, the flux rotates at synchronous speed, relative to rotor but with slip speed relative to space (*i.e.* stator), whereas in a normal induction motor, flux rotates synchronously relative to stator (*i.e.* space) but with slip speed relative to the rotor. (Art. 34.11).

Another point worth noting is that since at ***synchronous*** speed, magnetic field is stationary in space, the regulating winding acts as a d.c. armature and the direct current taken from the commutator flows in the secondary winding. Hence, Schrage motor then operates like a synchronous motor.

(*b*) Speed Control

It is quite easy to obtain speeds above as well as below synchronism in a Schrage motor. As shown in Fig. 35.42 (*b*) (*i*) when brush pairs are together on the same commutator segment (*i.e.* are electrically connected via commutator), the secondary winding is short-circuited and the machine

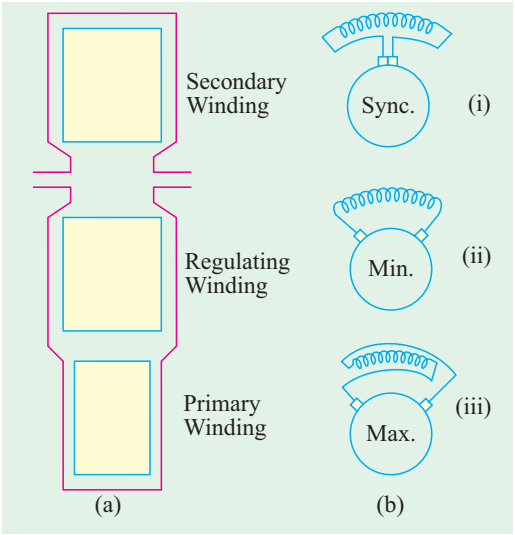


Fig. 35.42

operates as an inverted* plain squirrel-cage induction motor, running with a small positive slip. Parting the brushes in one direction, as shown in Fig. 35.42 (*b*) (*ii*) produces subsynchronous speeds, because in this case, regulating voltage injected into the secondary winding ***opposes*** the voltage induced in it from primary winding. However, when movement of brushes is reversed and they are parted in opposite directions, the direction of the regulating voltage is reversed and so motor speed ***increases*** to super synchronous (maximum) value, as shown in Fig. 35.41 (*b*) (*iii*) The commutator provides maximum voltage when brushes are separated by one pole pitch.

No-load motor speed is given by $N \cong N_s (1 - K \sin 0.5\beta)$ where β is brush separation in electrical degrees and K is a constant whose value depends on turn ratio of the secondary and regulating windings.

Maximum and minimum speeds are obtained by changing the magnitude of the regulating voltage. Schrage motors are capable of speed variations from zero to nearly twice the synchronous speed, though a speed range of 3:1 is sufficient for most applications. It is worth noting that Schrage motor is essentially a shunt machine, because for a particular brush separation, speed remains approximately constant as the load torque is increased as happens with dc shunt motors (Art 29.14).

(*c*) Power Factor Improvement

Power factor improvement can be brought about by changing the phase angle of the voltage injected into the secondary winding. As shown in Fig. 35.43, when one set of brushes is advanced more rapidly than the other is retarted, then injected voltage has a quadrature component which leads the rotor induced voltage. Hence, it results in the improvement of motor power factor. This differential movement of brush sets is obtained by coupling the racks driving the brush rockers to the hand wheel with gears having differing ratios. In Schrage motor, speed depends on angular distance between the individual brush sets (A_1 and A_2 in Fig. 35.41) but p.f. depends on the angular positions of the brushes as a whole.

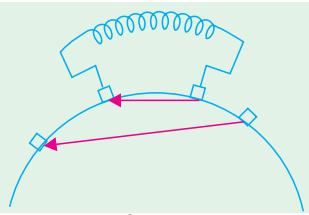


Fig. 35.43

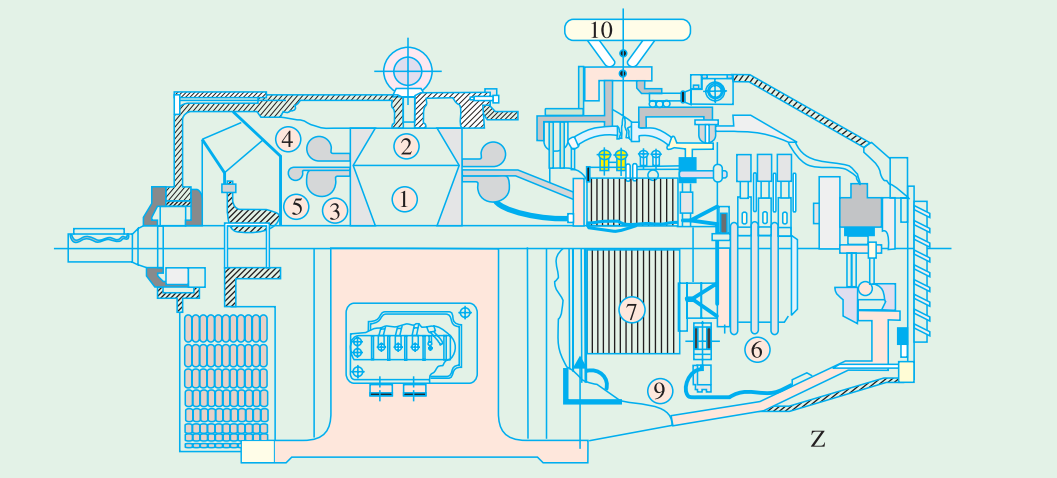


Fig. 35.44. Sectional drawing of a Schrage motor (*Courtesy : Elekrta Faurandou, Germany*)

(*d*) Starting

Schrage motors are usually started with brushes in the lowest speed position by direct-on contactor starters. Usually, interlocks are provided to prevent the contactor getting closed on the line when brushes are in any other position. One major disadvantage of this motor is that its operating voltage is limited to about 700 V because a.c. power has to be fed through slip-rings. It is available in sizes upto 40 kW and is designed to operate on 220, 440 and 550 V. It is ordinarily wound for four or six poles.

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* Inverted in the sense that primary is in the rotor and secondary in the stator - just opposite of that in the normal induction motor (Art. 34.3).



Fig. 35.45. Totally-enclosed surface cooled 750 W, 750 r.pm. high torque induction motor (*Courtesy : Jyoti Limited*)

Fig. 35.44 shows a sectional drawing of a Schrage motor. The details of different parts labelled in the diagram are as under:

1. rotor laminations **2.** stator laminations **3.** primary winding **4.** secondary winding **5.** regulating winding **6.** slip-ring unit **7.** commutator **8.** cable feed for outer brush yoke **9.** cable feed for inner brush yoke **10.** hand wheel.

35.21. Motor Enclosures

Enclosed and semi-enclosed motors are practically identical with open motors in mechanical construction and in their operating characteristics. Many different types of frames or enclosures are available to suit particular requirements. Some of the common type enclosures are described below:

(*i*) Totally-enclosed, Non-ventilated Type

Such motors have solid frames and end- shields, but no openings for ventilation. They get cooled by surface radiation only (Fig. 35.45). Such surface-cooled motors are seldom furnished in sizes above two or three kW, because higher ratings require frames of much larger sizes than fan-cooled motors of corresponding rating.

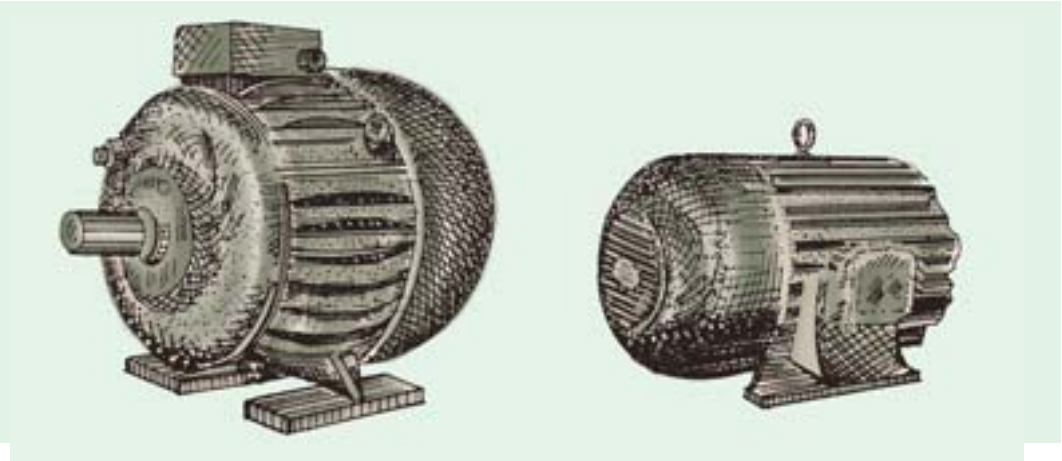


Fig. 35.46. Totally-enclosed, fan-cooled 10-kW 440/400-V, 1000 r.p.m. 50-Hz induction motor (*Courtesy : Jyoti Limited*)

Fig. 35.47. Squirrel-cage motor, showing cowl over the external fan. (*Courtesy : General Electric Co. of India*)

(*ii*) Splash-proof Type

In the frames of such motors, the ventilating openings are so constructed that the liquid drops or dust particles falling on the motor are coming towards it in a straight line at any angle not greater than 100° from the vertical are not able to enter the motor either directly or by striking and running along the surface.

(*iii*) Totally-enclosed, Fan-cooled Type

In such motors (Fig. 35.46), cooling air is drawn into the motor by a fan mounted on the shaft. This air is forced through the motor between the inner fully-enclosed frame and an outer shell, over the end balls and the stator laminations and is then discharged through openings in the opposite side. An internal fan carries the generated heat to the totally enclosing frame, from where it is conducted to

the outside. Because of totally enclosing frame, all working parts are protected against corrosive or abrasive effects of fumes, dust, and moisture.

(*iv*) Cowl-covered Motor

These motors are simplified form of fan-cooled motors (Fig. 35.47). These consist of totally-enclosed frame with a fan and cowl mounted at the end opposite to the driving end. The air is drawn into the cowl with the help of fan and is then forced over the frame. The contours of the cowl guide the cooling air in proper directions. These motors are superior to the usual fan-cooled motors for operation in extremely dusty atmosphere *i.e.* gas works, chemical works, collieries and quarries etc. because there are no air passages which will become clogged with dust.



Fig. 35.48. Squirrel cage motor

(*v*) Protected Type

This construction consists of perforated covers for the openings in both end shields

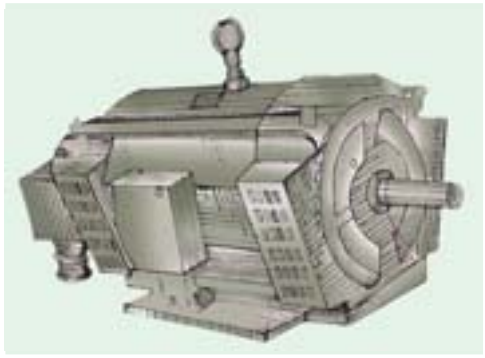


Fig. 35.49. Protected slip-ring motor with totally enclose slip -rings. (*Courtesy : General Electric Co. of India*)

(*vi*) Drip-proof Motors

The frames of such motors are so constructed that liquid drops or dust particles, falling on the machine at any angle greater than 15° from the vertical, cannot enter the motor, either directly or by striking and running along a horizontal or inwardly inclined smooth surface (Fig. 35.51).

(*vii*) Self (Pipe) Ventilated Type

The construction of such motors consists of enclosed shields with provision for pipe connection on both the shields. The motor fan circulates sufficient air through pipes which are of ample section.

(*viii*) Separately (Forced) Ventilated Type

These motors are similar to the self-ventilated type except that ventilation is provided by a separated blower.

35.22. Standard Types of Squirrel-cage Motors

Different types of 3-phase squirrel-cage motors have been standardized, according to their electric characteristics, into six types, designated as design *A*, *B*, *C*, *D*, *E* and *F* respectively. The original commercial squirrel-cage induction motors which were of shallow-slot type are designated as class *A*. For this reason, Class *A* motors are used as a reference and are referred to as 'normal starting-torque, normal starting-current, normal slip' motors.

(*i*) Class *A* - *N*ormal starting torque, normal starting current, normal slip

(*ii*) Class *B* - *N*ormal starting torque, low starting current, normal slip

(*iii*) Class *C* - High starting torque, low starting current, normal slip



Fig. 35.50. Squirrel cage A C induction motor

(*iv*) Class *D* - High starting torque, low starting current, high slip

(*v*) Class *E* - Low starting torque, normal starting current, low slip

(*vi*) Class *F* - Low starting torque, low starting current, normal slip

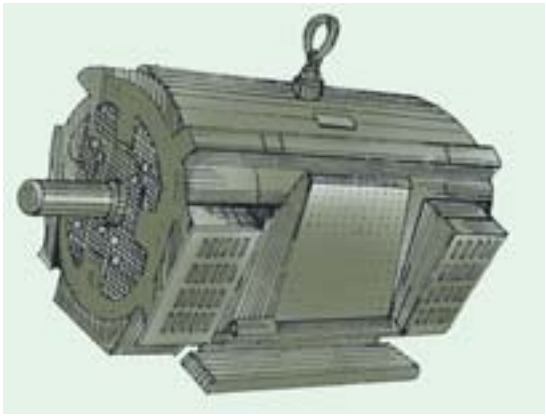


Fig. 35.51. Drip-proof slip-ring 50 up, 440/400-V, 50HZ, 1000 r.p.m. motor (courtesy : Jyoti Limited)

35.23. Class A Motors

It is the most popular type and employs squirrel cage having relatively low resistance and reactance. Its locked-rotor current with full voltage is generally more than 6 times the rated full-load current. For smaller sizes and number of poles, the starting torque with full voltage is nearly twice the full-load torque whereas for larger sizes and number of poles, the corresponding figure is 1.1 times the full-load torque. The full-load slip is less than 5 per cent. The general configuration of slot construction of such motors is shown in Fig. 35.52. As seen, the rotor bars are placed close to the surface so as to reduce rotor reactance.

Such motors are used for fans, pumps, compressors and conveyors etc. which are started and stopped in frequently and have low inertia loads so that the motor can accelerate in a few seconds.

35.24. Class B Motors

These motors are so built that they can be started at full-load while developing normal starting torque with relatively low starting current. Their locked-rotor current with full voltage applied is generally 5 to 5 1/2 times the full-load current. Their cages are of high reactance as seen from Fig. 35.53. The rotor is constructed with deep and narrow bars so as to obtain high reactance during starting.

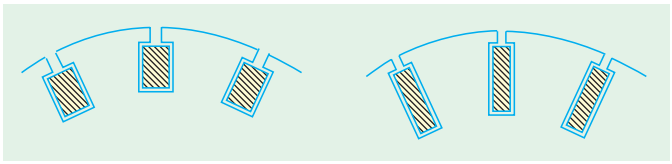


Fig. 35.52 **Fig. 35.53**

Such motors are well-suited for those applications where there is limitation on the starting current or if the starting current is still in excess of what can be permitted, then reduced voltage starting is employed. One of the common applications of such motors is large fans most of which have high moment of inertia. It also finds wide use in many machine tool applications, for pumps of centrifugal type and for driving electric generators.

35.25. Class C Motors

Such motors are usually of double squirrel-cage type (Fig. 35.54) and combine high starting torque with low starting current. Their locked-rotor currents and slip with full voltage applied are nearly the same as for class *B* motors. Their starting torque with full voltage applied is usually 2.75 times the full-load torque.

For those applications where reduced voltage starting does not give sufficient torque to start the load with either class *A* or *B* motor, class C motor, with its high inherent starting torque along with reduced starting current supplied by reduced-voltage starting may be used. Hence, it is frequently used for crushers, compression pumps, large refrigerators, coveyor equipment, textile machinery, boring mills and wood-working equipment etc.

35.26. Class D Motors

Such motors are provided with a high-resistance squirrel cage giving the motor a high starting torque with low starting current. Their locked-rotor currents with full voltage applied are of the same order as for class *C* motors. Their full-load slip varies from 5% to 20 per cent depending on the application. Their slot structure is shown in Fig. 35.55. For obtaining high starting torque with low starting current, thin rotor bars are used which make the leakage flux of the rotor low and the useful flux high.

Since these motors are used where extremely high starting torque is essential, they are usually used for bulldozers, shearing machines, punch presses, foundry equipment, stamping machines, hoists, laundry equipment and metal drawing equipment etc.

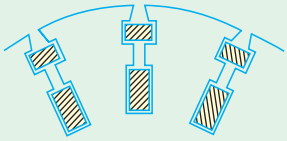


Fig. 35.54

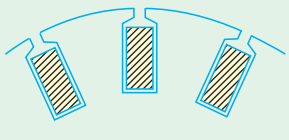
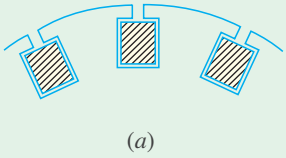


Fig. 35.55

35.27. Class E Motors

These motors have a relatively low slip at rated load. For motors above 5 kW rating, the starting current may be sufficiently high as to require a compensator or resistance starter. Their slot structure is shown in Fig. 35.56 (*a*).



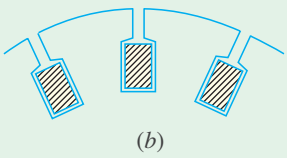


Fig. 35.56

35.28. Class F Motors

Such motors combine a low starting current with a low starting torque and may be started on full voltage. Their low starting current is due to the design of rotor which has high reactance during starting [Fig. 35.56 (*b*)]. The locked rotor currents with full voltage applied and the full-load slip are in the same range as those for class *B* and *C* motors. The starting torque with full voltage applied is nearly 1.25 times the full-torque.

QUESTIONS AND ANSWERS ON THREE-PHASE INDUCTION MOTORS

Q. 1. How do changes in supply voltage and frequency affect the performance of an induction motor ?

Ans. High voltage decreases both power factor and slip, but increases torque. Low voltage does just the opposite. Increase in frequency increases power factor but decreases the torque. However, per cent slip remains unchanged. Decrease in frequency decreases power factor but increases torque leaving per cent slip unaffected as before.

Q. 2. What is, in brief, the basis of operation of a 3-phase induction motor ?

Ans. The revolving magnetic field which is produced when a 3-phase stator winding is fed from a 3-phase supply.

Q. 3. What factors determine the direction of rotation of the motor ?

Ans. The phase sequence of the supply lines and the order in which these lines are connected to the stator winding.

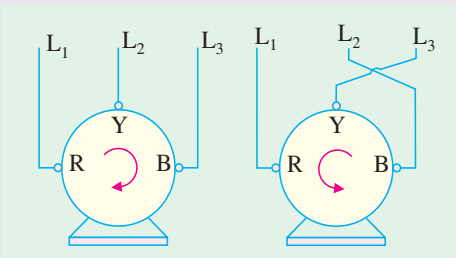


Fig. 35.57

Q. 4. How can the direction of rotation of the motor be reversed ?

Ans. By transposing or changing over any two line leads, as shown in Fig. 35.57.

Q. 5. Why are induction motors called asynchronous ?

Ans. Because their rotors can never run with the synchronous speed.

Q. 6. How does the slip vary with load ?

Ans. The greater the load, greater is the slip or slower is the rotor speed.

Q. 7. What modifications would be necessary if a motor is required to operate on voltage different from that for which it was originally designed ?

Ans. The number of conductors per slot will have to be changed in the same ratio as the change in voltage. If the voltage is doubled, the number of conductors per slot will have to be doubled.

Q. 8. Enumerate the possible reasons if a 3-phase motor fails to start.

Ans. Any one of the following reasons could be responsible :

1. one or more fuses may be blown.
2. voltage may be too low.
3. the starting load may be too heavy.
4. worn bearings due to which the armature may be touching field laminae, thus introducing excessive friction.

Q. 9. A motor stops after starting *i.e.* it fails to carry load. What could be the causes ?

Ans. Any one of the following:

1. hot bearings, which increase the load by excessive friction.
2. excessive tension on belt, which causes the bearings to heat.
3. failure of short cut-out switch.
4. single-phasing on the running position of the starter.

Q. 10. Which is the usual cause of blow-outs in induction motors ?

Ans. The commonest cause is single-phasing.

Q. 11. What is meant by 'single-phasing' and what are its causes ?

Ans. By single-phasing is meant the opening of one wire (or leg) of a three-phase circuit whereupon the remaining leg at once becomes single-phase. When a three-phase circuit functions normally, there are three distinct currents flowing in the circuit. As is known, any two of these currents use the third wire as the return path *i.e.* one of the three phases acts as a return path for the other two. Obviously, an open circuit in one leg kills two of the phases and there will be only one current or phase working, even though two wires are left intact. The remaining phase attempts to carry all the load. The usual cause of single-phasing is, what is generally referred to as ***running fuse***, which is a fuse whose current-carrying capacity is equal to the full-load current of the motor connected in the circuit. This fuse will blow-out whenever there is overload (either momentary or sustained) on the motor.

Q. 12. What happens if single-phasing occurs when the motor is running ? And when it is stationary ?

Ans. (*i*) If already running and carrying half load or less, the motor will continue running as a single-phase motor on the remaining single-phase supply, without damage because half loads do not blow normal fuses.

(*ii*) If motor is very heavily loaded, then it will stop under single-phasing and since it can neither restart nor blow out the remaining fuses, the burn-out is very prompt.

A stationary motor will not start with one line broken. In fact, due to heavy standstill current, it is likely to burn-out quickly unless immediately disconnected.

Q. 13. Which phase is likely to burn-out in a single-phasing delta-connected motor, shown in Fig. 35.58.

Ans. The *Y*-phase connected across the live or operative lines carries nearly three times its normal current and is the one most likely to burn-out.

The other two phases *R* and *B*, which are in series across L_2 and L_3 carry more than their full-load currents.

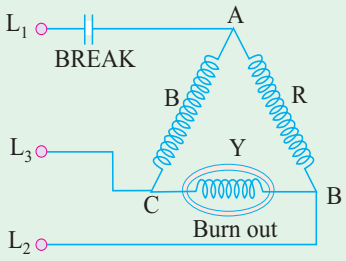


Fig. 35.58

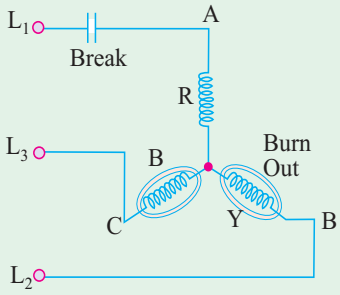


Fig. 35.59

Q. 14. What currents flow in single-phasing star-connected motor of Fig. 35.59.

Ans. With L_1 disabled, the currents flowing in L_2 and L_3 and through phases Y and B in series will be of the order of 250 per cent of the normal full-load current, 160 per cent on 3/4 load and 100 per cent on 1/2 load.

Q. 15. How can the motors be protected against single-phasing ?

Ans. (*i*) By incorporating a combined overload and single-phasing relay in the control gear.
(*ii*) By incorporating a phase-failure relay in the control gear. The relay may be either voltage or current-operated.

Q. 16. Can a 3-phase motor be run on a single-phase line ?

Ans. Yes, it can be. But a phase-splitter is essential.

Q. 17. What is a meant by a phase-splitter ?

Ans. It is a device consisting of a number of capacitors so connected in the motor circuit that it produces, from a single input wave, three output waves which differ in phase from each other.

Q. 18. What is the standard direction of rotation of an induction motor ?

Ans. Counterclockwise, when looking from the front end *i.e.* non-driving end of the motor.

Q. 19. Can a wound-motor be reversed by transposing any two leads from the slip-rings ?

Ans. No. There is only one way of doing so *i.e.* by transposing any two line leads.

Q. 20. What is jogging ?

Ans. It means inching a motor i.e. make it move a little at a time by constant starting and stopping.

Q. 21. What is meant by plugging ?

Ans. It means stopping a motor by instantaneously reversing it till it stops.

Q. 22. What are the indications of winding faults in an induction motor ?

Ans. Some of the indications are as under:
(*i*) excessive and unbalanced starting currents
(*ii*) some peculiar noises and (*iii*) overheating.

OBJECTIVE TESTS – 35

1. In the circle diagram for a 3-ϕ induction motor, the diameter of the circle is determined by
(*a*) rotor current
(*b*) exciting current
(*c*) total stator current
(*d*) rotor current referred to stator.

2. Point out the WRONG statement.
Blocked rotor test on a 3-ϕ induction motor helps to find
(*a*) short-circuit current with normal voltage
(*b*) short-circuit power factor
(*c*) fixed losses
(*d*) motor resistance as referred to stator.

3. In the circle diagram of an induction motor, point of maximum input lies on the tangent drawn parallel to
(*a*) output line
(*b*) torque line
(*c*) vertical axis
(*d*) horizontal axis.

4. An induction motor has a short-circuit current 7 times the full-load current and a full-load slip of 4 per cent. Its line-starting torque is ..... times the full-load torque.
(*a*) 7 (*b*) 1.96
(*c*) 4 (*d*) 49

5. In a SCIM, torque with autostarter is ..... times the torque with direct-switching.
(*a*) K^2 (b) K
(c) $1/K^2$ (d) $1/K$
where K is the transformation ratio of the autostarter.

6. If stator voltage of a SCIM is reduced to 50 per cent of its rated value, torque developed is reduced by ..... per cent of its full-load value.
(*a*) 50 (*b*) 25
(c) 75 (*d*) 57.7

7. For the purpose of starting an induction motor, a Y-Δ switch is equivalent to an auto-starter of ratio.....per cent.
(*a*) 33.3 (*b*) 57.7
(*c*) 73.2 (*d*) 60.

8. A double squirrel-cage motor (DSCM) scores over SCIM in the matter of
(*a*) starting torque
(b) high efficiency under running conditions
(c) speed regulation under normal operating conditions
(*d*) all of the above.

9. In a DSCM, outer cage is made of high resistance metal bars primarily for the purpose of increasing its
(*a*) speed regulation
(*b*) starting torque
(*c*) efficiency
(*d*) starting current.

10. A SCIM with 36-slot stator has two separate windings : one with 3 coil groups/ phase/pole and the other with 2 coil groups/phase/pole. The obtainable two motor speeds would be in the ratio of
(*a*) 3 : 2 (*b*) 2 : 3
(c) 2 : 1 (*d*) 1 : 2

11. A 6-pole 3-ϕ induction motor taking 25 kW from a 50-Hz supply is cumulatively-cascaded to a 4-pole motor. Neglecting all losses, speed of the 4-pole motor would be ..... r.p.m.
(*a*) 1500 (*b*) 1000
(c) 600 (*d*) 3000.
and its output would be ..... kW.
(*e*) 15 (*f*) 10
(g) 50/3 (*h*) 2.5.

12. Which class of induction motor will be well suited for large refrigerators?
(*a*) Class E (*b*) Class B
(*c*) Class F (*d*) Class C

13. In a Schrage motor operating at supersynchronous speed, the injected emf and the standstill secondary induced emf
(*a*) are in phase with each other
(*b*) are at 90° in time phase with each other
(*c*) are in phase opposition
(*d*) none of the above.
(Power App.-III, Delhi Univ. July 1987)

14. For starting a Schrage motor, 3-ϕ supply is connected to
(*a*) stator
(*b*) rotor via slip-rings
(*c*) regulating winding
(*d*) secondary winding via brushes.

15. Two separate induction motors, having 6 poles and 5 poles respectively and their cascade combination from 60 Hz, 3-phase supply can give the following synchronous speeds in rpm
(*a*) 720, 1200, 1500 and 3600
(*b*) 720, 1200 1800
(*c*) 600, 1000, 15000
(*d*) 720 and 3000
(Power App.-II, Delhi Univ.Jan 1987)

16. Mark the WRONG statement.
A Schrage motor is capable of behaving as a/ an
(*a*) inverted induction motor
(*b*) slip-ring induction motor
(*c*) shunt motor
(*d*) series motor
(*e*) synchronous motor.

17. When a stationary 3-phase induction motor is switched on with one phase disconnected
(*a*) it is likely to burn out quickly unless immediately disconnected
(b) it will start but very slowly
(c) it will make jerky start with loud growing noise
(d) remaining intact fuses will be blown out due to heavy inrush of current

18. If single-phasing of a 3-phase induction motor occurs under running conditions, it
(*a*) will stall immediately
(*b*) will keep running though with slightly increased slip
(*c*) may either stall or keep running depending on the load carried by it
(*d*) will become noisy while it still keeps running.

ANSWERS

1. *c* **2.** *c* **3.** *d* **4.** *b* **5.** *a* **6.** *c* **7.** *b* **8.** *d* **9.** *b* **10.** *a* **11.** *c, f* **12.** *d* **13.** *a* **14.** *b* **15.** *a* **16.** *d* **17.** *a* **18.** *c*

GO To FIRST

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CHAPTER 36

SINGLE-PHASE MOTORS

Learning Objectives

- Types of Single-phase Motors
- Single-phase Induction Motor
- Double-field Revoling Theory
- Making Single-phase Induction Motor Self-starting
- Equivalent Circuit of Single-phase Induction Motor without Core Loss
- Equivalent Cuicuit With Core Loss
- Types of Capacitors Start Motors
- Capacitor Start-and-Run Motor
- Shaded-pole Single-phase Motor
- Repulsion Type Motors
- Repulsion Motor
- Repulsion Principle
- Compensated Repulsion Motor
- Repulsion-start Induction-Run Motor
- Repulsion Induction Motor
- A.C. Series Motors
- Universal Motor
- Speed control of Universal Motors
- Reluctance Motor
- Hysteresis Motor



Single phase motor

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36.1. Types of Single-Phase Motors

Such motors, which are designed to operate from a single-phase supply, are manufactured in a large number of types to perform a wide variety of useful services in home, offices, factories, workshops and in business establishments etc. Small motors, particularly in the fractional kilo watt sizes are better known than any other. In fact, most of the new products of the manufacturers of space vehicles, aircrafts, business machines and power tools etc. have been possible due to the advances made in the design of fractional-kilowatt motors. Since the performance requirements of the various applications differ so widely, the motor-manufacturing industry has developed many different types of such motors, each being designed to meet specific demands.



Split-phase motor. Switch mechanism disconnects start winding when motor reached three-fourths of rated speed

Single-phase motors may be classified as under, depending on their construction and method of starting :

1. ***Induction Motors*** (split-phase, capacitor and shaded-pole etc.)
2. ***Repulsion Motors*** (sometime called Inductive-Series Motors)
3. ***A.C. Series Motor***
4. ***Un-excited Synchronous Motors***

36.2. Single-phase Induction Motor

Constructionally, this motor is, more or less, similar to a polyphase induction motor, except that ***(i)*** its stator is provided with a single-phase winding and ***(ii)*** a centrifugal switch is used in some types of motors, in order to cut out a winding, used only for starting purposes. It has distributed stator winding and a squirrel-cage rotor. When fed from a single-phase supply, its stator winding produces a flux (or field) which is only ***alternating i.e.*** one which alternates along one space axis only. It is not a synchronously revolving (or rotating) flux, as in the case of a two- or a three-phase stator winding, fed from a 2-or 3-phase supply. Now, an alternating or pulsating flux acting on a ***stationary*** squirrel-cage rotor cannot produce rotation (only a revolving flux can). That is why a single-phase motor is *not self-starting*.



Single-phase induction motor

However, if the rotor of such a machine is given an initial start by hand (or small motor) or otherwise, ***in either*** direction, then immediately a torque arises and the motor accelerates to its final speed (unless the applied torque is too high).

This peculiar behaviour of the motor has been explained in two ways : ***(i)*** by two -field or double-field revolving theory and ***(ii)*** by cross-field theory. Only the first theory will be discussed briefly.

36.3. Double-field Revolving Theory

This theory makes use of the idea that an alternating uni-axial quantity can be represented by two oppositely-rotating vectors of half magnitude. Accordingly, an ***alternating*** sinusoidal flux can be

represented by two ***revolving*** fluxes, each equal to half the value of the alternating flux and each rotating synchronously ($N_s = 120f/P$) in opposite direction*.

As shown in Fig. 36.1 (*a*), let the alternating flux have a maximum value of Φ_m. Its component fluxes *A* and *B* will each be equal to $\Phi_m/2$ revolving in anticlockwise and clockwise directions respectively.

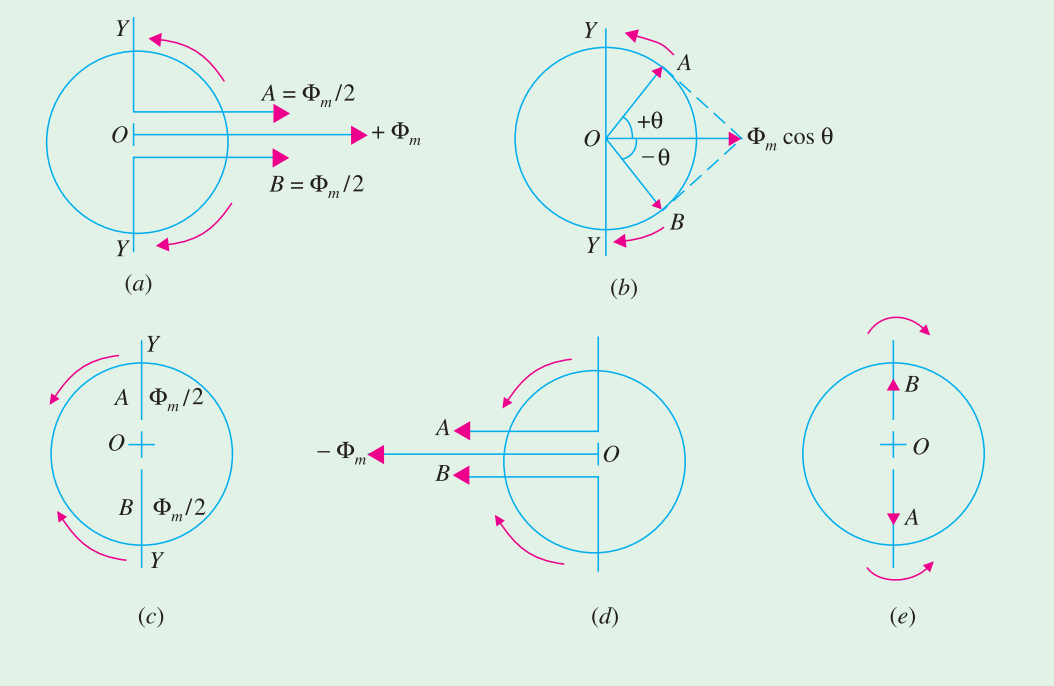


Fig. 36.1

After some time, when *A* and *B* would have rotated through angle + θ and − θ, as in Fig. 36.1 (*b*), the resultant flux would be

$$= 2 \times \frac{\Phi_m}{2} \cos \frac{2\theta}{2} = \Phi_m \cos \theta$$

After a quarter cycle of rotation, fluxes *A* and *B* will be oppositely-directed as shown in Fig. 36.1 (*c*) so that the resultant flux would be zero.

After half a cycle, fluxes *A* and *B* will have a resultant of $-2 \times \Phi_m/2 = -\Phi_m$. After three-quarters of a cycle, again the resultant is

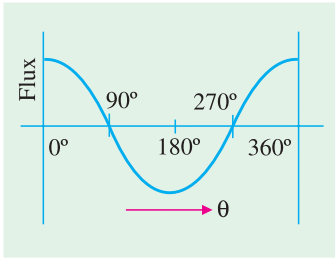


Fig. 36.2

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* For example, a flux given by $\Phi = \Phi_m \cos 2\pi ft$ is equivalent to two fluxes revolving in opposite directions, each with a magnitude of $1/2\Phi$ and an angular velocity of $2\pi f$. It may be noted that Euler's expressions for cos θ provides interesting justification for the decomposition of a pulsating flux. His expression is

$$\cos \theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

The term $e^{j\theta}$ represents a vector rotated clockwise through an angle θ whereas $e^{-j\theta}$ represents rotation in anticlockwise direction. Now, the above given flux can be expressed as

$$\phi_m \cos 2\pi ft = \frac{\phi_m}{2}\left(e^{j2\pi f t} + e^{-j2\pi f t}\right)$$

The right-hand expression represents two oppositely-rotating vectors of half magnitude.

zero, as shown in Fig. 36.1 (*e*) and so on. If we plot the values of resultant flux against θ between limits θ = 0° to θ= 360°, then a curve similar to the one shown in Fig. 36.2 is obtained. That is why an alternating flux can be looked upon as composed of two revolving fluxes, each of half the value and revolving synchronously in opposite directions.

It may be noted that if the slip of the rotor is s with respect to the forward rotating flux (*i.e.* one which rotates in the same direction as rotor) then its slip with respect to the backward rotating flux is (2 – *s*)*.

Each of the two component fluxes, while revolving round the stator, cuts the rotor, induces an e.m.f. and this produces its own torque. Obviously, the two torques (called forward and backward torques) are oppositely-directed, so that the net or resultant torques is equal to their difference as shown in Fig. 36.3.

Now, power developed by a rotor is $P_g = \left(\frac{1-s}{s}\right) I_2^2 \, R_2$

If N is the rotor r.p.s., then torque is given by $T_g = \frac{1}{2\pi N} \cdot \left(\frac{1-s}{s}\right) I_2^2 \, R_2$

Now, $N = N_s(1-s)$ $\therefore$ $T_g = \frac{1}{2\pi N_s} \cdot \frac{I_2^2 \, R_2}{s} = k \cdot \frac{I_2^2 \, R_2}{s}$

Hence, the forward and backward torques are given by

$$T_f = K \frac{I_2^2 \, R_2}{s} \quad \text{and} \quad T_b = -K \cdot \frac{I_2^2 \, R_2}{(2-s)}$$

or $T_f = \frac{I_2^2 \, R_2}{s}$ synch.watt and $T_b = -\frac{I_2^2 \, R_2}{(2-s)}$ synch. watt

Total torque $T = T_f + T_b$

Fig. 36.3 shows both torques and the resultant torque for slips between zero and +2. At standstill, $s = 1$ and $(2 - s) = 1$. Hence, T_f and T_b are numerically equal but, being oppositely directed, produce no resultant torque. That explains why there is no ***starting*** torque in a single-phase induction motor.

However, if the rotor is started somehow, say, in the clockwise direction, the clockwise torque starts increasing and, at the same time, the anticlockwise torque starts decreasing. Hence, there is a certain amount of net torque in the clockwise direction which accelerates the motor to full speed.

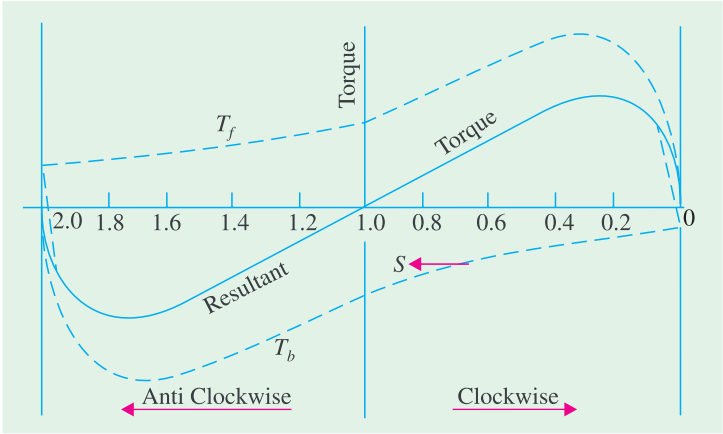


Fig. 36.3

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* It may be proved thus : If N is the r.p.m. of the rotor, then its slip with respect to forward rotating flux is

$$s = \frac{N_s - N}{N_s} = 1 - \frac{N}{N_s} \quad \text{or} \quad \frac{N}{N_s} = 1 - s$$

Keeping in mind the fact that the backward rotating flux rotates opposite to the rotor, the rotor slip with respect to this flux is

$$s_b = \frac{N_s - (-N)}{N_s} = 1 + \frac{N}{N_s} = 1 + (1 - s) = (2 - s)$$

36.4. Making Single-phase Induction Motor Self-starting

As discussed above, a single-phase induction motor is not self-starting. To overcome this drawback and make the motor self-starting, it is temporarily converted into a two-phase motor during starting period. For this purpose, the stator of a single-phase motor is provided with an extra winding, known as ***starting*** (or auxiliary) winding, in addition to the ***main or running winding***. The two windings are spaced 90° electrically apart and are connected in *parallel* across the single-phase supply as shown in Fig. 36.4.

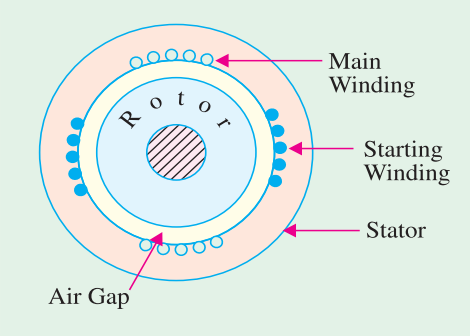


Fig. 36.4

It is so arranged that the phase-difference between the currents in the two stator windings is very large (ideal value being 90°). Hence, the motor behaves like a two-phase motor. These two currents produce a revolving flux and hence make the motor self-starting.

There are many methods by which the necessary phase-difference between the two currents can be created.

(i) In *split*-phase machine, shown in Fig. 36.5 (*a*), the main winding has low resistance but high reactance whereas the starting winding has a high resistance, but low reactance. The resistance of the starting winding may be increased either by connecting a high resistance *R* in series with it or by choosing a high-resistance fine copper wire for winding purposes.



Single-phase motor.

Hence, as shown in Fig. 36.5 (*b*), the current I_s drawn by the starting winding lags behind the applied voltage *V* by a small angle whereas current I_m taken by the main winding lags behind *V* by a very large angle. Phase angle between I_s and I_m is made as large as possible because the starting torque of a split-phase motor is proportional to sin α. A centrifugul switch *S* is connected in series with the starting winding and is located inside the motor. Its function is to automatically disconnect the starting winding from the supply when the motor has reached 70 to 80 per cent of its full-load speed.

In the case of split-phase motors that are hermetically sealed in refrigeration units, instead of internally-mounted centrifugal switch, an electromagnetic type of relay is used. As shown in Fig. 36.6, the relay coil is connected in series with main winding and the pair of contacts which are normally open, is included in the starting winding.

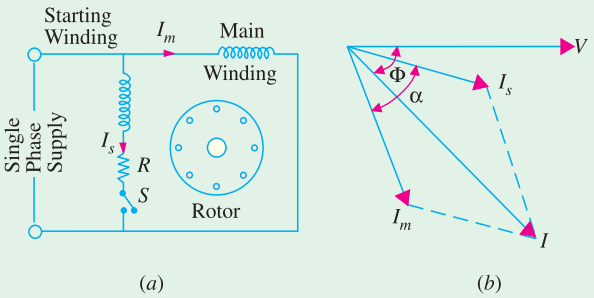


Fig. 36.5

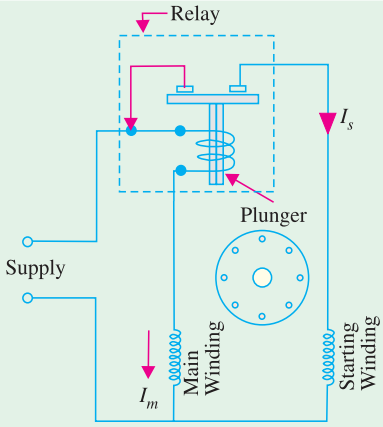


Fig. 36.6

During starting period, when I_m is large, relay contacts close thereby allowing I_s to flow and the motor starts as usual. After motor speeds up to 75 per cent of full-load speed, I_m drops to a value that is low enough to cause the contacts to open.

A typical torque/speed characteristic of such a motor is shown in Fig. 36.7. As seen, the starting torque is 150 to 200 per cent of the full-load torque with a starting current of 6 to 8 times the full-load current. These motors are often used in preference to the costlier capacitor-start motors. Typical applications are : fans and blowers, centrifugal pumps and separators, washing machines, small machine tools, duplicating machines and domestic refrigerators and oil burners etc. Commonly available sizes range from 1/20 to 1/3 h.p. (40 to 250 W) with speeds ranging from 3,450 to 865 r.p.m.

As shown in Fig. 36.8, the direction of rotation of such motors can be reversed by reversing the connections of one of the two stator windings (not both). For this purpose, the four leads are brought outside the frame.

As seen from Fig. 36.9, the connections of the starting winding have been reversed.

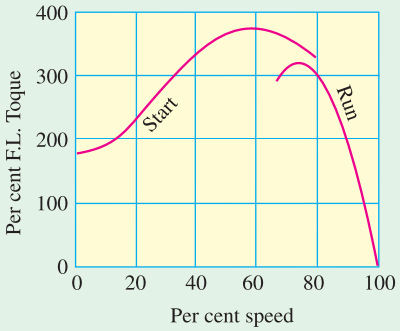


Fig. 36.7

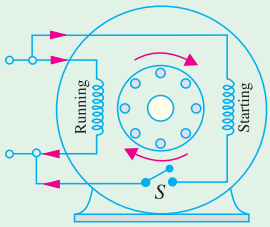


Fig. 36.8

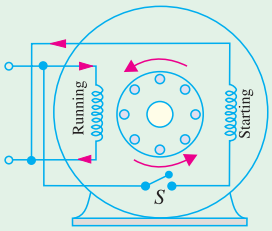


Fig. 36.9

The speed regulation of standard split-phase motors is nearly the same as of the 3-phase motors. Their speed varies about 2 to 5% between no load and full-load. For this reason such motors are usually regarded as practically constant-speed motors.

Note. Such motors are sometimes referred to as resistance-start split-phase induction motors in order to distinguish them from capacitor-start induction run and capacitor start-and-run motors described later.

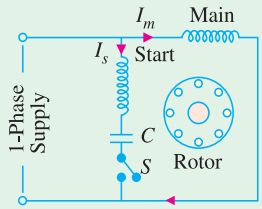


Fig. 36.10

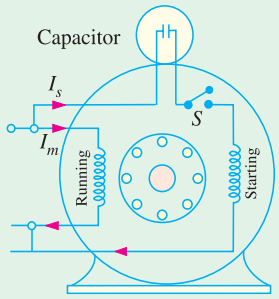


Fig. 36.11

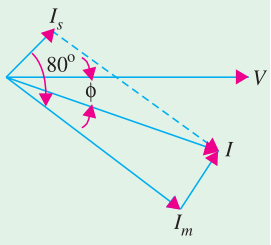


Fig. 36.12

***(ii)* Capacitor-start Induction-run motors.** In these motors, the necessary phase difference between I_s and I_m is produced by connecting a capacitor in series with the starting winding as shown in Fig. 36.10. The capacitor is generally of the electrolytic type and is usually mounted on the outside of the motor as a separate unit (Fig. 36.11).

The capacitor is designed for extremely short-duty service and is guaranteed for not more than 20 periods of operation per hour, each period not to exceed 3 seconds. When the motor reaches about

75 per cent of full speed, the centrifugal switch S opens and cuts out both the starting winding and the capacitor from the supply, thus leaving only the running winding across the lines. As shown in Fig. 36.12, current I_m drawn by the main winding lags the supply voltage V by a large angle whereas I_s leads V by a certain angle. The two currents are out of phase with each other by about 80° (for a 200-W 50-Hz motor) as compared to nearly 30° for a split-phase motor. Their resultant current I is small and is almost in phase with V as shown in Fig. 36.12.



Capacitor start/capacitor-run motor.

Since the torque developed by a split-phase motor is proportional to the sine of the angle between I_s and I_m, it is obvious that the increase in the angle (from 30° to 80°) alone increases the starting torque to nearly twice the value developed by a standard split-phase induction motor. Other improvements in motor design have made it possible to increase the starting torque to a value as high as 350 to 450 per cent.

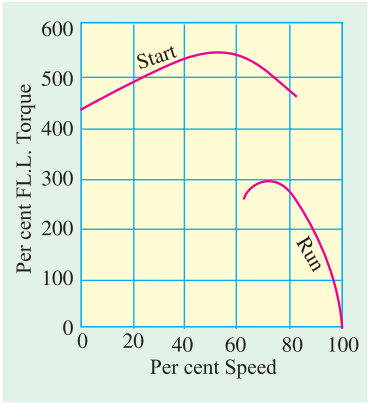


Fig. 36.13

Typical performance curve of such a motor is shown in Fig. 36.13.

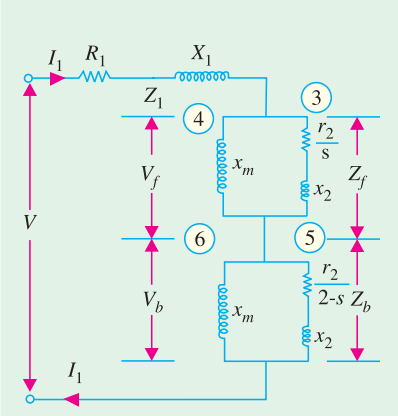


Fig. 36.14

36.5. Equivalent Circuit of a Single-phase Induction Motor–Without Core Loss

A single-phase motor may be looked upon as consisting of two motors, having a common stator winding, but with their respective rotors revolving in opposite directions. The equivalent circuit of such a motor based on double-field revolving theory is shown in Fig. 36.14. Here, the single-phase motor has been imagined to be made-up of (*i*) one stator winding and (*ii*) two imaginary rotors. The stator impedance is $Z = R_1 + j\ X_1$. The impedance of each rotor is $(r_2 + jx_2)$ where r_2 and x_2 represent half the actual rotor values in stator terms (*i.e.* x_2 stands for half the standstill reactance of the rotor, as referred to stator). Since iron loss has been neglected, the exciting branch is shown consisting of exciting reactance only. Each rotor has been assigned half the magnetising reactance* (*i.e* x_m represents half the actual reactance). The impedance of 'forward running' rotor is

$$Z_f = \frac{j\,x_m\left(\frac{r_2}{s} + j\,x_2\right)}{\frac{r_2}{s} + j\,(x_m + x_2)}$$

and it runs with a slip of s. The impedance of 'backward' running rotor is

* In fact, full values are shown by capital letters and half values by small letters.

$$Z_b = \frac{j x_m \left(\frac{r_2}{2-s} + j\, x_2 \right)}{\frac{r_2}{2-s} + j\,(x_m + x_2)}$$

and it runs with a slip of $(2 - s)$. Under standstill conditions, $V_f = V_b$, but under running conditions V_f is almost 90 to 95% of the applied voltage.

The forward torque in synchronous watts is $T_f = I_3^2\, r_2 / s$. Similarly, backward torque is $T_b = I_5^2\, r_2 / (2 - s)$

The total torque is $T = T_f - T_b$.

36.6. Equivalent Circuit–With Core Loss

The core loss can be represented by an equivalent resistance which may be connected either in parallel or in series with the magnetising reactance as shown in Fig. 36.15.

Since under running conditions V_f is very high (and V_b is correspondingly, low) most of the iron loss takes place in the 'forward motor' consisting of the common stator and forward-running rotor. Core-loss current I_w = core loss / V_f. Hence, half value of core-loss equivalent resistance is $r_c = V_f / I_w$. As shown in Fig. 36.15 (*a*), r_c has been connected in parallel with x_m in each rotor.

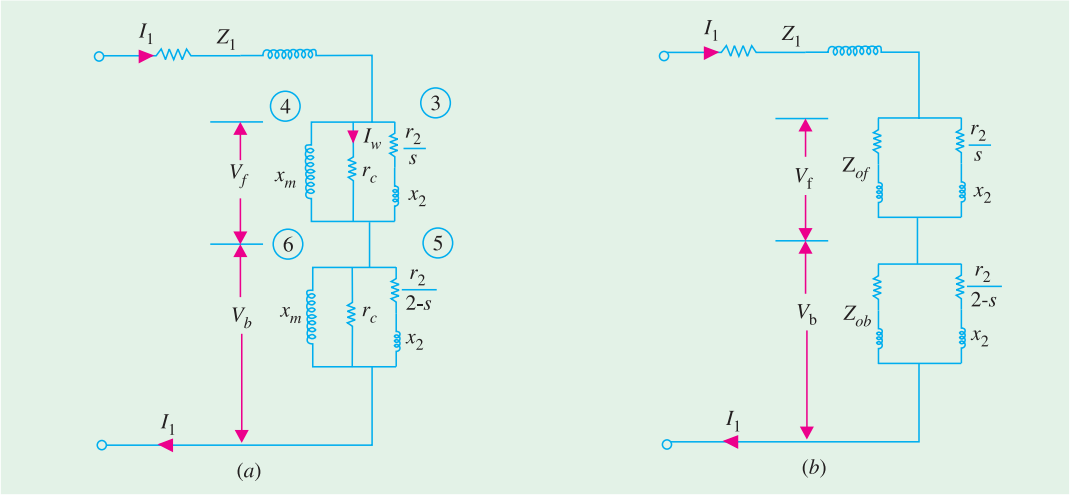


Fig. 36.15

Example 36.1. *Discuss the revolving field theory of single-phase induction motors. Find the mechanical power output at a slip of 0.05 of the 185-W, 4-pole, 110-V, 60-Hz single-phase induction motor, whose constants are given below:*

Resistance of the stator main winding	$R_1 = 1.86$ *ohm*
Reactance of the stator main winding	$X_1 = 2.56$ *ohm*
Magnetizing reactance of the stator main winding	$X_m = 53.5$ *ohm*
Rotor resistance at standstill	$R_2 = 3.56$ *ohm*
Rotor reactance at standstill	$X_2 = 2.56$ *ohm*

(Elect. Machines, Nagpur Univ. 1991)

Solution. Here, $X_m = 53.5\ \Omega$, hence $x_m = 53.5/2 = 26.7\ \Omega$

Similarly, $r_2 = R_2 / 2 = 3.56 / 2 = 1.78\ \Omega$ and $x_2 = X_2 / 2 = 2.56 / 2 = 1.28\ \Omega$

$$\therefore \quad Z_f = \frac{j x_m \left(\frac{r_2}{s} + j\, x_2\right)}{\frac{r_2}{s} + j\,(x_2 + x_m)} = x_m \frac{\frac{r_2}{s} \cdot x_m + j\left[(r_2/s)^2 + x_2\, x_0\right]}{(r_2/s)^2 + x_0^2} \quad \text{where } x_0 = (x_m + x_2)$$

$$\therefore \quad Z_f = 26.7\, \frac{(1.78/0.05) \times 26.7 + j\,[(1.78/0.05)^2 + 1.28 \times 27.98]}{(1.78/0.05)^2 + (27.98)^2}$$

$$= 12.4 + j\,17.15 = 21.15 \angle 54.2^\circ$$

$$\text{Similarly,} \quad Z_b = \frac{j x_m \left(\frac{r_2}{2-s} + j\, x_2\right)}{\frac{r_2}{2-s} + j\,(x_2 + x_m)} = x_m \frac{\left(\frac{r_2}{2-s}\right) x_m + j\left[\left(\frac{r_2}{2-s}\right)^2 + x_0\, x_2\right]}{\left(\frac{r_2}{2-s}\right)^2 + x_0^2}$$

$$= 26.7\, \frac{(1.78/1.95) \times 26.7 + j\,[(1.78/1.95)^2 + 1.28 \times 27.98]}{(1.78/1.95)^2 + (27.98)^2}$$

$$= 0.84 + j\,1.26 = 1.51 \angle 56.3^\circ$$

$$\mathbf{Z_1} = R_1 + j\, X_1 = 1.86 + j\,2.56 = 3.16 \angle 54^\circ$$

Total circuit impedance is

$$\mathbf{Z_{01}} = \mathbf{Z_1 + Z_f + Z_b} = (1.86 + j\,2.56) + (12.4 + j\,17.15) + (0.84 + j\,1.26)$$

$$= 15.1 + j\,20.97 = 25.85 \angle 54.3^\circ$$

Motor current $I_1 = 110/25.85 = 4.27$ A

$V_f = I_1 Z_f = 4.27 \times 21.15 = 90.4$ V; $V_b = I_1 Z_b = 4.27 \times 1.51 = 6.44$ V

$$Z_3 = \sqrt{\left(\frac{r_2}{s}\right)^2 + x_2^2} = 35.7\ \Omega, \quad Z_5 = \sqrt{\left(\frac{r_2}{2-s}\right)^2 + x_2^2} = 1.57\ \Omega$$

$I_3 = V_f / Z_3 = 90.4/35.7 = 2.53$ A, $I_5 = V_b / Z_5 = 6.44/1.57 = 4.1$ A

$T_f = I_3^2\, R_2 / s = 228$ synch. watts, $T_5 = I_5^2\, r_2/(2-s) = 15.3$ synch. watts.

$T = T_f - T_b = 228 - 15.3 = 212.7$ synch. watts

Output = synch. watt $\times (1 - s) = 212.7 \times 0.95 =$ **202 W**

Since friction and windage losses are not given, this also represents the net output.

Example 36.2. *Find the mechanical power output of 185-W, 4 pole, 110-V, 50-Hz single-phase induction motor, whose constants are given below at a slip of 0.05.*

$R_1 = 1.86\ \Omega$ $X_1 = 2.56\ \Omega$ $X_\phi = 53.5\ \Omega$ $R_2 = 3.56\ \Omega$ $X_2 = 2.56\ \Omega$

Core loss = 3.5 W, Friction and windage loss = 13.5 W.

(Electrical Machines-III, Indore Univ. 1987)

Solution. It would be seen that major part of this problem has already been solved in Example 36.1. Let us, now, assume that V_f = 82.5% of 110 V = 90.7 V. Then the core-loss current I_c = 35/90.7 = 0.386 A ; r_c = 90.7/0.386 = 235 Ω.

Motor I

conductance of core-loss branch = $1/r_c = 1/235 = 0.00426$ S

susceptance of magnetising branch = $-j / x_m = -$ j/26.7 = $-j\,0.0374$ S

$$\text{admittance of branch 3} = \frac{(r_2/s) - j\, x_2}{(r_2/s)^2 + x_2^2} = 0.028 - j\,0.00101 \text{ S}$$

admittance of 'motor' I is $\mathbf{Y_f} = 0.00426 - j\,0.0374 + 0.028 - j\,0.00101$

$= 0.03226 - j\,0.03841$ S

impedance $\mathbf{Z_f} = \mathbf{1/Y_f} = 12.96 + j\,15.2$ or $19.9\ \Omega$

Motor II

$$\text{admittance of branch 5} = \frac{\frac{r_2}{2-s} - j\,x_2}{\left(\frac{r_2}{2-s}\right)^2 + x_2^2} = \frac{0.91 - j\,1.28}{2.469} = 0.369 - j\,0.517$$

admittance of 'motor' **II**, $\mathbf{Y_b} = 0.00426 - j\,0.0374 + 0.369 - j\,0.517$

$= 0.3733 - j\,0.555$ S

Impedance of 'motor' II, $\mathbf{Z_b} = 1/\mathbf{Y_b} = 0.836 + j\,1.242$ or $1.5\ \Omega$

Impedance of entire motor (Fig. 36.16) $Z_{01} = \mathbf{Z_1} + \mathbf{Z_f} + \mathbf{Z_b} = 15.66 + j\,19$ or $24.7\ \Omega$

$I_1 = V/Z_{01} = 110/24.7 = 4.46$ A

$V_f = I_1 Z_f = 4.46 \times 19.9 = 88.8$ V

$V_b = 4.46 \times 1.5 = 6.69$ V

$I_3 = 88.8/35.62 = 2.5$ A

$I_5 = 6.69/1.57 = 4.25$ A

$T_f = I_3^2(r_2/s) = 222$ synch. watt

$T_b = I_5^2\left(\frac{r_2}{2-s}\right) = 16.5$ synch. watt

$T = T_f - T_b = 205.5$ synch. watt

Watts converted = synch. watt $(1 - s)$

$= 205.5 \times 0.95 = 195$ W

Net output $= 195 - 13.5 =$ **181.5 W.**

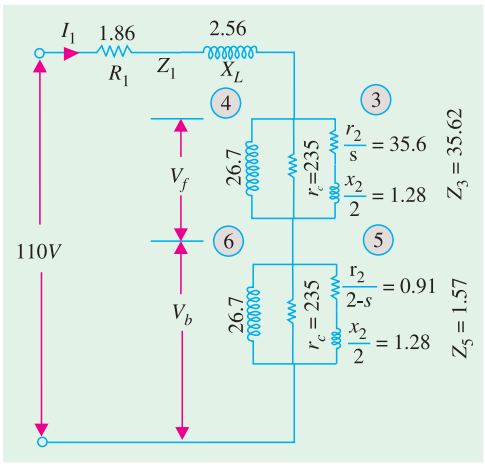


Fig. 36.16

Example 36.3. *A 250-W, 230-V, 50-Hz capacitor-start motor has the following constants for the main and auxiliary windings: Main winding, $Z_m = (4.5 + j\,3.7)$ ohm. Auxiliary winding $Z_a = (9.5 + j\,3.5)$ ohm. Determine the value of the starting capacitor that will place the main and auxiliary winding currents in quadrature at starting.*

(Electrical Machines-III, South Gujarat Univ. 1985)

Solution. Let X_C be the reactance of the capacitor connected in the auxiliary winding.

Then $\mathbf{Z_a} = 9.5 + j\,3.5 - j\,X_C = (9.5 + jX)$

where X is the net reactance.

Now, $\mathbf{Z_m} = 4.5 + j\,3.5 = 5.82 \angle 39.4^\circ$ ohm

Obviously, I_m lags behind V by 39.4°

Since, time phase angle between I_m and I_a has to be 90°, I_a must lead V by

$(90^\circ - 39.4^\circ) = 50.6^\circ$.

For auxiliary winding, $\tan \phi_a = X/R$ or $\tan 50.6^\circ = X/9.5$

or $X = 9.5 \times 1.217 = 11.56\ \Omega$ (capacitive)

$\therefore\ X_C = 11.56 + 3.5 = 15.06\ \Omega$ $\therefore$ $15.06 = 1/314\ C$; $C =$ **211 μF.**

Tutorial Problem No. 36.1.

1. A 1-ϕ, induction motor has stator windings in space quadrature and is supplied with a single-phase voltage of 200 V at 50 Hz. The standstill impedance of the main winding is (5.2 + *j* 10.1) and of the auxiliary winding is (19.7 + *j* 14.2). Find the value of capitance to be inserted in the auxiliary winding for maximum starting torque. ***(Electrical Machines-III, Indore Univ. July, 1977)***
2. A 230-V, 50-Hz, 6-pole, single-phase induction motor has the following constants.

   $r_1 = 0.12\ \Omega$, $r_2 = 0.14\ \Omega$, $x_1 = x_2 = 0.25\ \Omega$, $x_m = 15\ \Omega$.

   If the core loss is 250 W and friction and windage losses are 500 W, determine the efficiency and torque at $s = 0.05$. ***(Electrical Machines-IV, Bangalore Univ. Aug. 1978)***
3. Explain how the pulsating mmf of a single-phase induction motor may be considered equivalent to two oppositely-rotating fields. Develop an expression for the torque of the motor.

   A 125-W, 4-pole, 110-V, 50-Hz single-phase induction motor has the no-load rotational loss of 25 watts and total rotor copper loss at rated load of 25 watts at a slip of 0.06. The rotor I^2R loss may be neglected.

   At a slip s = 0.06, what is the power input to the machine ?

   (Electrical Machines-III, Indore Univ. July 1977)

36.7. Types of Capacitor–start Motors

Some of the important types of such motors are given below :

1. Single-voltage, externally-reversible type

In this motor, four leads are brought outside its housing; two from the main winding and two from the starting-winding circuit. These four leads are necessary for external reversing. As usual, internally, the starting winding is connected in series with the electrolytic capacitor and a centrifugal switch. The direction of rotation of the motor can be easily reversed externally by reversing the starting winding leads with respect to the running winding leads.

2. Single-voltage, non-reversible type

In this case, the starting winding leads are connected internally to the leads of the running winding. Consequently, there are only two external leads in such motors. Obviously, direction of rotation cannot be reversed unless the motor is taken apart and leads of the starting winding reversed.

3. Single-voltage reversible and with thermostat type

Many motors are fitted with a device called thermostat which provides protection against overload, overheating, and short-circuits etc. The thermostat usually consists of a bimetallic element that is connected in series with the motor and is often mounted on the outside of the motor.

The wiring diagram of a capacitor-start motor fitted with this protective device is shown in Fig. 36.17. When due to some reasons, excessive current flows through the motor, it produces abnormal heating of the bimetallic strip with the result that it bends and opens the contact points thus disconnecting the motor from the supply lines. When the thermostat element cools, it automatically closes the contacts*.

In the case of capacitor-start motors used for refrigerators, generally a terminal block is attached to the motor. Three out of the four block terminals are marked *T*, *TL* and *L* as shown in Fig. 36.18. Thermostat is connected to *T* and *TL*, capacitor between *L* and the unmarked terminal and the supply lines to *TL* and *L*.

---

* However, in some thermal units, a reset button has to be operated manually to restore the motor to operation.

In certain types of thermal units, a heating element is used for heating the bimetallic strip. In that case, the heating element is connected in the line and the element or bimetallic strip is placed either inside the heating unit or besides it.

4. Single-voltage, non-reversible with magnetic switch type

Such motors are commonly used in refrigerators where it is not possible to use a centrifugal switch. The circuit diagram is similar to that shown in Fig. 36.6. Since their application requires just one direction of rotation, these motors are not connected for reversing.

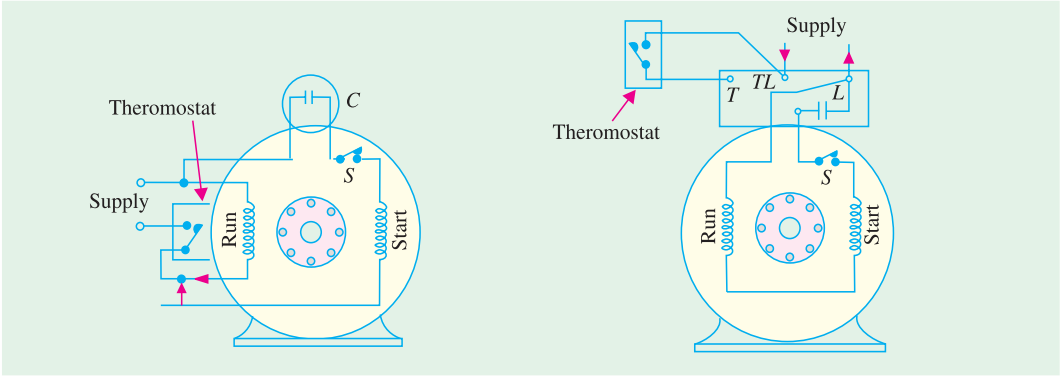


Fig. 36.17 Fig. 36.18

One disadvantage of a capacitor-start motor having magnetic switch lies in the possibility that slight overloads may operate the plunger thereby connecting the starting winding circuit to the supply. Since this winding is designed to operate for very short periods (3 seconds or less) it is likely to be burnt out.

5. Two-voltage, non-reversible Type

These motors can be operated from two a.c. voltage either 110 V and 220 V or 220 V and 440 V. Such motors have two main windings (or one main winding in two sections) and one starting winding with suitable number of leads brought out to permit changeover from one voltage to another.

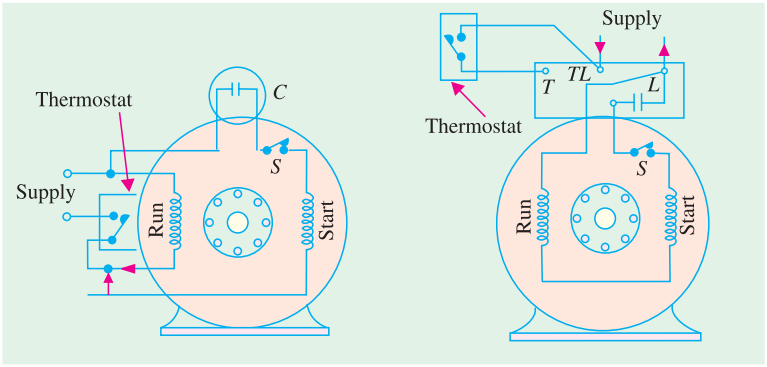


Fig. 36.19 Fig. 36.20

When the motor is to operate from lower voltage, the two main windings are connected in parallel (Fig. 36.19). Whereas for higher voltage, they are connected in series (Fig. 34.20). As will be seen from the above circuit diagrams, the starting winding is always operated on the low-voltage for which purpose it is connected across one of the main windings.

6. Two-voltage, reversible type

External reversing is made possible by means of two additional leads that are brought out from the starting winding.

Fig. 36.21 and 36.22 show connections for clockwise and anticlockwise rotations respectively when motor is operated from lower voltage. Similar wiring diagram can be drawn for higher voltage supply.

7. Single-voltage, three-lead reversible type

In such motors, a two-section running winding is used. The two sections R_1 and R_2 are internally connected in series and one lead of the starting winding is connected to the mid-point of R_1 and R_2.

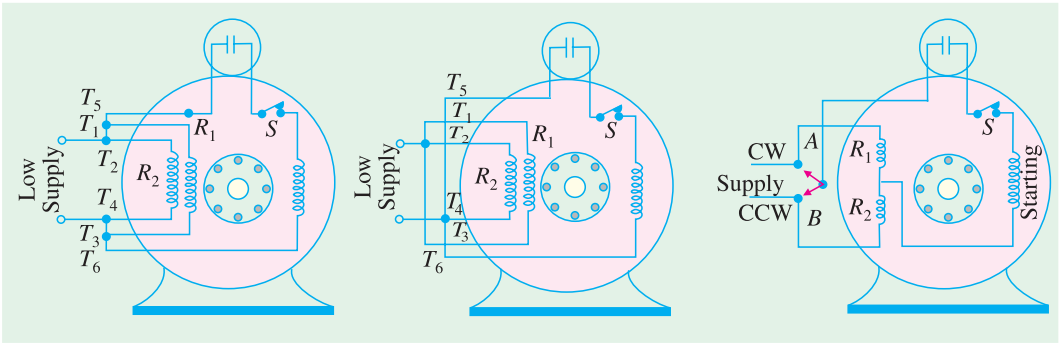


Fig. 36.21 Fig. 36.22 Fig. 36.23

The second lead of the starting winding and both leads of the running winding are brought outside as shown in Fig. 36.23. When the external lead of the starting winding is connected to point A, the winding is connected across R_1 and the motor runs clockwise. When the lead of starter winding is connected to point B, it is connected across R_2. Since current flowing through starting winding is reversed, the motor runs in counter-clockwise direction.

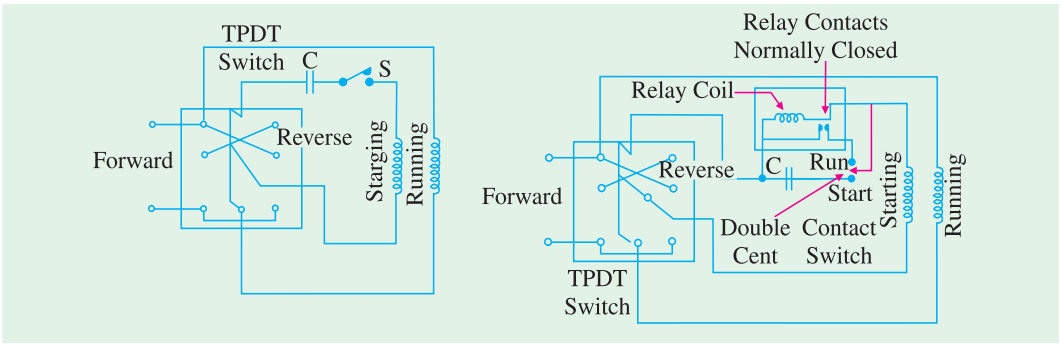


Fig. 36.24 Fig. 36.25

8. Single-voltage, instantly-reversible type

Normally, a motor must be brought to complete rest before it can be started in the reverse direction. It is so because the centrifugal switch cannot close unless the motor has practically stopped. Since starting winding is disconnected from supply when the motor is running, reversal of starting winding leads will not affect the operation of the motor. This reversal is achieved by a triple-pole, double-throw (*TPDT*) switch as shown in Fig. 36.24. The switch consists of three blades or poles which move together as one unit in either of the two positions. In one position of the switch (shown in one figure) motor runs clockwise and in the other, in counter-clockwise direction. Obviously, in this type of arrangement, it is necessary to wait till motor stops.

In certain applications where instant reversal is necessary while the motor is operating at full speed, a relay is fitted in the circuit to short-circuit the centrifugal switch and connect the starting winding in the circuit in the reversed direction (Fig. 36.25).

It will be seen that when at rest, the double-contact centrifugal switch is in the `start' position. In this position, two connections are made :

(*i*) the starting winding and capacitor *C* are placed in series across the supply line and

(*ii*) the coil of the normally-closed relay is connected across *C*

With the manual *TPDT* switch in the 'forward' position (*a*) running winding is connected across the line (*b*) starting winding and *C* are in series across the line and (*c*) relay coil is connected across *C*. The voltage developed across *C* is applied across the relay coil which results in opening of the relay contacts. With increase in the speed of the motor, the centrifugal switch is thrown in the 'running' position. This cuts out *C* from the circuit and leaves starting winding in series with the relay coil. Since relay coil has high resistance, it permits only enough current through the starting winding as to keep the relay contacts open.

During the fraction-of-a-second interval while *TPDT* switch is shifted from 'forward' to 'reverse' position, no current flows through the relay coil as a result of which the relay contacts close. When *TPDT* switch reaches the 'reverse' position, current flows through the now-closed relay contacts to the starting winding but in opposite direction. This produces a torque which is applied in a direction opposite to the rotation. Hence, **(*i*)** rotor is immediately brought to rest and **(*ii*)** centrifugal switch falls to the 'start' position. As before, *C* is put in series with the starter winding and the motor starts rotating in the opposite direction.

9. Two-speed type

Speed can be changed by changing the number of poles in the winding for which purpose two separate running windings are placed in the slots of the stator, one being 6-pole winding and the other, 8-pole winding. Only one starting winding is used which always acts in conjunction with the higher-speed running winding. The double-action or transfer type centrifugal switch *S* has two contact

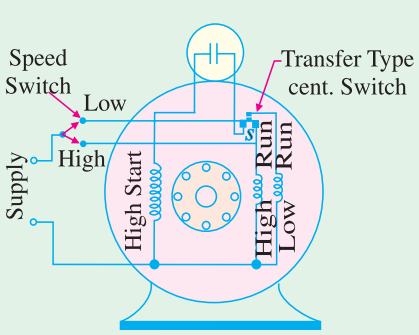


Fig. 36.26

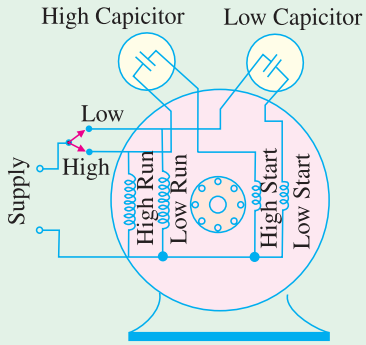


Fig. 36.27

points on the 'start' side and one on the 'run' side. As shown in Fig. 36.26, an external speed switch is used for changing the motor speed. The motor will always start on high speed irrespective of whether the speed switch is on the 'high' or 'low' contact. If speed switch is set on 'low', then as soon as the motor comes up to speed, the centrifugal switch

(*a*) cuts out the starting winding and high-speed running winding and

(*b*) cuts in the low-speed running winding.

10. Two-speed with two-capacitor type

As shown in Fig. 36.27, this motor has two running windings, two starting windings and two capacitors. One capacitor is used for high-speed operation and the other for low-speed operation. A double centrifugal switch *S* is employed for cutting out the starting winding after start.

36.8. Capacitor Start-and-Run Motor

This motor is similar to the capacitor-start motor [Art.36.4 (*ii*)] except that the starting winding and capacitor are connected in the circuit at ***all times***. The advantages of leaving the capacitor permanently in circuit are **(*i*)** improvement of over-load capacity of the motor **(*ii*)** a higher power factor **(*iii*)** higher efficiency and **(*iv*)** quieter running of the motor which is so much desirable for small power drives in offices and laboratories. Some of these motors which start and run with one value of capacitance in the circuit are called ***single-value*** capacitor-run motors. Other which start with high value of capacitance but run with a low value of capacitance are known as ***two-value*** capacitor-run motors.



Capacitor starts and run motor

(*i*) Single-value capacitor-Run Motor

It has one running winding and one starting winding in series with a capacitor as shown in Fig. 36.28. Since capacitor remains in the circuit permanently, this motor is often referred to as permanent-

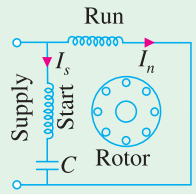


Fig. 36.28

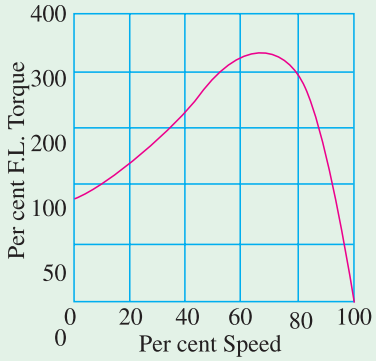


Fig. 36.29

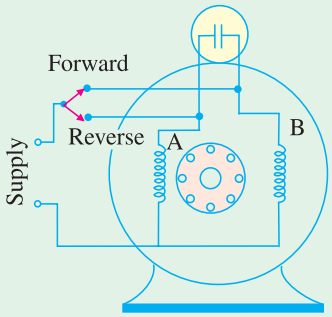


Fig. 36.30

split capacitor-run motor and behaves practically like an unbalanced 2-phase motor. Obviously, there is no need to use a centrifugal switch which was necessary in the case of capacitor-start motors. Since the same capacitor is used for starting and running, it is obvious that neither optimum starting nor optimum running performance can be obtained because value of capacitance used must be a compromise between the best value for starting and that for running. Generally, capacitors of 2 to 20 μF capacitance are employed and are more expensive oil or pyranol-insulated foil-paper capacitors because of continuous-duty rating. The low value of the capacitor results in small starting torque which is about 50 to 100 per cent of the rated torque (Fig. 36.29). Consequently, these motors are used where the required starting torque is low such as air - moving equipment *i.e.* fans, blowers and voltage regulators and also oil burners where quiet operation is particularly desirable.

One unique feature of this type of motor is that it can be easily reversed by an external switch provided its ***running and starting windings are identical***. One serves as the running winding and the other as a starting winding for one direction of rotation. For reverse rotation, the one that previously served as a running winding becomes the starting winding while the former starting winding serves as the running winding. As seen from Fig. 36.30 when the switch is in the forward position, winding *B* serves as running winding and *A* as starting winding. When switch is in 'reverse' position, winding *A* becomes the running winding and *B* the starting winding.

Such reversible motors are often used for operating devices that must be moved back and forth very frequently such as rheostats, induction regulations, furnace controls, valves and arc-welding controls.

(*ii*) Two-value capacitor-Run Motor

This motor starts with a high capacitor in series with the starting winding so that the starting torque is high. For running, a lower capacitor is substituted by the centrifugal switch. Both the running and starting windings remain in circuit.

The two values of capacitance can be obtained as follows:

1. by using two capacitors in parallel at the start and then switching out one for low-value run. (Fig. 36.31) or
2. by using a step-up auto-transformer in conjunction with one capacitor so that effective capacitance value is increased for starting purposes.

In Fig. 36.31, *B* is an electrolytic capacitor of high capacity (short duty) and *A* is an oil capacitor of low value (continuous duty). Generally, starting capacitor *B* is 10 to 15 times the running capacitor *A*. At the start, when the centrifugal switch is closed, the two capacitors are put in parallel, so that their combined capacitance is the sum of their individual capacitances. After the motor has reached 75 per cent full-load speed, the switch opens and only capacitor *A* remains in the starting winding circuit. In this way, both optimum starting and running performance is achieved in such motors. If properly designed, such motors have operating characteristics very closely resembling those displayed by two-phase motors. Their performance is characterised by

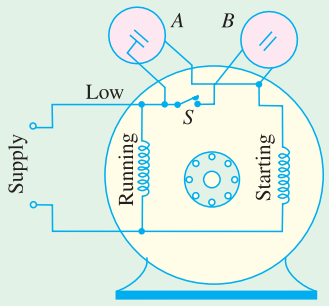


Fig. 36.31

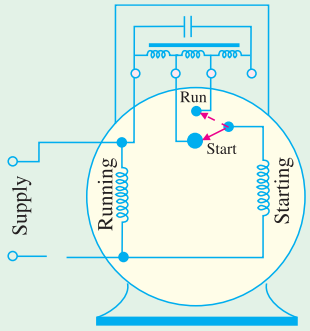


Fig.36.32

1. ability to start heavy loads
2. extremely quiet operation
3. higher efficiency and power factor
4. ability to develop 25 per cent overload capacity

Hence, such motors are ideally suited where load requirements are severe as in the case of compressors and fire strokers etc.



Auto-transformer

The use of an auto-transformer and single oil-type capacitor is illustrated in Fig.36.32. The transformer and capacitor are sealed in a rectangular iron box and mounted on top of the motor. The idea behind using this combination is that a capacitor of value *C* connected to the secondary of a step-up transformer, appears to the primary as though it had a value of K^2C where *K* is voltage transformation ratio. For example, if actual value of $C = 4\ \mu F$ and $K = 6$, then low-voltage primary acts as if it had a 144 μF ($= 6^2 \times 4$) capacitor connected across its terminals. Obviously, effective value of capacitance has increased 36 times. In the 'start' position of the switch, the connection is made to the

mid-tap of the auto-transformer so that $K = 2$. Hence, effective value of capacitance at start is 4 times the running value and is sufficient to give a high starting torque. As the motor speeds up, the centrifugal switch shifts the capacitor from one voltage tap to another so that the voltage transformation ratio changes from higher value at starting to a lower value for running. The capacitor which is actually of the paper-tinfoil construction is immersed in a high grade insulation like wax or mineral oil.

36.9. Shaded-pole Single-phase Motor

In such motors, the necessary phase-splitting is produced by induction. These motors have salient poles on the stator and a squirrel-cage type rotor Fig. 36.33 shows a four-pole motor with the field poles connected in series for alternate polarity. One pole of such a motor is shown separately in Fig. 36.34. The laminated pole has a slot cut across the laminations approximately one-third distance from one edge. Around the small part of the pole is placed a short-circuited Cu coil known as ***shading coil***. This part of the pole is known as ***shaded*** part and the other as ***unshaded*** part. When an alternating current is passed through the exciting (or field) winding surrounding the whole pole, the axis of the pole shifts from the unshaded part a to the shaded part *b*. This shifting of the magnetic axis is, in effect, equivalent to the actual physical movement of the pole. Hence, the rotor starts rotating in the direction of this shift *i.e.* from unshaded part to the shaded part.



Shaded pole single phase motor

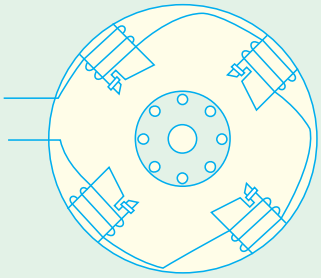


Fig. 36.33

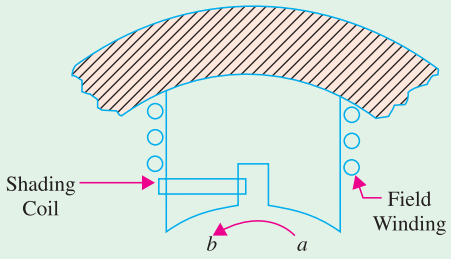


Fig. 36.34

Let us now discuss why shifting of the magnetic axis takes place. It is helpful to remember that the shading coil is highly inductive. When the alternating current through exciting coil tends to increase, it induces a current in the shading coil by transformer action in such a direction as to oppose its growth. Hence, flux density decreases in the shaded part when exciting current increases. However, flux density increases in the shaded part when exciting current starts decreasing (it being assumed that exciting current is sinusoidal).

In Fig. 36.35 (*a*) exciting current is rapidly increasing along *OA* (shown by dots). This will produce an e.m.f. in the shading coil. As shading coil is of low resistance, a large current will be set up in such a direction (according to Lenz's law) as to oppose the rise of exciting current (which is responsible for its production). Hence, the flux mostly shifts to the unshaded part and the magnetic axis lies along the middle of this part *i.e.* along *NC*.

Next, consider the moment when exciting current is near its peak value *i.e.* from point *A* to *B* [Fig. 36.35 (*b*)]. Here, the change in exciting current is very slow. Hence, practically no voltage and, therefore, no current is induced in the shading coil. The flux produced by exciting current is at its maximum value and is uniformly distributed over the pole face. So the magnetic axis shifts to the centre of the pole *i.e.* along positions *ND*.

Fig. 36.35 (*c*) represents the condition when the exciting current is rapidly decreasing from *B* to *C*. This again sets up induced current in the shading coil by transformer action. This current will flow in such a direction as to oppose this decrease in exciting current, with the result that the flux is strengthened in the shaded part of the pole. Consequently, the magnetic axis shifts to the middle part of the shaded pole *i.e.* along *NE*.

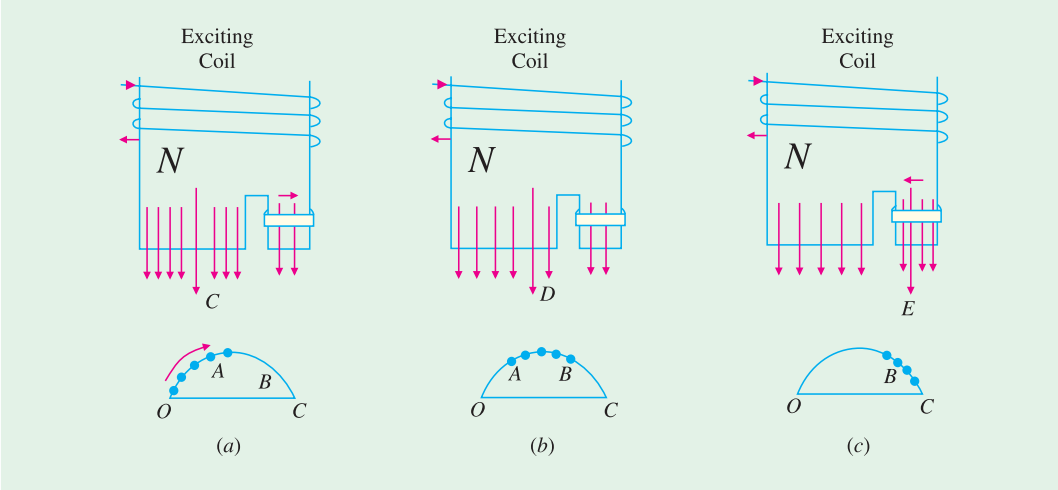


Fig. 36.35

From the above discussion we find that during the positive half-cycle of the exciting current, a *N*-pole shifts along the pole from the unshaded to the shaded part. During the next negative half-cycle of the exciting current, a *S*-pole trails along. The effect is as if a number of real poles were actually sweeping across the space from left to right.

Shaded pole motors are built commercially in very small sizes, varying approximately from 1/250 h.p. (3W) to 1/6 h.p. (125 W). Although such motors are simple in construction, extremely rugged, reliable and cheap, they suffer from the disadvantages of ***(i)*** low starting torque ***(ii)*** very little overload capacity and ***(iii)*** low efficiency. Efficiencies vary from 5% (for tiny sizes) to 35 (for higher ratings). Because of its low starting torque, the shaded-pole motor is generally used for small fans, toys, instruments, hair dryers, ventilators, circulators and electric clocks. It is also frequently used for such devices as churns, phonograph turntables and advertising displays etc. The direction of rotation of this motor cannot be changed, because it is fixed by the position of copper rings.

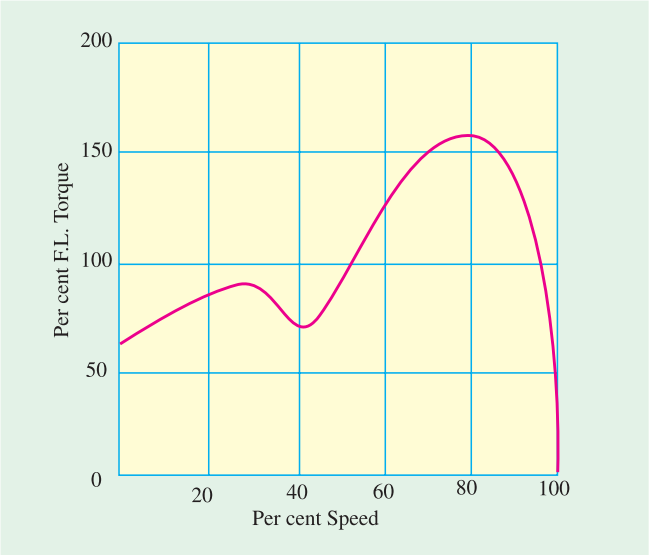


Fig. 36.36

A typical torque / speed curve for such a motor is shown in Fig. 36.36.

36.10. Repulsion Type Motors



Repulsion induction motor

These can be divided into the following four distinct categories :

1. **Repulsion Motor.** It consists of **(*a*)** one stator winding **(*b*)** one rotor which is wound like a d.c. armature **(*c*)** commutator and **(*d*)** a set of brushes, which are short-circuited and remain in contact with the commutator at all times. It operates continuously on the 'repulsion' principle. No short-circuiting mechanism is required for this type.
2. **Compensated Repulsion Motor.** It is identical with repulsion motor in all respects, except that **(*a*)** it carries an additional stator winding, called compensating winding **(*b*)** there is another set of two brushes which are placed midway between the usual short-circuited brush set. The compensating winding and this added set are connected in series.
3. **Repulsion-start Induction-run Motor.** This motor starts as a repulsion motor, but normally runs as an induction motor, with constant speed characteristics. It consists of **(*a*)** one stator winding **(*b*)** one rotor which is similar to the wire-wound d.c. armature **(*c*)** a commutator and **(*d*)** a centrifugal mechanism which short-circuits the commutator bars all the way round (with the help of a short-circuiting necklace) when the motor has reached nearly 75 per cent of full speed.
4. **Repulsion Induction Motor.** It works on the combined principle of repulsion and induction. It consists of **(*a*)** stator winding **(*b*)** two rotor windings : one squirrel cage and the other usual d.c. winding connected to the commutator and **(*c*)** a short-circuited set of two brushes.

It may be noted that repulsion motors have excellent characterstics, but are expensive and require more attention and maintenance than single-phase motors. Hence, they are being replaced by two-value capacitor motors for nearly all applications.

36.11. Repulsion Motor

Constructionally, it consists of the following :

1. Stator winding of the distributed non-salient pole type housed in the slots of a smooth-cored stator (just as in the case of split-phase motors). The stator is generally wound for four, six or eight poles.
2. A rotor (slotted core type) carrying a distributed winding (either lap or wave) which is connected to the commutator. The rotor is identical in construction to the d.c. armature.
3. A commutator, which may be one of the two types : an axial commutator with bars parallel to the shaft or a radial or vertical commutator having radial bars on which brushes press horizontally.
4. Carbon brushes (fitted in brush holders) which ride against the commutator and are used for conducting current through the armature (*i.e.* rotor) winding.

36.12. Repulsion Principle

To understand how torque is developed by the repulsion principle, consider Fig. 36.37 which shows a 2-pole salient pole motor with the magnetic axis vertical. For easy understanding, the stator winding has been shown with concentrated salient-pole construction (actually it is of distributed non-salient type). The basic functioning of the machine will be the same with either type of construction. As mentioned before, the armature is of standard d.c. construction with commutator and brushes (which are short-circuited with a low-resistance jumper).

Suppose that the direction of flow of the alternating current in the exciting or field (stator) winding is such that it creates a *N*-pole at the top and a *S*-pole at the bottom. The alternating flux produced by the stator winding will induce e.m.f. in the armature conductors by transformer action. The direction of the induced e.m.f. can be found by using Lenz's law and is as shown in Fig. 36.37 (*a*). However, the direction of the ***induced*** currents in the armature conductors will depend on the ***positions of the short-circuited brushes***. If brush axis is colinear with magnetic axis of the main poles, the directions of the induced currents (shown by dots and arrows) will be as indicated in Fig. 36.37 (*a*)*. As a result, the armature will become an electromagnet with a *N*-pole on its top, directly under the main *N*-pole and with a *S*-pole at the bottom, directly over the main *S*-pole. Because of this face-to-face positioning of the main and induced magnetic poles, no torque will be developed. The two forces of repulsion on top and bottom act along *Y Y′* in direct opposition to each other.**

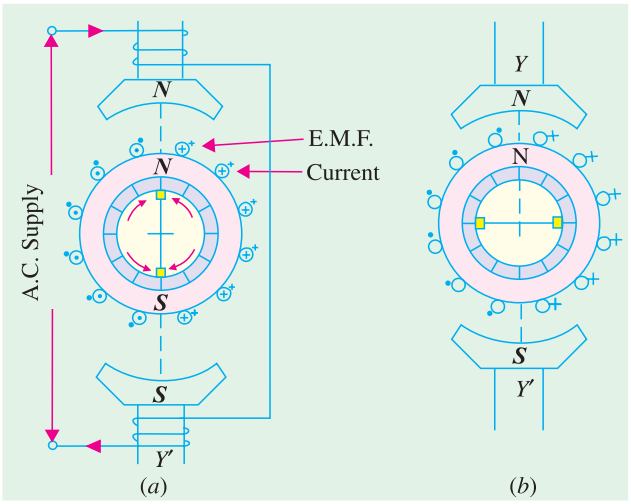


Fig. 36.37

If brushes are shifted through 90° to the position shown in Fig. 36.37 (*b*) so that the brush axis is at right angles to the magnetic axis of the main poles, the directions of the induced voltages at any time in the respective armature conductors are exactly the same as they were for the brush position of Fig. 36.37 (*a*). However, with brush positions of Fig. 36.37 (*b*), the voltages induced in the armature conductors in each path between the brush terminals will neutralize each other, hence there will be no net voltage across brushes to produce armature current. If there is no armature current, obviously, no torque will be developed.

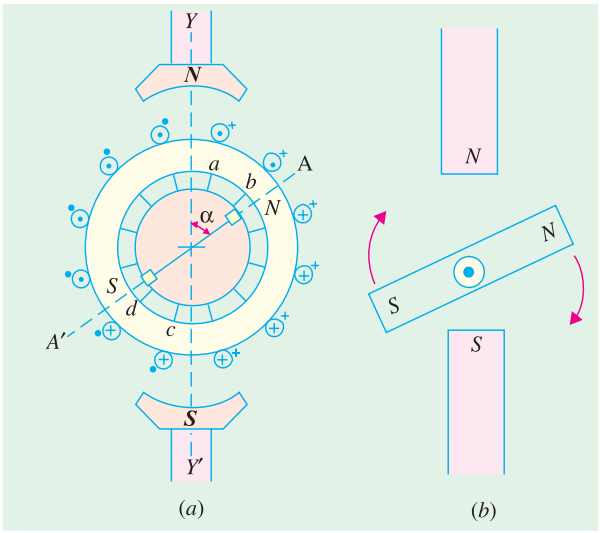


Fig. 36.38

If the brushes are set in position shown in Fig. 36.38 (*a*) so that the brush axis is neither in line with nor 90° from the magnetic axis *Y Y′* of the main poles, a net voltage*** will be induced between the brush terminals which will produce armature current. The armature will again act as an elec-

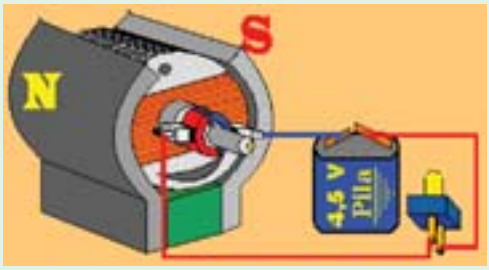
* It should be noted that during the next half-cycle of the supply current, the directions of the respective voltages will be in the opposite directions.

** Alternatively, the absence of the torque may be explained by arguing that the torques developed in the four quadrants neutralize each other.

*** It will be seen from Fig. 36.38 (*a*) that the induced voltages in conductors *a* and *b* oppose the voltages in other conductors lying above brush-axis. Similarly, induced voltages in conductors *c* and *d* oppose the voltages in other conductors, lying below the brush-axis. Yet the net voltage across brush terminals will be sufficient to produce current which will make the armature a powerful magnet.

tromagnet and develop its own *N*-and *S*-poles which, in this case, will not directly face the respective main poles. As shown in Fig. 36.38 (*a*), the armature poles lie along *A A′* making an angle of α with *Y Y′*.

Hence, *rotor N*-pole will be repelled by the main *N*-pole and the rotor *S*-pole will, similarly, be repelled by the ***main*** *S*-pole. Consequently, the rotor will rotate in clockwise direction [Fig.36.38 (*b*)]. Since the forces are those of ***repulsion***, it is appropriate to call the motor as repulsion motor.



Repulsion motor

It should be noted that if the brushes are shifted counter-clockwise from *Y Y′*, rotation will also be counter-clockwise. Obviously, direction of rotation of the motor is determined by the position of brushes with respect to the main magnetic axis.

It is worth noting that the value of starting torque developed by such a motor will depends on the ***amount*** of brush-shift whereas direction of rotation will depend on the ***direction*** of shift [Fig. 36.39 (*a*)]. Maximum starting torque is developed at some position where brush axis makes, an angle lying between 0° and 45° with the magnetic axis of main poles. Motor speed can also be controlled by means of brush shift. Variation of starting torque of a repulsion motor with brush-shift is shown in Fig. 36.39 (*b*).

A straight repulsion type motor has high starting torque (about 350 per cent) and moderate starting current (about 3 to 4 times full-load value).

Principal shortcomings of such a motor are :

1. speed varies with changing load, becoming dangerously high at no load.
2. low power factor, except at high speeds.
3. tendency to spark at brushes.

36.13. Compensated Repulsion Motor

It is a modified form of the straight repulsion motor discussed above. It has an additional stator winding, called compensating winding whose purpose is ***(i)*** to improve power-factor and ***(ii)*** to provide better speed regulation. This winding is much smaller than the stator winding and is usually wound in the inner slots of each main pole and is connected in series with the armature (Fig. 36.40) through an additional set of brushes placed mid-way between the usual short-circuited brushes.

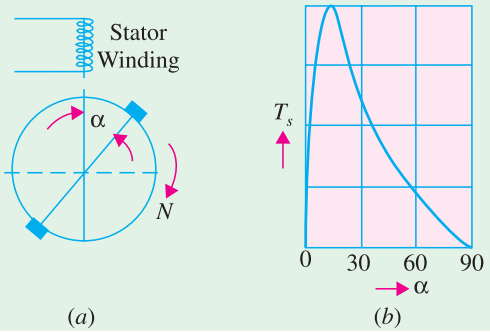


Fig. 36. 39

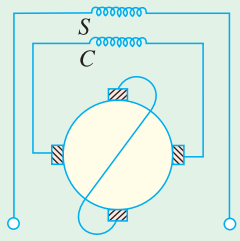


Fig. 36. 40

36.14. Repulsion-start Induction-Run Motor

As mentioned earlier, this motor starts as an ordinary repulsion motor, but after it reaches about 75 per cent of its full speed, centrifugal short-circuiting device short-circuits its commutator. From

then on, it runs as an induction motor, with a short-circuited squirrel-cage rotor. After the commutator is short-circuited, brushes do not carry any current, hence they may also be lifted from the commutator, in order to avoid unnecessary wear and tear and friction losses.

Repulsion-start motors are of two different designs :

1. Brush-lifting type in which the brushes are automatically lifted from the commutator when it is short-circuited. These motors generally employ radial form of commutator and are built both in small and large sizes.
2. Brush-riding type in which brushes ride on the commutator at all times. These motors use axial form of commutator and are always built in small sizes.

The starting torque of such a motor is in excess of 350 per cent with moderate starting current. It is particularly useful where starting period is of comparatively long duration, because of high inertia loads. Applications of such motors include machine tools, commercial refrigerators, compressors, pumps, hoists, floor-polishing and grinding devices etc.

36.15. Repulsion Induction Motor

In the field of repulsion motor, this type is becoming very popular, because of its good all-round characteristics which are comparable to those of a compound d.c. motor. It is particularly suitable for those applications where the load can be removed entirely by de-clutching or by a loose pulley.

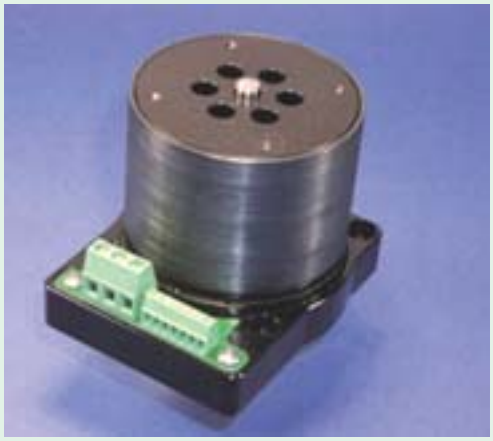
This motor is a combination of the repulsion and induction types and is sometimes referred to as ***squirrel-cage repulsion motor***. It possesses the desirable characteristics of a repulsion motor and the constant-speed characteristics of an induction motor.

It has the usual stator winding as in all repulsion motors. But there are two separate and independent windings in the rotor (Fig. 36.41).

(i) a squirrel-cage winding and

(ii) commutated winding similar to that of a d.c. armature.

Both these windings function during the *entire period* of operation of the motor. The commutated winding lies in the outer slots while squirrel-cage winding is located in the inner slots*. At start, the commutated winding supplies most of the torque, the squirrel-cage winding being practically inactive because of its high reactance. When the rotor accelerates, the squirrel-cage winding takes up a larger portion of the load.



Brushless d.c. seromotor

The brushes are short-circuited and ride on the commutator continuously. One of the advantages of this motor is that it requires no centrifugal short-circuiting mechanism. Sometimes such motors are also made with compensating winding for improving the power factor.

As shown in Fig. 36.42, its starting torque is high, being in excess of 300 per cent. Moreover, it has a fairly constant speed regulation. Its field of application includes house-hold refrigerators, garage air pumps, petrol pumps, compressors, machine tools, mixing machines, lifts and hoists etc.

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* Hence, commutated winding has low resistance whereas the squirrel-cage winding has inherently a high reactance.

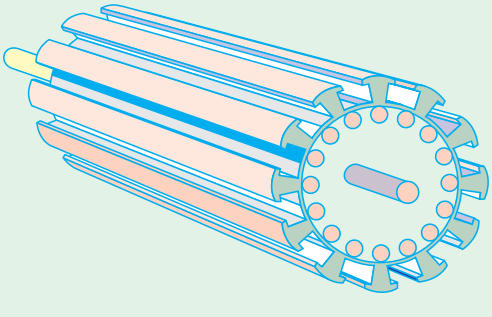


Fig. 36.41

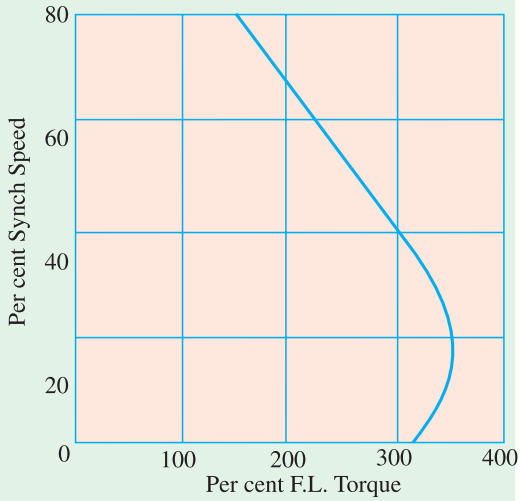


Fig. 36.42

These motors can be reversed by the usual brush-shifting arrangement.

36.16. A.C. Series Motors

If an ordinary d.c. series motor were connected to an a.c. supply, it will rotate and exert unidirectional torque* because the current flowing both in the armature and field reverses at the same time. But the performance of such a motor will not be satisfactory for the following reasons :

1. the alternating flux would cause excessive eddy current loss in the yoke and field cores which will become extremely heated.
2. vicious sparking will occur at brushes because of the huge voltage and current induced in the short-circuited armature coils during their commutation period.
3. power factor is low because of high inductance of the field and armature circuits.



Induction type AC sevro motor and control box

However, by proper modification of design and other refinements, a satisfactory single-phase motor has been produced.

The eddy current loss has been reduced by laminating the entire iron structure of the field cores and yoke.

Power factor improvement is possible only by reducing the magnitudes of the reactances of the field and armature windings. Field reactance is reduced by reducing the number of turns on the field windings. For a given current, it will reduce the field m.m.f. which will result in reduced air- gap flux. This will tend to increase the speed but reduce motor torque. To obtain the same torque, it will now be necessary to increase the number of armature turns proportionately. This will, however, result in increased inductive reactance of the armature, so that the overall reactance of the motor will not be

* However, torque developed is not of constant magnitude (as in d.c. series motors) but pulsates between zero and maximum value each half-cycle.

significantly decreased. Increased armature m.m.f. can be neutralized effectively by using a compensating winding. In conductively-compensated motors, the compensating winding is connected in series with the armature [Fig. 36.43 (*a*)] whereas in inductively-compensated motors, the compensating winding is short-circuited and has no interconnection with the motor circuit [Fig. 36.43 (*b*)]. The compensating winding acts as a short-circuited secondary of a transformer, for which the armature winding acts as a primary. The current in the compensating winding will be proportional to the armature current and 180° out of phase with it.

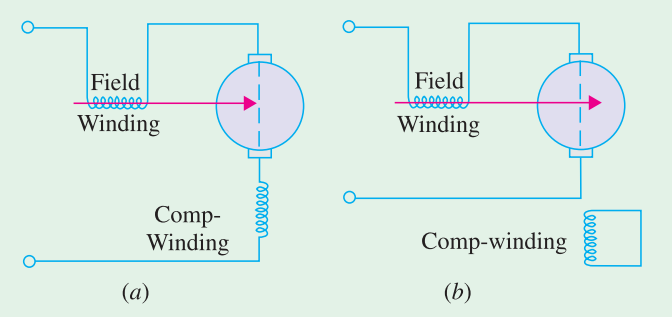


Fig. 36.43

Generally, all d.c. series motors are 'provided' with commutating poles for improving commutation (as in d.c. motors). But commutating poles alone will not produce satisfactory commutation, unless something is done to neutralize the huge voltage induced in the short-circuited armature coil by transformer action (this voltage is not there in d.c. series motor). It should be noted that in an a.c. series motor, the flux produced by the field winding is alternating and it induces voltage (by transformer action) in the short-circuited armature coil during its commutating period. The field winding, associated with the armature coil undergoing commutation, acts as primary and the armature coil during its commutating period acts as a short-circuited secondary. This transformer action produces heavy current in the armature coil as it passes through its commutating period and results in vicious sparking, unless the transformer voltage is neutralized. One method, which is often used for large motors, consists of shunting the winding of each commutating pole with a non-inductive resistance, as shown in Fig. 36.44 (*a*).

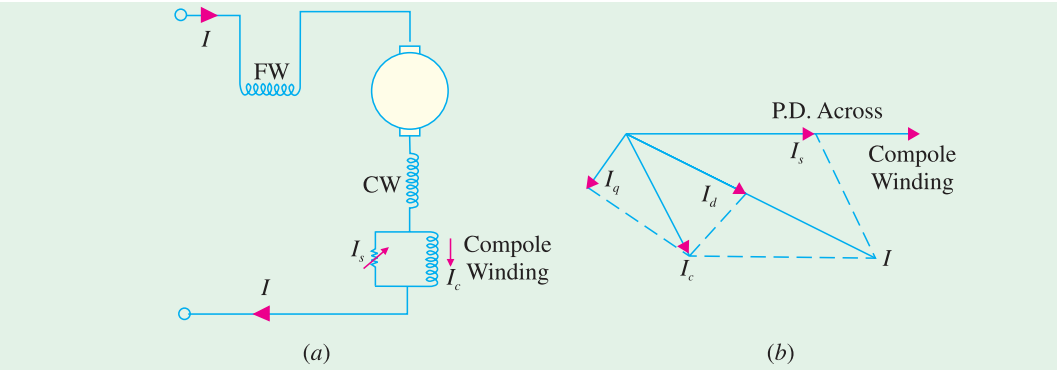


Fig. 36.44

Fig. 36.44. (*b*) shows the vector diagram of a shunted commutator pole. The current I_c through the commutating pole (which lags the total motor current) can be resolved into two rectangular components I_d and I_q as shown. I_d produces a flux which is in phase with total motor current I whereas flux produced by I_q lags I by 90°. By proper adjustment of shunt resistance (and hence I_s), the speed voltage generated in a short-circuited coil by the cutting of the 90° lagging component of the commutating pole flux may be made to neutralize the voltage induced by transformer action.

36.17. Universal Motor

A universal motor is defined as a motor which may be operated either on direct or single-phase

a.c. supply at approximately the same speed and output.

In fact, it is a smaller version (5 to 150 W) of the a.c. series motor described in Art. 36.16. Being a series-wound motor, it has high starting torque and a variable speed characteristic. It runs at dangerously high speed on no-load. That is why such motors are usually built into the device they drive.

Generally, universal motors are manufactured in two types:

1. ***concentrated-pole, non-compensated type*** (low power rating)
2. ***distributed-field compensated type*** (high power rating)

The non-compensated motor has two salient poles and is just like a 2-pole series d.c. motor except that whole of its magnetic path is laminated (Fig. 36.45). The laminated stator is necessary because the flux is alternating when motor is operated from a.c. supply. The armature is of wound type and similar to that of a small d.c. motor. It consists essentially of a laminated core having either straight or skewed slots and a commutator to which the leads of the armature winding are connected. The distributed-field compensated type motor has a stator core similar to that of a split-phase motor and a wound armature similar to that of a small d.c. motor. The compensating winding is used to reduce the reactance voltage present in the armature when motor runs on a.c. supply. This voltage is caused by the alternating flux by transformer action (Art. 36.16).



Universal motor.

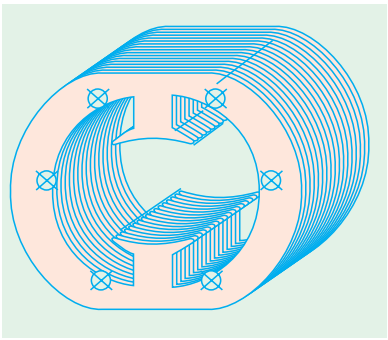


Fig. 36.45

In a 2-pole non-compensated motor, the voltage induced by transformer action in a coil during its commutation period is not sufficient to cause any serious commutation trouble. Moreover, high-resistance brushes are used to aid commutation.

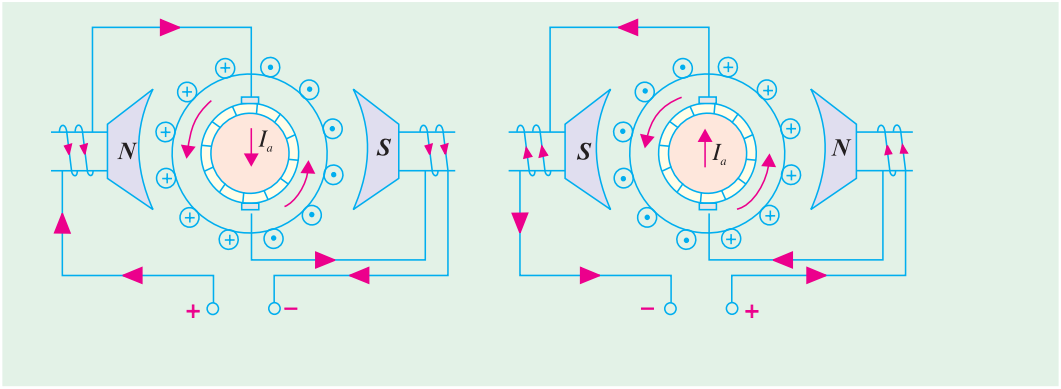


Fig. 36.46

Fig.36.47

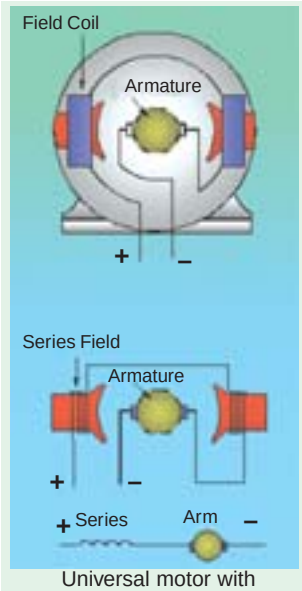
***(a)* Operation.** As explained in Art. 36.16, such motors develop unidirectional torque, regardless of whether they operate on d.c. or a.c. supply. The production of unidirectional torque, when the motor runs on a.c. supply can be easily understood from Fig. 36.46. The motor works on the same principle as a d.c. motor *i.e.* force between the main pole flux and the current-carrying armature conductors. This is true regardless of whether the current is alternating or direct (Fig. 36.47).

(*b*) Speed/Load Characteristic. The speed of a universal motor varies just like that of a d.c. series motor *i.e.* low at full-load and high on no-load (about 20,000 r.p.m. in some cases). In fact, on no-load the speed is limited only by its own friction and windage load. Fig. 36.48 shows typical torque characteristics of a universal motor both for d.c. and a.c. supply. Usually, gear trains are used to reduce the actual load speeds to proper values.

(*c*) Applications. Universal motors are used in vacuum cleaners where actual motor speed is the load speed. Other applications where motor speed is reduced by a gear train are : drink and food mixers, portable drills and domestic sewing machine etc.

(*d*) Reversal of Rotation. The concentrated-pole (or salient-pole) type universal motor may be reversed by reversing the flow of current through either the armature or field windings. The usual method is to interchange the leads on the brush holders (Fig.36.49).

The distributed-field compensated type universal motor may be reversed by interchanging either the armature or field leads and shifting the brushes against the direction in which the motor will rotate. The extent of brush shift usually amounts to several commutator bars.



Universal motor with interchangeable parts for mixing solids, liquids semi-solids and coating.

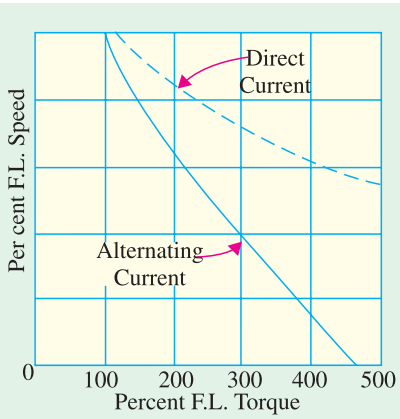


Fig. 36.48

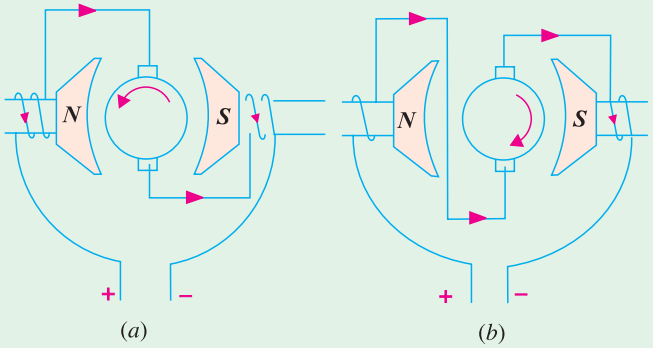


Fig.36.49

36.18. Speed Control of Universal Motors

The following methods are usually employed for speed-control purposes :

(*i*) Resistance Method. As shown in Fig. 36.50, the motor speed is controlled by connecting a variable resistance *R* in series with the motor. This method is employed for motors used in sewing machines. The amount of resistance in the circuit is changed by means of a foot-pedal.

(*ii*) Tapping-field Method. In this method, a field pole is tapped at various points and speed is controlled by varying the field strength (Fig. 36.51). For this purpose, either of the following two arrangements may be used :

(*a*) The field pole is wound in various sections with different sizes of wire and taps are brought out from each section.

(*b*) Nichrome resistance wire is wound over one field pole and taps are brought out from this wire.

***(iii)* Centrifugal Mechanism.** Universal motors, particularly those used for home food and drink mixers, have a number of speeds.Selection is made by a centrifugal device located inside the motor and connected, as shown in Fig. 36.52. The switch is adjustable by means of an external lever. If the motor speed rises above that set by the lever, the centrifugal device opens two contacts and inserts resistance R in the circuit, which causes the motor speed to decrease. When motor runs slow, the two contacts close and short-circuit the resistance, so that the motor speed rises. This process is repeated so rapidly that variations in speed are not noticeable.



Universal motor used for home food and drink mixers

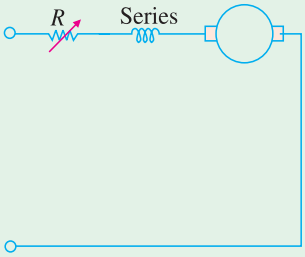


Fig. 36.50

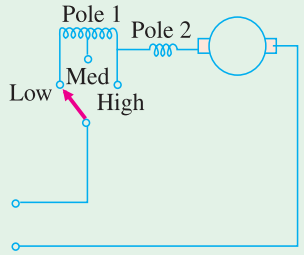


Fig. 36.51

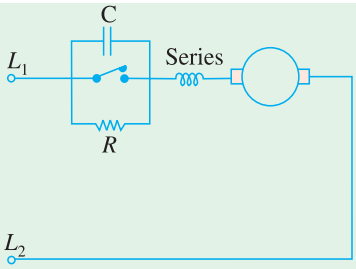


Fig.36.52

The resistance R is connected across the governor points as shown in Fig. 36.52. A capacitor C is used across the contact points in order to reduce sparking produced due to the opening and closing of these points. Moreover, it prevents the pitting of contacts.

Example 36.4. *A 250-W, single-phase, 50-Hz, 220-V universal motor runs at 2000 rpm and takes 1.0 A when supplied from a 220-V dc. supply. If the motor is connected to 220-V ac supply and takes 1.0 A (r.m.s), calculate the speed, torque and power factor. Assume $R_a = 20\ \Omega$ and $L_a = 0.4$ H.*

Solution. DC Operation : $E_{b.dc} = V - I_a R_a$

$= 220 - 20 \times 1 = 200$ V

AC Operation

$X_a = 2\pi \times 50 \times 0.4 = 125.7\ \Omega.$

As seen from Fig.36.53.

$V^2 = (E_{b.ac} + I_a R_a)^2 + (I_a \times X_a)^2$

$\therefore \quad E_{b.ac} = -I_a R_a + \sqrt{V^2 - (I_a X_a)^2}$

$= -1 \times 20 + \sqrt{220^2 - (125.7 \times 1)^2} = 160.5$ V

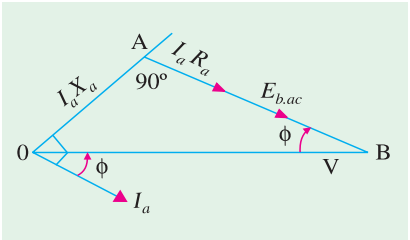


Fig. 36.53

Since armature current is the same for both dc and ac excitations, hence

$\frac{E_{b.dc}}{E_{b.ac}} = \frac{N_{dc}}{N_{ac}}$; $\quad \therefore \quad N_{ac} = 2000 \times \frac{160.5}{200} = 1605$ rpm

$\cos\phi = AB/OB = (E_{b.ac} + I_a R_a) / V = (160.5 + 20)/220 = 0.82$ lag

$$P_{mech} = E_{b.ac} \cdot I_a = 160.5 \times 1 = 160.5 \text{ W}$$
$$T = 9.55 \times 160.5/1605 = \mathbf{0.955 \text{ N-m}}$$

Example 36.5. *A universal series motor has resistance of 30 Ω and an inductance of 0.5 H. When connected to a 250 V d.c. supply and loaded to take 0.8 A, it runs at 2000 r.p.m. Estimate its speed and power factor, when connected to a 250-V, 50-Hz a.c. supply and loaded to take the same current.* **(Elect. Machine, A.M.I.E. Sec. B, 1992)**

Solution. A.C. Operation

$$X_a = 2\pi \times 50 \times 0.5 = 157\ \Omega,$$
$$R_a = 30\ \Omega$$
$$I_a R_a = 0.8 \times 30 = 24\ \Omega$$
$$I_a X_a = 0.8 \times 157 = 125.6 \text{ V}$$

The phasor diagram is shown in Fig. 36.54 (*b*)

$$V^2 = (E_{b.ac} + I_a R_a)^2 + (I_a X_a)^2$$
$$250^2 = (E_{b.ac} + 24)^2 + 125.6^2$$
$$\therefore \quad E_{b.as} = 192.6$$

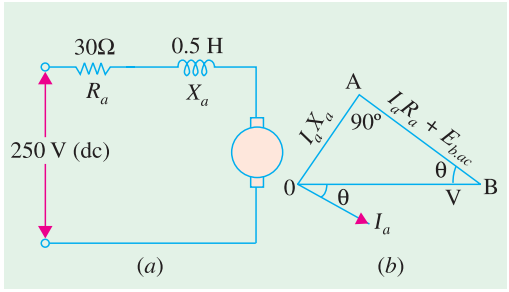


Fig. 36.54

DC Operation

$$E_{b.dc} = 250 - 0.8 \times 30 = 226 \text{ V}$$

Now, $\dfrac{E_{b.ac}}{E_{b.dc}} = \dfrac{N_{ac}}{N_{dc}}$ or $\dfrac{192.12}{226} = \dfrac{N_{ac}}{2000}$;

$$N_{ac} = 1700 \text{ rpm}$$
$$\cos\phi = (E_{b.ac} + I_a R_a)/V = 236.12/250 = \mathbf{0.864 \text{ lag}}$$

36.19. Unexcited Single-phase Synchronous Motors

These motors

1. operate from a single-phase a.c. supply
2. run at a constant speed – the synchronous speed of the revolving flux
3. need no d.c. excitation for their rotors (that is why they are called *unexcited*)
4. are self-starting.

These are of two types (*a*) reluctance motor and (*b*) hysteresis motor.

36.20. Reluctance Motor

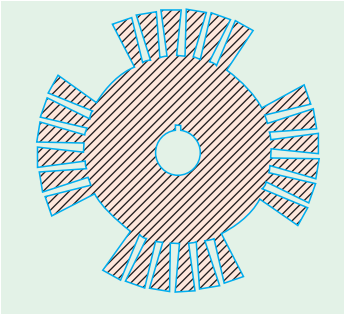


Fig. 36.55

It has either the conventional split-phase stator and a centrifugal switch for cutting out the auxiliary winding (split-phase type reluctance motor) or a stator similar to that of a permanent-split capacitor-run motor (capacitor-type reluctance motor). The stator produces the revolving field.

The squirrel-cage rotor is of unsymmetrical magnetic construction. This type of unsymmetrical construction can be achieved by removing some of the teeth of a symmetrical squirrel-cage rotor punching. For example, in a 48-teeth, four-pole rotor following teeth may be cut away :

1, 2, 3, 4, 5, 6–13, 14, 15, 16, 17, 18–25, 26, 27, 28, 29, 30–37, 38, 39, 40, 41, 42.

This would leave four projecting or salient poles (Fig. 36.55) consisting of the following sets of teeth : 7–12; 19–24; 31–36 and 43–48. In this way, the rotor offers variable magnetic reluctance to the stator flux, the reluctance varying with the position of the rotor.

Working


Reluctance brushless motor

For understanding the working of such a motor one basic fact must be kept in mind. And it is that when a piece of magnetic material is located in a magnetic field, a force acts on the material, tending to bring it into the most dense portion of the field. The force tends to align the specimen of material in such a way that the reluctance of the magnetic path that lies through the material will be minimum.

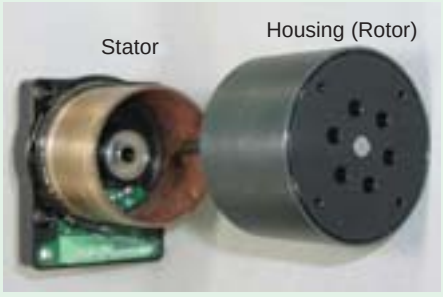
When the stator winding is energised, the revolving magnetic field exerts reluctance torque on the unsymmetrical rotor tending to align the salient pole axis of the rotor with the axis of the revolving magnetic field (because in this position, the reluctance of the magnetic path is minimum). If the reluctance torque is sufficient to start the motor and its load, the rotor will pull into step with the revolving field and continue to run at the speed of the revolving field.*

However, even though the rotor revolves synchronously, its poles lag behind the stator poles by a certain angle known as torque angle, (something similar to that in a synchronous motor). The reluctance torque increases with increase in torque angle, attaining maximum value when $\alpha = 45^\circ$. If α increases beyond 45°, the rotor falls out of synchronism. The average value of the reluctance torque is given by $T = K\,(V/f)^2 \sin 2\delta$ where K is a motor constant.

It may be noted that the amount of load which a reluctance motor could carry at its ***constant speed*** would only be a fraction of the load that the motor could normally carry when functioning as an induction motor. If the load is increased beyond a value under which the reluctance torque cannot maintain synchronous speed, the rotor drops out of step with the field. The speed, then, drops to some value at which the slip is sufficient to develop necessary torque to drive the load by induction-motor action.

The constant-speed characteristic of a reluctance motor makes it very suitable for such applications as signalling devices, recording instruments, many kinds of timers and phonographs etc.

36.21. Hysteresis Motor



Hysteresis motor

The operation of this motor depends on the presence of a ***continuously-revolving*** magnetic flux. Hence, for the split-phase operation, its stator has two windings which remain connected to the single-phase supply continuously both at starting as well as during the running of the motor. Usually, shaded-pole principle is employed for this purpose giving shaded-pole hysteresis motor. Alternatively, stator winding of the type used in capacitor-type motor may be used giving capacitor-type shaded-pole motor. Obviously, in either type, no centrifugal device is used.

The rotor is a smooth chrome-steel cylinder** having

* Actually, the motor starts as an induction motor and after it has reached its maximum speed as an induction, motor, the reluctance torque pulls its rotor into step with the revolving field so that the motor now runs as a synchronous motor by virtue of its saliency.

** Rotors of ceramic permanent magnet material are used whose resistivity approaches that of an insulator. Consequently, it is impossible to set up eddy currents in such a rotor. Hence, there is no eddy current loss but only hysteresis loss.

high retentivity so that the hysteresis loss is high. It has no winding. Because of high retentivity of the rotor material, it is very difficult to change the magnetic polarities once they are induced in the rotor by the revolving flux. The rotor revolves synchronously because the rotor poles magnetically lock up with the revolving stator poles of opposite polarity. However, the rotor poles always lag behind the stator poles by an angle α. Mechanical power developed by rotor is given by $P_m = P_h \left(\frac{1-s}{s}\right)$ where P_h is hysteres loss in rotor. Also $T_h = 9.55\, P_m/N_s$. It is seen that hysteresis torque depends solely on the area of rotor's hysteresis loop.

The fact that the rotor has no teeth or winding of any sort, results in making the motor extremely quiet in operation and free from mechanical and magnetic vibrations. This makes the motor particularly useful for driving tape-decks, tape-decks, turn-tables and other precision audio equipment. Since, commercial motors usually have two poles, they run at 3,000 r.p.m. at 50-Hz single-phase supply. In order to adopt such a motor for driving an electric clock and other indicating devices, gear train is connected to the motor shaft for reducing the load speed. The unit accelerates rapidly, changing from rest to full speed almost instantaneously. It must do so because it cannot accelerate gradually as an ordinary motor it is either operating at synchronous speed or not at all.

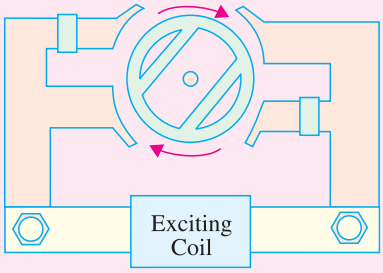


Fig. 36.56

Some unique features of a hysteresis motor are as under :

(i) since its hysteresis torque remains practically constant from locked rotor to synchronous speed, a hysteresis motor is able to synchronise any load it can accelerate–something no other motor does.

(ii) due to its smooth rotor, the motor operates quietly and does not suffer from magnetic pulsations caused by slots/salient-poles that are present in the rotors of other motors.

In Fig. 36.56. is shown a two-pole shaded-pole type hysteresis motor used for driving ordinary household electric clocks. The rotor is a thin metal cylinder and the shaft drives a gear train.

Example 36.6. *A 8-kW, 4-pole, 220-V, 50-Hz reluctance motor has a torque angle of 30° when operating under rated load conditions. Calculate* ***(i)*** *load torque* ***(ii)*** *torque angle if the voltage drops to 205 V and* (*iii*) *will the rotor pulled out of synchronism ?*

Solution. ***(i)*** $N_s = 120 \times 50/4 = 1500$ rpm; $T_{sh} = 9.55 \times \text{output}/N = 9.55 \times 8000/1500 = 51N\text{-}m$

(ii) With the same load torque and constant frequency,

$$V_1 \sin 2\alpha_1 = V_2^{\,2} \sin 2\alpha_2$$

$$\therefore \quad 220^2 \times \sin(2 \times 30^\circ) = 205^2 \times \sin 2\alpha; \qquad \therefore \quad \alpha = 42.9^\circ$$

(iii) since the new load angle is less than 45°, the rotor will not pull out of synchronous.

Tutorial Problems-36.2.

1. A 230-V, 50-Hz, 4-pole, class-A, single-phase induction motor has the following parameters at an operating temperature 63°C :

$r_{1m} = 2.51$ ohms, $r_2' = 7.81$ ohm, $X_m = 150.88$ ohm, $X_{im} = 4.62$ ohm, $X_2' = 4.62$ ohms

Determine stator main winding current and power factor when the motor is running at a slip of 0.05 at the specified temperature of 63°C.

[3.74 ∠48.24°, 0.666] (*AMIE Sec. B Elect. Machines (E-B) Summer 1991*)

2. A fractional horse-power universal motor has armature circuit resistance of 20 ohm and inductance of 0.4 H. On being connected to a 220-V d.c. supply, it draws 1.0 A from the mains and runs at 2000 r.p.m. Estimate the speed and power factor of the motor, when connected to a 230-V, 50-Hz supply drawing the same armature current. Draw relevant phasor diagram.

[1726 rpm, 0.84] (*AMIE Sec. B Elect. Machines 1991*)

3. A universal series motor, when operating on 220 V d.c. draws 10 A and runs at 1400 r.p.m. Find the new speed and power factor, when connected to 220 V, 25 Hz supply, the motor current remaining the same. The motor has total resistance of 1 ohm and total inductance of 0.1 H.

[961 rpm; 0.7] (*AMIE Sec. B Elect. Machines 1990*)

QUESTIONS AND ANSWERS ON SINGLE-PHASE MOTORS

Q.1. How would you reverse the direction of rotation of a capacitor start-induction-run motor ?

Ans. By reversing either the running or starting-winding leads where they are connected to the lines. Both must not be reversed.

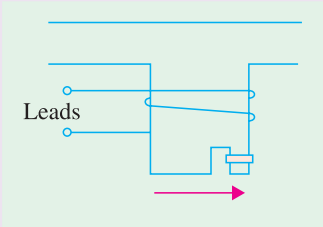


Fig. 36.57

Q.2. In which direction does a shaded-pole motor run ?

Ans. It runs from the unshaded to the shaded pole (Fig.36.57)

Q.3. Can such a motor be reversed ?

Ans. Normally, such motors are not reversible because that would involve mechanical dismantling and re-assembly. However, special motors are made having two rotors on a common shaft, each having one stator assembly for rotation in opposite direction.

Q.4. What is a universal motor ?

Ans. It is built like a series d.c. motor with the difference that both its stator and armature are laminated. They can be used either on d.c. or a.c. supply although the speed and power are greater on direct current. They cannot be satisfactorly made to run at less than about 2000 r.p.m.

Q.5. How can a universal motor be reversed ?

Ans. By reversing either the field leads or armature leads but not both.

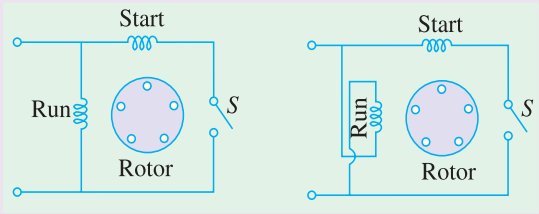


Fig. 36.58

Q.6. How can we reverse the direction of rotation of repulsion, repulsion-induction and repulsion-induction and repulsion- start-induction-run motors ?

Ans. By shifting the brush positions by about 15° electrical.

Q.7. How can we reverse the rotation of a 1-phase, split-phase motor ?

Ans. By reversing the leads to either the running or starter winding (Fig. 36.58) but not both.

Q.8. What could be the reasons if a repulsion-induction motor fails to start ?

Ans. Any one of the following :

1. no supply voltage
2. low voltage
3. excessive overload
4. the bearing lining may be stuck or 'frozen' to the shaft
5. armature may be rubbing
6. brush yoke may be incorrectly located
7. brush spacing may be wrong.

Q.9. What could be the reasons if a split-phase motor fails to start and hums loudly ?

Ans. It could be due to the starting winding being open or grounded or burnt out.

Q.10. What could be the reasons if a split-phase motor runs too slow ?

Ans. Any one of the following factors could be responsible :

1. wrong supply voltage and frequency
2. overload
3. grounded starting and running windings
4. short-circuited or open winding in field circuit.

OBJECTIVE TESTS – 36

1. The starting winding of a single-phase motor is placed in the
(*a*) rotor (*b*) stator
(*c*) armature (*d*) field.

2. One of the characteristics of a single- phase motor is that it
(*a*) is self-starting
(*b*) is not self-starting
(*c*) requires only one winding
(*d*) can rotate in one direction only.

3. After the starting winding of a single- phase induction motor is disconnected from supply, it continues to run only on .....winding.
(*a*) rotor (*b*) compensating
(*c*) field (*d*) running

4. If starting winding of a single-phase induction motor is left in the circuit, it will
(*a*) draw excessive current and overheat
(*b*) run slower
(*c*) run faster
(*d*) spark at light loads.

5. The direction of rotation of a single-phase motor can be reversed by
(*a*) reversing connections of both windings
(*b*) reversing connections of starting winding
(*c*) using a reversing switch
(*d*) reversing supply connections.

6. If a single-phase induction motor runs slower than normal, the more likely defect is
(*a*) improper fuses
(*b*) shorted running winding
(*c*) open starting winding
(*d*) worn bearings.

7. The capacitor in a capacitor-start induction- run ac motor is connected in series with ..... winding.
(*a*) starting (*b*) running
(*c*) squirrel-cage (*d*) compensating

8. A permanent-split single-phase capacitor motor does not have
(*a*) centrifugal switch
(*b*) starting winding
(*c*) squirrel-cage rotor
(*d*) high power factor.

9. The starting torque of a capacitor-start induction-run motor is directly related to the angle α between its two winding currents by the relation
(*a*) cos α (*b*) sin α
(*c*) tan α (*d*) sin $\alpha/2$.

10. In a two-value capacitor motor, the capacitor used for running purposes is a/an
(*a*) dry-type ac electrolytic capacitor
(*b*) paper-spaced oil-filled type
(*c*) air-capacitor
(*d*) ceramic type.

11. If the centrifugal switch of a two-value capacitor motor using two capacitors fails to open, then
(*a*) electrolytic capacitor will, in all probability, suffer breakdown
(*b*) motor will not carry the load
(*c*) motor will draw excessively high current
(*d*) motor will not come upto the rated speed.

12. Each of the following statements regarding a shaded-pole motor is true **except**
(*a*) its direction of rotation is from un- shaded to shaded portion of the poles
(*b*) it has very poor efficiency
(*c*) it has very poor p.f.
(*d*) it has high starting torque.

13. Compensating winding is employed in an ac series motor in order to

(*a*) compensate for decrease in field flux
(*b*) increase the total torque
(*c*) reduce the sparking at brushes
(*d*) reduce effects of armature reaction.

14. A universal motor is one which
(*a*) is available universally
(*b*) can be marketed internationally
(*c*) can be operated either on dc or ac supply
(d) runs at dangerously high speed on no-load.

15. In a single-phase series motor the main purpose of inductively-wound compensating winding is to reduce the
(*a*) reactance emf of commutation
(*b*) rotational emf of commutation
(*c*) transformer emf of commutation
(*d*) none of the above.
(Power App.-II, Delhi Univ. Jan. 1987)

16. A repulsion motor is equipped with
(*a*) a commutator (*b*) slip-rings
(*c*) a repeller
(*d*) neither (*a*) nor (*b*).

17. A repulsion-start induction-run single- phase motor runs as an induction motor only when
(*a*) brushes are shifted to neutral plane
(*b*) short-circuiter is disconnected
(*c*) commutator segments are short- circuited
(*d*) stator winding is reversed.

18. If a dc series motor is operated on ac supply, it will
(*a*) have poor efficiency
(*b*) have poor power factor
(*c*) spark excessively
(*d*) all of the above
(*e*) none of the above.

19. An outstanding feature of a universal motor is its
(*a*) best performance at 50 Hz supply
(*b*) slow speed at all loads
(*c*) excellent performance on dc. supply
(*d*) highest output kW/kg ratio.

20. The direction of rotation of a hysteresis motor is determined by the
(*a*) retentivity of the rotor material
(*b*) amount of hysteresis loss
(*c*) permeability of rotor material
(*d*) position of shaded pole with respect to the main pole.

21. Speed of the universal motor is
(*a*) dependent on frequency of supply
(*b*) proportional to frequency of supply
(*c*) independent of frequency of supply
(*d*) none of the above.
(Elect. Machines, A.M.I.E. Sec. B, 1993)

22. In the shaded pole squirrel cage induction motor the flux in the shaded part always
(*a*) leads the flux in the unshaded pole segment
(*b*) is in phase with the flux in the unshaded pole segment
(*c*) lags the flux in the unshaded pole segment
(*d*) none of the above.
(Elect. Machines, A.M.I.E. Sec. B, 1993)

23. Which of the following motor is an interesting example of beneficially utilizing a phenomenon that is often considered undesirable ?
(*a*) hysteresis motor
(*b*) reluctance motor
(*c*) stepper motor
(*d*) shaded-pole motor.

24. Usually, large motors are more efficient than small ones. The efficiency of the tiny motor used in a wrist watch is approximately..... per cent.
(*a*) 1
(*b*) 10
(*c*) 50
(*d*) 80

ANSWERS

1. *b* **2.** *b* **3.** *d* **4.** *a* **5.** *b* **6.** *d* **7.** *a* **8.** *a* **9.** *b* **10.** *b* **11.** *a* **12.** *d* **13.** *d* **14.** *c* **15.** *d* **16.** *a* **17.** *c* **18.** *d* **19.** *d* **20.** *d* **21.** *a* **22.** *c* **23.** *a* **24.** *a*

ROUGH WORK

GO To FIRST

CONTENTS

CHAPTER 37

ALTERNATORS

Learning Objectives

- Basic Principle
- Stationary Armature
- Rotor
- Armature Windings
- Wye and Delta Connections
- Distribution or Breadth Factor or Winding Factor or Spread Factor
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CONTENTS



Alternator

37.1. Basic Principle



A.C. generators or alternators (as they are usually called) operate on the same fundamental principles of electromagnetic induction as d.c. generators. They also consist of an armature winding and a magnetic field. But there is one important difference between the two. Whereas in d.c. generators, the ***armature rotates*** and the field system is ***stationary***, the arrangement in alternators is just the reverse of it. In their case, standard construction consists of armature winding mounted on a stationary element called ***stator*** and field windings on a rotating element called rotor. The details of construction are shown in Fig. 37.1.

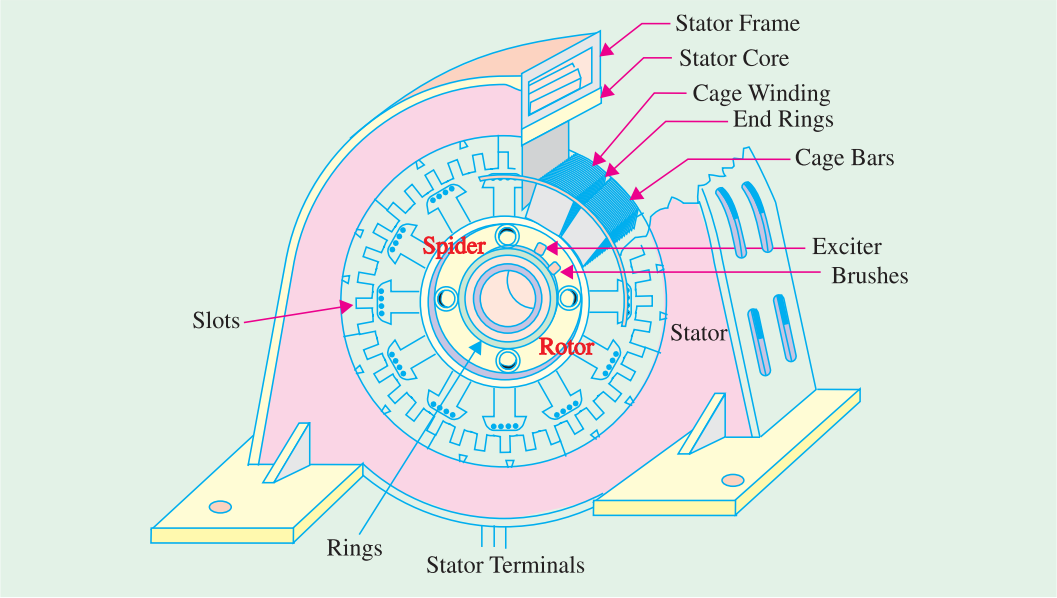


Fig. 37.1

The stator consists of a cast-iron frame, which supports the armature core, having slots on its inner periphery for housing the armature conductors. The rotor is like a flywheel having alternate *N* and *S* poles fixed to its outer rim. The magnetic poles are excited (or magnetised) from direct current supplied by a d.c. source at 125 to 600 volts. In most cases, necessary exciting (or magnetising) current is obtained from a small d.c. shunt generator which is belted or mounted on the shaft of the alternator itself. Because the field magnets are rotating, this current is supplied through two sliprings. As the exciting voltage is relatively small, the slip-rings and brush gear are of light construction. Recently, brushless excitation systems have been developed in which a 3-phase a.c. exciter and a group of rectifiers supply d.c. to the alternator. Hence, brushes, slip-rings and commutator are eliminated.

When the rotor rotates, the stator conductors (being stationary) are cut by the magnetic flux, hence they have induced e.m.f. produced in them. Because the magnetic poles are alternately *N* and *S*, they induce an e.m.f. and hence current in armature conductors, which first flows in one direction and then in the other. Hence, an alternating e.m.f. is produced in the stator conductors ***(i)*** whose frequency depends on the

number of N and S poles moving past a conductor in one second and ***(ii)*** whose direction is given by Fleming's Right-hand rule.

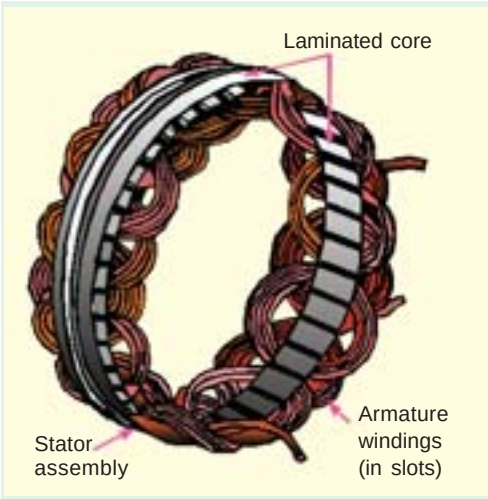
37.2. Stationary Armature

Advantages of having stationary armature (and a rotating field system) are :

1. The output current can be led directly from fixed terminals on the stator (or armature windings) to the load circuit, without having to pass it through brush-contacts.



Alternators



Stationary armature windings

2. It is easier to insulate stationary armature winding for high a.c. voltages, which may have as high a value as 30 kV or more.
3. The sliding contacts *i.e.* slip-rings are transferred to the low-voltage, low-power d.c. field circuit which can, therefore, be easily insulated.
4. The armature windings can be more easily braced to prevent any deformation, which could be produced by the mechanical stresses set up as a result of short-circuit current and the high centrifugal forces brought into play.

37.3. Details of Construction

1. Stator Frame

In d.c. machines, the outer frame (or yoke) serves to carry the magnetic flux but in alternators, it is not meant for that purpose. Here, it is used for holding the armature stampings and windings in position. Low-speed large-diameter alternators have frames which because of ease of manufacture, are cast in sections. Ventilation is maintained with the help of holes cast in the frame itself. The provision of radial ventilating spaces in the stampings assists in cooling the machine.

But, these days, instead of using castings, frames are generally fabricated from mild steel plates welded together in such a way as to form a frame having a box type section.

In Fig. 37.2 is shown the section through the top of a typical stator.

2. Stator Core

The armature core is supported by the stator frame and is built up of laminations of

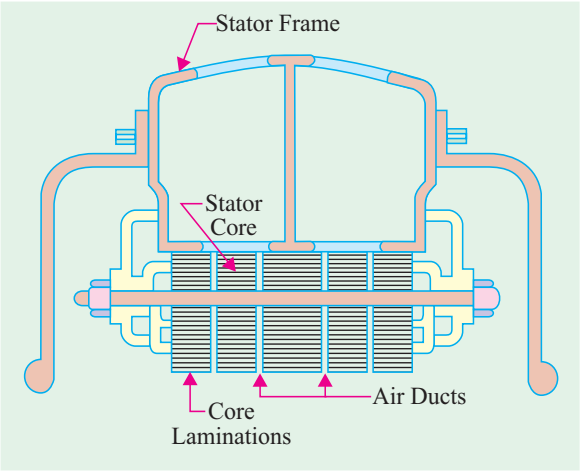


Fig. 37.2

special magnetic iron or steel alloy. The core is laminated to minimise loss due to eddy currents. The laminations are stamped out in complete rings (for smaller machine) or in segments (for larger machines). The laminations are insulated from each other and have spaces between them for allowing the cooling air to pass through. The slots for housing the armature conductors lie along the inner periphery of the core and are stamped out at the same time when laminations are formed. Different shapes of the armature slots are shown in Fig. 37.3.



Streetbike stator

The wide-open type slot (also used in d.c. machines) has the advantage of permitting easy installation of form-wound coils and their easy removal in case of repair. But it has the disadvantage of distributing the air-gap flux into bunches or tufts, that produce ripples in the wave of the generated e.m.f. The semi-closed type slots are better in this respect, but do not allow the use of form-wound coils. The wholly-closed type slots or tunnels do not disturb the air-gap flux but ***(i)*** they tend to increase the inductance of the windings ***(ii)*** the armature conductors have to be threaded through, thereby increasing initial labour and cost of winding and ***(iii)*** they present a complicated problem of end-connections. Hence, they are rarely used.

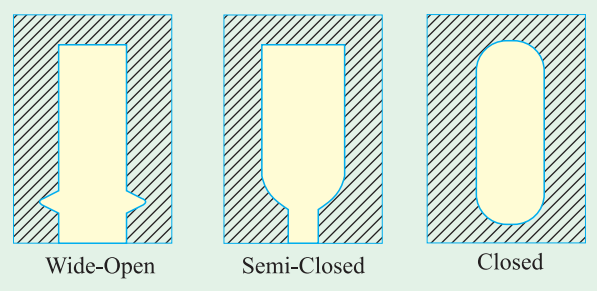
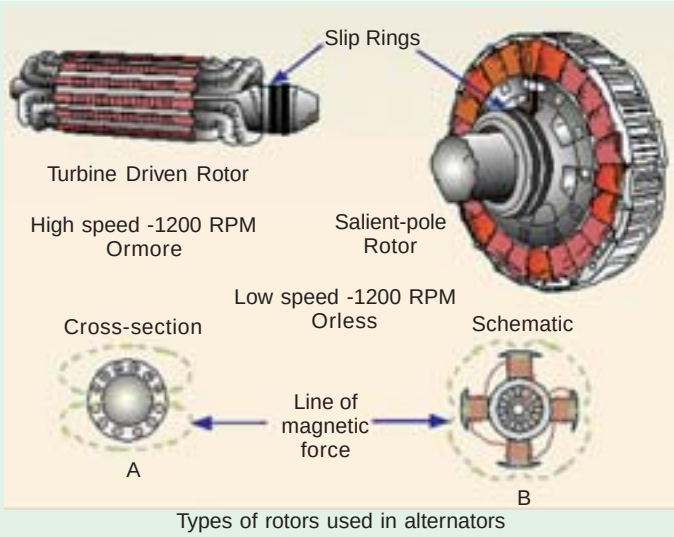


Fig. 37.3

37.4. Rotor

Two types of rotors are used in alternators ***(i)*** salient-pole type and ***(ii)*** smooth-cylindrical type.

(*i*) Salient (or projecting) Pole Type



Types of rotors used in alternators

It is used in low-and medium-speed (engine driven) alternators. It has a large number of projecting (salient) poles, having their cores bolted or dovetailed onto a heavy magnetic wheel of cast-iron, or steel of good magnetic quality (Fig. 37.4). Such generators are characterised by their large diameters and short axial lengths. The poles and pole-shoes (which cover 2/3 of pole-pitch) are laminated to minimize heating due to eddy currents. In large machines, field windings consist of rectangular copper strip wound on edge.

(*ii*) Smooth Cylindrical Type

It is used for steam turbine-driven alternators *i.e.* turbo-alternators, which run at very high speeds. The rotor consists of a smooth solid forged steel cylinder, having a number of slots milled out at intervals along the outer periphery (and parallel to the shaft) for accommodating field coils. Such rotors are designed mostly for 2-pole (or 4-pole) turbo-generators running at 3600 r.p.m. (or 1800 r.p.m.). Two (or four) regions corresponding to the central polar areas are left unslotted, as shown in Fig. 37.5 (*a*) and (*b*).

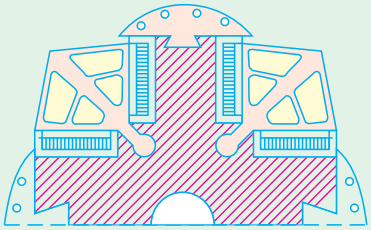


Fig. 37.4



Turbine alternator

The central polar areas are surrounded by the field windings placed in slots. The field coils are so arranged around these polar areas that flux density is maximum on the polar central line and gradually falls away on either side. It should be noted that in this case, poles are non-salient *i.e.* they do not project out from the surface of the rotor. To avoid excessive peripheral velocity, such rotors have very small diameters (about 1 metre or so). Hence, turbo-generators are characterised by small diameters and very long axial (or rotor) length. The cylindrical construction of the rotor gives better balance and quieter-operation and also less windage losses.

37.5. Damper Windings

Most of the alternators have their pole-shoes slotted for receiving copper bars of a grid or damper winding (also known as squirrel-cage winding). The copper bars are short-circuited at both ends by heavy copper rings (Fig. 37.6). These dampers are useful in preventing the hunting (momentary speed fluctuations) in generators and are needed in synchronous motors to provide the starting torque. Turbo-generators usually do not have these damper windings (except in special case to assist in synchronizing) because the solid field-poles themselves act as efficient dampers. It should be clearly understood that under normal running conditions, damper winding does not carry any current because rotor runs at synchronous speed.

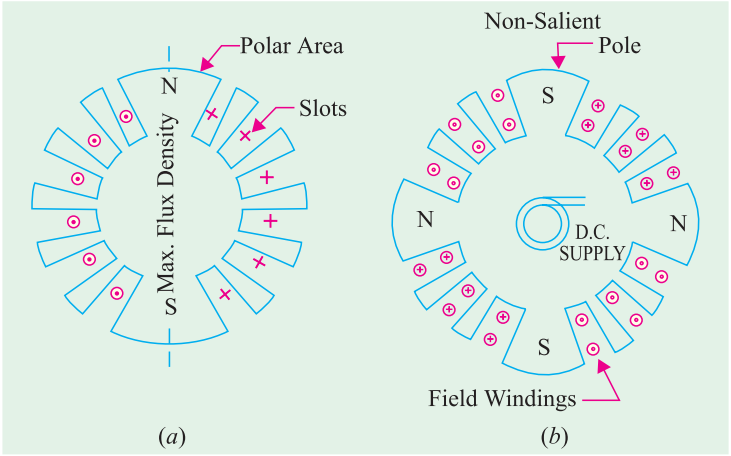


Fig. 37.5

The damper winding also tends to maintain balanced 3-ϕ voltage under unbalanced load conditions.

37.6. Speed and Frequency

In an alternator, there exists a definite relationship between the rotational speed (N) of the rotor, the frequency (f) of the generated e.m.f. and the number of poles P.

Consider the armature conductor marked X in Fig. 37.7 situated at the centre of a N-pole rotating in clockwise direction. The conductor being situated at the place of maximum flux density will have maximum e.m.f. induced in it.

The direction of the induced e.m.f. is given by Fleming's right hand rule. But while applying this rule, one should be careful to note that the thumb indicates the direction of the motion of the conductor relative to the field. To an observer stationed on the clockwise revolving poles, the conductor would seem to be rotating anti-clockwise. Hence, thumb should point to the left. The direction of the induced e.m.f. is downwards, in a direction at right angles to the plane of the paper.

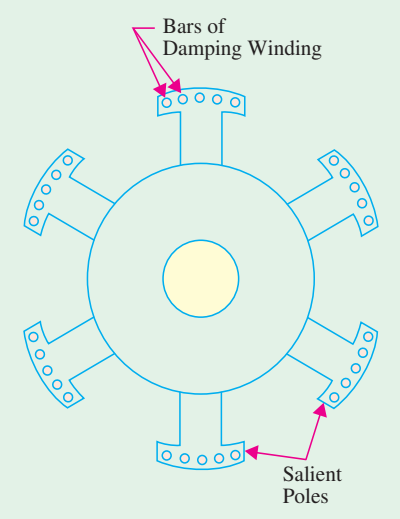


Fig. 37.6

When the conductor is in the interpolar gap, as at A in Fig. 37.7, it has minimum e.m.f. induced in it, because flux density is minimum there. Again, when it is at the centre of a S-pole, it has maximum e.m.f. induced in it, because flux density at B is maximum. But the direction of the e.m.f. when conductor is over a N-pole is opposite to that when it is over a S-pole.

Obviously, one cycle of e.m.f. is induced in a conductor when one pair of poles passes over it. In other words, the e.m.f. in an armature conductor goes through one cycle in angular distance equal to twice the pole-pitch, as shown in Fig. 37.7.

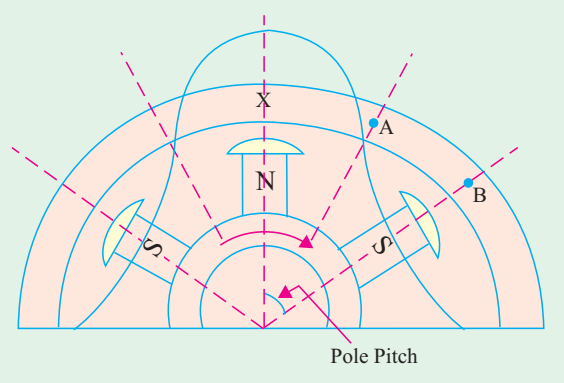


Fig. 37.7

Let P = total number of magnetic poles

N = rotative speed of the rotor in r.p.m.

f = frequency of generated e.m.f. in Hz.

Since one cycle of e.m.f. is produced when a pair of poles passes past a conductor, the number of cycles of e.m.f. produced in one revolution of the rotor is equal to the number of pair of poles.

$\therefore$ No. of cycles/revolution = $P/2$ and No. of revolutions/second = $N/60$

$$\therefore \quad \text{frequency} = \frac{P}{2} \times \frac{N}{60} = \frac{PN}{120} \text{ Hz}$$

or
$$f = \frac{PN}{120} \text{ Hz}$$

N is known as the synchronous speed, because it is the speed at which an alternator must run, in order to generate an e.m.f. of the required frequency. In fact, for a given frequency and given number of poles, the speed is fixed. For producing a frequency of 60 Hz, the alternator will have to run at the following speeds :

No. of poles	2	4	6	12	24	36
Speed (r.p.m.)	3600	1800	1200	600	300	200

Referring to the above equation, we get $P = 120f/N$

It is clear from the above that because of slow rotative speeds of engine-driven alternators, their number of poles is much greater as compared to that of the turbo-generators which run at very high speeds.



D.C. Armature

37.7. Armature Windings

The armature windings in alternators are different from those used in d.c. machines. The d.c. machines have closed circuit windings but alternator windings are open, in the sense that there is no closed path for the armature currents in the winding itself. One end of the winding is joined to the neutral point and the other is brought out (for a star-connected armature).

The two types of armature windings most commonly used for 3-phase alternators are :

(i) single-layer winding

(ii) double-layer winding

Single-layer Winding

It is variously referred to as concentric or chain winding. Sometimes, it is of simple bar type or wave winding.

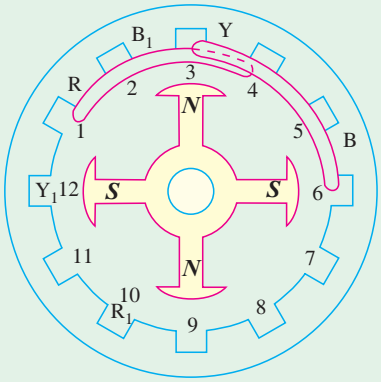
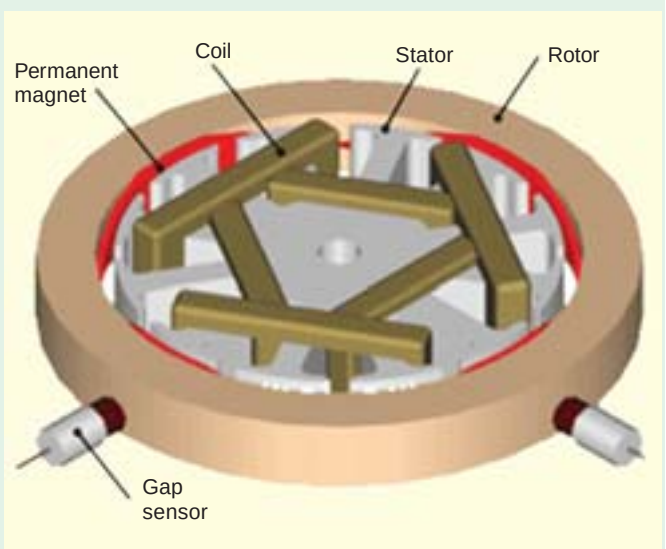


Fig. 37.8

The fundamental principle of such a winding is illustrated in Fig. 37.8 which shows a single-layer, one-turn, full-pitch winding for a four-pole generator. There are 12 slots in all, giving 3 slots per pole or 1 slot/phase/pole. The pole pitch is obviously 3. To get maximum e.m.f., two sides of a coil should be one pole-pitch apart *i.e.* coil span should be equal to one pole pitch. In other words, if one side of the coil is under the centre of a *N*-pole, then the, other side of the same *coil* should be under the centre of *S*-pole *i.e.* 180° (electrical) apart. In that case, the e.m.fs. induced in the two sides of the coil are added together. It is seen from the above figure, that *R* phase starts at slot No. 1, passes through slots 4, 7 and finishes at 10. The *Y*-phase starts 120° afterwards *i.e.* from slot No. 3 which is two slots away from the start of *R*-phase (because when 3 slots correspond to 180° electrical degrees, two slots correspond to an angular displacement of 120° electrical). It passes through slots 6, 9 and finishes at 12. Similarly, *B*-phase starts from slot No.5 *i.e.* two slots away from the start of *Y*-phase. It passes through slots 8, 11 and finishes at slot No. 2, The developed diagram is shown in Fig. 37.9. The ends of the windings are joined to form a star point for a *Y*-connection.



Single layer winding

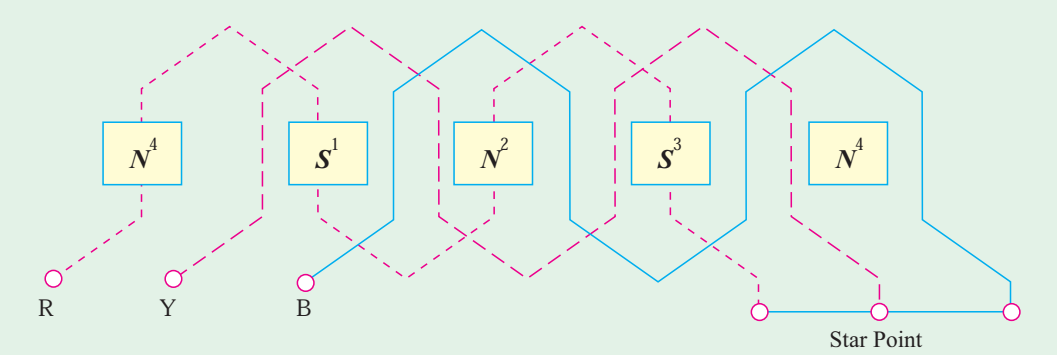


Fig. 37.9

37.8. Concentric or Chain Windings

For this type of winding, the number of slots is equal to twice the number of coils or equal to the number of *coil* sides. In Fig. 37.10 is shown a concentric winding for 3-phase alternator. It has one coil per pair of poles per phase.

It would be noted that the polar group of each phase is 360° (electrical) apart in this type of winding

1. It is necessary to use two different shapes of coils to avoid fouling of end connections.
2. Since polar groups of each phase are 360 electrical degrees apart, all such groups are connected in the same direction.
3. The disadvantage is that short-pitched coils cannot be used.

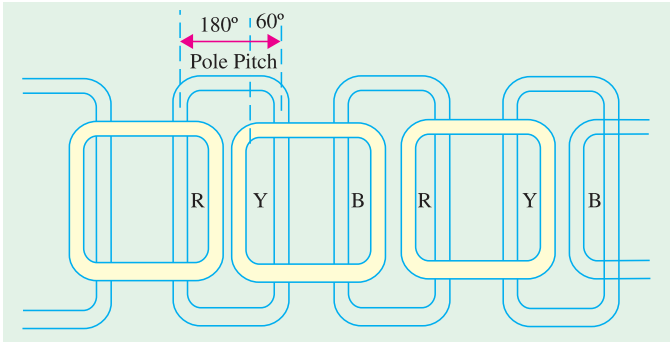


Fig. 37.10

In Fig. 37.11 is shown a concentric winding with two coils per group per pole. Different shapes of coils are required for this winding.

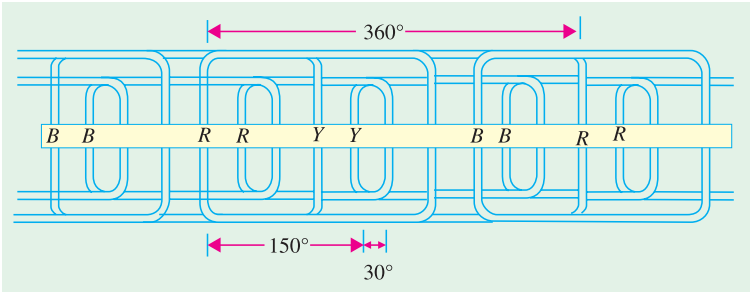


Fig. 37.11

All coil groups of phase *R* are connected in the same direction. It is seen that in each group, one coil has a pitch of 5/6 and the other has a pitch of 7/6 so that pitch factor (explained later) is 0.966. Such windings are used for large high-voltage machines.

37.9. Two-Layer Winding

This winding is either of wave-wound type or lap-wound type (this being much more common especially for high-speed turbo-generators). It is the simplest and, as said above, most commonly-used not only in synchronous machines but in induction motors as well.

Two important points regarding this winding should be noted :

(*a*) Ordinarily, the number of slots in stator (armature) is a multiple of the number of poles and the number of phases. Thus, the stator of a 4-pole, 3-phase alternator may have 12, 24, 36, 48 etc. slots all of which are seen to be multiple of 12 (*i.e.* 4×3).

(*b*) The number of stator slots is equal to the number of coils (which are all of the same shape). In other words, each slot contains two coil sides, one at the bottom of the slot and the other at the top. The coils overlap each other, just like shingles on a roof top.

For the 4-pole, 24-slot stator machine shown in Fig. 37.12, the pole-pitch is 24/4 = 6. For maximum voltage, the coils should be full-pitched. It means that if one side of the coil is in slot No.1, the other side should be in slot No.7, the two slots 1 and 7 being one pole-pitch or 180° (electrical) apart. To make matters simple, coils have been labelled as 1, 2, 3 and 4 etc. In the developed diagram of Fig. 37.14, the coil number is the number of the slot in which the left-hand side of the coil is placed.

Each of the three phases has 24/3 = 8 coils, these being so selected as to give maximum voltage when connected in series. When connected properly, coils 1, 7, 13 and 19 will add directly in phase. Hence, we get 4 coils for this phase. To complete eight coils for this phase, the other four selected are 2, 8, 14 and 20 each of which is at an angular displacement of 30° (elect.) from the adjacent coils of the first. The coils 1 and 2 of this phase are said to constitute a polar group (which is defined as the group of coils/phase/pole). Other polar groups for this phase are 7 and 8, 13 and 14, 19 and 20 etc. After the coils are placed in slots, the polar groups are joined. These groups are connected together with alternate poles reversed (Fig. 37.13) which shows winding for one phase only.

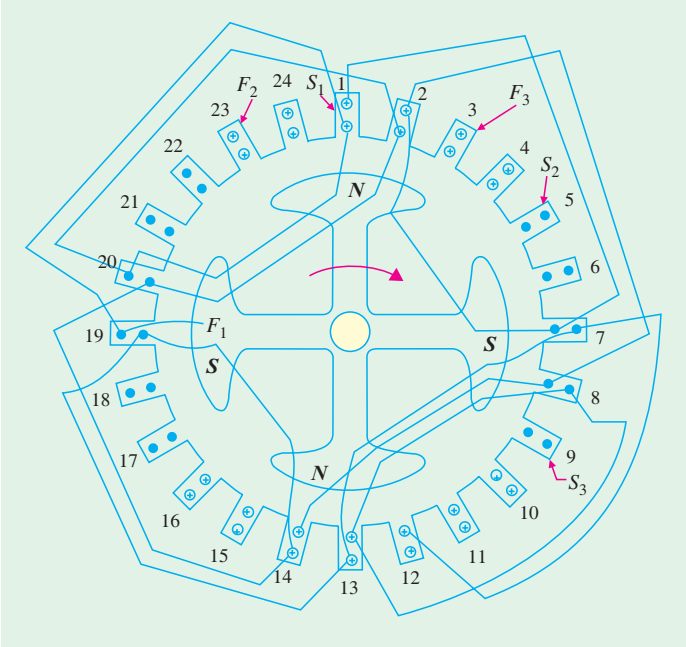


Fig. 37.12

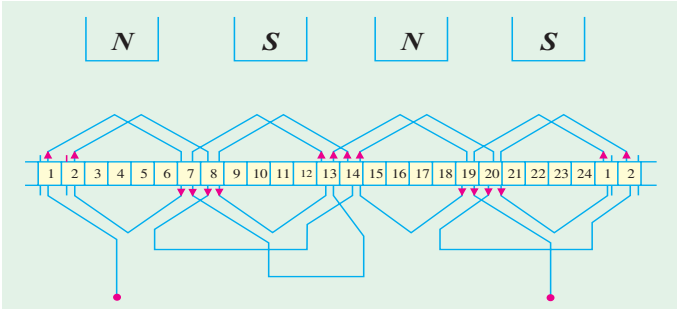


Fig. 37.13

Now, phase *Y* is to be so placed as to be 120° (elect.) away from phase *R*. Hence, it is started from slot 5 *i.e.* 4 slots away (Fig. 37.14). It should be noted that angular displacement between slot No. 1 and 5 is $4 \times 30 = 120°$ (elect). Starting from coil 5, each of the other eight coils of phase *Y* will be placed 4 slots to the right of corresponding coils for phase *R*. In the same way, *B* phase will start from coil 9. The complete wiring diagram for three phases is shown in Fig. 37.14. The terminals R_2, Y_2 and B_2 may be connected together to form a neutral for *Y*-connection.

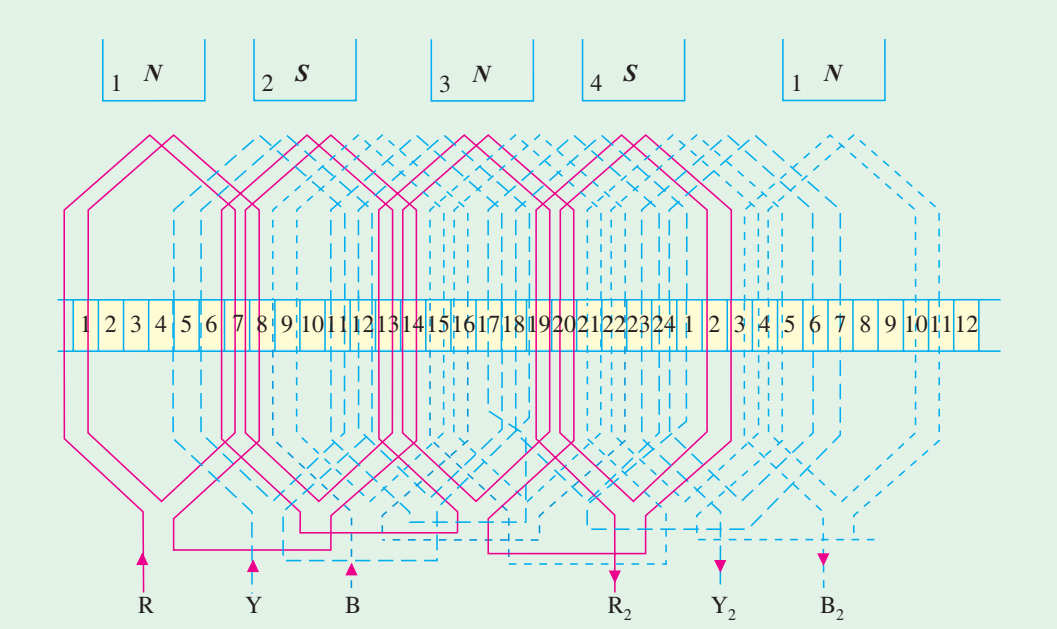


Fig. 37.14 (*a*)

A simplified diagram of the above winding is shown below Fig. 37.14. The method of construction for this can be understood by closely inspecting the developed diagram.

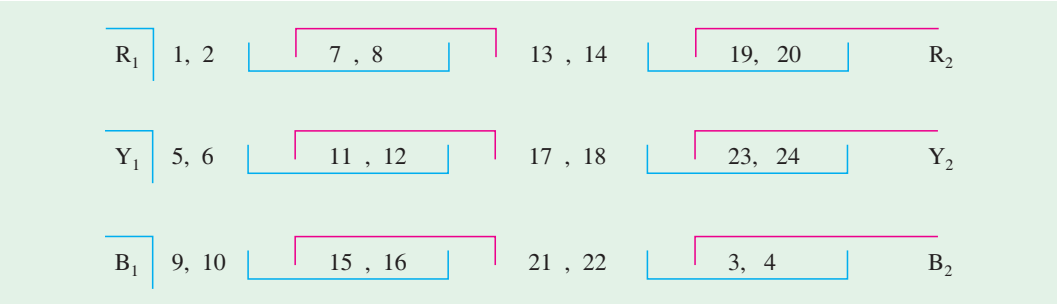


Fig. 37.14 (*b*)

37.10. Wye and Delta Connections

For *Y*-connection, R_1, Y_1 and B_1 are joined together to form the star-point. Then, ends R_2, Y_2 and B_2 are connected to the terminals. For delta connection, R_2 and Y_1, Y_2 and B_1 B_2 and R_1 are connected together and terminal leads are brought out from their junctions as shown in Fig. 37.15 (*a*) and (*b*).

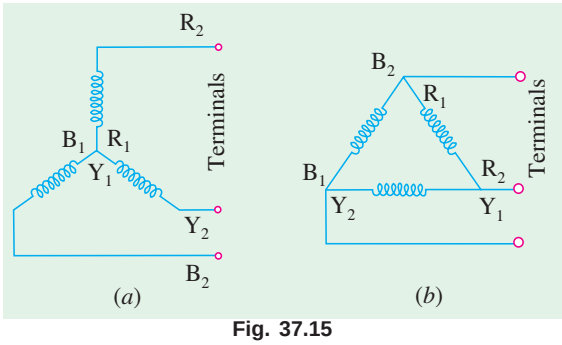


Fig. 37.15

37.11. Short-pitch Winding : Pitch factor/chording factor

So far we have discussed full-pitched coils *i.e.* coils having span which is equal to one pole-pitch *i.e.* spanning over 180° (electrical).

As shown in Fig. 37.16, if the coil sides are placed in slots 1 and 7, then it is full-pitched. If the coil sides are placed in slots 1 and 6, then it is short-pitched or fractional-pitched because coil span is equal to 5/6 of a pole-pitch. It falls short by 1/6 pole-pitch or by 180°/6 = 30°. Short-pitched coils are deliberately used because of the following advantages:

1. They save copper of end connections.
2. They improve the wave-form of the generated e.m.f. *i.e.* the generated e.m.f. can be made to approximate to a sine wave more easily and the distorting harmonics can be reduced or totally eliminated.
3. Due to elimination of high frequency harmonics, eddy current and hysteresis losses are reduced thereby increasing the efficiency.

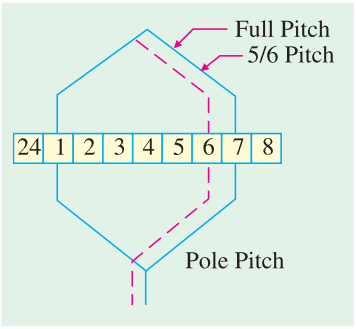


Fig. 37.16

But the disadvantage of using short-pitched coils is that the total voltage around the coils is somewhat reduced. Because the voltages induced in the two sides of the short-pitched coil are slightly out of phase, their resultant vectorial sum is less than their arithmetical sum.

The pitch factor or coil-span factor k_p or k_c is defined as

$$= \frac{\text{vector sum of the induced e.m.fs. per coil}}{\text{arithmetic sum of the induced e.m.fs. per coil}}$$

It is always less than unity.

Let E_S be the induced e.m.f. in each side of the coil. If the coil were full-pitched *i.e.* if its two sides were one pole-pitch apart, then total induced e.m.f. in the coil would have been = $2E_S$ [Fig. 37.17 (*a*).

If it is short-pitched by 30° (elect.) then as shown in Fig. 37.17 (*b*), their resultant is E which is the vector sum of two voltage 30° (electrical) apart.

$$\therefore \quad E = 2E_S \cos 30°/2 = 2E_S \cos 15°$$

$$k_c = \frac{\text{vector sum}}{\text{arithmetic sum}} = \frac{E}{2E_S} = \frac{2E_S \cos 15°}{2E_S} = \cos 15° = 0.966$$

Hence, pitch factor, $k_c = 0.966$.

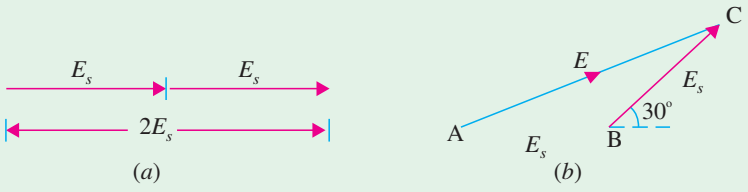


Fig. 37.17

In general, if the coil span falls short of full-pitch by an angle α (electrical)*,

then $k_c = \cos \alpha/2$.

Similarly, for a coil having a span of 2/3 pole-pitch, $k_c = \cos 60°/2 = \cos 30° = 0.866$.

It is lesser than the value in the first case.

Note. The value of α will usually be given in the question, if not, then assume $k_c = 1$.

* This angle is known as chording angle and the winding employing short-pitched coils is called chorded winding.

Example 37.1. *Calculate the pitch factor for the under-given windings : **(a)** 36 stator slots, 4-poles, coil-span, 1 to 8 **(b)** 72 stator slots, 6 poles, coils span 1 to 10 and **(c)** 96 stator slots, 6 poles, coil span 1 to 12. Sketch the three coil spans.*

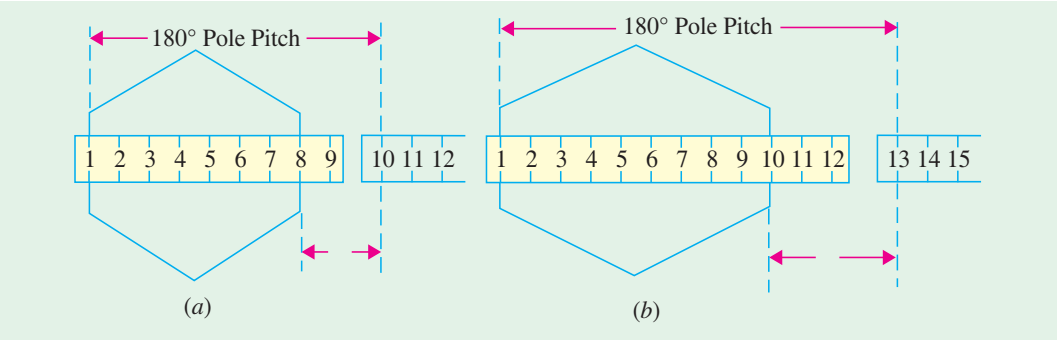


Fig. 37.18

Solution. **(a)** Here, the coil span falls short by $(2/9) \times 180° = 40°$

$$\alpha = 40°$$

$\therefore\ k_c = \cos 40°/2 = \cos 20° = \mathbf{0.94}$

(b) Here $\alpha = (3/12) \times 180° = 45°$ $\quad \therefore\ k_c = \cos 45°/2 = \cos 22.5° = \mathbf{0.924}$

(c) Here $\alpha = (5/16) \times 180° = 56°\ 16'$ $\quad \therefore\ k_c = \cos 28°8' = \mathbf{0.882}$

The coil spans have been shown in Fig. 37.18.

37.12. Distribution or Breadth Factor or Winding Factor or Spread Factor

It will be seen that in each phase, coils are not concentrated or bunched in one slot, but are distributed in a number of slots to form polar groups under each pole. These coils/phase are displaced from each other by a certain angle. The result is that the e.m.fs. induced in coil sides constituting a polar group are not in phase with each other but differ by an angle equal to angular displacement of the slots.

In Fig. 37.19 are shown the end connections of a 3-phase single-layer winding for a 4-pole alternator. It has a total of 36 slots *i.e.* 9 slots/pole. Obviously, there are 3 slots / phase / pole. For example, coils 1, 2 and 3 belong to *R* phase. Now, these three coils which constitute one polar group are not bunched in one slot but in three different slots. Angular displacement between any two adjacent slots = 180°/9 = 20° (elect.)

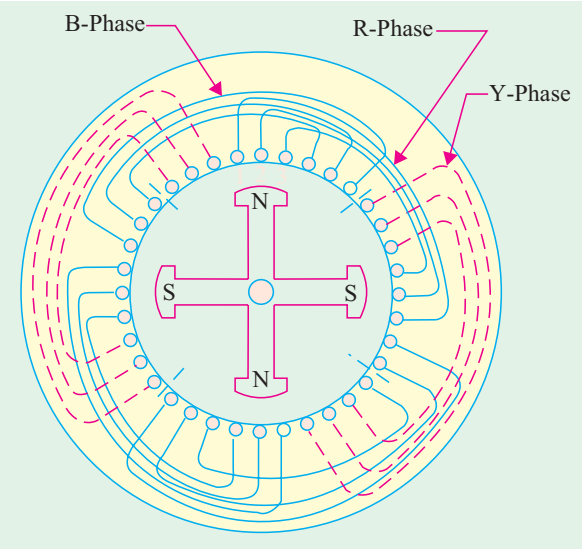


Fig. 37.19

If the three coils were bunched in one slot, then total e.m.f. induced in the three sides of the coil would be the arithmetic sum of the three e.m.f.s. *i.e.* = $3 E_S$, where E_S is the e.m.f. induced in one coil *side* [Fig.37.20 (*a*)].

Since the coils are distributed, the individual e.m.fs. have a phase difference of 20° with each other. Their vector sum as seen from Fig. 35.20 (*b*) is

$$E = E_S \cos 20° + E_S + E_S \cos 20°$$
$$= 2E_S \cos 20° + E_S$$
$$= 2E_S \times 0.9397 + E_S = 2.88\, E_S$$

The distribution factor (k_d) is defined as

$$= \frac{\text{e.m.f. with distributed winding}}{\text{e.m.f. with concentrated winding}}$$

In the present case

$$k_d = \frac{\text{e.m.f. with winding in 3 slots/pole/phase}}{\text{e.m.f. with winding in 1 slots/pole/phase}} = \frac{E}{3E_S} = \frac{2.88\, E_S}{3E_S} = 0.96$$

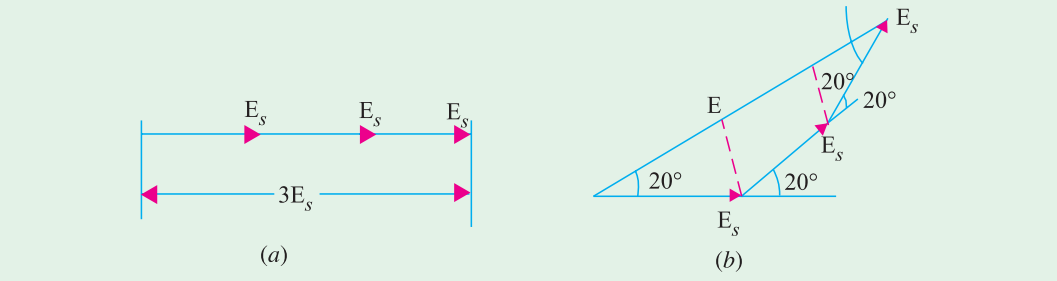


Fig. 37.20

General Case

Let β be the value of angular displacement between the slots. Its value is

$$\beta = \frac{180°}{\text{No. of slots/pole}} = \frac{180°}{n}$$

Let m = No. of slots/phase/pole

$m\beta$ = phase spread angle

Then, the resultant voltage induced in one polar group would be mE_S

where E_S is the voltage induced in one coil side. Fig. 37.21 illustrates the method for finding the vector sum of m voltages each of value E_S and having a mutual phase difference of β (if m is large, then the curve $ABCDE$ will become part of a circle of radius r).

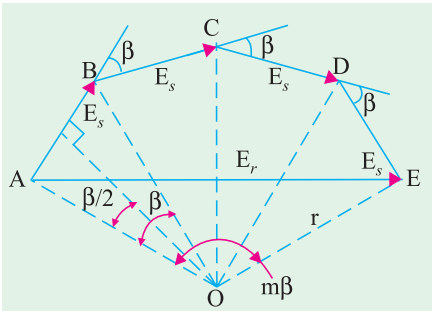


Fig. 37.21

$$AB = E_S = 2r \sin \beta/2$$

Arithmetic sum is $= mE_S = m \times 2r \sin \beta/2$

Their vector sum $= AE = E_r = 2r \sin m\beta/2$

$$k_d = \frac{\text{vector sum of coils e.m.fs.}}{\text{arithmetic sum of coil e.m.fs.}}$$

$$= \frac{2r \sin m\beta/2}{m \times 2r \sin \beta/2} = \frac{\sin m\beta/2}{m \sin \beta/2}$$

The value of distribution factor of a 3-phase alternator for different number of slots/pole/phase is given in table No. 37.1.

Table 37.1

Slots per pole	*m*	β°	*Distribution factor* k_d
3	1	60	1.000
6	2	30	0.966
9	3	20	0.960
12	4	15	0.958
15	5	12	0.957
18	6	10	0.956
24	8	7.5	0.955

In general, when β is small, the above ratio approaches

$$= \frac{\text{chord}}{\text{arc}} = \frac{\sin m\beta/2}{m\beta/2}$$ — angle $m\beta/2$ in radians.

Example 37.2. *Calculate the distribution factor for a 36-slots, 4-pole, single-layer three-phase winding.* **(Elect. Machine-I Nagpur Univ. 1993)**

Solution. $n = 36/4 = 9; \beta = 180°/9 = 20°; m = 36/4 \times 3 = 3$

$$kd = \frac{\sin m\beta/2}{m \sin \beta/2} = \frac{\sin 3 \times 20°/2}{3 \sin 20°/2} = \mathbf{0.96}$$

Example. 37.3. *A part of an alternator winding consists of six coils in series, each coil having an e.m.f. of 10 V r.m.s. induced in it. The coils are placed in successive slots and between each slot and the next, there is an electrical phase displacement of 30º. Find graphically or by calculation, the e.m.f. of the six coils in series.*

Solution. By calculation

Here $\beta = 30° : m = 6 \therefore k_d = \frac{\sin m\beta/2}{m \sin \beta/2} = \frac{\sin 90°}{6 \times \sin 15°} = \frac{1}{6 \times 0.2588}$

Arithmetic sum of voltage induced in 6 coils = $6 \times 10 = 60$ V

Vector sum $= k_d \times$ arithmetic sum $= 60 \times 1/6 \times 0.2588 = \mathbf{38.64V}$

Example 37.4. *Find the value of* k_d *for an alternator with 9 slots per pole for the following cases :*

(i) *One winding in all the slots* ***(ii)*** *one winding using only the first 2/3 of the slots/pole (iii) three equal windings placed sequentially in 60° group.*

Solution. Here, β = 180°/9 = 20° and values of *m i.e.* number of slots in a group are 9, 6 and 3 respectively.

(i) $m = 9, \quad \beta = 20°, \quad k_d = \frac{\sin 9 \times 20°/2}{9 \sin 20°/2} = \mathbf{0.64} \left[\text{or } k_d = \frac{\sin \pi/2}{\pi/2} = 0.637\right]$

(ii) $m = 6, \quad \beta = 20°, \quad k_d = \frac{\sin 6 \times 20°/2}{6 \sin 20°/2} = \mathbf{0.83} \left[\text{or } k_d = \frac{\sin \pi/3}{\pi/3} = 0.827\right]$

(iii) $m = 3, \quad \beta = 20°, \quad k_d = \frac{\sin 3 \times 20°/2}{3 \sin 20°/2} = \mathbf{0.96} \left[\text{or } k_d = \frac{\sin \pi/6}{\pi/6} = 0.955\right]$

37.13. Equation of Induced E.M.F.

Let Z = No. of conductors or coil sides in series/phase
= $2T$ — where T is the No. of coils or turns per phase (remember one turn or coil has two sides)
P = No. of poles
f = frequency of induced e.m.f. in Hz
Φ = flux/pole in webers

k_d = distribution factor = $\dfrac{\sin m\beta/2}{m \sin \beta/2}$

k_c or k_p = pitch or coil span factor = $\cos \alpha/2$
k_f = from factor = **1.11** —if e.m.f. is assumed sinusoidal
N = rotor r.p.m.

In one revolution of the rotor (*i.e.* in 60./N second) each stator conductor is cut by a flux of ΦP webers.

$\therefore$ $d\Phi = \Phi P$ and $dt = 60/N$ second

$\therefore$ Average e.m.f. induced per conductor = $\dfrac{d\Phi}{dt} = \dfrac{\Phi P}{60/N} = \dfrac{\Phi NP}{60}$

Now, we know that $f = PN/120$ or $N = 120\, f/P$

Substituting this value of N above, we get

Average e.m.f. per conductor = $\dfrac{\Phi P}{60} \times \dfrac{120\, f}{P} = 2f\, \Phi$ volt

If there are Z conductors in series/phase, then Average e.m.f./phase = $2f\, \Phi Z$ volt = $4\, f \Phi T$ volt

R.M.S. value of e.m.f./phase = $1.11 \times 4f \Phi T = 4.44 f \Phi T$ volt*.

This would have been the actual value of the induced voltage if all the coils in a phase were ***(i)*** full-pitched and ***(ii)*** concentrated or bunched in one slot (instead of being distributed in several slots under poles). But this not being so, the actually available voltage is reduced in the ratio of these two factors.

$\therefore$ Actually available voltage/phase = $4.44\, k_c\, k_d\, f \Phi\, T = 4\, k_f\, k_c k_d\, f \Phi\, T$ volt.

If the alternator is star-connected (as is usually the case) then the line voltage is $\sqrt{3}$ times the phase voltage (as found from the above formula).

37.14. Effect of Harmonics on Pitch and Distribution Factors

(a) If the short-pitch angle or chording angle is α degrees (electrical) for the fundamental flux wave, then its values for different harmonics are

for 3rd harmonic = 3α ; for 5th harmonic = 5α and so on.

$\therefore$ pitch-factor, k_c = $\cos \alpha/2$ —for fundamental
= $\cos 3\alpha/2$ —for 3rd harmonic
= $\cos 5\alpha/2$ —for 5th harmonic etc.

(b) Similarly, the distribution factor is also different for different harmonics. Its value becomes

$k_d = \dfrac{\sin m\beta/2}{m \sin \beta/2}$ where n is the order of the harmonic

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* It is exactly the same equation as the e.m.f. equation of a transformer. (Art 32.6)

for fundamental, $n = 1$ $k_{d1} = \dfrac{\sin m\beta/2}{m \sin \beta/2}$

for 3rd harmonic, $n = 3$ $k_{d3} = \dfrac{\sin 3m\beta/2}{m \sin 3\beta/2}$

for 5th harmonic, $n = 5$ $k_{d5} = \dfrac{\sin 5m\beta/2}{m \sin 5\beta/2}$

(*c*) Frequency is also changed. If fundamental frequency is 50 Hz *i.e.* $f_1 = 50$ Hz then other frequencies are :

3rd harmonic, $f_3 = 3 \times 50 = 150$ Hz, 5th harmonic, $f_5 = 5 \times 50 = 250$ Hz etc.

Example 37.5. *An alternator has 18 slots/pole and the first coil lies in slots 1 and 16. Calculate the pitch factor for **(i)** fundamental **(ii)** 3rd harmonic **(iii)** 5th harmonic and **(iv)** 7th harmonic.*

Solution. Here, coil span is $= (16 - 1) = 15$ slots, which falls short by 3 slots.

Hence, $\alpha = 180° \times 3/18 = 30°$

(*i*) $k_{c1} = \cos 30°/2 = \cos 15° =$ **0.966** **(*ii*)** $k_{c3} = \cos 3 \times 30°/2 =$ **0.707**

(*iii*) $k_{c5} = \cos 5 \times 30°/2 = \cos 75° =$ **0.259** **(*iv*)** $k_{c7} = \cos 7 \times 30°/2 = \cos 105° = \cos 75° =$ **0.259.**

Example 37.6. *A 3-phase, 16-pole alternator has a star-connected winding with 144 slots and 10 conductors per slot. The flux per pole is 0.03 Wb, Sinusoidally distributed and the speed is 375 r.p.m. Find the frequency rpm and the phase and line e.m.f. Assume full-pitched coil.*

(Elect. Machines, AMIE Sec. B, 1991)

Solution. $f = PN/120 = 16 \times 375/120 =$ **50 Hz**

Since k_c is not given, it would be taken as unity.

$n = 144/16 = 9; \beta = 180°/9 = 20°; m = 144/16 \times 3 = 3$

$k_d = \sin 3 \times (20°/2)/3 \sin (20°/2) = 0.96$

$Z = 144 \times 10 / 3 = 480; T = 480/2 = 240$ / phase

$E_{ph} = 4.44 \times 1 \times 0.96 \times 50 \times 0.03 \times 240 =$ **15.34 V**

Line voltage, $E_L = \sqrt{3}\, E_{ph} = \sqrt{3} \times 1534 =$ **2658 V**

Example 37.7. *Find the no-load phase and line voltage of a star-connected 3-phase, 6-pole alternator which runs at 1200 rpm, having flux per pole of 0.1 Wb sinusoidally distributed. Its stator has 54 slots having double layer winding. Each coil has 8 turns and the coil is chorded by 1 slot.*

(Elect. Machines-I, Nagput Univ. 1993)

Solution. Since winding is chorded by one slot, it is short-pitched by 1/9 or 180°/9 = 20°

$\therefore$ $k_c = \cos 20°/2 = 0.98$; $f = 6 \times 1200/120 = 60$ Hz

$n = 54/6 = 9$; $\beta = 180°/9 = 20°$, $m = 54/6 \times 3 = 3$

$k_d = \sin 3 \times (20°/2)/3 \sin (20°/2) = 0.96$

$Z = 54 \times 8/3 = 144$; $T = 144/2 = 72$, $f = 6 \times 1200/120 = 60$ Hz

$E_{ph} = 4.44 \times 0.98 \times 0.96 \times 60 \times 0.1 \times 72 = 1805$ V

Line voltage, $E_L = \sqrt{3} \times 1805 =$ **3125 V.**

Example 37.8. *The stator of a 3-phase, 16-pole alternator has 144 slots and there are 4 conductors per slot connected in two layers and the conductors of each phase are connected in series. If the speed of the alternator is 375 r.p.m., calculate the e.m.f. inducted per phase. Resultant flux in the air-gap is 5×10^{-2} webers per pole sinusoidally distributed. Assume the coil span as 150° electrical.*

(Elect. Machine, Nagpur Univ. 1993)

Solution. For sinusoidal flux distribution, $k_f = 1.11$; $\alpha = (180° - 150°) = 30°$ (elect)

$$k_c = \cos 30°/2 = 0.966^*$$

No. of slots / pole, $n = 144/16 = 9$;

$\beta = 180°/9 = 20°$

m = No. of slots/pole/phase $= 144/16 \times 3 = 3$

$$\therefore \quad k_d = \frac{\sin m\beta/2}{m \sin \beta/2} = \frac{\sin 3 \times 20°/2}{3 \sin 20°/2} = 0.96; \; f = 16 \times 375/120 = 50 \text{ Hz}$$

No. of slots / phase = 144/3 = 48; No of conductors / slot = 4

$\therefore$ No. of conductors in series/phase $= 48 \times 4 = 192$

$\therefore$ turns / phase = conductors per phase/2 = 192/2 = 96

$$E_{ph} = 4 k_f k_c k_d f \Phi T$$

$$= 4 \times 1.11 \times 0.966 \times 0.96 \times 50 \times 5 \times 10^{-2} \times 96 = \mathbf{988 \text{ V}}$$

Example 37.9. *A 10-pole, 50-Hz, 600 r.p.m. alternator has flux density distribution given by the following expression*

$$B = \sin \theta + 0.4 \sin 3\theta + 0.2 \sin 5\theta$$

The alternator has 180 slots wound with 2-layer 3-turn coils having a span of 15 slots. The coils are connected in 60° groups. If armature diameter is = 1.2 m and core length = 0.4 m, calculate

(*i*) *the expression for instantaneous e.m.f. / conductor*

(*ii*) *the expression for instantaneous e.m.f./coil*

(*iii*) *the r.m.s. phase and line voltages, if the machine is star-connected.*

Solution. For finding voltage/conductor, we may either use the relation *Blv* or use the relation of Art. 35-13.

Area of pole pitch $= (1.2\pi/10) \times 0.4 = 0.1508 \text{ m}^2$

Fundamental flux/pole, ϕ_1 = av. flux density × area $= 0.637 \times 1 \times 0.1508 = 0.096$ Wb

(a) RMS value of fundamental voltage per conductor,

$$= 1.1 \times 2 f\phi_1 = 1.1 \times 2 \times 50 \times 0.096 = \mathbf{10.56 \text{ V}}$$

Peak value $= \sqrt{2} \times 10.56 = 14.93$ V

Since harmonic conductor voltages are in proportion to their flux densities,

3rd harmonic voltage $= 0.4 \times 14.93 = 5.97$ V

5th harmonic voltage $= 0.2 \times 14.93 = 2.98$ V

Hence, equation of the instantaneous e.m.f./conductor is

$$\mathbf{e = 14.93 \sin \theta + 5.97 \sin 3\theta + 2.98 \sin 5\theta}$$

(b) Obviously, there are 6 conductors in a 3-turn coil. Using the values of k_c found in solved Ex. 37.5, we get

fundamental coil voltage $= 6 \times 14.93 \times 0.966 = 86.5$ V

3rd harmonic coil voltage $= 6 \times 5.97 \times 0.707 = 25.3$ V

5th harmonic coil voltage $= 6 \times 2.98 \times 0.259 = 4.63$ V

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* Since they are not of much interest, the relative phase angles of the voltages have not been included in the expression.

Hence, coil voltage expression is*

$$e = 86.5 \sin \theta + 25.3 \sin 3\theta + 4.63 \sin 5\theta$$

(c) Here, $m = 6, \beta = 180°/18 = 10°$; $k_{d1} = \dfrac{\sin 6 \times 10°/2}{6 \sin 10°/2} = 0.956$

$$k_{d3} = \frac{\sin 3 \times 6 \times 10°/2}{6 \sin 3 \times 10°/2} = 0.644 \qquad k_{d5} = \frac{\sin 5 \times 6 \times 10°/2}{6 \sin 5 \times 10°/2} = 0.197$$

It should be noted that number of coils per phase = 180/3 = 60

Fundamental phase e.m.f.= $(86.5/\sqrt{2}) \times 60 \times 0.956$ = 3510 V

3rd harmonic phase e.m.f.= $(25.3/\sqrt{2}) \times 60 \times 0.644$ = 691 V

5th harmonic phase e.m.f.= $(4.63/\sqrt{2}) \times 60 \times 0.197$ = 39 V

RMS value of phase voltage = $(3510^2 + 691^2 + 39^2)^{1/2}$ = **3577 V**

RMS value of line voltage = $\sqrt{3} \times (3510^2 + 39^2)^{1/2}$ = **6080 V**

Example 37.10. *A 4-pole, 3-phase, 50-Hz, star-connected alternator has 60 slots, with 4 conductors per slot. Coils are short-pitched by 3 slots. If the phase spread is 60°, find the line voltage induced for a flux per pole of 0.943 Wb distributed sinusoidally in space. All the turns per phase are in series.* **(Electrical Machinery, Mysore Univ. 1987)**

Solution. As explained in Art. 37.12, phase spread = $m\beta = 60°$ **—given**

Now, $m = 60/4 \times 3 = 5 \quad \therefore \quad 5\beta = 60°, \beta = 12°$

$$k_{d`} = \frac{\sin 5 \times 12°/2}{5 \sin 12°/2} = 0.957; \alpha = (3/15) \times 180° = 36°; k_c = \cos 18° = 0.95$$

$Z = 60 \times 4/3 = 80$; $T = 80/2 = 40$; $\Phi = 0.943$ Wb; $k_f = 1.11$

$\therefore \quad E_{ph} = 4 \times 1.11 \times 0.95 \times 0.975 \times 50 \times 0.943 \times 40 = 7613$ V

$E_L = \sqrt{3} \times 7613 =$ **13,185 V**

Example 37.11. *A 4-pole, 50-Hz, star-connected alternator has 15 slots per pole and each slot has 10 conductors. All the conductors of each phase are connected in series' the winding factor being 0.95. When running on no-load for a certain flux per pole, the terminal e.m.f. was 1825 volt. If the windings are lap-connected as in a d.c. machine, what would be the e.m.f. between the brushes for the same speed and the same flux/pole. Assume sinusoidal distribution of flux.*

Solution. Here $k_f = 1.11, k_d = 0.95, k_c = 1$ (assumed)

$f = 50$ Hz; e.m.f./phase = $1825/\sqrt{3}$ V

Total No. of slots = $15 \times 4 = 60$

$\therefore$ No. of slots/phase = 60/3 = 20; No. of turns/phase = $20 \times 10/2 = 100$

$\therefore \quad 1825/\sqrt{3} = 4 \times 1.11 \times 1 \times 0.95 \times \Phi \times 50 \times 100 \quad \therefore \Phi = 49.97$ mWb

When connected as a d.c. generator

$E_g = (\Phi ZN/60) \times (P/A)$ volt

$Z = 60 \times 10 = 600, \qquad N = 120\, f/P = 120 \times 50/4 = 1500$ r.p.m.

$$\therefore \quad E_g = \frac{49.97 \times 10^{-3} \times 600 \times 1500}{60} \times \frac{4}{4} = \textbf{750 V}$$

* Also $k_d = \sin 150°/2 = \sin 75° = 0.966$

Example 37.12. *An alternator on open-circuit generates 360 V at 60 Hz when the field current is 3.6 A. Neglecting saturation, determine the open-circuit e.m.f. when the frequency is 40 Hz and the field current is 2.4 A.*

Solution. As seen from the e.m.f. equation of an alternator,

$$E \propto \Phi f \quad \therefore \frac{E_1}{E_2} = \frac{\Phi_1 f_1}{\Phi_2 f_2}$$

Since saturation is neglected, $\Phi \propto I_f$ where I_f is the field current

$$\therefore \quad \frac{E_1}{E_2} = \frac{I_{f1} \cdot f_1}{I_{f2} \cdot f_2} \quad \text{or} \quad \frac{360}{E_2} = \frac{3.6 \times 60}{2.4 \times 40}; \; E_2 = \textbf{160 V}$$

Example 37.13. *Calculate the R.M.S. value of the induced e.m.f. per phase of a 10-pole, 3-phase, 50-Hz alternator with 2 slots per pole per phase and 4 conductors per slot in two layers. The coil span is 150°. The flux per pole has a fundamental component of 0.12 Wb and a 20% third component.* **(Elect. Machines-III, Punjab Univ. 1991)**

Solution. Fundamental E.M.F.

$$\alpha = (180° - 150°) = 30°; \; k_{c1} = \cos \alpha/2 = \cos 15° = 0.966$$

$$m = 2 \text{ ; No. of slots/pole} = 6 \text{ ; } \beta = 180°/6 = 30°$$

$$\therefore \quad k_{d1} = \frac{\sin m\beta/2}{m \sin \beta/2} = \frac{\sin 2 \times 30°/2}{2 \sin 30°/2} = 0.966$$

$$Z = 10 \times 2 \times 4 = 80 \text{ ; turn/phase, } T = 80/2 = 40$$

$\therefore$ Fundamental E.M.F./phase $= 4.44 \, k_c k_d f \Phi T$

$$\therefore \quad E_1 = 4.44 \times 0.966 \times 0.966 \times 50 \times 0.12 \times 40 = 995 \text{V}$$

Hormonic E.M.F.

$$K_{c3} = \cos 3\alpha/2 = \cos 3 \times 30°/2 = \cos 45° = 0.707$$

$$k_{d3} = \frac{\sin mn\beta/2}{m \sin n\beta/2} \text{ where } n \text{ is the order of the harmonic } i.e. \; n = 3$$

$$\therefore \quad k_{d3} = \frac{\sin 2 \times 3 \times 30°/2}{2 \sin 3 \times 30°/2} = \frac{\sin 90°}{2 \sin 45°} = 0.707, \; f_2 = 50 \times 3 = 150 \text{ Hz}$$

$$\Phi_3 = (1/3) \times 20\% \text{ of fundamental flux} = (1/3) \times 0.02 \times 0.12 = 0.008 \text{ Wb}$$

$$\therefore \quad E_3 = 4.44 \times 0.707 \times 0.707 \times 150 \times 0.008 \times 40 = 106 \text{ V}$$

$$\therefore \quad E \text{ per phase} = \sqrt{E_1^2 + E_3^2} = \sqrt{995^2 + 106^2} = \textbf{1000 V}$$

Note. Since phase e.m.fs. induced by the 3rd, 9th and 15th harmonics etc. are eliminated from the line voltages, the line voltage for a *Y*-connection would be $= 995 \times \sqrt{3}$ volt.

Example 37.14. *A 3-phase alternator has generated e.m.f. per phase of 230 V with 10 per cent third harmonic and 6 per cent fifth harmonic content. Calculate the r.m.s. line voltage for* ***(a)*** *star connection* ***(b)*** *delta-connection. Find also the circulating current in delta connection if the reactance per phase of the machine at 50-Hz is 10* Ω. **(Elect. Machines-III, Osmania Univ. 1988)**

Solution. It should be noted that in both star and delta-connections, the third harmonic components of the three phases cancel out at the line terminals because they are co-phased. Hence, the line e.m.f. is composed of the fundamental and the fifth harmonic only.

(a) **Star-connection**

$$E_1 = 230 \text{ V} \text{ ; } E_5 = 0.06 \times 230 = 13.8 \text{ V}$$

$$\text{E.M.F./phase} = \sqrt{E_1^2 + E_5^2} = \sqrt{230^2 + 13.8^2} = 230.2\ V$$

R.M.S. value of line e.m.f. $= \sqrt{3} \times 230.2 =$ **3.99 V**

(*b*) Delta-connection

Since for delta-connection, line e.m.f. is the same as the phase e.m.f.

R.M.S. value of line e.m.f. = **230.2 V**

In delta-connection, third harmonic components are additive round the mesh, hence a circulating current is set up whose magnitude depends on the reactance per phase at the third harmonic frequency.

R.M.S. value of third harmonic e.m.f. per phase $= 0.1 \times 230 = 23$ V

Reactance at triple frequency $= 10 \times 3 = 30\ \Omega$

Circulating current $= 23/30 =$ **0.77 A**

Example 37.15 (*a*). *A motor generator set used for providing variable frequency a.c. supply consists of a three-phase, 10-pole synchronous motor and a 24-pole, three- phase synchronous generator. The motor-generator set is fed from a 25 Hz, three-phase a.c. supply. A 6-pole, three-phase induction motor is electrically connected to the terminals of the synchronous generator and runs at a slip of 5%. Determine :*

(i) *the frequency of the generated voltage of the synchronous generator.*

(ii) *the speed at which the induction motor is running.* **(U.P. Technical University 2001)**

Solution. Speed of synchronous motor $= (120 \times 25)/10 = 300$ rpm.

(i) At 300 rpm, frequency of the voltage generated by 24-pole synchronous generator

$$= \frac{24 \times 300}{120} = 60 \text{ Hz}$$

Synchronous speed of the 6-pole induction motor fed from a 60 Hz supply

$$= \frac{120 \times 60}{6} = 1200 \text{ rpm}$$

(*ii*) With 5 % slip, the speed of this induction motor $= 0.95 \times 1200 = 1140$ rpm.

Further, the frequency of the rotor-currents $= s f = 0.05 \times 60 = 3$ Hz.

Example 37.15 (*b*). *Find the no load line voltage of a star connected 4-pole alternator from the following :*

Flux per pole = *0.12 Weber, Slots per pole per phase = 4*

Conductors/ slot = *4, Two layer winding, with coil span = 150°*

[Bharthithasan University, April 1997]

Solution. Total number of slots = $4 \times 3 \times 4 = 48$, Slot pitch = 15° electrical

Total number of conductors = $48 \times 4 = 192$, Total number of turns = 96

No. of turns in series per phase = 32

For a 60° phase spread,

$$k_b = \frac{\sin (60°/2)}{4 \times \sin 7.5^o} = 0.958$$

For 150° coil-span, pitch factor k_p = cos 15° = 0.966, and for 50 Hz frequency,

$E_{ph} = 4.44 \times 50 \times 0.12 \times 0.958 \times 0.966 \times 32 = 789$ volts

$E_{\text{line}} = 789 \times 1.732 = 1366.6$ volts

Tutorial Problems 37.1

1. Find the no-load phase and line voltage of a star-connected, 4-pole alternator having flux per pole of 0.1 Wb sinusoidally distributed; 4 slots per pole per phase, 4 conductors per slot, double-layer winding with a coil span of 150°.
[Assuming f = 50 Hz; 789 V; 1366 V] (*Elect. Technology-I, Bombay Univ. 1978*)
2. A 3-ϕ, 10-pole, *Y*-connected alternator runs at 600 r.p.m. It has 120 stator slots with 8 conductors per slot and the conductors of each phase are connected in series. Determine the phase and line e.m.fs. if the flux per pole is 56 mWb. Assume full-pitch coils.
[1910 V; 3300 V] (*Electrical Technology-II, Madras Univ. April 1977*)
3. Calculate the speed and open-circuit line and phase voltages of a 4-pole, 3-phase, 50-Hz, star-connected alternator with 36 slots and 30 conductors per slot. The flux per pole is 0.0496 Wb and is sinusoidally distributed. **[1500 r.p.m.; 3,300 V; 1,905 V]** (*Elect. Engg-II, Bombay Univ. 1979*)
4. A 4-pole, 3-phase, star-connected alternator armature has 12 slots with 24 conductors per slot and the flux per pole is 0.1 Wb sinusoidally distributed.
Calculate the line e.m.f. generated at 50 Hz. **[1850 V]**
5. A 3-phase, 16-pole alternator has a star-connected winding with 144 slots and 10 conductors per slot. The flux per pole is 30 mWb sinusoidally distributed. Find the frequency, the phase and line voltage if the speed is 375 rpm. **[50 Hz; 1530 V; 2650 V] (*Electrical Machines-I, Indore Univ. April 1977*)**
6. A synchronous generator has 9 slots per pole. If each coil spans 8 slot pitches, what is the value of the pitch factor ? **[0.985] (*Elect. Machines, A.M.I.E. Sec. B. 1989*)**
7. A 3-phase, *Y*-connected, 2-pole alternator runs at 3,600 r.p.m. If there are 500 conductors per phase in series on the armature winding and the sinusoidal flux per pole is 0.1 Wb, calculate the magnitude and frequency of the generated voltage from first principles. **[60 Hz; 11.5 kV]**
8. One phase of a 3-phase alternator consists of twelve coils in series. Each coil has an r.m.s. voltage of 10 V induced in it and the coils are arranged in slots so that there is a successive phase displacement of 10 electrical degrees between the e.m.f. in each coil and the next. Find graphically or by calculation, the r.m.s. value of the total phase voltage developed by the winding. If the alternator has six pole and is driven at 100 r.p.m., calculate the frequency of the e.m.f. generated. **[108 V; 50 Hz]**
9. A 4-pole, 50-Hz, 3-phase, Y-connected alternator has a single-layer, full-pitch winding with 21 slots per pole and two conductors per slot. The fundamental flux is 0.6 Wb and air-gap flux contains a third harmonic of 5% amplitude. Find the r.m.s. values of the phase e.m.f. due to the fundamental and the 3rd harmonic flux and the total induced e.m.f.
[3,550 V; 119.5 V; 3,553 V] (*Elect. Machines-III, Osmania Univ. 1977*)
10. A 3-phase, 10-pole alternator has 90 slots, each containing 12 conductors. If the speed is 600 r.p.m. and the flux per pole is 0.1 Wb, calculate the line e.m.f. when the phases are (*i*) star connected (*ii*) delta connected. Assume the winding factor to be 0.96 and the flux sinusoidally distributed.
[(i) 6.93 kV (ii) 4 kV] (*Elect. Engg-II, Kerala Univ. 1979*)
11. A star-connected 3-phase, 6-pole synchronous generator has a stator with 90 slots and 8 conductors per slot. The rotor revolves at 1000 r.p.m. The flux per pole is 4×10^{-2} weber. Calculate the e.m.f. generated, if all the conductors in each phase are in series. Assume sinusoidal flux distribution and full-pitched coils. **[Eph = 1,066 V] (*Elect. Machines, A.M.I.E. Summer, 1979*)**
12. A six-pole machine has an armature of 90 slots and 8 conductors per slot and revolves at 1000 r.p.m. the flux per pole being 50 milli weber. Calculate the e.m.f. generated as a three-phase star-connected machine if the winding factor is 0.96 and all the conductors in each phase are in series.
[1280 V] (*Elect. Machines, AMIE, Sec. B, (E-3), Summer 1992*)
13. A 3-phase, 16 pole alternator has a star connected winding with 144 slots and 10 conductors per slot. The flux/pole is 0.04 wb (sinusoidal) and the speed is 375 rpm. Find the frequency and phase and line e.m.f. The total turns/phase may be assumed to series connected.
[50 Hz, 2035 Hz, 3525 V] (*Rajiv Gandhi Technical University, Bhopal, 2000*)

37.15. Factors Affecting Alternator Size

The efficiency of an alternator always increases as its power increases. For example, if an alternator of 1 kW has an efficiency of 50%, then one of 10 MW will inevitably have an efficiency of about 90%. It is because of this improvement in efficiency with size that alternators of 1000 MW and above possess efficiencies of the order of 99%.



Light weight alternator

Another advantage of large machines is that power output per kilogram increases as the alternator power increases. If 1 kW alternator weighs 20 kg (*i.e.* 50 W/kg), then 10MW alternator weighing 20,000 kg yields 500 W/kg. In other words, larger alternators weigh relatively less than smaller ones and are, consequently, cheaper.

However, as alternator size increases, cooling problem becomes more serious. Since large machines inherently produce high power loss per unit surface area (W/m^2), they tend to overheat.



Large weight alternator

To keep the temperature rise within acceptable limits, we have to design efficient cooling system which becomes ever more elaborate as the power increases. For cooling alternators of rating upto 50 MW, circulating cold-air system is adequate but for those of rating between 50 and 300 MW, we have to resort to hydrogen cooling. Very big machines in 1000 MW range have to be equipped with hollow water-cooled conductors. Ultimately, a point is reached where increased cost of cooling exceeds the saving made elsewhere and this fixes the upper limit of the alternator size.

So for as the speed is concerned, low-speed alternators are always bigger than high speed alternators of the same power. Bigness always simplifies the cooling problem. For example, the large 200-rpm, 500-MVA alternators installed in a typical hydropower plant are air-cooled whereas much smaller 1800-r.p.m., 500-MVA alternators installed in a steam plant are hydrogen cooled.

37.16. Alternator on Load

As the load on an alternator is varied, its terminal voltage is also found to vary as in d.c. generators. This variation in terminal voltage V is due to the following reasons:

1. voltage drop due to armature resistance R_a
2. voltage drop due to armature leakage reactance X_L
3. voltage drop due to armature reaction

(*a*) Armature Resistance

The armature resistance/phase R_a causes a voltage drop/phase of IR_a which is in phase with the armature current I. However, this voltage drop is practically negligible.

(*b*) Armature Leakage Reactance

When current flows through the armature conductors, fluxes are set up which do not cross the air-gap, but take different paths. Such fluxes are known as *leakage fluxes*. Various types of leakage fluxes are shown in Fig. 37.22.

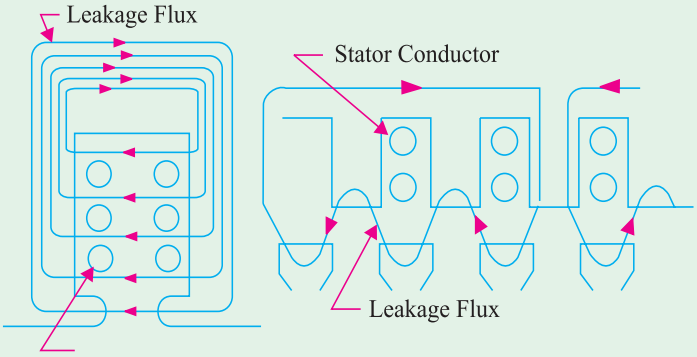


Fig. 37.22

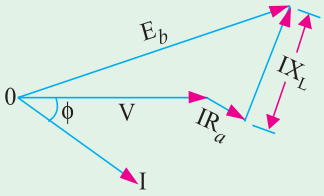


Fig. 37.23

The leakage flux is practically independent of saturation, but is dependent on I and its phase angle with terminal voltage V. This leakage flux sets up an e.m.f. of self-inductance which is known as *reactance e.m.f.* and which is ahead of I by 90°. Hence, armature winding is assumed to possess leakage reactance X_L (also known as Potier rectance X_P) such that voltage drop due to this equals IX_L. A part of the generated e.m.f. is used up in overcoming this reactance e.m.f.

$$\therefore \qquad E = V + I\,(R + jX_L)$$

This fact is illustrated in the vector diagram of Fig. 37.23.

(*c*) Armature Reaction

As in d.c. generators, armature reaction is the effect of armature flux on the main field flux. ***In the case of alternators, the power factor of the load has a considerable effect on the armature reaction.*** We will consider three cases : ***(i)*** when load of p.f. is unity ***(ii)*** when p.f. is zero lagging and ***(iii)*** when p.f. is zero leading.

Before discussing this, it should be noted that in a 3-phase machine the combined ampere-turn wave (or m.m.f. wave) is sinusoidal which moves synchronously. This amp-turn or m.m.f. wave is fixed relative to the poles, its amplitude is proportional to the load current, but its position depends on the p.f. of the load.

Consider a 3-phase, 2-pole alternator having a single-layer winding, as shown in Fig. 37.24 (*a*). For the sake of simplicity, assume that winding of each phase is concentrated (instead of being distributed) and that the number of turns per phase is N. Further suppose that the alternator is loaded with a resistive load of unity power factor, so that phase currents I_a, I_b and I_c are in phase with their respective phase voltages. Maximum current I_a will flow when the poles are in position shown in Fig. 37.24 (*a*) or at a time t_1 in Fig. 37.24 (*c*). When I_a has a maximum value, I_b and I_c have one-half their maximum values (the arrows attached to I_a , I_b and I_c are only polarity marks and are not meant to give the instantaneous directions of these currents at time t_1). The instantaneous directions of currents are shown in Fig. 37.24 (*a*). At the instant t_1, I_a flows in conductor α whereas I_b and I_c flow out.

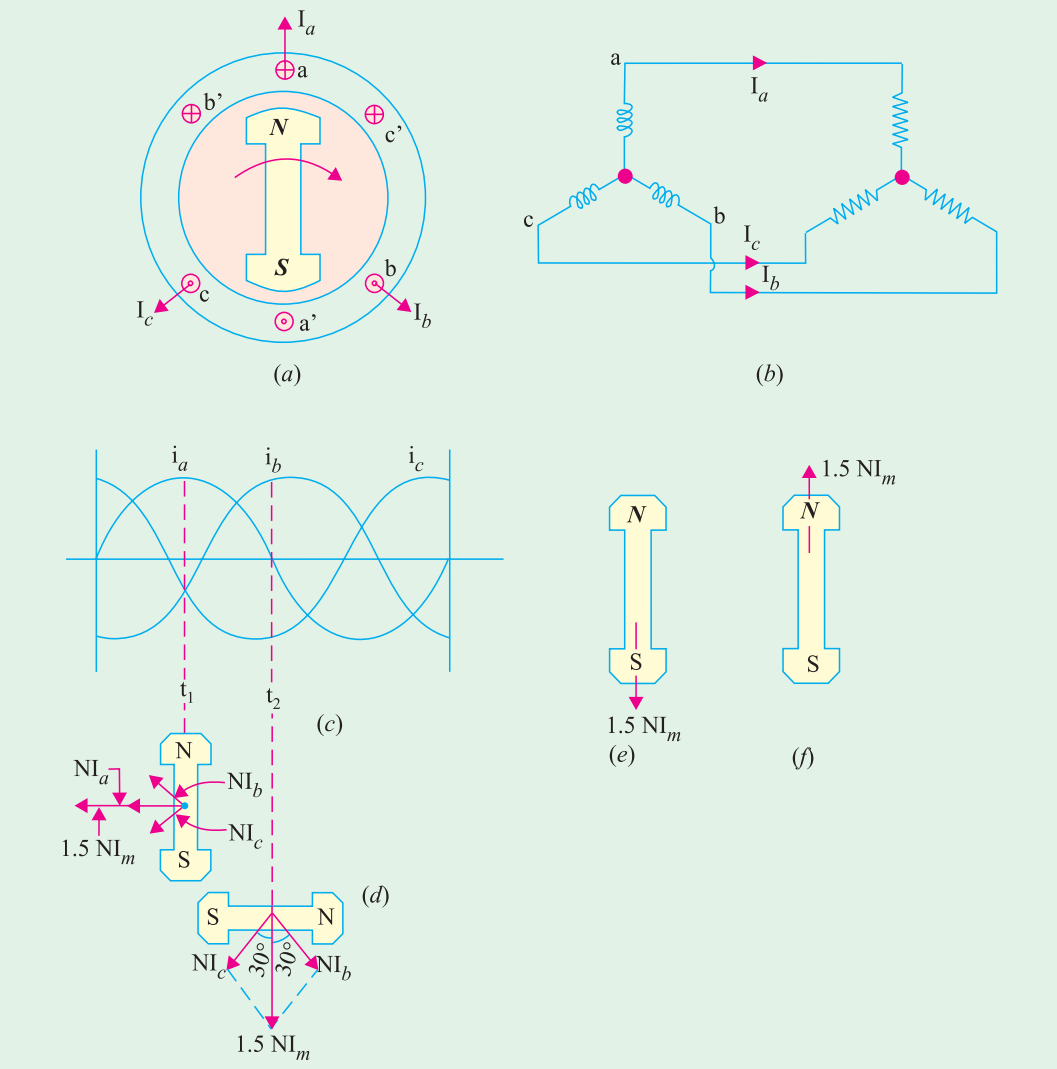


Fig. 37.24

As seen from Fig. 37.24 (*d*), the m.m.f. (= NI_m) produced by phase *a*-*a*′ is horizontal, whereas that produced by other two phases is (I_m/2) *N* each at 60° to the horizontal. The total armature m.m.f. is equal to the vector sum of these three m.m.fs.

∴ Armature m.m.f. $= NI_m + 2.(1/2\ NI_m) \cos 60° = 1.5\ NI_m$

As seen, at this instant t_1, the m.m.f. of the main field is upwards and the armature m.m.f. is behind it by 90 electrical degrees.

Next, let us investigate the armature m.m.f. at instant t_2. At this instant, the poles are in the horizontal position. Also $I_a = 0$, but I_b and I_c are each equal to 0.866 of their maximum values. Since I_c has not changed in direction during the interval t_1 to t_2, the direction of its m.m.f. vector remains unchanged. But I_b has changed direction, hence, its m.m.f. vector will now be in the position shown in Fig. 37.24 (*d*). Total armature m.m.f. is again the vector sum of these two m.m.fs.

∴ Armature m.m.f. $= 2 \times (0.866\ NI_m) \times \cos 30° = 1.5\ NI_m$.

If further investigations are made, it will be found that.

1. armature m.m.f. remains constant with time

2. it is 90 space degrees behind the main field m.m.f., so that it is only distortional in nature.
3. it rotates synchronously round the armature *i.e.* stator.

For a lagging load of zero power factor, all currents would be delayed in time 90° and armature m.m.f. would be shifted 90° with respect to the poles as shown in Fig. 37.24 (*e*). Obviously, armature m.m.f. would demagnetise the poles and cause a reduction in the induced e.m.f. and hence the terminal voltage.

For leading loads of zero power factor, the armature m.m.f. is advanced 90° with respect to the position shown in Fig. 37.24 (*d*). As shown in Fig. 37.24 (*f*), the armature m.m.f. strengthens the main m.m.f. In this case, armature reaction is wholly magnetising and causes an increase in the terminal voltage.

The above facts have been summarized briefly in the following paragraphs where the matter is discussed in terms of 'flux' rather than m.m.f. waves.

1. Unity Power Factor

In this case [Fig. 37.25 (*a*)] the armature flux is cross-magnetising. The result is that the flux at the leading tips of the poles is reduced while it is increased at the trailing tips. However, these two effects nearly offset each other leaving the average field strength constant. In other words, armature reaction for unity p.f. is distortional.

2. Zero P.F. lagging

As seen from Fig. 37.25 (*b*), here the armature flux (whose wave has moved backward by 90°) is in direct opposition to the main flux.

Hence, the main flux is decreased. Therefore, it is found that armature reaction, in this case, is wholly ***demagnetising***, with the result, that due to weakening of the main flux, less e.m.f. is generated. To keep the value of generated e.m.f. the same, field excitation will have to be increased to compensate for this weakening.

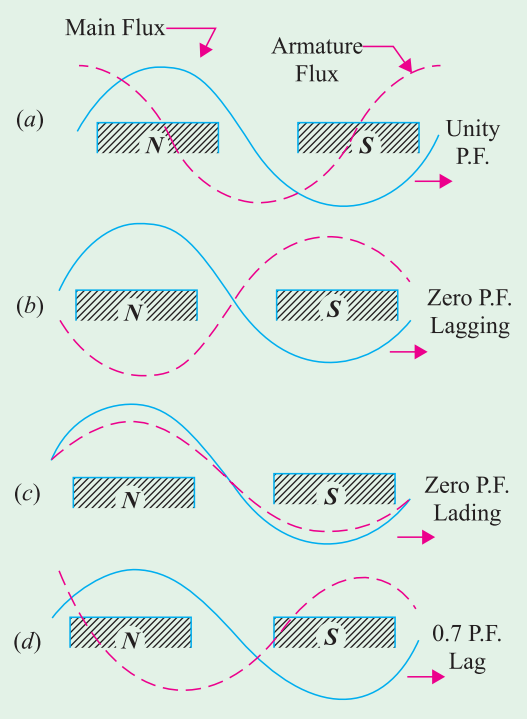


Fig. 37.25

3. Zero P.F. leading

In this case, shown in Fig. 37.25 (*c*) armature flux wave has moved forward by 90° so that it is in phase with the main flux wave. This results in added main flux. Hence, in this case, armature reaction is wholly *magnetising*, which results in greater induced e.m.f. To keep the value of generated e.m.f. the same, field excitation will have to be reduced somewhat.

4. For intermediate power factor [Fig. 37.25 (*d*)], the effect is partly distortional and partly demagnetising (because p.f. is lagging).

37.17. Synchronous Reactance

From the above discussion, it is clear that for the same field excitation, terminal voltage is decreased from its no-load value E_0 to V (for a lagging power factor). This is because of

1. drop due to armature resistance, IR_a
2. drop due to leakage reactance, IX_L
3. drop due to armature reaction.

The drop in voltage due to armature reaction may be accounted for by assumiung the presence of a fictitious reactance X_a in the armature winding. The value of X_a is such that IX_a represents the voltage drop due to armature reaction.

The leakage reactance X_L (or X_P) and the armature reactance X_a may be combined to give synchronous reactance X_S.

Hence $X_S = X_L + X_a$*

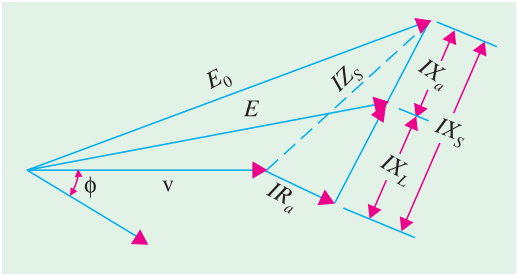


Fig. 37.26

Therefore, total voltage drop in an alternator under load is = $IR_a + jIX_S = I(R_a + jX_S) = IZ_S$ where Z_S is known as synchronous impedance of the armature, the word 'synchronous' being used merely as an indication that it refers to the working conditions.

Hence, we learn that the vector difference between no-load voltage E_0 and terminal voltage V is equal to IZ_S, as shown in Fig. 37.26.

37.18. Vector Diagrams of a Loaded Alternator

Before discussing the diagrams, following symbols should be clearly kept in mind.

E_0 = No-load e.m.f. This being the voltage induced in armature in the absence of three factors discussed in Art. 37.16. Hence, it represents the maximum value of the induced e.m.f.

E = Load induced e.m.f. It is the induced e.m.f. after allowing for armature reaction. E is vectorially less than E_0 by IX_a. Sometimes, it is written as E_a (Ex. 37.16).

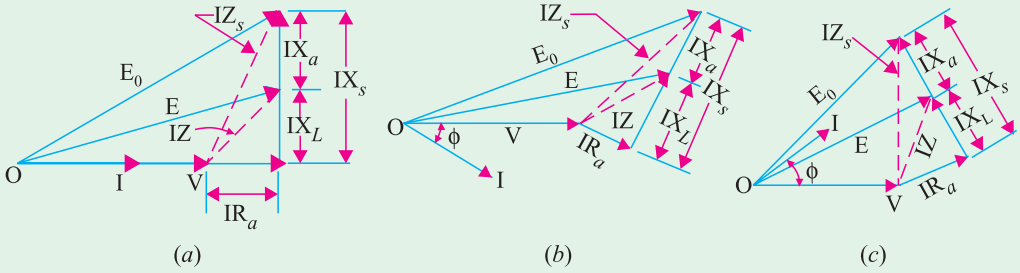


Fig. 37.27

V = Terminal voltage, It is vectorially less than E_0 by IZ_S or it is vectorially less than E by I_Z where

Z = $\sqrt{(R_a^2 + X_L^2)}$. It may also be written as Z_a.

I = armature current/phase and ϕ = load p.f. angle.

In Fig. 37.27 **(*a*)** is shown the case for unity p.f., in Fig. 37.27 **(*b*)** for lagging p.f. and in Fig. 37.27 **(*c*)** for leading p.f. All these diagrams apply to one phase of a 3-phase machine. Diagrams for the other phases can also be drawn similary.

Example 37.16. *A 3-phase, star-connected alternator supplies a load of 10 MW at p.f. 0.85 lagging and at 11 kV (terminal voltage). Its resistance is 0.1 ohm per phase and synchronous reactance 0.66 ohm per phase. Calculate the line value of e.m.f. generated.*

(Electrical Technology, Aligarh Muslim Univ. 1988)

* The ohmic value of X_a varies with the p.f. of the load because armature reaction depends on load p.f.

Solution. F.L. output current $= \dfrac{10\times10^6}{\sqrt{3}\times11,000\times0.85} = 618\ A$

IR_a drop $= 618\times0.1 = 61.8$ V

IX_S drop $= 618\times0.66 = 408$ V

Terminal voltage/phase $= 11,000/\sqrt{3} = 6,350$ V

$\phi = \cos^{-1}(0.85) = 31.8°$; $\sin\phi = 0.527$

As seen from the vector diagram of Fig. 37.28 where I instead of V has been taken along reference vector,

$$E_0 = \sqrt{(V\cos\phi + IR_a)^2 + (V\sin\phi + IX_S)^2}$$

$$= \sqrt{(6350\times0.85 + 61.8)^2 + (6350\times0.527 + 408)^2}$$

$= 6,625$ V

Line e.m.f. $= \sqrt{3}\times6,625 =$ **11,486 volt**

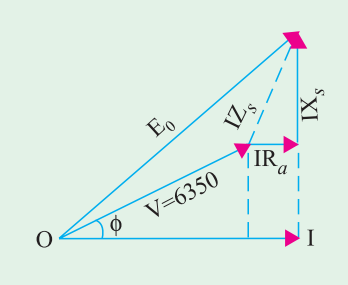


Fig. 37.28

37.19. Voltage Regulation

It is clear that with change in load, there is a change in terminal voltage of an alternator. The magnitude of this change depends ***not only on the load but also on the load power factor.***

The voltage regulation of an alternator is defined as "the rise in voltage when full-load is removed (field excitation and speed remaining the same) divided by the rated terminal voltage."

$\therefore$ % regulation 'up' $= \dfrac{E_0 - V}{V}\times100$

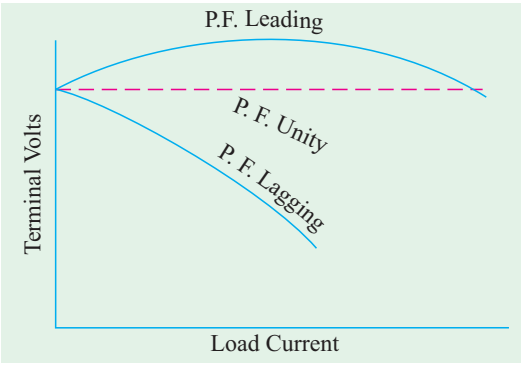


Fig. 37.29

Note. ***(i)*** $E_0 - V$ is the arithmetical difference and not the vectorial one.

(ii) In the case of *leading* load p.f., terminal voltage will fall on removing the full-load. Hence, regulation is negative in that case.

(iii) The rise in voltage when full-load is thrown off is not the same as the fall in voltage when full-load is applied.

Voltage characteristics of an alternator are shown in Fig. 37.29.

37.20. Determination of Voltage Regulation

In the case of small machines, the regulation may be found by direct loading. The procedure is as follows :

The alternator is driven at synchronous speed and the terminal voltage is adjusted to its rated value V. The load is varied until the wattmeter and ammeter (connected for the purpose) indicate the rated values at desired p.f. Then the entire load is thrown off while the speed and field excitation are kept constant. The open-circuit or no-load voltage E_0 is read. Hence, regulation can be found from

$$\%\ \text{regn} = \frac{E_0 - V}{V}\times100$$

In the case of large machines, the cost of finding the regulation by direct loading becomes prohibitive. Hence, other indirect methods are used as discussed below. It will be found that all these methods differ chiefly in the way the no-load voltage E_0 is found in each case.

1. ***Synchronous Impedance or E.M.F. Method.*** It is due to Behn Eschenberg.
2. ***The Ampere-turn or M.M.F. Method.*** This method is due to Rothert.
3. ***Zero Power Factor or Potier Method.*** As the name indicates, it is due to Potier.

All these methods require—

1. Armature (or stator) resistance R_a
2. Open-circuit/No-load characteristic.
3. Short-circuit characteristic (but zero power factor lagging characteristic for Potier method).

Now, let us take up each of these methods one by one.

(*i*) Value of Ra

Armature resistance R_a per phase can be measured directly by voltmeter and ammeter method or by using Wheatstone bridge. However, under working conditions, the effective value of R_a is increased due to 'skin effect'*. The value of R_a so obtained is increased by 60% or so to allow for this effect. Generally, a value 1.6 times the d.c. value is taken.

(*ii*) O.C. Characteristic

As in d.c. machines, this is plotted by running the machine on no-load and by noting the values of induced voltage and field excitation current. It is just like the *B-H* curve.

(*iii*) S.C. Characteristic

It is obtained by short-circuiting the armature (*i.e.* stator) windings through a low-resistance ammeter. The excitation is so adjusted as to give 1.5 to 2 times the value of full-load current. During this test, the speed which is not necessarily synchronous, is kept constant.

Example 37.17 (*a*). *The effective resistance of a 2200V, 50Hz, 440 KVA, 1-phase, alternator is 0.5 ohm. On short circuit, a field current of 40 A gives the full load current of 200 A. The electro-motive force on open-circuits with same field excitation is 1160 V. Calculate the synchronous impedance and reactance.* **(Madras University, 1997)**

Solution. For the 1-ph alternator, since the field current is same for O.C. and S.C. conditions

$$Z_S = \frac{1160}{200} = 5.8 \text{ ohms}$$

$$X_S = \sqrt{5.8^2 - 0.5^2} = 5.7784 \text{ ohms}$$

Example 37.17 (*b*). *A 60-KVA, 220 V, 50-Hz, 1-ϕ alternator has effective armature resistance of 0.016 ohm and an armature leakage reactance of 0.07 ohm. Compute the voltage induced in the armature when the alternator is delivering rated current at a load power factor of **(a)** unity **(b)** 0.7 lagging and **(c)** 0.7 leading.* **(Elect. Machines-I, Indore Univ. 1981)**

Solution. Full load rated current $I = 60{,}000/220 = 272.2$ A

$$IR_a = 272.2 \times 0.016 = 4.3 \text{ V};$$
$$IX_L = 272.2 \times 0.07 = 19 \text{ V}$$

***(a)* Unity p.f.** — Fig. 37.30 (*a*)

$$E = \sqrt{(V + IR_a)^2 + (I\,X_L)^2} = \sqrt{(220 + 4.3)^2 + 19^2} = \mathbf{225\ V}$$

* The 'skin effect' may sometimes increase the effective resistance of armature conductors as high as 6 times its d.c. value.

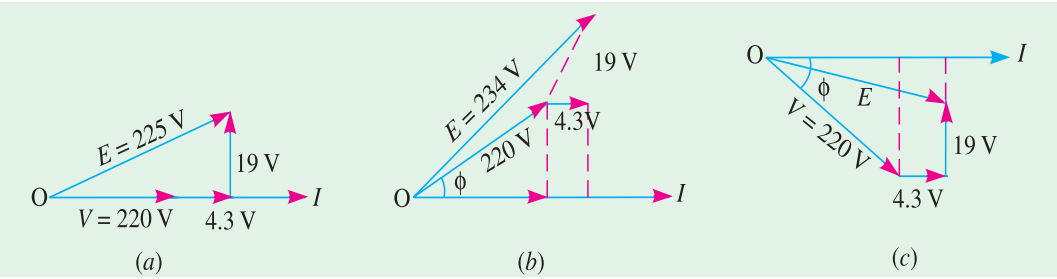


Fig. 37.30

(b) p.f. 0.7 (lag) —Fig. 37.30 (*b*)

$$E = [V\cos\phi + IR_a)^2 + (V\sin\phi + IX_L)^2]^{1/2}$$
$$= [(220 \times 0.7 + 4.3)^2 + (220 \times 0.7 + 19)^2]^{1/2} = \mathbf{234\ V}$$

(c) p.f. = 0.7 (lead) —Fig. 37.30 (*c*)

$$E = [(V\cos\phi + IR_a)^2 + (V\sin\phi - IX_L)^2]^{1/2}$$
$$= [(220 \times 0.7 + 4.3)^2 + (220 \times 0.7 - 19)^2]^{1/2} = \mathbf{208\ V}$$

Example 37.18 (*a*). *In a 50-kVA, star-connected, 440-V, 3-phase, 50-Hz alternator, the effective armature resistance is 0.25 ohm per phase. The synchronous reactance is 3.2 ohm per phase and leakage reactance is 0.5 ohm per phase. Determine at rated load and unity power factor :*

***(a)** Internal e.m.f. E_a **(b)** no-load e.m.f. E_0 **(c)** percentage regulation on full-load **(d)** value of synchronous reactance which replaces armature reaction.*

(Electrical Engg. Bombay Univ. 1987)

Solution. (*a*) The e.m.f. E_a is the vector sum of (*i*) terminal voltage V **(*ii*)** IR_a and **(*iii*)** IX_L as detailed in Art. 37.17. Here,

$$V = 440 / \sqrt{3} = 254\ \text{V}$$

F.L. output current at u.p.f. is

$$= 50{,}000/\sqrt{3} \times 440 = 65.6\ \text{A}$$

Resistive drop = $65.6 \times 0.25 = 16.4$ V

Leakage reactance drop $IX_L = 65.6 \times 0.5 = 32.8$ V

$$\therefore \quad E_a = \sqrt{(V + IR_a)^2 + (IX_L)^2}$$
$$= \sqrt{(254 + 16.4)^2 + 32.8^2} = 272\ \text{volt}$$

Line value $= \sqrt{3} \times 272 = \mathbf{471\ volt.}$

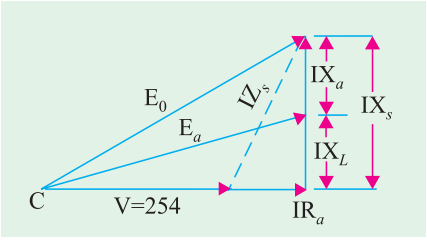


Fig. 37.31

(*b*) The no-load e.m.f. E_0 is the vector sum of **(*i*)** V **(*ii*)** IR_a and **(*iii*)** IX_S or is the vector sum of V and IZ_S (Fig. 37.31).

$$\therefore \quad E_0 = \sqrt{(V + IR_a)^2 + (IX_S)^2} = \sqrt{(254 + 16.4)^2 + (65.6 \times 3.2)^2} = 342\ \text{volt}$$

Line value $= \sqrt{3} \times 342 = \mathbf{592\ volt}$

(*c*) % age regulation 'up' $= \dfrac{E_0 - V}{V} \times 100 = \dfrac{342 - 254}{254} \times 100 = \mathbf{34.65\ per\ cent}$

(*d*) $X_a = X_S - X_L = 3.2 - 0.5 = \mathbf{2.7\ \Omega}$ ...Art. 37.17

Example 37.18 (*b*). *A 1000 kVA, 3300-V, 3-phase, star-connected alternator delivers full-load current at rated voltage at 0.80 p. f. Lagging. The resistance and synchronous reactance of the*

machine per phase are 0.5 ohm and 5 ohms respectively. Estimate the terminal voltage for the same excitation and same load current at 0.80 p. f. leading. **(Amravati University, 1999)**

Solution. $V_{ph} = \frac{3300}{\sqrt{3}} = 1905$ volts

At rated load, $I_{ph} = \frac{1000 \times 1000}{\sqrt{3} \times 3300} = 175$ amp

From phasor diagram for this case [Fig. 37.32 (*a*)]

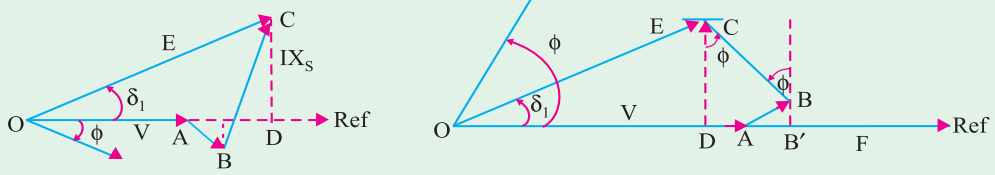
Component of E along Ref $= OD = OA + AB \cos \phi + BC \sin \phi$

$= 1905 + (87.5 \times 0.80) + (875 \times 0.60) = 2500$

Component of E along perpendicular direction

$= CD = -AB \sin \phi + BC \cos \phi$

$= 87.5 \times 0.6 + 875 \times 0.80 = 647.5$ volts



(*a*) Phasor diagram at lagging P.f. **(*b*)** Phasor diagram for leading P.F.

Fig. 37.32

$OA = 1950, AB = I_r = 87.5, BC = IX_S = 875$

$OC = E = \sqrt{OD^2 + DC^2} = \sqrt{2500^2 + 647.5^2} =$ **2582.5 volts**

$\delta_1 = \sin^{-1} \frac{CD}{OC} = \sin^{-1} (647.5 / 2582.5) = 14.52°$

Now, for *E* kept constant, and the alternator delivering rated current at 0.80 leading p.f., the phasor diagram is to be drawn to evaluate *V*.

Construction of the phasor diagram starts with marking the reference. Take a point *A* which is the terminating point of phasor *V* which starts from *O*. *O* is the point yet to be marked, for which the other phasors have to be drawn.

$AB = 87.5, BC = 875$

$BAF = 36.8°$

BC perpendicular to *AB*. From *C*, draw an arc of length *E*, *i.e.* 2582.5 volts to locate *O*.

Note. Construction of Phasor diagram starts from known *AE*, *V* is to be found.

Along the direction of the current, *AB* = 87.5, $\angle BAF = 36.8°$, since the current is leading. *BC* = 875 which must be perpendicular to *AB*. Having located *C*, draw a line *CD* which is perpendicular to the reference, with point *D*, on it, as shown.

Either proceed graphically drawing to scale or calculate geometrically :

$CD = AB \sin \phi + BC \cos \phi = (87.5 \times 0.60) + (875 \times 0.80) = 752.5$ volts

Since $CD = E \sin \delta_2$, $\sin \delta_2 = 752.5/2582.5$ giving $\delta_2 = 17°$

$OD = E \cos \delta = 2470$ volts

$DA = DB' - AB'$

$= BC \sin \phi - AB \cos \phi - 875 \times 0.6 - 87.5 \times 0.8 = 455$ volts

Terminal voltage, $V = OA = OD + DA = 2470 + 455 = 2925$ volts/phase

Since the alternator is star connected, line voltage $\sqrt{3} \times 2925 = 5066$ volts

Check : While delivering lagging p.f. current,

Total power delivered= (100 kVA) × 0.80 = 800 kW

In terms of E and δ_1 referring to the impedance-triangle in Fig. 37.32 (*c*)

total power delivered

$$= 3\left[\frac{VE}{Z_S}\cos(\theta - \delta_1) - \frac{V^2}{Z_S}\cos\theta\right]$$

$$= 3\left[\frac{1905 \times 2582.5}{5.025}\cos(84.3° - 14.52°) - 1905^2 \times \cos 84.3°\right]$$

$$= 800 \text{ kW} \quad \text{...checked}$$

$$OA = 0.5, \quad AB = 5$$

$$OB = \sqrt{0.5^2 + 5^2} = 5.025\ \Omega$$

$$\theta = \angle BOA = \tan^{-1}(X_S/R) = \tan^{-1} 10 = 84.3°$$

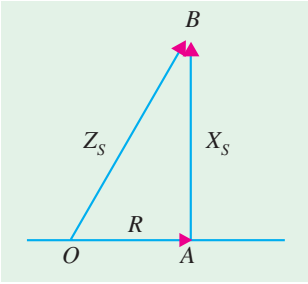


Fig. 37.32 (*c*) Impedance triangle for the Alternator

While delivering leading p.f. current the terminal voltage is 5.066 kV line to line.

Total power delivered in terms of V and I

$$= \sqrt{3} \times 5.066 \times 175 \times 0.8 \text{ kW} = 1228.4 \text{ kW}$$

In terms of E and with voltages expressed in volts,

total power output

$$= 3\left[\frac{VE}{Z_S}\cos(\theta - \delta_2) - \frac{V^2}{Z_S}\cos\theta\right] \times 10^{-3} \text{ kW}$$

$$= 3\left[\frac{2925 \times 2582.5}{5.025}\cos(84.3 - 17°) - \frac{2925^2}{5.025} \times \cos 84.3°\right] \times 10^{-3} \text{ kW}$$

$$= 3\left[\frac{2925 \times 2582.5}{5.025} \times \cos(67.3°) - \frac{2925 \times 2925}{5.025}\cos 84.3°\right] \text{kW}$$

$$= 3\,(580.11 - 169.10) = 1233 \text{ kW},$$ which agrees fairly closely to the previous figure and hence checks our answer.

37.21. Synchronous Impedance Method

Following procedural steps are involved in this method:

1. O.C.C is plotted from the given data as shown in Fig. 37.33 (*a*).

2. Similarly, S.C.C. is drawn from the data given by the short-circuit test. It is a straight line passing through the origin. Both these curves are drawn on a common field-current base.

Consider a field current I_f. The O.C. voltage corresponding to this field current is E_1. When winding is short-circuited, the terminal voltage is zero. Hence, it may be assumed that the whole of this voltage E_1 is being used to circulate the armature short-circuit current I_1 against the synchronous impedance Z_S.

$$\therefore \qquad E_1 = I_1 Z_S \qquad \therefore Z_S = \frac{E_1 \text{ (open-circuit)}}{I_1 \text{ (short-circuit)}}$$

3. Since R_a can be found as discussed earlier, $X_S = \sqrt{(Z_S^2 - R_a^2)}$

4. Knowing R_a and X_S, vector diagram as shown in Fig. 37.33 (*b*) can be drawn for any load and any power factor.

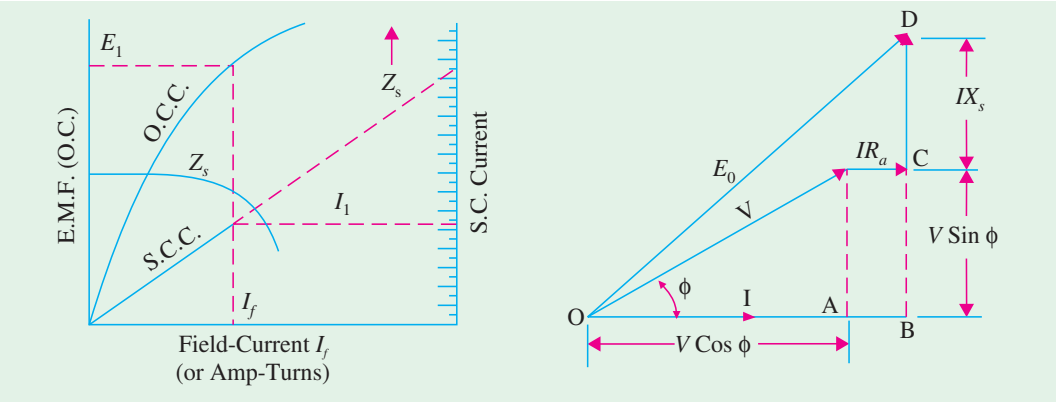


Fig. 37.33 (a) **Fig. 37.33 (b)**

Here $OD = E_0 \quad \therefore E_0 = \sqrt{(OB^2 + BD^2)}$

or $E_0 = \sqrt{[(V \cos \phi + IR_a)^2 + (V \sin \phi + IX_S)^2]}$

$\therefore$ % regn. 'up' $= \dfrac{E_0 - V}{V} \times 100$

Note. (*i*) Value of regulation for unity power factor or leading p.f. can also be found in a similar way.

(*ii*) This method is not accurate because the value of Z_S so found is always more than its value under normal voltage conditions and saturation. Hence, the value of regulation so obtained is always more than that found from an actual test. That is why it is called pessimistic method. The value of Z_S is not constant but varies with saturation. At low saturation, its value is larger because then the effect of a given armature ampere-turns is much more than at high saturation. Now, under short-circuit conditions, saturation is very low, because armature m.m.f. is directly demagnetising. Different values of Z_S corresponding to different values of field current are also plotted in Fig. 37.33 (*a*).

(*iii*) The value of Z_S usually taken is that obtained from full-load current in the short-circuit test.

(*iv*) Here, armature reactance X_a has not been treated separately but along with leakage reactance X_L.

Example 37.19. *Find the synchronous impedance and reactance of an alternator in which a given field current produces an armature current of 200 A on short-circuit and a generated e.m.f. of 50 V on open-circuit. The armature resistance is 0.1 ohm. To what induced voltage must the alternator be excited if it is to deliver a load of 100 A at a p.f. of 0.8 lagging, with a terminal voltage of 200V.* **(Elect. Machinery, Banglore Univ. 1991)**

Solution. It will be assumed that alternator is a single phase one. Now, for same field current,

$$Z_S = \frac{\text{O.C. volts}}{\text{S.C. current}} = \frac{50}{200} = \mathbf{0.25\ \Omega.}$$

$$X_S = \sqrt{Z_S^2 - R_a^2} = \sqrt{0.25^2 - 0.1^2} = \mathbf{0.23\ \Omega.}$$

Now, $IR_a = 100 \times 0.1 = 10$ V, $IX_S = 100 \times 0.23 = 23$ V;

$\cos \phi = 0.8$, $\sin \phi = 0.6$. As seen from Fig. 37.34.

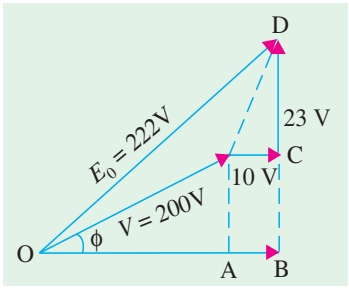


Fig. 37.34

$$E_0 = \sqrt{(V\cos\phi + IR_a)^2 + (V\sin\phi + IX_S)^2}$$
$$= [(200\times 0.8 + 10)^2 + (200\times 0.6 + 23)^2]^{1/2} = \mathbf{222\ V}$$

Example 37.20. *From the following test results, determine the voltage regulation of a 2000-V, 1-phase alternator delivering a current of 100 A at (i) unity p.f. (ii) 0.8 leading p.f. and (iii) 0.71 lagging p.f.*

Test results : Full-load current of 100 A is produced on short-circuit by a field excitation of 2.5A. An e.m.f. of 500 V is produced on open-circuit by the same excitation. The armature resistance is 0.8Ω.

(Elect. Engg.-II, M.S. Univ. 1987)

Solution.

$$Z_S = \frac{\text{O.C. volts}}{\text{S.C. current}}$$ —for same excitation

for same excitation

$$= 500/100 = 5\ \Omega$$

$$X_S = \sqrt{Z_S^2 - R_a^2} = \sqrt{5^2 - 0.8^2} = 4.936\ \Omega$$

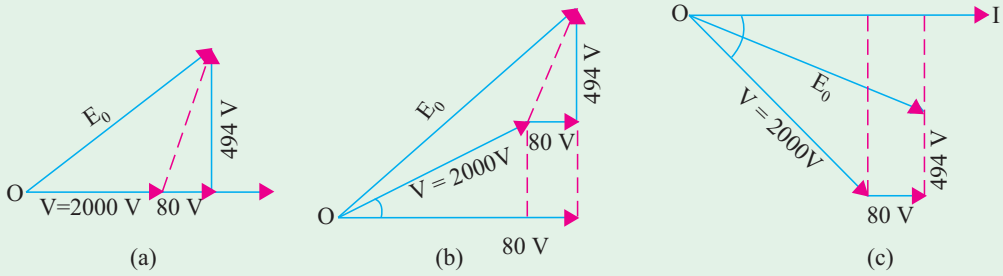


Fig. 37.35

(*i*) Unity p.f. (Fig. 37.35 (*a*)]

$$IR_a = 100\times 0.8 = 80\ \text{V};\quad IX_S = 100\times 4.936 = 494\ \text{V}$$

$$\therefore\quad E_0 = \sqrt{(2000+80)^2 + 494^2} = 2140\ V$$

$$\%\ \text{regn} = \frac{2140 - 2000}{2000}\times 100 = \mathbf{7\%}$$

(*ii*) p.f. = 0.8 (lead) [Fig. 37.35 (*c*)]

$$E_0 = [(2000\times 0.8 + 80)^2 + (2000\times 0.6 - 494)^2]^{1/2} = 1820\ \text{V}$$

$$\%\ \text{regn} = \frac{1820 - 2000}{2000}\times 100 = \mathbf{-9\%}$$

(*iii*) p.f. = 0.71(lag) [Fig.37.35 (*b*)]

$$E_0 = [(2000\times 0.71 + 80)^2 + (2000\times 0.71 + 494)^2]^{1/2} = 2432\ \text{V}$$

$$\%\ \text{regn} = \frac{2432 - 2000}{2000}\times 100 = \mathbf{21.6\%}$$

Example 37.21. *A 100-kVA, 3000-V, 50-Hz 3-phase star-connected alternator has effective armature resistance of 0.2 ohm. The field current of 40 A produces short-circuit current of 200 A and an open-circuit emf of 1040 V (line value). Calculate the full-load voltage regulation at 0.8 p.f. lagging and 0.8 p.f. leading. Draw phasor diagrams.*

(Basic Elect. Machines, Nagpur Univ. 1993)

Solution. $Z_S = \dfrac{\text{O.C. voltage/phase}}{\text{S.C. current/phase}}$ — for same excitation

$$= \frac{1040/\sqrt{3}}{200} = 3\ \Omega$$

$$X_S = \sqrt{Z_S^2 - R_a^2} = \sqrt{3^2 - 0.2^2}$$

$$= 2.99\ \Omega$$

F.L. current,

$I = 100{,}000/\sqrt{3} \times 3000$

$= 19.2$ A

$IR_a = 19.2 \times 0.2 = 3.84$ V

$IX_S = 19.2 \times 2.99 = 57.4$ V

Voltage/phase

$= 3000/\sqrt{3} = 1730$V

$\cos\phi = 0.8$; $\sin\phi = 0.6$

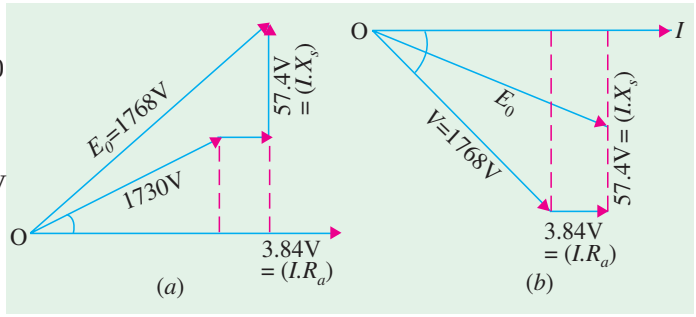


Fig. 37.36

(*i*) p.f. = 0.8 lagging

—Fig. 37.36 (*a*)

$$E_0 = [(V\cos\phi + IR_a)^2 + (V\sin\phi + IX_S)^2]^{1/2}$$

$$= (1730 \times 0.8 + 3.84)^2 + (1730 \times 0.6 + 57.4)^2]^{1/2} = 1768\ \text{V}$$

$$\%\ \text{regn. 'up'} = \frac{(1768 - 1730)}{1730} \times 100 = \mathbf{2.2\%}$$

(*ii*) 0.8 p.f. leading—Fig. 37.36 (*b*)

$$E_0 = [(V\cos\phi + IR_a)^2 + (V\sin\phi - IX_S)^2]^{1/2}$$

$$= [(1730 \times 0.8 + 3.84)^2 + (1730 \times 0.6 - 57.4)^2]^{1/2}$$

$$= 1699\ \text{V}$$

$$\%\ \text{regn.} = \frac{1699 - 1730}{1730} \times 100 = \mathbf{-1.8\%}$$

Example 37.22. *A 3-phase, star-connected alternator is rated at 1600 kVA, 13,500 V. The armature resistance and synchronous reactance are 1.5 Ω and 30 Ω respectively per phase. Calculate the percentage regulation for a load of 1280 kW at 0.8 leading power factor.*

(Advanced Elect. Machines AMIE Sec. B, 1991)

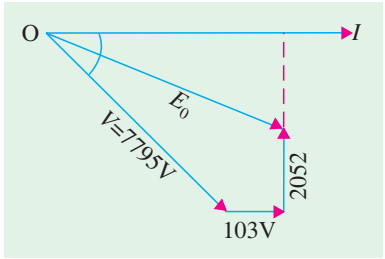


Fig. 37.37

Solution.

$$1280{,}000 = \sqrt{3} \times 13{,}500 \times I \times 0.8;$$

$$\therefore \quad I = 68.4\ \text{A}$$

$IR_a = 68.4 \times 1.5 = 103$ V ; $IX_S = 68.4 \times 30 = 2052$

Voltage/phase $= 13{,}500/\sqrt{3} = 7795$ V

As seen from Fig. 37.37.

$$E_0 = [(7795 \times 0.8 + 103)^2 + (7795 \times 0.6 - 2052)]^{1/2} = 6663\ \text{V}$$

$$\%\ \text{regn.} = (6663 - 7795)/7795$$

$$= -0.1411 \text{ or } \mathbf{-14.11\%}$$

Example 37.23. *A 3-phase, 10-kVA, 400-V, 50-Hz, Y-connected alternator supplies the rated load at 0.8 p.f. lag. If arm. resistance is 0.5 ohm and syn. reactance is 10 ohms, find the power angle and voltage regulation.* **(Elect. Machines-I Nagpur Univ. 1993)**

Solution. F.L. current, $I = 10,000/\sqrt{3} \times 400 = 14.4\ A$

$IR_a = 14.4 \times 0.5 = 7.2$ V

$IX_S = 14.4 \times 10 = 144$ V

Voltage/phase $= 400/\sqrt{3} = 231$ V

$\phi = \cos^{-1} 0.8 = 36.87°$, as shown in Fig. 37.38.

$E_0 = [(V\cos\phi + IR_a)^2 + (V\sin\phi + IX_S)^2]^{1/2}$

$= (231\times 0.8 + 7.2)^2 + (231 \times 0.6 + 144)^2]^{1/2}$

$= 342$ V

% regn. $= \dfrac{342 - 231}{231} \times 100 = 0.48$ or 48%

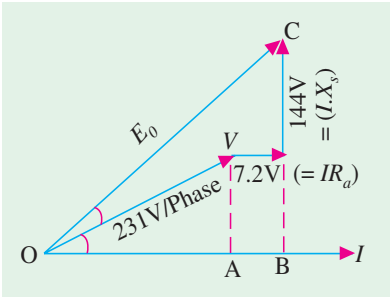


Fig. 37.38

The power angle of the machine is defined as the angle between V and E_0 *i.e.* angle δ

As seen from Fig. 37.38, $\tan(\phi + \delta) = \dfrac{BC}{OB} = \dfrac{231\times 0.6 + 144}{231\times 0.8 + 7.2} = \dfrac{282.6}{192} = 1.4419$;

$\therefore \quad (\phi + \delta) = 55.26°$

$\therefore$ power angle $\quad \delta = 55.26° - 36.87° =$ **18.39°**

Example 37.24. *The following test results are obtained from a 3-phase, 6,000-kVA, 6,600 V, star-connected, 2-pole, 50-Hz turbo-alternator:*

With a field current of 125 A, the open-circuit voltage is 8,000 V at the rated speed; with the same field current and rated speed, the short-circuit current is 800 A. At the rated full-load, the resistance drop is 3 per cent. Find the regulation of the alternator on full-load and at a power factor of 0.8 lagging. **(Electrical Technology, Utkal Univ. 1987)**

Solution. $Z_S = \dfrac{\text{O.C. voltage/phase}}{\text{S.C. current/phase}} = \dfrac{8000/\sqrt{3}}{800} = 5.77\ \Omega$

Voltage/phase $= 6,600\sqrt{3} = 3,810$ V

Resistive drop $= 3\%$ of $3,810$ V $= 0.03 \times 3,810 = 114.3$ V

Full-load current $= 6,000 \times 10^3/\sqrt{3} \times 6,600 = 525$ A

Now $IR_a = 114.3$V

$\therefore \quad R_a = 114.3/525 = 0.218\ \Omega$

$X_S = \sqrt{Z_S^2 - R_a^2} = \sqrt{5.77^2 - 0.218^2} = 5.74\ \Omega$ (approx.)

As seen from the vector diagram of Fig. 37.33, (*b*)

$E_0 = \sqrt{[3,810\times 0.8 + 114.3)^2 + (3,810\times 0.6 + 525\times 5.74)^2]} = 6,180$ V

$\therefore$ regulation $= (6,180 - 3,810) \times 100/3,810 =$ **62.2%**

Example 37.25. *A 3-phase 50-Hz star-connected 2000-kVA, 2300 V alternator gives a short-circuit current of 600 A for a certain field excitation. With the same excitation, the open circuit voltage was 900 V. The resistance between a pair of terminals was 0.12 Ω. Find full-load regulation at **(i)** UPF **(ii)** 0.8 p.f. lagging.* **(Elect. Machines, Nagpur Univ. 1993)**

Solution. $Z_S = \dfrac{\text{O.C. volts/phase}}{\text{S.C. current/phase}} = \dfrac{900/\sqrt{3}}{600} = 0.866\ \Omega$

Resistance between the terminals is 0.12 Ω. It is the resistance of two phases connected in series.

∴ Resistance/phase = 0.12/2 = 0.06 Ω;

effective resistance/phase = $0.06 \times 1.5 = 0.09\ \Omega$;

$$X_S = \sqrt{0.866^2 - 0.09^2} = 0.86\ \Omega$$

F.L. $I = 2000,000/\sqrt{3} \times 2300 = 500$ A

$IR_a = 500 \times 0.06 = 30$ V ;

$IX_S = 500 \times 0.86 = 430$ V

rated voltage/phase

$= 2300/\sqrt{3} = 1328$ V

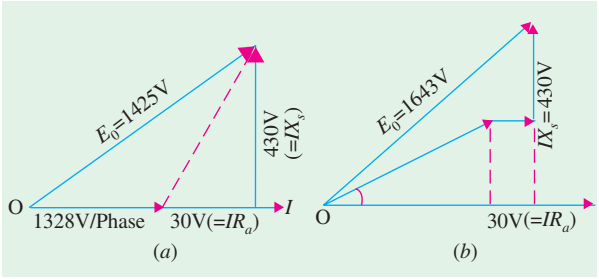


Fig. 37.39

(*i*) U.P.F. —Fig. 37.39 (*a*),

$$E_0 = [(V\cos\phi + IR_a)^2 + (IX_S)^2]^{1/2}$$

$$= \sqrt{(1328 + 30)^2 + 430^2} = 1425\ \text{V}$$

% regn. = (1425 – 1328)/1328 = 0.073 or **7.3%**

(*ii*) 0.8 p.f. lagging —Fig. 37.39 (*b*)

$$E_0 = [(V\cos\phi + IR_a)^2 + (V\sin\phi + IX_s)^2]^{1/2}$$

$$= [(1328 \times 0.8 + 30)^2 + (1328 \times 0.6 + 430)^2]^{1/2} = 1643\ \text{V}$$

∴ % regn. = (1643 – 1328)/1328 = 0.237 or **23.7%.**

Example 37.26. *A 2000-kVA, 11-kV, 3-phase, star-connected alternator has a resistance of 0.3 ohm and reactance of 5 ohm per phase. It delivers full-load current at 0.8 lagging power factor at rated voltage. Compute the terminal voltage for the same excitation and load current at 0.8 power factor leading.* **(Elect. Machines, Nagpur Univ. 1993)**

Solution. (*i*) At 0.8 p.f. lagging

F.L. $I = 2000,000/\sqrt{3} \times 11,000 = 105$ A

Terminal voltage = $11,000/\sqrt{3} = 6350$ V

$IR_a = 105 \times 0.3 = 31.5$ V;

$IX_S = 105 \times 5 = 525$ V

As seen from Fig. 37.40 (*a*)

$E_0 = [6350 \times 0.8 + 31.5)^2 + (6350 \times 0.6 + 525)^2]^{1/2} = 6700$ V

As seen from Fig. 37.40 (*b*), now, we are given $E_0 = 6700$ V and we are required to find the terminal voltage V at 0.8 p.f. leading.

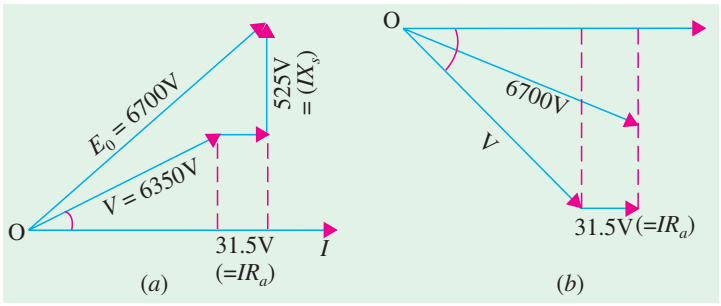


Fig. 37.40

$6700^2 = (0.8V + 31.5)^2 + (0.6V - 525)^2$; V = **6975 V**

Example 37.27. *The effective resistance of a 1200-kVA, 3.3-kV, 50-Hz, 3-phase, Y-connected alternator is 0.25 Ω/phase. A field current of 35 A produces a current of 200 A on short-circuit and 1.1 kV (line to line) on open circuit. Calculate the power angle and p.u. change in magnitude of the terminal voltage when the full load of 1200 kVA at 0.8 p.f. (lag) is thrown off. Draw the corresponding phasor diagram.* **(Elect. Machines, A.M.I.E. Sec. B, 1993)**

Solution. $Z_s = \dfrac{\text{O.C. voltage}}{\text{S.C. voltage}}$ —same excitation

$$= \frac{1.1\times10^3/\sqrt{3}}{200} = 3.175\ \Omega$$

$$X_S = \sqrt{3.175^2 - 0.25^2} = 3.165\ \Omega$$

$$V = 3.3\times10^3/\sqrt{3} = 1905\ \text{V}$$

$$\tan\theta = X_S/R_a = 3.165/0.25,\ \theta = 85.48°$$

$$\therefore\quad Z_s = 3.175\angle 85.48°$$

$$\text{Rated}\quad I_a = 1200\times10^3/\sqrt{3}\times3.3\times10^3$$

$$= 210\ \text{A}$$

Let, $V = 1905\angle 0°$, $I_a = 210\angle -36.87°$

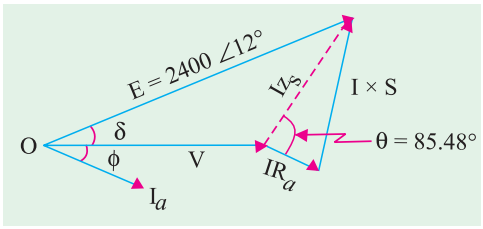


Fig. 37.41

As seen from Fig. 37.41,

$E = V + I_aZ_s = 1905 + 210\angle -36.87°\times 3.175\angle 85.48° = 2400\angle 12°$

Power angle $= \delta = 12°$

Per unit change in terminal voltage is

$= (2400 - 1905)/1905 =$ **0.26**

Example 37.28. *A given 3-MVA, 50-Hz, 11-kV, 3-ϕ, Y-connected alternator when supplying 100 A at zero p.f. leading has a line-to-line voltage of 12,370 V; when the load is removed, the terminal voltage falls down to 11,000 V. Predict the regulation of the alternator when supplying full-load at 0.8 p.f. lag. Assume an effective resistance of 0.4 Ω per phase.*

(Elect. Machines, Nagpur Univ. 1993)

Solution. As seen from Fig. 37.42 (*a*), at zero p.f. leading

$$E_0^2 = (V\cos\phi + IR_a)^2 + (V\sin\phi - IX_S)^2$$

Now, $E_0 = 11{,}000/\sqrt{3} = 6350\ \text{V}$

$V = 12370/\sqrt{3} = 7{,}142\ \text{V}$,

$\cos\phi = 0,\ \sin\phi = 1$

$$\therefore\quad 6350^2 = (0 + 100\times0.4)^2 + (7142 - 100\,X_S)^2$$

$$\therefore\quad 100\,X_S = 790 \text{ or } X_S = 7.9\ \Omega$$

F.L. current

$$I = \frac{3\times10^6}{\sqrt{3}\times11{,}000} = 157\ \text{A}$$

$$IR_a = 0.4\times157 = 63\ \text{V};\ IX_S$$

$$= 157\times7.9 = 1240\ \text{V}$$

$$\therefore\quad E_0 = [(6350\times0.8 + 63)^2 + (6350\times0.6 + 1240)^2]^{1/2} = 7210\ \text{V/phase}$$

$$\therefore\quad \%\ \text{regn} = \frac{7210 - 6350}{6350}\times100 = \textbf{13.5\%}$$

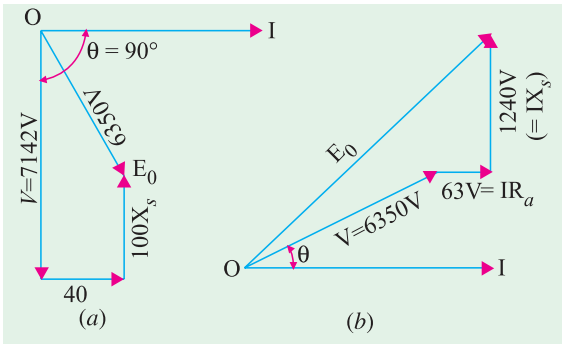


Fig. 37.42

Example 37.29. *A straight line law connects terminal voltage and load of a 3-phase star-connected alternator delivering current at 0.8 power factor lagging. At no-load, the terminal voltage is 3,500 V and at full-load of 2,280 kW, it is 3,300 V. Calculate the terminal voltage when delivering current to a 3-ϕ, star-connected load having a resistance of 8 Ω and a reactance of 6 Ω per phase. Assume constant speed and field excitation.* **(London Univ.)**

Solution. No-load phase voltage $= 3{,}500/\sqrt{3} = 2{,}021$ V

Phase voltage on full-load and 0.8 power factor $= 3{,}300/\sqrt{3} = 1905$ V

Full-load current is given by

$$\sqrt{3}\, V_L\, I_L \cos\phi = 2{,}280 \times 1000 \qquad \therefore \quad I_L = \frac{2{,}280 \times 1000}{\sqrt{3} \times 3{,}300 \times 0.8} = 500 \text{ A}$$

drop in terminal voltage/phase for 500 A = 2,021 – 1,905 = 116 V

Let us assume that alternator is supplying a current of x ampere.

Then, drop in terminal voltage per phase for x ampere is = $116\,x / 500 = 0.232\,x$ volt

$\therefore$ terminal p.d./phase when supplying x amperes at a p.f. of 0.8 lagging is

$$= 2{,}021 - 0.232\,x \text{ volt}$$

Impedance of connected load/phase $= \sqrt{(8^2 + 6^2)} = 10\ \Omega$

load p.f. $= \cos\phi = 8/10 = 0.8$

When current is x, the applied p.d. is $= 10\,x$

$\therefore \quad 10\,x = 2021 - 0.232\,x \quad$ or $\quad x = 197.5$ A

$\therefore$ terminal voltage/phase $= 2021 - (0.232 \times 197.5) = 1975.2$ V

$\therefore$ terminal voltage of alternator $= 1975.2 \times \sqrt{3} =$ **3,421 V**

Tutorial Problem No. 37.2

1. If a field excitation of 10 A in a certain alternator gives a current of 150 A on short-circuit and a terminal voltage of 900 V on open-circuit, find the internal voltage drop with a load current of 60A. **[360 V]**
2. A 500-V, 50-kVA, 1-ϕ alternator has an effective resistance of 0.2 Ω. A field current of 10 A produces an armature current of 200 A on short-circuit and an e.m.f. of 450 V on open-circuit. Calculate the full-load regulation at p.f. 0.8 lag. **[34.4%]**
   (Electrical Technology, Bombay Univ. 1978)
3. A 3-ϕ star-connected alternator is rated at 1600 kVA, 13,500 V. The armature effective resistance and synchronous reactance are 1.5 Ω and 30 Ω respectively per phase. Calculate the percentage regulation for a load of 1280 kW at power factors of (*a*) 0.8 leading and (*b*) 0.8 lagging. **[(*a*) –11.8% (*b*) 18.6%] *(Elect. Engg.-II, Bombay Univ. 1977)***
4. Determine the voltage regulation of a 2,000-V, 1-phase alternator giving a current of 100 A at 0.8 p.f. leading from the test results. Full-load current of 100 A is produced on short-circuit by a field excitation of 2.5 A. An e.m.f. of 500 V is produced on open-circuit by the same excitation. The armature resistance is 0.8 Ω. Draw the vector diagram.
   [– 8.9%] *(Electrical Machines-I, Gujarat Univ. Apr. 1976)*
5. In a single-phase alternator, a given field current produces an armature current of 250 A on short-circuit and a generated e.m.f. of 1500 V on open-circuit. Calculate the terminal p.d. when a load of 250 A at 6.6 kV and 0.8 p.f. lagging is switched off. Calculate also the regulation of the alternator at the given load.
   [7,898 V; 19.7%] *(Elect. Machines-II, Indore Univ. Dec. 1977)*
6. A 500-V, 50-kVA, single-phase alternator has an effective resistance of 0.2 Ω. A field current of 10 A produces an armature current of 200 A on short-circuit and e.m.f. of 450 V on open circuit. Calculate (a) the synchronous impedance and reactance and (*b*) the full-load regulation with 0.8 p.f. lagging.
   [(*a*) 2.25 Ω, 2.24 Ω, (b) 34.4%] *(Elect. Technology, Mysore Univ. 1979)*

7. A 100-kVA, 3,000-V, 50-Hz, 3-phase star-connected alternator has effective armature resistance of 0.2 Ω. A field current of 40 A produces short-circuit current of 200 A and an open-circuit e.m.f. of 1040 V (line value). Calculate the full-load percentage regulation at a power factor of 0.8 lagging. How will the regulation be affected if the alternator delivers its full-load output at a power factor of 0.8 leading?

[24.4% – 13.5%] (*Elect. Machines-II, Indore Univ. July 1977*)

8. A 3-ϕ, 50-Hz, star-connected, 2,000 kVA, 2,300-V alternator gives a short-circuit current of 600 A for a certain field excitation. With the same excitation, the O.C. voltage was 900 V. The resistance between a pair of terminals was 0.12 Ω. Find full-load regulation at (*a*) u.p.f. (*b*) 0.8 p.f.lagging (*c*) 0.8 p.f. leading.

[(*a*) 7.3% (b) 23.8% (*c*) –13.2%] (*Elect. Machinery-III, Bangalore Univ. Aug. 1979*)

9. A 3-phase star-connected alternator is excited to give 6600 V between lines on open circuit. It has a resistance of 0.5 Ω and synchronous reactance of 5 Ω per phase. Calculate the terminal voltage and regulation at full load current of 130 A when the P.F. is (*i*) 0.8 lagging, (*ii*) 0.6 leading. **[*Rajive Gandhi Technical University, Bhopal, 2000*]**

[(*i*) 3318 Volts/Ph, + 14.83% (*ii*) 4265 Volts/Ph, - 10.65%]

37.22. Rothert's M.M.F. or Ampere-turn Method

This method also utilizes O.C. and S.C. data, but is the converse of the E.M.F. method in the sense ***that armature leakage reactance is treated as an additional armature reaction***. In other words, it is assumed that the change in terminal p.d. on load is due entirely to armature reaction (and due to the ohmic resistance drop which, in most cases, is negligible). This fact is shown in Fig. 37.43.

Now, field A.T. required to produce a voltage of V on full-load is the vector sum of the following :

(i) Field A.T. required to produce V (or if R_a is to be taken into account, then $V + I R_a \cos\phi$) on *no-load*. This can be found from O.C.C. and

(ii) Field A.T. required to overcome the demagnetising effect of armature reaction on full-load. This value is found from short-circuit test. The field A.T. required to produce full-load current on short-circuit balances the armature reaction and the ***impedance drop.***

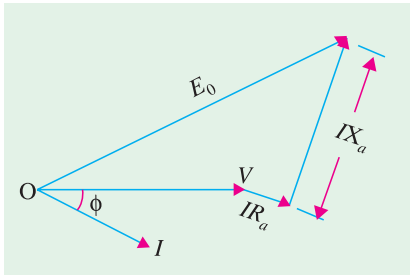


Fig. 37.43

The impedance drop can be neglected because R_a is usually very small and X_S is also small under short-circuit conditions. Hence, p.f. on short-circuit is almost zero lagging and the field A.T. are used entirely to overcome the armature reaction which is wholly demagnetising (Art. 37.15). ***In other words, the demagnetising armature A.T. on full-load are equal and opposite to the field A.T. required to produce full-load current on short-circuit.***

Now, if the alternator, instead of being on short-circuit, is supplying full-load current at its normal voltage and zero p.f. lagging, then total field A.T. required are the vector sum of

(i) the field A.T. = OA necessary to produce normal voltage (as obtained from O.C.C.) and

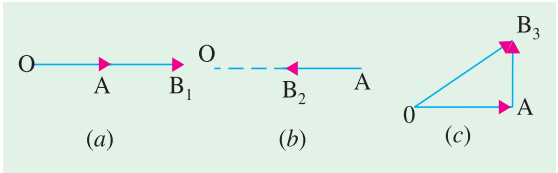


Fig. 37.44

(ii) the field A.T. necessary to neutralize the armature reaction AB_1. The total field A.T. are represented by OB_1 in Fig. 37.44 (*a*) and equals the vector sum of OA and AB_1

If the p.f. is zero leading, the armature reaction is *wholly magnetising*. Hence, in that case, the field A.T. required is OB_2 which is less than OA by the field A.T. = AB_2 required to produce full-load current on short-circuit [Fig. 37.44 (*b*)]

If p.f. is unity, the armature reaction is *cross-magnetising i.e.* its effect is distortional only. Hence, field A.T. required is OB_3 *i.e.* vector sum of OA and AB_3 which is drawn at right angles to OA as in Fig. 37.44 (*c*).

37.23. General Case

Let us consider the general case when the p.f. has any value between zero (lagging or leading) and unity. Field ampere-turns OA corresponding to V(or $V + IR_a \cos \phi$) is laid off horizontally. Then AB_1, representing full-load short-circuit field A.T. is drawn at an angle of $(90° + \phi)$ for a lagging p.f. The total field A.T. are given by OB_1 as in Fig. 37.45. (*a*). For a leading p.f., short-circuit A.T. = AB_2 is drawn at an angle of $(90° - \phi)$ as shown in Fig. 37.45 (*b*) and for unity p.f., AB_3 is drawn at right angles as shown in Fig. 37.45 (*c*).

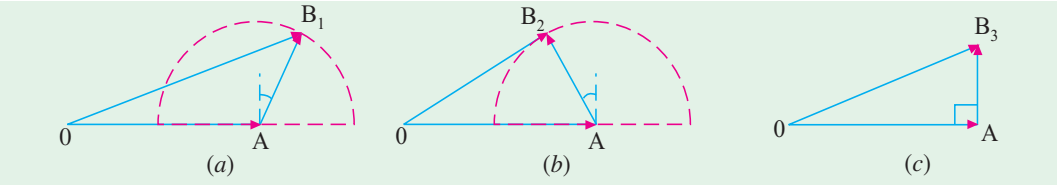


Fig. 37.45

In those cases where the number of turns on the field coils is not known, it is usual to work in terms of the field current as shown in Fig. 37.46.

In Fig. 37.47. is shown the complete diagram along with O.C. and S.C. characteristics. OA represents field current for normal voltage V. OC represents field current required for producing full-load current on *short-circuit*. Vector $AB = OC$ is drawn at an angle of $(90° + \phi)$ to OA (if the p.f. is lagging). The total field current is OB for which the corresponding O.C. voltage is E_0

$$\therefore \quad \% \text{ regn.} = \frac{E_0 - V}{V} \times 100$$

It should be noted that this method gives results which are less than the actual results, that is why it is sometimes referred to as optimistic method.

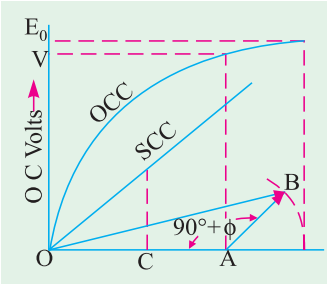


Fig. 37.46

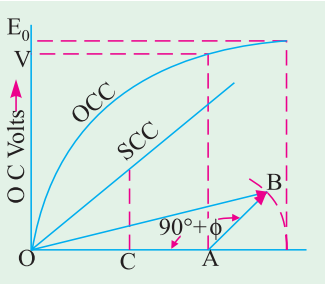


Fig. 37.47

Example 37.30. *A 3.5-MVA, Y-connected alternator rated at 4160 volts at 50-Hz has the open-circuit characteristic given by the following data :*

Field Current (Amps)	*50*	*100*	*150*	*200*	*250*	*300*	*350*	*400*	*450*
E.M.F. (Volts)	*1620*	*3150*	*4160*	*4750*	*5130*	*5370*	*5550*	*5650*	*5750*

A field current of 200 A is found necessary to circulate full-load current on short-circuit of the alternator. Calculate by ***(i)*** *synchronous impedance method and* ***(ii)*** *ampere-turn method the full-load voltage regulation at 0.8 p.f. lagging. Neglect resistance. Comment on the results obtained.*

(Electrical Machines-II, Indore Univ. 1984)

Solution. (*i*) As seen from the given data, a field current of 200 A produces O.C. voltage of 4750 (line value) and full-load current on short-circuit which is

$$= 3.5\times10^6/\sqrt{3}\times4160 = 486 \text{ A}$$

$$Z_S = \frac{\text{O.C. volt/phase}}{\text{S.C. current/phase}} = \frac{4750/\sqrt{3}}{486} = \frac{2740}{486} = 5.64\ \Omega/\text{phase}$$

Since $R_a = 0,\ X_S = Z_S \quad \therefore \quad IR_a = 0,\ IX_S = IZ_S = 486\times5.64 = 2740$ V

F.L. Voltage/phase $= 4160/\sqrt{3} = 2400$ V, $\cos\phi = 0.8, \sin\phi = 0.6$

$$E_0 = (V\cos\phi + IR_a)^2 + (V\sin\phi + I, X_S)^2]^{1/2}$$

$$= [(2400\times0.8 + 0)^2 + (2400\times0.6 + 2740)^2]^{1/2} = 4600 \text{ V}$$

$$\%\text{ regn. up} = \frac{4600-2400}{2400}\times100 = \mathbf{92.5\%}$$

(*ii*) It is seen from the given data that for normal voltage of 4160 V, field current needed is 150 A. Field current necessary to circulate F.L. current on short-circuit is 200 A.

In Fig. 37.48, *OA* represents 150 A. The vector *AB* which represents 200 A is vectorially added to *OA* at $(90° + \phi) = (90° + 36°52') = 126°52'$. Vector *OB* represents excitation necessary to produce a terminal p.d. of 4160 *V* at 0.8 p.f. lagging at *full-load*.

$$OB = [150^2 + 200^2 + 2\times150\times200\times\cos(180° - 126°52')]^{1/2}$$

$$= 313.8 \text{ A}$$

The generated phase e.m.f. E_0, corresponding to this excitation as found from *OCC* (if drawn) is 3140 V. Line value is $3140\times\sqrt{3} = 5440$ V.

$$\%\text{ regn.} = \frac{5440-4160}{4160}\times100 = \mathbf{30.7\%}$$

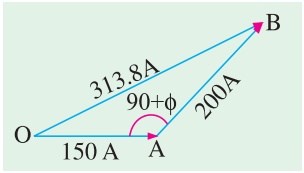


Fig. 37.48

Example 37.31. *The following test results are obtained on a 6,600-V alternator:*

Open-circuit voltage :	*3,100*	*4,900*	*6,600*	*7,500*	*8,300*
Field current (amps) :	*16*	*25*	*37.5*	*50*	*70*

A field current of 20 A is found necessary to circulate full-load current on short-circuit of the armature. Calculate by (i) the ampere-turn method and (ii) the synchronous impedance method the full-load regulation at 0.8 p.f. (lag). Neglect resistance and leakage reactance. State the drawbacks of each of these methods. **(Elect. Machinery-II, Bangalore Univ. 1992)**

Solution. (*i*) Ampere-turn Method

It is seen from the given data that for the normal voltage of 6,600 V, the field current needed is 37.5 A.

Field-current for full-load current, on short-circuit, is given as 20 A.

In Fig. 37.49, *OA* represents 37.5 A. The vector *AB*, which represents 20 A, is vectorially added to *OA* at $(90° + 36°52') = 126°52'$. Vector *OB* represents the excitation necessary to produce a terminal p.d. of 6,600 V at 0.8 p.f. lagging on ***full-load***

$$OB = \sqrt{37.5^2 + 20^2 + 2\times3.75\times20\times\cos53°8'} = 52 \text{ A}$$

The generated e.m.f. E_0 corresponding to this excitation, as found from O.C.C. of Fig. 37.49 is 7,600 V.

$$\text{Percentage regulation} = \frac{E_0 - V}{V} \times 100 = \frac{7,600 - 6,600}{6,600} \times 100 = \mathbf{15.16\%}$$

(*ii*) Synchronous Impedance Method

Let the voltage of 6,600 V be taken as 100 per cent and also let 100 per cent excitation be that which is required to produce 6,600 V on open-circuit, that is, the excitation of 37.5 A.

Full-load or 100 per cent armature current is produced on short-circuit by a field current of 20 A. If 100 per cent field current were applied on short-circuit, then S.C. current would be $100 \times 37.5/20 = 187.5$ per cent.

$$\therefore \quad Z_S = \frac{\text{O.C. voltage}}{\text{S.C. current}} \Bigg| \text{same excitation}$$

$$= 100/187.5 \text{ or } 0.533 \text{ or } 53.3\%$$

The impedance drop IZ_S is equal to 53.3% of the normal voltage. When the two are added vectorially (Fig. 37.50), the value of voltage is

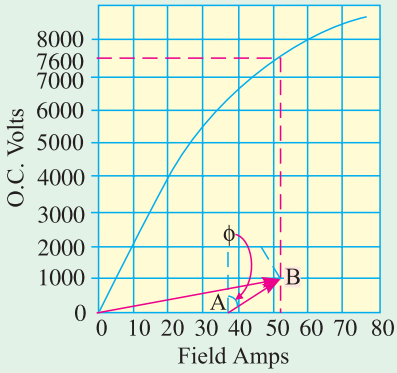


Fig. 37.49

$$E_0 = \sqrt{[100 + 53.3 \cos (90 - \phi)]^2 + [53.3 \sin (90 - \phi)]^2}$$

$$= \sqrt{(100 + 53.3 \times 0.6)^2 + (53.3 \times 0.8)^2} = 138.7\%$$

$$\% \text{ reg} = \frac{138.7 - 100}{100} \times 100 = \mathbf{38.7\%}$$

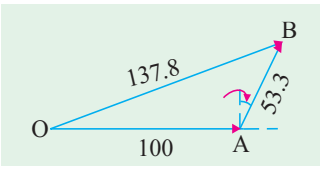


Fig. 37.50

The two value of regulation, found by the two methods, are found to differ widely from each other. The first method gives somewhat lesser value, while the other method gives a little higher value as compared to the actual value. However, the first value is more likely to be nearer the actual value, because the second method employs Z_S, which does not have a constant value. Its value depends on the field excitation.

Example 37.32. *The open-and short-circuit test readings for a 3-ϕ, star-connected, 1000-kVA, 2000 V, 50-Hz, synchronous generator are :*

Field Amps ;	*10*	*20*	*25*	*30*	*40*	*50*
O.C. Terminal V	*800*	*1500*	*1760*	*2000*	*2350*	*2600*
S.C. armature current in A:	—	*200*	*250*	*300*	—	—

The armature effective resistance is 0.2 Ω per phase. Draw the characteristic curves and estimate the full-load percentage regulation at (a) 0.8 p.f. lagging (b) 0.8 p.f. leading.

Solution. The O.C.C. and S.C.C. are plotted in Fig. 37.51

The phase voltages are : 462, 866, 1016, 1155, 1357, 1502.

Full-load phase voltage $= 2000/\sqrt{3} = 1155$ V

Full-load current $= 1,000,000/2000 \times \sqrt{3} = 288.7$ A

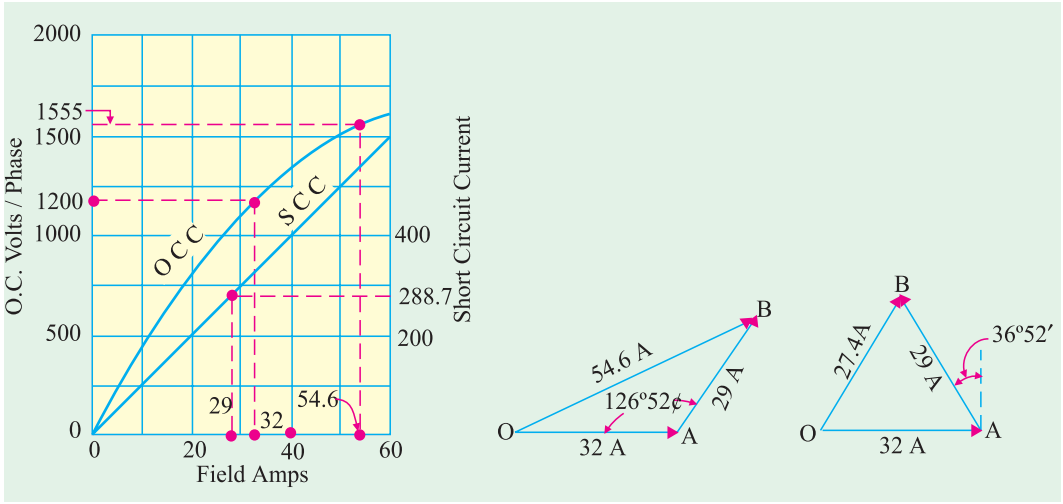


Fig. 37.51 Fig. 37.52 Fig. 37.53

Voltage/phase at full-load at 0.8 p.f. = $V + IR_a \cos\phi = 1155 + (288.7 \times 0.2 \times 0.8) = 1200$ volt

Form open-circuit curve, it is found that field current necessary to produce this voltage = 32 A.

From short-circuit characteristic, it is found that field current necessary to produce full-load current of 288.7 A is = 29 A.

(a) $\cos\phi = 0.8$, $\phi = 36°52'$ (lagging)

In Fig. 37.52, $OA = 32$ A, $AB = 29$ A and is at an angle of $(90° + 36°52') = 126°52'$ with OA. The total field current at full-load 0.8 p.f. lagging is $OB = 54.6$ A

O.C. volt corresponding to a field current of 54.6 A is = 1555 V

% regn. $= (1555 - 1155) \times 100/1155 = 34.6\%$

(b) In this case, as p.f. is leading, AB is drawn with OA (Fig. 37.53) at an angle of $90° - 36°\,52' = 53°8'$. $OB = 27.4$ A.

O.C. voltage corresponding to 27.4 A of field excitation is 1080 V.

$$\% \text{ regn.} = \frac{1080 - 1155}{1155} \times 100 = \mathbf{-6.4\%}$$

Example 37.33. *A 3-phase, 800-kV A, 3,300-V, 50-Hz alternator gave the following results:*

Exciting current (A)	*50*	*60*	*70*	*80*	*90*	*100*
O.C. volt (line)	*2560*	*3000*	*3300*	*3600*	*3800*	*3960*
S.C. current	*190*	—	—	—	—	—

The armature leakage reactance drop is 10% and the resistance drop is 2% of the normal voltage. Determine the excitation at full-load 0.8 power factor lagging by the m.m.f. method.

Solution. The phase voltages are : 1478, 1732, 1905, 2080, 2195, 2287

The O.C.C. is drawn in Fig. 37.54.

Normal phase voltage $= 3300/\sqrt{3} = 1905$ V; IR_a drop = 2% of 1905 = 38.1 volt

Leakage reactance drop = 10% of 1905 = 190.5 Volt

$$\therefore \quad E = \sqrt{[(1905 \times 0.8 \times + 38.1)^2 + (1905 \times 0.6 + 190.5)^2]} = 2{,}068 \text{ V}$$

The exciting current required to produce this voltage (as found from O.C.C.) is 82 A.

$$\text{Full load current} = 800.000/\sqrt{3} \times 3300 = 140\text{A}$$

As seen from S.C.C., the exciting current required to produce this full-load current of 140 A on short-circuit is 37 A.

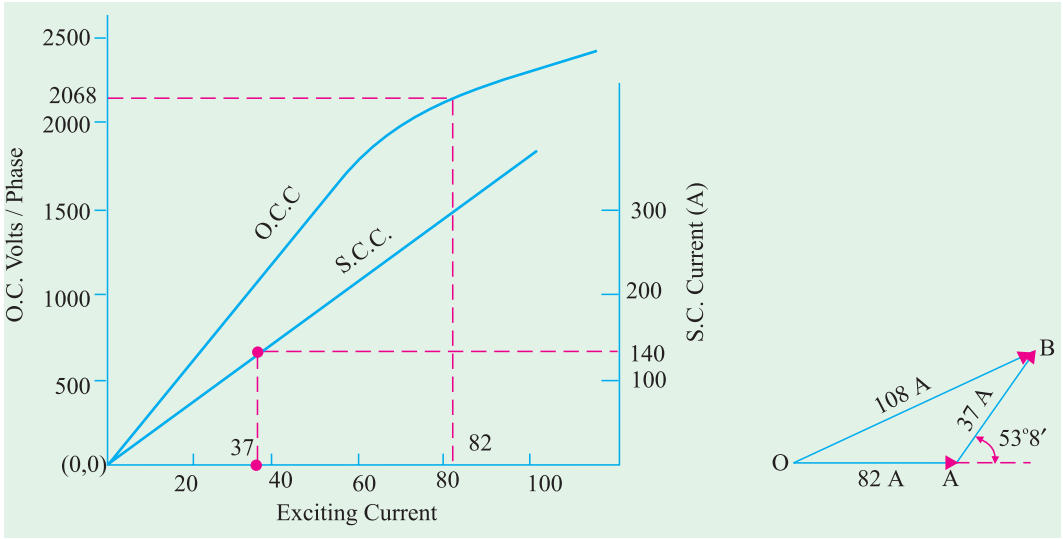


Fig. 37.54

Fig. 37.55

In Fig. 37.55, *OB* gives the excitation required on full-load to give a terminal phase voltage of 1905 V (or line voltage of 3300 V) at 0.8 p.f. lagging and its value is

$$= \sqrt{82^2 + 37^2 + 2 \times 82 \times 37 \times \cos 53°8'} = \mathbf{108\ A}$$

Tutorial Problem No. 37.3

1. A 30-kVA, 440-V, 50-Hz, 3-ϕ, star-connected synchronous generator gave the following test data :

Field current (A)	:	2	4	6	7	8	10	12	14
Terminal volts	:	155	287	395	440	475	530	570	592
S.C. current	:	11	22	34	40	46	57	69	80

Resistance between any two terminals is 0.3 Ω

Find regulation at full-load 0.8 p.f. lagging by (*a*) synchronous impedance method and (*b*) Rothert's ampere-turn method. Take Z_S corresponding to S.C. current of 80 A. **[(*a*) 51% (b) 29.9%]**

37.24. Zero Power Factor Method or Potier Method

This method is based on the separation of armature-leakage reactance drop and the armature reaction effects. Hence, it gives more accurate results. It makes use of the first two methods to some extent. The experimental data required is ***(i)*** no-load curve and ***(ii)*** full-load zero power factor curve (not the short-circuit characteristic) also called wattless load characteristic. It is the curve of terminal volts against excitation when armature is delivering F.L. current at zero p.f.

The reduction in voltage due to armature reaction is found from above and ***(ii)*** voltage drop due to armature leakage reactance X_L (also called Potier reactance) is found from both. By combining these two, E_0 can be calculated.

It should be noted that if we vectorially add to V the drop due to resistance and leakage reactance X_L, we get E. If to E is further added the drop due to armature reaction (assuming lagging p.f.), then we get E_0 (Art. 37.18).

The zero p.f. lagging curve can be obtained.

(a) if a similar machine is available which may be driven at no-load as a synchronous motor at practically zero p.f. or

(b) by loading the alternator with pure reactors

(c) by connecting the alternator to a 3-ϕ line with ammeters and wattmeters connected for measuring current and power and by so adjusting the field current that we get full- load armature current with zero wattmeter reading.

Point *B* (Fig. 37.56) was obtained in this manner when wattmeter was reading zero. Point *A* is obtained from a short-circuit test with full-load armature current. Hence, *OA* represents field current which is equal and opposite to the demagnetising armature reaction and for balancing leakage reactance drop at full-load (please refer to A.T. method). Knowing these two points, full-load zero p.f. curve *AB* can be drawn as under.

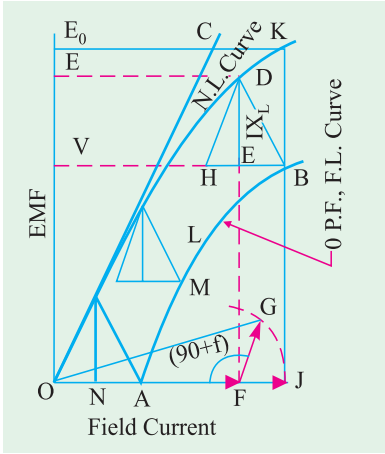


Fig. 37.56

From *B*, *BH* is drawn equal to and parallel to *OA*. From *H*, *HD* is drawn parallel to initial straight part of *N-L* curve *i.e.* parallel to *OC*, which is tangential to *N-L* curve. Hence, we get point *D* on no-load curve, which corresponds to point *B* on full-load zero p.f. curve. The triangle *BHD* is known as Potier triangle. This triangle is constant for a given armature current and hence can be transferred to give us other points like *M*, *L* etc. Draw *DE* perpendicular to *BH*. The length *DE* represents the drop in voltage due to armature leakage reactance X_L *i.e.* $I.X_L$. *BE* gives field current necessary to overcome demagnetising effect of armature reaction at full-load and *EH* for balancing the armature leakage reactance drop *DE*.

Let *V* be the terminal voltage on full-load, then if we add to it vectorially the voltage drop due to armature leakage reactance alone (neglecting R_a), then we get voltage $E = DF$ (and not E_0). Obviously, field excitation corresponding to *E* is given by *OF*. *NA* (= *BE*) represents the field current needed to overcome armature reaction. Hence, if we add *NA* vectorially to *OF* (as in Rothert's A.T. method) we get excitation for E_0 whose value can be read from *N-L* curve.

In Fig. 37.56, *FG* (= *NA*) is drawn at an angle of (90° + ϕ) for a lagging p.f. (or it is drawn at an angle of 90° – ϕ for a leading p.f.). The voltage corresponding to this excitation is $JK = E_0$

$$\therefore \quad \% \text{ regn.} = \frac{E_0 - V}{V} \times 100$$

The vector diagram is also shown separately in Fig. 37.57.

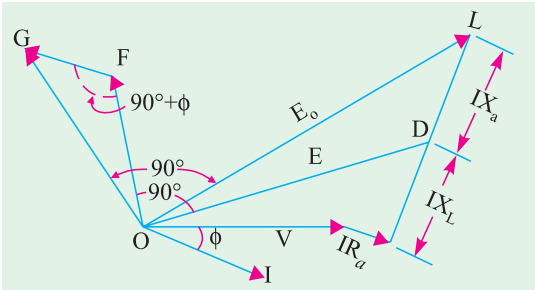


Fig. 37.57

Assuming a lagging p.f. with angle ϕ, vector for *I* is drawn at an angle of ϕ to *V*. IR_a is drawn parallel to current vector and IX_L is drawn perpendicular to it. *OD* represents voltage *E*. The excitation corresponding to it *i.e.*. *OF* is drawn at 90° ahead of it. *FG* (= *NA* = *BE* in Fig. 37.56) representing field current equivalent of full-load armature reaction, is drawn parallel to current vector *OI*. The closing side *OG* gives field excitation for E_0. Vector for E_0 is 90° lagging behind *OG*. *DL* represents voltage drop due to armature reaction.

37.25. Procedural Steps for Potier Method 1.

1. Suppose we are given *V*-the terminal voltage/phase.
2. We will be given or else we can calculate armature leakage reactance X_L and hence can calculate IX_L.

3. Adding IX_L (and IR_a if given) vectorially to V, we get voltage E.
4. We will next find from N-L curve, field excitation for voltage E. Let it be i_{f1}.
5. Further, field current i_{f2} necessary for balancing armature reaction is found from Potier triangle.
6. Combine i_{f1} and i_{f2} vertorially (as in *A.T.* method) to get i_f.
7. Read from N-L curve, the e.m.f. corresponding to i_f. This gives us E_0. Hence, regulation can be found.

Example 37.34. *A 3-phase, 6,00-V alternator has the following O.C.C. at normal speed :*

Field amperes	:	14	18	23	30	43
Terminal volts	:	4000	5000	6000	7000	8000

With armature short-circuited and full-load current flowing the field current is 17 A and when the machine is supplying full-load of 2,000 kVA at zero power factor, the field current is 42.5 A and the terminal voltage is 6,000 V.

Determine the field current required when the machine is supplying the full-load at 0.8 p.f. lagging. **(A.C. Machines-I, Jadavpur Univ. 1988)**

Solution. The *O.C.C.* is drawn in Fig. 37.58 with phase voltages which are

2310, 2828, 3465 4042 4620

The full-load zero p.f. characteristic can be drawn because two points are known *i.e.* (17, 0) and (42.5, 3465).

In the Potier ΔBDH, line DE represents the leakage reactance drop ($= IX_L$) and is (by measurement) equal to 450 V. As seen from Fig. 37.59.

$$E = \sqrt{(V\cos\phi)^2 + (V\sin\phi + IX_L)^2}$$

$$= \sqrt{(3465\times 0.8)^2 + (3465\times 0.6 + 450)^2}$$

$$= 3750 \text{ V}$$

From *O.C.C.* of Fig. 37.58, it is found that field amperes required for this voltage = 26.5 *A*.

Field amperes required for balancing armature reaction = BE = 14.5 A (by measure-ment from Potier triangle BDH).

As seen from Fig. 37.60, the field currents are added vectorially at an angle of $(90° + \phi) = 126° 52'$.

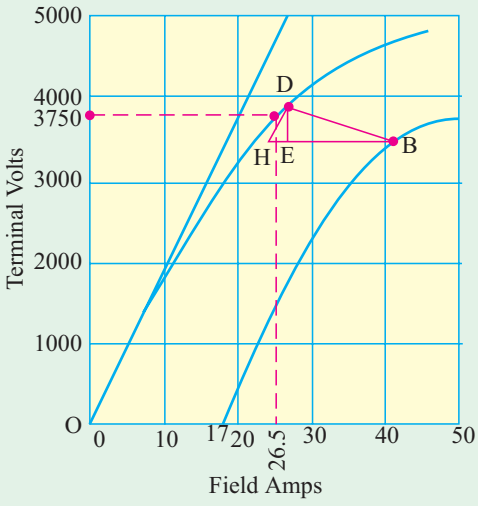


Fig. 37.58

Resultant field current is $OB = \sqrt{26.5^2 + 14.5^2 + 2\times 26.5\times 14.4\cos 53°8'}$ = **37.2 A**

Example 37.35. *An 11-kV, 1000-kVA, 3-phase, Y-connected alternator has a resistance of 2 Ω per phase. The open-circuit and full-load zero power factor characteristics are given below. Find the voltage regulation of the alternator for full load current at 0.8 p.f. lagging by Potier method.*

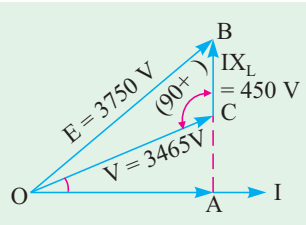


Fig. 37.59

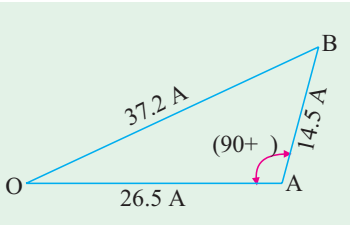


Fig. 37.60

Field current (A)	:	*40*	*50*	*110*	*140*	*180*
O.C.C. line voltage	:	*5,800*	*7,000*	*12,500*	*13,750*	*15,000*
Line volts zero p.f.		*0*	*1500*	*8500*	*10,500*	*12,500*

(Calcutta Univ. 1987 and S. Ramanandtirtha Univ. Nanded, 2001)

Solution. The O.C.C. and full-load zero p.f. curve for phase voltage are drawn in Fig. 37.61. The corresponding phase voltages are :

O.C.C. phase voltage	3350	4040	7220	7940	8660
Phase voltage zero p.f.	0	866	4900	6060	7220

Full-load current $= 1000 \times 1000/\sqrt{3} \times 11{,}000 = 52.5$ A

Phase voltage $= 11{,}000/\sqrt{3} = 6{,}350$ A

In the Potier ΔABC, $AC = 40$ A, CB is parallel to the tangent to the initial portion of the *O.C.C.* and BD is $\perp$ to AC.

BD = leakage reactance drop $IX_L = 1000$ V – by measurement

$AD = 30$ A — field current required to overcome demagnetising effect of armature reaction on full-load.

As shown in Fig. 37.62,

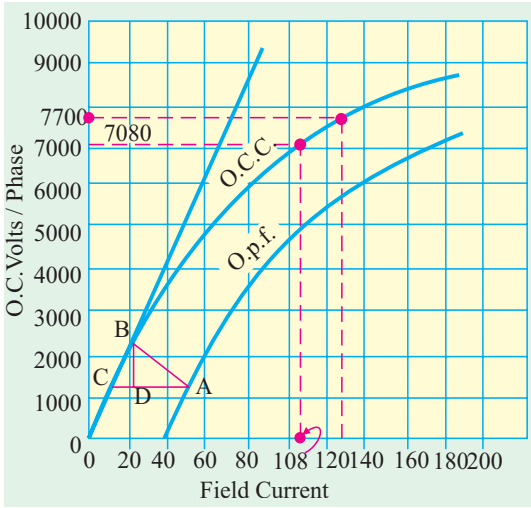


Fig. 37.61

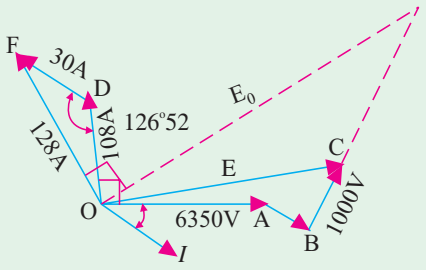


Fig. 37.62

$OA = 6{,}350$ V; $AB = IR_a = 52.5 \times 2 = 105$ V

$IX_L = BC = 1000$ V —by measurement

$$OC = E = \sqrt{(V\cos\phi + IR_a)^2 + (V\sin\phi + IX_L)^2}$$

$$= \sqrt{(6350 \times 0.8 + 105)^2 + (6350 \times 0.6 + 1000)^2}\ ;\ E = 7{,}080 \text{ V}$$

As seen from O.C.C., field current required for 7,080 V is 108 A. Vector OD (Fig. 37.62) represents 108 A and is drawn $\perp$ to OC. DF represents 30 A and is drawn parallel to OI or at $(90° + 36° 52')$ = 126° 52′ with OD. Total field current is OF.

$$OF = \sqrt{108^2 + 30^2 + 2 \times 108 \times 30 \cos 53°8'} = 128 \text{ A}$$

From O.C.C., it is found that the e.m.f. corresponding to this field current is 7,700V

$$\therefore \quad E_0 = 7{,}700 \text{ V; regulation} = \frac{7{,}700 - 6{,}350}{6{,}350} \times 100 = \mathbf{21.3\ per\ cent}$$

Example 37.36. *The following test results were obtained on a 275-kW, 3-ϕ, 6,600-V non-salient pole type generator.*

Open-circuit characteristic :

Volts	:	*5600*	*6600*	*7240*	*8100*
Exciting amperes	:	*46.5*	*58*	*67.5*	*96*

Short-circuit characteristic : Stator current 35 A with an exciting current of 50 A. Leakage reactance on full-load = 8%. Neglect armature resistance. Calculate as accurately as possible the exciting current (for full-load) at power factor 0.8 lagging and at unity. **(City & Guilds, London)**

Solution. First convert the O.C. line volts into phase volts by dividing the given terminal values by $\sqrt{3}$.

$\therefore$ O.C. volts (phase) : 3233, 3810, 4180, 4677.

O.C.C. is plotted in Fig. 37.63. For plotting S.C.C., we need two points. One is (0, 0) and the other is (50 A, 35 A). In fact, we can do without plotting the S.C.C. because it being a straight line, values of field currents corresponding to any armature current can be found by direct ratio.

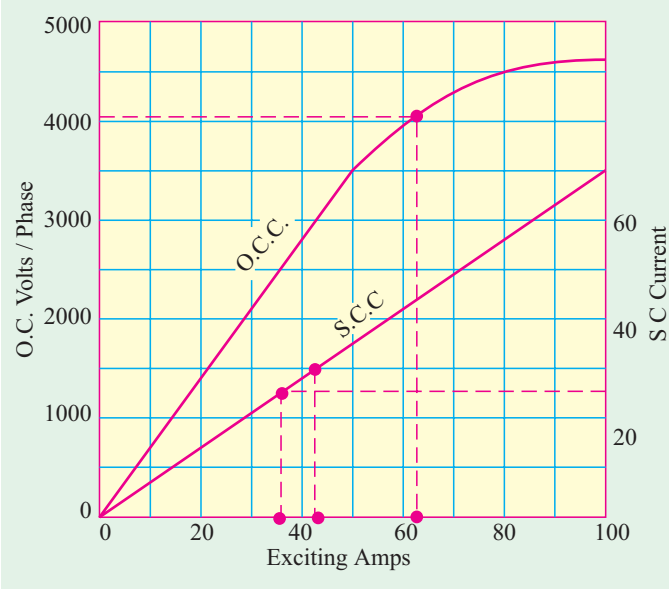


Fig. 37.63

Leakage reactance drop

$$= \frac{3810 \times 8}{100} = 304.8 \text{ V}$$

Normal phase voltage

$$= 6,600/\sqrt{3} = 3,810 \text{ V}$$

In Fig. 37.64, *OA* = 3810 *V* and at an angle ϕ ahead of current vector *OI*.

AB = 304.8 V is drawn at right angles to *OI*. Resultant of the two is *OB* = 4010 V.

From O.C.C., field current corresponding to 4,010 V is 62 A.

Full-load current at 0.8 p.f. = $275,000/\sqrt{3} \times 6600 \times 0.8 = 30$ A

35 A of armature current need 50 A of field current, hence 30 A of armature current need $30 \times 50/35 = 43$ A.

In Fig. 37.64, *OC* = 62 A is drawn at right angles to *OB*. Vector *CD* = 43 A is drawn parallel to *OI*. Then, *OD* = **94.3 A**

Note. Here. 43 A pf field excitation is assumed as having all been used for balancing armature reaction. In fact, a part of it is used for balancing armature leakage drop of 304.8 V. This fact has been clarified in the next example.

At Unity p.f.

In Fig. 37.65, *OA* again represents *V* = 3810 V, *AB* = 304.8 V and at right angles to *OA*.

The resultant $OB = \sqrt{(3810^2 + 304.8^2)} = 3830$ V

Field current from O.C.C. corresponding to this voltage = 59.8 A.

Hence, *OC* = 59.8 A is drawn perpendicular to *OB* (as before)

Full-load current at u.p.f. = $275,000/\sqrt{3} \times 6600 \times 1 = 24$ A

Now, 35 A armature current corresponds to a field current of 50 A, hence 24 A of armature current corresponds to $50 \times 24/35 = 34.3$ A.

Hence, CD = 34.3 A is drawn || to OA (and ⊥ to OC approximately).*

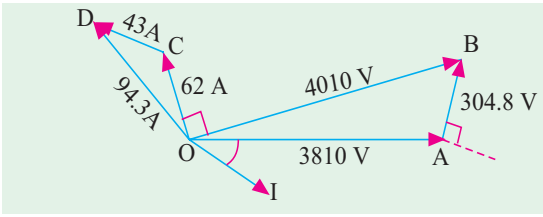


Fig. 37.64

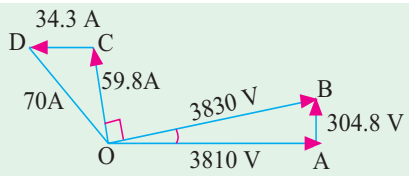


Fig. 37.65

$\therefore$ $OD = \sqrt{(59.8^2 + 34.3^2)} =$ **70 A**

Example 37.37. *A 600-kVA, 3,300-V, 8-pole, 3-phase, 50-Hz alternator has following characteristic :*

Amp-turns/pole :	*4000*	*5000*	*7000*	*10,000*
Terminal E.M.F. :	*2850*	*3400*	*3850*	*4400*

There are 200 conductor in series per phase.

Find the short-circuit characteristic, the field ampere-turns for full-load 0.8 p.f. (lagging) and the voltage regulation, having given that the inductive drop at full-load is 7% and that the equivalent armature reaction in amp-turns per pole = 1.06 × ampere-conductors per phase per pole.

(London Univ.)

Solution. O.C. terminal voltages are first converted into phase voltages and plotted against field amp-turns, as shown in Fig. 37.66.

Full-load current

$$= \frac{600,000}{\sqrt{3} \times 3300} = 105 \text{ A}$$

Demagnetising amp-turns per pole per phase for full-load at zero p.f.

$$= 1.06 \times 105 \times 200/8$$

$$= 2,780$$

Normal phase voltage

$$= 3300 / \sqrt{3} = 1910 \text{ volt}$$

Leakage reactance drop

$$= \frac{3300 \times 7}{\sqrt{3} \times 100} = 133 \text{ V}$$

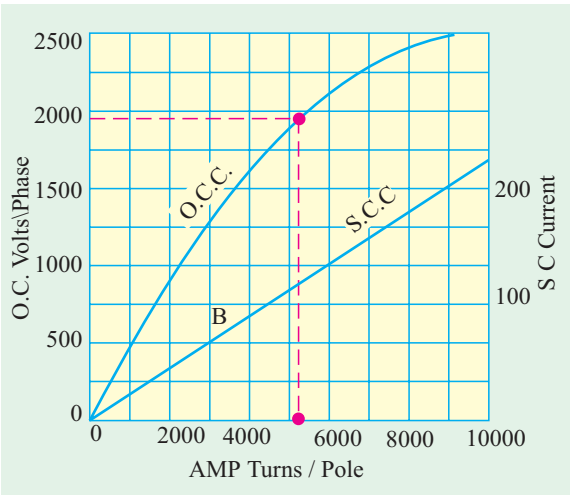


Fig. 37.66

In Fig. 37.67, OA represents 1910 V.

AB = 133 V is drawn ⊥ OI, OB is the resultant voltage E (not E_0).

$\therefore$ $OB = E$ = 1987 volt

From O.C.C., we find that 1987 V correspond to 5100 field amp-turns. Hence, OC = 5100 is drawn ⊥ to OB. CD = 2780 is || to OI. Hence, OD = 7240 (approx). From O.C.C. it is found that this

---

* It is so because angle between OA and OB is negligibly small. If not, then CD should be drawn at an angle of $(90 + \alpha)$ where α is the angle between OA and OB.

corresponds to an O.C. voltage of 2242 volt. Hence, when load is thrown off, the voltage will rise to 2242 V.

$$\therefore \quad \% \text{ regn.} = \frac{2242 - 1910}{1910} \times 100$$
$$= \mathbf{17.6\ \%}$$

How to deduce S.C.C. ?

We have found that field amp-turns for balancing armature reaction only are 2,780. To this should be added field amp-turns required for balancing the leakage reactance voltage drop of 133 V.

Field amp-turns corresponding to 133 volt on O.C. are 300 approximately. Hence, with reference to Fig. 37.56, $NA = 2780$, $ON = 300$

$\therefore$ Short-circuit field amp-turns
$= OA = 2780 + 300$
$= 3080$

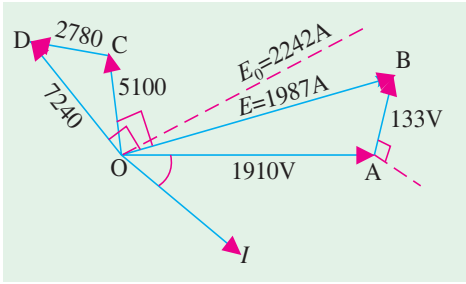


Fig. 37.67

Hence, we get a point B on S.C.C. *i.e.* (3080, 105) and the other point is the origin. So S.C.C. (which is a straight line) can be drawn as shown in Fig. 37.66.

Example 37.38. *The following figures give the open-circuit and full-load zero p.f saturation curves for a 15,000-kVA. 11,000 V, 3-ϕ, 50-Hz, star-connected turbo-alternator:*

Field AT in 10^3	:	10	18	24	30	40	45	50
O.C. line kV	:	4.9	8.4	10.1	11.5	12.8	13.3	13.65
Zero p.f. full-load line kV	:	—	0	—	—	—	10.2	—

Find the armature reaction, the armature reactance and the synchronous reactance. Deduce the regulation for full-load at 0.8 power lagging.

Solution. First, O.C.C. is drawn between phase voltages and field amp-turns, as shown in Fig. 37.68.

Full-load, zero p.f. line can be drawn, because two points are known *i.e.* A (18, 0) and C (45, 5890). Other points on this curve can be found by transferring the Potier triangle. At point C, draw $CD \parallel$ to and equal to OA and from D draw $DE \parallel$ to ON. Join EC. Hence, CDE is the Potier triangle.

Line EF is $\perp$ to DC

CF = field amp-turns for balancing armature-reaction only
= 15,700

$EF = GH$ = 640 volt = leakage reactance drop/phase

Short-circuit A.T. required = OA = 18,000

$$\text{Full-load current} = \frac{15,000 \times 1000}{\sqrt{3} \times 11,000} = 788 \text{ A}$$

$\therefore \quad 640 = I \times X_L \quad \therefore \quad X_L = 640/788 = 0.812\ \Omega$

From O.C.C., we find that 18,000 A.T. correspond to an O.C. voltage of $8,400/\sqrt{3} = 4,850$ V.

$$\therefore \quad Z_S = \frac{\text{O.C. volt}}{\text{S.C. cuerrent}} = \frac{4,850}{788}$$
$$= 6.16\ \Omega$$

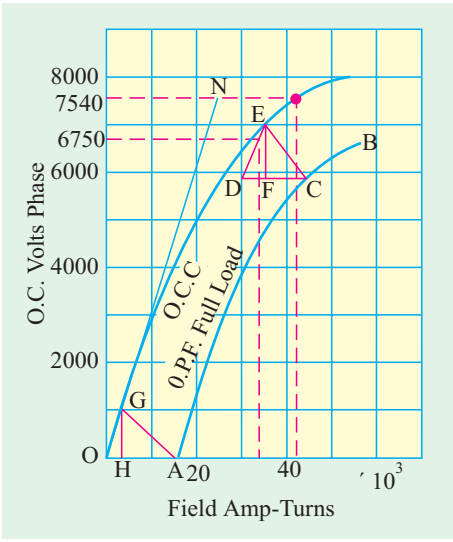


Fig. 37.68

(Art. 37.21)

As R_a is negligible, hence Z_S equals X_S.

Regulation

In Fig. 37.69, OA = phase voltage = $11{,}000/\sqrt{3}$

= 6,350 V

AB = 640 V and is drawn at right angles to OI or at $(90° + \phi)$ to OA.

Resultant is OB = 6,750 V

Field A.T. corresponding to O.C voltage of 6,750 V is = OC = 30,800 and is drawn $\perp$ to OB.

CD = armature reaction at F.L. = 15,700 and is drawn || to OI or at $(90° + \phi)$ to OC.

Hence, OD = 42,800.

From O.C.C., e.m.f. corresponding to 42,800 A.T. of rotor = 7,540 V

$\therefore$ % regn. up = (7,540 – 6,350)/6,350 = **0.187 or 18.7%**

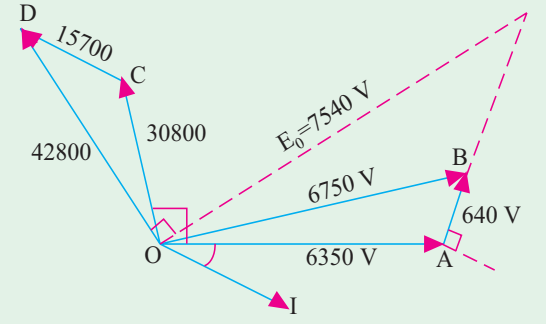


Fig. 37.69

Tutorial Problem No. 37.4

1. The following data relate to a 6,600-V, 10,000-kVA, 50-Hz, 3-ϕ, turbo- alternator:

O.C. kilovolt	4.25	5.45	6.6	7.3	8	9
Exciting A.T. in 10^3	60	80	100	120	145	220

Excitation needed to circulate full-load current on short circuit : 117,000 A.T. Inductive drop in stator winding at full-load = 15%. Find the voltage regulation at full-load 0.8 power factor.

[34.4%] (*City & Guilds, London*)

2. Deduce the exciting current for a 3-ϕ, 3300-V generator when supplying 100 kW at 0.8 power factor lagging, given magnetisation curve on open-circuit :

Line voltage :	3300	3600	3900
Exciting current :	80	96	118

There are 16 poles, 144 slots, 5 conductors/slot, single-circuit, full-pitched winding, star- connected. The stator winding has a resistance per phase of 0.15 Ω and a leakage reactance of 1.2 Ω. The field coils have each 108 turns. **[124 A] (*London Univ.*)**

3. Estimate the percentage regulation at full-load and power factor 0.8 lagging of a 1000-kVA, 6,600-V, 3-ϕ, 50-Hz, star-connected salient-pole synchronous generator. The open-circuit characteristic is as follows :

Terminal volt	4000	6000	6600	7200	8000
Field A.T.	5200	8500	10,000	12,500	17,500

Leakage reactance 10%, resistance 2%. Short-circuit characteristic : full-load current with a field excitation of 5000 A.T. Take the permeance to cross armature reaction as 35% of that to direct reaction. **[20% up]**

4. A 1000-kVA, 11,000-V, 3-ϕ, 50-Hz, star-connected turbo-generator has an effective resistance of 2Ω /phase. The O.C.C. and zero p.f. full-load data is as follows :

O.C. volt	5,805	7,000	12,550	13,755	15,000
Field current A	40	50	110	140	180
Terminal volt at F.L. zero p.f.	0	1500	8,500	10,500	12,400

Estimate the % regulation for F.L. at 0.8 p.f. lagging. **[22 %]**

5. A 5-MVA, 6.6 kV, 3-ϕ, star-connected alternator has a resistance of 0.075 Ω per phase. Estimate the regulation for a load of 500 A at p.f. (*a*) unity and (*b*) 0.9 leading (*c*) 0.71 lagging from the following open-circuit and full-load zero power factor curve.

Field current (A)	Open-circuit terminal voltage (V)	Saturation curve zero p.f.
32	3100	0
50	4900	1850
75	6600	4250
100	7500	5800
140	8300	7000

[(a) 6.3% (b) –7.9% (c) 20.2%] (*Electrical Machines-II, Indore Univ. Feb. 1978*)

37.26. Operation of a Salient Pole Synchronous Machine

A multipolar machine with cylindrical rotor has a uniform air-gap, because of which its reactance remains the same, irrespective of the spatial position of the rotor. However, a synchronous machine with salient or projecting poles has non-uniform air-gap due to which its reactance varies with the rotor position. Consequently, a cylindrical rotor machine possesses one axis of symmetry (pole axis or direct axis) whereas salient-pole machine possesses two axes of geometric symmetry ***(i)*** field poles axis, called direct axis or *d*-axis and ***(ii)*** axis passing through the centre of the interpolar space, called the quadrature axis or *q*-axis, as shown in Fig. 37.70.

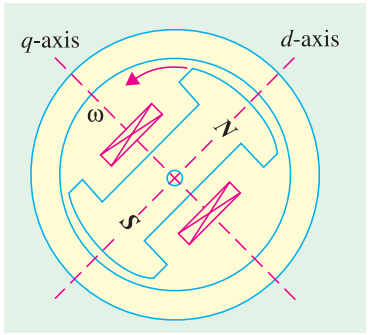


Fig. 37.70

Obviously, two mmfs act on the *d*-axis of a salient-pole synchronous machine *i.e.* field m.m.f. and armature m.m.f. whereas only one m.m.f., *i.e.* armature mmf acts on the *q*-axis, because field mmf has no component in the *q*-axis. The magnetic reluctance is low along the poles and high between the poles. The above facts form the basis of the two-reaction theory proposed by Blondel, according to which

(i) armature current I_a can be resolved into two components *i.e.* I_d perpendicular to E_0 and I_q along E_0 as shown in Fig. 37.71 (*b*).

(ii) armature reactance has two components *i.e.* *q*-axis armature reactance X_{ad} associated with I_d and *d*-axis armature reactance X_{aq} linked with I_q.

If we include the armature leakage reactance X_l which is the same on both axes, we get

$$X_d = X_{ad} + X_l \text{ and } X_q = X_{aq} + X_l$$

Since reluctance on the *q*-axis is higher, owing to the larger air-gap, hence,

$$X_{aq} < X_{ad} \text{ or } X_q < X_d \text{ or } X_d > X_q$$

37.27. Phasor Diagram for a Salient Pole Synchronous Machine

The equivalent circuit of a salient-pole synchronous generator is shown in Fig. 37.71 (*a*). The component currents I_d and I_q provide component voltage drops $jI_d X_d$ and $j I_q X_q$ as shown in Fig. 37.71(*b*) for a lagging load power factor.

The armature current I_a has been resolved into its rectangular components with respect to the axis for excitation voltage E_0.The angle ψ between E_0 and I_a is known as the internal power factor angle. The

vector for the armature resistance drop $I_a R_a$ is drawn parallel to I_a. Vector for the drop $I_d X_d$ is drawn perpendicular to I_d whereas that for $I_q \times X_q$ is drawn perpendicular to I_q. The angle δ between E_0 and V is called the power angle. Following phasor relationships are obvious from Fig. 37.71 (*b*)

$$E_0 = V + I_a R_a + jI_d X_d + jI_q X_q \text{ and } I_a = I_d + I_q$$

If R_a is neglected the phasor diagram becomes as shown in Fig. 37.72 (*a*). In this case,

$$E_0 = V + jI_d X_d + jI_q X_q$$

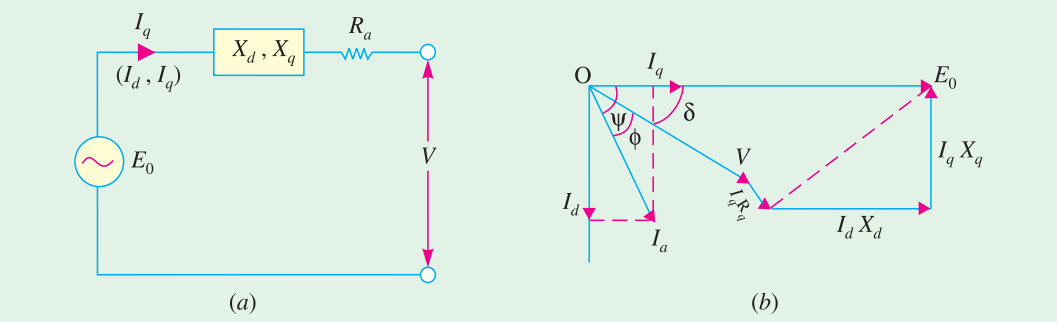


Fig. 37.71

Incidentally, we may also draw the phasor diagram with terminal voltage V lying in the horizontal direction as shown in Fig. 37-72 (*b*). Here, again drop $I_a R_a$ is $\parallel I_a$ and $I_d X_d$ is $\perp$ to I_d and drop $I_q X_q$ is $\perp$ to I_q as usual.

37.28. Calculations from Phasor Diagram

In Fig. 37.73, dotted line *AC* has been drawn perpendicular to I_a and *CB* is perpendicular to the phasor for E_0. The angle $ACB = \psi$ because angle between two lines is the same as between their perpendiculars. It is also seen that

$$I_d = I_a \sin\psi \; ; I_q = I_a \cos\psi \; ; \text{ hence, } I_a = I_q/\cos\psi$$

In ΔABC, $\quad BC/AC = \cos\psi$ or $AC = BC/\cos\psi = I_q X_q / \cos\psi = I_a X_q$

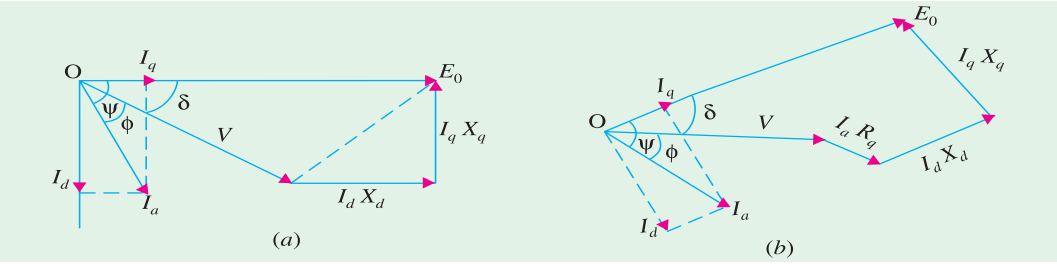


Fig. 37.72

From ΔODC, we get

$$\tan\psi = \frac{AD + AC}{OE + ED} = \frac{V\sin\phi + I_a X_q}{V\cos\phi + I_a R_a} \quad \text{—generating}$$

$$= \frac{V\sin\phi - I_a X_q}{V\sin\phi - I_a R_a} \quad \text{—motoring}$$

The angle ψ can be found from the above equation. Then, $\delta = \psi - \phi$ (generating) and $\delta = \phi - \psi$ (motoring)

As seen from Fig. 37.73, the excitation voltage is given by

$E_0 = V\cos\delta + I_q R_a + I_d X_d$ —generating

$= V\cos\delta - I_q R_a - I_d X_d$ —motoring

Note. Since angle ϕ is taken positive for lagging p.f., it will be taken negative for leading p.f.

If we neglect the armatrue resistance as shown in Fig. 37.72, then angle δ can be found directly as under :

$\psi = \phi + \delta$ (generating)

and $\psi = \phi - \delta$ (motoring).

In general, $\psi = (\phi \pm \delta)$.

$I_d = I_a \sin\psi$

$= I_a \sin(\phi \pm \delta)$; $I_q = I_a \cos\psi = I_a \cos(\phi \pm \delta)$

As seen from Fig. 37.73, $V \sin\delta = I_q X_q = I_a X_q \cos(\phi \pm \delta)$

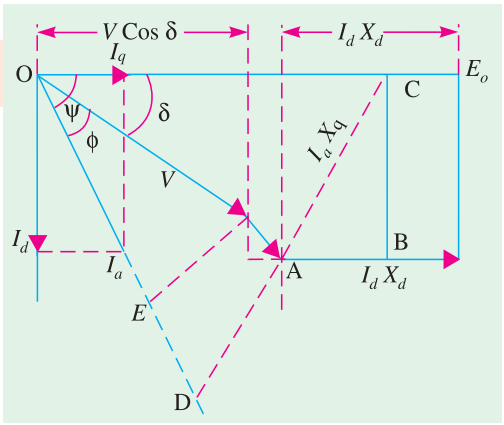


Fig. 37.73

$$\therefore \quad V\sin\delta = I_a X_q(\cos\phi\cos\delta \pm \sin\phi\sin\delta)$$

$$\text{or} \quad V = I_a X_q \cos\phi\cot\delta \pm I_a X_q \sin\phi$$

$$\therefore \quad I_a X_q \cos\phi\cot\delta = V \pm I_a X_q \sin\phi$$

$$\therefore \quad \tan\delta = \frac{I_a X_q \cos\phi}{V \pm I_a X_q \sin\phi}$$

In the above expression, plus sign is for synchronous generators and minus sign for synchronous motors.

Similarly, when R_a is neglected, then,

$$E_0 = V\cos\delta \pm I_d X_d$$

However, if R_a and hence $I_a R_a$ drop is not negligible then,

$$E_0 = V\cos\delta + I_q R_a + I_d X_d \quad \text{—generating}$$

$$= V\cos\delta - I_q R_a - I_d X_d \quad \text{—motoring}$$

37.29. Power Developed by a synchronous Generator

If we neglect R_a and hence Cu loss, then the power developed (P_d) by an alternator is equal to the power output (P_{out}). Hence, the per phase power output of an alternator is

$$P_{out} = VI_a\cos\phi = \text{power developed } (p_d) \quad \ldots(i)$$

Now, as seen from Fig., 37.72 (*a*), $I_q X_q = V\sin\delta$; $I_d X_d = E_0 - V\cos\delta$ ...(*ii*)

Also, $I_d = I_a \sin(\phi + \delta)$; $I_q = I_a\cos(\phi + \delta)$ ...(*iii*)

Substituting Eqn. (*iii*) in Eqn. (*ii*) and solving for $I_a\cos\phi$, we get

$$I_a\cos\phi = \frac{V}{X_d}\sin\delta + \frac{V}{2X_q}\sin 2\delta - \frac{V}{2X_d}\sin 2\delta$$

Finally, substituting the above in Eqn. (*i*), we get

$$P_d = \frac{E_0 V}{X_d}\sin\delta + \frac{1}{2}V^2\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\sin 2\delta = \frac{E_0 V}{X_d}\sin\delta + \frac{V^2(X_d - X_q)}{2X_d X_q}\sin 2\delta$$

The total power developed would be three times the above power.

As seen from the above expression, the power developed consists of two components, the first term represents power due to field excitation and the second term gives the reluctance power *i.e.*

power due to saliency. If $X_d = X_q$ *i.e.* the machine has a cylinderical rotor, then the second term becomes zero and the power is given by the first term only. If, on the other hand, there is no field excitation *i.e.* $E_0 = 0$, then the first term in the above expression becomes zero and the power developed is given by the second term. It may be noted that value of δ is positive for a generator and negative for a motor.

Example 37.39. *A 3-phase alternator has a direct-axis synchronous reactance of 0.7 p.u. and a quadrature axis synchronous reactance of 0.4 p.u. Draw the vector diagram for full-load 0.8 p.f. lagging and obtain therefrom* ***(i)*** *the load angle and* ***(ii)*** *the no-load per unit voltage.*

(Advanced Elect. Machines, AMIE Sec. B 1991)

Solution. $V = 1$ p.u.; $X_d = 0.7$ p.u.; $X_q = 0.4$ p.u.;

$\cos\phi = 0.8$; $\sin\phi = 0.6$; $\phi = \cos^{-1} 0.8 = 36.9°$; $I_a = 1$ p.u.

(i) $$\tan\delta = \frac{I_a X_q \cos\phi}{V + I_q \sin\phi} = \frac{1 \times 0.4 \times 0.8}{1 + 0.4 \times 0.6} = 0.258, \delta = 16.5°$$

(ii) $$I_d = I_a \sin(\phi + \delta) = 1 \sin(36.9° + 14.9°) = 0.78 \text{ A}$$

$$E_0 = V\cos\delta + I_d X_d = 1 \times 0.966 + 0.78 \times 0.75 = \mathbf{1.553}$$

Example 37.40. *A 3-phase, star-connected, 50-Hz synchronous generator has direct-axis synchronous reactance of 0.6 p.u. and quadrature-axis synchronous reactance of 0.45 p.u. The generator delivers rated kVA at rated voltage. Draw the phasor diagram at full-load 0.8 p.f. lagging and hence calculate the open-circuit voltage and voltage regulation. Resistive drop at full-load is 0.015 p.u.*

(Elect. Machines-II, Nagpur Univ. 1993)

Solution. $I_a = 1$ p.u.; $V = 1$ p.u.; $X_d = 0.6$ p.u.; $X_q = 0.45$ p.u. ; $R_a = 0.015$ p.u.

$$\tan\psi = \frac{V\sin\phi + I_a X_q}{V\cos\phi + I_a R_a} = \frac{1 \times 0.6 + 1 \times 0.45}{1 \times 0.8 + 1 \times 0.015} = 1.288; \quad \psi = 52.2°$$

$$\delta = \psi - \phi = 52.2° - 36.9° = 15.3°$$

$$I_d = I_a \sin\psi = 1 \times 0.79 = 0.79 \text{ A} \; ; \; I_q = I_a \cos\psi = 1 \times 0.61 = 0.61\text{A}$$

$$E_0 = V\cos\delta + I_q R_a + I_d X_d$$

$$= 1 \times 0.965 + 0.61 \times 0.015 + 0.79 \times 0.6 = 1.448$$

$$\therefore \quad \%\text{ regn.} = \frac{1.448 - 1}{1} \times 100 = \mathbf{44.8\%}$$

Example 37.41. *A 3-phase, Y-connected syn. generator supplies current of 10 A having phase angle of 20° lagging at 400 V. Find the load angle and the components of armature current I_d and I_q if $X_d = 10$ ohm and $X_q = 6.5$ ohm. Assume arm. resistance to be negligible.*

(Elect. Machines-I, Nagpur Univ. 1993)

Solution. $\cos\phi = \cos 20° = 0.94$; $\sin\phi = 0.342$; $I_a = 10$ A

$$\tan\delta = \frac{I_a X_q \cos\phi}{V + I_a X_q \sin\phi} = \frac{10 \times 6.5 \times 0.94}{400 + 10 \times 6.5 \times 0.342} = 0.1447$$

$$\delta = 8.23°$$

$$I_d = I_a \sin(\phi + \delta) = 10 \sin(20° + 8.23°) = 4.73 \text{ A}$$

$$I_q = I_a \cos(\phi + \delta) = 10 \cos(20° + 8.23°) = 8.81 \text{ A}$$

Incidentally, if required, voltage regulation of the above generator can be found as under:

$$I_d X_d = 4.73 \times 10 = 47.3 \text{ V}$$

$$E_0 = V\cos\delta + I_d X_d = 400 \cos 8.23° + 47.3 = 443 \text{ V}$$

$$\% \text{ regn.} = \frac{E_0 - V}{V} \times 100$$

$$= \frac{443 - 400}{400} \times 100 = \mathbf{10.75\%}$$

Tutorial Problem No. 37.5.

1. A 20 MVA, 3-phase, star-connected, 50-Hz, salient-pole has $X_d = 1$ p.u.; $X_q = 0.65$ p.u. and $R_a = 0.01$ p.u. The generator delivers 15 MW at 0.8 p.f. lagging to an 11-kV, 50-Hz system. What is the load angle and excitation e.m.f. under these conditions? **[18°; 1.73 p.u]**
2. A salient-pole synchronous generator delivers rated kVA at 0.8 p.f. lagging at rated terminal voltage. It has $X_d = 1.0$ p.u. and $X_q = 0.6$ p.u. If its armature resistance is negligible, compute the excitation voltage under these conditions. **[1.77 p.u]**
3. A 20-kVA, 220-V, 50-Hz, star-connected, 3-phase salient-pole synchronous generator supplies load at a lagging power factor angle of 45°. The phase constants of the generator are $X_d = 4.0\ \Omega$; $X_q = 2\ \Omega$ and $R_a = 0.5\ \Omega$. Calculate (*i*) power angle and (*ii*) voltage regulation under the given load conditions. **[(*i*) 20.6° (*ii*) 142%]**
4. A 3-phase salient-pole synchronous generator has $X_d = 0.8$ p.u.; $X_q = 0.5$ p.u. and $R_a = 0$. Generator supplies full-load at 0.8 p.f. lagging at rated terminal voltage. Compute (*i*) power angle and (*ii*) no-load voltage if excitation remains constant. **[(*i*) 17.1° (*ii*) 1.6 p.u]**

37.30. Parallel Operation of Alternators

The operation of connecting an alternator in parallel with another alternator or with common bus-bars is known as ***synchronizing***. Generally, alternators are used in a power system where they are in parallel with many other alternators. It means that the alternator is connected to a live system of constant voltage and constant frequency. Often the electrical system to which the alternator is connected, has already so many alternators and loads connected to it that no matter what power is delivered by the incoming alternator, the voltage and frequency of the system remain the same. In that case, the alternator is said to be connected to ***infinite*** bus-bars.

It is never advisable to connect a stationary alternator to live bus-bars, because, stator induced e.m.f. being zero, a short-circuit will result. For proper synchronization of alternators, the following three conditions must be satisfied :

1. The terminal voltage (effective) of the incoming alternator must be the same as bus-bar voltage.
2. The speed of the incoming machine must be such that its frequency (= $PN/120$) equals bus-bar frequency.
3. The phase of the alternator voltage must be identical with the phase of the bus-bar voltage. It means that the switch must be closed at (or very near) the instant the two voltages have correct phase relationship.

Condition **(1)** is indicated by a voltmeter, conditions **(2)** and **(3)** are indicated by synchronizing lamps or a synchronoscope.

37.31. Synchronizing of Alternators

(*a*) Single-phase Alternators

Suppose machine 2 is to be synchronized with or 'put on' the bus-bars to which machine 1 is already connected. This is done with the help of two lamps L_1 and L_2 (known as synchronizing lamps) connected as shown in Fig. 37.74.

It should be noted that E_1 and E_2 are in-phase relative to the external circuit but are in direct phase opposition in the local circuit (shown dotted).

If the speed of the incoming machine 2 is not brought up to that of machine 1, then its frequency will also be different, hence there will be a phase-difference between their voltages (even when they are equal in magnitude, which is determined by field excitation). This phase-difference will be continously changing with the changes in their frequencies. The result is that their resultant voltage will undergo changes similar to the frequency changes of beats produced, when two sound sources of nearly equal frequency are sounded together, as shown in Fig. 37.75.

Sometimes the resultant voltage is maximum and some other times minimum. Hence, the current is alternatingly maximum and minimum. Due to this changing current through the lamps, a flicker will be produced, the frequency of flicker being ($f_2 \sim f_1$). Lamps will dark out and glow up alternately. Darkness indicates that the two voltages E_1 and E_2 are in exact phase opposition relative to the local circuit and hence

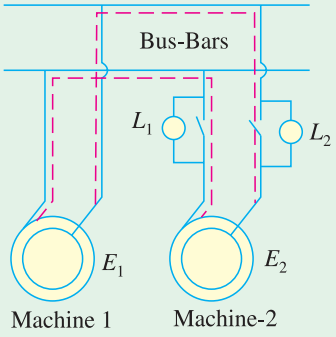


Fig. 37.74

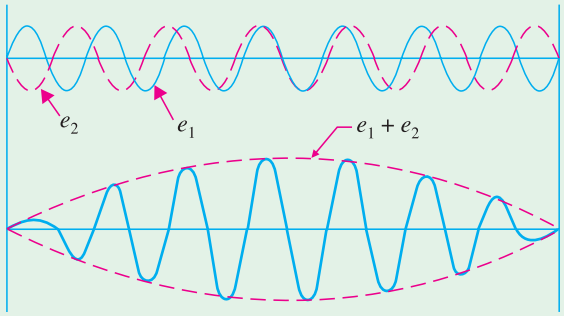


Fig. 37.75

there is no resultant current through the lamps. Synchronizing is done at the middle of the dark period. That is why, sometimes, it is known as 'lamps dark' synchronizing. Some engineers prefer 'lamps bright' synchronization because of the fact the lamps are much more sensitive to changes in voltage at their maximum brightness than when they are dark. Hence, a sharper and more accurate synchronization is obtained. In that case, the lamps are connected as shown in Fig. 37.76. Now, the lamps will glow brightest when the two voltages are in-phase with the bus-bar voltage because then voltage across them is twice the voltage of each machine.

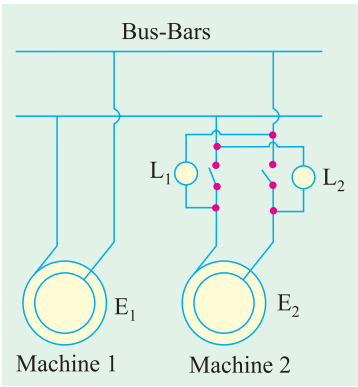


Fig. 37.76

(*b*) Three-phase Alternators

In 3-ϕ alternators, it is necessary to synchronize one phase only, the other two phases will then be synchronized automatically. However, first it is necessary that the incoming alternator is correctly 'phased out' *i.e.* the phases are connected in the proper order of *R, Y, B* and not *R, B, Y* etc.

In this case, three lamps are used. But they are deliberately connected asymmetrically, as shown in Fig. 37.77 and 37.78.

This transposition of two lamps, suggested by Siemens and Halske, helps to indicate whether the incoming machine is running too slow. If lamps were connected symmetrically, they would dark out or glow up simultaneously (if the phase rotation is the same as that of the bus-bars).

Lamp L_1 is connected between R and R', L_2 between Y and B' (not Y and Y') and L_3 between B and Y' (and not B and B'), as shown in Fig. 37.78.

Voltage stars of two machines are shown superimposed on each other in Fig. 37.79.

Two sets of star vectors will rotate at unequal speeds if the frequencies of the two machines are different. If the incoming alternator is running faster, then voltage star *R′Y′B′* will appear to rotate anticlockwise with respect to the bus-bar voltage star *RYB* at a speed corresponding to the difference between their frequencies. With reference to Fig. 37.79, it is seen that voltage across L_1 is *RR′* and is seen to be increasing from zero, that across L_2 is *YB′* which is decreasing, having just passed through its maximum, that across L_3 is *BY′* which is increasing and approaching its maximum. Hence, the lamps will light up one after the other in the order 2, 3, 1 ; 2, 3, 1 or 1, 2, 3.



The rotor and stator of 3-phase generator

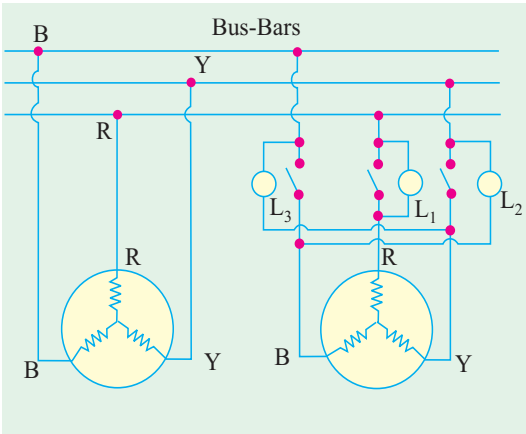


Fig. 37.77

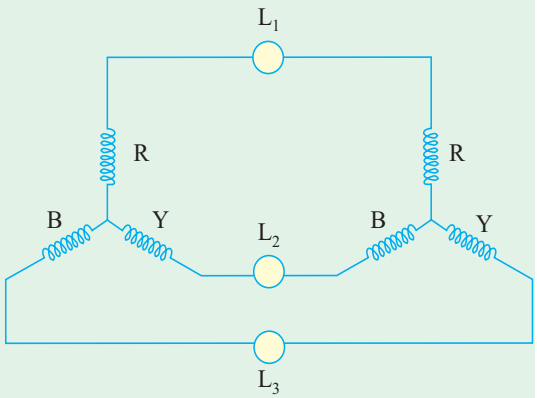


Fig. 37.78

Now, suppose that the incoming machine is slightly slower. Then the star R′Y′B′ will appear to be rotating clockwise relative to voltage star *RYB* (Fig. 37.80). Here, we find that voltage across L_3 *i.e. Y′B* is decreasing having just passed through its maximum, that across L_2 *i.e. YB′* is increasing and approaching its maximum, that across L_1 is decreasing having passed through its maximum earlier. Hence, the lamps will light up one after the other in the order 3, 2, 1 ; 3, 2, 1, etc. which is just the reverse of the first order. Usually, the three lamps are mounted at the three corners of a triangle and the apparent direction of rotation of light

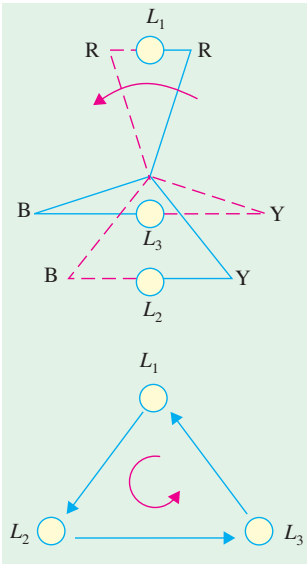


Fig. 37.79

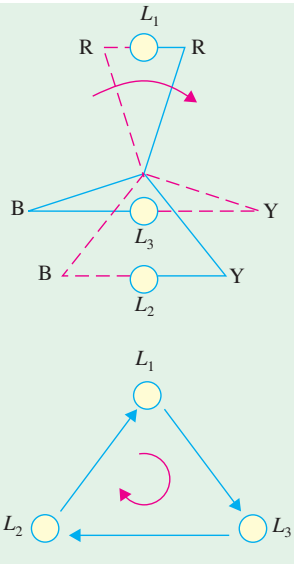


Fig. 37.80

indicates whether the incoming alternator is running too fast or too slow (Fig. 37.81). Synchronization is done at the moment the uncrossed lamp L_1 is in the middle of the dark period. When the alternator voltage is too high for the lamps to be used directly, then usually step-down transformers are used and the synchronizing lamps are connected to the secondaries.

It will be noted that when the uncrossed lamp L_1 is dark, the other two 'crossed' lamps L_2 and L_3 are dimly but equally bright. Hence, this method of synchronizing is also sometimes known as 'two bright and one dark' method.

It should be noted that synchronization by lamps is not quite accurate, because to a large extent, it depends on the sense of correct judgement of the operator. Hence, to eliminate the element of personal judgment in routine operation of alternators, the machines are synchronized by a more accurate device called a synchronoscope. It consists of 3 stationary coils and a rotating iron vane which is attached to a pointer. Out of three coils, a pair is connected to one phase of the line and the other to the corresponding machine terminals, potential transformer being usually used. The pointer moves to one side or the other from its vertical position depending on whether the incoming machine is too fast or too slow. For correct speed, the pointer points vertically up.

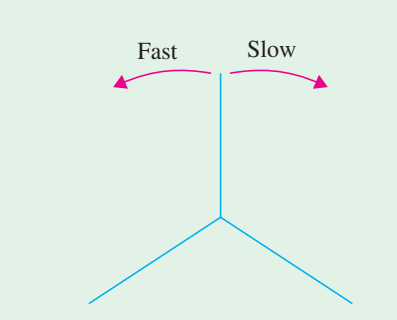


Fig. 37.81

Example 37.42. *In Fig. 37.74, E_1 = 220 V and f_1 = 60 Hz, whereas E_2 = 222 V and f_2 = 59 Hz. With the switch open; calculate*

(*i*) *maximum and minimum voltage across each lamp.*

(*ii*) *frequency of voltage across the lamps.*

(*iii*) *peak value of voltage across each lamp.*

(*iv*) *phase relations at the instants maximum and minimum voltages occur.*

(*v*) *the number of maximum light pulsations/minute.*

Solution. (*i*) E_{max}/lamp = (220 + 222)/2 = **221 V**

E_{min}/lamp = (222 – 220)/2 = **1.0 V**

(*ii*) $f = (f_1 - f_2) = (60 - 59) =$ **1.0 Hz**

(*iii*) $E_{peak} = 221/0.707 =$ **313 V**

(*iv*) in-phase and anti-phase respectively in the local circuit.

(*v*) No. of pulsation/min = (60 – 59) × 60 = **60.**

37.32. Synchronizing Current

Once synchronized properly, two alternators continue to run in synchronism. Any tendency on the part of one to drop out of synchronism is immediately counteracted by the production of a synchronizing torque, which brings it back to synchronism.

When in exact synchronism, the two alternators have equal terminal p.d.'s and are in exact phase opposition, so far as the local circuit (consisting of their armatures) is concerned. Hence, there is no current circulating round the local circuit. As shown in Fig. 37.82 (*b*) e.m.f. E_1 of machine No. 1 is in exact phase opposition to the e.m.f. of machine No. 2 *i.e.* E_2. It should be clearly understood that the two e.m.f.s. are in opposition, so far as their local circuit is concerned but are in the same direction with respect to the external circuit. Hence, there is no resultant voltage (assuming $E_1 = E_2$ in magnitude) round the local circuit.

But now suppose that due to change in the speed of the governor of second machine, E_2 falls back* by a phase angle of α electrical degrees, as shown in Fig. 37.82 (*c*) (though still $E_1 = E_2$). Now, they have a resultant voltage E_r, which when acting on the local circuit, circulates a current known as synchronizing current. The value of this current is given by $I_{SY} = E_r / Z_S$ where Z_S is the synchronous impedance of the phase windings of both the machines (or of one machine only if it is connected to infinite bus-bars**). The current I_{SY} lags behind E_r by an angle θ given by $\tan \theta = X_S / R_a$ where X_S is the combined synchronous reactance of the two machines and R_a their armature resistance. Since R_a is negligibly small, θ is almost 90 degrees. So I_{SY} lags E_r by 90° and is almost in phase with E_1. It is seen that I_{SY} is generating current with respect to machine No.1 and motoring current with respect to machine No. 2 (remember when the current flows in the same direction as e.m.f., then the alternator acts as a generator, and when it flows in the opposite direction, the machine acts as a motor). This current I_{SY} sets up a synchronising torque, which tends to retard the generating machine (*i.e.* No. 1) and accelerate the motoring machine (*i.e.* No. 2).

Similarly, if E_2 tends to advance in phase [Fig. 37.82 (*d*)], then I_{SY}, being generating current for machine No. 2, tends to retard it and being motoring current for machine No. 1 tends to accelerate it. Hence, any departure from synchronism results in the production of a synchronizing current I_{SY} which sets up synchronizing torque. This re-establishes synchronism between the two machines by retarding the leading machine and by accelerating the lagging one. This current I_{SY}, it should be noted, is superimposed on the load currents in case the machines are loaded.

37.33. Synchronizing Power

Consider Fig. 37.82 (*c*) where machine No. 1 is generating and supplying the synchronizing power $= E_1 I_{SY} \cos \phi_1$ which is approximately equal to $E_1 I_{SY}$ ($\because$ ϕ_1 is small). Since $\phi_1 = (90° - \theta)$, synchronizing power $= E_1 I_{SY} \cos \phi_1 = E_1 I_{SY} \cos (90° - \theta) = E_1 I_{SY}$, $\sin \theta \cong E_1 I_{SY}$ because $\theta \cong 90°$ so that

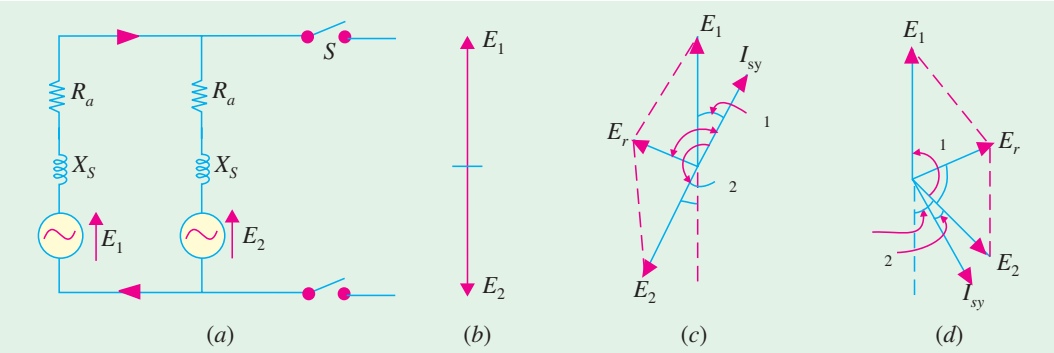


Fig. 37.82

$\sin \theta \cong 1$. This power output from machine No. 1 goes to supply (*a*) power input to machine No. 2 (which is motoring) and (*b*) the Cu losses in the local armature circuit of the two machines. Power input to machine No. 2 is $E_2 I_{SY} \cos \phi_2$ which is approximately equal to $E_2 I_{SY}$.

$$\therefore \quad E_1 I_{SY} = E_2 I_{SY} + \text{Cu losses}$$

Now, let $E_1 = E_2 = E$ (say)

Then, $E_r = 2E \cos [(180° - \alpha)/2]$*** $= 2E \cos [90° - (\alpha/2)]$

---

* Please remember that vectors are supposed to be rotating anticlockwise.

** Infinite bus-bars are those whose frequency and the phase of p.d.'s are not affected by changes in the conditions of any one machine connected in parallel to it. In other words, they are constant-frequency, constant-voltage bus-bars.

*** Strictly speaking, $E_r = 2E \sin \theta$. $\sin \alpha/2 \cong 2E \sin \alpha/2$.

$= 2E \sin \alpha/2 = 2E \times \alpha/2 = \alpha E$ ($\because$ α is small)

Here, the angle α is in electrical radians.

Now, $$I_{SY} = \frac{E_r}{\text{synch. impedance } Z_S} \cong \frac{E_r}{2X_S} = \frac{\alpha E}{2X_S}$$

—if R_a of both machines is negligible

Here, X_S represents synchronous reactance of one machine and not of both as in Art. 37.31 Synchronizing power (supplied by machine No. 1) is

$$P_{SY} = E_1 I_{SY} \cos \phi_1 = E I_{SY} \cos (90° - \theta) = EI_{SY} \sin \theta \cong EI_{SY}$$

Substituting the value of I_{SY} from above,

$$P_{SY} = E.\alpha E/2 Z_S = \alpha E^2/2Z_S \cong \alpha E^2/2X_S$$ —per phase

(more accurately, $P_{SY} = \alpha E^2 \sin \theta/2X_S$)

Total synchronizing power for three phases

$= 3P_{SY} = 3 \alpha E^2/2X_S$ (or $3 \alpha E^2 \sin \theta/2X_S$)

This is the value of the synchronizing power when two alternators are connected in parallel and are on no-load.

37.34. Alternators Connected to Infinite Bus-bars

Now, consider the case of an alternator which is connected to infinite bus-bars. The expression for P_{SY} given above is still applicable but with one important difference *i.e.* impedance (or reactance) of only that one alternator is considered (and not of two as done above). Hence, expression for synchronizing power in this case becomes

$E_r = \alpha E$ —as before

$I_{SY} = E_r/Z_S \cong E_r/X_S = \alpha E/X_S$ —if R_a is negligible

$\therefore$ Synchronizing power $P_{SY} = E I_{SY} = E.\alpha E/Z_S = \alpha E^2/Z_S \cong \alpha E^2/X_S$ — per phase

Now, $E/Z_S \cong E/X_S$ = S.C. current I_{SC}

$\therefore$ $P_{SY} = \alpha E^2/XS = \alpha E.E/X_S = \alpha E.I_{SY}$ —per phase

(more accurately, $P_{SY} = \alpha E^2 \sin \theta/X_S = \alpha E.I_{SC}.\sin \theta$)

Total synchronizing power for three phases = $3P_{SY}$

37.35. Synchronizing Torque T_{SY}

Let T_{SY} be the synchronizing torque per phase in newton-metre (N-m)

(*a*) When there are two alternators in parallel

$$\therefore \quad T_{SY} \times \frac{2\pi N_S}{60} = P_{SY} \quad \therefore \quad T_{SY} = \frac{P_{SY}}{2\pi N_S/60} = \frac{\alpha E^2/2X_S}{2\pi N_S/60} \text{ N-m}$$

Total torque due to three phases. $= \frac{3P_{SY}}{2\pi N_S/60} = \frac{3\alpha E^2/2X_S}{2\pi N_S/60}$ N-m

(*b*) Alternator connected to infinite bus-bars

$$T_{SY} \times \frac{2\pi N_S}{60} = P_{SY} \text{ or } T_{SY} = \frac{P_{SY}}{2\pi N_S/60} = \frac{\alpha E^2/X_S}{2\pi N_S/60} \text{ N-m}$$

Again, torque due to 3 phase $= \frac{3P_{SY}}{2\pi N_S/60} = \frac{3\alpha E^2/X_S}{2\pi N_S/60}$ N-m

where N_S = synchronous speed in r.p.m. $= 120 f/P$

37.36. Effect of Load on Synchronizing Power

In this case, instead of $P_{SY} = \alpha E^2/X_S$, the approximate value of synchronizing power would be $\cong \alpha EV/X_S$ where V is bus-bar voltage and E is the alternator induced e.m.f. per phase. The value of $E = V + IZ_S$

As seen from Fig. 37.83, for a lagging p.f.,

$E = (V \cos \phi + IR_a)^2 + (V \sin \phi + IX_S)^2]^{1/2}$

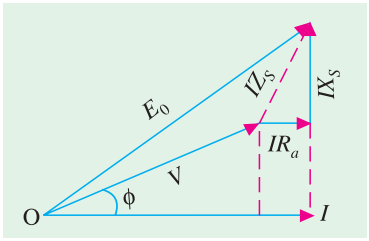


Fig. 37.83

Example 37.43. *Find the power angle when a 1500-kVA, 6.6 kV, 3-phase, Y-connected alternator having a resistance of 0.4 ohm and a reactance of 6 ohm per phase delivers full-load current at normal rated voltage and 0.8 p.f. lag. Draw the phasor diagram.*

(Electrical Machinery-II, Bangalore Univ. 1981)

Solution. It should be remembered that angle α between V and E is known as power angle (Fig. 37.84)

Full-load $I = 15 \times 10^5/\sqrt{3} \times 6600 = 131$ A

$IR_a = 131 \times 0.4 = 52.4$ V, $IX_S = 131 \times 6$

$= 786$ V

V/phase $= 6600/\sqrt{3} = 3810$ V;

$\phi = \cos^{-1}(0.8) = 36°50'$.

As seen from Fig. 37.84

$$\tan(\phi + \alpha) = \frac{AB}{OA} = \frac{V \sin \phi + IX_S}{V \cos \phi + I R_a}$$

$$= \frac{3810 \times 0.6 + 786}{3810 \times 0.8 + 52.4} = 0.991$$

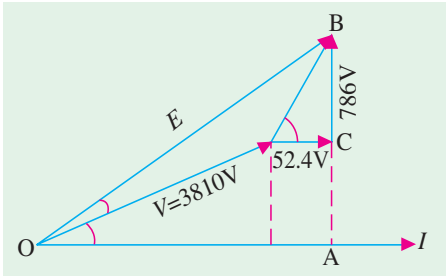


Fig. 37.84

$\therefore \quad (\phi + \alpha) = 44° \quad \therefore \alpha = 44° - 36°50' = 7°10'$

The angle α is also known as load angle or torque angle.

37.37. Alternative Expression for Synchronizing Power

As shown in Fig. 37.85, let V and E (or E_0) be the terminal voltage and induced e.m.f. per phase of the rotor. Then, taking $V = V \angle 0°$, the load current supplied by the alternator is

$$I = \frac{E - V}{Z_S} = \frac{E \angle \alpha - V \angle 0°}{Z_S \angle \theta}$$

$$= \frac{E}{Z_S} \angle \alpha - \theta - \frac{V}{Z_S} \angle -\theta$$

$$= \frac{E}{Z_S} [\cos(\theta - \alpha) - j \sin(\theta - \alpha)]$$

$$= -\frac{V}{Z_S} (\cos \theta - j \sin \theta)$$

$$= \left[\frac{E}{Z_S} \cos(\theta - \alpha) - \frac{V}{Z_S} \cos \theta\right] - j \left[\frac{E}{Z_S} \sin(\theta - \alpha) - \frac{V}{Z_S} \sin \theta\right]$$

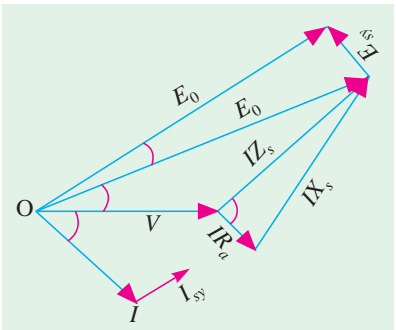


Fig. 37.85

These components represent the $I \cos \phi$ and $I \sin \phi$ respectively. The power P converted internally is given be the sum of the product of corresponding components of the current with $E \cos \alpha$ and $E \sin \alpha$.

$$\therefore \quad P = E\cos\alpha\left[\frac{E}{Z_S}\cos(\theta-\alpha)-\frac{V}{Z_S}\cos\theta\right]-E\sin\alpha\left[\frac{E}{Z_S}\sin(\theta-\alpha)-\frac{V}{Z_S}\sin\theta\right]$$

$$= E\left[\frac{E}{Z_S}\cos\theta\right]-E\left[\frac{V}{Z_S}.\cos(\theta+\alpha)\right]=\frac{E}{Z_S}\left[E\cos\theta-V(\cos\theta+\alpha)\right] \quad \text{—per phase*}$$

Now, let, for some reason, angle α be changed to $(\alpha \pm \delta)$. Since V is held rigidly constant, due to displacement $\pm\delta$, an additional e.m.f. of divergence *i.e.* $I_{SY} = 2E.\sin\alpha/2$ will be produced, which will set up an additional current I_{SY} given by $I_{SY} = E_{SY}/Z_S$. The internal power will become

$$P' = \frac{E}{Z_s}\left[E\cos\theta - V\cos(\theta+\alpha\pm\delta)\right]$$

The difference between P' and P gives the synchronizing power.

$$\therefore \quad P_{SY} = P' - P = \frac{EV}{Z_s}\left[\cos(\theta+\alpha)-\cos(\theta+\alpha\pm\delta)\right]$$

$$= \frac{EV}{Z_s}\left[\sin\delta.\ \sin(\theta+\alpha)\pm 2\cos(\theta+\alpha)\sin^2\delta/2\right]$$

If δ is very small, then $\sin^2(\delta/2)$ is zero, hence P_{SY} per phase is

$$P_{SY} = \frac{EV}{Z_S}.\sin(\theta+\alpha)\sin\delta \qquad ...(i)$$

(i) In large alternators, R_a is negligible, hence $\tan\theta = X_S/R_a = \infty$, so that $\theta \cong 90°$. Therefore, $\sin(\theta+\alpha) = \cos\alpha$.

$$\therefore \quad P_{SY} = \frac{EV}{Z_S}.\cos\alpha\sin\delta \quad \text{— per phase} \qquad ...(ii)$$

$$= \frac{EV}{X_S}\cos\alpha\sin\delta \quad \text{—per phase} \qquad ...(iii)$$

(ii) Consider the case of synchronizing an unloaded machine on to a constant-voltage bus-bars. For proper operation, $\alpha = 0$ so that E coincides with V. In that case, $\sin(\theta+\alpha) = \sin\theta$.

$$\therefore \quad P_{SY} = \frac{EV}{Z_S}\sin\theta\sin\delta \quad \text{—from } (i) \text{ above.}$$

Since δ is very small, $\sin\delta = \delta$,

$$\therefore \quad P_{SY} = \frac{EV}{Z_S}\delta\sin\theta = \frac{EV}{X_S}\delta\sin\theta$$

Usually, $\sin\theta \cong 1$, hence

$$\therefore \quad P_{SY} = \frac{EV}{Z_S}.\delta^{**} = V\left(\frac{E}{Z_S}\right)\delta = V\left(\frac{E}{X_S}\right)\delta = VI_{SC}.\delta \quad \text{—per phase}$$

37.38. Parallel Operation of Two Alternators

Consider two alternators with identical speed/load characteristics connected in parallel as shown in Fig. 37.86. The common terminal voltage **V** is given by

$$\mathbf{V} = \mathbf{E}_1 - \mathbf{I}_1\mathbf{Z}_1 = \mathbf{E}_2 - \mathbf{I}_2\mathbf{Z}_2$$

$$\therefore \quad \mathbf{E}_1 - \mathbf{E}_2 = \mathbf{I}_1\mathbf{Z}_1 - \mathbf{I}_2\mathbf{Z}_2$$

Also $\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2$ and $\mathbf{V} = \mathbf{IZ}$

$$\therefore \quad \mathbf{E}_1 = \mathbf{I}_1\mathbf{Z}_1 + \mathbf{IZ} = \mathbf{I}_1(\mathbf{Z}+\mathbf{Z}_1) + \mathbf{I}_2\mathbf{Z}$$

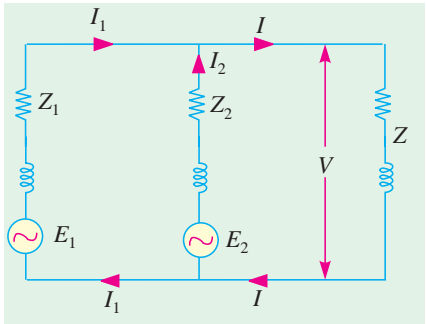


Fig. 37.86

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* In large machines, R_a is very small so that $\theta = 90°$, hence $P = \frac{E}{Z_S}V\cos(90°-\alpha) = \frac{E}{Z_S}V\sin\alpha = \alpha\ EV/Z_S$ —*if α is so small that $\sin\alpha = \alpha$*

** With $E = V$, the expression becomes $P_{SY} = \frac{V^2}{Z_S}\delta = \frac{\delta V^2}{X_S}$ It is the same as in Art. 37.33 —*per phase*

$$E_2 = I_2Z_2 + IZ = I_2(Z + Z_2) + I_1Z$$

$$\therefore \quad I_1 = \frac{(E_1 - E_2)\,Z + E_1Z_2}{Z\,(Z_1 + Z_2) + Z_1Z_2}$$

$$I_2 = \frac{(E_2 - E_1)\,Z + E_2Z_1}{Z\,(Z_1 + Z_2) + Z_1Z_2};$$

$$I = \frac{E_1Z_2 + E_2Z_1}{Z\,(Z_1 + Z_2) + Z_1Z_2}$$

$$V = IZ = \frac{E_1\,Z_2 + E_2Z_1}{Z_1 + Z_2 + (Z_1Z_2/Z)}; I_1 = \frac{E_1 - V}{Z_1}; I_2 = \frac{E_2 - V}{Z_2}$$

The circulating current under no-load condition is $I_C = (E_1 - E_2)/(Z_1 + Z_2)$.

Using Admittances

The terminal Voltage may also be expressed in terms of admittances as shown below:

$$V = IZ = (I_1 + I_2)Z \qquad \therefore \; I_1 + I_2 = V/Z = VY \qquad ...(i)$$

Also $I_1 = (E_1 - V)/Z_1 = (E_1 - V)Y_1$; $I_2 = (E_2 - V)/Z_2 = (E_2 - V)Y_2$

$$\therefore \quad I_1 + I_2 = (E_1 - V)\,Y_1 + (E_2 - V)Y_2 \qquad ...(ii)$$

From Eq. (*i*) and (*ii*), we get

$$VY = (E_1 - V)Y_1 + (E_2 - V)Y_2 \quad \text{or} \quad V = \frac{E_1\,Y_1 + E_2Y_2}{Y_1 + Y_2 + Y}$$

Using Parallel Generator Theorem

$$V = IZ = (I_1 + I_2)\,Z = \left(\frac{E_1 - V}{Z_1} + \frac{E_2 - V}{Z_2}\right)Z$$

$$= \left(\frac{E_1}{Z_1} + \frac{E_2}{Z_2}\right)Z - V\left(\frac{1}{Z_1} + \frac{1}{Z_2}\right)Z$$

$$\therefore \quad V\left(\frac{1}{Z} + \frac{1}{Z_1} + \frac{1}{Z_2}\right) = \frac{E_1}{Z_1} + \frac{E_2}{Z_2} = I_{SC1} + I_{SC2} = I_{SC}$$

where I_{SC1} and I_{SC2} are the short-circuit currents of the two alternators.

If $\dfrac{1}{Z_0} = \left(\dfrac{1}{Z} + \dfrac{1}{Z_1} + \dfrac{1}{Z_2}\right)$; then $V \times \dfrac{1}{Z_0} = I_{SC}$ or $V = Z_0\,I_{SC}$

Example 37.44. *A 3,000-kVA, 6-pole alternator runs at 1000 r.p.m. in parallel with other machines on 3,300-V bus-bars. The synchronous reactance is 25%. Calculate the synchronizing power for one mechanical degree of displacement and the corresponding synchronizing torque.*
(Elect. Machines-I, Gwalior Univ. 1984)

Solution. It may please be noted that here the alternator is working in parallel with many alternators. Hence, it may be considered to be connected to infinite bus-bars.

Voltage/phase = $3{,}300/\sqrt{3} = 1905$ V

F.L. current $I = 3 \times 10^6/\sqrt{3} \times 3300 = 525$ A

Now, IX_S = 25% of 1905 $\quad \therefore \; X_S = 0.25 \times 1905/525 = 0.9075\ \Omega$

Also, $P_{SY} = 3 \times \alpha E^2/X_S$

Here $\alpha = 1°$ (mech.); α (elect.) $= 1 \times (6/2) = 3°$

$\therefore \quad \alpha = 3 \times \pi/180 = \pi/60$ elect. radian.

$$\therefore \quad P_{SY} = \frac{3\times\pi\times1905^2}{60\times0.9075\times1000} = 628.4 \text{ kW}$$

$$T_{SY} = \frac{60.P_{SY}}{2\pi N_S} = 9.55\frac{P_{SY}}{N_S} = 9.55\frac{628.4\times10^3}{1000} = \textbf{6,000 N-m}$$

Example 37.45. *A 3-MVA, 6-pole alternator runs at 1000 r.p.m on 3.3-kV bus-bars. The synchronous reactance is 25 percent. Calculate the synchronising power and torque per mechanical degree of displacement when the alternator is supplying full-load at 0.8 lag.*
(Electrical Machines-1, Bombay Univ. 1987)

Solution. $V = 3{,}300/\sqrt{3} = 1905$ V/phase, F.L. $I = 3\times10^6/\sqrt{3}\times3{,}300 = 525$ A

IX_S = 25% of 1905 = 476 V; $X_S = 476/525 = 0.9075\ \Omega$

Let, $\mathbf{I} = 525\angle0°$, then, $\mathbf{V} = 1905\,(0.8 + j0.6) = 1524 + j1143$

$\mathbf{E_0} = \mathbf{V + IX_S} = (1524 + j1143) + (0 + j476) = (1524 + j1619) = 2220\angle46°44'$

Obviously, E_0 leads I by 46°44′. However, V leads I by $\cos^{-1}(0.8) = 36°50'$.

Hence, $\alpha = 46°44' - 36°50' = 9°54'$

α = 1° (mech.), No. of pair of poles = 6/2 = 3 $\quad\therefore\ \alpha = 1\times3 = 3°$ (elect.)

$$P_{SY}\text{ per phase} = \frac{EV}{X_S}\cos\alpha\sin\delta = \frac{2220\times1905}{0.9075}\times\cos 9°54'\sin 3° = 218 \text{ kW}$$

P_{SY} for three phases = $3\times218 = \textbf{654 kW}$

$T_{SY} = 9.55\times P_{SY}/N_S = 9.55\times654\times10^2/1000 = 6245$ N-m

Example 37.46. *A 750-kVA, 11-kV, 4-pole, 3-ϕ, star-connected alternator has percentage resistance and reactance of 1 and 15 respectively. Calculate the synchronising power per mechanical degree of displacement at **(a)** no-load **(b)** at full-load 0.8 p.f. lag. The terminal voltage in each case is 11 kV.*
(Electrical Machines-II, Indore Univ. 1985)

Solution. F.L. Current $I = 75\times10^3/\sqrt{3}\times11\times10^3 = 40$ A

$V_{ph} = 11{,}000/\sqrt{3} = 6{,}350$ V, IR_a = 1% of 6,350 = 63.5

or $40\,R_a = 63.5$, $R_a = 1.6\ \Omega$; $40\times X_S$ = 15% of 6,350 = 952.5 V

$\therefore\quad X_S = 23.8\ \Omega$; $Z_S = \sqrt{1.6^2 + 23.8^2} \cong 23.8\ \Omega$

(a) No-load

α (mech) = 1° : α (elect) = $1\times(4/2) = 2°$

$= 2\times\pi/180 = \pi/90$ elect. radian.

$$P_{SY} = \frac{\alpha E^2}{Z_S} \cong \frac{\alpha E^2}{X_S} = \frac{(\pi/90)\times6350^2}{23.8}$$

= 59,140 W = **59.14 kW/phase.**

On no-load, V has been taken to be equal to E.

(b) F.L. 0.8 p.f.

As indicated in Art. 37.35, $P_{SY} = \alpha EV/X_S$. The value of E (or E_0) can be found from Fig. 37.87.

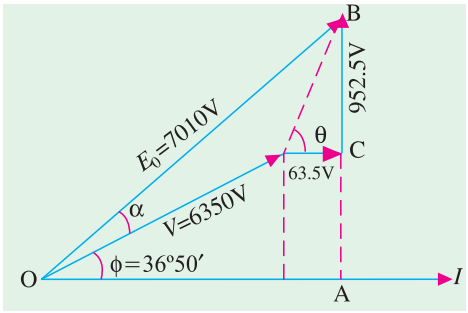


Fig. 37.87

$$E = [(V\cos\phi + IR_a)^2 + (V\sin\phi + IX_S)^2]^{1/2}$$
$$= [(6350\times0.8 + 63.5)^2 + (6350\times0.6 + 952.5)^2]^{1/2} = 7010 \text{ V}$$

$$P_{SY} = \frac{\alpha EV}{X_S} = \frac{(\pi/90)\times7010\times6350}{23.8} = 65{,}290 \text{ W}$$
$$= \textbf{65.29 kW/phase}$$

More Accurate Method [Art. 37.35]

$$P_{SY} = \frac{EV}{X_S}\cos\alpha\sin\delta$$

Now, E = 7010 V, V = 6350 V, $\delta = 1° \times (4/2) = 2°$ (elect)

As seen from Fig. 37.87, $\sin(\phi + \alpha) = AB/OB = (6350 \times 0.6 + 952.5)/7010 = 0.6794$

$\therefore \quad (\phi + \alpha) = 42°30'$; $\alpha = 42°30' - 36°50' = 5°40'$

$$\therefore \quad P_{SY} = \frac{7010 - 6350}{23.8} \times \cos 5°40' \times \sin 2°$$

$$= 7010 \times 6350 \times 0.9953 \times 0.0349/23.8 = 64{,}970 \text{ W} = \mathbf{64.97\ kW/phase}$$

Note. It would be instructive to link this example with Ex. 38.1 since both are concerned with synchronous machines, one generating and the other motoring.

Example 37.47. *A 2,000-kVA, 3-phase, 8-pole alternator runs at 750 r.p.m. in parallel with other machines on 6,000 V bus-bars. Find synchronizing power on full-load 0.8 p.f. lagging per mechanical degree of displacement and the corresponding synchronizing torque. The synchronous reactance is 6 ohm per phase.* **(Elect. Machines-II, Bombay Univ. 1987)**

Solution. Approximate Method

As seen from Art. 37.37 and 38, $P_{SY} = \alpha EV/X_S$ —per phase

Now α = 1°(mech); No. of pair of poles = 8/2 = 4;

$\alpha = 1 \times 4 = 4°$ (elect)

$= 4\pi/180 = \pi/45$ elect. radian

$V = 6000/\sqrt{3} = 3{,}465$ —assuming *Y*-connection

F.L. current $I = 2000 \times 10^3/\sqrt{3} \times 6000 = 192.4\ A$

As seen from Fig. 37.88,

$E_0 = [(V\cos\phi)^2 + (V\sin\phi + IX_S)^2]^{1/2} = 4295$ V

$= [(3465 \times 0.8)^2 + (3465 \times 0.6 + 192.4 \times 6)^2]^{1/2}$

$= 4295$ V

$P_{SY} = (\pi/45) \times 4295 \times 3465/6 = 173{,}160$ W

$= \mathbf{173.16\ kW/phase}$

P_{SY} for three phases $= 3 \times 173.16 = 519.5$ kW

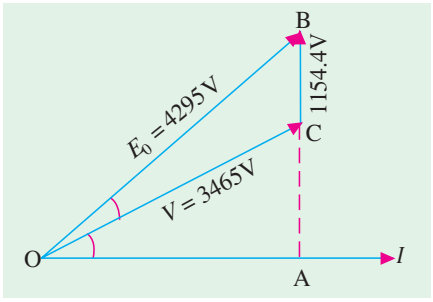


Fig. 37.88

If T_{SY} is the total synchronizing torque for three phases in N-m, then

$T_{SY} = 9.55\ P_{SY}/N_S = 9.55 \times 519{,}500/750 = \mathbf{6{,}614\ N\text{-}m}$

Exact Method

As shown in the vector diagram of Fig. 37.89, I is full-load current lagging V by $\phi = \cos^{-1}(0.8) = 36°50'$. The reactance drop is IX_S and its vector is at right angles to (lag.)*. The phase angle between E_0 and V is α.

F.L. current $I = 2{,}000{,}000/\sqrt{3} \times 6{,}000$

$= 192.4$ A

Let, $I = 192.4 \angle 0°$

V $= 3{,}465\ (0.8 + j\ 0.6) = 2{,}772 + j\ 2{,}079$

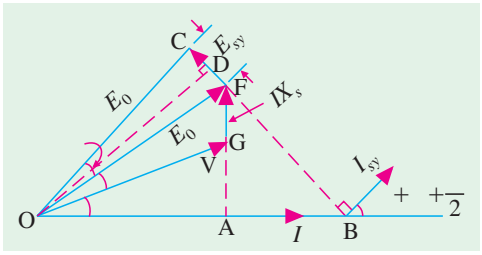


Fig. 37.89

* Earlier, we had called this e.m.f. as *E* when discussing regulation.

IX_S = 192.4 × 6 = 1154 V = (0 + *j* 1154) V

$\mathbf{E_0}$ = $\mathbf{V + IX_S}$

= (2,772 + *j* 2,079) + (0 + *j* 1154)

= 2,772 + *j* 3,233 = 4,259 ∠ 49°24′

α = 49°24′ – 36°50′ = 12°34′

E_{SY} = $2E_0 \sin \delta/2 = 2E_0 \sin (4°/2)$

= 2 × 4,259 × 0.0349 = 297.3 V

I_{SY} = 297.3/6 = 49.55 A

As seen, *V* leads *I* by ϕ and I_{SY} leads *I* by (ϕ + α + δ/2), hence I_{SY} leads *V* by (α + δ/2) = 12°34′ + (4°/2) = 14°34′.

∴ P_{SY}/phase = VI_{SY} cos 14°34′ = 3465 × 49.55 × cos 14°34′ = 166,200 W = 166.2 kW

Synchronising power for three phases is = 3 × 166.2 = **498.6 kW**

If T_{SY} is the total synchronizing torque, then $T_{SY} \times 2\pi \times 750/60 = 498{,}600$

∴ T_{SY} = 9.55 × 498,600/750 = **6,348 N-m**

Alternative Method

We may use Eq. (*iii*) of Art. 37.36 to find the total synchronizing power.

$$P_{SY} = \frac{EV}{X_s} \cos \alpha \sin \delta \quad \text{—per phase}$$

Here, E = 4,259 V ; V = 3,465 V; α = 12° 34′ ; δ = 4° (elect.)

∴ P_{SY}/phase = 4,259 × 3, 465 ′ cos 12° 34′ × sin 4°/6

= 4,259 × 3,465 × 0.976 × 0.0698/6 = 167,500 W = 167.5 kW

P_{SY} for 3 phases = 3 × 167.5 = **502.5 kW**

Next, T_{SY} may be found as above.

Example 37.48. *A 5,000-kV A, 10,000 V, 1500-r.p.m., 50-Hz alternator runs in parallel with other machines. Its synchronous reactance is 20%. Find for* ***(a)*** *no-load* ***(b)*** *full-load at power factor 0.8 lagging, synchronizing power per unit mechanical angle of phase displacement and calculate the synchronizing torque, if the mechanical displacement is 0.5°.*

(Elect. Engg. V, M.S. Univ. Baroda, 1986)

Solution. Voltage / phase = $10{,}000/\sqrt{3} = 5{,}775$ V

Full -load current = $5{,}000{,}000/\sqrt{3} \times 10{,}000 = 288.7$ A

$$X_S = \frac{20}{100} \times \frac{5{,}775}{288.7} = 4\ \Omega,\ P = \frac{120\ f}{N_S} = \frac{120 \times 50}{1500} = 4$$

α = 1° (mech.) ; No. of pair of poles = 2 ∴ α = 1 × 2 = 2° (elect.) = 2 π/180 = π / 90 radian

(*a*) At no-load

$$P_{SY} = \frac{3\alpha E^2}{X_S} = 3 \times \frac{\pi}{90} \times \frac{5{,}775^2}{4 \times 1000} = \mathbf{873.4\ kW}$$

∴ T_{SY} = 9.55 × (873.4 × 10^3)/1500 = 5,564 N-m

∴ T_{SY} for 0.5° = 5564/2 = **2,782 N-m**

(*b*) At F.L. p.f. 0.8 lagging

Let $\mathbf{I}$ = 288.7 ∠ 0°. Then $\mathbf{V}$ = 5775 (0.8 + *j* 0.6) = 4620 + *j* 3465

$\mathbf{I.X_S}$ = 288.7 ∠ 0° × 4 ∠ 90° = (0 + *j* 1155)

$\mathbf{E_0}$ = $\mathbf{V + IX_S}$ = (4620 + *j*3465) + (0 + *j*1155)

$= 4620 + j4620 = 6533 \angle 45°$

$\cos \phi = 0.8, \phi = \cos^{-1}(0.8) = 36°50'$

Now, E_0 leads I by 45° and V leads I by 36° 50′. Hence, E_0 leads V by (45° – 36°50′) =8°10′ *i.e.*α = 8°10′. As before, $\delta = 2°$ (elect).

As seen from Art. 37.36, $P_{SY} = \frac{EV}{X_S} \cos \alpha \sin \delta$ —per phase

$= 6533 \times 5775 \times \cos 8°\ 10' \times \sin 2°/4 = 326$ kW

P_{SY} for three phases $= 3 \times 326 = 978$ kW

T_{SY}/ unit displacement $= 9.55 \times 978 \times 10^3/1500 = 6{,}237$ N-m

T_{SY} for 0.5° displacement $= 6{,}237/2 =$ **3118.5 N-m**

(c) We could also use the approximate expression of Art. 37.36

P_{SY} per phase $= \alpha E V/X_S = (\pi/90) \times 6533 \times 5775/4 =$ **329.3 kW**

Example 37.49. *Two 3-phase, 6.6-kW, star-connected alternators supply a load of 3000 kW at 0.8 p.f. lagging. The synchronous impedance per phase of machine A is (0.5 + j10) Ω and of machine B is (0.4 + j 12) Ω. The excitation of machine A is adjusted so that it delivers 150 A at a lagging power factor and the governors are so set that load is shared equally between the machines.*

Determine the current, power factor, induced e.m.f. and load angle of each machine.

(Electrical Machines-II, South Gujarat Univ. 1985)

Solution. It is given that each machine carries a load of 1500 kW. Also, $V = 6600/\sqrt{3} = 3810$ V. Let $V = 3810 \angle 0° = (3810 + j\,0)$.

For machine No. 1

$\sqrt{3} / 6600 \times 150 \times \cos \phi_1 = 1500 \times 10^3$;

$\cos \phi_1 = 0.874, \phi_1 = 29°; \sin \phi_1 = 0.485$

Total current $I = 3000/\sqrt{3} \times 6.6 \times 0.8 = 328$ A

or $I = 828\,(0.8 - j\,0.6) = 262 - j\,195$

Now, $\mathbf{I_1} = 150\,(0.874 - j\,0.485) = 131 - j\,72.6$

$\therefore$ $\mathbf{I_2} = (262 - j\,195) - (131 - j\,72.6)$

$= (131 - j\,124.4)$

or $I_2 =$ **181 A,** $\cos \phi_2 = 131/181 =$ **0.723 (lag).**

$\mathbf{E_A} = \mathbf{V} + \mathbf{I_1 Z_1} = 3810 + (131 - j\,72.6)\,(0.5 + j10)$

$= 4600 + j1270$

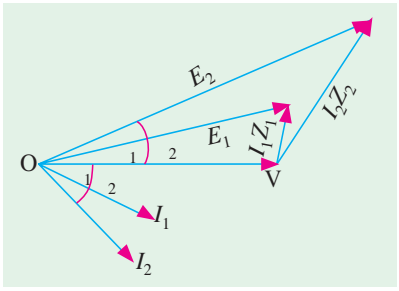


Fig. 37.90

Line value of e.m.f. $= \sqrt{3}\sqrt{(4600^2 + 1270^2)} =$ **8,260 V**

Load angle $\alpha_1 = (1270/4600) =$ **15.4°**

$\mathbf{E_B} = \mathbf{V} + \mathbf{I_2 Z_2} = 3810 + (131 - j124.4)\,(0.4 + j12)$

$= 5350 + j1520$

Line value of e.m.f $= \sqrt{3}\sqrt{5350^2 + 1520^2} =$ **9600 V**

Load angle $\alpha_2 = \tan^{-1}(1520/5350) =$ **15.9°**

Example 37.50. *Two single-phase alternator operating in parallel have induced e.m.fs on open circuit of 230 ∠ 0° and 230 ∠ 10° volts and respective reactances of j2 Ω and j3 Ω. Calculate* ***(i)*** *terminal voltage* ***(ii)*** *currents and* ***(iii)*** *power delivered by each of the alternators to a load of impedance 6 Ω (resistive).* **(Electrical Machines-II, Indore Univ. 1987)**

Solution. Here, $\mathbf{Z_1} = j2, \mathbf{Z_2} = j.3, \mathbf{Z} = 6; \mathbf{E_1} = 230 \angle 0°$ and

$$\mathbf{E_2} = 230 \angle 10° = 230\,(0.985 + j\,0.174) = (226.5 + j\,39.9), \text{ as in Fig. 37.90}$$

(*ii*) $$\mathbf{I_1} = \frac{(E_1 - E_2)\,Z + E_1 Z_2}{Z\,(Z_1 + Z_2) + Z_1 Z_2} = \frac{[(230 + j0) - (226.5 + j39.9)] \times 6 + 230 \times j3}{6\,(j2 - j3) + j2 \times j3}$$

$$= 14.3 - j3.56 = 14.73 \angle -14° \qquad \text{—Art. 37.38}$$

$$\mathbf{I_2} = \frac{(E_2 - E_1)\,Z + E_2 Z_1}{Z\,(Z_1 + Z_2) + Z_1 Z_2} = \frac{(-3.5 + j39.9) + (222.5 + j39.9) \times j2}{6\,(j2 + j3) + j2 \times j3}$$

$$= 22.6 - j\,1.15 = 22.63 \angle -3.4°$$

(*i*) $$\mathbf{I} = \mathbf{I_1} + \mathbf{I_2} = 36.9 - j\,4.71 = 37.2 \angle -7.3°$$

$$\mathbf{V} = \mathbf{IZ} = (36.9 - j4.71) \times 6 = 221.4 - j28.3 = 223.2 \angle -7.3°$$

(*iii*) $$P_1 = V I_1 \cos \phi_1 = 223.2 \times 14.73 \times \cos 14° = \mathbf{3190\ W}$$

$$P_2 = V I_2 \cos \phi_1 = 223.2 \times 22.63 \times \cos 3.4° = \mathbf{5040\ W}$$

Tutorial Problem No. 37.6.

1. Calculate the synchronizing torque for unit mechanical angle of phase displacement for a 5,000-kVA, 3-ϕ alternator running at 1,500 r.p.m. when connected to 6,600-volt, 50-Hz bus-bars. The armature has a short-circuit reactance of 15%. **[43,370 kg-m] (*City & Guilds, London*)**
2. Calculate the synchronizing torque for one mechanical degree of phase displacement in a 6,000-kVA, 50-Hz, alternator when running at 1,500 r.p.m with a generated e.m.f. of 10,000 volt. The machine has a synchronous impedance of 25%.
   [544 kg.m] (*Electrical Engineering-III, Madras Univ. April 1978; Osmania Univ. May 1976*)
3. A 10,000-kVA, 6,600-V, 16-pole, 50-Hz, 3-phase alternator has a synchronous reactance of 15%. Calculate the synchronous power per mechanical degree of phase displacement from the full load position at power factor 0.8 lagging. **[10 MW] (*Elect.Machines-I, Gwalior Univ. 1977*)**
4. A 6.6 kV, 3-phase, star-connected turbo-alternator of synchronous reactance 0.5 ohm/phase is applying 40 MVA at 0.8 lagging p.f. to a large system. If the steam supply is suddenly cut off, explain what takes place and determine the current the machine will then carry. Neglect losses.
   [2100 A] (*Elect. Machines (E-3) AMIE Sec. B Summer 1990*)
5. A 3-phase 400 kVA, 6.6 kV, 1500 rpm., 50 Hz alternator is running in parallel with infinite bus bars. Its synchronous reactance is 25%. Calculate (*i*) for no load (*ii*) full load 0.8 p.f. lagging the synchronizing power and torque per unit mechanical angle of displacement.
   [*Rajive Gandhi Technical University, 2000*] [(*i*) 55.82 kW, 355 Nw-m (*ii*) 64.2 kW, 409 Nw-m]

37.39. Effect of Unequal Voltages

Let us consider two alternators, which are running exactly in-phase (relative to the external circuit) but which have slightly unequal voltages, as shown in Fig. 37.91. If E_1 is greater than E_2, then their resultant is $E_r = (E_1 - E_2)$ and is in-phase with E_1. This E_r or E_{SY} set up a local synchronizing current I_{SY} which (as discussed earlier) is almost 90° behind E_{SY} and hence behind E_1 also. This lagging current produces demagnetising effect (Art. 37.16) on the first machine, hence E_1 is reduced. The other machine runs as a synchronous motor, taking almost 90° leading current. Hence, its field is strengthened due to magnetising effect of armature reaction (Art. 37.16). This tends to increase E_2. These two effects act together and hence lessen the inequalities between the two voltages and tend to establish stable conditions.

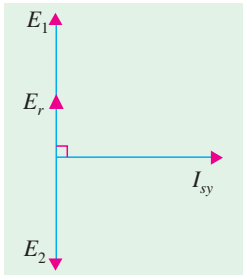


Fig. 37.91

37.40. Distribution of Load

It will, now be shown that the amount of load taken up by an alternator running, in parallel with other

machines, is solely determined by its driving torque *i.e.* by the power input to its prime mover (by giving it more or less steam, in the case of steam drive). Any alternation in its excitation merely changes its kVA output, but not its kW output. In other words, it merely changes the power factor at which the load is delivered.

(*a*) Effect of Change in Excitation

Suppose the initial operating conditions of the two parallel alternators are identical *i.e.* each alternator supplies one half of the active load (kW) and one-half of the reactive load (kVAR), the operating power factors thus being equal to the load p.f. In other words, both active and reactive powers are divided equally thereby giving equal apparent power triangles for the two machines as shown in Fig. 37.92 (*b*). As shown in Fig. 37.92 (*a*), each alternator supplies a load current I so that total output current is $2I$.

Now, let excitation of alternator No. 1 be increased, so that E_1 becomes greater than E_2. The difference between the two e.m.fs. sets up a circulating current $I_C = I_{SY} = (E_1 - E_2)/2Z_S$ which is confined to the local path through the armatures and round the bus-bars. This current is superimposed on the original current distribution. As seen, I_C is vectorially added to the load current of alternator No. 1 and subtracted from that of No. 2. The two machines now deliver load currents I_1 and I_2 at respective power factors of cos ϕ_1 and cos ϕ_2. These changes in load currents lead to changes in power factors, such that cos ϕ_1 is reduced, whereas cos ϕ_2 is increased. However, effect on the

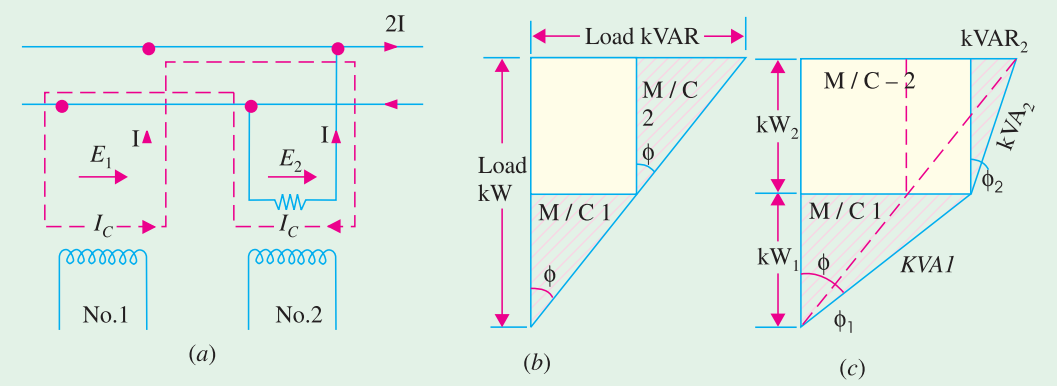


Fig. 37.92

kW loading of the two alternators is negligible, but kVAR$_1$ supplied by alternator No. 1 is increased, whereas kVAR$_2$ supplied by alternator No. 2 is correspondingly decreased, as shown by the kVA triangles of Fig. 37.92 (*c*).

(*b*) Effect of Change in Steam Supply

Now, suppose that excitations of the two alternators are kept the same but steam supply to alternator No. 1 is increased *i.e.* power input to its prime mover is increased. Since the speeds of the two machines are tied together by their synchronous bond, machine No. 1 cannot overrun machine No 2. Alternatively, it utilizes its increased power input for carrying

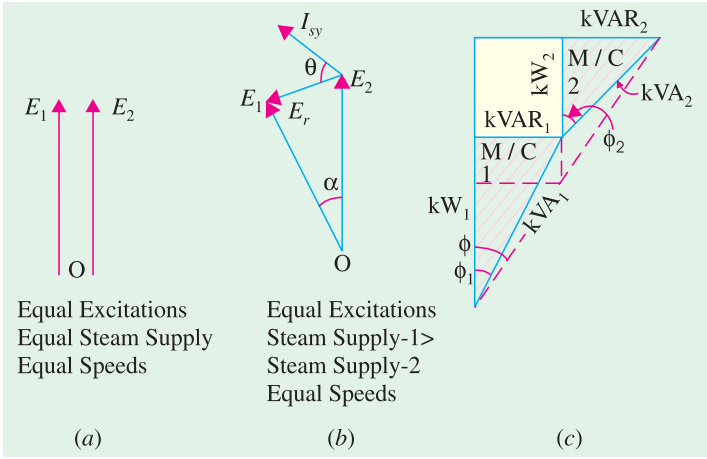


Fig. 37.93

more load than No. 2. This can be made possible only when rotor No. 1 advances its angular position with respect to No. 2 as shown in Fig. 37.93 (*b*) where E_1 is shown advanced ahead of E_2 by an angle α. Consequently, resultant voltage E_r (or E_{sy}) is produced which, acting on the local circuit, sets up a current I_{sy} which lags by almost 90° behind E_r but is almost in phase with E_1 (so long as angle α is small). Hence, power per phase of No. 1 is increased by an amount = $E_1 I_{sy}$ whereas that of No. 2 is decreased by the same amount (assuming total load power demand to remain unchanged). Since I_{sy} has no appreciable reactive (or quadrature) component, the increase in steam supply does not disturb the division of reactive powers, but it increases the active power output of alternator No. 1 and decreases that of No. 2. Load division, when steam supply to alternator No. 1 is increased, is shown in Fig. 37.93 (*c*).

So, it is found that by increasing the input to its prime mover, an alternator can be made to take a greater share of the load, though at a different power factor.

The points worth remembering are :

1. The load taken up by an alternators directly depends upon its driving torque or in other words, upon the angular advance of its rotor.

2. The excitation merely changes the p.f. at which the load is delivered without affecting the load so long as steam supply remains unchanged.

3. If input to the prime mover of an alternator is kept constant, but its excitation is changed, then kVA component of its output is changed, not kW.

Example 37.51. *Two identical 3-phase alternators work in parallel and supply a total load of 1, 500 kW at 11 kV at a power factor of 0.867 lagging. Each machine supplies half the total power. The synchronous reactance of each is 50 Ω per phase and the resistance is 4 Ω per phase. The field excitation of the first machine is so adjusted that its armature current is 50 A lagging. Determine the armature current of the second alternator and the generated voltage of the first machine.*

(Elect. Technology, Utkal Univ. 1983)

Solution. Load current at 0.867 p.f. lagging is

$$= \frac{1,500 \times 1,000}{\sqrt{3} \times 11,000 \times 0.887} = 90.4 \text{ A}; \cos\phi = 0.867; \sin\phi = 0.4985$$

Wattful component of the current = $90.4 \times 0.867 = 78.5$ A

Wattless component of the current = $90.4 \times 0.4985 = 45.2$ A

Each alternator supplies half of each of the above two component when conditions are identical (Fig. 37.94).

Current supplied by each machine = 90.4/2 = 45.2 A

Since the steam supply of first machine is not changed, the working components of both machines would remain the same at 78.5 / 2 = 39.25 A. But the wattless or reactive components would be redivided due to change in excitation. The armature current of the first machine is changed from 45.2 A to 50 A.

$\therefore$ Wattless component of the 1st machine $= \sqrt{50^2 - 39.25^2} = 31$ A

Wattless component of the 2nd machine $= 45.2 - 31 = 14.1$ A

The new current diagram is shown in Fig. 37.95 (*a*)

(*i*) Armative current of the 2nd alternator, $I_2 = \sqrt{39.25^2 + 14.1^2} = 41.75$ A

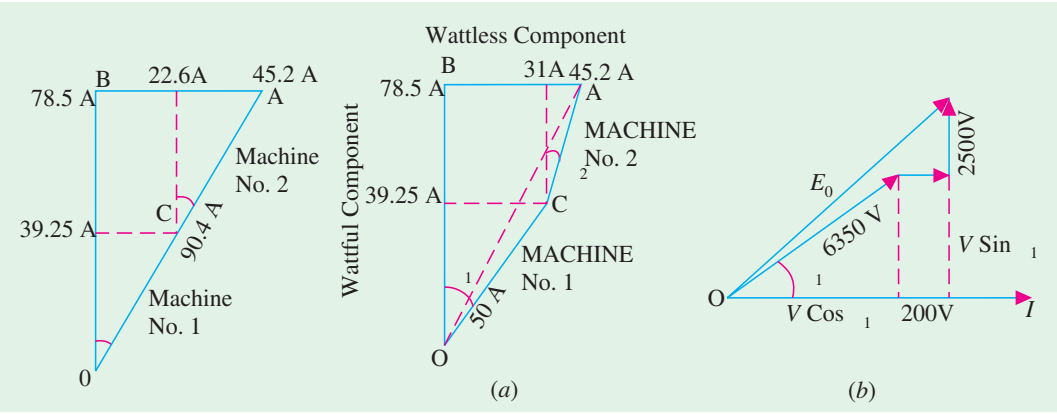


Fig. 37.94 **Fig. 37.95**

(ii) Terminal voltage / phase = $11,000 / \sqrt{3} = 6350$ V

Considering the first alternator,

IR drop = $4 \times 50 = 200$ V ; IX drop = $50 \times 50 = 2,500$ V

$\cos \phi_1 = 39.25/50 = 0.785$; $\sin \phi_1 = 0.62$

Then, as seen from Fig. 37.95 (*b*)

$$E = \sqrt{(V \cos \phi_1 + IR)^2 + (V \sin \phi_1 + IX)^2}$$

$$= \sqrt{(6,350 \times 0.785 + 200)^2 + (6,350 \times 0.62 + 2,500)^2} = 8,350 \text{ V}$$

Line voltage = $8,350 \times \sqrt{3}$ = **14,450 V**

Example 37.52. *Two alternators A and B operate in parallel and supply a load of 10 MW at 0.8 p.f. lagging* ***(a)*** *By adjusting steam supply of A, its power output is adjusted to 6,000 kW and by changing its excitation, its p.f. is adjusted to 0.92 lag. Find the p.f. of alternator B.*

(b) *If steam supply of both machines is left unchanged, but excitation of B is reduced so that its p.f. becomes 0.92 lead, find new p.f. of A.*

Solution. ***(a)*** $\cos \phi = 0.8$, $\phi = 36.9°$, $\tan \phi = 0.7508$; $\cos \phi_A = 0.92$, $\phi_A = 23°$; $\tan \phi_A = 0.4245$

load kW = 10,000, load kVAR = $10,000 \times 0.7508$ = 7508 (lag)

kW of A = 6,000, kVAR of A = $6,000 \times 0.4245$ = 2547 (lag)

Keeping in mind the convention that lagging kVAR is taken as negative we have,

kW of B = (10,000 – 6,000) = 4,000 : kVAR of B = (7508 – 2547) = 4961 (lag)

$\therefore$ kVA of $B = 4,000 - j4961 = 6373 \angle -51.1°$; $\cos \phi_B = \cos 51.1° =$ **0.628**

(b) Since steam supply remains unchanged, load kW of each machine remains as before but due to change in excitation, kVARs of the two machines are changed.

kW of B = 4,000, new kVAR of B = $4000 \times 0.4245 = 1698$ (lead)

kW of A = 6,000, new kVAR of A = $-7508 - (+1698) = -9206$ (lag.)

$\therefore$ new kVA of $A = 6,000 - j9206 = 10,988 \angle -56.9°$; $\cos \phi_A =$ **0.546 (lag)**

Example 37.53. *A 6,000-V, 1,000-kVA, 3-ϕ alternator is delivering full-load at 0.8 p.f. lagging. Its reactance is 20% and resistance negligible. By changing the excitation, the e.m.f. is increased by 25% at this load. Calculate the new current and the power factor. The machine is connected to infinite bus-bars.*

Solution. Full-load current $I = \dfrac{1,000,000}{\sqrt{3} \times 6,600} = 87.5$ A

Voltage/phase = $6,600/\sqrt{3} = 3,810$ V

Reactance = $\dfrac{3810 \times 20}{87.5 \times 100} = 8.7\ \Omega$

IX = 20% of 3810 = 762 V

In Fig. 37.96, current vector is taken along X-axis. ON represents bus-bar or terminal voltage and is hence constant.

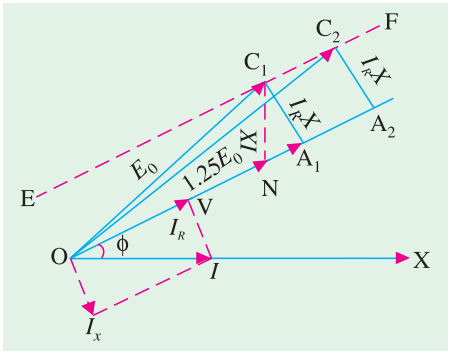


Fig. 37.96

Current I has been split up into its active and reactive components I_R and I_X respectively.

$NA_1 = I_X.X = 52.5 \times 7.8 = 457$ V

$A_1C_1 = I_R..X = 70 \times 8.7 = 609$ V

$$E_0 = OC_1 = \sqrt{[(V + I_X X)^2 + (I_R X)^2]}$$

$$= \sqrt{[(3,810 + 457)^2 + 609^2]} = 4,311 \text{ V}$$

When e.m.f. is increased by 25%, then E_0 becomes equal to $4,311 \times 1.25 = 5,389$ V

The locus of the extremity of E_0 lies on the line EF which is parallel to ON. Since the kW is unchanged, I_R and hence $I_R.X$ will remain the same. It is only the $I_X.X$ component which will be changed. Let OC_2 be the new value of E_0. Then $A_2C_2 = A_1C_1 = I_RX$ as before. But the $I_X X$ component will change. Let I' be the new line current having active component I_R (the same as before) and the new reactive component I_X'. Then, $I_X'X = NA_2$

From right-angled triangle OC_2A_2

$$OC_2^{\,2} = OA_2^{\,2} + A_2C_2^{\,2};\ 5,389^2 = (3810 + VA_2)^2 + 609^2$$

$\therefore$ $VA_2 = 1546\ V$ or $I_X'\ X = 1546$

$\therefore$ $I_X' = 1546/8.7 = 177.7$ A

$\therefore$ New line current $I' = \sqrt{(70^2 + 177.7^2)} =$ **191 A**

New angle of lag, $\phi' = \tan^{-1}(177.7/70) = 68°\ 30'$; $\cos\phi' = \cos 68°30' =$ **0.3665**

As a check, the new power $= \sqrt{3} \times 6,600 \times 191 \times 0.3665 = 800$ kW

It is the same as before $= 1000 \times 0.8 = 800$ kW

Example 37.54. *A 6,600-V, 1000-kVA alternator has a reactance of 20% and is delivering full-load at 0.8 p.f. lagging. It is connected to constant-frequency bus-bars. If steam supply is gradually increased, calculate* ***(i)*** *at what output will the power factor become unity* ***(ii)*** *the maximum load which it can supply without dropping out of synchronism and the corresponding power factor.*

Solution. We have found in Example 37.52 that

$I = 87.5\ A$, $X = 8.7\ \Omega$, V/phase = 3,810 V

$E_0 = 4,311\ V$, $I_R = 70\ A$, $I_X = 52.5\ A$ and

$IX = 87.5 \times 8.7 = 762$ V

Using this data, vector diagram of Fig. 37.97 can be constructed.

Since excitation is constant, E_0 remains constant, the extremity of E_0 lies on the arc of a circle of radius E_0 and centre O. Constant power lines have been shown dotted and they are all parallel to OV. Zero power output line coincides with OV. When p.f. is unity, the current vector lies along OV, I_1Z is $\perp$ to OV and cuts the arc at B_1. Obviously

$VB_1 = \sqrt{(OB_1^2 - OV^2)}$

$= \sqrt{(4,311^2 - 3,810^2)} = 2018$ V

Now $Z = X$ $\therefore$ $I_1X = 2,018$ V

$\therefore I_1 = 2018/8.7 = 232$ A

(i) $\therefore$ power output at u.p.f.

$$= \frac{\sqrt{3} \times 6,600 \times 232}{1000} = \mathbf{2,652\ kW}$$

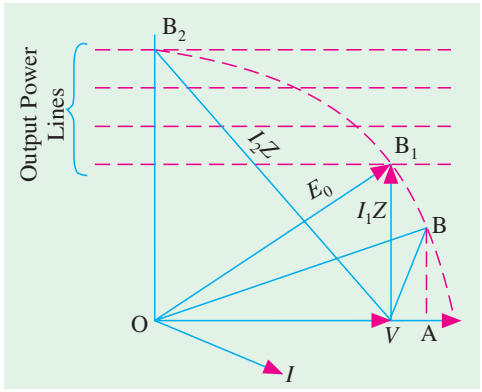


Fig. 37.97

(ii) As vector OB moves upwards along the arc, output power goes on increasing *i.e.*, point B shifts on to a higher output power line. Maximum output power is reached when OB reaches the position OB_2 where it is vertical to OV. The output power line passing through B_2 represents the maximum output for that excitation. If OB is further rotated, the point B_2 shifts down to a lower power line *i.e.* power is decreased. Hence, $B_2V = I_2Z$ where I_2 is the new current corresponding to maximum output.

From triangle OB_2V, it is seen that

$$B_2V = \sqrt{(OV^2 + OB_2^2)} = \sqrt{(3,810^2 + 4,311^2)} = 5,753 \text{ V}$$

$\therefore$ $I_2 Z = 5,753$ V $\therefore$ $I_2 = 5753/8.7 = 661$ A

Let I_{2R} and I_{2X} be the power and wattless components of I_2, then

$I_{2R} X = OB_2 = 4311$ and $I_{2R} = 4311/8.7 = 495.6$ A

Similarly $I_{2X} = 3810/8.7 = 438$ A; $\tan \phi_2 = 438/495.6 = \mathbf{0.884}$

$\therefore$ $\phi_2 = 41°28'$; $\cos \phi_2 = 0.749$

$\therefore$ Max. power output $= \dfrac{\sqrt{3} \times 6,600 \times 661 \times 0.749}{1000} = \mathbf{5,658\ kW}$

Example 37.55. *A 3-phase, star-connected turbo-alternator, having a synchronous reactance of 10 Ω per phase and negligible armature resistance, has an armature current of 220 A at unity p.f. The supply voltage is constant at 11 kV at constant frequency. If the steam admission is unchanged and the e.m.f. raised by 25%, determine the current and power factor.*

If the higher value of excitation is maintained and the steam supply is slowly increased, at what power output will the alternator break away from synchronism ?

Draw the vector diagram under maximum power condition.

(Elect.Machinery-III, Banglore Univ. 1992)

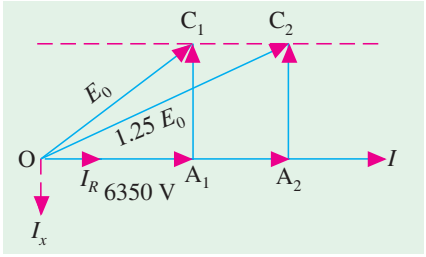


Fig. 37.98

Solution. The vector diagram for unity power factor is shown in Fig. 37.98. Here, the current is wholly active.

$$OA_1 = 11,000/\sqrt{3} = 6,350 \text{ V}$$

$$A_1C_1 = 220 \times 10 = 2,200 \text{ V}$$

$$E_0 = \sqrt{(6350^2 + 2,200^2)} = 6,810 \text{ V}$$

When e.m.f. is increased by 25%, the e.m.f. becomes $1.25 \times 6,810 = 8,512$ V and is represented by OC_2. Since the kW remains unchanged, $A_1C_1 = A_2C_2$. If I' is the new current, then its active component

I_R would be the same as before and equal to 220 A. Let its reactive component be I_X. Then

$$A_1A_2 = I_X.X_S = 10\ I_X$$

From right-angled $\Delta\ OA_2C_2$, we have

$$8{,}512^2 = (6350 + A_1A_2)^2 + 2{,}200^2$$

$$\therefore \quad A_1A_2 = 1870\ \text{V} \quad \therefore \quad 10\ I_X = 1870 \quad I_X = 187\ \text{A}$$

Hence, the new current has active component of 220 A and a reactive component of 187 A.

$$\text{New current} = \sqrt{220^2 + 187^2} = \mathbf{288.6\ A}$$

$$\text{New power factor} = \frac{\text{active component}}{\text{total current}} = \frac{220}{288.6} = \mathbf{0.762\ (lag)}$$

Since excitation remains constant, E_0 is constant. But as the steam supply is increased, the extremity of E_0 lies on a circle of radius E_0 and centre O as shown in Fig. 37.99.

The constant-power lines (shown dotted) are drawn parallel to OV and each represents the locus of the e.m.f. vector for a constant power output at varying excitation. Maximum power output condition is reached when the vector E_0 becomes perpendicular to OV. In other words, when the circular e.m.f. locus becomes tangential to the constant-power lines *i.e.*at point B. If the steam supply is increased further, the alternator will break away from synchronism.

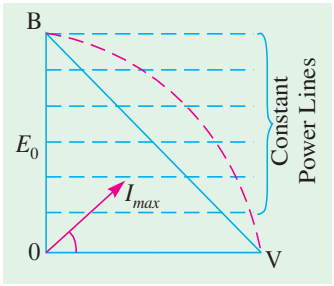


Fig. 37.99

$$B.V. = \sqrt{6350^2 + 8{,}512^2} = 10{,}620\ \text{V}$$

$$\therefore \quad I_{\max} \times 10 = 10{,}620 \text{ or } I_{\max} = 1{,}062\ \text{A}$$

If I_R and I_X are the active and reactive components of $I_{\max}$, then

$$10\ I_R = 8{,}512 \quad \therefore \quad I_R = 851.2\ \text{A};\ 10\ I_X = 6{,}350 \quad \therefore \quad I_X = 635\ \text{A}$$

Power factor at maximum power output = 851.2/1062 = 0.8 (lead)

$$\text{Maximum power output} = \sqrt{3} \times 11{,}000 \times 1062 \times 0.8 \times 10^{-3} = \mathbf{16{,}200\ kW}$$

Example 37.56. *Two 20-MVA, 3-ϕ alternators operate in parallel to supply a load of 35MVA at 0.8 p.f. lagging. If the output of one machine is 25 MVA at 0.9 lagging, what is the output and p.f. of the other machine?* **(Elect. Machines, Punjab Univ. 1990)**

Solution. Load MW = $35 \times 0.8 = 28$; load MVAR = $35 \times 0.6 = 21$

First Machine $\cos\phi_1 = 0.9$, $\sin\phi_1 = 0.436$; $\text{MVA}_1 = 25$, $\text{MW}_1 = 25 \times 0.9 = 22.5$

$\text{MVAR}_1 = 25 \times 0.436 = 10.9$

Second Machine $\text{MW}_2 = \text{MW} - \text{MW}_1 = 28 - 22.5 = 5.5$

$\text{MVAR}_2 = \text{MVAR} - \text{MVAR}_1 = 21 - 10.9 = 10.1$

$$\therefore \quad \text{MVA}_2 = \sqrt{\text{MW}_2^2 + \text{MVAR}_2^2} = \sqrt{5.5^2 + 10.1^2} = \mathbf{11.5}$$

$\cos\phi_2 = 5.5/11.5 = \mathbf{0.478\ (lag)}$

Example 37.57. *A lighting load of 600 kW and a motor load of 707 kW at 0.707 p.f. are supplied by two alternators running in parallel. One of the machines supplies 900 kW at 0.9 p.f. lagging. Find the load and p.f. of the second machine.*

(Electrical Technology, Bombay Univ. 1988 & Bharatiar University, 1997)

Solution.

	Active Power	*kVA*	*Reactive Power*
(*a*) Lighting Load (unity P.f.)	600 kW	600	—
(*b*) Motor, 0.707 P.f.	707 kW	1000	707 k VAR
Total Load :	1307	By Phasor addition	707 k VAR

One machine supplies an active power of 900 kW, and due to 0.9 lagging p.f., kVA = 1000 kVA and its k VAR = $1000 \times \sqrt{(1 - 0.9^2)}$ = 436 kVAR. Remaining share will be catered to by the second machine.

Active power shared by second machine = 1307 – 900 = **407 kW**

Reactive power shared by second machine = 707 – 436 = **271 kVAR**

Example 37.58. *Two alternators, working in parallel, supply the following loads :*

(i) *Lighting load of 500 kW* ***(ii)*** *1000 kW at p.f. 0.9 lagging*

(iii) *800 kW at p.f. 0.8 lagging* ***(iv)*** *500 kW at p.f. 0.9 leading*

One alternator is supplying 1500 kW at 0.95 p.f. lagging. Calculate the kW output and p.f of the other machine.

Solution. We will tabulate the kW and kVAR components of each load separately :

Load	*kW*	*kVAR*
(i)	500	
(ii)	1000	$\frac{1000}{0.9} \times 0.436 = 485$
(iii)	800	$\frac{800 \times 0.6}{0.8} = 600$
(iv)	500	$\frac{500 \times 0.436}{0.9} = -242$
Total	2800	+ 843

For 1st machine, it is given : kW = 1500, kVAR = (1500/0.95) × 0.3123 = 493

∴ kW supplied by other machine = 2800 – 1500 = **1300**

kVAR supplied = 843 – 493 = 350 ∴ tan ϕ 350/1300 = 0.27 ∴ cos ϕ = **0.966**

Example 37.59 *Two 3-ϕ synchronous mechanically-coupled generators operate in parallel on the same load. Determine the kW output and p.f. of each machine under the following conditions: synchronous impedance of each generator : 0.2 + j2 ohm/phase. Equivalent impedance of the load : 3 + j4 ohm/phase. Induced e.m.f. per phase, 2000 + j0 volt for machine I and 2,2000 + j 100 for II.* **[London Univ.]**

Solution. Current of 1st machine $= \mathbf{I}_1 = \frac{\mathbf{E}_1 - \mathbf{V}}{0.2 + j2}$ or $\mathbf{E}_1 - \mathbf{V} = \mathbf{I}_1 (0.2 + j2)$

Similarly $\mathbf{E}_2 - \mathbf{V} = \mathbf{I}_2 (0.2 + j2)$

Also $\mathbf{V} = (\mathbf{I}_1 + \mathbf{I}_2)(3 + j4)$ where 3 + *j*4 = load impedance

Now $\mathbf{E}_1 = 2{,}000 + j\,0$, $\mathbf{E}_2 = 2{,}200 + j100$

Solving from above, we get $\mathbf{I}_1 = 68.2 - j\,102.5$

Similarly $\mathbf{I}_2 = 127 - j\,196.4$; $\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = 195.2 - j299$

Now $\mathbf{V} = \mathbf{IZ} = (192.2 - j\,299)(3 + j\,4) = 1781 - j\,115.9$

Using the method of conjugate for power calculating, we have for the first machine

$P_{VA1} = (1781 - j\,115.9)(68.2 + j\,102.5) = 133{,}344 + j\,174{,}648$

∴ $kW_1 = 133.344$ kW/phase = 3 × 133.344 = **400 kW** —for 3 phases

Now $\tan^{-1}(102.5/68.2) = 56°24'$; $\tan^{-1}(115.9/1781) = 3°43'$

∴ for 1st machine ; $\cos(56°24' - 3°43') =$ **0.6062**

$PVA_2 = (1781 - j115.9)\,(127 + j\,196.4) = 248{,}950 + j\,335{,}069$

$\therefore \quad kW_2 = 248.95$ kW/phase $= 746.85$ kW —for 3 phases

$\tan^{-1}(196.4/127) = 57°6'$; $\cos\phi = \cos(57°6' - 3°43') = \mathbf{0.596}$

Example 37.60. *The speed regulations of two 800-kW alternators A and B, running in parallel, are 100% to 104% and 100% to 105% from full-load to no-load respectively. How will the two alternators share a load of 1000 kW? Also, find the load at which one machine ceases to supply any portion of the load.* **(Power Systems-I, A.M.I.E. 1989)**

Solution. The speed / load characteristics (assumed straight) for the alternators are shown in Fig. 37.100. Out of the combined load $AB = 1000$ kW, A's share is AM and B's share is BM. Hence, $AM + BM = 1000$ kW. PQ is the horizontal line drawn through point C, which is the point of intersection.

From similar Δs GDA and CDP, we have

$$\frac{CP}{GA} = \frac{PD}{AD} \text{ or}$$

$$CP = GA.\ \frac{PD}{AD}$$

Since,

$PD = (4 - h)$

$\therefore \quad CP = 800\,(4 - h)/4$

$= 200(4 - h)$

Similarly, from similar Δs BEF and QEC, we get

$$\frac{QC}{BF} = \frac{QE}{BE} \text{ or}$$

$QC = 800(5 - h)/5$

$= 160\,(5 - h)$

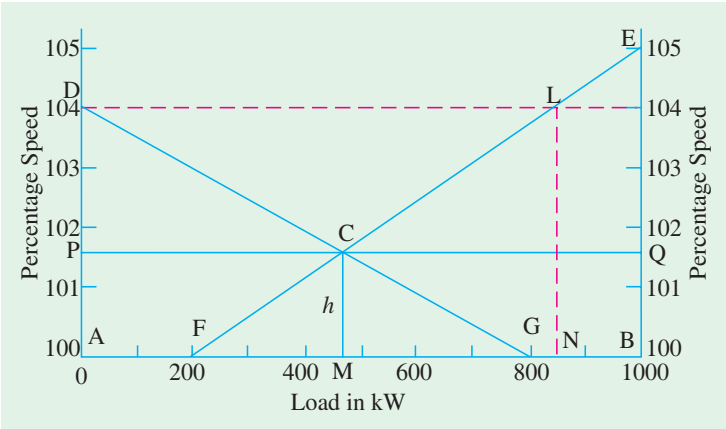


Fig. 37.100

$\therefore \quad CP + QC = 1000$ or $200\,(4 - h) + 160\,(5 - h) = 1000$ or $h = 5/3$

$\therefore \quad CP = 200(4 - 5/3) = 467$ kW, $QC = 160\,(5 - 5/3) = 533$ kW

Hence, alternator A supplies **467 kW** and B supplies **533 kW.**

Alternator A will cease supplying any load when line PQ is shifted to point D. Then, load supplied by alternator B ($= BN$) is such that the speed variation is from 105% to 104%.

Knowing that when its speed varies from 105% to 100%, alternator B supplies a load of 800 kW, hence load supplied for speed variation from 105% to 100% is (by proportion)

$= 800 \times 1/5 =$ **160 kW (= BN)**

Hence, when load drops from 1000 kW to 160 kW, alternator A will cease supplying any portion of this load.

Example 37.61. *Two 50-MVA, 3-ϕ alternators operate in parallel. The settings of the governors are such that the rise in speed from full-load to no-load is 2 per cent in one machine and 3 per cent in the other, the characteristics being straight lines in both cases. If each machine is fully loaded when the total load is 100 MW, what would be the load on each machine when the total load is 60 MW?* **(Electrical Machines-II, Punjab Univ. 1991)**

Solution. Fig. 37.101 shows the speed/load characteristics of the two machines, NB is of the first machine and MA is that of the second. Base AB shows equal load division at full-load and speed. As the machines are running in parallel, their frequencies must be the same. Let CD be drawn through x% speed where total load is 60 MW.

$$CE = 50 - AP = 50 - \frac{50}{3}x$$

$$ED = 50 - QB = 50 - \frac{50}{2}x$$

$$\therefore \quad CD = 50 - (50/3)x + 50 - 25x$$

$$\therefore \quad 60 = 50 - (50/3)x + 50 - 25x$$

$$x = \frac{24}{25}; \therefore \; LE = 100\,\frac{24}{25}\,\%$$

Load supplied by 1st machine

$$= ED = 50 - 25 \times \frac{24}{25} = \mathbf{26\ MW}$$

Load supplied by 2nd machine.

$$= CE = 50 - \left(\frac{50}{3}\right) \times \frac{24}{25} = \mathbf{34\ MW}$$

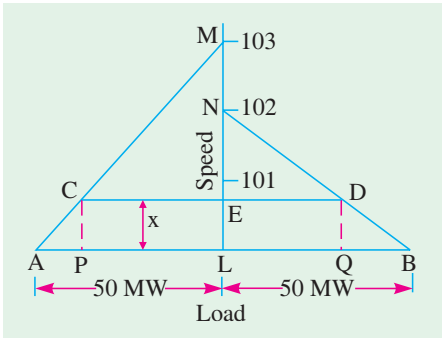


Fig. 37.101

Example 37.62. *Two identical 2,000 -kVA alternators operate in parallel. The governor of the first machine is such that the frequency drops uniformly from 50-Hz on no-load to 48-Hz on full-load. The corresponding uniform speed drop of the second machines is 50 to 47.5 Hz* ***(a)*** *How will the two machines share a load of 3,000 kW?* ***(b)*** *What is the maximum load at unity p.f. that can be delivered without overloading either machine ?* **(Electrical Machinery-II, Osmania Univ. 1989)**

Solution. In Fig. 37.102 are shown the frequency/load characteristics of the two machines, *AB* is that of the second machine and *AD* that of the first. Remembering that the frequency of the two machines must be the same at any load, a line *MN* is drawn at a frequency *x* as measured from point *A* (common point).

Total load at that frequency is

$NL + ML = 3000$ kW

From Δs *ABC* and *ANL*, $NL/2000 = x / 2.5$

$$\therefore \quad NL = 2000\,x/2.5 = 800\,x$$

Similarly, $ML = 2000\,x/2 = 1000x$

$$\therefore \quad 1800\,x = 3000 \text{ or } x = 5/3$$

Frequency $= 50 - 5/3 = 145/3$ Hz.

(a) $NL = 800 \times 5/3 =$ **1333 kW (assuming u.p.f.)**

$ML = 1000 \times 5/3 =$ **1667 kW (assuming u.p.f.)**

(b) For getting maximum load, *DE* is extended to cut *AB* at F. Max. load = *DF*.

Now, $EF = 2000 \times 2/3.5 = 1600$ kW

$$\therefore \quad \text{Max. load} = DF = 2{,}000 + 1{,}600$$

$$= \mathbf{3{,}600\ kW.}$$

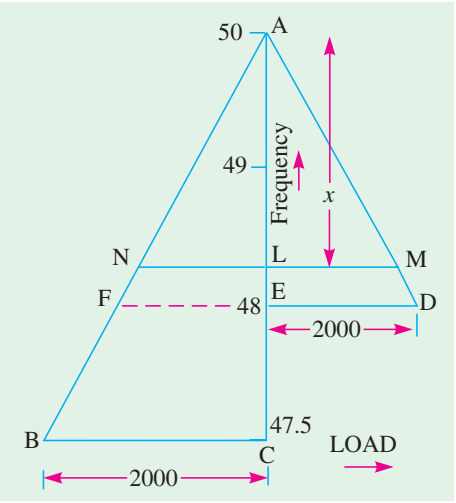


Fig. 37.102

Tutorial Problem No. 37.7

1. Two similar 6,600-V, 3-ϕ, generators are running in parallel on constant-voltage and frequency bus-bars. Each has an equivalent resistance and reactance of 0.05 Ω and 0.5 Ω respectively and supplies one half of a total load of 10,000 kW at a lagging p.f. of 0.8, the two machines being similarly excited. If the excitation of one machine be adjusted until the armature current is 438 A and the steam supply to the turbine remains unchanged, find the armature current, the e.m.f. and the p.f. of the other alternator. **[789 A, 7200 V, 0.556] (*City & Guilds, London*)**

2. A single-phase alternator connected to 6,600-V bus-bars has a synchronous impedance of 10Ω and a resistance of 1 Ω. If its excitation is such that on open circuit the p.d. would be 5000 V, calculate the maximum load the machine can supply to the external circuit before dropping out of step and the corresponding armature current and p.f. **[2864 kW, 787 A, 0.551] (*London Univ.*)**

3. A turbo-alternator having a reactance of 10 Ω has an armature current of 220 A at unity power factor when running on 11,000 V, constant-frequency bus-bars. If the steam admission is unchanged and the e.m.f. raised by 25%, determine graphically or otherwise the new value of the machine current and power factor. If this higher value of excitation were kept constant and the steam supply gradually increased, at what power output would the alternator break from synchronism? Find also the current and power factor to which this maximum load corresponds. State whether this p.f. is lagging or leading.

[360 A at 0.611 p.f. ; 15.427 kW ; 1785 A at 0.7865 leading] (*City & Guilds, London*)

4. Two single-phase alternators are connected to a 50-Hz bus-bars having a constant voltage of $10 \angle 0°$ kV. Generator *A* has an induced e.m.f. of $13 \angle 22.6°$ kV and a reactance of 2 Ω; generator *B* has an e.m.f. of $12.5 \angle 36.9°$ kV and a reactance of 3 Ω. Find the current, kW and kVAR supplied by each generator. **(*Electrical Machine-II, Indore Univ. July 1977*)**

5. Two 15-kVA, 400-V, 3-ph alternators in parallel supply a total load of 25 kVA at 0.8 p.f. lagging. If one alternator shares half the power at unity p.f., determine the p.f. and kVA shared by the other alternator. **[0.5548; 18.03 kVA] (*Electrical Technology-II, Madras Univ. Apr. 1977*)**

6. Two 3-ϕ, 6,600-V, star-connected alternators working in parallel supply the following loads :

(*i*) Lighting load of 400 kW **(*ii*)** 300 kW at p.f. 0.9 lagging

(*iii*) 400 kW at p.f. 0.8 lagging **(*iv*)** 1000 kW at p.f. 0.71 lagging

Find the output, armature current and the p.f. of the other machine if the armature current of one machine is 110 A at 0.9 p.f. lagging. **[970 kW, 116 A, 0.73 lagging]**

7. A 3-ϕ, star-connected, 11,000-V turbo-generator has an equivalent resistance and reactance of 0.5 Ω and 8 Ω respectively. It is delivering 200 A at u.p.f. when running on a constant-voltage and constant-frequency bus-bars. Assuming constant steam supply and unchanged efficiency, find the current and p.f. if the induced e.m.f. is raised by 25%. **[296 A, 0.67 lagging]**

8. Two similar 13,000-V, 3-ph alternators are operated in parallel on infinite bus-bars. Each machine has an effective resistance and reactance of 0.05 Ω and 0.5 Ω respectively. When equally excited, they share equally a total load of 18 MW at 0.8 p.f. lagging. If the excitation of one generator is adjusted until the armature current is 400 A and the steam supply to its turbine remains unaltered, find the armature current, the e.m.f. and the p.f. of the other generator.

[774.6 A; 0.5165 : 13,470 V] (*Electric Machinery-II, Madras Univ. Nov. 1977*)

37.41. Time-period of Oscillation

Every synchronous machine has a natural time period of free oscillation. Many causes, including the variations in load, create phase-swinging of the machine. If the time period of these oscillations coincides with natural time period of the machine, then the amplitude of the oscillations may become so greatly developed as to swing the machine out of synchronism.

The expression for the natural time period of oscillations of a synchronous machine is derived below :

Let T = torque per mechanical radian (in N-m/mech. radian)

$J = \Sigma m r^2$ —moment of inertia in kg-m^2.

The period of undamped free oscillations is given by $t = 2\pi \sqrt{\frac{J}{T}}$.

We have seen in Art. 37.32 that when an alternator swings out of phase by an angle α (electrical radian), then synchronizing power developed is

$P_{SY} = \alpha E^2/Z$ — α in elect. radian

$$= \frac{E^2}{Z} \text{ per electrical radian per phase.}$$

Now, 1 electrical radian $= \frac{P}{2} \times$ mechanical radian—where P is the number of poles.

$\therefore$ P_{SY} per mechanical radian displacement $= \frac{E^2 P}{2Z}$.

The synchronizing or restoring torque is given by

$$T_{SY} = \frac{P_{SY}}{2\pi N_S} = \frac{E^2 P}{4\pi Z N_S} \quad \text{—}N_S \text{ in r.p.s.}$$

Torque for three phases is $T = 3T_{SY} = \frac{3E^2 P}{4\pi Z N_S}$ where E is e.m.f. per phase ...(*i*)

Now E/Z = short-circuit current $= I_{SC}$

$f = PN_S/2$; hence $P/N_S = 2f/N_S^2$

Substituting these values in (*i*) above, we have

$$T_{SY} = \frac{3}{4\pi} \cdot \left(\frac{E}{Z}\right) \cdot E \cdot \frac{P}{N_S} = \frac{3}{4\pi} \cdot I_{SC} \cdot E \cdot \frac{2f}{N_S^2} = 0.477 \frac{EI_{SC} f}{N_S^2}$$

Now, $$t = 2\pi \sqrt{\frac{J}{0.477\, E\, I_{SC}\, f / N_S^2}} = 9.1\, N_S \sqrt{\frac{J}{E \cdot I_{SC} \cdot f}} \text{ second}$$

$$= 9.1\, N_S \sqrt{\frac{J}{\frac{1}{\sqrt{3}} \cdot E_L \cdot I \cdot (I_{SC}/I) \cdot f}}$$

$$= 9.1\, N_S \sqrt{\frac{J}{\frac{1}{2} \cdot \sqrt{3}\, E_L \cdot I \cdot (I_{SC}/I) \cdot f}}$$

$$= 9.1\, N_S \sqrt{\frac{J}{\frac{1000}{3} \cdot \frac{\sqrt{3} \cdot E_L \cdot I}{1000} \left(\frac{I_{SC}}{I}\right) \cdot f}}$$

$$= \frac{9.1 \times \sqrt{3}}{\sqrt{1000}} \cdot N_S \cdot \sqrt{\frac{J}{kVA \cdot (I_{SC}/I)\, f}}$$

$\therefore$ $$t = 0.4984\, N_S \sqrt{\frac{J}{kVA \cdot (I_{SC}/I) \cdot f}}$$

where kVA = full-load kVA of the alternator; N_S = r.p.s. of the rotating system

If N_S represents the speed in r.p.m., then

$$t = \frac{0.4984}{60} \cdot N_S \cdot \sqrt{\frac{J}{kVA \cdot (I_{SC}/I) \cdot f}} = 0.0083\, N_S \sqrt{\frac{J}{kVA\,(I_{SC}/I) \cdot f}} \text{ second}$$

Note. It may be proved that I_{SC}/I = 100/percentage reactance = 100/% X_S.

Proof. Reactance drop $= I.X_S = \frac{V \times \%\, X_S}{100}$ $\therefore$ $X_S = \frac{\text{reactance drop}}{\text{full-load current}} = \frac{V \times \%\, X_S}{100 \times I}$

Now $I_{SC} = \frac{V}{X_S} = \frac{V \times 100 \times I}{V \times \%\ X_S} = \frac{100}{\%X_S} \times I$; or $\frac{I_{SC}}{I} = \frac{100}{\%\ X_S}$

For example, if synchronous reactances is 25 per cent, then

I_{SC}/I =100/25 = 4 (please see Ex. 37.64)

Example 37.63. *A 5,000-kVA, 3-phase, 10,000-V, 50-Hz alternate runs at 1500 r.p.m. connected to constant-frequency, constant-voltage bus-bars. If the moment of inertia of entire rotating system is 1.5×10^4 kg.m^2 and the steady short-circuit current is 5 times the normal full-load current, find the natural time period of oscillation.* **(Elect. Engg. Grad. I.E.T.E. 1991)**

Solution. The time of oscillation is given by

$$t = 0.0083\, NS \sqrt{\frac{J}{kVA\,.\,(I_{SC}/I)\,f}}\ .\ \text{second}$$

Here, N_S = 1500 r.p.m. ; $I_{SC}/I = 5$* ; $J = 1.5 \times 10^4$ kg-m^2 ; f = 50 Hz

$$t = 0.0083 \times 1500 \sqrt{\frac{1.5 \times 10^4}{5000 \times 5 \times 50}} = \mathbf{1.364\ s.}$$

Example 37.64. *A 10,000-kVA, 4-pole, 6,600-V, 50-Hz, 3-phase star-connected alternator has a synchronous reactance of 25% and operates on constant-voltage, constant frequency bus-bars. If the natural period of oscillation while operating at full-load and unity power factor is to be limited to 1.5 second, calculate the moment of inertia of the rotating system.*

(Electric Machinery-II, Andhra Univ. 1990)

Solution. $t = 0.0083\, N_S \sqrt{\frac{J}{kVA\,(I_{SC}/I)\,f}}$ second.

Here $I_{SC}/I = 100/25 = 4$; $N_S = 120 \times 50/4 = 1500$ r.p.m.

$\therefore$ $1.5 = 0.0083 \times 1500 \sqrt{\frac{J}{10,000 \times 4 \times 50}} = 12.45 \times \frac{\sqrt{J}}{10^3 \times \sqrt{2}}$

$\therefore$ $J = (1.5 \times 10^3 \times \sqrt{2}/12.45)^2 = \mathbf{2.9 \times 10^4\ kg\text{-}m^2}$

Example 37.65. *A 10-MVA, 10-kV, 3-phase, 50-Hz, 1500 r.p.m. alternator is paralleled with others of much greater capacity. The moment of inertia of the rotor is 2×10^5 kg-m2 and the synchronous reactance of the machine is 40%. Calculate the frequency of oscillation of the rotor.*

(Elect. Machinery-III, Bangalore Univ. 1992)

Solution. Here, I_{SC}/I = 100/40 = 2.5

$$t = 0.0083 \times 1500 \sqrt{\frac{2 \times 10^5}{10^4 \times 2.5 \times 50}} = 5 \text{ second}$$

Frequency = 1/5 = **0.2 Hz.**

---

* It means that synchronous reactance of the alternator is 20 %.

Tutorial Problem No. 37.8

1. Show that an alternator running in parallel on constant-voltage and frequency bus-bars has a natural time period of oscillation. Deduce a formula for the time of one complete oscillation and calculate its value for a 5000-kVA, 3-phase, 10,000 V machine running at 1500 r.p.m. on constant 50-Hz bus-bars.

   The moment of inertia of the whole moving system is 14112 kg-m^2 and the steady short-circuit current is five times the normal full-load value. **[1.33 second]**

2. A 10,000-kVA, 5-kV, 3-phase, 4-pole, 50-Hz alternator is connected to infinite bus-bars. The short-circuit current is 3.5 times the normal full-load current and the moment of inertia of the rotating system is 21,000 kg-m^2. Calculate its normal period of oscillation. **[1.365 second]**

3. Calculate for full-load and unity p.f., the natural period of oscillation of a 50-Hz, 10,000-kVA, 11-kV alternator driven at 1500 r.p.m. and connected to an infinite bus-bar. The steady short-circuit current is four times the full-load current and the moment of the inertia of the rotating masses is 17,000 kg-m^2. **[1.148 s.] (*Electrical Machinery-II, Madras Univ. Apr. 1976*)**

4. Calculate the rotational inertia in kg-m^2 units of the moving system of 10,000 kVA, 6,600-V, 4-pole, turbo-alternator driven at 1500 r.p.m. for the set to have a natural period of 1 second when running in parallel with a number of other machines. The steady short-circuit current of the alternator is five times the full-load value. **[16,828 kg-m2] (*City & Guilds, London*)**

5. A 3-ϕ, 4-pole, 6,000 kVA, 5,000-V, 50-Hz star-connected alternator is running on constant-voltage and constant-frequency bus-bars. It has a short-circuit reactance of 25% and its rotor has a moment of inertia of 16,800 kg-m^2. Calculate its natural time period of oscillation. **[1.48 second]**

37.42. Maximum Power Output

For given values of terminal voltage, excitation and frequency, there is a maximum power that the alternator is capable of delivering. Fig. 37.103 (*a*) shows full-load conditions for a cylindrical rotor where IR_a drop has been neglected*.

The power output per phase is

$$P = VI\cos\phi = \frac{VIX_S\cos\phi}{X_S}$$

Now, from ΔOBC, we get

$$\frac{IX_S}{\sin\alpha} = \frac{E}{\sin(90+\phi)} = \frac{E}{\cos\phi}$$

$$IX_S\cos\phi = E\sin\alpha$$

$$\therefore \quad P = \frac{EV\sin\alpha}{X_S}$$

Power becomes maximum when $\alpha = 90°$, if V, E and X_S are regarded as constant (of course, E is fixed by excitation).

$$\therefore \quad P_{max} = EV/X_S$$

It will be seen from Fig. 37.103 (*b*) that under maximum power output conditions, I leads V by ϕ and since IX_S leads I by 90°, angle ϕ and hence $\cos\phi$ is fixed = E/IX_S.

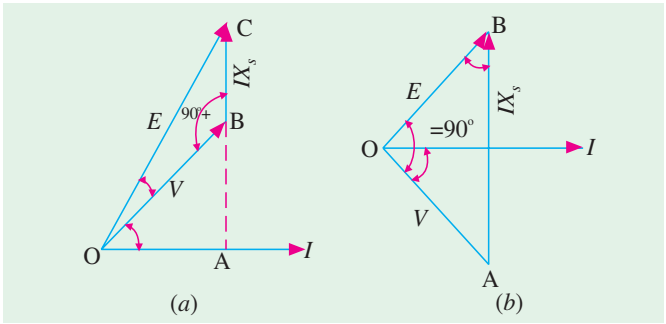


Fig. 37.103

* In fact, this drop can generally be neglected without sacrificing much accuracy of results.

Now, from right-angled ΔAOB, we have that $IX_S = \sqrt{E^2 + V^2}$. Hence, p.f. corresponding to maximum power output is

$$\cos\phi = \frac{E}{\sqrt{E^2 + V^2}}$$

The maximum power output per phase may also be written as

$$P_{max} = VI_{max}\cos\phi = VI_{max}\frac{E}{\sqrt{E^2 + V^2}}$$

where I_{max} represents the current/phase for maximum power output.

If I_f is the full-load current and $\%\, X_S$ is the percentage synchronous reactance, then

$$\%\, X_S = \frac{I_f X_S}{V}\times 100 \quad \therefore \quad \frac{V}{X_S} = \frac{I_f \times 100}{\%\, X_S}$$

Now, $$P_{max} = VI_{max}\frac{E}{\sqrt{E^2 + V^2}} = \frac{EV}{X_S} = \frac{EI_f \times 100}{\%\, X_S}$$

Two things are obvious from the above equations.

(i) $$I_{max} = \frac{100\, I_f}{\%\, X_S}\times\frac{\sqrt{E^2 + V^2}}{V}$$

Substituting the value of $\%X_S$ from above,

$$I_{max} = \frac{100\, I_f}{100\, I_f X_S}\times V\times\frac{\sqrt{E^2 + V^2}}{V} = \frac{\sqrt{E^2 + V^2}}{X_S}$$

(ii) $$P_{max} = \frac{100\, EI_f}{\%\, X_S} = \frac{E}{V}\cdot\frac{100}{\%\, X_S}\times VI_f \text{ per phase}$$

$$= \frac{E}{V}\cdot\frac{100}{\%\, X_S}\times \text{ F.L. power output at u.p.f.}$$

Total maximum power output of the alternator is

$$= \frac{E}{V}\cdot\frac{100}{\%\, X_S}\times \text{ F.L. power output at u.p.f.}$$

Example 37.66. *Derive the condition for the maximum output of a synchronous generator connected to infinite bus-bars and working at constant excitation.*

A 3-ϕ, 11-kV, 5-MVA, Y-connected alternator has a synchronous impedance of (1 + j 10) ohm per phase. Its excitation is such that the generated line e.m.f. is 14 kV. If the alternator is connected to infinite bus-bars, determine the maximum output at the given excitation

(Electrical Machines-III, Gujarat Univ. 1984)

Solution. For the first part, please refer to Art. 37.41

P_{max} per phase $= \frac{EV}{X_S}$ — if R_a is neglected $= \frac{V}{Z_s}(E - V\cos\theta)$ — if R_a is considered

Now, $E = 14{,}000/\sqrt{3} = 8{,}083$ V ; $V = 11{,}000/\sqrt{3} = 6352$ V

$\cos\theta = R_a/Z_S = 1\sqrt{1^2 + 10^2} = 1/10.05$

$\therefore \quad P_{max}$ per phase $= \frac{8083\times 6352}{10\times 1000} = 5{,}135$ kW

Total $P_{max} = 3\times 5{,}135 =$ **15,405 kW**

More accurately, P_{max}/phase $= \frac{6352}{10.05}\left(8083 - \frac{6352}{10.05}\right) = \frac{6352}{10.05}\times\frac{7451}{1000} = 4{,}711$ kW

Total $P_{max} = 4{,}711\times 3 =$ **14,133 kW.**

Example 37.67. *A 3-phase, 11-kVA, 10-MW, Y-connected synchronous generator has synchronous impedance of (0.8 + j 8.0) ohm per phase. If the excitation is such that the open circuit voltage is 14 kV, determine (i) the maximum output of the generator (ii) the current and p.f. at the maximum output.*

(Electrical Machines-III, Gujarat Univ. 1987)

Solution. (*i*) If we neglect R_a*, the P_{max} per phase = EV/X_S

where V is the terminal voltage (or bus-bar voltage in general) and **E** the e.m.f. of the machine.

$$\therefore \quad P_{max} = \frac{(11,000/\sqrt{3}) \times (14,000/\sqrt{3})}{8} = \frac{154,000}{24} \text{ kW/phase}$$

Total $P_{max} = 3 \times 154{,}000/24 = 19{,}250 \text{ kW} = \mathbf{19.25\ MW}$

Incidentally, this output is nearly twice the normal output.

(ii) $$I_{max} = \frac{\sqrt{E^2+V^2}}{X_S} = \frac{\sqrt{[(14,000/\sqrt{3})^2 + (11,000/\sqrt{3})^2]}}{8} = \mathbf{1287\ A}$$

(iii) $$\text{p.f.} = \frac{E}{\sqrt{E^2+V^2}} = \frac{14000/\sqrt{3}}{\sqrt{(14,000/\sqrt{3})^2 + (11,000/\sqrt{3})^2}} = \mathbf{0.786\ (lead).}$$

QUESTIONS AND ANSWERS ON ALTERNATORS

Q. 1. What are the two types of turbo-alternators ?

Ans. Vertical and horizontal.

Q. 2. How do you compare the two ?

Ans. Vertical type requires less floor space and while step bearing is necessary to carry the weight of the moving element, there is very little friction in the main bearings. The horizontal type requires no step bearing, but occupies more space.

Q. 3. What is step bearing ?

Ans. It consists of two cylindrical cast iron plates which bear upon each other and have a central recess between them. Suitable oil is pumped into this recess under considerable pressure.

Q. 4. What is direct-connected alternator ?

Ans. One in which the alternator and engine are directly connected. In other words, there is no intermediate gearing such as belt, chain etc. between the driving engine and alternator.

Q. 5. What is the difference between direct-connected and direct-coupled units ?

Ans. In the former, alternator and driving engine are directly and permanently connected. In the latter case, engine and alternator are each complete in itself and are connected by some device such as friction clutch, jaw clutch or shaft coupling.

Q. 6. Can a d.c. generator be converted into an alternator ?

Ans. Yes.

Q. 7. How ?

Ans. By providing two collector rings on one end of the armature and connecting these two rings to two points in the armature winding 180° apart.

Q. 8. Would this arrangement result in a desirable alternator ?

Ans. No.

* If R_a is not neglected, then $P_{max} = \frac{V}{Z_S}(E - V\cos\theta)$ where $\cos\theta = R_a/Z_S$ (Ex. 37.66)

Q. 9. How is a direct-connected exciter arranged in an alternator ?

Ans. The armature of the exciter is mounted on the shaft of the alternator close to the spider hub. In some cases, it is mounted at a distance sufficient to permit a pedestal and bearing to be placed between the exciter and the hub.

Q. 10. Any advantage of a direct-connected exciter ?

Ans. Yes, economy of space.

Q. 11. Any disadvantage ?

Ans. The exciter has to run at the same speed as the alternator which is slower than desirable. Hence, it must be larger for a given output than the gear-driven type, because it can be run at high speed and so made proportionately smaller.

OBJECTIVE TESTS – 37

1. The frequency of voltage generated by an alternator having 4-poles and rotating at 1800 r.p.m. is .....hertz.

(*a*) 60 (*b*) 7200

(*c*) 120 (*d*) 450.

2. A 50-Hz alternator will run at the greatest possible speed if it is wound for ..... poles.

(*a*) 8 (*b*) 6

(*c*) 4 (*d*) 2.

3. The main disadvantage of using short-pitch winding in alterators is that it

(*a*) reduces harmonics in the generated voltage

(*b*) reduces the total voltage around the armature coils

(*c*) produces asymmetry in the three phase windings

(*d*) increases Cu of end connections.

4. Three-phase alternators are invariably Y-connected because

(*a*) magnetic losses are minimised

(*b*) less turns of wire are required

(*c*) smaller conductors can be used

(*d*) higher terminal voltage is obtained.

5. The winding of a 4-pole alternator having 36 slots and a coil span of 1 to 8 is short-pitched by ..... degrees.

(*a*) 140 (*b*) 80

(*c*) 20 (*d*) 40.

6. If an alternator winding has a fractional pitch of 5/6, the coil span is ..... degrees.

(*a*) 300 (*b*) 150

(*c*) 30 (*d*) 60.

7. The harmonic which would be totally eliminated from the alternator e.m.f. using a fractional pitch of 4/5 is

(*a*) 3rd (*b*) 7th

(*c*) 5th (*d*) 9th.

8. For eliminating 7th harmonic from the e.m.f. wave of an alternator, the fractional-pitch must be

(*a*) 2/3 (*b*) 5/6

(*c*) 7/8 (*d*) 6/7.

9. If, in an alternator, chording angle for fundamental flux wave is α, its value for 5th harmonic is

(*a*) 5α (*b*) $\alpha/5$

(*c*) 25α (*d*) $\alpha/25$.

10. Regarding distribution factor of an armature winding of an alternator which statement is false?

(*a*) it decreases as the distribution of coils (slots/pole) increases

(*b*) higher its value, higher the induced e.m.f. per phase

(*c*) it is not affected by the type of winding either lap, or wave

(*d*) it is not affected by the number of turns per coil.

11. When speed of an alternator is changed from 3600 r.p.m. to 1800 r.p.m., the generated e.m.f./phases will become

(*a*) one-half (*b*) twice

(*c*) four times (*d*) one-fourth.

12. The magnitude of the three voltage drops in an alternator due to armature resistance, leakage reactance and armature reaction is solely determined by

(*a*) load current, I_a

(*b*) p.f. of the load
(*c*) whether it is a lagging or leading p.f. load
(*d*) field construction of the alternator.

13. Armature reaction in an alternator primarily affects
(*a*) rotor speed
(*b*) terminal voltage per phase
(*c*) frequency of armature current
(*d*) generated voltage per phase.

14. Under no-load condition, power drawn by the prime mover of an alternator goes to
(*a*) produce induced e.m.f. in armature winding
(*b*) meet no-load losses
(*c*) produce power in the armature
(*d*) meet Cu losses both in armature and rotor windings.

15. As load p.f. of an alternator becomes more leading, the value of generated voltage required to give rated terminal voltage
(*a*) increases
(*b*) remains unchanged
(*c*) decreases
(*d*) varies with rotor speed.

16. With a load p.f. of unity, the effect of armature reaction on the main-field flux of an alternator is
(*a*) distortional (*b*) magnetising
(*c*) demagnetising (*d*) nominal.

17. At lagging loads, armature reaction in an alternator is
(*a*) cross-magnetising (*b*) demagnetising
(*c*) non-effective (*d*) magnetising.

18. At leading p.f., the armature flux in an alternator ..... the rotor flux.
(*a*) opposes (*b*) aids
(*c*) distorts (*d*) does not affect.

19. The voltage regulation of an alternator having 0.75 leading p.f. load, no-load induced e.m.f. of 2400V and rated terminal voltage of 3000V is ..... percent.
(*a*) 20 (*b*) – 20
(*c*) 150 (*d*) – 26.7

20. If, in a 3-ϕ alternator, a field current of 50A produces a full-load armature current of 200 A on short-circuit and 1730 V on open circuit, then its synchronous impedance is ..... ohm.
(*a*) 8.66 (*b*) 4
(*c*) 5 (*d*) 34.6

21. The power factor of an alternator is determined by its
(*a*) speed
(*b*) load
(*c*) excitation
(*d*) prime mover.

22. For proper parallel operation, a.c. polyphase alternators must have the same
(*a*) speed (*b*) voltage rating
(*c*) kVA rating (*d*) excitation.

23. Of the following conditions, the one which does not have to be met by alternators working in parallel is
(*a*) terminal voltage of each machine must be the same
(*b*) the machines must have the same phase rotation
(*c*) the machines must operate at the same frequency
(*d*) the machines must have equal ratings.

24. After wiring up two 3-ϕ alternators, you checked their frequency and voltage and found them to be equal. Before connecting them in parallel, you would
(*a*) check turbine speed
(*b*) check phase rotation
(*c*) lubricate everything
(*d*) check steam pressure.

25. Zero power factor method of an alternator is used to find its
(*a*) efficiency
(*b*) voltage regulation
(*c*) armature resistance
(*d*) synchronous impedance.

26. Some engineers prefer `lamps bright' synchronization to 'lamps dark' synchronization because
(*a*) brightness of lamps can be judged easily
(*b*) it gives sharper and more accurate synchronization
(*c*) flicker is more pronounced
(*d*) it can be performed quickly.

27. It is never advisable to connect a stationary alternator to live bus-bars because it
(*a*) is likely to run as synchronous motor
(*b*) will get short-circuited
(*c*) will decrease bus-bar voltage though momentarily
(*d*) will disturb generated e.m.fs. of other alternators connected in parallel.

28. Two identical alternators are running in parallel and carry equal loads. If excitation of one alternator is increased without changing its steam supply, then

(*a*) it will keep supplying almost the same load

(*b*) kVAR supplied by it would decrease

(*c*) its p.f. will increase

(*d*) kVA supplied by it would decrease.

29. Keeping its excitation constant, if steam supply of an alternator running in parallel with another identical alternator is increased, then

(*a*) it would over-run the other alternator

(*b*) its rotor will fall back in phase with respect to the other machine

(*c*) it will supply greater portion of the load

(*d*) its power factor would be decreased.

30. The load sharing between two steam-driven alternators operating in parallel may be adjusted by varying the

(*a*) field strengths of the alternators

(*b*) power factors of the alternators

(*c*) steam supply to their prime movers

(*d*) speed of the alternators.

31. Squirrel-cage bars placed in the rotor pole faces of an alternator help reduce hunting

(*a*) above synchronous speed only

(*b*) below synchronous speed only

(*c*) above and below synchronous speeds both

(*d*) none of the above.

(Elect. Machines, A.M.I.E. Sec. B, 1993)

32. For a machine on infinite bus active power can be varied by

(*a*) changing field excitation

(*b*) changing of prime cover speed

(*c*) both (*a*) and (*b*) above

(*d*) none of the above .

(Elect. Machines, A.M.I.E. Sec. B, 1993)

ANSWERS

1. *a*	**2.** *d*	**3.** *b*	**4.** *d*	**5.** *d*	**6.** *b*	**7.** *c*	**8.** *d*	**9.** *a*	**10.** *b*	**11.** *a*
12. *a*	**13.** *d*	**14.** *b*	**15.** *c*	**16.** *a*	**17.** *d*	**18.** *b*	**19.** *b*	**20.** *c*	**21.** *b*	**22.** *b*
23. *d*	**24.** *b*	**25.** *b*	**26.** *b*	**27.** *b*	**28.** *a*	**29.** *c*	**30.** *c.*	**31.** *c*	**32.** *b*	

ROUGH WORK

GO To FIRST

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CHAPTER 38

SYNCHRONOUS MOTOR

Learning Objectives

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- Methods of Starting
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- Comparison between Synchronous and Induction Motors
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Rotary synchronous motor for lift applications

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38.1. Synchronous Motor—General

A synchronous motor (Fig. 38.1) is electrically identical with an alternator or a.c. generator. In fact, a given synchronous machine may be used, at least theoretically, as an alternator, when driven mechanically or as a motor, when driven electrically, just as in the case of d.c. machines. Most synchronous motors are rated between 150 kW and 15 MW and run at speeds ranging from 150 to 1800 r.p.m.



Synchronous motor

Some characteristic features of a synchronous motor are worth noting :

1. It runs either at synchronous speed or not at all *i.e.* while running it maintains a constant speed. The only way to change its speed is to vary the supply frequency (because $Ns = 120\,f / P$).

2. It is not inherently self-starting. It has to be run upto synchronous (or near synchronous) speed by some means, before it can be synchronized to the supply.

3. It is capable of being operated under a wide range of power factors, both lagging and leading. Hence, it can be used for power correction purposes, in addition to supplying torque to drive loads.

38.2. Principle of Operation

As shown in Art. 34.7, when a 3-ϕ winding is fed by a 3-ϕ supply, then a magnetic flux of constant magnitude but ***rotating*** at synchronous speed, is produced. Consider a two-pole stator of Fig. 38.2, in which are shown two stator poles (marked N_S and S_S) rotating at synchronous speed, say, in clockwise direction. With the rotor position as shown, suppose the stator poles are at that instant situated at points *A* and *B*. The two similar poles, *N* (of rotor) and N_S (of stator) as well as *S* and S_S will repel each other, with the result that the rotor tends to rotate in the anticlockwise direction.

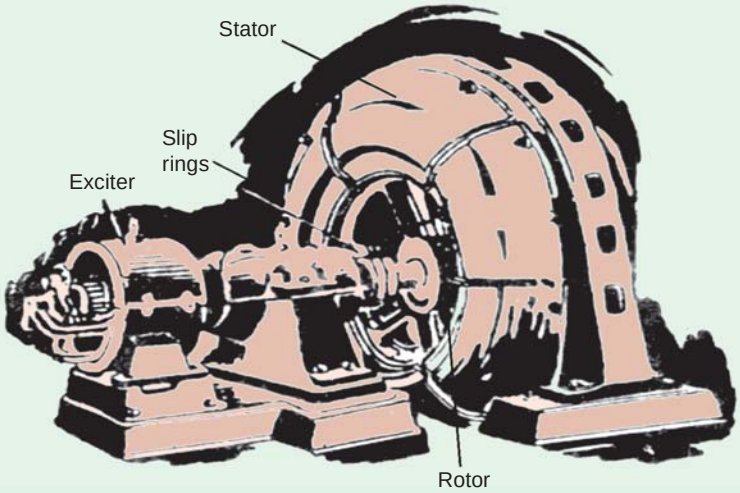


Fig. 38.1

But half a period later, stator poles, having rotated around, interchange their positions *i.e.* N_S is at point *B* and S_S at point *A*. Under these conditions, N_S attracts *S* and S_S attracts *N*. Hence, rotor tends to rotate clockwise (which is just the reverse of the first direction). Hence, we find that due to continuous and rapid rotation of stator poles, the rotor is subjected to a torque which is rapidly reversing *i.e.*, in quick succession, the rotor is subjected to torque which tends to move it first in one direction and then in the opposite direction. Owing to its large inertia, the rotor cannot instantaneously respond to such quickly-reversing torque, with the result that it remains stationary.

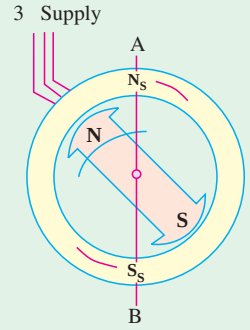
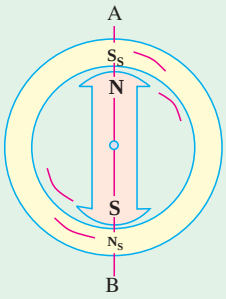
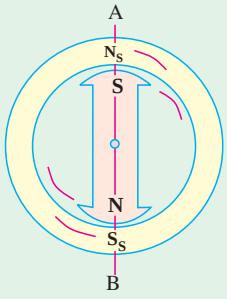


Fig. 38.2



(*a*)

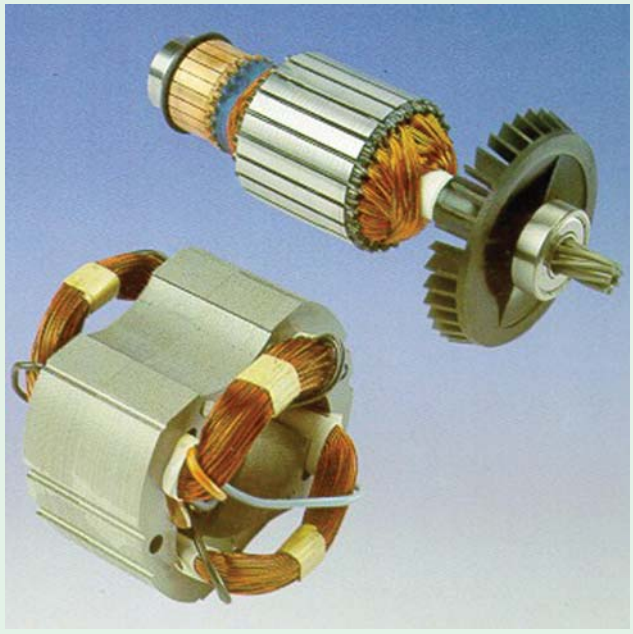
Fig. 38.3



(*b*)

Fig. 38.3

Now, consider the condition shown in Fig. 38.3 (*a*). The stator and rotor poles are attracting each other. Suppose that the rotor is not stationary, but is rotating clockwise, with such a speed that it turns through one pole-pitch by the time the stator poles interchange their positions, as shown in Fig. 38.3 (*b*). Here, again the stator and rotor poles attract each other. It means that if the rotor poles also shift their positions along with the stator poles, then they will continuously experience a unidirectional torque *i.e.*, clockwise torque, as shown in Fig. 38.3.



The rotor and the stator parts of motor.

38.3. Method of Starting

The rotor (which is as yet unexcited) is speeded up to synchronous / near synchronous speed by some arrangement and then excited by the d.c. source. The moment this (near) synchronously rotating rotor is excited, it is magnetically locked into position with the stator *i.e.*, the rotor poles are engaged with the stator poles and both run synchronously in the same direction. It is because of this interlocking of stator and rotor poles that the motor has either to run synchronously or not at all. The synchronous speed is given by the usual relation $N_S = 120\, f / P$.

However, it is important to understand that the arrangement between the stator and rotor poles is ***not an absolutely rigid one***. As the load on the motor is increased, the rotor progressively tends to fall back ***in phase*** (but *not* in speed as in d.c. motors) by some angle (Fig. 38.4) ***but it still continues to run synchronously***.The value of this load angle or coupling angle (as it is called) depends on the amount of load to be met by the motor. In other words, the torque developed by the motor depends on this angle, say, α.

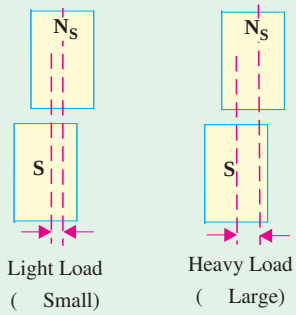


Fig. 38.4

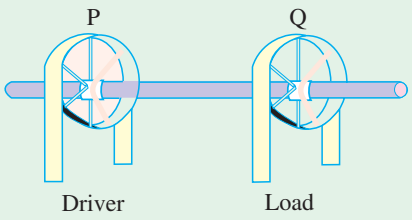


Fig. 38.5

The working of a synchronous motor is, in many ways, similar to the transmission of mechanical power by a shaft. In Fig. 38.5 are shown two pulleys *P* and *Q* transmitting power from the driver to the load. The two pulleys are assumed to be keyed together (just as stator and rotor poles are interlocked) hence they run at exactly the same (average) speed. When *Q* is loaded, it slightly falls behind owing to the twist in the shaft (twist angle corresponds to α in motor), the angle of twist, in fact, being a measure of the torque transmitted. It is clear that unless *Q* is so heavily loaded as to break the coupling, both ***pulleys must run at exactly the same (average) speed***.

38.4. Motor on Load with Constant Excitation

Before considering as to what goes on inside a synchronous motor, it is worthwhile to refer briefly to the d.c. motors. We have seen (Art. 29.3) that when a d.c. motor is running on a supply of, say, V volts then, on rotating, a back e.m.f. E_b is set up in its armature conductors. The resultant voltage across the armature is $(V - E_b)$ and it causes an armature current $I_a = (V - E_b)/ R_a$ to flow where R_a is armature circuit resistance. The value of E_b depends, among other factors, on the speed of the rotating armature. The mechanical power developed in armature depends on $E_b I_a$ (E_b and I_a being in opposition to each other).

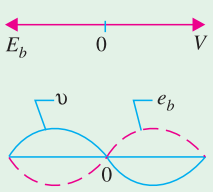


Fig. 38.6

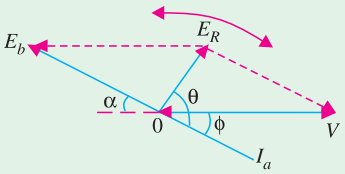


Fig. 38.7

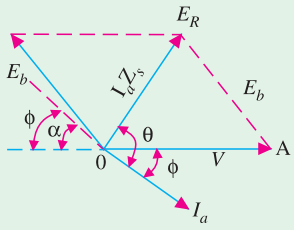
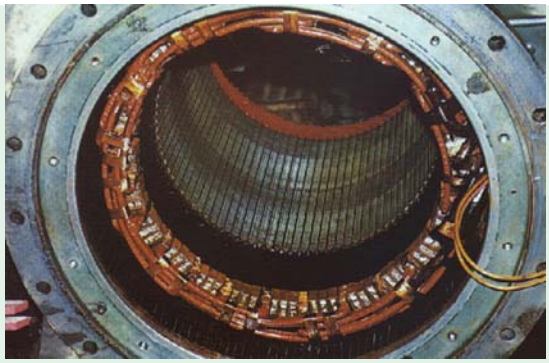


Fig. 38.8

Similarly, in a synchronous machine, a back e.m.f. E_b is set up in the armature (stator) by the rotor flux which opposes the applied voltage V. This back e.m.f. depends on rotor excitation only (and not on speed, as in d.c. motors). The net voltage in armature (stator) is the ***vector difference*** (not arithmetical, as in d.c. motors) of V and E_b. Armature current is obtained by dividing this *vector* difference of voltages by armature impedance (not resistance as in d.c. machines).

Fig. 38.6 shows the condition when the motor (properly synchronized to the supply) is running on ***no-load*** and has ***no losses***.* and is having field excitation which makes $E_b = V$. It is seen that vector difference of E_b and V is zero and so is the armature current. Motor intake is zero, as there is neither load nor losses to be met by it. In other words, the motor just floats.



Stator of synchronous motor

If motor is on no-load, but it has losses, then the vector for E_b falls back (vectors are rotating anti-clockwise) by a certain small angle α (Fig. 38.7), so that a resultant voltage E_R and hence current I_a is brought into existence, which supplies losses.**

If, now, the motor is loaded, then its rotor will further fall back ***in phase*** by a greater value of angle α – called the load angle or coupling angle (corresponding to the twist in the shaft of the pulleys). The resultant voltage E_R is increased and motor draws an increased armature current (Fig. 38.8), though at a slightly decreased power factor.

38.5. Power Flow within a Synchronous Motor

Let R_a = armature resistance / phase ; X_S = synchronous reactance / phase

then $\mathbf{Z_S} = \mathbf{R}_a + j\,X_S$; $\mathbf{I}_a = \dfrac{\mathbf{E_R}}{\mathbf{Z_S}} = \dfrac{\mathbf{V} - \mathbf{E_b}}{\mathbf{Z_S}}$; Obviously, $\mathbf{V} = \mathbf{E}_b + \mathbf{I}_a \mathbf{Z_S}$

The angle θ (known as internal angle) by which I_a lags behind E_R is given by $\tan\theta = X_S / R_a$.

If R_a is negligible, then $\theta = 90^\circ$. Motor input = $V\,I_a \cos\phi$ —per phase

Here, V is applied voltage / phase.

Total input for a star-connected, 3-phase machine is, $P = \sqrt{3}\,V_L\,.\,I_L \cos\phi$.

The mechanical power developed in the rotor is

P_m = back e.m.f. × armature current × cosine of the angle between the two *i.e.*, angle between I_a and E_b reversed.

$= E_b\,I_a \cos(\alpha - \phi)$ per phase ...Fig. 38.8

Out of this power developed, some would go to meet iron and friction and excitation losses. Hence, the power available at the shaft would be less than the developed power by this amount.

Out of the input power / phase $V\,I_a \cos\phi$, and amount $I_a^2\,R_a$ is wasted in armature***, the rest ($V.\,I_a \cos\phi - I_a^2\,R_a$) appears as mechanical power in rotor; out of it, iron, friction and excitation losses are met and the rest is available at the shaft. If power input / phase of the motor is P, then

$$P = P_m + I_a^2\,R_a$$

or mechanical power in rotor $P_m = P - I_a^2\,R_a$ —per phase

For three phases $P_m = \sqrt{3}\,V_L\,I_L \cos\phi - 3\,I_a^2\,R_a$

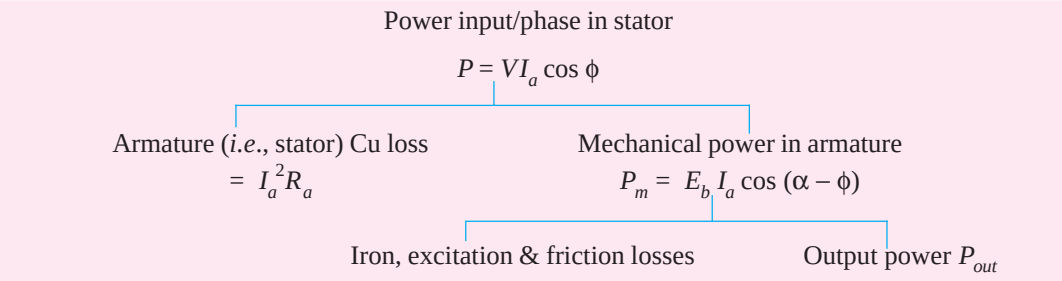
The per phase power development in a synchronous machine is as under :

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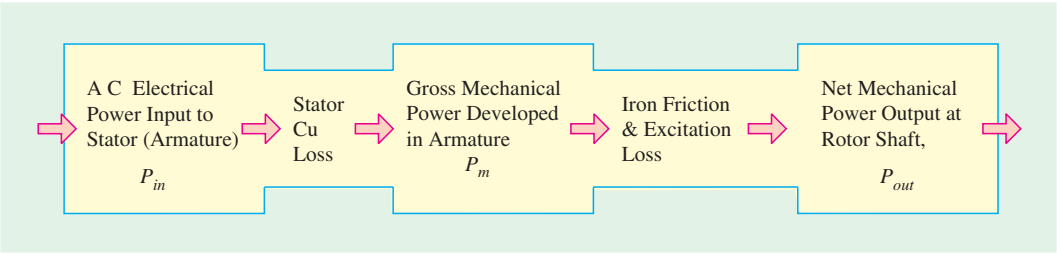
* This figure is exactly like Fig. 37.74 for alternator except that it has been shown horizontally rather than vertically.

** It is worth noting that magnitude of E_b does not change, only its phase changes. Its magnitude will change only when rotor dc excitation is changed *i.e.*, when magnetic strength of rotor poles is changed.

*** The Cu loss in rotor is not met by motor ac input, but by the dc source used for rotor excitation.



Different power stages in a synchronous motor are as under :



38.6. Equivalent Circuit of a Synchronous Motor

Fig. 38.9 (*a*) shows the equivalent circuit model for one armature phase of a cylindrical rotor synchronous motor.

It is seen from Fig. 38.9 (*b*) that the phase applied voltage V is the vector sum of reversed back e.m.f. *i.e.*, $-E_b$ and the impedance drop $I_a Z_S$. In other words, $V = (-E_b + I_a Z_S)$. The angle α* between the phasor for V and E_b is called the load angle or power angle of the synchronous motor.

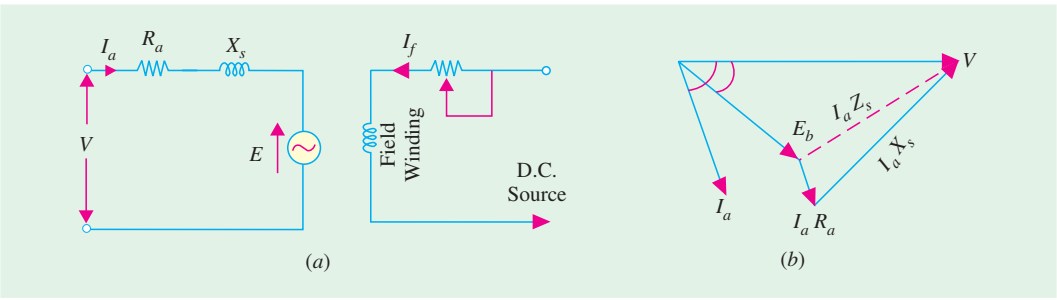


Fig. 38.9

38.7. Power Developed by a Synchronous Motor

Except for very small machines, the armature resistance of a synchronous motor is negligible as compared to its synchronous reactance. Hence, the equivalent circuit for the motor becomes as shown in Fig. 38.10 (*a*). From the phasor diagram of Fig. 38.10 (*b*), it is seen that

$$AB = E_b \sin \alpha = I_a X_S \cos \phi$$

or $$VI_a \cos \phi = \frac{E_b V}{X_S} \sin \alpha$$

Now, $VI_a \cos \phi =$ motor power input/phase

---

* This angle was designated as δ when discussing synchronous generators.

$$\therefore \quad P_{in} = \frac{E_b V}{X_S} \sin \alpha \quad \text{...per phase*}$$

$$= 3\frac{E_b V}{X_S} \sin \alpha \quad \text{... for three phases}$$

Since stator Cu losses have been neglected, P_{in} also represents the gross mechanical power $\{P_m\}$ developed by the motor.

$$\therefore \quad P_m = \frac{3E_b V}{X_S} \sin \alpha$$

The gross torque developed by the motor is $T_g = 9.55\, P_m / N_s$ N–m ...Ns in rpm.

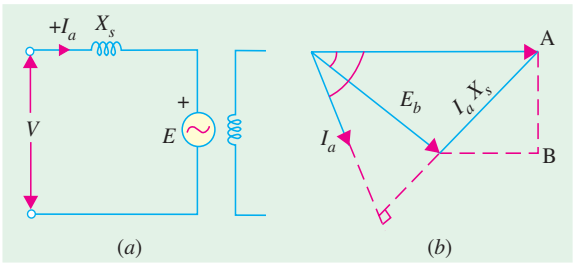


Fig. 38.10

Example 38.1. *A 75-kW, 3-ϕ, Y-connected, 50-Hz, 440-V cylindrical rotor synchronous motor operates at rated condition with 0.8 p.f. leading. The motor efficiency excluding field and stator losses, is 95% and X_S = 2.5 Ω. Calculate* ***(i)*** *mechanical power developed* ***(ii)*** *armature current* ***(iii)*** *back e.m.f.* ***(iv)*** *power angle and* ***(v)*** *maximum or pull-out torque of the motor.*

Solution. $N_S = 120 \times 50/4 = 1500$ rpm = 25 rps

(i) $P_m = P_{in} = P_{out} / \eta = 75 \times 10^3/0.95 =$ **78,950 W**

(ii) Since power input is known

$\therefore \quad \sqrt{3} \times 440 \times I_a \times 0.8 = 78{,}950; \quad I_a =$ **129 A**

(iii) Applied voltage/phase = $440/\sqrt{3} = 254$ V. Let V = 254 ∠0° as shown in Fig. 38.11.

Now, $V = E_b + j\, IX_S$ or $E_b = V - j\, I_a X_S = 254 \angle 0° - 129 \angle 36.9° \times 2.5 \angle 90° = 250 \angle 0° - 322 \angle 126.9° = 254 - 322$ (cos 126.9° + j sin 126.9°) = 254 – 322 (– 0.6 + j 0.8) = **516 ∠–30°**

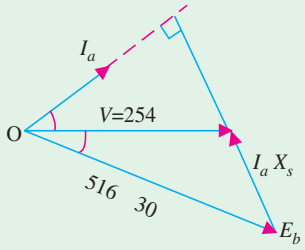


Fig. 38.11

(iv) $\therefore \quad \alpha =$ **–30°**

(v) pull-out torque occurs when α = 90°

$$\text{maximum } P_m = 3\frac{E_b V}{X_S} \sin \delta = 3\,\frac{256 \times 516}{2.5} = \sin 90° = 157{,}275 \text{ W}$$

$\therefore$ pull-out torque = 9.55 × 157, 275/1500 = **1,000 N-m**

38.8. Synchronous Motor with Different Excitations

A synchronous motor is said to have normal excitation when its $E_b = V$. If field excitation is such that $E_b < V$, the motor is said to be ***under-excited***. In both these conditions, it has a lagging power factor as shown in Fig. 38.12.

On the other hand, if d.c. field excitation is such that $E_b > V$, then motor is said to be ***over-excited*** and draws a leading current, as shown in Fig. 38.13 (*a*). There will be some value of excitation for which armature current will be in phase with V, so that power factor will become unity, as shown in Fig. 38.13 (*b*).

---

* Strictly speaking, it should be $P_{in} = \dfrac{-E_b V}{X_S} \sin \alpha$

The value of α and back e.m.f. E_b can be found with the help of vector diagrams for various power factors, shown in Fig. 38.14.

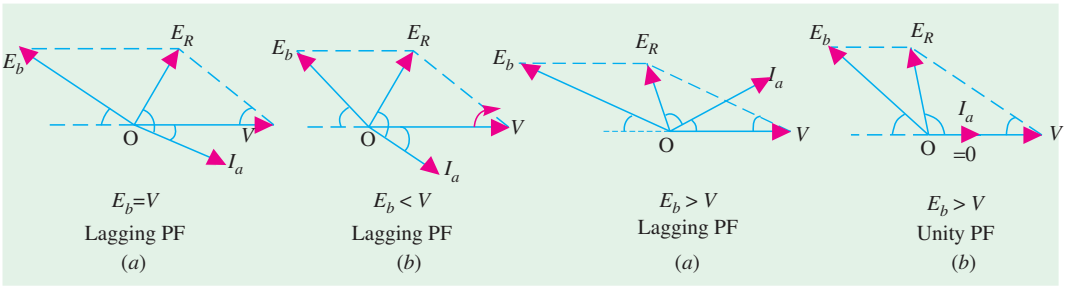


Fig. 38.12

Fig. 38.13

(*i*) Lagging p.f. As seen from Fig. 38.14 (*a*)

$$AC^2 = AB^2 + BC^2 = [V - E_R \cos(\theta - \phi)]^2 + [E_R \sin(\theta - \phi)]^2$$

$$\therefore \quad E_b = \sqrt{[V - I_a Z_S \cos(\theta - \phi)]^2 + [I_a Z_S \sin(\theta - \phi)]^2}$$

$$\text{Load angle} \quad \alpha = \tan^{-1}\left(\frac{BC}{AB}\right) = \tan^{-1}\left[\frac{I_a Z_S \sin(\theta - \phi)}{V - I_a Z_S \cos(\theta - \phi)}\right]$$

(*ii*) Leading p.f. [38.14 (*b*)]

$$E_b = V + I_a Z_S \cos[180^\circ - (\theta + \phi)] + j I_a Z_S \sin[180^\circ - (\theta + \phi)]$$

$$\alpha = \tan^{-1}$$

(*iii*) Unity p.f. [Fig. 38.14 (*c*)]

Here, $OB = I_a R_a$ and $BC = I_a X_S$

$$\therefore \quad E_b = (V - I_a R_a) + j I_a X_S \; ; \; \alpha = \tan^{-1}$$

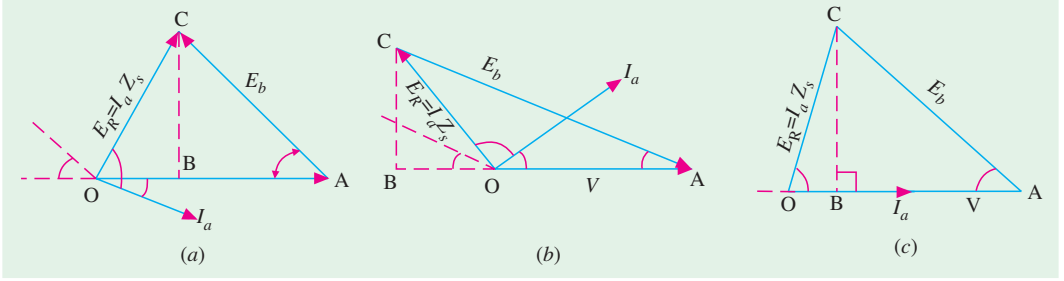


Fig. 38.14

38.9. Effect of Increased Load with Constant Excitation

We will study the effect of increased load on a synchronous motor under conditions of normal, under and over-excitation (ignoring the effects of armature reaction). With normal excitation, $E_b = V$, with under excitation, $E_b < V$ and with over-excitation, $E_b > V$. Whatever the value of excitation, it would be kept ***constant*** during our discussion. It would also be assumed that R_a is negligible as compared to X_S so that phase angle between E_R and I_a *i.e.*, $\theta = 90^\circ$.

(*i*) Normal Excitation

Fig. 38.15. (*a*) shows the condition when motor is running with light load so that **(*i*)** torque angle

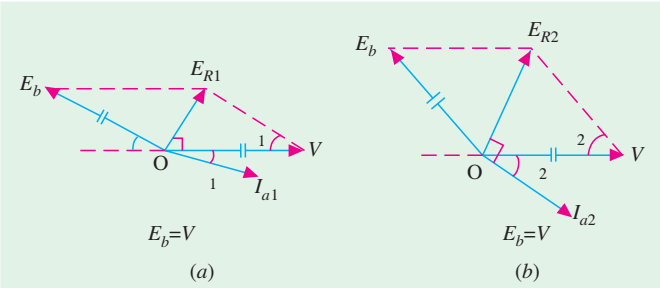


Fig. 38.15

α_1 is small ***(ii)*** so E_{R1} is small ***(iii)*** hence I_{a1} is small and (*iv*) ϕ_1 is small so that cos ϕ_1 is large.

Now, suppose that load on the motor is ***increased*** as shown in Fig. 38.15 (*b*). For meeting this extra load, motor must develop more torque by drawing more armature current. Unlike a d.c. motor, a synchronous motor cannot increase its I_a by decreasing its speed and hence E_b because both are constant in its case. What actually happens is as under :

1. rotor falls back ***in phase i.e.***, load angle increases to α_2 as shown in Fig. 38.15 (*b*),

2. the resultant voltage in armature is increased ***considerably*** to new value E_{R2},

3. as a result, I_{a1} increases to I_{a2}, thereby increasing the torque developed by the motor,

4. ϕ_1 increases to ϕ_2, so that power factor ***decreases*** from cos ϕ_1 to the new value cos ϕ_2.



Geared motor added to synchronous servo motor line offers a wide range of transmission ratios, and drive torques.

Since increase in I_a is much greater than the ***slight*** decrease in power factor, the torque developed by the motor is ***increased*** (on the whole) to a new value sufficient to meet the extra load put on the motor. It will be seen that essentially it is by increasing its I_a that the motor is able to carry the extra load put on it.

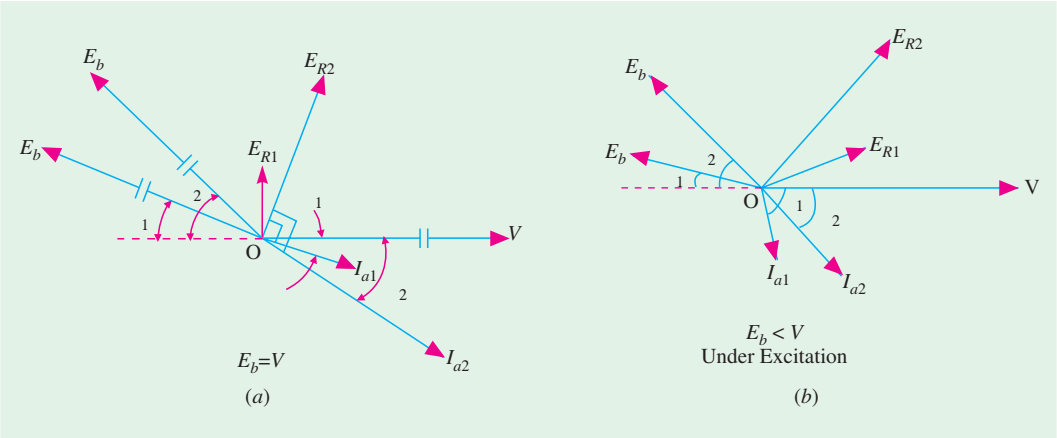


Fig. 38.16

A phase summary of the effect of increased load on a synchronous motor at ***normal excitation*** is shown in Fig. 38.16 (a) It is seen that there is a comparatively much greater ***increase*** in I_a than in ϕ.

(*ii*) Under-excitation

As shown in Fig. 38.16 (*b*), with a small load and hence, small torque angle α_1, I_{a1} lags behind V by a ***large*** phase angle ϕ_1 which means poor power factor. Unlike normal excitation, a much larger armature current must flow for developing the same power because of poor power factor. That is why I_{a1} of Fig. 38.16 (*b*) is larger than I_{a1} of Fig. 38.15 (*a*).

As load increases, E_{R1} increases to E_{R2}, consequently I_{a1} increases to I_{a2} and p.f. angle ***decreases*** from ϕ_1 to ϕ_2 or p.f. ***increases*** from cos ϕ_1 to cos ϕ_2. Due to increase both in I_a and p.f., power generated by the armature increases to meet the increased load. As seen, in this case, ***change in power factor is more than the change in I_a***.

(*iii*) Over-excitation

When running on light load, α_1 is small but I_{a1} is comparatively larger and ***leads V*** by a larger angle ϕ_1. Like the under-excited motor, as more load is applied, the power factor improves and ***approaches*** unity. The armature current also increases thereby producing the necessary increased armature power to meet the increased applied load (Fig. 38.17). However, it should be noted that in this case, power factor angle ϕ decreases (or p.f. increases) at a faster rate than the armature current thereby producing the necessary increased power to meet the increased load applied to the motor.

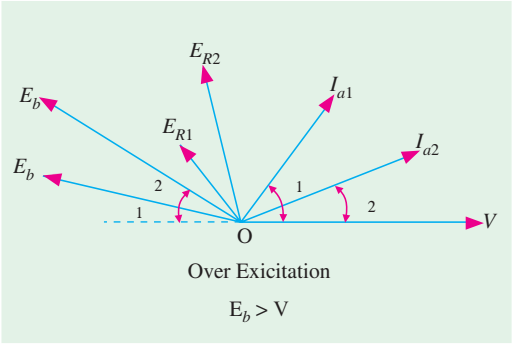


Fig. 38.17

Summary

The main points regarding the above three cases can be summarized as under :

1. As load on the motor increases, I_a ***increases regardless of excitation***.
2. For under-and over-excited motors, p.f. tends to approach unity with increase in load.
3. Both with under-and over-excitation, change in p.f. is greater than in I_a with increase in load.
4. With normal excitation, when load is increased change in I_a is greater than in p.f. which tends to become increasingly lagging.

Example 38.2. *A 20-pole, 693-V, 50-Hz, 3-ϕ, Δ-connected synchronous motor is operating at no-load with normal excitation. It has armature ressistance per phase of zero and synchronous reactance of 10 Ω. If rotor is retarded by 0.5° (mechanical) from its synchronous position, compute.*

(*i*) *rotor displacement in electrical degrees*

(*ii*) *armature emf / phase*

(*iii*) *armature current / phase*

(*iv*) *power drawn by the motor*

(*v*) *power developed by armature*

How will these quantities change when motor is loaded and the rotor displacement increases to 5° (mechanical) ?

(Elect. Machines, AMIE Sec. B, 1993)

Solution. (*a*) 0.5° (mech) Displacement [Fig 38.18 (*a*)]

(*i*) α (elect.) $= \frac{P}{2} \times \alpha$ (mech)

$\therefore$ α (elect)

$= \frac{20}{2} \times 0.5 =$ **5° (elect)**

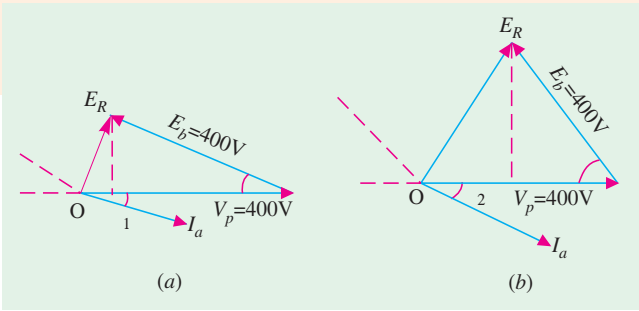


Fig. 38.18

(ii) $V_p = V_L / \sqrt{3} = 693 / \sqrt{3}$
$= 400$ V,
$E_b = V_p = 400$ V
$\therefore \quad E_R = (V_p - E_b \cos \alpha) + j E_b \sin \alpha = (400 - 400 \cos 5^\circ + j\, 400 \sin 5^\circ)$
$= 1.5 + j\, 35 =$ **$35 \angle 87.5$ V/phase**

(iii) $Z_S = 0 + j10 = 10 \angle 90^\circ$; $I_a = E_R / Z_S = 35 \angle 87.5^\circ / 10 \angle 90^\circ =$ **$3.5 \angle -2.5^\circ$ A/phase**
Obviously, I_a lags behind V_p by 2.5°

(iv) Power input/phase $V_p I_a \cos \phi = 400 \times 3.5 \times \cos 2.5^\circ = 1399$ W
Total input power $= 3 \times 1399 =$ **4197 W**

(v) Since R_a is negligible, armature Cu loss is also negligible. Hence 4197 W also represent power developed by armature.

(b) 5° (mech) Displacement – Fig. 38.18 (*b*)

(i) α (elect) $= \frac{20}{2} \times 5^\circ =$ **50°**

(ii) $E_R = (400 - 400 \cos 50^\circ) + j400 \sin 50^\circ = 143 + j\, 306.4 =$ **$338.2 \angle 64.9^\circ$**

(iii) $I_a = 338.2 \times 64.9^\circ / 10 \angle 90^\circ =$ **$33.8 \angle -25.1^\circ$ A/phase**

(iv) motor power/phase $= V_p I_a \cos \phi = 400 \times 33.8 \cos 25.1^\circ = 12{,}244$ W
Total power $= 3 \times 12{,}244 = 36{,}732$ W = **36.732 kW**

It is seen from above that as motor load is increased

1. rotor displacement increases from 5° (elect) to 50° (elect) *i.e.* E_b falls back in phase considerably.
2. E_R increases from 35 V to 338 V/phase
3. I_a increases from 3.5 A to 33.8 A
4. angle ϕ increases from 2.5° to 25.1° so that p.f. ***decreases*** from 0.999 (lag) to 0.906 (lag)
5. increase in power is almost ***directly proportional to increase in load angle***.

Obviously, increase in I_a is much more than decrease in power factor.

It is interesting to note that not only power but even I_a, E_R and ϕ also increase almost as many times as α.

Special Illustrative Example 38.3

Case of Cylindrical Rotor Machine :

A 3-Phase synchronous machine is worked as follows: Generator - mode : 400 V/Ph, 32 A/Ph, Unity p.f. $X_S = 10$ ohms. Motoring - mode : 400 V/Ph, 32 A/Ph, Unity p.f. , $X_S = 10$ ohms. Calculate E and δ in both the cases and comment.

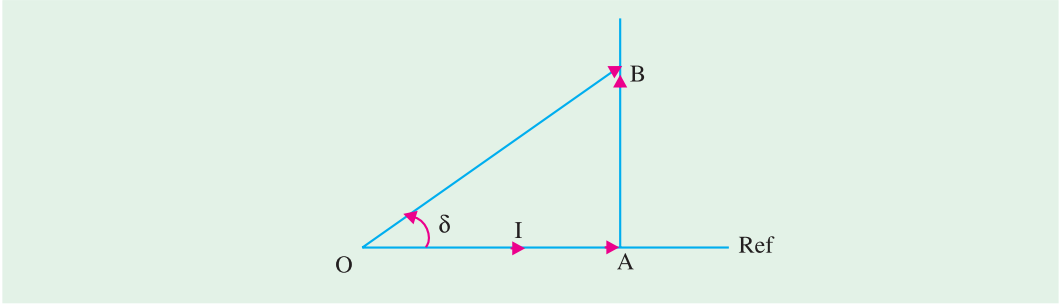


Fig. 38.19 (*a*) Generator-mode

Solution. In Fig. 38.19 (*a*), $V = OA = 400$, $IX_S = AB = 320$ V

$$E = OB = 512.25,\ \delta = \tan^{-1}\frac{320}{400} = 38.66^\circ$$

Total power in terms of parameters measurable at terminals (*i.e.*, V, I, and ϕ)

$$= 3\,V_{ph}\,I_{ph}\cos\phi = 3 \times 400 \times 32 = 38.4 \text{ kW}$$

Total power using other parameters $= 3 \times \left[\frac{VE}{X_S}\sin\delta\right] \times 10^{-3}$ kW

$$= 3 \times \frac{400 \times 512.25}{10} \times (\sin 38.66^\circ) \times 10^{-3} = 38.4 \text{ kW}$$

Since losses are neglected, this power is the electrical output of generator and also is the required mechanical input to the generator.

For motoring mode : $V = OA = 400$, $-IX_S = AB = 320$

$E = OB = 512.25$, as in Fig. 38.19 (*b*)

Hence, $|\delta| = 38.66°$, as before.

Comments : The change in the sign of δ has to be noted in the two modes. It is +ve for generator and –ve for motor. E happens to be equal in both the cases due to unity p.f. At other p.f., this will be different.

As before, power can be calculated in two ways and it will be electrical power input to motor and also the mechanical output of the motor.

Naturally, Power = 38.4 kW

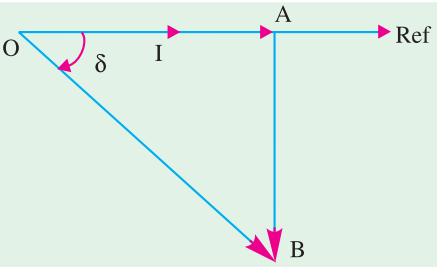


Fig. 38.19 (*b*) Motoring mode

38.10. Effect of Changing Excitation on Constant Load

As shown in Fig. 38.20 (*a*), suppose a synchronous motor is operating with normal excitation ($E_b = V$) at unity p.f. with a given load. If R_a is negligible as compared to X_S, then I_a lags E_R by 90° and is in phase with V because p.f. is unity. The armature is drawing a power of $V.I_a$ per phase which is enough to meet the mechanical load on the motor. Now, let us discuss the effect of decreasing or increasing the field excitation when the load applied to the motor ***remains constant***.

(*a*) Excitation Decreased

As shown in Fig. 38.20 (*b*), suppose due to decrease in excitation, back e.m.f. is reduced to Eb_1 at the ***same load angle*** α_1. The resultant voltage E_{R1} causes a lagging armature current I_{a1} to flow. Even though I_{a1} is larger than I_a in magnitude it is incapable of producing necessary power VI_a for carrying the ***constant*** load because $I_{a1}\cos\phi_1$ component is less than I_a so that $VI_{a1}\cos\phi_1 < VI_a$.

Hence, it becomes necessary for load angle to ***increase*** from α_1 to α_2. It increases back e.m.f. from E_{b1} to E_{b2} which, in turn, increases resultant voltage from E_{R1} to E_{R2}. Consequently, armature current increases to I_{a2} whose in-phase component produces enough power ($VI_{a2}\cos\phi_2$) to meet the constant load on the motor.

(b) Excitation Increased

The effect of increasing field excitation is shown in Fig. 38.20 (*c*) where increased E_{b1} is shown at the original load angle α_1. The resultant voltage E_{R1} causes a ***leading*** current I_{a1} whose in-phase component is larger than I_a. Hence, armature develops more power than the load on the motor. Accordingly, load angle ***decreases*** from α_1 to α_2 which decreases resultant voltage from E_{R1} to E_{R2}. Consequently, armature current decreases from I_{a1} to I_{a2} whose in-phase component $I_{a2} \cos \phi_2 = I_a$. In that case, armature develops power sufficient to carry the constant load on the motor.

Hence, we find that variations in the excitation of a synchronous motor running with a ***given*** load produce variations in its ***load angle only***.

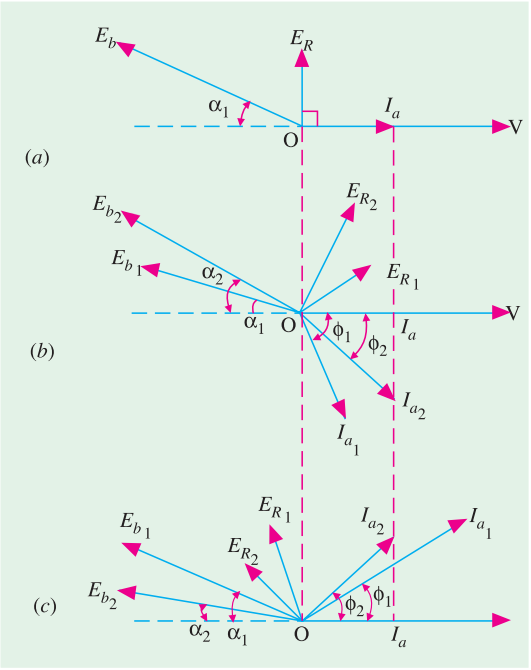


Fig. 38.20

38.11. Different Torques of a Synchronous Motor

Various torques associated with a synchronous motor are as follows:

1. starting torque
2. running torque
3. pull-in torque and
4. pull-out torque

(a) Starting Torque

It is the torque (or turning effort) developed by the motor when full voltage is applied to its stator (armature) winding. It is also sometimes called ***breakaway*** torque. Its value may be as low as 10% as in the case of centrifugal pumps and as high as 200 to 250% of full-load torque as in the case of loaded reciprocating two-cylinder compressors.



Torque motors are designed to privide maximum torque at locked rotor or near stalled conditions

(b) Running Torque

As its name indicates, it is the torque developed by the motor under running conditions. It is determined by the horse-power and speed of the ***driven*** machine. The peak horsepower determines the maximum torque that would be required by the driven machine. The motor must have a breakdown or a maximum running torque greater than this value in order to avoid stalling.

(c) Pull-in Torque

A synchronous motor is started as induction motor till it runs 2 to 5% below the synchronous speed. Afterwards, excitation is switched on and the rotor pulls into step with the synchronously-rotating stator field. The amount of torque at which the motor will pull into step is called the pull-in torque.

(*d*) Pull-out Torque

The maximum torque which the motor can develop without pulling out of step or synchronism is called the pull-out torque.

Normally, when load on the motor is increased, its rotor progressively tends to fall back ***in phase*** by some angle (called load angle) behind the synchronously-revolving stator magnetic field though it keeps running synchronously. Motor develops maximum torque when its rotor is retarded by an angle of 90° (or in other words, it has shifted backward by a distance equal to half the distance between adjacent poles). Any further increase in load will cause the motor to pull out of step (or synchronism) and stop.

38.12. Power Developed by a Synchronous Motor

In Fig. 38.21, OA represents supply voltage/phase and $I_a = I$ is the armature current, AB is back e.m.f. at a load angle of α. OB gives the resultant voltage $E_R = IZ_S$ (or IX_S if R_a is negligible). I leads V by ϕ and lags behind E_R by an angle $\theta = \tan^{-1}(X_S / R_a)$. Line CD is drawn at an angle of θ to AB. AC and ED are $\perp$ to CD (and hence to AE also).

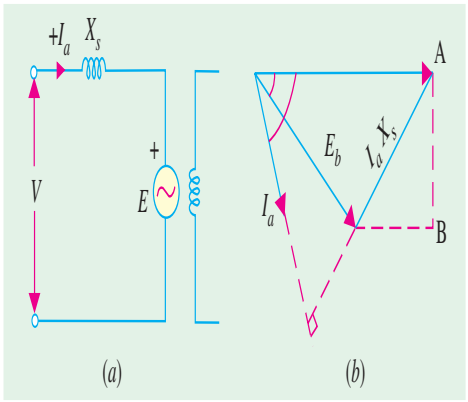


Fig. 38.21

Mechanical power per phase developed in the rotor is

$$P_m = E_b I \cos\psi \qquad ...(i)$$

In ΔOBD, $BD = I Z_S \cos\psi$

Now, $BD = CD - BC = AE - BC$

$$I Z_S \cos\psi = V\cos(\theta - \alpha) - E_b \cos\theta$$

$$\therefore \quad I\cos\psi = \frac{V}{Z_S}\cos(\theta-\alpha) - \frac{E_b}{Z_S}\cos\theta$$

Substituting this value in (i), we get

$$P_m \text{ per phase} = E_b\left[\frac{V}{Z_S}\cos(\theta-\alpha) - \frac{E_b}{Z_S}\cos\theta\right] = \frac{E_b V}{Z_S}\cos(\theta-\alpha) - \frac{E_b^2}{Z_S}\cos\theta^{*} \qquad ...(ii)$$

This is the expression for the mechanical power developed in terms of the load angle (α) and the internal angle (θ) of the motor for a constant voltage V and E_b (or excitation because E_b depends on excitation only).

If T_g is the gross armature torque developed by the motor, then

$$T_g \times 2\pi N_S = P_m \text{ or } T_g = P_m/\omega_s = P_m / 2\pi N_S \qquad -N_S \text{ in rps}$$

$$T_g = \frac{P_m}{2\pi N_S/60} = \frac{60}{2\pi}\cdot\frac{P_m}{N_S} = 9.55\frac{P_m}{N_S} \qquad -N_S \text{ in rpm}$$

Condition for maximum power developed can be found by differentiating the above expression with respect to load angle and then equating it to zero.

$$\therefore \quad \frac{dP_m}{d\alpha} = -\frac{E_b V}{Z_S}\sin(\theta-\alpha) = 0 \quad \text{or} \quad \sin(\theta-\alpha) = 0 \quad \therefore \quad \theta = \alpha$$

---

* Since R_a is generally negligible, $Z_S = X_S$ so that $\theta \cong 90°$. Hence

$$P_m = \frac{E_b V}{X_S}\cos(90° - \alpha) = \frac{E_b V}{X_S}\sin\alpha$$

This gives the value of mechanical power developed in terms of α – the basic variable of a synchronous machine.

$\therefore$ value of maximum power $(P_m)_{max} = \dfrac{E_b V}{Z_S} - \dfrac{E_b^2}{Z_S} \cos\alpha$ or $(P_m)_{max} = \dfrac{E_b V}{Z_S} - \dfrac{E_b^2}{Z_S} \cos\theta$. ...***(iii)***

This shows that the maximum power and hence torque ($\because$ speed is constant) depends on V and E_b *i.e.*, excitation. Maximum value of θ (and hence α) is 90°. For all values of V and E_b, this limiting value of α is the same but maximum torque will be proportional to the maximum power developed as given in equation ***(iii)***. Equation ***(ii)*** is plotted in Fig. 38.22.

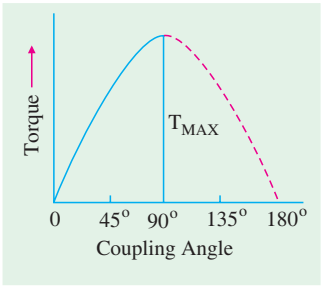


Fig. 38.22

If R_a is neglected, then $Z_S \cong X_S$ and $\theta = 90°$ $\therefore \cos\theta = 0$

$P_m = \dfrac{E_b V}{X_S} \sin\alpha$ ...(*iv*) $(P_m)_{max} = \dfrac{E_b V}{X_S}$ ... from equation ***(iii)****

The same value can be otained by putting $\alpha = 90°$ in equation ***(iv)***. This corresponds to the 'pull-out' torque.

38.13. Alternative Expression for Power Developed

In Fig. 38.23, as usual, OA represents the supply voltage per phase *i.e.*, V and AB (= OC) is the induced or back e.m.f. per phase *i.e.*, E_b at an angle α with OA. The armature current I (or I_a) lags V by ϕ.

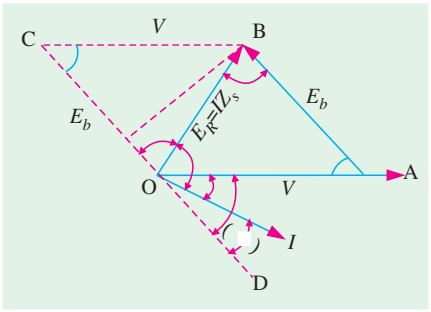


Fig. 38.23

Mechanical power developed is,

P_m = E_b . I × cosine of the angle between E_b and I

$= E_b I \cos \angle DOI$

$= E_b I \cos(\pi - \angle COI)$

$= -E_b I \cos(\theta + \gamma)$

$= -E_b \left(\dfrac{E_R}{Z_S}\right)(\cos\theta\cos\gamma - \sin\theta\sin\gamma)$ ...***(i)***

Now, E_R and functions of angles θ and γ will be eliminated as follows :

From ΔOAB ; $V/\sin\gamma = E_R / \sin\alpha$ $\therefore$ $\sin\gamma = V\sin\alpha / E_R$

From ΔOBC ; $E_R\cos\gamma + V\cos\alpha = E_b$ $\therefore$ $\cos\gamma = (E_b - V\cos\alpha)/E_R$

Also $\cos\theta = R_a / Z_S$ and $\sin\theta = X_S / Z_S$

Substituting these values in Eq. (*i*) above, we get

$$P_m = -\frac{E_b \,.\, E_R}{Z_S}\left(\frac{R_a}{Z_S}\cdot\frac{E_b - V\cos\alpha}{E_R} - \frac{X_S}{Z_S}\cdot\frac{V\sin\alpha}{E_R}\right)$$

$$= \frac{E_b V}{Z_{S^2}}(R_a\cos\alpha + X_S\sin\alpha) - \frac{E_b^2 R_a}{Z_{S^2}} \quad ...(ii)$$

It is seen that P_m varies with E_b (which depends on excitation) and angle α (which depends on the motor load).

Note. If we substitute $R_a = Z_S\cos\theta$ and $X_S = Z_S\sin\theta$ in Eq. (*ii*), we get

$$P_m = \frac{E_b V}{Z_{S^2}}(Z_S\cos\theta\cos\alpha + Z_S\sin\theta\sin\alpha) - \frac{E_b{}^2 Z_S\cos\theta}{Z_{S^2}} = \frac{E_b V}{Z_S}\cos(\theta - \alpha) - \frac{E_{b^2}}{Z_S}\cos\theta$$

It is the same expression as found in Art. 38.10.

* It is the same expression as found for an alternator or synchronous generator in Art. 37.37.

38.14. Various Conditions of Maxima

The following two cases may be considered :

(*i*) Fixed E_b, V, R_a and X_S. Under these conditions, P_m will vary with load angle α and will be maximum when $dP_m / d\alpha = 0$. Differentiating Eq. (*ii*) in Art. 38.11, we have

$$\frac{dP_m}{d\alpha} = \frac{E_b V}{Z_s^2} (X_S \cos\alpha - R_a \sin\alpha) = 0 \quad \text{or} \quad \tan\alpha = X_S / R_a = \tan\theta \quad \text{or} \quad \alpha = \theta$$

Putting $\alpha = \theta$ in the same Eq. (*ii*), we get

$$(P_m)_{max} = \frac{E_b V}{Z_s^2} (R_a \cos\theta + X_S \sin\theta) - \frac{E_b^2 R_a}{Z_s^2} =$$

$$= \frac{E_b V}{Z_s^2}\left(\frac{R_a^2 + X_s^2}{Z_S}\right) - \frac{E_b^2 R_a}{Z_s^2} = \frac{E_b V}{Z_S} - \frac{E_b^2 R_a}{Z_s^2} \qquad \text{...(i)}$$

This gives the value of power at which the motor falls out of step.

Solving for E_b from Eq. (*i*) above, we get

$$E_b = \frac{Z_S}{2R_a}\left[V \pm \sqrt{V^2 - 4R_a \,.\, (P_m)_{max}}\right]$$

The two values of E_b so obtained represent the excitation limits for any load.

(*ii*) Fixed V, R_a and X_S. In this case, P_m varies with excitation or E_b. Let us find the value of the excitation or induced e.m.f. E_b which is necessary for maximum power possible. For this purpose, Eq. **(*i*)** above may be differentiated with respect to E_b and equated to zero.

$$\therefore \qquad \frac{d\,(P_m)_{max}}{d\,E_b} = \frac{V}{Z_S} - \frac{2R_a E_b}{Z_{S^2}} = 0\,; \qquad E_b = \frac{V\,Z_S{}^*}{2R_a} \qquad \text{...(ii)}$$

Putting this value of E_b in Eq. (*i*) above, maximum power developed becomes

$$(P_m)_{max} = \frac{V^2}{2R_a} - \frac{V^2}{4R_a} = \frac{V^2}{4R_a}$$

38.15. Salient Pole Synchronous Motor

Cylindrical-rotor synchronous motors are much easier to analyse than those having salient-pole rotors. It is due to the fact that cylindrical-rotor motors have a uniform air-gap, whereas in salient-pole motors, air-gap is much greater between the poles than along the poles. Fortunately, cylindrical rotor theory is reasonably accurate in predicting the steady-state performance of salient-pole motors. Hence, salient-pole theory is required only when very high degree of accuracy is needed or when problems concerning transients or power system stability are to be handled.

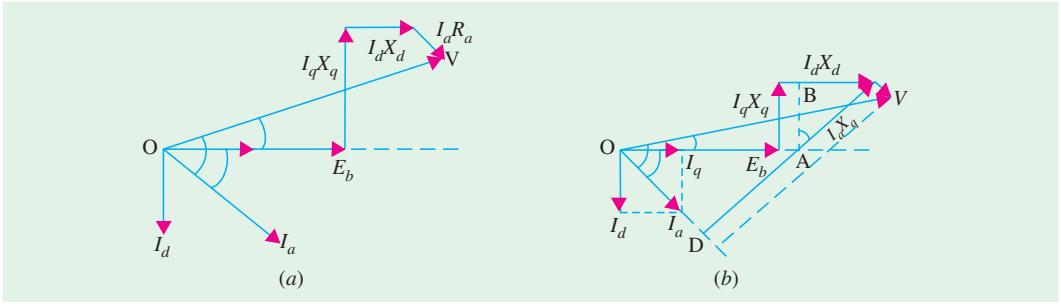


Fig. 38.24

* This is the value of induced e.m.f. to give maximum power, but it is not the maximum possible value of the generated voltage, at which the motor will operate.

The *d-q* currents and reactances for a salient-pole synchronous motor are exactly the same as discussed for salient-pole synchronous generator. The motor has *d*-axis reactance X_d and *q*-axis reactance X_q. Similarly, motor armature current I_a has two components : I_d and I_q. The complete phasor diagram of a salient-pole synchronous motor, for a lagging power factor is shown in Fig. 38.24 (*a*).

With the help of Fig. 38.24 (*b*), it can be proved that $\tan \psi = \dfrac{V \sin \phi - I_q X_q}{V \cos \phi - I_a R_a}$

If R_a is negligible, then $\tan \psi = (V \sin \phi + I_a X_q) / V \cos \phi$

For an overexcited motor *i.e.*, when motor has leading power factor,

$$\tan \psi = (V \sin \phi + I_a X_q) / V \cos \phi$$

The power angle α is given by $\alpha = \phi - \psi$

The magnitude of the excitation or the back e.m.f. E_b is given by

$$E_b = V \cos \alpha - I_q R_a - I_d X_d$$

Similarly, as proved earlier for a synchronous generator, it can also be proved from Fig. 38.24 (*b*) for a synchronous motor with $R_a = 0$ that

$$\tan \alpha = \frac{I_a X_q \cos \phi}{V - I_a X_q \sin \phi}$$

In case R_a is not negligible, it can be proved that

$$\tan \alpha = \frac{I_a X_q \cos \phi - I_a R_a \sin \phi}{V - I_a X_q \sin \phi - I_a R_a \cos \alpha}$$

38.16. Power Developed by a Salient Pole Synchronous Motor

The expression for the power developed by a salient-pole synchronous generator derived in Chapter 35 also applies to a salient-pole synchronous motor.

$$\therefore \quad P_m = \frac{E_b V}{X_d} \sin \alpha + \frac{V^2 (X_d - X_q)}{2 X_d X_q} \sin 2\alpha \quad \text{... per phase}$$

$$= 3 \times \left[\frac{E_b V}{X_d} \sin \alpha + \frac{V^2 (X_d - X_q)}{2 X_d X_q} \sin 2\alpha \right] \text{... per three phases}$$

$$T_g = 9.55 P_m / N_S \quad \text{...} N_S \text{ in rps.}$$

As explained earlier, the power consists of two components, the first component is called excitation power or magnet power and the second is called reluctance power (because when excitation is removed, the motor runs as a reluctance motor).

Example 38.4. *A 3-ϕ, 150-kW, 2300-V, 50-Hz, 1000-rpm salient-pole synchronous motor has X_d = 32 Ω / phase and X_q = 20 Ω / phase. Neglecting losses, calculate the torque developed by the motor if field excitation is so adjusted as to make the back e.m.f. twice the applied voltage and $\alpha = 16^o$.*

Solution. $V = 2300 / \sqrt{3} = 1328$ V ; $E_b = 2 \times 1328 = 2656$ V

Excitation power / phase $= \dfrac{E_b V}{X_d} \sin \alpha = \dfrac{2656 \times 1328}{32} \sin 16^o = 30{,}382$ W

Reluctance power / phase $= \dfrac{V^2 (X_d - X_q)}{2 X_d X_q} \sin 2\alpha = \dfrac{1328^2 (32 - 20)}{2 \times 32 \times 20} \sin 32^o = 8760$ W

Total power developed, $P_m = 3 (30382 + 8760) = 117, 425$ W

$T_g = 9.55 \times 117{,}425/1000 =$ **1120 N-m**

Example 38.5. *A 3300-V, 1.5-MW, 3-ϕ, Y-connected synchronous motor has $X_d = 4\Omega$ / phase and $X_q = 3\ \Omega$ / phase. Neglecting all losses, calculate the excitation e.m.f. when motor supplies rated load at unity p.f. Calculate the maximum mechanical power which the motor would develop for this field excitation.* **(Similar Example, Swami Ramanand Teertha Marathwada Univ. Nanded 2001)**

Solution. $V = 3300/\sqrt{3} = 1905$ V; $\cos\phi = 1$; $\sin\phi = 0$; $\phi = 0^\circ$

$$I_a = 1.5 \times 10^6/\sqrt{3} \times 3300 \times 1 = 262 \text{ A}$$

$$\tan\psi = \frac{V\sin\phi - I_a X_q}{V\cos\phi} = \frac{1905\times 0 - 262\times 3}{1905} = -0.4125;\ \psi = -22.4^\circ$$

$$\alpha = \phi - \psi = 0 - (-22.4^\circ) = 22.4^\circ$$

$$I_d = 262 \times \sin(-22.4^\circ) = -100 \text{ A};\ I_q = 262\cos(-22.4^\circ) = 242 \text{ A}$$

$$E_b = V\cos\alpha - I_d X_d = 1905\cos(-22.4^\circ) - (-100\times 4) = 2160 \text{ V}$$

$$= 1029\sin\alpha + 151\sin 2\alpha$$

$$P_m = \frac{E_b V}{X_d}\sin\alpha + \frac{V^2(X_d - X_q)}{2X_d X_q}\sin 2\alpha \quad \text{... per phase}$$

$$= \frac{2160\times 1905}{4\times 1000} + \frac{1905^2(4-3)}{2\times 4\times 3\times 1000}\sin 2\alpha \quad \text{...kW/phase}$$

$$= 1029\sin\alpha + 151\sin 2\alpha \quad \text{...kW/phase}$$

If developed power has to achieve maximum value, then

$$\frac{dP_m}{d\alpha} = 1029\cos\alpha + 2\times 151\cos 2\alpha = 0$$

$$\therefore\quad 1029\cos\alpha + 302(2\cos^2\alpha - 1) = 0 \quad \text{or} \quad 604\cos^2\alpha + 1029\cos\alpha - 302 = 0$$

$$\therefore\quad \cos\alpha = \frac{-1029 \pm \sqrt{1029^2 + 4\times 604\times 302}}{2\times 604} = 0.285;\ \alpha = 73.4^\circ$$

$$\therefore\quad \text{maximum } P_m = 1029\sin 73.4^\circ + 151\sin 2\times 73.4^\circ = 1070 \text{ kW/phase}$$

Hence, maximum power developed for three phases = $3\times 1070 =$ **3210 kW**

Example 38.6. *The input to an 11000-V, 3-phase, star-connected synchronous motor is 60 A. The effective resistance and synchronous reactance per phase are respectively 1 ohm and 30 ohm. Find* ***(i)*** *the power supplied to the motor* ***(ii)*** *mechanical power developed and* ***(iii)*** *induced emf for a power factor of 0.8 leading.* **(Elect. Engg. AMIETE (New Scheme) June 1990)**

Solution. ***(i)*** Motor power input $= \sqrt{3}\times 11000\times 60\times 0.8 = 915$ kW

(ii) stator Cu loss/phase $= 60^2\times 1 = 3600$ W; Cu loss for three phases $= 3\times 3600 = 10.8$ kW

$P_m = P_2 -$ rotor Cu loss $= 915 - 10.8 = 904.2$ kW

$V_p = 11000/\sqrt{3} = 6350$ V; $\phi = \cos^{-1} 0.8 = 36.9^\circ$;

$\theta = \tan^{-1}(30/1) = 88.1^\circ$

$Z_S \cong 30\ \Omega$; stator impedance drop / phase $= I_a Z_S$

$= 60\times 30 = 1800$ V

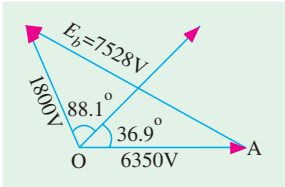


Fig. 38.25

As seen from Fig. 38.25,

$$E_b^2 = 6350^2 + 1800^2 - 2\times 6350\times 1800\times\cos(88.1^\circ + 36.9^\circ)$$

$$= 6350^2 + 1800^2 - 2\times 6350\times 1800\times -0.572$$

$$\therefore\quad E_b = 7528 \text{ V; line value of } E_b = 7528\times\sqrt{3} = \mathbf{13042}$$

Special Example 38.7. Case of Salient - Pole Machines

A synchronous machine is operated as below :

As a Generator : 3 -Phase, V_{ph} = 400, I_{ph} = 32, *unity p.f.*

As a Motor : 3 - Phase, V_{ph} = 400, I_{ph} = 32, *unity p.f.*

Machine parameters : X_d = 10 Ω, X_q = 6.5 Ω

Calculate excitation emf and δ *in the two modes and deal with the term power in these two cases.*

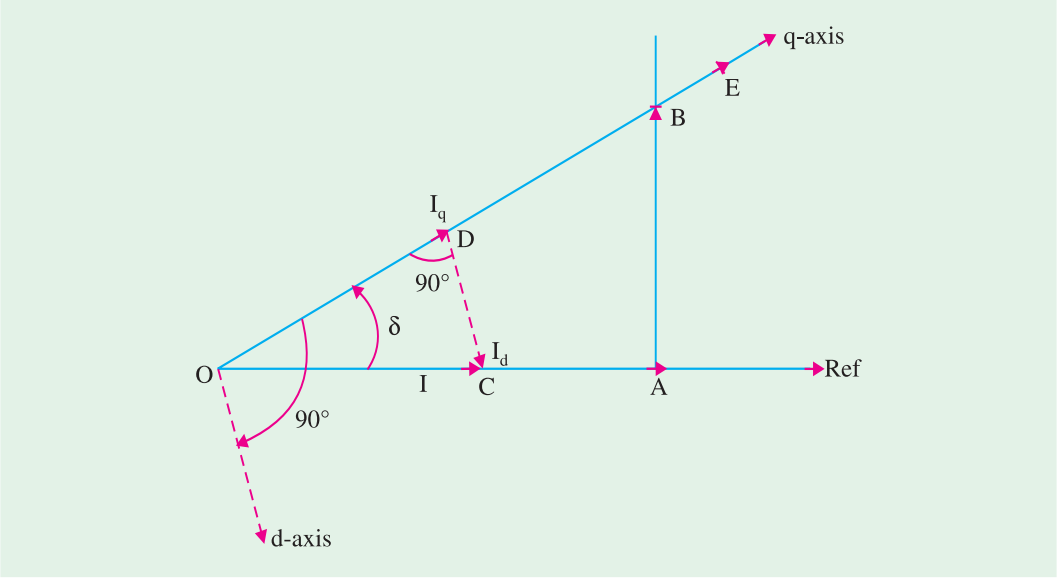


Fig. 38.26 (*a*) Generator-action

Solution.

Generating Mode :

Voltages :

$$OA = 400 \text{ V}, AB = IX_q$$
$$= 32 \times 6.5 = 208 \text{ V}$$

$$OB = \sqrt{400^2 + 208^2} = 451 \text{ V},$$

$$\delta = \tan^{-1}\frac{AB}{OA} = \tan^{-1}\frac{208}{400} = 27.5^\circ$$

$$BE = I_d(X_d - X_q)$$
$$= 140 \times 3.5 = 51.8 \text{ V},$$

$$E = OE = OB + BE = 502.8 \text{ V}$$

Currents : $I = OC = 32$, $I_q = I\cos\delta = OD = 28.4$ amp., $I_d = DC = I\sin\delta = 14.8$ amp. E leads V in case of generator, as shown in Fig. 38.26 (*a*)

$$\text{Power (by one formula)} = 3 \times 400 \times 32 \times 10^{-3} = 38.4 \text{ kW}$$

or
$$\text{Power (by another formula)} = 3 \times \left[\frac{400 \times 502.8}{10}\sin 27.5^\circ + \frac{400^2}{2} \times \left(\frac{3.5}{65}\right) \times \sin 55^\circ\right]$$
$$= 38.44 \text{ kW}$$

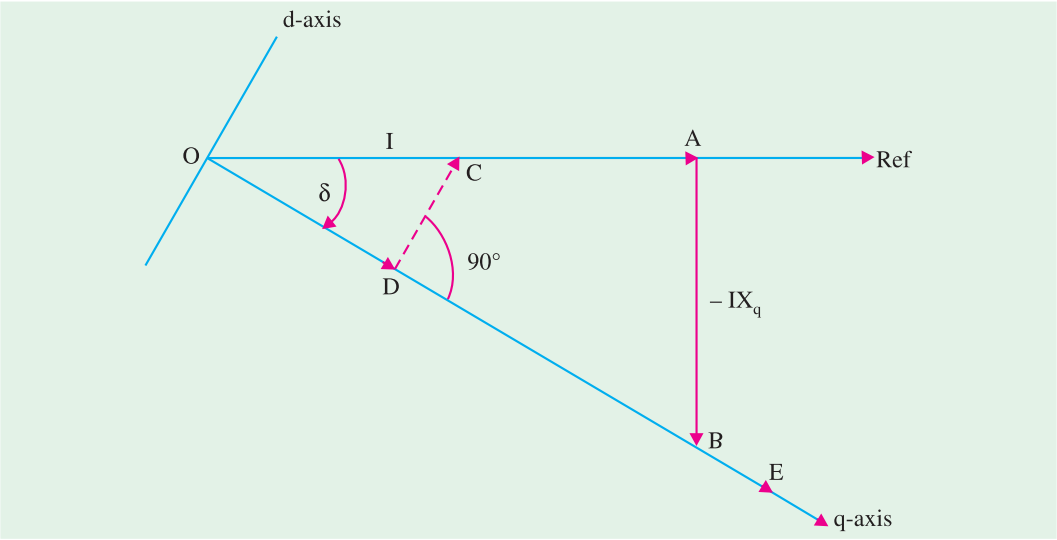


Fig. 38.26 (*b*) Phasor diagram : Motoring mode

Motoring mode of a salient pole synchronous machine

Voltages : $OA = 400$ V, $AB = -IX_q = 208$ V

$OB = \sqrt{400^2 + 208^2} = 451$ V

$\delta = \tan^{-1}\frac{AB}{OA} = \tan^{-1}\frac{208}{400} = 27.5^\circ$ as before but now E lags behind V.

$BE = I_d(X_d - X_q) = 51.8$ V in the direction shown. $OE = 502.8$ V as before

Currents : $OC = 32$ amp. $OD = 28.4$ amp. $DC = 14.8$ amp. Naturally, $I_q = 28.4$ amp. and $I_d = 14.8$ amp

Power (by one formula) = 38.4 kW

Power (by another formula) = 38.44 kW

Note. Numerical values of E and δ are same in cases of generator-mode and motor-mode, due to unity p.f. δ has different signs in the two cases.

Example 38.8. *A 500-V, 1-phase synchronous motor gives a net output mechanical power of 7.46 kW and operates at 0.9 p.f. lagging. Its effective resistance is 0.8 Ω. If the iron and friction losses are 500 W and excitation losses are 800 W, estimate the armature current. Calculate the commercial efficiency.* **(Electrical Machines-I, Gujarat Univ. 1988)**

Solution. Motor input $= VI_a \cos\phi$; Armature Cu loss $= I_a R_a^2$

Power developed in *armature* is $P_m = VI_a \cos\phi - I_a^2 R_a$

$$\therefore \quad I_a^2 R_a - VI_a \cos\phi + P_m = 0 \quad \text{or} \quad I_a = \frac{V\cos\phi \pm \sqrt{V^2\cos^2\phi - 4R_a P_m}}{2R_a}$$

Now, $P_{out} = 7.46$ kW $= 7{,}460$ W

$P_m = P_{out}$ + iron and friction losses + excitation losses

$= 7460 + 500 + 800 = 8760$ W ... Art. 38.5

$$I_a = \frac{500\times 0.9 \pm \sqrt{(500\times 0.9)^2 - 4\times 0.8\times 3760}}{2\times 0.8}$$

$$= \frac{450 \pm \sqrt{202{,}500 - 28{,}030}}{1.6} = \frac{450 \pm 417.7}{1.6} = \frac{32.3}{1.6} = \mathbf{20.2\ A}$$

Motor input = $500 \times 20.2 \times 0.9 = 9090$ W

η_c = net output / input = 7460 / 9090 = 0.8206 or **82.06%.**

Example 38.9. *A 2,300-V, 3-phase, star-connected synchronous motor has a resistance of 0.2 ohm per phase and a synchronous reactance of 2.2 ohm per phase. The motor is operating at 0.5 power factor leading with a line current of 200 A. Determine the value of the generated e.m.f. per phase.* **(Elect. Engg.-I, Nagpur Univ. 1993)**

Solution. Here, $\phi = \cos^{-1}(0.5) = 60^\circ$ (lead)

$\theta = \tan^{-1}(2.2/0.2) = 84.8^\circ$

$\therefore \quad (\theta + \phi) = 84.8^\circ + 60^\circ = 144.8^\circ$

$\cos 144.8^\circ = -\cos 35.2^\circ$

$V = 2300/\sqrt{3} = 1328$ volt

$Z_S = \sqrt{0.2^2 + 2.2^2} = 2.209\ \Omega$

$IZ_S = 200 \times 2.209 = 442$ V

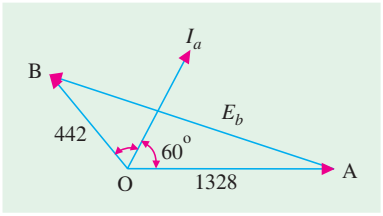


Fig. 38.27

The vector diagram is shown in Fig. 38.27.

$$E_b = \sqrt{V^2 + E_{R^2} - 2V.\ E_R \cos(\theta + \phi)}$$

$$= \sqrt{1328^2 + 442^2 + 2 \times 1328 \times 442 \times \cos 35.2^\circ} = \textbf{1708 Volt / Phase}$$

Example 38.10. *A 3-phase, 6,600-volts, 50-Hz, star-connected synchronous motor takes 50 A current. The resistance and synchronous reactance per phase are 1 ohm and 20 ohm respectively. Find the power supplied to the motor and induced emf for a power factor of* ***(i)*** *0.8 lagging and* ***(ii)*** *0.8 leading.* **(Eect. Engg. II pune Univ. 1988)**

Solution. ***(i)*** **p.f. = 0.8 lag** (Fig. 38.28 (*a*)).

Power input $= \sqrt{3} \times 6600 \times 50 \times 0.8 = 457{,}248$ W

Supply voltage / phase = $6600/\sqrt{3} = 3810$ V

$\phi = \cos^{-1}(0.8) = 36^\circ 52'$; $\theta = \tan^{-1}(X_S / R_a) = (20/1) = 87.8'$

$Z_S = \sqrt{20^2 + 1^2} = 20\ \Omega$ (approx.)

Impedance drop = $I_a Z_S = 50 \times 20 = 1000$ V/phase

$\therefore \quad E_b^2 = 3810^2 + 1000^2 - 2 \times 3810 \times 1000 \times \cos(87^\circ 8' - 36^\circ 52') \quad \therefore \quad E_b = 3263$ V / phase

Line induced e.m.f. = $3263 \times \sqrt{3} =$ **5651 V**

(ii) Power input would remain the same.

As shown in Fig. 38.28 (*b*), the current vector is drawn at a ***leading*** angle of

$\phi = 36^\circ 52'$

Now, $(\theta + \phi) = 87^\circ 8' + 36^\circ 52' = 124^\circ$, $\cos 124^\circ = -\cos 56^\circ$

$\therefore \quad E_b^2 = 3810^2 + 1000^2 - 2 \times 3810 \times 1000 \times -\cos 56^\circ \quad \therefore \quad E_b = 4447$ V / phase

Line induced e.m.f. = $\sqrt{3} \times 4447$

= **7,700 V**

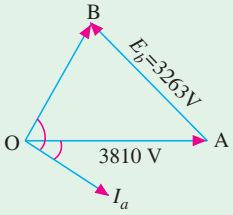


Fig. 38.28 (*a*)

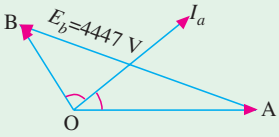


Fig. 38.28 (*b*)

Note. It may be noted that if $E_b > V$, then motor has a leading power factor and if $E_b < V$.

Example 38.11. *A synchronous motor having 40% reactance and a negligible resistance is to be operated at rated load at* **(i)** *u.p.f.* **(ii)** *0.8 p.f. lag* **(iii)** *0.8 p.f. lead. What are the values of induced e.m.f. ? Indicate assumptions made, if any.* **(Electrical Machines-II, Indore Univ. 1990)**

Solution. Let V = 100 V, then reactance drop = $I_a X_S$ = 40 V

(*i*) At unity p.f.

Here, $\theta = 90°$, $E_b = \sqrt{100^2 + 40^2} = 108$ V ...Fig. 38.29 (*a*)

(*ii*) At p.f. 0.8 (lag.) Here $\angle BOA = \theta - \phi = 90° - 36°54' = 53°6'$

$E_b^2 = 100^2 + 40^2 - 2 \times 100 \times 40 \times \cos 53°6'$; E_b = **82.5 V**, as in Fig. 38.29 (*b*)

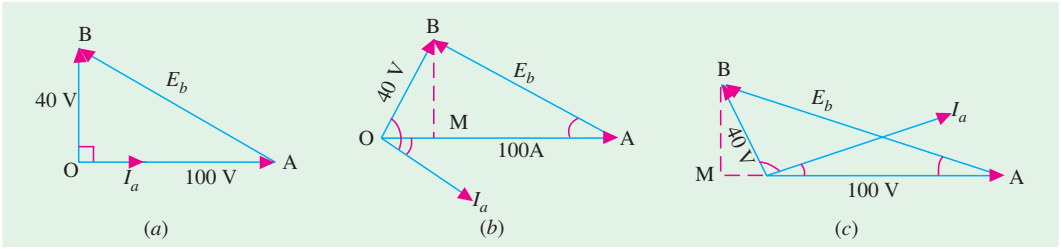


Fig. 38.29

Alternatively, $E_b = AB = \sqrt{AM^2 + MB^2} = \sqrt{76^2 + 32^2}$ = **82.5 V**

(*iii*) At p.f. 0.8 (lead.) Here, $(\theta + \phi) = 90° + 36.9° = 126.9°$

$E_b^2 = 100^2 + 40^2 - 2 \times 40 \times \cos 126.9°$ = **128 V**

Again from Fig. 38.29 (*c*), $E_b^2 = (OM + OA)^2 + MB^2 = 124^2 + 32^2$; E_b = **128 V.**

Example 38.12. *A 1,000-kVA, 11,000-V, 3-ϕ, star-connected synchronous motor has an armature resistance and reactance per phase of 3.5 Ω and 40 Ω respectively. Determine the induced e.m.f. and angular retardation of the rotor when fully loaded at* **(*a*)** *unity p.f.* **(*b*)** *0.8 p.f. lagging* **(*c*)** *0.8 p.f. leading.* **(Elect. Engineering-II, Bangalore Univ. 1992)**

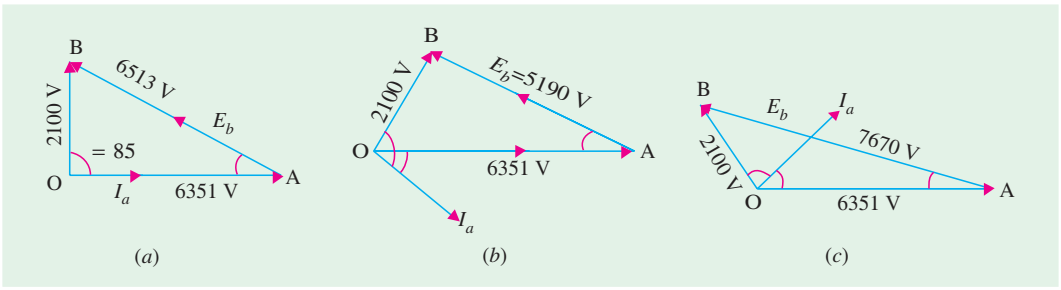


Fig. 38.30

Solution. Full-load armature current = $1{,}000 \times 1{,}000 / \sqrt{3} \times 11{,}000 = 52.5$ A

Voltage / phase = $11{,}000 / \sqrt{3} = 6{,}351$ V ; $\cos\phi = 0.8$ $\therefore$ $\phi = 36°53'$

Armature resistance drop / phase = $I_a R_a = 3.5 \times 52.5 = 184$ V

reactance drop / phase = $I_a X_S = 40 \times 52.5 = 2{,}100$ V

$\therefore$ impedance drop / phase = $I_a Z_S = \sqrt{(184^2 + 2100^2)} = 2{,}100$ V (approx.)

$\tan\theta = X_S / R_a$ $\therefore$ $\theta = \tan^{-1}(40 / 3.5) = 85°$

(*a*) At unity p.f. Vector diagram is shown in Fig. 38.30 (*a*)

$E_b^{\,2} = 6{,}351^2 + 2{,}100^2 - 2 \times 6{,}351 \times 2{,}100 \cos 85^\circ$; $E_b = 6{,}513$ V per phase

Induced line voltage $= 6{,}513 \times \sqrt{3} =$ **11,280 V**

From ΔOAB, $\dfrac{2100}{\sin \alpha} = \dfrac{6153}{\sin 85^\circ} = \dfrac{6153}{0.9961}$

$\sin \alpha = 2{,}100 \times 0.9961 / 6{,}513 = 0.3212$ $\therefore$ $\alpha =$ **18°44′**

(*b*) At p.f. 0.8 lagging – Fig. 38.30 (*b*)

$\angle BOA = \theta - \phi = 85^\circ - 36^\circ 53' = 48^\circ 7'$

$E_b^{\,2} = 6{,}351^2 + 2{,}100^2 - 2 \times 6{,}351 \times 2{,}100 \times \cos 48^\circ 7'$

$E_b = 5{,}190$ V per phase

Induced line voltage $= 5{,}190 \times \sqrt{3} =$ **8,989 V**

Again from the ΔOAB of Fig. 36.30 (*b*)

$\dfrac{2100}{\sin \alpha} = \dfrac{5190}{\sin 48^\circ 7'} = \dfrac{5.190}{0.7443}$

$\therefore$ $\sin \alpha = 2100 \times 0.7443/5190 = 0.3012$ $\therefore$ $\alpha =$ **17°32′**

(*c*) At p.f. 0.8 leading [Fig. 38.30 (*c*)]

$\angle BOA = \theta + \phi = 85^\circ + 36^\circ 53' = 121^\circ 53'$

$\therefore$ $E_b^{\,2} = 6{,}351^2 + 2{,}100^2 - 2 \times 6{,}351 \times 2{,}100 \times \cos 121^\circ 53'$

$\therefore$ $E_b = 7{,}670$ volt per phase.

Induced line e.m.f $= 7{,}670 \times \sqrt{3} =$ **13,280 V**

Also, $\dfrac{2{,}100}{\sin \alpha} = \dfrac{7{,}670}{\sin 121^\circ 53'} = \dfrac{7{,}670}{0.8493}$

$\therefore$ $\sin \alpha = 2{,}100 \times 0.8493 / 7{,}670 = 0.2325$ $\therefore$ $\alpha =$ **13°27′**

Special Example 38.13. *Both the modes of operation : Phase - angle = 20° Lag*

Part (a) : A three phase star-connected synchronous generator supplies a current of 10 A having a phase angle of 20° lagging at 400 volts/phase. Find the load angle and components of armature current (namely I_d and I_q) if X_d = 10 ohms, X_q = 6.5 ohms. Neglect r_a. Calculate voltage regulation.

Solution. The phasor diagram is drawn in Fig 38.31 (*a*)

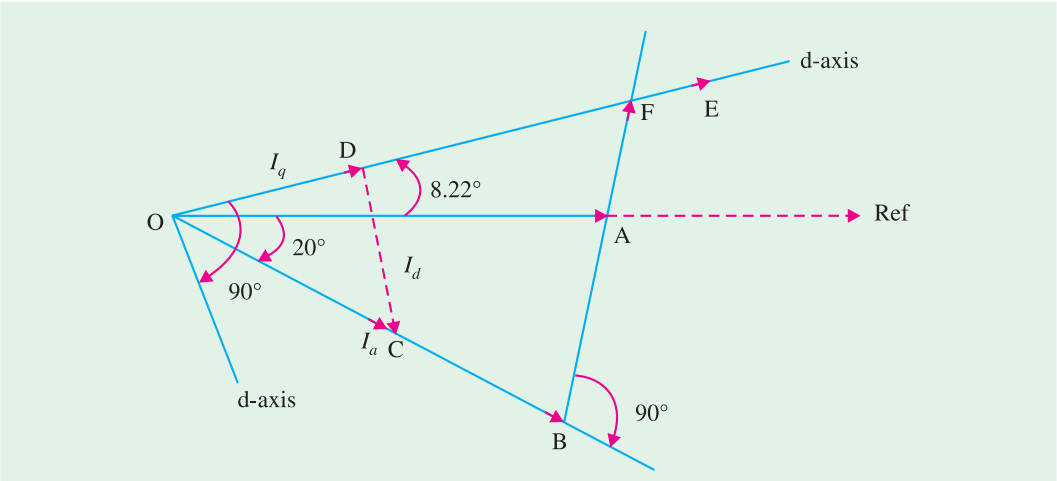


Fig. 38.31 (*a*) : Generator-mode

OA = 400 V, $OB = 400 \cos 20° = 376$ V, $AB = 400 \sin 20° = 136.8$ V

$AF = IX_q = 10 \times 6.5 = 65$ V, $BF = BA + AF = 201.8$ V

$OF = \sqrt{376^2 + 201.8^2} = 426.7$ V, $\delta = 8.22°$

$DC = I_d = I_a \sin 28.22° = 4.73$ amp, DC perpendicular to OD,

$OD = I_q = I_a \cos 28.22° = 8.81$ amp

$FE = I_d (X_d - X_q) = 4.73 \times 3.5 = 16.56$ V. This is along the direction of '$+q$' –axis

$E = OE = OF + FE = 426.7 + 16.56 = 443.3$ V

$$\% \text{ Regulation} = \frac{443 - 400}{400} \times 100\% = \mathbf{10.75\ \%}$$

If the same machine is now worked as a synchronous motor with terminal voltage, supply-current and its power-factor kept unaltered, find the excitation emf and the load angle.

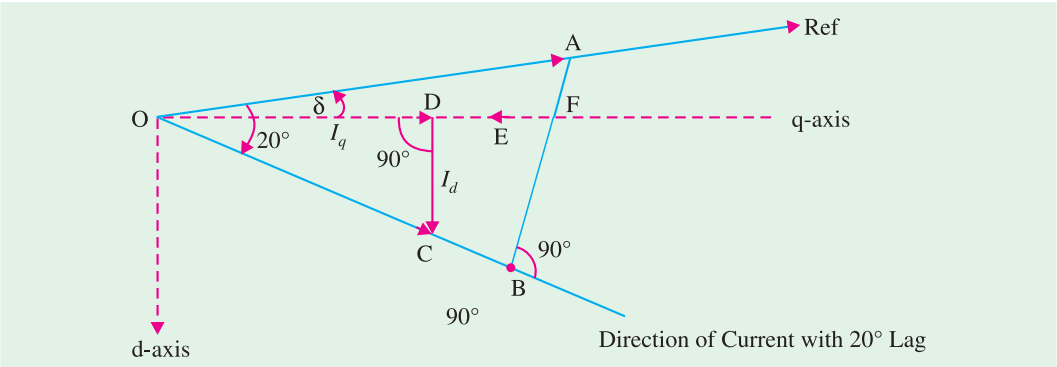


Fig. 38.31 (*b*) Motoring-mode

$AF = -I_a X_q = -65$ V, $AB = 136.8$ V, FB = 71.8 V

$OB = 400 \cos 20° = 376$ V

$OF = \sqrt{376^2 + 71.8^2} = 382.8$ V

$20° - \delta = \tan^{-1} BF/OB = \tan^{-1} 71.8/376 = 10.8°$, $\delta = 9.2°$

$FE = -I_d (X_d - X_q) = -1.874 \times (3.5) = -6.56$ volts, as shown in Fig.38.31 (*b*)

$E = OE = OF + FE = 382.8 - 6.56 = 376.24$ volts

Currents :

$I_a = OC = 10$ amp

$I_q = OD = 10 \cos 10.8° = 9.823$ amp, $I_d = DC = \sin 10.8° = 1.874$ amp

Note. I_d is in downward direction.

Hence, $-I_d (X_d - X_q)$ will be from F towards O *i.e.*, along '$-q$' direction.

Thus, Excitation emf = 376.24 Volts, Load angle = 9.2°

(**Note.** With respect to the generator mode, E has decreased, while δ has increased.)

Power (by one formula) = 11276 watts, as before

Power (by another formula)

$= 3 [(V E/X_d) \sin \delta + (V^2/2) \{(1/X_q) \sin 2\delta\}]$

$= 3 [(400 \times 376.24/10) \sin 9.2° + (400 \times 400/2) (3.5/65) \sin 18.4°]$

$= 3 \times [2406 + 1360] = 11298$ watts.

[This matches quite closely to the previous value calculated by other formula.]

Example 38.14. *A 1-ϕ alternator has armature impedance of (0.5 + j0.866). When running as a synchronous motor on 200-V supply, it provides a net output of 6 kW. The iron and friction losses amount to 500 W. If current drawn by the motor is 50 A, find the two possible phase angles of current and two possible induced e.m.fs.* **(Elec. Machines-I, Nagpur Univ. 1990)**

Solution. Arm. Cu loss/phase $= I_a^2 R_a = 50^2 \times 0.5 = 1250$ W

Motor intake $= 6000 + 500 + 1250 = 7750$ W

p.f. $= \cos\phi$ = Watts / VA $= 7750 / 200 \times 50 = 0.775$ $\therefore \phi = 39^\circ$ **lag or lead.**

$\theta = \tan^{-1}(X_S / R_a) = \tan^{-1}(0.866 / 0.5) = 60^\circ$;

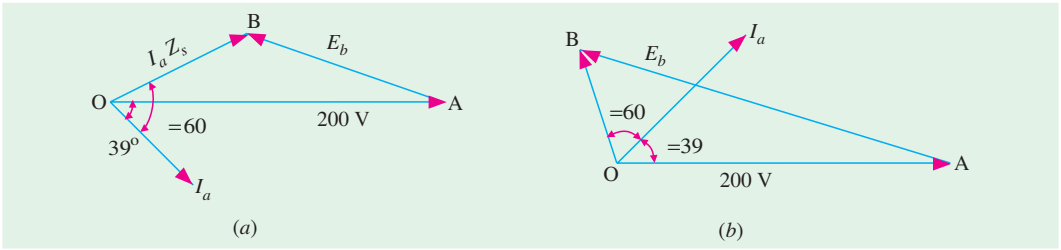


Fig. 38.32

$\angle BOA = 60^\circ - 39^\circ = 21^\circ$ – Fig. 38.32 (*a*)

$Z_S = \sqrt{0.5^2 + 0.866^2} = 1\ \Omega$; $I_a Z_S = 50 \times 1 = 50$ V

$AB = E_b = \sqrt{200^2 + 50^2 - 2 \times 200 \times 50 \cos 21^\circ}$; $E_b =$ **154 V.**

In Fig. 38.32 (*b*), $\angle BOA = 60^\circ + 39^\circ = 99^\circ$

$\therefore$ $AB = E_b = \sqrt{(200^2 + 50^2) - 2 \times 200 \times 50 \cos 99^\circ}$; $E_b =$ **214 V.**

Example 38.15. *A 2200-V, 3-ϕ, Y-connected, 50-Hz, 8-pole synchronous motor has Z_S = (0.4 + j 6) ohm/phase. When the motor runs at no-load, the field excitation is adjusted so that E is made equal to V. When the motor is loaded, the rotor is retarded by 3° mechanical.*

Draw the phasor diagram and calculate the armature current, power factor and power of the motor. What is the maximum power the motor can supply without falling out of step?

(Power Apparatus-II, Delhi Univ. 1988)

Solution. Per phase $E_b = V = 2200/\sqrt{3} = 1270$ V

$\alpha = 3^\circ$ (mech) $= 3^\circ \times (8/2) = 12^\circ$ (elect).

As seen from Fig 38.33 (*a*).

$E_R = (1270^2 + 1270^2 - 2 \times 1270 \times 1270 \times \cos 12^\circ)^{1/2}$

$= 266$ V; $Z_S = \sqrt{0.4^2 + 6^2} = 6.013\ \Omega$

$I_a = E_R / Z_S = 266 / 6.013 =$ **44.2 A.** From ΔOAB,

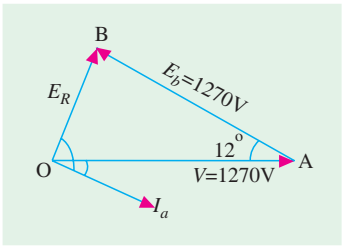


Fig. 38.33 (a)

we get, $\dfrac{1270}{\sin(\theta - \phi)} = \dfrac{266}{\sin 12^\circ}$

$\therefore$ $\sin(\theta - \phi) = 1270 \times 0.2079/266 = 0.9926$ $\therefore$ $(\theta - \phi) = 83^\circ$

Now, $\theta = \tan^{-1}(X_S / R_a) = \tan^{-1}(6 / 0.4) = 86.18^\circ$

$\phi = 86.18^\circ - 83^\circ = 3.18^\circ$ $\therefore$ p.f. $= \cos 3.18^\circ =$ **0.998(lag)**

Total motor power input $= 3 V I_a \cos\phi = 3 \times 1270 \times 44.2 \times 0.998 = 168$ kW

Total Cu loss $= 3 I_a^2 R_a = 3 \times 44.2^2 \times 0.4 = 2.34$ kW

Power developed by motor $= 168 - 2.34 =$ **165.66 kW**

$$P_{m(max)} = \frac{E_b V}{Z_S} - \frac{E_b^2 R_a}{Z_s^2} = \frac{1270 \times 1270}{6.013} - \frac{1270^2 \times 0.4}{6.013^2} =$$ **250 kW**

Example 38.16. *A 1–ϕ, synchronous motor has a back e.m.f. of 250 V, leading by 150 electrical degrees over the applied voltage of 200 volts. The synchronous reactance of the armature is 2.5 times its resistance. Find the power factor at which the motor is operating and state whether the current drawn by the motor is leading or lagging.*

Solution. As induced e.m.f. of 250 V is greater than the applied voltage of 200 V, it is clear that the motor is over-excited, hence it must be working with a leading power factor.

In the vector diagram of Fig. 38.33 (*b*), OA represents applied voltage, AB is back e.m.f. at an angle of 30° because $\angle AOC = 150°$ and $\angle COD = \angle BAO = 30°$. OB represents resultant of voltage V and E_b *i.e.* E_R

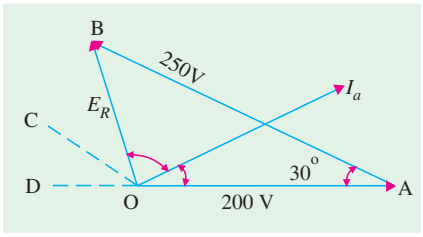


Fig. 38.33 (*b*)

In ΔOBA,

$$E_R = \sqrt{(V^2 + E_b^2 - 2V E_b \cos 30°)}$$

$$= \sqrt{(220^2 + 250^2 - 2 \times 200 \times 250 \times 0.866)} = 126 \text{ V}$$

Now, $\frac{E_R}{\sin 30°} = \frac{E_b}{\sin(\theta + \phi)}$ or $\frac{126}{0.5} = \frac{250}{\sin(\theta + \phi)}$

$\therefore \quad \sin(\theta + \phi) = 125/126$ (approx.) $\therefore \quad (\theta + \phi) = 90°$

Now $\tan \theta = 2.5 \quad \therefore \quad \theta = 68°12' \quad \therefore \quad \phi = 90° - 68°12' = 21°48'$

$\therefore$ p.f. of motor $= \cos 21°48' =$ **0.9285 (leading)**

Example 38.17. *The synchronous reactance per phase of a 3-phase star-connected 6,600 V synchronous motor is 10 Ω. For a certain load, the input is 900 kW and the induced line e.m.f. is 8,900 V. (line value) . Evaluate the line current. Neglect resistance.*

(Basic Elect. Machines, Nagpur Univ. (1993)

Solution. Applied voltage / phase = $6,600 / \sqrt{3} = 3,810$ V

Back e.m.f. / phase $= 8,900 / \sqrt{3} = 5,140$ V

Input $= \sqrt{3}\, V_L \,.\, I \cos \phi = 900,000$

$\therefore \quad I \cos \phi = 9 \times 105 / \sqrt{3} \times 6,600 = 78.74$ A

In ΔABC of vector diagram in Fig. 38.34, we have $AB^2 = AC^2 + BC^2$

Now $OB = I \,.\, X_S = 10\, I$

$BC = OB \cos \phi = 10\, I \cos \phi$

$= 10 \times 78.74 = 787.4$ V

$\therefore \quad 5,140^2 = 787.4^2 + AC^2 \quad \therefore \quad AC = 5,079$ V

$\therefore \quad OC = 5,079 - 3,810 = 1,269$ V

$\tan \phi = 1269/787.4 = 1.612$; $\phi = 58.2°$, $\cos \phi = 0.527$

Now $I \cos \phi = 78.74$; I = 78.74/0.527 = **149.4 A**

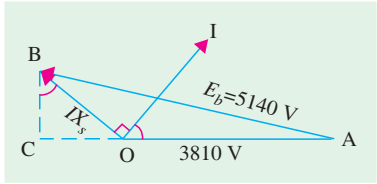


Fig. 38.34

Example 38.18. *A 6600-V, star-connected, 3-phase synchronous motor works at constant voltage and constant excitation. Its synchronous reactance is 20 ohms per phase and armature resistance negligible when the input power is 1000 kW, the power factor is 0.8 leading. Find the power angle and the power factor when the input is increased to 1500 kW.*

(Elect. Machines, AMIE Sec. B 1991)

Solution. When Power Input is 1000 kW (Fig. 38.35 (*a*))

$$\sqrt{3} \times 6600 \times I_{a1} \times 0.8 = 1000{,}000;\ I_{a1}=109.3 \text{ A}$$

$$Z_S = X_S = 20\ \Omega\ ;\ I_{a1} Z_S = 109.3 \times 20 = 2186 \text{ V}\ ;\ \phi_1 = \cos^{-1} 0.8 = 36.9^\circ;\ \theta = 90^\circ$$

$$E_b^2 = 3810^2 + 2186^2 - 2 \times 3810 \times 2186 \times \cos (90^\circ + 36.9^\circ)$$

$$= 3810^2 + 2186^2 - 2 \times 3810 \times 2186 \times - \cos 53.1^\circ;\ \therefore\ E_b = 5410 \text{ V}$$

Since excitation remains constant, E_b in the second case would remain the same *i.e.*, 5410 V.

When Power Input is 1500 kW :

$\sqrt{3} \times 6600 \times I_{a2} \cos \phi_2 = 1500{,}000;\ I_{a2} \cos \phi_2 = 131.2$ A

As seen from Fig. 38.35 (*b*), $OB = I_{a2} Z_S = 20\ I_a$

$BC = OB \cos \phi_2 = 20\ I_{a2}$

$\cos \phi_2 = 20 \times 131.2 = 2624$ V

In ΔABC, we have, $AB^2 = AC^2 + BC^2$ or $5410^2 = AC^2 + 2624^2$

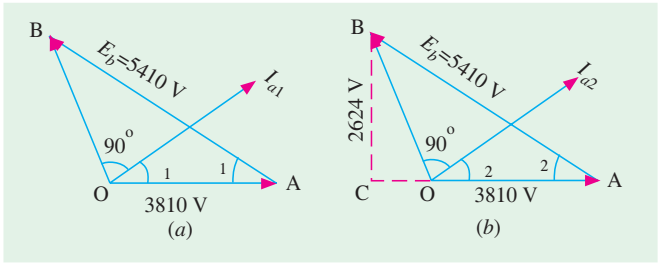


Fig. 38.35

$$\therefore \quad AC = 4730 \text{ V};\ OC = 4730 - 3810 = 920 \text{ V}$$

$$\tan \phi_2 = 920 / 2624\ ;\ \phi_2 = 19.4^\circ;\ \text{p.f.} = \cos \phi_2 = \cos 19.4^\circ = 0.9432 \text{ (lead)}$$

$$\tan \alpha_2 = BC / AC = 2624/4730;\ \alpha_2 = \mathbf{29^\circ}$$

Example 38.19. *A 3-phase, star-connected 400-V synchronous motor takes a power input of 5472 watts at rated voltage. Its synchronous reactance is 10 Ω per phase and resistance is negligible. If its excitation voltage is adjusted equal to the rated voltage of 400 V, calculate the load angle, power factor and the armature current.* **(Elect. Machines AMIE Sec. B, 1990)**

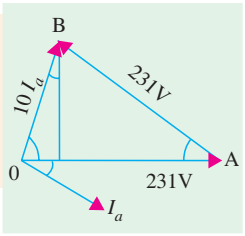


Fig. 38.36

Solution. $\sqrt{3} \times 400 \times I_a \cos \phi = 5472\ ;\ I_a \cos \phi = 7.9$ A

$$Z_S = 10\ \Omega;\ E_R = I_a Z_S = 10\ I_a$$

As seen from Fig. 38.36, $BC = OB \cos \phi = 10,\ I_a \cos \phi = 79$ V

$$AC = \sqrt{231^2 - 79^2} = 217 \text{ V};\ OC = 231 - 217 = 14 \text{ V}$$

$$\tan \phi = 14 / 79;\ \phi = 10^\circ;\ \cos \phi = 0.985 \text{ (lag)}$$

$$I_a \cos \phi = 7.9;\ I_a = 7.9 / 0.985 = 8 \text{ A};\ \tan \alpha = BC / AC = 79 / 217;\ \alpha = \mathbf{20^\circ}$$

Example 38.20. *A 2,000-V, 3-phase, star-connected synchronous motor has an effective resistance and synchronous reactance of 0.2 Ω and 2.2 Ω respectively. The input is 800 kW at normal voltage and the induced e.m.f. is 2,500 V. Calculate the line current and power factor.*

(Elect. Engg. A.M.I.E.T.E., June 1992)

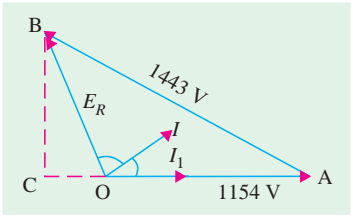


Fig. 38.37

Solution. Since the induced e.m.f. is greater than the applied voltage, the motor must be running with a leading p.f. If the motor current is I, then its in-phase or power component is $I \cos \phi$ and reactive component is $I \sin \phi$.

Let $I \cos \phi = I_1$ and $I \sin \phi = I_2$ so that $\boldsymbol{I} = (I_1 + j I_2)$

$I \cos \phi = I_1 = 800{,}00 / \sqrt{3} = 231$ A

Applied voltage / phase $= 2{,}000 / \sqrt{3} = 1{,}154$ V

Induced e.m.f. / phase $= 2500 / \sqrt{3} = 1{,}443$ V

In Fig. 38.37 $OA = 1154$ V and

$AB = 1443$ V, OI leads OA by ϕ

$E_R = I Z_S$ and $\theta = \tan^{-1} (2.2 / 0.2) = 84.8^\circ$

BC is $\perp AO$ produced.

Now, $\mathbf{E_R} = \mathbf{I Z_S} = (I_1 + j I_2)(0.2 + j2.2)$

$= (231 + jI_2)(0.2 + j2.2) = (46.2 - 2.2 I_2) + j (508.2 + 0.2 I_2)$

Obviously, $OC = (46.2 - 2.2 I_2)$; $BC = j (508.2 + 0.2 I_2)$

From the right-angled ΔABC, we have

$AB^2 = BC^2 + AC^2 = BC^2 + (AO + OC)^2$

or $1443^2 = (508.2 + 0.2 I_2)^2 + (1154 + 46.2 - 2.2 I_2)^2$

Solving the above quadratic equation, we get $I_2 = 71$ A

$I = \sqrt{I_1^2 + I_2^2} = \sqrt{231^2 + 71^2} = \mathbf{242\ A}$

p.f. $= I_1/I = 231/242 = \mathbf{0.95\ (lead)}$

Example 38.21. *A 3 phase, 440-V, 50 Hz, star-connected synchronous motor takes 7.46 kW from the three phase mains. The resistance per phase of the armature winding is 0.5 ohm. The motor operates at a p. f. of 0.75 lag. Iron and mechanical losses amount to 500 watts. The excitation loss is 650 watts. Assume the source for excitation to be a separate one.*

Calculate. ***(i)*** *armature current,* ***(ii)*** *power supplied to the motor,* ***(iii)*** *efficiency of the motor*

(Amravati University 1999)

Solution. A 3 -phase synchronous motor receives power from two sources :

(a) 3-phase a. c. source feeding power to the armature.

(b) D.C. source for the excitation, feeding electrical power only to the field winding.

Thus, power received from the d. c. source is utilized only to meet the copper-losses of the field winding.

3 Phase a.c. source feeds electrical power to the armature for following components of power:

(i) Net mechanical power output from the shaft

(ii) Copper-losses in armature winding

(iii) Friction, and armature-core-losses.

In case of the given problem

$\sqrt{3} \times I_a \times 440 \times 0.75 = 7460$

$I_a = 13.052$ amp

Total copper-loss in armature winding $= 3 \times 13.052^2 \times 0.50 = 255$ watts

Power supplied to the motor $= 7460 + 650 = \mathbf{8110\ watts}$

efficiency of the motor $= \dfrac{\text{Output}}{\text{Input}}$

Output from shaft = (Armature Input) – (Copper losses in armature winding) – (friction and iron losses)

$= 7460 - 255 - 500 = 6705$ watts

Efficiency of the motor $= \dfrac{6705}{8110} \times 100\% = \mathbf{82.7\%}$

Example 38.22. *Consider a 3300 V delta connected synchronous motor having a synchronous reactance per phase of 18 ohm. It operates at a leading pf of 0.707 when drawing 800 kW from mains. Calculate its excitation emf and the rotor angle (= delta), explaining the latter term.* **(Elect. Machines Nagpur Univ. 1993)**

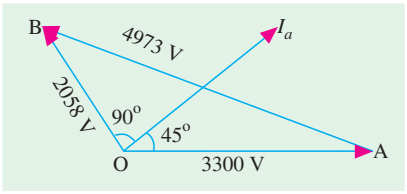


Fig. 38.38

Solution. $\sqrt{3} \times 3300 \times I_a \times 0.707 = 800{,}000$

$\therefore$ Line current = 198 A, phase current, $I_a = 198 / \sqrt{3} = 114.3$ A;

$$Z_S = 18\ \Omega\ ;\ I_a Z_S = 114.3 \times 18 = 2058\ \text{V}$$
$$\phi = \cos^{-1} 0.707;\ \phi = 45^\circ;\ \theta = 90^\circ;$$
$$\cos(\theta + \phi) = \cos 135^\circ = -\cos 45^\circ = -0.707$$

From Fig. 38.38, we find

$$E_b^2 = 3300^2 + 2058^2 - 2 \times 3300 \times 2058 \times -0.707$$
$$\therefore \quad E_b = \mathbf{4973\ V}$$

From ΔOAB, we get $2058/\sin\alpha = 4973/\sin 135^\circ$. Hence, $\alpha = \mathbf{17^\circ}$

Example 38.23. *A 75-kW, 400-V, 4-pole, 3-phase star connected synchronous motor has a resistance and synchronous reactance per phase of 0.04 ohm and 0.4 ohm respectively. Compute for full-load 0.8 p.f. lead the open circuit e.m.f. per phase and mechanical power developed. Assume an efficiency of 92.5%.* **(Elect. Machines AMIE Sec. B 1991)**

Solution. Motor input = 75,000 / 0.925 = 81,080 Ω

$$I_a = 81{,}080 / \quad \times 400 \times 0.8 = 146.3\ \text{A}\ ;\ Z_S = \sqrt{0.04^2 + 0.4^2} = 0.402\ \Omega$$

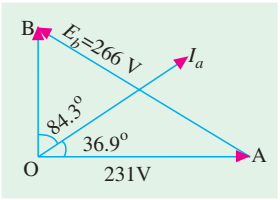


Fig. 38.39

$I_a Z_S = 146.3 \times 0.402 = 58.8$ V; $\tan\phi = 0.4 / 0.04 = 10$;
$\theta = 84.3^\circ$; $\phi = \cos^{-1} 0.8$; $\phi = 36.9^\circ$; $(\theta + \phi) = 121.2^\circ$;

$V_{ph} = 400 / \sqrt{3} = 231$ V

As seen from Fig. 36.39,

$E_b^2 = 231^2 + 58.8^2 - 2 \times 231 \times 58.8 \times \cos 121.2$; E_b / phase = 266 V

Stator Cu loss for 3 phases = $3 \times 146.3^2 \times 0.04 = 2570$ W;

$N_s = 120 \times 50/40 = 1500$ r.p.m.

$P_m = 81080 - 2570 = \mathbf{78510\ W}$; $T_g = 9.55 \times 78510/1500 = \mathbf{500\ N\text{-}m.}$

Example 38.24. *A 400-V, 3-phase, 50-Hz, Y-connected synchronous motor has a resistance and synchronous impedance of 0.5 Ω and 4 Ω per phase respectively. It takes a current of 15 A at unity power factor when operating with a certain field current. If the load torque is increased until the line current is increased to 60 A, the field current remaining unchanged, calculate the gross torque developed and the new power factor.* **(Elect. Machines, AMIE Sec. B 1992)**

Solution. The conditions corresponding to the first case are shown in Fig. 38.40.

$$\text{Voltage/phase} = 400/\sqrt{3} = 231\ \text{V}\ ;\ I_a Z_S = OB = 15 \times 4 = 60\ \text{V}$$
$$X_S = \sqrt{Z_S^2 - R_a^2} = \sqrt{4^2 - 0.5^2} = 3.968\ \Omega$$
$$\theta = \tan^{-1}(3.968/0.5) = \tan^{-1}(7.936) = 81.8^\circ$$
$$E_b^2 = 231^2 + 60^2 - 2 \times 231 \times 60 \times \cos 81^\circ 48'; \quad E_b = 231\ V$$

It is obvious that motor is running with normal excitation because $E_b = V$

When the motor load is increased, the phase angle between the applied voltage and the induced (or back) e.m.f. is increased. Art (38.7). The vector diagram is as shown in Fig. 38.41.

Let ϕ be the new phase angle.

$I_a Z_S = 60 \times 4 = 240$ V

$\angle BOA = (81°48' - \phi)$.

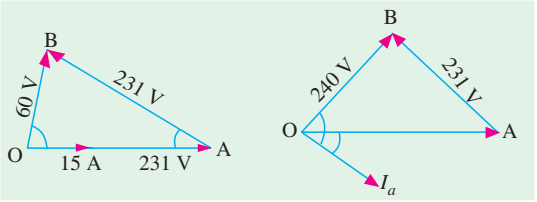


Fig. 38.40 Fig. 38.41

Since the field current remains constant, the value of E_b remains the same.

$\therefore \quad 231^2 = 231^2 + 240^2 - 2 \times 231 \times 240 \cos (81°48' - \phi)$

$\therefore \quad \cos (81°48' - \phi) = 0.4325$ or $81° 48' - \phi = 64°24'$

$\therefore \quad \phi = 81°48' - 64°24' = 17°24'$. New p.f. = cos 17°24′ = **0.954 (lag)**

Motor input $= \sqrt{3} \times 400 \times 60 \times 0.954 = 39{,}660$ W

Total armature Cu loss $= 3 \times 60^2 \times 0.5 = 5{,}400$ W

Electrical power converted into mechanical power = 39,660 – 5,400 – 34,260 W

$N_S = 120 \times 50/6 = 1000$ r.p.m. $\quad T_g = 9.55 \times 34{,}260/1000 =$ **327 N-m**

Example 38.25. *A 400-V, 10 h.p. (7.46 kW), 3-phase synchronous motor has negligible armature resistance and a synchronous reactance of 10* W */ phase. Determine the minimum current and the corresponding induced e.m.f. for full-load conditions. Assume an efficiency of 85%.*

(A.C. Machines-I, Jadavpur Univ. 1987)

Solution. The current is minimum when the power factor is unity *i.e.*, when cos $\phi = 1$. The vector diagram is as shown in Fig. 38.42.

Motor input = 7460 / 0.85 = 8,775 W

Motor line current = 8,775 / $\sqrt{3} \times 400 \times 1$ = **12.67 A**

Impedance drop = $I_a X_S = 10 \times 12.67 = 126.7$ V

Voltage / phase = 400 / $\sqrt{3}$ = 231 V

$E_b = \sqrt{231^2 + 126.7^2}$ = **263.4 V**

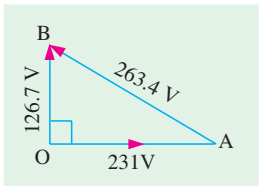


Fig. 38.42

Example 38.26. *A 400-V, 50-Hz, 3-phase, 37.5 kW, star-connected synchronous motor has a full-load efficiency of 88%. The synchronous impedance of the motor is (0.2 + j 1.6) ohm per phase. If the excitation of the motor is adjusted to give a leading power factor of 0.9, calculate the following for full load :*

(i) *the excitation e.m.f.*

(ii) *the total mechanical power developed* **(Elect.Machines, A.M.I.E. Sec. B, 1989)**

Solution. Motor input = 37.5/0.88 = 42.61 kW; $I_a = 42{,}610 / \sqrt{3} \times 400 \times 0.9 = 68.3$ A

$V = 400 / \sqrt{3} = 231$ V; $Z_S = 0.2 + j\,1.6 = 1.612 \angle 82.87°$

$E_R = I_a Z_S = 68.3 \times 1.612 = 110$ V;

$\phi = \cos^{-1}(0.9) = 25.84°$

Now, $(\phi + \theta) = 25.84° + 82.87° = 108.71°$

$\cos (\phi + \theta) = \cos 108.71° = -0.32$

(a) $\therefore \quad E_b^2 = V^2 + E_R^2 - 2 V E_R \cos 108.71°$ or $E_b = 286$ V

Line value of excitation voltage = $\sqrt{3} \times 285 =$ **495 V**

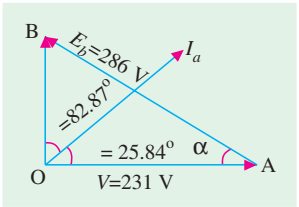


Fig. 38.43

(b) From ΔOAB, (Fig. 38.43) $E_R / \sin \alpha = E_b / \sin (\phi + \theta)$, $\alpha = 21.4°$

$$P_m = 3 \frac{E_b V}{Z_S} \sin \alpha = 3 \frac{286 \times 231}{1.612} \sin 21.4° = \textbf{14,954 W}$$

Example 38.27. *A 6600-V, star-connected, 3-phase synchronous motor works at constant voltage and constant excitation. Its synchronous reactance is 20 ohm per phase and armature resistance negligible. When the input power is 1000 kW, the power factor is 0.8 leading. Find the power angle and the power factor when the input is increased to 1500 kW.*

(Elect. Machines, A.M.I.E., Sec. B, 1991)

Solution. $V = 6600/\sqrt{3} = 3810$ V, $I_a = 1000 \times 10^3/\sqrt{3} \times 6600 \times 0.8 = 109.3$ A

The phasor diagram is shown in Fig. 38.44. Since R_a is negligible, $\theta = 90^\circ$

$$E_R = I_a X_S = 109.3 \times 20 = 2186 \text{ V}$$
$$\cos = 0.8, \phi = 36.87^\circ$$
$$E_b^2 = V^2 + E_R^2 - 2E_bV \cos(90^\circ + 36.87^\circ);$$
$$E_b = 5410 \text{ V}$$

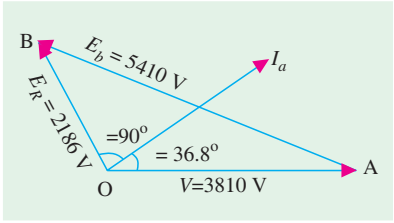


Fig. 38.44

Now, excitation has been kept constant but power has been increased to 1500 kW

$$\therefore \quad 3\frac{E_bV}{Z_S}\sin\alpha = P;\ 3 \times \frac{5410 \times 3810}{20}\sin\alpha = 1500 \times 10^3;\ \alpha = 29^\circ$$

Also,
$$\frac{E_b}{(\sin 90^\circ + \phi)} = \frac{V}{\sin[180^\circ - (\alpha + 90 + \phi)]} = \frac{V}{\cos(\alpha + \phi)}$$

or
$$\frac{E_b}{\cos\phi} = \frac{V}{\cos(\alpha + \phi)} \quad \text{or} \quad \frac{V}{E_b} = \frac{\cos(29^\circ + \phi)}{\cos\phi} = 0.3521$$

$\therefore \quad \phi = 19.39^\circ$, $\cos\phi = \cos 19.39^\circ =$ **0.94 (lead)**

Example 38.28. *A 400-V, 50-Hz, 6-pole, 3-phase, Y-connected synchronous motor has a synchronous reactance of 4 ohm/phase and a resistance of 0.5 ohm/phase. On full-load, the excitation is adjusted so that machine takes an armature current of 60 ampere at 0.866 p.f. leading.*

Keeping the excitation unchanged, find the maximum power output. Excitation, friction, windage and iron losses total 2 kW. **(Electrical Machinery-III, Bangalore Univ. 1990)**

Solution. $V = 400/\sqrt{3} = 231$ V/phase; $Z_S = 0.5 + j4 = 4.03 \angle 82.9^\circ$; $\theta = 82.9^\circ$

$$I_aZ_S = 60 \times 4.03 = 242 \text{ V}; \cos\phi = 0.866$$
$$\phi = 30^\circ \text{ (lead)}$$

As seen from Fig. 36.45,

$$E_b^2 = 231^2 + 242^2 - 2 \times 231 \times 242 \cos 112.9^\circ$$
$$E_b = \textbf{394 V}$$
$$(P_m)_{max} = \frac{E_bV}{Z_S} - \frac{E_b^2 R_a}{Z_{S^2}} = \frac{394 \times 231}{4.03} - \frac{394^2 \times 0.5}{4.03^2}$$
$$= 17{,}804 \text{ W/phase.} \quad \text{–Art. 38.12}$$

Maximum power developed in armature for 3 phases

$$= 3 \times 17{,}804 = 52{,}412 \text{ W}$$

Net output $= 52{,}412 - 2{,}000 = 50{,}412$ W = **50.4 kW**

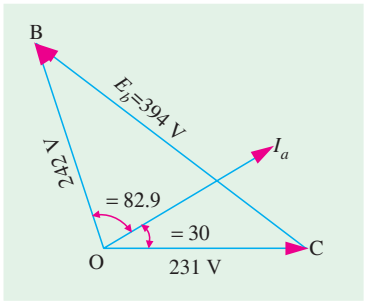


Fig. 38.45

Example 38.29. *A 6-pole synchronous motor has an armature impedance of 10 Ω and a resistance of 0.5 Ω. When running on 2,000 volts, 25-Hz supply mains, its field excitation is such that the e.m.f. induced in the machine is 1600 V. Calculate the maximum total torque in N-m developed before the machine drops out of synchronism.*

Solution. Assuming a three-phase motor,

$V = 2000$ V, $E_b = 1600$ V ; $R_a = 0.5\ \Omega$; $Z_S = 10\ \Omega$; $\cos\theta = 0.5/10 = 1/20$

Using equation (*iii*) of Art. 37-10, the total max. power for 3 phases is

$$(P_m)_{max} = \frac{E_b V}{Z_S} - \frac{E_b^2}{Z_S}\cos\theta = \frac{2000\times1600}{10} - \frac{1600^2\times1}{10\times20} = 307{,}200 \text{ watt}$$

Now, $N_S = 120\,f/P = 120\times25/6 = 500$ r.p.m.

Let $T_{g\,.\,max}$ be the maximum gross torque, then

$$T_{g\,max} = 9.55\times\frac{307200}{500} = \mathbf{5{,}868\ N\text{-}m}$$

Example. 38.30. *A 2,000-V, 3-phase, 4-pole, Y-connected synchronous motor runs at 1500 r.p.m. The excitation is constant and corresponds to an open-circuit terminal voltage of 2,000 V. The resistance is negligible as compared with synchronous reactance of 3 Ω per phase. Determine the power input, power factor and torque developed for an armature current of 200 A.*

(Elet. Engg.-I, Nagpur Univ. 1993)

Solution.

Voltage/phase = $2000/\sqrt{3} = 1150$ V

Induced e.m.f. = 1150 V – given

Impedance drop = $200\times3 = 600$ V

As shown in Fig. 38.46, the armature current is assumed to lag behind V by an angle ϕ. Since R_a is negligible, $\theta = 90°$.

$\angle BOA = (90° - \phi)$

Considering ΔBOA, we have

$1150^2 = 1150^2 + 600^2 - 2\times600\times1150\cos(90-\phi°)$

$\sin\phi = 0.2605$; $\phi = 16.2°$; p.f. $= \cos 16.2° = \mathbf{0.965\ (lag)}$

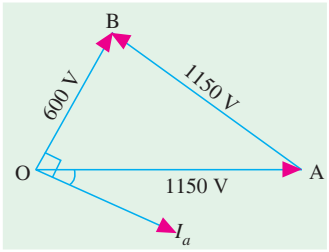


Fig. 38.46

Power input = $\sqrt{3}\times2{,}000\times200\times0.965 = \mathbf{668.5\ kW}$

$N_S = 1500$ r.p.m. $\quad\therefore\quad T_g = 9.55\times66{,}850/1500 = \mathbf{4{,}255\ N\text{-}m.}$

Example 38.31. *A 3-ϕ, 3300-V, Y-connected synchronous motor has an effective resistance and synchronous reactance of 2.0 Ω and 18.0 Ω per phase respectively. If the open-circuit generated e.m.f. is 3800 V between lines, calculate* ***(i)*** *the maximum total mechanical power that the motor can develop and* ***(ii)*** *the current and p.f. at the maximum mechanical power.*

(Electrical Machines-III. Gujarat Univ. 1988)

Solution. $\theta = \tan^{-1}(18/2) = 83.7°$; $V_{ph} = 3300/\sqrt{3} = 1905$ V; $E_b = 3800/\sqrt{3} = 2195$ V

Remembering that $\alpha = \theta$ for maximum power development (Ar. 38-10)

$E_R = (1905^2 + 2195^2 - 2\times1905\times2195\times\cos83.7°)^{1/2} = 2744$ volt per phase

$\therefore\quad I_a Z_S = 2{,}744$; Now, $Z_S = \sqrt{2^2+18^2} = 18.11\ \Omega$

$\therefore\quad I_a = 2744/18.11 = 152$ A/phase ; line current = **152 A**

$$(P_m)_{max}\text{ per phase} = \frac{E_b V}{Z_S} - \frac{E_b^2 R_a}{Z_{S^2}} = \frac{2195\times1905}{18.11} - \frac{2195^2\times2}{18.11^2}$$

$= 230{,}900 - 29{,}380 = 201520$ W per phase

Maximum power for three phases that the motor can develop *in its armature*

$= 201{,}520\times3 = \mathbf{604{,}560\ W}$

Total Cu losses = $3\times152^2\times2 = 138{,}700$ W

Motor input = $604{,}560 + 138{,}700 = 743{,}260$ W

$\therefore\quad \sqrt{3}\times3300\times152\times\cos\phi = 743{,}260 \quad\therefore\quad \cos\phi = \mathbf{0.855\ (lead).}$

Example 38.32. *The excitation of a 415-V, 3-phase, mesh-connected synchronous motor is such that the induced e.m.f. is 520 V. The impedance per phase is (0.5 + j4.0) ohm. If the friction and iron losses are constant at 1000 W, calculate the power output, line current, power factor and efficiency for maximum power output.* **(Elect. Machines-I, Madras Univ. 1987)**

Solution. As seen from Art. 38-12, for fixed E_b, V, R_a and X_S, maximum power is developed when $\alpha = \theta$.

Now, $\theta = \tan^{-1}(4/0.5) = \tan^{-1}(8) = 82.90^\circ = \alpha$

$$E_R = \sqrt{415^2 + 520^2 - 2 \times 415 \times 520 \times \cos 82.9^\circ} = 625 \text{ V per phase}$$

Now, $IZ_S = 625$; $Z_S = \sqrt{4^2 + 0.5^2} = 4.03\ \Omega$ $\therefore$ $I = 625/4.03 = 155$ A

Line current $= \times 155 = 268.5$ A

$$(P_m)_{max} = \frac{E_b V}{Z_S} - \frac{E_b^2 R_a}{Z_S^2} = \frac{520 \times 415}{4.03} - \frac{520^2 \times 0.5}{16.25} = 45{,}230 \text{ W}$$

Max. power for 3 phases $= 3 \times 45{,}230 = 135{,}690$ W

Power output = power developed – iron and friction losses

$= 135{,}690 - 1000 = 134{,}690$ W = **134.69 kW**

Total Cu loss $= 3 \times 155^2 \times 0.5 = 36{,}080$ W

Total motor input $= 135{,}690 + 36{,}080 = 171{,}770$ W

$\therefore$ $\sqrt{3} \times 415 \times 268..5 \times \cos\phi = 171{,}770$; $\cos\phi =$ **0.89 (lead)**

Efficiency $= 134{,}690/171{,}770 = 0.7845$ or **78.45%**

Tutorial Problems 38.1

1. A 3-phase, 400-V, synchronous motor takes 52.5 A at a power factor of 0.8 leading. Calculate the power supplied and the induced e.m.f. The motor impedance per phase is (0.25 + *j*3.2) ohm.
[29.1 kW; 670V]
2. The input to a 11-kV, 3ϕ, Y-connected synchronous motor is 60 A. The effective resistance and synchronous reactance per phase are $I\ \Omega$ and 30 Ω respectively. Find (*a*) power supplied to the motor and (*b*) the induced e.m.f. for a p.f. of 0.8 leading.
[(*a*) 915 kW (*b*) 13kV] (*Grad. I.E.T.E. Dec. 1978*)
3. A 2,200-V, 3-phase, star-connected synchronous motor has a resistance of 0.6 Ω and a synchronous reactance of 6 Ω. Find the generated e.m.f. and the angular retardation of the motor when the input is 200 kW at (*a*) power factor unity and (*b*) power factor 0.8 leading.
[(*a*) 2.21 kV; 14.3° (*b*) 2.62 kV; 12.8°]
4. A 3-phase, 220-V, 50-Hz, 1500 r.p.m., mesh-connected synchronous motor has a synchronous impedance of 4 ohm per phase. It receives an input line current of 30 A at a leading power factor of 0.8. Find the line value of the induced e.m.f. and the load angle expressed in mechanical degrees.
If the mechanical load is thrown off without change of excitation, determine the magnitude of the current under the new conditions. Neglect losses. **[268 V; 6°, 20.6 A]**
5. A 400-V, 3-phase, Y-connected synchronous motor takes 3.73 kW at normal voltage and has an impedance of (1 + *j*8) ohm per phase. Calculate the current and p.f. if the induced e.m.f. is 460 V.
[6.28 A; 0.86 lead] (*Electrical Engineering, Madras Univ. April 1979*)
6. The input to 6600-V, 3-phase, star-connected synchronous motor is 900 kW. The synchronous reactance per phase is 20 Ω and the effective resistance is negligible. If the generated voltage is 8,900 V

(line), calculate the motor current and its power factor.

[Hint. See solved Ex. 38.17] (*Electrotechnics, M.S. Univ. April 1979*)

7. A 3-phase synchronous motor connected to 6,600-V mains has a star-connected armature with an impedance of (2.5 + *j*15) ohm per phase. The excitation of machine gives 7000 V. The iron, friction and excitation losses are 12 kW. Find the maximum output of the motor. **[153.68 kW]**

8. A 3300-V, 3-phase, 50-Hz, star-connected synchronous motor has a synchronous impedance of (2 + *j*15) ohm. Operating with an excitation corresponding to an e.m.f. of 2,500 V between lines, it just falls out of step at full-load. To what open-circuit e.m.f. will it have to be excited to stand a 50% excess torque. **[4 kV]**

9. A 6.6 kV, star-connected, 3-phase, synchronous motor works at constant voltage and constant excitation. Its synchronous impedance is (2.0 + *j* 20) per phase. When the input is 1000 kW, its power factor is 0.8 leading. Find the power factor when the input is increased to 1500 kW (solve graphically or otherwise).

[0.925 lead] (*AMIE Sec. B Advanced Elect. Machines (E-9) Summer 1991*)

10. A 2200-V, 373 kW, 3-phase, star-connected synchronous motor has a resistance of 0.3 Ω and a synchronous reactance of 3.0 Ω per phase respectively. Determine the induced e.m.f. per phase if the motor works on full-load with an efficiency of 94 per cent and a p.f. of 0.8 leading.

[1510 V] (*Electrical Machinery, Mysore Univ. 1992*)

11. The synchronous reactance per phase of a 3-phase star-connected 6600 V synchronous motor is 20 Ω. For a certain load, the input is 915 kW at normal voltage and the induced line e.m.f. is 8,942 V. Evaluate the line current and the p.f. Neglect resistance. **[97 A; 0.8258 (lead)]**

12. A synchronous motor has an equivalent armature reactance of 3.3 Ω. The exciting current is adjusted to such a value that the generated e.m.f. is 950 V. Find the power factor at which the motor would operate when taking 80 kW from a 800-V supply mains. **[0.965 leading] (*City & Guilds, London*)**

13. The input to an 11000 V, 3-phase star-connected synchronous motor is 60 A. The effective resistance and synchronous reactance per phase are respectively 1 ohm and 30 ohms. Find the power supplied to the motor and the induced electromotive force for a power factor of 0.8 leading.

[914.5 kW, 13 kV] (*Elect. Machines, A.M.I.E. Sec. B, 1990*)

14. A 400-V, 6-pole, 3-phase, 50-Hz, star-connected synchronous motor has a resistance and synchronous reactance of 0.5 ohm per phase and 4-ohm per phase respectively. It takes a current of 15 A at unity power factor when operating with a certain field current. If the load torque is increased until the line current is 60 A, the field current remaining unchanged, find the gross torque developed, and the new power factor. **[354 Nm; 0.93] (*Elect. Engg. AMIETE Dec. 1990*)**

15. The input to a 11,000-V, 3-phase, star-connected synchronous motor is 60 amperes. The effective resistance and synchronous reactance per phase are respectively 1 ohm and 30 ohm. Find the power supplied to the motor and the induced e.m.f. for power factor of 0.8 (*a*) leading and (*b*) lagging.

[915 kW (a) 13 kV (*b*) 9.36 kV] (*Elect. Machines-II, South Gujarat Univ. 1981*)

16. Describe with the aid of a phasor diagram the behaviour of a synchronous motor starting from no-load to the pull-out point.

What is the output corresponding to a maximum input to a 3-ϕ delta-connected 250-V, 14.92 kW synchronous motor when the generated e.m.f. is 320 V ? The effective resistance and synchronous reactance per phase are 0.3 Ω and 4.5 Ω respectively. The friction, windage, iron and excitation losses total 800 watts and are assumed to remain constant. Give values for (*i*) output (*ii*) line current (*iii*) p.f.

[(*i*) 47.52 kW (*ii*) 161 A (*iii*) 0.804] (*Elect. Machines, Indore Univ. Feb. 1982*)

17. A synchronous motor takes 25 kW from 400 V supply mains. The synchronous reactance of the motor is 4 ohms. Calculate the power factor at which the motor would operate when the field excitation is so adjusted that the generated EMF is 500 volts.

[0.666 Leading] (*Rajiv Gandhi Technical University, Bhopal, 2000*)

38.17. Effect of Excitation on Armature Current and Power Factor

The value of excitation for which back e.m.f. E_b is equal (in magnitude) to applied voltage V is known as 100% excitation. We will now discuss what happens when motor is either over-excited or under-exicted although we have already touched this point in Art. 38-8.

Consider a synchronous motor in which the mechanical load is constant (and hence output is also constant if losses are neglected).

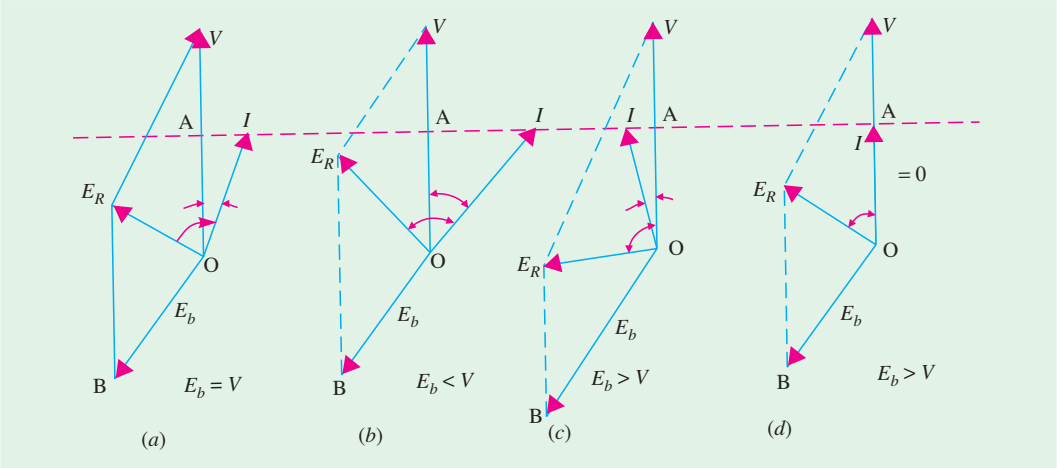


Fig. 38.47

Fig. 38.47 (*a*) shows the case for 100% excitation *i.e.*, when $E_b = V$. The armature current I lags behind V by a small angle ϕ. Its angle θ with E_R is fixed by stator constants *i.e.* $\tan\theta = X_S / R_a$.

In Fig. 38.47 (*b*)* excitation is less than 100% *i.e.*, $E_b < V$. Here, E_R is advanced clockwise and so is armature current (because it lags behind E_R by fixed angle θ). We note that the magnitude of I is increased but its power factor is decreased (ϕ has increased). Because input as well as V are constant, hence the power component of I *i.e.*, $I \cos\phi$ remains the same as before, but wattless component $I \sin\phi$ is increased. Hence, as excitation is decreased, I will increase but p.f. will decrease so that power component of I *i.e.*, $I \cos\phi = OA$ will remain constant. In fact, the locus of the extremity of current vector would be a straight horizontal line as shown.

Incidentally, it may be noted that when field current is reduced, the motor pull-out torque is also reduced in proportion.

Fig. 38.47 (*c*) represents the condition for overexcited motor *i.e.* when $E_b > V$. Here, the resultant voltage vector E_R is pulled anticlockwise and so is I. It is seen that now motor is drawing a leading current. It may also happen for some value of excitation, that I may be in phase with V *i.e.*, p.f. is unity [Fig. 38.47 (*d*)]. At that time, the current drawn by the motor would be ***minimum***.

Two important points stand out clearly from the above discussion :

(i) The magnitude of armature current varies with excitation. The current has large value both for low and high values of excitation (though it is lagging for low excitation and leading for higher excitation). In between, it has minimum value corresponding to a certain excitation. The variations of I with excitation are shown in Fig. 38.48 (*a*) which are known as '*V*' curves because of their shape.

(ii) For the same input, armature current varies over a wide range and so causes the power factor also to vary accordingly. When over-excited, motor runs with leading p.f. and with lagging p.f. when under-excited. In between, the p.f. is unity. The variations of *p.f.* with excitation

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* These are the same diagrams as given in Fig. 38.7 and 8 expect that vector for V has been shown vertical.

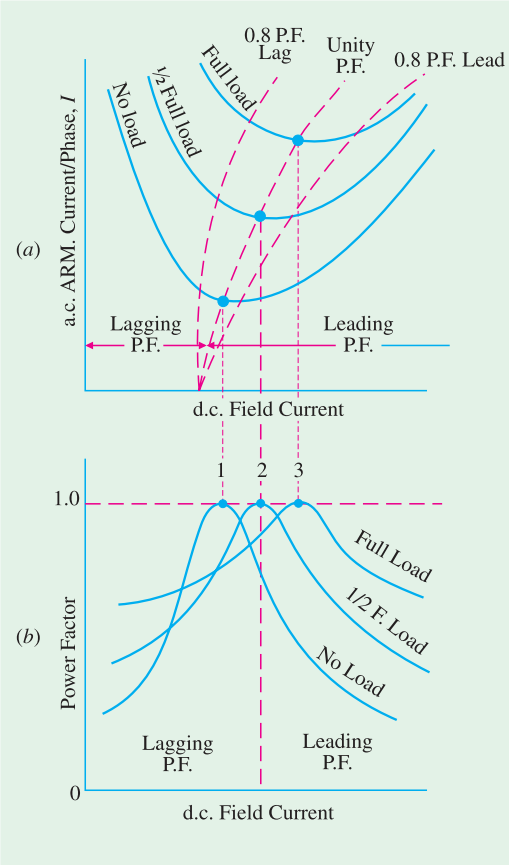


Fig. 38.48



Inductor motor

are shown in Fig. 38.48 (*b*). The curve for p.f. looks like inverted '*V*' curve. It would be noted that ***minimum armature current corresponds to unity power factor***.

It is seen (and it was pointed out in Art. 38.1) that an over-excited motor can be run with leading power factor. This property of the motor renders it extremely useful for phase advancing (and so power factor correcting) purposes in the case of industrial loads driven by induction motors (Fig. 38.49) and lighting and heating loads supplied through transformers. Both transformers and induction motors draw lagging currents from the line. Especially on light loads, the power drawn by them has a large reactive component and the power factor has a very low value. This reactive component, though essential for operating the electric machinery, entails appreciable loss in many ways. By using synchronous motors in conjunction with induction motors and transformers, the lagging reactive power required by the latter is supplied locally by the leading reactive component taken by the former, thereby relieving the line and generators of much of the reactive component. Hence, they now supply only the active component of the load current. When used in this way, a synchronous motor is called a ***synchronous capacitor***, because it draws, like a capacitor, leading current from the line. Most synchronous capacitors are rated between 20 MVAR and 200 MVAR and many are hydrogen-cooled.

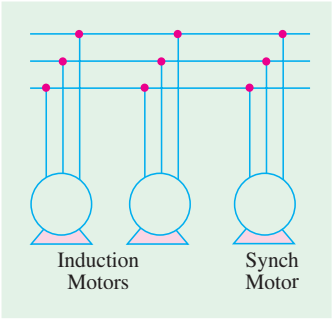


Fig. 38.49

Example 38.33. *Describe briefly the effect of varying excitation upon the armature current and p.f. of a synchronous motor when input power to the motor is maintained constant.*

A 400-V, 50-Hz, 3-ϕ, 37.3 kW, star-connected synchronous motor has a full-load efficiency of 88%. The synchronous impedance of the motor is (0.2 + j 1.6) Ω per phase. If the excitation of the motor is adjusted to give a leading p.f. of 0.9, calculate for full-load ***(a)*** *the induced e.m.f.* ***(b)*** *the total mechanical power developed.*

Solution. Voltage / phase = 400 / $\sqrt{3}$ = 231 V;

$Z_S = \sqrt{(1.6^2 + 0.2^2)} = 1.61\ \Omega$;

Full-load current $= \dfrac{37,300}{\sqrt{3} \times 400 \times 0.88 \times 0.9}$

$= 68$ A

$\therefore \quad I Z_S = 1.61 \times 68 = 109.5$ V

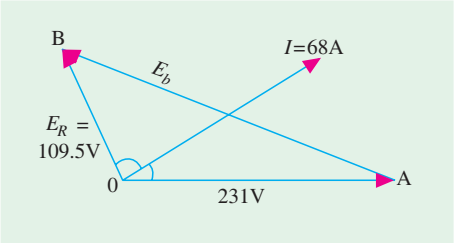


Fig. 38.50

With reference to Fig. 38.50

$\tan \theta = 1.6 / 0.2 = 8, \theta = 82°54'$

$\cos \phi = 0.9, \phi = 25°50'$.

$\therefore \quad (\theta + \phi) = 82°54' + 25°50' = 108°44'$

Now $\cos 108°44' = -0.3212$

(*a*) In $\Delta\, OAB$, $E_b^2 = 231^2 + 109.5^2 - 2 \times 231 \times 109.5 \times (-0.3212) = 285.6^2$; $E_b = 285.6$ V

Line value of $E_b = \sqrt{3} \times 285.6 =$ **495 V**

(*b*) Total motor input = 37,300 / 0.88 = 42,380 W

Total Cu losses $= 3 \times I^2 R_a = 3 \times 68^2 \times 0.2 = 2,774$ W

$\therefore$ Electric power converted into mechanical power = 42,380 – 2,774 = **39.3 kW.**

Example 38.34. *A 3-ϕ, star-connected synchronous motor takes 48 kW at 693 V, the power factor being 0.8 lagging. The induced e.m.f. is increased by 30%, the power taken remaining the same. Find the current and the p.f. The machine has a synchronous reactance of 2 W per phase and negligible resistance.*

Solution. Full-load current

$= 48.000/\sqrt{3} \times 693 \times 0.8 = 50$ A

Voltage/phase $= 693/\sqrt{3} = 400$ V

$Z_S = X_S = 2\ \Omega \quad \therefore \quad IZ_S = 50 \times 2 = 100$ V

$\tan \theta = 2/0 = \infty \quad \therefore\ \theta = 90°$; $\cos \phi = 0.8$, $\sin \phi = 0.6$

The vector diagram is shown in Fig. 38.51. In $\Delta\, OAB$,

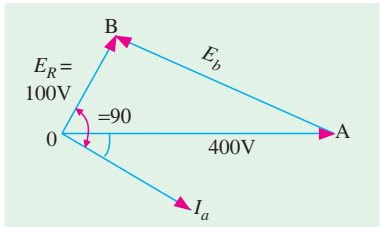


Fig. 38.51

$E_b^2 = 400^2 + 100^2 - 2 \times 400 \times 100 \times \cos(90° - \phi)$

$= 400^2 + 100^2 - 2 \times 400 \times 100 \times 0.6 = 349^2 \quad \therefore\ E_b = 349$ V.

The vector diagram for increased e.m.f. is shown in Fig. 38.52. Now, $E_b = 1.3 \times 349 = 454$ V. It can be safely assumed that in the second case, current is leading V by some angle ϕ'.

Let the new current and the leading angle of current by I' and ϕ' respectively. As power input remains the same and V is also constant, $I \cos \phi$ should be the same far the same input.

$\therefore \quad I \cos \phi = 50 \times 0.8 = 40 = I' \cos \phi'$

In $\Delta ABC, AB^2 = AC^2 + BC^2$

Now $BC = I' X_S \cos \phi' \quad (\because OB = I' X_S)$

$= 40 \times 2 = 80$ V

$\therefore \quad 454^2 = AC^2 + 80^2$ or $AC = 447$ V

$\therefore \quad OC = 447 - 400 = 47$ V

$\therefore \quad \tan \phi' = 47/80, \phi' = 30°26'$

$\therefore$ New p.f. $= \cos 30°26'$

$=$ **0.8623 (leading)**

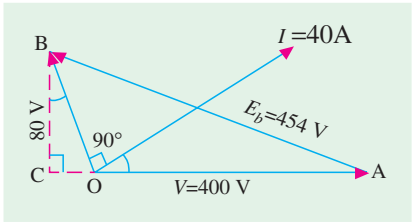


Fig. 38.52

Also, $I' \cos \phi' = 40 \quad \therefore \quad I' = 40/0.8623 =$ **46.4 A.**

Example 38.35. *A synchronous motor absorbing 60 kW is connected in parallel with a factory load of 240 kW having a lagging p.f. of 0.8. If the combined load has a p.f. of 0.9, what is the value of the leading kVAR supplied by the motor and at what p.f. is it working ?*

(Electrical Engineering-II, Banglore Univ. 1990)

Solution. Load connections and phase relationships are shown in Fig. 38.53.

Total load = 240 + 60 = 300 kW; combined p.f. = 0.9 (lag)

ϕ = 25.8°, tan ϕ = 0.4834, combined kVAR = 300 × 0.4834 = 145(lag)

Factory Load

$\cos \phi_L$ = 0.8, ϕ_L = 36.9°, tan ϕ_L = 0.75, load kVAR = 240 × 0.75 = 180 (lag)

or load kVA = 240/0.8 = 300, kVAR = 300 × sin ϕ_L = 300 × 0.6 = 180

∴ *leading* kVAR supplied by synchronous motor = 180 – 145 = **35.**

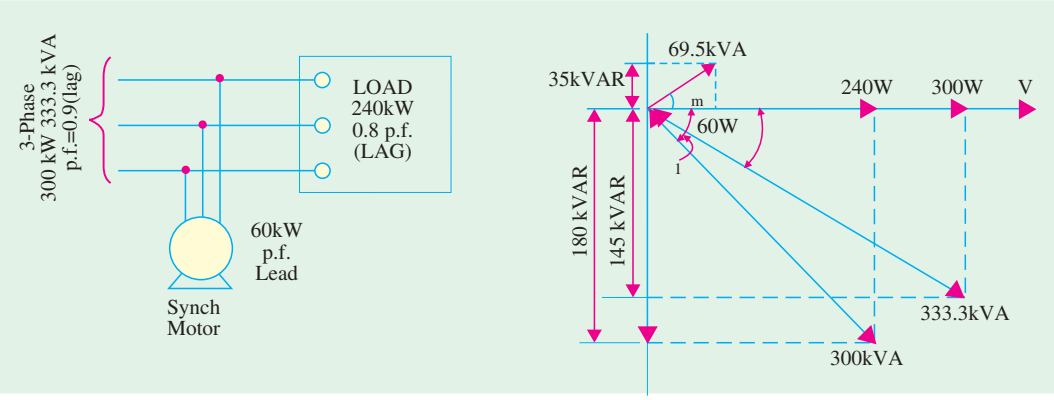


Fig. 38.53

For Synchronous Motor

kW = 60, leading kVAR = 35, tan ϕ_m = 35/60 ; ϕ_m = 30.3° ; cos 30.3° = 0.863

∴ motor p.f. = **0.863 (lead).** Incidentally, motor kVA = $\sqrt{60^2 + 35^2}$ = **69.5.**

38.18. Constant-power Lines

In Fig. 38.54, OA represents applied voltage / phase of the motor and AB is the back e.m.f. / phase, E_b. OB is their resultant voltage E_R. The armature current is OI lagging behind E_R by an angle $\theta = \tan^{-1} X_S / R_a$. Value of $I = E_R / Z_S$. Since Z_S is constant, E_R or vector OB represents (to some suitable scale) the main current I. OY is drawn at an angle ϕ with OB (or at an angle θ with CA). BL is drawn perpendicular to OX which is at right angles to OY. Vector OB, when referred to OY, also represents, on a different scale, the current both in magnitude and phase.

Hence, $OB \cos \phi = I \cos \phi = BL$

The power input / phase of the motor

$= VI \cos \phi = V \times BL.$

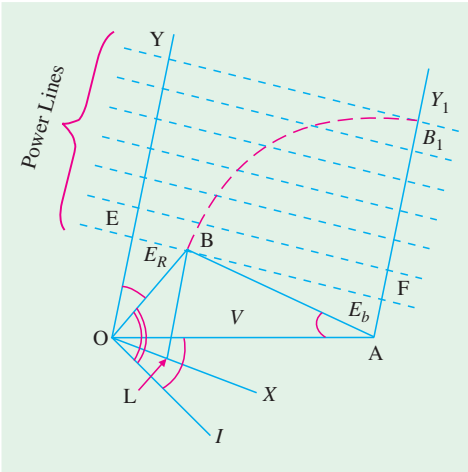


Fig. 38.54

As V is constant, power input is dependent on BL. If motor is working with a constant intake, then locus of B is a straight line || to OX and $\perp$ to OY *i.e.* line EF for which BL is constant. Hence, EF, represents a constant-power input line for a given voltage but varying excitation. Similarly, a series of such parallel lines can be drawn each representing a definite power intake of the motor. As regards these constant-power lines, it is to be noted that

1. for equal increase in intake, the power lines are parallel and equally-spaced
2. zero power line runs along OX
3. the perpendicular distance from B to OX (or zero power line) represents the motor intake
4. If excitation is fixed *i.e.* AB is constant in length, then as the load on motor is increased, increases. In other words, locus of B is a circle with radius = AB and centre at A. With increasing load, B goes on to lines of higher power till point B_1 is reached. Any further increase in load on the motor will bring point B down to a lower line. It means that as load increases beyond the value corresponding to point B_1, the motor intake decreases which is impossible. The area to the right of AY_1 represents unstable conditions. For a given voltage and excitation, the maximum power the motor can develop, is determined by the location of point B_1 beyond which the motor pulls out of synchronism.

38.19. Construction of V-curves

The V-curves of a synchornous motor show how armature current varies with its field current when motor ***input is kept constant***. These are obtained by plotting a.c. armature current against d.c. field current while motor input is kept constant and are so called because of their shape (Fig. 38.55). There is a family of such curves, each corresponding to a definite power intake.

In order to draw these curves experimentally, the motor is run from constant voltage and constant-frequency bus-bars. Power input to motor is kept constant at a definite value. Next, field current is increased in small steps and corresponding armature currents are noted. When plotted, we get a V-curve for a particular constant motor input. Similar curves can be drawn by keeping motor input constant at different values. A family of such curves is shown in Fig. 38.55.

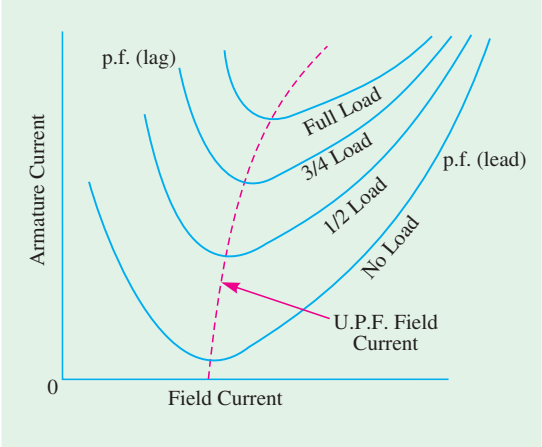


Fig. 38.55

Detailed procedure for graphic construction of V-curves is given below :

1. First, constant-power lines are drawn as discussed in Art. 38.14.
2. Then, with A as the centre, concentric circles of different radii AB, AB_1, AB_2, etc. are drawn where AB, AB_1, AB_2, etc., are the back e.m.fs corresponding to different excitations. The intersections of these circles with lines of constant power give positions of the working points for specific loads and excitations (hence back e.m.fs). The vectors OB, OB_1, OB_2 etc., represent different values of E_R (and hence currents) for different excitations. Back e.m.f. vectors AB, AB_1 etc., have not been drawn purposely in order to avoid confusion (Fig. 38.56).

3. The different values of back e.m.fs like AB, AB_1, AB_2, etc., are projected on the magnetisation and corresponding values of the field (or exciting) amperes are read from it.
4. The field amperes are plotted against the corresponding armature currents, giving us 'V' curves.

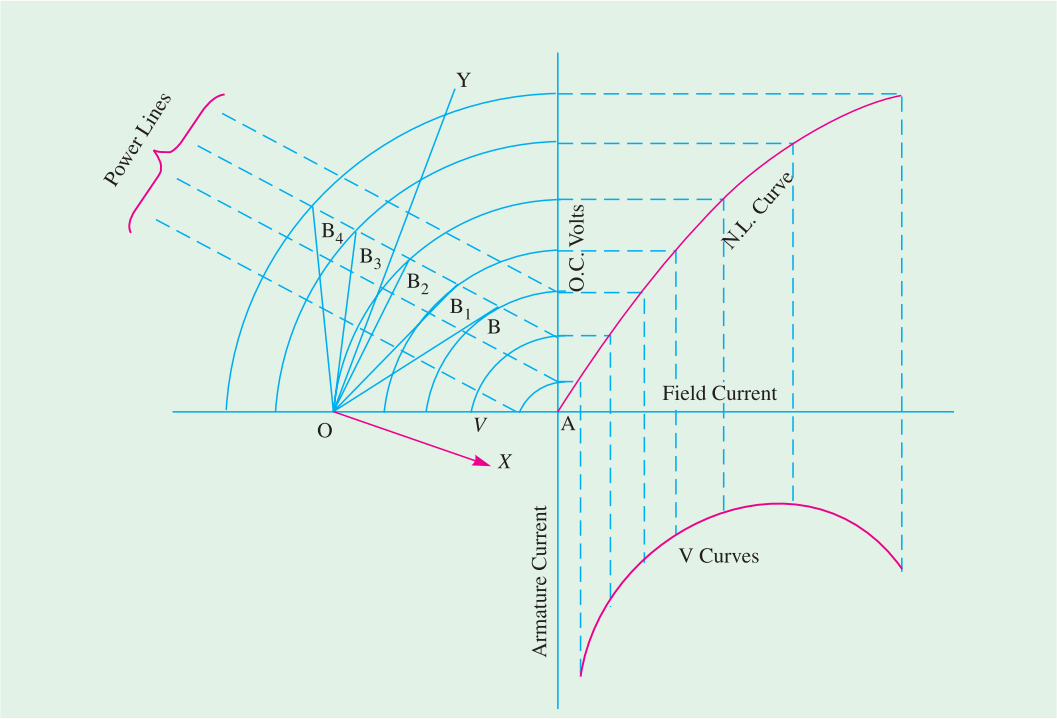


Fig. 38.56

38.20. Hunting or Surging or Phase Swinging

When a synchronous motor is used for driving a varying load, then a condition known as hunting is produced. Hunting may also be caused if supply frequency is pulsating (as in the case of generators driven by reciprocating internal combustion engines).

We know that when a synchronous motor is loaded (such as punch presses, shears, compressors and pumps etc.), its rotor falls back in phase by the coupling angle α. As load is progressively increased, this angle also increases so as to produce more torque for coping with the increased load. If now, there is sudden decrease in the motor load, the motor is immediately pulled up or advanced to a new value of α corresponding to the new load. But in this process, the rotor overshoots and hence is again pulled back. In this way, the rotor starts oscillating (like a pendulum) about its new position of

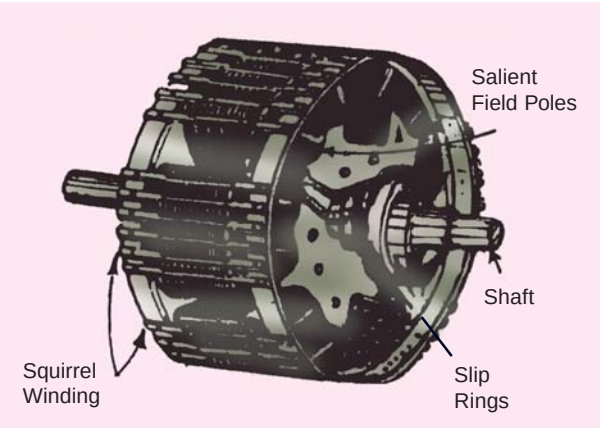
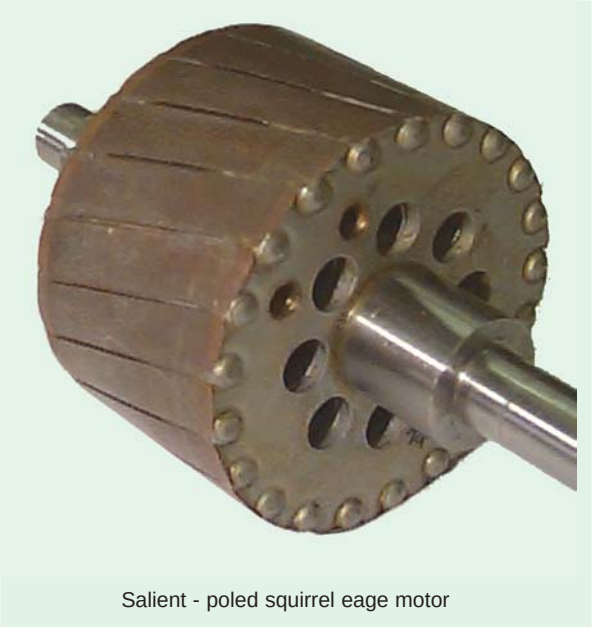


Fig. 38.57



Salient - poled squirrel eage motor

equilibrium corresponding to the new load. If the time period of these oscillations happens to be equal to the natural time period of the machine (refer Art. 37.36) then mechanical resonance is set up. The amplitude of these oscillations is built up to a large value and may eventually become so great as to throw the machine out of synchronism. To stop the build-up of these oscillations, dampers or damping grids (also known as squirrel-cage winding) are employed. These dampers consist of short-circuited Cu bars embedded in the faces of the field poles of the motor (Fig. 38.57). The oscillatory motion of the rotor sets up eddy currents in the dampers which flow in such a way as to suppress these oscillations.

But it should be clearly understood that dampers do not completely prevent hunting because their operation depends upon the presence of some oscillatory motion. Howover, they serve the additional purpose of making the synchronous motor self-starting.

38.21. Methods of Starting

As said above, almost all synchronous motors are equipped with dampers or squirrel cage windings consisting of Cu bars embedded in the pole-shoes and short-circuited at both ends. Such a motor starts readily, acting as an induction motor during the starting period. The procedure is as follows :

The line voltage is applied to the armature (stator) terminals and the field circuit is left ***unexcited***. Motor starts as an induction motor and while it reaches nearly 95% of its synchronous speed, the d.c. field is excited. At that moment the stator and rotor poles get engaged or interlocked with each other and hence pull the motor into synchronism.

However, two points should be noted :

1. At the beginning, when voltage is applied, the rotor is stationary. The rotating field of the stator winding induces a very large e.m.f. in the rotor during the starting period, though the value of this e.m.f. goes on decreasing as the rotor gathers speed.

   Normally, the field windings are meant for 110-V (or 250 V for large machines) but during starting period there are many thousands of volts induced in them. Hence, the rotor windings have to be highly insulated for withstanding such voltages.

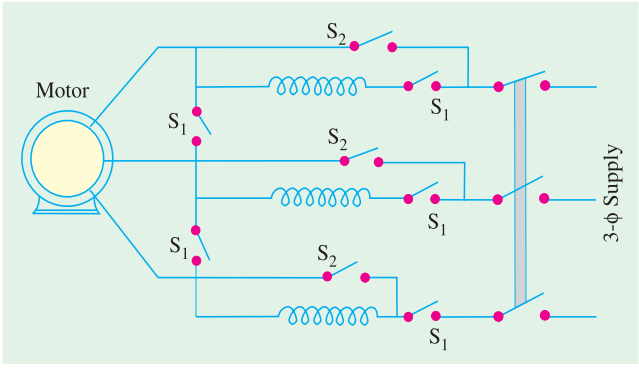


Fig. 38.58

2. When full line voltage is switched on to the armature at rest, a very large current, usually 5 to 7 times the full-load armature current is drawn by the motor. In some cases, this may not be objectionable but where it is, the applied voltage at starting, is reduced by using auto-transformers (Fig. 38.58). However, the voltage should not be reduced to a very low value because the starting torque of an induction motor varies approximately as the square of the applied voltage. Usually, a value of 50% to 80% of the full-line voltage is satisfactory.

   Auto-transformer connections are shown in Fig. 38.58. For reducing the supply voltage, the switches S_1 are closed and S_2 are kept open. When the motor has been speeded-up, S_2 are closed and S_1 opened to cut out the transformers.

38.22. Procedure for Starting a Synchronous Motor

While starting a modern synchronous motor provided with damper windings, following procedure is adopted.

1. First, main field winding is short-circuited.
2. Reduced voltage with the help of auto-transformers is applied across stator terminals. The motor starts up.
3. When it reaches a steady speed (as judged by its sound), a weak d.c. excitation is applied by removing the short-circuit on the main field winding. If excitation is sufficient, then the machine will be pulled into synchronism.
4. Full supply voltage is applied across stator terminals by cutting out the auto-transformers.
5. The motor may be operated at any desired power factor by changing the d.c. excitation.

38.23. Comparison Between Synchronous and Induction Motors

1. For a given frequency, the synchronous motor runs at a constant average speed whatever the load, while the speed of an induction motor falls somewhat with increase in load.
2. The synchronous motor can be operated over a wide range of power factors, both lagging and leading, but induction motor always runs with a lagging p.f. which may become very low at light loads.
3. A synchronous motor is inherently not self-starting.
4. The changes in applied voltage do not affect synchronous motor torque as much as they affect the induction motor torque. The breakdown torque of a synchronous motor varies approximately as the first power of applied voltage whereas that of an induction motor depends on the square of this voltage.
5. A d.c. excitation is required by synchronous motor but not by induction motor.
6. Synchronous motors are usually more costly and complicated than induction motors, but they are particularly attractive for low-speed drives (below 300 r.p.m.) because their power factor can always be adjusted to 1.0 and their efficiency is high. However, induction motors are excellent for speeds above 600 r.p.m.
7. Synchronous motors can be run at ultra-low speeds by using high power electronic converters which generate very low frequencies. Such motors of 10 MW range are used for driving crushers, rotary kilns and variable-speed ball mills etc.

38.24. Synchronous Motor Applications

Synchronous motors find extensive application for the following classes of service :

1. Power factor correction
2. Constant-speed, constant-load drives
3. Voltage regulation

(*a*) Power factor correction

Overexcited synchronous motors having leading power factor are widely used for improving power factor of those power systems which employ a large number of induction motors (Fig. 38.49) and other devices having lagging p.f. such as welders and flourescent lights etc.

(*b*) Constant-speed applications

Because of their high efficiency and high-speed, synchronous motors (above 600 r.p.m.) are well-suited for loads where constant speed is required such as centrifugal pumps, belt-driven reciprocating compressors, blowers, line shafts, rubber and paper mills etc.

Low-speed synchronous motors (below 600 r.p.m.) are used for drives such as centrifugal and screw-type pumps, ball and tube mills, vacuum pumps, chippers and metal rolling mills etc.

(*c*) Voltage regulation

The voltage at the end of a long transmission line varies greatly especially when large inductive loads are present. When an inductive load is disconnected suddenly, voltage tends to rise considerably above its normal value because of the line capacitance. By installing a synchronous motor with a field regulator (for varying its excitation), this voltage rise can be controlled.

When line voltage decreases due to inductive load, motor excitation is increased, thereby raising its p.f. which compensates for the line drop. If, on the other hand, line voltage rises due to line capacitive effect, motor excitation is decreased, thereby making its p.f. lagging which helps to maintain the line voltage at its normal value.

QUESTIONS AND ANSWERS ON SYNCHRONOUS MOTORS

Q. 1. Does change in excitation affect the synchronous motor speed ?
Ans. No.
Q. 2. The power factor ?
Ans. Yes.
Q. 3. How ?
Ans. When over-excited, synchronous motor has leading power factor. However, when underexcited, it has lagging power factor.
Q. 4. For what service are synchronous motors especially suited ?
Ans. For high voltage service.
Q. 5. Which has more efficiency; synchronous or induction motor ?
Ans. Synchronous motor.
Q. 6. Mention some specific applications of synchronous motor ?
Ans. **1.** constant speed load service **2.** reciprocating compressor drives
3. power factor correction **4.** voltage regulation of transmission lines.

Q. 7. What is a synchronous capacitor ?

Ans. An overexcited synchronous motor is called synchronous capacitor, because, like a capacitor, it takes a leading current.

Q. 8. What are the causes of faulty starting of a synchronous motor ?

Ans. It could be due to the following causes :

1. voltage may be too low – at least half voltage is required for starting
2. there may be open-circuit in one phase – due to which motor may heat up
3. static friction may be large – either due to high belt tension or too tight bearings
4. stator windings may be incorrectly connected
5. field excitation may be too strong.

Q. 9. What could be the reasons if a synchronous motor fails to start ?

Ans. It is usually due to the following reasons :

1. voltage may be too low
2. some faulty connection in auxiliary apparatus
3. too much starting load
4. open-circuit in one phase or short-circuit
5. field excitation may be excessive.

Q. 10. A synchronous motor starts as usual but fails to develop its full torque. What could it be due to ?

Ans. **1.** exciter voltage may be too low **2.** field spool may be reversed **3.** there may be either open-circuit or short-circuit in the field.

Q. 11. Will the motor start with the field excited ?

Ans. No.

Q. 12. Under which conditions a synchronous motor will fail to pull into step ?

Ans. **1.** no field excitation **2.** excessive load **3.** excessive load inertia

OBJECTIVE TESTS – 38

1. In a synchronous motor, damper winding is provided in order to
(*a*) stabilize rotor motion
(*b*) suppress rotor oscillations
(*c*) develop necessary starting torque
(*d*) both (*b*) and (*c*)

2. In a synchronous motor, the magnitude of stator back e.m.f. E_b depends on
(*a*) speed of the motor
(*b*) load on the motor
(*c*) both the speed and rotor flux
(*d*) d.c. excitation only

3. An electric motor in which both the rotor and stator fields rotates with the same speed is called a/an .....motor.
(*a*) d.c.
(*b*) chrage
(*c*) synchronous
(*d*) universal

4. While running, a synchronous motor is compelled to run at synchronous speed because of
(*a*) damper winding in its pole faces
(*b*) magnetic locking between stator and rotor poles
(*c*) induced e.m.f. in rotor field winding by stator flux
(*d*) compulsion due to Lenz's law

5. The direction of rotation of a synchronous motor can be reversed by reversing
(*a*) current to the field winding
(*b*) supply phase sequence
(*c*) polarity of rotor poles
(*d*) none of the above

6. When running under no-load condition and with normal excitation, armature current I_a drawn by a synchronous motor
(*a*) leads the back e.m.f. E_b by a small angle
(*b*) is large
(*c*) lags the applied voltage V by a small angle
(*d*) lags the resultant voltage E_R by 90°.

7. The angle between the synchronously-rotating stator flux and rotor poles of a synchronous motor is called..... angle.
(*a*) synchronizing
(*b*) torque
(*c*) power factor
(*d*) slip

8. If load angle of a 4-pole synchronous motor is 8° (elect), its value in mechanical degrees is .....
(*a*) 4
(*b*) 2
(*c*) 0.5
(*d*) 0.25

9. The maximum value of torque angle a in a synchronous motor is .....degrees electrical.
(*a*) 45
(*b*) 90
(*c*) between 45 and 90
(*d*) below 60

10. A synchronous motor running with normal excitation adjusts to load increases *essentially* by increase in its
(a) power factor
(b) torque angle
(c) back e.m.f.
(d) armature current.

11. When load on a synchronous motor running with normal excitation is increased, armature current drawn by it increases because
(a) back e.m.f. E_b becomes less than applied voltage V
(*b*) power factor is decreased
(*c*) net resultant voltage E_R in armature is increased
(*d*) motor speed is reduced

12. When load on a normally-excited synchronous motor is increased, its power factor tends to
(*a*) approach unity
(*b*) become increasingly lagging
(*c*) become increasingly leading
(*d*) remain unchanged.

13. The effect of increasing load on a synchronous motor running with normal excitation is to
(*a*) increase both its I_a and p.f.
(*b*) decrease I_a but increase p.f.
(*c*) increase I_a but decrease p.f.
(*d*) decrease both I_a and p.f.

14. Ignoring the effects of armature reaction, if excitation of a synchronous motor running with constant load is increased, its torque angle must necessarily
(*a*) decrease
(*b*) increase
(*c*) remain constant
(*d*) become twice the no-load value.

15. If the field of a synchronous motor is under-excited, the power factor will be
(*a*) lagging (*b*) leading
(*c*) unity (*d*) more than unity

16. Ignoring the effects of armature reaction, if excitation of a synchronous motor running with constant load is decreased from its normal value, it leads to
(*a*) increase in but decrease in E_b
(*b*) increase in E_b but decrease in I_a
(*c*) increase in both I_a and p.f. which is lagging
(*d*) increase in both I_a and ϕ

17. A synchronous motor connected to infinite bus-bars has at *constant* full-load, 100% excitation and unity p.f. On changing the excitation only, the armature current will have
(*a*) leading p.f. with under-excitation
(*b*) leading p.f. with over-excitation
(*c*) lagging p.f. with over-excitation
(*d*) no change of p.f.
(Power App.-II, Delhi Univ. Jan 1987)

18. The *V*-curves of a synchronous motor show relationship between
(*a*) excitation current and back e.m.f.
(*b*) field current and p.f.
(*c*) d.c. field current and a.c. armature current
(*d*) armature current and supply voltage.

19. When load on a synchronous motor is increased, its armature currents is increas- ed provided it is
(*a*) normally-excited
(*b*) over-excited
(*c*) under-excited
(*d*) all of the above

20. If main field current of a salient-pole synchronous motor fed from an infinite bus and running at no-load is reduced to zero, it would
(*a*) come to a stop
(*b*) continue running at synchronous speed
(*c*) run at sub-synchronous speed
(*d*) run at super-synchronous speed

21. In a synchronous machine when the rotor speed becomes more than the synchronous speed during hunting, the damping bars develop
(*a*) synchronous motor torque
(*b*) d.c. motor torque
(*c*) induction motor torque
(*d*) induction generator torque
(Power App.-II, Delhi Univ. Jan. 1987)

22. In a synchronous motor, the rotor Cu losses are met by
(*a*) motor input
(*b*) armature input
(*c*) supply lines
(*d*) d.c. source

23. A synchronous machine is called a doubly-excited machine because
(*a*) it can be overexcited
(*b*) it has two sets of rotor poles
(*c*) both its rotor and stator are excited
(*d*) it needs twice the normal exciting current.

24. Synchronous capacitor is
(*a*) an ordinary static capacitor bank
(*b*) an over-excited synchronous motor driving mechanical load
(*c*) an over-excited synchronous motor running without mechanical load
(*d*) none of the above 623
(Elect. Machines, A.M.I.E. Sec. B, 1993)

ANSWERS

1. *d*	**2.** *d*	**3.** *c*	**4.** *b*	**5.** *b*	**6.** *c*	**7.** *b*	**8.** *a*	**9.** *b*	**10.** *d*	**11.** *c*
12. *b*	**13.** *c*	**14.** *a*	**15.** *a*	**16.** *d*	**17.** *b*	**18.** *c*	**19.** *d*	**20.** *b*	**21.** *d*	**22.** *d*
23. *c*	**24.** *c*									

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SPECIAL MACHINES

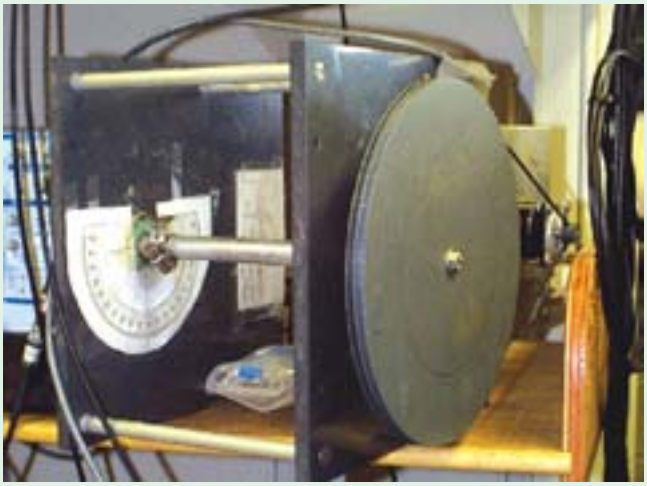


Stepper motor

39.1. Introduction

This chapter provides a brief introduction to electrical machines which have special applications. It includes machines whose stator coils are energized by electronically switched currents. The examples are: various types of stepper motors, brushless d.c. motor and switched reluctance motor etc. There is also a brief description of d.c./a.c. servomotors, synchro motors and resolvers. These motors are designed and built primarily for use in feedback control systems.

39.2. Stepper Motors



Stepper Motor

These motors are also called stepping motors or step motors. The name stepper is used because this motor rotates through a fixed angular step in response to each input current pulse received by its controller. In recent years, there has been wide-spread demand of stepping motors because of the explosive growth of the computer industry. Their popularity is due to the fact that they can be controlled directly by computers, microprocessors and programmable controllers.

As we know, industrial motors are used to convert electric energy into mechanical energy but they cannot be used for precision positioning of an object or precision control of speed without using closed-loop feedback. Stepping motors are ideally suited for situations where either precise positioning or precise speed control or both are required in automation systems.

Apart from stepping motors, other devices used for the above purposes are synchros and resolvers as well as dc/ac servomotors (discussed later).

The unique feature of a stepper motor is that its output shaft rotates in a series of discrete angular intervals or steps, one step being taken each time a command pulse is received. When a definite number of pulses are supplied, the shaft turns through a definite known angle. This fact makes the motor well-suited for open-loop position control because no feedback need be taken from the output shaft.

Such motors develop torques ranging from 1 μN-m (in a tiny wrist watch motor of 3 mm diameter) upto 40 N-m in a motor of 15 cm diameter suitable for machine tool applications. Their power output ranges from about 1 W to a maximum of 2500 W. The only moving part in a stepping motor is its rotor which has no windings, commutator or brushes. This feature makes the motor quite robust and reliable.

Step Angle

The angle through which the motor shaft rotates for each command pulse is called the step angle β. Smaller the step angle, greater the number of steps per revolution and higher the resolution or accuracy of positioning obtained. The step angles can be as small as 0.72° or as large as 90°. But the most common step sizes are 1.8°, 2.5°, 7.5° and 15°.

The value of step angle can be expressed either in terms of the rotor and stator poles (teeth) N_r and N_s respectively or in terms of the number of stator phases (m) and the number of rotor teeth.

$$\beta = \frac{(N_s - N_r)}{N_s \,.\, N_r} \times 360^o$$

or
$$\beta = \frac{360^o}{m\, N_r} = \frac{360^o}{\text{No. of stator phases} \times \text{No. of rotor teeth}}$$

For example, if $N_s = 8$ and $N_r = 6$, $\beta = (8 - 6) \times 360 / 8 \times 6 = 15^o$

Resolution is given by the number of steps needed to complete one revolution of the rotor shaft. Higher the resolution, greater the accuracy of positioning of objects by the motor

$\therefore$ Resolution = No. of steps / revolution = $360^o / \beta$

A stepping motor has the extraordinary ability to operate at very high stepping rates (upto 20,000 steps per second in some motors) and yet to remain fully in synchronism with the command pulses. When the pulse rate is high, the shaft rotation seems continuous. Operation at high speeds is called 'slewing'. When in the slewing range, the motor generally emits an audible whine having a fundamental frequency equal to the stepping rate. If f is the stepping frequency (or pulse rate) in pulses per second (pps) and β is the step angle, then motor shaft speed is given by

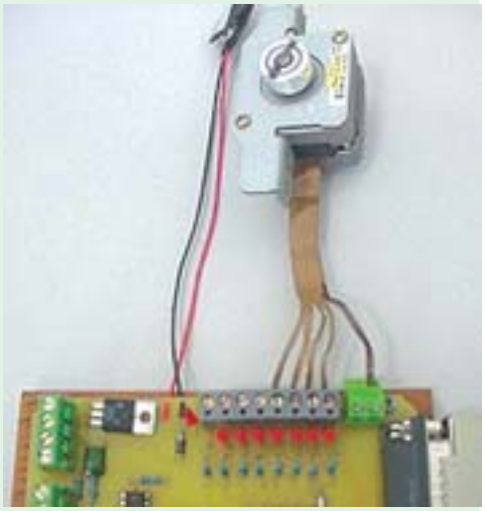
$$n = \beta \times f / 360 \text{ rps} = \text{pulse frequency resolution}$$

If the stepping rate is increased too quickly, the motor loses synchronism and stops. Same thing happens if when the motor is slewing, command pulses are suddenly stopped instead of being progressively slowed.

Stepping motors are designed to operate for long periods with the rotor held in a fixed position and with rated current flowing in the stator windings. It means that stalling is no problem for such motors whereas for most of the other motors, stalling results in the collapse of back emf (E_b) and a very high current which can lead to a quick burn-out.

Applications :

Such motors are used for operation control in computer peripherals, textile industry, IC fabrications and robotics etc. Applications requiring incremental motion are typewriters, line printers, tape drives, floppy disk drives, numerically-controlled machine tools, process control systems and X-Y plotters. Usually, position information can be obtained simply by keeping count of the pulses sent to the motor thereby eliminating the need for expensive position sensors and feedback controls. Stepper motors also perform countless tasks outside the computer industry. It includes commercial, military and medical applications where these motors perform such functions as mixing, cutting, striking, metering, blending and purging. They also take part in the manufacture of packed food stuffs, commercial end-products and even the production of science fiction movies.



Connecting a stepper motor to the interface

Example 39.1. *A hybrid VR stepping motor has 8 main poles which have been castleated to have 5 teeth each. If rotor has 50 teeth, calculate the stepping angle.*

Solution. $N_s = 8 \times 5 = 40$; $N_r = 50$

$\therefore$ $\beta = (50 - 40) \times 360 / 50 \times 40 = 1.8^o$.

Example 39.2. *A stepper motor has a step angle of 2.5°. Determine* ***(a)*** *resolution* ***(b)*** *number of steps required for the shaft to make 25 revolutions and* ***(c)*** *shaft speed, if the stepping frequency is 3600 pps.*

Solution. **(*a*)** Resolution = 360º / β = 360º / 2.5º = 144 steps / revolution.

(*b*) Now, steps / revolution = 144. Hence, steps required for making 25 revolutions = 144 × 25 = 3600.

(*c*) $n = \beta \times f / 360^\circ = 2.5 \times 3600 / 360^\circ = 25$ rps

39.3. Types of Stepper Motors

There is a large variety of stepper motors which can be divided into the following three basic categories :

(*i*) Variable Reluctance Stepper Motor

It has wound stator poles but the rotor poles are made of a ferromagnetic material as shown in Fig.39.1 (*a*). It can be of the single stack type (Fig.39.2) or multi-stack type (Fig.39.5) which gives smaller step angles. Direction of motor rotation is independent of the polarity of the stator current. It is called variable reluctance motor because the reluctance of the magnetic circuit formed by the rotor and stator teeth varies with the angular position of the rotor.

(*ii*) Permanent Magnet Stepper Motor

It also has wound stator poles but its rotor poles are permanently magnetized. It has a cylindrical rotor as shown in Fig. 39.1 (*b*). Its direction of rotation depends on the polarity of the stator current.



Permanent magnet stepper motor

(*iii*) Hybrid Stepper Motor

It has wound stator poles and permanently-magnetized rotor poles as shown in Fig.39.1(*c*). It is best suited when small step angles of 1.8º, 2.5º etc. are required.

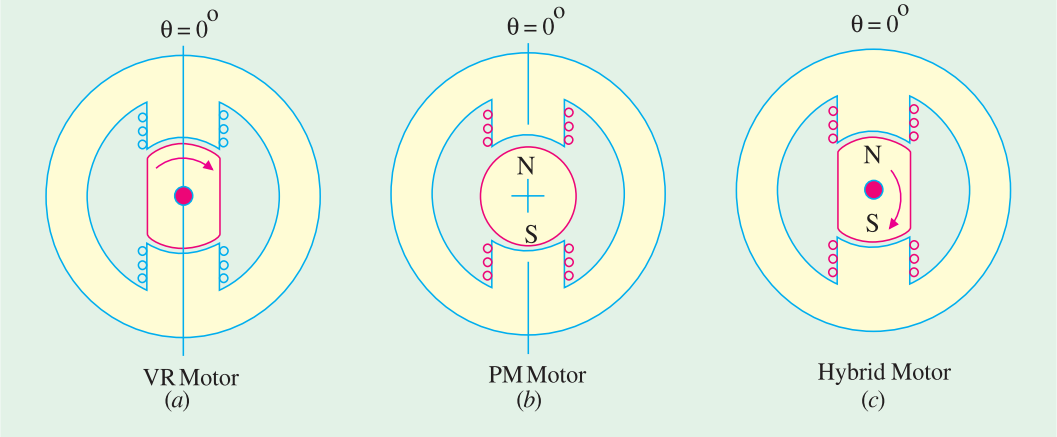


Fig. 39.1

As a variable speed machine, *VR* motor is sometime designed as a switched-reluctance motor. Similarly, *PM* stepper motor is also called variable speed brushless dc motor. The hybrid motor combines the features of *VR* stepper motor and *PM* stepper motor. Its stator construction is similar to the single-stack *VR* motor but the rotor is cylindrical and is composed of radially magnetized permanent magnets. A recent type uses a disc rotor which is magnetized axially to give a small stepping angle and low inertia.

39.4. Variable Reluctance Stepper Motors

Construction : A variable-reluctance motor is constructed from ferromagnetic material with salient poles as shown in Fig. 39.2. The stator is made from a stack of steel laminations and has six equally-spaced projecting poles (or teeth) each wound with an exciting coil. The rotor which may be solid or laminated has four projecting teeth of the same width as the stator teeth. As seen, there are three independent stator circuits or phases *A*, *B* and *C* and each one can be energised by a direct current pulse from the drive circuit (not shown in the figure).



Variable reluctance motor

A simple circuit arrangement for supplying current to the stator coils in proper sequence is shown in Fig. 39.2 (*e*). The six stator coils are connected in 2-coil groups to form three separate circuits called phases. Each phase has its own independent switch. Diametrically opposite pairs of stator coils are connected in series such that when one tooth becomes a *N*-pole, the other one becomes a *S*-pole. Although shown as mechanical switches in Fig. 39.2 (*e*), in actual practice, switching of phase currents is done with the help of solid-state control. When there is no current in the stator coils, the rotor is completely free to rotate. Energising one or more stator coils causes the rotor to step forward (or backward) to a position that forms a path of least reluctance with the magnetized stator teeth. The step angle of this three-phase, four rotor teeth motor is $\beta = 360/4 \times 3 = 30^{o}$.

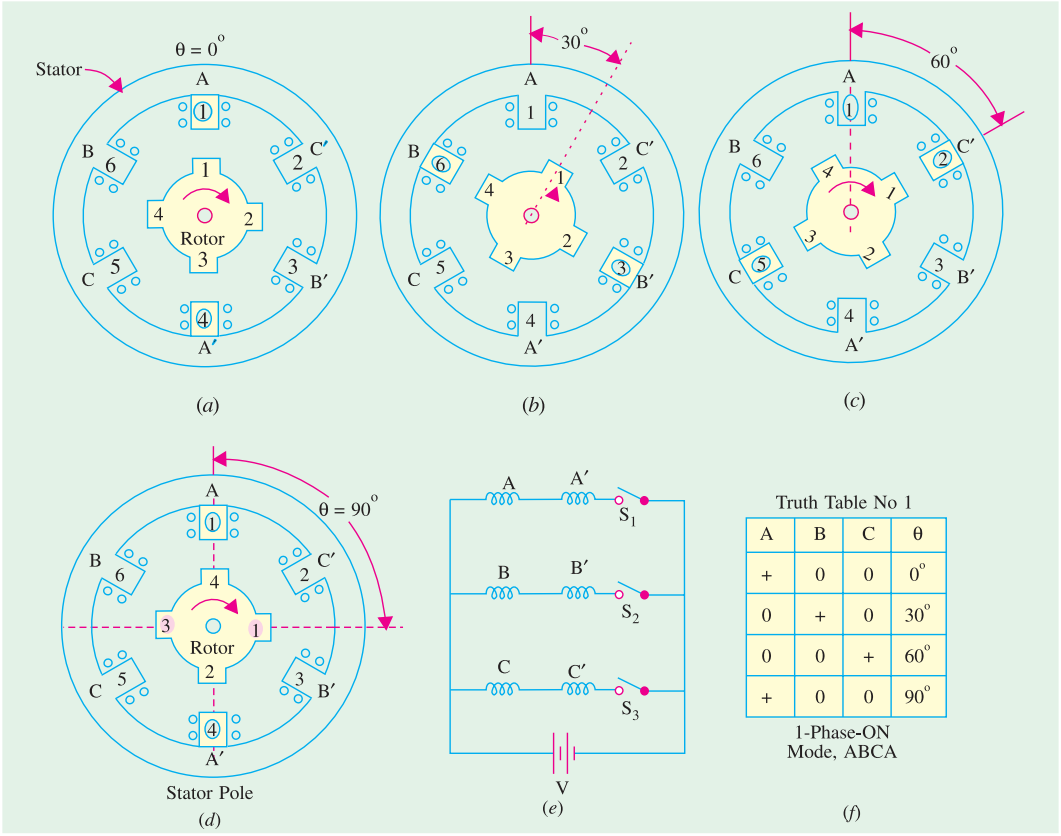


Fig. 39.2

Working. The motor has following modes of operation :

(*a*) 1-phase-ON or Full-step Operation

Fig. 39.2 (*a*) shows the position of the rotor when switch S_1 has been closed for energising phase *A*. A magnetic field with its axis along the stator poles of phase *A* is created. The rotor is therefore, attracted into a position of minimum reluctance with diametrically opposite rotor teeth 1 and 3 lining up with stator teeth 1 and 4 respectively. Closing S_2 and opening S_1 energizes phase *B* causing rotor teeth 2 and 4 to align with stator teeth 3 and 6 respectively as shown in Fig. 39.2 (*b*). The rotor rotates through full-step of 30° in the clockwise (CW) direction. Similarly, when S_3 is closed after opening S_2, phase *C* is energized which causes rotor teeth 1 and 3 to line up with stator teeth 2 and 5 respectively as shown in Fig. 39.2 (*c*). The rotor rotates through an additional angle of 30° in the clockwise (CW) direction. Next if S_3 is opened and S_1 is closed again, the rotor teeth 2 and 4 will align with stator teeth 4 and 1 respectively thereby making the rotor turn through a further angle of 30° as shown in Fig. 39.2 (*d*). By now the total angle turned is 90°. As each switch is closed and the preceding one opened, the rotor each time rotates through an angle of 30°. By repetitively closing the switches in the sequence 1-2-3-1 and thus energizing stator phases in sequence *ABCA* etc., the rotor will rotate clockwise in 30° steps. If the switch sequence is made 3-2-1-3 which makes phase sequence *CBAC* (or *ACB*), the rotor will rotate anticlockwise. This mode of operation is known as 1-phase-ON mode or full-step operation and is the simplest and widely-used way of making the motor step. The stator phase switching truth table is shown in Fig. 39.2 (*f*). It may be noted that the direction of the stator magnetizing current is not significant because a stator pole of either magnetic polarity will always attract the rotor pole by inducing opposite polarity.

(*b*) 2-phase-ON Mode

In this mode of operation, two stator phases are excited simultaneously. When phases *A* and *B* are energized together, the rotor experiences torques from both phases and comes to rest at a point mid-way between the two adjacent full-step positions. If the stator phases are switched in the sequence *AB, BC, CA, AB* etc., the motor will take full steps of 30° each (as in the 1-phase-*ON* mode) but its equilibrium positions will be interleaved between the full-step positions. The phase switching truth table for this mode is shown in Fig. 39.3 (*a*).

Truth Table No. 2

A	B	C	
+	+	0	15°
0	+	+	45°
+	0	+	75°
+	+	0	105°

2 Phase-ON Mode
AB, BC, CA, AB

Truth Table No. 3

	A	B	C		
A	+	0	0	0°	
	+	+	0	15°	AB
B	0	+	0	30°	
	0	+	+	45°	BC
C	0	0	+	65°	
	+	0	+	75°	CA
A	+	0	0	90°	

Half-Stepping Alternate
1-Phase-On &
2-Phase-on Mode
A, AB, B, BC, C, CA, A

Fig. 39.3

The 2-phase-ON mode provides greater holding torque and a much better damped single-stack response than the 1-phase-ON mode of operation.

(*c*) Half-step Operation

Half-step operation or 'half-stepping' can be obtained by exciting the three phases in the sequence *A*, *AB*, *B*, *BC*, *C* etc. *i.e.* alternately in the 1-phase-ON and 2-phase-ON modes. It is sometime known as 'wave' excitation and it causes the rotor to advance in steps of 15° *i.e.* half the full-step angle. The truth table for the phase pulsing sequence in half-stepping is shown in Fig. 39.3 (*b*).

Half-stepping can be illustrated with the help of Fig. 39.4 where only three successive pulses have been considered. Energizing only phase *A* causes the rotor position shown in Fig. 39.4 (*a*). Energising phases *A* and *B* simultaneously moves the rotor to the position shown in Fig. 39.4 (*b*) where rotor has moved through half a step only. Energising only phase *B* moves the rotor through another half-step as shown in Fig. 39.4 (*c*). With each pulse, the rotor moves 30 / 2 = 15° in the CCW direction.

It will be seen that in half-stepping mode, the step angle is halved thereby doubling the resolution. Moreover, continuous half-stepping produces a smoother shaft rotation.

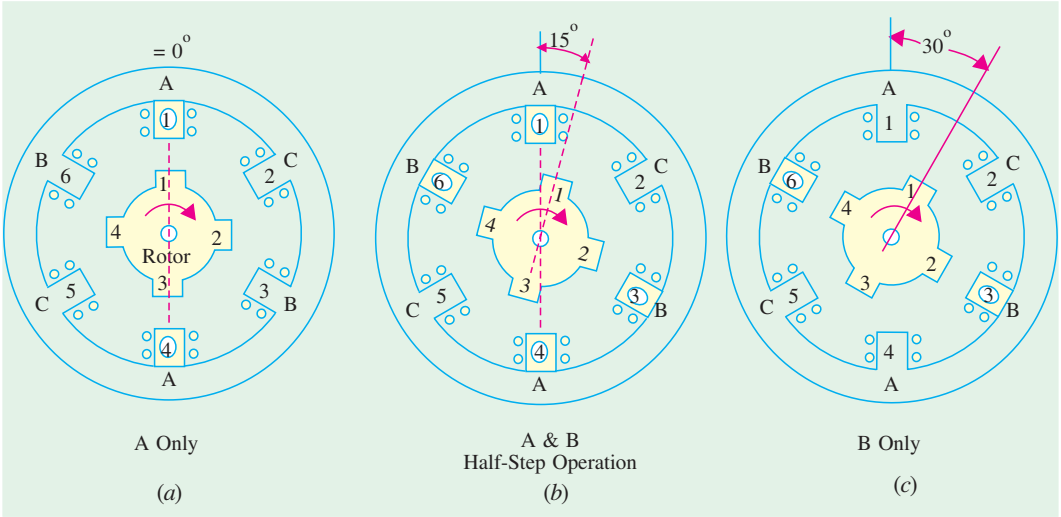


Fig. 39.4

(*d*) Microstepping

It is also known as mini-stepping. It utilizes two phases simultaneously as in 2-phase-*ON* mode but with the two currents deliberately made unequal (unlike in half-stepping where the two phase currents have to be kept equal). The current in phase *A* is held constant while that in phase *B* is increased in very small increments until maximum current is reached. The current in phase *A* is then reduced to zero using the same very small increments. In this way, the resultant step becomes very small and is called a microstep. For example, a *VR* stepper motor with a resolution of 200 steps / rev ($\beta = 1.8°$) can with microstepping have a resolution of 20,000 steps / rev ($\beta = 0.018°$). Stepper motors employing microstepping technique are used in printing and phototypesetting where very fine resolution is called for. As seen, microstepping provides smooth low-speed operation and high resolution.

Torque. If I_a is the d.c. current pulse passing through phase *A*, the torque produced by it is given by $T = (1 / 2)\, I_a^2\, dL / d\theta$. *VR* stepper motors have a high (torque / inertia) ratio giving high rates of acceleration and fast response. A possible disadvantage is the absence of detent torque which is necessary to retain the rotor at the step position in the event of a power failure.

39.5. Multi-stack VR Stepper Motor

So, far, we have discussed single-stack *VR* motors though multi-stack motors are also available which provide smaller step angles. The multi-stack motor is divided along its axial length into a number of magnetically-isolated sections or stacks which can be excited by a separate winding or phase. Both stator and rotor have the same number of poles. The stators have a common frame while rotors have a common shaft as shown in Fig. 39.5 (*a*) which represents a three-stack *VR* motor. The teeth of all the rotors are perfectly aligned with respect to themselves but the stator teeth of various stacks have a progressive angular displacement as shown in the developed diagram of Fig. 39.5 (*b*) for phase excitation.

Three-stack motors are most common although motors with upto seven stacks and phases are available. They have step angles in the range of 2º to 15º. For example, in a six-stack *VR* motor having 20 rotor teeth, the step angle $\beta = 360º / 6 \times 20 = 3º$.

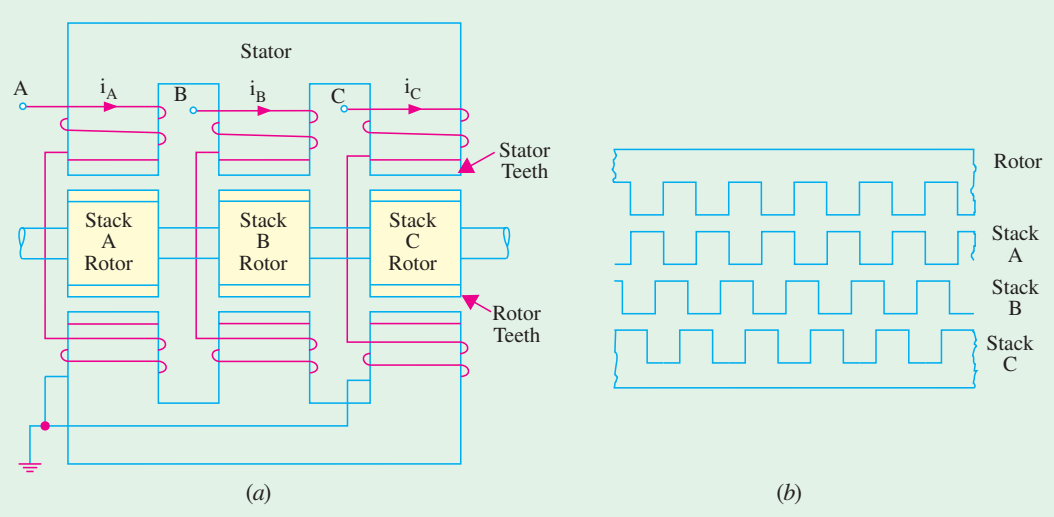
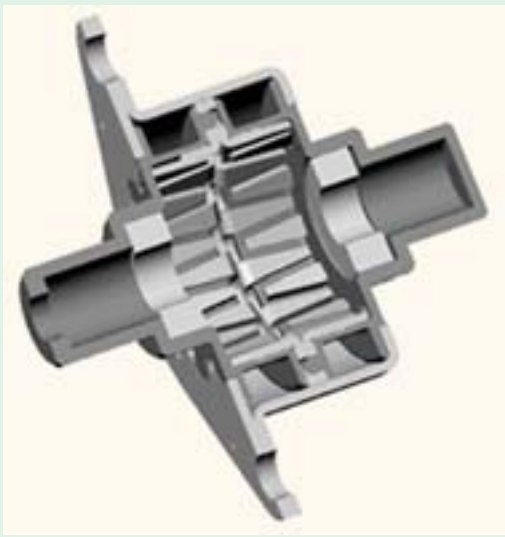


Fig. 39.5

39.6. Permanent-Magnet Stepping Motor

(*a*) Construction. Its stator construction is similar to that of the single-stack *VR* motor discussed above but the rotor is made of a permanent-magnet material like magnetically 'hard' ferrite. As shown in the Fig. 39.6 (*a*), the stator has projecting poles but the rotor is cylindrical and hasradially magnetized permanent magnets. The operating principle of such a motor can be understood with the help of Fig. 39.6 (*a*) where the rotor has two poles and the stator has four poles. Since two stator poles are energized by one winding, the motor has two windings or phases marked *A* and *B*. The step angle of this motor $\beta = 360º / mN_r = 360º / 2 \times 2 = 90º$ or $\beta = (4 - 2) \times 360º / 2 \times 4 = 90º$.



Permanent magnet stepper motor

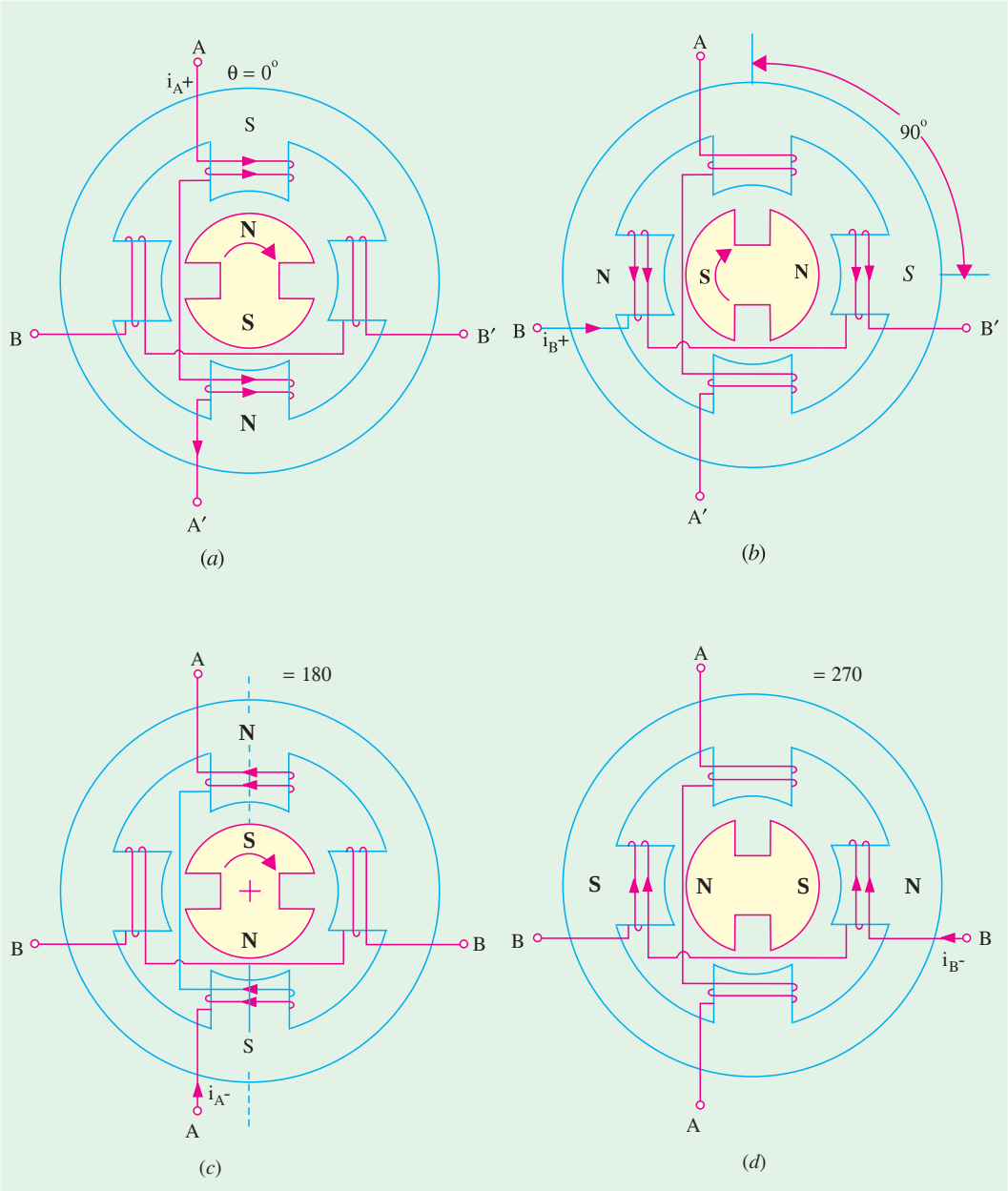


Fig. 39.6

(b)* Working.** When a particular stator phase is energized, the rotor magnetic poles move into alignment with the excited stator poles. The stator windings *A* and *B* can be excited with either polarity current (A^+ refers to positive current i_{A^+} in the phase *A* and A^- to ***negative current i_{A^-}). Fig.39.6 (*a*) shows the condition when phase *A* is excited with positive current i_{A^+}. Here, θ = 0º. If excitation is now switched to phase *B* as in Fig. 39.6 (*b*), the rotor rotates by a full step of 90º in the clockwise direction. Next, when phase *A* is excited with negative current i_{A^-}, the rotor turns through another 90º in *CW* direction as shown in Fig. 39.6 (*c*). Similarly, excitation of phase *B* with i_{B^-} further turns the rotor through another 90º in the same direction as shown in Fig. 39.6 (*d*). After this, excitation of phase *A* with i_{A^+} makes the rotor turn through one complete revolution of 360º.

Truth Table No. 1

A	B	
+	0	0°
0	+	90°
	0	180°
0		270°
+	0	0°

1-Phase-ON Mode

Truth Table No. 2

A	B	
+	+	45°
	+	135°
		225°
+		315°
+	+	45°

1-Phase-ON Mode

Truth Table No. 3

A	B	
+	0	0°
+	+	45°
0	+	90°
	+	135°
	0	180°
		225°
0		270°
+		315°
+	0	0°

Alternate 1-Phase-On & 2-Phase-On Modes

Fig. 39.7

It will be noted that in a permanent-magnet stepper motor, the direction of rotation depends on the polarity of the phase currents as tabulated below :

i_{A^+}; i_{B^+}; i_{A^-}; i_{B^-}; i_{A^+}, ...
$A^+; B^+; A^-; B^-; A^+;$ ..... for clockwise rotation

i_{A^+}; i_{B^-}; i_{A^-}; i_{B^+}; i_{A^+}; .....
$A^+; B^-; A^-; B^+; A^+;$ ..... for CCW rotation

Truth tables for three possible current sequences for producing clockwise rotation are given in Fig. 39.7. Table No.1 applies when only one phase is energized at a time in 1-phase-*ON* mode giving step size of 90º. Table No.2 represents 2-phase-*ON* mode when two phases are energised simultaneously. The resulting steps are of the same size but the effective rotor pole positions are midway between the two adjacent full-step positions. Table No.3 represents half-stepping when 1-phase-*ON* and 2-phase-*ON* modes are used alternately. In this case, the step size becomes half of the normal step or one-fourth of the pole-pitch (*i.e.* 90º / 2 = 45º or 180º / 4 = 45º). Microstepping can also be employed which will give further reduced step sizes thereby increasing the resolution.

(*c*) Advantages and Disadvantages. Since the permanent magnets of the motor do not require external exciting current, it has a low power requirement but possesses a high detent torque as compared to a *VR* stepper motor. This motor has higher inertia and hence slower acceleration. However, it produces more torque per ampere stator current than a *VR* motor. Since it is difficult to manufacture a small permanent-magnet rotor with large number of poles, the step size in such motors is relatively large ranging from 30º to 90º. However, recently disc rotors have been manufactured which are magnetized axially to give a small step size and low inertia.

Example 39.3. *A single-stack, 3-phase VR motor has a step angle of 15º. Find the number of its rotor and stator poles.*

Solution. Now, $\beta = 360^\circ / mN_r$ or $15^\circ = 360^\circ / 3 \times N_r$; $\therefore N_r = 8$.

For finding the value of N_s, we will use the relation $\beta = (N_s - N_r) \times 360^\circ / N_s \,.\, N_r$

(i) **When $N_s > N_r$.** Here, $\beta = (N_s - N_r) \times 360^\circ / N_s \,.\, N_r$

or $15^\circ = (N_s - 8) \times 360^\circ / 8\, N_s$; $\therefore N_s = \mathbf{12}$

(ii) **When $N_s < N_r$.** Here, $15^\circ = (8 - N_s) \times 360^\circ / 8\, N_s$; $\therefore N_s = \mathbf{6.}$

Example 39.4. *A four-stack VR stepper motor has a step angle of 1.8°. Find the number of its rotor and stator teeth.*

Solution. A four-stack motor has four phases. Hence, $m = 4$.

$\therefore 1.8^\circ = 360^\circ / 4 \times N_r$; $\therefore N_r = \mathbf{50.}$

Since in multi-stack motors, rotor teeth equal the stator teeth, hence $N_s = \mathbf{50.}$

39.7. Hybrid Stepper Motor



Hybrid stepper motor

(a) **Construction.** It combines the features of the variable reluctance and permanent-magnet stepper motors. The rotor consists of a permanent-magnet that is magnetized axially to create a pair of poles marked *N* and *S* in Fig. 39.8 (*b*). Two end-caps are fitted at both ends of this axial magnet. These end-caps consist of equal number of teeth which are magnetized by the respective polarities of the axial magnet. The rotor teeth of one end-cap are offset by a half tooth pitch so that a tooth at one end-cap coincides with a slot at the other. The cross-sectional views perpendicular to the shaft along *X*-*X*′ and *Y*-*Y*′ axes are shown in Fig. 39.8 (*a*) and (*c*) respectively. As seen, the stator consists of four stator poles which are excited by two stator windings in pairs. The rotor has five *N*-poles at one end and five *S*-poles at the other end of the axial magnet. The step angle of such a motor is $= (5 - 4) \times 360^\circ / 5 \times 4 = 18^\circ$.

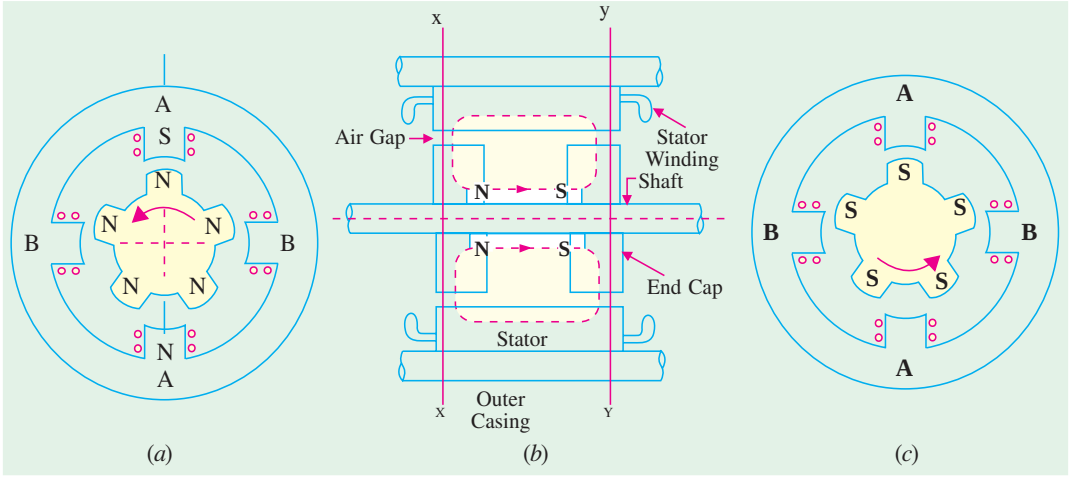


Fig. 39.8

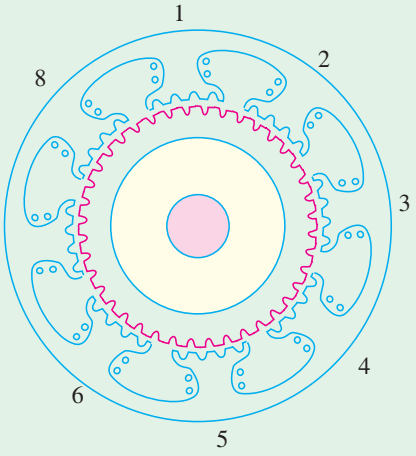
(b) **Working.** In Fig.39.8 (*a*), phase *A* is shown excited such that the top stator pole is a *S*-pole so that it attracts the top *N*-pole of the rotor and brings it in line with the *A*-*A*′ axis. To turn the rotor,

Truth Table

A	B	
+	0	0°
0	+	18°
–	0	36°
0	–	54°
+	0	72°

1-Phase ON
Full-Step Mode

(*a*)



(*b*)

Fig. 39.9

phase *A* is denergized and phase *B* is excited positively. The rotor will turn in the CCW direction by a full step of 18°.

Next, phase *A* and *B* are energized negatively one after the other to produce further rotations of 18° each in the same direction. The truth table is shown in Fig. 39.9 (*a*). For producing clockwise rotation, the phase sequence should be $A^+; B^-; A^-; B^+; A^+$ etc.

Practical hybrid stepping motors are built with more rotor poles than shown in Fig. 39.9 in order to give higher angular resolution. Hence, the stator poles are often slotted or castleated to increase the number of stator teeth. As shown in Fig. 39.9 (*b*), each of the eight stator poles has been alloted or castleated into five smaller poles making $N_s = 8 \times 5 = 40°$. If rotor has 50 teeth, then step angle = $(50 - 40) \times 360° / 50 \times 40 = 1.8°$. Step angle can also be decreased (and hence resolution increased) by having more than two stacks on the rotor.

This motor achieves small step sizes easily and with a simpler magnet structure whereas a purely *PM* motor requires a multiple permanent-magnet. As compared to *VR* motor, hybrid motor requires less excitation to achieve a given torque. However, like a *PM* motor, this motor also develops good detent torque provided by the permanent-magnet flux. This torque holds the rotor stationary while the power is switched off. This fact is quite helpful because the motor can be left overnight without fear of its being accidentally moved to a new position.

39.8. Summary of Stepper Motors

1. A stepper motor can be looked upon as a digital electromagnetic device where each pulse input results in a discrete output *i.e.* a definite angle of shaft rotation. It is ideally-suited for open-loop operation because by keeping a count of the number of input pulses, it is possible to know the exact position of the rotor shaft.

2. In a *VR* motor, excitation of the stator phases gives rise to a torque in a direction which minimizes the magnetic circuit reluctance. The reluctance torque depends on the square of the phase current and its direction is independent of the polarity of the phase current. A *VR* motor can be a single-stack or multi-stack motor. The step angle $\beta = 360° / mN_r$ where N_r is the number of rotor teeth and m is the number of phases in the single-stack motor or the number of stacks in the multi-stack motor.

3. A permanent-magnet stepper motor has a permanently-magnetized cylindrical rotor. The direction of the torque produced depends on the polarity of the stator current.

4. A hybrid motor combines the features of *VR* and *PM* stepper motors. The direction of its torque also depends on the polarity of the stator current. Its step angle $\beta = 360° / mN_r$.

5. In the 1-phase *ON* mode of excitation, the rotor moves by one full-step for each change of excitation. In the 2-phase-*ON* mode, the rotor moves in full steps although it comes to rest at a point midway between the two adjacent full-step positions.

6. Half-stepping can be achieved by alternating between the 1-phase-*ON* and 2-phase-*ON* modes. Step angle is reduced by half.

7. Microstepping is obtained by deliberately making two phase currents unequal in the 2-phase-*ON* mode.

Tutorial Problems 39.1

1. A stepper motor has a step angle of 1.8º. What number should be loaded into the encoder of its drive system if it is desired to turn the shaft ten complete revolutions ? **[2000]**
2. Calculate the step angle of a single-stack, 4-phase, 8/6-pole *VR* stepper motor. What is its resolution ? **[15º; 24 steps/rev]**
3. A stepper motor has a step angle of 1.8º and is driven at 4000 pps. Determine (*a*) resolution (*b*) motor speed (*c*) number of pulses required to rotate the shaft through 54º. **[(*a*) 200 steps/rev (*b*) 1200 rpm (*c*) 30]**
4. Calculate the pulse rate required to obtain a rotor speed of 2400 rpm for a stepper motor having a resolution of 200 steps/rev. **[4000 pps]**
5. A stepper motor has a resolution of 500 steps/rev in the 1-phase-*ON* mode of operation. If it is operated in half-step mode, determine (*a*) resolution (*b*) number of steps required to turn the rotor through 72º. **[(*a*) 1000 steps/rev (*b*) 200]**
6. What is the required resolution for a stepper motor that is to operate at a pulse frequency of 6000 pps and a travel 180º in 0.025 s ? **[300 steps/rev]**

39.9. Permanent-Magnet DC Motor

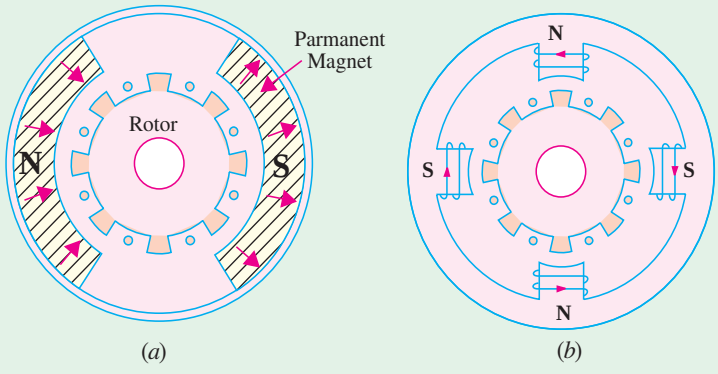
A permanent-magnet d.c. (PMDC) motor is similar to an ordinary d.c. shunt motor except that its field is provided by permanent magnets instead of salient-pole wound-field structure. Fig. 39.10 (*a*) shows 2-pole PMDC motor whereas Fig. 39.10 (*b*) shows a 4-pole wound-field d.c. motor for comparison purposes.



Permanent magnet DC - motor

(*a*) Construction

As shown in Fig. 39.10 (*a*), the permanent magnets of the PMDC motor are supported by a cylindrical steel stator which also serves as a return path for the magnetic flux. The rotor (*i.e.* armature) has winding slots, commutator segments and brushes as in conventional d.c. machines.



(*a*) (*b*)

Fig. 39.10

There are three types of permanent magnets used for such motors. The materials used have residual flux density and high coercivity.

(*i*) Alnico magnets – They are used in motors having ratings in the range of 1 kW to 150 kW.

(*ii*) Ceramic (ferrite) magnets – They are much economical in fractional kilowatt motors.

(*iii*) Rare-earth magnets – Made of samarium cobalt and neodymium iron cobalt which have the highest energy product. Such magnetic materials are costly but are best economic choice for small as well as large motors.

Another form of the stator construction is the one in which permanent-magnet material is cast in the form of a continuous ring instead of in two pieces as shown in Fig. 39.10 (*a*).

(*b*) Working

Most of these motors usually run on 6 V, 12 V or 24 V dc supply obtained either from batteries or rectified alternating current. In such motors, torque is produced by interaction between the axial current-carrying rotor conductors and the magnetic flux produced by the permanent magnets.

(*c*) Performance

Fig. 39.11 shows some typical performance curves for such a motor. Its speed-torque curve is a straight line which makes this motor ideal for a servomotor. Moreover, input current increases linearly with load torque. The efficiency of such motors is higher as compared to wound-field dc motors because, in their case, there is no field Cu loss.

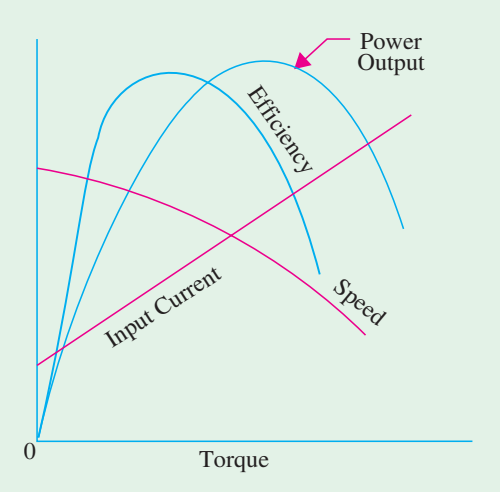


Fig. 39.11

(*d*) Speed Control

Since flux remains constant, speed of a PMDC motor cannot be controlled by using Flux Control Method (Art 33.2). The only way to control its speed is to vary the armature voltage with the help of an armature rheostat (Art 33.2) or electronically by using *x*-choppers. Consequently, such motors are found in systems where speed control below base speed only is required.

(*e*) Advantages

(*i*) In very small ratings, use of permanent-magnet excitation results in lower manufacturing cost.

(*ii*) In many cases a PMDC motor is smaller in size than a wound-field d.c. motor of equal power rating.

(*iii*) Since field excitation current is not required, the efficiency of these motors is generally higher than that of the wound-field motors.

(*iv*) Low-voltage PMDC motors produce less air noise.

(*v*) When designed for low-voltage (12 V or less) these motors produced very little radio and TV interference.

(*f*) Disadvantages

(*i*) Since their magnetic field is active at all times even when motor is not being used, these motors are made totally enclosed to prevent their magnets from collecting magnetic junk from neighbourhood. Hence, as compared to wound-field motors, their temperature

tends to be higher. However, it may not be much of a disadvantage in situations where motor is used for short intervals.

(*ii*) A more serious disadvantage is that the permanent magnets can be demagnetized by armature reaction mmf causing the motor to become inoperative. Demagnetization can result from **(*a*)** improper design **(*b*)** excessive armature current caused by a fault or transient or improper connection in the armature circuit **(*c*)** improper brush shift and **(*d*)** temperature effects.

(*g*) Applications

(*i*) Small, 12-V PMDC motors are used for driving automobile heater and air conditioner blowers, windshield wipers, windows, fans and radio antennas etc. They are also used for electric fuel pumps, marine engine starters, wheelchairs and cordless power tools.

(*ii*) Toy industry uses millions of such motors which are also used in other appliances such as the toothbrush, food mixer, ice crusher, portable vacuum cleaner and shoe polisher and also in portable electric tools such as drills, saber saws and hedge trimmers etc.

39.10. Low-inertia DC Motors

These motors are so designed as to make their armature mass very low. This permits them to start, stop and change direction and speed very quickly making them suitable for instrumentation applications. The two common types of low-inertia motors are **(*i*)** shell-type motor and **(*ii*)** printed-circuit (PC) motor.

39.11. Shell-type Low-intertia DC Motor

Its armature is made up of flat aluminium or copper coils bonded together to form a hollow cylinder as shown in Fig. 39.12. This hollow cylinder is not attached physically to its iron core which is stationary and is located inside the shell-type rotor. Since iron does not form part of the rotor, the rotor inertia is very small.

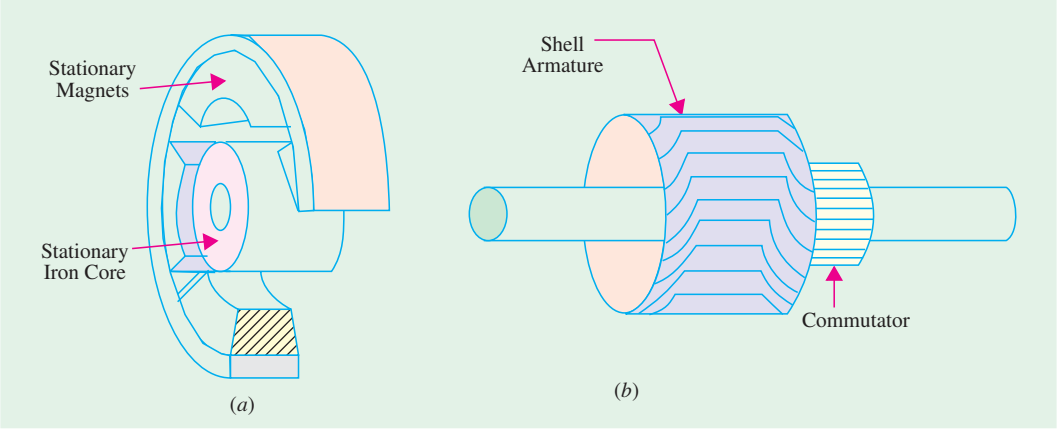


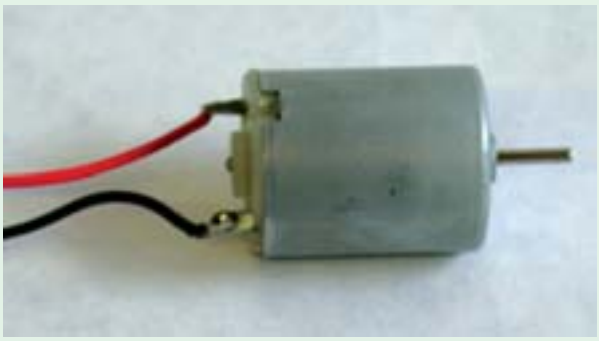
Fig. 39.12

39.12. Printed-circuit (Disc) DC Motor

(*a*) Constructional Details

It is a low-voltage dc motor which has its armature (rotor) winding and commutator printed on a thin disk of non-magnetic insulating material. This disk-shaped armature contains no iron and etched-copper conductors are printed on its both sides. It uses permanent magnets to produce the necessary

magnetic field. The magnetic circuit is completed through the flux-return plate which also supports the brushes. Fig. 39.13 shows an 8-pole motor having wave-wound armature. Brushes mounted in an axial direction bear directly on the inner parts of the armature conductors which thus serve as a commutator. Since the number of armature conductors is very large, the torque produced is uniform even at low speeds. Typical sizes of these motors are in the fractional and subfractional horsepower ranges. In many applications, acceleration from zero to a few thousand rpm can be obtained within 10 ms.



Low voltage DC motor

(*b*) Speed Control

The speed can be controlled by varying either the applied armature voltage or current. Because of their high efficiency, fan cooling is not required in many applications. The motor brushes require periodic inspection and replacement. The rotor disk which carries the conductors and commutator, being very thin, has a limited life. Hence, it requires replacing after some time.

(*c*) Main Features

The main features of this motor are **(*i*)** very low-inertia (*ii*) high overload current capability **(*iii*)** linear speed-torque characteristic **(*iv*)** smooth torque down to near-zero speed **(*v*)** very suitable for direct-drive control applications **(*vi*)** high torque/inertia ratio.

(*d*) Advantages

(*i*) High efficiency **(*ii*)** Simplified armature construction **(*iii*)** Being of low-voltage design, produces minimum of radio and TV interference.

(*e*) Disadvantages

(*i*) Restricted to low voltages only **(*ii*)** Short armature life **(*iii*)** Suited for intermittent duty cycle only because motor overheats in a very short time since there is no iron to absorb excess heat **(*v*)** liable to burn out if stalled or operated with the wrong supply voltage.

(*f*) Applications

These low-inertia motors have been developed specifically to provide high performance characteristics when used in direct-drive control applications. Examples are :

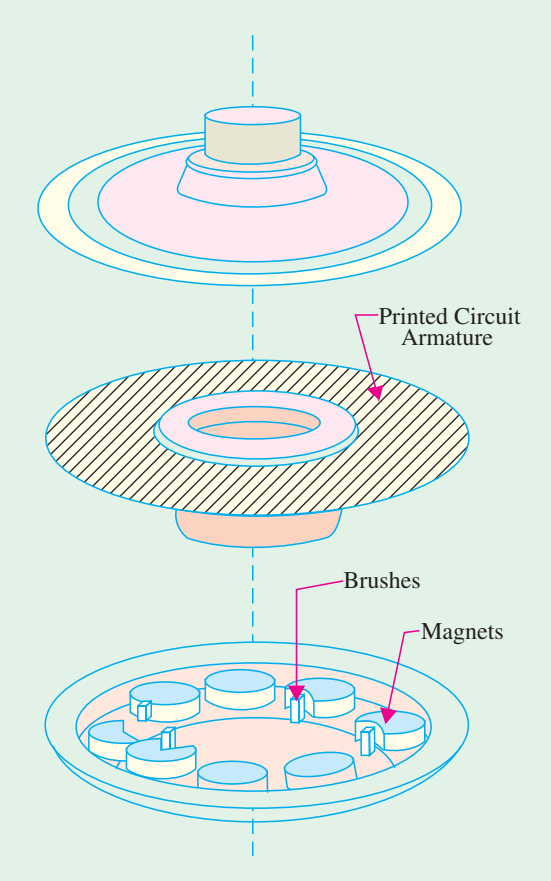


Fig. 39.13

(*i*) high speed paper tape readers **(*ii*)** oscillographs **(*iii*)** *X*-*Y* recorders **(*iv*)** layer winders **(*v*)** point-to-point tool positioners *i.e.* as positioning servomotors **(*vi*)** with in-built optical position encoder, it competes with stepping motor **(*vii*)** in high rating is being manufactured for heavy-duty drives such as lawn mowers and battery-driven vehicles etc.

39.13. Permanent-Magnet Synchronous Motors

(*a*) Construction and Performance

Such motors have a cage rotor having rare-earth permanent magnets instead of a wound field. Such a motor starts like an induction motor when fed from a fixed-frequency supply. A typical 2-pole and 4-pole surface-mounted versions of the rotor are shown in Fig. 39.14. Since no d.c. supply is needed for exciting the rotor, it can be made more robust and reliable. These motors have outputs ranging from about 100 W upto 100 kW. The maximum synchronous torque is designed to be around

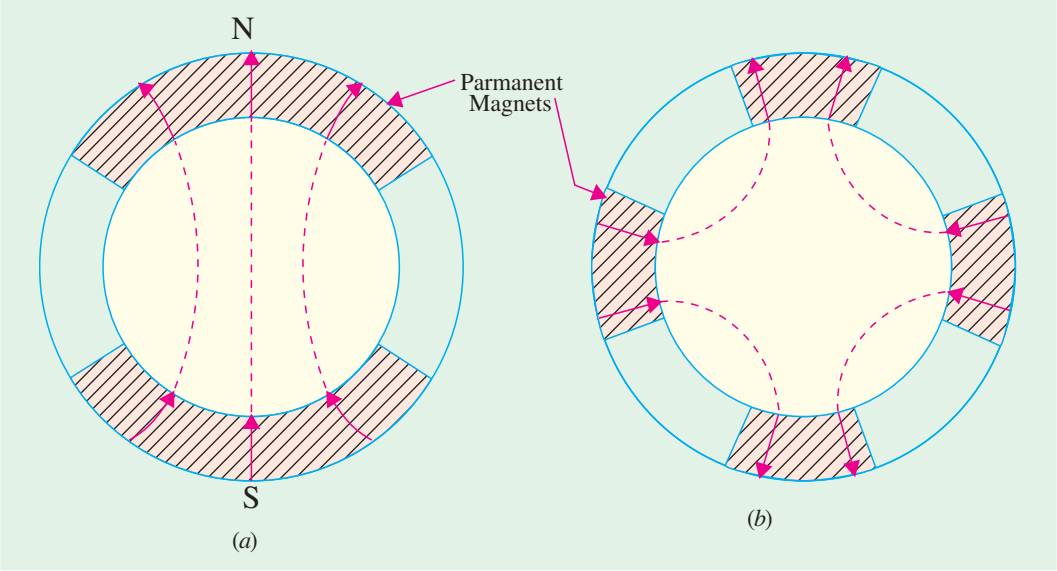


Fig. 39.14

150 per cent of the rated torque. If loaded beyond this point, the motor loses synchronism and will run either as an induction motor or stall.

These motors are usually designed for direct-on-line (DOL) starting. The efficiency and power factor of the permanent-magnet excited synchronous motors are each 5 to 10 points better than their reluctance motor counterparts.

(*b*) Advantages

Since there are no brushes or slip-rings, there is no sparking. Also, brush maintenance is eliminated. Such motors can pull into synchronism with inertia loads of many times their rotor inertia.

(*c*) Applications

These motors are used where precise speed must be maintained to ensure a consistent product. With a constant load, the motor maintains a constant speed.



Permanent magnetic synchronous motor

Hence, these motors are used for synthetic-fibre drawing where constant speeds are absolutely essential.

39.14. Synchros

It is a general name for self-synchronizing machines which, when electrically energized and electrically interconnected, exert torques which cause two mechanically independent shafts either to run in synchronism or to make the rotor of one unit follow the rotor position of the other. They are also known by the trade names of selsyns and autosyns. Synchros, in fact, are small cylindrical motors varying in diameter from 1.5 cm to 10 cm depending on their power output. They are low-torque devices and are widely used in control systems for transmitting shaft position information or for making two or more shafts to run in synchronism. If a large device like a robot arm is to be positioned, synchros will not work. Usually, a servomotor is needed for a higher torque.

39.15. Types of Synchros

There are many types of synchros but the four basic types used for position and error-voltage applications are as under :

(*i*) Control Transmitter (denoted by CX) – earlier called generator **(*ii*)** Control Receiver (CR) – earlier called motor **(*iii*)** Control-Transformer (CT) and **(*iv*)** Control Differential (CD). It may be further subdivided into control differential transmitter (CDX) and control differential receiver (CDR).

All of these synchros are single-phase units except the control differential which is of three-phase construction.



Synchros

(*a*) Constructional Features

1. Control Transmitter

Its constructional details are shown in Fig. 39.15 (*a*). It has a three-phase stator winding similar to that of a three-phase synchronous generator. The rotor is of the projecting-pole type using dumbell construction and has a single-phase winding. When a single-phase ac voltage is applied to the rotor through a pair of slip rings, it produces an alternating flux field along the axes of the rotor. This alternating flux induces three unbalanced single phase/voltage in the three stator windings by transformer action. If the rotor is aligned with the axis of the stator winding 2, flux linkage of this stator winding is maximum and this rotor position is defined as the electrical zero. In Fig. 39.15 (*b*), the rotor axis is displaced from the electrical zero by an angle displaced 120° apart.

(*b*) Control Receiver (CR)

Its construction is essentially the same as that of the control transmitter shown in Fig. 39.15 (*a*). It has three stator windings and a single-phase salient-pole rotor. However, unlike a *CX*, a *CR* has a mechanical viscous damper on the shaft which permits *CR* rotor to respond without overshooting its mark. In normal use, both the rotor and stator windings are excited with single-phase currents. When the field of the rotor conductors interacts with the field of the stator conductors, a torque is developed which produces rotation.

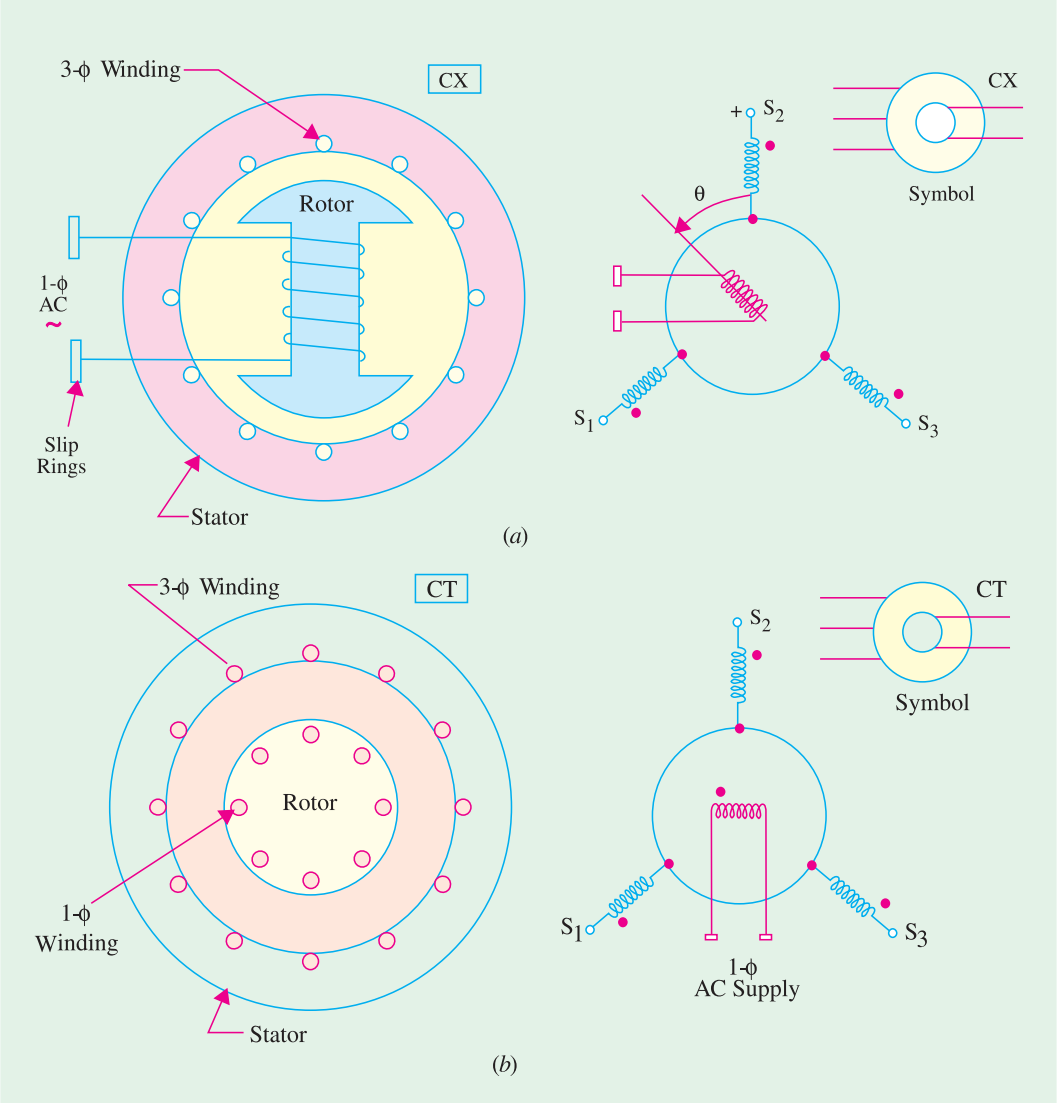


Fig. 39.15

(*c*) Control Transformer (CT)

As shown in Fig. 39.15 (*b*) its stator has a three-phase winding whereas the cylindrical rotor has a single-phase winding. In this case, the electrical zero is defined as that position of the rotor that makes the flux linkage with winding 2 of the stator zero. This rotor position has been shown in Fig. 39.15 (*b*) and is different from that of a control transmitter.

(*d*) Control Differential (CD)

The differential synchro has a balanced three-phase distributed winding in both the stator and the rotor. Moreover, it has a cylindrical rotor as shown in Fig. 39.16 (*a*). Although three-phase windings are involved, it must be kept in mind that these units deal solely with single-phase voltages. The three winding voltages are not polyphase voltages. Normally, the three-phase voltages are identical in magnitude but are separated in phase by 120°. In synchros, these voltages are in phase but differ in magnitude because of their physical orientation.

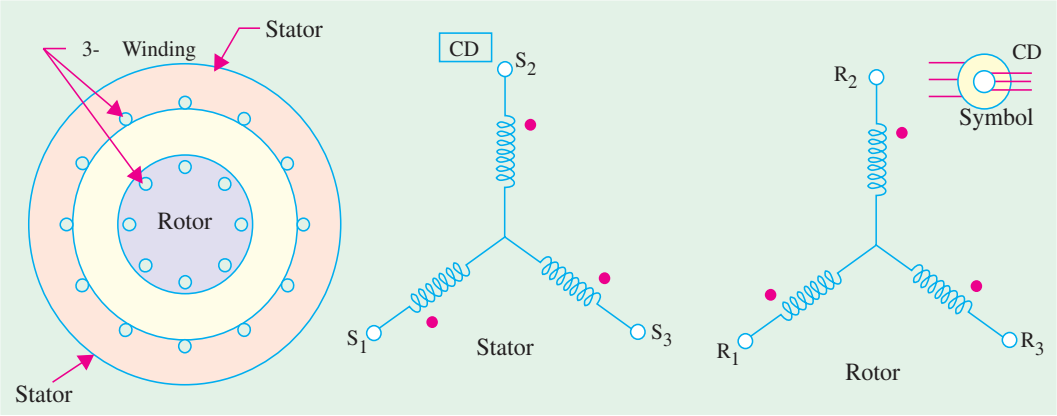


Fig. 39.16

(e) Voltage Relations

Consider the control transmitter shown in Fig. 39.17. Suppose that its rotor winding is excited by a single-phase sinusoidal ac voltage of rms value E_r and that rotor is held fast in its displaced position from the electrical zero. If K = stator turns / rotor turns, the rms voltage induced in the stator winding is $E = KE_r$. However, if we assume $K = 1$, then $E = E_r$.

The rms value of the induced emf in stator winding 2 when the rotor displacement is 'a' is given by

$$E_{2s} = E_r \cos \alpha.$$

Since the axis of the stator winding 1 is located 120° ahead of the axis of winding 2, the rms value of the induced emf in this winding is

$$E_{1s} = E_r \cos (\alpha - 120^\circ).$$

In the same way since winding 3 is located behind the axis of winding 2 by 120°, the expression for the induced emf in winding 3 becomes

$$E_{3s} = E_r \cos (\alpha + 120^\circ).$$

We can also find the values of terminal induced voltages as

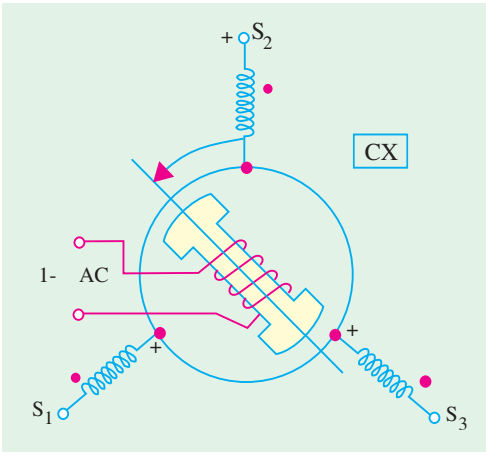


Fig. 39.17

$$E_{12} = E_{1s} + E_{s2} = E_{1s} - E_{2s}$$
$$= E_r \cos \alpha \cos 120^\circ + E_r \sin \alpha \sin 120^\circ - E_r \cos \alpha$$
$$= E_r \left(-\frac{3}{2} \cos \alpha + \frac{\sqrt{3}}{2} \sin \alpha \right)$$
$$= \sqrt{3}\, E_r \left(-\frac{1}{2} \cos \alpha + \frac{1}{2} \sin \alpha \right)$$
$$= \sqrt{3}\, E_r \cos (\alpha - 150^\circ)$$
$$E_{23} = E_{2s} + E_{s3} = E_{2s} - E_{3s}$$
$$= E_r \left(\frac{3}{2} \cos \alpha + \frac{\sqrt{3}}{2} \sin \alpha \right) = \sqrt{3}\, E_r \left(\frac{\sqrt{3}}{2} \cos \alpha + \frac{1}{2} \sin \alpha \right) = \sqrt{3}\, E_r \cos (\alpha - 30^\circ)$$
$$E_{31} = E_{3s} + E_{s1} = E_{3s} - E_{1s}$$
$$= E_r \cos (\alpha + 120^\circ) - E_r \cos (\alpha - 120^\circ)$$
$$= -\sqrt{3}\, E_r \sin \alpha = \sqrt{3}\, E_r \cos (\alpha + 90^\circ)$$

Example 39.5. *The rotor of a control transmitter (CX) is excited by a single-phase ac voltage of rms value 20 V. Find the value of E_{1s}, E_{2s} and E_{3s} for rotor angle $\alpha = +40^o$ and -40^o. Assume the stator/rotor turn ratio as unity. Also, find the values of terminal voltages when $\alpha = +30^o$.*

Solution. Since $K = 1$, the voltage relations derived in will be used.

(a) $\alpha = +40^o$

$E_{2s} = E_r \cos \alpha = 20 \cos 40^o = 15.3$ V

$E_{1s} = E_r \cos (\alpha - 120^o) = 20 \cos (40^o - 120^o) =$ **3.5 V**

$E_{3s} = E_r \cos (\alpha + 120^o) = 20 \cos 160^o =$ **– 18.8 V**

(b) $\alpha = -40^o$

$E_{2s} = 20 \cos (-40^o) = 15.3$ V

$E_{1s} = 20 \cos (-40^o - 120^o) = 20 \cos (-160^o) = -18.8$ V

$E_{3s} = 20 \cos (-40^o + 120^o) = 20 \cos 80^o =$ **3.5 V**

(c) $E_{12} = \sqrt{3} \times 20 \times \cos (30^o - 150^o) = -17.3$ V

$E_{23} = \sqrt{3}\ E_r \cos (\alpha - 30^o) = \sqrt{3}\ E_r \cos (30^o - 30^o) = 34.6$ V

$E_{31} = \sqrt{3}\ E_r \cos (\alpha + 90^o) = \sqrt{3} \times 20 \times \cos (30^o + 90^o) =$ **– 17.3 V**

39.16. Applications of Synchros

The synchros are extensively used in servomechanism for torque transmission, error detection and for adding and subtracting rotary angles. We will consider these applications one by one.

(a) Torque Transmission

Synchros are used to transmit torque over a long distance without the use of a rigid mechanical connection. Fig. 39.18 represents an arrangement for maintaining alignment of two distantly-located shafts. The arrangement requires a control transmitter (CX) and a control receiver (CR) which acts as a torque receiver. As *CX* is rotated by an angle α, *CR* also rotates through the same angle α. As shown, the stator windings of the two synchros are connected together and their rotors are connected to the same single-phase ac supply.

Working. Let us suppose that *CX* rotor is displaced by an angle α and switch SW_1 is closed to energize the rotor winding. The rotor winding flux will induce an unbalanced set of three single-phase voltages (in time phase with the rotor voltage) in the *CX* stator phase windings which will circulate currents in the *CR* stator windings. These currents produce the *CR* stator flux field whose axis is fixed by the angle α. If the *CR* rotor winding is now energized by closing switch SW_2, its flux field will interact with the flux field of the stator winding and thereby produce a torque. This torque will rotate the freely-moving *CR* rotor to a position which exactly corresponds with the *CT* rotor *i.e.* it will be displaced by the same angle α as shown in Fig. 39.18. It should be noted that if the two rotors are in the same relative positions, the stator voltages in the two synchros will be exactly equal and opposite. Hence, there will be no current flow in the two stator windings and so no torque will be produced and the system will achieve equilibrium. If now, the transmitter rotor angle changes to a new value, then new set of voltages would be induced in the transmitter stator windings which will again drive currents through the receiver stator windings. Hence, necessary torque will be produced which will turn the *CR* rotor through an angle corresponding to that of the *CT* rotor. That is why the transmitter rotor is called the master and the receiver rotor as the slave, because it follows its master. It is worth noting that this master-slave relationship is reversible because when the receiver rotor is displaced through a certain angle, it causes the transmitter rotor to turn through the same angle.

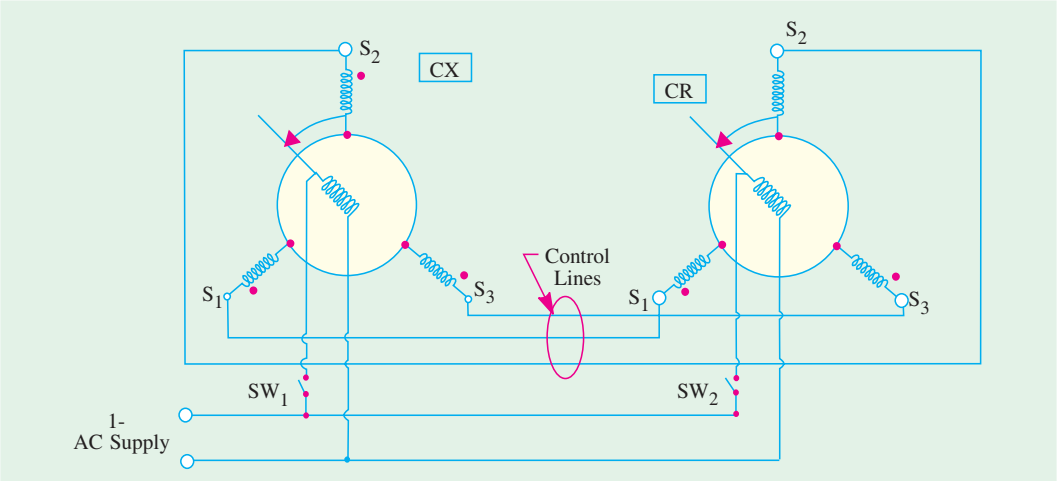


Fig. 39.18

(*b*) Error Detection

Synchros are also used for error detection in a servo control system. In this case, a command in the form of a mechanical displacement of the *CX* rotor is converted to an electrical voltage which appears at the *CT* rotor winding terminals which can be further amplified by an amplifier.

For this purpose, we require a *CX* synchro and a *CT* synchro as shown in Fig. 39.19. Only the *CX* rotor is energized from the single-phase ac voltage supply which produces an alternating air-gap flux field. This time-varying flux field induces voltages in the stator windings whose values for $\alpha = 30^{o}$ are as indicated in the Fig. 39.19. The *CX* stator voltages supply magnetizing currents in the *CT* stator

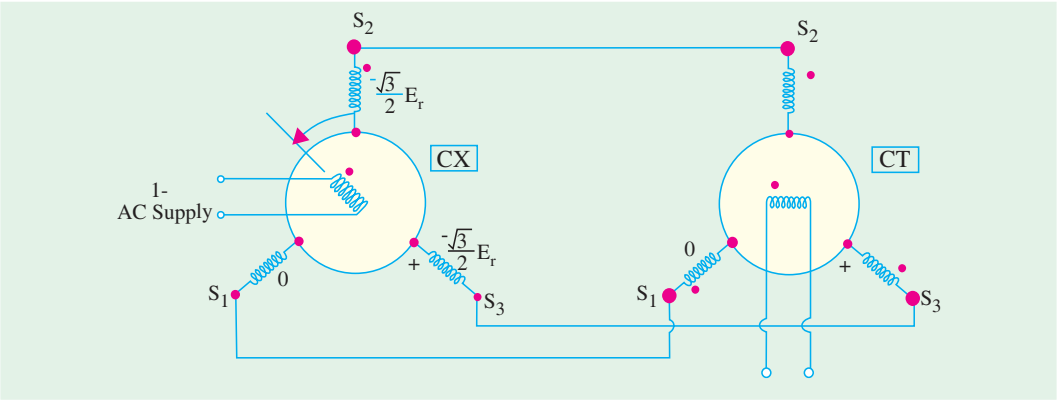


Fig. 39.19

windings which, in turn, create an alternating flux field in their own air-gap. The values of the *CT* stator phase currents are such that the air-gap flux produced by them induces voltages that are equal and opposite to those existing in the *CX* stator. Hence, the direction of the resultant flux produced by the *CX* stator phase currents is forced to take a position which is exactly identical to that of the rotor axis of the *CT*.

If the *CT* rotor is assumed to be held fast in its electrical zero position as shown in Fig.39.19, then the rms voltage induced in the rotor is given by $E = E_{max} \sin \alpha$, where E_{max} is the maximum voltage induced by the *CT* air-gap flux when coupling with the rotor windings is maximum and α is the displacement angle of the *CT* rotor.

In general, the value of the rms voltage induced in the *CT* rotor winding when the displacement of the *CX* rotor is α_x and that of the *CT* rotor is α_T is given by

$$E = E_{max} \sin (\alpha_x - \alpha_T)$$

39.17. Control Differential Transmitter

It can be used to produce a rotation equal to the sum of difference of the rotations of two shafts. The arrangement for this purpose is shown in Fig. 39.20 (*a*). Here, a CDX is coupled to a control transmitter on one side and a control receiver on the other. The *CX* and *CR* rotor windings are energized from the same single-phase voltage supply.

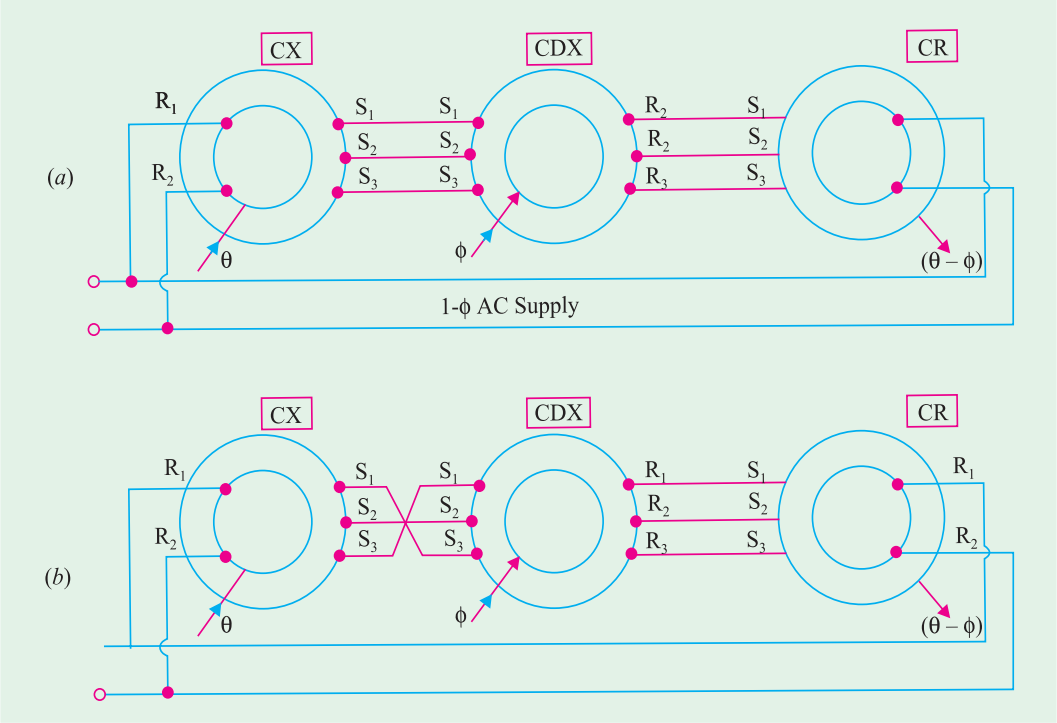


Fig. 39.20

It has two inputs : Mechanical θ and Electrical ϕ and the output is Machnical (θ – ϕ). The mechanical input (θ) to *CX* is converted and applied to the *CDX* stator. With a rotor input (ϕ), the electrical output of the *CDX* is applied to the *CR* stator which provides the mechanical output (θ – ϕ).

As shown in Fig. 39.20 (*b*), if any two stator connections between *CX* and *CDX* are transposed, the electrical input from *CX* to *CDX* becomes –θ, hence the output becomes (–θ – ϕ) = – (θ + ϕ).

39.18. Control Differential Receiver

In construction, it is similar to a *CDX* but it accepts two electrical input angles and provide the difference angle as a mechanical output (Fig. 39.21).

The arrangement consists of two control transmitters coupled to a *CDR*. The two control transmitters provide inputs to the CDX, one (θ) to the stator and the other (ϕ) to the rotor. The *CDX* output is the difference of the two inputs *i.e.* (θ – ϕ).

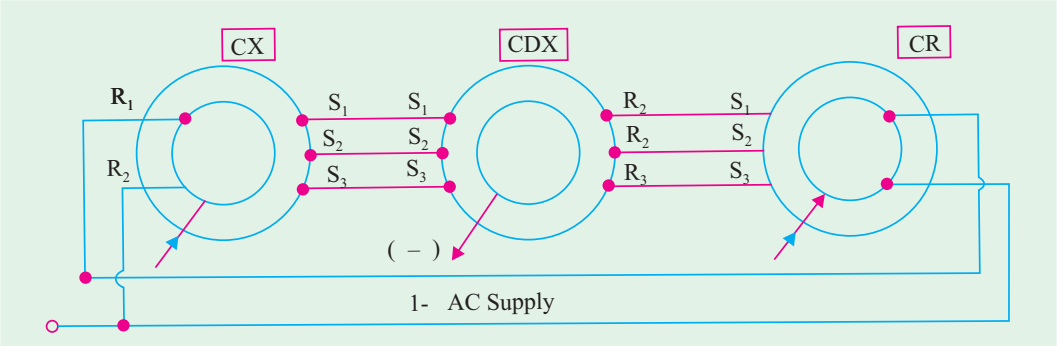


Fig. 39.21

39.19. Switched Reluctance Motor

The switched reluctance (SR) motor operates on the same basic principle as a variable reluctance stepper motor (Art. 39.4).

(*a*) Construction

Unlike a conventional synchronous motor, both the rotor and stator of a SR motor have salient poles as shown in Fig. 39.22. This doubly-salient arrangement is very effective for electromagnetic energy conversion.

The stator carries coils on each pole, the coils on opposite poles being connected in series. The eight stator coils shown in Figure are grouped to form four phases which are independently energized from a four-phase converter. The laminated rotor has no windings or magnets and is, therefore cheap to manufacture and extremely robust. The motor shown in Fig. 39.22 has eight stator poles and six rotor poles which is a widely-used arrangement although other pole combinations (like 6/4 poles) are used to suit different applications.

(*b*) Working

Usual arrangement is to energize stator coils sequentially with a single pulse of current at high speed. However, at starting and low speed, a current-chopper type control is used to limit the coil current.



Switched reluctance motor

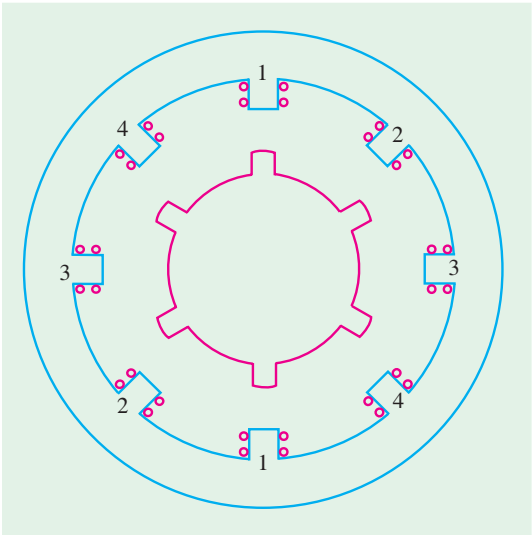


Fig. 39.22

The motor rotates in the anticlockwise direction when the stator phases are energized in the sequence 1, 2, 3, 4 and in clockwise direction when energized in the sequence 1, 4, 3, 2. When the stator coils are energized, the nearest pair of rotor poles is pulled into alignment with the appropriate stator poles by reluctance torque.

Closed-loop control is essential to optimize the switching angles of the applied coil voltages. The stator phases are switched by signals derived from a shaft-mounted rotor position detectors such as Hall-effect devices or optical sensors Fig. (39.23). This causes the behaviour of the SR motor to resemble that of a dc motor.

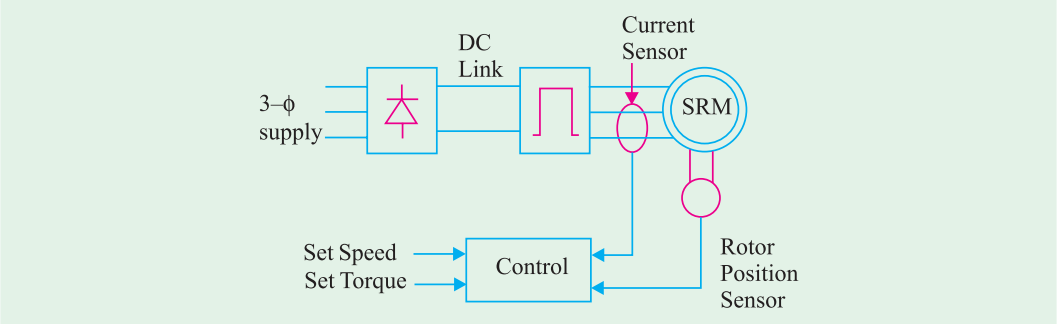


Fig. 39.23

(*c*) Advantages and Disadvantages

Although the newest arrival on the drives scene, the SR motor offers the following advantages:

(i) higher efficiency ***(ii)*** more power per unit weight and volume ***(iii)*** very robust because rotor has no windings or slip rings ***(iv)*** can run at very high speed (upto 30,000 rpm) in hazardous atmospheres ***(v)*** has versatile and flexible drive features and ***(vi)*** four-quadrant operation is possible with appropriate drive circuitry.

However, the drawbacks are that it is ***(i)*** relatively unproven ***(ii)*** noisy and ***(iii)*** not well-suited for smooth torque production.

(*d*) Applications

Even though the SR technology is still in its infancy, it has been successfully applied to a wide range of applications such as ***(i)*** general purpose industrial drives ***(ii)*** traction ***(iii)*** domestic appliances like food processors, vacuum cleaners and washing machines etc., and ***(iv)*** office and business equipment.

39.20. Comparison between *VR* Stepper Motor and SR Motor

VR Stepper Motor	*SR Motor*
1. It rotates in steps.	It is meant for continuous rotation.
2. It is designed first and foremost for open-loop operation.	Closed-loop control is essential for its optimal working.
3. Its rotor poles are made of ferromagnetic material.	Its rotor poles are also made of ferromagnetic material.
4. It is capable of half-step operation and microstepping.	It is not designed for this purpose.
5. Has low power rating.	Has power ratings upto 75 kW (100 hp).
6. Has lower efficiency.	Has higher overall efficiency.

39.21. The Resolver

In many ways, it is similar to a synchro but differs from it in the following respects : ***(i)*** Electrical displacement between stator windings is 90° and not 120° ***(ii)*** It has two stator windings and two rotor windings (Fig. 39.24) ***(iii)*** Its input can be either to the stator or to the rotor ***(iv)*** They are usually not used as followers because their output voltage is put to further use.

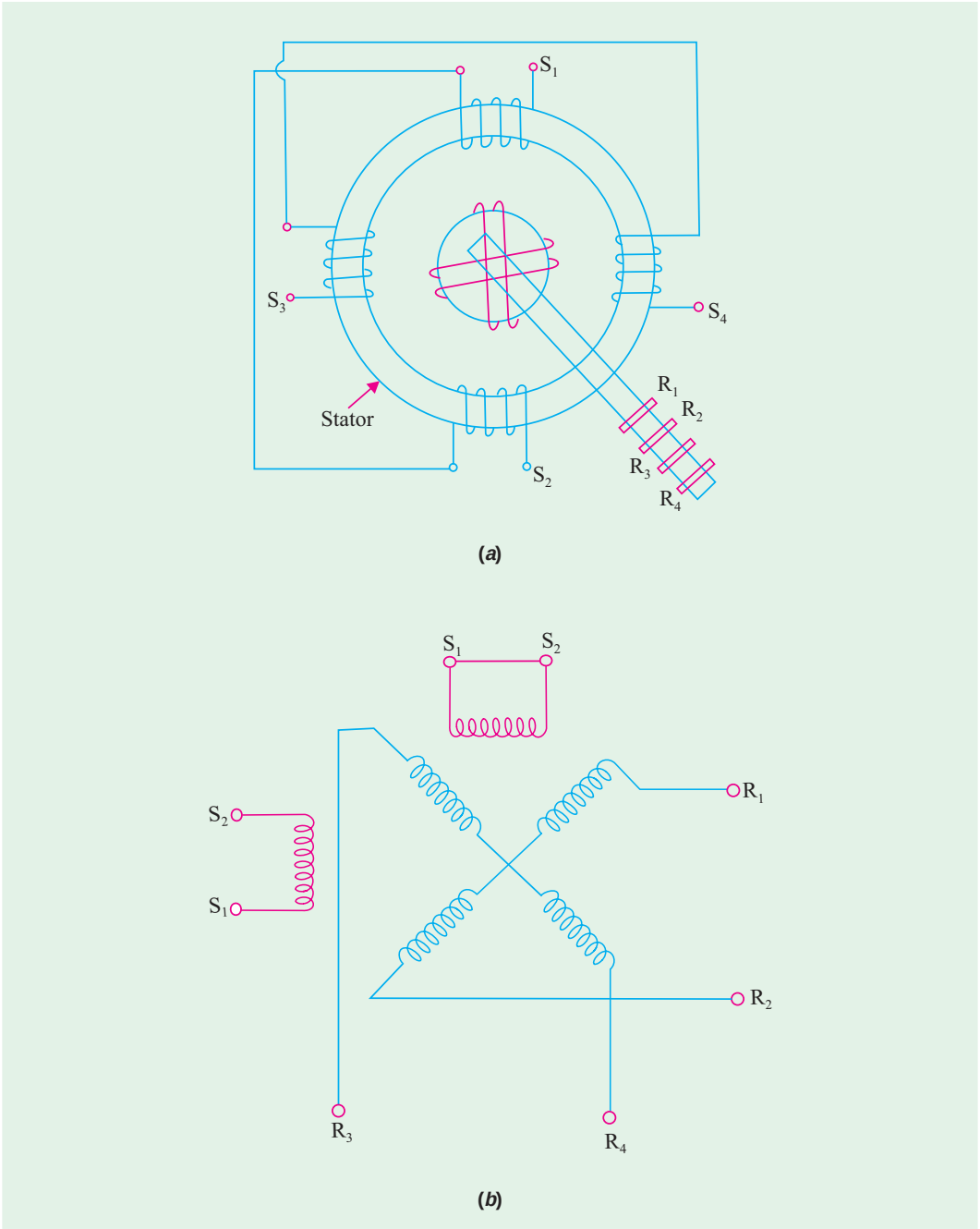


Fig. 39.24

(*a*) Construction

The main constructional features and the symbol for a resolver are shown in Fig. 39.24. There are two stator windings which are wound 90° apart. In most applications, only one stator winding is used, the other being short-circuited. The two rotor winding connections are brought out through slip rings and brushes.

(*b*) Applications

Resolvers find many applications in navigation and height determination as shown in Fig. 39.25 (*a*) and (*c*) where Fig. 39.25 (*b*) provides the key.

(*i*) Navigation Application

As shown in Fig. 39.25 (*a*), the purpose is to determine the distance *D* to the destination. Suppose the range *R* to a base station as found by a radar ranging device is 369 km. The angle θ is also determined directly. If the amplifier scale is 4.5 V per 100 km, the range would be represented by $369 \times (4.5 / 100) = 16.6$ V. Further suppose that angle θ is found to be 52.5°. Now, set the resolver at 52.5° and apply 16.6 V to rotor terminals $R_3 R_4$. The voltage which appears at terminals $S_1 S_2$ represents *D*. If we assume *K* = stator turns / rotor turns = 1, the voltage available at $S_1 S_2$ will be = 16.6 / cos 52.5° = 16.6 / 0.6088 = 27.3 V. Since 4.5 V represents 100 km, 27.3 V represents $27.3 \times 100 / 4.5 =$ 607 km.

(*ii*) Height Determination

Suppose the height *H* of a building is to be found. First of all, the oblique distance *D* to the top of the building is found by a range finder. Let *D* = 210 m and the scale of the amplifier to the resolver stator be 9 V per 100 m. The equivalent voltage is $9 \times 210 / 100 = 18.9$ V. This voltage is applied to stator terminals is $S_1 S_2$ of the resolver. Suppose the angle θ read from the resolver scale is 61.3°. The height of the building is given in the form of voltage which appears across the rotor terminals $R_1 R_2$. Assuming stator / rotor turn ratio as unity and the same amplifier ratio for the rotor output, the voltage across $R_1 R_2 = 18.9 \times \sin 61.3° = 16.6$ V. Hence, $H = 16.6 \times 100 / 9 = 184$ m. It would be seen that in using the resolver, there is no need to go through trigonometric calculations because the answers come out directly.

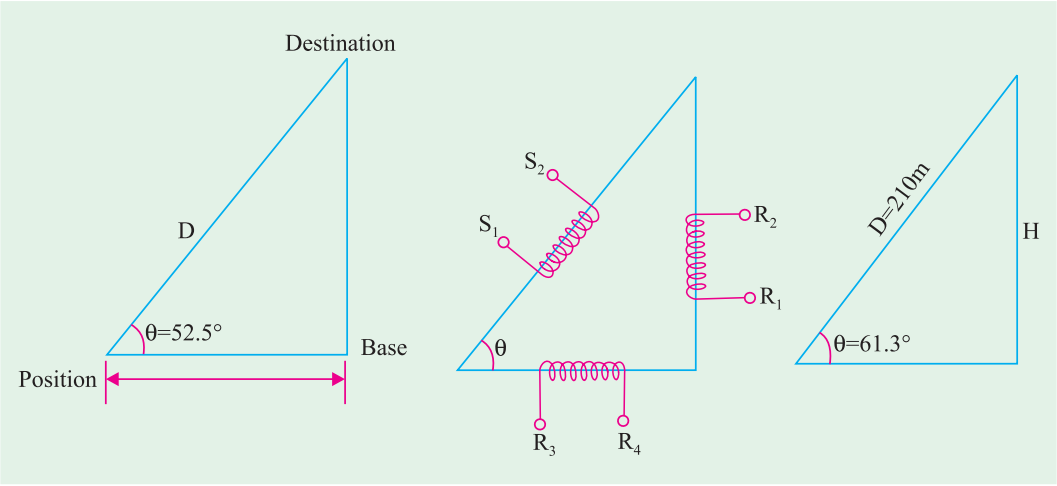


Fig. 39.25

39.22. Servomotors

They are also called control motors and have high-torque capabilities. Unlike large industrial motors, they are not used for continuous energy conversion but only for precise speed and precise position control at high torques. Of course, their basic principle of operation is the same as that of other electromagnetic motors. However, their construction, design and mode of operation are different. Their power ratings vary from a fraction of a watt upto a few 100 W. Due to their low-inertia, they have high speed of response. That is why they are smaller in diameter but longer in length. They generally operate at vary low speeds or sometimes zero speed. They find wide applications in radar, tracking and guidance systems, process controllers, computers and machine tools. Both dc and a.c. (2-phase and 3-phase) servomotors are used at present.

Servomotors differ in application capabilities from large industrial motors in the following respects :

1. They produce high torque at all speeds including zero speed.
2. They are capable of holding a static (*i.e.* no motion) position.
3. They do not overheat at standstill or lower speeds.
4. Due to low-inertia, they are able to reverse directions quickly.
5. They are able to accelerate and deaccelerate quickly.
6. They are able to return to a given position time after time without any drift.

These motors look like the usual electric motors. Their main difference from industrial motors is that more electric wires come out of them for power as well as for control. The servomotor wires go to a controller and not to the electrical line through contactors. Usually, a tachometer (speed indicating device) is mechanically connected to the motor shaft. Sometimes, blower or fans may also be attached for motor cooling at low speeds.



DC servo motor

39.23. DC Servomotors

These motors are either separately-excited dc motors or permanent-magnet dc motors. The schematic diagram of a separately-excited d.c. motor alongwith its armature and field MMFs and torque/speed characteristics is shown in Fig. 39.26. The speed of d.c. servomotors is normally controlled by varying the armature voltage. Their armature is deliberately designed to have large resistance so that torque-speed characteristics are linear and have a large negative slope as shown in Fig. 39.26 (*c*). The negative slope serves the purpose of providing the viscous damping for the servo drive system.

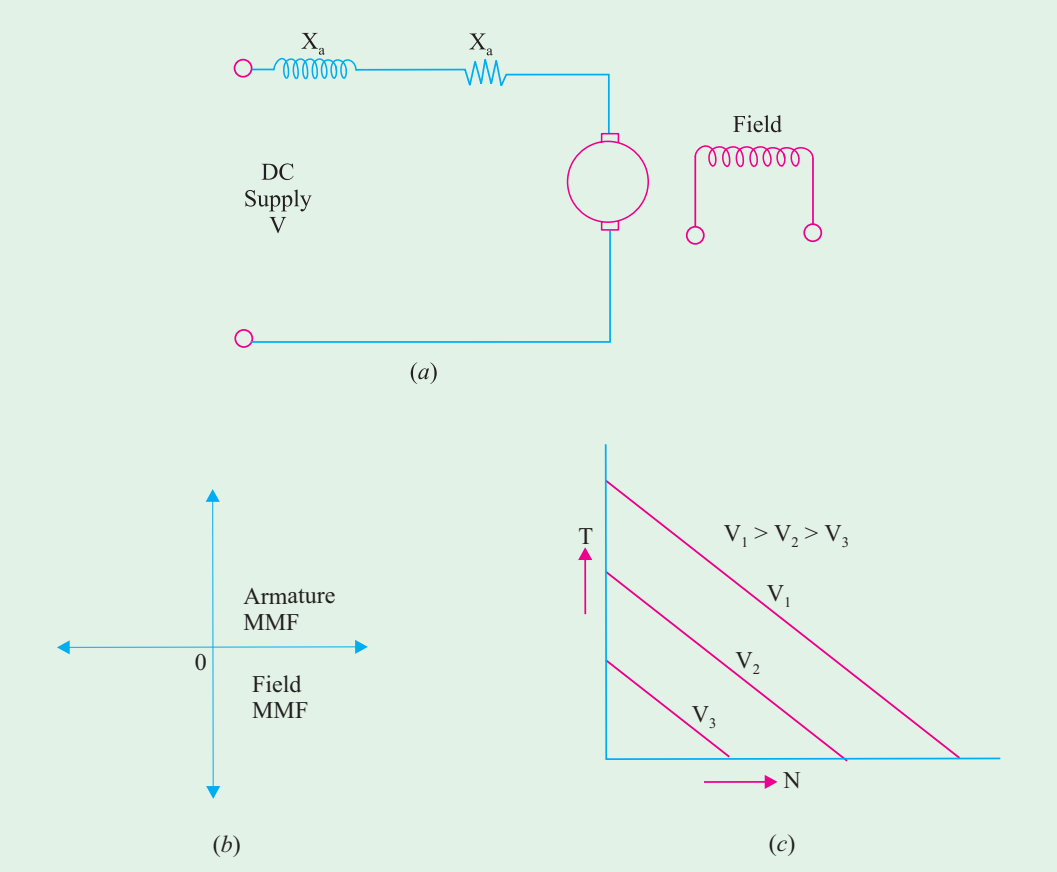


Fig. 39.26

As shown in Fig. 39.26 (*b*), the armature m.m.f. and excitation field mmf are in quadrature. This fact provides a fast torque response because torque and flux become decoupled. Accordingly, a step change in the armature voltage or current produces a quick change in the position or speed of the rotor.



Permanent magnet stepper motor

39.24. AC Servomotors

Presently, most of the ac servomotors are of the two-phase squirrel-cage induction type and are used for low power applications. However, recently three-phase induction motors have been modified for high power servo systems which had so far been using high power d.c. servomotors.

(*a*) Two-phase AC Servomotor

Such motors normally run on a frequency of 60 Hz or 400 Hz (for airborne systems). The stator has two distributed windings which are displaced from each other by 90° (electrical). The main

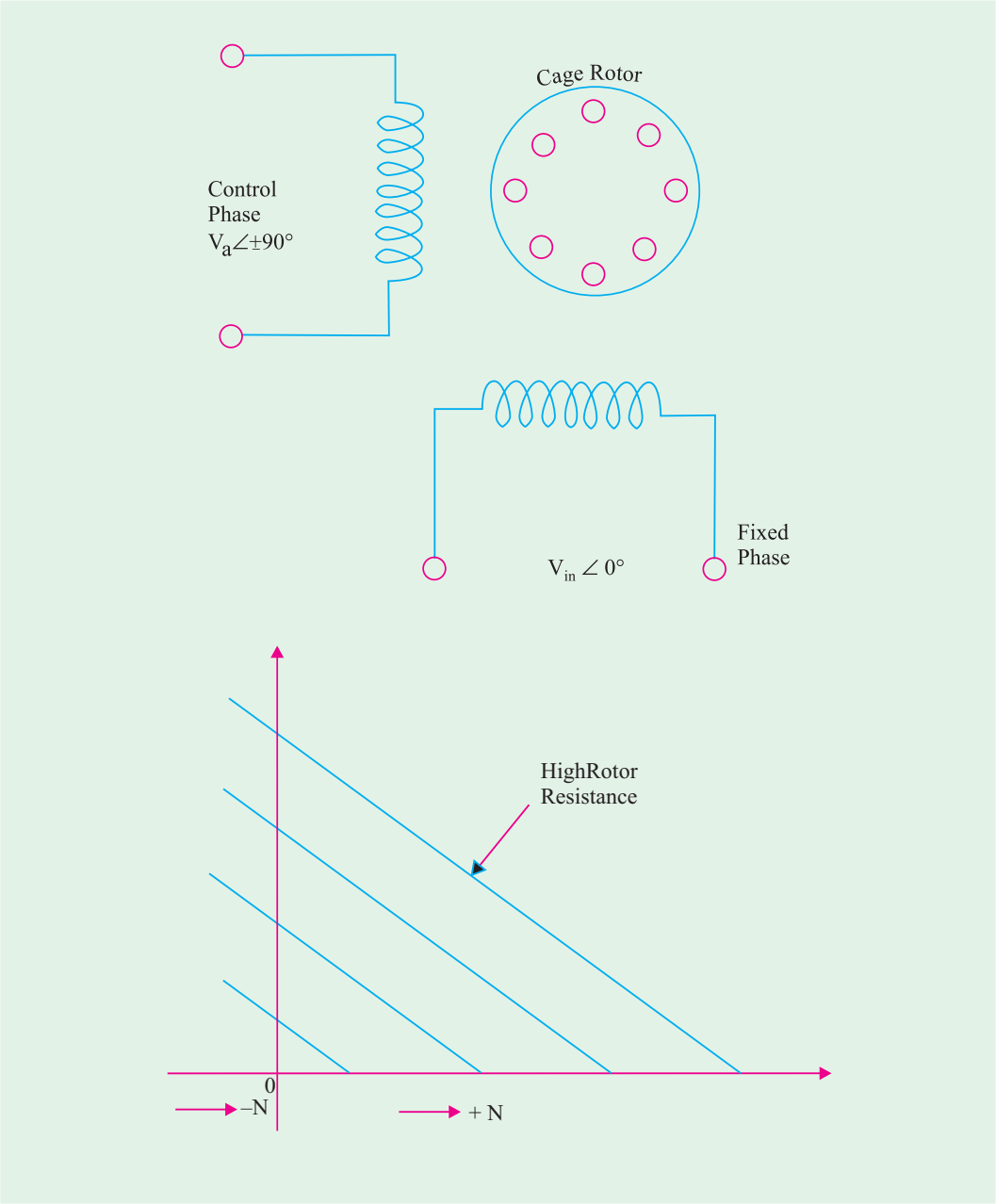


Fig. 39.27

winding (also called the reference or fixed phase) is supplied from a constant voltage source, $V_m \angle 0^o$ (Fig. 39.27). The other winding (also called the control phase) is supplied with a variable voltage of the same frequency as the reference phase but is phase-displaced by 90º (electrical). The control-phase voltage is controlled by an electronic controller. The speed and torque of the rotor are controlled by the phase difference between the main and control windings. Reversing the phase difference from leading to lagging (or ***vice-versa***) reverses the motor direction.

Since the rotor bars have high resistance, the torque-speed characteristics for various armature voltages are almost linear over a wide speed range particularly near the zero speed. The motor operation can be controlled by varying the voltage of the main phase while keeping that of the reference phase constant.

(*b*) Three-phase AC Servomotors

A great deal of research has been to modify a three-phase squirrel-cage induction motor for use in high power servo systems. Normally, such a motor is a highly non-linear coupled-circuit device. Recently, this machine has been operated successfully as a linear decoupled machine (like a d.c. machine) by using a control method called vector control or field oriented control. In this method, the currents fed to the machine are controlled in such a way that its torque and flux become decoupled as in a dc machine. This results in a high speed and a high torque response.

OBJECTIVE TESTS – 39

1. A single-stack, 4-phase, 6-pole VR stepper motor will have a step angle of
(*a*) 15°
(*b*) 30°
(*c*) 45°
(*d*) 90°

2. In a three-stack 12/8-pole VR motor, the rotor pole pitch is
(*a*) 15°
(*b*) 30°
(*c*) 45°
(*d*) 60°

3. A three-stack VR stepper motor has a step angle of 10°. What is the number of rotor teeth in each stack ?
(*a*) 36
(*b*) 24
(*c*) 18
(*d*) 12

4. If a hybrid stepper motor has a rotor pitch of 36° and a step angle of 9°, the number of its phases must be
(*a*) 4
(*b*) 2
(*c*) 3
(*d*) 6

5. What is the step angle of a permanent-magnet stepper motor having 8 stator poles and 4 rotor poles ?
(*a*) 60°
(*b*) 45°
(*c*) 30°
(*d*) 15°

6. A stepping motor is a ..... device.
(*a*) mechanical
(*b*) electrical
(*c*) analogue
(*d*) incremental

7. Operation of stepping motors at high speeds is referred to as
(*a*) fast forward
(*b*) slewing
(*c*) inching
(*d*) jogging

8. Which of the following phase switching sequence represents half-step operation of a VR stepper motor ?
(*a*) A, B, C, A .....
(*b*) A, C, B, A .....
(*c*) AB, BC, CA, AB .....
(*d*) A, AB, B, BC .....

9. The rotational speed of a given stepper motor is determined solely by the
(*a*) shaft load
(*b*) step pulse frequency
(*c*) polarity of stator current
(*d*) magnitude of stator current.

10. A stepper motor may be considered as a ..... converter.
(*a*) dc to dc
(*b*) ac to ac
(*c*) dc to ac
(*d*) digital-to-analogue

11. The rotor of a stepper motor has no

(*a*) windings
(*b*) commutator
(*c*) brushes
(*d*) all of the above.

12. Wave excitation of a stepper motor results in
(*a*) microstepping
(*b*) half-stepping
(*c*) increased step angle
(*d*) reduced resolution.

13. A stepper motor having a resolution of 300 steps/rev and running at 2400 rpm has a pulse rate of— pps.
(*a*) 4000
(*b*) 8000
(*c*) 6000
(*d*) 10,000

14. The torque exerted by the rotor magnetic field of a PM stepping motor with unexcited stator is called .....torque.
(*a*) reluctance
(*b*) detent
(*c*) holding
(*d*) either (*b*) or (*c*)

15. A variable reluctance stepper motor is constructed of ..... material with salient poles.
(*a*) paramagnetic
(*b*) ferromagnetic
(*c*) diamagnetic
(*d*) non-magnetic

16. Though structurally similar to a control transmitter, a control receiver differs from it in the following way :
(*a*) it has three-phase stator winding
(*b*) it has a rotor of dumbell construction
(*c*) it has a mechanical damper on its shaft
(*d*) it has single-phase rotor excitation.

17. The control ..... synchro has three-phase winding both on its stator and rotor.
(*a*) differential
(*b*) transformer
(*c*) receiver
(*d*) transmitter

18. Regarding voltages induced in the three stator windings of a synchro, which statement is false ?
(*a*) they depend on rotor position.
(*b*) they are in phase.
(*c*) they differ in magnitude.
(*d*) they are polyphase voltages.

19. The low-torque synchros cannot be used for
(*a*) torque transmission
(*b*) error detection
(*c*) instrument servos
(*d*) robot arm positioning.

20. Which of the following synchros are used for error detection in a servo control system ?
(*a*) control transmitter
(*b*) control transformer
(*c*) control receiver
(*d*) both (*a*) and (*b*).

21. For torque transmission over a long distance with the help of electrical wires only, which of the following two synchros are used ?
(*a*) *CX* and *CT*
(*b*) *CX* and *CR*
(*c*) *CX* and *CD*
(*d*) *CT* and *CD*.

22. The arrangement required for producing a rotation equal to the sum or difference of the rotation of two shafts consists of the following coupled synchros.
(*a*) control transmitter
(*b*) control receiver
(*c*) control differential transmitter
(*d*) all of the above.

23. Which of the following motor would suit applications where constant speed is absolutely essential to ensure a consistent product ?
(*a*) brushless dc motor
(*b*) disk motor
(*c*) permanent-magnet synchronous motor
(*d*) stepper motor.

24. A switched reluctance motor differs from a *VR* stepper motor in the sense that it
(*a*) has rotor poles of ferromagnetic material
(*b*) rotates continuously
(*c*) is designed for open-loop operation only
(*d*) has lower efficiency.

25. The electrical displacement between the two stator windings of a resolver is
(*a*) 120°
(*b*) 90°
(*c*) 60°
(*d*) 45°.

26. Which of the following motor runs from a low dc supply and has permanently magnetized salient poles on its rotor ?

(*a*) permanent-magnet d.c. motor

(*b*) disk d.c. motor

(*c*) permanent-magnet synchronous motor

(*d*) brushless d.c. motor.

27. A dc servomotor is similar to a regular d.c. motor except that its design is modified to cope with

(*a*) electronic switching

(*b*) slow speeds

(*c*) static conditions

(*d*) both (*b*) and (*c*).

28. One of the basic requirements of a servomotor is that it must produce high torque at all

(*a*) loads

(*b*) frequencies

(*c*) speeds

(*d*) voltages.

29. The most common two-phase ac servomotor differs from the standard ac induction motor because it has

(*a*) higher rotor resistance

(*b*) higher power rating

(*c*) motor stator windings

(*d*) greater inertia.

30. Squirrel-cage induction motor is finding increasing application in high-power servo systems because new methods have been found to

(*a*) increase its rotor resistance

(*b*) control its torque

(*c*) decrease its intertia

(*d*) decouple its torque and flux.

ANSWERS

1. *a* **2.** *c* **3.** *d* **4.** *a* **5.** *b* **6.** *d* **7.** *b* **8.** *d* **9.** *b* **10.** *d* **11.** *d* **12.** *b* **13.** *c* **14.** *d* **15.** *b* **16.** *c* **17.** *a* **18.** *d* **19.** *d* **20.** *d* **21.** *b* **22.** *d* **23.** *c* **24.** *b* **25.** *b* **26.** *a* **27.** *d* **28.** *c* **29.** *a* **30.** *d*.

QUESTIONS AND ANSWERS ON SPECIAL MACHINES

Q.1. Do stepper motors have internal or external fans ?

Ans. No. Because the heat generated in the stator winding is conducted through the stator iron to the case which is cooled by natural conduction, convection and radiation.

Q.2. Why do hybrid stepping motors have many phases sometime more than six ?

Ans. In order to obtain smaller step angles.

Q.3. Any disadvantage(s) of having more phases?

Ans. Minor ones are: more leads have to be brought out from the motor, more interconnections are required to the drive circuit and more switching devices are needed.

Q.4. What is the main attraction of a multi-stack *VR* stepper motor ?

Ans. It is well-suited to high stepping rates.

Q.5. You are given a *VR* motor and a hybrid stepper motor which look exactly similar. How would you tell which is which ?

Ans. Spin the rotor after short-circuiting the stator winding. If there is no mechanical resistance to rotation, it is a *VR* motor and if there is resistance, then it is a hybrid motor.

Q.6. How do you explain it ?

Ans. Since *V R* motor has magnetically neutral rotor, it will not induce any e.m.f. in the short-circuited winding *i.e*. the machine will not act as a generator and hence experience no drag on its rotation. However, the rotor of a hybrid motor has magnetic poles, hence it will act as a generator and so experience a drag.

Q.7. Will there be any harm if the rotor of a hybrid stepper motor is pulled out of its stator ?

Ans. Yes. The rotor will probably become partially demagnetized and, on reassembling, will give less holding torque.

GO To FIRST

VOLUME – III

TRANSMISSION, DISTRIBUTION AND UTILIZATION

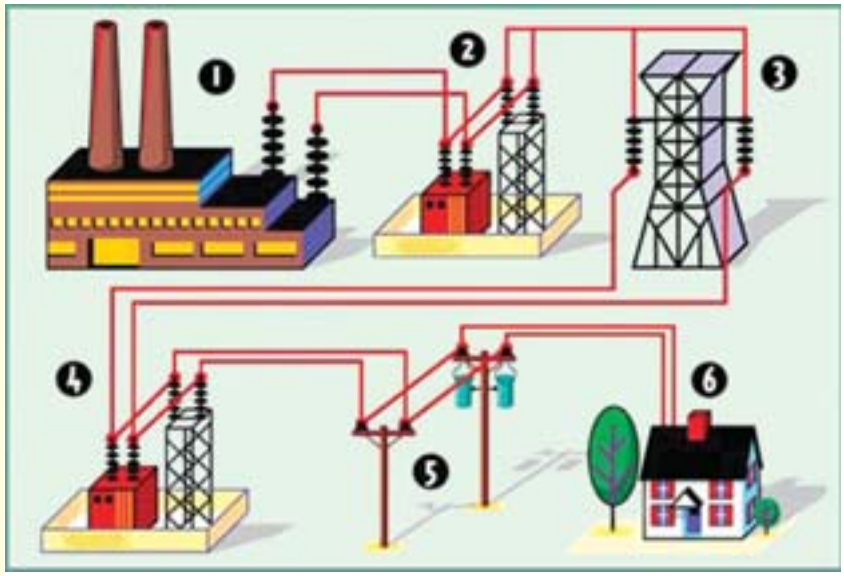
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CHAPTER 40

D.C. TRANSMISSION AND DISTRIBUTION

Learning Objectives

- Transmission and Distribution of D.C. Power
- Two-wire and Three-wire System
- Voltage Drop and Transmission Efficiency
- Methods of Feeding Distributor
- D.C.Distributor Fed at One End
- Uniformaly Loaded Distributor
- Distributor Fed at Both Ends with Equal Voltage
- Distributor Fed at Both ends with Unequal Voltage
- Uniform Loading with Distributor Fed at Both Ends
- Concentrated and Uniform Loading with Distributor Fed at One End
- Ring Distributor
- Current Loading and Load-point Voltage in a 3-wire System
- Three-wire System
- Balancers
- Boosters
- Comparison of 2-wire and 3-wire Distribution System



(1) Electricity leaves the power plant, (2) Its voltage is increased at a step-up transformer, (3) The electricity travels along a transmission line to the area where power is needed, (4) There, in the substation, voltage is decreased with the help of step-down transformer, (5) Again the transmission lines carry the electricity, (6) Electricity reaches the final consumption points

CONTENTS

40.1. Transmission and Distribution of D.C. Power

By transmission and distribution of electric power is meant its conveyance from the central station where it is generated to places, where it is demanded by the consumers like mills, factories, residential and commercial buildings, pumping stations etc. Electric power may be transmitted by two methods.

(i) By overhead system or ***(ii)*** By underground system—this being especially suited for densely-populated areas though it is somewhat costlier than the first method. In over-head system, power is conveyed by bare conductors of copper or aluminium which are strung between wooden or steel poles erected at convenient distances along a route. The bare copper or aluminium wire is fixed to an insulator which is itself fixed onto a cross-arm on the pole. The number of cross-arms carried by a pole depends on the number of wires it has to carry. Line supports consist of ***(i)*** pole structures and ***(ii)*** tower. Poles which are made of wood, reinforced concrete or steel are used up to 66 kV whereas steel towers are used for higher voltages.

The underground system employs insulated cables which may be single, double or triple-core etc.

A good system whether overhead or underground should fulfil the following requirements :

1. The voltage at the consumer's premises must be maintained within ± 4 or ± 6% of the declared voltage, the actual value depending on the type of load*.
2. The loss of power in the system itself should be a small percentage (about 10%) of the power transmitted.
3. The transmission cost should not be unduly excessive.
4. The maximum current passing through the conductor should be limited to such a value as not to overheat the conductor or damage its insulation.
5. The insulation resistance of the whole system should be very high so that there is no undue leakage or danger to human life.

It may, however, be mentioned here that these days all production of power is as a.c. power and nearly all d.c. power is obtained from large a.c. power systems by using converting machinery like synchronous or rotary converters, solid-state converters and motor-generator sets etc. There are many sound reasons for producing power in the form of alternating current rather than direct current.

(i) It is possible, in practice, to construct large high-speed a.c. generators of capacities up to 500 MW. Such generators are economical both in the matter of cost per kWh of electric energy produced as well as in operation. Unfortunately, d.c. generators cannot be built of ratings higher than 5 MW because of commutation trouble. Moreover, since they must operate at low speeds, it necessitates large and heavy machines.

(ii) A.C. voltage can be efficiently and conveniently raised or lowered for economic transmission and distribution of electric power respectively. On the other hand, d.c. power has to be generated at comparatively low voltages by units of relatively low power ratings. As yet, there is no economical method of raising the d.c. voltage for transmission and lowering it for distribution.

Fig. 40.1 shows a typical power system for obtaining d.c. power from a.c. power. Other details such as instruments, switches and circuit breakers etc. have been omitted.

Two 13.8 kV alternators run in parallel and supply power to the station bus-bars. The voltage is stepped up by 3-phase transformers to 66 kV for transmission purposes** and is again stepped down to 13.8 kV at the sub-station for distribution purposes. Fig. 40.1 shows only three methods commonly used for converting a.c. power to d.c. power at the sub-station.

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* According to Indian Electricity Rules, voltage fluctuations should not exceed ± 5% of normal voltage for L.T. supply and ± $12\frac{1}{2}$ % for H.T. supply.

** Transmission voltages of upto 400 kV are also used.

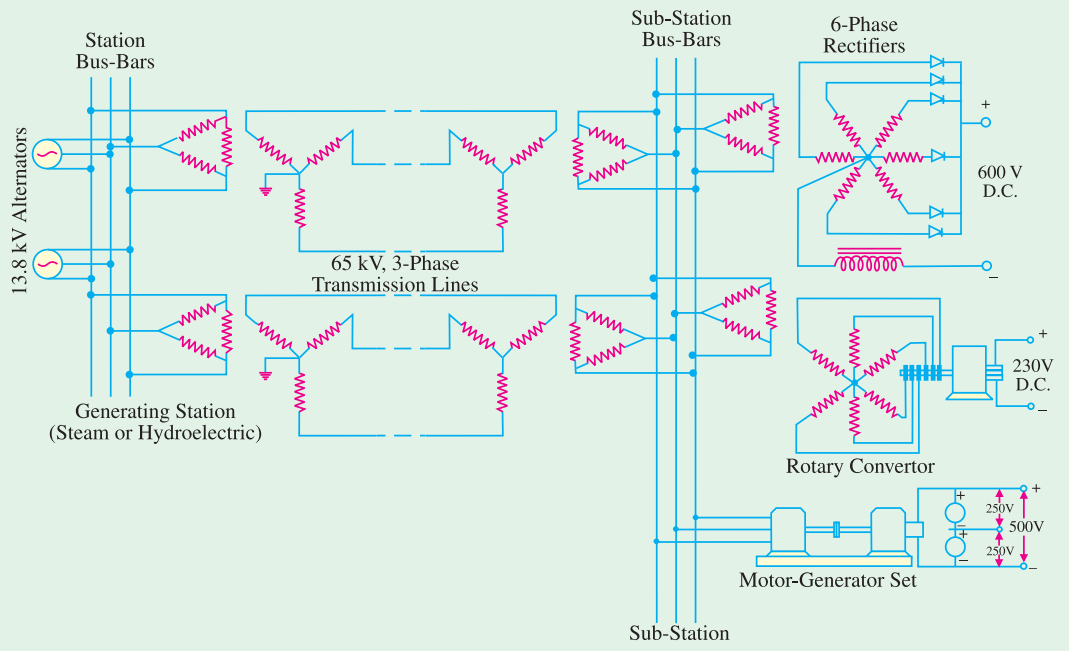


Fig. 40.1

(*a*) a 6-phase mercury-arc rectifier gives 600 V d.c. power after the voltage has been stepped down to a proper value by the transformers. This 600-V d.c. power is generally used by electric railways and for electrolytic processes.

(*b*) a rotary converter gives 230 V d.c. power.

(*c*) a motor-generator set converts a.c. power to 500/250 d.c. power for 3-wire distribution systems.



The above figure shows a motor-generator set. Nowadays, we use solid-state devices, called rectifiers, to convert standard AC to DC current. Back in the olden days, they needed a "motor dynamo" set to make the conversion as shown above. An AC motor would turn a DC Generator, as pictured above

In Fig. 40.2 is shown a schematic diagram of ***low tension*** distribution system for d.c. power. The whole system consists of a network of cables or conductors which convey power from central station to the consumer's premises. The station bus-bars are fed by a number of generators (only two shown in the figure) running in parallel. From the bus-bars, the power is carried by many ***feeders*** which radiate to various parts of a city or locality. These feeders deliver power at certain points to a distributor which runs along the various streets. The points *FF*, as shown in the figure, are known as ***feeding points***. Power connections to the various consumers are given ***from this distributor and not directly from the feeder***. The wires which convey power from the distributor to the consumer's premises are known as ***service mains (S)***. Sometimes when there is only one distributor in a locality, several sub-distributors (*SD*) branching off from the distributor are employed and service mains are now connected to them instead of distributor as shown in the figure.

Obviously, a feeder is designed on the basis of its current-carrying capacity whereas the design of distributor is based on the voltage drop occurring in it.

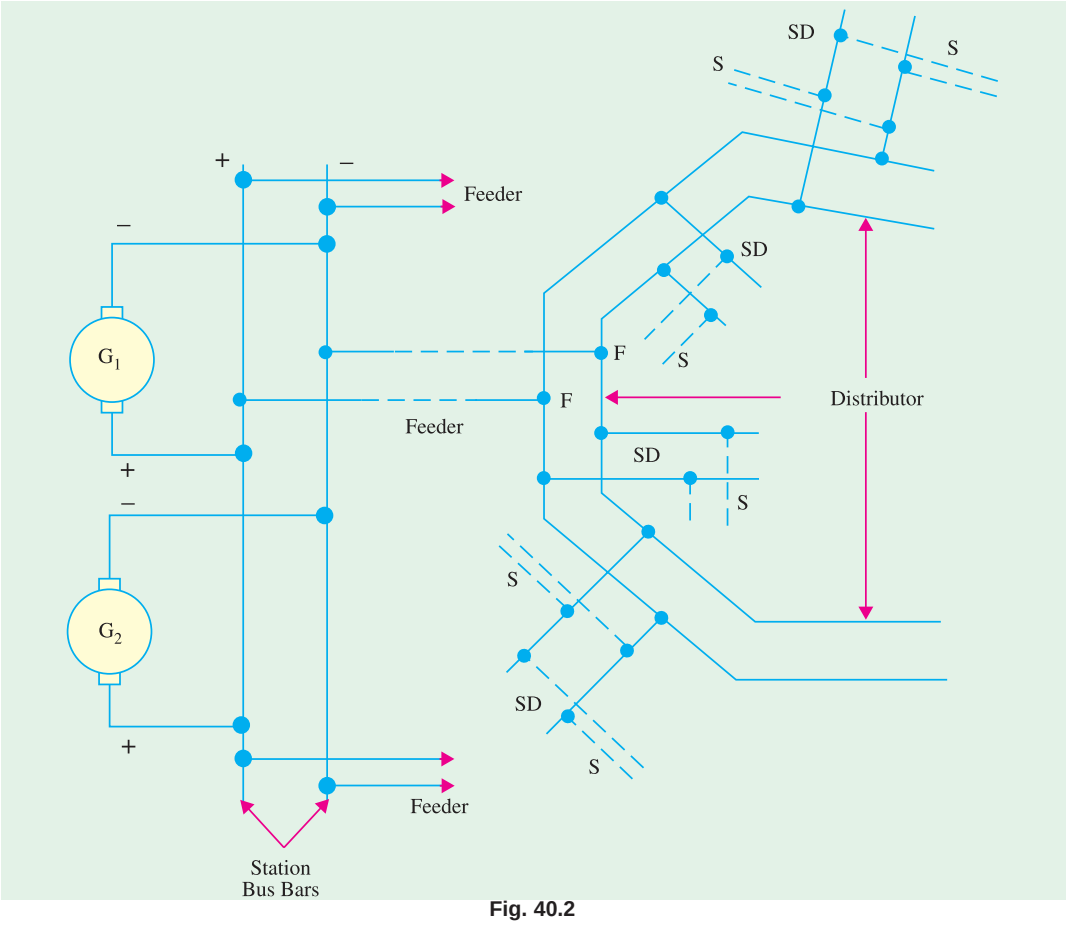


Fig. 40.2

40.2. Two-wire and Three-wire Systems

In d.c. systems, power may be fed and distributed either by ***(i) 2-wire system*** or ***(ii) 3-wire system***.

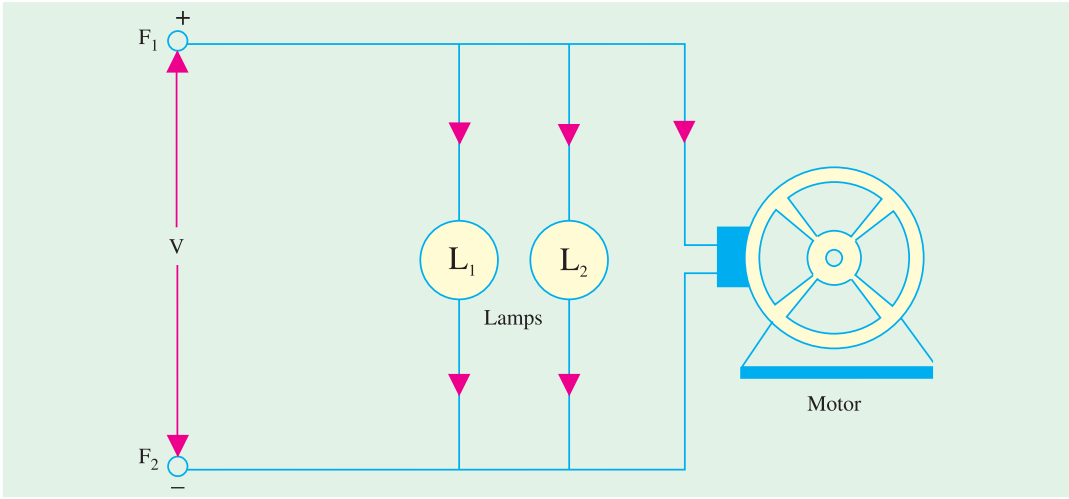


Fig. 40.3

In the 2-wire system, one is the outgoing or positive wire and the other is the return or negative wire. In the case of a 2-wire distributor, lamps, motors and other electrical apparatus are connected in parallel between the two wires as shown in Fig. 40.3. As seen, the potential difference and current have their maximum values at feeding points F_1 and F_2. The standard voltage between the conductors is 220 V.

The 2-wire system when used for transmission purposes, has much lower efficiency and economy as compared to the 3-wire system as shown later.

A 3-wire has not only a higher efficiency of transmission (Fig. 40.4) but when used for distribution purposes, it makes available two voltages at the consumer's end (Fig. 40.5). This 3-wire system consists of two '***outers***' and a ***middle*** or ***neutral*** wire which is earthed at the generator end. Its potential is midway between that of the outers *i.e.* if the p.d. between the outers is 460 V, then the p.d. of positive outer is 230 V ***above*** the neutral and that of negative outer is 230 V ***below*** the neutral. Motors requiring higher voltage are connected across the outers whereas lighting and heating circuits requiring less voltage are connected between any one of the outers and the neutral.

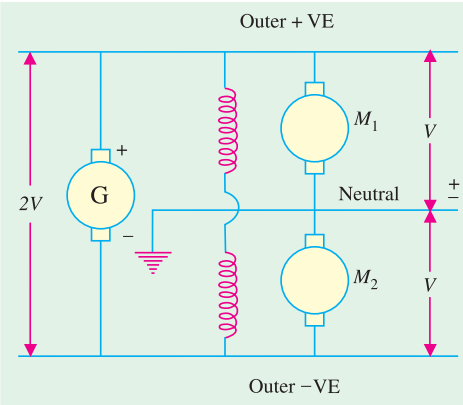


Fig. 40.4

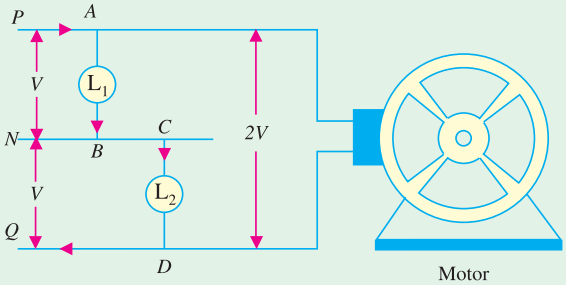


Fig. 40.5

The addition of the middle wire is made possible by the connection diagram shown in Fig. 40.4. G is the main generator which supplies power to the whole system. M_1 and M_2 are two identical shunt machines coupled mechanically with their armatures and shunt field winding joined in series across the outers. The junction of their armatures is earthed and the neutral wire is taken out from there.

40.3 Voltage Drop and Transmission Efficiency

Consider the case of a ***2-wire feeder*** (Fig. 40.6). AB is the sending end and CD the receiving end. Obviously, the p.d. at AB is higher than at CD. The difference in potential at the two ends is the potential drop or 'drop' in the cable. Suppose the transmitting voltage is 250 V, current in AC is 10 amperes, and resistance of each feeder conductor is 0.5 Ω, then drop in each feeder conductor is $10 \times 0.5 = 5$ volt and drop in both feeder conductor is $5 \times 2 = 10$ V.

$\therefore$ *P.d.* at Receiving end CD is = $250 - 10 = 240$ V

Input power at AB = $250 \times 10 = 2{,}500$ W

Output power at CD = $240 \times 10 = 2{,}400$ W

$\therefore$ power lost in two feeders = $2{,}500 - 2{,}400 = 100$W

The above power loss could also be found by using the formula

Power loss = $2I^2R = 2 \times 10^2 \times 0.5 = 100$ W

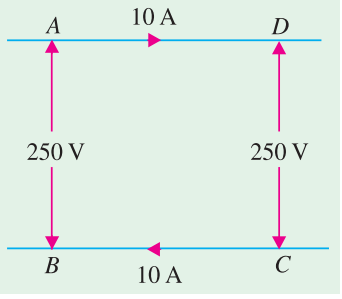


Fig. 40.6

The efficiency of transmission is defined, like any other efficiency, as the ratio of the output to input

$\therefore$ efficiency of transmission $= \dfrac{\text{power delivered by the line}}{\text{power received by the line}}$

In the present case, power delivered by the ***feeder*** is = 2500 W and power received by it as 2400 W.

$\therefore \quad \eta = 2400 \times 100/2500 = 96\%$

In general, if V_1 is the voltage at the sending end and V_2 at the receiving end and I the current delivered, then

$$\text{Input} = V_1 I, \qquad \text{output} = V_2 I \qquad \therefore \quad \eta = \frac{V_2 I}{V_1 I} = \frac{V_2}{V_1}$$

Now $V_2 = V_1 - \text{drop in both conductors}$

$= V_1 - IR$, where R is the resistance of both conductors

$$\therefore \quad \eta = \frac{V_1 - IR}{V_1} = 1 - \frac{IR}{V_1} \qquad \text{...(i)}$$

$$\text{or} \quad \%\ \text{efficiency} = 100 \times \left(1 - \frac{IR}{V_1}\right) = 100 - \left(\frac{IR}{V_1} \times 100\right)$$

Now, $(IR/V_1) \times 100$ represents the voltage drop in both conductors expressed as a percentage of the voltage at the sending end. It is known as percentage drop.

$\therefore \quad \%\ \eta = 100 - \%$ drop

In the present case total drop is 10 volt which expressed as percentage becomes $(10/250) \times 100$ = 4%. Hence % η = 100 – 4 = 96% **...as before**

It is seen from equation **(*i*)** above that for a given drop, transmission efficiency can be considerably increased by increasing the voltage at the transmitting end. *i.e.*, V_1. Moreover, the cross-section of copper in the cables is decreased in proportion to the increase in voltage which results in a proportionate reduction of the cost of copper in the cables.



A DC power distribution system consists of a network of cables or conductors which conveys power from central station to the consumer's premises.

The calculation of drop in a ***feeder*** is, as seen from above, quite easy because of the fact that current is constant throughout its length. But it is not so in the case of ***distributors*** which are tapped off at various places along their entire lengths. Hence, their different sections carry different currents over different lengths. For calculating the total voltage drop along the entire length of a distributor, following information is necessary.

(*i*) value of current tapped at each load point.

(*ii*) the resistance of each section of the distributor between tapped points.

Example 40.1. *A DC 2-wire feeder supplies a constant load with a sending-end voltage of 220 V. Calculate the saving in copper if this voltage is doubled with power transmitted remaining the same.*

Solution. Let l = length of each conductor in metre

σ = current density in A/m^2

P = power supplied in watts

(*i*) 220 V Supply

Current per feeder conductor $I_1 = P/220$

Area of conductor required $A_1 = I_1/\sigma = P/220\,\sigma$

Volume of Cu required for both conductors is

$$V_1 = 2A_1 l = \frac{2Pl}{220\sigma} = \frac{Pl}{110\sigma}$$

(*ii*) 440 V Supply

$$V_2 = \frac{Pl}{220\sigma}$$

$$\%\text{age saving in Cu} = \frac{V_1 - V_2}{V_1} \times 100 = \frac{\frac{Pl}{110\,\sigma} - \frac{Pl}{220\,\sigma}}{Pl/110\,\sigma} \times 100 = 50\%$$

40.4. Methods of Feeding a Distributor

Different methods of feeding a distributor are given below :

1. feeding at one end
2. feeding at both ends with equal voltages
3. feeding at both ends with unequal voltages
4. feeding at some intermediate point

In adding, the nature of loading also varies such as

(*a*) concentrated loading **(*b*)** uniform loading **(*c*)** combination of **(*a*)** and **(*b*)**.

Now, we will discuss some of the important cases separately.

40.5. D.C. Distributor Fed at One End

In Fig. 40.7 is shown one conductor AB of a distributor with concentrated loads and fed at one end.

Let i_1, i_2, i_3 etc. be the currents tapped off at points C, D, E and F and I_1, I_2, I_3 etc. the currents in the various sections of the distributor. Let r_1, r_2, r_3 etc. be the ohmic resistances of these various sections and ***R_1, R_2, R_3 etc. the total resistance from the feeding end A to the successive tapping points.***

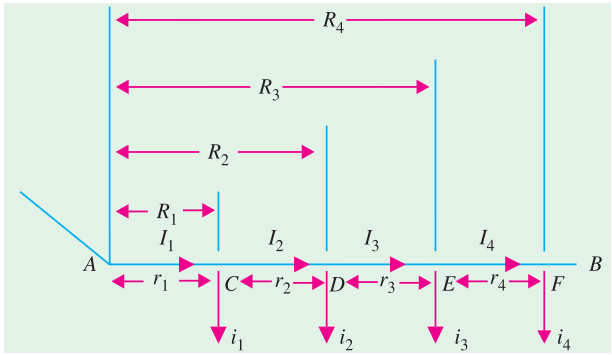


Fig. 40.7

Then, total drop in the distributor is

$$= r_1 I_1 + r_2 I_2 + r_3 I_3 + ...$$

Now $I_1 = i_1 + i_2 + i_3 + i_4 + ...; I_2 = i_2 + i_3 + i_4 + ...; I_3 = i_3 + i_4 + ...$

$\therefore$ $= r_1(i_1 + i_2 + i_3 + ...) + r_2\,(i_2 + i_3 + i_4 + ...) + r_3\,(i_3 + i_4 + ...)$

$= i_1 r_1 + i_2(r_1 + r_2) + i_3(r_1 + r_2 + r_3) + ... = i_1 R_1 + i_2 R_2 + i_3 R_3 + ...$

$=$ ***sum of the moments of each load current about feeding point A.***

1. Hence, the drop at ***the far end*** of a distributor fed at one end is given by the sum of the moments of various tapped currents about the feeding point *i.e.* $v = \Sigma\, i\, R$.

2. It follows from this that the total voltage drop is the same as that produced by a single load equal to the sum of the various concentrated loads, acting at the centre of gravity of the load system.

3. Let us find the drop at any intermediate point like E. The value of this drop is

$$= i_1 R_1 + i_2 R_2 + i_3 R_3 + R_3(i_4 + i_5 + i_6 + ...)^*$$

* The reader is advised to derive this expression himself.

$$= \begin{pmatrix}\text{sum of moments}\\ \text{upto } E\end{pmatrix} + \begin{pmatrix}\text{moment of load beyond}\\ E \text{ assumed acting at } E\end{pmatrix}$$

In general, the drop at any intermediate point is ***equal to the sum of moments of various tapped currents upto that point plus the moment of all the load currents beyond that point assumed to be acting at that point.***

The total drop over both conductors would, obviously, be twice the value calculated above.

40.6. Uniformly Loaded Distributor

In Fig. 40.8 is shown one conductor AB of a distributor fed at one end A and uniformly loaded with i amperes per unit length. Any convenient unit of length may be chosen *i.e.* a metre or 10 metres but at every such unit length, the load tapped is the same. Hence, let

i = current tapped off per unit length l = total length of the distributor

r = resistance per unit length of the distributor

Now, let us find the voltage drop at a point C (Fig. 40.9) which is at a distance of x units from feeding end A. The current at point C is $(il - ix) = i(l - x)$.

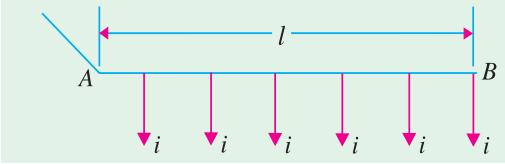


Fig. 40.8

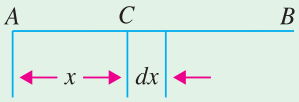


Fig. 40.9

Consider a small section of length dx near point C. Its resistance is (rdx). Hence, drop over length dx is

$$dv = i(l - x)(rdx) = (ilr - ixr)dx$$

The total drop up to point x is given by integrating the above quantity between proper limits.

$$\therefore \quad \int_0^v dx = \int_0^x (ilx - ixr)\,dx \qquad \therefore \quad v = ilrx - \frac{1}{2}irx^2 = ir\left(lx - \frac{x^2}{2}\right)$$

The drop at point B can be obtained by putting $x = l$ in the above expression.

$$\therefore \quad \text{drop at point } B = ir\left(l^2 - \frac{l^2}{2}\right) = \frac{irl^2}{2} = \frac{(i \times l) \times (r \times l)}{2} = \frac{1}{2}IR = I \times \frac{1}{2}R$$

where $i \times l = I$ – total current entering at point A; $r \times l = R$–the total resistance of distributor AB.

Total drop in the distributor $AB = \frac{1}{2}IR$

(i) It follows that in a uniformly loaded distributor, total drop is equal to that produced by the whole of the load assumed concentrated at the middle point.

(ii) Suppose that such a distributor is fed at both ends A and B with ***equal voltages***. In that case, the point of minimum potential is obviously the middle point. We can thus imagine as if the distributor were cut into two at the middle point, giving us two uniformly-loaded distributors each fed at one end with equal voltages. The resistance of each is $R/2$ and total current fed into each distributor is $I/2$. Hence, drop at the middle point is

$$= \frac{1}{2} \times \left(\frac{I}{2}\right) \times \left(\frac{R}{2}\right) = \frac{1}{8}IR$$

It is 1/4th of that of a distributor fed at one end only. The advantage of feeding a distributor at both ends, instead of at one end, is obvious.

The equation of drop at point C distant x units from feeding point $A = irlx - \frac{1}{2}irx^2$ shows that the diagram of drop of a uniformly-loaded distributor fed at one end is a parabola.

Example 40.2. *A uniform 2-wire d.c. distributor 200 metres long is loaded with 2 amperes/metre. Resistance of single wire is 0.3 ohm/kilometre. Calculate the maximum voltage drop if the distributor is fed (a) from one end (b) from both ends with equal voltages.*

(Elect. Technology ; Bombay Univ.)

Solution. **(*a*)** Total resistance of the distributor is $R = 0.3 \times (200/1000) = 0.06$ W

Total current entering the distributor is $I = 2 \times 200 = 400$ A

Total drop on the whole length of the distributor is (Art. 40.6)

$$= \frac{1}{2} IR = \frac{1}{2} \times 400 \times 0.06 = \mathbf{12\ V}$$

(*b*) Since distributor is fed from both ends, total voltage drop is

$$= \frac{1}{8} IR = \frac{1}{8} \times 400 \times 0.06 = \mathbf{3\ V}$$

As explained in Art. 40.6 the voltage drop in this case is one-fourth of that in case **(*a*)** above.

Example 40.3. *A 2-wire d.c. distributor AB is 300 metres long. It is fed at point A. The various loads and their positions are given below :*

At point	*distance from A in metres*	*concentrated load in A*
C	*40*	*30*
D	*100*	*40*
E	*150*	*100*
F	*250*	*50*

If the maximum permissible voltage drop is not to exceed 10 V, find the cross-sectional area of the distributor. Take $\rho = (1.78 \times 10^{-8})$ *Ω-m.* **(Electrical Power-I ; Bombay Univ.)**

Solution. The distributor along with its tapped currents is shown in Fig. 40.10.

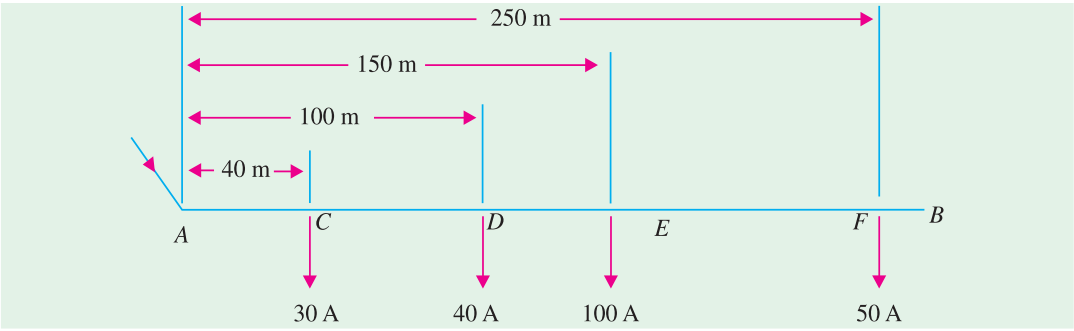


Fig. 40.10

Let 'A' be cross sectional area of distributor

As proved in Art. 40.5 total drop over the distributor is

$$v = i_1R_1 + i_2R_2 + i_3R_3 + i_4R_4 \quad \text{—for one conductor}$$

$$= 2(i_1R_1 + i_2R_2 + i_3R_3 + i_4R_4) \quad \text{—for two conductors}$$

where

R_1 = resistance of $AC = 1.78 \times 10^{-8} \times 40/A$

R_2 = resistance of $AD = 1.78 \times 10^{-8} \times 100/A$

R_3 = resistance of $AE = 1.78 \times 10^{-8} \times 150/A$

R_4 = resistance of $AF = 1.78 \times 10^{-8} \times 250/A$

Since $v = 10$ V, we get

$$10 = \frac{2 \times 1.78 \times 10^{-8}}{A} (30 \times 40 + 40 \times 100 + 100 \times 150 + 50 \times 250)$$

$$= \frac{2 \times 1.78 \times 10^{-8}}{A} \times 32,700$$

$$\therefore \quad A = 3.56 \times 327 \times 10^{-7} \text{ m}^2 = \mathbf{1,163 \text{ cm}^2}$$

40.7. Distributor Fed at Both Ends with Equal Voltages

It should be noted that in such cases

(i) the maximum voltage drop must always occur at one of the load points and

(ii) if both feeding ends are at the same potential, then the voltage drop between each end and this point must be the same, which in other words, means that the sum of the moments about ends must be equal.

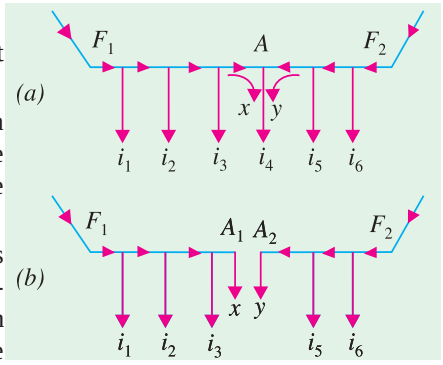


Fig. 40.11

In Fig. 40.11 (*a*) is shown a distributor fed at two points F_1 and F_2 with equal voltages. The potential of the conductor will gradually fall from F_1 onwards, reach a minimum value at one of the tapping, say, A and then rise again as the other feeding point F_2 is approached. All the currents tapped off between points F_1 and A will be supplied from F_1 while those tapped off between F_2 and A will be supplied from F_2. The current tapped at point A itself will, in general, be partly supplied by F_1 and partly by F_2. Let the values of these currents be x and y respectively. If the distributor were actually cut off into two at A—the point of minimum voltage, with x amperes tapped off from the left and y amperes tapped off from the right, then potential distribution would remain unchanged, showing that we can regard the distributor as consisting of two separate distributors each fed from one end only, as shown in Fig. 40.11 (*b*). The drop can be calculated by locating point A and then values of x and y can be calculated.

This can be done with the help of the following pair of equations :

$$x + y = i_4 \quad \text{and} \quad \text{drop from } F_1 \text{ to } A_1 = \text{drop from } F_2 \text{ to } A_2.$$

Example 40.4. *A 2-wire d.c. distributor $F_1 F_2$ 1000 metres long is loaded as under :*

Distance from F_1 (in metres) :	100	250	500	600	700	800	850	920
Load in amperes :	20	80	50	70	40	30	10	15

The feeding points F_1 and F_2 are maintained at the same potential. Find which point will have the minimum potential and what will be the drop at this point ? Take the cross-section of the conductors as 0.35 cm^2 and specific resistance of copper as (1.764×10^{-6}) Ω-cm.

Solution. The distributor along with its tapped currents is shown in Fig. 40.12. The numbers along the distributor indicate length in metres.

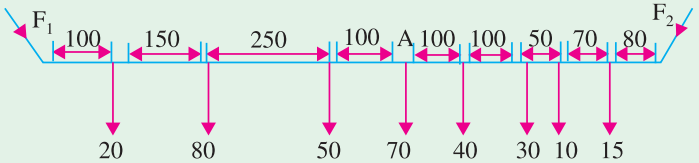


Fig. 40.12

The easiest method of locating the point of minimum potential is to take the moments about the two ends and then by comparing the two sums make a guess at the possible point. The way it is done is as follows :

It is found that 4th point from F_1 is the required point *i.e.* point of minimum potential. Using the previous two equations, we have

$$x + y = 70 \quad \text{and} \quad 47{,}000 + 600x = 20{,}700 + 400y$$

Solving the two equations, we get $x = 1.7$ A, $y = 70 - 1.7 = 68.3$ A

Drop at A per conductor = $47{,}000 + 600 \times 1.7 = 48{,}020$ ampere-metre.

$$\text{Resistance/metre} = \frac{\rho l}{A} = \frac{1.764 \times 10^{-8} \times 1}{0.35 \times 10^{-4}} = 50.4 \times 10^{-5}\ \Omega/\text{m}$$

Hence, drop per conductor $= 48{,}020 \times 50.4 \times 10^{-5} = 24.2$ V

Reckoning both conductors, the drop at A is = **48.4 V**

Moments about F_1 in ampere-metres	*Sum*	*Moments about F_2 in ampere-metres*	*Sum*
$20 \times 100 = 2{,}000$	2,000	$15 \times 80 = 1{,}200$	1,200
$80 \times 250 = 20{,}000$	22,000	$10 \times 150 = 1{,}500$	2,700
$50 \times 500 = 25{,}000$	47,000	$30 \times 200 = 6{,}000$	8,700
		$40 \times 300 = 12{,}000$	20,700
		$70 \times 400 = 28{,}000$	48,700

Alternative Solution

The alternative method is to take the total current fed at one end, say, F_1 as x and then to find the current distribution as shown in Fig. 40.13 (*a*). The drop over the whole distributor is equal to the sum of the products of currents in the various sections and their resistances. For a distributor fed at both ends with ***equal voltages***, this drop equals zero. In this way, value of x can be found.

Resistance per metre single = $5.04 \times 10^{-4}\ \Omega$

Resistance per metre double = $10.08 \times 10^{-4}\ \Omega$

$\therefore$ $10.08 \times 10^{-4}\,[100x + 150(x - 20) + 250\,(x - 100) + 100\,(x - 150) + 100\,(x - 200) + 100(x - 260) + 50(x - 290) + 70(x - 300) + 80\,(x - 315)] = 0$ or $1000x = 151{,}700$

$\therefore$ $x = 151.7$ A

This gives a current distribution as shown in Fig. 40.13 (*b*). Obviously, point A is the point of minimum potential.

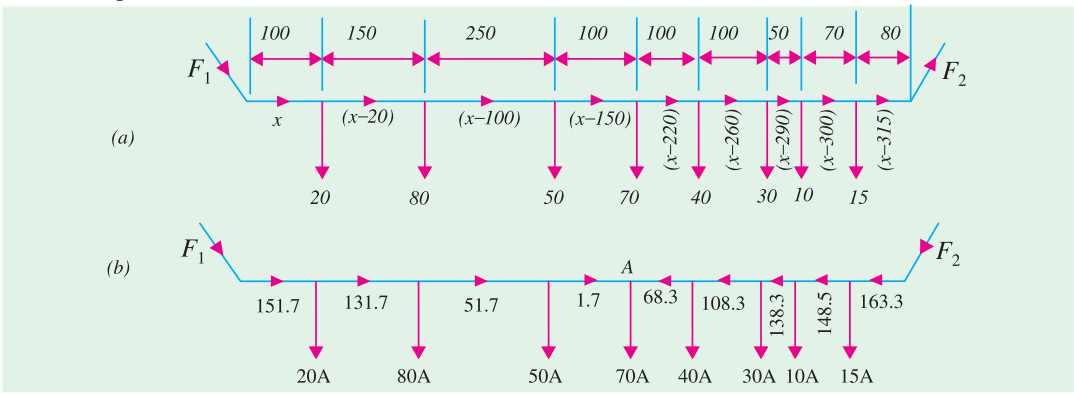


Fig. 40.13

Drop at A (considering both conductors) is

$= 10.08 \times 10^{-4}\,(100 \times 151.7 + 150 \times 131.7 + 250 \times 51.7 + 100 \times 1.7)$

$= 10.08 \times 10^{-4} \times 48{,}020 =$ **48.4 V** —**as before.**

Example 40.5. *The resistance of a cable is 0.1 Ω per 1000 metre for both conductors. It is loaded as shown in Fig. 40.14 (a). Find the current supplied at A and at B. If both the ends are supplied at 200 V.* **(Electrical Technology-II, Gwalior Univ.)**

Solution. Let the current distribution be as shown in Fig. 40.14 (*b*). Resistance for both conductors = 0.1/1000=10^{-4} Ω/m. The total drop over the whole cable is zero because it is fed at both ends by equal voltages.

$\therefore \quad 10^{-4}[500\,i + 700\,(i - 50) + 300\,(i - 150) + 250\,(i - 300)] = 0$

$i = \mathbf{88.6\ A}$

This gives the current distribution as shown in Fig. 40.14 (*c*).

Current in AC = 88.6 A

Current in CD = (88.6 – 50) = 38.6 A

Current in DE = (38.6 – 100) = – 61.4 A

Current in EB = (–61.4 – 150) = –211.4 A

Hence, current entering the cable at point B is = **211.4 A.**

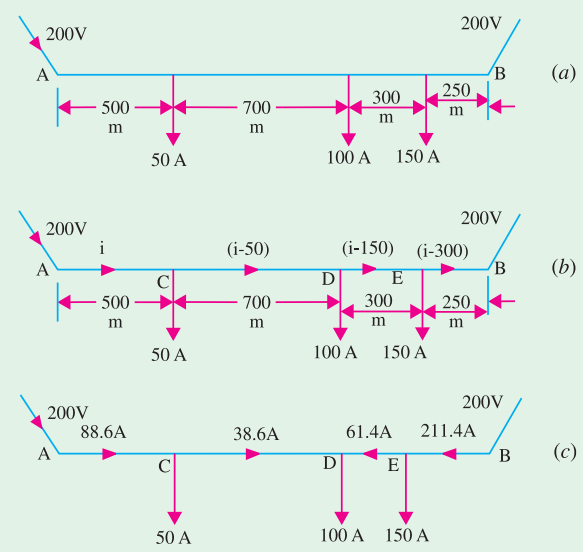


Fig. 40.14

Example 40.6. *The resistance of two conductors of a 2-conductor distributor shown in Fig. 39.15 is 0.1 Ω per 1000 m for both conductors. Find* ***(a)*** *the current supplied at A* ***(b)*** *the current supplied at B* ***(c)*** *the current in each section* ***(d)*** *the voltages at C, D and E. Both A and B are maintained at 200 V.* **(Electrical Engg. Grad. I.E.T.E.)**

Solution. The distributor along with its tapped currents is shown in Fig. 40.15. Let the current distribution be as shown. Resistance per metre double is = 0.1/1000 = 10^{-4} Ω.

The total drop over the whole distributor is zero because it is fed at both ends by equal voltages.

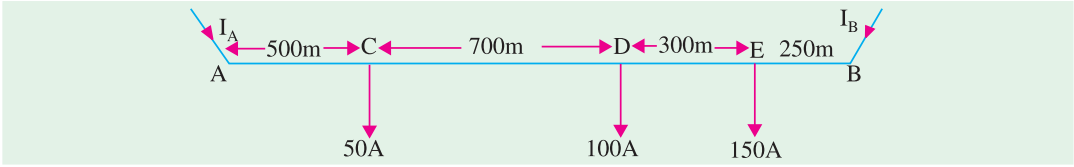


Fig. 40.15

$\therefore \quad 10^{-4}[500i + 700\,(i - 50) + 300\,(i - 150) + 250\,(i - 300)] = 0$

or $\quad 1750i = 155{,}000 \quad$ or $\quad i = 88.6$ A

This gives the current distribution shown in Fig. 40.16.

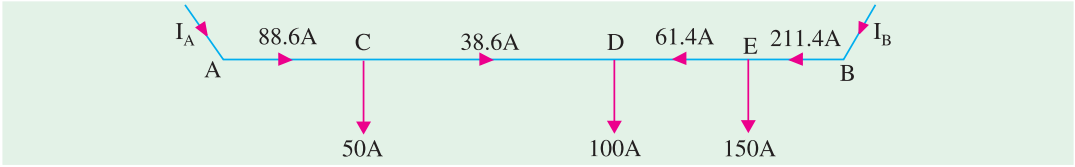


Fig. 40.16

(a) $I_A = \mathbf{88.6\ A}$ ***(b)*** $I_B = \mathbf{-211.4\ A}$

(c) Current in section AC = **88.6 A**; Current in CD = (88.6 – 50) = **38.6 A**

Current in section DE = (31.6 – 100) = **– 61.4 A**

Current in EB = (– 61.4 – 150) = **– 211.4 A**

(d) Drop over AC = $10^{-4} \times 500 \times 88.6$ = 4.43 V; Drop over CD = $10^{-4} \times 700 \times 38.6$ = **2.7 V**

Drop over $DE = 10^{-4} \times 300 \times -61.4 = -1.84$ V $\therefore$ Voltage at $C = 200 - 4.43 =$ **195.57 V**

Voltage at $D = (195.57 - 2.7) =$ **192.87 V**, Voltage at $E = 192.87 - (-1.84) =$ **194.71 V**

Example 40.7. *A 200 m long distributor is fed from both ends A and B at the same voltage of 250 V. The concentrated loads of 50, 40, 30 and 25 A are coming on the distributor at distances of 50, 75, 100 and 150 m respectively from end A. Determine the minimum potential and locate its position. Also, determine the current in each section of the distributor. It is given that the resistance of the distributor is 0.08 Ω per 100 metres for go and return.*

(Electric Power-I (Trans & Dist) Punjab Univ. 1993)

Solution. As shown in Fig. 40.17, let current fed at point A be i. The currents in various sections are as shown. Resistance per metre of the distributor (go and return) is 0.08/100 = 0.0008 Ω.

Since the distributor is fed at both ends with equal voltages, total drop over it is zero.

Hence, $0.0008\,[50i + 25(i - 50) + 25(i - 90) + 50(i - 120) + 50(i - 145)] = 0$

or $200i = 16{,}750$ or $i = 83.75$ A

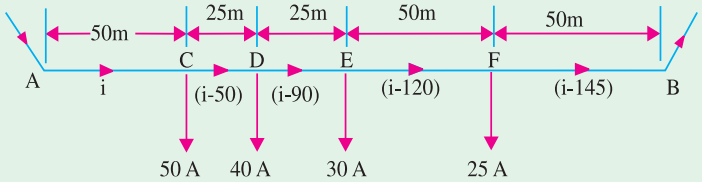


Fig. 40.17

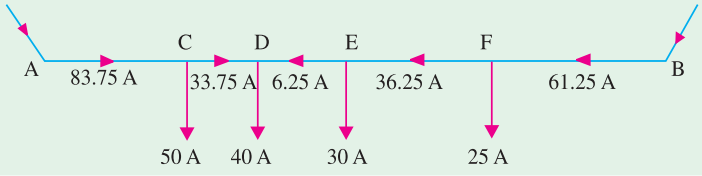


Fig 40.18

The actual current distribution is shown in Fig. 40.18. Obviously, point D is the point of minimum potential.

Drop at point D is $= 0.0008(50 \times 83.75 + 25 \times 33.75) = 4.025$ V

$\therefore$ minimum potential $= 250 - 4.025 =$ **245.98 V**

40.8. Distributor Fed at Both Ends with Unequal Voltages

This case can be dealt with either by taking moments (in amp-m) about the two ends and then making a guess about the point of minimum potential or by assuming a current x fed at one end and then finding the actual current distribution.

Consider the case already discussed in Ex. 40.4. Suppose, there is a difference of ν volts between ends F_1 and F_2 with F_1 being at higher potential. Convert ν volts into ampere-metres with the help of known value of resistance/metre. Since F_2 is at a lower potential, these ampere-metres appear in the coloumn for F_2 as initial drop.

If, for example, ν is 4 volts, then since resistance/metre is 5.04×10^{-4} Ω, initial ampere-metres for F_2 are

$$= \frac{4 \times 10^4}{5.04} = 7{,}938 \text{ amp-metres}$$

The table of respective moments will be as follows :

Moments about F_1	*Sum*	*Moments about F_2*	*Sum*
$20 \times 100 = 2000$	2,000	initial = 7,938	
$80 \times 250 = 20,000$	22,000	$15 \times 80 = 1,200$	9,138
$50 \times 500 = 25,000$	47,000	$10 \times 150 = 1,500$	10,638
		$30 \times 200 = 6,000$	16,638
		$40 \times 300 = 12,000$	28,638
		$70 \times 400 = 28,000$	56,638

As seen, the dividing point is the same as before.

$\therefore \quad x + y = 70$ and $47,000 + 600x = 28,638 + 400y$

Solving for x and y, we get $x = 9.64$ A and $y = 60.36$ A

After knowing the value of x, the drop at A can be calculated as before.

The alternative method of solution is illustrated in Ex. 40.12.

40.9. Uniform Loading with Distributor Fed at Both Ends

Consider a distributor PQ of length l units of length, having resistance per unit length of r ohms and with loading per unit length of i amperes. Let the difference in potentials of the two feeding points be v volts with Q at the lower potential. The procedure for finding the point of minimum potential is as follows:

Let us assume that point of minimum potential M is situated at a distance of x units from P. Then drop from P to M is $= irx^2/2$ volts (Art. 40.6)

Since the distance of M from Q is $(l - x)$, the drop from Q to M is $= \dfrac{ir\,(l-x)^2}{2}$ volt.

Since potential of P is greater than that of Q by v volts,

$$\therefore \quad \frac{irx^2}{2} = \frac{ir\,(l-x)^2}{2} + v \quad \text{or} \quad x = \frac{l}{2} + \frac{v}{irl}$$



A circuit-board as shown above uses DC current

40.10. Concentrated and Uniform Loading with Distributor Fed at One End

Such cases are solved in two stages. First, the drop at any point due to concentrated loading is found. To this add the voltage drop due to uniform loading as calculated from the relation.

$$ir\left(lx - \frac{x^2}{2}\right)$$

As an illustration of this method, please look up Ex. 40.9 and 40.13.

Example 40.8. *Each conductor of a 2-core distributor, 500 metres long has a cross-sectional area of 2 cm^2. The feeding point A is supplied at 255 V and the feeding point B at 250 V and load currents of 120 A and 160 A are taken at points C and D which are 150 metres and 350 metres respectively from the feeding point A. Calculate the voltage at each load. Specific resistance of copper is 1.7 $\times$ 10^{-6} Ω-cm.* **(Elect. Technology-I, Bombay Univ.)**

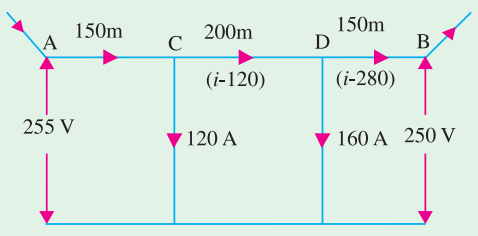


Fig. 40.19

Solution. $\rho = 1.7 \times 10^{-6}$ Ω-cm

$= 1.7 \times 10^{-8}$ Ω.m

Resistance per metre single $= \rho \frac{l}{A} = 1.7 \times 10^{-8}/2 \times 10^{-4} = 85 \times 10^{-6}$ m

Resistance per metre double $= 2 \times 85 \times 10^{-6} = 17 \times 10^{-5}$ Ω

The dorp over the whole distributor = 255 – 250 = 5 V

$\therefore \quad 17 \times 10^{-5} [150i + 200 (i - 120) + 150 (i - 280)] = 5$

$\therefore \quad i = 190.8$ A — **(Fig. 40.19)**

Current in section AC = 190.8 A; Current in section CD = (190.8 – 120) = 70.8 A

Current in section DB = (190.8 – 280) = – 89.2 A (from B to D)

Voltage at point C = 255 – drop over AC

$= 255 - (17 \times 10^{-5} \times 150 \times 190.8) =$ **250.13 V**

Voltage at point D = 250 – drop over BD

$= 250 - (17 \times 10^{-5} \times 150 \times 89.2) =$ **247.72 V**

Example 40.9. *A 2-wire distributor 500 metres long is fed at P at 250 V and loads of 40A, 20A, 60A, 30A are tapped off from points A, B, C and D which are at distances of 100 metres, 150 metres, 300 metres and 400 metres from P respectively. The distributor is also uniformly loaded at the rate of 0.1 A/m. If the resistance of the distributor per metre (go and return) is 0.0005 Ω, calculate the voltage at* ***(i)*** *point Q and* ***(ii)*** *point B.*

Solution. First, consider drop due to concentrated load only.

Drop in $PA = 150 \times (100 \times 0.0005) = 7.5$ V

Drop in $AB = 110 \times (50 \times 0.0005) = 2.75$ V

Drop in $BC = 90 \times (150 \times 0.0005) = 6.75$ V

Drop in $CD = 30 \times (100 \times 0.0005) = 1.5$ V

$\therefore$ total drop due to this load = 18.5 V

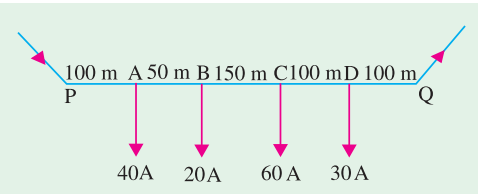


Fig. 40.20

Now, let us consider drop due to uniform load only.

Drop over length $l = ir\frac{l^2}{2} = 0.1 \times 0.0005 \times 500^2/2 = 6.25$ V

(i) $\therefore$ potential of point Q = 250 – (18.5 + 6.25) = **225.25 V**

(ii) Consider point B at the distance of 150 metres from P.

Drop due to concentrated loading = 7.5 + 2.75 = 10.25 V.

Drop due to uniform loading = $ir\,(lx - x^2/2)$; Here l = 500 m ; x = 150 m

$\therefore \quad \text{drop} = 0.1 \times 0.0005 \left(500 \times 150 - \frac{150^2}{2} \right) = 3.1875$ V

$\therefore$ potential of point P = 250 – (10.25 + 3.1875) = **13.44 V**

Example 40.10. *A distributor AB is fed from both ends. At feeding point A, the voltage is maintained at 236 V and at B at 237 V. The total length of the distributor is 200 metres and loads are tapped off as under :*

(i) *20 A at 50 metres from A* ***(ii)*** *40 A at 75 metres from A*

(iii) *25 A at 100 metres from A* ***(iv)*** *30 A at 150 metres from A*

The resistance per kilometre of one conductor is 0.4Ω. Calculate the currents in the various sections of the distributor, the minimum voltage and the point at which it occurs.

(Electrical Technology, Calcutta Univ.)

Solution. The distributor along with currents in its various sections is shown in Fig. 40.21.

Resistance/metre single = $0.4/1000 = 4 \times 10^{-4}\ \Omega$

Resistance/metre double = $8 \times 10^{-4}\ \Omega$

Voltage drop on both conductors of 200-metre long distributor is

$$= 8 \times 10^{-4} [50i + 26\,(i - 20) + 25(i - 60) + 50(i - 85) + 50(i - 115)]$$

$$= 8 \times 10^{-4}\,(200i - 12{,}000) \text{ volt}$$

This drop must be equal to the potential difference between A and B.

$$\therefore \quad 8 \times 10^{-4}\,(200i - 12{,}000) = 236 - 237 = -1$$

$$\therefore \quad i = 53.75 \text{ A}$$

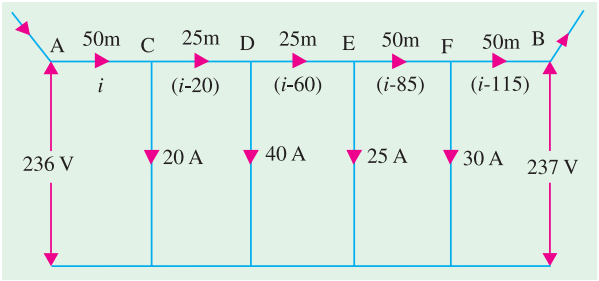


Fig. 40.21

Current in section AC = **53.75 A**

Current in section CD = 53.75 – 20 = **33.75 A**

Current in section DE = 53.75 – 60 = **– 6.25 A**

Current in section EF = 53.75 – 85 = **– 31.25 A**

Current in section FB = 53.75 – 115 = **– 61.25 A**

The actual current distribution is as shown in Fig. 40.22.

Obviously, minimum voltage occurs at point D *i.e.* 75 ***metre from point*** A (or 125 m from B)

Voltage drop across both conductors of the distributor over the length AD is

$$= 8 \times 10^{-4}\,(50 \times 53.75 + 25 \times 33.75) = 2.82 \text{ V}$$

$\therefore$ potential of point

D = 236 – 2.82 = **233.18 V**

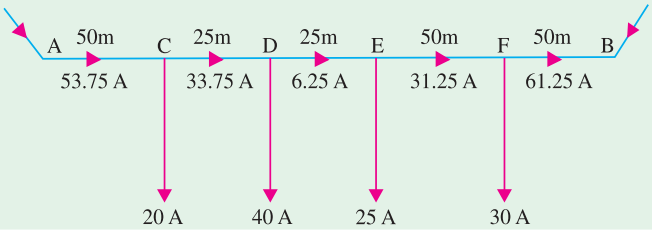


Fig. 40.22

Example 40.11. *A distributor cable AB is fed at its ends A and B. Loads of 12, 24, 72 and 48 A are taken from the cable at points C, D, E and F. The resistances of sections AC, CD, DE, EF and FB of the cable are 8, 6, 4, 10 and 5 milliohm respectively (for the go and return conductors together).*

The p.d. at point A is 240 V, the p.d. at the load F is also to be 240 V. Calculate the voltage at the feeding point B, the current supplied by each feeder and the p.d.s. at the loads C,D and E.

(Electrical Technology ; Utkal Univ.)

Solution. Let the current fed at the feeding point A be i. The current distribution in various sections becomes as shown in Fig. 40.23.

Voltage drop on both sides of the distributor over the section AF is

$$= [8i + 6(i - 12) + 4(i - 36) + 10(i - 108)] \times 10^{-3} = (28i - 1296) \times 10^{-3} \text{ V}$$

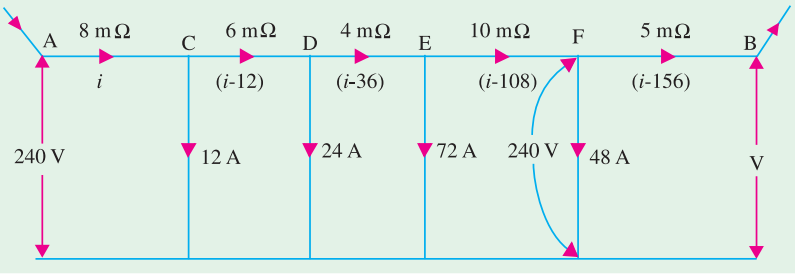


Fig. 40.23

Since points A and F are at the same potential, the p.d. between A and F is zero.

$\therefore \quad (28i - 1296) \times 10^{-3} = 0 \quad$ or $\quad i = 46.29$ A

Current in section $AC = 46.29$ A

Current in section $CD = 46.29 - 12 = 34.29$ A

Current in section $DE = 46.29 - 36 = 10.29$ A

Current in section $EF = 46.29 - 108 = -61.71$ A

Current in section $FB = 46.29 - 156 = -109.71$ A

The actual current distribution is as shown in Fig. 40.24.

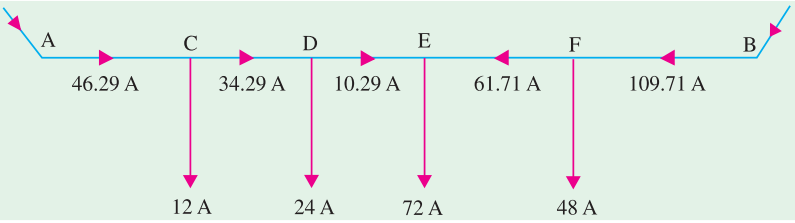


Fig. 40.24

Current applied by feeder at point A is 46.29 A and that supplied at point B is 109.71 A.

Voltage at feeding point $B = 240 -$ drop over $FB = 240 - (-5 \times 10^{-3} \times 109.71) =$ **240.55 V.**

Voltage at point $C = 240 -$ drop over $AC = 240 - (8 \times 10^{-3} \times 46.29) =$ **239.63 V**

Voltage at point $D = 239.63 -$ drop over $CD = 239.63 - (6 \times 10^{-3} \times 34.29) =$ **239.42 V**

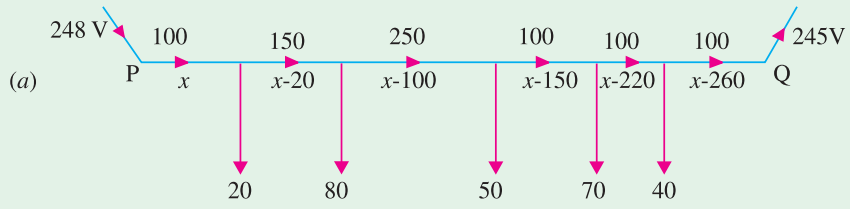
Voltage at point $E = 239.42 -$ drop over $DE = 239.42 - (4 \times 10^{-3} \times 10.29) =$ **239.38 V**

Example 40.12. *A two-wire, d.c. distributor PQ, 800 metre long is loaded as under :*

Distance from P (metres) :	*100*	*250*	*500*	*600*	*700*
Loads in amperes :	*20*	*80*	*50*	*70*	*40*

The feeding point at P is maintained at 248 V and that at Q at 245 V. The total resistance of the distributor (lead and return) is 0.1 Ω. Find ***(a)*** *the current supplied at P and Q and* ***(b)*** *the power dissipated in the distributor.*

Solution. As shown in Fig. 40.25 (*a*), let x be the current supplied from end P. The other currents in the various sections of the distributor are as shown in the figure.



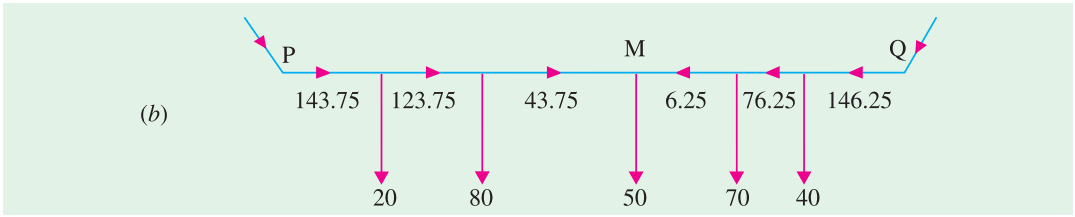


Fig. 40.25

Now, the total drop over PQ should equal the potential difference between ends P and Q *i.e.*, = 248 – 245 = 3.

Resistance/metre of both conductors = 0.1/800 = 1/8000 Ω

$$\therefore \quad \frac{1}{8000}[100x + 150(x - 20) + 250(x - 100) + 100(x - 150) + 100(x - 220) + 100(x - 246)] = 3$$

or $800x = 115{,}000 \quad \therefore \quad x = 143.75$ A

The actual distribution is shown in Fig. 40.25 (b) from where it is seen that point M has the minimum potential.

$$I_P = \mathbf{143.75\ A.} \qquad I_Q = \mathbf{116.25\ A}$$

$$\text{Power loss} = \Sigma I^2R = \frac{1}{8000}[143.75^2 \times 100 + 123.75^2 \times 150 + 43.75^2 \times 250 +$$

$$6.25^2 \times 100 + 76.25^2 \times 100 + 116.25^2 \times 100] = \mathbf{847.3\ W}$$

Example 40.13. *The two conductors of a d.c. distributor cable 1000 m long have a total resistance of 0.1 Ω. The ends A and B are fed at 240 V. The cable is uniformly loaded at 0.5 A per metre length and has concentrated loads of 120 A, 60 A, 100 A and 40 A at points distant 200, 400, 700 and 900 m respectively from the end A. Calculate* ***(i)*** *the point of minimum potential on the distributor* ***(ii)*** *the value of minimum potential and* ***(iii)*** *currents fed at the ends A and B.*

(Power System-I, AMIE, Sec. B, 1993)

Solution. Concentrated loads are shown in Fig. 40.26. Let us find out the point of minimum potential and currents at points A and B.

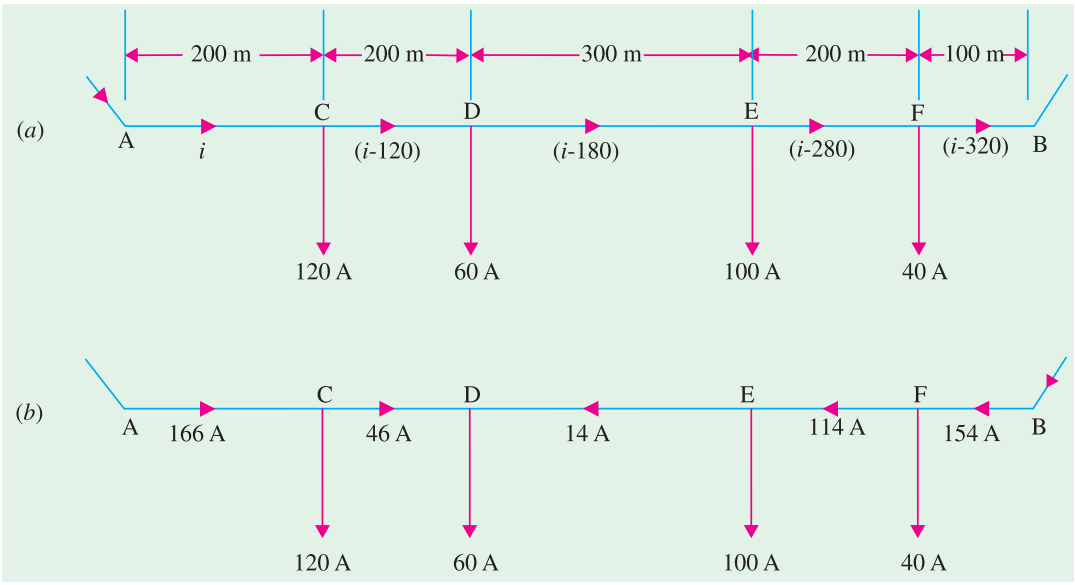


Fig. 40.26

It should be noted that location of point of minimum potential is not affected by the uniformly-spread load of 0.5 A/m. In fact, let us, for the time being, imagine that it is just not there. Then, assuming i to be the input current at A, the different currents in various sections are as shown. Since points A and B are fed at equal voltages, total drop over the distributor is zero. Distributor resistance per metre length (go and return) is $0.1/1000 = 10^{-4}\ \Omega$.

$\therefore \quad 10^{-4}\,[200i + 200(i - 120) + 300(i - 180) + 200(i - 280) + 100(i - 320)] = 0$

or $\qquad 1000i = 166{,}000 \qquad \therefore \quad i = 166$ A

Actual current distribution is shown in Fig. 40.26 (b) from where it is seen that point D is the point of minimum potential. The uniform load of 0.5 A/m upto point D will be supplied from A and the rest from point B.

Uniform load from A to $D = 400 \times 0.5 = 200$ A. Hence, $I_A = 166 + 200 =$ **366 A.**

Similarly, $\qquad I_B = 154 + (600 \times 0.5) =$ **454 A.**

Drop at D due to concentrated load is $= 10^{-4}\,(166 \times 200 + 46 \times 200) =$ **4.24 V.**

Drop due to uniform load can be found by imagining that the distributor is cut into two at point D so that AD can be looked upon as a distributor fed at one end and loaded uniformly. In that case, D becomes the other end of the distributor.

$\therefore$ drop at D due to uniform load (Art. 40.6)

$$= irl^2/2 = 0.5 \times 10^{-4} \times 400^2/2 = 4 \text{ V}$$

$\therefore$ total drop at $D = 4.24 + 4 = 8.24$ V $\quad \therefore$ potential of $D = 240 - 8.24 =$ **231.76 V**

Example 40.14. *It is proposed to lay out a d.c. distribution system comprising three sections—the first section consists of a cable from the sub-station to a point distant 800 metres from which two cables are taken, one 350 metres long supplying a load of 22 kW and the other 1.5 kilometre long and supplying a load of 44 kW. Calculate the cross-sectional area of each cable so that the total weight of copper required shall be minimum if the maximum drop of voltage along the cable is not to exceed 5% of the normal voltage of 440 V at the consumer's premises. Take specific resistance of copper at working temperature equal to 2 μ Ω-cm.*

Solution. Current taken from 350-m section is $I_1 = 22{,}000/440 = 50$ A

Current taken from 1.5 km section, $I_2 = 44{,}000/440 = 100$ A

$\therefore$ Current in first section $\quad I = 100 + 50 = 150$ A

Let $\qquad V =$ voltage drop across first section ; $R =$ resistance of the first section

$\qquad A =$ cross-sectional area of the first section

Then $\qquad R = V/I = V/150\ \Omega$

Now, $\qquad A = \dfrac{\rho l}{R} = \rho l \dfrac{I}{V} = 80{,}000 \times 2 \times 10^{-6} \times 150/V = 24/V \text{cm}^2$

Now, $\quad$ maximum allowable drop $= 5\%$ of $440 = 22$ V

$\therefore$ voltage drop along other sections $= (22 - V)$ volt

Hence, cross-sectional area of 350-m section is

$$A_1 = 35{,}000 \times 2 \times 10^{-6} \times 50/(22 - V) = 3.5/(22 - V)\ \text{cm}^2$$

Also, cross-sectional area of 1500-m section is

$$A_2 = 150{,}000 \times 2 \times 10^{-6} \times 100/(22 - V) = 30/(22 - V)\ \text{cm}^2$$

Now, total weight of copper required is proportional to the total volume.

$\therefore \qquad W = K[(800 \times 24/V) + 350 \times 3.5/(22 - V) + 1500 \times 30/(22 - V)]$

$$= K[1.92/V + 4.62/(22 - V)] \times 10^4$$

Weight of copper required would be minimum when $dW/dV = 0$

$$\therefore \quad \frac{dW}{dV} = K\left[\frac{-1.92}{V^2} + \frac{4.62}{(22-V)^2}\right] \times 10^4 = 0$$

$$\text{or} \quad \frac{1.92}{V^2} = \frac{4.62}{(22-V)^2} \quad \text{or} \quad (22-V)^2 = 2.4\,V^2$$

$$\text{or} \quad V = 22/2.55 = \mathbf{8.63 \text{ volt}} \qquad \therefore \quad A = 24/8.63 = \mathbf{2.781 \text{ cm}^2}$$

$$A_1 = 3.5/(22-8.63) = \mathbf{0.2618 \text{ cm}^2} \qquad A_2 = 30/(22-8.63) = \mathbf{2.246 \text{ cm}^2}$$

Example 40.15. *A d.c. two-wire distributor AB is 450 m long and is fed at both ends at 250 volts. It is loaded as follows : 20A at 60 m from A, 40A at 100m from A and a uniform loading of 1.5 A/m from 200 to 450 m from A. The resistance of each conductor is 0.05 Ω/km. Find the point of minimum potential and its potential.* **(Electrical Power-II, Bangalore Univ. 1993)**

Solution. In Fig. 40.27, let D be the point of minimum potential and let i be the current in distributor section CD. Then, current supplied to load D from end B is $(40 - i)$. If r is the resistance of the distributor/metre (both go and return), then

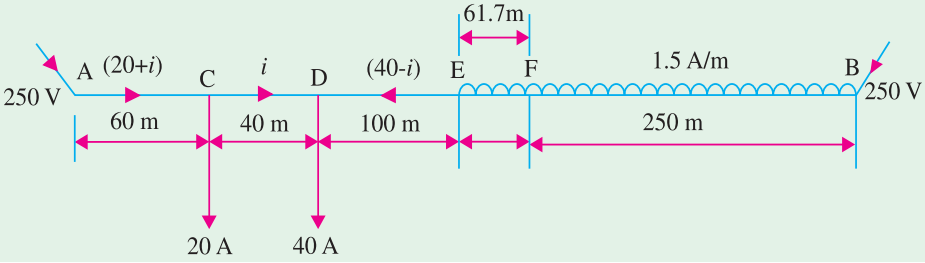


Fig. 40.27

$$\text{Drop over } AD = (20 + i) \times 60r + i \times 40r$$

$$\text{Drop over } BD = \text{drop due to concentrated load + drop due to distributed load}$$

$$= (40 - i) \times 350r + 1.5 \times r \times 250^2/2 \qquad \textbf{—Art. 40.6}$$

Since the two feeding points A and B are at the same potential of 250 V, the two drops must be equal.

$$\therefore \quad (20 + i) \times 60r + i \times 40r = (40 - i) \times 350r + 1.5 \times r \times 250^2/2 \quad \therefore \quad i = 132.6 \text{ Å}$$

Since $(40 - i)$ comes out to be negative, it means that D is not the point of minimum potential. The required point is somewhat nearer the other end B. Let it be F. Obviously, current in section DF = 132.6 – 40 = 92.6 A. Hence, distance of minimum potential point F from end A is

$$= 60 + 40 + 100 + 92.6/1.5 = \mathbf{261.7 \text{ m}}$$

Total voltage drop over section AF is

$$= 2 \times 0.05 \times 10^{-3}\,(152.6 \times 60 + 132.6 \times 40 + 92.6 \times 100 + 92.6 + 61.7/4) = 2.65 \text{ V}$$

$\therefore$ potential of point F = 250 – 2.65 = **247.35 V.**

Example 40.16. *A two-wire d.c. distributor AB, 1000 metres long, is supplied from both ends, 240 V at A and 242 V at B. There is a concentrated load of 200 A at a distance of 400 metre from A and a uniformly distributed load of 1.0 A/m between the mid-point and end B. Determine* ***(i)*** *the currents fed at A and B* ***(ii)*** *the point of minimum potential and* ***(iii)*** *voltage at this point. Take cable resistance as 0.005 Ω per 100 metre each core.*

Solution. The resistance per 100 metres of both cores = 0.005 × 2 = 0.01 Ω.

Let us take 100 m as the unit of length. Let current fed at end B be I_B as shown in Fig. 40.28.

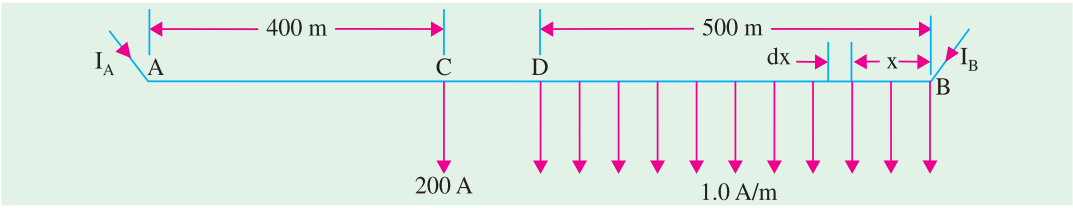


Fig. 40.28

Consider any element dx anywhere within distance BD *i.e.* distance over which uniformly distributed load is applied. Suppose it is at a distance of x units (of 100 metre each). The current through dx is $I = (I_B - 100 \times 1.0 \times x) = (I_B - 100x)$

$\therefore$ voltage drop over $dx = (I_B - 100x) \times 0.01 \times dx$

Voltage drop over distance BD

$$= \int_0^5 (I_B - 100x) \times 0.01 \times dx = 0.01 \int_0^5 (I_B - 100x)\, dx$$

$$= 0.01 \left| (I_B x - 500x^2) \right|_0^5 = (0.05\, IB - 12.5) \text{ V}$$

Voltage drop over $DC = (I_B - 500) \times 0.01$; Voltage drop over $CA = (I_B - 700) \times 0.01 \times 4$

Since total drop over $AB = 242 - 240 = 2$ volt

$\therefore$ $0.05\, I_B - 12.5 + 0.01\, I_B - 5 + 0.04\, I_B - 28 = 2$ $\therefore$ $I_B =$ **455 A**

Now, total current is = 500 + 200 = 700 A $\therefore$ $I_A = 700 - 455 =$ **245 A**

It is obvious that (245 – 200) = 45 A is fed into the distributed load at D. Hence, point of minimum potential M will be 45/1.0 = 45 metres from D. Its distance from $B = 500 - 45 = 455$ metres or 4.55 units of length.

Voltage drop upto this point from end $B = \int_0^{4.55} (455 - 100x) \times 0.01 \times dx = 10.35$ V

$\therefore$ potential of $M = 242 - 10.35 =$ **231.65 V**

Tutorial Problem No. 40.1

1. A 2-wire direct-current distributor PQ is 500 metres long. It is supplied by three feeders entering at P, Q and R where R is midway between P and Q. The resistance of the distributor, go and return, is 0.05 Ω per 100 metres. The distributor is loaded as follows :

Point	*A*	*B*
Distance from P (metres)	*100*	*350*
Load (ampere)	*30*	*40*

In addition, a distributed load of 0.5 A/metre exists from P to Q. Calculate (*a*) the current supplied by each feeder if P and R are maintained at 220 V while Q is at 215 V (*b*) the voltage at a point between P and Q and 50 metres from P.

[(*a*) current at P = 18 A ; at Q = 28.5 A ; at R = 98.5 A (*b*) voltage at stated point = 214.8 V]

2. A section of a 2-wire distributor network is 1,200 metres long and carries a uniformly distributed load of 0.5 A/ metre. The section is supplied at each end by a feeder from a distribution centre at which the voltage is maintained constant. One feeder is 900 and the other 600 metres long and each has a cross-sectional area 50% greater than that of the distributor. Find the current in each feeder cable and the distance from one end of the distributor at which p.d. is a minimum. ***(London Univ.)***

[Current in 900 metres: 272.7 A; current in 600 metres: 327.3 A; 545.5 metres from the 900 metre feeder point]

3. A pair of distributing mains of uniform cross-section 1,000 metres in length having a resistance of 0.15 Ω each, are loaded with currents of 50, 100, 57.5, 10 and 75 A at distances measured from one end where the voltage between mains is 211.6, of 100, 300, 540, 740 and 850 metres respectively. If the voltage of the other end is maintained at 210, calculate the total current entering the system at each end of the mains and the position of the point of minimum potential. ***(I.E.E. London)***

[160.63 A and 131.87 A ; 540 metre load]

4. A 2-core distribution cable *AB*, 400 metre long, supplies a uniformly distributed lighting load of 1 A/m. There are concentrated loads of 120, 72, 48 and 120 A at 40, 120, 200 and 320 metres respectively from end *A*. This cable has a resistance of 0.15 Ω per 1,000 m run. Calculate the voltage and the position of the lowest run lamp when the cable is fed at 250 Volts (*a*) from both ends *A* and *B* (*b*) from end *A* only. **[(*a*) 239.1 volt at 200 m (*b*) 207.6 volt at B]**

5. A 2-wire d.c. distributor *AB*, 300 m long, is fed at both ends *A* and *B*. The distributor supplies a uniformly distributed load of 0.25 A per m together with concentrated loads of 40 A at *C* and 60 A at *D*, *AC* and *BD* being 120 m each. *A* and *B* are maintained at 300 V, the loop resistance of the distributor is 0.1 ohm/100 m. Determine the current fed at *A* and *B* and also the potential of points *C* and *D*. **[85.5 A, 89.5 A, 291.54 V, 291.06 V]**

40.11. Ring Distributor

A ring distributor is a distributor which is arranged to form a closed circuit and which is fed at one or more than one points. For the purpose of calculating voltage distribution, it can be looked upon as consisting of a series of open distributors fed at both ends. By using a ring distributor fed properly, great economy in copper can be affected.

If the ring distributor is fed at one point then, for the purposes of calculation, it is equivalent to a straight distributor fed at both ends with equal voltages (Ex. 40.17).

Example 40.17. *A 400-metre ring distributor has loads as shown in Fig. 40.29 (a) where distances are in metres. The resistance of each conductor is 0.2 Ω per 1,000 metres and the loads tapped off at points B, C and D are as shown. If the distributor is fed at A, find voltages at B,C and D.*

Solution. Let us assume a current of *I* in section *AD* [Fig. 40.29 (*a*)] and then find the total drop which should be equated to zero.

$\therefore \quad 70\,I + 90\,(I - 50) + 80\,(I - 120) + 60\,(I - 220) = 0 \quad \therefore \quad 300\,I = 27{,}300 \text{ or } I = 91 \text{ A}$

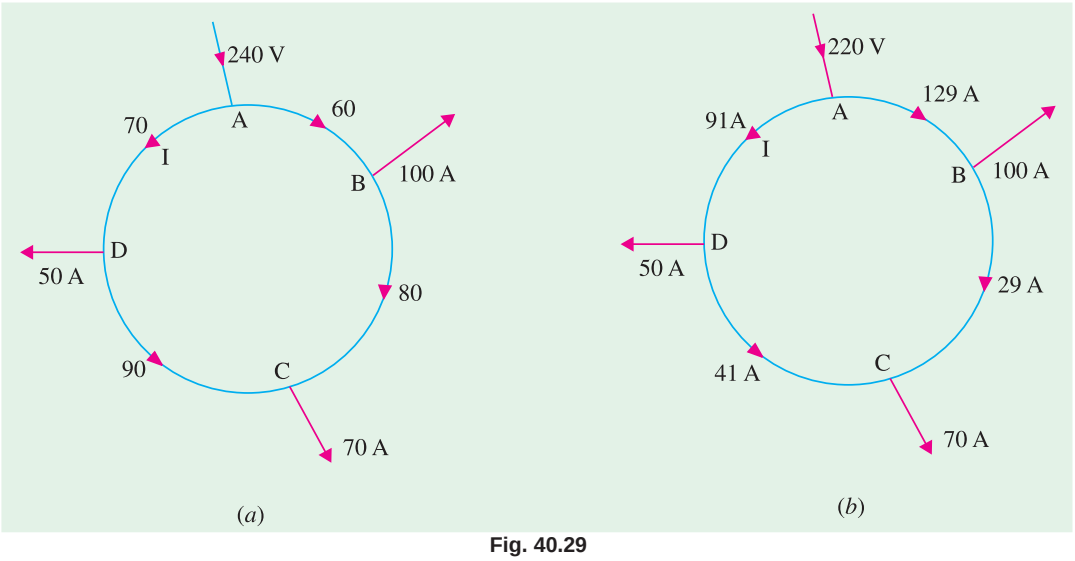


Fig. 40.29

The current distribution becomes as shown in Fig. 40.29 (*b*) from where it is seen that *C* is the point of minimum potential.

Drop in AD = $2(91 \times 70 \times 0.2/1{,}000) = 2.55$ V ; Drop in $DC = 2(41 \times 90 \times 0.2/1{,}000) = 1.48$ V

Drop in CB = $2(29 \times 80 \times 0.2/1{,}000) = 0.93$ V ; Drop in $BA = 2(129 \times 60 \times 0.2/1{,}000) = 3.1$ V

Voltage at D = $240 - 2.55 =$ **237.45 V** ; Voltage at $C = 237.45 - 1.48 =$ **235.97 V**

Voltage at $B = 240 - 3.1 =$ **236.9 V**

Example 40.18. *In a direct current ring main, a voltage of 400 V is maintained at A. At B, 500 metres away from A, a load of 150 A is taken and at C, 300 metres from B, a load of 200 A is taken. The distance between A and C is 700 metres. The resistance of each conductor of the mains is 0.03 Ω per 1,000 metres. Find the voltage at B and C and also find the current in the section BC.*

(Elect. Technology, Kerala Univ.)

Solution. Let us assume a current of I in section AB, then find the total drop round the ring main and equate it to zero. As seen from Fig. 40.30 (*a*).

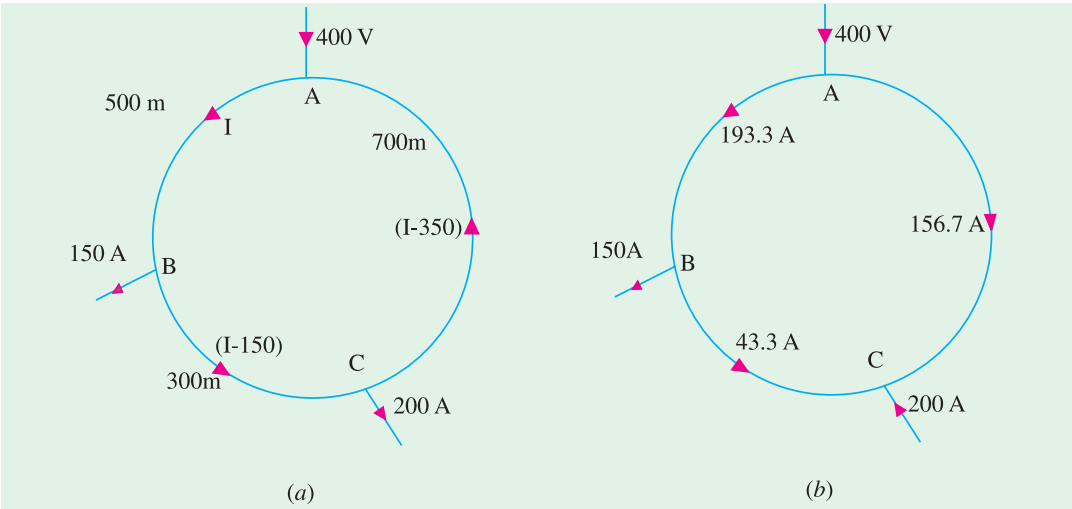


Fig. 40.30

$500\,I + 300\,(I - 150) + 700\,(I - 350) = 0$ $\quad\therefore\quad I = 193.3$ A

The current distribution becomes as shown in Fig. 40.30 (*b*) from where it is seen that *C* is the point of minimum potential.

Drop over AB = $2(193.3 \times 50 \times 0.03/1{,}000) = 5.8$ V

Drop over BC = $2(43.3 \times 300 \times 0.03/1{,}000) = 0.78$ V

Voltage at B = $400 - 5.8 = 394.2$ V ; Voltage at $C = 394.2 - 0.78 =$ **393.42 V**

The current in section BC is as shown in Fig. 40.30 (*b*).

Example 40.19. *A d.c. ring main ABCDE is fed at point A from a 220-V supply and the resistances (including both lead and return) of the various sections are as follows (in ohms) : AB = 0.1 ; BC = 0.05 ; CD = 0.01 ; DE = 0.025 and EA = 0.075. The main supplies loads of 10 A at B ; 20 A at C ; 30 A at D and 10 A at E. Find the magnitude and direction of the current flowing in each section and the voltage at each load point.*

If the points C and E are further linked together by a conductor of 0.05 Ω resistance and the output currents from the mains remain unchanged, find the new distribution of the current and voltage in the network. **(London Univ.)**

Solution. The ring main is shown in Fig. 40.31.

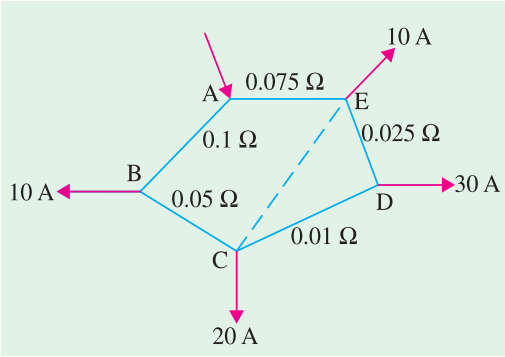


Fig. 40.31

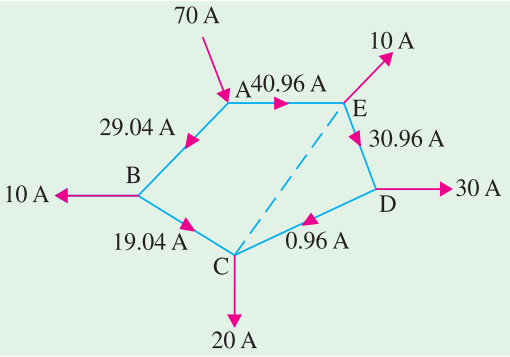


Fig. 40.32

Let us assume a current I amperes in section AB and put the total drop round the ring equal to zero.

$\therefore \quad 0.1\,I + 0.05(I - 10) + 0.01(I - 30) + 0.025(I - 60) + 0.075(I - 70) = 2$ or $I = 29.04$ A

Current distribution now becomes as shown in Fig. 40.32.

Drop in AB = $29.04 \times 0.1 = 2.9$ V; Drop in BC = $19.04 \times 0.05 = 0.95$ V

Drop in ED = $30.96 \times 0.025 = 3.77$V; Drop in AE = $40.96 \times 0.075 = 3.07$ V

$\therefore$ Potential of B = 217.1 V, Potential of C = **216.15 V**

Potential of E = 216.93 V, Potential of D = **216.16 V**

The interconnector between points C and E is shown in Fig. 40.31. It may be noted here that the function of the interconnector is to reduce the drop of voltage in various sections. For finding current in the interconnector, first p.d. across its ends is calculated. Then we calculate the resistance, viewed from points E and C of the network composed resistances of the distribution lines only, ignoring the load (Art, 2-22). Then current through the interconnector is given by

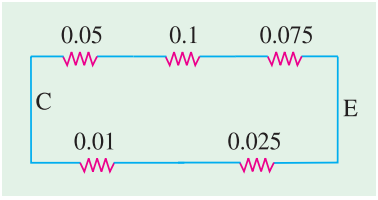


Fig. 40.33

$$I = \frac{\text{p.d. between points } E \text{ and } C}{\text{resistance of distribution network + interconnector}}$$

P.D. between points E and C = 216.93 – 216.15 = 0.78 V

To determine the resistance viewed from C and E, the network is drawn as shown in Fig. 40.33. Since the two branches are in parallel, the combined resistance is

$= 0.225 \times 0.035/(0.225 + 0.035) = 0.03\ \Omega$

$\therefore$ current in interconnector $= 0.78/(0.03 + 0.05) = 9.75$ A from E to C.

The currents in the other sections must be calculated again.

Let us assume a current I_1 and ED, then since the voltage round the closed mesh EDC is zero, hence

$-0.025\,I_1 - 0.01\,(I_1 - 30) + 0.05 \times 9.75 = 0$ or $0.035\,I_1 = 0.7875$ $\therefore$ $I_1 =$ **22.5 A**

Current in AE = $10 + 22.5 + 9.75 =$ **42.25 A;** Current in $AB = 70 - 42.25 =$ **27.75 A**

Drop in AB = $27.75 \times 0.1 =$ **2.775 V ;** Drop in $BC = 17.75 \times 0.05 =$ **0.888 V**

Drop in ED = $32.25 \times 0.025 =$ **0.806 V ;** Drop in $AE = 42.25 \times 0.075 =$ **3.169 V**

Potential of B = $220 - 2.775 =$ **217.225 V;** Potential of $C = 217.225 - 0.888 =$ **216.337 V**

Potential of E = $220 - 3.169 =$ **216.83 V ;** Potential of $D = 216.83 - 0.806 =$ **216.024 V**

Tutorial Problem No. 40.2

1. Four power loads *B*, *C*, *D* and *E* are connected in this order to a 2-core distributor cable, arranged as a ring main, and take currents of 20, 30, 25 and 30 A respectively. The ring is supplied from a substation at the point *A* between *B* and *E*. An interconnector cable joins the points *C* and *E* and from a point *F* on this inter-connector cable a current of 20 A is taken.
The total resistance of the cable between the load points is : *AB* = 0.04 Ω ; *BC* = 0.03 Ω ; *CD* = 0.02 Ω ; *DE* = 0.03 Ω ; *EA* = 0.04 Ω ; *CF* = 0.02 Ω and *EF* = 0.01 Ω. Calculate the current in each section of the ring and the interconnector. ***(London Univ.)***
[AB = 53.94 A ; BC = 33.94 A ; CD = 8.35 A ; ED = 16.65 A ; AE = 71.06 A ; FC = 4.42 A ; EF = 24.42 A]

2. A 2-core ring feeder cable *ABCDEA* is connected to a sub-station at *A* and supplies feeding points to a distribution network at *B*, *C*, *D* and *E*. The points *C* and *E* are connected by an inter-connecter *CFE* and a load is taken at *F*. The total resistance in ohms of both conductors in the several sections is *AB* = 0.05 ; *BC* = 0.4 ; *CD* = 0.03 ; *DE* = 0.04 ; *EA* = 0.05 ; *CF* = 0.02 ; *FE* = 0.1. The currents taken at the load points are *B* = 12 A ; *C* = 15 A ; *D* = 12 A; *E* = 15 A and *F* = 10 A. Calculate the current in each section of the cable and the p.d. at each load point, if the p.d. at *A* is maintained constant at 250 V. ***(City & Guilds, London)***
[Currents : AB = 27.7 A ; FC = 3.3 A ; P.D. s at B = 248.6 V ; C = 248 V ; D = 247.87 V ; E = 248.18 V ; F = 248 V]

3. A distributor cable in the form of a ring main *ABCDEA*, supplies loads of 20, 60, 30, and 40 A taken at the points *B*, *C*, *D* and *E* respectively, the feeding point being at *A*. The resistances of the sections are *AB* = 0.1 Ω, *BC* = 0.15 Ω, *CD* = 0.1 Ω, *DE* = 0.05 Ω and *EA* = 0.1 Ω. The points *E* and *C* are connected by a cable having a resistance of 0.3 Ω. Calculate the current in each section of the network. ***(City & Guilds, London)***
[A to B : 60 A ; B to C : 40 A ; E to C : 10 A ; D to C : 10 A ; E to D : 40 A ; A to E : 90 A]

40.12. Current Loading and Load-point Voltages in a 3-wire System

Consider a 3-wire, 500/250-V distributor shown in Fig. 40.34. The motor requiring 500 V is connected across the outers whereas other loads requiring lower voltage of 250 V are connected on both sides of the neutral.

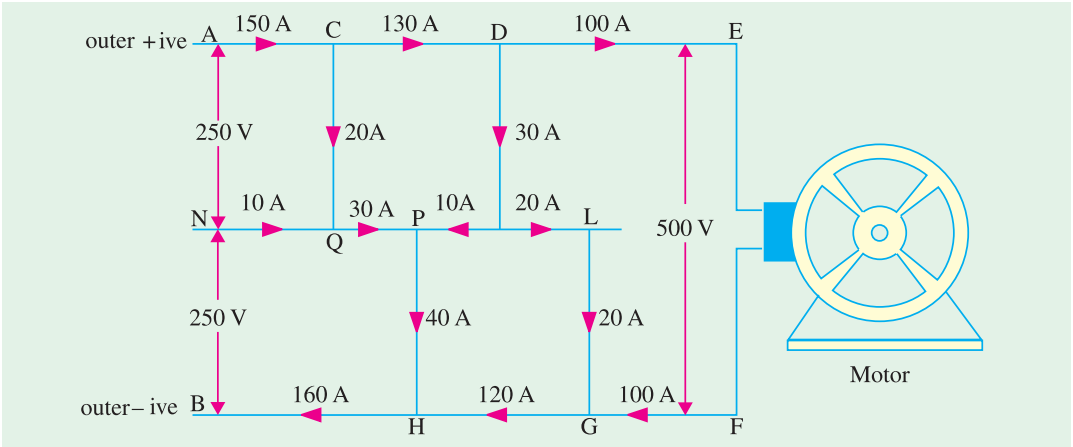


Fig. 40.34

The current in positive outer *AE* flows from the left to right and in negative outer *FB* from right to left. The current through the various sections of the neutral wire *NL* may flow in either direction depending on the load currents on the positive side and negative side but is independent of the loads connected between outers.

Since 150 A enters the +ve outer but 160 A comes out of the network, it means that a current of 10 A must flow into the neutral at point *N*. Once the direction and magnitude of current in *NQ* is known, the directions and magnitudes of currents in other sections of the neutral can be found very easily. Since *PH* takes 40 A, currents meeting at *P* should add up to 40 A. As seen, 20 A of *CQ* and 10A of *NQ* flow towards *P*, the balance of 10 A flows from point *M*. Then, 30 A current of *DM* is divided into two parts, 10 A flowing along *MP* and the other 20 A flowing along *ML* to feed the load *LG*.

Knowing the values of currents in the various conductors, voltage drops can be calculated provided resistances are known. After that, voltages at different load points can be calculated as illustrated in Ex. 40.20.

Example 40.20. *In a 3-wire distribution system, the supply voltage is 250 V on each side. The load on one side is a 3 ohm resistance and on the other, a 4 ohm resistance. The resistance of each of the 3 conductors is 0.05 Ω. Find the load voltages.*

(Elements of Elect. Engg-I, Bangalore Univ.)

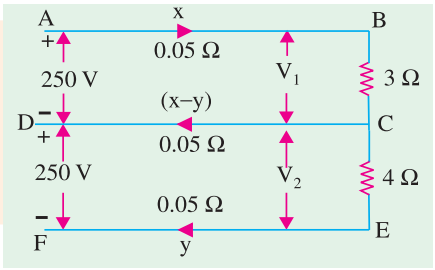


Fig. 40.35

Solution. Let the assumed directions of unknown currents in the three conductors be as shown in Fig. 40.35. Applying KVL (Art. 2.2) to closed circuit *ABCDA*, we have

$$-3.05x - 0.05(x - y) + 250 = 0$$

or $$310x - 5y = -25{,}000 \quad ...(i)$$

Similarly, circuit *DCEFD* yields

$$0.05\,(x - y) - 4.05y + 250 = 0$$

or $$5x - 410y = -25{,}000 \quad ...(ii)$$

From **(*i*)** and **(*ii*)**, we get $x = 81.64$ A, $y = 61.97$ A. Since both currents come out to be positive, it means that their assumed directions of flow are correct.

$$\therefore \quad V_1 = 250 - 0.05 \times 81.64 - 0.05\,(81.64 - 61.97) = \mathbf{244.9\ V}$$

$$V_2 = 250 + 0.05(81.64 - 61.97) - 0.05 \times 61.97 = \mathbf{247.9\ V}$$

Example 40.21. *A 3-wire d.c. distributor PQ, 250 metres long, is supplied at end P at 500/250 V and is loaded as under :*

Positive side: 20 A 150 metres from P ; 30 A 250 metres from P.

Negative side: 24 A 100 metres from P ; 36 A 220 metres from P.

The resistance of each outer wire is 0.02 Ω per 100 metres and the cross-section of the middle wire is one-half that of the outer. Find the voltage across each load point.

Solution. The current loading is shown in Fig. 40.36.

The current flowing into the positive side is 30 + 20 = 50 A. Since current flowing out of the negative side is 36 + 24 = 60 A, it means that 10 A must be flowing in the section *NC* of the neutral. To make up 24 A in load *CD*. 10 A are contributed by NC and 14 A by *BC*. The balance of 6 A flows through *BF* and adds up with 30 A of *KF* to make up 36A through load *FE*.

It is given that the resistance of outers is 0.02 Ω per 100 m. Since neutral is of half the cross-section, its resistance is 0.04 Ω per 100 m. Knowing them, voltage at load points can be determined as under. Let us see how we will calculate voltage across load *A B*.

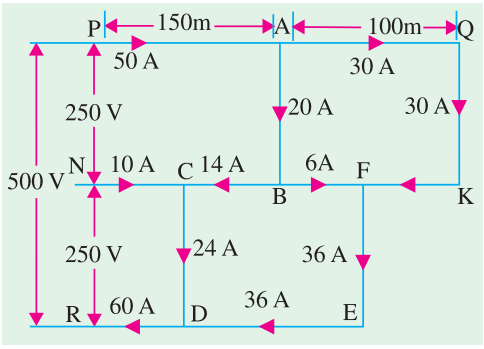


Fig. 40.36

Voltage at AB = 250 – drop in PA – drop in BC + drop in CN.

It should be particularly noted that drop in CN has been added instead of being ***subtracted.*** The reason is this : as we start from P and go round $PABCN$, we go along the current over section PA *i.e.* we go 'downstream' hence drop is taken negative, but along CN we go 'upstream', hence the drop as taken as positive*. Proceeding in this way, we tabulate the currents, resistances and voltage drops in various sections as given below :

Section	*Resistance (W)*	*Current (A)*	*Drop (V)*
PA	0.03	50	1.5
AQ	0.02	30	0.6
KF	0.012	30	0.36
BC	0.02	14	0.28
BF	0.028	6	0.168
NC	0.04	10	0.4
ED	0.024	36	0.864
DR	0.02	60	1.2

P.D. across AB = 250 – 1.5 – 0.28 + 0.4 = **248.62 V**

P.D. across QK = 248.62 – 0.6 – 0.36 + 0.168 = **247.83 V**

P.D. across CD = 250 – 0.4 – 1.2 = **248.4 V**

P.D. across FE = 248.4 + 0.28 – 0.168 – 0.864 = **247.65 V**

Tutorial Problem No. 40.3

1. A 3-wire system supplies three loads (*a*) 10 (*b*) 20 and (*c*) 30 amperes situated at distances of 100, 150 and 300 metres respectively from the supply point on the positive side of the neutral wire. Connected between negative and neutral wires are two loads (*d*) 30 A and (*e*) 20A, situated 120 and 200 metres respectively from the supply point. Give a diagram showing the values and directions of the currents in various parts of the neutral wire.

If the resistance of the outers is 0.05 Ω per 1000 metres and that of the neutral 0.1 Ω per 1000 metres, calculate the potential difference at the load points (*b*) and (*e*); the pressure at the supply point being 100 V between outers and neutral. **[10A and 5A; Volts at (*b*) = 99.385 V ; at (*e*) = 99.86 V]**

2. A 3-wire d.c. distributor 400 metres long is fed at both ends at 235 volts between each outer and neutral. Two loads P and Q are connected between the positive and neutral and two loads R and S are connected between the negative outer and the neutral. The loads and their distances from one end (X) of the distributor are as follows : Load P, 50 A, 100 metres from X ; Load Q, 70 A, 300 metres from X; Load R, 60 A, 150 metres from X; Load S, 60 A,350 metres from X. Determine the p.d. at each load point and the current at each feeding point. The resistance of each outer is 0.25 Ω per 1000 metres and that of the neutral is 0.5 W per 1000 metres.

[Current into the +ve outer at X = 55 A ; other end Y = 65 A; Out from –ve outer at X = 45 A; other end Y = 75 A ; P.D. s at P = 233.125 V; Q = 232.375 V; R = 232.812 V; S = 233.812 V]

40.13. Three-wire System

As already mentioned briefly in Art. 40.2, it consists of the 'outer' conductors (between which the voltage is twice the normal value for lighting) and the third wire which is called the middle or neutral wire. It is of half the cross-section as compared to any one of the two outers and is earthed at the generator end. The voltage of the neutral is thus approximately half way between that of the outers.

* Hence, sign convention in this : While going 'upstream' take the drop as positive and while going 'down-stream' take the drop as negative. (Art. 2-3).

If the total voltage between outers is 460 V, then the positive outer is 230 V ***above*** the neutral and the negative outer 230 V ***below*** the neutral. Motor loads requiring higher voltage are connected across the outers directly whereas lighting and heating loads requiring lesser voltage of 230 V are connected between any one of the outers and the neutral. If the loads on both sides of the neutral are equal *i.e.* balanced as shown in Fig. 40.37, then there is no current in the middle wire and the effect is as if the different loads were connected in series across the outers. However, in practice, although effort is made to distribute the various loads equally on the two sides of the neutral, yet it is difficult to achieve exact balance, with the result that some out-of-balance current does flow in the neutral as shown in Fig. 40.38. In the present case as viewed from generator end, the load is equivalent to a resistance of 25/3 ohms in series with a resistance of 25/2 ohms (Fig. 40.39). Obviously, the voltages across the two will becomes unequal. The voltage across the positive side will fall to $\frac{500 \times 25/2}{(25/2 + 25/3)}$ = 200 V and on the negative side, it will rise to $\frac{500 \times 25/3}{(25/2 + 25/3)}$ = 300 V. This difficulty of unequal voltages with unbalanced loads is removed by using balancers as discussed below in Art. 40.14.

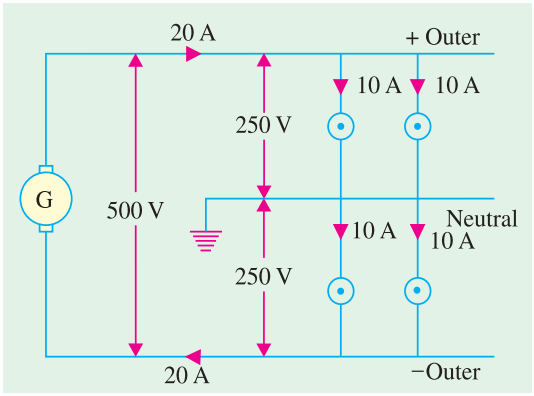


Fig. 40.37

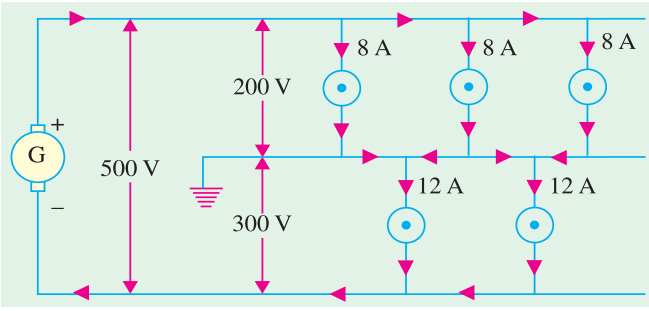


Fig. 40.38

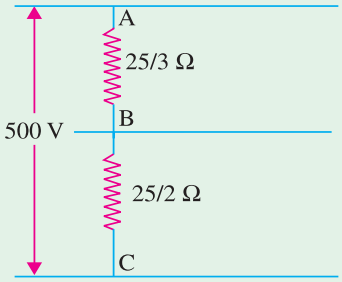


Fig. 40.39

40.14. Balancers

In order to maintain p.d.s on the two sides of neutral equal to each other, a balancer set is used. The commonest form of balancer consists of two identical shunt-wound machines which are not only coupled mechanically but have their armatures and field circuits joined in series across the outers. The neutral is connected to the junction of the armatures as shown. When the system is unloaded or when the loads on the two sides are balanced, then

1. both machines run as unloaded motors and
2. since their speeds and field currents are equal, their back e.m.fs. are the same.

When the two sides are unbalanced *i.e.* when the load supplied by + ve outer is different from that

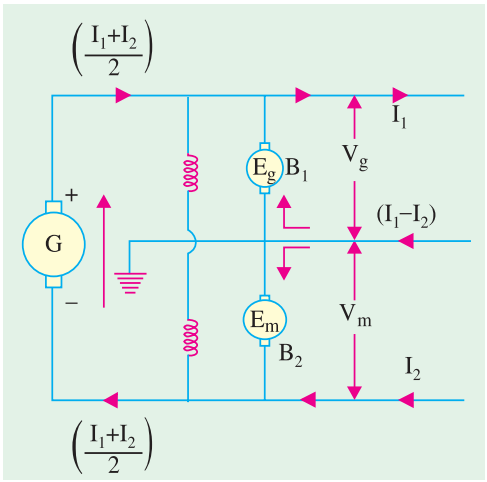


Fig. 40.40

supplied by negative outer, then out-of-balance current $(I_1 - I_2)$ flows through the mid-wire to the balancers where it divides into two halves each equal to $(I_1 - I_2)/2$ as shown in Fig. 40.40.

If I_1 is greater than I_2, then +ve side is more heavily loaded (than –ve side) hence the p.d. on this side tends to fall below the e.m.f. of the balancer on this side so that B_1 runs as a generator. However, the p.d. on the lightly-loaded side rises above the e.m.f. of the balancers on that side, hence B_2 runs as a motor. In this way, energy is transferred through balancers from the lightly-loaded side to the heavily-loaded side. In Fig. 40.40, machine B_2 is running as a motor and driving machine B_1 as a generator.

Let R_a = armature resistance of each machine
V_g = terminal p.d. of machine running as a generator *i.e.* B_1
E_g = induced e.m.f. of B_1
V_m = terminal p.d. of motoring machine *i.e.* B_2
E_m = induced e.m.f. of B_2

then $V_g = E_g - \dfrac{(I_1 - I_2)}{2} R_a$ and $V_m = E_m + \dfrac{(I_1 - I_2)}{2} R_a$

$\therefore \quad V_m - V_g = (E_m - E_g) + (I_1 - I_2) R_a$ ...**(*i*)**

Since the speed and excitation of the two machines are equal,

$\therefore \quad E_g = E_m \quad \therefore \quad V_m - V_g = (I_1 - I_2) R_2$ ...**(*ii*)**

Hence, we find that the difference of voltages between the two sides of the system is proportional to —

(*i*) out of balance current $(I_1 - I_2)$ and **(*ii*)** armature resistance of the balancer.

For this reason, R_a is kept very small and effort is made to arrange the loads on the two sides such that out-of-balance current is as small as possible.

The value $(V_m - V_g)$ can be still further reduced *i.e.* the voltages on the two sides can be more closely balanced by cross-connecting the balancer fields as shown in Fig. 40.41. In this way, generator draws its excitation from the lightly-loaded side which is at a higher voltage, hence E_g is increased. The motoring machine draws its excitation from the heavily loaded side which is at a little lower voltage, hence E_m is decreased. In this way, the difference $(E_m - E_g)$ is decreased and so is $(V_m - V_g)$. Further, regulation of the voltage can be accomplished by connecting an adjustable regulator in series with the two balancer fields as shown in Fig. 40.42.

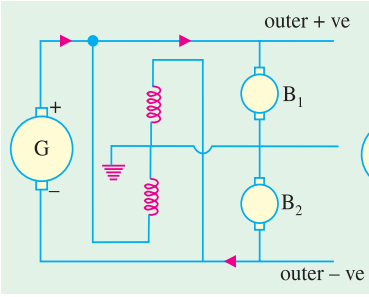


Fig. 40.41

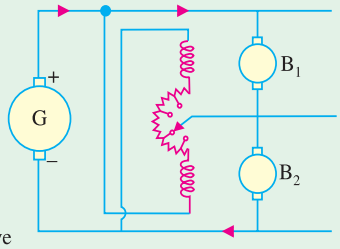


Fig. 40.42

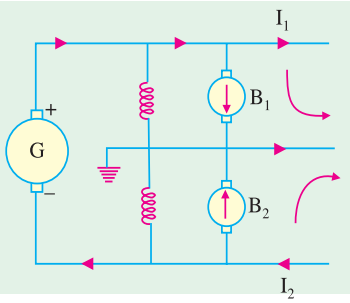


Fig. 40.43

It should be noted that since machine B_1 is running as a generator and B_2 as a motor, the directions of currents in B_1, B_2 and the neutral are as shown in Fig. 40.40. If, however, B_2 runs as a generator and B_1 as a motor *i.e.* if negative side is more heavily loaded than the +ve side, then directions of currents through B_1, B_2 and the neutral are as shown in Fig. 40.43. In particular, the change in the direction of the current through midwire should be noted.

Example 40.22. *A d.c. 3-wire system with 500-V between outers has lighting load of 100 kW on the positive and 50 kW on the negative side. If, at this loading, the balancer machines have each a loss of 2.5 kW, calculate the kW loading of each balancer machine and the total load on the system.*

Solution. The connections are shown in Fig. 40.44.

Total load on main generator = 100 + 50 + (2 × 2.5)
= **155 kW**
Output current of main generator = 155 × 1000/500
= 310 A
Load current on +ve side, I_1 = 100 × 1000/250
= 400 A
Load current on – ve side, I_2 = 50 × 1000/250
= 200A
Out-of-balance current = 400 – 200 = 200 A

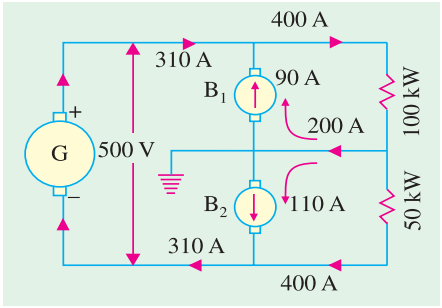


Fig. 40.44

Since +ve side is more heavily loaded , B_1 is working as a generator and B_2 as motor.

∴ current of B_1 = 400 – 310 = 90 A
current of B_2 = 200 – 90 = 110 A
Loading of B_1 = 250 × 90/1000 = **22.5 kW**
Loading of B_2 = 250 × 110/1000 = **27.5 kW**

Example 40.23. *In a 500/250-V d.c. 3-wire system, there is a current of 2000 A on the +ve side, 1600 A on the negative side and a load of 300 kW across the outers. The loss in each balancer set is 8 kW. Calculate the current in each armature of the balancer set and total load on the main generator.*

Solution. Connections are shown in Fig. 40.45.

It should be noted that loading across 'outers' directly in no way determines the current in the neutral.

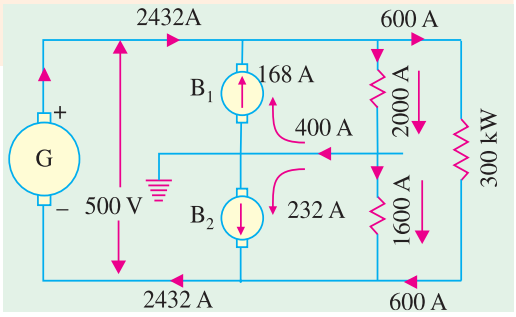


Fig. 40.45

+ve loading = 2000 × 250/1000
= 500 kW
– ve loading = 1600 × 250/1000
= 400 kW

Total loading on main generator is
= 500 + 400 + 300 + (2 × 8) = **1216 kW**
∴ Current of main generator = 1216 × 1000/500 = 2,432 A
Out-of-balance current = 2000 – 1600 = 400 A
Current through B_1 = 2,600 – 2,432 = **168 A**
Current through B_2 = 400 – 168 = **232 A**

Example 40.24. *On a 3-wire d.c. distribution system with 500 V between outers, there is a load of 1500 kW on the positive side and 2,000 kW on the negative side. Calculate the current in the neutral and in each of the balancer armatures and the total current supplied by the generator. Neglect losses.*

(Electrical Engineering ; Madras Univ.)

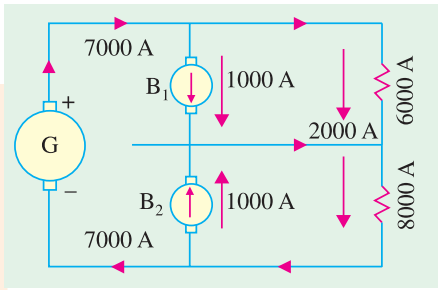


Fig. 40.46

Solution. Since negative side is more heavily loaded than the positive side, machine B_2 runs as a generator and B_1 as a motor. The directions of current through B_1 and B_2 are as shown in Fig. 40.46. Total loading on the main generator
= 2,000 + 1,500 = 3,500 kW
Current supplied by main generator = 3,500 × 1000/500 = **7,000 A**
Current on +ve side = 1500 × 1000/250 = **6,000 A**
Current on –ve side = 2000 × 1000/250 = **8,000 A**
Out-of-balance current = 8,000 – 6000 = **2,000 A**
Current through the armature of each machine = **1000 A**

Example 40.25. *A 125/250 V, 3-wire distributor has an out-of-balance current of 50 A and larger load of 500 A. The balancer set has a loss of 375 W in each machine. Calculate the current in each of the balancer machines and output of main generator.*

(Electrical Technology-II, Gwalior Univ.)

Solution. As shown in Fig. 40.47, let larger load current be I_1 = 500 A. Since $(I_1 - I_2) = 50$

$\therefore \quad I_2 = 450$ A

Larger load = $500 \times 125/1000 = 62.5$ kW

Smaller load = $450 \times 125/1000 = 56.25$ kW

Balancer loss = $2 \times 375 = 0.75$ kW

Output of main generator

= 62.5 + 56.25 + 0.75 = **119.5 kW**

Current of main generator

= $119.5 \times 1000/250 = 478$ A

As seen from Fig. 40.47, current of

B_1 = (500 – 478) = **22 A**

and current of B_2 = (50 – 22) = **28 A**

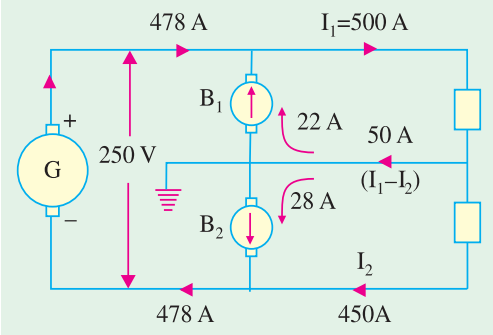


Fig. 40.47

Example 40.26. *The load on a d.c. 3-wire system with 500 V between outers consists of lighting current of 1500 A on the positive side and 1300 A on the negative side while motors connected across the outers absorb 500 kW. Assuming that at this loading, the balancer machines have each a loss of 5 kW, calculate the load on the main generator and on each of the balancer machines.*

(Electrical Engineering ; Madras Univ.)

Solution. Connections are shown in Fig. 40.48.

Positive loading = $1500 \times 250/1000 = 375$ kW

Negative loading = $1300 \times 250/1000 = 325$ kW

Total load on the main generator is

= 375 + 325 + 500 + (2 × 5) = **1210 kW**

Current supplied by the main generator is

= $1210 \times 1000/500 = 2{,}420$ A

Out-of-balance current = 1500 – 1300 = 200 A

Current through B_1 = 2500 – 2420 = 80 A

Current through B_2 = 200 – 80 = 120 A

Loading of B_1 = $80 \times 250/1000$ = **20 kW**

Loading of B_2 = $120 \times 250/1000$ = **30 kW**

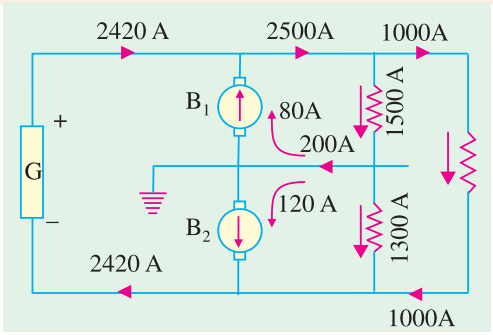


Fig. 40.48

Example 40.27. *A d.c. 3-wire system with 480 V across outers supplies 1200 A on the positive and 1000 A on the negative side. The balancer machines have each an armature resistance of 0.1W and take 10 A on no-load. Find*

(a) *the voltage across each balancer and*

(b) *the total load on the main generator and the current loading of each balancer machine.*

The balancer field windings are in series across the outers.

Solution. As shown in Fig. 40.49, B_1 is generating and B_2 is motoring.

The out-of-balance current is (1200 – 1000) = 200 A. Let current through the motoring machine be I_m, then that through the generating machine is $(200 - I_m)$. Let V_g and V_m be p.ds. of the two machines.

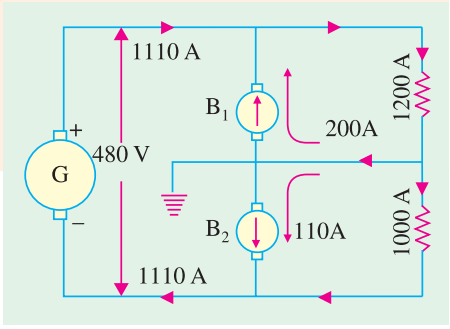


Fig. 40.49

Since B_2 is driving B_1, output of the motor supplies the losses in the set plus the output of the generator. Total losses in the set

$$= \text{no-load losses} + \text{Cu losses in two machines}$$
$$= 480 \times 10 + 0.1\, I_m^{\,2} + (200 - I_m)^2 \times 0.1$$
$$V_m I_m = V_g (200 - I_m) + \text{total losses}$$
$$= V_g (200 - I_m) + 4800 + 0.1\, I_m^2 + (200 - I_m)^2 \times 0.1$$

Now $V_m = E_b + I_m R_a$ and $V_g = E_b - I_g R_a$

Now, back e.m.f. $E_b = (240 - 0.1 \times 10) = 239$ V

$\therefore\ V_m = (239 + 0.1\, I_m)$ and $V_g = 239 - (200 - I_m) \times 0.1$

$\therefore\ (239 + 0.1\, I_m)\, I_m = [239 - (200 - I_m) \times 0.1]\,(200 - I_m)^2 + 4800 + 0.1\, I_m^{\,2} + (200 - I_m)^2 \times 0.1$

$\therefore\ I_m = 110$ A and $I_g = 200 - 110 = 90$ A

(a) $\therefore\ V_m = 239 + 0.1 \times 110 =$ **250 V** and $V_g = 239 - (0.1 \times 90) =$ **230 V**

(b) Load on main generator = $1200 - 90 =$ **1110 A**

Example 40.28. *A d.c. 3-wire system with 460 V between outers supplies 250 kW on the positive and 400 kW on the negative side, the voltages being balanced. Calculate the voltage on the positive and negative sides respectively, if the neutral wire becomes disconnected from the balancer set.*

(Electrical Power-III, Bangalore Univ.)

Solution. Before the disconnection of the mid-wire, voltages on both sides of the system are equal *i.e.* 230 V. The loads are 250 kW on the +ve side and 400 kW on the –ve side. If R_1 and R_2 are the resistances of the two loads, then

$$230^2/R_1 = 250{,}000\ ; \quad R_1 = 0.2116\ \Omega$$

Similarly, $R_2 = 2302/400{,}000 = 0.1322\ \Omega$

When the mid-wire is disconnected from the balancer set *i.e.* from the generator side, then the two resistances R_1 and R_2 are put in series across 460 V as shown in Fig. 40.50 (*b*).

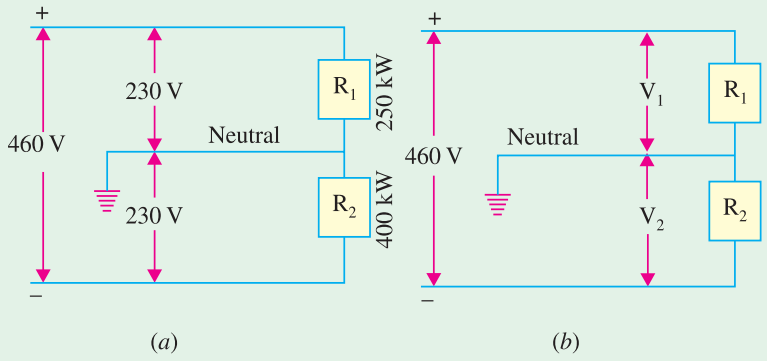


Fig. 40.50

$$\therefore \quad V_1 = \frac{R_1}{R_1 + R_2} \times 460 = \frac{0.02116 \times 460}{(0.2116 + 0.1322)} = \textbf{283 V}$$

$$V_2 = \frac{R_1}{R_1 + R_2} \times 460 = \frac{0.1322 \times 460}{0.3438} = \textbf{177 V}\ (\text{or } V_2 = 460 - V_1)$$

Tutorial Problem No. 40.4

1. In a 500/250-V d.c. 3-wire system there is an out-of-balance load of 125 kW on the positive side. The loss in each balancer machine is 7.5 kW and the current in the negative main is 1500 A. Calculate the total load on the generator and the current in each armature of the balancer set.

[890 kW ; 220 A ; 280 A]

2. A 460-V d.c. 2-wire supply is converted into 3-wire supply with the help of rotary balancer set, each machine having a no-load loss of 2.3 kW. If the load on the positive side of 69 kW and on the negative side 57.5 kW, calculate the currents flowing in each of the balancer machines. **[15 A ; 35 A]**
3. In a 500/250 volt 3-wire d.c. system there is an out-of-balance load of 200 kW on the positive side. The loss in each balancer is 10 kW and the current in the negative main is 2800 A. Calculate the current in each armature of the balancer set and the total load on the generators. ***(I.E. London)***

[Motoring machine = 440 A; Generating machine = 360 A ; Total load = 1620 kW]

40.15. Boosters

A booster is a generator whose function is to add to or inject into a circuit, a certain voltage that is sufficient to compensate for the I_R drop in the feeders etc.

In a d.c. system, it may sometimes happen that a certain feeder is much longer as compared to others and the power supplied by it is also larger. In that case, the voltage drop in this particular feeder will exceed the allowable drop of 6% from the declared voltage. This can be remedied in two ways ***(i)*** by increasing the cross-section of the feeder, so that its resistance and hence I_R drop is decreased ***(ii)*** or by increasing the voltage of the station bus-bars.

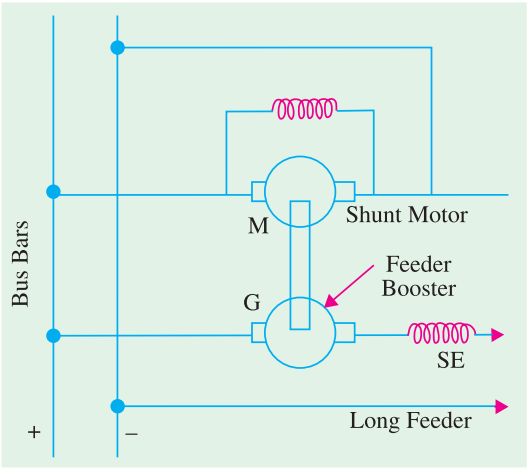


Fig. 40.51

The second method is not practicable because it will disturb the voltage of other feeders, whereas the first method will involve a large initial investment towards the cost of increased conductor material.

To avoid all these difficulties, the usual practice is to install a booster in series with this longer feeder as shown in Fig. 40.51. Since it is used for compensating drop in a feeder, it is known as feeder booster. It is a (series) generator connected in series with the feeder and driven at a constant speed by a shunt-motor working from the bus-bars. The drop in a feeder is proportional to the load current, hence the voltage injected into the feeder by the booster must also be proportional to the load current, if exact compensation is required. In other words, the booster must work on the straight or linear portion of its voltage characteristic.

Example 40.29. *A 2-wire system has the voltage at the supply end maintained at 500. The line is 3 km long. If the full-load current is 120 A, what must be the booster voltage and output in order that the far end voltage may also be 500 V.*

Take the resistance of the cable at the working temperature as 0.5 ohm/kilometre.

(Elect. Machinery-I, Calcutta Univ.)

Solution. Total resistance of the line is = $0.5 \times 3 = 1.5\ \Omega$

Full-load drop in the line is = $1.5 \times 120 = 180$ V

Hence, the terminal potential difference of the booster is **180 V** (*i.e.* 180/120 = 1.5 volt per ampere of line current).

Booster-output = $120 \times 180/1000 =$ **21.6 kW**

40.16. Comparison of 2-wire and 3-wire Distribution Systems

We will now compare the 2-wire and 3-wire systems from the point of view of economy of conductor material. For this comparison, it will be assumed that—

1. the amount of power transmitted is the same in both cases.
2. the distance of transmission is the same.

3. the efficiency of transmission (and hence losses) is the same.
4. voltage at consumer's terminals is the same.
5. the 3-wire system is balanced and
6. in the 3-wire system, the mid-wire is of half the cross-section of each outer.

Let W be the transmitted power in watts and V the voltage at the consumer's terminals. Also, let

R_2 = resistance in ohms of each wire of 2-wire system.
R_3 = resistance in ohms of each outer in 3-wire system.

The current in 2-wire system is W/V and the losses are $2\,(W/V)^2\,R_2$.

In the case of 3-wire system, voltage between outers is $2\,V$, so that current through outers is $(W/2V)$, because there is no current in the neutral according to our assumption (5) above. Total losses in the two outers are $2(W/2V)^2\,R_3$.

Since efficiencies are the same, it means the losses are also the same.

$$\therefore \quad 2(W/V)^2 R_2 = 2(W/2V)^2 \times R_3 \quad \text{or} \quad \frac{R_3}{R_2} = \frac{4}{1}$$

Since the cross-section and hence the volume of a conductor of given length, is inversely proportional to its resistance,

$$\therefore \quad \frac{\text{volume of each 3-wire conductor}}{\text{volume of each 2-wire conductor}} = \frac{1}{4}$$

Let us represent the volume of copper in the 2-wire system by 100 so that volume of each conductor is 50.

Then, volume of each outer in 3-wire system = 50/4 = 12.5
volume of neutral wire ,, ,, = 12.5/2 = 6.25
$\therefore$ total volume of copper in 3-wire system = 12.5 + 6.25 + 12.5 = 31.25

$$\therefore \quad \frac{\text{total coper vol. in 3-wire feeder}}{\text{total copper vol. in 2-wire feeder}} = \frac{31.25}{100} = \frac{5}{6}$$

Hence, a 3-wire system requires only 5/16th (or 31.25%) as much copper as a 2-wire system.

OBJECTIVE TESTS – 40

1. If in a d.c. 2-wire feeder, drop per feeder conductor is 2%, transmission efficiency of the feeder is ..... percent.
   (*a*) 98 (*b*) 94
   (*c*) 96 (*d*) 99
2. Transmitted power remaining the same, if supply voltage of a dc 2-wire feeder is increased by 100 percent, saving in copper is ..... percent.
   (*a*) 50 (*b*) 25
   (*c*) 100 (*d*) 75
3. A uniformly-loaded d.c. distributor is fed at both ends with equal voltages. As compared to a similar distributor fed at one end only, the drop at middle point is
   (*a*) one-half (*b*) one-fourth
   (*c*) one-third (*d*) twice.
4. In a d.c. 3-wire distributor using balancers and having unequal loads on the two sides
   (*a*) both balancers run as motors
   (*b*) both balancers run as generators
   (*c*) balancer connected to heavily-loaded side runs as a motor
   (*d*) balancer connected to lightly-loaded side runs as a motor.
5. As compared to a dc 2-wire distributor, a 3-wire distributor with same maximum voltage to earth uses only..... percent of copper.
   (*a*) 66.7 (*b*) 33.3
   (*c*) 31.25 (*d*) 150
6. In a d.c. 3-wire distribution system, balancer fields are cross-connected in order to
   (*a*) make both machines run as unloaded motors.
   (*b*) equalize voltages on the positive and negative outers
   (*c*) boost the generated voltage
   (*d*) balance loads on both sides of the neutral.

ANSWERS

1. (*c*) **2.** (*a*) **3.** (*b*) **4.** (*d*) **5.** (*c*) **6.** (*a*)

GO To FIRST

CONTENTS

CHAPTER 41

A.C. TRANSMISSION AND DISTRIBUTION

Learning Objectives

- General Layout of the System
- Power System and System Networks
- Systems of A.C. Distribution
- Effect of Voltage on Transmission Efficiency
- Comparison of Conductor Materials Required for Various Overhead Systems
- Reactance of an Isolated Single-Phase Transmission Line
- A.C. Distribution Calculations
- Load Division Between Parallel Lines
- Suspension Insulators
- Calculation of Voltage Distribution along Different Units
- Interconnectors
- Voltage Drop Over the Interconnectors
- Sag and Tension with Support at Unequal Levels
- Effect of Wind and Ice



The above figure shows a mini AC power plant. AC has distinct advantages over DC in generation as well as transmission.

CONTENTS

41.1. General Layout of the System

The conductor system by means of which electric power is conveyed from a generating station to the consumer's premises may, in general, be divided into two distinct parts *i.e.* transmission system and distribution system. Each part can again be sub-divided into two—primary transmission and secondary transmission and similarly, primary distribution and secondary distribution and then finally the system of supply to individual consumers. A typical layout of a generating, transmission and distribution network of a large system would be made up of elements as shown by a single-line diagram of Fig. 41.1 although it has to be realized that one or more of these elements may be missing in any particular system. For example, in a certain system, there may be no secondary transmission and in another case, when the generating station is nearby, there may be no transmission and the distribution system proper may begin at the generator bus-bars.

Now-a-days, generation and transmission is almost exclusively three-phase. The secondary transmission is also 3-phase whereas the distribution to the ultimate customer may be 3-phase or single-phase depending upon the requirements of the customers.

In Fig. 41.1, *C.S.* represents the central station where power is generated by 3-phase alternators at 6.6 or 11 or 13.2 or even 32 kV. The voltage is then stepped up by suitable 3-phase transformers for transmission purposes. Taking the generated voltage as 11 kV, the 3-phase transformers step it up to 132 kV as shown. Primary or high-voltage transmission is carried out at 132 kV*. The transmission voltage is, to a very large extent, determined by economic considerations. High voltage transmission requires conductors of smaller cross-section which results in economy of copper or aluminium. But at the same time cost of insulating the line and other expenses are increased. Hence, the economical voltage of transmission is that for which the saving in copper or aluminium is not offset **(*i*)** by the increased cost of insulating the line **(*ii*)** by the increased size of transmission-line structures and **(*iii*)** by the increased size of generating stations and sub-stations. A rough basis of determining the most economical transmission voltage is to use 650 volt per km of transmission line. For example, if transmission line is 200 km, then the most economical transmission voltage will be $200 \times 650 \cong 132{,}000$ V or 132 kV.

The 3-phase, 3-wire overhead high-voltage transmission line next terminates in step-down transformers in a sub-station known as Receiving Station (*R.S.*) which usually lies at the outskirts of a city because it is not safe to bring high-voltage overhead transmission lines into thickly-populated areas. Here, the voltage is stepped down to 33 kV. It may be noted here that for ensuring continuity of service ***transmission is always by duplicate lines***.

From the Receiving Station, power is next transmitted at 33 kV by underground cables (and occasionally by overhead lines) to various sub-stations (*SS*) located at various strategic points in the city. This is known as secondary or low-voltage transmission. From now onwards starts the primary and secondary distribution.

At the sub-station (*SS*) voltage is reduced from 33kV to 3.3kV 3-wire for primary distribution. Consumers whose demands exceeds 50 kVA are usually supplied from *SS* by special 3.3 kV feeders.

The secondary distribution is done at 400/230 V for which purpose voltage is reduced from 3.3 kV to 400 V at the distribution sub-stations. Feeders radiating from distribution sub-station supply power to distribution networks in their respective areas. If the distribution network happens to be at a great distance from sub-station, then they are supplied from the secondaries of distribution transformers which are either pole-mounted or else housed in kiosks at suitable points of the distribution networks. The most common system for secondary distribution is 400/230-V, 3-phase 4-wire system. The single-phase residential lighting load is connected between any one line and the neutral

* High voltages like 750 kV are in use in USSR (Konakovo-Moscow line) and 735 kV in Canada (Montreal-Manicoagan Scheme).

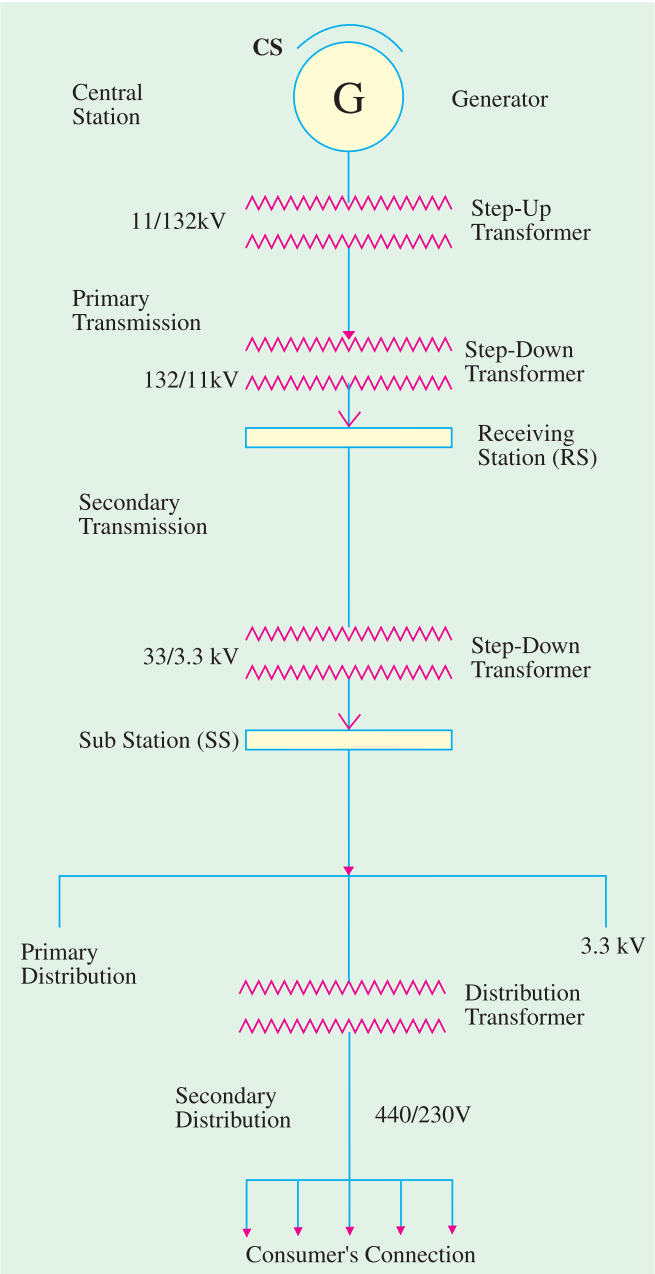


Fig. 41.1

whereas 3-phase, 400-V motor load is connected across 3-phase lines directly.

It should be noted that low-voltage distribution system is sub-divided into feeders, distributors and service mains. No consumer is given direct connection from the feeders, instead consumers are connected to distribution network through their service mains. The a.c. distributors are, in many ways, similar to the d.c. distributors as regards their constructional details and restrictions on drops in voltage.

Summarizing the above, we have

1. Generating voltage : 6.6, 11, 13.2 or 33 kV.
2. High voltage transmission : 220 kV, 132 kV, 66kV.
3. High voltage or primary distribution : 3.3, 6.6 kV.
4. Low-voltage distribution : A.C. 400/230, 3-phase, 4-wire D.C. 400/230 ; 3-wire system

The standard frequency for a.c. working is 50 Hz or 60 Hz (as in U.S.A.). For single-phase traction work, frequencies as low as 25 or 16 2/3Hz are also used.

41.2. Power Systems and System Networks

It is a common practice now-a-days to interconnect many types of generating stations (thermal and hydroelectric etc.) by means of a common electrical network and operate them all in parallel.

This combination of generating stations forms what is known as ***power system***. The various elements of such a system like generating stations, transmission lines, the substations, feeders and distributors etc. become tied into a whole by the integrated process of continuous generation and consumption of electric energy.

A system network (or grid) is the name given to that part of power system which consists of the sub-stations and transmission lines of various voltage rating.

Fig. 41.2 shows single-line diagram representing the main connections of a power system consisting of a heating and power central station (*HCS*), a large-capacity hydro-electric station (*HS*) and

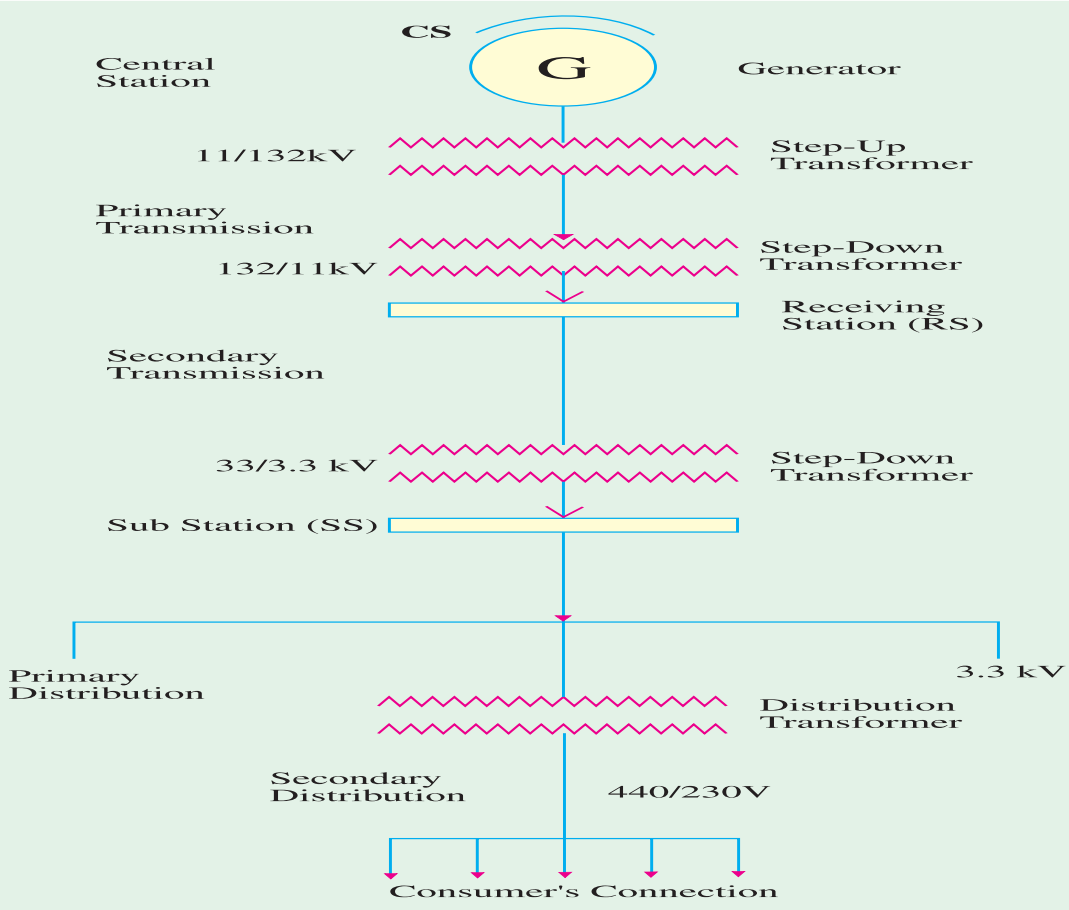


Fig. 41.2

two regional thermal power stations (*RTS*-1 and *RTS*-2). The stations *HS*, *RTS*-1 and *RTS*-2 are situated at large distances from the consumers, hence voltage of the electric power generated by them has to be stepped up by suitable transformers before it is fed into the system transmission network.

As shown in Fig. 41.2, *HS* is connected with the main 110-kV network of the system with the help of ***(i)*** two-220 kV transmission lines *L*-1 and ***(ii)*** main (regional) sub-station *A* which houses two 220/110 kV, 2-winding transformers for interconnecting the two transmission lines.

Transmission lines *L*-2, *L*-3 and *L*-4 constitute a high-voltage loop or ring mains. As seen, disconnnection of any one of the lines will not interrupt the connections between the elements of the system. Station *RTS*-1 feeds directly into the 110-kV line loop whereas *RTS*-2 is connected to the main network of the system by lines *L*-5 and *L*-6 through the buses of substations *B* and *C*. *HCS* is interconnected with 110-kV system through the buses of substation A by means of 10/110-kV transformers and line *L*-7.

It may be pointed out here that the main substations of the system are *A* and *B*. Substation *B* houses 3-winding transformers. The 35-kV supply is provided over quite large areas whereas 6-10 kV supply is supplied to consumers situated with a limited radius from the substations. Substations *C* and *D* are equipped with 2-winding transformers which operate at voltages indicated in the diagram. Substation C is known as a ***through substation*** whereas *D* is known as a *spur* or ***terminal*** substation.

Obviously, Fig. 41.2 shows only part of 220-kV and 110-kV lines and leaves out the 35, 10 and 6-kV circuits originating from the buses of the substations. Also, left out are low-voltage circuits for transmission and distribution (see Fig. 41.9).

41.3. Systems of A.C. Distribution

As mentioned earlier, a.c. power transmission is always at high voltage and mostly by 3-phase system. The use of single-phase system is limited to single-phase electric railways. Single-phase power transmission is used only for short distances and for relatively low voltages. As will be shown later, 3-phase power transmission requires less copper than either single-phase or 2-phase power transmission.

The distribution system begins either at the sub-station where power is delivered by over-head transmission lines and stepped down by transformers or in some cases at the generating station itself. Where a large area is involved, primary and secondary distributions may be used.

With respect to phases, the following systems are available for the distribution of a.c. power.

1. Single-phase, 2-wire system.
2. Single-phase, 3-wire system.
3. Two-phase, 3-wire system.
4. Two-phase, 4-wire system.
5. Three-phase, 3-wire system.
6. Three-phase, 4-wire system.



The above figure shows an electrical substation. A substation is a high-voltage electric system facility. It is used to switch generators, equipment, and circuits or lines in and out of a system. It is also used to change AC voltages from one level to another, and/or change alternating current to direct current or direct current to alternating current. Some substations are small with little more than a transformer and associated switches.

41.4. Single-phase, 2-wire System

It is shown in Fig. 41.3 (*a*) and (*b*). In Fig. 41.3 (*a*), one of the two wires is earthed whereas in Fig. 41.3 (*b*) mid-point of the phase winding is earthed.

41.5. Single-phase, 3-wire System

The 1-phase, 3-wire system is identical in principle with the 3-wire d.c. system. As shown in Fig. 41.4, the third wire or neutral is connected to the centre of the transformer secondary and earthed for protecting personnel from electric shock should the transformer insulation break down or the secondary main contact high voltage wire.

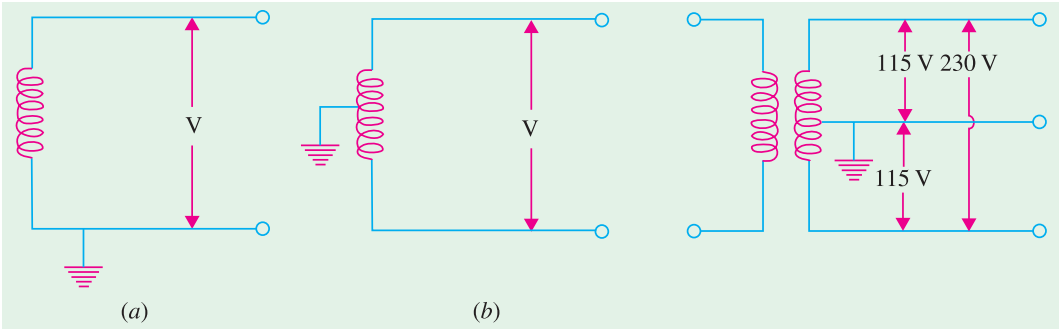


Fig. 41.3

Fig. 41.4

41.6. Two-phase, 3-wire System

This system is still used at some places. The third wire is taken from the junction of the two-phase windings I and II, whose voltages are in quadrature with each other as shown in Fig. 41.5. If the voltage between the third or neutral wire and either of the two wires is V, then the voltage between the outer wires is V as shown. As compared to 2-phase, 4-wire system, the 3-wire system suffers from the defect that it produces voltage unbalance because of the unsymmetrical voltage drop in the neutral.

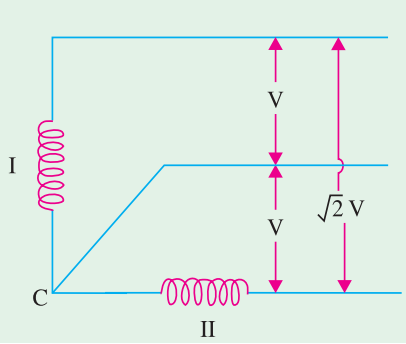


Fig. 41.5

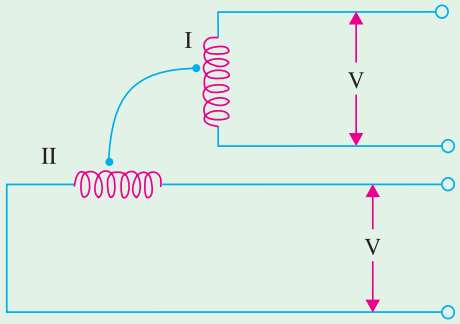


Fig. 41.6

41.7. Two-phase, 4-wire System

As shown in Fig. 41.6, the four wires are taken from the ends of the two-phase windings and the mid-points of the windings are connected together. As before, the voltage of the two windings are in quadrature with each other and the junction point may or may not be earthed. If voltage between the two wires of a phase winding be V, then the voltage between one wire of phase I and one wire of phase II is 0.707 V.

41.8. Three-phase, 3-wire System

Three-phase systems are used extensively. The 3-wire system may be delta-connected or star-connected whose star point is usually earthed. The voltage between lines is V in delta-connection and $\sqrt{3}\ V$ in case of star connection where V is the voltage of each phase as shown in Fig. 41.7 (*a*) and (*b*) respectively.

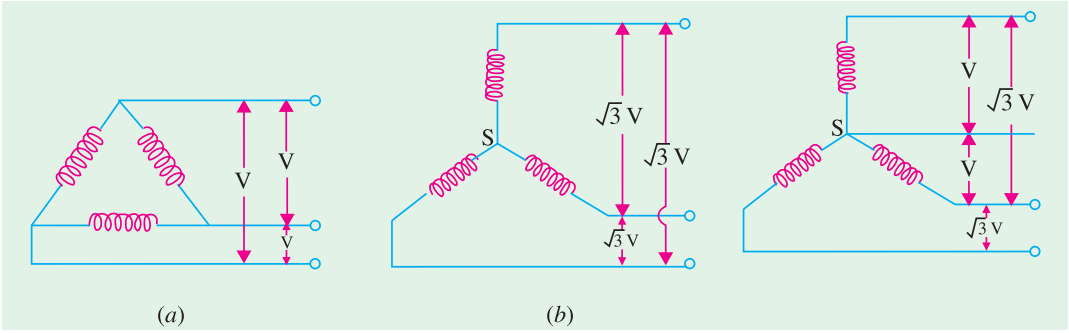


Fig. 41.7

Fig. 41.8

41.9. Three-phase, 4-wire System

The 4th or neutral wire is taken from the star point of the star-connection as shown in Fig. 41.8 and is of half the cross-section of the outers or line conductors. If V is the voltage of each winding,

then line voltage is $\sqrt{3}\ V$. Usually, phase voltage *i.e.* voltage between any outer and the neutral for a symmetrical system is 230 V so that the voltage between any two lines or outers is $\sqrt{3} \times 230 = 400$ V.

Single-phase residential lighting loads or single-phase motors which run on 230 V are connected between the neutral and any one of the line wires. These loads are connected symmetrically so that line wires are loaded equally. Hence, the resultant current in the neutral wire is zero or at least minimum. The three phase induction motors requiring higher voltages of 400 V or so are put across the lines directly.

41.10. Distribution

The distribution system may be divided into feeders, distributors, sub-distributors and service mains. As already explained in Art. 41.1, feeders are the conductors which connect the sub-station (in some cases the generating station) to the distributors serving a certain allotted area. From distributors various tappings are taken. The connecting link between the distributors and the consumers' terminals are the service mains. The essential difference between a feeder and a distributor is that whereas the current loading of a feeder is the same throughout its length, the distributor has a distributed loading which results in variations of current along its entire length. In other words, no direct tappings are taken from a feeder to a consumer's premises.

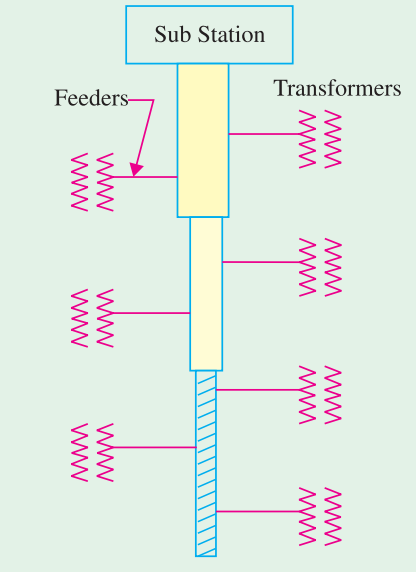


Fig. 41.9

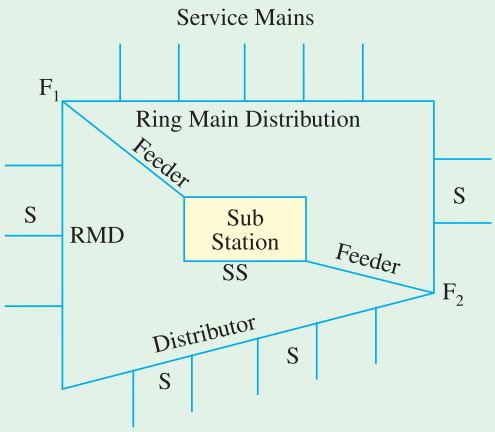


Fig. 41.10

In early days, ***radial*** distribution of tree-system type, as shown in Fig. 41.9, was used. In this system, a number of independent feeders branch out radially from a common source of supply *i.e.* a sub-station or generating station. The distribution transformers were connected to the taps along the length of the feeders. One of the main disadvantages of this system was that the consumer had to depend on one feeder only so that if a fault or breakdown occurred in his feeder, his supply of power was completely cut off till the fault was repaired. Hence, there was no absolute guarantee of continuous power supply.

For maintaining continuity of service, ring-main distributor (*R.M.D.*) system as shown in Fig. 41.10, is employed almost universally. *SS* represents the sub-station from which two feeders supply power to the ring-main distributor at feeding points F_1 and F_2. The ring-main forms a complete loop and has ***isolating*** switches provided at the poles at strategic points for isolating a particular section in case of fault. In this way, continuity of service can be maintained to other consumers on

healthy sections of the ring-main. The number of feeders of the ring-main depends on **(*i*)** the nature of loading—heavy or light **(*ii*)** the total length of the *R.M.D.* and **(*iii*)** on the permissible/allowable drop of voltage. Service mains (*S*) are taken off at various points of the *R.M.D.* Sometimes sub-distributors are also used. Since a loop or closed ring-main can be assumed to be equivalent to a number of straight distributors fed at both ends, the voltage drop is small which results in economy of conductor material. The service mains are the connecting link between the consumer's terminals and the *R.M.D.* or sub-distributor.

41.11. Effect of Voltage on Transmission Efficiency

Let us suppose that a power of W watt is to be delivered by a 3-phase transmission line at a line voltage of E and power factor cos ϕ.

The line current $I = \dfrac{W}{\sqrt{3}\, E \cos\phi}$

Let l = length of the line conductor ; σ = current density

ρ = specific resistance of conductor material, A = cross-section of conductor

then $A = \dfrac{I}{\sigma} = \dfrac{W}{\sqrt{3}\, E.\sigma.\cos\phi}$

Now $R = \dfrac{\rho l}{A} = \dfrac{\sqrt{3}\sigma\, \rho l\, E \cos\phi}{W}$

Line loss = 3 × loss per conductor = $3 I^2 R$

$$= 3\frac{W^2}{3E^2\cos^2\phi} \times \frac{\sqrt{3}\sigma\, \rho l \cos\phi}{W} = \frac{\sqrt{3}\sigma\rho l W}{E\cos\phi} \quad \text{...(1)}$$

$$\text{Line intake or input} = \text{output + losses} = W + \frac{\sqrt{3}\rho\sigma l W}{E\cos\phi} = W\left(1 + \frac{\sqrt{3}\sigma\rho l}{E\cos\phi}\right)$$

$\therefore$ efficiency of transmission

$$= \frac{\text{output}}{\text{input}} = \frac{W}{W\left(1 + \dfrac{\sqrt{3}\sigma\rho l}{E\cos\phi}\right)} = \left(1 - \frac{\sqrt{3}\sigma\rho l}{E\cos\phi}\right) \text{approx} \quad \text{...(2)}$$

$$\text{Voltage drop per line} = IR = \frac{\sqrt{3}\,\sigma\rho l\, E\cos\phi}{W} \times \frac{W}{\sqrt{3}\, E\cos\phi} = \sigma\rho l \quad \text{...(3)}$$

$$\text{Volume of copper} = 3\,lA = \frac{3Wl}{\sqrt{3}\,\sigma E\cos\phi} = \frac{\sqrt{3}Wl}{\sigma E\cos\phi} \quad \text{...(4)}$$

For a given value of transmitted power W, line length l, current density σ and specific resistance ρ of the given conductor material, the effect of supply voltage on transmission can be seen as follows :

1. From equation (1), line loss is inversely proportional to E. It is also inversely proportional to power factor, cos ϕ.
2. Transmission efficiency increases with voltage of transmission and power factor as seen from equation (2).
3. As seen from equation (3) for a given current density, the resistance drop per line is constant (since ρ and l have been assumed fixed in the present case). Hence, percentage drop is decreased as E is increased.
4. The volume of copper required for a transmission line is inversely proportional to the voltage and the power factor as seen from equation (4).

It is clear from the above that for long distance transmission of a.c. power, high voltage and high power factors are essential. But as pointed out earlier in Art. 41.1, economical upper limit of voltage is reached when the saving in cost of copper or aluminium is offset by the increased cost of insulation and increased cost of transformers and high-voltage switches etc. Usually, 650 volt per route km is taken as a rough guide.

41.12. Comparison of Conductor Materials Required for Various Overhead Systems

We will now calculate the amounts of conductor material required for various systems of a.c. power transmission. To do it without prejudice to any system, we will make the following assumptions :

1. Amount of power transmitted by each system is the same.
2. Distance of transmission is the same in each case.
3. Transmission efficiency is the same *i.e.* the losses are the same in each case.
4. Loads are balanced in the case of 3-wire systems.
5. Cross-section of the neutral wire is half that of any outer.
6. Maximum voltage to earth is the same in all cases.

By way of illustration a few cases will be compared, although the reader is advised to attempt others as well.

(*i*) Two-wire d.c. System and Three-phase, 3-wire System

Let in both cases, E be the maximum voltage between conductors and W the power transmitted.

In d.c. system, let R_1 be the resistance of each conductor and I_1 the current, then $I_1 = (W/E)$, hence losses in the two conductors are $= 2I_1^2 R_1 = 2\,(W/E)^2 R_1$ ...(*i*)

For a.c. system, let resistance per conductor be R_2 and power factor cos ϕ, then since E is the maximum voltage, the *R.M.S.* voltage is $E/\sqrt{2}$.

Current in each line, $$I_2 = \frac{W}{\sqrt{3}\,(E/\sqrt{2})\cos\phi}$$

Total losses in three conductors are,

$$3I_2^2 R_2 = \left[\frac{W}{\sqrt{3}\,(E/\sqrt{2})\cos\phi}\right]^2 R_2 = \frac{2W^2}{E^2}\times\frac{1}{\cos^2\phi}\cdot R_2 \quad \text{...(ii)}$$

Since transmission efficiency and hence losses are the same, therefore equating (*i*) and (*ii*), we get

$$\frac{2W^2 R_1}{E^2} = \frac{2W^2}{E^2}\cdot\frac{1}{\cos^2\phi}R_2 \quad \text{or} \quad \frac{R_1}{R_2} = \frac{1}{\cos^2\phi}$$

Since area of cross-section is inversely proportional to the resistance.

$$\therefore \quad \frac{\text{area of one a.c. conductor}}{\text{area of one d.c. conductor}} = \frac{1}{\cos^2\phi}$$

Now, for a given length, volumes are directly proportional to the areas of cross-section.

$$\therefore \quad \frac{\text{volume of one a.c. conductor}}{\text{volume of one d.c. conductor}} = \frac{1}{\cos^2\phi}$$

Keeping in mind the fact, that there are two conductors in d.c. system and three in a.c. system under consideration, we have

$$\frac{\text{total volume in 3-wire a.c. system}}{\text{total volume in 2-wire d.c. system}} = \frac{1}{\cos^2\phi}\cdot\frac{3}{2} = \frac{1.5}{\cos^2\phi}$$

(*ii*) Three-phase, 4-wire and 3-wire d.c. Systems

The neutral conductor of each system is earthed. Let E be the maximum voltage to earth and W the power to be transmitted in both cases.

For d.c. 3-wire system, voltage between outers is $2E$. If I_1 and R_1 are the current in and resistance of each conductor, then $I_1 = W/2E$. Assuming a balanced load (in which case there would be no current flowing in the neutral conductor), the value of loss in two conductors is

$$2I_1^2R_1 = 2(W/2E)^2R_1 = W^2R_1/2E^2$$

In the case of a.c. system, if E is the maximum voltage between any wire and neutral, then its R.M.S. value is $E/\sqrt{2}$. Hence, the line voltage = $\sqrt{3} \cdot E/\sqrt{2}$. The line current is

$$I_2 = \frac{W}{\sqrt{3} \times (\sqrt{3}\, E/\sqrt{2}) \cos \phi}$$

where $\cos \phi$ is the power factor. If R_2 is the resistance of each line, then total loss in 3 lines is

$$= 3I_{2R_2}^2 = 3 \times \left[\frac{W}{\sqrt{3}\,(\sqrt{3}E/\sqrt{2}) \cos \phi}\right]^2 \times R_2 = \frac{2\,W^2\,R}{3E^2 \cos^2 \phi}$$

For equal transmission efficiencies, the two losses should be the same.

$$\therefore \quad \frac{W^2R_1}{2E^2} = \frac{2\,W^2R_1}{3E^2 \cos^2 \phi} \quad \text{or} \quad \frac{R_1}{R_2} = \frac{4}{3 \cos^2 \phi}$$

Since volume is directly proportional to the area of cross-section which is itself inversely proportional to the resistance, hence

$$\frac{\text{volume of one a.c. conductor}}{\text{volume of one d.c. conductor}} = \frac{4}{3 \cos^2 \phi}$$

Assuming the neutral wires of each system to have half the cross-section of the outers, there is 3.5 times the quantity of one conductor in a.c. and 2.5 times in the d.c. system. Hence

$$\frac{\text{total volume in the a.c. system}}{\text{total volume in the d.c. system}} = \frac{3.5}{2.5} \times \frac{4}{3\cos^2 \phi} = \frac{1.867}{\cos^2 \phi}$$

In Table 41.1 are given the ratios of copper in any system as compared with that in the corresponding d.c. 2-wire system which has been alloted a basic number of 100.

Table No. 41.1

System	*With same maximum voltage to earth*	*With same maximum voltage between conductors*
D.C. 2-wire	100	100
D.C. 2-wire Mid-point earthed	25	100
D.C. 3-wire Neutral = ½ outer	31.25	125
D.C. 3-wire Neutral = ½ outer	37.5	150
Single-phase, 2-wire	$200/\cos^2 \phi$	$200/\cos^2 \phi$
Single-phase, 2-wire Mid-point earthed	$50/\cos^2 \phi$	$200/\cos^2 \phi$
Single-phase, 3-wire Neutral = ½ outer	$62.5/\cos^2 \phi$	$250/\cos^2 \phi$
Two-phase, 4-wire	$50/\cos^2 \phi$	$200/\cos^2 \phi$

Two-phase, 3-wire	$146/\cos^2\phi$	$291/\cos^2\phi$
Three-phase, 3-wire	$50/\cos^2\phi$	$150/\cos^2\phi$
Three-phase, 4-wire Neutral = outer	$67/\cos^2\phi$	$200/\cos^2\phi$

A comparison of costs for underground systems employing multicore cables can also be worked out on similar lines. In these case, however, maximum voltage between the conductors is taken as the criterion.

It would be clear from the above discussion, that for a.c. systems, the expression for volume contains $\cos^2\phi$ in denominator. Obviously, reasonable economy in copper can be achieved only when $\cos\phi$ approaches unity as closely as possible. For equal line loss, the volume of copper required for $\cos\phi = 0.8$ will be $1/0.8^2 = 1.6$ times that required for $\cos\phi = 1$.

Since the cost of distribution system represents a large percentage of the total capital investment of power supply companies, it is quite reasonable that a consumer should be penalized for low power factor of his load. This point is fully dealt with in Chapt. 47.

Example 41.1. *A 3-phase, 4-wire system is used for lighting. Compare the amount of copper required with that needed for a 2-wire D.C. system with same line voltage. Assume the same losses and balanced load. The neutral is one half the cross-section of one of the respective outers.*

Solution. (*a*) Two-wire *DC* Let V = voltage between conductors

P = power delivered, R_1 = resistance/conductor

Current $I_1 = P/V$

power loss $= 2I_1^2R_1 = 2P^2R_1/V^2$

(*b*) Three-phase, 4-wire

Let V be the line-to-neutral voltage and R_2 resistance of each phase wire.

$$P = 3VI_2\cos\phi = 3VI_2 \quad \text{—if } \cos\phi = 1$$

$$\text{Power loss} = 3I_2^2R_2 = 3(P/3V)^2R_2 = P^2R_2/3V^2$$

Since power losses in both systems are equal

$$\therefore \quad 2P^2R_1/V^2 = P^2R_2/3V^2 \quad \text{or} \quad R_1/R_2 = 1/6$$

If A_1 and A_2 are the cross-sectional areas of conductors in the two systems, then $A_1/A_2 = 6$. Because $R \propto l/A$

$$\therefore \quad \text{Cu reqd. for 2-wire system} = 2A_1l$$

$$\text{Cu reqd. for 3-}\phi\text{, 4-wire system} = (3A_2l + A_2l/2)$$

$$\therefore \quad \frac{\text{Cu for } 3-\phi \text{ system}}{\text{Cu for d.c. system}} = \frac{3.5\,A_2l}{2\,A_1l} = \frac{3.5}{2}\times\frac{1}{6} = 0.292$$

$$\text{Cu for 3-}\phi\text{ system} = \mathbf{0.292 \times Cu \text{ for d.c. system.}}$$

Example 41.2. *Estimate the weight of copper required to supply a load of 100 MW at upf by a 3-phase, 380-kV system over a distance of 100 km. The neutral point is earthed. The resistance of the conductor is 0.045 ohm/cm^2/km. The weight of copper is 0.01 kg/cm^3. The efficiency of transmission can be assumed to be 90 percent.* **(Power Systems, AMIE, Sec. B, 1994)**

Solution. Power loss in the line = $(1 - 0.9) \times 100 = 10$ MW

$$\text{Line current} = 100 \times 10^6/\sqrt{3} \times 380 \times 10^3 \times 1 = 152 \text{ A}$$

Since I^2R loss in 3-conductors is 10×106 W, loss per conductor is $= 10 \times 10^6/3 = 3.333 \times 10^6$ W

Resistance per conductor $= 10 \times 10^6/3 \times 152^2 = 144.3\ \Omega$

Resistance per conductor per km = 144.3/100 = 1.443 Ω

$\therefore$ Conductor cross-section = $0.045/1.443 = 0.03\ m^3$

Volume of copper per meter run = $0.03 \times 100 = 3\ cm^3$

Weight of copper for 3-conductor for 100 km length = $3 \times (3 \times 0.01) \times 100 \times 1000 =$ **9000 kg**

Example 41.3. *A d.c. 2-wire distribution system is converted into a.c. 3-phase, 3-wire system by adding a third conductor of the same size as the two existing conductors. If voltage between conductors and percentage power loss remain the same, calculate the percentage additional balanced load which can now be carried by the conductors at 0.95 p.f.*

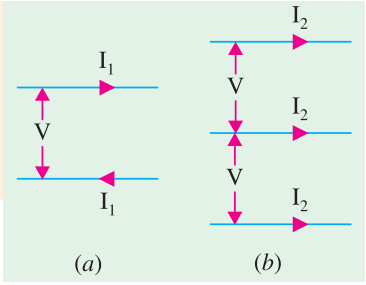


Fig. 41.11

Solution. (*a*) *DC* 2-wire system [Fig. 40.11 (*a*)]

If R is the resistance per conductor, then power transmitted is $P = VI_1$ and power loss = $2I_1 2R$. Percentage power loss

$= 2I_1^2R \times 100/VI_1$

$= 2I_1R \times 100/V$

(*b*) as 3-phase, 3-wire system [Fig. 40.11 (*b*)]

$P_2 = \sqrt{3}\, VI_2 \cos\phi$, power loss = $3I_2^2R$

% power loss $= 3I_2^2R/P_2 = \sqrt{3}I_2R \times 100\ V \cos\phi$

Since losses in the two cases are the same

$\therefore \quad 2I_1R \times 100/V = \sqrt{3}I_2R \times 100/V \cos\phi$ or $I_2 = 2\cos\phi \times I_1/\sqrt{3}$

$\therefore \quad P_2 = \sqrt{3}.V.2\cos^2\phi \times I_1/\sqrt{3} = 2VI_1\cos^2\phi = 2VI_1(0.95)^2 = 1.8\ VI_1$

Percentage additional power transmitted in a 3-phase, 3-wire system

$$= \frac{P_2 - P_1}{P_1} \times 100 = \frac{1.8\,VI_1 - VI_1}{VI_1} \times 100 = \mathbf{80\%}$$

Example 41.4. *A 2-phase, 3-wire a.c. system has a middle conductor of same cross-sectional area as the outer and supplies a load of 20 MW. The system is converted into 3-phase, 4-wire system by running a neutral wire. Calculate the new power which can be supplied if voltage across consumer terminal and percentage line losses remain the same. Assume balanced load.*

Solution. The two systems are shown in Fig. 41.12. Let R be the resistance per conductor.

$P_1 = 2VI_1 \cos\phi$; Cu loss, $W_1 = 2I_1^2R$

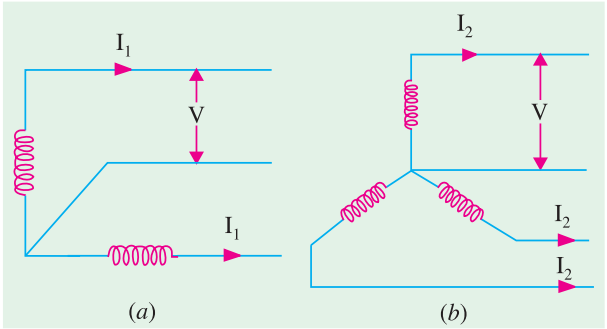


Fig. 41.12

Percentage Cu loss

$= (W_1/P_1) \times 100$

$= (2I^2R/2VI_1 \cos\phi) \times 100$

$= I_1R \times 100/V \cos\phi$

$P_2 = 3VI_2 \cos\phi$

$W_2 = 3I_2^2R$

% line loss $= (W_2/P_2) \times 100$

$= (3I_2^2R/3VI_2 \cos\phi) \times 100$

$= I_2R \times 100/V \cos\phi$

Since percentage line losses are the same in both cases

$\therefore \quad I_1R \times 100/V \cos\phi = I_2R \times 100/V \cos\phi \quad \therefore \quad I_1 = I_2$

Now, $P_1 = 2VI_1 \cos\phi = \mathbf{20\ MW}$; $\quad \therefore \quad VI_1 \cos\phi = 10$ MW

$P_2 = 3VI_2 \cos\phi = 3VI_1 \cos\phi = 3 \times 10 = \mathbf{30\ MW}$

41.13. Constants of a Transmission Line

A transmission line not only has an ohmic resistance but also inductance and capacitance between its conductors. These are known as the constants of a transmission line. While calculating the drop in a.c. transmission and distribution circuits, we will have to consider ***(i)*** resistive or ohmic drop—in phase with the current ***(ii)*** inductive drop—leading the current by 90° and ***(iii)*** the capacitive drop and charging current taken by the capacitance of the line. The capacitance and hence the charging current is usually negligible for short transmission lines.

41.14. Reactance of an Isolated Single-Phase Transmission Line

In Fig. 41.13 (*a*) are shown the cross-sections of two conductors of a single-phase line spaced *D* from centre to centre.

Since currents through the two conductors will, at any time, always be flowing in opposite directions, the fields surrounding them always reinforce each other in the space between them as shown in Fig. 41.13.

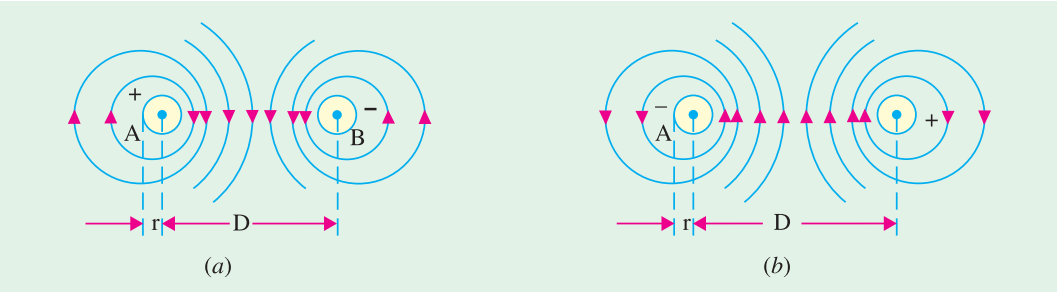


Fig. 41.13

The two parallel conductors form a rectangular loop of one turn through which flux is produced by currents in the two conductors. Since this flux links the loop, the loop possesses inductance. It might be thought that this inductance is negligible because the loop has only one turn and the entire flux-path lies through air of high reluctance. But as the cross-sectional area of the loop is large, from 1 to 10 metre wide and several km long, even for a small flux density, the total flux linking the loop is large and so inductance is appreciable.

It can be proved that inductance per loop metre (when $r \leq D$) is ...***(i)***

$$L = \frac{\mu}{\pi} \log_e D/r + \frac{\mu_i}{4\pi} \text{ henry/metre}$$

where μ = absolute permeability of the surrounding medium

μ_i = absolute permeability of the conductor material

Now, $\mu = \mu_0 \mu_r$ and $\mu_i = \mu_0 \mu_r$ where μ_r and μ_r' are the relative permeabilities of the surrounding medium and the conductor material. If surrounding medium is air, then $\mu_r = 1$. Also, if conductor is made of copper, then $\mu_r' = 1$. Hence, the above expression becomes.

$$L = \left(\frac{\mu_0 \mu_r}{\pi} \log h\ D/r + \frac{\mu_0 \mu_r'}{4\pi} \right) \text{H/m}$$

$$= \frac{\mu_o}{4\pi} (\log h\,D/r + 1)\ H/m = 10^{-7} (1 + \log h\,Dr)\ \text{H/m}$$

Loop reactance $X = 2\pi f L$ ohm/metre

Obviously, the inductance of each single conductor is half the above value*.

$$\text{inductance/conductor} = \frac{1}{2} (1 + \log h\,D/r) \times 10^{-7}\ \text{H/m} \qquad ...(ii)$$

$$\text{reactance/conductor} = 2\pi f \times \frac{1}{2} (1 + \log h\,D/r) \times 10^{-7}\ \Omega/\text{m}$$

41.15. Reactance of 3-phase Transmission Line

In 3-phase transmission, it is more convenient to consider the reactance of each conductor instead of the looped line or of the entire circuit. Two cases will be considered for 3-phase lines.

(*i*) Symmetrical Spacing

In Fig. 41.14 (*a*) are shown the three conductors spaced symmetrically *i.e.* each conductor is at the apex of the same equilateral triangle *ABC* of side *D*. The inductance per conductor per metre is found by using equation **(*ii*)** in Art 41.14 because it can be shown that inductance per km of one conductor of 3-phase circuit is the same as the inductance per conductor per km of single-phase circuit with equivalent spacing.

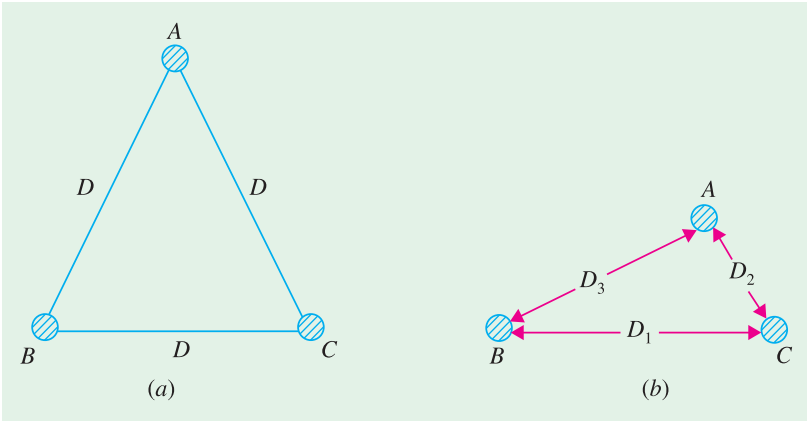


Fig. 41.14

(*ii*) Unsymmetrical Spacing

In Fig. 41.14 (*b*) is shown a 3-phase circuit with conductors placed unsymmetrically. In this case also, the inductance is given by equation **(*ii*)** of Art. 41.14 with the only modification that *D* is put equal to $\sqrt[3]{(D_1\ D_2\ D_3)}$.

41.16. Capacitance of a Single-phase Transmission Line

We know that any two conductors which are separated by an insulating medium constitute a capacitor. When a potential difference is established across two such conductors, the current flows in at one conductor and out at the other so long as that p.d. is maintained. The conductors of an overhead transmission line fulfil these conditions, hence when an alternating potential difference is applied across a transmission line, it draws a leading current even when it is unloaded. This leading current is in quadrature with the applied voltage and is known as the ***charging current***. Its value depends upon voltage, the capacitance of the line and the frequency of alternating current.

As shown in Art 5.10, the capacitance between conductors of a single-phase line is approximately given by

* It may be noted that standard conductors have a slightly higher inductance.

$$C = \frac{\pi \in}{\log h\, D/r} = \frac{\pi \in_0 \in_r}{2.3 \log_{10} D/r} \qquad F/m = \frac{0.0121 \in_r}{\log_{10} D/r}\ \mu F/\text{km}$$

Here, D is the distance between conductor centres and r the radius of each conductor, both expressed in the same units (Fig. 41.15).

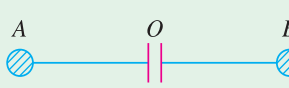


Fig. 41.15

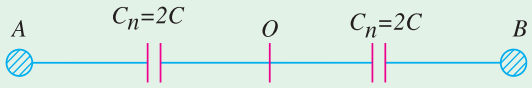


Fig. 41.16

As shown in Fig. 41.16, the capacitance to neutral $C_n = 2C$ where point O is the neutral of the system. Obviously, the total capacitance between conductors A and B is given by finding the resultant of two capacitances each of value C_n joined in series, the resultant, obviously, being equal to C.

It is important to remember that if capacitance to neutral is used for calculating the charging current, then voltage to neutral must also be used.

$$\therefore \qquad C_n = 2C = \frac{2\pi \in}{\log_e D/r}\ \text{F/m} \qquad \text{...(ii)}$$

$$\therefore \qquad \text{line charging current} = \frac{V}{X_C} = \frac{V}{1/(2\pi f C_n)} = 2\pi\ f C_n V\ \text{A/m}$$

where V is the ***voltage to neutral***.

However, it may be noted that ground effect has been neglected while deriving the above expression. This amounts to the tacit assumption that height h of the conductors is very large as compared to their spacing 'd'. In case ground effect is to be taken into account, the expression for capacitance becomes.

$$C_n = \frac{2\pi \in}{\log h \dfrac{d}{r\sqrt{1 + d^2/4h^2}}}\ \text{F/m} \qquad \text{(Ex. 41.7)}$$

Example 41.5. *What is the inductance per loop metre of two parallel conductors of a single phase system if each has a diameter of 1 cm and their axes are 5 cm apart when conductors have a relative permeability of* ***(a)*** *unity and* ***(b)*** *100. The relative permeability of the surrounding medium is unity in both cases. End effects may be neglected and the current may be assumed uniformly distributed over cross-section of the wires.*

Solution. ***(a)*** Here, $\mu = \mu_0 = 4\pi \times 10^{-7}$ H/m ; $\mu_i = 1$

$$\therefore \qquad L = \frac{4\pi \times 10^{-7}}{\pi}\left(\log h^5/0.5 + \frac{1}{4}\right) = \mathbf{1.02\ \mu H/m}$$

(b) Here, $\mu = \mu_0 = 4\pi \times 10^{-7}$; $\mu_i = 100\ \mu_0$

$$L = \frac{4\pi \times 10^{-7}}{\pi}\left(\log h^5/0.5 + \frac{100}{4}\right) = \mathbf{10.9\ \mu H/m}$$

Example 41.6. *A 20-km single-phase transmission line having 0.823 cm diameter has two line conductors separated by 1.5 metre. The conductor has a resistance of 0.311 ohm per kilometre. Find the loop impedance of this line at 50 Hz.* **(Gen. Trans. & Dist. Bombay Univ. 1992)**

Solution. Loop length $= 20\ \text{km} = 2 \times 10^4$ m

Total loop inductance is $L = 2 \times 10^4 \left(\dfrac{\mu}{\pi} \log_e D/r + \dfrac{\mu_i}{4\pi}\right)$ henry

Here, $D = 1.5$ m ; $r = 0.823/2 = 0.412$ cm $= 4.12 \times 10^{-3}$ m

Assuming $\mu_r = 1$ for copper conductors since they are non-magnetic and also taking $\mu_r = 1$ for air, we have

$$L = 2\times10^4\left(\frac{\mu}{\pi}\log_e D/r + \frac{\mu_i}{4\pi}\right) = \frac{2\times10^4\times4\pi\times10^{-7}}{\pi}\left(\log_e^{1.5/4.12\times10^{-3}} + \frac{1}{4}\right)\text{H}$$

$$= 8\times10^{-3}\,(5.89 + 0.25) = 49.12\times10^{-3}\text{ H}$$

Reactance $X = 2\pi\times50\times49.12\times10^{-3} = 15.44\ \Omega$; Loop resistance $= 2\times20\times0.311 = 12.44\ \Omega$

$$\text{Loop impedance} = \sqrt{12.44^2 + 15.44^2} = \mathbf{19.86\ \Omega}$$

Example 41.7. *The conductors in a single-phase transmission line are 6 m above ground. Each conductor has a diameter of 1.5 cm and the two conductors are spaced 3 m apart. Calculate the capacitance per km of the line* ***(i)*** *excluding ground effect and* ***(ii)*** *including the ground effect.*

Solution. The line conductors are shown in Fig. 41.17.

(i) $$C_n = \frac{2\pi\epsilon_0}{\log h\,d/r} = \frac{2\pi\times8.854\times10^{-12}}{\log h\,3/0.75\times10^{-2}}\text{ F/m}$$

$$= 9.27\times10^{-12}\text{ F/m} = \mathbf{9.27\times10^{-3}\ \mu F/km}$$

(ii) In this case,

$$C_n = \frac{2\pi\epsilon_0}{\log h\dfrac{d}{r\sqrt{1+d^2/4h^2}}}\text{ F/m}$$

$$= \frac{2\pi\times8.854\times10^{-12}}{\log h\dfrac{3}{0.75\times10^{-12}\sqrt{1+3^2/4\times6^2}}}$$

$$= 9.33\times10^{-12}\text{ F/m} = \mathbf{9.33\times10^{-3}\ mF/km}$$

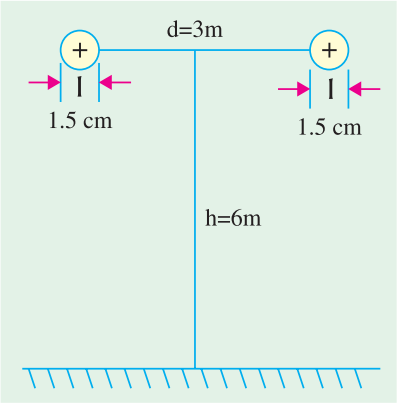


Fig. 41.17

It is seen that line capacitance when considering ground effect is more (by about 0.64% in the present case).

41.17. Capacitance of a Three-phase Transmission Line

In Fig. 41.18 (*a*) are shown three conductors spaced symmetrically and having a capacitance of *C* between each pair. The value of ***C i.e. line-to-line*** capacitance is the same as given by equation ***(i)*** in Art. 41.16.

Considering the two conductors *A* and *B*, their line capacitance *C* can be regarded as consisting of two line-to-neutral capacitances connected in series, so that for the line-to-neutral capacitance of *A* with respect to neutral plane 1, as in Art. 41.16, we have

$$C_{n1} = \frac{2\pi\epsilon}{\log_e D/r}\text{ F/m}$$

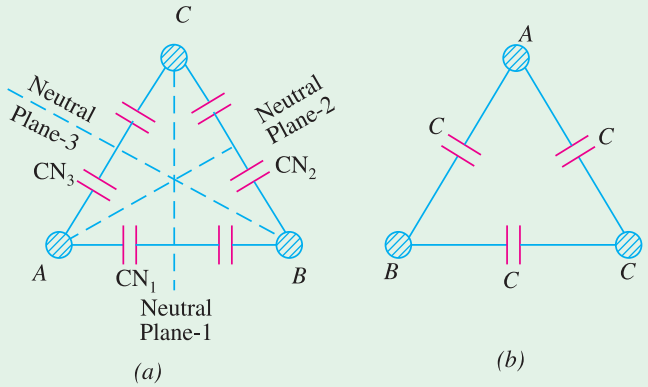


Fig. 41.18

Similarly, line-to-neutral capacitance of conductor A with respect to neutral plane 3 is

$$C_{n3} = \frac{2\pi\epsilon}{\log_e D/r} \text{ F/m}$$

The vector sum of these capacitances is equal to either because their phase angle is 120° *i.e.* the phase angle of the voltages which cut across them.

Hence, the capacitance to neutral per metre of one conductor of a 3-phase transmission line is

$$C_n = \frac{2\pi\epsilon}{\log_e D/r} \text{ F/m}$$

In case the conductors are placed asymmetrically, the distance used is $D = \sqrt[3]{(D_1 D_2 D_3)}$.

41.18. Short Single-phase Line Calculations

Single-phase circuits are always short and work at relatively low voltages. In such cases, the line constants can be regarded as 'lumped' instead of being distributed and capacitance can be neglected.

Let E_S = voltage at sending end ; E_R = voltage at receiving end

I = line current $\cos\phi_R$ = power factor at receiving end

R = resistance of both conductors; X = reactance of both conductors = ωL

Then, the equivalent circuit of a short single-phase line is as shown in Fig. 41.19.

Resistive drop = IR —in phase with I

Reactive drop = IX —in quadrature with I

The vector diagram is shown in Fig. 41.20. From the right-angled triangle OMA, we get

$$OM^2 = OA^2 + AM^2 = (OK + KA)^2 + (AC + CM)^2$$

$$E_S = \sqrt{[(E_R \cos\phi_R + IR)^2 + (E_R \sin\phi_R + IX)^2]}$$

An approximate expression for the voltage drop is as follows :

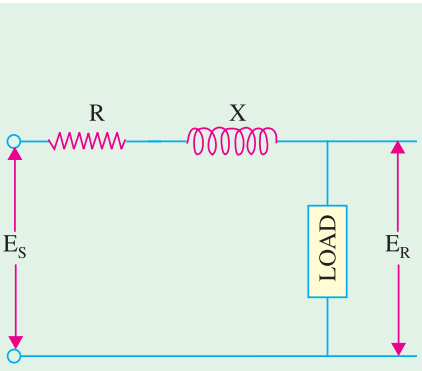


Fig. 41.19

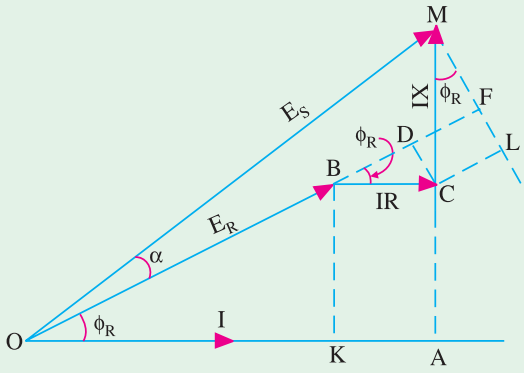


Fig. 41.20

Draw the various dotted lines as in Fig. 41.20. Here, ML is $\perp OF$, CL is $\| OF$ and CD is $\perp OF$. As seen

$OM = OF$ (approximately).

$= OD + DF = OB + BD + DF$

or $E_S = E_R + BD + DF = E_R + BD + CL = E_R + IR\cos\phi_R + IX\sin\phi_R$

or $E_S - E_R = IR\cos\phi_R + IX\sin\phi_R \quad \therefore$ drop $= I(R\cos\phi_R + X\sin\phi_R)$ (approx.)

Voltage regn. of line $= [(E_S - E_R)/E_R] \times 100$

Solution in Complex Notation

Let us take E_R as reference vector as shown in Fig. 41.21 so that $E_R = (E_R + j_0)$

As seen from ΔOAB, E_S is equal to the vector sum of E_R and IZ or $\mathbf{E_S = E_R + IZ}$

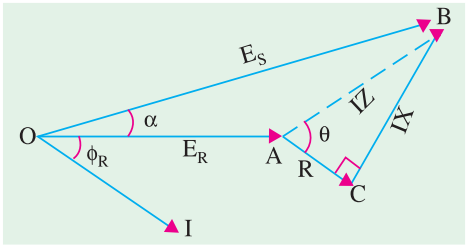


Fig. 41.21

Now, $\mathbf{I} = I \angle -\phi_R = I(\cos\phi_R - j\sin\phi_R)$

Similarly, $\mathbf{Z} = Z \angle \theta = (R + jX)$

$\therefore \quad \mathbf{E_S} = \mathbf{E_R} + \mathbf{I_Z} \angle \theta - \phi_R$

or $\mathbf{E_S} = \mathbf{E_R} + \mathbf{I}(\cos\phi_R - j\sin\phi_R)(R + jX)$

$= \mathbf{E_R} + (IR\cos\phi_R + IX\sin\phi_R) + j(IX\cos\phi_R - I_R\sin\phi_R)^2$

$\therefore \quad E_S = \sqrt{(E_R + IR\cos\phi_R + IX\sin\phi_R)^2 + (IX\sin\phi_R - IR\sin\phi_R)^2}$

If the p.f. is leading, then

$\mathbf{I} = I \angle \phi_R = I(\cos\phi_R + j\sin\phi_R)$

$\therefore \quad \mathbf{E_S} = \mathbf{E_R} + \mathbf{IZ} \angle\theta + \phi_R = \mathbf{E_R} + I(\cos\phi_R + j\sin\phi_R)(R + jX)$

It may be noted that the sending end power factor angle is $\phi_s = (\phi_R + \alpha)$.

Example 41.8. *A single-phase line has an impedance of $5\angle 60°$ and supplies a load of 120 A, 3,300 V at 0.8 p.f. lagging. Calculate the sending-end voltage and draw a vector diagram.*

(City & Guides, London)

Solution. The vector diagram is similar to that shown in Fig. 41.21. Here

$\mathbf{E_R} = 3{,}300 \angle 0°, \quad \mathbf{Z} = 5 \angle 60°$

Since $\phi_R = \cos^{-1}(0.8) = 36°52', \quad \therefore \quad \mathbf{I} = 120 \angle -36°52'$

Voltage drop $= \mathbf{IZ} = 120 \times 5 \angle 60° - 36°52'$

$= 600 \angle 23°8' = 600(0.9196 + j0.3928) = 551.8 + j\,235.7$

$\therefore \quad \mathbf{E_S} = (3{,}300 + j_0) + (551.8 + j235.7) = 3{,}851.8 + j235.7$

$\mathbf{E_S} = \sqrt{3851.8^2 + 235.7^2} = \mathbf{3{,}860\ V}$

Example 40.9. *An overhead, single-phase transmission line delivers 1100 kW at 33 kV at 0.8 p.f. lagging. The total resistance of the line is 10 Ω and total inductive reactance is 15 Ω. Determine **(i)** sending-end voltage **(ii)** sending-end p.f. and **(iii)** transmission efficiency.*

(Electrical Technology-I, Bombay Univ.)

Solution. Full-load line current is $I = 1100/33 \times 0.8 = 41.7$ A

Line loss $= I^2R = 41.7^2 \times 10 = 17{,}390$ W $= 17.39$ kW

(iii) Transmission efficiency $= \dfrac{\text{output}}{\text{output + losses}} = \dfrac{1100 \times 100}{1100 + 17.39} = \mathbf{98.5\%}$

(i) Line voltage drop $= \mathbf{IZ} = 41.7(0.8 - j\,0.6)(10 + j\,15) = 709 + j\,250$

Sending-end voltage is

$\mathbf{E_S} = \mathbf{E_R} + \mathbf{I_Z} = (33{,}000 + j\,0) + (709 + j\,250)$

$= 33{,}709 + j\,250 = 33{,}710 \angle 25°$

Hence, sending-end voltage is 33.71 kV

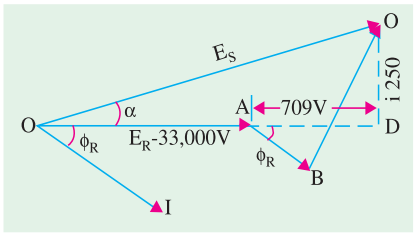


Fig. 41.22

(ii) As seen from Fig. 41.22, $\alpha = 0°25'$

Sending-end p.f. angle is

$\theta + \alpha = 36°52' + 0°25' = 37°17'$

$\therefore \quad$ p.f. $= \cos 37°17' = \mathbf{0.795\ (lag).}$

Note. As seen from Fig. 41.22, approximate line drop is

$= I(R\cos\phi_R + X\sin\phi_R) = 41.7\,(10\times0.8 + 15\times0.6) = 709$ V

$\therefore \quad E_S = 33{,}000 + 709 = 33{,}709$ V —as above

Example 41.10. *What is the maximum length in km for a 1-phase transmission line having copper conductors of 0.775 cm^2 cross-section over which 200 kW at unity power factor and at 3300 V can be delivered ? The efficiency of transmission is 90 per cent. Take specific resistance as (1.725×10^{-8}) Ω-m.* **(Electrical Technology, Bombay Univ.)**

Solution. Since transmission efficiency is 90 per cent, sending-end power is 200/0.9 kW.

Line loss $= (200/0.9 - 200) = 22.22$ kW $= 22{,}220$ W

Line current $= 200{,}000/3300 \times 1 = 60.6$ A

If R is the resistance of one conductor, then

$2I^2R$ = line loss or $2\times60.62\times R = 22{,}220$ or $R = 3.03$ W

Now, $R = \rho\dfrac{1}{A}$ $\therefore$ $3.03 = 1.725\times10^{-8}\times l/0.775\times10^{-4}$ $\therefore$ $l = 13{,}600$, m = **13.6 km**

Example 41.11. *An industrial load consisting of a group of induction motors which aggregate 500 kW at 0.6 power factor lagging is supplied by a distribution feeder having an equivalent impedance of (0.15 + j0.6) ohm. The voltage at the load end of the feeder is 2300 volts.*

(a) *Determine the load current.*

(b) *Find the power, reactive power and voltampere supplied to the sending end of the feeder.*

(c) *Find the voltage at the sending end of the feeder.*

(Electrical Technology, Vikram Univ., Ujjain)

Solution. *(a)* $I = 500\times103/\sqrt{3} = 209$ A

(c) Let $V_R = (2300 + j\,0)$; $I = 209\,(0.6 - j\,0.8)$

Voltage drop $= \mathbf{IZ} = 209\,(0.6 - j\,0.8)\,(0.15 + j\,0.6) = 119 + j50$

$\mathbf{V_S} = \mathbf{V_R + IZ} = 2300 + 119 + j\,50 = 2419 + j\,50 = 2420\angle1.2'$

(b) Sending power $= \sqrt{3}\times2420\times209\times0.5835 =$ **511.17 kW**

Sending-end reactive power $= \sqrt{3}\times2420\times209\times0.8121 =$ **711.42 kVAR**

Sending-end volt ampere kVA $= \sqrt{3}\times2420\times209 =$ **876 kVA**

or kVA $= \sqrt{\text{kW}^2 + \text{kVAR}^2} = \sqrt{511.17^2 + 711.42^2} =$ **876**

41.19. Short Three-phase Transmission Line Constants

Three-phase calculations are made in the same way as single-phase calculations because a 3-phase unit can be regarded as consisting of 3 single-phase units each transmitting one-third of the total power. In Fig. 41.23 is shown a 3-phase system in which each conductor has a resistance of $R\Omega$ and inductance of $X\Omega$. In the case of a 3-phase system, it is advantageous to work in phase instead of in line-to-line values. In Fig. 41.24, one phase is shown separately. Since in a balanced load, there is no current in the neutral wire hence no current flows through the ground.

The resistive and reactive drops are calculated as before. By adding these drops ***vectorially*** to the receiving-end voltage, the sending-end voltage can be calculated. The line voltage at the sending-end can be found by multiplying this value by $\sqrt{3}$.

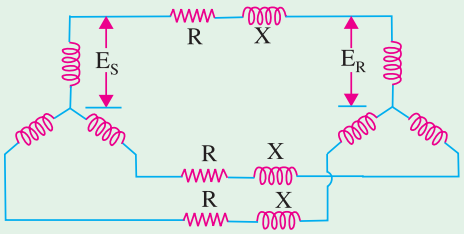


Fig. 41.23

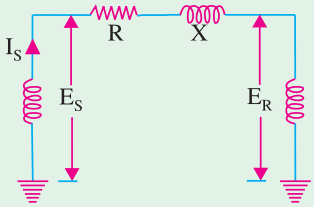


Fig. 41.24

In the figure, a star-connected load is assumed. But if it is a delta-connected load then it can be replaced by an equivalent star-connected load.

Example 41.12. *A 33-kV, 3-phase generating station is to supply 10 MW load at 31 kV and 0.9 power factor lagging over a 3-phase transmission line 3 km long. For the efficiency of the line to be 96% , what must be the resistance and reactance of the line?*

(Electrical Power-III, Bangalore Univ.)

Solution. Power output = 10 MW ; $\eta = 0.96$; Power input = 10/0.96 = 10.417 MW

Total loss = 0.417 MW

Now, $I = 10 \times 106/\sqrt{3} \times 31 \times 103 \times 0.9 = 207$ A

If R is the resistance per phase, then $3 \times 2072 \times R = 0.417 \times 106$ $\therefore$ $R = \mathbf{3.24\ \Omega}$

Now, V_S per phase = $33/\sqrt{3} = 19.052$ kV and V_R per phase = $31/\sqrt{3} = 17.898$ kV

Using the approximate relation of Art. 36-18, we get

$$V_S = V_R + I(R\cos\phi_R + X\sin\phi_R)$$

$$19{,}052 = 17{,}898 + 207(3.24 \times 0.9 + X \times 0.4368) \quad \therefore X = \mathbf{6.1\ \Omega/phase.}$$

Example 41.13. *A balanced Y-connected load of (300 + j100) Ω is supplied by a 3-phase line 40 km long with an impedance of (0.6 + j0.7) Ω per km (line-to-neutral). Find the voltage at the receiving end when the voltage at the sending end is 66 kV. What is the phase angle between these voltages? Also, find the transmission efficiency of the line.*

(Elect. Power Systems, Gujarat Univ.)

Solution. The circuit connections are shown in Fig. 41.25.

Resistance for 40 km conductor length = $40 \times 0.6 = 24\ \Omega$

Reactance for 40 km conductor length = $40 \times 0.7 = 28\ \Omega$

Total resistance/phase = $300 + 24 = 324\ \Omega$

Total reactance/phase = $100 + 28 = 128\ \Omega$

Total impedance/phase = $\sqrt{324^2 + 128^2} = 348\ \Omega$

Line current = $\dfrac{66{,}000/\sqrt{3}}{348} = \dfrac{38{,}100}{348} = 110$ A

Now, $\tan\phi_R$ = 00/300 ; $\phi_R = 18.4'$; $\cos\phi_R = 0.949$, $\sin\phi_R = 0.316$

Voltage drop in conductor resistance = $110 \times 24 = 2640$ V

Voltage drop in conductor reactance = $110 \times 28 = 3080$ V

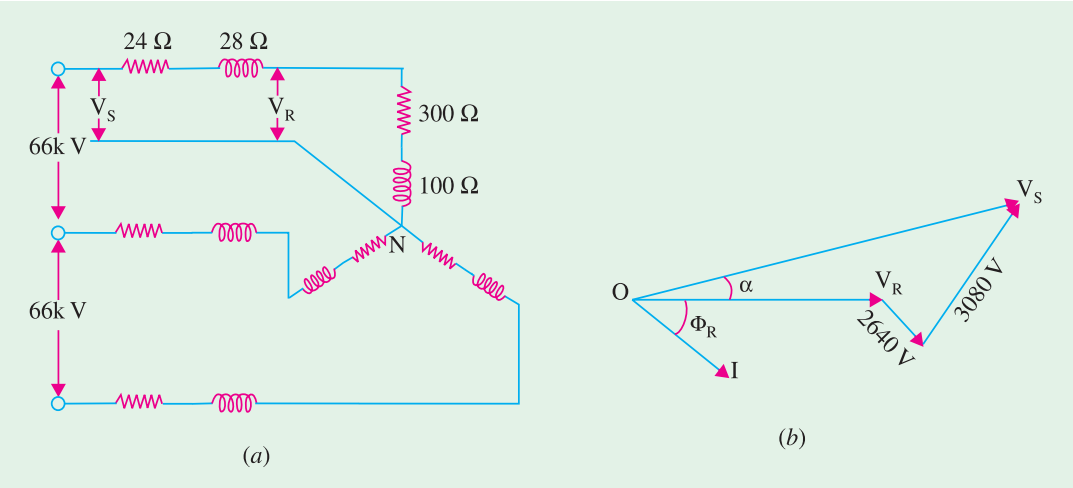


Fig. 41.25

It is seen from Art. 41.18 that

$$\mathbf{V_S} = (V_R + IR\cos\phi_R + IX\sin\phi_R) + j\,(IX\cos\phi_R - IR\sin\phi_R)$$
$$= (V_R + 2640 \times 0.949 + 3080 \times 0.316) + j\,(3080 \times 0.949 - 2640 \times 0.316)$$
$$= (V_R + 3475) + j\,2075$$

Now, $\mathbf{V_S} = 66000/\sqrt{3} = 38{,}100$ V

$\therefore \quad 38{,}100^2 = (V_R + 3475)^2 + 2075^2, \quad V_R = 34{,}585$ V

Line voltage across load $= 34{,}585 \times \sqrt{3} =$ **59.88 kV**

$\therefore \quad \mathbf{V_S} = (34{,}585 + 3475) + j\,2075 = 38{,}060 + j\,2075 = 38{,}100 \angle 3.1°$

Obviously, V_S leads V_R by 3.1° as shown in Fig. 41.25 (*b*).

Power delivered to the load/phase $= 110^2 \times 300$ W

Power transmitted/phase $= 110^2 \times 324$ W

$\therefore$ transmission efficiency $\eta = (300/324) \times 100 =$ **92.6%**

Example 41.14. *Define 'regulation' and 'efficiency' of a short transmission line.*

A 3-phase, 50-Hz, transmission line having resistance of 5Ω per phase and inductance of 30 mH per phase supplies a load of 1000 kW at 0.8 lagging and 11 kV at the receiving end. Find.

***(a)** sending end voltage and power factor **(b)** transmission efficiency **(c)** regulation.*

(Electrical Engineering-III. Poona Univ. 1990)

Solution. VR per phase $= 11{,}000/\sqrt{3} = 6{,}350$ V

Line current, $= 1000 \times 10^3/\sqrt{3} \times 11{,}000 \times 0.8 = 65.6$ A

$X_L = 2\pi \times 50 \times 30 \times 10^{-3} = 9.4\ \Omega$, $R = 5\Omega$; $Z_{ph} = (5 + j\,9.4)\ \Omega$

$\therefore$ drop per conductor $= 65.6\,(0.8 - j\,0.6)\,(5 + j\,9.4) = 632 + j\,296$

(a) $V_S = 6350 + (632 + j\,296) = 6{,}982 + j\,296 = 6{,}988 \angle 2.4°$

Sending-end line voltage $= 6{,}988 \times \sqrt{3} = 12{,}100$ V = **12.1 kV**

As seen from Fig. 41.25 (*b*), $\alpha = 2.4°$, Now, $\cos\phi_R = 0.8$, $\phi_R = 36.9°$

$\phi_S = 36.9° + 2.4° = 39.3°$, $\cos\phi_S =$ **0.774 (lag).**

(b) Total power loss $= 3\,I^2R = 3 \times 65.62 \times 5 = 64{,}550$ W $= 64.55$ kW

Input power $= 1000 + 64.55 = 1064.55$ kW

$\therefore$ power transmission $\eta = 1000/1064.55 = 0.9394$ or **93.94%**

(*c*) Now, line regulation is defined as the rise in voltage when full-load is thrown off the line divided by voltage at the load end.

$$\therefore \quad \% \text{ regn.} = \frac{(12.1 - 11)}{11} \times 100 = \textbf{10\%}$$

Example 41.15. *A short 3-ϕ line with an impedance of (6 + j8) Ω per line has sending and receiving end line voltages of 120 and 110 kV respectively for some receiving-end load at a p.f. of 0.9. Find the active power and the reactive power at the receiving end.*

(Transmission and Distribution, Madras Univ.)

Solution. As seen from Art. 41.18, considering phase values, we have

$V_S = V_R + I(R\cos\phi R + X\sin\phi R)$

Now, V_S per phase $= 120/\sqrt{3} = 69{,}280$ V

Similarly, V_R per phase $= 110/\sqrt{3} = 63{,}507$ V

$\therefore \quad 69{,}280 = 63{,}507 + I(6 \times 0.9 + 8 \times 0.435)$

$\therefore$ Line current $I = 5{,}773/8.88 = 650$ A

Active power at receiving end $= \sqrt{3} V_L I_L \cos\phi = \sqrt{3} \times 110 \times 650 \times 0.9 =$ **111,400 kW**

Reactive power at receiving end $= \sqrt{3}\, V_L I_L \sin\phi = \sqrt{3} \times 110 \times 650 \times 0.435 =$ **53,870 kVAR**

Example 41.16. *A 3-phase, 20 km line delivers a load of 10 MW at 11 kV having a lagging p.f. of 0.707 at the receiving end. The line has a resistance of 0.02 Ω/km phase and an inductive reactance of 0.07 Ω/km/phase. Calculate the regulation and efficiency of the line. If, now, the receiving-end p.f. is raised to 0.9 by using static capacitors, calculate the new value of regulation and efficiency.*

(Electrical Engg. ; Bombay Univ.)

Solution. (*i*) When p.f. = 0.707 (lag)

Line current $= 10 \times 10^6/\sqrt{3} \times 11{,}000 \times 0.707 = 743$ A

V_R per phase $= 11{,}000/\sqrt{3} = 6{,}352$ V

Total resistance/phase for 20 km $= 20 \times 0.02 = 0.4$ W

Total reactance/phase for 20 km $= 20 \times 0.07 = 1.4$ W

$\therefore$ Total impedance/phase $= (0.4 + j\,1.4)$ Ω

If V_R is taken as the reference vector, then drop per phase

$= 743\,(0.707 - j\,0.707)\,(0.4 + j\,1.4) = (945 + j\,525)$

$\therefore \quad V_S = 6{,}352 + 945 + j\,525 = 7{,}297 + j\,525$

or $V_S = \sqrt{7297^2 + 525^2} = 7{,}315$ V

$\therefore \quad \% \text{ regulation} = \dfrac{7{,}315 - 6{,}352}{6{,}352} \times 100 =$ **15.1 %**

Total line loss $= 3I^2R = 3 \times 7432 \times 0.4 = 662$ kW

Total output $= 10 + 0.662 = 10.662$ MW $\quad \therefore \quad \eta = 10 \times 100/10.662 =$ **94%**

(*ii*) When p.f. = 0.9 (lag)

Line current $= 10^7/\sqrt{3} \times 11{,}000 \times 0.9 = 583$ A

Drop/phase $= 583\,(0.9 - j\,0.435)\,(0.4 + j\,1.4) = 565 + j\,633$

$V_S = 6{,}352 + (565 + j\,633) = 6{,}917 + j\,633$

$\therefore \quad V_S = \sqrt{6{,}917^2 + 633^2} = 6{,}947$ V

$\therefore$ % regulation $= \dfrac{6,947 - 6,352}{6,352} \times 100 =$ **9.37%**

Total line loss $= 3 \times 5832 \times 0.4 = 408$ kW ; Total output = 10.408 MW

$\therefore \quad \eta = 10 \times 100/10.408 =$ **96.1%**

Example 41.17. *A load of 1,000 kW at 0.8 p.f. lagging is received at the end of a 3-phase line 10 km long. The resistance and inductance of each conductor per km are 0.531 W and 1.76 mH respectively. The voltage at the receiving end is 11 kV at 50 Hz. Find the sending-end voltage and the power loss in the line. What would be the reduction in the line loss if the p.f. of the load were improved to unity?* **(Elect. Power Systems, Gujarat Univ.)**

Solution. Line current $= 1,000 \times 1,000/\sqrt{3} \times 11 \times 1,000 \times 0.8 = 65.6$ A

Voltage/phase $= 11,000/\sqrt{3} = 6,352$ V

$X = 2\pi \times 50 \times 176 \times 10\sqrt{3} \times 10 = 5.53\ \Omega$; $R = 0.531 \times 10 = 5.31\ \Omega \quad \therefore \quad Z = (5.31 + j\,5.53)$

Voltage drop/phase $= 65.6\,(0.8 - j\,0.6)\,(5.31 + j\,5.53) = 496.4 + j\,81.2$

$\therefore \quad V_S = 6,352 + 496.4 + j\,81.2 = 6,848 + j\,81 = 6,848\ V$ numerically (approx.)

$\therefore$ line-to-line sending-end voltage $= 6,848 \times \sqrt{3} = 11,860\ V =$ **11.86 kV**

Total loss $= 3 \times 5.31 \times 65.62 = 68,550$ W = **68.55 kW**

Line current for unity p.f. $= 1,000/11 \times \sqrt{3} = 52.49$ A

$\therefore$ New losses $= 3 \times 5.31 \times 52.492 = 43.89$ kW

$\therefore$ reduction $= 68.55 - 43.89 =$ **24.66 kW**

Example 41.18. *Estimate the distance over which a load of 15,000 kW at 0.85 p.f. can be delivered by a 3-phase transmission line having conductors of steel-cored aluminium each of resistance 0.905 W per kilometre. The voltage at the receiving end is to be 132 kV and the loss in transmission is to be 7.5% of the load.* **(Transmission and Distribution, Madras Univ.)**

Solution. Line current $= 15,000/132 \times \sqrt{3} \times 0.85 = 77.2$ A

Total loss $= 7.5\%$ of $15,000 = 1,125$ kW

If R is the resistance of one conductor, then

$3\,I^2R = 1,125,000$ or $3 \times (77.2)^2 \times R = 1,125,000$; $R = 62.94\ \Omega$

Length of the line $= 62.94/0.905 =$ **69.55 km.**

Example 41.19. *A 3-ϕ line has a resistance of 5.31Ω and inductance of 0.0176 H. Power is transmitted at 33 kV, 50-Hz from one end and the load at the receiving end is 3,600 kW at 0.8 p.f. lagging. Find the line current, receiving-end voltage, sending-end p.f. and efficiency of transmisson.* **(Transmission and Distribution-I, Madras Univ.)**

Solution. $V_S = V_R + I_R \cos\phi_R + IX \sin\phi_R$ approximately

Now, power delivered/phase $= V_R\, I \cos\phi_R$

$\therefore \quad 1,200 \times 1,000 = VR\,.\,I. \times 0.8 \quad \therefore \quad I = 15 \times 105/V_R$

Also, V_S per phase $= 33,000/\sqrt{3} = 19,050$ V

$R = 5.31\ \Omega$; $X = 0.0176 \times 314 = 5.54\ \Omega$

$$\therefore \quad 19,050 = V_R + \frac{15 \times 10^5}{V_R} \times 5.31 \times 0.8 + \frac{15 \times 10^5}{V_R} \times 5.54 \times 0.6$$

$$\therefore \quad V_R^2 - 19,050\ V_R + 11,358,000 = 0 \text{ or } V_R = \frac{19.050 \pm 17,810}{2} = \frac{36.860}{2} = 18,430 \text{ V}$$

Line voltage at the receiving end $= 18,430 \times \sqrt{3} =$ **32 kV**

$I = 15 \times 105/18,430 =$ **81.5 A**

Now, $V_S = V_R + I(\cos\phi_R - j\sin\phi_R)(R + jX)$

$= 18{,}430 + 81.5(0.8 - j\,0.6)(5.31 + j\,5.54)$

$= 18{,}430 + 615 + j\,100 = 19{,}050 \angle 0.3°$

$\phi_R = \cos^{-1}(0.8) = 36°52'$;

$\phi_S = 36°52' + 18' = 37°10'$

$\therefore$ sending-end p.f. $= \cos\phi_S = \cos 37°10' =$ **0.797 A**

Power lost in line $= 3I^2R = 3 \times 81.52 \times 5.31 = 106$ kW

Power at sending end $= 3{,}600 + 106 = 3{,}706$ kW

transmission $\eta = 3{,}600 \times 100/3{,}706 =$ **97.2%**

Example 41.20. *A 3-phase short transmission line has resistance and reactance per phase of 15Ω and 20 Ω respectively. If the sending-end voltage is 33 kV and the regulation of the line is not to exceed 10%, find the maximum power in kW which can be transmitted over the line. Find also the k VAR supplied by the line when delivering the maximum power.*

Solution. As seen from Art. 41.18

$$V_S^2 = (V_R + I_R\cos\phi_R + IX\sin\phi_R)^2 + (IX\cos\phi_R - I_R\phi_R)^2$$

$$= V_R^2 + 2IV_R(R\cos\phi_R + X\sin\phi_R) + I^2(R^2 + X^2)$$

real power/phase, $P = V_R I\cos\phi_R$; reactive power/ phase, $Q = V_R I\sin\phi_R$

$\therefore \quad P^2 + Q^2 = V_R^2 I^2$ or $I^2 = (P^2 + Q^2)/V_R^2$

Substituting this value above, we get

$$V_S^2 = V_R^2 + 2PR + 2QX + \frac{P^2 + Q^2}{V_R^2}(R^2 + X^2)$$

or $$V_S^2 = V_R^2 - 2PR - 2QX - \frac{P^2 + Q^2}{V_R^2}(R^2 + X^2) = 0 \qquad ...(i)$$

To find the maximum power transmitted by the line, differentiate the above equation *w.r.t.* Q and put $dP/dQ = 0$ (treating V_S and V_R as constants).

$$\therefore \quad -2R\frac{dP}{dQ} - 2X - 2P\cdot\frac{(R^2 + X^2)}{V_R}\cdot\frac{dP}{dP} - 2Q\frac{(R^2 + X^2)}{V_R^2} = 0$$

Since $dP/dQ = 0 \;\therefore\; 2X + 2Q\frac{(R^2 + X^2)}{V_R^2} = 0$ or $Q = \frac{-V_R^2 X}{R^2 + X^2} = \frac{-V_R^2 X}{Z^2}$

Putting this value of Q in Eq. ***(i)*** above we have

$$\frac{P^2Z^2}{V_R^2} + 2PR - V_S^2 + V_R^2 - \frac{V_R^2 X^2}{Z^2} = 0$$

or $$\frac{P^2Z^2}{V_R^2} + 2PR - V_S^2 + V_R^2\left(1 - \frac{X^2}{Z^2}\right) = 0$$

or $$\frac{P^2Z^2}{V_R^2} + 2PR + \frac{V_S^2 + V_R^2}{Z^2} - V_S = 0$$

Solving for P, we get $P_{max} = \frac{V_R^2}{Z^2}\left(Z\cdot\frac{V_S}{V_R} - R\right)$ watts/phase

In the present case, $V_S = 33{,}000/\sqrt{3} = 19{,}050$ V ; Since regulation is limited to 10%,

$\therefore \quad V_R + 10\%\,V_R = V_S$ or $1.1\,V_R = 19{,}050$; $V_R = 17{,}320$ V ; $Z = \sqrt{15^2 + 20^2} = 25\ \Omega$

$$\therefore \quad P_{max} \text{ per phase} = \left(\frac{17.320}{25}\right)^2 \left(25 \times \frac{19,050}{17,320} - 15\right) = 6 \times 10^6 \text{ W} = 6 \text{ MW}$$

Total maximum power $= 3 \times 6 =$ **18 MW**

kVAR supplied/phase $Q = \frac{V_R^2 X}{Z^2} = -\frac{17,320^2 \times 20}{25^2} \times 10^{-3} = 9,598$

Total kVAR supplie $= 3 \times 9,598 =$ **28,794.**

Example 41.21. *A 3- ϕ, 50-Hz generating station supplies a load of 9,900 kW at 0.866 p.f. (lag) through a short overhead transmission line. Determine the sending-end voltage if the receiving-end voltage is 66 kV and also the efficiency of transmission. The resistance per km is 4Ω and inductance 40 mH. What is the maximum power in kVA that can be transmitted through the line if both the sending and receiving-end voltages are kept at 66 kV and resistance of the line is negligible.*

Solution. $R = 4\ \Omega$; $X = 40 \times 10^{-3} \times 314 = 12.56\ \Omega$

Line current $I = 9,900/\sqrt{3} \times 66 \times 0.866 = 100$ A

$V_R = 66,000/\sqrt{3} = 38,100$ V ; $\cos\ \phi_R = 0.866$; $\sin\ \phi_R = 0.5$

$V_S = V_R + I(R \cos \phi_R + X \sin\ \phi_R) = 38,100 + 100(4 \times 0.866 + 12.56 \times 0.5) = 39,075$ V

Line value of sending-end voltage $= 39,075 \times \sqrt{3} =$ **67.5 kV**

Total line loss $= 3I^2R = 3 \times 100^2 \times 4 = 120$ kW

$\therefore \quad \eta = 9,900/(9,900 + 120) = 0.988$ or 98.8%

Max, value of Q for 3-phases $= \frac{3.V_R^2}{Z^2}.X$

Now, $V_S = V_R = 38,100$ V and resistance is negligible.

$\therefore$ Max. value of $Q = \frac{3.V_R^2}{Z^2}.X\ \ 10^{-3} = 3 \times 3,810 \times 10^{-3}/12.56 =$ **3,48,000 kVA**

Example 41.22. *A 3-phase load of 2,000 kVA, 0.8 p.f. is supplied at 6.6 kV, 50-Hz by means of a 33 kV transmission line 20 km long and a 5 : 1 transformer. The resistance per km of each conductor is 0.4 Ω and reactance 0.5Ω. The resistance and reactance of the transformer primary are 7.5 Ω and 13.2 Ω, whilst the resistanceof the secondary is 0.35 Ω and reactance 0.65 Ω. Find the voltage necessary at the sending end of transformission line when 6.6 kV is maintained at the load-end and find the sending-end power factor. Determine also the efficiency of transmission.*

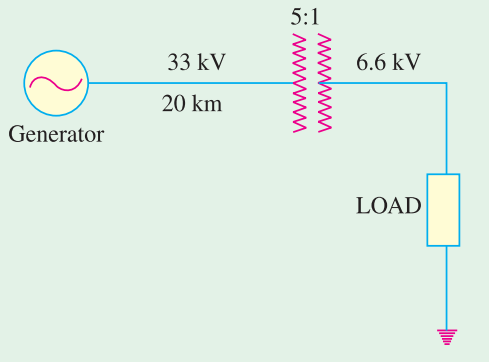


Fig. 41.26

Solution. One phase of the system is shown in Fig. 41.26. Impedance per phase of high voltage line = (8 + *j* 10)

Impedance of the transformer primary

$= (7.5 + j\ 13.2)$ ohm

Total impedance on the high-tension side

$= (8 + j\ 10) + (7.5 + j\ 13.2) = 15.5 + j\ 23.2$

This impedance can be transferred to secondary side by using the relation given in Art. 30.12.

Hence, impedance as referred to secondary side is

$= (15.5 + j\ 23.2)/52 = 0.62 + j\ 0.928$

Adding to them the impedance of the transformer secondary, we get the total impedance as referred to low-voltage side

$= (0.62 + j\,0.928) + (0.35 + j\,0.65) = 0.97 + j\,1.578$

Now, kVA load per phase $= 2{,}000/3.0 = 667$

Receiving-end voltage/phase $= 6.6/\sqrt{3} = 3.81$ kV

$\therefore$ current in the line $= 667/3.81 = 175$ A

Drop per conductor $= I(R\cos\phi + X\sin\phi) = 175(0.97 \times 0.8 + 1.578 \times 0.6) = 302$ V

Now, $E_S = E_R + I(R\cos\phi + X\sin\phi)$

Hence, sending-end voltage (phase to neutral as referred to the lower voltage side) is 3,810 + 302 = 4,112 V. As referred to high-voltage side, its value = $4{,}112 \times 5 = 20{,}560$ V

Line voltage $= 20{,}560 \times \sqrt{3}/1000 = 35.6$ kV

If ϕ_S is the power factor angle at the sending-end, then

$$\tan\phi_S = \frac{\sin\phi + (IX/E_B)}{\cos\phi + (IR/E_R)} = \frac{0.6 + (175 \times 1.578/3810)}{0.8 + (175 \times 0.97/3.10)} = 0.796$$

$\therefore \quad \phi_S = \tan^{-1}(0.796) = 38°31'$ $\quad\therefore\quad \cos\phi_S = \cos 38°31' =$ **0.782**

Power loss/phase $= 1752 \times 0.97/1000 = 29.7$ kW

Power at the receiving end/phase $= 2000 \times 0.8/3 = 533.3$ kW

$\therefore$ transmission efficiency $= \dfrac{533.3 \times 100}{533.3 + 29.7} =$ **94.7 %**

Tutorial Problem No. 41.1

1. 500 kW at 11kV are received from 3-phase transmission line each wire of which has a resistance of 1.2 Ω and a reactance of 1 Ω. Calculate the supply pressure when the power factor of the load is (*i*) unity and (*ii*) 0.5 leading. **[(*i*) 11,055 V (*ii*) 10,988 V]**

2. What load can be delivered by a 3-phase overhead line 5 km long with a pressure drop of 10%. Given that the station voltage is 11 kV, resistance per km of each line 0.09 Ω, reactance per km 0.08 Ω and the power factor of the load 0.8 lagging. **[14,520 kW]**

3. Estimate the distance over which a load of 15,000 kW at 0.85 power factor can be delivered by a 3-phase transmission line having conductors of steel-cored aluminium each of resistance 0.56 Ω per km. The p.d. at the receiving end is to be 132 kV and the loss in transmission is not to exceed 7.5% **[121.9 km] (*I.E.E. London*)**

4. A d.c. 2-wire system is to be converted into 3-phase, 3-wire a.c. system by adding a third conductor of the same size as the two existing conductors. Calculate the percentage additional balanced load that can now be carried by the conductors at 0.96 p.f. lagging. Assume the same voltage between the conductors and the same percentage power loss. **[84%]**

5. A 3-phase short transmission line of resistance 8 Ω and reactance 11 Ω per phase is supplied with a voltage of 11 kV. At the end of the line is a balanced load of P kW per phase at a p.f. of 0.8 leading. For what value of P is the voltage regulation of the line zero? **[210 kW] (*Electrical Technology, M.S. Univ. Baroda*)**

6. A 3-ph, 50-Hz transmission line 10 km long delivers 2,500 kVA at 10 kV. The p.f. of the load is 0.8 (lag). The resistance of each conductor is 0.3 W/km and the inductance 1.82 mH/km. Find (*a*) the voltage and p.f. at the sending end (*b*) the efficiency of transmission and (*c*) the percentage regulation of the line. **[(*a*) 11.48 kV ; 0.763 (*b*) 91.4% (*c*) 14.8%]**

7. The conductors in a single-phase transmission line are 6 m above the ground. Each conductor has 1.5 cm diameter and the conductors are spaced 3 m apart. Starting from the fundamentals, determine the capacitance per kilometre of the line first including and then excluding the effect of ground. What do you conclude ? **[4.60×10^{-3} μF/km ; 4.63×10^{-3} μF/km] (*Ranchi Univ.*)**

41.20. Effect of Capacitance

So far we have neglected the effect of capacitance on the line regulation because the capacitances of short lines transmitting at relatively low voltages (up to 20 kV) are negligible. But as the voltage and length of the transmission line increase, the capacitance gradually becomes of greater importance. Similarly, the leakage across insulators also assumes greater importance. Hence, exact calculations of regulation for long lines take into consideration the capacitance and leakage reactance of the lines and are quite elaborate, the amount of elaboration depending on the transmitting voltage.

(i) In the case of short lines, ordinarily, the capacitance is negligible. But if in a problem, the line capacitance is given and if the line is less than 80 km, then the line capacitance can be lumped at the receiving or load end as shown in Fig. 41.27 (*a*) although this method of localizing the line capacitance at the load end over-estimates the effect of capacitance. In that case, the line current $\mathbf{I}_S$ is the vector sum of the load current $\mathbf{I}_R$ and the charging current $\mathbf{I}_C$ of the capacitance. Hence, $\mathbf{I}_S = \mathbf{I}_R + \mathbf{I}_C$.

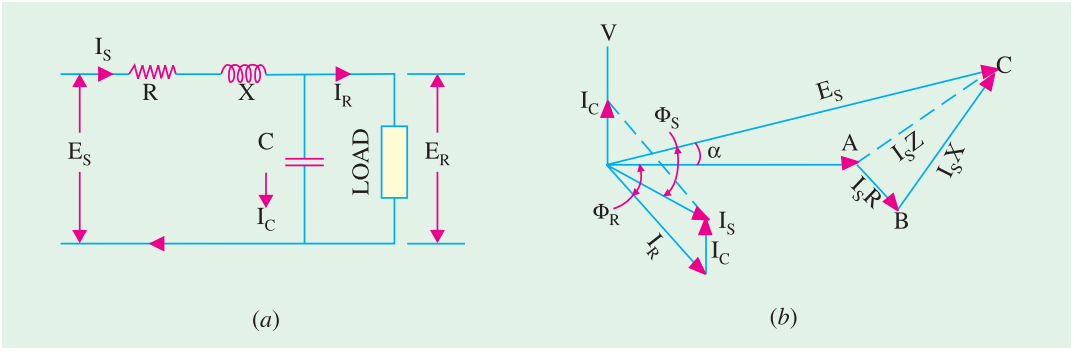


Fig. 41.27

Now, charging current $\mathbf{I}_C = j\omega CE_R$ $\therefore$ $\mathbf{I}_R = I_R(\cos\phi_R - j\sin\phi_R)$

$\therefore$ $\mathbf{I}_S = I_R\cos\phi_R - jI_R\sin\phi_R + j$ $CE_R = I_R\cos\phi R + j(-I_R\sin\phi_R + \omega CE_R)$

Line drop $= I_S(R + jX)$ $\therefore$ $\mathbf{E}_S = \mathbf{E}_R + \mathbf{I}_S(\boldsymbol{R + J_X})$

The vector diagram is shown in Fig. 41.27 (*b*).

(ii) In the case of lines with voltages up to 100 kV and 150 km in length,satisfactory solutions can be obtained by the so-called *T*-method and π-or pi method as described below.

Example 41.23. *A (medium) single-phase transmission line 50 km long has the following constants :*

resistance/km $= 0.5\ \Omega$; *reactance/km* $= 1.6\ \Omega$

susceptance/km $= 28 \times 10^{-6}$ S; *receiving-end line voltage* $= 66{,}000$ V

Assuming that total capacitance of the line is located at receiving end alone, determine the sending-end voltage, the sending-end current and regulation. The line is delivering 15,000 kW at 0.8 p.f. lagging. Draw a vector diagram to illustrate your answer.

Solution. Let E_S and E_R be sending-end and receiving-end voltages respectively as shown in Fig. 41.28.

Load current at the receiving-end is $I_R = 15 \times 10^4/66 \times 10^3 \times 0.8 = 284$ A

Total resistance $= 0.5 \times 50 = 25\ \Omega$; Total resistance $= 1.6 \times 50 = 80\ \Omega$

Susceptance $B = 28 \times 10^{-6} \times 50 = 14 \times 10^{-4}$ Siemens

Capacitive admittance $Y = B = 14 \times 10^{-4}$ Siemens

As seen from vector diagram of Fig. 41.29, sending-end current I_S is the vector sum of load current I_R and capacitive current I_C.

Now, $I_C = E_R\ Y = 66{,}000 \times 14 \times 10^{-4} = 92$ A

Let $\mathbf{E}_R = (66{,}000 + j\ 0)$

$\mathbf{I}_R = 284\ (0.8 - j\ 0.6) = 227 - j190$; $\mathbf{I}_C = j\ 92$

$\mathbf{I}_S = \mathbf{I}_R + \mathbf{I}_C = (227 - j\ 190 + j\ 92) = 240 \angle -18°57'$; $Z = 25 + j\ 80 = 84 \angle 72°36'$

Line drop $= \mathbf{I}_S\mathbf{Z} = 240 \angle -18°57' \times 84 \angle 72°36' = 20{,}160 \angle 53°39' = 11{,}950 + j\ 16{,}240$

$\mathbf{E}_S = \mathbf{E}_R + \mathbf{I}_S\mathbf{Z} = 66{,}000 + (11{,}950 + j\ 16{,}240) = 79{,}500 \angle 11°44'$

Regulation $= \dfrac{E_S - E_R}{E_S} \times 100 = \dfrac{79{,}500 - 66{,}000}{66{,}000} \times 100 =$ **20.5%**

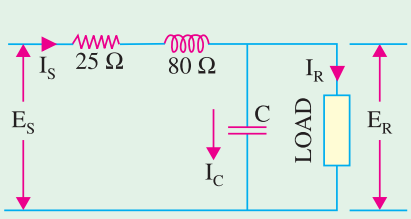


Fig. 41.28

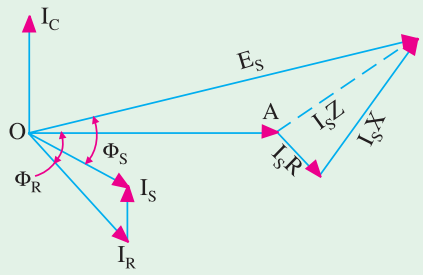


Fig. 41.29

41.21. "Nominal" T-method

In this *T*-method, also known as mid-capacitor method, the whole of the line capacitance is assumed to be concentrated at the middle point of the line and half the line resistance and reactance are lumped on its either side as shown in Fig. 41.30.

It is seen that $\mathbf{E}_1 = \mathbf{E}_R + \mathbf{I}_R\mathbf{Z}_{BC}$. Knowing E_1, we can find $\mathbf{I}_C$ as under.

$\mathbf{I}_C = j\omega C_{E1}$ $\quad\therefore\quad \mathbf{I}_S = \mathbf{I}_C + \mathbf{I}_R$

Obviously, current through portion $A\,B$ is I_S ; hence voltage drop $= I_S\ Z_{AB}$

$\therefore$ $\quad \mathbf{E}_S = \mathbf{E}_1 + \mathbf{I}_S\,\mathbf{Z}_{AB}$

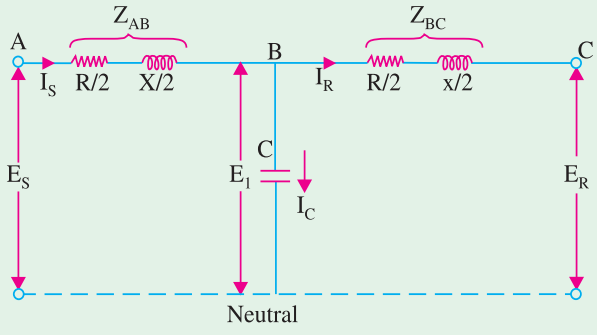


Fig. 41.30

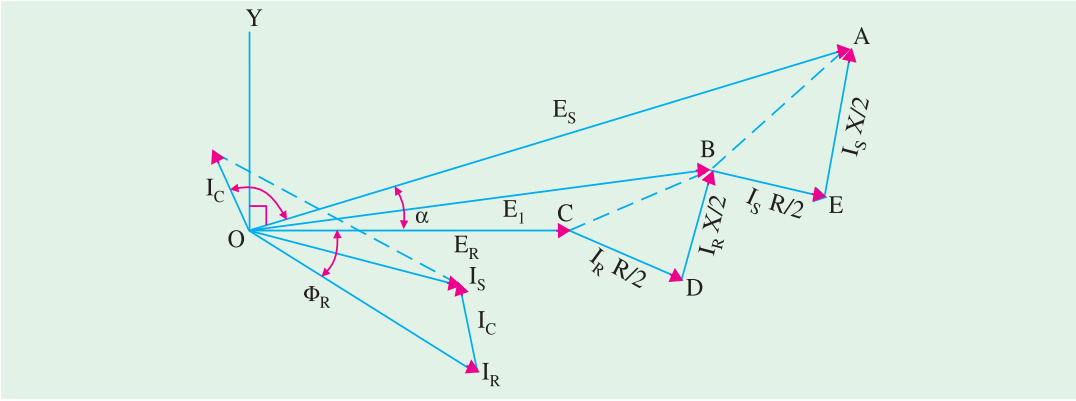


Fig. 41.31

The vector diagram is shown in Fig. 41.31. Receiving-end voltage E_R is taken as the reference vector. It may be pointed out here that all values are phase values *i.e.* line to neutral values. I_R is the load current lagging E_R by ϕ_R, $CD = I_R.R/2$ and parallel to I_R. $BD = I_R\ X/2$ and perpendicular to I_R. OB represents E_1 *i.e.* the voltage across the middle capacitor. BE represents $I_S.R/2$ and is parallel to I_S. Similarly, $EA = I_S.X/2$ and perpendicular to I_S. OA represents the voltage at the sending end.

It may be noted that if leakage is appreciable, then leakage conductance G can be assumed to be concentrated at the middle point of the line and can be represented by non-inductive conductance G shunting the middle capacitor as shown in Fig. 41.32.

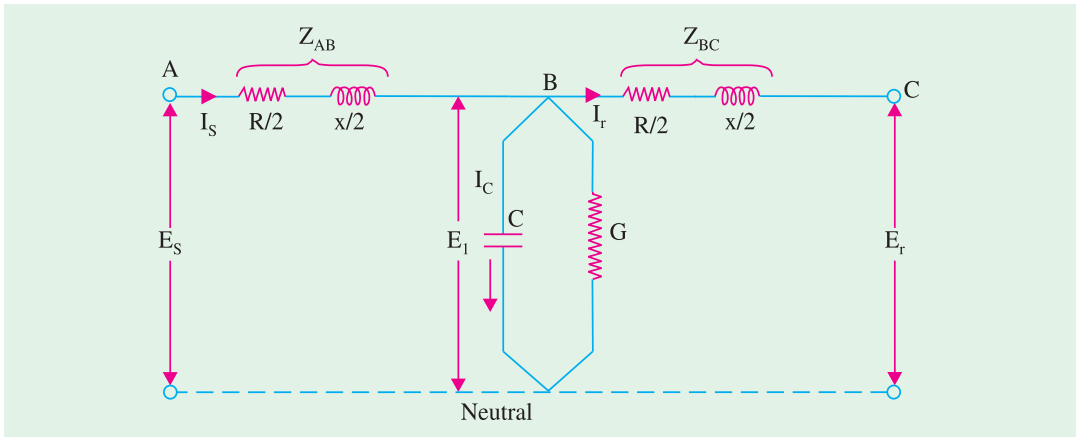


Fig. 41.32

Example 41.24. *A 3-phase, 50-Hz overhead transmission line 100 km long with 132 kV between lines at the receiving end has the following constants :*

resistance/km/phase = 0.15 Ω ; inductance/km/phase = 1.20 mH

capacitance/km/phase = 0.01 mF

Determine, using an approximate method of allowing for capacitance, the voltage, current and p.f. at the sending end when the load at the receiving end is 72 MW at 0.8 p.f. lagging. Draw vector diagram for the circuit assumed. **(Electrical Power System ; Gujarat Univ.)**

Solution. For a 100-km length of the line,

$$R = 0.15 \times 100 = 15\ \Omega\ ;\ X_L = 314 \times 1.2 \times 10^{-3} \times 100 = 37.7\ \Omega$$

$$X_C = 106/314 \times 0.01 \times 100 = 3187\ \Omega$$

Using the nominal T-method, the equivalent circuit is shown in Fig. 41.33 (a)

$V_R = 132/\sqrt{3} = 76.23$ kV $= 76{,}230$ V. Hence, $\mathbf{V}_R = 76{,}230 + j\,0$

Load current, $\mathbf{I}_R = 72 \times 102/\sqrt{3} \times 132 \times 103 \times 0.8 = 394$ A

$\therefore \quad \mathbf{I}_R = 394\,(0.8 - j\,0.6) = 315 - j\,236$ A ; $\mathbf{Z}_{BC} = (7.5 + j\,18.85)$ W

Drop/phase over BC $= \mathbf{I}_R\mathbf{Z}_{BC} = (315 - j\,236)\,(7.5 + j\,18.85) = 6802 + j\,4180$

$\mathbf{V}_1 = \mathbf{VR} + \mathbf{I}_R\mathbf{Z}_{BC} = (76{,}230 + j0) + (6802 + j\,4180) = 88{,}030 + j\,4180$

$$\mathbf{I}_C = \frac{\mathbf{V}_1}{\mathbf{X}_C} = \frac{83{,}030 + j\,4180}{-j\,3187} = -1.31 + j\,26$$

$\mathbf{I}_S = \mathbf{I}_C + \mathbf{I}_R = (-1.31 + j\,26) + (315 - j\,236) = (313.7 - j\,210) = 377.3 \angle -33.9°$

Drop/phase over AB $= \mathbf{I}_S\mathbf{Z}_{AB} = (313.7 - j\,210)\,(7.5 + j\,18.85) = 6320 + j\,4345$

$\therefore \quad \mathbf{V}_S = \mathbf{V}_1 + \mathbf{I}_S\mathbf{Z}_{AB} = (83{,}030 + j\,4180) + (6320 + j\,4345) = 89{,}350 + j\,8525 = 89{,}750 \angle 5.4°$

Line value of sending-end voltage $= \sqrt{3} \times 89{,}750 \times 10^{-3} =$ **155.7 kV**

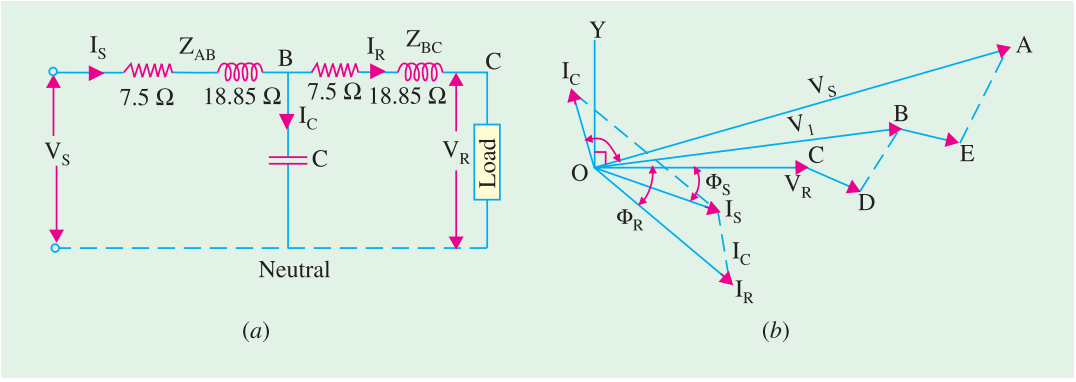


Fig. 41.33

Phase difference between V_S and I_S = 33.9° + 5.4° = 39.3° with current lagging as shown in Fig. 41.33 (b) $\cos \phi_S = \cos 39.3° =$ **0.774 (lag)**

Example 41.25. *A 3-phase, 50-Hz transmission line, 100 km long delivers 20 MW at 0.9 p.f. lagging and at 110 kV. The resistance and reactance of the line per phase per km are 0.2 Ω and 0.4 Ω respectively while the capacitive admittance is 2.5×10^{-6} S per km. Calculate* ***(a)*** *the voltage and current at the sending end and* ***(b)*** *the efficiency of transmission. Use the nominal T-method.*

(Electrical Power-I, M.S. Univ. Baroda)

Solution. Resistance for 100 km $= 0.2 \times 100 = 20\ \Omega$

Reactance for 100 km $= 0.4 \times 100 = 40\ \Omega$

Capacitive admittance for 100 km $= 2.5 \times 10^{-6} \times 100 = 2.5 \times 10^{-4}$ S

Let us take the receiving-end voltage E_R as reference vector.

$E_R = 110/\sqrt{3} = 63.5$ kV ; $I_R = 20 \times 106/\sqrt{3} \times 110 \times 103 \times 0.9 = 116.6$ A

$\cos \phi_R = 0.9$; $\sin \phi_R = 0.435$ (from tables)

With reference to Fig. 41.34, we have

$\mathbf{E}_R = (63.5 + j\,0)$ kV ; $\mathbf{I}_R = 116.6(0.9 - j\,0.\,435) = 105 - j\,50.7$; $\mathbf{Z}_{BC} = (10 + j\,20)$

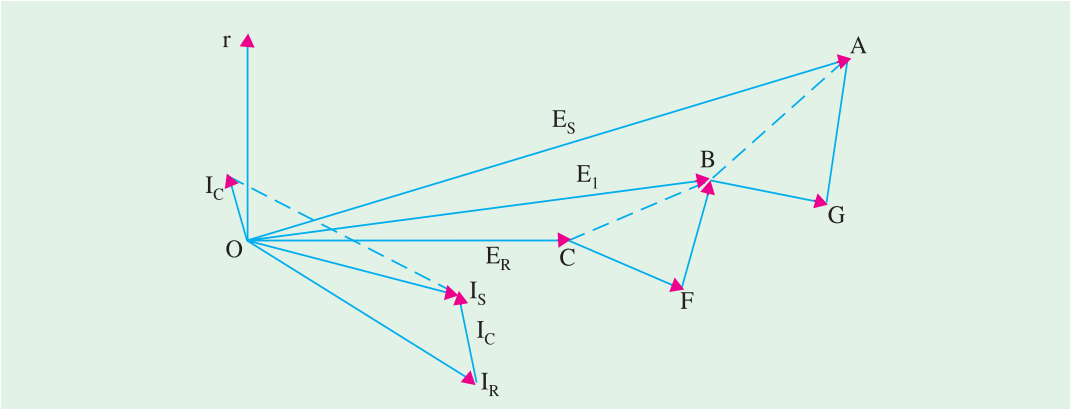


Fig. 41.34

Voltage drop between points *B* and *C* is

$$\mathbf{V}_{BC} = \mathbf{I}_R\mathbf{Z}_{BC} = (105 - j\,50.7)\,(10 + j\,20) = (2{,}064 + j\,1{,}593)\text{ V}$$

$$\mathbf{E}_1 = \mathbf{E}_R + \mathbf{I}_R\mathbf{Z}_{BC} = (63{,}500 + 2{,}064 + j\,1{,}593) = 65{,}564 + j\,1{,}593$$

$$\mathbf{I}_C = \mathbf{E}_1\mathbf{Y} = (65{,}564 + j\,1{,}593) \times j\,2.5 \times 10^{-4} = (-0.4 + j\,16.4)\text{ A}$$

$$\mathbf{I}_S = \mathbf{I}_R + \mathbf{I}_C = (105 - j\,50.7) + (-0.4 + j\,16.4) = (104.6 - j\,34.3) = 110.1\,\angle 18^\circ\,9$$

Drop between points *A* and *B* is

$$\mathbf{V}_{AB} = \mathbf{I}_S\mathbf{Z}_{AB} = (104.6 - j\,34.3)\,(10 + j\,20) = 1{,}732 + j\,1{,}749$$

$$\mathbf{E}_S = \mathbf{E}_1 + \mathbf{V}_{AB} = (65{,}564 + j\,1{,}593) + (1{,}732 + j\,1{,}749) = 67{,}296 + j\,3{,}342 = 67{,}380\,\angle 2^\circ\,51$$

(a) Sending-end voltage (line value) is $67{,}380 \times \sqrt{3} = 116{,}700$ V = **116.7 kV**

Sending-end current = **110.1 A**

Note. Phase difference between E_S and I_S is $(-18^\circ 9 - 2^\circ 51) = -21^\circ$ with current lagging. Hence, p.f. at sending-end is $\cos\phi_S = \cos 21^\circ = 0.934$ (lag).

(b) Copper loss for three phases between points *B* and *C* (Fig. 41.32) is

$$= 3 \times 116.6^2 \times 10 = 0.408\text{ MW}$$

Copper loss for three phases between points *A* and *B* is $= 3 \times 110.1^2 \times 10 = 0.363$ MW

Total Cu loss for 100 km of line length = 0.408 + 0.363 = 0.771 MW

Transmission $\eta = 20 \times 100/20.771 =$ **96.27 %**

Note. Line losses could also be computed by finding the line input which = $\sqrt{3}\,E_S.I_S\cos\phi_S$.

41.22. "Nominal" π-method

In this method, the line-to-neutral capacitance is divided into two halves ; one half being concentrated or localized at the sending-end and the other half at the receiving-end as shown in Fig. 41.35 (*a*). The capacitance at the sending or generating end has no effect on line drop or line regulation but its charging current must be added to the line current in order to obtain the total sending-end current $\mathbf{I}_S$.

It is obvious that I_S is the vector sum of I_{C2} and I_L where I_L is the vector sum of I_{C1} and I_R. The vector diagram is shown in Fig. 41.35 (*b*). E_R is taken as the reference vector. Current I_L is vector sum of I_R and I_{C1} (which is ahead of E_R by 90°). The drop $AB = I_L R$ is in phase with vector for I_L and reactive drop $BC = I_L X$ is in quadrature with I_L. *OC* represents the sending-end voltage. The sending-end current I_S is the vector sum of I_L and I_{C2} (which itself is ahead of E_S by 90°). It may be noted that

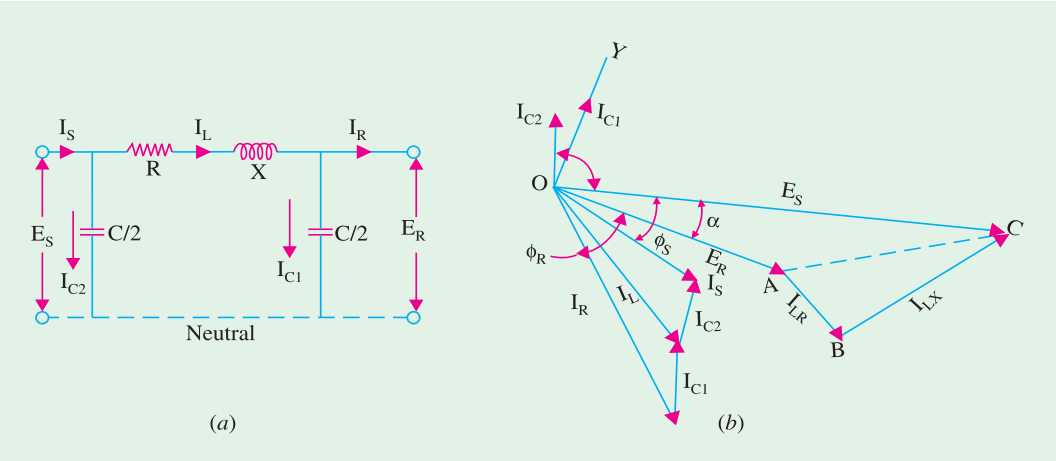


Fig. 41.35

if leakage reactance is not negligible, then leakage conductance G can also be divided into two equal halves and put at both ends in parallel with the capacitors as shown in Fig. 41.36.

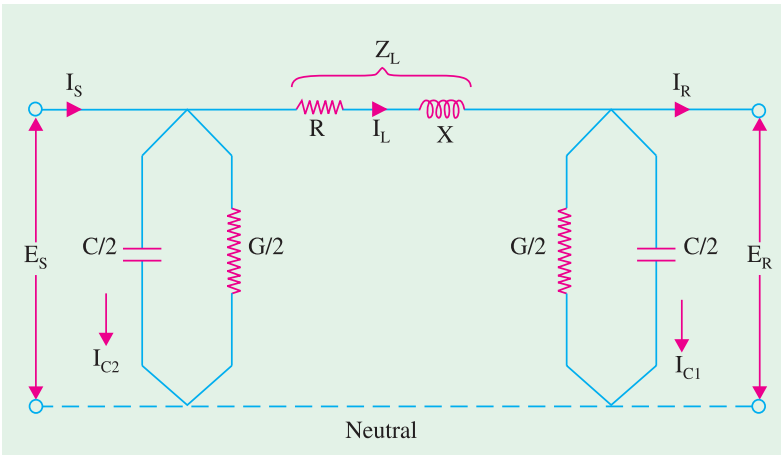


Fig. 41.36

41.23. Ferranti Effect

A long or medium transmission line has considerable capacitance and so draws leading charging current from the generating-end ***even when unloaded***. Moreover, receiving-end voltage V_R under no-load condition is found to be ***greater*** than sending-end voltage V_S. This phenomenon is known as ***Ferranti effect***.

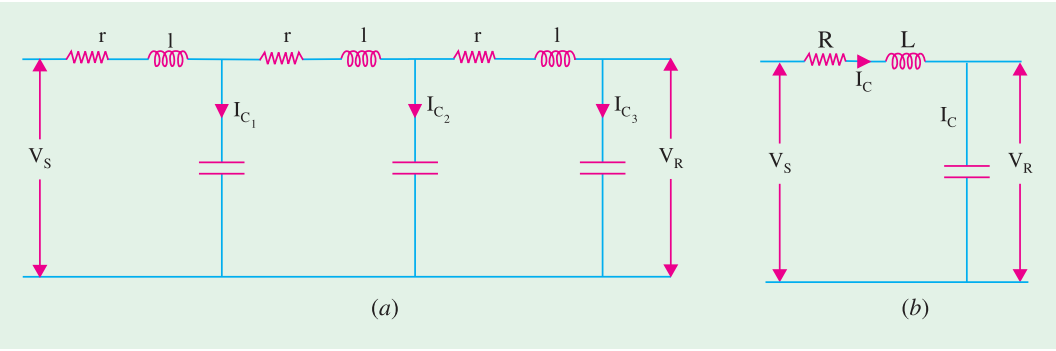


Fig. 41.37

Fig. 41.37 (*a*) shows the distributed parameters of such a line. It may be replaced by the circuit of Fig, 41.37 (*b*) where these distributed parameters have been lumped. As shown in the phasor

diagram of Fig. 41.38, the charging current I_C leads V_R by 90° and produces a phase voltage drop = $I_C Z = I_C (R + j X_L)$.

Obviously, $V_S < V_R$ Now, $I_C = V_R \omega C$

As seen from Fig. 41.38

$$V_S = \sqrt{(V_R - I_C X_L)^2 + (I_C R)^2}$$

If R is negligible, then

$$V_S = (V_R - I_C X_L) \text{ or } V_R = V_S + I_C X_L$$

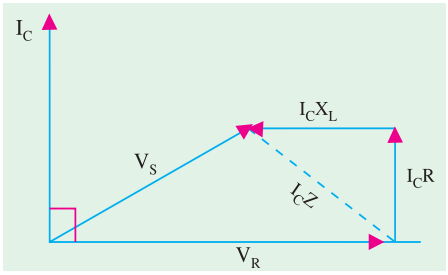


Fig. 41.38

41.24. Charging Current and Line Loss of an Unloaded Transmission Line

Fig. 41.39 shows the distribution of capacitance in a long transmission line of length l. Obviously, charging current I_C has maximum value at the sending end and linearly falls to zero at the receiving end. Accordingly, value of charging current at distance x from the sending end is proportionally equal to $I_C (l - x)/l$. The RMS value I of this current is given by

$$I^2 = \frac{1}{l}\int_0^l \frac{I_C^2 (l-x)^2}{l^2}.dx = \frac{I_C^2}{l^3}\int_0^l (l^2 + x^2 - 2lx)\, dx$$

$$= \frac{I_C^2}{l^3}\left[l^2 x + \frac{x^3}{3} - lx^2\right]_0^l = \frac{I_C^2}{3}$$

$$\therefore \quad I = I_C/\sqrt{3}$$

If R is the resistance of the line per phase, then total power loss in the line is

$$= 3 I^2 R = 3\left(\frac{I_C}{\sqrt{3}}\right)^2 R = I_C^2 R.$$

Example 41.26. *A 3-phase transmission line, 100 km long has following constants: resistance per km per phase = 0.28 Ω; inductive reactance per km per phase = 0.63 Ω. Capacitive susceptance per km per phase = 4×10^{-6} siemens. If the load at the receiving end is 75 MVA at 0.8 p.f. lagging with 132 kV between lines calculate sending-end voltage, current and p.f. Use nominal-π-method.*

(Power System-I, AMIE, Sec. B. 1994)

Solution. For a line of length 100 km,

resistance/phase = $0.28 \times 100 = 28\ \Omega$;

inductive reactance/phase = $0.63 \times 100 = 63\ \Omega$

Capacitive susceptance/phase = $4 \times 10^{-6} \times 100 = 4 \times 10^{-4}$ S

Capacitive susceptance at each end = 2×10^{-4} S

$\mathbf{V}_R = 132 \times 10^3/\sqrt{3} = 76{,}230$ V ; $\mathbf{V}_R = (76{,}230 + j\,0)$; $\mathbf{I}_R = 75 \times 10^6/\sqrt{3} \times 132 \times 10^3 \times 0.8 = 410$

$\mathbf{I}_R = 410\,(0.8 - j\,0.6) = 328 - j\,246$

$\mathbf{Y}_{C1} = j\,2 \times 10^{-4}$ S ; $\boldsymbol{I}_{C1} = \mathbf{V}_R \cdot \mathbf{Y}_{C1}$

$= 76{,}230 \times j\,2 \times 10^{-4} = j\,15.25$ A

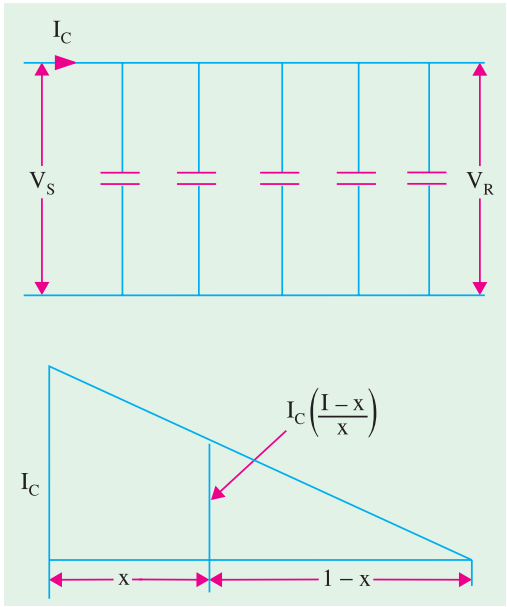


Fig. 41.39

$\mathbf{I}_L = \mathbf{I}_R + \mathbf{I}_{C1} = 328 - j\,231$ A

Drop per conductor = $\mathbf{I}_L \cdot \mathbf{Z}_L = (328 - j\,231)(28 + j\,63) = 23{,}737 + j\,14{,}196$

$\mathbf{V}_S = \mathbf{V}_R + \mathbf{I}_L \mathbf{Z}_L = 76{,}230 + 23{,}737 + j\,14{,}196 = 99{,}967 + j\,14{,}196 = 100{,}970 \angle 8.1^\circ$

Line value of sending-end voltage = $100{,}970 \times 10^{-3} \times \sqrt{3}$ = **174.9 kW**

$\mathbf{I}_C = \mathbf{Y}_S \mathbf{Y}_{C2} = 100{,}970 \angle 8.1^\circ \times 2 \times 10^{-4} \angle 90^\circ = 20.19 \angle 98.1^\circ = -2.84 + j\,20$

$\mathbf{I}_S = \mathbf{I}_C^2 + \mathbf{I}_L = (-2.84 + j\,20) + (328 - j\,231) = 325.2 - j\,211 = 388 \angle -33^\circ$

Angle between V_S and I_S = $8.1^\circ + 33^\circ = 41.4^\circ$; $\cos\phi$ = **0.877 (lag)**

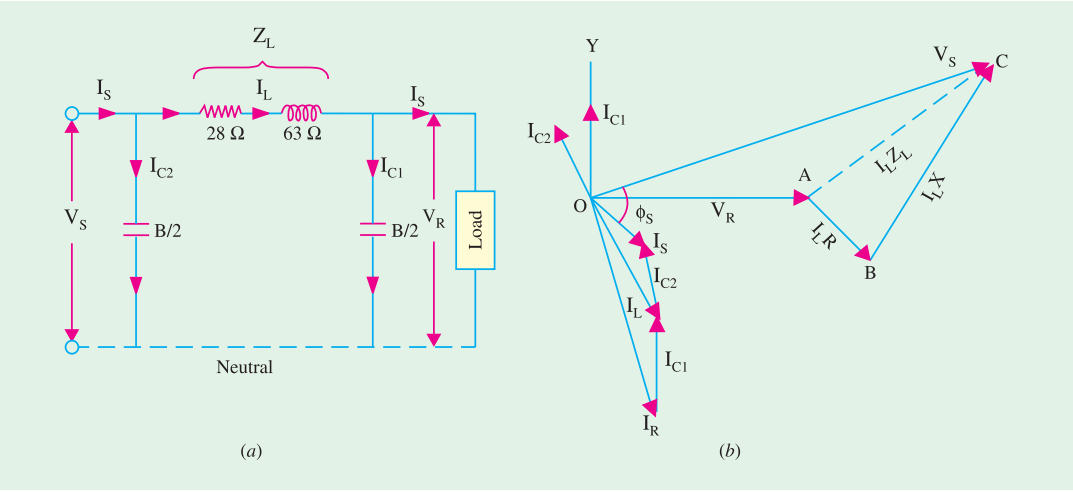


Fig. 41.40

Example 41.27. *A 100-km long, three-phase, 50-Hz transmission line has resistance/phase/km = 0.1 Ω ; reactance/phase/km = 0.5 Ω ; susceptance/phase/km = 10×10^{-6} siemens.*

If the line supplies a load of 20 MW at 0.9 p.f. lagging at 66 kV at the receiving end, calculate by nominal 'p' method, the regulation and efficiency of the line. Neglect leakage.

(Electrical Power-I, Bombay Univ. 1991)

Solution. For a 100 km line

resistance/phase = $0.1 \times 100 = 10\ \Omega$; inductive reactance/phase = $0.5 \times 100 = 50\ \Omega$

Capacitive susceptance/phase = $10 \times 10^{-4} \times 100 = 10 \times 10^{-4}$ siemens

Susceptance at each end = 5×10^{-4} siemens

$V_R = 66 \times 10^3/\sqrt{3} = 38{,}100$ V $\quad\therefore\quad \mathbf{V}_R = (38{,}100 + j\,0)$

$I_R = 20 \times 10^6/\sqrt{3} \times 66 \times 10^3 \times 0.9 = 195$ A ; $\mathbf{I}_R = 195(0.9 - j\,0.435) = 176 - j\,85$

$\mathbf{I}_{C1} = 38100 \times j\,5 \times 10^{-4} = j\,19.1$ A ; $\mathbf{I}_L = \mathbf{I}_R + \mathbf{I}_{C1} = 176 - j\,66$

drop/conductor $= \mathbf{I}_L \mathbf{Z}_L = (176 - j\,66)(10 + j\,50) = 5060 + j\,8140$

$\mathbf{V}_S = \mathbf{V}_R + \mathbf{I}_L \mathbf{Z}_L = 38{,}100 + 5060 + j\,8140 = 43{,}160 + j\,8140 = 43{,}920 \angle 10.7^\circ$

Line value $= 43{,}920 \times 10^{-3} \times \sqrt{3} = 76$ kV

$\mathbf{I}_{C2} = \mathbf{V}_S \cdot \mathbf{Y}_{C2} = 43{,}920 \angle 10.7^\circ \times 5 \times 10^{-4} \angle 90^\circ = 21.96 \angle 100.7^\circ = -4.1 + j\,21.6$

$\mathbf{I}_S = \mathbf{I}_{C2} + \mathbf{I}_L = 172 - j\,44.4 = 177.6 \angle -14.5^\circ$; $\phi_S = 14.5^\circ + 10.7^\circ = 25.2^\circ$

$\cos\phi_S = 0.905$ (lag)

(i) $\%\ \text{regulation} = \dfrac{76 - 66}{66} \times 100 =$ **15.15%**

(*ii*) line input $= \sqrt{3} \times 76 \times 103 \times 177.6 \times 0.905 = 21.5 \times 10^6\ \Omega = 21.15$ MW

$\therefore$ transmission $\eta = 20 \times 100/21.15 =$ **94.56%**

Example 41.28. (*a*) *A 50-Hz, 3-phase, 100-km long line delivers a load of 40 MVA at 110 kV and 0.7 p.f. lag. The line constants (line to neutral) are : resistance of 11 ohms, inductive reactance of 38 ohms and capacitive susceptance of 3×10^{-4} siemens. Find the sending-end voltage, current, power factor and efficiency of power transmission.*

(*b*) *draw the vector diagram.*

(*c*) *If the sending-end voltage is held constant and load is removed, calculate the receiving-end voltage and current.* **(Electrical Power-II, Bangalore Univ.)**

Solution. (*a*) Nominal π-method will be used to solve this problem. Capacitive susceptance (or admittance) at each end $= Y/2 = 3 \times 10^{-4}/2 = 1.5 \times 10^{-4}$ siemens as shown in Fig. 41.41.

$\mathbf{V}_R = 110/\sqrt{3} = 63.5$ kV/phase ; $\mathbf{I}_R = 40 \times 10^6/\sqrt{3} \times 110 \times 10^3 = 210$ A

Let $\mathbf{V}_R = (63{,}500 + j\,0)$; $\mathbf{I}_R = 210\,(0.7 - j\,0.714) = (147 - j\,150)$

$\mathbf{Y}_{C1} = 1.5 \times 10^{-4}$ siemens ; $\mathbf{I}_{C1} = \mathbf{V}_R \mathbf{Y}_{C1} = 63{,}500 \times j1.5 \times 10^{-4} = j\,9.5$ A

As seen from Fig. 41.36

$\mathbf{I}_L = \mathbf{I}_R + \mathbf{I}_{C1} = (147 - j\,140.5)$ A

Voltage drop/phase $= \mathbf{I}_L \mathbf{Z}_L = (147 - j\,140.5)\,(11 + j\,38) = 6956 + j\,4041$

$\mathbf{V}_S = \mathbf{V}_R + \mathbf{I}_L \mathbf{Z}_L = 63{,}500 + 6{,}956 + j4041 = 70{,}456 + j4041 = 70{,}570\ \angle 3.2^\circ$

Line value of sending-end voltage $= \sqrt{3} \times 70{,}570 = 122.2$ kV

$\mathbf{I}_{C1} = \mathbf{V}_S \mathbf{Y}_{C_2}$

$= (70{,}570 + j\,4041) \times j\,1.5 \times 10^{-4}$

$= -0.6 + j\,10.6 = 10.6\ \angle 93.2^\circ$

$\mathbf{I}_S = \mathbf{I}_{C_2} + \mathbf{I}_L$

$= (-0.6 + j\,10.6) + (147 - j\,140.5)$

$= 146.4 - j\,129.9 = 195.7\ \angle -41.6^\circ$

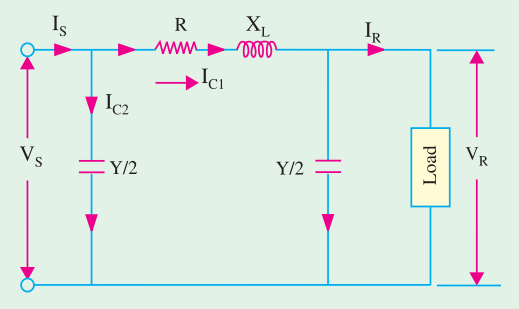


Fig. 41.41

Angle between V_S and $I_S = 41.6^\circ + 3.2^\circ = 44.8^\circ$; $\cos \phi_S = \cos 44.8^\circ = 0.71$ (lag)

Power input $= \sqrt{3} \times 122.2 \times 10^3 \times 195.7 \times 0.7 = 29.41$ MW

Power output $= 40 \times 0.7 = 28$ MW

$\therefore$ power transmission $\eta = 28/29.41 = 0.952$ or **95.2%**

(*b*) vector diagram is similar to that shown in Fig. 41.35 (*b*).

(*c*) As seen from Fig. 41.41 under no-load condition, current in the conductor is

$= \mathbf{I}_{C_1} = j\,9.5$ A

Drop/phase $= \mathbf{I}_{C_1} \mathbf{Z}_L = j\,9.5(11 + j\,38) = -361 + j\,105$

$\mathbf{V}_S = 63{,}500 + (-361 + j\,105) = 63{,}140 + j\,105 = 63{,}140\ \angle 0.1^\circ$

$\mathbf{I}_{C2} = (63{,}140 + j\,105) \times j\,1.5 \times 10^{-4} = (-0.016 + j\,9.47)$ A

$\mathbf{I}_S = \mathbf{I}_{C_1} + \mathbf{I}_{C_2} = j\,9.5 - 0.016 + j\,9.47 = (-0.016 + j\,18.97)$ A

Tutorial Problem No. 41.2

1. A 50-Hz, 3-phase line 100 km long delivers a load of 40,000 kVA at 110 kV and a lagging power factor of 0.7. The line constants (line-to-neutral) are : resistance 11 Ω ; inductive reactance 38 Ω; capacitive susceptance 3×10^{-4} S (half at each end), leakage negligible. Find the sending-end voltage, current, power factor and power input. **[122 kV ; 195.8 A ; 0.715 ; 29.675 kW]**

2. A 50-Hz, 3-phase transmission line has the following constants (line-to-neutral). Resistance 11 Ω, reactance 38 Ω; susceptance 3×10^{-4} S, leakage negligible. The capacitance can be assumed located half at each end of the line. Calculate the sending-end voltage, the line current and the efficiency of transmission when the load at the end of the line is 40,000 kVA at 110 kV power factor 0.7 lagging.
[122 kV ; 203.35 A ; 95.5%]

3. A 3-phase transmission line has the following constants (line-to-neutral) ; R = 10 Ω ; inductive reactance = 20 Ω, capacitive reactance = 2.5 k Ω. Using the nominal T-method, calculate the voltage, line current and power factor at sending-end and the efficiency of transmission when the transmission line supplies a balanced load of 10 MW at 66 kV and power factor 0.8 lagging.
[69.5 kV ; 100 A ; 0.85 lag ; 96.8 %] (*Electrical Technology ; M.S. Univ. Baroda*)

4. A 3-phase line has a resistance of 5.31 ohms and inductance of 0.0176 henry. Power is transmitted at 33 kV, 50 Hz from one end and the load at the receiving end is 3600 kW at 80 per cent power factor. Find the line current, receiving-end voltage, sending-end power factor and the efficiency of transmission. **[81.5 A, 31.9 kV, 0.796, 97.2%] (*U.P.S.C.*)**

5. Calculate the % regulation of 6.6 kV single-phase A.C. transmission line delivering 40 amps current at 0.8 lagging power factor. The total resistance and reactance of the line are 4.0 ohm and 5.0 ohm per phase respectively. **[3.76%]**

6. A-3 phase load of 2,000 kVA at 0.8 power factor is supplied at 6.6 kV, 50 Hz by means of a 33 kV transmission line, 32 km long in association with a 33/6.6 kV step-down transformer. The resistance and reactance of each conductor per km are 0.4 ohm and 0.5 ohm respectively. The resistance and reactance of the transformer primary are 7.5 ohm and 13.2 ohm while those of secondary are 0.35 ohm and 0.65 Ω respectively. Find the voltage necessary at the sending end of the transmission line when the voltage is maintained at 6.6 kV at the receiving end. Determine also the sending-end power and efficiency of transmission. **[35.4 kV, 1,689.5 kW, 94.7%]**

7. The generalised A and B constants of a transmission line are $0.96 \angle 1^\circ$ and $120 \angle 80^\circ$ respectively. If the line to line voltages at the sending and receiving ends are both 110 kV and the phase angle between them is 30°, find the receiving-end power factor and the current.

With the sending-end voltage maintained at 110 kV, if the load is suddenly thrown off, find the corresponding receiving-end voltage. **[0.868 (lead), 326 A,114.5 kV]**

8. A 3-phase, 50-Hz, 150 km line has a resistance, inductive reactance and capacitive shunt admittance of 0.1 Ω, 0.5 Ω and 3×10^{-6} siemens/km/phase. If the line delivers 50 MW at 110 kV and 0.8 p.f. lag, determine the sending-end voltage and current. Assume a nominal π-circuit for the line.
[143.7 kV, 306.3 A]

41.25. Generalised Circuit Constants of a Transmission Line

For any 4-terminal network *i.e.* one having two input and two output terminals (like a transmission line) the sending-end voltage per phase and the currents at the receiving and sending-end can be expressed by the following two equations :

$$\mathbf{V_S = AV_R + BI_R} \quad \text{...(}i\text{)}$$

$$\mathbf{I_S = CV_R + DI_R} \quad \text{...(}ii\text{)}$$

where **A**, **B**, **C** and **D** are the constants known as '*generalized circuit constants*' of the transmission line. Their values depend on the particular method adopted for solving the transmission network. Let us consider the following cases.

***(i)* Short Line.** In the case of lines up to 50 km, the effect of capacitance on the line performance is negligible. Hence, such a line can be represented as shown in Fig. 41.42 (*a*).

Here, $\mathbf{I_S = I_R}$ and $\mathbf{V_S = V_R + I_R Z}$

Comparing these with Eq. ***(i)*** and ***(ii)*** above, we get that

$$\mathbf{A} = 1 ; \quad \mathbf{B = Z}, \quad \mathbf{C} = 0 \quad \text{and} \quad \mathbf{D} = 1$$

Incidentally, it may be noted that $\mathbf{AD - BC} = 1$

(*ii*) Medium Line–Nominal-T Method

The circuit is shown in Fig. 41.42 (*b*).

It is seen that, $\mathbf{V}_S = \mathbf{V}_1 + \mathbf{I}_S \mathbf{Z}/2$ ...(*iii*)

Also $\mathbf{V}_1 = \mathbf{V}_R + \mathbf{I}_R\mathbf{Z}/2$ ...(*iv*)

Now, $\mathbf{I}_C = \mathbf{I}_S - \mathbf{I}_R = \mathbf{V}_1\mathbf{Y} = \mathbf{Y}\,(\mathbf{V}_R + \mathbf{I}_R\mathbf{Z}/2)$

$\therefore$ $\mathbf{I}_S = \mathbf{V}_R\mathbf{Y} + \mathbf{I}_R\left(1 + \frac{\mathbf{YZ}}{2}\right)$ ...(*v*)

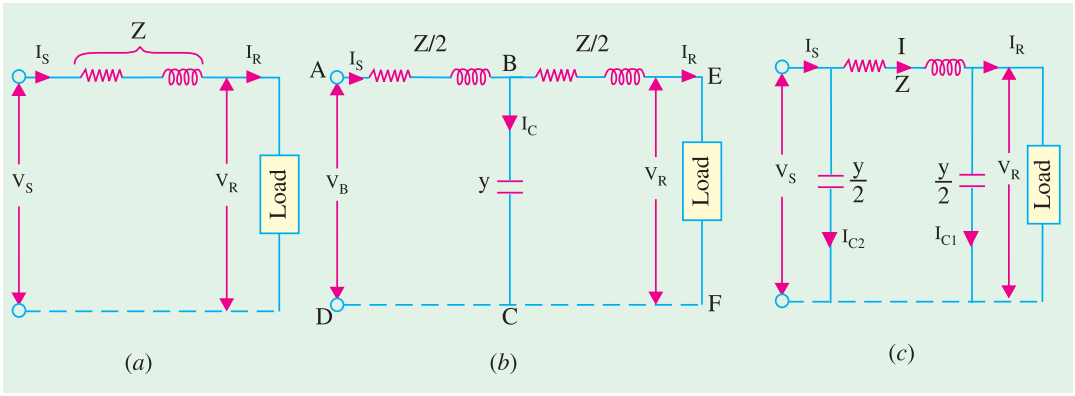


Fig. 41.42

Eliminating $\mathbf{V}_1$ from Eq. ***(iii)*** and ***(iv)*** we get

$$\mathbf{V}_S = \mathbf{V}_R + \frac{\mathbf{I}_S\mathbf{Z}}{2} + \frac{\mathbf{I}_R\mathbf{Z}}{2} \quad \text{...(vi)}$$

Substituting the value of I_S, we get

$$\mathbf{V}_S = \left(1 + \frac{\mathbf{YZ}}{2}\right)\mathbf{V}_R + \left(\mathbf{Z} + \frac{\mathbf{YZ}^2}{4}\right)\mathbf{I}_R \quad \text{...(vii)}$$

Comparing Eq. ***(vii)*** and ***(v)*** with Eq. ***(i)*** and ***(ii)*** respectively, it is found that

$$\mathbf{A} = \mathbf{D} = 1 + \frac{\mathbf{YZ}}{2}\ ;\ \mathbf{B} = \left(1 + \frac{\mathbf{YZ}}{4}\right) \text{ and } \mathbf{C} = \mathbf{Y}$$

It can again be proved that $\mathbf{AD - BC} = 1$.

(*iii*) Medium Line–Nominal- π Method

The circuit is shown in Fig. 41.42 (*c*). Here, series impedance per phase = $(R + j\,X)$ and admittance is $\mathbf{Y} = j\omega\,C$.

As seen, $\mathbf{I}_S = \mathbf{I} + \mathbf{I}_{C2} = \mathbf{I} + \mathbf{V}_S\,\mathbf{Y}/2$ ...(*viii*)

Also $\mathbf{I} = \mathbf{I}_{C1} + \mathbf{I}_R = \mathbf{I}_R + \mathbf{V}_R\,\mathbf{Y}/2$ ...(*ix*)

Now, $\mathbf{V}_S = \mathbf{V}_R + \mathbf{IZ} = \mathbf{V}_R + \mathbf{Z}\left(\mathbf{I}_R + \frac{\mathbf{V}_R\,\mathbf{Y}}{2}\right)$

$$= \left(1 + \frac{\mathbf{YZ}}{2}\right)\mathbf{V}_R + \mathbf{ZI}_R \quad \text{...(x)}$$

Eliminating $\mathbf{I}$ from Eq. (*viii*) and (*ix*), we get

$$\mathbf{I}_S = \mathbf{I}_R + \frac{\mathbf{V}_R\,\mathbf{Y}}{2} + \frac{\mathbf{V}_S\,\mathbf{Y}}{2}$$

Now, substituting the value of $\mathbf{V}_S$, we have

$$\mathbf{I}_S = \mathbf{I}_R + \frac{\mathbf{V}_R \mathbf{Y}}{2} + \frac{\mathbf{Y}}{2}\left[\left(1 + \frac{\mathbf{YZ}}{2}\right)\mathbf{V}_R + \mathbf{Z}\mathbf{I}_R\right]$$

or $$\mathbf{I}_S = \mathbf{Y}\left(1 + \frac{\mathbf{YZ}}{4}\right)\mathbf{V}_R + \left(1 + \frac{\mathbf{YZ}}{2}\right)\mathbf{I}_R \quad \text{...(xi)}$$

Comparing Eq. **(x)** and **(ii)** with Eq. **(i)** and **(xi)** above, we get,

$$\mathbf{A} = \mathbf{D} = \left(1 + \frac{\mathbf{YZ}}{2}\right) ; \mathbf{B} = \mathbf{Z}, \mathbf{C} = \mathbf{Y}\left(1 + \frac{\mathbf{YZ}}{4}\right)$$

Again, it can be shown that **AD – BC = 1**

Example 41.29. *Find the following for a single-circuit transmission line delivering a load of 50 MVA at 110 kV and p.f. 0.8 lagging :*

(i) *sending-end voltage,* ***(ii)*** *sending-end current,* ***(iii)*** *sending-end power, (iv) efficiency of transmission. (Given A = D = 0.98 ∠3° , B = 110 ∠75° ohm, C = 0.0005 ∠80° ohm)*

(Power Systems, AMIE, Sec. B, 1993)

Solution. Receiving-end voltage, $V_R = 110/\sqrt{3} = 63.5$ kV

Taking this voltage as reference voltage, we have

$\mathbf{V}_R = (63{,}500 + j\,0)$

$\mathbf{I}_R = 50 \times 10^6/\sqrt{3} \times 110 \times 10^3 = 262.4$ A

$\mathbf{I}_R = 262.4 \angle \cos^{-1} 0.8 = 262.4\,(0.8 - j\,0.6) = (210 - j\,157.5)$ A

$\mathbf{AV}_R = 0.98 \angle 3° \times 63{,}500 \angle 0° = 62{,}230 \angle 3° = (62145 + j\,3260)$ V

$\mathbf{BI}_R = 110 \angle 75° \times 262.4 \angle 36.86° = 28{,}865 \angle 111.8° = (-10{,}720 + j\,26{,}800)$ V

(i) $\mathbf{V}_S = \mathbf{AV}_R + \mathbf{BI}_R = 51{,}425 + j\,30{,}060 = 59{,}565, \angle 30.3°$ V

$\mathbf{CV}_R = 0.0005 \angle 80° \times 63{,}500 \angle 0° = 31.75 \angle 80° = (5.5 + j\,31.3)$ A

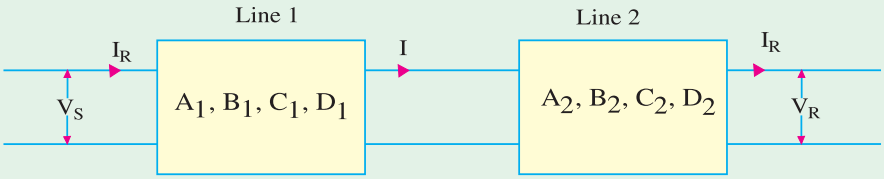


Fig. 41.43

$\mathbf{DI}_R = 0.98 \angle 3° \times 262.4 \angle 36.86° = 257.75 \angle 39.86° = 197.4 + j\,164.8$

(ii) $\mathbf{I}_S = \mathbf{CV}_R + \mathbf{DI}_R = 203 + j\,196 = \mathbf{282 \angle 44°\,A}$

(iii) Sending-end power $= 3V_S I_S \cos \phi_S = 3 \times 59.565 \times 282 \times \cos \angle 44.30° - 30.3° =$ **48.96 MW**

Receiving end power $= 50 \times 0.8 = 40$ MW

(iv) Transmission $\eta = 40 \times 100/48.96 =$ **81.7%**

Example 41.30. *A 150 km, 3-ϕ, 110-V, 50-Hz transmission line transmits a load of 40,000 kW at 0.8 p.f. lag at receiving end.*

resistance/km/phase = 0.15 Ω, reactance/km/phase = 0.6 Ω ; susceptance/km/phase = 10^{-5} S

(a) *determine the A, B, C and D constants of the line* ***(b)*** *find regulation of the line.*

(Power System-I, AMIE, 1993)

Solution. We will use the nominal-π method to solve the problem. For a length of 150 km :

$\mathbf{R} = 0.15 \times 150 = 22.5\ \Omega$; $\mathbf{X} = 0.6 \times 150 = 90\ \Omega$; $\mathbf{Y} = 150 \times 10^{-5}$ S $= 15 \times 10^{-4}$ S

$\mathbf{Z} = (\mathbf{R} + j\,\mathbf{X}) = (22.5 + j\,90) = 92.8 \angle 75°$ ohm ; $\mathbf{Y} = 15 \times 10^{-4} \angle 90°$ S

$$(a)\quad \mathbf{A} = \mathbf{D} = \left(1+\frac{\mathbf{YZ}}{2}\right) = 1 + \frac{j\,15\times10^{-4}}{2}\,(22.5 + j\,90) = (0.9675 + j\,0.01688)$$
$$= 0.968 \angle 1.0^\circ$$
$$\mathbf{B} = \mathbf{Z} = 92.8 \angle 7.5^\circ\ ;\ \mathbf{C} = \mathbf{Y}\left(1+\frac{\mathbf{YZ}}{4}\right) = j\,15\times10^{-4}\left[1+\frac{1}{4}.\,j\,15\times10^{-4}\,(22.5 + j90)\right]$$
$$= -\,0.00001266 + j\,0.00145 = 0.00145 \angle 90.5^\circ$$

(b) Now, the regulation at the receiving-end is defined as the change in voltage when full-load is thrown off, the sending-end voltage being held constant.

Now, when load is thrown off, $\mathbf{I}_R = 0$. Hence, putting this value in $\mathbf{V}_S = \mathbf{AV}_R + \mathbf{BI}_R$, we get

$$\mathbf{V}_S = \mathbf{AV}_{RO} \text{ or } \mathbf{V}_{RO} = \frac{\mathbf{V}_S}{\mathbf{A}} \quad \therefore \quad \%\ \text{regn.} = \frac{V_{RO} - V_R}{V_R}\times 100 = \frac{(V_S/A - V_R)}{V_R}\times 100$$

Now, $V_R = 100/\sqrt{3}$ kV = 63,520 V—at load

$\mathbf{I}_R = 40\times10^6/\sqrt{3}\times 110\times10^3\times 0.8 = 263$ A ; $\mathbf{I}_R = 263\,(0.8 - j\,0.6) = (210 - j\,158)$

$\mathbf{V}_S = \mathbf{AV}_R + \mathbf{BI}_R = 63{,}520\,(0.9675 + j\,0.01688) + (22.5 + j\,90)\,(210 - j\,158)$

$= 80{,}450 + j\,16{,}410 = 82{,}110 \angle 11.5^\circ$

$V_S/A = 82{,}110/0.968 = 84{,}800$; regn. $= \dfrac{84{,}800 - 63{,}520}{63{,}520}\times 100 = \mathbf{33.5\%}$

Example 41.31. *A 132-kV, 50-Hz, 3-phase transmission line delivers a load of 50 MW at 0.8 p.f. lagging at receiving-end.*

The generalised constants of the transmission line are

$A = D = 0.95 \angle 1.4^\circ$; $B = 96 \angle 7.8^\circ$; $C = 0.0015 \angle 90^\circ$

Find the regulation of the line and the charging current. Use nominal T-method.

(Electrical Power-I, Bombay Univ.)

Solution. $\mathbf{I}_R = 50\times10^6/\sqrt{3}\times 132\times10^3\times 0.8 = 273$ A

$\mathbf{I}_R = 273\,(0.8 - j\,0.6) = 218 - j\,164 = 273 \angle -36.9^\circ$

$\mathbf{V}_R = 132{,}000/\sqrt{3} = 76{,}230$; $\mathbf{V}_R = 76{,}230 + j\,0$

$\mathbf{V}_S = \mathbf{AV}_R + \mathbf{BI}_R = 76{,}230 \angle 0^\circ \times 0.95 \angle 1.4^\circ + 96 \angle 78^\circ.\ 273 \angle -36.9^\circ$

$= 72{,}418 \angle 1.4 + 26{,}208 \angle 41.1^\circ = 89{,}000 + j\,21{,}510 = 92{,}150 \angle 13^\circ$

$\mathbf{I}_S = \mathbf{CV}_R + \mathbf{DI}_R = 76{,}230\times 0.0015 \angle 90^\circ + 0.95 \angle 1.4^\circ.\ 273 \angle -36.9^\circ$

$= j\,114 + 259.3 \angle -35.5^\circ = 211 - j\,36$

$V_{RO} = V_S/A = 92{,}150/0.95 = 97{,}000$; % regn $= \dfrac{97{,}000 - 76{,}230}{76{,}230} = 27.1$

$\mathbf{I}_C = \mathbf{I}_S - \mathbf{I}_R = (211 - j\,36) - (218 - j\,164) = -7 + j\,128 = 128.2 \angle 93.1^\circ$

Example 41.32. *A 3-phase transmission line consists of two lines 1 and 2 connected in series, line 1 being at the sending end and 2 at the receiving end. The respective auxiliary constants of the two lines are : A_1, B_1, C_1, D_1 and A_2, B_2, C_2, D_2. Find the A, B, C, D constants of the whole line which is equivalent to two series-connected lines.*

Solution. The two series-connected lines along with their constants as shown in Fig. 41.43.

For line No. 1

$$\mathbf{V}_S = \mathbf{A}_1\mathbf{V} + \mathbf{B}_1\mathbf{I} \quad ; \quad \mathbf{I}_S = \mathbf{C}_1\mathbf{V} + \mathbf{D}_1\mathbf{I} \qquad \ldots(i)$$

For line No. 2

$$\mathbf{V} = \mathbf{A}_2\mathbf{V}_R + \mathbf{B}_2\mathbf{I}_R \quad ; \quad \mathbf{I} = \mathbf{C}_2\mathbf{V}_R + \mathbf{D}_2\mathbf{I}_2 \qquad \ldots(ii)$$

Substituting the values of **V** and **I** from Eq. ***(ii)*** into Eq. ***(i)***, we get

$$V_S = A_1(A_2V_R + B_2I_R) + B_1(C_2V_R + D_2I_R) = (A_1A_2 + B_1C_2)V_R + (A_1B_2 + B_1D_2)I_R$$

and $$I_S = C_1(A_2V_R + B_2I_R) + D_1(C_2V_R + D_2I_R) = (C_1A_2 + D_1C_2)V_R + (C_1B_2 + D_1D_2)I_R$$

Hence, the two lines connected in series have equivalent auxiliary constants of

$$\mathbf{A} = \mathbf{A_1A_2 + B_1C_2}, \qquad \mathbf{B} = \mathbf{A_1B_2 + B_1D_2}$$

$$\mathbf{C} = \mathbf{C_1A_2 + D_1C_2} \quad \text{and} \quad \mathbf{D} = \mathbf{C_1B_2 + D_1D_2}$$

41.26. Corona

When an alternating potential difference is applied across two conductors whose spacing is large as compared to their diameter, there is no apparent change in the condition of the atmospheric air surrounding the wires if voltage is low. However, when the p.d. is increased, then a point is reached when a faint luminous glow of bluish colour appears along the lengths of conductors and at the same time a hissing sound is heard. This bluish discharge is known as ***corona***. Corona is always accompanied by the production of ozone which is readily detected because of its characteristic odour. If the p.d. is further increased, then the glow and hissing both increase in intensity till a spark-over between the conductors takes place due to the break-down of air insulation. If the conductors are smooth and polished, the corona glow is uniform along their length but if there is any roughness, they will be picked up by relatively brighter illumination. In the case of conductors with a spacing shorter as compared to their diameters, sparking may take place without any visible glow. If the p.d. between wires is direct instead of alternating, there is a difference in the appearance of the two wires. The positive wire has a smooth glow about it whereas the glow about the negative wire is spotty.

Corona occurs when the electrostatic stress in the air around the conductors exceeds 30 kVA (maximum)/cm or 21.1 kV (r.m.s.)/cm. The ***effective disruptive critical voltage*** to neutral is given by the relation.

$$V_C = m_0 g_0 \, \delta \, r \log_h D/r \text{ kV/phase} = 2.3 \, m_0 g_0 \, \delta \, r \log_{10} D/r \text{ kV/phase}$$

where m_0 = irregularity factor which takes into account the surface conditions of the conductor.

D = distance between conductors in cm.

r = radius of the conductor in cm or the radius of the circumscribing circle in a stranded cable.

g_0 = breakdown strength or disruptive gradient of air at 76 cm of mercury and 25°C

= 30 kV (max.)/cm = 21.1 kV (r.m.s.)/cm

δ = air density factor

Substituting the value of g_0, we have

$$V_C = 21.1 \, m_0 \, \delta \, r \log_h D/r \text{ kV/phase} = 21.1 \, m_0 \, \delta \, r \times 2.3 \log_{10} D/r \text{ kV/phase}$$

$$= 48.8 \, m_0 \, \delta \, r \log_{10} D/r \text{ kV/phase}$$

The value of δ is given by $\delta = \dfrac{3.92\,b}{273 + t}$

where b = barometric pressure in cm of Hg and t = temp. in degrees centigrade.

when b = 76 cm and t = 25°, C δ = 1

The irregularity factor m_0 depends on the shape of cross-section of the wire and the state of its surface. Its value is unity for an absolutely smooth wire of one strand of circular section and less than unity for wires roughened due to weathering as shown below :

Irregularity Factor

Polished wires	...	1.0
Weathered wires	...	0.93 to 0.98
7-strand cables, concentric lay	...	0.83 to 0.87
Cables with more than 7-strands	...	0.80 to 0.85

41.27. Visual Critical Voltage

This voltage is higher than the disruptive critical voltage V_C. It is the voltage at which corona appears all along the line. The ***effective*** or r.m.s. value of visual critical voltage is given by the following empirical relation.

$$V_v = m_v g_0 \delta r \left(1+\frac{0.3}{\sqrt{\delta r}}\right) \log_h D/r \text{ kV/phase}$$

$$= 21.1\, m_v \delta r \left(1+\frac{0.3}{\sqrt{\delta r}}\right) \times 2.3 \log_{10} D/r \text{ kV/phase}$$

$$= 48.8\, m_v \delta r \left(1+\frac{0.3}{\sqrt{\delta r}}\right) \log_{10} D/r \text{ kV/phase}$$

where m_v is another irregularity factor having a value of 1.0 for smooth conductors and 0.72 to 0.82 for rough conductors.

41.28. Corona Power

Formation of corona is always accompanied by dissipation of energy. This loss will have some effect on efficiency of the line but will not have any appreciable effect on the line regulation. This loss is affected both by atmospheric and line conditions. Soon after the critical voltage is reached, the corona loss increases as the square of the excess voltage. The loss for voltage of V kilovolt to neutral is given by the following empirical relation.

$$P = 241 \frac{(f+25)}{\delta} \sqrt{\left(\frac{r}{D}\right)} (V - V_c)^2 \times 10^{-5} \text{ kW/km/phase}$$

where f is the frequency of the a.c. in Hz.

Obviously, total loss equals 3 times the above.

41.29. Disadvantages of Corona

1. There is a definite dissipation of power although it is not so important except under abnormal weather conditions like storms etc.
2. Corrosion due to ozone formation.
3. The current drawn by the line due to corona losses is non-sinusoidal in character, hence it causes non-sinusoidal drop in the line which may cause some interference with neighbouring communication circuits due to electromagnetic and electrostatic induction. Such a shape of corona current tends to introduce a large third harmonic component.

   However, it has been found that corona works as a safety valve for surges.
4. Particularly intense corona effects are observed at a working voltage of 35 kV or higher. Hence, designs have to be made to avoid any corona on the bus-bars of substations rated for 35 kV and higher voltages during their normal operation. Corona discharge round bus-bars is extremely undesirable because the intense ionization of the air reduces its dielectric strength, makes it easier for the flashover to occur in the insulators and between phases particularly when the surfaces concerned are dirty or soiled with other deposits. The ozone produced due to corona discharge aggressively attacks the metallic components in the substations and switchgear, covering them with oxides. Moreover, the crackling sound of the corona discharge in a substation masks other sounds like light crackling noise due to arcing in a loose contact, the sound of an impending breakdown or creepage discharge in the equipment, the rattling noise due to the loosening of steel in a transformer core etc. The timely detection of such sounds is very important if any serious breakdown is to be avoided.

Example 41.33. *Find the disruptive critical voltage for a transmission line having :*
conductor spacing = 1 m ; conductor (stranded) radius = 1 cm
barometric pressure = 76 cm of Hg ; temperature = 40°C
Air break-down potential gradient (at 76 cm of Hg and at 25°C) = 21.1 kV (r.m.s.)/cm.
(Electric Power Systems-I, Gujarat Univ.)

Solution. $V_C = 2.3\, m_0\, g_0\, r \log_{10} D/r$ kV/phase

Here, $g_0 = 21.1$ kV (r.m.s.)/cm ; $m_0 = 0.85$ (assumed)

$\delta = 3.92 \times 76/(273 + 40) = 0.952$; $\log_{10} D/r = \log_{10} 100/1 = 2$

$\therefore \quad V_c = 2.3 \times 0.85 \times 21.1 \times 0.952 \times 1 \times 2 = 78.54$ kV (r.m.s.)/phase

Line value $= 78.54 \times \sqrt{3} =$ **136 kV (r.m.s.)/phase**

Example 41.34. *Find the disruptive critical and visual corona voltage of a grid-line operating at 132 kV.*
conductor dia = 1.9 cm ; conductor spacing = 3.81 m
temperature = 44°C ; barometric pressure = 73.7 cm
conductor surface factor :
fine weather = 0.8 ; rough weather = 0.66.
(Electrical Power Systems-III, Gujarat Univ.)

Solution. $V_c = 48.8\, m_0\, \delta\, r \log_{10} D/r$ kV/phase

Here, $m_0 = 0.8$; $\delta = 3.92 \times 73.7/(273 + 44) = 0.91$

$\log_{10} 381/1.9 = \log_{10} 200.4 = 2.302$

$\therefore \quad V_c = 48.8 \times 0.8 \times 0.91 \times 1.9 \times 2.302 = 155.3$ kV/phase

$$V_v = 48.8\, m_v\, \delta\, r \left(1 + \frac{0.3}{\sqrt{\delta r}}\right) \log_{10} D/r \text{ kV/phase}$$

Here, $m_v = 0.66$; $\delta = 0.91$; $\sqrt{\delta r} = \sqrt{0.91 \times 1.9} = 1.314$

$\therefore \quad V_v = 48.8 \times 0.66 \times 0.91 \times 1.9 \left(1 + \frac{0.3}{1.314}\right) 2.302 =$ **157.5 kV/phase**

Example 41.35. *A certain 3-phase equilateral transmission line has a total corona loss of 53 kW at 106 kV and a loss of 98 kW at 110.9 kV. What is the disruptive critical voltage between lines? What is the corona loss at 113 kV?* **(Electrical Power Systems-II, Gujarat Univ.)**

Solution. As seen from Art. 41.28, the total corona loss for three phases is given by

$$P = 3 \times \frac{241\,(f + 25)}{\delta} \cdot \sqrt{\frac{r}{D}}\,(V - V_c)^2 \times 10^{-5} \text{ kW/km}$$

Other things being equal, $P \propto (V - V_c)^2$

$\therefore \quad 53 \propto \left(\frac{106}{\sqrt{3}} - V_c\right)^2 \propto (61.2 - V_c)^2$ —**1st case**

$98 \propto \left(\frac{110.9}{\sqrt{3}} - V_c\right)^2 \propto (64 - V_c)^2$ — **2nd case**

$\therefore \quad \frac{98}{53} = \frac{(64 - V_c)^2}{(61.2 - V_c)^2}$ or $V_c =$ **54.2 kV/km**

Similarly, $W \propto \left(\frac{113}{\sqrt{3}} - V_c\right)^2 \propto (65.2 - V_c)^2$

$$\frac{W}{98} = \frac{(65.2 - V_c)^2}{(64 - V_c)^2} = \frac{(65.2 - 54.2)^2}{(64 - 54.2)^2} \quad \therefore \quad W = \mathbf{123.4\ kW}$$

Example 41.36. *A 3-phase, 50-Hz, 220-kV transmission line consists of conductors of 1.2 cm radius spaced 2 metres at the corners of an equilateral triangle. Calculate the corona power loss per km of the line at a temperature of 20°C and barometric pressure of 72.2 cm. Take the surface factors of the conductor as 0.96.* **(Electrical Power-II, Bangalore Univ.)**

Solution. As seen from Art. 41.28, corona loss per phase is

$$P = 241 \frac{(f + 25)}{\delta} \sqrt{(r/D)} \,.\, (V - V_c)^2 \times 10^{-5} \text{ kW/km/phase}$$

Here, $$\delta = \frac{3.92\, b}{273 + t} = \frac{3.92 \times 72.2}{273 + 20} = 0.966$$

$$V_c = 48.8\, m_0\, \delta\, r \log_{10} D/r = 48.8 \times 0.96 \times 0.966 \times 1.2 \times \log_{10} \frac{200}{1.2}$$

$$= 120.66 \text{ kV/phase}$$

$$V = 220/\sqrt{3} = 127 \text{ kV/phase}$$

$\therefore$ $$P = 241 \times \frac{75}{0.966} \times \sqrt{\left(\frac{1.2}{200}\right)} \times (127 - 120.66)^2 \times 10^{-5} = 0.579 \text{ kW/km/phase}$$

Total loss for 3 phase = $3 \times 0.579 = \mathbf{1.737\ kW/km}$

41.30. Underground Cables

Underground cables are used where overhead lines are not possible as in large cities despite the fact that in their case, cost per kW per km is much more as compared to overhead transmission lines. Another advantage of overhead system for distributors is that tapping can be made at any time without any disturbance, which is of great importance in rapidly developing areas. However, underground cables are more advantageous for feeders which are not likely to be disturbed for tapping purposes because, being less liable to damage through storms or lighting or even wilful damage, they offer a safer guarantee of supply. But this advantage may be offset by the cost of trenching and expensive jointing necessary in case of repairs.



Underground cables are used where overhead transmission is difficult

However, cables score over overhead lines in cases where voltage regulation is more important, because, due to very small spacing of their conductors, they have a very low inductance and hence low inductive drops.

Cables may be classified in two ways according to ***(i)*** the type of insulating material used in their manufacture or ***(ii)*** the voltage at which they transmit power. The latter method of classification is, however, more generally used according to which cables are divided into three groups :

1. Low-tension cables–up to 1000 V
2. High-tension cables–up to 23,000 V
3. Super-tension cables–from 66 kV to 132 kV

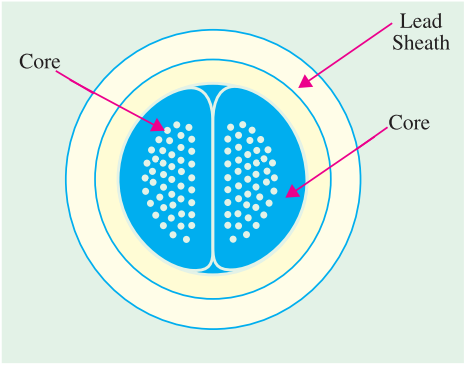


Fig. 41.44

For all cables, the conductor is tinned stranded copper of high conductivity. Stranding is done to secure flexibility and the number of conductors in a core is generally 3, 7, 19 and 37 etc. Except for 3-strand, all numbers have a centrally-disposed conductor with all others surrounding it. A cable may have one or more than one core depending on the type of service for which it is intended. It may be ***(i)*** single-core ***(ii)*** two-core ***(iii)*** three-core and ***(iv)*** four-core etc.

In Fig. 41.44 is shown a section through a twin-cored, high-tension lead-covered underground cable sheathed with a continuous tube of pure lead whereas Fig. 41.45 shows a section of a typical concentric type 2-core cable used for single-phase distribution. The cores are arranged concentrically. The outer core is arranged in the form of hollow tubing. Both cores are of stranded copper and paper-insulated and are protected by a lead-sheath.

In Fig. 41.46 is shown a section through a 3-core extra-high-tension paper-insulated lead-covered and steel-wire armoured cable.

The cores are surrounded by insulation or impregnated paper, varnished cambric or vulcanised bitumen.

The insulation is, in turn, surrounded by a metal sheath made of lead or a lead alloy and prevents the entry of moisture into the inner parts. On the sheath is applied the bedding which consists of two compounded paper tapes along with suitable compounded fibrous materials.

Next comes 'armouring' which is placed over the bedding and consists of either galvanized steel wires or two layers of steel tape. Armouring may not be done in the case of some cables.

Next comes 'serving' which consists of compounded fibrous material (jute etc.) placed over the armouring in the case of armoured cables or over the metal sheath in the case of unarmoured cables-in which case serving consists of two layers of compounded paper tapes and a final covering of compounded fibrous material.

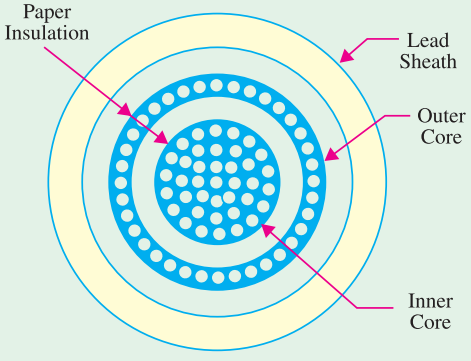


Fig. 41.45

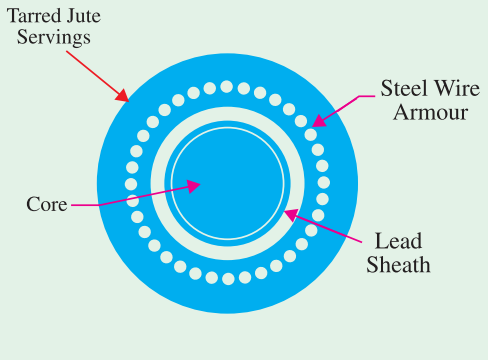


Fig. 41.46

41.31. Insulation Resistance of a Single-core Cable

The expression for the insulation resistance of such a cable, as derived in Art. 5.14.

$$= \frac{2.3\rho}{2\pi l} \log_{10} \frac{r_2}{r_1} \text{ ohm}$$

The value of specific resistance for paper is approximately 500 Ω-m.

41.32. Capacitance and Dielectric Stress

This has already been dealt with in Art. 5.9 and 5.13.

41.33. Capacitance of 3-core Belted Cables

The capacitance of a cable is much more important than of the overhead wires of the same length because in cables ***(i)*** conductors are nearer to each other and the sheath and ***(ii)*** they are separated by a dielectric medium of higher permittivity as compared to air. Fig. 41.47 shows a system of capacitances

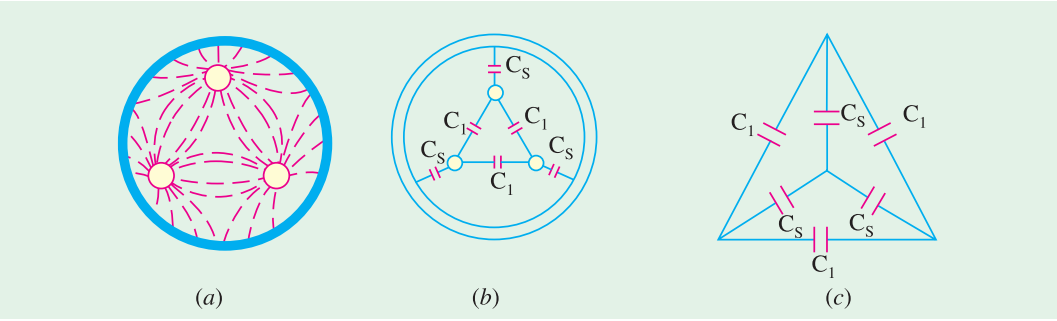


Fig. 41.47

in a belted 3-core cable used for 3-phase system. It can be regarded as equivalent to three-phase cables having a common sheath. Since there is a p.d. between pairs of conductors and between each conductor and sheath, there exist electrostatic fields as shown in Fig. 41.47 (*a*) which shows average distribution of electrostatic flux, though, actually, the distribution would be continually changing because of changing potential difference between conductors themselves and between conductors and sheath. Because of the existence of this electrostatic coupling, there exist six capacitances as shown in Fig. 41.47 (*b*). The three capacitances between three cores are delta-connected whereas the other three between each core and the sheath are star-connected, the sheath forming the star-point [Fig. 41.47 (*c*)].

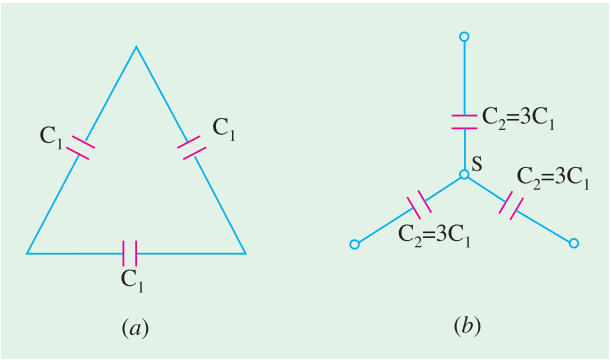


Fig. 41.48

The three delta-connected capacitances each of value C_1 can be converted into equivalent star-capacitance C_2 which will be three times the delta-capacitances C_1 as shown in Fig. 41.48.

The two star capacitances can now be combined as shown in Fig. 41.49 (*a*). In this way, the whole cable is equivalent to three star-connected capacitors each of capacitance $C_n = 3C_1 + C_S$ as shown in Fig. 41.49 (*b*).

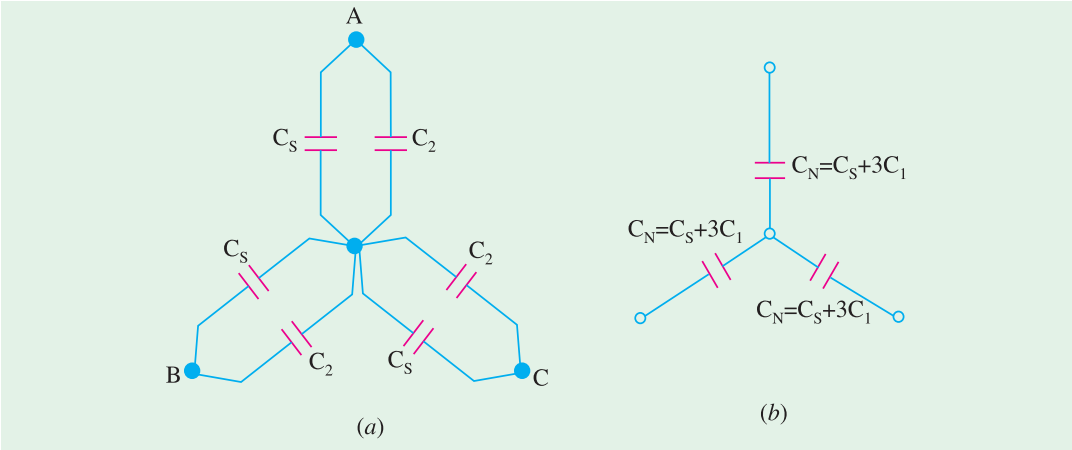


Fig. 41.49

If *Vp* is the phase voltage, then the charging current is given by

$$I_C = V_P (C_S + 3C_1)\omega = V_P \omega C_n$$

41.34. Tests for Three-phase Cable Capacitance

The following three tests may be used for the measurement of C_1 and C_S :

(i) In the first test, all the three cores are bunched together and then capacitance is measured by usual methods between these bunched cores and the earthed sheath. This gives 3 C_S because the three capacitances are in parallel.

(ii) In the second method, two cores are bunched with the sheath and capacitance is measured between these and the third core. It gives $2C_1 + C_S$. From this value, C_1 and C_S can be found.

(iii) In the third method, the capacitance C_L between two cores or lines is measured with the third core free or shorted to earth. This gives

$$\left(C + \frac{C_1}{2} + \frac{C_S}{2}\right) = \frac{1}{2}(3C_1 + C_S) = \frac{C_n}{2}$$

Hence, C_n is twice the measured value *i.e.* $C_n = 2C_L$

Therefore, charging current $I_C = \omega V_P C_n = \frac{2}{\sqrt{3}} . \omega V_L C_L$

where V_L is the line voltage (not phase voltage).

Example 41.37. *A single-core lead-covered cable is to be designed for 66-kV to earth. Its conductor radius is 1.0 cm and its three insulating materials A, B, C have permittivities of 5.4 and 3 respectively with corresponding maximum safe working stress of 38 kV per cm (r.m.s. value), 26-kV per cm and 20-kV per cm respectively. Find the minimum diameter of the lead sheath.*

Solution.

$$g_{1max} = \frac{Q}{2\pi\epsilon_0\epsilon_{r1} r} \text{ or } 38 = \frac{Q}{2\pi\epsilon_0 \times 5 \times 1} \quad ...(i)$$

$$g_{2max} = \frac{Q}{2\pi\epsilon_0\epsilon_{r2} r_1} \text{ or } 26 = \frac{Q}{2\pi\epsilon_0 \times 4 \times r_1} \quad ...(ii)$$

$$g_{3max} = \frac{Q}{2\pi\epsilon_0\epsilon_{r3} r_2} \text{ or } 20 = \frac{Q}{2\pi\epsilon_0 \times 3 \times r_2} \quad ...(iii)$$

From ***(i)*** and ***(ii)***, we get, $38/26 = 4r_1/5$, $r_1 = 1.83$ cm

Similarly, from ***(i)*** and ***(ii)***,

$$38/20 = 3r_2/5 \; ; \; r_2 = 3.17 \text{ cm}$$

$$V_1 = g_{1max} \times r \times 2.3 \log_{10} r_1/r$$

$$= 38 \times 1 \times 2.3 \log_{10} \frac{1.83}{1} = 22.9 \text{ kV}$$

$$V_2 = g_{2max} \times r_1 \times 2.3 \log_{10} \frac{r_2}{r_1}$$

$$= 26 \times 1.83 \times 2.3 \log_{10} \frac{3.17}{1.83}$$

$$= 26.1 \text{ kV}$$

$$V_3 = g_{3max} \times r_2 \times 2.3 \log_{10} \frac{r_3}{r_2}$$

$$= 20 \times 3.17 \times 2.3 \log_{10} \frac{r_3}{3.17}$$

$$= 145.82 \log_{10} \frac{r_3}{3.17}$$

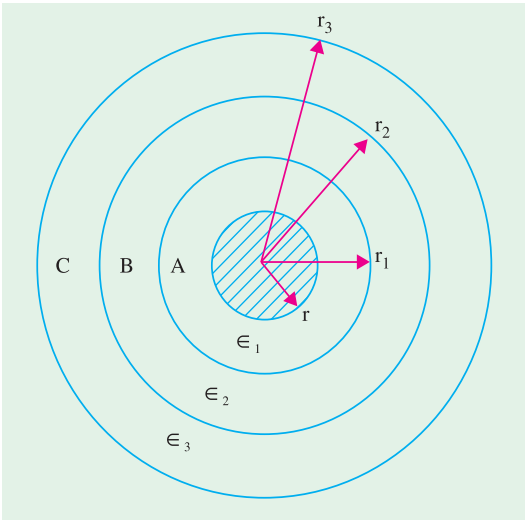


Fig. 41.50

Now, $V = V_1 + V_2 + V_3$

$\therefore \quad 66 = 22.9 + 26.1 + 145.82 \log_{10} \frac{r_3}{3.17}$

$\therefore \quad r_3 = 4.15$ cm

$\therefore$ diameter of the sheath $= 2r_3 = 2 \times 4.15 =$ **8.3 cm**

Example 41.38. *The capacitances per kilometer of a 3-phase cable are 0.63 μF between the three cores bunched and the sheath and 0.37 μF between one core and the other two connected to sheath. Calculate the charging current taken by eight kilometres of this cable when connected to a 3-phase, 50-Hz, 6,600-V supply.*

Solution. As shown in Art. 40.33, $0.63 = 3C_S \quad \therefore \quad C_S = 0.21$ μ F/km

From the second test,

$$0.37 = 2C_1 + C_2 = 2C_1 + 0.21 \quad \therefore \quad C_1 = 0.08 \text{ μ F/km}$$

$\therefore \quad C_S$ for 8 km $= 0.21 \times 8 = 1.68$ μ F ; C_1 for 8 km $= 0.08 \times 8 = 0.64$ μ F

$$C_n = C_S + 3C_1 = 1.68 + (3 \times 0.64) = 3.6 \text{ μ F}$$

Now, $V_p = (6{,}600/\sqrt{3})$; $\quad \omega = 314$ rad/s

$$I_C = \omega V_p C_n = (6{,}600/\sqrt{3}) \times 3.6 \times 10^{-6} \times 314 = \textbf{4.31 ampere}$$

Example 41.39. *A 3-core, 3-phase belted cable tested for capacitance between a pair of cores on single phase with the third core earthed, gave a capacitance of 0.4 mF per km. Calculate the charging current for 1.5 km length of this cable when connected to 22 kV, 3-phase, 50-Hz supply.*

Solution. $C_L = 0.4$ μF, $\quad V_L = 22{,}000$ V, $\quad \omega = 314$ rad/s

$$I_C = \frac{2}{\sqrt{3}} \quad \omega V_L C_L = \frac{2}{\sqrt{3}} \times 22{,}000 \times 0.4 \times 10^{-6} \times 314 = 3.2 \text{ A}$$

Charging current for 15 km $= 3.2 \times 15 =$ **48 A**

Example 41.40. *A 3-core, 3-phase metal-sheathed cable has **(i)** capacitance of 1 μF between shorted conductors and sheath and **(ii)** capacitance between two conductors shorted with the sheath and the third conductor 0.6 μF. Find the capacitance **(a)** between any two conductors **(b)** between any two shorted conductors and the third conductor.* **(Power Systems-I, AMIE, Sec. B, 1993)**

Solution. (*a*) The capacitance between two cores or lines when the third core is free or shorted to earth is given by $1/2\ (3C_1 + C_S)$

Now, **(*i*)** we have $3C_S = 1$ μ F or $C_s = 1/3$ μ F $= 0.333$ μ F

From *(ii)* we get, $2C_1 + C_S = 0.6\ \mu F$, $2C_1 = 0.6 - 0.333 = 0.267\ \mu F$,

$\therefore \quad C_1 = 0.133\ \mu F$

(b) The capacitance between two shorted conductors and the other is given by

$$2C_1 + \frac{2C_S \times C_S}{3C_S} = 2C_1 + \frac{2}{3}C_S = 2 \times 0.133 + \frac{2}{3} \times 0.333 = \mathbf{0.488\ mF}$$

41.35. A.C. Distributor Calculations

These calculations are similar to those for d.c. distributor but with the following differences :

1. The loads tapped off will be at different power factors. Each power factor is taken with respect to the voltage at the feeding point which is regarded as a reference vector.
2. The currents in the sections of the distributor will be given by the vector sum of load currents and not by their arithmetic sum as in a d.c. distributor. Currents can be added algebraically only when they are expressed in the complex notation.
3. The voltage drop, in the case of a.c. circuits, is not only due to ohmic resistance but due to inductive reactance as well (neglecting capacitive reactance if any).

   It has already been shown that voltage drop in an inductive circuit is given by

   $$I\,(R\cos\phi + X\sin\phi)$$

   The total drop will be given by $\Sigma\, I\,(R\cos\phi + X\sin\phi)$.

Questions on a.c. distributors may be solved in the following three ways :

1. Express voltages, currents and impedances in complex notation and then proceed exactly as in d.c. distributors.
2. Split the various currents into their active and reactive components. Now, the drop in the case of active components will be due to resistance only and in the case of reactive components due to reactance only. Find out these two drops and then add the two to find the total drop.
3. In cases where approximate solutions are sufficient, quick results can be obtained by finding the "distribution centre" or centre of gravity of the load.

All these three methods are illustrated by Ex. 41.42 given on the next page :

Example 41.41. *A 2-wire a.c. feeder 1 km long supplies a load of 100 A at 0.8 p.f. lag 200 volts at its far end and a load of 60 A at 0.9 p.f. lag at its mid-point. The resistance and reactance per km (lead and return) are 0.06 ohm and 0.08 ohm respectively. Calculate the voltage drop along the distributor from sending end to mid-point and from mid-point to far end.*

(Power Systems-I, AMIE, Sec. B, 1993)

Solution. Fig. 41.51 shows the feeder AC 1 km long having B as its mid-point and A as its sending-end point.

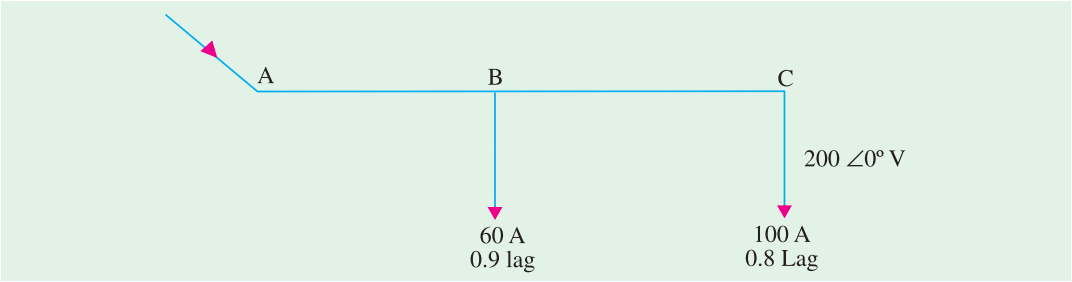


Fig. 41.51

Let the voltage of point C be taken as reference voltage.

$V_C = 200 + j_0$, $I_C = 100(0.8 - j\,0.6) = (80 - j\,60)$ A

Loop impedance of feeder BC (lead and return) $= (0.06 + j\,0.08)/2 = (0.03 + j\,0.04)$ ohm

Voltage drop in $BC = (80 - j\,60)(0.03 + j\,0.04) =$ **$(4.8 + j1.4)$ V**

$\therefore$ $V_B = 200 + 4.8 + j\,1.4 = (204.8 + j\,1.4)$ V

Current $I_B = 60\,(0.9 - j\,0.4357) = (54 - j\,26.14)$ A

Current in feeder AB *i.e.* $I_{AB} = I_C + I_B = (80 - j\,60) + (54 - j\,26.14) = (134 - j\,86.14)$ A

Drop in section $AB = (134 - j\,86.14)(0.03 + j\,0.04) = (7.46 + j\,2.78)$ V

Voltage drop from point A to point $B =$ **$(7.46 + j\,2.78)$ V**

Example 41.42. *A single-phase a.c. distributor 500 m long has a total impedance of $(0.02 + j\,0.04)\,\Omega$ and is fed from one end at 250V. It is loaded as under :*

(i) *50 A at unity power factor 200 m from feeding point.*

(ii) *100 A at 0.8 p.f. lagging 300 m from feeding point.*

(iii) *50 A at 0.6 p.f. lagging at the far end.*

Calculate the total voltage drop and voltage at the far end. **(Power System-I, AMIE, 1994)**

Solution. First Method

Current in section AD (Fig. 41.52) is the vector sum of the three load currents.

$\therefore$ current in AD

$= 50 + 100\,(0.8 - j\,0.6) + 50\,(0.6 - j\,0.8)$

$= 160 - j100$

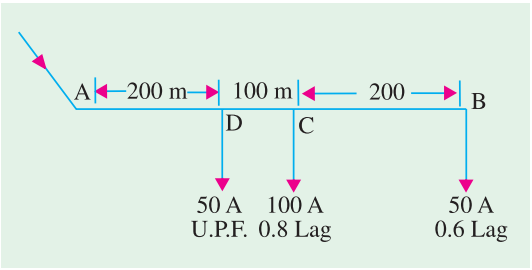


Fig. 41.52

Impedance of section AD

$= (200/500)(0.02 + j\,0.04)$

$= (0.008 + j\,0.016)$ W

Voltage drop in section $AD = (160 - j\,100) \times (0.008 + j\,0.016) = (2.88 + j\,1.76)$V

Current in section $DC = (160 - j100) - 50 = (110 - j100)$ A

Impedance of $DC = (0.004 + \text{j}\ 0.008)\ \Omega$

Drop in $CD = (100 - j100)(0.004 + j\,0.008) = (1.24 + j\,0.48)$ V

Current in $CB = 50\,(0.6 - j0.8) = (30 - j\,40)$ A

Impedence of $CB = (0.008 + j\,0.016)\ \Omega$

$\therefore$ drop in $CB = (30 - j40)(0.008 + j\,0.016) = (0.88 + j\,0.16)$ V

Total drop $= (2.88 + j\,1.76) + (1.24 + j\,0.48) + (0.88 + j\,0.16) = (5 + j\,2.4)$ V

Voltage at far end $= (250 + j\,0) - (5 + j\,2.4) = 245 - j\,2.4$ volt

Its magnitude is $= \sqrt{245^2 + 2.4^2} = 245$ V (approx)

Second Method

We will split the currents into their active and reactive components as under :

$50 \times 1 = 50$ A ; $\quad 100 \times 0.8 = 80$ A; $\quad 50 \times 0.6 = 30$ A

These are shown in Fig. 41.51 (*a*). The reactive or wattless components are

$50 \times 0 = 0$; $\quad 100 \times 0.6 = 60$ A ; $\quad 50 \times 0.8 = 40$ A

These are shown in Fig. 41.53 (*b*). The resistances and reactances are shown in their respective figures.

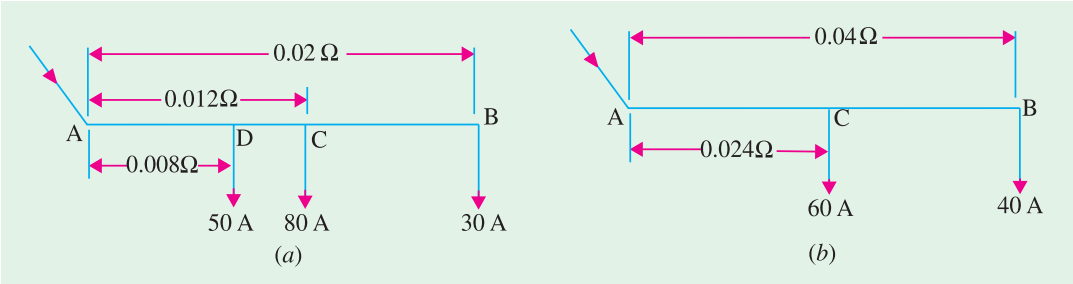


Fig. 41.53

Drops due to active components of currents are given by taking moments

$= 50 \times 0.008 + 80 \times 0.012 + 30 \times 0.02 = 1.96$ V

Drops due to reactive components $= 60 \times 0.024 + 40 \times 0.04 = 3.04$

Total drop $= 1.96 + 3.04 = 5$ V

This is approximately the same as before.

Third Method

The centre of gravity (C.G.) of the load is at the following distance from the feeding end

$$\frac{50 \times 200 + 100 \times 300 + 50 \times 500}{200} = 325 \text{ m}$$

Value of resistance upto C.G. $= 325 \times 0.02/500 = 0.013\ \Omega$.

Value of reactance upto C.G. $= 325 \times 0.04/500 = 0.026\ \Omega$.

Average p.f. $= \dfrac{50 \times 1 + 100 \times 0.8 + 50 \times 0.6}{200} = 0.8$

$\cos \phi_{av} = 0.8$; $\sin \phi_{av} = 0.6$

Drop $= 200(0.013 \times 0.8 + 0.026 \times 0.6) = 5.2$ V

This is approximately the same as before.

Example 41.43. *A single-phase distributor, one km long has resistance and reactance per conductor of 0.2 Ω and 0.3 Ω respectively. At the far end, the voltage $V_B = 240$ V and the current is 100 A at a power factor of 0.8 lag. At the mid-point A of the distributor current of 100 A is tapped at a power factor of 0.6 lag with reference to the voltage V_A at the mid-point. Calculate the supply voltage V_S for the distributor and the phase angle between V_S and V_B.*

Solution. As shown in Fig. 41.54 (*a*), let *SB* be the distributor with *A* as the mid point. Total impedance of the distributor is = (0.4 + *j* 0.6) Ω.

Let the voltage V_B at point *B* be taken as the reference voltage.

$\therefore \quad \mathbf{V}_B = (240 + j_0)$V ; $\mathbf{I}_B = 100\ (0.8 - j\ 0.6) = 80 - j\ 60$ A

Drop in section *AB* $= (80 - j\,60)\ (0.2 + j\,0.3) = (34 + j\,12)$ V

$\mathbf{V}_A = \mathbf{V}_B$ + drop over $\boldsymbol{AB} = (240 + j\ 0) + (34 + j12) = (274 + j12)$ V

The phase difference between V_A and V_B is = $\tan^{-1}(12/274) = 2°30'$

The load current I_A has lagging power factor of 0.6 with respect to V_A. It lags V_A by an angle $\phi = \cos^{-1}(0.6) = 53°8'$.

Hence, it lags behind V_B by an angle of $(53°8' - 2°30') = 50°38'$ as shown in the vector diagram of Fig. 41.54 (*b*).

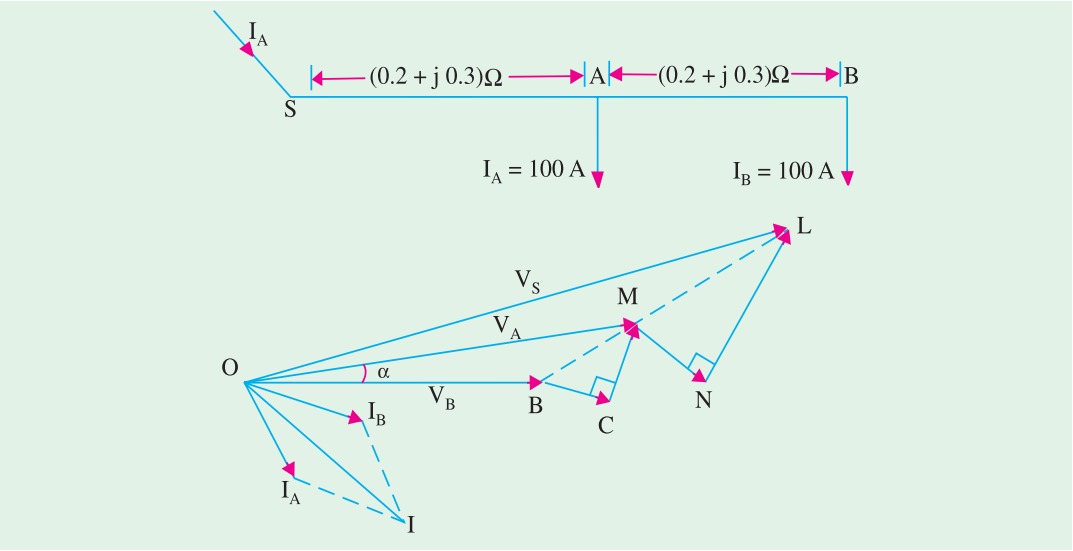


Fig. 41.54

$$\mathbf{I}_A = 100\,(\cos 50°38' - j \sin 50°38') = (63.4 - j77.3)\text{ A}$$

$$\mathbf{I} = \mathbf{I}_A + \mathbf{I}_B = (80 - j\,60) + (63.4 - j77.3) = (143.4 - j137.3)\text{ A}$$

Drop in section $S_A = (143.4 - j137.3)\,(0.2 + j\,0.3) = (69.87 + j\,15.56)$

$$\mathbf{V}_S = \mathbf{V}_A + \text{drop in section } S_A$$

$$= (274 + j12) + (69.87 + j15.56) = 343.9 + j\,27.6 = 345 \angle 5°28' \text{ V}$$

Hence, supply voltage is 345 V and lead V_B by 5°28′

Example 41.44. *A 1-phase ring distributor ABC is fed at A. The loads at B and C are 20 A at 0.8 p.f. lagging and 15 A at 0.6 p.f. lagging respectively, both expressed with reference to voltage at A. The total impedances of the sections AB, BC and CA are (1 + j1), (1 + j2) and (1 + j3) ohm respectively. Find the total current fed at A and the current in each section.*

(Transmission and Distribution-II, Madras Univ.)

Solution. Thevenin's theorem will be used to solve this problem. The ring distributor is shown in Fig. 41.55 (*a*). Imagine feeder *BC* to be removed [Fig. 41.55 (*b*)].

Current in $AB = 20(0.8 - j0.6) = (16 - j12)$ A Current in section $AC = 15(0.6 - j0.8) = (9 - j12)$A
Drop over $AB = (16 - j12)(1 + j1) = (28 + j\,4)$ V Drop over $AC = (9 - j\,12)(1 + j\,3) = (45 + j15)$V

Obviously, point *C* is at a lower potential as compared to point *B*.

p.d. between *B* and *C* = $(45 + j\,15) - (28 + j\,4) = (17 + j\,11)$ V

Impedance of the network as looked into from points *B* and *C* is = $(1 + j\,1) + (1 + j\,3) = (2 + j\,4)\Omega$.. The equivalent Thevenin's source is shown in Fig. 41.55 (*c*) with feeder *BC* connected across it.

$$\text{Current in } BC = \frac{17 + j\,11}{(2 + j4) + (1 + j2)} = (2.6 - j1.53)\text{ A}$$

Current in $AB = (16 - j12) + (2.6 - j1.53) = 18.6 - j\,13.53 = 23 \angle -36°$ A

Current in $BC = (9 - j12) - (2.6 - j1.53) = 6.4 - j11.5 = 13.2 \angle -60.9°$ A

Total current fed at point $A = (16 - j12) + (9 - j12) = 25 - j24 = 34.6 \angle -43.8°$ A.

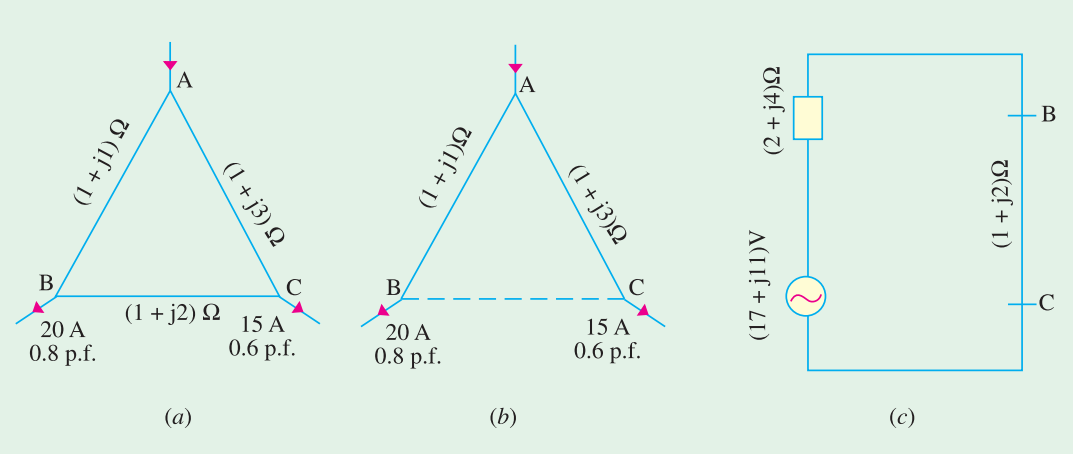


Fig. 41.55

Example 41.45. *A 2-wire ring distributor ABC is supplied at A at 400 V. Point loads of 20 A at a p.f. of 0.8 lagging and 30 A at a p.f. 0.6 lagging are tapped off at B and C respectively. Both the power factors refer to the voltage at A. The respective go-and-return impedances of sections AB,BC and CA are (1 + j2) ohm, (2 + j3) ohm and (1 + j3) ohm. Calculate the current flowing through each section and the potentials at B and C. Use Superposition theorem.*

(Power Systems-I, M.S. Univ. Baroda 1992)

Solution. The distributor circuit is shown in Fig. 41.56 (*a*). Currents in various sections are as shown. First, consider the load at point *B* acting alone as in Fig. 41.56 (*b*). The input current at point A divides in the inverse ratio of the impedances of the two paths *AB* and *ACB*.

Let the currents be $\mathbf{I}_1'$ and $\mathbf{I}_2'$

$$\mathbf{I}_1' = (16 - j12) \times \frac{(1 + j3) + (2 + j3)}{(1 + j3)(2 + j3) + (1 + j3)} = (16 - j12) \times \frac{(3 + j6)}{4 + j8} = (12 - j9)\ A$$

$$\mathbf{I}_2' = (16 - j12) - (12 - j9) = (4 - j3)\ A$$

Now, consider the load at point *C* to act alone as shown in Fig. 40.56 (c). Let the currents now be $\mathbf{I}_1''$ and $\mathbf{I}_2''$.

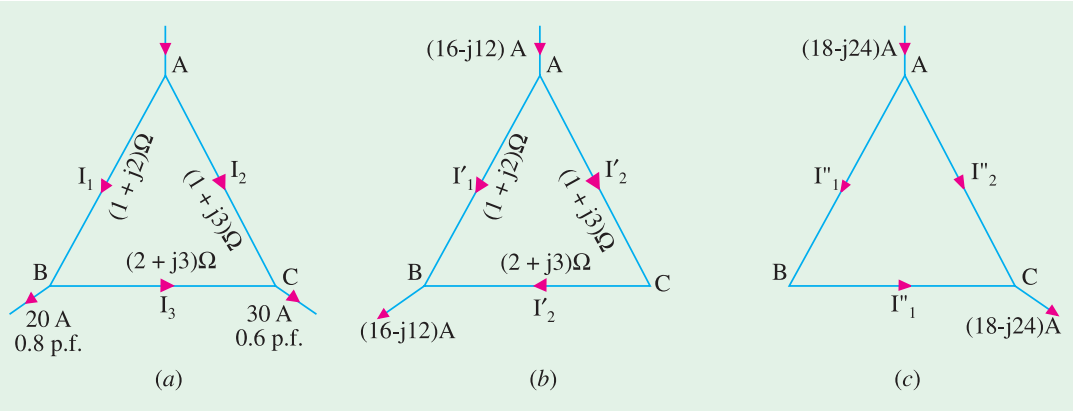


Fig. 41.56

$$\mathbf{I}_1'' = (18 - j24) \times \frac{1 + j3}{4 + j8} = 7.5 - j7.5;\ \mathbf{I}_2'' = (18 - j24) - (7.5 - j7.5) = (10.5 - j16.5)\ A$$

As per Superposition theorem, the current $\mathbf{I}_1$ in section AB is the vector sum of $\mathbf{I}_1'$ and $\mathbf{I}_1''$.

$\therefore \quad \mathbf{I}_1 = \mathbf{I}_1' + \mathbf{I}_1'' = (12 - j9) + (7.5 - j7.5) = (19.5 - j16.5)$ A

$\mathbf{I}_2 = \mathbf{I}_2' + \mathbf{I}_2'' = (4 - j3) + (10.5 - j16.5) = (14.5 - j19.5)$ A

$\mathbf{I}_3 = \mathbf{I}_1'' - \mathbf{I}_2' = (7.5 - j7.5) - (4 - j3) = (3.5 - j4.5)$ A

Potential of B = 400 – drop over AB

= $400 - (19.5 - j16.5)(1 + j2) = 347.5 - j22.5 = 348 \angle -3.7°$V

Potential of C = $400 - (14.5 - j19.5)(1 + j3) = 327 - j24 = 328 \angle -4.2°$V

Example 41.46. *A 3-phase ring main ABCD, fed from end A, supplies balanced loads of 50 A at 0.8 p.f. lagging at B, 120 A at u.p.f. at C and 70 A at 0.866 p.f. lagging at D, the load currents being referred to the voltage at point A.*

The impedance per phase of the various line sections are :

section AB = $(1 + j0.6)\ \Omega$; *section BC* = $(1.2 + j0.9)\ \Omega$

section CD = $(0.8 + j0.5)\ \Omega$; *section DA* = $(3 + j2)\ \Omega$

Determine the currents in the various sections. **(Electrical Power-I, Bombay Univ.)**

Solution. One phase of the ring main is shown in Fig. 41.57. Let the current in section $AB = (x + jy)$ A.

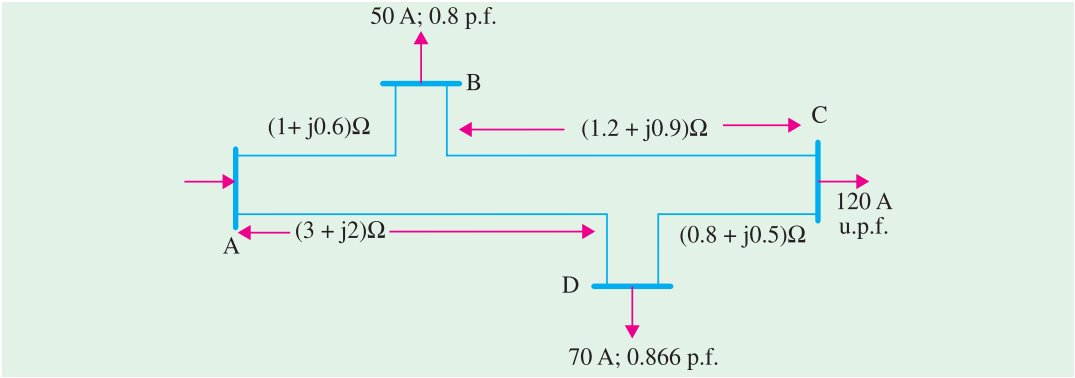


Fig. 41.57

Current in $BC = (x + jy) - 50(0.8 - j0.6) = (x - 40) + j(y + 30)$

Current in $CD = (x - 40) + j(y + 30) - (120 + j0) = (x - 160) + j(y + 30)$

Current in $DA = (x - 160) + j(y + 30) - 70(0.866 - j0.5) = (x - 220.6) + j(y + 65)$

Applying Kirchhoff's voltage law to the closed loop $ABCDA$, we have

$(1 + j0.6)(x + jy) + (1.2 + j0.9)$

$[(x - 40) + j(y + 30)] + (0.8 + j0.5)$

$[(x - 160) + j(y + 30)] + (3 + j2)$

$[(x - 220.6) + j(y + 65)] = 0$

or $(6x - 4y + 1009.8) + j(4x + 6y - 302.2) = 0$

Since the real (or active) and imaginary (or reactive) parts have to be separately zero.

$\therefore \quad 6x - 4y + 1009.8 = 0$ and $4x + 6y - 302.2 = 0$

Solving for x and y, we get

$x = 139.7$ and $y = -42.8$

$\therefore$ Current in section $AB = (139.7 - j42.8)$ A

Current in section $BC = (139.7 - 40) + j(-42.8 + 30) = (99.7 - j12.8)$ A

Current in section $CD = (139.7 - 160) + j(-42.8 + 30) = (-20.3 - j12.8)$ A

Current in section $DA = (139.7 - 220.6) + j(-42.8 + 65) = (-80.9 + j22.2)$ A

41.36. Load Division Between Parallel Lines

It is common practice to work two or more cables or overhead lines in parallel when continuity of supply is essential. In the case of a fault developing in one line of cable, the other lines or cables carry the total load till the fault is rectified."

Let us take the case of two lines in parallel and having impedances of $\mathbf{Z}_1$ and $\mathbf{Z}_2$. Their combined impedance is

$$\mathbf{Z} = \frac{\mathbf{Z}_1 \times \mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}$$

If $\mathbf{I}$ is the current delivered to both lines, then total drop $= \mathbf{IZ} = \mathbf{I} \times \frac{\mathbf{Z}_1 \mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}$

If $\mathbf{I}_1$ and $\mathbf{I}_2$ are the respective currents flowing in the two lines, then

$$\mathbf{I}_1 = \frac{\text{voltage drop}}{\mathbf{Z}_1} = \frac{\mathbf{IZ}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}; \quad \text{Similarly, } \mathbf{I}_2 = \frac{\mathbf{IZ}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}$$

It may be noted that in the case of two impedances in parallel, it is convenient to take voltage vector as the reference vector.

Example 41.47. *A total load of 12,000 kW at a power factor of 0.8 lagging is transmitted to a substation by two overhead three-phase lines connected in parallel. One line has a conductor resistance of 2 Ω per conductor and reactance (line to neutral) of 1.5 Ω, the corresponding values for the other line being 1.5 and 1.2 Ω respectively. Calculate the power transmitted by each overhead line.* **(London Univ.)**

Solution. Let us assume a line voltage of 1000 kV for convenience.

$\mathbf{Z}_1 = (2 + j1.5)$; $\mathbf{Z}_2 = (1.5 + j1.2)$ Ω

Total load current $\mathbf{I} = 12{,}000/\sqrt{3} \times 1000 \times 0.8 = 8.66$ A

Taking voltage along reference vector, we have $\mathbf{I} = 8.66\,(0.8 - j0.6)$

$$\mathbf{I}_1 = \frac{8.66\,(0.8 - j\,0.6)\,(1.5 + j\,1.2)}{(2 + j\,1.5)\,(1.5 + j\,1.2)} = 4.437(6.882 - j5.39) \text{ A}$$

$\therefore$ power transmitted by 1st line is

$$W_1 = 10^6 \times \frac{\sqrt{3}}{1000} \times 4.437 \times 6.882 = \mathbf{5{,}280\ kW}$$

Similarly $$\mathbf{I}_2 = \frac{8.66\,(0.8 - j\,0.6)\,(2 + j\,1.5)}{(2 + j\,1.5)\,(1.5 + j\,1.2)} = 4.437(8.75 - j6.75) \text{ A}$$

$$W_2 = 10^6 \times \frac{\sqrt{3}}{1000} \times 4.437 \times 8.75 = \mathbf{6{,}720\ kW}$$

As a check, total power $= 5{,}280 + 6{,}720 = \mathbf{12{,}000\ kW}$

41.37. Suspension Insulators

Suspension insulators are used when transmission voltage is high. A number of them are connected in series to form a chain and the line conductor is carried by the bottom most insulator.

As shown in Fig. 41.58 a 'cap' type suspension insulator consists of a single disc-shaped piece of porcelain grooved on the under surface to increase the surface leakage path. A galvanized cast iron cap is cemented at the top of the insulator. In the hollow cavity of the insulator is cemented a galvanized forged steel pin, the lower enlarged end of which fits into the cavity of the steel cap of the lower suspension insulator and forms a ball and socket connection.

A string of suspension insulators consists of many units, the number of units depending on the value of the transmission voltage. There exists mutual capacitance between different units and, in addition, there is capacitance to ground of each unit because of the nearness of the tower, the cross-arm and the line. Due to this capacitance to ground, the total system voltage is not equally distributed over the different units of the string. The unit nearest to the line conductor carries the maximum percentage of voltage, the figure progressively decreasing as the unit nearest to the tower is approached. The inequality of voltage distribution between individual units becomes more pronounced as the number of insulators increases and it also depends on the ratio (capacitance of insulator/capacitance of earth).

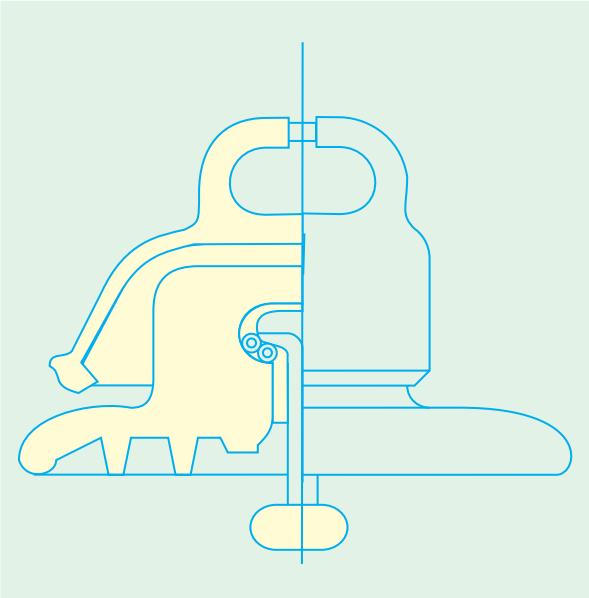


Fig. 41.58

String Efficiency

If there are n units in the string, then its efficiency is given by

$$\% \text{ string } \eta = \frac{\text{total voltage across the string}}{n \times \text{voltage across unit adjacent the line}} \times 100$$

41.38. Calculation of Voltage Distribution along Different Units

Let, C = capacitance to ground

kC = mutual capacitance between units

The current and voltage distribution is as shown in Fig. 41.59. It is seen that

$$I_1 = \frac{V_1}{1/\omega kC} = \omega k C V_1 \text{ ; Similarly } i_1 = \frac{V_1}{1/\omega C} = \omega C V_1$$

Now, $I_2 = I_1 + i_1 = \omega C V_1 (1 + k)$ and $V_2 = \frac{I_2}{\omega kC} = \frac{V_1 (1 + k)}{k}$

The current i_2 is produced by the voltage combination of $(V_1 + V_2)$

Now, $V_1 + V_2 = V_1\left(1 + \frac{1+k}{k}\right) = V_1\left(1 + \frac{1+2k}{k}\right)$; $\therefore\ i_2 = \omega C V_1\left(1 + \frac{1+2k}{k}\right)$

At junction B, we have

$$I_3 = I_2 + i_2 = \omega C V_1 \left\{1 + k\,\frac{(1+2k)}{k}\right\}$$

$$= \omega C V_1 \frac{(1 + 3k + k^2)}{k}$$

However, $I_3 = \omega k C V_3$; $\quad \therefore \quad V_3 = \frac{V_1 (1 + 3k + k^2)}{k^2} = V_1\left(1 + \frac{3}{k} + \frac{1}{k^2}\right)$

The current i_3 is produced by the voltage combination of $(V_1 + V_2 + V_3)$

Now, $V_1 + V_2 + V_3$

$$= V_1\left[1 + \frac{(1+k)}{k} + \frac{(1+3k+k^2)}{k^2}\right]$$

$$\therefore \quad i_3 = \omega C V_1\left(\frac{1+4k+3k^2}{k^2}\right)$$

At junction C, we have

$$I_4 = I_3 + i_3 = \omega C V_1$$

$$\left[\frac{(1+3k+k^2)}{k^2} + \frac{(1+4k+3k^2)}{k^2}\right]$$

Now, $I_4 = \omega k C V_4$,

$$\therefore \quad V_4 = V_1\left[\frac{(1+3k+k^2)}{k^2} + \frac{(1+4k+3k^2)}{k^2}\right]$$

$$= V_1\left(1 + \frac{6}{k} + \frac{5}{k^2} + \frac{1}{k^2}\right)$$

For the fifth insulator from the top, we have

$$V_5 = \left(1 + \frac{10}{k} + \frac{15}{k^2} + \frac{7}{k^3} + \frac{1}{k^4}\right)$$

and so on.

If the string has n units, the total voltage is given by

$$V = (V_1 + V_2 + V_3 + V_4 + \ldots\ldots\ldots\ldots + V_n)$$

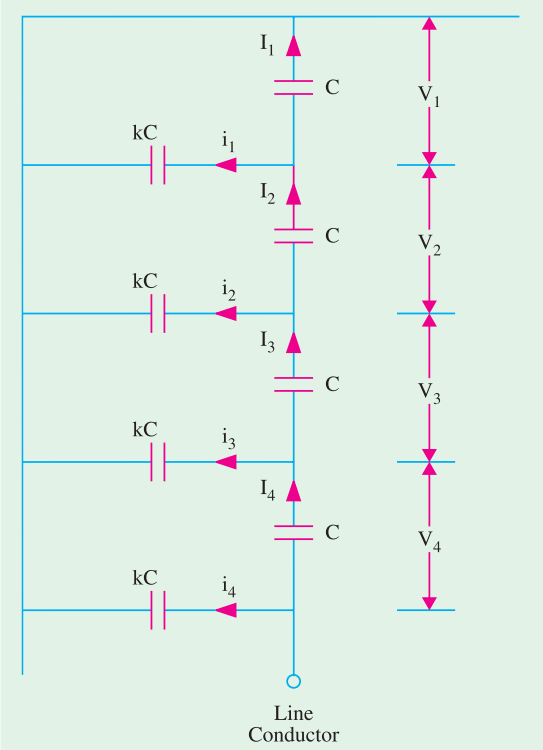


Fig. 41.59

Example 41.48. *For a string insulator with four discs, the capacitance of the disc is ten times the capacitance between the pin and earth. Calculate the voltage across each disc when used on a 66-kV line. Also, calculate the string efficiency.* **(Power Systems-I, AMIE Sec. B, 1994)**

Solution. Let C be the self-capacitance of each disc and kC the capacitance between each link pin and earth. We are given that $k = 10$

As seen from Art. 41.38,

$$V_2 = \frac{(1+k)}{k}V_1 = \frac{11}{10}V_1$$

$$V_3 = V_1\left(1 + \frac{3}{k} + \frac{1}{k^2}\right) = \frac{131}{100}V_1$$

$$V_4 = V_1\left(1 + \frac{6}{k} + \frac{5}{k^2} + \frac{1}{k^3}\right) = \frac{1561}{100}V_1$$

$$V = V_1 + V_2 + V_3 + V_4 = V_1 + \frac{11}{10}V_1 + \frac{131}{100}V_1 + \frac{1561}{1000}V_1 = \frac{4971}{1000}V_1$$

$$\therefore \quad V_1 = V\frac{1000}{4971} = \frac{66}{\sqrt{3}} \times \frac{1000}{4971} = \mathbf{7.66\ kV}$$

$$V_2 = \frac{11}{10}V_1 = \frac{11}{10} \times 7.66 = \mathbf{8.426\ kV}$$

$$V_3 = \frac{131}{100}V_1 = \frac{131}{100} \times 7.66 = \mathbf{10.03\ kV}$$

$$V_4 = \frac{1561}{1000} V_1 = \frac{1561}{1000} \times 7.66 = \mathbf{11.96\ kV}$$

$$\text{String } \eta = \frac{V}{4 \times V_4} \times 100 = \frac{66 \times 100}{\sqrt{3} \times 4 \times 11.96} = \mathbf{79.65\ \%}$$

Example 41.49. *Explain what is meant by string efficiency and how it can be improved. Each line of a 3-phase, 33-kV system is suspended by a string of 3 identical insulator discs. The capacitance of each disc is 9 times the capacitance to ground. Find voltage distribution across each insulator and the string efficiency. Suggest a method for improving the string efficiency.*

(Power Systems-I, AMIE, Sec. B, 1993)

Solution. Let C be the self-capacitance of each disc and kC the mutual capacitance between the units. We are given that $k = 9$.

$$V_2 = V_1 \frac{(1+k)}{k} = \frac{10}{9} V_1;$$

$$V_3 = V_1 \frac{(1+3k+k^2)}{k^2} = \frac{109}{81} V_1$$

Total voltage, $$V = V_1 + V_2 + V_3 = V_1 + \frac{10}{9} V_1 + \frac{109}{81} V_1 = \frac{280}{81} V_1$$

$\therefore$ $$\frac{33}{\sqrt{3}} = \frac{280}{81} V_1; \quad \therefore V_1 = \frac{33 \times 81}{\sqrt{3} \times 280} = \mathbf{5.51\ kV}$$

$$V_2 = (10/9)\, V_1 = \mathbf{6.12\ kV}\ ;\ V_3 = (109/81) V_1 = \mathbf{7.41\ kV}$$

String efficiency $$= \frac{V}{3 \times V_3} \times 100 = \frac{33/\sqrt{3}}{3 \times 7.41} \times 100 = \mathbf{85\%}$$

The string efficiency can be improved by providing a guard ring surrounding the lower-most unit which is connected to the metal work at the bottom. This ring increases the capacitance between the metal work and the line and helps in equalising the voltage distribution along the different units. The efficiency can also be improved by grading the insulators and by making the ratio (capacitance to earth/capacitance per insulator) as small as possible.

41.39. Interconnectors

An interconnector is a tieline which enables two generating stations to operate in parallel. It facilitates the flow of electric power in either direction between the two stations.

41.40. Voltage Drop Over the Interconnector

Let station 1 supply a current of 1 to station 2 along an interconnector having a resistance of RW and reactance of XTW per phase as shown in Fig. 41.60 (*a*). If the receiving end p.f. is cos ϕ lagging, then the vector diagram will be as shown in Fig. 41.60 (*b*).

Voltage drop over the interconnector

$$= I(\cos\phi - j\sin\phi)(R + jX)$$

$$= I(R\cos\phi + X\sin\phi) + jI(X\cos\phi - R\sin\phi)$$

% voltage drop $$= \left\{\frac{I(R\cos\phi + X\sin\phi) + jI(X\cos\phi - R\sin\phi)}{E_2}\right\} \times 100$$

Let $I\cos\phi = I_d$ and $I\sin\phi = I_q$, where I_d and I_q are the in-phase and quadrature components.

$\therefore$ voltage drop $$= \left\{\frac{(I_d R + I_q X) + j(I_d X - I_q R)}{E_2}\right\} \times 100$$

$$\therefore \quad \text{in-phase voltage drop} = \frac{(I_d R + I_q X)}{E_2} \times 100$$

$$\text{and quadrature voltage drop} = \frac{(I_d X^2 - I_q R)}{E_2} \times 100$$

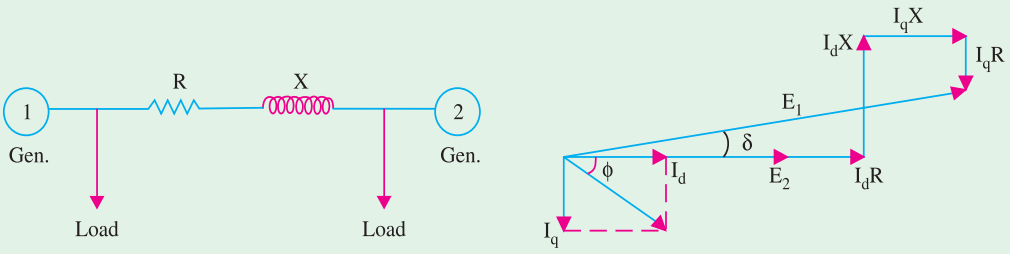


Fig. 41.60

Example 41.50. *The bus-bar voltages of two stations A and B are 33 kV and are in phase. If station A sends 8.5 MW power at u.p.f. to station B through an interconnector having an impedance of (3 + j4)Ω, determine the bus-bar voltage of station A and the phase angle shift between the bus-bar voltages.*

Solution. With reference to Fig. 41.61,

Voltage of station B, $V_B = 33{,}000/\sqrt{3} = 19{,}050$ V/phase

Since power transferred to station B is 8.5 MW, current in the interconnector is

$$I = 8.5 \times 10^3/\sqrt{3} = 148.7 \text{ A}$$

Taking V_B as the reference vector,

$$V_A = V_B + I(\cos\phi - j\sin\phi)(R + jX)$$
$$= 19{,}050 + 148.7\,(1 - j0)\,(3 + j4)$$
$$= 19{,}496 + j595 = 19{,}502 \angle 1.75°$$

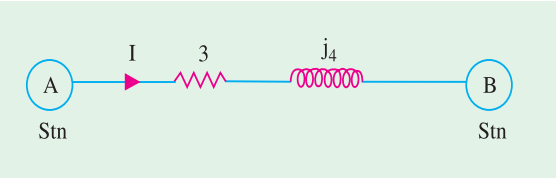


Fig. 41.61

The line-to-line bus-bar voltage of station A is

$$= 19{,}502 \times \sqrt{3} = \mathbf{33.78\ kV}$$

The phase-shift angle between bus-bar voltages is **1.75°**

41.41. Sag and Stress Analysis

The conductors of a transmission line are attached to suitable insulators carried on supports of wood, iron, steel or reinforced concrete. Obviously, the supports must be strong enough to withstand not only the dead weight of the conductors themselves but also the loads due to ice and sleet that may adhere to them and to wind pressure. Moreover, the minimum factor of safety for the conductors should be 2.0 based on ultimate strength.

Sag and stresses vary with temperature on account of thermal expansion and contraction of the line conductors. The value of sag as well as tension of a conductor would now be calculated when ***(i)*** supports are at equal levels and ***(ii)*** supports are at unequal levels.

41.42. Sag and Tension with Supports at Equal Levels

Fig. 41.62 shows a span of a wire with the two supports at the same elevation and separated by a horizontal distance $2l$. It can be proved that the conductor AB forms a catenary with the lowest point O forming the mid-point (where the curve is straight).

Let W be the weight of the wire per unit length and let point O be chosen as the reference point for measuring the co-ordinates of different points on the wire. Consider a point P having co-ordinates of x and y. The tension T at point P can be resolved into two rectangular components. T_x and T_y so that $T = \sqrt{T_x^2 + T_y^2}$. If S is the length of the arc OP, then its weight is WS which acts vertically downward through the centre of gravity of OP. There are four forces acting on OP—two vertical and two horizontal. Since OP is in equilibrium, the net force is zero. Equating the horizontal and vertical components, we have,

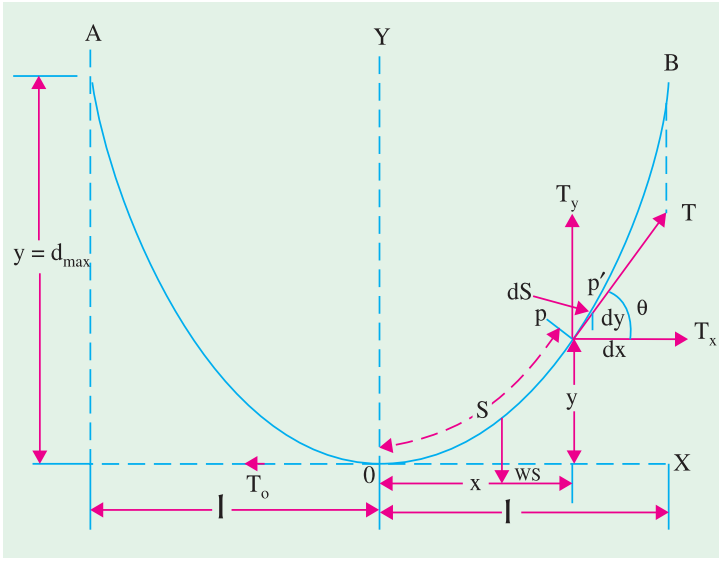


Fig. 41.62

$$T_0 = T_x \text{ and } T_y = W_S.$$

It may be noted that the horizontal component of tension is constant throughout the length of the wire :

Since line PT is tangential to the curve OB at point P, $\tan\theta = T_y / T_x$

It is also seen from the elementary piece PP' of the line that $\tan\theta = dy/dx$

$$\therefore \quad dy/dx = \tan\theta = \frac{T_y}{T_x} \text{ or } dy/dx = WS/T_0 \qquad \text{...(i)}$$

If $PP' = dS$, then $dS = \sqrt{(dx)^2 + (dy)^2} = dx\sqrt{1 + (dy/dx)^2}$

or $$\frac{dS}{dx} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \sqrt{1 + \left(\frac{WS}{T_0}\right)^2} \text{ or } dx = \frac{dS}{\sqrt{1 + \left(\frac{W^2 S^2}{T_0^2}\right)}}$$

Integrating both sides, we have $x = \left(\frac{T_0}{W}\right)\sinh^{-1}\left(\frac{WS}{T_0}\right) + C$

where C is the constant of integration.

Now, when $x = 0$, $S = 0$. Putting these values above, we find that $C = 0$.

$$\therefore \quad x = \left(\frac{T_0}{W}\right)\sinh^{-1}\left(\frac{WS}{T_0}\right) \text{ or } S = \frac{T_0}{W}\sinh\left(\frac{Wx}{T_0}\right) \qquad \text{...(ii)}$$

Substituting this value of S in Eq. (i), we get

$$\frac{dy}{dx} = \sinh\left(\frac{Wx}{T_0}\right) \text{ or } dy = \sinh\left(\frac{Wx}{T_0}\right)dx$$

$$\therefore \qquad y = \int \sinh\left(\frac{Ex}{T_0}\right)dx = \left(\frac{T_0}{W}\right)\cosh\left(\frac{Wx}{T_0}\right) + D$$

where D is also the constant of integration. At the origin point O, $x = 0$ and $y = 0$. Hence, the above equation becomes

$$0 = \left(\frac{T_0}{W}\right)\cosh 0 + D + \left(\frac{T_0}{W}\right) + D \qquad \therefore \quad D = -\frac{T_0}{W}$$

Substituting this value of D in the above equation, we get

$$y = \left(\frac{T_0}{W}\right)\cosh\left(\frac{Wx}{T_0}\right) - \frac{T_0}{W} = \frac{T_0}{W}\left[\cosh\left(\frac{Wx}{T_0}\right) - 1\right] \qquad \text{...(iii)}$$

This is the equation of the curve known as catenary. Hence, when a wire is hung between two supports, it forms a catenary

(a) The tension at point $P(x, y)$ is given by

$$T^2 = T_{x2} + T_{y2} = T_{02} + W^2S^2 = T_{02} + T_{02}\sinh^2\left(\frac{Wx}{T_0}\right) \qquad \text{—from Eq. (iii)}$$

$$= T_{02}\left[1 + \sinh^2\left(\frac{Wx}{T_0}\right)\right] = T - 2\cosh^2\left(\frac{Wx}{T_0}\right) \quad \therefore \; T = T_0\cosh\left(\frac{Wx}{T_0}\right) \qquad \text{...(iv)}$$

(b) Tension at points A and B where $x = \pm l$ is given by $T = T_0 \cosh(W_1/T_0)$ ...(v)

(c) The maximum sag is represented by the value of y at either of the two points A and B for which $x = +l$ and $x = -l$ respectively. Writing $y = d_{max}$ and putting $x = \pm l$ in Eq. (iii), we get,

$$d_{max} = \frac{T_0}{W}\left[\cosh\left(\frac{Wl}{T_0}\right) - 1\right] \qquad \text{...(vi)}$$

(d) The length of the wire or conductor in a half span is as seen from Eq. (ii) above,

$$S = \frac{T_0}{W}\sinh\left(\frac{Wl}{T_0}\right)$$

Approximate Formulae

The hyperbolic sine and cosine functions can be expanded into the following series

$$\sinh z = z + \frac{z^3}{3!} + \frac{z^5}{5!} + \frac{z^7}{7!} + \ldots \text{ and } \cosh z = 1 + \frac{z^2}{2!} + \frac{z^4}{4!} + \frac{z^6}{6!} + \ldots$$

Using the above, the approximate values of T d and S points at A and B may be found as follows :

(i)
$$T = T_0\cosh\left(\frac{Wl}{T_0}\right) = T_0\left(1 + \frac{W^2l^2}{2T_0^2} + \ldots\right) \qquad \text{—neglecting higher powers}$$

$$= T_0 + \frac{W^2l^2}{2T_0} \cong T_0 \qquad \text{—if } \frac{W^2l^2}{2T_0} \text{ is negligible}$$

$\therefore$ $T = T_0$ — *i.e.* tension at the supports is very approximately equal to the horizontal tension acting at any point on the wire.

(ii)
$$d = \frac{T_0}{W}\left[\cosh\left(\frac{Wl}{T_0}\right) - 1\right] = \frac{T_0}{W}\left[\left(1 + \frac{W^2l^2}{2T_0^2} + \ldots\right) - 1\right]$$

$$= \frac{Wl^2}{2T_0} = \frac{Wl^2}{2T} \text{ (approx)} \qquad \therefore \quad d = \frac{Wl^2}{2T}$$

It should be noted that W and T should be in the same units *i.e.* kg-wt or newton.

(iii) $S = \frac{T_0}{W}\sinh + \left(\frac{Wl}{T_0}\right) = \frac{T_0}{W}\left(\frac{Wl}{T_0} + \frac{W^3l^3}{6T_0^3} + \ldots\right) = l + \frac{W^2l^3}{6T_0^2}$ —neglecting higher terms.

The total length of the wire along the curve is $L = 2S = 2l + \frac{W^2l^2}{3T^2}$

This length consists of the unstertched length L_u and the stretch or extension.

$\therefore$ unstretched length $L_u = L$ —extension ΔL

Now, $E = \frac{T/A}{\Delta l/2l}$ or $\Delta l = \frac{2l.T}{EA}$

Substituting the value of T from the relation $d = Wl^2/2T$, we have

$$\Delta l = \frac{2l}{EA}\cdot\frac{Wl^2}{2d} = \frac{Wl^3}{EAd} \quad \therefore \quad L_u = L - \frac{Wl^3}{EAd}$$

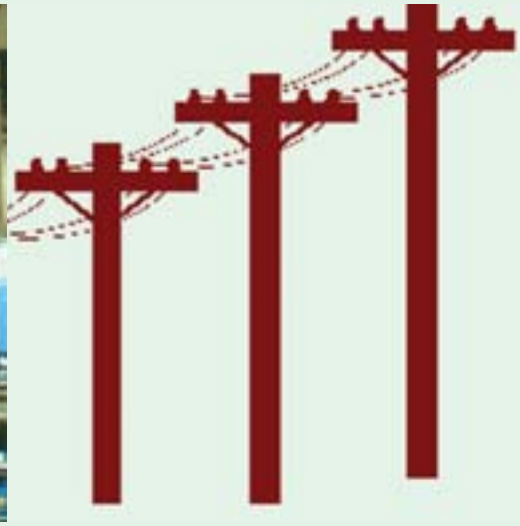
(iv) $$y = \frac{T_0}{W}\left[\cosh\left(\frac{Wx}{T_0}\right) - 1\right] = \frac{T_0}{W}\left[\left(1 + \frac{W^2x^2}{2T_0^2} + \ldots\right) - 1\right]$$

or $y = \frac{Wx^2}{2T}$ —**the equation of a parabola**

It shows that the catenary curve formed by the sagging is very approximately like a parabola in shape.

The above formulae are sufficiently accurate for all practical purposes provided sag is less than 10% of the span.





Overhead cables are economical and easy to maintain, which are usually made of bare copper or aluminum wires

41.43. Sag and Tension with Supports at Unequal Levels

Fig. 41.63 shows a span between two supports A and B whose elevations differ by η, their horizontal spacing being $2l$ as before. Such spans are generally met with in a hilly country. Let O be the lowest point of the catenary AOB. Obviously, OA is a catenary of half-span x_1 and OB of half-span x_2.

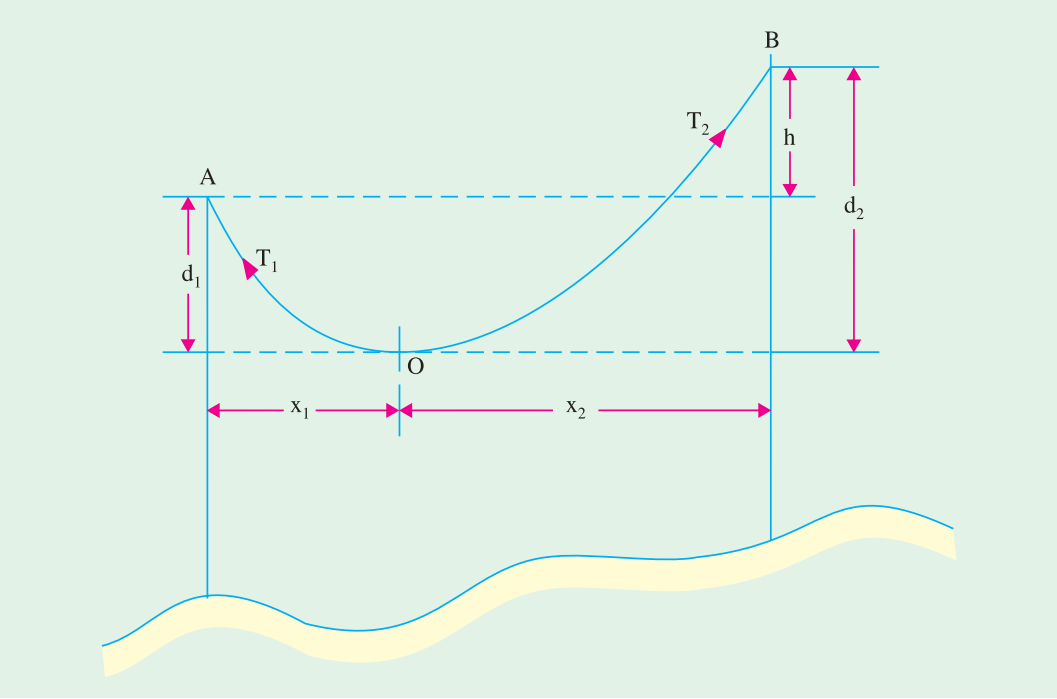


Fig. 41.63

The equations derived in Art. 41.42 also apply to this case.

(*i*) As seen from Eq. (*vi*) of Art. 41.42.

$$d_1 = \frac{T_0}{W}\left[\cosh\left(\frac{Wx_1}{T_0}\right)-1\right] \quad \text{—putting } l = x_1$$

$$d_2 = \frac{T_0}{W}\left[\cosh\left(\frac{Wx_2}{T_0}\right)-1\right] \quad \text{—putting } l = x_2$$

(*ii*) $$T_1 = T_0 \cosh\left(\frac{Wx_1}{T_0}\right) \text{ and } T_2 = T_0 \cosh\left(\frac{Wx_2}{T_0}\right)$$

It is obvious that maximum tension occurs at the higher support *B*.

$$\therefore \quad \text{max. permissible tension} = T_0 \cosh\left(\frac{Wx_2}{T_0}\right)$$

Approximate Formulae

Is is obvious that

$$d_2 - d_1 = \frac{T_0}{W}\left[\cosh\left(\frac{Wx_2}{T_0}\right) - \cosh\left(\frac{Wx_2}{T_0}\right)\right] = h \text{ and } x_1 + x_2 = 2l$$

Using approximations similar to those in Art. 41.42, we get

$$d_1 = \frac{Wx_1^2}{2T_0} = \frac{Wx_1^2}{2T} \quad \text{and} \quad d_2 = \frac{Wx_2^2}{2T_0} = \frac{Wx_2^2}{2T}$$

(it has been assumed, as before, that $T = T_0$)

$$\therefore \quad d_2 - d_1 = h = \frac{W}{2T}\left(x_2^{\,2} - x_1^{\,2}\right) = \frac{W}{2T}(x_2 + x_1)(x_2 - x_1) = \frac{Wl}{T}(x_2 - x_1)$$

$$\therefore \quad x_2 - x_1 = \frac{hT}{Wl}; \quad \text{Now, } x_2 - x_1 = hT/Wl \text{ and } x_2 + x_1 = 2l$$

$$\therefore \quad x_1 = 1 - \frac{hT}{2Wl} \quad \text{and} \quad x_2 = 1 + \frac{hT}{2Wl} \ *$$

Having found x_1 and x_2, values of d_1 and d_2 can be easily calculated. It is worth noting that in some cases, x_1 may be negative which means that there may be no horizontal point (like point O) in the span. Such a thing is very likely to happen when the line runs up a steep mountain side.

41.44. Effect of Wind and Ice

In the formulae derived so far, effect of ice and wind loading has not been taken into account. It is found that under favourable atmospheric conditions, quite an appreciable thickness of ice is formed on transmission lines. The weight of ice acts vertically downwards *i.e.* in the same direction as the weight of the conductor itself as shown in Fig. 41.64. In addition, there may be high wind which exerts considerable force on the conductor. This force is supposed to act in a horizontal direction. If W_i is the weight of ice per unit length of the conductor and W_u the force per unit length exerted by the wind, then total weight of the conductor per unit length is



Effects of wind, ice and snow are important considerations while designing electric cables

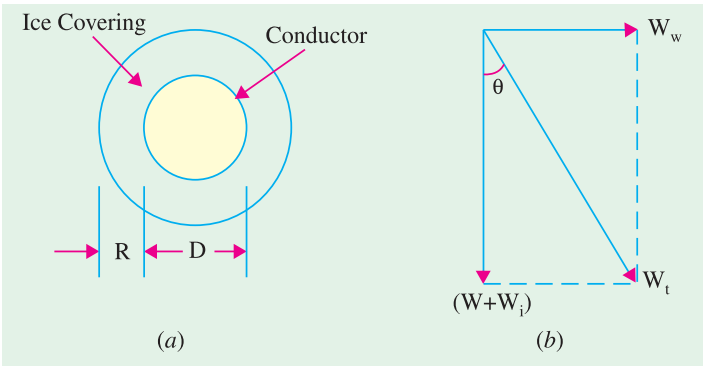


Fig. 41.64

$$Wt = \sqrt{(W + W_i)^2 + W_w^2}$$

It is obvious that in all equations derived in Art. 41.42 and 41.43, W should be replaced by Wt. As seen from Fig. 41.64 (*b*), the conductor sets itself in a plane at an angle of $\theta = \tan^{-1}[W_\omega/(W + W_i)]$ to the vertical but keeps the shape of a catenary (or a parabola approximately).

Note. (*i*) If P is the wind pressure per unit projected area of the conductor, then wind load or force per unit length of the ice-covered conductor is

$$W_\omega = P \times (D + 2R) \times l = P \times D \quad \text{—if conductor is without ice}$$

* If we put $h = 0$ *i.e.* assume supports at the same level, then as expected $x_1 = x_2 = l$

(ii) Ice is in the form of a hollow cylinder of inner diameter = D and outer diameter = $(D + 2R)$. Hence, volume per unit length of such a cylinder is

$$= \frac{\pi}{4}[(D+2R)^2 - D^2] \times 1 = \frac{\pi}{4}(4DR + 4R^2) = \pi R(D+R)$$

If ρ is the ice density, then weight of ice per unit length of the conductor is

$$W_i = \rho \times \text{volume} = \pi\rho\, R(D+R)$$

When ice and wind loads are taken into account, then the approximate formulae derived in Art. 41.42 become

$$d = \frac{W_t l^2}{2T}; \quad S = 1 + \frac{W_t^2 l^3}{6T^2} \quad \text{and} \quad y = \frac{W_t x^2}{2T}$$

Here 'd' represents the slant sag in a direction making an angle of θ with the vertical. The vertical sag would be = $d \cos\theta$.

Similarly, formulae given in Art. 41.43 become

$$d_1 = \frac{W_t \cdot x_1^2}{2T}; \quad d_2 = \frac{W_t \cdot x_2^2}{2T}; \quad x_1 = 1 - \frac{hT}{2W_t \cdot l} \text{ and } x_2 = 1 + \frac{hT}{2W_t \cdot l}$$

Example 41.51. *A transmission line conductor at a river crossing is supported from two towers at heights of 70 m above water level. The horizontal distance between towers is 300 m. If the tension in conductor is 1,500 kg, find the clearance at a point midway between the towers. The size of conductor is 0.9 cm^2. Density of conductor material is 8.9 g/cm^3 and suspension length of the string is 2 metres.*

Solution. Sag, $d = Wl^2/2T$

$l = 150$ m, $W = lA\rho = 2 \times (0.9 \times 10^{-4}) \times (8.9 \times 10^3) = 0.8$ kg wt; $T = 1500$ kg wt

$d = Wl^2/2T = 0.8 \times 150^2/2 \times 1500 = 6$ m

Clearance between conductor and water at mid-way between the towers

$$= 70 - 6 - 2 = 62 \text{ m}$$

Example 41.52. *The effective diameter of a line is 1.96 cm and it weighs 90 kg per 100 metre length. What would be the additional loading due to ice of radial thickness 1.25 cm and a horizontal wind pressure of 30 kg/m^2 of projected area? Also, find the total weight per metre run of the line. Density of ice is 920 kg/m^3.*

Solution. It should be noted that weights of the conductor and ice act vertically downwards whereas wind pressure is supposed to act horizontally. Hence, the total force on one metre length of the conductor is found by adding the horizontal and vertical forces vectorially.

$\therefore$ total weight $W_t = \sqrt{(W + W_i)^2 + W_w^2}$, $W_i = \pi\rho R\,(D + R)$

Here $\rho = 920$ kg/m^3, $R = 0.0125$ m; $D = 0.0196$ m

$W_w = P(D + 2R) = 30(0.0196 + 2 \times 0.0125) = 1.34$ kg wt/m; $W = 90/100 = 0.9$ kg wt/m

$\therefore$ $W_i = \pi \times 920 \times 0.0125(0.0196 + 0.0125) = 1.16$ kg wt/m

$\therefore$ $W_t = \sqrt{(W + W_i)^2 + W_w^2} = \sqrt{(0.9 + 1.16)^2 + 1.34^2} =$ **2.46 kg wt/m**

Example 41.53. *A transmission line has a span of 150 metres between supports, the supports being at the same level. The conductor has a cross-sectional area of 2 cm^2. The ultimate strength is 5,000 kg/cm^2. The specific gravity of the material is 8.9. If the wind pressure is 1.5 kg/m length of the conductor, calculate the sag at the centre of the conductor if factor of safety is 5.*

(Electrical Technology-I, Bombay Univ.)

Solution. Safety factor $= \dfrac{\text{breaking or ultimate stress}}{\text{working stress}}$

$\therefore$ working stress = 5,000/5 $= 10^3$ kg/cm^2; Working tension, $T = 10^2 \times 2 = 2{,}000$ kg wt.

Vol. of one metre length of conductor $= 2 \times 100 = 200$ cm^3

Wt. of 1 m of material $= 8.9 \times 200\ g$ or $W = 8.9 \times 200/1{,}000 = 1.98$ kg-wt.

Total Wt. per metre $= \sqrt{W^2 + W_w^2} = \sqrt{1.98^2 + 1.5^2} = 2.48$ kg-wt

Now $d = Wl^2/2T = 2.48 \times (150/2)^2/2 \times 2{,}000 =$ **3.5 metre**

Example 41.54. *A transmission line has a span of 200 metres between level supports. The conductor has a cross-sectional area of 1.29 cm^2, weighs 1,170 kg/km and has a breaking stress of 4,218 kg/cm^2. Calculate the sag for a factor of safety of 5 allowing a wind pressure of 122 kg per m^2 of projected area. What is the vertical sag?*

(Transmission and Distribution-II, Kerala Univ.)

Solution. Safety factor $= \dfrac{\text{breaking or ultimate stress}}{\text{working stress}}$

working stress $= 4{,}218/5 = 843.6$ kg/cm^2

Working tension $T = 843.6 \times 1.29 = 1{,}088$ kg.wt

$\therefore$ $W = 1{,}170$ kg/km $= 1.17$ kg.wt/metre

Let us now find diameter of the conductor from the equation

$\pi d^2/4 = 1.29$ or $d = 1.28$ cm

$\therefore$ Projected area of the conductor per metre length is $= 1.28 \times 10^{-2} \times 1 = 1.28 \times 10^{-2}$ m^2

$W_w = 122 \times 1.28 \times 10^{-2} = 1.56$ kg-wt/m

$\therefore$ $W_t = \sqrt{W^2 + W_w^2} = \sqrt{1.17^2 + 1.56^2} = 1.95$ kg-wt/m

Slant sag $d = W_t l^2/2T = 1.95 \times 100^2/2 \times 1088 =$ **8.96 m**

Now, $\tan\theta = W_w/W = 1.56/1.17 = 1.333$; $\theta = 53.2°$

$\therefore$ vertical sag $= d \cos 53.2° = 8.96 \times 0.599 =$ **5.3 m**

Example 41.55. *A transmission line has a span of 214 metres. The line conductor has a cross-section of 3.225 cm^2 and has an ultimate breaking strengh of 2,540 kg/cm^2. Assuming that the line is covered with ice and provides a combined copper and ice load of 1.125 kg/m while the wind pressure is 1.5 kg/m run* ***(i)*** *calculate the maximum sag produced. Take a factor of safety of 3* ***(ii)*** *also determine the vertical sag.* **(Electrical Power Systems, Gujarat Univ.)**

Solution. ***(i)*** Maximum sag in a direction making an angle of θ (Fig. 41.64) is $d = W_t l^2/2T$

Here, $W + Wi = 1.125$ kg-wt/m ; $Ww = 1.5$ kg-wt/m

$W_t = \sqrt{1.125^2 + 1.5^2} = 1.874$ kg-wt/m; $l = 214/2 = 107$ m

Safety factor $= \dfrac{\text{ultimate breaking stress}}{\text{working stress}}$

$\therefore$ working stress $= 2{,}540/3 = 847$ kg-wt/cm^2

Permissible tension $T = 847 \times 3.225 = 2{,}720$ kg-wt

$\therefore$ $d = 1.874 \times 107^2/2 \times 2{,}720 =$ **3.95 m**

(ii) Now $\tan\theta = W_w/(W_i + W) = 1.5/1.125 = 0.126$; $\theta = 53.2°$; $\cos\theta = 0.599$

$\therefore$ vertical sag $= d \cos\theta = 3.95 \times 0.599 =$ **2.35 m**

Example 41.56. *Two towers of height 30 and 90 m respectively support a transmission line conductor at water crossing. The horizontal distance between the towers is 500 m. If the tension in the conductor is 1,600 kg, find the minimum clearance of the conductor and the clearance of the conductor mid-way between the supports. Weight of the conductor is 1.5 kg/m. Bases of the towers can be considered to be at the water level.* **(Electrical Power-I, Bombay Univ.)**

Solution. $x_1 = l - \frac{hT}{2Wl}$

Here, $l = 500/2 = 250$ m

$h = 90 - 30 = 60$ m

$T = 1{,}600$ kg-wt ;

$W = 1.5$ kg-wt/m

$\therefore \quad x = 250 - \frac{60 \times 1{,}600}{2 \times 1.5 \times 250}$

$= 250 - 128 = 122$ m

$x_1 = 250 + 128 = 378$ m

$d_1 = \frac{W_{x1}^2}{2T} = \frac{1.5 \times 122^2}{2 \times 1600} = 7$ m

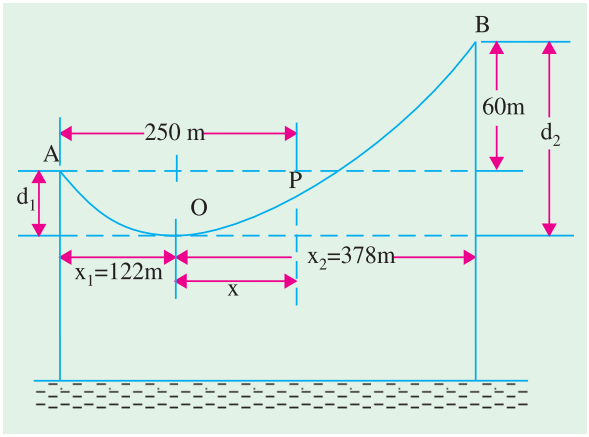


Fig. 41.65

As seen from Fig. 41.65, clearance of the lowest point O from the water level

$= 30 - 7 = 23$ m

The horizontal distance of mid-point P from the reference point O is $x = (250 - 122) = 128$m*. The height of the point P above O is

$$d_{mid} = \frac{Wx_1^2}{2T} = \frac{1.5 \times 128^2}{2 \times 1{,}600} = 7.68 \text{ m}$$

Hence, clearance of mid-point above water level is $23 + 7.68 =$ **30.68 m**

Example 41.57. *An overhead transmission line at a river crossing is supported from two towers at heights of 50 m and 100 m above the water level, the horizontal distance between the towers being 400 m. If the maximum allowable tension is 1,800 kg and the conductor weighs 1 kg/m, find the clearance between the conductor and water at a point mid-way between the towers.*

(Power System-I, AMIE, Sec. B, 1994)

Solution. Here, $h = 100 - 50 = 50$ m ;

$l = 400/2 = 200$ m

$T = 1800$ kg-wt

$W = 1$ kg-wt/m

$x_1 = l - \frac{hT}{2Wl}$

$= 200 - \frac{50 \times 1800}{2 \times 1 \times 200}$

$= -25$ m

$x_2 = l + \frac{hT}{2Wl}$ 200 + 225

$= 425$ m

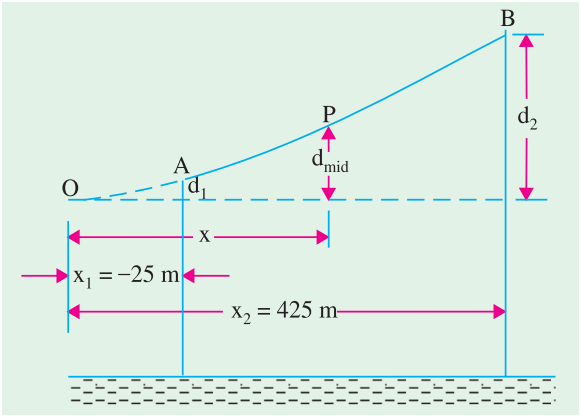


Fig. 41.66

* Or $x = \frac{1}{2}(x_2 - x_1)$. In this case, $x = \frac{1}{2}(378 - 122) = 128$ m

Since x_1 turns out to be negative, point A lies on the same side of O as B (Fig. 41.66). Distance of mid-point P from O is $x = (425 + 25) = 225$ m. Hence, height of P above O is

$$d_{mid} = \frac{Wx^2}{T} = \frac{1 \times 225^2}{2 \times 1,800} = 14 \text{ m}$$

Now, $$d_2 = \frac{Wx_2^2}{2T} = \frac{1 \times 425^2}{2 \times 1,800} = 50.2 \text{ m}$$

Hence, P is (50.2–14) = 36.2 m below point B. It means that mid-point P is (100 – 36.2) = 63.8 m above water level.

Example 41.58. *A conductor is strung across a river, being supported at the two ends at heights of 20 m and 16 m respectively, from the bed of the river. The distance between the supports is 375 m and the weight of the conductor = 1.2 kg/m.*

If the clearance of the conductor from the river bed be 9 m, find the horizontal tension in the conductor. Assume a parabolic configuration and that there is no wind or ice loading.

(Electrical Power-I, Bombay Univ.)

Solution. Here, $l = 375/2 = 187.5$ m; $h = 20 - 16 = 4$ m; $W = 1.2$ kg-wt/m; $T = ?$

$$x_1 = l - \frac{hT}{2Wl} = 187.5 - \frac{4 \times T}{2 \times 1.2 \times 187.5} = 187.5 - \frac{T}{112.5}$$

or $$x_1 = (187.5 - m) \quad \text{where} \quad m = T/112.5$$

Also, $$d_1 + 9 = 16 \quad \text{or} \quad d_1 = 7 = \frac{Wx_1^2}{2T}$$

$\therefore$ $$7 = \frac{1.2\,(187.5 - m)^2}{225\,m} \qquad (\because 2T = 2m \times 112.5 = 225 \text{ m})$$

or $1.2\,m^2 - 2,025\,m + 42,180 = 0$ or $m = 1,677$ or 10.83

Rejecting the bigger value which is absurd, we have

$$m = 10.83 \quad \text{or} \quad T/112.5 = 10.83 \quad \text{or} \quad T = \mathbf{1,215 \text{ kg-wt/m}}$$

Tutorial Problem No. 41.3

1. Show diagrammatically the distribution of electrostatic capacitance in a 3-core, 3-phase lead-sheathed cable.

   The capacitance of such a cable measured between any two of the conductors, the sheathing being earthed, is 0.3 μF per km. Find the equivalent star-connected capacitance and the kVA required to keep 10 km of the cable charged when connected to 20,000-V, 50 Hz bus-bars. **[0·6 μ F ; 754 kVA]**

2. The 3-phase output from a hydro-electric station is transmitted to a distributing centre by two overhead lines connected in parallel but following different routes. Find how a total load of 5,000 kW at a p.f. of 0.8 lagging would divide between the two routes if the respective line resistances are 1.5 and 1.0 Ω and their reactances at 25 Hz are 1.25 and 1.2 Ω.

   [2,612 kW ; 2,388 kW](*City & Guilds, London*)

3. Two 3-phase cables connected in parallel supply a 6,600-V, 1,000-kW load at a lagging power factor of 0.8. The current in one of the cables is 70 A and it delivers 600 kW. Calculate its reactance and resistance, given that the other cable has reactance of 2.6 Ω and a resistance of 2 Ω.

   [R = .795 Ω ; X = 1.7 Ω] (*London Univ.*)

4. A concentric cable has a conductor diameter of 1 cm and an insulation thickness of 1.5 cm. Find the maximum field strength when the cable is subjected to a test pressure of 33 kV.

   [47.6 kV/cm (r.m.s.) or 67.2 kV/cm (peak)] (*London Univ.*)

5. A single-phase ring distributor XYZ is fed at X. The loads at Y and Z are 20 A at 0.8 p.f. lagging and 15 A at 0.6 p.f. lagging respectively, both expressed with reference to voltage at X. The total impedances of the three sections $X\,Y$, YZ and ZX are $(1 + j1)$, $(1 + j2)$ and $(1 + j3)$ ohms respectively. Find the total current fed at X and the current in each section with respect to suply voltage at X.

(Allahabad Univ.)

[$34.6 \angle -43.8°$ A ; AXY = $23.1 \angle -32°$ A, AXZ = $13.1 \angle -60.8°$. YZ = $3.01 \angle -30.5°$A]

6. A single-phase distributor has a resistance of 0.2 ohm and reactance 0.3 ohm. At the far end, the voltage V_B = 240 volts, the current is 100 A and the power factor is 0.8. At the mid point A, current of 100 A is supplied at a power factor 0.6 with reference to voltage V_A at point A. Find supply voltage V_S and the phase angle between V_S and V_B. **[292 V, 2.6°]**

7. Estimate the corona loss for a 3-phase 110-kV, 50-Hz, 150-km long transmission line consisting of three conductors, each of 10 mm diameter and spaced 2.5 metre apart in an equilateral triangle formation. The temperature of air is 30°C and the atmospheric pressure is 750 mm of mercury. Take the irregularity factor as 0.85. **[385 kW](*AMIE*)**

8. A transmission line conductor at a river crossing is supported from two towers at heights of 20 m and 60 m above water level. The horizontal distance between the towers is 300 m. If the tension in the conductor is 1800 kg and the conductor weighs 1.0 kg per metre, find the clearance between the conductor and the water level at a point mid-way between the towers. Use approximate method.

(18.75 m) (*AMIE*)

9. Show that the positive and negative sequence impedances of transmission lines are same where as its zero sequence impedance is higher than positive sequence impedance.

(Nagpur University, Summer 2004)

10. A 132 kV, 3 phase, 50 Hz overhead line of 100 km length has a capacitance to earth of each line of 0.01 μF/km. Determine inductance and kVA rating of the arc suppression coil suitable for this system. ***(Nagpur University, Summer 2004)***

OBJECTIVE TESTS – 41

1. With same maximum voltage between conductors, the ratio of copper volumes in 3-phase, 3-wire system and 1-phase, 2-wire system is

(*a*) 4/3 (*b*) 3/4

(*c*) 5/3 (*d*) 3/5

2. The volume of copper required for an a.c. transmission line is inversely proportional to

(*a*) current (*b*) voltage

(*c*) power factor (*d*) both (*b*) and (*c*)

(*e*) both (*a*) and (*c*).

3. For a.c. transmission lines less than 80 km in length, it is usual to lump the line capacitance at

(*a*) the receiving end

(*b*) the sending end

(*c*) the mid-point

(*d*) any convenient point.

4. Corona occurs between two transmission wires when they

(*a*) are closely-spaced

(*b*) are widely-spaced

(*c*) have high potential difference

(*d*) carry d.c. power.

5. The only advantage of corona is that it

(*a*) makes line current non-sinusoidal

(*b*) works as a safety-valve for surges

(*c*) betrays its presence by hissing sound

(*d*) produces a pleasing luminous glow.

6. The sag produced in the conductor of a transmission wire depends on

(*a*) weight of the conductor per unit length

(*b*) tension in the conductor

(*c*) length of the conductor

(*d*) all of the above

(*e*) none of the above.

7. Suspension insulators are used when transmission voltage is

(*a*) high (*b*) low

(*c*) fluctuating (*d*) steady

8. The string efficiency of suspension insulators can be increased by

(*a*) providing a guard ring

(*b*) grading the insulators

(*c*) using identical insulator disc
(*d*) both (*a*) & (*b*).

9. An interconnector between two generating stations facilitates to
(*a*) keep their voltage constant
(*b*) run them in parallel
(*c*) transfer power in either direction
(*d*) both (*b*) & (*c*)

10. The effective disruptive critical voltage of a transmission line does NOT depend on
(*a*) irregularity factor
(*b*) conductor radius
(*c*) distance between conductors
(*d*) material of the conductors.

11. By which of the following systems electric power may be transmitted?
(*a*) Overhead system
(*b*) Underground system
(*c*) Both (a) and (b)
(*d*) None of the above

12. ..... are the conductors, which cannect the consumer's terminals to the distribution
(*a*) Distributors
(*b*) Service mains
(*c*) Feeders
(*d*) None of the above

13. The underground system cannot be operated above
(*a*) 440 V (*b*) 11 kV
(*c*) 33 kV (*d*) 66 kV

14. Overhead system can be designed for operation upto
(*a*) 11 kV (*b*) 33 kV
(*c*) 66 kV (*d*) 400 kV

15. If variable part of annual cost on account of interest and depreciation on the capital outlay is equal to the annual cost of electrical energy wasted in the conductors, the total annual cost will be minimum and the corresponding size of conductor will be most economical. This statement is known as
(*a*) Kelvin's law (*b*) Ohm's law
(*c*) Kirchhoff's law (*d*) Faraday's law
(*e*) none of the above

16. The wooden poles well impregnated with creosite oil or any preservative compound have life
(*a*) from 2 to 5 years
(*b*) 10 to 15 years
(*c*) 25 to 30 years
(*d*) 60 to 70 years

17. Which of the following materials is not used for transmission and distribution of electrical power?
(*a*) Copper
(*b*) Aluminium
(*c*) Steel
(*d*) Tungsten

18. Galvanised steel wire is generally used as
(*a*) stay wire
(*b*) earth wire
(*c*) structural components
(*d*) all of the above

19. The usual spans with R.C.C. poles are
(*a*) 40–50 metres
(*b*) 60–100 metres
(*c*) 80–100 metres
(*d*) 300–500 metres

20. The corona is considerably affected by which of the following?
(*a*) Size of the conductor
(*b*) Shape of the conductor
(*c*) Surface condition of the conductor
(*d*) All of the above

21. Which of the following are the constants of the transmission lines?
(*a*) Resistance
(*b*) Inductance
(*c*) Capacitance
(*d*) All of the above

22. %age regulation of a transmission line is given by

(*a*) $\frac{V_R - V_S}{V_{R^2}} \times 100$

(*b*) $\frac{V_S - V_R}{V_R} \times 100$

(*c*) $\frac{V_S - V_R}{V_S} \times 100$

(*d*) $\frac{V_S - V_R}{V_{R^2}} \times 100$

where V_S and V_R are the voltages at the sending end and receiving and respectively.

23. The phenomenon of rise in voltage at the receiving end of the open-circuited or lightly loaded line is called the
(*a*) Seeback effect
(*b*) Ferranti effect
(*c*) Raman effect
(*d*) none of the above

24. The square root of the ratio of line impedance and shunt admittance is called the
(*a*) surge impedance of the line
(*b*) conductance of the line
(*c*) regulation of the line
(*d*) none of the above

25. Which of the following is the demerit of a 'constant voltage transmission system'?
(*a*) Increase of short-circuit current of the system
(*b*) Availability of steady voltage at all loads at the line terminals
(*c*) Possibility of better protection for the line due to possible use of higher terminal reactances
(*d*) Improvement of power factor at times of moderate and heavy loads
(*e*) Possibility of carrying increased power for a given conductor size in case of long-distance heavy power transmission

26. Low voltage cables are meant for use up to
(*a*) 1.1 kV
(*b*) 3.3 kV
(*c*) 6.6 kV
(*d*) 11 kV

27. The operating voltage of high voltage cables is upto
(*a*) 1.1 kV
(*b*) 3.3 kV
(*c*) 6.6 kV
(*d*) 11 kV

28. The operating voltage of supertension cables is upto
(*a*) 3.3 kV
(*b*) 6.6 kV
(*c*) 11 kV
(*d*) 33 kV

29. The operating voltage of extra high tension cables is upto
(*a*) 6.6 kV
(*b*) 11 kV
(*c*) 33 kV
(*d*) 66 kV
(*e*) 132 kV

30. Which of the following methods is used for laying of underground cables?
(*a*) Direct laying
(*b*) Draw-in-system
(*c*) Solid system
(*d*) All of the above

31. Which of the following is the source of heat generation in the cables?
(*a*) Dielectric losses in cable insulation
(*b*) I^2R losses in the conductor
(*c*) Losses in the metallic sheathings and armourings
(*d*) All of the above

32. Due to which of the following reasons the cables should not be operated too hot?
(*a*) The oil may loose its viscosity and it may start drawing off from higher levels
(*b*) Expansion of the oil may cause the sheath to burst
(*c*) Unequal expansion may create voids in the insulation which will lead to ionization
(*d*) The thermal instability may rise due to the rapid increase of dielectric losses with temperature

33. Which of the following D.C. distribution system is the simplest and lowest in first cost?
(*a*) Radial system
(*b*) Ring system
(*c*) Inter-connected system
(d) Non of the above

34. A booster is a
(*a*) series wound generator
(*b*) shunt wound generator
(*c*) synchronous generator
(*d*) none of the above

35. Besides a method of trial and error, which of the following methods is employed for solution of network problems in interconnected system?
(*a*) Circulating current method
(*b*) Thevenin's theorem
(*c*) Superposition of currents
(*d*) direct application of Kirchhoff's laws
(*e*) All of the above

36. Which of the following faults is most likely to occur in cables?
(*a*) Cross or short-circuit fault
(*b*) Open circuit fault
(*c*) Breakdown of cable insulation
(*d*) all of the above

37. The cause of damage to the lead sheath of a cable is
(*a*) crystallisation of the lead through vibration
(*b*) chemical action on the lead when
(*c*) mechanical damage
(*d*) all of the above

38. The voltage of the single phase supply to residential consumers is
(*a*) 110 V
(*b*) 210 V
(*c*) 230 V
(*d*) 400 V

39. Most of the high voltage transmission lines in India are
(*a*) underground
(*b*) overhead

(*c*) either of the above
(*d*) none of the above

40. The distributors for residential areas are
(*a*) single phase
(*b*) three-phase three wire
(*c*) three-phase four wire
(*d*) non of the above

41. The conductors of the overhead lines are
(*a*) solid
(*b*) stranded
(*c*) both solid and stranded
(*d*) none of the above

42. High voltage transmission lines use
(*a*) suspension insulators
(*b*) pin insulators
(*c*) both (*a*) and (*b*)
(*d*) none of the above

43. Multicore cables generally use
(*a*) square conductors
(b) circular conductors
(c) rectangular conductors
(d) sector-shaped conductors
(e) none of the above

44. Distributio lines in India generally use
(*a*) wooden poles
(*b*) R.C.C. poles
(*c*) steel towers
(*d*) none of the above

45. The material commonly used for insulation in high voltage cables is
(*a*) lead
(*b*) paper
(*c*) rubber
(*d*) none of the above

46. The loads on distributors systems are generally
(*a*) balanced
(*b*) unbalanced
(*c*) either of the above
(*d*) none of the above

47. The power factor of industrial loads is generally
(*a*) unity
(*b*) lagging
(*c*) leading
(*d*) zero

48. Overhead lines generally use
(*a*) copper conductors
(*b*) all aluminium conductors
(*c*) A.C.S.R. conductors
(*d*) none of these

49. In transmission lines the cross-arms are made of
(*a*) copper
(*b*) wood
(*c*) R.C.C.
(*d*) steel

50. The material generally used for armour of high voltage cables is
(*a*) aluminium
(*b*) steel
(*c*) brass
(*d*) copper

51. Transmission line insulators are made of
(*a*) glass
(*b*) porcelain
(*c*) iron
(*d*) P.V.C.

52. The material commonly used for sheaths of underground cables is
(*a*) lead
(*b*) rubber
(c) copper
(d) iron

53. The minimum clearance between the ground and a 220 kV line is about
(*a*) 4.3 m
(*b*) 5.5 m
(*c*) 7.0 m
(*d*) 10.5 m

54. The spacing between phase conductors of a 220 kV line is approximately equal to
(*a*) 2 m
(*b*) 3.5 m
(*c*) 6 m
(*d*) 8.5 m

55. Large industrial consumers are supplied electrical energy at
(*a*) 400 V
(*b*) 11 kV
(*c*) 66 kV
(*d*) 400 kV

56. In a D.C. 3-wire distribution system, balancer fields are cross-connected in order to
(*a*) boost the generated voltage
(*b*) balance loads on both sides of the neutral
(*c*) make both machines run as unloaded motors
(*d*) equalize voltages on the positive and negative outers

57. In a D.C. 3-wire distributor using balancers and having unequal loads on the two sides
(*a*) both balancers run as generators
(*b*) both balancers run as motors
(*c*) balancer connected to lightly-loaded side runs as a motor
(*d*) balancer connected to heavily-loaded side runs as a motor

58. Transmitted power remaining the same, if supply voltae of a D.C. 2-wire feeder is increased 100 percent, saving in copper is

(*a*) 25 percent
(*b*) 50 percent
(*c*) 75 percent
(*d*) 100 percent

59. A uniformly-loaded D.C. distributor is fed at both ends with equal voltages. As compared to a similar distributor fed at one end only, the drop at a middle point is
(*a*) one-fourth
(*b*) one-third
(*c*) one-half
(*d*) twice
(*e*) none of the above

60. As compared to a 2-wire D.C. distributor, a 3-wire distributor with same maximum voltage to earth uses only
(*a*) 31.25 percent of copper
(*b*) 33.3 percent of copper
(*c*) 66.7 percent of copper
(*d*) 125 percent of copper

61. Which of the following is usually not the generating voltage?
(*a*) 6.6 kV
(*b*) 8.8 kV
(*c*) 11 kV
(*d*) 13.2 kV

62. For an overhead line, the surge impedance is taken as
(*a*) 20–40 ohms
(*b*) 70–80 ohms
(*c*) 100–200 ohms
(*d*) 500–1000 ohms
(*e*) none of the above

63. The presence of ozone due to corona is harmful because it
(*a*) reduces power factor
(*b*) corrodes the material
(*c*) gives odour
(*d*) transfer energy to the ground
(*e*) none of the above

64. A feeder, in a transmission system, feeds power to
(*a*) distributors
(*b*) generating stations
(*c*) service mains
(*d*) all of the above

65. The power transmitted will be maximum when
(*a*) corona losses are minimum
(*b*) reactance is high
(*c*) sending end voltage is more
(*d*) receiving end voltage is more

66. A 3-phase 4 wire system is commonly used on
(*a*) primary transmission
(*b*) secondary transmission
(*c*) primary distribution
(*d*) secondary distribution

67. Which of the following materials is used for overhead transmission lines?
(*a*) Steel cored aluminium
(*b*) Galvanised steel
(*c*) Cadmium copper
(*d*) Any of the above

68. Which of the following is not a constituent for making porcelain insulators?
(*a*) Quartz
(*b*) Kaolin
(*c*) Felspar
(*d*) Silica

69. There is a greater possibility of occurence of corona during
(*a*) dry weather
(*b*) winter
(*c*) summer heat
(*d*) humid weather
(*e*) none of the above

70. Which of the following relays is used on long transmission lines?
(*a*) Impedance realy
(*b*) Mho's relay
(*c*) Reactance relay
(*d*) None of the above

71. The steel used in steel cored conductors is usually
(*a*) alloy steel
(*b*) stainless steel
(*c*) mild steel
(*d*) high speed steel
(*e*) all of the above

72. Which of the following distribution system is more reliable?
(*a*) Radial system
(*b*) Tree system
(*c*) Ring main system
(*d*) All are equally reliable

73. Which of the followin characteristics should the line supports for transmission lines possess?
(*a*) Low cost
(*b*) High mechanical strength
(*c*) Longer life
(*d*) All of the above

74. Transmission voltage of 11 kV is normally used for distance upto
(*a*) 20–25 km
(*b*) 40–50 km
(*c*) 60–70 km
(*d*) 80–100 km

75. which of the following regulations is considered best?
(*a*) 50%
(*b*) 20%

(*c*) 10%
(*d*) 2%

76. Skin effect is proportional to
(*a*) (conductor diameter)^4
(*b*) (conductor diameter)^3
(*c*) (conductor diameter)^2
(*d*) (conductor diameter)$^{1/2}$
(*e*) none of the above

77. A conductor, due to sag between two supports, takes the form of
(*a*) semi-circle
(*b*) triangle
(*c*) ellipse
(*d*) catenary

78. In A.C.S.R. conductors, the insulation between aluminium and steel conductors is
(*a*) insulin
(*b*) bitumen
(*c*) varnish
(*d*) no insulation is required

79. Which of the following bus-bar schemes has the lowest cost?
(*a*) Ring bus-bar scheme
(*b*) Single bus-bar scheme
(*c*) Breaker and a hlaf scheme
(*d*) Main and transfer scheme

80. Owing to skin effect
(*a*) current flows through the half cross-section of the conductor
(*b*) portion of the conductor near the surface carries more current and core of the conductor carries less current
(*c*) portion of the conductor near the surface carries less current and core of the conductor carries more current
(*d*) none of the above

81. By which of the following methods string efficiency can be improved?
(*a*) Using a guard ring
(*b*) Grading the insulator
(*c*) Using long cross arm
(*d*) Any of the above
(*e*) None of the above

82. In aluminium conductors, steel core is provided to
(*a*) compensate for skin effect
(*b*) neutralise proximity effect
(*c*) reduce line inductance
(*d*) increase the tensile strength

83. By which of the following a bus-bar is rated?
(*a*) Current only
(*b*) Current and voltage
(*c*) Current, voltage and frequency
(*d*) Current, voltage, frequency and short time current

84. A circuit is disconnected by isolators when
(*a*) line is energize
(*b*) there is no current in the line
(*c*) line is on full load
(*d*) circuit breaker is not open

85. For which of the following equipment current rating is not necessary?
(*a*) Circuit breakers
(*b*) Isolators
(*c*) Load break switch
(*d*) Circuit breakers and load break switches

86. In a substation the following equipment is not installed
(*a*) exciters
(*b*) series capacitors
(*c*) shunt reactors
(*d*) voltage transformers

87. Corona usuall occurs when the electrostatic stress in air around the conductor exceeds
(*a*) 6.6 kV (r.m.s. value)/cm
(*b*) 11 kV (r.m.s. value)/cm
(*c*) 22 kV (maximum value)/cm
(*d*) 30 kV (maximum value)/cm

88. The voltage drop, for constant voltage transmission is compensated by installing
(*a*) inductors
(*b*) capacitors
(*c*) synchronous motors
(*d*) all of above
(*e*) none of the above

89. The use of strain type insulators is made where the conductors are
(*a*) dead ended
(*b*) at intermediate anchor towers
(*c*) any of the above
(*d*) none of the above

90. The current drawn by the line due to corona losses is
(*a*) non-sinusoidal
(*b*) sinusoidal
(*c*) triangular
(*d*) square

91. Pin type insulators are generally not used for voltages beyond
(*a*) 1 kV
(*b*) 11 kV
(*c*) 22 kV
(*d*) 33 kV

92. Aluminium has a specific gravity of
(*a*) 1.5
(*b*) 2.7
(*c*) 4.2
(*d*) 7.8

93. For transmission of power over a distance of 200 km, the transmission voltage should be

(*a*) 132 kV
(*b*) 66 kV
(*c*) 33 kV
(*d*) 11 kV

94. For aluminium, as compared to copper, all the following factors have higher values except
(*a*) specific volume
(*b*) electrical conductivity
(*c*) co-efficient of linear expansion
(*d*) resistance per unit length for same cross-section

95. Which of the following equipment, for regulating the voltage in distribution feeder, will be most economical?
(*a*) Static condenser
(b) Synchronous condenser
(*c*) The changing transformer
(*d*) Booster transformer

96. In a tap changing transformer, the tappings are provided on
(*a*) primary winding
(*b*) secondary winding
(*c*) high voltage winding
(*d*) any of the above

97. Constant voltage transmission entails the following disadvantage
(*a*) large conductor area is required for same power transmission
(*b*) short-circuit current of the system is increased
(*c*) either of the above
(*d*) none of the above

98. On which of the following factors skin effect depends?
(*a*) Frequency of the current
(*b*) Size of the conductor
(*c*) Resistivity of the conductor material
(*d*) All of the above

99. The effect of corona can be detected by
(*a*) presence of zone detected by odour
(b) hissing sound
(*c*) faint luminous glow of bluish colour
(*d*) all of the above

100. for transmission of power over a distance of 500 km, the transmission voltage should be in the range
(*a*) 150 to 220 kV
(*b*) 100 to 120 kV
(*c*) 60 to 100 kV
(*d*) 20 to 50 kV

101. In the analysis of which of the following lines shunt capacitance is neglected?
(*a*) Short transmission lines
(*b*) Medium transmission lines
(*c*) Long transmission lines
(*d*) Modium as well as long transmission lines

102. When the interconnector between two stations has large reactance
(*a*) the transfer of power will take place with voltage fluctuation and noise
(*b*) the transfer of power will take place with least loss
(*c*) the stations will fall out of step because of large angular displacement between the stations
(*d*) none of the above

103. The frequency of voltage generated, in case of generators, can be increased by
(*a*) using reactors
(*b*) increasing the load
(*c*) adjusting the governor
(*d*) reducing the terminal voltage
(*e*) none of the above

104. When an alternator connected to the bus-bar is shut down the bus-bar voltage will
(*a*) fall
(*b*) rise
(*c*) remain uncharged
(*d*) none of the above

105. The angular displacement between two interconnected stations is mainly due to
(*a*) armature reactance of both alternators
(*b*) reactance of the interconnector
(*c*) synchronous reactance of both the alternators
(*d*) all of the above

106. Electro-mechanical voltage regulators are generally used in
(*a*) reactors
(*b*) generators
(*c*) transformers
(*d*) all of the above

107. Series capacitors on transmission lines are of little use when the load VAR requirement is
(*a*) large
(*b*) small
(*c*) fluctuating
(*d*) any of the above

108. The voltage regulation in magnetic amplifier type voltage regulator is effected by
(*a*) electromagnetic induction
(*b*) varying the resistance
(*c*) varying the reactance
(*d*) variable transformer

109. when a conductor carries more current on the surface as compared to core, it is due to
(*a*) permeability variation
(*b*) corona
(*c*) skin effect
(*d*) unsymmetrical fault

(*e*) none of the above

110. The following system is not generally used
(*a*) 1-phase 3 wire
(*b*) 1-phase 4 wire
(*c*) 3-phase 3 wire
(*d*) 3-phase 4 wire

111. The skin effect of a conductor will reduce as the
(*a*) resistivity of conductor material increases
(*b*) permeability of conductor material increases
(*c*) diameter increases
(*d*) frequency increases

112. When a live conductor of public electric wupply breaks down and touches the earth which of the following will happen?
(*a*) Current will flow to earth
(*b*) Supply voltage will drop
(*c*) Supply voltage will increase
(*d*) No current will flow in the conductor

113. 310 km line is considered as
(*a*) a long line
(*b*) a medium line
(*c*) a short line
(*d*) any of the above

114. The conductors are bundled primarily to
(*a*) increase reactance
(*b*) reduce reactance
(*c*) reduce ratio interference
(*d*) none of the above

115. The surge impedance in a transmission line having negligible resistance is given as
(*a*) $\sqrt{LC}$
(*b*) $\sqrt{L/C}$
(*c*) $\sqrt{1/LC}$
(*d*) $\sqrt{L+C}$
(*e*) none of the above

116. The top most conductor in a high transmission line is
(*a*) earth conductor
(*b*) R-phase conductor
(*c*) Y-phase conductor
(*d*) B-phase conductor

117. In A.C.S.R. conductor the function of steel is to
(*a*) provide additional mechanical strength
(*b*) prevent corona
(*c*) take care of surges
(*d*) reduce inductance and subsequently improve power factor

118. In transmission and distribution system the permissible voltage variation is
(*a*) ±1 percent
(*b*) ± 10 percent
(*c*) ± 20 percent
(*d*) ± 30 percent
(*e*) none of the above

119. By which of the following methods voltage of transmission can be regulated?
(*a*) use of series capacitors to neutralies the effect of series reactance
(*b*) Switching in shunt capacitors at the receiving end during have loads
(*c*) Use of tap changing transformers
(*d*) Any of the above methods

120. Which of the following distribution systems is the most economical?
(*a*) A.C. 1-phase system
(*b*) A.C. 3-phase 3 wire system
(*c*) A.C. 3-phase 4 wire system
(*d*) Direct current system

121. Which of the following is the main advantage of A.C. transmission system over D.C. transmission system?
(*a*) Less instability problem
(*b*) Less insulation problems
(*c*) Easy transformation
(*d*) Less losses in transmission over long distances

122. A tap changing transformer is used to
(*a*) supply low voltage current for instruments
(*b*) step up the voltage
(*c*) step down the voltage
(*d*) step up as well as step down the voltage

123. Which of the following bar schemes is the most expensive?
(*a*) Double bus-bar double breaker
(*b*) Ringbus-bar scheme
(*c*) Single bus-bar scheme
(*d*) Main and transfer scheme

124. By which of the following methods the protection agains direct lightning strokes and high voltage sweep waves is provided?
(*a*) Lightening arresters
(*b*) Ground wire
(*c*) Lightening arresters and ground wires
(*d*) Earthing of neutral
(*d*) None of the above

125. In which of the following voltage regulators the effect of dead zero is found?
(*a*) Electromagnetic type
(*b*) Magnetic amplifier
(*c*) Electronic type using integrated circuits
(*e*) all of the above

126. Corona results in

(*a*) radio interference
(*b*) power factor improvement
(*c*) better regulation
(*d*) none of the above

127. Which of the following has least effect on corona?
(*a*) Atmospheric temperature
(*b*) Number of ions
(*c*) Size and charge per ion
(*d*) Mean free path

128. In context of corona, if the conductors are polished and smooth, which of the following statements is correct?
(*a*) Hissing sound will be more intense
(*b*) Power loss will be least
(*c*) Corona glow will be uniform along the length of the conductor
(*d*) Corona glow will not occur

129. Power loss due to corona is not directly proportional to
(*a*) spacing between conductors
(*b*) supply voltage frequency
(*c*) phase-neutral voltage
(*d*) all of the above

130. Poles which carry transformers are usually
(*a*) circular
(*b*) I-type
(*c*) A-type
(*d*) H-type
(*e*) none of the above

131. Out of the following which type of poles are bulky?
(*a*) Transmission towers
(*b*) Concrete poles
(*c*) Tubular steel poles
(*d*) Wooden poles

132. The effect of ice on transmission line conductors is to increase the
(*a*) transmission losses
(*b*) weight of the conductor
(*c*) tendency for corona
(*d*) resistance to flow of current

133. If the height of transmission tower is increased
(*a*) the line capacitance will decrease but line inductance will remain uncharged
(*b*) the line capacitance and inductance will not change
(*c*) the line capacitance will increase but line inductance will decrease
(*d*) the line capacitance will decrease and line inductance will increase

134. If staring efficiency is 100 percent it means that
(*a*) potential across each disc is zero
(*b*) potential across each disc is same
(*c*) one of the insulator discs is shorted
(*d*) none of the above

135. In a 70/6 A.C.S.R. conductor there are
(*a*) 35 aluminium conductors and 3 steel conductors
(*b*) 70 aluminium conductors and 6 steel conductors
(*c*) 70 steel conductors and 6 aluminium conductors
(*d*) none of the above

136. On which of the following does the size of a feeder depend?
(*a*) Voltage drop
(*b*) Voltage
(*c*) Frequency
(*d*) Current carrying capacity

137. Which of the following are connected by the service mains?
(*a*) Transformer and earth
(*b*) Distributor and relay system
(*c*) Distributor and consumer terminals
(*d*) Distributor and transformer

138. In the design of a distributor which of the following is the major consideration?
(*a*) Voltage drop
(*b*) Current carrying capacity
(*c*) Frequency
(*d*) kVA of system
(*e*) None of the above

139. In a distribution system major cost is that of
(*a*) earthing system
(*b*) distribution transformer
(*c*) conductors
(*d*) meters

140. A booster is connected in
(*a*) parallel with earth connection
(*b*) parallel with the feeder
(*c*) series with the feeder
(*d*) series with earth connection

141. With which of the following are step-up substations associated?
(*a*) Concentrated load
(*b*) Consumer location
(*c*) Distributors
(*d*) Generating stations
(*e*) None of the above

142. Which of the following equipment should be installed by the consumers having low power factor?
(*a*) Synchronous condensers
(*b*) Capacitor bank
(*c*) Tap changing transformer
(*d*) Any of the above
(*e*) None of the above

143. Which of the following equipment is used to

limit short-circuit current level in a substation?
(*a*) Isolator
(*b*) Lightning switch
(*c*) Coupling capacitor
(*d*) Series reactor

144. Steepness of the travelling waves is alternated by ..... of the line
(*a*) capacitance
(*b*) inductance
(*c*) resistance
(*d*) all of the above

145. The limit of distance of transmission line may be increased by the use of
(*a*) series resistances
(*b*) shunt capacitors and series reactors
(*c*) series capacitors and shunt reactors
(*d*) synchronous condensers
(*e*) none of the above

146. By which of the following feactors is the sag of a transmission line least affected?
(*a*) Current through the conductor
(*b*) Ice deposited on the conductor
(*c*) Self weight of conductor
(*d*) Temperature of surrounding air
(*e*) None of the above

147. Which of the following cause transient disturbances?
(*a*) Faults
(*b*) Load variations
(*c*) Switching operations
(*d*) Any of the above

148. A gay wire
(*a*) protects conductors against shortcircuiting
(*b*) provides emergency earth route
(*c*) provides protection against surges
(*d*) supports the pole

149. Which of the following is neglected in the analysis of short transmission lines?
(*a*) Series impedance
(*b*) Shunt admittance
(*c*) I^2R loss
(*d*) None of the above
(*e*) All of the above

150. Basically the boosters are
(*a*) synchronous motors
(*b*) capacitors
(*c*) inductors
(*d*) transformers

151. Which of the following is a static exciter?
(*a*) Rectifier
(*b*) Rotorol
(*c*) Amplidyne
(*d*) D.C. separately excited generator

152. For exact compensation of voltage drop in the feeder the booster
(*a*) must be earthed
(*b*) must work on line voltage
(*c*) must work on linear portion of its V-I characteristics
(*d*) must work on non-linear portion of its V-I characteristics

153. The purpose of using a booster is to
(*a*) increase current
(*b*) reduce current
(*c*) reduce voltage drop
(*d*) compensate for voltage drop
(e) none of the above

154. Induction regulators are used for voltage control in
(*a*) alternators
(*b*) primary distribution
(*c*) secondary distribution
(*d*) none of the above

155. A synchronous condenser is generally installed at the ..... of the transmission line
(*a*) receiving end
(*b*) sending end
(*c*) middle
(*d*) none of the above

156. The area of cross-section of the neutral in a 3-wire D.C. system is generally ..... the area of cross-section of main conductor
(*a*) same as
(*b*) one-fourth
(*c*) one half
(*d*) double

157. For which of the following, the excitation control method is satisfactory?
(*a*) Low voltage lines
(*b*) High voltage lines
(*c*) Short lines
(*d*) Long lines

158. In which of the following cases shunt capacitance is negligible?
(*a*) Short transmission lines
(*b*) Medium transmission lines
(*c*) Long transmission lines
(*d*) All transmission lines

159. A lightning arrester is usually located nearer to
(*a*) transformer
(*b*) isolator
(*c*) busbar
(*d*) circuit breaker
(*e*) none of the above

160. The material used for the manufacture of grounding wires is
(*a*) cast iron
(*b*) aluminium

(*c*) stainlesssteel
(*d*) galvanised steel

161. Surge absorbers protect against ..... oscillations
(*a*) high voltage high frequency
(*b*) high voltage low frequency
(*c*) low voltage high frequency
(*d*) low voltage low frequency

162. Skin effect is noticeable only at ..... frequencies
(*a*) audio
(*b*) low
(*c*) high
(*d*) all

163. Per system stability is least affected by
(*a*) reactance of generator
(*b*) input torque
(*c*) losses
(*d*) reactance of transmission line

164. When the load at the receiving end of a long transmission line is removed, the sending end voltage is less than the receiving end voltage. This effect is known as
(*a*) Ferranti effect
(*b*) Proximity effect
(*c*) Kelvin effect
(*d*) Faraday effect
(*e*) Skin effect

165. In medium transmission lines the shunt capacitance is taken into account in
(*a*) T-method
(*b*) π-method
(*c*) steinmetz method
(*d*) all of the above

166. System grounding is done so that
(*a*) inductive interference between power and communication circuits can be controlled
(*b*) the floating potential on the lower voltage winding for a transformer is brought down to an insignificant value
(*c*) the arcing faults to earth would not set up dangerously high voltage on healthy phases
(*d*) for all above reasons

167. Which of the following can be used for bus-bars?
(*a*) Tubes
(*b*) Rods
(*c*) Bars
(*d*) Any of the above

168. If the height of transmission tower is increased, which of the following parameters is likely to change?
(*a*) Capacitance
(*b*) Inductance
(*c*) Resistance
(*d*) All of the above
(*e*) None of the above

169. A.C.S.R. conductor having 7 steel stands surrounded by 25 aluminium conductors will be specified as
(*a*) 25/7
(*b*) 50/15
(*c*) 7/25
(*d*) 15/50

170. Impedance relay is used on ..... transmission lines
(*a*) short
(*b*) medium
(*c*) long
(*d*) all

171. Corona is likely to occur maximum in
(*a*) transmission lines
(*b*) distribution lines
(*c*) domestic wiring
(*d*) all of the above

172. The effect of wind pressure is more predominant on
(*a*) supporting towers
(*b*) neutral wires
(*c*) transmission lines
(*d*) insulators

173. As compared to cables, the disadvantages of transmission lines is
(*a*) inductive interference between power and communication circuits
(*b*) exposure to lightning
(*c*) exposure to atmospheric hazards like smoke, ice, etc.
(*d*) all of the above

174. In overhead transmission lines the effect of capacitance cand be neglected when the length of line is less than
(*a*) 80 km
(*b*) 110km
(*c*) 150 km
(*d*) 210 km

175. The effective resistance of a conductor will be the same as 'ohmic resistance' when
(*a*) power factor is unity
(*b*) current is uniformly distributed in the conductor cross-section
(*c*) voltage is low
(*d*) current is in true sine wave from

176. Conductors for high voltage transmission lines are suspended from towers to
(*a*) increase clearance from ground
(*b*) reduce clearance from ground
(*c*) take care of extension in length during summer
(*d*) reduce wind and snow loads

(*e*) none of the above

177. To increase the capacity of a transmission line for transmitting power which of the following must be decreased?
(*a*) Capacitance
(*b*) Line inductance
(*c*) Voltage
(*d*) All of the above

178. By using bundled conductors which of the following is reduced?
(*a*) Power loss due to corona
(*b*) Capacitance of the circuit
(*c*) Inductance of the circuit
(*d*) None of the above
(*e*) All of the above

179. Which of the following short-circuits is most dangerous?
(*a*) Dead short-circuit
(*b*) Line to ground short-circuit
(*c*) Line to line short-circuit
(*d*) Line to line and ground short-circuit
(*e*) all of the above

180. Due to which of the following reasons aluminium is being favoured as busbar material?
(*a*) Low density
(*b*) Low cost
(c) Ease of fabrication
(*d*) None of the above

181. In case of transmission line conductors with the increase in atmospheric temperature
(*a*) length decreases but stress increases
(*b*) length increases but stress decreases
(*c*) both the length and stress increases
(*d*) both the length and stress decrease

182. Skin effect exists only in
(*a*) a.c. transmission
(*b*) high voltage d.c. overhead transmission
(*c*) low voltage d.c. overhead transmission
(*d*) cables carrying d.c. current

183. Floating neutral, in 3-phase supply, is undesirable because it causes
(*a*) low voltage across the load
(*b*) high voltage across the load
(*c*) unequal line voltages across the load
(*d*) none of the above

184. The surge resistance of cables is
(*a*) 20 ohms
(*b*) 50 ohms
(*c*) 200 ohms
(*d*) 300 ohms

185. The electrostatic stress in underground cables is
(*a*) zero at the conductor as well as on the sheath
(*b*) same at the conductor and sheath
(*c*) minimum at the conductor and minimum at the sheath
(*d*) maximum at the conductor and minimum at the sheath

186. The ground ring transmission lines are used to
(*a*) reduce the transmission losses
(*b*) reduce the earth capacitance of the lowest unit
(*c*) increase the earth capacitance of the lowest unit
(*d*) none of the above

187. The string efficiency of an insulator can be increased by
(*a*) correct grading of insulators of various capacitances
(*b*) reducing the number of strings
(*c*) increasing the number of strings in the insulator
(*d*) none of the above

188. High voltages for transmitting power is economically available from
(*a*) d.c. currents
(*b*) a.c. currents
(*c*) carrier currents
(*d*) none of the above

189. High voltage is primarily used, for long distance power transmission, to
(*a*) reduce the time of transmission
(*b*) reduce the transmission losses
(*c*) make the system reliable
(*d*) none of the above

190. By using bundle conductors, the critical voltage for the formation of corona will
(*a*) remain same
(*b*) decrease
(*c*) increase
(*d*) not occur

191. If the voltage is increased x times, the size of the conductor would be
(*a*) reduced to $1/x^2$ times
(*b*) reduced to 1/x times
(*c*) increased x times
(*d*) increased to x^2 times
(*e*) none of the above

192. The colour of the neutral of three-core flexible cable is
(*a*) blue
(*b*) brown
(*c*) red
(*d*) black

193. In the cablessheaths are used to
(*a*) prevent the moisture from entering the cable

(*b*) provide strength to the cable
(*c*) provide proper insulation
(*d*) none of the above

194. The charging current in the cables
(*a*) leads the voltage by 180^o
(*b*) leads the voltageby 90^o
(*c*) lags the voltage by 90^o
(*d*) lags the voltage by 180^o

195. Ground wire is used to
(*a*) avoid overloading
(*b*) give the support to the tower
(*c*) give good regulation
(*d*) connect a circuit conductor or other device to an earth-plate

196. Earthing is necessary to give protection against
(*a*) danger of electric shock
(*b*) voltage fluctuation
(*c*) overloading
(*d*) high temperature of the conductors

197. Resistance grounding isused for voltage between
(*a*) 3.3kV to 11 kV
(*b*) 11 kV to 33 kV
(*c*) 33 kV to 66 kV
(*d*) none of the above

198. Solid grounding is adopted for voltages below
(*a*) 100 V
(*b*) 200 V
(*c*) 400 V
(*d*) 660 V

199. The size of the earth wire is determined by
(*a*) the atmospheric conditions
(*b*) the voltage of the service wires
(*c*) the ampere capacity of the service wires
(*d*) none of the above

200. Transmission lines link
(*a*) generating station to receiving and station
(*b*) receiving and station to distribution transformer
(*c*) distribution transformer to consumer premises
(*d*) service points to consumer premises
(*e*) none of the above

ANSWERS

1. (*b*)	**2.** (*d*)	**3.** (*a*)	**4.** (*c*)	**5.** (*b*)	**6.** (*d*)	**7.** (*a*)	**8.** (*d*)	**9.** (*d*)	**10.** (*d*)
11. (*c*)	**12.** (*b*)	**13.** (*d*)	**14.** (*d*)	**15.** (*a*)	**16.** (*c*)	**17.** (*d*)	**18.** (*d*)	**19.** (*c*)	**20.** (*d*)
21. (*d*)	**22.** (*b*)	**23.** (*b*)	**24.** (*a*)	**25.** (*a*)	**26.** (*c*)	**27.** (*d*)	**28.** (*d*)	**29.** (*d*)	**30.** (*d*)
31. (*d*)	**32.** (*e*)	**33.** (*a*)	**34.** (*a*)	**35.** (*e*)	**36.** (*d*)	**37.** (*d*)	**38.** (*c*)	**39.** (*b*)	**40.** (*c*)
41. (*b*)	**42.** (*a*)	**43.** (*d*)	**44.** (*b*)	**45.** (*b*)	**46.** (*b*)	**47.** (*b*)	**48.** (*c*)	**49.** (*d*)	**50.** (*b*)
51. (*b*)	**52.** (*a*)	**53.** (*c*)	**54.** (*c*)	**55.** (*c*)	**56.** (*d*)	**57.** (*c*)	**58.** (*b*)	**59.** (*a*)	**60.** (*a*)
61. (*b*)	**62.** (*c*)	**63.** (*b*)	**64.** (*a*)	**65.** (*c*)	**66.** (*d*)	**67.** (*d*)	**68.** (*d*)	**69.** (*d*)	**70.** (*b*)
71. (*c*)	**72.** (*c*)	**73.** (*d*)	**74.** (*a*)	**75.** (*b*)	**76.** (*c*)	**77.** (*d*)	**78.** (*d*)	**79.** (*b*)	**80.** (*b*)
81. (*d*)	**82.** (*d*)	**83.** (*d*)	**84.** (*b*)	**85.** (*b*)	**86.** (*a*)	**87.** (*d*)	**88.** (*c*)	**89.** (*c*)	**90.** (*a*)
91. (*d*)	**92.** (*b*)	**93.** (*a*)	**94.** (*b*)	**95.** (*d*)	**96.** (*b*)	**97.** (*b*)	**98.** (*d*)	**99.** (*d*)	**100.** (*a*)
101. (*a*)	**102.** (*c*)	**103.** (*c*)	**104.** (*c*)	**105.** (*b*)	**106.** (*b*)	**107.** (*b*)	**108.** (*c*)	**109.** (*c*)	**110.** (*b*)
111. (*a*)	**112.** (*a*)	**113.** (*a*)	**114.** (*b*)	**115.** (*b*)	**116.** (*a*)	**117.** (*a*)	**118.** (*b*)	**119.** (*d*)	**120.** (*d*)
121. (*d*)	**122.** (*d*)	**123.** (*a*)	**124.** (*c*)	**125.** (*a*)	**126.** (*a*)	**127.** (*a*)	**128.** (*c*)	**129.** (*a*)	**130.** (*d*)
131. (*b*)	**132.** (*b*)	**133.** (*a*)	**134.** (*b*)	**135.** (*b*)	**136.** (*d*)	**137.** (*c*)	**138.** (*a*)	**139.** (*b*)	**140.** (*c*)
141. (*d*)	**142.** (*b*)	**143.** (*d*)	**144.** (*c*)	**145.** (*c*)	**146.** (*a*)	**147.** (*d*)	**148.** (*d*)	**149.** (*b*)	**150.** (*d*)
151. (*a*)	**152.** (*c*)	**153.** (*d*)	**154.** (*b*)	**155.** (*a*)	**156.** (*c*)	**157.** (*c*)	**158.** (*a*)	**159.** (*a*)	**160.** (*d*)
161. (*c*)	**162.** (*c*)	**163.** (*c*)	**164.** (*a*)	**165.** (*d*)	**166.** (*d*)	**167.** (*d*)	**168.** (*a*)	**169.** (*a*)	**170.** (*b*)
171. (*a*)	**172.** (*a*)	**173.** (*d*)	**174.** (*a*)	**175.** (*b*)	**176.** (*a*)	**177.** (*b*)	**178.** (*a*)	**179.** (*a*)	**180.** (*b*)
181. (*b*)	**182.** (*a*)	**183.** (*c*)	**184.** (*b*)	**185.** (*d*)	**186.** (*b*)	**187.** (*a*)	**188.** (*b*)	**189.** (*b*)	**190.** (*c*)
191. (*a*)	**192.** (*a*)	**193.** (*a*)	**194.** (*b*)	**195.** (*d*)	**196.** (*a*)	**197.** (*a*)	**198.** (*d*)	**199.** (*c*)	**200.** (*a*)

GO To FIRST

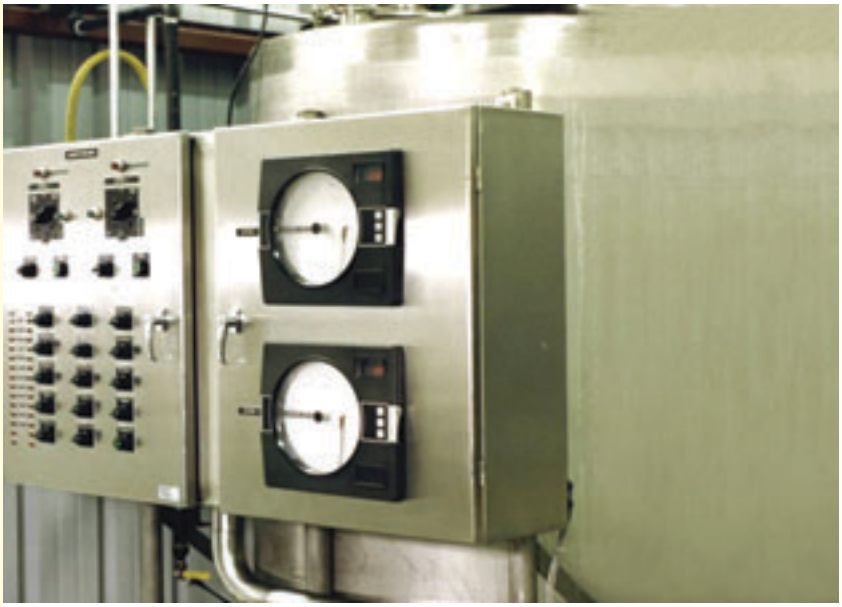
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CHAPTER 42

DISTRIBUTION AUTOMATION

Learning Objectives

- Introduction
- Need Based Energy Management (NBEM)
- Advatanges of NBEM
- Automated System
- Sectionalizing Switches
- Remote Terminal Units (RTU's)
- Data Acquisition System (DAS)
- Communication Interface
- Radio Communication
- Machine Interface
- A Typical SCADA System
- Load Management in DMS Automated Distribution System
- Substation Automation
- Control System
- Protective System
- Feeder Automation
- Distribution Equipment
- Automation Equipment
- Consumer Side Automation
- Energy Auditing
- Reduced Line Loss
- Power Quality
- Deferred Capital Expenses
- Energy Cost Reduction
- Optimal Energy Use



An automated electric distribution system ensures that the power is transmitted efficiently and without interruptions. Faults in transmission are identified and rectified with minimum human intervention

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42.1. Introduction

The power industry may seem to lack competition. This thought arises because each power company operates in a geographic region not served by other companies. Favorable electric rates are a compelling factor in the location of an industry, although this factor is much less important in times when costs are rising rapidly and rates charged for power are uncertain than in periods of stable economic conditions. Regulation of rates by State Electricity Boards, however, places constant pressure on companies to achieve maximum economy and earn a reasonable profit in the face of advancing costs of production.

Power shortage in India is endemic. Despite heavy investments in the power sector and additional power generation every year, there appears to be no chance of the shortage syndrome easing-up in the near future. Barring one or two small States with hydro-potential, all other States and Union Territories have perpetual power deficits. There seems to be no respite from these crippling deficits as seen from the figures that are being published from time to time.

In power sector, it has almost become a ritual to insistently clamor for more generation. This school of thought which is predominant majority, believes in classical concept of load management which emphasizes continuous and copious supply of energy to all sectors of consumers at all times. This ideal proposition could be achieved, only if unlimited resources at our command are available. But in actual practice, these resources are not only limited but scarce. These scarce resources are to be distributed throughout the country uniformly and called for immediate attention. A demand based energy management would therefore only result in shortage syndrome repeating itself endlessly since the ever-growing 'demand' could never be met fully and satisfactorily. A better alternative would be to replace the Demand Based Energy Management with a Need Based one.

42.2. Need Based Energy Management (NBEM)

In power sector, there is a distinct difference between 'demand' and 'need'. Consumers of electric power could be classified into five broad categories: the industrial users, agricultural sector, commercial organizations, domestic consumers and essential services. Industrial users could be further sub-divided as shift based industries and continuous process industries. Agricultural sector would include irrigation tubewells and rural industries. Out of these several groups and sub-groups, only three - viz - continuous process industries, domestic consumers and essential services need power round the clock, others may demand power for 24 hours of the day, but they don't need it.

A Need Based Energy Management would :

(i) Identify the needs of various consumers
(ii) Forecast the generation requirement based on the need
(iii) Plan power generation as per forecast
(iv) Laydown a suitable transmission and distribution network
(v) Regulate distribution as per need
(vi) Monitor matching of need with supply

The greatest bugbear of NBEM is distribution network. Consumers cannot be supplied power as per their need since the distribution network is not designed for that purpose. Present day power distribution network is full of constraints and is clumsy to the core. The ills of the system are many – poor reliability, high line losses, low voltage profiles, overloading of transformers, poor maintenance, absence of conservation measures, stealing of power, haphazard layouts, whimsical load connections, inadequate clearances etc. With a single feeder connected to all types of consumers, there is no load discipline and the distribution network is exposed to several malpractices and distortions. Generation of power also suffers and as a result, engineers incharge of power generation in state electricity boards are not enthusiastic about optimizing power generation because they feel that their efforts are wasted due to the chaotic and sub-optimal distribution system. So if power problem is to be solved, distribution holds the key.

Figure 42.1 shows utility system with SCADA (Supervisory Control and Data Acquisition) and Distribution Automation.

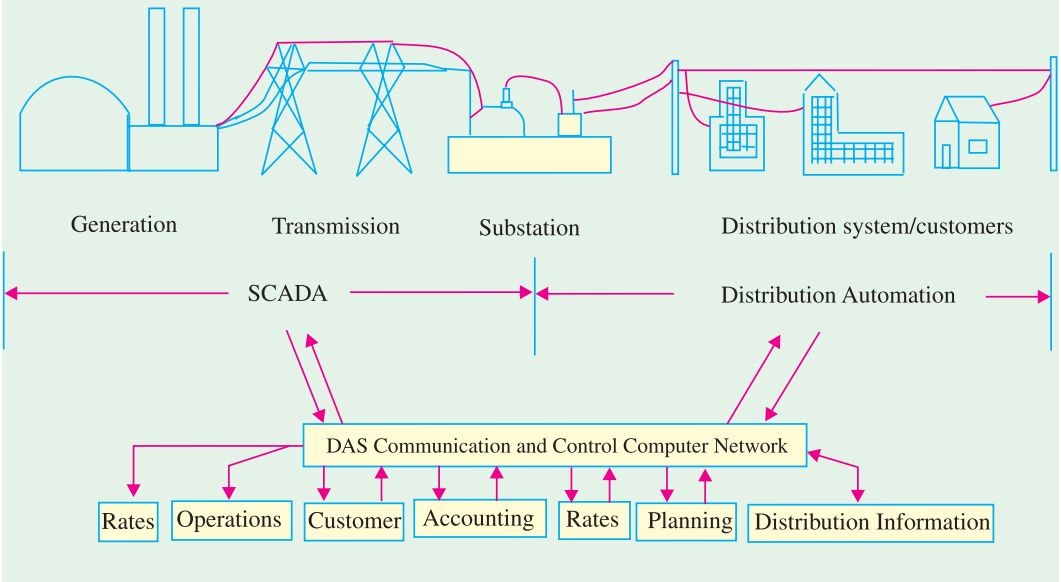


Fig. 42.1 Utility systems with SCADA and Distribution Automation

42.2.1. Advantages of NBEM

The distinct advantages of NBEM are:

(1) It ensures high reliability of supply to consumers meeting the specific demand effectively for periods of actual requirements.

(2) The system losses can be substantially reduced since line and equipment does not get overloaded at any point of time.

(3) The voltage profile at all levels is improved thus safeguarding the customer's equipment from losing their efficiency and performance at low voltage.

(4) The scheme facilitates the adoption of energy conservation schemes and energy audit policy.

(5) Power cuts are reduced and quality of power improves leading to better industrial and agricultural health and productivity.

42.3. Conventional Distribution Network

The power system network, which generally concerns (or which is in close proximity of) the common man, is the distribution network of 11 KV lines or feeders downstream of the 33 KV substation. Each 11 KV feeder, which emanates from the 33 KV substation branches further into several subsidiary 11 KV feeders to carry power close to the load points, where it is further step-down to either 230 V or 415 V.

The present structure of the distribution feeders doesn't support quick fault detection, isolation of faulty region and restoration of supply to the maximum outage area, which is healthy. In the absence of switches at different points in the distribution network, it is not possible to isolate certain loads for load shedding as and when required. The only option available in the present distribution network is the circuit breaker (one each for every main 11 KV feeder) at the 33 KV substation. However, these circuit breakers are actually provided as a means of protection to completely isolate the downstream network in the event of a fault. Using this as a tool for load management is not desirable, as it disconnects the power supply to a very large segment of consumers. Clearly, there is a need to put in place a system that can achieve a finer resolution in load management.

In the event of a fault on any feeder section downstream, the circuit breaker at the 33 KV substation trips (opens). As a result, there is a blackout over a large section of the distribution network. If the faulty segment could be precisely identified, it would be possible to substantially reduce the blackout area, by re-routing the power to the healthy feeder segments through the operation of sectionalizing switches, placed at strategic locations in various feeder segments.

Thus, lack of information at the base station (33 KV sub-station) of the loading and health status of the 11 KV/ 415 V distribution transformers and associated feeders is one primary cause of inefficient power distribution. Also, due to absence of monitoring, overloading occurs, which results in low voltage at the customer end and increases the risk of frequent breakdowns of transformers and feeders.

42.4. Automated System

The inefficient operation of the conventional distribution system can be mainly attributed to the frequent occurrence of faults and the uncertainty in detecting them. To enhance the electrical power distribution reliability, sectionalizing switches are provided along the way of primary feeders. Thus, by adding fault detecting relays to the sectionalizing switches along with circuit breaker and protective relays at the distribution substations, the system is capable to determine fault sections. To reduce the service disruption area in case of power failure, normally open (NO) sectionalizing switches called as route (tie) switches are used for supply restoration process. The operation of these switches is controlled from the control center through the Remote Terminal Units (RTU'S).

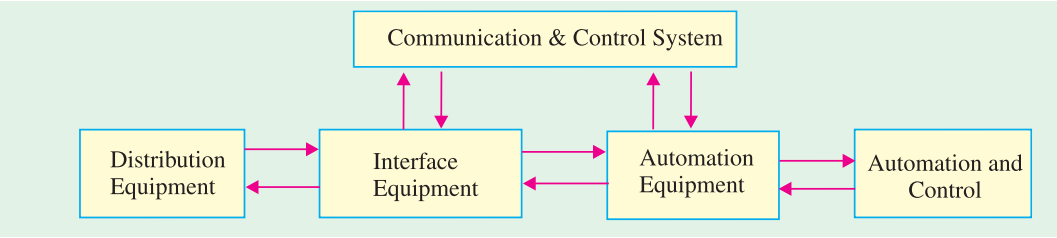


Fig. 42.2. Interconnection of distribution, control and communication system.

In distribution automation (DA) system, the various quantities (*e.g.*, voltage, current, switch status, temperature and oil level, etc.) are recorded in the field at the distribution transformers and feeders, using a data acquisition device called Remote Terminal Unit. These quantities are transmitted on-line to the base station through a communication media. The acquired data is processed at the base station for display at multiple computers through a Graphic user interface (GUI). In the event of a system quantity crossing a pre-defined threshold, an alarm is generated for operator intervention. Any control action, for opening or closing of the switch or circuit breaker, is initiated by the operator and transmitted from the 33 KV base station through the communication channel to the remote terminal unit associated with the corresponding switch or CB. The desired switching takes place and the action is acknowledged back to the operator.

Interconnection of distribution, control and communication system is shown in Figure 42.2.

All the above mentioned functions of data collection, data transmission, data monitoring, data processing, man-machine interface, etc. are realized using an integrated distribution SCADA (Supervisory Control And Data Acquisition) system.

The implementation of SCADA system in the electric utility involves the installation of following units :

1. Sectionalizing Switches
2. Remote Terminal Unit
3. Data Acquisition System
4. Communication Interface
5. Control Computer

42.5. Sectionalizing Switches

These sectionalizing switches are basically either air-brake switches or Load Break Switches (LBS) or Moulded Case Circuit Breaker (MCCB). These are remotely operable switches designed specifically for 11 KV and 415 V feeders. However, switches of appropriate rating corresponding to the rated feeder current can also be chosen.

Generally, 11 KV Vacuum break line sectionalizers are installed away from the substation and on the pole. A fault-indicating device is provided for location of fault at any section. To avoid opening of switches due to transient voltage drop or any other mal-operation, generally, 1.5 - 3 seconds delay is provided. Benefits obtained from installing these sectionalizing switches include :

1. Immediate isolation with indication of faulty section
2. Immediate restoration of supply to the healthy section
3. No down time for transient faults
4. Improvement of revenue due to lesser outage of section and low down time
5. Requires less man-power in the system and
6. Better Reliability of power supply

42.6. Remote Terminal Units (RTU's)

A typical SCADA system consists of remote terminal units, to record ad check, measured values and meter readings, before transmitting them to control station and in the opposite direction, to transmit commands, set point values and other signals to the switchgear and actuator.

The functions of RTU's can be given as following :

(*a*) Acquisition of information such as measured values, signals, meter readings, etc.

(*b*) Transmit commands or instructions (binary plus type or continuous), set points, control variables, etc. including their monitoring as a function of time.

(*c*) Recognition of changes in signal input states plus time data allocation and sequential recording of events by the master control station.

(*d*) Processing of information transmitted to and from the telecommunication equipment such as data compression, coding and protection.

(*e*) Communication with master control station.

A typical architecture RTU interfacing is shown in Fig 42.3.

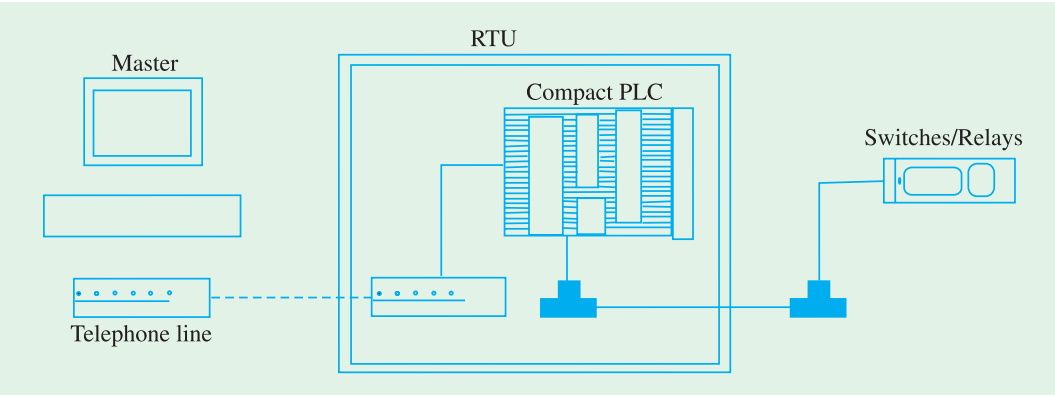


Fig. 42.3. Typical Architecture of RTU Interfacing

42.7. Data Acquisition System (DAS)

The data regarding the complete network consists of electrical and mechanical variables, on / off states, analog quantities, digital quantities, changes of state, sequence of events, time of occurrence

and several other data, which the control room operators will like to know.

Data is acquired by means of current transformers (CTs), potential transformers (PTs), transducers and other forms of collecting information. Transducers convert the data into electrical form to enable easy measurement and transmission. Data may be collected at low level or high level. Then it is amplified in signal amplifier and conditioned in data signal conditioner. The data is transmitted from the process location to the control room and from the control room to the control center.

The large number of electrical, mechanical and other data are scanned at required interval, recorded and displayed as per the requirement. Some of the data is converted from analog to digital form through A/D (Analogue to Digital) converters.

The data loggers perform the following functions :

1. Input Scanning
2. Signal Amplification and A/D Conversion
3. Display, Recording and Processing

The input scanner is generally a multi-way device, which selects input signals at regular periodic intervals in a sequence decided by the rate of change of input data. Slow varying quantities are scanned with a lower period of time-intervals. Output of scanner is given to A/D converter. Digital signals are obtained through DSP by micro-controllers or the control computer. This acquired signal can be displayed, recorded and processed for appropriate actions to be performed later.

42.8. Communication Interface

A good data communication system to transmit the control commands and data between Distribution Control Centre (DCC) and a large number of device remotely located on the distribution network is a pre-requisite for the good performance of Distribution Automation System (DAS). The communication requirements of each DAS is unique, depending upon the Distribution Automation functions selected for the implementation. A wide range of communication technologies are available to perform the tasks of DAS. The choice of communication technology also has a big impact on the cost of DAS.

RTU's communicate with the control room through a communication interface, which could be any of the following :

(*a*) Power line carrier communication (PLCC) :

Each end of the transmission line is provided with identical carrier equipment in the frequency range of 30 to 500 kHz. The high frequency signals are transmitted through power lines. The carrier current equipment compromises the coupling capacitor and the tuning circuit.

(*b*) Fibre optics data communication :

Application of fibre optic communication is presently in infant stage and has a vast scope due to freedom from electromagnetic interference and enormous data handling capacity of a single pair of optical fibre. The information is exchanged in the form of digitised light signals transmitted through optical fibres.

Fibre optics, with its explicit downward cost trend in terms of product as well as installation and maintenance costs has become a widely accepted choice, as it offers both technical and commercial advantages over conventional systems that use metallic cables and radio links. The communication basically consists of a transmitter and a receiver for information signals coming from the user's device which is connected through copper wire to the switching center or exchange, where it is changed into a digital signal like 1s or 0 s for easy handling. The signal is then transferred to the transmitter. In the transmitter, the information signal which is electrical, drives an optical source : Laser or a light emitting diode (LED), which in turn, optically modulates the information signal,

which gets coupled into the optical fibre. The receiver located at the other side of the link detects the original signal and demodulates or converts back the optical signal to the original information signal (Electrical). The signal is then connected through copper wires to the switching device or exchange for selection and connection to the proper user or user device. This has advantage of high data rate (9600 bits and much more) and immunity from noise.

(*c*) Radio communication :

Radio communication utilizes frequency bands between 85 MHz to 13 GHz, Point to point radio links, Multi terminal radio communication facility, Limited area radio scheme, Mobile radio sets, Emergency radio communication etc. are the types of radio communication facilities used in electrical power systems.



Nowadays even telephone connections are via wireless systems. The figure shows the transmission tower from where radio waves can be transmitted to other stations and satellites

(*d*) Public telephone communication :

Dial-up and dedicated leased telephone lines are often used for Distribution Automation. The dial-up lines are suitable for infrequent data trans-mission. The leased line are suitable for continuous communication but are expensive. The reliability of communication varies greatly and is dependent upon the telephone company. Telephone communication service through packet switching network, cellular radio are viable and may have the advantages of providing services in other-wise inaccessible places.

(*e*) Satellite communication :

A satellite communi-cation system using very small aperture terminal (VSAT) is suitable for DAS. VSAT is a point to multi-point star network like TDMA. It consists of one single Hub and number of remote Personal Earth Stations (PES). The communication between transponder and Hub is TDM access and between Hub and PES is TDMA access. The communi-cation system between Hub remote PES is through two separate radio links. The link from remote PES to Hub is called inroute and from Hub to the remote PES is called as outroute. No end user transmission either originates or terminates at the satellite. In India extended 'C' band with up link in the 6.15/ 6.815 and down link in the 4.09/ 4.59 KHz range is used. Fig. 42.4 shows schematic diagram of VSAT network.

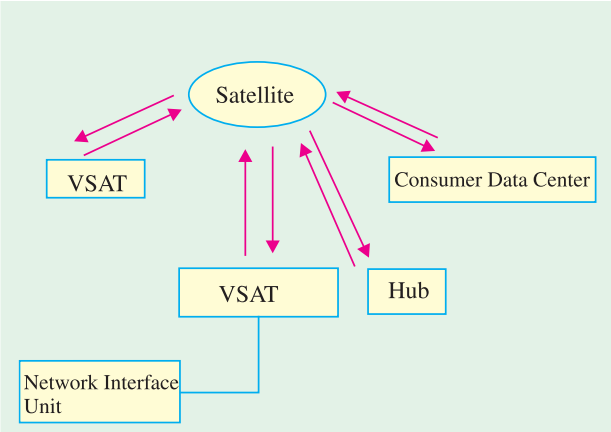
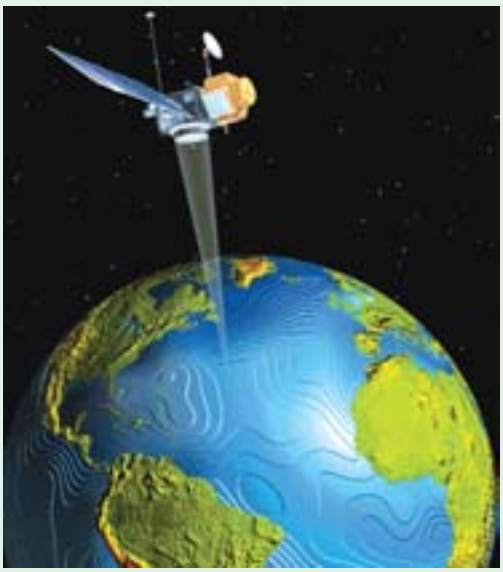


Fig. 42.4 Schematic diagram of VSAT network



A satellite helps to develop instant communication links with the stations spread across the globe

(f) Polling scheme :

SCADA systems intended for electric system operations almost universally use a polling scheme between the central master and individual RTUs.

The master station controls all activities and RTUs respond only to polling requests. Fig. 42.5 illustrates the most common communication arrangement. Multiple two or four wire telephone grade circuits radiate from the master. The media for these circuits may be leased telephone circuits from a common carrier, private microwave, fibre optic cable systems, two-way cable TV, power line carrier, or even satellite. Polling and command requests and RTU responses are time multiplexed on each circuit. Each circuit terminating at the master station is independently serviced on an asynchronous basis by the master station. The most commonly used information rates is 1200 bits/sec. using asynchronous byte-oriented message formats.

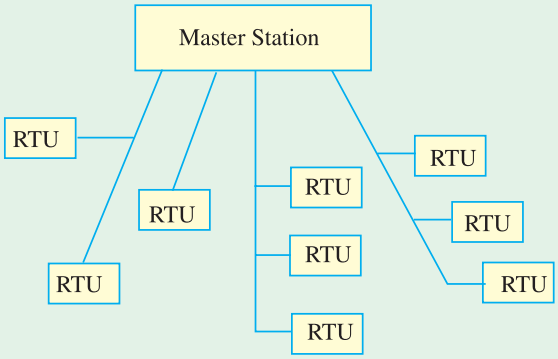


Fig. 42.5. Typical multidrop communication system

42.9. Distribution SCADA

Distribution SCADA involves collecting and analyzing information to take decisions, implementing the appropriate decisions and then verifying whether the desired results are achieved. Fig 42.6 gives flow diagram of the SCADA functions.

Data acquisition in an electric utility SCADA system concentrates on the power system performance quantities like bus line volts, transformer currents, real and reactive power flow, C.B. status (circuit breaker status), isolator status and secondary quantities such as transformer temperature, insulating gas pressure, tank oil levels, flow levels etc. Often transformer tap positions, usual positions or other multiple position quantities are also transmitted in analog format.

The usual reason for installing supervisory control is to provide the system with sufficient information and control to operate the power system or some part of it in a safe, secure and economic manner.

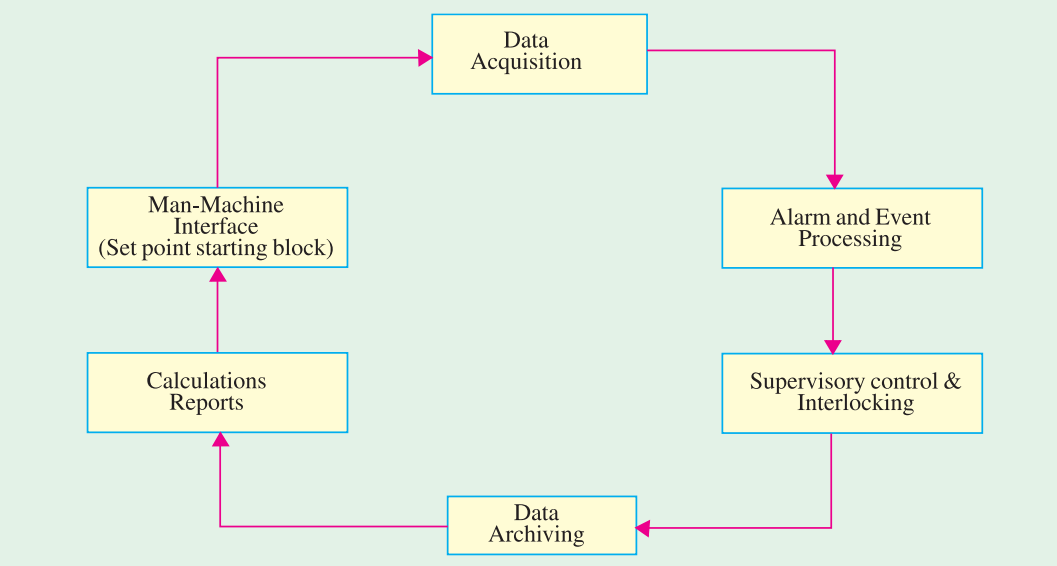


Fig. 42.6. SCADA functions

42.10. Man – Machine Interface

The implementation of SCADA system in an electric utility requires the installation of remote terminal units. RTU's are designed to acquire data and transfer the same to the master station through a communication link. They collect data from transducers, transmitters, connect input from equipment/ instruments, meter readings etc., performs analogue/ digital conversions, check data scaling and corrections (typically at I/O levels) performs preprocessing tasks and send/ receive messages from/ to master stations via interfaces. Fig 42.7 shows the flow diagram for man-machine interface.

Man-Machine Interface (MMI) is the interface between man and technology for control of the technical process. The computer system at master control centre or central control room integrates with RTU over the communication link with its transmission protocol, acquires the remote substation or distribution transformer/ feeder data and transfers the same to the computer system for man-machine interface.

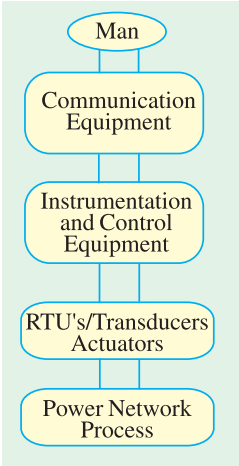


Fig. 42.7. Man - Machine Interface

42.11. A Typical SCADA System

A typical SCADA system may therefore comprises hardware and the software.
The hardware may include :

1. User friendly man-machine interface
2. Work station
3. Service having a particular function
4. Communication sub system
5. RTU's

All the above components communicate with each other through a local area network (LAN) with internationally standardised protocols. A flexible redundancy is provided, assigning hot standby server to any server fulfilling time critical functions.

A software must offer the following features :

1. Use of high level programming language
2. Modular structure with clear and standardised interfaces between software modules and the database
3. Inter module communication only via a 'soft bus' independent of their computer residency
4. Easy addition of further application programs
5. Comfortable on-line diagnostics, development tools and file editors.

Artificial Intelligence (AI) plays a pivotal role in the development of software, rule based experts systems, logic based systems like fuzzy logic, neural networks etc. fall under this category. Few applications of the recent times are stated on the next page :

1. Application of expert system to power system restoration using flow control rules, energisation rules, line switching rules, load shedding rules, voltage correction rules etc.
2. Knowledge-based expert systems for fault location and diagnosis on electric power distribution feeders.
3. Fuzzy based logic for reconfiguration of distribution networks.

The application of model based expert system results in an intelligent power distribution system with learning self organizing and diagnostic capabilities. In addition, it advises the control center operations in cases of emergency. Distribution automated systems can therefore be operated in two operational modes: Online and Off-line. The Online operation permits faulty equipment identification, restoration planning, network maintenance scheduling and emergency operations. The Off-line simulator mode allows the user to verify the validity of acquired knowledge by setting an imaginary fault on the system and provides a convenient way of training the inexperienced operator.

42.12. Distribution Automation

Distribution Automation functions provide a means to more effectively manage minute by minute continuous operation of a distribution system. Distribution Automation provides a tool to achieve a maximum utilization of the utility's physical plant and to provide the highest quality of service to its customers. Obviously, both the utility and its customers are beneficiaries of successful Distribution Automation.

Distribution Automation systems are modular, hence they may be implemented in stages, commencing from a modest degree of capability and complexity and growing as necessary to achieve tangible and intangible economic benefits. For example, a utility may start with a limited capability SCADA System for sub station monitoring and control, extend this to the feeders and finally implement a complete integration of automation functions. Systems implemented in this fashion must be designed to accommodate future expansion.

Distribution Automation System offers an integrated 'Distributed Management System' (DMS). The functions of DMS are shown in Fig 42.8. As in any other SCADA system, Distribution SCADA involves

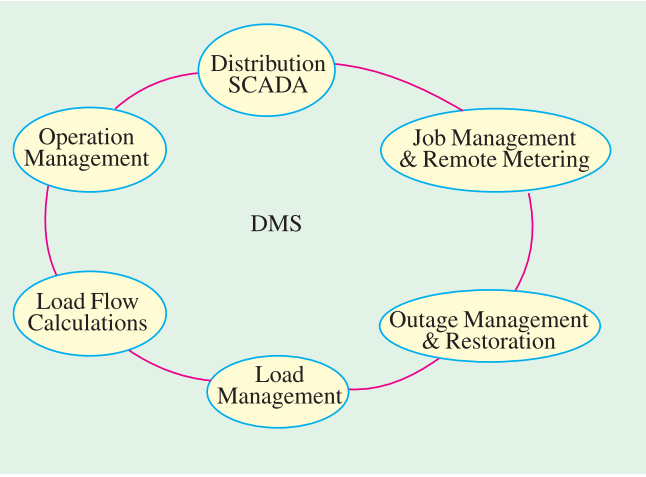


Fig. 42.8. Functions of distributed management system

collecting and analyzing information to take decisions, implementing the appropriate decisions and then verifying that the desired results are achieved. Operation management supports the analysis of distribution network. It models the load profile to present state of the network. Load flow calculations estimate voltage levels and power flows at each feeder. Job management makes switching order handling easier and work protection tagging ensures the safety of repair crew on duty. Outage management and service restoration facilitates to reduce outage time thereby increasing the reliability of the supply. Remote metering provides for the appropriate selection of energy registers where time-of-use rates are in effect, thus improving energy metering services to be more accurate and more frequent.

42.13. Load Management in DMS

This involves controlling system loads by remote control of individual customer loads. Control includes suppressing or biasing automatic control of cyclic loads, as well as load switching. Load Management can also be effected by inducing customers to suppress loads during utility selected daily periods by means of time-of-day rate incentives. Distribution Automation provides the control and monitoring ability required for both the load management scenarios -viz - direct control of customers loads and the monitoring necessary to verify that programmed levels are achieved. Execution of load management provides several possible benefits to the utility and its customers. Maximum utilization of the existing distribution system can lead to deferrals of capital expenditure. This is achieved by :

- **(*i*)** Shaping the daily (or monthly, annual) load characteristic by suppressing loads at peak times and encouraging energy consumption at off-peak times.
- **(*ii*)** Minimizing the requirement for more costly generation or power purchases by suppressing loads.
- **(*iii*)** Relieving the consequences of significant loss of generation or similar emergency situations by suppressing load.
- **(*iv*)** Reducing cold load pick-up during re-energization of circuits using devices with cold load pick-up features.

The effectiveness of direct control of customer loads is obviously enhanced by selecting the larger and more significant customer loads. These include electric space and water heating, air conditioning, washing machines, dryers and others of comparable magnitude.

More sophisticated customer's activated load management strategies are under study, taking advantage of the capabilities of Distribution Automation and of customers installed load control PC. With such an arrangement, the utility could vary rates throughout the day, reflecting actual generation costs and any system supply capability constraints and broadcast this information to all customer's PC. Each PC could then control it's loads to confirm to a customer selected cost bias. This is sometimes called as 'spot pricing'.

42.14. Automated Distribution System

In the conventional distribution system the abnormal conditions are detected manually which costs lots of time and money to both consumers and power industry. In order to maintain high service quality and reliability and minimize loss in revenues, automation is required. Automation may be applied to the power distribution system so that problems on the distribution network may be detected and operated upon so as to minimize the outage time.

The equipments (either fixed wired or /and programmable), which are used for distribution automation, include :

1. Data collection equipment
2. Data transmission (telemetering) equipment

3. Data monitoring equipment
4. Data processing equipment
5. Man-Machine interface.

All the above equipments are integrated through distribution SCADA system.

Distribution SCADA involves collecting and analyzing information to take decisions and then implementing them and finally verifying whether desired results are achieved .

The implementation of SCADA system in the electric utility involves the installation of following units :

(*a*) Data acquisition unit : The basic variables (data) required for control, monitoring and protection include current, voltage, frequency, time, power factor, reactive power and real power. The data may be tapped in analog or digital form as required. For data collection purpose CT's and PT's are used. Transducers may be needed to convert the data into electrical form to enable easy measurement and transmission .The data is amplified in signal amplifier and conditioned in data signal conditioner. The data is then transmitted from the process location to the transmission room. In the control room data processing and data logging are performed which includes input scanning at required intervals, recording, programming and display by microprocessor, PLC, PC etc.

(*b*) Remote terminal unit (RTU) : RTU is used to record and check signals, measured values and meter readings before transmitting them to control station and in the opposite direction to transmit commands, set point values and other signals to the switchgear and actuator.

(*c*) Communication unit : A good data communication system to transmit the control command and data between distribution control centre and large number of remotely located devices is a prerequisite for a good performance of DA system. Wide range of communication technologies are available to perform the task of DA system which include public telephone communication (leased or dedicated line), power line carrier communication (PLCC), UHF MARS (ultra high frequency multi address radio system) and VHF Radio, as discussed earlier.

Distribution Automation can be broadly classified as :

1. Substation Automation.
2. Feeder Automation.
3. Consumer side Automation

42.15. Substation Automation

Substation automation is the cutting edge technology in electrical engineering. It means having an intelligent, interactive power distribution network including :

1. Increased performance and reliability of electrical protection.
2. Advanced disturbance and event recording capabilities, aiding in detailed electrical fault analysis.
3. Display of real time substation information in a control center.
4. Remote switching and advanced supervisory control.
5. Increased integrity and safety of the electrical power network including advanced interlocking functions.
6. Advanced automation functions like intelligent load-shedding.

42.15.1 Requirements

The general requirements for selecting an automation system while designing a new substation are :

1. The system should be adaptable to any vendor's hardware.
2. It should incorporate distributed architecture to minimize wiring.
3. It should be flexible and easily set up by the user.

4. The substation unit should include a computer to store data and pre-process information.

42.15.2. Functioning

Bus voltages and frequencies, line loading, transformer loading, power factor, real and reactive power flow, temperature, etc. are the basic variables related with substation control and instrumentation. The various supervision, control and protection functions are performed in the substation control room. The relays, protection and control panels are installed in the controlled room. These panels along with PC aids in automatic operation of various circuit breakers, tap changers, autoreclosers, sectionalizing switches and other devices during faults and abnormal conditions. Thus, primary control in substation is of two categories.

1. Normal routine operation by operator's command with the aid of analog and digital control system.
2. Automatic operation by action of protective relays , control systems and PC.

The automated substation functioning can be treated as integration of two subsystems, as discussed below :

(*a*) Control System : The task of control system in a substation includes data collection, scanning, event reporting and recording; voltage control, power control, frequency control, other automatic and semiautomatic controls etc. The various switching actions like auto reclosing of line circuit breakers, operation of sectionalizing switches, on-load tap changers are performed by remote command from control room. The other sequential operations like load transfer from one bus to another, load shedding etc. are also taken care by control center.

(*b*) Protective System : The task of protective system includes sensing abnormal condition, annunciation of abnormal condition, alarm, automatic tripping, back-up protection, protective signaling.

The above two systems work in close co-operation with each other. Most of the above functions *i.e.* automatic switching sequences, sequential event recording, compiling of energy and other reports, etc. are integrated in software in the substation computer. This software is of modular design, which facilitates addition of new functions.

The communication between circuit breakers, autoreclosers and sectionalizing switches in the primary and secondary distribution circuits located in the field and the PC in distribution substation control room is through radio telecontrol or fibre optic channel or power line carrier channel as is feasible.

42.16. Feeder Automation

Automating the fault diagnosis and supply restoration process significantly reduces the duration of service interruptions. The key objective behind automating the service restoration process is to restore supply to maximum loads in out-of-service zones. This is achieved by reconfiguring the network such that the constraints of the system are not violated. Providing timely restoration of supply to outage areas of the feeder enhances the value of service to customers and retains the revenue for the power industry.

The system data consisting of the status signals and electrical analog quantities are obtained using a suitable Data Acquisition System and processed by the control computer for typical functions of fault detection, isolation and network reconfiguration for supply restoration. The equipments normally required in Feeder Automation are discussed below :

(*a*) Distribution equipment :

This includes transformers, breakers, load break switches and motor operators, power reclosures, voltage regulators, capacitor banks etc.

(b) Interface equipment :

Interface equipment is required for the purpose of data acquisition and control. Potential transformers, current transformer, watt, var meter and voltage transducers, relays are some examples.

(c) Automation equipment :

Automation Equipment includes a DAS, communication equipment, substation remote terminal units (RTU) and distribution feeder RTU, current-to-voltage converters, etc.

42.17. Consumer Side Automation

Consumer side automation is very important for a distribution company as almost 80% of all the losses are taking place on distribution side alone. It is needed to evaluate the performance of a specific area in the distribution system and judge the overall losses.

42.17.1. Energy Auditing

Energy audit has a very wide range of applications in the electrical systems. It means overall accounting of energy generated, transmitted and distributed. As far as distribution side is concerned energy audit would mean overall accounting of energy supplied to and utilized by the consumers. Energy audit can also be used for rethinking about billing strategy, usefulness of an individual subscriber, loading of a given feeder etc.

Remote metering is used in energy auditing in which the energy used by a consumer is billed from a remote (distant) location without actually going to the place. In remote metering, the concept of TOD (time of day) metering can be introduced wherein the electronic meters at consumer's service entrance point are programmed to read the following meter readings on monthly basis.

1. KW hrs. consumed during calendar month by the consumer during low tariff and high tariff hours.
2. KVA maximum demand by the consumer during the calendar month (based on maximum demand lasting for 30 minutes duration).
3. Low tariff for off peak hour consumption.

These readings are telemetered to the control room for the purpose of monthly billing and cash collection through the various modes of communication available (- viz - Telephonic and wireless communication) depending on the load condition of the consumer. Tampering of energy - meters (if done) is also telemetered for taking action/penalty and disconnection of service.

Thus energy audit, though a very cumbersome and tedious job, can make a non-profitable business of distribution into a highly profitable one.

42.18. Advantages of Distribution Automation

More and more electric utilities are looking to distribution automation as an answer to the three major economic challenges facing the industry : the rising cost of adding generating capacity, increased saturation of existing distribution networks and greater sensitivity to customer service. Therefore, utilities that employ distribution automation expect both cost and service benefits. These benefits accumulate in areas that are related to investments, interruptions and customer service, as well as in areas related to operational cost savings, as given below :

(a) Reduced line loss :

The distribution substation is the electrical hub for the distribution network. A close coordination between the substation equipment, distribution feeders and associated equipment is necessary to increase system reliability. Volt/VAR control is addressed through expert algorithms which monitors and controls substation voltage devices in coordination with down-line voltage devices to reduce line loss and increase line throughout.

(b) Power quality :

Mitigation equipment is essential to maintain power quality over distribution feeders. The substation RTU in conjunction with power monitoring equipment on the feeders monitors, detects,

and corrects power-related problems before they occur, providing a greater level of customer satisfaction.

(*c*) Deferred capital expenses :

A preventive maintenance algorithm may be integrated into the system. The resulting ability to schedule maintenance, reduces labour costs, optimizes equipment use and extends equipment life.

(*d*) Energy cost reduction :

Real-time monitoring of power usage throughout the distribution feeder provides data allowing the end user to track his energy consumption patterns, allocate usage and assign accountability to first line supervisors and daily operating personnel to reduce overall costs.

(*e*) Optimal energy use :

Real-time control, as part of a fully-integrated, automated power management system, provides the ability to perform calculations to reduce demand charges. It also offers a load-shedding/ preservation algorithm to optimize utility and multiple power sources, integrating cost of power into the algorithm.

(*f*) Economic benefits :

Investment related benefits of distribution automation came from a more effective use of the system. Utilities are able to operate closer to the edge to the physical limits of their systems. Distribution automation makes this possible by providing increased availability of better data for planning, engineering and maintenance. Investment related benefits can be achieved by deferring addition of generation capacity, releasing transmission capacity and deferring the addition, replacement of distribution substation equipment. Features such as voltage/VAR control, data monitoring and logging and load management contribute to capital deferred benefits.

Distribution automation can provide a balance of both quantitative and qualitative benefits in the areas of interruption and customer service by automatically locating feeder faults, decreasing the time required to restore service to unfaulted feeder sections, and reducing costs associated with customer complaints.

(*g*) Improved reliability :

On the qualitative side, improved reliability adds perceived value for customer and reduce the number of complaints. Distribution automation features that provide interruption and customer service related benefits include load shedding and other automatic control functions.

Lower operating costs are another major benefits of distribution automation. Operating cost reduction are achieved through improved voltage profiles, controlled VAR flow, repairs and maintenance savings, generation fuel savings from reduced substation transformer load losses, reduced feeder primary and distribution transformer losses, load management and reduced spinning reserve requirements. In addition, data acquisition and processing and remote metering functions play a large role in reducing operating costs and should be considered an integral part of any distribution automation system.

Through real time operation, the control computer can locate the faults much faster and control the switches and reclosures to quickly reroute power and minimize the total time-out, thus increasing the system reliability.

(*h*) Compatibility :

Distribution automation spans many functional and product areas including computer systems, application software, RTUs, communication systems and metering products. No single vendor provides all the pieces. Therefore, in order to be able to supply a utility with a complete and integrated system, it is important for the supplier to have alliances and agreements with other vendors.

An effective distribution automation system combines complementary function and capabilities and require an architecture that is flexible or "open" so that it can accommodate products from different vendors. In addition, a distribution automation system often requires interfaces with existing system in order to allow migration and integration, still monitoring network security.

Tutorial Problem No. 42.1

1. Why we need Transmission System Interconnection? *(Nagpur University, Summer 2004)*
2. Describe different distribution automation methods and their advantages and disadvantages.

OBJECTIVE TESTS – 42

1. Need based energy management is better than demand based energy management
   (*a*) true (*b*) false
2. Round the clock power is required to
   (*a*) agricultural uses
   (*b*) shift based industries
   (*c*) commercial organisations
   (*d*) essential services
3. Need based energy management would include
   (*a*) plan power generation as per forecast
   (*b*) monitor matching of need with supply
   (*c*) identify the needs of various consumers
   (*d*) all of the above
4. Existing Distribution systems are
   (*a*) properly designed
   (*b*) chaotic
   (*c*) optimally operated
   (*d*) automated
5. Distribution systems are not radial
   (*a*) true (*b*) false
6. The major aim of distribution automation is to keep minimum area in dark for a minimum time
   (*a*) true (*b*) false
7. Distribution automation is used for
   (*a*) electrical quantities only
   (*b*) physical quantities only
   (*c*) both of the above
8. Sectionalizing switches are circuit breakers
   (*a*) true (*b*) false
9. RTU is a -
   (*a*) transmitter (*b*) receiver
   (*c*) trans-receiver
10. The main problem in implanting Distri-bution Automation in metros is -
    (*a*) RTUs (*b*) communication
    (*c*) D.A.S. (*d*) control computers
11. Distribution SCADA means –
    (*a*) data acquisition
    (*b*) man – machine interface
    (*c*) data archiving
    (*d*) all of the above
12. Now - a - days the trend is towards –
    (*a*) manned substation
    (*b*) unmanned substation
13. Artificial Intelligence plays a pivotal data in distribution automation
    (*a*) true (*b*) false
14. Distribution Automation means –
    (*a*) substation Automation
    (*b*) feeder Automation
    (*c*) consumer side Automation
    (*d*) all of the above
15. Distribution Automotive system are modular
    (*a*) true (*b*) false
16. Existing distribution systems are controlled
    (*a*) automatically
    (*b*) manually
17. Distribution Automotive systems operate in real time
    (*a*) true (*b*) false
18. Until recent years, much of the investment and technical exploitation was done in –
    (*a*) generation
    (*b*) transmission
    (*c*) generation and transmission
    (*d*) distribution
19. Demand supply gap in power sector is expected to grow by - - - - percent every year.
    (*a*) 5 (*b*) 10
    (*c*) 15 (*d*) 20
20. Major revenue losses in distribution system occur in –
    (*a*) domestic sector
    (*b*) industrial sector
    (*c*) agricultural sector
    (*d*) commercial sector

ANSWERS

1. (*a*) **2.** (*d*) **3.** (*d*) **4.** (*b*) **5.** (*b*) **6.** (*a*) **7.** (*c*) **8.** (*b*) **9.** (*c*) **10.** (*b*)
11. (*d*) **12.** (*b*) **13.** (*a*) **14.** (*d*) **15.** (*a*) **16.** (*b*) **17.** (*a*) **18.** (*c*) **19.** (*d*) **20.** (*c*)

GO To FIRST

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CHAPTER 43

ELECTRIC TRACTION

Learning Objectives

- General
- Traction System
- Direct Steam Engine Drive
- Advantages of Electric Traction
- Saving in High Grade Coal
- Disadvantages of Electric Traction
- System of Railway Electrification
- Three Phase Low-Frequency A.C. System
- Block Diagram of an AC Locomotive
- The Tramways
- Collector Gear for OHE
- Confusion Regarding Weight and Mass of Train
- Tractive Efforts for Propulsion of a Train
- Power Output from Driving Axles
- Energy Output from Driving Axles
- Specific Energy Output
- Evaluation of Specific Energy Output
- Energy Consumption
- Specific Energy Consumption
- Adhesive Weight
- Coefficient of Adhesion



In electric traction, driving force (or tractive force) is generated by electricity, using electric motors. Electric trains, trams, trolley, buses and battery run cars are some examples where electric traction is employed.

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43.1. General

By electric traction is meant locomotion in which the driving (or tractive) force is obtained from electric motors. It is used in electric trains, tramcars, trolley buses and diesel-electric vehicles etc. Electric traction has many advantages as compared to other non-electrical systems of traction including steam traction.

43.2. Traction Systems

Broadly speaking, all traction systems may be classified into two categories :

(*a*) non-electric traction systems

They do not involve the use of electrical energy at ***any stage***. Examples are : steam engine drive used in railways and internal-combustion-engine drive used for road transport.



The above picture shows a diesel train engine. These engines are now being rapidly replaced by electric engines

(*b*) electric traction systems

They involve the use of electric energy at some stage or the other. They may be further subdivided into two groups :

1. First group consists of self-contained vehicles or locomotives. Examples are : battery-electric drive and diesel-electric drive etc.
2. Second group consists of vehicles which receive electric power from a distribution network fed at suitable points from either central power stations or suitably-spaced sub-stations. Examples are : railway electric locomotive fed from overhead ac supply and tramways and trolly buses supplied with dc supply.

43.3. Direct Steam Engine Drive

Though losing ground gradually due to various reasons, steam locomotive is still the most widely-adopted means of propulsion for railway work. Invariably, the reciprocating engine is employed because

1. it is inherently simple.
2. connection between its cylinders and the driving wheels is simple.
3. its speed can be controlled very easily.

However, the steam locomotive suffers from the following disadvantages :

1. since it is difficult to install a condenser on a locomotive, the steam engine runs non-condensing and, therefore, has a very low thermal efficiency of about 6-8 percent.
2. it has strictly limited overload capacity.
3. it is available for hauling work for about 60% of its working days, the remaining 40% being spent in preparing for service, in maintenance and overhaul.

43.4. Diesel-electric Drive

It is a self-contained motive power unit which employs a diesel engine for direct drive of a dc generator. This generator supplies current to traction motors which are geared to the driving axles.

In India, diesel locomotives were introduced in 1945 for shunting service on broad-guage (BG) sections and in 1956 for high-speed main-line operations on metre-guage (MG) sections. It was only in 1958 that Indian Railways went in for extensive main-line dieselisation.*

Diesel-electric traction has the following advantages :

1. no modification of existing tracks is required while converting from steam to diesel-electric traction.
2. it provides greater tractive effort as compared to steam engine which results in higher starting acceleration.
3. it is available for hauling for about 90% of its working days.
4. diesel-electric locomotive is more efficient than a steam locomotive (though less efficient than an electric locomotive).

Disadvantages

1. for same power, diesel-electric locomotive is costlier than either the steam or electric locomotive.
2. overload capacity is limited because diesel engine is a constant-kW output prime mover.
3. life of a diesel engine is comparatively shorter.
4. diesel-electric locomotive is heavier than plain electric locomotive because it carries the main engine, generator and traction motors etc.
5. regenerative braking cannot be employed though rheostatic braking can be.

43.5. Battery-electric Drive

In this case, the vehicle carries secondary batteries which supply current to dc motors used for driving the vehicle. Such a drive is well-suited for shunting in railway yards, for traction in mines, for local delivery of goods in large towns and large industrial plants. They have low maintenance cost and are free from smoke. However, the scope of such vehicles is limited because of the small capacity of the batteries and the necessity of charging them frequently.



The above picture shows a battery run car. Battery run vehicles are seen as alternatives for future transport due to their pollution-free locomotion

43.6. Advantages of Electric Traction

As compared to steam traction, electric trac-

* The Diesel Locomotive Works at Varanasi turns out 140 locomotives of 2700 hp (2015 kW) annually. Soon it will be producing new generation diesel engines of 4000 hp (2985 kW).

tion has the following advantages :

1. Cleanliness. Since it does not produce any smoke or corrosive fumes, electric traction is most suited for underground and tube railways. Also, it causes no damage to the buildings and other apparatus due to the absence of smoke and flue gases.

2. Maintenance Cost. The maintenance cost of an electric locomotive is nearly 50% of that for a steam locomotive. Moreover, the maintenance time is also much less.

3. Starting Time. An electric locomotive can be started at a moment's notice whereas a steam locomotive requires about two hours to heat up.

4. High Starting Torque. The motors used in electric traction have a very high starting torque. Hence, it is possible to achieve higher accelerations of 1.5 to 2.5 km/h/s as against 0.6 to 0.8 km/h/s in steam traction. As a result, we are able to get the following additional advantages:

(*i*) high schedule speed

(*ii*) increased traffic handling capacity

(*iii*) because of (*i*) and (*ii*) above, less terminal space is required—a factor of great importance in urban areas.

5. Braking. It is possible to use regenerative braking in electric traction system. It leads to the following advantages :

(*i*) about 80% of the energy taken from the supply during ascent is returned to it during descent.

(*ii*) goods traffic on gradients becomes safer and speedier.

(*iii*) since mechanical brakes are used to a very small extent, maintenance of brake shoes, wheels, tyres and track rails is considerably reduced because of less wear and tear.

6. Saving in High Grade Coal. Steam locomotives use costly high-grade coal which is not so abundant. But electric locomotives can be fed either from hydroelectric stations or pit-head thermal power stations which use cheap low-grade coal. In this way, high-grade coal can be saved for metallurgical purposes.

7. Lower Centre of Gravity. Since height of an electric locomotive is much less than that of a steam locomotive, its centre of gravity is comparatively low. This fact enables an electric locomotive to negotiate curves at higher speeds quite safely.

8. Absence of Unbalanced Forces. Electric traction has higher coefficient of adhesion since there are no unbalanced forces produced by reciprocating masses as is the case in steam traction. It not only reduces the weight/kW ratio of an electric locomotive but also improves its riding quality in addition to reducing the wear and tear of the track rails.

43.7. Disadvantages of Electric Traction

1. The most vital factor against electric traction is the initial high cost of laying out overhead electric supply system. Unless the traffic to be handled is heavy, electric traction becomes uneconomical.

2. Power failure for few minutes can cause traffic dislocation for hours.

3. Communication lines which usually run parallel to the power supply lines suffer from electrical interference. Hence, these communication lines have either to be removed away from the rail track or else underground cables have to be used for the purpose which makes the entire system still more expensive.

4. Electric traction can be used only on those routes which have been electrified. Obviously, this restriction does not apply to steam traction.

5. Provision of a negative booster is essential in the case of electric traction. By avoiding the

flow of return currents through earth, it curtails corrosion of underground pipe work and interference with telegraph and telephone circuits.

43.8. Systems of Railway Electrification

Presently, following four types of track electrification systems are available :

1. Direct current system—600 V, 750 V, 1500 V, 3000 V
2. Single-phase ac system—15-25 kV, $16\frac{2}{3}$, 25 and 50 Hz
3. Three-phase ac system—3000-3500 V at $16\frac{2}{3}$ Hz
4. Composite system—involving conversion of single-phase ac into 3-phase ac or dc.

43.9. Direct Current System

Direct current at 600-750 V is universally employed for tramways in urban areas and for many suburban railways while 1500-3000 V dc is used for main line railways. The current collection is from third rail (or conductor rail) up to 750 V, where large currents are involved and from overhead wire for 1500 V and 3000 V, where small currents are involved. Since in majority of cases, track (or running) rails are used as the return conductor, only one conductor rail is required. Both of these contact systems are fed from substations which are spaced 3 to 5 km for heavy suburban traffic and 40-50 km for main lines operating at higher voltages of 1500 V to 3000 V. These sub-stations themselves receive power from 110/132 kV, 3-phase network (or grid). At these substations, this high-voltage 3-phase supply is converted into low-voltage 1-phase supply with the help of Scott-connected or V-connected 3-phase transformers (Art. 31.9). Next, this low ac voltage is converted into the required dc voltage by using suitable rectifiers or converters (like rotary converter, mercury-arc, metal or semiconductor rectifiers). These substations are usually automatic and are remote-controlled.

The dc supply so obtained is fed via suitable contact system to the traction motors which are either dc series motors for electric locomotive or compound motors for tramway and trolley buses where regenerative braking is desired.

It may be noted that for ***heavy suburban service***, low voltage dc system is undoubtedly superior to 1-phase ac system due to the following reasons :

1. dc motors are better suited for frequent and rapid acceleration of heavy trains than ac motors.
2. dc train equipment is lighter, less costly and more efficient than similar ac equipment.
3. when operating under similar service conditions, dc train consumes less energy than a 1-phase ac train.
4. the conductor rail for dc distribution system is less costly, both initially and in maintenance than the high-voltage overhead ac distribution system.
5. dc system causes no electrical interference with overhead communication lines.

The only disadvantage of dc system is the necessity of locating ac/dc conversion sub-stations at relatively short distances apart.

43.10. Single-Phase Low-frequency AC System

In this system, ac voltages from 11 to 15 kV at $16\frac{2}{3}$ or 25 Hz are used. If supply is from a generating station exclusively meant for the traction system, there is no difficulty in getting the electric supply of $16\frac{2}{3}$ or 25 Hz. If, however, electric supply is taken from the high voltage transmission lines at 50 Hz, then in addition to step-down transformer, the substation is provided with a frequency

converter. The frequency converter equipment consists of a 3-phase synchronous motor which drives a I-phase alternator having or 25 Hz frequency.

The 15 kV $16\frac{2}{3}$ or 25 Hz supply is fed to the electric locomotor via a single over-head wire (running rail providing the return path).

A step-down transformer carried by the locomotive reduces the 15-kV voltage to 300-400 V for feeding the ac series motors. Speed regulation of ac series motors is achieved by applying variable voltage from the tapped secondary of the above transformer.

Low-frequency ac supply is used because apart from improving the commutation properties of ac motors, it increases their efficiency and power factor. Moreover, at low frequency, line reactance is less so that line impedance drop and hence line voltage drop is reduced. Because of this reduced line drop, it is feasible to space the substations 50 to 80 km apart. Another advantage of employing low frequency is that it reduces telephonic interference.

41.11. Three-phase Low-frequency AC System

It uses 3-phase induction motors which work on a 3.3 kV, $16\frac{2}{3}$ Hz supply. Sub-stations receive power at a very high voltage from 3-phase transmission lines at the usual industrial frequency of 50 Hz. This high voltage is stepped down to 3.3 kV by transformers whereas frequency is reduced from 50 Hz to $16\frac{2}{3}$ Hz by frequency converters installed at the sub-stations. Obviously, this system employs *two* overhead contact wires, the track rail forming the third phase (of course, this leads to insulation difficulties at the junctions).

Induction motors used in the system are quite simple and robust and give trouble-free operation. They possess the merits of high efficiency and of operating as a generator when driven at speeds above the synchronous speed. Hence, they have the property of automatic regenerative braking during the descent on gradients. However, it may be noted that despite all its advantages, this system has not found much favour and has, in fact, become obsolete because of its certain inherent limitations given below :

1. the overhead contact wire system becomes complicated at crossings and junctions.
2. constant-speed characteristics of induction motors are not suitable for traction work.
3. induction motors have speed/torque characteristics similar to dc shunt motors. Hence, they are not suitable for parallel operation because, even with little difference in rotational speeds caused by unequal diameters of the wheels, motors will becomes loaded very unevenly.

43.12. Composite System

Such a system incorporates good points of two systems while ignoring their bad points. Two such composite systems presently in use are :

1. 1-phase to 3-phase system also called Kando system
2. 1-phase to dc system.

43.13. Kando System

In this system, single-phase 16-kV, 50 Hz supply from the sub-station is picked up by the locomotive through the single overhead contact wire. It is then converted into 3-phase ac supply at the same frequency by means of phase converter equipment carried on the locomotives. This 3-phase supply is then fed to the 3-phase induction motors.

As seen, the complicated overhead two contact wire arrangement of ordinary 3-phase system is replaced by a single wire system. By using silicon controlled rectifier as inverter, it is possible to get variable-frequency 3-phase supply at 1/2 to 9 Hz frequency. At this low frequency, 3-phase motors develop high starting torque without taking excessive current. In view of the above, Kando system is likely to be developed further.

43.14. Single-phase AC to DC System

This system combines the advantages of high-voltage ac distribution at industrial frequency with the dc series motors traction. It employs overhead 25-kV, 50-Hz supply which is stepped down by the transformer installed in the locomotive itself. The low-voltage ac supply is then converted into dc supply by the rectifier which is also carried on the locomotive. This dc supply is finally fed to dc series traction motor fitted between the wheels. The system of traction employing 25-kV, 50-Hz, 1-phase ac supply has been adopted for all future track electrification in India.

43.15. Advantages of 25-kV, 50-Hz AC System

Advantages of this system of track electrification over other systems particularly the dc system are as under :

1. Light Overhead Catenary

Since voltage is high (25 kV), line current for a given traction demand is less. Hence, cross-section of the overhead conductors is reduced. Since these small-sized conductors are light, supporting structures and foundations are also light and simple. Of course, high voltage needs higher insulation which increases the cost of overhead equipment (OHE) but the reduction in the size of conductors has an overriding effect.

2. Less Number of Substations

Since in the 25-kV system, line current is less, line voltage drop which is mainly due to the resistance of the line is correspondingly less. It improves the voltage regulation of the line which fact makes larger spacing of 50-80 km between sub-stations possible as against 5-15 km with 1500 V dc system and 15-30 km with 3000 V dc sysem. Since the required number of substations along the track is considerably reduced, it leads to substantial saving in the capital expenditure on track electrification.

3. Flexibility in the Location of Substations

Larger spacing of substations leads to greater flexibility in the selection of site for their proper location. These substations can be located near the national high-voltage grid which, in our country, fortunately runs close to the main railway routes. The substations are fed from this grid thereby saving the railway administration lot of expenditure for erecting special transmission lines for their substations. On the other hand, in view of closer spacing of dc substations and their far away location, railway administration has to erect its own transmission lines for taking feed from the national grid to the substations which consequently increases the initial cost of electrification.

4. Simplicity of Substation Design

In ac systems, the substations are simple in design and layout because they do not have to install and maintain rotary converters or rectifiers as in dc systems. They only consist of static transformers alongwith their associated switchgear and take their power directly from the high-voltage national grid running over the length and breadth of our country. Since such sub-stations are remotely controlled, they have few attending personnel or even may be unattended.

5. Lower Cost of Fixed Installations

The cost of fixed installations is much less for 25 kV ac system as compared to dc system. In fact, cost is in ascending order for 25 kV ac, 3000 V dc and 1500 V dc systems. Consequently, traffic densities for which these systems are economical are also in the ascending order.

6. Higher Coefficient of Adhesion

The straight dc locomotive has a coefficient of adhesion of about 27% whereas its value for ac rectifier locomotive is nearly 45%. For this reason, a lighter ac locomotive can haul the same load as a heavier straight dc locomotive. Consequently, ac locomotives are capable of achieving higher speeds in coping with heavier traffic.

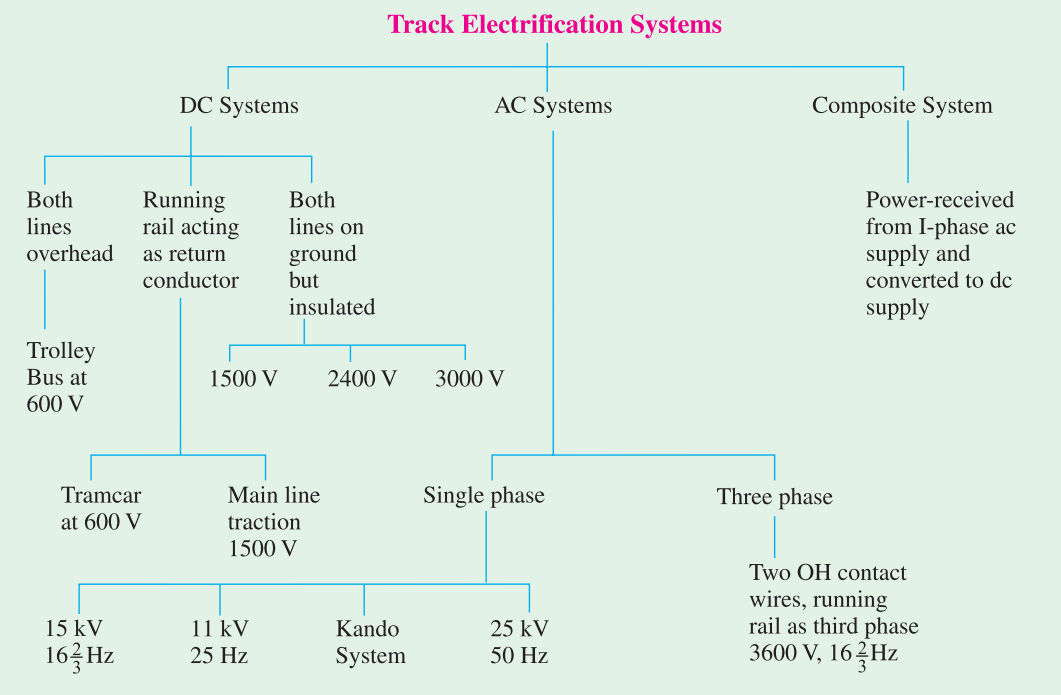
7. Higher Starting Efficiency

An ac locomotive has higher starting efficiency than a straight dc locomotive. In dc locomotive supply voltage at starting is reduced by means of ohmic resistors but by on-load primary or secondary tap-changer in ac locomotives.

43.16. Disadvantages of 25-kV AC System

1. Single-phase ac system produces both current and voltage unbalancing effect on the supply.
2. It produces interference in telecommunication circuits. Fortunately, it is possible at least to minimize both these undesirable effects.

Different track electrification systems are summarised below :



43.17. Block Diagram of an AC Locomotive

The various components of an ac locomotive running on single-phase 25-kV, 50-Hz ac supply are numbered in Fig. 43.1.

1. OH contact wire

2. pantograph
3. circuit breakers
4. on-load tap-changers
5. transformer
6. rectifier
7. smoothing choke
8. dc traction motors.

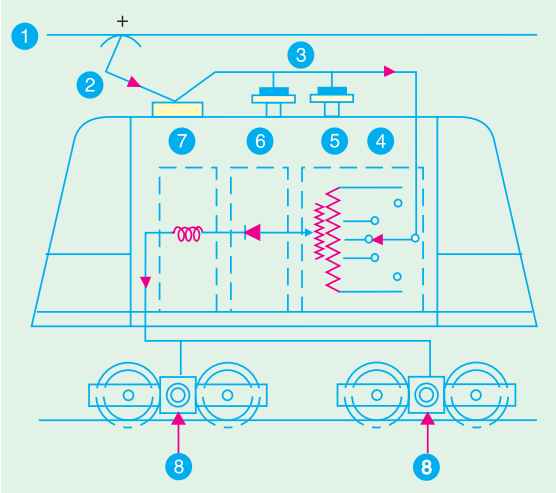


Fig. 43.1

As seen, power at 25 kV is taken via a pantograph from the overhead contact wire and fed to the step-down transformer in the locomotive. The low ac voltage so obtained is converted into pulsating dc voltage by means of the rectifier. The pulsations in the dc voltage are then removed by the smoothing choke before it is fed to dc series traction motors which are mounted between the wheels.

The function of circuit breakers is to immediately disconnect the locomotive from the overhead supply in case of any fault in its electrical system. The on-load tap-changer is used to change the voltage across the motors and hence regulate their speed.

43.18. The Tramways

It is the most economical means of transport for very dense traffic in the congested streets of large cities. It receives power through a bow collector or a grooved wheel from an overhead conductor at about 600 V dc, the running rail forming the return conductor. It is provided with at least two driving axles in order to ***(i)*** secure necessary adhesion ***(ii)*** start it from either end and ***(iii)*** use two motors with series-parallel control. Two drum-type controllers, one at each end, are used for controlling the tramcar. Though these controllers are connected in parallel, they have suitable interlocking arrangement meant to prevent their being used simultaneously.

Tramcars are being replaced by trolley-buses and internal-combustion-engined omnibuses because of the following reasons :

1. tramcars lack flexibility of operation in congested areas.
2. the track constitutes a source of danger to other road users.



Trolley Bus

43.19. The Trolleybus

It is an electrically-operated pneumatic-tyred vehicle which needs ***no track in the roadway***. It receives its power at 600 V dc from two overhead contact wires. Since adhesion between a rubber-tyred wheel and ground is sufficiently high, only a single driving axle and, hence, a single motor is used. The trolleybus can manoeuvre through traffic a metre or two on each side of the centre line of the trolley wires.

43.20. Overhead Equipment (OHE)

Broadly speaking, there are two systems of current collection by a traction unit :

(*i*) third rail system and (*ii*) overhead wire system.

It has been found that current collection from overhead wire is far superior to that from the third rail. Moreover, insulation of third rail at high voltage becomes an impracticable proposition and endangers the safety of the working personnel.

The simplest type of OHE consists of a single contact wire of hard drawn copper or silico-bronze supported either by bracket or an overhead span. To facilitate connection to the supports, the wire is grooved as shown in Fig. 43.2. Because there is appreciable sag of the wire between supports, it limits the speed of the traction unit to about 30 km/h. Hence, single contact wire system is suitable for tramways and in complicated yards and terminal stations where speeds are low and simplicity of layout is desirable.

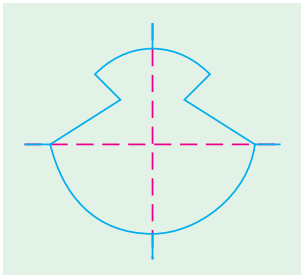


Fig. 43.2

For collection of current by high-speed trains, the contact (or trolley) wire has to be kept level without any abrupt changes in its height between the supporting structures. It can be done by using the single catenary system which consists of one catenary or messenger wire of steel with high sag and the trolley (or contact) wire supported from messenger wire by means of droppers clipped to both wires as shown in Fig. 43.3.

43.21. Collector Gear for OHE

The most essential requirement of a collector is that it should keep continuous contact with trolley wire at all speeds. Three types of gear are in common use :

1. trolley collector **2.** bow collector and **3.** pantograph collector.

To ensure even pressure on OHE, the gear equipment must, be flexible in order to follow variations in the sag of the contact wire. Also, reasonable precautions must be taken to prevent the collector from leaving the overhead wire at points and crossings.

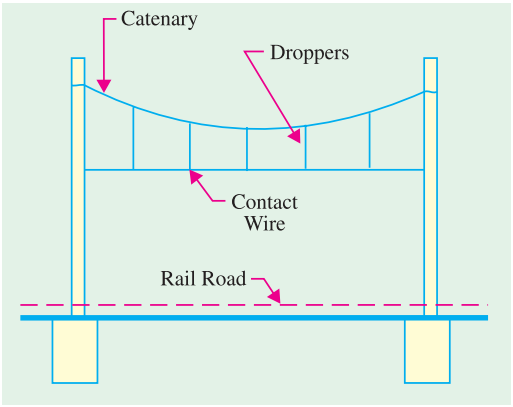


Fig. 43.3

43.22. The Trolley Collector

This collector is employed on tramways and trolley buses and is mounted on the roof of the vehicle. Contact with the OH wire is made by means of either a grooved wheel or a sliding shoe carried at the end of a light trolley pole attached to the top of the vehicle and held in contact with OH wire by means of a spring. The pole is hinged to a swivelling base so that it may be reversed for reverse running thereby making it unnecessary for the trolley wire to be accurately maintained above the centre of the track. Trolley collectors always operate in the trailing position.

The trolley collector is suitable for low speeds upto 32 km/h beyond which there is a risk of its jumping off the OH contact wire particularly at points and crossing.

43.23. The Bow Collector

It can be used for higher speeds. As shown in Fig. 43.4, it consists of two roof-mounted trolley poles at the ends of which is placed a light metal strip (or bow) about one metre long for current collection. The collection strip is purposely made of soft material (copper, aluminium or carbon) in order that most of the wear may occur on it rather than on the trolley wire. The bow collector also

operates in the trailing position. Hence, it requires provision of either duplicate bows or an arrangement for reversing the bow for running in the reverse direction. Bow collector is not suitable for railway work where speeds up to 120 km/h and currents up to 3000 A are encountered. It is so because the inertia of the bow collector is too high to ensure satisfactory current collection.

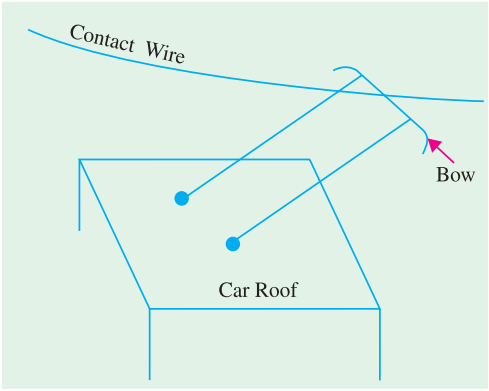


Fig. 43.4

43.24. The Pantograph Collector

Its function is to maintain link between overhead contact wire and power circuit of the electric locomotive at different speeds under all wind conditions and stiffness of OHE. It means that positive pressure has to be maintained at all times to avoid loss of contact and sparking but the pressure must be as low as possible in order to minimize wear of OH contact wire.

A 'diamond' type single-pan pantograph is shown in Fig. 43.5. It consists of a pentagonal framework of high-tensile alloy-steel tubing. The contact portion consists of a pressed steel pan fitted with renewable copper wearing strips which are forced against the OH contact wire by the upward action of pantograph springs. The pantograph can be raised or lowered from cabin by air cylinders.

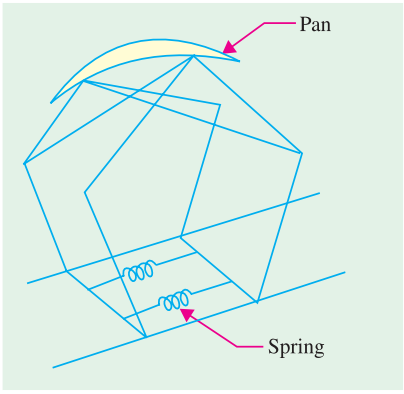


Fig. 43.5

43.25. Conductor Rail Equipment

The conductor rails may be divided into three classes depending on the position of the contact surface which may be located at the top, bottom or side of the rail. The top contact rail is adopted universally for 600 V dc electrification. The side contact rail is used for 1200 V dc supply. The under contact rail has the advantage of being protected from snow, sleet and ice.

Fig. 43.6 shows the case when electric supply is collected from the top of an insulated conductor rail *C* (of special high-conductivity steel) running parallel to the track at a distance of 0.3 to 0.4 m from the running rail (*R*) which forms the return path. *L* is the insulator and *W* is the wooden protection used at stations and crossings.



The pantograph mechanism helps to maintain a link between the overhead contact wire and power circuit of the electric locomotive

The current is collected from top surface of the rail by flat steel shoes (200 mm × 75 mm), the necessary contact pressure being obtained by gravity. Since it is not always possible to provide conductor rail on the same side of the track, shoes are provided on both sides of the locomotive or train. Moreover two shoes are provided on each side in order to avoid current interruption at points and crossings where there are gaps in the running rail.

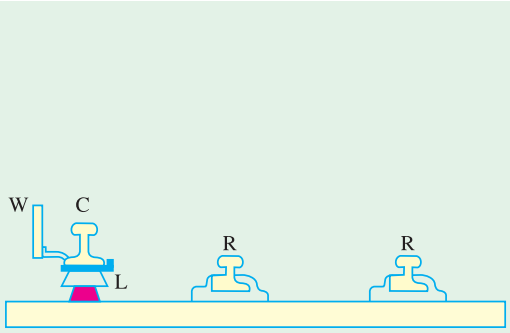


Fig. 43.6

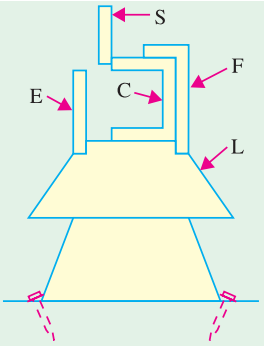


Fig. 43.7

Fig. 43.7 shows the side contact rail and the method of the mounting. The conductor rail (*C*) rests upon a wooden block recessed into the top of the procelain insulator *L*. Current is collected by steel shoes (*S*) which are kept pressed on the contact rail by springs. *E* and *F* are the guards which rest upon ledges on the insulator.

43.26. Types of Railway Services

There are three types of passenger services offered by the railways :

1. City or Urban Service. In this case, there are frequent stops, the distance between stops being nearly 1 km or less. Hence, high acceleration and retardation are essential to achieve moderately high schedule speed between the stations.

2. Suburban Service. In this case, the distance between stops averages from 3 to 5 km over a distance of 25 to 30 km from the city terminus. Here, also, high rates of acceleration and retardation are necessary.

3. Main Line Service. It involves operation over long routes where stops are infrequent. Here, operating speed is high and accelerating and braking periods are relatively unimportant.

On goods traffic side also, there are three types of services ***(i)*** main-line freight service ***(ii)*** local or pick-up freight service and ***(iii)*** shunting service.

43.27. Train Movement

The movement of trains and their energy consumption can be conveniently studied by means of speed/time and speed/distance curves. As their names indicate, former gives speed of the train at various ***times*** after the start of the run and the later gives speed at various ***distances*** from the starting point. Out of the two, speed/time curve is more important because

1. its slope gives acceleration or retardation as the case may be.
2. area between it and the horizontal (*i.e.* time) axis represents the distance travelled.
3. energy required for propulsion can be calculated if resistance to the motion of train is known.

43.28. Typical Speed/Time Curve

Typical speed/time curve for electric trains operating on passenger services is shown in Fig. 43.8. It may be divided into the following **five** parts :

1. Constant Acceleration Period (0 to t_1)

It is also called notching-up or starting period because during this period, starting resistance of the motors is gradually cut out so that the motor current (and hence, tractive effort) is maintained nearly constant which produces constant acceleration alternatively called 'rheostatic acceleration' or 'acceleration while notching'.

2. Acceleration on Speed Curve (t_1 to t_2)

This acceleration commences after the starting resistance has been all cut-out at point t_1 and full supply voltage has been applied to the motors. During this period, the motor current and torque decrease as train speed increases. Hence, acceleration gradually ***decreases*** till torque developed by motors exactly balances that due to resistance to the train motion. The shape of the portion *AB* of the speed/time curve depends primarily on the torque/speed characteristics of the traction motors.

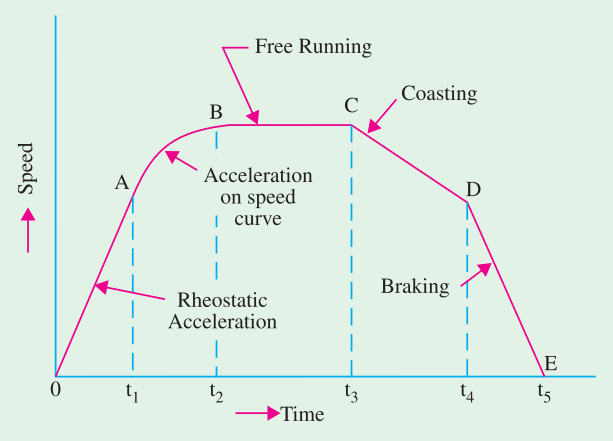


Fig. 43.8

3. Free-running Period (t_2 to t_3)

The train continues to run at the speed reached at point t_2. It is represented by portion *BC* in Fig. 43.8 and is a constant-speed period which occurs on level tracks.

4. Coasting (t_3 to t_4)

Power to the motors is cut off at point t_3 so that the train runs under its momentum, the speed gradually falling due to friction, windage etc. (portion *CD*). During this period, retardation remains practically constant. Coasting is desirable because it utilizes some of the kinetic energy of the train which would, otherwise, be wasted during braking. Hence, it helps to reduce the energy consumption of the train.

5. Braking (t_4 to t_5)

At point t_4, brakes are applied and the train is brought to rest at point t_5.

It may be noted that coasting and braking are governed by train resistance and allowable retardation respectively.

43.29. Speed/Time Curves for Different Services

Fig. 43.9 (*a*) is representative of city service where relative values of acceleration and retardation are high in order to achieve moderately high average speed between stops. Due to short

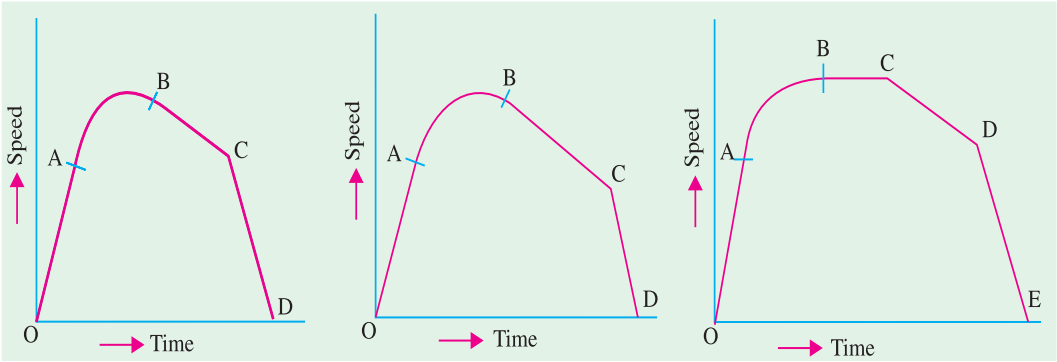


Fig. 43.9

distances between stops, there is no possibility of free-running period though a short coasting period is included to save on energy consumption.

In suburban services [Fig. 43.9 (*b*)], again there is no free-running period but there is comparatively ***longer*** coasting period because of longer distances between stops. In this case also, relatively

high values of acceleration and retardation are required in order to make the service as attractive as possible.

For main-line service [Fig. 43.9 (c)], there are long periods of free-running at high speeds. The accelerating and retardation periods are relatively unimportant.

43.30. Simplified Speed/Time Curve

For the purpose of comparative performance for a given service, the actual speed/time curve of Fig. 43.8 is replaced by a simplified speed/time curve which does not involve the knowledge of motor characteristics. Such a curve has simple geometric shape so that simple mathematics can be used to find the relation between acceleration, retradation, average speed and distance etc. The simple curve would be fairly accurate provided it ***(i) retains the same acceleration and retardation and (ii) has the same area as the actual speed/time curve***. The simplified speed/time curve can have either of the two shapes :

(i) trapezoidal shape OA_1B_1C of Fig. 43.10 where speed-curve running and coasting periods of the actual speed/time curve have been replaced by a constant-speed period.

(ii) quadrilateral shape OA_2B_2C where the same two periods are replaced by the extensions of initial constnat acceleration and coasting periods.

It is found that trapezodial diagram OA_1B_1C gives simpler relationships between the principal quantities involved in train movement and also gives closer approximation of actual energy consumed during ***main-line service on level track***. On the other hand, quadrilateral diagram approximates more closely to the actual conditions in ***city and suburban services***.

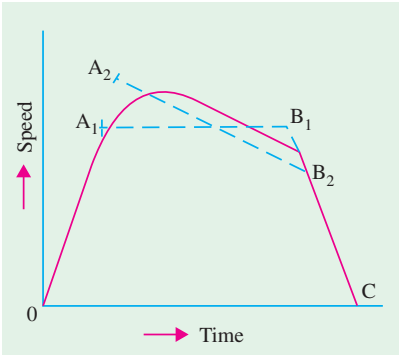


Fig. 43.10

43.31. Average and Schedule Speed

While considering train movement, the following three speeds are of importance :

1. **Crest Speed.** It is the maximum speed (V_m) attained by a train during the run.
2. **Average Speed** $= \dfrac{\text{distance between stops}}{\text{actual time of run}}$

In this case, only running time is considered but *not the stop time*.

3. **Schedule Speed** $= \dfrac{\text{distance between stops}}{\text{actual time of run + stop time}}$

Obviously, schedule speed can be obtained from average speed by including the duration of stops. For a given distance between stations, higher values of acceleration and retardation will mean lesser running time and, consequently, higher schedule speed. Similarly, for a given distance between stations and for fixed values of acceleration and retardation, higher crest speed will result in higher schedule speed. For the same value of average speed, increase in duration of stops decreases the schedule speed.

43.32. SI Units in Traction Mechanics

In describing various quantities involved in the mechanics of train movement, only the latest SI system will be used. Since SI system is an 'absolute system', only absolute units will be used while gravitational units (used hitherto) will be discarded.

1. **Force.** It is measured in newton (N)
2. **Mass.** Its unit is kilogram (kg). Commonly used bigger units is tonne (t), 1 tonne = 1000 kg

3. **Energy.** Its basic unit is joule (J). Other units often employed are watt-hour (Wh) and kilowatt-hour (kWh).

$$1\ \text{Wh} = 1\frac{\text{J}}{\text{s}} \times 3600\ \text{s} = 3600\ \text{J} = 3.6\ \text{kJ}$$

$$1\text{kWh} = 1000 \times 1\frac{\text{J}}{\text{s}} \times 3600\ \text{s} = 36 \times 10^5\ \text{J} = 3.6\ \text{MJ}$$

4. **Work.** Its unit is the same as that of energy.

5. **Power.** Its unit is watt (W) which equals 1 J/s. Other units are kilowatt (kW) and megawatt (MW).

6. **Distance.** Its unit is metre. Other unit often used is kilometre (km).

7. **Velocity.** Its absolute unit is metre per second (m/s). If velocity is given in km/h (or km.ph), it can be easily converted into the SI unit of m/s by multiplying it with a factor of (1000/3600) = 5/18 = 0.2778. For example, 72 km.ph = $72 \times 5/18 = 72 \times 0.2778 = 20$ m/s.

8. **Acceleration.** Its unit is metre/second^2 (m/s^2). If acceleration is given in km/h/s (or km-ph.ps), then it can be converted into m/s^2 by simply multiplying it by the factor (1000/3600) = 5/18 = 0.2778 *i.e.* the same factor as for velocity.
For example, 1.8 km.ph.ps = $1.8 \times 5/18 = 1.8 \times 0.2778 = 0.5$ m/s^2

43.33. Confusion Regarding Weight and Mass of a Train

Many students often get confused regarding the correct meaning of the terms 'weight' and 'mass' and their units while solving numericals on train movement particularly when they are not expressed clearly and consistently in their absolute units. It is primarily due to the mixing up of absolute units with gravitational units. There would be no confusion at all if ***we are consistent in using only absolute units*** as required by the SI system of units which disallows the use of gravitational units.

Though this topic was briefly discussed earlier, it is worth repeating here.

1. **Mass (M).** It is the quantity of matter contained in a body.
Its absolute unit is kilogram (kg). Other multiple in common use is tonne.

2. **Weight (W).** It is the **force** with which earth pulls a body downwards.

The weight of a body can be expressed in (*i*) the ***absolute*** unit of newton (N) or (*ii*) the ***gravitational*** unit of kilogram-weight (kg. wt) which is often writing as 'kgf' in engineering literature.

Another still bigger ***gravitational*** unit commonly used in traction work is tonne-weight (t-wt)

$$1\ \text{t-wt} = 1000\ \text{kg-wt} = 1000 \times 9.8\ \text{N} = 9800\ \text{N}$$

(*i*) Absolute Unit of Weight

It is called newton (N) whose definition may be obtained from Newton's Second Law of Motion.

Commonly used multiple is kilo-newton (kN). Obviously, 1 kN = 1000 N = 10^3 N.

For example, if a mass of 200 kg has to be given an acceleration of 2.5 m/s^2, force required is $F = 200 \times 2.5 = 500$ N.

If a train of mass 500 tonne has to be given an acceleration of 0.6 m/s^2, force required is

$$F = ma = (500 \times 1000) \times 0.6 = 300{,}000\ \text{N} = 300\ \text{kN}$$

(*ii*) Gravitational Unit of Weight

It is 'g' times bigger than newton. It is called kilogram-weight (kg.wt.)

$$1\ \text{kg.wt} = g\ \text{newton} = 9.81\ \text{N} \cong 9.8\ \text{N}$$

Unfortunately, the word 'wt' is usually omitted from kg-wt when expressing the weight of the body on the assumption that it can be understood or inferred from the language used.

Take the statement "a body has a *weight* of 100 kg". It looks as if the weight of the body has been

expressed in terms of the mass unit 'kg'. To avoid this confusion, statement should be 'a body has a weight of 100 kg. wt.' But the first statement is justified by the writers on the ground that since the word 'weight' has already been used in the statement, it should be automatically understood by the readers that 'kg' is not the 'kg' of mass but is kg-wt. It would be mass kg if the statement is 'a body has a mass of 100 kg'. Often kg-wt is written as 'kgf' where 'f' is the first letter of the word force and is added to distinguish it from kg of mass.

Now, consider the statement "a body *weighing* 500 kg travels with a speed of 36 km/h....."

Now, weight of the body W = 500 kg.wt. = 500×9.8 N

Since we know the weight of the body, we can find its mass from the relation $W = mg$. But while using this equation, it is essential that we must ***consistently* use** the ***absolute*** units only. In this equation, W must be in newton (not in kg. wt), m in kg and g in m/s^2.

$\therefore \quad 500 \times 9.8 = m \times 9.8$; $\quad \therefore \quad m = 500$ kg

It means that a body which *weighs* 500 kg (wt) has a *mass* of 500 kg.

As a practical rule, weight of a body in **gravitational** units is numerically equal to its mass in **absolute** units. This simple fact must be clearly understood to avoid any confusion between weight and mass of a body.

A train which weighs 500 tonne has a mass of 500 tonne as proved below :

train weight, $W = 500$ tonne-wt $= 500 \times 1000$ kg-wt $= 500 \times 1000 \times 9.8$ N

Now, $W = mg$; $\quad \therefore \quad 500 \times 1000 \times 9.8 = m \times 9.8$

$\therefore \quad m = 500 \times 1000$ kg $= 500 \times 1000/1000 = 500$ tonne

To avoid this unfortunate confusion, it would be helpful to change our terminology. For example, instead of saying "a train weighing 500 tonne is....." it is better to say "a 500-t train is ....." or "a train having a mass of 500 t is ....."

In order to remove this confusion, SI system of units has disallowed the use of gravitational units. There will be no confusion if ***we consistently use only absolute units***.

43.34. Quantities Involved in Traction Mechanics

Following principal quantities are involved in train movement :

D	= distance between stops	M	= dead mass of the train
M_e	= effective mass of the train	W	= dead weight of the train
W_e	= effective weight of the train	α	= accelaration during starting period
β_c	= retardation during coasting	β	= retardation during braking
V_a	= average speed	V_m	= maximum (or crest) speed.
t	= total time for the run	t_1	= time of acceleration
t_2	= time of free running = $t - (t_1 + t_3)$	t_3	= time of braking
F_t	= tractive effort	T	= torque

43.35. Relationship Between Principal Quantities in Trapezoidal Diagram

As seen from Fig. 43.11.

$\alpha = V_m / t_1 \quad$ or $\quad t_1 = V_m / \alpha$

$\beta = V_m / t_3 \quad$ or $\quad t_3 = V_m / \beta$

As we know, total distance D between the two stops is given by the area of trapezium $OABC$.

$$\therefore \quad D = \text{area } OABC$$
$$= \text{area } OAD + \text{area } ABED + \text{area } BCE$$
$$= \frac{1}{2} V_m t_1 + V_m t_2 + \frac{1}{2} V_m t_3$$

$$= \frac{1}{2} V_m t_1 + V_m[t - (t_1 + t_3)] + \frac{1}{2} V_m t_3$$

$$= V_m \left[\frac{t_1}{2} + t - t_1 - t_3 + \frac{t_3}{2}\right]$$

$$= V_m \left[t - \frac{1}{2}(t_1 + t_3)\right]$$

$$= V_m \left[t - \frac{V_m}{2}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)\right]$$

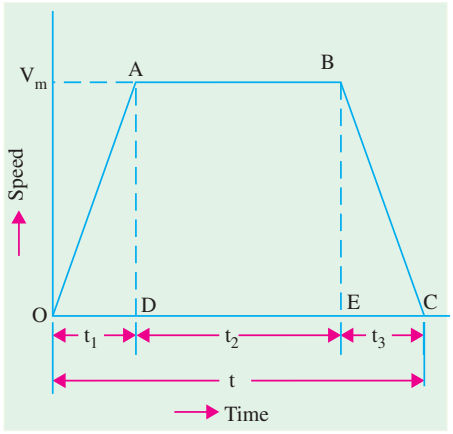


Fig. 43.11

Let, $K = \frac{1}{2}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$. Substituting this value of K in the above equation, we get

$$D = V_m (t - KV_m)$$

or $KV_m^2 - V_m t + D = 0$ ...(*i*)

$$\therefore \quad V_m = \frac{t \pm \sqrt{t^2 - 4KD}}{2K}$$

Rejecting the positive sign which gives impracticable value, we get

$$V_m = \frac{t \pm \sqrt{t^2 - 4KD}}{2K}$$

From Eq. (*i*) above, we get

$$KV_m^2 = V_m t - D \quad \text{or} \quad K = \frac{t}{V_m} - \frac{D}{V_m^2} = \frac{D}{V_m^2}\left(V_m \cdot \frac{t}{D} - 1\right)$$

Now, $V_a = \frac{D}{t}$ $\quad \therefore \quad K = \frac{D}{V_m^2}\left(\frac{V_m}{V_a} - 1\right)$

Obviously, if V_m, V_a and D are given, then value of K and hence of α and β can be found (Ex. 43.2).

43.36. Relationship Between Principal Quantities in Quadrilateral Diagram

The diagram is shown in Fig. 43.12. Let β_c represent the retardation during coasting period. As before,

$t_1 = V_1/\alpha$, $t_2 = (V_2 - V_1)/\beta_c$ and $t_3 = V_2/\beta$

D = area $OABC$

= area OAD + area $ABED$ + area BCE

$$= \frac{1}{2} V_1 t_1 + t_2\left(\frac{V_1 + V_2}{2}\right) + \frac{1}{2} V_2 t_3$$

$$= \frac{1}{2} V_1 (t_1 + t_2) + \frac{1}{2} V_2 (t_2 + t_3)$$

$$= \frac{1}{2} V_1 (t - t_3) + \frac{1}{2} V_2 (t - t_1)$$

$$= \frac{1}{2} t (V_1 + V_2) - \frac{V_1 t_1}{2} - \frac{V_1 t_3}{2}$$

$$= \frac{1}{2} t (V_1 + V_2) - \frac{1}{2} V_1 V_2 \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$$

$$= \frac{1}{2} t (V_1 + V_2) - KV_1 V_2$$

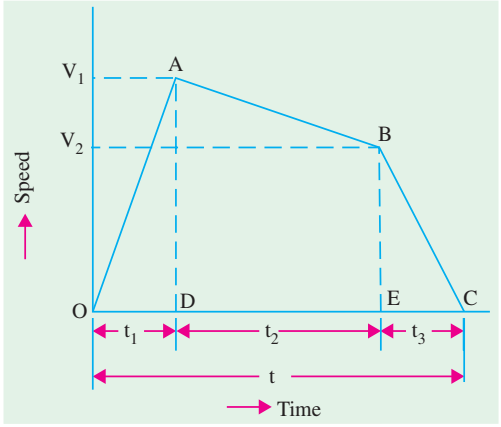


Fig. 43.12

where $K = \frac{1}{2}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)=\frac{\alpha+\beta}{2\alpha\beta}$ Also, $\beta_c = \frac{(V_1 - V_2)}{t_2}$

$\therefore \quad V_2 = V_1 - \beta_c t_2 = V_1 - \beta_c (t - t_1 - t_3)$

$= V_1 - \beta_c \left(t - \frac{V_1}{\alpha} - \frac{V_2}{\beta}\right) = V_1 \beta_c \left(t - \frac{V_1}{\alpha}\right) + \beta_c \frac{V_2}{\beta}$

or $V_2\left(1 - \frac{\beta_c}{\beta}\right) = V_1 - \beta_c\left(t - \frac{V_1}{\alpha}\right)$ $\therefore \quad V_2 = \frac{V_1 - \beta_c (t - V_1/\alpha)}{(1 - \beta_c/\beta)}$

Example 43.1. *A suburban train runs with an average speed of 36 km/h between two stations 2 km apart. Values of acceleration and retardation are 1.8 km/h/s and 3.6 km/h/s.*

Compute the maximum speed of the train assuming trapezoidal speed/time curve.

(Electric Traction, Punjab Univ. 1994)

Solution. Now, V_a = 36 km/h = 36 × 5/18 = 10 m/s

α = 1.8 km/h/s = 1.8 × 5/18 = 0.5 m/s^2, β = 3.6 km/h/s = 3.6 × 5/18 = 1.0 m/s^2

$t = D/V_a$ = 2000/10 = 200 s ; $K = (\alpha + \beta)/2\alpha\beta = (0.5 + 1.0)/2 \times 0.5 \times 1 = 1.5$

$$V_m = \frac{t - \sqrt{t^2 - 4KD}}{2K} = \frac{200 - \sqrt{200^2 - 4 \times 1.5 \times 2000}}{2 \times 1.5}$$

= 11 m/s = 11 × 18/5

= **39.6 km/h**

Example 43.2. *A train is required to run between two stations 1.5 km apart at a schedule speed of 36 km/h, the duration of stops being 25 seconds. The braking retardation is 3 km/h/s. Assuming a trapezoidal speed/time curve, calculate the acceleration if the ratio of maximum speed to average speed is to be 1.25*

(Elect. Power, Bombay Univ. 1980)



Electric traction provides high starting torque and low maintenance costs making it the best choice for trains

Solution. Here, D = 1500 m ; schedule speed = 36 km/h = 36 × 5/18 = 10 m/s

β = 3 km/h/s = 3 × 5/18 = 5/6 m/s^2

Schedule time of run = 1500/10 = 150 s ; Actual time of run = 150 – 25 = 125 s

$\therefore \quad V_a$ = 1500/125 = 12 m/s ; V_m = 1.25 × 12 = 15 m/s

Now, $K = \frac{D}{V_m^2}\left(\frac{V_m}{V_a} - 1\right) = \frac{1500}{15^2}(1.25 - 1) = \frac{5}{3}$

Also, $K = \frac{1}{2}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)$ or $\frac{5}{3} = \frac{1}{2}\left(\frac{1}{\alpha}+\frac{6}{5}\right)$

$\therefore \quad \alpha$ = 0.47 m/s^2 = 0.47 × 18/5 = **1.7 km/h/s**

Example 43.3. *Find the schedule speed of an electric train for a run of 1.5 km if the ratio of its maximum to average speed is 1.25. It has a braking retardation of 3.6 km/h/s, acceleration of 1.8 km/h/s and stop time of 21 second. Assume trapezoidal speed/time curve.*

Solution. $\alpha = 1.8 \times 5/18 = 0.5$ m/s^2, $\beta = 3.6 \times 5/18 = 1.0$ m/s^2

$D = 1.5$ km $= 1500$ m

$K = \frac{1}{2}\left(\frac{1}{0.5} + \frac{1}{1}\right) = \frac{3}{2}$ Now, $K = \frac{D}{V_{m^2}}\left(\frac{V_m}{V_a} - 1\right)$

or $V_m^2 = \frac{D}{K}\left(\frac{V_m}{V_a} - 1\right)$ $\therefore$ $V_m^2 = \frac{1500}{3/2}(1.25 - 1) = 250$; $V_m = 15.8$ m/s

$V_a = V_m/1.25 = 15.8/1.25 = 12.6$ m/s

Actual time of run = $1500/12.6 = 119$ seconds

Schedule time = $119 + 21 = 140$ second

$\therefore$ Schedule speed = $1500/140 = 10.7$ m/s = **38.5 km/h**

Example 43.4. *A train runs between two stations 1.6 km apart at an average speed of 36 km/h. If the maximum speed is to be limited to 72 km/h, acceleration to 2.7 km/h/s, coasting retardation to 0.18 km/h/s and braking retardation to 3.2 km/h/s, compute the duration of acceleration, coasting and braking periods.*

Assume a simplified speed/time curve.

Solution. Given : $D = 1.6$ km $= 1600$ m, $V_a = 36$ km/h $= 10$ m/s

$V_1 = 72$ km/h $= 20$ m/s ; $\alpha = 2.7$ km/h/s $= 0.75$ m/s^2

$\beta_c = 0.18$ km/h/s $= 0.05$ m/s^2 ; $\beta = 3.6$ km/h/s $= 1.0$ m/s^2

With reference to Fig. 43.12, we have

Duration of acceleration, $t_1 = V_1/\alpha = 20/0.75 = 27$ s

Actual time of run, $t = 1600/10 =$ **160 s**

Duration of braking, $t_3 = V_2/1.0 = V_2$ second

Duration of coasting, $t_2 = (V_1 - V_2)/\beta_c = (20 - V_2)/0.05 = (400 - 20\ V_2)$ second

Now, $t = t_1 + t_2 + t_3$ or $160 = 27 + (400 - 20\ V_2) + V_2$ $\therefore$ $V_2 = 14$ m/s

$\therefore$ $t_2 = (20 - 14)/0.05 = 120$ s ; $t_3 = 14/1.0 =$ **14 s**

43.37. Tractive Effort for Propulsion of a Train

The tractive effort (F_t) is the force developed by the traction unit at the rim of the driving wheels for moving the unit itself and its train (trailing load). The tractive effort required for train propulsion on a *level track* is

$$F_t = F_a + F_r$$

If gradients are involved, the above expression becomes

$F_t = F_a + F_g + F_r$ — for ascending gradient

$= F_a - F_g + F_r$ — for descending gradient

where F_a = force required for giving linear acceleration to the train

F_g = force required to overcome the effect of gravity

F_r = force required to overcome resistance to train motion.

(a) Value of F_a

If M is the dead (or stationary) mass of the train and a its linear acceleration, then

$$F_a = Ma$$

Since a train has rotating parts like wheels, axles, motor armatures and gearing etc., its *effective* (or accelerating) mass M_e is more (about 8 – 15%) than its stationary mass. These parts have to be given angular acceleration at the same time as the whole train is accelerated in the linear direction. Hence, $F_e = M_e a$

(*i*) If M_e is in kg and α in m/s^2, then $F_a = M_e\, a$ newton

(*ii*) If M_e is in tonne and α in km/h/s, then converting them into absolute units, we have

$$F_a = (1000\, M_e) \times (1000/3600)\, a = 277.8\, M_e\, a \text{ newton}$$

(*b*) Value of F_g

As seen from Fig. 43.13, $F_g = W \sin\theta = Mg \sin\theta$

In railway practice, gradient is expressed as the rise (in metres) a track distance of 100 m and is called percentage gradient.

$$\therefore \quad \%\, G = \frac{BC}{AC/100} = 100\,\frac{BC}{AC} = 100 \sin\theta$$

Substituting the value of $\sin\theta$ in the above equation, we get

$$F_g = Mg\, G/100 = 9.8 \times 10^{-2}\, MG$$

(*i*) When M is in kg, $F_g = 9.8 \times 10^{-2}\, MG$ newton

(*ii*) When M is given in tonne, then

$$F_g = 9.8 \times 10^{-2}\, (1000\, M)\, G = 98\, MG \text{ newton}$$

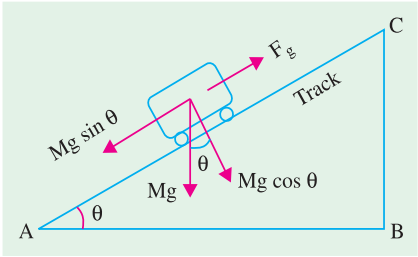


Fig. 43.13

(*c*) Value of F_r

Train resistance comprises all those forces which oppose its motion. It consists of mechanical resistance and wind resistance. Mechanical resistance itself is made up of internal and external resistances. The internal resistance comprises friction at journals, axles, guides and buffers etc. The external resistance consists of friction between wheels and rails and flange friction etc. Mechanical resistance is almost independent of train speed but depends on its weight. The wind friction varies directly as the square of the train speed.

If r is specific resistance of the train *i.e.* resistance offered per unit mass of the train, then $F_r = M.r$.

(*i*) If r is in newton per kg of train mass and M is the train mass in kg, then

$$F_r = M.r \text{ newton}$$

(*ii*) If r is in newton per tonne train mass (N/t) and M is in tonne (t), then

$$F_r = M \text{ tonne} \times r = M_r \text{ newton}^*$$

Hence, expression for total tractive effort becomes

$$F_t = F_a \pm F_g + F_r = (277.8\, \alpha\, M_e \pm 98\, MG + Mr) \text{ newton}$$

Please remember that here M is in tonne, α in km/h/s, G is in metres per 100 m of track length (*i.e.* % G) and r is in newton/tonne (N/t) of train mass.

The positive sign for F_g is taken when motion is along an ascending gradient and negative sign when motion is along a descending gradient.

43.38. Power Output from Driving Axles

If F_t is the tractive effort and v is the train velocity, then

$$\text{output power} = F_t \times v$$

(*i*) If F_t is in newton and v in m/s, then

$$\text{output power} = F_t \times v \text{ watt}$$

(*ii*) If F_t is in newton and v is in km/h , then converting v into m/s, we have

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* If r is in kg (wt) per tonne train mass and M is in tonne, then $F_r = M$ tonne $\times (r \times 9.8)$ newton/tonne $= 9.8\, Mr$ newton.

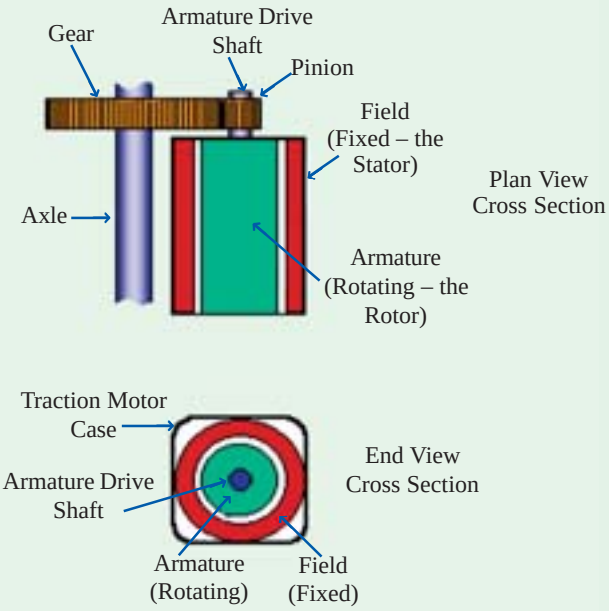


Diagram Shows how a DC motor drives the axle through a pinion and gearwheel

Traction motor schematic diagram

$$\text{output power} = F_t \times \left(\frac{1000}{3600}\right) v \text{ watt} = \frac{F_t\, v}{3600} \text{ kW}$$

If η is the efficiency of transmission gear, then power output of motors is

$$= F_t \,.\, v/\eta \text{ watt} \qquad \text{— } v \text{ in m/s}$$

$$= \frac{F_t\, v}{3600\, \eta} \text{ kW} \qquad \text{— } v \text{ in km/h}$$

43.39. Energy Output from Driving Axles

Energy (like work) is given by the product of power and time.

$$E = (F_t \times v) \times t = F_t \times (v \times t) = F_t \times D$$

where D is the distance travelled in the direction of tractive effort.

Total energy output from driving axles for the run is

E = energy during acceleration + energy during free run

As seen from Fig. 43.11

$$E = F_t \times \text{area } OAD + F_t' \times \text{area } ABED = F_t \times \frac{1}{2} V_m t_1 + F_t' \times \frac{1}{2} V_m t_2$$

where F_t is the tractive effort during accelerating period and F_t' that during free-running period. Incidentally, F_t will consist of all the three components given in Art. 43.37 whereas F_t' will consist of (98 MG + Mr) provided there is an ascending gradient.

43.40. Specific Energy Output

It is the energy output of the driving wheel expressed in watt-hour (Wh) per tonne-km (t-km) of

the train. It can be found by first converting the energy output into Wh and then dividing it by the mass of the train in tonne and route distance in km.

Hence, unit of specific energy output generally used in railway work is : Wh/tonne-km (Wh/t-km).

43.41. Evaluation of Specific Energy Output

We will first calculate the total energy output of the driving axles and then divide it by train mass in tonne and route length in km to find the specific energy output. It will be presumed that :

(i) there is a gradient of G throughout the run and

(ii) power remains ON upto the end of free run in the case of trapezoidal curve (Fig. 43.11) and upto the accelerating period in the case of quadrilateral curve (Fig. 43.12).

Now, output of the driving axles is used for the following purposes :

1. for accelerating the train **2.** for overcoming the gradient

3. for overcoming train resistance.

(*a*) Energy required for train acceleration (E_a)

As seen from trapezoidal diagram of Fig. 43.11,

$$E_a = F_a \times \text{distance } OAD = 277.8\ \alpha\ M_e \times \frac{1}{2}\ V_m.t_1 \text{ joules}$$

$$= 277.8\ \alpha\ M_e \times \frac{1}{2}\ V_m \times \frac{V_m}{\alpha} \text{ joules} \qquad \left(\because t_1 = \frac{V_m}{\alpha}\right)$$

$$= 277.8\ \alpha\ M_e \times \left[\frac{1}{2}.\frac{V_m \times 1000}{3600} \times \frac{V_m}{\alpha}\right] \text{ joules}$$

It will be seen that since V_m is in km/h, it has been converted into m/s by multiplying it with the conversion factor of (1000/3600). In the case of (V_m /t), conversion factors for V_m and a being the same, they cancel out. Since 1 Wh = 3600 J.

$$\therefore \quad E_a = 277.8\ \alpha\ M_e \left[\frac{1}{2}.\frac{V_m \times 1000}{3600} \times \frac{V_m}{\alpha}\right] \text{Wh} = 0.01072\ \frac{V_m^2}{M_e}\ \text{Wh}$$

(*b*) Energy required for over coming gradient (E_g)

$$E_g = F_g \times D'$$

where 'D'' is the ***total distance over which power remains*** ON. Its maximum value equals the distance represented by the area *OABE* in Fig. 43.11 *i.e.* from the start to the end of free-running period in the case of trapezoidal curve [as per assumption (*i*) above].

Substituting the value of F_g from Art. 43.37, we get

$$E_g = 98\ MG.\ (1000\ D') \text{ joules} = 98{,}000\ MGD' \text{ joules}$$

It has been assumed that D' is in km.

When expressed in Wh, it becomes

$$E_g = 98{,}000\ MGD' \frac{1}{3600}\ \text{Wh} = 27.25\ MGD'\ \text{Wh}$$

(*c*) Energy required for overcoming resistance (E_r)

$$E_r = F_r \times D' = M\ .\ r \times (1000\ D') \text{ joules} \qquad \text{— } D' \text{ in km}$$

$$= \frac{1000\ Mr\ D'}{3600}\ \text{Wh} = 0.2778\ Mr\ D'\ \text{Wh} \qquad \text{— } D' \text{ in km}$$

$\therefore$ total energy output of the driving axles is

$$E = E_a + E_g + E_r$$
$$= (0.01072\, V_m^2/M_e + 27.25\, MGD' + 0.2778\, Mr\, D' \text{ Wh}$$

Specific energy output

$$E_{spo} = \frac{E}{M \times D} \quad \text{— } D \text{ is the } \textit{total} \text{ run length}$$
$$= \left(0.01072 \frac{V_m^2}{D} . \frac{M_e}{M} + 27.25\, G \frac{D'}{D} + 0.2778\, r \frac{D'}{D}\right) \text{Wh/t-km}$$

It may be noted that *if there is no gradient*, then

$$E_{spo} = \left(0.01072 \frac{V_m^2}{D} . \frac{M_e}{M} + 0.2778\, r \frac{D'}{D}\right) \text{Wh/t-km}$$

Alternative Method

As before, we will consider the trapezoidal speed/time curve. Now, we will calculate energy output not **force-wise** but **period-wise**.

(*i*) Energy output during accelerating period

E_a = $F_t \times$ distance travelled during accelerating period
= $F_t \times$ area OAD —Fig. 43.11

$$= F_t \times \frac{1}{2} V_m\, t_1 = \frac{1}{2} F_t . V_m . \frac{V_m}{\alpha}$$
$$= \frac{1}{2} . F_t \left(\frac{1000}{3600} . V_m\right) . \frac{V_m}{\alpha} \text{ joules}$$
$$= \frac{1}{2} . F_t \left(\frac{1000}{3600} . V_m\right) . \frac{V_m}{\alpha} . \frac{1}{3600} \text{ Wh}$$

Substituting the value of F_t, we get

$$E_a = \frac{1000}{(3600)^2} . \frac{V_m^2}{2\alpha} (277.8\, \alpha\, M_e + 98\, MG + Mr) \text{ Wh}$$

It must be remembered that during this period, ***all the three forces are at work*** (Art. 43.37)

(*ii*) Energy output during free-running period

Here, work is required only against two forces *i.e.* gravity and resistance (as mentioned earlier).

Energy E_{fr} = $F_t' \times$ area $ABED$ —Fig. 43.11

$$= F_t' \times (V_m \times t_2) = F_t' \times \left(\frac{1000}{3600} V_m\right) . t_2 \text{ joules}$$
$$= F_t' \times \left(\frac{1000}{3600} V_m\right) \times t_2 \times \frac{1}{3600} \text{ Wh} = \left(\frac{1000}{3600}\right) F_t' \times V_m\, t_2 . \frac{1}{3600} \text{ Wh}$$
$$= \left(\frac{1000}{3600}\right) . F_t' \times D_{fr} \text{ Wh} = \left(\frac{1000}{3600}\right) (98\, MG + Mr)\, D_{fr} \text{ Wh}$$

where D_{fr} is the distance in km travelled during the free-running period*

Total energy required is the sum of the above two energies.

$\therefore$ $E = E_a + E_{fr}$

$$= \frac{1000}{(3600)^2} \frac{V_m^2}{2\alpha} (277.8\, \alpha\, .\, M_e + 98\, MG + Mr) + \frac{1000}{3600} (98\, MG + Mr)\, D_{fr} \text{ Wh}$$

---

* D_{fr} = velocity in km/h × time in hours
= $V_m \times (t_2/3600)$ because times are always taken in seconds.

$$= \frac{1000}{(3600)^2}\frac{V_m^2}{2\alpha} 277.8\,\alpha M_e + \frac{1000}{(3600)^2}\cdot\frac{V_m^2}{2\alpha}(98\,MG + Mr) + \frac{1000}{3600}(98\,MG + Mr)\,.\,D_{fr}\ \text{Wh}$$

$$= 0.01072\,V_m^2\,.\,M_e + \frac{1000}{3600}(98\,MG + Mr)\left(\frac{V_m^2}{2\alpha\times 3600} + D_{fr}\right)\text{Wh}$$

Now, $$\frac{V_m^2}{2\alpha\times 3600} = \frac{1}{2}\left(\frac{V_m}{3600}\right)\cdot\frac{V_m}{\alpha} = \frac{1}{2}\left(\frac{V_m}{3600}\right)\cdot t_1$$

= distance travelled during accelerating period *i.e.* D_a

$\therefore$ $$E = 0.01072\,V_m^2\,.\,M_e + \frac{1000}{3600}(98\,MG + Mr)(D_a + D_{fr})\ \text{Wh}$$

$$= 0.01072\,V_m^2\,.\,M_e + (27.25\,MG + 0.2778\,Mr)\,D'\ \text{Wh}$$

It is the same expression as found above.

43.42. Energy Consumption

It equals the total energy input to the traction motors from the supply. It is usually expressed in Wh which equals 3600 J. It can be found by dividing the energy output of the driving wheels with the combined efficiency of transmission gear and motor.

$$\therefore \quad \text{energy consumption} = \frac{\text{output of driving axles}}{\eta_{motor}\times\eta_{gear}}$$

43.43. Specific Energy Consumption

It is the energy consumed (in Wh) per tonne mass of the train per km length of the run.

Specific energy consumption,

$$E_{spc} = \frac{\text{total energy consumed in Wh}}{\text{train mass in tonne}\times\text{run length in km}} = \frac{\text{specific energy output}}{\eta}$$

where η = overall efficiency of transmission gear and motor = $\eta_{gear}\times\eta_{motor}$

As seen from Art. 43.41, specific energy consumption is

$$E_{spc} = \left(0.01072\,.\,\frac{V_{m^2}}{\eta D}\,.\,\frac{M_e}{M} + 27.25\,\frac{G}{\eta}\,.\,\frac{D'}{D} + 0.2778\,\frac{r}{\eta}\,.\,\frac{D'}{D}\right)\text{Wh/t-km}$$

If no gradient is involved, then specific enrgy consumption is

$$E_{spc} = \left(0.01072\,.\,\frac{V_{m^2}}{\eta D}\,.\,\frac{M_e}{M} + 0.2778\,\frac{r}{\eta}\,.\,\frac{D'}{D}\right)\text{Wh/t-km}$$

The specific energy consumption of a train running at a given schedule speed is influenced by

1. Distance between stops **2.** Acceleration **3.** Retardation **4.** Maximum speed **5.** Type of train and equipment **6.** Track configuration.

43.44. Adhesive Weight

It is given by the total weight carried on the driving wheels. Its value is $W_a = x\,W$, where W is dead weight and x is a fraction varying from 0.6 to 0.8.

43.45. Coefficient of Adhesion

Adhesion between two bodies is due to interlocking of the irregularities of their surfaces in contact. The adhesive weight of a train is ***equal to the total weight to be carried on the driving***

wheels. It is less than the dead weight by about 20 to 40%.

If $x = \dfrac{\text{adhesive weight, } W_a}{\text{dead weight } W}$, then, $W_a = x\,W$

Let, F_t = tractive effort to slip the wheels

or

= maximum tractive effort possible without wheel slip

Coefficient of adhesion, $\mu_a = F_t / W_a$

$\therefore \quad F_t = \mu_a W_a = \mu_a xW = \mu_a x Mg$

If M is in tonne, then

$$F_t = 1000 \times 9.8\, x\, \mu_a M = 9800\, \mu_a\, x\, M \text{ newton}$$

It has been found that tractive effort can be increased by increasing the motor torque but only upto a certain point. Beyond this point, any increase in motor torque does not increase the tractive effort but merely causes the driving wheels to slip. It is seen from the above relation that for increasing F_t, it is not enough to increase the kW rating of the traction motors alone but the weight on the driving wheels has also to be increased.

Adhesion also plays an important role in braking. If braking effort exceeds the adhesive weight of the vehicle, skidding takes place.

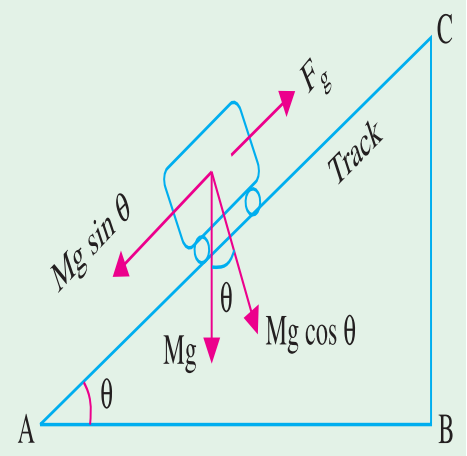


Fig. 43.14

43.46. Mechanism of Train Movement

The essentials of driving mechanism in an electric vehicle are illustrated in Fig. 43.14. The armature of the driving motor has a pinion which meshes with the gear wheel keyed to the axle of the driving wheel. In this way, motor torque is transferred to the wheel through the gear.

Let, T = torque exerted by the motor

F_1 = tractive effort at the pinion

F_t = tractive effort at the wheel

γ = gear ratio

Here, d_1, d_2 = diameters of the pinion and gear wheel respectively

D = diameter of the driving wheel

η = efficiency of power transmission from the motor to driving axle

Now, $T = F_1 \times d_1/2$ or $F_1 = 2T/d_1$

Tractive effort transferred to the driving wheel is

$$F_t = \eta F_1 \left(\frac{d_2}{D}\right) = \eta \cdot \frac{2T}{d_1}\left(\frac{d_2}{D}\right) = \eta\, T \left(\frac{2}{D}\right)\left(\frac{d_2}{d_1}\right) = 2\,\gamma\eta\,\frac{T}{D}$$

For obtaining motion of the train without slipping, $F_t \le \mu_a W_a$ where μ_a is the coefficient of adhesion (Art. 43.45) and W_a is the adhesive weight.

Example 43.5. *The peripheral speed of a railway traction motor cannot be allowed to exceed 44 m/s. If gear ratio is 18/75, motor armature diameter 42 cm and wheel diameter 91 cm, calculate the limiting value of the train speed.*

Solution. Maximum number of revolutions per second made by armature

$$= \frac{\text{armature velocity}}{\text{armature circumference}} = \frac{44}{0.42\,\pi} = \frac{100}{3} \text{ rps.}$$

Maximum number of revolutions per second made by the driving wheel

$$= \frac{100}{3} \times \frac{18}{75} = 8 \text{ rps.}$$

Maximum distance travelled by the driving wheel in one second

$$= 8 \times 0.91\,\pi \text{ m/s} = 22.88 \text{ m/s}$$

Hence, limiting value of train speed

$$= 22.88 \text{ m/s} = 22.88 \times 18/5 = \mathbf{82 \text{ km/h}}$$

Example 43.6. *A 250-tonne motor coach driven by four motors takes 20 seconds to attain a speed of 42 km/h, starting from rest on an ascending gradient of 1 in 80. The gear ratio is 3.5, gear efficiency 92%, wheel diameter 92 cm train resistance 40 N/t and rotational inertia 10 percent of the dead weight. Find the torque developed by each motor.*

Solution. $F_t = (277.8 \times M_e a + 98\,MG + Mr)$ newton

Now, $\alpha = V_m / t_1 = 42/20 = 2.1$ km/h/s Since gradient is 1 in 80, it becomes 1.25 in 100. Hence, percentage gradient $G = 1.25$. Also, $M_e = 1.1\,M$. The tractive effort at the driving wheel is

$$F_t = 277.8 \times (1.1 \times 250) \times 2.1 + 98 \times 250 \times 1.25 + 250 \times 40$$
$$= 160{,}430 + 30{,}625 + 10{,}000 = 201{,}055 \text{ N}$$

Now, $F_t = 2\gamma\eta T/D$ or $201{,}055 = 2 \times 3.5 \times 0.92 \times T/0.92$ $\therefore T = 28{,}744$ N–m

Torque developed by each motor = 28,744/4 = **7,186 N–m**

Example 43.7. *A 250-tonne motor coach having 4 motors, each developing a torque of 8000 N-m during acceleration, starts from rest. If up-gradient is 30 in 1000, gear ratio 3.5, gear transmission efficiency 90%, wheel diameter 90 cm, train resistance 50 N/t, rotational inertia effect 10%, compute the time taken by the coach to attain a speed of 80 km/h.*

If supply voltage is 3000 V and motor efficiency 85%, calculate the current taken during the acceleration period.

Solution. Tractive effort (Art. 43.46) at the wheel

$$= 2\gamma\eta T/D = 2 \times 3.5 \times 0.9 \times (8000 \times 4)/0.9 = 224{,}000 \text{ N}$$

Also, $F_t = (277.8\,a\,M_e + 98\,MG + Mr)$ newton

$$= (277.8 \times (1.1 \times 250) \times a + 98 \times 250 \times 3 + 250 \times 50 \text{ N}$$
$$= (76{,}395\,a + 86{,}000) \text{ N}$$

Equating the two expression for tractive effort, we get

$$224{,}000 = 76{,}395\,a + 86{,}000 \; ; \; a = 1.8 \text{ km/h/s}$$

Time taken to achieve a speed of 80 km/h is

$$t_1 = V_m / a = 80/1.8 = \mathbf{44.4 \text{ second}}$$

Power taken by motors (Art. 41.36) is

$$= \frac{F_t \times v}{\eta} = \frac{F_t \times V_m}{\eta} = F_t \cdot \left(\frac{1000}{3600}\right) \cdot \frac{V_m}{\eta} \text{ watt}$$
$$= 22{,}000 \times 0.2778 \times 80/0.85 = 58.56 \times 10^5 \text{ W}$$

Total current drawn $= 55.56 \times 10^5/3000 = 1952$ A

Current drawn/motor $= 1952/4 = \mathbf{488 \text{ A.}}$

Example 43.8. *A goods train weighing 500 tonne is to be hauled by a locomotive up an ascending gradient of 2% with an acceleration of 1 km/h/s. If coefficient of adhesion is 0.25, train resistance 40 N/t and effect of rotational inertia 10%, find the weight of locomotive and number of axles if load is not to increase beyond 21 tonne/axle.*

Solution. It should be clearly understood that a train weighing 500 tonne has a mass of 500 (Art. 43.33).

Tractive effort required is

$$F_t = (277.8\, a\, M_e + 98\, MG + Mr) \text{ newton} = M\left(277.8\, a\, .\frac{M_e}{M} + 98\, G + r\right) \text{ newton}$$

$$= M\,(277.8 \times 1 \times 1.1 + 98 \times 2 + 40) = 541.6\, M \text{ newton}$$

If M_L is the mass of the locomotive, then

$$F_t = 541.6\,(M + M_L) = 541.6\,(500 + M_L) \text{ newton}$$

Maximum tractive effort (Art. 43.45) is given by

$$F_t = 1000\, \mu_a\, M_L\, .\, g = 1000 \times 0.25\, M_L \times 9.8 \qquad \text{— } x = 1$$

$$\therefore \quad 541.6\,(500 + M_L) = 1000 \times 0.25\, M_L \times 9.8 \qquad \therefore \quad M_L = \mathbf{142 \text{ tonne}}$$

Hence, weight of the locomotive is 142 tonne. Since, weight per axle is not to exceed 21 tonne, the number of axles required is = 142/21 = 7.

Example 43.9. *An electric locomotive weighing 100 tonne can just accelerate a train of 500 tonne (trailing weight) with an acceleration of 1 km/h/s on an up-gradient of 0.1%. Train resistance is 45 N/t and rotational inertia is 10%. If this locomotive is helped by another locomotive of weight 120 tonne, find :*

(i) *the trailing weight that can now be hauled up the same gradient under the same conditions.*

(ii) *the maximum gradient, if the trailing hauled load remains unchanged.*

Assume adhesive weight expressed as percentage of total dead weight as 0.8 for both locomotives. **(Utilization of Elect. Power ; AMIE, Summer)**

Solution. Dead weight of the train and locomotive combined = (100 + 500) = 600 tonne. Same is the value of the dead mass.

$$\therefore \quad F_t = (277.8a\, M_e + 98\, MG + Mr) \text{ newton}$$

$$= 277.8 \times 1 \times (1.1 \times 600) + 98 \times 600 \times 0.1 + 600 \times 45 = 216{,}228 \text{ N}$$

Maximum tractive effort (Art. 43.45) of the first locomotive

$$= 9800x\, \mu_a M_L = 9800 \times 0.8 \times \mu_a \times 1000 = 784{,}000\, \mu_a$$

$$\therefore \quad 784{,}000\mu_a = 216{,}288\ ; \qquad \mu_a = 0.276$$

With two locomotive, $M_L' = (100 + 120) = 220$ tonne

$$\therefore \quad F_t = 9800\, x\, \mu_a M_L' = 9800 \times 0.8 \times 0.276 \times 220 = 476{,}045 \text{ N}$$

(i) Let trailing load which the two combined locomotives can haul be M tonne. In that case, total dead mass becomes $M = (100 + 120 + M) = (220 + M)$ tonne. Tractive effort required is

$$= (277.8\, M_e' + 98\, M'G + M'r) \text{ newton}$$

$$= M'\,(277.8 \times 1 \times 1.1 + 98 \times 0.1 + 45) = 360.4\, M' \text{ newton}$$

$$\therefore \quad 360.4\, M' = 476{,}045;\ M' = 1321 \text{ tonne} \qquad \therefore \quad \text{trailing load, } M = 1321 - 220 = \mathbf{1101 \text{ tonne}}$$

(ii) Total hauled load = 500 + 100 + 120 = 720 tonne

Let G be the value of maximum percentage gradient. Then

$$F_t = (277.8\, a\, M_e + 98MG + Mr) \text{ newton} = M\left(277.8\, a\, \frac{M_e}{M} + 98G + r\right) \text{ newton}$$

$= 720\,(277.8 \times 1 \times 1.1 + 98G + 45)$ newton $= (252{,}418 + 70{,}560\,G)$ newton

Equating it with the combined tractive effort of the two locomotive as calculated above, we have,

$476{,}045 = 252{,}418 + 70{,}560\,G \quad \therefore \quad G =$ **3.17 percent**

Example 43.10. *The average distance between stops on a level section of a railway is 1.25 km. Motor-coach train weighing 200 tonne has a schedule speed of 30 km/h, the duration of stops being 30 seconds. The acceleration is 1.9 km/h/s and the braking retardation is 3.2 km/h/s. Train resistance to traction is 45 N/t. Allowance for rotational inertia is 10%. Calculate the specific energy output in Wh/t-km. Assume a trapezoidal speed/time curve.* **(Elect. Power, Bombay Univ.)**

Solution. $\alpha = 1.9 \times 5/18 = 9.5/18$ m/s^2 : $\beta = 3.2 \times 5/18 = 8/9$ m/s^2

$K = (\alpha + \beta)/2\alpha\beta = 1.5$; $D = 1.25$ km $= 1250$ m

Schedule time $= 1.25 \times 3600/30 = 150$ s. Running time $= 150 - 30 = 120$ s

$$V_m = \frac{t - \sqrt{t^2 - 4KD}}{2K} = \frac{120 - \sqrt{120^2 - 4 \times 1.5 \times 1250}}{2 \times 1.5} = 10.4 \text{ m/s} = \textbf{37.4 km/h}$$

Braking distance $D = V_m^2/2\beta = 10.42/2 \times (8/9) = 0.06$ km

$\therefore \quad D' = D$ – braking distance $= 1.25 - 0.06 = 1.19$ km

$$\text{Specific energy output} = 0.01072\,\frac{V_m^2}{D} \cdot \frac{M_e}{M} + 0.2778\,r\,\frac{D'}{D}$$

$$= 0.01072 \times \frac{37.4^2}{1.25} \times 1.1 + 0.2778 \times 50 \times \frac{1.19}{1.25} \text{ Wh/t-km}$$

$= 16.5 + 13.2 =$ **29.7 Wh/t-km**

Example. 43.11. *A 300-tonne EMU is started with a uniform acceleration and reaches a speed of 40 km/h in 24 seconds on a level track. Assuming trapezoidal speed/time curve, find specific energy consumption if rotational inertia is 8%, retardation is 3 km/h/s, distance between stops is 3 km, motor efficiency is 0.9 and train resistance is 40 N/tonne.*

(Elect. Traction, AMIE Summer)

Solution. First of all, let us find D' – the distance upto which energy is consumed from the supply. It is the distance travelled upto the end of free-running period. It is equal to the total distance minus the distance travelled during braking.

Braking time, $t_2 = V_m/\beta = 40/3 = 13.33$ second

Distance travelled during braking period

$$= \frac{1}{2} V_m t_3 = \frac{1}{2} \times 40 \times \left(\frac{13.33}{3600}\right) = 0.074 \text{ km}$$

$\therefore \quad D' = D$ – braking distance $= 3 - 0.074 = 2.926$ km

Since, $M_e/M = 1.08$, using the relation derived in Art. 43.43, we get the value of specific energy consumption as

$$= \left(0.01072\,\frac{V_m^2}{\eta D} \cdot \frac{M_e}{M} + 0.2778\,\frac{r}{\eta}\,\frac{D'}{D}\right) \text{ Wh/t-km}$$

$$= \left(0.01072 \times \frac{40^2}{0.9 \times 3} \times 1.08 + 0.2778 \times \frac{49}{0.9} \times \frac{2.926}{3}\right) = \textbf{21.6 Wh/t-km.}$$

Example 43.12. *An electric train accelerates uniformly from rest to a speed of 50 km/h in 25 seconds. It then coasts for 70 seconds against a constant resistance of 60 N/t and is then braked to rest with uniform retardation of 3.0 km/h/s in 12 seconds. Compute*

(i) *uniform acceleration* ***(ii)*** *coasting retardation*

(iii) *schedule speed if station stops are of 20-second duration*

Allow 10% for rotational inertia. How will the schedule speed be affected if duration of stops is reduced to 15 seconds, other factors remaining the same ?

Solution. ***(i)*** As seen from Fig. 43.15, $\alpha = V_1/t_1 = 50/25 =$ **2 km/h/s**

(ii) The speeds at points *B* and *C* are connected by the relation

$$0 = V_2 + \beta t_3 \quad \text{or} \quad 0 = V_2 + (-3) \times 12 \qquad \therefore \quad V_2 = 36 \text{ km/h}$$

Coasting retardation, $\beta_c = (V_2 - V_1)/t_2 = (36 - 50)/70 =$ **– 0.2 km/h/s**

(iii) Distance travelled during acceleration

$$= \frac{1}{2} V_1 t_1 = \frac{1}{2} \times 50 \frac{\text{km}}{\text{h}} \times \frac{25}{3600} \text{ h}$$

$$= 0.174 \text{ km}$$

Distance travelled during coasting can be found from the relation

$$V_{22} - V_{12} = 2 \beta_c D \qquad \text{or}$$

$$D = (362 - 502)/2 \times -0.2 \times 3600$$

$$= 0.836 \text{ km}$$

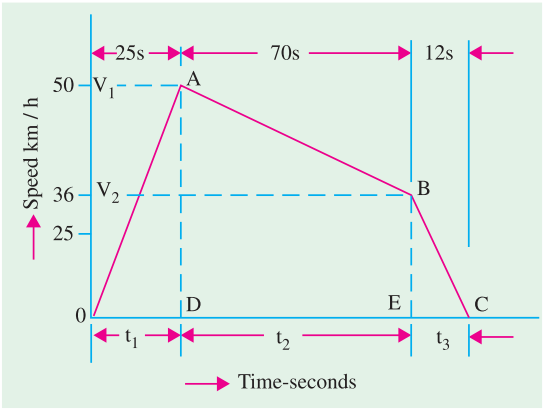


Fig. 43.15

Distance covered during braking

$$= \frac{1}{2} V_2 t_3 = \frac{1}{2} \times 36 \frac{\text{km}}{\text{h}} \times \frac{12}{3600} \text{ h}$$

$$= 0.06 \text{ km}$$

Total distance travelled from start to stop

$$= 0.174 + 0.836 + 0.06 = 1.07 \text{ km}$$

Total time taken including stop time

$$= 25 + 70 + 12 + 20 = 127 \text{ second}$$

Schedule speed $= 1.07 \times 3600/127 =$ **30.3 km/s**

Schedule speed with a stop of 15 s is $= 1.07 \times 3600/122 =$ **31.6 km/h**

Example 43.13. *A 350-tonne electric train runs up an ascending gradient of 1% with the following speed/time curves :*

1. *uniform acceleration of 1.6 km/h/s for 25 seconds*
2. *constant speed for 50 seconds*
3. *coasting for 30 seconds*
4. *braking at 2.56 km/h/s to rest.*

Compute the specific energy consumption if train resistance is 50 N/t, effect of rotational inertia 10%, overall efficiency of transmission gear and motor, 75%.

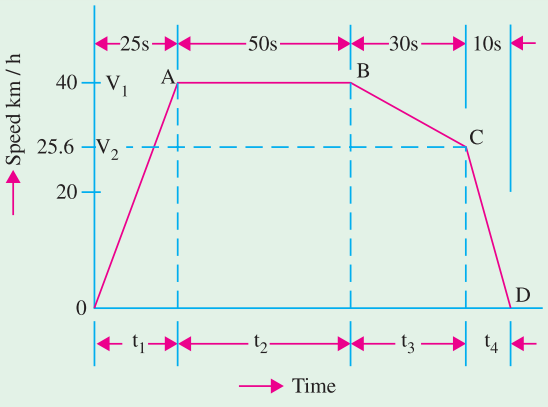


Fig. 43.16

Solution. As seen from Fig. 43.16, $V_1 = \alpha \, . \, t_1 = 1.6 \times 25 = 40$ km/h

Tractive force during coasting is

$$F_t = (98 MG + M.r)$$

$$= M (98 \times 1 + 50)$$

$$= 148 M \text{ newton}$$

Also, $F_t = 277.8 \, M_e \, \beta_c$ during coasting. Equating the two expressions, we get $277.8 \, M_e$

. $\beta_c = 148\ M$

$$\therefore\ \beta_c = \frac{148}{277.8} \times \frac{M}{M_e} = \frac{148}{277.8} \times \frac{1}{1.1}\ ;$$

$$\beta_c = 0.48 \text{ km/h/s}$$

Now, $V_2 = V_1 + \beta_c t_3$

$= 40 + (-0.48) \times 30$

$= 25.6$ km/h

$t_4 = V_2/\beta = 25.6/2.56 = 10$ second

Distance travelled during acceleration period $= \frac{1}{2} \times 40 \frac{\text{km}}{\text{h}} \times \frac{25}{3600}\ \text{h} = 0.139$ km

Distance travelled during constant speed period is $= V_1 \times t_2 = 40 \times 50/3600 = 0.555$ km

Distance travelled during coasting $= \left(\frac{V_1 + V_2}{2}\right) \times t_3 = \frac{40 + 25.6}{2} \times \frac{30}{3600} = 0.273$ km

Distance travelled during braking $= \frac{1}{2} V_2 t_4 = \frac{1}{2} \times 25.6 \times \frac{10}{3600} = 0.035$ km

Total distance between stops $= 0.139 + 0.555 + 0.273 + 0.035 = 1.002$ km

Distance travelled during acceleration and free-running period is

$D' = 0.139 + 0.555 = 0.694$ km

Specific energy consumption (Art. 43.43) is

$$= \left(0.01072 \frac{V_m^2}{\eta D} \cdot \frac{M_e}{M} + 27.25 \frac{G}{\eta} \cdot \frac{D'}{D} + 0.2778 \frac{r}{\eta} \cdot \frac{D'}{D}\right) \text{Wh/t-km}$$

$$= \left(0.01072 \times \frac{40^2}{0.75 \times 1.002} \times 1.1 + 27.25 \times \frac{1}{0.75} \times \frac{0.694}{1.002} + 0.2778 \times \frac{50}{0.75} \times \frac{0.694}{1.002}\right)$$

$= 25.1 + 25.2 + 12.8 =$ **63.1 Wh/t-km**

Example 43.14. *An ore-carrying train weighing 5000 tonne is to be hauled down a gradient of 1 : 50 at a maximum speed of 30 km/h and started on a level track at an acceleration of 0.29 km/h/s. How many locomotives, each weighing 75 tonne, will have to be employed ?*

Train resistance during starting = 29.4 N/t, Train resistance at 30 km/h = 49 N/t

Coefficient of adhesion = 0.3, Rotational inertia = 10%

(Utilization of Elect. Power, AMIE)

Solution. Downward force due to gravity

$= Mg \sin\theta = (5000 \times 1000) \times 9.8 \times 1/50 = 980{,}000$ N

Train resistance $= 49 \times 5000 = 245{,}000$ N

Braking force to be supplied by brakes $= 980{,}000 - 245{,}000 = 735{,}000$ N

Max. braking force which one locomotive can provide

$= 1000\ \mu_a\ Mg$ newton —M in tonne

$= 1000 \times 0.3 \times 75 \times 9.8 = 220{,}500$ N

No. of locomotives required for braking $= 735{,}000/220{,}500 = 3.33$

Since fraction is meaningless, it means that 4 locomotives are needed.

Tractive effort required to haul the train on level track

$= (277.8\ \alpha\ M_e + Mr)$ newton

$= 277.8 \times (5000 \times 1.1) \times 0.29 + 5000 \times 29.4 = 590{,}090$ N

No. of locomotives required = 590,090/220,500 = 2.68 ≅ **3**

It means that **4 locomotives** are enough to look after braking as well as starting.

Example 43.15. *A 200-tonne electric train runs according to the following quadrilateral speed/time curve:*

1. *uniform acceleration from rest at 2 km/h/s for 30 seconds*
2. *coasting for 50 seconds*
3. *duration of braking : 15 seconds*

If up-gradient is = 1%, train resistance = 40 N/t, rotational inertia effect = 10%, duration of stops = 15 s and overall efficiency of gear and motor = 75%, find

(i) *schedule speed* ***(ii)*** *specific energy consumption* ***(iii)*** *how will the value of specific energy consumption change if there is a down-gradient of 1% rather than the up-gradient ?*

(Electric Traction Punjab Univ. 1993)

Solution. $V_1 = \alpha . t_1 = 2 \times 30 = 60$ km/h/s

During coasting, gravity component and train resistance will cause coasting retardation β_c.

Retarding force = $(98\ MG + Mr)$ newton = $(98 \times 200 \times 1.0 + 200 \times 40) = 27{,}600$ N

As per Art, 43.37, $277.8\ M_e . \beta_c = 27{,}600$ or $277.8 \times (200 \times 1.1) \times \beta_c = 27{,}600$

$\therefore \quad \beta_c = 0.45$ km/h/s

Now, $V_2 = V_1 + \beta_c\ t_2$ or $V_2 = 60 + (-0.45) \times 50 = 37.5$ km/h

Braking retardation $\beta = V_2/t_3 = 37.5/15 = 2.5$ km/h/s

Distance travelled during acceleration (area *OAD* in Fig. 43.17)

$$= \frac{1}{2} V_1 t_1 = \frac{1}{2} \times 60 \times \left(\frac{30}{3600}\right) = 0.25 \text{ km}$$

Distance travelled during coasting

$$= \text{area } ABED = \left(\frac{V_1 + V_2}{2}\right) \times t_2 = \left(\frac{60 + 37.5}{2}\right) \times \frac{50}{3600} = 0.677 \text{ km}$$

Distance travelled during braking

$$= \text{area } BCE = \frac{1}{2} V_2 t_3 = \frac{1}{2} \times 37.5 \times \frac{15}{3600} = 0.078 \text{ km}$$

Total distance travelled, $D = 0.25 + 0.677 + 0.078 = 1.005$ km

Total schedule time = 30 + 50 + 15 + 15 = 110 s

(i) $\therefore$ Schedule speed $= \dfrac{1.005}{110/3600} =$ **32.9 km/h**

(ii) As per Art. 43.43, specific energy consumption

$$= \left(0.01072 \frac{V_m^2}{\eta D} . \frac{M_e}{M} + 27.25 \frac{G}{\eta} . \frac{D'}{D} + 0.2778 . \frac{r}{\eta} . \frac{D'}{D}\right) \text{Wh/t-km}$$

$$= \left(0.01072 \times \frac{60^2}{0.75 \times 1.005} \times 1.1 + 27.25 \times \frac{1}{0.75} \times \frac{0.25}{1.005} + 0.2778 \times \frac{40}{0.75} \times \frac{0.25}{1.005}\right) \text{Wh/t-km}$$

(iii) the speed/time curve for this case is shown in Fig. 43.18. As before, $V_1 = 60$ km/h. Here, we will take $G = -1\%$

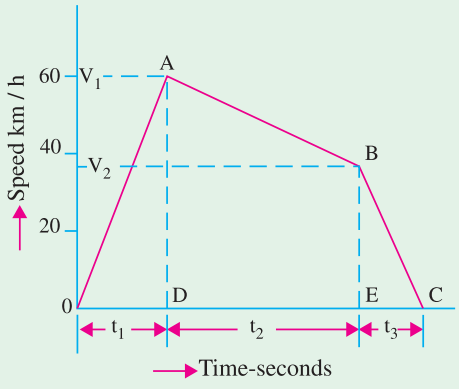


Fig. 43.17

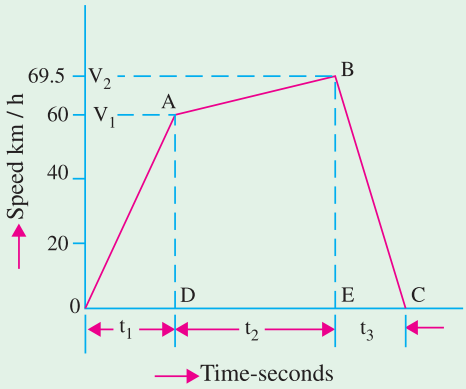


Fig. 43.18

$\therefore$ Retarding force = $(98\,MG + Mr)$ newton = $98 \times 200 \times (-1.0) + 200 \times 40 = -11{,}600$ N

The negative sign indicates that instead of being a retarding force, it is, in fact, an accelerating force. If α_c is the acceleration produced, then

$$11{,}600 = 277.8 \times (200 \times 1.1) \times \alpha_c$$

$$\alpha_c = 0.19 \text{ km/h/s}$$

Also, $V_2 = V_1 + \alpha_c t_2 = 60 + 0.19 \times 50 = 69.5$ km/h

$\beta = V_2 / t_3 = 69.5/15 = 4.63$ km/h/s

Distance travelled during acceleration = 0.25 km —as before

Distance travelled during coasting $= \dfrac{60 + 69.5}{2} \times \dfrac{50}{3600} = 0.9$ km

Distance travelled during braking $= \dfrac{1}{2} \times 69.5 \times \dfrac{15}{3600} = 0.145$ km

$\therefore$ $D = 0.25 + 0.9 + 0.145 = 1.295$ km

Hence, specific energy consumption is

$$= \left(0.01072 \times \frac{60^2}{0.75 \times 1.295} \times 1.1 - 27.25 \times \frac{1}{0.75} \times \frac{0.25}{1.295} + 0.2778 \times \frac{40}{0.75} \times \frac{0.25}{1.295} \right) \text{Wh/t-km}$$

$= 43.7 - 7.01 + 2.86 =$ **39.55 Wh/t-km**

As seen, energy consumption has decreased from 69 to 39.55 Wh/t-km.

Example 43.16. *An electric train has an average speed of 45 kmph on a level track between stops 1500 m apart. It is accelerated at 1.8 kmphps and is braked at 3 kmphps. Draw the speed - time curve for the run.*

Solution.

Acceleration $\alpha = 1.8$ kmphps

Retardation $\beta = 3.0$ kmphps

Distance of run $S = 1.5$ km

Average speed $V_a = 45$ kmph

Time of run, $T = \dfrac{S}{V_a} \times 3600 = \dfrac{1.5}{45} \times 3600 = 120$ seconds.

Using equation $V_m = \dfrac{T}{2K} - \sqrt{\dfrac{T^2}{4K^2} - \dfrac{3600\,S}{K}}$

Where $K = \frac{1}{2\alpha} + \frac{1}{2\beta} = \frac{1}{2\times 1.8} + \frac{1}{2\times 3.0} = 0.4444$

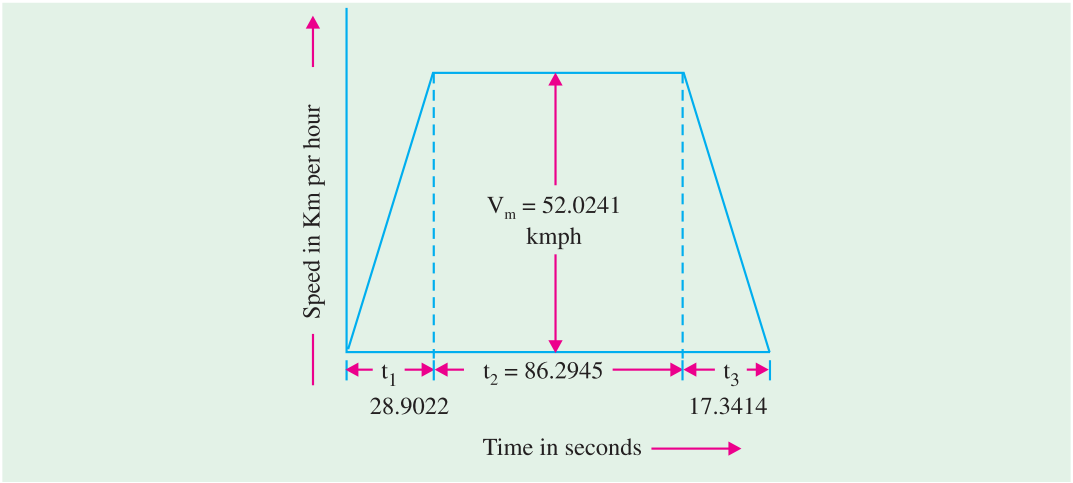


Fig. 43.19

$\therefore$ Maximum speed, $V_m = \frac{120}{2\times 0.4444} - \sqrt{\frac{(120)^2}{(2\times 0.4444)^2} - \frac{3600\times 1.5}{0.4444}}$

$\therefore$ $V_m = 52.0241$ kmph

Acceleration period, $t_1 = \frac{V_m}{\alpha} = \frac{52.0241}{1.8} = 28.9022$ seconds

Braking period, $t_3 = \frac{V_m}{\beta} = \frac{52.0241}{3.0} = 17.3414$ seconds

Free running period, $t_2 = T - (t_1 + t_3) = 120 - (28.9622 + 17.3414) = 86.2945$ seconds

Example 43.17. *A train has schedule speed of 60 km per hour between the stops which are 9 km apart. Determine the crest speed over the run, assuming trapezoidal speed – time curve. The train accelerates at 3 kmphps and retards at 4.5 kmphps. Duration of stops is 75 seconds.*

Solution.

Acceleration $\alpha = 3$ kmphps

Retardation $\beta = 4.5$ kmphps

Distance of run, $S = 9$ km

Schedule speed, $V_s = 60$ kmph

Schedule time, $T_s = \frac{S}{V_s} \times 3{,}600$ seconds $= \frac{9}{60} \times 3{,}600 = 540$ seconds

Actual time of run, $T = T_s$ – Time of stop $= 540 - 75 = 465$ seconds

Using the equation

$$V_m = \frac{T}{2K} - \sqrt{\frac{T^2}{4K^2} - \frac{3{,}600\,S}{K}}$$

where $K = \frac{1}{2\alpha} + \frac{1}{2\beta} = \frac{1}{6} + \frac{1}{9} = 0.2777$

$\therefore$ Maximum speed, $V_m = \frac{465}{2\times 0.2777} - \sqrt{\frac{(465)^2}{4\times(0.2777)^2} - \frac{3{,}600\times 9}{0.2777}}$

$$V_m = 837 - \sqrt{700569 - 116640}$$

$$\therefore \quad V_m = \mathbf{72.8475\ kmph}$$

Example 43.18. *An electric train is to have acceleration and braking retardation of 1.2 km/hour/sec and 4.8 km/hour/sec respectively. If the ratio of maximum to average speed is 1.6 and time for stops 35 seconds, find schedule speed for a run of 3 km. Assume simplified trapezoidal speed-time curve.*

Solution.

Acceleration $\alpha = 1.2$ kmphps

Retardation $\beta = 4.8$ kmphps

Distance of run, $S = 3$ km

Let the actual time of run be T seconds

Average speed, $V_a = \dfrac{3,600\ S}{T} = \dfrac{3,600 \times 3}{T} = \dfrac{10800}{T}$ kmph

Maximum speed, $V_m = 1.6 \times \dfrac{10800}{T} = \dfrac{17280}{T}$ kmph

Since $V_m^2 \left(\dfrac{1}{2\alpha} + \dfrac{1}{2\beta} \right) - V_m T + 3,600\ S = 0$

$$\therefore \quad V_m^2 = \frac{V_m T - 3,600\ S}{\frac{1}{2\alpha} + \frac{1}{2\beta}} = \frac{\frac{17280}{T} T - 3600 \times 3}{\frac{1}{2 \times 1.2} + \frac{1}{2 \times 4.8}}$$

or $V_m = 111.5419$ kmph

and $V_a = \dfrac{V_m}{1.5} = \dfrac{111.5419}{1.5} = 74.3613$ kmph

Actual time of run $T = \dfrac{3,600\ S}{V_a} = \dfrac{3600 \times 3}{74.3613} = 145.2369$ seconds

Schedule time, T_s = Actual time of run + Time of stop = 145.2369 + 35 j 180 seconds

$\therefore$ Schedule speed, $V_s = \dfrac{S \times 3,600}{T_s} = \dfrac{3 \times 3600}{180} = \mathbf{60\ kmph}$

Example 43.19. *An electric train has a schedule speed of 25 kmph between stations 800 m apart. The duration of stop is 20 seconds, the maximum speed is 20 percent higher than the average running speed and the braking retardation is 3 kmphps. Calculate the rate of acceleration required to operate this service.*

Solution.

Schedule speed, $V_s = 25$ kmph

Distance of run, $S = 800$ metres $= 0.8$ km

Retardation, $\beta = 3$ kmphps

Schedule time of run, $T_s = \dfrac{3,600 \times S}{T} = \dfrac{3600 \times 0.8}{25} = 115.2$ seconds

Actual time of run, $T = T_s$ – duration of stop $= 115.2 - 20 = 95.2$ seconds

Average speed, $V_a = \dfrac{3,600 \times S}{T} = \dfrac{3600 \times 0.8}{95.2} = 30.25$ kmphs

Maximum speed, $V_m = 1.2\ V_a = 1.2 \times 30.25 = 36.3$ kmph

Since $V_m^2\left(\frac{1}{2\alpha}+\frac{1}{2\beta}\right)-V_m T+3,600\,S=0$

$$\therefore \quad \frac{1}{2\alpha}+\frac{1}{2\beta} = \frac{V_m T-3,600\,S}{V_m^2}$$

$$\text{or} \quad \frac{1}{2\alpha}+\frac{1}{2\times 3} = \frac{36.3\times 95.2-3,600\times 0.8}{(36.3)^2} = \mathbf{0.4369}$$

$$\text{or} \quad \alpha = 1.85 \text{ kmphps}$$

Example 43.20. *A suburban electric train has a maximum speed of 80 kmph. The schedule speed including a station stop of 35 seconds is 50 kmph. If the acceleration is 1.5 kmphps, find the value of retardation when the average distance between stops is 5 km.*

Solution.

Schedule speed, $V_s = 50$ kmph

Distance of run, $S = 5$ km

Acceleration, $\alpha = 1.5$ kmphps

Maximum speed, $V_m = 80$ kmph

Duration of stop = 35 seconds

$$\text{Schedule time of run, } T_s = \frac{3,600\times S}{V_s} = \frac{3,600\times 5}{50} = 360 \text{ seconds}$$

$$\text{Actual time of run, } T = \frac{T_s - \text{duration of stop}}{V_m^2} = 360-30 = 330 \text{ seconds}$$

Since $V_m^2\left(\frac{1}{2\alpha}+\frac{1}{2\beta}\right)-V_m T+3,600\,S=0$

$$\text{or} \quad \frac{1}{2\alpha}+\frac{1}{2\beta} = V_m T-3,600\,S = \frac{80\times 330-3,600\times 5}{(80)^2} = 1.3125$$

$$\text{or} \quad \frac{1}{2\beta} = 1.3125-\frac{1}{2\alpha} = 1.3125-\frac{1}{2\times 1.5} = 0.9792$$

$$\text{or} \quad \beta = \frac{1}{2\times 0.9792} = \mathbf{0.51064 \text{ kmphps}}$$

Example 43.21. *A train is required to run between two stations 1.6 km apart at the average speed of 40 kmph. The run is to be made to a simplified quadrilateral speed-time curve. If the maximum speed is to be limited to 64 kmph, acceleration to 2.0 kmphps and coasting and braking retardation to 0.16 kmphps and 3.2 kmphps respectively, determine the duration of acceleration, coasting and braking periods.*

Solution.

Distance of run, $S = 1.6$ km

Average speed, $V_a = 40$ kmph

Maximum speed, $V_m = 64$ kmph

Acceleration, $\alpha = 2.0$ kmphps

Coasting retardation, $\beta_C = 0.16$ kmphps

Braking retardation, $\beta = 3.2$ kmphps

$$\text{Duration of acceleration, } t_1 = \frac{V_m}{\alpha} = \frac{64}{2.0} = \mathbf{32 \text{ seconds}}$$

Actual time of run, $T = \frac{3,600\ S}{V_a} = \frac{3,600 \times 1.6}{40} = 144$ seconds

Let the speed before applying brakes be V_2

then duration of coasting, $t_2 = \frac{V_m - V_2}{\beta_c} = \frac{64 - V_2}{0.16}$ seconds

Duration of braking, $t_3 = \frac{V_2}{\beta} = \frac{V_2}{3.2}$ seconds

Since actual time of run, $T = t_1 + t_2 + t_3$

$\therefore \quad 144 = 32 + \frac{64 - V_2}{0.16} + \frac{V_2}{3.2}$

or $V_2 \left(\frac{1}{0.16} - \frac{1}{3.2}\right) = 32 + 400 - 144$

or $V_2 = \frac{288}{6.25 - 0.3125} = 48.5$ kmph

Duration of coasting, $t_2 = \frac{V_m - V_2}{\beta_c} = \frac{64 - 48.5}{0.16} =$ **96.85 seconds**

Duration of braking, $t_3 = \frac{V_2}{\beta} = \frac{48.5}{3.2} =$ **15.15 seconds**

43.47. General Features of Traction Motor

Electric Features

- High starting torque
- Series Speed - Torque characteristic
- Simple speed control
- Possibility of dynamic/ regenerative braking
- Good commutation under rapid fluctuations of supply voltage.

Mechanical Features

- Robustness and ability to withstand continuous vibrations.
- Minimum weight and overall dimensions
- Protection against dirt and dust

No type of motor completely fulfills all these requirements. Motors, which have been found satisfactory are D.C. series for D.C. systems and A.C. series for A.C. systems. While using A.C. three phase motors are used. With the advent of Power Electronics it is very easy to convert single phase A.C. supply drawn from pantograph to three phase A.C.

43.48. Speed - Torque Characteristic of D.C. Motor

$$V = E_b + I_a R_a$$
$$V \cdot I_a = E_b \cdot I_a + I_a^2 R_a$$

where $E_b\ I_a$ = Power input to armature = Electrical power converted into mechanical power at the shaft of motor.

Mechanical Power $= T \cdot \omega = T \times \frac{2\pi N}{60}$

$\therefore \quad \frac{2\pi NT}{60} = E_b \cdot I_a \therefore \quad T = \frac{60\ E_b\ I_a}{2\pi\ N} = 9.55\ \frac{E_b\ I_a}{N}$

But $E_b = \frac{\phi\ ZNP}{60\ A}$

$$\therefore \quad T = 9.55 \frac{\phi ZNP}{60\,A} \frac{I_a}{N} = 9.55 \frac{\phi ZP}{60} \frac{I_a}{A}$$

$$= 0.1592 \times \phi \times \left[Z \frac{I_a}{A} \right] P \text{ Nw-m}$$

$\therefore$ Torque T = 0.1592 × flux per pole × armature amp. conductors × Number of poles

Also speed 'N' can be calculated as:

$$E_b = \frac{\phi ZNP}{60\,A} \qquad \therefore \quad N = \frac{(E_b)}{\phi ZP} 60\,A$$

$$N = \frac{(V - I_a R_a) 60\,A}{\phi ZP} \qquad \therefore \quad N \propto \frac{V - I_a R_a}{\phi}$$

But $T = 9.55 \frac{\phi ZP}{60} \frac{I_a}{A}$ from the equation of torque

$$\therefore \quad \frac{T}{I_a} = \frac{9.55\,\phi ZP}{60\,A} \Rightarrow \frac{9.55\,I_a}{T} = \frac{60\,A}{\phi ZP}$$ Put this value in the above equation of N

$$\therefore \quad N = \frac{(V - I_a R_a) \times 9.55\,I_a}{T}$$

Speed $$N = \frac{9.55\,(V - I_a R_a)}{T / I_a}$$

The torque - current and speed - torque curves for D.C. motors are shown in Fig. 43.20 (*a*) and (*b*) respectively.

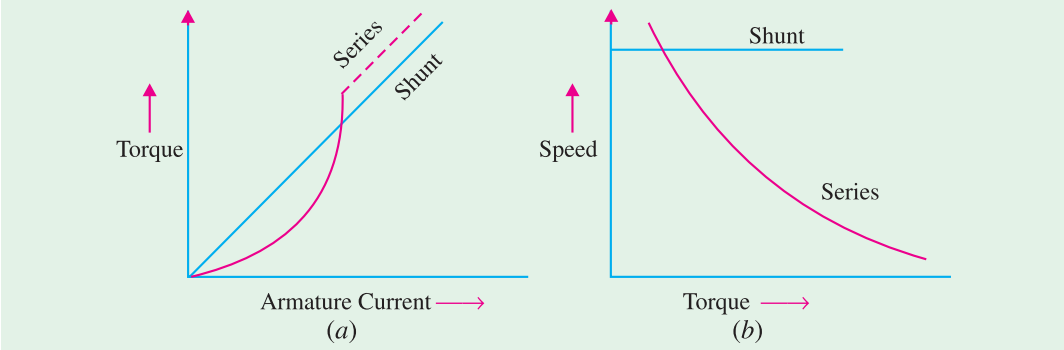


Fig. 43.20

43.49. Parallel Operation of Series Motors with Unequal Wheel Diameter

An electric locomotive uses more than one motor. Each motor drives different set of axles and wheels. Due to wear and tear the diameter of wheels become different, after a long service. But the linear speed of locomotive and wheels will be the same. Therefore, motor speeds will be different due to difference in diameter of wheels driven by them as shown in Fig. 43.21. Therefore, when the motors are connected in parallel they will not share the torques equally, as the current shared by them will be different.

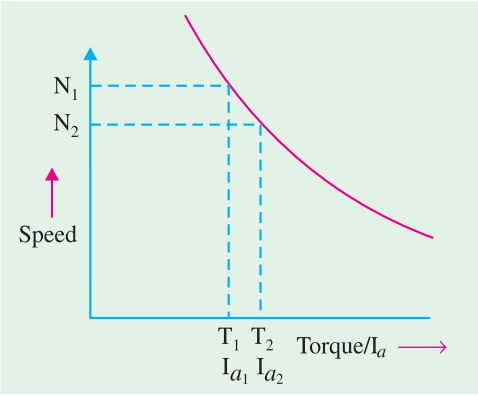


Fig. 43.21

Let the motor wheels ratio is 1.04 : 1 *i.e.* speed of rotation of motor-1 is 1.04 times that of motor-2, as shown in Fig. 43.21.

Let motor 1 drives wheel with 100 c.m. dia. and motor 2 drives wheel with 104 c.m. dia. Then speed of rotation of motor -1 will be $\frac{104}{100}$ = 1.04 times that of 2 *i.e.* $N_2 = 1.04\, N_1$, for a given speed of locomotive.

43.50. Series operation of Series Motor with unequal wheel diameter

Let the motors '*A*' and '*B*' be identical having armature resistance *R* in series, as shown in Fig. 43.22.

Since they are in series, the same current '*I*' will flow through both. But due to unequal wheel diameter; they deliver different loads *i.e.* voltage across each will be different

$$V = V_A + V_B \qquad \text{and} \qquad N \propto V - IR$$

$$\therefore \quad \frac{N_A}{N_B} = \frac{V_A - IR}{V_B - IR}$$

$$\therefore \quad V_A - IR = \frac{N_A}{N_B}(V_B - IR)$$

$$V_A = \frac{N_A}{N_B}(V_B - IR) + IR$$

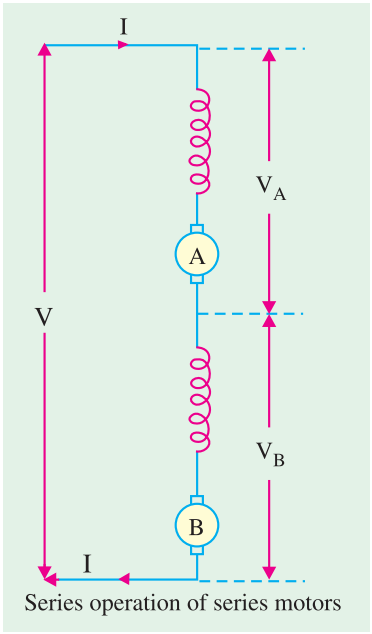
$$V_A = \frac{N_A}{N_B}(V - V_A - IR) + IR$$

$$V_A = \frac{N_A}{N_B}(V - IR) + IR - \frac{N_A}{N_B}V_A$$

$$V_A + \frac{N_A}{N_B}V_A = \frac{N_A}{N_B}(V - IR) + IR$$

$$V_A\left(1 + \frac{N_A}{N_B}\right) = \frac{N_A}{N_B}(V - IR) + IR$$

$$V_A = \frac{\frac{N_A}{N_B}(V - IR) + IR}{1 + \frac{N_A}{N_B}}$$



Series operation of series motors

Fig. 43.22

Similarly,

$$V_B = \frac{\frac{N_B}{N_A}(V - IR) + IR}{1 + \frac{N_B}{N_A}}$$

43.51. Series Operation of Shunt Motors with Unequal Wheel Diameter

It is similar to the case of series operation of series motors, and hence the same equation holds good.

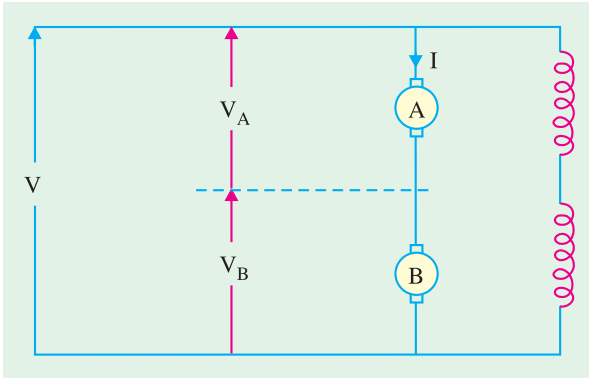


Fig. 43.23

43.52. Parallel Operation of Shunt Motors with Unequal Wheel Diameter

As seen from the Fig. 43.24, a small difference in speeds of two motors, causes motors to be loaded very unequally due to flat speed current curve of D.C. shunt motor.

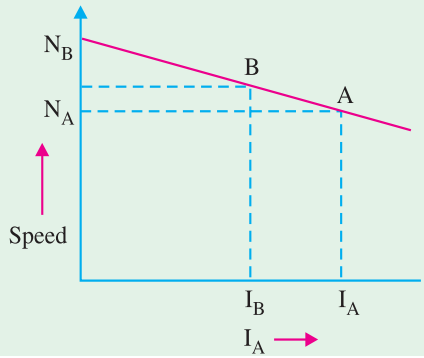


Fig. 43.24

Example 43.22. *The torque-armature current characteristics of a series traction motor are given as:*

Armature Current (amp) : 5 10 15 20 25 30 35 40

Torque (N-m) : 20 50 100 155 215 290 360 430

The motor resistance is 0.3Ω. If this motor is connected across 230 V, deduce the speed armature current characteristics.

Solution.

Supply voltage, $V = 230$ V.

Total Resistance of series motor $R_m = R_a + R_{se} = 0.3\ \Omega$.

Armature current, I_a in amperes	5	10	15	20	25	30	35	40
Torque, T in N-m	20	50	100	155	215	290	360	430
Back e.m.f., E_b = $(V - I_a R_m)$ in volts	228.5	227.0	225.5	224.0	222.5	221.0	219.5	218.0
Speed, N_a $\frac{955\ (e - I_a R_a)}{\phi}$ in R.P.M.	545	434	323	276	247	218	204	194

The deduced speed-armature current characteristic is shown in Fig. 43.25.

Example 43.23. *The following figures give the magnetization curve of d.c. series motor when working as a separately excited generator at 600 rpm.:*

Field Current (amperes) : 20 40 60 80

E.M.F. (volts) : 215 381 485 550

The total resistance of the motor is 0.8 ohm. Deduce the speed – torque curve for this motor when operating at a constant voltage of 600 volts.

Solution.

Voltage applied across the motor, $V = 600$ volts

Resistance of the motor, $R_m = (R_a + R_{se}) = 0.8\ \Omega$

Speed, $N_1 = 600$ r.p.m.

Field Current (amperes)	20	40	60	80
Back e.m.f., E_1 (Volts) at speed N_1 (600 r.p.m.) = e.m.f. generated by the armature (given)	215	381	485	550
Back e.m.f., E (Volts) at speed N (to be determined) = $V - I_a R_m$	584	568	552	536
Speed, N (to be determined in r.p.m.) = $N_1/E_1 \times (V - I_a R_m)$	1,630	895	683	585
Torque, $T = \frac{9.55\,(V - I_a R_m)\, I_a}{N}$ N-m	68.4	240	462	700

The deduced speed-torque curve for the motor is shown in Fig. 43.26.

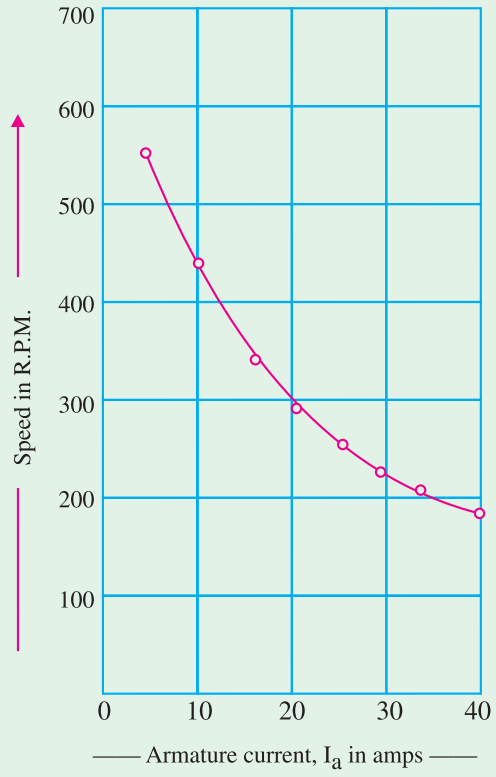


Fig. 43.25

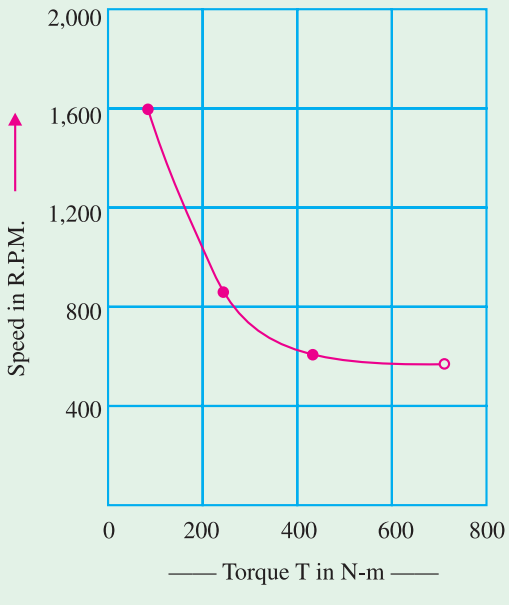


Fig. 43.26

Example 43.24. *Two d.c. traction motors run at a speed of 900 r.p.m. and 950 r.p.m. respectively when each takes a current of 50 A from 500 V mains. Each motor has an effective resistance of 0.3 W. Calculate the speed and voltage across each machine when mechanically coupled and electrically connected in series and taking a current of 50 A from 500 V mains, the resistance of each motor being unchanged.*

Solution.

Let the two motors be *A* and *B* of speed N_A = 900 rpm. And N_B = 950 r.p.m. respectively.

Resistance of each motor R_m = 0.3 Ω

Applied voltage across each motor, $V = 500$ V.

Back e.m.f. of motor A when taking a current of 50 A

$$E_{b_A} = V - I R_m = 500 - 50 \times 0.3 = 485 \text{ V}$$

Back e.m.f. of motor B when taking a current of 50 A

$$E_{b_B} = V - I R_m = 500 - 50 \times 0.3 = 485 \text{ V}$$

When the machines are mechanically coupled and connected in series, the speed of each motor will be same, say N, current will be same and equal to 50 A (given) and the sum of voltage across the two motors will be equal to 500 V.

Let the voltage across motors A and B be V_A and V_B respectively

Now $V_A + V_B = 500$ ...(*i*)

Back e.m.f. of motor A, $E_{b'_A} = E_{b_A} \times \frac{N}{N_A} = \frac{485}{900} \times N$

Voltage across motor A, $V_A = E_{b'_A} + I R_m = \frac{485}{900} \times N + 50 \times 0.3 = \frac{485}{900} \times N + 15$

Back e.m.f. of motor B, $E_{b'_B} = E_{b_B} \times \frac{N}{N_B} = \frac{485}{950} N$

Voltage across motor B, $V_B = E_{b'_B} + I R_m = \frac{485}{950} N + 15$

Substituting $V_A = \frac{485}{900} N + 15$ and $V_B = \frac{485}{950} N + 15$ in expression (*i*) we get

$$\frac{485}{900} N + 15 + \frac{485}{950} N + 15 = 500$$

or $$\left(\frac{485}{900} + \frac{485}{950}\right) N = 470$$

or $$N \left(\frac{1}{900} + \frac{1}{950}\right) = \frac{470}{485}$$

$\therefore$ $$N = \mathbf{447.87 \text{ r.p.m.}}$$

P.D. across machine A, $V_A = \frac{485\, N}{900} + 15 = \mathbf{256.35 \text{ V}}$

P.D. across machine B, $V_B = \frac{485\, N}{950} + 15 = \mathbf{243.65 \text{ V}}$

Example 43.25. *A tram car is equipped with two motors which are operating in parallel. Calculate the current drawn from the supply main at 500 volts when the car is running at a steady speed of 50 kmph and each motor is developing a tractive effort of 2100 N. The resistance of each motor is 0.4 ohm. The friction, windage and other losses may be assumed as 3500 watts per motor.*

Solution.

Voltage across each motor, $V = 500$ volts

Maximum speed, $V_m = 50$ kmph

Tractive effort, $F_t = 2100$ Newtons

Motor resistance, $R_m = 0.4$ W

Losses per motor = 3500 watts

Power output of each motor $= \frac{F_t \times V_m \times 1000}{3600}$ watts

$$= \frac{2100 \times 50 \times 1000}{3600} \text{ watts} = 29166.67 \text{ watts.}$$

Constant losses = 3500 watts

Copper losses = $I^2 R_m = 0.4\, I^2$

Where I is the current drawn from supply mains

$$\text{Input to motor} = \text{Motor output} + \text{constant losses} + \text{copper losses}$$

$$VI = 29166.67 + 3500 + 0.4\, I^2$$

$$0.4\, I^2 - 500\, I + 32666.67 = 0$$

$$I = \frac{500 \pm \sqrt{(500)^2 - 4 \times 0.4 \times 32666.67}}{2 \times 0.4}$$

$$I = 69.16 \text{ A} \quad \text{or} \quad 1180.84 \text{ A}$$

Current drawn by each motor = 69.16A

∈ 1180.84 A being unreasonably high can not be accepted

Total current drawn from supply mains = 69.16×2 = **138.32 A**

Example 43.26. *A motor coach is being driven by two identical d.c. series motors. First motor is geared to driving wheel having diameter of 90 cm and other motor to driving wheel having diameter of 86 cm. The speed of the first motor is 500 r.p.m. when connected in parallel with the other across 600 V supply. Find the motor speeds when connected in series across the same supply. Assume armature current to remain same and armature voltage drop of 10% at this current.*

Solution.

Speed of first motor, $N_1 = 500$ r.p.m.

Back e.m.f., $E_{b_1} = 600 - \frac{10}{100} \times 600$

$= 540$ volts.

When the motors are connected in series across 600 V supply, as shown in Fig. 43.27.

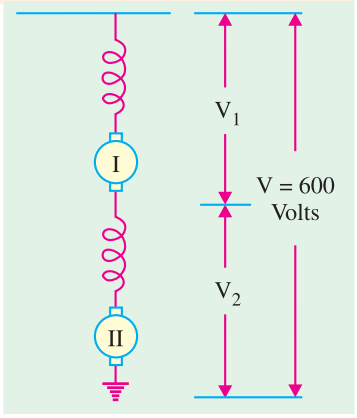


Fig. 43.27

Let the supply voltage across motors I and II be V_1 and V_2 volts and speed N'_1 and N'_2 respectively.

Since speed, $N \propto \frac{V - IR}{\phi}$

Current through the motors remains the same, therefore flux produced by it also remains the same and $N \propto (V - IR)$

$$\therefore \quad \frac{N'_1}{N'_2} = \frac{V_1 - I\,R}{V_2 - I\,R} = \frac{V_1 - \frac{10}{100} \times 600}{V_2 - \frac{10}{100} \times 600} = \frac{V_1 - 60}{V_2 - 60} \qquad \text{...(i)}$$

And also $N'_1\, D_1 = N'_2 D_1 = N'_2 D_2$

$$\therefore \quad \frac{N'_1}{N'_2} = \frac{D_2}{D_1} = \frac{86}{90}$$

$$\frac{V_1 - 60}{V_2 - 60} = \frac{86}{90}$$

(*ii*) Since peripheral speed is equal

or $90\, V_1 - 5{,}400 = 86\, V_2 - 5{,}160$

or $90\, V_1 - 86\, V_2 = 5{,}400 - 5{,}160 = 240$ ...(*iii*)

and also $V_1 + V_2 = 600$ V ...(*iv*)

Solving expressions (*iii*) and (*iv*)

$$V_1 = 294.55 \text{ V}$$

And $$V_2 = 305.45 \text{ V}$$

Now the speeds of the motors can be calculated as follows :

$$\frac{N_1'}{N_2'} = \frac{\text{ }}{\text{ }}$$

or $$N'_1 = N_1 \times \frac{E'_{b_1}}{E_b} = 500 \times \frac{294.55 - 60}{600 - 60} = 217 \text{ r.p.m}$$

and $$N'_2 = N_1' \times \frac{D_1}{D_2} = 217 \times \frac{90}{86} = \mathbf{277 \text{ r.p.m}}$$

Example 43.27. *Two similar series type motors are used to drive a locomotive. The supply fed to their parallel connection is 650 V. If the first motor 'A' is geared to drive wheels of radius 45 cms. and other motor 'B' to 43 cms. And if the speed of first motor 'A' when connected in parallel to 2nd motor 'B' across the main supply lines is 400 rpm., find voltages and speeds of motors when connected in series. Assume I_a to be constant and armature voltage drop of 10% at this current.*

Solution.

$N \propto V - IR$ as flux ϕ is constant, since I_a is constant

$N_A = V_A - IR \qquad N_B = V_B - IR \qquad$ Also $V = V_A + V_B$

Assume $$\frac{N_A}{N_B} = \rho$$

$$V_A = \frac{\rho\,(V - IR) + IR}{1 + \rho}$$

Armature voltage drop = 10% of 650 V $\qquad \therefore \quad IR = 65$

But $$\frac{N_A}{N_B} = \frac{r_B}{r_A} = \frac{43}{45} = \rho$$

$$V_A = \frac{43/45\,(650 - 65) + 65}{1 + 43/45} = 320 \text{ V}$$

$$V_B = V - V_A = 650 - 320 = 330 \text{ V}$$

Speed N_A of motor A is 400 rpm with a supply of 650 V.

$\therefore$ Speed N'_A of motor A with supply voltage of 320 V will be

$$\frac{N_A'}{N_A} = \frac{320 - IR}{650 - IR} = \frac{320 - 65}{650 - 65} = \frac{255}{585}$$

$\therefore$ $$N'_A = \frac{255}{585} N_A = \frac{255}{585} \times 400 = \mathbf{175 \text{ r.p.m.}}$$

$$\frac{N_A}{N_B} = \frac{r_B}{r_A} = \frac{N_A'}{N_B'} = \frac{43}{45}$$

$$N'_B = \frac{45}{43} N'_A = \frac{45}{43} \times 175 = \mathbf{184 \text{ r.p.m.}}$$

43.53. Control of D.C. Motors

The starting current of motor is limited to its normal rated current by starter during starting. At the instant of switching on the motor, back e.m.f. $E_b = 0$

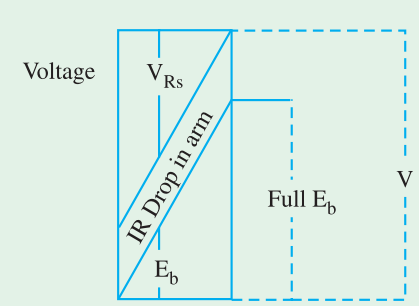
$\therefore$ Supply voltage = V = IR + Voltage drop across R_s.

At any other instant during starting

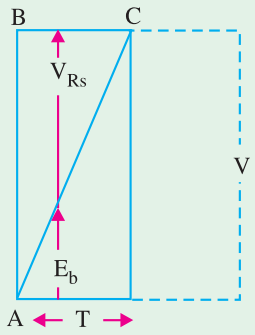
$V = IR +$ Voltage across $R_s + E_b$

At the end of accelerating period, when total R_s is cut-off

$$V = E_b + IR$$



(*a*) Voltage during starting of D.C. motor



(*b*) Voltage during starting neglecting IR drop

Fig. 43.28

If T is the time in sec. for starting and neglecting IR drop, total energy supplied = $V.I.T.$ watt-sec

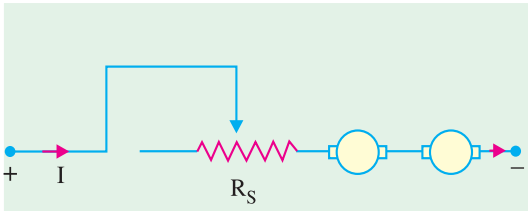
From Fig. 43.28 (*b*) Energy wasted in R_s = Area of triangle $ABC \times I$ = ½. $T.V.I.$ watt - sec. = ½ VIT watt - sec. But total energy supplied = $V.I.T$ watt - sec.

∴ Half the energy is wasted in starting

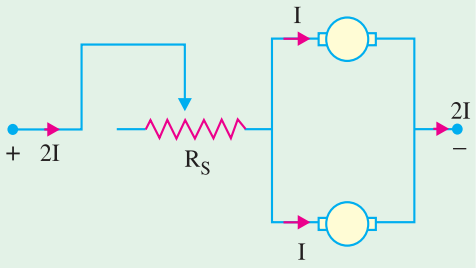
∴ $\eta_{starting} = 50\%$

43.54. Series - Parallel Starting

With a 2 motor equipment ½ the normal voltage will be applied to each motor at starting as shown in Fig. 43.29 (*a*) (Series connection) and they will run upto approximate ½ speed, at which instant they are switched on to parallel and full voltage is applied to each motor. R_s is gradually cut-out, with motors in series connection and then reinserted when the motors are connected in parallel, and again gradually cut-out.



(*a*) Motors in series



(*b*) Motors in parallel

Fig. 43.29

In traction work, 2 or more similar motors are employed. Consider 2 series motors started by series parallel method, which results in saving of energy.

(*a*) Series operation. The 2 motors, are started in series with the help of R_s. The current during starting is limited to normal rated current 'I' per motor. During series operation, current 'I' is drawn

from supply. At the instant of starting $OA = AB = IR$ drop in each motor. OK = Supply voltage 'V'. The back e.m.fs. of 2 motors jointly develop along OM as shown in Fig. 43.30 (a). At point. E, supply voltage V = Back e.m.fs of 2 motors + IR drops of 2 motor. Any point on the line BC represents the sum of Back e.m.fs. of 2 motors + IR drops of 2 motors + Voltage across resistance R_s of 2 motors

OE = time taken for series running.

At pt 'E' at the end of series running period, each motor has developed a back e.m.f.

$$= \frac{V}{2} - IR$$

$$EL = ED - LD$$

(*b*) Parallel operation. The motors are switched on in parallel at the instant 'E', with R_s reinserted as shown in Fig. 43.29 (b). Current drawn is $2I$ from supply. Back e.m.f. across each motor = EL. So the back e.m.f. now develops along LG. At point 'H' when the motors are in full parallel, ($R_s = 0$ and both the motors are running at rated speed)

Supply voltage $= V = HF = HG + GF$

= Normal Back e.m.f. of each motor + IR drop in each motor.

43.55. To find t_s, t_p and η of starting

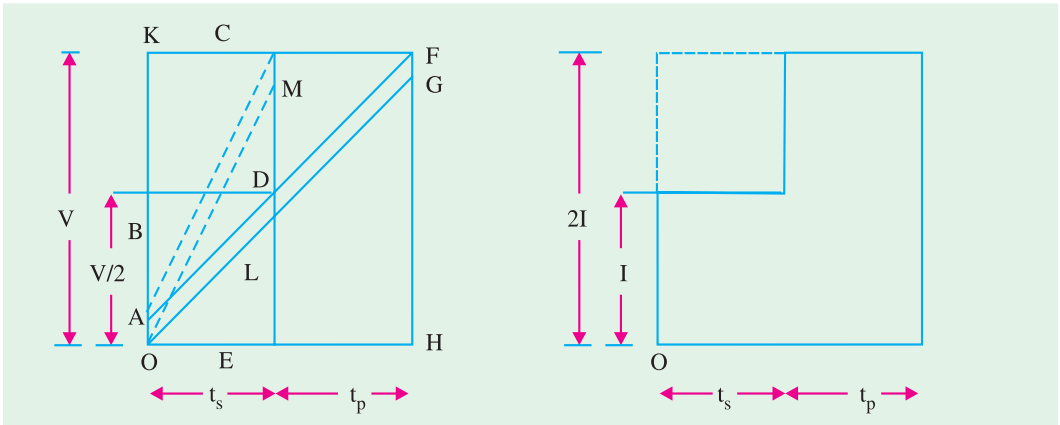
The values of time t_s during which the motors remain in series and t_p during which they are in parallel can be determined from Fig. 43.30 (a), (c). From Fig. 43.30 (a), triangles OLE and OGH are similar

$$\therefore \quad \frac{OE}{OH} = \frac{LE}{GH} \quad \therefore \quad \frac{t_s}{T} = \frac{DE - DL}{FH - FG} = \frac{V/2 - IR}{V - IR}$$

$$\therefore \quad t_s = \frac{1}{2}\left(\frac{V - 2IR}{V - IR}\right)T$$

$$t_p = T - t_s = T - \left\{\frac{1}{2}\left(\frac{V - 2IR}{V - IR}\right)T\right\}$$

$$t_p = T\left\{1 - \frac{1}{2}\left(\frac{V - 2IR}{V - IR}\right)T\right\}$$



(a) Voltage built-up in series-parallel starting (b) Variation of current in series-parallel starting

Fig. 43.30

To calculate η of starting, neglect *IR* drop in armature circuit.

This modifies Fig. 43.30 (*a*) to Fig. 43.30 (*c*). '*D*' is midpoint of *CE* and back e.m.f. develops along *DF* in parallel combination. *KC* = *CF* *i.e.* time for series combination = time for parallel combination

i.e. $t_s = t_p = t$ and average starting current = *I* per motor.

Energy lost in R_s = Area under triangle *OKC* + Area under triangle *CDF*

$$= \left(\frac{1}{2} V I\right) \times t + \left(\frac{1}{2} \frac{V}{2} 2I\right) \times t = VIt$$

But total energy supplied

$$= I V t + 2 I V t$$

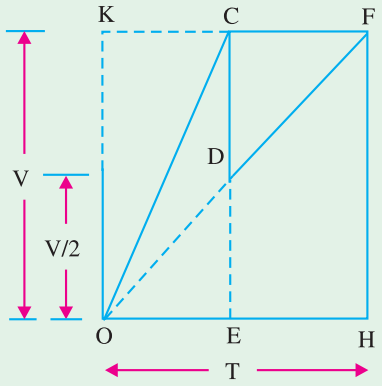
(Series) (Parallel)

$$= 3 V I t$$

$$\therefore \eta \text{ of starting} = \frac{3VIt - VIt}{3VIt}$$

$$= \frac{2}{3} = 66.6\%$$

$\therefore$ η is increased by 16.66% as compared to pervious case. If there are 4 motors then $\eta_{starting}$ = 73%. So there is saving of energy lost in R_s, during starting period as compared with starting by both motors in parallel.



(*c*) Efficiency of starting by series-parallel method

Fig. 43.30

Example 43.28. *Two motors of a motor coach are started on series - parallel system, the current per motor being 350 A (Considered as being maintained constant) during the starting period which is 18 sec. If the acceleration during starting period is uniform, the line voltage is 600 V and resistance of each motor is 0.1 W. Find* ***(a)*** *the time during which the motors are operated in series.* ***(b)*** *the energy loss in the rheostat during starting period.* **[Nagpur University, Summer 2002]**

Solution.

Time during which motors are in series is given by

$$t_s = \frac{1}{2}\left(\frac{V - 2IR}{V - IR}\right)T = \frac{1}{2}\left(\frac{600 - 2 \times 350 \times 0.1}{600 - 350 \times 0.1}\right)18$$

$$t_s = \mathbf{8.44 \text{ sec.}}$$

Time during which motors are in parallel.

$$t_p = T - T_s = 18 - 8.44 = 9.56 \text{ sec.}$$

Back e.m.f. E_b of each motor, in series operation (from Fig. 43.30*a*)

$$E_{b_s} = \frac{V}{2} - IR = \frac{600}{2} - 350\,(0.1) = 265 \text{ V.}$$

When 2 motors are in series,

Total $E_b = 265 + 265 = 530$ V

$$E_{b_p} = V - IR = 600 - 350\,(0.1) = 565 \text{ V}$$

Energy lost when motors are connected in series

$$= \frac{1}{2} E_b \; I \; t_s = \frac{1}{2} \times 530 \times 350 \times \frac{8.44}{3600} = 217 \text{ watt - hours}$$

Energy lost when motors are connected in parallel

$$\frac{1}{2}\frac{E_b}{2} 2 I t_P = \frac{1}{2} \times \frac{565}{2} \times 2 \times 350 \times \frac{9.56}{3600} = 262.5 \text{ watt - hour}$$

$\int$ 263 watt - hours

$\therefore$ Total energy lost = (217 + 263) watt - hours = **480 watt - hours**

43.56. Series Parallel Control by Shunt Transition Method

The various stages involved in this method of series – parallel control are shown in Fig. 43.31

In steps 1, 2, 3, 4 the motors are in series and are accelerated by cutting out the R_s in steps. In step 4, motors are in full series. During transition from series to parallel, R_s is reinserted in circuit– step 5. One of the motors is bypassed -step 6 and disconnected from main circuit – step 7. It is then connected in parallel with other motor -step 8, giving 1st parallel position. R_s is again cut-out in steps completely and the motors are placed in full parallel.

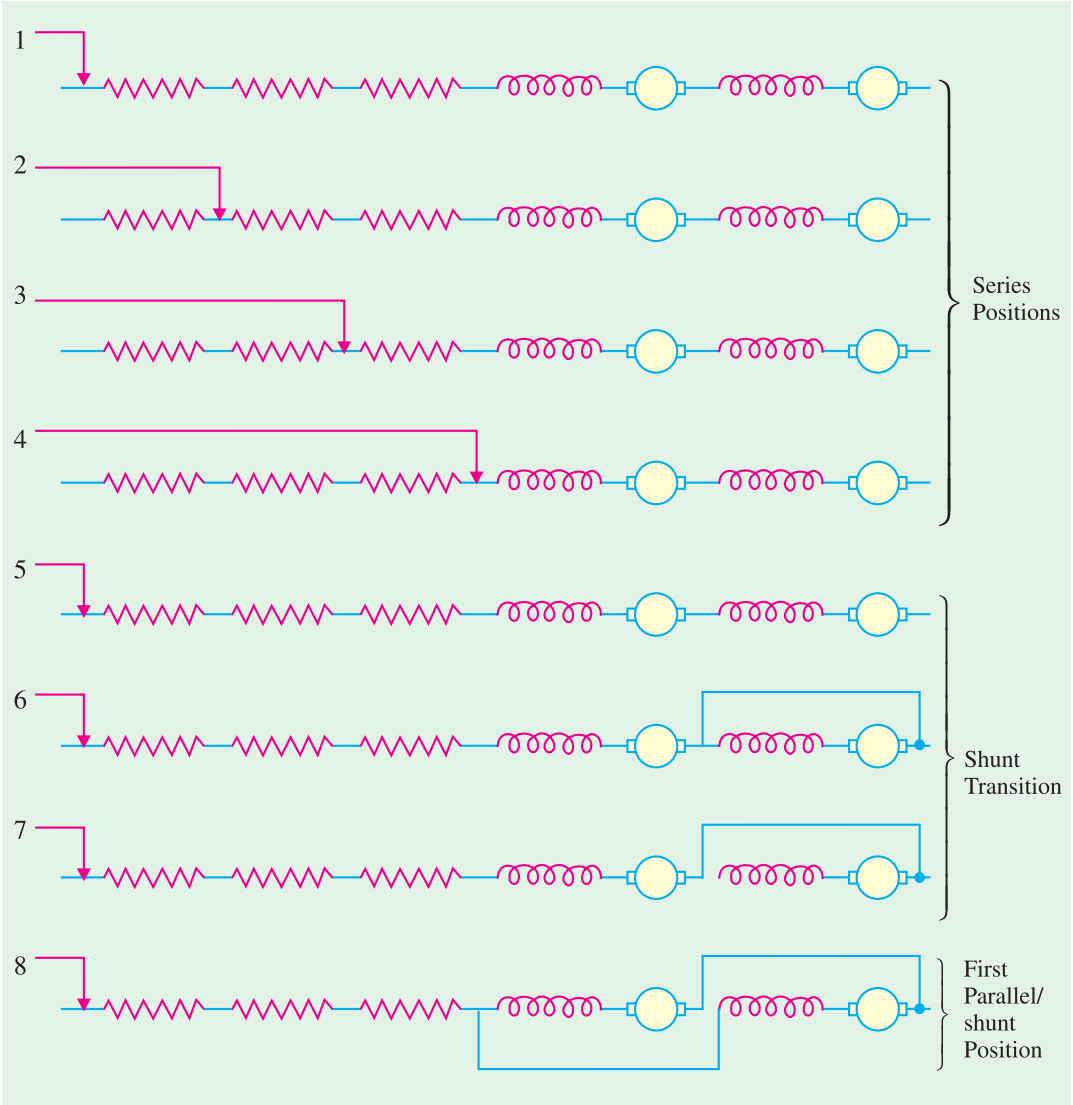


Fig. 43.31

The main difficulty with series parallel control is to obtain a satisfactory method of transition from series to parallel without interrupting the torque or allowing any heavy rushes of current.

In shunt transition method, one motor is short circuited and the total torque is reduced by about 50% during transition period, causing a noticeable jerk in the motion of vehicle.

The Bridge transition is more complicated, but the resistances which are connected in parallel with or 'bridged' across the motors are of such a value that current through the motors is not altered in magnitude and the total torque is therefore held constant and hence it is normally used for railways. So in this method it is seen that, both motors remain in circuit through-out the transition. Thus the jerks will not be experienced if this method is employed.

43.57. Series Parallel Control by Bridge Transition

(*a*) At starting, motors are in series with R_s *i.e.* link *P* in position = AA'

(*b*) Motors in full series with link *P* in position = BB' (No R_s in the circuit)

The motor and R_s are connected in the form of Wheatstone Bridge. Initially motors are in series with full R_s as shown in Fig. 43.32 (*a*). *A* and *A'* are moved in direction of arrow heads. In position *BB'* motors are in full series, as shown in Fig. 43.32 (*b*), with no R_s present in the circuit.

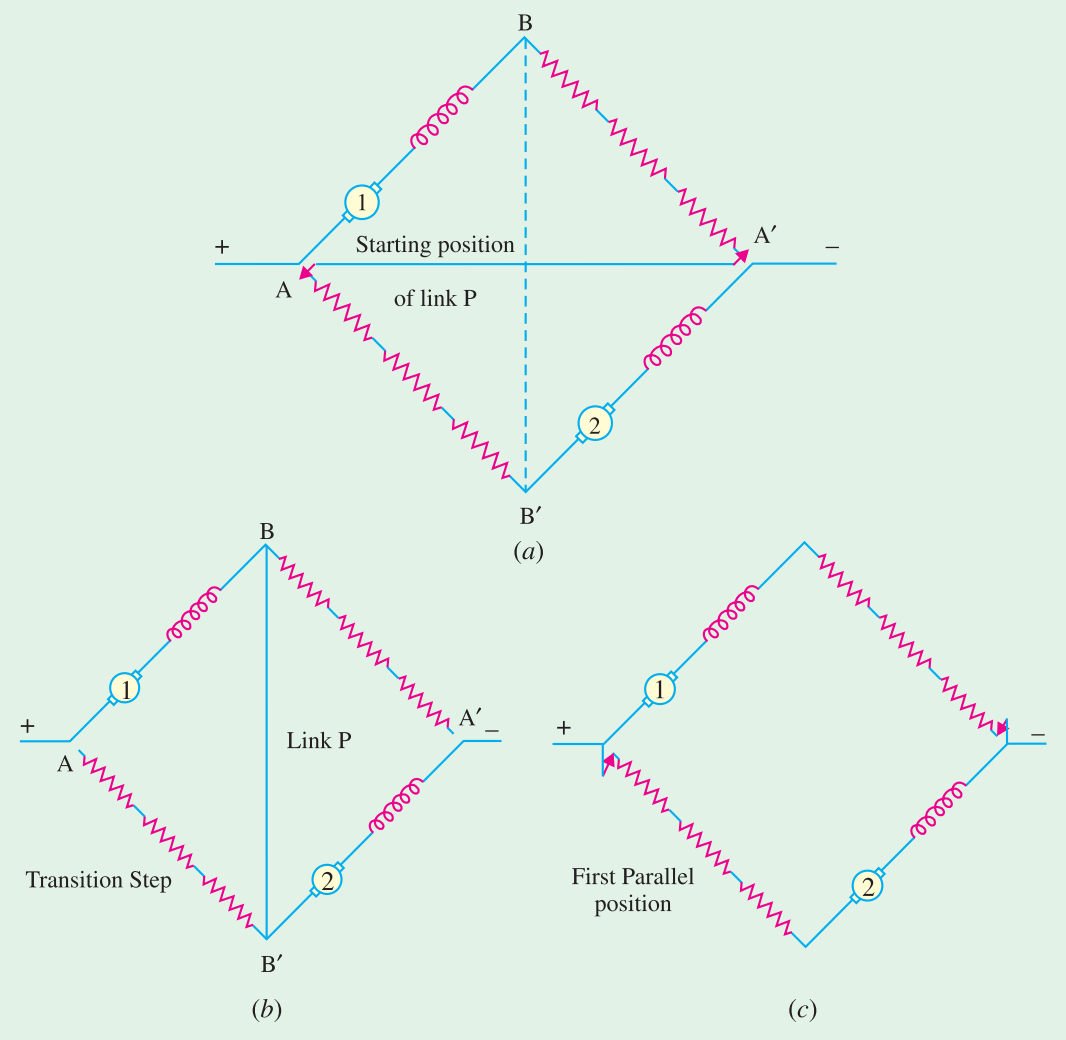


Fig. 43.32

In transition step the R_s is reinserted.

In Ist parallel step, link P is removed and motors are connected in parallel with full R_s as shown in Fig. 43.32 (*c*). Advantage of this method is that the normal acceleration torque is available from both the motors, through - out starting period. Therefore acceleration is smoother, without any jerks, which is very much desirable for traction motors.

43.58. Braking in Traction

Both electrical and mechanical braking is used. Mechanical braking provides holding torque. Electric Braking reduces wear on mechanical brakes, provides higher retardation, thus bringing a vehicle quickly to rest. Different types of electrical braking used in traction are discussed.

43.59. Rheostatic Braking

(a) Equalizer Connection ***(b)*** Cross Connection

(*a*) Equalizer Connection

For traction work, where 2 or more motors are employed, these are connected in parallel for braking, because series connection would produce too high voltage. K.E. of the vehicle is utilized in driving the machines as generators, which is dissipated in braking resistance in the form of heat.

To ensure that the 2 machines share the load equally, an equalizer connection is used as shown in Fig. 43.33 (*a*). If it is not used, the machine whose acceleration builts-up first would send a current through the 2nd machine in opposite direction, causing it to excite with reverse voltage. So that the 2 machines would be short circuited on themselves. The current would be dangerously high. Equalizer prevents such conditions. Hence Equalizer connection is important during braking in traction.

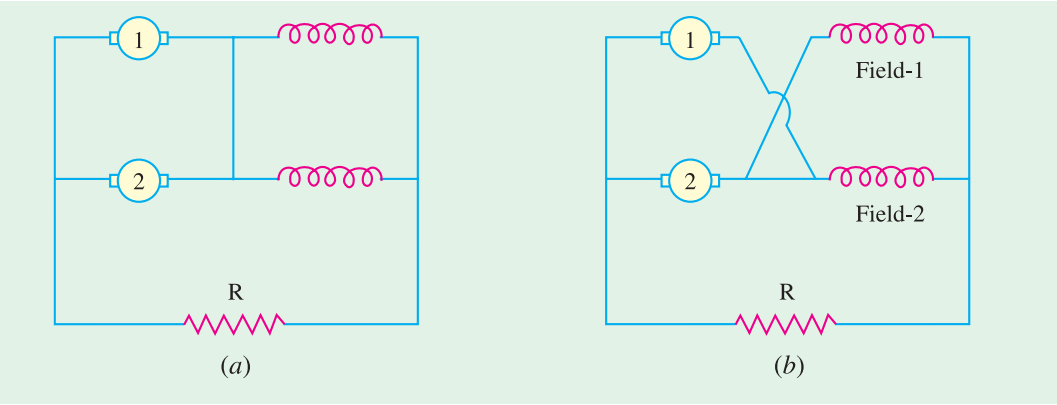


Fig. 43.33

(*b*) Cross Connection

In cross connection the field of machine 2 is connected in series with armature of machine 1 and the field of machine 1 is connected in series with armature of machine 2 as shown in Fig. 43.33 (*b*). Suppose the voltage of machine 1 is greater than that of 2. So it will send greater current through field of machine 2, causing it to excite to higher voltage. At the same time machine 1 excitation is low, because of lower voltage of machine 2. Hence machine 2 will produce more voltage and machine 1 voltage will be reduced. Thus automatic compensation is provided and the 2 machines operate satisfactorily.

Because of cross - connection during braking of traction motors, current in any of the motor will not go to a very high value.

43.60. Regenerative Braking with D.C. Motors

In order to achieve the regenerative braking, it is essential that ***(i)*** the voltage generated by the machine should exceed the supply voltage and ***(ii)*** the voltage should bez kept at this value, irrespective of machine speed. Fig. 43.34 (*a*) shows the case of 4 series motors connected in parallel during normal running *i.e.* motoring.

One method of connection during regenerative barking, is to arrange the machines as shunt machines, with series fields of 3 machines connected across the supply in series with suitable resistance. One of the field winding is still kept in series across the 4 parallel armatures as shown in figure 43.34 (*b*).

The machine acts as a compound generator. (with slight differential compounding) Such an arrangement is quiet stable; any change in line voltage produces a change in excitation which produces corresponding change in e.m.f. of motors, so that inherent compensation is provided *e.g.* let the line voltage tends to increase beyond the e.m.f. of generators. The increased voltage across the shunt circuit increases the excitation thereby increasing the generated voltage. Vice-versa is also true. The arrangement is therefore self compensating.

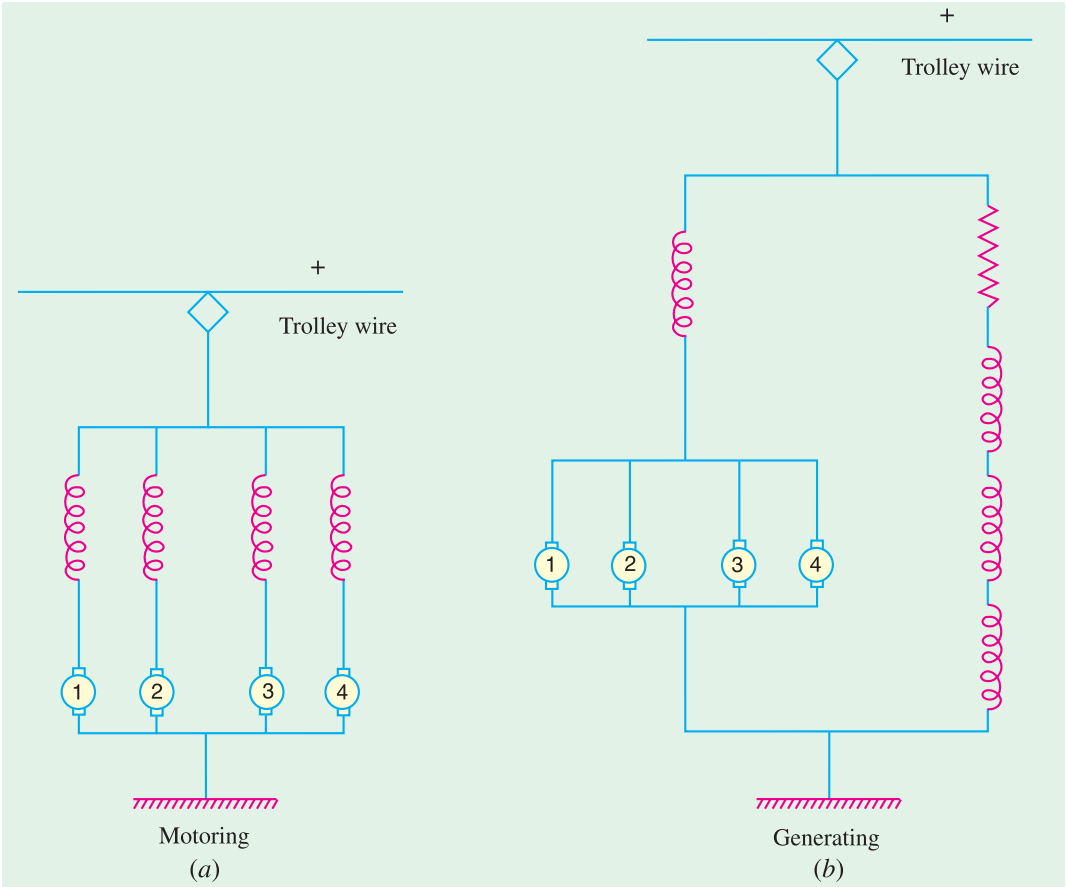


Fig. 43.34

D.C. series motor can't be used for regenerative braking without modification for obvious reasons. During regeneration current through armature reverses; and excitation has to be maintained. Hence field connection must be reversed.

Example 43.29. *Two 750 V D.C. series motors each having a resistance of 0.1 W are started on series - parallel system. Mean current through - out the starting period is 300 A. Starting period is 15 sec. and train speed at the end of this period is 25 km/hr. Calculate*

(i) *Rheostatic losses during series and parallel combination of motors*

(ii) *Energy lost in motor*

(iii) *Motor output*

(iv) *Starting* η

(v) *Train speed at which transition from series to parallel must be made.*

[Nagpur University, Summer 2000]

Solution.

(i) $$t_s = \frac{1}{2}\left[\frac{V - 2IR}{V - IR}\right]T$$

$$t_s = \frac{1}{2}\left[\frac{750 - 2(300)\,0.1}{750 - (300)\,0.1}\right]15 = 7.1875 \text{ rec.}$$

$\therefore$ $$t_p = T - t_s = 7.8125 \text{ sec.}$$

Energy lost in Rheostat $= \frac{1}{2} E_{b_s} I t_s + \frac{1}{2}\frac{E_{b_p}}{2} 2 I.t_p$

$$= \frac{1}{2}\left[2\times\left[\frac{V}{2} - IR\right]\right] I \,.\, t_s + \frac{1}{2}[[V - IR]/2]\, 2I \,.\, t_p$$

$$= \frac{1}{2}\left[2\times\left[\frac{750}{2} - 300\,(0.1)\right]\right] 300\times 7.1875 + \frac{1}{2}\left[\frac{750 - 300\,(0.1)}{2}\right]\times 2(300)\times 7.1825$$

$$= 743906.25 + 843750$$

$$= 1587656.25 \text{ watt – sec.}$$

$$= \frac{1587656.25}{3600} = \textbf{441.00 watt - hrs.}$$

(ii) Total Energy supplied $= V.I.\, t_s + 2\, I \,.\, V \,.\, t_p$

$$= 750 \times 300\,(7.1875) + 2\,(300)\,750\,(7.8125)$$

$$= 1617187.5 + 3515625$$

$$= 5132812.5 \text{ watt-sec} = 1425.7812 \text{ watt – hrs.}$$

Energy lost in 2 Motors $= (I_a^2 \times R_a) \times 2 \times 15$

$$= (300^2 \times 0.1) \times 2 \times 15 = 270000 \text{ watt - sec.} = \textbf{75 watt - hrs.}$$

(iii) Motor O/P = Total Energy supplied – Energy lost in Rheostat – Energy lost in armature

$$= 1425.7812 - 441 - 75$$

$$= \textbf{909.7812 watt – hrs.}$$

(iv) $$\eta \text{ starting} = \frac{\text{Total Energy Supplied} - \text{Energy lost in Rheostat}}{\text{Total Energy Supplied}}$$

$$= \frac{1425.7812 - 441.00}{1425.7812}\times 100$$

$$= \textbf{69.0605\%}$$

(v) Acceleration is uniform during starting period of 15 sec. Therefore speed after which series to parallel transition must be made is given as –

$$= \frac{\text{Speed after starting period}}{\text{Total starting period}} \times t_s$$

$$= \frac{25}{15} \times 7.1875$$

$$= \mathbf{11.9791\ km/hr.}$$

Example 43.30. *Two 600-V motors each having a resistance of 0.1Ω are started on the series-parallel system, the mean current per motor throughtout the starting period being 300A. The starting period is 20 seconds and the train speed at the end of this period is 30 km per hour. Calculate* ***(i)*** *the rheostatic losses (in kwh) during* ***(a)*** *the series and* ***(b)*** *the parallel combinations of motors* ***(ii)*** *the train speed at which transition from series to parallel must be made.*

Solution.

Number of motors operating = 2
Line voltage, V = 600 volts
Current per motor, I = 300 amperes
Starting period, T_s = 20 seconds
Motor resistance, R = 0.1 Ω
Maximum speed, V_m = 30 kmph.
Back e.m.f. of each motor in full series position,

$$E_{b_s} = \frac{V}{2} - IR = \frac{600}{2} - 300 \times 0.1 = 270 \text{ volts.}$$

Back e.m.f. of each motor in full parallel position,

$$E_{b_p} = V - IR - 300 \times 0.1 = 570 \text{ volts}$$

Assuming smooth acceleration, back e.m.f. will be built up at constant rate.

Since motors take 20 seconds to build up 570 volts, therefore time taken to build up 270 volts e.m.f. will be :

$$T_{\text{series}} = 20 \times \frac{270}{570} = 9.4737 \text{ seconds}$$

$$T_{\text{parallel}} = 20 - 9.4737 = 10.5263 \text{ seconds}$$

(i) ***(a)*** Voltage drop in the starting rheostat in series combination at the starting instant

$$= V - 2IR = 600 - 2 \times 300 \times 0.1 = 540 \text{ volts,}$$

which reduces to zero in full series position

Energy dissipated in starting resistance during series combination

$$= \frac{(V - 2IR) + 0}{2} \times I \times \frac{T_{\text{series}}}{3600} = \frac{540 + 0}{2} \times 300 \times \frac{9.4737}{3600}$$

$$= \mathbf{213.1579\ watt\text{ - }hours}$$

(b) Voltage drop across the starting resistance in first parallel position is equal to $V/2$ *i.e.* 300 volts which gradually reduces to zero.

Energy dissipated in starting resistance during parallel combination

$$= \frac{\frac{V}{2} + 0}{2} \times 21 \times \frac{T_{\text{parallel}}}{3600} = \frac{\frac{600}{2} + 0}{2} \times 2 \times 300 \times \frac{10.5263}{3600}$$

$$= \mathbf{263.1579\ watt - hours}$$

(ii) Acceleration, $\alpha = \dfrac{\text{Maximum speed}}{\text{Starting period}} = \dfrac{V_m}{T_s} = \dfrac{30}{20} = 1.5$ kmphps.

Speed at the end of series period = $\alpha\ T_{\text{series}} = 1.5 \times 9.4737 = \mathbf{14.21\ km/hour}$

Example 43.31. *Two d.c. series motors of a motor coach have resistance of 0.1 W each. These motors draw a current of 500 A from 600 V mains during series – parallel starting period of 25 seconds. If the acceleration during starting period remains uniform, determine:*

(i) *time during which the motors operate in (a) series (b) parallel.*

(ii) *the speed at which the series connections are to be changed if the speed just after starting period is 80 kmph.*

Solution.

Number of motors operating = 2

Line voltage, V = 600 V

Current per motor, I = 500 A

Motor resistance, R = 0.1 Ω

Maximum speed, V_m = 80 kmph.

Back e.m.f. of each motor in full series position.

$$E_{b_s} = \frac{V}{2} - IR = \frac{600}{2} - 500 \times 0.1 = 250 \text{ V}$$

Back e.m.f. each motor in full parallel operation,

$$E_{b_p} = V - IR = 600 - 500 \times 0.1 = 550 \text{ V}$$

Since motors take 25 seconds to build up 550 V, therefore, time taken to build up 250 V, will be: (assuming smooth acceleration and building up of e.m.f. at constant rate.)

(i) Period of series operation, $T_{\text{series}} = 25 \times \frac{255}{550}$ = **11.3636 seconds**

Period of parallel operation, $T_{\text{parallel}} = T - T_{se} = 25 - 11.3636$ = **13.6363 seconds**

(iii) Speed at which the series connections are to be changed

$$= \alpha\, T_{\text{series}} = \frac{V_m}{T} \cdot T_{\text{series}} = \frac{80}{25} \times 11.3636 = \textbf{36.3636 kmph}$$

Example 43.32. *The following figures refer to the speed-current and torque – current characteristics of a 600 V d.c. series traction motor.*

Current, amperes :	*50*	*100*	*150*	*200*	*250*
Speed, kmph :	*73.6*	*48*	*41.1*	*37.3*	*35.2*
Torque, N-m :	*150*	*525*	*930*	*1,335*	*1,750*

Determine the braking torque at a speed of 48 kmph when operating as self excited d.c. generator. Assume resistance of motor and braking rheostat to be 0.6Ω and 3.0 Ω respectively.

Solution.

As motor :

Terminal voltage, V = 600 volts.

The motor current at a speed of 48 kmph (from speed-current characteristic curve),

I = 100 A

Back e.m.f. developed by the motor, $E_b = V - IR_m = 600 - 100 \times 0.6$ = **540 V**

As Generator:

At the instant of applying rheostatic braking at speed of 48 kmph, the terminal voltage of machine will be equal to e.m.f. developed by the machine *i.e.* 540 volts.

Total resistance in the circuit = $R_m + R_{\text{rheostat}} = 0.6 + 3 = 3.6\ \Omega$

Current delivered by the machine, $I = \frac{540}{3.6}$ = **150 amps**

The braking torque (the torque corresponding to 150 amperes from torque-current curve)

= **930 N-m**

Tutorial Problem No. 43.1

1. A train weighs 500 tonnes. What is its mass in (*i*) tonnes and (*ii*) kilograms.
[(*i*) 500 t (*ii*) 500,000 kg]

2. A train has a mass of 200 tonnes. What is its weight in (*i*) newtons and (*ii*) kg-wt (*iii*) tonnes-wt.
[(*i*) 19.6×10^5 N (*ii*) 200,000 kg. wt (*iii*) 200 t-wt)]

3. A train has a speed of 100 km/h. What is its value in m/s ? **[27.78 m/s]**

4. A certain express train has an acceleration of 3.6 km/h/s. What is its value in m/s^2 ? **[1.0 m/s^2]**

5. If there is an ascending gradient of 15 m in a track length of 1 km, what is the value of percentage gradient ? **[1.5%]**

6. A train runs at an average speed of 45 km per hour between stations 2.5 km apart. The train accelerates at 2 km/h/s and retards at 3 km/h/s. Find its maximum speed assuming a trapezoidal speed/time curve. Calculate also the distance travelled by it before the brakes are applied.
[50.263 km/h, 2.383 km] (*Elect. Traction and Utilization B.H.U.*)

7. The schedule speed with a 200 tonne train on an electric railway with stations 777 metres apart is 27.3 km/h and the maximum speed is 20 percent higher than the average running speed. The braking rate is 3.22 km/h/s and the duration of stops is 20 seconds. Find the acceleration required. Assume a simplified speed-time curve with free running at the maximum speed.
[2.73 km/h/s] (*Traction and Utilization of Elect. Power, Agra Univ.*)

8. A suburban electric train has a maximum speed of 65 km/h.. The schedule speed including a station stop of 30 seconds is 43.5 km/h. If the acceleration is 1.3 km/h/s, find the value of retardation when the average distance between stops is 3 km.
[β = 1.21 km/h/s] (*Utilization of Elect. Power and Traction, Gorakhpur Univ.,*)

9. An electric train is accelerated uniformly from rest to a speed of 40 km/h, the period of acceleration being 20 seconds. If it coasts for 60 seconds against a constant resistance of 50 N/t and is brought to rest in a further period of 10 seconds by braking, determine :
(*i*) the acceleration (*ii*) the coasting retardation (*iii*) the braking retardation (*iv*) distance travelled and (*v*) schedule speed with station stops of 10 seconds duration.
Allow 10 percent for rotational inertia. **(*Elect. Traction, Punjab Univ.*)**
[α = 2 km/h/s, β_c = 0.1636 km/h/s, β = 3 km/h/s., *D* = 0.736 km., *V* = 27.5 km/h]

10. The speed-time curve of an electric train on a uniform rising gradient of 1 in 100 comprises :
(*i*) uniform acceleration from rest at 2 km/h/s/ for 30 seconds.
(*ii*) coasting with power off for 70 seconds.
(*iii*) braking at 3 km/h/s to a standstill.
The weight of the train is 250 tonnes, the train resistance on level track being 49 N/tonne and allowance for rotary inertia 1%.
Calculate the maximum power developed by traction motors and total distance travelled by the train. Assume transmission efficiency as 97%.
[3,3258 kW, 1.12 km] (*Traction and Utilization of Elect. Power, Agra Univ.*)

11. A 400-tonne goods train is to be hauled by a locomotive up a gradient of 2% with an acceleration of 1 km/h/s. Co-efficient of adhesion is 20%, track resistance 40 N/tonne and effective rotating masses 10% of the dead weight. Find the weight of the locomotive and number of axles if the axle load is not increased beyond 22 tonnes.**[152.6 tonnes, 7] *(Traction and Utilization of Elect. Power, Agra Univ.)***

12. A 500-tonne goods train is to be hauled by a locomotive up a gradient of 20% with an acceleration of 1.2 km/h/s. Co-efficient of adhesion is 25%, track resistance 40 N/ tonne and effective rotating masses 10% of dead weight. Find the weight of the locomotive and number of axles if axle load is not to exceed 20 tonnes. **[160 tonnes, 8] (*Utilization of Elect. Power, A.M.I.E. Winter*)**

13. Determine the maximum adhesive weight of a loco required to start a 2340 tonne weight (inclusive of loco) on 1 : 150 gradient and accelerate it at 0.1 km/h/s. Assume co-efficient of adhesion as 0.25, train resistance 39.2 N/tonne and rotary inertia as 8%. **[128.5 tonnes]**
(Elect. Traction, A.M.I.E., May)

14. Ore carrying trains weighing 5000 tonne each are to be hauled down a gradient of 1 in 60 at a maximum speed of 40 km/h and started on a level track at an acceleration of 0.1 m/s^2. How many locomotives, each weighing 75 tonne, will have to be employed ?

Train resistance during starting = 29.4 N/tonne
Train resistance at 40 km/h = 56.1 kg/tonne
Co-efficient of adhesion = 1/3 ; Rotational inertia = 1/10

[3 Loco] (*Engg. Service Examination U.P.S.C.*)

15. A locomotive accelerates a 400-tonne train up a gradient of 1 in 100 at 0.8 km/h/s. Assuming the coefficient of adhesion to be 0.25, determine the minimum adhesive weight of the locomotive. Assume train resistance of 60 N/tonne and allow 10% for the effect of rotational inertia.
[65.7 t] (*Elect. Traction and Utilization, Nagpur Univ.*)

16. Calculate the specific energy consumption if a maximum speed of 12.20 metres/sec and for a given run of 1525 m an acceleration of 0.366 m/s^2 are desired. Train resistance during acceleration is 52.6 N/1000 kg and during coasting is 6.12 N/1000 kg, 10% being allowable for rotational inertia. The efficiency of the equipment during the acceleration period is 50%. Assume a quadrilateral speed-time curve. **[3.38 Wh/kg-m] (*Util. of Elect. Power, A.M.I.E. Sec. B*)**

17. An electric locomotive of 100 tonne can just accelerate a train of 500 tonne (trailing weight) with an acceleration of 1 km/h/s on an upgradient of 1/1000. Tractive resistance of the track is 45 N per tonne and the rotational inertia is 10%. If this locomotive is helpled by another locomotive of 120 tonnes, find, (*i*) the trailing weight that can be hauled up the same gradient under the same conditions and (*ii*) the maximum gradient, the trailing weight hauled remaining unchanged.
Assume adhesive weight expressed as percentage of total dead weight to be the same for both the locomotive. **[(*i*) 1120 t (*ii*) 3.15%] (*Util. of Elect. Power, A.M.I.E. Sec. B.*)**

18. An electric train has quadrilateral speed-time curve as follows :
(*i*) uniform acceleration from rest at 2 km/h/s for 30 sec,
(*ii*) coasting for 50 sec.
(*iii*) uniform braking to rest for 20 seconds.
If the train is moving uniform upgradient of a 10/1000, train resistance is 40 N/tonne, rotational inertia effect 10% of dead weight and duration of stop 30 seconds, find the schedule speed.
[28.4 km/h] (*Util. of Elect. Power, A.M.I.E. Sec. B.*)

19. The schedule speed with a 200 tonne train on an electric railway with stations 777 metres apart is 27.3 km/h and the maximum speed is 20% higher than the average running speed. The braking rate is 3.22 km/h/s and the duration of stops is 20 seconds. Find the acceleration required. Assume a simplified speed-time curve with the free running at the maximum speed.
[2.73 km/h/s] (*Traction & Util. of Elect. Power, Agra Univ.*)

20. An electric train has an average speed of 42 km/h on a level track between stops 1,400 metre apart. It is accelerated at 1.7 km/h/s and is braked at 3.3 km/h/s. Draw the speed-time curve for the run. Estimate the sp. energy consumption. Assume tractive resistance as 50 N/t and allow 10% for rotational inertia. **[39.48 Wh/t-km] (*Util. of Elect. Power, A.M.I.E. Sec. B.*)**

21. An electric train weighing 200 tonne has eight motors geared to driving wheels, each wheel is 90 cm diameter. Determine the torque developed by each motor to accelerate the train to a speed of 48 km/h in 30 seconds up a gradient of 1 in 200. The tractive resistance is 50 N/t, the effect of rotational inertia is 10% of the train weight, the gear ratio is 4 to 1 and gearing efficiency is 80%.
[2,067 N-m] (*Traction & Util. of Elect. Power, Agra Univ.*)

22. An electric train accelerates uniformly from rest to a speed of 48 km/h in 24 seconds. It then coasts for 69 seconds against a constant resistance of 58 N/t and is braked to rest at 3.3 km/h/s in 11 seconds.

Calculate (*i*) the acceleration (*ii*) coasting retardation and (*iii*) the schedule speed, if the station stops are of 20 seconds duration. What would be the effect on schedule speed of reducing the station stops to 15 second duration, other conditions remaining the same? Allow 10% for the rotational inertia.

[(*i*) 2 km/h/s (*ii*) 0.19 km/h/s (*iii*) 30.25 km/h] (*Util. of Elect. Power, A.M.I.E. Sec. B.*)

23. An electric train accelerates uniformly from rest to a speed of 50 km/h in 25 seconds. It then coasts for 1 minute 10 seconds against a constant resistance of 70 N/t and is braked to rest at 4 km/h/s in 10 seconds. Calculate the schedule speed, if the station stops are of 15 second duration.

[31.125 km/h] (*Util. of Elect. Power, A.M.I.E. Sec. B*)

24. An electric train has a quadrilatural speed-time curve as follows :

(*i*) uniform acceleration from rest at 2.5 km/h/s for 25 second

(*ii*) coasting for 50 second (*iii*) duration of braking 25 second.

If the train is moving along a uniform upgradient of 1 in 100 with a tractive resistance of 45 N/t, rotational inertia 10% of dead weight, duration of stops at stations 20 second and overall efficiency of transmission gear and motor 80%, calculate the schedule speed and specific energy consumption of run. **[69 km/h, 26.61 Wh/t-km] (*Util. of Elect. Power, A.M.I.E. Sec. B*)**

25. An ore-carrying train weighing 5000 tonne is to be hauled down a gradient of 1 in 50 at a maximum speed of 30 km/h and started on a level track at an acceleration of 0.08 m/s^2. How many locomotives, each weighing 75 tonne, will have to be employed ?

Train resistance during starting =3 kg/t

Train resistance at 30 km/h = 5 kg/t

Co-efficient of adhesion = 0.3, Rotational inertia = 10%.

[4 loco] (*Util. of Elect. Power, A.M.I.E. Sec. B.*)

26. A train with an electric locomotive weighing 300 tonne is is to be accelerated up a gradient of 1 in 33 at an acceleration of 1 km/h/s. If the train resistance, co-efficient of adhesion and effect of rotational inertia are 80 N/t, 0.25 and 12.5% of the dead weight respectively, determine the minimum adhesive weight of the locomotive. **[88 t] (*Util. of Elect. Power, A.M.I.E. Sec. B.)***

27. A train weighing 400 tonne has speed reduced by regenerative braking from 40 to 20 km/h over a distance of 2 km at a down gradient of 20%. Calculate the electrical energy and average power returned to the line. Tractive resistance is 40 N/t and allow rotational inertia of 10% and efficiency of conversion 75%. **[324 kW/h, 4860 kW] (*Util. & Traction Power, Agra Univ.*)**

28. A 250-tonne motor coach having 4 motors, each developing 5,000 N-m torque acceleration, starts from rest. If upgradient is 25 in 1000, gear ratio 5, gear transmission efficiency 88%, wheel radius 44 cm, train resistance 50 N/t addition of rotational inertia 10%, calculate the time taken to reach a speed of 45 km/h.

If the supply voltage were 1500 V d.c. and efficiency of motor is 83.4%, determine the current drawn per motor during notching period. **[27.25 s, 500 A] (*Util. of Elect. Power, A.M.I.E. Sec. B.*)**

29. An electric train weighing 100 tonne has a rotational inertia of 10%. This train while running between two stations which are 2.5 km apart has an average speed of 50 km/h. The acceleration and retardation during braking are respectively 1 km/h/s and 2 km/h/s. The percentage gradient between these two stations is 1% and the train is to move up the incline. The track resistance is 40 N/t. If the combined efficiency of the electric train is 60%, determine (*i*) maximum power at driving axle (*ii*) total energy consumption and (*iii*) specific energy consumption. Assume that journey estimation is being made in simplified trapezoidal speed-time curve.

[(*i*) 875 kW (*ii*) 23.65 kW/h (*iii*) 94.6 Wh/t-km] (*Util. of Elect. Power, A.M.I.E. Sec. B.*)

30. A 500-tonne goods train is to be hauled by a locomotive up a gradient of 1 in 40 with an acceleration of 1.5 km/h/s. Determine the weight of the locomotive and number of axles, if axle load is not to exceed 24 tonne. Co-efficient of adhesion is 0.31, track resistance 45 N/t and effective rotating masses 10% of dead weight. **[7] (*Util. of Elect. Power, A.M.I.E. Sec. B.*)**

31. Two d.c. series motors of a motor coach have resistance of 0.1 W each. These motors draw a current of 500 A from 600V mains during series-parallel starting period of 20 seconds. If the acceleration during starting period remains uniform, determine :

(*i*) time during which motor operates in (*a*) series, (*b*) parallel

(*ii*) the speed at which the series connections are to be changed if the speed just after starting period is 70 km/h.

[(*i*) 9.098, 10.971 Sec. (*ii*) 31.82 km/h] (*Utili. of Elect. Power and Traction, Agra Univ.*)

32. Explain how series motors are ideally suited for traction service.
(Nagpur University, Summer 2004)

33. Explain any one method for regenerative braking of D.C. motor for traction.
(Nagpur University, Summer 2004)

34. Discuss the effect of unequal wheel diameters on the parallel operation of traction motors.
(Nagpur University, Summer 2004)

35. Explain the various modes of operation in traction services with neat speed-time curve.
(Nagpur University, Summer 2004)

36. A 100 tonne motor coach is driven by 4 motors, each developing a torque of 5000 N-m during acceleration. If up-gradient is 50 in 1000, gear ratio a = 0.25, gear transmission efficiency 98%, wheel radius 0.54 M, train resistance 25 N/tonne, effective mass on account of rotational inertia is 10% higher, calculate the time taken to attain a speed of 100 kmph.
(Nagpur University, Summer 2004)

37. What are the requirements of an ideal traction system?
(J.N. University, Hyderabad, November 2003)

38. What are the advantages and disadvantages of electric traction?
(J.N. University, Hyderabad, November 2003)

39. Write a brief note on the single phase a.c. series motor and comment upon it's suitability for traction services. How does it compare in performance with the d.c. Services motor.
(J.N. University, Hyderabad, November 2003)

40. Draw the speed-time curve of a main line service and explain.
(J.N. University, Hyderabad, November 2003)

41. A train hs a scheduled speed of 40 km/hr between two stops, which are 4 kms apart. etermine the crest speed over the run, if the duration of stops is 60 sec and acceleration and retardation both are 2 km/hr/sec each. Assume simplified trapezoidal speed-time curve.
(J.N. University, Hyderabad, November 2003)

42. What are the various electric traction systems in India? Compare them.
(J.N. University, Hyderabad, November 2003)

43. Give the features of various motors used in electric traction.
(J.N. University, Hyderabad, November 2003)

44. Draw the speed-time curve of a suburban service train and explain.
(J.N. University, Hyderabad, November 2003)

45. A train accelerates to a speed of 48 km/hr in 24 sec. then it coasts for 69 sec under a constant resistance of 58 newton/tonne and brakes are applied at 3.3 km/hr/sec in 11 sec. Calculate (*i*) the acceleration (*ii*) the coasting retardation (*iii*) the scheduled speed if station stoppage is 20 secs. What is theeffect of scheduled speed if station stoppage is reduced to 15 sec duration, other conditions remaining same. Allow 10% for rotational inertia.
(J.N. University, Hyderabad, November 2003)

46. Derive an expression for specific energy output on level track using a simplified speed-time curve. What purpose is achieved by this quantity? ***(J.N. University, Hyderabad, November 2003)***

47. A 400 tonne goods train is to be hauled by a locomotive up a gradient of 2% with acceleration of 1 km/hr/sec, coefficient of adhesion 20%, track resistance 40 newtons/tonne and effective rotating masses 10% of the dead weight. Find the weight of the locomotive and the number of axles if the axle load is not to increase beyond 22 tonnes.
(J.N. University, Hyderabad, November 2003)

48. A motor has the following load cycle :
Accelerating period 0-15 sec Load rising uniformly from 0 to 1000 h.p.
Full speed period 15-85 sec Load constant at 600 h.p.
Decelerating period 85-100 sec h.p. returned to line falls uniformly from 200 to zero
Decking period 100-120 sec Motor stationary. Estimate the size of the motor.
(J.N. University, Hyderabad, November 2003)

49. Explain the characteristics of series motors and also explain how they are suitable of electric traction work? ***(J.N. University, Hyderabad, November 2003)***

50. For a trapezoidal speed-time curve of a electric train, derive expression for maximum speed and distance between stops. ***(J.N. University, Hyderabad, November 2003)***

51. A mail is to be run between two stations 5 kms apart at an average speed of 50 km/hr. If the maximum speed is to be limited to 70 km/hr, acceleration to 2 km/hr/sec, braking retardation to 4 km/hr/sec and coasting retardation to 0.1 km/hr/sec, determine the speed at the end of coasting, duration of coasting period and braking period. ***(J.N. University, Hyderabad, November 2003)***

52. Discuss the merits and demerits of the D.C. and 1–ϕ A.C. systems for the main and suburban line electrification of the railways. ***(J.N. University, Hyderabad, April 2003)***

53. Which system do you consider to be the best for the suburban railways in the vicinity of large cities? Given reasons for your answer. ***(J.N. University, Hyderabad, April 2003)***

54. Derive expression for the tractive effort for a train on a level track.
(J.N. University, Hyderabad, April 2003)

55. The maximum speed of a suburbanelectric train is 60 km/hr. Its scheduled speed is 40 km/hr and duration of stops is 30 sec. If the acceleration is 2 km/hr/sec and distance between stops is 2 kms, determine the retardation. ***(J.N. University, Hyderabad, April 2003)***

56. What are various types of traction motors? ***(J.N. University, Hyderabad, April 2003)***

57. What are the advantages of series parallel control of D.C. motors?
(J.N. University, Hyderabad, April 2003)

58. Describe about duplication of railway transmission lines.
(J.N. University, Hyderabad, April 2003)

59. Write a note on feeding and distributing system on A.C. Traction and for d.c. tram ways.
(J.N. University, Hyderabad, April 2003)

60. For a quadrilateral speed-time curve of a electric train, derive expression for the distance between stops and speed at the end of the coasting period. ***(J.N. University, Hyderabad, April 2003)***

61. A train is required to run between stations 1.6 kms apart at an average speed of 40 km/hr. The runis to be made from a quadilateral speed-time curve. The acceleration is 2 km/hr/sec. The coasting and braking retardations are 0.16 km/hr/sec and 3.2 km/hr/sec respectively. Determine the duration of acceleration, coasting and braking and the distance covered in each period.
(J.N. University, Hyderabad, April 2003)

62. Explain the characteristics of D.C. compound motors and explain its advantage over the series motor. ***(J.N. University, Hyderabad, April 2003)***

63. What are the requirements to be satisfied by an ideal traction system?
(J.N. University, Hyderabad, April 2003)

64. What are the advantages and disdvantages of electrification of track?
(J.N. University, Hyderabad, April 2003)

65. Discuss why a D.C. series motor is ideally suited for traction services.
(J.N. University, Hyderabad, April 2003)

66. An electric locomotive of 100 tonnes can just accelerate a train of 500 tonnes (trailing weight) with an acceleration of 1 km/hr/sec on an up gradient 1 in 1000. Tractive resistance of the track is 45 newtons/tonne and the rotational inertia is 10%. If this locomotive is helped by another locomotive of 120 tonnes, find (i) the trailing weight that can be hauled up the same gradient, under the same condition (ii) the maximum gradient, the trailing hauled load remaining unchanged. Assume adhesive weight expressed as percentage of total dead weight to be same for both the locomotives. ***(J.N. University, Hyderabad, April 2003)***

67. Explain how electric regeneration braking is obtained with a D.C. locomotive. How is the braking torque varied? ***(J.N. University, Hyderabad, April 2003)***

68. Explain why a series motor is preferred for the electric traction.
(J.N. University, Hyderabad, April 2003)

69. The characteristics of a series motor at 525 – V are as follows :

Current (A)	50	100	150	200
Speed (RPM)	1200	952	840	745

Determine the current when working as a generator at 1000 R.P.M. and loaded with a resistance of 3 ohms. The resistance of the motor is 0.5 ohms. ***(J.N. University, Hyderabad, April 2003)***

70. Briefly explain the a.c. motors used in traction. ***(J.N. University, Hyderabad, April 2003)***

71. The scheduled speed of a trolley service is to be 53 km/hr. The distance between stops is 2.8 km. The track is level and each stop is of 30 sec duration. Using simplified speed-time curve, calculate the maximum speed, assuming the acceleration to be 2 km/hr/sec, retardation 3.2 km/hr/sec, the dead weight of the car as 16 tonnes, rotational inertia as 10% of the dead weight and track resistance as 40 newtons/tonne. If the overall efficiency is 80%, calculate (*i*) the maximum power output from the driving axles (*ii*) the specific energy consumption in watt-hr/tonne-km.
(J.N. University, Hyderabad, April 2003)

72. Discuss various traction systems you know of?
(J.N. University, Hyderabad, December 2002/January 2003)

73. Explain the requirements for ideal traction and show which drive satisfies almost all the requirements.
(J.N. University, Hyderabad, December 2002/January 2003)

74. Define the adhesive weight of a locomotive which accelerates up a gradient of 1 in 100 at 0.8 kmphps. The self weight of locomotive is 350 Tonnes. Coefficient of adhesion is 0.25. Assume a trainresistance of 45 N–m/Tonne and allow 10% for the effect of rotational inertia.
(J.N. University, Hyderabad, December 2002/January 2003)

75. State Factors affecting specific energy consumption.
(J.N. University, Hyderabad, December 2002/January 2003)

76. Explain with the help of a diagram, the four quadrant speed-torque characteristic of an induction motor when running in (*i*) forward direction (*ii*) reverse direction.
(J.N. University, Hyderabad, December 2002/January 2003)

77. Explain the general features of traction motors.
(J.N. University, Hyderabad, December 2002/January 2003)

78. A 250 tonne electric train maintains a scheduled speed of 30 kmph between stations situated 5 km apart, with station stops of 30 sec. The acceleration is 1.8 kmph ps and the braking retardation is 3 kmph ps. Assuming a trapezoidal speed-time curve, calculate (*i*) maximum speed of the train (*ii*) energy output of the motors if the tractive resistance is 40 NW per tonne.
(J.N. University, Hyderabad, December 2002/January 2003)

79. Discuss the relative merits of electric traction and the factors on which the choice of traction system depends. ***(J.N. University, Hyderabad, December 2002/January 2003)***

80. Explain the terms (*i*) tractiveeffort (*ii*) coefficient of adhesion (*iii*) specific energy consumption of train (*iv*) tractive resistance. ***(J.N. University, Hyderabad, December 2002/January 2003)***

81. Existing traction systems in India. ***(J.N. University, Hyderabad, December 2002/January 2003)***

82. Explain the terms tractive effort, coefficient of adhesion, train resistance and specific energy consumption of train. ***(J.N. University, Hyderabad, December 2002/January 2003)***

83. An electric train maintains a scheduled speed of 40 kmph between stations situated at 1.5 km apart. If is accelerated at 1.7 kmph.ps and is braked at 3.2 kmph.ps. Draw the speed-time curve for the run. Estimate the energy consumption at the axle of the train. Assume tractive resistance constants at 50 NW per tonne and allow 10% for the effect of rotation inertia. ***(J.N. University, Hyderabad, December 2002/January 2003)***

84. Explain the advantages of series parallel control of starting as compared to the rheostatic starting for a pair of dc traction motors. ***(J.N. University, Hyderabad, December 2002/January 2003)***

85. Discuss the main features of various train services. What type of services correspond to trapezoidal and quadrilateral speed-time curves. ***(J.N. University, Hyderabad, December 2002/January 2003)***

86. Existing electric traction system in India. ***(J.N. University, Hyderabad, December 2002/January 2003)***

87. Briefly explain the controlling of D.C. Motor. ***(Anna Univ., Chennai 2003)***

OBJECTIVE TESTS – 43

1. Diesel electric traction has comparatively limited overload capacity because

(*a*) diesel electric locomotive is heavier than a plain electric locomotive

(*b*) diesel engine has shorter life span

(*c*) diesel engine is a constant-kW output prime mover

(*d*) regenerative braking cannot be employed.

2. The most vital factor against electric traction is the

(*a*) necessity of providing a negative booster

(*b*) possibility of electric supply failure

(*c*) high cost of its maintenance

(*d*) high initial cost of laying out overhead electric supply system.

3. The direct current system used for tramways has a voltage of about .....volt.

(*a*) 750

(*b*) 1500

(*c*) 3000

(*d*) 2400

4. In electric traction if contact voltage exceeds 1500 V, current collection is invariably via a

(*a*) contact rail

(*b*) overhead wire

(*c*) third rail

(*d*) conductor rail.

5. For the single-phase ac system of track electrification, low frequency is desirable because of the following advantages

(*a*) it improves commutation properties of ac motors

(*b*) it increases ac motor efficiency

(*c*) it increases ac motor power factor

(*d*) all of the above.

6. In Kando system of track electrification, ...is converted into .....

(*a*) 1-phase ac, dc

(*b*) 3-phase ac, 1-phase ac

(*c*) 1-phase ac, 3-phase ac

(*d*) 3-phase ac, dc.

7. The main reason for choosing the composite 1-phase ac-to-dc system for all future track electrification in India is that it

(*a*) needs less number of sub-stations

(*b*) combines the advantages of high-voltage ac distribution at 50 Hz with dc series traction motors

(*c*) provides flexibility in the location of sub-stations

(*d*) requires light overhead catenary.

8. Ordinary, tramway is the most economical means of transport for

(*a*) very dense traffic of large city

(*b*) medium traffic densities

(*c*) rural services

(*d*) suburban services.

9. Unlike a tramway, a trolleybus requires no

(*a*) overhead contact wire

(*b*) driving axles

(*c*) hand brakes

(*d*) running rail.

10. The current collector which can be used at different speeds under all wind conditions and stiffness of OHE is called ..... collector.
(*a*) trolley
(*b*) bow
(*c*) pantograph
(*d*) messenger.

11. The speed/time curve for city service has no..... period.
(*a*) coasting
(*b*) free-running
(*c*) acceleration
(*d*) braking.

12. For the same value of average speed, increase in the duration of stops..... speed.
(*a*) increases the schedule
(*b*) increases the crest
(*c*) decreases the crest
(*d*) decreases the schedule.

13. A train weighing 490 tonne and running at 90 km/h has a mass of ..... kg and a speed of ..... m/s.
(*a*) 50,000, 25
(*b*) 490,000, 25
(*c*) 490, 25
(*d*) 50, 324.

14. A train has a mass of 500 tonne. Its weight is
(*a*) 500 t.wt
(*b*) 500,000 kg-wt
(*c*) 4,900,000 newton
(*d*) all of the above
(*e*) none of the above.

15. The free-running speed of a train does NOT depend on the
(*a*) duration of stops
(*b*) distance between stops
(*c*) running time
(*d*) acceleration.

16. A motor coach weighing 100 tonnes is to be given an acceleration of 1.0 km/h/s on an ascending gradient of 1 percent. Neglecting rotational inertia and train resistance, the tractive force required is ..... newton.
(*a*) 109,800
(*b*) 37,580
(*c*) 28,760
(*d*) 125,780.

17. In a train, the energy output of the driving axles in used for
(*a*) accelerating the train
(*b*) overcoming the gradient
(*c*) overcoming train resistance
(*d*) all of the above.

18. Longer coasting period for a train results in
(*a*) higher acceleration
(*b*) higher retardation
(*c*) lower specific energy consumption
(*d*) higher schedule speed.

19. Tractive effort of an electric locomotive can be increased by
(*a*) increasing the supply voltage
(*b*) using high kW motors
(*c*) increasing dead weight over the driving axles
(*d*) both (*b*) and (*c*) (*e*)both (*a*) and (*b*).

20. Skidding of a vehicle always occurs when
(*a*) braking effort exceeds its adhesive weight
(*b*) it negotiates a curve
(*c*) it passes over points and crossings
(*d*) brake is applied suddenly.

21. Which of the following is an advantage of electric traction over other methods of traction?
(*a*) Faster acceleration
(*b*) No pollution problems
(*c*) Better braking action
(*d*) All of the above

22. Which of the following is the voltage for single phase A.C. system?
(*a*) 22 V
(*b*) 440 V
(*c*) 5 kV
(*d*) 15 kV
(*e*) None of the above

23. Long distance railways use which of the following?
(*a*) 200 V D.C.
(*b*) 25 kV single phase A.C.
(*c*) 25 kV two phace A.C.
(*d*) 25 kV three phase A.C.

24. The speed of a locomotive is controlled by
(*a*) flywheel
(*b*) gear box
(*c*) applying brakes
(*d*) regulating steam flow to engine

25. Main traction system used in India are, those using
(*a*) electric locomotives
(*b*) diesel engine locomotives
(*c*) steam engine locomotives
(*d*) diesel electric locomotives
(*e*) all of the above

26. In India diesel locomotives are manufactured at
(*a*) Ajmer
(*b*) Varanasi
(*c*) Bangalore
(*d*) Jamalpur

27. For diesel locomotives the range of horsepower is
(*a*) 50 to 200
(*b*) 500 to 1000
(*c*) 1500 to 2500
(*d*) 3000 to 5000

28. ..... locomotive has the highest operational availability.
(*a*) Electric
(*b*) Diesel
(*c*) Steam

29. The horsepower of steam locomotives is
(*a*) upto 1500
(*b*) 1500 to 2000
(*c*) 2000 to 3000
(*d*) 3000 to 4000

30. The overall efficiency of steam locomotive is around
(*a*) 5 to 10 percent
(*b*) 15 to 20 percent
(*c*) 25 to 35 percent
(*d*) 35 to 45 percent

31. In tramways which of the following motors is used?
(*a*) D.C. shunt motor
(*b*) D.C. series motor
(*c*) A.C. three phase motor
(*d*) A.C. single phase capacitor start motor

32. In a steam locomotive electric power is provided through
(*a*) overhead wire
(*b*) battery system
(*c*) small turbo-generator
(*d*) diesel engine generator

33. Which of the following drives is suitable for mines where explosive gas exists?
(*a*) Steam engine
(*b*) Diesel engine
(*c*) Battery locomotive
(*d*) Any of the above

34. In case of locomotives the tractive power is provided by
(*a*) single cylinder double acting steam engine
(*b*) double cylinder, single acting steam engine
(*c*) double cylinder, double acting steam engine
(*d*) single stage steam turbine

35. Overload capacity of diesel engines is usually restricted to
(*a*) 2 percent
(*b*) 10 percent
(*c*) 20 percent
(*d*) 40 percent

36. In case of steam engines the steam pressure is
(*a*) 1 to 4 kgf/cm^2
(*b*) 5 to 8 kgf/cm^2
(*c*) 10 to 15 kgf/cm^2
(*d*) 25 to 35 kgf/cm^2

37. The steam engine provided on steam locomotives is
(*a*) single acting condensing type
(*b*) single acting non-condensing type
(*c*) double acting condensing type
(*d*) double acting non-condensing type

38. Electric locomotives in India are manufactured at
(*a*) Jamalpur
(*b*) Bangalore
(*c*) Chittranjan
(*d*) Gorakhpur

39. The wheels of a train, engine as well as bogies, are slightly tapered to
(*a*) reduce friction
(*b*) increase friction
(*c*) facilitate braking
(*d*) facilitate in taking turns

40. Automatic signalling is used for which of the following trains?

(*a*) Mail and express trains
(*b*) Superfast trains
(*c*) Suburban and Urban electric trains
(*c*) All trains

41. The efficiency of diesel locomotives is nearly
(*a*) 20 to 25 percent
(*b*) 30 to 40 percent
(*c*) 45 to 55 percent
(*d*) 60 to 70 percent

42. The speed of a superfast train is
(*a*) 60 kmph
(*b*) 75 kmph
(*c*) 100 kmph
(*d*) more than 100 kmph

43. The number of passenger coaches that can be attached to a diesel engine locomotive on broad gauge is usually restricted to
(*a*) 5
(*b*) 10
(*c*) 14
(*d*) 17

44. Which of the following state capitals is not on broad gauge track?
(*a*) lucknow
(*b*) Bhopal
(*c*) Jaipur
(*d*) Chandigarh

45. Which of the following is the advantage of electric braking?
(*a*) It avoids wear of track
(*b*) Motor continues to remain loaded during braking
(*c*) It is instantaneous
(*d*) More heat is generated during braking

46. Which of the following braking systems on the locomotives in costly?
(*a*) Regenerative braking on electric locomotives
(*b*) Vacuum braking on diesel locomotives
(*c*) Vacuum braking on steam locomotives
(*d*) All braking systems are equally costly

47. Tractive effort is required to
(*a*) overcome the gravity component of train mass
(*b*) overcome friction, windage and curve resistance
(*c*) accelerate the train mass
(*d*) do all of the above

48. For given maximum axle load tractive efforts of A.C. locomotive will be
(*a*) less than that of D.C. locomotive
(*b*) more than that of D.C. locomotive
(*c*) equal to that of D.C. locomotive
(*d*) none of the above

49. Co-efficient of adhesion reduces due to the presence of which of the following?
(*a*) Sand on rails
(*b*) Dew on rails
(*c*) Oil on the rails
(*d*) both (b) and (c)

50. Due to which of the following co-efficient of adhesion improves?
(*a*) Rust on the rails
(*b*) Dust on the rails
(*c*) Sand on the rails
(*d*) All of the above

51. Quadrilateral speed-time curve pertains to which of the following services?
(*a*) Main line service
(*b*) Urban service
(*c*) Sub-urban service
(*d*) Urban and sub-urban service

52. Which of the following is the disadvantage of electric traction over other systems of traction?
(*a*) Corrosion problems in the underground pip work
(*b*) Short time power failure interrupts traffic for hours
(*c*) High capital outlay in fixed installations beside route limitation
(*d*) Interference with communication lines
(*e*) All of the above

53. Co-efficient of adhesion is
(*a*) high in case of D.C. traction than in the case of A.C. traction
(*b*) low in case of D.C. traction that in the case of A.C. traction
(*c*) equal in both A.C. and D.C. traction
(*d*) any of the above

54. Speed-time curve of main line service differs from thoseof urban and suburban services on following account
(*a*) it has longer free running period
(*b*) it has longer coasting period

(*c*) accelerating and braking periods are comparatively smaller
(*d*) all of the above

55. The rate of acceleration on suburban or urban services is restricted by the consideration of
(*a*) engine power
(*b*) track curves
(*c*) passanger discomfort
(*d*) track size

56. The specific energy consumption of a train depends on which of the following?
(*a*) Acceleration and retardation
(*b*) Gradient
(*c*) Distance covered
(*d*) all of the above

57. The friction at the track is proportional to
(*a*) 1/speed
(*b*) $1/(\text{speed})^2$
(*c*) speed
(*d*) none of the above

58. The air resistance to the movement of the train is proportional to
(*a*) speed
(*b*) $(\text{speed})^2$
(*c*) $(\text{speed})^3$
(*d*) 1/speed

59. The normal value of adhesion friction is
(*a*) 0.12
(*b*) 0.25
(*c*) 0.40
(*d*) 0.75

60. The pulsating torque exerted by steam locomotives causes which of the following?
(*a*) Jolting and skidding
(*b*) Hammer blow
(*c*) Pitching
(*d*) All of the above

61. Which of the following braking systems is used on steam locomotives?
(*a*) Hydraulic system
(*b*) Pneumatic system
(*c*) Vacuum system
(*d*) None of the above

62. Vacuum is created by which of the following?
(*a*) Vacuum pump
(*b*) Ejector
(*c*) Any of the above
(*d*) None of the above

63. The resistance encountered by a train in motion is on account of
(*a*) resistance offered by air
(*b*) friction at the track
(*c*) friction at various parts of the rolling stock
(*d*) all of the above

64. Battery operated trucks are used in
(*a*) steel mills
(*b*) power stations
(*c*) narrow gauge traction
(*d*) factories for material transportation

65. ..... method can bring the locomotive todead stop.
(*a*) Plugging braking
(*b*) Rheostatic braking
(*c*) Regenerative braking
(*d*) None of the above

66. The value of co-efficient of adhesion will be high when rails are
(*a*) greased
(*b*) wet
(*c*) sprayed with oil
(*d*) cleaned with sand

67. The voltage used for suburban trains in D.C. system is usually
(*a*) 12 V
(*b*) 24 V
(*c*) 220 V
(*d*) 600 to 750 V

68. For three-phase induction motors which of the following is the least efficient method of speed control?
(*a*) Cascade control
(*b*) Pole changing
(*c*) Rheostatic control
(*d*) Combination of cascade and pole changing

69. Specific energy consumption becomes
(*a*) more on steeper gradient
(*b*) more with high train resistance
(*c*) less if distance between stops is more
(*d*) all of the above

70. In main line service as compared to urban and suburban service

(*a*) distance between the stops is more
(*b*) maximum speed reached is high
(*c*) acceleration and retardation rates are low
(*d*) all of the above

71. Locomotive having monomotor bogies
(*a*) has better co-efficient of adhesion
(*b*) are suited both for passanger as well as freight service
(*c*) has better riding qualities due to the reduction of lateral forces
(*d*) has all above qualities

72. Series motor is not suited for traction duty due to which of the following account?
(*a*) Less current drain on the heavy load torque
(*b*) Current surges after temporary switching off supply
(*c*) self relieving property
(*d*) Commutating property at heavy load

73. When a bogie negotiates a curve, reduction in adhesion occurs resulting in sliding. Thus sliding is acute when
(*a*) wheel base of axles is more
(*b*) degree of curvature is more
(*c*) both (a) and (b)
(*c*) none of the above

74. Energy consumption in propelling the train is required for which of the following?
(*a*) Work against the resistance to motion
(*b*) Work against gravity while moving up the gradient
(*c*) Acceleration
(*d*) All of the above

75. An ideal traction system should have .....
(*a*) easy speed control
(*b*) high starting tractive effort
(*c*) equipment capable of with standing large temporary loads
(*d*) all of the above

76. ..... have maximum unbalanced forces
(*a*) Diesel shunters
(*b*) Steam locomotives
(*c*) Electric locomotives
(*d*) Diesel locomotives

77. Specific energy consumption is affected by which of the following factors?
(*a*) Regardation and acceleration values
(*b*) Gradient
(*c*) Distance between stops
(*d*) All of the above

78. In case of ..... free running and coasting periods are generally long.
(*a*) main-line service
(*b*) urban wervice
(*c*) sub-urban service
(*d*) all of the above

79. Overhead lines for power supply to tramcars are at a minimum height of
(*a*) 3 m
(b) 6 m
(*c*) 10 m
(*d*) 20 m

80. The return circuit for tram cars is through .....
(*a*) neutral wire
(*b*) rails
(*c*) cables
(*d*) common earthing

81. Specific energy consumption is least in ..... service.
(*a*) main line
(*b*) urban
(*c*) suburban

82. Locomotives with monometer bogies have
(*a*) uneven distribution of tractive effect
(*b*) suitability for passanger as well as freight service
(*c*) lot of skidding
(*d*) low co-efficient of adhesion

83. ..... was the first city in India to adopt electric traction.
(*a*) Delhi
(*b*) Madras
(*c*) Calcutta
(*d*) Bombay

84. ..... frequency is not common in low frequency traction system
(*a*) 40 Hz
(*b*) 25 Hz
(*c*) 16Hz

85. For 25 kV single phase system power supply frequency is .....
(*a*) 60 Hz
(*b*) 50 Hz
(*c*) 25 Hz
(*d*) $16\frac{2}{3}$ Hz

86. Power for lighting in passenger coach, in a long distance electric train, is provided
(*a*) directly through overhead electric
(*b*) through individual generator of bogie and batteries
(*c*) through rails
(*d*) through locomotive

87. In India, electrification of railway track was done for the first time in which of the following years?
(*a*) 1820–1825
(*b*) 1880–1885
(*c*) 1925–1932
(*d*) 1947–1954

88. Suri transmission is .....
(*a*) electrical-pneumatic
(*b*) mechanical-electrical
(*c*) hydro-mechanical
(*d*) hydro-pneumatic

89. In case of a steam engine an average coat consumption per km is nearly
(*a*) 150 to 175 kg
(*b*) 100 to 120 kg
(*c*) 60 to 80 kg
(*d*) 28 to 30 kg

90. Which of the following happens in Kando system?
(*a*) Three phase A.C. is converted into D.C.
(*b*) Single phase A.C. is converted into D.C.
(*c*) Single phase supply is converted into three phase system
(*d*) None of the above

91. For which of the following locomotives the maintenance requirements are the least?
(*a*) Steam locomotives
(*b*) Diesel locomotives
(*c*) Electric locomotives
(*d*) Equal in all of the above

92. Which of the following methods is used to control speed of 25 kV, 50 Hz single phase traction?
(*a*) Reduced current method
(*b*) Tapchanging control of transformer
(*c*) Series parallel operation of motors
(*d*) All of the above

93. If the co-efficient of adhesion on dry rails is 0.26, which of the following could be the value for wet rails?
(*a*) 0.3
(*b*) 0.26
(*c*) 0.225
(*d*) 0.16

94. ..... watt-hours per tonne km is usually the specific energy consumption for suburban services.
(*a*) 15–20
(*b*) 50–75
(*c*) 120–150
(*d*) 160–200

95. The braking retardation is usually in the range
(*a*) 0.15 to 0.30 km phps
(*b*) 0.30 to 0.6 km phps
(*c*) 0.6 to 2.4 km phps
(*d*) 3 to 5 km phps
(e) 10 to 15 km phps

96. The rate of acceleration on suburban or urban service is in the range
(*a*) 0.2 to 0.5 km phps
(*b*) 1.6 to 4.0 km phps
(*c*) 5 to 10 km phps
(*d*) 15 to 25 km phps

97. The coasting retardation is around
(*a*) 0.16 km phps
(b) 1.6 km phps
(*c*) 16 km phps
(*d*) 40 km phps

98. which of the following track is electrified
(*a*) Delhi–Bombay
(*b*) Delhi–Madras
(*c*) Delhi–Howrah
(*d*) Delhi–Ahmedabad

99. ..... is the method of braking in which motor armature remains connected to the supply and draws power from it producing torque opposite to the direction of motion.
(*a*) Rheostatic braking
(*b*) Regerative braking
(*c*) Plugging

100. For 600 V D.C. line for tramcars, brack is connected to .....
(*a*) positive of the supply
(*b*) negative of the supply
(*c*) mid voltage of 300 V
(*d*) none of the above

ANSWERS

1. (c)	2. (d)	3. (a)	4. (b)	5. (d)	6. (c)	7. (b)	8. (a)	9. (d)	10. (c)
11. (b)	12. (d)	13. (b)	14. (d)	15. (a)	16. (b)	17. (d)	18. (c)	19. (d)	20. (a)
21. (d)	22. (d)	23. (b)	24. (d)	25. (e)	26. (b)	27. (c)	28. (a)	29. (a)	30. (a)
31. (b)	32. (c)	33. (c)	34. (c)	35. (b)	36. (c)	37. (d)	38. (c)	39. (d)	40. (c)
41. (a)	42. (d)	43. (d)	44. (c)	45. (a)	46. (a)	47. (d)	48. (b)	49. (d)	50. (d)
51. (d)	52. (e)	53. (b)	54. (d)	55. (c)	56. (d)	57. (c)	58. (b)	59. (b)	60. (a)
61. (c)	62. (c)	63. (d)	64. (d)	65. (a)	66. (d)	67. (d)	68. (c)	69. (d)	70. (d)
71. (d)	72. (b)	73. (c)	74. (d)	75. (d)	76. (b)	77. (d)	78. (a)	79. (c)	80. (b)
81. (a)	82. (b)	83. (d)	84. (a)	85. (b)	86. (b)	87. (c)	88. (c)	89. (d)	90. (c)
91. (c)	92. (b)	93. (d)	94. (b)	95. (d)	96. (b)	97. (a)	98. (c)	99. (c)	100. (b)

ROUGH WORK

GO To FIRST

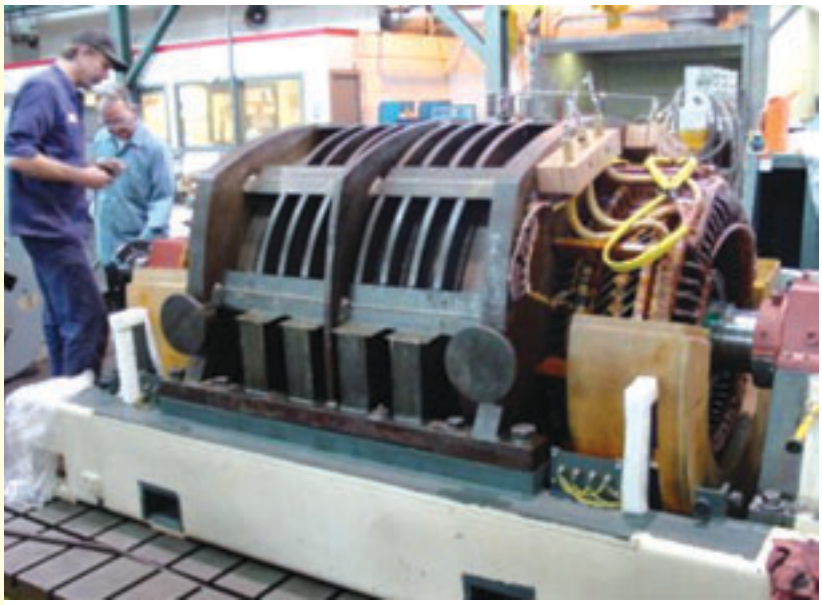
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CHAPTER 44

INDUSTRIAL APPLICATIONS OF ELECTRIC MOTORS

Learning Objectives

- Advantages of Electric Drive
- Classification of Electric Drives
- Advantages of Individual Drives
- Selection Drive
- Electric Characteristics
- Types of Enclosures
- Bearings Transmission of Power
- Noise
- Size and Rating
- Estimation of Motor Rating
- Different Types of Industrial Loads
- Motors for Different Industrial Drives
- Types of Electric Braking
- Plugging Applied to DC Motors
- Plugging of Induction Motors
- Rheostatic Braking
- Rheostatic Braking of DC Motors
- Rheostatic Braking Torque
- Rheostatic Braking of Induction Motors
- Regenerative Braking
- Energy Saving in Regenerative Braking



The above figure shows a squirrel cage motor. In industries electric drive is preferred over mechanical drive, because electric drive has the advantages of quick start, high torques and comparatively hassle free operation

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44.1. Advantages of Electric Drive

Almost all modern industrial and commercial undertakings employ electric drive in preference to mechanical drive because it possesses the following advantages :

1. It is simple in construction and has less maintenance cost
2. Its speed control is easy and smooth
3. It is neat, clean and free from any smoke or flue gases
4. It can be installed at any desired convenient place thus affording more flexibility in the layout
5. It can be remotely controlled
6. Being compact, it requires less space
7. It can be started immediately without any loss of time
8. It has comparatively longer life.

However, electric drive system has two inherent disadvantages :

1. It comes to stop as soon as there is failure of electric supply and
2. It cannot be used at far off places which are not served by electric supply.

However, the above two disadvantages can be overcome by installing diesel-driven dc generators and turbine-driven 3-phase alternators which can be used either in the absence of or on the failure of normal electric supply.

44.2. Classification of Electric Drives

Electric drives may be grouped into three categories : group drive, individual drive and multimotor drive.

In group drive, a single motor drives a number of machines through belts from a common shaft. It is also called line shaft drive. In the case of an indvidual drive, each machine is driven by its own separate motor with the help of gears, pulley etc. In multi-motor drives separate motors are provided for actuating different parts of the driven mechanism. For example, in travelling cranes, three motors are used : one for hoisting, another for long travel motion and the third for cross travel motion. Multimotor drives are commonly used in paper mills, rolling mills, rotary printing presses and metal working machines etc.

Each type of electric drive has its own advantages and disadvantages. The group drive has following advantages :

1. It leads to saving in initial cost because one 150-kW motor costs much less than ten 15-kW motors needed for driving 10 separate machines.
2. Since all ten motors will seldomly be required to work simultaneously, a single motor of even 100-kW will be sufficient to drive the main shaft. This diversity in load reduces the initial cost still further.
3. Since a single large motor will always run at full-load, it will have higher efficiency and power factor in case it is an induction motor.
4. Group drive can be used with advantage in those industrial processes where there is a sequence of continuity in the operation and where it is desirable to stop these processes simultaneously as in a flour mill.

However, group drive is seldom used these days due to the following disadvantages :

1. Any fault in the driving motor renders all the driven equipment idle. Hence, this system is unreliable.
2. If all the machines driven by the line shaft do not work together, the main motor runs at reduced load. Consequently, it runs with low efficiency and with poor power factor.

3. Considerable amount of power is lost in the energy transmitting mechanism.
4. Dlexibility of layout of different machines is lost since they have to be so located as to suit the position of the line shaft.
5. The use of line shaft, pulleys and belts etc. makes the drive look quite untidy and less safe to operate.
6. It cannot be used where constant speed is required as in paper and textile industry.
7. Noise level at the worksite is quite high.

44.3. Advantages of Individual Drive

It has the following advantages :

1. Since each machine is driven by a separate motor, it can be run and stopped as desired.
2. Machines not required can be shut down and also replaced with a minimum of dislocation.
3. There is flexibility in the installation of different machines.
4. In the case of motor fault, only its connected machine will stop whereas others will continue working undisturbed.
5. The absence of belts and line shafts greatly reduces the risk of accidents to the operating personnel.
6. Ach operator has full control of the machine which can be quickly stopped if an accident occurs.
7. Maintenance of line shafts, bearings, pulleys and belts etc. is eliminated. Similarly there is no danger of oil falling on articles being manufactured–something very important in textile industry.

The only disadvantage of individual drive is its initial high cost (Ex 44.1). However, the use of individual drives and multimotor drives has led to the introduction of automation in production processes which, apart from increasing the productivity of various undertakings, has increased the reliability and safety of operation.

Example 44.1. *A motor costing Rs. 10,000/- is used for group drive in a certain installation. How will its total annual cost compare with the case where four individual motors each costing Rs. 4000/- were used ? With group drive, the energy consumption is 50 MWh whereas it is 45 MWh for individual drive. The cost of electric energy is 20 paise/kWh. Assume depreciation, maintenance and other fixed charges at 10% in the case of group drive and 15 per cent in the case of individual drive.*

Solution. Group Drive

Capital cost	= Rs. 10,000/-
Annual depreciation, maintenance and other fixed charges	= 10% of Rs. 10,000 = Rs. 1,000/-
Annual cost of energy	= Rs. $50 \times 10^3 \times (20/100)$ = Rs. 10,000/-
Total annual cost	= Rs. 1,000 + Rs. 10,000 = **Rs. 11,000/-**

Individual Drive

Capital cost	= $4 \times$ Rs. 4000 = Rs. 16,000/-
Annual depreciation, maintenance and other fixed charges	= 15% of Rs. 16,000 = Rs. 2400/-
Annual cost of energy	= Rs. $45 \times 10^3 \times (20/100)$ = Rs. 9000/-
Total annual cost	= Rs. 9000 + Rs. 2400= **Rs. 11,400/-**

It is seen from the above example that individual drive is costlier than the group drive.

44.4. Selection of a Motor

The selection of a driving motor depends primarily on the conditions under which it has to operate and the type of load it has to handle. Main guiding factors for such a selection are as follows :

(*a*) Electrical characteristics

1. Starting characteristics
2. Running characteristics
3. Speed control
4. Braking

(*b*) Mechanical considerations

1. Type of enclosure
2. Type of bearings
3. Method of power transmission
4. Type of cooling
5. Noise level

(*c*) Size and rating of motors

1. Requirement for continuous, intermittent or variable load cycle
2. Overload capacity

(*d*) Cost

1. Capital cost
2. Running cost

In addition to the above factors, one has to take into consideration the type of current available whether alternating or direct. However, the basic problem is one of matching the mechanical output of the motor with the load requirement *i.e.* to select a motor with the correct speed/torque characteristics as demanded by the load. In fact, the complete selection process requires the analysis and synthesis of not only the load and the proposed motor but the complete drive assembly and the control equipment which may include rectification or frequency changing.

44.5. Electrical Characteristics

Electrical characteristics of different electric drives have been discussed in Vol. II of this book entitled "A.C. and D.C. Machines".

44.6. Types of Enclosures

The main function of an enclosure is to provide protection not only to the working personnel but also to the motor itself against the harmful ingress of dirt, abrasive dust, vapours and liquids and solid foreign bodies such as a spanner or screw driver etc. At the same time, it should not adversely affect the proper cooling of the motor. Hence, different types of enclosures are used for different motors depending upon the environmental conditions. Some of the commonly used motor enclosures are as under :

1. Open Type. In this case, the machine is open at both ends with its rotor being supported on pedestal bearings or end brackets. There is free ventilation since the stator and rotor ends are in free contact with the surrounding air. Such, machines are housed in a separate neat and clean room. This type of enclosure is used for large machines such as d.c. motors and generators.

2. Screen Protected Type. In this case, the enclosure has large openings for free ventilation. However, these openings are fitted with screen covers which safeguard against accidental contacts and rats entering the machine but afford no protection from dirt, dust and falling water. Screen-protected type motors are installed where dry and neat conditions prevail without any gases or fumes.

3. Drip Proof Type. This enclosure is used in very damp conditions. *i.e.* for pumping sets. Since motor openings are protected by over-hanging cowls, vertically falling water and dust are not able to enter the machine.

4. Splash-proof Type. In such machines, the ventilating openings are so designed that liquid or dust particles at an angle between vertical and 100° from it cannot enter the machine. Such type of motors can be safely used in rain.

5. Totally Enclosed (TE) Type. In this case, the motor is completely enclosed and no openings are left for ventilation. All the heat generated due to losses is dissipated from the outer surface which is finned to increase the cooling area. Such motors are used for dusty atmosphere *i.e.* sawmills, coal-handling plants and stone-crushing quarries etc.

6. Totally-enclosed Fan-cooled (TEFC) Type. In this case, a fan is mounted on the shaft external to the totally enclosed casing and air is blown over the ribbed outer surfaces of the stator and endshields (Fig. 44.1). Such motors are commonly used in flour mills, cement works and sawmills etc. They require little maintenance apart from lubrication and are capable of giving years of useful service without any interruption of production.



Fig. 44.1. A three-phase motor

7. Pipe-ventilated Type. Such an enclosure is used for very dusty surroundings. The motor is totally enclosed but is cooled by neat and clean air brought through a separate pipe from outside the dust-laden area. The extra cost of the piping is offset by the use of a smaller size motor on account of better cooling.

8. Flame-proof (FLP) Type. Such motors are employed in atmospheres which contain inflammable gases and vapours *i.e.* in coal mines and chemical plants. They are totally enclosed but their enclosures are so constructed that any explosion within the motor due to any spark does not ignite the gases outside. The maximum operating temperature at the surface of the motor is much less than the ignition temperature of the surrounding gases.

44.7. Bearings

These are used for supporting the rotating parts of the machines and are of two types :

1. Ball or roller bearings **2.** Sleeve or bush bearings

(*a*) Ball Bearings

Upto about 75kW motors, ball bearings are preferred to other bearings because of their following advantages :

1. They have low friction loss

2. They occupy less space

3. They require less maintenance

4. Their use allows much smaller air-gap between the stator and rotor of an induction motor

5. Their life is long.



A ball-bearing

Their main disadvantages are with regard to cost and noise particularly at high motor speeds.

(*b*) Sleeve Bearings

These are in the form of self-aligning pourous bronze bushes for fractional kW motors and in the

form of journal bearings for larger motors. Since they run very silently, they are fitted on super-silent motors used for driving fans and lifts in offices or other applications where noise must be reduced to the absolute minimum.



A sleeve bearing

44.8. Transmission of Power

There are many ways of transmitting mechanical power developed by a motor to the driven machine.

1. Direct Drive. In this case, motor is coupled directly to the driven machine with the help of solid or flexible coupling. Flexible coupling helps in protecting the motor from sudden jerks. Direct drive is nearly 100% efficient and requires minimum space but is used only when speed of the driven machine equals the motor speed.

2. Belt Drive. Flat belts are extensively used for line-shaft drives and can transmit a maximum power of about 250 kW. Where possible, the minimum distance between the pulley centres should be 4 times the diameter of the larger pulley with a maximum ratio between pulley diameters of 6 : 1. The power transmitted by a flat belt increases in proportion to its width and varies greatly with its quality and thickness. There is a slip of 3 to 4 per cent in the belt drive.

3. Rope Drive. In this drive, a number of ropes are run in V-grooves over the pulleys. It has negligible slip and is used when the power to be transmitted is beyond the scope of belt drive.



Fig. 44.2. Geared Motor Unit

4. Chain Drive. Though somewhat more expensive, it is more efficient and is capable of transmitting larger amounts of power. It is noiseless, slipless and smooth in operation.

5. Gear Drive. It is used when a high-speed motor is to drive a low-speed machine. The coupling between the two is through a suitable ratio gear box. In fact motors for low-speed drives are manufactured with the reduction gear incorporated in the unit itself. Fig. 44.2 shows such a unit consisting of a flange motor bolted to a high-efficiency gear box which is usually equipped with feet, the motor being overhung.

44.9. Noise

The noise produced by a motor could be magnetic noise, windage noise and mechanical noise. Noise level must be kept to the minimum in order to avoid fatigue to the workers in a workshop. Similarly, motors used for domestic and hospital appliances and in offices and theatres must be almost noiseless. Tranmission of noise from the building where the motor is installed to another building can be reduced if motor foundation is flexible *i.e.* has rubber pads and springs.

44.10. Motors for Different Industrial Drives

1. D.C. Series Motor. Since it has high starting torque and variable speed, it is used for heavy duty applications such as electric locomotives, steel rolling mills, hoists, lifts and cranes.

2. D.C. Shunt Motor. It has medium starting torque and a nearly constant speed. Hence, it is used for driving constant-speed line shafts, lathes, vacuum cleaners, wood-working machines, laundry washing machines, elevators, conveyors, grinders and small printing presses etc.

3. Cumulative Compound Motor. It is a varying-speed motor with high starting torque and

is used for driving compressors, variable-head centifugal pumps, rotary presses, circular saws, shearing machines, elevators and continuous conveyors etc.



Heavy duty hydraulic motor for high torque and low speeds

4. Three-phase Synchronous Motor. Because its speed remains constant under varying loads, it is used for driving continuously-operating equipment at constant speed such as ammonia and air compressors, motor-generator sets, continuous rolling mills, paper and cement industries.

5. Squirrel Cage Induction Motor. This motor is quite simple but rugged and possesses high over-load capacity. It has a nearly constant speed and poor starting torque. Hence, it is used for low and medium power drives where speed control is not required as for water pumps, tube wells, lathes, drills, grinders, polishers, wood planers, fans, blowers, laundary washing machines and compressors etc.

6. Double Squirrel Cage Motor. It has high starting torque, large overload capacity and a nearly constant speed. Hence, it is used for driving loads which require high starting torque such as compressor pumps, reciprocating pumps, large refrigerators, crushers, boring mills, textile machinery, cranes, punches and lathes etc.

7. Slip-ring Induction Motor. It has high starting torque and large overload capacity. Its speed can be changed upto 50% of its normal speed. Hence, it is used for those industrial drives which require high starting torque and speed control such as lifts, pumps, winding machines, printing presses, line shafts, elevators and compressors etc.

8. Single-phase Synchronous Motor. Because of its constant speed, it is used in teleprinters, clocks, all kinds of timing devices, recording instruments, sound recording and reproducing systems.

9. Single-phase Series Motor. It possesses high starting torque and its speed can be controlled over a wide range. It is used for driving small domestic appliances like refrigerators and vacuum cleaners etc.

10. Repulsion Motor. It has high starting torque and is capable of wide speed control. Moreover, it has high speed at high loads. Hence, it is used for drives which require large starting torque and adjustable but constant speed as in coil winding machines.

11. Capacitor-start Induction-run Motor. It has fairly constant speed and moderately high starting torque. Speed control is not possible. It is used for compressors, refrigerators and small portable hoists.

12. Capacitor-start-and-run Motor. Its operating characteristics are similar to the above motor except that it has better power factor and higher efficiency. Hence, it is used for drives requiring quiet operations.

44.11. Advantages of Electrical Braking Over Mechanical Braking

1. In mechanical braking; due to excessive wear on brake drum, liner etc. it needs frequent and costly replacement. This is not needed in electrical braking and so electrical braking is more economical than mechanical braking.
2. Due to wear and tear of brake liner frequent adjustments are needed thereby making the maintenance costly.
3. Mechanical braking produces metal dust, which can damage bearings. Electrical braking has no such problems.
4. If mechanical brakes are not correctly adjusted it may result in shock loading of machine or machine parts in case of lift, trains which may result in discomfort to the occupants.
5. Electrical braking is smooth.

6. In mechanical braking the heat is produced at brake liner or brake drum, which may be a source of failure of the brake. In electric braking the heat is produced at convenient place, which in no way is harmful to a braking system.
7. In regenerative braking electrical energy can be returned back to the supply which is not possible in mechanical braking.
8. Noise produced is very high in mechanical braking.

Only disadvantage in electrical braking is that it is ineffective in applying holding torque.

44.12. Types of Electric Braking

There are three types of electric braking as applicable to electric motors in addition to eddy-current braking. These have already been discussed briefly in Art. 44.7.

1. Plugging or reverse-current braking.
2. Rheostatic or dynamic braking.
3. Regenerative braking.

In many cases, provision of an arrangement for stopping a motor and its driven load is as important as starting it. For example, a planing machine must be quickly stopped at the end of its stroke in order to achieve a high rate of production. In other cases, rapid stops are essential for preventing any danger to operator or damage to the product being manufactured. Similarly, in the case of lifts and hoists, effective braking must be provided for their proper functioning.

44.13. Plugging Applied to D.C. Motors

As discussed earlier in Art. 42.7, in this case, armature connections are reversed whereas ***field winding connections remains unchanged***. With reversed armature connections, the motor develops a torque in the ***opposite*** direction. When speed reduces to zero, motor will accelerate in the opposite direction. Hence, the arrangement is made to disconnect the motor from the supply as soon as it comes to rest. Fig. 44.3 shows running and reversed connections for shunt motors whereas Fig. 44.4 shows similar conditions for series motors.

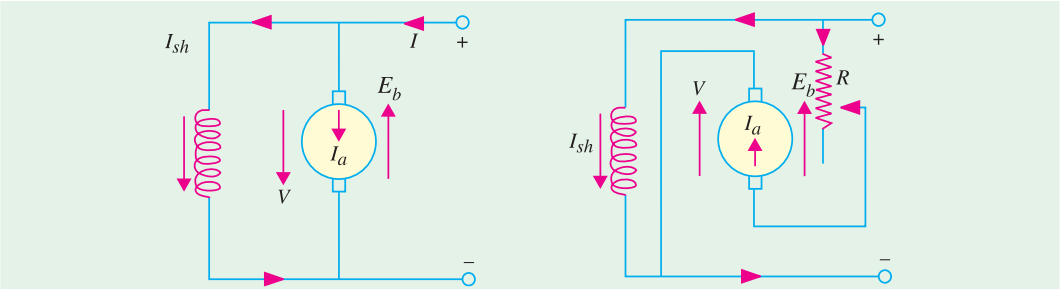


Fig. 44.3

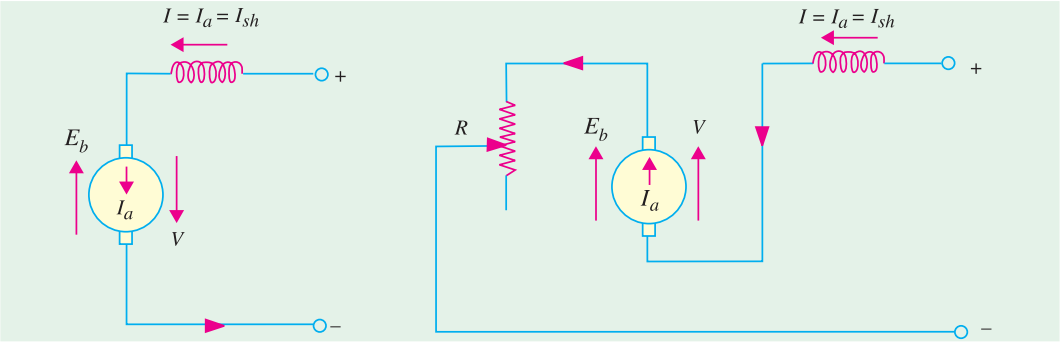


Fig. 44.4

Since with reversed connection, V and E_b are in the same direction, voltage across the armature is almost double of its normal value. In order to avoid excessive current through the armature, additional resistance R is connected in series with armature.

This method of braking is wasteful because in addition to wasting kinetic energy of the moving parts, it draws additional energy from the supply during braking.

Braking Torque. The electric braking torque is given by

$$T_B \propto \Phi I_a = k_1 \Phi I_a; \text{ Now, } I_a = (V + E_b)/R$$

$$\therefore \quad T_B = K_1 \Phi \cdot \frac{V + E_b}{R} = k_1 \Phi \frac{V + k_2 \Phi N}{R} \qquad (\because E_b \propto \Phi N)$$

$$= \frac{K_1 \Phi V}{R} + \frac{k_1 k_2 \Phi^2 N}{R} = k_3 \Phi + k_4 \Phi^2 N$$

Shunt Motor

Since in the case, Φ is practically constant, $T_B = k_5 + k_6 N$.

Series Motor

$$T_B = k_3 \Phi + k_4 \Phi^2 N = k_5 I_a + k_6 N I_a^2 \qquad (\because \phi \propto I_a)$$

The value of braking torque can be found with the help of magnetisation curve of a series motor.

Example 44.2. *A 40-kW, 440-V, d.c. shunt motor is braked by plugging. Calculate* ***(i)*** *the value of resistance that must be placed in series with the armature circuit to limit the initial braking current to 150 A* ***(ii)*** *the braking torque and* ***(iii)*** *the torque when motor speed falls to 360 rpm.*

Armature resistance R_a = 0.1W, full-load I_a = 100 A, full-load speed = 600 rpm.

(Electric Drives & Util. Punjab Univ. : 1994)

Solution. Full-load $E_b = 440 - 100 \times 0.1 = 430$ V

Voltage across the armature at the start of braking $= V + E_b = 440 + 430 = 870$ V

(i) Since initial braking current is limited to 150 A, total armature circuit resistance required is

$$R_t = 870 / 150 = 5.8\ \Omega \quad \therefore \quad R = R_t - R_a = 5.8 - 0.1 = \mathbf{5.7\ \Omega}$$

(ii) For a shunt motor, $T_B \propto \Phi I_a \propto I_a \quad \therefore \quad \Phi$ is constant

$$\text{Now, } \frac{\text{initial braking torque}}{\text{full-load toruqe}} = \frac{\text{initial braking current}}{\text{full-load current}}$$

Full-load torque $= 40 \times 103 / 2\pi\ (600/60) = 636.6$ N-m

$\therefore$ initial braking torque $= 636.6 \times 150 / 100 =$ **955 N-m**

(iii) The decrease in E_b is directly proportional to the decrease in motor speed.

$\therefore$ E_b at 360 rpm $= 430 \times 360/600 = 258$ V

I_a at 360 rpm $= (440 + 258) / 5.8 = 120$ A

T_B at 360 rpm $= 636.6 \times 120/100 =$ **764 N-m**

44.14. Plugging of Induction Motors

This method of braking is applied to an induction motor by transposing any of its two line leads as shown in Fig. 44.5. It reverses the direction of rotation of the synchronously-rotating magnetic field which produces a torque in the reverse direction, thus applying braking on the motor. Hence, at the ***first instant*** after plugging, the rotor is running in a direction opposite to that of the stator field. It means that speed of the rotor relative to the magnetic field is $(N_s + N) \cong 2N_s$ as shown in Fig. 44.6.

In Fig 44.6. ordinate BC represents the braking torque at the instant of plugging. As seen, this torque gradually increases as motor approaches standstill condition after which motor is disconnected from the supply (otherwise it will start up again in the reverse direction).

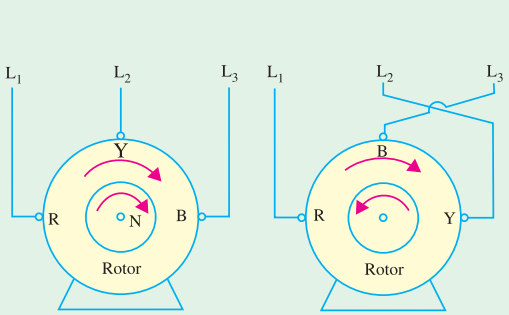


Fig. 44.5

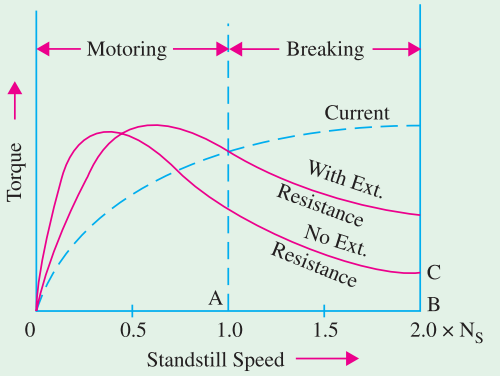


Fig. 44.6

As compared to squirrel cage motors, slip-ring motors are more suitable for plugging because, in their case, external resistance can be added to get the desired braking torque.

Example 44.3. *A 30-kW, 400-V, 3-phase, 4-pole, 50-Hz induction motor has full-load slip of 5%. If the ratio of standstill reactance to resistance per motor phase is 4, estimate the plugging torque at full speed.* **(Utilisation of Elect. Energy, Punjab Univ.)**

Solution. $N_s = 120\,f/P = 120 \times 50/4 = 1500$ rpm

Full-load speed, $N_f = N_s\,(1-s) = 1500\,(1 - 0.05) = 1425$ rpm

Full-load torque, $T_f = \dfrac{30\times10^3}{2\pi\times1425/60} = 200$ N-m

Since, $T \propto \dfrac{sR_2\,E_2^2}{R_2^2 + s^2\,X_2^2} \quad \therefore \quad \dfrac{T_2}{T_1} = \dfrac{s_2\,R_2\,E_2^2/(R_2^2 + s_2^2\,X_2^2)}{s_1R_2E_2^2/(R_2^2 + s_1^2\,X_2^2)}$

$$= \frac{s_2(R_2^2 + s_1^2\,X_2^2)}{s_1\,(R_2^2 + s_2^2\,X_2^2)} = \frac{s_2}{s_1}\cdot\frac{1 + s_1^2\,(X_2/R_2)^2}{1 + s_2^2\,(X_2/R_2)^2}$$

$$= \frac{s_1}{s_2}\cdot\frac{1+16s_1^2}{1+16s_2^2}\left(\frac{X_2}{R_2} = 4\right)$$

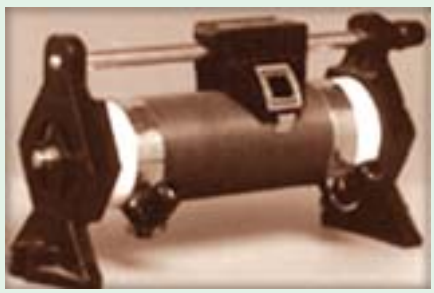
Slip, $S_p = 2 - 0.05 = 1.95$

$\therefore$ plugging torque, $T_p = \dfrac{1.95}{0.05}\cdot\dfrac{1+16\times(0.05)^2}{1+16\,(1.95)^2}\times T_f$

$$= 39\times\frac{1.04}{61.84}\times 200 = \mathbf{131\ N\text{-}m.}$$

44.15. Rheostatic Braking

In this method of electric braking, motor is disconnected from the supply though its field continues to be energised in ***the same direction***. The motor starts working as a generator and all the kinetic energy of the equipment to be braked is converted into electrical energy and is further dissipated in the variable external resistance R connected across the motor during the braking period. This external resistance must be less than the critical resistance otherwise there will not be enough current for generator excitation (Art. 44.3).



Rheostat

D.C. and synchronous motors can be braked this way but induction motors require separate d.c. source for field excitation.

This method has advantage over plugging because, in this case, no power is drawn from the supply during braking.

44.16. Rheostatic Braking of D.C. Motors

Fig. 44.7 shows connections for a d.c. shunt motor. For applying rheostatic braking armature is disconnected from the supply and connected to a variable external resistance R while the field remains on the supply. The motor starts working as a generator whose induced emf E_b depends upon its speed. At the start of braking, when speed is high, E_b is large, hence I_a is large. As speed decreases, E_b decreases, hence I_a decreases. Since $T_b \propto \Phi I_a$, it will be high at high speeds but low at low speeds. By gradually cutting out R, I_a and, hence, T_B can be kept constant throughout. Value of $I_a = E_b (R + R_a)$.

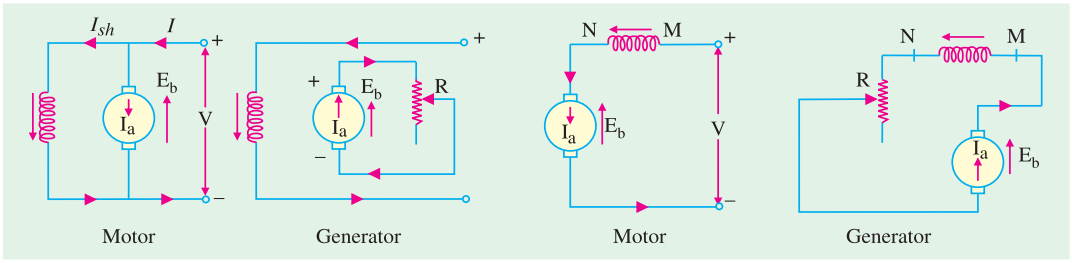


Fig. 44.7 **Fig. 44.8**

Fig. 44.8 shows running and braking conditions for a d.c. series motor. In this case also, for rheostatic braking, the armature is disconnected from the supply and, at the same time, is connected across R. However, connections are so made that current keeps flowing through the series field ***in the same direction*** otherwise no braking torque would be produced. The motor starts working as a series generator provided R is less than the critical resistance.

44.17. Rheostatic Braking Torque

$T_B \propto \Phi I_a$. Now, $I_a = E_b/(R + R_a) = E_b/Rt$

Since $E_b \propto \Phi N$, $I_a \propto \Phi N/R_t$. $\therefore$ $T_B \propto \Phi^2 N/R_t = k_1 \Phi^2 N$

1. For D.C. shunt motors and synchronous motors, Φ is constant. Hence

$$T_B = k_1 N$$

2. In the case of series motors, flux depends on current. Hence, braking torque can be found from its magnetisation curve.

When rheostatic braking is to be applied to the series motors used for traction work, they are connected in parallel (Fig. 44.9) rather than in series because series connection produces excessive voltage across the loading rheostats.

However, it is essential to achieve electrical stability in parallel operation of two series generators. It can be achieved either by equalizing the exciting currents *i.e.* by connecting the two fields in parallel [Fig. 44.9 (*a*)] or by cross-connection [Fig. 44.9 (*b*)] where field of one machine is excited by the armature current of the other. If equalizer is not used, then the machine which happens to build up first will send current through the other ***in the opposite direction*** thereby exciting it with reverse voltage. Consequently, the two machines would be short-circuited upon themselves and may burn out on account of excessive voltage and, hence, current.

In the cross-connection of Fig. 44.9 (*b*), suppose the voltage of machine No. 1 is greater than that of No. 2. It would send a larger current through F_2, thereby exciting it to a higher voltage. This results in stability of their parallel operation because stronger machine always helps the weaker one.

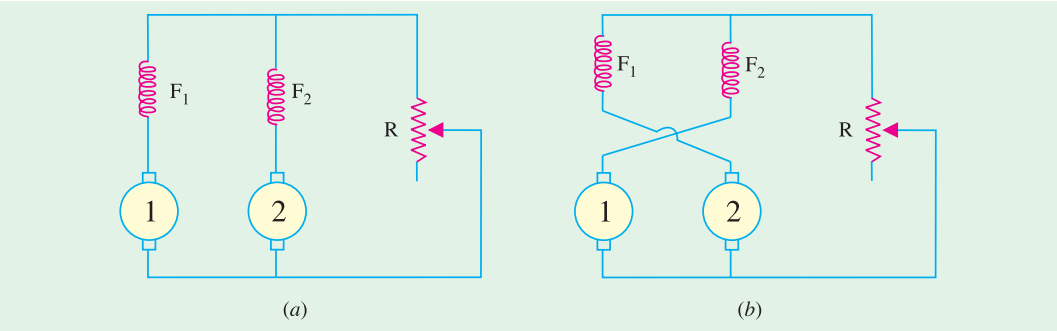


Fig. 44.9

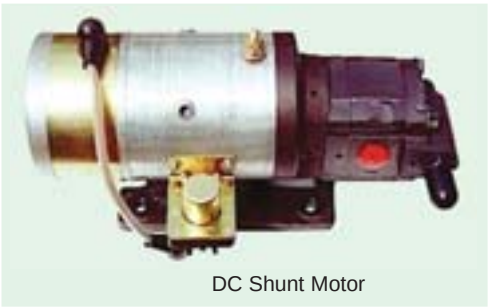
The cross-connection method has one special advantage over equalizer-connection method. If due to any reason (say, a run-back on a gradient) direction of rotation of the generators is reversed, no braking effect would be produced with connections of Fig. 44.9 (*a*) since the machines will fail to excite. However, with cross-excited fields, the machines will build up in series and being short-circuited upon themselves, will provide an emergency braking and would not allow the coach/car to run back on a gradient.

44.18. Rheostatic Braking of Induction Motors

If an induction motor is disconnected from the supply for rheostatic braking, there would be no magnetic flux and, hence, no generated emf in the rotor and no braking torque. However, if after disconnection, direct current is passed through the stator, steady flux would be set up in the air-gap which will induce current, in the short-circuited rotor. This current which is proportional to the rotor speed, will produce the required braking torque whose value can be regulated by either controlling d.c. excitation or varying the rotor resistance.

44.19. Regenerative Braking

In this method of braking, motor is not disconnected from the supply but is made to run as a generator by utilizing the kinetic energy of the moving train. Electrical energy is fed back to the supply. The magnetic drag produced on account of generator action offers the braking torque. It is the most efficient method of braking. Take the case of a shunt motor. It will run as a generator whenever its E_b becomes greater than V. Now, E_b can exceed V in two ways :


DC Shunt Motor

1. by increasing field excitation
2. by increasing motor speed beyond its normal value, field current remaining the same. It happens when load on the motor has overhauling characteristics as in the lowering of the cage or a hoist or the down-gradient movement of an electric train.

Regenerative braking can be easily applied to d.c. shunt motors though not down to very low speeds because it is not possible to increase field current sufficiently.

In the case of d.c. series motors, reversal of current necessary to produce regeneration would

cause reversal of the field and hence of E_b. Consequently, modifications are necessary if regenerative braking is to be employed with d.c. series motors used in electric traction.

It may, however, be clearly understood that regenerative braking cannot be used for stopping a motor. Its main advantages are ***(i)*** reduced energy consumption particularly on main-line railways having long gradients and mountain railways ***(ii)*** reduced wear of brake shoes and wheel tyres and ***(iii)*** lower maintenance cost for these items.

44.20. Energy Saving in Regenerative Braking

We will now compute the amount of energy recuperated between any two points on a level track during which regenerative braking is employed. The amount of energy thus recovered and then returned to the supply lines depends on :

(i) initial and final velocities of the train during braking

(ii) efficiency of the system and ***(iii)*** train resistance.

Suppose regenerative braking is applied when train velocity is V_1 km/h and ceases when it is V_2 km/h. If M_e tonne is the effective mass of the train, then

$$\text{K.E. of the train at } V_1 = \frac{1}{2} M_e V_1^2 = \frac{1}{2} (1000\, M_e) \times \left(\frac{1000\, V_1}{3600} \right)^2 \text{ joules}$$

$$= \frac{1}{2} (1000\, M_e) \left(\frac{1000\, V_1}{3600} \right)^2 \times \frac{1}{3600} \text{ Wh}$$

$$= 0.01072\, M_e V_1^{\,2} \text{ Wh} = 0.01072\, \frac{M_e}{M} V_1^2 \text{ Wh/tonne}$$

$$\text{K.E. at } V_2 = 0.01072\, \frac{M_e}{M} V_2^2 \text{ Wh/tonne}$$

Hence, energy available for recovery is $= 0.01072\, \frac{M_e}{M} (V_1^{\,2} - V_2^{\,2})$ Wh/tonne

If $r\,N/t$ is the specific resistance of the train, then total resistance = rM newton.

If d km is the distance travelled during braking, then

energy spent = $rM \times (1000\, d)$ joules = $rMd \propto \frac{1000}{3600}$ Wh = $0.2778\, rd$ Wh/tonne

Hence, net energy recuperated during regenerative braking is

$$= 0.01072\, \frac{M_e}{M} (V_1^{\,2} - V_2^{\,2}) - 0.2778\, rd \text{ Wh/tonne}$$

Gradient. If there is a *descending* gradient of G per cent over the same distance of d km, then downward force is = 98 MG newton

Energy provided during braking

$$= 98\, MG \times (1000\, d) \text{ joules} = 98\, MG\, d\, (1000 / 3600) \text{ Wh} = 27.25\, Gd \text{ Wh/tonne}$$

Hence, net energy recuperated in this case is

$$= \left[0.01072\, \frac{M_e}{M} (V_1^2 - V_2^2) - 0.2778\, rd + 2725\, Gd \right] \text{ Wh/tonne}$$

$$= 0.01072\, \frac{M_e}{M} (V_1^2 - V_2^2) + d\, (27.75\, G - 0.2778\, r) \text{ Wh/tonne}$$

If η is the system efficiency, net energy returned to the line is

(i) **level track**

$$= \eta \left[0.01072\, \frac{M_e}{M} (V_1^2 - V_2^2) - 0.2778\, rd \right] \text{ Wh/tonne}$$

(*ii*) descending gradient

$$= \eta\left[0.01072\,\frac{M_e}{M}\,(V_1^2 - V_2^2) + d\,(27.25\,G - 0.2778\,r)\right] \text{ Wh/tonne}$$

Example 44.4. *A 500-t electric trains travels down a descending gradient of 1 in 80 for 90 seconds during which period its speed is reduced from 100 km/h to 60 km/h by regenerative braking. Compute the energy returned to the lines of kWh if tractive resistance = 50 N/t; allowance for rotational inertia = 10% ; overall efficiency of the system = 75 %.*

Solution. Here $G = 1 \times 100 / 80 = 1.25\%$ $\quad M_e/M = 1.1$

$$d = \left(\frac{V_1 + V_2}{2}\right) \times t = \left(\frac{100 + 60}{2}\right) \times \frac{90}{3600} = 2 \text{ km}$$

Hence, energy returned to the supply line

$$= 0.75\,[(0.01072 \times 1.1\,(1002 - 602) + 2\,(27.25 \times 1.25 - 0.2778 \times 50)] \text{ Wh/t}$$

$$= 0.75\,[75.5 + 2\,(34 - 13.9)] = 86.77 \text{ Wh/t}$$

$$= 86.77 \times 500 \text{ Wh} = 86.77 \times 500 \times 10^{-3} \text{ kWh} = \mathbf{43.4 \text{ kWh}}$$

Example 44.5. *A 350-t electric train has its speed reduced by regenerative braking from 60 to 40 km/h over a distance of 2 km along down gradient of 1.5%. Caculate* ***(i)*** *electrical energy and* ***(ii)*** *average power returned to the line. Assume specific train resistance = 50 N/t ; rotational inertia effect = 10% ; conversion efficiency of the system = 75%.* **(Elect. Power, Bombay Univ.)**

Solution . ***(i)*** Energy returned to the line is

$$= 0.75\,[0.01072 \times 1.1\,(60^2 - 40^2) + 2\,(27.25 \times 1.5 - 0.2778 \times 50)] \text{ Wh/t}$$

$$= 58.2 \text{ Wh/t} = 58.2 \times 350 \times 10^{-3} = \mathbf{20.4 \text{ kWh}}$$

(ii) Average speed $= (60 + 40)/2 = 50$ km/h ; time taken $= 2/50$ h $= 1/25$ h

$$\therefore \quad \text{power returned} = \frac{20.4 \text{ kWh}}{1/25\text{h}} = \mathbf{510 \text{ kW}}$$

Example 44.6. *If in Example 42.4, regenerative braking is applied in such a way that train speed on down gradient remains constant at 60 km/h, what would be the power fed into the line?*

Solution : Since no acceleration is involved, the down-gradient tractive effort which drives the motors as generators is

$$F_t = (98\,MG - Mr) \text{ newton} = (98 \times 350 \times 1.5 - 350 \times 50) = 33{,}950 \text{ N}$$

Power that can be recuperated is

$$= F_t \times \left(\frac{1000}{3600}\right) V = 0.2778\,F_t\,V \text{ watt} = 0.2778 \times 33{,}950 \times 60 = 565{,}878 \text{ W}$$

Since $\eta = 0.75$, the power that is actually returned to the line is

$$= 0.75 \times 565{,}878 \times 10^{-3} = \mathbf{424.4 \text{ kW}}$$

Example 44.7. *A train weighing 500 tonne is going down a gradient of 20 in 1000. It is desired to maintain train speed at 40 km/h by regenerative braking. Calculate the power fed into the line. Tractive resistance is 40 N/t and allow rotational intertia of 10% and efficiency of conversion of 75%.* **(Util. of Elect. Power, A.M.I.E. Sec. B.)**

Solution. Down-gradient tractive effort which drives the motors as generators is

$$F_t = (98\,MG - Mr) = (98{,}000 \times 500 \times 2 - 500 \times 40) = 78{,}000 \text{ N}$$

Power that can be recuperated is $= 0.2778\,F_tV = 0.2778 \times 78{,}000 \times 40 = 866{,}736$ W

Since $\eta = 0.75$, the power that is actually fed into the lines is

$$= 0.75 \times 866{,}736 \times 10^{-3} = \mathbf{650 \text{ kW.}}$$

Example 44.8. *A 250-V d.c. shunt motor, taking an armature current of 150 A and running at 550 r.p.m. is braked by reversing the connections to the armature and inserting additional resistance in series with it. Calculate :*

(a) the value of series resistance required to limit the initial current to 240 A.

(b) the initial value of braking torque.

(c) the value of braking torque when the speed has fallen to 200 r.p.m.

The armature resistance is 0.09 Ω. Neglect winding friction and iron losses.

(Traction and Util. of Elect. Power, Agra Univ.)

Solution. Induced emf at full-load, $E_b = 250 - 150 \times 0.09 = 236.5$ V

Voltage across the armature at braking instant = $V + E_b = 250 + 236.5 = 486.5$ V

(a) Resistance required in the armature circuit to limit the initial current to 240 A

$$= \frac{486.5}{240} = \mathbf{2.027\ \Omega}$$

Resistance to be added in the armature circuit = 2.027 – 0.09 = **1.937 Ω**

(b) F.L. Torque, $T_f = VI/2\pi$ (N/60) = 250 × (550/60) = 650 N-m

Initial braking torque = $T_f \dfrac{\text{initial braking current}}{\text{full-load current}} = \dfrac{650 \times 240}{150}$ = **1040 N-m**

(c) When speed falls to 200 r.p.m., back emf also falls in the same proportion as the speed.

$\therefore \quad E_b = E_b \times 200/550 = 236.5 \times 200.550 = 94.6$ V

$\therefore$ current drawn = (250 + 94.6)/2.027 = 170 A

$\therefore$ braking torque = 650 × 170/150= **737 N-m**

Example 44.9. *A 400 V 3-ph squirrel cage induction motor has a full load slip of 4%. A stand-still impedance of 1.54Ω and the full load current = 30A. The maximum starting current which may be taken from line is 75A. What taping must be provided on an auto-transformer starter to limit the current to this value and what would be the starting torque available in terms of full load torque ?*

[Nagpur University, Winter 1994]

Solution.

$$\frac{V_2}{V_1} = \frac{I_1}{I_2} = X$$

or $$V_1 I_1 = V_2 I_2 = X$$

$$I_1 = 75\text{A}$$

$$V_1 = \frac{400}{\sqrt{3}} = 231 \text{ V} \quad \text{and} \quad I_1 = I_2 X$$

$$I_1 = \frac{V_2}{Z} X$$

$$I_1 = \frac{X V_1}{1.54} X$$

$$I_1 = \frac{X^2 \times 231}{1.54}$$

$\Rightarrow$ $$75 = \frac{X^2 \times 231}{1.54}$$

$\Rightarrow$ $$X^2 = \frac{75 \times 1.54}{231}$$

$$X = 0.7071$$

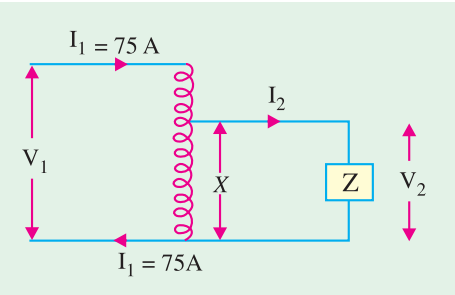


Fig. 44.10

$$\frac{T_s}{T_{FL}} = X^2 \left(\frac{I_S}{I_{FL}}\right)^2 \times \text{Slip}_{(FL)}$$

Now $I_2 = I_s = \frac{I_1}{X} = \frac{75}{0.708} = 106\ A$

$s_{FL} = 0.04. \quad I_s = 106\ \text{A} \quad I_{FL} = 30\ \text{A}.$

$\therefore \quad \frac{T_s}{T_{HL}} = (0.701)^2 \left(\frac{106}{30}\right)^2 \times 0.04$

$\therefore \quad T_s = 0.25\ T_{FL}$

Example 44.10. *A 220V, 10 H.P. shunt motor has field and armature resistances of 122W and 0.3W. respectively. Calculate the resistance to be inserted in the armature circuit to reduce the speed to 80% assuming motor η at full load to be 80%.*

(a) *When torque is to remain constant.*

(b) *When torque is proportional to square of the speed.* **[Nagpur University, Winter 1994]**

Solution. $I_f = \frac{220}{112} = 1.8$ Amp.

Motor $O/P = 10 \times 746 = 7460$ W

$\therefore$ Motor $I/P = \frac{7460}{0.8} = 9300$ W

Line current $I_L = \frac{9300}{220} = 42.2$ Amp.

$\therefore \quad I_a = 42.2 - 1.8 = 40.4$ A

$\therefore \quad E_{b_1} = 220 - 40.4 \times 0.3 = 208$ V

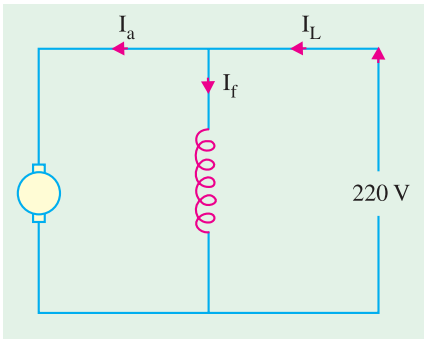


Fig. 44.11

Now $\frac{N_2}{N_1} = \frac{E_{b_2}}{E_{b_1}} \quad \because \phi$ is constant

$0.8 = \frac{E_{b_2}}{E_{b_1}} \quad \therefore 0.8 = \frac{E_{b_2}}{208} \quad \Rightarrow \quad E_{b_2} = \mathbf{166.4\ V}$

(a) $\because$ Torque remains constant and ϕ is constant

$\therefore\ I_a$ at reduced speed will also remain same

$\therefore \quad E_{b_2} = V - I_{a_2} R$ where R is total resistance

$166.4 = 220 - 40.4 \times R \quad \therefore \quad R = \frac{220 - 166.4}{40.0} = 1.34\ \Omega$

$\therefore$ Additional resistance in armature circuit $= 1.34 - 0.3 = \mathbf{1.04\ \Omega}$

(b) $\frac{T_2}{T_1} = \left(\frac{N_2}{N_1}\right)^2$

$T \propto I_a$ also $T \propto \phi I_a$

$\because \quad T \propto I_a \quad (\because \phi$ is constant $)$

$\frac{T_2}{T_1} = (0.8)^2 = 0.64$

$\therefore \quad \frac{T_2}{T_1} = \frac{I_{a_2}}{I_{a_2}} \quad \therefore 0.64 = \frac{I_{a_2}}{40.4} \quad \Rightarrow \quad I_{a_2} = 25.3$ A

$E_{b_2} = 220 - 25.3 \times R$

$166.4 = 220 - 25.5936 \times R$

$\therefore \quad R = \mathbf{2.0943\ \Omega}$

$\therefore$ Additional resistance $= 2.0943 - 0.3 = \mathbf{1.7943\ \Omega}$

Example 44.11. *A 37.5 H.P., 220 V D.C. shunt motor with a full load speed of 535 r.p.m. is to be braked by plugging. Estimate the value of resistance which should be placed in series with it to limit the initial braking current to 200 amps. What would be the initial value of the electric braking torque and the value when the speed had fallen to half its full load value? Armature resistance of motor is 0.086 Ω and full load armature current is 140 amps.*

Solution. $E = V - I_a R_a$

Back e.m.f. of motor $= E = 220 - 140 \times 0.086 = 220 - 12 = 208$ Volts.

Total voltage during braking $= E + V$

$= 220 + 208 = 428$ V

$R = \dfrac{V}{I}$

Resistance required $= \dfrac{428}{200} = 2.14\ \Omega$

There is already 0.086 Ω present in aramature.

$\therefore$ Resistance to be added $= 2.14 - 0.086 = 2.054\ \Omega$

Torque $\propto \phi\ I$ or Torque $\propto I$ ($\because$ ϕ is constant for shunt motor)

$$\frac{\text{Initial braking torque}}{\text{Initial braking current}} = \frac{\text{F.L. toruqe}}{\text{F.L. current}}$$

Power $=$ Torque $\times\ \omega$

$\omega = \dfrac{2\pi N}{60}$ rad/sec.

$37.5 \times 746 = T \times \dfrac{2\pi \times 535}{60}$

Full load torque $= 499.33$ Nw-m.

Initial braking torque $= 499.33 \times \dfrac{200}{140} = 713.328$ Nw-m.

$\rightarrow$ At half – speed back e.m.f. falls to half its original value $= 208/2 = 104$ V

Current $= \dfrac{220 + 104}{2.14} = 151$ Amps.

Electric braking torque at $\frac{1}{2}$ speed $= 499.33 \times \dfrac{151}{140} = \mathbf{538.56\ Nw\text{-}m.}$

Example 44.12. *A 500 V series motor having armature and field resistances of 0.2 and 0.3 Ω, runs at 500 r.p.m. when taking 70 Amps. Assuming unsaturated field find out its speed when field diverter of 0.684 Ω is used for following load whose torque*

(a) *remains constant*

(b) *varies as square of speed.*

Solution. When no diverter connected, $E_{b_1} = 500 - 70\ (0.2 + 0.3) = 465$ V

(a) If I_{a_2} be the armature current when diverter is used, then current flowing through

field $= I_{f2} = I_{a_2} \times \dfrac{0.684}{0.3 + 0.684} = 0.695\ I_{a_2}$

$\because$ Load torque is constant

$\therefore \quad I_{a_1}\ \phi_1 = I_{a_2}\ \phi_2 \qquad (\because \phi\ \alpha\ I_a)$

$$\therefore \quad I_{a_1}\,\phi_1 = I_{a_2}\,(0.695)\,I_{a_2} \Rightarrow I_{a_2} = \frac{I_{a_1}}{\sqrt{0.695}} = \frac{70}{\sqrt{0.695}} = 84 \text{ A}$$

$$\therefore \text{ Field current} = I_{f_2} = 0.695\ I_{a_2} = 0.695 \times 84 = 58.4 \text{ A}$$

$$\text{Resistance of field with diverter} = \frac{0.3 \times 0.684}{0.3 + 0.684} = 0.208\ \Omega$$

Total field and armature resistance = 0.2 + 0.208 = 0.408 Ω

$$E_{b_2} = 500 - 84\ (0.408) = 465.8 \text{ V}$$

$$\frac{N_1}{N_2} = \frac{E_{b_1}}{E_{b_2}} \times \frac{\phi_2}{\phi_1}$$

$$\frac{500}{N_2} = \frac{465}{465.8} \times \frac{58.4}{70} \Rightarrow N_2 = \mathbf{600\ r.p.m}$$

(b)

$$\frac{T_1}{T_2} = \left(\frac{N_1}{N_2}\right)^2 \quad \because \frac{T_1}{T_2} = \frac{I_{a_1}\,\phi_1}{I_{a_2}\,\phi_2} = \frac{I_{a_1} \cdot I_{a_1}}{I_{a_2} \times 0.695\ I_{a_2}}$$

$$\therefore \quad \left(\frac{N_1}{N_2}\right)^2 = \frac{I_{a_1} \cdot I_{a_1}}{Ia_2^2 \times 0.695} \Rightarrow \frac{N_1}{N_2} = \frac{I_{a_1}}{I_{a_2}\sqrt{0.695}} = \frac{70}{I_{a_2}\sqrt{0.695}}$$

$$\frac{N_1}{N_2} = \frac{E_{b_1}}{E_{b_2}} \times \frac{\phi_2}{\phi_1}$$

$$\frac{70}{I_{a_2}\sqrt{0.695}} = \frac{465}{500 - I_{a_2}\,(0.2)\,0.208)} \times \frac{0.695\ I_{a_2}}{70}$$

$$I_{a_2}^2 + 7.42\ I_{a_2} - 9093 = 0$$

$$\Rightarrow \quad I_{a_2} = 91.7 \text{ A} \qquad \because \text{ negative value is absurd.}$$

$$\frac{N_1}{N_2} = \frac{70}{I_{a_2}\sqrt{0.695}} \Rightarrow \frac{500}{N_2} = \frac{70}{91.7\sqrt{0.695}}$$

$$\therefore \quad N_2 = \mathbf{546\ r.p.m.}$$

Example 44.13. *A 200 V series motor runs at 1000 r.p.m. and takes 20 Amps. Armature and field resistance is 0.4 W . Calculate the resistance to be inserted in series so as to reduce the speed to 800 r.p.m., assuming torque to vary as cube of the speed and unsaturated field.*

Solution.

$$\frac{T_1}{T_2} = \left(\frac{N_1}{N_2}\right)^3 = \left(\frac{1000}{800}\right)^3 = \frac{125}{64}$$

$$\therefore \quad \frac{T_1}{T_2} = \frac{I_{a_1}\,\phi_1}{I_{a_2}\,\phi_2} = \frac{20 \times 20}{I_{a_2} \times I_{a_2}} \qquad \because \ \phi \propto I_a \text{ for series motor.}$$

$$\frac{125}{64} = \frac{20^2}{I_{a_2}^2}\ I_{a_2}\ I_{a_2} = 14.3 \text{ Amp}$$

$$E_{b_1} = 200 - 20 \times 0.4 = 192 \text{ V.}$$

$$\frac{E_{b_1}}{E_{b_2}} = \frac{N_1}{N_2} \times \frac{\phi_1}{\phi_2}$$

$$\frac{192}{E_{b_2}} = \frac{1000}{800} \times \frac{20}{14.3}$$

$$E_{b_2} = 110 \text{ V}; \quad E_{b_2} = V - IR$$

$$110 = 200 - 14.3 \times R \; ; \; R = \frac{90}{14.3} = 6.3\ \Omega$$

Additional resistance required = $6.3 - 0.4 = \mathbf{5.9\ \Omega}$

Example 44.14. *A 220V, 500 r.p.m. D.C. shunt motor with an armature resistance of 0.08Ω and full load armature current of 150 Amp. is to be braked by plugging. Estimate the value of resistance which is to be placed in series with the armature to limit initial braking current to 200 Amps. What would be the speed at which the electric braking torque is 75% of its initial value.*

Solution. Back e.m.f. of motor $= E_{b_1} = V - I_a R_a$

$= 220 - 150 \times 0.08 = 208$ V

Voltage across armature when braking starts

$= 220 + 208 = 428$ V

Initial braking current to be limited to 200 A.

$\therefore$ Resistance in armature circuit $= \frac{428}{200} = 2.14\ \Omega$

$\therefore$ External resistance required $= 2.14 - 0.08 = 2.06\ \Omega$

Since field Flux ϕ is constant therefore 75% torque will be produced when armature current is 75% of 200 Amp. *i.e.* 150Amp.

Let N_2 be the speed in r.p.m. at which 75% braking torque is produced. At this speed generated e.m.f. in armature

$$\because \quad \frac{E_{b_1}}{E_{b_2}} = \frac{N_1}{N_2} \; ; \; \frac{208}{E_{b_2}} = \frac{500}{N_2} \quad \therefore \quad E_{b_2} = \frac{208}{500} N_2$$

Voltage across armature when braking starts

$$150 \times 2.14 = \left(220 + \frac{208}{500} N_2\right) \text{Volts}$$

$$\therefore \quad N_2 = \mathbf{243\ r.p.m.}$$

Example 44.15. *A D.C. series motor operating at 250 V D.C. mains and draws 25 A and runs at 1200 r.p.m.* $R_a = 0.1\ \Omega$ *and* $R_{se} = 0.3\ \Omega$.

A resistance of 25 Ω is placed in parallel with the armature of motor. Determine:

(i) *The speed of motor with the shunted armature connection, if the magnetic circuit remains unsaturated and the load torque remains constant.*

(ii) *No load speed of motor.* **[Nagpur University Winter 1995]**

Solution. $$\frac{N_2}{N_1} = \frac{E_{b_2}}{E_{b_1}} \times \frac{\phi_1}{\phi_2}$$

Voltage across diverter $= 250 - 0.3\ I_2$

$$I_{\text{div}} = \frac{250 - 0.3 I_2}{25}$$

$$I_{a_2} = I_2 - \frac{250 - 0.3 I_2}{25} = 1.012\ I_2 - 10$$

As T is constant $\therefore \quad \phi_1\ I_{a_1} = \phi_2\ I_{a_2}$

$$\therefore \quad I_{a_1}^2 = I_2\ (I_{a_2})$$

$$(25)^2 = I_2\ (1.012\ I_2 - 10)$$

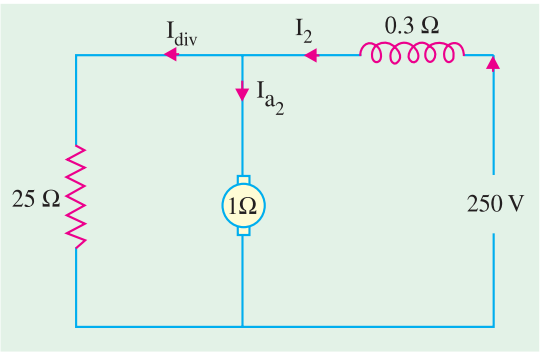


Fig. 44.12

$$1.012\, I_2^2 - 10 \times I_2 - 625 = 0 \implies I_2 = 30.27\, A \text{ (neglecting negative value)}$$

$$(25)^2 = (30.27)\ I_{a_2} \implies I_{a_2} = 20.65 \text{ A}$$

$$E_{b_1} = V - I_{a_1}\ (R_a + R_{se}) = 250 - 25\ (0.4) = 240 \text{ Volts}$$

$$E_{b_2} = V - I_2\ (R_{se}) - I_{a_2}\ (R_a) = 250 - 30.27\ (0.3) - 0.1\ (20.65) = 238.85 \text{ V.}$$

$$\frac{N_2}{N_1} = \frac{E_{b_2}}{E_{b_1}} \times \frac{\phi_1}{\phi_2}$$

$$\frac{N_2}{1200} = \frac{238.85}{240} \times \frac{2.5}{20.27} \quad \therefore \quad N_2 = \mathbf{986\ r.p.m.}$$

(ii) Series motor on no load.

Series motor can't be started on no load. When flux is zero, motor tries to run at infinite speed, which is not possible. So in the process, it tries to draw very high current from supply and fuse blows-out.

Example 44.16. *A 4 pole, 50Hz, slip ring Induction Motor has rotor resistance and stand still reactance referred to stator of 0.2 Ω and 1 Ω per phase respectively. At full load, it runs at 1440 r.p.m. Determine the value of resistance to be inserted in rotor in ohm/ph to operate at a speed of 1200 r.p.m., if :*

(i) *Load torque remains constant.* ***(ii)*** *Load torque varies as square of the speed.*

Neglect rtator resistance and leakage reactance.

Solution. ***(i)*** Load torque constant

$$\implies T \propto \frac{s}{R_2} \qquad N_s = 1500 \text{ rpm}$$

$$\therefore \quad T_1 \propto \frac{s_1}{R_2} \qquad T_2 \propto \frac{s_2}{(R_1 + r)}$$

$$s_1 = \frac{1500 - 1400}{1500} = 0.04 \qquad s_2 = \frac{1500 - 1240}{1500} = 0.2$$

As $T_1 = T_2 \quad \therefore \quad \dfrac{s_1}{R_2} = \dfrac{s_2}{R_1 + r}$

$$\frac{0.04}{0.2} = \frac{0.2}{0.2 + r}$$

$$\therefore \quad r = 0.8\ \Omega$$

(ii) Load torque varies as square of the speed.

$$\frac{T_1}{T_2} = \left[\frac{N_1}{N_2}\right]^2 = \left[\frac{1440}{1200}\right]^2 = 1.44$$

$$\frac{T_1}{T_2} = 1.44 = \frac{\dfrac{R_2 s_1}{R_2^2 + (s_1\ X_2)^2}}{\dfrac{(R_2 + r)\ s_2}{(R_2 + r)^2 + (s_2\ X_2)^2}} = \frac{\dfrac{0.2 \times 0.04}{0.2^2 + (0.04 \times 1)^2}}{\dfrac{(0.02 + r)\ (0.2)}{(0.2 + r)^2 + (0.2 \times 1)^2}}$$

Substituting $R_2 + r = R$

$$1.44 = \frac{0.1923}{\dfrac{0.2\ R}{R^2 + 0.04}}$$

$$\therefore \quad 0.1923\, R^2 - 0.288\, R + 0.007652 = 0$$

$$\implies R = 1.47 \text{ and } 0.0272, \text{ But } R > 0.2 \quad \therefore \quad R = 1.47\ \Omega$$

$$\therefore \quad R = 0.2 + r\ ;\ 1.47 = 0.2 + r \implies r = 1.27\ \Omega$$

Tutorial Problem No. 44.1

1. The characteristics of a series traction motor at 525 V are as follows :

current	:	50	70	80	90	A
speed	:	33.8	26.9	25.1	23.7	km/h
Gross torque	:	217	352	423	502	N-m

Determine the gross braking torque at a speed of 25.7 km/h when operating as self-excited series generator and loaded with an external resistance of 6 Ω. Resistance of motor = 0.5 Ω.

[382.4 N-m] (*London Univ.*)

2. The characteristics of a series motor at 525 V are as follows :

current	:	75	125	175	225	A
speed	:	1200	950	840	745	r.p.m.

Calculate the current when operating as a generator at 1000 r.p.m. and loaded on a rheostat having a resistance of 3.25 Ω. The resistance of motor is 3.5 Ω. **[150 A] (*London Univ.*)**

3. A train weighing 400 tonne travels a distance of 10 km down a gradient of 2%, getting its speed reduced from 40 to 20 km/h, the train resistance is = 50 N/t, allowance for rotational inertia = 10% and overall efficiency = 72%. Estimate (*i*) power and (*ii*) energy returned to the line.

[(*i*) 363 kW (*ii*) 121 kWh] (*Elect. Power, Bombay Univ.*)

4. A 400-tonne train travels down a gradient of in 100 for 20 seconds during which period its speed is reduced from 80 km/h to 50 km/h by regenerative braking. Find the energy returned to the lines if the tractive resistance is 49 N/t and allowance for rotational inertia is 7.5%. Overall efficiency of motors is 75%. **[28.2 kWh] (*A.M.I.E.*)**

5. A 400-tonne train travels down a gradient of 1 in 70 for 120 seconds during which period its speed is reduced from 80 km/h to 50 km/h by regenerative braking. Find the energy returned to the line if tractive resistance is 49 N/t and allowance for rotational inertia is 7.5%. Overall efficiency of motors is 75%. **[30.12 kWh] (*A.M.I.E.*)**

6. A train weighing 500 tonne is going down a gradient of 20 in 1000. It is desired to maintain train speed at 40 km/h by regenerative braking. Calculate the power fed into the line. Tractive resistance is 40 N/t and allow rotational inertia of 10% and efficiency of conversion of 75%.

[650 kW] (*Utilization of Elect. Power, A.M.I.E.*)

7. A 18.65 kW, 220-V D.C. shunt motor whith a full-load speed of 600 r.p.m. is to be braked by plugging. Estimate the value of the resistance which should be placed in series with it to limit the current to 130A. What would be the initial value of the electric braking torque and value when speed has fallen to half of its full-load vlaue ? Armature resistance of motor is 0.1 W. Full-load armature current is 95 A. **[3.211 Ω, 400.5 N-m, 302.57 N-m] (*Util of Elect. Power, A.M.I.E. Sec. B.*)**

8. A 400-tonne train travels down a gradient of 1 in 70 for 120 seconds during which period its speed is reduced from 80 km/h to 50 km/h by regenerative braking. Find the energy returned to the lines if tractive resistance is 5 kg / tonne and allowance for rotational inertia is 7.5 %. Overall efficiency of motor is 75%. **[30.64%]**

9. What are the advantages of Electrical Drive over other Drives? What are the mainfeatures of Group Drive and an Individual Drive? ***(Nagpur University, Summer 2004)***

10. What are the essential requirements of starting of any motor? With the help of neat diagram explain 'open circuit transition' and 'closed circuit transition' in Auto transformer starting of Induction Motor. ***(Nagpur University, Summer 2004)***

11. What is the principle of speed control of D.C. motors for, below the base speed and above the base speed.Explain with neat N-T characteristics. ***(Nagpur University, Summer 2004)***

12. A 400 V, 25 h.p., 450 rpm, D.C. shunt motor is braked by plugging when running on full load. Determine the braking resistance necessary if the maximum braking current is not to exceed twice the full load current. Determine also the maximum braking torque and the braking torque when the motor is just reaching zero speed. The efficiency of the motor is 74.6% and the armature resistance is 0.2 Ω. ***(Nagpur University, Summer 2004)***

13. Mention the Advantage of PLC over conventional motor control.

(Nagpur University, Summer 2004)

14. Suggest the motors required for following Drives :-
(*i*) Rolling mills (*ii*) Marine drive (*iii*) Home appliances (*iv*) Pump
(*v*) Refrigeration and air-conditioning (*vi*) Lifts. ***(Nagpur University, Summer 2004)***

15. Explain with neat block diagram the digital control of Electrical Drives.
(Nagpur University, Summer 2004)

16. Explain Series parallel control of traction motor. ***(Nagpur University, Summer 2004)***

17. Write short Notes on Speed reversal by contactor and relay. ***(Nagpur University, Summer 2004)***

18. Write short Notes on Ratings of contactors. ***(Nagpur University, Summer 2004)***

19. Write short Notes on Magnetic time-delay relay. ***(Nagpur University, Summer 2004)***

20. Discuss the advantages and disadvantages of electric drive over other drives.
(J.N. University, Hyderabad, November 2003)

21. Though a.c. is superior to d.c. for electric drives, sometimes d.c. is preferred. Give the reasons and mention some of the applications. ***(J.N. University, Hyderabad, November 2003)***

22. A d.c. series motor drives a load, the torque of which varies as the square of the speed. The motor takes current of 30 amps, when the speed is 600 r.p.m. Determine the speed and current when the field winding is shunted by a diverter, the resistance of which is 1.5 times that of the field winding. The losses may be neglected. ***(J.N. University, Hyderabad, November 2003)***

23. State the condition under which regenerative braking with d.c. services motor is possible and with the aid of diagrams of connection, explain the various methods of providing regeneration.
(J.N. University, Hyderabad, November 2003)

24. Explain what you mean by ''Individual drive'' and ''Group drive''. Discuss their relative merits and demerits. ***(J.N. University, Hyderabad, November 2003)***

25. A 500 V d.c. series motor runs at 500 r.p.m. and takes 60 amps. The resistances of the field and the armature are 0.3 and 0.2 Ohms, respectively. Calculate the value of the resistance to be shunted with the series field winding in order that the speed may be increased to 600 r.p.m., if the torque were to remain constant. Saturation may be neglected.
(J.N. University, Hyderabad, November 2003)

26. A motor has the following duty cycle :
Load rising from 200 to 400 h.p. – 4 minutes
Uniform load 300 h.p. – 2 minutes
Regenerative braking h.p. Returned to supply from 50 to zero – 1 minute.
Remains idle for 1 minute.
Estimate the h.p. of the motor. ***(J.N. University, Hyderabad, November 2003)***

27. What are various types of electric braking used? ***(J.N. University, Hyderabad, November 2003)***

28. Explain how rheostatic braking is done in D.C. shunt motors and series motors.
(J.N. University, Hyderabad, November 2003)

29. Describe how plegging, rheostics braking and regenerative braking are employed with D.C. series motor. ***(J.N. University, Hyderabad, November 2003)***

30. Where is the use of Individual drive recommended and why?
(J.N. University, Hyderabad, November 2003)

31. The speed of a 15 h.p. (Metric) 400 V d.c. shunt motor is to be reduced by 25% by the use of a controller. The field current is 2.5 amps and the armature resistance is 0.5 Ohm. Calculate the resistance of the controller, if the torque remains constant and the efficiency is 82%.
(J.N. University, Hyderabad, November 2003)

32. Explain regenerative braking of electric motors. ***(J.N. University, Hyderabad, November 2003)***

33. ''If a high degree of speed control is required, d.c. is perferable to a.c. for an electric drive''. Justify.
(J.N. University, Hyderabad, April 2003)

34. A 200 V shunt motor has an armature resistance of 0.5 ohm. It takes a current of 16 amps on full load and runs at 600 r.p.m. If a resistance of 0.5 ohm is placed in the armature circuit, find the

ratio of the stalling torque to the full load torque. *(J.N. University, Hyderabad, April 2003)*

35. What are the requirements of good electric braking? *(J.N. University, Hyderabad, April 2003)*

36. Explain the method of rheostatic braking. *(J.N. University, Hyderabad, April 2003; Anna University, Chennai 2003)*

37. Mean horizontal Candlepower *(J.N. University, Hyderabad, April 2003)*

38. Mean hemispherical Candlepower *(J.N. University, Hyderabad, April 2003)*

39. Luminous flux. *(J.N. University, Hyderabad, April 2003)*

40. Define : (*i*) Luminous intensity (*ii*) Point source (*iii*) Lumen and (*iv*) Uniform point source. *(J.N. University, Hyderabad, April 2003)*

41. Prove that Luminous intensity of a point source is equal to the luminous flux per unit solid angle. *(J.N. University, Hyderabad, April 2003)*

42. Discuss the various factors that govern the choice of a motor for a given service. *(J.N. University, Hyderabad, April 2003)*

43. A 6 pole, 50 Hz slip ring induction motor with a rotor resistance per phase of 0.2 ohm and a stand still reactance of 1.0 ohm per phase runs at 960 r.p.m. at full load. Calculate the resistance to be inserted in the rotor circuit to reduce the speed to 800 r.p.m., if the torque remains unaltered. *(J.N. University, Hyderabad, April 2003)*

44. Compare the features of individual and group drives. *(J.N. University, Hyderabad, April 2003)*

45. What is an electric drive? Classify various types of electric drives and discuss their merits and demerits. *(J.N. University, Hyderabad, December 2002/January 2003)*

46. Suggest, with reasons the electric drive used for the following applications. (*i*) Rolling mills (*ii*) Textile mills (*iii*) Cement mills (*iv*) Paper mills (*v*) Coal mining (*vi*) Lift, Cranes, Lathes and pumps. *(J.N. University, Hyderabad, December 2002/January 2003)*

47. A 100 hp, 500 rpm d.c. shunt motor is driving a grinding mill through gears. The moment of inertia of the mill is 1265 kgm^2. If the current taken by the motor must not exceed twice full load current during starting, estimate the minimum time taken to run the mill upto full speed. *(J.N. University, Hyderabad, December 2002/January 2003)*

48. Explain the different methods of electric braking of a 3 phase induction motor. *(J.N. University, Hyderabad, December 2002/January 2003)*

49. A 50 hp, 400V, 750 rpm synchronous motor has a moment of inertia 20 kgm^2 and employs rheostatic braking for obtaining rapid stopping in case of emergency when the motor is running at full load, star connected braking resistor of 2 ohm per phase is switched on. Determine the time taken and the number of revolutions made before the motor is stopped. Assume as efficiency of 90% and a full load power factor of 0.95. *(J.N. University, Hyderabad, December 2002/January 2003)*

50. Explain regenerative braking of induction motor. *(J.N. University, Hyderabad, December 2002/January 2003)*

51. What is dynamic braking? *(Anna University, Chenni, Summer 2003)*

52. What is regenerative braking? *(Anna University, Chenni, Summer 2003)*

53. What are braking systems applicable to a DC shunt motor? *(Anna University, Chenni, Summer 2003)*

54. What for Series motor Regenerative Braking is not suited? *(Anna University, Chenni, Summer 2003)*

55. What are the important stages in controlling an electrical drive. *(Anna University, Chenni, Summer 2003)*

56. Explain rheostatic braking of D.C. motors. *(Anna University, Chennai 2003)*

OBJECTIVE TESTS – 44

1. A steel mill requires a motor having high starting torque, wide speed range and precise speed control. Which one of the following motors will you choose ?

(*a*) d.c. shunt motor

(*b*) synchronous motor

(*c*) d.c. series motor

(*d*) slip-ring induction motor.

2. Heavy-duty steel-works cranes which have wide load variations are equipped with ..... motor.

(*a*) double squirrel-cage

(*b*) d.c. series
(*c*) slip-ring induction
(*d*) cumulative compound.

3. A reciprocating pump which is required to start under load will need .....motor.
(*a*) repulsion
(*b*) squirrel-cage induction
(*c*) synchronous
(*d*) double squirrel-cage induction.

4. Motors used in wood-working industry have ..... enclosure.
(*a*) screen protected (*b*) drip proof
(*c*) TEFC (*d*) TE

5. Single-phase synchronous motors are used in teleprinters, clocks and all kinds of timing devices becasue of their
(*a*) low starting torque
(*b*) high power factor
(*c*) constant speed
(*d*) over-load capacity.

6. Which motor is generally used in rolling mills, paper and cement industries ?
(*a*) d.c. shunt motor
(*b*) double squirrel-cage motor
(*c*) slip-ring induction motor
(*d*) three-phase synchronous motor

7. Direct drive is used for power transmission only when
(*a*) negligible slip is required
(*b*) large amount of power is invloved
(*c*) speed of the driven machine equals the motor speed
(*d*) high-speed motor is to drive a low-speed machine.

8. Which type of enclosure will be most suitable for motors employed in atmospheres containing inflammable gases and vapours ?
(*a*) pipe-ventilated
(*b*) totally enclosed, fan-cool
(*c*) flame proof
(*d*) screen-protected.

9. While plugging d.c. motors, ... connections are reversed
(*a*) supply
(*b*) armature
(*c*) field
(*d*) both armature and field

10. During rheostatic braking of a d.c., motor,
(*a*) its field is disconnected from the supply
(*b*) its armature is reverse-connected
(*c*) it works as a d.c. generator
(*d*) direction of its field current is reversed.

11. Rheostatic braking may be applied to an induction motor provided
(*a*) separate d.c. source for field excitation is available
(*b*) it is a squirrel cage type
(*c*) it is slip-ring type
(*d*) variable external resistance is available

12. During regenerative braking of electric motors, they are
(*a*) disconnected from the supply
(*b*) reverse-connected to the supply
(*c*) made to run as generators
(*d*) made to stop.

13. Regenerative braking
(*a*) can be used for stopping a motor
(*b*) cannot be easily applied to d.c. series motors
(*c*) can be easily applied to d.c. shunt motors
(*d*) cannot be used wen motor load has over-hauling characteristics

14. Net energy saved during regenerative braking of an electric train
(*a*) increases with increase in specific resistance
(*b*) is high with high down gradient
(*c*) decreases with reduction in train speed due to braking
(*d*) is independent of the train weight.

15. The selection of an electric motor for any application depends on which of the following factors?
(*a*) Electrical characteristics
(*b*) Mechanical characteristics
(*c*) Size and reating of motors
(*d*) cost
(*e*) All of teh above

16. For a particular application the type of electric and control gear are determined by which of the following considerations?
(*a*) Starting torque
(*b*) Conditions of environment
(*c*) Limitation on starting current
(*d*) Speed control range and its nature
(*e*) all of the above

17. Which of the following motors is preferred for traction work?
(*a*) Universal motor
(*b*) D.C. series motor
(*c*) Synchronous motor
(*d*) three-phase induction motor

18. Which of the following motors always starts

on load?
(*a*) Conveyor motor (*b*) Floor mill motor
(*c*) Fan motor (*d*) All of the above

19. ..... is preferred for automatic drives.
(*a*) Squirrel cage induction motor
(*b*) Synchronous motors
(*c*) Ward-Leonard controlled D.C. motors
(*d*) Any of the above

20. When the load is above ..... a synchronous motor is found to be more economical.
(*a*) 2 kW (*b*) 20 kW
(*c*) 50 kW (*d*) 100 kW

21. The load cycle for a motor driving a power press will be .....
(*a*) variable load
(*b*) continuous
(*c*) continuous but periodical
(*d*) intermittent and variable load

22. Light duty cranes are used in which of the following?
(*a*) Power houses
(*b*) Pumping station
(*c*) Automobile workshops
(*d*) all of the above

23. While selecting an electric motor for a floor mill, which electrical characteristics will be of least significance?
(*a*) Running characteristics
(*b*) Starting charasteristics
(*c*) Efficiency
(*d*) Braking

24. Which of the following motors are preferred for overhead travelling cranes?
(*a*) Slow speed motors
(*b*) Continuous duty motors
(*c*) Short time rated motors
(*d*) None of the above

25. ..... is preferred for synthetic fibre mills.
(*a*) Synchronous motor
(*b*) Reluctance motor
(*c*) Series motor
(*d*) Shunt motor

26. Ward-Leonard controlled D.C. drives are generally used for ..... excavators.
(*a*) Light duty (*b*) Medium duty
(*c*) Heavy duty (*d*) All of the above

27. Which of the following motors is used for elevators?
(*a*) Induction motor
(*b*) Synchronous motor
(*c*) Capacitor start single phase motor
(*d*) Any of the above

28. Which part of a motor needs maximum attention for maintenance?
(*a*) Frame (*b*) Bearing
(*c*) Stator winding (*d*) Rotor winding

29. ..... need frequent starting and stopping of electric motors.
(*a*) Paper mills
(*b*) Grinding mills
(*c*) Air-conditioners
(*d*) Lifts and hoists

30. Which feature, while selecting a motor for centrifugal pump, will be of least significance?
(*a*) Starting characteristics
(*b*) Operating speed
(*c*) Horse power
(*d*) Speed control

31. ..... motor is a constant speed motor.
(*a*) Synchronous motor
(*b*) Schrage motor
(*c*) Induction motor
(*d*) Universal motor

32. The starting torque is case of centrifugal pumps is generally
(*a*) less than running torque
(*b*) same as running torque
(*c*) slightly more than running torque
(*d*) double the running torque

33. Which of the following motors are best for the rolling mills?
(*a*) Single phase motors
(*b*) Squirrel cage induction motors
(*c*) Slip ring induction motors
(*d*) D.C. motors

34. ..... is not a part of ball bearing?
(*a*) Inner race (*b*) Outer race
(*c*) Cage (*d*) Bush

35. The starting torque of a D.C. motor is independent of which of the following?
(*a*) Flux
(*b*) Armature current
(*c*) Flux and armature current
(*d*) Speed

36. Rotor of a motor is usually supported on ..... bearings.
(*a*) ball or roller (*b*) needle
(*c*) bush (*d*) thrust

37. For which of the following applications D.C. motors are still preferred?
(*a*) High efficiency operation
(*b*) Reversibility

(*c*) Variable speed drive
(*d*) High starting torque

38. In a paper mill where constant speed is required
(*a*) synchronous motors are preferred
(*b*) A.C. motors are preferred
(*c*) individual drive is preferred
(*d*) group drive is preferred

39. A reluctance motor .....
(*a*) is provided with slip rings
(*b*) requires starting gear
(*c*) has high cost
(*d*) is compact

40. The size of an excavator is usually expressed in terms of
(*a*) 'crowd' motion (*b*) angle of swing
(*c*) cubic metres (*d*) travel in metres

41. For blowers which of the following motors in preferred?
(*a*) d.C. series motor
(*b*) D.C. shunt motor
(*c*) Squirrel cage induction motor
(*d*) Wound rotor induction motor

42. Belted slip ring induction motor is almost invariably used for
(*a*) water pumps
(*b*) jaw crushers
(*c*) centrifugal blowers
(*d*) none of the above

43. Which of the following is essentially needed while selecting a motor?
(*a*) Pulley (*b*) Starter
(*c*) Foundation pedal (*d*) Bearings

44. Reluctance motor is a .....
(*a*) variable torque motor
(*b*) low torque variable speed motor
(*c*) self starting type synchronous motor
(*d*) low noise, slow speed motor

45. ..... method of starting a three phase induction motor needs six terminals.
(*a*) Star-delta
(*b*) Resistance starting
(*c*) Auto-transformer
(*d*) None of the above

46. In ..... method of starting three phase induction motors the starting voltage is not reduced.
(*a*) auto-transformer
(*b*) star-delta
(*c*) slip ring
(*d*) any of the above

47. In jaw crushers a motor has to often start against ..... load.
(*a*) heavy (*b*) medium
(*c*) normal (*d*) low

48. For a motor-generator set which of the following motors will be preferred?
(*a*) Synchronous motor
(*b*) Slip ring induction motor
(*c*) Pole changing induction motor
(*d*) Squirrel cage induction motor

49. Which of the following motors is usually preferred for kiln drives?
(*a*) Cascade controlled A.C. motor
(*b*) slip ring induction motor
(*c*) three phase shunt would commutator motor
(*d*) Any of the above

50. Heat control switches are used in .....
(*a*) transformers
(*b*) cooling ranges
(*c*) three phase induction motors
(*d*) single phase

51. ..... has relatively wider range of speed control
(*a*) Synchronous motor
(*b*) Ship ring induction motor
(*c*) Squirrel cage induction motor
(*d*) D.C. shunt motor

52. In squirrel cage induction motors which of the following methods of starting cannot be used?
(*a*) Resistance in rotor circuit
(*b*) Resistance in stator circuit
(*c*) Auto-transformer starting
(*d*) Star-delta starting

53. In which of the following applications the load on motor changes in cyclic order?
(*a*) Electric shovels
(*b*) Cranes
(*c*) Rolling mills
(*d*) All of the above

54. Flame proof motors are used in
(*a*) paper mills
(*b*) steel mills
(*c*) moist atmospheres
(*d*) explosive atmospheres

55. Which of the following machines has heavy fluctuation of load?
(*a*) Printing machine
(*b*) Punching machine
(*c*) Planer
(*d*) Lathe

56. For derries and winches which of the following drives can be used?
(*a*) Pole changing squirrel cage motors
(*b*) D.C. motors with Ward-leonard control

(*c*) A.C. slip ring motors with variable resistance
(*d*) Any of the above

57. Battery operated scooter for braking uses
(*a*) plugging
(*b*) mechanical braking
(*c*) regenerative braking
(*d*) rheostatic braking

58. ..... has least range of speed control.
(*a*) Slip ring induction motor
(*b*) Synchronous motor
(*c*) D.C. shunt motor
(*d*) Schrage motor

59. ..... has the least value of starting torque to full load torque ratio.
(*a*) D.C. shunt motor
(*b*) D.C. series motor
(*c*) Squirrel cage induction motor
(*d*) Slip ring induction motor

60. In case of ..... speed control by injecting e.m.f. in the rotor circuit is possible.
(*a*) d.c. shunt motor
(*b*) schrage motor
(*c*) synchronous motor
(*d*) slip ring induction motor

61. A pony motor is used for the starting which of the following motors?
(*a*) Squirel cage induction motor
(*b*) Schrage motor
(*c*) Synchronous motor
(*d*) None of the above

62. In ..... the speed can be varied by changing the position of brushes.
(*a*) slip ring motor
(*b*) schrage motor
(*c*) induction motor
(*d*) repulsion motor

63. In which of the following applications variable speed operation is preferred?
(*a*) Exhaust fan
(*b*) Ceiling fan
(*c*) Refrigerator
(*d*) Water pump

64. Heavy duty cranes are used in
(*a*) ore handling plants
(*b*) steel plants
(*c*) heavy engineering workshops
(*d*) all of the above

65. the travelling speed of cranes varies from
(*a*) 20 to 30 m/s
(*b*) 10 to 15 m/s
(*c*) 5 to 10 m/s
(*d*) 1 to 2.5 m/s

66. Besides a constant speed a synchronous rotor possesses which of the following advantages?
(*a*) Lower cost
(*b*) Batter efficiency
(*c*) High power factor
(*d*) All of the above

67. By the use of which of the following D.C. can be obtained from A.C.?
(*a*) Silicon diodes
(*b*) Mercury are rectifier
(*c*) Motor generator set
(*d*) any of the above

68. Which of the following motors is preferred when quick speed reversal is the main consideration?
(*a*) Squirrel cage induction motor
(*b*) Wound rotor induction motor
(*c*) Synchronous motor
(*d*) D.C. motor

69. Which of the following motors is preferred when smooth and precise speed control over a wide range is desired?
(*a*) D.C. motor
(*b*) Squirrel cage induction motor
(*c*) Wound rotor induction motor
(*d*) Synchronous motor

70. For crane travel which of the following motors is normally used?
(*a*) Synchronous motor
(*b*) D.C. differentially compound motor
(*c*) Ward-Leonard controlled D.C. shunt motor
(*d*) A.C. slip ring motor

71. The capacity of a crane is expressed in terms of
(*a*) type of drive
(*b*) span
(*c*) tonnes
(*d*) any of the above

72. the characteristics of drive for crane hoisting and lowering are which of the following?
(*a*) Precise control
(*b*) Smooth movement
(*c*) Fast speed control
(*d*) All of the above

73. Which of the following motors is preferred for boom hoist of a travelling crane?
(*a*) Single phase motor
(*b*) Synchronous motor
(*c*) A.C. slip ring motor

(*d*) Ward-Leonard controlled D.C. shunt motor

74. A wound rotor induction motor is preferred, as compared to squirrel cage induction motor, when major consideration is
(*a*) slop speed operation
(*b*) high starting torque
(*c*) low windage losses
(*d*) all of the above

75. Which of the following motors has series characteristics?
(*a*) Shadel pole motor
(*b*) Repulsion motor
(*c*) Capacitor start motor
(*d*) None of the above

76. Which of the following happens when star-delta starter is used?
(*a*) Starting voltage is reduced
(*b*) Starting current is reduced
(*c*) Both (*a*) and (*b*)
(*d*) None of the above

77. For a D.C. shunt motor which of the following is incorrect?
(*a*) Unsuitable for heavy duty starting
(*b*) Torque varies as armature current
(*c*) Torque-armature current is a straight line
(*d*) Torque is zero for zero armature current

78. For which of the following applications motor has to start with high acceleration?
(*a*) Oil expeller
(*b*) Floor mill
(*c*) Lifts and hoists
(*d*) centrifugal pump

79. Which of the following types of motor enclosure is safest?
(*a*) totally enclosed
(*b*) Totally enclosed fan cooled
(*c*) Open type
(*d*) Semi closed

80. While selecting motor for an air conditioner which of the following characteristics is of great importance?
(*a*) Type of bearings
(*b*) Type of enclosure
(*c*) Noise
(*d*) Arrangement for power transmission
(*e*) None of the above

81. The diameter of the rotor shaft for an electric motor depends on which of the following?
(*a*) r.p.m. only
(*b*) Horse power only
(*c*) Horse power and r.p.m.
(*d*) Horse power, r.p.m. and power factor

82. Which of the following alternatives will be cheaper?
(*a*) A 100 H.P. A.C. three phase motor
(*b*) Four motors of 25 H.P. each
(*c*) Five motors of 20 H.P. each
(*d*) Ten motors of 10 H.P. each

83. The cost of an induction motor will increase as
(*a*) horsepower rating increases but r.p.m. decreases
(*b*) horsepower rating decreases but r.p.m. increases
(*c*) horsepower rating and operating speed increases
(*d*) horsepower rating and operating speed decreases

84. in series motor which of the following methods can be used for changing the flux per pole?
(*a*) Tapped field control
(*b*) Diverter field control
(*c*) Series-parallel control
(*d*) Any of the above

ANSWERS

1. (*c*)	**2.** (*b*)	**3.** (*d*)	**4.** (*c*)	**5.** (*c*)	**6.** (*d*)	**7.** (*c*)	**8.** (*c*)	**9.** (*b*)	**10.** (*c*)
11. (*a*)	**12.** (*c*)	**13.** (*c*)	**14.** (*b*)	**15.** (*e*)	**16.** (*e*)	**17.** (*b*)	**18.** (*d*)	**19.** (*c*)	**20.** (*d*)
21. (*d*)	**22.** (*d*)	**23.** (*d*)	**24.** (*c*)	**25.** (*b*)	**26.** (*c*)	**27.** (*a*)	**28.** (*b*)	**29.** (*d*)	**30.** (*d*)
31. (*a*)	**32.** (*a*)	**33.** (*d*)	**34.** (*d*)	**35.** (*d*)	**36.** (*a*)	**37.** (*c*)	**38.** (*c*)	**39.** (*d*)	**40.** (*c*)
41. (*c*)	**42.** (*b*)	**43.** (*b*)	**44.** (*c*)	**45.** (*a*)	**46.** (*c*)	**47.** (*a*)	**48.** (*a*)	**49.** (*d*)	**50.** (*b*)
51. (*d*)	**52.** (*a*)	**53.** (*d*)	**54.** (*d*)	**55.** (*b*)	**56.** (*d*)	**57.** (*b*)	**58.** (*b*)	**59.** (*c*)	**60.** (*d*)
61. (*c*)	**62.** (*b*)	**63.** (*b*)	**64.** (*d*)	**65.** (*d*)	**66.** (*c*)	**67.** (*d*)	**68.** (*c*)	**69.** (*a*)	**70.** (*d*)
71. (*c*)	**72.** (*d*)	**73.** (*c*)	**74.** (*b*)	**75.** (*b*)	**76.** (*c*)	**77.** (*a*)	**78.** (*c*)	**79.** (*b*)	**80.** (*c*)
81. (*c*)	**82.** (*a*)	**83.** (*a*)	**84.** (*d*)						

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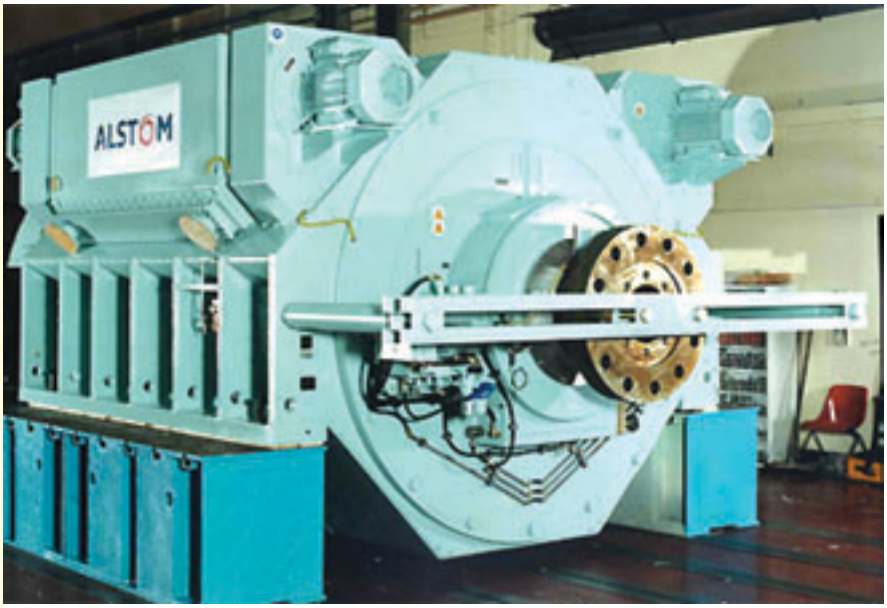
CONTENTS

CHAPTER 45

Learning Objectives

- Size and Rating
- Estimation of Motor Rating
- Different Types of Industrial Loads
- Heating of Motor or Temperature Rise
- Equation for Heating of Motor
- Heating Time Constant
- Equation for Cooling of Motor or Temperature Fall
- Cooling Time Constant
- Heating and Cooling Curves
- Load Equalization
- Use of Flywheels
- Flywheel Calculations
- Load Removed (Flywheel Accelerating)
- Choice of Flywheel

RATING AND SERVICE CAPACITY



Generator converts mechanical energy into electrical energy using electromagnetic induction

CONTENTS

45.1. Size and Rating

The factors which govern the size and rating of motor for any particular service are its maximum temperature rise under given load conditions and the maximum torque required. It is found that a motor which is satisfactory from the point of view of maximum temperature rise usually satisfies the requirement of maximum torque as well. For class-A insulation, maximum permissible temperature rise is 40°C whereas for class – B insulation, it is 50°C. This temperature rise depends on whether the motor has to run continuously, intermittently or on variable load.

Different ratings for electrical motors are as under:

1. Continuous Rating. It is based on the maximum load which a motor can deliver for an indefinite period without its temperature exceeding the specified limits and also possesing the ability to take 25% overload for a period of time not exceeding two hours under the same conditions.

For example, if a motor is rated continuous 10 KW, it means that it is capable of giving an output of 10 KW continuously for an indefinite period of time and 12.5 KW for a period of two hours without its temperature exceeding the specified limits.

2. Continuous Maximum Rating. It is the load capacity as given above but without overload capacity. Hence, these motors are a little bit inferior to the continuous-rated motors.

3. Intermittent Rating. It is based on the output which a motor can deliver for a specified period, say one hour or ½ hour or ¼ hour without exceeding the temperature rise.

This rating indicates the maximum load of the motor for the specified time followed by a no-load period during which the machine cools down to its original temperature.

45.2. Estimation of Motor Rating

Since primary limitation for the operation of an electric motor is its temperature rise, hence motor rating is calculated on the basis of its average temperature rise. The average temperature rise depends on the average heating which itself is proportional to the square of the current and the time for which the load persists.

For example, if a motor carries a load L_1 for time t_1 and load L_2 for time t_2 and so on, then

$$\text{Average heating} \propto L_1^2 t_1 + L_2^2 t_2 + ... + L_n^2 t_n$$

In fact, heating is proportional to square of the current but since load can be expressed in terms of the current drawn, the proportionality can be taken for load instead of the current.

$$\therefore \text{ size of the motor } = \sqrt{\frac{L_1^2 t_1 + L_2^2 t_2 + ... + L_n^2 t_n}{t_1 + t_2 + ...t_n}}$$

Generally, load on a motor is expressed by its load cycle. Usually, there are periods of no-load in the cycle. When motor runs on no-load, heat generated is small although heat dissipation continues at the same rate as long as the machine is running. Hence, there is a difference in the heating of a motor running at no-load and when at rest. It is commonly followed practice in America to consider the period at rest as one – third while calculating the size of motor. It results in giving a higher motor rating which is advantageous and safe.

Example 45.1 *An electric motor operates at full-load of 100 KW for 10 minutes, at ¾ full load for the next 10 minutes and at ½ load for next 20 minutes, no-load for the next 20 minutes and this cycle repeats continuously. Find the continuous rating of the suitable motor.*

Solution.

$$\text{Size of the motor required} = \sqrt{\frac{100^2 \times 10 + 75^2 \times 10 + 50^2 \times 20 + 0 \times 20}{10 + 10 + 20 + 20}}$$

$$= 61 \text{ KW}$$

According to American practice, we will consider the period of rest as (20/3) minutes. In that case, the motors size is

$$= \sqrt{\frac{100^2 \times 10 + 75^2 \times 10 + 50^2 \times 20 + 0 \times 20}{10 + 10 + 20 + (20/3)}}$$

= 66 KW

Example 45.2. *An electric motor has to be selected for a load which rises uniformly from zero to 200 KW in 10 minutes after which it remains constant at 200 KW for the next 10 minutes, followed by a no-load period of 15 minutes before the cycle repeats itself. Estimate a suitable size of continuously rated motor.*

Solution.

$$\text{Motor size} = \sqrt{\frac{(200/2)^2 \times 10 + (200)^2 \times 10 + 0 \times 15}{10 + 10 + (15 \times 1/3)}}$$

= 140 KW

According to American practice, no-load has been taken as one third.

Example 45.3. *A certain motor has to perform the following duty cycle:*

100 KW for 10 minutes *No-load for 5 minutes*

50 KW for 8 minutes *No-load for 4 minutes*

The duty cycle is repeated indefinitely. Draw the curve for the load cycle. Assuming that the heating is proportional to the square of the load, determine suitable size of a continuously-rated motor. **[Utilisation of Electric Power A.M.I.E.]**

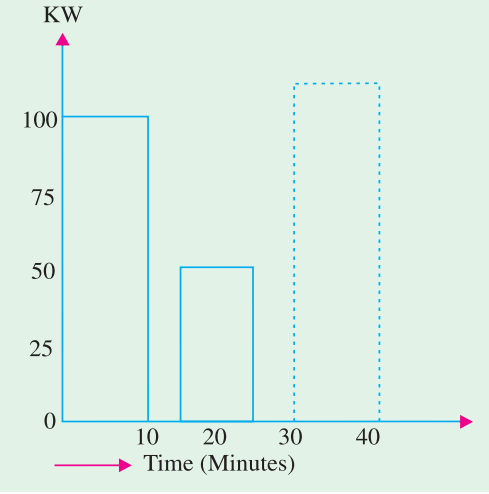


Fig. 45.1

Solution.

As explained above, heating is prorortional to the square of the current and hence, to the square of the load.

∴ size of the continuously-rated motor

$$\sqrt{\frac{100^2 \times 10 + 50^2 \times 8}{10 + 5 + 8 + 4}}$$

= 66.67 kW

Hence, the motor of **70 KW** would be adequate. The curve of the load cycle is shown in Fig. 45.1

The ultimate usefulness of the above factors is to select a motor of as small a size as possible compatible with temperature rise and to ensure that the motor has ample overload torque to cater for maximum-load conditions. Obviously, over-motoring of any industrial drive will result in a waste of electrical energy, a low power factor and unnecessarily high capital cost for the motor and control gear.

45.3. Different Types of Industrial Loads

The three different types of industrial loads under which electric motors are required to work are as under:

(i) continuous load ***(ii)*** intermittent load and ***(iii)*** variable or fluctuating load.

The size of the motor depends on two factors. Firstly, on the temperature rise which, in turn,

will depend on whether the motor is to operate on continuous, intermittent or variable load. Secondly, it will depend on the maximum torque to be developed by the motor. Keeping in mind the load torque requirements, the rating of the motor will be decided by the load conditions as described below.

(*i*) Continuous Load. In such cases, the calculation of motor size is simpler because the loads like pumps and fans require a constant power input to keep them operating. However, it is essential to calculate the KW rating of the motor correctly. If the KW rating of the motor is less than what is required, the motor will overheat and consequently burn out. If, on the other hand, KW rating is more than what is needed by the load, the motor will remain cool but will operate at lower efficiency and power.

(*ii*) Intermittent Loads. Such loads can be of the following two types:

(*a*) In this type of load, motor is loaded for a short time and then shut of for a sufficient by long time, allowing the motor to cool down to room temperature as shown in Fig. 45.2. In such cases, a motor with a short time rating is used as in a kitchen mixie.



Torque motors are designed to provide maximum torque at locked rotor or near stalled conditions. Their applications are in servo and positioning systems, tension reels, automatic door openers, and filament winding equipment.

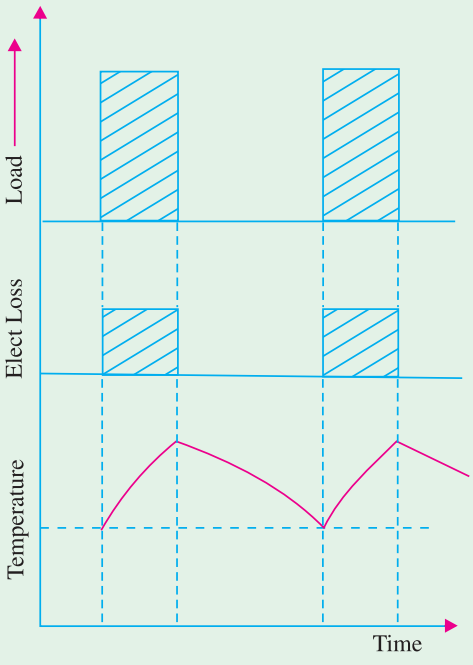


Fig. 45.2

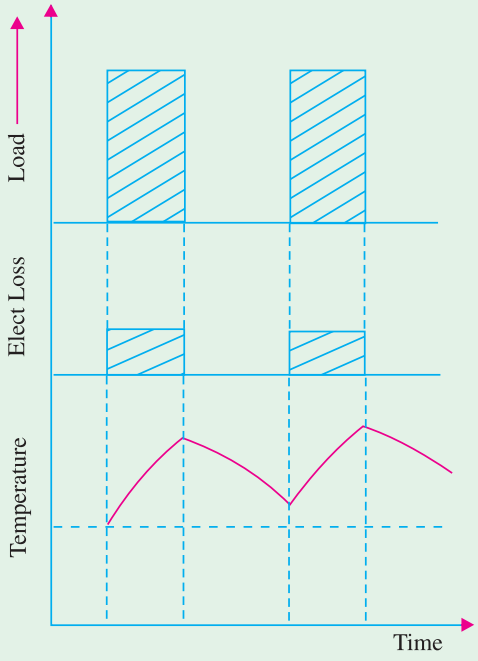


Fig. 45.3

(*b*) In this type of load, motor is loaded for a short time and then it is shut off for a short time. The shut off time is so short that the motor cannot cool down to the room temperature as shown in Fig.45.3. In such cases, a suitable continuous or short-time rated motor is chosen which, when operating on a given load cycle, will not exceed the specified temperature limit.

(*iii*) Variable Loads. In the case of such loads, the most accurate method of selecting a suitable motor is to draw the heating and cooling curves as per the load fluctuations for a number of motors. The smallest size motor which does not exceed the permitted temperature rise when operating on the particular load cycle should be chosen for the purpose.

However, a simpler but sufficiently accurate method of selection of a suitable rating of a motor is to assume that heating is proportional to the square of the current and hence the square of the load. The suitable continuous rating of the motor would equal the r.m.s. value of the load current.

Example 45.4. *A motor has to perform the following duty cycle*

100 H.P.	*For*	*10 min*
No Load	*”*	*5 min*
60 H.P.	*”*	*8 min*
No Load	*”*	*4 min*

which is repeated infinitely. Determine the suitable size of continuously rated motor.

Solution.

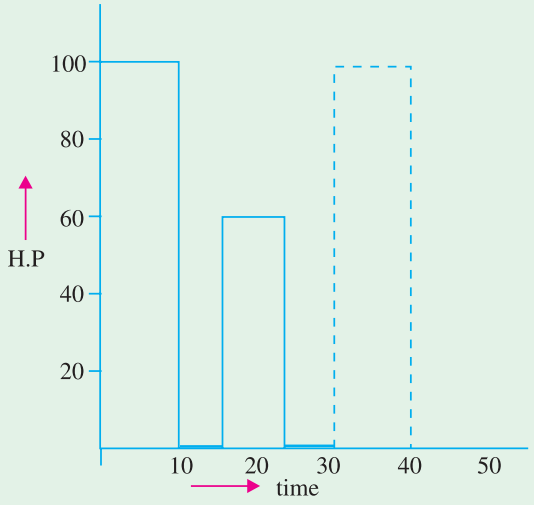


Fig. 45.4

$$\text{R.M.S. H.P.} = \sqrt{\left\{\frac{1}{\text{Time for one cycle}}\right\}\int \text{HP}^2\,dt}$$

$$\text{R.M.S. H.P.} = \left[\frac{\sum \text{HP}^2 \times \text{time}}{\text{Time for one cycle}}\right]^{\frac{1}{2}}$$

$$= \sqrt{\frac{100^2 \times 10 + 50^2 \times 8}{10+5+8+4}} = 69.07 \text{ H.P.}$$

$\simeq$ **75 H.P. motor can be used.**

Example 45.5. *A motor working in a coal mine has to exert power starting from zero and rising uniformly to 100 H.P. in 5 min after which it works at a constant rate of 50 H.P. for 10 min. Then, a no load period of 3 min. The cycle is repeated indefinitely, estimate suitable size of motor.*
[Nagpur University Summer 2000]

Solution.

(*a*) For time period : 0 - 5 min

$$\Rightarrow \quad y = mx + c$$

$$\text{Slope} = \frac{(100-0)}{5}$$

$$m = 20 \text{ HP/min}$$

$$\therefore \quad y = 20\,x + 0$$

$$\boldsymbol{y = 20\,x}$$

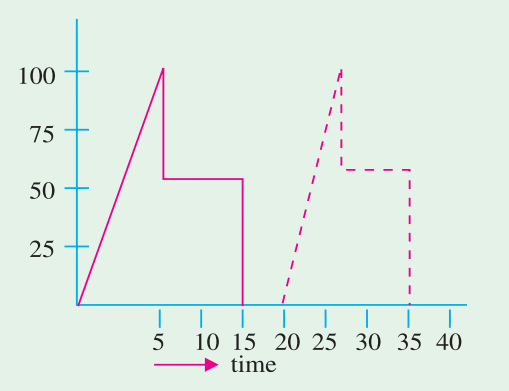


Fig. 45.5

(*b*) For total time period : 0 - 18 min

R.M.S. H P^2

$$= \left\{ \left[\int_0^5 y^2\,dx \right] + 50^2 \times 10 + 0^2 \times 3 \right\} / 18$$

$$\Rightarrow \text{H.P.}^2 \times 18 = \left[\int_0^5 (20\,x)^2\,dx \right] + 25000 = \left[\frac{400\,x^3}{3} \right]_0^5 + 25000$$

$$\therefore \text{H.P.}^2 \times 18 = \frac{400 \times 125}{3} + 25000$$

$$\Rightarrow \text{H.P.} = \sqrt{\frac{41666.67}{18}} = 48.11 \text{H.P.} \simeq 50 \text{ H.P. motor can used}$$

or Same problem can be solved by Simpson's 1/3rd Rule of Integration

$$\text{H. P.} = \sqrt{\frac{\frac{1}{3} \times 100^2 \times 5 + 50^2 \times 10}{18}}$$

H. P. = 48.11 H.P. ; $\therefore$ **H. P. $\simeq$ 50 H.P.**

Example 45.6. *A motor has following duty cycle*

Load rising from 200 to 400 H.P.	-	*4 min.*
Uniform load 300 H. P.	-	*2 min.*
Regenerative braking - H.P. returned to supply from 50 to zero	-	*1 min.*
Remaining idle for	-	*1 min.*

Estimate suitable H. P. rating of the motor. motor can be used.
[Nagpur University Winter 1994]

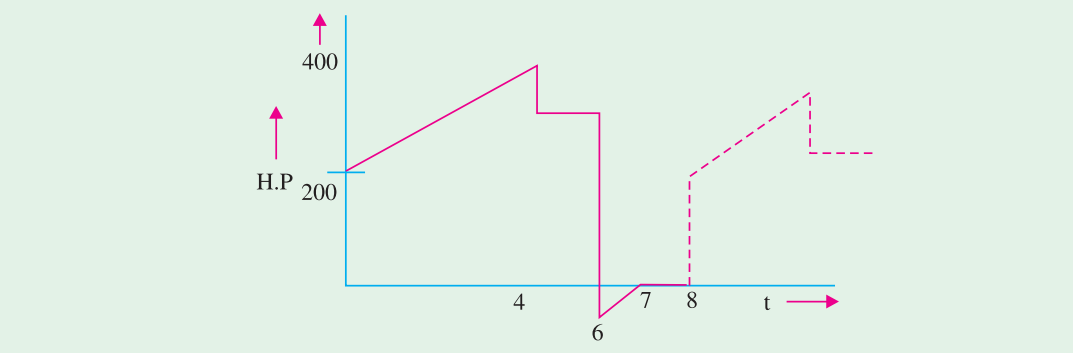


Fig. 45.6

Solution.

$$\text{H. P.} = \sqrt{\frac{\frac{1}{3}(H_1^2 + H_1H_2 + H_2^2)\,t_1 + H_3^2\,t_2 + \frac{1}{3}H_4^2\,t_3}{8}}$$

$$= \sqrt{\frac{\frac{1}{3}(200^2 + 200\times400 + 400^2)\times4 + 300^2\times2 + \frac{1}{3}50^2\times1}{8}}$$

$$= \sqrt{\frac{1662500}{24}} = \mathbf{263\ H.\ P.}$$

Note. During regenerative braking, even though H.P. is returned to line, machine will be carrying current. So far heating is concerned, it is immaterial whether machine is taking current from or giving current to line.

This problem can be solved by another method as follows:-

(a) For time period : 0 - 4 min

$$\rightarrow \int_0^4 (50x+200)^2\,dx = \int_0^4 (250x^2 + 20000x + 40000)\,dx$$

$$= 2500\left[\frac{x^3}{3}\right]_0^4 + 20000\left[\frac{x^2}{2}\right]_0^4 + 40000x$$

$$= 2500\left[\frac{4^3}{3}\right] + 20000\left[\frac{4^2}{2}\right] + 40000\times4 = \mathbf{373333.3\ H.P.}$$

(b) For time period : 4 - 6 min

$$\rightarrow (300)^2 \times 2 = \mathbf{180000\ H.P.}$$

(c) For time period : 6 - 7 min

$$\rightarrow \int_0^1 (50x)^2\,dx = 2500\left[\frac{x^3}{3}\right]_0^1 = \frac{2500}{3} = 833.33\,\text{H.P}$$

$$\therefore \quad \text{R.M.S. H.P.} = \sqrt{\frac{373333.33 + 180000 + 833.33}{8}} = 263.1\ \text{H.P}$$

$$\simeq \quad \mathbf{300\ H.P.\ motor\ will\ be\ suitable}$$

Example 45.7. *The load cycle of a motor for 15 min. in driving some equipment is as follows :*

0 - 5 min	-	*30 H. P.*
5 - 9 min	-	*No Load*
9 - 12 min	-	*45 H. P.*
12 - 15 min	-	*No Load*

The load cycle is repeated indefinitely. Suggest a suitable size of continuously rated motor.

Solution.

$$\text{R.M.S. H.P} = \left[\frac{30^2 \times 5 + 45^2 \times 3}{15}\right]^{\frac{1}{2}}$$

$$= 26.55 \text{ H. P.}$$

$\therefore$ R.M.S. H.P $\simeq$ **30 H. P. motor will be suitable.**

Example 45.8. *A motor driving a colliery winder has the following acceleration period*

load cycle 0 - 15 sec.	*:*	*Load rising uniformly from 0 - 1000 H.P.*
Full speed period	*:*	*15 - 85 sec. Load const. at 600 H.P.*
Decceleration period	*:*	*85 to 95 sec. regenerative braking the H. P. returned uniformly from 200 to 0 H. P.*
95 - 120 sec.	*:*	*Motor stationary.*

Estimate the size of continuously rated motor.

Solution.

$$\text{R.M.S. H. P.} = \left[\frac{\frac{1}{3}(1000)^2 \times 15 + 600^2 \times 70 + \frac{1}{3}(200)^2 \times 10}{120}\right]^{\frac{1}{2}}$$

$$= 502 \text{ H. P.}$$

$\cong$ **505 H. P. motor can be used.**

45.4. Heating of Motor or Temperature Rise

The rise in temperature of a motor results from the heat generated by the losses and an expression for this temperature rise is obtained by equating the rate at which heat is being generated by these losses to the rate at which heat is being absorbed by the motor for raising the temperature of motor and in dissipation from the surfaces exposed to cooling media.

So long as the temperature of machine rises, the generated heat will be stored in body and the rest will be dissipated to cooling medium depending upon the temperature difference. This is called as unstable or transient situation.

If the temperature of body rises, it has to store heat. The amount of heat *i.e.* stored depends upon the heat capacity of the body. If the temperature of the machine remains constant *i.e.* it doesn't rise, then no further storage of heat takes place and all the heat *i.e.* generated must be dissipated. So rate of heat generation in motor equals rate of heat dissipation from the cooling surface. This is called a stable situation.

45.5. Equation for Heating of Motor

Let,

$W \rightarrow$ Heat generated in motor due to powerloss in watts.

$G \rightarrow$ Weight of motor (kg)

S → Average specific heat in (Watt - Sec.) to raise the temperature of unit weight through 1°C.

$G \times S$ → Heat required to raise the temperature of motor through 1°C (Watt - Sec.)

θ → Temperature rise above cooling medium in °C.

θ_f → Final temperature rise in °C.

A → Cooling surface area of motor.

λ → Rate of heat dissipation from the cooling surface.
[(Watts/Unit area/°C rise in temperature.) above cooling medium]

$A\lambda$ → Rate of heat dissipation in Watts /°C rise in temperature for a motor.

Assumptions

1. Loss 'W' remains constant during temperature rise.
2. Heat dissipation is proportional to the temperature difference between motor and cooling medium.
3. Temperature of cooling medium remains constant.

{Rate of heat generation in motor}
= {Rate of heat absorption by the motor} + {Rate of heat dissipation from cooling surface}

$$\rightarrow \quad W = GS\frac{d\theta}{dt} + A\lambda\theta$$

$$\text{or} \quad W - A\lambda\theta = GS\frac{d\theta}{dt}$$

$$\rightarrow \quad \frac{W}{A\lambda} - \theta = \frac{GS}{A\lambda}\frac{d\theta}{dt}$$

$$\rightarrow \quad \frac{d\theta}{\left(\frac{W}{A\lambda} - \theta\right)} = \frac{dt}{\frac{GS}{A\lambda}}$$

By integrating,

$$\log_e\left(\frac{W}{A\lambda} - \theta\right) = -\frac{A\lambda}{GS}t + C \qquad \text{..... (2)}$$

At $t = 0$, $\theta = \theta_1$ [Initial temperature rise *i.e.* difference between the temperature of cooling medium and temperature of motor, during starting]

If starting from cold position, $\theta_1 = 0$

Substituting the values of t and θ in above equation.

$$C = \log_e\left(\frac{W}{A\lambda} - \theta_1\right)$$

$$\therefore \quad (2) \Rightarrow \log_e\left[\frac{\left(\frac{W}{A\lambda} - \theta\right)}{\left(\frac{W}{A\lambda} - \theta_1\right)}\right] = -\frac{A\lambda}{GS}t$$

by taking antilog, $$\frac{\left(\frac{W}{A\lambda} - \theta\right)}{\left(\frac{W}{A\lambda} - \theta_1\right)} = e^{-\frac{A\lambda}{GS}t}$$

$$\therefore \quad \theta = \frac{W}{A\lambda} - \left(\frac{W}{A\lambda} - \theta_1\right) e^{-\frac{A\lambda}{GS}t} \qquad \text{..... (3)}$$

When, the final temperature rise of θ_f is reached, all the heat generated is dissipated from the cooling surface so that,

equation (1) becomes $W = A\lambda\ \theta_f$ or $\theta_f = \frac{W}{A\lambda}$

And $\frac{GS}{A\lambda}$ = Heating time constant

$$\therefore \quad \frac{A\lambda}{GS} = \frac{1}{T}$$

Then equation (3) becomes;

$$\theta = \theta_f - (\theta_f - \theta_1)\, e^{\frac{-t}{T}}$$

If starting from cold, then $\theta_1 = 0$

$$\therefore \quad \theta = \theta_f\,(1 - e^{\frac{-t}{T}})$$

45.6. Heating Time Constant

Heating time constant of motor is defined as the time required to heat up the motor upto 0.633 times its final temperature rise.

$\theta = (1 - e^{-t/T})$

At $t = T$, $\theta = 0.633\,\theta_f$

After time $t = T$ θ reaches to 63.3 % of θ_f

$t = 2T$ θ reaches to 86.5 % of θ_f

$t = 3T$ θ reaches to 95 % of θ_f

$t = 4T$ θ reaches to 98.2 % of θ_f

$t = 5T$ θ reaches to 99.3 % of θ_f

T = Heating time constant.

= 90 min for motors upto 20 H.P.

= 300 min for larger motors.

45.7. Equation for Cooling of Motor or Temperature Fall

If rate of heat generation is less than rate of heat dissipation, cooling will take place.

$\therefore$ {Rate of heat generation in motor}+{Rate of heat absorption by motor}={Rate of heat dissipation from cooling surface}

$$W + GS\frac{d\theta}{dt} = A\lambda'\theta$$ where λ' = Rate of heat dissipation during cooling surface

$$W - A\lambda'\theta = -GS\frac{d\theta}{dt} \qquad \Rightarrow \frac{W}{A\lambda'} - \theta = -\frac{GS}{A\lambda'}\frac{d\theta}{dt}$$

$$\theta - \frac{W}{A\lambda'} = \frac{GS}{A\lambda'}\frac{d\theta}{dt} \quad \Rightarrow \quad \frac{d\theta}{\theta - \frac{W}{A\lambda'}} = \frac{dt}{\frac{GS}{A\lambda'}}$$

$$\therefore \quad \int \frac{d\theta}{\theta - \frac{W}{A\lambda'}} = \int \frac{dt}{\frac{GS}{A\lambda'}}$$

$$\log_e\left(\theta - \frac{W}{A\lambda'}\right) = -\frac{A\lambda'}{GS}t + C$$

At $t = 0$ Let $\theta = \theta_o$ Difference of temperature between cooling medium and motor (Temperature rise at which cooling starts.)

$$\therefore \quad C = \log_e\left(\theta_0 - \frac{W}{A\lambda'}\right)$$ Put this value of C in the abve equation.

$$\therefore \quad \log_e \frac{\theta - \frac{W}{A\lambda'}}{\theta_0 - \frac{W}{A\lambda'}} = -\frac{A\lambda'}{GS}t$$

If θ_f' is final temperature drop (above that of cooling medium), then at this temperature whatever heat is generated will be dissipated.

$$\therefore \quad W = A\lambda'\theta_f' \quad \Rightarrow \quad \theta_f' = \frac{W}{A\lambda'}$$

$$\therefore \quad \log_e\left\{\frac{\theta - \theta_f'}{\theta_0 - \theta_f}\right\} = -\frac{t}{T'}$$ Where T' is cooling time constant $= \frac{GS}{A\lambda'}$

$$\therefore \quad \frac{\theta - \theta_f'}{\theta_0 - \theta_f'} = e^{-t/T} \quad \Rightarrow \quad \left(\theta - \theta_f'\right) = \left(\theta_0 - \theta_f'\right)e^{\frac{-t}{T'}}$$

$$\theta = \theta_f' + \left(\theta_0 - \theta_f'\right)e^{\frac{-t}{T'}}$$

If motor is disconnected from supply, there will be no losses taking place and so final temperature reached will be ambient temperature. Hence $\theta_f' = 0$ ($\because W = 0$)

$$\therefore \quad \theta = \theta_0 . e^{\frac{-t}{T'}}$$

If $t = T'$, then $\theta = \theta_0 . e^{-1} \Rightarrow \theta = \frac{\theta_0}{e} = 0.368\ \theta_0$; $\therefore \theta = 0.368\ \theta_0$

45.8. Cooling Time Constant

Cooling time constant is defined as the time required to cool machine down to 0.368 times the initial temperature rise above ambient temperature.

By putting different values of T' in $\theta = \theta_0 . e^{-t/T'}$

$\therefore$ After time t	=	T'	θ has fallen to	36.8% of θ_0
t	=	$2T'$	θ has fallen to	13.5% of θ_0
t	=	$3T'$	θ has fallen to	5% of θ_0

t = $4T'$ θ has fallen to 1.8% of θ_0

t = $5T'$ θ has fallen to 0.7% of θ_0

45.9. Heating and Cooling Curves

(*a*) Motor continuously worked on Full Load.

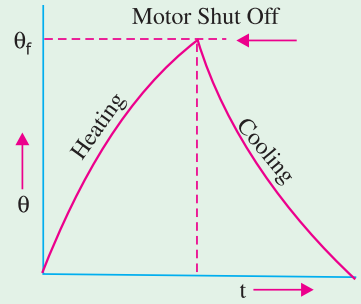


Fig. 45.7

Maximum permissible temperature rise.

Motor reaches final temperature rise and then cooling is carried out to ambient temperature.

(*b*) Motor Run for short time

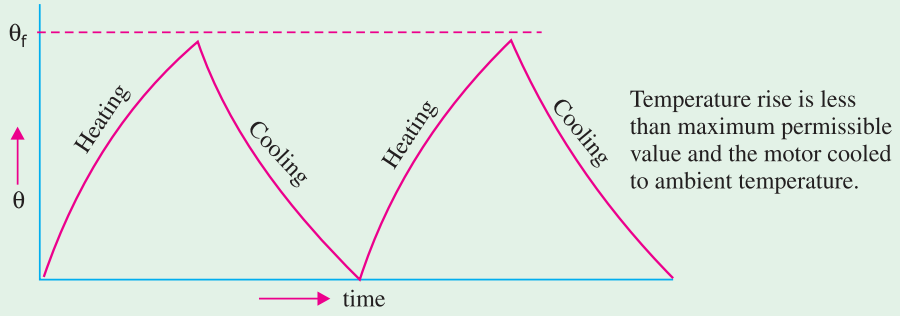


Fig. 45.8

(*c*) Cooling period not sufficient to cool down the motor to its ambient temperature.

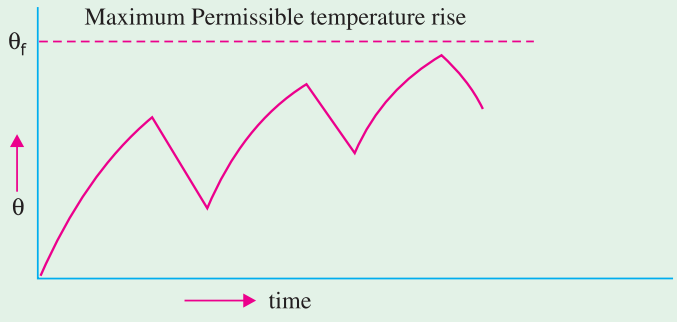


Fig. 45.9

* For intermittent loads, a motor of smaller rating can be used without exceeding maximum permissible temperature rise.

Example 45.9. *A 40 KW motor when run continuously on full load, attains a temperature of 35°C, above the surrounding air. Its heating time constant is 90 min. What would be the 1/2 hour rating of the motor for this temperature rise? Assume that the machine cools down completely between each load period and that the losses are proportional to square of the load,*

Solution.

Let '*P*' KW be the ½ hour rating of the motor

θ_f – Final temperature rise at P K W

θ_f' – Final temperature rise at 40 KW

$\therefore$ Losses at P KW $\propto P^2$

Losses at 40 K W $\propto 40^2$

$$\Rightarrow \quad \frac{\theta_f}{\theta_f'} = \frac{\text{Losses at } P\text{KW}}{\text{Losses at } 40\,\text{KW}} = \left(\frac{P}{40}\right)^2 \quad \therefore \quad \theta_f = \left(\frac{P}{40}\right)^2 \theta_f'$$

As the machine cools down completely, for '*P*' KW the equation will be

$$\theta = \theta_f \left(1 - e^{\frac{1}{T}}\right) \quad \text{where} \qquad \theta_f = \left(\frac{P}{40}\right)^2 \times 35$$

$$\Rightarrow \quad 35 = \left(\frac{P}{40}\right)^2 \times 35\left(1 - e^{-\frac{0.5}{1.5}}\right); \qquad \therefore \text{ P = } \mathbf{75.13\ KW}$$

Example 45.10. *Determine the one - hour rating of a 15 H.P. motor having heting time constant of 2 hours. The motor attains the temperature rise of 40°C on continuous run at full load. Assume that the losses are proportional to square of the load and the motor is allowed to cool down to the ambient temperature before being loaded again.* **[Nagpur University Summer 2001]**

Solution.

Let 'P' H. P be one - hour rating of the motor

$$\text{Losses at this load} = \text{Original losses} \times \left(\frac{P}{15}\right)^2.$$

Let θ_f be the final temperature rise at P H.P. and θ_f' at 15 H P

$$\therefore \quad \frac{\theta_f}{\theta_f'} = \frac{\text{Losses at P H.P.}}{\text{Original Losses}} = \left(\frac{P}{15}\right)^2$$

$$\theta_f = \theta_f' \left(\frac{P}{15}\right)^2 = 40\left(\frac{P}{15}\right)^2$$

$$\theta = \theta_f \left(1 - e^{\frac{-t}{T}}\right)$$

$$40 = 40\left[\frac{P}{15}\right]^2 \left(1 - e^{\frac{-1}{2}}\right)$$

P = 23.96 H.P.

$P \simeq$ **24 H.P.**

Example 45.11. *The heating and cooling time constants of a motor are 1 hour and 2 hours respectively. Final temperature rise attained is 100°C. This motor runs at full load for 30 miniutes and then kept idle for 12 min. and the cycle is repeated indefinitely. Determine the temperature rise of motor after one cycle.* **[Nagpur University Winter 1997]**

Solution. $\theta = \theta_f \left(1 - e^{\frac{-t}{T}}\right)$

$= 100\left(1 - e^{\frac{-30}{60}}\right) = 39.34°$

$\theta = \theta_0 e^{-\frac{t}{T'}} = 39.34 e^{\frac{-12}{120}} = \mathbf{35.6°C}$

= Temperature rise of motor after 1 cycle.

Example 45.12. *Calculate the maximum overload that can be carried by a 20 KW output motor, if the temperature rise is not to exceed 50°C after one hour on overload. The temperature rise on full load, after 1 hour is 30°C and after 2 hours is 40°C. Assume losses proportional to square of load.*

Solution. $\theta = \theta_f \left(1 - e^{\frac{-t}{T}}\right)$

$30 = \theta_f \left(1 - e^{\frac{-1}{T}}\right)$ and $40 = \theta_f \left(1 - e^{-\frac{2}{T}}\right)$

$\frac{1 - e^{\frac{-2}{T}}}{1 - e^{\frac{-1}{T}}} = \frac{40}{30}$ Put $x = e^{\frac{-1}{T}}$

$\therefore \quad \frac{1 - x^2}{1 - x} = \frac{4}{3} \qquad \therefore \quad \frac{(1-x)(1+x)}{(1-x)} = \frac{4}{3}$

$\Rightarrow \quad 1 + x = \frac{4}{3}$

$\therefore \quad x = \frac{1}{3} = e^{-1/T} \quad \Rightarrow \quad T = 0.91$ hrs.

To find $\theta_f \Rightarrow \quad 30 = \theta_f \left(1 - e^{\frac{-1}{T}}\right) \Rightarrow 30 = \theta_f \left(1 - \frac{1}{3}\right) \Rightarrow \theta_f = 45°C$

After 1 hr. $\quad 50 = \theta_f \left(1 - \frac{1}{e^T}\right)$

$50 = \theta_f \left(1 - \frac{1}{3}\right) \qquad \therefore \quad \theta_f = 75°C$

Let the maximum overload capacity of 20 KW motor is P KW

$\therefore \quad \frac{\theta_f}{\theta'_f} = \frac{\text{Losses at PK W}}{\text{Original Losses}} = \left(\frac{P}{20}\right)^2$

$\therefore \quad \frac{75}{45}=\left(\frac{P}{20}\right)^2 \qquad \therefore \quad P=25.8 \text{ kW}$

Example 45.13. *In a transformer the temperature rise is 25°C after 1 hour and 37.5°C after 2 hours, starting from cold conditions. Calculate its final steady temperature rise and the heating time constant. If the transformer temperature falls from the final steady value to 40°C in 1.5 hours when disconnected, calculate its cooling time constant. Ambient temperature is 30°C.*

Solution. $\theta=\theta_f\left(1-e^{\frac{-1}{T}}\right)$

$25=\theta_f\left(1-e^{\frac{-1}{T}}\right)$ and $37.5=\theta_f\left(1-e^{\frac{-2}{T}}\right)$

$\therefore \quad \frac{37.5}{25}=\frac{1-e^{\frac{-2}{T}}}{1-e^{\frac{-1}{T}}}, \qquad$ Put $x=e^{-\frac{1}{T}}$

$1.5=\frac{1-x^2}{1-x}, \qquad 1.5=\frac{(1-x)(1+x)}{(1-x)}$

$1.5 = 1 + X \qquad \therefore \quad X = 0.5$

$\therefore \quad e^{-\frac{1}{T}}=0.5 \qquad \therefore \quad T = 1.44 \text{ hrs}$

$25=\theta_f\left(1-e^{\frac{-1}{T}}\right) \qquad \Rightarrow \quad 25=\theta_f\left(1-e^{\frac{-1}{1.44}}\right) \Rightarrow \quad \theta_f=50°\text{C}$

Cooling : Temperature rise after 1.5 hours above ambient temperature = 40 – 30 = 10°C.

$\because$ The transformer is disconnected $\theta=\theta_0 e^{\frac{-t}{T'}}$

$10=50e^{\frac{-1.5}{T'}} \qquad \therefore \quad T' = 0.932 \text{ hrs}$

Example 45.14. *The initial temperature of machine is 45°C. Calculate the temperature of machine after 1.2 hours, if its final steady temperature rise is 85°C and the heating time constant is 2.4 hours. Ambient temperature is 25°C*

Solution.

$\theta=\theta_f-(\theta_f-\theta_1)e^{-t/T} \qquad \theta = 85 - (85 - 20)\ e^{-12/24}$

θ = 45.54°C - Temperature rise above cooling medium

$\therefore$ Temperature of machine after 1.2 hours is = 45 . 54 + 25 = **70 .54°C**

Example 45.15. *The following rises were observed in a temperature rise test on a D.C. machine at full loads.*

After 1 hour — 15°C

After 2 hours — 25°C

Find out ***(i)*** *Final steady temperature rise and time constant.*

(ii) *The steady temperature rise after 1 hour at 50% overload, from cold. Assume that the final temperature rise on 50% overload is 90°C.*

[Nagpur University Summer 1998]

Solution. $\theta = \theta_f (1 - e^{\frac{-t}{T}})$, as motor is starting from cold.

$15 = \theta_f (1 - e^{\frac{-1}{T}})$ and $25 = \theta_f (1 - e^{-\frac{2}{T}})$

$$\therefore \quad \frac{25}{15} = \frac{\theta_f \left(1 - e^{\frac{-2}{T}}\right)}{\theta_f \left(1 - e^{\frac{-1}{T}}\right)}$$

$$\therefore \quad e^{\frac{-1}{T}} = \frac{25}{15} - 1 \quad , \quad e^{\frac{-1}{T}} = \frac{2}{3}$$

$\therefore T$ = 2.466 hours, $15 = \theta_f \left(1 - e^{\frac{-1}{T}}\right)$

by putting value of T,

$$15 = \theta_f \left(1 - e^{\frac{-1}{2.466}}\right)$$

$\Rightarrow$ θ_f = **45°C**

(ii) On 50% overload θ_f = 90°C

$\therefore$ Final temperature rise after 1 hour at 50% overload is $\theta = \theta_f (1 - e^{\frac{-t}{T}})$

$$\theta = 90\left(1 - e^{\frac{-1}{2.466}}\right) = \mathbf{30^\circ C}$$

45.10. Load Equalization

If the load fluctuates between wide limits in space of few seconds, then large peak demands of current will be taken from supply and produce heavy voltage drops in the system. Large size of conductor is also required for this.

Process of smoothing out these fluctuating loads is commonly referred to as load equalization and involves storage of energy during light load periods which can be given out during the peak load period, so that demand from supply is approximately constant. Tariff is also affected as it is based on M.D. (Maximum Demand)

For example, in steel rolling mill, when the billet is in between the rolls it is a peak load period and when it comes out it is a light load period, when the motor has to supply only the friction and internal losses, as shown in figure 45.10.

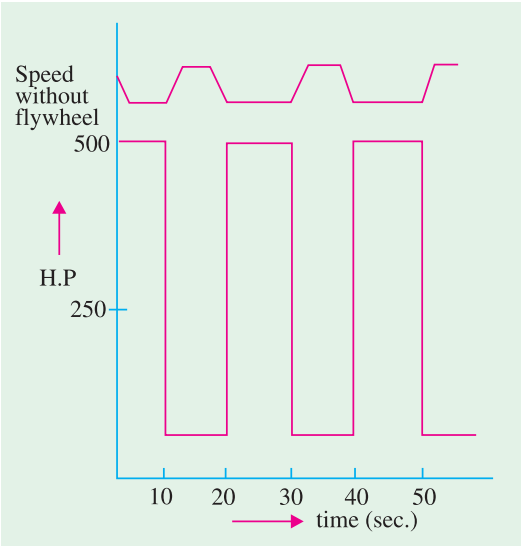


Fig. 45.10

45.11. Use Of Flywheels

The method of Load Eqalization most commonly employed is by means of a flywheel. During peak load period, the flywheel deccelarates and gives up its stored kinetic energy, thus reducing the load demanded from the supply. During light load periods, energy is taken from supply to accelerate flywheel, and replenish its stored energy ready for the next peak. Flywheel is mounted on the motor shaft near the motor. The motor must have drooping speed characteristics, that is, there should be a drop in speed as the load comes to enable flywheel to give up its stored energy. When the Ward - Leonard system is used with a flywheel, then it is called as Ward - Leonard Ilgner control.

45.12. Flywheel Calculations

The behaviour of flywheel may be determined as follows.

Fly wheel Decelerating :- (or Load increasing)

Let T_L → Load torque assumed constant during the time for which load is applied in kg-m

T_f → Torque supplied by flywheel in kg-m

T_o → Torque required on no load to overcome friction internal losses etc., in kg-m

T_m → Torque supplied by the motor at any instant, in kg-m

ω_o → No Load speed of motor in rad/sec.

ω → Speed of motor at any instant in rad/sec.

s → motor slip speed (ω_o - ω) in rad/sec.

I → Moment of inertia of flywheel in kg-m^2

g → Acceleration due to gravity in m/sec^2

t → time in sec.

When the flywheel deacelerates, it gives up its stored energy.

$$\rightarrow \quad T_m = T_L - T_f \qquad \text{or} \quad T_L = T_m + T_f \qquad \text{..... (1)}$$

Energy stored by flywheel when running at speed 'ω' is 1/2 Iω^2/g.

If speed is reduced from ω_0 to ω.

The energy given up by flywheel is

$$= \frac{1}{2}\frac{I}{g}\left(\omega_0^2 - \omega^2\right)$$

$$= \frac{1}{2}\frac{I}{g}\left(\omega_0 + \omega\right)(\omega_0 - \omega) \qquad \text{.... (2)}$$

$\left(\frac{\omega_0 + \omega}{2}\right)$ = mean speed. Assuming speed drop of not more than 10%., this may be assumed equal to ω.

$$\therefore \quad \left(\frac{\omega_0 + \omega}{2}\right) \simeq \omega \qquad \text{Also} \quad (\omega_0 - \omega) = s$$

$\therefore$ From equation (2), Energy given up $= \frac{I}{g}\omega.s$

Power given up $= \frac{I}{g}\omega.\frac{ds}{dt}$

but Torque $= \frac{\text{Power}}{\omega}$

$\therefore$ Torque supplied by flywheel.

$$T_f = \frac{I}{g}\frac{ds}{dt}$$

$\therefore$ From equation (1), $T_m = T_L - \frac{I}{g}\frac{ds}{dt}$

For values of slip speed upto 10% of No - load speed, slip is proportional to Torque

or $s = K T_m$

$\therefore \quad T_m = T_L - \frac{I}{g} K \frac{dT_m}{dt}$

This equation is similar to the equation for heating of the motor $W - A\lambda\theta = G.S. \frac{d\theta}{dt}$

i.e. $(T_L - T_m) = \frac{I}{g} K \frac{dT_m}{dt} \quad \Rightarrow \quad g\frac{dt}{IK} = \frac{dT_m}{(T_L - T_m)}$

By integrating both sides.

$$-ln(T_L - T_m) = \frac{tg}{IK} + C_1 \qquad \text{.... (3)}$$

At $t = 0$, when load starts increasing from no load *i.e.* $T_m = T_o$

Hence, at $t = 0$ $T_m = T_o$

$\therefore C_1 = -\ln (T_L - T_o)$

By substituting the value of C_1 above, in equation (3) $-\text{in } (T_L - T_m) = \frac{tg}{IK} - \text{In } (T_L - T_0)$

$\therefore \quad ln\left(\frac{T_L - T_m}{T_L - T_0}\right) = -\frac{tg}{IK} \quad \Rightarrow \quad \left(\frac{T_L - T_m}{T_L - T_0}\right) = e^{\frac{-tg}{Ik}}$

$\Rightarrow \quad (T_L - T_m) = (T_L - T_0)\, e^{\frac{-tg}{Ik}} \quad \therefore \quad T_m = T_L - (T_L - T_0)\, e^{\frac{-tg}{Ik}}$

If the Load torque falls to zero between each rolling period, then $T_m = T_L - \left(1 - e^{\frac{-tg}{IK}}\right) (\because T_0 = 0)$

45.13. Load Removed (Flywheel Accelerating)

Slip speed is decreasing and therefore $\frac{ds}{dt}$ is negative

$$T_m = T_0 + T_f = T_0 - \frac{I}{g}\frac{ds}{dt} \quad \Rightarrow \quad T_0 - T_m = \frac{I}{g} K \frac{dT_m}{dt}$$

$$\frac{g\,dt}{I\,K} = \frac{dT_m}{T_0 - T_m}$$

After integrating both sides,

$-\text{In } (T_0 - T_m) = \frac{tg}{IK} + C$ At $t = 0$, $T_m = T_m'$ motor torque at the instant, when load is removed

$\therefore \; C = -\ln (T_o - T_m')$ Putting this value of C in the above equation

$$-\ln (T_0 - T_m) = \frac{tg}{IK} - \text{In } (T_0 - T_m')$$

$$\therefore \quad \ln\left(\frac{T_0 - T_m}{T_0 - T_m'}\right) = \frac{-tg}{IK} \qquad \therefore \quad T_0 - T_m = (T_0 - T_m')\, e^{\frac{-tg}{IK}}$$

$$\therefore \quad T_m = T_0 + (T_m' - T_0)\, e^{\frac{-tg}{IK}}$$

Where T_m' = the motor torque, at the instant the load is removed.

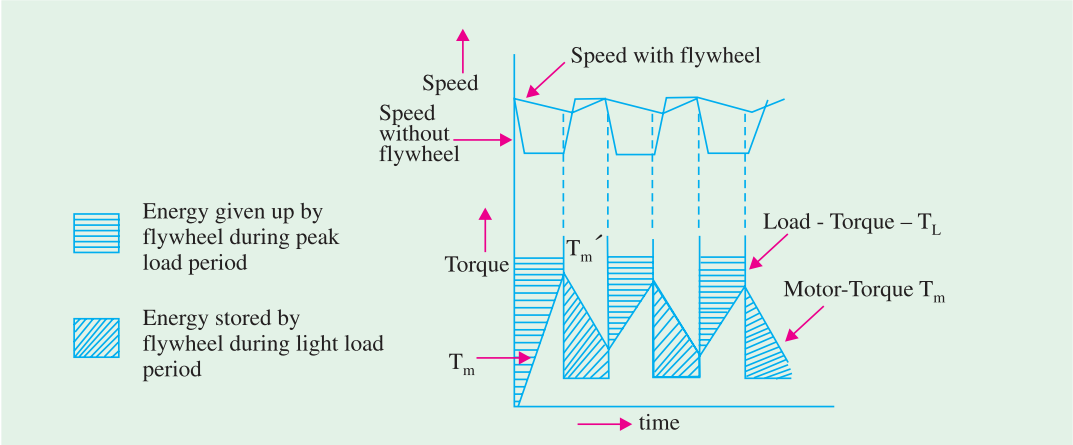


Fig. 45.11. Rolling mill drive with Flywheel

45.14. Choice of Flywheel

There are two choices left for selecting a flywheel to give up its maximum stored energy:

1. Large drop in speed and small flywheel (But with this the quality of production will suffer, since a speed drop of 10 to 15% for maximum load is usually employed).
2. Small drop in speed and large flywheel. (This is expensive and creates additional friction losses. Also design of shaft and bearing of motor is to be modified.) So compromise is made between the two and a proper flywheel is chosen.



The above figure shows the flywheel of a motor as a separate part

Example 45.16. *The following data refers to a 500 H.P. rolling mill, induction motor equipped with a flywheel.*

No load speed → *40 r.p.m.*
Slip at full load (torque) → *12%*
Load torque during actual rolling → *41500 kg - m*
Duration of each rolling period → *10 sec.*

Determine inertia of flywheel required in the above case to limit motor torque to twice its full load value. Neglect no - load losses and assume that the rolling mill torque falls to zero between each rolling period. Assume motor slip proportional to full load torque.

[Nagpur University Summer 1996]

Solution. $\omega = \frac{2\pi N}{60} = \frac{2\pi \times 40}{60} = 4.189$ rad/sec

given, $T_0 = 0$ $\qquad T_m = T_L - (T_L - T_0)\, e^{\frac{-tg}{IK}}$

$t = 10$ sec.

$T_L = 41500$ kg - m

$T_m = 2 \times T_{\text{Full Load}}$

Now $T_{Full\ Load} = \frac{500 \times 735.5}{0.88 \times 4.189}$ N-m. $\quad \because\ s = 12\%$

$(1 - s) = 0.88$

$= 99765$ N-m

$= 10169.7$ kg-m

$\therefore \quad T_m = 2 \times 10169.7$

$= 20339.5$ kg

$s = \omega_0 - \omega$

$= \frac{2\pi}{60}(N_0 - N)$

$= \frac{2\pi}{60}(40 - 0.88(40))$

$= \frac{2\pi}{60}(4.8) = 0.503$ rad/ sec.

$s = K\ T_{FL}$

$0.503 = K\,(10169.7) \implies K = 4.91 \times 10^{-5}$

$T_m = T_L - (T_L - T_0)\, e^{\frac{-tg}{IK}}$

$\because\ T_0 = 0 \qquad \therefore T_m = T_L \left(1 - e^{\frac{-tg}{IK}}\right)$

$20339.5 = 41500 \left(1 - \frac{-10 \times 981}{e^{1 \times 4.91 \times 10^{-5}}}\right)$

$\therefore$ I = **2.9663 × 10^6 kg - m^2**

Example 45.17. *A 6 pole, 50 Hz Induction Motor has a flywheel of 1200 kg-m^2 as moment of inertia. Load torque is 100 kg - m. for 10 sec. No load period is long enough for the flywheel, to regain its full speed. Motor has a slip of 6% at a torque of 50 kg-m. Calculate*

(i) Maximum torque exerted by motor.

(ii) Speed at the end of deaceleration period. **[Nagpur University Winter 1996]**

Solution. **(i)** $T_m = T_L - (T_L - T_0)\, e^{\frac{-tg}{IK}}$

Assume $T_0 = 0, \quad \rightarrow \quad T_m = T_L (1 - e^{\frac{-tg}{IK}})$

$T_L = 100$ kg – m, $\quad t = 10$ sec, $\quad g = 9.81$ m/sec^2, $\quad I = 1200$ kg – m^2 $\quad s = \omega_0 - \omega$

$$s = KT \quad \therefore \quad \text{slip speed} = s = \frac{2\pi}{60}(N_0 - N)$$

$$N_S = \frac{120\,f}{p} = \frac{120\times 50}{6} = 1000 \text{ rpm} = N_0$$

N = 0.94 × 1000 = 940 rpm.

$$\therefore \quad S = \frac{2\pi}{60}(1000 - 940) = 2\pi \text{ rad/sec} \quad \text{or} \quad s = \frac{2\pi N}{60} = \frac{2\pi \times 1000 \times 0.06}{60}$$

$$s = 2\pi = 6.283 \text{ rad/sec}$$

$$\therefore \; K = \frac{S}{T} = \frac{2\pi}{50} = 0.04\pi = 0.125$$

$$\therefore \quad T_m = T_L\,(1 - e^{\frac{-tg}{IK}}) = \left(1 - e^{\frac{-10\times 9.81}{1200\times 0.04\pi}}\right)$$

T_m = **47.83 kg - m**

(ii) Slip speed = 0.04 π × 47.8 rad / sec

$s = KT_m$

$s = 0.125\ (47.83)$

$s = 5.98$ rad/sec

$$s = 5.98\frac{60}{2\pi} \text{ rpm} = 57.5 \text{ rpm} = \text{slip speed}$$

∴ Actual speed = 1000 – 57.5 = **942.5 rpm**

Example 45.18. *An Induction Motor equipped with a flywheel is driving a rolling mill which requires a Load Torque of 1900 N - m for 10 sec. followed by 250 N - m for 30 sec. This cycle being repeated indefinitely. The synchronous speed of motor is 750 r.p.m and it has a slip of 10% when delivering 1400 N-m Torque. The total Moment of Inertia of the flywheel and other rotating parts is 2100 kg-m². Draw the curves showing the torque exerted by the motor and the speed for five complete cycles, assuming that initial torque is zero.* **[Nagpur University Summer 1998]**

Solution.

T_L = 1900 N- m for 10 sec.
T_L = 250 N- m for 30 sec.
T_0 = 0 (assumed)
Slip = 10% at 1400 N – m torque
Slip = 750 × 0.1 = 75 r.p.m.

Ns = 750 r.p.m.
s = 10%
I = 2100 Kg- m², T_m = 1400 N – m

$$= \frac{75\times 2\pi}{60} = 7.85 \text{ rad/sec.}$$

$$s = K\,T_m \; ; K = \frac{S}{T_m} = \frac{7.85}{1400} = 0.0056$$

(i) **During 1st cycle :**

(a) Flywheel de-acelerating :

$$T_m = T_L - (T_L - T_0)\,e^{\frac{-t}{IK}}$$ [When torque is taken in N - m.]

→ After 10 sec

$T_m = 1900 - (1900 - 0)\, e^{-0.085 \times 10}$ $\quad \because \quad \frac{1}{IK} = 0.085$

T_m = 1088 N- m

Slip = 0.0056 × 1088 = 6.08 rad/sec

Slip = 58 r.p.m.

Speed = 750 – 58 = **692 r.p.m.**

(*b*) Flywheel acacelerating (Off Load Period)



Flywheel

$T_m = T_0 + (T_m' - T_0)\, e^{-\frac{t}{IK}}$

T_0 = No load torque = 280 N- m

T_m' = 1088 N-m (T_m at the beginning of the period *i.e.* the motor torque at the instant when load is removed)

After 30 sec.,

$T_m = 280 + (1088 - 280)\ e^{-0.085 \times 30}$

T_m = 343 N-m

$\therefore$ Slip at this T_m = 0.0056 × 343 = 1.92 rad/ sec = 18.34 r.p.m

$\therefore$ Speed = (750 – 18.34) r.p.m. = **731.6 r.p.m.**

(*ii*) During 2nd cycle :

(*a*) Flywheel deacelerating T_0 = 343 N-m.

T_m = 1900 – (1900 – 343) $e^{-0.085 \times 30}$

= 1235 N-m.

$\therefore$ Slip at this T_m = 0.0056 × 1235 = 6.92 rad/ sec

= 66 r.p.m.

$\rightarrow$ speed = 750 – 66 = **684 r.p.m.**

(*b*) Off Load Period:

$T_m = 280 + (1235 - 280)\, e^{-0.085 \times 30}$

= 354.6 N-m.

$\therefore$ Slip at this T_m = 0.0056 × 354.6 = 1.99 rad/sec

= 19 r.p.m.

speed = 750 – 19 = 731.0 r.p.m.

(*iii*) During 3rd Cycle :

(*a*) On Load period : T_m can be found as above.

T_m = 1263 N – m

Speed = **683.6 r.p.m.**

(*b*) Off Load period

T_m = 354.6 N – m

speed = **731.0 r.p.m.**

Initial condition at the beginning of the 3rd peak load are thus practically the same as that at the beginning of 2nd. Therefore Motor Torque in this and all succeeding load cycles will follow a similar curve to that in second period.

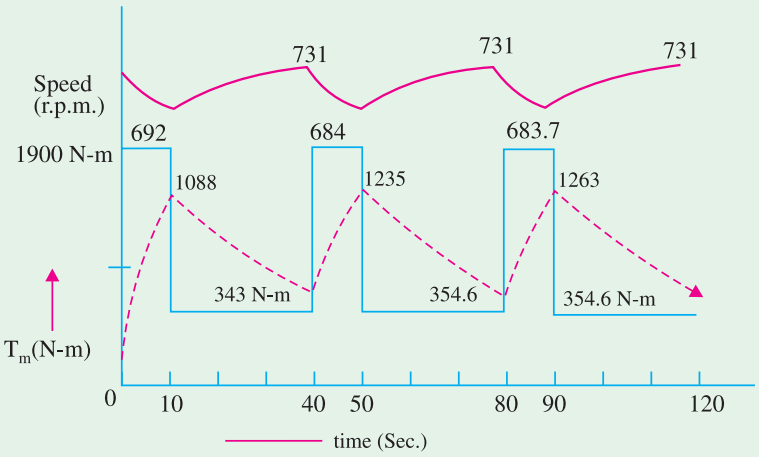


Fig. 45.12

Example 45.19. *A motor fitted with a flywheel supplies a load torque of 150 kg-m for 15 sec. During the no-load period, the flywheel regains its original speed. The motor torque is required to be limited to 85 kg-m. Determine moment of inertia of flywheel. The no - load speed of motor is 500 r.p.m. and it has a slip of 10% on full load.*

Solution.

$$T_m = T_L - (T_L - T_0)e^{\frac{-tg}{IK}}$$

$$T_m = T_L\left(1 - e^{\frac{-tg}{IK}}\right), \qquad \because\ T_0 = 0\,\text{kg} - \text{m}$$

T_m = 85 kg-m, T_L = 150 kg-m, T_0 = 0 kg-m, t = 15 sec., I = ?, g = 9.81 m/sec^2

$s = \text{K}\,T_{\text{F.L.}}$, where $s = \omega_0 - \omega$

$$\frac{2\pi(500)\times 0.1}{60} = \text{K}\times 85 \implies \text{K} = 0.0617$$

$$\therefore\ 85 = 150\left(1 - e^{\frac{-15\times 9.81}{I\times 0.627}}\right); \qquad \therefore\ \text{I} = \textbf{2884 kg- m}^2$$

Example 45.20. *A 3 – ϕ, 50 KW, 6 pole, 960 r.p.m. induction motor has a constant load torque of 300 N-m and at wide intervals additional torque of 1500 N-m for 10 sec. Calculate*

(a) *The moment of inertia of the flywheel used for load equalization, if the motor torque is not to exceed twice the rated torque.*

(b) *Time taken after removal of additional load, before the motor torque becomes 700 N-m.*

Solution.

(a) $P = T\times\omega \qquad \therefore\ T = P/\omega$

$$\therefore\ T_{F.L.} = \frac{50\times 10^3}{\frac{2\pi\times 960}{60}} = 497.36\,\text{N-m.}$$

$$\therefore\ T_m = 2\times T_{FL} = 2\times 497.36 = 994.72\,\text{N} - \text{m}$$

$$T_L = 1500 + 300 = 1800 \text{ N-m}$$

$$N_S = \frac{120\,f}{P} = 1000 \text{ r.p.m.}$$

$\therefore$ F.L. slip $= 1000 - 960 = 40$ rpm $= 40$ r.p.m. $= 4\%$

$\therefore$ $s = K\ T_{F.L.}$

$$\frac{2\pi(40)}{60} = K \times 497.36 \qquad \therefore \quad K = 8.42 \times 10^{-3}$$

$$T_m = T_L - (T_L - T_0)\,e^{\frac{-t}{IK}} \text{ as torque is in N - m}$$

$$994.72 = 1800 - (1800 - 300) \times e^{\frac{-10}{I \times 8.42 \times 10^{-3}}}$$

$\therefore$ $I =$ **1909 kg– m²**

(b)

$$T_m = T_0 + (T_m' - T_0)\,e^{\frac{-t}{IK}}$$

$$700 = 300 + (994.72 - 300)\,e^{\frac{-t}{1909 \times 8.42 \times 10^{-3}}}$$

$\therefore$ $t =$ **8.87 sec.**

Example 45.21. *A 3-phase, 8 pole, 50 c.p.s. Induction Motor equipped with a flywheel supplies a constant load torque of 100 N-m and at wide intervals an additional load torque of 300 N-m for 6 sec. The motor runs at 735 r.p.m., at 100 N-m torque. Find moment of inertia of the flywheel, if the motor torque is not to exceed 250 N-m.*

Solution.

$T_0 = 100\ N - m.$ $\quad \therefore \quad T_L = 100 + 300 = 400 \text{ N - m}$

$$N_S = \frac{120\,f}{P} = \frac{120\,f}{P} = 750 \text{ rpm.}$$

Slip at 100 N-m torque = 750 – 735 = 15 r.p.m.

$s = K\ T_m$

$$\frac{2\pi}{60}(15) = K(100) \qquad \therefore \quad K = 0.0157 \quad ; \qquad T_m = T_L - (T_L - T_0)\,e^{\frac{-t}{IK}}$$

$$250 = 400 - (400 - 100)\,e^{\frac{-6}{I(0.0157)}}$$

$\therefore$ $I =$ **552 kg - m²**

Example 45.22. *A 6 pole, 50 Hz, 3 - ϕ wound rotor Induction Motor has a flywheel coupled to its shaft. The total moment of inertia is 1000 kg-m². Load torque is 1000 N-m for 10 sec. followed by a no load period which is long enough for the motor to reach its no – load speed. Motor has a slip of 5% at a torque of 500 N-m. Find*

(a) *Maximum torque developed by motor*

(b) *Speed at the end of deaceleration period.* **[Nagpur University Winter 1996]**

Solution.

(a)

$$T_m = T_L\left(1 - e^{\frac{-t}{IK}}\right)$$

$s = KT_m$ But $N_S = \frac{120\,f}{P} = 1000$ r.p.m.

$\frac{2\pi}{60}$ (1000 × 0.05) = K (500) ; K = 6.2×10^{-3}

$T_m = 1000\left(1 - e^{\frac{-10}{1000 \times 6.2 \times 10^{-3}}}\right)$; T_m = **796.39 N – m**

(b) $s = K\,T_{F.L.}$

$\frac{2\pi}{60}(100 - N) = 6.2 \times 10^{-3} \times 790.39$

∴ N = **952.2 r.p.m.**

Example 45.23. *A motor fitted with a flywheel supplies a load torque of 1000 N-m. for 2 sec. During no load period, the flywheel regains its original speed. The motor torque is to be limited to 500 N-m. Find moment of inertia of the flywheel. No load speed of the motor is 500 r.p.m. and its full load slip is 10%.*

Solution.

$s = K\,T_{F.L.}$

$\frac{2\pi}{60}(500 \times 0.1) = K\,500$, K = 0.0104 ; $T_m = T_L\left(1 - e^{\frac{-t}{IK}}\right)$

$500 = 1000\left\{1 - e^{\frac{-2}{I(0.0104}}\right\}$; I = **277.44 kg- m²**

Tutorial Problem No. 45.1

1. A motor driving a colliery winding equipment has to deliver a load rising uniformly from zero to a maximum of 1500 KW in 20 sec. during the accelerating period, 750 KW for 40 sec. during the full speed period and during the deceleration period of 10 sec., when regenerative braking is taking place from an initial value of 250 KW to zero and then a no load period of 20 sec. Estimate remittable KW rating of the motor. **[648 KW]**
2. A constant speed drive has the following duty cycle:

Load rising from 0 to 400 KW	–	5 minutes
Uniform load of 400 KW	–	5 minutes
Regenerative power of 400 KW returned to supply	–	4 minutes
Remains idle for	–	2 minutes

Estimate power rating of motor. **[380 H. P.]**
[*Nagpur University Winter 96*]

3. Determine the rated current of a transformer for the following duty cycle:
   - 500 A for 3 minutes
   - Sharp increase to 1000 A and constant at this value for 1 minute
   - Gradually decreasing to 200 A. for 2 minutes
   - Constant at this value for 2 minutes
   - Gradually increasing to 500 A during 2 minutes repeated indefinitely. **[540 A]**
4. An induction motor has to perform the following duty cycle:
   75 KW for 10 miniutes, No load for 5 minutes
   45 KW for 8 minutes, No load for 4 minutes

which is repeated indefinitely.
Determine suitable capacity of a continuously rated motor. **[70 H. P.]**

5. A 25 H.P. motor has heating time constant of 90 min. and when run continuously on full load attains a temperature of 45^{0}C. above the surrounding air. What would be the half hour rating of the motor for this temperature rise, assuming that it cools down completely between each load period and that the losses are proportional to square of the load. **[47 H.P.]**

6. At full load of 10 H.P., temperature rise of a motor is 25^0 C. after 1 hr. and 40^{o}C after 2 hrs. Find (*a*) Heating time constant of motor, (*b*) Final temperature rise on full load.
[T = 1. 96 hrs, θ_f = 62.5°C]

7. A totally enclosed motor has a temperature rise of 20^{o}C after half an hour and 35^{0}C after one hour on full load. Determine temperature rise after 2 hours on full load. **[54.68°C]**

8. A 25 H.P., 3-ϕ, 10 pole, 50 c.p.s. induction motor provided with a flywheel has to supply a load torque of 800 N-m for 10 sec, followed by a no load period, during which the flywheel regains its full speed. Full load slip of motor is 4% and torque-speed curve may be assumed linear over the working range. Find moment of inertia of flywheel, if the motor torque is not to exceed twice the full load torque. Assume efficiency = 90%. **[718 kg-m^2]**

9. A motor fitted with a flywheel has to supply a load torque of 200 kg-m for 10 sec, followed by a no load period. During the no load period, the motor regains its speed. It is desired to limit the motor torque to 100 kg-m. What should be the moment of inertia of flywheel. No load speed of motor is 500 r.p.m. and has a slip of 10% at a torque of 100 kg-m. . **[I = 2703 kg-m^2]**

10. A 50 Hz., 3-ϕ, 10 pole, 25H.P., induction motor has a constant load torque of 20 kg-m and at wide intervals additional torque of 100 kg-m for 10 sec. Full load slip of the motor is 4% and its efficiency is 88%. Find -
(*a*) Moment of inertial of flywheel , if motor torque not to exceed twice full load torque.
(*b*) Time taken after removal of additional load, before motor torque is 45 kg- m.
[I = 1926 kg-m^2, t = 9.99 sec.]

11. Define the following terms regarding the ratings of motor :-
(*i*) Continuous rating (*ii*) short time rating (*iii*) Intermittent rating.
(Nagpur University, Summer 2004)

12. With the help of heating and cooling curves define and explain the terms :
(*i*) Heating time constant (*ii*) Cooling time constant. ***(Nagpur University, Summer 2004)***

13. What do you mean by 'load-equilisation' it is possible to apply this scheme for reversible drive? Why? ***(Nagpur University, Summer 2004)***

14. A motor is equipped with the flywheel has to supply a load torque of 600 N-m for 10 seconds, followed by no load period long enought for flywheel to regain its full speed. It is desired to limit the motor torque of 450 N-m. What should be moment of inertia of the flywheel? the no load speed of the motor is 600 rpm and has 8% slip at a torque of 450 N-m. The speed-torque characteristics of the motor can be assumed to be a straight line in the region of interest.
(Nagpur University, Summer 2004)

15. A motor has the following load cycle :
Accelerating period 0-15 sec Load rising uniformly from 0 to 1000 h.p.
Full speed period 15-85 sec Load constant at 600 h.p.
Decelerating period 85-100 sec h.p. returned to line falls uniformly 200 to zero
Decking period 100-120 sec Motor stationary. Estimate the size the motor.
(J.N. University, Hyderabad, November 2003)

16. A motor driving a load has to deliver a load rising uniformly from zero to maximum of 2000 h.p. in 20 sec during the acceleration period, 1000 h.p. for 40 sec during the full speed period and during the deceleration period of 10 sec when regenerating braking taking place the h.p. returned to the supply falls from 330 to zero. The interval for decking before the next load cycle starts is 20 sec. Estimate the h.p. Rating of the motor. ***(J.N. University, Hyderabad, November 2003)***

17. Draw and explain the output vs. time characteristics of any three types of loads.
(J.N. University, Hyderabad, November 2003)

18. Discuss series and parallel operation of series and shunt motors with unequal wheeldiameters. Comment on the load sharing in each case. ***(J.N. University, Hyderabad, November 2003)***

19. Discuss the various factors that govern the size and the rating of a motor for a particular service. ***(J.N. University, Hyderabad, April 2003)***

20. A motor has to deliver a load rising uniformly from zero to a maximum of 1500 Kw in 20 sec during the acceleration period, 1,000 Kw for 50 sec during the full load period and during the deceleration period of 10 sec when regenerative braking takes place the Kw returned to the supply falls from an initial value of 500 to zero uniformly. The interval for decking before the next load cycle starts is 20 sec. Estimate the rating of the motor. ***(J.N. University, Hyderabad, April 2003)***

21. Derive an expression for the temperature rise of an equipment in terms of the heating time constant. ***(J.N. University, Hyderabad, April 2003)***

22. At full load of 10 h.p., the temperature rise of a motor is 25 degree C after one hour, and 40 degree C after 2 hours. Find the final temperature rise on full load. Assume that the iron losses are 80% of full load copper losses. ***(J.N. University, Hyderabad, April 2003)***

23. Explain what you mean by Lord Equalization and how it is accomplished. ***(J.N. University, Hyderabad, April 2003)***

24. A motor fitted with a flywheel supplies a load torque of 150 kg-m for 15 sec. During the no load period the flywheel regains its originalspeed. The motor torque is required to be limited to 85 kg-m. Determine the moments of inertia of the flywheel. The no load speed of the motor is 500 r.p.m. and it has a slip of 10% on full load. ***(J.N. University, Hyderabad, April 2003)***

25. Discuss the various losses that occur in magnetic conductors which cause the temperature rise in any electrical apparatus and suggest how they can be reduced. ***(J.N. University, Hyderabad, April 2003)***

26. The outside of a 12 h.p. (metric) motor is equivalent to a cylinder of 65 cms diameter and 1 meter length. The motor weighs 400 Kg and has a specific heat of 700 Joules per kg per degree C. The outer surface is capable of heat dissipation of 12 W per meter square per degree C. Find the final temperature rise and thermal constant of the motor when operating at full load with an effciency of 90%. ***(J.N. University, Hyderabad, April 2003)***

27. ''A flywheel is not used with a synchronous motor for load equalization''. Discuss. ***(J.N. University, Hyderabad, April 2003)***

28. A 25 h.p. 3-phase 10 pole, 50 Hz induction motor fitted with flywheel has to supply a load torque of 750 Nw-m for 12 sec followed by a no load period during which the flywheel regains its original speed. Full load slip of the motor is 4% and the torque-speed curve is linear. Find the moment of inertia of the flywheel if the motor torque is not to exceed 2 times the full load torque. ***(J.N. University, Hyderabad, April 2003)***

29. Explain what do you mean by Load Equalization and how it is accomplished. ***(J.N. University, Hyderabad, April 2003)***

30. A motor fitted with a flywheel supplies a load torque of 150 kg-m for 15 sec. During the no load period the flywheel regains its original speed. The motor torque is required to be limited to 85 kg-m. Determine the moments of inertia of the flywheel. The no load speed of the motor is 500 r.p.m. and it has aslip of 10% on full load. ***(J.N. University, Hyderabad, April 2003)***

31. A 100 hp motor has atemperature rise of 50^{o}C when running continuously on full load. It has a time constant of 90 minutes. Determine 1/2 hr rating of the motor for same temperature rise. Assume that the losses are proportional to the square of the load and motor cools completely between each load period. ***(J.N. University, Hyderabad, December 2002/January 2003)***

32. Explain 'load equalisation'. How this can be achieved in industrial drives. ***(J.N. University, Hyderabad, December 2002/January 2003)***

33. Obtain the expression for temperature rise of a electrical machine. State the assumptions made if any. *(J.N. University, Hyderabad, December 2002/January 2003)*

34. A 75 kW, 500 rpm dc shunt motor is used to drive machinery for which the stored energy per kW is 5400 Joules. Estimate the time taken to start the motor, if the load torque is equal to full load torque during the starting period and the current is limited to 1 1/2 times the full load current. *(J.N. University, Hyderabad, December 2002/January 2003)*

OBJECTIVE TESTS – 45

1. Heat dissipation is assumed proportional to
(*a*) Temperature difference
(*b*) Temperature difference between motor and cooling medium
(*c*) Temperature of cooling medium

2. Temperature of cooling medium is assumed
(*a*) constant (*b*) variable

3. When the motor reaches final temperature rise its temperature remains
(*a*) constant
(*b*) falls
(*c*) rises.

4. For intermittent load, a motor of smaller rating can be used
(*a*) true (*b*) false

5. If motor is disconnected from supply, final temperature reached will be the ambient temperature
(*a*) true (*b*) false

6. Final temperature rise is theoretically attained only after
(*a*) fixed time
(*b*) variable time
(*c*) infinite time

7. Motor is derated when taken at altitude
(*a*) Yes (*b*) No

8. The rolling mill load
(*a*) is constant
(*b*) fluctuates widely within long intervals of time
(*c*) fluctuates widely within short intervals of time
(*d*) varies

9. Size of motor is decided by
(*a*) load
(*b*) current
(*c*) heat produced in motor
(*d*) torque

10. Tariff is affected by sudden load drawn by motor
(*a*) true (*b*) false

11. Flywheel helps in smoothing only
(*a*) speed fluctuations
(*b*) current fluctuations
(*c*) both of the above

12. To use flywheel, motor should have
(*a*) constant speed characteristics
(*b*) drooping speed characteristics
(*c*) variable speed characteristics

13. During light load period
(*a*) flywheel absorbs energy
(*b*) flywheel gives up energy
(*c*) flywheel does nothing

14. During peak load periods
(*a*) flywheel absorbs energy
(*b*) flywheel gives up energy
(*c*) flywheel does nothing

15. Large size of flywheel
(*a*) can be used practically
(*b*) can't be used practically

ANSWERS

1. (*b*) **2.** (*a*) **3.** (*a*) **4.** (*a*) **5.** (*a*) **6.** (*c*) **7.** (*a*) **8.** (*c*) **9.** (*c*) **10.** (*a*)
11. (*c*) **12.** (*b*) **13.** (*a*) **14.** (*b*) **15.** (*b*)

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CHAPTER 46

ELECTRONIC CONTROL OF A.C. MOTORS

Learning Objectives

- Classes of Electronic AC Drives
- Variable Frequency Speed Control of a SCIM
- Variable Voltage Speed Control of a SCIM
- Chopper Speed Control of a WRIM
- Electronic Speed Control of Synchronous Motors
- Speed Control by Current-fed D.C. Link
- Synchronous Motor and Cycloconverter



Efficient control of motors becomes critical where high precision, accuracy, flexibility, reliability and faster response are of paramount importance. Electronic and digital controls are employed in such conditions

CONTENTS

46.1. Classes of Electronic A.C. Drives

AC motors, particularly, the squirrel-cage and wound-rotor induction motors as well as synchronous motors lend themselves well to electronic control of their speed and torque. Such a control is usually exercised by varying voltage and frequency. Majority of the electronic a.c. drives can be grouped under the following broad classes :

1. **static frequency changers** like cyclo-converters which convert incoming high line frequency directly into the desired low load frequency. Cyclo-converters are used both for synchronous and squirrel-cage induction motors.
2. **variable-voltage controllers** which control the speed and torque by varying the a.c. voltage with the help of SCRs and gate turn-off thyristors (GTOs).
3. **rectifier-inverter systems** with natural commutation.
4. **rectifier-inverter systems** with self-commutation.

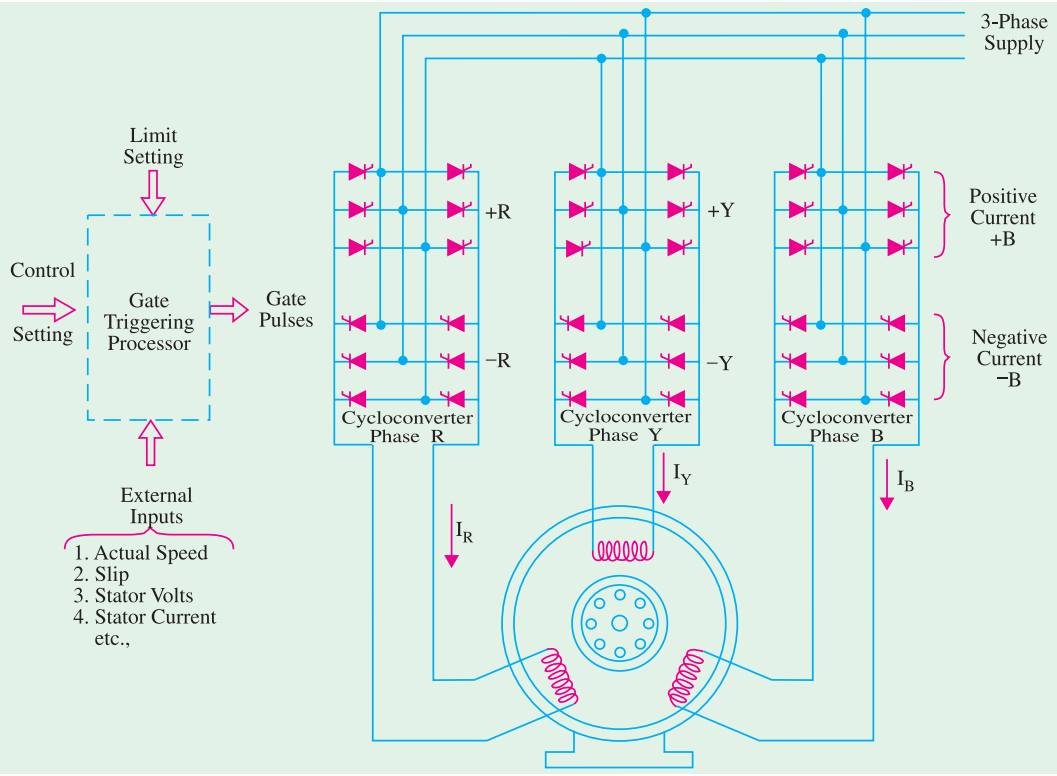


Fig. 46.1

46.2. Variable-frequency Speed Control of a SCIM

Fig. 46.1 shows a 3-phase SCIM connected to the outputs of three 3-phase cycloconverters. As seen, each cyclo-converter consists of two 3-phase thyristor bridges, each fed by the same 3-phase, 50-Hz line. The $+R$ bridge generates the positive half-cycle for R-phase whereas $-R$ generates the negative half. The frequency of the cycloconverter output can be reduced to any value (even upto zero) by controlling the application of firing pulses to the thyristor gates. This low frequency permits excellent speed control. For example, the speed of a 4-pole induction motor can be varied from zero to 1200 rpm on a 50-Hz line by varying the output frequency of the cycloconverter from zero to 40 Hz. The stator voltage is adjusted in proportion to the frequency in order to maintain a constant flux in the motor.

This arrangement provides excellent torque/speed characteristics in all 4-quadrants including regenerative braking. However, such cycloconverter-fed motors run about 10°C hotter than normal and hence require adequate cooling. A small part of the reactive power required by SCIM is provided by the cycloconverter, the rest being supplied by the 3-phase line. Consequently, power factor is poor which makes cycloconverter drives feasible only on small and medium power induction motors.

46.3. Variable Voltage Speed Control of a SCIM

In this method, the speed of a SCIM is varied by varying the stator voltage with the help of three sets of SCRs connected back-to back (Fig. 46.2). The stator voltage is reduced by delaying the firing (or triggering) of the thyristors. If we delay the firing pulses by 100°, the voltage obtained is about 50% of the rated voltage which decreases the motor speed considerably.

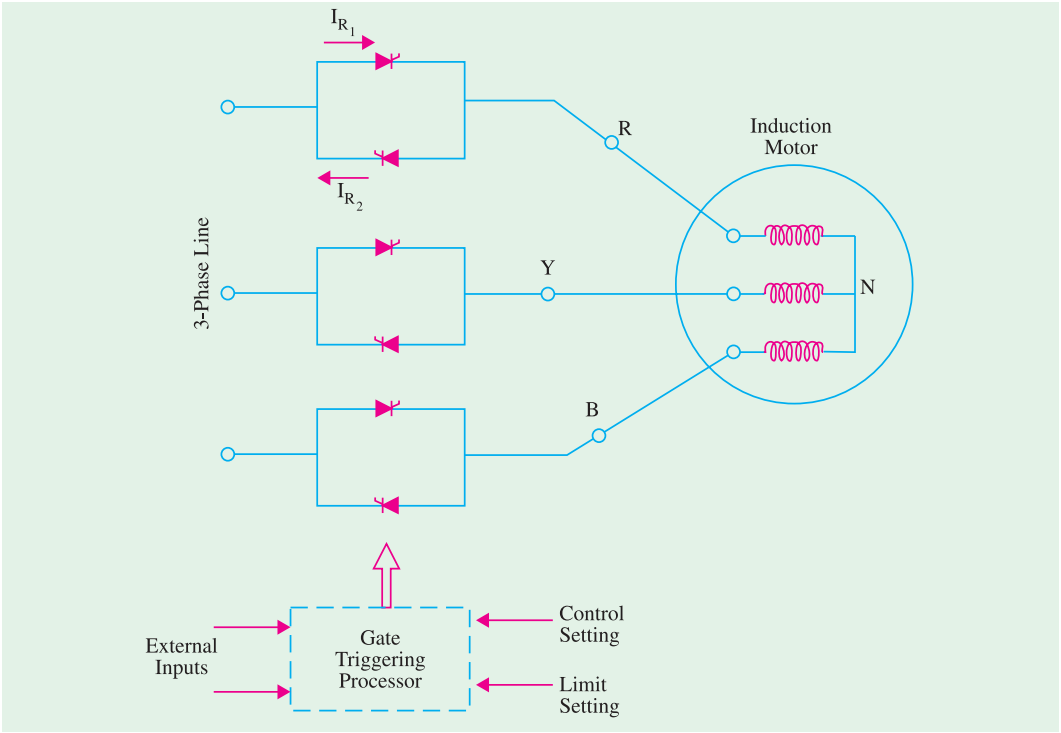


Fig. 46.2

Unfortunately, I^2R losses are considerable due to distortion in voltage. Moreover, p.f. is also low due to large lag between the current and voltage. Hence, this electronic speed control method is feasible for motors rated below 15 kW but is quite suitable for small hoists which get enough time to cool off because of intermittent working. Of course, p.f. can be improved by using special thyristors called gate turn-off thyristors (GTOs) which force the current to flow almost in phase with the voltage (or even lead it).



A commonly used electronic power inverter

46.4. Speed Control of a SCIM with Rectifier-Inverter System

A rectifier-inverter system with a d.c. link is used to control the speed of a SCIM. The inverter used is a self-commuted type (different from a naturally commutated type) which converts d.c. power into a.c. power at a frequency determined by the frequency of the

pulses applied to the thyristor gates. The rectifier is connected to the 3-phase supply line whereas the inverter is connected to the stator of the SCIM.

Two types of links are used :

1. constant-current d.c. link —for speed control of *individual* motors.
2. constant-voltage d.c. link —for speed control of several motors.

As shown in Fig. 46.3, the constant-current link supplies constant current to the inverter which then feeds it sequentially (through proper switching sequence) to the three phases of the motor. Similarly, the constant-voltage dc link (Fig. 46.4) provides a constant voltage to the inverter which is switched from one phase of the motor to the next in a proper sequence.

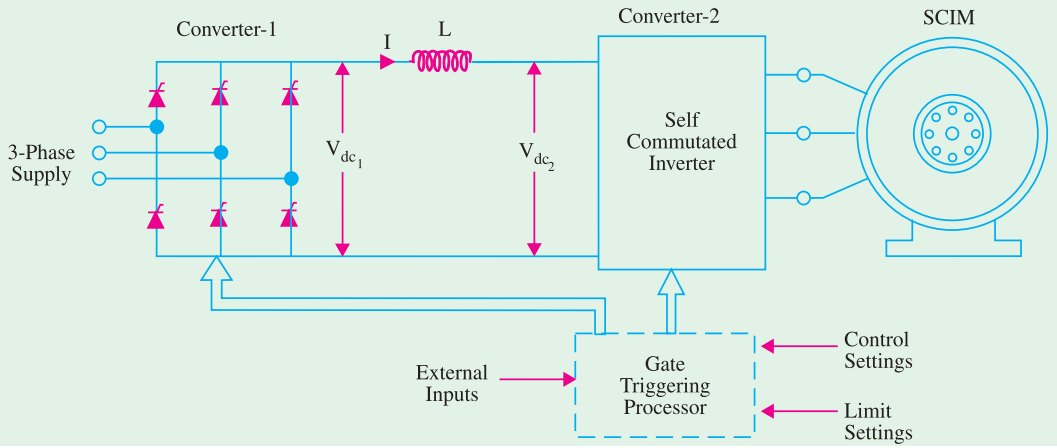


Fig. 46.3

The arrangement of Fig. 46.3 gives speed control with high efficiency in all 4 quadrants in addition to the facility of regenerative braking. Heavy inertia loads can be quickly accelerated because motor develops full break-down torque right from the start. The output frequency of the inverter varies over a range of 20 : 1 with a top frequency of about 1 kHz. The a.c. voltage supplied by the inverter is changed in proportion to the frequency so as to maintain the stator flux constant.

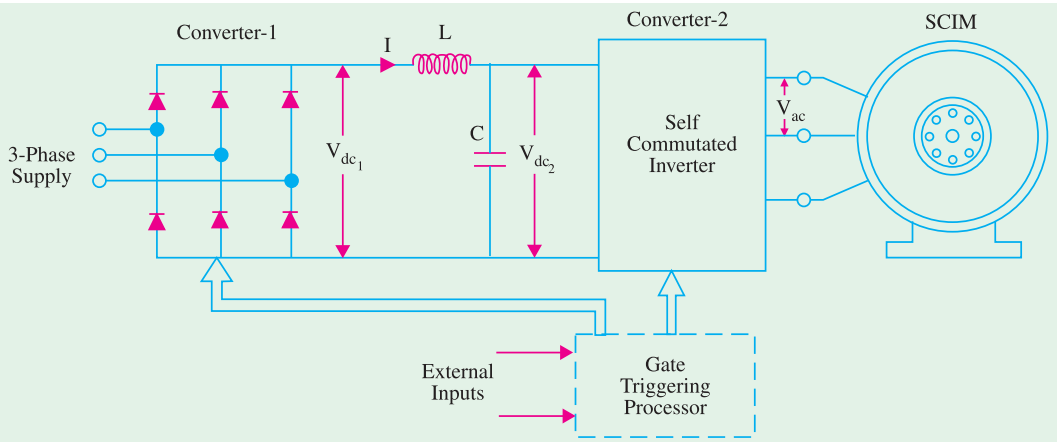


Fig. 46.4

Consequently, d.c. link voltage V_{dc1} has to be reduced as the motor speeds up. This is accomplished by increasing the firing angle of the thyristors in converter 1. Unfortunately, this leads to increase in the reactive power drawn from the 3-phase line which results in poor power factor. To improve the p.f., use of capacitors is necessary. The direction of rotation can be changed by altering the phase sequence of the pulses that trigger the gates of converter 2.

The voltage-fed frequency converter of Fig. 46.4 consists of a rectifier and a self-commutated inverter connected by a d.c. link and is often used for group drives in textile mills. The 3-phase bridge rectifier produces d.c. voltage V_{dc1} which is smoothened up by the LC filter before being applied to the inverter. The inverter successively switches its output ac voltage V_{ac} to the three phases of the motor. This voltage is varied in proportion to the frequency so as to maintain constant flux in the motor. Since, V_{ac} depends on V_{dc2} which itself depends on V_{dc1}, it is V_{dc1} which is changed as frequency varies. In this system, motor speed can be controlled from zero to maximum while developing full breakdown torque.



A simple electronic control system

46.5. Chopper Speed Control of a WRIM

As discussed in Art. 30.18 (*d*), the speed of a WRIM can be controlled by inserting three variable resistors in the rotor circuit. The all-electronic control of speed can be achieved by connecting a 3-phase bridge rectifier across the rotor terminals and then feed the rotor output to a single fixed resistor or R_0 via a chopper (Fig. 46.5).

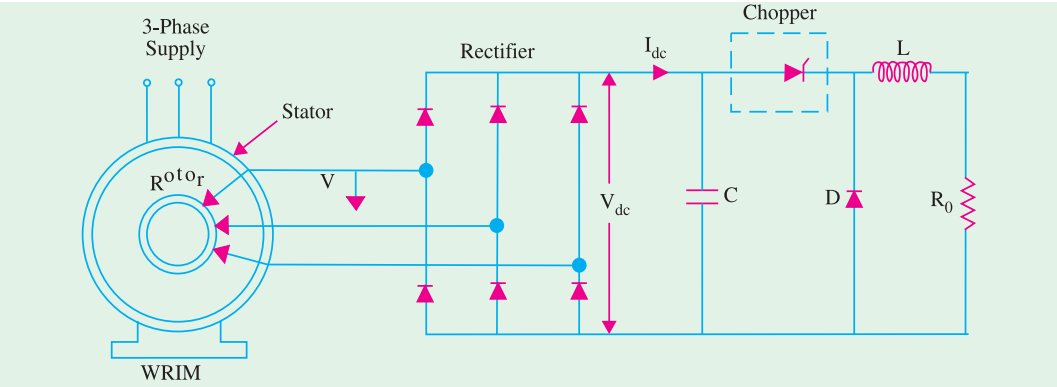


Fig. 46.5

The capacitor C supplies the high current pulses drawn by the chopper. The inductor L and free-wheeling diode D serve the same purpose as discused in Art. 28.14. By varying the chopper on-time TON, the apparent resistance R_a across the bridge rectifier can be made either high or low. The value of apparent resistance is given by $R_a = R_0/f^2 T_{ON}^2$ where f is the OFF/ON switching frequency of the chopper. The resulting torque/speed characteristic is similar to the one obtained with a 3-phase rheostat.

Example 46.1. *The wound-rotor induction motor of Fig.43.5 is rated at 30-kW, 975 rpm, 440-V, 50 Hz. The open-circuit line voltage is 400 V and the load resistance is 0.5 Ω. If chopper frequency is 200 Hz, calculate* T_{ON} *so that the motor develops a gross torque of 200 N-m at 750 rpm. Also, calculate the magnitude of the current pulses drawn from the capacitor.*

Solution. Obviously, N_S = 1000 rpm. Hence, slip at 750 rpm is = (1000 – 750)/1000 = 0.25. The rotor line voltage at 750 rpm is = $sE_2 = 0.25 \times 400 = 100$ V.

The d.c. voltage of 3-phase bridge rectifier is $V_{dc} = 1.35$ V $= 1.35 \times 100 = 135$ V.

Now, $T_g = P^2/2\pi N_s; P^2 = T_g \times 2\pi N_s = 200 \times 2\pi \times (1000/60) = 20{,}950$ W

Part of P^2 dissipated as heat = $sP^2 = 0.25 \times 20{,}950 = 5{,}238$ W

The power is actually dissipated in R_0 and is, obviously, equal to the rectifier output $V_{dc}.I_{dc}$.

$\therefore \quad V_{dc}.I_{dc} = 5238$ or $I_{dc} = 5238/135 = 38.8$ A

The apparent resistance at the input to the chopper is

$$R_a = V_{dc}/I_{dc} = 135/38.8 = 3.5\ \Omega$$

Now, $R_a = R_0/f^2$ क्त्2 or $T_{ON} = \sqrt{R_0/f^2 R_a} = \sqrt{0.5/200^2 \times 3.5} =$ **1.9 ms**

Current in R_0 can be found from the relation

$$I_0^2 R_0 = 5238 \quad \text{or} \quad I_0 = \sqrt{5238/0.5} = \textbf{102 A}$$

As seen, capacitor delivers current pulses of magnitude क्त्2 A and a pulse width of 1.9 ms at the rate of 200 pulses/second. However, the rectifier continuously charges C with a current of 38.8 A.

46.6. Electronic Speed Control of Synchronous Motors

The speed of such motors may be controlled efficiently by using ***(i)*** current-fet delink and ***(ii)*** cycloconverter as discussed below :

46.7. Speed Control by Current-fed DC Link

As shown in Fig. 46.6, the typical circuit consists of three converters two of which are connected between the three-phase source and the synchronous motor whereas the third converter (bridge rectifier) supplies dc field excitation for the rotor. Converter-1 (*C*-1) acts as a controlled rectifier and feeds d.c. power to converter-2 (*C*-2). The converter-2 behaves as a naturally commutated inverter whose a.c. voltage and frequency are established by the motor. The function of the smoothing inductor *L* is to maintain a ripple-free current in the d.c. link between the two converters. Converter-1 acts as a current source and controls *I*.

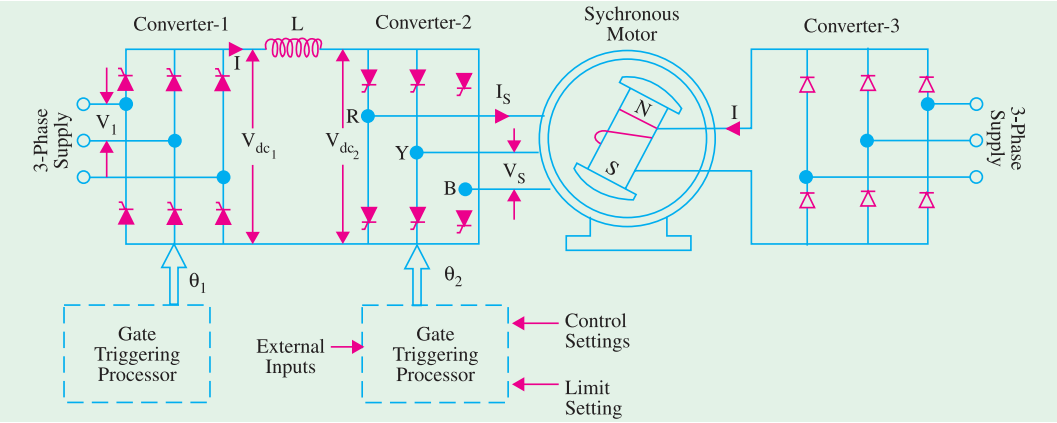


Fig. 46.6

The converter-2 is naturally commutated by voltage V_s induced across motor terminals by its revolving magnetic flux. The revolving flux which depends on the stator currents and the d.c. field exciting current I_{f1} is usually kept constant. Consequently, V_s is proportional to motor speed.

As regards various controls, information picked off from various points is processed in the gate-triggering processors which then send out appropriate gate firing pulses to converters 1 and 2. The processors receive information about the desired rotor speed, its actual speed, instantaneous rotor position, field current, stator voltage and current etc. The processors check whether these inputs represent normal or abnormal conditions and send appropriate gate firing pulses either to correct the situation or meet a specific demand.

Gate triggering of C-1 is done at line frequency (50 Hz) whereas that of C-2 is done at motor frequency. In fact, gate pulses of C-2 are controlled by rotor position which is sensed by position transducers mounted at the end of the shaft. The motor speed can be increased by increasing either d.c. link current *I* or exciting current I_f.

Now, $V_{dc2} = 1.35\ V_s \cos \alpha_1$ and $V_{dc1} = 1.35\ V_s \cos \alpha_1$

where V_{dc2} = d.c. voltage generated by *C*-2, V_{dc1} = d.c. voltage supplied by *C*-1



Special features of A C Synchronous motors: 1. Bi-directional, 2. Instantaneous Start, Stop and Reverse, 3. Identical Starting and Running Currents, 4. Residual Torque always present, 5. No damage due to stalling, 6. Low speed of 60 rpm. Applications of AC Synchronous Motors are found in: 1. Actuators, 2. Remote control of switches 3. Winding machines, 4. Machine tool applications, 6. Valve controls, 6. Printing machines, 7. Automatic welding machines, 8. Medical equipment, 9. Conveyor systems, 10. Paper feeders

α_2 = firing angle of *C*-2 ; α_1 = firing angle of *C*-1

The firing angle α_1 is automatically controlled and supplies *I* which is sufficient to develop the required torque. This method of speed control is applied to motors ranging from 1 kW to several MW. Permanent-magnet synchronous motors used in textile industry and brushless synchronous motors for nuclear reactor pumps are controlled by this method.

46.8. Synchronous Motor and Cycloconverter

As shown in Fig. 46.7, the arrangement consists of three cycloconverters connected to the three

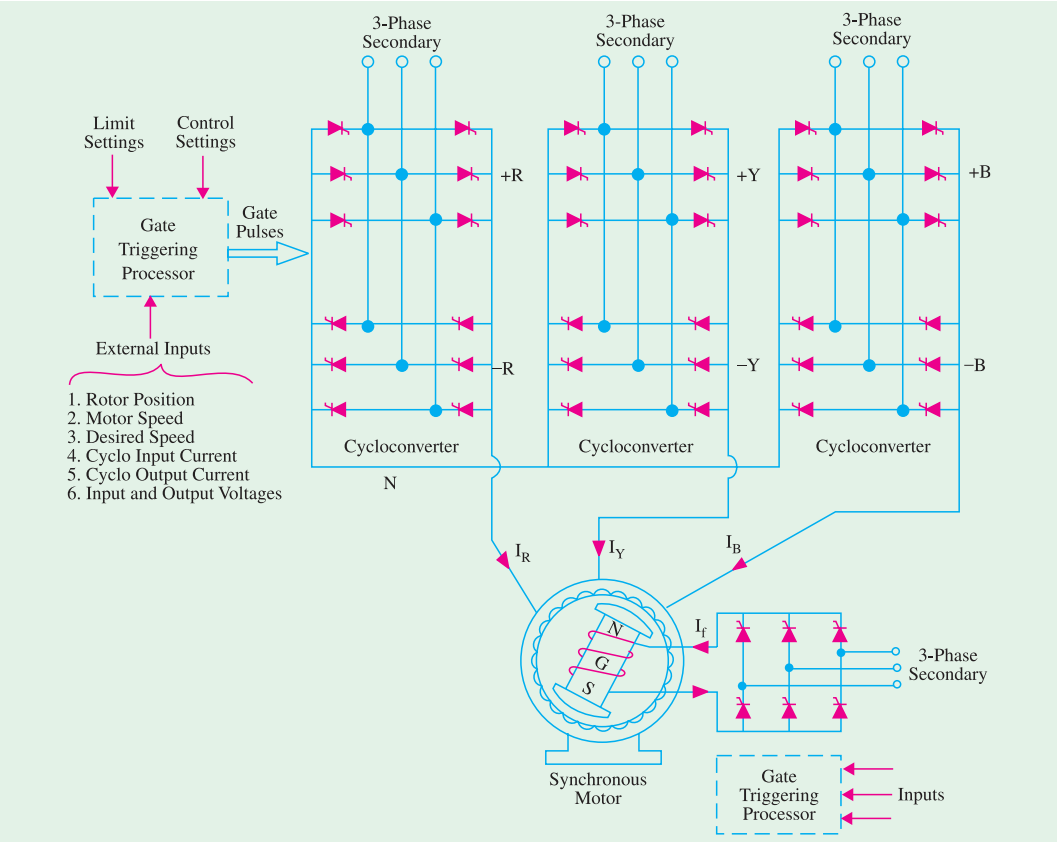


Fig. 46.7

one phases of the synchronous motor and one controlled rectifier for supplying field exciting current I_f to the rotor. Each cycloconverter is composed of two three-phase bridges and supplies a single-phase output. As is well known, a cycloconverter can directly convert a.c. power at higher frequency to one at a lower frequency. With a line frequency of 50 Hz, the cycloconverter output frequency can be varied from zero to 10 Hz.

The cycloconverters and the controlled rectifier function as current sources. The air-gap flux is kept constant by controlling the magnitude of the stator currents and exciting current I_f. By proper timing of gate pulses, motor can be made to operate at unity power factor.

The speed of cycloconverter-driven large slow-speed synchronous motors can be continuously varied from zero to 15 rpm. Such low speeds permit direct drive of the ball mill without using a gear reducer. Such high-power low-speed systems are also being introduced as propeller drives on board the ships.

46.9. Digital Control of Electric Motors

Advantages of Digital Control

1. High precision and accuracy
2. Better speed regulation
3. Faster response
4. Flexibility
5. Better time response
6. Improved performance
7. Economical
8. Easy software control
9. Reliability
10. The greatest advantage of the digital control is that by changing the program, desired control technique can be implemented without any change in the hardware.

The speed information can be fed into microcomputer using a D.C. Tacho (Speed encoder) and A/D converter (Speed *I/P* module). The motor current data is usually fed into the computer through a fast A/D converter. A synchronizing circuit interface (Line synchronizing circuit) is required so that the micro-computer can synchronize the generation of the firing pulse data with the supply line frequency. The gate pulse generator is shown as receiving a firing signal from microcomputer.

A set of instruction (Program) is stored in memory and those are executed by computer for proper functioning of a drive. A typical program flow - chart for this drive system is shown in figure (46.9). The sequence of instructions allows the computer to process data for speed regulation, current regulation and reversal operation.

46.10. Application of Digital Control

The above operations can be clearly understood by considering one of the applications of Digital Control system, such as Digital Control System for Speed Control of D.C. drives using a Micro-computer :

Various components and their operations shown in Fig. 46.8 are discussed below :

(i) Thyristor Converter

PC based control systems can be built where a phase-controlled rectifier supplies a D.C. motor. The main control to be handled is to turn on & off SCRs. Thyristor power converter in this case is a dual converter – one for forward and other for reverse direction.

(ii) Gate Pulse Generator and Amplifier

PC is used for firing angle control of dual converter. It can be programmed using suitable software to perform the function of firing range selection, firing pulse generation, etc. The firing pulses so obtained are amplified, if needed to turn *ON* the SCR reliably. Changeover signal decides whether to

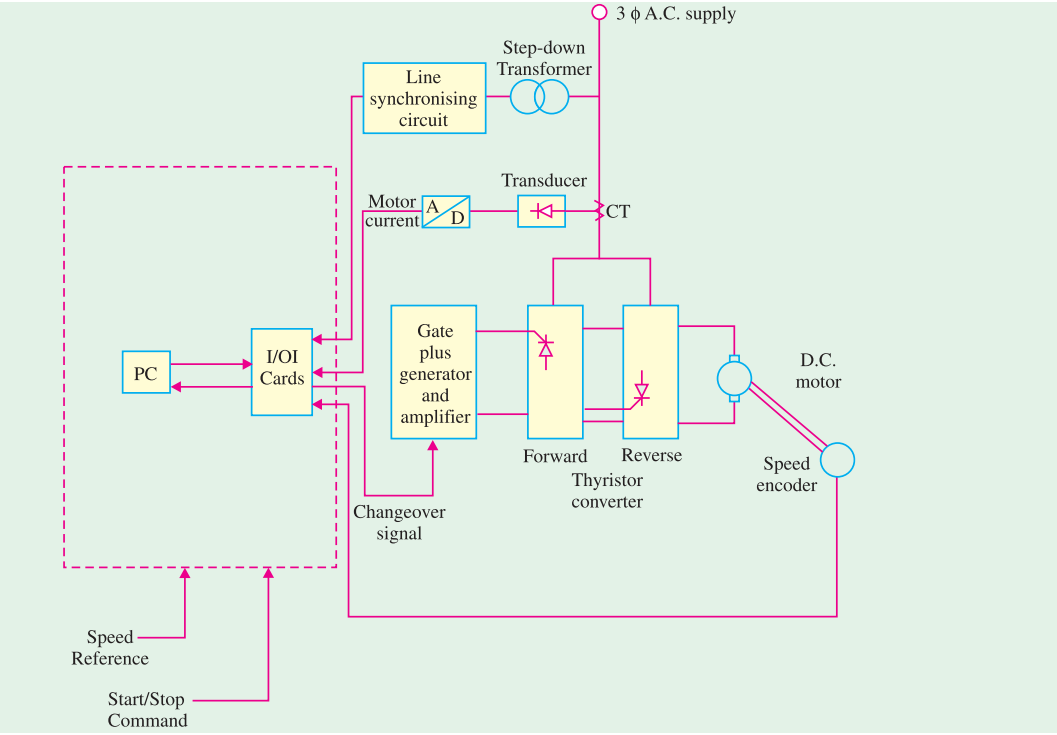


Fig. 46.8

switch *ON* forward or reverse group of SCRs. The gate pulse generator is shown as receiving a firing signal from *PC*.

(iii) Speed Encoder and Input Module

The speed information can be fed to PC through speed input module. The speed measurement is done digitally by means of speed/shaft encoder. It consists of a disc with definite number of holes drilled on it. This disc is fixed on to the shaft. Using a light source and a phototransistor; a series of pulses is obtained, as the shaft rotates. This pulse train is processed and shaped. These optically coded pulses are counted to get actual speed of motor.

(iv) A/D Converter and Transducer

The motor current drawn from supply is stepped down with the help of current transformer. It is converted to D.C. voltage output with the help of current transducer. As PC can't process analog signals, this analog current signal is fed to A/D converter to obtain digital signal which is fed to PC.

(v) Line Synchronizing Circuit

This is required so that PC can synchronize the generation of firing pulse data, with supply line frequency.

(vi) I/O Cards

Input/ Output cards are required to interface PC with the outside world.

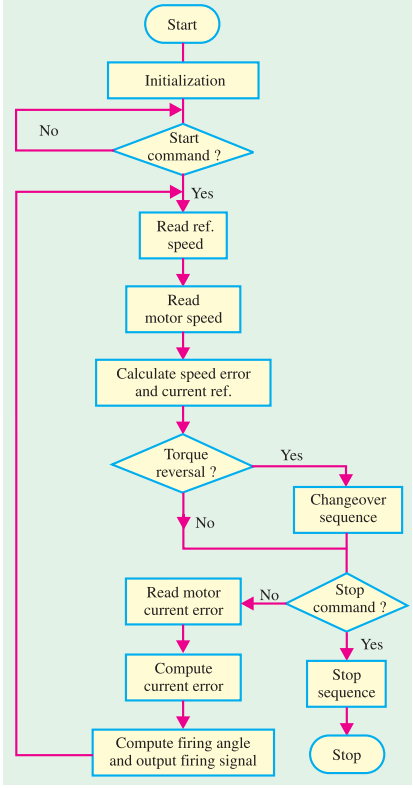


Fig. 46.9. Program flow chart for digital control of motor.

OBJECTIVE TESTS – 46

1. The function of a cycloconverter is to convert
(*a*) ac power into d.c. power
(*b*) direct current into alternating current
(*c*) high ac frequency directly to low ac frequency
(*d*) a sine wave into a rectangular wave.

2. Major disadvantage of using three sets of SCRs for variable-voltage speed control of a SCIM is the
(*a*) considerable I^2R loss
(*b*) poor power factor
(*c*) long delay of thyristor firing pulses
(*d*) necessity of using a processor.

3. In the current-fed frequency converter arrangement for controlling the speed of an individual SCIM, the direction of rotation can be reversed by
(*a*) changing the output frequency of the inverter
(*b*) altering the phase sequence of pulses that trigger converter-2
(*c*) interchanging any two line leads
(*d*) reversing the d.c. link current.

4. In the chopper speed control method for a WRIM, the motor speed inversely depends on
(*a*) fixed resistor across the rectifier
(*b*) chopper switching frequency
(*c*) chopper ON time TON
(*d*) both (*b*) and (*c*).

5. In the synchronous motor drive using current-fed dc link
(*a*) converter-2 functions as a self-commutated inverter
(*b*) converter-1 works as an uncontrolled rectifier
(*c*) converter-3 is a controlled rectifier
(*d*) gate triggering of converter-2 is done at motor frequency.

6. In the three cycloconverter drive of a synchronous motor
(*a*) each cycloconverter produces a 3-phase output
(*b*) all cycloconverters act as voltage sources
(*c*) a 3-phase controlled rectifier provides field exciting current.
(*d*) air-gap flux is kept constant by controlling stator currents only.

ANSWERS

1. (*c*) **2.** (*a*) **3.** (*b*) **4.** (*d*) **5.** (*d*) **6.** (*c*)

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CHAPTER 47

ELECTRIC HEATING

Learning Objectives

- Introduction
- Advantages of Electric Heating
- Different Methods of Heat Transfer
- Methods of Electric Heating
- Resistance Heating
- Requirement of a Good Heating Element
- Resistance Furnaces or Ovens
- Temperature Control of Resistance Furnaces
- Design of Heating Element
- Arc Furnaces
- Direct Arc Furnace
- Indirect Arc Furnace
- Induction Heating
- Core Type Induction Furnace
- Indirect Core Type Induction Furnace
- Coreless Induction Furnace
- High Frequency Eddy Current Heating
- Dielectric Heating
- Dielectric Loss
- Advantages of Dielectric Heating
- Applications of Dielectric Heating
- Choice of Frequency
- Infrared Heating



The above figure shows an electric arc furnace, producing steel. Electric heating is widely used in furnaces in metallurgical and chemical industries

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47.1. Introduction

Electric heating is extensively used both for domestic and industrial applications. Domestic applications include **(*i*)** room heaters **(*ii*)** immersion heaters for water heating **(*iii*)** hot plates for cooking **(*iv*)** electric kettles **(*v*)** electric irons **(*vi*)** pop-corn plants **(*vii*)** electric ovens for bakeries and **(*viii*)** electric toasters etc.

Industrial applications of electric heating include **(*i*)** melting of metals **(*ii*)** heat treatment of metals like annealing, tempering, soldering and brazing etc. **(*iii*)** moulding of glass **(*iv*)** baking of insulators **(*v*)** enamelling of copper wires etc.

47.2. Advantages of Electric Heating

As compared to other methods of heating using gas, coal and fire etc., electric heating is far superior for the following reasons :

(*i*) Cleanliness. Since neither dust nor ash is produced in electric heating, it is a clean system of heating requiring minimum cost of cleaning. Moreover, the material to be heated does not get contaminated.

(*ii*) No Pollution. Since no flue gases are produced in electric heating, no provision has to be made for their exit.

(*iii*) Economical. Electric heating is economical because electric furnaces are cheaper in their initial cost as well as maintenance cost since they do not require big space for installation or for storage of coal and wood. Moreover, there is no need to construct any chimney or to provide extra heat installation.

(*iv*) Ease of Control. It is easy to control and regulate the temperature of an electric furnace with the help of manual or automatic devices. Temperature can be controlled within ± 5°C which is not possible in any other form of heating.

(*v*) Special Heating Requirement. Special heating requirements such as uniform heating of a material or heating one particular portion of the job without affecting its other parts or heating with no oxidation can be met only by electric heating.

(*vi*) Higher Efficiency. Heat produced electrically does not go away waste through the chimney and other by products. Consequently, most of the heat produced is utilised for heating the material itself. Hence, electric heating has higher efficiency as compared to other types of heating.

(*vii*) Better Working Conditions. Since electric heating produces no irritating noises and also the radiation losses are low, it results in low ambient temperature. Hence, working with electric furnaces is convenient and cool.

(*viii*) Heating of Bad Conductors. Bad conductors of heat and electricity like wood, plastic and bakery items can be uniformly and suitably heated with dielectric heating process.

(*ix*) Safety. Electric heating is quite safe because it responds quickly to the controlled signals.

(*x*) Lower Attention and Maintenance Cost. Electric heating equipment generally will not require much attention and supervision and their maintenance cost is almost negligible. Hence, labour charges are negligibly small as compared to other forms of heating.

47.3. Different Methods of Heat Transfer

The different methods by which heat is transferred from a hot body to a cold body are as under:

1. Conduction

In this mode of heat transfer, one molecule of the body gets heated and transfers some of the

heat to the adjacent molecule and so on. There is a temperature gradient between the two ends of the body being heated.

Consider a solid material of cross-section A sq.m. and thickness x metre as shown in Fig. 47.1. If T_1 and T_2 are the temperatures of the two sides of the slab in °K, then heat conducted between the two opposite faces in time t seconds is given by:

$$H = \frac{KA(T_1 - T_2)t}{x}$$

where K is thermal conductivity of the material.

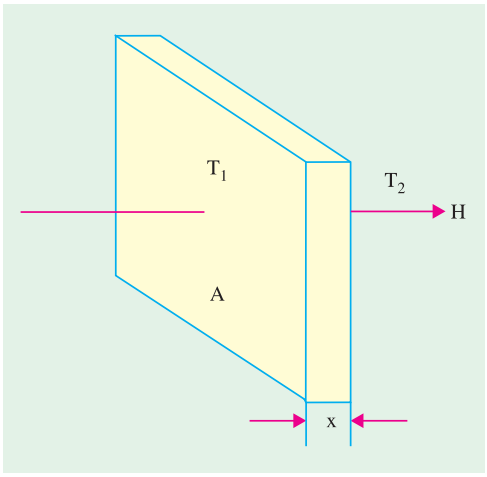


Fig. 47.1

2. Convection

In this process, heat is transferred by the flow of hot and cold air currents. This process is applied in the heating of water by immersion heater or heating of buildings. The quantity of heat absorbed by the body by convection process depends mainly on the temperature of the heating element above the surroundings and upon the size of the surface of the heater. It also depends, to some extent, on the position of the heater. The amount of heat dissipated is given by

$H = a(T_1 - T_2)$, where a and b are constants and T_1 and T_2 are the temperatures of the heating surface and the fluid in °K respectively.

In electric furnaces, heat transferred by convection is negligible.

3. Radiation

It is the transfer of heat from a hot body to a cold body in a straight line without affecting the intervening medium. The rate of heat emission is given by Stefan's law according to which

Heat dissipated, $H = 5.72\, eK\left[\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4\right]$ W/m^2

where K is radiating efficiency and e is known as emissivity of the heating element.

If d is the diameter of the heating wire and l its total length, then its surface area from which heat is radiated = $\pi \times d \times l$. If H is the power radiated per m^2 of the heating surface, then total power radiated as heat = $H \times \pi\, d\, l$. If P is the electrical power input to the heating element, then $P = \pi dl \times H$.

47.4. Methods of Electric Heating

Basically, heat is produced due to the circulation of current through a resistance. The current may circulate directly due to the application of potential difference or it may be due to induced eddy currents. Similarly, in magnetic materials, hysteresis losses are used to create heat. In dielectric heating, molecular friction is employed for heating the substance. An arc established between an electrode and the material to be heated can be made a source of heat. Bombarding the surface of material by high energy particles can be used to heat the body.



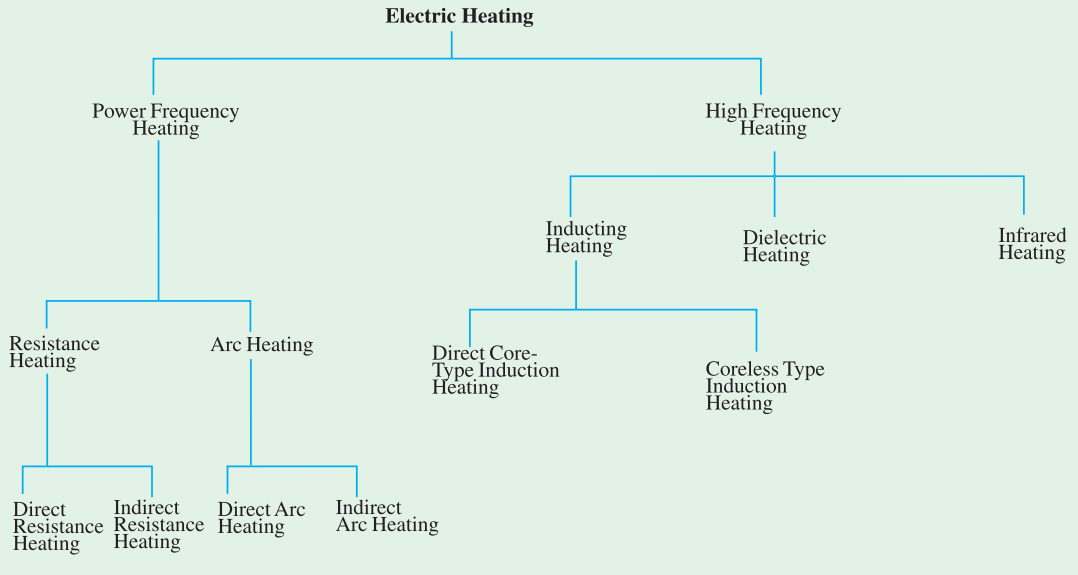
Room heater is a familiar appliance where electric heating is employed

Different methods of producing heat for general industrial and domestic purposes may be classified below :

47.5. Resistance Heating

It is based on the I^2R effect. When current is passed through a resistance element I^2R loss takes place which produces heat. There are two methods of resistance heating.

(*a*) Direct Resistance Heating. In this method the material (or charge) to be heated is treated as a resistance and current is passed through it. The charge may be in the form of powder, small solid pieces or liquid. The two electrodes are inserted in the charge and connected to either a.c. or d.c.



supply (Fig. 47.2). Obviously, two electrodes will be required in the case of d.c. or single-phase a.c. supply but there would be three electrodes in the case of 3-phase supply. When the charge is in the form of small pieces, a powder of high resistivity material is sprinkled over the surface of the charge to avoid direct short circuit. Heat is produced when current passes through it. This method of heating has high efficiency because the heat is produced in the charge itself.

(*b*) Indirect Resistance Heating. In this method of heating, electric current is passed through a resistance element which is placed in an electric oven. Heat produced is proportional to I^2R losses in the heating element. The heat so

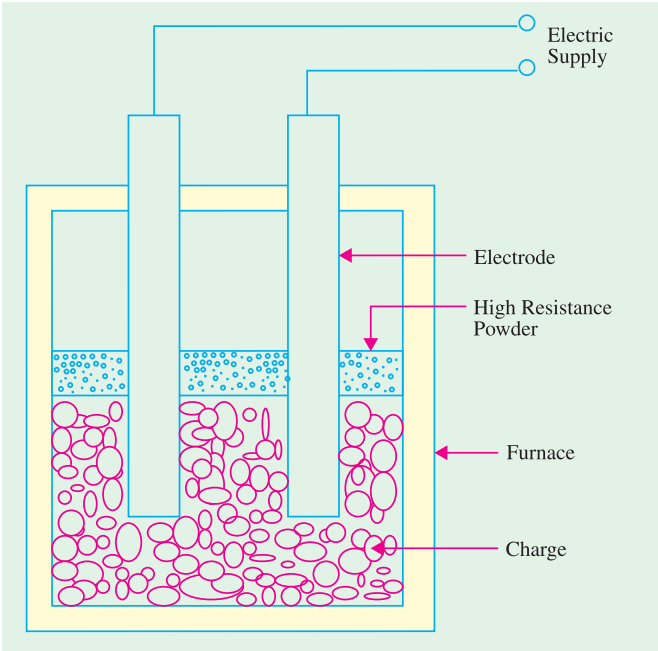


Fig. 47.2

produced is delivered to the charge either by radiation or convection or by a combination of the two.

Sometimes, resistance is placed in a cylinder which is surrounded by the charge placed in the jacket as shown in the Fig. 47.3. This arrangement provides uniform temperature. Moreover, automatic temperature control can also be provided.

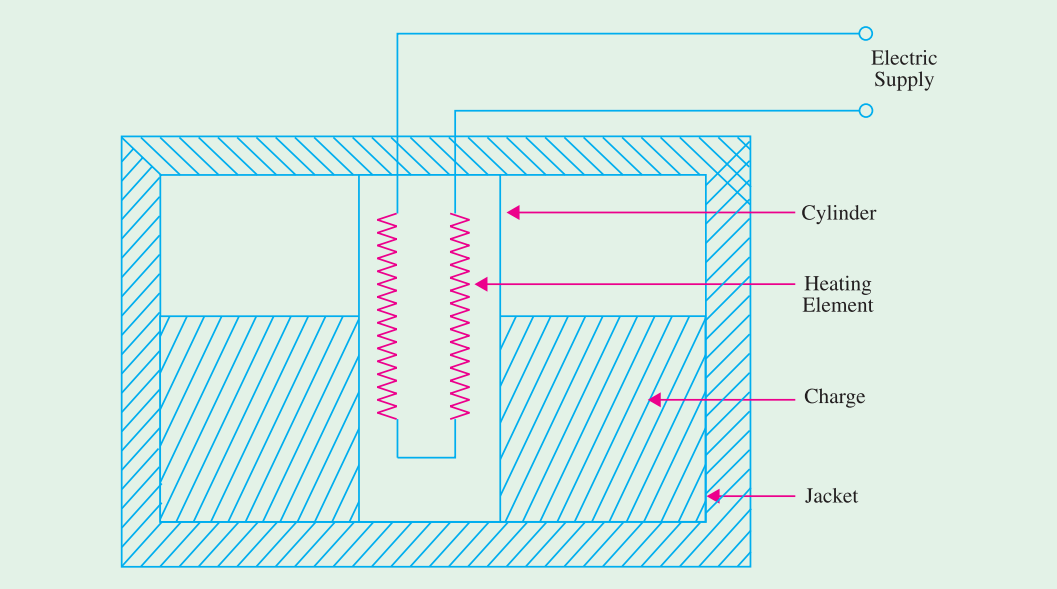


Fig. 47.3

47.6. Requirement of a Good Heating Element

Indirect resistance furnaces use many different types of heating elements for producing heat. A good heating element should have the following properties :

(1) High Specific Resistance. When specific resistance of the material of the wire is high, only short length of it will be required for a particular resistance (and hence heat) or for the same length of the wire and the currrent, heat produced will be more.

(2) High Melting Temperature. If the melting temperature of the heating element is high, it would be possible to obtain higher operating temperatures.

(3) Low Temperature Coefficient of Resistance. In case the material has low temperature coefficient of resistance, there would be only small variations in its resistance over its normal range of temperature. Hence, the current drawn by the heating element when cold (*i.e.*, at start) would be practically the same when it is hot.

(4) High Oxidising Temperature. Oxidisation temperature of the heating element should be high in order to ensure longer life.

(5) Positive Temperature Coefficient of Resistance. If the temperature coefficient of the resistance of heating element is negative, its resistance will decrease with rise in temperature and it will draw more current which will produce more wattage and hence heat. With more heat, the resistance will decrease further resulting in instability of operation.

(6) Ductile. Since the material of the heating elements has to have convenient shapes and sizes, it should have high ductility and flexibility.

(7) Mechanical Strength. The material of the heating element should posses high mechanical strength of its own. Usually, different types of alloys are used to get different operating temperatures. For example maximum working temperature of ***constant*** an (45% Ni, 55% Cu) is 400°C, that of

nichrome (50%, Ni 20% Cr) is 1150°C, that of ***Kantha*** (70% Fe, 25% Cr, 5% Al) is 1200° C and that of ***silicon carbide*** is 1450°C.

With the passage of time, every heating element breaks open and becomes unserviceable. Some of the factors responsible for its failure are :

(1) Formation of hot spots which shine brighter during operation, **(2)** Oxidation **(3)** Corrosion **(4)** Mechanical failure.

47.7. Resistance Furnaces or Ovens

These are suitably-insulated closed chambers with a provision for ventilation and are used for a wide variety of purposes including heat treatment of metals like annealing and hardening etc., stoving of enamelled wares, drying and baking of potteries, vulcanizing and hardening of synthetic materials and for commercial and domestic heating. Temperatures upto 1000°C can be obtained by using heating elements made of nickel, chromium and iron. Ovens using heating elements made of graphite can produce temperatures upto 3000°C. Heating elements may consist of circular wires or rectangular ribbons. The ovens are usually made of a metal framework having an internal lining of fire bricks. The heating element may be located on the top, bottom or sides of the oven. The nature of the insulating material is determined by the maximum temperature required in the oven.

An enclosure for charge which is heated by radiation or convection or both is called a ***heating chamber***.

47.8. Temperature Control of Resistance Furnaces

The temperature of a resistance furnace can be changed by controlling the I^2R or V^2/R losses.

Following different methods are used for the above purpose :

(1) Intermittent Switching. In this case, the furnace voltage is switched ON and OFF intermittently. When the voltage supply is switched off, heat production within the surface is stalled and hence its temperature is reduced. When the supply is restored, heat production starts and the furnace temperature begins to increase. Hence, by this simple method, the furnace temperature can be limited between two limits.



Electric resistance furnace

(2) By Changing the Number of Heating Elements. In this case, the number of heating elements is changed without cutting off the supply to the entire furnace. Smaller the number of heating elements, lesser the heat produced.

In the case of a 3-phase circuit, equal number of heating elements is switched off from each phase in order to maintain a balanced load condition.

(3) Variation in Circuit Configuration. In the case of 3-phase secondary load, the heating elements give less heat when connected in a star than when connected in delta because in the two cases, voltages across the elements is different (Fig. 47.4). In single-phase circuits, series and parallel grouping of the heating elements causes change in power dissipation resulting in change of furnace temperature.

As shown in Fig. 47.5 heat produced is more when all these elements are connected in parallel than when they are connected in series or series-parallel.

(4) Change of Applied Voltage. (*a*) Obviously, lesser the magnitude of the voltage applied to the load, lesser the power dissipated and hence, lesser the temperature produced. In the case of a furnace transformer having high voltage primary, the tapping control is kept in the primary winding because the magnitude of the primary current is less. Consider the multi-tap step-down transformer shown in Fig. 47.6.



Resistance heating tube furnace

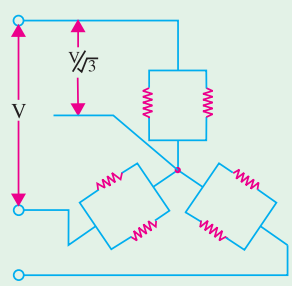
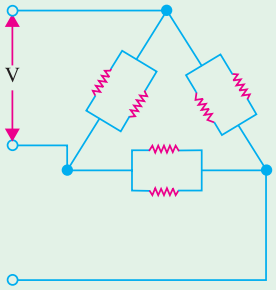


Fig. 47.4



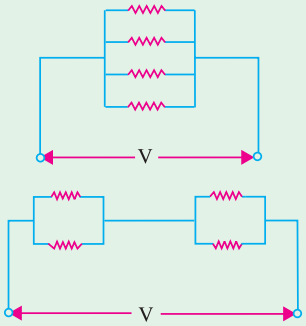


Fig. 47.5

Let the four tappings on the primary winding have 100%, 80%, 60% and 50%. When 100% primary turns are used, secondary voltage is given by $V_2 = (N_2/N_1)V_i$ where V_i is the input voltage. When 50% tapping is used, the number of primary turns involved is $N_1/2$. Hence, available secondary voltage $V_2 = (2N_2/N_1)V_i$. By selecting a suitable primary tapping, secondary voltage can be increased or decreased causing a change of temperature in the furnace.

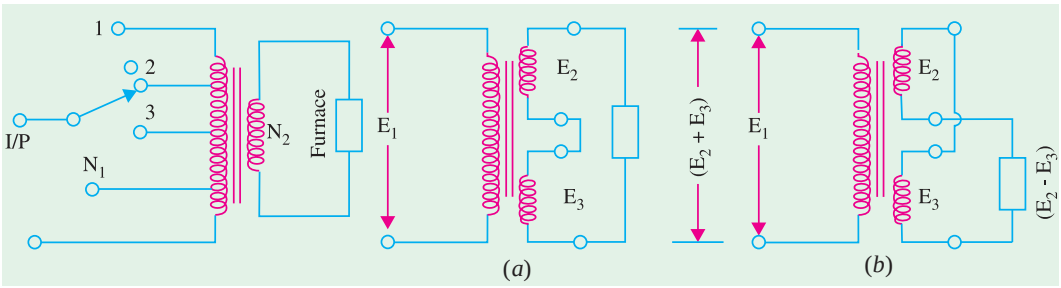


Fig. 47.6

Fig. 47.7

(*b*) Bucking-Boosting the Secondary Voltage. In this method, the transformer secondary is wound in two sections having unequal number of turns. If the two sections are connected in series-aiding, the secondary voltage is boosted *i.e.*, increased to $(E_2 + E_3)$ as shown in Fig. 47.7 (*a*).

When the two sections are connected in series-opposing [Fig. 47.7 (*b*)] the secondary voltage is reduced *i.e.*, there is bucking effect. Consequently, furnace voltage becomes $(E_2 - E_3)$ and, hence, furnace temperature is reduced.

(*c*) Autotransformer Control. Fig. 47.8 shows the use of tapped autotransformer used for decreasing the furnace voltage and, hence, temperature of small electric furnaces. The required voltage can be selected with the help of a voltage selector.

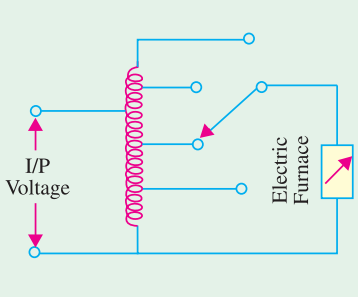


Fig. 47.8

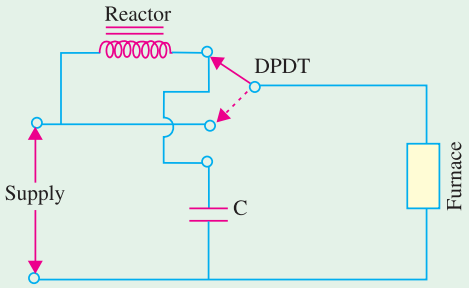


Fig. 47.9

(*d*) Series Reactor Voltage. In this case, a heavy-duty core-wounded coil is placed in series with the furnace as and when desired. Due to drop in voltage across the impedence of the coil, the voltage available across the furnace is reduced. With the help of D.P.D.T. switch, high/low, two-mode temperature control can be obtained as shown in the Fig. 47.9. Since the addition of series coil reduces the power factor, a power capacitor is simultaneously introduced in the circuit for keeping the p.f. nearly unity. As seen, the inductor is connected in series, whereas the capacitor is in parallel with the furnace.

47.9. Design of Heating Element

Normally, wires of circular cross-section or rectangular conducting ribbons are used as heating elements. Under steady-state conditions, a heating element dissipates as much heat from its surface as it receives the power from the electric supply. If P is the power input and H is the heat dissipated by radiation, then $P = H$ under steady-state conditions.

As per Stefan's law of radiation, heat radiated by a hot body is given by

$$H = 5.72\, eK\left[\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4\right] \text{W/m}^2$$

where T_1 is the temperature of hot body in °K and T_2 that of the cold body (or cold surroundings) in °K

$$\text{Now,} \quad P = \frac{V^2}{R}, \quad \text{and} \quad R = \rho\frac{l}{A} = \rho\frac{l}{\pi d^2/4} = \frac{4\rho l}{\pi d^2}$$

$$\therefore \quad P = \frac{V^2}{4\rho l/\pi d^2} = \frac{\pi d^2 V^2}{4\rho l} \quad or \quad \frac{l}{d^2} = \frac{\pi V^2}{4l\rho} \qquad \text{...(i)}$$

Total surface area of the wire of the element = $(\pi d) \times l$

If H is the heat dissipated by radiation per second per unit surface area of the wire, then heat radiated per second

$$= (\pi d) \times l \times H \qquad \text{...(ii)}$$

Equating **(*i*)** and **(*ii*)**, we have

$$P = (\pi d) \times l \times H \quad \text{or} \quad \frac{\pi d^2 V^2}{4\rho l} = (\pi d) \times H \text{ or } \frac{d}{l^2} = \frac{4\rho H}{V^2} \qquad \text{... (iii)}$$

We can find the values of l and d from Eq. **(*i*)** and **(*iii*)** given above.

Ribbon Type Element

If w is the width of the ribbon and t its thickness, then

$$P = \frac{V^2}{R} = \frac{V^2}{\rho l/A} = \frac{V^2}{\rho l/Twt} = \frac{wtV/^2}{\rho l} \quad or \quad \frac{t}{wt} = \frac{V^2}{\rho P} \qquad \text{...(iv)}$$

Heat lost from ribbon surface = $2wl\,H$ (neglecting the side area $2tl$)

$$\therefore \quad \frac{wtV^2}{\rho l}=2wlH \qquad \text{or} \qquad \frac{t}{l^2}=\frac{2\rho H}{V^2} \qquad \text{...(v)}$$

Values of l and w for a given ribbon of thickness t can be found from Eqn. **(iv)** and **(v)** given above.

Example 47.1. *A resistance oven employing nichrome wire is to be operated from 220 V single-phase supply and is to be rated at 16 kW. If the temperature of the element is to be limited to 1,170°C and average temperature of the charge is 500°C, find the diameter and length of the element wire.*

Radiating efficiency = 0.57, Emmissivity=0.9, Specific resistance of nichrome=(109 ×10^{-8}) ohm-m. **(Utili. of Elect. Energy, Punjab Univ.)**

Solution. P = 16 kW = 16,000 W

From Article 47.9, $\frac{l}{d^2}=\frac{\pi V^2}{4\rho P}=\frac{\pi\times(220)^2}{4\times109\times10^{-8}\times16,000}=2,179,660$ ... **(i)**

Now, $H=5.72eK\left[\left(\frac{T_1}{100}\right)^4-\left(\frac{T_2}{100}\right)^4\right]\ W/m^2=5.72\times0.9\times0.57\left[\left(\frac{1443}{100}\right)^4-\left(\frac{773}{100}\right)^4\right]$

= 116,752 W/m^2

Now, total heat dissipated/s = electrical power input

$\therefore \quad (\pi d)\times l\times116,752=16,000; \quad \therefore \quad dl=0.0436$

or $d^2 l^2 = 0.0019$ ...**(ii)**

From Eqn. (*i*) and (*ii*), l^3 = 2,179,660 × 0.0019 = 4141

$\therefore \quad l$ = **16.05 m**

d = 0.0436/16.05 = 2.716 × 10^{-3} m = **2.716 mm**

Example 47.2. *A 30-kW, 3-ϕ, 400-V resistance oven is to employ nickel-chrome strip 0.254 mm thick for the three star-connected heating elements. If the wire temperature is to be 1,100°C and that of the charge to be 700°C, estimate a suitable width for the strip. Assume emissivity = 0.9 and radiating efficiency to be 0.5 and resistivity of the strip material is 101.6 × 10^{-8} Ω - m. What would be the temperature of the wire if the charge were cold ?* **(Utili. of Elect. Power A.M.I.E. Sec. B)**

Solution. Power/phase = 30 × 1000/3 = 10,000 W, $V_{ph}=400/\sqrt{3}=231$V

If R is the resistance of the strip, $R = V_{ph}^2/P = 231^2/10,000 = 5.34\ \Omega$

Resistance of the strip, $R=\frac{\rho l}{wt}$ *or* $\frac{l}{w}=\frac{5.34\times0.245\times10^{-3}}{101.6\times10^{-8}}=1335$ ...**(i)**

Heat dissipated from surface of the strip,

$$H=5.72\times0.9\times0.5\left[\left(\frac{1373}{100}\right)^4-\left(\frac{973}{100}\right)^4\right]=68,400\ \text{W/m}^2$$

Surface area of the strip = $2wl$; Total heat dissipated = $2wl \times H$

$\therefore$ 68,400 × 2 × wl = 10,000 or wl = 0.0731 ...**(ii)**

From Eqn. **(i)** and **(ii)**, we get w = 0.0731/1335 or w = **7.4 mm**

Example 47.3. *A cubic water tank has surface area of 6.0 m^2 and is filled to 90% capacity six times daily. The water is heated from 20°C to 65°C. The losses per square metre of tank surface per 1°C temperature difference are 6.3 W. Find the loading in kW and the efficiency of the tank. Assume specific heat of water = 4,200 J/kg/°C and one kWh = 3.6 MJ.* **(A.M.I.E. Sec. B)**

Solution. If l is the side of the tank, then total surface area of the tank = $6l^2$

$\therefore \quad 6l^2 = 6 \qquad$ or $\qquad l = 6/6 = 1\ m^2$

Volume of the tank = $l^3 = 1 m^3$

Volume of water to be heated daily = $6 \times (1 \times 0.9) = 5.4\ m^3$

Since $1 m^3$ of water weighs 1000 kg, mass of water to be heated daily = $5.4 \times 1000 = 5400$ kg

Heat required to raise the temperature of water = $5400 \times 4200\ (65 - 20) = 1020$ MJ = $1020/3.6 = 283.3$ kWh

Daily loss from the surface of the tank = $6.3 \times 6 \times (65 - 20) \times 24/1000 = 40.8$ kWh

Energy supplied per day = 283.3 + 40.8 = 324.1 kWh

Loading in kW = 324.1/24 = **3.5 kW**

Efficiency of the tank = $283.3 \times 100/324.1 = 87.4\%$

47.10. Arc Furnaces

If a sufficiently high voltage is applied across an air-gap, the air becomes ionized and starts conducting in the form of a continuous spark or arc thereby producing intense heat. When electrodes are made of carbon/graphite, the temperature obtained is in the range of 3000°C-3500C. The high voltage required for striking the arc can be obtained by using a step-up transformer fed from a variable a.c. supply as shown in Fig. 47.10 (*a*).

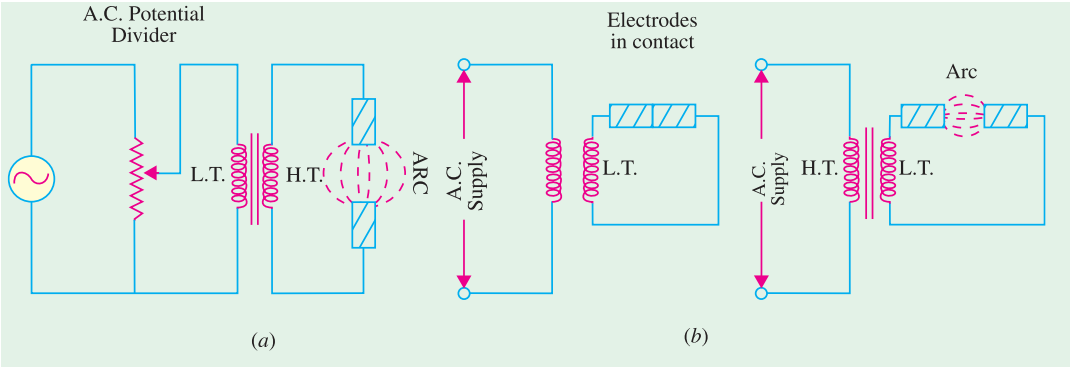


Fig. 47.10

An arc can also be obtained by using low voltage across two electrodes initially in contact with each other as shown in Fig. 47.10 (*b*). The low voltage required for this purpose can be obtained by using a step-down transformer. Initially, the low voltage is applied, when the two electrodes are in contact with each other. Next, when the two electrodes are gradually separated from each other, an arc is established between the two.

Arc furnaces can be of the following two types :

1. Direct Arc Furnace

In this case, arc is formed between the two electrodes and the charge in such a way that electric current passes through the body of the charge as shown in Fig. 47.11 (*a*). Such furnaces produce very high temperatures.

2. Indirect Arc Furnace

In this case, arc is formed between the two electrodes and the heat thus produced is passed on to the charge by radiation as shown in Fig. 47.11 (*b*).

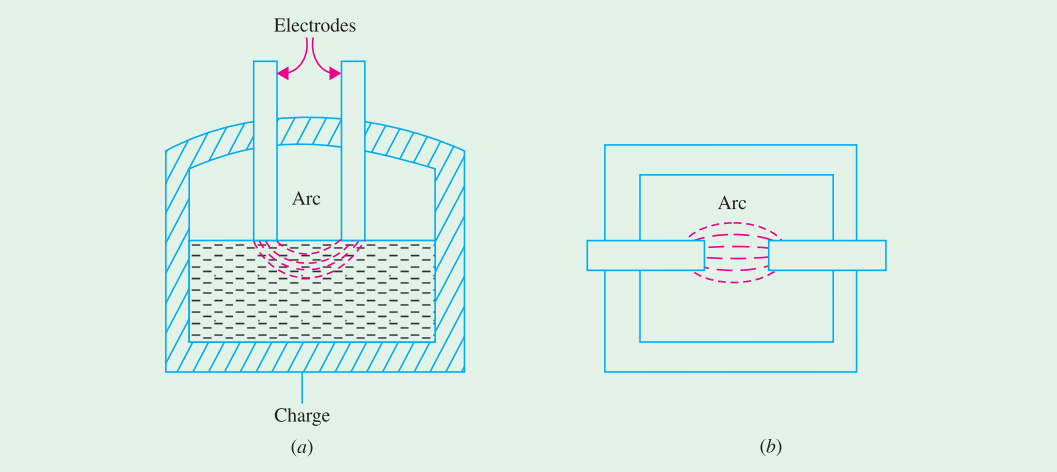


Fig. 47.11

47.11. Direct Arc Furnace

It could be either of conducting-bottom type [Fig. 47.12 (*a*)] or non-conducting bottom type [Fig. 47.12 (*b*)].

As seen from Fig. 47.12 (*a*), bottom of the furnace forms part of the electric circuit so that current passes through the body of the charge which offers very low resistance. Hence, it is possible

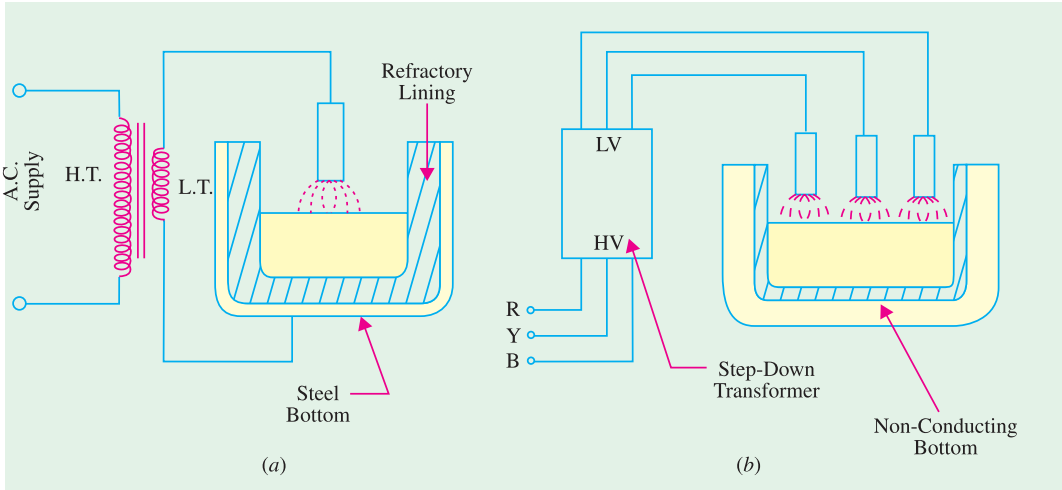


Fig. 47.12

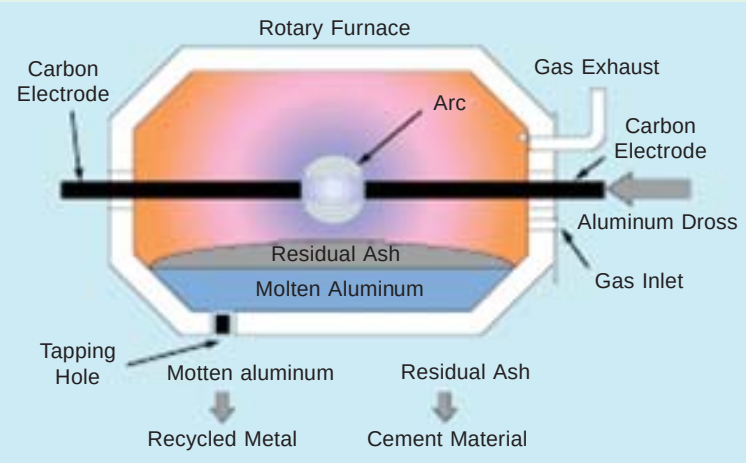
to obtain high temperatures in such furnaces. Moreover, it produces uniform heating of charge without stirring it mechanically. In Fig. 47.12 (*b*), no current passes through the body of the furnace.

Most common application of these furnaces is in the production of steel because of the ease with which the composition of the final product can be controlled during refining.

Most of the furnaces in general use are of non-conducting bottom type due to insulation problem faced in case of conducting bottom.

47.12. Indirect Arc Furnace

Fig. 47.13 shows a single-phase indirect arc furnace which is cylindrical in shape. The arc is struck by short-circuiting the electrodes manually or automatically for a moment and then, withdrawing them apart. The heat from the arc and the hot refractory lining is transferred to the top layer of the charge by radiation. The heat from the hot top layer of the charge is further transferred to other parts of the charge by conduction.



Schematic view of an indirect electric arc furnace

Since no current passes through the body of the charge, there is no inherent stirring action due to electro-magnetic forces set up by the current. Hence, such furnaces have to be rocked continuously in order to distribute heat uniformly by exposing different layers of the charge to the heat of the arc. An electric motor is used to operate suitable grinders and rollers to impart rocking motion to the furnace. Rocking action provides not only thorough mixing of the charge, it also increases the furnace efficiency in addition to increasing the life of the refractory lining material. Since in this furnace, charge is heated by radiation only, its temperature is lower than that obtainable in a direct arc furnace. Such furnaces are mainly used for melting non-ferrous metals although they can be used in iron foundaries where small quantities of iron are required frequently.

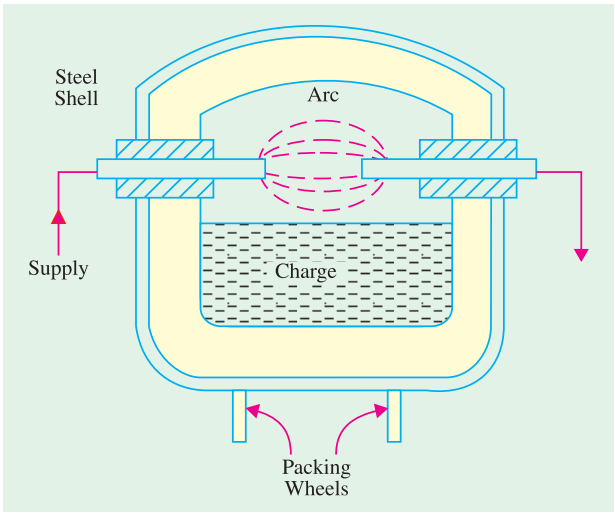


Fig. 47.13

Example 47.4. *A 4-phase electric arc furnace has the following data :*

Current drawn = 5000 A ; Arc voltage = 50 V
Resistance of transformer referred to secondary = 0.002 Ω
Resistance of transformer referred to secondary = 0.004 Ω

(i) *Calculate the power factor and kW drawn from the supply.*

(ii) *If the overall efficiency of the furnace is 65%, find the time required to melt 2 tonnes of steel if latent heat of steel = 8.89 kcal/kg, specific heat of steel = 0.12, melting point of steel = 1370°C and initial temperature of steel = 20°C.*

(Utilisation of Elect. Power, A.M.I.E. Sec. B)

Solution. Voltage drop due to transformer resistance = 5000 × 0.002 = 10 V
Voltage drop due to transformer reactance = 5000 × 0.004 = 20 V.

Since arc voltage drop is resistive in nature, it is vectorially added to the transformer resistance drop.

Open circuit transformer secondary voltage/phase = $\sqrt{(50+10)^2+20^2} = 63.25\,V$

(i) Supply p.f.= (50 + 10)/63.25 = **0.9487**

Power drawn/phase by the secondary = 5000 × 63.25 × 0.9487 = 300,000 W = 300 kW

Total power drawn from the supply = 3 × 300 = **900 kW**

(ii) Energy required to melt 2 tonnes of steel = $ms\,(t_2 - t_1)$ + mL = 2000 × 0.12 × (1370 – 20) + 2000 × 8.89 = 341,780 kcal.= 341,780/860 = 397.4 kWh

Power actually utilised = 900 × 0.65 = 585 kW

Time required for melting steel = 397.4/585 = 0.679 hours = **40 minutes 46 seconds.**

Example. 47.5. *If a 3-phase arc furnace is to melt 10 tonne steel in 2 hours, estimate the average input to the furnace if overall efficiency is 50%. If the current input is 9,000 A with the above kW input and the resistance and reactance of furnace leads (including transformer) are 0.003 Ω and 0.005 Ω respectively, estimate the arc voltage and total kVA taken from the supply*
Specific heat of steel = 444 J $kg^{-1}°C^{-1}$
Latent heat of fusion of steel = 37.25 kJ/kg
Melting point of steel = 1,370 °C **(Utilisation of Elect. Energy, Punjab Univ. 1989.)**

Solution. Energy required to melt 10 tonnes of steel = 10,000 × 444 (1370 – 20) + 10,000 × 37,250 = 6366.5 × 106 J = 1768.5 kWh

It has been assumed in the above calculation that the initial melting temperature of steel is 20°C.

Since time taken is two hours, average output power = 1768.5/2 = 884.25 kW

Average input power = 884.25/0.5=1768.5 kW

Voltage drop due to the resistance of the furnace leads (including transformer) = 9000 × 0.003 = 27 V

Voltage drop due to reactance of the furnace leads (including transformer) = 9000 × 0.005 = 45 V

If VA is the arc drop (which is assumed resistive in nature) then

O.D. secondary voltage/phase $= \sqrt{(V_A+27)^2+45^2}$

$$P.F. = \frac{V_A+27}{(V_A+27)^2+(45)^2}$$

Total power input = 3 × power drawn/phase

$\therefore \quad 1768.5 \times 10^3 = 3 \times 9000\ \sqrt{(V_A+27)^2+(35)^2} \times (V_A+27)/\sqrt{(V_A+27)^2+(45)^2}$

$\therefore \quad V_A + 27 = 65.5 \quad$ or $\quad V_A = 65.5 - 27 =$ **38.5 V**

O.C. secondary voltage/phase = $\sqrt{(V_A+27)^2 + (45)^2} = 79.5\,V$

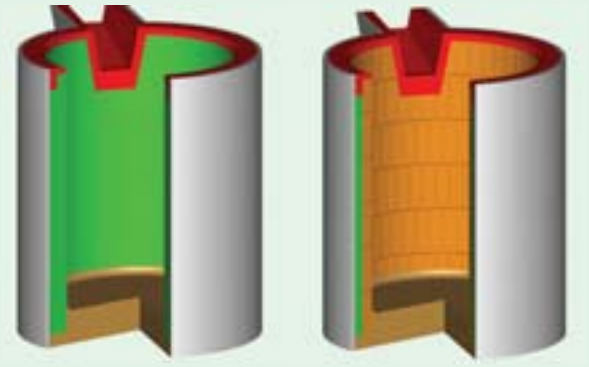
Total kVA taken from supply line = 3 × 9000 × 79.5 × 10^{-3} = **2145 kVA.**

47.13. Induction Heating

This heating process makes use of the currents induced by the electro-magnetic action in the charge to be heated. In fact, induction heating is based on the principle of transformer working. The primary winding which is supplied from an a.c. source is magnetically coupled to the charge which acts as a short circuited secondary of single turn. When an a.c. voltage is applied to the primary, it induces voltage in the secondary *i.e.* charge. The secondary current heats up the charge in the same way as any electric current does while passing through a resistance. If V is the voltage induced in the charge and R is the charge resistance, then heat produced = V^2/R. The value of current induced in the charge depends on ***(i)*** magnitude of the primary current ***(ii)*** turn ratio of the transformer

(iii) co-efficient of magnetic coupling.

Low-frequency induction furnaces are used for melting and refining of different metals. However, for other processes like case hardenning and soldering etc., high-frequency eddy-current heating is employed. Low-frequency induction furnaces employed for the melting of metals are of the following two types :



(*a*) (*b*)

Inside views of an induction furnaces: (*a*) Rammed induction furnace and (*b*) bricked induction furnace

(*a*) Core-type Furnaces — which operate just like a two winding transformer. These can be further sub-divided into ***(i)*** Direct core-type furnaces ***(ii)*** Vertical core-type furnaces and ***(iii)*** Indirect core-type furnaces.

(*b*) Coreless-type Furnaces — in which an inductively-heated element is made to transfer heat to the charge by radiation.

47.14. Core Type Induction Furnace

It is shown in Fig. 47.14 and is essentially a transformer in which the charge to be heated forms a single-turn short-circuited secondary and is magnetically coupled to the primary by an iron core. The furnace consists of a circular hearth which contains the charge to be melted in the form of an annular ring. When there is no molten metal in the ring, the secondary becomes open-circuited there-by cutting off the secondary current. Hence, to start the furnace, molted metal has to be poured in the annular hearth. Since, magnetic coupling between the primary and secondary is very poor, it results in high leakage and low power factor. In order to nullify the effect of increased leakage reactance, low primary frequency of the order of 10 Hz is used. If the transformer secondary current density exceeds 500 A/cm^2 then, due to the interaction of secondary current with the alternating magnetic field, the molten metal is squeezed to the extent that secondary circuit is interrupted. This effect is known as "pinch effect".

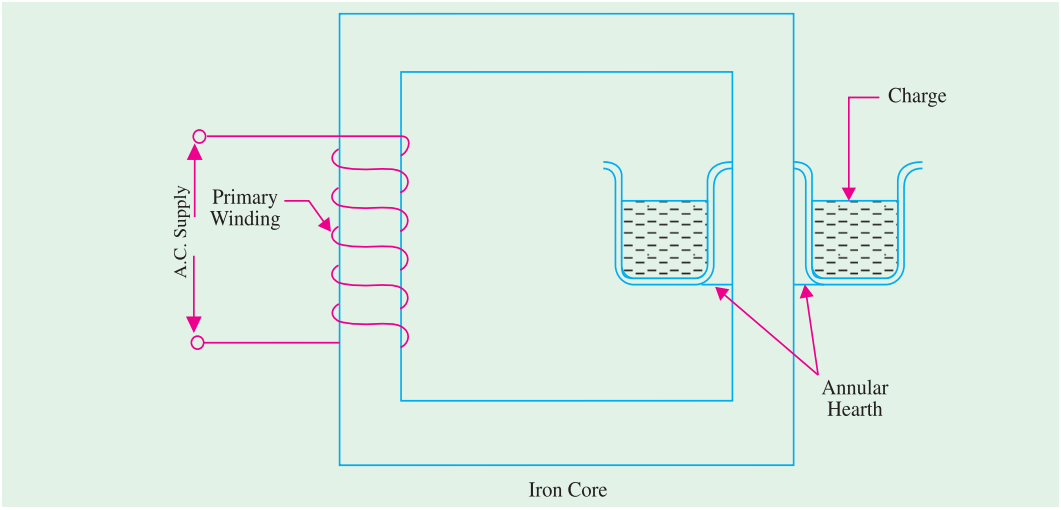


Fig. 47.14

This furnace suffers from the following drawbacks :

1. It has to be run on low-frequency supply which entails extra expenditure on motor-generator set or frequency convertor.
2. It suffers from pinching effect.
3. The crucible for charge is of odd shape and is very inconvenient for tapping the molten charge.
4. It does not function if there is no molten metal in the hearth *i.e.* when the secondary is open. Every time molten metal has to be poured to start the furnace.
5. It is not suitable for intermittent service.

However, in this furnace, melting is rapid and clean and temperature can be controlled easily. Moreover, inherent stirring action of the charge by electro-magnetic forces ensures greater uniformity of the end product.

47.15. Vertical Core-Type Induction Furnace

It is also known as Ajax-Wyatt furnace and represents an improvement over the core-type furnace discussed above. As shown in Fig. 47.15, it has vertical channel (instead of a horizontal one) for the charge, so that the crucible used is also vertical which is convenient from metallurgical point of view. In this furnace, magnetic coupling is comparatively better and power factor is high. Hence, it can be operated from normal frequency supply. The circulation of the molten metal is kept up round the Vee portion by convection currents as shown in Fig. 47.15.

As Vee channel is narrow, even a small quantity of charge is sufficient to keep the secondary circuit closed. However, Vee channel must be kept full of charge in order to maintain continuity of secondary circuit. This fact makes this furnace suitable for continuous operation. The tendency of the secondary circuit to rupture due to pinch-effect is counteracted by the weight of the charge in the crucible.

The choice of material for inner lining of the furnace depends on the type of charge used. Clay lining is used for yellow brass. For red brass and bronze, an alloy of magnetia and alumina or corundum is used. The top of the furnace is covered with an insulated cover which can be removed for charging. The furnace can be tilted by the suitable hydraulic arrangement for taking out the molten metal.

This furnace is widely used for melting and refining of brass and other non-ferrous metals. As said earlier, it is suitable for continuous operation. It has a p.f. of 0.8-0.85. With normal supply frequency, its efficiency is about 75% and its standard size varies from 60-300 kW, all single-phase.

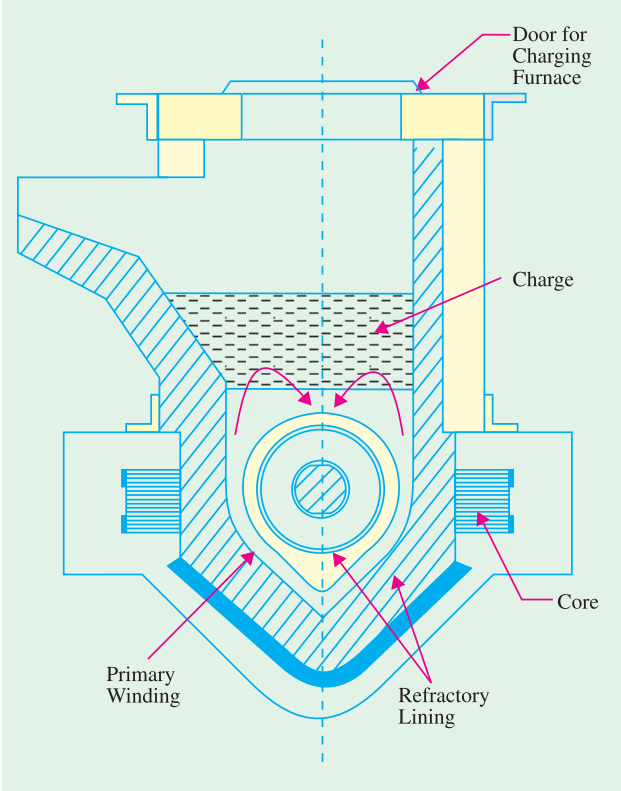


Fig. 47.15

47.16. Indirect Core-Type Induction Furnace

In this furnace, a suitable element is heated by induction which, in turn, transfers the heat to the charge by radiation. So far as the charge is concerned, the conditions are similar to those in a resistance oven.

As shown in Fig. 47.16, the secondary consists of a metal container which forms the walls of the furnace proper. The primary winding is magnetically coupled to this secondary by an iron core. When primary winding is connected to a.c. supply, secondary current is induced in the metal container by transformer action which heats up the container. The metal container transfers this heat to the charge. A special advantage of this furnace is that its temperature can be automatically controlled without the use of an external equipment. The part AB of the magnetic circuit situated inside the oven chamber consists of a special alloy which loses its magnetic properties at a particular temperature but regains them when cooled back to the same temperature. As soon as the chamber attains the critical temperature, reluctance of the magnetic circuit increases manifold thereby cutting off the

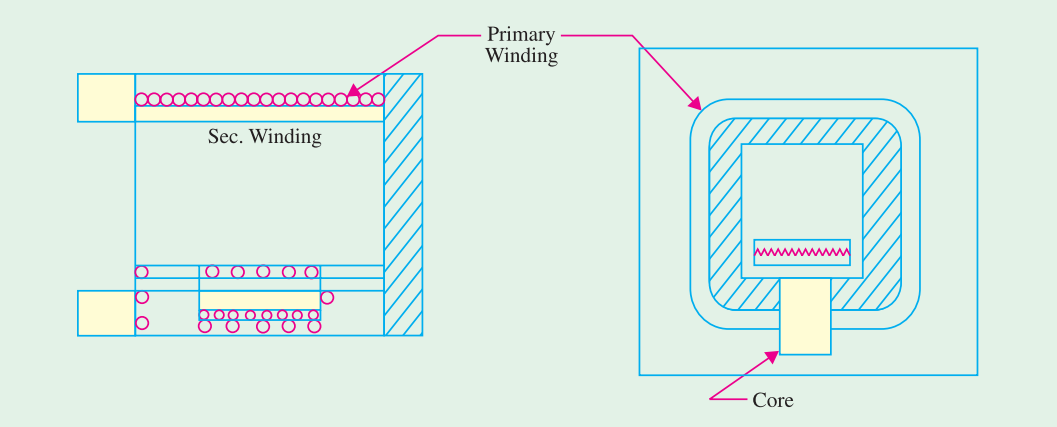


Fig. 47.16

heat supply. The bar AB is detachable and can be replaced by other bars having different critical temperatures.

47.17. Coreless Induction Furnace

As shown in Fig. 47.17, the three main parts of the furnace are ***(i)*** primary coil ***(ii)*** a ceramic crucible containing charge which forms the secondary and ***(iii)*** the frame which includes supports and tilting mechanism. The distinctive feature of this furnace is that it contains no heavy iron core with the result that there is no continuous path for the magnetic flux. The crucible and the coil are relatively light in construction and can be conveniently tilted for pouring.

The charge is put into the crucible and primary winding is connected to a high-frequency a.c. supply. The flux produce by the primary sets up eddy-currents in the charge and heats it up to the melting point. The charge need not be in the molten state at the start as was required by core-type furnaces. The eddy- currents also set up electromotive forces which produce stirring action which is essential for obtaining uniforms quality of metal. Since flux density is low (due to the absence of the magntic core) high frequency supply has to be used because eddy-current loss $W_e \propto B^2 f^2$. However, this high frequency increases the resistance of the primary winding due to skin effect, thereby increasing primary Cu losses. Hence, the primary winding is not made of Cu wire but consists of hollow Cu tubes which are cooled by water circulating through them.

Since magnetic coupling between the primary and secondary windings is low, the furnace p.f. lies between 0.1 and 0.3. Hence, static capacitors are invariably used in parallel with the furnace to improve its p.f.

Such furnaces are commonly used for steel production and for melting of non-ferrous metals like brass, bronze, copper and aluminum etc., along with various alloys of these elements. Special application of these furnaces include vacuum melting, melting in a controlled atmosphere and melting for precision casting where high frequency induction heating is used. It also finds wide use in electronic industry and in other industrial activities like soldering, brazing hardening and annealing and sterilizing surgical instruments etc.

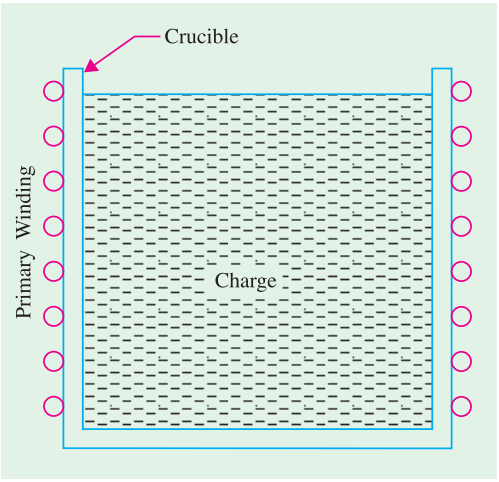


Fig. 47.17

Some of the advantages of coreless induction furnaces are as follows :

(1) They are fast in operation.

(2) They produce most uniform quality of product.

(3) They can be operated intermittenly.

(4) Their operation is free from smoke, dirt, dust and noises.

(5) They can be used for all industrial applications requiring heating and melting.

(6) They have low erection and operating costs.

(7) Their charging and pouring is simple.

47.18. High Frequency Eddy-current Heating

For heating an article by eddy-currents, it is placed inside a high frequency a.c. current-carrying coil (Fig. 47.18). The alternating magnetic field produced by the coil sets up eddy-currents in the article which, consequently, gets heated up. Such a coil is known as heater coil or work coil and the material to be heated is known as ***charge or load***. Primarily, it is the eddy-current loss which is responsible for the production of heat although hysteresis loss also contributes to some extent in the case of non-magnetic materials.

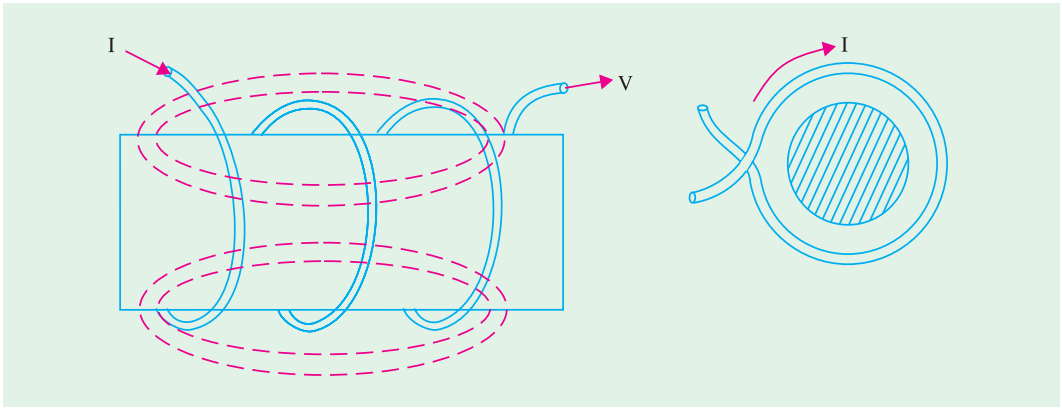


Fig. 47.18

The eddy-current loss $W_e \propto B^2 f^2$. Hence, this loss can be controlled by controlling flux density B and the supply frequency f. This loss is greatest on the surface of the material but decreases as we go deep inside. The depth of the material upto which the eddy-current loss penetrates is given by

$$d = \frac{1}{2\pi}\sqrt{\frac{\rho \times 10^9}{\mu_r . f}}$$

where ρ = resistivity of the molten metal

f = supply frequency

μ_r = relative permeability of the charge



Eddy Current Heater

Advantages of Eddy-current Heating

(1) There is negligible wastage of heat because the heat is produced in the body to be heated.

(2) It can take place in vacuum or other special environs where other types of heating are not possible.

(3) Heat can be made to penetrate any depth of the body by selecting proper supply frequently.

Applications of Eddy-current Heating

(1) Surface Hardening. The bar whose surface is to be hardened by heat treatment is placed within the working coil which is connected to an a.c. supply of high frequency. The depth upto which the surface is to be hardened can be obtained by the proper selection of frequency of the coil current. After a few seconds, when surface has reached the proper temperature, a.c. supply is cut off and the bar is at once dipped in water.

(2) Annealing. Normally, annealing process takes long time resulting in scaling of the metal which is undesirable. However, in eddy-current heating, time taken is much less so that no scale formation takes place.

(3) Soldering. Eddy-current heating is economical for precise high-temperature soldering where silver, copper and their alloys are used as solders.

Example. 47.6. *Determine the efficiency of a high-frequency induction furnace which takes 10 minutes to melt 2 kg of a aluminium initially at a temperature of 20°C. The power drawn by the furnace is 5 kW, specific heat of aluminium = 0.212, melting point of alumimium = 660° C and latent heat of fusion of alumimium. = 77 kcal/kg.*

Solution. Heat required to melt aluminium = 2 × 77 = 154 kcal

Heat required to raise the temperature of aluminium from 20°C to 660°C

= 2 × 0.212 × (660 – 20) = 2 × 0.212 × 640 = 271.4 kcal

Total heat required = 154 + 271.4 = 425.4 kcal

Heat required per hour = 425.4 × 60/10 = 2552.4 kcal

Power delivered to aluminium = 2552.4/860 = 2.96 kW

∴ efficiency = output/input = 2.97/5 = **0.594 or 59.4%**

Example 47.7. *A low-frequency induction furnace has a secondary voltage of 20V and takes 600 kW at 0.6 p.f. when the hearth is full. If the secondary voltage is kept constant, determine the power absorbed and the p.f. when the hearth is half-full. Assume that the resistance of the secondary circuit is doubled but the reactance remains the same.*

Solution. Secondary current = $600 \times 10^3/20 \times 0.6 = 5 \times 10^4$ A

If this current is taken as the reference quantity, then secondary voltage is

$V_2 = 20(0.6 + j\,0.8) = (12 + j16)$V

Hence, secondary impedence, $Z_2 = (12 + j\,16)/5 \times 10^4 = (2.4 + j\,3.2) \times 10^{-4}$ ohm

Now, if the secondary resistance is double, then total impedence when the hearth is half-full is

$$= Z_2 = (4.8 + j3.2) \times 10^{-4} \text{ ohm}$$

Now, secondary current I_2 $= 20/(4.8 + j3.2) \times 10^{-4}$

$$= 20/5.77 \angle 33.7° \times 10^4 = 3.466 \angle -33.7° \times 10^4 \text{ A}$$

Now p.f. = cos 33.7° = 0.832

Hence, power absorbed = $20 \times 3.466 \times 10^4 \times 0.832 \times 10^{-3}$ = **580 kW**

Example 47.8. *Estimate the energy required to melt 0.5 tonne of brass in a single-phase induction furnace. If the melt is to be carried out in 0.5 hour, what must be the average power input to the furnace?*

Specific heat of brass = *0.094*
Latent heat of fusion of brass = *30 kilocal/kg*
Melting point of brass = *920°C*
Furnace efficiency = *60.2%*
The temperature of the cold charge may be taken as 20°C

Solution. Total amount of heat required to melt 0.5 kg of brass.

$$= (0.5 \times 1000) \times 39 + 500 \times 0.094 \times (920 — 20)$$

$$= 61{,}800 \text{ kcal} = 61{,}800/860 = 71.86 \text{ kWh}$$

Total furnace input = 71.86/0.602 = **119.4 kWh**

Example 47.9. *A low-frequency induction furnace whose secondary voltage is maintained constant at 10 V, takes 400 kW at 0.6 p.f. when the hearth is full. Assuming the resistance of the secondary circuit to vary inversely as the height of the charge and reactance to remain constant, find the height upto which the hearth should be filled to obtain maximum heat.*

(Utili. of Elect. Power and Traction, Gorakhpur Univ.)

Solution. Secondary current $I_2 = P/V_2 \cos \phi$

$$= 400 \times 10^3/10 \times 0.6 = 6.667 \times 10^4 \text{ A}$$

Impedence of the secondary circuit when hearth is full

$$Z_2 = V_2/I_2 = 10/6.667 \times 10^4 = 1.5 \times 10^{-4} \ \Omega$$

Secondary resistance when hearth is full, $R_2 = Z_2 \cos \phi$

$$= 1.5 \times 10^{-4} \times 0.6 = 0.9 \times 10^{-4} \ \Omega$$

Reactance of the secondary circuit, $X_2 = Z_2 \sin \phi$

$$= 1.5 \times 10^{-4} \times 0.8 = 1.2 \times 10^{-4} \ \Omega$$

In the second, let the height of the charge be x times of the full hearth *i.e.* $h = x$H

Since resistance varies inversely as the height of the charge

$$= R_2 = R_2/x = 0.9 \times 10^{-4}/x \ \Omega$$

Power drawn and hence heat produced will be maximum where secondary resistance equals its reactance.

$$\therefore \quad 0.9 \times 10^{-4}/x = 1.2 \times 10^{-4} \quad \text{or} \quad x = \mathbf{3/4}$$

Hence, maximum heat would be produced in the charge when its height is three-fourth the height of the hearth.

47.19. Dielectric Heating

It is also called high-frequency capacitative heating and is used for heating insulators like wood, plastics and ceramics etc. which cannot be heated easily and uniformly by other methods. The supply frequency required for dielectric heating is between 10-50 MHz and the applied voltage is upto 20 kV. The overall efficiency of dielectric heating is about 50%.

47.20. Dielectric Loss

When a practical capacitor is connected across an a.c. supply, it draws a current which leads the voltage by an angle ϕ, which is a little less than 90° or falls short of 90° by an angle δ. It means that there is a certain component of the current which is in phase with the voltage and hence produces some loss called dielectric loss. At the normal supply frequency of 50 Hz, this loss is negligibly small but at higher frequencies of 50 MHz or so, this loss becomes so large that it is sufficient to heat the dielectric in which it takes place. The insulating material to be heated is placed between two conducting plates in order to form a parallel-plate capacitor as shown in Fig. 47.19 (*a*).

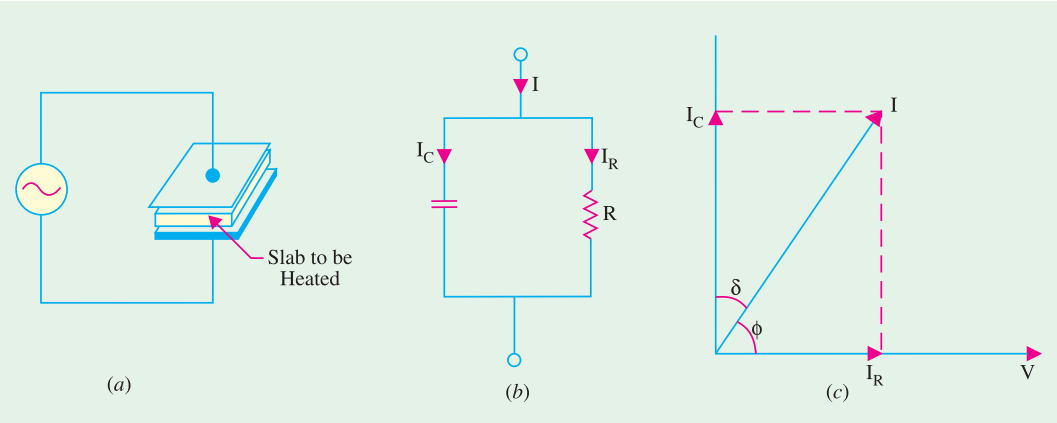


Fig. 47.19

Fig. 47.19 (*b*) shows the equivalent circuit of the capacitor and Fig. 47.19 (*c*) gives its vector diagram.

Power drawn from supply $= VI \cos \phi$

Now, $I_c = I = V/X_c = 2\pi f CV$

$\therefore$ $P = V(2\pi fCV) \cos \phi = 2\pi fCV^2 \cos \phi$

Now, $\phi = (90 - \delta)$, $\cos \phi = \cos (90 - \delta) = \sin \delta = \tan \delta = \delta$

where δ is very small and is expressed in radians.

$$P = 2\pi fCV^2 \delta \text{ watts}$$

Here, $C = \in_0 \in_r \frac{A}{d}$ where d is the thickness and A is the surface area of the dielectric slab.

This power is converted into heat. Since for a given insulator material, C and δ are constant, the dielectric loss is directly proportional to $V^2 f$. That is why high-frequency voltage is used in dielectric heating. Generally, a.c. voltage of about 20 kV at a frequency of 10-30 MHz is used.

47.21. Advantages of Dielectric Heating

1. Since heat is generated within the dielectric medium itself, it results in uniform heating.
2. Heating becomes faster with increasing frequency.
3. It is the only method for heating bad conductors of heat.
4. Heating is fastest in this method of heating.
5. Since no naked flame appears in the process, inflammable articles like plastics and wooden products etc., can be heated safely.
6. Heating can be stopped immediately as and when desired.

47.22. Applications of Dielectric Heating

Since cost of dielectric heating is very high, it is employed where other methods are not possible or are too slow. Some of the applications of dielectric heating are as under :



Dielectric heating

1. For gluing of multilayer plywood boards.
2. For baking of sand cores which are used in the moulding process.
3. For preheating of plastic compounds before sending them to the moulding section.
4. For drying of tobacco after glycerine has been mixed with it for making cigarettes.
5. For baking of biscuits and cakes etc. in bakeries with the help of automatic machines.
6. For electronic sewing of plastic garments like raincoats etc. with the help of cold rollers fed with high-frequency supply.
7. For dehydration of food which is then sealed in air-tight containers.
8. For removal of moistures from oil emulsions.
9. In diathermy for relieving pain in different parts of the human body.
10. For quick drying of glue used for book binding purposes.

Example. 47.10. *A slab of insulating material 150 cm^2 in area and 1 cm thick is to be heated by dielectric heating. The power required is 400 W at 30 MHz. Material has relative permittivity of 5 and p.f. of 0.05. Determine the necessary voltage. Absolute permittivity = 8.854×10^{-12} F/m.* **(Utilisation of Elect. Energy, Punjab Univ.)**

Solution. $P = 400\ W$, p.f. = 0.05, $f = 30 \times 10^6$ Hz

$C = \in_0 \in_r A/d = 8.854 \times 10^{-12} \times 5 \times 150 \times 10^{-4}/1 \times 10^{-2} = 66.4 \times 10^{-12}$ F

Now, $P = 2\pi f CV^2 \cos\phi$ or $400 = 2\pi \times 30 \times 10^6 \times 66.4 \times 10^{-12} \times V^2 \times 0.05$ or $V =$ **800 V**

Example. 47.11. *An insulating material 2 cm thick and 200 cm^2 in area is to be heated by dielectric heating. The material has relative permitivity of 5 and power factor of 0.05. Power required is 400 W and frequency of 40 MHz is to be used. Determine the necessary voltage and the current that will flow through the material. If the voltage were to be limited to 700 V, what will be the frequency to get the same loss?* **(A.M.I.E. Sec. B)**

Solution. $C = 8.854 \times 10^{-12} \times 5 \times 200 \times 10^{-4}/2 \times 10^{-2} = 44.27 \times 10^{-12}$ F

$$P = 2\pi fCV^2 \cos\phi \quad \text{or} \quad V = \sqrt{400/2\pi \times 40 \times 10^6 \times 44.27 \times 10^{-12} \times 0.05} = \textbf{848 V}$$

Current flowing through the material,

$$I = P/V \cos\phi = 400/848 \times 0.05 = \textbf{9.48 A}$$

Heat produced $\propto V^2 f$

$$\therefore \quad V_2^2 f_2 = V_1^2 f_1 \quad \text{or} \quad f_2 = f_1 (V_1/V_2)^2 = 40 \times 10^6 (848/700)^2 = \textbf{58.7 MHz}$$

Example 47.12. *A plywood board of 0.5 × 0.25 × 0.02 metre is to be heated from 25 to 125°C in 10 minutes by dielectric heating employing a frequency of 30 MHz. Determine the power required in this heating process. Assume specific heat of wood 1500/J/kg/°C; weight of wood 600 kg/m^3 and efficiency of process 50%.* **(Utilisation of Traction, B.H.U.)**

Solution. Volume of plywood to be heated = $0.5 \times 0.25 \times 0.02 = 0.0025\ m^3$

Weight of plywood = $0.0025 \times 600 = 1.5$ kg

Heat required to raise the temperature of plywood board from 25° to 125°

$= 1.5 \times 1500\ (125 - 25) = 2,25,000$ J or W-s

or $H = 2,25,000/60 \times 60 = 62.5$ Wh

Since heating is to be done in 10 minutes, power required $= 62.5/(10/60) = 375$ W

Since efficiency is 50%, power input $= 375/0.5 =$ **700 W**

47.23. Choice of Frequency

The selection of frequency for heating is important because it has a great bearing on the work to be heated and the method of its heating whether by induction heating or dielectric heating. Furnaces running on power frequency of 50 Hz can be of 1 MW capacity whereas those running on medium frequencies (500 Hz to 1000 Hz) have a capacity of 50 kW and those running on high frequency (1 MHz to 2 MHz) have capacities ranging from 200 kW to 500 kW.

1. Induction Heating. While choosing frequency for induction heating, the following factors are considered :

(*a*) Thickness of the surface to be heated. Higher the frequency, thinner the surface that will get heated.

(*b*) The time of continuous heating. Longer the duration of heating, deeper the penetration of heat in the work due to conduction.

(*c*) The temperature to be obtained. Higher the temperature, higher the capacity of the generator required.

2. Dielectric Heating. The power consumed during dielectric heating, $P = 2\pi f C V^2 \cos \phi$. As seen, $P \propto f \times C \times V^2 \times \cos \phi$. Hence, rate of heat production can be increased by increasing voltage or voltage across any specimen is limited by its thickness or because of the consideration of potential gradient, breakdown voltage and safety etc., Voltages ranging from 600 V to 3000 V are used for dielectric heating, although voltages of 20 kV or so are also used sometimes.

Rate of heat production can also be increased by applying high potential but it is also limited because of the following considerations :

(*a*) Possibility of formation of standing waves between the surface of two electrodes having wavelength nearly equal to or more than one quarter of the wavelength of the particular frequency used.

(*b*) Necessity of employing special matching circuit at higher frequencies due to the fact that maximum power transfer takes place when the oscillator impedence equals the load impedence.

(*c*) At higher frequencies it is difficult for tuning inductance to resonate with the charge capacitance.

(*d*) At higher frequencies, it is almost impossible to get uniform voltage distribution.

(*e*) Since higher frequencies disturb near-by radio station services, special arrangement has to be made to stop radiations from the high-frequency generator used for the purpose.

47.24. Infrared Heating

When tungsten filament lamps are operated at about 2300°C (instead of 3000°C), they produce plenty of heat radiations called ***infrared radiations***. With the help of suitable reflectors, these infrared radiations are focused on the surface to be heated. The lamps so employed have ratings varying from 250 W to 1000 W operating at 115 W. Lower voltage results in robust filaments. With this arrangement, the charge temperature obtain is between 200°C and 300°C. The heat emission intensity obtained is about 7000 W/m^2 as compared to 1500 W/m^2 obtained with ordinary resistance furnaces. In this type of heating, heat absorption remains practically constant whatever the charge temperature whereas

it falls rapidly as the temperature of charge rises in the ordinary resistance furnace.

Infrared heating is used for paint drying and for drying foundary moulds, for low temperature heating of plastics and for various dehydration and other processes.

Tutorial Problem No. 47.1

1. A slab of insulating material 150 cm^2 in area and 1 cm. thick is to be heated by dielectric heating. The power required is 400W at 30 MHz. Material has permitivity of 5 and p.f. 0.05. Determine the voltage necessary. Absolute permittivity = 8.854×10^{-12} F/m.

[800 V] ***(Utilisation of Elect. Energy, Punjab Univ.)***

2. A 20-kW, 1-phase, 220-V resistance oven employs circular nickel chrome wire for its heating elements. If the wire temperature is not to exceed 1107° C and the temperature of the charge is to be 500°C, calculate the length and size of wire required. Assume a radiating efficiency of 0.6 and the specific resistance of nickel chrome as 100×10^{-6} Ω cm and emissivity 0.9.

[*l* =17.1 m, d = 0.3 cm] ***(Utilisation of Elect. Power, A.M.I.E.)***

3. An electric toaster consists of two resistance elements each of 190 ohm. Calculate the power drawn from 250 V a.c. single-phase supply, when the elements are connected in (*i*) parallel and (*ii*) series.

[(*i*) 658 W (*ii*) 164.5 W]

4. A 15-kW, 220-V, single-phase resistance oven employs nickel-chrome wire for its heating elements. If the wire temperature is not to exceed 1,000°C and the temperature of the charge is to be 600°C, calculate the diameter and length of the wire. Assume radiating efficiency to be 0.6 and emissivity as 0.9. For nickel chrome resistivity is 1.016×10^{-6} Ω–m. **[3.11 mm, 24.24 m]**

(Utili. of Elect. Power, A.M.I.E. Sec. B)

5. A 30-kW, 3-phase, 400-V resistance oven is to employ nickel-chrome strip 0.025 cm thick for the 3-phase star-connected heating elements. If the wire temperature is to be 1100°C and that of charge is to be 700°C, estimate a suitable width for the strip. Assume radiating efficiency as 0.6 and emissivity as 0.90. The specific resistance of the nichromealloy is 1.03×10^{-6} Ω–m. State any assumptions made. **[6.86 mm]** ***(Utili. of Elect. Power, A.M.I.E. Sec. B.)***

6. Estimate the efficiency of a high-frequency induction furnace which takes 10 minutes to melt 1.8 kg of a aluminium, the input to the furnace being 5 kW and initial temperature 15°C. Given :

Specific heat of aluminium = 880 *J*/kg/°C

Melting point of aluminium = 660°C

Latent heat of fusion of aluminium = 32 kJ/kg and 1J = 2.78×10^{-7} kWh.

[36 %] ***(Elect. Drives and Util. of Elect. Energy, Punjab Univ.)***

7. A 3-phase electric arc furnace has the following data :

Current drawn = 5,000 A ; Arc voltage = 50 V

Resistance of transformer referred to primary =0.002 Ω

Resistance of transformer referred to secondary=0.004 Ω

(*i*) Calculate the power factor and kW drawn from the supply.

(*ii*) If the overall efficiency of furnace is 65%, find the time required to melt 2 tonnes of steel when latent heat of steel = 8.89 kcal/kg. Specific heat of steel = 0.12, melting point of steel = 1370°C and initial temperature of steel = 20°C.

[(*i*) 0.9847; 900 kW, (*ii*) 40 min. 46 s.] ***(Utili. of Elect. Power, A.M.I.E. Sec. B.)***

8. Dielectric heating is to be employed to heat a slab of insulating material 20 mm thick and 1530 mm^2 in area. Power required is 200 W and a frequency of 3 MHz is to be used. The material has a permitivity of 5 and p.f. of 0.05. Determine the voltage necessary and the current which will flow through the material. **[8000 V ; 0.5 A]** ***(Elect. Drives and Utili of Elect. Energy, Punjab Univ.)***

9. What are the advantages of electrically produced heat? What are the properties to be possessed by the element used in resistance oven? ***(J.N. University, Hyderabad, November 2003)***

10. A 20 kW single-phase, 220 V resistance oven employs circular nichrome wire for its heating element, if the wire temperature is not to exceed 1227^o and the temperature of the charge is to be 427^oC, calculate the size and length of the wire required. Assume emissivity = 0.9, radiating efficiency = 0.6 and specific resistance of wire = 1.09×10^{-6}Ω–m.

(J.N. University, Hyderabad, November 2003)

11. Discuss the various modes of heat dissipation. ***(J.N. University, Hyderabad, November 2003)***

12. Explain in brief how heating is done in the following cases?
(*i*) Resistance heating, (*ii*) Induction heating, (*iii*) Dielectric heating.
(J.N. University, Hyderabad, November 2003)

13. 90 Kg of tin is to smelt during an hour in smelting furnace. Determine the suitable rating of teh furnace, if melting temperature = 230°C, specific heat = 0.055, latent heat of liquidification is 13.3 Kcal/kg. Take the initial temperature of the metal as 35°C.
(J.N. University, Hyderabad, November 2003)

14. Explain the principle of Induction heating, What are the applications ofinduction heating.
(J.N. University, Hyderabad, November 2003)

15. With a neat sketch explain the working principle of coreless type induction furnace.
(J.N. University, Hyderabad, November 2003)

16. Explain with a neat sketch the principle of coreless type induction furnace.
(J.N. University, Hyderabad, November 2003)

17. 100 kg of tin is to smelt in one hour in a smelting furnance. Determine the suitable rating of furnace if smelting temperature of tin is 235°C; specific heat is 0.055, latent heat of liquidification 13.3 Kcal/kg. Take initial temperature of metal as 35°C.***(J.N. University, Hyderabad, November 2003)***

18. A low frequency Induction Furnace whose secondary voltage is maintained constant at 12 Volts takes 300 Kw at 0.65 p.f. when the head of the charge and reactance to remain constant, find the height upto which the hearth should be filled to obtain maximum heat.
(J.N. University, Hyderabad, November 2003)

19. Give relative advantages and disadvantages of direct and indirect electric arc furnances.
(J.N. University, Hyderabad, April 2003)

20. An electric arc furnace consuming 5 KW takes 15 minutes to just melt 1.5 Kgs of aluminium, the initial temperature being 15°C. Find the efficiency of the furnace. Specific heat of aluminium is 0.212, melting point 658°C and latent heat of fusion is 76.8 Cal per gram.
(J.N. University, Hyderabad, April 2003)

21. What are the causes of failure of heating elements? ***(J.N. University, Hyderabad, April 2003)***

22. Six resistances each of 40ohms are used as heating elements in furnace. Find the power of the furnace for various connections to a three phase 230 V supply.
(J.N. University, Hyderabad, April 2003)

23. What are different methods of heat transfer Explain in brief ?
(J.N. University, Hyderabad, April 2003)

24. What are the advantages of radiant heating ? ***(J.N. University, Hyderabad, April 2003)***

25. Describe various types of electric heating equipment. ***(J.N. University, Hyderabad, April 2003)***

26. What are the causes of failure of heating elements? ***(J.N. University, Hyderabad, April 2003)***

27. Six resistances each of 40 ohms are used as heating elements in furnace. Find the power of the furnace for various connections to a three phase 230 V supply.
(J.N. University, Hyderabad, April 2003)

28. Explain why very high frequencies should not be used for dielectric hearing.
(J.N. University, Hyderabad, December 2002/January 2003)

29. A wooden board 30 cms × 15 cms × 2 cms is to be heated from 20°C to 180°C in 10 minutes by dielectric heating using 40 MHz supply. Specific heat of wood 0.35 and density 0.55 gm/cc. $\varepsilon_r = 5$ and p.f. 0.05. Estimate the voltage across the specimen and current during heating. Assume loss of energy by conduction, convection and radiation as 10%.
(J.N. University, Hyderabad, December 2002/January 2003)

30. Write short Notes on The Ajax-yatt furnace.
(J.N. University, Hyderabad, December 2002/January 2003)

31. Discuss the different methods of electric heating and their relative merits
(J.N. University, Hyderabad, December 2002/January 2003)

32. Dielectric heating is to be employed to heat a slab of insulating meterial of 20 mm thick and 1500 mm^2 in area. The power required is 200 watts at a frequency of 30 MHz. The material has a permittivity of 5 and a power factor of 0.05. Determine the voltage necessary and the current which flows through the material. ***(J.N. University, Hyderabad, December 2002/January 2003)***

33. State the advantages of electric heating.
(J.N. University, Hyderabad, December 2002/January 2003)

34. Brifely explain the different methods of electric heating.
(J.N. University, Hyderabad, December 2002/January 2003)

35. Estimate the energy required to melt 500 kg of brass in a single phase Ajax-wyatt furnace.Ifthe melting is to be carried out in 3/4 hour, what must be the avarage power input to the furnace.
(J.N. University, Hyderabad, December 2002/January 2003)

OBJECTIVE TESTS – 47

1. As compared to other methods of heating using gas and coal etc, electric heating is far superior because of its.
(*a*) cleanliness
(*b*) ease of control
(*c*) higher efficiency
(*d*) all of the above.

2. Magnetic materials are heated with the help of
(*a*) hysteresis loss (*b*) electric arc
(*c*) electric current (*d*) radiation.

3. In the indirect resistance heating method, heat is delivered to the charge
(*a*) direcly (*b*) by radiation
(*c*) by convection (*d*) both (*b*) and (*c*).

4. The main requirements of a good heating element used in a resistance furnaces are
(*a*) high resistivity
(*b*) high melting-temperature
(*c*) positive resistance-temperature coefficient
(*d*) all of the above.

5. Electric ovens using heating elements of ..... can produce tempera- tures up to 3000°C
(*a*) nickel (*b*) graphite
(*c*) chromium (*d*) iron.

6. The temperature of resistance furnaces can be controlled by changing the
(*a*) applied voltage
(*b*) number of heating elements
(*c*) circuit configuration
(*d*) all of the above.

7. Which of the following heating method is based on the transformer principle ?
(*a*) resistance heating
(*b*) eddy-current heating
(*c*) induction heating
(*d*) dielectric heating.

8. When graphite electrodes are used in arc furnaces, the temperature obtained is in the range of .....degree centi-grade.
(*a*) 3000-3500 (*b*) 2500-3000
(*c*) 2000-2500 (*d*) 1500-2000

9. Which of the following furnace suffers from pinch effect?
(*a*) resistance furnace
(*b*) core type induction furnace
(*c*) coreless induction furnace
(*d*) vertical core type induction furnace.

10. Which of the following induction furnace has the lowest power factor?
(*a*) vertical core type
(*b*) indirect core type
(*c*) coreless type
(*d*) core type.

11. The coreless induction furnace uses high-frequency electric supply in order to obtain high
(*a*) flux density
(*b*) eddy-current loss
(*c*) primary resistance
(*d*) power factor.

12. Inflammable articles like plastic and wooden products etc, can be safely heated by using..... heating.
(*a*) eddy-current (*b*) dielectric
(*c*) induction (*d*) resistance

13. Which of the following is an advantages of heating by electricity?
(*a*) Quicker operation
(*b*) Higher efficiency
(*c*) Absence of flue gases
(*d*) All of the above

14. ..... has the highest value of thermal conductivity.
(*a*) Copper (*b*) Aluminium
(*c*) Brass (*d*) Steel

15. Which of the following heating methods has maximum power factor?
(*a*) Arc heating
(*b*) Dielectric heating
(*c*) Induction heating
(*d*) Resistance heating

16. ..... method has leading power factor
(*a*) Resistance heating
(*b*) Dielectric heating
(*c*) Arc heating
(*d*) Induction heating

17. ..... is used for heating non-conducting meterials.
(*a*) Eddy current heating
(*b*) Arc heating
(*c*) Induction heating
(*d*) Dielectric heating

18. Which of the following methods of heating is not dependent on the frequency of supply?
(*a*) Induction heating
(*b*) Dielectric heating
(*c*) Electric resistance heating
(*d*) All of the above

19. When a body reflects entire radiation incident on it, then it is known as
(*a*) white body (*b*) grey body
(*c*) black body (*d*) transparent body

20. For the transmission of heat from one body to another
(*a*) temperature of the two bodies must be different
(*b*) both bodies must be solids
(*c*) both bodies must be in contact
(*d*) at least one of the bodies must have some source of heating

21. Heat transfer by condition will not occur when
(*a*) bodies are kept in vacuum
(*b*) bodies are immersed in water
(*c*) bodies are exposed to thermal radiations
(*d*) temperatures of the two bodies are identical

22. A perfect black body is one that
(*a*) transmits all incident radiations
(*b*) absorbs all incident radions
(*c*) reflects all incident radiations
(*d*) absorbs, reflects and transmits all incident radiations

23. Heat is transferred simultaneously by condition, convection and radiation
(*a*) inside boiler furnaces
(*b*) during melting of ice
(*c*) through the surface of the insulted pipe carring steam
(*d*) from refrigerator coils to freezer of a refrigerator

24. The process of heat transfer during the re-entry of satellites and missiles, at very high speeds, into earth's atmosphere is known as
(*a*) ablation
(*b*) radiation
(*c*) viscous dissipation
(*d*) irradiation

25. Which of the following has the highest value of thermal conductivity?
(*a*) Water (*b*) Steam
(*c*) Solid ice (*d*) Melting ice

26. Induction heating process is based on which of the following principles?
(*a*) Thermal ion release principle
(*b*) Nucleate heating principle
(*c*) Resistance heating principle
(*d*) Electro-magnetic induction principle

27. Which of the following insulating materials is suitable for low temperature applications?
(*a*) Asbestos paper
(*b*) Diatomaceous earth
(*c*) 80 percent magnesia
(*d*) Cork

28. A non-dimensional number generally associated with natural convection heat transfer is
(*a*) Prandtl number
(*b*) Grash off number
(*c*) Pecelet number
(*d*) Nusselt number

29. The temperature inside a furnace is usually measured by which of the following?
(*a*) Optical pyrometer
(*b*) Mercury thermometer
(*c*) Alcohol thermometer
(*d*) Any of the above

30. Which of the following will happen if the thickness of refractory wall of furnace is increased?
(*a*) Heat loss through furnace wall will increase
(*b*) Temperature inside the furnace will fall
(*c*) Temperature on the outer surface of furnace walls will drop
(*d*) Energy consumption will increase

31. The material of the heating element for a furnace should have
(*a*) lower melting point
(*b*) higher temperature co-efficient
(*c*) high specific resistance
(*d*) all of the above

32. In a resistance furnace the atmosphere is .....
(*a*) oxidising (*b*) deoxidising
(*c*) reducing (*d*) neutral

33. By which of the following methods the temperature inside a resistance furnance can be varied?
(*a*) By disconnecting some of the heating elements
(*b*) By varying the operating voltage
(*c*) By verying the current through heating elements
(*d*) By any of the above method

34. In induction heating ..... is abnormally high.
(*a*) phase angle (*b*) frequency
(*c*) current (*d*) voltage

35. By the use of which of the following high frequency power supplyv for induction furnaces can be obtained?
(*a*) Coreless transformers
(*b*) Current transformers
(*c*) Motor-generator set
(*d*) Multi-phase transformer

36. Induction furnaces are employed for which of the following?
(*a*) Heat treatment of castings
(*b*) Heating of insulators
(*c*) Melting aluminium
(*d*) None of the above

37. In an electric room heat convector the method of heating used is
(*a*) arc heating
(*b*) resistance heating
(*c*) induction heating
(*d*) dielectric heating

38. In a domestic cake baking oven the temperature is controlled by
(*a*) voltage variation
(*b*) thermostat
(*c*) auto-transformer
(*d*) series-parallel operation

39. In an electric press mica is used
(*a*) as an insulator
(*b*) as a device for power factor improvement
(*c*) for dielectric heating
(*d*) for induction heating

40. Induction heating takes place in which of the following?
(*a*) Insulating materials
(*b*) Conducting materials which are magnetic
(*c*) Conducting materialswhich are non-magnetic
(*d*) Conducting materials which may or may not be magnetic

41. For heating element high resistivity material is chosen to
(*a*) reduce the length of heating element
(*b*) increase the life of the heating element
(*c*) reduce the effect of oxidation
(*d*) produce large amount of heat

42. In resistance heating highest working temperature is obtained from heating elements made of .....
(*a*) nickel copper (*b*) nichrome
(*c*) silicon carbide (*d*) silver

43. For intermittent work which of the following furnaces is suitable?
(*a*) Indirect arc furnace
(*b*) Core less furnace
(*c*) Either of the above
(*d*) None of the above

44. Due to which of the following reasons it is desirable to have short arc length?
(*a*) To achieve better heating
(*b*) To increase the life of roof refractory
(*c*) To have better stirring action
(*d*) To reduce problem of oxidation
(*e*) All of the above

45. In the indirect resistance heating method, maximum heat-transfer takes place by
(*a*) radiation (*b*) convection
(*c*) conduction (*d*) any of the above

46. Property of low temperature co-efficient of heating element is desired due to which of the following reasons?
(*a*) To avoid initial rush of current
(*b*) To avoid change in kW rating with temperature
(*c*) Both (*a*) and (*b*)
(*d*) Either (*a*) or (*b*)

47. Which of the following methods is used to control temperature in resistance furnaces?
(*a*) Variation of resistance
(*b*) Variation of voltage
(*c*) Periodical switching on and off of the supply
(*d*) All of the above methods

48. It is desirable to operate the arc furnace at power factor of
(*a*) zero (*b*) 0.707 lagging
(*c*) unity (*d*) 0.707 leading

49. Radiations from a black body are proportional to
(*a*) T (*b*) T^2
(*c*) T^3 (*d*) T^4

50. In arc furnace the function of choke is
(*a*) to stabilize the arc
(*b*) to improve power factor
(*c*) to reduce severity of the surge
(*d*) none of the above

51. Ajax Wyatt furnace is started when
(*a*) it is filled below core level
(*b*) it is filled above core level
(*c*) it is fully empty
(*d*) none of the above

52. In electric press, mica is used because it is ..... conductor of heat but/and ..... conductor of electricity.
(*a*) bad, good (*b*) bad, bad
(*c*) good, bad (*d*) good, good

53. Resistance variation method of temperature control is done by connecting resistance elements in

(*a*) series
(*b*) parallel
(*c*) series-parallel connections
(*d*) star-delta connections
(*e*) all of the above ways

54. Hysteresis loss and eddy current loss are used in
(*a*) induction heating of steel
(*b*) dielectric heating
(*c*) induction heating of brass
(*d*) resistance heating

55. In heating the ferromagnetic material by induction heating, heat is produced due to
(*a*) induced current flow through the charge
(*b*) hysteresis loss taking place below curie temperature
(*c*) due to hysteresis loss as well as eddy current loss taking place in the charge
(*d*) one of the above factors

56. Radiant heating is used for which of the following?
(*a*) Annealing of metals
(*b*) Melting of ferrous metals
(*c*) Heating of liquids in electric kettle
(*d*) Drying of paints and varnishes

57. Which of the following devices is necessarily required for automatic temperature control in a furnace?
(*a*) Thermostat
(*b*) Thermocouple
(*c*) auto-transformer
(*d*) Heating elements of variable resistance material

58. For radiant heating around 2250°C, the heating elements are made of
(*a*) copper alloy (*b*) carbon
(*c*) tungsten alloy (*d*) stainless steel alloy

59. Which of the following is an advantage of eddy current heating?
(*a*) The amount of heat generated can be controlled accurately
(*b*) Heat at very high rate can be generated
(*c*) The area of the surface over which heat is produced can be accurately controlled
(*d*) All of the above

60. The electrode of a direct arc furnace is made of
(*a*) tungsten
(*b*) graphite
(*c*) silver
(*d*) copper

61. Direct arc furnaces have which of the following power factors?
(*a*) Unity
(*b*) Low, lagging
(*c*) Low, leading
(*d*) Any of the above

62. In direct arc furnace, which of the following has high value?
(*a*) Current
(*b*) Voltage
(*c*) Power factor
(*d*) All of the above

ANSWERS

1. (*d*)	**2.** (*a*)	**3.** (*d*)	**4.** (*d*)	**5.** (*b*)	**6.** (*d*)	**7.** (*c*)
8. (*a*)	**9.** (*b*)	**10.** (*d*)	**11.** (*b*)	**12.** (*b*)	**13.** (*d*)	**14.** (*a*)
15. (*d*)	**16.** (*b*)	**17.** (*d*)	**18.** (*c*)	**19.** (*a*)	**20.** (*a*)	**21.** (*d*)
22. (*b*)	**23.** (*a*)	**24.** (*a*)	**25.** (*c*)	**26.** (*d*)	**27.** (*b*)	**28.** (*b*)
29. (*a*)	**30.** (*c*)	**31.** (*c*)	**32.** (*a*)	**33.** (*d*)	**34.** (*b*)	**35.** (*c*)
36. (*a*)	**37.** (*b*)	**38.** (*b*)	**39.** (*a*)	**40.** (*d*)	**41.** (*a*)	**42.** (*c*)
43. (*a*)	**44.** (*e*)	**45.** (*a*)	**46.** (*c*)	**47.** (*d*)	**48.** (*b*)	**49.** (*d*)
50. (*a*)	**51.** (*d*)	**52.** (*c*)	**53.** (*e*)	**54.** (*a*)	**55.** (*c*)	**56.** (*d*)
57. (*b*)	**58.** (*c*)	**59.** (*d*)	**60.** (*b*)	**61.** (*b*)	**62.** (*a*)	

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CHAPTER 48

ELECTRIC WELDING

Learning Objectives

- Definition of Welding
- Welding Processes
- Four Positions of Arc Welding
- Electrodes for Metal Arc Welding
- Advantages of Coated Electrodes
- Arc Welding Machines
- V-I Characteristics of Arc Welding D.C. Machines
- D.C. Welding Machines with Motor Generator Set
- AC Rectified Welding Unit
- AC Welding Machines
- Carbon Arc Welding
- Submerged Arc Welding
- Gas Shield Arc Welding
- TIG Welding
- MIG Welding
- MAG Welding
- Resistance Welding
- Spot Welding
- Seam Welding
- Projection Welding
- Butt Welding
- Flash Butt Welding
- Upset Welding
- Stud Welding
- Electrogas Welding
- Electron Beam Welding



Electricity is used to generate heat necessary to melt the metal to form the necessary joints

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48.1. Definition of Welding

It is the process of joining two pieces of metal or non-metal at faces rendered plastic or liquid by the application of heat or pressure or both. Filler material may be used to effect the union.

48.2. Welding Processes

All welding processes fall into two distinct categories :

1. Fusion Welding—It involves melting of the parent metal. Examples are:

(*i*) Carbon arc welding, metal arc welding, electron beam welding, electroslag welding and electrogas welding which utilize electric energy and

(*ii*) Gas welding and thermit welding which utilize chemical energy for the melting purpose.

2. Non-fusion Welding—It does not involve melting of the parent metal. Examples are:

(*i*) Forge welding and gas non-fusion welding which use chemical energy.

(*ii*) Explosive welding, friction welding and ultrasonic welding etc., which use mechanical energy.

(*iii*) Resistance welding which uses electrical energy.

Proper selection of the welding process depends on the **(*a*)** kind of metals to be joined **(*b*)** cost involved **(*c*)** nature of products to be fabricated and **(*d*)** production techniques adopted.
The principal welding processes have been tabulated in Fig. 48.1

48.3. Use of Electricity in Welding

Electricity is used in welding for generating heat at the point of welding in order to melt the material which will subsequently fuse and form the actual weld joint. There are many ways of producing this localised heat but the two most common methods are as follows :

1. resistance welding—here current is passed through the inherent resistance of the joint to be welded thereby generating the heat as per the equation I^2Rt/J kilocalories.

2. arc welding—here electricity is conducted in the form of an arc which is established between the two metallic surfaces

48.4. Formation and Characteristics of Electric Arc

An electric arc is formed whenever electric current is passed between two metallic electrodes which are separated by a short distance from each other. The arc is started by momentarily touching the positive electrode (anode) to the negative metal (or plate) and then withdrawing it to about 3 to 6 mm from the plate. When electrode first touches the plate, a large short-circuit current flows and as it is later withdrawn from the plate, current continues to flow in the form of a spark across the air gap so formed. Due to this spark (or discharge), the air in the gap becomes ionized *i.e.* is split into negative electrons and positive ions. Consequently, air becomes conducting and current is able to flow across the gap in the form of an arc.

As shown in Fig. 48.2, the arc consists of ***lighter*** electrons which flow from cathode to anode and ***heavier*** positive ions which flow from anode to cathode. Intense heat is generated when high-velocity electrons strike the anode. Heat generated at the cathode is much less because of the low velocity of the impinging ions. It is found that nearly **two-third** of the heat is developed at the anode which burns into the form of a crater where temperature rises to a value of 3500-4000°C. The remaining one-third of the heat is developed near the cathode. The above statement is true in all d.c. systems of welding where positive side of the circuit is the hottest side. As a result, an electrode connected to the positive end of the d.c. supply circuit will burn 50% faster than if connected to the negative end. This fact can be used for obtaining desired penetration of the base metal during welding.

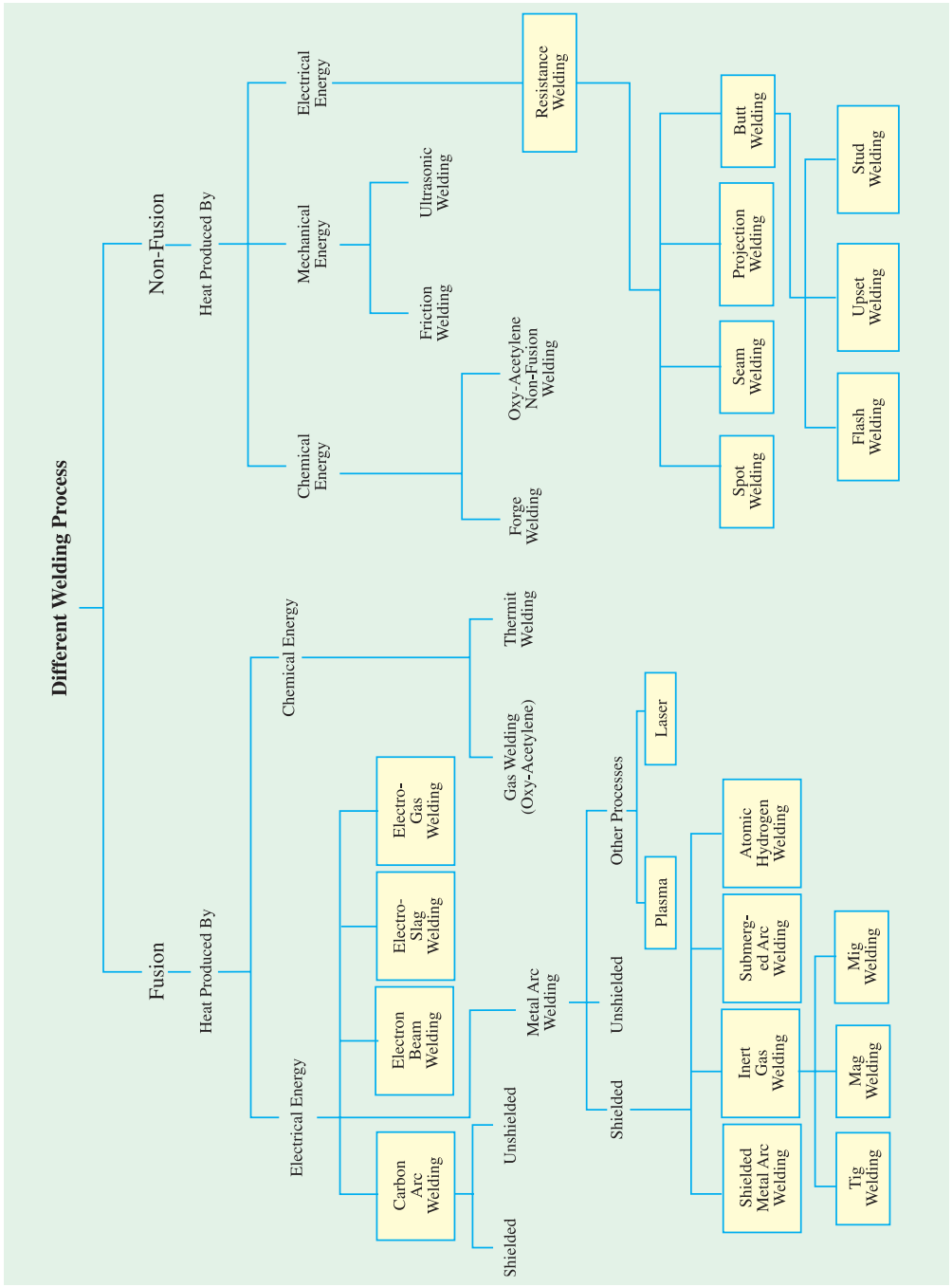


Fig. 48.1

If positive supply end is connected to the base metal (which is normally grounded), penetration will be greater due to more heat and, at the same time, the electrode will burn away slowly [Fig. 48.3 (*a*)] since it is connected to the negative end of the supply. If supply connections are reversed, the penetration of heat zone in the base metal will be comparatively shallow and, at the same time, electrode will burn fast [Fig. 48.3 (*b*)]. AC supply produces a penetration depth that is nearly halfway between that achieved by the d.c. positive ground and negative ground as shown in Fig. 48.3 (*c*).

It may be noted that with a.c. supply, heat is developed equally at the anode and cathode due to rapid reversal of their polarity. The arc utilized for arc welding is a low-voltage high-current discharge. The voltage required for striking the arc is higher than needed for maintaining it. Moreover, amperage increases as voltage decreases after the arc has been established. Fig 48.4 shows V/I characteristics of an electric arc for increasing air-gap lengths. The voltage required to strike a d.c. arc is about 50-55 V and that for a.c. arc is 80-90 V. The voltage drop across the arc is nearly 15-20 V. It is difficult to maintain the arc with a voltage less than 14 V or more than 40 V.

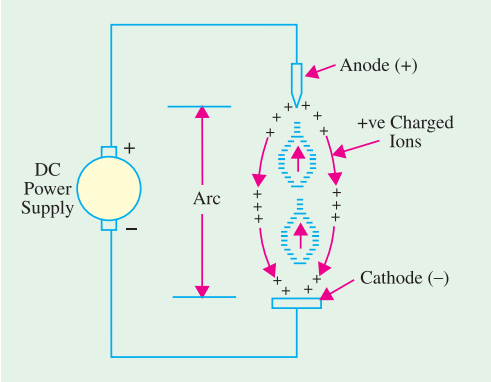


Fig. 48.2

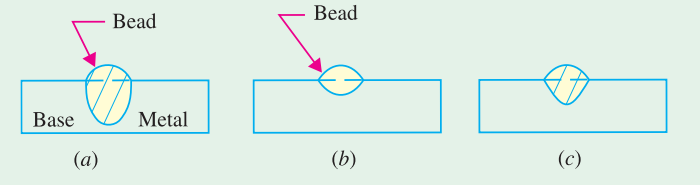


Fig. 48.3

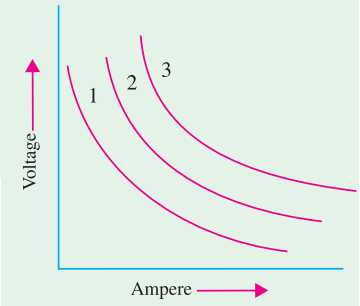


Fig. 48.4

48.5. Effect of Arc Length

In metal arc welding, a fairly short arc length is necessary for getting good welds. Short arc length permits the heat to be concentrated on the workpiece, is more stable because effect of magnetic blow is reduced and the vapours from the arc surround the electrode metal and the molten pool thereby preventing air from destroying the weld metal. When arc length is long

1. large amount of heat is lost into the surrounding area thus preventing good penetration and fusion;
2. arc flame is very unstable since effect of magnetic blow is increased. Hence, arc flame will have a tendency to blow out;
3. air is able to reach the molten globule of metal as it passes from the electrode to the weld and weld pool. It leads to the contamination of the weld due to absorption of oxygen and nitrogen;
4. weld deposits have low strength, poor ductility, high porosity, poor fusion and excessive spatter.

The length of arc required for welding will depend on the kind of electrode used, its coating, its diameter, position of welding and the amount of current used. Usually, shorter arc length are necessary for vertical, horizontal and overhead welding than for flat welding.



An electric arc is produced when electricity is passed between two electrodes

48.6. Arc Blow

An arc column can be considered as a flexible current-carrying conductor which can be easily deflected by the magnetic field set up in its neighbourhood by the positive and negative leads from the d.c. welding set. The

two leads carry currents in the opposite directions and hence, set up a repulsive magnetic force which pulls the arc away from the weld point particularly when welding corners where field concentration is maximum. The deflection of the arc is called **arc blow**. This condition is encountered only with d.c. welding sets and is especially noticeable when welding with bare electrodes. It is experienced most when using currents above 200 A or below 40 A.

Due to arc blow, heat penetration in the required area is low which leads to incomplete fusion and bead porosity apart from excessive weld spatter.

Arc blow can be avoided by using a.c. rather than d.c. welding machines because reversing currents in the welding leads produce magnetic fields which cancel each other out thereby eliminating the arc blow. However, with d.c. welding machines, arc blow effects can be minimized by ***(i)*** welding away from the earth ground connection, ***(ii)*** changing the position of the earth connection on the work, ***(iii)*** wrapping the welding electrode cable a few turns around the work, ***(iv)*** reducing the welding current or electrode size, ***(v)*** reducing the rate of travel of the electrode and ***(vi)*** shortening the arc column length etc.

48.7. Polarity in DC Welding

Arc welding with the electrode connected to the positive end of the d.c. supply is called reverse polarity.* Obviously, the workpiece is connected to the negative end.

A better name for d.c. reverse polarity (DCRP) *is* ***electrode-positive*** as shown in Fig. 48.5 (*a*). As stated earlier in Art. 48.4, two-third of the arc heat is developed at the anode. Hence, in DCRP welding, electrode is the hottest whereas workpiece is comparatively cooler. Consequently, electrode burns much faster but weld bead is relatively shallow and wide. That is why thick and heavily-coated electrodes are used in DCRP welding because they require more heat for melting.

Arc welding with the electrode connected to the negative end of the d.c. supply is called **straight polarity**.** Obviously, the workpiece is connected to the positive end as shown in Fig. 48.5 (*b*). A better name for d.c. straight polarity (DCSP) is ***electrode-negative***.

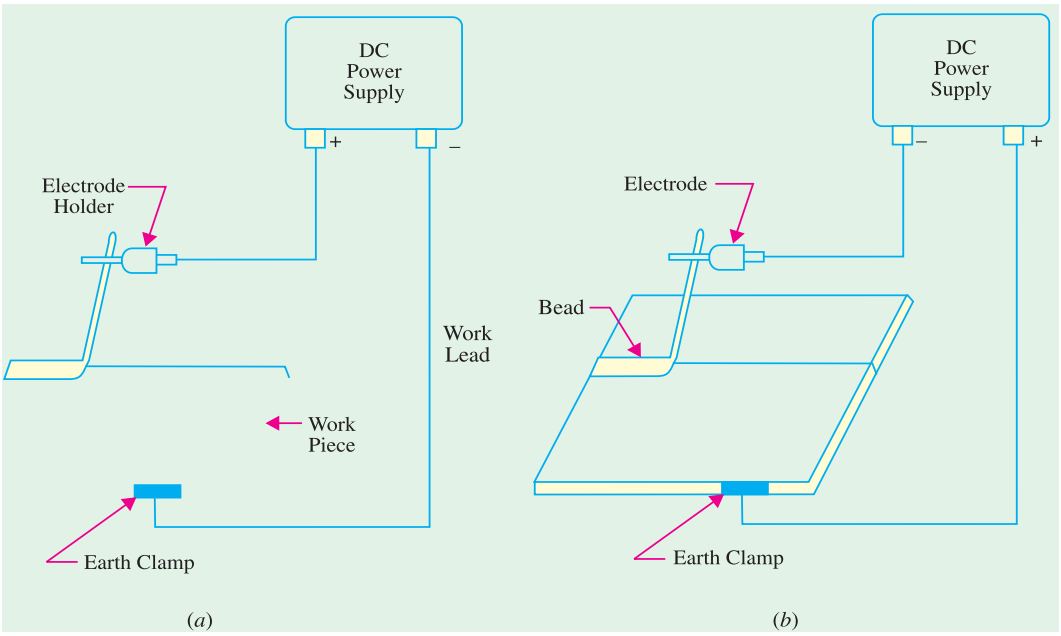


Fig. 48.5

* In British literature, it is called straight polarity.
** In British literature, it is called reverse polarity.

In DCSP welding, workpiece is the hottest, hence base metal penetration is narrow and deep. Moreover, bare and medium-coated electrodes can be used in this welding as they require less amount of heat for melting.

It is seen from the above discussion that polarity necessary for the welding operation is determined by the type of electrode used.

It is also worth noting that in a.c. welding, there is no choice of polarity because the circuit becomes alternately positive, first on one side and then on the other. In fact, it is a combination of D CSP and D CRP.

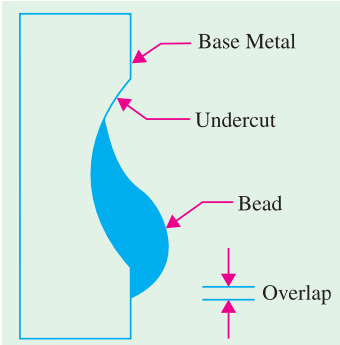


Fig. 48.6

48.8. Four Positions of Arc Welding

There are four basic positions in which manual arc welding is done.

1. Flat position. It is shown in Fig. 48.7 (*a*). Of all the positions, flat position is the easiest, most economical and the most used for all shielded arc welding. It provides the strongest weld joints. Weld beads are exceedingly smooth and free of slag spots. This position is most adaptable for welding of both ferrous and non-ferrous metals particularly for cast iron.

2. Horizontal Position. It is the second most popular position and is shown in Fig. 48.7 (*b*). It also requires a short arc length because it helps in preventing the molten puddle of the metal from sagging. However, major errors that occur while welding in horizontal position are under-cutting and over-lapping of the weld zone (Fig. 48.6).

3. Vertical Position. It is shown in Fig. 48.7 (*c*). In this case, the welder can deposit the bead either in the uphill or downhill direction. Downhill welding is preferred for thin metals because it is faster than the uphill welding. Uphill welding is suited for thick metals because it produces stronger welds.

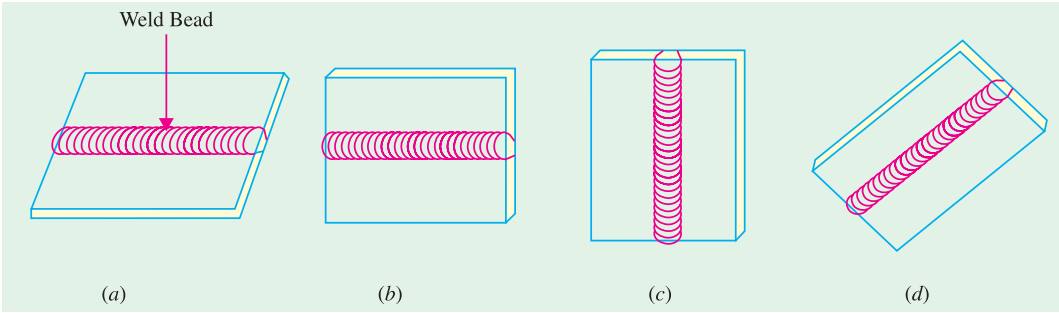


Fig. 48.7.

4. Overhead Position. It is shown in Fig. 48.7 (*d*). Here, the welder has to be very cautious otherwise he may get burnt by drops of falling metal. This position is thought to be the most hazardous but not the most difficult one.

48.9. Electrodes for Metal Arc Welding

An electrode is a filler metal in the form of a wire or rod which is either bare or coated uniformly with flux. As per IS : 814-1970, the contact end of the electrode is left bare and clean to a length of 20-30 mm. for inserting it into electrode holder (Fig. 48.8).

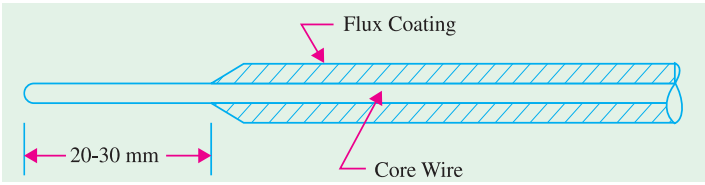


Fig. 48.8

Metal arc welding was originally done with bare electrodes which consisted of a piece of wire or rod of the same metal as the base metal. However, due to atmospheric contamination, they produced brittle and poor quality welds.

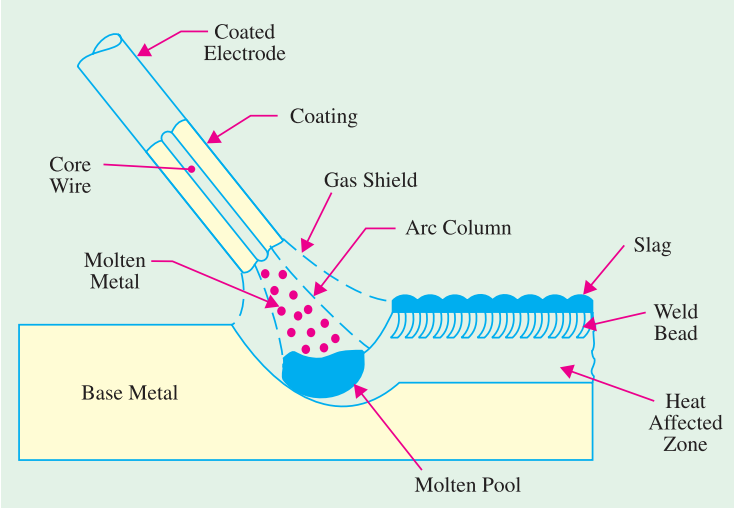


Fig. 48.9

Hence, bare wire is no longer used except for automatic welding in which case arrangement is made to protect the weld area from the atmosphere by either powdered flux or an inert gas. Since 1929, coated electrodes are being extensively used for shielded arc welding. They consist of a metal core wire surrounded by a thick flux coating applied by extrusion, winding or other processes. Depending on the thickness of the flux coating, coated electrodes may be classified into ***(i)*** lightly-dusted (or dipped) electrodes and ***(ii)*** semi-coated (or heavy-coated) electrodes. Materials commonly used for coating are ***(i)*** titanium oxide ***(ii)*** ferromanganese ***(iii)*** silica flour ***(iv)*** asbestos clay ***(v)*** calcium carbonate and ***(vi)*** cellulose with sodium silicate often used to hold ingredients together.

Electrode coating contributes a lot towards improving the quality of the weld. Part of the coating burns in the intense heat of the arc and provides a gaseous shield around the arc which prevents oxygen, nitrogen and other impurities in the atmosphere from combining with the molten metal to cause a poor quality brittle and weak weld. Another portion of the coating flux melts and mixes with the impurities in the molten pool causing them to float to the top of the weld where they cool in the form of slag (Fig. 48.9). This slag improves the bead quality by protecting it from the contaminating effects of the atmosphere and causing it to cool down more uniformly. It also helps in controlling the basic shape of the weld bead.

The type of electrode used depends on the type of metal to be welded, the welding position, the type of electric supply whether a.c. or d.c. and the polarity of the welding machine.

48.10. Advantages of Coated Electrodes

The principal advantages of using electrode coating are as under :

1. It stablizes the arc because it contains ionizing agents such as compounds of sodium and potassium.
2. It fluxes away impurities present on the surface being welded.
3. It forms slag over the weld which ***(i)*** protects it from atmospheric contamination ***(ii)*** makes it cool uniformly thereby reducing the changes of brittleness and ***(iii)*** provides a smoother surface by reducing 'ripples' caused by the welding operation.
4. It adds certain materials to the weld metal to compensate for the loss of any volatile alloying elements or constituents lost by oxidization.
5. It speeds up the welding operation by increasing the rate of melting.
6. It prevents the sputtering of metal during welding.

7. it makes it possible for the electrode to be used on a.c. supply. In a.c. welding, arc tends to cool and interrupt at zero-current positions. But the shielding gases produced by the flux keep the arc space ionized thus enabling the coated electrodes to be used on a.c. supply.

It is worth noting that efficiency of all coated (or covered) electrodes is impaired by dampness. Hence, they must always be stored in a dry space. If dampness is suspected, the electrodes should be dried in a warm cabinet for a few hours.

48.11. Types of Joints and Types of Applicable Welds

Bureau of Indian Standards (B.I.S.) has recommended the following types of joints and the welds applicable to each one of them (Fig. 48.10).

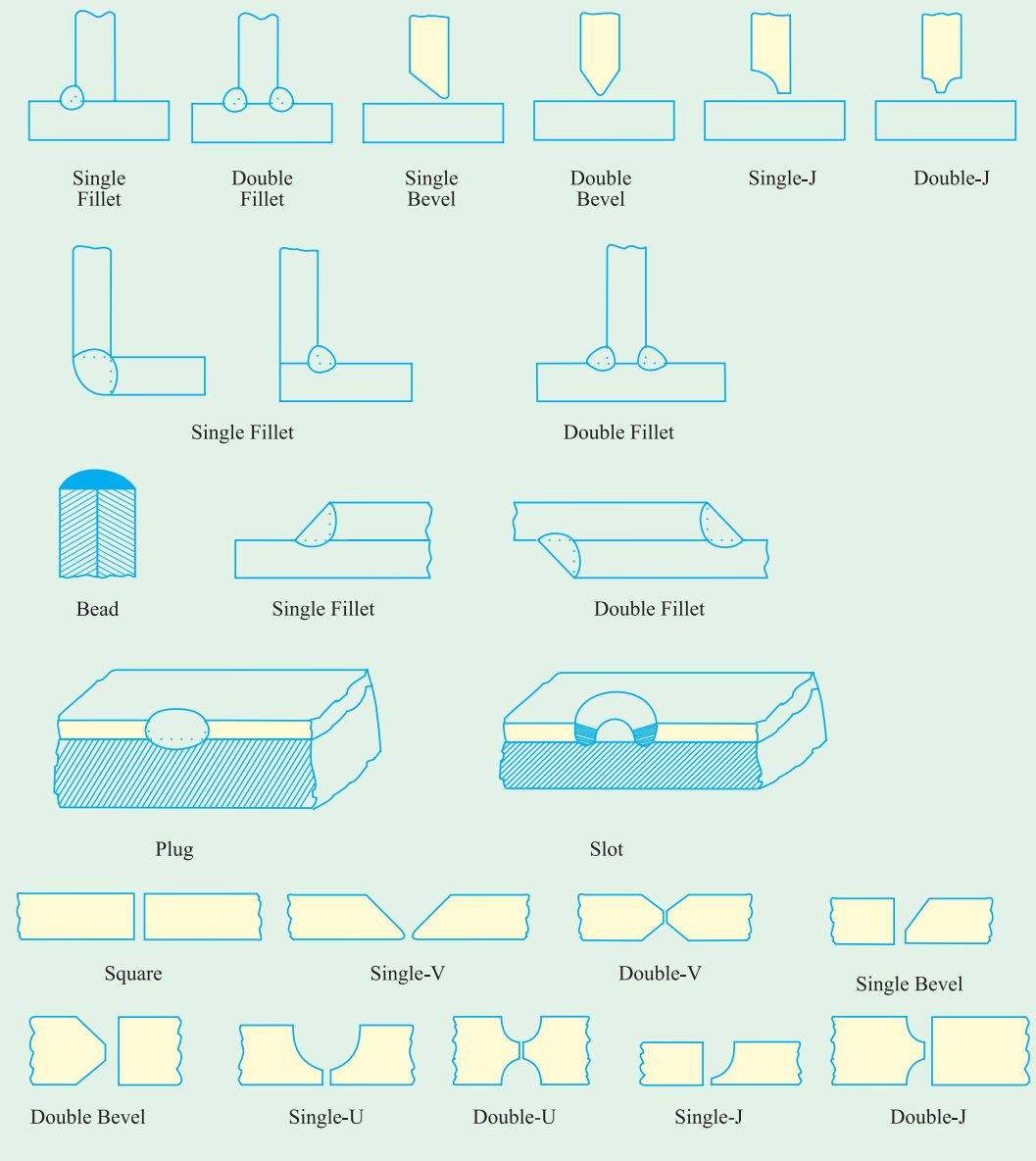


Fig. 48.10

1. Tee joint	—	with six types of welds.
2. Corner joint	—	with two types of welds.
3. Edge joint	—	with one type of weld.
4. Lap joint	—	with four types of welds.
5. Butt joint	—	with nine types of welds.

48.12. Arc Welding Machines

Welding is never done directly from the supply mains. Instead, special welding machines are used which provided currents of various characteristics. Use of such machines is essential for the following reasons :

1. To convert a.c. supply into d.c. supply when d.c. welding is desired.
2. To reduce the high supply voltage to a safer and suitable voltage for welding purposes.
3. To provide high current necessary for arc welding without drawing a corresponding high current from the supply mains.
4. To provide suitable voltage/current relationships necessary for arc welding at minimum cost.

There are two general types of arc welding machines :

(*a*) d.c. welding machines

(*i*) motor-generator set

(*ii*) a.c. transformers with rectifiers

(*b*) a.c. welding machines

48.13. V-I Characteristics of Arc Welding DC Machines

It is found that during welding operation, large fluctuations in current and arc voltage result from the mechanism of metal transfer and other factors. The welding machine must compensate for such changes in arc voltage in order to maintain an even arc column. There are three major voltage/current characteristics used in modern d.c. welding machines which help in controlling these current fluctuations :

1. drooping arc voltage (DAV).
2. constant arc voltage (CAV).
3. rising arc voltage (RAV).

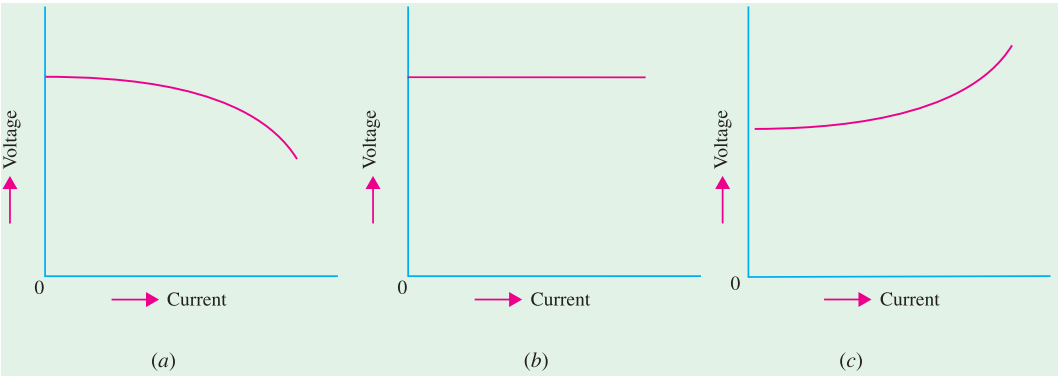


Fig. 48.11

The machines with DAV characteristics have high open-circuit voltage which drops to a minimum when arc column is started. The value of current rises rapidly as shown in Fig. 48.11 (*a*). This type of characteristic is preferred for manual shield metal arc welding.

The CAV characteristic shown in Fig. 48.11 (*b*) is suitable for semi-automatic or automatic welding processes because voltage remains constant irrespective of the amount of current drawn.

Because of its rising voltage characteristic, RAV has an advantage over CAV because it maintains a constant arc gap even if short circuit occurs due to metal transfer by the arc. Moreover, it is well-adopted to fully automatic process.

DC welding machines can be controlled by a simple rheostat in the exciter circuit or by a combination of exciter regulator and series of field taps. Some arc welding are equipped with remote-controlled current units enabling the operator to vary voltage-amperage requirement without leaving the machines.



DC and AC welding machines

48.14. DC Welding Machines with Motor Generator Set

Such a welding plant is a self-contained single-operator motor-generator set consisting of a reverse series winding d.c. generator driven by either a d.c. or an a.c. motor (usually 3-phase). The series winding produces a magnetic field which opposes that of the shunt winding. On open-circuit, only shunt field is operative and provides maximum voltage for striking the arc. After the arc has been established, current flows through the series winding and sets up a flux which opposes the flux produced by shunt winding. Due to decreases in the net flux, generator voltage is decreased (Art. 48.13). With the help of shunt regulator, generator voltage and current values can be adjusted to the desired level. Matters are so arranged that despite changes in arc voltage due to variations in arc length, current remains practically constant. Fig. 48.12 shows the circuit of a d.c. motor-generator type of welding machine.

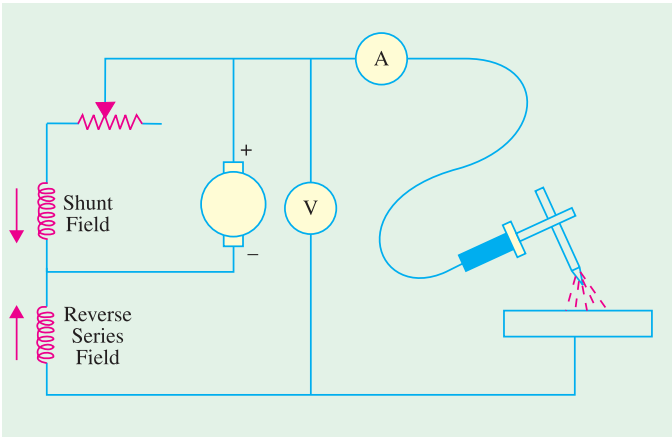


Fig. 48.12

Advantages. Such a d.c. welder has the following advantages :

1. It permits portable operation.
2. It can be used with either straight or reverse polarity.
3. It can be employed on nearly all ferrous and non-ferrous materials.
4. It can use a large variety of stick electrodes.
5. It can be used for all positions of welding.

Disadvantages

1. It has high initial cost.
2. Its maintenance cost is higher.
3. Machine is quite noisy in operation.
4. It suffers from arc blow.

48.15. AC Rectified Welding Unit

It consists of a transformer (single-or three-phase) and a rectifier unit as shown in Fig. 48.13. Such a unit has no moving parts, hence it has long life. The only moving part is the fan for cooling the transformer. But this fan is not the basic part of the electrical system. Fig. 48.13 shows a single-phase full-wave rectified circuit of the welder. Silicon diodes are used for converting a.c. into d.c. These diodes are hermetically sealed and are almost ageless because they maintain rectifying characteristics indefinitely.

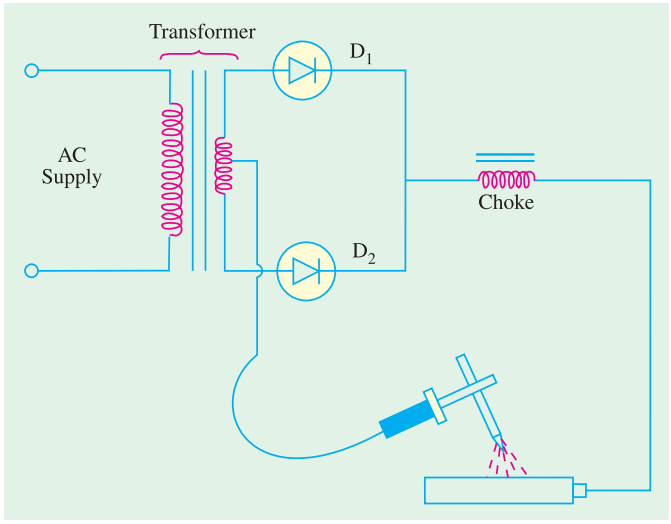


Fig. 48.13

Such a transformer-rectifier welder is most adaptable for shield arc welding because it provides both d.c. and a.c. polarities. It is very efficient and quiet in operation. These welders are particularly suitable for the welding of ***(i)*** pipes in all positions ***(ii)*** non-ferrous metals ***(iii)*** low-alloy and corrosion-heat and creep-resisting steel ***(iv)*** mild steels in thin gauges.

48.16. AC Welding Machines

As shown in Fig. 48.14, it consists of a step-down transformer with a tapped secondary having an adjustable reactor in series with it for obtaining drooping V/I characteristics. The secondary is tapped to give different voltage/current settings.

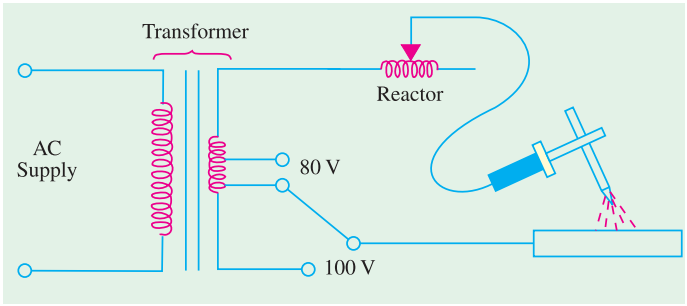


Fig. 48.14

Advantages. This a.c. welder which can be operated from either a single-phase or 3-phase supply has the following advantages :

(i) Low initial cost
(ii) Low operation and maintenance cost
(iii) Low wear
(iv) No arc blow

Disadvantages. ***(i)*** its polarity cannot be changed ***(ii)*** it is not suitable for welding of cast iron and non-ferrous metals.

48.17. Duty Cycle of a Welder

The duty cycle of an arc welder is based on a working period of 10 minutes. For example, if a welder is operated for 2 minutes in a period of 10 minutes, then its percentage duty cycle is (2/10) × 100 = 20 percent. Conversely, a 10 percent duty cycle would mean that the welder would

be operated for 10 percent of 10 minutes *i.e.* for one minute only in a period of 10 minutes.

Usually, values of maximum amperage and voltage are indicated along with the duty cycle. It is advisable to adhere to these values. Suppose a welding machine has maximum amperage of 300A and voltage of 50 V for a duty cycle of 60 percent. If this machine is operated at higher settings and for periods longer than 6 minutes, then its internal insulation will deteriorate and cause its early failure.

48.18. Carbon Arc Welding

(*a*) General

Carbon arc welding was the first electric welding process developed by a French inventor Auguste de Meritens in 1881. In this process, fusion of metal is accomplished by the heat of an electric arc. No pressure is used and generally, no shielding atmosphere is utilized. Filler rod is used only when necessary. Although not used extensively these days, it has, nevertheless, certain useful fields of application.

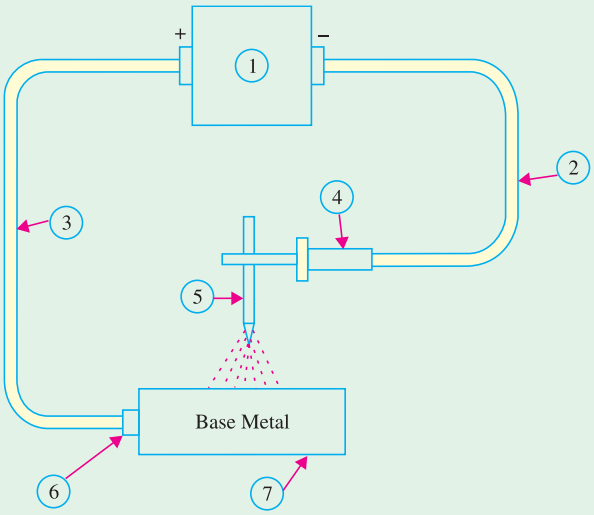
Carbon arc welding differs from the more common shield metal arc welding in that ***it uses non-consumable carbon or graphic electrodes*** instead of the consumable flux-coated electrodes.

(*b*) Welding Circuit

The basic circuit is shown in Fig. 48.15 and can be used with d.c. as well as a.c. supply. When direct current is used, the electrode is mostly negative (DCSP). The process is started by adjusting the amperage on the d.c. welder, turning welder ON and bringing the electrode into contact with the workpiece. After the arc column starts, electrode is withdrawn 25 – 40 mm away and the arc is maintained at this distance. The arc can be extinguished by simply removing the electrode from the workpiece completely. The only function of the carbon arc is to supply heat to the base metal. This heat is used to melt the base metal or filler rod for obtaining fusion weld Depending on the type and size of electrodes, maximum current values range from 15 A to 600 A for single-electrode carbon arc welding.

(*c*) Electrodes

These are made of either carbon or graphite, are usually 300 mm long and 2.5 – 12 mm in diameter. Graphite electrodes are harder, more brittle and last longer than carbon electrodes. They can withstand higher current densities but their arc column is harder to control. Though considered non-consumable, they do disintegrate gradually due to vaporisation and oxidisation.



1. welding plant
2. electrode cable
3. ground cable
4. electrode holder
5. electrode
6. earth clamp
7. work piece

Fig. 48.15

(d) Applications

1. The joint designs that can be used with carbon arc welding are butt joints, bevel joints, flange joints, lap joints and fillet joints.
2. This process is easily adaptable for automation particularly where amount of weld deposit is large and materials to be fabricated are of simple geometrical shapes such as water tanks.
3. It is suitable for welding galvanised sheets using copper-silicon-manganese alloy filler metal.
4. It is useful for welding thin high-nickel alloys.
5. Monel metal can be easily welded with this process by using a suitable coated filler rod.
6. Stainless steel of thinner gauges is often welded by the carbon-arc process with excellent results.

(e) Advantages and Disadvantages

1. The main advantage of this process is that the temperature of the molten pool can be easily controlled by simply varying the arc length.
2. It is easily adaptable to automation.
3. It can be easily adapted to inert gas shielding of the weld and
4. It can be used as an excellent heat source for brazing, braze welding and soldering etc.

Its disadvantages are as under :

1. A separate filler rod has to be used if any filler material is required.
2. Since arc serves only as a heat source, it does not transfer any metal to help reinforce the weld joint.
3. The major disadvantage of the carbon-arc process is that blow holes occur due to magnetic arc blow especially when welding near edges of the workpiece.

48.19. Submerged Arc Welding

In this ***fusion*** process, welding is done under a blanket of granulated flux which shields the weld from all bad effects of atmospheric gases while a consumable electrode is continuously and mechanically fed into the arc. The arc, the end of the bare metal electrode and the molten weld pool are all submerged under a thick mound of finely-divided granulated powder that contains deoxidisers, cleansers and other fluxing agents. The fluxing powder is fed from a hopper that is carried on the welding head itself (Fig. 48.16). This hopper spread the powder in a continuous mound ahead of the

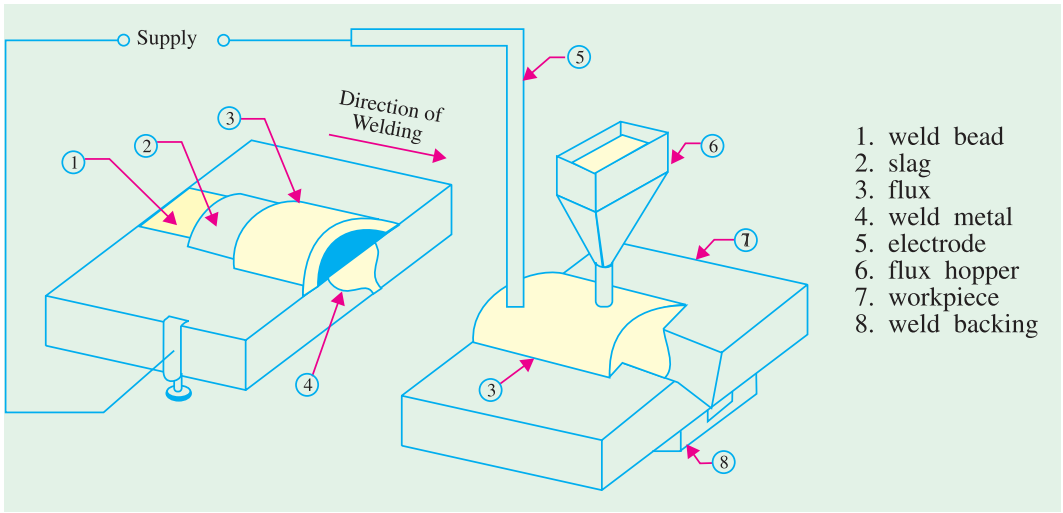


Fig. 48.16

electrode in the direction of welding. Since arc column is completely submerged under the powder, there is no splatter or smoke and, at the same time, weld is completely protected from atmospheric contamination. Because of this protection, weld beads are extremely smooth. The flux adjacent to the arc column melts and floats to the top of the molten pool where it solidifies to form slag. This slag is easy to remove. Often it cracks off by itself as it cools. The unused flux is removed and is reused again and again.

The electrode is either a bare wire or has a slight mist of copper coated over it to prevent oxidation. In automatic or semi-automatic submerged arc welding, wire electrode is fed mechanically through an electrically contacting collet. Though a.c. power supply may be used, yet d.c. supply is more popular because it assures a simplified and positive control of the welding process. This process requires high current densities about 5 to 6 times of those used in ordinary manual stick electrode welding. As a result, melting rate of the electrode as well as welding speed become much higher. Faster welding speed minimizes distortion and warpage.

The submerged arc process is suitable for

1. Welding low-alloy, high-tensile steels.

2. Welding mild, low-carbon steels.

3. Joining medium-carbon steel, heat-resistant steels and corrosion-resistant steels etc.

4. Welding nickel, Monel and other non-ferrous metals like copper.

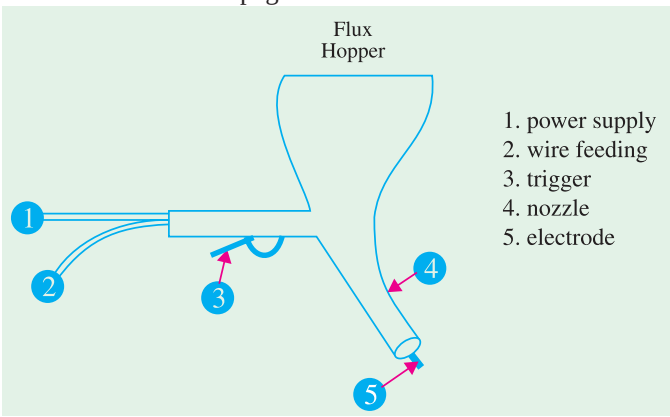


Fig. 48.17

This process has many industrial applications such as fabrication of pipes, boiler pressure vessels, railroad tank cars, structural shapes etc. which demand welding in a straight line. Welds made by this process have high strength and ductility. A major advantage of this process is that fairly thick sections can be welded in a single pass without edge preparation.

Submerged arc welding can be done manually where automatic process is not possible such as on curved lines and irregular joints. Such a welding gun is shown in Fig. 48.17. Both manual and automatic submerged arc processes are most suited for flat and slightly downhill welding positions.

48.20. Twin Submerged Arc Welding

As shown in Fig. 48.18, in this case, two electrodes are used simultaneously instead of one. Hence, weld deposit size is increased considerably. Moreover, due to increase in welding current (upto 1500 A), much deeper penetration of base metal is achieved.

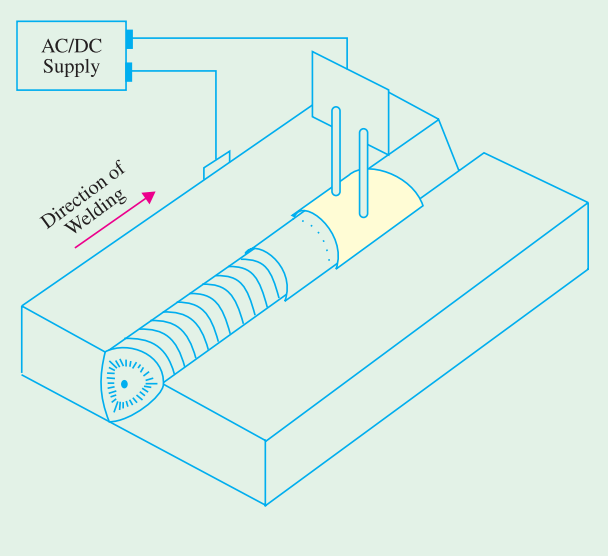


Fig. 48.18

48.21. Gas Shield Arc Welding

In this fusion process, welding is done with bare electrodes but weld zone is shielded from the atmosphere by a gas which is piped to the arc column. Shielding gases used are carbon dioxide, argon, helium, hydrogen and oxygen. No flux is required. Different processes using shielding gas are as follows.

(*a*) Tungsten inert-gas (TIG) Process

In this process, non-consumable tungsten electrode is used and filler wire is fed separately. The weld zone is shielded from the atmosphere by the inert gas (argon or helium) which is ducted directly to the weld zone where it surrounds the tungsten and the arc column.

(*b*) Metal inert-gas (MIG) Process

It is a refinement of the TIG process. It uses a bare consumable (*i.e.* fusible) wire electrode which acts as the source for the arc column as well as the supply for the filler material. The weld zone is shielded by argon gas which is ducted directly to the electrode point.

48.22. TIG Welding

(*a*) Basic Principle

It is an electric process which uses a bare non-consumable tungsten electrode for striking the arc only (Fig. 48.19). Filler material is added separately. It uses an inert gas to shield the weld puddle from atmospheric contamination. This gas is ducted directly to the weld zone from a gas cylinder.

(*b*) Welding Equipment

The usual TIG welding system consists of the following (Fig. 48.20).

1. A standard shield arc welding machine complete with cables etc.
2. A supply of inert gas complete with hose, regulators etc.
3. A source of water supply (in the case of water-cooled torches).
4. A TIG torch with a control switch to which all the above are connected.

(*c*) Electrodes

The electrodes are made of either pure tungsten or zirconiated or thoriated tungsten. Addition of zironium or thorium (0.001 to 2%) improves electron emission tremendously.

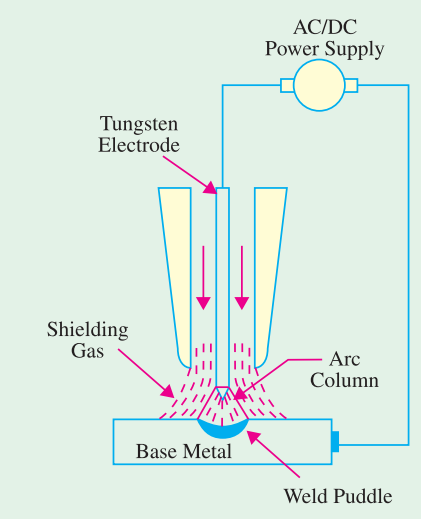


Fig. 48.19

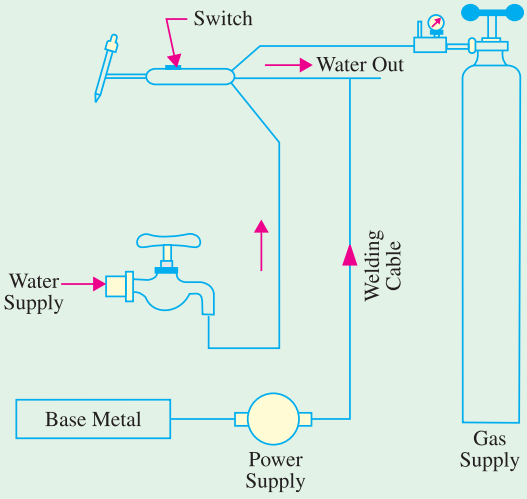


Fig. 48.20

(*d*) Power Supply

The three basic power supplies used in TIG operation are :

1. DCSP power supply–here electrode is negative, runs cooler and, hence, can be thin.
2. DCRP power supply–here electrode is positive and hot. Hence, it has to be large.
3. A.C. high frequency (ACHF) power supply–it is a combination of standard a.c. supply of 50 Hz and high-voltage high-frequency d.c. supply. The function of this d.c. supply is to sustain the arc when a.c. supply is at zero current positions.

(*e*) Advantages of TIG Welding

1. It provides maximum protection to weld bead from atmospheric contamination.
2. TIG welds are stronger, more ductile and more corrosion-resistant than those of shield metal arc welding.
3. Since no flux is used, there is no flux entrapment in the bead.
4. Since no flux is required, a wider variety of joint designs can be used.
5. No post-weld cleansing is necessary.
6. There is no weld splatter or sparks that could damage the surface of the base metal.
7. It gives relatively fast welding speeds.
8. It is suitable for welding food or medical containers where entrapment of any decaying organic matter could be extremely harmful.
9. It is suitable for all welding positions–the flat, horizontal, vertical and overhead positions. The joints suitable for TIG welding process are ***(i)*** butt joint ***(ii)*** lap joint ***(iii)*** T-joint, ***(iv)*** corner joint and ***(v)*** edge joint.

(*f*) Applications

1.	aluminium and its alloys	—	AC/DCRP
2.	magnesium and its alloys	—	ACHF
3.	stainless steel	—	DCSP
4.	mild steel, low-alloy steel, medium	—	DCSP
	-carbon steel and cast iron	—	DCSP
5.	copper and allows	—	DCSP
6.	nickel and alloys	—	DCSP

TIG welding is also used for dissimilar metals, hardfacing and surfacing of metals. Special industrial applications include manufacture of metal furniture and air-conditioning equipment.

Fig. 48.21 shows Phillips 400-D compact fan-cooled DC TIG welding set which has an open-circuit voltage of 80 V and a welding current of 400 A with 60% duty cycle and 310 A with 100% duty cycle.



Fig. 48.21 (TIG welding set)

48.23. MIG Welding

(*a*) Basic Principle

It is also called inert-gas consumable- electrode process. The fusible wire electrode is driven by the drive wheels. Its function is two-fold: to produce arc column and to provide filler material. This process uses inert gas for shielding the weld zone from atmospheric contamination. Argon is used to weld non-ferrous metals though helium gives better control of porosity and arc stability. This

process can deposit large quantities of weld metal at a fast welding speed. The process is easily adaptable to semi-automatic or fully automatic operations.

(*b*) Welding Equipment

The basic MIG welding system (Fig. 48.23) consists of the following :

1. Welding power supply
2. Inert gas supply with a regulator and flow meter
3. Wire feed unit containing controls for wire feed, gas flow and the ON/OFF switch for MIG torch
4. MIG torch
5. Depending on amperage, a water cooling unit.

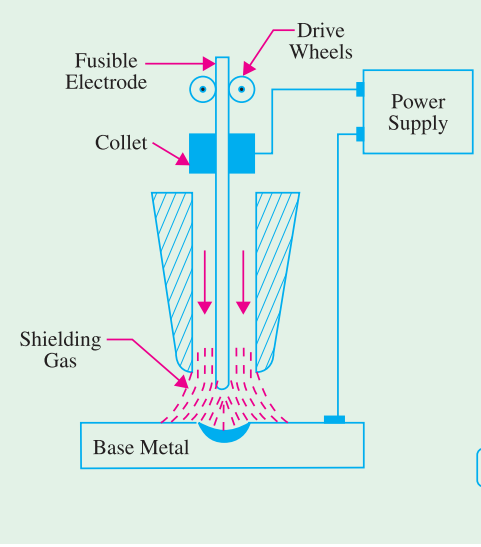


Fig. 48.22

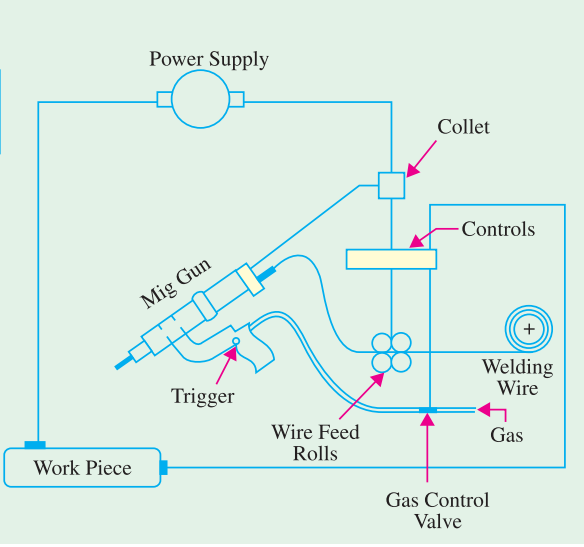


Fig. 48.23

(*c*) Electrode

It is a bare wire fed to the MIG gun by a suitable wire-feed mechanism.

(*d*) Power Supply

The major power supply used for MIG welding is DCRP and the machines which provide this supply are motor-generator sets or a.c. transformers with rectifiers (Art. 48.14). They have either CAV or RAV characteristics (Art. 48.12). The CAV supply gives the operator great latitude in arc length and is helpful in preventing the wire electrode from stubbing. A DCRP current produces deeper penetration and a cleaner weld surface than other types of current.

The RAV machines are more suitable for automatic operation. They are capable of handling large diameter wires than CAV machines.

Fig. 48.24 shows semi-automatic forced-air cooled arc welding set MIG-400. It consists of

(*i*) Indarc 400 MMR rectifier which is basically a 3-phase transformer rectifier with silicon



Fig. 48.24.
MIG-400 Welding Set. (Courtesy : Indian Oxygen Ltd. Calcutta)

diodes and a constant potential output. It provides maximum current of 400 A at 40 V for 75% duty cycle and 350 A at 42 V for 100% duty cycle.

(*ii*) Indarc Wire Feeder which has a twin roll drive system, designed to feed 0.8 to 2.4 mm diameter welding wires to a hand-operated MIG welding torch.

(*iii*) MIG Torches which are available in both air-cooled and water-cooled varieties. Fig. 48.25 (*a*) and (*b*) show light-weight swan-necked torches which are designed to operate upto 360 A and 400 A with CO_2 as shielding gas. Fig. 48.25 (*c*) shows a heavy-duty water-cooled torch designed to operate upto 550 A with CO_2/mixed shielding gases at 100% duty cycle.

(*iv*) CO_2 Kit for hard wire applications and Argon Kit for soft wire applications.

(*e*) Advantages of MIG Welding

1. Gives high metal deposit rates varying from 2 to 8 kg/h.
2. Requires no flux.
3. Requires no post-welding cleaning.
4. Gives complete protection to weld bead from atmospheric contamination.
5. Is adaptable for manual and automatic operations.
6. Can be used for a wide range of metals both ferrous and non-ferrous.
7. Is easy to operate requiring comparatively much less operating skill.
8. Is especially suited for horizontal, vertical and overhead welding positions.

(*f*) Applications

With inert gas shielding, this process is suitable for fusion welding of **(*i*)** aluminium and its alloys **(*ii*)** nickel and its alloys **(*iii*)** copper alloys **(*iv*)** carbon steels **(*v*)** low-alloy steels **(*vi*)** high strength steels and **(*vii*)** titanium.

48.24. MAG Welding

As discussed earlier, in MIG welding process, the shielding gas used is monoatomic (argon or helium) and is inert *i.e.* chemically inactive and metal transfer takes place by axial pulverization. In MAG (metal-active-gas) process, shielding, gas used is chemically active *i.e.* carbon dioxide or its mixture with other gases. Transfer of metal takes place in big drops.

48.25. Atomic Hydrogen Welding

(*a*) General

It is a non-pressure fusion welding process and the welder set is used only as heat supply for the base metal. If additional metal is required, a filler rod can be melted into the joint. It uses two tungsten electrodes between which an arc column (actually, an arc fan) is maintained by an a.c. supply.

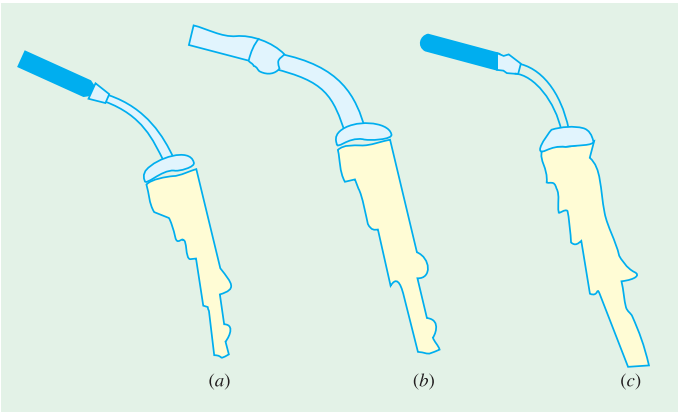


Fig. 48.25

(*b*) Basic Principle

As shown in Fig. 48.26, an arc column is struck between two tungsten electrodes with an a.c. power supply. Soon after, normal molecular hydrogen (H_2) is forced through this arc column. Due to intense heat of the arc column, this diatomic hydrogen is dissociated into atomic hydrogen (H).

However, atomic hydrogen being unstable, recombines to form stable molecular hydrogen. In so doing, it releases intense heat at about 3750°C which is used to fuse the metals.

(*c*) Welding Equipment

The welding equipment essentially consists of the following :

1. Standard welding machine consisting of a step-down transformer with tapped secondary (not shown in Fig. 48.27) energised from normal a.c. supply. Amperage requirement ranges from 15 A to 150 A
2. Hydrogen gas supply with an appropriate regulator
3. Atomic hydrogen welding torch having an ON-OFF switch and a trigger for moving the two tungsten electrodes close together for striking and maintaining the arc column.

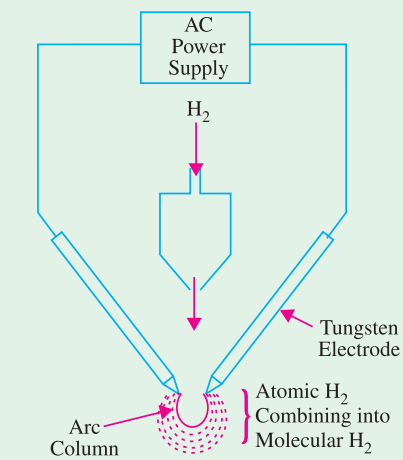


Fig. 48.26

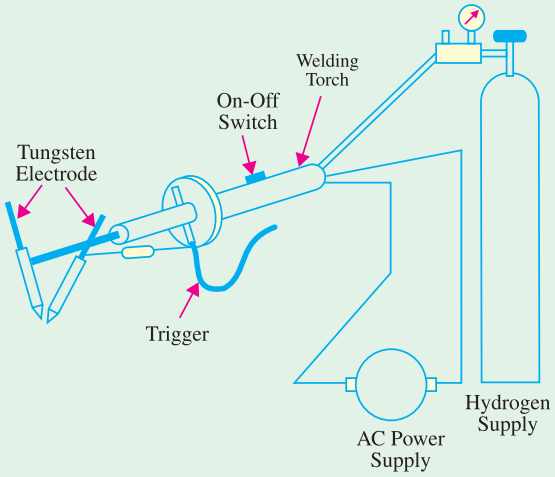


Fig. 48.27

(*d*) Method of Welding

The torch is held in the right hand with first finger resting lightly on the trigger. The arc is struck either by allowing the two tungsten electrodes to touch and separate or by drawing the separated electrodes over a carbon block. At the same time, a stream of hydrogen is allowed to flow through the arc. As soon as the arc strikes, an intensely hot flame extends fanwise between the electrodes. When this fan touches the workpiece, it melts it down quickly. If filler material is required, it can be added from the rod held in the left hand as in gas welding.

(*e*) Advantages

1. Arc and weld zone are shrouded by burning hydrogen which, being an active reducing agent, protects them from atmospheric contamination.
2. Can be used for materials too thin for gas welding.
3. Can weld quite thick sections.
4. Gives strong, ductile and sound welds.
5. Can be used for welding of mild steel, alloy steels and stainless steels and aluminium alloys.
6. Can also be used for welding of most non-ferrous metals such as nickel, monel, brass, bronze, tungsten and molybdenum etc.

48.26. Resistance Welding

It is fundamentally a heat and squeeze process. The term ***'resistance welding'*** denotes a group

of processes in which welding heat is produced by the resistance offered to the passage of electric current through the two metal pieces being welded. These processes differ from the fusion processes in the sense that no extra metal is added to the joint by means of a filler wire or electrode. According to Joule's law, heat produced electrically is given by $H = I^2Rt/J$. Obviously, amount of heat produced depends on.

(*i*) square of the current **(*ii*)** the time of current and **(*iii*)** the resistance offered.

As seen, in simple resistance welding, high-amperage current is necessary for adequate weld. Usually, R is the contact resistance between the two metals being welded together. The current is passed for a suitable length of time controlled by a timer. The various types of resistance welding processes may be divided into the following four main groups :

(*i*) spot welding **(*ii*)** seam welding **(*iii*)** projection welding and **(*iv*)** butt welding which could be further subdivided into flash welding, upset welding and stud welding etc.

Advantages

Some of the advantages of resistance welding are as under :

1. Heat is localized where required
2. Welding action is rapid
3. No filler material is needed
4. Requires comparatively lesser skill
5. Is suitable for large quantity production
6. Both similar and dissimilar metals can be welded
7. Parent metal is not harmed
8. Difficult shapes and sections can be welded.

Only disadvantages are with regard to high initial as well as maintenance cost.

It is a form of resistance welding in which the two surfaces are joined by spots of fused metal caused by fused metal between suitable electrodes under pressure.

48.27. Spot Welding

The process depends on two factors :

1. Resistance heating of small portions of the two workpieces to plastic state and

2. Application of forging pressure for welding the two workpieces.

Heat produced is $H = I^2Rt/J$. The resistance R is made up of **(*i*)** resistance of the electrodes and metals themselves **(*ii*)** contact resistance between electrodes and workpieces and **(*iii*)** contact resistance between the two workpieces. Generally, contact resistance between the two workpieces is the greatest.

As shown in 48.28 (*b*), mechanical pressure is applied by the tips of the two electrodes. In fact, these electrodes not only provide the forging pressure but also carry the welding current and concentrate the welding heat on the weld spot directly below them.



Spot welding machine

Fig. 48.28 (*a*) shows diagrammatically the basic parts of a modern spot welding. It consists of a step-down transformer which can supply huge currents (upto 5,000 A) for short duration of time.

The lower arm is fixed whereas the upper one is movable. The electrodes are made of low-resistance, hard-copper alloy and are either air cooled or butt-cooled by water circulating through the rifled drillings in the electrode. Pointed electrodes [Fig. 48.29 (*a*)] are used for ferrous materials

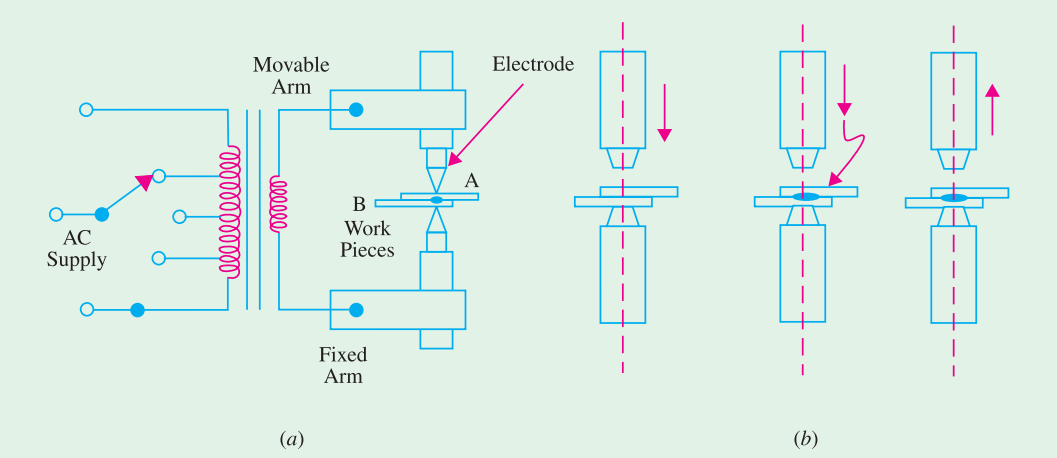


Fig. 48.28

whereas domed electrodes [Fig. 48.25 (*b*)] are used for non-ferrous materials. Flat domes are used when spot-welding deformation is not desired. The weld size is determined by the diameter of the electrode.

The welding machine is cycled in order to produce the required heat timed to coincide with the pressure exerted by the electrodes as shown in Fig. 48.28 (*a*). As the movable electrode comes down and presses the two workpieces *A* and *B* together, current is passed through the assembly. The metals under the pressure zone get heated upto about 950°C and fuse together. As they fuse, their resistance is reduced to zero, hence there is a surge of current. This surge is made to switch off the welding current automatically. In motor-driven machines, speeds of 300 strokes/minute are common.

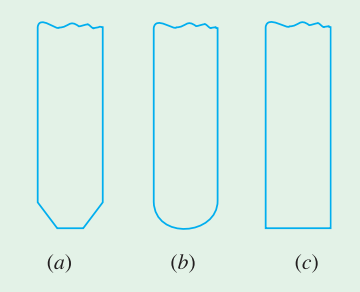


Fig. 48.29

Spot welders are of two different types. One is a stationary welder which is available in different sizes. The other has a stationary transformer but the electrodes are in a gun form.

Electric resistance spot welding is probably the best known and most widely-used because of its low cost, speed and dependability. It can be easily performed by even a semi-skilled operator. This process has a fast welding rate and quick set-up time apart from having low unit cost per weld.

Spot welding is used for galvanized, tinned and lead-coated sheets and mild steel sheet work. This technique is also applied to non-ferrous materials such as brass, aluminium, nickel and bronze etc.

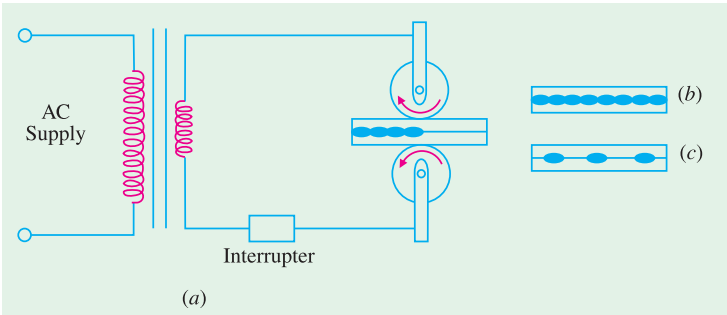


Fig. 48.30

48.28. Seam Welding

The seam welder differs from ordinary spot welder only

in respect of its electrodes which are of disc or roller shape as shown in Fig. 48.30 (*a*). These copper wheels are power driven and rotate whilst gripping the work. The current is so applied through the wheels that the weld spots either overlap as in Fig. 48.30 (*b*) or are made at regular intervals as in Fig. 48.30 (*c*). The continuous or overlapped seam weld is also called ***stitch weld*** whereas the other is called roll weld.

Seam welding is confined to welding of thin materials ranging in thickness from 2 mm to 5 mm. It is also restricted to metals having low hardenability rating such as hot-rolled grades of low-alloy steels. Stitch welding is commonly used for long water-tight and gas-tight joints. Roll welding is used for simple joints which are not water-tight or gas-tight. Seam welds are usually tested by pillow test.

48.29. Projection Welding

It can be regarded as a mass-production form of spot welding. Technically, it is a cross between spot welding and butt welding. It uses the same equipment as spot welding. However, in this process, large-diameter flat electrodes (also called platens) are used. This welding process derives its name from the fact that, prior to welding, projections are raised on the surfaces to be welded [Fig. 48.31 (*a*)]. As seen, the upper and lower platens are connected across the secondary of a step-down transformer and are large enough to cover all the projections to be welded at one stroke of the machine. When platen *A* touches the workpiece, welding current flows ***through each projection***.

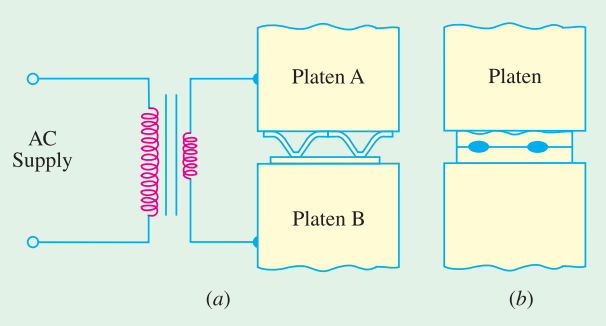


Fig. 48.31

The welding process is started by first lowering the upper platen *A* on to the work-piece and then applying mechanical pressure to ensure correctly-forged welds. Soon after, welding current is switched on as in spot welding. As projection areas heat up, they collapse and union takes place at all projections simultaneously [Fig. 48.31 (*b*)].

It is seen that projections serve many purposes :

1. They increase the welding resistance of the material locally.
2. They accurately locate the positions of the welds.
3. They speed up the welding process by making it possible to perform several small welds simultaneously.
4. They reduce the amount of current and pressure needed to form a good bond between two surfaces.
5. They prolong the life of the electrode considerably because the metal itself controls the heat produced.

Projection welding is used extensively by auto manufactures for joining nuts, bolts and studs to steel plates in car bodies. This process is especially suitable for metals like brass, aluminium and copper etc. manily due to their high thermal conductivity.

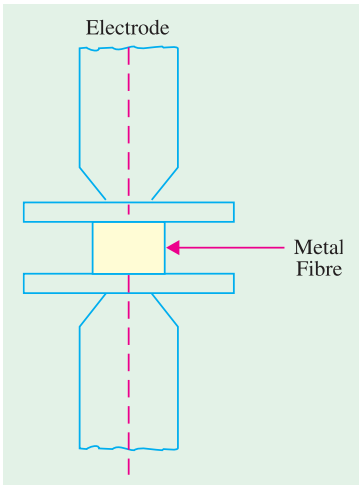


Fig. 48.32

A variation of projection welding is the metal fibre welding which uses a metal fibre rather than a projection point (Fig. 48.32). This metal fibre is generally a felt material. Instead of projections, tiny elements of this felt material are placed between the two metals which are then projection-welded in the usual way.

48.30. Butt Welding

In this case, the two workpieces are brought into contact end-to-end and the butted ends are heated by passing a heavy current through the joint. As in other forms of resistance welding, the weld heat is produced mainly by the electrical resistance of the joint faces. In this case, however, the electrodes are in the form of powerful vice clamps which hold the work-pieces and also convey the forging pressure to the joint [Fig. 48.33].

This process is useful where parts have to be joined end-to-end or edge-to-edge. *i.e.* for welding pipes, wires and rods. It is also employed for making continuous lengths of chain.

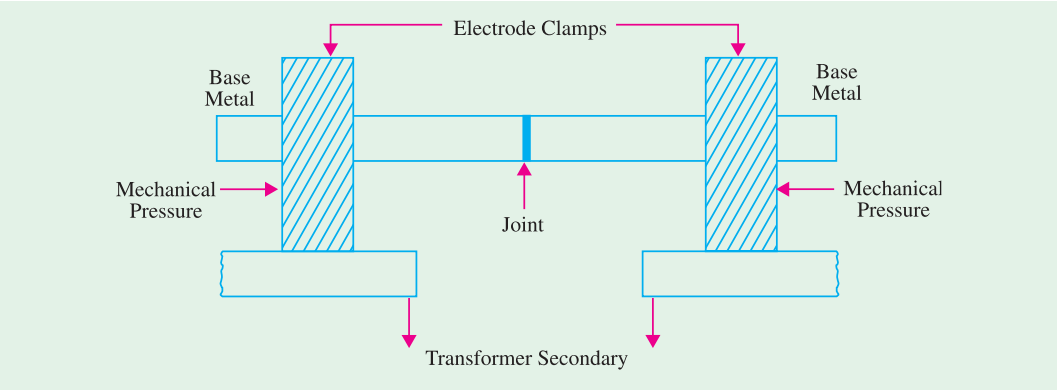


Fig. 48.33

48.31. Flash Butt Welding

It is also called by the simple name of ***flash welding***. It is similar to butt welding but with the difference that here current is applied when ends of the two metal pieces are quite close to each other ***but do not touch intimately***. Hence, an arc or flash is set up between them which supplies the necessary welding heat. As seen, in the process heat is applied ***before*** the two parts are pressed together.

As shown in Fig. 48.34 (*a*), the workpieces to be welded are clamped into specially designed electrodes one of which is fixed whereas the other is movable. After the flash has melted their faces, current is cut off and the movable platen applies the forging pressure to form a fusion weld. As shown in Fig. 48.34 (*b*), there is increase in the size of the weld zone because of the pressure which forces the soft ends together.

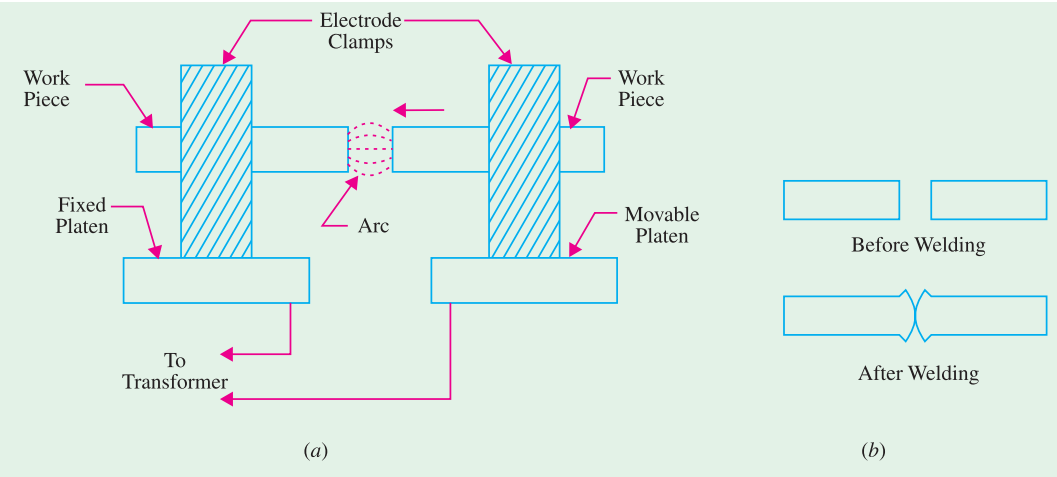


Fig. 48.34

Advantages

1. Even rough or irregular ends can be flash-welded. There is no need to level them by machining and grinding because all irregularities are burnt away during flashing period.
2. It is much quicker than butt welding.
3. It uses considerably less current than butt welding.
4. One of its major advantages is that dissimilar metals with different welding temperatures can be flash-welded.

Applications

1. To assemble rods, bars, tubings, sheets and most ferrous metals.
2. In the production of wheel rims for automobiles and bicycles.
3. For welding tubular parts such as automobile break cross-shafts.
4. For welding tube coils for refrigeration plants etc.

48.32. Upset Welding

In this process, ***no flash is allowed to occur*** between the two pieces of the metals to be welded. When the two base metals are brought together to a single interface, heavy current is passed between them which heats them up. After their temperature reaches a value of about 950°C, the two pieces of base metal are pressed together more firmly. This pressing together is called ***upsetting***. This upsetting takes place ***while current is flowing and continues even after current is switched off***. This upsetting action mixes the two metals homogeneously while pushing out many atmospheric impurities.

48.33. Stud Welding

(*a*) Basic Principle

It is similar to flash welding because it incorporates a method of drawing an arc between the stud (a rod) and the surface of the base metal. Then, the two molten surfaces are brought together under pressure to form a weld. Stud welding eliminates the need for drilling holes in the main structure.

(*b*) Welding Equipment

The stud welding equipment consists of a stud welding gun, a d.c. power supply capable of giving currents upto 400 A, a device to control current and studs and ferrules which are used not only as arc shields but also as containing walls for the molten metal.

(*c*) Applications

It is a low-cost method of fastening extensions (studs) to a metal surface. Most of the ferrous and non-ferrous metals can be stud-welded successfully. Ferrous metals include stainless steel, carbon steel and low-alloy steel. Non-ferrous metals include aluminium, lead-free brass, bronze and chrome-plated metals.

Stud welding finds application in the installations of conduit pipe hangers, planking and corrugated roofings.

This process is also used extensively in shipbuilding, railroad and automotive industries.

48.34. Plasma Arc Welding

(*a*) Basic Principle

It consists of a high-current electronic arc which is forced through a small hole in a water-coled metallic nozzle [Fig. 48.35 (*a*)]. The plasma gas itself is used to protect the nozzle from the extreme heat of the arc. The plasma arc is shielded by inert gases like argon and helium which are pumped through an extra passageway within the nozzle of the plasma torch. As seen, plasma arc consists of electronic arc, plasma gas and gases used to shield the jet column. The idea of using the nozzle is to

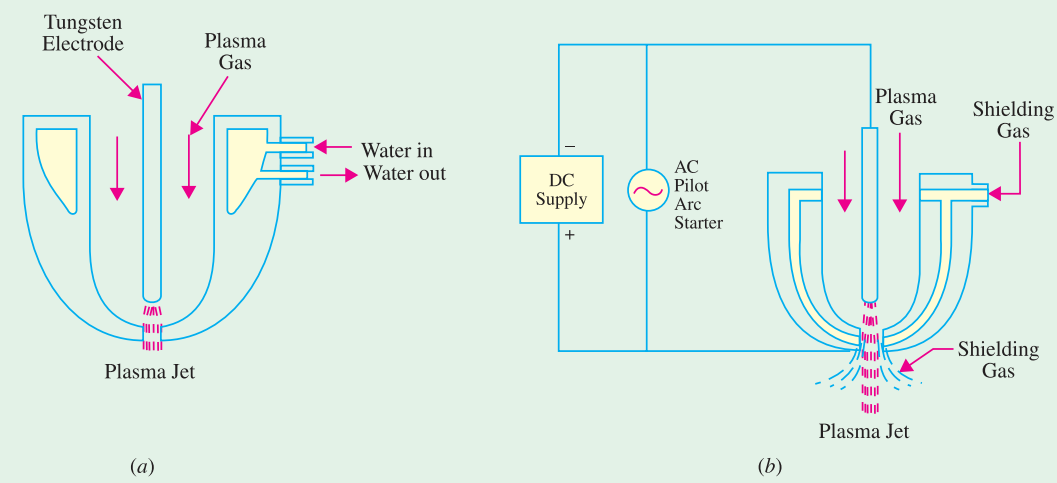


Fig. 48.35

constrict the arc thereby increasing its pressure. Collision of high-energy electrons with gas molecules produces the plasma which is swept through the nozzle and forms the current path between the electrode and the workpiece. Plasma jet torches have temperature capability of about 35,000°C.

(*b*) Electrodes

For stainless steel welding and most other metals, straight polarity tungsten electrodes are used. But for aluminium welding, reverse polarity water-cooled copper electrodes are used.



Plasma arc welding

(*c*) Power Supply

Plasma arc welding requires d.c. power supply which could be provided either by a motor-generator set or transformer-rectifier combination. The latter is preferred because it produces better arc stability. The d.c. supply should have an open-circuit voltage of about 70V and drooping voltage-ampere characteristics. A high-frequency pilot arc circuit is employed to start the arc [Fig. 48.35 (*b*)].

(*d*) Method of Welding

Welding with plasma arc jet is done by a process called 'keyhole' method. As the plasma jet strikes the surface of the workpiece, it burns a hole through it. As the torch progresses along the work-piece, this hole also progresses alongwith but is filled up by the molten metal as it moves along. obviously, 100 percent penetration is achieved in this method of welding. Since plasma jet melts a large surface area of the base metal, it produces a weld bead of wineglass design as shown in Fig. 48.36. The shape of the bead can be changed by changing the tip of the nozzle of the torch. Practically, all welding is done mechanically.



Fig. 48.36

(*e*) Applications

1. Plasma arc welding process has many aerospace applications.
2. It is used for welding of reactive metals and thin materials.
3. It is capable of welding high-carbon steel, stainless steel, maraging steel, copper and copper alloys, brass alloys, aluminium and titanium.
4. It is also used for metal spraying.

5. It can be modified for metal cutting purposes. It has been used for cutting aluminium, carbon steel, stainless steel and other hard-to-cut steels. It can produce high-quality dross-free aluminium cuts 15 cm deep.

(*f*) Disadvantages

1. Since it uses more electrical equipment, it has higher electrical hazards.
2. It produces ***ultra-violet and infra-red*** radiations necessitating the use of tinted lenses.
3. It produces high-pitched noise (100 dB) which makes it necessary for the operator to use ear plugs.

48.35. Electroslag Welding

(*a*) General

It is a metal-arc welding process and may be considered as a further development of sub-merged-arc welding.

This process is used for welding joints of thick sections of ferrous metals in a single pass and without any special joint preparation. Theoretically, there is no upper limit to the thickness of the weld bead. It is usually a vertical uphill process.

It is called ***electroslag*** process because heat is generated by passing current through the molten slag which floats over the top of the metal.

(*b*) Welding Equipment

As shown in Fig. 48.37, two water-cooled copper shoes (or dams) are placed on either side of the joint to be welded for the purpose of confining the molten metal in the joint area. The electrode is fed into the weld joint almost vertically from special wire guides. There is a mechanical device which raises the shoes and wire-feed mechanism as the weld continues upwards till it is completed. An a.c. welding machine has 100 percent duty cycle and which can supply currents upto 1000 A if needed.

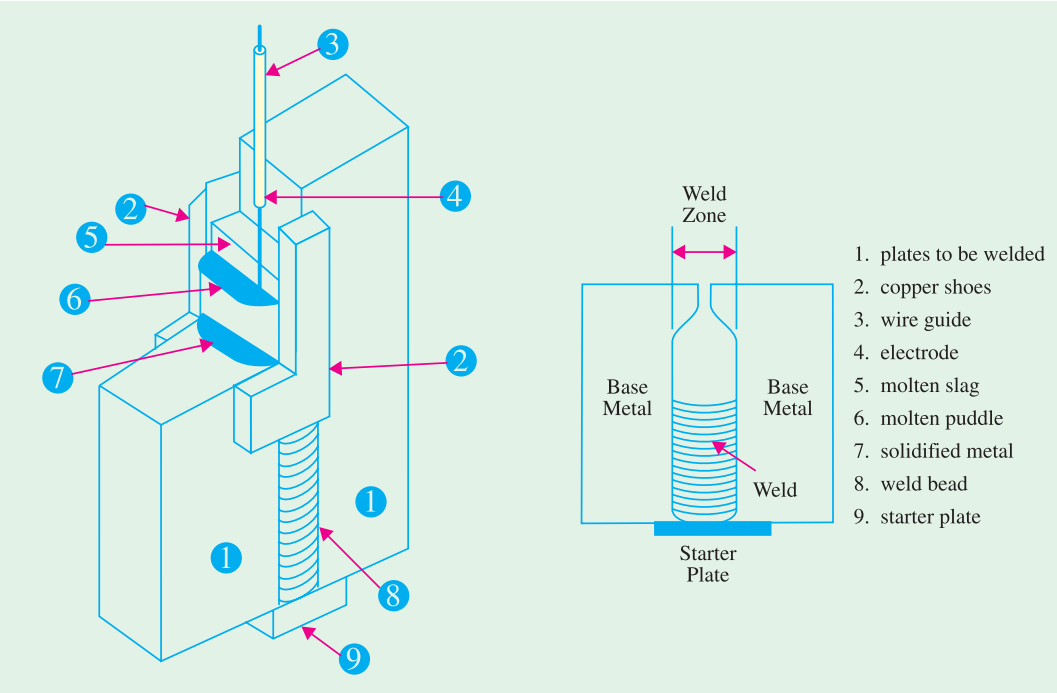


Fig. 48.37

(*c*) Welding Process

The electroslag process is initiated just like submerged arc process by starting an electric arc beneath a layer of granular welding flux. When a sufficient thick layer of hot flux or molten slag is formed, arc action stops and from then onwards, current passes from the electrode to workpiece through the molten slag. At this point, the process becomes truly electroslag welding. A starting plate is used in order to build up proper depth of conductive slag before molten pool comes in contact with the workpieces.

The heat generated by the resistance to the flow of current through the molten slag is sufficient to melt the edges of the workpiece and the filler electrode. The molten base metal and filler metal collect at the bottom of the slag pool forming the weld pool. When weld pool solidfies, weld bead is formed which joins the faces of the base metal as shown in Fig. 48.33 (*b*).

As welding is continued upwards, flux flows to the top in the form of molten slag and cleanses the impurities from the molten metal. A mechanism raises the equipment as the weld is completed in the uphill vertical position.

(*d*) Advantages

1. It needs no special joint preparation.
2. It does welding in a single pass rather than in costly multiple passes.
3. There is theoretically no maximum thickness of the plate it can weld.
4. There is also no theoretical upper limit to the thickness of the weld bead. Weld beads upto 400 mm thick have been performed with the presenttly-available equipment.
5. This process requires less electrical power per kg of deposited metal than either the submerged arc welding process or the shield arc process.
6. It has high deposit rate of upto 20 kg of weld metal per hour.
7. It has lower flux consumption.
8. Due to uniform heating of the weld area, distortion and residual stresses are reduced to the minimal amounts.

However, for electroslag welding, it is necessary to have only a square butt joint or a square edge on the plates to be welded.

(*e*) Applications

It is commonly used in the fabrication of large vessels and tanks. Low-carbon steels produce excellent welding properties with this process.

48.36. Electrogas Welding

This process works on the same basic principle as the electroslag process but has certain additional features of submerged arc welding. Unlike electroslag process, the electrogas process uses an inert gas for shielding the weld from oxidation and there is a continuous arc (as in submerged arc process) to heat the weld pool.

48.37. Electron Beam Welding

In this process, welding operation is performed in a vacuum chamber with the help of a sharply-focussed beam of high-velocity electrons. The electrons after being emitted from a suitable electrode are accelerated by the high anode voltage and are then focussed into a fine beam which is finally directed to the workpiece. Obviously, this process needs no electrodes. The electron beam produces intense local heat which can melt not only the metal but can even boil it. A properly-focussed electron beam can completly penetrate through the base metal thereby creating a small hole whose walls are molten. As the beam moves along the joint, it melt the material coming in contact with it. The molten metal flows back to the previously-melted hole where it fuses to make a perfect weld for the entire depth of penetration.

Electron-beam welding has following advantages :

1. It produces deep penetration with little distortion.
2. Its input power is small as compared to other electrical welding devices.
3. Electron-beam weld is much narrower than the fusion weld.
4. It is especially suitable for reactive metals which become contaminated when exposed to air because this process is carried out in vacuum.
5. It completely eliminates the contamination of the weld zone and the weld bead because operation is performed in a vacuum chamber.



Electron beam welding facility

6. It is especially suited to the welding of beryllium which is being widely used in the fabrication of industrial and aerospace components.
7. Its high deposition rate produces welds of excellent quality with only a single pass.
8. It is the only process which can join high temperature metals such as columbium.

At present, its only serious limitations are that it is extremely expensive and is not available in portable form. However, recently a non-vacuum electron-beam welder has been developed.

48.38. Laser Welding

It uses an extremely concentrated beam of coherent monochromatic light *i.e.* light of only one colour (or wavelength). It concentrates tremendous amount of energy on a very small area of the workpiece to produce fusion. It uses solid laser (ruby, saphire), gas laser (CO_2) and semiconductor laser. Both the gas laser and solid laser need capacitor storage to store energy for later injection into the flash tube which produces the required laser beam.

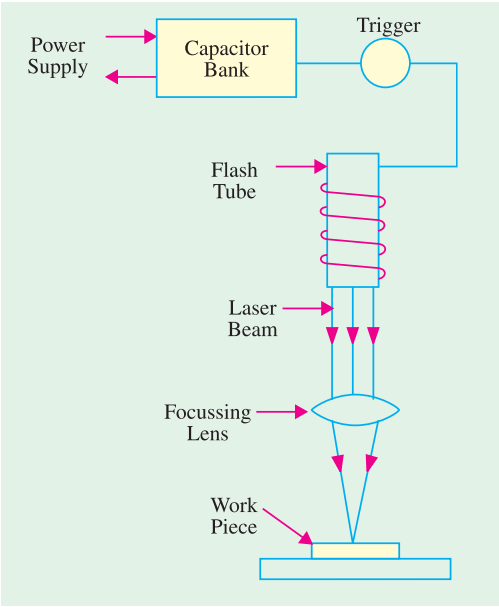


Fig. 48.38

The gas laser welding equipment consists of **(*i*)** capacitor bank for energy storage **(*ii*)** a triggering device **(*iii*)** a flash tube that is wrapped with wire **(*iv*)** lasing material **(*v*)** focussing lens and **(*vi*)** a worktable that can rotate in the three X, Y and Z directions.

When triggered, the capacitor bank supplies electrical energy to the flash tube through the wire. This energy is then converted into short-duration beam of laser light which is pin-pointed on the workpiece as shown in Fig. 48.38. Fusion takes place immediately and weld is completed fast.

Since duration of laser weld beam is very short (2 ms or so), two basic welding methods have been adopted. In the first method, the workpiece is moved so fast that the entire joint is welded in a single burst of the light. The other method uses a number of pulses one after the other to form the weld joint similar to that formed in electric resistance seam welding (Art 48.31).

Laser welding is used in the aircraft and electronic industries for lighter gauge metals.

Some of the advantages of laser welding process are as follows :

1. It does not require any electrode.
2. It can make welds with high degree of precision and on materials as thin as 0.025 mm.
3. It does not heat the workpiece except at one point. In fact, heat-affected zone is virtually non-existent.
4. Liquidus is reached only at the point of fusion.
5. It can produce glass-to-metal seals as in the construction of klystron tubes.
6. Since laser beam is small in size and quick in action, it keeps the weld zone uncontaminated.
7. It can weld dissimilar metals with widely varying physical properties.
8. It produces minimal thermal distortion and shrinkage because area of heat-affected zone is the minimum possible.
9. It can easily bond refractory materials like molybdenum, titanium and tantalium etc.

However, the major disadvantage of this process is its slow welding speed. Moreover, it is limited to welding with thin metals only.

Tutorial Problem No. 48.1

1. Describe various types of electric arc welding processes.
(J.N. University, Hyderabad, December 2002/January 2003)
2. Compare resistance welding and arc welding.
(J.N. University, Hyderabad, December 2002/January 2003)
3. Briefly explain the different methods of electric welding and state their relative merits.
(J.N. University, Hyderabad, December 2002/January 2003)
4. Give the comparison between A.C. and D.C. welding.
(J.N. University, Hyderabad, December 2002/January 2003)
5. Explain the different methods of electric welding and their relative advantages.
(J.N. University, Hyderabad, December 2002/January 2003)

OBJECTIVE TESTS – 48

1. The basic *electrical* requirement in arc welding is that there should be
(*a*) coated electrodes
(*b*) high open-circuit voltage
(*c*) no arc blow
(*d*) d.c. power supply.
2. Welding is not done directly from the supply mains because
(*a*) it is customary to use welding machines
(*b*) its voltage is too high
(*c*) its voltage keeps fluctuating
(*d*) it is impracticable to draw heavy currents.
3. A.C. welding machine cannot be used for welding
(*a*) MIG
(*b*) atomic hydrogen
(*c*) resistance
(*d*) submerged arc.
4. In electric welding, arc blow can be avoided by
(*a*) using bare electrodes
(*b*) welding away from earth ground connection
(*c*) using a.c. welding machines
(*d*) increasing arc length.

5. In DCSP welding
   (*a*) electrode is the hottest
   (*b*) workpiece is relatively coole
   (*c*) base metal penetration is deep
   (*d*) heavily-coated electrodes are used.
6. Overhead welding position is thought to be the most
   (*a*) hazardous
   (*b*) difficult
   (*c*) economical
   (*d*) useful.
7. The *ultimate* aim of using electrode coating is to
   (*a*) provide shielding to weld pool
   (*b*) prevent atmospheric contamination
   (*c*) improve bead quality
   (*d*) cleanse the base metal.
8. In electrode-positive welding ..... of the total heat is produced at the electrode.
   (*a*) one-third
   (*b*) two-third
   (*c*) one-half
   (*d*) one-fourth.
9. Submerged arc process is characterised by
   (*a*) deep penetration
   (*b*) high welding current
   (*c*) exceptionally smooth beads
   (*d*) all of the above.
10. The major disadvantage of carbon arc welding is that
    (*a*) there is occurrence of blow holes
    (*b*) electrodes are consumed fast
    (*c*) separate filler rod is needed
    (*d*) bare electrodes are necessary.
11. In atomic hydrogen welding, electrodes are long-lived because
    (*a*) two are used at a time
    (*b*) arc is in the shape of a fan
    (*c*) of a.c. supply
    (*d*) it is a non-pressure process.
12. Unlike TIG welding, MIG welding
    (*a*) requires no flux
    (*b*) uses consumable electrodes
    (*c*) provides complete protection from atmospheric contamination
    (*d*) requires no post-weld cleansing.
13. The major power supply used in MIG welding is
    (*a*) a.c. supply
    (*b*) DCSP
    (*c*) electrode-negative
    (*d*) DCRP.
14. MIG welding process is becoming increa- singly popular in welding industry mainly because of
    (*a*) its easy operation
    (*b*) its high metal deposit rate
    (*c*) its use in both ferrous and non-ferrous metals.
    (*d*) both (*a*) and (*b*).
15. A weld bead of wineglass design is produced in ..... welding.
    (*a*) plasma arc
    (*b*) electron beam
    (*c*) laser
    (*d*) MAG.
16. Spot welding process basically depends on
    (*a*) Ohmic resistance
    (*b*) generation of heat
    (*c*) application of forging pressure
    (*d*) both (*b*) and (*c*).
17. Electric resistance seam welding uses ..... electrodes.
    (*a*) pointed
    (*b*) disc
    (*c*) flat
    (*d*) domed.
18. Projection welding can be regarded as a mass production form of ..... welding.
    (*a*) seam
    (*b*) butt
    (*c*) spot
    (*d*) upset.
19. In the process of electroslag welding, theoretically there is no upper limit to the
    (*a*) thickness of weld bead
    (*b*) rate of metal deposit
    (*c*) slag bath temperature
    (*d*) rate of slag consumption.
20. High temperature metals like columbium can be easily welded by..... welding.
    (*a*) flash
    (*b*) MIG
    (*c*) TIG
    (*d*) electron beam.
21. During resistance welding heat produced at the joint is proportional to
    (*a*) I^2R
    (*b*) kVA
    (*c*) current
    (*d*) voltage
22. Grey iron in usually welded by ..... welding
    (*a*) gas
    (*b*) arc

(*c*) resistance
(*d*) MIG

23. The metal surfaces, for electrical resistance welding must be.....
(*a*) lubricated
(*b*) cleaned
(*c*) moistened
(*d*) rough

24. In a welded joint poor fusion is due to which of the following?
(*a*) Improper current
(*b*) High welding speed
(*c*) Uncleaned metal surface
(*d*) Lack of flux

25. For arc welding, D.C. is produced by which of the following?
(*a*) motor-generator set
(*b*) regulator
(*c*) transformer
(*d*) none of the above

26. ..... welding process uses consumable electrodes.
(*a*) TIG
(*b*) MIG
(*c*) Laser
(*d*) All of the above

27. Which of the following equipment is generally used for arc welding?
(*a*) single phase alternator
(*b*) two phase alternator
(*c*) three phase alternator
(*d*) transformer

28. Which of the following is not an inert gas?
(*a*) argon
(*b*) carbon dioxide
(*c*) helium
(*d*) all of the above

29. Electronic components are joined by which of the following methods?
(*a*) brazing
(*b*) soldering
(*c*) seam welding
(*d*) spot welding
(*e*) none of the above

30. Resistance welding cannot be used for
(*a*) dielectrics
(*b*) ferrous materials
(*c*) non-ferrous metals
(*d*) any of the above

31. Electric arc welding process produces temperature upto
(*a*) $1000^{o}C$
(*b*) $1500^{o}C$
(*c*) $3500^{o}C$
(*d*) $5550^{o}C$

32. Increased heat due to shorter arc is harmful on account of
(*a*) under-cutting of base material
(*b*) burn through
(*c*) excessive porosity
(*d*) all of the above

33. Arc blow results in which of the following?
(*a*) Non-uniform weld beads
(*b*) Shallow weld puddle given rise to weak weld
(*c*) Splashing out of metal from weld puddle
(*d*) All of the above defects

34. Inseam welding
(*a*) the work piece is fixed and disc electrodes move
(*b*) the work piece moves but rotating electrodes are fixed
(*c*) any of the above
(*d*) none of the above

35. In arc welding major personal hazards are
(*a*) flying sperks
(*b*) weld spatter
(*c*) harmful infrared and ultra-violet rays from the arc
(*d*) all of the above

36. In spot welding composition and thickness of the base metal decides
(*a*) the amount of squeeze pressure
(*b*) hold time
(*c*) the amount of weld current
(*d*) all above

37. Helium produces which of the following?
(*a*) deeper penetration
(*b*) faster welding speeds
(*c*) narrower heat affected zone in base metal
(*d*) all of the above

38. Due to which of the following reasons aluminium is difficult to weld?
(*a*) it has an oxide coating
(*b*) it conducts away heat very rapidly
(*c*) both (a) and (b)
(*d*) none of the above

39. Welding leads have
(*a*) high flexibility
(*b*) high current handling capacity
(*c*) both (a) and (b)
(*d*) none of the above

40. Air craft body is
(*a*) spot welded

(*b*) gas welded
(*c*) seam welded
(*d*) riveted

41. For arc welding current range is usually
(*a*) 10 to 15 A
(*b*) 30 to 40 A
(*c*) 50 to 100 A
(*d*) 100 to 350 A

42. Spot welding is used for
(*a*) thin metal sheets
(*b*) rough and irregular surfaces
(*c*) costings only
(*d*) thick sections

43. Galvanising is a process of applying a layer of
(*a*) aluminium
(*b*) lead
(*c*) copper
(*d*) zinc

44. A seamless pipe has
(*a*) steam welded joint
(*b*) spot welded joint
(*c*) arc welded joint
(*d*) no joint

45. Motor-generator set for D.C. arc welding has generator of
(*a*) series type
(*b*) shunt type
(*c*) differentially compound type
(*d*) level compound type

46. Plain and butt welds may be used on materials upto thickness of nearly
(*a*) 5 mm
(*b*) 10 mm
(*c*) 25 mm
(*d*) 50 mm

47. In argon arc welding argon is used as a
(*a*) flux
(*b*) source of heat
(*c*) agent for heat transfer
(*d*) shield to protect the work from oxidation

48. During arc welding as the thickness of the metal to be welded increases
(*a*) current should decrease, voltage should increase
(*b*) current should increases, voltage remaining the same
(*c*) current should increase, voltage should decrease
(*d*) voltage should increase, current remaining the same

49. In D.C. arc welding
(*a*) electrode is made positive and workpiece negative
(*b*) electrode is made negative and workpiece positive
(*c*) both electrode as well as workpiece are made positive
(*d*) both electrode as well as workpiece are made negative

50. The purpose of coating on arc welding electrodes is to
(*a*) stabilise the arc
(*b*) provide a protecting atmosphere
(*c*) provide slag to protect the molten metal
(*d*) all of the above

51. 50 percent duty cycle of a welding machine means
(*a*) machine input is 50 percent of rated input
(*b*) machine efficiency is 50 percent
(*c*) machine work on 50 percent output
(*d*) machine works for 5 minutes in a duration of 10 minutes

ANSWERS

1. (*b*)	**2.** (*d*)	**3.** (*a*)	**4.** (*c*)	**5.** (*c*)	**6.** (*a*	**7.** (*c*)	**8.** (*b*)	**9.** (*d*)
10. (*a*)	**11.** (*c*)	**12.** (*b*)	**13.** (*d*)	**14.** (*d*)	**15.** (*a*)	**16.** (*d*)	**17.** (*b*)	**18.** (*c*)
19. (*a*)	**20.** (*d*)	**21.** (*a*)	**22.** (*a*)	**23.** (*b*)	**24.** (*a*)	**25.** (*a*)	**26.** (*b*)	**27.** (*d*)
28. (*b*)	**29.** (*b*)	**30.** (*a*)	**31.** (*d*)	**32.** (*d*)	**33.** (*d*)	**34.** (*c*)	**35.** (*d*)	**36.** (*d*)
37. (*d*)	**38.** (*c*)	**39.** (*d*)	**40.** (*c*)	**41.** (*d*)	**42.** (*a*)	**43.** (*d*)	**44.** (*d*)	**45.** (*d*)
46. (*c*)	**47.** (*d*)	**48.** (*b*)	**49.** (*b*)	**50.** (*d*)	**51.** (*d*)			

GO To FIRST

CONTENTS

CHAPTER 49

ILLUMINATION

Learning Objectives

- Radiations from a Hot Body
- Solid Angle
- Definitions
- Calculation of Luminance (L)
- Laws of Illumination or Illuminance
- Polar Curves of C.P. Distribution
- Uses of Polar Curves
- Determination of M.S.C.P and M.H.C.P. from Polar Diagrams
- Integrating Sphere or Photometer
- Diffusing and Reflecting Surfaces
- Lighting Schemes
- Illumination Required for Different Purposes
- Space / Height Ratio
- Design of Lighting Schemes and Layouts
- Utilisation Factor ([h]
- Depreciation Factor (P)
- Floodlighting
- Artificial Source of Light
- Incandescent Lamp
- Filament Dimensions
- Incandescent Lamp Characteristics
- Clear and Inside
- Frosted Gas-filled Lamps
- Discharge Lamps
- Sodium Vapour Lamp



When some materials are heated above certain temperatures, they start radiating energy in the form of light. This phenomenon is called luminance. Electric lamps are made based on this phenomenon.

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49.1. Radiations From a Hot Body

The usual method of producing artificial light consists in raising a solid body or vapour to incandescence by applying heat to it. It is found that as the body is gradually heated above room temperature, it begins to radiate energy in the surrounding medium in the form of electromagnetic waves of various wavelengths. The ***nature*** of this radiant energy depends on the temperature of the hot body. Thus, when the temperature is low, radiated energy is in the form of heat waves only but when a certain temperature is reached, light waves are also radiated out in addition to heat waves and the body becomes luminous. Further increase in temperature produces an increase in the amount of both kinds of radiations but the colour of light or visible radiation changes from bright red to orange, to yellow and then finally, if the temperature is high enough, to white. As temperature is increased, the wavelength of the visible radiation goes on becoming shorter. It should be noted that heat waves are identical to light waves except that they are of longer wavelength and hence produce no impression on the retina. Obviously, from the point of view of light emission, heat energy represents so much wasted energy.

The ratio $\frac{\text{energy radiated out in the form of light}}{\text{total energy radiated out by the hot body}}$ is called the ***radiant*** efficiency of the luminous source and, obviously, depends on the temperature of the source. As the temperature is increased beyond that at which light waves were first given off, the radiant efficiency increases, because light energy will increase in greater proportion than the total radiated energy. When emitted light becomes white *i.e.*, it includes all the visible wavelengths, from extreme red to extreme violet, then a further increase in temperature produces radiations which are of wavelength smaller than that of violet radiations. Such radiations are invisible and are known as ultra-violet radiations. It is found that maximum radiant efficiency would occur at about 6200°C and even then the value of this maximum efficiency would be 20%. Since this temperature is far above the highest that has yet been obtained in practice, it is obvious that the actual efficiency of all artificial sources of light *i.e.* those depending on ***temperature incandescence***, is low.

As discussed above, light is radiant energy which is assumed to be propagated in the form of transverse waves through an invisible medium known as ether. These light waves travel with a velocity of 2.99776×10^8 m/s or 3×10^8 m/s approximately but their wavelengths are different. The wavelength of red light is nearly 0.000078 cm and that of violet light 0.000039 cm. Since these wavelengths are very small, instead of using 1 cm as the unit for their measurement, a submultiple 10^{-8} cm is used. This submultiple is known as Angstrom Unit (A.U.)

$$1 \text{ A.U.} = 10^{-8} \text{ cm} = 10^{-10} \text{ m}$$

Hence, the wave-length of red light becomes $\lambda_r = 7800 \times 10^{-10}$ m or 7800 A.U. and $\lambda_v = 3900 \times 10^{-10}$ m or 3900 A.U. The sensation of colour is due to the difference in the wavelengths and hence frequencies of the light radiations.

49.2. Solid Angle

Consider an area A which is part of a sphere of radius r (Fig. 49.1). Let us find the solid angle ω subtended by this area at the centre C of the sphere. For this purpose, let point C be joined to every point on the edges of the area A. Then, the angle enclosed by the cone at point C gives the solid angle. Its value is

$$\omega = \frac{A}{r^2} \text{ steradian}$$

The unit of solid angle is ***steradian*** (sr). If, in the above equation, $A = r^2$, then $\omega = 1$ steradian. Hence, steradian is defined as the angle subtended at the centre of a sphere by a part of its surface having an area equal to (radius)^2.

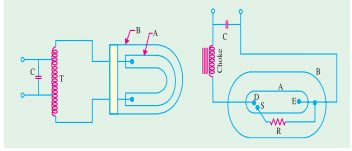


Fig. 49.1

Obviously, the solid angle subtended at the centre by whole of the spherical surface = $4\pi r^2/r^2 = 4\pi$ steradian (sr).



Fireworks radiate light energy of different frequencies, which appear in different colours

49.3. Definitions

Before proceeding further, definitions of a few principal terms employed in connection with illumination, are given below :

1. Candela. It is the unit of luminous intensity of a source. It is defined as 1/60th of the luminous intensity per cm^2 of a black body radiator at the temperature of solidification of platinum (2045°K).

A source of one candela (*cd*) emits one lumen per steradian. Hence, total flux emitted by it allround is $4\pi \times 1 = 4\pi$ lumen.

2. Luminous Flux (F or Φ). It is the light energy radiated out per second from the body in the form of luminous light waves.

Since, it is a rate of flow of energy, it is a sort of *power* unit. Unit of luminous flux is *lumen* (lm). It is defined as the ***flux contained per unit solid angle of a source of one candela or standard candle*** (Fig. 49.2).

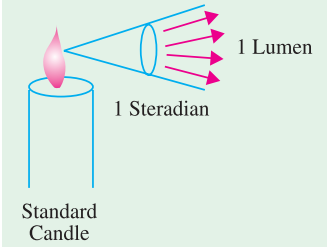


Fig. 49.2

Approximate relation between lumen and electric unit of power *i.e.* watt is given as

$$1 \text{ lumen} = 0.0016 \text{ watt (approx.)}$$

3. Lumen-hour. It is the quantity of light delivered in one hour by a flux of one lumen.*

4. Luminous Intensity (I) or Candle-power of a point source in any particular direction is given by the ***luminous flux radiated out per unit solid angle in that direction***. In other words, it is solid angular flux density of a source in a specified direction.

If $d\Phi$ is the luminous flux radiated out by a source within a solid angle of $d\omega$ steradian in any particular direction, then $I = d\Phi/d\omega$.

If flux is measured in lumens and solid angle in steradian, then its unit is lumen/steradian (lm/sr) or candela (*cd*).

If a source has an average luminous intensity of I lm/sr (or I candela), then total flux radiated by it all around is $\Phi = \omega I = 4\pi I$ lumen.

Generally, the luminous intensity or candle power of a source is different in different directions. The average candle-power of a source is the average value of its candle power in all directions. Obviously, it is given by total flux (in lm) emitted in all directions in all planes divided by 4π. This average candle-power is also known as ***mean spherical candle-power*** (M.S.C.P.).

$$\therefore \quad \text{M.S.C.P.} = \frac{\text{total flux in lumens}}{4\pi}$$

If the average is taken over a hemisphere (instead of sphere), then this average candle power is known as ***mean hemispherical candle-power*** (M.H.S.C.P.).

It is given by the total flux emitted in a hemisphere (usually the lower one) divided by the solid angle subtended at the point source by the hemisphere.

$$\therefore \quad \text{M.H.S.C.P.} = \frac{\text{flux emitted in a hemisphere}}{2\pi}$$

* It is similar to watt-hour (Wh)

5. Reduction Factor of a source is given by the ratio, f = M.S.C.P./M.H.C.P. where M.H.C.P. is the mean horizontal candle power.

It is also referred to as spherical reduction factor.

6. Illuminance or Illumination (E). When the luminous flux falls on a surface, it is said to be illuminated. The illumination of a surface is measured by the normal luminous flux per unit area received by it.

If $d\Phi$ is the luminous flux incident normally on an area dA, then $E = d\Phi/dA$ or $E = \Phi/A$.

Unit. Since flux Φ is measured in lumens and area in m^2, unit of E is lm/m^2 or lux. The alternative name is metre-candle (m-cd). Let us see why ? Imagine a sphere of radius of one metre around a point source of one candela. Flux radiated out by this source is 4π lumen. This flux falls normally on the curved surface of the sphere which is = $4\pi m^2$. Obviously, illumination at every point on the inner surface of this sphere is $4\pi\ lm/4\pi\ m^2 = 1\ lm/m^2$. However, the term lm/m^2 is to be perferred to metre-candle.

7. Luminance (L) of an Extended Source. Suppose ΔA is an element of area of an ***extended*** source and Δt its luminous intensity when viewed in a direction making an angle ϕ with the perpendicular to the surface of the source (Fig. 49.3), then luminance of the source element is given by

$$L = \frac{\Delta I}{\Delta A \cos\phi} = \frac{\Delta I}{\Delta A'}\ cd/m^2 \quad ...(i)$$

where $\Delta A' = \Delta A \cos\phi$

= area of the source element projected onto a plane perpendicular to the specified direction.

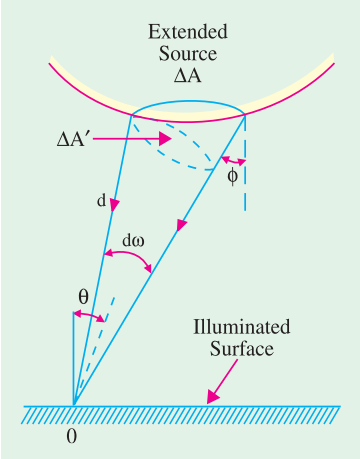


Fig. 49.3

As will be seen from Art. 49.5.

$$E = \frac{I\cos\theta}{d^2} \quad \text{or} \quad \Delta E = \frac{\Delta I}{d^2}\cos\theta$$

Substituting the value of ΔI from Eq. (i) above, we get

$$\Delta E = \frac{L.\Delta A'}{d^2}\cos\theta = L\cos\theta.\, d\omega$$

where $d\omega = \Delta A'/d^2$ steradian

$$E = \int L\cos\theta\,.\,d\omega = L\int \cos\theta\,.\,d\omega$$

—if L is constant.

8. Luminous Exitance (M) of a Surface. The luminous exitance (M) at a point on a surface is defined as luminous flux emitted per unit area in all directions. If an element of an illuminated area ΔA emits a total flux of $\Delta\Phi$ in all directions (over a solid angle of 2π steradian) then

$$M = \Delta\Phi/\Delta A \qquad lm/m^2$$

It can be proved that $M = \pi L$ in the case of a uniform diffuse ***source***.

9. Transmittance (T) of an Illuminated Diffuse Reflecting Surface. It is defined as the ratio of the total luminous flux transmitted by it to the total flux incident on it.

The relation between luminous exitance (M) of a surface transmitting light and illuminance (E) on the other side of it is

$$M = TE \quad \text{or} \quad T = M/E$$

Since light falling on a surface is either transmitted, reflected or absorbed the following relation holds good

$$T + \rho + \alpha = 1$$

where α is the absorptance of the surface.

10. Reflection Ratio or Coefficient of Reflection or Reflectance (ρ). It is given by the luminous flux reflected from a small area of the surface to the total flux incident upon it

$$\rho = M/E \text{ } i.e. \text{ ratio of luminous exitance and illuminance.}$$

It is always less than unity. Its value is zero for ideal 'black body' and unity for a perfect reflector.

11. Specific Output or Efficiency of a lamp is the ratio of luminous flux to the power intake. Its unit is lumen/watt (lm/W). Following relations should be taken note of :

(*a*) $$\frac{\text{lumen}}{\text{watt}} = \frac{4\pi \times \text{M.S.C.P.}}{\text{watt}}$$

or $$\frac{\text{lm}}{\text{W}} = \frac{4\pi}{\text{watt/M.S.C.P.}}$$

(*b*) since f = M.S.C.P./M.H.C.P. $\therefore$ $\text{lm/W} = \dfrac{4\pi f}{\text{watt/M.S.C.P.}}$

(*c*) Obviously, $\text{watts/M.S.C.P.} = \dfrac{4\pi}{\text{1m/W}} = \dfrac{\text{watt/M.H.C.P.}}{f}$

(*d*) Also $\text{watts/M.H.C.P.} = \dfrac{4\pi f}{\text{1m/W}} = f \times \text{watts/M.S.C.P.}$

12. Specific Consumption. It is defined as the ratio of the power input to the average candle-power. It is expressed in terms of watts per average candle or watts/M.S.C.P.

The summary of the above quantities along with their units and symbol is given in Table 49.1.

Table 49.1

Name of Qty	*Unit*	*Symbols*
Luminous Flux	Lumen	F or Φ
Luminous Intensity (candle-power)	Candela	I
Illumination or Illuminance	lm/m^2 or lux	E
Luminance or Brightness	cd/m^2	L or B
Luminous Exitance	lm/m^2	M

49.4. Calculation of Luminance (L) of a Diffuse Reflecting Surface

The luminance (or brightness) of a surface largely depends on the character of the surface, if it is itself not the emitter. In the case of a polished surface, the luminance depends on the angle of viewing. But if the surface is matt and diffusion is good, then the luminance or brightness is practically independent of the angle of viewing. However, the reflectance of the surface reduces the brightness proportionately. In Fig. 49.4 is shown a perfectly diffusing surface of small area A. Suppose that at point M on a hemisphere with centre O and radius R, the illuminance is L cd/m^2. Obviously, ***luminous intensity*** at point M is $= L \times A \cos\theta$ candela (or lumen/steradian). Now, the hemisphere can be divided into a number of zones as shown. Consider one such zone MN between θ and $(\theta + d\theta)$. The width of this zone is $R.d\theta$ and length $2\pi R \sin\theta$ so that its area (shown shaded) is $= 2\pi R^2 \sin\theta . d\theta$. Hence, it subtends a solid angle $= 2\pi R^2 \sin\theta . d\theta / R^2 = 2\pi \sin\theta . d\theta$ steradian at point O. The luminous flux passing through this zone is

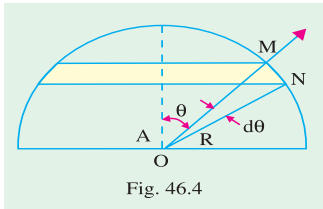


Fig. 46.4

Fig. 49.4

$$d\Phi = L\ A \cos\theta \times 2\pi \sin\theta . d\theta = \pi L\ A \times 2 \sin\theta \cos\theta\ d\theta = \pi L A \sin 2\theta\ d\theta \text{ lumen}$$

Total flux passing through the whole hemisphere is

$$\Phi = \int_0^{\pi/2} \pi L A \sin 2\theta . d\theta = \pi L A \text{ lumen}$$

If the ***illumination*** of the surface (produced by a light source) is E lm/m^2 and ρ is its reflection coefficient, then $\Phi = \rho A E$ lumen.

Equating the two values of flux, we have $\pi L A = \rho A E$

or $$L = \rho E/\pi \text{ cd/m}^2 = \rho E \text{ lm/m}^2$$

For example, consider a perfectly diffusing surface having $\rho = 0.8$ and held at a distance of 2 metres from a source of luminous intensity 100 candela at right angles to the direction of flux. Then

$$E = 100/22 = 25 \text{ lm/m}^2$$

$$L = \rho E/\pi = 25 \times 0.8/\pi = 6.36 \text{ cd/m}^2 = 636 \times \pi = 20 \text{ lm/m}^2$$

49.5. Laws of Illumination or Illuminance

The illumination **(*E*)** of a surface depends upon the following factors. The source is assumed to be a point source or is otherwise sufficiently away from the surface to be regarded as such.

(*i*) ***E*** is directly proportional to the luminous intensity **(*I*)** of the source or $E \propto I$

(*ii*) ***Inverse Square Law***. The illumination of a surface is inversely proportional to the square of the distance of the surface from the source.

In other words, $E \propto 1/r^2$

Proof

In Fig. 49.5 are shown portions of the surfaces of three spheres whose radii are in the ratio 1 : 2 : 3. All these portions, obviously, subtend the same solid angle at the source and hence receive the same amount of total flux. However, since their areas are in the ratio of 1 : 4 : 9, their illuminations are in the ratio $1 : \frac{1}{4} : \frac{1}{9}$.

(*iii*) ***Lambert's Cosine Law***. According to this law, E is directly proportional to the cosine of the ***angle made by the normal to the illuminated surface with the direction of the incident flux***.

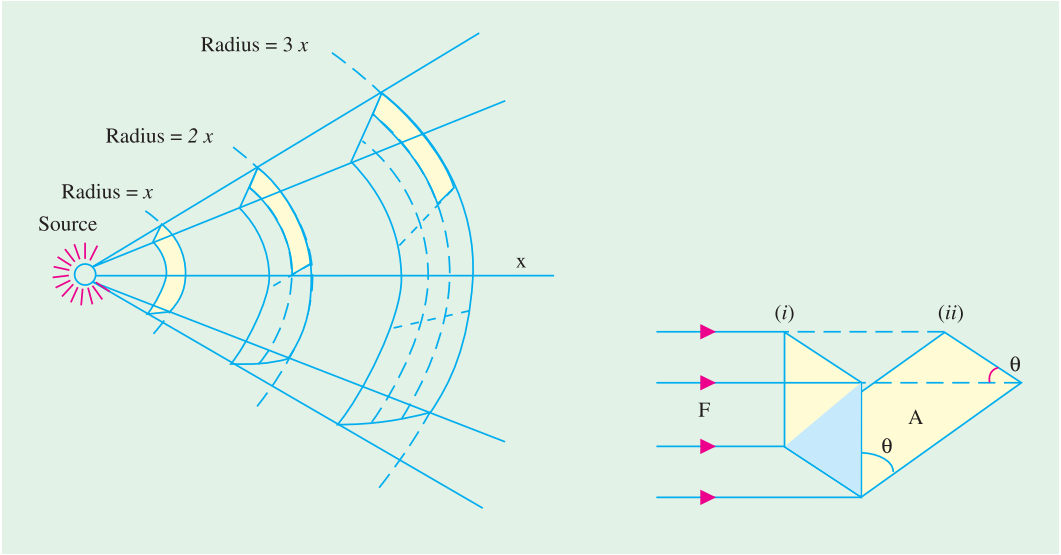


Fig. 49.5 **Fig. 49.6**

Proof

As shown in Fig. 49.6, let Φ be the flux incident on the surface of area A when in position 1. When this surface is turned back through an angle θ, then the flux incident on it is $\Phi \cos \theta$. Hence, illumination of the surface when in position 1 is $E_1 = \Phi/A$. But when in position 2.

$$E_2 = \frac{\Phi \cos\theta}{A} \qquad \therefore \qquad E_2 = E_1 \cos\theta$$

Combining all these factors together, we get $E = I \cos\theta / r^2$. The unit is lm/m^2.

The above expression makes the determination of illumination possible at a given point provided the position and the luminous intensity or candle power (in the given direction) of the source (or sources) by which it is illuminated, are known as illustrated by the following examples.

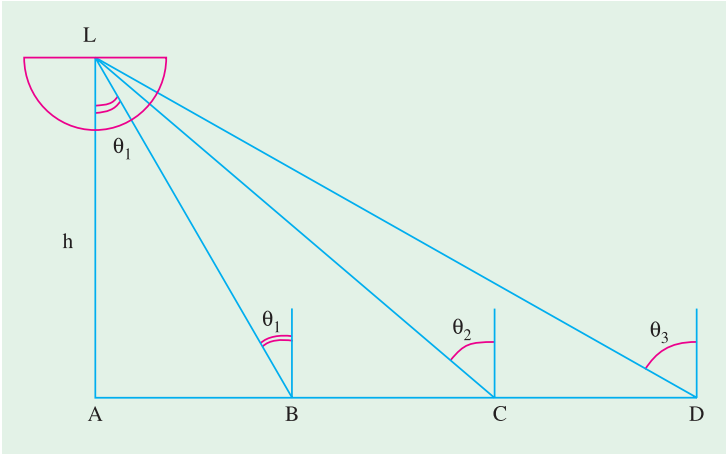


Fig. 49.7

Consider a lamp of uniform luminous intensity suspended at a height h above the working plane as shown in Fig. 49.7. Let us consider the value of illumination at point A immediately below the lamp and at other points B,C,D etc., lying in the working plane at different distances from A.

$$E_A = \frac{I}{h^2} \quad \text{—since } \theta = 0 \text{ and } \cos\theta = 1$$

$$E_B = \frac{I}{LB^2} \times \cos\theta_1. \qquad \text{Since,} \quad \cos\theta_1 = h / LB$$

$$\therefore \qquad E_B = \frac{I}{LB^2} \times \frac{h}{LB} = I \times \frac{h}{LB^2} = \frac{1}{h^2} \cdot \frac{h^3}{LB^3} = \frac{1}{h^2} \left(\frac{h}{LB}\right)^3$$

Now $\dfrac{1}{h^2} = E_A$ and $\left(\dfrac{h}{LB}\right)^3 = \cos^3\theta_1$

$\therefore$ $E_B = E_A \cos^3\theta_1$

Similarly, $E_C = E_A . \cos^3\theta_2$ and $E_D = E_A \cos^3\theta_3$ and so on.

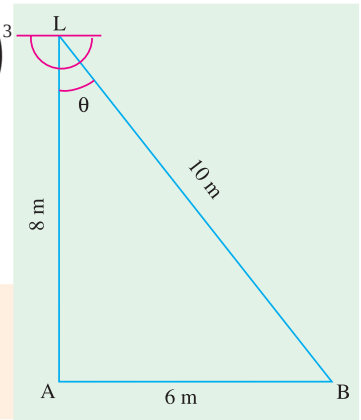


Fig. 49.8

Example 49.1. *A lamp giving out 1200 lm in all directions is suspended 8 m above the working plane. Calculate the illumination at a point on the working plane 6 m away from the foot of the lamp.* **(Electrical Technology, Aligarh Muslim Univ.)**

Solution. Luminous intensity of the lamp is

$$I = 1200/4\pi = 95.5 \text{ cd}$$

As seen from Fig. 49.8.

$$L_B = \sqrt{8^2 + 6^2} = 10 \text{ m} \; ; \; \cos\theta = 8/10 = 0.8$$

Now, $E = I \cos\theta / r^2$

$\therefore$ $E_B = 95.5 \times 0.8/10^2 =$ **0.764 lm/m^2**

Example 49.2. *A small light source with intensity uniform in all directions is mounted at a height of 10 metres above a horizontal surface. Two points A and B both lie on the surface with point*

A directly beneath the source. How far is B from A if the illumination at B is only 1/10 as great as at A ? **(A.M.I.E.)**

Solution. Let the intensity of the lamp be I and the distance between A and B be x metres as shown in Fig. 49.9.

Illumination at point $A, E_A = I/10^2 = I/100$ lux

Illumination at point B,

Illumination at point B,

$$E_B = \frac{I}{10^2} \times \left[\frac{10}{\sqrt{10^2 + x^2}}\right]^2 = \frac{10\,I}{(100 + x^2)^{3/2}}$$

Since $E_B = \frac{E_A}{10}$,

$$\therefore \quad \frac{10\,I}{(100 + x^2)^{3/2}} = \frac{1}{10} \times \frac{I}{100}, \qquad \therefore \quad x = \mathbf{19.1\ m}$$

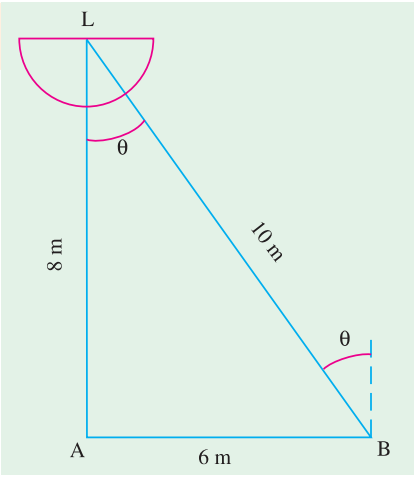


Fig. 49.9

Example 49.3. *A corridor is lighted by 4 lamps spaced 10 m apart and suspended at a height of 5 m above the centre line of the floor. If each lamp gives 200 C.P. in all directions below the horizontal, find the illumination at the point on the floor mid-way between the second and third lamps.*

(Electrical Engineering, Bombay Univ.)

Solution. As seen from 49.10, illumination at point C is due to all the four lamps. Since point C is symmetrically situated between the lamps, illumination at this point is twice that due to L_1 and L_2.

(i) illumination due to $L_1 = I \cos\theta_1 / L_1C^2$ $L_1C = \sqrt{5^2 + 15^2} = 15.8$ m

$$\cos\theta = 5/15.8$$

$$\text{illumination due to } L_1 = \frac{220 \times (5/15.8)}{250} = \mathbf{0.253\ lm/m^2}$$

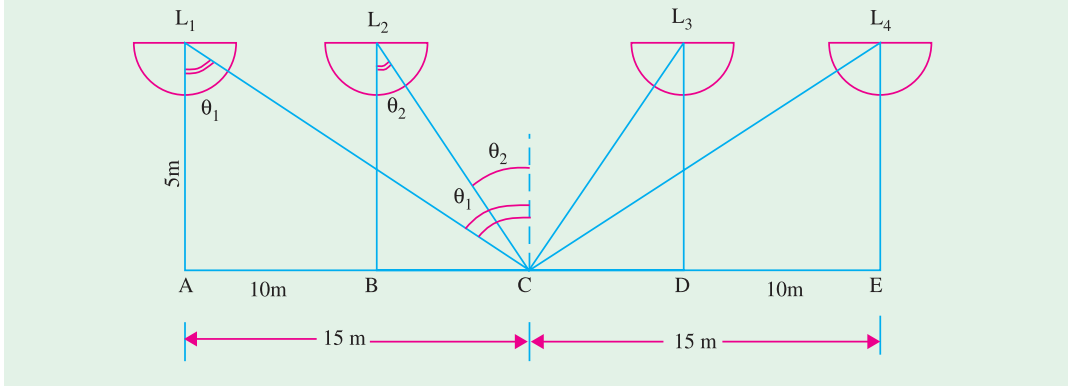


Fig. 49.10

(ii) $L_2C = 5/\sqrt{2}$ m ; $\theta_2 = 45°$; $\cos\theta_2 = 1/\sqrt{2}$

$$\text{Illumination due to } L_2 = \frac{200 \times 1/\sqrt{2}}{50} = 2.83\ \text{lm/m}^2$$

$\therefore$ illumination at C due to L_1 and $L_2 = 3.08$ lm/m^2

Illumination at C due to all the four lamps, $E_C = 2 \times 3.08 = \mathbf{6.16\ lm/m^2}$

Example 49.4. *Two lamps A and B of 200 candela and 400 candela respectively are situated 100 m apart. The height of A above the ground level is 10 m and that of B is 20 m. If a photometer is placed at the centre of the line joining the two lamp posts, calculate its reading.*

(Electrical Technology, Gujarat Univ.)

Solution. When the illumination photometer is placed at the centre point, it will read the value of combined illumination produced by the two lamps (Fig. 49.11).

Now, $L_1C = \sqrt{10^2 + 50^2}$

$= 51$ m

$L_2C = \sqrt{20^2 + 50^2}$

$= 53.9$ m

$\cos \theta_1 = 10/51$;

$\cos \theta_2 = 20/53.9$

Illumination at point C due to lamp L_1

$$= \frac{200 \times 10}{51 \times 2600}$$

$$= \mathbf{0.015\ lm/m^2}$$

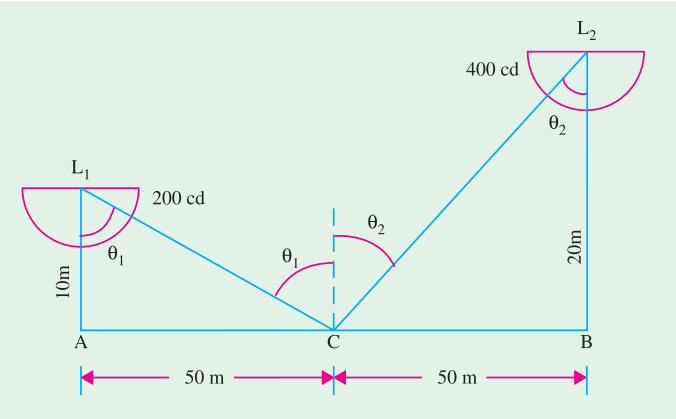


Fig. 49.11

Similarly, illumination due to lamp L_2

$$= \frac{400 \times 20/53.9}{2900} = \mathbf{0.051\ lm/m^2}$$

$\therefore \quad E_C = 0.015 + 0.051$

$= \mathbf{0.066\ lm/m^2\ or\ lux}$

Example 49.5. *The average luminous output of an 80-W fluorescent lamp 1.5 metre in length and 3.5 cm diameter is 3300 lumens. Calculate its average brightness. If the auxiliary gear associated with the lamp consumes a load equivalent to 25 percent of the lamp, calculate the cost of running a twin unit for 2500 hours at 30 paise per kWh.*

Solution. Surface area of the lamp $= \pi \times 0.035 \times 1.5 = 0.165\ m^2$

Flux emitted per unit area $= 3300/0.165 = 2 \times 10^4\ lm/m^2$

$$\therefore \quad B = \frac{\text{flux emitted per unit erea}}{\pi}\ cd/m^2 = 2 \times \frac{10^2}{\pi} = \mathbf{6{,}382\ cd/m^2}$$

Total load of twin fitting $= 2[80 + 0.25 \times 80] = 200$ W

Energy consumed for 2500 hr $= 2500 \times 200 \times 10^{-3} = 500$ kWh

cost $=$ Rs. $500 \times 0.3 =$ **Rs. 150.**

Example 49.6. *A small area 7.5 m in diameter is to be illuminated by a lamp suspended at a height of 4.5 m over the centre of the area. The lamp having an efficiency of 20 lm/w is fitted with a reflector which directs the light output only over the surface to be illuminated, giving uniform candle power over this angle. Utilisation coefficient = 0.40. Find out the wattage of the lamp. Assume 800 lux of illumination level from the lamp.* **(A.M.I.E.)**

Solution. $A = \pi d^2/4 = 44.18\ m^2, \quad E = 800$ lux

Luminous flux reaching the surface $= 800 \times 44.18 = 35{,}344$ lm

Total flux emitted by the lamp = 35,344/0.4 = 88,360 lm

Lamp wattage = 88,360/20 = **4420 W**

Example 49.7. *A lamp of 100 candela is placed 1 m below a plane mirror which reflects 90% of light falling on it. The lamp is hung 4 m above ground. Find the illumination at a point on the ground 3 m away from the point vertically below the lamp.*

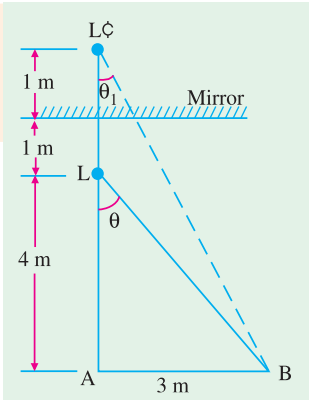


Fig. 49.12

Solution. The lamp and mirror arrangement is shown in Fig. 49.12. The lamp L produces an image L' as far behind the mirror as it is in front. Height of the image from the ground is (5 + 1) = 6m. L' acts as the secondary source of light and its candle power is = 0.9 × 100 = 90 candela.

Illumination at point B equals the sum of illumination due to L and that due to L'.

$$\therefore \quad E_B = \frac{100}{(LB)^2} \times \cos\theta + \frac{90}{(L'B)^2} \cos\theta_1$$

$$= \frac{100}{5^2} \times \frac{4}{5} \times \frac{90}{45} \times \frac{6}{\sqrt{45}} = \mathbf{5\ lux}$$

Example. 49.8. *A light source having an intensity of 500 candle in all directions is fitted with a reflector so that it directs 80% of its light along a beam having a divergence of 15°. What is the total light flux emitted along the beam? What will be the average illumination produced on a surface normal to the beam direction at a distance of 10 m?* **(A.M.I.E.)**

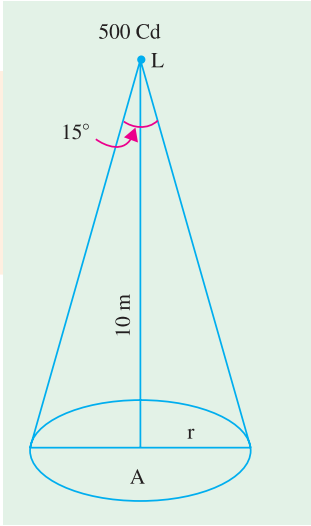


Fig. 49.13

Solution. Total flux emitted along the beam = 0.8 (4π × 500) = **5,227 lm**

Beam angle, θ = 15°, l = 10 m

Radius of the circle to be illuminated, $r = l \tan\theta/2$

= 10 tan 15°/2 = 1.316 m

Area of the surface to be illuminated, $A = \pi r^2 = \pi \times 1.316^2$

= 5.44 m^2

$\therefore$ Average illumination = 5227/5.44 = **961 lux**

Example 49.9. *A lamp has a uniform candle power of 300 in all directions and is fitted with a reflector which directs 50% of the total emitted light uniformly on to a flat circular disc of 20 m diameter placed 20 m vertically below the lamp. Calculate the illumination (a) at the centre and (b) at the edge of the surface without the reflector. Repeat these two calculations with the reflector provided.* **(Electrical Engg., Grad I.E.T.E.)**

Solution. It should be noted that the formula $E = I \cos\theta/r^2$ will not be applicable when the reflector is used. Moreover, with reflector, illumination would be uniform.

Without Reflector

(a) $E = 300 \times 1/20^2 = \mathbf{0.75\ lm/m^2}$

(b) Here, $\theta = \tan^{-1}(10/20) = 26.6°$, $\cos\theta = 0.89$; $x^2 = 10^2 + 20^2 = 500$

$\therefore$ $E = 300 \times 0.89/500 = \mathbf{0.534\ lm/m^2}$

With Reflector

Luminous output of the lamp = 300 × 4π lumen

Flux directed by the reflector = $0.5 \times 1200\ \pi = 600\ \pi$ lm

Illumination produced on the disc = $600\ \pi/100\ \pi =$ **6 lm/m^2**

It is the same at every point of the disc.

Example 49.10. *A light is placed 3 m above the ground and its candle power is 100 cos θ in any downward direction making an angle q with the vertical. If P and Q are two points on the grond, P being vertically under the light and the distance PQ being 3 m, calculate.*

(a) *the illumination of the ground at P and also at Q.*

(b) *the total radiations sent down by the lamp.*

Solution. With reference to Fig. 49.14

(*a*) C.P. along $LP = 100 \times \cos 0° = 100$ cd $\quad \therefore \quad E_P = 100/3^2 =$ **11.1 lm/m^2**

C.P. along $LQ = 100 \times \cos 45° = 70.7 \quad \therefore \quad E_Q = 70.7 \times \cos 45°/18 =$ **1.39 lm/m^2**

(*b*) Consider an imaginary hemisphere of radius r metre at whose centre lies the given lamp (Fig. 49.15).

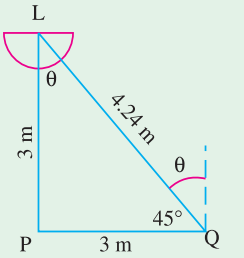


Fig. 49.14

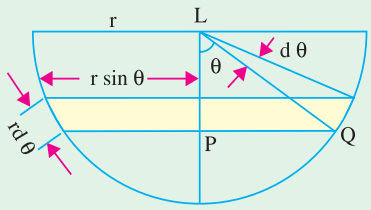


Fig. 49.15

C.P. along $LQ = 100 \cos \theta \quad \therefore \quad E_Q = 100 \cos \theta / r^2$

The area of the elementary strip at an angular distance θ from the vertical and of width $PQ = r.\ d\theta$ is $= (2\pi\ r \sin \theta) \times r.d\theta = 2\pi r^2 \sin \theta\ d\theta$.

Flux incident on the shaded area

$$= \frac{100 \cos \theta}{r^2} 2\pi\ r^2 \sin \theta.\ d\theta = 100\ \pi.\ 2 \sin \theta.\ \cos \theta\ d\theta = 100\ \pi \sin 2\theta\ d\theta.$$

Total flux over the hemisphere can be obtained by integrating the above expression between proper limits.

$$\therefore \quad \text{total flux} = \int_0^{\pi/2} 100\ \pi \sin 2\theta\ d\theta = 100\ \pi \left| -\frac{\cos 2\theta}{2} \right|_0^{\pi/2} = \frac{100\ \pi}{2}$$

$$= 100\ \pi = \textbf{314.lm}.$$

Example 49.11. *A drawing office containing a number of boards and having a total effective area of 70 m^2 is lit by a number of 40 W incandescent lamps giving 11 lm/W. An illumination of 80 lux is required on the drawing boards. Assuming that 60% of the total light emitted by the lamps is available for illuminating the drawing boards, estimate the number of lamps required.*

Solution. Let N be the number of 40 W lamps required.

Output/lamp = $40 \times 11 = 440$ lm ; Total flux = $440\ N$ lm

Flux actually utilized = $0.6 \times 440\ N = 264\ N$ lm

Illumination required = 80 lux = 80 lm/m^2

Total flux required at the rate of 80 lm/m^2 = 80×70 lm = 5600 lm

$264\ N = 5600 \quad \therefore \quad N =$ **21**

Example 49.12. *A perfectly diffusing surface has a luminous intensity of 10 candles at an angle of 60° to the normal. If the area of the surface is 100 cm^2, determine the brightness and total flux radiated.*

Solution. Brightness B is defined as the luminous intensity divided by the projected area (Fig. 49.16).

Luminous intensity = 10 candela

Projected area = $A \cos \theta$

$= 100 \times \cos 60^\circ$

$= 50$ cm^2

$\therefore \quad B = 10/50 \times 10^{-4}$

$= 2 \times 10^3$ cd/m^2

$= \mathbf{2\pi \times 10^3}$ **lm/m^2**—Art. 46.4

Total flux radiated $= 2\pi \times 10^3 \times 100 \times 10^{-4}$

$=$ **68.2 lm**

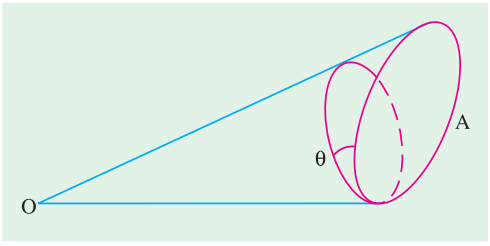


Fig. 49.16

Example 49.13. *Calculate the brightness (or luminance) of snow under an illumination of* ***(a)*** *44,000 lux and* ***(b)*** *0.22 lux. Assume that snow behaves like a perfect diffusor having a reflection factor of 85 per cent.*

Solution. ***(a)*** $L = \rho E/\pi = 44{,}000 \times 0.85/\pi$ cd/m^2 = **1.19×10^4 cd/m^2**

(b) $L = \dfrac{0.22 \times 0.85}{\pi}$ = **5.9×10^{-2} cd/m^2**

Example 49.14. *A 21 cm diameter globe of dense opal glass encloses a lamp emitting 1000 lumens and has uniform brightness of 4×10^3 lumen/m^2 when viewed in any direction. What would be the luminous intensity of the globe in any direction? Find what percentage of the flux emitted by the lamp is absorbed by the globe.*

Solution. Surface area of the globe $= \pi d^2 = \pi \times 21^2 = 1{,}386$ cm^2 $= 0.1386$ m^2

Flux emitted by the globe is $= 0.1386 \times 4 \times 10^3 = 554.4$ lumen

Now, 1 candela $= 4\pi$ lumens

Hence, luminous intensity of globe is $= 554.4/4\pi =$ **44 cd**

Flux absorbed by the globe is $= 1000 - 554.4 = 445.6$ lm.

$\therefore$ percentage absorption $= 445.6 \times 100/1000 =$ **44.56%**

Example 49.15. A *2.5 cm diameter disc source of luminance 1000 cd/cm^2 is placed at the focus of a specular parabolic reflector normal to the axis. The focal length of the reflector is 10 cm, diameter 40 cm and reflectance 0.8. Calculate the axial intensity and beam-spread. Also show diagrammatically what will happen if the source were moved away from the reflector along the axis in either direction.* **(A.M.I.E. Sec. B, Winter 1991)**

Solution. The axial or beam intensity I depends upon

(i) luminance of the disc source *i.e.* L

(ii) aperture of the reflector *i.e.* A

(iii) reflectivity of the reflector *i.e.* r

$\therefore \quad I = \rho A L$ candela

Now, $L = 1000$ cd/cm^2 $= 10^7$ cd/m^2

$A = \pi d^2/4 = \pi \times 0.4^2/4 = 125.7 \times 10^{-3}$ m^2

$\therefore \quad I = 0.8 \times 125.7 \times 10^{-3} \times 10^7 =$ **1,005,600 cd**

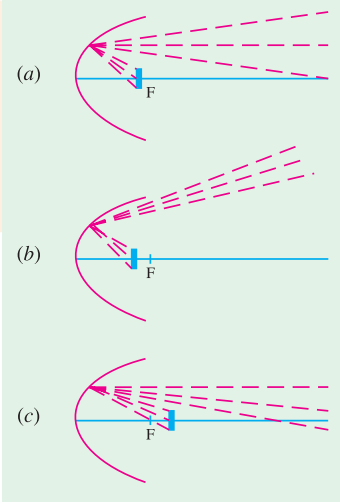


Fig. 49.17

To a first approximation, the beam spread for disc source is determined by reflector focal length and the size of the disc source. If θ is the total beam spread when the source is at the focus of the reflector [Fig. 49.17] (*a*)] then

$$\theta = 2 \tan^{-1} (r/f)$$

Here, $r = 2.5/2 = 1.25$ cm ; $f = 10$ cm

$\therefore \quad \theta = 2 \tan^{-1}(1.25/10) = 2 \times 7°7' =$ **14°14′**

The effect of axial movement of the source is shown in Fig. 49.17 (*b*) and (*c*).

Example 49.16. *A 22 diameter globe of opal glass encloses a lamp of uniform luminous intensity 120 C.P. Thirty per cent of light emitted by the lamp is absorbed by globe. Determine* ***(a)*** *luminance of globe* ***(b)*** *C.P. of globe in any direction.*

Solution.

(a) Flux emitted by source $= 120 \times 4\pi$ lm ; flux emitted by globe $= 0.7 \times 480\ \pi$ lm

$$\therefore \quad L = \frac{0.7 \times 480\ \pi}{\pi \times 0.22^2} = \textbf{6,940 lm/m}^2$$

(b) Since 1 candela $= 4\ \pi$ lm

$\therefore$ candle-power or luminous intensity of the globe is

$$= \frac{\text{flux in lumens}}{4\ \pi} = \frac{0.7 \times 480\ \pi}{\pi \times 4} = \textbf{84 cd}$$

Example 49.17. *A 0.4 m diameter diffusing sphere of opal glass (20 percent absorption) encloses an incandescent lamp with a luminous flux of 4850 lumens. Calculate the average luminance of the sphere.* **(A.M.I.E. Sec. B, Summer 1993)**

Solution. Flux emitted by the globe 80% or 4850 = 3880 lm

Surface area of the globe $= 4\pi r^2 = \pi d^2 \text{m}^2$

$$B = \frac{\text{flux emitted}}{\text{surfacr area}} = \frac{3880}{\pi \times 0.4^2} = \textbf{7,720 Im/m}^2$$

Tutorial Problem No. 49.1

1. A high-pressure mercury-vapour lamp is mounted at a height of 6 m in the middle of a large road crossing. A special reflector directs 100 C.P. maximum in a cone of 70° to the vertical line. Calculate the intensity of illumination on the road surface due to this beam of 100 C.P. ***(Electrical Engineering, Bombay Univ.)***
2. A room 6m × 4 m is illuminated by a single lamp of 100 C.P. in all directions suspended at the centre 3 m above the floor level. Calculate the illumination (*i*) below the lamp and (*ii*) at the corner of the room. ***(Mech. & Elect. Engg. : Gujarat Univ.)***
3. A lamp of 100 candle-power is placed at the centre of a room 10 m × 6m × 4 m high. Calculate the illumination in each corner of the floor and at a point in the middle of a 6 m wall at a height of 2 m from the floor. ***(Utilization of Elect. Power A.M.I.E.)***
4. A source of 5000 lumen is suspended 6.1 m. above ground. Find out the illumination (*i*) at a point just below the lamp and (*ii*) at a point 12.2 m away from the first, assuming uniform distribution of light from the source. **[(*i*) 10.7 lux (*ii*) 0.96 lux]** ***(A.M.I.E. Sec. B)***
5. Determine the average illumination of a room measuring 9.15 m by 12.2 m illuminated by a dozen 150 W lamps. The luminous efficiency of lamps may be taken as 14 lm/W and the co-efficient of utilisation as 0.35. **[79 lux]** ***(A.M.I.E. Sec. B)***
6. Two lamps are hung at a height of 9 m from the floor level. The distance between the lamps is one metre. Lamp one is of 500 candela. If the illumination on the floor vertically below this lamp is 20 lux, find the candle power of the lamp number two. **[1140 candela]** ***(Utili. of Elect. Power A.M.I.E.)***

7. Two powerful street lamps of 1,000 candela and 800 candela (assumed uniform in all directions) are mounted 12.5 m above the road level and are spaced 25 metres apart. Find the intensity of horizontal illumination produced at a point on the ground in-between the lamp posts and just below the lamp posts. **[4.07 lux, 6.86 lux, 5.69 lux] (*A.M.I.E.*)**

8. It is required to provide an illumination of 100 lux in a factory hall 30 m by 15 m. Assume that the depreciation factor is 0.8, coefficient of utilisation is 0.4 and efficiency of lamp is 14 lm/W. Suggest the number of lamps and their ratings. The sizes of the lamps available are 100, 250, 400 and 500 W.
[40 lamps of 250 W in 5 rows]

9. It is required to provide an illumination of 100 lm/m^2 in a workshop hall 40m × 10m and efficiency of lamp is 14 lm/W. Calculate the number and rating of lamps and their positions when trusses are provided at mutual distance of 5m. Take coefficient of utilisation as 0.4 and depreciation factor as 0.8. **[14 lamps of 750 W each]**

10. A drawing hall 30 m by 15 m with a ceiling height of 5 m is to be provided with a general illumination of 120 lux. Taking a coefficient of utilization of 0.5 and depreciation factor of 1.4, determine the number of flourscent tubes required, their spacing, mounting height and total wattage.
Taking luminous efficiency of flourescent tube as 40 Im/W for 80 W tubes.
[48, 24 twin-tube units each tube of 80 W; row spacing of 5 m and unit spacing of 3.75m, 3840 W]
(*Utilisation of Elect. Power, A.M.I.E.*)

49.6. Laws Governing Illumination of Different Sources

The laws applicable to the illumination produced by the following three types of sources will be considered.

(*i*) Point Source

As discussed in Art. 49.5, the law governing changes in illumination due to point source of light is $E = I \cos \theta / d^2$.

(*ii*) Line Source

Provided the line source is of infinite length and of uniform intensity, the illumination at a point lying on a surface parallel to and facing the line source is given by

$$E = \frac{\pi I}{2 d} \text{ lm/m}^2$$

where I = luminous intensity normal to the line source (in candles per-meter length of the sources)

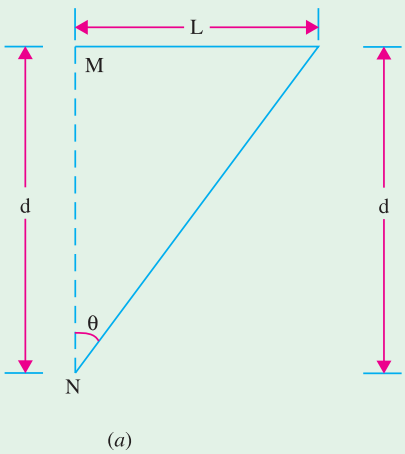


Fig. 49.18

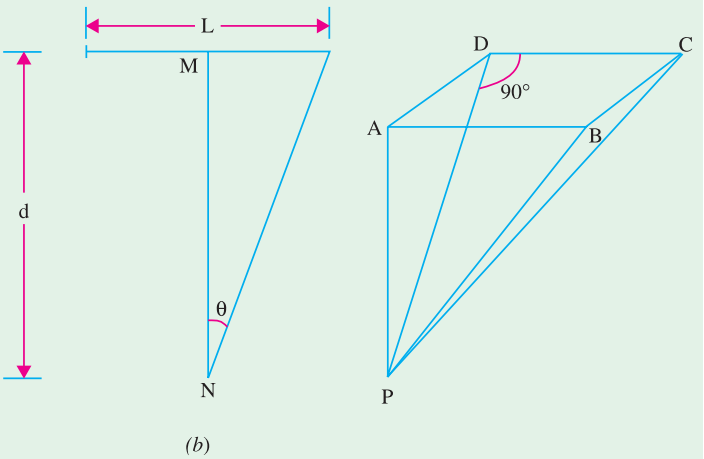


Fig. 49.19

However, in practice, the line sources are of finite length, so that the following law applies

$$E = \frac{I}{4d}(\sin 2\theta + 2\theta) \text{ lm/m}^2 \quad \text{—Fig. 49.18 } (a)$$

$$= \frac{I}{2d}(\sin 2\theta + 2\theta) \text{ lm/m}^2 \quad \text{—Fig. 49.18 } (b)$$

where I = candle power per metre length in a direction normal to the line source

$$= \frac{\Phi}{\pi^2 L} \text{ cd/m}$$

where Φ is the total flux of the source in lumens and L is the length of the line source in metres.

(*iii*) Surface Source

Provided the surface source is of infinite area and of uniform brightness, illumination at any point facing the source is independant of the distance between the point and the surface source. Mathematically, its value is $E = \pi L$ lm/m^2 where L is the luminance of the surface source in cd/m^2.

In case the surface source is limited and rectangular in shape (Fig. 49.19), the law governing the illumination at a point P is

$$E = \frac{L}{2}(\alpha' \sin\beta + \beta' \sin a)$$

where $\alpha = \angle APD$; $\alpha' = \angle BPC$; $\beta = \angle APB$; $\beta' = \angle DPC$

Note. (*i*) In case, distance d is more than 5 times the greatest dimension of the source, then irrespective of its shape, the illumination is found to obey inverse square law. This would be the case for illumination at points 5 metres or more away from a fluorescent tube of length one metre.

(*ii*) In the case of surface sources of large area, such as luminous daylights covering the whole ceiling of a large room, illumination is found to be practically constant irrespective of the height of the working place.

It may be noted that a point source produces deep shadows which may, however, be cancelled by installing a large number of fittings. Usually, glare is present. However, point sources are of great practical importance where accurate light control is required as in search-lights.

Line sources give more diffusion but cast shadows of objects lying parallel to them thus hindering vision.

Large-area surface sources though generally of low brightness, produce minimum glare but no shadows. However, the final effect is not liveliness but one of deadness.

Example 49.18. *A show case is lighted by 4 metre of architectural tubular lamps arranged in a continuous line and placed along the top of the case. Determine the illumination produced on a horizontal surface 2 metres below the lamps in a position directly underneath the centre of the 4 m length of the lamps on the assumption that in tubular lamps emit 1,880 lm per metre run. Neglect the effect of any reflectors which may be used.*

Solution. As seen in Art 49.10

$$E = \frac{I}{2d}(\sin 2\theta + 2\theta) \text{ lm/m}^2 \text{ and } I = \frac{\Phi}{\pi^2 L} \text{ cd/m}$$

As seen from Fig. 49.15

$$\theta = \tan^{-1}(L/2d)$$

$$= \tan{-}1(4/2) = 45°$$

$$I = 4 \times 1{,}880/\pi^2 \times 4$$

$$= \mathbf{188 \text{ cd/m}}$$

$\therefore$ $$E = \frac{188}{2 \times 2}\left(\sin 90° + \frac{\pi}{2}\right) = \mathbf{121 \text{ lm/m}^2}$$

49.7. Polar Curves of C.P. Distribution

All our calculations so far were based on the tacit assumption that the light source was of equal luminous intensity or candle-power in all directions. However, lamps and other sources of light, as a rule, do not give uniform distribution in the space surrounding them.

If the actual luminous intensity of a source in various directions be plotted to scale along lines radiating from the centre of the source at corresponding angles, we obtain the polar curve of the candle power.

Suppose we construct a figure consisting of large number of spokes radiating out from a point —the length of each spoke representing to some scale the candle power or luminous intensity of the source in that particular direction. If now we join the ends of these spokes by some suitable material, say, by linen cloth, then we get a surface whose shape will represent to scale the three dimensional candle power distribution of the source placed at the centre. In the ideal case of a point source having equal distribution in all directions, the surface would be spherical.

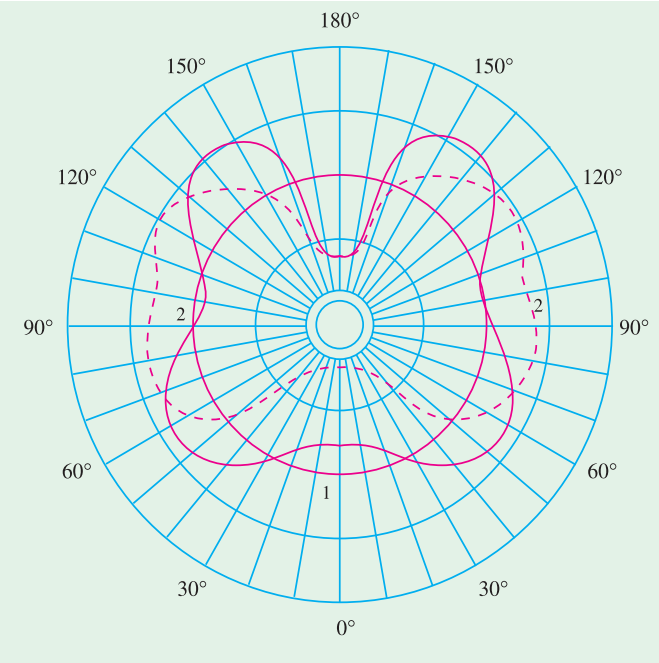


Fig. 49.20

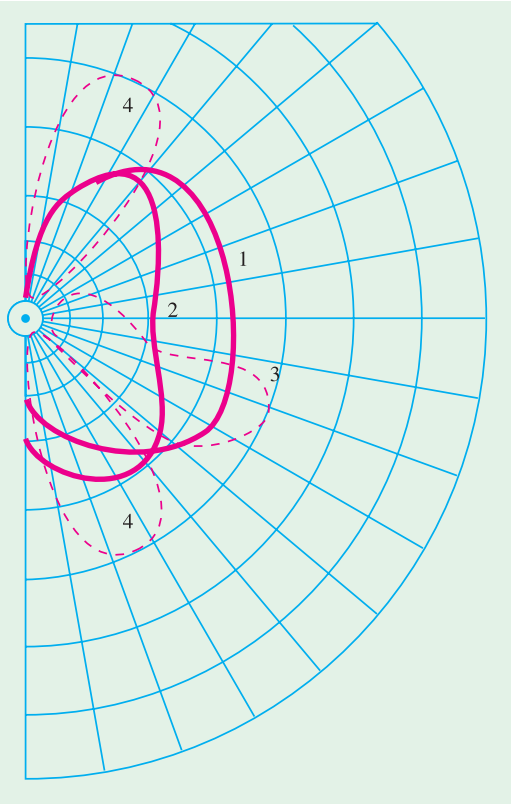


Fig. 49.21

It would be realized that it is difficult to give a graphic representation of such a 3-dimensional model in a plane surface. Therefore, as with engineering drawings, it is usual to draw only one or more elevations and a plan of sections through the centre of the source. Elevations represent c.p. distribution in the ***vertical*** plane and the plans represent c.p. distribution in ***horizontal*** plane. The number of elevations required to give a complete idea of the c.p. distribution of the source in all directions depends upon the shape of the plan *i.e.* on the horizontal distribution. If the distribution is uniform in every horizontal plane *i.e.* if the polar curve of horizontal distribution is a circle, then only one vertical curve is sufficient to give full idea of the space distribution.

In Fig. 49.20 are shown two polar curves of c.p. distribution in a vertical plane. Curve 1 is for

vacuum type tungsten lamp with zig-zag filament whereas curve 2 is for gas filled tungsten lamp with filament arranged as a horizontal ring.

If the polar curve is symmetrical about the vertical axis as in the figures given below, then it is sufficient to give only the polar curve within one semicircle in order to completely define the distribution of c.p. as shown in Fig. 49.21.

The curves 1 and 2 are as in Fig. 49.20, curves 3 is for d.c. open arc with plain carbons and curve 4 is for a.c. arc with plain carbons. However, if the source and/or reflector are not symmterical about vertical axis, it is impossible to represent the space distribution of c.p. by a single polar diagram and even polar diagrams for two planes at right angles to each other give no definite idea as to the distribution in the intermediate planes.

Consider a filament lamp with a helmet-type reflector whose axis is inclined and cross-section elliptical—such reflectors are widely used for lighting shop windows. Fig. 49.22 represents the distribution of luminous intensity of such source and its reflector in two planes at right angles to each other.

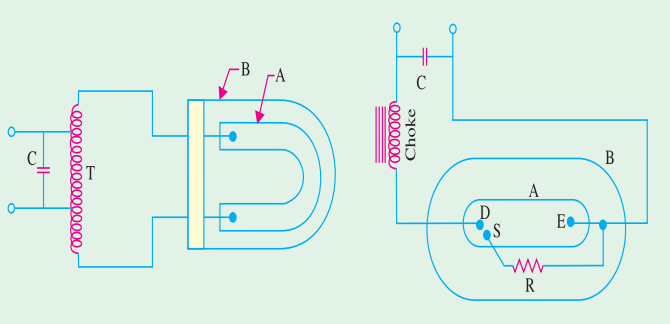


Fig. 49.22

The importance of considering the polar curves in different planes when the c.p. distribution in asymmetrical is even more strikingly depicted by the polar curves in YY plane and XX plane of a lamp with a special type of reflector designed for street lighting purposes (Fig. 49.23).

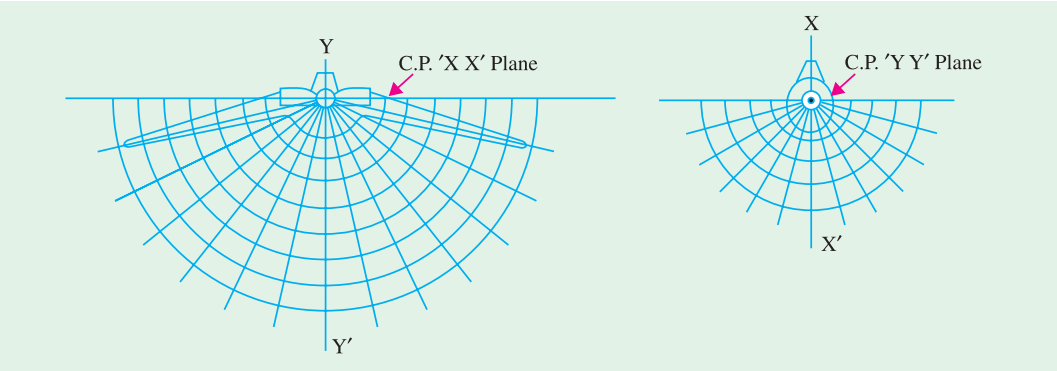


Fig. 49.23

It would be realized from above that the polar distribution of light from any source can be given any desired form by using reflectors and/or refractors of appropriate shape.

In Fig. 49.24 is shown the polar curve of c.p. distribution of a straight type of lamp in a horizontal plane.

49.8. Uses of Polar Curves

Polar curves are made use of in determining the M.S.C.P. etc. of a source. They are also used in determining the actual illumination of a surface *i.e.* while calculating the illumination in a particular direction, the c.p. in that particular direction as read from the vertical polar curve, should be employed.

49.9. Determination of M.S.C.P. and M.H.C.P. from Polar Diagrams

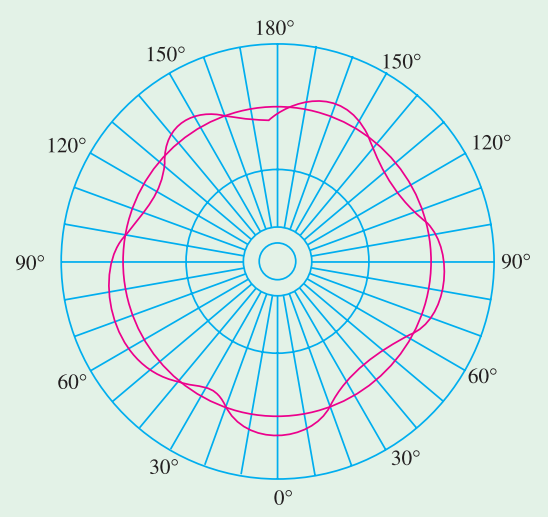


Fig. 49.24

In Fig. 49.25 (*a*) is shown the polar distribution curve of a filament lamp in a horizontal plane and the polar curve in Fig. 49.25 (*b*) represents the c.p. distribution in a vertical plane. It will be seen that the horizontal candle-power is almost uniform in all directions in that plane. However, in the vertical plane, there is a large variation in the candle power which falls to zero behind the cap of the lamp. The curve in Fig. 49.25 (*a*) has been drawn with the help of a photometer while the lamp is rotated about a vertical axis, say, 10° at a time. But the curve in Fig. 49.25 (*b*) was drawn while the lamp was rotated in a vertical plane about a horizontal axis passing through the centre of the filament.

The M.H.C.P. is taken as the mean of the readings in Fig. 49.25 (*a*). However, a more accurate result can be obtained by plotting candle power on an angular base along the rectangular axes and by determining the mean height of the curve by the mid-ordinate or by Simpson's rule.

The M.S.C.P. of the lamp can be obtained from the vertical polar curve of Fig. 49.25 (*b*) by Rousseau's construction as explained below :

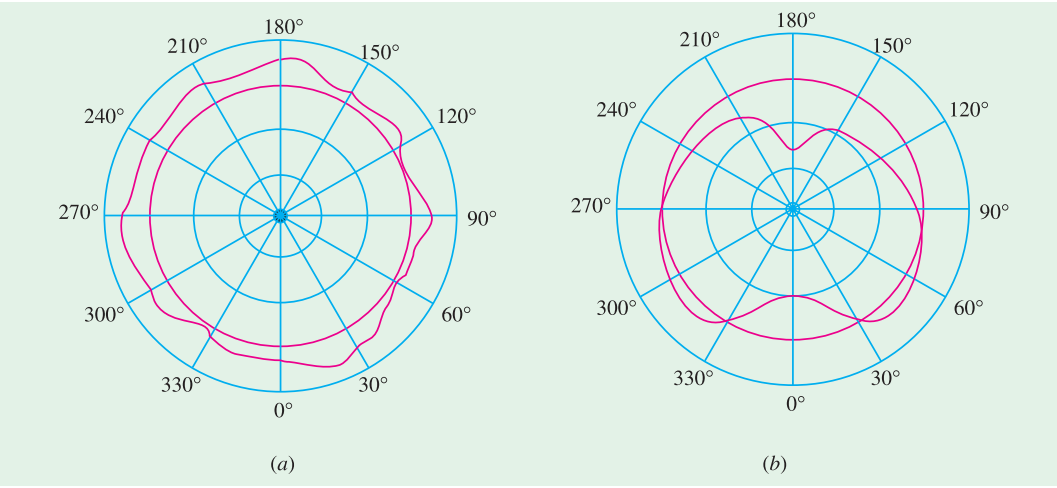


Fig. 49.25

Only half of the vertical polar curve is shown in the figure (Fig. 49.26) since it is symmetrical about the vertical axis. With *O* is the centre and radius *OR* equal to the maximum radius of the polar curve, a semi-circle *LRM* is drawn. A convenient number of points on this semi-circle (say 10° points) are projected onto any vertical plane as shown. For example, points *a*,*b*,*c* etc. are projected to *d*,*e*,*f* and so on. From point *d*, the horizontal line *dg* is drawn equal to the intercept *OA* of the polar diagram on the radius *oa*. Similarly, *eh* = *OB*, *fk* = *OC* and so on. The points *g*, *h*, *k* etc., define the Rousseau figure. The average width *w* of this figure represents the M.S.C.P. to the same scale as that of the candle powers in the polar curve. The average width is obtained by dividing the Rousseau area by the base of the Rousseau figure *i.e.* length lm which is the projection of the semi-circle *LM* on the vertical axis. The area may be determined by Simpson's rule or by using a planimeter.

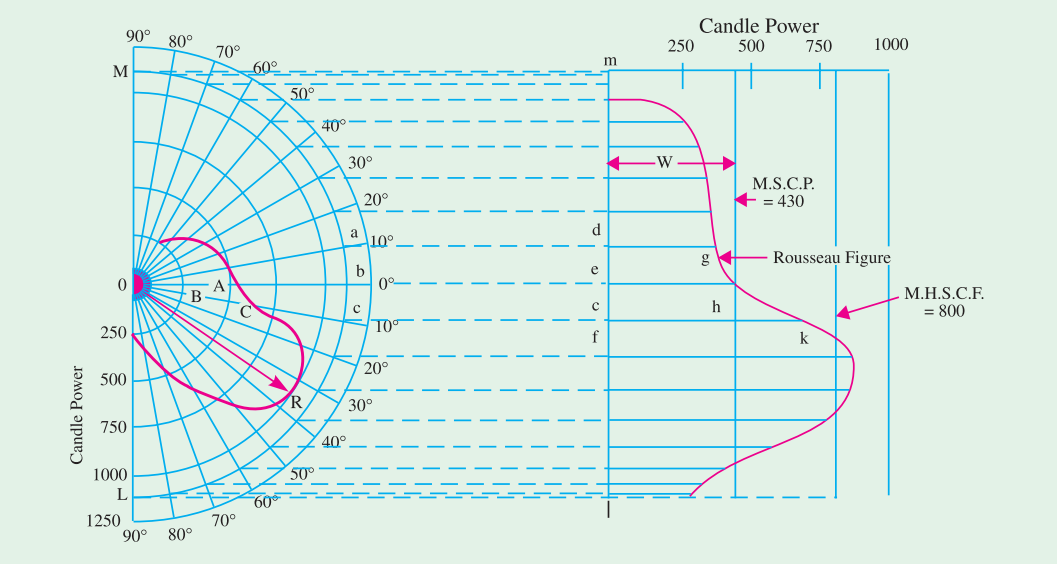


Fig. 49.26

$$\therefore \quad \text{M.S.C.P.} = \frac{\text{area of Rousseau figure}}{\text{length of the base}}$$

As explained earlier, the M.H.C.P. of an incandescent lamp can be easily obtained by mounting the lamp with its axis vertical and taking photometer readings in the horizontal plane while the lamp is rotated about its axis in steps of 10° or so. A definite ratio exists between the M.H.C.P. and M.S.C.P. of each particular type of filament. M.S.C.P. of a lamp can be found by multiplying M.H.C.P. by a factor known as spherical reduction factor which, as defined earlier, is

$$\text{Spherical reduction factor} \quad f = \frac{\text{M.S.C.P.}}{\text{M.H.C.P.}} \qquad \therefore \quad \text{M.S.C.P.} = f \times \text{M.H.C.P.}$$

For the particular lamp considered, $f = 430/80 = 0.54$ (approx.)

Typical values of this factor are :

Ordinary vacuum-type tungsten lamp having zig-zag filament	0.76 – 0.78
Gas-filled tungsten lamp with filament in the form of broad shallow *V*'s	0.85 – 0.94
Gas-filled tungsten lamp with filament in the shape of a horizontal ring	1.0 – 1.2

The total lumen output is given by the relation ; lumen output = $4\pi \times$ M.S.C.P.

In the present case, lumen output = $4\pi \times 430$

= 5,405 lm

49.10. Integrating Sphere or Photometer

The M.S.C.P. is usually measured by means of an integrating photometer, the most accurate form of which consists of a hollow sphere (as originally proposed by Ulbricht) whose diameter is large (at least 6 times) as compared to that of the lamp under test. The interior surface of the hollow sphere is whitened by means of a special matt white paint. When the lamp is placed inside the sphere (not

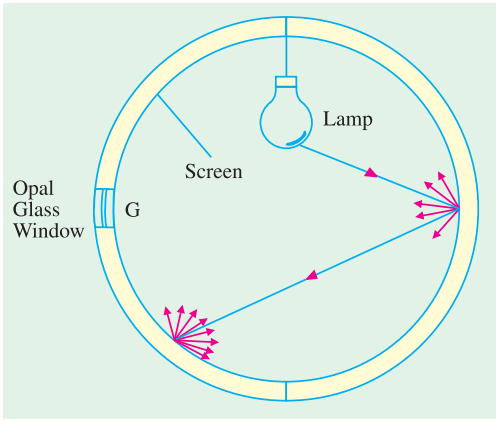


Fig. 49.27

necessarily at its centre) then due to successive reflections, its light is so diffused as to produce a uniform illumination over the whole surface. At some point, a small matt opal-glass window, shaded from the direct rays of the source, is made in the hollow sphere.

The brightness of the matt opal glass is proportional to that of the interior surface of the sphere. By using a suitable illumination photometer, the illumination of the window can be measured which can be used to find out the total flux emitted by the source.

$$\text{Total flux} = \text{illumination } (1\text{m/m}^2) \times \text{surface area of the sphere } (\text{m}^2)$$

$$\therefore \quad \text{M.S.C.P.} = \frac{\text{total flux}}{4\pi} \text{ candela}$$

Theory. In Fig. 49.28 is shown a light source L of luminous intensity I candela and having a total flux output of F_L placed at the centre of an integrating sphere of radius r and reflection factor ρ. Let E_A and E_B represent the illuminations at two points A and B, each of infinitely small area da and db respectively and distance m apart. We will now consider total illumination (both direct and reflected) at point A.

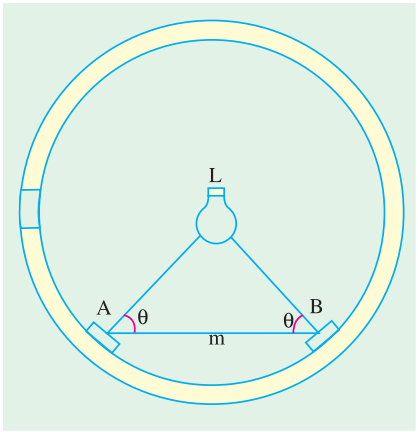


Fig. 49.28

Obviously, E_A directly due to $L = I/r^2$

E_B directly due to $L = I/r^2$

Luminous intensity of B in the direction of A is

$$I_B = \frac{\rho . E_B . A_B}{\pi} \text{ candela} \qquad \text{—Art. 45.4}$$

where A_B = projected area of B at right angles to the line $BA = db.\cos\theta$

$$\therefore \quad I_B = \frac{\rho . I . db \cos\theta}{\pi r^2} \text{ candela}$$

Hence, illumination of A due to B is $= \dfrac{I_B \cos\theta}{m^2} = \dfrac{\rho I . db \cos^2\theta}{\pi r^2 \times m^2}$

Now, as seen from Fig. 49.28, $m = 2r\cos\theta$

$\therefore$ illumination of A due to B becomes

$$= \frac{\rho . I . db \cos^2\theta}{\pi r^2 \times 4r^2 \cos^2\theta} = \frac{\rho}{4\pi r^2} \times \frac{I}{r^2} \times db = \frac{\rho}{S} . E_B . db = \frac{\rho F_B}{D}$$

where F_B = flux incident on B and A = surface area of the sphere

Hence, total illumination due to first reflection $= \sum \dfrac{\rho F_B}{S} = \dfrac{\rho F_B}{S}$

Now, consider any other point C. Illumination on B due to point $C = \rho F_L/S$. The illumination on A due to C as reflected from B.

$$= \left[\rho . \left(\frac{\rho F_L}{S}\right) \times \frac{db\cos\theta}{\pi}\right] \times \frac{\cos\theta}{m^2} = \frac{\rho F_L}{S} \times \frac{\rho . db\cos\theta}{\pi} \times \frac{\cos\theta}{4r^2\cos^2\theta}$$

$$= \frac{\rho F_L}{S} \times \frac{\rho . db}{S}$$

Total illumination due to two reflections $= \sum \dfrac{\rho F_L}{S} \times \dfrac{\rho . db}{S} = \dfrac{\rho^2 F_L}{S}$ $(\because \Sigma ab = S)$

Continuing this way, it can be proved that total illumination at point A from all reflections from all points

$$= \frac{\rho F_L}{S}(1 + \rho^2 + \rho^3 + \ldots\ldots\ldots + \rho^{n-1}) = \frac{\rho F_L}{S}\left(\frac{1}{1-\rho}\right)$$

Hence, total illumination at A from direct and reflected lights is

$$= E_A + \frac{\rho F_L}{S}\left(\frac{1}{1-\rho}\right)$$

If A is shielded from lamp L, then its illumination is proportional to F_L because $\frac{\rho}{S}\left(\frac{1}{1-\rho}\right)$ is a constant factor. Obviously, if either brightness or illumination at one point in the sphere is measured, it would be proportional to the light output of the source. This fact is made use of while using this sphere as a globe photometer.

Example 49.19. *If an integrating sphere 0.6 m in diameter whose inner surface has a reflection coefficient of 0.8 contains a lamp producing on the portion of the sphere, screened from direct radiation, a luminance of 1000 cd/m^2, what is the luminous flux yield of the source ?*

(A.M.I.E. Sec. B. Summer 1986)

Solution. Obviously, the screened portion of the sphere receives light by reflection only. Reflection illumination of the screened portion is

$$E = \frac{\rho F_1}{S}\left(\frac{1}{1-\rho}\right) = \frac{0.8\,F_L}{\pi \times 0.6^2}\left(\frac{1}{1-0.8}\right) = \frac{100\,F_L}{9\pi}\ \text{lm/m}^2$$

Also $L = \rho E/\pi$ — Art. 49.4

$\therefore$ $1000 = \frac{100\,F_L}{9\pi} \times \frac{0.8}{\pi}$ or $F_L = \mathbf{1/25\ lm}$

49.11. Diffusing and Reflecting Surfaces : Globes and Reflectors

When light falls on polished metallic surfaces or silvered surfaces, then most of it is reflected back according to the laws of reflection *i.e.* the angle of incidence is equal to the angle of reflection. Only a small portion of the incident light is absorbed and there is always the image of the source. Such reflection is known as ***specular*** reflection.

However, as shown in Fig. 49.29 (*b*), if light is incident on coarse surfaces like paper, frosted glass, painted ceiling etc., then it is scattered or diffused in all directions, hence no image of the source is formed. Such reflection of light is called ***diffuse reflection***. A perfect diffuser is one that scatters light uniformly in all directions and hence appears equally bright from whatever direction it is viewed. A white blotting paper is the nearest approach to a diffuser.

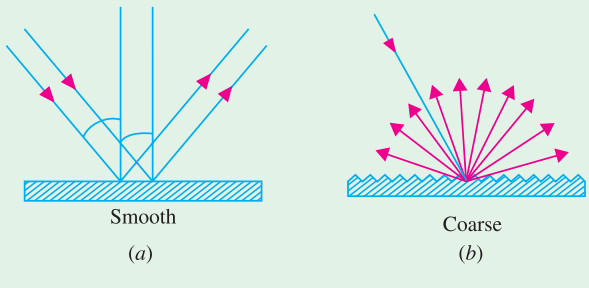


Fig. 49.29

By reflecting factor of a surface is meant the ratio $= \frac{\text{reflected light}}{\text{incident light}}$

It is also known as reflection ratio or coefficient of reflection of a surface.

If the light is incident on a transparent surface, then some of it is absorbed and greater percentage of it passes through and emerges on the other side.

To avoid direct glare from electric arcs and incandescent filament lamps, they are surrounded more or less completely by diffusing shades or globes. In addition, a reflector may also be embodied to prevent the escape of light in directions where it serves no useful purpose. In that case, so far as the surroundings are concerned, the diffusing globe is the source of light. Its average brilliancy is lower

the more its diffusing area. Depending on the optical density, these globes absorb 15 to 40% of light emitted by the encircled bulb. The bulbs may also be frosted externally by etching or sand-blasting but internal frosting is better because there is no sharp scratching or cracks to weaken the glass.

In domestic fittings, a variety of shades are used whose main purpose is to avoid glare. Properly designed and installed prismatic glass shades and holophane type reflectors have high efficiency and are capable of giving accurate predetermined distribution of light.

Regular metallic reflection is used in search-light mirrors and for general lighting purposes. But where it is used for general lighting, the silvered reflectors are usually fluted to make the illumination as uniform as possible.

Regular cleaning of all shades, globes, and reflectors is very important otherwise the loss of light by absorption by dust etc., collected on them becomes very serious.

Various types of reflectors are illustrated in Figs. 49.30 to 49.34. Fig. 49.30 shows a holophane stiletto reflector used where extensive, intensive or focussing light distribution is required.

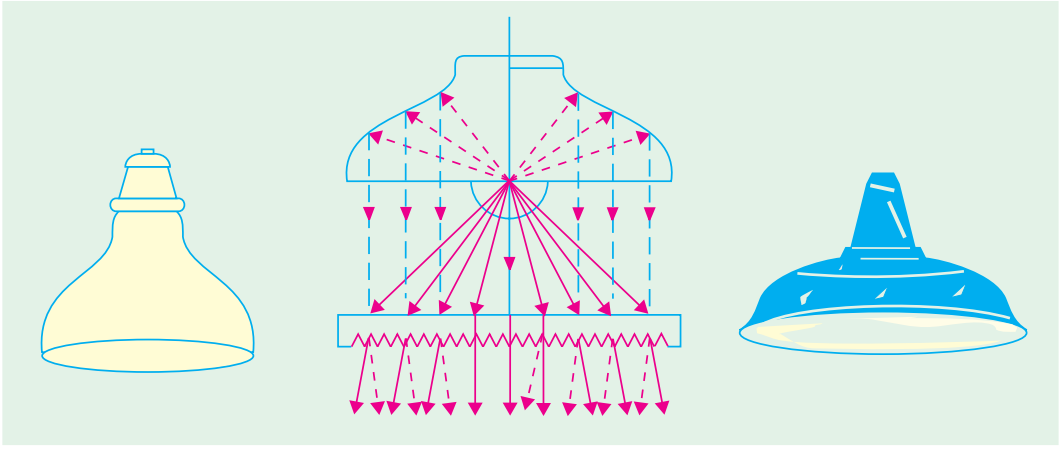


Fig. 49.30 Fig. 49.31 Fig. 49.32

The optical combination of a lamp, reflector and a lens plate, as shown in Fig. 49.31, provides a high degree of light control. Multiple panels can be conveniently incorporated in fittings suited to different architectural schemes.

The dispersive reflector of Fig. 49.32 is suitable for practically all classes of industrial installations. The reflector is a combination of concave and cylindrical reflecting surfaces in the form of a deep bowl of wide dispersive power. It gives maximum intensity between 0° and 45° from the vertical.

The concentrating reflector of parabolic form shown in Fig. 49.33 is suitable for situations requiring lofty installations and strongly-concentrated illumination as in public halls, foundries and power stations etc. They give maximum intensity in zones from 0° to 25° from the vertical.

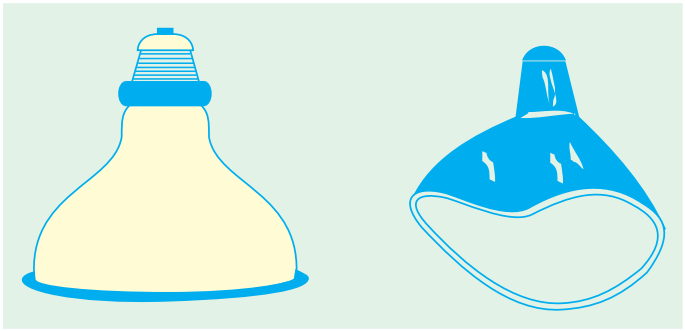


Fig. 49.33 Fig. 49.34

The elliptical angle reflector shown in Fig. 49.34 is suitable for the side lighting of switchboards, show windows etc., because they give a forward projection of light in the vertical plane and spread the light in the horizontal plane.

49.12. Lighting Schemes

Different lighting schemes may be classfied as ***(i)*** direct lighting ***(ii)*** indirect lighting and ***(iii)*** semi-direct lighting ***(iv)*** semi-indirect lighting and ***(v)*** general diffusing systems.

(i) Direct Lighting

As the name indicates, in the form of lighting, the light from the source falls directly on the object or the surface to be illuminated (Fig. 49.35). With the help of shades and globes and reflectors of various types as disscussed in Art. 49.11, most of the light is directed in the lower hemisphere and also the brilliant source of light is kept out of the direct line of vision. Direct illumination by lamps in suitable reflectors can be supplemented by standard or bracket lamps on desk or by additional pendant fittings over counters.

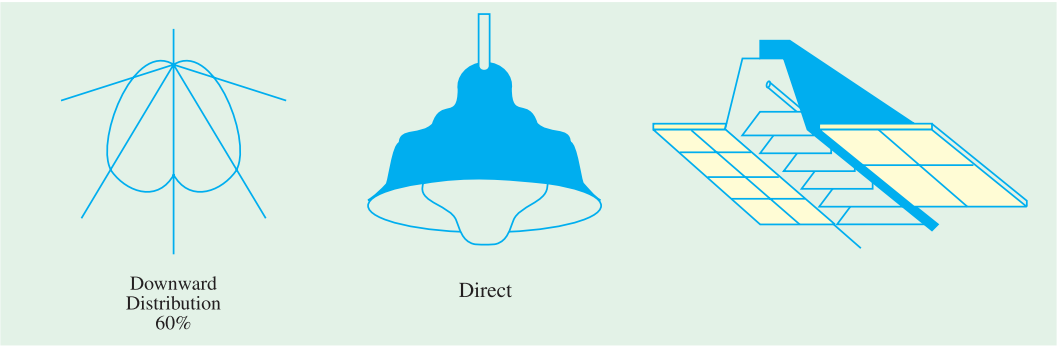


Fig. 49.35

The fundamental point worth remembering is planning any lighting installation is that sufficient and sufficiently uniform lighting is to be provided at the working or reading plane. For this purpose, lamps of suitable size have to be so located and furnished with such fittings as to give correct degree and distribution of illumination at the required place. Moreover, it is important to keep the lamps and fittings clean otherwise the decrease in effective illumination due to dirty bulbs or reflectors may amount to 15 to 25% in offices and domestic lighting and more in industrial areas as a result of a few weeks neglect.

Direct lighting, though most efficient, is liable to cause glare and hard shadows.

(ii) Indirect Lighting

In this form of lighting, light does not reach the surface directly from the source but indirectly by diffuse reflection (Fig. 49.36). The lamps are either placed behind a cornice or in suspended ***opaque*** bowls. In both cases, a silvered reflector which is corrugated for eliminating striations is placed beneath the lamp.

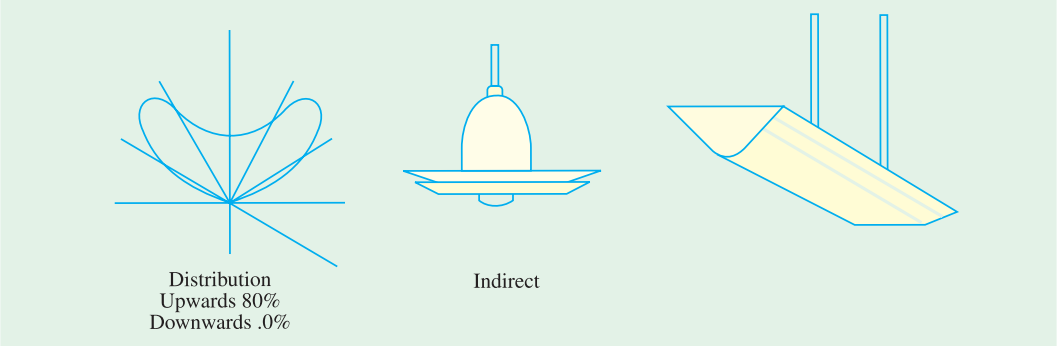


Fig. 49.36

In this way, maximum light is thrown upwards on the ceiling from which it is distributed all over the room by diffuse reflection. Even gradation of light on the ceiling is secured by careful adjustement of the position and the number of lamps. In the cornice and bowl system of lighting, bowl fittings are generally suspended about three-fourths the height of the room and in the case of cornice lighting, a frieze of curved profile aids in throwing the light out into the room to be illuminated. Since in indirect lighting whole of the light on the working plane is received by diffuse reflection, it is important to keep the fittings clean.

One of the main characteristics of indirect lighting is that it provides shadowless illumination which is very useful for drawing offices, composing rooms and in workshops especially where large machines and other obstructions would cast troublesome shadows if direct lighting were used.

However, many people find purely indirect lighting flat and monotonous and even depressive. Most of the users demand 50 to 100% more light at their working plane by indirect lighting than with direct lighting. However, for appreciating relief, a certain proportion of direct lighting is essential.

(*iii*) Semi-direct System

This system utilizes luminaries which send most of the light downwards directly on the working plane but a considerable amount reaches the ceilings and walls also (Fig. 49.37).

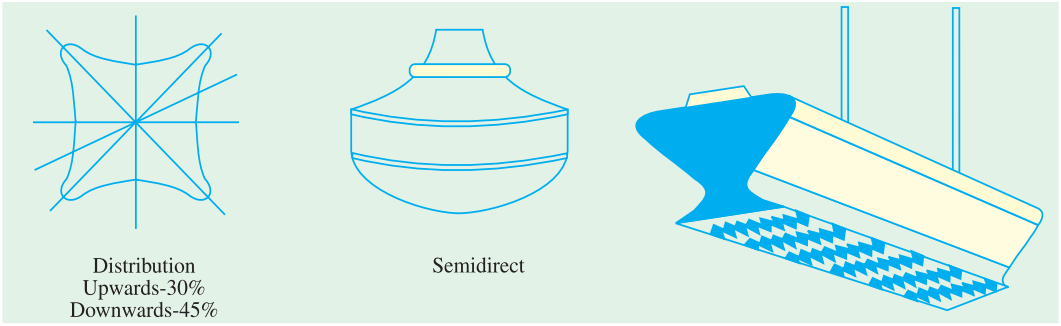


Fig. 49.37

The division is usually 30% upwards and 45% downwards. Such a system is best suited to rooms with high ceilings where a high level of uniformly-distributed illumination is desirable. Glare in such units is avoided by using diffusing globes which not only improve the brightness towards the eye level but improve the efficiency of the system with reference to the working plane.

(*iv*) Semi-indirect Lighting

In this system which is, in fact, a compromise between the first two systems, the light is partly received by diffuse reflection and partly direct from the source (Fig. 49.38). Such a system, therefore, eliminates the objections of indirect lighting mentioned above. Instead of using opaque bowls with reflectors, translucent bowls without reflector are used. Most of the light is, as before, directed upwards to the ceiling for diffuse reflection and the rest reaches the working plane directly except for some absorption by the bowl.

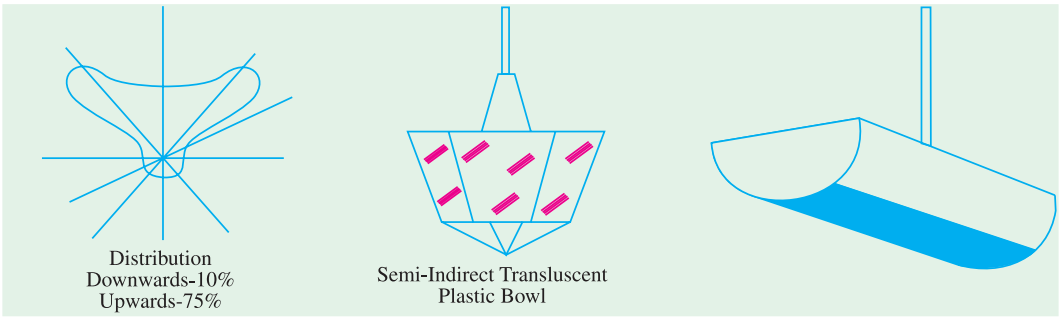


Fig. 49.38

(v) General Diffusing System

In this system, luminaries are employed which have almost equal light distribution downwards and upwards as shown in Fig. 49.39.

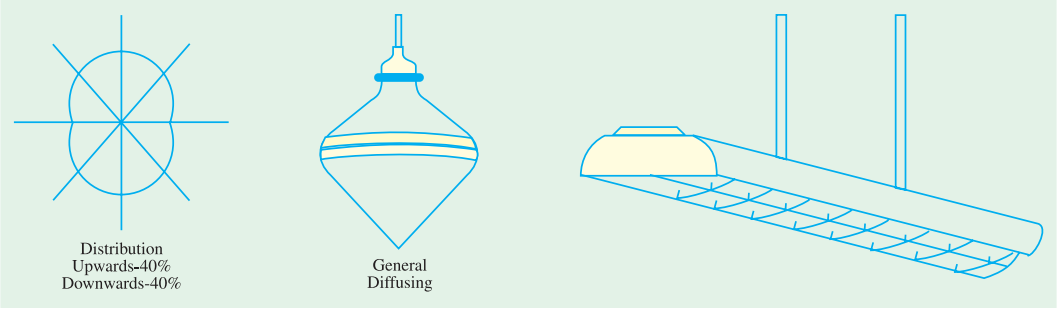


Fig. 49.39

49.13. Illumination Required for Different Purposes

There has been a steady movement towards higher intensities for artificial illumination during the last few decades. The movement is likely to continue because the highest intensities in average installations are much less than those of the diffused daylight. The human eye posses a tremendous power of accommodation and it can work comfortably within an enormous range of illuminations.

For example, at full noon, sun provides about 120,000 lm/m^2, diffuse day-light near a window is of the order of 600 lm/m^2 (value varying widely) and full moon-light gives 0.1 to 0.3 lm/m^2. For reading, usually 20 to 30 lm/m^2 is generally considered sufficient, though daylight illumination is much higher.

Some persons can read without much strain even when illumination is as low as 3 lm/m^2. Because of this, it is difficult to lay down definite values of illumination for various purposes but the following summary will be found useful :

Purpose and Places	*lm/m^2*
Precision work, displays, tasks requiring rapid discrimination	above 500
Extra fine machine work, around needles of sewing machines, fine engraving, inspection of fine details having low contrast	200-500
Proof-reading, drawing, sustained reading, fine assembling, skilled bench-work	100-200
Drawing offices, art exhibition, usual reading	60-100
In museums, drill halls, for work of simple nature not involving close attention to fine details	40-60
Usual observation as in bed-rooms, waiting rooms, auditoriums and general lighting in factories	20-40
Hospital wards, yards, railway platforms and corridors	5-10

49.14. Space/Height Ratio

It is given by the ratio : $\frac{\text{horizontal distance between two lamps}}{\text{mounting height of lamps}}$

This ratio depends on the nature of the polar curve of a lamp ***when used along with its reflector***. A reflector has tremendous influence on the shape of the polar curve of the lamp, hence the value of space/height ratio, in fact, depends entirely on the type of reflector used. For obtaining uniform illumination on the working plane, it is essential to choose a correct value for this ratio.

In other words, it means that a reflector gives uniform illumination for a definite value of this ratio only. The ratio may be found easily if the polar curve of the type of fixture used is known. For reflectors normally used in indoor lighting, the value of this ratio lies between 1 and 2.

49.15. Design of Lighting Schemes and Lay-outs

A well-designed lighting scheme is one which

(i) provides adequate illumination ***(ii)*** avoids glare and hard shadows ***(iii)*** provides sufficiently uniform distribution of light all over the working plane.

Before explaining the method of determining the number, size and proper arrangement of lamps in order to produce a given uniform illumination over a certain area, let us first consider the following two factors which are of importance in such calculations.

49.16. Utilization Factor or Coefficient of Utilization (η)

It is the ratio of the lumens actually received by a particular surface to the total lumens emitted by a luminous source.

$$\therefore \quad \eta = \frac{\text{lumens actually received on working plane}}{\text{lumens emitted by the light source}}$$

The value of this factor varies widely and depends on the following factors :

1. the type of lighting system, whether direct or indirect etc.
2. the type and mounting height of the fittings
3. the colour and surface of walls and ceilings and
4. to some extent on the shape and dimensions of the room.

For example, for direct lighting, the value of η varies between 0.4 and 0.6 and mainly depends on the shape of the room and the type and mounting height of fittings but very little on the colour of walls and ceiling. For indirect lighting, its value lies between 0.1 and 0.35 and the effect of walls and ceiling, from which light is reflected on the working plane, is much greater. Exact determination of the value of utilization factor is complicated especially in small rooms where light undergoes multiple reflections.

Since the light leaving the lamp in different directions is subjected to different degrees of absorption, the initial polar curve of distribution has also to be taken into account. Even though manufacturers of lighting fittings supply tables giving utilization factors for each type of fitting under specified conditions yet, since such tables apply only to the fittings for which they have been compiled, a good deal of judgment is necessary while using them.

49.17. Depreciation Factor (*p*)

This factor allows for the fact that effective candle power of all lamps or luminous sources deteriorates owing to blackening and/or accumulation of dust or dirt on the globes and reflectors etc. Similarly, walls and ceilings etc., also do not reflect as much light as when they are clean. The value of this factor may be taken as 1/1.3 if the lamp fittings are likely to be cleaned regularly or 1/1.5 if there is much dust etc.

$$p = \frac{\text{illumination under actual condtions}}{\text{illumination when everything is perfectly clean}}$$

Since illumination is specified in lm/m^2, the area in square metre multiplied by the illumination required in lm/m^2 gives the total useful luminous flux that must reach the working plane. Taking into consideration the utilization and depreciation or maintenance factors, the expression for the gross lumens required is

Total lumens, $$\Phi = \frac{E \times A}{\eta \times p}$$

where E = desired illumination in lm/m^2 ; A = area of working plane to be illuminated in m^2
p = depreciation or maintenance factor ; η = utilization factor.

The size of the lamp depends on the number of fittings which, if uniform distribution is required, should not be far apart. The actual spacing and arrangement is governed by space/height values and by the layout of ceiling beams or columns. Greater the height, wider the spacing that may be used, although the larger will be the unit required. Having settled the number of units required, the lumens per unit may be found from (total lumens/number of units) from which the size of lamp can be calculated.

Example 49.20. *A room 8 m × 12 m is lighted by 15 lamps to a fairly uniform illumination of 100 lm/m^2. Calculate the utilization coefficient of the room given that the output of each lamp is 1600 lumens.*

Solution. Lumens emitted by the lamps = 15 × 1600 = 24,000 lm
Lumens received by the working plane of the room = 8 × 12 × 100 = 9600 lm
Utilization coefficient = 9600/24,000 = **0.4 or 40%.**

Example 49.21. *The illumination in a drawing office 30 m × 10 m is to have a value of 250 lux and is to be provided by a number of 300-W filament lamps. If the coefficient of utilization is 0.4 and the depreciation factor 0.9, determine the number of lamps required. The luminous efficiency of each lamp is 14 lm/W.* **(Elect. Drives & Utilization, Punjab Univ. Dec. 1994)**

Solution. $\Phi = EA/\eta p$; E = 250 lm/m^2, $A = 30 \times 10 = 300$ m^2; $\eta = 0.4$, $p = 0.9$
$\therefore \quad \Phi = 250 \times 300/0.4 \times 0.9 = 208{,}333$ lm
Flux emitted/lamp = 300 × 14 = 4200 lm; No. of lamps reqd.= 208,333/4200 = **50.**

Example 49.22. *Find the total saving in electrical load and percentage increase in illumination if instead of using twelve 150 W tungsten-filament lamps, we use twelve 80 W fluorescent tubes. It may be assumed that **(i)** there is a choke loss of 25 per cent of rated lamp wattage **(ii)** average luminous efficiency throughout life for each lamp is 15 lm/W and for each tube 40 lm/W and **(iii)** coefficient of utilization remains the same in both cases.*

Solution. Total load of filament lamps = 12 × 150 = 1800 W
Total load of tubes = 12 (80 + 0.25 × 80) = 1200 W
Net saving in load = 1800 – 1200 = **600 W**
If A is the room area and η the corfficient of utilization, then

illumination with lamps, $E_1 = \dfrac{12 \times 150 \times 15\eta}{A} =$ **27,000 η/A lm/m^2**

illumination with tubes, $E_2 = \dfrac{12 \times 80 \times 40\eta}{A} =$ **38,400 η/A lm/m^2**

increase in illumination $= \dfrac{38{,}400 - 27{,}000}{27{,}000} =$ **0.42 or 42%**

Example 49.23. *A football pitch 120 m × 60 m is to be illuminated for night play by similar banks of equal 1000 W lamps supported on twelve towers which are distributed around the ground to provide approximately uniform illumination of the pitch. Assuming that 40% of the total light emitted reaches the playing pitch and that an illumination of 1000 lm/m^2 is necessary for television purposes, calculate the number of lamps on each tower. The overall efficiency of the lamp is to be taken as 30 lm/W.* **(Elect. Technology-I, Bombay Univ.)**

Solution. Area to be illuminated = 120 × 60 = 7,200 m^2

Flux required = $7{,}200 \times 1{,}000 = 7.2 \times 10^6$ lm

Since only 40% of the flux emitted reaches the ground, the total luminous flux required to be produced is = $7.2 \times 10^6/0.4 = 18 \times 10^6$ lm

Flux contributed by each tower bank = $18 \times 10^6/12 = 1.5 \times 10^6$ lm

Output of each 1000-W lamp = $30 \times 1000 = 3 \times 10^4$ lm

Hence, number of such lamps on each tower is = $1.5 \times 10^6/3 \times 10^4 =$ **50**

Example 49.24. *Design a suitable lighting scheme for a factory 120 m × 40 m with a height of 7 m. Illumination required is 60 lux. State the number, location and mounting height of 40 W fluorescent tubes giving 45 lm/W.*

Depreciation factor = 1.2 ; utilization factor = 0.5

(Electric Drives & Util. Punjab Univ. 1993)

Solution. $\Phi = \dfrac{60 \times 120 \times 40}{0.5 \times (1/1.2)} = 691{,}200$ lm; Flux per tube = $45 \times 40 =$ **1800 lm.**

No. of fluorescent tubes reqd. = 691,200/1800 = 384. If twin-tube fittings are employed, then number of such fittings required. = 384/2 = **192.**

These can be arranged in 8 rows of 24 fittings each. Assuming that the working plane is 1 metre above the floor level and the fittings are fixed 1 metre below the ceiling, we get a space/height factor of unity both along the length as well as width of the factory bay.

Example 49.25. *A drawing hall in an engineering college is to be provided with a lighting installation. The hall is 30 m × 20 m × 8 m (high). The mounting height is 5 m and the required level of illumination is 144 lm/m^2. Using metal filament lamps, estimate the size and number of single lamp luminaries and also draw their spacing layout. Assume :*

Utilization coefficient = 0.6; maintenance factor = 0.75; space/height ratio=1

lumens/watt for 300-W lamp = 13,lumens/watt for 500-W lamp = 16.

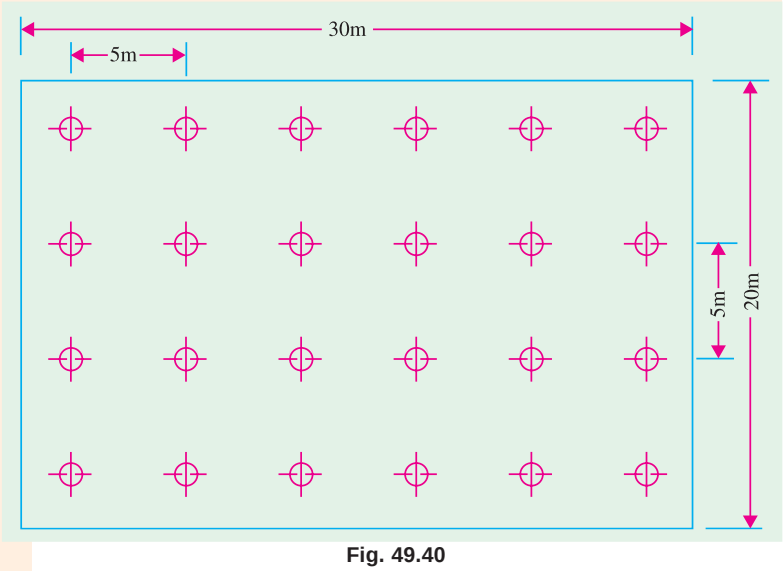


Fig. 49.40

Solution. Flux is given by $\Phi = EA/\eta p = 30 \times 20 \times 144/0.6 \times 0.75 = 192{,}000$ lm

Lumen output per 500-W lamp = $500 \times 16 = 8{,}000$

$\therefore$ No. of 500-W lamps required = 192,000/8000 = **24**

Similarly, No. of 300-W lamps required = 192,000/3900 = **49**

The 300-W lamps cannot be used because their number cannot be arranged in a hall of 30 m × 20 m with a space/height ratio of unity. However, 500-W lamps can be arranged in 4 rows of 6 lamps each with a spacing of 5 m both in the width and the length of the hall as shown in Fig. 49.40.

Example 49.26. *Estimate the number and wattage of lamps which would be required to illuminate a workshop space 60 × 15 metres by means of lamps mounted 5 metres above the working plane. The average illumination required is about 100 lux.*

Coefficient of utilization=0.4 ; Luminous efficiency=16 lm/W.

Assume a spacing/height ratio of unity and a candle power depreciation of 20%.

(Utilization of Electrical Energy, Madras Univ.)

Solution. Luminous flux is given by $\Phi = \frac{EA}{\eta p} = \frac{100 \times (60 \times 15)}{0.4 \times 1/1.2} = 27 \times 10^4$ lm

Total wattage reqd. $= 27 \times 10^4/16 =$ **17,000 W**

For a space/height ratio of unity, only three lamps can be mounted along the width of the room. Similarly, 12 lamps can be arranged along the length of the room. Total number of lamps required is $12 \times 3 =$ **36**. Wattage of each lamp is = 17,000/36 = 472 W. We will take the nearest standard lamp of **500 W**. These thirty-six lamps will be arranged as shown in Fig. 49.41.

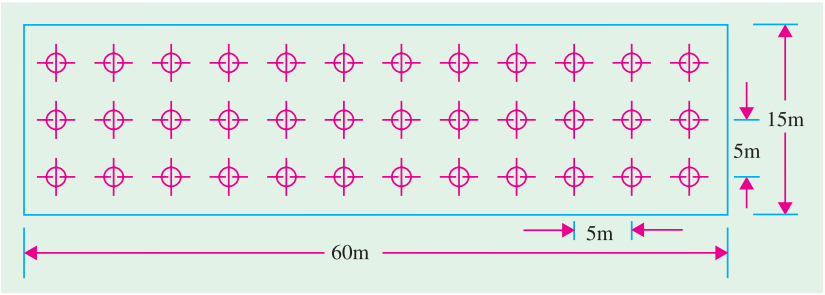


Fig. 49.41

Example 49.27. *A drawing hall 40 m × 25 m × 6 high is to be illuminated with metal-filament gas-filled lamps to an average illumination of 90 lm/m^2 on a working plane 1 metre above the floor. Estimate suitable number, size and mounting height of lamps. Sketch the spacing layout.*

Assume coefficient of utilization of 0.5, depreciation factor of 1.2 and spacing/height ratio of 1.2

Size of lamps	*200 W*	*300 W*	*500 W*
Luminous efficiency (in lm/W)	*16*	*18*	*20*

(Elect. Technology, Bombay Univ.)

Solution. Total flux required is $\Phi = \frac{40 \times 25 \times 90}{0.5 \times 1/1.2} = 216{,}000$ lm

Lumen output of each 200-W lamp is 3200 lm, of 300-W lamp is 5,400 lm and of 500-W lamp is 10,000 lm.

No. of 200-W lamps reqd. $= \frac{216{,}000}{3{,}200} =$ **67**; No. of 300-W lamps reqd.$= \frac{216{,}000}{5{,}400} =$ **40**

No. of 500-W lamps reqd. = 216,000/10,000 = **22**

With a spacing/height ratio of 1.2, it is impossible to arrange both 200-W and 300-W lamps. Hence, the choice falls on 500-W lamp. If instead of the calculated 22, we take 24 lamps of 500 wattage, they can be arranged in four rows each having six lamps as shown in Fig. 49.42. Spacing along the length of the hall is 40/6 = 6.67 m and that along the width is 25/4 = 6.25 m.

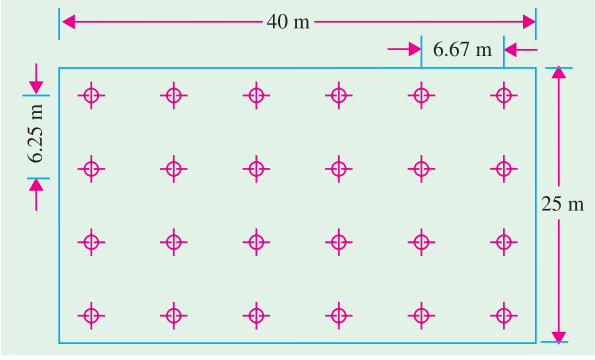


Fig. 49.42

Since mounting height of the lamps is 5 m above the working plane, it gives a space/height ratio of 6.7/5 = 1.5 along the length and 6.25/5 = 1.4 along the width of the hall.

Example 49.28. *A school classroom, 7 m × 10 m × 4 m high is to be illuminated to 135 lm/m^2 on the working plane. If the coefficient of utilization is 0.45 and the sources give 13 lumens per watt, work out the total wattage required, assuming a depreciation factor of 0.8 . Sketch roughly the plan of the room, showing suitable positions for fittings, giving reasons for the positions chosen.*

Solution. Total flux required is $\Phi = EA/\eta p$

Now $E = 135$ lm/m^2 ;

$A = 7 \times 10 = 70$ m^2 ;

$\eta = 0.45$; $p = 0.8$

$F = 135 \times 7 \times 10/0.45 \times 0.8 = 26{,}250$ lm

Total wattage reqd. $= 26{,}250/13 =$ **2020 W**

Taking into consideration the dimensions of the room, light fitting of 200 W would be utilized.

No. of fittings required $= 2020/200 =$ **10**

As shown in Fig. 49.43, the back row of fittings has been located 2/3 m from the rear wall so as to ***(i)*** provide adequate illumination on the rear desk and ***(ii)*** to minimise glare from paper because light would be incident practically over the shoulders of the students. The two side fittings help eliminate shadows while writing. One fitting has been provided at the chalk board end of the classroom for the benefit of the teacher. The fittings should be of general diffusing pendant type at a height of 3 m from the floor.

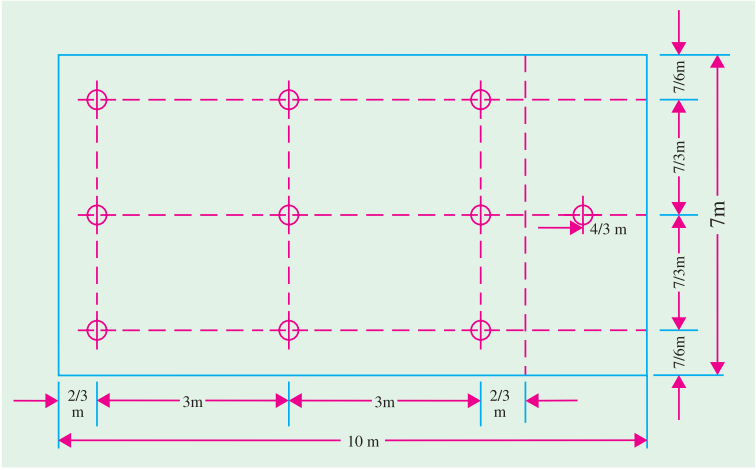


Fig. 49.43

Example 49.29. *A hall 30 m long and 12 m wide is to be illuminated and the illumination required is 50 lm/m^2. Calculate the number, the wattage of each unit and the location and mounting height of the units, taking a depreciation factor of 1.3 and utilization factor of 0.5, given that the outputs of the different types of lamp are as under :*

Watts :	*100*	*200*	*300*	*500*	*1000*
Lumens :	*1615*	*3650*	*4700*	*9950*	*21500*

(Utili. of Elect. Power, A.M.I.E.)

Solution. Area to be illuminated, $A = 30 \times 12 = 360$ m^2

Illumination required, $E = 50$ lm/m^2, $p = 1/1.3$, $\eta = 0.5$

Now, $\Phi = EA/\eta p = 50 \times 360/0.5 \times (1/1.3) = 46{,}800$ lm

If 100-W lamps are used, No. reqd. = 46,800/1615 = 29

If 200-W lamps are used, No. reqd. = 46,800/3650 = 13

If 300-W lamps are used, No. reqd. = 6,800/4700 = 10

If 500-W lamps are used, No. reqd. = 46,800/9950 = 5

If 1000-W lamps are used, No. reqd. = 46,800/21500 = 2

If we take the mounting height of 5 m, then 300 W lamps would be suitable. The No. of lamps required would be 10, arranged in two rows, each row having 5 lamps thus giving a spacing of 6 m in lengths as well as width and space/height ratio of 6/5 = 1.2.

If we use lamps of low power, their number would be large thereby increasing the number of fittings and hence cost. Lamps of higher voltage would be few in number but will not give a desirable space/height ratio.

Tutorial Problem No. 49.2

1. A room 30 m × 15 m is to be illuminated by 15 lamps to give an average illumination of 40 lm/m^2. The utilization factor is 4.2 and the depreciation factor is 1.4. Find the M.S.C.P. of each lamp.
[561 cd] (*Elect. Technology-1. Bombay Univ.*)
2. A factory space of 33 m × 13 m is to be illuminated with an average illumination of 72 lm/m^2 by 200 W lamps. The coefficient of utilization is 0.4 and the depreciation factor is 1.4. Calculate the number of lamps required. The lumen output of a 200-W lamp is 2,730 lm.
[40] (*Elect. Technology-I, Bombay Univ.*)
3. A drawing hall 30 m × 15 m with a ceiling height of 5 m is to be provided with an illumination of 120 lux. Taking the coefficient of utilization of 0.5, depreciation factor of 1.4, determine the No. of fluorescent tubes required and their spacing, mounting height and total wattage. Take luminous efficiency of fluorescent tube as 40 lm/W for 80-W tube. ***(A.M.I.E. Sec. B, Summer)***
4. A room 40 m × 15 m is to be illuminated by 1.5 m 80-W fluorescent tubes mounted 3.5 m above the working plane on which an average illumination of 180 lm/m^2 is required. Using maintenance factor of 0.8 and the utilization factor of 0.5, design and sketch a suitable layout. The 80-W fluorescent tube has an output of 4,500 lm. ***(Electrical Technology, Bombay Univ.)***
5. A hall is to be provided with a lighting installation. The hall is 30 m × 20 m × 8 m (high). The mounting height is 5 m and the required level of illumination is 110 lux. Using metal filament lamps, estimate the size and number of single lamp luminaries and draw their spacing layout. Assume depreciation factor = 0.8, utilization coefficient = 0.6 and space/height ratio = 1.

Watt :	200	300	500
Lumen/watt :	10	12	12.3

[24 lamps, 500 W] (*Services & Equipment-II, Calcutta Univ.*)
6. It is required to provide an illumination of 100 lux in a factory hall 30 m × 12 m. Assume that the depreciation factor is 0.8, the coefficient of utilization 0.4 and the efficiency of proposed lamps 14 lm/W. Calculate the number of lamps and their disposition.
(Utilization of Elect. Energy, Madras Univ.)
7. Define the terms : (*i*) Lux (*ii*) Luminous Flux (*iii*) Candle Power.
A workshop 100 m × 50 m is to be illuminated with intensity of illumination being 50 lux. Design a suitable scheme of lighting if coefficient of utilization = 0.9 ; Depreciation factor = 0.7 and efficiency of lamps = 80 lm/W. Use 100-W lamps. ***(Electrical Engineering-III, Poona Univ.)***

49.18. Floodlighting

It means 'flooding' of large surfaces with the help of light from powerful projectors. Flooding is employed for the following purposes :

1. For aesthetic purposes as for enhancing the beauty of a building by night *i.e.* flood lighting of ancient monuments, religious buildings on important festive occassions etc.
2. For advertising purposes *i.e.* flood lighting, huge hoardings and commercial buildings.
3. For industrial and commercial purposes as in the case of railway yards, sports stadiums and quarries etc.

Usually, floodlight projectors having suitable reflectors fitted with standard 250-, 500-, or 1,000-watt gas-filled tungsten lamps, are employed. One of the two typical floodlight installations often used is as shown in Fig. 49.44 (*a*). The projector is kept 15 m to 30 m away from the surface to be floodlighted and provides approximately parallel beam having beam spread of 25° to 30°. Fig. 49.44 (*b*) shows the case when the projector cannot be located away from the building. In that case, an asymmetric reflector is used which directs more intense light towards the top of the building.

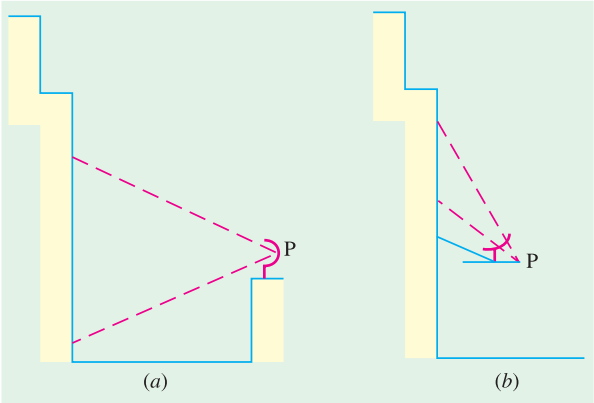


Fig. 49.44

The total luminous flux required to floodlight a building can be found from the relation, $\Phi = EA/\eta \times p$.

However, in the case of flood-lighting, one more factor has to be taken into account. That factor is known as waste-light factor (W). It is so because when several projectors are used, there is bound to be a certain amount of overlap and also because some light would fall beyond the edges of the area to be illuminated. These two factors are taken into account by multiplying the theoretical value of the flux required by a waste-light factor which has a value of nearly 1.2 for regular surfaces and about 1.5 for irregular objects like statues etc. Hence, the formula for calculation of total flux required for floodlighting purposes is

$$\Phi = \frac{EAW}{\eta p}$$

Example 49.30. *It is desired to floodlight the front of a building 42 m wide and 16 m high. Projectors of 30° beam spread and 1000-W lamps giving 20 lumen/watt are available. If the desired level of illumination is 75 lm/m^2 and if the projectors are to be located at ground level 17 m away, design and show a suitable scheme. Assume the following :*

Coefficient of utilization = 0.4 ; Depreciation factor = 1.3; Waste-light factor = 1.2.

(Electrical Power-II ; M.S. Univ. Baroda)

Solution. $\Phi = \frac{EAW}{\eta p}$

Here $E = 75$ lm/m^2 ; $A = 42 \times 16 = 672$ m^2 ; $W = 1.2$; $\eta = 0.4$; $p = 1/1.3$

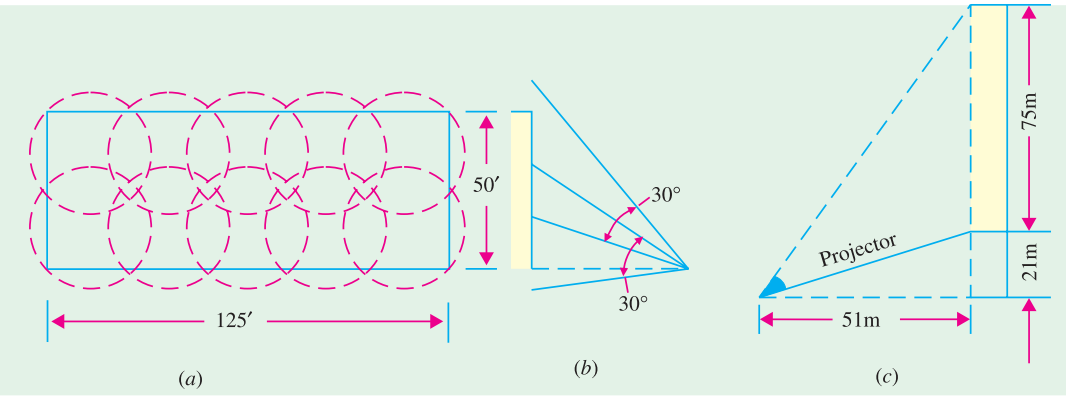


Fig. 49.45

$$\therefore \quad \Phi = \frac{7.5 \times 672 \times 1.2}{0.4 \times 1/1.3} = 196{,}500 \text{ lm}$$

Lumen output of each 1,000-W lamp = $1{,}000 \times 20 = 20{,}000$ lm

No. of lamps required = 196,500/20,000 = **10.**

With a beam spread* of 30°, it is possible to cover the whole length and width of the building by arranging the 10 projectors in two rows as shown in Fig. 49.45 (*a*).

Example 49.31. *Estimate the number of 1000-W floodlight projectors required to illuminate the up per 75 m of one face of a 96 m tower of width 13 m if approximate initial average luminance is to be 6.85 cd/m^2. The projectors are mounted at ground level 51m from base of the tower. Utilization factor is = 0.2; reflection factor of wall = 25% and efficiency of each lamp = 18 lm/W.*

(A.M.I.E. Sec. B., Winter 1992)

Solution. $B = 6.85$ cd/m^2 Now, $B = \rho E/\pi$ cd/m^2 —Art 49.4

$\therefore \quad E = pB/\rho = 6.85\,\pi/0.25 = 27.4\,\pi$ lm/m^2

Area to be floodlighted $= 13 \times 75 = 975$ m^2

$\therefore$ flux required $= 27.4\,\pi \times 975$ lm

Taking utilization factor into account, the flux to be emitted by all the lamps

$= 27.4\,\pi \times 975/0.2$ lm

Flux emitted by each lamp $= 18 \times 1000 = 18{,}000$ lm

$\therefore$ No. of lamps reqd. $= \dfrac{27.4\,\pi \times 975}{0.2 \times 18{,}000} =$ **24 (approx.)**

49.19. Artificial Sources of Light

The different methods of producing light by electricity may, in a board sense, be divided into three groups.

1. By ***temperature incandescence***. In this method, an electric current is passed through a filament of thin wire placed in vacuum or an inert gas. The current generates enough heat to raise the temperature of the filament to luminosity.

Incandescent tungsten filament lamps are examples of this type and since their output depends on the temperature of their filaments, they are known as ***temperature radiators***.

2. By establishing an arc between two carbon electrodes. The source of light, in their case, is the incandescent electrode.

3. ***Discharge Lamps***. In these lamps, gas or vapour is made luminous by an electric discharge through them. The colour and intensity of light *i.e.* candle-power emitted depends on the nature of the gas or vapour only. It should be particularly noted that these discharge lamps are luminiscent-light lamps and do not depend on temperature for higher efficiencies. In this respect, they differ radically from incandescent lamps whose efficiency is dependent on temperature. Mercury vapour lamp, sodium-vapour lamp, neon-gas lamp and fluorescent lamps are examples of light sources based on discharge through gases and vapours.

49.20. Incandescent Lamp

An incandescent lamp essentially consists of a fine wire of a high-resistance metal placed in an evacuated glass bulb and heated to luminosity by the passage of current through it. Such lamps were

* It indicates the divergence of a beam and may be defined as the angle within which the minimum illumination on a surface normal to the axis of the beam is 1/10th of the maximum.

produced commercially for the first time by Edison in 1879. His early lamps had filaments of carbonized paper which were, later on, replaced by carbonized bamboo. They had the disadvantage of negative temperature coefficient of resistivity. In 1905, the metallized carbon-filament lamps were put in the market whose filaments had a positive temperature coefficient of resistivity (like metals). Such lamps gave 4 lm/W.



An incandescent lamp

At approximately the same time, osmium lamps were manufactured which had filaments made of osmium which is very rare and expensive metal. Such lamps had a very fair maintenance of candle-power during their useful life and an average efficiency of 5 lm/W. However, osmium filaments were found to be very fragile.

In 1906 tantalum lamps having filaments of tantalum were produced which had an initial efficiency of 5 lm/watt.

All these lamps were superseded by tungsten lamps which were commercially produced in about 1937 or so. The superiority of tungsten lies mainly in its ability to withstand a high operating temperature without undue vaporisation of the filament. The necessity of high working temperature is due to the fact that the amount of visible radiation increases with temperature and so does the radiant efficiency of the luminous source. The melting temperature of tungsten is 3655°K whereas that of osmium is 2972°K and that of tantalum is 3172°K. Actually, carbon has a higher melting point than tungsten but its operating temperature is limited to about 2073°K because of rapid vaporization beyond this temperature.

In fact, the ideal material for the filament of incandescent lamps is one which has the following properties :

1. A high melting and hence operating temperature
2. A low vapour pressure
3. A high specific resistance and a low temperature coefficient
4. Ductility and
5. Sufficient mechanical strength to withstand vibrations.

Since tungsten possesses practically all the above mentioned qualities, it is used in almost all modern incandescent lamps. The earlier lamps had a square-cage type filament supported from a central glass stem enclosed in an evacuated glass bulb. The object of vacuum was two fold :

(a) to prevent oxidation and

(b) to minimize loss of heat by convection and the consequent lowering of filament temperature. However, vacuum favoured the evaporation of the filament with the resulting blackening of the lamp so that the operating temperature had to be kept as low as 2670° K with serious loss in luminous efficiency.

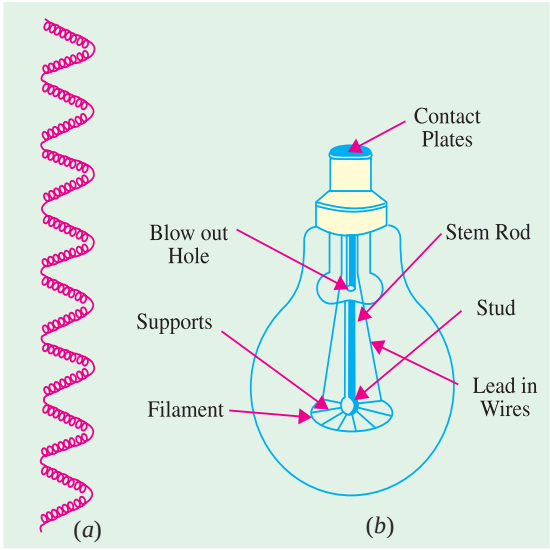


Fig. 49.46

It was, later on, found that this difficulty could be solved to a great extent by inserting a chemically inert gas like nitrogen or argon. The presence of these gases within the glass bulb decreased the evaporation of the filament and so lengthened its life. The filament could now be run at a relatively higher temperature and hence higher luminous efficiency could be realized. In practice, it was found that an admixture of 85% argon and about 15 percent nitrogen gave the best results.

However, introduction of gas led to another difficulty *i.e.* loss of heat due to convection which offsets the additional increase in efficiency. However, it was found that for securing greater efficiency, a concentrated filament having a tightly-wound helical construction was necessary. Such a coiled filament was less exposed to circulating gases, its turns supplying heat to each other and further the filament was mechanically stronger. The latest improvement is that the coiled filament is itself 'coiled' resulting in 'coiled-coil' filament Fig. 49.46 (*a*) which leads to further concentrating the heat, reducing the effective exposure to gases and allows higher temperature operation, thus giving greater efficiency. The construction of a modern coiled-coil gas-filled filament lamp is shown in Fig. 49.46 (*b*). The lamp has a 'wreath' filament *i.e.* a coiled filament arranged in the form of a wreath on radial supports.



A electric filament is a metallic wire, usually made of tungsten, when heated to luminance by passing electricity, radiates light

49.21. Filament Dimensions

There is found to be a definite relation between the diameter of a given filament and the current. Consider a filament operating at a fixed temperature and efficiency. Then since no heat is being utilized for further raising the temperature, all the heat produced in a given time is mostly lost by radiation (if vacuum is good). In other words,

Heat produced per second = heat lost per second by radiation.

$$\text{Now, power intake} = I^2 R = I^2 \times \frac{\rho l}{A} = \frac{I^2 \rho l}{\pi d^2/4} = I^2 \left(\frac{4\rho l}{\pi d^2} \right)$$

where, I = filament current in amperes, l = filament length
A = filament cross-section d = filament diameter
r = resistivity of filament material at the working temperature.

Heat radiated per second from the surface is proportional to the area of the surface and emissivity of the material

$\therefore$ heat lost/second $\propto$ surface area $\times$ emissivity σ

$\therefore$ $I^2 (4rl/\pi d^2) \propto l \times \pi d \times \sigma$ or $I^2 \propto d^3$ ...(*i*)

$\therefore$ $I \propto d^{1.5}$ or $d \propto I^{2/3}$

In general, for two filaments of the same material working at the same temperature and efficiency, the relation as seen from (*i*) above is

$$\left(\frac{I_1}{I_2} \right)^2 = \left(\frac{d_1}{d_2} \right)^3$$

It would be noticed that the above expressions are similar to those concerning fusing current of a given material under stated conditons (Preece's Rule).

Moreover, for two filaments working at the same temperature, the flux per unit area is the same. Denoting their lengths by l_1 and l_2 and their diameters by d_1 and d_2 respectively, we have, Lumen output $\propto l_1 d_1 \propto l_2 d_2$ or $l_1 d_1 = l_2 d_2$ = constant.

Example 49.32. *If the filament of a 32 candela, 100-V lamp has a length l and diameter d, calculate the length and diameter of the filament of a 16 candela 200-V lamp, assuming that the two lamps run at the same intrinsic brilliance.*

Solution. Using the above relation $32 \propto l_1 d_1$ and $16 \propto l_2 d_2$

$$\therefore \quad l_2 d_2 = \frac{1}{2} l_1 d_1$$

Assuming that the power intakes of the two lamps are proportional to their outputs, we have

$$32 \propto 100\, I_1 \text{ and } 16 \propto 200\, I_2 \quad \therefore \quad I_2 = I_1 \times (16/200) \times (100/32) = \frac{1}{4} l_1$$

$$\text{Also } I_1 \propto d_1^{3/2} \text{ and } I_2 \propto d_2^{3/2} \quad \therefore \quad (d_2/d_1)^{3/2} = I_2/I_1 = \frac{1}{4}$$

$$\therefore \quad d_2 = 0.4\, d_1 \text{ (approx.)}$$

$$\therefore \quad l_2 = \frac{1}{2} l_1 \times (d_1/d_2) = \frac{1}{2} \times (1/0.3968) \times l_1 = 1/26\, l_1.$$

Actually, this comparison is not correct because a thicker filament can be worked at a somewhat higher temperature than a thinner one.

Example 49.33. *An incandescent lamp has a filament of 0.005 cm diameter and one metre length. It is required to construct another lamp of similar type to work at double the supply voltage and give half the candle power. Assuming that the new lamp operates at the same brilliancy, determine suitable dimensions for its filament.* **(Elect. Technology, Utkal Univ.)**

Solution. Let I_1 and I_2 be the luminous intensities of the two lamps. Then

$$I_1 \propto l_1 d_1 \text{ and } l_2 \propto l_2 d_2 \quad \therefore \quad \frac{l_2 d_2}{l_1 d_1} = \frac{I_2}{I_1} = \frac{1}{2} \text{ or } l_2 d_2 = \frac{1}{2} l_1 d_1$$

Assuming that the power intakes of the two lamps are proportional to their outputs, we have

$$I_1 \propto V_1 i_1 \text{ and } I_2 \propto V_2 i_2 \quad \therefore \quad \frac{V_2 i_2}{V_1 i_1} = \frac{I_2}{I_1}$$

$$\therefore \quad i_2 = i_1 (V_1/V_2)(I_2/I_1) = i_1 \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} i_1 \quad \text{Now, } i_1 \propto d_1^{3/2} \text{ and } i_2 \propto d_2^{3/2}$$

$$\therefore \quad (d_2/d_1)^{3/2} = (i_2/i_1) = \frac{1}{4} \quad \therefore \quad d_2 = 0.3968\, d_1$$

$$\therefore \quad l_2 = \frac{1}{2} l_1 \frac{d_1}{d_2} = \frac{1}{2} l_1 \times \left(\frac{1}{0.3968} \right) = 1.26\, l_1$$

Now, $d_1 =$ **0.005 cm ;** $l_1 =$ **100 cm**

$\therefore \quad d_2 = 0.3968 \times 0.005 =$ **0.001984 cm.** $\quad l_2 = 1.26 \times 100 =$ **126 cm.**

Example 49.34. *A 60 candle power, 250-V metal filament lamp has a measured candle power of 71.5 candela at 260 V and 50 candela at 240 V.*

(a) *Find the constant for the lamp in the expression $C = aV^b$ where C = candle power and V = voltage.*

(b) *Calculate the change of candle power per volt at 250 V. Determine the percentage variation of candle power due to a voltage variation of æ 4% from the normal value.* **(A.M.I.E. Sec. B)**

Solution. **(*a*)** $C = aV^b \quad \therefore \quad 71.5 = a \times 260^b$ and $50 = a \times 240^b$

$\therefore \quad 71.5/50 = (260/240)^b$, $b = \mathbf{4.5}$

Substituting this value of b in the above equation, we get

$a = 50/240^{4.5}$, $\quad a = \mathbf{0.98 \times 10^{-9}}$

Hence, the expression for the candle power of the lamp becomes $C = 0.98 \times 10^{-9}\, V^{4.5}$ candela

(*b*) Differentiating the above expression and putting $V = 250$ V, we get

$$\frac{dC}{dV} = 0.98 \times 10^{-9} \times 4.5 \times 250^{3.5} = \textbf{4.4 candela per volt}$$

When voltage increases by 4%, $C_2/C_1 = 1.04^{4.5}$

$$\% \text{ change in candle power} = \frac{C_2 - C_1}{C_1} \times 100 = (1.04^{4.5} - 1) \times 100 = 19.3$$

When voltage falls by 4%, $C_2/C_1 = 0.96^{4.5}$

$\therefore$ % change in candle power $= (0.96^{4.5} - 1) \times 100 = \mathbf{-16.8}$

49.22. Incandescent Lamp Characteristics

The operating characteristics of an incandescent lamp are materially affected by departure from its normal working voltage. Initially, there is a rapid heating up of the lamp due to its low thermal capacity, but then soon its power intake becomes steady. If the filament resistance were not dependent on its temperature, the rate of generation of heat would have been directly proportional to the square of voltage applied across the lamp. However, because of ***(i)*** positive temperature coefficient of resistance and ***(ii)*** complex mechanism of heat transfer from filament to gas, the relations between the lamp characteristics and its voltage are mostly experimental. Some of the characteristics of gas-filled lamps are given below.

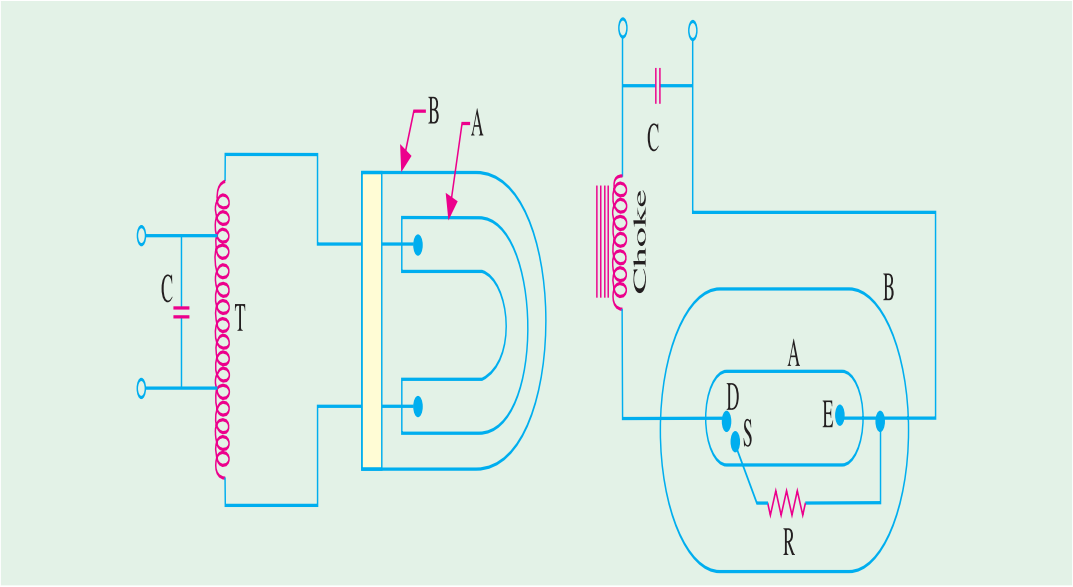


Fig. 49.47

(i) It is found that candle power or lumen output of the lamp varies with the voltage as lumen output $\propto V^{3.3}$.

(*ii*) Variation of lumen output in terms of current is given by : lumen output $\propto I^5$

(*iii*) Life of the lamp is given by : life $\propto 1/V^{13}$

(*iv*) Wattage is given by $W \propto V^{1.43}$

(*v*) Its lumen/watt is given by : lm/watt $\propto V^2$

The characteristic curves are plotted in Fig. 49.47. The life characteristic is very revealing. Even a small undervoltage considerably increases its life whereas overvoltage of as small a value as 5% shortens its life by 50%.

49.23. Clear and Inside-frosted Gas-filled Lamps

The advantages of clear glass-filled lamps are that they facilitate light control and are necessary for use in lighting units where accurate distribution is required such as in flood-lights for buildings, projectors and motor-car headlights. However, they produce hard shadows and glare from filaments. Inside-frosted gas-filled lamps have luminous output nearly 2 per cent less than clear glass lamps of the same rating, but they produce softer shadows and practically eliminate glare from filaments. Such lamps are ideal for use in industrial open fittings located in the line of sight at low mounting heights and in diffuse fittings of opal glass type in order to avoid the presence of filament striations on the surface of the glassware etc.

Another new type of incandescent lamp is the inside-silica coated lamp which, due to the fine coating of silica on the inside of its bulb, has high diffusion of light output. Hence, the light from the filament is evenly distributed over the entire bulb surface thus eliminating the noticeably-bright spot around the filament area of an inside-frosted lamp. Such lamps are less glaring, soften shadows and minimize the brightness of reflections from specular (shiny) surfaces.

49.24. Discharge Lamps

In all discharge lamps, an electric current is passed through a gas or vapour which renders it luminous. The elements most commonly used in this process of producing light by gaseous conduction are neon, mercury and sodium vapours. The colours (*i.e.* wavelength) of light produced depends on the nature of gas or vapour. For example, the neon discharge yields orange-red light of nearly 6,500 A.U. which is very popular for advertising signs and other spectacular effects. The pressure used in neon tubes is usually from 3 to 20 mm of Hg. Mercury-vapour light is always bluish green and deficient in red rays, whereas sodium vapour light is orange-yellow.

Discharge lamps are of two types. The first type consists of those lamps in which the colour of light ***is the same as produced by the discharge through the gas or vapour***. To this group belong the neon gas lamps, mercury vapour (M.V.) and sodium vapour lamps. The other type consists of vapour lamps which use the phenomenon of fluorescence. In their case, the discharge through the vapour produces ultra-violet waves which cause fluorescence in certain materials known as phosphors. The radiations from the mercury discharge (especially 2537 A° line) impinge on these phosphors which absorb them and then re-radiate them at longer wave-lengths of visible spectrum. The inside of the fluorescent lamp is coated with these phosphors for this purpose. Different phosphors have different exciting ranges of frequency and give lights of different colours as shown in table 49.2.

Table 49.2

Phosphor	*Lamp Colour*	*Exciting range A°*	*Emitted wavelenght A°*
Calcium Tangstate	Blue	2200 - 3000	4400
Zinc Silicate	Green	2200 - 2960	5250
Cadmium Borate	Pink	2200 - 3600	6150
Cadmium Silicate	Yellow-pink	2200 - 3200	5950

49.25. Sodium Vapour Lamp



Sodium vapour lamp

One type of low-pressure sodium-vapour lamp along with its circuit connection is shown in Fig. 49.48. It consists of an inner U-tube A made of a special sodium-vapour-resisting glass. It houses the two electrodes and contains sodium together with the small amount of neon-gas at a pressure of about 10 mm of mercury and one per cent of argon whose main function is to reduce the initial ionizing potential. The discharge is first started in the neon gas (which gives out redish colour). After a few minutes, the heat of discharge through the neon gas becomes sufficient to vaporise sodium and then discharge passes through the sodium vapour. In this way, the lamp starts its normal operation emitting its characteristic yellow light.

The tungsten-coated electrodes are connected across auto-transformer *T* having a relatively high leakage reactance. The open-circuit voltage of this transformer is about 450 V which is sufficient to initiate a discharge through the neon gas. The leakage reactance is used not only for starting the current but also for limiting its value to safe limit. The electric discharge or arc strikes immediately after the supply is switched on whether the lamp is hot or cold. The normal burning position of the lamp is horizontal although two smaller sizes of lamp may be burnt vertically. The lamp is surrounded by an outer glass envelope *B* which serves to reduce the loss of heat from the inner discharge tube *A*. In this way, *B* helps to maintain the necessary high temperature needed for the operation of a sodium vapour lamp irrespective of draughts. The capacitor *C* is meant for improving the power factor of the circuit.

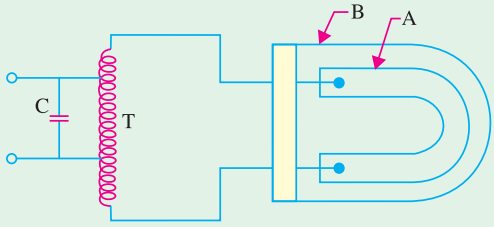


Fig. 49.48

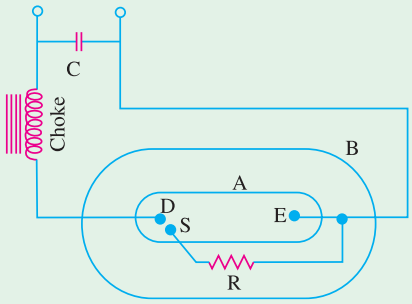


Fig. 49.49

The light emitted by such lamps consists entirely of yellow colour. Solid objects illuminated by sodium-vapour lamp, therefore, present a picture in monochrome appearing as various shades of yellow or black.

49.26. High-pressure Mercury Vapour Lamp

Like sodium-vapour lamp, this lamp is also classified as electric discharge lamp in which light is produced by gaseous conduction. Such a lamp usually consists of two bulbs — an arc-tube containing the electric discharge and an outer bulb which protects the arc-tube from changes in temperature. The

inner tube or arc tube *A* is made of quartz (or hard glass) the outer bulb *B* of hard glass. As shown in Fig. 49.49, the arc tube contains a small amount of mercury and argon gas and houses three electrodes *D*, *E* and *S*. The main electrodes are *D* and *E* whereas *S* is the auxiliary starting electrode. *S* is connected through a high resistance *R* (about 50 kΩ) to the main electrode situated at the outer end of the tube. The main electrodes consist of tungsten coils with electron-emitting coating or elements of thorium metal.

When the supply is switched on, initial discharge for the few seconds is established in the argon gas between *D* and *S* and then in the argon between *D* and *E*. The heat produced due to this discharge through the gas is sufficient to vaporise mercury. Consequently, pressure inside *A* increases to about one or two atmospheres and the p.d. across *D* and *E* grows from about 20 to 150 V, the operation taking about 5-7 minutes. During this time, discharge is established through the mercury vapours which emit greenish-blue light.

The choke serves to limit the current drawn by the discharge tube *A* to a safe limit and capacitor *C* helps to improve the power factor of the circuit.

True colour rendition is not possible with mercury vapour lamps since there is complete absence of red-light from their radiations. Consequently, red objects appear black, all blues appear mercury-spectrum blue and all greens the mercury-spectrum green with the result that colour values are distorted.

Correction for colour distortion can be achieved by

1. Using incandescent lamps (which are rich in red light) in combination with the mercury lamps.
2. Using colour-corrected mercury lamps which have an inside phosphor coat to add red colour to the mercury spectrum.

Stroboscopic (Flickering) effect in mercury vapour lamps is caused by the 100 on and off arc strikes when the lamps are used on the 50-Hz supply. The effect may be minimized by

1. Using two lamps on lead-lag transformer
2. Using three lamps on separated phases of a 3-phase supply and
3. Using incandescent lamps in combination with mercury lamps.

In the last few years, there has been tremendous improvement in the construction and operation of mercury-vapour lamps, which has increased their usefulness and boosted their application for all types of industrial lighting, floodlighting and street lighting etc. As compared to an incandescent lamp, a mercury-vapour lamp is (*a*) smaller in size (*b*) has 5 to 10 times longer operating life and (*c*) has 3 times higher efficiency *i.e.* 3 times more light output for given electrical wattage input.

Typical mercury-vapour lamp applications are :

1. High-bay industrial lighting — where high level illumination is required and colour rendition is not important.
2. Flood-lighting and street-lighting
3. Photochemical applications — where ultra-violet output is useful as in chlorination, water sterilization and photocopying etc.
4. For a wide range of inspection techniques by ultra-violet activation of fluorescent and phosphorescent dyes and pigments.
5. Sun-tan lamps — for utilizing the spectrum lines in the erythemal region of ultra-violet energy for producing sun-tan.

49.27. Fluorescent Mercury-vapour Lamps

Basically, a fluorescent lamp consists of a long glass tube internally coated with a suitable fluorescent powder. The tube contains a small amount of mercury along with argon whose function is to facilitate the starting of the arc. There are two sealed-in electrodes at each end of the tube. Two basic types of electrodes are used in fluorescent lamps :

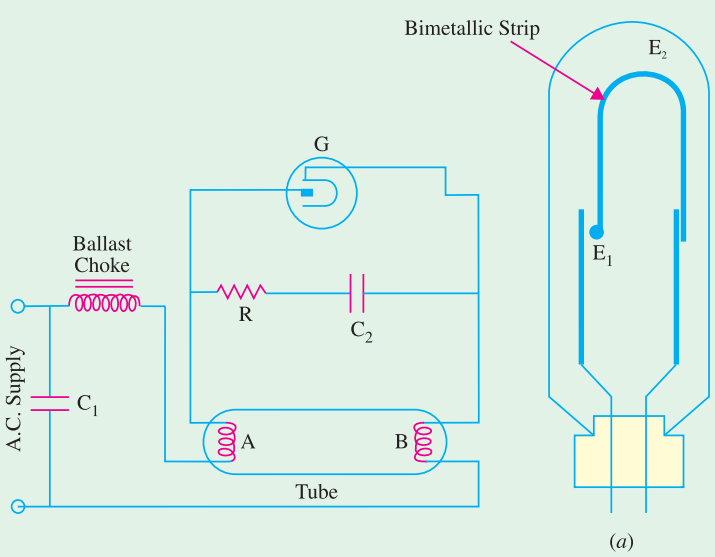


Fig. 49.50 Fig. 49.51

1. The coated-coil tungsten wire type. This type is used in standard pre-heat, rapid-start, instant-start lamps etc.

2. The inside-coated metal cylinder type which operates at a lower and more even temperature than the tungsten type and is called 'cold cathode'.*

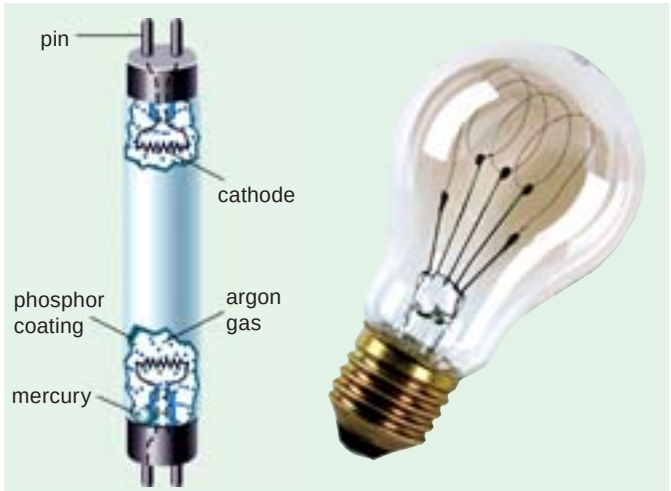
Circuits employed for the control of fluorescent lamps can be divided into two main groups ***(i)*** switch-start circuits and ***(ii)*** startless circuits requiring no starter. There are two types of starters ***(a)*** glow type — which is a voltage-operated device and ***(b)*** thermal switch — which is a current-operated device. Fig. 49.50 shows a flourescent tube fitted with a glow starter *G*. As shown separately in Fig. 49.51 (*a*), the glow switch consists of two electrodes enclosed in a glass bulb filled with a mixture of helium and hydrogen or argon or neon at low pressure. One electrode E_1 is fixed whereas the other E_2 is movable and is made of a U-shaped bimetallic strip. To reduce radio interference, a small capacitor C_2 is connected across the switch. The resistor *R* checks capacitor surges and prevents the starter electrodes or contacts from welding together. The complete starter switch along with the capacitor and resistor is contained in an aluminium casing is shown in Fig. 49.51 (*b*). Normally, the contacts are open and when supply is switched on, the glow switch receives almost full mains voltage**. The voltage is sufficient to start a glow discharge between the two electrodes E_1 and E_2 and the heat generated is sufficient to bend the bimetallic strip E_2 till it makes contact with the fixed electrode E_1, thus closing the contacts. This action completes the main circuit through the choke and the lamp electrodes *A* and *B* (Fig. 49.50). At the same time, since the glow between E_1 and E_2 has been shorted out, the bimetallic strip cools and the contacts E_1 and E_2 open. By this time, lamp electrodes *A* and *B* become heated to incandescence and the argon gas in their immediate vicinity is ionized. Due to opening of the glow switch contacts, a high inductive e.m.f. of about 1000 volts is induced in the choke. This voltage surge is sufficient to initiate a discharge in the argon gas lying between electrodes *A* and *B*. The heat thus produced is sufficient to vaporize mercury and the p.d. across the fluorescent tube falls to about 100 or 110 V which is not sufficient to restart the glow in

* A cold cathode fluorescent lamp requires higher operating voltage than the other type. Although cold cathode lamps have less efficiency, they have much longer life than other lamps.

** It is so because only the small discharge current flows and voltage drop across the choke is negligible.

G. Finally, the discharge is established through the mercury vapour which emits ultra-violet radiations. These radiations impinge on the fluorescent powder and make it emit visible light.

The function of the capacitor C_1 is to improve the power factor of the circuit. It may be noted that the function of the highly-inductive choke (also called ballast) is ***(i)*** to supply large potential for starting the arc or discharge and ***(ii)*** to limit the arc current to a safe value.



A fluorescent lamp radiates more light compared to an incandescent filament bulb, for the same amount of power consumed

49.28. Fluorescent Lamp Circuit with Thermal Switch

The circuit arrangement is shown in Fig. 49.52. The switch has a bimetallic strip close to a resistance *R* which produces heat. The switch is generally enclosed in hydrogen-filled glass bulb *G*. The two switch electrodes E_1 and E_2 are normally closed when the lamp is not in operation. When normal supply is switched on, the lamp filament electrodes *A* and *B* are connected together through the thermal switch and a large current passes through them. Consequently, they are heated to incandescence. Meanwhile heat produced in resistance *R* causes the bimetallic strip E_2 to break contact. The inductive surge of about 1000 V produced by the choke is sufficient to start discharge through mercury vapours as explained in Art. 49.27. The heat produced in *R* keeps the switch contacts E_1 and E_2 open during the time lamp is in operation.

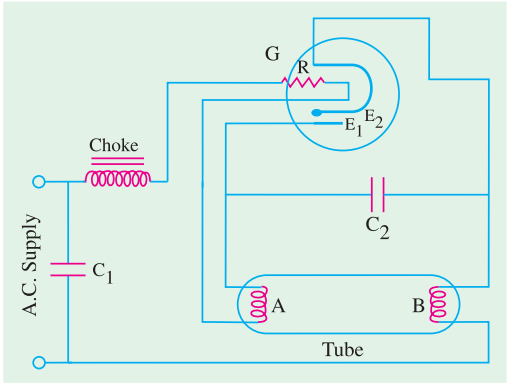


Fig. 49.52

49.29. Startless Fluorescent Lamp Circuit

Such a circuit (Fig. (49.53) which does not require the use of a starter switch is commercially known as 'instant-start' or 'quick-start'. In this case, the normal starter is replaced by a filament heating transformer whose secondaries *SS* heat up the lamp electrodes *A* and *B* to incandescence in a fraction of a second. This combination of pre-heating and application of full supply voltage across lamp electrodes *A* and *B* is sufficient to start ionization in the neighbourhood of the electrodes which further spreads to the whole tube. An earthed strip *E* is used to ensure satisfactory starting.

The advantages of startless method are

1. It is almost instantaneous starting.
2. There is no flickering and no false starts.
3. It can start and operate at low voltage of 160-180 V.
4. Its maintenance cost is lower due to the elimination of any starter-switch replacements.
5. It lengthens the life of the lamp.

49.30. Stroboscopic Effect of Fluorescent Lamps

Stroboscopic or flickering effect produced by fluorescent lamps is due to the periodic fluctuations in the light output of the lamp caused by cyclic variations of the current on a.c. circuits. This phenomenon creates multiple-image appearance of moving objects and makes the movement appear jerky. In this connection, it is worth noting that

1. This flicker effect is more pronounced at lower frequencies.
2. Frequency of such flickers is twice the supply frequency.
3. The fluorescent powder used in the tube is slightly phosphorescent, hence stroboscopic effect is reduced to some extent due to after-glow.

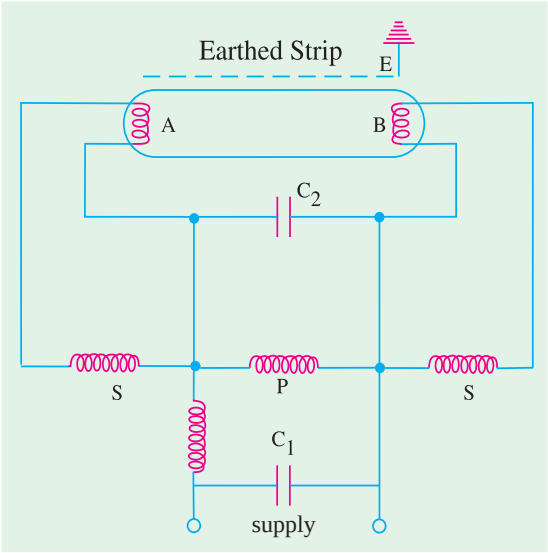


Fig. 49.53

Stroboscopic effect is very troublesome in the following cases :

1. When an operator has to move objects very quickly particularly those having polished finish. These objects would appear to move with jerky motion which over a long period would produce visual fatigue.
2. In the case of rotating machines whose frequency of rotation happens to be a multiple of flicker frequency, the machines appear to decrease in speed of rotation or be stationary. Sometimes the machines may even seem to rotate in the opposite direction.

Some of the methods employed for minimizing stroboscopic effect are given below :

1. By using three lamps on the seperate phases of a 3-phase supply. In this case, the three light waves reaching the working plane would overlap by 120° so that the resultant fluctuation will be very much less than from a single fluorescent lamp.
2. By using a 'twin lamp' circuit on single-phase supply as shown in Fig. 49.54, one of the chokes has a capacitor in series with it and the lamp. In this way, a phase displacement of nearly 120° is introduced between the branch currents and also between the two light waves thereby reducing the resultant fluctuation.
3. By operating the lamps from a high frequency supply. Obviously, stroboscopic effect will entirely disappear on d.c. supply.

49.31. Comparison of Different Light Sources

1. Incandescent Lamps. They have instantaneous start and become momentarily off when the supply goes off. The colour of their light is very near the natural light. Their initial cost of installation is minimum but their running cost is maximum. They work equally well both on d.c. and a.c. supply and frequent switching does not affect their life of operation. Change of supply voltage affects their efficiency, output and life in a very significant

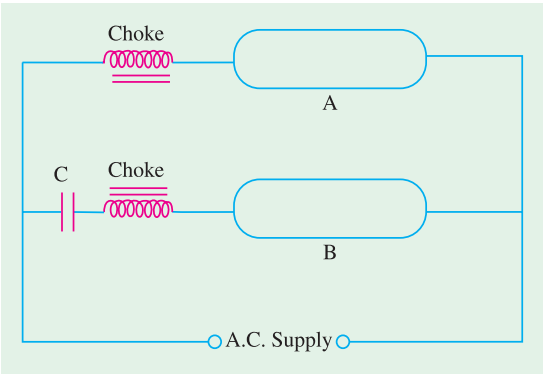


Fig. 49.54

way. They have an average working life of 1000 hours and luminous efficiency of 12 lm/W. Since their light has no stroboscopic effects, the incandescent lamps are suitable for domestic, industrial, street lighting and floodlights etc. They are available in a wide range of voltage ratings and, hence are used in automobiles, trains, emergency lights, aeroplanes and signals for railways etc.

2. Flourescent Lamps. They have a reaction time of one second or a little more at the start. They go off and restart when the supply is restored. The colour of their light varies with the phosphor coating. Their initial cost of installation is maximum but running cost is minimum. Since stroboscopic effect is present, they are suitable for semi-direct lighting, domestic, industrial, commercial, roads and halls etc. Change in voltage affects their starting although light output does not change as remarkably as in the case of incandescent lamps. Colour of their light changes with the different phosphor coating on the inner side of the tube. Frequent switching affects their life period. They have quite high utility but their voltage rating is limited. Hence, their use is confined to mains voltage or complicated inverter circuits which convert 12 V d.c. into high volt d.c.. They have an average working life of 4000 hours and a luminous efficiency of 40 lm/W.

3. Mercury Vapour Lamps. They take 5 to 6 minutes for starting. They go off and cannot be restarted after the recovery of the voltage till the pressure falls to normal. They suffer from high colour distortion. Their initial cost of installation is high but lesser than that of flourescent lamps. Their running cost is much less than incandescent lamps but higher than flourescent tubes for the same levels of illumination. Stroboscopic effect is present in their light. They are suitable for open space like yards, parks and highway lighting etc. Change in voltage effects their starting time and colour of radiations emitted by them. Switching does not affect their life period. They have very limited utility that too on mains voltage. They are suitable for vertical position of working. They have an average working life of 3000 hours and an efficiency of 40 lm/W.

4. Sodium Vapour Lamps. They have a starting time of 5 to 6 minutes. They go off and cannot be restarted after the recovery of the voltage till its value falls to the normal value. The colour of their light is yellowish and produces colour distortion.Their initial cost of installation is maximum although their running cost is less than for filament lamps but more than for flourescent lamps. They have stroboscopic effect and are suitable for use in open spaces, highways and street lighting etc. Change in voltage affects their starting time and colour of their radiations. They work on a.c. voltage and frequent switching affects their life. They are not suitable for local lighting. The colour of their light cannot be changed. They are very suitable for street lighting purposes. Their position of working is horizontal. They have a working life of about 3000 hours and efficiency of 60-70 lm/W.

Tutorial Problem No. 49.3

1. Explain $\cos^3 \phi$ law. ***(J.N. University, Hyderabad, November 2003)***
2. A lamp of 500 candle power is placed at the centre of a room,(20 m × 10 m × 5 m.) Calculate the illumination in each corner of the floor and a point in the middle of a 10 m wall at a height of 2 m from floor. ***(J.N. University, Hyderabad, November 2003)***
3. Enumerate the various factors, which have to be considered while designing any lighting scheme. ***(J.N. University, Hyderabad, November 2003)***
4. Prove that 1 candle/sq. feet = ($\pi fl - L$.) ***(J.N. University, Hyderabad, November 2003)***
5. A lamp giving 300 c.p. in all directions below the horizontal is suspended 2 metres above the centre of a square table of 1 metre side. Calculate the maximum and minimum illumination on the surface of the table. ***(J.N. University, Hyderabad, November 2003)***
6. Explain the various types of lighting schemes with relevant diagrams. ***(J.N. University, Hyderabad, November 2003)***
7. Discuss inverse square law? Corire law of Illustration. ***(J.N. University, Hyderabad, November 2003)***

8. A lamp fitted with 120 degrees angled cone reflector illuminates circular area of 200 meters in diameter.The illumination of the disc increases uniformly from 0.5 metre-candle at the edge to 2 meter-candle at the centre.
Determine :
(*i*) the total light received (*ii*) Average illumination of the disc (*iii*) Average c.p. of the source.
(J.N. University, Hyderabad, November 2003)

9. Discuss the flood lighting with suitable diagrams. ***(J.N. University, Hyderabad, November 2003)***

10. Explain the measurement techniques used for luminous intensity.
(J.N. University, Hyderabad, November 2003)

11. Write short notes on :
(*i*) Bunsen photometer head (*ii*) Lummer-Brodherm photometer head (*iii*) Flicker photometer head.
(J.N. University, Hyderabad, November 2003)

12. What do you understand by polar curves as applicable to light source? Explain.
(J.N. University, Hyderabad, November 2003)

13. Mean spherical Candlepower. ***(J.N. University, Hyderabad, April 2003)***

14. Explain how you will measure the candlepower of a source of light.
(J.N. University, Hyderabad, April 2003)

15. Explain the Rosseau's construction for calculating M.S.C.P. of a lamp.
(J.N. University, Hyderabad, April 2003)

16. What do you mean by International Luminosity curve? Explain.
(J.N. University, Hyderabad, April 2003)

17. Explain in detail the primary standard of luminous intensity with relevant diagram.
(J.N. University, Hyderabad, April 2003)

18. Explain with sketches the constructional features of a filament lamp.
(J.N. University, Hyderabad, April 2003)

19. Explain how the standard lamps canbe calibrated w.r.t. primary and secondary standards.
(J.N. University, Hyderabad, April 2003)

20. Briefly explain the various laboratory standards used in Illumination.
(J.N. University, Hyderabad, April 2003)

21. Write short notes on :
(*a*) High pressure mercury vapour lamp (*i*) M.A. Type (*ii*) M.T. Type
(*b*) Mercury fluorescent lamp.
(J.N. University, Hyderabad, April 2003)

22. Explain with connection diagram the operation of the low pressure fluorescent lamp and state its advantage. ***(J.N. University, Hyderabad, April 2003)***

23. Explain clearly the following :
Illumination, Luminous efficiency, MSCP, MHCP and solid angle.
(J.N. University, Hyderabad, December 2002/January 2003)

24. A small light source with uniform intensity is mounted at a height of 10 meters above a horizontal surface. Two points A and B both lie on the surface with point A directly beneath the source. How far is B from A if the illumination at B is only 1/15th of that at A?
(J.N. University, Hyderabad, December 2002/January 2003)

25. Discuss the : (*i*) Specular reflection principle (*ii*) Diffusion principle of street lighting.
(J.N. University, Hyderabad, December 2002/January 2003)

26. Determine the height at which a light source having uniform spherical distribution should be placed over a floor in order that the intensity of horizontal illumination at a given distance from its vertical line may be greatest. ***(J.N. University, Hyderabad, December 2002/January 2003)***

27. What is a Glare? ***(J.N. University, Hyderabad, December 2002/January 2003)***

28. With the help of a neat diagram, explain the principle of operation of fluorescent lamp.
(J.N. University, Hyderabad, December 2002/January 2003)

29. A machine shop 30 m long and 15 m wide is to have a general illumination of 150 lux on the work plane provided by lamps mounted 5 m above it. Assuming a coefficient of utilization of 0.55, determine suitable number and position of light. Assume any data if required.
(J.N. University, Hyderabad, December 2002/January 2003)

30. Laws of illumination. ***(J.N. University, Hyderabad, December 2002/January 2003)***

31. Working principle of sodium vapour lamp.
(J.N. University, Hyderabad, December 2002/January 2003)

32. Explain the principle of operation of sodium vapour lamp and its advantages.
(J.N. University, Hyderabad, December 2002/January 2003)

33. A corridor is lighted by lamps spaced 9.15 m apart and suspended at a height of 4.75 m above the centre line of the floor. If each lamp gives 100 candle power in all directions, find the maximum and minimum illumination on the floor along the centre line. Assume and data if required.
(J.N. University, Hyderabad, December 2002/January 2003)

34. Discuss the laws of illumination and its limitations in practice.
(J.N. University, Hyderabad, December 2002/January 2003)

35. State the functions of starter and choke in a fluorescent lamp.
(J.N. University, Hyderabad, December 2002/January 2003)

36. Mercury Vapour Lamp. ***(J.N. University, Hyderabad, December 2002/January 2003)***

37. Describe briefly (*i*) conduit system (*ii*) C.T.S. system of writing.
(Bangalore University, January/February 2003)

38. With a neat circuit diagram, explain the two way control of a filament lamp.
(Belgaum Karnataka University, February 2002)

39. With a neat sketch explain the working of a sodium vapour lamp.
(Belgaum Karnataka University, February 2002)

40. Mention the different types of wiring. With relevant circuit diagrams and switching tables, explain two-way and three-way control of lamps.
(Belgaum Karnataka University, January/February 2003)

41. With a neat sketch explain the working of a fluorescent lamp.
(Belgaum Karnataka University, January/February 2003)

OBJECTIVE TESTS – 49

1. Candela is the unit of
(*a*) flux
(*b*) luminous intensity
(*c*) illumination
(*d*) luminance.

2. The unit of illuminance is
(*a*) lumen
(*b*) cd/m^2
(*c*) lux
(*d*) steradian.

3. The illumination at various points on a horizontal surface illuminated by the same source varies as
(*a*) $\cos^3\theta$
(*b*) $\cos\theta$
(*c*) $1/r^2$
(*d*) $\cos^2\theta$.

4. The M.S.C.P. of a lamp which gives out a total luminous flux of 400 π lumen is ..... candela.
(*a*) 200
(*b*) 100
(*c*) 50
(*d*) 400.

5. A perfect diffuser surface is one that
(*a*) diffuses all the incident light
(*b*) absorbs all the incident light
(*c*) transmits all the incident light
(*d*) scatters light uniformly in all directions.

6. The direct lighting scheme is most efficient but is liable to cause
(*a*) monotony
(*b*) glare
(*c*) hard shadows
(*d*) both (*b*) and (*c*).

7. Total flux required in any lighting scheme depends inversely on
(*a*) illumination

(*b*) surface area
(*c*) utilization factor
(*d*) space/height ratio.

8. Floodlighting is NOT used for ..... purposes.
(*a*) reading
(*b*) aesthetic
(*c*) advertising
(*d*) industrial.

9. Which of the following lamp has minimum initial cost of installation but maximum running cost ?
(*a*) incandescent
(*b*) fluorescent
(*c*) mercury vapour
(*d*) sodium vapour.

10. An incandescent lamp can be used
(*a*) in any position
(*b*) on both ac and dc supply
(*c*) for street lighting
(*d*) all of the above.

11. The average working life of a fluorescent lamp is about..... hours.
(*a*) 1000
(*b*) 4000
(*c*) 3000
(*d*) 5000.

12. The luminous efficiency of a sodium vapour lamp is about ..... lumen/watt.
(*a*) 10
(*b*) 30
(*c*) 50
(*d*) 70

13. Which of the following statements is correct?
(*a*) Light is a form of heat energy
(*b*) Light is a form of electrical energy
(*c*) Light consists of shooting particles
(*d*) Light consists of electromagnetic waves

14. Luminous efficiency of a fluorescent tube is
(*a*) 10 lumens/watt
(*b*) 20 lumens/watt
(*c*) 40 lumens/watt
(*c*) 60 lumens/watt

15. Candela is the unit of which of the following?
(*a*) Wavelength
(*b*) Luminous intensity
(*c*) Luminous flux
(*d*) Frequency

16. Colour of light depends upon
(*a*) frequency
(*b*) wave length
(*c*) both (a) and (b)
(*d*) speed of light

17. Illumination of one lumen per sq. metre is called .....
(*a*) lumen metre
(*b*) lux
(*c*) foot candle
(*d*) candela

18. A solid angle is expressed in terms of .....
(*a*) radians/metre
(*b*) radians
(*c*) steradians
(*d*) degrees

19. The unit of luminous flux is .....
(*a*) watt/m^2
(*b*) lumen
(*c*) lumen/m^2
(*d*) watt

20. Filament lamps operate normally at a power factor of
(*a*) 0.5 lagging
(*b*) 0.8 lagging
(*c*) unity
(*d*) 0.8 leading

21. The filament of a GLS lamp is made of
(*a*) tungsten
(*b*) copper
(*c*) carbon
(*d*) aluminium

22. Find diameter tungsten wires are made by
(*a*) turning
(*b*) swaging
(*c*) compressing
(*d*) wire drawing

23. What percentage of the input energy is radiated by filament lamps?
(*a*) 2 to 5 percent
(*b*) 10 to 15 percent
(*c*) 25 to 30 percent
(*d*) 40 to 50 percent

24. Which of the following lamps is the cheapest for the same wattage?
(*a*) Fluorescent tube
(*b*) Mercury vapour lamp
(*c*) GLS lamp
(*d*) Sodium vapour lamp

25. Which of the following is not the standard rating of GLS lamps?
(*a*) 100 W
(*b*) 75 W
(*c*) 40 W
(*d*) 15 W

26. In houses the illumination is in the range of
(*a*) 2–5 lumens/watt
(*b*) 10–20 lumens/watt

(*c*) 35–45 lumens/watt
(*d*) 60–65 lumens/watt

27. ''The illumination is directly proportional to the cosine of the angle made by the normal to the illuminated surface with the direction of the incident flux''. Above statement is associated with
(*a*) Lambert's cosine law
(*b*) Planck's law
(*c*) Bunsen's law of the illumination
(*d*) Macbeth's law of illumination

28. The colour of sodium vapour discharge lamp is
(*a*) red
(*b*) pink
(*c*) yellow
(*d*) bluish green

29. Carbon arc lamps are commonly used in
(*a*) photography
(*b*) cinema projectors
(*c*) domestic lighting
(*d*) street lighting

30. Desired illumination level on the working plane depends upon
(*a*) age group of observers
(*b*) whether the object is stationary or moving
(*c*) Size of the object to be seen and its distance from the observer
(*d*) whether the object is to be seen for longer duration or shorter duration of time
(*e*) all above factors

31. On which of the following factors dies the depreciation or maintenance factor depend?
(*a*) Lamp cleaning schedule
(*b*) Ageing of the lamp
(*c*) Type of work carried out at the premises
(*d*) All of the above factors

32. In lighting installating using filament lamps 1% voltage drop results into
(*a*) no loss of light
(*b*) 1.5 percent loss in the light output
(*c*) 3.5 percent loss in the light output
(*d*) 15 percent loss in the light output

33. For the same lumen output, the running cost of the fluorescent lamp is
(*a*) equal to that of filament lamp
(*b*) less than that of filament lamp
(*c*) more than that of filament lamp
(*d*) any of the above

34. For the same power output
(*a*) high voltage rated lamps will be more sturdy
(*b*) low voltage rated lamps will be more sturdy
(*c*) both low and high voltage rated lamps will be equally sturdy

35. The cost of a fluorescent lamp is more than that of incandescent lamp because of which of the following factors?
(*a*) More labour is required in its manufacturing
(*b*) Number of components used is more
(*c*) Quantity of glass used is more
(*d*) All of the above factors

36. Filament lamp at starting will take current
(*a*) less than its full running current
(*b*) equal to its full running current
(*c*) more than its full running current

37. A reflector is provided to
(*a*) protect the lamp
(*b*) provide better illumination
(*c*) avoid glare
(*d*) do all of the above

38. The purpose of coating the fluorescent tube from inside with white power is
(*a*) to improve its life
(*b*) to improve the appearance
(*c*) to change the colour of light emitted to white
(*d*) to increase the light radiations due to secondary emissions

39. ..... will need lowest level of illumination.
(*a*) Audiotoriums
(*b*) Railway platform
(*c*) Displays
(*d*) Fine engravings

40. Due to moonlight, illumination is nearly
(*a*) 3000 lumens/m^2
(*b*) 300 lumens/m^2
(*c*) 30 lumens/m^2
(*d*) 0.3 lumen/m^2

41. Which of the following instruments is used for the comparison of candle powers of different sources?
(*a*) Radiometer
(*b*) Bunsen meter
(*c*) Photometer
(*d*) Candle meter

42. ..... photometer is used for comparing the lights of different colours?
(*a*) Grease spot
(*b*) Bunsen
(*c*) Lummer brodhum
(*d*) Guilds flicker

43. In the fluorescent tube circuit the function of choke is primarily to
(*a*) reduce the flicker

(*b*) minimise the starting surge
(*c*) initiate the arc and stabilize it
(*d*) reduce the starting current

44. ..... cannot sustain much voltage fluctuations.
(*a*) Sodium vapour lamp
(*b*) Mercury vapour lamp
(*c*) Incandescent lamp
(*d*) Fluorescent lamp

45. The function of capacitor across the supply to the fluorescent tube is primarily to
(*a*) stabilize the arc
(*b*) reduce the starting current
(*c*) improve the supply power factor
(*d*) reduce the noise

46. ..... does not have separate choke
(*a*) Sodium vapour lamp
(*b*) Fluorescent lamp
(*c*) Mercury vapour lamp
(*d*) All of the above

47. In sodium vapour lamp the function of the leak transformer is
(*a*) to stabilize the arc
(*b*) to reduce the supply voltage
(*c*) both (*a*) and (*b*)
(*d*) none of the above

48. Most affected parameter of a filament lamp due to voltage change is
(*a*) wattage
(*b*) life
(*c*) luminous efficiency
(*d*) light output

49. In electric discharge lamps for stabilizing the arc
(*a*) a reactive choke is connected in series with the supply
(*b*) a condenser is connected in series to the supply
(*c*) a condenser is connected in parallel to the supply
(*d*) a variable resistor is connected in the circuit

50. For precision work the illumination level required is of the order of
(*a*) 500-1000 lumens/m^2
(*b*) 200-2000 lumens/m^2
(*c*) 50-100 lumens/m^2
(*d*) 10-25 lumens/m^2

51. ..... is a cold cathode lamp.
(*a*) Fluorescent lamp
(*b*) Neon lamp
(*c*) Mercury vapour lamp
(*d*) Sodium vapour lamp

52. In case of ..... least illumination level is required.
(*a*) skilled bench work
(*b*) drawing offices
(*c*) hospital wards
(*d*) find machine work

53. For normal reading the illumination level required is around
(*a*) 20-40 lumens/m^2
(*b*) 60-100 lumens/m^2
(*c*) 200-300 lumens/m^2
(*d*) 400-500 lumens/m^2

54. In electric discharge lamps light is produced by
(*a*) cathode ray emission
(*b*) ionisation in a gas or vapour
(*c*) heating effect of current
(*d*) magnetic effect of current

55. A substance which change its electrical resistance when illuminated by light is called .....
(*a*) photoelectric
(*b*) photovoltaic
(*c*) phatoconductive
(*d*) none of the above

56. In case of ..... power factor is the highest.
(*a*) GLS lamps
(*b*) mercury arc lamps
(*c*) tube lights
(*d*) sodium vapour lamps

57. A mercury vapour lamp gives ..... light.
(*a*) white
(*b*) pink
(*c*) yellow
(*d*) greenish blue

58. Sometimes the wheels of rotating machinery, under the influence of fluorescent lamps appear to be stationary. This is due to the
(*a*) low power factor
(*b*) stroboscopic effect
(*c*) fluctuations
(*d*) luminescence effect

59. Which of the following bulbs operates on least power?
(*a*) GLS bulb
(*b*) Torch bulb
(*c*) Neon bulb
(*d*) Night bulb

60. The flicker effect of fluorescent lamps is more pronounced at
(*a*) lower frequencies
(*b*) higher frequencies
(*c*) lower voltages
(*d*) higher voltages

61. Which of the following application does not need ultraviolet lamps?
(*a*) Car lighting
(*b*) Medical purposes
(*c*) Blue print machine
(*d*) Aircraft cockpit dashboard lighting

62. Which gas can be filled in GLS lamps?
(*a*) Oxygen
(*b*) Carbon dioxide
(*c*) Xenon
(*d*) Any inert gas

ANSWERS

1.	(*b*)	**2.**	(*c*)	**3.**	(*a*)	**4.**	(*b*)	**5.**	(*d*)	**6.**	(*d*)	**7.**	(*c*)	**8.**	(*a*)	**9.**	(*a*)	**10.**	(*d*)
11.	(*b*)	**12.**	(*d*)	**13.**	(*d*)	**14.**	(*d*)	**15.**	(*b*)	**16.**	(*c*)	**17.**	(*b*)	**18.**	(*c*)	**19.**	(*b*)	**20.**	(*c*)
21.	(*a*)	**22.**	(*d*)	**23.**	(*b*)	**24.**	(*c*)	**25.**	(*b*)	**26.**	(*d*)	**27.**	(*a*)	**28.**	(*c*)	**29.**	(*b*)	**30.**	(*e*)
31.	(*d*)	**32.**	(*c*)	**33.**	(*b*)	**34.**	(*b*)	**35.**	(*d*)	**36.**	(*c*)	**37.**	(*d*)	**38.**	(*d*)	**39.**	(*b*)	**40.**	(*d*)
41.	(*c*)	**42.**	(*d*)	**43.**	(*c*)	**44.**	(*c*)	**45.**	(*c*)	**46.**	(*a*)	**47.**	(*c*)	**48.**	(*b*)	**49.**	(*a*)	**50.**	(*a*)
51.	(*b*)	**52.**	(*c*)	**53.**	(*b*)	**54.**	(*b*)	**55.**	(*c*)	**56.**	(*a*)	**57.**	(*d*)	**58.**	(*b*)	**59.**	(*b*)	**60.**	(*a*)
61.	(*a*)	**62.**	(*d*)																

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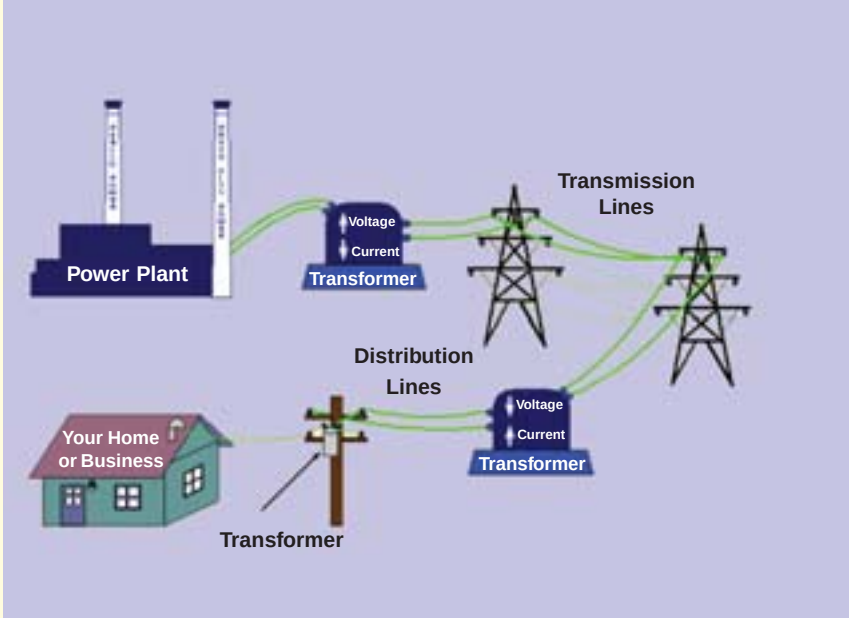
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CHAPTER 50

TARIFFS AND ECONOMIC CONSIDERATIONS

Learning Objectives

- Economic Motive
- Depreciation
- Indian Currency
- Factors Influencing Cost and Tariffs of Electric Supply
- Demand
- Average Demand
- Maximum Demand
- Demand Factor
- Diversity of Demand
- Diversity Factor
- Load Factor
- Plant Factor or Capacity Factor
- Utilization Factor (or Plant use Factor)
- Connected Load Factor
- Tariffs
- Flat Rate
- Sliding Scale
- Two-part Tariff
- Kelvin's Law
- Effect of Cable Insulation
- Note on Power Factor
- Disadvantages of Low Power Factor
- Economics of Power Factor
- Economical Limit of Power Factor Correction



For the successful running of an electricity production, transmission and distribution system, it is necessary to properly account for the various direct and indirect costs involved, before fixing the final kWh charges for the consumers

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50.1. Economic Motive

In all engineering projects with the exception of the construction of works of art or memorial buildings, the question of cost is of first importance. In fact, in most cases the cost decides whether a project will be undertaken or not although political and other considerations may intervene sometimes. However, the design and construction of an electric power system is undertaken for the purpose of producing electric power to be sold at a profit. Hence, every effort is made to produce the power as cheaply as possible. The problem of calculating the cost of any scheme is often difficult because the cost varies considerably with time, tariffs and even with convention. In general, the cost of producing electric power can be roughly devided into the following two portions :

(*a*) Fixed Cost. These do not vary with the operation of the plant *i.e.* these are independent of the number of units of electric energy produced and mainly consist of :

1. Interest on capital investment,
2. Allowance for depreciation (*i.e.* wearing out of the depreciable parts of the plant augmented by obsolescence, buildings, the transmission and distribution system etc.),
3. Taxes and insurance, 4. most of the salaries and wages, 5. small portion of the fuel cost.

(*b*) Running or Operating Costs. These vary with the operation of the plant *i.e.* these are proportional to the number of units of electric energy generated and are mostly made up of :

1. most of the fuel cost, 2. small portion of salaries and wages, 3. repair and maintenance.

50.2. Depreciation

It is obvious that from the very day the construction of a generating plant is completed, deterioration starts and due to wear and tear from use and the age and physical decay from lapse of time, there results a reduction in the value of the plant — a loss of some part of the capital investment in the perishable property. The rate of wear and disintegration is dependent upon **(*i*)** conditions under which the plant or apparatus is working, **(*ii*)** how it is protected from elements and **(*iii*)** how promptly the required repairs are carried out.

Hence, as the property decreases from its original cost when installed, to its final scrap or salvage value at the end of its useful life, it is essential that the owner will have in hand at any given time as much money as represents the shrinkage in value and at the time of actual retirement of the plant, he must certainly have in hand the full sum of the depreciable part of the property. By adding this amount to the net salvage value of the plant, the owner can rebuild the same type of property as he did in the first instance or he can build some other property of an equivalent earning capacity.

The useful life of the apparatus ends when its repair becomes so frequent and expensive that it is found cheaper to retire the equipment and replace it by a new one.

It may be pointed out here that in addition to depreciation from wear and tear mentioned above, there can also be depreciation of the apparatus due to the inadequacy from obsolescence, both sentimental and economic, from the requirements of the regulating authorities and from accidental damages and if any of these factors become operative, they may force the actual retirement of the apparatus much before the end of its normal useful life and so shorten the period during which its depreciation expenses can be collected. These factors will necessitate increased depreciation rate and the consequent build up of the depreciation reserve as to be adequate for the actual retirement.

Some of the important methods of providing for depreciation are :

1. Straight-line method,
2. Diminishing-value method,
3. Retirement-expense method,
4. Sinking-fund method.

In the straight-line method, provision is made for setting aside each year an equal proportional part of the depreciable cost based on the useful life of the property. Suppose a machine costs

Rs. 45,000 and its useful life is estimated as ten years with a scrap value of Rs. 5,000, then the annual depreciation value will be 1/10 of Rs. 40,000 *i.e.* Rs. 4000. This method is extremely simple and easy to apply when the only causes for retirement of the machine are the wear and tear or the slow action of elements. But it is extremely difficult to estimate when obsolescence or accidental damage may occur to the machine. This method ignores the amount of interest earned on the amount set aside yearly.

In the diminishing value method, provision is made for setting aside each year a fixed rate, first applied to the original cost and then to the diminishing value; such rate being based upon the estimated useful life of the apparatus. This method leads to heaviest charges for depreciation in early years when maintenance charges are lowest and so evens out the total expense on the apparatus for depreciation plus the maintenance over its total useful life. This method has the serious disadvantage of imposing an extremely heavy burden on the early years of a new plant which has as yet to develop its load and build up its income as it goes along.

The retirement expense method which is not based on the estimated life of the property, aims at creating an adequate reserve to take care of retirement before such retirements actually occurs. Because of many objections raised against this method, it is no longer used now.

In the sinking-fund method, provision is made for setting aside each year such a sum as, invested at certain interest rate compounded annually, will equal the amount of depreciable property at the end of its useful life. As compared to straight-line method, it requires smaller annual amounts and also the amounts for annuity are uniform. This method would be discussed in detail in this chapter.

Suppose P is the capital outlay required for an installation and r p.a. is the interest per unit (6% is equivalent to $r = 0.06$). The installation should obviously provide rP as annual interest which is added to its annual running cost. Were the installation to last forever, then this would have been the only charge to be made. But as the useful life of the installation has a definite value, it is necessary to provide a sinking fund to produce sufficient amount at the end of the estimated useful life to replace the installation by a new one. Let the cost of replacement be denoted by Q. This Q will be equal to P if the used installation has zero scrap value, less than P if it has positive scrap value and greater than P if it has a negative scrap value. If the useful life is n years, then the problem is to find the annual charge q to provide a sinking fund which will make available an amount Q at the end of n years. Since amount q will earn an annual interest rq, hence its value after one year becomes $q + rq = q\,(1 + r)$. This sum will earn an interest of $r \times q\,(1 + r)$ and hence its value at the end of two years will become $q\,(1 + r) + qr\,(1 + r)$ or $q(1 + r)^2$. Similarly, its value at the end of three years is $q(1 + r)^3$. *i.e.* its value is multiplied by $(1 + r)$ every year so that the first payment becomes worth $q(1 + r)^n$ at the end of n years. The second payment to the sinking-fund is made at the beginning of the second year, hence its value at the end of the useful life of the installation becomes $q(1 + r)^{n-1}$ because this amount earns interest only for $(n - 1)$ years. The total sum available at the end of n years is therefore

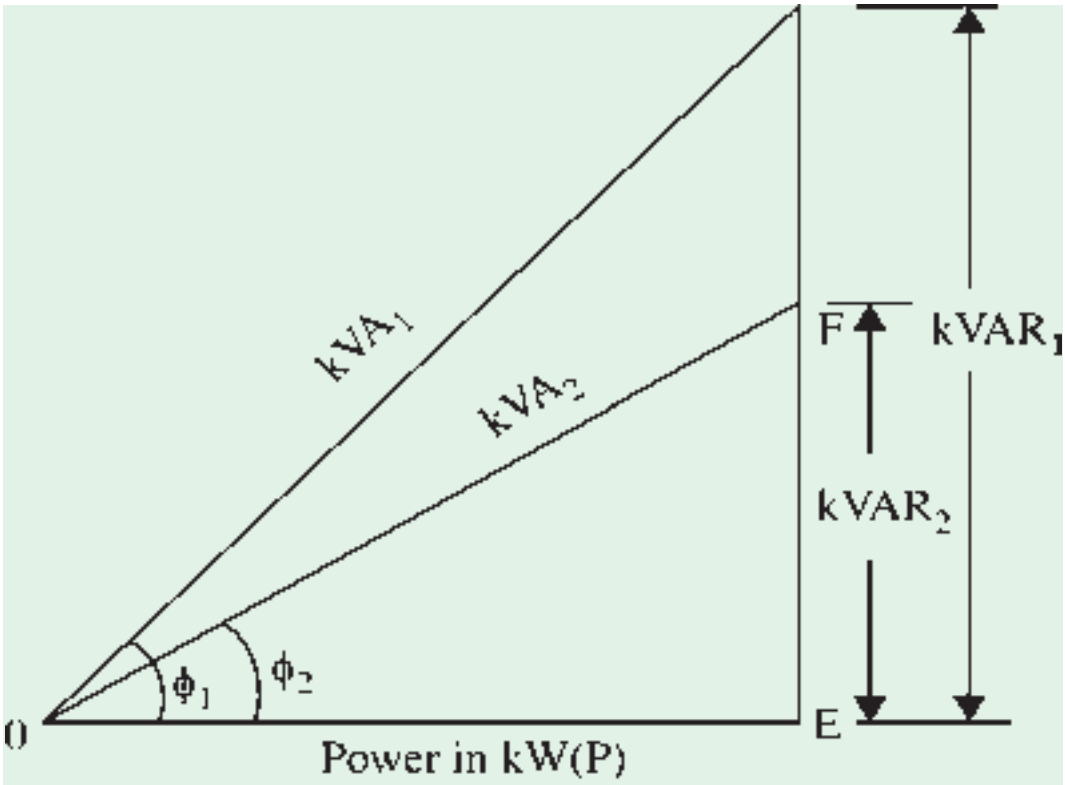
$$= q\,(1 + r)^n + q\,(1 + r)^{n-1} + \ldots\ldots\ldots q\,(1 + r)^2 + q\,(1 + r)$$

$$= q\,\frac{(1 + r)^{n+1} - (1 + r)}{(1 + r) - 1} = q\,\frac{1 + r}{r}[(1 + r)^n - 1]$$

This sum must, obviously, be equal to the cost of renewal Q.

$$\therefore \qquad Q = q\,[(1 + r)^n - 1] \text{ or } q = Q\,\frac{r}{1 + r} \div [(1 + r)^n - 1]$$

Hence, the total annual charge on the installation is $(rP + q)$ *i.e.* the plant should bring in so much money every year.



An overview of a combined cycle power plant

50.3. Indian Currency

The basic unit of Indian currency is rupee (Re). Its plural form is rupees (Rs.) One rupee contain 100 paisa. Higher multiples of rupees in common use are :

1 lakh (or lac) = Rs. 100,000 = Rs. 10^5 = Rs. 0.1 million
1 crore = 100 lakh = Rs. 10^7 = Rs. 10 million

Example 50.1. *Find the total annual charge on an installation costing Rs. 500,000 to buy and install, the estimated life being 30 years and negligible scrap value. Interest is 4% compounded annually.*

Solution. Since scrap value is negligible, $Q = P$. Now Q = Rs. 500,000; $r = 0.04$, $n = 30$ years.

$$\therefore \quad q = 500,000 \times \frac{0.04}{1.04} \div [1.04^{30} - 1] = \frac{500,000 \times 0.04}{1.04 \times 2.236} = 8,600$$

Hence, the total annual charge on the installation is

$$= rP + q = (0.04 \times 500,000) + 8,600 = \textbf{Rs. 28,600}$$

Example 50.2. *A power plant having initial cost of Rs. 2.5 lakhs has an estimated salvage value of Rs. 30,000 at the end of its useful life of 20 years. What will be the annual deposit necessary if it is calculated by :*

***(i)** straight-line depreciation method. **(ii)** sinking-fund method with compound interest at 7%.*

(Electrical Engineering-III, Poona Univ.)

Solution. Here, $Q = P$ – scrap value = Rs. 250,000 – Rs. 30,000 = Rs. 220,000

$r = 0.07, n = 20$

(*i*) Total depreciation of 20 years = Rs. 220,000

$\therefore$ annual depreciation = Rs. 220,000/20 = Rs. 11,300.

$\therefore$ annual deposit = $rP + q = 0.07 \times 250{,}000 + 11{,}000$ = **Rs. 28,500**

(*ii*) $q = Q\dfrac{r}{r+1} \div [(1+r)^n - 1]$

$= \text{Rs. } 220{,}000\,[1.0720 - 1] = \text{Rs. } 5015.$

Annual deposit = $0.07 \times 250{,}000 + 5015$ = **Rs. 22,515**

Example 50.3. *A plant initially costing Rs. 5 lakhs has an estimated salvage value of Rs. 1 lakh at the end of its useful life of 20 years. What will be its valuation half-way through its life* ***(a)*** *on the basis of straight-line depreciation and* ***(b)*** *on the sinking-fund basis at 8% compounded annually?*

Solution. ***(a)*** In this method, depreciation is directly proportional to time.

Total depreciation in 20 years = Rs. (5 – 1) = Rs. 4 lakhs

$\therefore$ depreciation in 10 years = Rs. 4/2 = Rs. 2 lakhs

$\therefore$ its value after 10 years = (5 – 2) = **Rs. 3 lakhs.**

(*b*) Now, Q = 5 – 1 = Rs. 4 lakhs ; $r = 0.08$, $n = 20$

The annual charge is $q = Q\dfrac{r}{r+1} \div [(1+r)^n - 1]$

$\therefore$ $q = 4\times10^5 \times \dfrac{0.08}{1.08}[1.08^{20} - 1] = \text{Rs. } 8095$

At the end of 10 years, the amount deposited in the sinking fund would become

$$= q\frac{1+r}{r}[(1+r)^n - 1] = 8095 \times \frac{1.08}{0.08} \times (1.08^{10} - 1) = \text{Rs. } 126{,}647$$

$\therefore$ value at the end of 10 years = Rs. 500,000 – Rs. 126,647

= Rs. 373,353 = **Rs. 3.73353 lakhs.**

50.4. Factors Influencing Costs and Tariffs of Electric Supply

In the succeeding paragraphs we will discuss some of the factors which determine the cost of



Automation of electricity production, transmission and distribution helps in the effective cost management

generating electric energy and hence the rates or tariffs of charging for this energy. The cost is composed of ***(i) standing charges which are independent of the output*** **and** ***(ii) running or operating charges which are proportional to the output.*** The size or capacity of the generating plant and hence the necessary capital investment is determined by the maximum demand imposed on the generating plant.

50.5. Demand

By '***demand***' of a system is meant its load requirement (usually in kW or kVA) averaged over a suitable and specified *interval* of time of short duration.

It should be noted that since '***demand***' means the load averaged over an ***interval*** of time, there is no such thing as ***instantaneous*** demand.

50.6 Average Demand

By ***average*** demand of an installation is meant its average power requirement during some specified period of time of considerable duration such as a day or month or year giving us daily or monthly or yearly average power respectively.

Obviously, the average power demand of an installation during a specific period can be obtained by dividing the energy consumption of the installation in kWh by the number of hours in the period.

In this way, we get the arithmetical average.

$$\text{Average power} = \frac{\text{kWh consumed in the period}}{\text{hours in the period}}$$

50.7. Maximum Demand

The maximum demand of an installation is defined as the greatest of all the demands which have occurred during a given period.

It is measured, according to specifications, over a prescribed time interval during a certain period such as a day, a month or a year.

It should be clearly understood that it is not the greatest instantaneous demand but the greatest average power demand occuring during any of the relatively short intervals of 1-minute, 15-minute or 30 minute duration within that period.

In Fig. 50.1 is shown the graph of an imaginary load extending over a period of 5 hours. The maximum demand on 30 min. interval basis occurs during the interval *AB i.e.* from 8-30 p.m to 9-00 p.m. Its value as calculated in Fig. 50.1 is 288 kW. A close inspection of the figure shows that average load is greater during the 30 min. interval *AB* than it is during any other 30-min interval during this period of 5 hours. The average load over 30-min. interval *AB* is obtained first by scaling kW instantaneous demands at five equidistant points between ordinates *AC* and *BD* and then by taking arithmetic average of these values as shown. Hence, 30 min. maximum demand from the above load graph is 288 kW.

It may be noted that the above method of averaging can be made to yield more accurate results by ***(i)*** considering a large number of ordinates and ***(ii)*** by scaling the ordinates more precisely.

It may also be noted that if the maximum demand were to be based on a 15 min. interval, then it will occur during the 15-min. interval *MN* and its value will be 342 kW as shown in Fig. 50.1. It is seen that not only has the position of maximum demand changed but its value has also changed. The 30-min. maximum demand has lesser value than 15-min. max. demand. In the present case, 1-min. max. demand will have still greater value and will occur somewhere near point *M*.

From the above discussion, it should be clear that the unqualified term "maximum demand" is indefinite and has no specific meaning. For example, a statement that "maximum demand is 150 kW" carries no specific meaning. To render any statement of maximum demand meaningful, it is necessary

(*i*) to indicate the *period* of load duration under consideration and **(*ii*)** to specify the ***time interval*** used *i.e.* 15-min. or 30-min. etc. and also **(*iii*)** the method used for averaging the demand during that interval.

Now, let us see why it is the *average maximum demand* over a definite *interval* of time that is of interest rather than the *instantaneous maximum demand*.

Maximum demand determinations are mostly used for estimating the capacity (and hence cost) of the generator and other electrical apparatus required for serving a certain specific load.

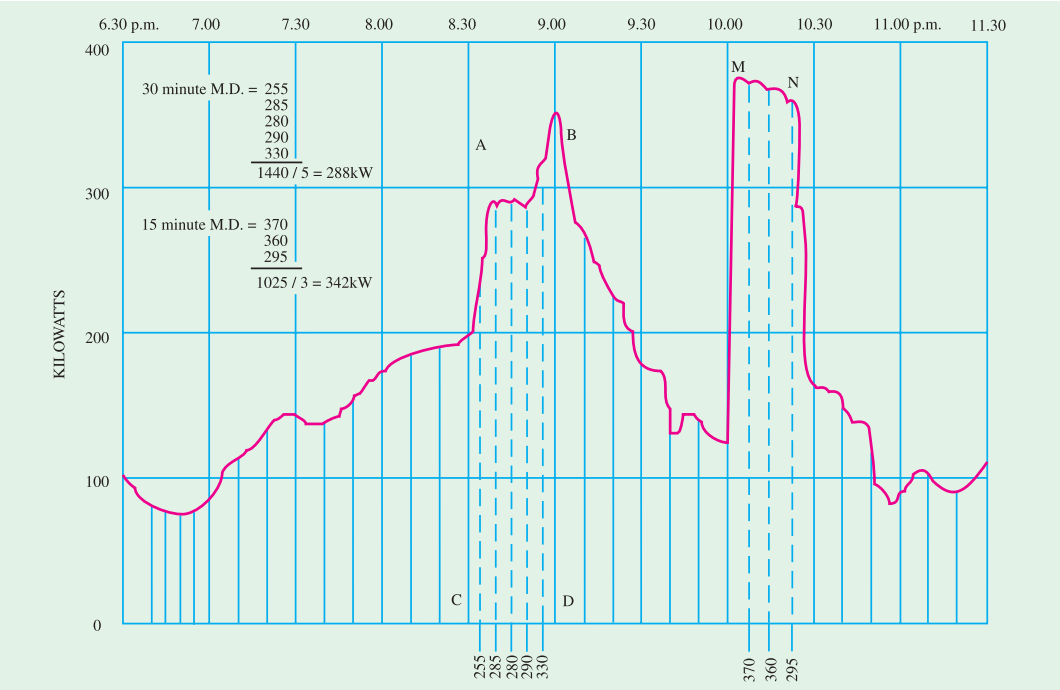


Fig. 50.1

The one main reason why maximam demand values are important is because of the direct bearing they have in establishing the capacity of the generating equipment or indirectly, the initial investment required for serving the consumers. The amount of this investment will have a further effect on fixing of rates for electric service. Since all electric machines have ample overload capacity *i.e.* they are capable of taking 100% or more overloads for short periods without any permanent adverse effects, it is not logical or economically desirable to base the continuous capacity requirements of generators on instantaneous maximum loads which will be imposed on them only momentarily or for very short periods.

Consider the graph of the power load (Fig. 50.2) to be impressed on a certain generator. Let it be required to find the rating of a generator capable of supplying this load. It is seen that there are peak loads of short durations at point *A*, *B*, *C* and *D* of values 250, 330, 230 and 260 kW. However, during the interval *EF* a demand of 210 kW persists for more than half an hour. Hence, in this particular case, the capacity of the generator required, as based on 30-min. maximum demand, should be 210 kW, it being of course, assumed that 4-hour load conditions graphed in Fig. 50.2 are typical of the conditions which exist during any similar period of generator's operation.

In the end, it may be remarked that the exact time interval for maximum demand determinations, over which the greatest demand is averaged varies not only with the characteristics of the load but with the policy of the firm measuring the load. However, 15-min. interval is now most generally

used, peak load of shorter durations being considered as temporary overloads.

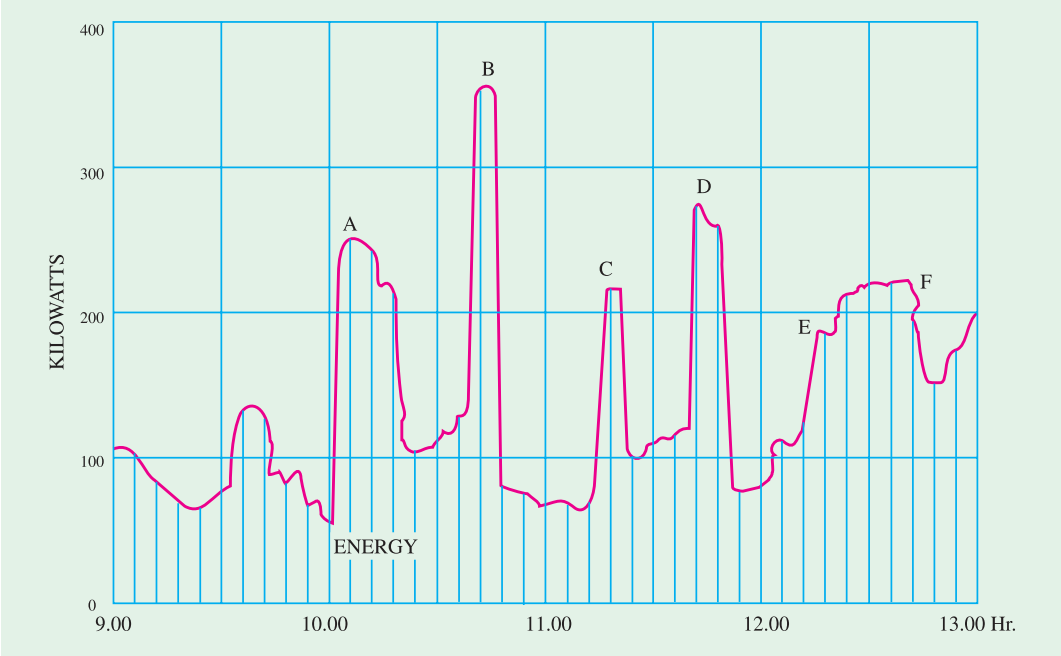


Fig. 50.2

50.8. Demand Factor

Demand factors are used for estimating the proportion of the total connected load which will come on the power plant at one time. It is defined as the ***ratio of actual maximum demand made by the load to the rating of the connected load.***

$$\text{Demand factor} = \frac{\text{maximum demand}}{\text{connected load}}$$

The idea of a demand factor was introduced because of the fact that normally the kW or kVA maximum demand of a group of electrical devices or 'receivers' is always less than the sum of the kW or kVa ratings or capacities of these receivers. There are two reasons for the existence of this condition **(*i*)** the electrical apparatus is usually selected of capacity somewhat greater than that actually required in order to provide some reserve or overload capacity and **(*ii*)** in a group of electrical devices it very rarely happens that all devices will, at the same time, impose their maximum demands which each can impose *i.e.* rarely will all 'receivers' be running full-load simultaneously.

The demand factor of an installation can be determined if **(*i*)** maximum demand and **(*ii*)** connected load are known.

Maximum demand can be determined as discussed in Art. 50.7 whereas connected load can be calculated by adding together the name-plate ratings of all the electrical devices in the installation. The value of demand factor is always less than unity.

Demand factors are generally used for determining the capacity and hence cost of the power equipment required to serve a given load. And because of their influence on the required investment, they become important factors in computing rate schedules.

As an example, suppose a residence has the following connected load : three 60-W lamps; ten 40-W lamps; four 100-W lamps and five 10-W lamps. Let us assume that the demand meter indicates a 30-min. maximum demand of 650 W. The demand factor can be found as follows :

Connected load = $(3 \times 60) + (10 \times 40) + (4 \times 100) + (5 \times 10) = 1{,}030$ W

30-min. max.demand = 650 W

Hence, the demand factor of this lighting installation is given as

$$= \frac{\text{max. demand}}{\text{connected load}} = \frac{650}{1.030} = \mathbf{0.631 \text{ or } 63.1\%}$$

Demand factors of lighting installations are usually fairly constant because lighting loads are not subject to such sudden and pronounced variations as like power loads.

50.9. Diversity of Demand

In central-station parlance, diversity of demand implies that maximum demands of various consumers belonging to different classes and the various circuit elements in a distribution system are not coincident. In other words, the maximum demands of various consumers occur at different times during the day and not simultaneously. It will be shown later that from the economic angle, it is extremely fortunate that there exists a diversity or non-simultaneity of maximum demand of various consumers which results in lower costs of electric energy.

For example, residence lighting load is maximum in the evening whereas manufacturing establishments require their maximum power during daytime hours. Similarly, certain commercial establishments like department stores usually use more power in day-time than in the evening whereas some other stores like drug stores etc. use more power in the evening.

The economic significance of the concept of diversity of demand can only be appreciated if one considers the increase in the capacity of the generating and distributing plant (and hence the corresponding increase in investment) that would be necessary, if the maximum demands of all the consumers occurred simultaneously. It is of great concern to the engineer because he has to take it into consideration while planning his generating and distributing plant. Also diversity is an important element in fixing the rates of electric service. If it were not for the fact that the coincident maximum demand imposed on a certain station is much less than the sum of maximum demands of all the consumers fed by that station, the investment required for providing the electric service would have been far in excess of that required at present. Because of the necessity of increase in investment, that cost of electric supply would also have been increased accordingly.

50.10. Diversity Factor

The non-coincidence of the maximum demands of various consumers is taken into consideration in the so-called diversity factor which is defined as the ratio of the sum of the individual maximum demands of the different elements of a load during a specified period to the simultaneous (or coincident) maximum demand of all these elements of load during the same peirod.

$$\text{Diversity factor}^* = \frac{\text{maximum demand}}{\text{connected load}}$$

Its value is usually much greater than unity. It is clear that if all the loads in a group impose their maximum demands simultaneously, then diversity factor is equal to unity. High value of diversity factor means that more consumers can be supplied for a given station maximum demand and so lower prices can be offered to consumers. Usually domestic load gives higher value of diversity factor than industrial load. As shown in Fig. 50.3, suppose that the maximum demands of six elements of a load as observed from their maximum demand meters M_1 and M_2 etc. are 620 W, 504 W, 435 W, 380 W, 160 W and 595 W respectively.

* Sometimes, the diversity factor is given by certain authors as the reciprocal of the value so obtained.

Also, suppose that the (coincident) maximum demand of the whole group as observed by the maximum demand meter *MT* is only 900 W. It is so because the maximum demands of all load elements did not occur simultaneously.

Sum of individual maximum demands

$= 620 + 504 + 435 + 380 + 160 + 595 = 2694 \text{ W} = 2.694 \text{ kW}$

$\therefore$ diversity factor $= 2.694/0.9 = \mathbf{2.99}$

It may be noted here that because of the diversity of demand, the maximum demand on a transformer is less than the sum of the maximum demands of the consumers supplied by that transformer. Further, the maximum demand imposed on a feeder is less than the sum of the maximum demands of transformers connected to that feeder. Similarly, the maximum demand imposed on the generating station is less than the sum of the maximum demands of all the feeders suplied station is less than the sum of the maximum demands of all the feeders supplied from the station. The effective demand of a consumer on a generator is given as follows :

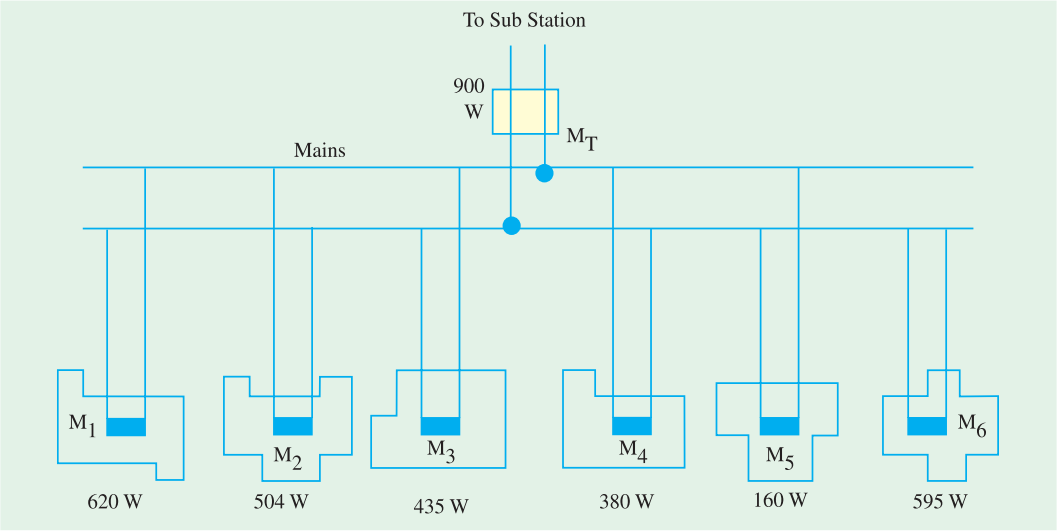


Fig. 50.3

Multiply this connected load by demand factor and then divide the product by diversity factor for consumer to generator.

As an example, let us find the total diversity factor for a residence lighting system whose component diversity factors are : between consumers 2.6 ; between transformers 1.32 ; between feeders 1.13 and between sub-stations 1.1. The total diversity factor between the consumers and the generating equipment would be the product of these component factors *i.e.*

$= 2.6 \times 1.32 \times 1.13 \times 1.1 = 4.266$

The factor may now be used in determining the effective demand of consumers on the generator.

It may be proved that generating equipment can be economized by grouping on one supply source different elements of load having high diversity factor. In fact, the percentage of the generating equipment which can be eliminated is equal to 100 percent minus the reciprocal of diversity factor expressed as a percentage. Suppose four loads of maximum demand 120, 360, 200 and 520 kVA respectively are to be supplied. If each of these loads were supplied by a separate transformer, then aggregate transformer capacity required would be = 120 + 360 + 200 + 520 = 1200 kVA. Suppose these loads had a diversity factor of 2.5 among themselves, then (coincident) maximum demand of the whole group would be 1200/2.5 = 480 kVA.

In other words, a single 480 kVA transformer can serve the combined load. The saving = 1200 – 480 = 720 kVA which expressed as a percentage is 720 × 100/1200 = 60%. Now, reciprocal of diversity factor = 1/2.5 = 0.4 or 40 %. The percentage saving in the required apparatus is also = 100 – 40 = 60% which proves the statement made above.

50.11. Load Factor

It is defined ***as the ratio of the average power to the maximum demand***.

It is necessary that in each case the time interval over which the maximum demand is based and the ***period**** over which the power is averaged must be definitely specified.

If, for example, the maximum demand is based on a 30-min. interval and the power is averged over a month, then it is known as 'half-hour monthly' load factor.

Load factors are usually expressed as percentages. The average power may be either generated or consumed depending on whether the load factor is required for generating equipment or receiving equipment.

When applied to a generating station, annual load factor is

$$= \frac{\text{No. of units actually supplied/year}}{\text{Max. possible No. of units that can be supplied}}$$

It may be noted that *maximum* in this definition means the value of the maximum peak load and *not the maximum* kW *installed capacity of the plant equipment of the station.*

$$\therefore \quad \text{annual load factor} = \frac{\text{No. of units actually supplied/year}}{\text{Max. possible demand} \times 8760}$$

$$\text{Monthly load factor} = \frac{\text{No. of units actually supplied/month}}{\text{Max. possible demand} \times 24 \times 30}$$

When applied to a consuming equipment

$$\text{annual load factor} = \frac{\text{No. of units consumed/year}}{\text{Max. demand} \times 8760}$$

$$\text{monthly load factor} = \frac{\text{No. of units consumed/month}}{\text{Max. demand} \times 24 \times 30}$$

$$\text{Daily load factor} = \frac{\text{No. of units consumed/day}}{\text{Max. demand} \times 24}$$

$$\text{In general, load factor} = \frac{\text{Average power}}{\text{Max. demand}} \text{ per year or per month or per day}$$

The value of maximum demand can be found by using a maximum demand meter set for 30-min. or 15-min. interval as already explained in Art. 50.7. The average power can also be found either by graphic method explained below or by using a planimeter.

In the graphic method, momentary powers are scaled or read from the load-graph at the end of a number of suitable and equal time intervals over the entire time comprehended by the graph. Then these are added up. Average power is obtained by dividing this sum by the number of periods into which the total time was apportioned.

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* If not specified, it is assumed to be one year of 24 × 365 = 8,760 hours.

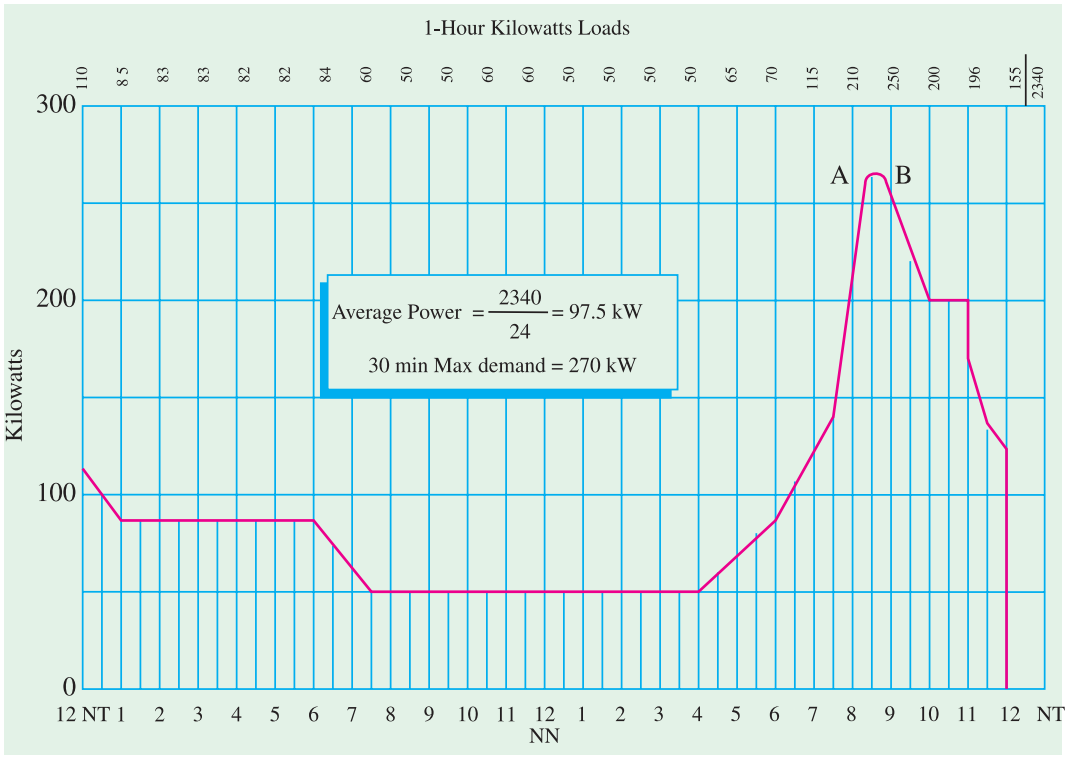


Fig. 50.4

The number of time intervals into which the entire time is apportioned is determined by the contour of the graph and the degree of accuracy required.

The general, the accuracy will increase with the increase in the number of the intervals. For a graph having a smooth contour and comprehending 24 hours, sufficient accuracy can be obtained by taking 1 hour intervals. However, in the case of a graph which has extremely irregular contours and comprehends short time intervals, reasonable accuracy can only be obtained if 15-min. or even 1 min. intervals are used.

As an example, let us find the load factor of a generating equipment whose load graph (imaginary one) is shown in Fig. 50.4. For calculating average power over a period of 24 hours, let us take in view of the regularity of the curve, a time interval of 1 hour as shown. The average power is 97.5 kW. Now, the 30-min. maximum demand is 270 kW and occurs during 30-min. interval of *A B*. Obviously load factor = $97.5 \times 100/270 = 36.1\%$.

50.12. Significance of Load Factor

Load factor is, in fact, an index to the proportion of the whole time a generator plant or system is being worked to its full capacity. The generating equipment has to be selected on the basis of maximum power demand that is likely to be imposed on it. However, because of general nature of things, it seldom happens that a generating equipment has imposed on it during all the 8,760 hrs of a year the maximum load which it can handle. But whether the equipment is being worked to its full capacity or not, there are certain fixed charges (like interest, depreciation, taxes, insurance, part of staff salaries etc.) which are adding up continuously. In other words, the equipment is costing money to its owner whether working or idle. The equipment earns a net profit only during those hours when it is fully loaded and the more it is fully loaded, the more is the profit to the owner. Hence, from the standpoint

of economics, it is desirable to keep the equipment loaded for as much time as possible i.e. it is economical to obtain high load factors.

If the load factor is poor *i.e.* kWh of electric energy produced is small, then charge per kWh would obviously be high. But if load factor is high *i.e.* the number of kWh generated is large, then cost of production and hence charge per kWh are reduced because now the standing charges are distributed over a larger number of units of energy.

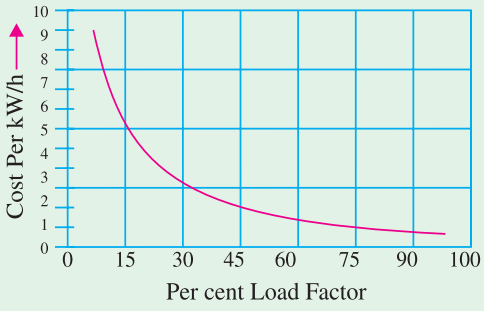


Fig. 50.5

The fact that fixed charges per kWh increase with decreasing load factor and vice versa is brought out in Ex. 50.15 and is graphically shown in Fig. 50.5.

It may be remarked here that increase of diversity in demand increases the load factor almost in direct proportion.

Load factor of a generating plant may be improved by seeking and accepting off-peak loads at reduced rates and by combining lighting, industrial and inter-urban railway loads.

Example 50.4. *A consumer has the following connected load : 10 lamps of 60 W each and two heaters of 1000 W each. His maximum demand is 1500 W. On the average, he uses 8 lamps for 5 hours a day and each heater for 3 hours a day. Find his total load, montly energy consumption and load factor.* **(Power Systems-I, AMIE, Sec. B, 1993)**

Solution. Total connected load = $10 \times 60 + 2 \times 1000 =$ **2600 W**.

Daily energy consumption is = $(8 \times 60 \times 5) + (2 \times 1000 \times 3) = 8400$ Wh = **8.4 kWh**

Monthly energy consumption = $8.4 \times 30 =$ **252 kWh**

$$\text{Monthly load factor} = \frac{252}{1500 \times 10^{-3} \times 24 \times 30} = \textbf{0.233 or 23.3 \%}$$

Example 50.5. *The load survey of a small town gives the following categories of expected loads.*

	Type	*Load in kW*	*% D.F.*	*Group D.F.*
1.	*Residential lighting*	*1000*	*60*	*3*
2.	*Commercial lighting*	*300*	*75*	*1.5*
3.	*Street lighting*	*50*	*100*	*1.0*
4.	*Domestic power*	*300*	*50*	*1.5*
5.	*Industrial power*	*1800*	*55*	*1.2*

What should be the kVA capacity of the S/S assuming a station p.f. of 0.8 lagging ?

Solution. (*i*) Residential lighting. Total max. demand = $1000 \times 0.6 =$ **600 kW**

Max. demand of the group = 600/3 = **200 kW**

(*ii*) Commercial lighting Total max. demand = $300 \times 0.75 =$ **225 kW**

Max. demand of the group = 225/1.5 = **150 kW**

(*iii*) Street lighting Total max. demand = **50 kW**

Max. demand of the group = 50/1 = **50 kW**

(*iv*) Domestic power Total max. demand = $300 \times 0.5 =$ **150 kW**

Max. demand of the group = 150/1.5 = **100 kW**

(v) Industrial power Total max. demand = 1800×0.55 = **990 kW**

Max. demand of the group = 990/1.2 = **825 kW**

Total max. demand at the station = 200 + 150 + 50 + 100 + 825 = **1325 kW.**

Capacity of the sub-station required at a p.f. of 0.8 lagging = 1325 / 0.8 = **1656 kVA**

Example 50.6. *A consumer has the following load-schedule for a day :*

From midnight (12 p.m.) to 6 a.m. = 200 W ; From 6 a.m. to 12 noon = 3000 W

From 12 noon to 1 p.m. = 100 W ; From 1 p.m. to 4 p.m. = 4000 W

From 4 p.m. to 9 p.m. = 2000 W ; From 9 p.m. to mid-night (12 p.m.) = 1000 W

Find the load factor.

If the tariff is 50 paisa per kW of max. demand plus 35 paisa per kWh, find the daily bill the consumer has to pay. **(Electrical Engineering-III, Poona Univ.)**

Solution. Energy consumed per day *i.e.* in 24 hours

$= (200 \times 6) + (3000 \times 6) + (100 \times 1) + (4000 \times 3) + (2000 \times 5) + (1000 \times 3) = 44{,}300$ Wh

Average power = 44,300/24 = 1846 W = 1.846 kW

$$\text{Daily load factor} = \frac{\text{average power}}{\text{max. power demand}} = \frac{1846}{4000} = 0.461 \text{ or } \mathbf{46.1\%}$$

Since max. demand = 4 kW M.D. charge = $4 \times 1/2$ = Rs. 2/-

Energy consumed = 44.3 kWh Energy charge = Rs. $44.3 \times 35/100$ = Rs. 15.5/-

∴ daily bill of the consumer = Rs. 2 + Rs. 15.5 = **Rs. 17.5**

Example 50.7. *A generating station has a connected load of 43,000 kW and a maximum demand of 20,000 kW, the units generated being 61,500,000 for the year. Calculate the load factor and demand factor for this case.*

Solution. $\text{Demand factor} = \frac{\text{maximum demand}}{\text{connected load}} = 0.465$ or **46.5%**

Average power = 61,500,000/8,760 = 7020 W ($\because$ 1 year = 8760 hr)

$$\therefore \quad \text{Load factor} = \frac{\text{average power}}{\text{max. power demand}} = \frac{7020}{20{,}000} = 0.351 \text{ or } \mathbf{35.1\%}$$

Example 50.8. *A 100 MW power station delivers 100 MW for 2 hours, 50 MW for 6 hours and is shut down for the rest of each day. It is also shut down for maintenance for 45 days each year. Calculate its annual load factor.* **(Generation and Utilization, Kerala Univ.)**

Solution. The station operates for (365 – 45) = 320 days in a year. Hence, number of MWh supplied in one year = $(100 \times 2 \times 320) + (50 \times 6 \times 320) = 160{,}000$ MWh

Max. No. of MWh which can be supplied per year with a max. demand of 100 MW is

$= 100 \times (320 \times 24) = 768{,}000$ MWh

$$\therefore \quad \text{load factor} = \frac{160{,}000}{760{,}000} \times 100 = \mathbf{20.8\%}$$

Example 50.9. *Differentiate between fixed and running charges in the operation of a power company.*

Calculate the cost per kWh delivered from the generating station whose

(i) *capital cost = Rs. 10^6,* ***(ii)*** *annual cost of fuel = Rs. 10^5,*

(iii) *wages and taxes = Rs. 5×10^5,* ***(iv)*** *maximum demand laod = 10,000 kW,*

(v) *rate of interest and depreciation = 10%* ***(vi)*** *annual load factor = 50%.*

Total number of hours in a year is 8,760. **(Electrical Technology-I, Bombay Univ.)**

Solution. Average power demand = max. load × load factor = 10,000 × 0.5 = 5,000 kW

Units supplied/year = 5,000 × 8,760 = 438 × 10^5 kWh

Annual cost of the fuel plus wages and taxes = Rs. 6 × 10^5

Interest and depreciation charges/year = 10% of Rs. 106 = Rs. 10^5

Total annual charges = Rs. 7 × 10^5 ; Cost / kWh = Rs. 7 × 10^5/438 × 10^5 = **1.6 paisa**

Example 50.10. *A new colony of 200 houses is being established, with each house having an average connected load of 20 kW. The business centre of the colony will have a total connected load of 200 kW. Find the peak demand of the city sub-station given the following data.*

	Demand factor	*Group D.F.*	*Peak D.F.*
Residential load	*50%*	*3.2*	*1.5*
Business load	*60%*	*1.4*	*1.2*

Solution. The three demand factors are defined as under :

$$\text{Demand factor} = \frac{\text{max. demand}}{\text{connected load}}$$

$$\text{group D.F.} = \frac{\text{sum of individual max. demands}}{\text{actual max. demand of the group}}$$

$$\text{Peak D.F.} = \frac{\text{max. demand of consumer group}}{\text{demand of consumer group at the time of system peak demand}}$$

Max. demand of each house = 2 × 0.5 = 1.0 kW

Max. demand of residential consumer = 1 × 200/3.2 = 62.5 kW

Demand of the residential consumer at the time of the system peak = 62.5 / 1.5 = 41.7 kW

Max. demand of commercial consumer = 200 × 0.6 = 120 kW

Max. demand of commercial group = 120/1.4 = 85.7 kW

Commercial demand at the time of system peak = 85.7 / 1.2 = 71.4 kW

Total demand of the residential and commercial consumers at the time of system peak

= 41.7 + 71.4 = **113 kW**

Example 50.11. *In Fig. 50.6 is shown the distribution network from main sub-station. There are four feeders connected to each load centre sub-station. The connected loads of different feeders and their maximum demands are as follows :*

Feeder No.	*Connected load, kW*	*Maximum Demand, kW*
1.	*150*	*125*
2.	*150*	*125*
3.	*500*	*350*
4.	*750*	*600*

If the actual demand on each load centre is 1000 kW, what is the diversity factor on the feeders? If load centres B, C and D are similar to A and the diversity factor between different load centres is 1.1, calculate the maximum demand of the main sub-station. What would be the kVA capacity of the transformer required at the main sub-station if the overall p.f. at the main sub-station is 0.8 ?

Solution. Diversity factor of the feeders

$$= \frac{\text{total of max. demand of different feeders}}{\text{simultaneous max. demand}} = \frac{125 + 125 + 350 + 600}{1000} = 1.2$$

Total max. demand of all the 4 load centre sub-stations = 4×1000 = **4000 kW**

Diversity factor of load centres = 1.1

Simultaneous max. demand on the main sub-station = 4000/1.1 = 3636 kW

The kVA capacity of the transformer to be used at the sub-station = 3636/0.8 = **4545**

Example 50.12. *If a generating station had a maximum load for the year of 18,000 kW and a load factor of 30.5% and the maximum loads on the sub-stations were 7,500, 5,000, 3,400, 4,600 and 2,800 kW, calculate the units generated for the year and the diversity factor.*

Solution. Load factor $= \dfrac{\text{average power}}{\text{maximum power demand}}$ $\therefore$ $0.305 = \dfrac{\text{average power}}{18{,}000}$

$\therefore$ average power $= 18{,}000 \times 0.305$ kW

kWh generated per year $= 8{,}000 \times 0.305 \times 8.760 =$ **48.09×10^6 kWh**

Sum of individual maximum demands = 7,500 + 5,000 + 3,400 + 4,600 + 2,800 = 23,300

$\therefore$ diversity factor=23,300/18,000 = **1.3 (approx.)**

Example 50.13. *A power station is supplying four regions of load whose peak loads are 10MW, 5 MW, 8 MW and 7 MW. the diversity factor of the load at the station is 1.5 and the average annual load factor is 60%. Calculate the maximum demand on the station and the annual energy supplied from the station. Suggest the installed capacity and the number of units taking all aspects into account.* **(A.M.I.E. Sec. B, Winter 1990)**

Solution. Diversity factor $= \dfrac{\text{sum of individual max. demands}}{\text{max. demands of the whole load}}$

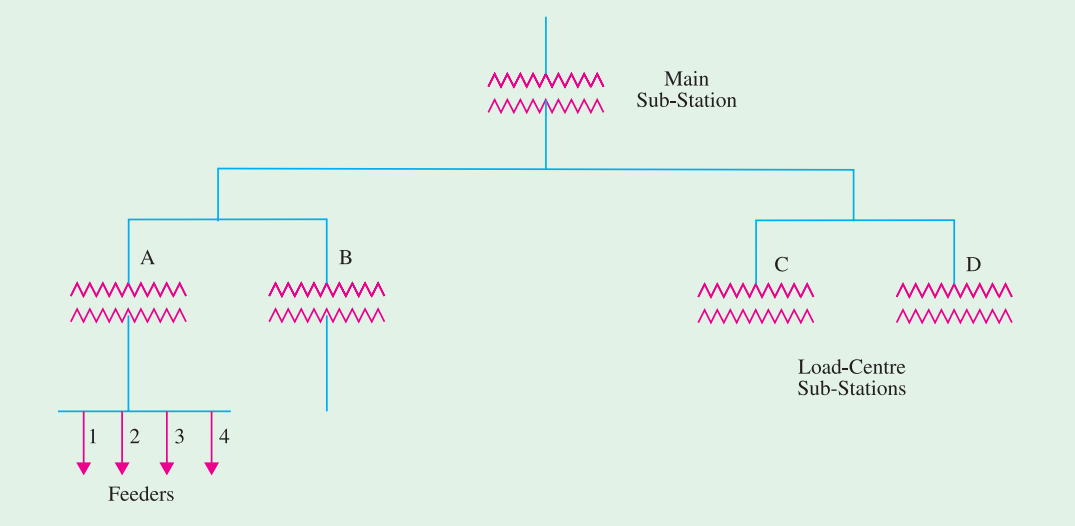


Fig. 50.6

$\therefore$ Max. demand of the whole load imposed on the station = (10 + 5 + 8 + 7)/1.5 = **20 MW**

Now, annual load factor $= \dfrac{\text{No. of units supplied/year}}{\text{Max. demand} \times 8760}$

$\therefore$ No. of units supplied / year $= 0.6 \times 20 \times 10^3 \times 8760 =$ **105.12×10^6 kWh**

Provision for future growth in load may be made by making installed capacity 50% more than the maximum demand of the whole load. Hence, installed capacity is $20 \times 1.5 = 30$ MW. Four generators, two of **10 MW** each and other two of **5 MW** each may be installed.

Example 50.14. *The capital cost of 30 MW generating station is Rs. 15×10^6. The annual expenses incurred on account of fuel, taxes, salaries and maintenance amount to Rs. 1.25×10^6. The station operates at an annual load factor of 35%. Determine the generating cost per unit delivered, assuming rate of interest 5% and rate of depreciation 6%.* **(Electrical Power-I, Bombay Univ.)**

Solution. Average power = max. power × load factor = $30 \times 10^6 \times 0.35 = 10{,}500$ kW

Units produced/year = $10{,}500 \times 8{,}760 = 91.98 \times 10^6$ kWh; Annual expenses = Rs. 1.25×10^6

Depreciation plus interest = 11% of capital cost = 11% of Rs. 15×10^6 = Rs. 1.65×10^6

Total expenses/year = Rs. 1.25×10^6 + Rs. 1.65×10^6 = Rs. 2.9×10^6

$\therefore$ cost/kWh = Rs. $2.9 \times 10^6 / 91.98 \times 10^6$ = **3.15 paisa / kWh.**

Example 50.15. *A generating plant has a maximum capacity of 100 kW and costs Rs.300,000. The fixed charges are 12% consisting of 5% interest, 5% depreciation and 2% taxes etc. Find the fixed charges per kWh generated if load factor is **(i)** 100% and **(ii)** 25%.*

Solution. Annual fixed charges = Rs. 300,000 × 12/100 = Rs. 36,000

With a load factor of 100%, number of kWh generated per year = 100 × 1 × 8,760

= 876,000 kWh.

Similarly, units generated with a load factor of 25% = 100 × 0.25 × 8,760 = 219,000 kWh.

(*i*) Fixed charge / kWh = 36,000 × 100/876,000 = **4.1 paisa**

(*ii*) Fixed charge / kWh = 36,000 × 100/219,000 = **16.4 paisa**

As seen, the charge has increased four-fold. In fact, charge varies inversely as the load factor.

Example 50.16. *The annual working cost of a thermal station is represented by the formula Rs. (a + b kW + c kWh) where a, b and c are constants for that particular station, kW is the total installed capacity and kWh is the energy produced per annum.*

*Determine the values of a, b and c for a 100 MW station having annual load factor of 55% and for which **(i)** capital cost of buildings and equipment is Rs. 90 million, **(ii)** the annual cost of fuel, oil, taxation and wages and salaries of operating staff is Rs. 1,20,000, **(iii)** interest and depreciation on buildings and equipment are 10% p.a., **(iv)** annual cost of orginasation, interest on cost of site etc. is Rs. 80,000.*

Solution. In the given formula, *a* represents the fixed cost, *b* semi-fixed cost and *c* the running cost. Here, *a* = **Rs. 80,000**



An overview of a thermal power plant

Now, $b \times$ kW minimum demand = semi-fixed cost

or, $b \times 100 \times 10^3 = 90 \times 10^6 \quad \therefore b = \mathbf{900}$

Total units generated per annum = kW max. demand $\times$ load factor $\times$ 8760

$= 100 \times 10^3 \times 0.55 \times 8760 = 438 \times 10^6$ kWh

Since running cost is Rs. 1,20,000

$\therefore \quad c \times 438 \times 10^6 = 1,20,000$ or $c = \mathbf{0.00027}$

Example 50.17. *In a steam generating station, the relation between the water evaporated W kg and coal consumed C kg and power in kW generated per 8-hour shift is as follows :*

$$W = 28,000 + 5.4 \text{ kWh}; \; C = 6000 + 0.9 \text{ kWh}$$

What would be the limiting value of the water evaporated per kg of coal consumed as the station output increases ? Also, calculate the amount of coal required per hour to keep the station running at no-load.

Solution. For an 8-hour shift, Wt. of water evaporated per kg of coal consumed is

$$\frac{W}{C} = \frac{28,000 + 5.4 \text{ kWh}}{6,000 + 0.9 \text{ kWh}}$$

As the station output increases, the ratio W/C approaches the value 5.4 / 0.9 = 6

Hence, weight of water evaporated per kg of coal approaches a limiting value of **6 kg** as the station output increases.

Since at no-load, there is no generation of output power, kWh = 0. Substituting this value of kWh in the above ratio we get,

Coal consumption per 8-hour shift = 6000 kg

$\therefore$ coal consumption per hour on no-load = $\frac{6000}{8}$ **= 725 kg.**

Example 50.18. *Estimate the generating cost per kWh delivered from a generating station from the following data :*

Plant capacity = 50 MW ; annual load factor = 40%; capital cost = Rs. 3.60 crores; annual cost of wages, taxation etc. = Rs. 4 lakhs; cost of fuel, lubrication, maintenance etc. = 2.0 paise per kWh generated, interest 5% per annum, depreciation 5% per annum of initial value.

(Electrical Technology, M.S. Univ. Baroda)

Solution. Average power over a year = maximum power $\times$ load factor

$= 50 \times 10^6 \times 0.4 = 2 \times 10^7$ W $= 2 \times 10^4$ kW

Units produced/year = $20,000 \times 8,760 = 1,752 \times 10^5$ kWh

Depreciation plus interest = 10% of initial investment $= 0.1 \times 3.6 \times 10^7$

$= \mathbf{Rs.\ 3.6 \times 10^6}$

Annual wages and taxation etc. = Rs. 4 lakhs = Rs. 0.4×10^6

Total cost/year = Rs. $(3.6 + 0.4) \times 10^6$ = Rs. 4×10^6

Cost / kWh = $4 \times 10^6 \times 100 / 1,752 \times 10^5$ = **2.28 paisa**

Adding the cost of fuel, lubrication and maintenance etc., we get

Cost per kWh delivered = 2.0 + 2.28 = **4.28 paisa.**

Example 50.19. *The following data relate to a 1000 kW thermal station :*

Cost of Plant = *Rs. 1,200 per kW*

Interest, insurance and taxes = *5% p.a.*

Depreciation = *5% p.a.*

Cost of primary distribution system = *Rs. 4,00,000*
Interest, insurance, taxes and depreciaton = *5% p.a.*
Cost of coal including transportation = *Rs. 40 per tonne*
Operating cost = *Rs. 4,00,000 p.a.*
Plant maintenance cost : fixed = *Rs. 20,000 p.a.*
variable = *Rs. 30,000 p.a.*
Installed plant capacity = *10,000 kW*
Maximum demand = *9,000 kW*
Annual load factor = *60%*
Consumption of coal = *25,300 tonne*

Find the cost of power generation per kilowatt per year, the cost per kilowatt-hour generated and the total cost of generation per kilowatt-hour. Transmission/primary distribution is chargeable to generation. **(Power Systems-I, AMIE, Sec. B, 1993)**

Solution. Cost of the plant = Rs. 1200 per kW

Fixed cost per annum is as under :

(*i*) on account of capital cost = $(1200 \times 10{,}000) \times 0.1 + 400 \times 103 \times 0.05$

= **Rs. 1.22×10^6**

(*ii*) part of maintenance cost = Rs. 20,000 = Rs. 0.02×10^6

∴ total fixed cost = $1.22 \times 10^6 + 0.02 \times 10^6$ = **Rs. 1.24×10^6**

Running or variable cost per annum is as under : (*i*) operation cost = Rs. 4,00,000, (*ii*) part of maintenance cost = Rs. 30,000,

(*iii*) fuel cost = Rs. $25{,}300 \times 40$ = Rs. 10,12,000

Total cost = $4{,}00{,}000 + 30{,}000 + 10{,}12{,}000$ = Rs. 1.442×10^6

$$\text{Load factor} = \frac{\text{average demand}}{\text{maximum demand}} \quad \text{or} \quad 0.6 = \frac{\text{average demand}}{9{,}000 \times 8{,}760}$$

∴ average demand = **47,305 MWh**

Total cost per annum = $1.24 \times 10^6 + 1.442 \times 10^6$ = Rs. 2.682×10^6

Cost per kWh generated = $2.682 \times 10^6 / 47{,}305 \times 10^3$ = Rs. 0.0567 = **5.7 paisa.**

Since total installed capacity is 10,000 kW, the cost per kW per year

= $2.682 \times 10^6 / 10{,}000$ = **Rs. 268.2**

Example 50.20. *A consumer has an annual consumption of 176,400 kWh. The charge is Rs. 120 per kW of maximum demand plus 4 paisa per kWh.*

(i) *Find the annual bill and the overall cost per kWh if the load factor is 36%.*

(ii) *What is the overall cost per kWh, if the consumption were reduced 25% with the same load factor ?*

(iii) *What is the overall cost per kWh, if the load factor is 27% with the same consumption as in (i)* **(Utili. of Elect. Power, AMIE Sec. B)**

Solution. (*i*) Since load factor is 0.36 and there are 8760 hrs in a year,

Annual max. demand = $176{,}400/0.36 \times 8760$ = **55.94 kW**

The annual bill will be based on maximum annual demand charges plus the annual energy consumption charge.

∴ annual bill = Rs. $(55.94 \times 120 + 176{,}400 \times 0.04)$ = **Rs. 13,768**

Overall cost/kWh = Rs. 13,768 / 176,400 = **7.8 paisa.**

(ii) In this case, the annual consumption is reduced to $176,400 \times 0.75 = 132,300$ kWh but the load factor remains the same.

Annual max. demand = $132,300/0.36 \times 8760 = 41.95$ kW

$\therefore$ annual bill = Rs. $(41.95 \times 120 + 132,300 \times 0.04)$ = **Rs. 10,326**

Overall cost/kWh = $10,326/132,300$ = **7.8 paisa**

It will be seen that the annual max. demand charge is reduced but the overall cost per kWh remains the same.

(iii) Since, load factor has decreased to 0.27,

Annual max. demand = $176,400/0.27 \times 8760 = 74.58$ kW

Annual bill = Rs. $(74.58 \times 120 + 176,400 \times 0.04)$ = Rs. 16,006

Overall cost/kWh = Rs. $16,006/176,400$ = **9.1 paisa**

Here, it will be seen that due to decrease in load factor, the annual bill as well as cost per kWh have increased.

50.13. Plant Factor or Capacity Factor

This factor relates specifically to a generating plant unlike load factor which may relate either to generating or receiving equipment for the whole station.

It is defined as the ***ratio of the average load to the rated capacity of the power plant i.e. the aggregate rating of the generators***. It is preferable to use continuous rating while calculating the aggregate.

$$\therefore \quad \text{plant factor} = \frac{\text{average load}}{\text{rated capacity of plant}} = \frac{\text{average demand on station}}{\text{max. installed capacity of the station}}$$

It may be of interest to note that if the maximum load corresponds exactly to the plant ratings, then load factor and plant factor will be identical.

50.14. Utilization Factor (or Plant Use Factor)

It is given by the ratio of the kWh generated to the product of the capcity of the plant and the number of hours the plant has been actually used.

$$\text{Utilization factor} = \frac{\text{station output in kWh}}{\text{plant capacity} \times \text{hours of use}}$$

If there are three units in a plant of ratings kW_1, kW_2 and kW_3 and their operation hours are h_1, h_2 and h_3 respectively, then

$$\text{Utilization factor} = \frac{\text{station output in kWH}}{(kW_1 \times h_1) + (kW_2 \times h_2) + (kW_3 \times h_3)}$$

50.15. Connected Load Factor

The factor relates only to the receiving equipment and is defined as the ratio of the average power input to the connected load.

To render the above value specific, it is essential*

(i) to define the period during which average is taken and

(ii) to state the basis on which the connected load is computed.

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* Wherever feasible, it should be stated on continuous-rating basis. Lighting connected load is taken equal to the sum of the wattages of all lamps in the installation whereas motor connected load is equal to the sum of the name-plate outputs of all motors (and not their input ratings).

$$\text{Connected-load factor} = \frac{\text{average power input}}{\text{connected load}}$$

It can be proved that the connected-load factor of a ***receiving*** equipment is equal to the product of its demand factor and its load factor.

$$\text{Connected-load factor} = \frac{\text{average power input}}{\text{connected load}}$$

$$= \frac{\text{Average power}}{\text{Max. demand}} \times \frac{\text{max demand}}{\text{connected load}} = \text{load factor} \times \text{demand factor}$$

50.16. Load Curves of a Generating Station

The total power requirement of a generating station can be estimated provided variation of load with time is known. Following curves help to acquire this knowledge.

(*i*) Load Curve (or Chronological Curve)

It represents the load in its proper time sequence. As shown in Fig. 50.7 (*a*), this curve is obtained by plotting the station load (in kW) along *Y*-axis and the time when it occurs along *X*-axis. Usually, such curves are plotted for one day *i.e.* for 24 hours by taking average load (kW) on hourly basis. The area under the curve represents the total energy consumed by the load in one day. Following information can be obtained from the load curve :

(*a*) maximum load imposed on the station, **(*b*)** size of the generating unit required and **(*c*)** daily operating schedule of the station.

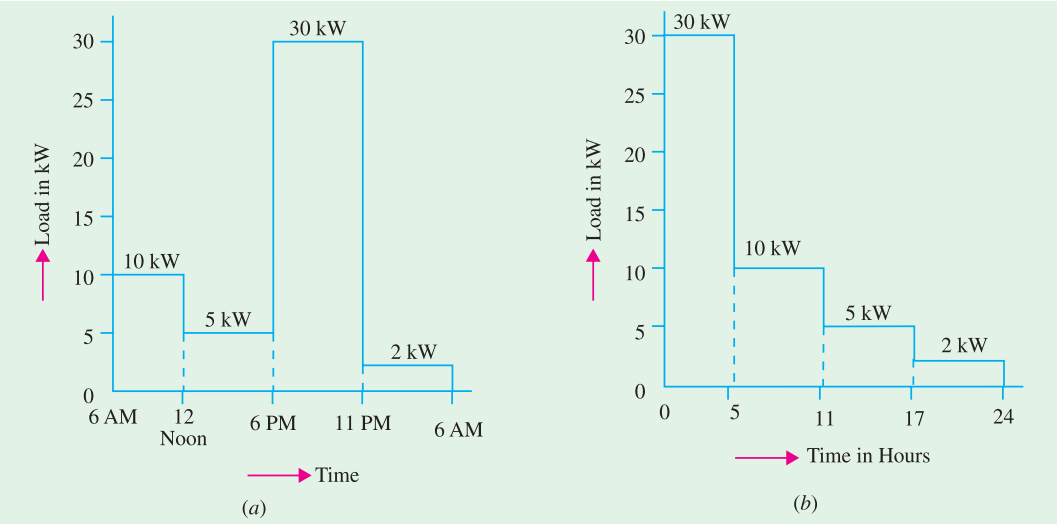


Fig. 50.7

(*ii*) Load Duration Curve

It represents the same data (*i.e.* load *vs* time) but the ordinates are rearranged in magnitude sequence (not time sequence). Here, greatest load is plotted on the left, lesser load towards the right and the least load on the extreme right. In other words, loads are plotted in descending order. As seen from Fig. 50.7 (*a*) maximum load on the station is 30 kW which lasts for 5 hours from 6 p.m. to 11 p.m. It is plotted first in Fig. 50.7 (*b*). The next lower load is 10 kW from 6 a.m. to 12 noon *i.e.* for 6 hours. It has been plotted next to the highest load. The other lesser loads are plotted afterwards. The areas under the load curve and load duration curve are equal and each represents the total units consumed during a day of 24 hours.

(*iii*) Load Energy Curve (or Integrated Load Duration Curve)

It represents the relation between a particular load on the station and the total number of kWhs produced ***at or below this load***. The load in kW is taken along the ordinate (Y-axis) and kWh generated ***upto this load*** along the abscissa (X-axis) as shown in Fig. 50.8. This curve is derived from the load duration curve. For example, for a load of 2 kW, the number of units generated is $2 \times 24 = 48$ kWh. It corresponds to point A on the curve. For a load of 5 kW, the units generated are $= 5 \times 17 + 2 \times 7 = 99$ kWh. It corresponds to point B. For a load of 10 kW, the units generated are $= 10 \times 11 + 5 \times 6 + 2 \times 7 = 154$ kWh (point C). Finally, for a load of 30 kW, the number of units generated is $= 30 \times 5 + 10 \times 6 + 5 \times 6 + 2 \times 7 = 254$ kWh (point D.)

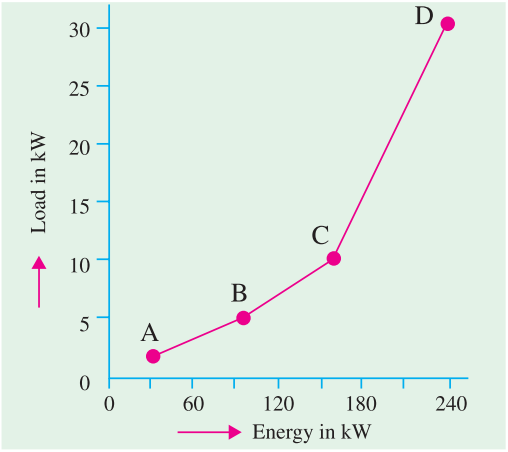


Fig. 50.8

Example 50.21. *A power station has a load cycle as under :*

60 MW for 6 hr; 200 MW for 8 hr ; 160 MW for 4 hr ; 100 MW for 6 hr

If the power station is equipped with 4 sets of 75 MW each, calculate the load factor and the capacity factor from the above data. Calculate the daily fuel requirement if the calorific value of the oil used were 10,000 kcal/kg and the average heat rate of the station were 2,860 kcal/kWh.

(Electric Power Systems-III, Gujarat Univ.)

Solution. Daily load factor $= \dfrac{\text{units actually supplied in a day}}{\text{max. demand} \times 24}$

Now, MWh supplied per day $= (260 \times 6) + (200 \times 8) + (160 \times 4) + (100 \times 6) = 4{,}400$

$\therefore$ station daily load factor $= \dfrac{4{,}440}{260 \times 24} = 0.704$ or **70.4%**

Capacity factor $= \dfrac{\text{average demand on station}}{\text{installed capacity of the station}}$

No. of MWh supplied/day $= 4{,}400$ $\therefore$ average power/day $= 4{,}400/24$ MW

Total installed capacity of the station $= 75 \times 4 = 300$ MW

$\therefore$ capacity factor $= \dfrac{4{,}400/24}{300} = 0.611$ or **61.1%**

Energy supplied/day $= 4{,}400$ MWh $= 44 \times 10^5$ kWh

Heat required/day $= 44 \times 10^5 \times 2{,}860$ kcal

Amount of fuel required/day $= 44 \times 2{,}860 \times 10^5 / 10^5$ kg $=$ **125 tonne.**

Example 50.22. *A generating station has two 50 MW units each running for 8,500 hours in a year and one 30 MW unit running for 1,250 hours in one year. The station output is 650×10^6 kWh per year. Calculate **(i)** station load factor, **(ii)** the utilization factor.*

Solution. **(*i*)** $kW_1 \times h_1 = 50 \times 10^3 \times 8{,}500 =$ **425×10^6 kWh**

(*ii*) $kW_2 \times h_2 = 50 \times 10^3 \times 8{,}500 =$ **425×10^6 kWh**

(*iii*) $kW_3 \times h_3 = 30 \times 10^3 \times 1{,}250 =$ **37.5×10^6 kWh**

$\therefore$ $\Sigma\,(kW) \times h = (2 \times 425 + 37.5) \times 10^6 =$ **887.5×10^6 kWh**

Total installed capacity of the station $= 2 \times 50 + 30 = 130 \times 10^3$ kW

(*i*) Assuming that maximum demand equals installed capacity of the station,

$$\text{annual load factor} = \frac{\text{units generated/year}}{\text{max. demand} \times 8{,}760} = \frac{650 \times 10^6}{130 \times 10^3 \times 8760} = \textbf{0.636 or 63.6\%}$$

Note. In view of the above assumption, this also represents the plant or capacity factor.

(ii) $$\text{utilization factor} = \frac{\text{station output in kWh}}{\Sigma(\text{kW}) \times h} = \frac{650 \times 10^6}{887.5 \times 10^6} = \textbf{0.732 or 73.2\%}$$

Example 50.23. *The yearly duration curve of a certain plant may be considered as a straight line from 40,000 kW to 8,000 kW. To meet this load, three turbo-generators, two rated at 20,000 kW each and one at 10,000 kW are installed. Determine **(a)** the installed capacity, **(b)** plant factor, **(c)** maximum demand, **(d)** load factor and **(e)** utilization factor.* **(Ranchi Univ.)**



Fig. 50.9

Solution. **(a)** installed capacity
$= 20 + 20 + 10 = \textbf{50 MW}$

(b) Average demand on plant $= (40{,}000 + 8{,}000)/2$
$= 24{,}000$ kW

plant factor = av. demand / installed capacity
$= 24{,}000/50{,}000 = \textbf{0.48\% or 48\%}$

(c) Max. demand, obviously, is **40,000 kW**

(d) From load duration curve, total energy generated/ year
$= 24{,}000 \times 8760$ kWh $= 21 \times 10^7$ kWh.

Load factor $= 21 \times 10^7 / 40{,}000 \times 8{,}760 =$ **0.6 or 60%**

(e) $\text{u.f.} = \frac{\text{max. demand}}{\text{plant capacity}} \times 100 = \frac{40{,}000}{50{,}000} \times 100 =$ **80%**

Example 50.24. *The load duration curve of a system is as shown in Fig. 50.10. The system is supplied by three stations; a steam station, a run-of-river station and a reservoir hydro-electric station. The ratios of number of units supplied by the three stations are as below :*

Steam	:	*Run of river*	:	*Reservoir*
7	:	*4*	:	*1*

The run-of-river station is capable of generating power continuosuly and works as a peak load station. Estimate the maximum demand on each station and also the load factor of each station. **(Ranchi Univ.)**

Solution. Here 100% time will be taken as 8760 hours.

Total units generated = area under the curve
$= \frac{1}{2}(160 + 80) \times 10^3 \times 8760 = 1051.2 \times 10^6$ kWh

From the given ratio, the number of units supplied by each station can be calculated

Units Generated

Run-of-river-station $= 1051.2 \times 10^6 \times 4/12 = 350.4 \times 10^6$ kWh

Steam station $= 1051.2 \times 10^6 \times 7/12 = 613.2 \times 10^6$ kWh

Reservoir HE station $= 1051.2 \times 10^6 \times 1/12 = 87.6 \times 10^6$ kWh

Max. demand of *ROR* station $= 350.4 \times 10^6 / 8760 =$ **40 MW**

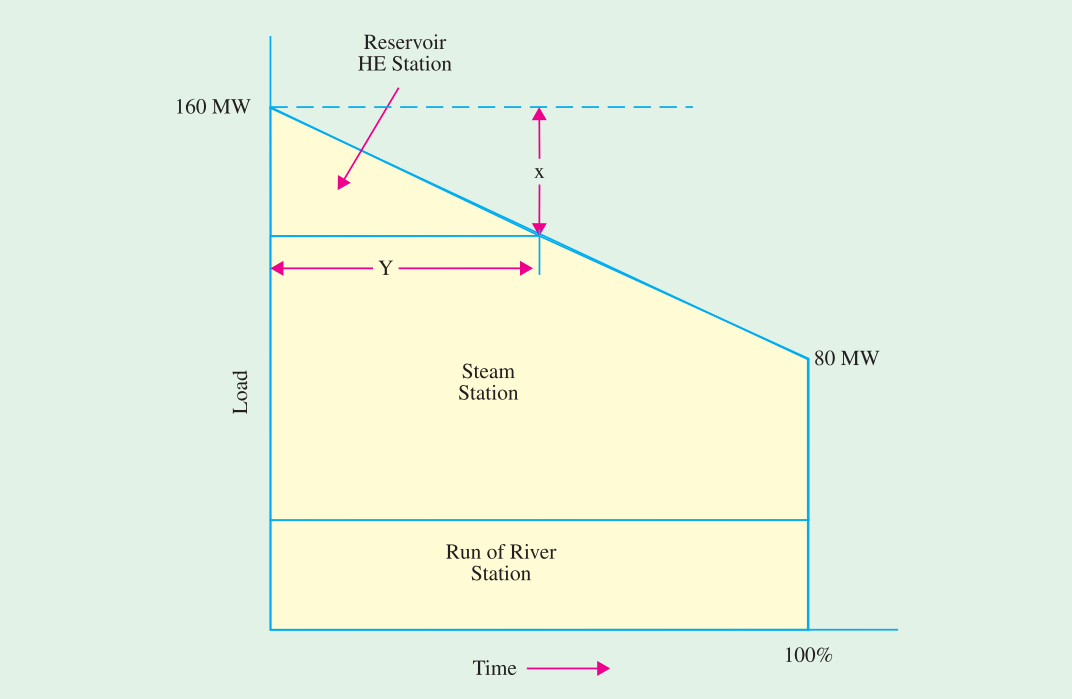


Fig. 50.10

Let x MW be the maximum demand of the reservoir plant. As shown in Fig. 50.10. let it operate for y hours.

Obviously, $y = \frac{x}{80} \times 8760$

Area under the curve for reservoir station = $\frac{1}{2} xy \times 10^3$ kWh

$$= \frac{1}{2} x \times \frac{x}{80} \times 8760 \times 10^3 = 54,750\ x^2$$

$\therefore$ $54,750\ x^2 = 87.6 \times 10^6$; $x =$ **40 MW**

$\therefore$ max. demand of steam station= 160 – (40 + 40) = **80 MW.**

Load factor

Since ROR station works continuously as a base load station, its load factor is **100%.**

Reservoir station = $87.6 \times 10^6 / 40 \times 10^3 \times 8760$ = **0.25 or 25%**

Steam station = $613.2 \times 10^6 / 80 \times 10^3 \times 8760$ = **0.875 or 87.5%.**

Example 47.25. *A load having a maximum value of 150 MW can be supplied by either a hydro-electric plant or a steam power plant. The costs are as follows :*

Capital cost of steam plant = Rs. 700 per kW installed

Capital cost of hydro-electric plant = Rs. 1,600 per kW installed

Operating cost of steam plant = Rs. 0.03 per kWh

Operating cost of hydro-electric plant = Rs. 0.006 per kWh

Interest on capital cost 8 per cent. Calculate the minimum load factor above which the hydro-electric plant will be more economical.

Solution. Let x be the total number of units generated per annum.

Steam Plant

Capital cost = Rs. $700 \times 150 \times 10^3$ = **Rs. 10.5×10^7**

Interest charges = 8% of Rs. 10.5×10^7 = **Rs. 8.4×10^6**

$\therefore$ fixed cost/unit = Rs. $8.4 \times 10^6 / x$; operating cost / unit = **Re. 0.03**

$\therefore$ total cost/unit generated = **Rs. $(8.4 \times 10^4 / x + 0.03)$**

Hydro Plant

Capital cost = Rs. $1600 \times 150 \times 10^3$ = **Rs. 24×10^7**

Interest charges = 8% of Rs. 24×10^7 = **Rs. 19.2×10^6**

Total cost/unit = **Rs. $(19.2 \times 10^6 / x + 0.006)$**

The two overall costs will be equal when

$(8.4 \times 10^6 / x) + 0.03 = (19.2 \times 10^6 / x) + 0.006$; $x = 45 \times 10^7$ kWh

Obviously, if units generated are more than 45×10^7 kWh, hydro-electric station will be cheaper.

Load factor = $45 \times 10^7 / 150 \times 10^3 \times 8760$ = **0.342 or 34.2%**

This represents the minimum load factor beyond which hydro-electric station would be economical.

Example 50.26. *A power system having maximum demand of 100 MW has a load 30% and is to be supplied by either of the following schemes :*

(a) *a steam station in conjunction with a hydro-electric station, the latter supplying 100×10^6 units per annum with a max. output of 40 MW,*

(b) *a steam station capable of supply the whole load,*

(c) *a hydro station capable of supplying the whole load,*

Compare the overall cost per unit generated assuming the following data :

	Steam	*Hydro*
Capital cost / kW	*Rs. 1,250*	*Rs. 2,500*
Interest and depreciation on the capital cost	*12%*	*10%*
Operating cost/kWh	*5 paise*	*1.5 paise*
Transmission cost/kWh	*Negligible*	*0.2 paise*

Show how overall cost would be affected in case (ii) and (iii) above if the system load factor were improved to 90 per cent. **(Elect. Power System-III, Gujarat Univ.)**

Solution. Average power = $100 \times 0.3 = 30$ MW = 3×10^4 kW

Units generated in one year = $3 \times 10^4 \times 8{,}760 = 262.8 \times 10^6$ kWh

(*a*) Steam Station in Conjunction with Hydro Station

Units supplied by hydro-station = **100×10^6 kWh**

Units supplied by steam station = $(262.8 - 100) \times 10^6$ = **162.8×10^6 kWh**

Since, maximum output of hydro-station is 40 MW, the balance (100 – 40) = 60 MW is supplied by the steam station.

(*i*) Steam Station

Capital cost = Rs. $60 \times 10^3 \times 1{,}250$ = **Rs. 75×10^6**

Annual interest and depreciation - Rs. $0.12 \times 75 \times 10^6$ = **Rs. 9×10^6**

Operating cost = Rs. $0.5 \times 162.8 \times 10^6 / 100$ = **Rs. 8.14×10^6**

Transmission cost = negligible

Total annual cost = Rs. $(9 + 8.14) \times 10^6$ = **Rs. 17.14×10^6**

(*ii*) Hydro Station

Capital cost = Rs. $40 \times 10^3 \times 2{,}500$ = Rs. 100×10^6

Annual interest and depreciation = Rs. $0.1 \times 100 \times 10^6$ = Rs. 10×10^6

Operating cost = Rs. $0.15 \times 100 \times 10^6/100$ = Rs. 1.5×10^6

Transmission cost = Rs. $.02 \times 100 \times 10^6 / 100$ = Rs. 0.2×10^6

Total annual cost = Rs. $(10 + 1.5 + 1.2) \times 10^6$ = Rs. 11.7×10^6

Combined annual charge for steam and hydro stations

= Rs. $(17.14 + 11.7) \times 10^6$ = Rs. 28.84×10^6

$\therefore$ Overall cost/kWh = Rs. $\dfrac{28.84 \times 10^6}{262.8 \times 10^6}$ = **10.97 paise**

(*b*) Steam Station Alone

Capital cost = Rs. $1{,}250 \times 100 \times 10^3$ = Rs. 125×10^6

Annual interest and depreciation = Rs. $0.12 \times 125 \times 10^6$ = Rs. 15×10^6

$\therefore$ fixed charge / unit = Rs. $15 \times 10^6/262.8 \times 10^6$ = 5.71 paise

Operating cost/unit = 5 paisa ; Transmission cost / unit = 0

$\therefore$ overall cost per unit = (5.71 + 5) = **10.71 paise**

(*c*) Hydro Station Alone

Annual interest and depreciation on capital cost

= Rs. $0.1\ (2{,}500 \times 100 \times 10^3)$ = Rs. 25×10^6

$\therefore$ fixed charge / unit = Rs. $25 \times 10^6 / 262.8 \times 10^6$ = 9.51 paise

Operating cost/unit = 1.5 paisa ; Transmission cost/unit = 0.2 paise

$\therefore$ overall cost/unit = (9.51 + 1.5 + 0.2) = **11.21 paise**

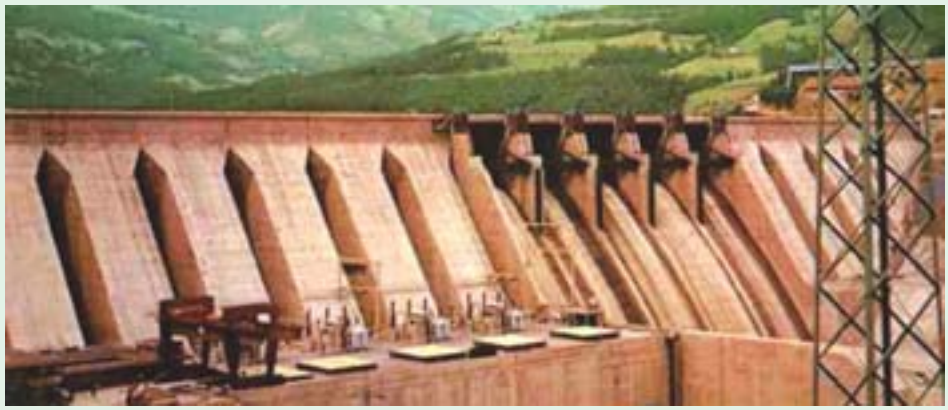
(*d*) (*i*) Steam Station

Since number of units generated will increase three-fold, fixed charge per unit will decrease to one-third of its previous value *i.e.* to 5.71/3 = 1.9 paisa. Since other charges are unaffected by change in load factor.

$\therefore$ Overall cost/unit = (1.9 + 5) = **6.9 paise**

(*ii*) Hydro Station

For same reasons, fixed cost per unit becomes 9.51/3 = 3.17 paise Overall cost/unit = (3.17 + 1.5 + 0.2) = **4.87 paise**



An overview of a hydroelectric plant

Example 50.27. *The capital cost of a hydro-power station of 50,000 kW capacity is Rs. 1,200 per kW. The annual charge on investment including depreciation etc. is 10%. A royality of Rs. 1 per kW per year and Rs. 0.01 per kWh generated is to be paid for using the river water for generation of power. The maximum demand is 40,000 kW and the yearly load factor is 80%. Salaries, maintenance charges and supplies etc. total Rs. 6,50,000. If 20% of this expense is also chargeable as fixed charges, determine the generation cost in the form of A per kW plus B per kWh.*

(A.M.I.E. Sec. B,)

Solution. Capital cost of station = Rs. $1200 \times 50{,}000$ = Rs. 6×10^7

Annual charge on investment including depreciaiton
= 10% of Rs. 6×10^7 = Rs. 6×10^6

Total running charges = 80% of Rs. 6,50,000 = Rs. 5,20,000

Fixed charges = 20% of Rs. 6,50,000 = Rs. 1,30,000

Total annual fixed charges = Rs. 6×10^6 + Rs. 0.13×10^6 = Rs. 6.13×10^6

Cost per M.D. kW due to fixed charges = Rs. 6.13×10^6 / 40,000 = Rs. 153.25

Cost per M.D. kW due to royalty = Rs. 1

Total cost per M.D. kW = Rs. 154.25

Total No. of units generated per annum = $40{,}000 \times 0.8 \times 8760 = 28 \times 10^7$ kWh

Cost per unit due to running charges = Rs. 5,20,000 / 28×10^7 = 0.18 p

Royalty cost/unit = 1 p

$\therefore$ total cost/unit = 1.18 p

$\therefore$ generation cost = Rs. 154.25 kWh + 1.18 p kWh

= **Rs. (154.25 kW + 1.18×10^{-2} kWh)**

Example 50.28. *The capital costs of steam and water power stations are Rs. 1,200 and Rs. 2,100 per kW of the installed capacity. The corresponding running costs are 5 paise and 3.2 paise per kWh respectively.*

The reserve capacity in the case of the steam station is to be 25% and that for the water power station is to be 33.33% of the installed capacity.

At what load factor would the overall cost per kWh be the same in both cases ? Assume interest and depreciation charges on the capital to be 9% for the thermal and 7.5% for the hydro-electric station. What would be the cost of generating 500 million kWh at this load factor ?

Solution. Let x be the maximum demand in kWh and y the load factor.

Total No. of units produced = $xy \times 8760$ kWh

Steam Station

installed capacity = 1.25 x (including reserve capacity)

capital cost = Rs. $1200 \times 1.25\,x$ = Rs. $1500\,x$

annual interest and depreciation = 9% of Rs. $1500\,x$ = Rs. $135\,x$

annual running cost = Rs. $8760\,xy \times 5/100$ = Rs. $438\,xy$

Total annual cost = Rs. $(135\,x + 438\,xy)$

$\therefore$ total cost / unit = **Rs. $(135\,x + 438\,xy / 8760\,xy$**

Hydro-electric Station

installed capacity = 1.33 x (including reserve capacity)

capital cost = Rs. $1.33\,x \times 2100$ = Rs. $2800\,x$

annual interest and depreciation = 7.5% of Rs. $2800\,x$ = Rs. $210\,x$

annual running cost = Rs. $8760xy \times 3.2/100$ = Rs. $280\,xy$

$\therefore$ total annual cost = Rs. $(210x + 280\,xy)$

$\therefore$ total cost/unit = Rs. $(210x + 280\,xy)/8760\,xy$

For the two costs to be the same, we have

$$\frac{135\,x + 438\,xy}{8760\,xy} = \frac{210\,x + 280\,xy}{8760\,xy}, \quad y = \mathbf{0.475 \text{ or } 47.5\%}$$

Cost of 500×10^6 Units

Now, maximum demand = $500 \times 10^6 / 8760 \times 0.475 = 12 \times 10^4$ kW

$\therefore$ $x = 12 \times 10^4$, $y = 0.475$

$\therefore$ generating cost = Rs. $(136 \times 12 \times 10^4 + 438 \times 12 \times 10^4 \times 0.475)$ = **41,166,000**

Example 50.29. *In a particular area, both steam and hydro-stations are equally possible. It has been estimated that capital cost and the running costs of these two types will be as follows:*

Capital cost/kW	*Running cost/kWh*	*Interest*
Hydro : Rs. 2,200	*1 Paise*	*5%*
Steam : Rs. 1,200	*5 Paise*	*5%*

If expected average load factor is only 10%, which is economical to operate : steam or hydro? If the load factor is 50%, would there be any change in the choice ? If so, indicate with calculation.

(Electric Power-II Punjab Univ. 1991)

Solution. Let x be the capacity of power station in kW.

Case I. Load factor = 10%

Total units generated/annum = $x \times 0.1 \times 8760 = 876\,x$ kWh

(*a*) Hydro Station

capital cost = Rs. $2200\,x$

annual fixed charges = 5% of Rs. $2200\,x$ = Rs. $110\,x$

annual running charges = Rs. $876x \times 1/100$ = Rs. $8.76\,x$

total annual charges = Rs. $(110 + 8.76)x$

total cost/unit = Rs. $(110 + 8.76)\,x / 876\,x$ = **13.5 p**

(*b*) Steam Station

capital cost = Rs. $1200\,x$

annual fixed charges = 5% of Rs. $1200\,x$ = Rs. $60\,x$

annual running charges = Rs. $876\,x \times 5/100$ = Rs. $43.8\,x$

total annual charges = Rs. $(60 + 43.8)x$

overall cost/unit = Rs. $103.8\,x/8.76\,x$ = **11.85 p**

Obviously, steam station is more economical to operate.

Case II. Load factor = 50%

Total units generated/annum = $x \times 8760 \times 0.5 = 4380\,x$

(*a*) Hydro Station

If we proceed as above, we find that total cost/unit = **3.5 p**

(*b*) Steam Station

total cost/unit = **6.35 p**

Obviously, in this case, hydro-station is more economical.

Example 50.30. *The annual working cost of a thermal station can be represented by a formula Rs. (a + b kW + c kWh) where a, b and c are constants for a particular station, kW is the total installed capactiy and kWh the energy produced per annum. Explain the significance of the constants a, b and c and the factors on which their values depend.*

Determine the values of a, b and c for a 60 MW station operating with annual load factor of 40% for which :

(i) *capital cost of buildings and equipment is Rs.* 5×10^5

(ii) *the annual cost of fuel, oil, taxation and wages and salaries of operating staff is Rs.90,000*

(iii) *the interest and depreciation on buildings and equipment are 10% per annum*

(iv) *annual cost of organisation and interest on cost of site etc. is Rs. 50,000.*

Solution. Here, *a* represents fixed charge due to the annual cost of the organisation, interest on the capital investment on land or site etc.

The constant *b* represents semi-fixed cost. The constant *b* is such that when multiplied by the maximum kW demand on the station, it equals the annual interest and depreciation on the capital cost of the buildings equipment and the salary of the charge engineer.

Constant *c* repesents running cost and its value is such that when multiplied by the annual total kWh output of the station, it equals the annual cost of the fuel, oil, taxation, wages and salaries of the operating staff.



An overview of a thermal power plant near Tokyo, Japan

(i) Here, $a = \mathbf{50{,}000}$

(ii) $b \times$ max. kW demand = annual interest on the capital cost of the buildings and equipment etc. $= 0.1 \times 5 \times 10^5$

$\therefore \quad b \times 60 \times 10^3 = 0.5 \times 10^5$ or $b = \mathbf{0.834}$

(iii) annual average power $= 0.5 \times 60 \times 10^3 = 30 \times 10^3$ kW

Units produced annually $= 30 \times 10^3 \times 8{,}760 = 262.8 \times 10^6$ kWh

$\therefore \quad c \times 262.8 \times 10^6 = 90{,}000$; $c = \mathbf{0.0034}$

50.17. Tariffs

The size and cost of installations in a generating station is determined by the maximum demand made by the different consumers on the station. Each consumer expects his maximum demand to be met at any time of the day or night. For example, he may close down his workshop or house for a month or so but on his return he expects to be able to switch on his light, motor and other equipment without any previous warning to the supply company. Since electric energy, unlike gas or water cannot be stored, but must be produced as and when required, hence the generating equipment has to be held in 'readiness' to meet every consumer's full requirement at all hours of the day.

This virtually amounts to allocating a certain portion of the generating plant and the associated distribution system to each consumer for his individual use. Hence, it is only fair that a consumer should pay the fixed charges on that portion of the plant that can be assumed to have been exclusively allocated to him plus the charges proportional to the units actually used by him.

Hence, any method of charging or tariff, in the fairness to the supply company, should take into account the two costs of producing the electric energy **(*i*)** fixed or standing cost proportional to the maximum demand and **(*ii*)** running cost proportional to the energy used. Such two-part tariffs are in common use. Some of the different ways of rate making are described below :

50.18. Flat Rate

This was the earliest type of tariff though it is not much used these days because, strictly speaking, it is not based on the considerations discussed above. In this system, charge is made at a simple flat rate per unit. But the lighting loads and power loads are metered separately and charged at different rates. Since the lighting load has a poor load factor *i.e.* the number of units sold is small in relation to the installed capacity of the generating plant, the fixed cost per kWh generated is high and this is taken into account by making the price per unit comparatively high. But since the power load is more predictable and has a high load factor, the cost per kWh generated is much lower which results in low rate per unit.

50.19. Sliding Scale

In this type of tariff, the fixed costs are collected by charging the first block of units at a higher rate and then reducing the rates, usually in many steps, for units in excess of this quantity.

50.20. Two-part Tariff

This tariff is based on the principles laid down in Art. 50.16. It consists of two parts **(*i*)** a fixed charge proportional to the maximum demand (but independent of the units used) and **(*ii*)** a low running charge proportional to the actual number of units used.

The maximum demand during a specified period, usually a quarter, is measured by a maximum demand indicator. The maximum demand indicator is usually a watt-hour meter which returns to zero automatically at the end of every half hour but is fitted with a tell-tale pointer which is left behind at the maximum reduing reached during the quarter under consideration.

This type of tariff is expressed by a first degree equation like Rs. $A \times$ kW + $B \times$ kWh where Rs. A is the charge per annum per kW of maximum demand and B is the price per kWh.

Sometimes, the customer is penalized for his poor load power factor by basing the fixed charges on kVA instead of per kW of maximum demand.

Example 50.31. *Compute the cost of electrical energy and average cost for consuming 375 kWh under 'block rate tariff' as under :*

First 50 kWh at 60 paisa per kWh ; next 50 kWh at 50 paisa per kWh; next 50 kWh at 40 paisa per kWh; next 50 kWh at 30 paisa per kWh.

Excess over 200 kWh at 25 paisa per kWh. **(Utilisation of Elect. Power, AMIE Sec. B)**

Solution. Energy charge for the first 50 kWh is = Rs. 0.6×50 = Rs. 30

Energy charge for the next 50 kWh at 50 paisa / kWh = Rs. 0.5×50 = Rs. 25

Energy charge for the next 50 kWh at 40 paisa / kWh = Rs. 0.4×50 = Rs. 20

Energy charge for the next 50 kWh at 30 paisa / kWh = Rs. 0.3×50 = Rs. 15

Energy charge for the rest (375 – 200) *i.e.* 175 kWh =Rs. 0.25×175 = Rs. 43.75

Total cost of energy for 375 kWh = Rs. (30 + 25 + 20 + 15 + 43.75) = **Rs. 133.75**

Average cost of electrical energy/kWh = Rs. 133.75 / 375 = **36 paise.**

Example 50.32. *The output of a generating station is 390×10^6 units per annum and installed capacity is 80,000 kW. If the annual fixed charges are Rs. 18 per kW of isntalled plant and running charges are 5 paisa per kWh, what is the cost per unit at the generating station ?*

(Eletrical Technology, Bombay Univ.)

Solution. Annual fixed charges = Rs. $18 \times 80{,}000$ = Rs. 1.44×10^6

Fixed charges/kWh = Rs. $1.44 \times 10^6 / 390 \times 10^6$ = 0.37 paisa

Running charges/kWh = 5 paisa

Hence, cost at the generating station is = 5 + 0.37 = **5.37 paise/kWh.**

Example 50.33. *A power station has an installed capacity of 20 MW. The capital cost of station is Rs. 800 per kW. The fixed costs are 30% of the cost of investment. On full-load at 100% load factor, the variable costs of the station per year are 1.5 times the fixed cost. Assume no reserve capacity and variable cost to be proportional to the energy produced, find the cost of generation per kWh at load factors of 100% and 20%. Comment on the results.* **(Ranchi University)**

Solution. Capital cost of the station = Rs. $800 \times 20{,}000$ = Rs. 16×10^6

(*a*) At 100% load factor

Fixed cost = Rs. $16 \times 10^6 \times 13/100$ = Rs. 2.08×10^6

Variable cost = Rs. $1.5 \times 2.08 \times 10^6$ = Rs. 3.12×10^6

Total operating cost per annum = Rs. $(2.08 + 3.12) \times 10^6$ = Rs. 5.2×10^6

Total No. of units generated = kW max. demand $\times$ LF $\times$ 8760 = $20{,}000 \times 1 \times 8760$

= 175.2×10^6 kWh

Cost of generation per kWh = $5.2 \times 10^6 \times 100/175.2 \times 10^6$ = **3.25 paise.**

(*b*) At 20% load factor :

Fixed cost = Rs. 2.08×10^6 as before

Total units generated per annum = $20{,}000 \times 0.2 \times 8760 = 35.04 \times 10^6$ kWh

Since variable cost is proportional generated, the variable = Rs. $3.12 \times 10^6 \times 35.04 \times 10^6 / 175.2 \times 10^6$ to kWh

cost at load of 20% is = Rs. 0.624×10^6

Total operating cost = Rs. $(2.08 + 0.624) \times 10^6 = 2.704 \times 10^6$

Cost of generation per kWh = $2.704 \times 10^6 \times 100/35.04 \times 10^6$ = **7.7 paise**

It is obvious from the above calculations that as the station load factor is reduced, the cost of electric generation is increased.

Example 50.34. *The annual output of a generating sub-station is 525.6×10^6 kWh and the average load factor is 60%. If annual fixed charges are Rs. 20 per kW installed plant and the annual running charges are 1 paisa per kWh, what would be the cost per kWh at the bus bars ?*

Solution. Average power supplied per annum = $525.6 \times 10^6 / 8760$ = 60,000 kW

Max. demand = $\frac{\text{average power}}{\text{load factor}} = \frac{60{,}000}{0.6}$ = 100,000 kW*

Annual fixed charges = Rs. $20 \times 100{,}000$ = Rs. 2×10^6

Fixed charges / kWh = $\frac{2 \times 10^6}{525.6 \times 10^6} = \frac{2 \times 100}{525.6}$ = 0.38 paisa

Annual running charges per kWh = 1 paisa. Hence, cost per kWh at the bus-bars = **1.38 paisa.**

* It has been assumed that the installed capacity is equal to the maximum demand of 100,000 kW.

Example 50.35. *A certain factory working 24 hours a day is charged at Rs. 10 per kVA of max. demand plus 5 paisa per kV ARh. The meters record for a month of 30 days; 135,200 kWh, 180,020 kV ARh and maximum demand 310 kW. Calculate*

(i) M.D. charges, (ii) monthly bill, (iii) load factor, (iv) average power factor.

(Electric Engineering-II, Bangalore Univ.)

Solution. Average deamnd = $135,200/24 \times 30 = 188$ kW

Average reactive power = $180,020/24 \times 30 = 250$ kVAR

Now, $\tan \phi$ = kVAR/kW = 250/88 ; $\phi = 53°$; $\cos \phi = 0.6$;

M.D. kVA = $310/0.6 = 517$

(i) M.D. charge = Rs. 10×517 = **Rs. 5170**

(ii) reactive energy charges = Rs. $5 \times 180,020/100$ = Rs. 9001

$\therefore$ monthly bill = **Rs. 14,171**

(iii) monthly load factor = 188/310 = 0.606 or 60.6% (*iv*) average p.f. = **0.6**

Example 50.36. *The cost data of a power supply company is as follows :*

Station maximum demand = 50 MW ; station load factor = 60% ; Reserve capacity = 20% ; capital cost = Rs. 2,000 per kW; interest and depreciation = 12% ; salaries (annual) = Rs. 5×10^5; fuel cost (annual) = Rs. 5×10^6 ; maintenance and repairs (annual) Rs. 2×10^5 ; losses in distribution = 8% ; load diversity factor = 1.7.

Calculate the average cost per unit and the two-part tariff, assuming 80 per cent of salaries and repair and maintenace cost to be fixed. **(Electrical Power-I, Bombay Univ.)**

Solution. Station capacity = 50 + 20% of 50 = 60 MW

Average power = $60 \times 0.6 = 36$ MW = 36×10^3 kW

Capital investment = Rs. $60 \times 10^3 \times 2000$ = Rs. 12×10^7

Interest + depreciation = Rs. $0.12 \times 12 \times 10^7$ = Rs. 14.4×10^6

Total cost both fixed and running

= Rs. 14.4×10^6 + Rs. 5×10^5 + Rs. 5×10^6 + Rs. 2×10^5 = Rs. 201×10^5

No. of units generated annually = $8760 \times 36 \times 10^3$

$\therefore$ overall cost/unit = Rs. $\dfrac{201 \times 10^5}{8760 \times 36 \times 10^3}$ = **6.37 paise**

Fixed charges

Annual interest and depreciation = Rs. 14.4×10^6

80% of salaries = Rs. $0.8 \times 5 \times 10^5$ = Rs. 4×10^5

80% of repair and maintenance cost = Rs. $0.8 \times 2 \times 10^5$ = Rs. 1.6×10^5

Total fixed charges = Rs. 149.6×10^5

Aggregate maximum demand of all consumers

= Max. demand on generating station $\times$ diversity factor

= $60 \times 10^3 \times 1.7 = 10^2 \times 10^3$ kW

$\therefore$ annual cost/kW of maximum demand = Rs. $149.6 \times 10^5 / 10^2 \times 10^3$ = **Rs. 149.6**

Running Charges

Cost of fuel = Rs. 50×10^5 ; 20% of salaries = Rs. 1×10^5

20% of maintenance = 0.4×10^5 ;

Total = Rs. 51.4×10^5

Considering distribution loss of 8%, cost per unit delivered to the consumer is

$$= \text{Rs. } 51.4 \times 105/0.92 \times 8760 \times 36 \times 10^3 = \textbf{1.77 paise}$$

Hence, two-part tariff is : **Rs. 149.6 per kW max.** demand plus **1.77 paisa per kWh** consumed.

Example 50.37. *A certain electric supply undertaking having a maximum demand of 110 MW generates 400×10^6 kWh per year. The supply undertaking supplies power to consumers having an aggregate demand of 170 MW. The annual expenses including capital charges are :*

Fuel = *Rs.* 5×10^6

Fixed expenses connected with generation = *Rs.* 7×10^6

Transmission and distribution expenses = *Rs.* 8×10^6

Determine a two-part tariff for the consumers on the basis of actual cost.

Assume 90% of the fuel cost as variable charges and transmission and distrbution losses as 15% of energy generated. **(Electrical Power-I. Bombay Univ.)**

Solution. Total fixed charges per annum are as under :

Fixed charges for generation = Rs. 7×10^6

Transmission and distribution expenses = Rs. 8×10^6

10% of annual fuel cost = Rs. 0.5×10^6 Total = Rs. 15.5×10^6

This cost has to be spread over the aggregate maximum demand of all consumers *i.e.* 170 MW.

$\therefore$ cost per kW of maximum deamnd = Rs. $15.5 \times 10^6 / 170 \times 10^3$ = **Rs. 91.2**

Running charges = 90% of fuel cost = Rs. 4.5×10^6

These charges have to be spread over the number of kWh acutally delivered.

No. of units delivered = 85% of No. of units generated

$= 0.85 \times 400 \times 10^6 = 340 \times 10^6$

$\therefore$ running cost / kWh = Rs. $4.5 \times 10^6 / 340 \times 10^6$ = **1.32 paise**

Hence, the two-part tariff for the consumer would be **Rs. 91.2 per kW** of maximum demand and **1.32 paisa per kWh** consumed.

Example 50.38. *A customer is offered power at Rs. 80 per annum per kVA of maximum demand plus 8 paisa per unit metered. He proposes to install a motor to carry his estimated maximum demand of 300 b.h.p. (223.8 kW). The motor available has a power factor of 0.85 at full-load. How many units will he require at 20% load factor and what willl be his annual bill ?*

(Electric Power II Punjab Univ. 1992)

Solution. Assuming a motor efficiency of 90%, the full-load power intake of motor = 223.8/0.9 = 746/3 kW. This represents the max. demand.

$$\text{Now, load factor} = \frac{\text{average}}{\text{max. demand}}$$

$\therefore$ average power = max. demand $\times$ load factor = $(746/3) \times 0.2$ = **149.2/3 kW**

$\therefore$ annual consumption = $149.2 \times 8{,}760/3 = 435{,}700$ kWh

Max. kVA of demand = $300 \times 746/0.9 \times 0.85 \times 1000 = 292.5$

$\therefore$ cost per kVA of maximum demand = 292.5×80 = Rs. 23,400

Cost of units consumed/annum = $435{,}700 \times 8/100$ = Rs. 34,856

$\therefore$ annual bill = Rs. 23,400 + Rs. 34,856 = **Rs. 58,256**

Example 50.39. *How two-part tariff is modified for penalising low p.f. consumers ?*

An industry consumes 4 million kWh/year with a maximum demand of 1000 kW at 0.8 p.f. What is its load factor ?

(a) *Calculate the annual energy charges if tariff in force is as under :*
Max. demand charge = Rs. 5 per kVA per month. Energy charges = Rs. 0.35 per kWh
(b) *Also calculate reduction in this bill if the maximum demand is reduced to 900 kW at 0.9 p.f. lagging.* **(Electrotechnics, Gujarat Univ.)**

Solution. Load factor $= 4 \times 10^6/1000 \times 8760 = 0.4566$ or 45.66%

(a) Max. kVA demand $= 1000/0.8 = 1250$

Annual M.D. charge/kVA $= 5 \times 12 =$ Rs. 60

M.D. charge for 1250 kVA $=$ Rs. $60 \times 1250 =$ Rs. 75×10^3

Annual energy charge $=$ Rs. $4 \times 10^6 \times 0.35 =$ Rs. 1400×10^3

Annual energy bill $=$ **Rs. 1475×10^3**

(b) Since load factor remains the same, annual average power $= 900 \times 0.4566$ kW

Units consumed annually $= 8760 \times 900 \times 0.4566 = 3.6 \times 10^6$

Max. kVA demand = 900/0.9 = 1000, M.D. charge for 1000 kVA = Rs. $60 \times 1000 =$ Rs. 60×10^3

Energy charge = Rs. $3.6 \times 10^6 \times 0.35 =$ Rs. 1260×10^3; Annual bill = Rs.$1{,}320 \times 10^3$

Annual saving $=$ Rs. $(1475 - 1320) \times 10^3 =$ **Rs. 155×10^3**

Example 50.40. *A supply is offered on the basis of fixed charges of Rs. 30 per annum plus 3 paise per unit or alternatively, at the rate of 6 paisa per unit for the first 400 units per annum and 5 paise per unit for all the additional units. Find the number of units taken per annum for which the cost under these two tariffs becomes the same.* **(Electrical Technology-I, Bombay Univ.)**

Solution. Let x kWh be the annual consumption of the consumer for which the two tariffs are equally advantageous.

Cost according to the first tariff $=$ Rs. $30 + 3x/100$

Charges according to the alternative tariff are

$=$ Rs. $6 \times 400/100 +$ Rs. $(x - 400) \times 5/100 =$ Rs. $[24 + (x - 400)/20]$

Since charges in both cases are to be equal

$$\therefore \quad 30 + \frac{3x}{100} = 24 + \frac{(x - 400)}{20}$$

or $x =$ **1300 kWh.**

Example. 50.41. *If power is charged for at the rate of Rs. 75 per kVA of maximum demand and 4 paisa per unit, what is the cost per unit at 25% yearly load factor* ***(a)*** *for unity power factor demand and* ***(b)*** *for 0.7 power factor demand.*

Solution. (*a*) At 25% load factor and unity power factor

$$\text{Maximum demand charge } \textit{per unit} = \frac{75 \times 100}{8760 \times 0.25} = 3.43 \text{ paise}$$

Energy charge per unit = 4 paisa ; Cost *per unit* = 4 + 3.43 = **7.43 paise**

(*b*) At 25% load factor and 0.7 power factor

$$\text{Maximum demand charge } \textit{per unit} = \frac{75 \times 100}{0.7 \times 0.25 \times 8760} = \textbf{4.9 paise}$$

Energy charge per unit = 4 paise ; Cost per unit = 4 + 4.9 = **8.9 paise**

Exmaple 50.42. *Explain different methods of tariff. A tariff in force is Rs. 50 per kVA of max. demand per year plus 10 p per kWh. A consumer has a max. demand of 10 kW with a load factor of 60% and p.f. 0.8 lag.*

(i) Calculate saving in his annual bill if he improves p.f. to 0.9 lag.

(ii) Show the effect of improving load factor to 80% with the same max. demand and p.f. 0.8 lag on the total cost per kWh. **(Electrical Engineering-III, Poona Univ.)**

Solution. (*i*) Max. kVA demand at 0.8 p.f. = 10/0.8 = 12.5

M.D. charges = Rs. 50 × 12.5 = Rs. 625

Max. kVA demand at 0.9 p.f. = 10/0.9 ; M.D. charge = Rs. 50 × 10/0.9 = Rs. 555.55

Since energy consumed remains constant, saving is due only to reduction in M.D. charges.

∴ saving = Rs. (625 – 555.55) = **Rs. 69.45**

(*ii*) With 60% load factor, average power = 10 × 0.6 = 6 kW. The number of units consumed annually = 6 × 8760 = 52,560 kWh. Also, annual M.D. charge = Rs. 50 × 12.5 = Rs. 625

M.D. charge/unit consumed = 625 × 100/52,560 = 1.19 paise

Total cost per unit = 10 + 1.19 = **11.19 paise**

With a laod p.f. of 80% average power = 10 × 0.8 = 8 kW

Number of units consumed annually = 8 × 8,760 = 70,080 kWh. The M.D. charge as before = Rs. 625.

∴ M.D. charge/unit consumed = 625 × 100/70,080 = 0.89 paise

∴ total cost/unit consumed = 10 + 0.89 = **10.89 paise**

Obviously, with improvement in load factor, total cost per unit is reduced.

Example 50.43. *A consumer has a maximum demand (M.D.) of 20 kW at 0.8 p.f. lagging and an annual load factor of 60%. There are two alternative tariffs (i) Rs. 200 per kVA of M.D. plus 3p per kWh consumed and (ii) Rs. 50 per kVA of M.D. plus 7p per kWh consumed. Determine which of the tariffs will be economical for him.* **(Electrical Engineering, Banaras Hindu Univ.)**

Solution. An M.D. of 20 kW at 0.8 p.f. = 20/0.8 = 25 kVA

Average power = M.D. × load factor = 20 × 0.6 = 12 kW

Energy consumed/year = 12 × 8760 = 105120 kWh

(*a*) M.D. charge = Rs. (200 × 25) = Rs. 5000

Energy charge = Rs. 3 × 105, 120/100 = Rs. 3153.60

Annual charges = Rs. 5000 + Rs. 3153.60 = **Rs. 8153.60**

(*b*) M.D. charge = Rs. (50 × 25) = Rs. 1250

Energy charge = Rs. $\frac{7 \times 105120}{100}$ = Rs. 7358.40

Annual charge = Rs. 1250 + Rs. 7358.40 = **Rs. 8608.40**

Obviously, tariff (*a*) is economical.

Example 50.44. *Determine the load factor at which the cost of supplying a unit of electricity from a Diesel station and from a steam station is the same if the respective annual fixed and running charges are as follows.*

Station	*Fixed charges*	*Running charges*
Diesel	*Rs. 300 per kW*	*25 paise/kWh*
Steam	*Rs. 1200 per kW*	*6.25 paise/kWh*

(Electrical Power-I, Gujarat Univ.)

Solution. (*i*) Diesel Station

Suppose that the energy supplied in one year is one unit *i.e.* one kWh.

∴ annual average power = 1 kWh/8760 *h* = 1/8760 kW

Now, annual load factor (L) = $\frac{\text{annual average power}}{\text{annual max. demand}}$

$\therefore$ max. demand = $\frac{\text{average power}}{L} = \frac{1}{8760\,L}$ kW

$\therefore$ fixed charge = Rs. $300 \times 1/8760\,L$ = $30{,}000 / 8760\,L$ paise

Running charge = $1 \times 25 = 25$ paise

Hence, fixed and running charges per kWh supplied by Diesel station = $\left(\frac{30{,}000}{8760\,L}\right)$ paise

(*ii*) Steam Station

In the same way, fixed charge = Rs. $1200/8760\,L = 120{,}000/8760\,L$ paise

Running charge = 6.25 paise

$\therefore$ fixed plus running charges for supplying one kWh of energy are

= $(120{,}000/8760\,L) + 6.25$ paise.

Since the two charges have to be the same

$\therefore$ $\frac{120{,}000}{8760\,L} + 6.25 = \frac{30{,}000}{8760\,L} + 25$; L = **0.55 or 55%.**

Example 50.45. *A factory has a maximum load of 300 kW at 0.72 p.f. with an annual consumption of 40,000 units, the tariff is Rs. 4.50 per kVA of maximum demand plus 2 paisa/unit. Find out the average price per unit. What will be the annual saving if the power factor be improved to units ?*

(Electrical Technology-I, Bombay Univ.)

Sulotion. kVA load at 0.72 p.f. = 300/0.72 = 1250/3

$\therefore$ max. kVA demand charge = Rs. $4.5 \times 1250/3$ = Rs. 1875

$\therefore$ M.D. charge per unit = Rs. 1875 / 40,000 = 4.69 paise

$\therefore$ total charge per unit = 2 + 4.69 = **6.69 paise**

Max. kVA demand at unity p.f. = 300

M.D. charge per unit = Rs. 300×4.5 = Rs. 1350

Annual saving = Rs. 1875 – Rs. 1350 = **Rs. 525.**

Example 50.46. *There is a choice of two lamps, one costs Rs. 1.2 and takes 100 W and the other costs Rs. 5.0 and takes 30 W ; each gives the same candle power and has the same useful life of 1000 hours. Which will prove more economical with electrical energy at Rs. 60 per annum per kW of maximum demand plus 3 paise per unit ? At what load factor would they be equally advantageous?*

Solution. (*i*) First Lamp

Initial cost of the first lamp per hour = Rs. 120/1000 = 0.12 paisa

Max. demand / hr. = 100/1000 = 0.1 kW

Max. demand charge/hr. = $(60 \times 100) \times 0.1/8760$ = 0.069 paisa

Energy charge/hr. = 3×0.1 = 0.3 paisa

$\therefore$ Total cost/hr. = 0.12 + 0.069 + 0.3 = **0.489 paisa**

(*ii*) Second Lamp

Initial cost/hr. = 500/1000 = 0.5 paisa ; Max. demand /hr ; = 30/1000 = 0.03 kW

Max. demand charge/hr. = $(60 \times 100) \times 0.03 / 8760$ = 0.02 paisa

Energy charge/hr. = 3×0.03 = 0.09 paisa $\therefore$ total cost/hr. = 0.5 + 0.02 + 0.09 = **0.61 paisa**

Hence, the first lamp would be more economical.

It would be seen that the only charge which will vary with load factor is the max. demand charge. Moreover the maximum demand charge varies inversely as the load factor. Let x be the load factor at which both lamps are equally advantageous. Then

$$0.12 + \frac{0.069}{x} + 0.3 = 0.05 + \frac{0.02}{x} + 0.09 \quad \therefore \quad x = \mathbf{0.29}$$

Hence, both lamps would be equally advantageous at **29%** load factor.

Example 50.47. *The following data refers to a public undertaking which supplies electric energy to its consumers at a fixed tariff of 11.37 paise per unit.*

Total installed capacity = 344 MVA ; Total capital investment = Rs. 22.4 crores;

Annual recurring expenses = Rs. 9.4 crores ; Interest charge = 6% ; depreciation charge = 5%

Estimate the annual load factor at which the system should operate so that there is neither profit nor loss to the undertaking. Assume distribution losses at 7.84% and the average system p.f. at 0.86.

(Electrical Power-I, Bombay Univ.)

Solution. Yearly load factor (L) $= \dfrac{\text{No. of kWh supplied in a year}}{\text{Max. No. of kWh which can be supplied}}$ *

$$= \frac{\text{kWh supplied/year}}{\text{Max. output in kW} \times 8760}$$

$\therefore$ kWh supplied/year $= \text{Max. output in kW} \times 8760 \times L$

$= (344{,}000 \times 0.86) \times 8760 \times L = L \times 25.92 \times 10^8$

Considering distribution losses of 7.84%, the units actually supplied

$= 92.16$ per cent of $(L \times 25.92 \times 10^8) = L \times 23.89 \times 10^8$

Amount collected @ of 11.37 paise / kWh = Rs. $L \times 23.89 \times 10^8 \times 11.37 / 100$

If there is to be no profit or gain, then this amount must just equal the fixed and running charges.

Annual interest and depreciation on capital investment

$= 11\%$ of Rs. $22.4 \times 10^7 =$ Rs. 2.46×10^7

Total annual expenses $=$ Rs. $2.46 \times 10^7 +$ Rs. $9.4 \times 10^7 =$ Rs. 11.86×10^7

$\therefore \quad L \times 23.89 \times 10^8 \times 11.37 / 100 = 11.86 \times 10^7$; $L =$ **0.437 or 4.37%**

Example 50.48. *An area has a M.D. of 250 MW and a load factor of 45%. Calculate the overall cost per unit generated by **(i)** steam power station with 30 per cent reserve generating capacity and **(ii)** nuclear station with no reserve capacity.*

Steam station : Capital cost per kW = Rs. 1000 ; interest and depreciation on capital costs = 15% ; operating cost per unit = 5 paise.

Nucelar station : capital cost per kW = Rs. 2000 ; interest and depreciation on capital cost = 12% ; operating cost per unit = 2 paise.

For which load factor will the overall cost in the two cases become equal ?

(Electrical Power-I, Bombay Univ.)

Solution. (*i*) Steam Station

Taking into consideration the reserve generating capacity, the installed capacity of the steam station $= 250 + (30\% \text{ of } 250) = 325 \text{ MW} = 325 \times 10^3$ kW

average power (annual) $= \text{M.D.} \times \text{load factor} = 325 \times 10^3 \times 0.45 = 146{,}250$ kW

$\therefore$ No. of units produced/year $= 8760 \times 146{,}250 = 128 \times 10^7$

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* In this case max. demand has been taken as equal to the installed capacity.

Capital investment = Rs. $325 \times 10^3 \times 1000$ = Rs. 325×10^6

Annual interest and depreciation = Rs. $325 \times 10^6 \times 0.15$ = Rs. 48.75×10^6

These fixed charges have to be spread over the total number of units produced by the station.

$\therefore$ fixed charges / unit = Rs. $48.75 \times 10^6 / 128 \times 10^7$ = 3.8 paise

$\therefore$ overall cost per unit generated = 3.8 + 5 = **8.8 paise**

(*ii*) Nuclear Station

Since there is no reserve capacity, installed capacity of the station equals the maximum demand of 250 MW = 25×10^4 kW.

Average power = $0.45 \times 25 \times 10^4$ = 112,500 kW

Units produced annually = $8760 \times 112{,}500 = 98.3 \times 10^7$

Capital investment = Rs. $2000 \times 250 \times 10^3$ = Rs. 5×10^8

Annual interest and depreciation = Rs. $0.12 \times 5 \times 10^8$ = Rs. 6×10^7

$\therefore$ fixed charges / unit = Rs. $6 \times 10^7 / 98.3 \times 10^7$ = 6.1 paise

Overall cost per unit generated = 6.1 + 2.2 = **8.3 paise**

Load Factor

Suppose L is the load factor at which the overall cost per unit generated is the same.

$$\text{Cost / unit for steam station} = \left(\frac{48.75\times10^8}{325\times10^3\times8760\,L}+5\right)=\left(\frac{1.712}{L}+5\right)\text{paise.}$$

Similarly, overall cost/unit for nuclear station is

$$=\left(\frac{6\times10^7\times100}{250\times10^3\times8760\,L}+2\right)=\left(\frac{2.74}{L}+2\right)\text{paise}$$

Equating the two, we get $[(1.712/L) + 5] = [2.74/L) + 2]$ $\therefore$ L = **0.34 or 34%**

Example 50.49. *The maximum demand of a customer is 25 amperes at 220 volt and his total energy consumption is 9750 kWh. If the energy is charged at the rate of 20 paise per kWh for 500 hours' use of the maximum demand plus 5 paise power unit for all additional units, estimate his annual bill and the equivalent flat rate.*

Solution. Charge at the max. demand rate.

Max. demand = $25 \times 220/1{,}000$ = 5.5 kW

Units consumed at maximum demand rate = 5.5×500 = 2,750 kWh

Max. demand charge = $2{,}750 \times 20/100$ = **Rs. 550**

Energy Charge

Units to be charged at lower rate = 9,750 – 2,750 = 7,000

$$\text{Charge} = \frac{7{,}000\times5}{100} = \text{Rs. } 350$$

$\therefore$ annual bill = Rs. 550 + 350 = **Rs. 900**

Equivalent flat rate = $900 \times 100/9{,}750$ = **9.2 paise**

Example 50.50. *A workshop having a number of induction motors has a maximum demand of 750 kW with a power factor of 0.75 and a load factor of 35%. If the tariff is Rs. 75 per annum per kVA of maximum demand plus 3 paise per unit, estimate what expenditure would it pay to incur to raise the power factor of 0.9.*

Solution. Max. kVA demand at 0.75 p.f. = 750/0.75 = 1000

M.D. charge = 75×1000 = Rs. 75,000 ; Max. kVA demand at 0.9 p.f. = 750/0.9

M.D. charge = (7,500/9) × 75 = Rs. 62,500

Difference in one year = 75,000 – 62,500 = **Rs. 12,500**

If the annual interest on money borrowed for purchasing the p.f. improvement plant is assumed 10%, then an expenditure of Rs. 125,000 is justified.

Example 50.51. *The owner of a new factory is comparing a private oil-engine generating station with public supply. Calculate the average price per unit his supply would cost him in each case, using the following data :*

Max. demand, 600 kW; load factor, 30% ; supply tariff, Rs. 70 per kW of maximum demand plus 3 paise per unit; capital cost of plant required for public supply, Rs. 10^5; capital cost of plant required for private generating station, Rs. 4×10^5 ; cost of fuel, Rs. 80 per tonne ; consumption of fuel oil; 0.3 kg per unit generated. Other work costs for private plant are as follows : lubricating oil, stores and water = 0.35 paise per unit generated ; wages 1.1 paise ; repairs and maintenance 0.3 paise per unit.

Solution. Public Supply Charges

Running charge/unit = 3.0 paise ; Average power consumption = 600 × 0.3 = 180 kW

Annual energy consumption = 180 × 8,760 kWh

Fixed annual charges = Rs. 70 × 600 = Rs. 42,000

Let us assume a capital charge rate of 10%. Hence, further annual amount to be charged from the customer is 0.1×10^5 = Rs. 10,000

∴ total fixed charges/annum = Rs. 42,000 + Rs. 10,000 = Rs. 52,000

∴ fixed charge per unit generated = $\frac{52,000 \times 100}{180 \times 8760}$ = 3.3 paise

∴ fixed plus running charges per unit generated = 3.0 + 3.3 = **6.3 paise.**

Private Supply Charges

Annual capital charges = Rs. $4 \times 10^5 \times 0.1$=Rs. 40,000

No. of units generated annually = 180 × 8,760 kWh

∴ fixed charges per unit generated = $\frac{4,000 \times 100}{180 \times 8760}$ = 2.54 paise

Cost of oil per unit generated = $\frac{80 \times 100 \times 0.3}{1,000}$ = 2.4 paise

Running charges per unit generated are :

Lubricating oil, stores, water = 0.35 paise; Wages = 1.1 paise

Repairs and maintenance = 0.3 paisa; Total = 0.35 + 1.1 + 0.3 = 1.75 paise

Total running charges = 2.4 + 1.75 = **4.15 paise**

Fixed plus running charges per unit generated = 2.54 + 4.15 = **6.69 paise**

Example 50.52. *Calculate the minimum two-part tariff to be charged to the consumers of a supply undertaking from the following data :*

Generating cost per kWh; 3.6 paise ; Generating cost per kW of maximum demand, Rs. 50

Total energy generated per year ; $4,380 \times 10^4$ kWh

Load factor at the generating station, 50%

Annual charges for distribution Rs. 125,000

Diversity factor for the distribution network, 1.25

Total loss between station and consumer, 10%.

Solution. Average generating power = $4{,}380 \times 10^4/8760 = 5{,}000$ kW

$\therefore$ maximum load on generator = 5,000/load factor = 5,000 / 0.5 = 10,000 kW

$\therefore$ annual fixed charges = Rs. 10,000 × 50 = Rs. 500,000

Fixed charges = Rs. 125,000 Total fixed charges = Rs. 625,000

Since diversity factor is known, consumer's max. demand = 10,000 × 1.25 = 12,500 kW

Hence, Rs. 625,000 have to be equally distributed over the 12,500 kW maximum demand.

$\therefore$ cost per kW max, demand = 625,000/12,500 = Rs. 50

$\therefore$ monthly kW maximum charges is = Rs. 50/12 = Rs. 4.17

Since losses are 10%, consumer gets 0.9 kWh for every kWh generated at the station. Hence cost per kWh to the consumer is 3.6/0.9 = 4 paise.

Therefore, minimum charges are **Rs. 4.17 per kW** of max. demand per month and **4 paise per kWh** consumed.

Example 50.53. *Two systems of tariffs are available for a factory working 8 hours a day for 300 working days in a year.*

(a) *High-voltage supply at 5 paise per unit plus Rs. 4.50 per month per kVA of maximum demand.*

(b) *Low-voltage supply at Rs. 5 per month per kVA of maximum demand plus 5.5 paise per unit.*

The factory has an average load of 200 kW at 0.8 power factor and a maximum demand of 250 kW at the same p.f.

The high-voltage equipment costs Rs. 50 per kVA and losses can be taken as 4 per cent. Interest and depreciation charges are 12 per cent. Calculate the difference in the annual cost between the two systems.

Solution. First, let us find the annual cost according to rate (*a*).

(a) Capacity of h.v. switchgear = (250/0.8) × (100/96) kVA

Annual interest on capital investment and depreciation

$$= \left(\frac{250}{0.5} \times \frac{100}{96}\right) \times 50 \times 0.12 = \text{Rs. } 1953$$

Annual charge due to kVA max. demand is

$$= \frac{250}{0.5} \times \frac{100}{96} \times 12 \times 4.5 = \text{Rs. } 17{,}580$$

Annual charge due to kWh consumption

$$= 200 \times \frac{100}{96} \times \frac{5}{100} \times (8 \times 300) = \text{Rs. } 25{,}000$$

$\therefore$ total charges = Rs. (1953 + 17,580 + 25,000) = **Rs. 44,533**

(b) The total annual cost due to rate ***(b)*** would be as under.

Annual charge due to kVA max. demand is = 250 × 5 × 12/0.8 = Rs. 18,750

Annual charge due to kWh consumption is $= 200 \times \frac{5.5}{100} \times 2{,}400 = \text{Rs. } 26{,}400$

$\therefore$ total charges = Rs. 18,750 + Rs. 26,400 = Rs. 45,150

Hence, high-voltage supply is cheaper by 45,150 – 44,533 = **Rs. 617.**

Example 50.54. *Estimate what the consumption must be in order to justify the following maximum demand tariff in preference to the flat rate if the maximum demand is 6 kW.*

On Maximum Demand Tariff. A max. demand rate of 37 paise per unit for the first 200 hr. at the maximum demand rate plus 3 paisa for all units in excess.

Flat-rate tariff, 20 paise per unit.

Solution. Let x = the number of units to be consumed (within a specific period)

On Max. Demand Tariff

Units consumed at max. demand rate = $6 \times 200 = 1200$ kWh

Units in excess of the max. demand units = $(x - 1200)$

Cost of max. demand units = $1200 \times 37 = 44{,}400$ paise

Cost of excess units = $3(x - 1200) = (3x - 3600)$ paise

$\therefore$ total cost on this tariff = $44{,}400 + 3x - 3600 = 3x + 40{,}800$ paise

Flat-rate Tariff

Total cost of units on this rate = $20x$ paise

$\therefore$ $20x = 3x + 40{,}800$

or $17x = 40{,}800$ $\therefore$ $x =$ **2,400 units**

Tutorial Problem No. 50.1

1. A plant costing Rs. 650,000 has a useful life of 15 years. Find the amount which should be saved annually to replace the equipment at the end of that time.

(*i*) by the straight line method and (*ii*) by the sinking fund method if the annual rate of compound interest is 15%.

Assume that the salvage value of equipment is Rs. 5000.

[(i) Rs. 4,000 (ii) Rs. 1,261] (*Elect. Generation, Punjab Univ.*)

2. A plant costs Rs. 7.56×10^5 and it is estimated that after 25 years, it will have to be replaced by a new one. At that instant, its salavage value will be Rs. 1.56×10^5. Calculate.

(*i*) the annual deposit to be made in order to replace the plant after 25 years and

(*ii*) value of the plant after 10 years on the 'reducing balance depreciation method'.

[(i) 0.0612 (ii) Rs. 4.02×10^5] (*Util. of Elect. Power, AMIE Sec. B,*)

3. From the following data, estimated the generating cost per unit delivered at the station.

Capacity of the generating plant = 10 MW; annual load factor = 0.4 ; capital cost = Rs. 5 million; annual cost of fuel, oil, wages, taxes and salaries = Rs. 2×10^5; Rate of interest = 5%; rate of depreciation = 5% of initial value. **[2 paisa/kWh]**

4. From the following data, find the cost of generation per unit delivered from the station.

Capacity of the plant installed = 100 MW; annual load factor = 35% ; capital cost of power plant = Rs. 1.25 crores ; Annual cost of fuel, oil, salaries and taxation = Rs. 0.15 crore; Interest and depreciation on capital = 12%.

If the annual load factor of the station is raised to 40%, find the percentage saving in cost per unit.

[1 paisa/kWh ; 12.2%]

5. The capital costs of steam and water power station are Rs. 1200 and Rs. 2100 per kW of the installed capacity. The corresponding running costs are 5 paise and 3.2 paise per kWh respectively.

The reserve-capacity in the case of the steam station is to be 25% and that for the water power station is to be 33.33% of the installed capacity.

At what load factor will the overall cost per kWh be the same in both cases ? Assume interest and depreciation charges on the capital to be 9% for the thermal and 7.5% for the hydroelectric station. What would be cost of generating 500 million kWh at this load factor ? **[47.5% ; Rs. 4.12 crores]**

6. In a particular area, both steam and hydro stations are equally possible. It has been estimated that capital costs and the running costs of these two types will be as follows :

	Capital cost /kW	*running cost/kW*	*interest*
Hydro	Rs. 2,200	1 paisa	5%
Steam	Rs. 1,200	5 paise	5%

If expected average load factor is only 10% which is economical to operate, steam or hydro ? If the load factor is 50%, would there be any changes in the choice ? If so, indicate with calculations.

[10% load factor ; hydro ; 13.5 paise / kWh ; Steam = 11.85 paise/ kWh50% load factor ; hydro ; 3.5 paise/kWh ; Steam = 6.35 paise/kWh]

7. A consumer takes a steady load of 200 kW at a power factor of 0.8 lagging for 10 hours per day and 300 days per annum. Estimate his annual payment under each of the following tariffs :

(*i*) 10 paisa per kWh plus Rs. 100 per annum per kVA

(*ii*) 10 paisa per kWh plus Rs. 100 per annum per kW plus 2 paisa per kVARh.

[(i) Rs. 85,000 (ii) Rs. 170,000.] (*Util. of Elect. Power, Punjab Univ.*)

8. A consumer requires one million units per year and his annual load factor is 50%. The tariff in force is Rs. 120 per kW per annum plus 5 paise per unit. Estimate the saving in his energy charges which would result if he improved his load factor to 100%.

[Rs. 13,698.60] (*Util. of Elect. Power, AMIE*)

9. A factory has a maximum demand of 1000 kW and a load factor of 40%. It buys power from a company at the rate of Rs. 45 per kW plus Rs. 0.025/kWh consumed. Calculate the annual electricity bill of the factory. Also work out the overall cost of one unit of electricity.

[Rs. 1,32,600, 3.78 paise] (*Util of Elect. Power, AMIE*)

10. A power station having a maximum demand of 100MW has a load factor of 30%. It is to be supplied by any of the following schemes :

(*a*) a steam power station in conjunction with a hydro station, the latter supplying 108 kWh/annum with a maximum output of 40 MW.

(*b*) a steam station capable of supplying whole load,

(*c*) a hydro station capable of supplying whole load.

	The following data may be assumed :	steam	hydro
(*i*)	Capital cost/kW installed capacity	Rs. 600	Rs. 1500
(*ii*)	Interest and depreciation on capital cost	12%	10%
(*iii*)	Operating cost/unit	5 p	1 p
(*iv*)	Transmission cost/unit negligible	0.25 p	

Neglect spares. Calculate the overall cost per unit generated in each case.

[(a) 7.5 P/ kWh (b) 7.74 P/kWh (c) 6.95 P/kWh]

11. An electricity undertaking having a maximum load of 100 MW generates 375 million kWh per annum and supplies consumers having an aggregate maximum demand of 165 MW.

The annual expenses including capital charges are : for fuel Rs. 30 lakhs, fixed charges concerning generation Rs. 40 lakhs. Fixed charges connected with transmission and distribution: Rs. 50 lakhs. Assume that 90% of the fuel cost is essential to running charges and with the losses in transmission and distribution as 15% of the kWh generated, deduce a two-part tariff to find the actual cost of supply to consumers. **[Rs. 56.3 per kW Max. demand ; 0.84 P/kWh]**

12. Obtain a tariff for the consumers of supply undertaking which generates 39×10^7 kWh/year and has maximum demand of 130 MW connected to it. The cost is distributed as follows :

Fuel : Rs. 37.5 lakhs : Generation : Rs. 18 lakhs ; Transmission : Rs. 37.5 lakhs ; Distribution : Rs.25.5 lakhs.

Of these items, 90%, 10% 5% and 7% respectively are allocated to running charges, the remainder being fixed charges. The total loss between the station and the consumers is 10% of the energy generated.

Find also the load factor and overall cost per unit.

[Rs. 61 per kW max. demand and 1.1 P/kWh ; 34.2% ; 3.38 P/kWh]

13. A generating station has got maximum demand of 80 kW. Calculate the cost per kWh delivered from the following data :

Capital cost = Rs. 16×10^6
Annual cost of fuel and oil = Rs. 14×10^5
Taxes, wages and salaries = Rs. 9×10^5
The rate of interest and depreciation is 12%; Annual load factor is 60%.

[10.036 paise] (*Utili. of Elect. Power. AMIE*)

14. A S/S supplies power by four feeders to its consumers. Feeder No. 1 supplies six consumers whose individual daily maximum demands are 70 kW, 90 kW, 20 kW, 50 kW, 10 kW and 20 kW, while the maximum demand on feeder is 200 kW. Feeder No. 2 supplies four consumers, whose daily max. demands are 60 kW, 40 kW, 70 kW, and 30 kW, while the maximum demand on feeder No. 2 is 160 kW. Feeders Nos. 3 and 4 have a daily maximum demand of 150 kW and 200 kW respectively, while the maximum demand on the station is 600 kW.

Determine the diversity factor for the consumers of feeder No. 1, feeder No. 2 and for the four feeders. **[1.3, 1.25, 1.18] (*AMIE*)**

15. A generating station has a M.D. of 75 MW and a yearly load factor of 40%. Generating costs inclusive of station capital cost are Rs. 60 per annum per kW demand plus 1 paisa/ kWh transmitted. The annual capital charge for the transmission system is Rs. 1.5 million, the respective diversity factors being 1.2 and 1.25 respectively. The efficiency of transmission system is 90% and that of the distribution system inclusive of substation losses 85%. Find the yearly cost per kW demand and the cost per kWh supplied (*a*) at the substation and (*b*) at the consumer's premises.

[(*a*) Rs. 72.2 per kW M.D. and 1.1. P/kWh (b) Rs. 71.1 per kW M.D. and 1.31 P/kWh]

50.21. Kelvin's Law

We will now consider the application of Kelvin's economy law to power transmission through feeder cables. Since, in feeders, voltage drop is not of vital importance, they can be designed on the basis of their current-carrying capacity and where feasible, of minimum financial losses.

The financial loss occuring in a current-carrying conductor is made up of two parts :

(*i*) interest on the capital cost of the conductor plus allowance for depreciation and

(*ii*) the cost of energy loss due to **(*a*)** ohmic resistance *i.e.*, I^2R loss, **(*b*)** losses in the metallic sheaths of sheathed cables and **(*c*)** in the case of high tension insulated cables, loss in the insulating material used.

For a given length of the cable, the weight and hence cost of copper required is directly proportional to the cable cross-section. The annual combined interest on capital cost and depreciation is also directly proportional to the cross-section of the cable and can be written as Rs. PA where P = *a* constant and A = cross-section of the cable.

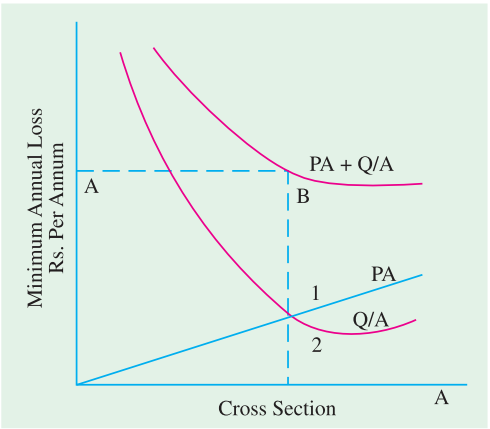


Fig. 50.11

Now, we will consider the cost of I^2R loss, neglecting at the moment losses other than ohmic. The ohmic resistance of the conductor is proportional to $1/A$. For a given annual load curve, the energy loss is proportional to resistance and so proportional to $1/A$. It can be written as Q/A where Q is another constant. If the load is variable, then current used in calculating I^2R loss will be the r.m.s. value of current reckoned over a period of one year.

$\therefore$ total annual financial loss is $L =$ Rs. $\left(PA + \frac{Q}{A}\right)$

Now, it can be proved that this annual financial loss will be minimum when $PA = Q/A$ *i.e.*, ***when annual charge on the capital outlay is equal to the annual value of the energy loss***.* The most economical cross-section which will make the above cost equal is given by :

$$PA = \frac{Q}{A} \text{ or } A^2 = Q/P \therefore A = \sqrt{Q/P}$$

The most economical cross section can be also found graphically as shown in Fig. 50.11. It is seen that graph of PA is a straight line (curve 1) and that of Q/A is a rectangular hyperbola (curve 2). The graph for total annual loss is obtained by combining curves 1 and 2 and is found to exhibit a minimum for that value of 'A' which corresponds to the intersection of the two curves.

Since we have neglected the dielectric and sheath losses while deriving the above condition of minimum financial loss, it is obvious that it applies only to ***bare conductors.***

In practice, Kelvin's law may have to be modified because the most economical size so calculated may not always be the practical one because it may be too small to carry the current.

Example 50.55. *If the cost of an overhead line is Rs. 2000 A (where A is the cross-sectional area in cm^2) and if the interest and depreciation charges on the line are 8%, estimate the most economical current density to use for a transmission requiring full-load current for 60% of the year. The cost of generating electric energy is 5 paise/kWh. The resistance of a conductor one kilometre long and 1 cm^2 cross-section is 0.18 Ω.*

Solution. Take one kilometre of the overhead line and let I be the full-load current, then if R is the resistance of each line, the full-load power loss in the line is

$$= 2I^2R = 2I^2 \times 0.18 \times \frac{1}{A} \text{ watt} = \frac{0.36\, I^2}{A} \times 10^{-3} \text{ kW}$$

Since the line works for 60% of the year *i.e.* for $(0.6 \times 365 \times 24)$ hours, hence total annual loss is $= \frac{0.35\, I^2}{A} \times 10^{-3} \times 0.6 \times 365 \times 24$ kWh

$\therefore$ annual cost of this loss = Rs. $\left(\frac{0.35\, I^2}{A} \times 10^{-3} \times 0.6 \times 365 \times 24 \times 0.05 \right) = 0.0946$ I2/A

The annual value of interest on capital outlay and depreciation = 8% of 2,000 A = Rs. 160 A

For minimum total financial loss : $160\, A = 0.0946\, I^2/A$ I/A $= \sqrt{(160/0.0946)}$ = **41.12 A/cm^2**

Example 50.56. *A 500-V, 2-core feeder cable 4 km long supplies a maximum current of 200 A and the demand is such that the copper loss per annum is such as would be produced by the full-load current flowiing for six months. The resistance of the conductor 1 km long and 1 sq cm. cross-sectional area is 0.17 Ω. The cost of cable including installation is Rs. (120 A + 24) per metre where A is the area of cross-section in sq. cm and interest and depreciation charges are 10%. The cost of energy is 4 paise per kWh. Find the most economical cross-section.*

(Electrical Technology, M.S. Univ. Baroda)

Solution. Let us consider one kilometer length of the feeder cable.**

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* For this expression to have minimum value $\frac{dL}{dA} = 0$

$$\frac{d}{dA}(P + Q/A) = 0 \quad \text{or} \quad P - \frac{Q}{A^2} = 0 \quad \therefore \quad P = \frac{Q}{A^2} \text{ or } PA = \frac{Q}{A} \text{ or } A = \sqrt{Q/P}$$

** Even through we are given 4 km length of the cable, so far as our calculations are concerned, any convenient unit of length can be taken.

Cable cost/metre = Rs. $(120A + 24)$

Cost of 1 km long cable = Rs. $120A \times 1000$ = Rs. $12 \times 10^4 A$

Interest and depreciation per annum is = 10% of Rs. $12 \times 10^4 A$ = Rs. $12 \times 10^3 A$

Resistance of one km long cable is R = 0.17/A ohm. —where A is in cm^2

Cu loss in the cable = $2I^2R = 2 \times 200^2 \times 0.17/A$ $W = 13.6 \times 10^3/AW = 13.6/A$ kW

Energy loss over six months = $(13.6/A) \times (8760/2)$ kWh = $59{,}568/A$ kWh

Cost of this energy loss = Rs. $59{,}568 \times 0.04/A$ = Rs. $2383/A$

For most economical cross-section ; 12×10^3 A = $2{,}383/A$, $A = \sqrt{0.1986}$ = **0.446 cm^2**

Example 50.57. *A 2-core, 11-kV cable is to supply 1 MW at 0.8 p.f. lag for 3000 hours in a year. Capital cost of the cable is Rs. (20 + 400a) per metre where a is the cross-sectional area of core in cm^2. Interest and depreciation total 10% and cost per unit of energy is 15 paise. If the length of the cable is 1 km, calculate the most economical cross-section of the conductor. The specific resistance of copper is 1.75×10^{-6} ohm-cm.* **(Power Systems-I, AMIE, Sec. B, 1993)**

Solution. Cost of 1 km length of the cable = Rs. $(20 + 400a) \times 1000 = (20{,}000 + 400 \times 10^3 a)$

If a is the cross-sectional area of each core of the cable, then resistance of 1 km cable length is

$= 1.75 \times 10^{-6} \times 1000 \times 100/a$ = $0.175/a$ Ω.; F.L. current = $1 \times 10^6/11 \times 10^3 \times 0.8 = 113.6$ A

Power loss in the cable = $2I^2R = 2 \times 113.6^2 \times (0.175/a) = 4516.7/a$ W

Annual cost of energy loss = Rs. $(4516.7/a) \times 3000 \times 10^{-3} \times (15/100)$ = Rs. $2032.5/a$.

Interest and depreciation per annum = $0.1 \times 400 \times 10^3 a = 40{,}000\ a$

As per Kelvin's law, the most economical cross-section would be given by

$40{,}000\ a = 2032.5/a$ $a = 0.2254$ cm^2 and

$d = \sqrt{0.2254 \times 4/\pi}$ = **0.536 cm.**

50.22. Effect of Cable Insulation

Let us now consider the effect of insulation in the case of an insulated cable. The cost of insulation would now be added to the annual value of interest and depreciation. Since for a given type of cable, a given type of armouring and for a given voltage of transmission, the insulation cost does not vary much with the cross-section of the cable, it is taken care of by merely adding a constant R to the term Rs. PA. Hence, the annual interest and depreciation, and insulation cost is represented by the term Rs. $(PA + R)$ where R is a constant representing insulation cost. The addition of term Rs. R results in merely raising the graph of Rs. PA vertically through a distance representing Rs. R, as shown in Fig. 50.12.

The total annual financial loss of insulated cable is thus represented by Rs. $(PA + R + Q/A)$. It should be noted that curve for Q/A is not disturbed at all. However, the graph for total loss *i.e.* $(PA + Q/A + R)$ is only shifted vertically through a distance equal to Rs.

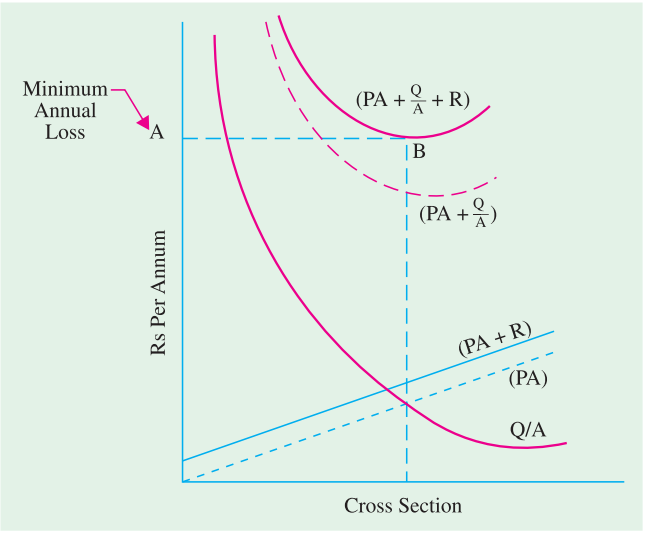


Fig. 50.12

R without any horizontal displacement of the point of minimum financial loss. In other words, the insulation cost does not affect the value of most economical cross-section. Hence, for an insulated cable, Kelvin's law is : *the most economical cross-section is that which makes the interest on the capital outlay plus depreciation due to the conductor in the cable equal to the annual cost of energy lost.*

Example 50.58. *The cost of a two-core feeder cable including insulation is Rs. (130 A + 24) per metre and the interest and depreciation charges 10% per annum. The cable is two km in length and the cost of energy is 4 paisa per unit. The maximum current in the feeder is 250 amperes and the demand is such that the copper loss is equal to that which would be produced by the full current flowing for six months. If the resistance of a conductor of 1 sq. cm cross-sectional area and one km in length be 0.18 Ω, find the most economical section of the same.*

Solution. Cable cost/m = Rs. $(130A + 24)$ where A is the cable cross-section in cm^2.

Cost of 1 km long conductor = Rs. $(130A \times 1000)$ = Rs. $(13 \times 10^4 A)$

Interest and depreciation per annum = $0.1 \times 13 \times 10^4 A$ = Rs. $(13 \times 10^3 A)$

Resistance of one km long conductor = 0.18/A ohm

Cu loss in the cable $= 2I^2R = 2 \times 250^2 \times \frac{0.18}{A}$ W $= 2 \times 250^2 \times \frac{0.18}{A} \times 10^{-3}$ kW

Annual value of this energy loss = Rs. $\left(2 \times 250^2 \times \frac{0.18}{A} \times 10^{-3} \frac{4}{100}\right) \times \frac{8760}{2}$

= Rs. 3941/A

For most economical cross-section 13×10^3 A = 3,942/A $\therefore$ $A^2 = 3942/13 \times 10^3$

$\therefore$ A = **0.505 cm^2**

Example 50.59. *An 11-kV, 3-core cable is to supply a works with 500-kW at 0.9 p.f. lagging for 2,000 hours p.a. Capital cost of the cable per core when laid is Rs. (10,000 + 32,00 A) per km where A is the cross-sectional area of the core in sq. cm. The resistance per km of conductor of 1 cm^2 cross-section is 0.16 Ω.*

If the energy losses cost 5 paise per unit and the interest and sinking fund is recovered by a charge of 8% p.a., calculate the most economical current density and state the conductor diameter.

(Power Systems-I, AMIE-1994)

Solution. The annual charge on cost of conductors per km is = $0.08 \times 32{,}000 A$ = Rs. 2,560 A

Current per conductor is = $500{,}000/\sqrt{3} \times 11{,}000 \times 0.9 = 29.2$ A

Losses in all the three cores = $3I^2R = 3 \times 29.2^2 \times \frac{0.16}{A} = \frac{409.3}{A} \times 10^{-3}$ kW

Annual cost of this loss = Rs. $\frac{409.3 \times 10^{-3}}{A} \times \frac{5}{100} \times 200$ = Rs. $\frac{40.93}{A}$

Obviously, the values of the two constants per km are P = 2,560 and Q = 40.93

The most economical cross-section is given by $A = \sqrt{Q/P} = \sqrt{(40.93/2{,}560)} = 0.1265$ cm^2

The current density is 29.2/0.1265 = **231 A/cm^2** and the conductor diameter is

$d = \sqrt{4 \times 0.1256/\pi}$ = **0.4 cm.**

Example 50.60. *Discuss limitations of the application of Kelvin's law.*

An industrial load is supplied by a 3-phase cable from a sub-station at a distance of 6 km. The voltage at the load is 11 kV. The daily load cycle for six days in a week for the entire year is as given below :

(i) 700 kW at 0.8 p.f. for 7 hours, (ii) 400 kW at 0.9 p.f. for 3 hours,

(iii) 88 kW at unity p.f. for 14 hours.

Compute the most economical cross-section of conductors for a cable whose cost is Rs. (5000 A + 1500) per km (including the cost of laying etc.). The tariff for the energy consumed at the load is Rs. 150 per annum per kVA of M.D. plus 5 paise per unit. Assume the rate of interest and depreciation as 15%. The resistance per km of the conductor is (0.173/A) Ω.

(Electrical Power-I ; Bombay Univ.)

Solution. Capital cost of the cable = 6 × Rs. (5000 A + 1500) = Rs. (30,000 A + 9000).

Annual cost of interest and depreciation = Rs. 0.15 (30,000 A + 9000) = Rs. (4500 A + 1350).

Resistance of each conductor = $6 \times 0.173/A = 1038/A$ ohm

Line currents due to different loads are :

(i) At 700 kW, 0.8 p.f.; $I = 700 \times 10^3/\sqrt{3} \times 11 \times 10^3 \times 0.8 =$ **45.9 A**

(ii) At 400 kW, 0.9 p.f.; $I = 400 \times 10^3/\sqrt{3} \times 11 \times 10^3 \times 0.9 =$ **23.3 A**

(iii) At 88 kW, u.p.f. $I = 88/ \times 11 =$ **4.62 A**

The corresponding energy losses per week in the cable are :

(i) loss = $3 \times 45.92 \times (1.038/A) \times (6 \times 7)/1000 = (276/A)$ kWh

(ii) loss = $3 \times 23.32 \times (1.038/A) \times (6 \times 3)/1000 = (30.4/A)$ kWh

(iii) loss = $3 \times 4.622 \times (1.038/A) \times (6 \times 14) \times 10^{-3} = (5.6/A)$ kWh

Total weekly loss = $(276/A) + (30.4/A) + (5.6/A) = (312/A)$ kWh

Taking 52 weeks in one year, we have

Annual Cu loss = Rs. $\frac{5}{100} \cdot \frac{312}{A} \times 52 =$ Rs. $\frac{811}{A}$

Max. voltage drop in each conductor = $45.9 \times 1.038/A = (47.64/A)$ volt

Max. kVA demand charge due to this drop for three conductors is

= Rs. $3 \times (47.64/A) \times 45.9 \times 10^{-3} \times 150 =$ Rs. $984/A$

Total annual charges due to cable loss = Rs. $(811/A)$ + Rs. $(984/A)$ = Rs. $(1795/A)$

For the most economical size of the cable ; $4500\,A = 1795/A$; $A = 0.63$ cm^2.

Note. Though not given, it has been assumed that A appearing in Rs. (5000 A + 1500) is in cm^2.

Tutorial Problem No. 50.2

1. The daily load cycle of a 3-phase, 110-kV transmission line is as follows :

(a) 6 hours–20 MW, (b) 12 hours–5 MW, (c) 10 hours – 6 MW. The load p.f. is 0.8 lag for all the three loads. Determine the most economical cross-section if the cost of the line including erection is Rs. (9,000 + 600 A) per km where A is the cross-section of each conductor in cm^2. The rate of interest and depreciation is 10% and energy cost is 6 paise/unit. The line is in use all the year. The resistance per km of each conductor is 0.176/A ohm. **[1.64 cm^2]**

2. Find the most economical cross-section of conductor for a system to transmit 120 A at 250 V all the year round with a cost of 10 paise per kWh for energy wasted. The two-core cable costs Rs. 20 A per metre where A is the cross-section of each core in cm^2. The length of feeder is one km. The interest and depreciation charges total 8% of total cost. One km of Cu wire 1 cm^2 in cross-section area has a resistance of 0.15 W.

[1.52 cm^2]

50.23. Note on Power Factor

Irrespective of the nature of voltage and current, the power factor may be defined as the ratio of true power consumed to the apparent power (product of voltage and current).

$$\therefore \quad \text{p.f.} = \frac{W}{VA} = \frac{kW}{kVA}$$

If voltage and current are both sinusoidal, then p.f. = kW/kVA = cos ϕ

where ϕ is the phase difference between voltage and current.

The power factor of all a.c. motors (except over-excited synchronous motors and certain types of commutator motors) and transformers is less than unity (lagging). Majority of industrial motors are induction motors which have a reasonable high power factor (0.9 or so) at full-load but very low power factor at light loads and at no-load as low a value as 0.1. Induction motor is, in fact, a simple resistance-inductance circuit. The current taken by such a circuit has two components ***(i)*** active or wattful component Iw which is in phase with the voltage. Its value is $I \cos \phi$ where I is total circuit current. It is this component which represents the useful power in the circuit and ***(ii)*** reactive or wattless or idle component I_μ which lags behind the applied voltage by 90°. Its value is $I \sin \phi$ and its purpose is to produce alternating magnetic flux. Hence, it is also known as wattless or magnetising current.

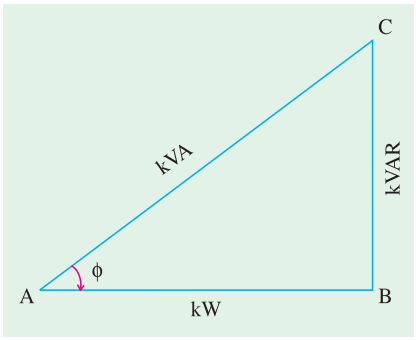


Fig. 50.13

Obviously $I^2 = I_w^{\,2} + I_\mu^{\,2}$

Multiplying both sides by V_2, we get

$$V^2I^2 = V^2I_w^{\,2} + V^2I_\mu^{\,2}$$

or $(VI)^2 = (VI_w)^2 + (VI_\mu)^2$

Now VI = volt-amperes written as VA ; VI_w = wattage W

VI_μ = reactive volt-ampere written as $VA\,R$.

Hence, in any inductive circuit as shown in Fig. 50.13

$$\text{kVA}^2 = \text{kW}^2 + \text{kVA R}^2$$

For example, suppose an R-L circuit draws a current of 100 A from a supply of 250 V at a power factor of 0.8, then total kilo-voltampere of the circuit is $250 \times 100/1000 = 25$ kVA. The true power is $250 \times 100 \times 0.8/1000 = 20$ kW and reactive voltampere is $250 \times 100 \times 0.6/1000 = 15$ kVAR.

Obviously, $\text{kVA} = \sqrt{20^2 + 15^2} = 25$

50.24. Disadvantages of Low Power Factor

Suppose a 3-phase balanced system is supplying a load W at a voltage of V and p.f. cos ϕ, then current flowing through the conductor is $I = W/\sqrt{3}\,.\,V\,.\cos\phi$.

A lower power factor, obviously, means a higher current and this affects in the following three ways :

1. Line losses are proportional to I^2 which means proportional to $1/\cos^2\phi$. Thus losses at cos ϕ = 0.8 are 1/0.82 = 1.57 times those at unity power factor.
2. Ratings of generators and transformers etc. are proportional to current, hence to 1/cos ϕ, therefore, larger generators and transformers are required.
3. Low lagging power factor causes a large voltage drop, hence extra regulation equipment is required to keep voltage drop within prescribed limits.

Explanation

Low power factor means higher wattless current which presents a serious problem from supply

viewpoint. Consider the following numerical example. Suppose an alternator (single-phase) is to have an output of 1500 kW at 10,000 V.

(*a*) Load at unity power factor $P = VI \times 1 = VI$ watt and $I = \frac{P}{V} = \frac{1,500,000}{10,000} = 150$ A

In this case kVA = kW = 1500

(*b*) If the load has a lagging p.f. of 0.6, then

$$I = \frac{P}{V \cos \phi} = \frac{1,500,000}{10,000 \times 0.6} = 250 \text{ A and kVA} = \mathbf{2500}$$

It is obvious that in case **(*b*)**, the current-carrying capacity of alternator winding will have to be (250-150)/150 = 0.67 or 67% greater. Now, the size and hence cost of a given type of alternator is decided by kVA output and not by kW.

In case **(*a*)**, kVA = kW1500. In case **(*b*)** kVA = 1500/0.6 = 2500.

It is seen that with a load p.f. of 0.6, the cost of an alternator to give the same power output and hence the earning capacity, is about 67% greater than when p.f. is unity. Moreover, the switchgear will have to carry 250 A instead of 150 A and so, will be correspondingly larger and more costly. Since transformers are almost always used in the line, they too will be more expensive.

The cables connecting the alternator with the load will also have to carry more current. If current density is the same, then cable cross-section and hence the weight of copper required will be increased.

Therefore, it is seen that a low power factor leads to a high capital cost for the alternator, switchgear, transformers and cables etc. Since the value of power factor is decided by the consuming devices like motors etc., the electric supply undertakings encourage the consumers to make their power factor as high as possible. As an inducement, they offer cheaper tariffs in different ways discussed in Art. 50.25.

The disadvantages of low power factor are summarized below :

In a transmission line (supplying an inductive load) it is only the in-phase component of line current which is active in the transmission of power. When power factor is low, then in-phase (or active) component is small, but reactive component is large hence unnecessarily large current is required to transmit a given amount of power. Large reactive component means large voltage drop and hence greater Cu losses with the result that the regulation is increased and efficiency is decreased.

Lighting and heating loads, being mainly resistive, have unity p.f. but electromagnetic machinery has a low p.f. especially on no-load or light loads.

A low power factor (lagging) system has the following disadvantages as compared to a system carrying the same power but at higher p.f.

1. As current is large and at a large lagging angle, it cause greater losses and requires higher excitation in the generators, thereby reducing the efficiency of generation.
2. Increased Cu losses are incurred in the cables and machinery because of large current. Both these factors raise the running cost of the system.
3. The ratings of the generators, transformers, switchgears and cables etc., have to be increased, which means additional capital charges with increased depreciation and interest.
4. Due to voltage drop in generators, transformers and cables etc. the regulation becomes poor. As supply authorities are usually bound to maintain the voltage at consumer's terminals within prescribed limits, they have to incur additional capital cost of tap-changing gear on transformers to compensate for the voltage drop. Hence, the supply authorities penalise industrial consumers for their low power factor by charging increased tariff for kVA maximum demand in addition to usual kWh charge. Obviously, it is advantageous for the consumer to improve his load p.f. with the help of phase-advancing equipment (*i.e.* synchronous capacitors) or static capacitors.

50.25. Economics of Power Factor

In order to induce the customers to keep their load p.f. as high as possible, tariff rates for a.c. power are such that the overall charge per kWh of the energy consumed depends, in some way or the other, on the load p.f. of the consumer. Some of the ways in which it is done are given below :

(i) The total bill for consumption is so adjusted as to make it depend on the deviation of the load p.f. from a standard value,say 0.9. The bill is increased by a constant percentage for each unit p.f. deviation from 0.9, say, for every decrease of 0.01 from 0.9. Similarly, there would be bonus for each increase of unit p.f. value.

(ii) The total power bill is adjusted in some way on the total reactive kilo-volt ampere-hour *i.e.* kVARh instead of kWh. Special meters are installed for measuring kVARh.

(iii) But the most commonly-used tariff is the two-part tariff (Art. 50.20). The fixed charges instead of being based on kW maximum demand are based on kVA maximum demand so that they become inversely proportional to the power factor (because kVA = kW/cos ϕ)

Explanation

Consider a single-phase, 1500-kW, 10,000-V alternator already discussed in Art. 50.24. Suppose whole of its output is taken up by one consumer and the two-part tariff is Rs. 60 per annum per kVA max. demand plus 2 paise per kWh. Let the load factor be 30%. Then,

Average power throughout the year = $0.3 \times 1500 = 450$ kW

$\therefore$ annual consumption = $450 \times 8{,}760$ kWh

Annual cost at 2 paise per kWh = Rs. $450 \times 8{,}760 \times 0.02$ = Rs. 78,840

For the fixed kVA charge, we have

Annual charge at unit p.f. = Rs. 60×1500 = Rs. 90,000

Annual charge at of 0.6 p.f. = Rs. $60 \times (1500/0.6)$ = Rs. 150,000

Total charges at unity p.f. = Rs. 90,000 + 78,840 = Rs. 168,840

Total charges of 0.6 p.f. = Rs. 150,000 + 78,840 = Rs. 228,840

It is seen that annual saving for unity p.f. is Rs. 60,000. Of course, this may not represent the net total saving when p.f. correction has been done either by phase-advancing equipment or static capacitors because the cost and maintenance of p.f. correcting apparatus has still to be taken into account. The cost will be by way of interest on capital required to install the p.f. improvement apparatus plus depreciation, maintenance expenses etc.

Consider the undergiven case. To illustrate how two-part tariff operates for different cases, suppose that rate of charging power to a large factory is Rs. 90 per annum per kVA maximum demand and 2 paise per kWh consumed. Also, suppose that the factory has 450 kW maximum load demand and an annual consumption of 10,000,000 kWh.

(i) If load p.f. were 0.9, then 450 kW max. demand is equal to 450/0.9 = 500 kVA max. demand.

(ii) If load p.f. were 0.7, then 450 kW max. demand is equal to 450/0.7 = 643 kVA max. demand.

Cost on kWh consumed will be the same for both power factors, it is only the maximum demand charge which will vary with the power factor.

At 0.7, the annual fixed charge for 643 kVA at Rs. 90 is Rs. 90×643 = Rs. 57,870

At 0.9, the annual fixed charge for 500 kVA = Rs. 90×500 = Rs. 45,000

$\therefore$ extra charge = Rs. 12,870

Now, let us assume that phase advancing equipment required to improve the power factor from 0.7 to 0.9 costs Rs. 50,000. Then, taking combined interest and depreciation at 12%, the annual charge will be Rs. $50{,}000 \times 0.12$ = Rs. 6,000.

Hence, the net annual saving comes to Rs. 12,870 – Rs. 6,000 = Rs. 6,870.

If, suppose, the p.f. correcting plant had cost Rs. 107,250, then interest and depreciation allowance on it would be = Rs. 107,250 × 0.12 = Rs. 12,870. In that case, there would be no point in installing such an apparatus because there would be no net saving.

It can be seen from the above example that higher the desired power factor improvement *i.e.* greater the kVAR reduction, the more costly the p.f. improvement plant and hence greater the charge on interest on capital outlay and depreciation. A point is reached, in practice, when any further improvement in power factor costs more than the saving in the power bill. Hence, it is necessary for the consumer to find out the value of power factor at which his net saving will be maximum. The value can be found if ***(i)*** annual charge per kVA maximum demand and ***(ii)*** the cost per kVA rating of phase advancing equipment are known.

50.26. Economical Limit of Power Factor Correction

Suppose a consumer is charged at Rs. A per kVA maximum demand plus a flat rate per kWh. Further, suppose that he is taking power of P kW at a power factor of $\cos\phi_1$. As shown in Fig. 50.14, his kVA_1 is $P/\cos\phi_1$ and his $kVAR_1$ is $P\tan\phi_1$. Suppose by installing static capacitors or synchronous capacitors, he improves his power factor to $\cos\phi_2$ (his power consumption P remaining the same). In that case, his kVA_2 is $P/\cos\phi_2$ and $kVAR_2$ is $P\tan\phi_2$.

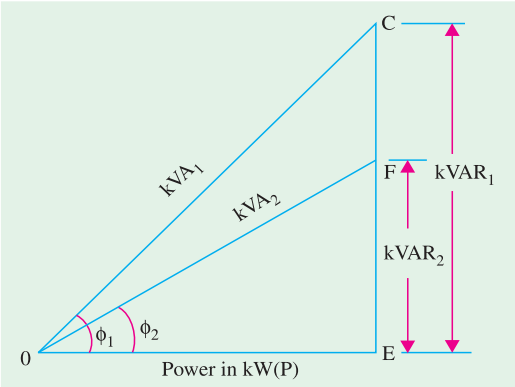


Fig. 50.14

Reduction in his kVA maximum demand is = $(kVA_1 - kVA_2) = (P/\cos\phi_1 - P/\cos\phi_2)$. Since charge is Rs. A per kVA maximum demand, his annual saving on this account is

$$= A\,(P/\cos\phi_1 - P/\cos\phi_2).$$

His kVAR is reduced from $kVAR_1$ to $kVAR_2$, the difference $(kVAR_1 - kVAR_2) = (P\tan\phi_1 - P\tan\phi_2)$ being neutralized by the leading kVAR supplied by the phase advancer.

$\therefore$ Leading kVAR supplied by phase advancer is = $(P\tan\phi_1 - P\tan\phi_2)$.

If the cost per kVAR of advancing plant is Rs. B and the rate of interest and depreciation is p percent per year, then its cost per annum is

$$= \frac{BP}{100}\,(P\tan\phi_1 - P\tan\phi_2)$$

$$= C\,(P\tan\phi_1 - P\tan\phi_2) \text{ where } C = BP/100$$

$\therefore$ Net annual saving $S = A\,(P/\cos\phi_1 - P/\cos\phi_2) - C(P\tan\phi_1 - P\tan\phi_2)$

This net saving is maximum when $dS/d\phi_2 = 0$

$$\therefore \quad \frac{dS}{d\phi_2} = \frac{d}{d\phi_2}[A\,(P/\cos\phi_1 - P/\cos\phi_2) - C\,(P\tan\phi_1 - P\tan\phi_2)]$$

$$= -AP\sec\phi_2\tan\phi_2 + CP\sec^2\phi_2$$

For maximum saving, $dS/d\phi_2 = 0$

$\therefore \quad -AP\sec\phi_2\tan\phi_2 + CP\sec^2\phi_2 = 0 \quad$ or $\quad \sin\phi_2 = C/A = BP/A$

From this expression, ϕ_2 and hence $\cos\phi_2$ can be found. It is interesting to note that the most economical angle of lag ϕ_2 is independent of the original value ϕ_1.

Obviously, most economical p.f. is

$$\cos\phi_2 = \sqrt{1-\sin^2\phi_2} = \sqrt{1-(C/A)^2} = \sqrt{1-(BP/100\,A)^2}$$

Example 50.61. *A 3-phase, 50-Hz, 3,000-V motor develops 600 h.p. (447.6 kW), the power factor being 0.75 lagging and the efficiency 0.93. A bank of capacitors is connected in delta across the supply terminals and power factor raised to 0.95 lagging. Each of the capacitance units is built of five similar 600-V capacitors. Determine capacitance of each capacitor.* **(London Univ.)**

Solution. Power input $P = 447{,}600/0.93 = 480{,}000$ W

$\cos\phi_1 = 0.75$; $\phi_1 = \cos^{-1}(0.75) = 41°24'$, $\cos\phi_2 = \cos^{-1}(0.95) = 18°12'$.

$\therefore \tan\phi_1 = \tan 41°24' = 0.8816$, $\tan\phi_2 = \tan 18°12' = 0.3288$

As shown in Fig. 50.14, leading VAR supplied by capacitor bank is

$= P(\tan\phi_1 - \tan\phi_2) = 480{,}000\,(0.8816 - 0.3288) = 48 \times 5{,}528$

Leading VAR supplied by each of the three sets $= 48 \times 5{,}528/3 = 16 \times 5{,}528$ ...**(*i*)**

Phase current of capacitor is $I_{CP} = \dfrac{V}{X_C} = \dfrac{3{,}000}{X_C} = \dfrac{3{,}000}{1/\omega C} = 3{,}000 \times 314\,C$

where C is the total capacitance in each phase.

$\therefore$ VAR of each phase $= VI_{cap} = 3{,}000 \times 3{,}000 \times 314\,C$ ...**(*ii*)**

Equating **(*i*)** and **(*ii*)** we get, $3{,}000 \times 3{,}000 \times 314\,C = 16 \times 5{,}528$ $\therefore$ $C = 31.22$ μF

Since it is the combined capacitance of five equal capacitors joined in series, the capacitance of each $= 5 \times 31.22 =$ **156 μF**.

Example 50.62. *A synchronous motor having a power consumption of 50 kW is connected in parallel with a load of 200 kW having a lagging power factor of 0.8. If the combined load has a p.f. of 0.9, what is the value of leading reactive kVA supplied by the motor and at what p.f. is it working?* **(I.E.E. London)**

Solution. Let ϕ_1 = p.f. angle of motor ; ϕ_2 = p.f. angle of load

ϕ_t = combined p.f. angle ; $\phi_2 = \cos^{-1}(0.8) = 36°52'$

$\tan\phi_2 = \tan 36°52' = 0.75$; $\phi_t = \cos^{-1}(0.9) = 25°51'$

$\tan\phi_t = \tan 25°51' = 0.4854$

Combined power $P = 200 + 50 = 250$ kW

Total kVAR $= P\tan\phi_t = 250 \times 0.4854 = 121.1$

Load kVAR $= 200 \times \tan\phi_2 = 200 \times 0.75 = 150$

$\therefore$ leading kVAR supplied by synchronous motor $= 150 - 121.1 =$ **28.9**

$\tan\phi_1 = 28.9/50$; $\phi_1 = 30.1°$, $\cos\phi_1 =$ **0.86 (lead)**

Example 50.63. *A generating station supplies power to the following, lighting load 100-kW ; an induction motor 400 h.p. (298.4 kW), power factor 0.8, efficiency, 0.92 ; a rotary converter giving 100 A at 500 V at an efficiency of 0.94. What must be the power factor of the rotary converter in order that the power factor of the supply station may be unity.*

Solution. Since, lighting load has no kVAR (unity p.f. assumed), the lagging kVAR of induction motor are neutralized by the leading kVAR of rotary converter only,

(*i*) Motor input $= 298.4/0.92 = 324.4$ kW

motor p.f. angle $\phi_m = \cos^{-1}(0.8) = 36°52'$ $\therefore$ $\tan\phi_m = \tan 36°52' = 0.75$

$\therefore$ lagging motor kVAR $= 324.4 \times \tan\phi_m = 324.4 \times 0.75 = 243.3$

$\therefore$ leading kVAR to be supplied by rotary converter $= 243.3$

Rotary converter intake $= 500 \times 100/0.94 \times 1000 = 53.2$ kW

$\tan\phi = \dfrac{\text{kVAR}}{\text{kW}} = \dfrac{243.3}{53.2} = 4.573$

$\therefore$ $\phi = 77°40'$ or $\cos\phi =$ **0.214 (leading)**

Example 50.64. *A factory has an average annual demand of 50 kW and an annual load factor of 0.5. The power factor is 0.75 lagging. The tariff is Rs. 100 per kVA maximum demand per annum plus five paise per kWh. If loss-free capacitors costing Rs. 600 per kVAR are to be utilized, find the value of the power factor at which maximum saving will result. The interest and depreciation together amount to ten per cent. Also, determine the saving affected by improving the power factor to this value.* **(Electrical Technology ; M.S. Univ. Baroda)**

Solution. The most economical power factor angle is given by $\sin \phi = C/A$

C = 10% of Rs. 600 = Rs. 60, A = Rs. 100 $\quad \therefore \sin \phi = 60/100 = 0.6$

$\phi = \sin^{-1}(0.6) = 36°52'$; New p.f. = $\cos 36°52' =$ **0.8**

The net annual saving due to improving in power factor can be found as follows :

Max. demand = 50/0.5 = 100 kW

At load p.f. of 0.75, maximum demand of 100 kW represents 100/0.75 = 400/3 kVA maximum demand.

At load p.f. of 0.8, 100 kW represent 100/0.8=125 kVA maximum demand.

Max. kVA demand charge

At 0.75 p.f. = Rs. $100 \times 400/3$ = Rs. 13,333.3

At 0.8 p.f. = Rs. 100×125 = Rs. 12,500

Annual saving = Rs. (13,333.3 – 12,500) = **Rs. 838.3**

Example 50.65. *For increasing the kW capacity of a plant working at 0.7 lag p.f. the necessary increase of power can be obtained by raising the p.f. to 0.85 or by installing additional plant. What is the maximum cost per kVA of p.f. correction apparatus to make its use more economical than additional plant at Rs. 500 kVA ?* **(Gen. Protect. and Switchgear, Madras Univ.)**

Solution. Let kVA_1 be the initial capacity of the plant and kVA_2 its increased capacity with extra or additional plant. As seen from Fig. 50.15.

$KVA_1 \cos \phi_2 = kVA_2 \cos \phi_1$

$\therefore \quad kVA_2 = kVA_1 \times \cos \phi_2 / \cos \phi_1$

$= kVA_1 \times 0.85/0.7 = (17/14)\ kVA_1$

kVA of the additional plant $= kVA_2 - kVA_1 = (17/14)\ kVA_1 - kVA_1 = (3/14)\ kVA_1$

Capital cost of additional plant = Rs. $(500 \times 3/14)\ kVA_1$ = Rs. $(750/7)\ kVA_1$

Now, $\cos \phi_1 = 0.7$, $\phi_1 = 45°34'$; $\sin \phi_1 = 0.714$

$\cos \phi_2 = 0.85$, $\phi_2 = 31.8°$; $\sin \phi_2 = 0.527$

$\therefore$ kVAR supplied by p.f. correction apparatus

$$= (kVA_2 \sin \phi_1 - kVA1 \sin \phi_s) = \left(\frac{7}{14} \times 0.714\ kVA_1 - kVA_1 \times 0.527\right) = 0.34\ kVA_1$$

If the capital cost of the p.f. correction apparatus be Rs. x per kVAR, the total cost is = $0.34x$ kVA_1.

If the annual cost of the additional plant is to be the same as that of the p.f. correction apparatus (assuming same annual interest and depreciation), then

$(750/7)\ kVA_1 = 0.34\ x\ kVA_1$ x = Rs. 315.2

If p.f. correction apparatus is loss-free, then its kVAR = kVA. Hence, the maximum cost per kVA of p.f. correction apparatus that can be paid is **Rs. 315.2.**

Example 50.66. *A consumer taking a steady load of 160 kW at a p.f. of 0.8 lag is charged at Rs. 80 per annum per kVA of maximum demand plus 5 paise per kWh consumed. Calculate the value to which he should improve the p.f. in order to affect the maximum saving if the leading kVA cost Rs.100 per kVA and interest and depreciation be at 12% per annum.*

Calculate also the saving.

(Electrical Technology ; Gujarat Univ.)

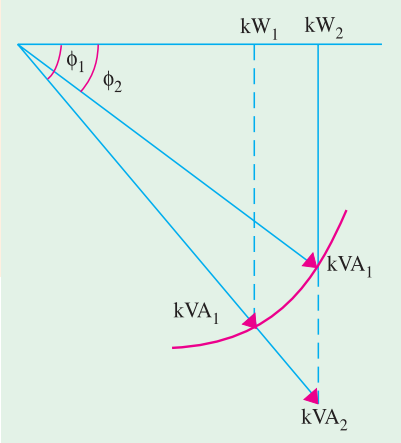


Fig. 50.15

Solution. With reference to Art. 50.26, $C = BP/100 = 12 \times 100/100 = 12$, $A = 80$

Most economical p.f. $\cos \phi_2 = \sqrt{1-(BP/100A)^2}$

$= \sqrt{1-(12/80)^2}$

$=$ **0.9887 (lag)**

Since load remains steady, maximum load is also 160 kW.

At load p.f. of 0.8, maximum load of 160 kW represents 160/0.8 = 200 kVA.

Similarly, at 0.9887 p.f., it represents 160/0.9887 = 162 kVA.

Max. kVA demand charge at 0.8 p.f. = Rs. 80×200 = Rs. 16000

Max. kVA demand charge at 0.9887 p.f. = Rs. 80×162 = Rs. 12,960

$\therefore$ annual saving = Rs. (16,000 – 12,960) = **Rs. 3,040.**

Example 50.67. *A consumer takes a steady load of 1500 kW at a p.f. of 0.71 lagging and pays Rs. 50 per annum per kVA of maximum demand. Phase advancing plant costs Rs. 80 per kVA. Determine the capacity of the phase advancing plant required for minimum overall annual expenditure. Interest and depreciation total 10%. What will be the value of the new power factor of the supply?*

(Electrical Power-III, Bangalore Univ.)

Solution. Minimum overall annual expenditure corresponds to most economical power factor given by $\cos \phi_2 = \sqrt{1-(BP/100A)^2}$ as shown in Art. 50.26.

Now, $B = 80$, $P = 10$, $A = 50$ $\cos \phi_2 = \sqrt{1-(80 \times 10/100 \times 50)^2}$ $=$ **0.987**

Now, $\cos \phi_1 = 0.71$, $\phi_1 = 44.7°$

tan $\phi_1 = 0.9896$; $\cos \phi_2 = 0.987$. $\phi_2 = 9.2°$, $\tan \phi_2 = 0.162$

kVA supplied by phase advancing plant = $P(\tan \phi_1 - \tan \phi_2) = 1500\ (0.9896 - 0.162) =$ **1240.**

Example 50.68. *A factory takes a load of 200 kW at 0.85 p.f. (lagging) for 2,500 hours per annum and buys energy on tariff of Rs. 150 per kVA plus 6 paise per kWh consumed. If the power factor is improved to 0.9 lagging by means of capacitors costing Rs. 525 per kVA and having a power loss of 100 W per kVA, calculate the annual saving affected by their use. Allow 8% per annum for interest and depreciation on the capacitors.*

Solution. Factory load = 200 kW

$\cos \phi_1 = 0.85$ (lagging)

$\therefore$ $\phi_1 = \cos^{-1}(0.85) = 31°48'$ $\tan \phi_1 = 0.62$

$\therefore$ lagging kVAR of factory load = $0.62 \times 200 = 124$

(or first find total kVA = 200/0.85 and then multiply it by $\sin \phi_1$ to get kVAR)

Suppose, x = capacitor's kVAR (leading), Total kVAR = $124 - x$

Because loss per kVA is 100 W *i.e.*, 1/10 kW per kVA

$\therefore$ capacitor's loss $= x/10$ kW

$\therefore$ Total kW $= 200 + (x/10)$

Now, overall p.f. $= \cos\phi = 0.9 \quad \therefore \quad \phi = 25°51' \quad \therefore \tan 25'51' = 0.4845$

Now $\tan\phi = \dfrac{\text{total kVAR}}{\text{total kW}} = \dfrac{124 - x}{200 + (x/10)} \quad \therefore \quad x = 25.9$ kVAR

Cost per annum before improvement

kVA = 200/0.85 = 235.3 Units consumed per annum = $200 \times 2500 = 5 \times 10^5$ kWh

$\therefore$ annual cost = Rs. $(235.3 \times 150 + 5 \times 105 \times 6/100)$ = Rs. 65,295

Cost per annum after improvement

kVA = 200/0.9 = 222.2, Cost = Rs. 222.2×150 = Rs. 33,330

Cost of energy = Rs. $5 \times 10^5 \times 6/100$ = Rs. 30,000 (as before)

Cost of losses occuring in capacitors = Rs. $\dfrac{25.9 \times 100 \times 2500 \times 6}{1000 \times 100}$ = Rs. 389

Annual interest and depreciation cost on capacitors =Rs. $525 \times 25.9 \times 8/100$ = Rs. 1088

$\therefore$ total annual cost is = Rs. (33,330 + 30,000 + 389 + 1088) = Rs. 64,807

$\therefore$ annual saving = Rs. (65,295 – 64,807) = **Rs. 488.**

Example 50.69. *A 30 h.p. (22.38 kW) induction motor is supplied with energy on a two-part tariff of Rs. 60 per kVA of maximum demand per annum plus 5 paise per unit. Motor (A) has an efficiency of 89% and a power factor of 0.83. Motor (B) with an efficiency of 90% and a p.f. of 0.91 costs Rs. 160 more. With motor (A) the p.f. would be raised to 0.91 (lagging) by installing capacitors at a cost Rs. 50 per kVA.*

If the service required from the motor is equivalent to 2,280 hr. per annum at full load, compare the annual charges in the two cases. Assume interest and depreciation charges to be % per annum for the motor and 8% per annum for the capacitors.

Solution. For Motor A

Full-load motor input = $22.38 \times 10^3/0.89 = 25.14$ kW ; kVA = 25.14/0.83 = 30.29

Now, when p.f. is raised to 0.91, then kVA demand of motor = 25.14/0.91 = 27.6

Annual cost of energy supplied to motor = $27.6 \times 60 + (25.14 \times 2{,}280 \times 5/100)$

= 1,656 + 2,866 = Rs. 4,522

By using capacitors, the phase angle of load is decreased from ϕ_1 ($\cos\phi_1 = 0.83$) to ϕ_2 ($\cos\phi_2 = 0.91$).

kVAR necessary for this improvement = load kW $(\tan\phi_1 - \tan\phi_2)$

= 25.14 (0.672 – 0.4557) = 5.439

Annual charges on capacitors = Rs. $50 \times 5.439 \times 8/100$ = Rs. 21.8

Total charges per annum including interest and depreciation on motor

= Rs. (4,522 + 21.8) = Rs. 4,544.

For Motor B

F.L. motor input = 22.38/0.9 = 24.86 kW ; kVA = 24.86/0.91 = 27.32

Annual cost of energy supplied to motor = $27.32 \times 60 + (24.86 \times 2280 \times 5/100)$ = Rs. 4,473

If we do not include the interest and depreciation on the motor itself, then *B* is cheaper than *A* by Rs. (4544 – 4473) i.e., by Rs. 71. But *B* cost Rs. 160 more than *A*. Hence, additional annual charges on *B* are % of Rs. 160 *i.e.*, Rs. 20. Hence, B is cheaper than *A* by Rs. (71 – 20) = Rs. 51 per annum.

Example 50.70. *The motor of a 22.5 kW condensate pump has been burnt beyond economical repairs. Two alternatives have been proposed to replace it by*

Motor A. Cost = Rs. 6000 ; η at full-load=90% ; at half-load = 86%.

Motor B. Cost = Rs. 4000 ; η at full-load=85% ; at half-load= 82%.

The life of each motor is 20 years and its salvage value is 10% of the initial cost. The rate of interest is 5% annually. The motor operates at full-load for 25% of the time and at half-load for the remaining period. The annual maintenance cost of motor A is Rs. 420 and that of motor B is Rs. 240. The energy rate is 10 paise per kWh.

Which motor will you recommend ? **(Power Plant Engg., AMIE)**

Solution. Motor A

Annual interest on capital cost = Rs. $6000 \times 5/100$ = Rs. 300

Annual depreciation charges = $\dfrac{\text{original cost-salvage value}}{\text{life of motor in years}}$

= Rs. $\dfrac{6000-(6000\times 10/100)}{20}$ = Rs. 270

Annual maintenance cost = Rs. 420

Energy input per annum = $\dfrac{22.5\times 0.25\times 8760}{0.9}+\dfrac{11.25\times 0.75\times 8760}{0.86}=1,40,695$ kWh

Annual energy cost = Rs. $0.1 \times 1,40,695$ = Rs. 14,069

Total annual cost = Rs. (300 + 270 + 420 + 14,069) = Rs. 15,059

Motor B

Annual interest on capital cost = Rs. $4000 \times 5/100$ = Rs. 200

Annual depreciation charges = Rs. $\dfrac{4000-(4000\times 10/100)}{20}$ = Rs. 180

Annual maintenance charges = Rs. 240

Energy input per annum = $\dfrac{22.5\times 0.25\times 8760}{0.85}+\dfrac{11.25\times 0.75\times 8760}{0.82}$ = 1,48,108 kWh

Annual energy cost = Rs. $0.1 \times 1,48,108$ = Rs. 14,811

Total annual cost = Rs. (200 + 180 + 240 + 14,811) = Rs. 15,431

Since the annual cost of Motor *B* is Rs. (15431 – 15059) = Rs. 372 more than that of Motor *A*, hence **motor A would be recommended.**

Example 50.71. *An industrial load takes 10^6 kWh a year, the power factor being 0.707 lagging. The maximum demand is 500 kVA. The tariff is Rs. 75 per annum per kVA maximum demand plus 3 paise per unit. Calculate the yearly cost of supply and find the annual saving in cost by installing phase advancing plant costing Rs. 45 per kVA which raises the plant power factor from 0.707 to 0.9 lagging. Allow 10% per annum on the cost of the phase advancing plant to cover all additional costs.*

Solution. Max. demand charge per annum = Rs. 500×75 = Rs. 37,500

Annual energy charges = Rs. $10^6 \times 3/100$ = Rs. 30,000

$\therefore$ yearly cost of supply = Rs. (37,500 + 30,000) = **Rs. 67,500**

Now, when p.f. is increased from 0.707 to 0.9, then maximum kVA demand is reduced to $500 \times 0.707/0.9 = 392.8$.

Now, annual cost of the supply = Rs. 75×392.8 + Rs. 30,000

= 29,460 + 30,000 = Rs. 59,460

kVAR to be supplied by the phase-advancing plant for improving the p.f. from 0.707 to 0.9 = kW demand $(\tan \phi_1 - \tan \phi_2)$ where $\phi_1 = \cos^{-1}(0.707)$ and $\phi_2 = \cos^{-1}(0.9)$.

$\therefore$ kVAR = $500 \times 0.707 \times 0.516 = 182.4$

Annual cost by way of interest and depreciation etc. on the phase advancing plant

= Rs. $182.4 \times 45 \times 0.1$ = Rs. 821

The annual saving would be due to reduction in the maximum demand charge only because units consumed remain the same throughout.

∴ yearly saving = initial charges for kVA – (new charges for kVA + annual cost of phase advancer) = 67,500 – (59,460 + 821) = **Rs. 7,219.**

Example 50.72. *It is necessary to choose a transformer to supply a load which varies over 24 hour period in the manner given below :*

500 kVA for 4 hours, 1000 kVA for 6 hours, 1500 kVA for 12 hours and 2000 kVA for the rest of the period.

Two transformers each rated at 1500 kVA have been quoted. Transformer I has iron loss of 2.7 kW and full-load copper loss of 8.1 kW while transformer II has an iron loss and full-load copper loss of 5.4 kW each.

(i) *Calculate the annual cost of supplying losses for each transformer if electrical energy costs 10 paise per kWh.*

(ii) *Determine which transformer should be chosen if the capital cost of the transformer I is Rs. 1000 more than that of the transformer II and annual charges of interest and depreciation are 10%.*

(iii) *What difference in capital cost will reverse the decision made in (ii) above ?*

(Utilisation of Elect. Power AMIE)

Solution. (i) Transformer No. 1

Iron loss/day = $2.7 \times 24 = 64.8$ kW

Cu loss/day = $8.1\,(500/1500)^2 \times 4 + 8.1 \times (1000/1500)^2 \times 6 + 8.1 \times (1500/1500)^2 \times 12 + 8.1 \times (2000/1500)^2 \times 2 = 151.2$ kWh

Annual energy loss = $365\,(64.8 + 151.2) = 78{,}840$ kWh

Annual cost of both losses = Rs. $0.1 \times 78{,}840$ = **Rs. 7884**

Transformer No. 2

Iron loss/day = $5.4 \times 24 = 129.6$ kWh

Cu loss/day = $5.4\,(500/1500)^2 \times 4 + 5.4\,(1000/1500)^2 \times 6 + 5.4\,(1500/1500)^2 \times 12 + 5.4\,(2000/1500)^2 \times 2 = 100.8$ kWh

Annual energy loss = $365\,(129.6 + 100.8) = 84{,}096$ kWh

Annual cost of both losses = Rs. $0.1 \times 84{,}096$ = **Rs. 8410**

(ii) Since the cost of transformer No. 1 is Rs. 1000 more than that of transformer No. 2, extra annual charge in case transformer No. 1 is selected = Rs. $1000 \times 10/100$ = Rs. 100

Annual saving in energy cost due to losses = Rs. (8410 – 7884) = Rs. 526

Since the annual saving in the energy cost of supplying losses is much than the extra annual charges due to the higher cost of the transformer No. 1, therefore transformer No. 1 will be selected.

(iii) Transformer No. 2 would be chosen in case annual charges due to extra capital cost of transformer No. 1 exceed the annual saving on the energy cost of supplying losses.

If x is the extra capital cost of transformer No. 1, then transformer No. 2 could be selected if

$x \times 10/100 > 526$ or $x >$ **Rs. 5260**

Example 50.73. *Three-phase 50-Hz power is supplied to a mill, the voltage being stepped down to 460-V before use.*

The monthly power rate is 7.50 per kVA. It is found that the average power factor is 0.745 while the monthly demand is 611 kVA.

To improve power factor, 210 kVA capacitors are installed in which there is negligible power loss. The installed cost of the equipment is Rs. 11,600 and fixed charges are estimated at 15% per year. What is the yearly saving introduced by the capacitors.

Solution. Monthly demand = 611 kVA ; p.f. = 0.745

$\phi = \cos^{-1}(0.745) = 41°5'$ $\therefore \sin\phi = \sin 41°5' = 0.667$

$\therefore$ kW component = kVA $\cos\phi = 611 \times 0.745 = 455$

kVAR component = kVA $\sin\phi = 611 \times 0.667 = 408$ (lagging)

leading kVAR supplied by capacitor = 210

Hence, kVAR after p.f. improvement = 408 – 210 = 198

$\therefore$ kVA after p.f. improvement = $\sqrt{455^2 + 198^2} = 491$

Reduction in kVA = 611 – 491 = 120

Hence, monthly saving on kVA charge = Rs. 120×7.5 = Rs. 900

Yearly saving on kVA charge = Rs. 12×900 = Rs. 10,800

Capital cost on capacitors = Rs. 11,600

Fixed charge per annum = Rs. $0.15 \times 11{,}600$ = Rs, 1,740

Hence, net saving per annum = Rs. (10,800 – 1,740) = **Rs. 9,060**

Example 50.74. *A supply undertaking is offering the following two tariffs to prospective customers :*

Tariff A : Lighting : 20 paise per unit; domestic power: 5 paise per unit, meter rent: 30 paise per meter per month.

Tariff B : 12 per cent on the rateable value of the customer's premises plus 3 paise per unit for all purposes.

If the annual rateable value of the customer's premises is Rs. 2,500 and his normal consumption for lighting per month is 40 units, determine what amount of domestic power consumption will make both the tariffs equally advantageous.

Solution. Let x = minimum number of power units consumed per month.

Tariff A

Total cost per month = meter rent + lighting charges + power charges

= $(2 \times 30) + (40 \times 20) + 5x = 860 + 5x$ paise

Tariff B

Total cost per month = 12% of 2500 + energy charges for all purposes

= $0.12 \times 2500 \times 100/12 + 3(40 + x)$

$\therefore$ $860 + 5x = 2500 + 120 + 3x$ $\therefore$ $2x = 1760$ $\therefore$ x = **880 units**

Example 50.75. *Transformers and low-tension motors of a certain size can be purchased at Rs. 12 per kVA of full output and Rs. 24 per kW output respectively. If their respective efficiencies are 98% and 90%, what price per kW output could be paid for high-tension motors of the same size but of average efficiency only 89%? Assume an annual load factor of 30%, the cost of energy per unit as 7 paise and interest and depreciation at the rate of 8% for low-tension motors.*

Solution. Let S = the price of h.t. motors per output kW in rupees.

Low-tension Motors with Transformers

(*a*) Interest and depreciation on the motor input kW = Rs. $\frac{24 \times 0.08}{0.9}$ = Rs. 2.13

(*b*) Let ∞ a p.f.or the mers and calculate their standing charges on the basis of their input expressed in kW.

$\therefore$ interest and depreciation on transformers per input kW = Rs. $\frac{12 \times 0.9}{0.99} \times 0.08$ = Rs. 0.88

(c) Running charges = $\frac{(8,760 \times 0.3) \times 7}{(0.98 \times 0.9) \times 100}$ = Rs. 208.6

Hence, total cost of transformers and low-tension motors per input kW

= 2.13 + 0.88 + 208.6 = Rs. 211.6

High Tension Motors

Standing charges/output kW = Rs. $\frac{S}{.89} \times .12$ = Rs. $\frac{12S}{89}$

Running cost = $\frac{(1 \times 8,760 \times 0.3) \times 7}{0.89 \times 100}$ = Rs. 206.7

Total cost of h.t. motors = (12 S/89) + 206.7.

$\therefore$ 12 S/89 + 206.7 = 211.6 $\therefore$ S = **Rs. 36.34 kW output.**

Example 50.76. *An industrial load can be supplied on the following alternative tariffs* ***(a)*** *high-voltage supply at Rs. 45 per kVA per annum plus 1.5 paise per kWh or* ***(b)*** *low-voltage supply at Rs. 50 per annum plus 1.8 paise per kWh. Transformers and switchgear etc. for the H.V. supply cost Rs. 35 per kVA, the full-load transformer losses being 2%. The fixed charges on the capital cost of the high-voltage plant are 25% and the installation works at full-load. If there are 50 working weeks in a year, find the number of working hours per week above which the H.V. supply is cheaper.*

Solution. Let x be the number or working hours per week above which H.V. supply is cheaper than the L.V. supply. Also, suppose that the load = 100 kW.

Rating of transformers and switchgear = 100/0.98=10,000/98 kW, because transformer losses are 2%.

Cost of switchgear and transformer = (10,000/98) × 35 = Rs. 3,571.5

Annual fixed charges = 3,571.5 × 0.25 = Rs. 892.9

Annual energy consumption = 100 × x × 50 = 5,000 x kWh

On the H.V., the total kWh which will have to be paid for is 5,000 x/0.98.

(a) H.V. Supply

Total annual cost = 45 × kVA + energy charges + charge on H.V. plant

= $45 \times \frac{100}{0.98} + \frac{1.5 \times 5,000\ x}{100 \times 0.98}$ + Rs. 892.9

= Rs. 4,592 + Rs. 76.54 x + Rs. 892.9 = **Rs. (5,485 + 76.54 x)**

(b) L.V. Supply

Total annual cost = Rs. 50 × kVA + energy charges = $50 \times 100 + \frac{1.8 \times 5,000\ x}{100}$

= **Rs. (5,000 + 90 x)**

If the two annual costs are to be equal, then

5,485 + 76.54 x = 5,000 + 90 x

$\therefore$ 13.46 x = 485 or x = **36 hr.**

Hence, above 36 hours per week, the H.V. supply would be cheaper.

Example 50.77. *For a particular drive in a factory requiring 10 h.p. (7.46 kW) motors, following tenders have been received. Which one will you select ?*

	cost	*efficiency*
Motor X	*Rs. 1,150*	*86%*
Motor Y	*Rs. 1,000*	*85%*

Electrical tariff is Rs. 50 per kW + 5 paise per kWh. Assume interest and depreciation as 10% and that the factory works for 10 hours a day for 300 days a year.

(Electrical Engineering, Bombay Univ.)

Solution. (*i*) Motor X

F.L. power input	= 7.46/0.86	= 8.674 kW
Units consumed/year	= 8760 × 8.674	= 75,980 kWh
kW charges	= Rs. 50 × 8.674	= Rs. 433.7
kWh charges	= Rs. 5 × 75,980/100	= Rs. 3,799
Fixed charges	= Rs. 0.1 × 1150	= Rs. 115
Total annual charges	= (3,799 + 433.7 + 115)	= **Rs. 4,347.7**

(*ii*) Motor Y

F.L. power input	= 7.46/0.85	= 8.776 kW
units consumed/year	= 8760 × 8.766	= 76,880 kWh
kW charges	= Rs. 50 × 8.766	= Rs. 438.8
kWh charges	= Rs. 5 × 76,880/100	= Rs. 3,844
Fixed charges	= Rs. 0.1 × 1000	= Rs. 100
Total annual charges	= (438.8 + 3,844 + 100)	= Rs. 4,382.8
Obviously, motor *X* is cheaper by Rs. (4,382.8 × 4,347.7)		= **Rs. 35.1**

Example 50.78. *A 200-h p. (149.2 kW) motor is required to operate at full-load for 1500 hr, at half-load for 3000 hr per year and to be shut down for the remainder of the time. Two motors are available.*

Motor A : efficiency at full load = 90% ; at half-load = 88%

Motor B : efficiency at full load = 90% ; at half-load = 89%

The unit of energy is 5 paise/kWh and interest and depreciation may be taken as 12 per cent per year. If motor A cost Rs. 9,000 ; what is the maximum price which could economically be paid for motor B ?

(Electrical Power-II, Bombay Univ.)

Solution. (*a*) Motor A

F.L. power input	= 149.2/0.9	= 165.8 kW
Half-load power input	= (149.2/2)/0.88	= 84.7 kW
Total energy consumed per year	= (1500 × 165.8) + (3000 × 84.7) = 502,800 kWh	
Cost of energy	= Rs. 5 × 502,800/100 =	Rs. 25,140
Interest and depreciation on motor	= Rs. 0.12 × 9000	= Rs. 1,080
Total charge	= Rs. 25,140 + Rs. 1,080 = **Rs. 26,220**	

(*b*) Motor B

F.L. input = 149.2/0.9 = 165.8 kW : Half-load input = (149.2/2)/0.89 = 83.8 kW

Total energy consumed per year	= (1500 × 165.8) + (3000 × 83.8) = 497,100 kWh	
Cost of energy consumed	= Rs. 5 × 497,100/100 = Rs. 24,855	

Let x be the maximum price which could be paid for motor B.

Annual interest and depreciation = Rs. 0.12 x

annual charges = Rs. (24,855 + 0.12x)

For the two charges to be equal = 24,855 + 0.12x = 26,220 ; x = Rs. 11,375

Example 50.79. *Two tenders A and B for a 1000-kVA, 0.8 power factor transformer are : A, full-load efficiency = 98.5% and iron loss = 6 kW at rated voltage ; B, 98.8% and iron loss 4 kW but costs Rs. 1,500 more than A. The load cycle is 2000 hours per annum at full-load, 600 hours at half-load and 400 hours at 25 kVA. Annual charges for interest and depreciation are 12.5% of capital cost and energy costs 3 paise per kWh. Which tender is better and what would be the annual saving.*

Solution. Tender A Transformer full-load output = 1000 × 0.8 = 800 kW

Efficiency = 98.5% Input = 800/0.985

Total losses = (800/0.985) – 800 = 12.2 kW ; F.L. Cu losses = 12.2 – 6 = 6.2 kW

Total losses per year for a running period of 3000 hr. are—

(i) Iron loss = 3,000 × 6 = 18,000 kWh

(ii) F.L. Cu losses for 2000 hours = 2,000 × 6.2 = 12,400 kWh

(iii) Cu loss at half-load for 600 hours = $\frac{1}{4} \times 6.2 \times 600 = 930$ kWh

(iv) Cu loss at 25 kVA load for 400 hours = $\left(\frac{25}{1000}\right)^2 \times 6.2 \times 400 = 1.5$ kWh

Total energy loss per year = 18,000 + 12,400 + 930 + 1.5 = 31,332 kWh

Cost = Rs. 31,332 × 3/100 = Rs. 939.96

Tender B.

Total loss of transformer = (800/0.988) – 800 = 9.7 kW

Iron loss = 4 kW ; Full-load Cu loss = 9.7 – 4 = 5.7 kW

Total losses for 3,000 hours per year are as follows :

(i) Iron loss = 3,000 × 4 = 12,000 kWh

(ii) F.L. Cu loss for 2000 hours = 5.7 × 2000 = 11,400 kWh

(iii) Cu loss at half-load for 600 hours = (5.7/4) × 600 = 855 kWh

(iv) Cu loss at 25 kVA for 400 hours = $(25/1000)^2 \times 5.7 \times 400 = 1.4$ kWh

Total yearly loss = 12,000 + 11,400 + 855 + 1.4 = 24,256 kWh

Cost = Rs. 24,256 × 3/100 = Rs. 727.68 ; Extra cost of B = Rs. 1500

Annual interest and depreciation etc. = 12.5% or 1500 = Rs. 187.50

∴ yearly cost of B = Rs. 727.68 + Rs. 187.50 = Rs. 915.18

Obviously, tender B is cheaper by Rs. (939.96 – 915.18) = **Rs. 24.78 annually.**

Example 50.80. *Transformer A has iron loss of 150 kWh and load loss of 140 kWh daily while the corresponding losses of transformer B are 75 kWh and 235 kWh. If annual charges are 12.5% of the capital costs and energy costs 5 paise per kWh, what should be the difference in the cost of the two transformers so as to make them equally economical ?*

Solution. Transformer A

Yearly loss = 365 × (150 + 140) = 365 × 290 kWh

Transformer B

Yearly loss = 365 × (75 + 235) = 365 × 310 kWh

Difference in yearly loss = 365 (310 – 290) = 7,300 kWh

Value of this loss = Rs. 7,300 × 5/100 = Rs. 365

Hence, transformer *B* has greater losses per year and so is costlier by Rs. 365. If it is to be equally advantageous, then let it cost Rs. *x* less than *A*. Annual interest and depreciation on it is Rs. 0.125 *x*.

$\therefore \quad 0.125x = 365$

or x = Rs. 2,920

Hence, transformer *B* should cost Rs. 2,920 less than *A*.

Example 50.81. *Quotations received from three sources for transformers are :*

	Price	*No-load loss*	*Full-load loss*
A	*Rs. 41,000*	*16 kW*	*50 kW*
B	*Rs. 45,000*	*14 kW*	*45 kW*
C	*Rs. 38,000*	*19 kW*	*60 kW*

If the transformers are kept energized for the whole of day (24 hours), but will be on load for 12 hours per day, the remaining period on no-load, the electricity cost being 5 paise per kWh and fixed charges Rs. 125 per kW of loss per annum and if depreciation is 10% of the initial cost, which of the transformers would be most economical to purchase ?

Solution. Transformer A

F.L. loss = 50 kW

No-load losses = 16 kW (this represents iron loss)

∴ F.L. Cu loss = 50 – 16 = 34 kW

F.L. Cu loss for 12 hours* = 12 × 34 = 408 kWh

Iron losses for 24 hours = 24 × 16 = 384 kWh

Total loss per day = 408 + 384 = 792 kWh

Annual loss = 792 × 365 kWh

Cost of this loss = Rs. 792 × 365 × 5/100 = Rs. 14,454

Annual fixed charges = Rs. 125 × 50 = Rs. 6,250

Annual depreciation = Rs. 0.1 × 41,000 = Rs. 4,100

Total annual charges for transformer *A* = Rs. 14,454 + Rs. 6,250 + Rs. 4,100 = Rs. 24,804

Transformer B

F.L. loss = 45 kW ; Iron or no-load loss = 14 kW

F.L. Cu loss = 45 – 14 = 31 kW; Iron loss/day = 24 × 14 = 336 kWh

F.L. Cu losss for 12 hours* = 12 × 31 = 372 kWh

Total loss/day = 708 kWh ; Annual loss = 708 × 365 kWh

Cost of this loss = 708 × 365 × 5/100 = Rs. 12,921

Fixed charges = Rs. 125 × 45 = Rs. 5,625

Annual depreciation = Rs. 0.1 × 45,000 = Rs. 4,500

Total annual charges for transformer *B* = Rs. 12,921 + Rs. 5,625 + Rs. 4,500 = Rs. 23,046

Transformer C

F.L. loss = 60 kW ; Iron or N.L. loss = 19 kW

F.L. Cu loss = 60 – 19 = 41 kW

* Cu loss is proportional to $(kVA)^2$

Iron loss for 24 hr. = $24 \times 19 = 456$ kWh

F.L. Cu loss for 12 hr = $12 \times 41 = 492$ kWh

Total losses per day = 948 kWh ; Annual losses = 948×365 kWh

Cost of these losses = Rs. $948 \times 365 \times 5/100$ = Rs. 17,301

Fixed charges = Rs. 125×60 = Rs. 7,500 ;

Annual depreciation = Rs. $0.1 \times 38{,}000$ = Rs. 3,800

Total annual charges on transformer C = Rs. 17,301 + Rs. 7,500 + Rs. 3,800 = Rs. 28,601

Obviously, transformer B is cheaper and should be purchased.

Example. 50.82. *A 37.3 kW induction motor has power factor 0.9 and efficiency 0.9 at full-load, power factor 0.6 and efficiency 0.7 at half-load. At no-load, the current is 25% of the full-load current and power factor 0.1. Capacitors are supplied to make the line power factor 0.8 at half-load. With these capacitors in circuit, find the line power factor at* ***(i)*** *full-load and* ***(ii)*** *no-load.*

(Utilisation of Elect. Power, AMIE)

Solution. Full-load motor input, $P_1 = 37.3/0.9 = 40.86$ kW

Lagging kVAR drawn by the motor at full-load, $\text{kVAR}_1 = P_1 \tan \phi_1 = 40.86 \tan (\cos^{-1} 0.9) = 19.79$.

Half-load motor input, $P_2 = (0.5 \times 37.3)/0.7 = 26.27$ kW

Lagging kVAR drawn by the motor at half-load,

$\text{kVAR}_2 = P_2 \tan \phi_2 = 26.27 \tan (\cos^{-1} 0.6) = 35.02$

Full-load current, $I_1 = 37.3 \times 10^3/\sqrt{3}\, V_L \times 0.9 \times 0.9 = 26,212/V_L$

Current at no-load, $I_0 = 0.25\, I_1 = 0.25 \times 26{,}212/V_L = 6553/V_L$

Motor input at no-load, $P_0 = \sqrt{3} V_L I_0 \cos \phi_0 = \sqrt{3} \times 6553 \times V_L \times 0.1/V_L = 1135$ W $= 1.135$ kW

Lagging kVAR drawn by the motor at no-load, $\text{kVAR}_0 = 1.135 \tan (\cos^{-1} 0.1) = 11.293$

Lagging kVAR drawn from the mains at half-load with capacitors,

$\text{kVAR}_{2C} = 26.27 \tan (\cos^{-1} 0.8) = 19.7$

kVAR supplied by capacitors, $\text{kVAR}_C = \text{kVAR}_2 - \text{kVAR}_{2C} = 35.02 - 19.7 = 15.32$

kVAR drawn from the main at full-load with capacitors $\text{kVAR}_{1C} = \text{kVAR}_K - \text{kVAR}_C = 19.79 - 15.32 = 4.47$.

(i) Line power factor at full-load = $\cos (\tan^{-1} \text{kVAR}_{1C}/P_1) = \cos (\tan^{-1} 4.47/40.86) =$ **0.994 lagging**

(ii) kVAR drawn from mains at no-load with capacitors = $11.293 - 15.32 = -4.027$

Line power factor at no-load = $\cos (\tan^{-1} - 4.027/1.135) = \cos (-74.26^\circ)$

= **0.271 leading.**

Tutorial Problem No. 50.3

1. A power plant is working at its maximum kVA capacity with a lagging p.f. of 0.7. It is now required to increase its kW capacity to meet the demand of additional load. This can be done by raising the p.f. to 0.85 by correction apparatus or by installing extra generating plant which costs Rs. 800 per kVA. Find the minimum cost per kVA of p.f. apparatus to make its use more economical than the additional generating plant. **[Rs. 502 per kVAR]**
2. An industrial load of 4 MW is supplied at 11 kV, the p.f. being 0.8 lag. A synchronous motor is required to meet additional load of 1500 h.p. (1,119 kW) and at the same time to raise the resultant p.f. to 0.95 lag. Find the kVA capacity of the motor and the p.f. at which it must operate. The efficiency of the motor is 80%. **[1875 kVA ; 0.7466 (lead)]**

3. A factory takes a load of 300 kW at 0.8 p.f. lag for 4800 hours per annum and buys energy at a tariff of Rs. 100 per kVA plus 3 p/kWh consumed. If p.f. is improved to 0.9 (lag) by means of capacitors costing Rs. 200 per kVA and having a power loss of 50 W per kVA, calculate the annual saving affected by their use. Allow 10% per annum for interest and depreciation of the capacitors.

[Rs. 693 per year]

4. The following two tenders are received in connection with the purchase of 1000-kVA transformer.

Transformer A : full-load efficiency = 98% ; core loss = 8 kW

Transformer B : full-load efficiency = 91.5% ; core loss = 6 kW

The service needed per year is 200 hours as 1000 kVA, 1000 hours at 500 kVA and 1000 hours at 200 kVA. If the transformer A is costing Rs. 25,000, estimate the maximum price that could be paid for transformer *B*. Take interest and depreciation at 10 per cent per annum and cost of energy 5 paise per kWh. Assume working at unity power factor. **[Max. Price for B = Rs. 21,425]**

5. Tenders *A* and *B* for a 1000-kVA, 0.8 p.f. transformer are :

A : full-load η = 98.3% ; core loss = 7 kW at rated voltage

B : full-load η = 98.7% ; core loss = 4 kW but costs Rs. 5000 more than *A*.

The service needed is 1800 hours per annum at 1000 kVA, 600 hours at 600 kVA, 400 hours at 25 kVA and remaining period shut down. Take the annual charges for interest and depreciation at 12.5 per cent of capital cost and energy costs at 8 paise/kWh ; which is the better tender and what will be annual saving ? **[Tender B is cheaper by Rs. 211]**

6. A 440-V, 50-Hz induction motor takes a line current of 45 A at a power factor of 0.8 (lagging). Three D-connected capacitors are installed to improve the power factor to 0.95 (lagging). Calculate the kVA of the capacitor bank and the capacitance of each capacitor.

[11.45 kVA ; 2.7 μ F] (*I.E.E. London*)

7. A generator station supplies power to the following : lighting 100 kW; an induction motor 400 h.p. (298.4 kW) power factor 0.8, efficiency 0.92, a rotary converter giving 100 A at 800 V at an efficiency of 0.94. What must be the p.f. of the rotary converter in order that the p.f. of the supply station may be unity. **[0.214 leading] (*C. & G. London*)**

8. A substation which is fed by a single feeder cable supplies the following loads : 1,000 kW at p.f. 0.85 lagging, 1,500 kW at 0.8 lagging, 2,000 kW at 0.75 lagging : 500 kW at 0.9 leading. Find the p.f. of the supply to the substation and the load the feeder cable could carry at unity p.f. with the same cable heating. **[0.84 ; 5,960 kW] (*I.E.E. London*)**

9. A 3-phase synchronous motor is connected in parallel with a load of 200 kW at 0.8 p.f. (lagging) and its excitation is adjusted until it raises the total p.f. to 0.9 (lagging). If the mechanical load on the motor, including losses, takes 50 kW, calculate the kVA input to the motor.

[57.8 kVA] (*I.E.E. London*)

10. A 3-phase, 3,000-V, 50-Hz motor develops 375 h.p. (223.8 kW), the p.f. being 0.75 lagging and the efficiency 92%. A bank of capacitors is star-connected in parallel with the motor and the total p.f. raised to 0.9 lagging. Each phase of the capacitor bank is made up of 3 capacitors joined in series. Determine the capacitance of each. **[118 μF] (*London Univ.*)**

11. For increasing the kW capacity of a power plant working at 0.7 lagging power factor, the necessary increase in power can be obtained by raising the power factor to 0.9 or by installing additional plant. What is the maximum cost per kVA of power factor correction apparatus to make its use more economical than additional plant at Rs. 800 per kVA?

[Rs. 525] (*Utili. of Elect. Power, AMIE*)

12. A 340-kW, 3300-V, 50-Hz, 3f star-connected induction motor has full-load efficiency of 85% and power factor of 0.8 lagging. It is desired to improve power factor to 0.96 lagging by using a bank of three capacitors. Calculate

(*i*) the kVA rating of the capacitor bank,

(*ii*) the capacitance of each limb of the capacitor bank connected in delta,

(*iii*) the capacitance of each capacitor, if each one of the limbs of the delta-connected capacitor bank is formed by using 6 similar 3300-V capacitors.

[(*i*) 183.33 kVAR (*ii*) 17,86 μF (*iii*) 2.977 μF] (*Util. of Elect. Power, AMIE*)

13. A system is working at its maximum kVA capacity with a lagging power factor of 0.71. An anticipated increase of load could be met by

(*i*) raising the power factor of the system to 0.87 by the installation of phase advancers,

(*ii*) installing extra generating plant, cables etc. to meet the increased power demand.

Estimate the limiting cost per kVA of phase advancing plant which would justify its use if the cost for generating plant is Rs. 60 per kVA. Interest and depreciation charges may be assumed to be 1% in each case. **[Rs. 36.5] (*Util. of Elect. Power, AMIE*)**

14. A consumer requires an induction motor of 36.775 kW. He is offered two motors of the following specifications :

Motor *A* : efficiency 88% and p.f. = 0.9 Motor *B* : efficiency 90% and p.f. = 0.81

The consumer is being charged on a two-part tariff of Rs. 70 per kVA of the maximum demand plus 5 paise per unit. The motor (*B*) power factor is raised to 0.89 by installing capacitors. The motor *B* costs Rs. 150 less than *A*.

The cost of capacitor is Rs.60 per kVAR. Determine which motor is more economical and by how much. Assume rate of interest and depreciation as 10% and working hours of motors as 2400 hours in a year. **[Motor B, Rs. 111.10] *(Power Systems-II, AMIE)***

15. A 250-V, 7.46 kW motor is to be selected for a workshop from two motors *A* and *B*. The cost of each motor is same but the losses at full load are different as given below :

	Motor A	Motor B
Stray loss	1000 W	900 W
Shunt field loss	250 W	200 W
Armature copper loss	300 W	450 W

The motor has to work on full-load for 8 hours, half-load for 4 hours and quarter load for 4 hours each day. Which motor should be selected ? **[Motor B] (*Util. of Elect. Power AMIE*)**

16. What is load duration curve? ***(Anna University, April 2002)***

17. How will you classify loads? ***(Anna University, April 2002)***

18. Why should there be diversity? ***(Anna University, April 2002)***

19. Define load factor. ***(Anna University, April 2002)***

20. What is M.D.? ***(Anna University, April 2002)***

21. Distinguish between a base load plant and a peak load plant. ***(Anna University, April 2002)***

OBJECTIVE TESTS – 50

1. While calculating the cost of electric power generation, which of the following is NOT considered a fixed cost ?

(*a*) interest on capital investment

(*b*) taxes and insurance

(*c*) most of the salaries and wages

(*d*) repair and maintenance.

2. Maximum demand of an installation is given by its

(*a*) instantaneous maximum demand

(*b*) greatest average power demand

(*c*) average maximum demand over a definite interval of time during a certain period.

(*d*) average power demand during an interval of 1-minute.

3. A diversity factor of 2.5 gives a saving of ..... percent in the generating equipment.

(*a*) 60

(*b*) 50

(*c*) 40

(*d*) 25.

4. Mark the WRONG statement.
High load factor of a generating equipment
(*a*) leads to lesser charges per kWh
(*b*) implies lower diversity in demand
(*c*) gives more profit to the owner
(*d*) can be obtained by accepting off-peak loads.

5. In a generating station, fixed, charges at 100% load factor are 6 paise/kWh. With 25% load factor, the charges would become ..... paise/kWh.
(*a*) 1.5
(*b*) 10
(*c*) 24
(*d*) 3

6. When considering the economics of power transmission, Kelvin's law is used for finding the
(*a*) cost of energy loss in bare conductors
(*b*) most economical cross-section of the con ductors
(*c*) interest on capital cost of the con- ductor
(*d*) the maximum voltage drop in feeders.

7. A 3-phase balanced system working at 0.9 lagging power factor has a line loss of 3600 kW. If p.f. is reduced to 0.6, the line loss would become ..... kW.
(*a*) 8100
(*b*) 1600
(*c*) 5400
(*d*) 2400.

8. Mark the WRONG statement.
While considering power factor improve- ment, the most economical angle of lag depends on the
(*a*) cost/kVA rating of phase advancer
(*b*) rate of interest on capital outlay
(*c*) rate of depreciation
(*d*) value of original lagging p.f. angle.

9. Load factor of a power station is defined as
(*a*) maximum demand / average load
(*b*) average load × maximum demand
(*c*) average load / maximum demand
(*d*) (average load × maximum demand)^{½}

10. Load factor of a power station is generally
(*a*) equal to unity
(*b*) less than unity
(*c*) more than unity
(*d*) equal to zero

11. Diversity factor is always
(*a*) equal to unity
(*b*) less than unity
(*c*) more than unity
(*e*) more than twenty

12. Load factor for heavy industries may be taken as
(*a*) 10 to 20%
(*b*) 25 to 40%
(*c*) 50 to 70%
(*d*) 70 to 80%

13. The load factor of domestic load is usually
(*a*) 10 to 15%
(*b*) 30 to 40%
(*c*) 50 to 60%
(*d*) 60 to 70%

14. Annual depreciation cost is calculated by
(*a*) sinking fund method
(*b*) straight line method
(*c*) both (*a*) and (*b*)
(*d*) none of the above

15. Depreciation charges are high in case of
(*a*) thermal plant
(*b*) diesel plant
(*c*) hydroelectric plant

16. Demand factor is defined as
(*a*) average load/maximum load
(*b*) maximum demand/connected load
(*c*) connected load/maximum demand
(*d*) average load × maximum load

17. High load factor indicates
(*a*) cost of generation per unit power is increased
(*b*) total plant capacity is utilised for most of the time
(*c*) total plant capacity is not properly utilised for most of the time
(*d*) none of the above

18. A load curve indicates
(*a*) average power used during the period
(*b*) average kWh (kW) energy consumption during the period
(*c*) either of the above
(*d*) none of the above

19. Approximate estimation of power demand can be made by
(*a*) load survey method
(*b*) statistical methods
(*c*) mathematical method
(*d*) economic parameters
(*e*) all of the above

20. Annual depreciation as per straight line method, is calculated by
(*a*) the capital cost divided by number of year of life
(*b*) the capital cost minus the salvage value, is divided by the number of years of life
(*c*) increasing a uniform sum of money per annum at stipulated rate of interest
(*d*) none of the above

21. A consumer has to pay lesser fixed charges in
(*a*) flat rate tariff
(*b*) two part tariff
(*c*) maximum demand tariff
(*d*) any of the above

22. In two part tariff, variation in load factor will affect
(*a*) fixed charges
(*b*) operating or running charges
(*c*) both (*a*) and (*b*)
(*d*) either (*a*) & (*b*)

23. In Hopkinson demand rate or two part tariff the demand rate for fixed charges are
(*a*) dependent upon the energy consumed
(*b*) dependent upon the maximum demand of the consumer
(*c*) both (*a*) and (*b*)
(*d*) neither (*a*) and (*b*)

24. Which plant can never have 100 percent load factor?
(*a*) Peak load plant
(*b*) Base load plant
(*c*) Nuclear power plant
(*d*) Hydro electric plant

25. The area under a load curve gives
(*a*) average demand
(*b*) energy consumed
(*c*) maximum demand
(*d*) none of the above

26. Different generating stations use following prime movers
(*a*) diesel engine
(*b*) hydraulic turbine
(*c*) gas turbine
(*d*) steam turbine
(*e*) any of the above

27. Diversity factor has direct effect on the
(*a*) fixed cost of unit generated
(*b*) running cost of unit generated
(*c*) both (*a*) and (*b*)
(*d*) neither (*a*) nor (*b*)

28. Following power plant has instant starting
(*a*) nuclear power plant
(*b*) hydro power plant
(*c*) both (*a*) and (*b*)
(*d*) none of the above

29. Which of the following generating station has minimum running cost?
(*a*) Nuclear
(*b*) Hydro
(*c*) Thermal
(*d*) Diesel

30. Power plant having maximum demand more than the installed rated capacity will have utilisation factor
(*a*) equal to unity
(*b*) less than unity
(*c*) more than unity
(*d*) none of the above

31. Load curve is useful in deciding the
(*a*) operating schedule of generating units
(*b*) sizes of generating units
(*c*) total installed capacity of the plant
(*d*) all of the above

32. Load curve of a power plant has always
(*a*) zero slope
(*b*) positive slope
(*c*) negative slope
(*d*) any combination of (*a*), (*b*) and (*c*)

33. Annual operating expenditure of a power plant consists of
(*a*) fixed charges
(*b*) semi-fixed charges
(*c*) running charges
(*d*) all of the above

34. Maximum demand on a power plant is
(*a*) the greatest of all "short time interval averaged" demand during a period
(*b*) instantaneous maximum value of kVA supplied during a period
(*c*) both (*a*) or (*b*)
(*d*) none of the above

35. Annual instalment towards depreciation reduces as rate of interest increases with
(*a*) sinking fund depreciation
(*b*) straight line depreciation
(*c*) reducing balances depreciation
(*d*) none of the above

36. Annual depreciation of the plant is proportional to the earning capacity of the plant vide

(*a*) sinking fund depreaciation
(*b*) straight line depreciation
(*c*) reducing balances depreciation
(*d*) none of the above

37. For high value of diversity factor, a power station of given installed capacity will be in a position to supply
(*a*) less number of consumers
(b) more number of consumers
(*c*) neither (*a*) nor (*b*)
(*d*) either (*a*) or (*b*)

38. Salvage value of the plant is always
(*a*) positive
(*b*) negative
(*c*) zero
(*d*) any of the above

39. Load curve helps in deciding
(a) total installed capacity of the plant
(b) size of the generating units
(c) operating schedule of generating units
(d) all of the above

40. ..... can generate power at unpredictable or uncontrolled times.
(*a*) Solar power plant
(*b*) Tidal power plant
(*c*) Wind power plant
(*d*) Any of the above

41. Direct conversion of heat into electric power is possible through
(*a*) fuel cell
(*b*) batteries
(*c*) thermionic converter
(*d*) all of the above

42. A low utilization factor for a plant indicates that
(*a*) plant is used for stand by purpose only
(*b*) plant is under maintenance
(*c*) plant is used for base load only
(*d*) plant is used for peak load as well as base load

43. Which of the following is not a source of power?
(*a*) Thermocouple
(*b*) Photovoltaic cell
(*c*) Solar cell
(*d*) Photoelectric cell

44. Which of the following should be used for extinguishing electrical fires?
(*a*) Water
(*b*) Carbon tetrachloride fire extinguisher
(*c*) Foam type fire extinguisher
(*d*) CO_2 fire extinguisher

45. Low power factor is usually not due to
(*a*) arc lamps
(*b*) induction motors
(*c*) fluorescent tubes
(*d*) incandescent lamp

46. Ships are generally powered by
(*a*) unclear power plants
(*b*) hydraulic turbines
(*c*) diesel engines
(*d*) steam accumulators
(*e*) none of the above

47. Direct conversion of heat into electrical energy is possible through
(*a*) fuel cells
(*b*) solar cells
(*c*) MHD generators
(*d*) none of the above

48. Which of the following place is not associated with nuclear power plants in India?
(*a*) Narora
(*b*) Tarapur
(*c*) Kota
(*d*) Benglore

49. During load shedding
(*a*) system power factor is changed
(*b*) some loads are switched off
(*c*) system voltage is reduced
(*d*) system frequency is reduced

50. Efficiency is the secondary consideration in which of the following plants?
(*a*) Base load plants
(*b*) Peak load plants
(*c*) Both (*a*) and (*b*)
(*d*) none of the above

51. Air will not be the working substance in which of the following?
(*a*) Closed cycle gas turbine
(*b*) Open cycle gas turbine
(*c*) Diesel engine
(*d*) Petrol engine

52. A nuclear power plant is invariably used as a
(*a*) peak load plant
(*b*) base load plant
(*c*) stand-by plant
(*d*) espinning reserve plant
(*e*) any of the above

53. .....power plant is expected to have the longest life.

(*a*) Steam
(*b*) Diesel
(*c*) Hydroelectric
(*d*) Any of the above

54. ..... power plant cannot have single unit of 100 MW.
(*a*) Hydroelectric
(*b*) Nuclear
(*c*) Steam
(*d*) Diseal
(*e*) Any of the above

55. Which of the following, in a thermal power plant, is not a fixed cost?
(*a*) Fuel cost
(*b*) Interest on capital
(*c*) Depreciation
(*d*) Insurance charges

56. ..... will offer the least load.
(*a*) Vacuum cleaner
(*b*) Television
(*c*) Hair dryer
(*d*) Electric shaver

57. In ..... fuel transportation cost is least.
(*a*) nuclear power plants
(*b*) diesel generating plants
(*c*) steam power stations

58. Which of the following equipment provides fluctuating load?
(*a*) Exhaust fan
(*b*) Lathe machine
(*c*) Welding transformer
(*d*) All of the above

59. The increased load during summer months is due to
(*a*) increased business activity
(*b*) increased water supply
(*c*) increased use of fans and air conditioners
(*d*) none of the above

60. ..... is the reserved generating capacity available for service under emergency conditions which is not kept in operation but in working order.
(*a*) Hot reserve
(*b*) Cold reserve
(*c*) Spinning reserve
(*d*) Firm power

61. Generating capacity onnected to the bus bars and ready to take load when switched on is known as .....
(*a*) firm power
(*b*) cold reserve
(*c*) hot reserve
(*d*) spinning reserve

62. ..... offers the highest electric load.
(*a*) Television set
(*b*) Toaster
(*c*) Vacuum cleaner
(*d*) Washing machine

63. ..... industry has the least power consumption per tonne of product.
(*a*) Soap
(*b*) Sugar
(*c*) Vegetable oil
(*d*) Caustic soda

64. With reference to a power station which of the following is not a fixed cost?
(*a*) Fuel cost
(*b*) Interest on capital
(*c*) Insurance changes
(*d*) Depreciation

65. ..... is invariably used as base load plant.
(*a*) Diesel engine plant
(*b*) Nuclear power plant
(*c*) Gas turbine plant
(*d*) Pumped storage plant

66. In a power plant if the maximum demand on the plant is equal to the plant capacity, then
(*a*) plant reserve capacity will be zero
(*b*) diversity factor will be unity
(*c*) load factor will be unity
(*d*) load factor will be nearly 60%

67. In case of ..... fuel transportation is the major problem.
(*a*) diesel power plants
(*b*) nuclear power plants
(*c*) hydro-electric power plants
(*d*) thermal power plants

68. Which of the following power plants need the least period for installation?
(*a*) Thermal power plant
(*b*) Diesel power plant
(*c*) Nuclear power plant
(*d*) Hydro-electric power plant

69. For which of the following power plants highly skilled engineers are required for running the plants?
(*a*) Nuclear power plants
(*b*) Gas turbine power plants
(*c*) Solar power plants
(*d*) Hydro-electric power plants

70. In which of the following power plants the maintenance cost is usually high?
(*a*) Nuclear power plant
(*b*) Hydro-electric power plants
(*c*) Thermal power plants
(*d*) Diesel engine power plants

71. ..... is inveriably used for peak load
(*a*) Nuclear power lant
(*b*) Steam turbine plant
(*c*) Pumped storage plant
(*d*) None of the above

72. Which of the following is not an operating cost?
(*a*) Maintenance cost
(*b*) Fuel cost
(*c*) Salaries of high officials
(*d*) Salaries of operating stall

73. Which of the following is the essential requirement of peak load plant?
(*a*) It should run at high speed
(*b*) It should produce high voltage
(*c*) It should be small in size
(*d*) It should be capable of starting quickly

74. Large capacity generators are invariably
(*a*) water cooled
(*b*) natural air cooled
(*c*) forced air cooled
(*d*) hydrogen cooled

75. By the use of which of the following power factor can be improved?
(*a*) Phase advancers
(*b*) Synchronous compensators
(*c*) Static capacitors
(*d*) Any of the above

76. An induction motor has relatively high power factor at
(*a*) rated r.p.m.
(*b*) no load
(*c*) 20 percent load
(*d*) near full load
(*e*) none of the above

77. Which of the following is the disadvantage due to low power factor?
(*a*) Poor voltage regulation
(*b*) Increased transmission losses
(*c*) High cost of equipment for a given load
(*d*) All of the above

78. In a distribution system, in order to improve power factor, the synchronous capacitors are installed
(*a*) at the receiving end
(*b*) at the sending end
(*c*) either (*a*) or (*b*)
(*d*) none of the above

79. Satic capacitors are rated in terms of
(*a*) kW
(*b*) kWh
(*c*) kVAR
(*d*) none of the above

80. Base load plants usually have ..... capital cost, ... operating cost and ..... load factor.
(*a*) high, high, high
(*b*) high, low, high
(*c*) low, low, low
(*d*) low, high, low

81. Which of the following is the disadvantage of a synchronous condenser?
(*a*) High maintenance cost
(*b*) Continuous losses in motor
(*c*) Noise
(*d*) All of the above

82. For a consumer the most economical power factor is generally
(*a*) 0.5 lagging
(*b*) 0.5 leading
(*c*) 0.95 lagging
(*d*) 0.95 leading

83. A synchronous condenser is virtually which of the following?
(*a*) Induction motor
(*b*) Underexcited synchronous motor
(*c*) Over excited synchronous motor
(*d*) D.C. generator
(*e*) None of the above

84. For a power plant which of the following constitutes running cost?
(*a*) Cost of wages
(*b*) Cost of fuel
(*c*) Cost of lubricants
(*d*) All of the above

85. In an interconnected system, the diversity factor of the whole system
(*a*) remains unchanged
(*b*) decreases
(*c*) increases
(*d*) none of the above

86. Generators for peak load plants are usually designed for maximum efficienty at
(*a*) 25 to 50 percent full load
(*b*) 50 to 75 percent full load
(*c*) full load
(*d*) 25 percent overload

87. ..... will be least affected due to charge in supply voltage frequency.
(*a*) Electric clock
(*b*) Mixer grinder
(*c*) Ceiling fan
(*d*) Room heater

88. For the same maximum demand, if load factor is decreased, the cost generation will
(*a*) remain unchanged
(*b*) decrease
(*c*) increase

89. The connected load of a domestic consumer is around
(*a*) 5 kW
(*b*) 40 kW
(*c*) 80 kW
(*d*) 120 kW

90. Which of the following is not necessarily an advantage of interconnecting various power stations?
(*a*) Improved frequency of power supplied
(*b*) Reduction in total installed capacity
(*c*) Increased reliability
(*d*) Economy in operation of plants

91. A power transformer is usually rated in
(*a*) kW
(*b*) kVAR
(*c*) kWh
(*d*) kVA

92. ..... public sector undertaking is associated with erection and sometimes running of thermal power plants
(*a*) NTPC
(*b*) SAIL
(*c*) BEL
(*d*) BHEL

93. Most efficient plants are normally used as
(*a*) peak load plants
(*b*) base load plants
(*c*) either (*a*) or (*b*)
(*d*) none of the above

94. For a diesel generating station the useful life is expected to be around
(*a*) 15 to 20 years
(*b*) 20 to 50years
(*c*) 50 to 75 years
(*d*) 75 to 100 years

95. Which of the following is not a method for estimating depreciation charges?
(*a*) Sinking fund method
(*b*) Straight line method
(*c*) Diminishin value method
(*d*) Halsey's 50–50 formula

96. The expected useful life of an hydroelectric power station is around
(*a*) 15 years
(*b*) 30 years
(*c*) 60 years
(*d*) 100 years

97. In a load curve the highest point represents
(*a*) peak demand
(*b*) average demand
(*c*) diversified demand
(*d*) none of the above

98. Which of the following source of power is least reliable?
(*a*) Solar energy
(*b*) Geothermal power
(*c*) Wind power
(*d*) MHD

99. In Indial production and distribution of electrical energy is confined to
(*a*) private sector
(*b*) public sector
(*c*) government sectors
(*d*) joint sector
(*e*) none of the above

100. A pilot exciter is provided on generators for which of the following reasons?
(*a*) To excite the poles of main exciter
(*b*) To provide requisite starting torque to main exciter

(*c*) To provide requisit starting torque to generator
(*d*) None of the above

101. The primary reason for low power factor is supply system is due to installation of
(*a*) induction motors
(*b*) synchronous motors
(*c*) single phase motors
(*d*) d.c. motors

102. an over excited synchronous motor on no-load is known as
(*a*) syachronous condenser
(*b*) generator
(*c*) induction motor
(*d*) alternator

103. Which of the following is an advantage of static capacitor for power factor improvement?
(*a*) Little maintenance cost
(*b*) Ease in installation
(*c*) Low losses
(*d*) All of the above

104. For any type of consumer the ideal tariff is
(*a*) two part tariff
(*b*) three part tariff
(*c*) block rate tariff
(*d*) any of the above

105. The efficiency of a plant is of least concern when it is selected as
(*a*) peak load plant
(*b*) casual run plant
(*c*) either (*a*) or (*b*)
(*d*) base load plant

106. Power generation cost reduces as
(*a*) diversity factor increases and load factor decreases
(*b*) diversity factor decreases and load factor increases
(*c*) both diversity factor as well as load factor decrease
(*d*) both diversity factor as well as load factor increase

107. The depreciation charges in diminishing value method are
(*a*) light in early years
(*b*) heavy in early years
(*c*) heavy in later years
(*d*) same in all years

108. The area under daily load curve divided by 24 hours gives
(*a*) average load
(*b*) least load
(*c*) peak demand
(*d*) total kWh generated

109. Maximum demand tariff is generally not applied to domestic consumers because
(a) they consume less power
(*b*) their load factor is low
(*c*) their maximum demand is low
(*d*) none of the above

110. A 130 MW generator is usually ..... cooled
(*a*) air
(*b*) oxygen
(*c*) nitrogen
(*d*) hydrogen

111. For cooling of large size generators hydrogen is used because
(*a*) it is light
(*b*) it offers reduced fire risk
(*c*) it has high thermal conductivity
(*d*) all of the above

112. Major share of power produced in India is through
(*a*) diesel power plants
(*b*) hydroelectric power plants
(*c*) thermal power plants
(*d*) nuclear power plants

113. Which of the following may not be the effect of low plant operating power factor?
(*a*) Improved illumination from lighting
(*b*) Reduced voltage level
(*c*) Over loaded transformers
(*d*) Overloaded cables

114. Which of the following plants is almost inevitably used as base load plant?
(*a*) Diesel engine plant
(*b*) Gas turbine plant
(*c*) Nuclear power plant
(*d*) Pumped storage plant

115. Which of the following component, in a steam power plant, needs maximum maintenance attention?
(*a*) Steam turbine
(*b*) Condenser
(*c*) Water treatment plant
(*d*) Bioler

116. For the same cylinder dimensions and speed, which of the following engine will produce least power?
(*a*) Supercharged engine
(*b*) Diesel engine

(*c*) Petrol engine
(*d*) All of the above engines will equal power

117. The least share of power is provided in India, by which of the following power plants?
(*a*) Diesel power plants
(*b*) Thermal power plants
(*c*) Hydro-electric power plants
(*d*) Nuclear power plants

118. Submarines for under water movement, are powered by which of the following?
(*a*) Steam accumulators
(*b*) Air motors
(*c*) Diesel engines
(*d*) Batteries

119. An alternator coupled to a ..... runs at slow speed, as compared to as compared to others.
(*a*) diesel engine
(*b*) hydraulic turbine
(*c*) steam turbine
(*d*) gas turbine

120. The effect of electric shock on human body depends on which of the following
(*a*) current
(*b*) voltage
(*c*) duration of cantact
(*d*) all of the above

121. Which lightening stroke is most dangerous?
(*a*) Direct stroke on line conductor
(*b*) Indirect stroke on conductor
(*c*) Direct stroke on tower top
(*d*) Direct stroke on ground wire

122. Which of the following devices may be used to provide protection against lightening over voltages?
(*a*) Horn gaps
(*b*) Rod gaps
(*c*) Surge absorbers
(*d*) All of the above

123. When the demand of consumers is not met by a power plant, it will resort to which of the following?
(*a*) Load shedding
(*b*) Power factor improvement at the generators
(*c*) Penalising high load consumers by increasing the charges for electricity
(*d*) Efficient plant operation.

124. Load shedding is possible through which of the following?
(*a*) Switching of the loads
(*b*) Frequency reduction
(*c*) Voltage reducteion
(*d*) Any of the above

125. In power plants insurance cover is provided for which of the following?
(*a*) Unskilled workers only
(*b*) Skilled workers only
(*c*) Equipment only
(*d*) All of the above

126. A company can raise funds through
(*a*) fixed deposits
(*b*) shares
(*c*) bonds
(*d*) any of the above

127. Which of the following are not repayable after a stipulated period?
(*a*) Shares
(*b*) Fixed deposits
(*c*) Cash certificates
(*d*) Bonds

128. The knowledge of diversity factor helps in determining
(*a*) plant capacity
(*b*) average load
(*c*) peak load
(*d*) kWh generated
(*e*) none of the above

129. Load shedding is done to
(*a*) improve power factor
(*b*) run the equipment efficiently
(*c*) repair the machine
(*d*) reduce peak demand

130. when a plant resorts to load shedding it can be concluted that
(*a*) peak demand is more than the installed capacity
(*b*) daily load factor is unity
(*c*) diversity factor is zero
(*d*) plant is under repairs

131. Which of the following is the disadvantage of static capacitor for power factor improvement?
(*a*) Easily damaged by high voltage
(*b*) Cannot be repaired
(*c*) Short service life
(*d*) All of the above

132. If the tariff for electrical energy charages provides incentive by way of reduced charges for higher consumption, then it can be concluede that
(*a*) Load factor is unity

(*b*) power is generated through hydroelectric plant
(*c*) plant has sufficient reserve capacity
(*d*) station has more than two generators

133. Anything having some heat value can be used as fuel in case of
(*a*) open cycle gas turbines
(*b*) closed cycle gas turbines
(*c*) petrol engines
(*d*) diesel engines

ANSWERS

1. (*d*)	**2.** (*c*)	**3.** (*a*)	**4.** (*b*)	**5.** (*c*)	**6.** (*b*)	**7.** (*a*)
8. (*d*)	**9.** (*c*)	**10.** (*b*)	**11.** (*c*)	**12.** (*d*)	**13.** (*a*)	**14.** (*c*)
15. (*a*)	**16.** (b)	**17.** (*b*)	**18.** (*b*)	**19.** (*e*)	**20.** (*b*)	**21.** (*c*)
22. (*b*)	**23.** (*b*)	**24.** (*a*)	**25.** (*b*)	**26.** (*e*)	**27.** (*a*)	**28.** (*d*)
29. (*b*)	**30.** (*c*)	**31.** (*d*)	**32.** (*d*)	**33.** (*d*)	**34.** (*a*)	**35.** (*a*)
36. (*c*)	**37.** (*b*)	**38.** (*d*)	**39.** (*d*)	**40.** (*d*)	**41.** (*c*)	**42.** (*a*)
43. (*a*)	**44.** (*b*)	**45.** (*d*)	**46.** (*c*)	**47.** (*c*)	**48.** (*d*)	**49.** (*b*)
50. (*b*)	**51.** (*a*)	**52.** (*b*)	**53.** (*c*)	**54.** (*d*)	**55.** (*a*)	**56.** (*d*)
57. (*a*)	**58.** (*c*)	**59.** (*c*)	**60.** (*b*)	**61.** (*d*)	**62.** (*b*)	**63.** (*c*)
64. (*a*)	**65.** (*b*)	**66.** (*a*)	**67.** (*d*)	**68.** (*b*)	**69.** (*a*)	**70.** (*c*)
71. (*c*)	**72.** (*c*)	**73.** (*d*)	**74.** (*d*)	**75.** (*d*)	**76.** (*d*)	**77.** (*d*)
78. (*a*)	**79.** (*c*)	**80.** (*b*)	**81.** (*d*)	**82.** (*c*)	**83.** (*c*)	**84.** (*d*)
85. (*c*)	**86.** (*b*)	**87.** (*f*)	**88.** (*c*)	**89.** (*a*)	**90.** (*a*)	**91.** (*d*)
92. (*a*)	**93.** (*b*)	**94.** (*a*)	**95.** (*d*)	**96.** (*d*)	**97.** (*a*)	**98.** (*c*)
99. (*b*)	**100.** (*a*)	**101.** (*a*)	**102.** (*a*)	**103.** (*d*)	**104.** (*b*)	**105.** (*c*)
106. (*d*)	**107.** (*b*)	**108.** (*a*)	**109.** (*c*)	**110.** (*d*)	**111.** (*d*)	**112.** (*c*)
113. (*a*)	**114.** (*c*)	**115.** (*d*)	**116.** (*d*)	**117.** (*a*)	**118.** (*d*)	**119.** (*b*)
120. (*d*)	**121.** (*a*)	**122.** (*d*)	**123.** (*a*)	**124.** (*d*)	**125.** (*c*)	**126.** (*d*)
127. (*d*)	**128.** (a)	**129.** (*d*)	**130.** (*a*)	**131.** (*d*)	**132.** (*c*)	**133.** (*b*)

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ELECTRONICS DEVICES AND CIRCUITS

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CHAPTER 51

SEMI-CONDUCTOR PHYSICS

Learning Objectives

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- Bohr's Atomic Model
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A worker at a hydro-electric power station is carrying out routine maintenance on a high-voltage insulator made of a ceramic material.

CONTENTS

51.1. The Atom

According to the model of atom proposed by Bohr in 1913, an atom is composed of a number of electrons moving in circular or elliptical orbits around a relatively heavy nucleus of protons and neutrons as shown in Fig. 51.1. Although, this simple model of an atom has been replaced by later models, yet it affords a convenient method of understanding the working of semi-conductor devices. Electron has a mass of nearly 9.1×10^{-31} kg and a charge of 1.6×10^{-19} C. The diameter of an atom is approximately 10^{-10} m and that of the nucleus about 10^{-15} m. The number of protons in the atom of an element gives its atomic number while the atomic mass number is determined by the number of protons and neutrons present in the nucleus.

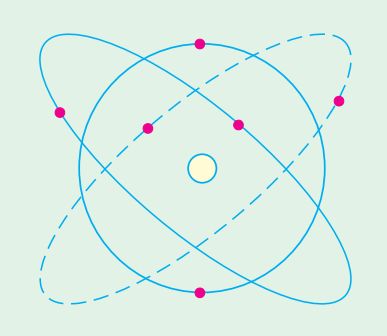
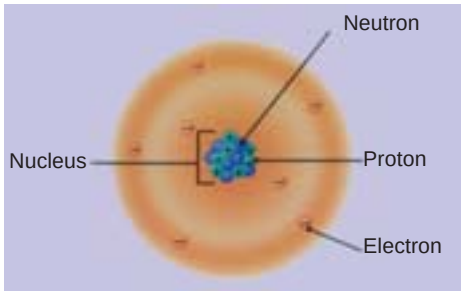


Fig. 51.1

51.2. Bohr's Atomic Model

The nuclear atomic model proposed by Rutherford in 1911 was found to suffer from two serious drawbacks concerning distribution of extra-nuclear electrons and stability of the atom as a whole. It was later on superseded by atomic model proposed by Bohr in 1913. Using Planck's Quantum Theory, Bohr made the following postulates :



1. The atom has a massive positively-charged nucleus;
2. The electrons revolve round their nucleus in circular orbits, the centrifugal force being balanced by the electrostatic pull between the nucleus and electrons;
3. An electron cannot revolve round the nucleus in any arbitrary orbit but in just certain definite discrete orbits. ***Only those orbits are possible (or permitted) for which the orbital angular momentum (i.e. moment of momentum) of the electron is equal to an integral multiple of $h/2\pi$ i.e. orbital angular momentum = $nh/2\pi$ where n is an integer and h is Planck's constant***. Such orbits are also known as ***stationary orbits***;
4. While revolving in these permitted stationary (or stable) orbits, the electron does not radiate out any electromagnetic energy. In other words, ***the permissible orbits are non-radiating paths of the electron***;
5. The atom radiates out energy only when an electron *jumps* from one orbit to another. If E_2 and E_1 are the energies corresponding to two orbits before and *after* the jump, the frequency of the emitted photon is given by the relation

$$E_2 - E_1 = hf \quad \text{or} \quad \Delta E = hf$$

where f is the frequency of the emitted radiations.



Neils Bohr
(1885–1962)

Explanation. If I is the moment of inertia of an electron and ω its angular velocity, then as per assumption (3) given above

$\omega I = nh/2\pi$ or $(mr^2)\,\omega = nh/2\pi$ or $(mr^2)v/r = nh/2\pi$ or $mvr = n.h/2\pi$

Alternatively, since the momentum of the revolving electron is mv, its moment about the nucleus is mvr (Fig. 51.2).

Hence, $mvr = nh/2\pi$

when n = 1, 2, 3 etc. for the first, second and third orbits respectively. It is called the ***principal quantum number*** and because it can take whole number values only, ***it fixes the size of the allowed orbits*** (also called Bohr's circular orbits).

Let the different permitted orbits have energies of E_1, E_2, E_3 etc. as shown in Fig. 51.3 (*a*). The electron can be raised from n = 1 orbit to any other higher orbit if it is given proper amount of energy.

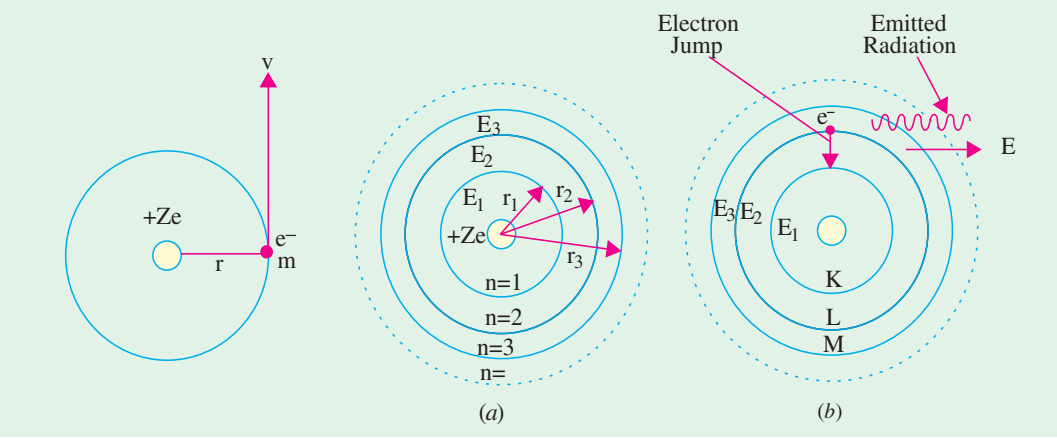


Fig. 51.2 **Fig. 51.3**

When it drops back to n = 1 orbit after a short interval of time, it gives out the energy difference ΔE in the form of a radiation as shown in Fig. 51.3 (*b*). The relation between the energy released and frequency of the emitted radiation is

$$E_2 - E_1 = hf \quad \text{or} \quad \Delta E = hf$$

51.3. Calculations Concerning Bohr's Atomic Model

The above postulates concerning Bohr's atomic model can be utilized to calculate not only the radii of different electron orbits but also the velocity and orbital frequency possessed by different electrons.

Now, the stability of the atom requires that the centrifugal force acting on the revolving electron be balanced by the electrostatic pull exerted by the positively charged nucleus on the electron.

$$\therefore \quad \frac{mv^2}{r} = \frac{Ze \cdot e}{4\pi\epsilon_0 r^2} \quad \text{or} \quad mv^2 r = \frac{Ze^2}{4\pi\epsilon_0}$$

Also, according to Bohr's postulates, $mvr = nh/2\pi$.

The above two equations may be used to find the radii of different Bohr's circular orbits.

(*a*) Radii of Orbits. Eliminating v from the above two equations, we get

$$r = \frac{\epsilon_0 n^2 h^2}{\pi mZe^2} \quad \text{or} \quad r_n = \frac{\epsilon_0 n^2 h^2}{\pi mZe^2}$$

It is seen that the radii of the permitted orbits vary as the square of the principal quantum number n. Also, $r_n = n^2 r_1$.

For hydrogen atom, $r_1 = 0.53 \times 10^{-10}$ m, $r_2 = 2^2\ r_1 = 2.12 \times 10^{-10}$ m. All values between r_1 and r_2 are forbidden.

(*b*) Velocity of Revolving Electrons. The velocity of a revolving electron as found from the above equations is

$$v = Ze^2/2\epsilon_0 nh = 9 \times 10^9 . 2\pi Ze^2/nh.$$

It shows that velocity is inversely proportional to n *i.e.* $v \propto 1/n$. Hence, the electron in the innermost orbit has the highest velocity (nearly 1/37 of the velocity of light).

(*c*) Orbital Frequency. The orbital ***rotational*** frequency* of an electron is

$$f = \frac{v}{2\pi r} = \frac{mZ^2e^4}{4\epsilon_0^2 n^3h^3}.$$ It is seen that $f \propto 1/n^3$

(*d*) Electron Energy. The orbital energy of a revolving electron is of two types :

(*i*) Kinetic energy due to the motion of the electron. Its value is = (1/2) mv^2.

As seen from above, $mv^2 = Ze^2/4\pi\epsilon_0 r$.

Hence, K.E. = (1/2) $mv^2 = Ze^2/8\pi\epsilon_0 r$. —as represented by curve 2 in Fig. 51.4.

Eliminating r from above,

K.E. = $mZ^2e^4/8\epsilon_0^2 n^2h^2$.

(*ii*) Potential energy—because the electron lies in the electric field of the positive nucleus.

Now, the potential at a point distant r from the nucleus is

$$V = Q/4\pi\epsilon_0 r = Ze/4\pi\epsilon_0 r$$

The potential energy of an electron (of charge—e) is

$$\text{P.E.} = V \times (-e) = -\frac{Ze^2}{4\pi\epsilon_0 r}$$

or $$= -\frac{mZ^2e^4}{4\epsilon_0^2 n^2h^2}$$ — curve 1 in Fig. 51.4

$\therefore$ Orbital energy = K.E. + P.E.

$$= \frac{Ze^2}{8\pi\epsilon_0 r} - \frac{Ze^2}{4\pi\epsilon_0 r}$$

$$= -\frac{Ze^2}{8\pi\epsilon_0 r}$$ — curve 3 in Fig. 51.4

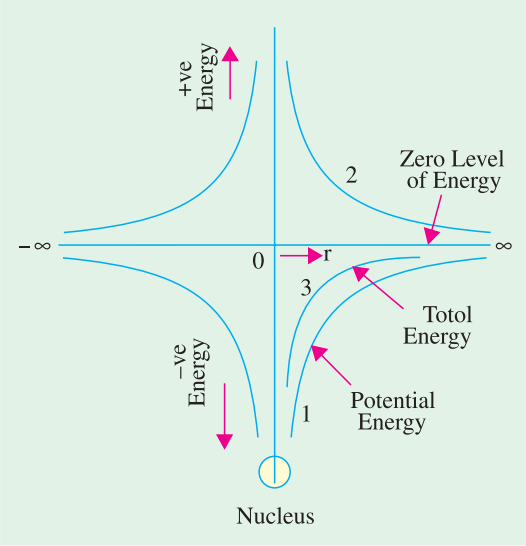


Fig. 51.4

Variations of potential energy and total energy are shown in Fig. 51.4.

If r is eliminated from the above equation, the orbital energy** is

$$E_n = -mZ^2e^4/8\epsilon_0^2 n^2h^2$$

where n = 1, 2, 3..... etc. for the energy states that it is possible for the electron to have. It will be seen that $E_n = E_1/n^2$.

51.4. Normal, Excited and Ionized Atom

Consider the case of the simplest atom *i.e.* hydrogen atom. When its only electron is in its innermost orbit (n = 1), then the atom is said to be in its normal (or unexcited) state. Generally, it is this condition in which most of the free hydrogen atoms in a gas are found to exist at normal room temperature and pressure. However, if spark is passed through hydrogen gas contained in a vessel, then high-speed electrons produced by the spark collide with hydrogen atoms and may either completely remove the n = 1 electron from them or raise it to higher permitted orbits having n = 2, 3, 4 etc.

When the electron is completely removed from the atom, the atom is said to be ionized. If, however, the electron is forced into an outer or higher n-value orbit, then the atom, is said to be

* It is the mechanical frequency of rotation and should not be confused with the frequency of emitted radiations.

** The negative sign only indicates that this much energy is required to remove the electron from the atom.

excited (or in an excited state). The atom does not remain in the excited state longer than 10^{-8} second because the electron under the attractive force of the nucleus jumps to the lower permitted orbit. In doing so, the electron loses the energy it had earlier gained during collision. However, the electron may return by several jumps, thereby emitting many different radiations of different frequencies.

51.5. Electron Energy Levels in Hydrogen Atom

As seen from Art. 51.3, orbital energy of an electron revolving in n_{th} orbit or shell is

$$E_n = -m Z^2 e^4/8\epsilon_0^2 n^2 h^2$$

In the case of hydrogen atom, $Z = 1$, hence

$$E_n = -\frac{me^4}{8\epsilon_0^2 n^2 h^2} = -\frac{me^4}{8\epsilon_0^2 h^2}\cdot\frac{1}{n^2} = -\frac{21.7\times10^{-19}}{n^2} \text{ joules}$$

$$= \frac{-21.7\times10^{-19}}{1.6\times10^{-19}}\cdot\frac{1}{n^2} = -\frac{13.6}{n^2}\,eV \qquad (\because 1\,eV = 1.6\times10^{-19} \text{ joules})$$

This expression gives the total energy of an electron when it occupies any one of the different orbits (or shells) of the hydrogen atom.

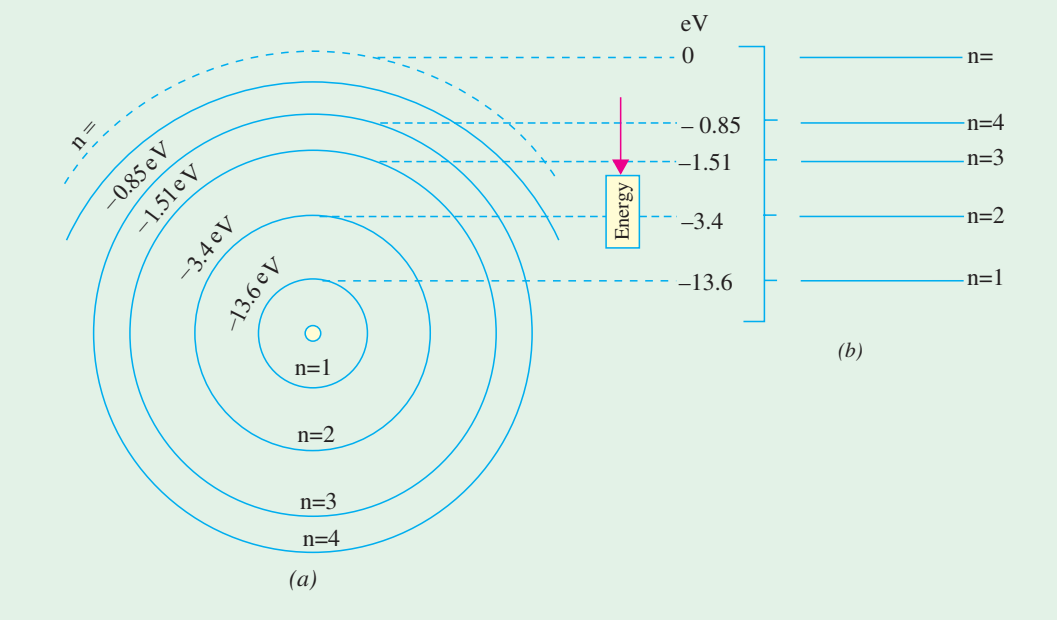


Fig. 51.5

For K-shell, $n = 1$, $E_1 = -13.6$ eV

For L-shell, $n = 2$, $E_2 = -13.6/2^2 = -3.4$ eV

For M-shell, $n = 3$, $E_3 = -13.6/3^2 = -1.51$ eV etc.

Instead of drawing various electron orbits to the scale of their radii as in Fig. 51.5 (*a*), it is customary to draw horizontal lines to an energy scale as shown in Fig. 51.5 (*b*) and such a diagram is called energy level diagram (*ELD*) of an atom. In this array of energies, the higher (*i.e.* less negative) energies are at the top while the lower (*i.e.* more negative) energies are at the bottom. The various electron jumps between allowed orbits now become vertical arrows between energy levels. Greater the length of the arrow, greater is the energy 'hf' of the radiated photon.

Example 51.1. *Calculate the value of the kinetic, potential and total energy of an electron revolving in Bohr's first orbit in a hydrogen atom.*

Solution. ***(i)*** $$K.E. = \frac{me^4}{8\epsilon_0^2 n^2 h^2} = \frac{9.1\times10^{-31}\times(1.6\times10^{-19})^4}{8\times(8.854\times10^{-12})^2\times1^2\times(6.625\times10^{-34})^2}$$

$$= 21.7\times10^{-19} \text{ joules} = \mathbf{13.6\ eV}$$

(ii) $$P.E. = \frac{me^4}{4\epsilon_0^2 n^2 h^2} = -43.4\times10^{-19} \text{ joules} = \mathbf{-27.2\ eV}.$$

(iii) Total energy = $K.E. + P.E. = 13.6 + (-27.2) = \mathbf{-13.6\ eV}$

51.6. Orbital (or Azimuthal) Quantum Number

According to Bohr's postulates (Art. 51.2), there is only one orbit (and hence one energy level) corresponding to each value of the principal quantum number n. However, subsequent experimental evidence revealed that all orbits (except $n = 1$ orbit) consist of more than one orbit called sub-orbits. This group of sub-orbits is collectively known as ***shell.*** For example, $n = 1$ shell or K-shell consists of only one orbit which may also be called its own sub-orbit or sub-shell. The $n = 2$ shell or L-shell consists of two sub-shells. Similarly, $n = 3$ shell or M-shell consists of three sub-shells. In other words, ***the number of sub-shells is equal to the n-value of the shell***. The $2n^2$ electrons of the shell now get divided between these sub-shells.

In order to distinguish between different sub-shells belonging to a given shell, a new quantum number called orbital (or azimuthal) quantum number l has been introduced. This quantum number can have integral values lying between zero and $(n - 1)$ *i.e.* $0 \le l \le (n - 1)$. It is helpful in the following ways :

(a) It gives the number of sub-shells which are contained in one shell. The ***number*** of sub-shells is equal to the number of values which l can have subject to the restriction $0 \le l \le (n - 1)$ as shown in Fig. 51.6 (*a*).

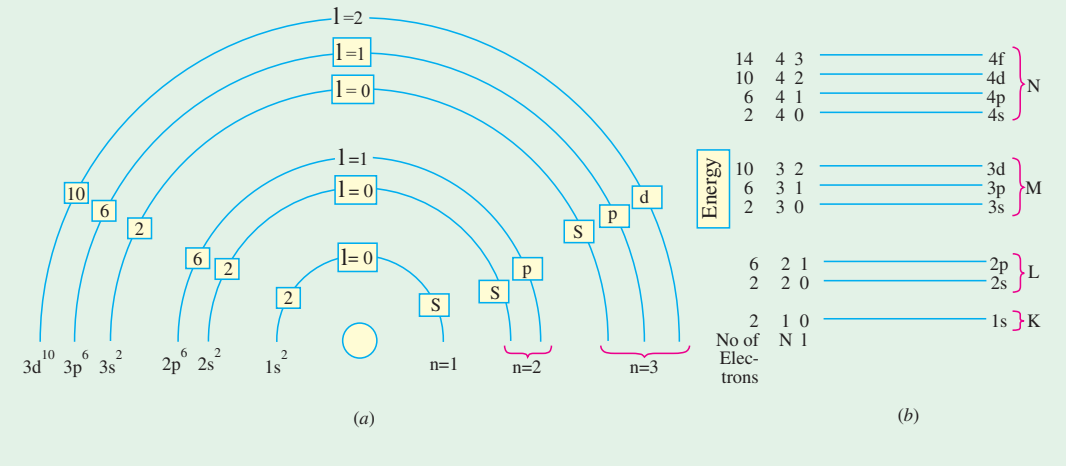


Fig. 51.6

(b) It helps to distinguish between different sub-shells of a shell by its different values for each sub-shell. Consider the following cases :

1. ***n* = 1 shell.** Here, l can have only one value *i.e.* zero. Hence, K-shell has only one sub-shell with two quantum numbers of $n = 1$ and $l = 0$.

2. ***n* = 2 shell.** Here, $l = 0, 1$. Hence, this shell has two sub-shells* which are distinguishable from each other by their different quantum numbers of $n = 2, l = 0$ and $n = 2, l = 1$. (Fig. 51.6).
3. ***n* = 3 shell.** Here $l = 0, 1, 2$, showing that *M*-shell has ***three*** sub-shells which differ in their *l*-values. The two quantum numbers for the three sub-shells are :

   $n = 3, l = 0$; $n = 3, l = 1$; $n = 3, l = 2$.

(*c*) It helps to determine the ***shape*** of different sub-shells of a given shell. Some sub-shells are circular whereas others are elliptical in shape. If a and b are semi-major and semi-minor axes respectively of an ellipse, n and l are its principal quantum number and orbital quantum number respectively, then the relation between them is

$$\frac{b}{a} = \frac{l+1}{n}$$

Consider the following cases :

1. **K-shell or n = 1 shell**

   Here, $l = 0$, hence $b/a = (0 + 1)/1$ or $b/a = 1$ or $b = a$. In other words, the first shell (also called sub-shell or *K*-shell) around the nucleus of an atom is circular in shape as shown in Fig. 51.7 (*a*).

2. **L-shell or n = 2 shell**

   As stated earlier and shown in Fig. 51.6, it has two more sub-shells.

   First sub-shell with $l = 0$. Here, $\frac{b}{a} = \frac{0+1}{2} = \frac{1}{2}$ or $b = \frac{a}{2}$. It is elliptical in shape as shown in Fig. 51.7 (*b*).

   Second sub-shell with $l = 1$. Here $\frac{b}{a} = \frac{1+1}{2} = 1$ or $b = a$.

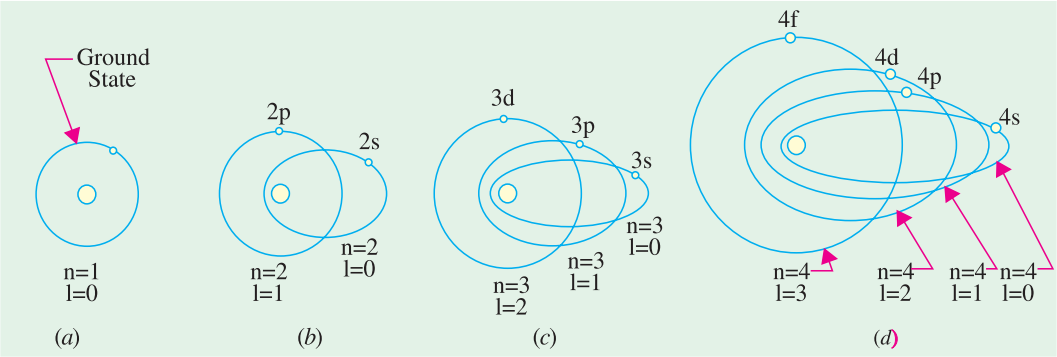


Fig. 51.7

This sub-shell has circular shape as shown in Fig. 51.7 (*b*).

3. **M-shell or n = 3 shell**

   It has three sub-shells with $l = 0, 1, 2$.

   First sub-shell with $l = 0$. Here, $b/a = (0 + 1)/3 = 1/3$. It is elliptical in shape.

   Second sub-shell with $l = 1$. Here, $b/a = (1 + 1)/3 = 2/3$. It is also elliptical in shape.

   Third sub-shell with $l = 2$. Here, $b/a = (2 + 1)/3 = 1$ or $b = a$. Obviously, this sub-shell is circular in shape.

   Fig. 51.7 shows to shapes of different sub-shells per values of n from 1 to 4.

(*d*) It determines the distribution of electrons in various sub-shells of a shell. The maximum

* Different sub-shells have been shown circular for the sake of simplicity although, in practice, most of them are elliptical.

number of electrons which a sub-shell of any n-value can accommodate is = 2 $(2l + 1)$. Consider the following :

1. **K-shell.** Here $l = 0$, hence it can have 2 $(2 \times 0 + 1) = 2$ electrons. They are designated as $1s^2$ electrons.
2. **L-shell.** First sub-shell with $l = 0$ can have 2 $(2 \times 0 + 1) = 2$ electrons which are designated as $2s^2$ electrons.
   Second sub-shell with $l = 1$ can have a maximum of 2 $(2 \times 1 + 1) = 6$ electrons which are designated as $2p^6$ electrons.
   Total number of electrons in the two sub-shells is 2 + 6 = 8 which tallies with $2n^2 = 2 \times 2^2 = 8$.
3. **M-shell.** First sub-shell with $l = 0$ has, as explained above, 2 electrons which are designated as $3s^2$ electrons.
   Second sub-shell $l = 1$ can have 6 electrons written as $3p^6$ electrons.
   Third sub-shell with $l = 2$ can accommodate a maximum of $2(2 \times 2 + 1) = 10$ electrons which are known as $3d^{10}$ electrons.
   Again, it will be seen that total number of electrons in the three sub-shells of $n = 3$ shell is = 2 + 6 + 10 = 18 which equals $2n^2 = 2 \times 3^2 = 18$.
4. **N-shell.** First sub-shell with $l = 0$ can have maximum of two electrons designated as $4s^2$ electrons. Second sub-shell can contain 6 electrons known as $4p^6$ electrons.
   Similarly, third sub-shell can have 10 electrons designated as $4d^{10}$ electrons.
   The fourth sub-shell can have a maximum of $2(2 \times 3 + 1) = 14$ electrons whose designation is $4f^{14}$ electrons.

The total number of electrons is = 2 + 6 + 10 + 14 = 32 which is in accordance with the number $2n^2 = 2 \times 4^2 = 32$.

Incidentally, it should be noted that different l values of 0, 1, 2, 3, 4..... etc. are identified as s, p, d, f, g..... etc.

The different sub-shells and maximum number of electrons they can accommodate are tabulated below.

Shell	K	L		M			N			
n	1	2		3			4			
l	0	0	1	0	1	2	0	1	2	3
sub-shell	s	s	p	s	p	d	s	p	d	f
Maximum No. of electrons $= 2(2l + 1)$	2	2	6	2	6	10	2	6	10	14
Total No. of electrons $= 2n^2$	2	8		18			32			

(e) The orbital quantum number l also quantizes the orbital angular momentum p_l associated with each sub-shell

$$p_l = l.h/2\pi \quad \text{where} \quad l = 0, 1, 2, 3, \text{.....etc.}^*$$

However, quantum mechanical considerations indicate that $p_l \neq l.h/2\pi$ as stated above but instead $p_l = l^*.h/2\pi$ where $l^* = [l\,(l + 1)]^{1/2}$.

---

* According to Bohr's theory, $p_1 = n\,.\,h/2\pi$.

51.7. Electron Configuration of Atoms

By electron configuration of an atom is meant the distribution of its electrons in its various sub-shells around the nucleus. Following three rules govern the electron distribution :

1. ***Maximum number*** of electrons a shell can have is = $2n^2$.
2. In the nth shell, there are n sub-shells having different values of l such as 0, 1, 2..... $(n - 1)$.
3. Each sub-shell can accommodate a maximum of $2(2l + 1)$ electrons.

Consider the following atoms :

(*i*) Sodium atom, Z = 11. It has 11 electrons. Hence, its electronic configuration is $1s^2$, $2s^2$, $2p^6$, $3s^1$. Obviously, *Na* has a single electron in its outermost sub-shell and hence is said to be monovalent. Same property is possessed by other alkali metals like Li, K, Rb and Cs. They have similar chemical properties and are, therefore, included in the same group in the periodic table.

The electron in the inner sub-shells are very tightly bound to the nucleus and cannot be easily removed. In other words, they have high ***binding energy***.

(*ii*) Copper atom, Z = 29. It has 29 electrons. Its electronic configuration is $1s^2$, $2s^2$, $2p^6$, $3s^2$, $3p^6$, $3d^{10}$, $4s^1$.

51.8. Orbital Magnetic Quantum Number (*ml*.)

It determines the spatial ***orientation*** of elliptical electron orbits with respect to an applied magnetic field. There are restrictions on the orientations of the electron orbits because of which they are said to be ***space quantized***. Whereas l determines the orbital angular momentum, m_l represents the magnitude of the component of angular momentum along the direction of the magnetic field.

Looked from a different angle, m_l determines the number of sub-subshells in a given shell.

This quantum number can have any one of the $(2l + 1)$ values ranging from $+ l$ to $- l$ including zero *i.e.* l, $(l - 1)$, $(l - 2)$.....2, 1, 0, – 1, – 2,..... $- (l - 2)$, $- (l - 1)$, $- l$.

Each sub-subshell can accommodate a maximum of 2 electrons, so that maximum number of electrons in a shell becomes $2(2l + 1)$ as stated earlier in Art. 51.6 (*d*).

51.9. Magnetic Spin Quantum Number (m_s)

It has been found that an electron spins around its own mechanical axis as it rotates in orbit around the nucleus. When the electron is subjected to a magnetic field, its spin axis orientates itself either parallel to or antiparallel to the direction of the field.

This quantum number arises out of quantization of the electron spin angular momentum. It determines the spin orientation up or down and has correspondingly two values of + 1/2 and – 1/2.

51.10. Pauli's Exclusion Principle

This principle which was enunciated by Pauli in 1925 states that in an atom, ***no two electrons can have the same set of values for its four quantum number n, l, m_l and m_s***. In other words, no two electrons can be described by an identical set of four quantum numbers. They may have at the most three numbers alike but at least one must be different. Consequently, it restricts the number of electrons an atom can have.

Consider the case of He atom which has two electrons. These electrons occupy K-shell ($n = 1$) and are designated as $1s^2$ electrons. Their four quantum numbers are as follows :

	n	l	m_l	m_s
1st electron :	1	0	0	+ 1/2
2nd electron :	1	0	0	– 1/2

As seen, the two electrons have different sets of four quantum numbers (1, 0, 0, +1/2) and (1, 0, 0, – 1/2) as required by Pauli's Exclusion Principle.

This principle affords a very rational theoretical justification for the electronic shell structure of the atoms deduced experimently.

Pauli's Exclusion Principle may be used to find the total number of electrons an atom can have in its various shells. Consider the following examples :

n = 1, K-shell

l = 0, 1s sub-shell, m_l = 0; m_s = ± 1/2	2 electrons
Total	2 electrons

n = 2, L-shell

(*i*) l = 0, 2s-shell, m_l = 0; m_s = ± 1/2	2 electrons
(*ii*) l = 1, 2p sub-shell, m_l = – 1, 0, + 1; m_s = ± 1/2	6 electrons
Total	8 electrons

n = 3, M-shell

(*i*) l = 0, 3s sub-shell, m_l = 0 ; m_s = ± 1/2	2 electrons
(*ii*) l = 1, 3p sub-shell, m_l = –1, 0, +1 ; m_s = ± 1/2	6 electrons
(*iii*) l = 2, 3d sub-shell, m_l = –2, – 1, 0, + 1, +2 ; m_s = ± 1/2	10 electrons
Total	18 electrons

n = 4, N-shell

(*i*) l = 0, 4s sub-shell, m_l = 0, m_s = ±1/2	2 electrons
(*ii*) l = 1, 4p sub-shell, m_l = –1, 0, 1 ; m_s = ±1/2	6 electrons
(*iii*) l = 2, 4d sub-shell, m_l = – 2, – 1, 0, + 1, + 2, m_s = ±1/2	10 electrons
(*iv*) l = 3, 4f sub-shell, m_l = – 3, – 2, – 1, 0, + 1, + 2, + 3, m_s = ± 1/2	14 electrons
Total	32 electrons

51.11. Energy Bands in Solids

In the case of an ***isolated single*** atom, there are ***single*** energy levels as shown for a hydrogen atom in Fig. 51.5. But there are significant changes in the energy levels when atoms are brought close together as in solids.

It is found that each of the energy levels of an atom splits into N levels of energy where N is the number of atoms in the crystal. Each original energy level becomes a ***band*** of very closely-spaced levels of slightly different energy. The individual energies within the band are so close together that, for many purposes, the energy band may be considered to be a continuous one.

Fig. 51.8 (a) shows the splitting of K, L and M levels as the distance between different atoms is reduced. At first, only valence level or M-level is affected as shown by dotted vertical line marked A, then as separation is reduced, inner shells also become affected as indicated by dotted vertical line B.

Consider the case of Na crystal which consists of an ordered array of many closely-packed sodium atoms usually referred to as crystal lattice. Each Na atom has 11 electrons arranged in different shells and sub-sheels as shown in Fig. 51.9 (a). As seen, the ls, 2s and 2p sub-shells are filled but 3s sub-shell is incomplete and could hold one electron more. The electrons in each sub-shell occurs specific energy levels as shown in Fig. 51.9 (b). For a small sodium crystal containing 10^{20} atoms, the band formed by splitting of s-subshell has 2 electronic levels (one with spin up and the others with spin down). Similarly in a p-band there will be 6×10^{20} closely packed levels because there are 6 electrons in a filled p subshell.

In general, in an assembly of N atoms, the number of possible energy states is N. Since only two electrons of opposite spin can occupy the same state (as per Pauli's Exclusion Principle discussed earlier), the maximum number of electrons which these N states can occupy is $2N$.

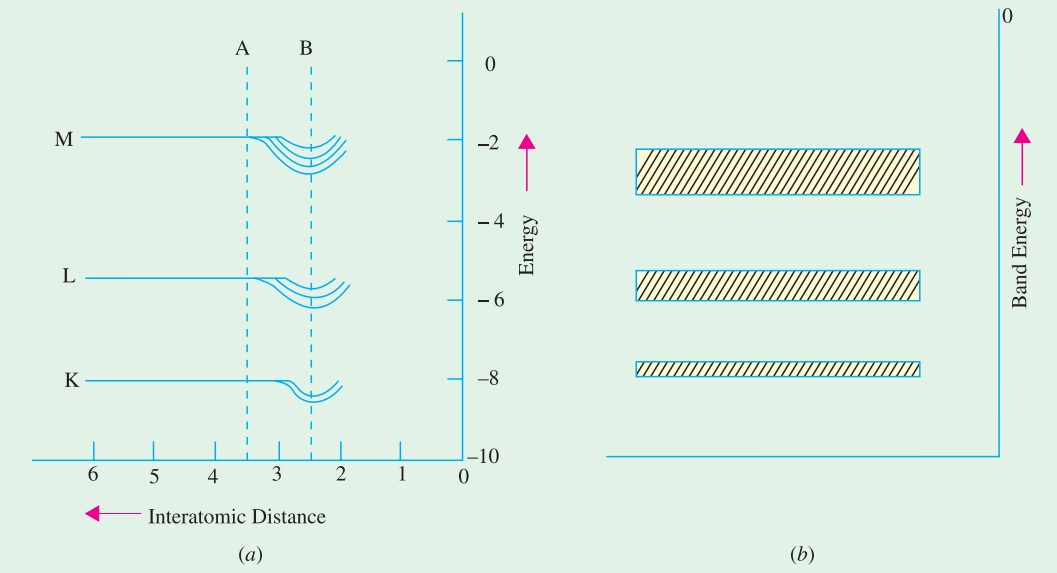


Fig. 51.8

51.12. Spacing Between Energy Levels of a Band

It will be quite interesting as well as instructive to calculate the spacing between different energy levels in an energy band. A crystal weighing one milligram contains about 10^{19} atoms. If we assume the valency band to be an s-band, it will contain 2×10^{19} levels. Suppose the width of the energy band is 2 eV. Then, it is obvious that 2×10^{19} levels per milligram are spread over an energy band width of 2 eV. Hence, spacing between different levels = $2/(2 \times 10^{19}) = 10^{-19}$ eV. It will be appreciated that even though energy levels are discrete, the picture of a band as a continuum of energy levels is a very good approximation. This splitting of the single energy level of an ***isolated*** atom into a band of

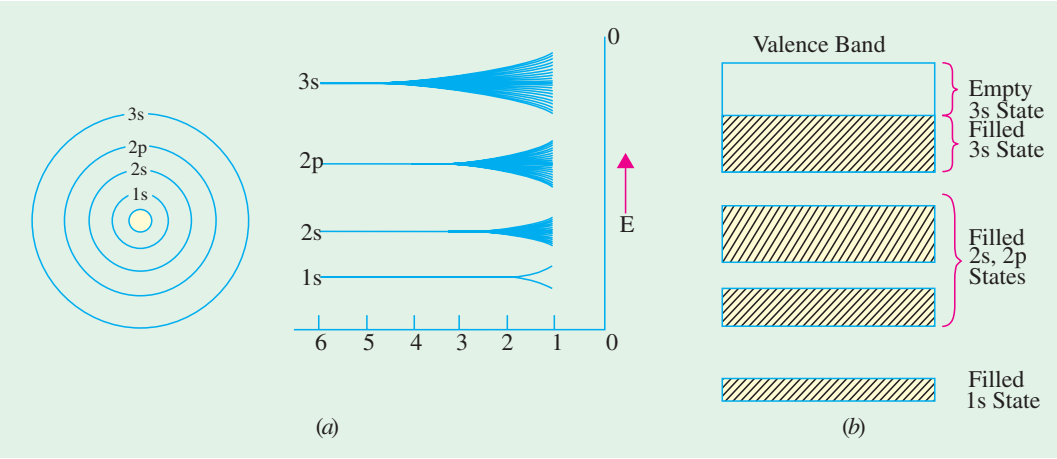


Fig. 51.9

energy in the case of a ***solid*** is responsible for most of the electrical, magnetic and optical properties of that solid. It is worth pointing out here that in gases under normal conditions of temperature and pressure, the atomic spacing is so great that there is no splitting of energy levels and hence no band formation.

51.13. Energy Bands in Lithium and their Occupancy

Consider the case of lithium metal—the simplest atom which forms a solid at ordinary temperature. Its atom has three electrons, two of which have the same energy and the third one has higher value of energy. In an isolated single atom, two electrons move round the electron orbit with $n = 1$ whereas the third occupies the orbit with $n = 2$ as shown in Fig. 51.10 (*a*). Now, consider a piece of lithium metal containing 100 atoms. It will be found that the lower level (with $n = 1$) forms a band of 200 electrons occupying 100 different energy states. The higher level (with $n = 2$) forms a wider band of 100 energy states which could, as before, accommodate 200 electrons. But as there are only 100 electrons available (one from each atom), this energy band remains half-filled [Fig. 51.10 (*b*)].

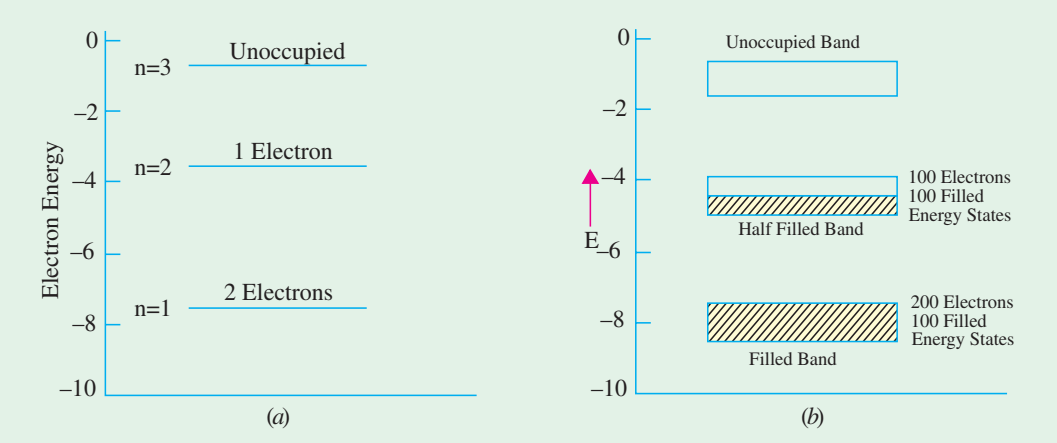


Fig. 51.10

51.14. Valence and Conduction Bands

The outermost electrons of an atom *i.e.* those in the shell furthermost from the nucleus are called ***valence*** electrons and have the ***highest*** energy* or least binding energy. It is these electrons which are most affected when a number of atoms are brought very close together as during the formation of a solid. The states of lower-energy electrons orbiting in shells nearer to the nucleus are little, if at all, affected by this atomic proximity.

The band of energy occupied by the valence electrons is called the ***valence band*** and is, obviously, the ***highest occupied band***. It may be completely filled or partially filled with electrons but never empty.

The next higher permitted energy band is called the ***conduction*** band and may either be ***empty*** *or* ***partially filled*** with electrons. In fact, it may be defined as the lowest unfilled energy band.

In conduction band, electrons can move freely and hence are known as ***conduction*** electrons. The gap between these two bands is known as the ***forbidden energy gap***.

It may be noted that the covalent force of the crystal lattice have their source in the valence band. If a valence electron happens to absorb enough energy, it jumps across the forbidden energy gap and enters the conduction band (Fig. 51.12). An electron in the conduction band can jump to an adjacent conduction band more readily than it can jump back to the valence band from where it had come earlier. However, if a conduction electron happens to radiate too much energy, it will suddenly reappear in the valence band once again.

When an electron is ejected from the valence band, a covalent bond is broken and a positively-charged hole is left behind. This hole can travel to an adjacent atom by acquiring an electron from

* In the algebraic sense only. Their binding energy is the least.

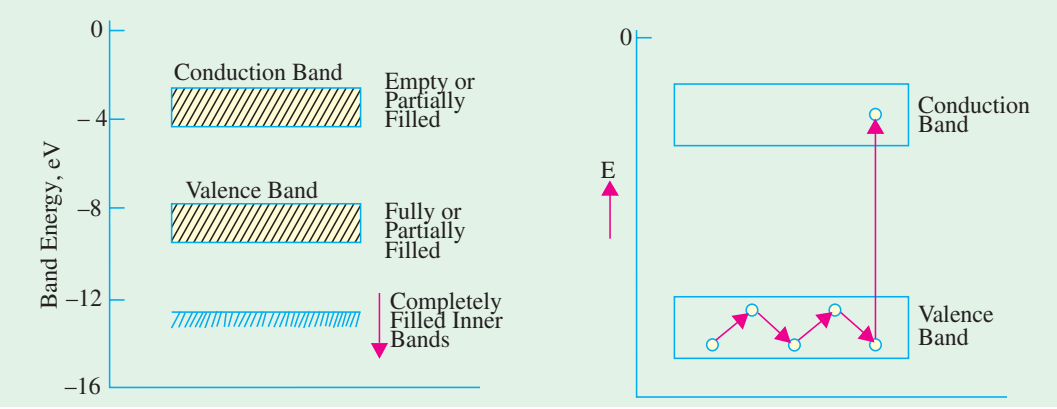


Fig. 51.11 Fig. 51.12

that atom which involves breaking an existing covalent bond and then re-establishing a covalent bond by filling up the hole. It is to be noted carefully that holes are filled by electrons which move fromadjacent atoms without passing through the forbidden energy gap.

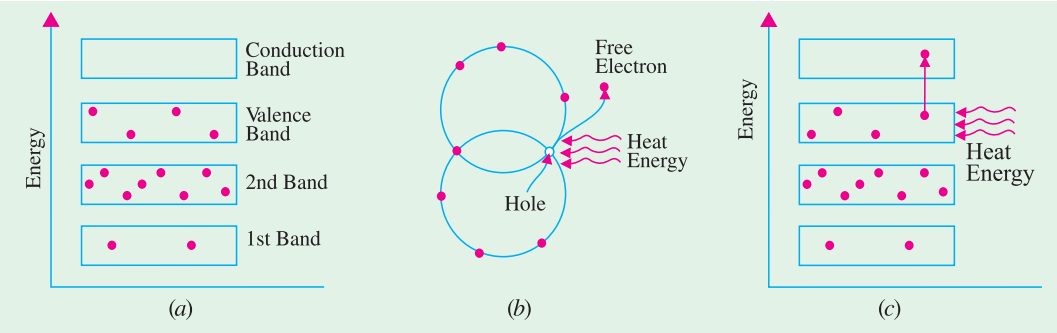


Fig. 51.13

It is simply another way of saying that conditions in the conduction band have nothing to do with the hole flow. It points to a very important distinction between the hole current and electron current—although holes flow with ease, they ***experience more opposition than electron flow in the conduction band***.

To summarize the above, it may be repeated that :

1. conduction electrons are found in and freely flow in the ***conduction*** band ;
2. holes exist in and flow in the valence band ;
3. conduction electrons move almost twice as fast as the holes.

Fig. 51.13 (*a*) shows the energy band diagram of an unexcited silicon atom ($Z = 14$) with its electronic distribution. When silicon crystal is given thermal or light energy from outside [Fig. 51.13 (*b*)], some electrons gain sufficient energy to jump the gap from the valence band into the conduction band thereby becoming free electrons [Fig. 51.13 (*c*)]. For every electron which jumps to conduction band, a hole is created in the valence band. In this way, an electron-hole pair is created.

51.15. Insulators, Conductors and Semiconductors

The electrical conduction properties of different elements and compounds can be explained in terms of the electrons having energies in the valence and conduction bands. The electrons lying in the lower energy bands, which are normally filled, play no part in the conduction process.

(i) **Insulators.** Stated simply, insulators are those materials in which valence electrons are

bound very tightly to their parents atoms, thus requiring very large electric field to remove them from the attraction of their nuclei. In other words, insulators have no free charge carriers available with them under normal conditions.

In terms of energy bands, it means that insulators (*a*) have a full valence band,

1. have an empty conduction band,
2. have a large energy gap (of several eV) between them and
3. at ordinary temperatures, the probability of electrons from full valence band gaining sufficient energy so as to surmount energy gap and thus become available for conduction in the conduction band, is slight.

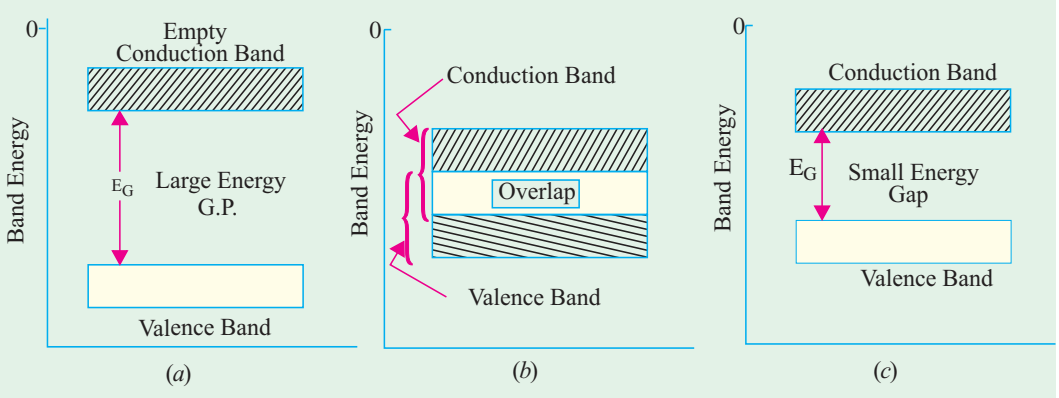


Fig. 51.14

This is shown in Fig. 51.14 (*a*). For conduction to take place, electrons must be given sufficient energy to jump from the valence band to the conduction band. Increase in temperature enables some electrons to go to the conduction band which fact accounts for the negative resistance-temperature coefficient of insulators.

***(ii)* Conductors.** Put in a simple way, conducting materials are those in which plenty of free electrons are available for electric conduction.

In terms of energy bands, it means that electrical conductors are those which have overlapping valence and conduction bands as shown in Fig. 51.14 (*b*).

In fact, there is no physical distinction ***between the two bands***. Hence, the availablity of a large number of conduction electrons.

Another point worth noting is that in the absence of forbidden energy gap in good conductors, there is no structure to establish holes. The ***total current in such conductors is simply a flow of electrons***. It is exactly for this reason that the existence of holes was not discovered until semi-conductors were studied thoughly.

***(iii)* Semiconductors.** A semiconductor material is one whose electrical properties lie in between those of insulators and good conductors. Examples are : germanium and silicon.



Insulators

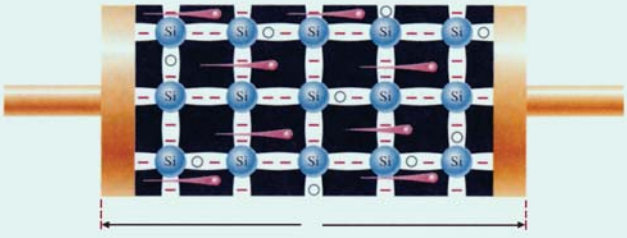
In terms of energy bands, semiconductors can be defined as those materials which have almost an empty conduction band and almost filled valence band with a very narrow energy gap (of the order of 1 eV) separating the two.

At 0°K, there are no electrons in the conduction band and the valence band is completely filled. However, with increase in temperature, width of the forbidden energy bands is decreased so that

some of the electrons are liberated into the conduction band. In other words, conductivity of semiconductors increases with temperature. Moreover, such departing electrons leave behind positive holes in the valence band (Fig. 51.12). Hence, semiconductor current is the sum of electron and hole currents flowing in opposite directions.

51.16. Crystal Structure

Semiconductors like germanium and silicon, have crystalline structure. That is the atoms are arranged in three-dimensional periodic fashion. The periodic arrangement of atoms in a crystal is called ***lattice***. In a crystal, an atom strays far from a single, fixed position. The thermal vibrations associated with the atom are centred about this position. For a given semiconductor (silicon or germanium), there is a ***unit cell*** that is representative of the entire lattice. By repeating the unit cell throughout the crystal, we can generate the entire lattice.



Current conduction through semiconductor

There are several different types of crystal lattice depending upon the symmetry and internal structure. One of them is the cubic crystal lattice. There are three basic types of unit cells in a cubic crystal lattice. These are simple cubic (SC), base centred cubic (BCC) and face centred cubic (FCC). Fig. 51.15 (*a*) shows a SC crystal. In this unit cell, each corner of the cubic lattice is occupied by an atom (indicated by a small sphere) that has six equidistant nearest neighbouring atoms. The dimension "*a*" is called the ***lattice constant***. Only ***podium*** is crystallised into the simple cubic lattice.

Fig. 51.15 (*b*) shows a BCC unit cell. In this unit cell, each atom has eight nearest neighbouring atoms. Crystals exhibiting the BCC lattices include those of ***sodium*** and ***tungsten***. Fig. 51.15 (*c*) shows a FCC unit cell. This unit cell has one atom at each of the six cubic faces in addition to the eight corner atoms. In an FCC lattice, each atom has 12 nearest neighbouring atoms. A large number of elements exhibit the FCC lattice form. These include ***aluminium, copper, gold*** and ***platinum***.

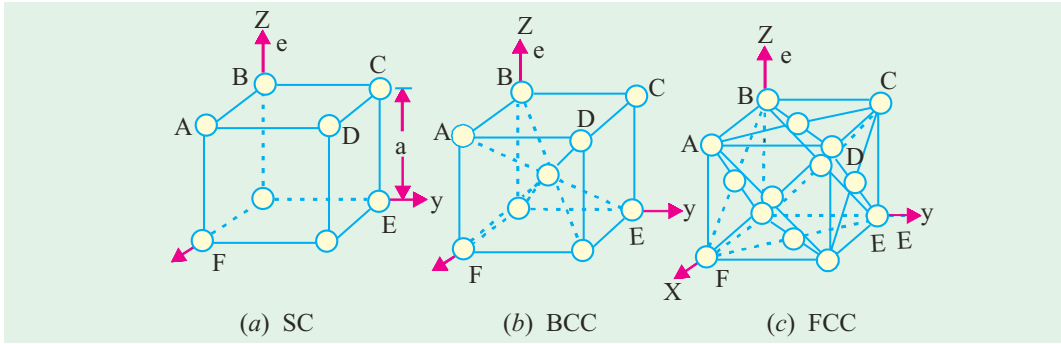


Fig. 51.15

Fig. 51.16 shows the crystal structure of silicon and germanium—the element semiconductors. This type of structure is called diamond crystal structure and it belongs to cubic crystal family. All atoms are identical in a diamond lattice. Note that each atom in diamond lattice is surrounded by four equidistant neighbours that lie at the corners of a tetrahedron.

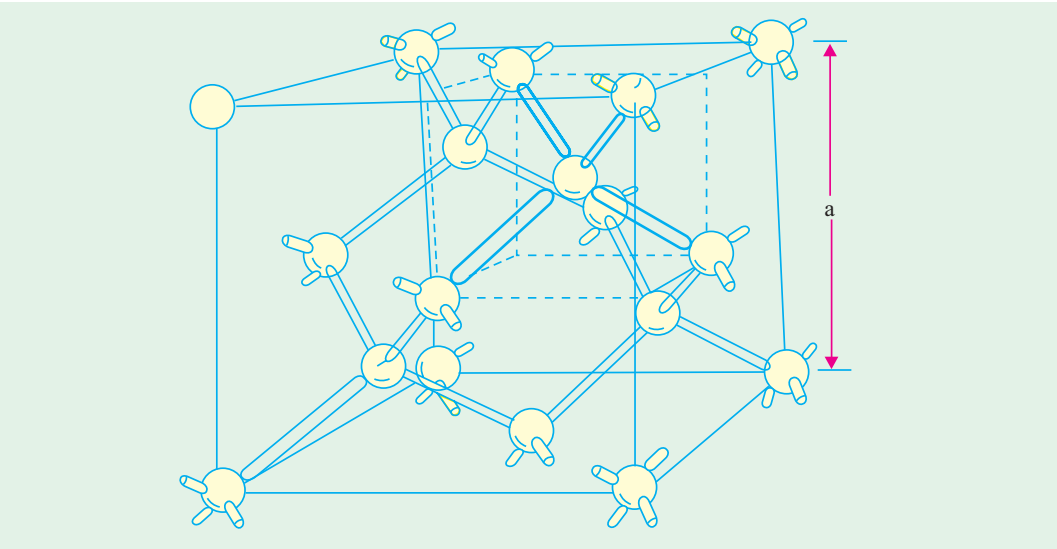


Fig. 51.16

51.17. Representation of Crystal Planes and Directions

Referring to Fig. 51.15 (*b*) again, we note that the there are four atoms in the ABCD plane and give atoms in ACEF plane (four atoms from the corners and one from the centre). Moreover, the atomic spacing are different for the two planes. Therefore the crystal properties along different planes are different. The electrical and other device characteristics are dependent on the crystal orientation. A convenient method of defining or representing the various planes in a crystal is to use ***Miller indices***. These indices are obtained using the following steps :

1. Find the intercepts of the plane on the three coordinate axes in terms of lattice constant.
2. Take the reciprocals of these numbers and reduce them to the smallest three integers having the same ratio.
3. Enclose the result in parentheses (*hk* 1) as the Miller indices for a single plane.

For example, consider the plane ABC having the intercept at *a*, 2*a* and 2*a* along the three rectangular coordinate axes as shown in Fig. 51.17. Taking the reciprocal of these intercepts, we get 1, 1/2, 1/2. Multiplying each fraction by the least common multiplier 2, we find that the smallest three integers having the same ratio are 2, 1 and 1. Thus the Miller indices from the plane are (211). The plane ABC can also be referred to as (211) plane.

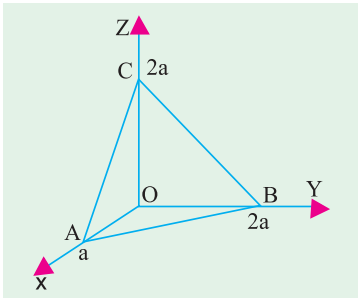


Fig. 51.17

Fig. 51.18 shows the Miller indices of important planes in cubic crystal. It may be noted that for a plane that intercepts the *X*-axis on the negative side of the origin, the Miller indices are represented by (hk1). Similarly for a plane that intercepts the *Y*-axis on the negative side of the origin, the Miller indices are represented by (hk1) and so on.

Sometimes it convenient to represent the Miller indices of the planes of equivalent symmetry by (hk1). For example, in cubic crystal, the planes (100), (010), (001), $(\bar{1}00)$, $(0\,\bar{1}0)$ and $(00\,\bar{1})$ have the same symmetry. Therefore, these planes can be represented by Miller indices as (100). Note that we have used the curly brackets to represent a set of planes with equivalent symmetry.

The line that originates from the origin and passes through the plane at right angles to it is called ***crystal direction***. The crystal direction is indicated by enclosing the Miller indices in a square brackets

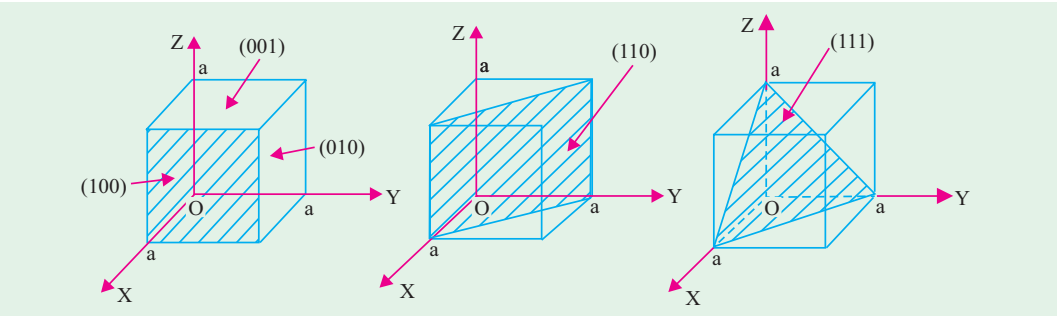
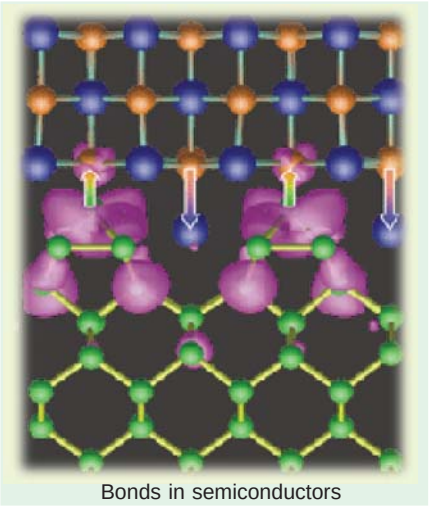


Fig. 51.18

i.e., [hk1]. For example, the direction idicated by [100] is a direction for the X-axis and it is perpendicualr to (100) plane. Similarly [111] direction is perpendicular to (111) plane. It is possible to represent a set of equivalent directions by < hk1 >. Notice the use of carat signs. Thus < 100 > represents a set of directions for [100], [010], [100], $[0\,\bar{1}0]$ and $[00\,\bar{1}]$.

51.18. Atomic Binding in Semiconductors

Semiconductors like germanium* and silicon, have crystalline structure. Their atoms are arranged in an ordered array known as crystal lattice. Both these materials are tetravalent *i.e.* each has four valence electrons in its outermost shell. The neighbouring atoms form ***covalent*** bonds by sharing four electrons with each other so as to achieve inert gas structure (*i.e.* 8 electrons in the outermost orbit). A two-dimensional view of the germanium crystal lattice is shown in Fig. 51.19 (*b*) in which circles represent atom cores consisting of the nuclei and inner 28 electrons. Each ***pair*** of lines represents a covalent bond. The dots represent the valence electrons. It is seen that each atom has 8 electrons under its influence.



Bonds in semiconductors

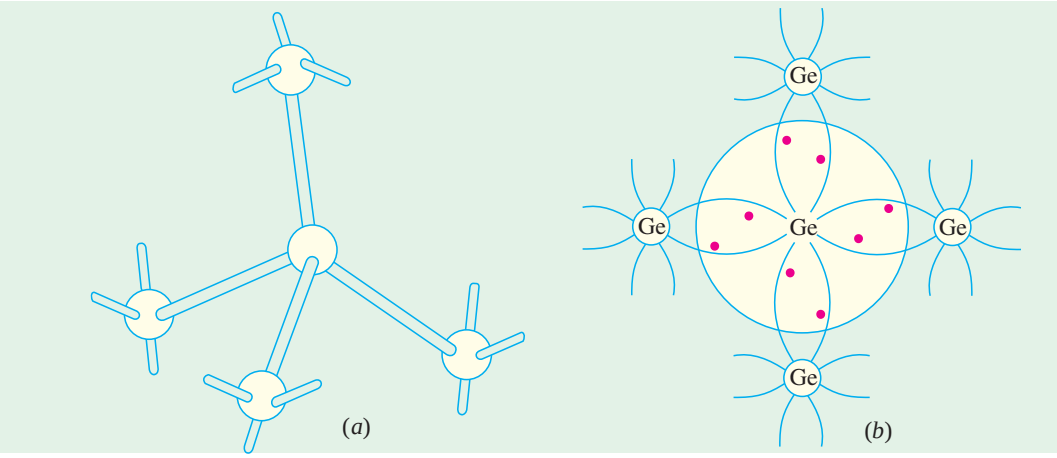


Fig. 51.19

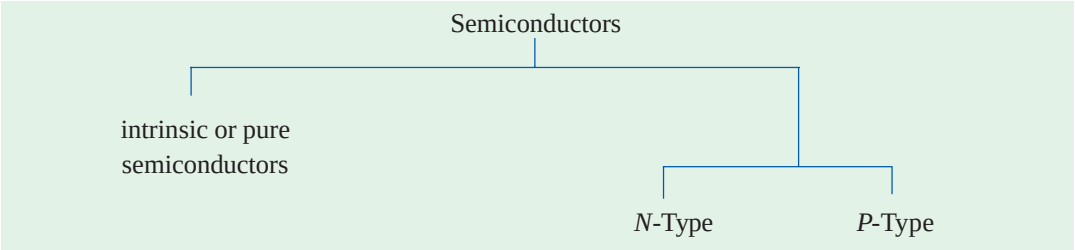
* A single germanium atom has 32 electrons out of which only four electrons take part in electrical properties of germanium, the remaining 28 electrons being tightly bound to the nucleus. The four electrons revolve in the outermost shell and are called valence electrons.

A 3-dimensional view of germanium crystal lattice is shown in Fig. 51.19 (*a*) where each atom is surrounded symmetrically by four other atoms forming a tetrahedral crystal. Each atom shares a valence electron with each of its four neighbours, thereby forming a stable structure.

In the case of pure (*i.e.* intrinsic) germanium, the covalent bonds have to be broken to provide electrons for conduction. There are many ways of rupturning the covalent bond and thereby setting the electrons free. One way is to increase the crystal temperature above 0°K.

It may be noted that covalent crystals are characterised by their hardness and brittleness. Their brittleness is due to the fact that in such crystals, adjacent atoms must remain in accurate alignment since the bond is strongly directional and formed along a line joining the atoms. The hardness is due to the great strength of the covalent bond itself.

51.19. Types of Semiconductors



Semiconductors can be classified as shown below :

51.20. Intrinsic Semiconductors

An intrinsic semiconductor is one ***which is made of the semiconductor material in its extremely pure form***.

Common examples of such semiconductors are : pure germanium and silicon which have forbidden energy gaps of 0.72 eV and 1.1 eV respectively. The energy gap is so small that even at ordinary room temperature, there are many electrons which possess sufficient energy to jump across the small energy gap between the valence and the conduction bands. However, it is worth noting that for each electron liberated into the conduction band, a positively charged hole is created in the valence band (Fig. 1.20). When an electric field is applied to an intrinsic semiconductor at a temperature greater than 0°K, conduction electrons move to the anode and the holes in the valence band move to the cathode. Hence semiconductor current consists of movement of electrons and holes in opposite directions. Electron current is due to movement of electrons in the conduction band whereas hole current is within the valence band as a result of the holes 'jumping' from one atom to another.

As stated above, in pure semiconductors, electric conduction is due to the thermally-generated electron hole pairs. Hence in pure semiconductors kept in the dark, thermally-generated charge carriers are the only means of conduction. The number of such charge carriers per unit volume (*i.e.* intrinsic carrier density) is given by

$$n_i = N \exp(-E_g / 2kT)$$

where N is constant for a given semiconductor, E_g is the band gap energy in joules, k is Boltzmann's constant and T is the temperature in °K.

Example 51.2. *Find the intrinsic carrier concentration in silicon at 300° K for which* $N = 3 \times 10^{25}\ m^{-3}$, $E_g = 1.1$ *eV,* $\mu_e = 0.14\ m^2$/V-s *and* $\mu_h = 0.05\ m^2$/V–s. *Also, find the conductivity of silicon.* **(Electronics-II, Madras Univ. 1993)**

Solution. The intrinsic carrier concentration in pure silicon is given by

$$n_i = N \exp(-E_g/2kT)$$

Now, $N = 3 \times 10^{25}\ m^{-3}$, $E_g = 1.1\ eV = 1.1 \times 1.6 \times 10^{-19} = 1.76 \times 10^{-19}\ J$

$k = 1.38 \times 10^{-23}$ J/K, $T = 300°K$

$\therefore \quad n_i = 3 \times 10^{25}\ (-1.76 \times 10^{-19})/2 \times 1.38 \times 10^{-23} \times 300) = \mathbf{2 \times 10^{16}\ m^{-3}}$

$\sigma = n_i e\ (\mu_e + \mu_h) = 2 \times 10^{16} \times 1.6 \times 10^{-19}\ (0.14 + 0.05) = \mathbf{0.61 \times 10^{-3}\ S/m}$

51.21. Hole Formation in Semiconductors

The formation of a hole which is a positive charge carrier is explained below :

As shown in Fig. 51.21, suppose the covalent bond is broken at *A* and the electron has moved through the crystal lattice leaving behind a hole in the covalent bond. An electron at *B* may jump into the vacant hole at *A* and later, an electron at *C* may jump into the hole at *B* and so on. In this way, by a succession of electron movements, a hole will appear at *G* and a negative charge would have moved from *G* to *A*. It would, however, be more convenient to regard positive charge to have moved from *A* to *G* and this conception gives rise to a hole as a positive charge carrier as if it were an electron with a positive charge. It should be clearly understood that these holes are due to the movement of electrons in the valence band and that each electron movement corresponds to a collision. The drift velocity of holes is, obviously, much less than the drift velocity of electrons.

Alternatively, an intrinsic semiconductor may be defined as one ***in which the number of conduction electrons is equal to the number of holes***.

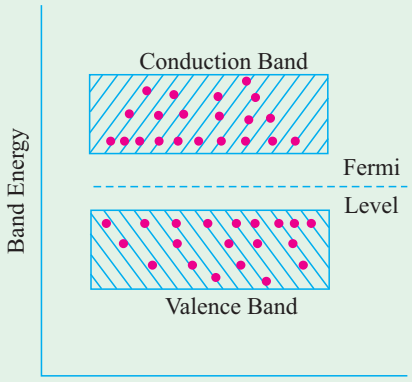


Fig. 51.20

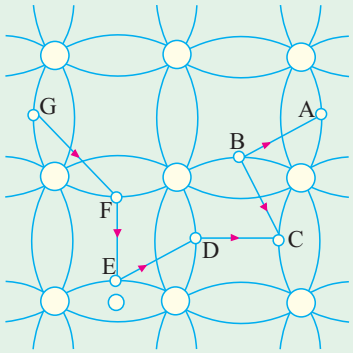


Fig. 51.21

Schematic energy band diagram of an intrinsic semiconductor at room temperature is shown in Fig. 51.20. Only two bands *i.e.* valence and conduction bands have been shown since lower filled bands are not of any consequence. Here, Fermi level* lies exactly in the middle of the forbidden energy gap.

51.22. Fermi Level in an Intrinsic Semiconductor

It can be proved that in an intrinsic semiconductor, Fermi energy level* E_F lies in the middle of the energy gap *i.e.* midway between the conduction and valence bands.

Let, at any temperature $T°K$

n_c = No. of electrons in the conduction band

n_v = No. of electrons in the valence band

* For the present discussion, Fermi level may be defined as the energy which corresponds to the centre of gravity of conduction electrons and holes weighted according to their energies.

$N = n_c + n_v$

= No. of electrons in both bands.

Further, let us make the following simplifying assumptions :

1. width of energy bands are small as compared to forbidden energy gap between them;
2. since band widths are small, all levels in a band have the same energy;
3. energies of all levels in valence band are zero as shown in Fig. 51.22;
4. energies of all levels in the conduction band are equal to E_g.

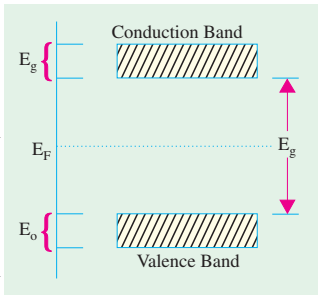


Fig. 51.22

In Fig. 51.22, the zero-energy reference level has been arbitrarily taken at the top of valence band.

Now, number of electrons in the conduction band is

$$n_c = N.P(E_g)$$

where $P(E_g)$ represents the probability of an electron having energy E_g. Its value can be found from Fermi-Dirac probability distribution function given by

$$P(E) = \frac{1}{1 + e^{(E - E_g)/kT}}$$

where $P(E)$ is the probability of finding an electron having any particular value of energy E and E_F is Fermi level.

$$\therefore \quad P(E_g) = \frac{1}{1 + e^{(E_g - E_F)/kT}} \qquad \therefore \quad n_c = \frac{N}{1 + e^{(E_g - E_F)/kT}}$$

Now, number of electrons in the valence band is $n_v = N.P(0)$

The probability $P(0)$ of an electron being found in the valence band with zero energy can again be calculated by putting $E = 0$ in the Femi-Dirac probability distribution function.

$$P(0) = \frac{1}{1 + e^{(0 - E_F)/kT}} = \frac{1}{1 + e^{-E_F/kT}}. \qquad \text{Hence, } n_c = \frac{N}{1 + e^{-E_F/kT}}$$

Now $N = n_c + n_v$ or $N = \frac{N}{1 + e^{(E_g - E_F)/kT}} + \frac{N}{1 + e^{-E_F/kT}}$

$$\therefore \quad 1 - \frac{1}{1 + e^{-E_F/kT}} = \frac{1}{1 + e^{(E_g - E_F)/kT}}$$

which on simplification gives $E_F = E_g/2$.

Hence, it shows that in an intrinsic semiconductor, the Fermi level lies midway between the conduction and valence bands. That this conclusion is physically possible can be best seen with the help of Fig. 51.23.

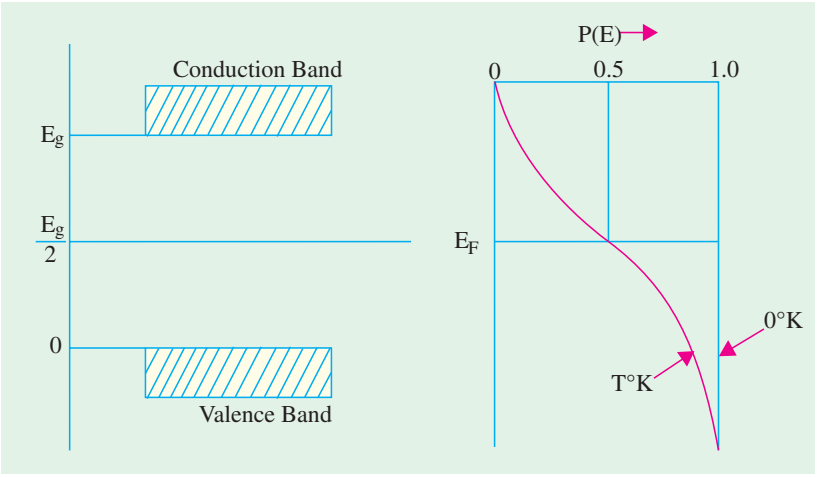


Fig. 51.23

51.23. Extrinsic Semiconductors

Those intrinsic semiconductors to which some suitable impurity or doping agent or dopant has been added in extremely small amounts (about 1 part in 10^8) are called ***extrinsic*** or impurity semiconductors.

The usual doping agents are :

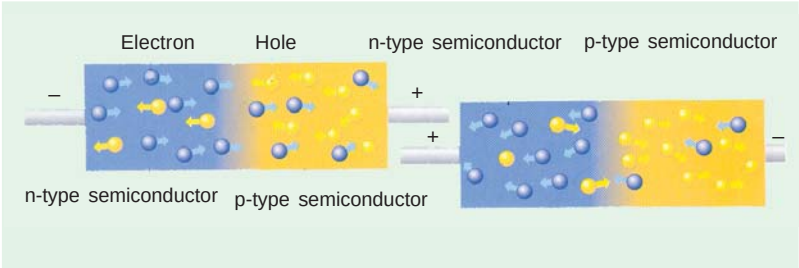
1. ***pentavalent*** atoms having five valence electrons (arsenic, antimony, phosphorus) or
2. ***trivalent*** atoms having three valence electrons (gallium, indium, aluminium, boron).

Pentavalent doping atom is known as ***donor*** atom because it donates or contributes one electron to the conduction band of pure germanium. The trivalent atom, on the other hand, is called ***acceptor*** atom because it accepts one electron from the germanium atom.

Depending on the type of doping material used, extrinsic semiconductors can be sub-divided into two classes :

(*i*) *N*-type semiconductors and **(*ii*)** *P*-type semiconductors.

(*a*) *N*-type Extrinsic Semiconductor. This type of semiconductor is obtained when a pentavalent material like antimonty (Sb) is added to pure germanium crystal. As shown in Fig. 51.24 (*a*), each antimony atom forms covalent bonds with the surrounding four germanium atoms with the help of four of its five electrons. The fifth electron is superfluous and is loosely bound to the antimony atom. Hence, it can be easily excited from the valence band to the conduction band by the application of electric field or increase in thermal energy. Thus, practically every antimony atom introduced into the germanium lattice, contributes one conduction electron into the germanium lattice ***without creating a positive hole***. Antimony is called ***donor*** impurity and makes the pure germanium an *N*-type (*N* for negative) extrinsic semi-conductor. As an aid to memory, the student should associate the **N** in do**N**or with **N** in the *N*-type material and *N* in Negative charge carrier.



It may be noted that by giving away its one valence electron, the donor atom becomes a positively-charged *ion*. But it cannot take part in conduction because it is firmly fixed or tied into the crystal lattice. It will be seen that apart from electrons and holes ***intrinsically available in germanium***, the addition of antimony greatly increases the number of conduction electrons. Hence, concentration of electrons in the conduction band is increased and exceeds the concentration of holes in the valence band. Because of this, Fermi level shifts upwards towards the bottom of the conduction band as shown in Fig. 51.24 (*b*),* because the number of charge carriers has become more in conduction band than in valence band.

In terms of energy levels, the fifth antimony electron has an energy level (called donor level) just below the conduction band. Usually, the donor level is 0.01 eV below conduction band for germanium and 0.054 eV for silicon.

It is seen from the above description that in *N*-type semiconductors, electrons ***are the majority carriers*** while holes constitute the minority carriers. Hence, *N*-type semiconductor conducts principally by electrons in the nearly empty conduction band and the process is called '***excess***' conduction.

Another point worth noting is that even though *N*-type semiconductor has excess of electrons,

* Since the number of electrons as compared to the number of holes increases with temperature, the position of Fermi level also changes considereably with temperature.

still it is electrically neutral. It is so because by the addition of donor impurity, number of electrons available for conduction purposes becomes more than the number of holes available intrinsically. But the total charge of the semiconductor does not change because the donor impurity brings in as much negative charge (by way of electrons) as positive charge (by way of protons in its nucleus).

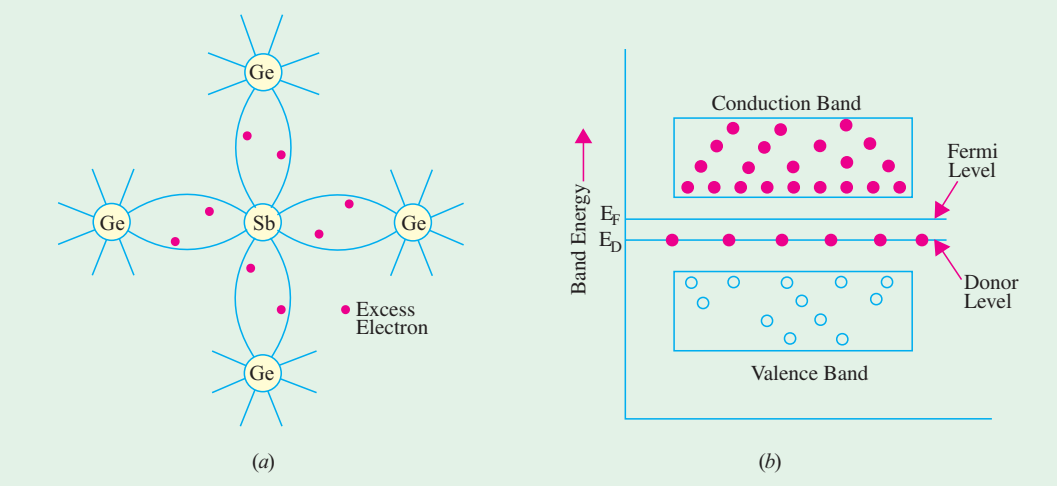


Fig. 51.24

(*b*) *P*-type Extrinsic Semiconductor. This type of semiconductor is obtained when traces of a trivalent like boron (*B*) are added to a pure germanium crystal.

In this case, the three valence electrons of boron atom form covalent bonds with four surrounding germanium atoms but one bond is left incomplete and gives rise to a hole as shown in Fig. 51.25 (*a*).

Thus, boron which is called an ***acceptor*** impurity causes as many positive holes in a germanium crystal as there are boron atoms thereby producing a *P*-type (*P* for positive) extrinsic semiconductor. As an aid to memory, the student should associate the **P** in acce**P**tor with **P** in *P*-type material and **P** with **P**ositive charge carrier.

In this type of semiconductor, conduction is by the movement of holes in the valence band. Accordingly, holes form the majority carriers ***whereas electrons constitute minority carriers***. The process of conduction is called 'deficit' conduction.

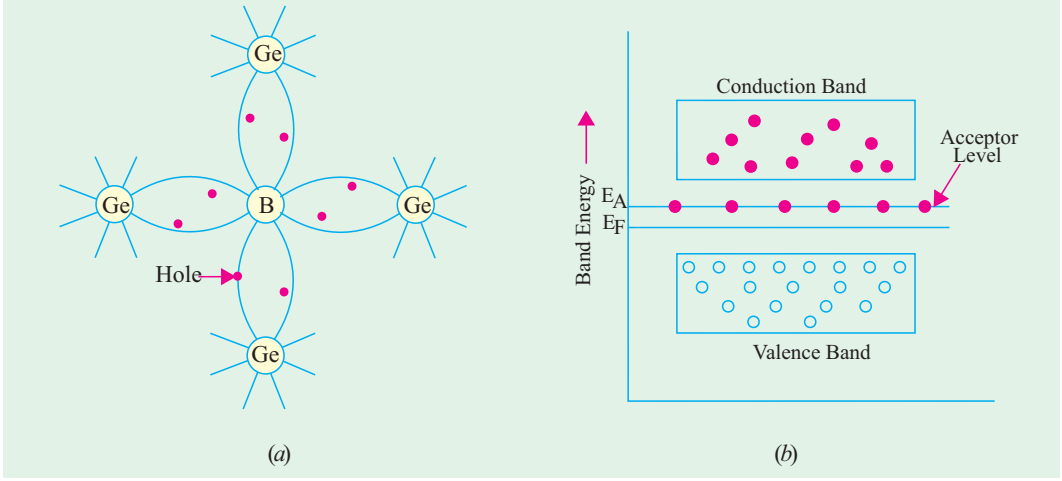


Fig. 51.25

Since concentration of holes in the valence band is more than the concentration of electrons in the conduction band, Fermi level shifts nearer to the valence band [Fig. 51.25 (*b*)]. The acceptor level lies immediately above the Fermi level. Conduction is by means of hole movement at the top of valence band, acceptor level readily accepting electrons from the valence band.

Again, it may be noted that even though *P*-type semiconductor has excess of holes for conduction purposes, on the whole it is electrically neutral for the same reasons as given above.

51.24. Majority and Minority Carriers

In a piece of ***pure*** germanium or silicon, no free charge carriers are available at 0°K. However, as its temperature is raised to room temperature, some of the covalent bonds are broken by heat energy and as a result, electron-hole pairs are produced. These are called thermally-generated charge carriers. They are also known as intrinsically-available charge carriers. Ordinarily, their number is quite small.

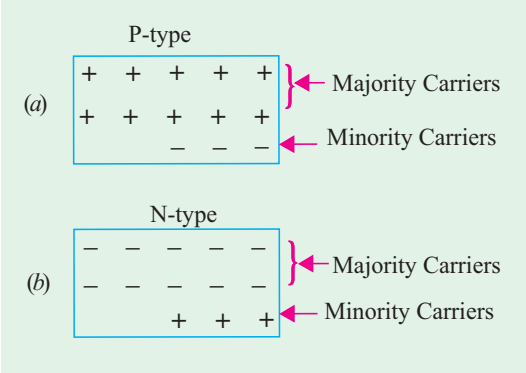


Fig. 51.26

An intrinsic of pure germanium can be converted into a *P*-type semiconductor by the addition of an acceptor impurity which adds a large number of holes to it. Hence, a *P*-type material contains following charge carriers :

(*a*) large number of positive holes—most of them being the added impurity holes with only a very small number of thermally generated ones;

(*b*) a very small number of thermally-generated electrons (the companions of the thermally generated holes mentioned above).

Obviously, in a *P*-type material, the number of holes (both added and thermally-generated) is much more than that of electrons. Hence, in such a material, holes constitute ***majority*** carriers and electrons form ***minority*** carriers as shown in Fig. 51.26 (*a*).

Similarly, in an *N*-type material, the number of electrons (both added and thermally-generated) is much larger than the number of thermally-generated holes. Hence, in such a material, electrons are majority carriers whereas holes are minority carriers as shown in Fig. 51.26 (*b*).

51.25. Mobile Charge Carriers and Immobile Ions

As discussed in Art. 51.23, *P*-type material is formed by the addition of acceptor impurity atoms like boron to the pure *Ge* or *Si* crystals. The number of holes added is equal to the number of boron atoms because each such atom contributes one hole. Now, when a hole moves ***away*** from its parent atom, the remaining atom becomes a ***negative ion***. Unlike the mobile and free-moving hole, this ion

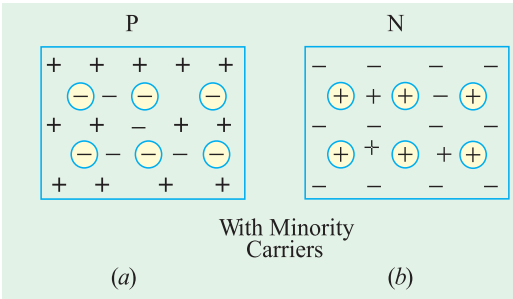


Fig. 51.27

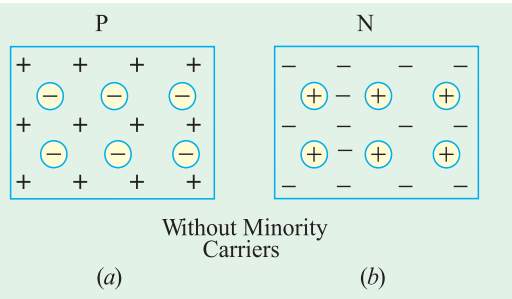


Fig. 51.28

cannot take part in conduction because it is fixed in the crystal lattice. In Fig. 51.27 (*a*), these immobile ions are shown by ***circled*** minus signs whereas free and mobile holes are shown by ***uncircled*** plus signs. Thermally-generated electrons (which form minority carriers) are shown by ***uncircled*** minus signs.

Similarly, addition of penta-valent atoms like antimony to pure *Ge* or *Si* crystal produces an *N*-type material. The number of free and mobile electrons which are added equals the number of donor *Sb* atoms. Again, when an electron moves ***away*** from its parent atom, it leaves behind a positive ion. This ion, being fixed in crystal structure, cannot take part in conduction. As shown in Fig. 51.27 (*b*); these immobile ions are represented by ***circled*** plus signs whereas free and mobile electrons are represented by ***uncircled*** minus signs. The thermally-generated holes (which form minority carriers in this case) are shown by ***uncircled*** plus signs. In Fig. 51.28, minority carriers of both types have been neglected. Hence, the figure does not show the small number of free electrons in the *P*-type material or the small number of holes in the *N*-type material.

51.26. Electron Conductivity of a Metal



According to free electron model of an atom, the valence electrons are not attached to individual atoms but are free to move about in all directions among the atoms. These electrons are called ***conduction*** electrons and are said to form 'free electron cloud' or free electron 'gas' or the Fermi gas. For example in copper there is one such free electron per atom, the other 28 electrons remaining bound to the copper nuclei to form positive ion cores.

When no external field is applied to the metal, the free electrons move randomly in all directions as shown in Fig. 51.29 (*a*). However, when an external electric field is applied to the metal, the free electron motion becomes directed a shown in Fig. 51.29 (*b*). This directed flow of electrons results in a net charge displacement in a definite direction. This type of motion is known as ***drift*** and the phenomenon is referred to as process of conduction by drift charge. The drift velocity (v) of the electrons is dependent upon the electron mobility (μ_e) and the applied electric field E. The actual relation is $v = \mu_e E$

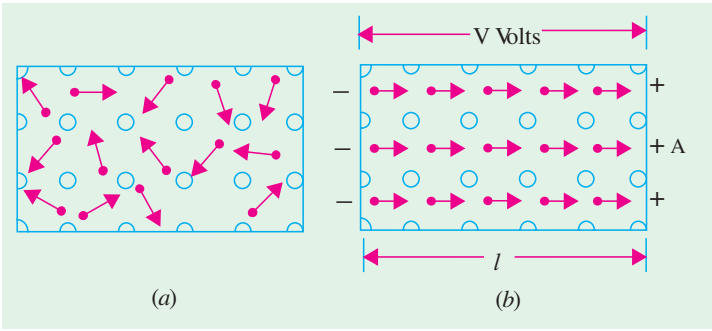


Fig. 51.29

Let, e = electron charge (coloumb)
v = electron drift velocity (m/s)
A = conductor cross-section (m^2)
n = number of free electrons per unit volume of the conductor
i.e. electron density (per m^3)
l = length of the conductor (m)
E = V/l — applied electric field (V/m)

Now, electric current flowing in any conductor is given by the amount of charge which flows in one second across any plane of the conductor (Fig. 51.30). The ***total*** number of electrons which cross the plane P of cross-section A in one second = $n \times (v \times A)$.

Charge carried by them per second is = $envA$. Hence, $I = \nu enA$.

Substituting the value of ν, we get

$$I = neA\mu_e E = nAe\mu_e V/l$$

$$\therefore \quad R = \frac{V}{I} = \frac{l}{A}\left(\frac{1}{ne\mu_e}\right) = \rho\frac{l}{A}$$

$\therefore$ resistivity $\rho = 1/ne\,\mu_e$ ohm-m and conductivity $\sigma = ne\mu_e$ Siemens/m

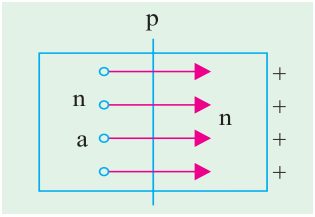


Fig. 51.30

Incidentally, it may be noted that conductivity of a semiconductor differs from that of a metal in one important respect *i.e.* in a semiconductor, charge carriers are both holes as well as electrons whereas in metals, electrons are the only charge carriers (Art. 51.27).

Example 51.3. *A copper wire of 2mm diameter with conductivity of 5.8×10^7 Siemens/m and electron mobility of 0.0032 m^2/V-s is subjected to an electric field of 20 V/m. Find (a) the charge density of free electrons, (b) the current density, (c) current flowing in the wire, (d) the electron drift velocity.* **(U.P.S.C. Engg. Services 2002)**

Solution. $d = 2\text{mm} = 2\times10^{-3}\text{m}$, $\sigma = 5.8\times10^7$ S/m,

$\mu_e = 0.0032\ \text{m}^2/\text{V-s}$, $E = 20\text{m V/m}$

$\sigma = qn\mu$

(a) $$n = \frac{\sigma}{q\mu_e} = \frac{5.8\times10^7}{1.6\times10^{-19}\times0.0032} = \mathbf{1.132\times10^{29}/m^3}$$

(b) $$J = \sigma E = (5.8\times10^7)\times(20\times10^{-3}) = \mathbf{1.16\times10^6\ A/m^2}$$

(c) Let I = current flowing through the wire.

Area of cross-section of a wire,

$$A = \frac{\pi d^2}{4} = \frac{\pi\times\left(2\times10^{-3}\right)^2}{4} = 3.16\times10^{-6}\text{m}^2$$

$$J = \frac{I}{A} \quad \text{or} \quad I = JA = (1.16\times10^6)\times(3.16\times10^{-6}) = \mathbf{3.67A}$$

(d) $$\upsilon = \mu E = 0.0032\times(20\times10^{-3}) = \mathbf{64\ m/s}$$

51.27. Conductivity of Intrinsic Semiconductors

In their case, current flow is due to the movement of electrons and holes in opposite directions. However, since their charges are of opposite sign, the current due to each is in the same direction. Even though the number of electrons equals the number of holes, hole mobility μ_h is practically half of electron mobility μ_e.

As shown in Fig. 51.31, the total current flow which is due to the sum of electron flow and hole flow, is given by

$$I = I_e + I_h$$

Let ν_e = drift velocity of electrons (m/s)

ν_h = drift velocity of holes (m/s)

n_i = density of free electrons in an intrinsic semiconductor (per m^3)

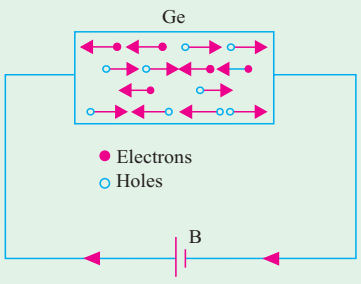


Fig. 51.31

p_i = density of holes in an intrinsic semiconductor (per m^3)
e = electron charge (coulomb)
A = cross-section of the semiconductor (m^2)

Since in an intrinsic semiconductor $n_i = p_i$

$\therefore \quad I = n_i e (\nu_e + \nu_h) A = n_i e (\mu_h + \mu_e) EA$

where μ_e = electron mobility = ν_e / E

μ_h = hole mobility = ν_h / E

Since $E = V/l$ where l is the length of the intrinsic semiconductor,

$$\therefore \quad I = n_i e (\mu_e + \mu_h) AV/l$$

$$\therefore \quad \frac{V}{I} = \frac{l}{A} \cdot \frac{1}{n_i e (\mu_e + \mu_h)} = \rho_i \frac{l}{A}$$

where ρ is the resistivity of the semoconductor. It is given by

$$\rho_i = \frac{1}{n_i e (\mu_e + \mu_h)} \text{ ohm-m}$$

The electrical conductivity which is the reciprocal of resistivity is given by

$$\sigma_i = n_i e (\mu_e + \mu_h) \text{ S/m}$$

Now, current density, $J = I/A \quad \therefore \quad J = n_i e (\mu_e + \mu_h) E = \sigma_i E \quad \therefore \quad \sigma_i = J/E$

Obviously, conductivity of semiconductors depends on two factors ***(i)*** number of current carriers present per unit volume and ***(ii)*** the mobility of the current carriers. It is found that with increase in temperature, n_i as well p_i increase and correspondingly, the conductivity of intrinsic semiconductors increases *i.e.* resistivity decreases.

51.28. Conductivity of Extrinsic Semiconductors

The general expression for current density (derived above) in the case of an extrinsic semiconductor when an electric field is employed is

$$J = (ne\mu_e + pe\mu_h) E \qquad \text{...(i)}$$

(a) If it is an *N*-type semiconductor, then the above expression becomes

$$J_n = e (n_n \mu_e + p_n \mu_h) E$$

where n_n and p_n represent the electron and hole densities in the *N*-type semiconductor ***after doping***.

(b) If it is a *P*-type semiconductor, then

$$J_p = e (n_p \mu_e + p_p \mu_h) E$$

where n_p and p_p represent similar quantities in a *P*-type semiconductor after doping.

The conductivity is given by $\sigma = J/E$

$\therefore \quad \sigma = ne\mu_e + pe\mu_h$

or $\sigma_n = e (n_n \mu_e + p_n \mu_h)$ — for *N*-type

and $\sigma_p = e (n_p \mu_e + p_p \mu_h)$ — for *P*-type

(i) In *N*-type semiconductors, electrons form the majority carriers although holes are also available as minority carriers.

The current density in such a semiconductor is given by Eq. (*i*) above. However, since electron density in such extrinsic semiconductors is much more than hole density *i.e.* $n_n \gg p_n$, the above expressions are simplified to

$$J_n = n_n e \mu_e E \quad \text{and} \quad \sigma_n = n_n e \mu_e \qquad \text{...(ii)}$$

(ii) In *P*-type semiconductors, conduction is by means of holes in the valence band which form majority carriers in this case although electrons are available as minority carriers.

Since in such extrinsic semiconductors, $n_p \ll p_p$, the above expressions become

$$J_p = p_p e \mu_h E \quad \text{and} \quad \sigma_p = p_p e \mu_h \qquad \text{...(iii)}$$

51.29. Conductivity when Intrinsic Charge Carrier Densities are Neglected

In case density of charge carriers available intrinsically is negligible as compared to the added impurity atoms (whether of donor or a acceptor type), then the formulae for conductivity given by Eq. ***(ii)*** and ***(iii)*** above will be changed as follows :

(*a*) For *N*-type semiconductor

As seen from Eq. ***(ii)*** above, the conductivity is given by $\sigma_n = n_n e\ \mu_e$ where n_n is electron density after doping.

In this relation, intrinsic hole density has already been neglected. The remaining electron density is also made up of the following two components :

1. intrinsic hole density due to holes available in a *pure* semiconductor ;
2. electron density N_d contributed by added donor impurity.

However, if we further neglect the intrinsic electron density, then $\sigma_n = N_d\ e\ \mu_e$

(*b*) For *P*-type semiconductor

As seen from Eq. ***(iii)*** above, the conductivity is given by $\sigma_p = p_p\ e\ \mu_h$ where p_p represents hole density. Again, in this relation, intrinsic electron density has been already neglected. This hole density further consists of the following two components :

1. intrinsic hole density due to holes available in a pure semiconductor ;
2. hole density (N_a) contributed by added acceptor impurity.

However, if we further neglect the intrinsic hole density, then $\sigma_p = N_a\ e\ \mu_h$

51.30. Conductivity of Pure and P-type Germanium

As shown in Art. 51.25, the conductivity of pure germanium is given by

$$\sigma = n_i e\ (\mu_e + \mu_h) = p_i e\ (\mu_e + \mu_h)$$

When germanium is doped by a trivalent impurity like indium, it becomes a *P*-type semiconductor. After doping, its conductivity depends on the number of charge carriers available in it. The law of Mass Action can be used for finding this number. For acceptor impurity, the law may be stated as follows :

$$n_p\ p_p = n_i\ p_i = n_i^2 = p_i^2$$

where n_p and p_p represent the 'free' electron and hole densities respectively in the semi-conductor after doping and n_i and p_i, the electron and hole densities ***before*** doping *i.e.* in an intrinsic semiconductor. Put in another way, it simply means that, at constant temperature, the product of the number of electron carriers and the number of hole carriers is independent of the density of acceptor atoms. In physical terms, it means that the introduction of *P*-type impurity fills some of the electron levels produced by thermal action.

By calculating n_p and p_p from above and knowing μ_e and μ_h, conductivity after doping can be found out as illustrated by the following example.

(*a*) Pure Germanium

$$\sigma = n_i\ e\ (\mu_e + \mu_h) = p_i\ e\ (\mu_e + \mu_h)$$

Let, $n_i = p_i = 2 \times 10^{19}$ per m^3, $\mu_e = 0.36\ m^2$/V-s

$\mu_h = 0.17\ m^2$ / V-s and $e = 1.6 \times 10^{-19}$ C

$\sigma = 2 \times 10^{19} \times 1.6 \times 10^{-19}\ (0.36 + 0.17) = 1.69$ S/m.

(*b*) *P*-type Germanium

here, $\sigma_p = e\ (n_p\ \mu_e + p_p\ \mu_h)$

Suppose, we add 10^{22} atoms/m^3 of indium and that $n_i = 2 \times 10^{19}$ charge carriers (either electrons or holes) per m^3.

Then, $n_p\ p_p = n_i^2 = (2 \times 10^{19})^2 = 4 \times 10^{38}$ and $p_p - n_p = 10^{20}$

$\therefore$ $p_p - 4 \times 10^{38}/p_p = 10^{20}$ or $p_p^2 - 10^{20}\ p_p - (4 \times 10^{38}) = 0$

Solving the above quadratic equation and taking positive value only.

$$p_p = 1.04 \times 10^{20} \quad \text{and} \quad n_p = 0.04 \times 10^{20}$$
$$\sigma_p = 1.6 \times 10^{-19} (0.04 \times 10^{20} \times 0.36 + 1.04 \times 10^{20} \times 0.17) = 3.1 \text{ S/m}^*$$

It is seen that conductivity is almost doubled.

Example 51.4. *What length of a round copper wire of diameter 1 mm will have a resistance of 1k Ω if copper conductivity is 60 MS/m. A cylindrical piece of silicon having a diameter of 1 mm is doped with 10^{20} m^{-3} atoms of phosphorous which are fully ionized. What length of this silicon would be required to give a resistance of 1 k Ω if electronic mobility in silicon is 0.1 m^2 / V-s ?*

(Electronic Devices & Circuits, Pune Univ. 1994)

Solution. $R = 1 \text{ k}\Omega = 1000\ \Omega$, $\sigma = 60 \times 10^6$ S/m, $A = \pi d^2/4 = \pi \times (1 \times 10^{-3})^2/4 \text{m}^2$

$R = l/\sigma A$

$l = \sigma AR = 60 \times 10^6 \times (\pi \times 10^{-6}/4) \times 1000 = 47{,}100$ m = **47.1 km**

For Silicon Wire

$$\sigma = n_i e \mu_e = 10^{20} \times 1.6 \times 10^{-19} \times 0.1 = 1.6 \text{ S/m}$$
$$l = \sigma A R = 1.6 \times (\pi \times 10^{-6}/4) \times 1000$$
$$= 1.26 \times 10^{-3} = \mathbf{1.26 \text{ mm}}$$

Example 51.5. *Calculate the intrinsic conductivity of silicon at room temperature if $n = 1.41 \times 10^{16}$ m^{-3}, $\mu_e = 0.145$ m^2/V-s, $\mu_h = 0.05$ m^2/V-s and $e = 1.6 \times 10^{-19}$ C. What are the individual contributions made by electrons and holes ?* **(Electronic Engg., Nagpur Univ. 1991)**

Solution. As seen from Art. 1.27, the conductivity of an intrinsic semiconductor is given by

$$\sigma_i = n_i e \mu_e + n_i e \mu_h$$
$$= 1.41 \times 10^{16} \times 1.6 \times 10^{-19} \times 0.145 + 1.41 \times 10^{16} \times 1.6 \times 10^{-19} \times 0.05$$
$$= 0.325 \times 10^{-3} + 0.112 \times 10^{-3} \text{ S/m} = \mathbf{0.437 \times 10^{-3} \text{ S/m}}$$

Contribution by electrons = $\mathbf{0.325 \times 10^{-3}}$ **S/m**

Contribution by holes = $\mathbf{0.112 \times 10^{-3}}$ **S/m**

Example 51.6. *Calculate the donor concentration in N-type germanium having resistivity of 100 Ω-m. Derive the formula you use. Take $e = 1.6 \times 10^{-19}$ C, $\mu_e = 0.36$ m^2 V^{-1} s^{-1}.*

(Electronics ; Nagpur Univ. 1990)

Solution. As seen from Art. 1.29, $\rho_n = 1/N_d e \mu_e$

$\therefore \quad 100 = 1/N_d \times 1.6 \times 10^{-19} \times 0.36$; $\quad N_d = \mathbf{1.74 \times 10^{17} \text{ atoms/m}^3}$

Example 51.7. *An N-type silicon has a resistivity of 1500 Ω-m at a certain temperature. Compute the electron-hole concentration given that $n_i = 1.5 \times 10^{16}$ m^{-3}, $\mu_e = 0.14$ m/V-s, $\mu_h = 0.05$ m^2/V-s and $e = 1.6 \times 10^{-19}$ C.*

Solution. Being *N*-type silicon, it is assumed that $n \gg p$

$\therefore \quad \sigma = e(n\mu_e + p\mu_h) = n_e \mu_e$

$\therefore \quad \rho = 1/n_e\mu_e$ or $15 = 1/n \times 1.6 \times 10^{-19} \times 0.14$ or $n = \mathbf{3.1 \times 10^{20} \text{ m}^{-3}}$

Now, $np = n_i^2$ or $p = n_i^2/n = (1.5 \times 10^{16})^2/(3.1 \times 10^{20}) = \mathbf{2 \times 10^{12} \text{ m}^{-3}}$

Example 51.8. *A specimen of pure germanium at 300° K has a density of charge carriers of $2.5 \times 10^{19}/m^3$. It is doped with donor impurity atoms at the rate of one impurity atom for every 10^6 atoms of germanium. All impurity atoms may be supposed to be ionized. The density of germanium atom is 4.2×10^{28} atoms/m^3.*

Find the resistivity of the doped germanium if electron mobility is 0.36 m^2/V-s.

Solution. Density of added impurity atoms is

$$N_d = 4.2 \times 10^{28}/10^6 = 4.2 \times 10^{22} \text{ atoms/m}^3$$

* It is the new name for the old unit of mho/m.

As seen, it is very large as compared to the intrinsic charge carrier density of $2.5 \times 10^{19}/m^3$ which will, therefore, be neglected. Now, as seen from Art. 1.29.

$$\sigma_n = N_d\, e\, \mu_e = 4.2 \times 10^{22} \times 1.6 \times 10^{-19} \times 0.36 = 2.42 \times 10^3 \text{ S/m}$$
$$\rho_n = 1/\sigma_n = 1/2.42 \times 10^3 = \mathbf{0.413 \times 10^{-3}\ \Omega\text{-m.}}$$

Example 51.9. *Compute the relative concentration of silicon atoms and electron-hole pairs at 300° K. Also, calculate intrinsic resistivity of silicon. Given Avogadro's number = 6.02×10^{23} atoms/g-atom, density = 2.33×10^6 g/m^3, atomic Wt. = 28.1, intrinsic carrier density = 1.5×10^{16} m^{-3}, $\mu_e = 0.14$ m^2/V-s, $\mu_h = 0.05$ m^2/V-s.*

Solution. n_A = Avogadro's No. × density/atomic Wt.

$$= 6.02 \times 10^{23} \times 2.33 \times 10^{16}/28.1 \cong 5 \times 10^{28} \text{ atoms/m}^3$$

Since intrinsic concentration n_i *i.e.*, electron-hole pairs/m^3 is 1.5×10^{16}

$$\therefore \quad \frac{n_A}{n_i} = \frac{5 \times 10^{28}}{1.5 \times 10^{16}} \cong \mathbf{3.3 \times 10^{12}}$$

It means that there are 3.3×10^{12} Si atoms for each electron-hole pair.

Since in a pure semiconductor, $n = p = n_i$, intrinsic conductivity is given by

$$\sigma_i = n_i e\,(\mu_e + \mu_h) = 1.5 \times 10^{16} \times 1.6 \times 10^{-19}\,(0.14 + 0.05) = 0.456 \times 10^{-3} \text{ S/m}$$

Hence, intrinsic resistivity is given by

$$\rho_i = 1/\sigma_i = 1/0.456 \times 10^{-3} = \mathbf{2193\ \Omega\text{-m}}$$

Example 51.10. *Silicon is doped with acceptor atoms to a density of 10^{22} m^{-3}. If it is assumed that all acceptor centres are ionized, calculate the conductivity of the extrinsic silicon. Given that intrinsic density is 1.4×10^{16} m^{-3}, $\mu_e = 0.145$ m^{-3} and $\mu_h = 0.05$ m^{-3}.*

(Electronic Devices & Circuits, Pune Univ. 1992)

Solution. The minority carrier density can be found from the equation given in Art. 1.30, *i.e* $np = n_i^2$.

Now, $p = 10^{22} \quad \therefore \quad n \times 10^{22} = (1.4 \times 10^{16})^2 ; \quad n = 1.96 \times 10^{10}/\text{m}^3$

Now, $\sigma = ne\,\mu_e + pe\,\mu_h$

$$= 1.96 \times 10^{10} \times 0.145 \times 1.6 \times 10^{-19} + 10^{22} \times 0.05 \times 1.6 \times 10^{-19} = \mathbf{80\ S/m}$$

51.31. Drift

Directed motion of charge carriers in semiconductors occurs through two mechanisms :

(i) charge drift under the influence of applied electric field and

(ii) diffusion of charge from a region of high charge density to one of low charge density.

Consider the drift phenomenon first. When no electric field is applied to the semiconductor which is above 0° K, the conduction electrons (as well as holes) move within the crystal with random motion and repeatedly collide with each other and the fixed ions. Due to randomness of their motion, the net average velocity of these charge carriers in any given direction is zero. Hence, no current exists in the crystal under this condition of no field.

Now, consider the case when an electric field is applied to the crystal. Under the influence of this field, the charge carriers attain a ***directed*** motion which is superimposed on their random thermal motion. This results in a net average velocity called ***drift*** velocity in the direction of the applied electric field. Of course, electrons and holes move in opposite directions but because of their opposite charges, both produce current in the same direction. In extrinsic semiconductors, this current is essentially a majority carrier flow.

The drift velocity is proportional to electric field strength E, the constant of proportionality being called mobility μ. The exact relation between the two is $v = \mu E$.

Let us find the value of drift current in a semiconductor.

(i) current density due to electron drift is $J_e = e\mu_e nE$ where μ_e is electron mobility, n is electron density and E is the electric field strength.

(ii) current density due to hole drift is $J_h = e\ \mu_h\ pE$ where p is hole density.

Total current density due to electron and hole drift is

$$J = J_e + J_h = e\mu_e nE + e\mu_h pE = e(n\mu_e + p\mu_h)E$$
$$= e\mu_h(p + bn)E \qquad \text{where } b = \mu_e/\mu_h$$

51.32. Diffusion

It is gradual flow of charge from a region of high density to a region of low density. It is a force-free process based on non-uniform distribution of charge carriers in a semiconductor crystal. It leads to an electric current without the benefit of an applied field. This flow or diffusion of carriers is proportional to the carrier density gradient, the constant of proportionality being called diffusion constant or diffusion coefficient D which has a unit of m^2/s.

Current density due to hole diffusion is $J_h = -eD_h\, dp/dx$. Similarly, current density due to electron diffusion is $J_e = eD_e\, dn/dx$.

where D_e, D_h = electron and hole diffusion constants respectively
dn/dx = density gradient of electrons
dp/dx = density gradient of holes.

It is obvious that diffusion depends on charge in homogeneity or on the presence of a space gradient of charge density. It can occur in regions free of electric field. On the other hand, drift current is a function of both electric field and charge density.

Incidentally, it may be noted that, generally, diffusion leads to redistribution of charges which further results in the development of potential difference between different parts of the semiconductor. The electric field due to this potential difference sets up drift current in opposition to diffusion current. Final equilibrium is achieved when the potential difference developed becomes sufficiently large so as to create a drift current equal and opposite to the diffusion current thus resulting in zero net flow of current.

51.33. Combined Drift and Diffusion Currents

In semiconductors, drift and diffusion processes may be present simultaneously. The expressions for total electron and hole densities become.

$$J_e = e\mu_e nE + eD_e\, dn/dx \text{ A/m}^2 \qquad \text{and} \qquad J_h = e\mu_h\, pE - eD_h dp/dx \text{ A/m}^2$$

51.34. Relation Between D and μ

Both diffusion constant and mobility are statistical thermodynamic phenomena and are related to each other by the following equation.

$$\mu_e = \frac{e}{kT} D_e \text{ and } \mu_h = \frac{e}{kT} D_h \quad \text{or} \quad \frac{D_e}{\mu_e} = \frac{D_h}{\mu_h} = \frac{kT}{e} = \frac{T}{11,600}$$

The relationship is known as Einstein's equation

At t = 23°C, T = 300°K, hence D/μ = 300/11,600 = 1/39 or μ = 39 D

Example 51.11. *Calculate diffusion constants for electrons and holes at 300°K in silicon if μ_e = 0.15 m^2/V-s and μ_h = 0.05 m^2/V-s.*

Solution. According to Einstein's equation,

$D = \mu kT/e$ or $D = \mu/39$ **m^2/s** — at 300°K
$D_e = \mu_e/39 = 0.15/39 =$ **3.85×10^{-3} m^2/s**
$D_h = \mu_h/39 = 0.05/39 =$ **6.4×10^{-5} m^2/s**

Example 51.12. *Find the diffusion coefficients of holes and electrons for germanium at 300 K. The carrier mobilities in cm^2/volt-sec at 300 K for electrons and holes are respectively 3600 and 1700. Density of carriers is 2.5×10^{13}. Boltzmann constant, $k = 1.38 \times 10^{23}$.*

(U.P.S.C. Engg. Services, 1996)

Solution. According to the Einstein's equation,

$$D = \mu kT/e \quad \text{or} \quad D = \mu/39 \text{ m}^2\text{/s} - \text{at } 300^\circ \text{ K}$$

$$D_e = \mu_e/39 = 3600 \text{ (cm}^2\text{/V-s)}/39 = 0.36 \text{ (m}^2\text{/V-s)}/39 = \mathbf{9.2 \times 10^{-3} \text{ m}^2\text{/s}}$$

$$D_h = \mu_h/39 = 1700 \text{ (cm}^2\text{/V-s)}/39 = 0.17 \text{ (m}^2\text{/V-s)}/39 = \mathbf{4.36 \times 10^{-3} \text{ m}^2\text{/s}}$$

51.35. Recombination

Apart from drift and diffusion, a third phenomenon which occurs in semiconductors is called ***recombination*** that results from the collision of an electron with a hole.

The process is essentially the return of a free conduction electron to the valence band and is accompanied by the emission of energy. Obviously, the recombination rate is directly proportional to the carrier concentration for the simple reason that larger the number of carriers, the more likely is the occurrence of electron-hole recombination. This phenomenon is important in describing minority carrier flow.

As is well-known, in a semiconductor, thermal generation of electron-hole pairs also takes place continuously. Hence, there is ***net recombination*** rate given by the difference between the recombination and generation rates.

51.36. Carrier Life Time

It is defined as the time for which, on an average, a charge carrier will exist before recombination with a carrier of opposite charge. Its value varies from nanoseconds (10^{-9}) to hundreds of microseconds (μs) and depends on temperature and impurity concentration in the semiconductor material.

51.37. Total Carrier Flow

The total carrier flow in a semiconductor is the sum of the three flow phenomena discussed above. Each type of carrier has to be treated separately and the number of electrons or holes leaving the sample being accounted for by drift or diffusion or net recombination. The current in the semiconductor is then the sum of the electron and hole currents.

51.38. P-N Junction

It is possible to manufacture a single piece of a semiconductor material half of which is doped by P-type impurity and the other half by *N*-type impurity as shown in Fig. 51.32. The plane dividing the two zones is called ***junction***. Theoretically, junction plane is assumed to lie where the density of donors and acceptors is equal. The *P-N* junction is fundamental to the operation of diodes, transistors and other solid-state devices.

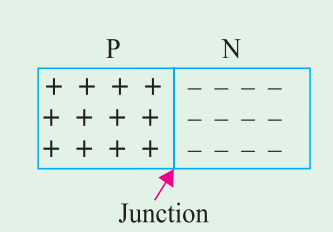
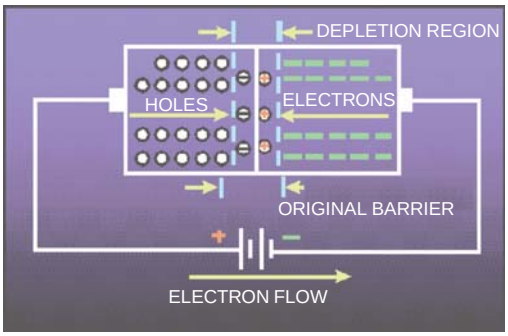


Fig. 51.32

Let us see if anything unusual happens at the junction. It is found that following three phenomena take place :

1. A thin ***depletion layer*** or region (also called space-charge region or transition region) is established on both sides of the junction and is so called because it is depleted of ***free charge carriers***. Its thickness is about 10^{-6} m.
2. A barrier potential or junction potential is developed across the junction.
3. The presence of depletion layer gives rise to junction and diffusion capacitances (Art. 51.5).

51.39. Formation of Depletion Layer



Suppose that a junction has just been formed. At that instant, holes are still in the *P*-region and electrons in the *N*-region. However, there is greater concentration of holes in *P*-region than in *N*-region (where they exist as minority carriers). Similarly, concentration of electrons is greater in *N*-region than in *P*-region (where they exist as minority carriers). This concentration differences establishes density gradient across the junction resulting in carrier diffusion. Holes diffuse from *P* to *N*-region and electrons from *N*-to *P*-region and terminate their existence by recombination [Fig. 51.33 (*a*)]. This recombination of free and mobile electrons and holes produces the narrow region at the junction called depletion layer. It is so named because this region is devoid of (or depleted of) ***free and mobile charge carriers like electrons and holes***—there being present only positive ions which are not free to move.

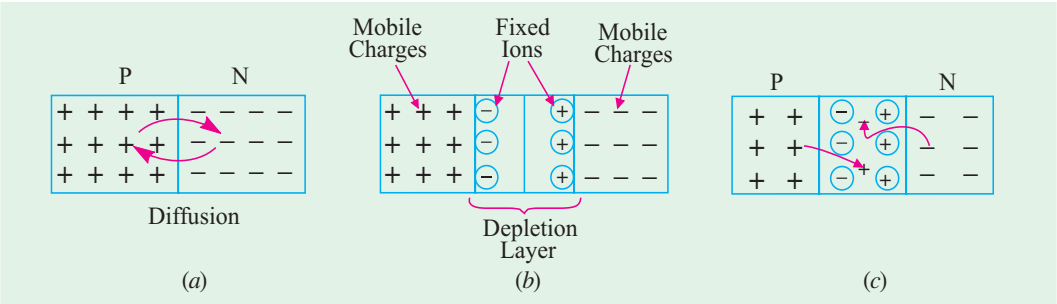
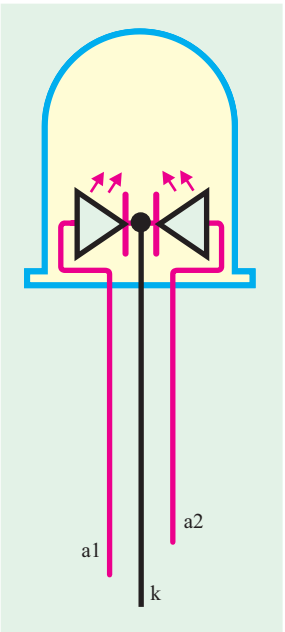


Fig. 51.33

It might seem from above that eventually all the holes from the *P*-side would diffuse to the *N*-side and all the electrons from the *N*-side would diffuse to the *P*-side but this does not occur due to the formation of ions on the two sides of the junction. The impurity atoms which provide these migratory electrons and holes are left behind in an ionized state bearing a charge which is opposite to that of the departed carrier. Also, these impurity ions, just like germanium atoms, are fixed in their positions in the crystal lattice in the *P*- and *N*- regions of the diode. Hence, as shown in Fig. 51.33 (*b*), they form parallel rows or 'plates' of opposite charges facing each other across the depletion layer. Obviously, row of ***fixed*** positive ions in the *N*-region is produced by the migration of electrons from the *N*- to *P*- region. Similarly, the row of ***fixed*** negative ions in the *P*-region is produced by the migration of holes from the *P*- to *N*-region.



If a majority carrier (either an electron or a hole) tries to cross into depletion layer, it can meet either of the following two facts :

(i) either it can be trapped or captured by the row of fixed impurity ions of opposite sign which guard its own region. For example, a hole trying to approach the depletion layer may be neutralized by the row of fixed negative ions situated in the *P*-region itself

at the edge of the depletion layer. So will be the case with the electron trying to approach the depletion layer from *N*-region [Fig. 51.33 (*c*)]; or

(ii) it may succeed in entering the depletion layer where it will be repelled by the row of similarly-charged impurity ions guarding the other region. But its life will be cut short by recombination with a majority carrier of opposite sign which has similarly entered the depletion layer from the other half of the diode.

Ultimately, an equilibrium condition is reached when depletion layer has widened to such an extent that no electrons or holes can cross the *P-N* junction.

51.40. Junction or Barrier Voltage

Even though depletion layer is cleared of charge carriers, it has oppositely-charged fixed rows of ions on its two sides. Because of this charge separation, an electric potential difference V_B is established across the junction even when the ***junction is externally isolated*** (Fig. 51.34). It is known as ***junction or barrier potential***. It stops further flow of carriers across the junction unless supplied by energy from an external source. At room temperature of 300°K, V_B is about 0.3 V for Ge and 0.7 V for Si.

The value of barrier voltage is given by $V_B = V_T \log_e N_a N_d / n_i^2$ where N_a, N_d, n_i^2 and V_T have the meanings explained in Art. 51.29 and 34. The value of V_T at room temperature of 300°K is given by

$$V_T = V_{300} = \frac{kT}{e} = \frac{1.38 \times 10^{-23} \times 300}{1.6 \times 10^{-19}} = 26 \text{ mV}$$

$$\therefore \quad V_B = 26 \log_e (N_a N_d / n_i^2) \text{ mV}$$

Barrier voltage depends on doping density, electronic charge and temperature. For a given junction, the first two factors are constant, thus making V_B dependent on temperature. With increase in temperature, more minority charge carriers are produced, leading to their increased drift across the junction. As a result, equilibrium occurs at a lower barrier potential. It is found that both for Ge and Si, V_B decreases by about 2 mV / °C.

$$\therefore \quad \Delta V_B = -0.002 \, \Delta t$$ where Δt is the rise in temperature in °C.

The strong field set up by V_B causes drift of carriers through depletion layer. As seen from |Fig. 51.35 (*b*), under the influence of this field, ***holes drift from N-to P-region and electrons from P- to -N region***. This drift current must be equal and opposite to the diffusion current [Fig. 51.35 (*a*)] because under condition of equilibrium and with no external supply, net current through the crystal is zero.

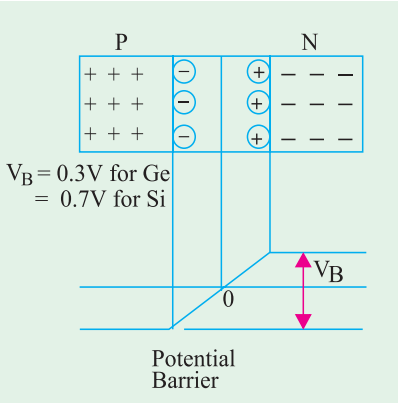


Fig. 51.34

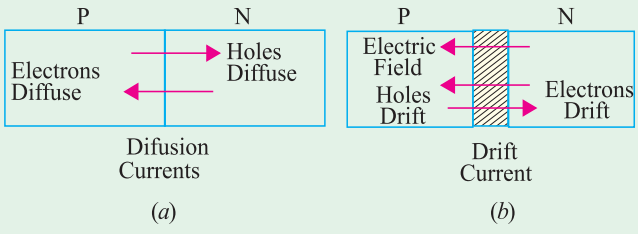


Fig. 51.35

To summarize the main points of Art. 51.39 and 51.40, we have :

1. As soon as *P-N* junction is formed, free electrons and holes start diffusing across the junction and recombining.

2. Their recombination leads to the appearance of a depletion layer across the junction which contains no ***mobile*** carriers but only immobile ions.
3. These immobile ions set up a barrier potential and hence an electric field which sets up drift current that is equal and opposite to the diffusion current when final equilibrium is reached.

Example 51.13. *Calculate the barrier potential at room temperature for P-N junction in silicon which is doped to a carrier density of 10^{21} m^{-3} on the P-side and 10^{22} m^{-3} on the N-side. The intrinsic carrier density for silicon is 1.4×10^{16} m^{-3}.* **(Electronics-I, Bangalore Univ. 1992)**

Solution. Using the relation given in Art 1.40, we have

$$V_B = 26 \log_e (N_a N_d / n_i^2) \text{ mV}$$
$$= 26 \log_e 10^{21} \times 10^{22}/(1.4 \times 10^{16})^2 = 641 \text{ mV} = \mathbf{0.641\ V}$$

Example 51.14. *Calculate the change in barrier potential of a P-N junction at 300°K if doping on the N-side is increased 1000 times while keeping doping on P-side unchanged.*

Solution. As seen from Art. 51.40, at 300°K

$$V_B = 26 \log_e (N_a N_d / n^2) \text{ mV}$$
$$\therefore \quad V_{B1} = 26 \ln (N_{a1} N_{d1}/n_i^2)\ ; \quad V_{B2} = 26 \ln (N_{a2} N_{d2} / n_i^2)$$
$$\therefore \quad V_{B2} - V_{B1} = 26 \log_e (N_{a2} N_{d2})/(N_{a1} N_{d1})$$
$$= 26 \log_e (N_{d2} / N_{d1}) = 26\ ln\ 1000 = \mathbf{179\ mV}$$

51.41. Energy Band Diagram of a P-N Junction

Now, let us consider the operation of a *P-N* junction in terms of its energy bands. At the instant of junction formation, energy bands of the trivalent impurity atoms in the *P*-region are at a slightly higher level than those of the pentavalent impurity atoms in the *N*-region as shown in Fig. 51.36. It is so because core attraction for valence electrons (+3) in a trivalent atom is less than the core attraction for valence electrons (+5) in a pentavalent atom. Consequently, trivalent valence electrons are in slightly higher orbit and, hence, at a higher energy level. However, there is some over-lap between respective bands of the two regions. Due to this reason, some high-energy electrons near the top of *N*-region conduction band diffuse into the lower part of the *P*-region conduction band. Soon after, they recombine with the holes in the valence band as shown in Fig. 51.36 (*a*). As diffusion continues, depletion layer begins to form. Another side-effect of this electron diffusion is that energy bands in

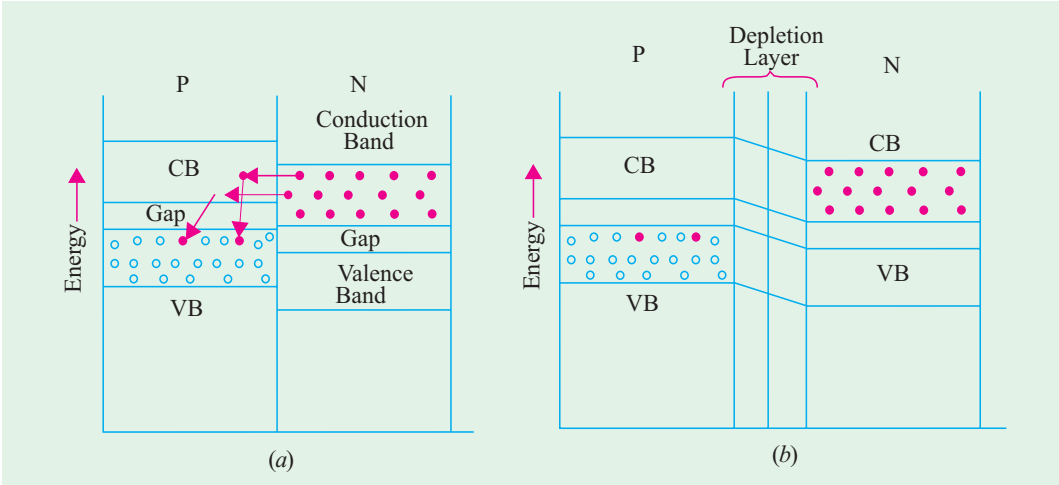


Fig. 51.36

the *N*-region shift downward due to loss of high-energy electrons. When the top of the conduction band in *N*-region reaches the same level as the bottom of the conduction band in *P*-region, further diffusion ceases and equilibrium condition is reached as shown in Fig. 51.36 (*b*). To an electron which might still try to diffuse across the junction, the path looks like a steep energy hill. It cannot climb this energy hill unless it receives energy from an external source.

51.42. Forward Biased P-N Junction

Suppose, positive battery terminal is connected to *P*-region of a semiconductor and the negative battery terminal to the *N*-region as shown in Fig. 51.37 (*a*). In that case the junction is said to be biased in the ***forward direction*** because it permits easy flow of current across the junction. This current flow may be explained in the following two ways :

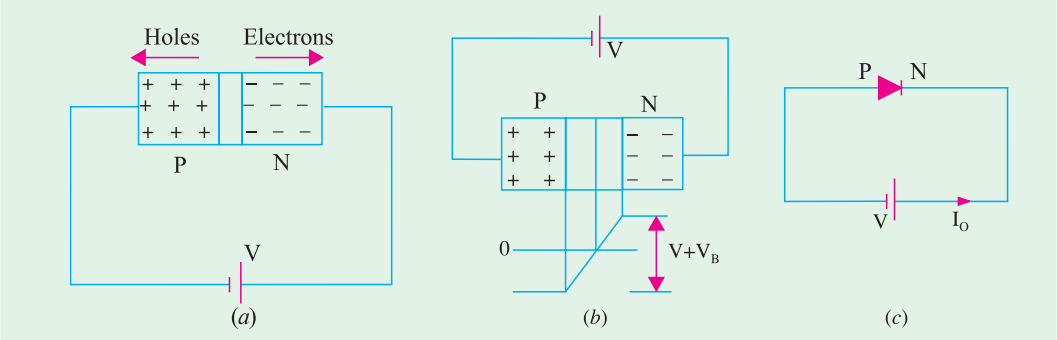


Fig. 51.37

(i) As soon as battery connection is made, holes are ***repelled*** by the positive battery terminal and electrons by the negative battery terminal with the result that both the electrons and the holes are driven ***towards*** the junction where they recombine. This ***en masse*** movement of electrons to the left and that of holes to the right of the junction constitutes a large current flow through the semiconductor. Obviously, the junction offers ***low resistance*** in the forward direction.

A more detailed picture of carrier flow is as under :

As free electrons move to the left, new ***free*** electrons are injected by the negative battery terminal into the *N*-region of the semiconductor. Thus, a flow of electrons is set up in the wire connected to the negative battery terminal. As holes are driven towards the junction, more holes are created in the *P*-region by the breakage of covalent bonds (Art. 51.21). These newly-created holes are driven towards the junction to keep up a continuous supply. But the electrons so produced are attracted to the left by the positive battery terminal from where they go to the negative terminal and finally to the *N*-region of the crystal.

Incidentally, it may be noted that though there is movement of both electrons and holes ***inside*** the crystal, ***only free electrons move in the external circuit i.e.*** in the battery-connecting wires.

Note. There is also present an extremely small amount of current I_0 due to minority carriers on either side of the junction. But it is negligible as compared to forward current which is due to majority carriers. This current I_0 depends on temperature but is almost independent of applied voltage.

(ii) Another way to explain current flow in forward direction is to say that forward bias of V volts lowers the barrier potential to $(V - V_B)$ which now allows more current to flow across the junction [Fig. 51.37 (*b*)].

Incidentally, it may be noted that forward bias reduces the thickness of the depletion layer as shown in Fig. 51.38.

Energy band diagram for forward bias is shown in Fig. 51.39. By comparing this figure with Fig.

51.36 it is seen that energy hill has been reduced. Because of this reduction, conduction electrons in *N*-region are able to cross over to *P*-region. After reaching there, each electron falls into a hole (path *A*) and becomes a valence electron. In this way, it is able to continue its journey towards the left end of the crystal.

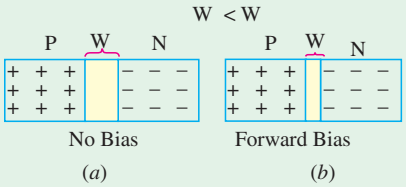


Fig. 51.38

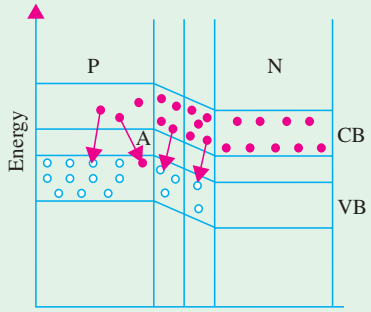


Fig. 51.39

51.43. Forward V/I Characteristic

A typical *V*/*I* characteristic for a forward-biased *P*-*N* junction is shown in Fig. 51.40. It is seen that forward current rises exponentially with the applied forward voltage. However, at ordinary room temperature, a p.d. of about 0.3 V is required before a reasonable amount of forward current starts flowing in a germanium junction. This voltage is known as ***threshold voltage*** (V_{th}) **or** ***cut-in voltage*** or ***knee voltage*** V_K. It is practically the same as barrier voltage V_B. Its value for silicon junction is about 0.7 volt. For $V < V_{th}$, current flow is negligible. But as applied voltage increases beyond the threshold value, the forward current increases sharply. If forward voltage is increased beyond a certain safe value, it will produce an extremely large current which may destroy the junction due to overheating.

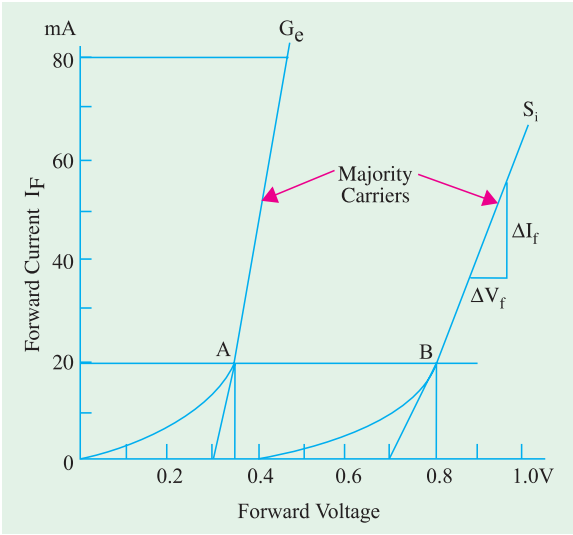


Fig. 51.40

Ge devices can stand junction temperatures around 100ºC whereas *Si* units can function upto 175ºC.

Obviously, the ***forward-biased junction has a low resistance***. For point *B* in Fig. 51.40, the forward resistance for Si is

$$R_F = 0.8\text{V}/20\text{ mA} = 40\ \Omega$$

Similarly, for point *A* on the *Ge* curve, $R_F = 0.36$ V/20 mA = 18 Ω

In practice, this static forward resistance is not used. Instead, the **dynamic** resistance or **incremental** resistance or **ac resistance** of the juncion is used. It is given by the reciprocal of the slope of the forward characteristic.

$$r_{ac} = \frac{1}{\Delta I_F/\Delta V_F} = \frac{\Delta V_F}{\Delta I_F}$$

Here, $\Delta V_F = 0.19$ V and $\Delta I_F = 37.6$ mA

$$\therefore \qquad r_{ac} = \frac{0.19}{37.6 \times 10^{-3}} \cong 5\ \Omega$$

51.44. Reverse Biased P-N Junction

When battery connections to the semiconductor are made as shown in Fig. 51.41 (*a*), the junction is said to ***reverse-biased***. In this case, holes are attracted by the negative battery terminal and electrons by the positive terminal so that both holes and electrons move ***away*** from the junction and ***away*** from each other. Since there is no electron-hole combination, no current flows and the junction offers high resistance.

Another way of looking at the process is that in this case, the applied voltage increases the barrier potential to $(V + V_B)$, thereby blocking the flow of majority carriers.

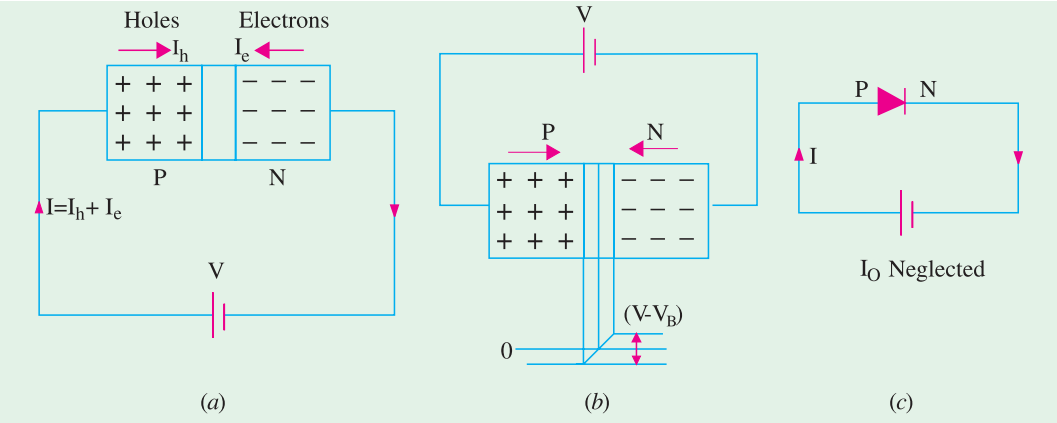


Fig. 51.41

Incidentally, it may be noted that under reverse bias condition, width of depletion layer is increased because of increased barrier potential as shown in Fig. 51.42.

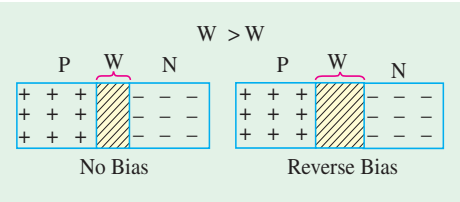


Fig. 51.42

Although, in this case, there is practically no current due to ***majority*** carriers, yet there is a small amount of current (a few μA only) due to the flow of ***minority*** carriers across the junction. As explained earlier in Art. 51.24, due to thermal energy, there are always generated some holes in the *N*-type region and some electrons in the *P*-type region of the semiconductor as shown in Fig. 51.28. The battery drives these minority carriers across the junction thereby producing a small current called ***reverse current or reverse saturation current*** I_0 or I_S. Since minority carriers are thermally-generated, I_0 is extremely ***temperature dependent***. For the same reason, forward current is also temperature dependent but to a much less degree because minority current forms a very small percentage of the majority current. The name saturation has been used because we cannot get minority current more than what is produced by thermal energy. In other words, I_S ***does not increase with increase in reverse bias***.

I_S is found to increase approximately 7 percent per 0°C rise in temperature both for Ge and Si. Since, $(1.07)^{10} = 2$, it means that reverse current ***doubles for every 10°C rise in temperature***. It is worth noting that reverse saturation current is also referred to as leakage current of the *P-N* junction diode.

With reverse bias, energy hill becomes too steep for majority carriers to go up the hill and cross over.

51.45. Reverse V/I Characteristic

As said earlier, the reverse saturation current is also referred to as ***leakage current*** of the *P-N* junction. Fig. 51.43 shows *V/I* characteristics of a reverse-biased *P-N* junction. It is seen that as reverse voltage is increased from zero, the reverse current quickly rises to its maximum or saturation value. Keeping temperature constant as the reverse voltage is increased, I_o is found to increase only slightly. This slight increase is due to the impurities on the surface of the semiconductor which behaves as a resistor and hence obeys Ohm's law. This gives rise to a very small current called ***surface leakage current***. Unlike the main leakage (or saturation) current, this surface leakage current is independent of temperature but depends on the magnitude of the reverse voltage.

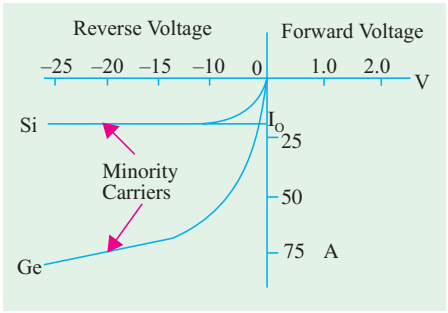


Fig. 51.43

A reverse-biased junction can be represented by a very large resistance. As seen from Fig. 51.43, in the case of Si, for a reverse voltage of about 15 V, I_0 = 10 μA. Hence, reverse resistance is R_R = 15 V/10 μA = 1.5 MΩ

51.46. Hall Effect

If a specimen (whether of a metal or a semiconductor) carrying a current *I* is placed in a ***transverse*** magnetic field of flux density *B*, an electric field is developed along a direction perpendicular to both *B* and *I*. This phenomenon is known as ***Hall effect*** and is used for the following purposes :

1. to determine whether a semiconductor is of *N*-type or *P*-type;
2. to find carrier concentration;
3. to measure the conductivity of the material;
4. to find carrier mobility;
5. to detect and measure magnetic fields one million times smaller than that of earth with the help of Hall-effect magnetometers.

As shown in Fig. 51.44 (*a*), a current *I* is flowing through the semiconductor in the direction *MN* under the influence of an external applied electric field E_L. Obviously, electrons comprising this current move along *NM* with a velocity of v_e. Consider one such electron shown in the figure. The direction of the force exerted on it by the magnetic field *B* can be found by using Fleming's left-hand rule and is as shown*. The magnitude of force is $Be\ v_e$.

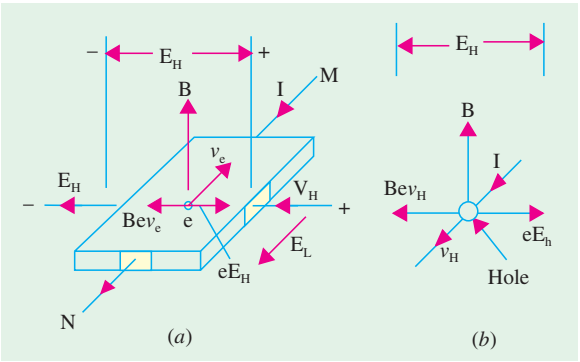


Fig. 51.44

Under the influence of this force, electrons will tend to crowd towards the left side of semiconductor. This collection of electrons to one side gives rise to electric potential difference V_H (called Hall voltage) and hence to an electric field E_H. This field begins to prevent additional electrons from arriving there. Force exerted by this field on the electrons is eE_H. Equilibrium is established when the two oppositely-directed magnetic and electric forces acting on the electron become equal in magnitude.

* While using the rule, the middle finger must point in the direction of conventional current *i.e.* along *MN* and not along the direction of electronic current *i.e.* along *NM*.

i.e. $Bev_e = eE_H$

or $Bv_e = E_H$ ...(*i*)

Also $v_e = \mu_e E_L$

$\therefore$ $B\mu_e E_L = E_H$ or $\mu_e B = E_H/E_L$

Hence, knowing E_L and B and measuring E_H, we can find electron mobility.

Now, current density $J = I/A = nev_e A/A = nev_e$

Substituting the value of υ_e from (*i*) above, we get

$J = neE_H / B$ or $n = JB / eE_H$

This equation may be used to find electron density.

Now, Hall coefficient, R_H = 1/charge density = $1/ne$ or $R_H = E_H/JB$

As shown in Art. 51.26. $\mu_e = 1/ne\,\rho = R_H/\rho$

where ρ is the resistivity of the semiconductor.

It may be noted that the above treatment is equally applicable to *P*-type semiconductors where current flow is made up of hole movement. If holes were to move in the same direction as that of the electrons in Fig. 51.44 (*a*), then polarity of Hall voltage E_H would be reversed as shown in Fig. 51.44 (*b*). In fact, it was the observation of the polarity of Hall voltage associated with *P*-type material which led to the concept of a positive hole.

Example 51.15. *The Hall-coefficient of a specimen of doped semiconductor is* 3.66×10^{-4} m^3 C^{-1} *and the resistivity of the specimen is* 8.93×10^{-3} Ω*–m. Determine the carrier mobility in* $m^2 v^{-1} s^{-1}$.

Solution. Using equation $\mu_e = R_H/\rho$

$$\mu_e = \frac{3.66 \times 10^{-4}}{8.93 \times 10^{-3}} = \mathbf{0.041\ m^2\ v^{-1}\ s^{-1}}$$

Example 51.16. *A sample of N-type semiconductor has a hall coefficient of 160* cm^3*/Coulomb. If is resistivity is 0.16* Ω*-cm, estimate the electron mobility in the sample,*

Solution. Using the relation, $\mu_e = R_H/\rho$, we have

$$\mu_e = \frac{160}{0.16} = \mathbf{1000\ cm^2/Volt\text{-}Sec.}$$

Example 51.17. *A current of 50 A is passed through a metal strip, which is subjected to a magnetic flux of density 1.2* Wb/m^2*. The magnetic field is directed at right angles to the current direction and the thickness of the strip in the direction of magnetic field is 0.5 mm. The Hall voltage is found to be 100 V. Calculate the number of conduction electrons per cubic metre in the metal.*

Solution. We know that $n = \dfrac{JB}{eE_H}$

Since $J = I/A = I/d.t$ where d is the distance between the two surfaces between the ends across which the elecric field is measured. The above equation can be rewritten as,

$$n = \frac{(I/A)\,B}{eE_H} = \frac{(I/d.t)\,B}{eV_H/d} = \frac{BI}{eV_H t} \qquad (\therefore\ E_H = V_H/d)$$

Substituting the values of V_H, t, B, I and e, we get

$$n = \frac{1.2 \times 50}{\left(1.6 \times 10^{-19}\right) \times 100 \times \left(0.5 \times 10^{-3}\right)} = \mathbf{7.51 \times 10^{21}/m^3}$$

Example 51.18. *An N-type semiconductor has a resistivity of* 20×10^{-2} *ohm-m. The mobility of electrons through a separate experiment was found to be* 100×10^{-4} m^2 $v^{-1}s^{-1}$*. Find the number of electron carriers per* m^3.

Solution. Using the equation, $\mu_e = 1/ne\,\rho$, we have

$$n = \frac{1}{e\mu_e \rho} = \frac{1}{1.6\times10^{-19}\times\left(100\times10^{-4}\right)\times\left(20\times10^{-2}\right)}$$

$$= \mathbf{3.1\times10^{21}/m^3}$$

Example 51.19. *The Hall-coefficient of a specimen of doped silicon is found to be 3.66×10^{-4} m^3C; the resistivity of the specimen is 8.93×10^{-3} Ω-m. Find the mobility and density of charge carriers assuming single carrier conduction.* **(U.P.S.C. Engg. Services 2002)**

Solution. $R_H = 3.66\times10^{-4}\ m^3C$; $\rho = 8.93\times10^{-3}$ Ω-m.

$$\mu = \sigma R_H = \frac{1}{\rho} R_H = \frac{1}{8.93\times10^{-3}}\times 3.66\times10^{-4} = \mathbf{0.041 m^2/V.s}$$

$$R_H = \frac{1}{ne} \text{ or } n = \frac{1}{R_H e} = \frac{1}{\left(3.66\times10^{-4}\right)\times\left(1.6\times10^{-19}\right)} = \mathbf{1.7\times10^{22}/m^3}$$

Example 51.20. *Resistivity of a sample semiconductor is 9m Ω-m. Its holes have mobility of 0.03 m^2/Vs. Calculate the value of Hall-coefficient of the sample.* **(U.P.S.C. Engg. Services 2002)**

Solution. $\rho = 9$ m Ω-m $= 9\times10^{-3}$ Ω-m, $\mu_p = 0.03 m^2$/V.s

$$\sigma = \frac{1}{\rho} = \frac{1}{9\times10^{-3}} = \mathbf{111.1 S/m}$$

$$\mu_p = \sigma R_H$$

$$R_H = \frac{\mu_p}{\sigma} = \frac{0.03}{111.1} = \mathbf{2.7\times10^{-4}\ m^3C}$$

Tutorial Problem No. 51.1

1. Compute the intrinsic conductivity of a specimen of pure silicon at room temperature given that $n_i = 1.4\times10^6\ m^{-3}$, $\mu_e = 0.145\ m^2$/V–s, $\mu_\mu = 0.05\ m^2$/V-*s* and $e = 1.6\times10^{-19}$ C. Also, calculate the individual contributions from electrons and holes.
[0.437×10^{-3} S/m ; 0.325×10^{-3} S/m ; 0.112×10^{-3} S/m]
2. Find (*i*) conductivity and (*ii*) resistance of a bar of pure silicon of length 1 cm and cross-sectional area 1 mm^2 at 300°K. Given : $n_i = 1.5\times10^{16}$ per m^3, $\mu_e = 0.13\ m^2$/V-s, $\mu_h = 0.05\ m^2$/V-s and $e = 1.6\times10^{-19}$ C. **[(*i*) 4.32×10^{-4} S/m (*ii*) 23.15 MΩ]**
3. A specimen of silicon is doped with acceptor impurity to a density of 10^{22} atoms per m^3. Given that $n_i = 1.4\times10^{16}$ per m^3, $\mu_e = 0.145\ m^2$/V–s, $\mu_h = 0.05\ m^2$/V-s, $e = 1.6\times10^{-19}$ C. All impurity atoms may be assumed to be ionized. **[nearly 80 S/m]**
4. Calculate the conductivity of a specimen of pure *Si* at room temperature of 300°K for which $n_i = 1.5\times10^{16}\ m^{-3}$, $\mu_e = 0.13\ m^2$/V-s, $\mu_h = 0.05\ m^2$/V-s, $e = 1.6\times10^{-19}$ C.
The *Si* specimen is now doped 2 parts per 10^8 of a donor impurity. If there are 5×10^{28} *Si* atoms/m^3, calculate its conductivity. By what factor has the conductivity increased ?
[4.32×10^{-4} S/m ; 20.8 S/m ; ≃ 48,000]
5. Mobilities of electrons and holes in a sample of intrinsic germanium at room temperature are 0.36 m^2 /V-s and 0.17 m^2 / V-s respectively. If the electron and hole densities are each equal to 2.5×10^{19} per m^3, calculate the conductivity. **[2.12 S/m]*(Electronic-I, Bangalore Univ.)***
6. A p.d. of 10 V is applied longitudinally to a rectangular specimen of intrinsic germanium of length 2.5 cm, width 0.4 cm and thickness 0.15 cm. Calculate at room temperature
(*i*) electron and hole drift velocities;
(*ii*) the conductivity of intrinsic G*e* if intrinsic carrier density is = $2.5\times10^{19}/m^3$;

(*iii*) the total current.

Given, $\mu_e = 0.38\ m^2\ V^{-1}s^{-1}$, $\mu_h = 0.18\ m^2V^{-1}s^{-1}$, $k = 1.38 \times 10^{-23}$ J-deg^{-1}, $e = 1.6 \times 10^{-19}$C

[(*i*) 152 m/s, 72 m/s (*ii*) 2.24 S/m (*iii*) 5.38 mA]

(Applied Electronics & Circuits, Grad. I.E.T.E. 1987)

5. The resistivity of a dopoed silicon material is 9×10^{-3}. The Hall coefficient is 3.6×10^{-4} Coulomb^{-1}. Assuring single carrier conduction, find the mobility and density of charge carriers. $e = 1.6 \times 10^{-19}$ Coulomb. ***(U.P.S.C. Engg. Services 1994)***

6. Consider an intrinsic Ge bar with material time constant of 100 μsec across a cross-section of 1mm^2 and length 1 cm. One side of the bar is illuminated with 10^{15} photons/sec. Assume that each incident photon generate one electron hole pair and these are uniformly distributed throughout the bar. Find the bar resistance under constant excitation at room temperature.

(Electronic Devices and Circuits, Nagpur Univ. Summer, 2004)

7. Find the concentration of holes and electrons in a *p*-type germanium at 300K, if the conductivity is 100 per ohm-cm. Also find these values for *n*-type silicon, if the conductivity is 0.1 per ohm-cms. Given that

for germanium $n_i = 2.5 \times 10^{13}/cm^3$

$M_n = 3800\ cm^2/V–s$, $M_p = 1800\ cm^2/V\text{-}s$

for silicon, $n_i = 1.5 \times 10^{10}$ per cm^3

$M_n = 1300\ cm^2/V\text{-}s$ and $M_p = 500\ cm^2/V–s$ ***(Electronics Engg; Bangalore Univ. 2004)***

8. A germanium P-N junction at 300 K has the following parameters; $N_D = 5 \times 10^{18}/cm^3$, $N_A = 6 \times 10^{16}/cm^3$, $n_i = 1.5 \times 10^{10}/cm^3$; Calculate the minority electron density in the P-region and the minority hole density in the N-region. ***(Electronics Engg. Bangalore Univ. 2003)***

OBJECTIVE TESTS – 51

1. The total energy of a revolving electron in an atom can

(*a*) have any value above zero

(*b*) never be positive

(*c*) never be negative

(*d*) not be calculated.

2. An atom is said to be ionised when any one of its orbiting electron

(*a*) Jumps from one orbit to another

(*b*) is raised to a higher orbit

(*c*) comes to the ground state

(*d*) is completely removed.

3. The maximum number of electrons which the M-shell of an atom can contain is

(*a*) 32 (*b*) 8

(*c*) 18 (*d*) 50.

4. Electronic distribution of an Si atom is

(*a*) 2, 10, 2 (*b*) 2, 8, 4

(c) 2, 7, 5 (*d*) 2, 4, 8.

5. Semiconductor materials have ..... bonds.

(*a*) ionic (*b*) covalent

(*c*) mutual (*d*) metallic.

6. The maximum number of electrons which the valence shell of an atom can have is

(*a*) 6 (*b*) 8

(*c*) 18 (*d*) 2

7. Silicon has Z = 14. Its outermost orbit is

(*a*) partially filled

(*b*) half filled

(*c*) completely occupied

(*d*) empty

8. Major part of the current in an intrinsic semiconductor is due to

(*a*) conduction-band electrons

(*b*) valence-band electrons

(*c*) holes in the valence band

(*d*) thermally-generated electron.

9. Conduction electrons have more mobility than holes because they

(*a*) are lighter

(*b*) experience collisions less frequently

(*c*) have negative charge

(*d*) need less energy to move them.

10. Doping materials are called impurities because they

(*a*) decrease the number of charge carriers
(*b*) change the chemical properties of semiconductors
(*c*) make semiconductors less than 100 percent pure
(*d*) alter the crystal structures of the pure semiconductors.

11. Current flow in a semiconductor depends on the phenomenon of
(*a*) drift (*b*) diffusion
(*c*) recombination (*d*) all of the above.

12. The process of adding impurities to a pure semiconductor is called
(*a*) mixing (*b*) doping
(*c*) diffusing (*d*) refining.

13. The most widely used semiconucting material in electronic devices is
(*a*) germanium (*b*) sillicon
(*c*) copper (*d*) carbon

14. Electon-hole pairs are produced by
(*a*) recombination (*b*) thermal energy
(*c*) ionization (*d*) doping

15. Recombination takes place when
(*a*) an electron falls into a hole
(*b*) a positive and a negative ion bond together
(*c*) a valence electron becomes a conduction
(*d*) a crystal is formed

16. When a *P-N* junction is formed, diffusion current causes
(*a*) mixing of current carriers
(*b*) forward bias
(*c*) reverse bias
(*d*) barrier potential.

17. The leakage current of a *P-N* diode is caused by
(*a*) heat energy (*b*) chemical energy
(*c*) barrier potential (*d*) majority carriers.

18. Electronic components which are made of a semiconductor material are often called ..... devices.
(*a*) solid-state (*b*) silicon
(*c*) germanium (*d*) intrinsic.

19. Any voltage that is connected across a *P-N* junction is called ..... voltage.
(*a*) breakdown (*b*) barrier
(*c*) bias (*d*) reverse.

20. The area within a semiconductor diode where no mobile current carriers exist when it is formed is called ..... region.
(*a*) depletion (*b*) saturation
(*c*) potential barrier (*d*) space charge.

21. The depletion region of a semiconductor diode is due to
(*a*) reverse biasing
(*b*) forward biasing
(*c*) crystal doping
(*d*) migration of mobile charge carriers.

22. If an intrinsic semiconductor is doped with a very small amount of boron, then in the extinsic semiconductor so formed, the number of electrons and holes will,
(*a*) decrease
(*b*) increase and decrease respectively
(*c*) increase
(*d*) decrease and increase respectively.

23. Two initially identical samples A and B of pure germanium are doped with donors to concentrations of 1×10^{20} and 3×10^{20} repectively. If the hole concentration in A is 9×10^{12}, then the hole concentration in B at the same temperature will be
(*a*) $3\ 10^{12}\ m^{-3}$ (*b*) $7 \times 10^{12}\ m^{-3}$
(*c*) $11 \times 10^{12}\ m^{-3}$ (*d*) $27 \times 10^{12}\ m^{-3}$

24. Hall effect is observed in a specimen when it (metal or a semiconductor) is carrying current and is placed in a magnetic field. The resultant electric field inside the specimen will be in
(*a*) a direction normal to both current and magnetic field
(*b*) the direction of current
(*c*) a direction antiparallel to the magnetic field
(*d*) an arbitrary direction depending upon the conductivity of the specimen.

ANSWERS

1. (*b*)	**2.** (*d*)	**3.** (*c*)	**4.** (*b*)	**5.** (*b*)	**6.** (*b*)	**7.** (*b*)
8. (*a*)	**9.** (*d*)	**10.** (*d*)	**11.** (*d*)	**12.** (*b*)	**13.** (*b*)	**14.** (*b*)
15. (*a*)	**16.** (*d*)	**17.** (*a*)	**18.** (*a*)	**19.** (*c*)	**20.** (*a*)	**21.** (*d*)
22. (*d*)	**23.** (*a*)	**24.** (*a*)				

GO To FIRST

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CHAPTER 52

P-N JUNCTION DIODE

Learning Objectives

- P-N Junction Diode
- Derivation of Junction Resistance
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- Junction Capacitance
- Equivalent Circuit of P-N Junction
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- Point Contact Junction
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- Diode Clipper and Clamper Circuits
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- Some Clipping Circuits
- Clampers
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Chemist, led the research for the molecular diode (In the semiconductor industry, called p-n junctions)

CONTENTS

52.1. P-N Junction Diode

(*a*) Construction

It is a two-terminal device consisting of a P-N junction formed either in Ge or Si crystal. Its circuit symbol is shown in Fig. 52.1 (*a*). The *P*-and *N*-type regions are referred to as anode and cathode respectively. In Fig. 52.1 (*b*), arrow-head indicates the conventional direction of current flow when forward-biased. It is the same direction in which hole flow takes place.

Commercially available diodes usually have some means to indicate which lead is *P* and which lead is *N*. Standard notation consists of type numbers preceded by 'IN' such as IN 240 and IN 1250. Here, 240 and 1250 correspond to colour bands. Fig. 52.2 (*a*) shows typical diodes having a variety of physical structures whereas Fig. 52.2 (*b*) illustrates terminal identifications. Also refer to the picture of two commercial diodes shown in Fig 52.1(*c*).

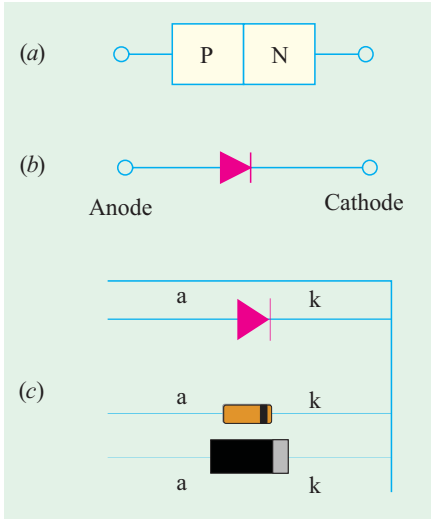


Fig. 52.1

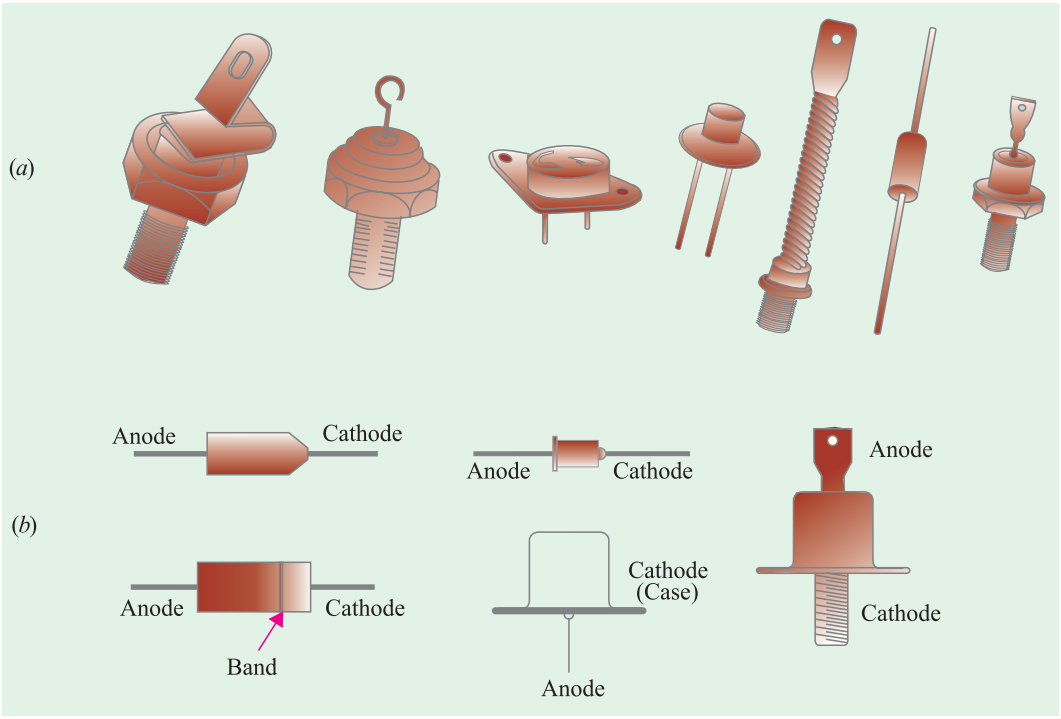


Fig. 52.2

The low-current diodes whose body is about 3 mm long can carry a forward current of about 100 mA, have saturation current of 5 μA at room temperature (25°C) and can withstand a reverse voltage of 75 V without breaking down. The medium-current diodes can pass a forward current of about 500 mA and can withstand a reverse voltage of 250 V. The high-current diodes or power diodes can pass a forward current of many amperes and can survive several hundred volts of reverse voltage.

(*b*) Diode Mounting

Low and medium-current diodes are usually mounted by soldering their leads to the connecting

terminals. The heat generated by these diodes (when operating) is small enough to be carried away by air convection and conduction along the connecting leads. However, high-current stud-mounted diodes generate large amounts of heat for which air convection is totally inadequate. For coolong, they need heat sinks made of metals such as copper or aluminium which are good conductors of heat. The sink absorbs heat from the device and then transfers it to the surrounding air by convection and radiation since it has large surface area.

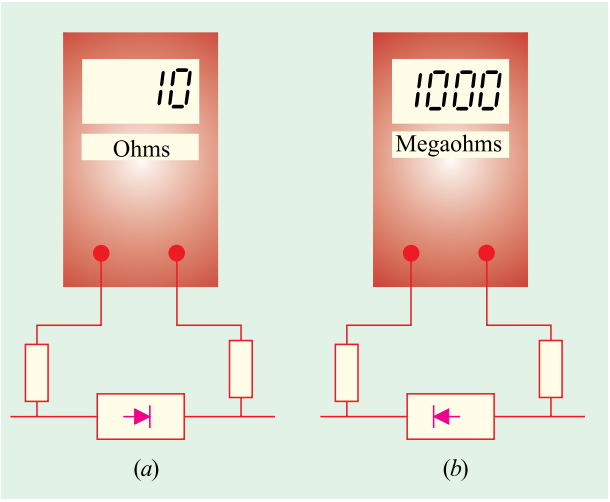


Fig. 52.3

(c) Working

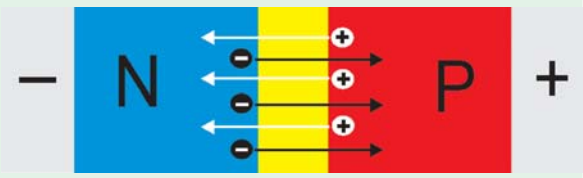
A P-N junction diode is one-way device offering low resistance when forward-biased [Fig. 52.3 (*a*)] and behaving almost as an insulator when reverse-biased [Fig. 52.3 (*b*)]. Hence, such diodes are mostly used as rectifiers *i.e.* for converting alternating current into direct current.

(d) V/I Characteristic

Fig. 52.4 shows the static voltage-current characteristics for a low-power P-N junction diode.

1. Forward Characteristic

When the diode is forward-biased and the applied voltage is increased from zero, hardly any current flows through the device in the beginning. It is so because the external voltage is being opposed by the internal barrier voltage V_B whose value is 0.7 V for Si and 0.3 V for Ge. As soon as V_B is neutralized, current through the diode increases rapidly with increasing applied battery voltage. It is found that as little a voltage as 1.0 V produces a forward current of about 50 mA. A burnout is likely to occur if forward voltage is increased beyond a certain safe limit.



Forward biased diode.

2. Reverse Characteristic

When the diode is reverse-biased, majority carriers are blocked and only a small current (due to minority carriers) flows through the diode. As the reverse voltage is increased from zero, the reverse current very quickly reaches its maximum or saturation value I_0 which is also known as ***leakage current***. It is of the order of nanoamperes (nA) for Si and microamperes (μA) for Ge. The value of I_0 (or I_s) is independent of the applied reverse voltage but depends on **(*a*)** temperature, **(*b*)** degree of doping and **(*c*)** physical size of the junction.



Reverse biased diode.

As seen from Fig. 52.4, when reverse voltage exceeds a certain value called break-down voltage V_{BR} (or Zener voltage V_z), the leakage current suddenly and sharply increases, the curve indicating zero resistance at this point. Any further increase in voltage is likely to produce burnout unless protected by a current-limiting resistor.

When P-N junction diodes are employed primarily because of this breakdown property as voltage regulators, they are called Zener diodes (Art. 54.1).

3. Equation of the Static Characteristic

The volt-ampere characteristics described above are called ***static*** characteristics because they describe the d.c. behaviour of the diode. The forward and reverse characteristics have been combined into a single diagram of Fig. 52.4.

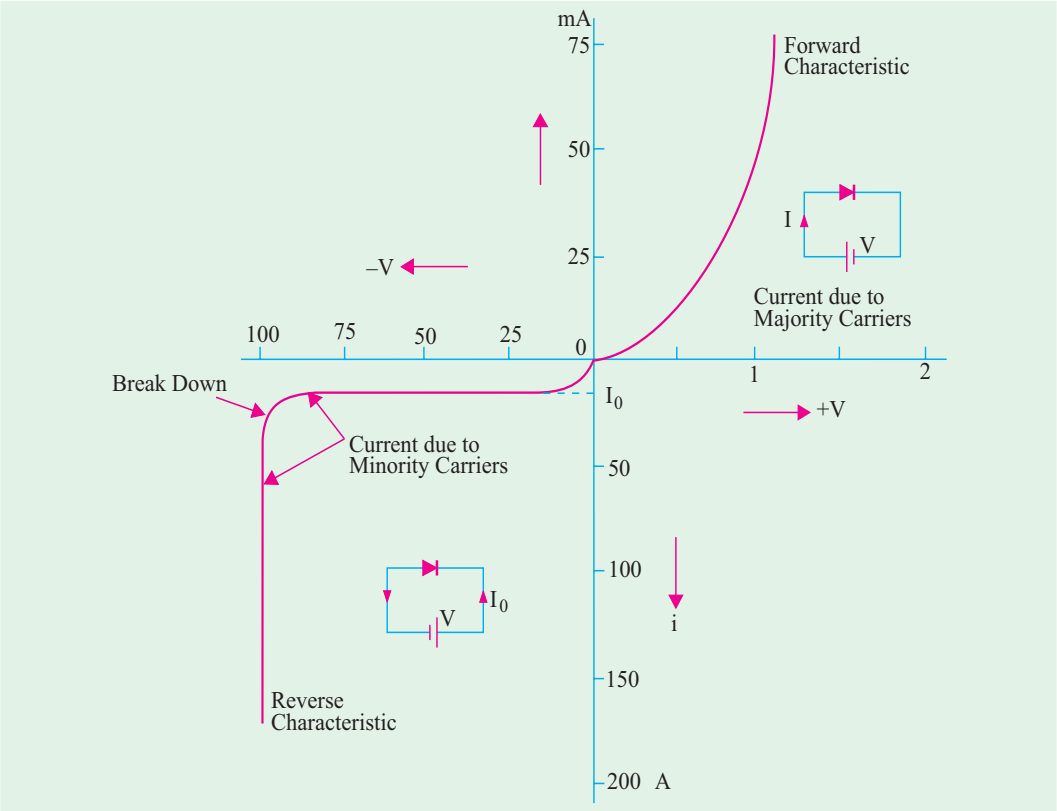


Fig. 52.4

These characteristics can be described by the analytical equation called ***Boltzmann diode equation*** given below :

$$I = I_0 \left(e^{\frac{eV}{kT}} - 1\right) \text{ ampere}$$

where I_0 = diode reverse saturation current

V = voltage across junction – positive for forward bias and negative for reverse bias.

k = Boltzmann constant = 1.38×10^{-23} J/°K

T = crystal temperature in °K

η = 1 – for germanium

= 2 – for silicon

Hence, the above diode equation becomes

$I = I_0 (e^{eV/kT} - 1)$. – for germanium

$I = I_0 (e^{eV/2kT} - 1)$ – for silicon

Now, e/k = 11,600 and putting $T/11{,}600 = V_T$, the above equation may be written as

$$I = I_0 (e^{11{,}600V/\eta T} - 1) = I_0 (e^{V/\eta V_T} - 1) \text{ ampere}$$

Now, at room temperature of (273 + 20) = 293°K, $V_T = 293/11{,}600 = 0.025$ V = 25 mV. Substituting

the value of η, we have

$$I = I_0 (e^{40\,V} - 1) \quad \text{– for Ge}$$
$$\cong I_0 e^{40\,V} \quad \text{– if } V > 1 \text{ volt}$$
$$I = I_0 (e^{20\,V} - 1) \quad \text{– for Si}$$
$$\cong I_0 e^{20\,V} \quad \text{– if } V > 1 \text{ volt}$$

We may also write the above diode equation as under

$$I = I_0 (e^{V_f/\eta V_T} - 1) \quad \text{– forward bias}$$
$$= I_0 (e^{V_R/\eta V_T} - 1) \quad \text{– reverse bias}$$

(e) Diode Parameters

The diode parameters of greatest interest are as under :

1. Bulk resistance (r_B)

It is the sum of the resistance values of the P-and N-type semiconductor materials of which the diode is made of.

$$\therefore \quad r_B = r_P + r_N \quad \text{– Fig. 52.5}$$

Usually, it is very small. It is given by

$$r_B = (V_F - V_B)/I_F$$

It is the resistance offered by the diode well above the barrier voltage *i.e.* when current is large. Obviously, this resistance is offered in the forward direction.

2. Junction resistance (r_j)

Its value for forward-biased junction depends on the magnitude of forward *dc* current.

$$r_j = 25 \text{ mV}/I_F(\text{mA}) \quad \text{– for Ge}$$
$$= 50 \text{ mV}/I_F(\text{mA}) \quad \text{– for Si}$$

Obviously, it is a *variable* resistance.

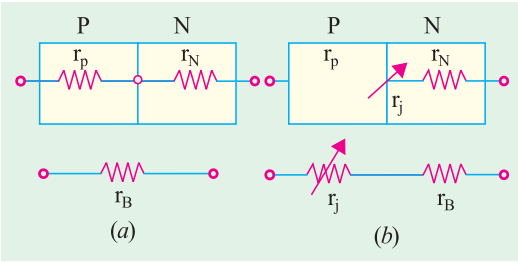


Fig. 52.5

3. Dynamic or ac resistance

$$r_{ac} \text{ or } r_d = r_B + r_j$$

For large values of forward current, r_j is negligible. Hence, $r_{ac} = r_B$. For small values of I_F, r_B is negligible as compared to r_j. Hence $r_{ac} = r_j$.

4. Forward voltage drop

It is given by the relation

$$\text{forward voltage drop} = \frac{\text{power dissipated}}{\text{forward dc current}}$$

5. Reverse saturation current (I_0). It has already been discussed in Art. 52.42.

6. Reverse breakdown voltage (V_{BR}). It is discussed in Art. 51.3.

7. Reverse dc resistance $R_R = \dfrac{\text{reverse voltage}}{\text{reverse current}}$

(f) Applications

The main applications of semiconductor diodes in modern electronic circuitry are as under :

1. As power or rectifier diodes. They convert ac current into dc current for dc power supplies of electronic circuits.
2. As signal diodes in communication circuits for modulation and demodulation of small signals.
3. As Zener diodes in voltage stabilizing circuits.
4. As varactor diodes–for use in voltage-controlled tuning circuits as may be found in radio and TV receivers. For this purpose, the diode is deliberately made to have a certain range of junction capacitance. The capacitance of the reverse-biased diode is given by $C = K/\sqrt{V_R}$

where V_R is the reverse voltage.

5. In logic circuits used in computers.

Example 52.1. *Using approximate Boltzmann's diode equation, find the change in forward bias for doubling the forward current of a germanium semiconductors at 290°K.*

(Basic Electronics, Osmania Univ. 1993)

Solution. The approximate Boltzmann's diode equation is given by $I = I_0 \exp(eV/kT)$

$$\therefore \quad I_1 = I_0 \exp(eV_1/kT) \text{ and } I_2 = I_0 \exp(eV_2/kT)$$

$$\therefore \quad \frac{I_2}{I_1} = \exp\left[\frac{e}{kT}(V_2 - V_1)\right]$$

$$\text{or} \quad (V_2 - V_1) = \frac{kT}{e} \ln\left(\frac{I_2}{I_1}\right) = 25 \ln\left(\frac{I_2}{I_1}\right) \text{ mV}$$

$$\text{Since} \quad I_2 = 2I_1 \quad \text{or} \quad I_2 / I_1 = 2$$

$$\therefore \quad (V_2 - V_1) = 25 \ln 2 = 25 \times 0.693 = \mathbf{17.3\ mV}$$

Example 52.2. *A certain P-N junction diode has a leakage current of 10^{-14} A at room temperature of 27°C and 10^{-9} A at 125°C. The diode is forward-biased with a constant-current source of 1 mA at room temperature. If current is assumed to remain constant, calculate the junction barrier voltage at room temperature and at 125°C.*

Solution. As given in Art. 52.1,

$$I = I_0 \exp(eV/kT - 1) \quad \text{or} \quad (I/I_0) + 1 = \exp(eV/kT)$$

Taking logarithm of both sides and solving for V, we get

$$V = \frac{kT}{e} \ln\left(\frac{I}{I_0} + 1\right)$$

Now, at 27°C or (273 + 27) = 300°K

$$\therefore \quad kT/e = 1.38 \times 10^{-23} \times 300 / 1.6 \times 10^{-19} = 26 \text{ mV}$$

$$\text{At} \quad (273 + 125) = 398°\text{K},$$

$$kT/e = 1.38 \times 10^{-23} \times 398 \times (1.6 \times 10^{-19}) = 36 \text{ mV}$$

$$\text{Hence, at 27°C,} \quad V_B = 26 \ln\left(\frac{10^{-3}}{10^{-14}} + 1\right) = 660 \text{ mV} = \mathbf{0.66\ V}$$

$$\text{At 125°C,} \quad V_B = 36 \text{ In}\left(\frac{10^{-3}}{10^{-9}} + 1\right) = 500 \text{ mV} = \mathbf{0.5\ V}$$

52.2. Derivation of Junction Resistance

Junction resistance is also known as ***incremental* or *dynamic resistance*** and is an important parameter particularly in connection with small-signal operations of the diode.

$$r_j = dV/dI \quad \text{or} \quad g_j = dI/dV$$

$$\text{Now,} \quad I = I_0(e^{V/\eta V_T} - 1) = I_0 e^{V/\eta V_T} - I_0$$

$$\therefore \quad g_j = \frac{dI}{dV} = \frac{I_0 e^{V/\eta V_T}}{\eta V_T} = \frac{I + I_0}{\eta V_T}$$

(*a*) Reverse bias

When reverse bias is greater than a few tenths of a volt *i.e.* when $|V/\eta V_T| \gg |$, then g_j is extremely small so that r_j is very large. That high value is also represented by R_R.

(*b*) Forward bias

Again, for a forward bias greater than a few tenths of a volt, $I \gg I_0$, hence, $g_j = I/\eta V_T$ and $r_j = \eta V_T/I$.

Now, at room temperature of 293°K, $V_T = T/11{,}600 = 293/11{,}600 = 25$ mV. Also $\eta = 1$ for Ge and 2 for Si.

$\therefore$ $r_j = 25\,mV/I\,mA$ – for germanium
$= 50\,mV/I\,mA$ – for silicon

Example 52.3. *A silicon diode has a forward voltage drop of 1.2 V for a forward dc current of 100 mA. It has a reverse current of 1 μA for a reverse voltage of 10 V. Calculate*
(a) *bulk and reverse resistance of the diode*
(b) *ac resistance at forward dc current of* ***(i)*** *2.5 mA and* ***(ii)*** *25 mA.*

Solution. (a) $r_B = \dfrac{V_F - V_B}{I_F} = \dfrac{1.2\text{ V} - 0.7\text{ V}}{100\text{ mA}} = 5\ \Omega$

$R_R = V_R / I_R = 10\text{ V}/1\ \mu\text{A} =$ **10 M**

(b) (i) $r_j = 25\text{ mV}/2.5\text{ mA} = 10\ \Omega$ $r_{ac} = r_B + r_j = 5 + 10 =$ **15 Ω**

(ii) $r_j = 25\text{ mV}/25\text{ mA} = 1\ \Omega$ $\therefore$ $r_{ac} = 5 + 1 =$ **6 Ω**

Example 52.4. *Using analytical expression for diode current, calculate the dynamic slope resistance of a germanium diode at 290 K when forward biased at current of* ***(i)*** *10 μA and* ***(ii)*** *5 mA.* **(Electronics-I, Mysore Univ. 1992)**

Solution. $I = I_0 (e^{eV/kT} - 1) \approx I_0 e^{eV/kT}$

$\therefore$ $dI = \dfrac{e}{kT} I_0 [e^{eV/kT}]\, dV = \dfrac{e}{kT}\, I\, dV$

$\therefore$ $r_d = \dfrac{dV}{dI} = \dfrac{kT}{eI} = \dfrac{25 \times 10^{-3}}{I}$ – I in ampere

(i) Now, $I = 10\ \mu\text{A} = 10 \times 10^{-6} = 10^{-5}$ A

$r_d = 25 \times 10^{-3} / 10^{-5} =$ **2500 Ω**

(ii) Now, $I = 5\text{ mA} = 5 \times 10^{-3}$ A

$\therefore$ $r_d = 25 \times 10^{-3}/5 \times 10^{-3} =$ **5 Ω**

Example 52.5. *Find the current through the 20 Ω resistor shown in Fig. 52.6 (a). Each silicon diode has a barrier potential of 0.7 V and a dynamic resistance of 2 Ω. Use the diode equivalent circuit technique.* **(Semiconductor Devices, Gujarat BTE, 1993)**

Solution. In Fig. 52.6 (*b*) each diode has been replaced by its equivalent circuit. It is seen that diodes D_1 and D_3 are forward-biased by 5 V battery whereas D_2 and D_4 are reverse-biased. Hence, the current will flow from point *A* to *B*, then to *C* via 20 Ω resistance and then back to the negative terminal of the 5 V battery.

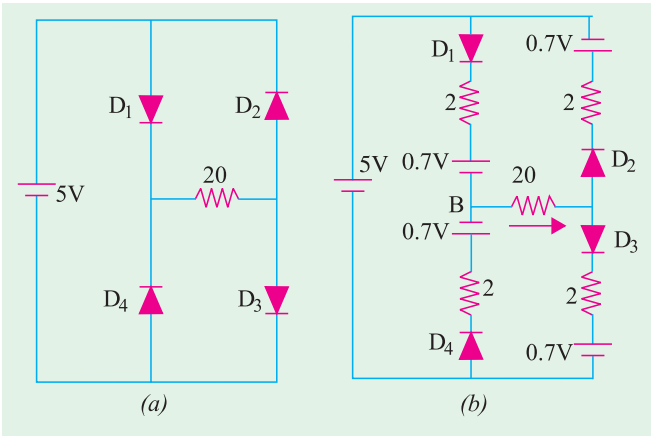


Fig. 52.6

The net voltage in the equivalent circuit is

$V_{net} = 5 - 0.7 - 0.7 = 3.6$ V

Total resistance seen by this net voltage is

$R_T = 2 + 20 + 2 = 24\ \Omega$

The circuit current $I = V_{net}/R_T = 3.6/24 =$ **0.15 A**

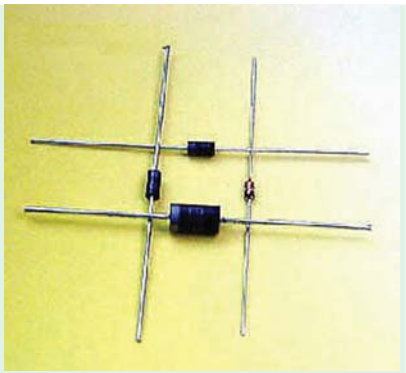
52.3. Junction Breakdown

If the reverse bias applied to a *P-N* junction is increased, a point is reached when the junction breaks down and reverse current rises sharply to a value limited only by the external resistance

connected in series with the junction (Fig. 52.7). This critical value of the voltage is known as **breakdown voltage** (V_{BR}). It is found that once breakdown has occured, very little further increase in voltage is required to increase the current to relatively high values. The junction itself offers almost zero resistance at this point.

The breakdown voltage depends on the width of the depletion region which, in turn, depends on the doping level.

The following two mechanism are responsible for breakdown under increasing reverse voltage:



Zener diodes

1. Zener Breakdown

This form of breakdown occurs in junctions which, being heavily doped, have narrow depletion layers. The breakdown voltage sets up a very strong electric field (about 10^8 V/m) across this narrow layer. This field is strong enough to ***break or rupture the covalent bonds*** thereby generating electron-hole pairs. Even a small further increase in reverse voltage is capable of producing large number of current carriers. That is why the junction has very low resistance in the break-down region.

2. Avalanche Breakdown

This form of breakdown occurs in junctions which being lightly-doped, have wide depletion layers where the electric field is not strong enough to produce Zener breakdown. Instead, the minority carriers (accelerated by this field) collide with the semiconductor atoms in the depletion region. Upon collision with valence electrons, covalent bonds are broken and electron-hole pairs are generated. These newly-generated charge carriers are also accelerated by the electric field resulting in more collisions and hence further production of charge carriers. This leads to an avalanche (or flood) of charge carriers and, consequently, to a very low reverse resistance. The two breakdown phenomena are shown in Fig. 52.7

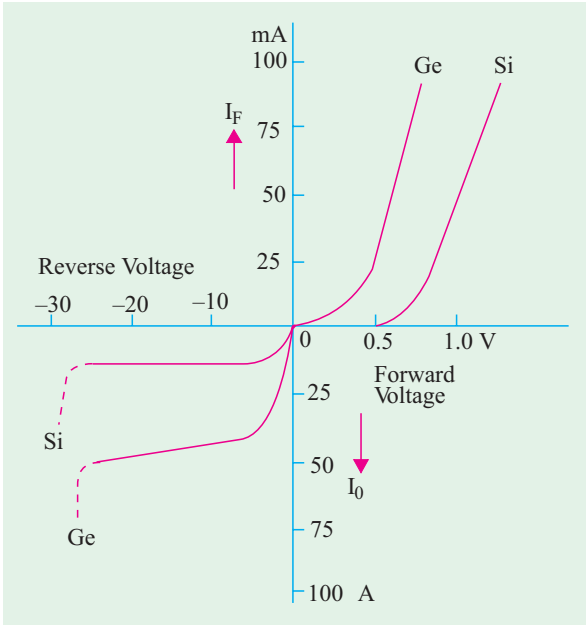


Fig. 52.7

52.4. Junction Capacitance

Capacitive effects are exhibited by *P-N* junctions when they are either forward-biased or reverse-biased.

(*a*) Transition Capacitance (C_T) or Space-charge Capacitance

When a *P-N* junction is reverse-biased, the depletion region acts like an insulator or as a dielectric material essential for making a capacitor. The *P*- and *N*-type regions on either side have low resistance and act as the plates. We, therefore, have all the components necessary for making a parallel-plate capacitor. This junction capacitance is called ***transition or space charge*** capacitance (C_{pn} or C_T). It may be calculated by the usual formula $C = \in A/d$. Its typical value is 40 pF. Since thickness of depletion (or transition) layer depends on the amount of reverse bias, capacitance C_T can be

controlled with the help of applied bias. This property of variable capacitance possessed by a reverse-biased *P-N* junction is used in the construction of a device known as ***varicap or varactor***.

This capacitance is voltage dependent as given by the relation

$$C_T = \frac{K}{(V_K + V_R)^n}$$

where V_K = knee voltage ; V_R = applied reverse voltage

K = constant depending on semiconductor material

$n = \frac{1}{2}$ – for alloy junction and = $\frac{1}{3}$ –for diffused junction

The voltage-variance capacitance of a reverse-biased *P-N* junction is used in many circuits one of which is automatic frequency control (AFC) in an FM tuner. Other applications include self-balancing bridge circuits, special type of amplifiers known as ***parametric*** amplifiers and electronic tuners in TV.

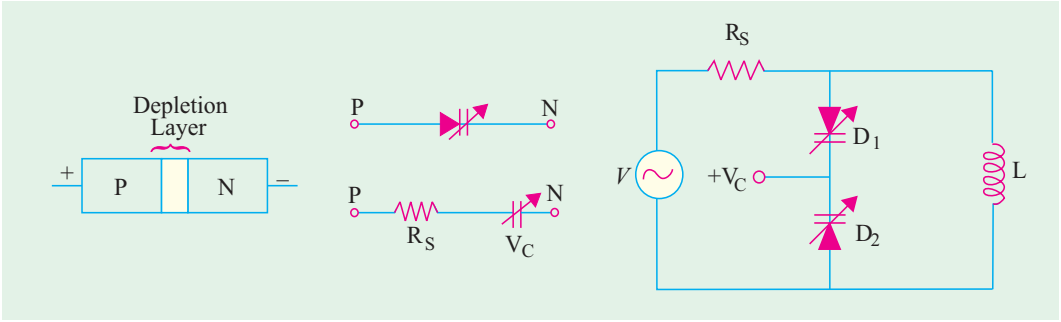


Fig. 52.8

Fig. 52.9

When used in such a role, the diodes are called ***varactors, varicaps* or *voltacaps***. The symbol of these diodes when used in this role is shown in Fig. 52.8 along with its equivalent circuit.

When used in a resonant circuit, varactor acts as a variable capacitor and allows the resonant frequency to be adjusted by a variable voltage level. In Fig. 52.8, two varactors have been used to provide total variable capacitance in a parallel resonant circuit. Here V_C is a variable dc voltage that controls the reverse bias and hence the capacitance of the diodes.

(*b*) Diffusion or Storage Capacitance (C_D)

This capacitive effect is present when the junction is ***forward-biased***. It is called diffusion capacitance to account for the time delay in moving charges across the junction by diffusion process.* Due to this fact, this capacitance cannot be identified in terms of a dielectric and plates. It varies directly with the magnitude of forward current as explained below in more details.

Consider a forward-biased junction which is carrying a forward current I_F. Suppose the applied voltage is suddenly reversed, then I_F ceases suddenly but leaves lot of majority charge carriers in the depletion region. These charge carriers must get out of the region which, to their bad luck, becomes wider under the reverse bias. Hence, it is seen that when a forward-biased *P-N* junction is suddenly reverse-biased, a reverse current flows which is large initially but gradually decreases to the level of saturation current I_0. This effect can be likened to the discharging current of a capacitor and is, therefore, rightly represented by a capacitance called ***diffusion capacitance C_D***. Since the number of charge carriers left in depletion layer is proportional to forward current, C_D is direcly proportional to I_F. Its typical value is 0.02 μF which is 5000 times C_T.

The capacitance assumes great significance in the operation of devices which are required to switch rapidly from forward to reverse bias. If C_D is large, this switchover cannot be rapid. It will delay both the switch-on and the switch-off. This effect of C_D is variously known as ***recovery time*** or carrier storage.

* In the case of forward bias, the diode current is almost entirely due to diffusion (drift current being negligible).

If τ is mean lifetime of charge carriers, then a flow of charge Q yields a diode current of

$$I = \frac{Q}{\tau} = I_0 (e^{V/\eta V_T} - 1) \cong I_0 \, e^{V/\eta V_T}, \; Q = I\tau = \tau I_0 \, e^{V/\eta V_T}$$

$$\therefore \quad C_D = \frac{dQ}{dV} = \frac{\tau I_0}{\eta V_T} e^{V/\eta V_T} \cong \frac{\tau I}{\eta V_T}$$

52.5. Equivalent Circuit of a P-N Junction

We have seen from above that a forward-biased junction offers ac resistance r_{ac} and possesses diffusion capacitance C_D (which comes into the picture only when frequency of the applied voltage is very high). Hence, it can be represented by the equivalent circuit of Fig. 52.10 (*a*). An opposing battery has been connected in series with r_{ac} to account for the junction barrier potential.

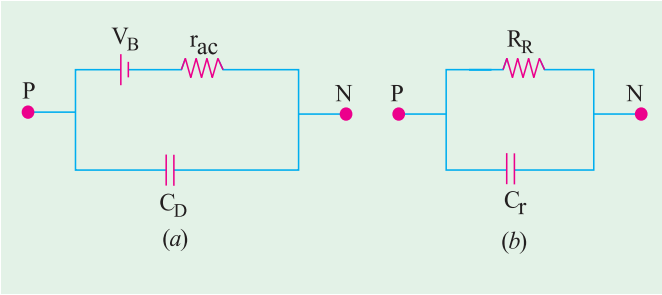


Fig. 52.10

As seen from Fig. 52.10 (*b*), a reverse-biased junction can be simply represented by a reverse resistance R_R connected in *parallel* with a capacitance C_T or C_{pn}.

Example 52.6. *Calculate the barrier potential for Si junction at (a) 100°C and (b) 0°C if its value at 25°C is 0.7 V.*

Solution. As seen from Art. 1.38.

$$\Delta V = -0.002. \quad \Delta t = -0.002 \, (t_2 - t_1)$$

(*a*) $\Delta t = (100 - 25) = 75°C$ $\quad \therefore \Delta V = -0.002 \times 75 = -0.15$ V

$\therefore$ V_B at 100°C $= 0.7 + (-0.15) =$ **0.55 V**

(*b*) $\Delta t = (0 - 25) = -25°C$ $\quad \therefore \Delta V = -0.002 \times (-25) = 0.05$ V

$\therefore$ V_B at 0°C $= 0.7 + 0.05 =$ **0.75 V**

Example 52.7. *A germanium diode draws 40 mA with a forward bias of 0.25 V. The junction is at room temperature of 293°K. Calculate the reverse saturation current of the diode.*

Solution. $I = I_0 (e^{40V} - 1)$ or $40 \times 10^{-3} = I_0 (e^{40 \times 0.25} - 1)$

$\therefore$ $I_0 = 40 \times 10^3/(22{,}027 - 1) =$ **1.82 μ A**

Example 52.8. *Calculate forward current in Ge diode at 20°C when forward voltage is 0.3 V. Compare this value with that after a temperature rise of 50°C. Assume that reverse saturation current doubles for every 10°C rise in temperature.* **(Electronics-I, Mysore Univ. 1991)**

Solution. According to Boltzmann's diode equation

$$I_{20} = I_0 (e^{40V} - 1) = I_0 (e^{40 \times 0.3} - 1) = I_0 (e^{12} - 1) = 162{,}755 \, I_0$$

At, $t = (20 + 50) = 70°C$ or $T = 70 + 273 = 343°K$

$V_T = 343/11{,}600 = 0.0296$ V $\cong 0.03$ V, hence, $V/\eta \, V_T = 0.3/1 \times 0.03 = 10$

The value of I_0 at 70°C is $I_0' = 2^5 \times I_0 = 32 \, I_0$ because it doubles for every 10°C rise in temperature.

$\therefore$ $I_{70} = I_0' (e^{10} - 1) = 32 \, I_0 (22{,}026 - 1) = 704{,}800 \, I_0$

It means that I_{70} = (704,800, 162,755) $I_{20} = 4.3 \, I_{20}$ *i.e.* forward current has increased only 4.3 times whereas I_0 has increased 32 times for the same rise in temperature.

Example 52.9. *The capacitance of a varactor varies from 5 to 50 pF. Two such varactor diodes are used in the tuning circuit of Fig. 52.11. If L = 10 mH, determine the tuning range of the circuit.*

Solution. It should be noted that the two varactor capacitances are connected in series (Fig. 52.11).

$$C_{T(min)} = 5/2 = 2.5 \text{ pF} \; ; \; C_{T(max)} = 50/2 = 25 \text{ pF}$$

$$f_{r(max)} = 1/2\,\pi\,\sqrt{LC} = 1/2\pi\sqrt{10\times10^{-3}\times2.5\times10^{-12}}$$

$$= \mathbf{1\ MHz}$$

$$f_{r(min)} = 1/2\,\pi\,\sqrt{10\times10^{-3}\times25\times10^{-12}} = \mathbf{318\ KHz}$$

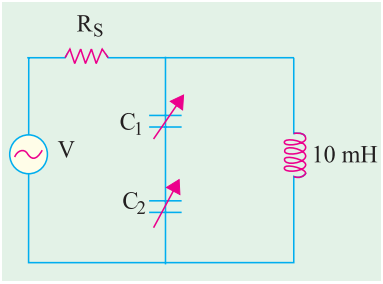
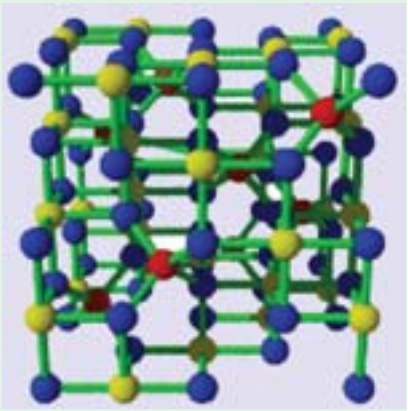


Fig. 52.11

52.6. Diode Fabrication

The electrical characteristics of a semiconductor diode depend on two factors ***(i)*** the material employed and ***(ii)*** the type of *P-N* junction used.

The two most commonly-used materials are germanium (Ge) and silicon (Si). Since Ge has higher electrical conduction than Si, it is often used in low- and medium-power diodes. On the other hand, Si has been found more suitable for high-power applications because it can be operated at higher temperatures than Ge. A new material called gallium-arsenide (GaAs) is found to combine desirable features of both Ge and Si and is finding ever-increasing use in many new applications.



Polycrystalline structures

The *P-N* junction may be produced by any one of the following methods :

1. grown junction **2.** alloy junction
3. diffused junction **4.** epitaxial growth **5.** point contact junction.

The first step in the manufacture of any semiconductor device is to obtain the semiconductor material in an extremely pure form. The accepted impurity level is less than one part of impurity in one billion (10^9) parts of the semiconductor material. To begin with, the raw material is subjected to a series of chemical reactions and then to a zone refining process which employs induction heating to reduce the impurity level of the polycrystalline structure. Next, the Czochralski or floating zone technique is used to form single crystals of Ge or Si for fabrication of diodes. These crystals are then cut into wafers as thin as 0.025 mm (nearly one fourth the thickness of this paper).

Now, we will briefly discuss the four basic processes commonly used in the manufacture of semiconductor diodes.

52.7. Grown Junction

Such junctions are produced by employing either the Czochralski or floating zone technique. The apparatus used for Czochralski technique is shown in Fig. 52.12. A single crystal seed of the desired impurity level is immersed in the molten semiconductor material contained in a crucible. Then, it is gradually withdrawn while the shaft holding the seed is slowly turning. When crystal is being pulled out, impurities of *P*- and *N*-type are alternately added to produce a *P-N* junction. This large area crystal is then cut into a large number of smaller-area diodes.

52.8. Alloy Junction

The alloy process produces junction diodes that have high PIV and current ratings but which

have large junction capacitance due to their large junction area.

In this process, a tiny dot (or pellet) of indium (or any other *P*-type impurity) is placed on the

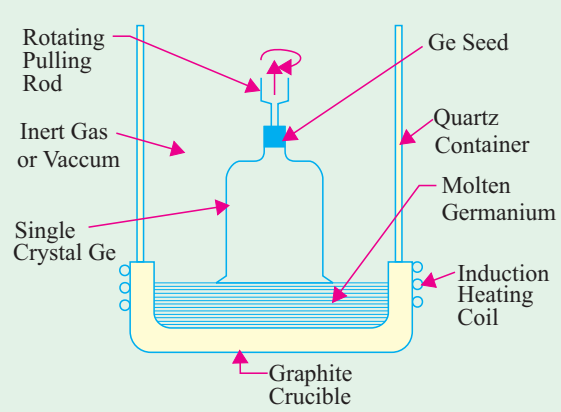


Fig. 52.12

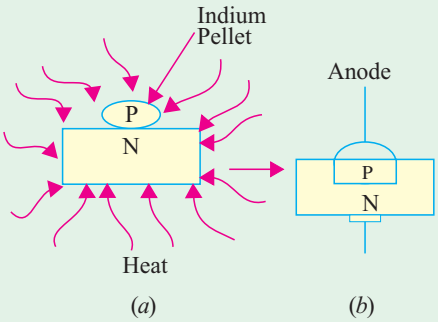


Fig. 52.13

surface of an *N*-type silicon wafer and the two are heated well above the melting temperature of indium (about 150°C) as shown in Fig. 52.13 (*a*). Consequently, indium melts and dissolves some of the silicon. The temperature is then lowered and silicon refreezes to form a single crystal having a *P*-*N* junction as shown in Fig. 52.13 (*b*).

52.9. Diffused Junction

The diffusion process employs either solid or gaseous diffusion. This process takes more time than alloy process but is relatively cheaper and more accurately controllable. In this process, particles from an area of high concentration drift to surrounding region of lesser concentration.

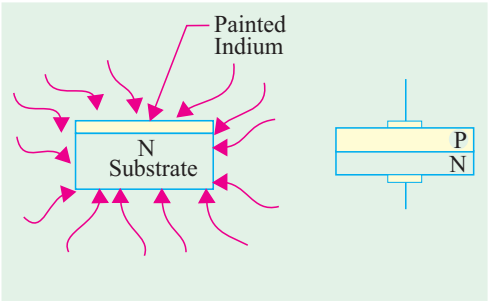


Fig. 52.14

(*i*) Solid Diffusion

The solid diffusion process starts with the 'painting' of a *P*-type impurity (say, indium) on an *N*-type substrate and heating the two until the impurity (say, indium) on an *N*-type substrate and heating the two until the impurity diffuses a short distance into the substrate to form *P*-type layer (Fig. 52.14).

(*ii*) Gaseous Diffusion

In the gaseous diffusion process, an *N*-type material is heated in a chamber containing a high concentration of an acceptor impurity in vapour form (Fig. 52.15). Some of the acceptor atoms are diffused (or absorbed) into the *N*-type substrate to form the *P*-type layer thus creating a *P*-*N* junction. By exposing only part of the *N*-type material during the diffusion process (the remainder being covered by a thin coating of SiO_2), the size of the *P*-region can be controlled. Finally, metal contacts are electroplated on the surface of each region for connecting the leads.

The diffusion technique enables simultaneous fabrication of many hundreds of diodes on one small disc of a semiconductor material. That is why it is the most frequently-used technique not only for the manufacture of semi-conductor diodes but also for the production of transistors and integrated circuits etc.

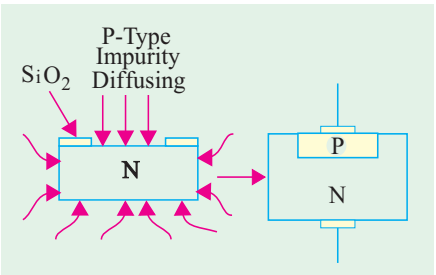


Fig. 52.15

52.10. Epitaxial Junction

This junction differs from the diffusion junction only in the manner in which the junction is fabricated. Such junctions are grown on top of an *N*-type wafer in a high temperature chamber. The growth proceeds atom by atom and hence is exactly similar to the crystal lattice of the wafer on which it is grown. Such junctions have the advantage of low resistance.

52.11. Point Contact Junction

It consists of an *N*-type germanium or silicon wafer about 1.25 mm square by 0.5 mm thick, one face of which is soldered to a metal base by radio-frequency heating as shown in Fig. 52.16 (*a*). The other face has a phosphor bronze (or tungsten) spring (called a cat's whisker) pressed against it. The *P-N* junction is formed by passing a large current for a second or two through the wire while the crystal face with wire point is kept positive. The heat so produced drives away some of the electrons from the atoms in the small region around the point of contact thereby leaving holes behind. This

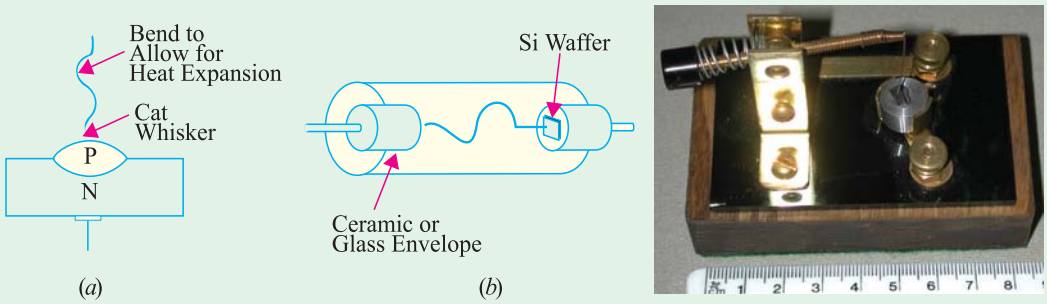


Fig. 52.16

small region of the *N*-type material is, consequently, converted into *P*-type material as shown in Fig. 52.16 (*a*). The small area of the *P-N* junction results in very low junction capacitance as mentioned earlier.

52.12. The Ideal Diode

There is no such thing as an ***ideal*** diode or perfect diode. The existence of such a diode is visualized simply as an aid in analysing the diode circuits.

An ideal diode may be defined as a two-terminal device which



Ideal diode

(*a*) conducts with zero resistance when forward-biased, and

(*b*) appears as an infinite resistance when reverse-biased.

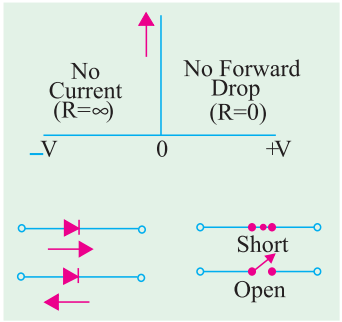


Fig. 52.17

In other words, such a device acts as a short-circuit in the forward direction and as an open-circuit in the reverse direction as shown in Fig. 52.17.

Also, in the forward direction, there is no voltage drop (even though current is there) since a short has zero resistance. On the other hand, there is no reverse current because reverse resistance is infinite.

It is helpful to think of an ideal diode as a bistable switch which is closed in the forward direction and open in the reverse direction. Hence, it has two stable states : ON or OFF.

Example 52.10. *Calculate the circuit current and power dissipated in the* ***(a)*** *ideal diode* ***(b)*** *6 Ω resistor of the circuit shown in Fig. 52.18 (a).*

Solution. The diode is an ideal one and is forward-biased. Hence, it can be replaced by a short (closed switch) as shown in Fig. 52.18 (*b*). The circuit current, as given by Ohm's law, is

$$I = 12/6 = \mathbf{2\ A}$$

(a) Since there is no voltage drop across the diode, power consumed by it is **zero**.

As we know, there is no power when either the voltage or current is zero. In the forward direction, there is current but no voltage drop, hence power dissipated by the ideal diode is zero. In the reverse direction, there is voltage but no current. Hence, power dissipated by the diode is again zero. In fact, an ideal diode never dissipates any power.

(b) power consumed by 6 Ω resistor = $2^2 \times 6 = \mathbf{24\ W.}$

Example 52.11. *Calculate the current in the circuit of Fig. 52.19 (a).*

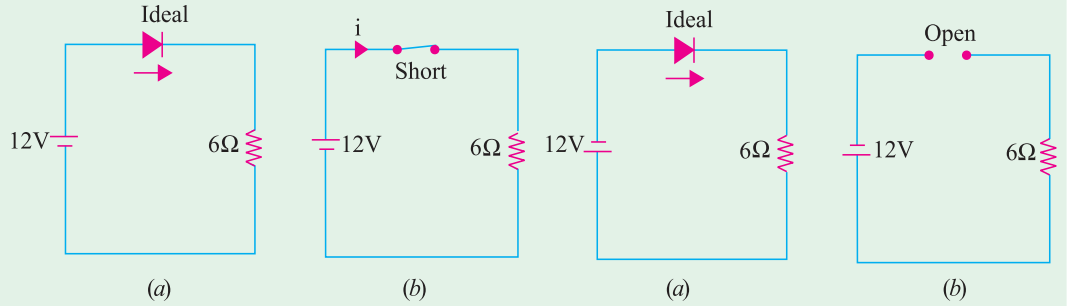


Fig. 52.18

Fig. 52.19

Solution. As seen from the polarity of the battery terminals, the diode is reverse-biased.

Hence, it acts like an open switch as shown in Fig. 52.18 (*b*). Obviously, current in such a circuit is zero.

Example 52.12. *Find the current, if any flowing in the circuit of Fig. 52.20 (a) which uses two oppositely-connected ideal diodes in parallel.*

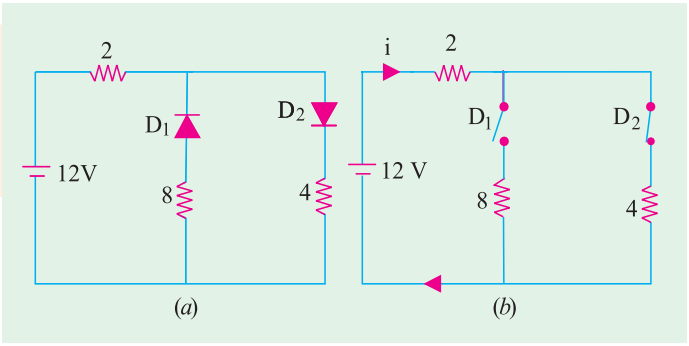


Fig. 52.20

Solution. The diode D_1 is reverse-biased and acts as an open switch as shown in Fig. 52.20 (*b*). So, there is no current through D_1 and the 8 Ω resistor.

However, D_2 is forward-biased and acts like a short-current or closed switch. The current drawn is $I = 12/(2 + 4) = \mathbf{2\ A}$.

Example. 52.13. *A sinusoidal voltage of peak value 50 V is applied to a diode as shown in Fig. 52.21. Sketch the waveform of voltage V_O treating the diode as an ideal one.*

Solution. First, consider the positive half-cycle of the input signal. The diode acts as a short in the forward direction and the moment V_{in} exceeds battery voltage of 10 V, current will start flowing through the circuit. The value of V_O will remain steady at 10 V, the balance of 40 V dropping across 5 K resistance. It is seen that value of V_O is set by the battery voltage.

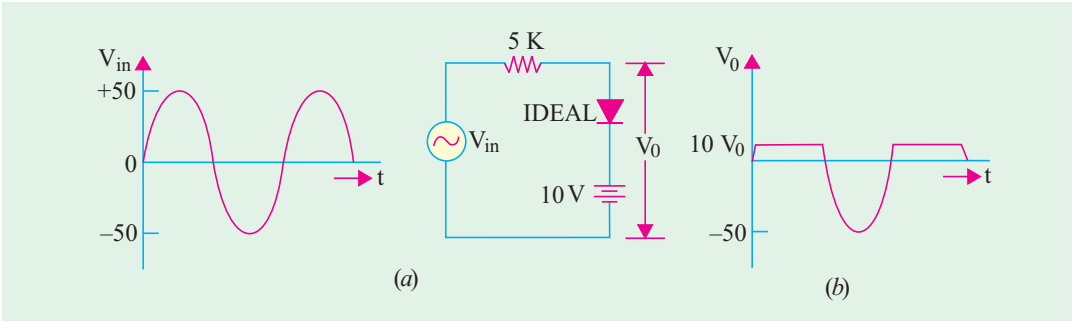


Fig. 52.21

Next, consider the negative input half-cycle. Now, the diode acts like an open switch. Consequently, there is no circuit current and thus no voltage drop across 5 K resistor. Hence, V_0 equals source voltage of peak value 50 V.

It is interesting to note that the above circuit acts as a ***positive*** clipper with a clipping level of 10 V *i.e.* equal to battery voltage. All positive parts of V_{in} above 10 V have been clipped off [Fig. 52.21 (*b*)].

Example 52.14. *Sketch the waveform of the output voltage V_O of the circuit shown in Fig. 52.22 (a)].*

Solution. It is obvious that diode D_1 and its 10 V battery act as positive clipper with positive clipping level at +10 V. Similarly, D_2 and its 10 V battery act as negative clipper with a clipping level at −10 V.

It is clear that output voltage V_0 would be clipped during both the positive and negative half-cycles as shown in Fig. 52.22 (*b*).

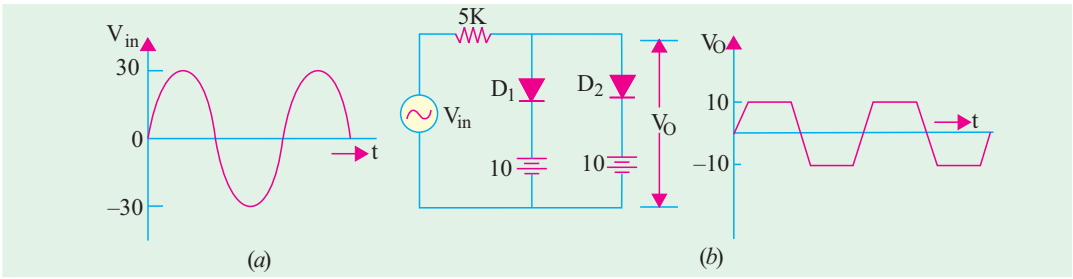


Fig. 52.22

Incidentally, the above example represents one way of obtaining an approximate square waveform from a sine wave. In fact, clipping takes place at +10 V and −10 V regardless of the shape of the input wave *i.e.* whether it is triangular or sawtooth etc.

Example 52.15. *In Fig 52.23 draw input and output waveforms to scale. Consider diode forward resistance of 50 ohms.* **(Nagpur Univ. Winter 2003)**

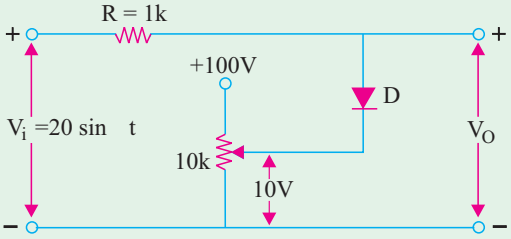


Fig. 52.23

Solution. It is obvious that the diode D and its 10V voltage drop across the 10k variable resistor act as positive clipper. Since the value of R>>R_f (*i.e.* forward resistance of a diode), therefore, the output voltage is clipped at 10V as shown in Fig 52.24.

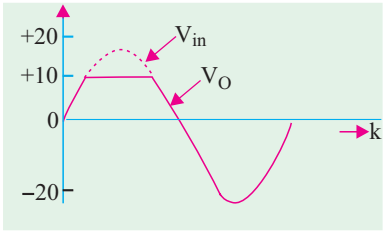


Fig. 52.24

52.13. The Real Diode

A real diode neither conducts in the forward direction with zero resistance nor it offers infinite resistance in the reverse direction.

(*a*) Forward Direction

In this case, we have to take two factors into account. One is that forward current does not start flowing until the voltage applied to the diode exceeds its threshold or knee voltage V_K (0.3 V for Ge and 0.7 V for Si). Hence, a real diode is shown as equivalent to an ideal diode in series with a small ***oppositely-connected*** battery of e.m.f. V_K as shown in Fig. 52.25 (*a*).

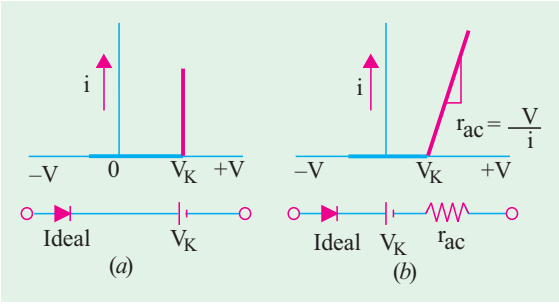


Fig. 52.25

The second factor to be considered is the forward dynamic or ac resistance (r_{ac}) offered by the circuit. So far, we considered this resistance to be zero implying that forward characteristic is a straight vertical line [Fig. 52.25 (*a*)]. If we take r_{ac} into account, the forward characteristic becomes as shown in Fig. 52.25 (*b*). Here, the reciprocal of the slope of this characteristic represents r_{ac}.

(*i*) Large Signal Operation

Large signal sources are those whose voltage is much greater than the diode knee voltage V_K (nearly equal to barrier potential V_B). Under such conditions, forward current would be large, so that r_j (Art. 52.1) would be negligible.

$$\therefore \qquad r_{ac} = r_j + r_B \cong r_B$$

(*ii*) Small Signal Operation

In this case, the signal voltage is much smaller than V_K (0.3 V for Ge and 0.7 V for Si). Since I_F would be small, r_j would be very large as compared to r_B.

$$\therefore \qquad r_{ac} = r_j + r_B \cong r_j$$

(*b*) Reverse Direction

An actual or real diode does not have infinite resistance in the reverse direction because it will always have some reverse saturation current prior to breakdown. For example, if with a V_R = 50 V, I_R is 10 μA, then $R_R = 5 \times 10^6\ \Omega = 5$ M. Silicon diodes have reverse resistance of many thousands of megohms. Hence, an actual diode in the reverse direction can be thought of as equivalent to a high resistor. This would be true only in the case of signals of low frequencies. For high-frequency signals, we will have to take into account the capacitive effects (Art 52.4).

Example 52.16. *Sketch the waveform for the output voltage V_0 in Fig. 52.26. Take V_K = 0.3V for germanium diode which has a forward current of 28 mA at 1 V.*

Solution. $r_B = (1 - 0.3)/28\ \text{mA} = 25\ \Omega$; $r_j = 25/28 = 0.9\ \Omega$ – negligible

$$\therefore \qquad r_{ac} = r_j + r_B \cong r_B = 25\ \Omega.$$

Hence, the equivalent circuit becomes as shown in Fig. 52.27. Whenever V_{in} exceeds 0.3 V in the forward direction, diode is shorted and the circuit offers a total resistance = (25 + 75) = 100 Ω.

The peak voltage = 20 – 0.3 = 19.7 V. Hence, peak value of forward current is = 19.7/100 = 197 mA.

$\therefore$ peak value of output voltage = $197 \times 10^{-3} \times 75 = 14.77$ V

Hence, the peak value of the half-wave rectified voltage is 14.77 V as shown in Fig. 52.27 (*b*). During the negative half-cycle of the applied voltage, diode acts as an open switch so that $V_0 = 0$. The waveform sketch of V_0 is shown in Fig. 52.27 (*b*).

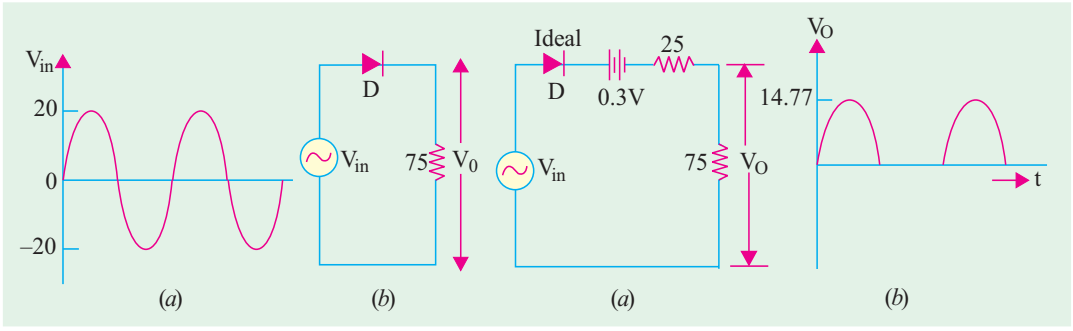


Fig. 52.26

Fig. 52.27

52.14. Diode Circuits with DC and AC Voltage Sources

We will often come across diode and transistor circuits which will contain both dc and ac voltage sources. Such circuits can be easily analysed by using Superposition Theorem (Art 52.15). We will first draw the dc equivalent circuit while neglecting ac-sources and find the required current and voltage values. Next, we will draw the ac equivalent circuit while neglecting the dc sources and again find the voltage and current values. Finally, we will superimpose the two sets of values to get the final result.

While drawing the equivalent circuits, following points must be remembered :

1. Direct current cannot flow 'through' a capacitor. Hence, all capacitors look ***like an open switch to a dc source.***

2. Usually, capacitors used in most circuits are large enough to ***look like a short to an ac source*** particularly one of very high frequency. Similarly, dc batteries would also act as short circuits unless they have very high internal resistances.

Example 52.17. *The silicon diode shown in Fig. 52.28 has a bulk resistance of 1 Ω. The frequency of the 10 mV (peak) signal is so high that the reactance of the coupling, capacitor may be taken as zero. Sketch the approximate waveform of the total voltage 'V' across the diode.*

(Basic Electronics, Punjab Univ.)

Solution. We will apply Superposition theorem to find *V*. First, voltage drop due to dc source would be found and then it would be added to the drop due to the ac source.

(*i*) DC Equivalent Circuit

It is seen from Fig. 52.26 that the circuit to the left of point *A* is 'open' to the dc source of 20 V because of capacitor *C*. Hence, the equivalent dc circuit is as shown in Fig. 52.29 (*a*). As seen, diode is forward-biased by the battery. Hence, only 0.7 V dc appear across the diode.

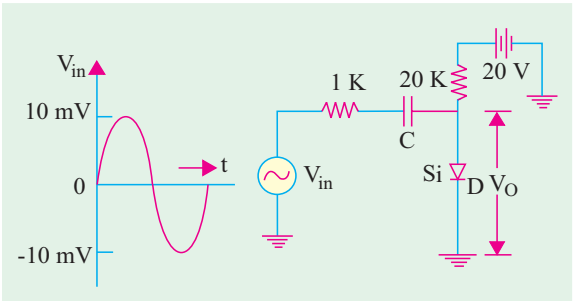


Fig. 52.28

The dc current $I = (20 - 0.7)/20\ \text{K} \cong 1\ \text{mA}$.

(*ii*) AC Equivalent Circuit

Here, the capacitor C and the 20-V battery would be treated as shorts thereby giving us the ac equivalent circuit of Fig. 52.29 (*b*).

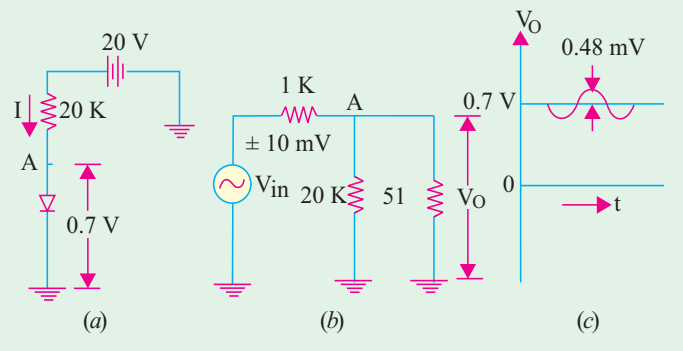


Fig. 52.29

Since, it is a silicon diode

$r_j = 50\ \text{mV}/1\ \text{mV} = 50\ \Omega$

$r_{ac} = r_j + r_B = 50 + 1 = 51\ \Omega$

As shown in Fig. 52.29 (*b*), so far as the signal source is concerned, 20 K resistor and ac resistance of the diode are connected in parallel at point A. Now, 20 K || 51 Ω = 51 Ω. Hence, 1 K and 51 Ω are put in series across the signal source of peak value 10 mV. The peak value of the ac voltage drop over 51 Ω resistance is

$$= 10 \times \frac{51}{51 + 1000} = \mathbf{0.48\ mV}$$

The total drop across the diode is the sum of the ac and dc drops. The combined voltage waveform is shown in Fig. 52.29 (*c*). It consists of a dc voltage of 0.7 V over which rides an ac voltage of peak value ± 0.48 mV.

Example 52.18. *The Ge diode shown in Fig. 52.30 has a bulk resistance of 2 Ω. The two capacitors offer negligible reactance to the a.c. signal. Sketch the waveform of the output voltage.*

Solution. As seen from Fig. 52.30 (*b*), no dc current can pass through 100 K resistor because it is blocked by C_2. Hence, no dc voltage appears across 100-K resistor. The battery current passes through 10 K resistor, through diode, through 20 K resistor and finally to ground.

$$I = (30 - 0.3)/(10 + 20) \times 10^3 = 0.99\ \text{mA} \cong 1\ \text{mA}$$

$$r_{ac} = r_j + r_B = (25\ \text{mV}/1\ \text{mA}) + 2 = \mathbf{27\ \Omega}$$

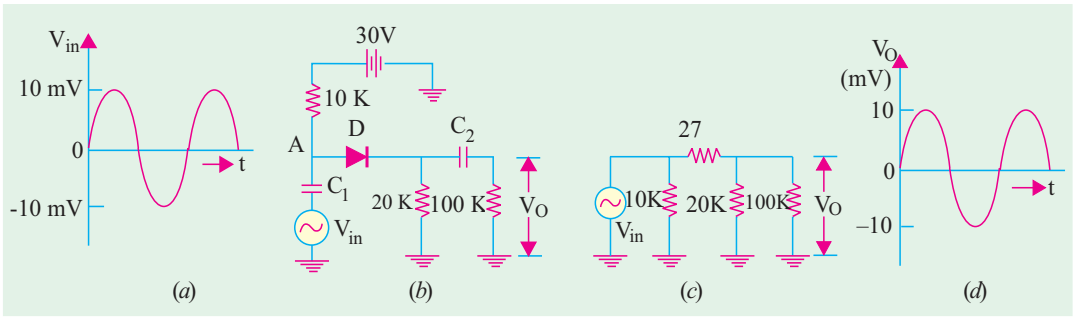


Fig. 52.30

AC equivalent circuit is shown in Fig. 52.30 (*c*) where capacitors and battery have been shorted. As seen, diode resistance of 27 Ω is in series with 20 K || 100 K. It means that only a very small part of ac. Source voltage drops across the diode. Hence, almost all a.c.source voltage appears across 100 K resistor. Obviously, V is practically the same as ac source voltage as shown in Fig. 52.30 (*d*).

52.15. Diode Clipper and Clamper Circuits

These are diode waveshaping circuits *i.e.* circuits meant to control the **shape** of the voltage and current waveforms to suit various purposes. Each performs the waveshaping function indicated by its

name. The output of the clipping circuit appears as if a portion of the input signal were ***clipped off***. But clamper circuits simply clams (*i.e.* lift up or down) the input signal to a different dc level.

52.16. Clippers

A clipping circuit requires a minimum of two components *i.e.* a diode and a resistor. Often, dc battery is also used to fix the clipping level. The input waveform can be clipped at different levels by simply changing the battery voltage and by interchanging the position of various elements. We will use an ideal diode which acts like a closed switch when forward-biased and as an open switch when reverse-biased.

Such circuit are used in radars and digital computers etc. when it is desired to remove signal voltages above or below a specified voltage level. Another application is in radio receivers for communication circuits where noise pulses that rise well above the signal amplitude are clipped down to the desired level.

Example 52.19. *For the simple parallel clipper of Fig. 52.31, find the shape of the output voltage V_O across the diode if the input sine wave signal is as shown in Fig. 52.31 (a). What will happen when diode and resistor are inter-changed ?*

Solution. When positive half-cycle of the signal voltage is applied to the clipper *i.e.* when *A* is positive with respect to *B*, the diode *D* is reverse-biased. Hence, it acts as an open switch. Consequently, the entire input voltage appears across it.

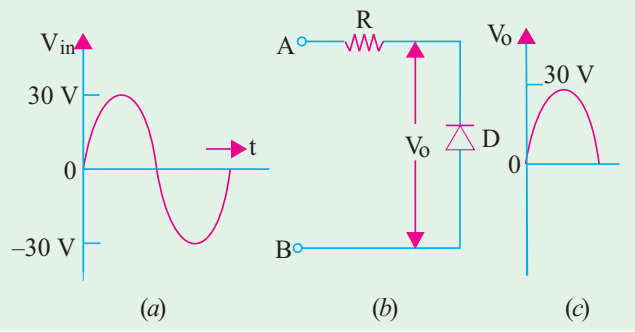


Fig. 52.31

During the negative half-cycle of the signal voltage when circuit terminal *B* becomes positive with respect to *A*, the diode is forward-biased. Hence, it acts like a closed switch (or short) across which no voltage is dropped. Hence, the waveshape of V_0 is as shown in Fig. 52.31 (*c*). It is seen that the negative portion of the signal voltage has been removed. Hence, such a circuit is called a **negative clipper**.

When Diode and Resistor are Interchanged

In this case, the circuit becomes as shown in Fig. 52.32 (*b*). Now, the output voltage V_0 is that which is dropped across *R*. During the positive half-cycle of the signal voltage, *D* acts as an open switch. Hence, all applied voltage drops across *D* and none across *R*. So, there is no output signal voltage.

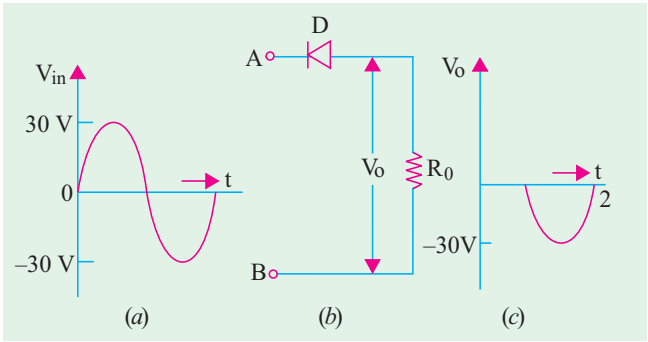


Fig. 52.32

During the negative input half-cycle, terminal *B* is positive and so it is forward-biases *B* which acts as a short. Hence, there is no voltage drop across *D*. Consequently, all the applied signal voltage drops across *R* and none across *D*. As a result, the negative half-cycle of the input signal is allowed to pass through the clipper circuit. Obviously, now the circuit acts as a **positive** clipper.

Example 52.20. *What would be the output waveform displayed by the oscilloscope in Fig. 52.33? The silicon diode has a barrier voltage of 0.7 V.*

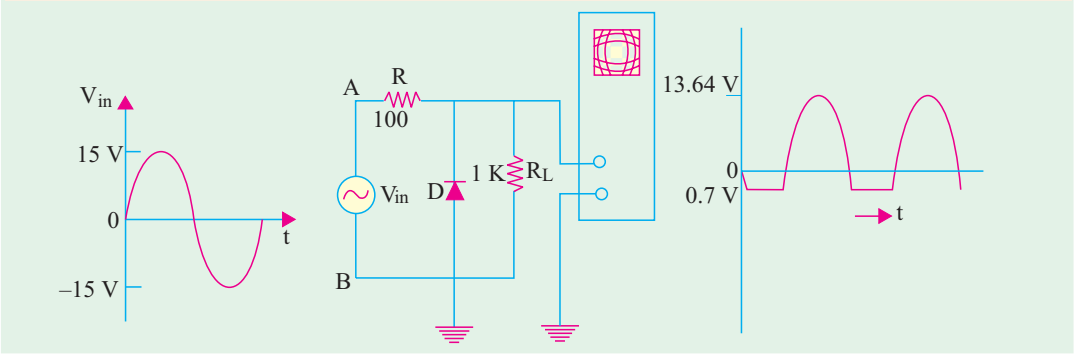


Fig. 52.33

Solution. Consider the negative input half-cycle first *i.e.* when point B is positive with respect to point A. The diode starts conducting when applied voltage exceeds 0.7 V. Since D and R_L are in parallel, voltage across them cannot exceed 0.7 V. Obviously, negative half-cycle beyond 0.7 V gets clipped. Hence, circuit behaves like a negative clipper.

During the positive input half-cycle when point A is positive, diode becomes reverse-biased and hence, becomes open-circuited. The applied voltage drops across the resistors R and R_L connected in series. The peak value of the output voltage is

$$= 15\left(\frac{R_L}{R+R_L}\right) = 15 \times \frac{1}{1.1} = 13.64 \text{ V}$$

Hence, the output voltage as displayed by the oscilloscope would be as shown in Fig. 52.33.

Example. 52.21. *With the sine wave signal input of Fig. 52.34 (a), find the shape of the output signal V_0 in the biased series clipper of Fig. 52.34 (b). What would happen if battery connections are reversed ?*

Solution. Let us consider the positive half-cycle of the singal *i.e.* when terminal A of the circuit becomes positive with respect to B. The diode appears as a short since it is forward-biased.

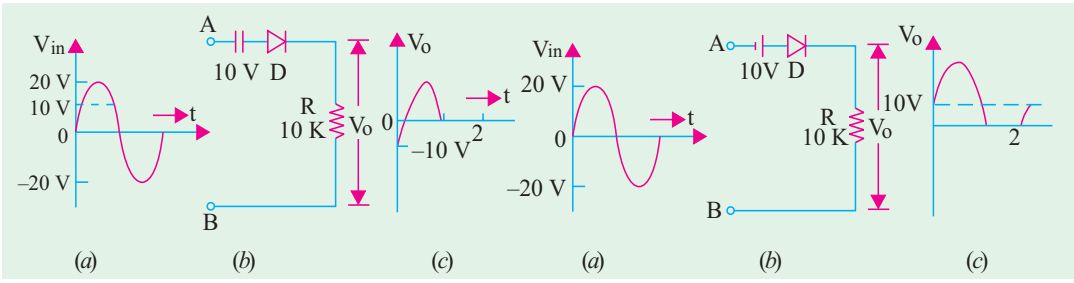


Fig. 52.34

Fig. 52.35

But no current flows till V_{in} exceeds the opposing battery voltage of 10 V. Hence, only upper part of the positive signal voltage passes through the clipper circuit and appears as V_0 across R. Its shape is shown in Fig. 52.34 (*c*). The negative half-cycle of the signal voltage is clipped off.

In fact, in this circuit, the entire input is clipped off except positive peak portions.

Reversed Battery Connections

The battery connections have been reversed in Fig. 52.35. In this case, during the positive half-cycle of the signal, the voltage across R would be the sum of the signal voltage and the battery voltage *i.e.* signal voltage would be lifted up by 10 V as shown in Fig. 52.35 (*c*).

During the negative input half-cycle, the lower peak portions of the signals would be clipped off because of the 10 V battery.

Example. 52.22. *The triangular voltage of Fig. 52.36 (a) is applied to the biased parallel clipper circuit of Fig 52.36 (b). Find the wave-shape of the output voltage.*

Solution. During the positive half-cycle, D_1 would conduct but D_2 will act as an open-circuit. However, value of V_0 cannot exceed 10V because points C and D are electrically connected across the 10 V battery since D_1 is shorted. Hence, signal voltage above 10 V level would be clipped off as shown in Fig. 52.36 (c).

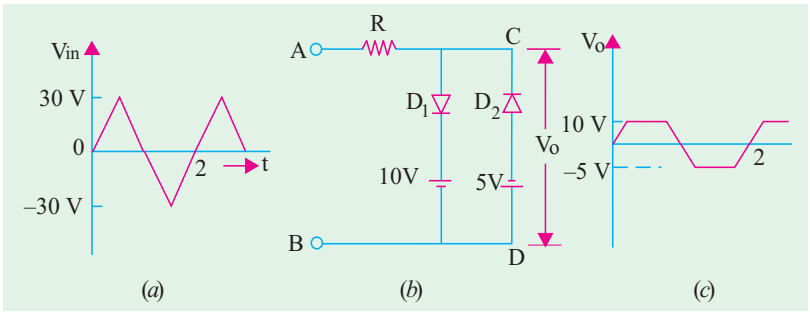


Fig. 52.36

During the negative half-cycle, D_1 is open but D_2 conducts. Again, V_0 cannot exceed 5 V since it is the voltage across points C and D whose value is fixed by the battery connected in that branch. Hence, signal voltage beyond 5 V is clipped off. The wave-shape of V_0 is as shown in Fig. 52.36 (c).

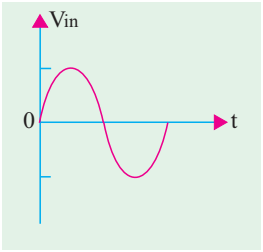


Fig. 52.37

52.17. Some Clipping Circuits

For the following circuits, a sinusoidal input signal as shown in Fig. 52.37 would be assumed.

(a) Biased Series Clippers

The output voltage has the waveform as shown in Fig. 52.38.

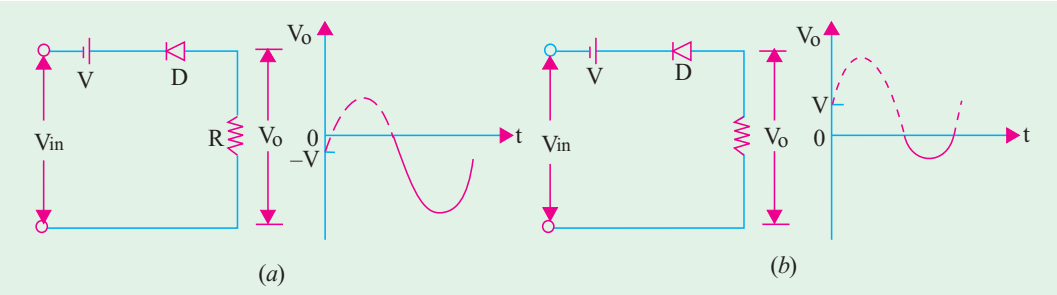


Fig. 52.38

(b) Biased Parallel Clippers

The waveforms of the output voltage are as shown in Fig. 52.39.

Clipping has been changed by changing the battery and diode connections.

52.18. Clampers

To put it simply, clamping is the process of introducing a dc level into an ac signal. Clampers are also sometimes known as dc restorers.

By way of illustration, consider the signal shown in Fig. 52.40 (a). It is a sine wave with equal positive and negative swings of ± 5 V about 0 V. Hence, its average value over one cycle is zero (it has no dc level).

In Fig. 52.40 (*b*), the signal waveform has been lifted up so as to just touch the horizontal axis. It is now said to have acquired a dc level of 5 V. This output wave-form is said to be ***positively*** clamped at 0 V. Fig. 52.40 (*c*) shows an output waveform which is negatively clamped at 0 V.

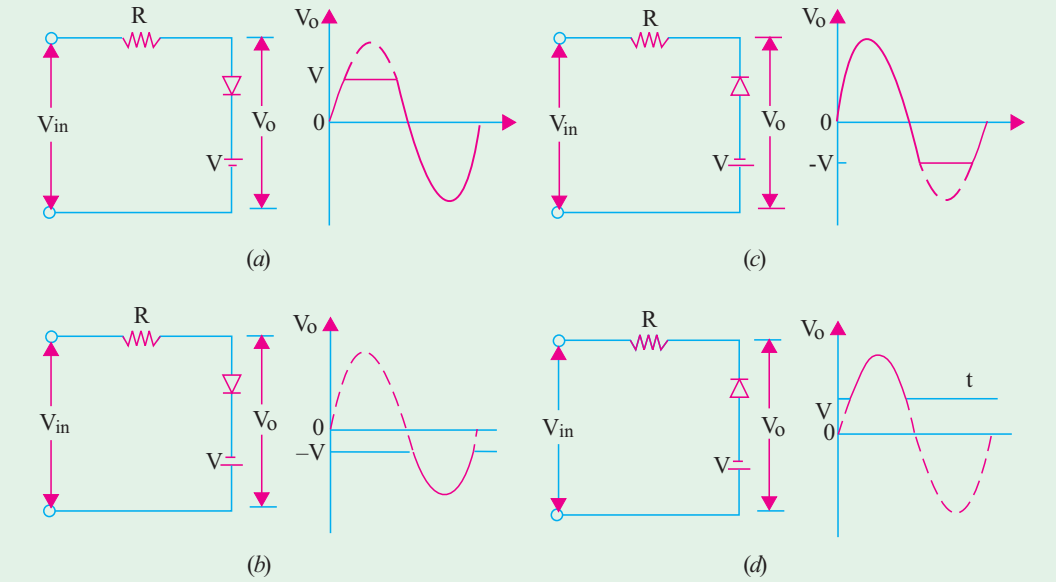


Fig. 52.39

A circuit capable of accepting the input signal shown in Fig. 52.40 (*a*) and delivering the output shown in Fig. 52.40 (*b*) or (*c*) is called a ***clamper***. Such a circuit has a minimum requirement of ***three*** elements.

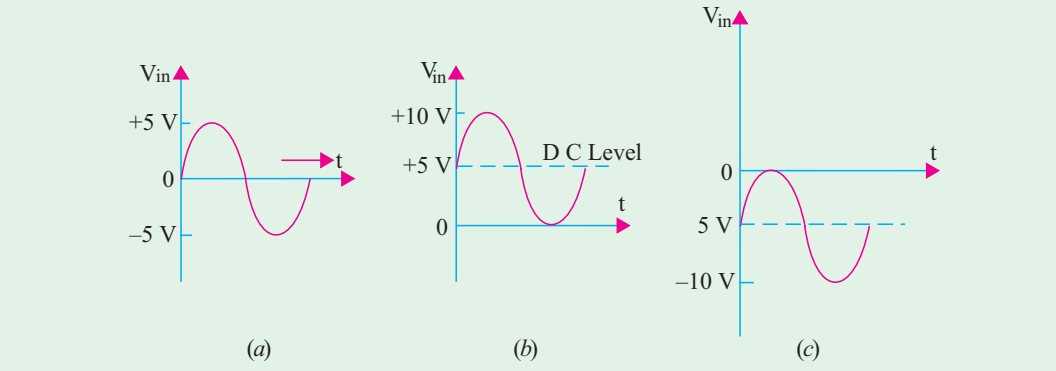


Fig. 52.40

1. a diode **2.** a capacitor and **3.** a resistor.

It will generally need a dc battery also. Following additional points regarding clamper circuits are worth keeping in mind.

1. both *R* and *C* affect the waveform.

2. values of *R* and *C* should produce a time constant ($\lambda = CR$) which is large enough to ensure that capacitor remains almost fully charged during the time-period of the signal. In other words, time constant $\lambda \gg T/2$ where *T* is the time-period of the input signal. For good clamping action, the *RC* time constant should be at least ten times the time-period of the input signal voltage.

3. it is advantageous to first consider the condition under which the diode becomes forward biased.
4. for all clamping circuits, voltage swing of the input and output waveforms is the same.
5. such circuits are often used in TV receivers as dc restorers. The incoming composite video signal is normally processed through capacitively-coupled amplifiers which eliminate the dc component thereby losing the black and white reference levels and the blanking level. These reference levels have to be restored before applying the video signal to the picture tube.

Example 52.23. *The input signal of Fig. 52.41 (a) is applied to the clamper circuit shown in Fig. 52.41 (b). Draw the waveform of the output voltage V_0. How will it change if R is made 100 Ω ?* **(Electronic circuits, Bangalore Univ.)**

Solution. As seen, time-period of the input signal is T = 1/1000 second = 1 ms

$\therefore \quad 0 \rightarrow t_1 = t_1 \rightarrow t_2 = t_2 \rightarrow t_2 = T/2$ ..... = 0.5 ms.

$\lambda = C_R = 1 \times 10^{-6} \times 10 \times 10^3 = 10$ ms

As seen, $\lambda \gg T/2$. Hence, once charged, the capacitor will have hardly any time to discharge by the time signal polarity reverses.

(*a*) Positive Input Half-cycle

When positive half-cycle of the input signal voltage is applied to the clamper circuit, its terminal *A* becomes positive with respect to terminal *B*. Hence, *D* acts like a short as shown in Fig. 52.41 (*c*). A steady positive voltage of 5 V remains applied to *A* for 0.5 ms. At the same time, *R* is also shorted out [Fig. 52.41 (*c*)] because it is in parallel with *D*. Hence, C will rapidly charge to 5 V. Being across a short, $V_0 = 0$ during positive half-cycle as shown in Fig. 52.41 (*d*).

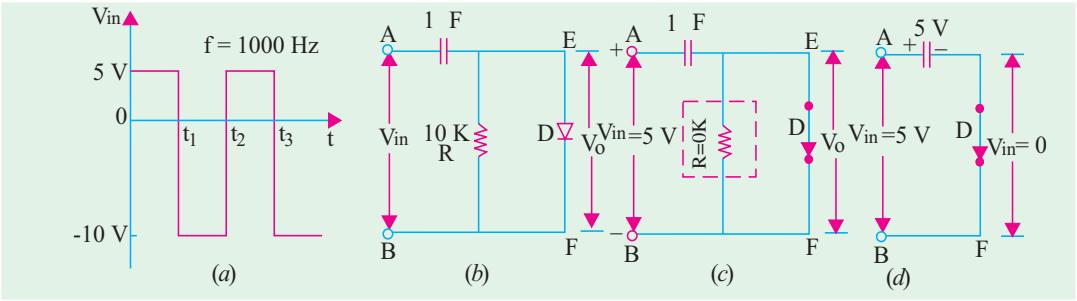


Fig. 52.41

(*b*) Negative Input Half-cycle

In this case, terminal *B* becomes positive and so reverse-biases *D* by 10 V. Hence, *D* acts like an open switch as shown in Fig. 52.42 (*a*). Now, *R* and *C* get connected in series so that their $\lambda = RC$ = 10 ms. As stated earlier, capacitor will take a time of 5λ = 50 ms to get fully discharged. But the input signal will allow it just 0.5 ms during which to discharge. Obviously, *C* would hardly get discharged in this extremely short time interval of 0.5 ms. Hence, it can be assumed to be still fully charged with the original polarity during this negative half-cycle.

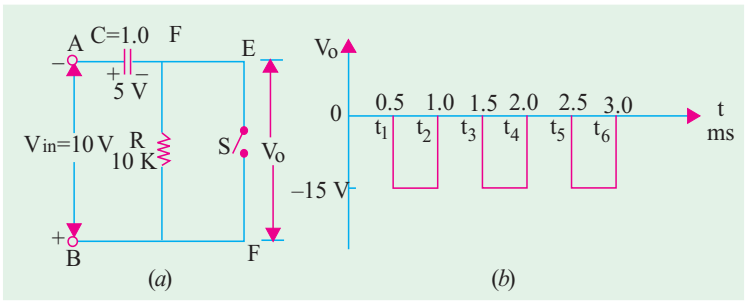


Fig. 52.42

The output voltage V_0 across the 'open' will be

= voltage from $E \rightarrow A \rightarrow B \rightarrow F$ – Fig. 52.40 (*a*)

= 5 + 10 = 15 V – with *E* negative

The waveform of the output voltage is shown in Fig. 52.42 (*b*). It has same frequency as that of the input signal. However, it has been clamped down in the negative region. It is seen that voltage swing of both input and output circuits is the same *i.e.* 15 V. It is never the case in clipping circuits.

When R = 100

Now, $\lambda = 100 \times 1 \times 10^{-6}$ ms = 0.1 ms. Hence, the capacitor which is almost instantaneously charged to +5 V during the positive input half-cycle, will be almost completely discharged during the negative half-cycle because, now, 5 λ (full discharge time) equals the half time-period (0.5 s) of the signal.

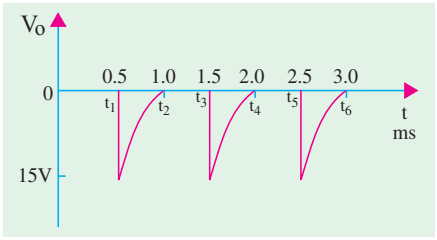


Fig. 52.43

Hence, in this case, V_0 would be momentarily equal to – 15 V at the beginning of the negative half-cycle but will fall off to almost 0 V before the signal reverses its polarity (Fig. 52.43). As seen, υ_0 consists of voltage spikes of amplitude – 15 V.

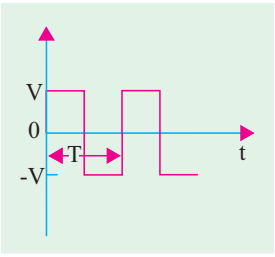


Fig. 52.44

52.19. Summary of Clamping Circuits

In the following clamping circuits, it would be assumed that the amount of the time $5\lambda = 5\,RC \gg T/2$ where T is the time-period of the input signal. For all circuits, we will take the same input signal shown in Fig. 52.44 with a peak value of V. We will also take note of the change in the output waveform when diode connections are reversed.

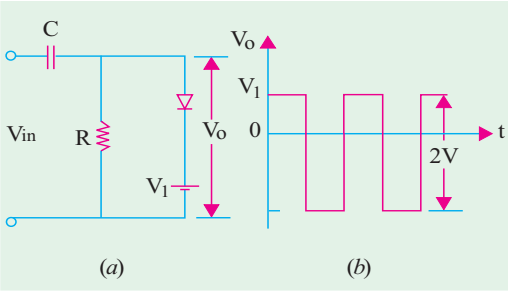


Fig. 52.45

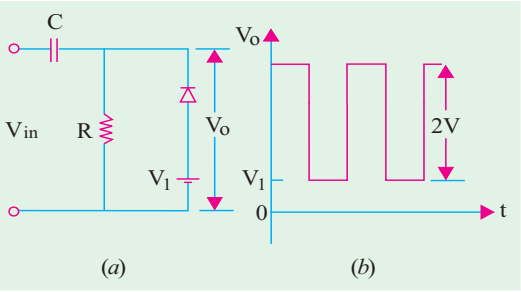


Fig. 52.46

It is seen from Fig. 52.44 and 52.45 that negative clamping has changed to positive clamping when the diode connections are reversed.

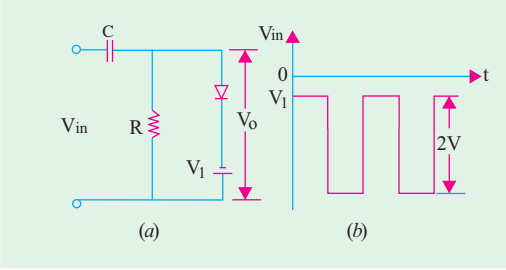


Fig. 52.47

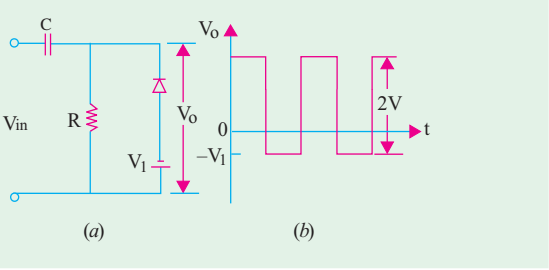


Fig. 52.48

The same thing happens in the case of clamping circuits shown in Fig. 52.47 and 52.48.

Tutorial Problems No. 52.1

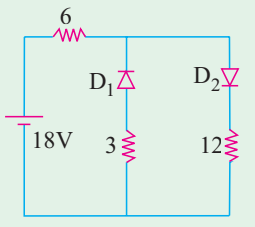


Fig. 52.49

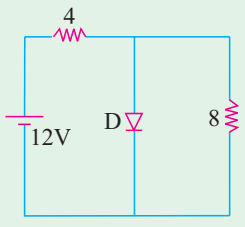


Fig. 52.50

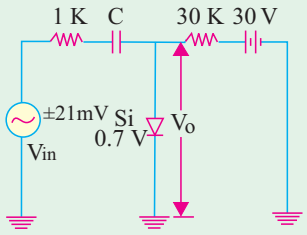


Fig. 52.51

1. Find the current supplied, if any, by the battery in the circuit of Fig. 52.49 which uses two oppositely-connected ideal diodes in parallel. **[1 A]**
2. What is the current supplied by the battery in Fig. 52.50 if D is an ideal diode. **[3 A]**
3. In Fig. 52.51, signal voltage has a peak value of ≠ 21 mV and its frequency is so high that reactance of the coupling capacitor can be neglected. If bulk resistance of the silicon diode is neglected, what

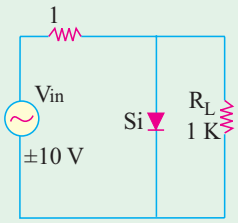


Fig. 52.52

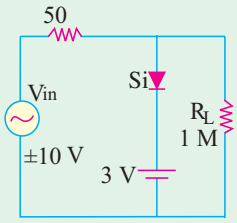


Fig. 52.53

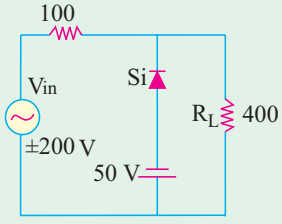


Fig. 52.54

would be the approximate waveform of the total voltage 'V_0' across the diode. **[0.7 V dc on which rides an ac voltage of peak value = 1 mV]**

4. Sketch the voltage across R_L in the clipper circuit of Fig. 52.52.
5. Sketch the voltage across R_L in the clipper circuit of Fig. 52.53.
6. What is the waveform of the voltage across R_L in Fig. 52.54 ?
7. What is the approximate dc component in the output of a positive diode clamper with a peak input of 20 V ? **[20 V]**

OBJECTIVE TESTS – 52

1. For a silicon diode, the value of the forward-bias voltage typically
   (*a*) must be greater than 0.3 V
   (*b*) must be greater than 0.7 V
   (*c*) depends on the width of the depletion region
   (*d*) depends on the concentration of majority carriers.
2. When forward biased, a diode
   (*a*) blocks current
   (*b*) conducts current
   (*c*) has a high resistance
   (*d*) drops a large voltage
3. The term bias means
   (*a*) the ratio of majority carriers to minority carriers
   (*b*) the amount of current across the P-N junction
   (*c*) a dc voltage applied across the P-N junction to control its operation
   (*d*) none of the above
4. To forward-bias a P-N junction, diode,
   (*a*) an external voltage is applied that is positive at the anode and negative at the cathode
   (*b*) an external voltage is applied that is negative at the anode and positive at the cathode.

(*c*) an external voltage is applied that is positive at the P-region and negative at the N-region

(*d*) a and c above

5. When a P-N junction is forward-biased

(*a*) the only current is the hole current

(*b*) the only current is the electron current

(*c*) the only currentis produced by majority carriers

(*d*) the current is produced by both holes and electrons.

6. A P-N junction diode's dynamic conductance is directly proportional to

(*a*) the applied voltage

(*b*) the temperature

(*c*) the current

(*d*) the thermal voltage

(Hint : Conductance is reciprocal of the resistance)

7. The junction capacitance of a linearly graded junction varies with the applied reverse bias, V_R as

(*a*) V_R^{-1} (*b*) $V_R^{-1/2}$

(*c*) $V_R^{-1/3}$ (*d*) $V_R^{1/2}$

(UPSC Engg. Services 2002)

8. The diffusion capacitance of a forward biased $P^+ N$ (P^+ indicates heavily doped *P*-region) junction diode with a steady current *I* depends on

(*a*) width of the depletion region

(*b*) mean life-time of holes

(*c*) mean life-time of electrons

(*d*) junction-area

(UPSC Engg. Services 2002)

9. The width of depletion layer of a P-N junction

(*a*) decreases with light doping

(*b*) increases with heavy doping

(*c*) is independent of applied voltage

(*d*) is increased under reverse bias.

10. At room temperature of 25°C, the barrier potential for silicon is 0.7 V. Its value at 125°C is ..... volt.

(*a*) 0.5 (*b*) 0.3

(*c*) 0.9 (*d*) 0.7.

11. Junction breakdown occurs

(*a*) under high temperature condition

(*b*) with forward bias

(*c*) under reverse bias

(*d*) because of manufacturing defect.

12. Avalanche breakdown is primarily dependent on the phenomenon of

(*a*) collision (*b*) doping

(*c*) ionization (*d*) recombination.

13. Reverse current in a silicon junction nearly doubles for every..... °C rise in temperature.

(*a*) 10 (*b*) 2

(*c*) 6 (*d*) 5.

14. In the forward region of its characteristic, a diode appears as a/an

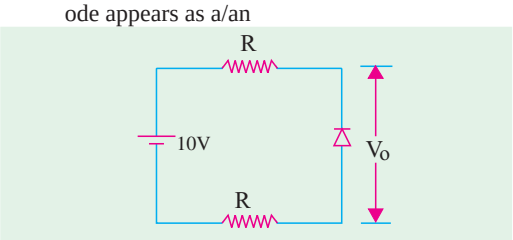


Fig. 52.55

(*a*) OFF switch (*b*) high resistance

(*c*) capacitor (*d*) ON switch.

15. The approximate value of V_O across the diode in Fig. 52.53 is

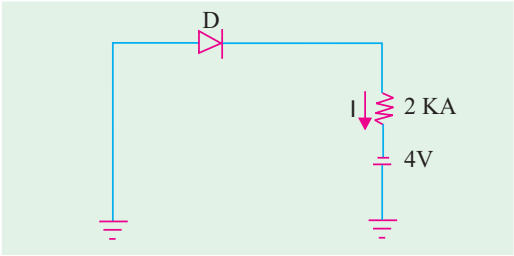


Fig. 52.56

(*a*) zero (*b*) 10 V

(*c*) 5 V

(*d*) dependent onvalue of R.

16. The diode 'D' is ideal in the network shown in Fig. 52.56. The current 'I' will be.

(*a*) Ä 2 n A (*b*) zero

(*c*) 2 m A (*d*) 4 mA

17. The voltages at V_1 and V_2 of the arrangement shown in Fig. 52.57 will be respectively.

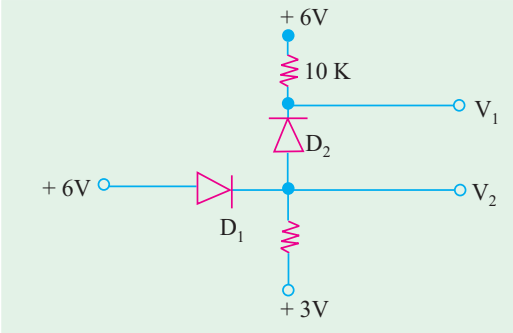


Fig. 52.57

(*a*) 6 V and 5.4 V (*b*) 5.4 V and 6 V

(*c*) 3 V and 5.4 V (*d*) 6 V and 5 V

18. Without a dc source, a clipper acts like a

(*a*) rectifier (*b*) clamper
(*c*) demodulator (*d*) chopper

19. The primary function of a clamper circuit is to

(*a*) suppress variations in signal voltage
(*b*) raise positive half-cycle of the signal
(*c*) lower negative half-cycle of the signal
(*d*) introduce a dc level into an ac signal

20. For an input $V_S = 5 \sin t$ (assuming ideal diode), the circuit shown in Fig. 52.58 will be behave as a

(*a*) clipper, sine wave clipped at –2 V
(*b*) clamper, sine wave clamped at –2 V
(*c*) clamper, sine wave clamped at zero volt
(*d*) clipped, sine wave clipped at 2 V

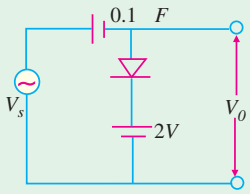


Fig. 52.58

21. A clipping circuit is shown in Fig. 52.59. Its transfer characteristic will be

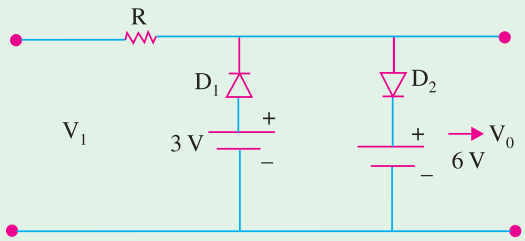
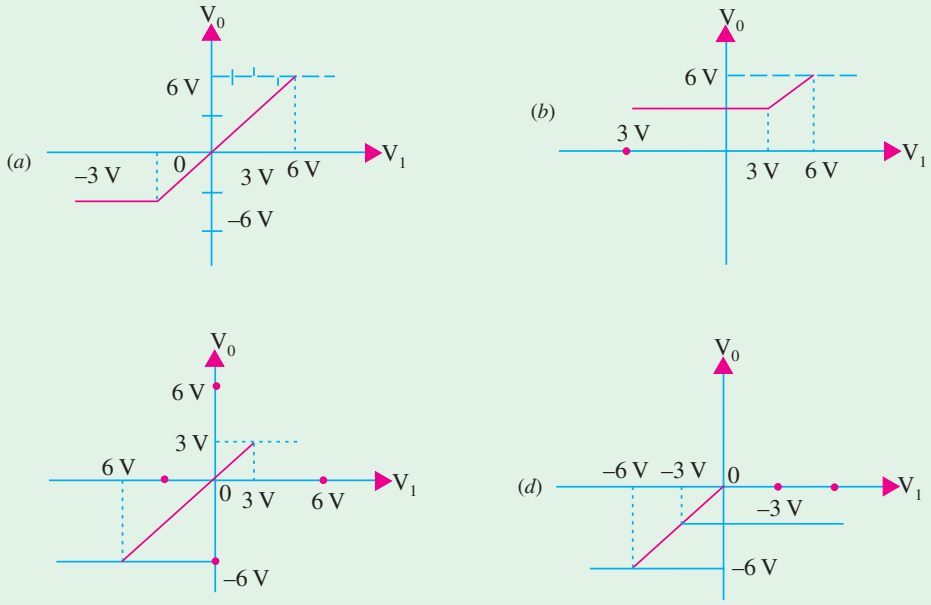


Fig. 52.59



ANSWERS

1. (*b*)	**2.** (*b*)	**3.** (*c*)	**4.** (*d*)	**5.** (*d*)	**6.** (*c*)	**7.** (*c*)
8. (*c*)	**9.** (*d*)	**10.** (*c*)	**11.** (*c*)	**12.** (*a*)	**13.** (*c*)	**14.** (*d*)
15. (*b*)	**16.** (*c*)	**17.** (*a*)	**18.** (*a*)	**19.** (*d*)	**20.** (*b*)	**21.** (*b*)

ROUGH WORK

GO To FIRST

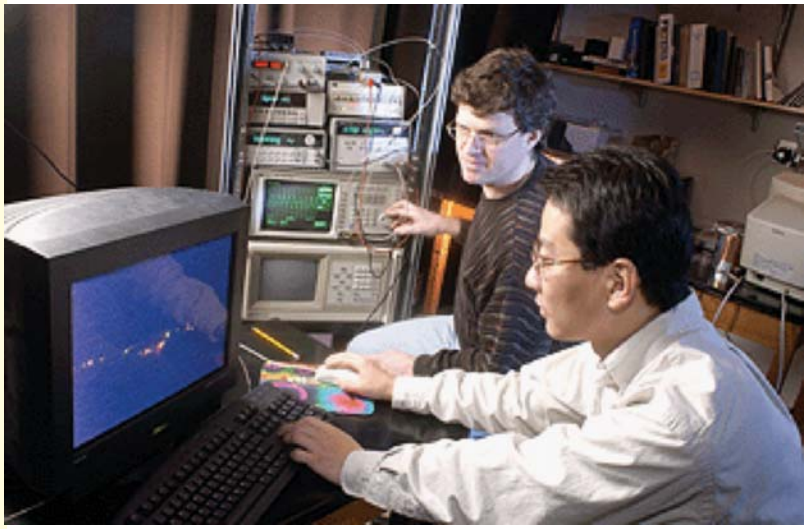
CONTENTS

CHAPTER 53

OPTO-ELECTRONIC DEVICES

Learning Objectives

- Fundamentals of Light
- Light Emitting Diode (LED)
- Use of LEDs in Facsimile Machines
- Liquid Crystal Displays
- P-N Junction Photodiode
- Dust Sensor
- Photoconductive Cell
- Phototransistor
- Photodarlington
- Photo voltaic or Solar Cell
- Laser Diode
- Optical Disks
- Read-only Optical Disk Equipment
- Printers Using Laser Diodes
- Hologram Scanners
- Laser Range Finder
- Light-activated SCR(LASCR)
- Optical Isolators
- Optical Modulators
- Optical Fibre Communication Systems
- Optical Fibre Data Links



Researchers have demonstrated a new type of nanometer scale optoelectronic device that performs addition and other complex logic operations

CONTENTS

53.1. Fundamentals of Light

According to the Quantum Theory, light consists of discrete packets of energy called photons. The energy contained in a photon depends on the ***frequency*** of the light and is given by the relation $E = hf$ where h is Plank's constant (6.625×10^{-34} Joule-second). In this equation, energy E is in Joules and frequency f is in hertz (Hz). As seen, photon energy is directly proportional to frequency: higher the frequency, greater the energy. Now, velocity of light is given by $c = f\lambda$ where c is the velocity of the light (3×10^8 m/s) and λ is the wavelength of light in metres. The wavelength of light determines its colour in the visible range and whether it is ultraviolet or infrared outside the visible range.

Now, $E = hf = hc/\lambda$ or $\lambda = hc/E$ metres

$\therefore \quad \lambda = (6.625 \times 10^{-34}) \times (3 \times 10^8)/E = (19.875 \times 10^{-26})/E$ —E in Joules

If E is in electron-volt (eV), then since 1 eV = 1.6×10^{-19} J

$\therefore \quad \lambda = (19.875 \times 10^{-26})/(E \times 1.6 \times 10^{-19}) = (12.42 \times 10^{-7})/E$ metre

or $\lambda = 1.242$ μm

In a forward-biased *P-N* junction, electrons and holes both cross the junction. In the process, some electrons and holes recombine with the result that electrons lose energy. The amount of energy lost is equal to the difference in energy between the conduction and valence bands, this being known as the semiconductor energy band gap E_g. The value of E_g for silicon is 1.1 eV, for GaAs is 1.43 eV and for InAs is 0.36 eV. For example, the wavelength of light emitted by silicon *P-N* junction is $\lambda = 1.242/E_g = 1.242/1.1 = 1.13$ μm.

53.2. Light Emitting Diode (LED)

(a) Theory

As the name indicates, it is a forward-biased *P-N* junction which emits visible light when energised. As discussed earlier (Art. 53.40), charge carrier recombination takes place when electrons from the *N*-side cross the junction and recombine with the holes on the *P*-side.

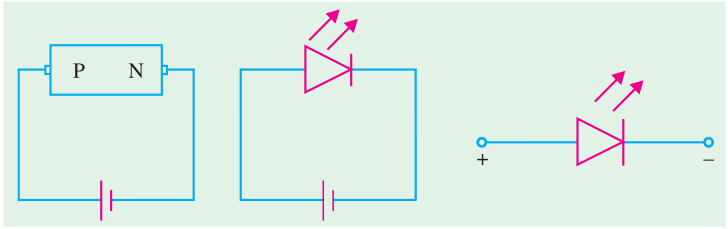
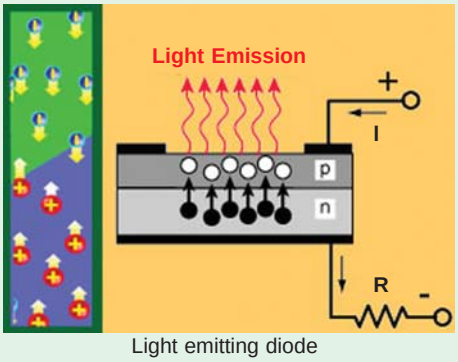


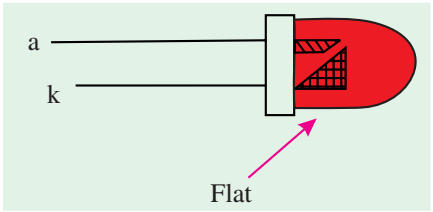
Fig. 53.1

Now, electrons are in the higher conduction band on the *N*-side whereas holes are in the lower valence band on the *P*-side. During recombination, some of the energy difference is given up in the form of heat and light (*i.e.* photons). For Si and Ge junctions, greater percentage of this energy is given up in the form of heat so that the amount emitted as light is insignificant. But in the case of other semiconductor materials like gallium arsenide (GaAs), gallium phosphide (GaP) and gallium-arsenide-phosphide (GaAsP), a greater percentage of energy released during recombination is given out in the form of light. If the semiconductor material is translucent, light is emitted and the junction becomes a light source *i.e.* a light-emitting diode (LED) as shown schematically in Fig. 53.1. The colour of the emitted light depends on the type of material used as given on the next page.



Light emitting diode

1. GaAs — infrared radiation (invisible).
2. GaP — red or green light.
3. GaAsP — red or yellow (amber) light.



LEDs that emit ***blue*** light are also available but red is the most common. LEDs emit no light when reverse-biased. In fact, operating LEDs in reverse direction will quickly destroy them. Fig. 53.1 shows a picture of LEDs that emits different colours of light.

(*b*) Construction

Broadly speaking, the LED structures can be divided into two categories :

1. ***Surface-emitting LEDs :*** These LEDs emit light in a direction **perpendicular** to the *PN* junction plane.
2. ***Edge-emitting LEDs :*** These LEDs emit light in a direction **parallel** to the *PN* junction plane.

Fig. 53.2 shows the construction of a surface-emitting LED. As seen from this figure, an *N*-type layer is grown on a substrate and a *P*-type layer is deposited on it by diffusion. Since carrier recombination takes place in the *P*-layer, it is kept upper most. The metal anode connections are made at the outer edges of the *P*-layer so as to allow more central surface area for the light to escape. LEDs are manufactured with domed lenses in order to lessen the reabsorption problem.

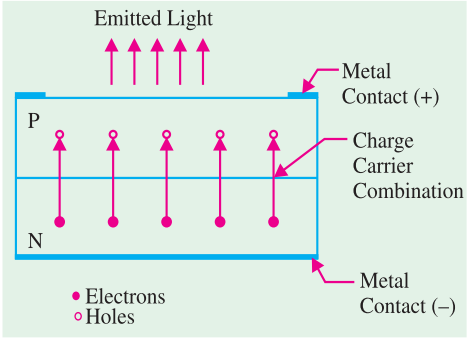


Fig. 53.2

A metal (gold) film is applied to the bottom of the substrate for reflecting as much light as possible to the surface of the device and also to provide cathode connection. LEDs are always encased in order to protect their delicate wires.

Being made of semiconductor material, it is rugged and has a life of more than 10,000 hours.

(*c*) Working

The forward voltage across an LED is considerably greater than for a silicon *PN* junction diode. Typically the maximum forward voltage for LED is between 1.2 V and 3.2 V depending on the device. Reverse breakdown voltage for an LED is of the order of 3 V to 10 V. Fig. 53.3 (*a*) shows a simple circuit to illustrate the working of an LED. The LED emits light in response to a sufficient forward current. The amount of power output translated into light is directly proportional to the forward current as shown in Fig. 53.3 (*b*). It is evident from this figure that greater the forward current, the greater the light output.

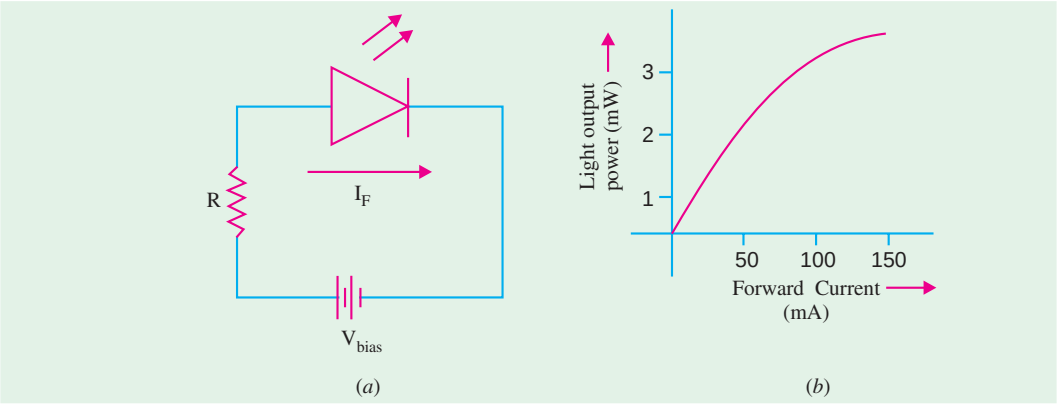


Fig. 53.3

(*d*) Applications

To chose emitting diodes for a particular application, one or more of the following points have to be considered : wavelength of light emitted, input power required, output power, efficiency, turn-on and turn-off time, mounting arrangement, light intensity and brightness etc.

Since LEDs operate at voltage levels from 1.5 V to 3.3 V, they are highly compatible with solid-state circuitry.

Their uses include the following :

1. LEDs are used in burglar-alarm systems;
2. for solid-state video displays which are rapidly replacing cathode-ray tubes (CRT);
3. in image sensing circuits used for 'picturephone';
4. in the field of optical fibre communication systems where high-radiance GaAs diodes are matched into the silica-fibre optical cable;
5. in data links and remote controllers;
6. in arrays of different types for displaying alphanumeric (letters and numbers) or supplying input power to lasers or for entering information into optical computer memories;
7. for numeric displays in hand-held or pocket calculators.

As shown in Fig. 53.4 (*a*) a seven-segment display consists of seven rectangular LEDs which can form the digits 0 to 9. The seven LED segments are labelled '*a*' to '*g*'. Each of this segments is

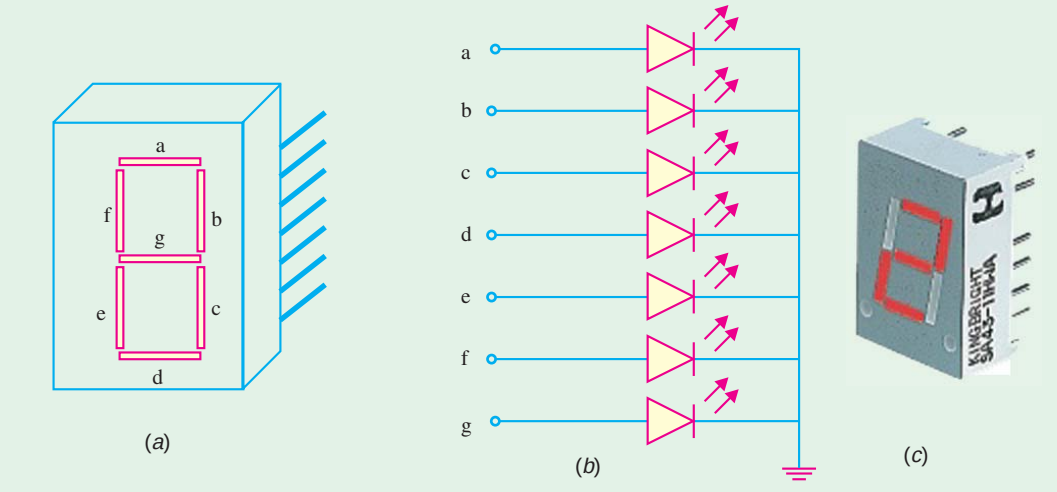
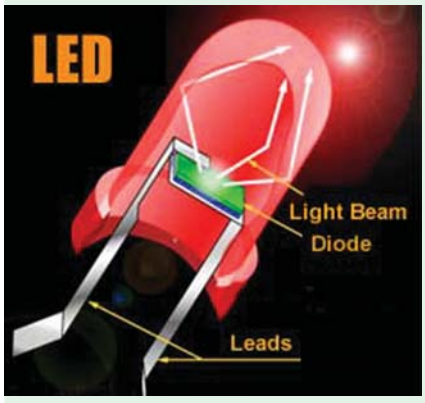


Fig. 53.4



LED

controlled through one of the display LEDs. Seven-segment displays come in two types, common-cathode and common-anode type. In the common-cathode type, all the cathodes of the diodes are tied together as shown in Fig. 53.4 (*b*). This makes it possible to light any segment by forward-biasing that particular LED. For example, to light number 5, segments *a*, *f*, *g*, *c* and *d* must be forward-biased. Since the cathodes are tied to ground, only 5 volt is to be applied to the anode of these segments to light them.

The common-anode seven-segment display has all its anodes tied together to +5 volt and ground is used to light the individual segments. Fig. 53.4(*c*) shows a picture of a seven-segment display.

(*e*) Multicoloured LEDs

LEDs are available which gives out light in either two or three colours. There are also blinking LEDs. A two-colour LED is a three-terminal device as shown in Fig. 3.5. The longest lead is the cathode and the remaining two leads are the anodes. When leads R and C are forward-biased, the LED emits red light and when leads G and C are forward-biased, LED emits green light. The tricolour LED looks similar to the ordinary LED but emits, red, green or yellow light depending on operating conditions. It has two leads and each of these acts as both anode and cathode. When dc current flows through it in one direction, LED emits red light but when current flows in the opposite direction, LED emits green light. However, with ac current, yellow light is given out.

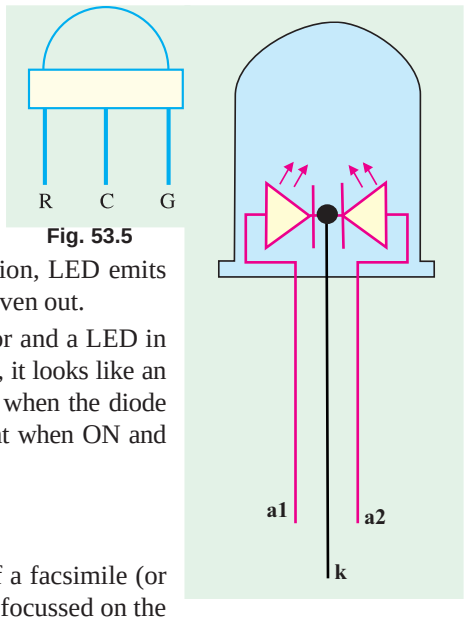


Fig. 53.5

The blinking LED is a combination of an oscillator and a LED in one package. Since it has an anode and a cathode lead, it looks like an ordinary LED. The blinking frequency is usually 3 Hz when the diode forward bias is 5 V. It conducts about 20 mA of current when ON and 0.9 mA when OFF.

53.3. Use of LEDs in Facsimile Machines

Fig. 53.6 shows a simplified schematic diagram of a facsimile (or fax) machine. As seen, the light from the LED array is focussed on the document paper. The light reflected at the paper is focussed on a charge-coupled device (CCD) by a combination of mirror and a lens. This causes the optical information to be converted into electrical information. The electrical information is then sent through the data-processing unit to its destination via telephone line.

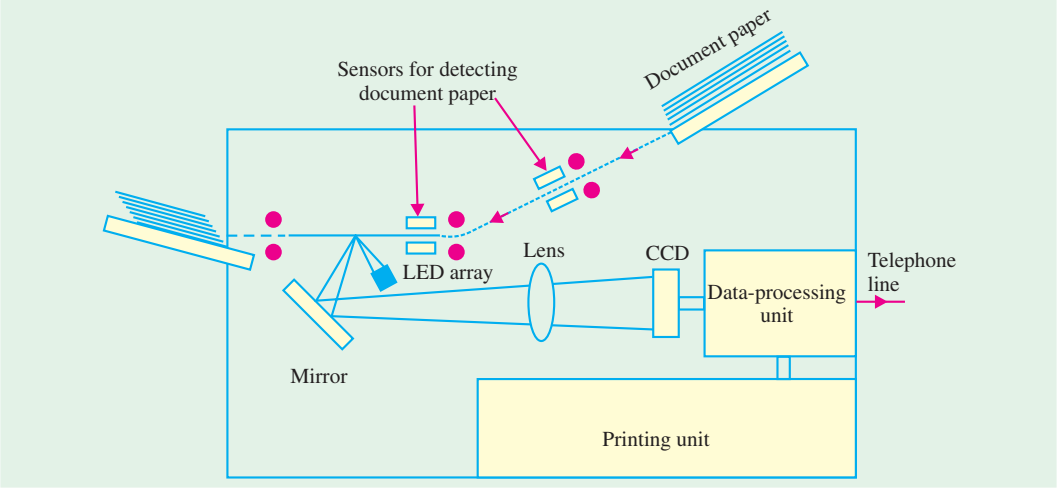


Fig. 53.6

53.4. Liquid Crystals Displays

(*a*) General

A liquid crystal is a material (usually, an organic compound) which flows like a liquid at room temperature but whose molecular structure has some properties normally associated with solids (examples of such compounds are : cholesteryl nonanoate and p-azoxyanisole). As is well-known,

the molecules in ordinary liquids have random orientation but in a liquid crystal they are oriented in a definite crystal pattern. Normally, a thin layer of liquid crystal is transparent to incident light but when an electric field is applied across it, its molecular arrangement is disturbed causing changes in its optical properties. When light falls on an activated layer of a liquid crystal, it is either absorbed or else is scattered by the disoriented molecules.

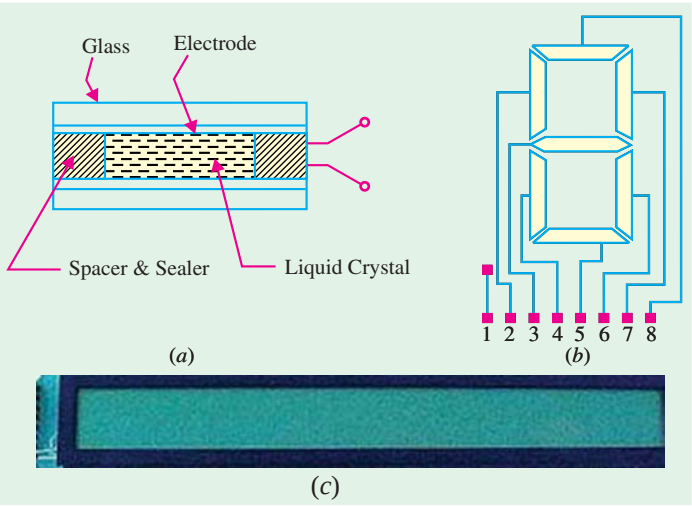


Fig. 53.7

(*b*) Construction

As shown in Fig. 53.7 (*a*), a liquid crystal 'cell' consists of a thin layer (about 10 μm) of a liquid crystal sandwiched between two glass sheets with transparent electrodes deposited on their inside faces. With both glass sheets transparent, the cell is known as ***transmittive*** type cell. When one glass is transparent and the other has a reflective coating, the cell is called ***reflective*** type. The LCD does not produce any illumination of its own. It, in fact, depends entirely on illumination falling on it from an external source for its visual effect.

(*c*) Working

The two types of display available are known as **(*i*) field-effect display** and **(*ii*) dynamic scattering display.** When field-effect display is energized, the energized areas of the LCD absorb the incident light and, hence give localized black display. When dynamic scattering display is energized, the molecules of energized area of the display become turbulent and scatter light in all directions. Consequently, the activated areas take on a frosted glass appearance resulting in a silver display. Of course, the un-energized areas remain translucent.

As shown in Fig. 53.7 (*b*), a digit on an LCD has a segment appearance. For example, if number 5 is required, the terminals 8, 2, 3, 6 and 5 would be energized so that only these regions would be activated while the other areas would remain clear.

(*d*) Advantages

An LCD has the distinct advantage of extremely low power requirement (about 10-15 μW per 7-segment display as compared to a few mW for a LED). It is due to the fact that it does not itself generate any illumination but depends on external illumination for its visual effect (colour depending on the incident light). They have a life-time of about 50,000 hours.

(*e*) Uses

1. Field-effect LCDs are normally used in watches and portable instruments where source of energy is a prime consideration.
2. Thousands of tiny LCDs are used to form the picture elements (pixels) of the screen in one type of B & W pocket TV receiver.
3. Recent desk top LCD monitors.
4. Note book computer display
5. Cellular phone display, to display data on personal digital assistant (PDAs) such as Palm V*x* etc.

The liquid crystal display (LCDs) commonly used on notebook computers and handheld PDAs are also appearing on desktop. These flat panel displays promise great clarity at increasingly high resolutions and are available in screen sizes upto 15 inches. The LCD monitor offers benefits and drawbacks. The first benefit is size. Because of the need to house the tube itself, cathode-ray tube (CRT) monitors are big and heavy. LCD monitors are only a few inches deep and they are much lighter in weight. However LCD monitors are expensive than CRTs at present. Another problem is the viewing angle. The optimal viewing angle of an LCD is from straight in front and as you move further to the side the screen becomes harder to read, much more so than with a CRT. Moreover screen resolutions generally reach only as high as $1{,}024 \times 768$, which is insufficient for some applications. Fig. 53.7(*c*) shows the picture of an LCD used in portable instrument.

53.5. P-N Junction Photodiode

It is a two-terminal junction device which is operated by first reverse-biasing the junction and then illuminating it. A reverse-biased *P-N* junction has a small amount of reverse saturation current I_s (or I_0) due to thermally-generated electron-hole pairs. In silicon, I_s is the range of nanoamperes. The number of these minority carriers depends on the intensity of light incident on the junction. When the diode is in glass package, light can reach the junction and thus change the reverse current.

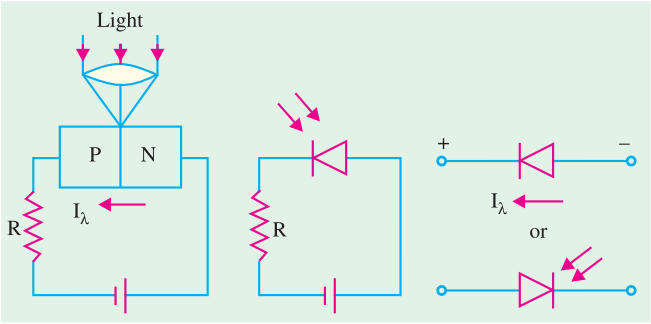
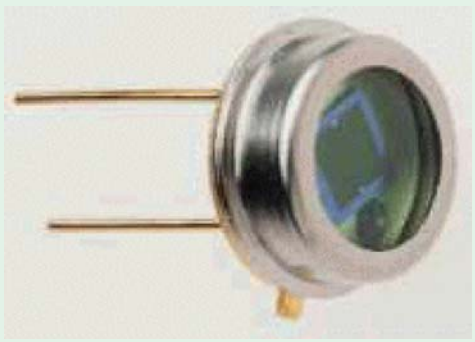


Fig. 53.8



Photodiode

The basic biasing arrangement, construction and symbols of a photodiode are shown in Fig. 53.8. As seen, a lens has been used in the cap of the unit to focus maximum light on the reverse-biased junction. The active diameter of these devices is about 2.5 mm but they are mounted in standard TO-5 packages with a window to allow maximum incident light.

The characteristics of Fig. 53.9 show that for a given reverse voltage, I_λ (or I_s) increases with increase in the level of illumination. The dark current refers to the current that flows when no light is incident. By changing the illumination level, reverse current can be changed. In this way, reverse resistance of the diode can be changed by a factor of nearly 20.

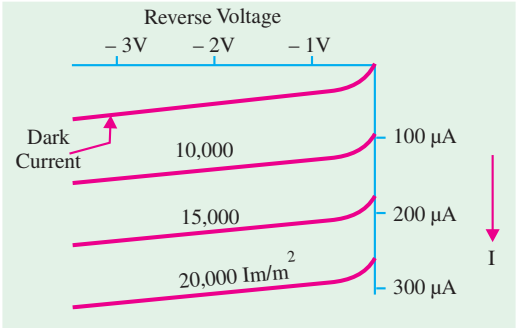


Fig. 53.9

A photodiode can turn its current ON and OFF in nanoseconds. Hence, it is one of the fastest photodetectors. It is used where it is required to switch light ON and OFF at a maximum rate. Applications of a photodiode include

1. detection, both visible and invisible ;
2. demodulation ;
3. switching ;
4. logic circuit that require stability and high speed ;
5. character recognition ;
6. optical communication equipment ;
7. encoders etc.

53.6. Dust Sensor

Fig. 53.10 shows a combination of an LED and a photodiode used as a dust sensor. As seen, the light emitted from the LED gets reflected by the dust particles. The reflected light is collected by the photodiode and is converted into an electrical signal. The dust sensor is employed in cleaners.

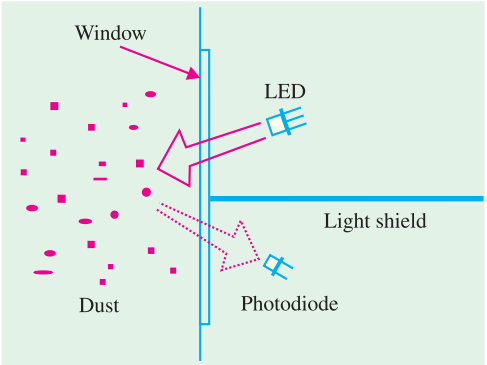
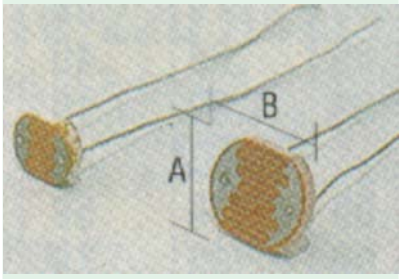


Fig. 53.10

The combination of an LED and a photodiode is also used as : **(1)** a paper sensor in facsimile machines, **(2)** as a tape-end sensor in videotape recorders/players, and **(3)** as a dirt detector for rinsing in washing machines.

53.7. Photoconductive Cell

It is a semiconductor device whose resistance varies inversely with the intensity of light that falls upon it. It is also known as ***photoresistive*** cell or ***photoresistor*** because it operates on the principle of photoresistivity.



CdS photo sensitive detectors

(*a*) Theory

The resistivity (and, hence, resistance) of a semiconductor depends on the number of free charge carriers available in it. When the semiconductor is not illuminated, the number of charge carriers is small and, hence, resistivity is high. But when light in the form of photons strikes the semiconductor, each photon delivers energy to it. If the photon energy is greater than the energy band gap of the semiconductor, free mobile charge carriers are liberated and, as a result, resistivity of the semiconductor is decreased.

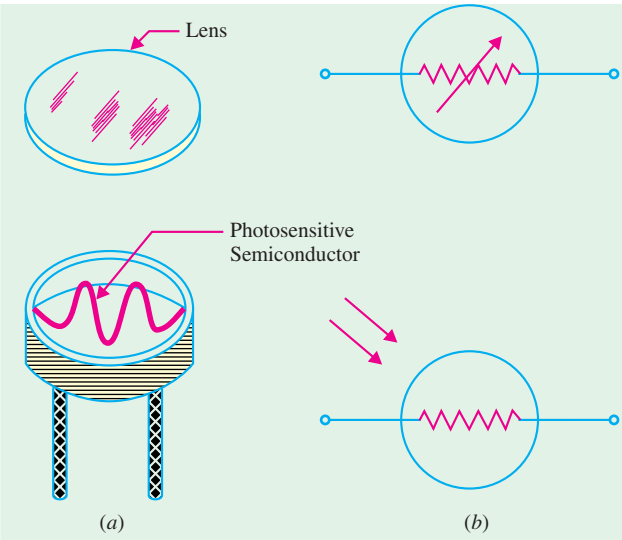


Fig. 53.11

(*b*) Construction and Working

Photoconductive cells are generally made of cadmium compounds such as cadmium sulphide (CdS) and cadmium selenide (CdSe). Spectral response of CdS cell is similar to the human eye, hence such cells are often used to simulate the human eye. That is why they find

use in light metering circuits in photographic cameras.

The construction of a typical photo conductive cell and its two alternative circuit symbols are shown in Fig. 53.11 (*a*) and (*b*) respectively. As seen, a thin layer of photosensitive semiconductor material is deposited in the form of a long strip zig-zagged across a disc-shaped ceramic base with protective sides. For added protection, a glass lens or plastic cover is used. The two ends of the strip are brought out to connecting pins below the base.

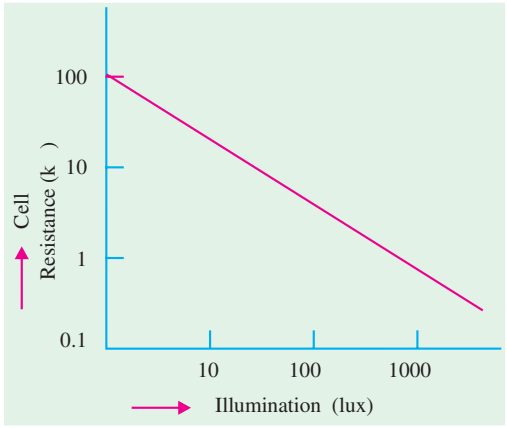


Fig. 53.12

The terminal characteristic of a photoconductive cell is shown in Fig. 53.12. It depicts how the resistance of the cell varies with light intensity. Typically, the dark resistance of the cell is 1 MΩ or larger. Under illumination, the cell resistance drops to a value between 1 and 100 kΩ depending on surface illumination.

(*c*) Applications

A photoconductive cell is an inexpensive and simple detector which is widely used in OFF/ON circuits, light-measurement and light-detecting circuits.

Example 53.1. *A relay is controlled by a photoconductive cell which has resistance of 100 kΩ when illuminated and 1 kΩ when in the dark. The relay is supplied with 10 mA from a 30-V supply when cell is illuminated and is required to be de-energized when the cell is in the dark. Sketch a suitable circuit and calculate the required series resistance and value of dark current.*

(Optoelectronic Devices, Gujarat Univ. 1993)

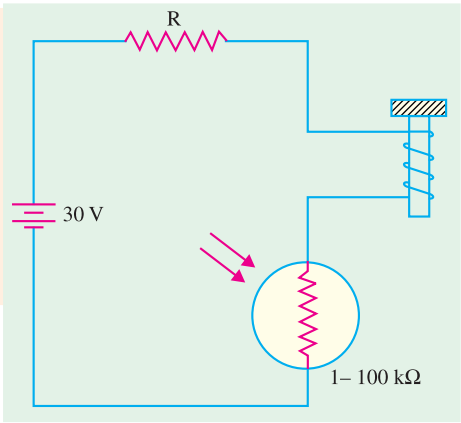


Fig. 53.13

Solution. The circuit is as shown in Fig. 30.13 where R is a current-limiting resistor.

$$I = 30/(R + r)$$

—where r is cell resistance

$$\therefore \quad R = (30/I) - r$$

When illuminated

$$R = (30/10 \times 10^{-3}) - 1 \times 10^{3} = 2 \times 10^{3} = \mathbf{2\ k\Omega}$$

Dark current is given by

$$I_d = 30/(2 + 100) \times 10^{3} = 0.3 \times 10^{-3}\ \text{A} = \mathbf{0.3\ mA}$$

53.8. Phototransistor

It is light-sensitive transistor and is similar to an ordinary bipolar junction transistor (BJT) except that it has no connection to the base terminal. Its operation is based on the photodiode that exists at the *CB* junction. Instead of the base current, the input to the transistor is provided in the form of light as shown in the schematic symbol of Fig. 53.14 (*a*).

Silicon NPNs are mostly used as photo transistors. The device is usually packed in a TO-type can with a lens on top although it is


Phototransistor

sometimes encapsulated in clear plastic. When there is no incident light on the *CB* junction, there is a small thermally-generated collector-to-emitter leakage current I_{CEO} which, in this case, is called dark current and is in the *nA* range.

When light is incident on the *CB* junction, a base current I_λ is produced which is directly proportional to the light intensity. Hence, collector current $I_C = \beta I_\lambda$

Typical collector characteristic curves of a phototransistor are shown in Fig. 53.14 (*b*). Each individual curve corresponds to a certain value of light intensity expressed in mW/cm^2. As seen, I_C increases with light intensity.

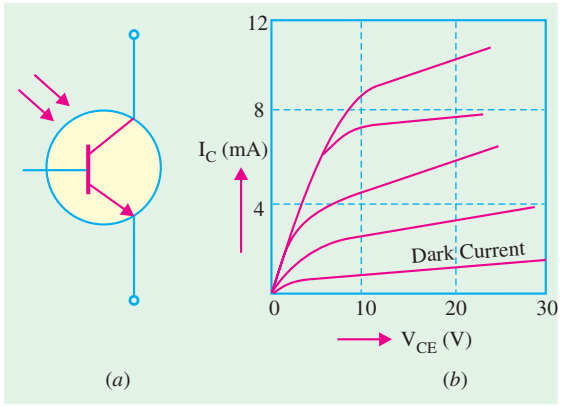


Fig. 53.14

The phototransistor has applications similar to those of a photodiode. Their main differences are in the current and response time. The photo-transistor has the advantages of greater sensitivity and current capacity than photodiodes. However, photodiodes are faster of the two, switching in less than a nanosecond.

53.9. Photodarlington

As shown in Fig. 53.15 a photodarlington consists of a phototransistor in a Darlington arrangement with a common transistor. It has a much greater sensitivity to incident radiant energy than a phototransistor because of higher current gain. However, its switching time of 50 μs is much longer than the phototransistor (2 μs) or the photodiode (1 ns). Its circuit symbol is shown in Fig. 53.15.

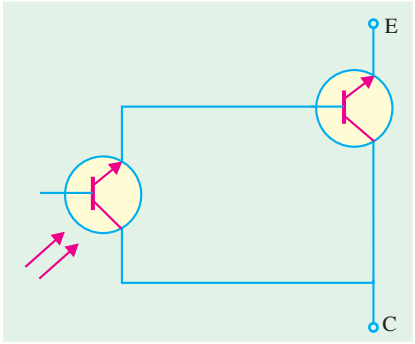


Fig. 53.15

Applications

Photodarlingtons are used in a variety of applications some of which are given below.

A light-operated relay is shown in Fig. 53.16 (*a*). The phototransistor T_1 drives the bipolar transistor T_2. When sufficient light falls on T_2, it is driven into saturation so that I_C is increased manifold. This collector current while passing through the relay coil energizes the relay.

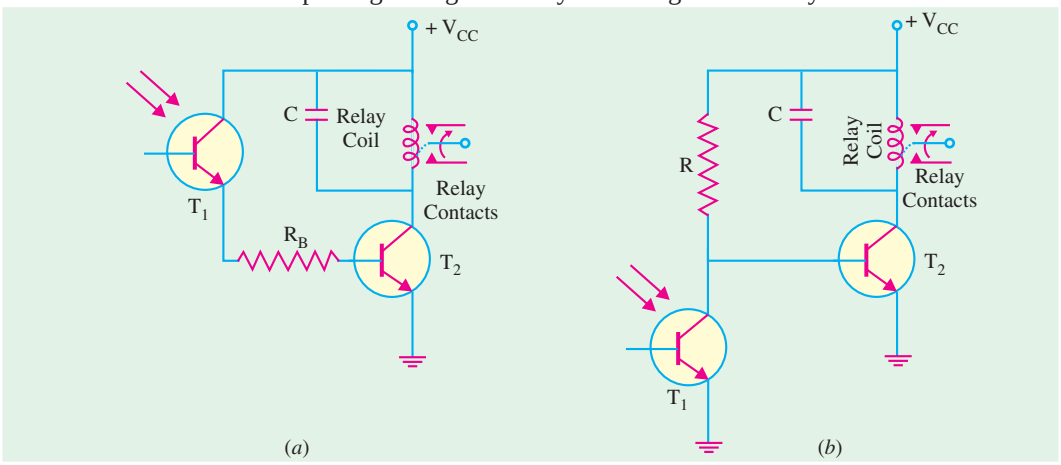


Fig. 53.16

Fig. 53.16 (*b*) shows a dark-operated relay circuit *i.e.* one in which relay is deenergized when light falls on the phototransistor. Here, T_1 and R form a potential divider across V_{CC}. With insufficient light incident on T_1, transistor T_2 is biased ON thereby keeping the relay energized. However, when there is sufficient light, T_1 turns ON and pulls the base of T_2 low thereby turning T_2 OFF and hence, deenergizing the relay.

Such relays are used in many applications such as ***(i)*** automatic door activators, ***(ii)*** process counters and ***(iii)*** various alarm systems for smoke or intrusion detection.

53.10. Photo voltaic or Solar Cell

Such cells operate on the principle of photovoltaic action *i.e.* conversion of light energy into electrical energy. This action occurs in all semiconductors which are constructed to absorb energy.

(*a*) Construction

As shown in Fig. 53.17 (*a*), a basic solar cell consists of *P*-type and *N*-type semiconductor material (usually, silicon or selenium) forming a *P-N* junction. The bottom surface of the cell (which is always away from light) covered with a continuous conductive contact to which a wire lead is attached. The upper surface has a maximum area exposed to light with a small contact either along the edge or around the perimeter. The surface layer of *P*-type material is extremely thin (0.5 mm) so that light can penetrate to the junction.

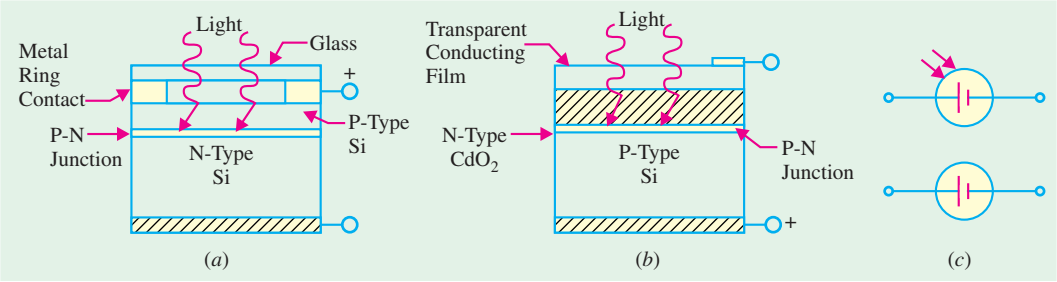
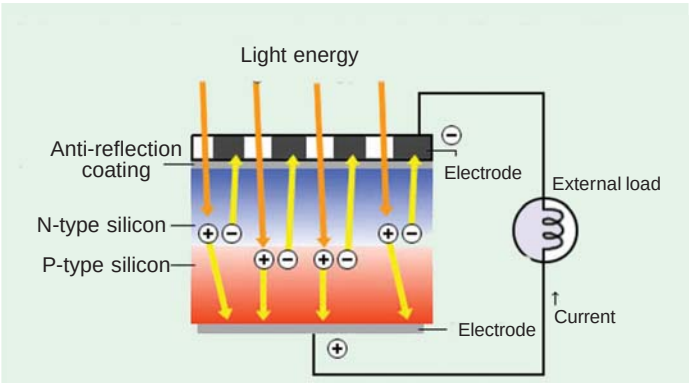


Fig. 53.17

Although silicon is commonly used for fabricating solar cells, another construction consists of *P*-type selenium covered with a layer of *N*-type cadmium oxide to form *P-N* junction as shown in Fig. 53.17 (*b*). Two alternative circuit symbols are shown in Fig. 53.17 (c). Power solar cells are also fabricated in flat strips to form efficient coverage of available surface area. Incidentally, the maximum efficiency of a solar cell in converting sunlight into electrical energy is nearly 15 per cent at the present.

(*b*) Theory

When the *P-N* junction of a solar cell is illuminated, electron-hole pairs are generated in much the same way, as in photovoltaic cell. An electric field is established near the *P-N* junction by the positive and negative ions created due to the production of electron-hole pairs which leads to the development of potential across the junction. Since the number of electron-hole pairs far exceeds the number needed for thermal equilibrium, many of the



A photovoltaic cell generates electricity when irradiated by sunlight

electrons are pulled across the junction by the force of the electric field. Those that cross the junction contribute to the current in the cell and through the external load. The terminal voltage of the cell is directly proportional to the intensity of the incident light. The voltage may be as high as 0.6 V depending on the external load. Usually a large number of cells are arranged in an array in order to obtained higher voltages and currents as shown in Fig. 53.18.

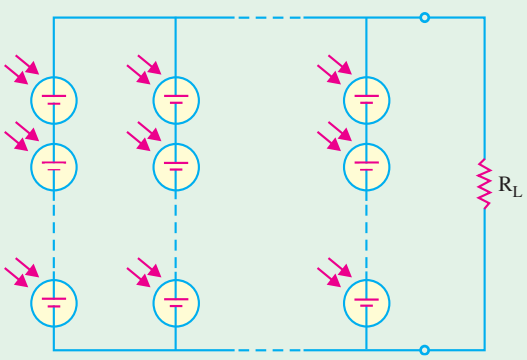


Fig. 53.18

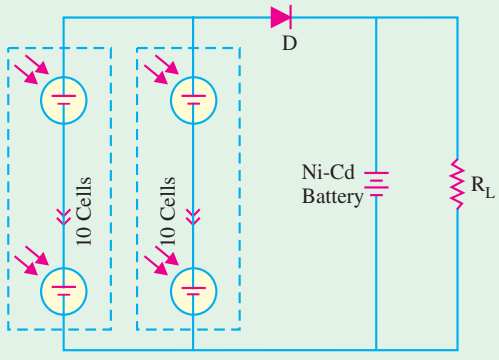


Fig. 53.19

Solar cells act like a battery when connected in series or parallel. Fig. 53.19 show two groups of 10 series cells connected in parallel with each other. If each cell provides 0.5 V at 150 mA, the overall value of the solar bank is 5 V at 150 mA. The two parallel solar banks provide 5 V at 300 mA. This solar power source supplies the load and also charges the Ni-Cd battery. The battery provides power in the absence of light. A blocking diode *D* is used to isolate the solar cells from the Ni-Cd battery otherwise in the absence of light, the battery will discharge through the cells thereby damaging them.

A solar cell operates with fair efficiency, has unlimited life, can be easily mass-produced and has a high power capacity per weight. It is because of these qualities that it has become an important source of power for earth satellites.

Example 53.2. *An earth satellite has on board 12-V battery which supplies a continuous current of 0.5 A. Solar cells are used to keep the battery charged. The solar cells are illuminated by the sun for 12 hours in every 24 hours. If during exposure, each cell gives 0.5 V at 50 mA, determine the number of cells required.* **(Optoelectronics Devices, Gujarat Univ. 1994)**

Solution. The solar cell battery-charging circuit is shown in Fig. 53.20. The cells must be connected in series to provide the necessary voltage and such groups must be connected in parallel to provide the necessary current. The charging voltage has to be greater than the battery voltage of 12 V. Allowing for different drops, let the solar bank voltage be 13.5 V.

Number of series connected solar cells = 13.5/0.5 = 27

The charge given out by batteries during a 24 hour period = 12 × 0.5 = 6 Ah. Hence, solar cells must supply this much charge over the same period. However, solar cells deliver current only when they illuminated *i.e.* for 12 hours in every 24 hours. Necessary charging current

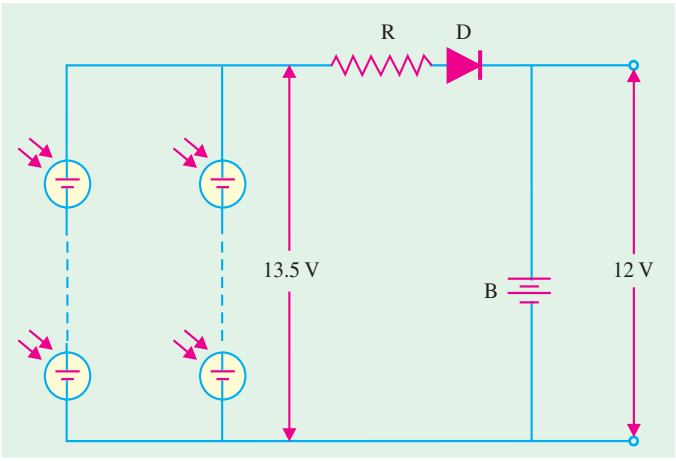


Fig. 53.20

required from the solar cells = 6 Ah/12 h = 0.5 A.

Total number of groups of solar cells required to be connected in parallel is

= output current / cell current = $0.5 / 50 \times 10^3 = 10$

$\therefore$ total number of solar cells required for the earth satellite = $27 \times 10 =$ **270**

53.11. Laser Diode

Like LEDs, laser diodes are typical *PN* junction devices used under a forward-bias. The word LASER is an acronym for ***Light Amplification*** by ***Stimulated Emission of Radiation.*** The use of laser is (becoming increasing common) in medical equipment used in surgery and in consumer products like compact disk (CD) players, laser printers, hologram scanners etc.

(*a*) Construction

Broadly speaking, the laser diode structure can be divided into two categories :

1. ***Surface-emitting laser diodes :*** These laser diodes emit light in a direction **perpendicular** to the PN junction plane.
2. ***Edge-emitting laser diodes :*** These laser diodes emit light in a direction **parallel** to the *PN* junction plane.

Fig. 53.21 (*a*) shows the structure of an edge-emitting laser diode. This type of structure is called Fabry-Perot type laser. As seen from the figure, a *P-N* junction is formed by two layers of doped gallium arsenide (GaAs). The length of the *PN* junction bears a precise relationship with the wavelength of the light to be emitted. As seen, there is a highly reflective surface at one end of the junction and a partially reflective surface at the other end. External leads provide the anode and cathode connections.

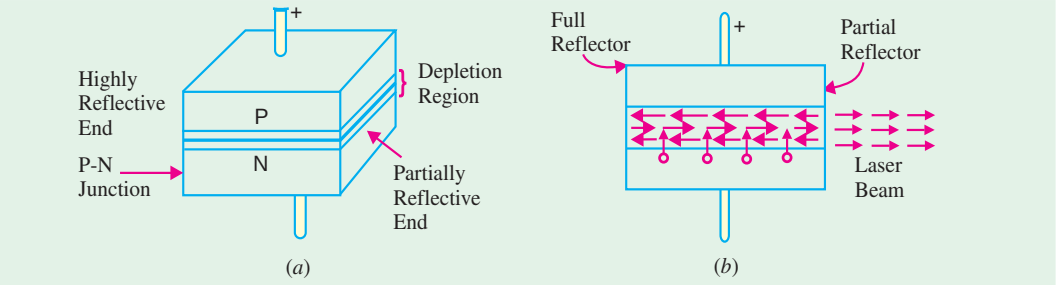
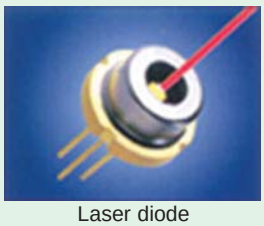


Fig. 53.21

(*b*) Theory


Laser diode

When the *P-N* junction is forward-biased by an external voltage source, electrons move across the junction and usual recombination occurs in the depletion region which results in the production of photons. As forward current is increased, more photons are produced which drift at random in the depletion region. Some of these photons strike the reflective surface perpendicularly. These reflected photons enter the depletion region, strike other atoms and release more photons. All these photons move back and forth between the two reflective surfaces. [Fig. 53.21 (*b*)] The photon activity becomes so intense that at some point, a strong beam of laser light comes out of the partially reflective surface of the diode.

(*c*) Unique Characteristics of Laser Light

The beam of laser light produced by the diode has the following unique characteristics :

1. It is coherent *i.e.* there is no path difference between the waves comprising the beam;
2. It is monochromatic *i.e.* it consists of one wavelength and hence one colour only;

3. It is collimated *i.e.* emitted light waves travel parallel to each other.

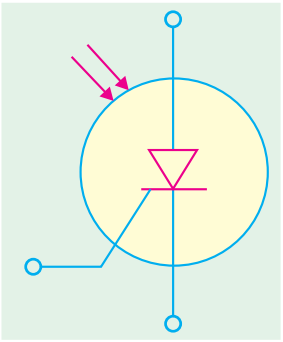


Fig. 53.22

Laser diodes have a threshold level of current above which the laser action occurs but below which the laser diode behaves like a LED emitting incoherent light. The schematic symbol of a laser diode is similar to that of LED. Incidentally, a filter or lens is necessary to view the laser beam.

(*d*) Applications

Laser diodes are used in variety of applications ranging from medical equipment used in surgery to consumer products like optical disk equipment, laser printers, hologram scanners etc. Laser diodes emitting visible light are used as pointers. Those emitting visible and infrared light are used to measure range (or distance). The laser diodes are also widely used in parallel processing of information and in parallel interconnections between computers. Some of these applications are discussed in the following articles.

53.12. Optical Disks

The major application field for laser diodes is in optical disk equipment. This equipment is used for reading or recording information and can be broadly divided into two groups :

1. Reading-only and **2.** Recording-and-reading type.

The optical disk equipment of either type make use of a laser diode, lenses and photodiodes. During recording, it changes electrical information into optical information and then records the information on the optical disk. During reading (or playback), the head optically reads the recorded information and changes the optical information into electrical information. Fig. 53.22 shows the different types of optical disks used in practice. The commercial systems make use of disks that are 90, 120, 130 and 300 mm in diameter. A mini disk, 64 mm in diameter is also used for digital audio.

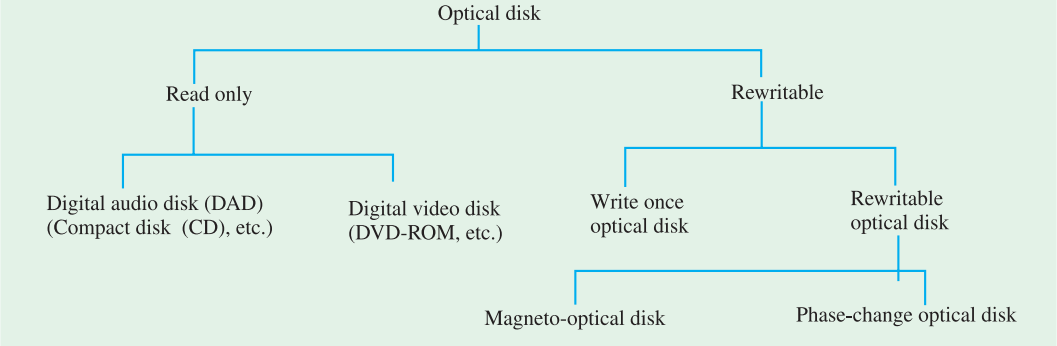


Fig. 53.23

The optical disks have several advantages over semiconductor memories. Some of these include their larger data storage capacity, shorter access time and smaller size. Therefore they are used in terminal equipment of computers as well as in audio visual equipment.

53.13. Read-only Optical Disks Equipment

Fig. 53.24 shows an optical equipment for reading data from digital audio (compact) disks. Compact disks (CDs) which are 120 mm in diameter are typical digital audio disks. Compact disks usually means digital audio compact disk, but it also includes the read-only memory (CD-ROM) for data memory and interactive compact disk (CD-I) for multimedia use.

Audio information (*i.e.* sound) is digitally recorded in stereo on the surface of a CD in the form of microscopic "pits" and "flats". As seen from Fig. 53.24, the light emitted from the laser diode passes through the lens and is focussed to a diameter of about 1 μm on the surface of a disk. As the CD rotates, the lens and beam follow the track under control of a servo motor. The laser light which is altered by the pits and flats along the recorded track is reflected back from the track through the lens and optical system to infrared photodiodes. The signal from the photodiodes is then used to reproduce the digitally recorded sound.



A CD-Rom

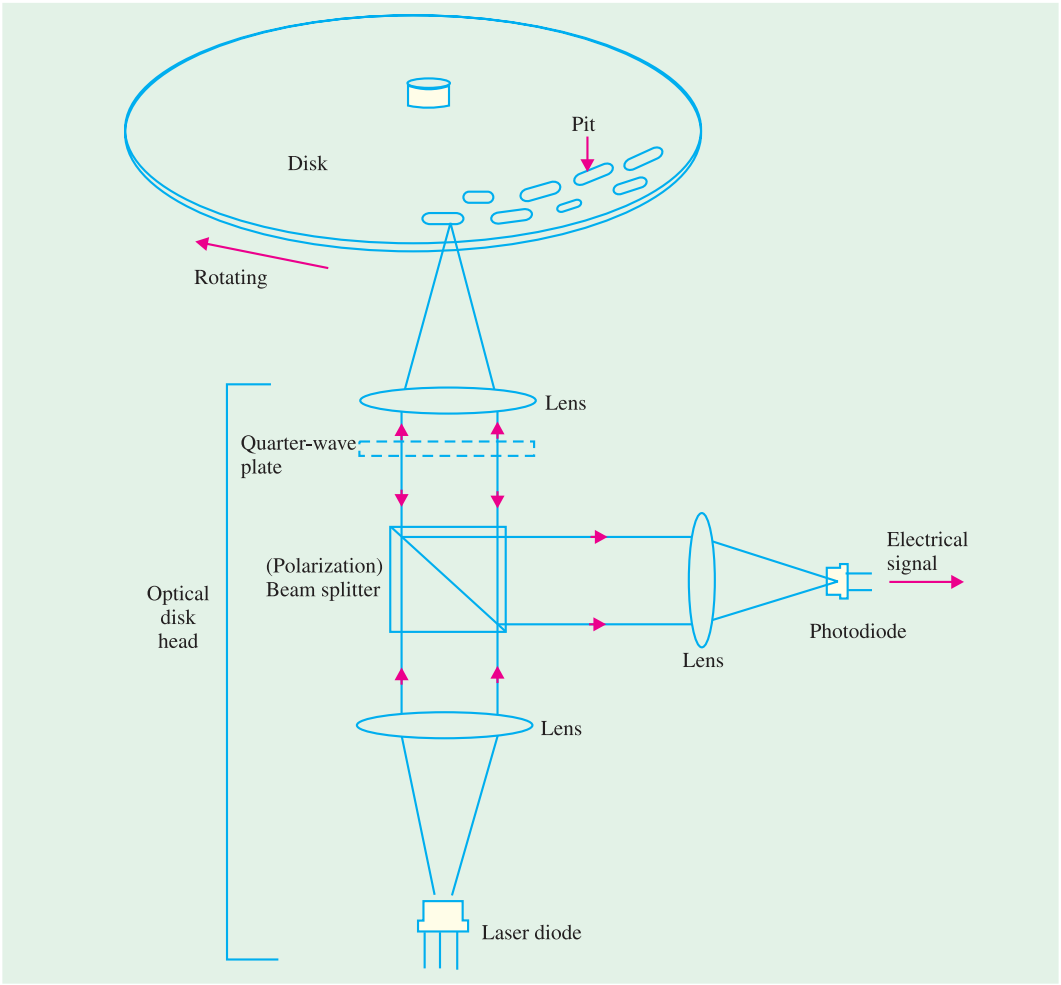


Fig. 53.24

53.14. Printers Using Laser Diodes

There are two types of optical sources usually used in printers ; **(1)** laser diodes and **(2)** LED arrays. The printers using laser diodes are called laser beam printers (or simply laser printers). These are one of the most attractive type of equipment in office automation in today's world. Words and figures can be printed rapidly and clearly more easily by a laser printer than by other types of printers.

(Courtesy optical semiconductor devices by M.Fukuda published by John Weliy & Sons Inc.)

Fig. 53.25 shows a simplified diagram of a laser printer. As seen the laser diode is driven by modulated signals from the computer. The optical beam after passing through the lens is reflected by the rotating polygon mirror and scanned on the photosensitive drum. The drum is homogeneously charged when it passes through the charging unit consisting of an LED array. The homogeneous electrification is partially erased in accordance with the scanned optical beam. This is because of the fact that the electrical resistance at the light-irradiated part decreases and the electric charge is released. This causes the signals (*i.e.* data) from the computer to be written on to the drum. At the developing unit, an electrically charged powder (called toner) is electrostatically attached to the written parts. At the transcribing unit, the powder is transferred to the paper. Next, the transferred pattern is fixed by heating and pressing at the fixing unit. The data from the computer is thus printed on the paper.

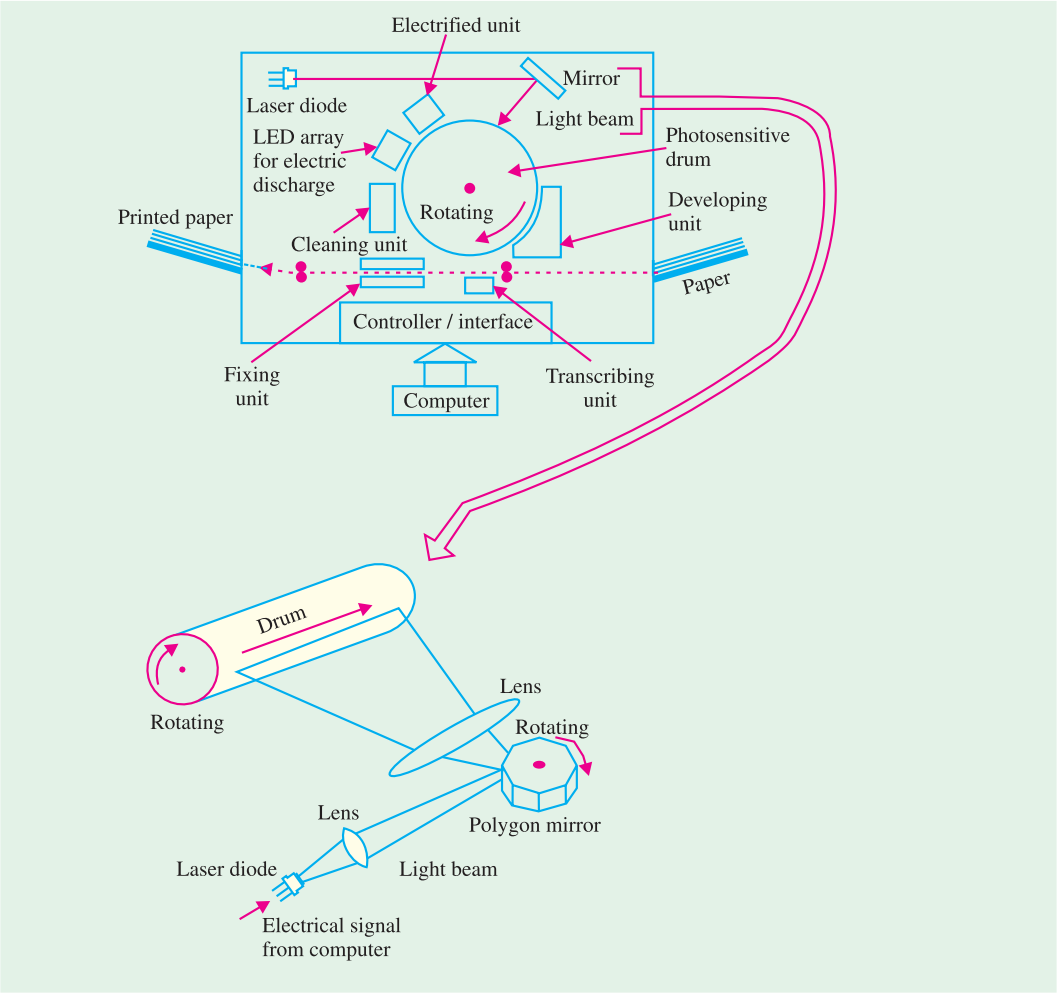


Fig. 53.25

53.15. Hologram Scanners

The hologram scanner is widely used in various equipment and is ordinarily used in bar-code readers in point-of-sale systems (such as super marked checkout counters). It is also used in laser printers for scanning the laser beam on the drum precisely.

Fig. 53.26 shows a simplified schematic of a hologram scanner. As seen, the optical beam for reading the bar-code is focussed by a lens through the hologram disk and scanned on the bar-code by rotating the hologram disk. Gratings with coaxial circles are formed on the hologram disk. This



Hologram scanners

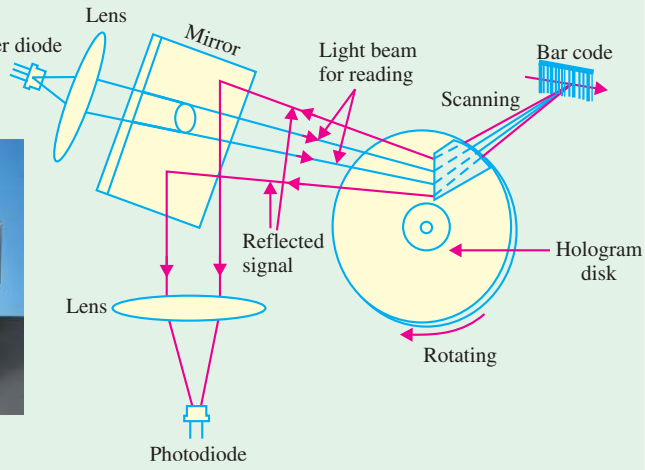


Fig. 53.26

causes the incident laser beam to bend at the grating by an amount determined by the grating pitch. The reflected light modulated according to the bar-code is reflected by the mirror and monitored by the photodiode. The monitored optical signal is then translated into an electrical signal.

53.16. Laser Range Finder

The laser diodes along with photodiodes can be used to measure the range (*i.e.* a distance) of an object. Fig. 53.27 shows a simple schematic of a laser range finder. As seen, the laser diode is modulated with high current pulses. The pulsed high-power beam is emitted in the direction of an object. The beam is reflected from the object. The reflected beam is detected with a photo detector (or photodiode). The range is calculated as the difference between the time the light was emitted from the laser diode and the time it was detected by the photodiode.

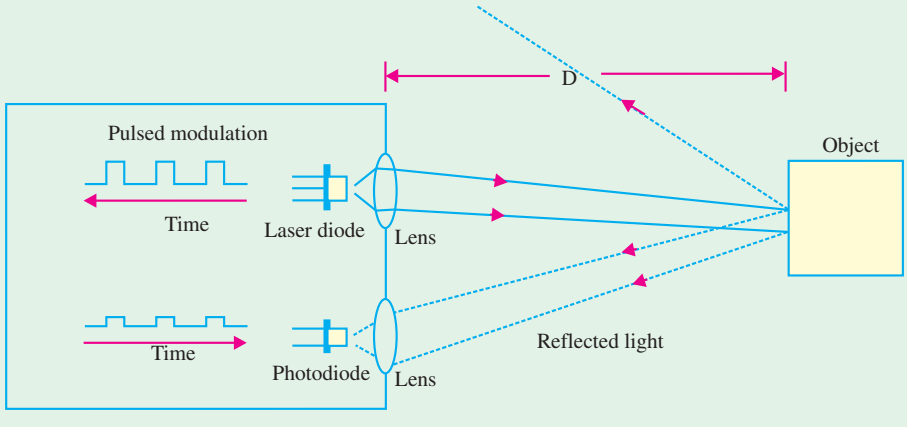


Fig. 53.27

Let D = distance between the laser range finder and the object.

ΔT = Time difference between the instance when the light was emitted from the laser diode and the instance when it was detected by the photo-diode

Then $D = \frac{1}{2} \times \text{speed of light} \times \Delta T$

A 2-dimensional array of laser diodes and photodetectors can be constructed. Such a system is used to obtain 3-D images of an object.

53.17. Light-activated SCR (LASCR)

The operation of a LASCR is essentially similar to that of a conventional SCR except that it is light-triggered (Fig.53.28). Moreover, it has a window and lens which focuses light on the gate junction area. The LASCR operates like a latch. It can be triggered ON by a light input on the gate area but does not turn OFF when light source is removed. It can be turned OFF only by reducing the current through it below its holding current. Depending on its size, a LASCR is capable of handling larger amount of current that can be handled by a photodiode or a photo-transistor.

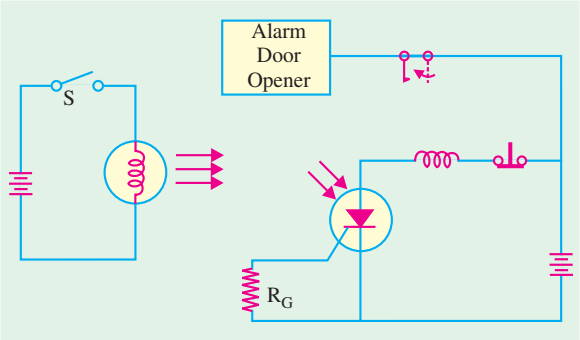


Fig. 53.28

Fig. 53.28 shows how a LASCR can be used for energizing a latching relay. The input dc source turns on the electric lamp and the resulting incident light triggers the LASCR into conduction. The anode current energizes the relay and closes the contact. It is seen that the input dc source is electrically isolated from the rest of the circuit.

53.18. Optical Isolators

Optical isolators are designed to electrically isolate one circuit from another while allowing one circuit to control the other. The usual purpose of isolation is to provide protection from high-voltage transients, surge voltages and low-level electrical noise that could possibly result in an erroneous output or damage to the device. Such isolators allow interfacing of circuits with different voltage levels and different grounds etc.

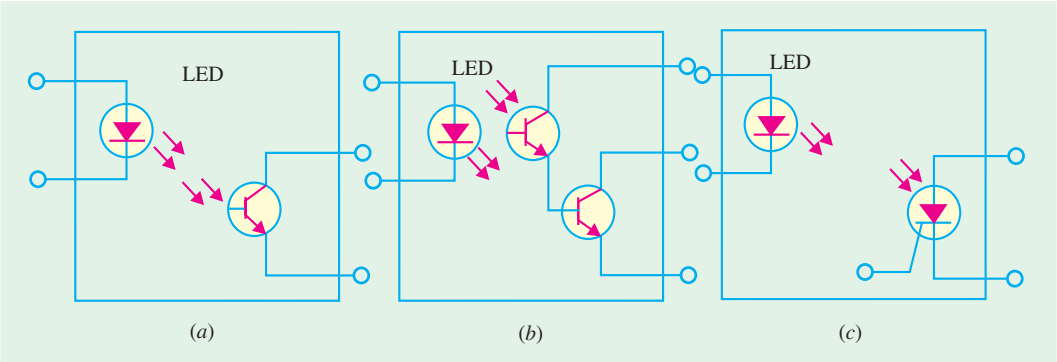


Fig. 53.29

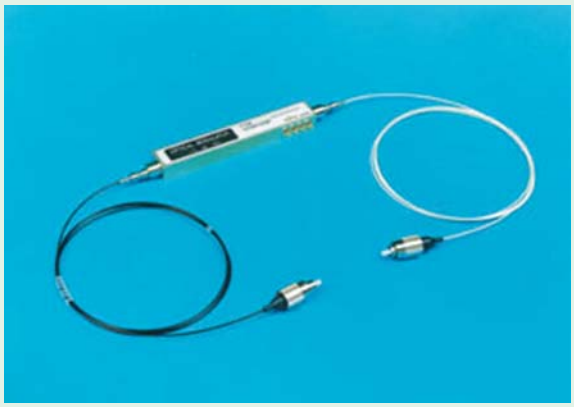
An optical isolator (or coupler) consists of a light source such as LED and a photodetector such as a photo transistor as shown in Fig. 53. 29 (*a*) and is available in a standard *IC* package.

When LED is forward-biased, the light produced by it is transferred to the phototransistor which is turned ON thereby producing current through the external load.

Fig. 53.29 (*b*) shows a Darlington transistor coupler which is used when increased output current capability is needed beyond that provided by the phototransistor output. The LASCR output coupler of Fig. 53.29 (*c*) can be used in applications where a low-level input voltage is required to latch a high voltage relay for activating some kind of electro-mechanical device.

53.19. Optical Modulators

Light emitting *PN* junction devices such as LEDs and laser diodes are easily modulated by superimposing signals on to the injected current. This is **direct modulation.** Laser diodes in high-bit rate and long-span optical communication systems are frequently used under direct modulation.



Optical modulator

However direct modulation results in ***chirping*** which limits transmission quality because of ***dispersion*** in optical fibres. An optical modulator can modulate the light output from laser diodes with little or no chirping. There are two types of optical modulators :

1. The semiconductor optical modulators
2. Optical modulators composed of dielectric materials such as ***lithium nitrate*** ($LiNO_3$)

The semiconductor optical modulators are *PN* junction diodes and can further be subdivided into two types :

1. Devices used under forward bias (as LEDs and laser diodes are used). The optical modulation in these devices is carried out by changing gain or loss within the modulators.
2. Devices used under reverse bias (*i.e.*, as photodiodes are used). Most high-performance semiconductor optical modulators are used under reverse bias. The reverse bias is needed to generate strong electric field. Optical modulation is basically performed by modulating the ***refractive index*** or ***optical absorption coefficient*** of the modulators. The devices which make use of refractive index phenomenon for modulation are called phase modulation type devices while those that use optical absorption coefficient phenomenon are called intensity modulation type devices.

There are several different types of optical modulators available today. But the waveguide type optical modulator is more common in use. Further there are several different waveguide type optical modulator structures possible. Fig. 53.30 shows a mesa type optical modulator structure.

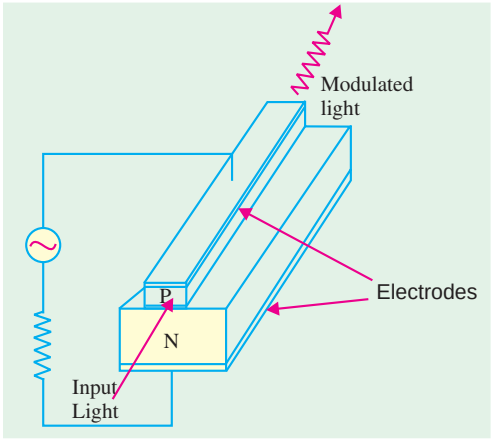


Fig. 53.30

It may be noted that although we have shown the structure making use of a simple *N*- and *P*-layer but in reality each layer (*N*-type or *P*-type) is made up of several different semiconductors.

53.20. Optical Fibre Communication Systems

The optical fibre communication systems (such as public communication networks and data links) are the basic infrastructure of the information hungry society. There are several advantages of the optical fibre system over metallic transmission systems as listed below :

1. Data can be transmitted at a very high-frequency over longer distances without much loss.
2. Electromagnetic induction (EMI) noise is never induced during transmission through optical fibre cables.
3. Optical fibre cable is light, flexible and economical.

Fig. 53.31 shows the public optical fibre communication system broadly divided into two groups: (1) Submarine systems, and (2) Land systems. Submarine systems have already been used to connect countries all over the world. The submarine systems help people to talk overseas without any time delay.

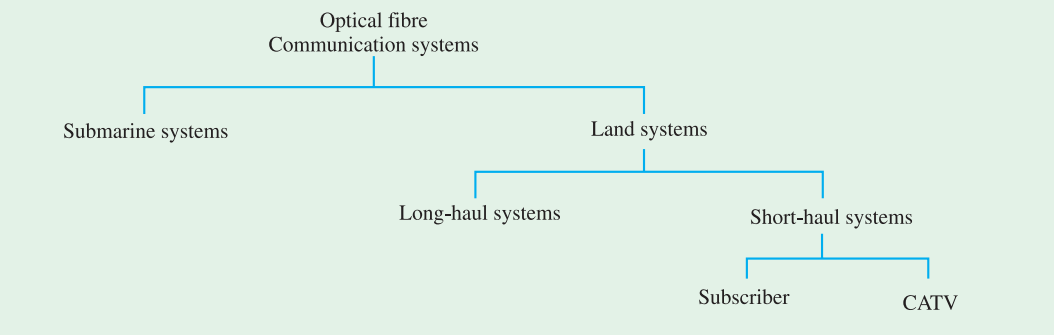


Fig. 53.31

In land systems, long-haul systems have been connected between large cities. The land systems also include systems such as subscriber systems and CATV (*i.e.* community or common antenna television, cable and telecommunication television system, or cable television system).

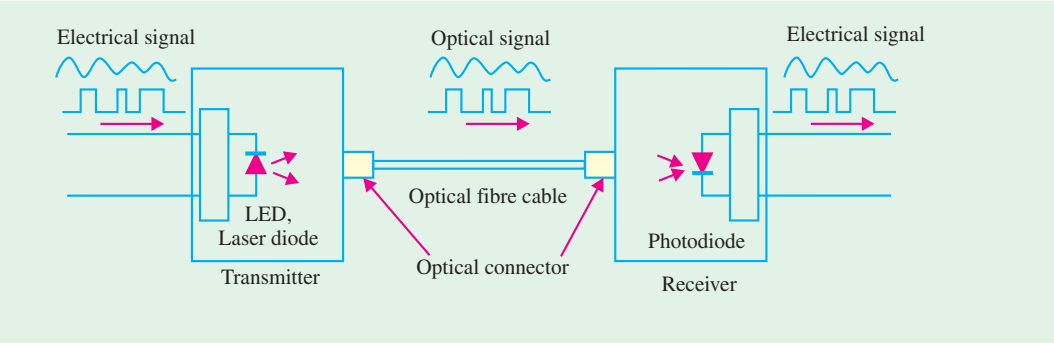


Fig. 53.32

Fig. 53.32 shows an application of LEDs, laser diodes and photodiodes in a simplified optical fibre communication systems. The LEDs and laser diodes emit light modulated with a signal. The optical signal is then transmitted through the optical fibre and is received with photodiodes on the destination side. In this type of a system LEDs or laser diodes emit the light directly through the optical fibre and therefore is referred to as direct modulation type systems. But in more recent

systems, the **optical modulators** modulate the light emitted from the laser diodes and then the modulated light is transmitted through the optical fibre [refer to Fig. 53.33].

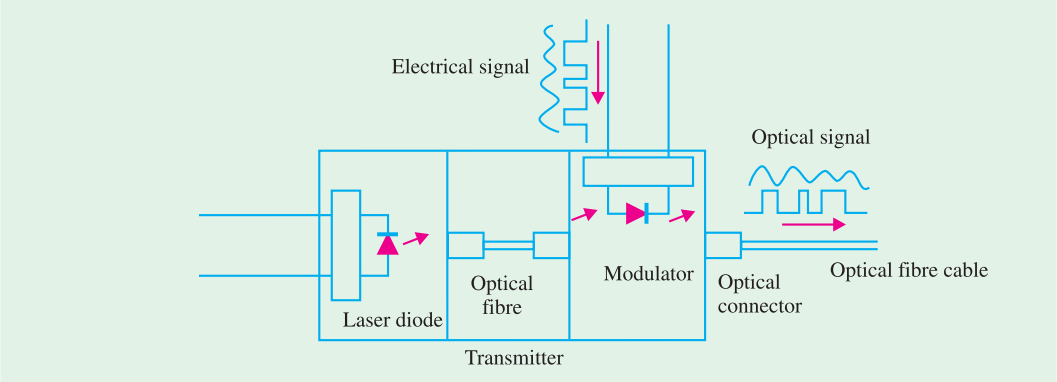


Fig. 53.33

In long-haul systems, **repeaters** (which include photodiodes and laser diodes and electronic circuits) are inserted. In the repeater, the weak optical signal being transmitted through the optical fibre is detected by the photodiode. The detected signal is reformed and amplified by the electronic circuits. The amplified signal is converted again into an optical signal by a laser diode and transmitted again through the optical fibre cable. Fig. 53.34 shows a simple schematic of a repeater.

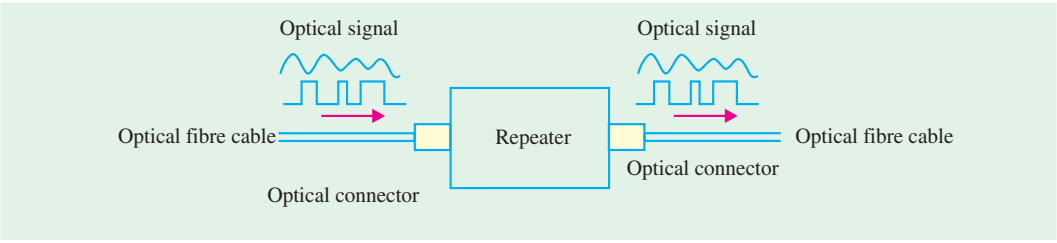


Fig. 53.34

From the modulation point of view, the optical fibre communication systems can be divided into ***digital systems*** and ***analog systems***. Most long-haul and large capacity optical fibre communication systems are digital systems. The analog systems are used for transmitting information over a short distance.

53.21. Optical Fibre Data Links

The use of optical fibre data links has wide spread in the past few decades. Its application ranges from local area networks (LANs) to the computer, digital audio and mobile fields. Several different types of LEDs and laser diodes emitting light at wavelengths ranging from visible to the infrared are used as optical sources. The transmission data rate is a function of transmission distance and varies from application to application. For computer links where the distance varies from 1 m to 100 m, the data transmission rate varies from 1 M bits/s to 100 M bits/s. For local-area-networks used in factory, office and building automation, the data transmission rate varies from 10 K bits/s to 5 M bits/s. In digital audio field, where the distance is below 1 m the data transmission rate varies from 500 bits/s to over 10 M bits/s. Similarly in mobile fields (such as ship, aircraft, train and automotive applications) where the distance could vary from 1 m to 100 m, the data transmission rate varies from 1 K bits/s to 1 M bits/s.

1. Optical fibre local area networks. The optical fibre local area networks (LANs) are similar to the public communication systems. Some of their advantages over the systems using metallic cables are : **(1)** high transmission capacity and bit rate and **(2)** longer transmission distance. However, the range of LANs is restricted. They are more commonly used within factories, offices, buildings etc. Computers, printers, facsimile machines and other office equipment are connected with each other by optical fibre cables as shown in Fig. 53.35.

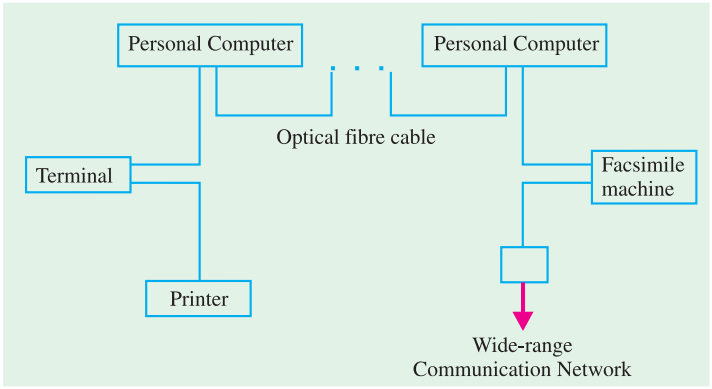


Fig. 53.35

The LEDs and laser diodes are used to transmit data through the optical fibre cable and photodiodes are used to receive data. The different types of equipment connected in the LAN could be one of the following two types : (1) An optical ethernet having a radial-shape network as shown in Fig. 53.36 (*a*) or (2) a fibre-distributed data interface (FDDI) having a ring-shape network as shown in Fig. 53.36 (*b*).

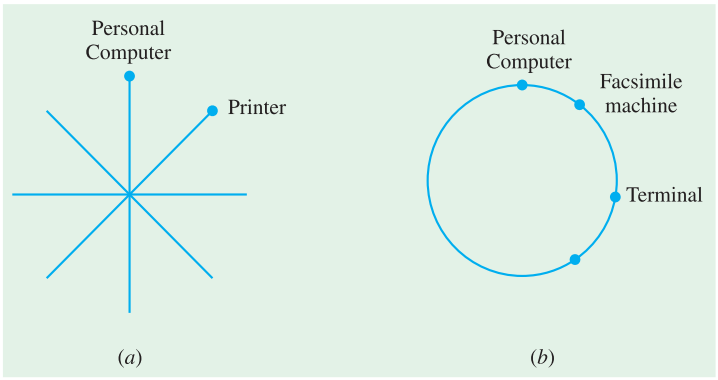


Fig. 53.36

2. Digital audio field. Fig. 53.37 shows an example of a data link in digital audio field. As seen, the optical fibre cable is used to connect compact disk (CD) player, laser disk (LD) player, digital audio tape (DAT) and tuner with the amplifier and speaker. The connection between the amplifier and everything except DAT is unidirectional. The audio digital signals from CD, LD player, DAT and tuner are converted into optical signals by LEDs or laser diodes at one end of the fibre optic cable and then transmitted through the cable to the opposite end. At the opposite end, the signals are received by photodiodes and converted into an electrical signal for amplification and finally speaker for reproduction to a sound.

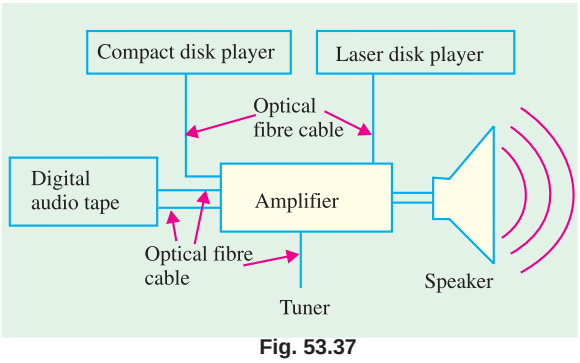


Fig. 53.37

3. Mobile fields. The optical data links are very suitable in mobile fields such as ship, aircraft, train, automotive etc. The reason is that optical data links are very compact, and light in weight than metallic data links. In addition to this, the optical data links are not subjected to noise induced by electromagnetic induction.

OBJECTIVE TESTS – 53

1. LEDs are commonly fabricated from gallium compounds like gallium arsenide and gallium phosphide because they
(*a*) are cheap
(*b*) are easily available
(*c*) emit more heat
(*d*) emit more light.

2. A LED is basically a ..... P-N junction.
(*a*) forward-biased
(*b*) reverse-biased
(*c*) lightly-doped
(*d*) heavily-doped.

3. As compared to a LED display, the distinct advantage of an LCD display is that it requires
(*a*) no illumination
(*b*) extremely low power
(*c*) no forward-bias
(*d*) a solid crystal

4. Before illuminating a P-N junction photodiode, it has to be
(*a*) reverse-biased
(*b*) forward-biased
(*c*) switched ON
(*d*) switched OFF.

5. In a photoconductive cell, the resistance of the semiconductor material varies ..... with the intensity of incident light.
(*a*) directly
(*b*) inversely
(*c*) exponentially
(*d*) logarithmically.

6. A photoconductive cell is known as ..... cell.
(*a*) phototransistor
(*b*) photoresistor
(*c*) photovoltaic
(*d*) both (*a*) and (*b*).

7. A phototransistor excels a photodiode in the matter of
(*a*) faster switching
(*b*) greater sensitivity
(*c*) higher current capacity
(*d*) both (*a*) and (*b*)
(*e*) both (*b*) and (*c*).

8. A photodarlington comprises of
(*a*) a phototransistor
(*b*) a transistor
(*c*) a photodiode
(*d*) both (*a*) and (*b*).

9. A solar cell operates on the principle of
(*a*) diffusion
(*b*) recombination
(*c*) photo voltaic action
(*d*) carrier flow.

10. Solar cells are used as source of power in earth satellites because they have
(*a*) very high efficiency
(*b*) unlimited life
(*c*) higher power capacity per weight
(*d*) both (*b*) and (*c*)
(*e*) both (*a*) and (*b*).

11. The device possessing the highest sensitivity is a
(*a*) photo conductive cell
(*b*) photovoltaic cell
(*c*) photodiode
(*d*) phototransistor

12. The unique characteristics of LASER light are that it is
(*a*) coherent
(*b*) monochromatic
(*c*) collimated
(*d*) all of the above

13. The LASCR operates like a
(*a*) latch (*b*) LED
(*c*) photodiode (*d*) phototransistor.

14. Optical couplers are designed to ..... one circuit from another.
(*a*) control (*b*) isolate
(*c*) disconnect (*d*) protect.

15. The main purpose of using optical isolators is to provide protection to devices from
(*a*) high-voltage transients
(*b*) surge voltages
(*c*) low-level noise
(*d*) all of the above.

16. A LED emits visible light when its .....
(*a*) P-N junction is reverse-biased
(*b*) depletion region widens
(*c*) holes and electrons recombine
(*d*) P-N junction becomes hot.

17. In LED, light is emitted because
(*a*) recombination of charge carriers takes place
(*b*) diode gets heated up
(*c*) light falling on the diode gets amplified
(*d*) light gets reflected due to lens action.

18. GaAs, LEDs emit radiation in the
(*a*) ultraviolet region
(*b*) violet-blue green range of the visible region
(*c*) visible region
(*d*) infra-red region

19. Phototransistors respond much like a conventional transistor except that, in their case, light energy is used to .....
(*a*) alter leakage current
(*b*) change base voltage
(*c*) switch it ON
(*d*) alter emitter current.

ANSWERS

1. *(d)* **2.** *(a)* **3.** *b* **4.** *(a)* **5.** *(b)* **6.** *(d)* **7.** *(e)* **8.** *(d)* **9.** *(c)* **10.** *(d)* **11.** *(d)* **12.** *(d)*
13. *(a)* **14.** *(b)* **15.** *(d)* **16.** *(c)* **17.** *(a)* **18.** *(d)* **19.** *(c)*

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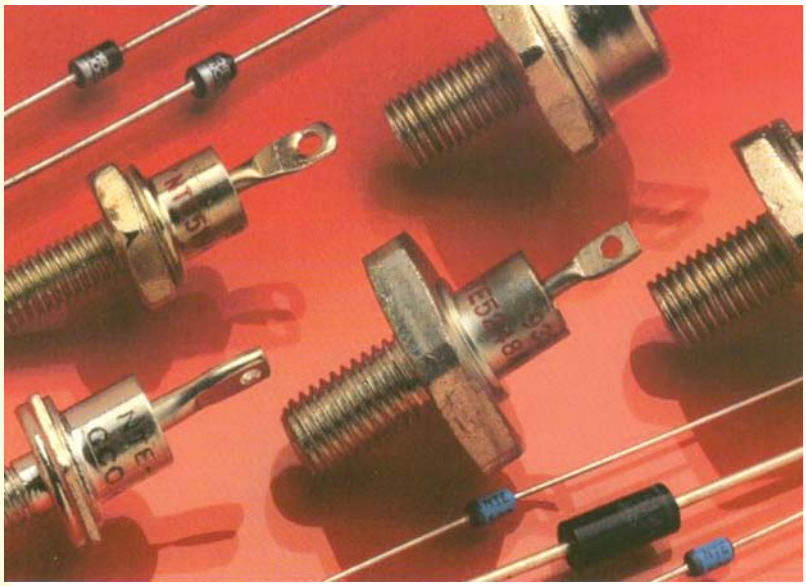
CONTENTS

CHAPTER 54

SPECIAL DIODES

Learning Objectives

- Zener Diode
- Voltage Regulation
- Zener Diode as Peak Clipper
- Meter Protection
- Zener Diode as a Reference Element
- Tunneling Effect
- Tunnel Diode
- Tunnel Diode Oscillator
- Varactor Diode
- PIN Diode
- Schottky Diode
- Step Recovery Diode
- Gunn Diode
- IMPATT Diode



A major application for zener diodes is voltage regulation in dc power supplies. Zener diode maintains a nearly constant dc voltage under the proper operating conditions.

CONTENTS

54.1. Zener Diode

It is a reverse-biased heavily-doped silicon (or germanium) *P-N* junction diode which is operated in the ***breakdown region*** where current is limited by both external resistance and power dissipation of the diode. Silicon is perferred to Ge because of its higher temperature and current capability. As seen from Art. 52.3, when a diode breaks down, both Zener and avalanche effects are present although usually one or the other predominates depending on the value of reverse voltage. At reverse voltages less than 6 V, Zener effect predominates whereas above 6 V, avalanche effect is predominant. Strictly speaking, the first one should be called Zener diode and the second one as avalanche diode but the general practice is to call both types as Zener diodes.

Zener breakdown occurs due to ***breaking of covalent bonds by the strong electric field set up in the depletion region by the reverse voltage.*** It produces an extremely large number of electrons and holes which constitute the reverse saturation current (now called Zener current, I_z) whose value is limited only by the external resistance in the circuit. ***It is independent of the applied voltage.*** Avalanche breakdown occurs at higher reverse voltages when thermally-generated electrons acquire sufficient energy to produce more carriers by collision.

(*a*) V/I Characteristic

A typical characteristic is shown by Fig. 54.1 in the negative quadrant. The forward characteristic is simply that of an ordinary forward-biased junction diode. The important points on the reverse characteristic are :

V_z = Zener breakdown voltage

$I_{z\,min}$ = minimum current to sustain breakdown

$I_{z\,max}$ = maximum Zener current limited by maximum power dissipation.

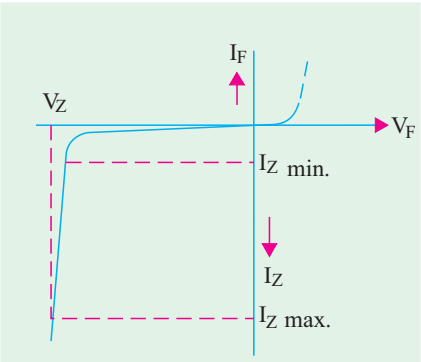


Fig. 54.1

Since its reverse characteristic is not exactly vertical, the diode possesses some resistance called ***Zener dynamic impedance****. However, we will neglect it assuming that the characteristic is truly vertical. In other words, we will assume an ideal Zener diode for which voltage ***does not change once it goes into breakdown.*** It means that V_z remains constant even when I_z increases considerably.

The schematic symbol of a Zener diode and its equivalent circuit are shown in Fig. 54.2 (*a*). The complete equivalent circuit is shown in Fig. 54.2 (*b*) and the approximate one in Fig. 54.2 (*c*) where it looks like a battery of V_z volts.

The schematic symbol of Fig. 54.2 (*a*) is similar to that of a normal diode except that the line representing the cathode is bent at both ends. With a little mental effort, the cathode symbol can be imagined to look like the letter *Z* for Zener.

(*b*) Zener Voltages

Zener diodes are available having Zener voltages of 2.4 V to 200 V. This voltage is temperature dependent. Their power dissipation is given by the product $V_z I_z$... maximum ratings vary from 150 mW to 50 W.

(*c*) Zener Biasing

For proper working of a Zener diode in any circuit, it is essential that it must

1. be reverse-biased;
2. have voltage across it greater than V_z;

* Its value is given by $Z_z = \Delta V_z / \Delta I_z$. It is negiligible as compared to large external resistance connected in the circuit.

3. be in a circuit where current is less than $I_{z\,max}$;

(*d*) Diode Identification

Physically, a Zener diode looks like any other diode and is recognized by its IN number such as IN 750 (10 W power) or IN 4000 (high power). Fig 54.2(*d*) shows a picture of a zener diode with V_z = 4.7V.

(*e*) Uses

Zener diodes find numerous applications in transistor circuitry. Some of their common uses are :

1. as voltage regulators;
2. as a fixed reference voltage in a network for biasing and comparison purposes and for calibrating voltmeters;
3. as peak clippers or voltage limiters;
4. for metre protection against damage from accidental application of excessive voltage;
5. for reshaping a waveform.

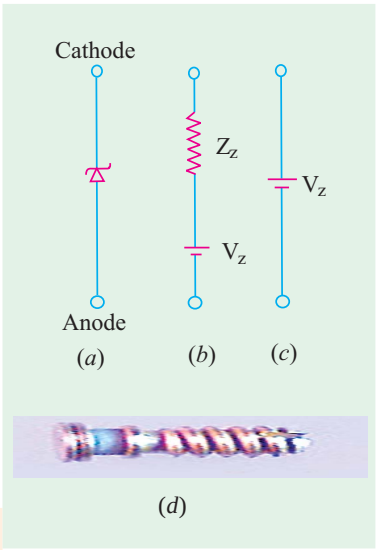


Fig. 54.2

Example 54.1. *Determine whether the ideal Zener diode of Fig. 54.3 is properly biased. Explain why ?*

Solution. Since positive battery terminal is connected to its cathode, the diode is reverse-biased.

Since applied voltage is less than V_z, the diode is not properly voltage-biased.

Example 54.2. *Find out if the Zener diode of Fig. 54.4 is properly-biased. If so, find diode current assuming it to be an ideal one.*

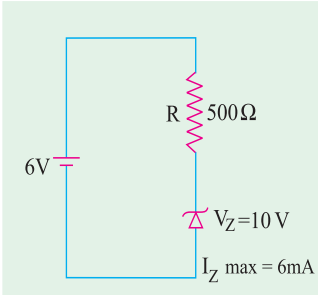


Fig. 54.3

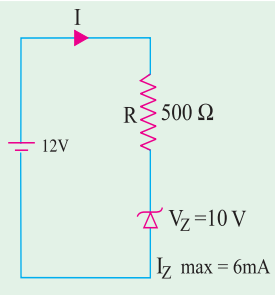


Fig. 54.4

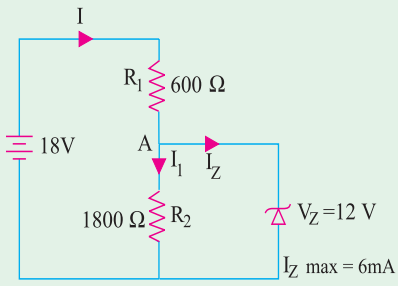


Fig. 54.5

Solution. Polarity-wise, the diode is properly-biased. Since applied voltage is greater than V_z, the diode is properly voltage-biased.

Drop across $R = 12 - 10 = 2$ V $\quad\therefore\quad I = 2/500 =$ **4 mA**

Since this current is less than the maximum diode current, the diode is properly-biased according to the criteria laid down in Art. 54.1 (*c*).

Example. 54.3. *Determine if the Zener diode of Fig. 54.5 is biased properly. If so, find I_z and the power dissipated by the diode.*

Solution. Since its anode is connected to the negative battery terminal, the Zener diode is correctly reverse-biased.

Now, $V_{AB} = V_Z = 12$ V. Hence, drop across $R_1 = 18 - 12 = 6$ V

$\therefore$ $I = 6/600 = 0.01$ A $= 10$ mA

$I_1 = 12/1800 = 6.7 \times 10^{-3}$ A $= 6.7$ mA

$I_z = I - I_1 = 10 - 6.7 = 3.3$ mA

Since I_z is less than $I_{z\,max}$, the diode is properly-biased in every respect as per Art 4.1. Power dissipated = $V_z I_z = 12 \times 3.3 =$ **39.6 mW.**

Example. 54.4. *Calculate the value of E_0 in the given circuit of Fig. 54.6. Given E_{in} = 6 V and 20 V.* **(Electrical Engg. II, Indore Univ.)**

Solution. When E_{in} is 6 V, the diode acts like an open circuit. It is so because 6 V is not enough to cause Zener break-down which will take place only when E_{in} exceeds 10 V. Hence, in this case, $E_0 = 0$.

When E_{in} = 20 V, breakdown occurs but voltage across diode remains constant at 10 V. The balance (20 – 10) = 10 V appears across 100 Ω resistor. Hence, E_0 = drop across R = **10 V.**

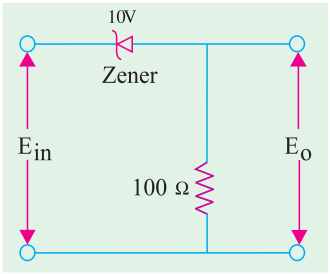


Fig. 54.6

54.2. Voltage Regulation

It is a measure of a circuit's ability to maintain a constant output voltage even when either input voltage or load current varies. A Zener diode, when working in the breakdown region, can serve as a voltage regulator. In Fig. 54.7, V_{in} is the input dc voltage whose variations are to be regulated. The Zener diode is reverse-connected across V_{in}. When p.d. across the diode is greater than V_z, it conducts and draws relatively large current through the series resistance R.

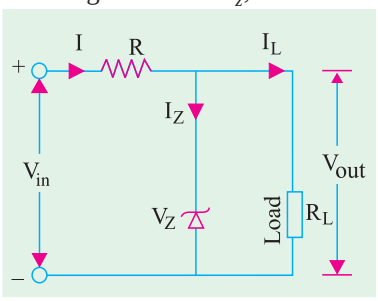


Fig. 54.7

The load resistance R_L across which a constant voltage V_{out} is required, is connected in parallel with the diode. The total current I passing through R equals the sum of diode current and load current *i.e.* $I = I_z + I_L$.

It will be seen that under all conditions $V_{out} = V_z$. Hence, $V_{in} = IR + V_{out} = IR + V_z$.

Case 1. Suppose R_L is kept fixed but supply voltage V_{in} is increased slightly. It will increase I. This increases in I will be absorbed by the Zener diode without affecting I_L. The increase in V_{in} will be dropped across R thereby keeping V_{out} constant. Conversely if supply voltage V_{in} falls, the diode takes a smaller current and voltage drop across R is reduced, thus againt keeping V_{out} constant. Hence, when V_{in} changes, I and IR drop change in such a way as to keep V_{out} (= V_z) constant.

Case 2. In this case, V_{in} is fixed but I_z is changed. When I_L increases, diode current I_z decreases thereby keeping I and hence IR drop constant. In this way, V_{out} remains unaffected.

Should I_L decrease, I_z would increase in order to keep I and hence IR drop constant. Again, V_{out} would remain unchanged because

$$V_{out} = V_{in} - IR = V_{in} - (I_z + I_L)R$$

Incidentally, it may be noted that $R = (V_{in} - V_{out}) / (I_z + I_L)$

It may also be noted that when diode current reaches its maximum value, I_L becomes zero. In that case

$$R = \frac{V_{in} - V_{out}}{I_{Zmax}}$$

In Fig. 54.7, only one reference voltage level is available. Fig. 54.8 shows the circuits for establishing two reference levels. Here, two diodes having different Zener voltages have been connected in series.

Example 54.5. *Calculate the battery current I, I_z and I_L in the circuit of Fig. 54.9. How will these values be affected if source voltage increases to 70 V? Neglect Zener resistance.*

(Industrial Electronics, Pune Univ.)

Solution. When V_{in} = **40 V**

Now, $V_{AB} = V_z = 10$ V

$\therefore$ $I = 30/3$ K = **10 mA**

$\therefore$ drop across 3K series (or line) resistor is = 40 – 10 = 30 V

$I_L = V_z/R_L = V_{AB}/R_L = 10/2$ K = **5 mA**

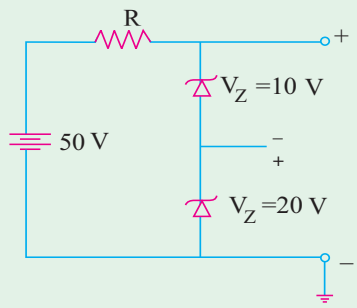


Fig. 54.8

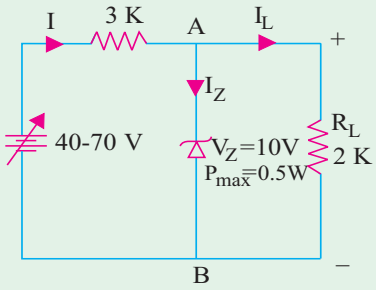


Fig. 54.9

$\therefore$ $I_z = I - I_L = 10 - 5 = \mathbf{5\ mA}$

Now, $P_{max} = V_z.I_{z\,(max)}$ or $0.5 = 10 \times I_{z\,(max)}$

or $I_{z\,(max)} = 0.5/10 = 0.05$ A = 50 mA

Obviously, diode current of 5 mA is very much within the current range of the diode.

(*b*) When, $\mathbf{V_{in} = 70\ V}$

Drop across $R = 70 - 10 = 60$ V $\therefore$ $I = 60/3$ K = **20 mA**

$I_L =$ **5 mA** (as before); $I_z = I - I_L = 20 - 5 =$ **15 mA**

Example 54.6. *Using the ideal Zener approximations, find current through the diode of Fig. 54.10 when load resistance R_L is **(i)** 30 K **(ii)** 5 K **(iii)** 3 K.* **(Electronics, Madurai Kamraj Univ.)**

Solution. (*i*) $\mathbf{R_L = 30\ K}$

$V_{AB} = V_z = 30$ V ; drop across $R = 60 - 30 = 30$ V

$\therefore$ $I = 30/3$ K = 10 mA ; $I_L = V_{AB}/R_L = 30/30$ K = 1 mA

$\therefore$ $I_z = I - I_L = 10 - 1 = 9$ mA

(*ii*) When $\mathbf{R_L = 5\ K}$

$I = 10$ mA — as before ; $I_L = 30/5\ K = 6$ mA $I_z = 10 - 6 =$ **4 mA**

(*iii*) When $\mathbf{R_L = 3K}$

$I = 10$ mA — as before ; $I_L = 30/3$ K = 10 mA $I_z = I - I_L = 10 - 10 = 0$

In this case, it is obvious that the diode is just on the ***verge of coming out of breakdown region***. If R_L is reduced further, the diode will come out of breakdown region and would no longer act as a voltage regulator.

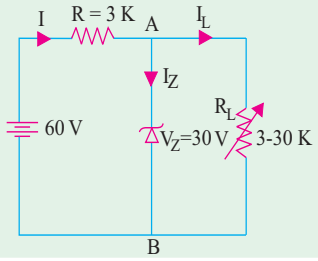


Fig. 54.10

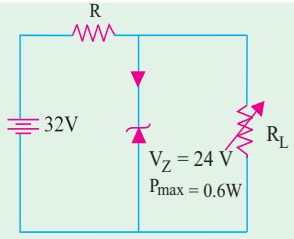


Fig. 54.11

Example 54.7. *A 24-V, 600-mW Zener diode is to be used for providing a 24-V stabilized supply to a variable load (Fig. 54.11). If input voltage is 32 V, calculate the **(i)** series resistance R required **(ii)** diode current when $R_L = 1200\ \Omega$.* **(Applied Electronics, Punjab Univ. 1991)**

Solution. (*i*) $V_z I_z(max) = 600$ mW ; $I_{z(max)} = 600/24 = 25$ mA

$\therefore$ $R = \dfrac{V_{in} - V_{out}}{I_{z\,(max)}} = \dfrac{32 - 24}{25 \times 10^{-3}} =$ **320 Ω**

(*ii*) When $R_L = 1200\ \Omega$, $I_L = V_z/R_L = 24/1200 = 20$ mA; $I_z = 25 - 20 =$ **5 mA**

54.3. Zener Diode as Peak Clipper

Use of Zener diodes in wave-shaping circuits is illustrated in Fig. 54.12. The two similar diodes D_1 and D_2 have been joined back-to-back across the input sine wave voltage of peak value ± 25 V. Both have V_z = 20 V. As seen, the output is a semi-square wave with a peak value of ± 20 V.

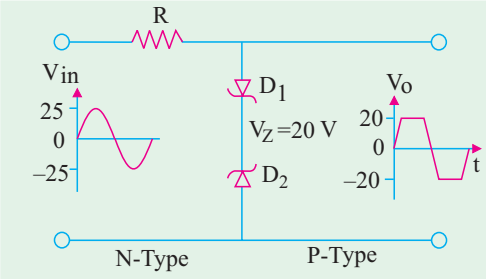


Fig. 54.12

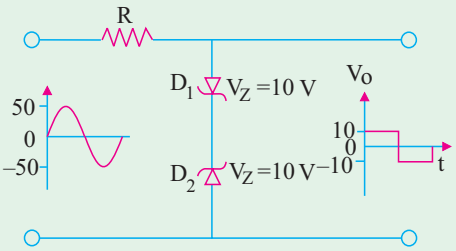


Fig. 54.13

It is well-known that a Zener diode acts like *a* 'short' (or very low resistance) in the forward direction and an 'open' in the reverse direction till it goes into breakdown at V_z. During positive input half-cycle, D_1 is shorted (being forward-biased) but D_2 acts like an open upto 20 V. Thereafter, it goes into breakdown and holds the output voltage constant till input voltage falls below 20 V in the later part of the half-cycle. At that point, D_2 comes out of the breakdown and again acts like an open across which the entire input voltage is dropped.

During the negative input half-cycle, roles of D_1 and D_2 are reversed. As a result, the output wave is clipped on both peaks as shown in Fig. 54.12.

If we increase the peak value of the input signal voltage and use Zener diodes of lesser V_z value, we can get an almost square output voltage wave from a sinusoidal input wave as shown in Fig. 54.13.

54.4. Meter Protection

Zener diodes are frequently used in volt-ohm-milliammeters (VOM) for protecting meter movement against burn-out from accidental overloads. If VOM is set to its 2.5 V range and the test leads are accidentally connected to a 25 V circuit, an unprotected meter will be burned out or at least get severely damaged.

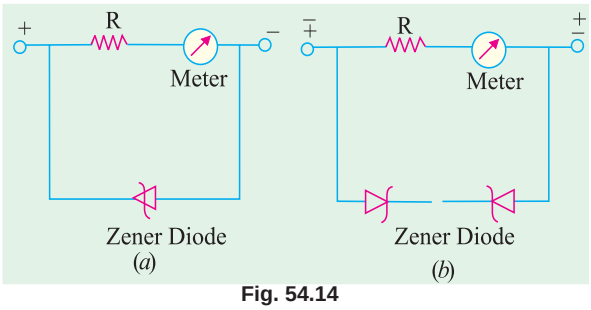


Fig. 54.14

This hazard can be avoided by connecting a Zener diode in parallel with the meter as shown in Fig. 54.14 (*a*). In the event of an accidental overload, most of the current will pass through the diode. Two Zener diodes connected as shown in Fig. 54.14 (*b*) can provide overload protection regardless of the applied polarity.

54.5. Zener Diode as a Reference Element

In many electronic circuits, it is desirable to maintain a constant voltage between two points and use it as a reference voltage for comparing other voltages against it. The difference between the two voltages is amplified and then used for performing some control function. This type of arrangement is used for power supply voltage regulator circuits, measurement circuits and servomechanism circuits. The constant-voltage characteristic in its breakdown region makes a Zener diode desirable for this application. Fig. 54.15 shows a circuit in which Zener diode is used as a reference element. The reference voltage equals the Zener breakdown voltage. The value of R is so chosen that the diode operates well within its breakdown region. The difference ($E_{in} - E_{ref}$) gives the control output.

54.6. Tunneling Effect

In a normally-doped P-N junction, the depletion layer is relatively wide and a potential barrier exists across the junction. The charge carriers on either side of the junction cannot cross over unless they possess sufficient energy to overcome this barrier (0.3 V for Ge and 0.7 V for Si). As is well-known, width of the depletion region depends directly on the doping density of the semiconductor. If a P-N junction is doped very heavily (1000 times or more)*, its depletion layer

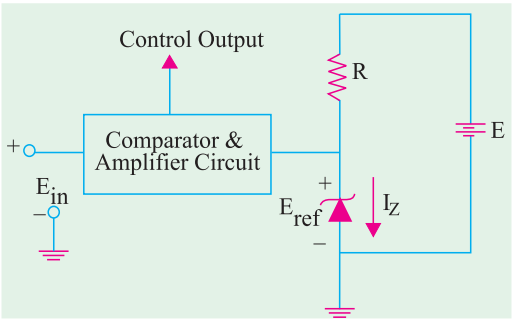


Fig. 54.15

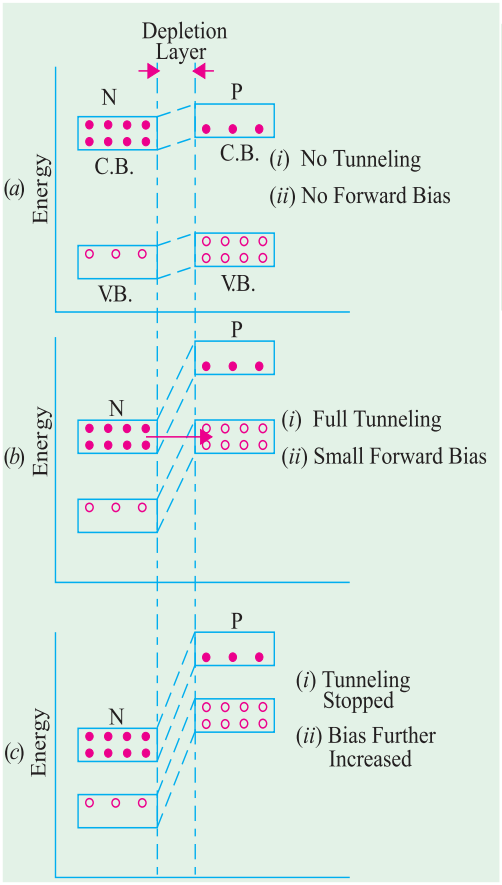


Fig. 54.17

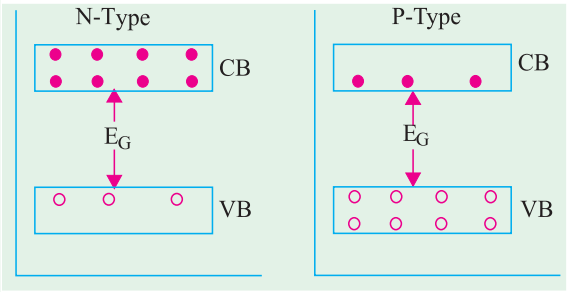


Fig. 54.16

becomes extremely thin (about 0.00001 mm). It is found that under such conditions, many carriers can 'punch through' the junction with the speed of light even when they do not possess enough energy to overcome the potential barrier. Consequently, large forward current is produced even when the applied bias is much less than 0.3 V.

This conduction mechanism in which charge carriers (possessing very little energy) bore through a barrier directly instead of climbing over it is called **tunneling.**

Explanation

Energy band diagrams (EBD) of *N*-type and *P*-type semiconductor materials can be used to explain this tunneling phenomenon. Fig. 54.16 shows the energy band diagram of the two types of silicon separately. As explained earlier (Art. 51.21), in the *N*-type semiconductor, there is increased concentration of electrons in the conduction band. It would be further increased under heavy doping. Similarly, in a *P*-type material, there is increased concentration of holes in the valence band for similar reasons.

(*a*) No Forward Bias

When the *N*-type and *P*-type materials are joined, the EBD under no-bias conditiion becomes as shown in Fig. 54.17 (*a*). The junction barrier produces only a rough alignment of the two materials and their respective valence and conduction bands. As seen, the depletion region between the two is extremely narrow due to very heavy doping on both sides of the junction. The potential hill is also increased as shown.

* Much more than even for a Zener diode.

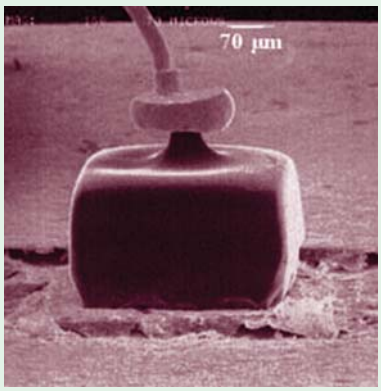
(*b*) Small Forward Bias

When a very small forward voltage (≅ 0.1 V) is applied, the EBDs become as shown in Fig. 54.17 (*b*). Due to the downward movement of the *N*-region, the *P*-region valence band becomes exactly aligned with the *N*-region conduction band. At this stage, electrons tunnel through the thin depletion layer with the velocity of light thereby giving rise to a large current called peak current I_p.

(*c*) Large Forward Bias

When the forward bias is increased further, the two bands get out of alignment as shown in Fig. 54.17 (*c*). Hence, tunneling of electrons stops thereby decreasing the current. Since current decreases with increase in applied voltage (*i.e.* *dV/dI* is negative), the junction is said to possess negative resistance at this stage. This resistance increases throughout the negative region.

However, it is found that when applied forward voltage is increased still further, the current starts increasing once again as in a normal junction diode.



Discrete commercial *Si* tunnel diode

54.7. Tunnel Diode

This diode was first introduced by Dr. Leo Easki in 1958.

(*a*) Construction

It is a high-conductivity two-terminal *P-N* junction diode having doping density about 1000 times higher as compared to an ordinary junction diode. This heavy doping produces following three unusual effects :

1. Firstly, it reduces the width of the depletion layer to an extremely small value (about 0.00001 mm).
2. Secondly, it reduces the reverse breakdown voltage to a very small value (approaching zero) with the result that the diode appears to be broken down for any reverse voltage.

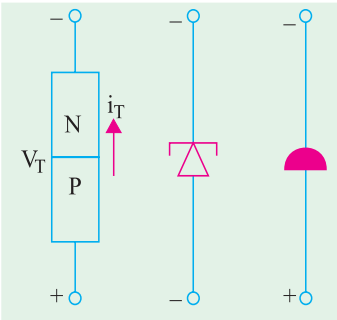


Fig. 54.18

3. Thirdly, it produces a negative resistance section on the *V/I* characteristic of the diode.

It is called a ***tunnel*** diode because due to its extremely thin depletion layer, electrons are able to ***tunnel through*** the potential barrier at relatively low forward bias voltage (less than 0.05 V). Such diodes are usually fabricated from germanium, gallium-arsenide (GaAs) and gallium antimonide (GaSb).

The commonly-used schematic symbols for the diode are shown in Fig. 54.18. It should be handled with caution because being a low-power device, it can be easily damaged by heat and static electricity.

(*b*) V/I Characteristic

It is shown in Fig. 54.19. As seen, forward bias produces immediate conduction *i.e.* as soon as forward bias is applied, significant current is produced. The current quickly rises to its peak value I_p when the applied forward voltage reaches a value V_p (point *A*). When forward voltage is increased further, diode current starts decreasing till it achieves its minimum value called valley current I_v corresponding to valley voltage V_v (point *B*). For voltages greater than V_v, current starts increasing again as in any ordinary junction diode.

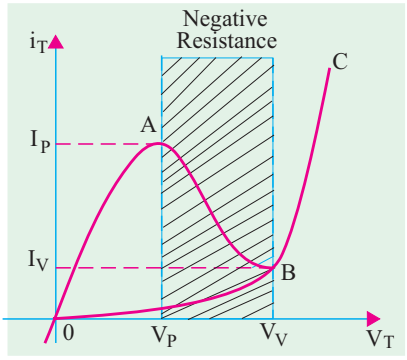


Fig. 54.19

As seen from Fig. 54.19, between the peak point A and valley point B, current decreases with increase in the applied voltage. In other words, tunnel diode possesses negative resistance (— R_N) in this region. In fact, this constitutes the most useful property of the diode. Instead of absorbing power, a negative resistance produces power. By offsetting losses in L and C components of a tank circuit, such a negative resistance permits oscillations. Hence, a tunnel diode is used as a very high frequency oscillator.

Another point worth noting is that this resistance increases as we go from point A to B because as voltage is increased, current keeps decreasing which means that diode negative resistance keeps increasing.

(*c*) Tunneling Theory

At zero forward bias, the energy levels of conduction electrons in N-region of the junction are slightly out of alignment with the energy levels of holes in the P-region. As the forward voltage is slightly increased, electron levels start getting aligned with the hole levels on the other side of junction thus permitting some electrons to cross over. This kind of junction crossing is called tunneling.

As voltage is increased to peak voltage (V_P), all conduction band electrons in the N-region are able to cross over to the valence band in the P-region because the two bands are exactly aligned. Hence, maximum current (called peak current I_P) flows in the circuit.

After V_P, as the applied voltage is increased, current starts decreasing because the two bands start gradually getting out of alignment. It reaches minimum value (called valley current) when the two are totally out of alignment at a forward bias of V_V (valley voltage).

For voltages greater than V_V, current starts increasing again exactly as it does in the case of an ordinary P-N junction diode.

Tunneling is much faster than normal crossing which enables a tunnel diode to switch ON and OFF much faster than an ordinary diode. That is why a tunnel diode is extensively used in special applications requiring very fast switching speeds like high-speed computer memories and high frequency oscillators etc.

(*d*) Diode Parameters

(*i*) Negative Resistance ($-R_N$). It is the resistance offered by the diode within the negative-resistance section of its characteristic (shown shaded in Fig. 54.19). It equals the reciprocal of the slope of the characteristic in this region.

It may also be found from the following relation $R_N = -dV/dI$.

Its value depends on the semiconductor material used (varying from – 10 Ω to – 200 Ω).

(*ii*) I_P/I_V Ratio

It is almost as important a factor (particularly for computer applications) as the negative resistance of the diode.

Silicon diodes have a low I_P/I_V ratio of 3 : 1 and their negative resistance can be approximated from $R_N = -200/I_P$. Such diodes are used mainly for switches operating in high ambient temperatures.

Germanium diodes have an I_P/I_V ratio of 6 : 1 and negative resistance formula $R_N = -120/I_P$. GaAs diodes (used exclusively in oscillators) have an I_P/I_V ratio of about 10 : 1 and a negative resistance nearly equal to that of silicon diodes.

The minimum I_P/I_V ratio for GaSb diode is about 12 : 1 and has the lowest resistance of all given by $R_N = -60/I_P$. Hence, such diodes have the lowest noise.

(*e*) Equivalent Circuit

The equivalent circuit of a tunnel diode is shown in Fig. 54.20. The capacitance C is the junction diffusion capacitance (1 to 10 pF) and ($-R_N$) is the negative resistance. The inductor L_S is due mainly to the terminal leads (0.1 to 4 nH). The resistance R_S is due to the leads, ohmic contact and semiconductor materials (1 – 5 Ω). These factors limit the frequency at which the diode may be used. They are also important in determining the switching-speed limit.

(*f*) Biasing the Diode

The tunnel diode has to be biased from some dc source for fixing its *Q*-point on its characteristic when used as an amplifier or as anoscillator and modulator.

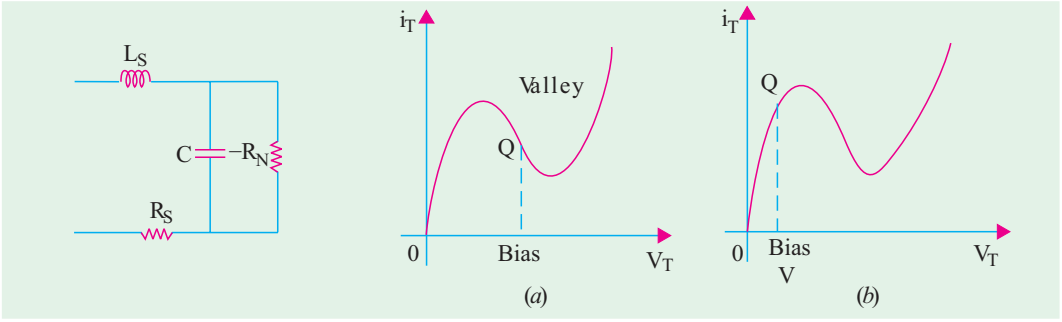


Fig. 54.20 Fig. 54.21

The diode is usually biased in the negative region [Fig. 54.21 (*a*)]. In mixer and relaxation oscillator applications, it is biased in the positive-resistance region nearest zero [Fig. 54.21 (*b*)].

(*g*) Applications

Tunnel diode is commonly used for the following purposes :

1. as an ultrahigh-speed switch-due to tunneling mechanism which essentially takes place at the speed of light. It has a switching time of the order of nanoseconds or even picoseconds;
2. as logic memory storage device – due to triple-valued feature of its curve for current.
3. as microwave oscillator at a frequency of about 10 GHz – due to its extremely small capacitance and inductance and negative resistance.
4. in relaxation oscillator circuits – due to its negative resistance. In this respect, it is very similar to the unijunction transistor.

(*h*) Advantages and Disadvantages

The advantages of a tunnel diode are :

1. low noise,
2. ease of operation,
3. high speed,
4. low power,
5. Insensitivity to nuclear radiations

The disadvantages are :

1. the voltage range over which it can be operated properly is 1 V or less;
2. being a two-terminal device, it provides no isolation between the input and output circuits.

54.8. Tunnel Diode Oscillator

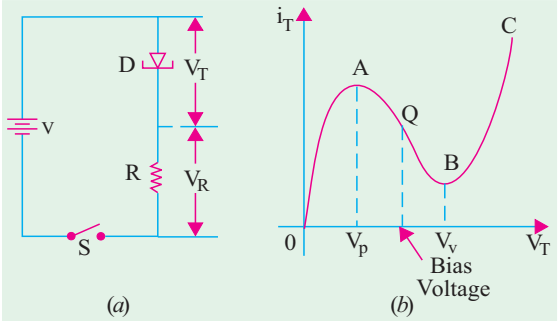


Fig. 54.22

The basic job of an oscillator is to convert dc power into ac power. Ordinarily, we do not expect an ac signal from a circuit which has no input ac source. But the circuit shown in Fig. 54.22 does exactly that as explained below.

The value of *R* is so selected as to bias the diode *D* in the negative-resistance region *A* –*B*. The working or quiescent point *Q* is almost at the centre of the curve *A* – *B*. When *S* is closed, the current immediately rises to a value determined by *R* and the diode resistance which are in series. The applied voltage *V* divides across *D* and *R* according to the ratio of their resistances.

However, as V_T exceeds V_P, diode is driven into the negative area and its resistance starts to increase (Art 4.7). Hence, V_T increases further till it becomes equal to valley voltage V_V (point *B*). At this point, further increase in V_T drives the diode into the positive- resistance region *BC* [Fig. 54.22

(b)]. The resulting increase in current now increases V_R but correspondingly decreases V_T, thereby bringing the diode back into the negative-resistance region. This decrease in V_T increases the circuit current till point A is reached when V_T equals V_P.

It describes one cycle of operation. In this way, the circuit will continue to oscillate back and forth through the negative-resistance region *i.e.* between points A and B on its characteristic. Its output across R is like a sine wave.

Fig. 54.23 shows a practical circuit drawn in two slightly different ways. Here, R_2 sets the proper bias level for the diode whereas R_1 (in parallel with the LC tank circuit) sets proper current level for it. The capacitor C_C is the coupling capacitor. As the switch S is closed, the diode is set into oscillations whose frequency equals the resonant frequency of the tank.

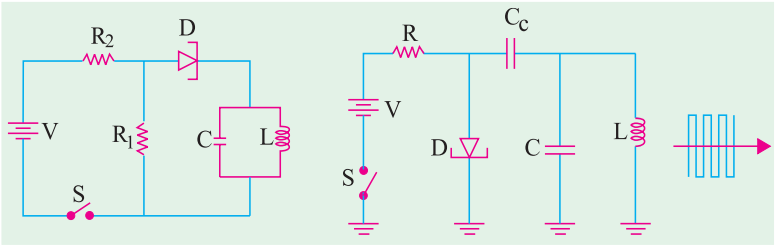


Fig. 54.23

54.9. Varactor Diode

The varactor diode is a semiconductor, voltage-dependent variable capacitor alternatively known as varicap or voltacap or voltage-variable capacitor (VVC) diode. Basically, it is just a reverse-biased junction diode whose mode of operation depends on its transition capacitance (C_T). As explained earlier in Art. 52.4, reverse-biased junctions behave like capacitors whose capacitance is $\propto 1/V_R^n$ where n varies from 1/3 to 1/2. As reverse voltage V_R is increased, depletion layer widens thereby decreasing the junction capacitance. Hence, we can change diode capacitance by simply changing V_R. Silicon diodes which are optimised for this variable capacitance effect are called varactors.

The picture, schematic symbol and a simple equivalent circuit for a varactor are shown in Fig. 54.24.

Varactors may be of two types as shown in Fig. 54.25. The doping profile of the abrupt-junction diode is shown in Fig. 54.25 (a) and that of the hyperabrupt-junction diode in Fig. 54.25 (b). The abrupt-junction diode has uniform doping and a capacitive tuning ratio (TR) of 4 : 1. For example, if its maximum transition capacitance is 100 pF and minimum 25 pF, then its TR is 4 : 1 which is not enough to tune a broadcast receiver over its entire frequency range of 550 to 1050 kHz.



Varactor diode

The hyperabrupt-junction diode has highest impurity concentration near the junction. It results in narrower depletion layer and larger capacitance. Also, changes in V_R produce larger capacitance changes. Such a diode has a tuning range of 10 : 1 enough to tune a broadcast receiver through its frequency range of nearly 3 : 1.

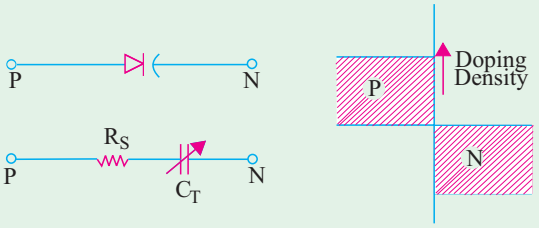


Fig. 54.24

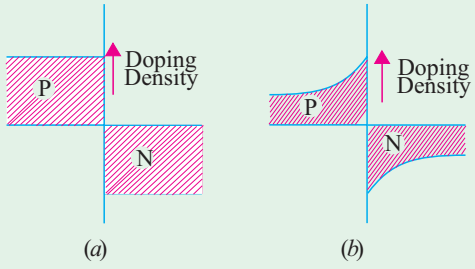


Fig. 54.25

Applications

Since the junction capacitance of a varactor is in the pF range, it is suitable for use in high-frequency circuits. Its main applications are as

1. automatic frequency control device,
2. FM modulator,
3. adjustable band-pass filter,
4. Parametric amplifier.

54.10. PIN Diode

(*a*) Construction

It is composed of three sections. These are the usual *P* and *N*-regions but sandwiched between them is an intrinsic layer or *I*-layer of pure silicon (Fig. 54.26). Being intrinsic (or undoped) layer, it offers relatively high resistance. This high-resistance region gives it two advantages as compared to an ordinary *P*-*N* diode. The advantages are :

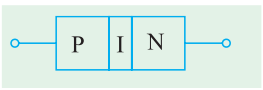


Fig. 54.26



Pin diode

1. decrease in capacitance C_{pn} because capacitance is inversely proportional to the seperation of *P*-and *N*-regions. It allows the diode a faster response time. Hence, PIN diodes are used at high frequencies (more than 300 MHz);
2. possibility of greater electric field between the *P*-and *N*-junctions. It enhances the electron-hole pair generation thereby enabling PIN diode to process even very weak input signals.

(*b*) Diode Resistance

1. When forward-biased, it offers a variable resitance $r_{ac} \cong 50/I$ where *I* is the dc current in mA (Art. 52.2). For large dc currents, it would look like a *short*.
2. When reverse-biased, it looks like an 'open' *i.e.* it offers infinite resistance in the reverse direction.

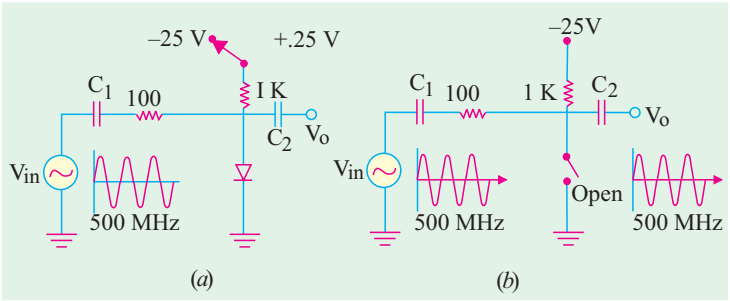


Fig. 54.27

(*c*) Operation

(*i*) High Frequency Switching. Its use in electronic high frequency switching is illustrated in Fig. 54.27.

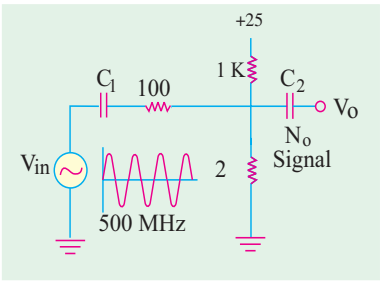


Fig. 54.28

When the diode is reverse-biased, it looks like an 'open' as shown in Fig. 54.27 (*b*). The 500-MHz input signal voltage divides across the series-connected 100 Ω resistance and the diode in proportion to their resistances. Since the diode has infinite resistance (being open), the entire input signal appears across it. Hence, the whole input signal passes out *via* coupling capacitor C_2 without any attenuation (or loss). When the diode is forward-biased by the + 25 V dc source, $I = 25/1\ K = 25$ mA. Hence, diode resistance $r_{ac} = 50/25 = 2\ \Omega$ as shown by its equivalent circuit in Fig. 54.28. Now, almost all the input signal voltage drops across 100 Ω resistance and practically none

across the 2 Ω resistance. Hence, there is hardly any signal output.

In practice instead of mechanically switching the diode-biasing supply from – 25 V to + 25 V, a transistor is used to do this switching operation. In this way, we can turn a very high frequency signal (MHz range) OFF and ON with the speed of a transistor switching circuit.

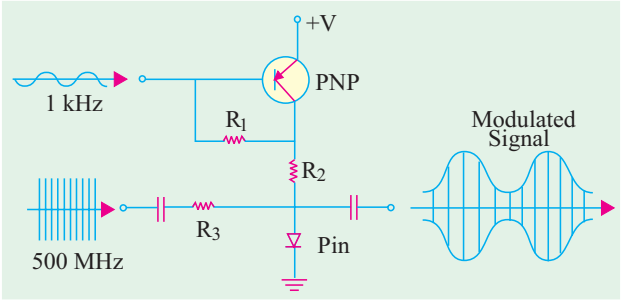


Fig. 54.29

(*ii*) Use as AM Modulator. The way in which the 500-MHz signal is modulated at 1 kHz rate is illustrated in Fig. 54.29.

A 1 kHz signal is fed into a *PNP* transistor where it varies its dc output current at the same rate. This varying dc current is applied as biasing current to the PIN diode as shown in Fig. 54.29. It varies the diode ac resistance as seen by the 500 MHz signal. Hence, the signal is modulated at 1 kHz rate as shown.

(*d*) Applications

(*i*) as a switching diode for signal frequencies upto GHz range;

(*ii*) as an AM modulator of very high frequency signals.

54.11. Schottky Diode

It is also called Schottky barrier diode or ***hot-carrier*** diode. It is mainly used as a rectifier at signal frequencies exceeding 300 MHz. It has more uniform junction region and is more rugged than PIN diode – its main rival.



Schottky barrier diode family expands to meet needs of power related application

(*a*) Construction

It is a metal-semiconductor junction diode with ***no depletion layer***. It uses a metal (like gold, silver, platinum, tungsten etc.) on the side of the junction and usually an *N*-type doped silicon semiconductor on the other side. The diode and its schematic symbol are shown in Fig. 54.30.

(*b*) Operation

When the diode is unbiased, electrons on the *N*-side have lower energy levels than electrons in the metal. Hence, they cannot surmount the junction barrier (called Schottky barrier) for going over to the metal.

When the diode is forward-biased, conduction electrons on *N*-side gain enough energy to cross the junction and enter the metal. Since these electrons plunge into the metal with very large energy, they are commonly called ***'hot-carriers'***. That is why this diode is often referred to as hot-carrier diode.

(*c*) Applications

This diode possesses two unique features as compared to an ordinary *P-N* junction diode :

1. it is a unipolar device because it has electrons as majority carriers on both sides of the junction. An ordianry *P-N* junction diode is a bipolar device because it has both electrons and holes as majority carriers;
2. since no holes are available in metal, there is ***no depletion layer or stored charges*** to worry about. Hence, Schottky diode can switch OFF faster than a bipolar diode.

Because of these qualities, Schottky diode can easily rectify signals of frequencies exceeding 300 MHz. As shown in Fig. 54.31, it can produce an almost perfect half-wave rectified output.

The present maximum current rating of the device is about 100 A. It is commonly used in switching power supplies that operate at frequencies of 20 GHz. Another big advantage of this diode is its low noise figure which is extremely important in communication receivers and radar units etc.

It is also used in clipping and clamping circuits, computer gating, mixing and detecting networks used is communication systems.

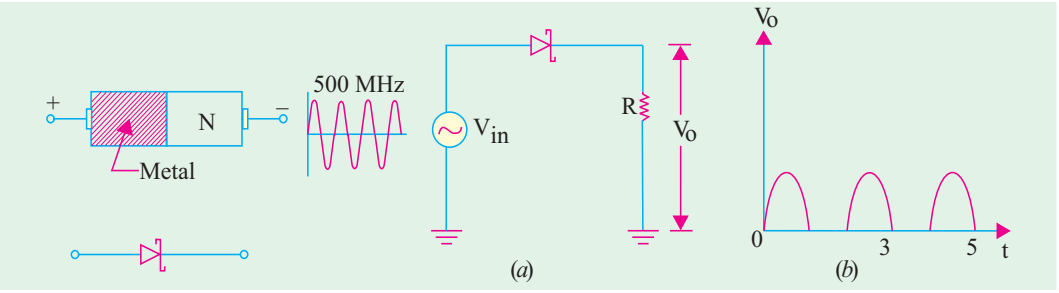


Fig. 54.30 Fig. 54.31

54.12. Step Recovery Diode

It is another type of VVC diode having a ***graded doping profile*** where doping density decreases near the junction as shown in Fig. 54.32. This results in the production of strong electric fields on both sides of the junction.

(*a*) Theory

At low frequencies, an ordinary diode acts as a rectifier. It conducts in the forward direction but not in the reverse direction *i.e.* it recovers immediately from ON state to the OFF state. However, it is found that when driven forward-to-reverse by a high-frequency signal (above a few MHz), the diode does not recover immediately. Even during the negative half-cycle of the input signal when the diode is reverse-biased, it keeps conducting for a while after which the reverse current ceases abruptly in one step. This reverse conduction is due to the fact that charges stored in the depletion layer during the period of forward bias take time to drain away from the junction.

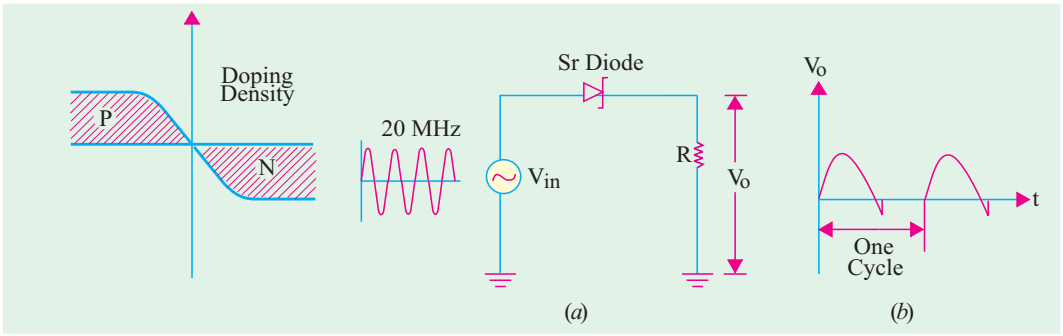


Fig. 54.32 Fig. 54.33

Fig. 54.33 (*a*) shows a step-recovery diode being driven by a 20-MHz signal source. As seen from Fig. 54.33 (*b*), it conducts in the forward direction like any diode. During the reverse half-cycle, we get reverse current due to the draining of the stored charge after which current suddenly drops to zero. It looks as though diode has suddenly ***snapped open*** during the early part of the reverse cycle. That is why it is sometimes called a ***snap diode.***

The step or sudden recovery from reverse current ON to reverse current OFF gives the diode its name.

(*b*) Applications

Its main use is in high-frequency harmonic generator circuits as a frequency multiplier as explained below.

It is found that whenever a waveform has sudden step or transition, it contains all the harmonics of the input signal (*i.e.* multiples of its fundamental frequency). For example, the output waveform of Fig. 54.33 (*b*) contains waves of frequencies 40 MHz, 60 MHz, 80 MHz and so on.

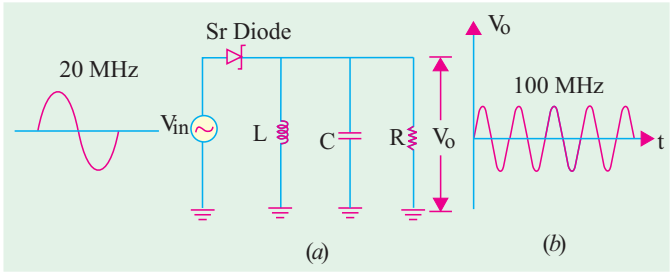


Fig. 54.34

Fig. 54.34 shows how the output of a step recovery diode can be used to drive a tuning circuit which can be made to tune out all harmonics except one *i.e.* fifth in this case (100 MHz). With an input signal of 20 MHz, the step recovery diode generates harmonics of different multiple frequencies listed above. However, the resonant *L*-*C* circuit is tuned to 5th harmonic of f = 100 MHz. Hence, all except this harmonic are filtered out of the circuit. The signal appearing across *R* is almost a pure sine wave with f = 100 MHz as shown separately in Fig. 54.34 (*b*).

Step-recovery diodes are also used in pulse and digital circuits for generating very fast pulses with rise time of less than 1 nanosecond.

54.13. Gunn Diode

It is a negative-resistance microwave device for oscillator applications.

As shown in Fig. 54.35, it consists of a thin slice of *N*-type gallium arsenide sandwiched between two metal conductors. The central section is *N*-gallium arsenide whereas the two outer sections are epitaxially grown from GaAs with increased doping and higher conductivity. As an oscillator, its frequencies range from 5 GHz and 100 mW output upto 35 GHz and 1 mW output. Efficiencies of 3 to 5 per cent are possible at present. Fig. 54.35(*b*) shows the picture of a Gunn diode.

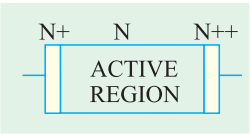


Fig. 54.35(*a*)

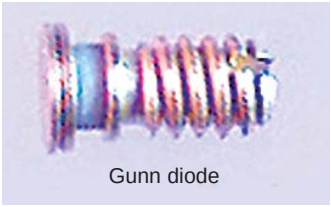


Fig. 54.35(*b*)

54.14. IMPATT Diode

IMPATT stands for ***impact avalanche and transit time*** diode. As the name indicates, it is a microwave diode that utilizes the delay time required for attaining an avalanche condition plus transit time to produce a negative-resistance characteristic. It is used as a microwave oscillator within a frequency range of 10-100 GHz.

Tutorial Problems 54.1

1. Is the ideal Zener diode shown in Fig. 54.36 properly biased ? If not, explain why ? **[No]**
2. Check up if the diode in Fig. 54.37 is biased properly for normal operation. What is the current taken by the diode ? **[Yes : 4 mA]**
3. Check up if Zener diode of Fig. 54.38 is reverse-biased as well as properly voltage-biased. Calculate diode current and power dissipation. **[6 mA, 96 mW]**
4. Using ideal Zener diode approximations, find the minimum and maximum currents through the diode in Fig. 54.39. **[4 mA, 8 mA]**

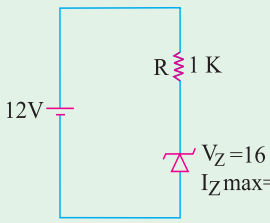


Fig. 54.36

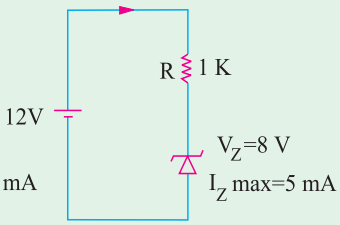


Fig. 54.37

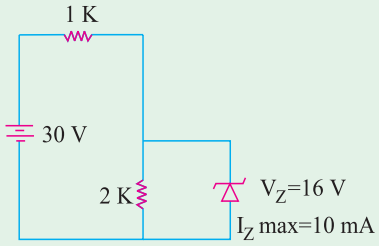


Fig. 54.38

5. A 9 V stabilized voltage supply is required to run a car stereo system from car's 12 V battery. A Zener diode with V_z = 9 V and P_{max} = 0.25 W is used as a voltage regulator as shown in Fig. 54.40.

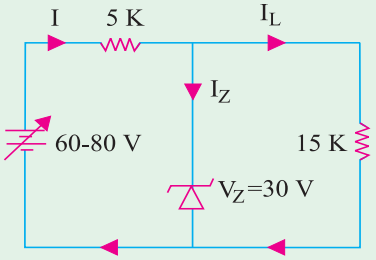


Fig. 54.39

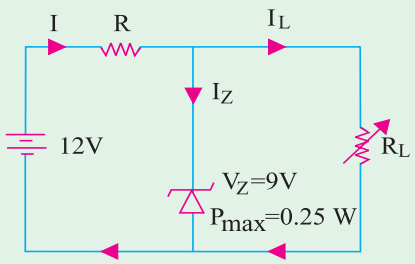


Fig. 54.40

Find the value of the series resistor R. **[120 Ω]**

6. A load of 1kW is connected across a 10 V Zener regulator as shown in Fig. 54. 41. The zener current can vary between 5mA to 55mA while maintaining the voltage constant. Find the minimum and maximum voltage level at input.

(Electronic Devices and Circuits, Nagpur Univ. Summer, 2004)

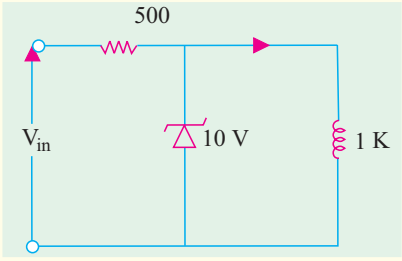


Fig. 54.41

7. A 24V, 600 mW zener diode is used for providing a 24V stabilized supply to a variable load. If the input voltage is 32V, calculate

(*i*) the value of series resistance required.
(*ii*) diode current when the load is 1200 Ω

(Electronics Engg., Bangalore Univ. 2003)

OBJECTIVE TESTS – 54

1. Silicon is preferred for manufacturing Zener diodes because it
(*a*) is relatively cheap
(*b*) needs lower doping level
(*c*) has higher temperature and current capacity
(*d*) has lower break-down voltage.

2. When used in a circuit, a Zener diode is always
(*a*) forward-biased
(*b*) connected in series
(*c*) troubled by overheating
(*d*) reverse-biased.

3. The main reason why electrons can tunnel through a *P-N* junction is that
(*a*) they have high energy
(*b*) barrier potential is very low
(*c*) depletion layer is extremely thin
(*d*) impurity level is low.

4. The I_P/I_V ratio of a tunnel diode is of primary importance in

(*a*) determining tunneling speed of electrons
(*b*) the design of an oscillator
(*c*) amplifier designing
(*d*) computer applications.

5. Mark the INCORRECT statement. A varactor diode
(*a*) has variable capacitance
(*b*) utilizes transition capacitance of a junction
(*c*) has always a uniform doping profile
(*d*) is often used as an automatic frequency control device.

6. The microwave device used as an oscillator within the frequency range 10-1000 GHz is ..... diode.
(*a*) Schottky
(*b*) IMPATT
(*c*) Gunn
(*d*) Step Recovery.

7. A PIN diode is frequently used as a
(*a*) peak clipper
(*b*) voltage regulator
(*c*) harmonic generator
(*d*) switching diode for frequencies upto GHz range.

8. Mark the WRONG statement. A Schottky diode
(*a*) has no depletion layer
(*b*) has metal-semiconductor junction
(*c*) has fast recovery time
(*d*) is a bipolar device
(*e*) is also called hot-carrier diode
(*f*) can easily rectify high-frequency signals.

9. A special purpose diode which uses metals like gold, silver or platinum on one side of the junction, *n*-type doped silicon on another side and has almost no charge storage in the junction, is a
(*a*) Schotty diode
(*b*) tunnel diode
(*c*) varactor diode
(*d*) zener diode

10. A step-recovery diode
(*a*) has an extremely short recovery time
(*b*) conducts equally well in both directions
(*c*) is mainly used a harmonic generator
(*d*) is an ideal rectifier of high-frequency signals.

11. A semiconductor device that resembles a voltage variable capacitor is called diode.
(*a*) tunnel
(*b*) PIN
(*c*) Schottky
(*d*) varactor

12. A diode that has no depletion layers and operates with hot carriers is called ..... diode.
(*a*) Schottky
(*b*) Gunn
(*c*) step recovery
(*d*) PIN

13. In switching devices, gold doping is used to
(*a*) improve bonding
(*b*) reduce storage time
(*c*) increase the mobility of the carrier
(*d*) protect the terminals against corrosion

14. When the reverse bias voltage of a varactor diode increases, its
(*a*) capacitance decreases
(*b*) lealkage current decreases
(*c*) negative resistance increases
(*d*) depletion zone decreases.

15. Which of the following are negative-resistance microwave diodes oscillator applications ?
(*a*) Gunn
(*b*) IMPATT
(*c*) step recovery
(*d*) both (*a*) and (*b*)
(*e*) both (*b*) and (*c*).

16. A negative-resistance microwave diode having a thin slice of a semiconductor material sandwiched between two metal conductors is called ..... diode.
(*a*) Schottky
(*b*) PIN
(*c*) Gunn
(*d*) varactor.

17. Zener diodes are used primarily as
(*a*) rectifiers
(*b*) voltage regulators
(*c*) oscillators
(*d*) amplifiers.

18. The diode which is often used for voltage regulation in electronic circuits is called ..... diode

(*a*) zener (*b*) varactor
(*c*) silicon (*d*) germanium.

19. Avalanche photodiodes are preferred over PIN diodes in optical communication systems because of

(*a*) speed of operation
(*b*) higher sensitivity
(*c*) larger bandwidth
(*d*) larger power handling capacity

20. The current through the Zener diode in Fig. 54. 42.

(*a*) 33 mA
(*b*) 3.3 mA
(*c*) 2 mA
(*d*) 0 mA

(GATE ; 2004)

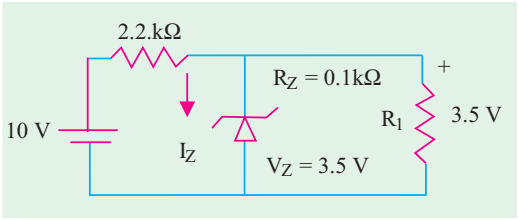


Fig. 54.42

ANSWERS

1. (*c*) **2.** (*d*) **3.** (*c*) **4.** (*d*) **5.** (*c*) **6.** (*b*) **7.** (*d*) **8.** (*d*) **9.** (*a*) **10.** (*c*)
11. (*d*) **12.** (*a*) **13.** (*b*) **14.** (*a*) **15.** (*d*) **16.** (*b*) **17.** (*b*) **18.** (*a*) **19.** (*a*) **20.** (*c*)

GO To FIRST

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CHAPTER 55

DC POWER SUPPLIES

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Low cost DC Power Supply

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55.1. Introduction

Most of the electronic devices and circuits require a dc source for their operation. Dry cells and batteries are one form of dc source. They have the advantage of being portable and ripple-free. However, their voltages are low, they need frequent replacement and are expensive as compared to conventional dc power supplies. Since the most convenient and economical source of power is the domestic ac supply, it is advantageous to convert this alternating voltage (usually, 220 V rms) to dc voltage (usually smaller in value). This process of converting ac voltage into dc voltage is called ***rectification*** and is accomplished with the help of a

(*i*) rectifier **(*ii*)** filter and **(*iii*)** voltage regulator circuit.

These elements put together constitute dc power supply.

55.2. Unregulated Power Supply

An unregulated power supply is one whose dc terminal voltage is affected significantly by the amount of load. As the load draws more current, the dc terminal voltage becomes less.

55.3. Regulated Power Supply

It is that dc power supply whose terminal voltage remains almost constant regardless of the amount of current drawn from it. An unregulated supply can be converted into a regulated power supply by adding a voltage regulating circuit to it (Art 56.5).

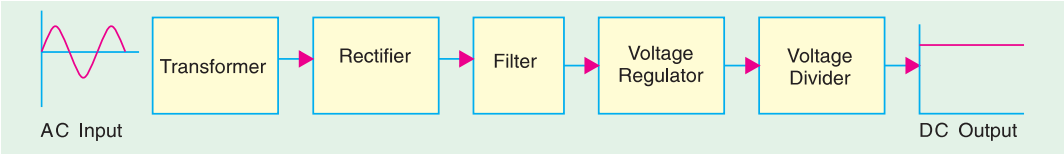


Fig. 55.1

A typical dc power supply consists of five stages as shown in Fig. 55.1.

1. Transformer. Its job is either to step up or (mostly) step down the ac supply voltage to suit the requirement of the solid-state electronic devices and circuits fed by the dc power supply. It also provides isolation from the supply line–an important safety consideration.

2. Rectifier. It is a circuit which employs one or more diodes to convert ac voltage into pulsating dc voltage.

3. Filter. The function of this circuit element is to remove the fluctuations or pulsations (called ripples) present in the output voltage supplied by the rectifier. Of course, no filter can, in practice, gives an output voltage as ripple-free as that of a dc battery but it approaches it so closely that the power supply performs as well.



Single filament Rectifier

4. Voltage Regulator. Its main function is to keep the terminal voltage of the dc supply constant even when

(*i*) ac input voltage to the transformer varies (deviations from 220 V are common); or

(*ii*) the load varies.

Usually, Zener diodes and transistors are used for voltage regulation purposes. Again, it is impossible to get 100% constant voltage but minor variations are acceptable for most of the jobs.

5. Voltage Divider. Its function is to provide different dc-voltages needed by different electronic circuits. It consists of a number of resistors connected in series across the output terminals of the voltage regulator. Obviously, it eliminates the necessity of providing separate dc power supplies to different electronic circuits working on different dc levels.

Comments. Strictly speaking, all that is really required for conversion from ac to dc is a transformer and a rectifier (in fact, even the transformer could be eliminated if no voltage transformation is required). The filter, voltage regulator and voltage divider are mere refinements of a dc power supply though they are essential for most applications except for battery charging and running small dc motors etc.

55.4. Rectifiers

We will consider the following circuits :

1. single-phase half-wave rectifier,
2. single-phase full-wave rectifier,
3. full-wave bridge circuit,
4. three-phase half-wave rectifier,
5. three-phase full-wave rectifier,
6. six-phase half-wave rectifier,
7. three-phase bridge circuit,
8. voltage multiplier circuits.

Many semiconductor devices or systems (like car stereo systems) require a negative dc source or both a negative and a positive dc source. For the sake of simplicity, we will analyse only the positive dc power supplies. However, a positive dc supply can be converted into a negative one by simply reversing the two leads in the same way as we reverse the polarity of a dry cell.

Quite a number of integrated circuits (*ICs*) require both positive and negative source with common ground. In that case, the polarised components in the negative portion of the supply will have to be reversed. For example, its rectifier, filter capacitor and voltage/current regulation devices will have to be reversed as compared to the positive supply.

55.5. Single-phase Half-Wave Rectifier

The basic circuit of a half-wave rectifier with a resistive load (but no filter circuit) is shown in Fig. 55.2 (*a*). The alternating secondary voltage is applied to a diode connected in series with a load resistor R_L. Let the equation of the alternating secondary voltage be $V_s = V_{sm}\, wt$.

(*a*) Working

During the positive half-cycle of the input ac voltage, the diode *D* is forward-biased (ON) and conducts. While conducting, the diode acts as a short-circuit so that circuit current flows and hence, positive half-cycle of the input ac voltage is dropped across R_L. It constitutes the output voltage V_L as shown in Fig. 55.2 (*b*). Waveform of the load voltage is also shown in Fig. 55.2 (*b*). It consists of half-wave rectified sinusoids of peak value V_{LM}.

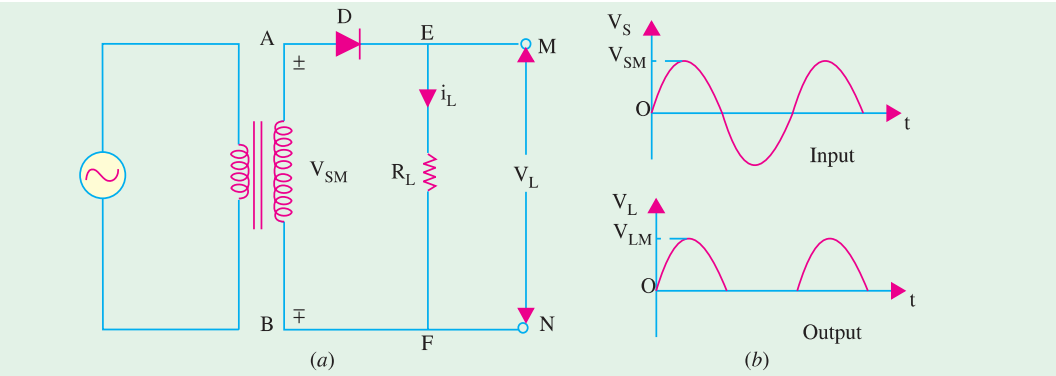


Fig. 55.2

During the negative input half-cycle, the diode is reverse-biased (OFF) and so, does not conduct *i.e.* there is no current flow. Hence, there is no voltage drop across R_L. In other words $i_L = 0$ and

V_L = 0. Obviously, the negative input half-cycle is suppressed *i.e.* it is not utilized for delivering power to the load. As seen, the output is not a steady dc but ***only a pulsating dc wave*** having a ripple frequency equal to that of the input voltage frequency. This wave can be observed by an oscilloscope connected across R_L. When measured by a dc meter, it will show some ***average*** positive value both for voltage and current. Since only one half-cycle of the input wave is used, it is called a half-wave rectifier. It should be noted that forward voltage drop across the diode has been neglected in the above discussion. We have, in fact, assumed an ideal diode (having zero forward resistance and infinite reverse resistance).

(*b*) Average and RMS Values

Let

V_{sm} = maximum value of transformer secondary voltage

V_s = rms value of secondary voltage

V_{LM} = maximum value of load voltage

= V_{sm} – diode drop – secondary resistance drop

V_L = rms value of load voltage

I_L = rms value of load current

$V_{L(dc)}$ = average value of load voltage

$I_{L(dc)}$ = average value of load current

I_{LM} = maximum value of load current

R_L = load resistance

R_S = transformer secondary resistance

r_d = diode forward resistance

$R_0 = R_S + r_d$

$$I_{LM} = \frac{V_{sm} - V_B}{(R_S + r_d) + R_L} = \frac{V_{sm} - V_B}{R_0 + R_L}, \qquad V_{LM} = I_{LM} . R_L$$

$$V_{L(dc)} = \frac{V_{LM}}{\pi} = 0.318\, V_{LM}, \; I_{L(dc)} = \frac{I_{LM}}{\pi} = 0.318\, I_{LM}$$

$$I_L = I_L M_n/2 = 0.5\, I_{LM} = 0.5\, V_{LM}/R_L$$

(*c*) Efficiency

The efficiency of rectification is given by the ratio of the output dc power to the total amount of input power supplied to the circuit. It is also called the conversion efficiency.

$$\eta = \frac{P_{dc}}{P_{in}} = \frac{\text{power in the load}}{\text{input power}}$$

Now,

$$P_{dc} = I_{L(dc)}^{\,2} R_L = \left(\frac{I_{LM}}{\pi}\right)^2 \qquad R_L = \frac{I_{LM}^{\,2}}{\pi^2} \cdot R_L$$

$$P_{in} = I_L^{\,2} (R_L + R_0) = \left(\frac{I_{LM}}{2}\right)^2 (R_L + R_0) = \frac{I_{LM}^{\,2}}{4} (R_L + R_0)$$

$\therefore$

$$\eta = \frac{P_{dc}}{P_{in}} = \left(\frac{4}{\pi^2}\right) \frac{R_L}{(R_L + R_0)} = \frac{0.406}{1 + R_0/R_L} = \frac{409.6\%}{(1 + R_0/R_L)}$$

If R_0 is neglected h = 40.6%. Obviously, it is the maximum possible effficiency of a half-wave rectifier.

(*d*) Frequency Components of H.W. Rectified Voltage and Current

As shown in Fig. 55.3, the load current I_L consist of a dc component $I_{L(dc)}$ and an ac component $I_{L(ac)}$. The Fourier series of the half-wave rectified current flowing through the load is found to be

$$i_L = I_{LM} \left(\frac{1}{\pi} + \frac{1}{2} \sin \omega t - \frac{2}{3\pi} \cos 2\omega t - \frac{2}{15\pi} \cos 4\, \omega t + \ldots\right)$$

As seen, the half-wave rectified current consists of a large number of ac components (which constitute the ripple) in addition to the dc component. The first term is I_{LM}/π which represents the dc component $I_{L(dc)}$. The second term $(I_{LM}/2) \sin \omega t$ has peak value of $(I_{LM}/2)$. It is called the fundamental or first harmonic component and its rms value is $I_{L1} = I_{LM}/2\sqrt{2}$.

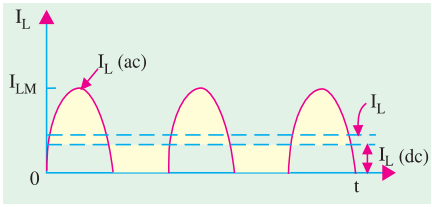


Fig. 55.3

The third term represents the second harmonic component whose frequency is double that of the supply frequency. The rms value is I_{L2} = peak value/$\sqrt{2} = 2 I_{LM}/3\pi\sqrt{2} = I_{LM}/3\pi$.

The fourth term represents the third harmonic component whose frequency is four times the supply frequency. Its rms value is $2 I_{LM}/15\pi \times \sqrt{2} = \sqrt{2} \cdot I_{LM}/15\pi$.

The rms values of other components can be similarly calculated. However, they are found to be of continuously diminishing value.

As discussed above, the rectified output (or load) current consists of

(i) dc component, $I_{L(dc)} = I_{LM}/\pi$ and

(ii) ac components of rms values I_{L1}, I_{L2} and I_{L3} etc. Their combined rms value is given by

$$I_{L(ac)} = \sqrt{I_{L1}^2 + I_{L2}^2 + I_{L3}^2 + ...}$$

The rms (or effective) value of the total load current is given by

$$I_L = \sqrt{I_{L(dc)}^{\ 2} + I_{L(ac)^2}} = \sqrt{I_{L(dc)}^{\ 2} + (I_{L1}^{\ 2} + I_{L2}^{\ 2} + I_{L3}^{\ 2} + ...}$$

Similarly, the Fourier series of the load voltage is given by

$$V_L = V_{LM}\left(\frac{1}{\pi} + \frac{1}{2}\sin \omega t - \frac{2}{3\pi}\cos 2\omega t - \frac{2}{15\pi}\cos 4\omega t.....\right)$$

It also consists of

(i) a dc component, $V_{L(dc)} = V_{LM}/\pi$

(ii) ac components of rms values VL_1, VL_2 and V_{L3} etc. which are given by

$$V_{L1} = V_{LM}/\sqrt{2}, V_{L2} = \sqrt{2} \cdot V_{LM}/3\pi ; V_{L3} = \sqrt{2}\, V_{LM}/15\pi \text{ etc.}$$

Again, $$V_{L(ac)} = \sqrt{V_{L1}^2 + V_{L2}^2 + V_{L3}^2 + ...}$$

The rms value of the entire load voltage is given by

$$V = \sqrt{V_{L(dc)}^{\ 2} + V_{L(ac)}^{\ 2}} = \sqrt{V_{L(dc)}^{\ 2} + V_{L1}^2 + V_{L2}^2 + V_{L3}^2 + ...}$$

(e) Ripple Factor

When defined in terms of voltage, it is given by

$$\gamma = \frac{\text{rms value of ac components}}{\text{dc value of load voltage}} = \frac{V_{L(ac)}}{V_{L(dc)}} = \frac{V_{r(ms)}}{V_{L(ms)}}$$

In terms of current, we have $\gamma = I_{L(ac)}/I_{L(dc)}$

As seen from above, $$I_{L(ac)} = \sqrt{I_L^2 - I_{L(dc)}^2}$$

$$\gamma = \frac{I_{L(ac)}}{I_{L(ac)}} = \frac{\sqrt{I_L^2 - I_{L(dc)}^2}}{I_{L(dc)}} = \sqrt{\left(\frac{I_L}{I_{dc}}\right)^2 - 1}$$

Now, $I_L/I_{L(dc)}$ = form factor K_f (Art 12.18) $\quad \therefore \quad \gamma = \sqrt{K_f^2 - 1}$

In the case of a half-wave rectifies with resistive load but no filter $K_f = \pi/2 = 1.57$

$\therefore \quad \gamma = \sqrt{1.57^2 - 1} = 1.21$

Alternatively, the value of γ could be found as under :

If we neglect fourth and higher harmonics in the load current, then as seen from above

$$I_{L(ac)} = \sqrt{I_{L1}^2 + I_{L2}^2 + I_{L3}^3 + ...}$$

$$= \sqrt{(I_{LM}/2\sqrt{2})^2 + (\sqrt{2}I_{LM}/3\pi)^2 + (\sqrt{2}\,.\,I_{LM}/15\pi)^2 + ...} = 0.385\, I_{LM}$$

$$\therefore \quad \gamma = \frac{I_{L(ac)}}{I_{L(dc)}} = \frac{0.385\, I_{LM}}{I_{LM}/\pi} = \frac{0.385\, I_{LM}}{0.318\, I_{LM}} = 1.21$$

(f) Peak Inverse Voltage (PIV)

It is the maximum voltage that occurs across the rectifying diode ***in the reverse direction***. As seen from Fig. 55.2, the diode is reverse-biased during the negative half-cycle and the maximum voltage applied across it equals the maximum secondary voltage *i.e.* V_{sm}.

(g) Transformer Utilization Factor (TUF)

While designing any power supply, it is necessary to determine the rating of the transformer. It can be done provided *TUF* is known. The value of *TUF* depends on the amount of power to be delivered to the load and the type of rectifier circuit to be used.

$$TUF = \frac{\text{dc power delivered to the load}}{\text{ac rating of transformer secondary}} = \frac{P_{dc}}{P_{ac\,.\,rated}} = \frac{P_{dc}}{P_{in\,.\,rated}}$$

At first sight it might appear as if the above ratio is the same as the conversion efficiency. Actually, it is not so because the rating of the transformer secondary is different from the actual power delivered by the secondary.

$$P_{dc} = V_{L(dc)} \cdot I_{L(dc)} = \frac{V_{LM}}{\pi} \cdot \frac{V_{LM}}{R_L} = \frac{V_{LM}^2}{\pi\, R_L}$$

$$= \frac{V_{sm}^2}{\pi\, R_L} \qquad \text{—if drop over } R_0 \text{ is neglected}$$

Now, the rated voltage of transformer secondary is $V_{sm}/\sqrt{2}$ but the actual current flowing through the secondary is $I_L = I_{LM}/2$ (and not $I_{LM}/\sqrt{2}$) since it is a half-wave rectified current.

$$P_{ac.rated} = \frac{V_{sm}}{\sqrt{2}} \cdot \frac{I_{LM}}{2} = \frac{V_{sm}}{\sqrt{2}} \cdot \frac{V_{LM}}{2\, R_L} = \frac{V_{sm}^2}{2\sqrt{2}\, R_L}$$

$$TUF = \frac{V_{sm}^2/\pi\, R_L}{V_{sm}^2/2\sqrt{2}\, R_L} = \frac{2\sqrt{2}}{\pi} = 0.287$$

However, due to saturation effects produced by the flow of direct current through the tranformer secondary, the value of *TUF* is further reduced to 0.2.

Obviously, dc power delivered to the load=ac transformer rating × *TUF*

If, we have a 1-kVA transformer, then the power which it would be able to deliver to a resistive load in a half-wave rectifier without over-heating would be = 0.2 × 1000 = 200 W.

Example 55.1. *In the half-wave rectifier circuit of Fig. 55.4, determine*

(i) *maximum and rms values of load voltage,*

(ii) *peak and rms values of load current,*

(iii) *power absorbed by the load,*

(iv) *PIV of the diode,*

(v) *rms value of ripple voltage.*

Neglect resistance of transformer secondary and that of the diode.

Solution. Here, $K = N_2/N_1 = 1/10$. Peak primary voltage is

$V_{pm} = 220\sqrt{2} = 310$ V.

Hence, $V_{sm} = KV_{pm}$

$= 310/10 = \mathbf{31\ V}$

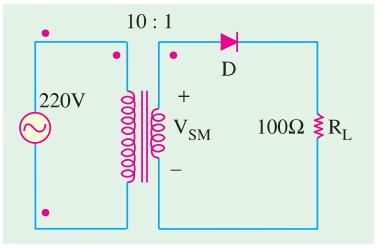


Fig. 55.4

(i) $V_{LM} = V_{sm} = \mathbf{31\ V}$

$V_L = V_{LM}/2 = 31/2 = \mathbf{15.5\ V}$

(ii) $I_{LM} = V_{LM}/R_L = 31/100 = \mathbf{0.31\ A}$

$I_L = I_{LM}/2 = 0.31/2 = \mathbf{0.155\ A}$

(iii) $P_L = V_L I_L = 15.5 \times 0.155 = \mathbf{2.4\ W}$

(iv) $P_{IV} = 2\,V_{sm} = 2 \times 31 = \mathbf{62\ V}$

(v) $V_{L(ac)} = \sqrt{V_L^2 - V_{L(dc)}^2}$

Now, $V_L = V_{LM}/2$ and $V_{L(dc)} = V_{LM}/\pi$

$\therefore \quad V_{L(ac)} = \sqrt{(V_{LM}/2)^2 - (V_{LM}/\pi)^2} = 0.385\,V_{LM}$

$\therefore \quad V_{r(rms)} = V_{L(ac)} = 0.385 \times 31 = \mathbf{11.9\ V}$

It represents the rms value of the ripple voltage.

55.6. Equivalent Circuit of a HW Rectifier

Such a circuit is shown in Fig. 55.5. Here, the diode has been replaced by its equivalent circuit (Art. 55.13). The transformer secondary of Fig. 55.2 has been replaced by an ac sinusodial generator having a peak value of V_{sm}. Resistance R_S represents transformer secondary resistance. Obvisouly.

$$I_{LM} = \frac{V_{sm} - V_B}{(R_S + r_d) + R_L} = \frac{V_{sm} - V_B}{R_0 + R_L}$$

$V_{LM} = I_{LM} \cdot R_L$

$V_{L(dc)} = V_{LM}/\pi,\ I_{L(dc)} = I_{LM}/\pi$

$V_L = V_{LM}/2$ and $I_L = I_{LM}/2$

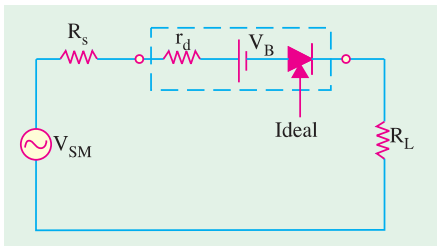


Fig. 55.5

(i) $\eta = \left(\frac{4}{\pi^2}\right)\frac{R_L}{(R_S + r_d) + R_L}$

$= \left(\frac{4}{\pi^2}\right)\frac{R_L}{R_0 + R_L} = \frac{40.6\%}{1 + R_0/R_L}$

(ii) Voltage regulation is given by

$$V_R = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100$$

(iii) Under no-load condition *i.e.* when no output current flows, the voltage has maximum value. When rectifier is fully loaded *i.e.* when output current flows, there is drop over R_0. Hence, output voltage is decreased by this much amount.

$$V_{FL} = V_{NL}\frac{R_L}{R_0 + R_L}$$

Substituting this value in the above equation, we get $V_R = R_0/R_L$

Example 55.2. *A half-wave rectifier using silicon diode has a secondary emf of 14.14 V (rms) with a resistance of 0.2 Ω. The diode has a forward resistance of 0.05 Ω and a threshold voltage of 0.7 V. If load resistance is 10 Ω, determine*

(i) dc load current (ii) dc load voltage (iii) voltage regulation and (iv) efficiency.
(Applied Electronics-I, Punjab University, 1992)

Solution. $V_{sm} = \sqrt{2} \times 14.14 = 20$ V, $R_0 = 0.2 + 0.05 = 0.25\ \Omega$

(i) $I_{LM} = \dfrac{V_{sm} - V_B}{R_0 + R_L} = \dfrac{20 - 0.7}{10.25} = 1.88$ A ; $I_{L(dc)} = \dfrac{I_{LM}}{\pi} = \dfrac{1.88}{\pi} = \mathbf{0.6\ A}$

(ii) $V_{L(dc)} = I_{L(dc)} \cdot R_L = 0.6 \times 10 = \mathbf{6\ V}$

(iii) $V_R = R_0/R_L = 0.25/10 = 0.025$ or **2.5%**

(iv) $\eta = \dfrac{40.6}{1 + 0.25/10} = \mathbf{39.6\%}$

55.7. Single-phase Full-wave Rectifier

In this case, both half-cycles of the input are utilized with the help of two diodes working alternately. For full-wave rectification, use of a transformer is essential (though it is optional for half-wave rectification).

The full-wave rectifier circuit using two diodes and a centre-tapped transformer shown in 55.6 (*a*). The centre-tap is usually taken as the ground or zero voltage reference point.

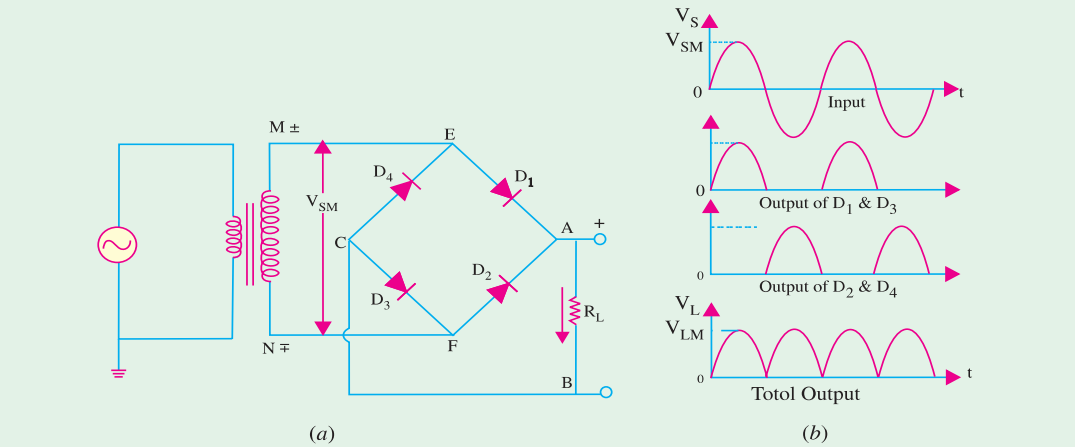


Fig. 55.6

Fig. 55.7 shows two different ways of drawing the circuit. In Fig. 55.7 (*a*), R_L becomes connected to point *G via* the earth whereas in Fig. 55.7 (*b*). It is connected directly to *G*.

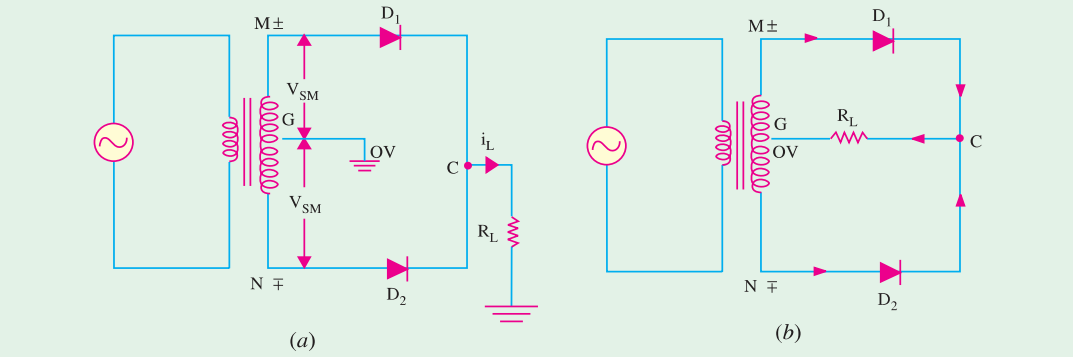


Fig. 55.7

(*a*) Working

When input ac supply is switched on, the ends *M* and *N* of the transformer secondary become +ve and – ve alternately. During the positive half-cycle of the ac input, terminal *M* is +ve, *G* is at zero potential and *N* is at –ve potential. Hence, being forward-biased, diode D_1 conducts (but not D_2 which is reverse-biased) and current flows along MD_1CABG. As a result, positive half-cycle of the voltage appears across R_L.

During the negative half-cycle, when terminal *N* becomes +ve, then D_2 conducts (but not D_1) and current flows along ND_2CABG. So, we find that current keeps on flowing through R_L in the *same* direction (*i.e.* from *A* to *B*) in both half-cycles of ac input. It means that both half-cycles of the input ac supply are utilized as shown in Fig. 55.6 (*b*). Also, the frequency of the rectified output voltage is twice the supply frequency. Of course, this rectified output consists of a dc component and many ac components of diminishing amplitudes.

(*b*) Average and RMS Values

As proved earlier in and now shown in Fig. 5.8

$$V_L = V_{LM}/\sqrt{2} = 0.707\, V_{LM}\,;\; V_{L(dc)} = 2\, V_{LM}/\pi = 0.636\ \text{V}$$

$V_{L(ac)}$ = rms value of ac components in the output voltage

$$= \sqrt{V_L^2 - V_{L(dc)}^2}$$

Similarly,

$$I_{LM} = \frac{V_{LM}}{R_L}\;;\quad I_L = \frac{I_{LM}}{\sqrt{2}} = 0.707\, I_{LM}$$

$$I_{L(dc)} = \frac{2\, I_{LM}}{\pi} = 0.636\, I_{LM}\,;\; I_{L(ac)} = \sqrt{I_L^2 - I_{L(dc)}^2}$$

Incidentally, $I_{L(ac)}$ is the same thing as $I_{r(rms)}$.

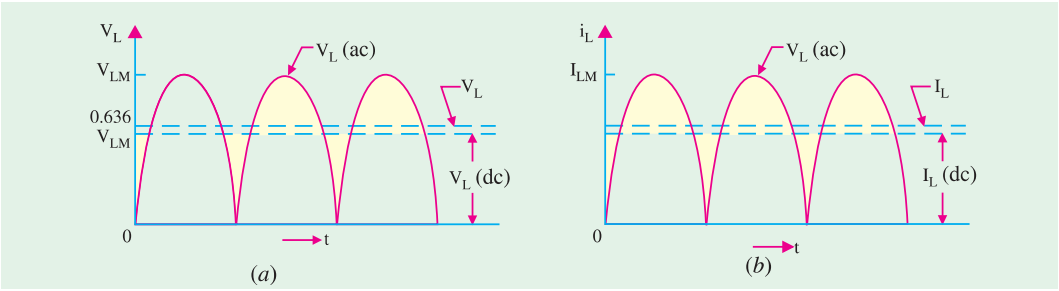


Fig. 55.8

(*c*) Efficiency

$$P_{in} = I_L^2\,(R_0 + R_L) = \left(\frac{I_{LM}}{\sqrt{2}}\right)^2 (R_0 + R_L) = \frac{1}{2}\, I_{LM}^2\,(R_0 + R_L)$$

$$P_{dc} = I_{L(dc)}^2\,(R_0 + R_L) = \left(\frac{2\, I_{LM}}{\pi}\right)^2 (R_0 + R_L) = \frac{4\, I_{LM}^2}{\pi^2}\,(R_0 + R_L)$$

$$\therefore \quad \eta = \frac{P_{dc}}{P_{in}} = \left(\frac{8}{\pi^2}\right)\left(\frac{R_L}{R_0 + R_L}\right) = \frac{0.812}{(1 + R_0/R_L)} = \frac{81.2\%}{(1 + R_0/R_L)}$$

It is twice the value for the half-wave rectifier for the simple reason that a full-wave rectifier utilizes both half-cycles of the input ac supply.

(*d*) Frequency Components

As in the case of a HW rectifier, the output of a full-wave rectifier also consists of **(*i*)** a dc

component and ***(ii)*** a number of ac components which form the ripple. The Fourier series for rectifier output voltage is

$$V_L = V_{LM}\left(\frac{2}{\pi} - \frac{4}{3\pi}\cos 2\omega t - \frac{4}{15\pi}\cos 4\omega t - \frac{4}{35}\cos 6\omega t - ....\right)$$

As seen, $V_{L(dc)} = \frac{2V_{LM}}{\pi}$; $V_{L1} = \frac{4V_{LM}}{\sqrt{2}\,3\pi}$, $V_{L2} = \frac{4V_{LM}}{\sqrt{2}.15\pi}$ etc.

$$V_{L(ac)} = \sqrt{V_{L1^2} + V_{L2^2}} = \sqrt{\left(\frac{4V_{LM}}{\sqrt{2}.3\pi}\right)^2 + \left(\frac{4V_{IM}}{\sqrt{2}.15\pi}\right)^2} = 0.305\,V_{LM}$$

Similalry, $I_{L(ac)} = I_{r(rms)} = \sqrt{I_{L1}^2 + I_{L2}^2} = 0.305\,I_{LM}$

(e) Ripple Factor

$$\gamma = \frac{V_{L(ac)}}{V_{L(dc)}} = \frac{V_{r(rms)}}{V_{L(rms)}} = \frac{0.305\,V_{LM}}{0.636\,V_{LM}} = 0.482$$

It is much less as compared to 1.21 for half-wave rectifier.

(f) PIV

Its value is = $2\,V_{sm}$

(g) TUF

Its value is found by considering the primary and secondary windings of the transformer separately. Its value is 0.693 (as compared to 0.287 for a half-wave rectifier). In such a rectifier, there is no problem due to dc saturation of flux in the core because the dc currents in the two halves of the secondary flow in opposite directions.

Example 55.3. *With reference to the full-wave rectifier of Fig. 55.9, determine*
(i) *peak, dc component, rms and ac component of load voltage,*
(ii) *peak, dc component, rms and ac component of load current,*
(iii) *ripple factor,*
(iv) *peak and average diode currents,*
(v) *total power supplied to the load.*
Neglect diode and secondary winding resistances.

Solution. Here, $K = 1/2$

$V_{pm} = 220 = 312$ V. Hence,

$V_{MN} = 312/2 = 156$ V so that

$V_{MG} = V_{GN} = 156/2 = 78$ V

(i) $V_{sm} = V_{LM} = \mathbf{78\ V}$

$V_{L(dc)} = 0.636\,V_{LM} = 0.636 \times 78 = \mathbf{49.6\ V}$

$V_L = 0.707\,V_{LM} = 0.707 \times 78 = \mathbf{55\ V}$

$V_{L(ac)} = \sqrt{55^2 - 49.6^2} = \mathbf{23.8\ V}$

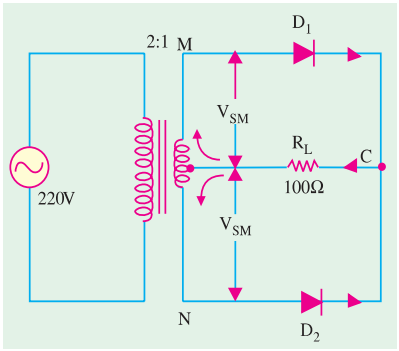


Fig. 55.9

It also represents $V_{r(rms)}$.

(ii) $I_{LM} = V_{LM}/R_L = 78/100 = \mathbf{0.78\ A}$

$I_{L(dc)} = 0.636 \times 0.78 = \mathbf{0.496\ A}$

$I_L = 0.707 \times 0.78 = \mathbf{0.55\ A}$

$I_{L(ac)} = \sqrt{0.55^2 - 49.6^2} = \mathbf{0.238\ A}$

It also represents $I_{r(rms)}$.

(iii) $\gamma = \frac{I_{L(ac)}}{I_{L(dc)}} = \frac{I_{r(max)}}{I_{L(max)}} = \frac{0.238}{0.496} = \mathbf{0.48}$

(iv) peak diode current = peak load current = **0.78 A**

For finding the average current of a diode, it must be remembered that each diode carries current for one half-cycle only.

$$I_{D(av)} = I_{D(max)}/\pi = 0.318 \times 0.78 = \mathbf{0.25\ A}$$

(v) $$PL = V_L I_L = 55 \times 0.55 = \mathbf{30.25\ W}$$

Example 55.4. *A 1-ϕ, full-wave rectifier supplies power to a 1 k W load. The ac voltage applied to the diode is 300-0-300 V (rms). If diode resistance is 25 W and that of the transformer secondary negligible, determine"*

(i) *average load current,* **(ii)** *average value of load voltage,*
(iii) *rms value of ripple,* **(iv)** *efficiency.*

(Applied Electronics, Bombay Univ.)

Solution. It may be noted that rms value of ac voltage across each secondary half is 300 V.

(i) $$V_{sm} = 300 = 424\ \text{V}, I_{LM} = V_{sm}/(r_d + R_L) = 424/1025 = 0.414\ \text{A}$$

$$I_{L(dc)} = I_{LM}/\pi = 2 \times 0.414/\pi = \mathbf{0.263\ A}$$

(ii) $$V_{L(dc)} = I_{L(dc)} \cdot R_L = 0.263 \times 1000 = \mathbf{263\ V}$$

(iii) $$\gamma = \frac{V_{L(ac)}}{V_{L(dc)}} = \frac{V_{r(rms)}}{V_{L(dc)}}$$

$\therefore$ $$V_{r(rms)} = \gamma \cdot V_{L(dc)} = 0.482 \times 263 = \mathbf{126.8\ V}$$

(iv) $$\eta = \frac{81.2\%}{1 + r_d / R_L} = \frac{81.2\%}{1 + 25/1000} = \mathbf{79.2\%}$$

Example 55.5. *A full-wave rectifier is built up by using the same components as in Ex. 55.2. Determine :*

(i) *dc load current,* **(ii)** *dc load voltage,*
(iii) *voltage regulation,* **(iv)** *circuit efficiency and*
(v) *diode PIV and current rating.*

Solution.

(i) $$I_{L(dc)} = 0.636 \times 1.88 = \mathbf{1.2\ A}$$

(ii) $$V_{L(dc)} = I_{L(dc)} \cdot R_L = 1.2 \times 10 = \mathbf{12\ V}$$

(iii) $$VR = 0.25/10 = 0.025 \text{ or } \mathbf{2.5\%}$$

(iv) $$\eta = \frac{81.2\%}{1 + R_0 / R_L} = \frac{81.2\%}{1 + 0.25/10} = \mathbf{79.2\ \%}$$

(v) $$PIV = 2\, V_{sm} = 2 \times 20 = \mathbf{40\ V}$$

With a safety factor of 1.5, $PIV = 40 \times 1.5 = 60$ V. A dc current rating of about 2 A would be satisfactory.

Example 55.6. *Silicon diodes are used in a two-diode full-wave rectifier circuit to supply a load with 12 volts D.C. Assuming ideal diodes and that the load resistance is 12 ohms, compute* ***(i)*** *the transformer secondary voltage,* ***(ii)*** *the load ripple voltage,* ***(iii)*** *the efficiency of the rectifier. Derive equations used.* **(Electronics-I, Bangalore Univ.)**

Solution.

(i) $V_{L(dc)} = 2\, V_{sm}/\pi$; $V_{sm} = \pi \times 12/2 = \mathbf{18.8\ V.}$

It is the maximum value of the voltage across one half of the secondary.

(ii) RMS value of ripple voltage = $\gamma\, V_{L(dc)} = 0.482 \times 12 = \mathbf{5.78\ V}$

(iii) Since diodes are ideal ones, their forward resistance is negligible. Hence,

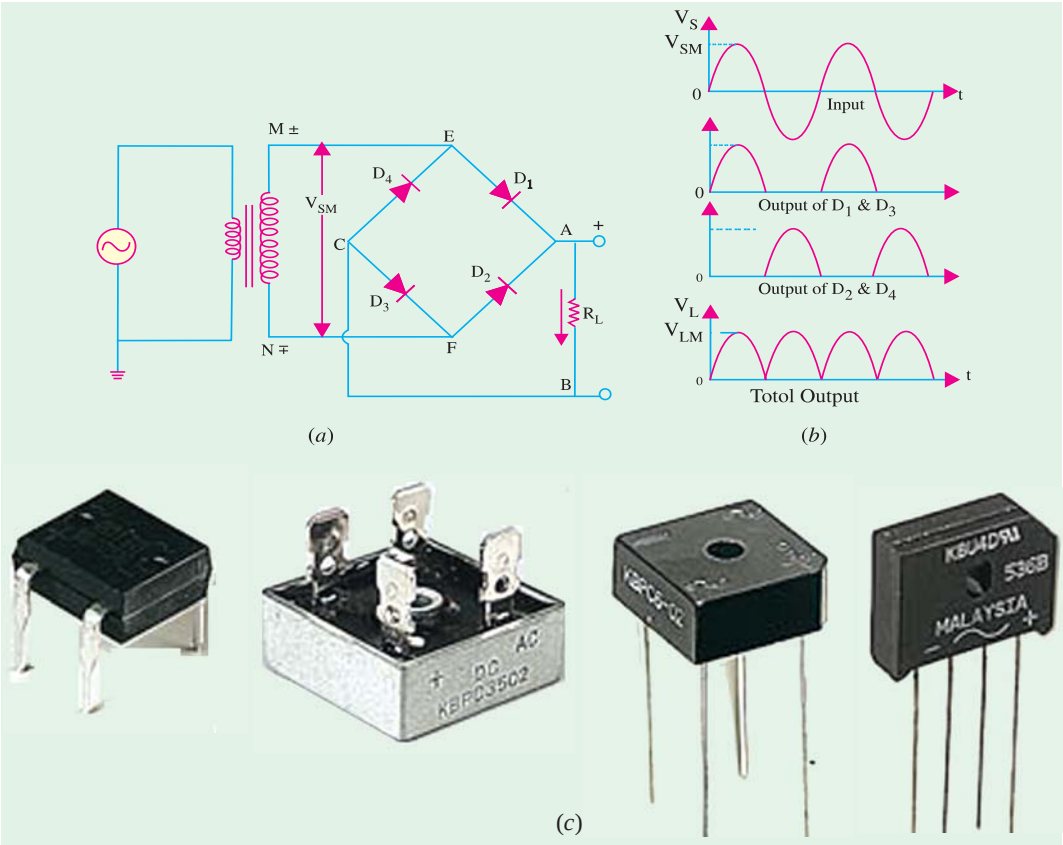


Fig. 55.10

$$\eta = \frac{81.2\%}{(1 + r_d / R_L)} = -\frac{81.2\%}{1 + 0/12} = \mathbf{81.2\%}$$

55.8. Full-Wave Bridge Rectifier

It is the most frequently-used circuit for electronic dc power supplies. It requires four diodes but the transformer used is not centre-tapped and has a maximum voltage of V_{sm}. The full-wave bridge-rectifier is available in three distinct physics forms.

1. four discrete diodes,
2. one device inside a four-terminal case,
3. as part of an array of diodes in an IC.

The circuit using four discrete diodes is shown in Fig. 55.10 (*a*) and 55.10 (*c*) shows some pictures of the bridge rectifier available as one device in a four terminal case.

(*a*) Working

During the positive input half-cycle, terminal *M* of the secondary is positive and *N* is negative as shown separately in Fig. 55.11 (*a*). Diodes D_1 and D_3 become forward-biased (ON) whereas D_2 and D_4 are reverse-biased (OFF). Hence, current flows along *MEABCFN* producing a drop across R_L.

During the negative input half-cycle, secondary terminal *N* becomes positive and *M* negative. Now, D_2 and D_4 are forward-biased. Circuit current flows along *NFABCEM* as shown in Fig. 55.11 (*b*). Hence, we find that current keeps flowing through load resistance R_L in the same direction *AB* during both half-cycles of the ac input supply. Consequently, point *A* of the bridge rectifier always

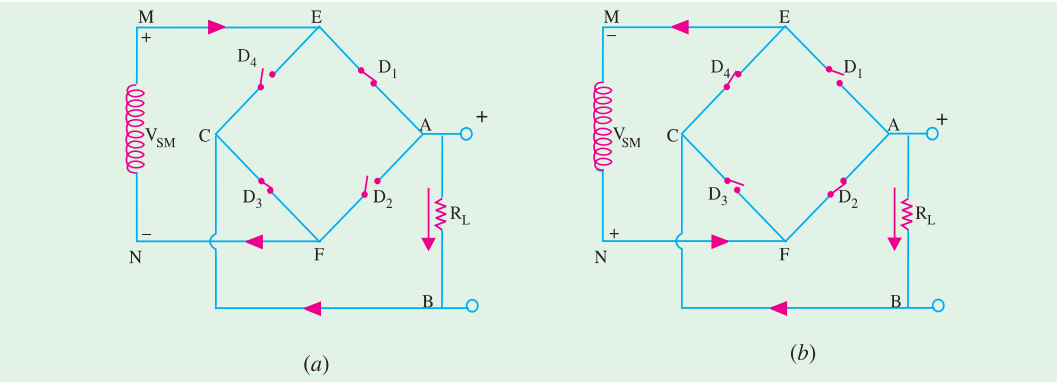


Fig. 55.11

acts as an anode and point *C* as cathode. The output voltage across R_L is as shown in Fig. 55.10 (*b*). Its frequency is twice that of the supply frequency.

(*b*) Average and RMS Values

These are the same as for the centre-tapped full-wave rectifier discussed in Art 55.7.

(*c*) Efficiency

$$\% \eta = \frac{81.2}{1 + 2\, r_d / R_L}$$

(*d*) Ripple Factor

It is the same as for a full-wave rectifier *i.e.* $\gamma = 0.482$

(*e*) PIV

The *PIV* rating of each of the four diodes is equal to V_{SM} —the entire voltage across the secondary.

When Secondary and Diode Resistances are considered

(*i*) $I_{LM} = \dfrac{V_{sm} - 2V_B}{(R_S + 2r_d) + R_L} = \dfrac{V_{sm} - 2V_B}{R_0 + R_L}$ **(*ii*)** $V_{LM} = I_{LM} . R_L$

(*iii*) $\eta = \left(\dfrac{8}{\pi^2}\right)\left(\dfrac{R_L}{(R_S + 2\, r_d + R_L}\right) = \left(\dfrac{8}{\pi^2}\right)\dfrac{R_L}{R_0 + R_L}$

(*iv*) $V_R = \dfrac{R_S + 2\, r_d}{R_L} = \dfrac{R_0}{R_L}$

(*f*) Advantages

After the advent of low-cost, highly-reliable and small-sized silicon diodes, bridge circuit has become much more popular than the centre-tapped transformer *FW* rectifier. The main reason for this is that for a bridge rectifier, a much smaller transformer is required for the same output because it utilizes the transformer secondary continuously unlike the 2-diode FW rectifier which uses the two halves of the secondary alternately.

So, the advantages of the bridge rectifier are :

1. no centre-tap is required on the transformer;
2. much smaller transformers are required;
3. it is suitable for high-voltage applications ;
4. it has less *PIV* rating per diode.

The obvious disadvantage is the need for twice as many diodes as for the centre-tapped transformer version. But ready availability of low-cost silicon diodes has made it more economical despite its requirement of four diodes.

55.9. Three-phase Half-wave Rectifier

Rectification of a 3-phase supply with the help of diodes is shown in Fig. 55.12 along with a smoothing circuit. The three diodes are connected to the three phases of star-connected secondary of a 3-phase transformer. Neutral point *N* of the secondary is the negative terminal for the rectified output and is earthed as shown.

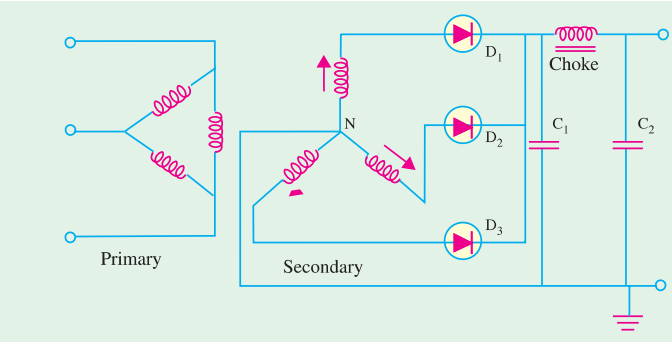


Fig. 55.12

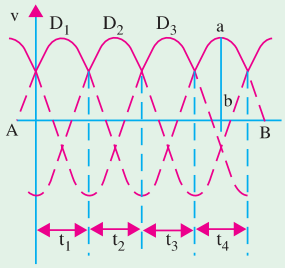


Fig. 55.13

The shape of the output is shown in Fig. 55.13. The horizontal line *AB* represents the potential of the negative d.c. terminal output and the sine waves 1, 2 and 3 each represent the anode potentials of the three diodes.

During one-third of the cycle *i.e.*, during time t_1, only diode D_1 will conduct. It will cease conducting at t_2 and then D_2 will conduct current upto t_2 after which D_2 will take over and will supply anode current till t_3. When one diode conducts, the other two remain inactive because then their anodes are more –ve than their cathodes. This process repeats itself during each ensuing cycle, with the conducting period of each diode being as indicated. The output is given by the vertical distance 'ab' between the upper envelope and the line *AB*. Obviously, the output fluctuates between the maximum and minimum values thrice in each cycle. The variations of output lie between V_{sm} and 0.5 V_{sm} (neglecting voltage drop in diodes) and has a mean value of $V_{dc} = 0.83\ V_{sm}$ or $1.17\ V_S$ where V_S is the r.m.s. value of the secondary phase voltage. Its maximum conversion $\eta = 96.5\%$ and $\gamma = 0.17$. But it should be noted that the magnitude of these fluctuations or pulsations is lesser than for a 1-phase, full-wave rectified output since the current never touches zero. It is further smoothened up by a *C-L-C* filter circuit as shown in Fig. 55.12.

It is seen that direct current of each diode appears in the secondary phase winding and so causes transformer saturation resulting in large primary current. It can be avoided by using zig-zag secondary.

55.10. Full-wave Rectification of 3-phase Currents

As shown in Fig. 55.14, a three-phase full-wave rectifier requires a transformer with six secondary windings connected to give two separate three-phase supplies 180° out of phase with each other. The centre taps are connected by an interphase transformer which enables the two rectifier units to operate independently of each

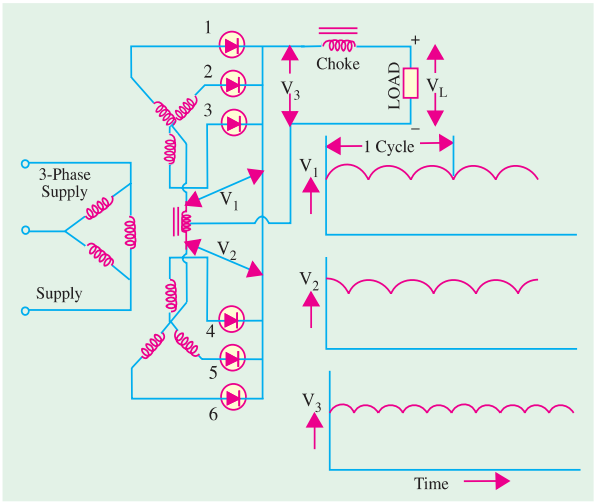


Fig. 55.14

other. Obviously, each diode conducts for one-third cycle. However, the addition of two outputs cancels the lowest frequency component of the ripple.

The output voltage has a mean value of 0.83 V_{sm} (less the diode voltage drop) and a ripple having a fundamental frequency six times the supply frequency.

Three-phase full-wave rectifier circuit is preferred for high powers because

(i) each secondary carries current for one-third of a cycle;

(ii) each primary carries current for two-thirds of a cycle;

(iii) Cu loss in the tranformer windings is comparatively lower.

55.11. Six-phase Half-wave Rectifier

Such a rectifier can be operated from a 3-phase supply using a transformer with three centre-tapped secondary windings with all the centre taps connected together as shown in Fig. 55.15. Obviously, each diode conducts for one-sixth of a cycle. The output voltage has a mean value of 0.955 V_{sm} (less the voltage drop in the diode). Ripple has a very small value and a fundamental frequency six times the supply frequency.

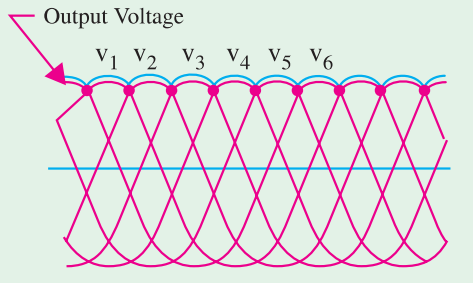


Fig. 55.15

55.12. Three-phase Bridge Circuit

This circuit is very frequently used because apart from being simple, it does not require centre-tap transformer. Only that diode supplies the load whose phase voltage is more positive than the others. For example, when D_1 is forward-biased, D_2 and D_3 are reverse-biased. The current returns to the supply *via* D_5 and D_6.

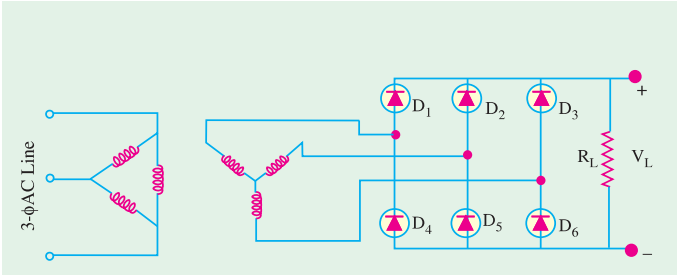


Fig. 55.16

In this circuit, I_{dc} is 0.955 times the peak current through each diode and only one-third of it flows through each diode (rather than one-sixth as in Fig. 5.15). Similarly, V_{dc} is twice of that in 3-ϕ, half-wave rectifier or $2 \times 1.17 = 2.34$ times the r.m.s. a.c. voltage across each secondary leg. Accordingly, r.m.s. voltage across each secondary leg need be only 1/2.34 = 0.428 times the desired d.c. output voltage.

Example 55.7. *The r.m.s. value of transformer secondary voltage per leg (V_S) in a full-wave, D/Y, 6-f rectifier is 150 V. If average value of load current is 2 A, find* ***(i)*** V_{dc} ***(ii)*** *peak and average current through each diode and* ***(iii)*** *average power delivered to the load i.e.* P_{dc}.

Solution. ***(i)*** $V_{dc} = 2.34 \times V_S = 2.34 \times 150 = \mathbf{351\ V}$

(ii) peak current/diode $= (1/0.955) \times I_{dc} = 1.05 \times 2 = 2.1$ A

average d.c. current/diode $= 2/3 = \mathbf{0.667\ A}$

(In *D/Y* half-wave rectifier, its value is 1/6th rather than 1/3rd)

(iii) $P_{dc} = V_{dc} I_{dc} = 351 \times 2 = 702$ W

55.13. Calculations with Resistive Load

Let us suppose that no filter is connected across the rectifier (Fig. 55.12) but only a load of

resistance R. Further, we will neglect transformer resistance and leakage and internal diode drops. In that case,

$$V_{dc} = 0.827\, V_{sm}\,;\; I_{dc} = 0.827\, V_{sm}/R$$

$$P_{dc} = V_{dc}\, I_{dc} = (0.827\, V_{sm})^2/R; \qquad P_{ac} = P_{in} = 0.706\, V^2{}_{sm}/R$$

$$\therefore \quad \text{rectifier } \eta = \frac{P_{dc}}{P_{ac}} = \frac{(0.827\, V_{sm})^2 / R}{0.706\, V_{sm}^2 / R} = 0.965 = 96.5\%$$

$$\gamma = \sqrt{(1.014)^2 - 1} = 0.17 \text{ or } 17\%.$$

Conclusions

Here, average d.c. current is 0.827 times the peak current as compared to 0.318 times for 1-ϕ, half-wave circuit and 0.636 times for 1-ϕ, full-wave circuit. Moreover, V_{dc} is also correspondingly high, it is 51.17 times the r.m.s. voltage of each secondary leg ($\because$ $1.414 \times 0.827 = 1.17$). Conversely, r.m.s. voltage (V_S) across each leg of the secondary need only be 1/1.17 = 0.855 times the average desired d.c. output voltage across the load.

55.14. Filters

The main function of a filter circuit (Fig. 55.17) is to minimize the ripple content in the rectifier output.

As seen, output of various rectifier circuits is pulsating. It has a dc value and some ac components called ripples. This type of output is not useful for driving sophisticated electronic circuits/ devices. In fact, these circuits require a very steady dc output that approaches the smoothness of a battery's output.

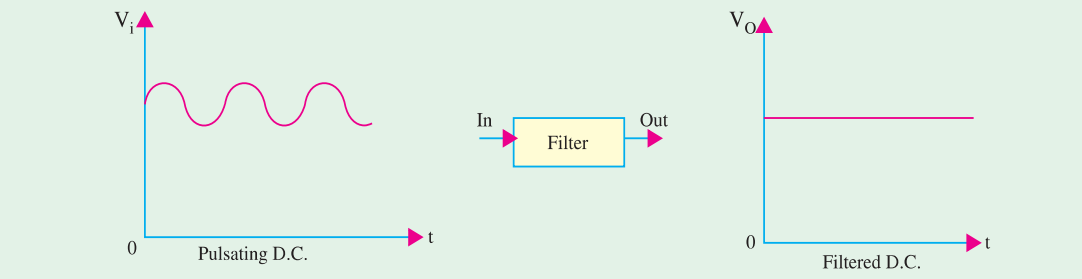


Fig. 55.17

A circuit that converts a pulsating output from a rectifier into a very steady dc level is known as filter because it filters out or smoothens out the pulsations in the output.

We will consider the following popular filter circuits :

1. Series capacitor filter,
2. series inductor filter,
3. *L*-*C* filter (or *L*-type),
4. *R*-*C* filter,
5. *R*-*L*-*C* filter.

55.15. Shunt Capacitor Filter

In this circuit, a suitable single capacitor C is connected across the rectifier and in parallel with the load R_L to achieve filtering action. This type of filter is known as ***capacitor input filter***.

This filter circuit depends for its operation on the property of a capacitor to charge up (*i.e.* store energy) during conducting half-cycle and to discharge (*i.e.* deliver energy) during the non-conducting half-cycle. In simple words, a capacitor opposes any change in voltage. When connected across a pulsating d.c. voltage, it tends to smoothen out or filter out the voltage pulsations (or ripples). The filtering action of the simple capacitor filter when used in a half-wave rectifier can be understood with the help of Fig. 55.18.

(*a*) Circuit Analysis

When positive half-cycle of the ac input is applied, the diode is forward-biased and hence is turned ON. This allows *C* to quickly charge up to peak value of input voltage V_{ip} [point *b* in Fig. 55.18 (*b*)] because charging time constant is almost zero. It is so because there is no resistance in the charging path except diode forward resistance which is negligible. Hence, capacitor follows the charging voltage as shown. After being fully charged, the capacitor holds the charge till input ac supply to the rectifier goes negative. During the negative half-cycle, the capacitor attempts to discharge. However, it cannot discharge through diode which, being now reverse-biased, is OFF. Hence, it discharges through R_L from point *b* to *c* in Fig. 55.18 (*c*) and its voltage decreases somewhat. The discharging time constant (= CR_L) is usually 100 times more than the charging time. Hence, *C* does not have sufficient time to discharge appreciably. It is seen that even during negative half-cycle of the input supply, the capacitor maintains a sufficiently large voltage across R_L.

During the next positive half-cycle, when rectifier voltage exceeds the capacitor voltage represented by point *c* in Fig. 55.18 (*c*), *C* is again charged quickly to V_{ip} as represented by point *d*. Once more, input voltage goes negative, opening the diode and forcing *C* to discharge through R_L during the interval *de*. In this way, R_L sees a nearly constant dc voltage across it at all times.

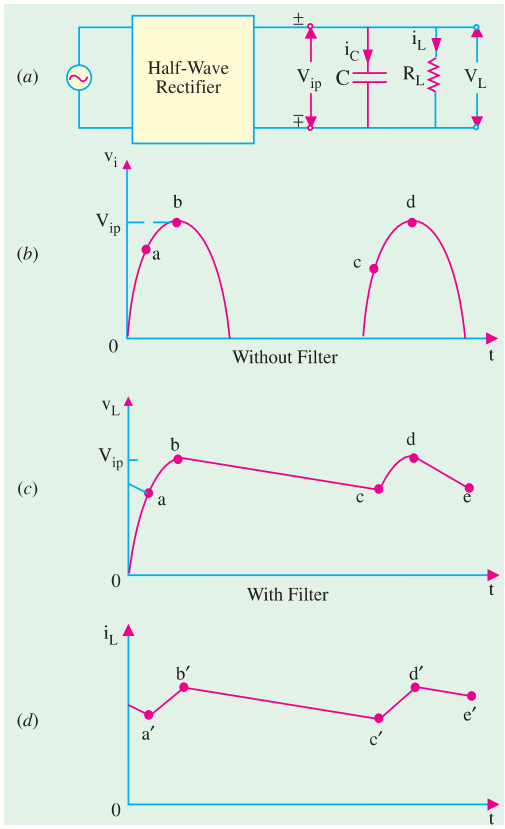


Fig. 55.18

The filtering action of this simple capacitor filter on a full-wave rectifier is shown in Fig. 55.19. It is seen that as compared to a HW rectifier.

(i) dc load voltage increases slightly towards V_{ip},

(ii) ripple voltage has been reduced by half.

The decreased ripple is because of shorter discharge time before the capacitor is reenergised by another pulse of current.

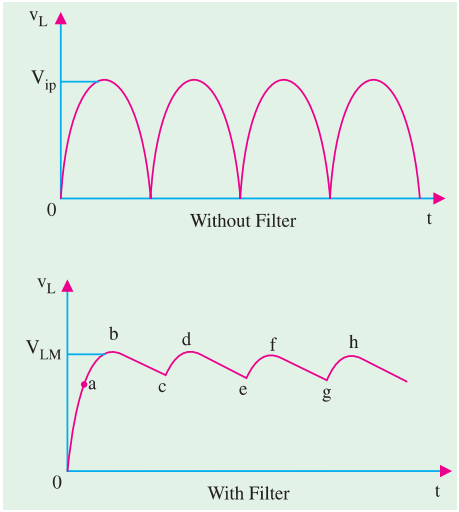


Fig. 55.19

(*b*) Load Current

The load current has the same wave-shape as v_L because load is purely resistive. It is shown in Fig. 55.18 (*d*). During periods *a′b′* and *c′d′* etc., current is supplied by the diode and during periods *b′c′* and *d′e′* etc. by the capacitor.

(*c*) Diode Current

Diode current flows during short intervals of time like *ab* and *cd* etc. in Fig. 55.18 (*c*) which is reproduced in Fig. 55.20. During these intervals, diode

output voltage is greater than the capacitor voltage which is also the load voltage. Hence, diode current is a surging current *i.e.* it takes the form of short-duration pulses as shown in Fig. 55.20. A small resistor is always connected in series with the diode to limit this surge current. It is known as surge limiting resistor.

The sole function of the diode is to recharge C and the sole function of C is to supply load current by discharge.

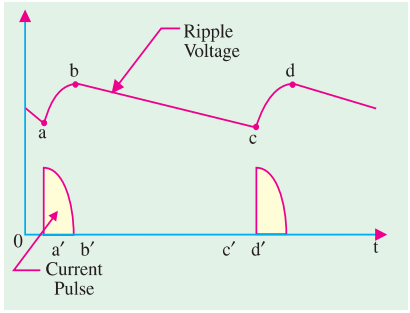


Fig. 55.20

55.16. Effect of Increasing Filter Capacitance

A capacitor has the basic property of opposing changes in voltage. Hence, a bigger capacitor would tend to reduce the ripple magnitude. It has been found that increasing the capacitor size.

1. increases V_{dc} towards the limiting value V_{ip};
2. reduces the magnitude of ripple voltage ;
3. reduces the time of flow of current pulse through the diode ;
4. increases the peak current in the diode.

55.17. Calculations of Shunt Capacitor Filter

Consider the rectifier and filter circuit of Fig. 55.21 where a capacitor has been connected across R_L. The output voltage waveforms of a half-wave rectifier with a shunt capacitor filter are shown in Fig. 55.22 (*a*) whereas Fig. 55.22 (*b*) shows those for a full-wave rectifier. The ripple voltage which occurs under light load conditions can be approximated by a triangular wave which has a peak-to-peak value of $V_{r(p\text{-}p)}$ and a time period of T_r* centred around the dc level.

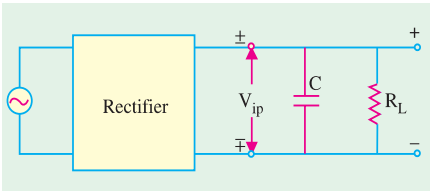


Fig. 55.21

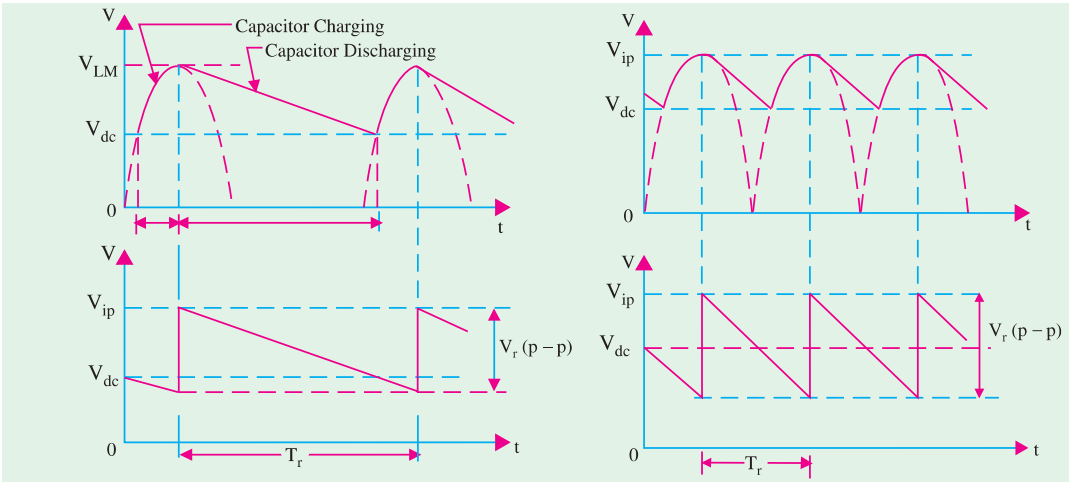


Fig. 55.22

In fact, $V_{r(p\text{-}p)}$ is the amount by which capacitor voltage falls during discharge period T_r. This discharge is actually exponential** but can be approximated by a straight line discharge if we assume

---

* Since charging time is negligibly small, the approximate discharging time represents the full time-period.

** given by $V_c = V_{ip}\, e^{-t}/CR_L$

the discharge rate to remain constant at the dc level I_{dc}. In that case, charge lost dQ in time T_r is $I_{dc}\,T_r$.

$$\therefore \quad V_{r(p-p)} = \frac{dQ}{C} = \frac{I_{dc}T_r}{C} = \frac{V_{dc}}{f_r CR_L} \qquad \left(\because\ I_{dc} = \frac{V_{dc}}{R_L}\right)$$

The triangular ripple has an rms value given by $V_{r(rms)} = V_{r(p-p)} / 2\sqrt{3}$

$$\therefore \quad V_{r(rms)} = \frac{V_{r(p-p)}}{2\sqrt{3}} = \frac{V_{dc}}{2\sqrt{3}f_r CR_L} \quad \therefore \quad \gamma = \frac{V_{r(rms)}}{V_{dc}} = \frac{1}{2\sqrt{3}f_r CR_L}$$

Now, f_r is the frequency of the ripple voltage. For a half-wave rectifier, f_r equals the rectifier line input frequency whereas it is double the line input frequency for a full-wave rectifier. If f is the line frequency, then

$$\gamma \cong \frac{1}{2\sqrt{3}f\,CR_L} \qquad \text{— for HW rectifier}$$

$$\cong \frac{1}{4\sqrt{3}f\,CR_L} \qquad \text{— for FW rectifier}$$

It can be further proved that

$$\gamma = \frac{1}{2\sqrt{3}f\,CR_L} = \frac{I_{dc}}{4\sqrt{3}f\,C}\left(\frac{1}{V_{ip}} - \frac{1}{V_{dc}}\right) \qquad \text{— half-wave rectifier}$$

$$= \frac{1}{4\sqrt{3}f\,CR_L} = \frac{I_{dc}}{4\sqrt{3}f\,CV_{ip}} \qquad \text{—for full-wave rectifier}$$

It is seen from above that ripple increases with increase in load (*i.e.* output) current.

Incidentally, $V_{dc} = V_{ip} - \dfrac{V_{r(p-p)}}{2}$

where V_{ip} = peak rectifier output voltage.

Substituting the value of $V_{r(p-p)} = V_{dc} / f_r CR_L$, we get

$$V_{dc} = V_{ip} - \frac{V_{dc}}{2f_r\,CR_L} \quad \text{or} \quad V_{dc} = -\frac{V_{ip}}{1 + \dfrac{1}{2f_r\,CR_L}}$$

$$\therefore \quad V_{dc} = V_{ip}\left(\frac{2fCR_L}{1+2fCR_L}\right) = \frac{V_{ip} - I_{dc}/4fC}{1 + I_{dc}/4fCV_{ip}} \qquad \text{—half-wave rectifier}$$

$$= V_{ip}\left(\frac{4fCR_L}{1+4fCR_L}\right) = \frac{V_{ip}}{1 + I_{dc}/4fCV_{ip}} \qquad \text{—full-wave rectifier}$$

Example 55.8. *A half-wave rectifier has a peak output voltage of 12.2 V at 50 Hz and feeds a resistive load of 100 Ω. Determine* ***(i)*** *the value of the shunt capacitor to give 1 percent ripple factor and* ***(ii)*** *the resulting dc voltage across the load resistor.*

Solution. ***(i)*** $\gamma = 1/2\sqrt{3}\ fCRL$ or $0.01 = 1/2\sqrt{3}$ $\therefore$ $C = 5770\ \mu\text{F}$

(ii) $$V_{dc} = \frac{V_{ip}}{1 + 1/2fCR_L} = \frac{12.2}{1 + 1/2 \times 5770 \times 10^{-6} \times 100} = \mathbf{12\ V}$$

Example 55.9. *Find the ripple factor and dc output voltage for the filtered bridge rectifier shown in Fig. 55.23. Each silicon diode has a threshold voltage of 0.7 V.*

Solution. Peak primary voltage = $230 \times \sqrt{3}$

$= 325$ V

Peak secondary voltage = 325 × 1/10
= 32.5 V

Peak full-wave rectified voltage at the filter input

$$V_{ip} = 32.5 - 2 \times 0.7 = 31.1 \text{ V}$$

$$\gamma = \frac{1}{4\sqrt{3}\, fCR_L}$$

$$= \frac{1}{4\sqrt{3} \times 50 \times 5 \times 10^{-6} \times 20 \times 10^{3}}$$

$$= 0.028 \text{ or } \mathbf{2.8\ \%}$$

$$V_{dc} = \frac{V_{ip}}{1 + \frac{1}{4\, fCR_L}} = \frac{31.1}{1 + \frac{1}{4 \times 50 \times 5 \times 10^{-6} \times 20 \times 10^{3}}} = \mathbf{29.6\ V}$$

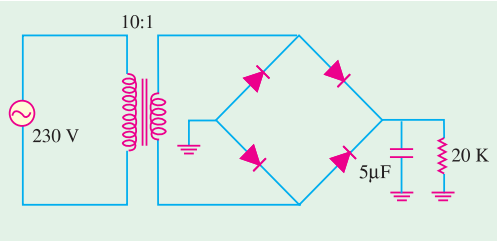


Fig. 55.23

Example 55.10. *Derive an expression for the ripple in the output of a full-wave rectifier circuit, with a simple capacitor element as the filter.*

The load current from the above circuit operating from 200-V, 50-Hz supply is 12 mA. Calculate minimum value of filter capacitor which is required to keep the ripple voltage below 2%.

(App. Electronics and Circuits ; Grad. I.E.T.E.)

Solution. As seen from Art. 5.17 $\gamma = I_{dc} / 4\sqrt{3}\, fCV_{ip}$

$$\therefore \quad C = I_{dc} / 4\sqrt{3}\, f\gamma\, V_{ip} = 12 \times 10^{-3} / 4\sqrt{3} \times 50 \times 0.02 \times 200 = 6\ \mu\text{F}$$

55.18. Series Inductor Filter

The filter consists of a choke in series with the load resistor R_L as shown in Fig. 52.24. The operation of such a filter depends on the fundamental property of an inductor to oppose any sudden changes in the current flowing through it. Since this inductor presents high impedance to the ac components in the filter output, it reduces their amplitude with respect to the dc component thereby producing only a small ripple as shown in Fig. 55.24 (*b*).

The Fourier series for the rectifier output voltage is

$$v_i = V_{ip}\left(\frac{2}{\pi} - \frac{4}{3\pi}\cos 2\omega t - \frac{4}{15\pi}\cos 4\omega t - \ldots.\right)$$

For finding the ripple factor, we will calculate the dc as well as ac drop over R_L. If we neglect choke resistance (R_C), then the entire dc component of filter output is available across R_L and its value is $V_{dc} = 2\, V_{ip} / \pi$ *.

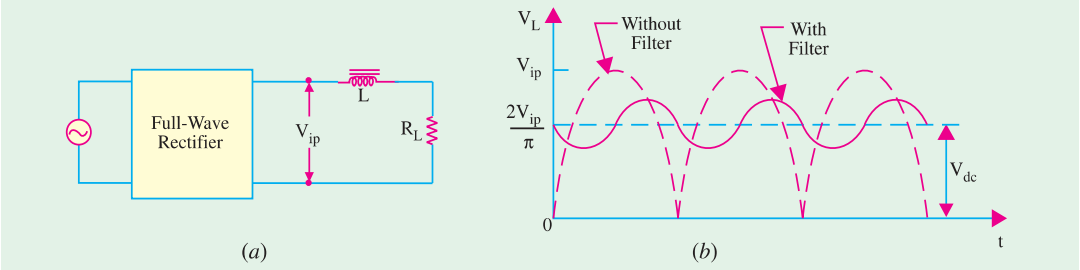


Fig. 55.24

* If R_C is taken into account, then filter output dc voltage drops partly on R_C and partly on R_L. The drop over R_L would be

$$= \frac{2V_{ip}}{\pi} \cdot \frac{R_L}{R_L + R_C}$$

We will consider only the second harmonic voltage $(4V_{ip}/3\pi)\cos 2\omega t$ of frequency 2ω and neglect higher harmonic voltages. This ac voltage partly drops over X_L and partly over R_L. Since choke and R_L are connected in series, the maximum value of drop over R_L is

$$= \frac{4V_{ip}}{3\pi} \cdot \frac{R_L}{\sqrt{R_L^2 + X_L^2}}$$

The rms value of this ac voltage drop across R_L is

$$V_{ac} = \frac{4V_{ip}}{\sqrt{2.3\pi}} \frac{R_L}{\sqrt{R_L^2 + X_L^2}}$$

$$\therefore \quad \gamma = \frac{V_{ac}}{V_{dc}} = \frac{4V_{ip}}{\sqrt{2.3\pi}} \frac{R_L}{\sqrt{R_L^2 + X_L^2}} \times \frac{\pi}{2V_{ip}}$$

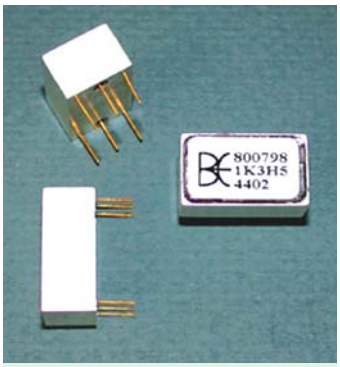
$$= \frac{\sqrt{2}\, R_L}{3\sqrt{(R_L^2 + X_L^2)}} = \frac{\sqrt{2}}{3\sqrt{1 + X_L^2 + R_L^2)^{1/2}}}$$

Since $X_L = 2\omega L$, hence

$$\gamma = \frac{\sqrt{2}}{3\,(1 + 4\omega^2 L^2 / R_L^2)^{1/2}}$$

If $4\omega^2 L^2/R^2_L \gg 1$, then $\gamma = \frac{\sqrt{2}R_L}{3\times 2\omega L} = \frac{R_L}{\sqrt{2.3\omega L}}$

It is seen that ripple decreases as R_L decreases or load current increases (just the opposite of what happens in the case of shunt capacitor filter).



L.C. Filter

55.19. The Choke Input or L-C Filter

It is a combination of two filters considered in Art. 55.15 and 55.18 and provides a lower ripple than is possible with either L or C alone. As is known, in an inductor filter, ripple increases with R_L but decreases in a capacitor filter. The combination of L and C (*i.e.* L–section) filter makes the ripple independent of R_L. Fig. 55.25 (*a*) shows the filter and (*b*) the voltage variations.

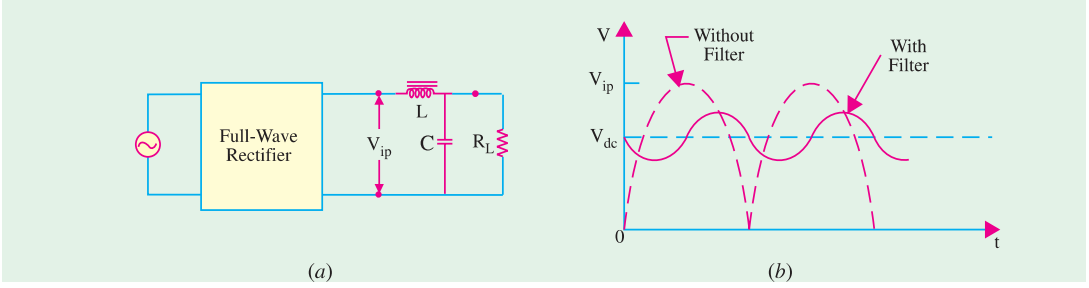


Fig. 55.25

Ripple Factor

If choke resistance R_C is neglected, then dc voltage available across $R_L = 2V_{ip}/\pi$. The ac drop over R_L is the same as across C. Since $X_C \ll R_L$, the parallel combination of R_L and X_C has impedance $\cong X_C$. The second harmonic voltage $(4V_{ip}/3\pi)\cos 2\omega t$ can be assumed to drop over the L–C series combination because R_L is effectively not there.

Maximum value of ac drop over C is

$$= \frac{4V_{ip}}{3\pi} \cdot \frac{X_C}{(X_L + X_C)}$$

$$RMS \text{ value} = \frac{4V_{ip}}{\sqrt{2}.3\pi} \cdot \frac{X_C}{(X_L + X_C)}$$

$$\therefore \quad \gamma = \frac{V_{ac}}{V_{dc}} \frac{4V_{ip}}{\sqrt{2}.3\pi} \cdot \frac{X_C}{X_L + X_C} \times \frac{\pi}{2V_{ip}} = \frac{\sqrt{2}\,X_C}{3X_L} \text{—if } X_C \ll X_L$$

$$= \frac{\sqrt{2}}{3(2\omega C)(2\omega L)} = \frac{\sqrt{2}}{12\omega^2 LC} = \frac{1.19}{LC} \text{—if } C \text{ is in } \mu\text{F and } L \text{ in henrys}$$

Now, $I_{dc} = 2V_{ip}/\pi RL$; maximum value of second harmonic current $I_{2h} = 4V_{ip}/3\pi \times 2\omega L$. The critical (or minimum) value of choke inductance essential for proper working of the filter is reached when $I_{dc} = I_{2h}$ or $2V_{ip}/\pi R_L = 4V_{ip}/3\pi.2\omega_L$ or $L = R_L/3\omega_L$. For $f = 50$ Hz, $L = R_L/940$.

Example 55.11. *A single-phase full-wave rectifier uses 300-0-300 V, 50-Hz transformer. For a load current of 60 mA, design an L-filter using 10 H coil and a suitable capacitor to ensure a ripple factor of not more than 1%.* **(Electronics-II, Bangalore Univ.)**

Solution.

$$\gamma = 1.19/L$$

$$0.01 = 1.19/LC\,; \quad \therefore \quad L_C = 119$$

Hence,

$$10 \times C = 119$$

$$C = 11.9\ \mu\text{F} \cong 12\ \mu\text{F}.$$

55.20. The R-C Filter

Such a filter is shown in Fig. 55.26. Suppose that it is connected to a full-wave rectifier having a filtered output voltage of V_{ip}. The dc component voltage which drops over R_L is

$$= \frac{2V_{ip}}{\pi} \cdot \frac{R_L}{R + R_L} \qquad \text{... (i)}$$

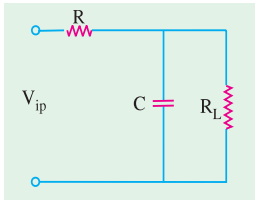


Fig. 55.26

Again, we would consider only the second harmonic voltage $(4V_{ip}/3\pi)\cos 2\omega t$. As before, it will be assumed that $X_C \ll R_L$ so that $R_L \| X_C \cong X_C$. In that case, ac voltage would be assumed to drop across R-C combination.

$$V_{ac} = \frac{1}{\sqrt{2}} \cdot \frac{4V_{ip}}{3\pi} \cdot \frac{X_C}{\sqrt{R^2 + X_C{}^2}} = \frac{4V_{ip}}{\sqrt{2}.3\pi} \cdot \frac{1}{\sqrt{1 + R^2/X_C{}^2}}$$

Same is the drop across R_L.

$$\therefore \quad \gamma = \frac{V_{ac}}{V_{dc}} = \frac{4V_{ip}}{\sqrt{2}.3\pi} \cdot \frac{1}{\sqrt{1 + R^2/X_C{}^2}} \times \frac{\pi(R + R_L)}{2V_{ip}.R_L}$$

$$= \frac{\sqrt{2}}{3} \cdot \frac{1 + R/R_L}{\sqrt{1 + R^2/X_C{}^2}} = \frac{(1 + R/R_L)}{3\sqrt{2}\,\omega CR} \qquad \text{—if } R^2/X_C{}^2 \gg 1$$

55.21. The C-L-C or Pi Filter

As shown in Fig. 55.27, it consists of one inductor and two capacitors connected across its each end. The three components are arranged in the shape of the Greek letter π. It is also called capacitor input π-filter. The input capacitor C_1 is selected to offer very low reactance to the ripple frequency. Hence, major part of filtering is done by C_1. Most of the remaining ripple is removed by the combined action of L and C_2.

The charging and discharging action of C_1 is exactly the same as described in Art. 55.15. The output voltage waveform is also like that shown in Fig. 55.25 (*b*).

This circuit gives much better filtering than LC filter circuit. However, C_1 is still directly connected across the supply and would need high pulses of current if load current is large. Since these high peak current pulses are likely to damage the rectifier diode, this filter is used ***with low-current equipment.***

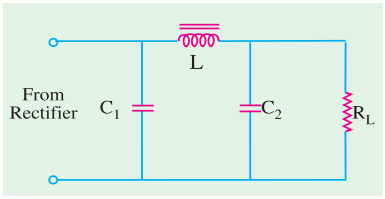


Fig. 55.27

Though this filter-gives somewhat higher output voltage, its voltage regulation is inferior to that of the LC filter.

The ripple factor of this filter is given by

$$\gamma = \sqrt{2}\,\frac{X_{C1}\,X_{C2}}{R_L X_L} = \frac{\sqrt{2}}{8\omega^3 C_1 C_2 L R_L} = \frac{5700}{L C_1 C_2 R_L} \qquad \text{—when } f = 50 \text{ Hz}$$

Here, C_1, C_2 are in μF, L in henrys and R_L in ohms.

55.22. Bleeder Resistor

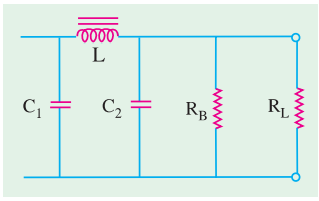


Fig. 55.28

Very often, a resistor (called bleeder resistor) is placed across the filter output (Fig. 55.28) because it provides the following advantages :

1. It improves voltage regulation of the supply.
   By acting as a pre-load on the supply, it causes an initial voltage drop. When the real load is connected, there is only a small amount of additional drop. In this way, difference between no-load and full-load voltage is reduced thereby improving the regulation.
2. It provides safety to the technicians handling the equipment.
   When power supply is switched off, it provides a path for the filter capacitor to discharge through. That is why it is called bleeder resistor. Without it, the capacitor will retain its charge for quite sometime even when the power supply is switched off. This high voltage can be dangerous for people working with the equipment.
3. By maintaining a minimum current through the choke, it improves its filtering action. Value of R_B should be such as to conduct 10 per cent of the total load current.

55.23. Voltage Dividers

Often more than one dc voltage is needed for the operation of electronic circuits. A single power supply can provide as many voltages as are needed by using a voltage divider. As shown in Fig. 55.29 a voltage divider is a single tapped resistor connected across the output terminals of the supply. The tapped resistor may consist of two or three resistors connected in series across the supply. In fact, bleeder resistor may also be used as a voltage divider.

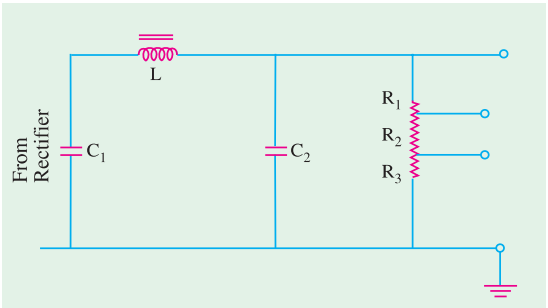
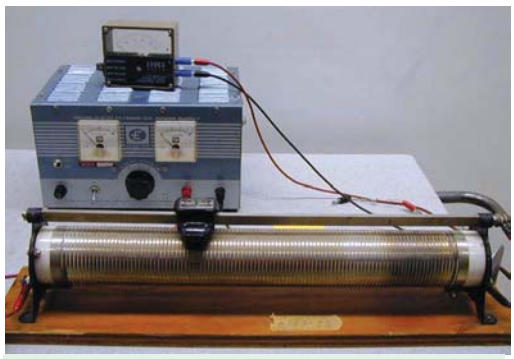


Fig. 55.29


Voltage Dividers

55.24. Complete Power Supply

Fig. 55.30 shows a complete solid-state power supply. From left to right, it consists of a transformer with a current-limiting resistor R_1, rectifier diodes for full-wave rectification, a π-type filter, a transistor series voltage regulator and a voltage divider.

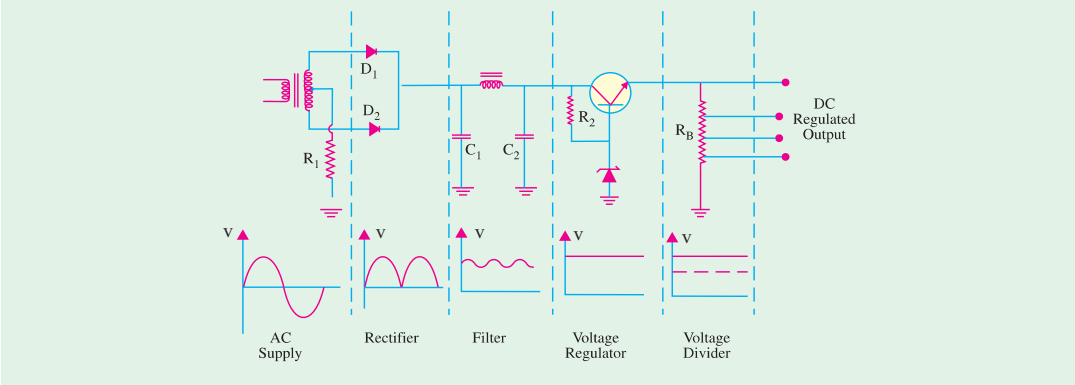


Fig. 55.30

As seen, unregulated ac voltage is fed from the transformer through a full-wave rectifier. It is then filtered by the *CLC* filter and finally regulated by a transistor regulator. The regulated dc supply becomes available across voltage divider resistance R_B. The output is practically ripple-free.

55.25. Voltage Multipliers

A voltage multiplier is a circuit which produces a greater dc output voltage than ac input voltage to the rectifiers. Multipliers are required in many circuit applications where it is necessary to have high voltages with low currents as for electron accelerating purposes in a cathode-ray tube (*CRT*).

We will consider the following circuit :

1. half-wave voltage doubler,
2. full-wave voltage doubler,
3. voltage tripler,
4. voltage quadrupler.

55.26. Half-wave Voltage Doubler

It is also known as cascade voltage doubler. The circuit is shown in Fig. 55.31.

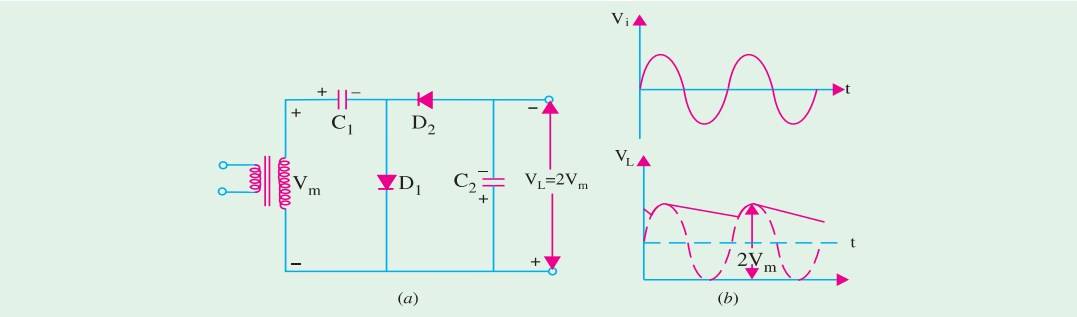


Fig. 55.31

Circuit Analysis

During the positive half-cycle of the input voltage, D_1 conducts (not D_2) and charges C_1 to peak value of secondary voltage (V_m) with the polarity as shown in Fig. 55.31 (*a*). During the negative half-cycle, D_2 conducts (not D_1) and charges C_2. The voltage across C_2 is the sum of peak supply

voltage and the voltage across C_1 (Fig. 55.31). It can be proved by applying *KVL* to the outer loop. Starting from the bottom of the transformer secondary in Fig. 55.32 and going clockwise, we get

$$-V_m - V_m + V_{C2} = 0$$

or $$V_{C2} = 2V_m = 2 \times \text{peak input voltage}$$

During the next positive half-cycle, D_2 is open and C_2 will discharge through the load if it is connected. If no load is connected across C_2, then both capacitors stay charged *i.e.* C_1 to V_m and C_2 to $2V_m$. If there is a load connected across C_2, it will discharge a little bit and, as a result of it, voltage across it will drop slightly. But it will get recharged in the next half-cycle.

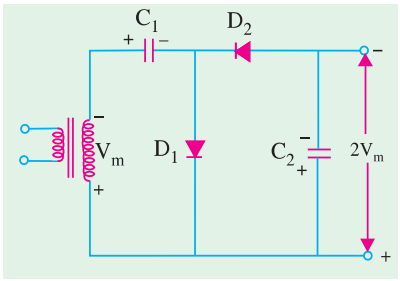


Fig. 55.32

The output waveform shown in Fig. 55.31 (*b*) is that of a half-wave rectifier filtered by a shunt capacitor. *PIV* across each diode is $2V_m$. Ripple frequency is equal to the supply frequency.

This circuit has very poor regulation and its ripple content is also high. This circuit has a common connection between the line and load (which a full-wave doubler does not have).

55.27. Full-wave Voltage Doubler

This circuit is shown in Fig. 55.33 (*a*). During the positive half-cycle of the input voltage, D_1 conducts (but not D_2) and charges capacitor C_1 to the peak voltage V_m with the polarity as shown.

During the negative half-cycle, D_2 conducts (but not D_1) charging C_2 to V_m. As far as the load is concerned, voltages across C_1 and C_2 are in series-aiding. If there is no load connected across the output, then load voltage $V_L = 2V_m$ as shown in Fig. 55.33 (*a*). For example, if 220-V, 50-Hz is the supply, then $V_{dc} = 2V_m = 2 \times \sqrt{2} = 620$ V. Of course, if a load is connected across the output terminals, then V_L would be less than $2V_m$.

The waveform of the output voltage is shown in Fig. 55.33 (*b*).

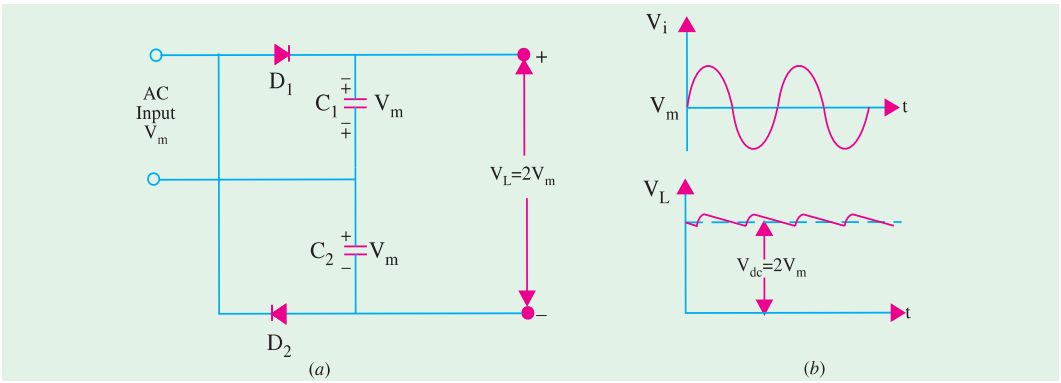


Fig. 55.33

The *PIV* rating of each diode is $2V_m$. Ripple frequency is twice the supply frequency. As seen, there is no common connection between the supply line and the load.

It is worth noting that where the expense of a line transformer is justified, it is preferable to use the superior conventional full-wave rectifier (Art. 55.7).

55.28. Voltage Tripler and Quadrupler Circuits

(*a*) General

The half-wave voltage doubler circuit (Fig. 55.31) can be extended to obtain any multiple of the peak input voltage (V_m) *i.e.* $3V_m$, $4V_m$, $5V_m$ etc. Theoretically speaking, there is no upper limit to the

amount of voltage multiplication that can be obtained. Though voltage triplers and quadruplers are commonly used, practical considerations limit additional multiplications. The main handicap is that total amount of capacitance becomes unduly large to maintain the desired dc output voltage for any thing except extremely light loads.

(b) Circuit

The circuit for different multipliers is shown in Fig. 55.34. It should be obvious from the repetitive pattern of the circuit connections how additional diodes and capacitors may be connected to the doubler circuit for obtaining higher multiplications of the peak output voltage V_m.

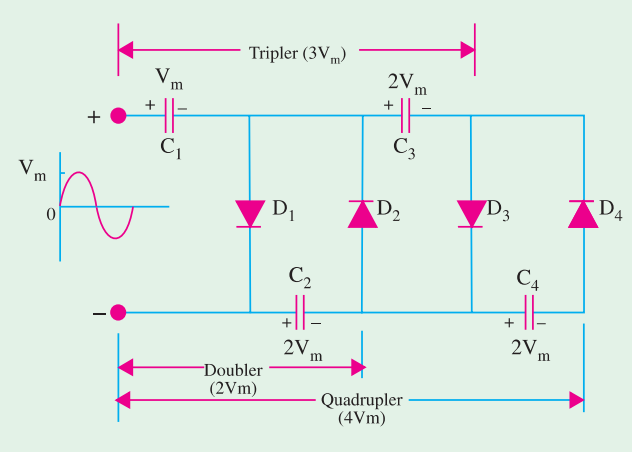


Fig. 55.34

(c) Analysis

During the first positive half-cycle, C_1 charges to V_m as diode D_1 conducts. During negative half-cycle, C_2 is charged through D_2 to $2V_m$ *i.e.* to the sum of voltage across C_1 and peak input voltage V_m (Art 55.26).

During the second positive half-cycle, D_3 conduct and voltage across C_2* charges C_3 to same voltage $2\ V_m$. During the negative half-cycle, diodes D_2 and D_4 conduct allowing C_3 to charge C_4 to the same peak voltage $2V_m$.

If is seen from Fig. 55.34 that voltage across C_2 is $2V_m$, across C_1 and C_3 is $3V_m$ and across C_2 and C_4 is $4V_m$.

If additional diodes and capacitors are used, each capacitor would be charged to a peak voltage of $2V_m$.

The *PIV* rating of each diode is $2V_m$ and ripple frequency is twice the line frequency. Generally, these circuits are used where both the supply voltage and load are maintained constant.

55.29. Troubleshooting Power Supplies

There are usually two types of problems with power supplies *i.e.* either no dc output or low dc output.

The situation of no dc output can occur due to any one of the following reasons :

1. when there is no output from the rectifiers,
2. when there is no ac input to power supply,
3. when filter choke is open,
4. when the first input capacitor shorts.

A low dc output can occur in the following situations :

1. decreased input ac voltage,
2. open input capacitor of the filter circuit,
3. partial short across the load.

55.30. Controlled Rectification

It is that rectification in which the output of a rectifier circuit can be varied by controlling the point in the ac cycle at which the circuit is turned ON. A thyristor or *SCR* can be used for the purpose

* C_1 cannot charge C_3 because it is shorted by D_1.

because under proper firing conditions, it can control the conduction angle of the rectifier circuit.

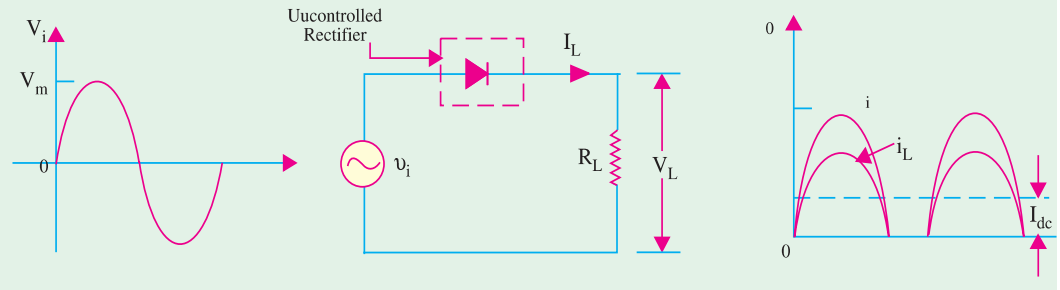


Fig. 55.35

In an ordinary diode rectifier circuit, current flows through the diode whenever instantaneous value of the ac supply voltage is greater than the voltage across the load at that instant.

For a resistive load shown in Fig. 55.35, load current flows at all times during the positive half-cycles of the supply.

In a controlled rectifier, on the other hand, load current flows only when a control signal is applied to turn on the rectifier at a specific point [like *A* in Fig. 55.36 (*b*)] in the ac cycle. Point *A* corresponds to the angle θ_1 so that conduction is delayed by this much period.

Once the HW rectifier is turned ON, it remains in conduction for the rest of the positive half-cycle *i.e.* upto 180°. Obviously, the firing point *A* is determined by the angle of delay in a plying the firing signal by the control circuit. As θ_1 increases, conduction occurs later in the cycle thereby decreasing the load current further.

The curve for voltage drop across rectifier diode is shown in Fig. 55.36 (*c*). During the positive half-cycle when the rectifier is fired into conduction, it acts like a short and voltage across it drops to zero (neglecting forward voltage of the diode). During negative half-cycle when diode is reverse-biased (and hence open) the full supply voltage appears across it as shown.

The curve for load voltage is shown in Fig. 55.36 (*d*). Since current obeys Ohm's law, it follows the load voltage.

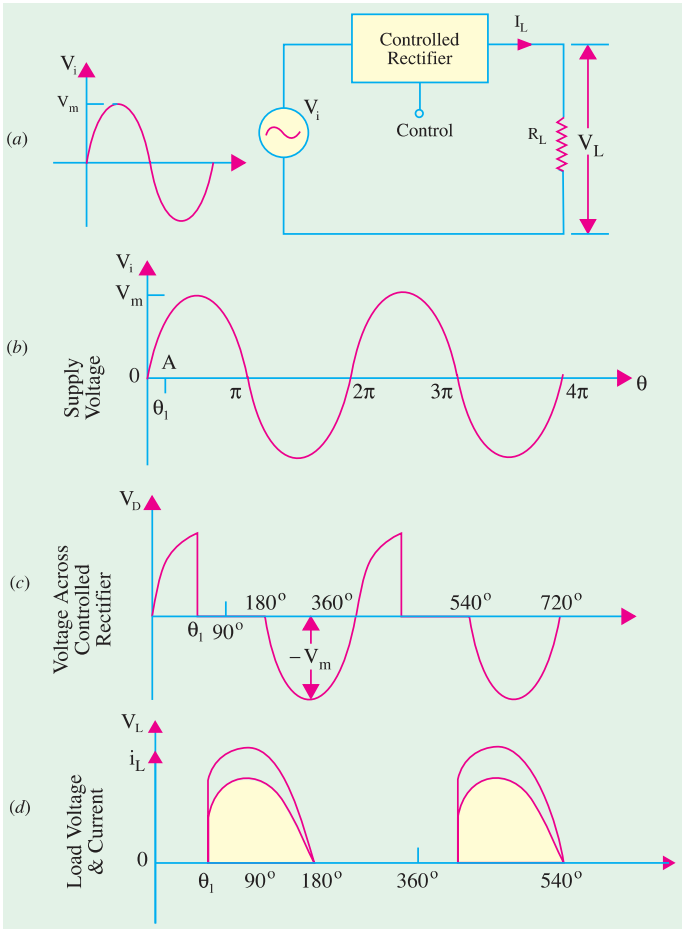


Fig. 55.36

Average Value of Load Voltage

Let the equation of the supply voltage be $V_i = V_m \sin \theta$

For a half-wave rectifier, average value is found by taking average over the whole cycle (2π) even though conduction takes

place only from θ_1–π in the entire cycle.

$$\therefore \quad V_L = \frac{1}{2\pi}\int_{\theta_1}^{\pi} V_i \, d\theta$$

$$= \frac{1}{2\pi}\int_{\theta_1}^{\pi} V_m \sin\theta \, d\theta$$

$$= \frac{V_m}{2\pi}\int_{\theta_1}^{\pi} \sin\theta \, d\theta$$

$$= \frac{V_m}{2\pi}\left|-\cos\theta\right|_{\theta_1}^{\pi}$$

$$= \frac{V_m}{2\pi}\left|(-\cos\pi) - (-\cos\theta_1)\right|$$

$$= \frac{V_m}{2\pi}(1+\cos\theta_1)$$

$$\therefore \quad V_{dc} = V_L = \frac{V_m}{2\pi}(1+\cos\theta_1)$$

For a resistive load,

$$I_{dc} = I_L = \frac{V_m}{2\pi R_L}(1+\cos\theta_1)$$

Note : If

$\theta_1 = 0$, then V_{dc}

$= \frac{V_m}{\pi}$ and $I_{dc} = \frac{V_m}{\pi RL}$

These are the same values as for an uncontrolled or ordinary HW rectifier (Art 55.6).

55.31. Output Waveforms for Different Firing Angles

In all waveforms given in Fig. 55.37 it has been assumed that the ac supply voltage is sinusoidal given by the equation $V_i = V_m \sin\theta$. Also, a resistive load has been assumed.

Following points must always be kept in mind while drawing these diagrams. When the diode is forward-biased, it conducts and behaves like a short. Hence, drop across it is almost zero. Instead, whole of the applied voltage drops across load resistance. When during the negative input half-cycle, diode is reverse-biased, it does not conduct and behaves like an 'open'. Hence, all the applied voltage appears across the rectifier diode and none across R_L. In other words, diode anode voltage and load voltage are mutually exclusive ***i.e. when one is there, the other is not.***

It is seen from Fig. 55.38 that as delay angle θ_1 is increased, the output keeps decreasing till for $\theta_1 = 180°$, ***the output is zero.***

55.32. Output Voltage and Current Values in Controlled Rectifiers

We will use the equation derived in Art 55.30 for a half-wave rectifier *i.e.*

$$V_{dc} = \frac{V_m}{2\pi}(1+\cos\theta_1)$$

$$I_{dc} = \frac{V_m}{2\pi R_L}(1+\cos\theta_1) = \frac{V_{dc}}{R_L}$$

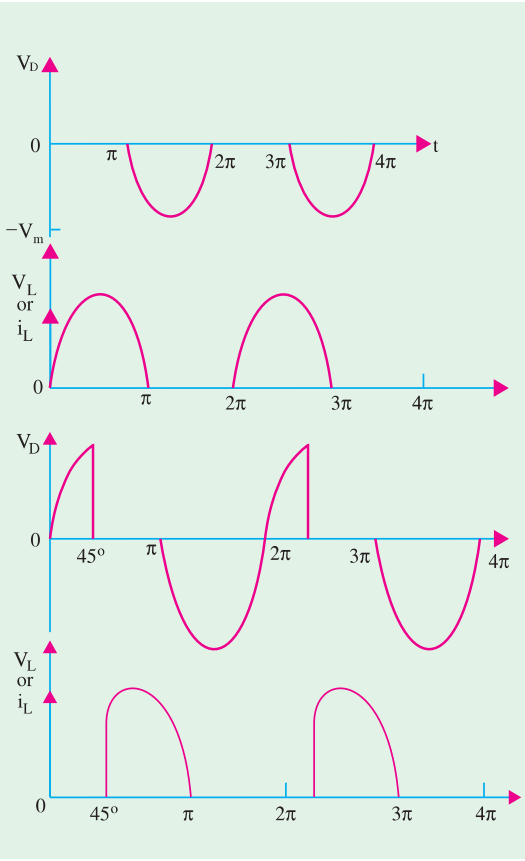


Fig. 55.37

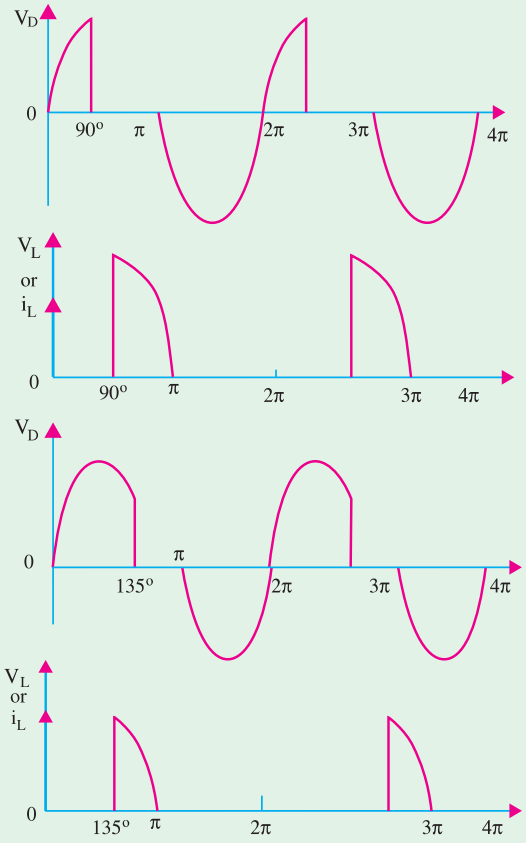


Fig. 55.38

(a) when $\theta_1 = 0$

$$\cos\theta_1 = \cos 0° = 1$$

$$V_{dc} = \frac{V_m}{2\pi}(1+1) = \frac{V_m}{\pi} = 0.318\, V_m$$

$$I_{dc} = \frac{V_{dc}}{R_L} = \frac{V_m}{\pi R_L} = 0.318\,\frac{V_m}{R_L}$$

(b) when $\theta_1 = 30°$

$$\cos\theta_1 = \cos 30° = 0.866$$

$$\therefore \quad V_{dc} = \frac{V_m}{2\pi}(1+0.866) = 0.297\, V_m \quad ; \quad I_{dc} = \frac{V_{dc}}{R_L} = 0.297\,\frac{V_m}{R_L}$$

(c) when $\theta_1 = 45°$

$$\cos\theta_1 = \cos 45° = 0.707$$

$$\therefore \quad V_{dc} = \frac{V_m}{2\pi}(1+0.707) = 0.27\, V_m \; ; I_{dc} = \frac{V_{dc}}{R_L} = 0.27\,\frac{V_m}{R_L}$$

(d) when $\theta_1 = 60°$

$$\cos\theta_1 = \cos 60° = 0.5$$

$$\therefore \quad V_{dc} = \frac{V_m}{2\pi}(1+0.5) = 0.239\, V_m \; ; I_{dc} = \frac{V_{dc}}{R_L} = 0.239\,\frac{V_m}{R_L}$$

(e) when $\theta_1 = 90°$

$$\cos\theta_1 = \cos 90° = 0$$

$$V_{dc} = \frac{V_m}{2\pi}(1+0) = \frac{V_m}{2\pi} = 0.159\,V_m$$

$$I_{dc} = \frac{V_{dc}}{R_L} = \frac{V_m}{2\pi R_L} = 0.159\frac{V_m}{R_L}$$

(f) when $\theta_1 = 120°$

$$\cos\theta_1 = \cos 120° = -0.5$$

$$\therefore \quad V_{dc} = \frac{V_m}{2\pi}(1-10.5) = 0.08\,V_m\,;\; I_{dc} = \frac{V_{dc}}{R_L} = 0.08\frac{V_m}{R_L}$$

(g) when $\theta_1 = 135°$

$$\cos\theta_1 = \cos 135° = -0.707$$

$$V_{dc} = \frac{V_m}{2\pi}(1-0.797) = 0.0466\,V_m\,;\; I_{dc} = 0.0466\frac{V_m}{R_L}$$

Example 55.12. *A 100–Ω load is connected to a peak supply of 300 V through a controlled half-wave diode rectifier. The load power is to be varied from 25 W to 80 W. What is the angular firing control required ? Neglect forward drop of the diode.* **(Basic Electronics, Pune Univ. 1990)**

Solution. $P = V_{dc} \cdot I_{dc}$

Now, $V_{dc} = \frac{V_m}{2\pi}(1+\cos\theta)$. $I_{dc} = \frac{V_m}{2\pi R_L}(1+\cos\theta)$

$$\therefore \quad P = \left(\frac{V_m}{2\pi}\right)^2 \cdot \frac{1}{R_L}(1+\cos\theta)^2 = \left(\frac{300}{2\pi}\right)^2 \cdot \frac{1}{100}(1+\cos\theta)^2$$

$$= 22.5\,(1+\cos\theta)^2$$

(i) when P = 80 W

$$\therefore \quad 80 = 22.5\,(1+\cos\theta)^2 \quad \text{or} \quad (1+\cos\theta) = 1.8856$$

$$\therefore \quad \cos\theta = 0.8856 \quad \text{or} \quad \theta = \mathbf{27.7°}$$

(ii) when P = 25 W

$$\therefore \quad 25 = 22.5\,(1+\cos\theta)^2 \quad \therefore \quad \cos\theta = 0.1111 \quad \text{or} \quad \theta = \mathbf{83.6°}$$

Example 55.13. *In a controlled half-wave rectifier, peak supply voltage is 200 V and the value of load resistor is 1 k Ω. Calculate the power delivered to the load circuit for firing angles of* ***(i)*** *0°* ***(ii)*** *45°* ***(iii)*** *90° and* ***(iv)*** *135°.* **(Solid State Devices and Ckts, BHU)**

Solution. (i) $V_{dc} = 0.318 \times 200 = 63.6$ V ; $I_{dc} = 63.6/1000 = 63.6$ mV

$P = V_{dc} \cdot I_{dc} = 63.6 \times 63.6 = 4045$ mW = **4.045 W**

(ii) $V_{dc} = 0.27 \times 200 = 54$ V ; $I_{dc} = 54/1000 = 54$ mA

$P = 54 \times 54 = 2916$ mW = **2.916 W**

(iii) $V_{dc} = 0.159 \times 200 = 31.8$ V ; $I_{dc} = 31.8$ mA

$P = 31.8 \times 31.8 = 1011$ mW = **1.011 W**

(iv) $V_{dc} = 0.0466 \times 200 = 9.3$ V ; $I_{dc} = 9.3$ mA

$P = 9.3 \times 9.3 = 86.5$ mW = **0.0865 W**

55.33. Average Values for FW Controlled Rectifier

In this case, the average values would be doubled because there are two half sinusoids to be averaged as shown in Fig. 55.39 for $\theta_1 = 60°$.

$$V_{dc} = 2 \times \frac{V_m}{2\pi}(1 + \cos\theta_1) = \frac{V_m}{\pi}(1 \cos\theta_1)$$

$$\left[I_{dc} = \frac{V_{dc}}{R_L} = \frac{V_m}{\pi R_L}(1 + \cos\theta_1) \right]$$

In the present case, $\theta_1 = 60°$, $\cos\theta_1 = 0.5$

$$\therefore \quad V_{dc} = \frac{V_m}{\pi}(1 + 0.5) = 0.477\, V_m$$

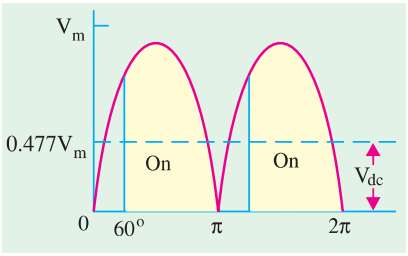


Fig. 55.39

55.34. Silicon Controlled Rectifier (*SCR*)

It is a trijunction *PNPN* device having three external connections : anode (*A*), cathode (*C*) and gate (*G*) as shown in Fig. 55.40.

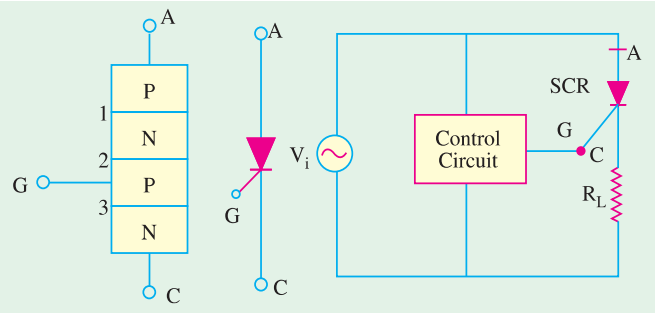


Fig. 55.40



SCR and diode tester

When *A* is made positive with respect to *C*, the junctions marked 1 and 3 are forward-biased but the centremost junction marked 2 is reverse-biased. Hence, the *SCR* does not conduct. However, if a sufficiently positive voltage pulse is applied to this junction via the gate *G*, the *SCR* starts conducting like an ordinary diode. Gate has no control over *SCR* once it has been triggered into conduction. Conduction can only be terminated by removing positive voltage from its anode.

As shown in Fig. 55.40 (*b*), *SCR* needs a control circuit which triggers it into conduction by a gate pulse. Two popular ways of turning ON an *SCR* are:

1. Amplitude firing–in which gate current flows for the whole period of conduction.
2. *UJT* oscillator firing–in this method, gate current is supplied only momentarily. This method allows control over the power delivered to *SCR*'s load from zero watt to fully ON.

Some of the simple ways of controlling the conduction of an *SCR* are discussed below.

55.35. Pulse Control of *SCR*

The simplest *SCR* control circuit of this type is shown in Fig. 55.41. If *SCR* were an ordinary rectifier, it would produce half-wave rectified ac voltage across R_L. However, *SCR* will not conduct even during positive half-cycle of the input ac voltage unless *G* is given positive voltage to forward-bias its centremost junction. By applying a trigger pulse at any time during the positive input half-cycle, it can be fired into conduction.

The resultant load waveform across R_L will consist of a portion of the positive half-cycle commencing at the instant at which the *SCR* is triggered [Fig. 55.41 (*c*)].

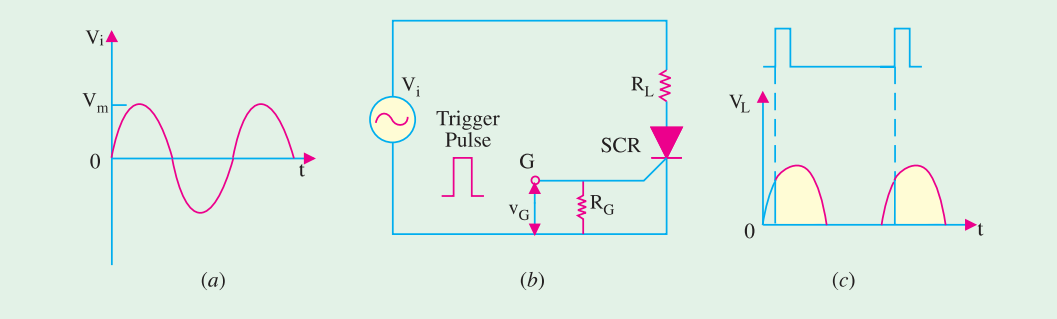


Fig. 55.41

55.36. 90° Phase Control of *SCR*

Fig. 55.42 shows a circuit which can trigger (or switch on) *SCR* any where from the commencement of the ac cycle to the peak of its positive half-cycle *i.e.* between 0° and 90°. As shown, gate current is derived from the ac supply (often it is from rectifier output as in Fig. 5.44) *via R*. If *R* is set at low value, the *SCR* will trigger almost at the commencement of the positive half-cycle of the ac input. On the other hand, if *R* is set at high resistance, the *SCR* may not switch ON until the peak positive half-cycle when sufficiently large gate current would become available.

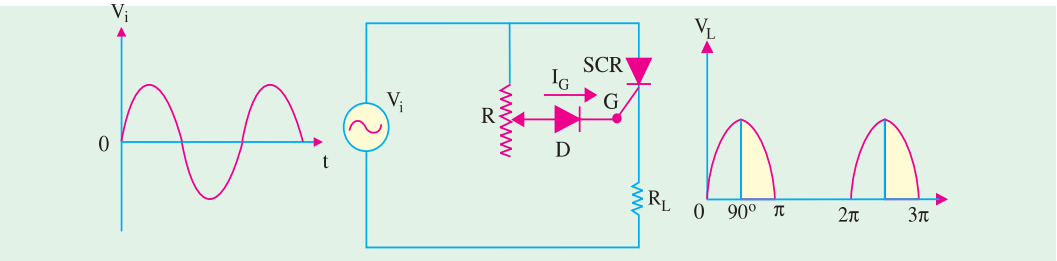


Fig. 55.42

If I_G is not large enough even at peak positive half-cycle, then *SCR* will not trigger at all because I_G has the greatest value at the peak input and then falls off as the voltage falls. The purpose of diode *D* is to protect the *SCR* gate from negative voltage during the negative input half-cycle. This method is also known as the amplitude firing of an *SCR*.

55.37. 180° Phase Control of *SCR*

The circuit shown in Fig. 55.43 can trigger the *SCR* from 0° to 180° of the input waveform.

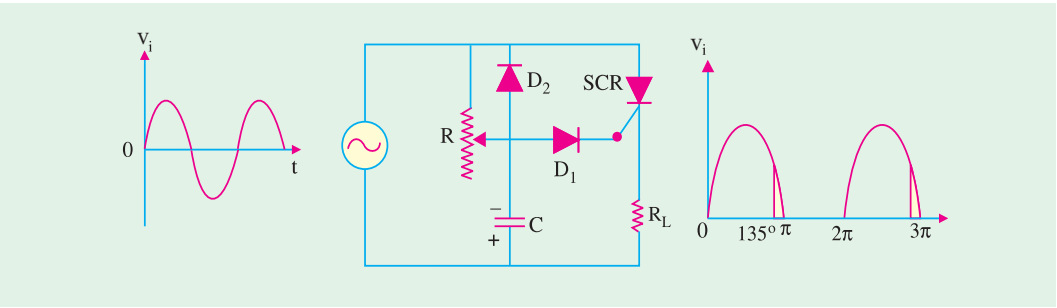


Fig. 55.43

The circuit is identical to that of Fig. 55.42 except for the addition of the diode D_2 and capacitor *C*.

We will start the analysis with the negative half-cycle. During the negative half-cycle of the input, C is charged immediately (with the polarity as shown) to the peak of the input voltage because D_2 is forward-biased. When peak of the negative half-cycle passes over, D_2 becomes reverse-biased because its anode (connected to C) becomes more negative than its cathode (connected to the supply). Hence, C starts to discharge through R. Depending on the time constant (= CR), C may almost be completely discharged at the commencement of the oncoming positive half-cycle or may retain partial charge until almost 180° of positive half-cycle has passed. So long as C remains negatively-charged, D_1 is reverse-biased and the gate cannot become positive to trigger the SCR into conduction. Hence, R and/or C can be adjusted to trigger the SCR anywhere from 0° to 180° of the input ac cycle.

55.38. *SCR* Controlled Load Circuit

We will now consider a circuit where an amplitude-fired SCR is used to control the power in a full-wave rectifier circuit.

In Fig. 55.44, R_L is the load resistance which may be a lamp load, heater or a small dc motor whose power we wish to control. The full-wave bridge rectifier furnishes the rectified output shown in Fig 55.45 (*a*). Obviously, control over both half-cycles of the ac input is possible.

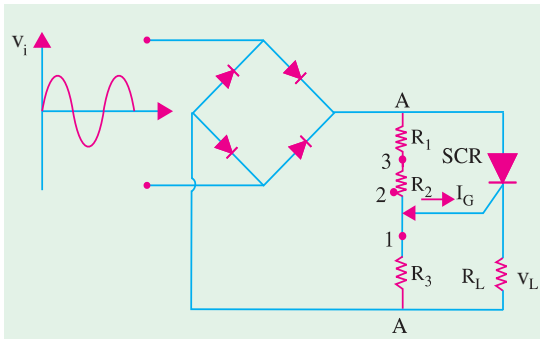


Fig. 55.44

The rectifier output voltage is dropped across the potentiometer $A\,A$. When potentiometer is set at point 1, drop across R_3 is not enough to provide sufficient gate current to switch ON the SCR. Hence, there is no load current and, consequently, no drop across the load. When potentiometer is set at point 2 and then 3, the SCR gets fired yielding load voltage waveforms shown in Fig. 55.45 (*b*) and (*c*). Hence, changing the moving contact on the potentiometer changes the conduction angle and hence the amount of power delivered to the load.

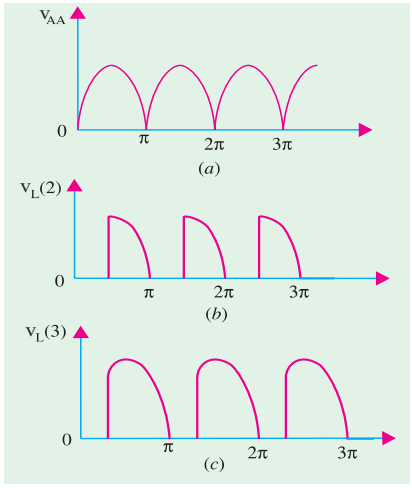


Fig. 55.45

55.39. UJT Controlled Load Circuit

This circuit employs a UJT oscillator to control the firing of the SCR as shown in Fig. 55.46. The full-wave rectified output voltage VAF is available across two parallel paths AF and BF. When SCR is not conducting (*i.e.* it acts as an open), then whole output voltage drops across it and none across R_L. When it conducts (*i.e.* acts as a short), whole of VAF drops across R_L and none across it.

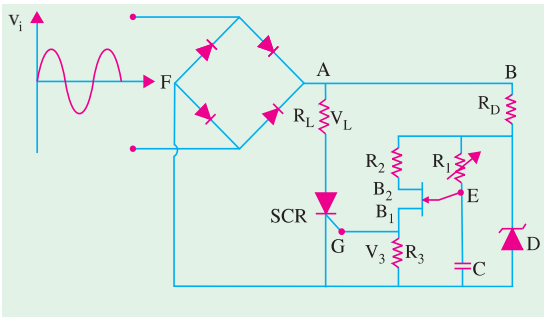


Fig. 55.46

Now, consider the other parallel circuit BF. R_D is the voltage dropping resistor so that a suitable voltage $V\upsilon$ is applied to the UJT oscillator circuit for the UJT to work without damage. The full-wave rectified voltage is clipped at a convenient level by Zener diode D for proper operation of UJT. Varying R_1 varies the time it takes for C to charge to the UJT's firing voltage. Charging of C always starts when V_U is at 0 V dc or at 0° of V_{AF}.

If R_1 is very small, C charges very quickly ($\because$ time constant CR_1 is very short) and reaches the

UJT firing voltage soon after V_{AF} starts to go positive. Hence, *UJT* fires early on every cycle of V_{AF}. When *UJT* fires, it becomes almost a short between its emitter (E) and B_1. Thus, it discharges C very quickly through R_3 which is in parallel with the *SCR*'s gate. This triggers the *SCR* early in the cycle of V_{AF} and the *SCR* latches itself ON until V_{AF} falls to zero. When V_{AF} goes to zero, the *SCR* is shut OFF. The cycle repeats itself with the *SCR* being turned ON early in the cycle thereby delivering nearly full-power to the load.

When R_1 is increased, V_C rises slowly and reaches the *UJT* firing voltage later during each cycle of *VAF*. Therefore, *SCR* is turned ON much later thereby considerably cutting down the power delivered to the load as shown in Fig. 55.47. Obviously, the *SCR* is being fired during the last few degrees of the cycle.

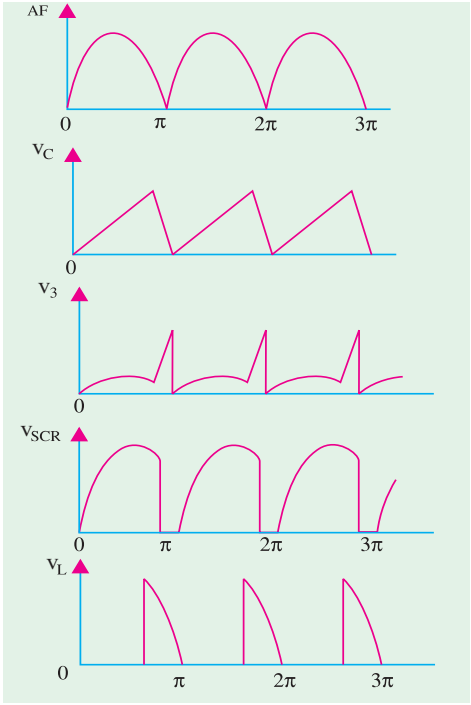


Fig. 55.47

55.40 Chopper

(a) Definition

To put it simply, a chopper is a ***dc-to-dc converter.*** It converts a given constant dc voltage into a variable average dc voltage across a load by placing a high-speed static switch between the dc source and the load. This high-speed static switch is called a chopper because it chops off the dc supply into ON and OFF periods of flow.

(b) Basic Circuit

A basic chopper circuit is shown in Fig. 55.48. When switch S is closed, the dc supply voltage V_{dc} is applied across the load and when it is open, the load is disconnected from the supply. By varying the ratio of the switch-closed time (T_{ON}) to the switch-open time (T_{OFF}) at a fixed frequency, the value of the average output dc voltage can be controlled.

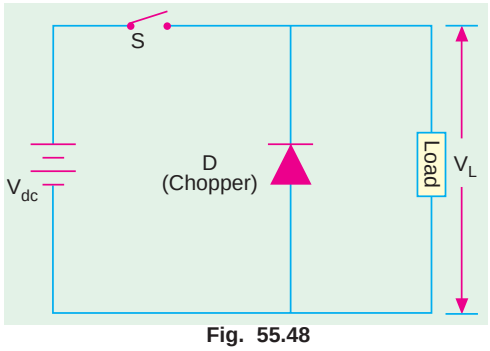


Fig. 55.48

The switch S in Fig. 55.48 could be either a transistor or an *SCR* depending on the amount of power involved. An *SCR* is used in high-power applications whereas transistors are used when power involved is low. A dc chopper circuit using an *SCR* is shown in Fig. 55.49. The *SCR* acts like a static switch and has two states of ON and OFF. The duration of ON and OFF states can be varied with the help of triggering and commutating circuits (not shown) respectively. By changing the values of ON and OFF periods, average dc load voltage can be changed.

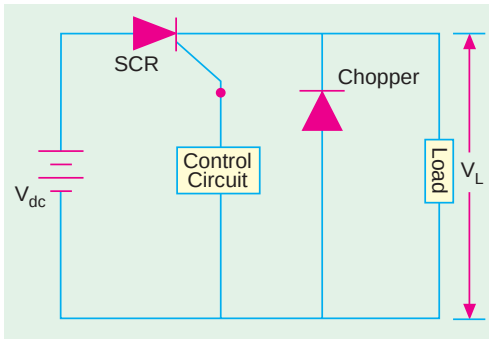


Fig. 55.49

(c) Working

When *SCR* is triggerred into conduction with the help of control circuitry, full dc voltage V_{dc} is applied across

the load for a period of T_{ON}. When the *SCR* is switched OFF by the control circuitry, there is no voltage across the load. The output load voltage is in the form of a square wave as shown in Fig. 55.50. The waveshapes of the load current and chopper current for an inductive load are as shown. The freewheeling diode *D* provides path for the stored inductive energy to flow.

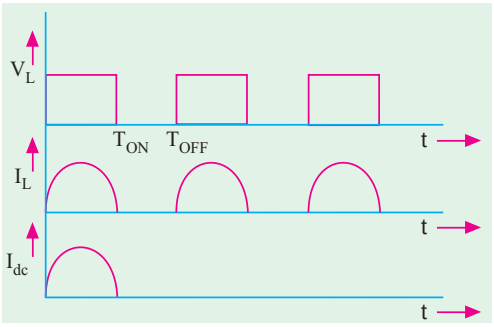


Fig. 55.50

(*d*) Calculations

If T_{ON} is the ON time and T_{OFF} is the OFF time of the chopper, the duty cycle of the chopper is given by

$$\text{duty cycle} = \frac{T_{ON}}{T} = \frac{T_{ON}}{T_{ON} + T_{OFF}}$$

The load voltage V_L is given by

$$V_L = V_{dc}\left(\frac{T_{ON}}{T}\right) = V_{dc} \times \text{duty cycle} = f\, V_{dc}\, T_{ON}$$

where f (= 1/T) is the switching frequency of the chopper.

It is seen from the above that V_L depends on the duty cycle since V_{dc} is constant. Hence, following different methods of controlling V_L are available.

1. keeping T_{OFF} constant, varying T_{ON} ;
2. keeping T_{ON} constant, varying T_{OFF} ;
3. varying the ratio T_{ON}/T_{OFF} ;
4. any combination of the above.

Example 55.14. *A dc chopper has ON time of 30 μs and OFF time of 10 μs. Calculate* ***(i)*** *chopper duty cycle,* ***(ii)*** *chopping frequency.*

Solution. (*i*) duty cycle $= T_{ON}/T = 30/(30 + 10) = 0.75$

(*ii*) chopping frequency, $f = 1/T = 1/(30 + 10) \times 10 - 6$

$= 25{,}000$ Hz = **25 kHz**

Example 55.15. *A chopper supplied by a 200 V dc has ON time of 30 ms and OFF time of 10 ms. Determine the value of the average dc output voltage.*

(Industrial Electronics, Mysore Univ. 1993)

Solution. $T_{ON} = 30 \times 10 - 3\ s, \quad T_{OFF} = 10 \times 10^{-3}\ s,$

$T = 40 \times 10 - 3\ s$

Duty cycle of the chopper $= 30 \times 10^{-3} / 40 \times 10^{-3} = 0.75$

$V_L = V_{dc} \times \text{duty cycle} = 200 \times 0.75 =$ **150 V**

55.41. Inverters



Power Inverter

An inverter is a device that ***changes dc power into ac power*** (just the opposite of converters). The inversion process can be achieved with the help of transistors, *SCR*s and tunnel diodes etc. For low and medium outputs, transistorised inverters are suitable but for high power outputs, *SCR* inverters are essential. For very low voltage and high current requirements, tunnel diode invertors are used.

For inverter applications, transistors have definite advantages over *SCR*s regarding the switching speed, simplicity of control circuitry, high efficiency and greater reliability. It is mainly due to this fact the *SCR* inverters require

complicated circuitry for triggerring and commutation.

The basic working principle of an inverter may be explained with the help of circuit shown in Fig. 55.51. It is called voltage-driven inverter because a dc voltage source is connected through semiconductor switches directly to the primary of a transformer.

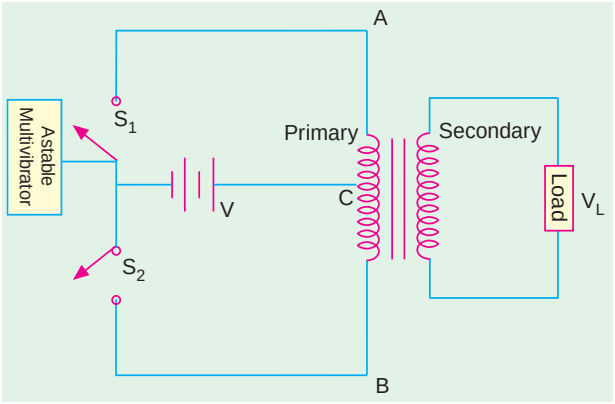


Fig. 55.51

In Fig. 55.51, S_1 and S_2 are switching devices (transistors of *SCR*s) which open and close alternately at regular intervals of time. The two switching devices are generally driven by an astable multi-vibrator operating at the desired frequency. When S_1 is closed, the entire dc source voltage V is applied across points A and B of the transformer primary. S_1 remains closed for a certain period of time after which it is cut off and S_2 closes. It also remains closed for the same period of time during which the source voltage V is impressed across points B and C of the primary. S_2 then opens out and S_1 closes. In this way, an alternating voltage is applied across the primary which induces an ac voltage in the secondary. Since dc supply voltage is connected directly across the primary, the output waveform of the secondary voltage is a square wave (Fig. 55.52) irrespective of the type of load and load power factor. However, the waveforms of both the primary and secondary currents depend on the type of load whether resistive, inductive or capacitive.

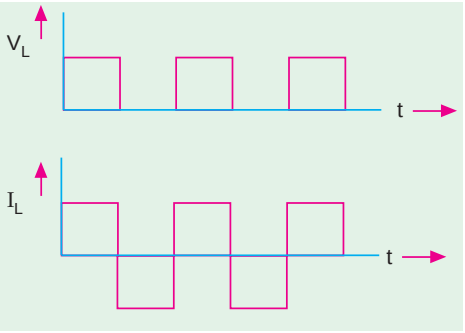


Fig. 55.52

55.42. Single-phase Inverter

Fig. 55.53 (*a*) shows a single-phase inverter with a load resistor using 4 *SCR*s working in pairs. The triggerring and commutating circuitry of the *SCR*s has not been shown in the figure. The two thyristors SCR_1 and SCR_4 are triggerred simultaneously so that load current passes through R_L from left to right. Exactly when these two *SCR*s are switched off by the commutating circuitry, thyristors SCR_2 and SCR_3 are triggerred into conduction thereby sending current through R_L from right to left. Hence, an ac voltage is developed across the load whose waveform is as shown in Fig. 55.53 (*b*).

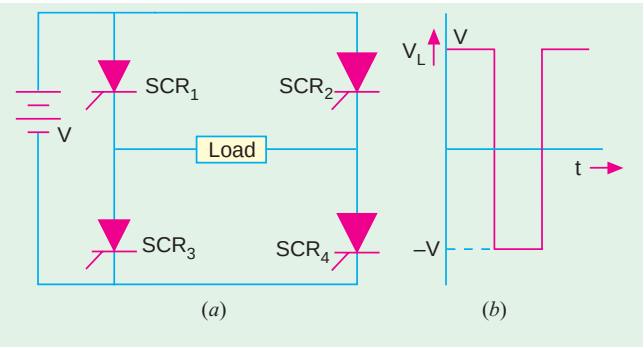


Fig. 55.53



Single-phase inverter

55.43. Push pull Inverter

Fig. 55.54 shows an inverter which

employs two *SCR*s and one transformer. These two *SCR*s are triggerred into conduction alternately for the same period of time. As a result, current through the primary becomes alternating which induces an ac voltage across the secondary and hence the load. As explained earlier in Art 55.42, the secondary ac voltage has a square waveform. The capacitor *C* is connected across the anodes of the two *SCR*s and provides commutation *i.e.* switching off of the *SCR*s. The capacitor charges to double the supply voltage as a result of transformer action between the two halves of the primary winding. This large voltage is sufficient to reverse-bias the *SCR*s and drive the holding current below its rated value.

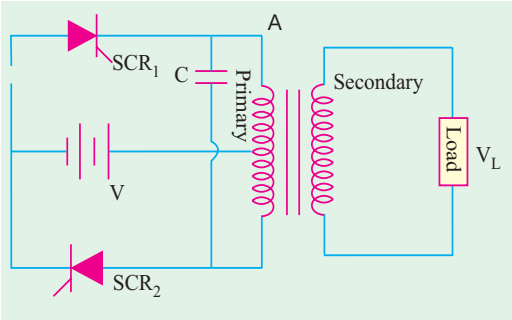


Fig. 55.54

Tutorial Problems No. 55.1

1. A 1-ϕ half-wave rectifier using a 10 : 1 transformer supplies power to a 9 Ω load. If the primary input voltage has an r.m.s. value of 200 V and forward diode resistance is 0.2 Ω and transformer secondary resistance is 0.8 W, determine
(*i*) $I_L(dc)$, (*ii*) r.m.s. ripple voltage and (*iii*) efficiency **[(*i*) 0.9 A(*ii*) 9.8 V(*iii*) 36.54%]**

2. A single-phase full-wave rectifier using a power transformer has secondary voltage of 100-0-100V (r.m.s.). It supplies a load of 1 K. Neglecting transformer losses and forward voltage drop of the diode, determine :
(*a*) dc output voltage, (*b*) dc output current, (*c*) ripple voltage and *PIV* rating of the diodes.
[(*a*) 90 V (*b*) 90 mA (*c*) 43.4 V (rms.) ; 282 V]

3. A single-phase half-wave rectifier supplies power to a 1 K load. The rectifier voltage is 200 V (r.m.s.). Neglecting diode resistance, calculate (*i*) dc load voltage, (*ii*) dc load current and (*iii*) r.m.s. ripple voltage. **[(*i*) 90 V(*ii*) 90 mA(*iii*) 1.09 V]**

4. A single-phase half-wave diode rectifier supplies power to a 2-kΩ resistive load. The input ac supply voltage has a peak value of 300 V. Neglecting forward drop of the diode, calculate
(*a*) V_{dc}, (*b*) I_{dc}, (*c*) power delivered to the load, (*d*) ripple voltage (rms value)
[(*a*) 95.4 V(*b*) 47.7 mA (*c*) 4550 mW (*d*) 115.4 V]

5. A full-wave diode rectifier supplies a load of 10 kΩ. The ac voltage applied to the diode is 300-0-300 V rms. It diode resistance is neglected, calculate: (*a*) V_{dc}, (*b*) I_{dc}, (*c*) I_{rms}, (*d*) form factor, (*e*) ripple voltage. **[(*a*) 270 V (*b*) 27 mA (*c*) 30 mA (*d*) 1.11 (*e*) 130.1 V]**

6. A dc and an ac voltmeter are used to measure the output voltage of a filter circuit. The readings of the two voltmeters are 50 V and 5 V respectively. Calculate the ripple factor of the filter. **[10%]**

7. In a controlled full-wave rectifier, peak supply voltage is 200 V and load resistance 1 kW. Calculate the power delivered to the load for firing angles of (*a*) 60° and (*b*) 120°. **[(*a*) 9.1 W (*b*) 1.01 W]**

OBJECTIVE TESTS – 55

1. The ripple factor of a power supply is a measure of
(*a*) its filter efficiency
(*b*) its voltage regulation
(*c*) diode rating
(*d*) purity of power output.

2. The basic reason why a FW rectifier has twice the efficiency of a HW rectifier is that
(*a*) it makes use of a transformer
(*b*) its ripple factor is much less
(*c*) it utilizes both half-cycle of the input
(*d*) its output frequency is double the line frequency.

3. The output of a half-wave rectifier is suitable only for
(*a*) running car radios
(*b*) running ac motors
(*c*) charging batteries
(*d*) running tape-recorders.

4. The ripple factor of a bridge rectifier is
(*a*) 0.406 (*b*) 0.812
(*c*) 1.21 (*d*) 1.11

5. The ripple factor of a power suply is given by (symobls have the usual meaning).

(*a*) $\frac{P_{dc}}{P_{ac}}$ (*b*) $\sqrt{\left(\frac{I_{rms}}{I_{dc}}\right)^2 - 1}$

(*c*) $\sqrt{\left(\frac{I_{dc}}{I_{rms}}\right)^2 - 1}$ (*d*) $\frac{I_{dc}}{I_{rms}}$

6. The PIV of a half-wave rectifier circuit with a shunt capacitor filter is

(*a*) $2V_{sm}$ (*b*) V_{sm}

(*c*) $V_{sm}/2$ (*d*) $3V_{sm}$.

7. The primary function of a rectifier filter is to

(*a*) minimise ac input variations

(*b*) suppress odd harmonics in the rectifier output

(*c*) stabilise dc level of the output voltage

(*d*) remove ripples from the rectified output

8. In a rectifier, larger the value of shunt capacitor filter

(*a*) larger the *p-p* value of ripple voltage

(*b*) larger the peak current in the recti- fying diode

(*c*) longer the time that current pulse flows through the diode

(*d*) smaller the dc voltage across the load.

9. In a LC filter, the ripple factor,

(*a*) increases with the load current

(*b*) increases with the load resistance

(*c*) remains constant with the load current

(*d*) has the lowest value.

10. The main reason why a bleeder resistor is used in a dc power supply is that it

(*a*) keeps the supply ON

(*b*) improves voltage regulation

(*c*) improves filtering action

(*d*) both (*b*) and (*c*).

11. Which stage of a dc power supply uses a Zener as the main component?

(*a*) rectifier (*b*) voltage divider

(*c*) regulator (*d*) filter.

12. Which rectifier requires four diodes ?

(*a*) half-wave voltage doubler

(*b*) full-wave voltage doubler

(*c*) full-wave bridge circuit

(*d*) voltage quadrupler.

13. For a half-wave controlled rectifier, the average value of output dc voltage is given by

(*a*) $V_{dc} = \frac{V_m}{2\pi}(1 - \cos\theta_1)$

(*b*) $V_{dc} = \frac{2V_m}{\pi}(1 + \cos\theta_1)$

(*c*) $V_{dc} = \frac{V_m}{\pi}(\cos\theta_1 - 1)$

(*d*) $V_{dc} = \frac{V_m}{2\pi}(\cos\theta_1 + 1)$.

14. If, by mistake, ac source in a bridge rectifier is connected across the dc termi- nals, it will burn out and hence short ..... diodes.

(*a*) one (*b*) two

(*c*) three (*d*) four.

15. The circuit in Fig. 55.55 shows a full-wave rectifier. The input voltage is (rms) single-phase ac. The peak reverse voltage across the diodes D1 and D2 .

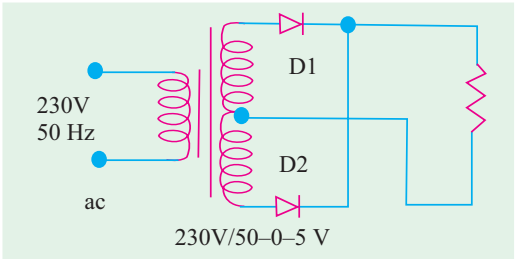


Fig. 55.55

(*a*) $100\sqrt{2}$ V (*b*) 100 V

(*c*) $50\sqrt{2}$ V (*d*) 50V

(GATE ; 2004)

16. The circuit in Fig. 55.56 shows a 3-phase half-wave rectifier. The source is a symmetrical, 3-phase four-wire system. The line-to-line voltage of the source is 100 V. The supply frequency is 400 Hz. The ripple frequency at the output is

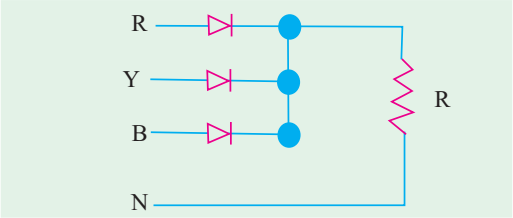


Fig. 55.56

(*a*) 400 Hz (*b*) 800 Hz

(*b*) 1200 Hz (*d*) 2400 Hz

(GATE ; 2004)

ANSWERS

1. *(a)* **2.** *(c)* **3.** *(b)* **4.** *(c)* **5.** *(b)* **6.** *(a)* **7.** *(d)* **8.** *(b)* **9.** *(c)* **10.** *(d)* **11.** *(c)* **12.** *(b)* **13.** *(c)* **14.** *(d)* **15.** *(a)* **16.** *(c)*

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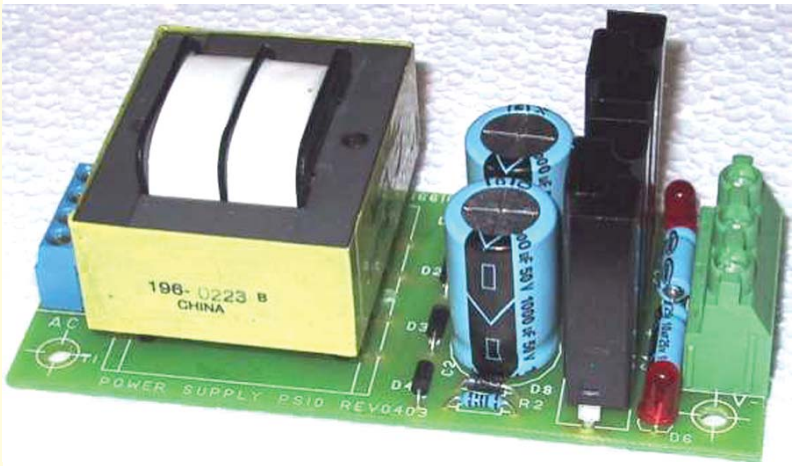
CHAPTER 56

REGULATED POWER SUPPLY

Learning Objectives

- General
- Voltage Regulation
- Zener Diode Shunt Regulator
- Transistor Series Voltage Regulator
- Controlled Transistor Series Regulator
- Transistor Shunt Voltage Regulator
- Transistor Current Regulator
- Variable Feedback Regulator
- Basic OP-AMP Series Regulator
- Basic OP-AMP Shunt Regulator
- Switching Regulators
- Step-down Switching Regulator
- Step-up Switching Regulator
- Inverting Switching Regulator
- IC Voltage Regulators
- Fixed Positive Linear Voltage Regulators
- Fixed Negative Linear Voltage Regulators
- Adjustable Positive Output Linear Voltage Regulators
- Adjustable Negative Output Linear Voltage Regulators
- Use of External Pass Transistor with Linear Voltage Regulators
- Use of Linear Voltage Regulator as a Current Regulator
- Switching Voltage IC Regulators

CONTENTS



Voltage regulator provides a constant dc output voltage that is essentially independent of the input voltage, output load current and temperature.

56.1. General

The various power-supply circuits considered in Chapter 5 suffer from the drawback that their dc output voltage changes with changes in load or input voltage. Such a dc power supply is called unregulated power supply. Regulated power supply can be obtained by using a voltage regulator circuit. A regulator is an electronic control circuit which is capable of providing a nearly constant dc output voltage even when there are variations in load or input voltage. A source of regulated dc power is essential for all communication, instrumentation, computers or any other electronic system.

We will consider both ***linear*** regulators and ***switching*** regulators which are also available in integrated circuit form. In linear regulators, the transistor operates somewhere between saturation and cut-off. It is always ON and dissipates power. Hence, its efficiency (output power/input power) is 50 per cent or less. In switching regulators, the transistor operates like a switch *i.e.* it is either saturated or cut-off. Hence, its power efficiency is 90 per cent or more.

The linear regulators are of two basic type *i.e.* series regulators and shunt regulators. Likewise switching regulators can be of three basic types **(*i*)** step-down type, **(*ii*)** step-up type and **(*iii*)** inverting type.

56.2. Voltage Regulation

As stated above, in an unregulated power supply, output voltage changes whenever input supply voltage or load resistance changes. It is never constant. The change in voltage from no-load to full-load condition is called ***voltage regulation.*** The aim of a voltage regulator circuit is to reduce these variations to zero or, at least, to the minimum possible value.

The percentage regulation or, simply, regulation of a power supply is given by

$$\% \text{ regulation} = \frac{V_{max} - V_{min}}{V_{max}} \times 100$$

where V_{max} = maximum dc output voltage and V_{min} = minimum dc output voltage.

When we say that 10 V regulated dc power supply has a regulation of 0.005 per cent, it means that dc output voltage will vary within an envelope 0.005 per cent of 10 V.

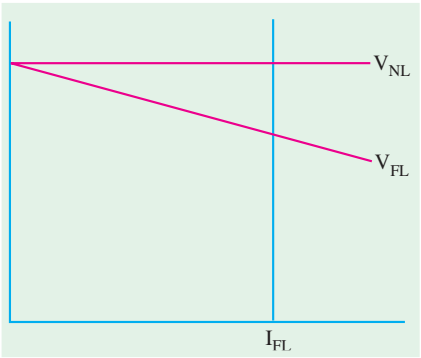


Fig. 56.1

Now, $0.005\% \text{ of } 10 \text{ V} = \frac{V_{max} - V_{min}}{V_{max}} \times 100$

$= 0.0005 \text{ V} = 0.5 \text{ mV}$

Hence, output voltage will vary by ± 0.25 mV. So, we see that instead of expressing voltage regulation by unwieldly expression 0.005 per cent, we can express it by a simple figure of ± 0.25 mV.

In general, $\% \text{ regn.} = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100$

where, V_{NL} = no-load or open-circuit terminal voltage of the supply (Fig. 56.1).

V_{FL} = full-load terminal voltage of the supply

In an ideal or perfectly regulated dc power supply, the percentage voltage regulation is zero. This voltage regulation is also called ***load regulation.***

56.3. Zener Diode Shunt Regulator

A simple shunt voltage regulating system using a zener diode is shown in Fig. 56.2. The

input voltage V_{in}, in fact, is the unregulated output of a rectifier. This simple regulator restricts output voltage variations within reasonable limits around V_z in the face of changing load current or changing input voltage. Obviously, the Zener diode will regulate so long as it is kept in reverse conduction.

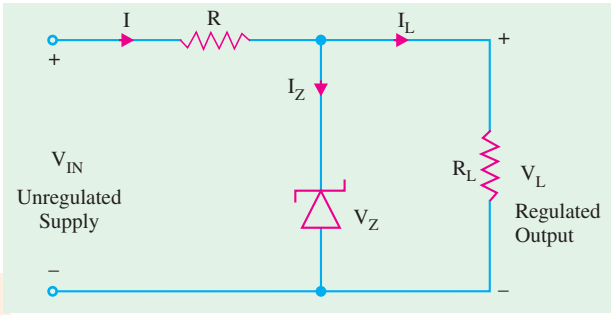


Fig. 56.2

Example 56.1. *The Zener diode of Fig. 56.2 has the following ratings :*

$V_z = 6.8\ V$ at $I_z = 50\ mA$

$r_z = 2\ \Omega$ at $I_z = 50\ mA$

$I_{z(min)} = 5\ mA$ $I_{z(max)} = 150\ mA$

What would be the load voltage when load current I_L varies from 10 mA to 120 mA ? Also, calculate voltage regulation of the regulator. **(Power Electronics-I, Punjab Univ. 1992)**

Solution. We will call V_z = 6.8 V and I_z = 50 mA as reference values and calculate changes in voltage with respect to these values.

(*i*) I_L = 120 mA

Obviously, $I_z = 150 - 120 = 30$ mA

Deviation from 50 mA $= 30 - 50 = -20$ mA

Drops across diode $= I_z.r_z = -20 \times 2 = -40$ mV

$V_L = V_z + I_Z.r_z = 6.8 + (-40 \times 10^{-3}) =$ **6.76 V**

(*ii*) I_L = 10 mA

Now, $I_z = 150 - 10 = 140$ mA

Deviation from reference value $= 140 - 50 = 90$ mA

Diode drop $= I_z r_z = 90 \times 10^{-3} \times 2 = 0.18$ V

$\therefore$ $V_L = V_z + I_z r_z = 6.8 + 0.18 =$ **6.98 V**

$$\% \text{ regn.} = \frac{6.98 - 6.76}{6.98} \times 100 = 3.15\ \%$$

For many applications, a change in load voltage of 3.15% is acceptable but, in some, it may be intolerable. This regulation can be reduced to 1% or less with the help of circuits discussed below.

56.4. Transistor Series Voltage Regulator

The circuit is shown in Fig. 56.3. It is also called ***emitter-follower*** regulator because the voltage at the emitter follows the base voltage. In this set-up, the transistor behaves like a variable resistor whose resistance is determined by the base current. It is called ***pass*** transistor because total current to be regulated passes through it.

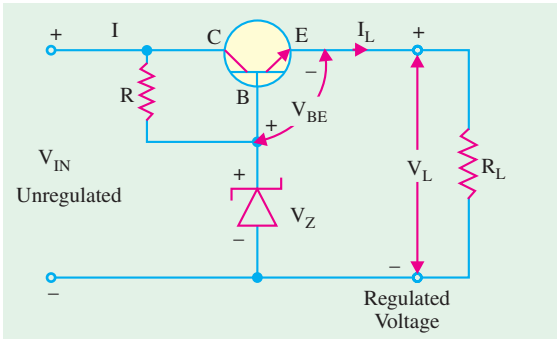


Fig. 56.3

Keeping in mind the polarities of different voltages, they are related by the equation derived from *KVL*

$$V_L + V_{BE} - V_z = 0$$

$$\therefore \quad V_{BE} \text{ (fixed)} = V_z - V_L$$

When current demand is increased by decreasing R_L, V_L tends to decrease. As seen from the above equation, it will increase V_{BE} because V_z is fixed. This will increase forward bias of the transistor thereby increasing its level of conduction. This, is turn, will lead to decrease in the collector-emitter resistance of the transistor which will slightly increase the input current in order to compensate for decrease in R_L so that $V_L = (I_L R_R)$ will remain at a constant value. Incidentally, R is used for limiting current passing through the Zener diode.

56.5. Controlled Transistor Series Regulator

The circuit employing a second transistor T_2 as a sensing element is shown in Fig. 56.4. It has the additional feature of control with the help of potentiometer $R_1 - R_2$. In the discussion to follow, it will be assumed that I is much greater than I_{B2}. Now, there is a drop of V_L on $(R_1 + R_2)$ and a drop of $(V_z + V_{BE2})$ across R_2.

$$\frac{V_L}{V_z + V_{BE2}} = \frac{R_1 + R_2}{R_2} \quad \text{or} \quad V_L = \frac{R_1 + R_2}{R_2}(V_z + V_{BE2})$$

Now, $(R_1 + R_2)$ and $(V_z + V_{BE2})$ both have constant values so that $V_L \propto 1/R_2$. If the potentiometer is adjusted so that R_2 decreases, then V_L increases and *vice versa*.

Suppose R_L is decreased. Then, I_L increases but V_L decreases. Decreases in V_L decreases I_{B2} and I_{C2}. Assuming I_3 to be relatively constant (or decreasing only slightly), I_{B1} is increased thereby decreasing the terminal (collector-emitter) resistance of T_1. This leads to decrease in V_{CE1} thereby offsetting the decrease in V_L which is, therefore, returned to its original value.

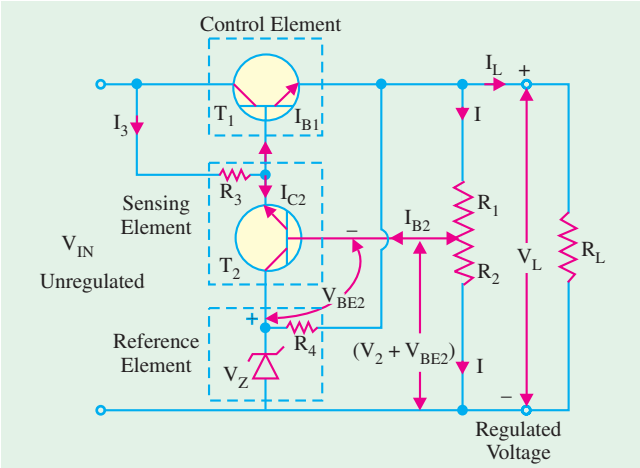


Fig. 56.4

In sequential logic, we have

$$V_L \downarrow \qquad I_{B2} \downarrow \qquad I_{C2} \downarrow \qquad I_{B1} \uparrow \qquad V_{CE1} \downarrow \qquad V_2 \uparrow$$

56.6. Transistor Shunt Voltage Regulator

It employs the transistor in shunt configuration as shown in Fig. 56.5.

Since path AB is in parallel across V_L, we have from Kirchhoff's Voltage Law

$$V_L - V_z - V_{BE} = 0$$

or $V_{BE} = V_L - V_z$ (fixed)

Since V_z is fixed, any decrease or increase in V_L will have a corresponding effect on V_{BE}. Suppose, V_L decreases, then as seen from the above relation, V_{BE} also decreases. As a result, I_B decreases, hence,

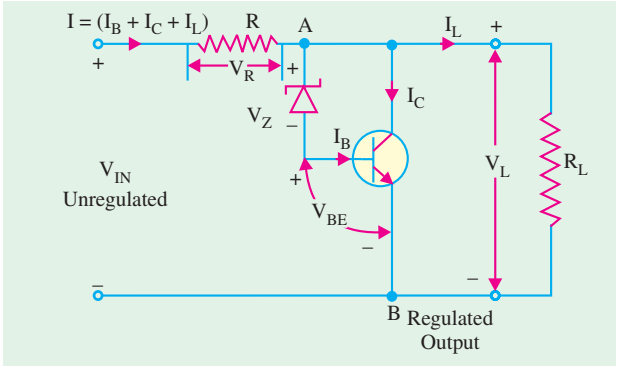


Fig. 56.5

I_C (= βI_B) decreases, thereby decreasing I and hence V_R (= IR). Consequently, V_L increases because at all times

$$V_{in} = V_R + V_L \quad \text{or} \quad V_L = V_{in} - V_R$$

In sequential logic, $V_z \downarrow$ $V_{BE} \downarrow$ $I_B \downarrow$ $I_C \downarrow$ $I_R \downarrow$ $V_R \downarrow$ $V_L \uparrow$

Same line of logic applies in case V_L tries to increase.

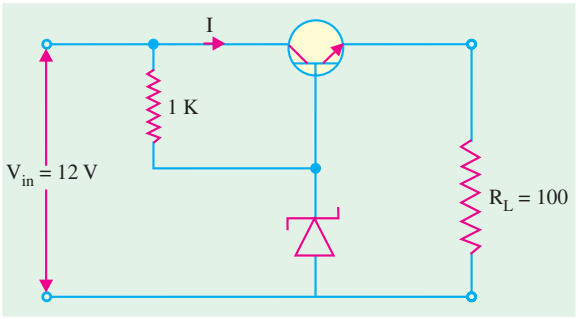


Fig. 56.6

Example 56.2. *In the NPN emitter-follower regulator circuit of Fig. 56.6, calculate* ***(i)*** V_L, ***(ii)*** V_{CE}, ***(iii)*** I_E *and* ***(iv)*** *power dissipated. Take* $V_{BE} = 0.7$ *V.*

Solution.

(i) $V_L = V_{out} = V_Z - V_{BE}$
$= 9 - 0.7 = \mathbf{8.3\ V}$

(ii) $V_{CE} = V_{in} - V_{out} = 12 - 8.3 = \mathbf{3.7\ V}$

(iii) $I_E = I_L = V_L/R_L$
$= 8.3/100 = \mathbf{83\ mA}$

(iv) Power dissipated $= V_{CE} I_E$
$= 3.7\ \text{V} \times 83\ \text{mA} = \mathbf{310\ mW}$

Example 56.3. *Compute the output voltage* V_{out} *for the op-amp series regulator shown in Fig. 56.7.* **(Industrial Electronics, Bombay Univ. 1992)**

Solution. We are given that V_{REF}
$= 6$ V and $R_2 = R_3 = 1$ K

$V_{out} = V_{REF}(1 + R_z/R_3)$
$= 6(1 + 10/10)$
$= 6 \times 2 = \mathbf{12\ V}$

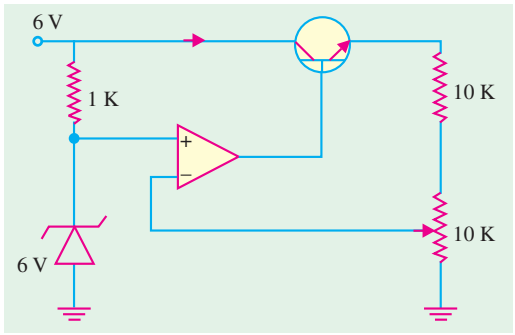


Fig. 56.7

56.7. Transistor Current Regulator

The main function of a current regulator is to maintain a fixed current through the load despite variations in the terminal voltage. Such a circuit employing a Zener diode and a *PNP* transistor is shown in Fig. 56.8. Suppose, due to drop in V_L, current I_L (= I_C) is decreased. This will decrease I_E (≅ I_C). Hence, drop across R_E *i.e.* V_{RE} will decrease. As per Kirchoff's Voltage Law

$$-V_{RE} - V_{BE} + V_z = 0, \quad \text{or} \quad V_{BE} = V_z - V_{RE}$$
(fixed)



Transistor current regulator

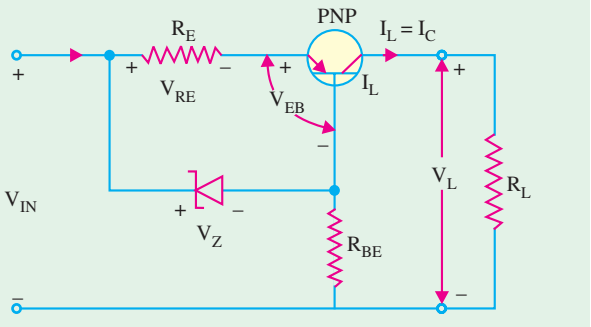


Fig. 56.8

Hence, a decrease in V_{RE} will increase V_{BE} and, hence, the conductivity of the transistor thereby keeping I_L at a fixed level.

A similar logic applies when there is increase in V_L.

56.8. Variable Feedback Regulator

The regulators considered so far provide a non-adjustable output voltage. This would be fine if only single value of regulated voltage is required. Fig. 56.9 shows a feedback regulator which provides different values of regulated dc voltage. In Fig. 56.9, T_1 is the pass transistor and T_2 is the feedback transistor whose job is to provide and sample output (*i.e.* load) voltage. It offsets any change in the output voltage. Since potentiometer R_3 is connected in parallel with Zener diode D, it has Zener voltage V_z applied across it. Voltage across the wiper varies from 0 to V_z. Capacitor C ensures that voltage across D and R_3 does not change suddenly.

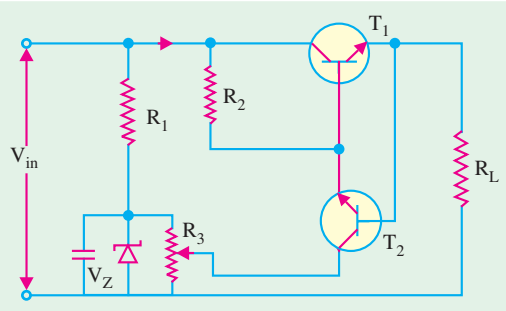


Fig. 56.9

Voltage at the base of T_2 is 0.7 V more positive than the voltage at its emitter. Its emitter voltage and hence the base voltage can be changed with the help of R_3. Since base of T_2 is tied to the output, it is responsible for providing output or load voltage. The voltage V_{CE1} across the pass transistor is given by the difference of input voltage and output voltage. The current through T_1 is equal to the load current. R_2 prevents saturation of transistors whereas R_1 limits the current flowing through D.

The working of feedback transistor can be explained as follows :

Since base voltage of T_2 is directly related to V_{out} it will change if V_{out} changes. The base and collector of T_2 are 180° out of phase with each other. If base voltage increases due to increase in V_{out} collector voltage would decrease. Now, collector of T_2 controls base of T_1. As the base voltage of T_1 decreases, its collector-emitter resistance increases which lowers the load current. This, in turn, lowers the output voltage thereby offsetting the attempted increases in V_{out}. The opposite of these steps provides the action of an attempted decrease in output voltage.

Example 56.4. *In the variable feedback regulator circuit of Fig. 56.8, V_{in} = 25 V, V_z = 15 V and R_L = 1 K. If the wiper of R_3 is adjusted half-way and assuming silicon transistor, compute* ***(i)*** V_{out} ***(ii)*** I_L ***(iii)*** I_{E1} ***(iv)*** P_1.

Solution. ***(i)*** V_{out} = voltage at wiper + V_{BE2} = (15/2) + 0.7 = **8.2 V**

(ii) $I_L = V_{out}/R_L$ = 8.2 V/1 K = **8.2 mA**

(iii) $I_{E1} = I_L$ = **8.2 mA**

(iv) $V_{CE1} = V_{in} - V_{out}$

= 25 – 8.2 = **16.8 V**

$\therefore$ P_1 = 16.8 V × 8.2 mA

= **140 mW**

56.9. Basic Op-amp Series Regulator

Its circuit is shown in Fig. 56.10 and its operation is as follows :

The potentiometer R_2-R_3 senses any change in out-

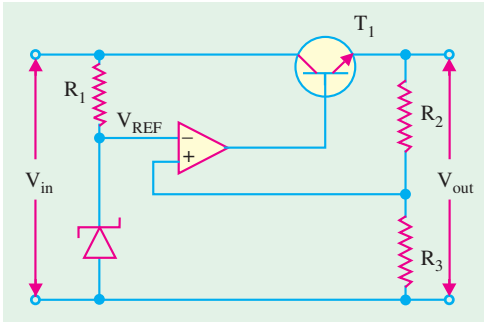


Fig. 56.10

put voltage V_{out}. When V_{out} attempts to decrease because of decrease in V_{in} or because of the increase in I_L, a proportional voltage decrease is applied to the inverting output of the op-amp by the potentiometer. Since, the other op-amp input is held by the Zener voltage at a fixed reference voltage V_{REF}, a small difference voltage (called error voltage) is developed across the two inputs of the op-amp. This difference voltage is amplified and op-amp's outout voltage increases. This increase in voltage is applied to the base of T_1 causing the emitter voltage (= V_{out}) to increase till the voltage to the inverting input again equals the reference (Zener) voltage. This action offsets attempted decrease in the output voltage thus keeping it almost constant. The opposite action occurs if the output voltage tries to increase.

Calculations

It will be seen that the op-amp of Fig. 56.10 is actually connected as a non-inverting amplifier where V_{REF} is the input at the noninverting terminal and the R_2/R_3 voltage divider forms the negative feed-back network. The closed-loop voltage gain is given by $A = 1 + (R_2/R_3)$. Neglecting base-emitter voltage of T_1, we get

$$V_{out} = V_{REF}(1 + R_2/R_3)$$

It is seen that V_{out} depends on Zener voltage and potential divider resistors R_2 and R_3 but is independent of input voltage V_{in}.

56.10. Basic Op-amp Shunt Regulator

Such a shunt type linear regulator is shown in Fig. 56.11. Here, the control element is a series resistor R_1 and a transistor T_1 in parallel with the load. In such a regulator, regulation is achieved by controlling the current through T_1.

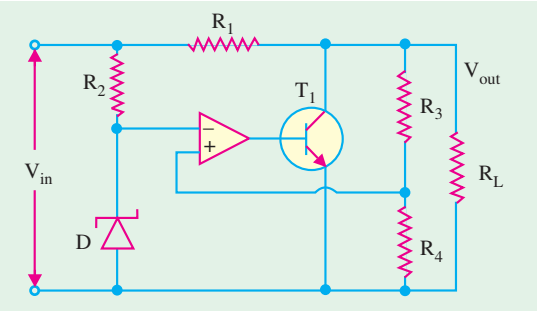


Fig. 56.11

Working

When output voltage tries to decrease due to change in either the input voltage or load current or temperature, the attempted decrease is sensed by R_3 and R_4 and applied to the non-inverting input of the op-amp. The resulting difference in voltage reduces the op-amp's output, driving T_1 less thus reducing its collector current (shunt current), and increasing its collector-to-emitter resistance. Since collector-to-emitter resistance acts as a voltage divider with R_1, this action offsets the attempted decrease in output voltage and hence, maintains it at a constant value. The opposite action occurs when output voltage tries to increase. The shunt regulator is less efficient than the series type but offers inherent short-circuit protection.

56.11. Switching Regulators

In the linear regulators considered so far, the control element *i.e.* the transistor conducts all the time, the amount of conduction varying with changes in output voltage or current. Due to continuous power loss, the efficiency of such a regulator is reduced to 50 per cent or less.

A switching regulator is different because its control element operates like a switch *i.e.* either it is saturated (closed) or cut-off (open). Hence, there is no unnecessary wastage of power which results in higher efficiency of 90% or more.

Switching regulators are of three basic types : ***(i)*** step-down regulator, ***(ii)*** step-up regulator and ***(iii)*** inverting regulator.

56.12. Step-down Switching Regulator

In this regulator (Fig. 56.12), V_{out} is always less than V_{in}. An unregulated positive dc voltage is applied to the collector of the *NPN* transistor. A series of pulses from an oscillator is sent to the base of transistor *T* which gets saturated (closed) on each of the positive pulses. It is so because an *NPN* transistor needs a positive voltage pulse on its base in order to turn ON. A saturated transistor acts as a closed switch, hence it allows V_{in} to send current through *L* and charge *C* to the value of output voltage during the on-time (T_{ON}) of the pulse. The diode D_1 is reverse-biased at this point and hence, does not conduct.

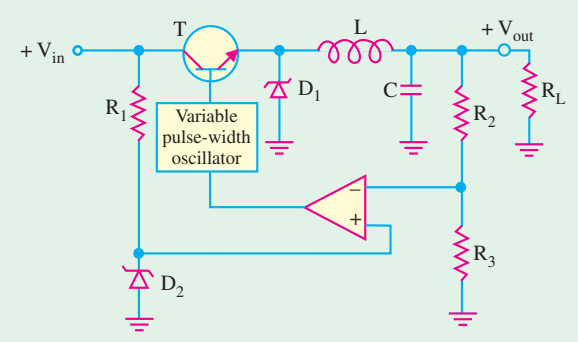
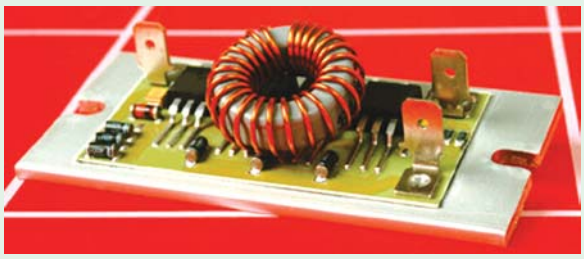


Fig. 56.12



Switching regulator in hybrid film technology

Eventually when positive pulse turns to zero, *T* is cut-off and acts like an open switch during the off period (T_{OFF}) of the pulse. The collapsing magnetic field of the coil produces self-induced voltage and keeps the current flowing by returning energy to the circuit.

The value of output voltage depends on input voltage and pulse width *i.e.* on-time of the transistor. When on-time is increased relative to off-time, *C* charges more thus increasing V_{out}. When T_{ON} is decreased, *C* discharges more thus decreasing V_{out}. By adjusting the duty cycle (T_{ON}/T) of the transistor, V_{out} can be varied.

$$\therefore \qquad V_{out} = V_{in}(T_{ON}/T)$$

where *T* is the period of the ON-OFF cycle of the transistor and is related to frequency by $T = 1/f$. Also, $T = T_{ON} + T_{OFF}$ and the ratio (T_{ON}/T) is called the ***duty cycle.***

The regulating action of the circuit is as follows :

When V_{out} tries to decrease, on-time of the transistor is increased causing an additional charge on the capacitor *C* to offset the attempted decrease. When V_{out} tries to increase, T_{ON} of the transistor is decreased causing *C* to discharge enough to offset the attempted increase.

56.13. Step-up Switching Regulator

The circuit is shown in Fig. 56.13. When transistor *T* turns ON on the arrival of the positive pulse at its base, voltage across L increases quickly to $V_{in} - V_{CE(sat)}$ and magnetic field of *L* expands quickly. During on-time of the transistor, V_L keeps decreasing from its initial maximum value. The longer transistor is ON, the smaller V_L becomes.

When transistor turns OFF, magnetic field of *L* collapses and its polarity reverses so that

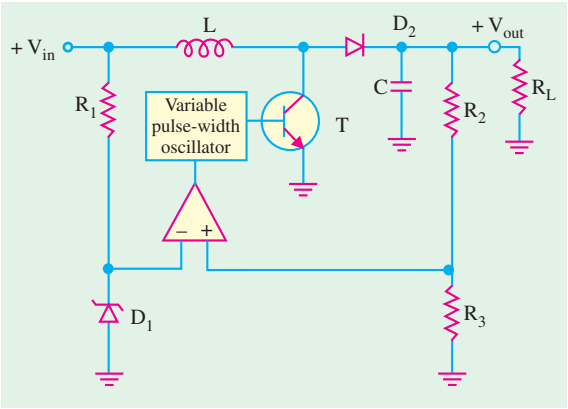


Fig. 56.13

its voltage adds to the input voltage thus producing an output voltage greater than the input voltage. During off-time of the transistor, D_2 is forward-biased and allows C to charge. The variations in V_{out} due to charging and discharging action are sufficiently smoothed by filtering action of L and C.

It may be noted that shorter the on-time of the transistor, greater the inductor voltage and hence greater the output voltage (because greater V_L adds to V_{in}). On the other hand, the longer the on-time, the smaller the inductor voltage and hence, lesser the output voltage (because smaller V_L adds to V_{in}).

The regulating action can be understood as follows :

When V_{out} tries to decrease (because of either increasing load or decreasing V_{in}), transistor on time decreases thereby offsetting attempted decrease in V_{out}. When V_{out} tries to increase, on-time increases and attempted increase in V_{out} is offset.

As seen, the output voltage is inversely related to the duty cycle.

$$\therefore \quad V_{out} = V_{in}(T/T_{ON})$$



Inverting Switching Regulator

56.14. Inverting Switching Regulator

The basic diagram of such a regulator is shown in Fig. 56.14. This regulator provides an output voltage that is opposite in polarity to the input voltage.

When transistor T turns ON by the positive pulse, the inductor voltage V_L jumps to $V_{in} - V_{CE(sat)}$ and the magnetic field of the inductor expands rapidly. When transistor is ON, the diode D_2 is reverse-biased and V_L decreases from its initial maximum value. When transistor turns OFF, the magnetic field collapses and inductor's polarity reverses. This forward-biases D_2, charges C and produces a negative output voltage. This repetitive ON-OFF action of the transistor produces a repetitive charging and discharging that is smoothed by LC filter action. As in the case of a step-up regulator, lesser the time for which transistor is ON, greater the output voltage and ***vice versa***.

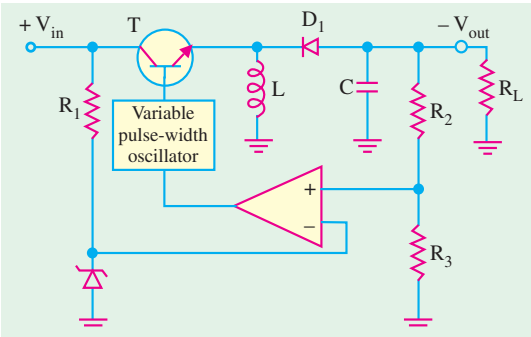


Fig. 56.14

56.15. IC Voltage Regulators

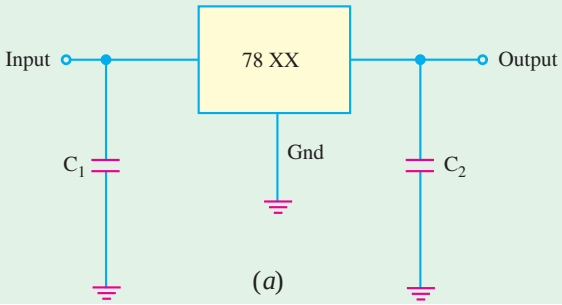
Due to low-cost fabrication technique, many commercial integrated-circuit (*IC*) regulators are available since the past two decades. These include fairly simple, fixed-voltage types of high-quality precision regulators. These *IC* regulators have much improved performance as compared to those made from discrete components. They have a number of unique build-in features such as current limiting, self-protection against overtemperature, remote control operation over a wide range of input voltages and foldback current limiting.

Now we will study the following types of *IC* voltage regulators : **(1)** fixed positive linear voltage regulators, **(2)** fixed negative linear voltage regulators, **(3)** adjustable positive linear voltage regulators, and **(4)** adjustable negative linear voltage regulators.

56.16. Fixed Positive Linear Voltage Regulators

There are many *IC* regulators available in the market that produces a fixed positive output voltage. But 7800 series of *IC* regulators is representative of three terminal devices that are available

with several fixed positive output voltages making them useful in a wide range of applications. Fig. 56.15 (*a*) shows a standard configuration of a fixed positive voltage *IC* regulator of 7800 series. Notice that it has three terminals labelled as input, output and ground. The last two digits (marked xx) in the part number designate the output voltage. For example, *IC* 7805 is a +5 V regulator. Similarly *IC* 7812 is a +12 V regulator and *IC* 7815 is a +15 V regulator. The capacitor C_1 (typically 0.33 µF) is required only if the power supply filter is located more than 3 inches from the *IC* regulator. The capacitor C_2 (typically 0.01 µF) acts basically as a line filter to improve transient response.



(*a*)

Part Number	Output Voltage
7805	+5.0 V
7806	+6.0 V
7808	+8.0 V
7809	+9.0 V
7812	+12.0 V
7815	+15.0 V
7818	+18.0 V
7824	+24.0 V

(*b*)

Fig. 56.15

Fig. 56.15 (*b*) shows the part number and the output voltage of 7800 series *IC* voltage regulators. As seen from this figure, the 7800 series has *IC* regulators that can produce output voltages ranging from +5.0 to +24.0 volt. It may be carefully noted that although these regulators are designed primarily to produce fixed output voltage but they can be used with external components to obtain adjustable output voltage and current.

Fig. 56.16 shows the circuit indicating the use of 78XX as an adjustable voltage regulator. The output voltage is given by the equation,

$$V_{out} = V_{fixed} + \left(\frac{V_{fixed}}{R_1} + I_Q\right) R_2$$

For example, for a 7805 *IC* regulator, $V_{fixed} = 5$ V. Let $R_1 = R_2 = 1$ kΩ and $I_Q = 5$ mA, then its output voltage is,

$$V_{out} = 5 + \left(\frac{5}{1\ \text{k}\Omega} + 5\ \text{mA}\right) \times 1\ \text{k}\Omega$$
$$= 15\ \text{V}$$

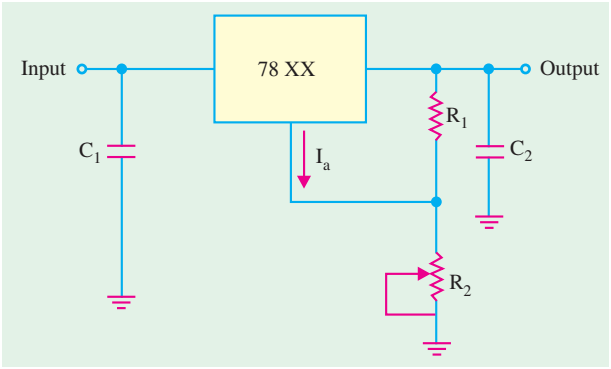
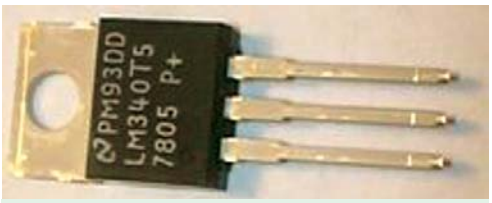


Fig. 56.16

Thus output voltage of *IC* 7805 regulator can be adjusted anywhere between 5 V to 15 V.

This example indicates that the output of *IC* 7805 regulator is adjusted to 15 V using external resistances R_1 and R_2.

The standard 7800 series can produce output current in excess of 1 A when used with adequate



IC 7805 regulator

heat sink. It is available in aluminium can package TO-3 (indicated by *K* in the part number) and plastic package TO-220 (indicated by *T* in the part number), The 78L00 series can provide up to 100 mA and is available in TO-92 and metal TO-39 low profile packages. The 78M00 series can provide upto 0·5 A and is available in plastic TO-202 package. Fig. 6.17 shows the typical metal and plastic packages for the *IC* voltage regulators.

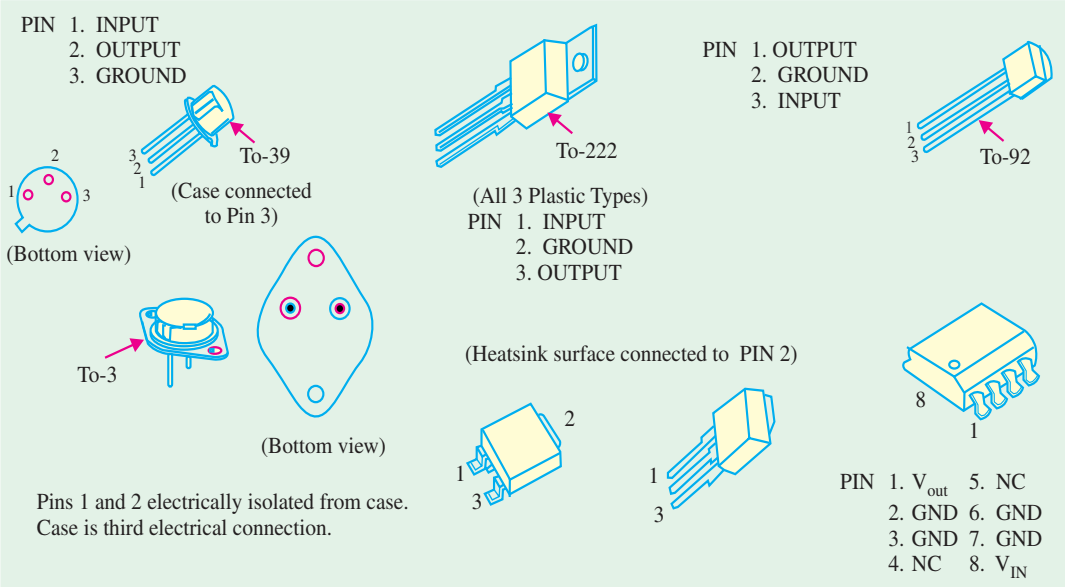


Fig. 56.17

It may be noted that the input voltage for the *IC* regulator must be at least 2 V above the output voltage. This is required in order to maintain regulation. The input voltage should not be more than 35 or 40 volts depending upon the part number. The circuit inside all the *IC* regulators have internal thermal overload protection and short-circuit current-limiting features. Thermal overload in a *IC* regulator occurs whenever the internal power dissipation becomes excessive and the temperature of the device exceeds a certain value.

In India, Bharat Electronic Limited, Bangalore manufactures the *IC* voltage regulators with output voltages of 5 and 12 V. These are available in the market with part numbers BEL 7805 and BEL 7812 respectively.

Fig 56.18 shows a picture of a transformer bridge rectifier and voltage regulator in a dc power supply.

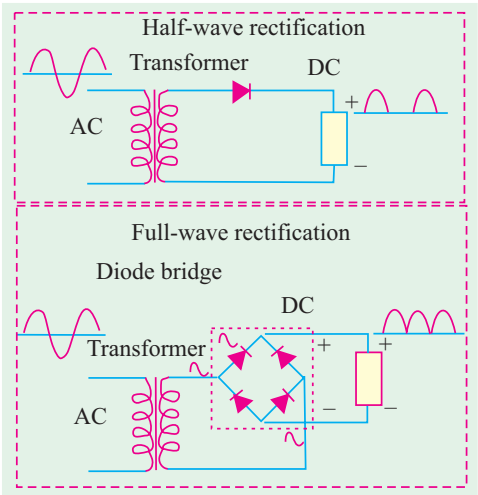
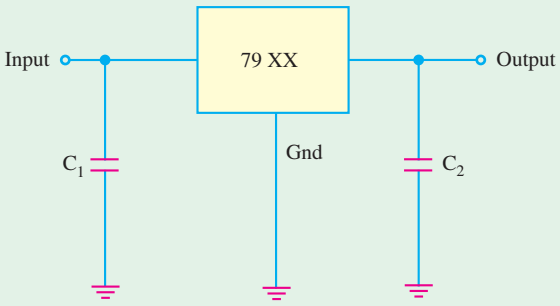


Fig. 56.18

56.17. Fixed Negative Linear Voltage Regulators

The 7900 series is typical of three-terminal *IC* regulators that provide a fixed negative output voltage. This series is a negative-voltage counterpart of the 7800 series and shares most of the same features, characteristics and package types. Fig. 56.19 (*a*) indicates the standard configuration and Fig. 56.19 (*b*), the part numbers with corresponding output voltages that are available in 7900 series.



(a)

Part Number	Output Voltage
7905	– 5.0 V
7905·2	– 5.2 V
7906	– 6.0 V
7908	– 8.0 V
7912	– 12.0 V
7915	– 15.0 V
7918	– 18.0 V
7924	– 24.0 V

(b)

Fig. 56.19

The capacitor C_1 (typically 0.22 μF) is required only if the power supply filter is located more than 3 inches away from the IC regulator. The capacitor C_2 (typically 1 μF) is required for stability of the output voltage. Both capacitors C_1 and C_2 must be solid tantalum capacitors.

Fig. 56.20 shows the use of 79XX to produce an adjustable output voltage. The capacitor C_3 (typically 25 μF) improves the transient response of the output voltage. The output voltage is given by the equation,

$$V_{out} = V_{fixed}\left(\frac{R_1 + R_2}{R_2}\right)$$

The recommended value of R_2, for 7905 is 300 Ω, for 7915, its value is 750 Ω and for 7915 is 1 kΩ.

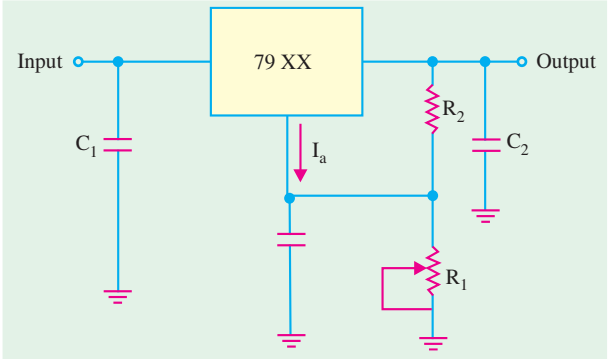


Fig. 56.20

In India, BEL manufactures *IC* regulators with output voltage of –5 V and –12 V. These are available in the market with part numbers BEL 7905 and BEL 7915 respectively.

56.18. Adjustable Positive Output Linear Voltage Regulators

We have already seen in Art. 56.16 that by adding external resistors, we can adjust the output voltage of 7800 series IC regulators higher than their fixed (or set) voltages. Fot example, the output voltage of 7805 can be adjusted higher than 5 V. But the performance and reliability of 7800 series to produce voltage higher than its fixed value is not considered to be good.

The LM 317 and LM 723 are IC regulators whose output voltage can be adjusted over a wide range. The output voltage of LM 317 can be adjusted from 1·2 V to 37 V, it can supply output current of 100 mA and is available in TO-92 package *i.e.*, it is also a 3 terminal IC regulator. On the other hand, the output voltage of LM 723 can be adjusted from 2 V to 37 V, it can supply output current of 150 mA without external transistor. But with the addition of external transistor, the output current capability can be increased in excess of 10 A. The LM 723 is available in dual-in-line package and in a metal can package.

Fig. 56.21 (*a*) shows the LM 317 connected to the external resistors R_1 and R_2 to produce an adjustable output voltage.

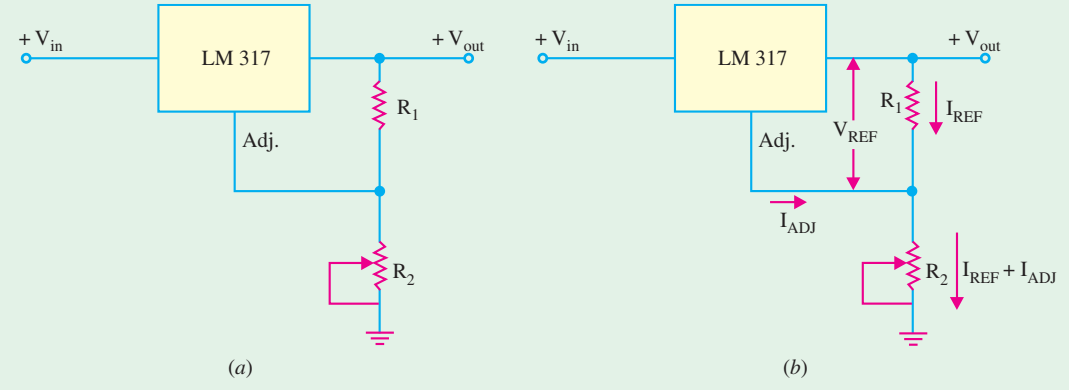


Fig. 56.21

In operation, the LM 317 develops a constant 1·25 V reference voltage (V_{REF}) between the output and adjustment terminal. This constant reference voltage produces a constant current, (I_{REF}) through R_1, regardless of the value of R_2 Fig. 56.20 (*b*). Notice that the value of current through R_2 is the sum of I_{REF} and I_{ADJ}, where I_{ADJ} is a very small current at the adjustment terminal. The value of I_{ADJ} is typically around 100 µA. It can be shown that the output voltage.

$$V_{out} = V_{REF}\left(1 + \frac{R_2}{R_1}\right) + I_{ADJ} \cdot R_2$$

It is evident from the above equation that the output voltage is a function of R_1 and R_2. Usually the value of R_1 is recommended to be around 220 Ω. Once the value of R_1 is set, the output voltage is adjusted by varying R_2.

Example 56.5. *Calculate the minimum and maximum output voltages for the IC voltage regulator shown in Fig. 56.22. Assume I_{ADJ} = 100 µA, V_{in} = + 35 V.*

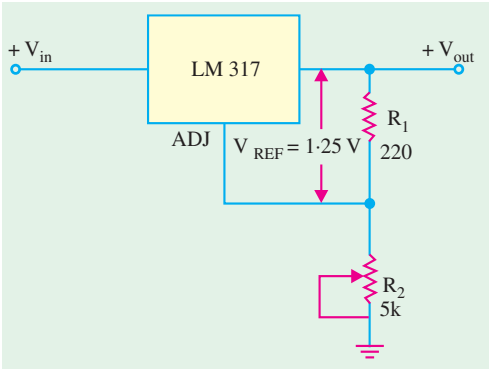


Fig. 56.22

Solution. The equation for output voltage of the IC voltage regulator is given by,

$$V_{out} = V_{REF}\left(1 + \frac{R_2}{R_1}\right) + I_{ADJ} \cdot R_2$$

When R_2 is set at its minimum value (*i.e.* 0 Ω), the output voltage,

$$V_{out(min)} = 1.25\left(1 + \frac{0}{220}\right) + (100 \times 10^{-6}) \times 0$$
$$= \mathbf{1.25\ V}$$

When R_2 is set at its maximum values (*i.e.* 5 kΩ), the output voltage,

$$V_{out(max)} = 1.25\left(1 + \frac{5000}{220}\right) + (100 \times 10^{-6}) \times 5000$$
$$= 29.66 + 0.5 = \mathbf{30.16\ V.}$$

56.19. Adjustable Negative Output Linear Voltage Regulators

A good example of this type of regulators is LM 337. The regulator is a negative output counterpart of LM 317. The LM 337 (like LM 317) requires two external resistors for adjustment of output

voltage as shown in Fig. 56.23. The output voltage can be adjusted anywhere from –1.2 V to – 37 V depending upon the external resistor values.

The LM 723 can also be used as an adjustable negative output voltage regulator. The output voltage of this *IC* regulator can be adjusted anywhere from –2.0 V to –37 V depending upon the external resistor values.

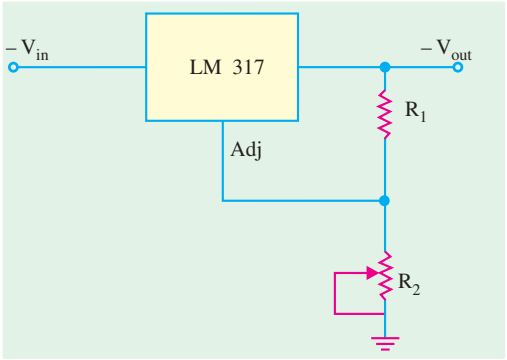


Fig. 56.23

56.20. Use of External Pass Transistor with Linear Voltage Regulators

We have already mentioned in the last two articles that a linear voltage regulator (7800 and 7900 series) is capable of delivering only a certain amount of output current to a load. For example, the 7800 series regulators can handle a maximum output current of at least 1.3 A and typical 2.5 A. If the load current exceeds the maximum allowable value, there will be a **thermal overload** and the regulator will **shut down.** ***A thermal overload condition means that there is excessive power dissipation inside the regulator.***

If an application requires a larger value of load current than the maximum current that the regulator can deliver, we will have to use an external pass transistor as shown in Fig. 56.24. The value of R_{ext} (current-sensing resistor) determines the value of current at which the external pass transister (T_{ext}) begins to conduct because it sets the base-to-emitter voltage of the transistor.

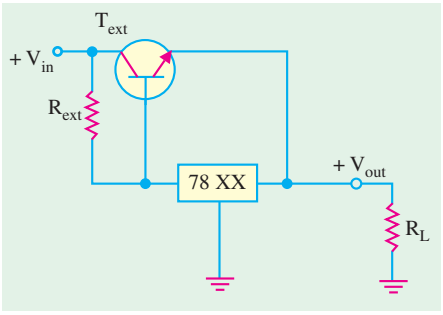


Fig. 56.24

As long as the current is less than the value set by R_{ext}, the transistor T_{ext} is off and the regulator operates normally. This is because the voltage drop across R_{ext} is less than 0.7 V (*i.e.* the base to emitter voltage required to turn T_{ext} on. The value of R_{ext} is determined by the equation $R_{ext} = 0.7\ V/I_{max}$ where I_{max} is the maximum value of current that the voltage regulator is to handle **internally**.

When the current is sufficient to produce at least a 0.7 V drop across R_{ext}, the transistor T_{ext} turns on and conducts any current in excess of I_{max}. The transistor T_{ext} will conduct current depending on the load requirement. For example, if the total load current is 5 A and I_{max} was selected to be 1 A, then the external pass Transistor (T_{ext}) will conduct 4 A of current through it.

It may be noted that the external pass transistor is a power transistor with heat sink that must be capable of handling a maximum power given by the equation,

$$P_{ext} = I_{ext}(V_{in} - V_{out})$$

One major drawback of the circuit shown in Fig. 56.24 is that the external pass transistor is not protected from excessive current, such as would result from a shorted output. This drawback can be overcome by using an additional current limiting circuit as shown in Fig. 56.25.

The operation of this circuit may be explained as follows. The current sensing resistor, R_{lim} sets the base-to-emitter voltage of T_{lim}. The base-to-emitter voltage of T_{ext} is now determined by $\left(V_{R_{ext}} - V_{R_{lim}}\right)$ because they have opposite polarities. So for normal operation, the drop across R_{ext} must be sufficient to overcome the opposing drop across R_{lim}.

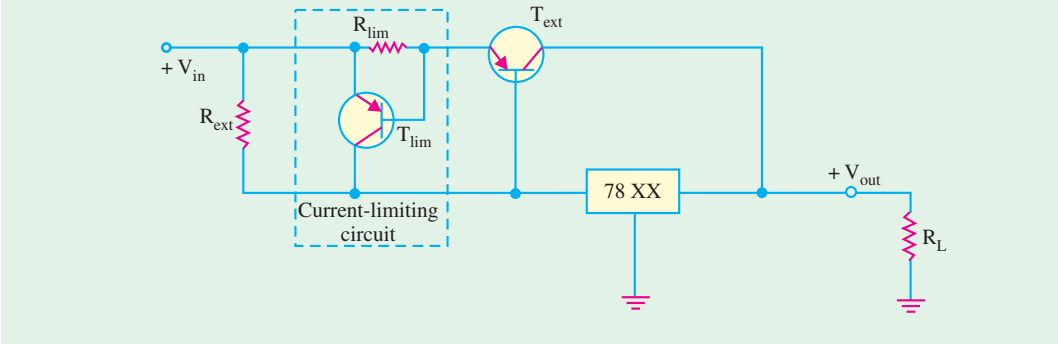


Fig. 56.25

If the current through T_{ext} exceeds a certain maximum value, ($I_{ext(max)}$) because of a shorted output or a faulty load, the voltage across R_{lim} reaches 0.7 V and turns T_{lim} on. As a result, T_{lim} now conducts current away from T_{ext} and through the regulator. This forces a thermal overload to occur and shut down the regulator. Remember, the *IC* voltage regulator circuitry is internally protected from thermal overload as part of its design. This way the external pass transistor is protected from excessive current.

56.21. Use of Linear Voltage Regulators as a Current Regulator

The 3-terminal linear voltage regulator can be used as a current source when an application requires that a constant current be supplied to a variable load. The basic circuit is shown in Fig. 56.26. Here R_1 is the current-setting resistor. The regulator provides a fixed output voltage, V_{out} between the ground terminal and the output terminal. However, it may be noted that the ground pin of the regulator is not connected to the circuit ground. The constant current supplied to the load, is given by the equation,

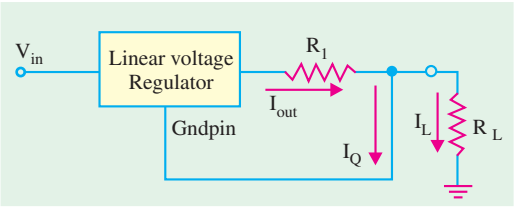


Fig. 56.26

$$I_L = \frac{V_{out}}{R_L} + I_Q$$

Usually the current, I_Q is very small as compared to the output current and hence can be neglected, therefore

$$I_L \approx \frac{V_{Out}}{R_L}$$

For example, if we use 7805 regulator to provide a constant current of 1 A to a variable load, then

$$R_1 = \frac{5}{1} = 5\ \Omega$$

Please note that input voltage must be at least 2 V greater than the output voltage. Thus for 5 V regulator, V_{in} must be greater than 7 V.

56.22. Switching Voltage IC Regulators

There are several switching voltage *IC* regulators available in the market. The choice depends upon the desired application and the cost. However, we will illustrate it with the *IC* 78S40. This device is a universal device that can be used with external components to provide step-up, step down and inverting operation.

Fig. 56.27 shows the internal circuitry of the *IC* 78C40. The circuitry can be compared to the basic switching regulators discussed in Art. 56.12, 56.13, and 56.14. As seen from this diagram, the

oscillator and comparator functions are directly comparable. The logic gate and flip-flop in the 78S40 were not included in the basic circuit of Fig. 56.12, but they provide additional regulation. Transistor T_1 and T_2 perform the same function as T in the basic circuit. The 1.25 V reference block in the 78S40 has the same purpose as that of the zener diode in the basic circuit and diode D_1 in the 78S40 corresponds to D_1 in the basic circuit.

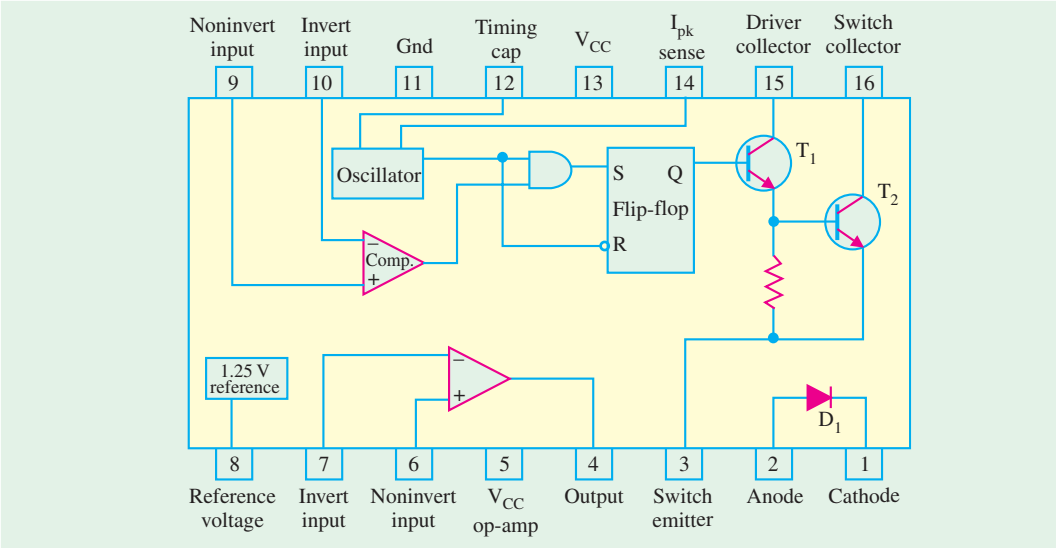


Fig. 56.27

The circuitry of 78S40 has also an uncommitted or unused OP-AMP. We require external circuitry to operate this device as a regulator.

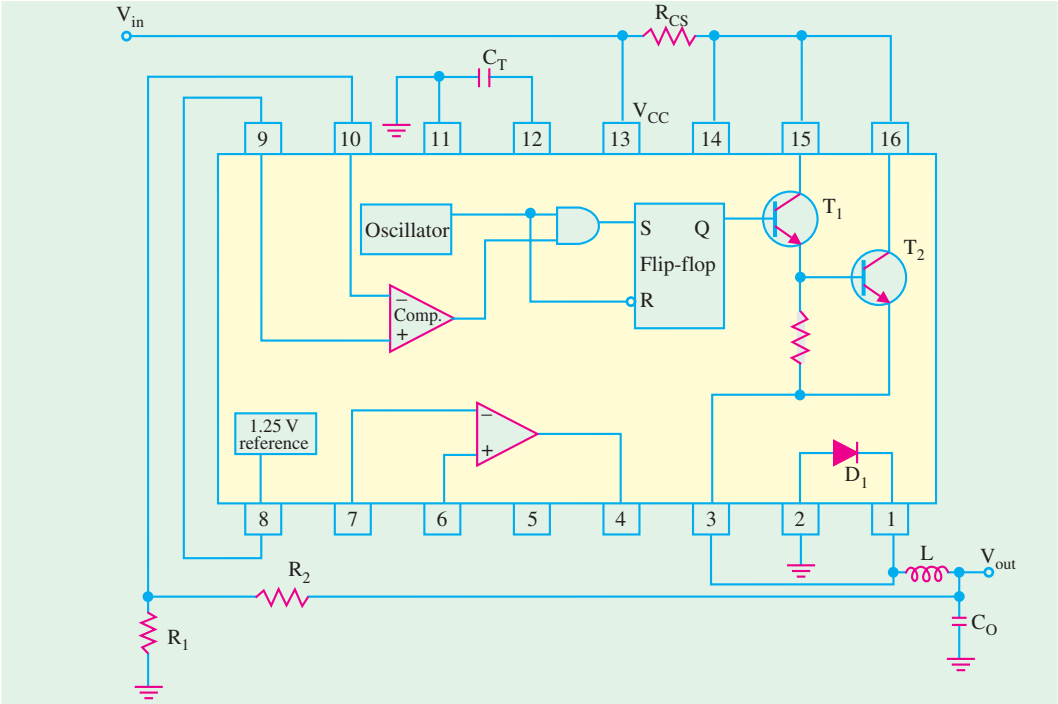


Fig. 56.28

Fig. 56.28 shows the external connections of the IC 78S40 for a step-down switching regulator configuration. Note that in this configuration the circuit produces an output voltage which is less than input voltage.

Fig. 56.28 shows the IC 78S40 connected to the external components for a step-up switching regulator configuration. In this case the output voltage is greater than the input voltage. An inverting configuration is also possible but is not shown here.

In both the circuits of Fig. 56.27 and Fig. 56.28, the capacitor C_T (called timing capacitor) controls the pulse width and frequency of the oscillator and thus establishes the on-time of transistor T_1. The voltage across the resistor R_{CS} (called current-sensing resistor) is used internally by the oscillator to vary the duty cycle based on the desired peak current. The voltage divider made up of R_1 and R_2 reduces the output voltage to a value equal to the reference voltage. If the output voltage (V_{out}) exceeds its set value, the output of the comparator switches to the low state, disabling the gate to turn T_2 off until the output decreases. This regulating action is in addition to that produced by the duty cycle variation of the oscillator.

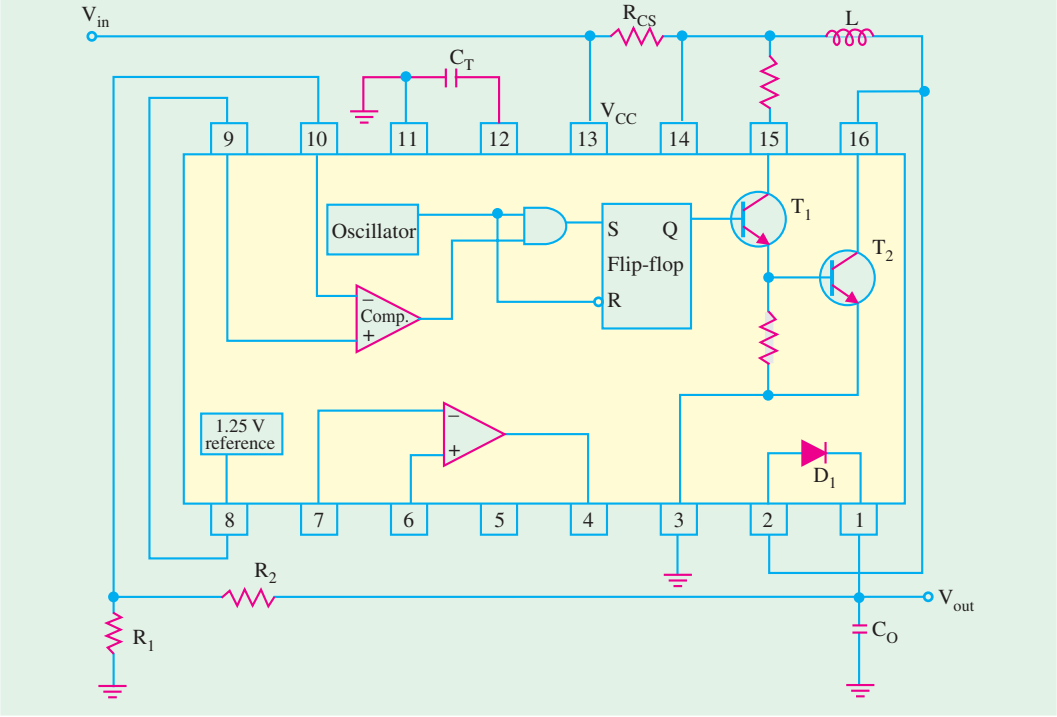


Fig. 56.29

Tutorial Problems No. 56.1

1. The Zener diode of Fig. 56.30 has the following ratings :

$V_z = 15$ V at $I_z = 17$ mA

$r_z = 14\ \Omega$ at $I_z = 17$ mA

$I_{z(min)} = 0.25$ mA $I_{z(max)} = 66$ mA

What would be the load voltage when load current I_L varies from 1 mA to 60 mA. Also calculate the voltage regulation of the regulator.

(14.846 V, 15.672 V, 5.27%)

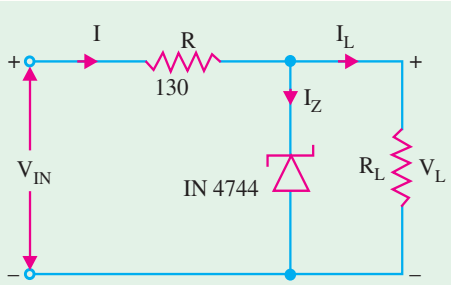


Fig. 56.30

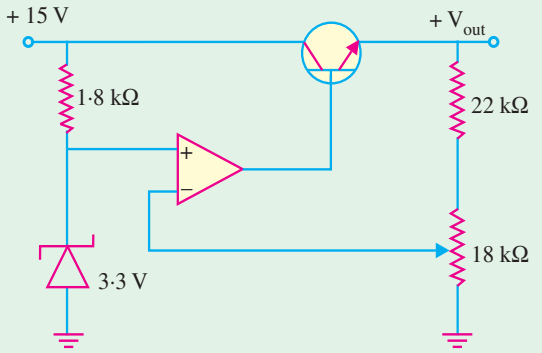


Fig. 56.31

2. Determine the output voltage, V_{out} for the op-amp series regulator shown in Fig. 56.31.

(7.3 V)

3. Calculate the minimum and maximum output voltages for the *IC* voltage regulator shown in Fig. 53.32. Assume I_{ADJ} = 50 µA, V_{in} = +35 V, V_{REF} = 1.2 V **(1.25 V, 12.71 V)**

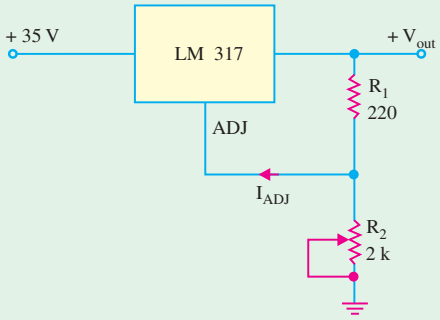


Fig. 56.32

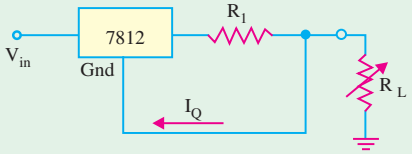


Fig. 56.33

4. Fig. 56.33 shows the circuit of a current regulator. What value of R_1 is necessary to provide a constant current of 1 A. **(12 Ω)**

OBJECTIVE TESTS – 56

1. The main job of a voltage regulator is to provide a nearly—output voltage.
(*a*) sinusoidal (*b*) constant
(*c*) smooth (*d*) fluctuating.

2. A 10-V dc regulator power supply has a regulation of 0.005 per cent. Its output voltage will vary within an envelope of ..... millivolt.
(*a*) ± 2.5 (*b*) ± 0.5
(*c*) ± 5 (*d*) ± 0.05

3. An ideal voltage regulator has a voltage regulation of
(*a*) 1 (*b*) 100
(*c*) 50 (*d*) 0.

4. In a Zener diode shunt voltage regulator, the diode regulates so long as it is kept in ..... condition.
(*a*) forward (*b*) reverse
(*c*) loaded (*d*) unloaded

5. The power efficiency of a switching voltage regulator is much higher than that of a linear regulator because it operates.
(*a*) in saturation (*b*) in cut-off
(*c*) like a switch (*d*) on high duty cycle.

6. A transistor series voltage regulator is called emitter-follower regulator because the emitter of the pass transistor follows the ..... voltage.
(*a*) output (*b*) input
(*c*) base (*d*) collector

7. In an op-amp series voltage regulator, output voltage depends on
(*a*) Zener voltage
(*b*) voltage divider resistors
(*c*) output voltage
(*d*) both (*a*) and (*b*)

8. In a feedback series regulator circuit, the output voltage is regulated by controlling the
(*a*) magnitude of input voltage
(*b*) gain of the feedback transistor
(*c*) reference voltage
(*d*) voltage drop across the series pass transistor

9. An op-amp shunt regulator differs from the series regulator in the sense that its control element is connected in
(*a*) series with line resistor
(*b*) parallel with line resistor
(*c*) parallel with load resistor
(*d*) parallel with input voltage.

10. A switching voltage regulator can be of the following type :
(*a*) step-down (*b*) step-up
(*c*) inverting (*d*) all of the above

11. In an inverting type switching regulator, output voltage is ..... input voltage.
(*a*) lesser than (*b*) greater than
(*c*) equal to (*d*) opposite to.

12. The output voltage of a step-down type switching voltage regulator depends on
(*a*) input voltage (*b*) duty cycle
(*c*) transistor on-time(*d*) all of the above.

13. As compared to voltage regulators made up of discrete components, IC regulators have the inherent advantage/s of
(*a*) self protection against over-temperature
(*b*) remote control
(*c*) current limiting
(*d*) all of the above

14. A 12 V monolithic regulator is adjusted to obtain a higher output voltage as shown in Fig. 56.34. The V_0 will be

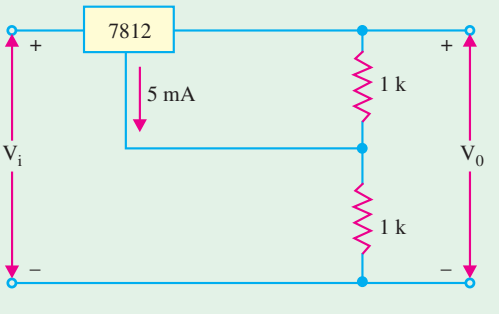


Fig. 56.34

(*a*) 12 V (*b*) 17 V
(*c*) 24 V (*d*) 29 V

15. A three terminal monolithic IC regulator can be used as
(*a*) an adjustable output voltage regulator alone
(*b*) an adjustable output voltage regulator and a current regulator
(*c*) a current regulator and a power switch
(*d*) a current regulator alone

ANSWERS

1. (*b*) **2.** (*a*) **3.** (*d*) **4.** (*b*) **5.** (*c*) **6.** (*c*) **7.** (*d*) **8.** (*d*) **9.** (*a*) **10.** (*d*) **11.** (*d*) **12.** (*d*) **13.** (*d*) **14.** (*d*) **15.** (*d*).

ROUGH WORK

GO To FIRST

CONTENTS

CHAPTER 57

BIPOLAR JUNCTION TRANSISTOR

Learning Objectives

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- Transistor Circuit Configurations
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- CE Configuration
- Relation between α and β
- CC Configuration
- Relation between Transistor Currents
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- Thermal Runaway
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- Common Base Test Circuit
- Common Base Static Characteristics
- Common Emitter Static Characteristics
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- Active Region DC Model of BJT
- BJT Switches
- Normal DC Voltage Transistor Indications
- Transistor Fault Location
- Increase/Decrease Notation



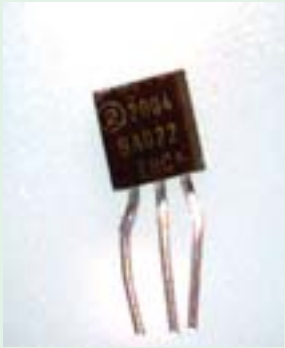
Bipolar junction transistor is used in two broad areas–as a linear amplifier to boost or amplify an electrical signal and as an electronic switch

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57.1. Bipolar Junction Transistor

The transistor was invented by a team of three scientists at Bell Laboratories, USA in 1947. Although the first transistor was not a bipolar junction device, yet it was the beginning of a technological revolution that is still continuing in the twenty first century. All of the complex electronic devices and systems developed or in use today, are an outgrowth of early developments in semiconductor transistors.

There are two basic types of transistors : **(1)** the bipolar junction transistor (BJT) which we will study in this chapter and the field-effect transistor (FET) which is covered in chapter 13. The bipolar junction transistor is used in two broad areas of electronics : **(1)** as a linear amplifier to boost an electrical signal and **(2)** as an electronic switch.



Bipolar junction transistor

Basically, the bipolar junction transistor consists of two back-to-back *P-N* junctions manufactured in a single piece of a semiconductor crystal. These two junctions give rise to three regions called ***emitter, base*** and ***collector.*** As shown in Fig. 57.1 (*a*) junction transistor is simply a sandwich of one type of semiconductor material between two layers of the other type. Fig. 57.1 (*a*) shows a layer of *N*-type material sandwiched between two layers of *P*-type material. It is described as a *PNP* transistor. Fig. 57.1 (*b*) shown an *NPN* – transistor consisting of a layer of *P*-type material sandwiched between two layers of *N*-type material.

The emitter, base and collector are provided with terminals which are labelled as *E*, *B* and *C*. The two junctions are : emitter-base (*E*/*B*) junction and collector-base (*C*/*B*) junction.

The symbols employed for *PNP* and *NPN* transistors are also shown in Fig. 57.1. The arrowhead is always at the emitter (not at the collector) and in each case, its direction indicates the ***conventional*** direction of current flow. For a *PNP* transistor, arrowhead points from emitter to base meaning that emitter is positive with respect to base (and also with respect to collector)* For *NPN* transistor, it points from base to emitter meaning that base (and collector as well)* is positive with respect to the emitter.

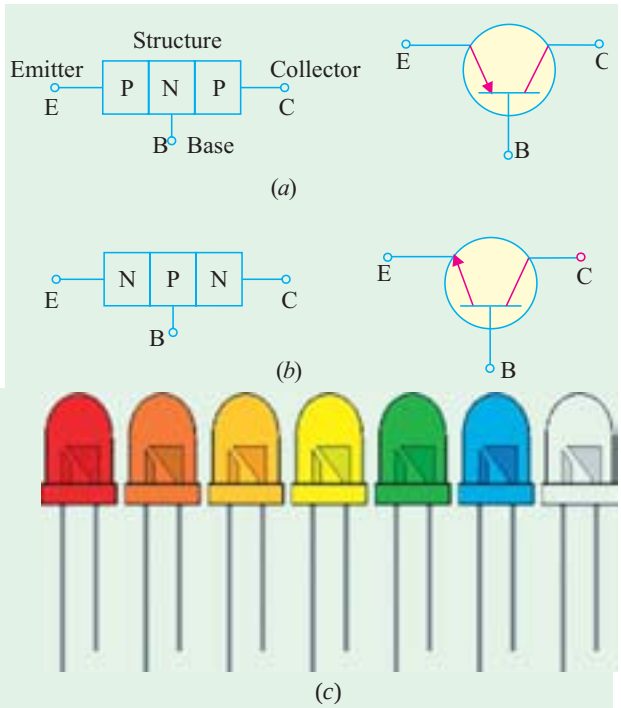


Fig. 57.1

1. Emitter

It is more heavily doped than any of the other regions because its main function is to supply majority charge carries (either electrons or holes) to the base.

2. Base

It forms the middle section of the transistor. It is very thin (10^{-6} m) as compared to either the emitter or collector and is very ***lightly-doped.***

3. Collector

Its main function (as indicated by its name) is to collect majority charge carriers coming from the emitter and passing through the base.

* In a transistor, for normal operation, collector and base have the same polarity with respect to the emitter (Art. 57.3)

In most transistors, collector region is made physically larger than the emitter region because it has to dissipate much greater power. Because of this difference, there is no possibility of inverting the transistor *i.e.* making its collector the emitter and its emitter the collector. Fig 57.1 (*c*), shows the picture of C1815 (front and the back view) transistor.

57.2. Transistor Biasing

For proper working of a transistor, it is essential to apply voltages of correct polarity across its two junctions. It is worthwhile to remember that for normal operation;

1. emitter-base junction is always forward-biased and

2. collector-base junction is always reverse-biased.

This type of biasing is known as *FR* biasing.

In Fig. 57.2, two batteries respectively provide the dc emitter supply voltage V_{EE} and collector supply voltage V_{CC} for properly biasing the two junctions of the transistor. In Fig. 57.2 (*a*), **P**ositive terminal of V_{EE} is connected to **P**-type emitter in order to repel or **P**ush holes into the base.

The negative terminal of V_{CC} is connected to the collector so that it may ***attract*** or ***pull*** holes through the base. Similar considerations apply to the *NPN* transistor of Fig. 57.2 (*b*). It must be remembered that a transistor will never conduct any current if its emitter-base junction is not forward-biased.* Also refer to the picture shown in Fig. 57.2 (*c*).

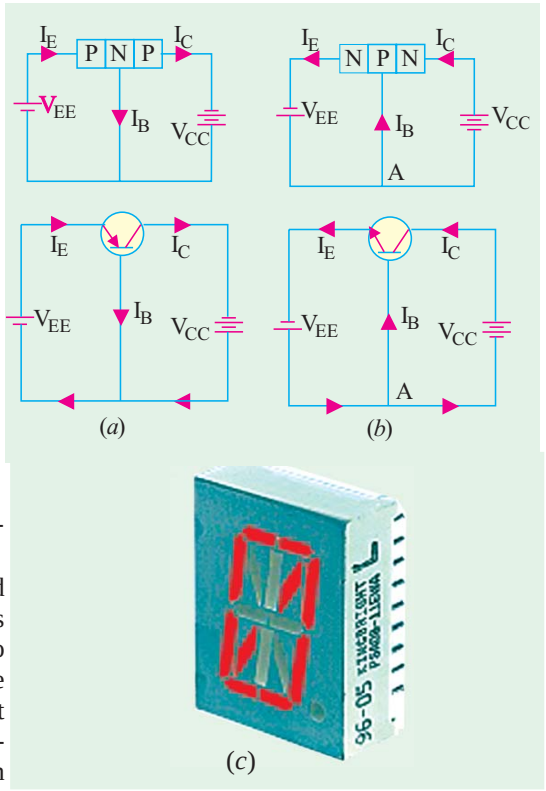


Fig. 57.2

57.3. Important Biasing Rule

For a *PNP* transistor, both collector and base are negative with respect to the emmitter (the letter *N* of Negative being the same as the middle letter of *PNP*). Of course, collector is ***more negative*** than base [Fig. 57.3 (*a*)]. Similarly, for *NPN* transistor, both collector and base are positive with respect to the emitter (the letter **P** of **P**ositive being the same as the middle letter of *NPN*). Again, collector is ***more positive*** than the base as shown in Fig. 57.3 (*b*).

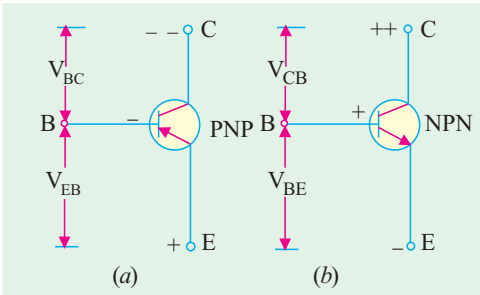


Fig. 57.3

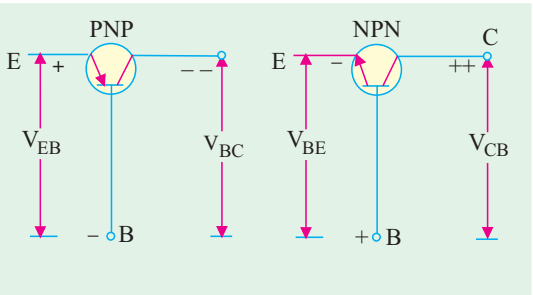


Fig. 57.4

* There would be no current due to majority charge carriers. However, there would be an extremely small current due to minority charge carriers which is called leakage current of the transistor (Art. 57.12).

It may be noted that different potentials have been designated by double subscripts. The first subscript always represents the point or terminal which is more positive (or less negative) than the point or terminal represented by the second subscript. For example, in Fig. 57.3 (*a*), the potential difference between emitter and base is written as V_{EB} (and not V_{BE}) because ***emitter is positive with respect to base.*** Now, between the base and collector themselves, collector is more negative than base. Hence, their potential difference is written as V_{BC} and not as V_{CB}. Same is the case with voltages marked in Fig. 57.4.

57.4. Transistor Currents

The three primary currents which flow in a properly-biased transistor are I_E, I_B and I_C. In Fig. 57.5 (*a*) are shown the directions of flow as well as relative magnitudes of these currents for a *PNP* transistor connected in the common-base mode. It is seen that again,

$$I_E = I_B + I_C$$

It means that a small part (about 1—2%) of emitter current goes to supply base current and the remaining major part (98—99%) goes to supply collector current.

Moreover, I_E flows into the transistor whereas both I_B and I_C flow out of it.

Fig. 57.5 (*b*) shows the flow of currents in the same transistor when connected in the common-emitter mode. It is seen that again, $I_E = I_B + I_C$

By normal convention, currents flowing ***into*** a transistor are taken as positive whereas those flowing ***out*** of it are taken as negative. Hence, I_E is positive whereas both I_B and I_C are negative. Applying Kirchhoff's Current Law, we have

$$I_E + (-I_B) + (-I_C) = 0 \quad \text{or} \quad I_E - I_B - I_C = 0 \quad \text{or} \quad I_E = I_B + I_C$$

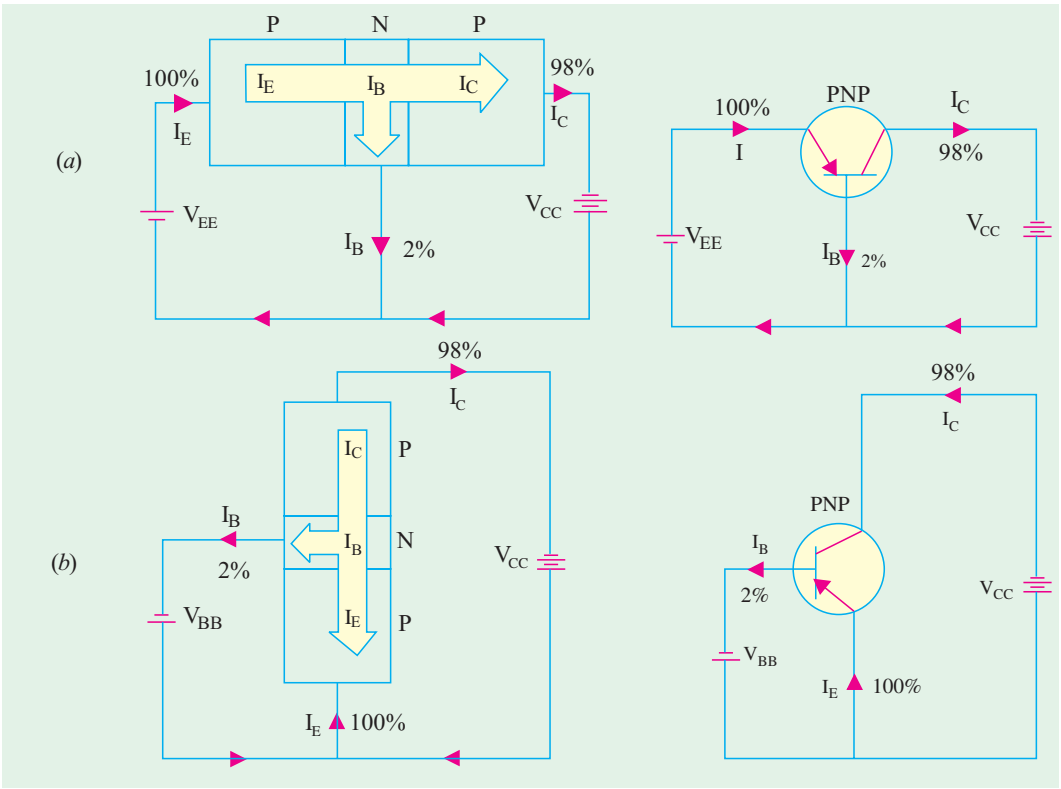


Fig. 57.5

This statement is true ***regardless of transistor type or transistor configuration.***

Note. For the time being, we have not taken into account the leakage currents which exist in a transistor (Art. 57.12).

57.5. Summing Up

The four basic guideposts about all transistor circuits are :

1. conventional current flows along the arrow whereas electrons flow against it;
2. *E/B* junction is always forward-biased;
3. *C/B* junction is always reverse-biased;
4. $I_E = I_B + I_C$.

57.6. Transistor Circuit Configurations

Basically, there are three types of circuit connections (called configurations) for operating a transistor.

1. common-base (*CB*), 2. common-emitter (*CE*), 3. common-collector (*CC*).

The term 'common' is used to denote the electrode that is common to the input and output circuits. Because the common electrode is generally grounded, these modes of operation are frequently referred to as grounded-base, grounded-emitter and grounded-collector configurations as shown in Fig. 57.6 for a *PNP* – transistor.

Since a transistor is a 3-terminal (and not a 4-terminal) device, one of its terminals has to be common to the input and output circuits.

57.7. CB Configuration

In this configuration, emitter current I_E is the input current and collector current I_C is the output current. The input signal is applied between the emitter and base whereas output is taken out from the collector and base as shown in Fig. 57.6 (*a*).

The ratio of the collector current to the emitter current is called dc alpha (α_{dc}) of a transistor.

$$\therefore \qquad \alpha_{dc}^{*} = \frac{-I_C}{I_E}$$

The negative sign is due to the fact that current I_E flows into the transistor whereas I_C flows out of it. Hence, I_E is taken as positive and I_C as negative.

$$\therefore \qquad I_C = -\alpha_{dc}.I_E$$

If we write adc simply as α**, then $\alpha = I_E/I_C$

It is also called forward current transfer ratio ($-h_{FB}$). In h_{FB}, subscript *F* stands for forward and *B* for common-base. The subscript d.c. on a signifies that this ratio is defined from dc values of I_C and I_E.

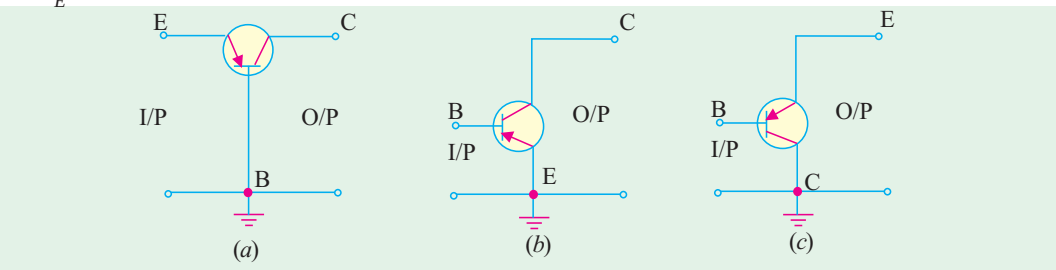


Fig. 57.6

The α of a tranistor is a measure of the quality of a transistor ; higher the value of α, better the transistor in the sense that collector current more closely equals the emitter current. Its value ranges

* More accurately, $\alpha_{dc} = \frac{I_C - I_{CBO}}{I_E}$ ...Art.57.12

** Negative sign has been omitted, since we are here concerned with only magnitudes of the currents involved.

from 0.95 to 0.999. Obviously, it applies only to *CB* configuration of a transistor. As seen from above and Fig. 57.7.

$I_C = \alpha I_E$. Now, $I_B = I_E - \alpha I_E = (1 - \alpha) I_E$

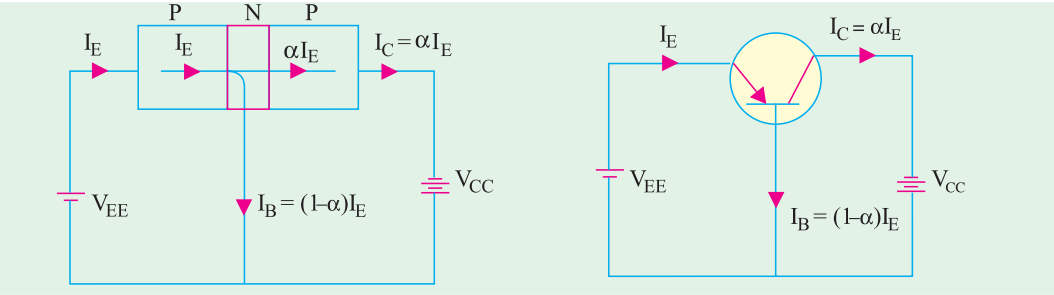


Fig. 57.7

Incidentally, there is also an a.c. α for a transistor. It refers to the ratio of ***change*** in collector current to the ***change*** in emitter current.

$$\therefore \quad \alpha_{ac} = \frac{-\Delta I_C}{\Delta I_E}$$

It is also, known as short-circuit gain of a transistor and is written as $-h_{fb}$. It may be noted that upper case subscript '*FB*' indicates dc value whereas lower case subscript '*fb*' indicates ac value. For all practical purposes, $\alpha_{dc} = \alpha_{ac} = \alpha$.

Example 57.1. *Following current readings are obtained in a transistor connected in CB configuration : I_E = 2 mA and I_B = 20 mA. Compute the values of α and I_C.*

(Electronics-II, Punjab Univ. 1992)

Solution. $I_C = I_E - I_B = 2 \times 10^{-3} - 20 \times 10^{-6} =$ **1.98 mA**

$\alpha = I_C / I_E = 1.98/2 =$ **0.99**

57.8. CE Configuration

Here, input signal is applied between the base and emitter and output signal is taken out from the collector and emitter circuit. As seen from Fig. 57.6 (*b*), I_B is the input current and I_C is the output current.

The ratio of the d.c. collector current to dc base current is called dc beta (β_{dc}) or just β of the transistor.

$$\therefore \quad \beta = -I_C / -I_B = I_C / I_B \quad \text{or} \quad I_C = \beta I_B$$ — Fig. 57.8 (*a*)

It is also called common-emitter d.c. ***forward transfer ratio*** and is written as h_{FE}. It is possible for β to have as high a value as 500.

While analysing ac operation of a transistor, we use ac β which is given by $\beta_{ac} = \Delta I_C / \Delta I_B$.

It is also written as h_{fe}.

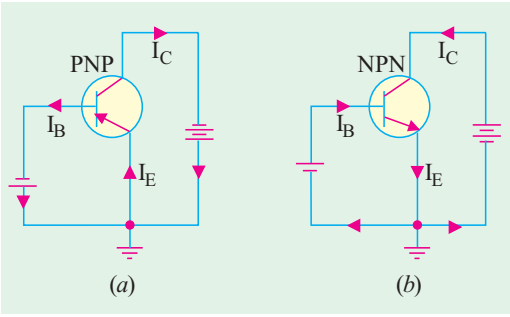


Fig. 57.8

The flow of various currents in a *CE* configuration both for *PNP* and *NPN* transistor is shown in Fig. 57.8. As seen

$$I_E = I_B + I_C = I_B + \beta I_B = (1 + \beta) I_B$$

57.9. Relation Between α and β

$$\beta = \frac{I_C}{I_B} \quad \text{and} \quad \alpha = \frac{I_C}{I_E} \quad \therefore \quad \frac{\beta}{\alpha} = \frac{I_E}{I_B}$$ — only numerical value of α

Now, $I_B = I_E - I_C$ $\therefore$ $\beta = \dfrac{I_C}{I_E - I_C} = \dfrac{I_C / I_E}{I_E / I_E - I_C / I_E}$ or $\beta = \dfrac{\alpha}{1-\alpha}$

Cross-multiplying the above equation and simplifying it, we get

$\beta (1 - a) = \alpha$ or $\beta = \alpha (1 + \beta)$ or $\alpha = \beta / (1 + \beta)$

It is seen from the about 2 equations that $1 - \alpha = 1/(1 + \beta)$

57.10. CC Configuration

In this case, input signal is applied between base and collector and output signal is taken out from emitter-collector circuit [Fig. 57.6 (*c*)]. Conventionally speaking, here I_B is the input current and I_E is the output current as shown in Fig. 57.9. The current gain of the circuit is

$$\frac{I_E}{I_B} = \frac{I_E}{I_C} \cdot \frac{I_C}{I_B} = \frac{\beta}{\alpha} = \frac{\beta}{\beta/(1+\beta)} = (1 + \beta)$$

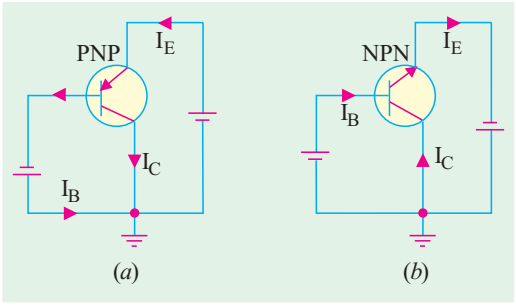


Fig. 57.9

The flow paths of various currents in a *CC* configuration are shown in Fig. 57.9. It is seen that

$I_E = I_B + I_C = I_B + \beta I_B = (1 + \beta) I_B$

$\therefore$ output current = $(1 + \beta) \times$ input current.

57.11. Relations Between Transistor Currents

While deriving various equations, following definitions should be kept in mind.

$$\alpha = \frac{I_C}{I_E}, \quad \beta = \frac{I_C}{I_B}, \quad \alpha = \frac{\beta}{(1+\beta)} \text{ and } \beta = \frac{\alpha}{(1-\alpha)}$$

(i) $I_C = \beta I_B = \alpha I_E = \dfrac{\beta}{1+\beta} I_E$

(ii) $I_B = \dfrac{I_C}{\beta} = \dfrac{I_E}{1+\beta} = (1-\alpha) I_E$

(iii) $I_E = \dfrac{I_C}{\alpha} = \dfrac{1+\beta}{\beta} I_C = (1+\beta)\ I_B = \dfrac{I_B}{(1-\alpha)}$

(iv) The three transistor d.c. currents always bear the following ratio*

$I_E : I_B : I_C \quad :: \quad 1 : (1 - \alpha) : \alpha$

Incidentally, it may be noted that for ac currents, small letters i_e, i_b and i_c are used.

57.12. Leakage Currents in a Transistor

(*a*) CB Circuit

Consider the *CB* transistor circuit shown in Fig. 57.11. The emitter current (due to majority carriers) initiated by the forward-biased emitter base junction is split into two parts :

(i) $(1 - \alpha) I_E$ which becomes base current I_B in the external circuit and

(ii) αI_E which becomes collector current I_C in the external circuit.

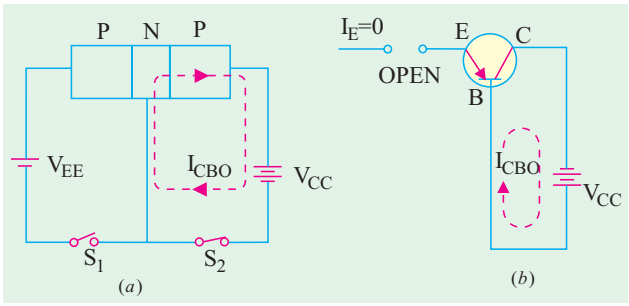


Fig. 57.10

* It reminds us of the power distribution relationship in an induction motor.

As mentioned earlier (Art. 57.2), though *C/B* junction is reverse-biased for majority charge carriers (*i.e.* holes in this case), it is forward-biased so far as thermally-generated minority charge carriers (*i.e.* electrons in this case) are concerned. This current flows even when emitter is disconnected from its dc supply as shown in Fig. 57.10 (*a*) where switch, S_1 is open. It flows in the ***same*** direction* as the collector current of majority carriers. It is called leakage current I_{CBO}. The subscripts *CBO* stand for 'Collector to Base with emitter Open.' Very often, it is simply written as I_{CO}.

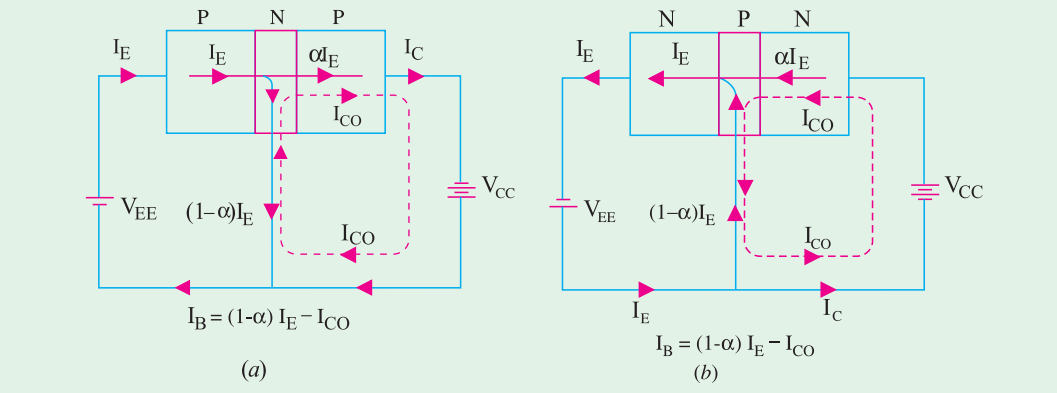


Fig. 57.11

It should be noted that

(i) I_{CBO} is exactly like the reverse saturation current I_S or I_0 of a reverse-biased diode discussed in Art. 57.1.

(ii) I_{CBO} is extremely temperature-dependent because it is made up of thermally-generated minority carriers. As mentioned earlier, I_{CBO} doubles for every 10°C rise in temperature for *Ge* and 6°C for *Si*.

If we take into account the leakage current, the current distribution in a *CB* transistor circuit becomes as shown in Fig. 57.11 both for *PNP* and *NPN* type transistors.

It is seen that total collector current is actually the sum of two components :

(i) current produced by normal transistor action *i.e.* component controlled by emitter current. Its value is a I_E and is due to majority carriers.

(ii) temperature-dependent leakage current I_{CO} due to minority carriers.

$$\therefore \quad I_C = \alpha I_E + I_{CO} \qquad ...(i) \qquad \therefore \quad \alpha = \frac{I_C - I_{CO}}{I_E}$$

Since $I_{CO} \ll I_C$, hence $\alpha \cong I_C / I_E$

(iii) Substituting the value of $I_E = (I_C + I_B)$ in Eq. (*i*) above, we get

$$I_C = (I_C + I_B) + I_{CO} \qquad \text{or} \qquad I_C(1-\alpha) = \alpha I_B + I_{CO}$$

$$\therefore \quad I_C = \frac{\alpha I_B}{1-\alpha} + \frac{I_{CO}}{1-\alpha}$$

(iv) Eliminating I_C from Eq. (i) above, we get

$$(I_E - I_B) = \alpha I_E + I_{CO} \qquad \text{or} \qquad I_B = (1-\alpha)\, I_E - I_{CO}$$

(*b*) CE Circuit

In Fig. 57.12 (*a*) is shown a common-emitter circuit of an *NPN* transistor whose base lead is

* Actually, electrons (which form minority charge carriers in collector) flow from negative terminal of collector battery, to collector, then to base through *C/B* junction and finally, to positive terminal of V_{CC}. However, conventional current flows in the opposite direction as shown by dotted lien in Fig. 57.10 (*a*)

open. It is found that despite $I_B = 0$, there is a leakage current from collector to emitter. It is called I_{CEO}, the subscripts *CEO* standing for 'Collector to Emitter with base Open'.

Taking this leakage current into account, the current distribution through a *CE* circuit becomes as shown in Fig. 57.12 (*c*).

$$I_C = \beta I_B + I_{CEO} = \beta I_B + (1 + \beta) I_{CO} = \beta I_B + I_{CO}/(1 - \alpha)$$

(*i*) $$\therefore \quad I_C = \frac{\alpha I_B}{1-\alpha} + \frac{I_{CO}}{1-\alpha}$$

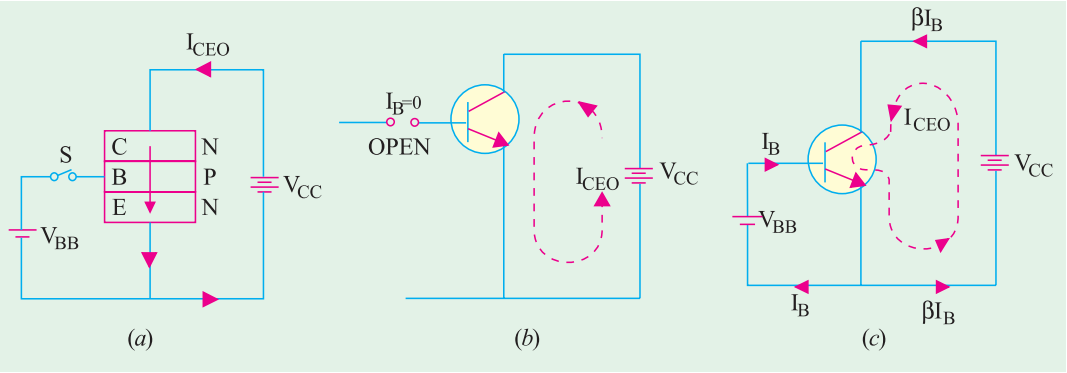


Fig. 57.12

Now, $\beta I_B = \alpha I_E$. Substituting this value above, we get,

$I_C = \alpha I_E + I_{CEO}$. Also, $I_B = I_E - I_C$

Substituting the value of I_C from above, we have

(*ii*) $I_B = I_E - \alpha I_E - I_{CEO} = (1 - \alpha) I_E - I_{CEO}$

57.13. Thermal Runaway

As seen from Art. 57.12, for a *CE* circuit

$$I_C = \beta I_B + (1 + \beta) I_{CO}$$

The leakage current is extremely temperature-dependent. It almost doubles for every 6°C rise in temperature in *Ge* and for every 10°C rise in *Si*. Any increase in I_{CO} is magnified $(1 + \beta)$ times *i.e.* 300 to 500 times. Even a slight increase in I_{CO} will affect I_C considerably. As I_C increases, collector power dissipation increases which raises the operating temperature that leads to further increase in I_C. If this succession of increases is allowed to continue, soon I_C will increase beyond safe operating value thereby damaging the transistor itself—a condition known as ***thermal runaway***. Hence, some form of stabilization is necessary to prevent this thermal runaway.

Example 57.2. *The reverse saturation current of an NPN transistor in common-base circuit is 12.5 μA. For an emitter current of 2 mA, collector current is 1.97 mA. Determine the current gain and base current.* **(Electronics-1, Gwalior Univ. 1988)**

Solution. Given : $I_{CBO} = 12.5\ \mu A$; $I_E = 2$ mA, $I_C = 1.97$ mA ; $\alpha = ?$, $I_B = ?$

$$I_C = \alpha I_E + I_{CBO} \qquad \therefore \quad \alpha = \frac{I_C - I_{CBO}}{I_E} = \frac{1.97 - 12.5 \times 10^{-3}}{2} = \mathbf{0.978}$$

$I_B = I_E - I_C = 2 - 1.97 = \mathbf{0.03\ mA}$.

Example. 57.3. *Derive an expression for forward current gain and leakage current of common-emitter configuration in terms of current gain and leakage current of common-base configuration. If a = 0.98, I_{CBO} = 5 mA, calculate b and ICEO.* **(Electronics-I, Mysore Univ. 1990)**

Solution. $\beta = \alpha / (1 - \alpha) = 0.98/(1 - 0.98) = \mathbf{49}$

$I_{CEO} = (1 + \beta)\, I_{CO} = (1 + 49) \times 5 \quad = 250\ \mu A = \mathbf{0.25\ mA}.$

Example 57.4. *For a transistor, $I_B = 100\ \mu A$, $\alpha_{dc} = 0.98$ and $I_{CO} = 5\ \mu A$. Find the values of I_C and I_E.*

Solution. As seen from Art. 57.12, $I_C = \dfrac{\alpha I_B}{1-\alpha} + \dfrac{I_{CO}}{1-\alpha} = \dfrac{0.98 \times 100}{1-0.98} + \dfrac{5}{1-0.98} = \mathbf{5.15\ mA}$

$I_E = I_C + I_B = 5.15 + 100 \times 10^{-3} = \mathbf{5.25\ mA}.$

Example 57.5. *A transistor operating in CB configuration has $I_C = 2.98$ mA, $I_E = 3.00$ mA and $I_{CO} = 0.01$ mA. What current will flow in the collector circuit of this transistor when connected in CE configuration with a base current of 30 μA.* **(Electronics-II, M.S. Univ. Vadodra 1990)**

Solution. For *CE* configuration, $I_C = \beta I_B + (1 + \beta)\, I_{CO}$

Let us find the value of β from data given for *CB* configuration. For such a circuit $I_C = \alpha I_E + I_{CO}$ or $2.98 = \alpha \times 3 + 0.01$; $\alpha = 0.99$; $\beta = \alpha/(1 - \alpha) = 0.99/(1 - 0.09) = 99$.

$\therefore$ For *CE* circuit, $I_C = 99 \times 0.03 + (1 + 99) \times 0.01 = \mathbf{3.97\ mA}$.

Example 57.6. *For a certain transistor, $I_C = 5.505$ mA, IB = 50 mA, $I_{CO} = 5$ mA. Determine **(i)** values of α, β and I_E **(ii)** the new level of I_B required to make $I_C = 10$ mA.*

Solution. (*i*) $I_C = \beta I_B + (1 + \beta)\, I_{CO}$ or $5.505 \times 10^3 = \beta \times 50 + (1 + \beta) \times 5 \ \therefore \beta = 100$

Now, $I_E = I_C + I_B = 5.505 + 50 \times 10^{-3} = \mathbf{5.555\ mA}$.

Also, $I_C = \alpha I_E + I_{CO}$; $5.505 = \alpha \times 5.555 + 5 \times 10^{-3} \quad \therefore \quad \alpha = 5.500/5.555 = \mathbf{0.99}$

(*ii*) As seen from Art. 7.12, $I_C = \beta I_B + (1 + \beta)\, I_{CO}$

$\therefore \quad 10 = 100\, I_B + 101 \times 5 \times 10^{-3}$; $\quad I_B = 0.09495\ \text{mA} = \mathbf{94.95\ \mu A}$.

Example. 57.7. *Discuss the operation of a PNP transistor.*

The reverse saturation current in a PNP germanium transistor type OC 71 is 8 μA. If the transistor common base current gain is 0.979, calculate the collector and emitter current for 40 μA base current. What is the collector current when base current is zero ?

(Electronics-1, Gwalior Univ. 1986)

Solution. Given : $I_{CO} = 8\ \mu A = 0.008\ \mu A$, $\alpha = 0.979$; $I_B = 40\ \mu A = 0.04$ mA

In a *CE* circuit: $I_C = \beta I_B + I_{CEO} = \beta I_B + I_C/(1 - \alpha)$.

Now, $\beta = \alpha/(1 - \alpha) = 0.979/(1 - 0.979) = 46.6$

$\therefore \quad I_C = 46.6 \times 0.04 + (1 + 46.6) \times 0.008 = \mathbf{1.9\ mA}$; $I_E = I_C + I_B = 1.9 + 0.04 = \mathbf{1.94\ mA}$

57.14. Transistor Static Characteristics

There are the curves which represents relationship between different d.c. currents and voltages of a transistor. These are helpful in studying the operation of a transistor when connected in a circuit. The three important characteristics of a transistor are :

1. Input characteristic, **2.** Output characteristic, **3.** Constant-current transfer characteristic.

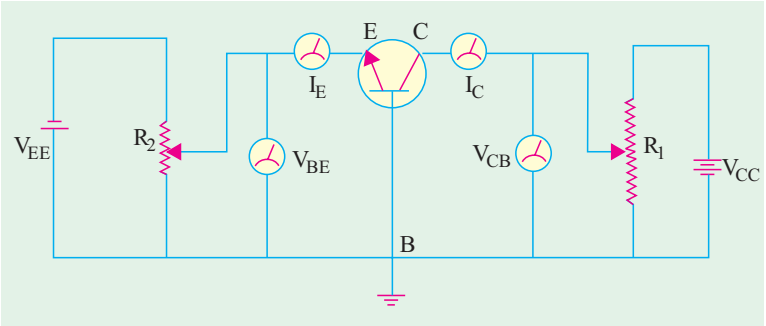


Fig. 57.13

57.15 Common Base Test Circuit

The static characteristics of an *NPN* transistor connected in common-base configuration can be determined by the use of test circuit shown in Fig. 57.13. Milliammeters are included in series with the emitter and collector circuits to measure I_E and I_C. Similarly, voltmeters are connected across E and B to measure voltage V_{BE} and across C and B to measure V_{CB}. The two potentiometer resistors R_1 and R_2 supply variable voltages from the collector and emitter dc supplies respectively.

57.16. Common Base Static Characteristics

(a) Input Characteristic

It shows how I_E varies with V_{BE} when voltage V_{CB} is held constant. The method of determining this characteristic is as follows :

First, voltage V_{CB} is adjusted to a suitable value with the help of R_1 (Fig. 57.13). Next, voltage V_{BE} is increased in a number of discrete steps and corresponding values of I_E are noted from the milliammeter connected for the purpose. When plotted, we get the input characteristic shown in Fig. 57.14, one for *Ge* and the other for *Si*. Both curves are exactly similar to the forward characteristic of a *P-N* diode which, in essence, is what the emitter-base junction is.

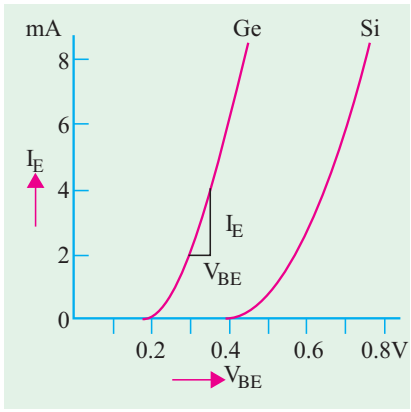


Fig. 57.14

This characteristic may be used to find the input resistance of the transistor. Its value is given by the reciprocal of its slope.

$$R_{in} = \Delta V_{BE} / \Delta I_E \qquad \text{— } V_{CB} \text{ constant.}$$

Since the characteristic is initially nonlinear, R_{in} will vary with the point of measurement. Its value over linear part of the characteristic is about 50 Ω but for low values of V_{BE}, it is considerably greater. This change in R_{in} with change in V_{BE} gives rise to distortion of signals handled by the transistor.

This characteristic is hardly affected by changes either in V_{CB} or temperature.

(b) Output Characteristic

It shows the way I_C varies with V_{CB} when I_E is held constant. The method of obtaining this characteristic is as follows:

First, movable contact, on R_2 (Fig. 57.13) is changed to get a suitable value of V_{BE} and hence that of I_E. While keeping I_E constant at this value, V_{CB} is increased from zero in a number of steps and the corresponding collector current I_C that flows is noted.

Next, V_{CB} is reduced back to zero, I_E is increased to a value a little higher than before and the whole procedure is repeated. In this way, whole family of curves is obtained, a typical family being shown in Fig. 57.15.

1. The reciprocal of the near horizontal part of the characteristic gives the output resistance R_{out} of the transistor which it would offer to an input signal. Since the characteristic is linear over most of its length (meaning that I_C is virtually independent of V_{CB}). R_{out} is very high, a typical value being 500 kΩ.

$$R_{out} = \frac{1}{\Delta I_C / \Delta V_{CB}} = \frac{\Delta V_{CB}}{\Delta I_C}$$

2. It is seen that I_C flows even when $V_{CB} = 0$. For example, it has a value = 1.8 mA corresponding to $V_{CB} = 0$ for $I_E = 2$ mA as shown in Fig. 57.15. It is due to the fact that electrons are being injected into the base under the action of forward-biased *E/B* junction and are being collected by the collector due to the action of the internal junction voltage at the *C/B* junction (Art. 57.2). For reducing I_C to zero, it is essential to neutralize this potential barrier by applying a small forward bias ac-ross *C/B* junction.

3. Another important feature of the characteristic is that a small amount of collector current flows even when emitter current $I_E = 0$. As we know (Art. 57.12), it is collector leakage current I_{CBO}.
4. This characteristic may be used to find α_{ac} of the transistor as shown in Fig. 57.15.

$$\alpha_{ac} = \frac{\Delta I_C}{\Delta I_E} = \frac{DE}{BC}$$

$$= \frac{6.2 - 4.3}{2} = 0.95$$

5. Another point worth noting is that although I_C is practically independent of V_{CB} over the working range of the transistor, yet if V_{CB} is permitted to increase beyond a certain value, I_C eventually increases rapidly due to avalanche breakdown as shown in Fig. 57.15.

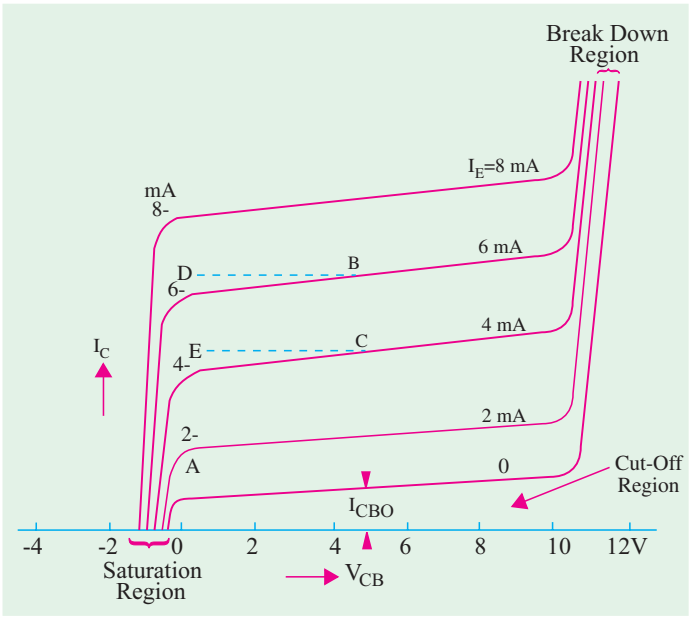


Fig. 57.15

(c) Current Transfer Characteristic

It shows how I_C varies with changes in I_E when V_{CB} is held constant. For drawing this characteristic, first V_{CB} is set to a convenient value and then I_E is increased in steps and corresponding values of I_C noted. A typical transfer characteristic is shown in Fig. 57.16 (*a*). Fig. 57.16 (*b*) shows a more detailed view of the portion near the origin.

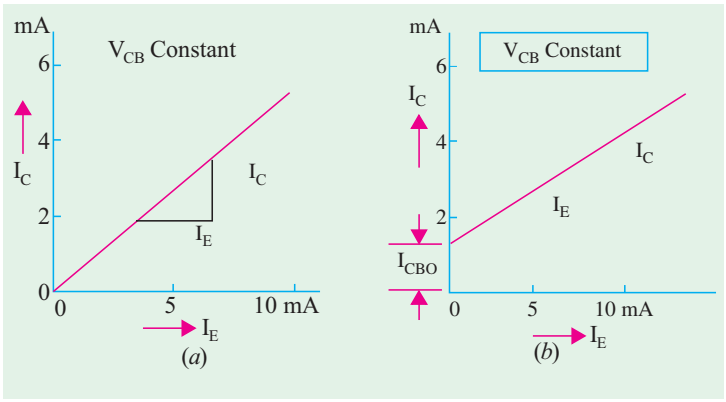


Fig. 57.16

As seen, α_{ac} may be found from the equation

$$\alpha_{ac} = \Delta I_C / \Delta I_E$$

Usually, α_{ac} is found from output characteristic than from this characteristic.

It may be noted in the end that *CB* connection is rarely employed for audio-frequency circuits because ***(i)*** its current gain is less than unity and ***(ii)*** its input and output resistances are so different.

57.17. Common Emitter Test Circuit

The static characteristics of an *NPN* transistor connected in *CE* configuration may be determined by the use of circuit diagram shown in Fig. 57.17. A milliammeter (or a microammeter in the case of a low-power transistor) is connected in series with the base to measure I_B. Similarly, a milliammeter is included in the collector circuit to measure I_C. A voltmeter with a typical range of 0 –1 V is connected across base and emitter terminals for measuring V_{BE}.

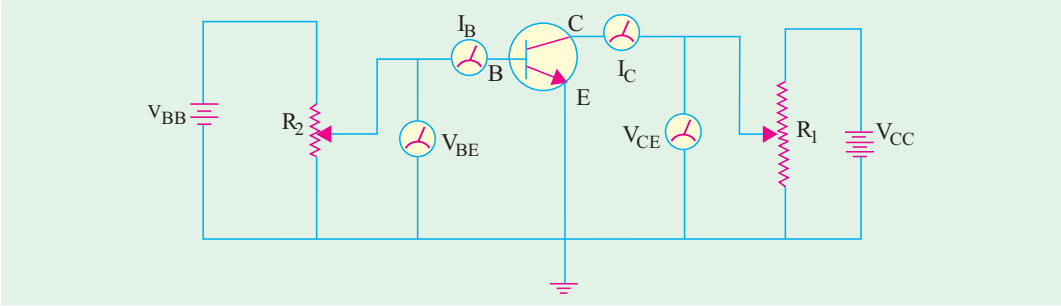


Fig. 57.17

Potentiometer R_2 connected across dc supply V_{BB} is used to vary I_B and V_{BE}. A second voltmeter with a typical range of 0–20 V is connected across collector-emitter terminals to measure the output collector-emitter voltage V_{CE}.

57.18. Common Emitter Static Characteristics

(a) Input Characteristic

It shows how I_B varies with changes in V_{BE} when V_{CE} is held constant at a particular value.

To begin with, voltage V_{CE} is maintained constant at a convenient value and then V_{BE} is increased in steps. Corresponding values of I_B are noted at each step. The procedure is then repeated for a different but constant value of V_{CE}. A typical input characteristic is shown in Fig. 57.18. Like *CB* connection, the overall shape resembles the forward characteristic of a *P-N* diode. The reciprocal of the slope gives the input resistance R_{in} of the transistor.

$$R_{in} = \frac{1}{\Delta I_B / \Delta V_{BE}} = \frac{\Delta V_{BE}}{\Delta I_B}$$

Due to initial non-linearity of the curve, R_{in} varies considerably from a value of 4 kΩ near the origin to a value of 600 Ω over the more linear part of the curve.

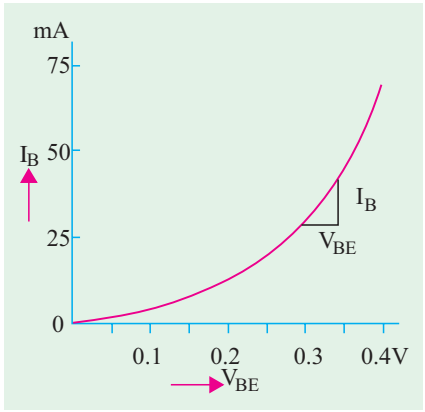


Fig. 57.18

(b) Output or Collector Characteristic

It indicates the way in which I_C varies with changes in V_{CE} when I_B is held constant.

For obtaining this characteristic, first I_B is set to a convenient value and maintained constant and then V_{CE} is increased from zero in steps, I_C being noted at each step. Next, V_{CE} is reduced to zero and I_B increased to another convenient value and the whole procedure repeated. In this way, a family of curves (Fig. 57.19) is obtained.

It is seen that as V_{CE} increases from zero, I_C rapidly increases to a near saturation level for a fixed value of I_B. As shown, a small amount of collector current flows even when $I_B = 0$. It is called I_{CEO} (Art. 57.12). Since main collector current is zero, the transistor is said to be **cut-off**.

It may be noted that if V_{CE} is allowed to increase too far, *C/B* junction completely breaks down and due to this avalanche breakdown, I_C increases rapidly and may cause damage to the transistor.

When V_{CE} has very low value (ideally zero), the transistor is said to be saturated and it operates in the saturation region of the characteristic. Here, change in I_B does not produce a corresponding change in I_C.

This characteristic can be used to find β_{ac} at a specific value of I_B and V_{CE}. It is given by $\beta_{ac} = \Delta I_C / \Delta I_B$.

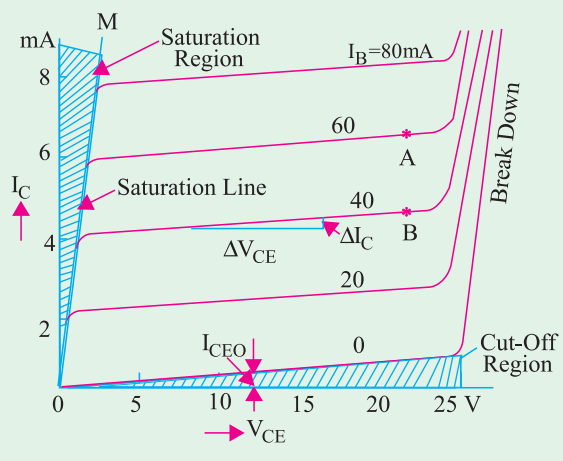


Fig. 57.19

We may select any two points *A* and *B* on the I_B = 60 μA and 40 μA lines respectively and measure corresponding values of I_C from the diagram for finding ΔI_C. Since ΔI_B = (60 – 40) = 20 μA, β_{ac} can be easily found.

The value of output resistance R_{out} (= $\Delta V_{CE}/\Delta I_C$) over the near horizontal part of the characteristic varies from 10 kΩ to 50 kΩ.

(*c*) Current Transfer Characteristic

It indicates how I_C varies with changes in I_B when V_{CE} is held constant at a given value.

Such a typical characteristic is shown in Fig. 57.20 (*a*). Its slope gives

$$\beta_{ac} = \Delta I_C/\Delta I_B$$

From Fig. 57.20 (*b*), it is seen that a small collector current flows even when I_B = 0. It is the common-emitter leakage current I_{CEO} = (1 + β) I_{CO}. Like I_{CO}, it is also due to the flow of minority carriers across the reverse-biased *C/B* junction.

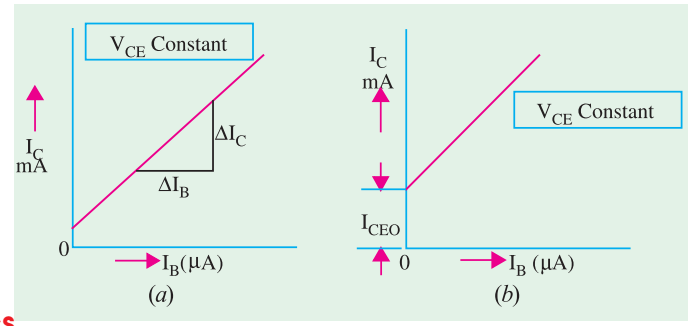


Fig. 57.20

57.19. Common Collector Static Characteristics

As shown in Fig. 57.21, in this case, collector terminal is common carrier to both the input (*CB*) and output (*CE*) carriers circuits.

The output characteristic is I_E versus V_{CE} for several fixed values of I_B. Since $I_C \cong I_E$, this characteristic is practically idential to that of the *CE* circuit and is shown in Fig. 57.22 (*a*).

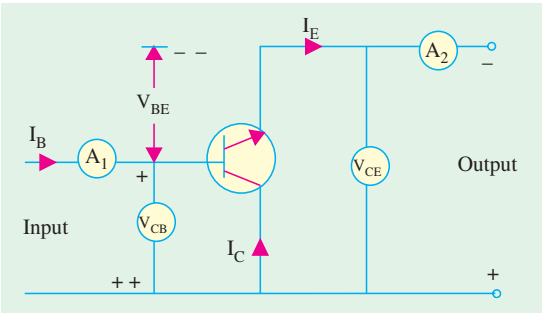


Fig. 57.21

Similarly, its current gain characteristic I_C versus I_B for different values of V_{CE} is similar to that of a *CE* circuit because $I_C \cong I_E$.

The *CC* input characteristic is a plot of V_{CB} versus I_B for different values of V_{CE} and is shown in figure 57.22 (*b*). It is quite different from those for *CB* or *CE* circuit. This difference is due to the fact that input voltage V_{CB} is largely determined by the value of *CE* voltage. Consider the input characteristic for I_B = 100 μA and V_{CE} = 2 V.

$$V_{CB} = V_{CE} - V_{BE} = 2 - 0.7 = 1.3 \text{ V} \quad \text{— for Si material}$$

Moreover, as V_{CB} is increased, V_{BE} is reduced thereby reducing I_B.

Now, consider the values V_{CE} = 4 V and I_B = 100 μA

$$V_{CB} = 4 - 0.7 = 3.3 \text{ V}$$

Again, as V_{CB} increases, I_B is decreased.

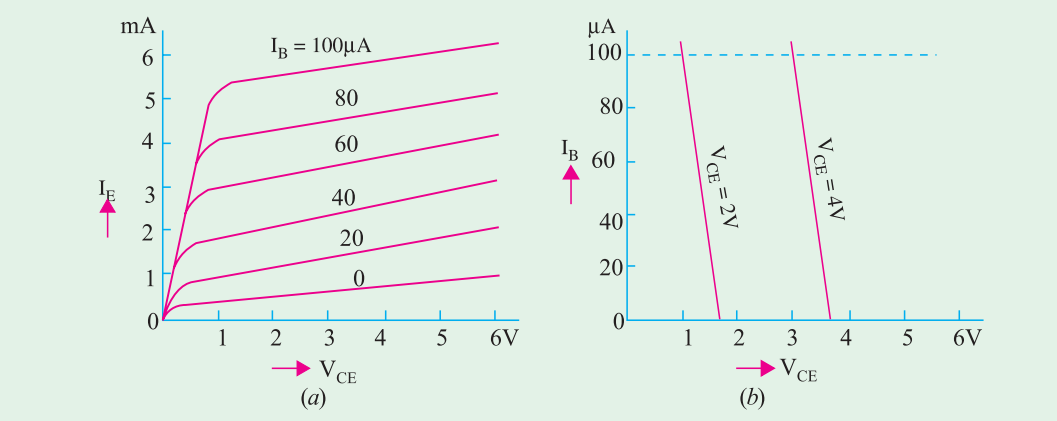


Fig. 57.22

57.20. Different Ways of Drawing Transistor Circuits

In Fig. 57.23 (*a*) is shown a *CB* transistor circuit which derives its voltage and current requirements from two independent power sources *i.e.* two different batteries. Correct battery connections can be done by remembering the transistor polarity rule (Art. 57.2) that in an *NPN* transistor, both collector and base have to be Positive with respect to the emitter. Of course, collector is a ***little bit more*** positive than base which means that between themselves, collector is at a ***slightly higher positive*** potential with the respect to the base. Conversely, base is at a little lower potential with respect to the collector.

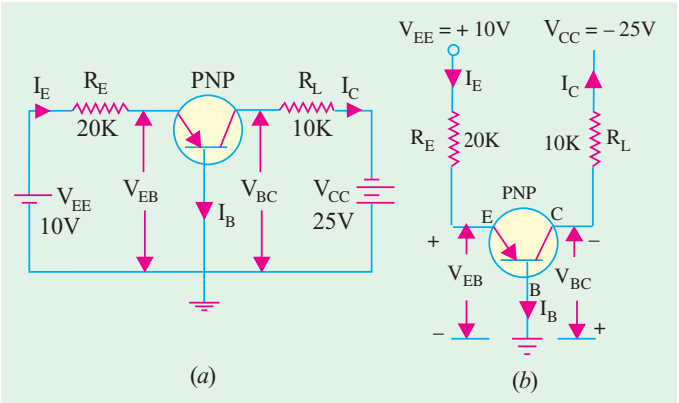


Fig. 57.23

Putting it in a slightly different way, we can say that collector is positive w.r.t. base and conversely, base is negative w.r.t. collector. That is why, potential difference between collector and base in written as V_{CB} (and not V_{BC}) because terminal at higher potential is mentioned first. Same reasoning applies to V_{BE}. Fig. 57.23 (*b*) shows another and more popular way of indicating power supply voltage. Only one terminal of the battery is shown, the other terminal is ***understood to be grounded so as to provide a complete path for the current.***

For example, negative terminal of V_{CC} and positive terminal of V_{EE} are supposed to be grounded (as is the base) even though not shown as such in the diagram.

Fig. 57.24 (*a*) shows an *NPN* transistor connected in *CE* configuration with volt-

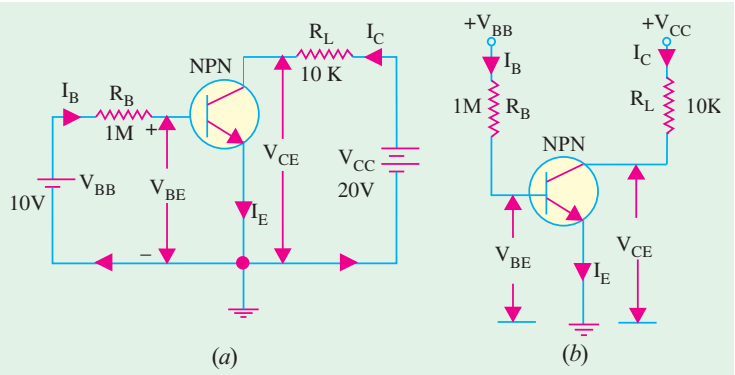


Fig. 57.24

ages and currents drawn from two independent power sources. As seen, battery con-nections and voltage markings are as per the rule given in Art. 57.2. Fig. 57.24 (*b*) shows the more popular way of indicating power supply voltages.

As seen, both collector and base are positive with respect to the common electrode *i.e.* emitter. Hence, a single battery can be used to get proper voltages across the two as shown in Fig. 57.25.

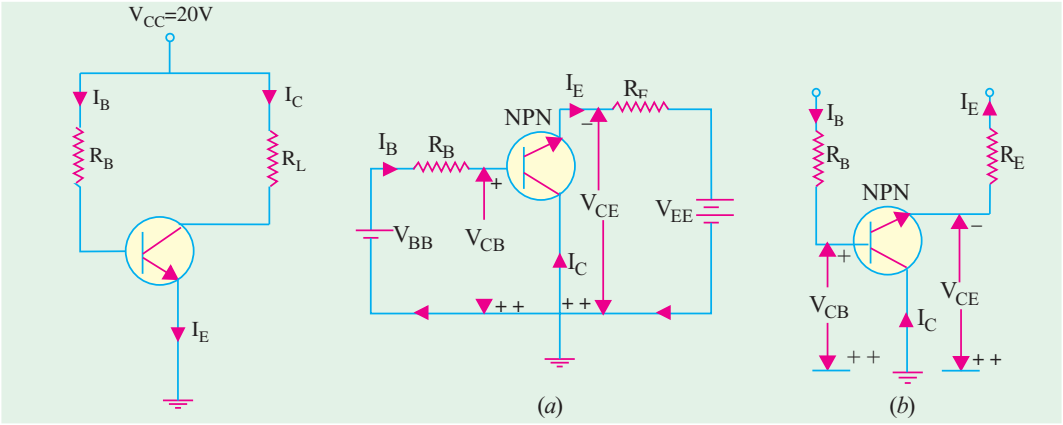


Fig. 57.25 **Fig. 57.26**

Fig. 57.26 (*a*) shows the *CC* configuration of an *NPN* transistor and Fig. 57.26 (*b*) shows the same circuit drawn differently.

57.21. Common Base Formulas

Let us find the values of different voltages and currents for the circuit in Fig. 57.23 (*b*). Consider the circuit *MEBM*. Applying Kirchhoff's voltage law and starting from point *B* (or ground) upwards, we get

(*a*) $-V_{BE} - I_E R_E + V_{EE}^{*} = 0$ or $I_E = \dfrac{V_{EE} - V_{BE}}{R_E}$

where V_{BE} = 0.3 V (for *Ge*) and 0.7 V (for *Si*)

Since, generally, $V_{EE} \gg V_{BE}$, we can simplify the above to $I_E \cong V_{EE}/R_E$ = 10 V/20 K = 0.5 mA (Fig. 57.23).

Taking V_{BE} into account and assuming silicon transistor

I_E = (10 – 0.7) V/20 K = 0.465 mA

(*b*) $I_C = \alpha I_E \cong I_E$ = 0.5 mA neglecting leakage current.

(*c*) From circuit *NCBN*, we get

$V_{CB} = V_{CC} - 1_C \cong V_{CC} - I_E R_L = 25 - 0.5 \times 10 = 20$ V $(\therefore I_C \cong I_E)$

Example 57.8. *In the circuit of Fig. 57.27* (*a*), what value of R_L causes V_{CB} = 5 V?

Solution. $I_E \cong V_{EE}/R_E$ = 10 V/10 K = 1 mA

$I_C = \alpha I_E \cong I_E$ = 1 mA

Now, $V_{CC} = I_{CRL} + V_{CB}$

$\therefore \quad R_L = \dfrac{V_{CC} - V_{CB}}{I_C} = \dfrac{20 - 5}{1\,\text{mA}} = \mathbf{15K}$

Example. 57.9. *For the circuit shown in Fig. 57.27 (b), find the value of R_E which causes V_{BC} = 10 V.*

* It is taken positive because we are going from the negative to the positive terminal of the emitter battery.

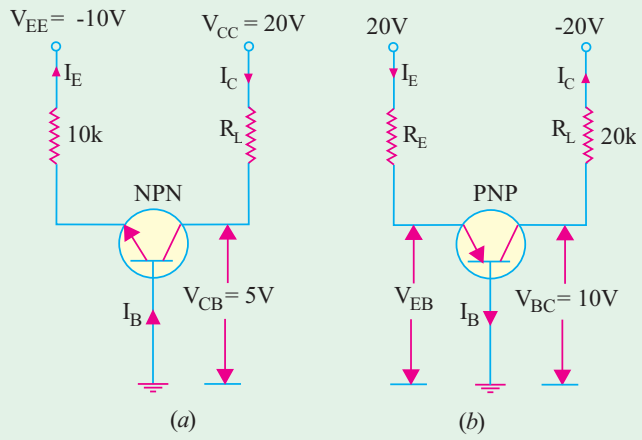


Fig. 57.27

Solution. $I_C = \dfrac{V_{CC} - V_{BC}}{R_L}$

$= \dfrac{20-10}{20\,\text{K}} = 0.5\,\text{mA}$

Now, $I_E = I_C/\alpha \cong I_C = 0.5$ mA

If we neglect V_{BE}, then entire V_{EE} = 20 V has to be dropped across R_E.

$\therefore \quad 0.5\,R_E = 20$

or $R_E = 20/0.5$ mA $= 40$ K

57.22. Common Emitter Formulas

Consider the *CE* circuit of Fig. 57.28. Taking the emitter-base circuit, we have

$$I_B = \frac{V_{BB} - V_{BE}}{R_B} \cong \frac{V_{BB}}{R_B}$$

$I_C = \beta\, I_B$ — neglecting leakage current I_{CEO}

$V_{CE} = V_{CC} - I_C R_L$

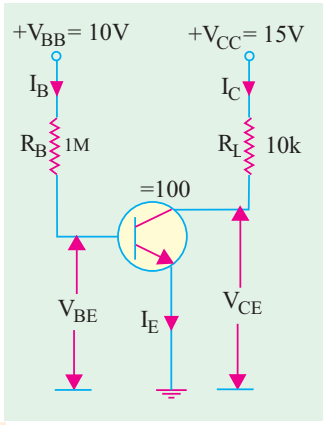


Fig. 57.28

Example 57.10. *For the circuit of Fig. 57.28, find* ***(i)*** I_B ***(ii)*** I_C ***(iii)*** I_E *and* ***(iv)*** V_{CE}. *Neglect* V_{BE}.

Sol. (*i*) $I_B \cong \dfrac{V_{BB}}{R_B} = \dfrac{10}{1M} =$ **10μA**

(*ii*) $I_C = \beta I_B = 100 \times 10\ \mu\text{A} =$ **1 mA**

(*iii*) $I_E = I_B + I_C = 1\text{mA} + 10\ \mu\text{A} =$ **1.01 mA**

(*iv*) $V_{CE} = V_{CC} - I_{CRC} = 15 - 1 \times 10 =$ **5 V**

Example 57.11. *Find the exact value of emitter current* I_E *in the two-supply emitter bias circuit of Fig. 57.29.*

(Electronics-1, Bangalore Univ. 1989)

Solution. Let us apply Kirchhoff´s voltage law to the loop containing R_B, R_E and V_{EE}. Starting from emitter and going clock-wise, we get

$$-I_E R_E + V_{EE} - I_B R_B - V_{BE} = 0$$

or $I_E R_E + I_B R_B = V_{EE} - V_{BE}$ ... ***(i)***

Now $\beta = I_C/I_B \cong I_E/I_B$ $\therefore \quad I_B \cong I_E/\beta$

Substituting this value in Eq. ***(i)*** above, we get

$$I_E R_E + \frac{I_E R_B}{\beta} = V_{EE} - V_{BE} \quad \text{or}$$

$$I_E = \frac{V_{EE} - V_{BE}}{R_E + R_B/\beta}$$

Since, in most cases, $(R_B/\beta) \ll R_E$

$\therefore \quad I_E = (V_{EE} - V_{BE})/R_E \cong V_{EE}/R_E$

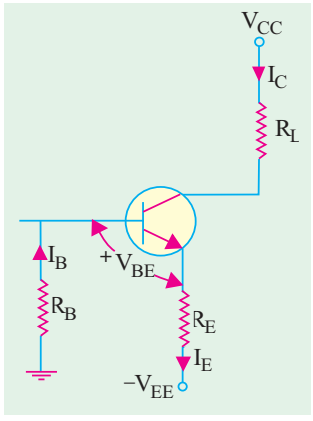


Fig. 57.29

Also, $I_B = I_E/(1+\beta) \cong I_E/\beta$

Example. 57.12. *In the circuit of Fig. 57.30, find* ***(i)*** I_E, ***(ii)*** I_B, ***(iii)*** I_C *and* ***(iv)*** V_{CE}. *Neglect* V_{BE} *and take* $\beta = 100$.

Sol. (*i*) $I_E = \dfrac{V_{EE}}{R_E + R_B/\beta} = \dfrac{30}{30 + 20/100} \cong$ **1 mA**

(*ii*) $I_B \cong I_E/\beta = 1/100 =$ **0.01 mA**

(*iii*) $I_C = I_E - I_B = 1 - 0.01 =$ **0.99 mA**

(*iv*) $V_{CE} = V_{CC} - I_C R_L = 30 - 10 \times 0.99 =$ **20.1 V.**

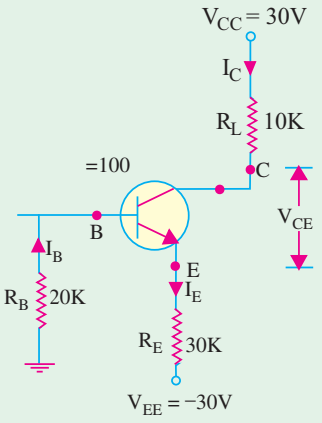


Fig. 57.30

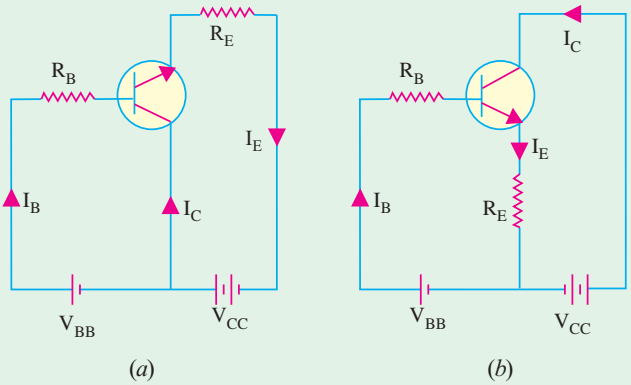


Fig. 57.31

57.23. Common Collector Formulas

The *CC* circuit with its proper d.c. biasing voltage sources is shown in Fig. 57.31 (*a*). The two circuits given in Fig. 57.31 represent the same thing.

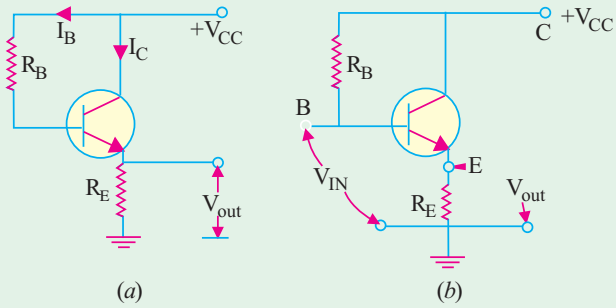


Fig. 57.32

Another way of drawing the same circuit is shown in Fig. 57.32 (*a*) where only one battery has been used. It should be noted that load resistor is not in the collector lead but in the emitter lead as shown.

Fig. 57.32 (*b*) makes the circuit connection quite clear. Input is between base and collector terminals whereas output is between emitter and collector terminals.

It is seen that

$$I_E = \frac{V_{CC} - V_{BE}}{R_E + R_B/\beta}; \quad V_{CC} = V_{CE} + I_E R_E; \quad I_E = \frac{V_{CC} - V_{BE}}{R_E + \beta R_E}; \quad I_C = \beta I_B$$

Example 57.13. *In the CC circuit of Fig. 57.33, find* ***(a)*** I_B , ***(b)*** I_E , ***(c)*** V_{CE} , ***(d)*** V_E and ***(e)*** V_B.

Take $\beta = 49$ and $V_{BE} = 0.7$ V.

Solution. **(*a*)** $I_B = \dfrac{V_{CC} - V_{BE}}{R_B + (1+\beta) R_E}$

$$= \frac{9.0 - 0.7}{100 + 50 \times 2} = \mathbf{41.5\ \mu A}$$

(*b*) $I_E = (1 + \beta)\, I_B = 50 \times 41.5 = \mathbf{2.075\ mA}$

(*c*) $V_{CE} = V_{CC} - I_E R_E = 9 - 2.075 \times 2 = \mathbf{5.85\ V}$

(*d*) $V_E = I_E R_E = 2.075 \times 2 = \mathbf{4.15\ V}$

(*e*) $V_B = V_{BE} + I_E R_E = 0.7 + 4.15 = \mathbf{4.85\ V}$

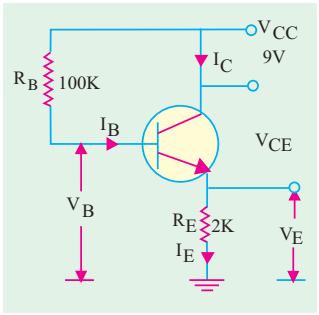


Fig. 57.33

57.24. The Beta Rule

According to this rule, resistance from one part of a transistor circuit can be referred to another of its parts (as we do with the primary and secondary winding impedances of a transformer). For example, resistance R_L in the collector circuit can be referred to the base circuit and ***vice versa.*** Similarly, R_E can be referred to the base circuit and, reciprocally, R_B can be referred to the emitter circuit. Since current through R_L is I_C (= β I_B), hence β-factor comes into the picture. Similarly, current through R_E is I_E which is (1 + β) times I_B, hence (1 + β) or approximately β-factor comes into the picture again, Use of this 'β-rule' makes transistor circuit calculations quite quick and easy. It makes the calculation of I_B quite simple.

The 'β-rule' may be stated as under :

1. When referring R_L or R_C to the base circuit, **multiply** it by β. When referring R_B to the collector circuit, **divide** it by β.
2. When referring R_E to base circuit, **multiply** it by (1 + β) or just β (as a close approximation).
3. Similarly, when referring R_B to emitter circuit, **divide** it by (1 + β) or β.

Before you apply this rule to any circuit, you must remember one very important point otherwise you are likely to get wrong answers. The point is that ***only those resistances are transferred which lie in the path of the current being calculated.*** Not otherwise. The utility of this rule will be demonstrated by solving the following problems.

Example 57.14. *Calculate the value of V_{CE} in the collector stabilisation circuit of Fig. 57.34.*

Solution. We will use β-rule to find I_C in the following two ways.

(*i*) First Method

Here, we will transfer R_L to the base circuit.

$$I_B = \frac{V_{CC}}{R_B + \beta R_L} = \frac{20}{1000 + 100(10)} = 10\,\text{mA}$$

$$I_C = \beta I_B = 100 \times 10 = 1000\ \text{mA} = 1\ \text{A}$$

$$V_{CE} \cong V_{CC} - I_C R_L = 20 - 1 \times 10 = \mathbf{10\ V}$$

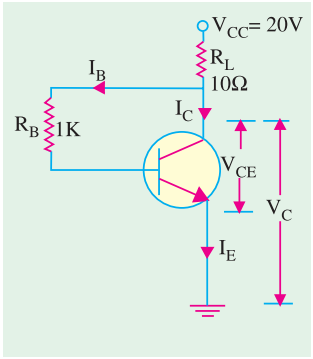


Fig. 57.34

(*ii*) Second Method

Now, we will refer R_B to collector circuit.

$$I_C \cong \frac{V_{CC}}{R_L + R_B/\beta} = \frac{20}{10 + 1000/100)} = 1\,\text{mA}$$

$V_{CE} = V_{CC} - I_C R_L = 10$ V — as above

It was a simple circuit because $R_E = 0$ and R_B was connected to V_{CC} through R_L and not directly (in which case, R_L would not lie in the path of I_B). Now, we will consider the case when R_E is present and R_L does not lie in the path of I_B.

Example 57.15. *Calculate the three transistor currents in the circuit of Fig. 57.35.*

Solution. (*i*) First Method

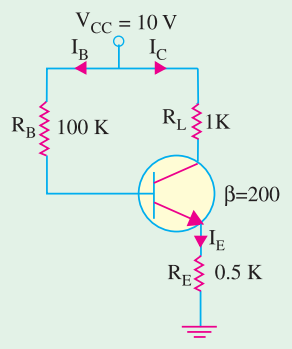


Fig. 57.35

Since R_E lies in the path of I_B

$$\therefore \quad I_B = \frac{V_{CC}}{R_L + \beta R_E} \qquad \text{— neglecting } V_{BE}$$

$$= \frac{10}{100 + 200\,(0.5)} = \mathbf{0.05\ mA}$$

$$I_C = \beta I_B = 200 \times 0.05 = \mathbf{10\ mA}, \quad I_E = I_B + I_C = \mathbf{10.05\ mA}$$

(*ii*) Second Method

Now, we will transfer R_B to emitter circuit and find I_E directly.

$$I_E = \frac{V_{CC}}{R_E + R_B/\beta} = \frac{10}{0.5 + 100/200} = 10\,\text{mA} \qquad \text{— as before}$$

$$I_B = I_C/\beta \cong 10/200 = \mathbf{0.05\ mA}$$

Example 57.16. *Calculate I_E in the circuit of Fig. 57.36.*

(Electronic & Commu., Ranchi Univ. 1990)

Solution. If we neglect V_{BE}, then as seen from the circuit of Fig. 57.36.

$$I_E = \frac{V_{EE}}{R_E + R_B/\beta} = \frac{10}{10 + 10/100} = \mathbf{0.99\ mA}$$

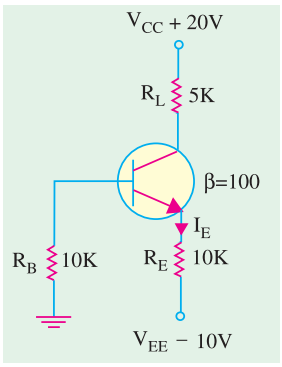


Fig. 57.36

57.25. Importance of V_{CE}

The voltage V_{CE} is very important in checking whether the transistor is

(*a*) defective, **(*b*)** working in cut-off,

(*c*) in saturation or well into saturation (Example 57.17 and 57.18)

When $V_{CE} = V_{CC}$, the transistor is in cut-off *i.e.* it is turned OFF. When $V_{CE} = 0$, the transistor is in saturation *i.e.* it is turned fully ON. When V_{CE} is less than zero *i.e.* negative, the transistor is said to be well into saturation. In practice, both these conditions are avoided. For amplifier operation, $V_{CE} = \frac{1}{2} V_{CC}$ *i.e.* transistor is operated at approximately $\frac{1}{2}$ ON. In this way, variations in I_B in either direction will control I_C in both directions. In other words, when I_B increases or decreases, I_C also increases or decreases. However, if I_B is OFF, I_C is also OFF. On the other hand, if collector has been turned fully ON, maximum I_C flows. Hence, no further increase in I_B can be reflected in I_C.

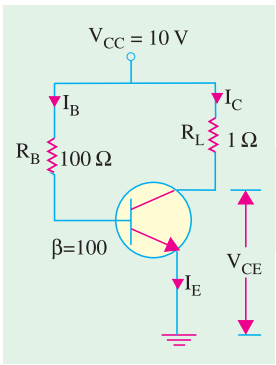


Fig. 57.37

Example 57.17. *For the CE circuit of Fig. 57.37, find the value of V_{CE}. Take $\beta = 100$ and neglect V_{BE}. Is the transistor working in cut-off or saturation ?*

Solution. $I_B = 10/100 = 0.1$ A

$I_C = \beta I_B = 100 \times 0.1 = 10$ A

$V_{CE} = V_{CC} - I_C R_L = 10 - 10 \times 1 = 0$

Obviously, the transistor is operating just at saturation and not well into saturation.

Example 57.18. *Find out whether the transistor of Fig. 57.38 is working in saturation or well into saturation. Neglect V_{BE}.* **(Basic Electronics, Bombay Univ.)**

Solution. $I_B = 10/10 = 1$ A

$I_C = 100 \times 1 = 100$ A

Obviously, I_C cannot be that large because its maximum value is given by $V_{CC}/R_L = 10/1 = 10$ A. However, let us assume that I_C takes this value temporarily. Then,

$$V_{CE} = V_{CC} - I_C R_L$$
$$= 10 - 100 \times 1 = -90 \text{ V}$$

It means that the transistor **is working well into saturation.**

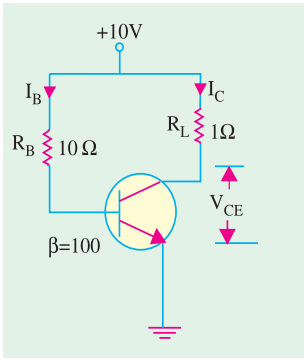


Fig. 57.38

57.26. Cut-Off And Saturation Points

Consider the circuit of Fig. 57.39 (*a*). As seen from Art 57.22,

$$V_{CE} = V_{CC} - I_C R_L.$$

Since, $I_B = 0$, $\therefore$ $I_C = 0$.

Hence, $V_{CE} = V_{CC}$

Under these conditions, the transistor is said to be cut-off for the simple reason that it does not ***conduct any current.*** This value of V_{CE} is written as $V_{CE\ (\text{cut-off})}$. Incidentally, a transistor when cut-off acts like an open switch.

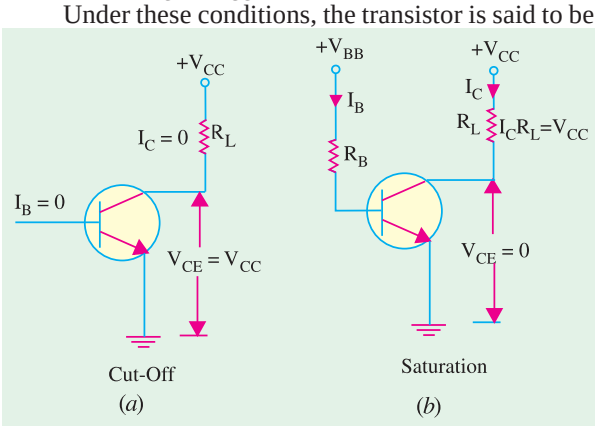


Fig. 57.39

If, in Fig. 57.39 (*b*), values of R_B and R_L are such that V_{CE} comes out to be zero, then transistor is said to be saturated. Putting $V_{CE} = 0$ in the above equation, we get

$$0 = V_{CC} - I_C R_L$$

or $$I_C = V_{CC} / R_L$$

It should be noted that a transistor, when saturated, acts as a closed switch of negligible resistance.

It is obvious that under saturation condition,

(i) whole of V_{CC} drops across R_L.

(ii) collector current has maximum possible value called $I_C(sat)$.

Normal operation of a transistor lies between the above two extreme conditions of cut-off and saturation.

Example. 57.19. *In a simple amplifier circuit (Fig. 57.40) with base resistance, $R_B = 50$ K, $R_E = 2$ K, $R_C = 3$K, $V_{CC} = 10$ V, $h_{FE} = 100$, determine whether or not the silicon transistor is in the saturation and find I_B and I_C. Explain the saturation region in common-emitter characteristics.*

(Electronics, MS. Univ. Baroda,)

Solution. Whether the transistor is in saturation or not will depend on the value of V_{CE}.

$$I_E = \frac{V_{BB} - V_{BE}}{R_E + R_B/\beta} \cong \frac{V_{BB}}{R_E + R_B/\beta}$$

$$= \frac{5}{2 + 50/100} = 2 \text{ mA}$$

$$I_C \cong I_E = 2 \text{ mA} \; ; \; I_B = I_C/\beta = 2/100$$
$$= 0.02 \text{ mA}$$

Now, $$V_{CC} = I_C R_C + V_{CE} + I_E R_E$$

or $$V_{CE} = 10 - (2 \times 3) - (2 \times 2) = 0$$

Obviously, the transistor has entered saturation.

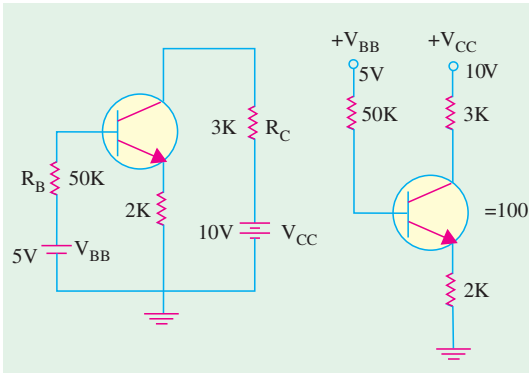


Fig. 57.40

57.27. BJT Operating Regions

A BJT has two junctions *i.e.* base-emitter and base-collector junctions either of which could be forward-biased or reverse-biased. With two junctions, there are four possible combinations of bias condition.

(*i*) both junctions reverse-biased,
(*ii*) both junctions forward-biased,
(*iii*) *BE* junction forward-biased, *BC* junction reverse-biased.
(*iv*) *BE* junction reverse-biased, *BC* junction forward-biased.

Since condition **(*iv*)** is generally not used, we will tabulate the remaining three conditions below.

Table No. 57.1: Transistor Operation Regions

BE Jn	BC Jn	Region
RB*	RB	cut-off
FB**	FB	saturation
FB	RB	active

* Reverse-biased, ** Forward-biased

(*a*) Cut-off

This condition corresponds to reverse-bias for both base-emitter and base-collector junctions. In fact, both diodes act like open circuits under these conditions as shown in Fig. 57.41, which is true for an ideal transistor. The revese leakage current (Art 57.12) has been neglected. As seen, the three transistor terminals are uncoupled from each other. In cut-off, $V_{CE} = V_{CC}$.

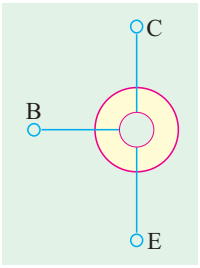


Fig. 57.41

(*b*) Saturation

This condition corresponds to forward-bias for both base-emitter and base-collector junctions. The transistor becomes saturated *i.e.* there is perfect short-circuit for both base-emitter and base-collector diodes. The ideal case is shown in Fig. 57.42, where the three transistor terminals have been connected together thereby acquiring equal potentials. In this case, $V_{CE} = 0$.

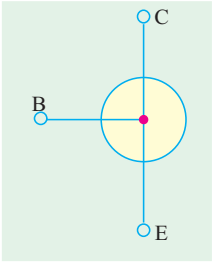


Fig. 57.42

(*c*) Active Region

This condition corresponds to forward-bias for base-emitter junction and reverse bias for base-collector junction. In this, $V_{CE} > 0$.

57.28. Active Region DC Model of a BJT

Such a model is used for predicting transistor operation in the active region. This condition is shown in Fig. 57.43 both for a *PNP* and an *NPN* transistor. A base-emitter junction voltage of 0.7 V has been assumed for silicon transistor. The *BE* junction is represented by a constant voltage source since it is forward-biased. As seen, in an *NPN* transistor, base is 0.7 V higher than the emitter terminal. However, in a *PNP* transistor, base is 0.7 V lower than the emitter terminal.

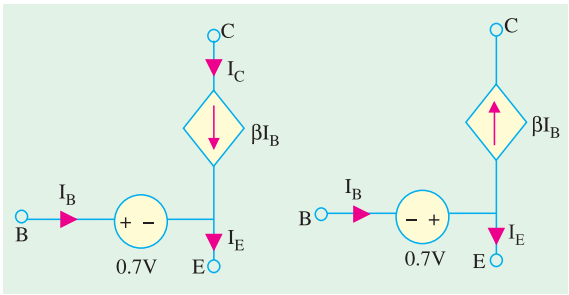


Fig. 57.43

To account for the effect of base control, a current source of βI_B is placed between collector and base terminals. It is called a dependent or controlled source because it is a function of a variable in another circuit. It may be noted that $I_E = (I_B + I_C)$ in both cases.

57.29. BJT Switches

Very often, bipolar junction transistors are used as electronic switches. With the help of such a switch, a given load can be turned ON or OFF by a small control signal. This control signal might be the one appearing at the output of a digital logic or a microprocessor. The power level of the control signal is usually very small and, hence, it is incapable of switching the load directly. However, such a control signal is certainly capable of providing enough base drive to switch a transistor ON or OFF and, hence, the transistor is made to switch the load.

When using BJT as a switch, usually two levels of control signal are employed. With one level, the transistor operates in the cut-off region (open) whereas with the other level, it operates in the saturation region and acts as a short-circuit. Fig. 57.44 (*b*) shows the condition when control signal $v_i = 0$. In this case, the *BE* junction is reverse-biased and the transistor is open and, hence acts as an open switch. However, as shown in Fig. 57.44 (*c*) if v_i equals a positive voltage of sufficient magnitude to produce saturation *i.e.* if $v_i = v_i$ the transistor acts as a closed switch.

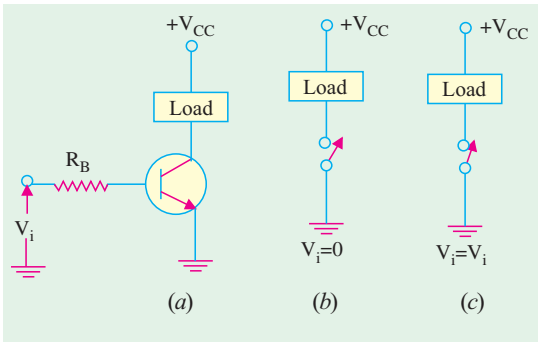


Fig. 57.44

Fig. 57.45 shows a form of series switching circuit utilizing an *NPN* transistor with a negative dc supply and a control signal voltage having levels of zero and $-v_i$.

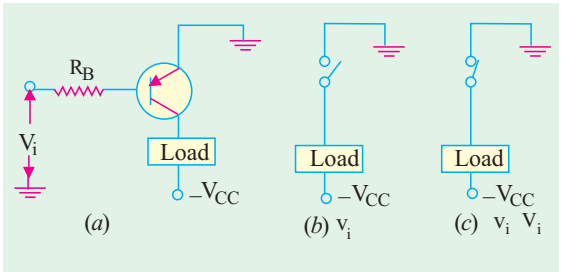


Fig. 57.45

Example 57.20. *The circuit of Fig. 57.46 is designed to produce nearly constant current through the variable collector load resistance. An ideal 6V source is used to establish the current. Determine* ***(a)*** *value of* I_C *and* V_E, ***(b)*** *range of* R_C *over which the circuit will function properly. Assume silicon transistor and a b large enough to justify the assumptions used.*

(Applied Electronics-II, Punjab Univ. 1993)

Solution. (*a*) $I_C \cong I_E = (6 - 0.7)/530 = \mathbf{10\ mA}$

$V_E = 6 - 530 \times (10 \times 10) = \mathbf{5.3\ V}.$

This voltage will remain constant so long as transistor operation is confined to active region.

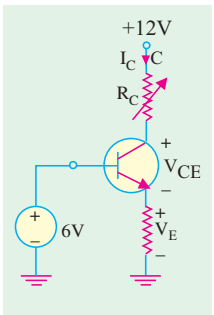


Fig. 57.46

(*b*) When $R_C = 0$

$V_{CE} = 12 - 5.3 = 6.7$ V

It is certainly well within the active region. As R_C increases, its drop increases and hence, V_{CE} decreases. There will be some value of R_C at which active region operation ceases.

Now, $V_{CE} = 12 - 5.3 - I_C R_C = 6.7 - I_C R_C$

Value of $R_{C(max)}$ can be found by puting $V_{CE} = 0$

$\therefore \quad 0 = 6.7 - I_C R_{C(max)}$

or $R_{C(max)} = 6.7/I_C = 6.7/0.01 = \mathbf{670\ \Omega}$

Hence, circuit will function as a constant current source so long as R_C is in the range $0 < R_C < 670\ \Omega$. When R_C exceeds 670 Ω, the *BJT* becomes saturated.

57.30. Normal DC Voltage Transistor Indications

For a transistor to operate as an amplifier, it is desirable that $V_{CE} = \frac{1}{2} V_{CC}$. However, in actual practice, wide tolerances are allowed. Generally, V_{CE} varies between 25% to 75% of V_{CC}. Any transistor amplifier with $V_{CE} = V_{CC}$ is either open or is operating in cut-off. When operating with V_{CE} near cut-off, the amplifier causes lot of distortion. Same is the case when V_{CE} is nearly zero. Hence, any transistor amplifier with V_{CE} more than 75% V_{CC} or less than 25% V_{CC} should be suspected of having a problem and further investigated.

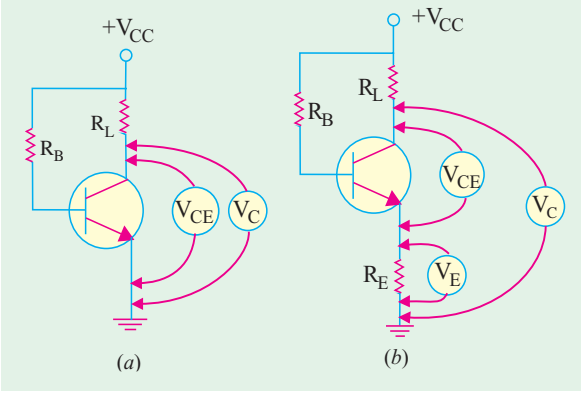


Fig. 57.47

In the circuit shown in Fig. 57.47 (*a*), V_{CE} should be in the range 25 –75% of V_{CC}. In the circuit of Fig. 57.47 (*b*), V_{CE} may be normal but either R_L or R_E could be shorted. Hence, V_C and V_E should be measured seperately. Moreover, V_{CE} could be found by subtracting V_E from V_C.

For the circuit of Fig. 57.47 (*b*), the normal voltmeter readings are

$$V_{CE} = \frac{1}{2} V_{CC} \; ; \qquad V_E = \frac{1}{4} V_{CC} \; ; \qquad V_E = \frac{3}{4} V_{CC}$$

If instead of R_L, there is a low-resistance coil in the circuit, then

$$V_{CE} = \frac{1}{2} V_{CC} \; ; \qquad V_E = \frac{1}{2} V_{CC} \; ; \qquad V_C = V_{CC}$$

57.31. Transistor Fault Location

Voltage measurements are employed in the vast majority of trouble situations because current measurements are comparatively difficult to make. Magnitude of V_{CE} is of great diagnostic value in finding and locating faults in a transistor circuit. Following possibilities are considered :

***(a)* $V_{CE} = 0$**

Possibilities are that the transistor is

1. shorted out, **2.** operating in saturation, **3.** disconnected from V_{CC}

***(b)* $V_{CE} = \frac{1}{2} V_{CC}$**

It shows that the circuit is operating normally and is well-designed.

***(c)* $V_{CE} = V_{CC}$**

Possibilities are that the transistor is

1. open-circuited,

2. operating in out-off

3. having all resistors in series with V_{CE} shorted.

Example 57.21. *Compute the value of V_{CE} for the CE circuit shown in Fig. 57.48.*

Solution. Since the collector is disconnected from the supply due to 'open' in the

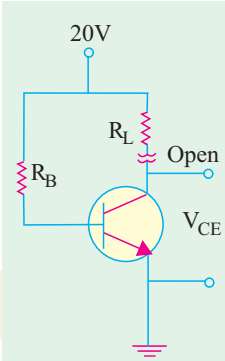


Fig. 57.48

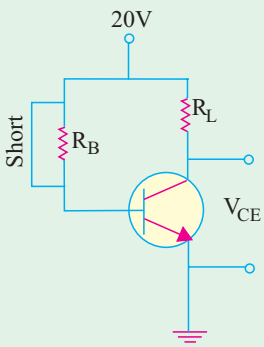


Fig. 57.49

circuit, $V_{CE} = 0$. It represents fault condition No. (*a*) 3 in Art. 57.31.

Example. 57.22. *What is the value of V_{CE} in the CE circuit of Fig. 57.49.*

Solution. Since R_B is shorted out, I_B, would increase and probably burn out the *E/B* junction. But this burn out is not indicated in the question. Hence, with high base current, the transistor is operating in saturation so that $V_{CE} = 0$.

Example 57.23. *What is the value of V_{CE} in the circuit of Fig. 57.50.*

Solution. Since R_L is the only resistor in series with the transistor and is shorted out, it means that there is no voltage drop anywhere. Hence, $V_{CE} = V_{CC}$. It represents fault No. (*c*) 3 stated in Art. 7.31 above.

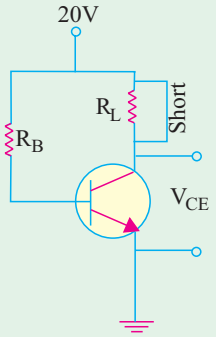


Fig. 57.50

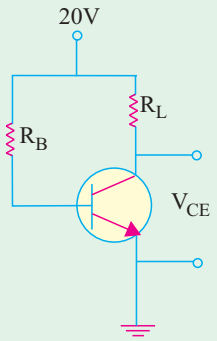


Fig. 57.51

Example 57.24. *Find the possible value of V_{CE}, V_C and V_E for the circuit shown in Fig. 57.51.*

Solution. In the circuit of Fig. 57.51, there is neither a short nor an open and the voltage polarities are correct for an *NPN* transistor. It looks like a well-designed circuit operating normally. Hence, according to Art. 57.30.

$$V_{CE} = \frac{1}{2}V_{CC} = \mathbf{10\ V};\ V_E = \frac{1}{4}V_{CC} = \mathbf{5\ V}$$

$$V_C = \frac{3}{4}V_{CC} = \mathbf{15\ V}$$

Example 57.25. *Find the values of V_C, V_E and V_{CE} in the circuit of Fig. 57.52.*

Solution. Since $I_B = 0$, transistor is cut off.

Hence, $V_E = \mathbf{0}$
Also $V_C = \mathbf{-20\ V}$
and $V_{CE} = \mathbf{-20\ V}$

Example 57.26. *What would be the values of V_C, V_E and V_{CE} for the circuit shown in Fig. 57.53.*

Solution. Since emitter is open, no current flows in any part of the circuit. The transistor is essentially cut off. Without IR drops, all points above the emitter are at 30 V.

$\therefore$ $V_E = \mathbf{30\ V}$;
$V_C = \mathbf{30\ V}$ and
$V_{CE} = \mathbf{0\ V}$

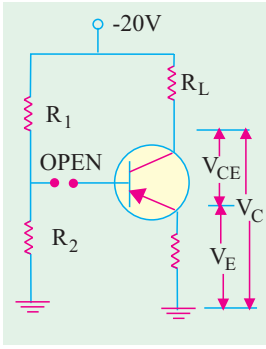


Fig. 57.52

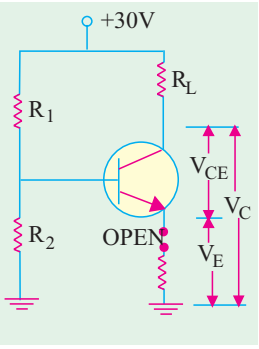


Fig. 57.53

57.32. Solving Universal

Stabilization Circuit

Such a circuit is shown in Fig. 57.54 in which ***R_E appears*** to be in parallel with R_2. But according to the β-rule (Art 57.24), R_2 is actually in parallel with βR_E. In a well-designed circuit, the resistance βR_E is much larger than R_2. Hence, their combined resistance = $R_2 \| \beta R_E \cong R_2$. On this assumption as well as another that I_B is practically zero, we can find voltage drop across R_2 by the Proportional Voltage Formula. Since V_{CC} is applied across $R_1 - R_2$ potential divider circuit, drop across R_2.

$$= V_{CC} \cdot R_2/(R_1 + R_2)$$

If we neglect V_{BE}, then this drop equals V_E.

$$\therefore \qquad V_E \cong V_{CC}\frac{R_2}{R_1+R_2}$$

and $$I_E \cong \frac{V_E}{R_E}$$

Having found I_E, other currents and voltage drops can be easily found.

$$V_{EE} = V_{CC} - I_C R_L - I_E R_E$$

Since, $I_E \cong I_C$

$$\therefore \quad V_{CE} = V_{CC} - I_E R_L - I_E R_E = V_{CC} - I_E (R_L + R_E)$$

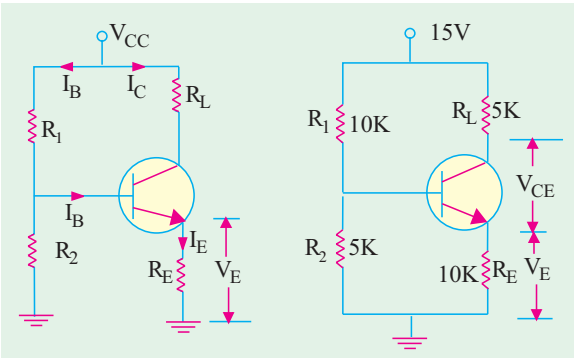


Fig. 57.54 Fig. 57.55

Example 57.27. *Find V_{CE} and V_E for the circuit shown in Fig. 57.55. Neglect V_{BE}.*

Solution. As explained above

$$V_E = V_2 = V_{CC}\frac{R_E}{R_1+R_2} = 15\times\frac{5}{15} = \mathbf{5\ V}$$

$$I_E = V_E/R_E = 5\text{ V}/10\text{ K} \quad = 0.5\text{ mA}$$

$$I_C \cong I_E = 0.5\text{ mA},\ V_{CE} = V_{CC} - I_E(R_L + R_E) = 15 - 0.5\times 15 = \mathbf{7.5\ V}$$

57.33. Notation for Voltages and Currents

In order to avoid confusion while dealing with dc and ac voltages and currents, following notation will be employed :

1. For d.c. or non-time-varying quantities

We will use capital letters with capital subscripts such as

I_E, I_B, I_C — for *dc* currents

V_E, V_B, V_C — for *dc* voltages to ground

V_{BE}, V_{CB}, V_{CE} — for *dc* potential differences

V_{EE}, V_{CC}, V_{BB} — for *dc* source or supply voltages

2. For ac quantities

We will use the following symbols :

i_e, i_b, i_c — for instantaneous values of ac currents

I_e, I_b, I_c — for r.m.s values of a.c. currents

v_e, v_b, v_c — for instantaneous values of a.c. voltages to ground

v_{be}, v_{eb}, v_{ce} — for a.c. voltage differences

3. Total ac and dc voltages and currents

In this case, we will use a hybrid notation. For example, i_E will be used to represent the total emitter current, *i.e.* sum of dc and ac currents in the emitter.

Fig. 57.56 illustrates the notation discussed above.

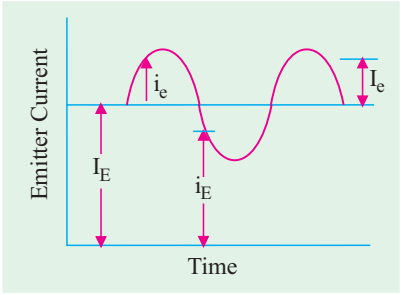


Fig. 57.56

57.34. Increase/Decrease Notation

This notation is very helpful in analysing transistor operation when ac signal is applied to it. It is simply this:

↑ means increases and ↓ means decrease.

As an illustration, consider the transistor circuit of Fig. 57.57.

If V_{BB} were increased ($\uparrow$), I_B would increase ($\uparrow$). This would increase I_C ($\uparrow$) because it equals βI_B. The drop $I_C R_L$ would increase ($\downarrow$) and, hence, V_{CE} will decrease ($\downarrow$) because $V_{CE} = V_C - I_C R_L$.

Using increase/decrease notation, the above sequence of changes can be written as

$$V_{BB} \uparrow , I_B \uparrow , I_C \uparrow , I_C R_L \uparrow V_{CE} \downarrow$$

At one look, we can straight away say that as input voltage is increased, output voltage is decreased.

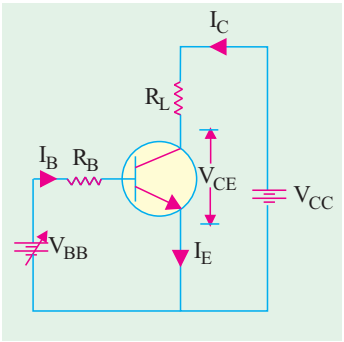


Fig. 57.57

57.35. Applying AC to a DC Biased Transistor

Suppose we want to apply an ac signal to the input emitter-base circuit of a properly-biased transistor shown in Fig. 57.58. If we apply the a.c. source directly across the *EBJ* as shown in Fig. 57.58 (*a*), it will upset the d.c. bias. It should be kept in mind that most ***ac signal sources are nearly a short to dc.*** Hence, nearly whole of I_B would pass through a.c. source rather than the base thereby spoiling the transistor bias.

In order to connect the ac source and at the same time not upset the d.c. bias, the ac source is connected via a coupling capacitor *C* as shown in Fig. 57.58 (*b*). This capacitor acts as an 'open' for dc but almost a short for ac source provided it is of sufficiently large capacitance.

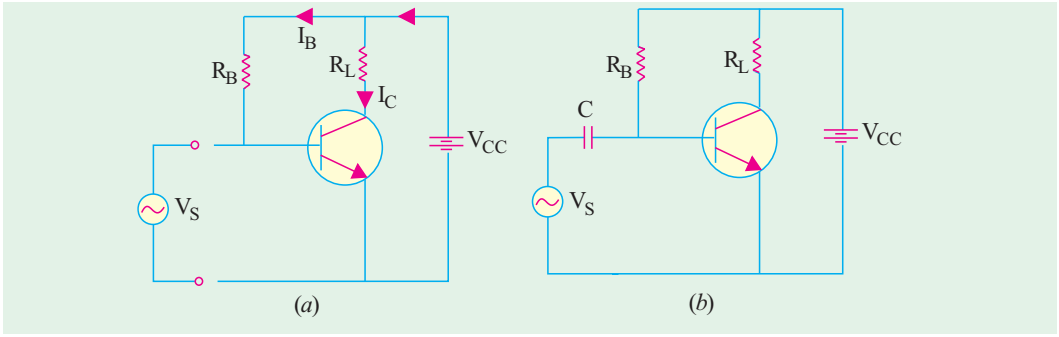


Fig. 57.58

57.36. Transistor AC/DC Analysis

In Fig. 57.59 is shown a *CE* amplifier circuit having an ac signal voltage v_{be}* applied across its *E/B* junction. This voltage will be added to the dc voltage V_{BE} as if the two were connected in series. The resultant voltage is shown in Fig. 57.59 (*b*) which shows ac voltage riding the d.c. level. The variations in the resultant output voltage V_{CE} [Fig. 57.59 (*b*)] can be expressed in terms of the increase/decrease notation. It will be assumed that V_{BE} is such as to bias V_{CE} at V_{CC} when no a.c. signal is applied.

(*i*) First Quarter Cycle

In the first quarter-cycle of the input signal, both V_{BE} and V_{BE} increase thereby giving rise to the following sequence of changes :

$$V_{BE} \uparrow , i_B \uparrow , \quad i_C \uparrow , \quad i_C R_L \uparrow \; V_{CE} \downarrow$$

Hence, output voltage decreases as shown in Fig. 57.59 (*c*)

(*ii*) Second Quarter Cycle

Here, V_{bc} as well as V_{BE} decrease. Hence,

$$V_{BE} \downarrow , \quad i_B \downarrow , \quad i_C \downarrow , \quad i_C R_L \downarrow , \quad V_{CE} \downarrow$$

* Normally, we will use the notation v_i or cin or c_i while discussing amplifiers.

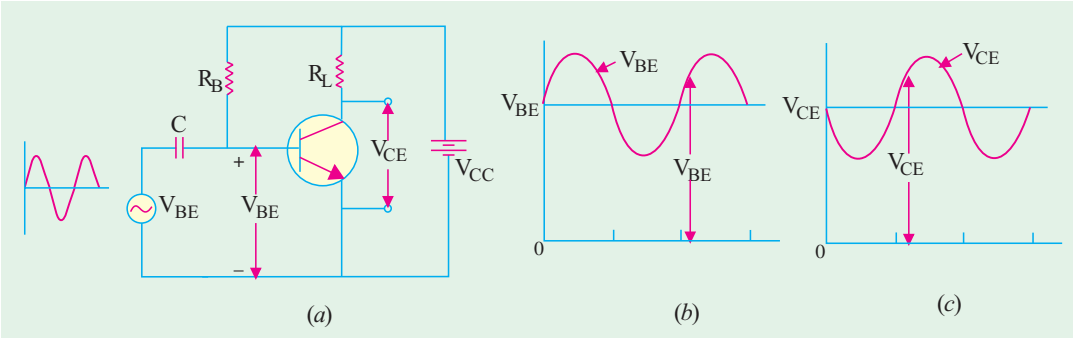


Fig. 57.59

Again, V_{CE} does the opposite of V_{BE}.

Same changes will happen in third quarter cycle as happened in the first quarter-cycle and so on. It is seen from Fig. 57.60 (*c*) that output *ac* voltage is 180° out of phase with the input voltage.

Example 57.28. *Calculate the value of V_{CE} in the circuit of Fig. 57.60 (a) if a.c. signal voltage is sinusoidal with a peak value of 0.01 V. Take voltage gain A_v of the circuit as 100 and β = 100. Depict the waveform of the output voltage separately.*

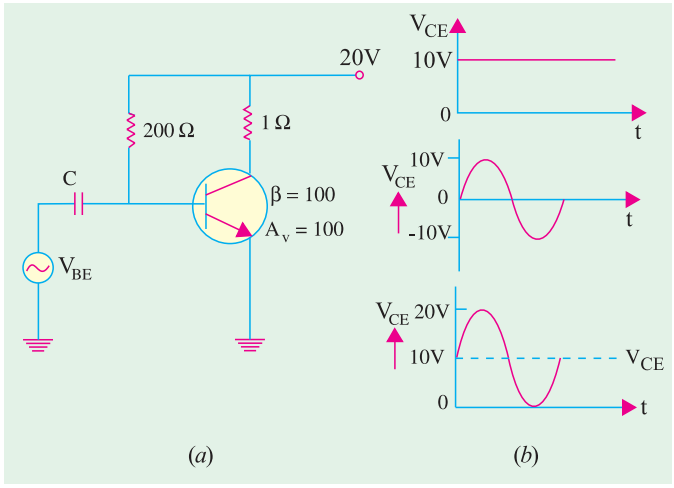


Fig. 57.60

Solution.

$$I_B = \frac{V_{CC}}{R_B} = \frac{20}{200} = 0.1\text{A}$$

$$I_C = \beta I_B = 100 \times 0.1 = 10\text{ A}$$

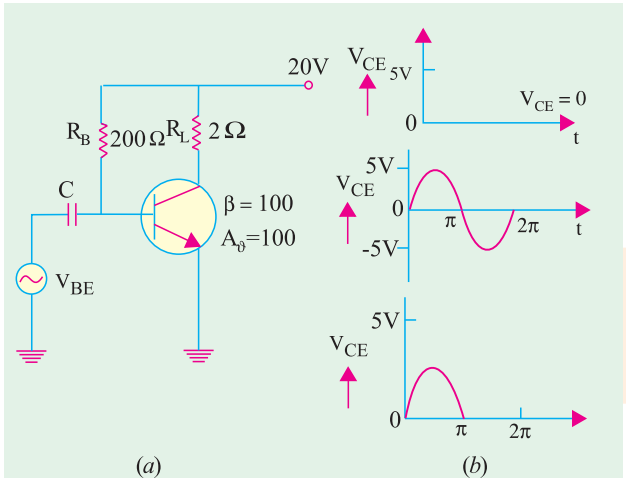


Fig. 57.61

$V_{CE} = V_{CC} - I_C R_L = 20 - 10 \times 1 = 10$ V;

$v_{CE} = A_V v_{BE} = 100 \times 0.01 = 10$ V

The combined output voltage v_{CE} is the sum of v_{CE} and v_{CE} and is shown graphically in Fig. 57.60 (*b*). It is seen that 100 times amplified ac signal rides the dc voltage.

Example 57.29. *Find v_{CE} in the circuit of Fig. 57.61 (a) and sketch its waveform. Take A_u = 100 and β = 100 and peak input signal voltage as 0.05 V.*

Solution. $I_B = 20/200 = 0.1$ A ; $I_C = 100 \times 0.1 = 10$ A ; $V_{CE} = 20 - (10 \times 2) = 0$.

Obviously, the transistor has been biased at saturation as shown in Fig. 57.61 (*b*).

The addition of v_{CE} and v_{CE}, is shown graphically in Fig. 57.61 (*b*). During the positive half-cycle of the signal, the transistor comes out of saturation and lets pass the half-cycle. However, during the negative half-cycle of input signal, transistor is further driven into saturation. Since it is already biased at V_{CC}´*s* most negative limit (0 volt), it cannot further go negative. Hence, the negative half-cycle of the signal is lost in saturation.

Tutorial Problems No. 57.1

1. A *CB*-connected transistor has $\alpha = 0.96$ and $I_E = 2$ mA. Find its I_C and I_B. **[1.92 mA, 80 μA]**

2. A *CB*-connected transistor has $I_B = 20$ μA and $I_E = 2$ mA. Compute the value of α and I_C. **[0.99, 1.98 mA]**

3. A *CE*-connected transistor has $\alpha = 100$ and $I_B = 50$ μA. Compute the values of α, I_C and I_E. **[0.99 ; 5 mA ; 5.05 mA]**

4. The following quantities are measured in a *CE* transistor : $I_C = 5$ mA ; $I_B = 100$ μA. Determine β and I_E. **[0.98 ; 50 ; 51 mA]**

5. A transistor has $\alpha = 0.98$, $I_{CBO} = 5$ μA and $I_B = 100$ μA. Find the values of I_C and I_E. **[5.15 mA ; 5.25 mA]**

6. Following measurements are made in a transistor ; $I_C = 5.202$ mA, $I_B = 50$ mA, $I_{CBO} = 2$ mA. Compute the values of α, β and I_E. **[0.99 ; 100 ; 5.252 mA]**

7. Following measurements were made in a certain transistor :

$I_C = 5.202$ mA; $I_B = 50$ mA ; $I_{CBO} = 2$ mA.

Determine (*i*) α, β and I_E (*ii*) new value of I_B required to make $I_C = 10$ mA.

[(*i*) 0.99 ; 100 ; 5.252 mA (*ii*) 97.98 A]

8. For the *CB* circuit of Fig. 57.62, find the value of V_{CB}. Neglect junction voltage V_{BE}. **[5 V]**

9. In the *CB* circuit of Fig. 57.63, what value of R_E causes $V_{BC} = 10$ V? Neglect V_{EB}. **[5 K]**

10. For the *CE* circuit of Fig. 57.64, calculate the values of I_B, I_C, I_E and V_{CE}. Take β = 50 and neglect V_{BE}. **[100 μA, 5 mA, 5.1 mA, 7.5 V]**

11. In the circuit of Fig. 57.65, calculate I_B, I_C, I_E and V_{CE}. Take transistor β = 50 and neglect V_{BE}. **[100 μA, 5 mA, 5.1 mA, 5V]**

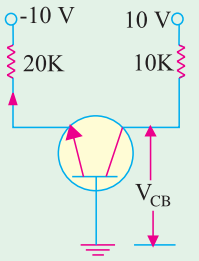


Fig. 57.62

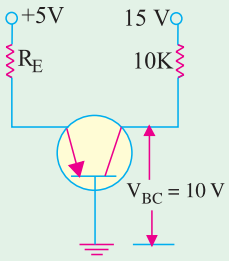


Fig. 57.63

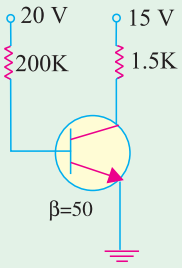


Fig. 57.64

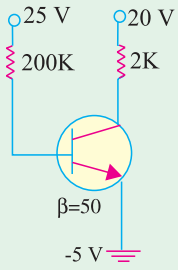


Fig. 57.65

12. In the *CC* circuit of Fig. 57.66, compute the values of I_E, I_B, I_C and V_{CE}. Neglect V_{BE}. **[0.25 mA, 2.48 μA, 0.248 mA, 5 V]**

13. In the *CC* circuit of Fig. 57.67, find IE, I_B, I_C and V_{CE}. Neglect V_{BE}. **[0.25 mA, 2.48 μA, 0.248 mA, 5 V]**

14. In the circuit of Fig. 57.68, find the drop across R_L. The transistor β = 100. **[5 V]**

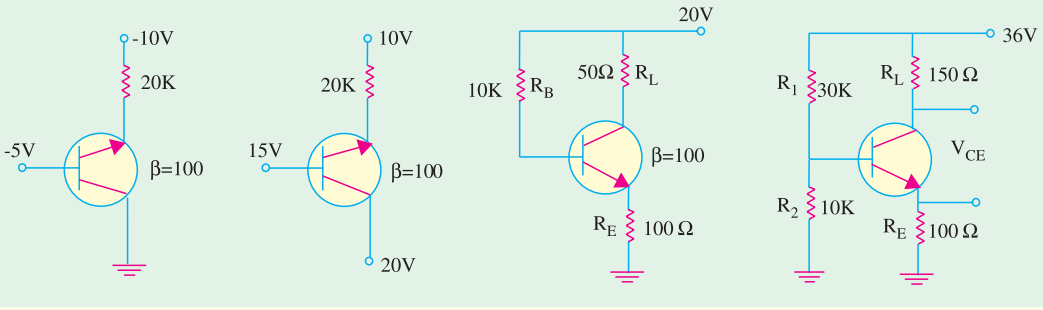


Fig. 57.66 **Fig. 57.67** **Fig. 57.68** **Fig. 57.69**

15. Find the value of V_{CE} for the universal stabilization circuit of Fig. 57.69.

[13.5 V]

16. ***(i)*** The reverse saturation current for the Ge transistor in Fig. 57.70 is 2μ A at room temperature (25°C) and increases by a factor of 2 for each temperature increase of 10°C. Bias voltage V_{BB} = 5V. Find the maximum allowable value for R_B if the transistor is to remain cutoff at a temperature of 75°C.

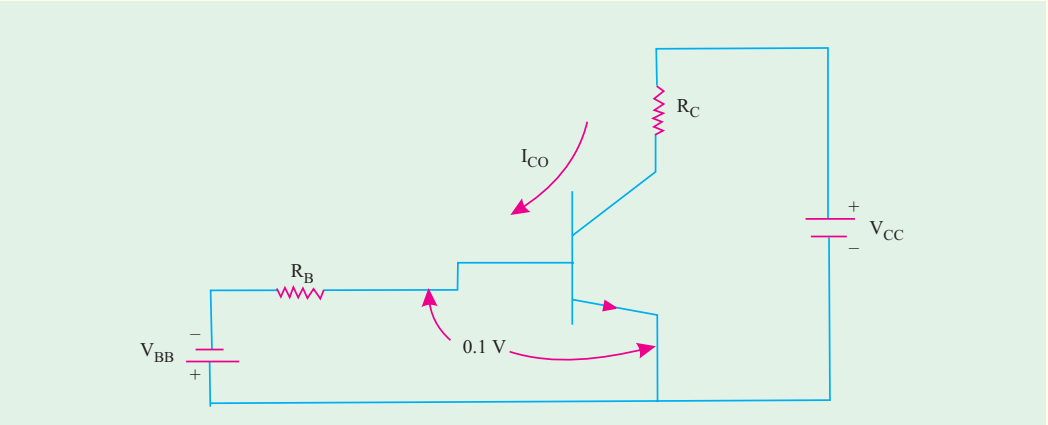


Fig. 57.70

(*ii*) If V_{BB} = 1.0V and R_B = 50 k-ohm, how high may the temperature increase before the transistor comes out of cut-off ***(Electronic Devices and Circuits Nagpur University Summer, 2004)***

17. Calculate the exact value of emitter current and V_{CE} in the circuit shown in Fig. 57.71 Assume transistor to be silicon and β= 100.

(Electronic Devices of Circuits, Nagpur University Summer, 2004)

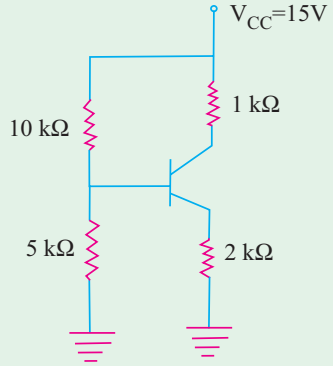


Fig. 57.71

OBJECTIVE TESTS – 57

1. The emitter of a transistor is generally doped the heaviest because it
(*a*) has to dissipate maximum power
(*b*) has to supply the charge carriers
(*c*) is the first region of the transistor
(*d*) must possess low resistance.

2. For current working of an *NPN* bipolar junction transistor, the different electrodes should have the following polarities with respect to emitter.
(*a*) collector +ve, base –ve
(*b*) collector –ve, base + ve
(*c*) collector – ve, base –ve
(*d*) collector + ve, base +ve

3. Select the CORRECT alternative.
In a bipolar transistor
(*a*) emitter region is of low/high resistivity matterial which is lightly/ heavily-doped.
(*b*) collector region is of lower/higher conductivity than emitter region
(*c*) base region is of high/low resistivity material which is only lightly/ heavily doped.

4. In a properly-biased *NPN* transistor, most of the electrons from the emitter
(*a*) recombine with holes in the base
(*b*) recombine in the emitter itself
(*c*) pass through the base to the collector
(*d*) are stopped by the junction barrier.

5. The following relationships between α and β are correct EXCEPT

(*a*) $\beta = \frac{\alpha}{1-\alpha}$ (*b*) $\alpha = \frac{\beta}{1-\beta}$

(*c*) $\alpha = \frac{\beta}{1+\beta}$ (*d*) $1-\alpha = \frac{1}{1+\beta}$

6. The value of total collector current in a *CB* circuit is
(*a*) $I_C = \alpha I_E$ (*b*) $I_C = \alpha I_E + I_{CO}$
(*c*) $I_C = \alpha I_E - I_{CO}$ (*d*) $I_C = \alpha I_E$.

7. In a junction transistor, the collector cut off current I_{CBO} reduces considerably by doping the
(*a*) emitter with high level of impurity
(*b*) emitter with low level of impurity
(*c*) collector with high level of impurity
(*d*) collector with low level of impurity

8. In a transistor amplifier, the reverse saturation current I_{CO}
(*a*) doubles for every 10°C rise in temperature
(*b*) doubles for every 1°C rise in temperature
(*c*) increases linearly with the temperature
(*d*) doubles for every 5°C rise in temperature

9. In the case of a bipolar transistor, α is
(*a*) positive and > 1
(*b*) positive and < 1
(*c*) negative and > 1
(*d*) negative and < 1.

10. The *EBJ* of a given transistor is forward- biased and its *CBJ* reverse-biased. If the base current is increased, then its
(*a*) I_C will decrease
(*b*) V_{CE} will increase
(*c*) I_C will increase
(*d*) V_{CC} will increase.

11. The collector characteristics of a *CE* - connected transistor may be used to find its
(*a*) input resistance
(*b*) base current
(*c*) output resistance
(*d*) voltage gain.

12. Which of the following approximations is often used in electronic circuits ?
(*a*) $I_C \cong I_E$ (*b*) $I_B \cong I_C$
(*c*) $I_B \cong I_E$ (*d*) $I_E \cong I_B + I_C$

13. When a transistor is fully switched ON, it is said to be
(*a*) shorted (*b*) saturated
(*c*) open (*d*) cut-off

14. If a change in base current does not change the collector current, the transistor amplifier is said to be
(*a*) saturated (*b*) cut-off
(*c*) critical (*d*) complemented.

15. When an *NPN* transistor is saturated, its V_{CE}
(*a*) is zero and I_C is zero
(*b*) is low and I_C is high
(*c*) equals V_{CC} and I_C is zero
(d) equals V_{CC} and I_C is high.

16. When an *NPN* transistor is cut-off, its V_{CC}
(*a*) equals V_{CC} and I_C is high
(*b*) equals V_{CC} and I_C is zero
(*c*) is low and I_C is high
(*d*) is high and I_C is low.

17. If, in a bipolar junction transistor, $I_B = 100\ \mu A$ and $I_C = 10$ mA, in what range does the value of its beta lie ?
(*a*) 0.1 to 1.0 (*b*) 1.01 to 10
(*c*) 10.1 to 100 (*d*) 100.1 to 1000.

18. In a *BJT*, largest current flow occurs
(*a*) in the emitter (*b*) in the collector
(*c*) in the base
(*d*) through *CB* junction.

19. In a properly-connected *BJT*, an increase in base current causes increase in
(*a*) I_C only (*b*) I_E only
(*c*) both I_C and I_E (*d*) leakage current.

20. When a *BJT* operates in cut-off
(*a*) $V_{CE} = 0$
(*b*) $V_{CE} = V_{cc}$
(*c*) V_{CE} has negative value
(*d*) I_C is maximum.

21. When a *BJT* is in saturation
(*a*) $I_C = 0$
(*b*) I_B controls I_C
(*c*) $V_{CE} = 0$
(*d*) V_{CE} has positive value.

22. The best approximation for V_C in the circuit shown in Fig. 54.72 will be (assume β to be high)
(*a*) 4 V (*b*) 6.8 V
(*c*) 8.7 V (*d*) 10.7 V

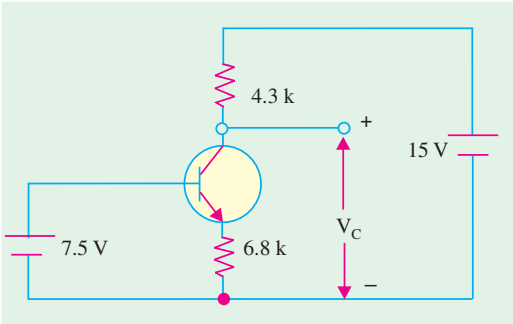


Fig. 57.72

23. Assume = 0.7 V and β = 50 for the transistor in the circuit shown in Fig. 57.73. For = 2V, the value of R_B is

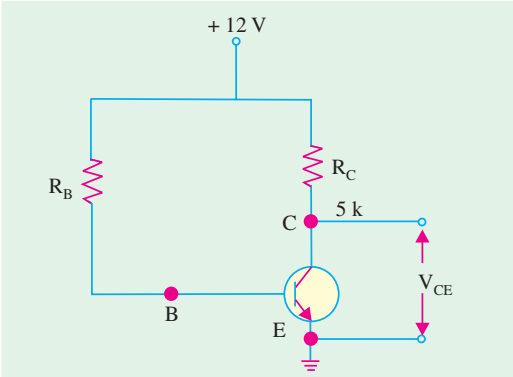


Fig. 57.73

(*a*) 200 Ω (*b*) 242 Ω
(*c*) 283 Ω (*d*) 300 Ω

24. In the circuit shown in Fig. 57.74, if $R_L = R_C =$ KΩ , then the value of V_0 will be

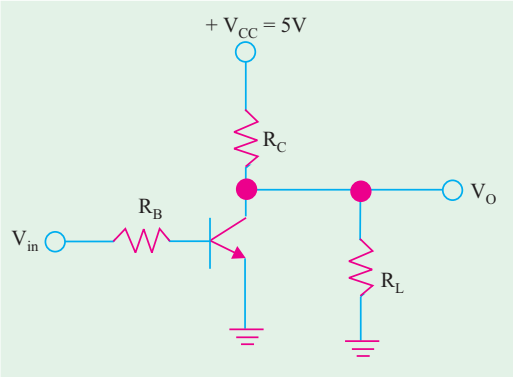


Fig. 57.74

(*a*) 4.55 V (*b*) 2.5 V
(*c*) 1 V (*d*) zero

25. A transistor is operated as a non-saturated switch to eliminate
(*a*) storage time
(*b*) turn-off time
(*c*) turn-on time
(*d*) delay time

26. Early-effect in BJT refers to
(*a*) avalanche break down
(*b*) thermal break down
(*c*) base narrowing
(*d*) zener break-down

(UPSC Engg. Services 2002)

(**Hint.** Early effect also called base-width modulation) is the variation of effective base width by the collector voltage)

27. A bipolar junction transistor (BJT) is used as power control switch by biasing it in the cut-off region (OFF state) or in the saturation region (ON state). In the ON state, for the BJT.
(*a*) both the base-emitter junction and base-collector junctions are reverse biased
(*b*) the base-emitter is reverse biased, and the base-collector junction is forward biased
(*c*) the base-emitter junction is forward biased, and the base-collector junction is reverse biased
(*d*) both the base-emitter and base-collector junctions are forward biased.

ANSWERS

1. *(a)* **2.** *(d)* **3.** *(a)* low, heavily *(b)* lower *(c)* high, lightly **4.** *(c)* **5.** *(b)* **6.** *(b)* **7.** *(d)*
8. *(a)* **9.** *(d)* **10.** *(c)* **11.** *(c)* **12.** *(a)* **13.** *(b)* **14.** *(a)* **15.** *(b)* **16.** *(b)* **17.** *(b)* **18.** *(a)*
19. *(c)* **20.** *(b)* **21.** *(c)* **22.** *(d)* **23.** *(d)* **24.** *(b)* **25.** *(a)* **26.** *(c)* **27.** *(d)*

GO To FIRST

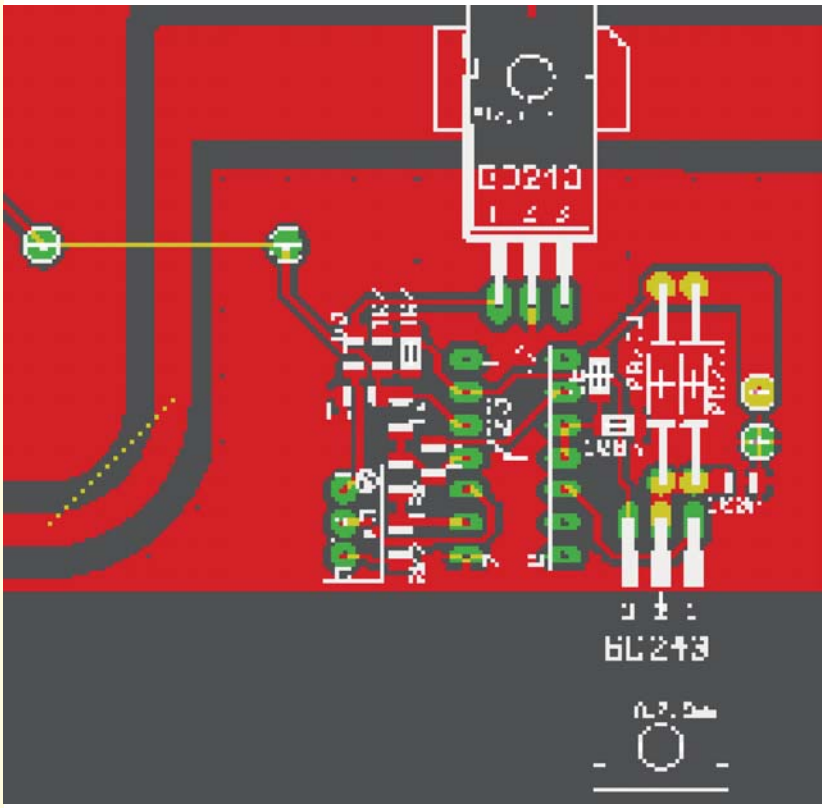
CONTENTS

CHAPTER 58

LOAD LINES AND DC BIAS CIRCUITS

Learning Objectives

- DC Load Line
- Q-Point and Maximum Undistorted Output
- Need for Biasing a Transistor
- Factor Affecting Bias Variations
- Stability Factor
- Beta Sensitivity
- Stability Factor for CB and CE Circuits
- Different Methods for Transistor Biasing
- Base Bias
- Base Bias with Emitter Feedback
- Base Bias with Collector Feedback
- Base Bias with Collector and Emitter Feedbacks
- Emitter Bias with two Supplies
- Voltage Divider Bias
- Load Line and Output Characteristics
- AC Load Line



DC biasing is used to establish a steady level of transistor current and voltage called the dc operating point or quescent point (Q-Point)

CONTENTS

58.1. D.C. Load Line

For drawing the dc load line of a transistor, one need to know only its cut-off and saturation points. It is a straight line jointing these two points. For the *CE* circuit of Fig. 58.28, the load line is drawn in Fig. 58.1. *A* is the cut-off point and *B* is the saturation point.

The voltage equation of the collector-emitter is

$$V_{CC} = I_C R_L + V_{CE} \quad \therefore \quad I_C = \frac{V_{CC}}{R_L} - \frac{V_{CE}}{R_L}$$

Consider the following two particular cases :

(*i*) when $I_C = 0$, $V_{CE} = V_{CC}$

— cut-off point A

(*ii*) when $V_{CE} = 0$, $I_C = V_{CC}/R_L$

— saturation point *B*

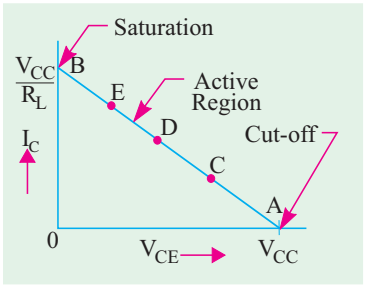


Fig. 58.1

Obviously, load line can be drawn if only V_{CC} and R_L are known.

Incidentally slope of the load line $AB = -1/R_L$

Note. The above given equation can be written as

$$I_C = \frac{V_{CE}}{R_L} + \frac{V_{CC}}{R_L}$$

It is a linear equation similar to $y = -mx + c$

The graph of this equation is a straight line whose slope is $m = -1/R_L$

Active Region

All operating points (like *C*, *D*, *E* etc. in Fig. 58.1) lying between cut-off and saturation points form the ***active region*** of the transistor. In this region, *E*/*B* junction is forward-biased and *C*/*B* junction is reverse-biased—conditions necessary for the proper operation of a transistor.

Quiescent Point

It is a point on the dc load line, which represents the values of I_C and V_{CE} that exist in a transistor circuit when ***no input signal is applied.***

It is also known as the ***dc operating point or working point.*** The best position for this point is midway between cut-off and saturation points where $V_{CE} = \frac{1}{2} V_{CC}$ (like point *D* in Fig. 58.1).

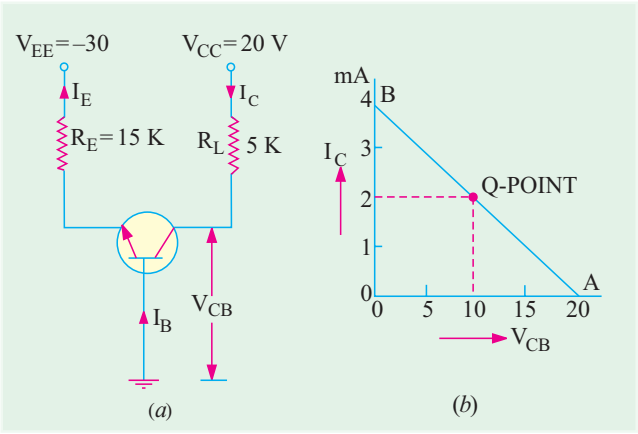


Fig. 58.2

Example 58.1. *For the circuit shown in Fig. 58.2 (a), draw the dc load line and locate its quiescent or dc working point.*

Solution. The cut-off point is easily found because it lies along *X*-axis where $V_{CE} = V_{CC} = 20$ V *i.e.* point *A* in Fig. 58.2 (*b*). At saturation point *B*, saturation value of collector current is $I_{C(sat)} = V_{CC}/R_L = 20$ V/5 K = 4 mA.

The line *A B* represents the load line for the given circuit. We will now find the actual operating point.

$I_E = V_{EE}/R_E = 30\text{ V}/15\text{ K} = 2\text{ mA}$ —neglecting V_{BE}

$I_C = \alpha I_E \cong I_E = 2\text{ mA}$; $\therefore$ $V_{CB} = V_{CC} - I_C R_L = 20 - 2 \times 5 = 10\text{ V}$

Hence, *Q*-point is located at **(10 V ; 2 mA)** as shown in Fig. 58.2 (*b*)

Example 58.2. *In the CB circuit of Fig. 8.3 (a), find*

(a) *dc operating point and dc load line*

(b) *maximum peak-to-peak unclipped signal*

(c) *the approximate value of ac source voltage that will cause clipping.*

(Electronics and Telecom Engg. Jadavpur Univ.)

Solution.

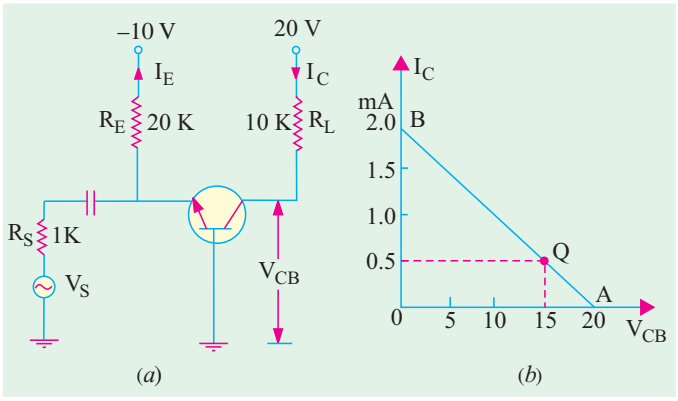


Fig. 58.3

(a) $I_{C(sat)} = \dfrac{V_{CC}}{R_L} = 2\text{ mA}$ — point B

V_{CB} at cut-off $= V_{CC} = 20\text{ V}$ —point A

Hence, *AB* is the dc load line and is shown in Fig. 58.3 (*b*).

Now, $I_E = 10/20 = 0.5\text{ mA}$

$\therefore$ $I_C \cong I_E = 0.5\text{ mA}$

$V_{CB} = V_{CC} - I_C R_L$

$= 20 - 0.5 \times 10$

$= 15\text{ V}$

The *Q*-point is located at **(15 V, 0.5 mA)**

(b) It is obvious from Fig. 58.3 (*b*) that maximum positive swing can be from 15 V to 20 V *i.e.* 5 V only. Of course, on the negative swing, the output swing can go from 15 V down to zero volt. The limiting factor being ***cut-off on positive half-cycle,*** hence maximum unclipped peak-to-peak voltage that we can get from this circuit is $2 \times 5 =$ **10 V**.

(c) The approximate voltage gain of the above circuit is

$$A_v = \frac{V_0}{V_S} = \frac{R_L}{R_S} = \frac{10\text{ K}}{1\text{K}} = 10$$

It means that signal voltage will be amplified 10 times. Hence, maximum value of source voltage for obtaining unclipped or undistorted output is

$$V_s = \frac{V_0}{10} = \frac{10 V_{p-p}}{10} = 1 V_{p-p}$$

Example 58.3. *For the CE circuit shown in Fig. 58.4 (a), draw the dc load line and mark the dc working point on it. Assume b=100 and neglect V.*

(Applied Electronics, Punjab Univ.)

Solution. Cut-off point *A* is located where, $I_C = 0$ and $V_{CE} = V_{CC} = 30\text{ V}$. Saturation point *B* is given where $V_{CE} = 0$ and

$I_{C(sat)} = 30\text{ V}/5\text{ K} = 6\text{ mA}$.

Line *AB* represents the load line in Fig. 58.4 (*b*).

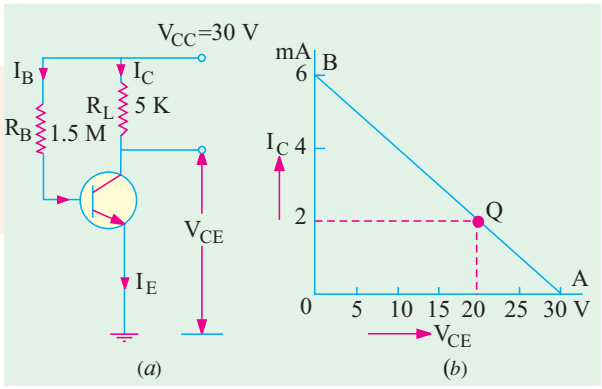


Fig. 58.4

Let us find the dc working point from the given values of resistances and supply voltage.

$I_B = 30\ V/1.5\ M = 20\ \mu A$; $I_C = \beta I_B = 100 \times 20 = 2000\ \mu A = 2\ mA$;

$V_C = V_{CC} - I_C R_L = 30 - 2 \times 5 = 20\ V$

Hence, Q–point is **(20 V ; 2 mA)** as shown in Fig. 58.4.

58.2. Q-Point and Maximum Undistorted Output

Position of the Q-point on the dc load line determines the maximum signal that we can get from the circuit before clipping occurs. Consider the cases shown in Fig. 58.5.

In Fig. 58.5 (*a*), when Q is located near cut-off point, signal first starts to clip at A. It is called ***cut-off clipping*** because the positive swing of the signal drives the transistor to cut-off. In fact, as seen from Fig. 58.5 (*a*), maximum positive swing is = $I_{CQ} R_{ac}$.

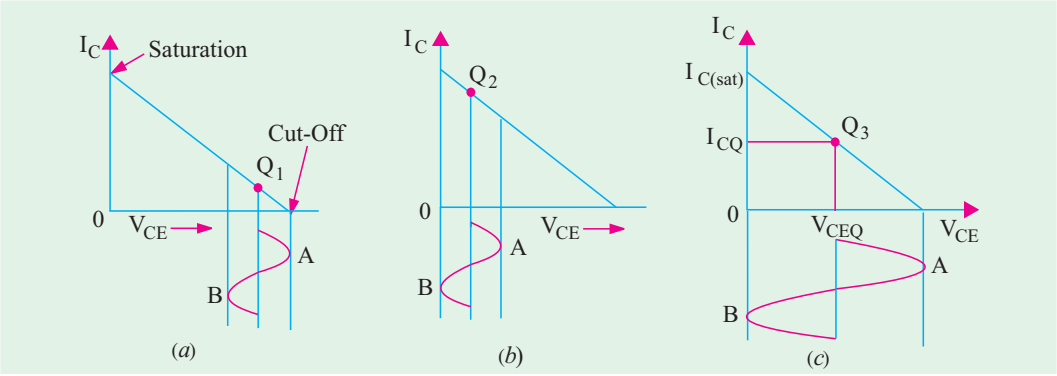


Fig. 58.5

If the Q-point Q_2 is located near saturation point, then clipping first starts at point B as shown in Fig. 58.5 (*b*). It is caused by saturation. The maximum negative swing = V_{CEQ}.

In Fig. 58.5 (*c*), the Q-point Q_3 is located at the centre of the load line. In this condition, we get the ***maximum possible output signal.*** The point Q_3 gives the optimum Q-point. The maximum undistorted signal = $2V_{CEQ}$.

In general, consider the case shown in Fig. 58.6. Since $A < B$, maximum possible peak-to-peak output signal = 2 A.

If the operating point were so located that $A > B$, then maximum possible peak-to-peak output signal = $2B$.

When operating point is located at the centre of the load line, then maximum undistorted peak-to-peak signal is = $2A = 2B = V_{CC} = 2\ V\ C_{EQ}$.

Under optimum working conditions corresponding to Fig. 58.5 (*c*), I_{CQ} is half the saturation value given by V_{CC}/R_L (Art. 8.1).

$$\therefore \quad I_{CQ} = \frac{1}{2} \cdot \frac{V_{CC}}{R_L} = \frac{V_{CC}}{2R}$$

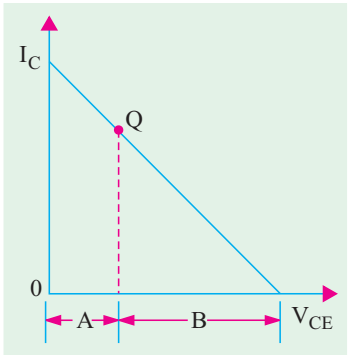


Fig. 58.6

Example 58.4. *Determine the value of R_B required to adjust the circuit of Fig. 58.7 to optimum operating point. Take $\beta = 50$ and $V_{BE} = 0.7\ V$.*

Solution. As seen from above

$$I_{CQ} = \frac{V_{CC}}{2R_L} = \frac{20}{2 \times 10} = 1\ mA$$

The corresponding base current is

$$I_{BQ} = \frac{I_{CQ}}{\beta} = \frac{1}{50} = 2\ \mu A$$

Now, $V_{CC} = I_B R_B + V_{BE}$

$$\therefore \quad R_B = \frac{V_{CC} - V_{BE}}{I_B} = \frac{20 - 0.7}{20 \times 10^{-6}} = \mathbf{965\ K}$$

58.3. Need For Biasing a Transistor

For normal operation of a transistor amplifier circuit, it is essential that there should be a

(a) forward bias on the emitter-base junction and

(b) reverse bias on the collector-base junction.

In addition, amount of bias required is important for establishing the *Q*-point which is dictated by the mode of operation desired.

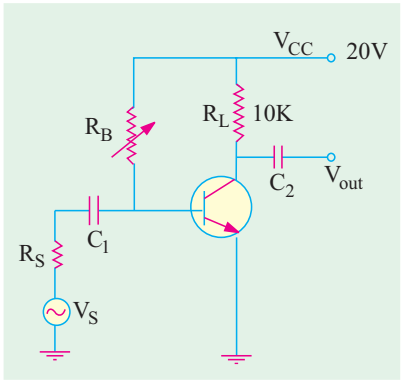


Fig. 58.7

If the transistor is not biased correctly, it would

1. work inefficiently and

2. produce distortion in the output signal.

It is desirable that once selected, the *Q*-point should remain stable *i.e.* should not shift its position due to temperature rise etc. Unfortunately, this does not happen in practice unless special efforts are made for the purpose.

58.4. Factors Affecting Bias Variations

In practice, it is found that even after careful selection, *Q*-point tends to shift its position. This bias instability is the direct result of thermal instability which itself is produced by cumulative increase in I_C that may, if unchecked, lead to thermal runaway (Art. 58.13).

The collector current for C_E circuit is given by

$$I_C = \beta I_B + I_{CEO} = \beta I_B + (1 + \beta)\, I_{CO}$$

This equation has three variables : β, I_B and I_{CO} all of which are found to increase with temperature. In particular, increase in I_{CO} produces significant increase in collector current I_C. This leads to increased power dissipation with further increase in temperature and hence I_C. Being a cumulative process, it can lead to thermal runaway which will destroy the transistor itself !

However, if by some circuit modification, I_C is made to decrease with temperature automatically, then decrease in the term βI_B can be made to neutralize the increase in the term $(1 + \beta)\, I_{CO}$, thereby keeping I_C constant. This will achieve thermal stability resulting in bias stability.

58.5. Stability Factor

The degree of success achieved in stabilizing I_C in the face of variations in I_{CO} is expressed in terms of current stability factor *S*. It is defined as the rate of change of I_C with respect to I_{CO} when both β and I_B (V_{BE}) are held constant.

$$\therefore \quad S = \frac{dI_C}{dI_{CO}} \qquad \text{— } \beta \text{ and } I_B \text{ constant}$$

Larger the value of *S*, greater the thermal instability and ***vice versa*** (in view of the above, this factor should, more appropriately, be called instability factor !).

The stability factor may be alternatively expressed by using the well-known equation $I_C = I\beta + (I + \beta)\, I_{CO}$ which, on differentiation with respect to I_C, yields.

$$I = \beta \frac{dI_B}{dI_C} + (1+\beta)\frac{dI_{CO}}{dI_C} = \beta \frac{dI_B}{dI_C} + (1+\beta)\frac{1}{S} \qquad \therefore \quad S = \frac{(1+\beta)}{1 - \beta(dI_B / dI_C)}$$

The stability factor of any circuit can be found by using the general formula

$$S = \frac{1 + R_B / R_E}{1 + (1-\alpha)\,(R_B / R_E)}$$

where R_B = ***total*** series parallel resistance in the base
R_E = ***total*** series dc resistance in the emitter
α = dc alpha of the transistor

58.6. Beta Sensitivity

By β–sensitivity of a circuit is meant the influence that the β-value has on its dc operating point. Variations in β-value are caused by variations in the circuit operating conditions or by the substitution of one transistor with another. Beta sensitivity K_b is given by

$$\frac{dI_C}{I_C} = K_\beta \frac{dI_B}{\beta} \qquad \therefore \qquad K_\beta = \frac{\beta}{I_C} \cdot \frac{dI_C}{dI_B}$$

Obviously, K_β is dimensionless ratio and can have values ranging from zero to unity.

58.7. Stability Factor for CB and CE Circuits

(*i*) CB Circuit

Here, collector current is given by

$$I_C = \alpha I_E + I_{CO} \qquad \therefore \qquad \frac{dI_C}{dI_{CO}} = 0 + 1 \qquad \text{or} \qquad S = 1$$

(*ii*) CE Circuit

$$I_C = \beta I_B + (1 + \beta)\, I_{CO} \qquad \therefore \quad \frac{dI_C}{dI_{CO}} = (1+\beta)$$ —treating I_B as a constant

$\therefore \quad S = (1 + \beta)$.

If $\beta = 100$, then $S = 101$ which means that I_C changes 101 times as much as I_{CO}.

58.8. Different Methods for Transistor Biasing

Some of the methods used for providing bias for a transistor are :

1. base bias or fixed current bias (Fig. 58.9)

It is not a very satisfactory method because bias voltages and currents do not remain constant during transistor operation.

2. base bias with emitter feedback (Fig. 58.10)

This circuit achieves good stability of dc operating point against changes in β with the help of emitter resistor which causes degeneration to take place.

3. base bias with collector feedback (Fig. 58.11)

It is also known as collector-to-base bias or collector feedback bias. It provides better bias stability.

4. base bias with collector and emitter feedbacks

It is a combination of (2) and (3) above.

5. emitter bias with two supplies (Fig. 58.13)

This circuit uses both a positive and a negative supply voltage. Here, base is at approximately 0 volt *i.e.* $V_B \cong 0$.

6. voltage divider bias (Fig. 58.15)

It is most widely used in linear discrete circuits because it provides good bias stability. It is also called universal bias circuit or base bias with one supply.

Each of the above circuits will now be discussed separately.

58.9. Base Bias

It has already been discussed in Art. 58.20 and is shown in Fig. 58.25. For such a circuit, $S = (1 + \beta) \cong \beta$ and $K_\beta = 1$.

58.10. Base Bias with Emitter Feedback

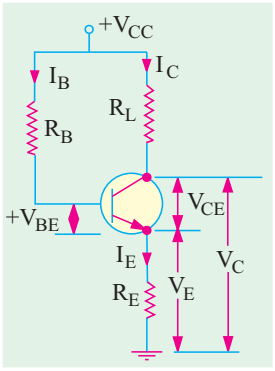


Fig. 58.8

This circuit is obtained by simply adding an emitter resistor to the base bias circuit as shown in Fig. 58.8.

1. At saturation, V_{CE} is essentially zero, hence V_{CC} is distributed over R_L and R_E.

$$\therefore \qquad I_{C(sat)} = \frac{V_{CC}}{R_E + R_L}$$

2. I_C can be found as follows :

Consider the supply, base, emitter and ground route. Applying Kirchhoff 's Voltage Law, we have

$$-I_B R_B - V_{BE} - I_E R_E + V_{CC} = 0$$

or $$V_{CC} = I_B R_B + V_{BE} + I_E R_E \qquad ...(i)$$

Now $I_B = I_C/\beta$ and $I_E \cong I_C$

Substituting these values in the above equation, we have

$$V_{CC} \cong \frac{I_C R_B}{\beta} + V_{BE} + I_C R_E$$

$$\therefore \qquad I_C \cong \frac{V_{CC} - V_{BE}}{R_E + R_B/\beta} \cong \frac{V_{CC}}{R_E + R_B/\beta}$$ —neglecting VBE

(we could have applied the β-rule given in Art. 57.24)

3. collector-to-ground voltage $V_C = V_{CC} - I_C R_L$

4. emitter-to-ground voltage $V_E = I_E R_E \cong I_C R_E$

5. $$S = \frac{1 + R_B/R_E}{1 + R_B/(1+\beta)R_E} = \frac{1 + R_B/R_E}{1 + R_B/\beta R_E}$$

6. The β-sensitivity of this circuit is $K_\beta = \dfrac{1}{1 + \beta R_E/R_B}$

Example 58.5. *For the circuit shown in Fig. 58.9, find* ***(i)*** $I_C(sat)$, ***(ii)*** I_C, ***(iii)*** V_C, ***(iv)*** V_E, ***(v)*** V_{CE} *and* ***(vi)*** K_β.

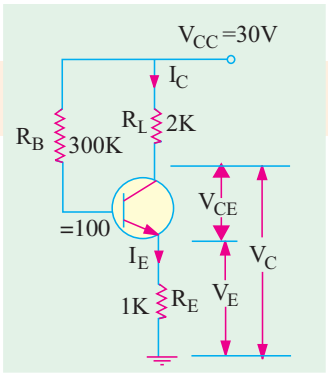


Fig. 58.9

Solution. ***(i)*** $I_{C(sat)} = \dfrac{V_{CC}}{R_E + R_L} = \dfrac{30}{1+2} =$ **10 mA**

(ii) actual $I_C \cong \dfrac{V_{CC}}{R_E + R_B/\beta} = \dfrac{30}{1+300/100} =$ **7.5 mA**

(iii) $V_C = V_{CC} \times I_C R_L = 30 - 2 \times 7.5 =$ **15 V**

(iv) $V_E \cong I_E R_E \cong I_C R_E = 7.5 \times 1 =$ **7.5 V**

(v) $V_{CE} = V_C - V_E = 15 - 7.5 =$ **7.5 V**

(vi) $K_\beta = \dfrac{1}{1+100\times 1/300} =$ **0.75**

Example 58.6. *The base-biased transistor circuit of Fig. 58.10 is subjected to increase in junction temperature from 25°C to 75°C. If β increases from 100 to 150 with rising temperature, calculate the percentage change in Q-point values (I_C, V_{CE}) over the temperature range. Assume that V_{BE} remains constant at 0.7 V.*

Solution. At 25°C $I_B = \dfrac{V_{CC} - V_{BE}}{R_B} = \dfrac{12 - 0.7}{100\times 10^3} = 0.113\,\text{mA}$

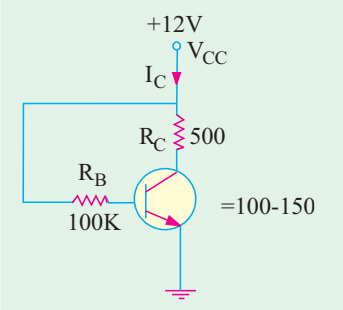


Fig. 58.10

$I_C = \beta I_B = 100 \times 0.113 = 11.3$ mA

$V_{CE} = I_C R_C = 12 - (11.3 \times 10{-}3 \times 500) = 6.35$ V

$V_{CE} = V_{CC} - I_C R_C = 12 \times (11.3 \times 10 \times 3 \times 500) = 6.35$ V

At 25°

$I_B = 0.113$ mA

$I_C = \beta I_B = 150 \times 0.113 = 16.95$ mA

$V_{CE} = 12 - (16.95 \times 10^{-3} \times 500) = 3.52$ V

$$\% \ \Delta I_C = \frac{16.95 - 11.3}{11.3} \times 100 = \textbf{50 (increase)}$$

$$\% \ \Delta V_{CE} = \frac{3.52 - 6.35}{6.35} \times 100 = \textbf{–44.57 (decrease)}$$

It is seen that *Q*-point is very much dependent on temperature and makes the base-bias arrangement very unstable.

58.11. Base Bias with Collector Feedback

This circuit (Fig. 58.11) is like the base bias circuit except that base resistor is returned to collector rather than to the V_{CC} supply. It derives its name from the fact that since voltage for R_B is derived from collector, there exists a negative feed back effect which tends to stabilise I_C against changes in β. To understand this action, suppose that somehow β increases. It will increase I_C as well as $I_C R_L$ but decrease V_C which is applied across R_B. Consequently, I_B will be decreased which will partially compensate for the original increase in β.

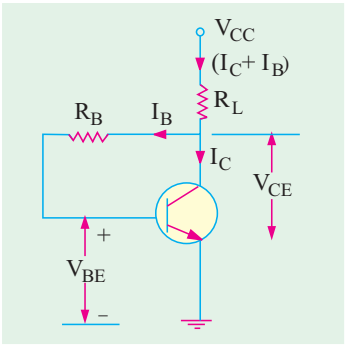


Fig. 58.11

(i) $I_{C(sat)} = V_{CC}/R_L$ —since $V_{CE} = 0$

(ii) $V_C = V_{CC} - (I_B + I_C) R_L \cong V_{CC} - I_C R_L$

Also, $V_C = I_B R_B + V_{BE}$

Equating the two expressions for V_C, we have

$I_B R_B + V_{BE} \cong V_{CC} \times I_C R_L$

Since $I_B = I_C/\beta$, we get

$$\frac{I_C}{\beta} . R_B + V_{BE} \cong V_{CC} - I_C R_L$$

$$\therefore \quad I_C = \frac{V_{CC} - V_{BE}}{R_L + R_B/\beta} \cong \frac{V_{CC}}{R_L + R_B/\beta}$$

This is also the approximate value of I_E (again, we could take the help of β-rule). The β-sensitivity factor is given by

$$K_\beta = \frac{1}{\beta R_L / R_B} = 1 - \frac{I_C}{I_{C(sat)}}$$

$$S = \frac{1 + R_B/R_L}{1 + R_B/(1+\beta)R_L} \cong \frac{V_{CC}}{R_L + R_B/\beta}$$

Example 58.7. *In Fig. 58.11, V_{CC} =12 V, V_{BE} = 0.7 V, R_L = 1K, R_B = 100K, β = 100. Find* ***(i)*** I_C, ***(ii)*** V_{CE}, ***(iii)*** I_B, ***(iv)*** K_β *and* ***(v)*** *S.*

Solution. ***(i)*** $I_C \cong I_E = \dfrac{12-0.7}{1+100/100} =$ **5.6 mA**

(ii) $V_{CE} \cong 12 - (5.65 \times 1) =$ **6.35 V** ***(iii)*** $I_B = I_C/\beta = 5.65/100 =$ **56.5 μA**

(iv) $K_B = \dfrac{1}{1+100\times 1/100} =$ **0.5** ***(v)*** $S = \dfrac{1+100/1}{1+100\times 1/101} =$ **50.5**

58.12. Base Bias with Collector and Emitter Feedbacks

In the circuit of Fig. 58.12, both collector and emitter feedbacks have been used in an attempt to reduce circuit sensitivity to changes in β. If β increases, emitter voltage increases but collector voltage decreases. It means that voltage across R_B is reduced causing I_B to decrease thereby partially off-setting the increase in β.

Under saturation conditions, V_{CC} is distributed over R_L and R_E. Assuming I_B to be negligible as compared to I_C, we get, $I_{C(sat)} = V_{CC}/(R_E + R_L)$.

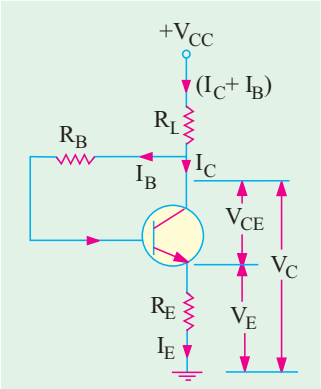


Fig. 58.12

Actual value of I_C is $= \dfrac{V_{CC} - V_{BE}}{R_E + R_L + R_B/\beta}$

—going *via* R_B because V_{CE} is unknown.

$V_C = V_{CC} - (I_C + I_B)$; $R_L \cong V_{CC} - I_C R_L$

$V_E = I_E R_E \cong I_C R_E$; $V_{CE} = V_C - V_E$

$V_{CE} \cong V_{CC} - I_C(R_L + R_E)$

$$S = \frac{1 + R_B/(R_E + R_L)}{1 + R_B/\beta(R_E + R_L)}$$

It can be proved that $K_\beta = \dfrac{1}{1+\beta(R_E+R_L)/R_B} = 1 - \dfrac{I_C}{I_{C(sat)}}$

Obviously, K_β will be degraded with increase in R_B.

Example 58.8. *For the circuit shown in Fig. 58.13, find* ***(a)*** $I_{C(sat)}$ ***(b)*** V_{CE} *and* ***(c)*** K_β. *Neglect* V_{BE} *and take* $\beta = 100$.

Solution. ***(a)*** $I_{C(sat)} = 15/(10 + 10) =$ **0.75 mA**

(b) $I_C = \dfrac{V_{CC}}{R_E + R_L + R_B/\beta} = \dfrac{15}{10+10+500/100} = 0.6\,\text{mA}$

$V_{CE} = 15 - 0.6\,(10 + 10) =$ **3 V**

(c) $K_\beta = \dfrac{1}{1+100(10+10)/50} = 0.2$

or $K_\beta = 1 - I_C/I_{C(sat)} = 1 - 0.6/0.75 = 0.2$

Obviously, K_β will be degraded with increase in R_B.

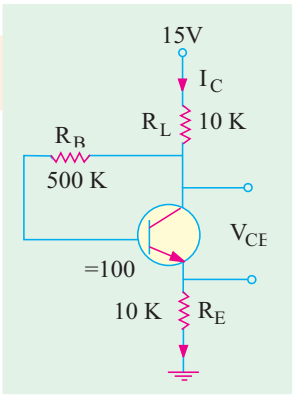


Fig. 58.13

58.13. Emitter Bias with Two Supplies

This circuit gives a reasonably stable *Q*-point and is widely used whenever two supplies (positive and negative) are available. Its popularity is due to the fact that I_C is essentially independent of β.

It can be shown that $V_B \cong 0$ and $V_E = -V_{BE}$

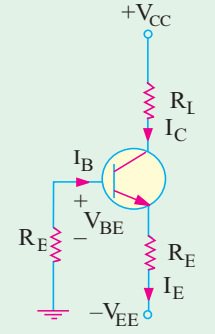


Fig. 58.14

Starting from ground and going clockwise round the base-emitter circuit, we get according to *KVL*.

$$-I_B R_B - V_{BE} - I_E R_E + V_{EE} = 0$$

or $$I_B R_B + I_E R_E = V_{EE} - V_{BE} \quad ...(i)$$

Now, $I_B = I_C/\beta \cong I_E/\beta$. Substituting this in ($i$) above we have

$$\frac{I_E R_B}{\beta} + I_E R_E = V_{EE} - V_{BE} \quad or \quad I_E = \frac{V_{EE} - V_{BE}}{R_E + R_B/\beta}$$

If $V_{EE} \gg V_{BE}$ and $R_E \gg R_B/\beta$, $I_E = V_{EE}/R_B$.

If V_E is the emitter to ground voltage, then

$$-I_B R_B - V_{BE} - V_E = 0$$

or $$V_E = -(I_B R_B + V_{BE}) = -(V_{BE} + I_C R_B/\beta) \cong -V_{BE}$$

For this circuit, $S = \dfrac{1 + R_B/R_E}{1 + R_B/\beta R_E}$ and $K_B = \dfrac{1}{1 + \beta R_E/R_B}$

Example 58.9. *For the circuit of Fig. 58.15, find* ***(i)*** I_E, ***(ii)*** I_C, ***(iii)*** V_C, ***(iv)*** V_E, ***(v)*** V_{CE}, ***(vi)*** *stability factor and* ***(vii)*** K_β *for a* β *of 50. Take* $V_{BE} = 0.7$ *V.*

(Electronics-II, Bangalore Univ. 1995)

Solution. ***(i)*** $I_E = \dfrac{V_{EE} - V_{BE}}{R_E + R_B/\beta} = \dfrac{10 - 0.7}{20 + 10/50} = 0.46$ **mA**

(ii) $I_C \cong I_E = 0.46$ **mA**

(iii) $V_C = V_{CC} - I_C R_L = 20 - 0.46 \times 10 = 15.4$ **V**

(iv) $V_E = -(V_{BE} + I_C R_B/\beta)$
$= (0.7 + 0.46 \times 10/50) = -0.8$ V

(v) $V_{CE} = V_C - V_E = 15.4 - (-0.8) = 16.2$ V

(vi) $S = \dfrac{1 + R_B/R_E}{1 + R_B/\beta R_B} = \dfrac{1 + 10/20}{1 + 10/50 \times 20} = 1.485$

(vii) $K_\beta = \dfrac{1}{1 + R_E/R_\beta} = \dfrac{1}{1 + 50 \times 20/10} \cong 0.01$

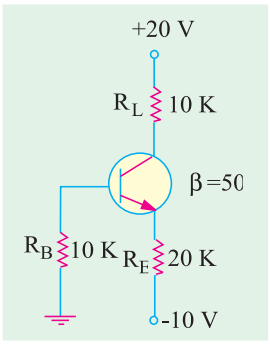


Fig. 58.15

58.14. Voltage Divider Bias

The arrangement is commonly used for transistors incorporated in integrated circuits (*ICs*).

The name 'voltage divider' is derived from the fact that resistors R_1 and R_2 form a potential divider across V_{CC} (Fig. 58.16)*. The voltage drop V_2 across R_2 forward-biases the emitter whereas V_{CC} supply reverse-biases the collector.

As per voltage divider theorem.

$$V_2 = V_{CC} \cdot R_2/(R_1 + R_2)$$

As seen, $$V_E = V_2 - V_{BE}$$

$$\therefore \quad I_E = \frac{V_E}{R_E} = \frac{V_2 - V_{BE}}{R_E} \cong \frac{V_2}{R_E}$$

Also, $$V_C = V_{CC} - I_C R_L$$

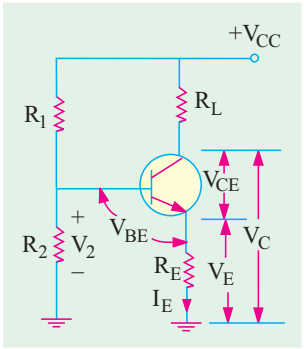


Fig. 58.16

* It is also known as Universal Bias Stabilization Circuit.

$$V_{CE} = V_C - V_E = V_{CC} - I_C R_L - I_E R_E$$
$$\cong V_{CC} - I_C (R_L + R_E) \quad \therefore \quad I_C \cong I_E$$

As before, $$I_{C(sat)} \cong \frac{V_{CC}}{R_L + R_E}$$

It is seen from above calculations that value of β was never used anywhere. The base voltage is set by V_{CC} and R_1 and R_2. The *dc* bias circuit is independent of transistor β. That is why it is such a very popular bias circuit.

$$K_\beta = \frac{1}{1 + \beta R_E / (R_1 \| R_2)}$$

$$S = \frac{1 + (R_1 \| R_2)/R_e}{1 + (R_1 \| R_2)/(1+\beta) R_E}$$

$$\cong \frac{1 + (R_1 \| R_2)/R_E}{1 + (R_1 \| R_2)/\beta R_E}$$

Using Thevenin's Theorem

More accurate results can be obtained by Thevenizing the voltage divider circuit as shown in Fig. 58.17. The first step is to open the base lead at point *A* and remove the transistor along with R_L and R_E thereby leaving the voltage divider circuit behind as in Fig. 58.17 (*a*) and (*b*).

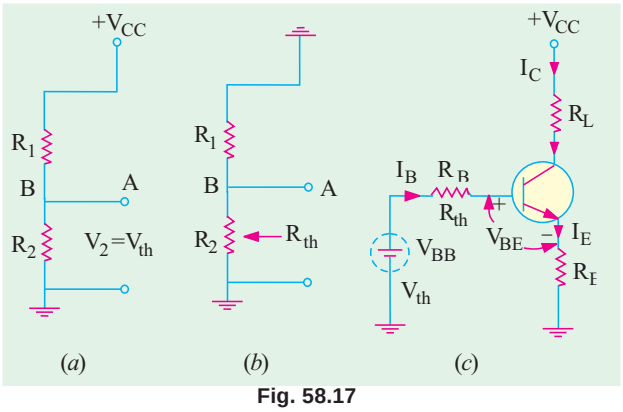


Fig. 58.17

$$V_{th} = V_2 = V_{CC} \cdot \frac{R_2}{R_1 + R_2} \quad \text{and} \quad R_{th} = R_1 \| R_2 = \frac{R_1 R_2}{(R_1 + R_2)}$$

The original circuit is reduced to that shown in Fig. 58.17 (*c*) where $V_{th} = V_{BB'}$;. $R_{th} = R_B'$. Now, applying *KVL* to the base-emitter loop, we get

$$V_{BB'} - I_B R_B' - V_{BE} - I_E R_E = 0$$

Substituting the value of $I_E = (1 + \beta) I_B$ in (*i*) above, we get

$$I_B = \frac{V_{BB}' - V_{BE}}{R_B' + (1+\beta) R_E}$$

However, if we substitute the value of $I_B = I_E/(1 + \beta)$, we get

$$I_E = \frac{V_{BB}' - V_{BE}}{R_E + R_B'/(1+\beta)}$$

$$V_{CE} = V_{CC} - I_C R_L - I_E R_E \cong V_{CC} - I_C (R_L + R_E)$$

The above results could also be obtained directly by applying β-rule (Art. 58.12)

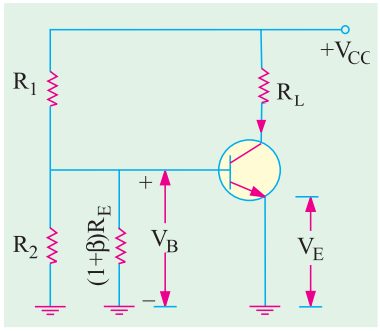


Fig. 58.18

Using β-rule

As per β-rule (Art. 58.12) when R_E is transferred to the base circuit, it becomes $(1 + \beta) R_E$ and is in parallel with R_2 as shown in Fig. 8.18. Now, V_{CC} drops over R_1 and $R_2 \| (1 + \beta) R_E$

$$\therefore V_B = V_{CC}\frac{R_2 \parallel (1+\beta)R_E}{R_1 + R_2 \parallel (1+\beta)R_E}$$

$$V_E = V_B - V_{BE};\ I_E = V_E/R_E \qquad \text{and so on.}$$

Example. 58.10. *For the circuit of Fig. 58.19, find (a) $I_C(sat)$, (b) I_C, (c) V_{CE}, (d) K_β. Neglect V_{BE} and take $\beta = 50$.* **(Electronics, Gorakhpur Univ.)**

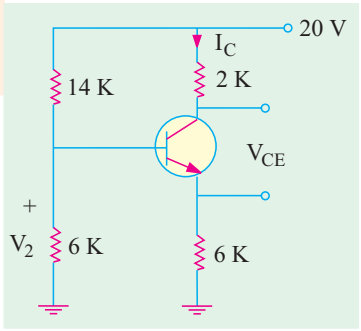


Fig. 58.19

Solution.

(a) $I_{C(sat)} = \dfrac{V_{CC}}{R_L + R_E} = \dfrac{20}{2+6} =$ **2.5 mA**

(b) $I_C \cong I_E \cong V_2/R_E = 6/6 =$ **1 mA**

(c) $V_{CE} = V_{CC} - I_C(R_L + R_E) = 20 - 1\ (2 + 6) =$ **12 V**

(d) $R_1 \parallel R_2 = 84/20 = 4.2$ K

$$K_\beta = \frac{1}{1 + 50 \times 6/4.2} = \textbf{0.0138}$$

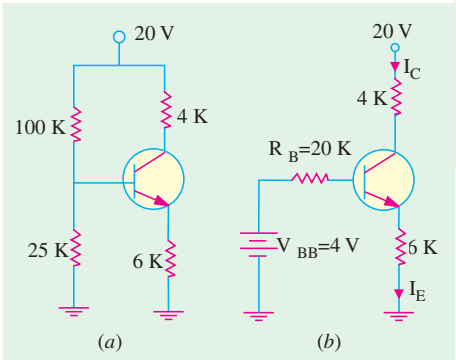


Fig. 58.20

Example 58.11. *For the circuit shown in Fig. 58.20 (a), draw the dc load line and mark the Q-point of the circuit. Assume germanium material with $V_{BB} = 0.3$ V and $\beta = 50$.* **(Electronics-I, M.S. Univ. 1991)**

Solution. $V_{CC(cut-off)} = V_{CC} = 20$ V

$$I_{C(sat)} = \frac{V_{CC}}{R_L + R_E} = \frac{20}{4+6} = 2 \text{ mA}$$

(a) Approximate Method

$$V_2 = V_{CC}\frac{R_2}{R_1 + R_2} = 20 \times \frac{25}{125} = 4 \text{ V}$$

$$I_E = \frac{V_2 - V_{BE}}{R_E} = \frac{4 - 0.3}{6} = 0.62 \text{ mA}$$

$\therefore \quad V_{CE} = V_{CC} - I_C(R_L + R_E) = 20 - 0.62 \times 10 =$ **13.8 V**

This *Q*-point **(13.8 V, 0.62 mA)** is shown in Fig. 58.21 (*a*).

As seen from Fig. 58.20 (*b*)

$V_{BB} = 20 \times 25/125$
$= 4$ V

$$I_B = \frac{V_{BB}' - V_{BE}}{R_{B'} + (1+\beta)R_E}$$

$$= \frac{4 - 0.3}{20 + 51 \times 6}$$

$= 11.3\ \mu$A

$I_C = \beta I_B = 50 \times 11.3$
$= 0.565$ mA

$I_E = (1 + \beta\ I_B$
$= 576\ \mu$A
$= 0.576$ mA

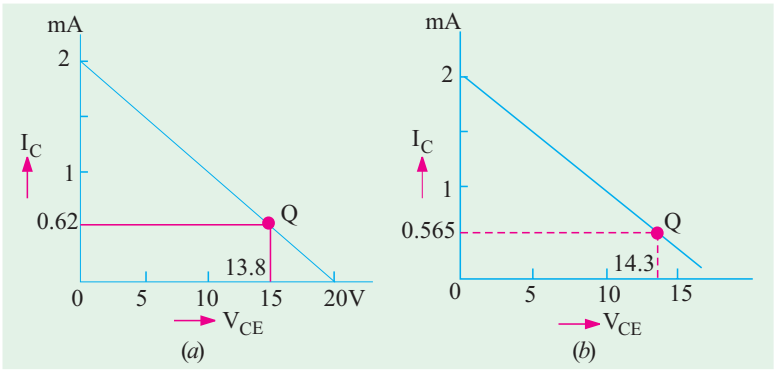


Fig. 58.21

$\therefore \quad V_{CE} = V_{CC} - I_C R_C - I_E R_E = 20 - 0.565 \times 4 - 0.576 \times 6 =$ **14.3 V**

The new and more accurate *Q*-point **(14.3 V, 0.565 mA)** is shown in Fig. 58.21 (*b*).

58.15. Load Line and Output Characteristics

In order to study the effect of bias conditions on the performance of a *CE* circuit, it is necessary to superimpose the dc load line on the transistor output (V_{CE}/I_C) characteristics. Consider a silicon *NPN* transistor which is connected in *CE* configuration (Fig. 58.22) and whose output characteristics are given in Fig. 58.23. Let its $\beta = 100$.

First, let us find the cut-off and saturation points for drawing the dc load line and then mark in the *Q*-point.

$I_{C(sat)} = 10/2 = 5$ mA —point *B* in Fig. 58.23

$V_{CE(cut\text{-}off)} = V_{CC} = 10$ V —point *A* in Fig. 58.23

The load line is drawn in Fig. 58.23 below.

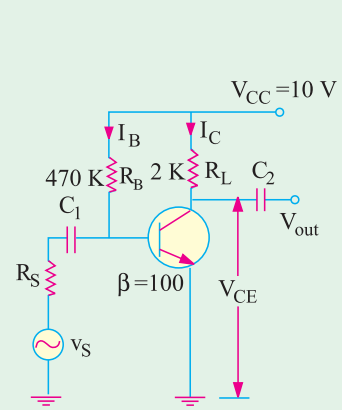


Fig. 58.22

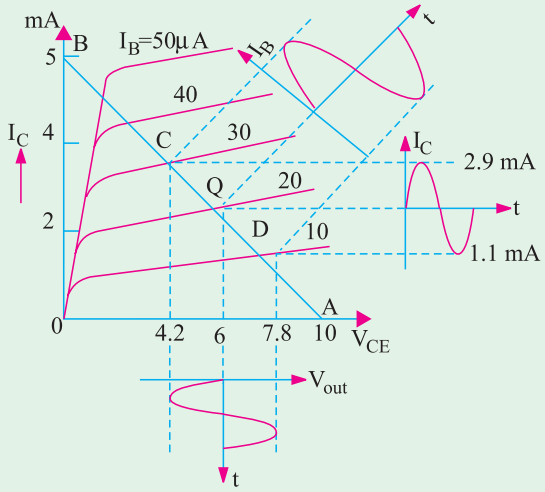


Fig. 58.23

Actual $I_B = \dfrac{V_{CC} - V_{BE}}{R_B} = \dfrac{10 - 0.7}{470} = 20\,\mu A$

$I_C = \beta I_B = 100 \times 20 = 2000\ \mu A = 2$ mA

$V_{CE} = V_{CC} \times I_C R_L = 10 - 2 \times 2 = 6$ V

This locates the *Q*-point in Fig. 58.23.

Suposse an ac input signal voltage injects a sinusoidal base current of peak value 10 μA into the circuit of Fig. 58.22. Obviously, it will swing the operating or *Q*-point up and down along the load line.

When positive half-cycle of I_B is applied, the *Q*-point shifts to point *C* which lies on the (20 + 10) = 30 mA line.

Similarly, during negative half-cycle of input base current, *Q*-point shifts to point *D* which lies on the (20 – 10) = 10 μA line.

By measurement, at point *C*, $I_C = 2.9$ mA. Hence, $V_{CE} = 10 - 2 \times 2.9 = 4.2$ V

Similarly, at point *D*, I_C measures 1.1 μA.Hence, $V_{CE} = 10 - 2 \times 1.1 = 7.8$ V

It is seen that V_{CE} decreases from 6 V to 4.2 V *i.e.* by a peak value of (6-4.2) = 1.8 V when base current goes positive. On the other hand, V_{CE} increases from 6 V to 7.8 V *i.e.* by a peak value of (7.8 – 6) = 1.8 V when input base current signal goes negative. Since changes in V_{CE} represent changes in output voltage, it means that when input signal is applied, I_B varies according to the signal

amplitude and causes I_C to vary, thereby producing voltage variations.

Incidentally, it may be noted that variations in voltage drop across R_L are exactly the same as in V_{CE}.

Steady drop across R_L when no signal is applied = 2 × 2 = 4 V. When base signal goes positive, drop across R_L = 2 × 2.9 = 5.8 V. When base signal goes negative, I_C = 1.1 mA and drop across R_L = 2 × 1.1 = 2.2 V.

Hence, voltage variation is = 5.8 – 4 = 1.8 V during positive input half-cycle and 4 – 2.2 = 1.8 V during negative input half-cycle. Obviously, rms voltage variation $1.8\sqrt{2}$ 1.27 V

Now, proper dissipated in RL by ac component of output voltage is

$P_{ac} = 1.27^2/2 = 0.81$ mW and $P_{dc} = I_C^2 R = 2^2 \times 2 = 8$ mW

Total power dissipated in R_L = 8.81 mW.

58.16. AC Load Line

It is the line along which *Q*-point shifts up and down when changes in output voltage and current of an amplifier are caused by an ac signal.

This line is steeper than the dc line but the two intersect at the *Q*-point determined by biasing dc voltage and currents.

AC load line takes into account the ac load resistance whereas *dc* load line considers only the dc load resistance.

(*i*) DC Load Line

The cut-off point for this line is where $V_{CE} = V_{CC}$. It is also written as $V_{CE(cut\text{-}off)}$.

Saturation point is given by

$I_C = V_{CC}/R_L$. It is also written as $I_{C(sat)}$.

It is represented by straight line *AQB* in Fig. 58.24.

(*ii*) AC Load Line

The cut-off point is given by $V_{CE(out\text{-}off)} = V_{CEQ} + I_{CQ} R_{ac}$ where Rac is the ac load resistance*.

Saturation point is given by

$$I_{C(sat)} = I_{CQ} + V_{CEQ}/R_{ac}.$$

It is represented by straight line *CQD* in Fig. 58.24.

The slope of the ac load line is given by $y = \times 1/R_{ac}$.

It is seen from Fig. 58.24 that maximum possible positive signal swing is = $I_{CQ} R_{ac}$. Similarly, maximum possible negative signal swing is V_{CEQ}. In other words, peak-signal handling capacity is limited to I_{CQ} Rac or V_{CEQ} whichever is smaller.

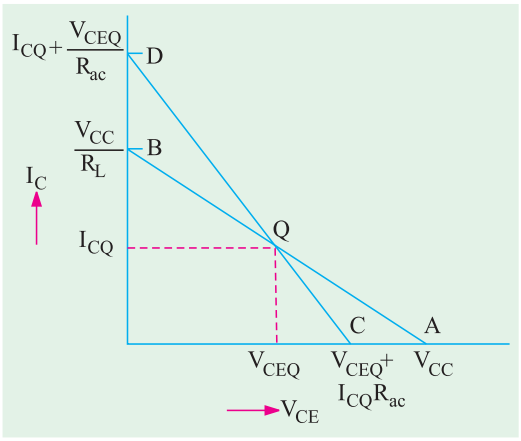


Fig. 58.24

Example 58.12. *Draw the dc and ac load lines for the CE circuit shown in Fig. 58.25 (a). What is the maximum peak-to-peak signal that can be obtained ?*

(Applied Electronics, Kerala Univ. 1991)

Solution. DC Load Line [Fig. 58.25 (*b*)]

$V_{CE(cut\text{-}off)} = V_{CC} = 20$ V (point A) and $I_{C(sat)} = V_{CC}/(R_1 + R_E) = 20/5 = 4$mA (point B). Hence, A_{QB} represents dc load line for the given circuit.

Approximate bias conditions can be quickly found by assuming that I_B is too small to affect the base bias in Fig. 58.25 (*a*).

* Written as r_L in Art. 59.4

$V_2 = 20 \times 4/(4 + 16) = 4$ V

If we neglect V_{BE}, $V_2 = V_E$; I_E,

$$I_E = \frac{V_E}{R_E} = \frac{V_2}{R_E} = \frac{4}{2}$$

$= 2$ mA

Also, $I_C \cong I_E = 2$ mA.

Hence, $I_{CQ} = 2$ mA

The corresponding value of V_{CEQ} can be found by drawing dotted line in Fig. 58.25 (*b*) or may be calculated as under :

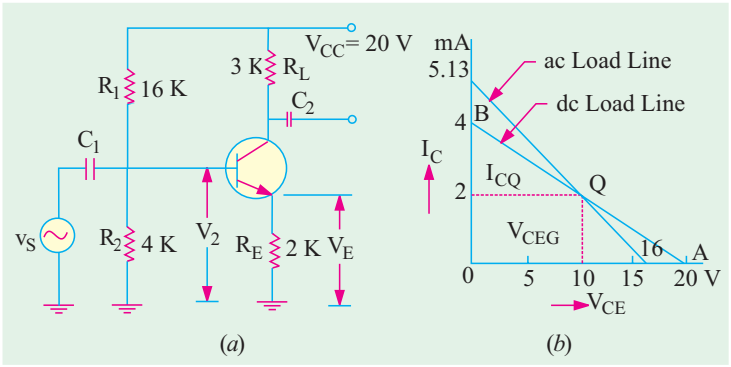


Fig. 58.25

AC Load Line

Cut-off pont,

$V_{CE(cut\text{-}off)} = V_{CEQ} + I_{CQ} R_{ac}$

Now, for the given circuit, ac load resistance is $R_{ac} = R_C = 3$K

Cut-off point = 10 + 2 × 3 = 16 V [Fig. 58.25 (*b*)].

Saturation point, $I_{c(sat)} = I_{CQ} + \frac{V_{CEQ}}{R_{ac}} = 2 + \frac{10}{3} = 5.13\text{mA}$

Hence, line joining 16-V point and 5.13 mA point gives ac load line as shown in Fig. 58.25 (*b*). As expected, this line passes through the *Q*-point.

Now, $I_{CQ}.\ R_{ac} = 2 \times 3 = 6$ V and $V_{CEQ} = 10$ V. Taking the smaller quantity, maximum peak output signal = 6 V. Hence, peak-to-peak value = 2 × 6 = 12 V.

Example 58.13. *Find the dc and ac load lines for CE circuit shown in Fig. 58.26 (a).*

Solution. The given circuit is identical to that shown in Fig. 58.25 (*a*) except for the addition of 6-K resistor. This makes R_{AC} = 3 K || 6 K = 2 K because collector feeds these two resistors in parallel. The dc loadline would remain unaffected. Change would occur only in the ac load line.

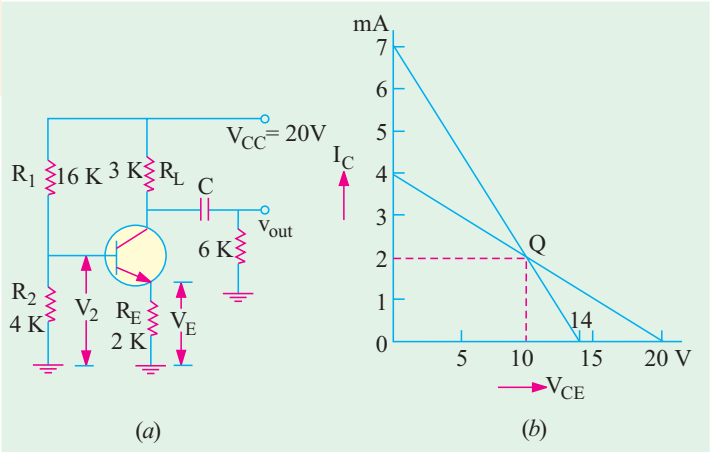


Fig. 58.26

AC Load Line

$V_{CE(cut\text{-}off)} = V_{CEO} + I_{CQ} R_{ac} =$ 10 + 2 × 2 = **14 V**

$I_{C(sat)} = I_{CO} + V_{CEO}/R_{ac} = 2 + 10/2 =$ **7 mA**

Example 58.14. *Draw the dc and ac load lines for the CB circuit shown in Fig. 58.27 (a). Which swing starts clipping first ?*

Solution. The dc load line passes through cut-off point of 30 V and saturation point of V_{CC}/R_L = 1 mA.

Now, $I_E \cong 20/40 = 0.5$ mA ; $I_C \cong I_E = 0.5$ mA; $I_{CQ} = 0.5$ mA

$V_{CB} = V_{CC} - I_C R_C = 30 - 0.5 \times 30 = 15$ V ; $V_{CBQ} = 15$ V

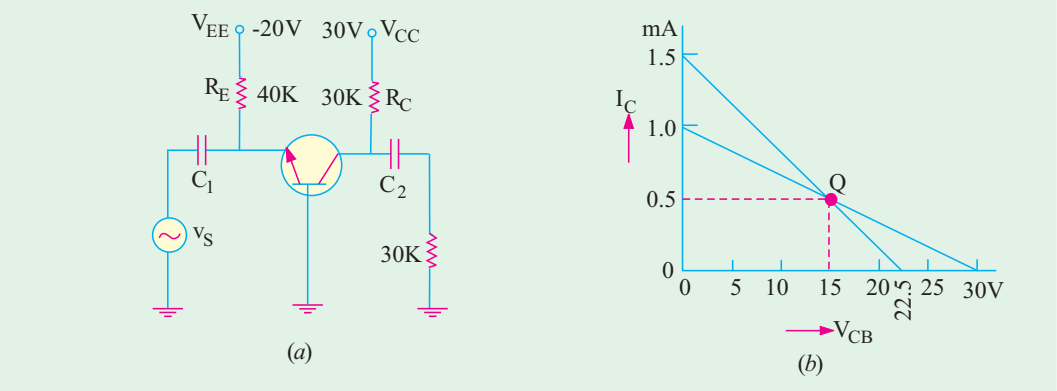


Fig. 58.27

Hence, *dc* operating point or *Q*-point is (15 V, 0.5 mA) as shown in Fig. 58.27 (*b*).

The cut-off point for ac load line is = $V_{CBQ} + I_{CQ}R_{AC}$

Since collector sees an ac load of two 30 K resistors in parallel, hence R_{ac} = 30 K || 30 K =15 K.

$\therefore \quad V_{CBQ} + I_{CQ}R_{ac} = 15 + 0.5 \times 15 = 22.5$ V

Saturation current for ac line is = $I_{CQ} + V_{CBQ}/R_{ac} = 0.5 + 15/15 = 1.5$ mA

The line joining these two points (and also passing through *Q*) gives the ac load line as shown in Fig. 58.27 (*b*).

Note. Knowing ac cut-off point and *Q*-point, we can draw the ac load line. Hence, we need not find the value of saturation current for this purpose.

As seen, positive swing starts clipping first because $I_{CQ}R_{ac}$ is less than V_{CBQ}. Obviously, maximum peak-to-peak signal that can be obtained from this circuit = 2 × 7.5 = 15 V.

Example. 58.15. *For the circuit shown in Fig. 58.27 (a), find the approximate value of source voltage us that will cause clipping. The voltage source has an internal resistance of 1 K.*

Solution. As found out in Ex. 58.14, the maximum swing of the unclipped output = 2 × 7.5 = 15 V.

$$\text{Now,} \quad A_v = \frac{V_{out}}{V_s} = \frac{R_{ac}}{R_S} = \frac{15}{1} = 15$$

$$\therefore \quad V_S = \frac{V_{out}}{A_v} = \frac{15V_{p-p}}{15} = 1V_{p\text{-}p}$$

Tutorial Problems No. 58.1

1. For the CB circuit shown in Fig. 58.28, find the approximate location of *Q*-point. **[10 V, 2 mA]**
2. For the circuit of Fig. 58.29, find
   (*a*) dc operating-point,
   (*b*) maximum peak-to-peak unclipped signal. **[(15 V, 0.5 mA) ; 10 V_{p-p}]**
3. What is the maximum peak-to-peak signal that can be obtained from the circuit of Fig. 58.30 ? **[20 V_{p-p}]**
4. Find the value of maximum peak-to-peak output of signal that can be obtained from the circuit of Fig. 58.31. **[10 V_{p-p}]**

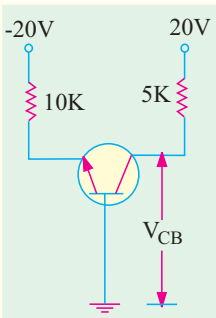


Fig. 58.28

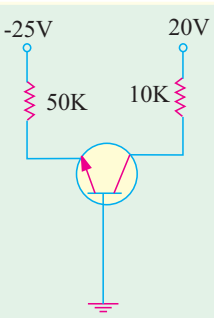


Fig. 58.29

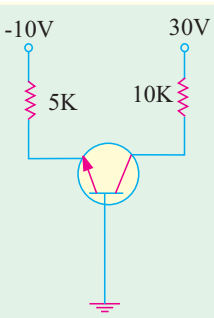


Fig. 58.30

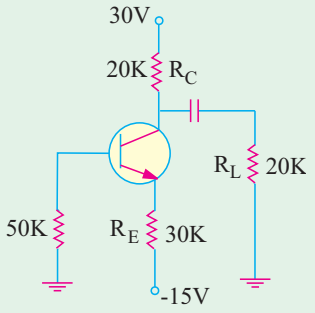


Fig. 58.31

OBJECTIVE TESTS — 58

1. The dc load line of a transistor circuit
(*a*) has a negative slope
(*b*) is a curved line
(*c*) gives graphic relation between I_C and I_B
(*d*) does not contain the *Q*-point.

2. The positive swing of the output signal in a transistor circuit starts clipping first when *Q*-point of the circuit moves
(*a*) to the centre of the load line
(*b*) two-third way up the load line
(*c*) towards the saturation point
(*d*) towards the cut-off point.

3. To avoid thermal run away in the design of analog circuit, the operating point of the *BJT* should be such that it satisfies the condition
(*a*) $V_{CE} = 1/2\ V_{CC}$
(*b*) $V_{CE} < 1/2\ V_{CC}$
(*c*) $V_{CE} > 1/2\ V_{CC}$
(*d*) $V_{CE} < 0.78\ V_{CC}$

4. In the case of a *BJT* amplifier, bias stability is achieved by
(*a*) keeping the base current constant
(*b*) changing the base current in order to keep the I_C and V_{CB} constant
(*c*) keeping the temperature constant
(*d*) keeping the temperature and the base current constant

5. For a transistor amplifier with self-biasing network, the following components are used :
$R_1 = 4\ \text{k}\Omega$, $R_2 = 4\ \text{k}\Omega$ and $R_E = 1\ \text{k}\Omega$
the approximate value of the stability factor '*S*' will be
(*a*) 4
(*b*) 3
(*c*) 2
(*d*) 1.5

6. A transistor circuit employing base bias with collector feedback has greater stability than the one without feedback because
(*a*) I_C decrease in magnitude
(*b*) V_{BE} is decreased
(*c*) of negative feedback effect
(*d*) I_C becomes independent of β.

7. The universal bias stabilization circuit is most popular because
(*a*) I_C does not depend on transistor characteristics
(*b*) its β-sensitivity is high
(*c*) voltage divider is heavily loaded by transistor base
(*d*) I_C equals I_E.

8. Improper biasing of a transistor circuit leads to
(*a*) excessive heat production at collector terminal
(*b*) distortion in output signal
(*c*) faulty location of load line
(*d*) heavy loading of emitter terminal.

9. The negative output swing in a transistor circuit starts clipping first when *Q*-point
(*a*) has optimum value
(*b*) is near saturation point
(*c*) is near cut-off point
(*d*) is in the active region of the load line.

10. When a *BJT* is employed as an amplifier, it operates

(*a*) in cut-off
(*b*) in saturation
(*c*) well into saturation
(*d*) over the active region.

11. Which of the following method used for biasing a *BJT* in integrated circuits is considered independent of transistor beta?
(*a*) fixed biasing
(*b*) voltage divider bias
(*c*) collector feedback bias
(*d*) base bias with collector feedback.

12. The voltage V_0 of the circuit shown in Fig. 58.32 is,

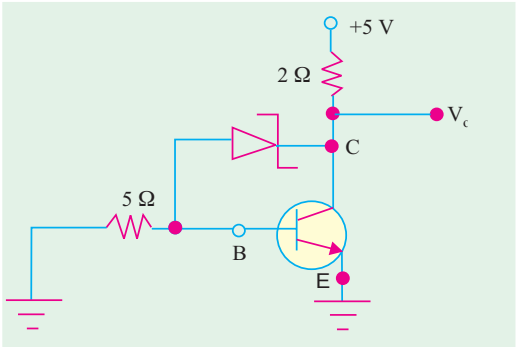


Fig. 58.32

(*a*) 5.1 V
(*b*) 3.1 V
(*c*) 2.5 V
(*d*) zero

13. The collector voltage V_C of the circuit shown in Fig. 58.33, is approximately
(*a*) 2 V
(*b*) 4.6 V
(*c*) 9.6
(*d*) 8.6 V

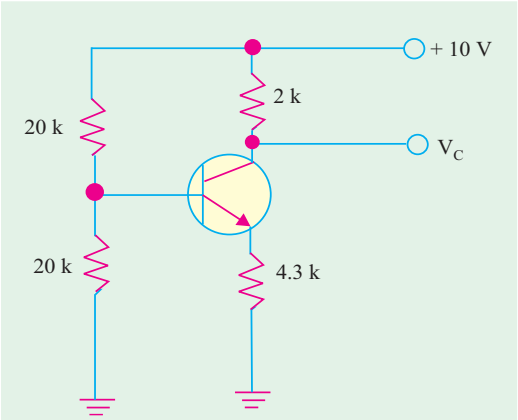


Fig. 58.33

ANSWERS

1.	*(a)*	**2.**	*(d)*	**3.**	*(c)*	**4.**	*(a)*	**5.**	*(b)*	**6.**	*(b)*
7.	*(a)*	**8.**	*(b)*	**9.**	*(b)*	**10.**	*(d)*	**11.**	*(b)*	**12.**	*(d)*
13.	*(a)*										

GO To FIRST

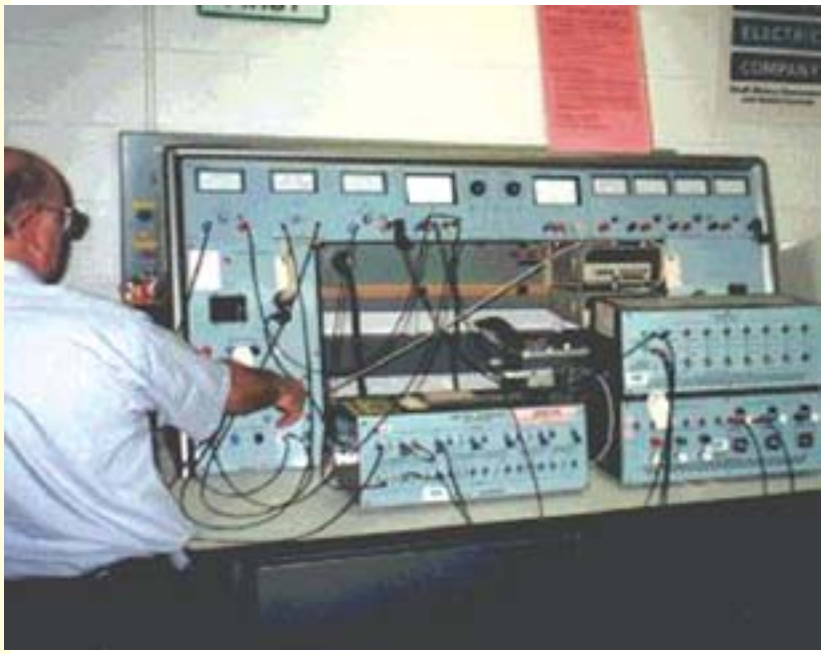
CONTENTS

CHAPTER 59

TRANSISTOR EQUIVALENT CIRCUITS AND MODELS

Learning Objectives

- General
- DC Equivalent Circuit
- AC Equivalent Circuit
- Equivalent Circuit of a CB Amplifier
- Effect of Source Resistance R_s on Voltage Gain
- Equivalent Circuit of a CE Amplifier
- Equivalent Circuit of a CC Amplifier
- Small-signal Low-frequency Model or Representation
- T-Model
- Formulas for T-Equivalent of a CC Circuit
- What are h-parameters ?
- Input Impedance of a Two Port Network
- Voltage Gain of a Two Port Network
- The h-parameters of an Ideal CB Transistor
- The h-parameters of an ideal CE Transistor
- Approximate Hybrid Equivalent Circuits
- Transistor Amplifier Formulae Using h-parameters
- Typical Values of Transistor h-parameters
- Approximate Hybrid Formulas
- Common Emitter h- parameter Analysis
- Common Collector h-parameter Analysis
- Conversion of h-parameters



Generalised hybrid parameter equivalent circuit

CONTENTS

59.1. General

We will begin by idealizing a transistor with the help of simple approximations that will retain its essential features while discarding its less important qualities. These approximations will help us to ***analyse transistor circuits easily and rapidly.***

We will discuss only the ***small signal*** equivalent circuits in this chapter. Small signal operation is that in which the ac input signal voltages and currents are in the order of ± 10 per cent of *Q*-point voltages and currents.

There are two prominent schools of thought today regarding the equivalent circuit to be substituted for the transistor. The two approaches make use of

(*a*) four *h*-parameters of the transistor and the values of circuit components,

(*b*) the beta (β) of the transistor and the values of the circuit components.

Since long, industrial and educational institutions have heavily relied on the hybrid parameters because they produce more accurate results in the analysis of amplifier circuits. In fact, hybrid-parameter equivalent circuit continues to be popular even to-day. But their use is beset with the following difficulties :

1. The values of *h*-parameters are not so readily or easily available.
2. Their values vary considerably with individual transistors ***even of the same type number.***
3. Their values are limited to a particular set of operating conditions for reasonably accurate results.

The second method which employs transistor beta and resistance values is gaining more popularity of late. It has the following advantages :

1. The required values are easily available;
2. The procedure followed is simple and easy to understand;
3. The results obtained are quite accurate for the study of amplifier circuit characteristics.

To begin with, we will consider the second method first.

59.2. DC Equivalent Circuit

(*a*) CB Circuit

In an ideal transistor, $\alpha = 1$ which means that $I_C = I_E$.

The emitter diode acts like any ***forward-biased ideal diode.*** However, due to transistor action, collector diode acts as a ***current source.*** In other words, for the purpose of drawing dc equivalent circuit, we can view an ideal transistor as nothing more than a rectifier diode in emitter and a current source in collector. In the dc equivalent circuit of Fig. 59.1 (*b*), current arrow always points in the direction of conventional current.

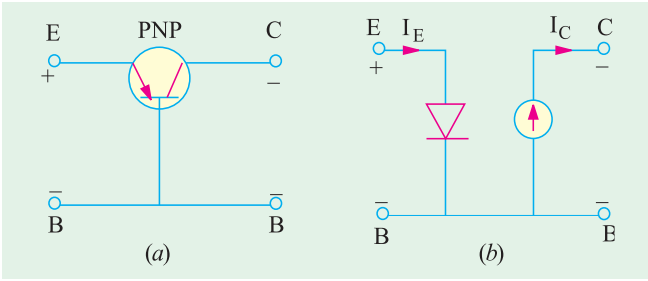


Fig. 59.1

As per the polarities of transistor terminals (Art. 59.3) shown in Fig. 59.1 (*a*), emitter current flow from *E* to *B* and collector current from *B* to *C*.

The dc equivalent circuit shown in Fig. 59.2 for an *NPN* transistor is exactly similar except that direction of current flow is opposite.

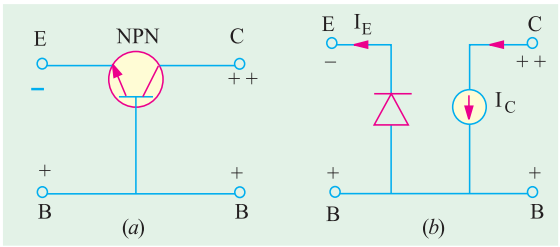


Fig. 59.2

(b) CE Circuit

Fig. 59.3 shows the dc equivalent circuit of an *NPN* transistor when connected in the *CE* configuration. Direction of current flow can be easily found by remembering the transistor polarity rule given in Art 59.3. In an ideal *CE* transistor, we disregard leakage current and take a.c beta equal to dc beta.

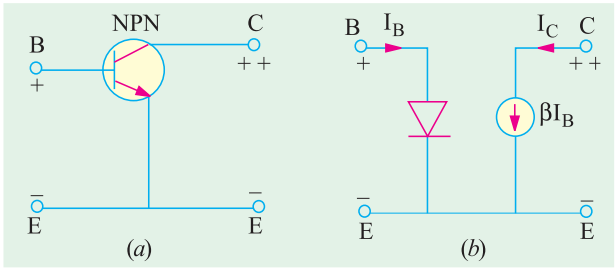


Fig. 59.3

59.3. AC Equivalent Circuit

(a) CB Circuit

In the case of ***small* input** ac signals, the emitter ***diode does not rectify*,** instead it offers resistance called ac ***resistance*.** As usual, collector diode acts as a current source.

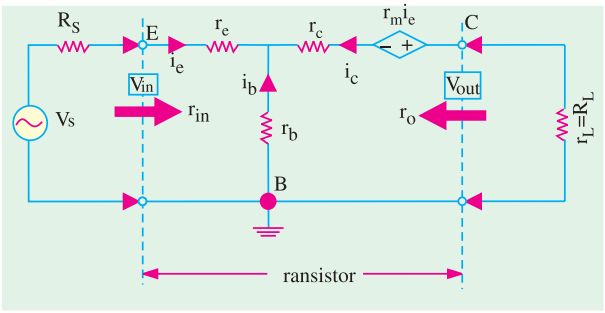


Fig. 59.4

Fig. 59.4 shows the ac equivalent circuit of a transistor connected in the *CB* configuration.* Here, ac resistance offered by the emitter diode is

r_{ac} = junction resistance (r_j) + base-spreading resistance (r_B)**

$= r_j$ r_B is negligible

$$= \frac{25\text{ mV}}{I_E}$$

— where I_E is dc emitter current in mA

It is written as r_e' signifying junction resistance of the emitter *i.e.* a.c resistance looking into the emitter.

$$r_e' \quad \frac{25\text{ mV}}{I_E}$$

Hence, the a.c equivalent circuit of a *CB* circuit becomes as shown in Fig. 59.5. Since changes in collector current are almost equal to changes in emitter current, $\Delta i_c = \Delta\, i_e$.

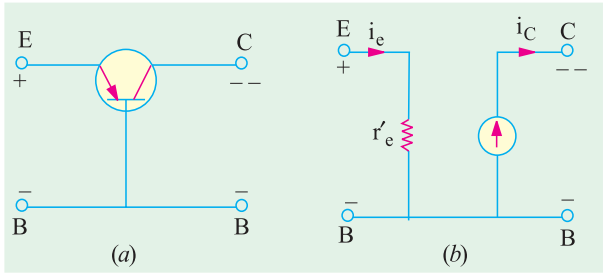


Fig. 59.5

(b) CE Circuit

Fig. 59.6 (*a*) shows the equivalent circuit when an *NPN* transistor has been connected in the *CE* configuration.

The a.c resistance ***looking into the base is***

$$r_{ac} \quad \frac{25\text{ mV}}{I_B}$$ — d.c current is I_B not I_E

---

* This circuit is valid both for *PNP* as well as *NPN* transistors because difference in the direction of ac current does not matter.

** Also called bulk-resistance.

$$\frac{25\,\text{mV}}{I_C/} \quad , \frac{25\,\text{mV}}{I_C}$$

$$, \frac{25\,\text{mV}}{I_E} \quad r_e'$$

Strictly speaking,

$r_{ac} = (1 + \beta)\, r' \cong \beta\, r_e'$ — Art 7.24

The a.c collector current is β times the base current *i.e.*, $i_c = \beta\, i_b$.

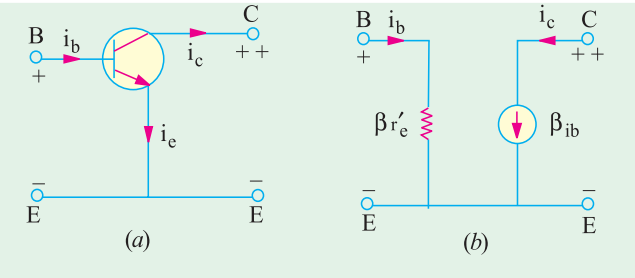


Fig. 59.6

59.4. Equivalent Circuit of a CB Amplifier

In Fig. 59.7 (*a*) is shown the circuit of a common-base amplifier. As seen, emitter is forward-biased by $-V_{EE}$ and collector is reverse-biased by $+ V_{CC}$. The a.c signal source voltage υ_s drives the

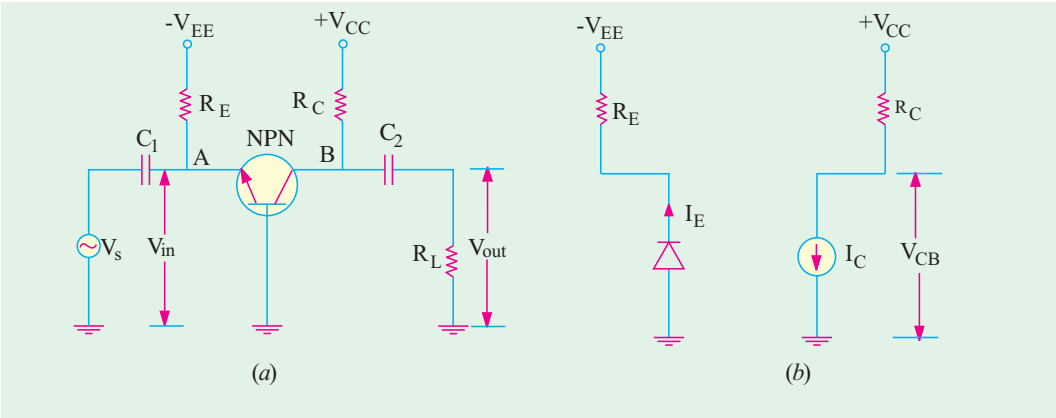


Fig. 59.7

emitter. It produces small fluctuations in transistor voltage and currents in the output circuit. It will be seen that ac output voltage is amplified because it is more than V_s.

(*a*) DC Equivalent Circuit

For drawing dc equivalent circuit, following procedure should be adopted :

(*i*) short all ac sources *i.e.* reduce them to zero,

(*ii*) open all capacitors because they block dc.

If we do this, then as seen from Fig. 59.7 (*a*), neither emitter current can pass through C_1 nor collector current can pass through C_2. These currents are confined to their respective resistances R_E and R_C (earlier we had been designating it as resistance R_L).

Here, $I_C \cong I_E$ and $V_{CB} = V_{CC} - I_C R_L$

Hence, the dc equivalent circuit becomes as shown in Fig. 59.7 (*b*).

(*b*) AC Equivalent Circuit

For drawing ac equivalent circuit, following procedure is adopted :

(*i*) all dc sources are shorted *i.e.* they are treated as ac ground,

(*ii*) all coupling capacitors like C_1 and C_2 in Fig. 59.7 (*a*) are shorted and

(*iii*) emitter diode is replaced by its a.c resistance $r_{ac} = r_e'$

$$r_e' \quad \frac{25\,\text{mV}}{I_E}$$

where I_E is dc emitter current in mA.

As seen by the input a.c signal, it has to feed R_E and r_e' in parallel [Fig. 59.8 (*a*)]. As looked

from point A in Fig. 59.7 (*a*), R_E is grounded through V_{EE} which has been shorted and r_e' is grounded *via* the base.

Similarly, collector has to feed R_C and R_L which are connected in parallel across it *i.e* at point *B* in Fig. 59.7 (*a*). The ac signal in collector sees an output load resistance of $r_L = R_C \| R_L$.

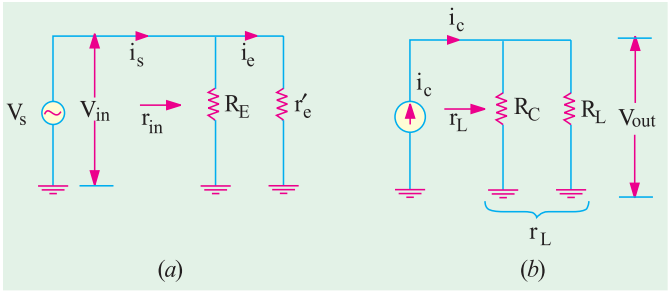


Fig. 59.8

Hence, ac equivalent circuit is as shown in Fig. 59.8 (*b*). Here, collector diode itself has been shown as a current source.

Following two points are worth noting :

(i) changes in collector signal current are very nearly equal to changes in emitter signal current. Hence, $\Delta i_c \cong \Delta i_e$.,

(ii) directions of ac currents shown in the circuit diagram are those which correspond to positive half-cycle of the a.c input voltage. That is why i_c is shown flowing upwards in Fig. 59.8 (*b*).

(*c*) Principal Operating Characteristics

1. Input Resistance

As seen from Fig. 59.8 (*a*), the input resistance of the circuit (or stage) is given by

$$r_{in} \quad R_E \| r_e' \quad \frac{r_e' R_E}{R_E \quad r_e'}$$

In practice, R_E is always much greater than r_e' so that parallel combination $R_E \| r_e' \cong r_e'$

$\therefore \; r_{in} \cong = r_e'$ input resistance of the emitter diode

2. AC Load Resistance

The collector load as seen by output ac signal consists of a parallel combination of R_L and R_C. This has already been designated as r_L.

$\therefore \qquad r_L = R_L \| R_C$

It should be carefully noted that it is the output resistance ***as seen by collector and not the ac output resistance when looking into the collector.***

Note. In case, R_L has not been connected, then $r_L = R_C$ (Ex. 59.1)

3. Current Gain

It is given by the ratio $A_i \quad \frac{i_c}{i_e}$

4. Voltage Gain

It is given by the ratio

$$A_v \quad \frac{V_{out}}{V_{in}}$$

Now, V_{in}* $= i_e r_{in} = (1 + \beta)\, i_b \,.\, r_{in}$

$V_{out} \quad = i_c r_L = \beta i_b r_L$

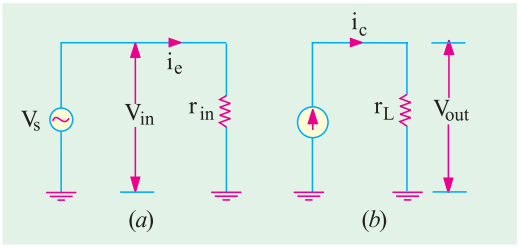


Fig. 59.9

* Here, V_{in} equals υ_s because there is no internal resistance of the source. In case it is there, the two will not be equal (Art 59.5).

$$\therefore \quad A_v \quad \frac{i_b\, r_L}{(1 \quad) i_b\, r_{in}}$$

Taking the ratio $\beta / (1 + \beta)$ as unity, $A_V \quad \frac{r_L}{r_{in}} \quad \frac{r_L}{r'_e}$

5. Power Gain

$$A_p = \quad A_v . A_i$$

When expressed in decibels, it is written as $G_p = 10 \log_{10} A_p \, dB$.

When expressed in terms of r_{in} and r_L, the a.c equivalent circuit becomes as shown in Fig. 59.9.

Example 59.1. *For the single-stage CB amplifier shown in Fig. 59.10 (a), find r_{in}, r_L, A_i, A_v and A_p. What would be the rms value of the signal voltage across the load if v_s has an rms value of 1.5 mV? Assume silicon material and transistor $\alpha = 0.98$.* **(Electronics-II, Madras Univ.)**

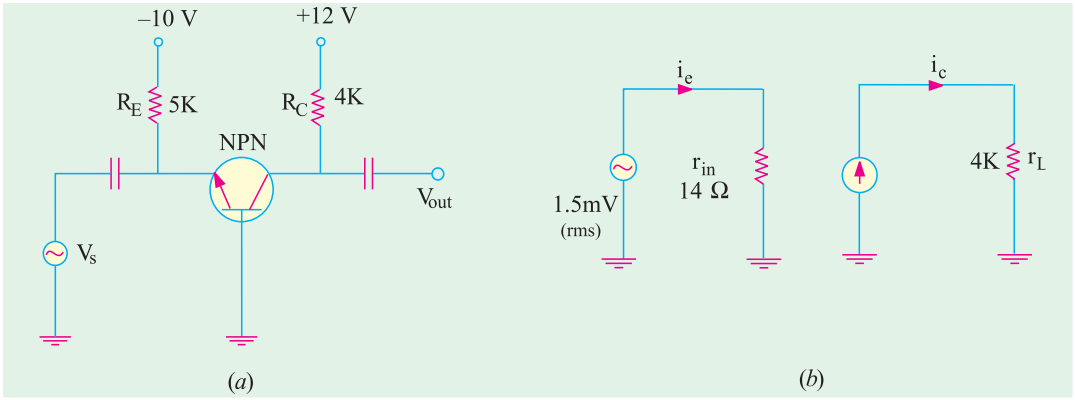


Fig. 59.10

Solution. For finding , let us first find I_E.

$$I_E \quad \frac{V_{EE} \quad V_{BE}}{R_E} \quad \frac{10 \quad 0.7}{5\text{K}} \quad 1.86\,\text{mA}$$

$$r'_e \quad \frac{25\,\text{mV}}{I_E\ \text{mA}} \quad \frac{25}{1.86} \quad 14$$

(i) $r_{in} = r'_e \parallel R_E = 14\ \Omega \parallel 5\ \text{K} \cong$ **14 Ω** **(ii)** $r_L = R_C =$ **4 K**

The ac equivalent circuit becomes as shown in Fig. 59.10 (*b*).

(iii) $A_i = \alpha =$ **0.98** **(iv)** $A_v \quad \frac{r_L}{r_{in}} \quad \frac{r_L}{r'_e} \quad \frac{4K}{14} \quad$ **286**

(v) $A_p = A_i A_v = 0.98 \times 286 = 280$ $G_p = 10 \log_{10} 280 =$ **24.5 dB**

Output ac voltage = $A_v \times$ input voltage

$\therefore \quad V_{out} \quad = A_v \times V_{in} = 286 \times 1.5 = 429\ \text{mV} =$ **0.429 V**

Example 59.2. *For the CB amplifier circuit shown in Fig. 59.11 (a), find*
(i) *stage r_{in}* **(ii)** r_L **(iii)** *a.c output voltage V_{out}* **(iv)** *voltage gain A_v.*
Take $V_{BE} = 0.7\ V$.

Solution. $I_E \quad = (20 - 0.7)/30\ \text{K} = 0.64\ \text{mA}$; $r'_e = 25/0.64 = 39\ \Omega$

(i) stage $r_{in} \quad = r'_e \parallel R_E' = 39\Omega \parallel 30\ \text{K} \cong 39\ \Omega$

(ii) $r_L \quad = R_L \parallel R_C = 30\ \text{K} \parallel 15\ \text{K} =$ **10 K**

The amplifier circuit alongwith its ac equivalent circuit is shown in Fig. 59.11.

(iii) We will first find the value of n_{out} as a drop across $r_L = R_C \| R_L$. For that purpose, we will employ rms values.

Now, $i_e \quad \frac{V_{in}}{r_{in}} \quad \frac{V_s}{r_{in}} \quad \frac{1.6}{3.9} \quad 0.04 \text{ mA}^*$

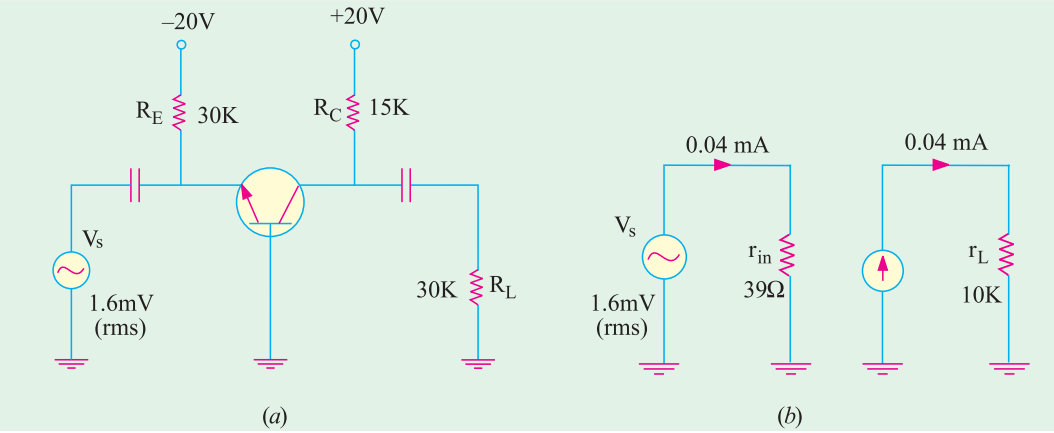


Fig. 59.11

$i_c = i_e = 0.04 \text{ mA} \; ; \; V_{out} = i_c r_L = 0.04 \times 10 \text{ K} = 0.4 \text{ V}$

(iv) $A_v \quad \frac{r_L}{r_{in}} \quad \frac{r_L}{r_e'} \quad \frac{r_L}{r_e'} \quad \frac{10 \text{ K}}{39} \quad$ **250**

The rms value of ac output voltage may be found with the help of A_v.

Now, $A_v \quad \frac{V_{out}}{V_{in}} \qquad V_{out} \quad A_v . V_{in} \quad 250 \quad 1.6 \quad 400 \text{ mV} \quad$ **0.4 V**

59.5. Effect of Source Resistance R_s on Voltage Gain

The voltage gain for the *CB* amplifier circuit has so far been calculated on the assumption that the resistance R_S of the ac signal source is negligible. The voltage gain will decrease as R_S increases because more and more of V_S will drop on R_S rather than on r_{in} and so will not appear in the output.

The basic circuit is shown in Fig. 59.12 where R_S is the internal resistance of the ac signal source.

The ac equivalent circuit is shown in Fig. 59.13 (*a*). Here, R_S is in series with $(r_e' \| R_E)$. The input ac signal voltage v_S drops on these two series resistors. Obviously, V_{in} is the drop across $r_e' \| R_E$ and is less than V_S. Using Proportional Voltage Formula,

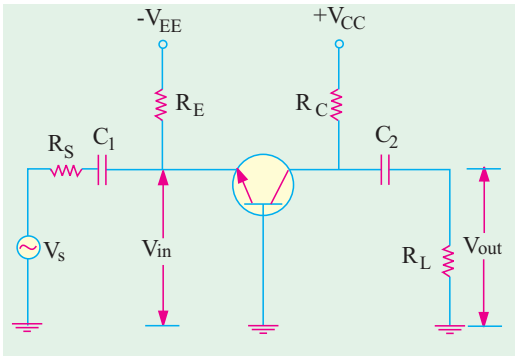


Fig. 59.12

* Strictly speaking, it is not i_e but i_s . a small part of the ac source current goes through $R_E = 30$ K and the balance (major part) goes through parallel resistance r_e'. However, if we neglect current through 30 K, then $i_e = i_s$

$$V_{in} \quad V_s \frac{r_e' \| R_E}{R_s \quad r_e' \| R_E}$$

In practice, $R_E >> r_e'$, so that $r_e' \| R_E \cong r_e'$

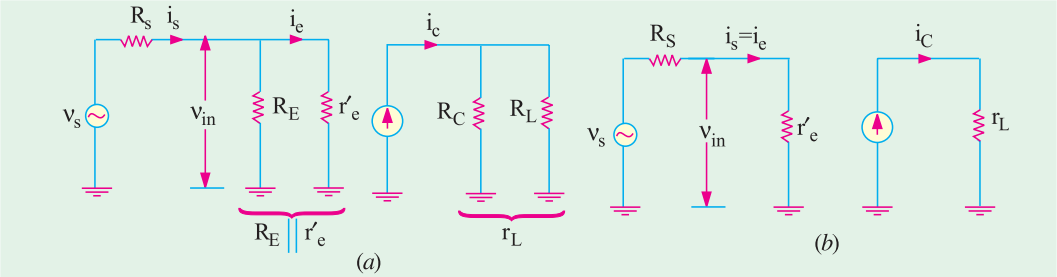


Fig. 59.13

$$\therefore \quad V_{in} \quad V_s \frac{r_e'}{R_s \quad r_e'} \quad \frac{V_s}{1 \quad \frac{R_s}{r_e'}}$$

As R_S increases in comparison to r_e', the ratio R_S / r_e' increases thereby making v_{in} increasingly less than V_s.

Moreover, as seen from the source, input resistance is

$$R_S + r_e' \,//\, R_E \cong R_S + R_e' \qquad \therefore \quad \frac{V_{out}}{V_s} \quad \frac{r_L}{(R_S \quad r_e')} \quad \frac{r_L}{R_S} \quad \text{— if } R_S >> r_e'$$

Making R_S much larger than r_e' is called swamping out the emitter diode. This makes voltage gain independent of r_e' and hence the particular transistor is used. The above gain is different from the voltage gain considered from emitter to collector which is

$$\frac{V_{out}}{V_{in}} \quad \frac{r_L}{r_{in}} \quad \frac{r_L}{r'_e}$$

Example. 59.3. *In the CB amplifier circuit of Fig. 59.14, find*
(a) voltage gain from source to output *(b) voltage gain from emitter to output*
(c) approximate value of V_{in} **(Electronics-I, Patna Univ.)**

Solution. (a) The voltage gain from source to output is given by

$$\frac{V_{out}}{V_S} \quad \frac{r_L}{(R_S \quad r_e')}$$

$r_L = R_C \| R_L = 10\text{ K} \| 2\text{M}$

$\cong 10\text{ K}$; $I_E = 25/25\text{ K} = 1\text{ mA}$

$r_e' = 25/1 = 25\ \Omega$

$$\frac{V_{out}}{V_s} \quad \frac{10\text{ K}}{(1\text{K} \quad 26\ )} \quad \mathbf{10} \qquad ...(i)$$

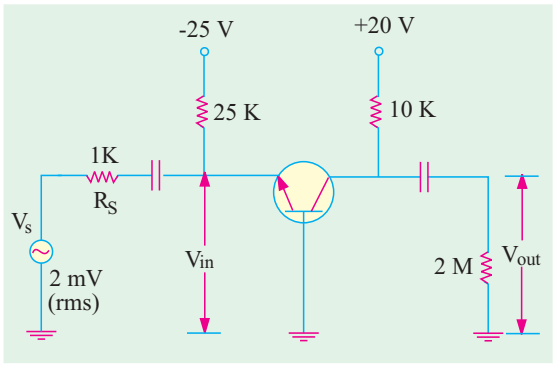


Fig. 59.14

(b) The voltage gain from emitter to output is

$$\frac{V_{out}}{V_s} \quad \frac{r_L}{r_{in}}$$

Now, $r_{in} = R_E // r_e' = 25\ \text{K} \parallel 25\ \Omega \cong 25\ \Omega$ $\therefore$ $\frac{V_{out}}{V_S}$ $\frac{10K}{25}$ **400** ...(*ii*)

(*c*) The value of V_{in} may be found by the following two ways :

(*i*) As seen from Eq. (*ii*) above, $V_{in} = \frac{V_{out}}{400}$, Now, from Eq. (*i*) above, we have

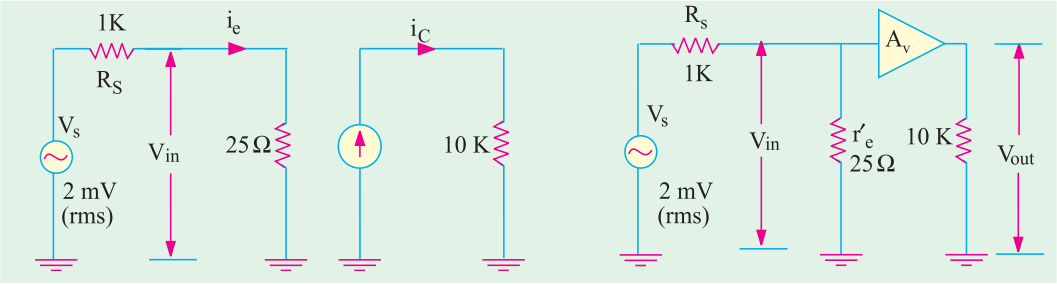


Fig. 59.15

$V_{out} = 10\ V_s = 10 \times 2 = 20\ \text{mV}$ $\therefore$ V_{in} $\frac{20}{400}$ 10^3 **50μV**

(*ii*) As seen from the ac equivalent circuit of the amplifier (Fig. 59.15), V_s is dropped proportionally on the two series resistances R_S and r_e' (strictly speaking $r_e' \parallel R_E$). The drop across r_{in} is called V_{in}.

$\therefore$ V_{in} $V_s \frac{r_e'}{R_s \quad r_e'}$ $2\,\text{m}V$ $\frac{25}{1000 \quad 25}$ **50μV**

59.6. Equivalent Circuit of a CE Amplifier

Consider the simple *CE* amplifier circuit of Fig. 59.16 (*a*) in which base bias has been employed.

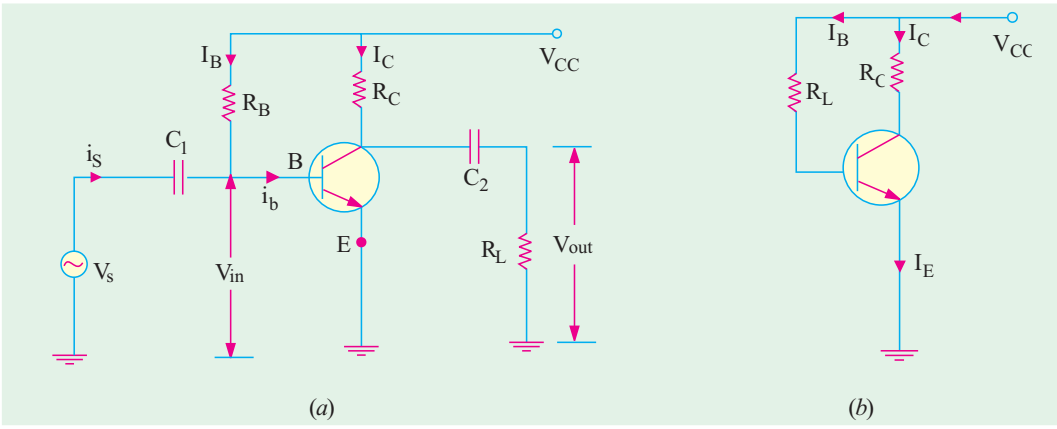


Fig. 59.16

(*a*) DC Equivalent Circuit

Fog drawing the dc equivalent circuit, same procedure is adopted as given in Art. 59.2. It is shown in Fig. 59.16 (*b*).

As seen, I_B $\frac{V_{CC} \quad V_{BE}}{R_B}$ $\frac{V_{CC}}{R_B}$; $I_C = \beta\ .\ I_B$; $I_E \cong I_C \cong \beta I_B$

(*b*) AC Equivalent Circuit

Let us now analyse the ac equivalent circuit given in Fig. 59.17.

As proved in Art. 59.3, the ac resistance as seen by the input signal when looking into the base is = $\beta\, r_e'$. It may be called $r_{in(base)}$ to distinguish it from r_{in} which is resistance of the ***CE stage*** as shown in Fig. 59.17 (*a*).

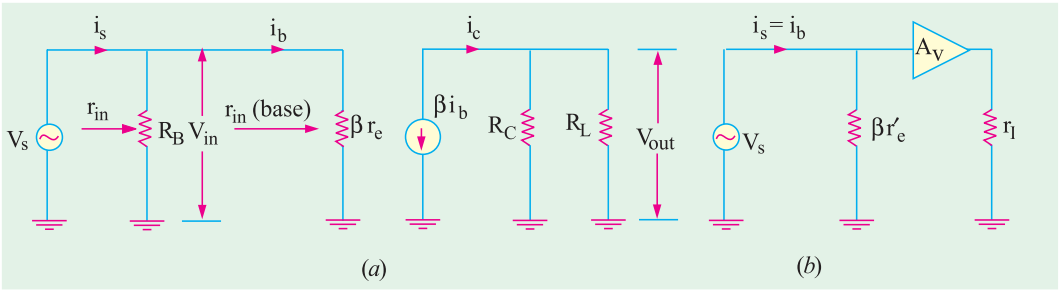


Fig. 59.17

It may be noted that in the absence of ac voltage source resistance R_S, whole of V_s acts across R_B as well as $\beta\, r_e'$ because the two are connected in parallel across it.

Another point worth noting is that major part of source current is passes through $\beta\, r_e'$ and an extremely small part (= v_s / R_B) passes through R_S. Since R_B is usually very large, current passing through it can be easily neglected. Hence, as shown in Fig. 59.17 (*b*), $i_b = i_s$.

(*c*) Principal Operating Characteristics

1. Input Resistance

As seen from Fig. 59.17, input resistance of the *stage* is

$r_{in} = R_B // \beta\, r_e' \cong \beta\, r_e'$ $\qquad R_B >> \beta\, r_e'$

= input resistance of the base

$r_{in(stage)} = r_{in(base)}$

2. AC Load Resistance. $r_L = R_C \| R_L$

3. Current Gain. $A_i \quad \dfrac{i_c}{i_b}$

4. Voltage gain. The voltage gain of the stage or circuit is $A_v \quad \dfrac{v_{out}}{v_{in}}$

Now, $v_{in} \quad i_b \quad r_e'$ and $v_{out} \quad i_c \,.\, r_L \quad i_b r_L$

$\therefore \quad A_v \quad \dfrac{i_b r_L}{i_b r_e'} \quad \dfrac{r_L}{r_e'}$ — if $R_B >> \beta\, r_s'$

5. Power Gain. $A_p = A_i \,.\, A_v$ and $G_p = 10 \log_{10} A_v \, dB$

Example. 59.4. *If in the CE circuit of Fig. 59.16 (a),* $V_{CC} = 20$ *V,* $R_C = 10$ *K,* $R_B = 1$ *M,* $R_L = 1$ *M,* $v_s = 2mV$ *and* $\beta = 50$, *find* **(*i*)** i_b and i_c **(*ii*)** r_{in} **(*iii*)** r_L **(*iv*)** A_n **(*v*)** A_p and G_p. **(Basic Electronics, Bombay Univ. 1991)**

Solution. $I_B = 20/1\ \text{M} = 20\ \mu\text{A}$; $I_C = \beta\, I_B = 50 \times 20\ \text{mA} = 1\ \text{mA}$

$r_e' = 25\ \text{mV}/1\ \text{mA} = 25\ \Omega$; $r_{in(base)} = \beta r_e' = 50 \times 20 = 1250\ \Omega$

(*i*) As seen from Fig. 59.17 (*a*), ac base current is given by

$$i_b \quad \frac{v_s}{r_{in(base)}} \quad \frac{2\,\text{mV}}{1250} \quad \textbf{1.6}\mu\textbf{A} \ ; \ i_c = \beta i_b = 50 \times 1.6 = \textbf{80}\mu\textbf{A}$$

(*ii*) stage $r_{in} = R_B // \beta r_e' = 1\ \text{M} \| 1250\ \Omega \cong$ **1250 Ω**

(*iii*) $r_L = R_L \| R_C = 1\ \text{M} \| 10\ \text{K} \cong$ **10 K**

(iv) $A_v = \frac{r_v'}{r_e'} = \frac{r_L'}{r_e'} = \frac{10\text{ K}}{25} = 400$

(v) $A_p = A_i \cdot A_v = 50 \times 400 = 20{,}000$; $G_p = 10 \log_{10} 20{,}000 = \mathbf{43\ dB}$

Example 59.5. *In the CE amplifier circuit of Fig. 59.18 employing emitter feedback, find :*
(i) r_{in} **(ii)** r_L **(iii)** A_v **(iv)** A_p and **(v)** G_p
Take transistor $\beta = 100$. *How will these values change if emitter bypass capacitor is removed ?*
(Electronics—II, Madras Univ.)

Solution. It should be carefully noted that emitter bypass capacitor C provides ***ac ground*** to the signal *i.e.* it shorts out R_E to ground so far as ***ac signal is concerned.*** However, it plays its normal role so far as dc quantities are concerned.

Hence, the ac equivalent circuit of the amplifier becomes as shown in Fig. 59.19.

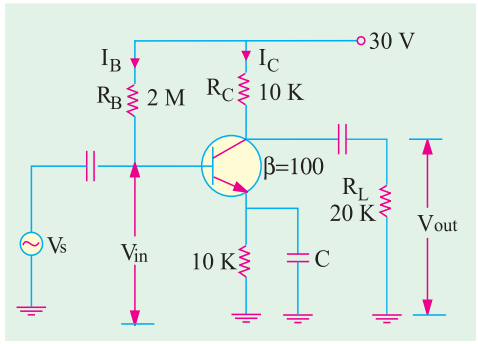


Fig. 59.18

Now, $I_B = \frac{V_{CC}}{R_B \quad R_E} = \frac{30}{2\text{M} \quad 100 \quad 10\text{K}}$

$= 10\ \mu\text{A}$; $I_C = \beta I_B$

$= 100 \times 10\ \text{mA} = 1\ \text{mA}$

$I_E \cong I_C = 1\ \text{mA}$, $r_e' = 25/1 = \mathbf{25\ \Omega}$;

$\beta r_e' = 100 \times 25 = \mathbf{2500\ \Omega}$

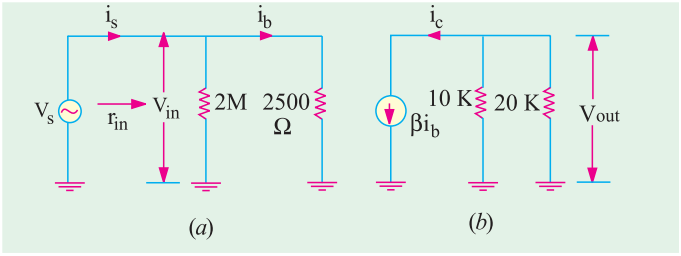


Fig. 59.19

(i) As seen from Fig. 59.19

$r_{in} = R_B \| \beta r_e' = 2\text{M} \| 2500\ \Omega \cong \mathbf{2500\Omega}$

(ii) $r_L = R_C \| R_L = 10\text{ K} \| 20\text{ K} = \mathbf{6.67\ K}$

(iii) $A_V = \frac{V_{out}}{V_{in}} = \frac{r_L'}{r_e'}$

$= \frac{6.67\text{ K}}{25} = \mathbf{267}$

(iv) $A_p = A_i \cdot A_v = 100 \times 267 = \mathbf{26{,}700}$; $G_p = 10 \log_{10} 26{,}700 = \mathbf{44.3\ dB}$

When Emitter Bypass Capacitor is Removed

When bypass capacitor is removed, ac ground is removed. Now, the ac signal will have to pass through R_E also. According to β-rule of Art. 57.24, the total emitter resistance referred to base will become $(1 + \beta)(r_e' + R_E) \cong \beta (r_e' + R_E)$ because r_e' is in series with R_E as shown in Fig. 59.20 (*a*).

Now, the ac equivalent circuit becomes as shown in Fig. 59.20 (*b*).

Stage $r_{in} = R_B \| \beta (r'_e + R_E) \cong R_B \| \beta R_E$

It is so because R_E is much greater than r_e'.

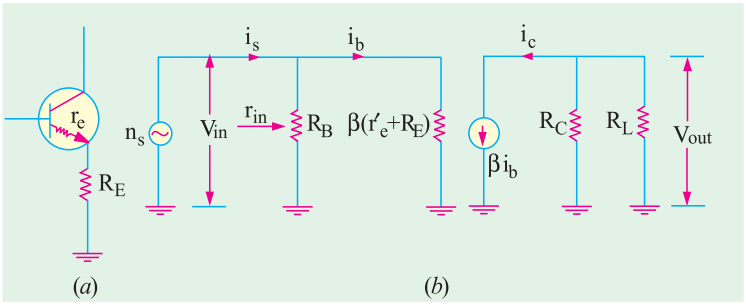


Fig. 59.20

(i) $r_e' = 25\ \Omega$ — as before

$\therefore \quad \beta(r_e' + R_E) \cong \beta R_E = 100 \times 10\ \text{K} = 1\ \text{M}$

$\therefore \quad r_{in} = R_B \| \beta R_E = 2\ \text{M} \| 1\text{M}$

$= \frac{2}{3}\text{M} = \mathbf{666.7\ K}$ — much greater than before

(ii) $r_L = \mathbf{6.67\ K}$ — it remains unchanged

(iii) $V_{in} = i_b \beta (r_e' + R_E)$; $V_{out} = i_c . r_L = \beta i_b r_L$

$\therefore \quad A_v \quad \frac{V_{out}}{V_{in}} \quad \frac{i_b r_L}{i_b . (r_e' \ R_E)} \quad \frac{r_L}{(r_e' \ R_E)} \quad \frac{r_L}{R_E}$

$\frac{6.67\ \text{K}}{10\ \text{K}} \quad \mathbf{0.667}$ —reduced drastically

(iv) $A_p = A_i A_v = 100 \times 0.667 = \mathbf{66.7}$ — reduced considerably

It is seen that by removing bypass capacitor, ***excessive degeneration has occurred in the amplifier circuit.***

Example 59.6. *For the circuit shown in Fig. 59.21, compute* ***(i)*** $r_{in\ (base)}$ ***(ii)*** V_{out} ***(iii)*** A_v ***(iv)*** r_{in}. *Neglect* V_{BE} *and take* $\beta = 200$.

Solution. It may be noted that voltage divider bias has been used in the circuit.

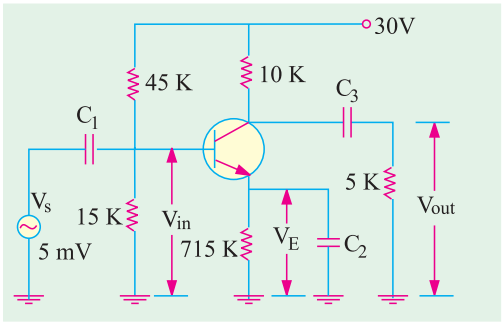


Fig. 59.21

$V_2 \quad \frac{15}{15 \quad 45} \quad 30 \quad 7.5\ \text{V}$

$V_E = V_2 - V_{BE} \cong V_2 = 7.5\ \text{V}$

$I_E = 7.5/7.5\ \text{K} = 1\ \text{mA}$

$r_e' = 25\ \text{mV}/1\ \text{mA} = 25\Omega$

The ac equivalent circuit is shown in Fig. 59.22. Since dc souce is shorted, 45 K, resistor is ac grounded. On the input side, three resistance become paralleled across v_s *i.e.* **(i)** 15 K **(ii)** 45 K and **(iii)** $\beta r_e'$ or $r_{in(base)}$.

Capacitor C_2 ac grounds the emitter resistance R_E, so does C_2 to 5 K and V_{CC} to 10 K.

(i) $r_{in(base)} = \beta r_e' = 200 \times 25 = \mathbf{5\ K}$ **(ii)** $i_b = 5\ \text{mV}/5\text{K} = 1\ \mu\text{A}$

(Obviously, i_b is not the current which leaves the source but that part of the source current is which enters the base). Now, collector load resistance is $i_c = \beta i_b = 200 \times 1 = 200\ \mu\text{A} = 0.2\ \text{A}$

$r_L = 10\ \text{K} \| 5\ \text{K} = 10/3\ \text{K} = 3.33\ \text{K}$, $V_{out} = i_c r_L = 0.2 \times 3.33 = \mathbf{0.667\ V}$

(iii) $A_v \quad \frac{V_{out}}{V_{in}} \quad \frac{0.667\ \text{V}}{5\ \text{m V}} \quad \mathbf{133}$

or $A_v \quad \frac{r_L}{r_e'} \quad \frac{3.33\ \text{K}}{25} \quad 133$

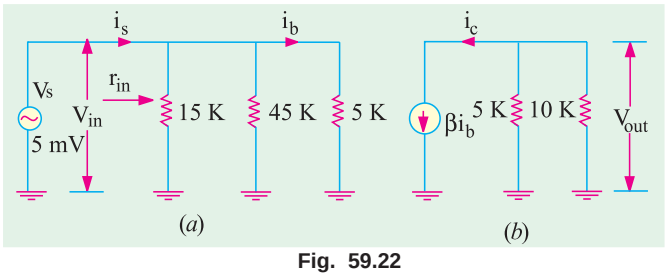


Fig. 59.22

(iv) r_{in} means the input ac resistance as seen from the source *i.e.* from point *A* in Fig. 59.22. It is different from $r_{in(base)}$. Obviously, r_{in} is equal to the equivalent resistance of three resistances connected in parallel. $r_{in} = 15\ \text{K} \| 45\ \text{K} \| 5\ \text{K} = 3.54\ \text{K}$.

Example 59.7. *For the CE amplifier circuit of Fig. 59.23, find out (i) r_{in} (ii) r_L (iii) A_v (iv) A_p and (v) V_{out}. Take transistor $\beta = 50$ and $R_S = 0$.* **(Applied Electronics-II, Punjab Univ. 1992)**

Solution.

$$I_E = \frac{V_{EE}}{R_E} = \frac{20}{40} = 0.5\,\text{mA}$$

$$r_e' = 25/0.5 = 50\ \Omega$$

$$\beta r_e' = 50 \times 50 = 2500\ \Omega$$

(i) $r_{in} = 10\text{ K} \parallel 2500\ \Omega = \mathbf{2000\ \Omega}$

(ii) $r_L = 20\text{ K} \parallel 20\text{ K} = \mathbf{10\text{ K}}$

(iii) $A_v = \frac{r_L}{r_e'} = \frac{10\,K}{50} = \mathbf{200}$

(iv) $A_p = A_v . A_i$

$= 200 \times 50 = 10{,}000$

$G_p = 10 \log 10\ 10000 = \mathbf{40\ dB}$

(v) $V_{out} = A_v \times V_{in} = 20 \times 2 = \mathbf{40\ mV\ (r.m.s.)}$

Since R_S is zero, whole of v_s appears across the diode base.

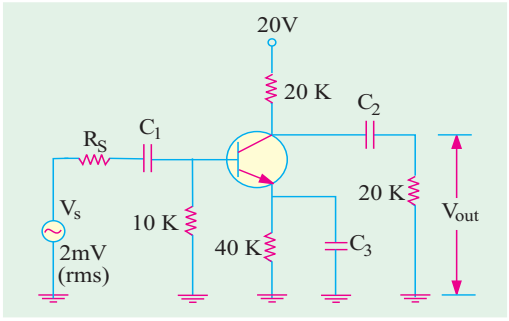


Fig. 59.23

Example. 59.8. *For the single-stage CE amplifier circuit of Fig. 59.24, find approximate value of (i) r_{in} (ii) r_L (iii) A_v (iv) A_p and G_p. Take transistor $\beta = 100$. Use $r_e' = 50\,mV/I_E$.* **(Electronics-II, Gujarat Univ. 1991)**

Solution. $V_2 = 20 \frac{5}{5\ 45} = 2\text{ V};$

$$I_E = \frac{V_2}{R_E} = \frac{2}{1} = 2\,\text{mA}$$

$$r_e' = 50/2 = 25\ \Omega;$$

$$\beta r_e' = 25 \times 100 = 2.5\text{ K}$$

(i) $r_{in} = R_1 \parallel R_2 \parallel \beta r_e' = 45 \parallel 5 \parallel 2.5 = \mathbf{1.6\text{ K}}$

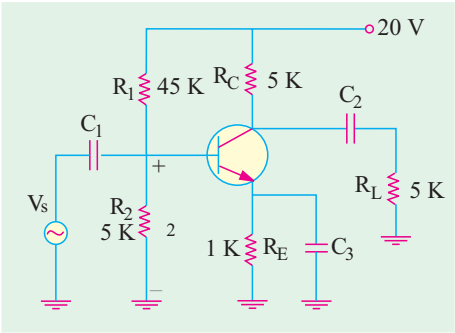


Fig. 59.24

(ii) $r_L = 5\text{ K} \parallel 5\text{ K} = 2.5\text{ K}$

(iii) $A_v = \frac{r_L}{r_e} = \frac{2.5\text{ K}}{25} = \mathbf{100}$

(iv) $A_P = A_v . A_i = 100 \times 100 = \mathbf{10{,}000}$

(v) $G_p = 10 \log_{10} 10{,}000 = \mathbf{40\ dB}$

Note. R_E did not come into picture because it was ac grounded by the bypass capacitor C_3.

Example 59.9. *Find the approximate values of the quantities of Ex. 59.8 in case bypass capacitor C_3 in Fig. 59.24 is removed.*

Solution. (i) In this case,

$$r_{in} = R_1 \parallel R_2 \parallel \beta (r_e' + R_E) = 45\text{ K} \parallel 5\text{ K} \parallel 100\ (25 + 1\text{ K})$$

$$\cong 45\text{ K} \parallel 5\text{ K} \parallel 100\text{ K} = \mathbf{5\text{ K}}$$

(ii) $r_L = 2.5\text{ K}$ — as before

(iii) $A_v = \frac{r_L}{(r_e' \quad R_E)} \quad \frac{r_L}{r_E} \quad \frac{2.5\text{ K}}{1\text{K}} \quad \mathbf{2.5}$

(Please note the reduction)

(iv) $A_p = 100 \times 2.5 = \mathbf{250}$

(v) $G_p = 10 \log_{10} 250 = \mathbf{24\ dB}$

59.7. Effect of Source Resistance R_s

Greater the internal resistance of the ac signal source, greater the internal voltage drop and hence lesser the value of V_{in} because

$V_{in} = V_s$ – drop across R_S.

Consider the CE circuit shown in Fig. 59.25 whose ac equivalent circuit is shown in Fig. 59.26.

As seen from Fig. 59.26, on the input side, R_S is in series with $R_B \| \beta r_E'$. Hence, V_S is divided between them in the direct ratio of their resistances.

If R_S is much less than $R_B \| \beta r_e'$, then

$V_{in} \cong V_s$

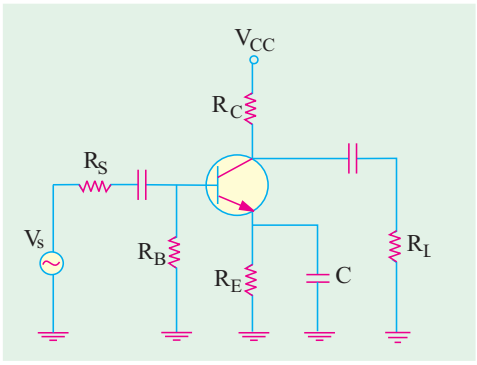


Fig. 59.25

The voltage gain from base to output is still given by $\frac{V_{out}}{V_{in}} \quad \frac{r_L}{r_e'}$

If, in any question, V_s is given, we will first find out how much of it appears across the base as v_{in}. Then, for determining V_{out}, we will simply multipy this V_{in} by the voltage gain.

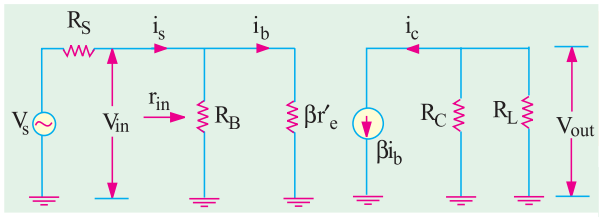


Fig. 59.26

Example 59.10. *In the CE amplifier circuit of Fig. 59.27, find the rms signal output voltage when R_S is* ***(a)*** *100 Ω and* ***(b)*** *3 K. Take b = 50.*

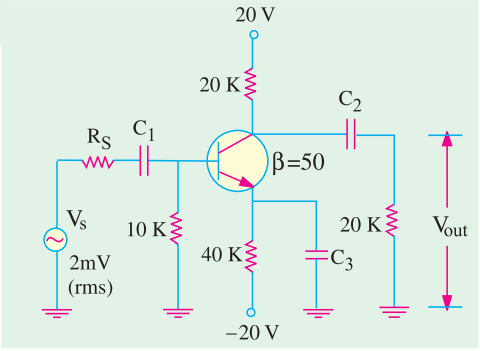


Fig. 59.27

Solution. This circuit is similar to that shown if Fig. 59.23 except for the addition of R_S.

As found in Ex. 9.8, $r_e' = 50\ \Omega$,

$\beta r_e' = 2500\ \Omega$.

$\therefore \quad R_B \| \beta r_e' = 10\ \text{K} \| 22500\ \Omega = 2000\ \Omega$

(a) When $R_S = 100\ \Omega$

In this case, 2 mV are dropped across a series combination of 100 Ω and 2000 Ω. Drop over 100 Ω is negligible as compared to that on 2000 Ω. Hence, it can be presumed that $V_s = V_{in} = 2$mV. We have already found that $A_v = 200$.

$\therefore \quad V_{out} = A_v \times V_{in} = 200 \times 2 =$ **400 mV (rms)**

(b) When $R_S = 3$ K

In this case, drop across $R_B \| \beta r_e' = 2$ K is V_{in} $V_{in} \frac{R_B \| r_e'}{R_s \quad (R_B \| r_e')} \quad 2 \quad \frac{2}{3 \quad 2} \quad 0.8\,\text{mV}$

$\therefore \quad V_{out} = 200 \times 0.8 =$ **160 mV**

Obviously, as R_S is increased, V_{out} is decreased.

59.8. Equivalent Circuit of a CC Amplifier

We will consider the two-supply emitter-bias circuit shown in Fig. 59.28 (*a*) in which the collector is placed at ac (not dc) ground. The ac input signal is coupled into the base and output signal is taken out of the emitter. This circuit is also called **emitter follower circuit** because the

emitter signal ***follows the signal at the base both in magnitude and phase.***

(a) DC Equivalent Circuit

It is drawn in the usual way by opening all the capacitors and shorting all ac sources. It is shown in Fig. 59.28 (*b*).

$$I_E \quad \frac{V_{EE} \quad V_{BE}}{R_E \quad R_B /} \quad \frac{V_{BE}}{R_E}$$

(b) AC Equivalent Circuit

It is obtained by shorting all ac sources and capacitors and is shown in Fig. 59.29.

(c) Principal Operating Characteristics

1. Input Resistance

The input resistance of the *CC* stage is given by the parallel combination of R_S and $r_{in(base)}$. Now, $r_{in(base)}$ is the input resistance looking into the base. It is found to be equal to $(1 + \beta)(r_e' + r_L) \cong \beta r_L$ *i.e.* β times the ac load seen by the emitter.

$$\therefore \quad r_{in} \text{ or } r_{in(stage)} = R_B \| r_{in(base)}$$
$$= R_B \| \beta (r_e' + r_L) \cong \beta (r_e' + r_L)$$
—when R_B is very large
$$\cong \beta r_L \quad \text{— when } r_e' << r_L$$
$$= \beta R_E \quad \text{— if } R_L = 0$$

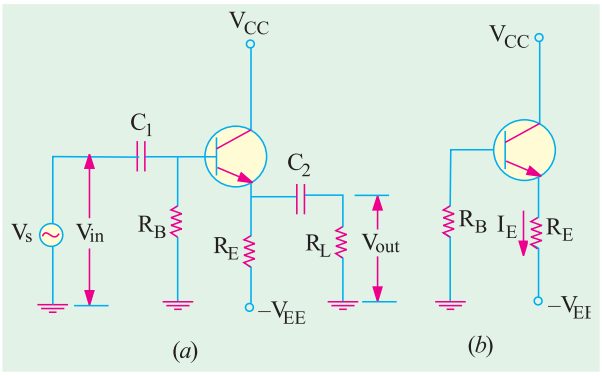


Fig. 59.28

2. AC Load Resistance : $r_L = R_E \| R_L$

It is the ac output resistance as seen by the emitter (and not the one when looking into the emitter).

3. Current Gain. $A_i \quad \frac{i_e}{i_b} \quad (1 \quad)$

4. Voltage Gain. $V_{in} = i_b . r_{in} = i_b . \beta (r_e' + r_L)$:
$V_{out} = i.e . r_L = \beta i_b . r_L$

$$A_v \quad \frac{V_{out}}{V_{in}} \quad \frac{i_b \, r_L}{i_b . \ (r_e' \quad r_L)} \quad \frac{r_L}{r_e' \quad r_L} \quad \frac{r_L}{r_L} \quad 1$$
— if $r_e' << r_L$

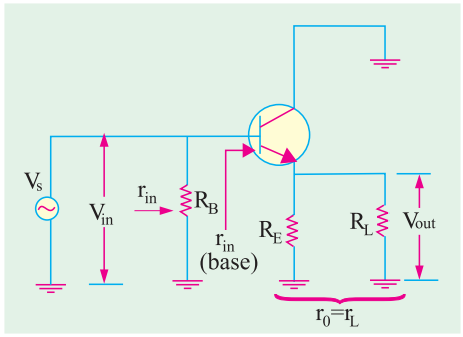


Fig. 59.29

It means that output signal from emitter has the **same magnitude** as the input signal at the base.

5. Power Gain $A_p = A_v . A_i = 1 \times (1 + \beta) = (1 + \beta) \cong \beta$

$$G_p = 10 \log_{10} A_p \text{ dB}$$

It would be noted from above that main usefulness of the emitter follower is to **step up the impedance level** *i.e.* it transforms load impedance to a much higher value. It does not increase the signal voltage. Hence, primary application of emitter follower or *CC* stage is as an impedance matching device. It offers a higher impedance at the input terminals *i.e.* r_{in} and a low output impedance (r_L) ***– something opposite of typical basic transistor amplifier.***

Another point worth keeping in mind is that the above formulas are not exact because we have used ideal transistor approximations. However, these formulas do help in rapidly grasping the essential features of an emitter follower. Moreover, they are adequate for preliminary analysis and design.

Example. 59.11. *For the emitter follower shown in Fig. 59.30, find*
(i) r_{in} *or* $r_{in(stage)}$ ***(ii)*** r_L ***(iii)*** A_v ***(iv)*** A_p. *Take transistor* $\beta = 50$.
(Electronics, Delhi Univ.)

Solution. $I_E = 20/20 = 1$ mA

$r_e' = 25/1 = 25\ \Omega$

$r_L = R_E \| R_L = 20 \| 5 = 4$ K

$r_{in(base)} = \beta(r_e' + r_L) \cong \beta\, r_L = 50 \times 4 = 200$ K

$\therefore$ (r_e' of 25 Ω has been neglected as compared to r_L of 4 K)

(i) $r_{in(stage)}$ or $r_{in} = R_B \| r_{in(base)}$

$= 400\text{ K} \| 200\text{ K}$

$= \mathbf{133.3\ K}$

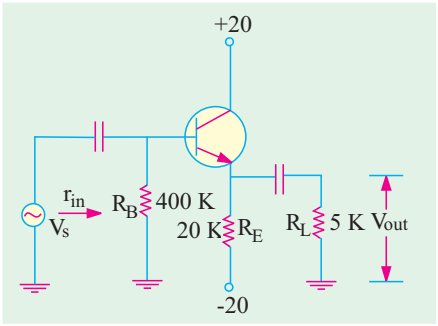


Fig. 59.30

(ii) $r_L = 20\text{ K} \| 5\text{ K} = \mathbf{4\ K}$

(iii) $A_v \cong 1$ since r_e' is negligible as compared to r_L. If it is not, then A_v could be even less than 1.

(iv) $A_p = A_i \,.\, A_v$ $\beta \times 1 = \beta = 50$; $G_p = 10 \log_{10} 50 = \mathbf{17\ dB}$

Example 59.12. *For the beta-stabilized emitter follower circuit of Fig. 59.31, find*
(i) $r_{in(base)}$ ***(ii)*** r_{in} or $r_{in(stage)}$ ***(iii)*** r_L and ***(iv)*** A_v. Take transistor $\beta = 100$.
(Electronics Technology, Mysore Univ.)

Solution. $V_2 \quad V_{CC} \dfrac{R_2}{R_1 \quad R_2} \quad 20 \quad \dfrac{10}{40} \quad 5$ V

$I_E \quad \dfrac{V_2}{R_E} \quad \dfrac{5}{5} \quad 1$ mA

$r_e' = 25/1 = 25\ \Omega$

$r_L = 5 \| 5 = 2.5$ K

(i) $r_{in(base)} = \beta\,(r_e' + r_L)$

$= 100\,(25 + 2500) = \mathbf{250\ K}$

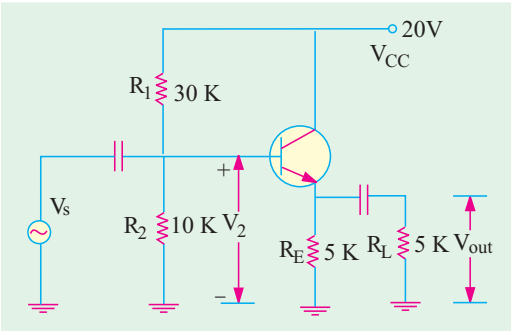


Fig. 59.31

(ii) $r_{in(base)} = R_1 \| R_2 \|$

$r_{in(base)} = 30\text{K} \| 10\text{ K} \| 250\text{ K} \cong 8$ K

(iii) $r_L = 5\text{ K} \| 5\text{ K} = \mathbf{2.5\ K}$

(iv) $A_p = A_i \,.\, A_v \cong 1 \times \beta = 100$; $G_p = 100 \log_{10} 100 = \mathbf{20dB}$

Example 59.13. *For the CE circuit of Fig. 59.32, find*
(i) r_{in} or $r_{in(stage)}$ ***(ii)*** A_v ***(iii)*** A_p. Take transistor $\beta = 200$

Solution . $I_E = 20/20 = 1$ mA $r_e' = 25/1 = 25\ \Omega$

$r_L = 20\text{ K} \| 50\ \Omega = 50\ \Omega$

$r_{in(base)} = \beta\,(r_e' + r_L) = 200\,(25 + 50) = 15$ K;

(i) r_{in} or $r_{in(stage)} = R_B \| r_{in(base)}$

$= 100\text{ K} \| 15\text{ K} = \mathbf{13\ K}$

(ii) Since r_e' is not negligible as compared to r_L, we will use the expression

$A_v \quad \dfrac{r_L}{r_e' \quad r_L} \quad \dfrac{50}{25 \quad 50} \quad \mathbf{0.667}$

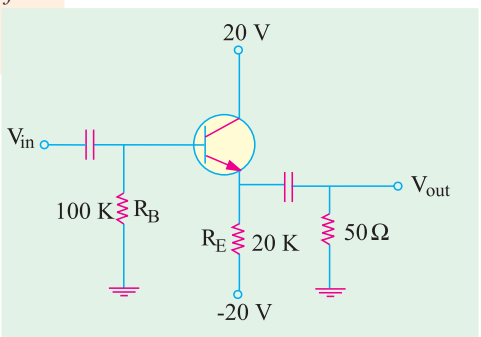


Fig. 59.32

(*iii*) $A_p = A_v \,.\, A_i \;\; = 0.667 \times 200 = \mathbf{133.3}$

59.9. Small-signal Low-frequency Model or Representation

Although many transistor representations or models have been suggested and widely used, the equivalent *T*-model is the easiest to understand because, in this representation, component parts retain their identity in all configurations leading to rapid appreciation of a given network.

59.10. T-Model

Such models are not in common use today because they do not take into account any gain between input and output.

(*a*) CB Circuit

In Fig. 59.33 is shown the low-frequency *T*-equivalent circuit of a transistor connected in *CB* configuration. It utilizes *T*- or *r*-parameters. All these parameters are ac parameters and are measured under open-circuit conditions. Here, r_e represents the ac resistance of the forward-biased emitter-base junction. Its value is

$$r_e \quad \frac{25\text{ mV}}{I_E} \quad \text{— for Ge}$$

$$\frac{50\text{ mV}}{I_E} \quad \text{— for Si}$$

This resistance is fairly small and depends on I_E. Also, r_e represents the ac resistance of the reverse-biased *C/B* junction. It is of the order of a few MΩ. Finally, r_b represents the resistance of the base region which is common to both junctions. Its value depends on the degree of doping. Usually, r_b is larger than r_e but much smaller than r_c.

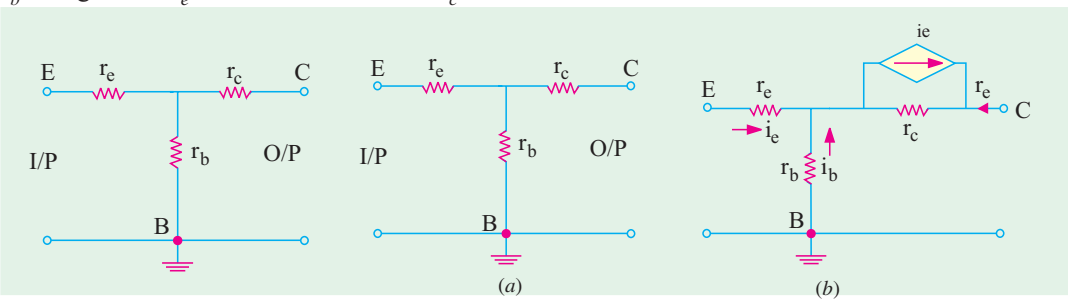


Fig. 59.33 Fig. 59.34

However, circuit shown in Fig. 59.34 (*a*) is not complete because it does not illustrate the forward current transfer ratio. Since current in the output of a transistor depends on the current at the input, a dependent current-generator in parallel with r_c must be included as shown in Fig. 59.34 (*b*). As it is the usual practice, all currents are shown flowing inwards even though some of them may actually be flowing in the opposite direction.

The current generator may be replaced by a voltage generator with the help of Thevenin's theorem as shown in Fig. 59.35 (*a*). In that case, the *T*-equivalent circuit becomes as shown in Fig. 59.35 (*b*). The generator has a voltage of $\alpha i_e r_c = r_{in} i_e$ where $r_{in} = \alpha r_e$.

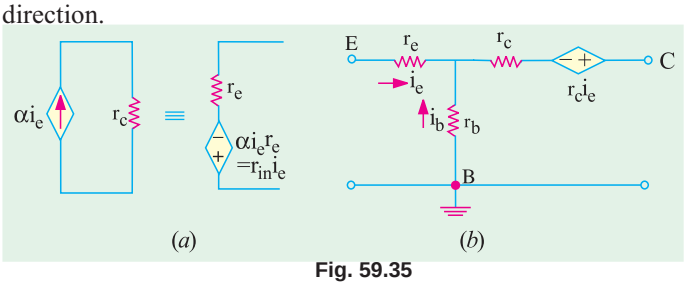


Fig. 59.35

Typical values of different parameters are :

$r_e = 25$ to $50\ \Omega$; $r_b = 100$ to $1000\ \Omega$; $r_c = 1\ \text{M}\Omega$

(*b*) CE Circuit

The *T*-equivalent circuit for such a configuration is shown in Fig. 59.36. Whereas circuit shown in Fig. 59.36 contains a parallel current-generator that shown in Fig. 59.37 contains a series-voltage generator.

(*c*) CC Circuit

The *T*-equivalent circuit for such a configuration is shown in Fig. 59.38.

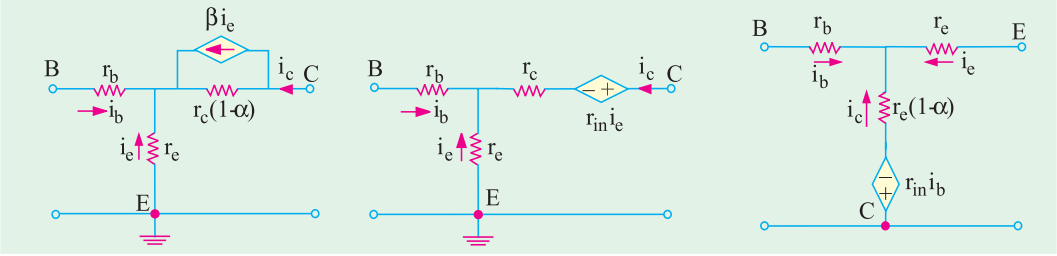


Fig. 59.36 Fig. 59.37 Fig. 59.38

59.11. Formulas for T-Equivalent of a CB Circuit

In Fig. 59.39 is shown a small-signal low-frequency *T*-equivalent circuit for *CB* configuration. The ac input signal source has a resistance of R_S and voltage of V_s.

Of course, dc biasing circuit has been omitted and only ac equivalent shown. The approximate expressions for input and output resistance and voltage and current gains as derived by applying *KVL* to the input and output loops are given below without derivation :

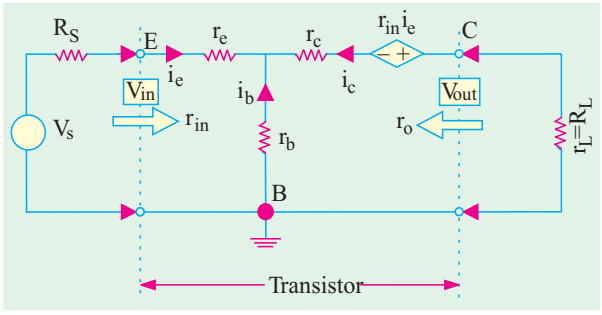


Fig. 59.39

1. $r_{in} \quad r_e \quad r_b(1 \quad) \quad r_e \quad \dfrac{r_b}{(1 \quad)} \quad r_e \quad r_b \dfrac{r_c(1 \quad) \quad R_L}{r_c \quad r_L}$

2. $r_o \quad r_c \quad \dfrac{r_b r_c}{r_e \quad r_b \quad R_S}$

3. $A_i = \alpha$

4. $A_v \quad \dfrac{v_{out}}{v_{in}} \quad \dfrac{R_L}{r_e \quad r_b(1 \quad)}$

   $A_v \quad \dfrac{V_{out}}{V_{in}} \quad \dfrac{R_L}{(r_e \quad R_s) \quad r_b(1 \quad)} \quad \dfrac{R_L}{(r_e \quad R_s)}$

5. $A_P = A_i \,.\, A_v$

   $\dfrac{^2 R_L}{(r_e \quad r_b(1 \quad)}$ — no source resistance

   $\dfrac{^2 R_L}{(r_e \quad R_s) \quad r_b(1 \quad)}$ — with source resistance

   $G_p = 10 \log_{10} A_p$

59.12. Formulas for T-Equivalent of a CE Circuit

The low-frequency small-signal *T*-equivalent circuit for such a configuration is shown in Fig. 59.40.

The approximate expressions for various resistances and gains as found by applying *KVL* to the input and output loops are given below :

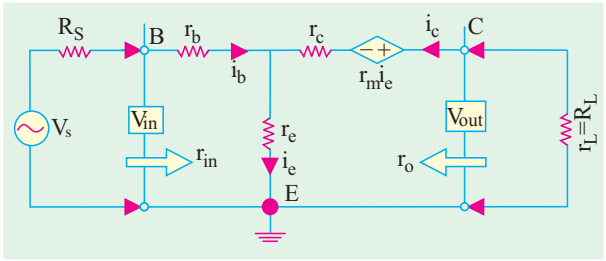


Fig. 59.40

1. $r_{in} \quad r_b \quad \frac{r_e}{((1 \quad)} = r_b + (1+\beta) r_e$

2. $r_o \quad r_e (1 \quad) \quad \frac{r_e r_e}{r_b \; R_L \; r_e \; R_S}$

3. $A_i = \beta \quad \overline{(1 \quad)}$

4. $A_v \quad \frac{V_{out}}{V_{in}} \quad \frac{}{r_e \quad r_b (1 \quad)}$

$A_{vs} \quad \frac{V_{out}}{V_s} \quad \frac{R_L}{r_e \quad (r_b \quad R_s)(1 \quad)}$

5. $A_p = A_v . A_i \quad \frac{^2 R_L}{(1 \quad)[(r_e \quad r_b (1 \quad)]}$ $\quad \frac{^2 R_L}{(1 \quad)[(r_e \quad (r_b \quad R_S)(1 \quad)]}$

59.13. Formulas for T-Equivalent of a CC Circuit

The low-frequency *T*-equivalent circuit for such a configuration is shown in Fig. 59.41.

The approximate expressions for various resistances and gains are given below :

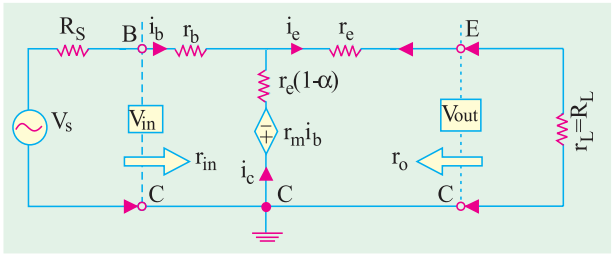


Fig. 59.41

1. $r_{in} \quad \frac{R_L}{1} \quad (1 \quad) R_L$

2. $r_v = r_e + (1-\alpha)(r_b + R_S)$

$r_e \quad \frac{(r_b \quad R_S)}{(1 \quad)}$

3. $A_i \quad \frac{1}{(1 \quad)} \quad (1 \quad)$

4. $A_n = 1$

5. $A_p \quad \frac{1}{(1 \quad)} \quad (1 \quad) \quad G_p = 10 A_p \text{ dB}$

Example 59.14. *A junction transistor has $r_e = 50\ \Omega$, $r_b = 1$ K, $r_c = 1$M and $\alpha = 0.98$. It is used in common-base circuit with a load resistance of 10 K. Calculate the current , voltage and power gains and the input resistance.* **(Applied Electronics, Punjab Univ. 1993)**

Solution. **(*i*)** $A_i = \alpha = 0.98$

(*ii*) $A_v \quad \frac{R_L}{r_e \quad r_b (1 \quad)} \quad \frac{0.98 \quad 10,000}{50 \quad 1000 (1-0.98)}$ **140** **(*iii*)** $A_p = A_v . A_i = 140 \times 0.98 = 137$

(*iv*) $r_{in} = r_e + r_b (1-\alpha) = 50 + 1000\ (1 - 0.98) =$ **70 Ω**

Example 59.15. *A P-N-P junction transistor is used as a voltage amplifier in the grounded-base circuit with a load resistance being 300 K and the internal resistance of the original source being 200 Ω.*

Derive an expression for the voltage gain of the amplifier and calculate its magnitude if the transistor T-network parameters are : $r_e = 18\ \Omega$, $r_b = 700\ \Omega$, $r_c = 1$ M and $r_m = 976$ K.

(Applied Electronics and Circuits, Grad. I.E.T.E.)

Solution. Here, $r_m = \alpha (r_c + r_b) \cong \alpha r_c \quad \therefore \alpha = r_m / r_c = 976 \text{ K} / 1000 \text{ K} = 0.976$

$$A_v \quad \frac{V_2}{V_1} \quad \frac{R_L}{r_c \quad r(1 \quad)} \quad \frac{0.976 \quad 300 \quad 10^3}{18 \quad 700(1-0.976)} \quad \textbf{8,410}$$

$$A_{vg} \quad \frac{V_2}{Vg} \quad \frac{R_L}{R_G \quad r_c \quad r_b(1 \quad)} \quad \frac{0.976 \quad 300 \quad 10^3}{200 \quad 18 \quad 700(1-0.976)} \quad \textbf{1247}$$

Example 59.16. *Calculate the input and output resistance, overall current, voltage and power gains for a CE connected transistor having following r-parameters :*

$r_b = 30\ \Omega$, $r_e = 400\ \Omega$, $r_c = 0.75$ M, $\alpha = 0.95$, $R_L = 10$ K *and* $R_S = 400\ \Omega$

Also, calculate the power gain in decibels.

(Electronics Technology, Bangalore Univ. 1999)

Solution. ***(i)*** $r_{in} \quad r_b \quad \frac{r_e}{(1- \quad)} \quad 30 \quad \frac{400}{(1-0.95)} \quad$ **8030 Ω**

(ii) $r_o \quad r_e(1 \quad) \quad \frac{r_c r_e}{r_b \quad r_c \quad R_S}$

$$750,000(1-0.95) \quad \frac{0.95 \quad 750,000 \quad 400}{(30 \quad 400 \quad 400)} \quad 380,870 \quad \textbf{0.38 M}$$

(iii) $A_i \quad \frac{}{1} \quad \frac{0.95}{(1 \quad 0.95)} \quad 19$

(iv) $A \quad \frac{R_L}{r_e \quad (r_b \quad R_s)(1 \quad)} \quad \frac{0.95 \times 10 \times 10^3}{400 + (30 + 400) \times 0.05} = \textbf{22.5}$

(v) $A_p = A_v \cdot A_i = 22.5 \times 19 = \textbf{427.5}$

Power gain in decibels, $G_p = 10 \log_{10} 427.5 = \textbf{26.3 dB}$

59.14. What are h-parameters ?

These are ***four*** constants which describe the behaviour of a two-port linear network. A linear network is one in which resistance, inductances and capacitances remain fixed when voltage across them is changed.

Consider an unknown linear network contained in a black box as shown in Fig. 59.42. As a matter of convention, currents flowing into the box are taken positive whereas those flowing out of it are considered negative.

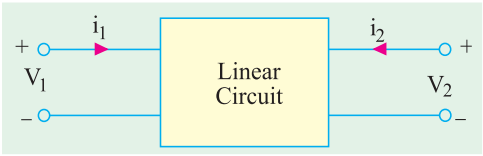


Fig. 59.42

Similarly, voltages are positive from the upper to the lower terminals and negative the other way around.

The electrical behaviour of such a circuit can be described with the help of four hybrid parameters or constants designated as $h_{11}, h_{12}, h_{21}, h_{22}$. In this type of double-number subscripts, it is implied that ***the first variable is always divided by the other.*** The subscript 1 refers to quantities on the input side and 2 to the quantities on the ***output side.*** The letter '*h*' has come from the word ***hybrid*** which means ***mixture*** of distinctly different items. These constants are hybrid because they have different units.

Out of the four *h*-parameters, two are found by short-circuiting the output terminals 2-2 and the other two by open-circuiting the input terminals 1-1 of the circuit.

(*a*) Finding h_{11} and h_{21} from Short-Circuit Test

As shown in Fig. 59.43, the output terminals have been shorted so that $v_2 = 0$, because no voltage can exist on a short. The linear circuit within the box is driven by an input voltage v_1. It

produces an input current i_1 whose magnitude depends on the type of circuit within the box.

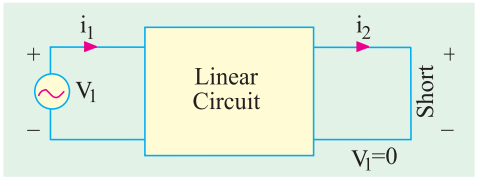


Fig. 59.43

h_{11} $\frac{V_1}{i_2}$ — output shorted

h_{21} $\frac{i_2}{i_1}$ — output shorted

These two constants are known as **forward** parameters.

The constant h_{11} represents input impedance with output shorted and has the unit of ohm. The constant h_{21} represents current gain of the circuit with output shorted and has no unit since it is the ratio of two similar quantities.

The voltages and currents of such a two-port network are related by the following sets of equations or *V/I* relations.

$V_1 = h_{11}\, i_1 + h_{12}\, V_2$ ...(*i*)

$i_2 = h_{21}\, i_1 + h_{22}\, V_2$ ... (*ii*)

Here, the h_s are constants for a given circuit but change if the circuit is changed. Knowlege of parameters enables us to find the voltages and currents with the help of the above two equations.

(*b*) Finding h_{12} and h_{22} from Open-circuit Test

As shown in Fig. 59.44, the input terminals are open so that $i_1 = 0$ but there does appear a voltage v_1 across them. The output terminals are driven by an ac voltage v_2 which sets up current i_2.

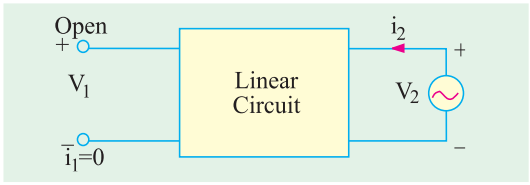


Fig. 59.44

h_{12} $\frac{V_1}{V_2}$ — input open

h_{22} $\frac{i_2}{V_2}$ — input open

As seen, h_{12} represents voltage gain (not forward gain which is v_2 / v_1). Hence, it has no units. The constant h_{22} represents admittance (which is reverse of resistance) and has the unit of mho or Siemens, *S*. It is actually the admittance looking into the output terminals with input terminals open. Generally, these two constants are also referred to as ***reverse parameters.***

Summary of *h*-parameters

h_{11} = input impedance] with output shorted

h_{21} = forward current gain]

h_{12} = reverse voltage gain] with input open

h_{22} = output admittance

59.15. Input Impedance of a Two Port Network

Consider the two-port linear network shown in Fig. 59.45 which has a load resistance r_L across its output terminals. The voltage source V_1 on the input side drives the circuit and sets up current i_1. As seen, $Z_{in} = V_1 / i_1$. Substituting the value of V_1 from Eq. (*i*) of Art. 59.14, we get

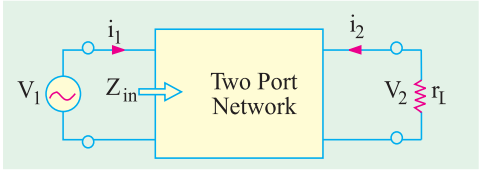


Fig. 59.45

Z_{in} $\frac{V_1}{i_1}$ $\frac{h_{11} i_1 \quad h_{12} v_2}{i_1}$

h_{11} $\frac{h_{12} V_2}{i_1}$ ... (*i*)

As seen from Fig. 59.45 above $i_2 = -v_2 / R_L$*
Substituting this value of I_2 in Eq. (*ii*) of Art 59.14, we have

$$\frac{-v_2}{R_L} \quad h_{22}\, i_1 \quad h_{22} v_2 \qquad \text{or} \qquad \frac{v_2}{i_1} \quad \frac{h_{21}}{h_{22} \quad 1/R_L}$$

Substituting this value in Eq. (*i*) above, we have

$$Z_{in} \quad h_{11} \quad \frac{h_{12}.h_{21}}{h_{22} \quad 1/R_L}$$

59.16. Voltage Gain of a Two Port Network

The voltage gain of such a circuit (Fig. 59.45) is $A_v = v_2 / v_1$. Now $v_1 = i_1 . Z_{in}$. Hence, $A_v = v_2 / i_1 . Z_{in}$.
Substituting the value of v_2 / i_1 as found earlier in Art. 59.15, we get

$$A_v \quad \frac{h_{21}}{Z_{in}(h_{22} \quad 1/R_L)}$$

Example. 59.17. *Find the h-parameters of the circuit shown in Fig. 59.46 (a).*
Solution. First of all, let us find the forward parameters *hu* and h_{21}. For that purpose, a short is put across the output terminals as shown in Fig. 59.46 (*b*).

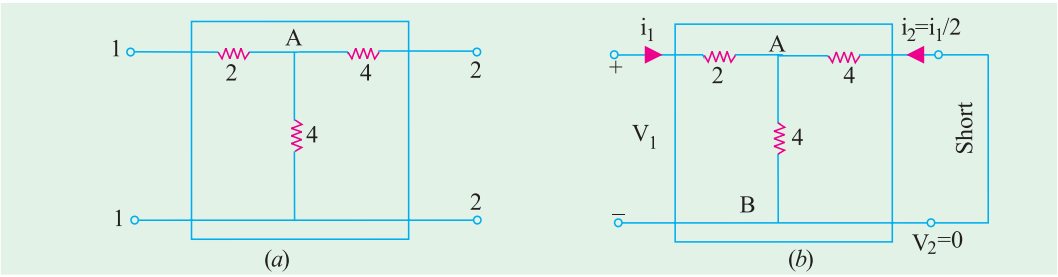


Fig. 59.46

(*i*) The input impedance of the network as viewed from input terminals is
$h_{11} = 2 + 4 \parallel 4 = \mathbf{4\ \Omega}$

(*ii*) As seen from Fig. 9.46 (*b*), input current i_1 divides into two equal parts at point A. The output current $i_2 = -i_1/2$ (negative sign has been taken because actually it is flowing out of the box).

$$\therefore \quad h_{21} \quad \frac{i_2}{i_1} \quad \frac{-i_1/2}{i_1} \quad \frac{1}{2} \quad \mathbf{-0.5}$$

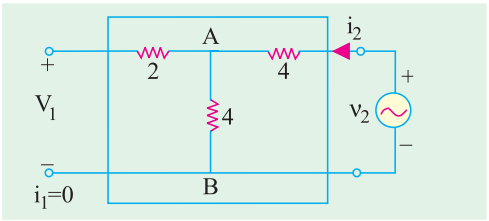


Fig. 59.47

(*iii*) Now, for finding reverse parameters, we will keep input terminals open and apply v_2 across output terminals as shown in Fig. 59.47. It will produce a current i_2 which will produce equal drops across the two 4 Ω resistors. The voltage which appears across input terminals as v_1 is the drop across the vertical 4 Ω resistor connected at point A. Hence, $v_1 = v_2 / 2$.

$$\therefore \quad h_{12} \quad \frac{v_1}{v_2} \quad \frac{v_2/2}{v_2} \quad \mathbf{0.5}$$

* The negative sign is used because actual load current is opposite to that shown in figure.

The input impedance of the network when viewed from output terminals with input terminals open is = 4 + 4 = 8 Ω.

$\therefore \quad h_{22} = 1/8 =$ **0.125 Siemens (*i.e.* mho)**

Hence, for the network shown in Fig. 59.46 (*a*), the *h*-parameters are as under :

$h_{11} = 4\ \Omega \qquad h_{21} = -0.5 \qquad h_{12} = 0.5$ and $\qquad h_{22} = 0.125$ S

59.17. The h-parameter Notation for Transistors

While using *h*-parameters for transistor circuits, their numerical subscripts are replaced by the first letters for defining them.

$h_{11} = h_i$ = input impedance
$h_{21} = h_f$ = forward current gain
] output shorted

$h_{12} = h_r$ = reverse voltage gain
$h_{22} = h_0$ = output admittance
] input open

A second subscript is added to the above parameters to indicate the particular configuration. For example, for *CE* connection, the four parameters are written as :

$h_{ie} \qquad h_{fe} \qquad h_{re}$ and h_{oe}

Similarly, for *CB* connection, these are written as h_{ib}, h_{fb}, h_{rb}, h_{op} and for *CC* connection as h_{ic}, h_{fc}, h_{rc} and h_{oc}.

59.18. The h-parameters of an Ideal Transistor

As stated earlier, every linear circuit has a set of parameters associated with it, which fully describe its behaviour. When small ac signals are involved, a transistor behaves like a linear device because its output ac signal ***varies directly as the input signal.*** Hence, for small ac signals, each transistor has its own characteristic set of *h*-parameters or constants.

The *h*-parameters depend on a number of factors such as

1. transistor type **2.** configuration **3.** operating point **4.** temperature **5.** frequency

These *h*-parameters can be found experimentally or graphically. The parameters h_i and h_r are determined from input characteristics of the *CE* transistor whereas h_f and h_0 are found from output characteristics.

59.19. The h-parameters of an Ideal CB Transistor

In Fig. 59.48 (*a*), a *CB*-connected transistor has been shown connected in a black box. Fig., 59.48 (*b*) gives its equivalent circuit. It should be noted that no external biassing resistors or any signal source has been shown connected to the transistor.

(*i*) Forward Parameters

The two forward h-parameters can be found from the circuit of Fig. 59.49 (*a*) where a short has been put across the output. The input impedance is simply r_e.

$\therefore \qquad h_{ib} = r_e$

The output current equals the input current *i.e.* Since it flows out of the box, it is taken as negative. The forward current gain is

$$h_{fb} \quad \frac{-i_e}{i_e} \quad 1$$

(It also called the ac α of the *CB* circuit.)

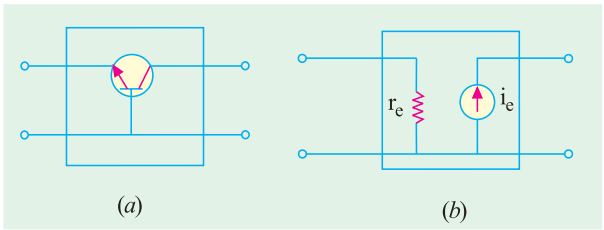


Fig. 59.48

(*ii*) Reverse Parameters

The two reverse parameters can be found from the circuit diagram of Fig. 59.49 (*b*). When input terminals are open, there can be no ac emitter current. It means that ac current source (inside the box)

has a value of zero and so appears as an 'open'. Because of this open, no voltage can appear across input terminals, however, large v_2 may be. Hence, $v_1 = 0$.

$$\therefore \quad h_{rb} \quad \frac{v_1}{v_2} \quad \frac{0}{v_2} \quad 0$$

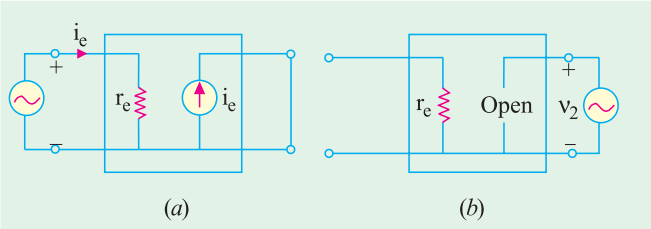


Fig. 59.49

Similarly, the impedance, looking into the output terminals is infinite. Consequently, its admittance (= 1/∞) is zero.

$$\therefore \quad h_{ob} = 0$$

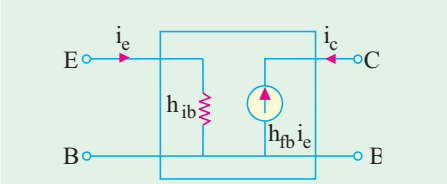


Fig. 59.50

Summary

The four *h*-parameters of an ideal transistor connected in *CB* configuration are

$$h_{ib} = r_e \; ; \quad h_{fb} = -1, \quad h_{rb} = 0 \; ; \quad h_{ob} = 0$$

The equivalent hybrid circuit is shown in Fig. 59.50.

Note. In an actual transistor, h_{rb} and h_{ob} are not zero but have some finite value though extremely small (ranging from 10^{-} to 10^{-6}).

In reality, output impedance is not infinity but very high so that h_{ob} is extremely small. Similarly, there is some amount of feedback between the output and the input circuits (even when open) though it is very small. Hence, h_{rb} is very small.

59.20. The *h*-parameters of an ideal CE Transistor

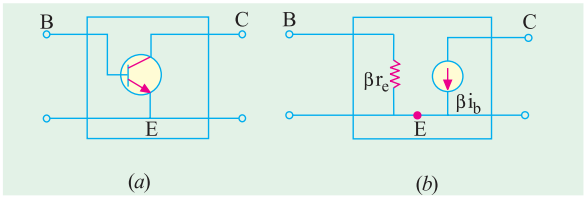


Fig. 59.51

Fig. 59.51 (*a*) shows a *CE*-connected ideal transistor contained in a black box whereas 59.51 (*b*) shows its ac equivalent circuit in terms of its β and resistance values.

(*a*) Forward Parameters

The two forward *h*-parameters can be found from the circuit of Fig. 59.52 (*a*) where output has been shorted. Obviously, the input impedance is simply βr_e.

$$\therefore \quad h_{ie} = \beta r_e$$

The forward current gain is given by

$$h_{fe} \quad \frac{i_2}{i_1} \quad \frac{i_b}{i_b}$$

(It is also called the ac beta of the *CE* circuit)

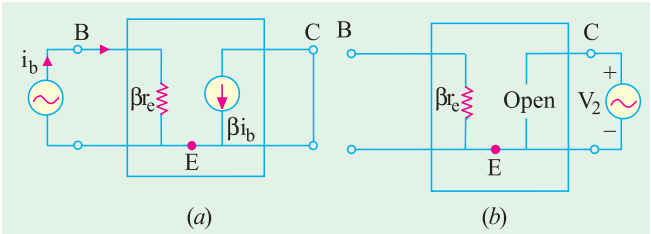


Fig. 59.52

(*b*) Reverse Parameters

These can be found by reference to the circuit of Fig. 59.52 (*b*) where input terminals are open but output terminals are driven by an are voltage source v_2. With input terminals open, there can be no base current so that $i_b = 0$. If $i_b = 0$, then collector current source has zero value and looks like an open. Hence, there can be no v_1 due to this open.

$$\therefore \qquad h_{re} \quad \frac{v_1}{v_2} \quad \frac{0}{v_2} \quad 0$$

Again, the impedance looking into the output terminals is infinite so that conductance is zero.

$$\therefore \qquad h_{oe} = 0$$

Hence, the four *h*-parameters of an ideal transistor connected in *CE* configuration are :

$$h_{ie} = \beta r_e; \quad h_{fe} = \beta, \quad hr_e = 0; \quad h_{oe} = 0.$$

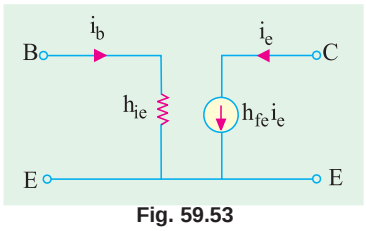


Fig. 59.53

The hybrid equivalent circuit of such a transistor is shown in Fig. 59.53.

Note. In practice, h_{re} and h_{oe} are not exactly zero but quite small for the same reasons as given in the Note to Art 59.19.

59.21. Approximate Hybrid Equivalent Circuits

So far, we did not take into account the following two factors which exist in an actual transistor (as opposed to an ideal one).

(*i*) because of the transistor's non-unilateral behaviour, there is a 'feedback' of the output voltage into the input voltage. This feedback is represented by a voltage-controlled generator $h_r v_2$ as shown in Fig. 59.54 and 59.55.

By definition, an ideal amplifier is one which responds only to signals applied to its input terminals. It should not do the reverse *i.e.* reproduce at the input any portion of the ac signal applied at the output. Such an ideal one-way device is called a unilateral device. A real transistor cannot be unilateral because of unaviodable interaction between its input and output circuits (after all, it consists of a single piece of a crystal). Therefore, not only its output responds to its input but, to a lesser degree, its input also responds to its output.

(*ii*) even when input circuits is open, there is some effective value of conductanece when looking into the transistor from its output terminals. It is represented by h_0.

We will now draw ***low-frequency small-signal*** hybrid equivalent circuits after taking into account the 'feedback' voltage generator and output admittance.

(*a*) Hybrid CB Circuit

In Fig. 59.54 (*a*) is shown an *NPN* transistor connected in *CB* configuration. Its ac equivalent circuit employing *h*-parameters is shown in Fig. 59.54 (*b*). The *V/I* relationships are given by the following two equations.

$$v_{eb} = h_{ib}\, i_e + h_{rb}\, v_{cb}$$
$$i_e = h_{fb}\, i_e + h_{ob}\, v_{eb}$$

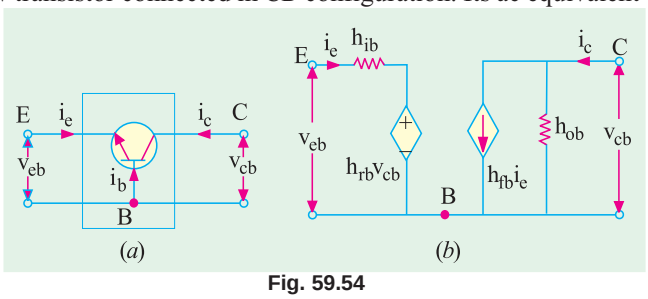


Fig. 59.54

These equations are self-evident because applied voltage across input terminals must equal the drop over h_{ib} and the generator voltage. Similarly, current i_c in the output terminals must equal the sum of two branch currents.

As per current convention stated earlier (Art. 59.14), collector i_e is shown flowing ***inwards*** though actually this current flows ***outwards*** as shown by the arrow inside the ac current source. Similarly, ac voltage polarities have been taken by considering upper terminal positive and lower one as negative (please remember that the dc biasing rule of Art. 59.12 does not apply here).

It may be noted that no external dc biasing resistor or ac voltage sources have been connected to the equivalent circuit as yet.

Incidentally, it may be noted that the ac equivalent circuit contains a Thevenin's circuit in the input and a Norton's circuit in the output. It is all the more a reason to call it a hybrid equivalent circuit.

(*b*) Hybrid CE Circuit

The hybrid equivalent of the transistor alone when connected in *CE* configuration is shown in Fig. 59.55 (*b*). Its *V/I* characteristics are described by the following two equations.

$$v_{be} = h_{ie}\, i_b + h_{re}\, v_{ec}$$
$$i_e = h_{fb}\, i_b + h_{oe}\, v_{ec}$$

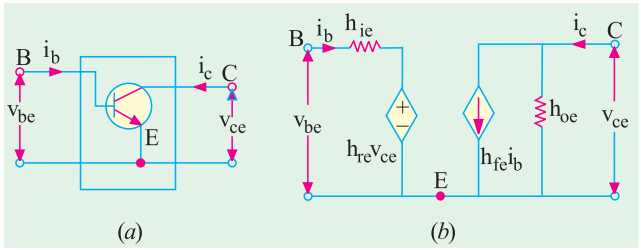


Fig. 59.55

We may connect signal input source across its input terminals and load resistance across output terminals (Art. 59.22).

(*c*) Hybrid CC Circuit

The hybrid equivalent of a transistor alone when connected in *CC* configuration is shown in Fig. 59.56 (*b*). Its *V/I* characteristics are defined by the following two equations :

$$v_{be} = h_{ie}\, i_b + h_{re}\, v_{ec}$$
$$i_e = h_{fe}\, i_b + h_{oc}\, v_{ce}$$

We may connect signal input source across ioutput terminals *BC* and load resistance across output terminals *EC* to get a *CC* amplifier.

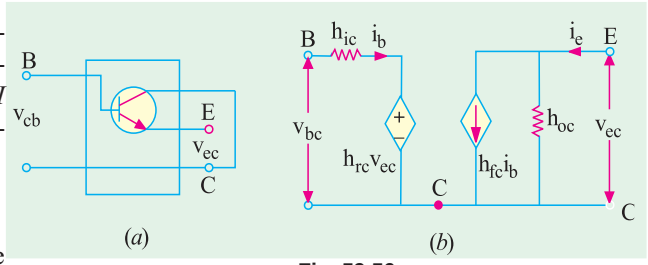


Fig. 59.56

59.22. Transistor Amplifier Formulae Using *h*-parameters

As shown in Fig. 59.57, if we add a signal source across input terminals 1-1 of a transistor and a load resistor across its output terminals 2–2, we get a small-signal, low-frequency hybrid model of a transistor amplifier. It is valid for all the three configurations and holds good for all types of load whether a resistance of an impedance. We will now find expressions for its gains and impedances.

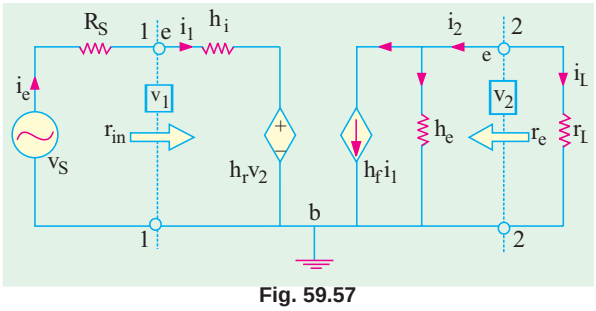


Fig. 59.57

Before undertaking the above derivations, let us consider different components in the hybrid model of Fig. 59.57. The input resistance looks like a resistance (h_i) in series with a voltage generator ($h_r v_2$). This generator represents the voltage feed-back from the output to the input circuit. It is known as voltage-controlled generator because its value is determined by v_2 (as h_r is a dimensionless constant). The output circuit also has two components **(*i*)** h_0 component which represents the conductance as seen from output terminals and **(*ii*)** the current-controlled generator ($h_f i_1$) which simulates the transistor's ability to amplify. The parameter h_f is a dimensionless constant.

The above model can be described mathematically by using the following two equations :

Input Circuit

v_1 = sum of voltage drops from *a* to *b*

$= h_i\, i_1 + h_r\, v_2$ ...(*i*)

Output Circuit

i_2 = sum of currents leaving junction *c*

$= h_f i_1 + h_0\, v_2$ ...(*ii*)

Now, $v_2 = -i_2 r_L$. Substituting this value in Eq. (*ii*) above, we have

$$i_2 = h_f i_1 - h_0 i_2 r_L \quad \text{... (iii)}$$

Eq. (*i*) and (*iii*) can now be used to find various gains of a transistor.

(*i*) Current Gain

It is given by $A_i = i_2/i_1$

Dividing both sides of Eq. (*iii*) by i_1, we get

$$\frac{i_2}{i_1} \quad h_f \quad h_0 \frac{i_2}{i_1} r_L \qquad \text{or} \qquad A_i = h_f - h_0 A_i r_L$$

$$\therefore \qquad A_i \quad \frac{h_f}{1 \quad h_0 r_L}$$

If $r_L = 0$ or $h_0 r_L << 1$, then $A_i = h_f$

Current Gain Taking R_S into Account

The source current is not the transistor input current because i_1 partly flows along R_S and partly along r_{in}.

To illustrate this point, consider the Norton's equivalent of the source (Fig. 59.58). The overall current gain A_{is} is given by

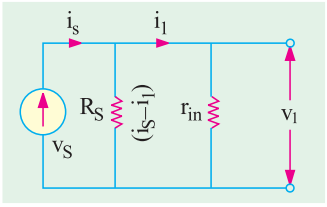


Fig. 59.58

$$A_{is} \quad \frac{i_2}{i_s} \quad \frac{i_2}{i_1} \quad \frac{i_1}{i_s} \qquad A_i \frac{i_1}{i_s}$$

As seen from Fig. 9.58

$$i_1 \quad \frac{i_s \; R_s}{r_{in} \quad R_s} \qquad \text{or} \qquad \frac{i_1}{i_s} \quad \frac{R_s}{r_{in} \quad R_s}$$

$$\therefore \qquad A_{is} = \quad A_i \,.\, R_S / (r_{in} + R_S)$$

(*ii*) Input Impedance

It is defined as the resistance when looking into the amplifier from its input terminals. Hence, $r_{in} = v_1/i_1$.

From Eq. (*i*) above, we have

$$r_{in} \quad \frac{1}{i_1} \quad \frac{h_i i_1 \quad h_r \; 2}{i_1} \qquad h_i \qquad h_r \,.\, \frac{2}{i_1}$$

Substituting the value of $v_2 = -i_2 r_L = -A_i \, i_1 \, r_L$, we get

$$r_{in} \quad h_i \quad h_i A_i r_L \quad h_i \quad \frac{h_f h_r r_L}{1 \quad h_0 r_L} \qquad h_i \quad \frac{h_f h_r}{h_0 \quad 1/r_L}$$

$$\frac{h_i \qquad h r_L}{1 \quad h_0 r_L} \qquad \text{where} \quad \Delta h = h_i h_0 - h_f h_r$$

$$\cong \; h_i \qquad \text{– if } h_r \text{ or } r_L \text{ is very small.}$$

It is seen that r_{in} depends on r_L *i.e.* ac resistance of the load across output terminals of the transistor.

(*iii*) Voltage Gain

$A_v = v_2 / v_1$. It is also known as the internal voltage gain of the transistor. It is different from $A_{vs} = v_2 / v_s$ which is the gain from the source to the output terminals and is known as stage gain or overall gain.

As seen from above, $v_2 = -A_i i_2 r_L$ and $v_1 = i_1 r_{in}$

$$\therefore \quad A_v \quad \frac{2}{1} \quad \frac{i_2 A_i r_L}{i_L r_{in}} \qquad A_i \frac{r_L}{r_{in}}$$

$$\frac{h_f}{1 \quad h_0 r_L} \qquad \frac{r_L (1 \quad h_0 . r_L)}{h_i \qquad h . r_L} \qquad \frac{h_f . r_L}{h_i \qquad h . r_L}$$

$$\frac{h_f r_L}{h_i (1 \quad h_0 r_L) \quad h_f h_r r_L} \qquad h_f \cdot \frac{r_L}{h_i}$$

Overall voltage gain is

$$A_{vs} \quad \frac{v_2}{s} \quad \frac{v_2}{v_1} \quad \frac{v_i}{v_s} \quad A_v \frac{v_i}{v_s}$$

Now, v_s drops over series combination of R_S and r_{in}.

Drop across r_{in} constitutes v_1. Hence, $v_1 = v_s \times r_{in}/(R_S + R_{in})$

$$\therefore \qquad \frac{v_1}{v_s} \quad \frac{r_{in}}{(R_S \quad r_{in})} \qquad \text{or} \qquad A_{vs} \quad A_v \frac{r_{in}}{(R_5 \quad r_{in})}$$

As seen, if $R_S = 0$, $A_{vs} = A_v$

Value of A_{vs} may also be obtained by adding R_S to h_i in the expression for A_v.

(*iv*) Output Impedance

It is defined as $r_0 \quad \frac{v_2}{i_2}\Big|_{v_s \quad 0}$ or $g_0 \quad \frac{i_2}{v_2}$

Dividing both sides of Eq. (*ii*) by v_2, we get

$$g_0 \quad h_f \cdot \frac{i_2}{v_2} \quad h_0 \qquad \text{..... (iv)}$$

Taking $v_s = 0$ and then applying KVL to the input circuit in Fig. 59.57, we get

$-i_1(h_i + R_S) - h_r v_2 = 0$ or $i_1/v_2 = -h_r/(h_i + R_S)$

Substituting this value in Eq. (*iv*) above, we have

$$g_0 \quad h_0 \cdot \frac{h_f h_r}{(h_i \quad R_S)} \quad \frac{h_o R_S \quad h}{(h_i \quad R_S)} \qquad \therefore \quad r_o \quad \frac{1}{g_o} \quad \frac{h_i \quad R_S}{h_o R_S \quad h}$$

It is seen that rin depends on r_L whereas ro depends on R_S.

If R_S is very large (*i.e.* circuit is driven by a current source) or h_r is negligible, then $r_o \cong 1/h_o$.

(*v*) Power Gain

$$A_p \quad \frac{P_2}{P_1} \quad \frac{v_2 i_2}{v_1 i_1} \quad A_v A_i \quad A_i^2 \frac{r_L}{r_{in}}$$

The above formulae are summarized below :

(*i*)

$$A_i \quad \frac{h_f}{1 \quad h_o r_L} \quad h_f$$

$$A_i \quad \frac{h_f R_S}{(1 \quad h_o r_L)(r_{in} \quad R_S)} \quad \frac{h_o R_S}{(r_{in} \quad R_S)}$$

(*ii*)

$$r_{in} \quad h_i \quad \frac{h_f h_r r_L}{1 \quad h_o r_L} \quad h_i$$

(*iii*)

$$A_v \quad \frac{h_f h_r}{h_i \quad h . r_L} \quad \frac{h_f r_L}{h_i} ; \; A_{vs} \quad \frac{h_f r_L}{(h_i \quad R_S)}$$

(*iv*) $r_o \quad \frac{h_i \quad R_S}{h_o R_S \quad h} \quad \frac{1 \quad h_i / R_S}{h_o \quad h / R_S} \quad \frac{1}{h_o}$

59.23. Typical Values of Transistor *h*-parameters

In the table below are given typical values for each parameter for the broad range of transistors available today in each of the three configurations.

Parameter	*CB*	*CE*	*CC*
h_i	25 Ω	1 K	1 K
h_r	3×10^{-4}	2.5×10^{-4}	$\cong 1$
h_f	–0.98	50	–50
h_o	0.5×10^{-6}S	25×10^{-6}S	25×10^{-6}S

59.24. Approximate Hybrid Formulas

The approximate hybrid formulas for the three connections are listed below. These are applicable when h_o and h_r are very small and R_S is very large. The given values refer to transistor terminals. The values of $r_{in(stage)}$ or r_{in}' and $r_{o(stage)}$ will depend on biasing resistors and load resistance respectively.

Item	*CE*	*CB*	*CC*
r_{in}	h_{ie}	h_{ib}	$h_{ic} + h_{fe}R_L$
ro	$\frac{1}{h_{oc}}$	$\frac{1}{h_{oB}}$	$\frac{h_{ie}}{h_{fc}}$
A_i	$h_{fe} = \beta$	$-h_{fb} \cong 1$	$-h_{fe} \cong \beta$
A_v	$\frac{h_{ie}R_C}{h_{is}}$	$\frac{f_{fb}}{h_{ib}}R_C$	1

59.25. Common Emitter *h*-parameter Analysis

The *h*-parameter equivalent of the *CE* circuit of Fig. 59.59 (*a*) is shown in Fig. 59.59 (*b*). In Fig. 59.59 (*a*), no emitter resistor has been connected.

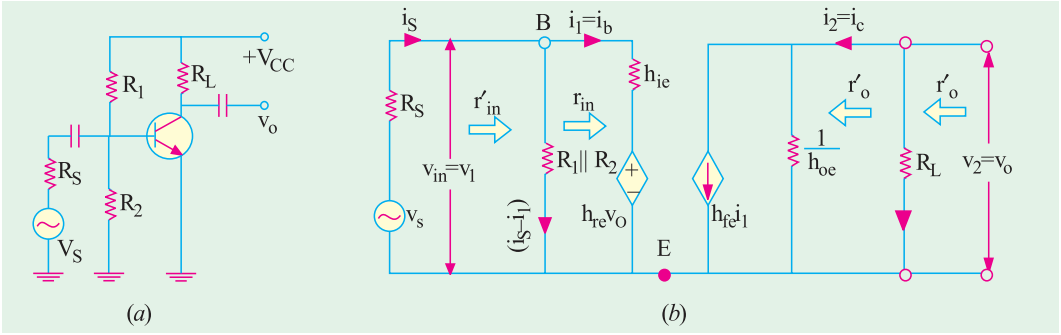


Fig. 59.59

However, Fig. 59.60 shows the *CE* circuit with an emitter resistor R_E.

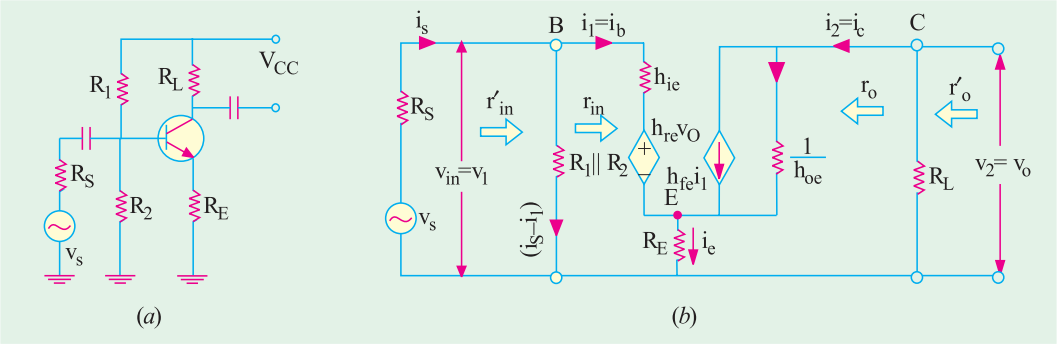


Fig. 59.60

We will now derive expressions for voltage and current gains for both these circuits.

1. Input Impedance

When looking into the base-emitter terminals of the transistor, h_{ie} is in series with h_{re} no. For a *CE* circuit, h_{re} is very small so that $h_{re}\, v_0$ is negligible as compared to the drop over h_{ie}. Hence, $r_{in} \cong h_{ie}$.

Now, consider the circuit of Fig. 59.60. Again ignoring $h_{re} v_o$, we have

$$v_1 = h_{ie} i_b + i_e R_E = h_{ie}\, i_b + (i_b + i)\, R_E$$
$$= h_{ie} i_b + i_b R_E + h_{fe}\, i_b R_E \qquad ((\ \ i_c \ \ h_{fe} i_b)$$
$$= i_b [h_{ie} + R_E (1 + h_{fe})]$$

$$\therefore \qquad r_{in} = r_{in(base)} = \frac{v_1}{i_1} \quad \frac{v_1}{i_b} \quad h_{ie} \quad (1 \quad h_{fe})\, R_E \ *$$

$$r_{in} \text{ or } r_{in(base)} = R_1 \| R_2 \| r_{in(base)}$$

2. Output Impedance

Looking back into the collector and emitter terminals of the transistor in Fig. 59.59 (*b*), $r_o \cong 1/h_{oe}$.

As seen, r_o' or $r_{o(stage)} = r_o \| R_L = (1/h_o) \| r_L \qquad (\quad r_L \quad R_L)$

Since $1/h_{oe}$ is typically 1 M or so and R_L is usually much smaller, $r_o' \cong R_L = r_L$

3. Voltage Gain

$$A_v \quad \frac{v_2}{v_1} \quad \frac{v_o}{v_{in}} \qquad \text{– Fig. 9.59 } (b)$$

Now, $\quad v_o = - i_c R_L \quad$ and $\qquad v_{in} \cong i_b h_{ie}$

$$\therefore \qquad A_v \quad \frac{i_s R_L}{i_b h_{ie}} \qquad \frac{i_c}{i_b} \cdot \frac{R_L}{h_{ie}} \qquad \frac{h_f R_L}{h_{ie}}$$

Now, consider Fig. 59.60 (*b*). Ignoring $h_{re}\, v_o$, we have from the input loop of the circuit

$$v_{in} \ = \ i_b [h_{ie} + R_E (1 + h_{fe})] \qquad \text{–proved above}$$

$$\therefore \qquad A_v \quad \frac{v_o}{v_{in}} \quad \frac{i_c R_L}{i_b [h_{ie} \quad R_E (1 \quad h_{fe})]} \quad \frac{h_{fe} R_L}{h_{fe} \quad (1 \quad h_{fe}) R_E}$$

$$\frac{R_L}{R_E} \qquad \text{— if } (1 + h_{fe})\, R_E >> h_{ie}$$

* The above result could also be obtained by applying β-rule (Art. 9..24)

In general, ———————————

4. Current Gain

$$A_i \quad \frac{i_2}{i_1} \quad \frac{h_{fe}}{1 \quad h_{oe} r_L} \quad h_{fe} \qquad \text{— if } h_{oe} r_L << 1$$

$$A_{is} \quad \frac{h_{fe} \cdot R_1 \| R_2}{r_{in} \quad R_1 \| R_2}$$

5. Power Gain

$$A_p = A_v \times A_i$$

59.26. Common Collector *h*-parameter Analysis

The *CC* transistor circuit and its *h*-parameter equivalent are shown in Fig. 59.61.

One can make quick approximations of *CC* gains and impedance if one remembers that $h_{re} = 1$ *i.e.* all of v_o is fed back to the input (Art. 59.23).

1. Input Impedance

$$v_{in} = i_b h_{ic} + h_{rc} v_o = i_b h_{ic} + v_o = i_b h_{ic} + i_e R_L$$
$$= i_b h_{ic} + h_{fe} i_b R_L = i_b (h_{ic} + h_{fe} R_L) \qquad i_c = h_{fe} i_b$$

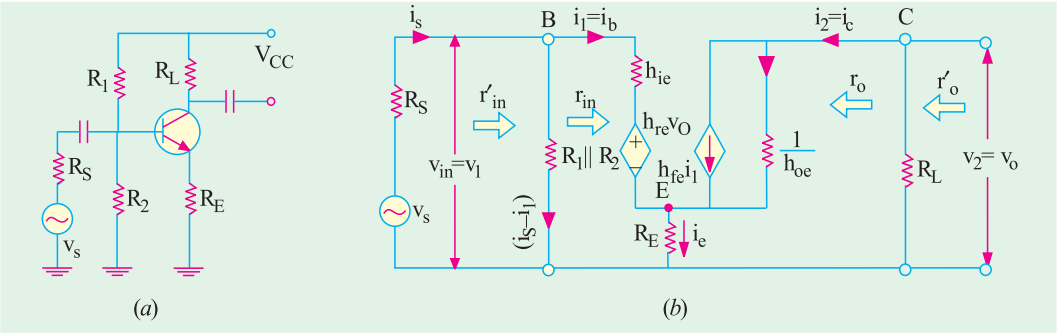


Fig. 59.61

$$\therefore \qquad r_{in} \quad \frac{v_{in}}{i_b} \quad h_{ic} \quad h_{fe} R_L$$

As seen, $r_{in(stage)} = r_{in(base)} \| R_1 \| R_2 = r_{in(base)} \| R_B$ where $R_B = R_1 \| R_2$

2. Output Impedance

$$r_o \quad \frac{v_2}{i_2}\Big|_{v_s \quad 0} \qquad \frac{v_o}{i_c}\Big|_{v_s \quad 0}$$

Now, $i_e \cong i_c = h_{fe} i_b = h_{fc} i_1$

Since $v_s = 0$, i_b is produced by $h_{rc} v_o = v_o$

Hence, considering the input circuit loop, we get

$$i_b \quad \frac{v_0}{h_{ic} \quad (R_S \| R_1 \| R_2)} \quad \frac{v_0}{h_{ic} \quad R_S \| R_B}$$

$$i_c \quad h_{fc} i_b \quad \frac{h_{fc} v_o}{h_{ic} \quad (R_S \| R_B)} \qquad \text{where } R_B = R_1 \| R_2$$

$$\therefore \quad r_o \quad \frac{V_o}{i_e} \quad \frac{h_{ic} \quad (R_S \| R_1 \| R_2)}{h_{fe}}$$

Also, r_o' or $r_{o(stage)} = r_o \| R_L$

3. Voltage Gain

$$A_v \quad \frac{v_2}{v_1} \quad \frac{v_o}{v_{in}}$$

Now, $v_o = i_e R_L = h_{fe} i_b R_L$ and $i_b = (v_{in} - v_o) / h_{ic}$

$$v_o \quad \frac{h_{fe} R_L}{h_{ie}} (v_{in} \quad v_o) \quad \text{or} \quad v_o \quad 1 \quad \frac{h_{fe} R_L}{h_{ic}} \quad \frac{h_{fe} R_L v_{in}}{h_{ic}}$$

$$\therefore \quad A_v \quad \frac{v_o}{v_{in}} \quad \frac{h_{fe} R_L / h_{ic}}{1 \quad h_{fe} R_L / h_{ic}} \quad 1$$

4. Current Gain

$$A_i \quad \frac{i_2}{i_1} \quad \frac{i_e}{i_b} \quad h_{fe} \; ; \; A_{is} \quad \frac{h_{fe} R_B}{r_{in} \quad R_B}$$

where $R_B = R_1 \| R_2$

59.27. Conversion of *h*-parameters

Transistor data sheets generally specify the transistor in terms of its *h*-parameters for *CB* connection *i.e.* h_{ib}, h_{fb}, h_{rb} and h_{ob}. If we want to use the transistor in *CE* or *CC* configuration we will have to convert the given set of parameters into a set of *CE* or *CC* parameters. Approximate conversion formulae are tabulated over leaf :

Table No. 59.2

From CB to CE	*From CE to CB*	*From CE to CC*
$h_{ie} \quad \frac{h_{ib}}{1 \quad h_{fb}}$	$h_{ie} \quad \frac{h_{ie}}{1 \quad h_{fe}}$	$h_{ic} = h_{ic}$
$h_{oe} \quad \frac{h_{ob}}{1 \quad h_{fb}}$	$h_{ob} \quad \frac{h_{oe}}{1 \quad h_{fe}}$	$h_{oc} = h_{oe}$
$h_{fe} \quad \frac{h_{fb}}{1 \quad h_{fb}}$	$h_{fb} \quad \frac{h_{fe}}{1 \quad h_{fe}}$	$h_{fe} = -(1 + h_{fe})$
$h_{re} \quad \frac{h_{ib} h_{ob}}{1 \quad h_{fb}} \quad h_{rb}$	$h_{rb} \quad \frac{h_{ie} h_{oe}}{1 \quad h_{fe}} \quad h_{re}$	$h_{re} = 1 - h_{re} \cong 1$

Example. 59.18. *A transistor used in CB circuit has the following set of parameters.*
$h_{ib} = 36\ \Omega$, $h_{fb} = 0.98$, $h_{rb} = 5 \times 10^{-4}$, $h_{ob} = 10^{-6}$ *Siemens*
With $R_S = 2$ *K and* $R_C = 10$ *K, calculate* **(i)** $r_{in(base)}$ **(ii)** r_{out} **(iii)** A_i and **(iv)** A_v.
(Applied Electronics-I; Punjab Univ. 1991)

Solution. Approximate Values

(i) $r_{in} = h_{ib} = 36\ \Omega$ **(ii)** $r_o \quad \frac{1}{h_{ob}} \quad \frac{1}{10^{\ 6}}$ **1M**

(iii) $A_i = h_{fb} = -0.98$ **(iv)** $A_v \quad \frac{h_{fb}}{h_{ib}} \quad R_C \quad \frac{0.98}{36} \quad 10\text{K}$ **272**

More Accurate Values

(i) $r_{in(base)} \quad h_{ib} \quad \dfrac{h_{rb}h_{fb}}{h_{ob} \quad 1/r_L}$ –Art 9.22

$= 36 \quad \dfrac{0.98 \quad 5 \quad 10^{4}}{10^{6} \quad 1/10 \quad 10^{3}}$ ($\therefore \quad r_L = R_C$ since there is no R_L)

$= 36 + 4.9 =$ **40.9 Ω**

It is the input resistance at transistor terminals.

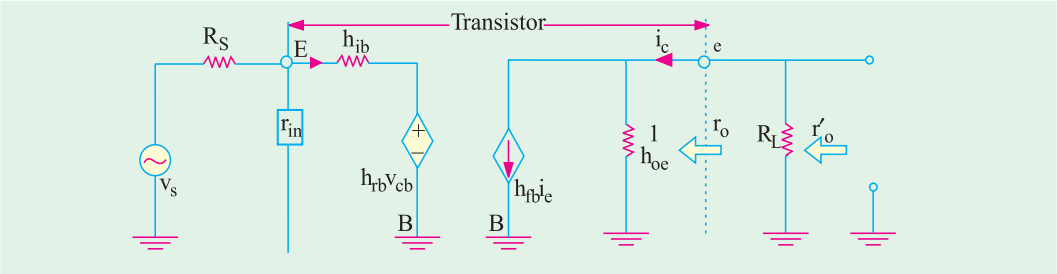


Fig. 59.62

(ii) $r_o \quad \dfrac{h_{ib} \quad R_s}{h_o(h_{ib} \quad R_S) \quad h_{fb}.h_{rb}} \quad \dfrac{36 \quad 2000}{10^{6}(36 \quad 2000) \quad (0.98) \quad 5 \quad 10^{4}}$ **0.8 M**

It is the output resistance at transistor terminals.

(iii) $A_i \quad \dfrac{h_{fb}}{1 \quad h_{ob}r_L} \quad \dfrac{0.98}{1 \quad 10^{6} \quad 10^{4}}$ **–0.97**

(iv) $A_v \quad \dfrac{h_{fb}r_L}{h_{ib}(1 \quad h_{ob}r_L) \quad h_{fb}\,h_{rb}.r_L}$ ($r_L \quad R_L$)

Here, ac load $r_L = R_L = 10$ K = 104 Ω

$\therefore \quad A_v \quad \dfrac{(0.98) \quad 10^{4}}{36(1 \quad 10^{6} \quad 10^{4}) \quad (0.98) \quad 5 \quad 10^{4} \quad 10^{4}}$ **249**

Example 59.19. *A transistor used in CE connection (Fig. 59.63) has the following set of h-parameters : $h_{ir} = 1$ K, $h_{fe} = 100$, $h_{re} = 5 \times 10^{-4}$ and $h_{oc} = 2 \times 10^{-5}$ S. With $R_S = 2$ K and $R_C = 5$ K, determine*
(i) r_{in} **(ii)** r_o **(iii)** A_i *and* **(iv)** A_v

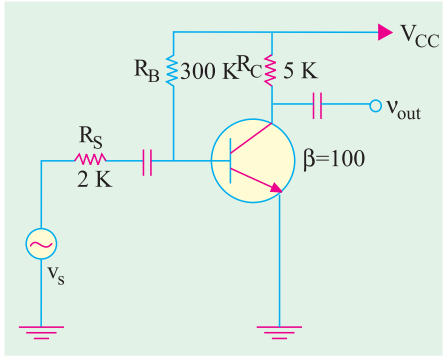


Fig. 59.63

Solution.

(i) $r_{in} \quad h_{ie} \quad \dfrac{h_{fb}r_L}{h_o \quad 1/r_L}$

$= 1000 \quad \dfrac{5 \quad 10^{4} \quad 100}{2 \quad 10^{5} \quad 1/5 \quad 10^{3}}$

= 723 Ω

(ii) $r_o \quad \dfrac{h_{ie} \quad R_s}{(h_{ie} \quad R_s)h_{oe} \quad h_{fe}.h_{re}} \quad \dfrac{1000 \quad 2000}{(1000 \quad 2000) \quad 10^{5} \quad 100 \quad 5 \quad 10^{4}}$

$= 30{,}000\Omega =$ **0.03 M**

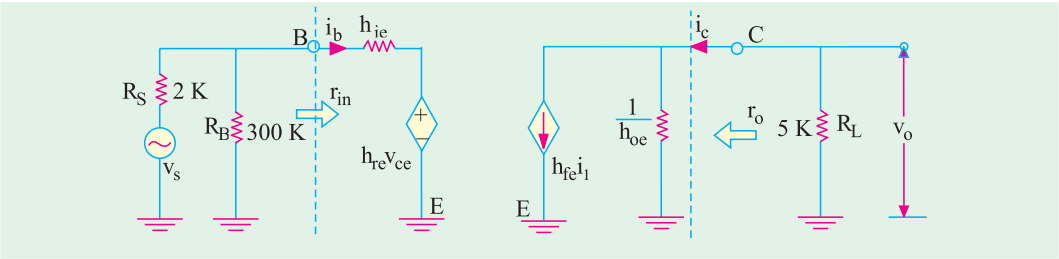


Fig. 59.64

(iii) $A_i \quad \frac{h_{fe}}{1 \quad h_{oe}r_L} \quad \frac{100}{1 \quad 2 \quad 10^{5} \quad 5 \quad 10^{3}}$ **91**

(iv) $A_i \quad \frac{h_{fe}r_L}{(h_{ie} \quad R_s)\,(1 \quad h_{oe}.r_L) \quad h_{fe}h_{re}r_L}$

$$\frac{100 \quad 5 \quad 10^{3}}{(100 \quad 2000)(1 \quad 2 \quad 10^{5} \quad 5 \quad 10^{3}) \quad 100 \quad 5 \quad 10^{4} \quad 5 \quad 10^{3}}$$ **–164**

The negative sign indicates that there is 180° phase shift between the input and output ac signals. Obviously, it is the overall (or circuit) voltage gain and not the voltage gain of the transistor alone.

Example 59.20. *In the CE circuit shown in Fig. 59.65, the transistor parameters are :*
h_{ie}=2 K, h_{fe}=100, $h_{re} = 5 \times 10^{-4}$, $h_{oe} = 2 \times 10^{-5}$ S
Calculate **(i)** $r_{in(base)}$, **(ii)** $r_{in(stage)}$, **(iii)** r_o, **(iv)** $r_{o(stage)}$, **(v)** A_i and **(vi)** A_v.
(Electronics–1, Karnataka Univ.)

Solution. The hybrid equivalent circuit is shown in Fig. 59.66. We will use the approximate formulas given in Art. 59.24.

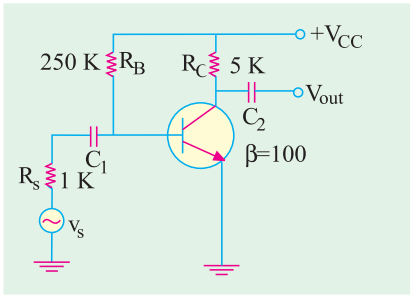


Fig. 59.65

(i) $r_{in(base)} = h_{ie}$ **= 2K**

(ii) $r_{in(stage)} = 2\text{ K} \parallel 250\text{ K} =$ **1.98 K**

(iii) $r_o = 1 / h_{oe} = 1/2 \times 10^{-5} =$ **50 K**

It is the output impedance of the transistor only.

(iv) $r_{o(stage)} = r_o' = 50\text{ K} \parallel 5\text{ K} =$ **4.54 K**

The impedance takes into account the collector load.

(v) $A_i \cong h_{fe} = 100$

(vi) $A_v \quad \frac{h_{fe}r_L}{h_{ie}} \quad \frac{100 \quad 5}{2}$ **–250**

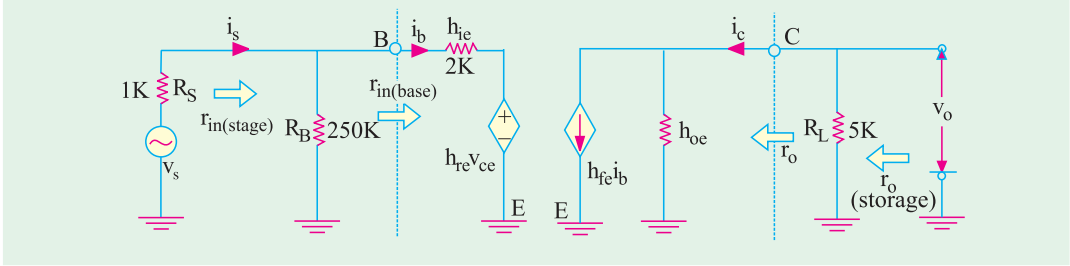


Fig. 59.66

Example 59.21. *Determine the various gains of the circuit of Fig. 59.67 if an emitter resistance of 0.5 K is included in the circuit.* **(Applied Electronics, Punjab Univ. 1991)**

Solution. The *CE* circuit with R_E included is shown in Fig. 59.47. Different performance characteristics of the circuit are as under :

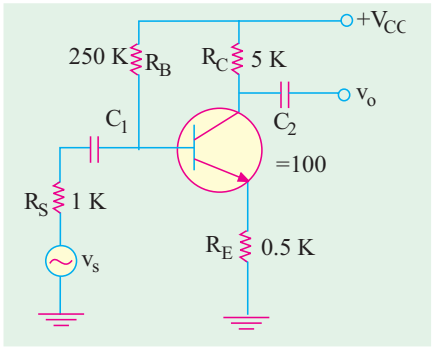


Fig. 59.67

(i) $r_{in(base)} \quad h_{ie} \quad (1 \quad) R_E$

$= 2 + 101 \times 0.5 = \mathbf{52.5\ K}$

(ii) $r_{in(stage)} = R_B \| r_{in} = 250 \| 52.5$

$= \mathbf{43.3K}$

(iii) $r_o = 1/h_{oe} = \mathbf{50\ K}$

(iv) $r_{o(stage)} = r_o' = 50\ \text{K} \| 5\ \text{K}$

$= \mathbf{4.54\ K}$

(v) $A_v = h_{fe} = \mathbf{100}$

(vi) $A_v \quad \dfrac{h_{fe} r_L}{h_{ie} \quad (1 \quad) R_E} \quad \dfrac{100 \quad 5}{2 \quad 50.5} \quad \mathbf{-9.5}$

The value is reduced from 250 to 9.5.

Example 59.22. *The transistor of Fig. 59.68 has the following set of h-parameters :*

$h_{ie} = 2\text{K}, h_{fe} = 50, h_{rs} = 4 \times 10^{-4}, h_{oe} = 25 \times 10^{-6}$ *Siemens*

Determine **(i)** $r_{in(base)}$ **(ii)** $r_{in(base)}$ **(iii)** r_o **(iv)** $r_{o(stage)}$ and **(v)** A_v.

(Electronics-I, Patna Univ. 1991)

Solution. we will use the formula derived in Art. 59.25.

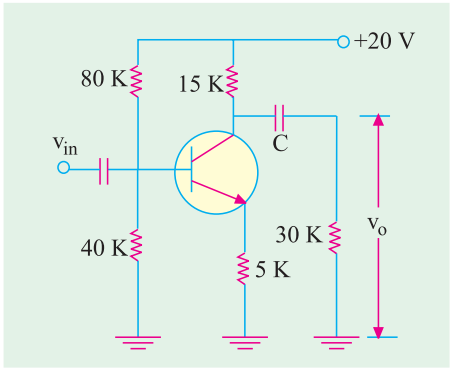


Fig. 59.68

(i) $r_{in(base)} \quad h_{ie} \quad (1 \quad h_{fe)R_E} \quad 2 \quad (1 \quad 50) \quad 5$

$= \mathbf{257\ K}$

(ii) $r_{in(stage)} = r_{in(base)} \| R_1 \| R_2 = 257 \| 80 \| 40$

$= \mathbf{24\ K}$

(iii) $r_o = 1/h_{oe} = 1/25 \times 10^{-6} = \mathbf{40\ K}$

(iv) $r_{o(stage)} = r_o \| r_L$ where $r_L = 15\ \text{K} \| 30\ \text{K}$

$= 10\ \text{K} = 40\ \text{K} \| 10 = \mathbf{8\ K}$

(v) $A_v \quad \dfrac{h_{fe} R_L}{h_{ie} \quad (1 \quad h_{fe}) R_E}$

$\dfrac{50 \quad 10}{257} \quad \mathbf{-2.9}$

Example 59.23. *Draw a hybrid small-signal model for a transistor in CE configuration.*

A single source with an open circuit voltage of 1 mV and internal impedance of 600 W is connected to the input of transistor AC 125 in CE configuration. The small-signal parameters measured at $V_{CB} = -5$ *V and* $I_E = 2$ *mA, are as follows :*

$h_{ie} = 1.7\ \text{K}, h_{re} = 6.5 \times 10^{-4}, h_{fe} = 125, h_{oe} = 80\ \mu\text{S}.$

Calculate the input and output impedances and signal amplification for a load of 5.6 K. Also, find the value of the signal voltage at the output. **(Electronics-I, Gwalior Univ.)**

Solution. $\Delta h = 1.7 \times 10^3 \times 80 \times 10^{-6} - 125 \times 6.5 \times 10^{-4} = 0.055$

(i) $r_{in} \quad \dfrac{h_{ie} \quad h.R_L}{1 \quad h_{oe} R_L} \quad \dfrac{1700 \quad 5.6 \quad 10^3 \quad 55 \quad 10^{\ 3}}{1 \quad 80 \quad 10^{\ 6} \quad 5.6 \quad 10^3}$ –Art. 59.22

(ii) $r_o \quad \dfrac{h_{ie} \quad R_S}{h_{oe} R_S \quad h} \quad \dfrac{1700 \quad 600}{80 \quad 10^{6} \quad 600 \quad 0.055} \quad \dfrac{2300}{0.103}$ **22.3 K**

(iii) $A_v \quad \dfrac{h_{fe} R_L}{h_{ie} \quad R_L . \quad h} \quad \dfrac{125 \quad 5600}{1700 \quad 308}$ **–348.6**

The negative sign merely indicates that there is phase reversal of 180° between the output and input voltages.

Now, 1 mV signal voltage is divided between r_{in} and R_S. The input voltage v_{in} is that which drops over $r_{in} = 1 \times r_{in}/(r_{in} + R_S) = 1 \times 1387/(1387 + 600) = 0.698$ mV

$\therefore$ output voltage = $-348.6 \times 0.698 =$ **–243.3 mV.**

Example 59.24. *The transistor of Fig. 59.69 has the following set of h-parameters :*

$h_{ie} = 2$ K, $h_{fe} = 100$, $h_{re} = 5 \times 10^{-4}$, $h_{oe} = 2.5 \times 10^{-5}$ S

Find the voltage gain and the ac impedance of the stage.

(Electronics-II, Bombay Univ. 1992)

Solution. Using somewhat exact formulas given in Art. 59.22, we have

$$r_{in(base)} \quad h_{ie} \quad \frac{h_{fe} h_{re}}{h_{oe} \quad 1/r_L}$$

Now, collector load

$$r_L = 10 \text{ K} \parallel 30 \text{ K} = 7.5 \text{ K}$$

$$\therefore \quad r_{in(base)} \quad 200 \quad \frac{100 \quad 5 \quad 10^{4}}{2.5 \quad 10^{5} \quad 1/7.5 \quad 10^{3}}$$

$= 2000 - 316 =$ **1684 Ω**

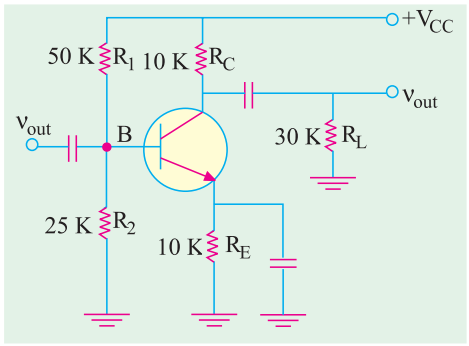


Fig. 59.69

The ac input impedance of the stage *i.e.* impedance when looking into point *B* is

$r_{in(base)} = r_{in(base)} \parallel R_1 \parallel R_2 = 1.684 \parallel 50 \parallel 25 =$ **1.53K**

$$A_v \quad \frac{h_{fe}}{r_{in(base)} (h_{oe} \quad 1/r_L)} \qquad \text{Now,} \quad r_L = 10 \text{ K} \parallel 30 \text{ K} = 7.5 \text{ K}$$

$$\therefore \quad A_v \quad \frac{100}{0184 \ (2.5 \quad 10^{5} \quad 1/7500)}$$ **– 375**

Obviously, R_E does not come into the ac picture because it is ac grounded by the bypass capacitor.

Example 59.25. *In the CC circuit of Fig. 59.70, the transistor parameter are $h_{ic} = 2K$ and $h_{fc} = 100$. Calculate the circuit input and output impedance and voltage, current and power gains.*

(Electronic Technology, Bangalore Univ.)

Solution. $r_{in} \cong h_{ic} + h_{fe} R_L = 2 + 100 \times 5 =$ **502 K**

$$r_{in(stage)} = R_1 \parallel R_2 \parallel r_{in}$$

$$= 10 \parallel 10 \parallel 502 = \textbf{4.95 K}$$

$$r_o \quad \frac{h_{ie} \quad (R_s \parallel R_1 \parallel R_2)}{h_{fe}}$$

$$\frac{2 \quad (1 \parallel 10 \parallel 100)}{100}$$ **28.3 Ω**

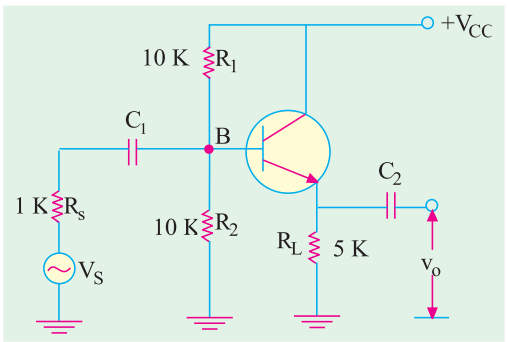


Fig. 59.70

$r_{o(stage)} = r_o \| R_L = 28.3\ \Omega \| 5K$

$= \mathbf{28.1\Omega}$

$A_v \cong 1$ and $A_i = h_{fe} = 100$

Example 59.26. *A transistor with $h_{ie} = 1.5$ K and $h_{fe} = 75$ is used in an emitter follower circuit where resistances R_1 and R_2 are used for normal biasing. Calculate* ***(i)*** *A_i* ***(ii)*** *r_{in}* ***(iii)*** *r_o and* ***(iv)*** *A_v if $R_E = 860\ \Omega$, $R_1 \| R_2 = 20K$ and $R_S = 1K$.* **(Electronic Engg.; Indore Univ. 1992)**

Solution. Let us first convert the given *CE* values of *h*-parameters into their equivalent *CC* values with the help of Table No. 59.2. It is seen that $h_{ic} = h_{ie} = 1.5$ K and $h_{fc} = (1 + h_{fe}) = 76$.

(i) $A_i = h_{fe} = 76$ ***(ii)*** $r_{in} = h_{ic} + h_{fc} R_L = 1.5 + 76 \times 0.860 = \mathbf{66.9\ K}$

(iii) $r_o \quad \frac{h_{ic}}{h_{fe}} \quad \frac{(R_s \| R_1 \| R_2)}{} \quad \frac{1.5}{76} \quad \frac{1 \| 20}{} \quad \mathbf{32.3\ \Omega}$ ***(iv)*** $A_v \cong 1$

Tutorial Problems No. 59.1

1. Using ideal transistor approximations for the single-stage *CB* amplifier of Fig. 59.71, find (*i*) stage r_{in} (*ii*) r_L (*iii*) Avand (*iv*) A_p. Take transistor $\alpha = -0.99$.

[(*i*) **25 Ω** (*ii*) **10 K** (*iii*) **400** (*iv*) **396**]

2. For the single-stage *CB* amplifier circuit shown in Fig. 59.72, find
(*i*) r_{in} (*ii*) r_L (*iii*) A_v and (*iv*) A_p

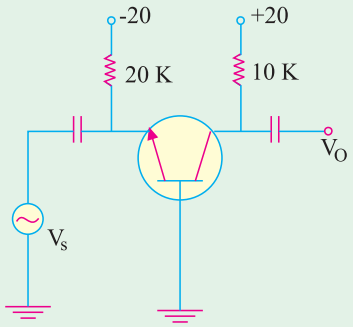


Fig. 59.71

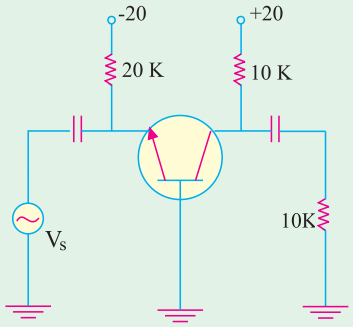


Fig. 59.72

Transistor $\alpha = -0.9$. Take G_e transistor material. [(*i*) **25 Ω** (*ii*) **5 K** (*iii*) **200** (*iv*) **196**]

3. For the *CB* amplifier circuit of Fig. 59.73, find approximate values of

(i) stage r_{in}
(ii) r_L
(iii) $A_v = v_{out} / v_{in}$
(iv) $A_{v\sigma} = v_{out} / v_s$
(v) A_p
(vi) G_p

Take transistor $\alpha = -0.98$

[(*i*) **25 Ω** (*ii*) **5K** (*iii*) **200** (*iv*) **22.2** (*v*) **196** (*vi*) **22.9 dB**]

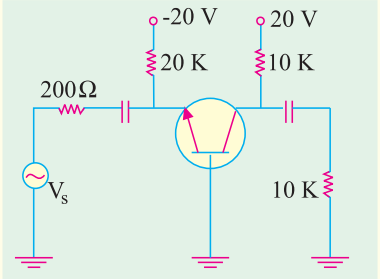


Fig. 59.73

4. Find r_{in}, r_L, A_i, A_v and G_p for the *CE* amplifier shown in Fig. 9.74.

[(*i*) **1250 Ω** (*ii*) **5K** (*iii*) **50** (*iv*) **200** (*v*) **30 dB**]

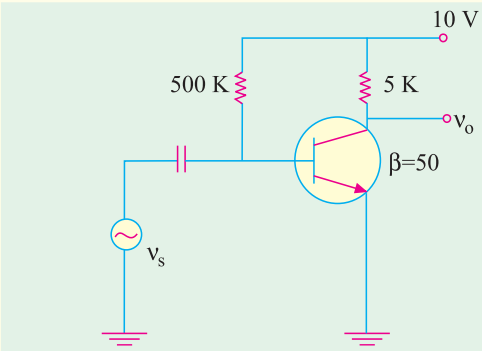


Fig. 59.74

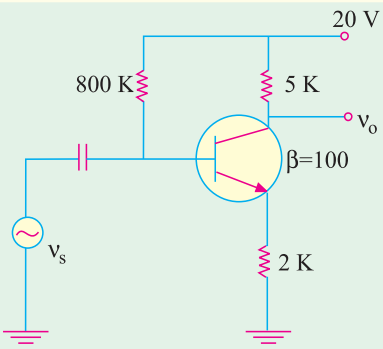


Fig. 59.75

5. For the single-stage *CE* amplifier circuit of Fig. 59.75, find
(*i*) r_{in} (*ii*) r_L (*iii*) A_v (*iv*) A_p and (*v*) G_p
Take β = 100 and use $r_e = 50\ mV/I_E$. **[(*i*) 160 K (*ii*) 5K (*iii*) 2.5 (*iv*) 250 (*v*) 24 dB]**

6. For the C_E amplifier of Fig. 59.76, find approximate values of
(*i*) r_{in} (*ii*) r_L (*iii*) A_v (*iv*) A_p and G_p.
Take transistor β = 50 and use $r_e = 50\ mV/I_E$. **[(*i*) 2K (*ii*) 10K (*iii*) 200 (*iv*) 10,000 (*v*) 40 dB]**

7. For the emitter follower circuit shown in Fig. 59.77, find
(*i*) $r_{in(base)}$ (*ii*) $r_{in(stage)}$
(*iii*) stage A_v (*iv*) A_p in decibles
Take β = 100 and use $r_e = 25\ mV/I_E$. **[(*i*) 10K (*ii*) 9.52K (*iii*) 0.75 (*iv*) 18.75 dB]**

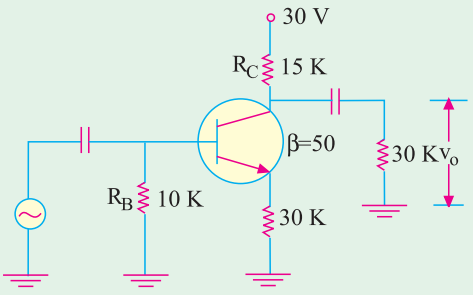


Fig. 59.76

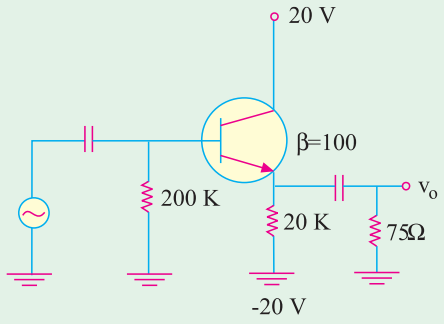


Fig. 59.77

8. An *NPN* silicon transistor is connected as a C_E amplifier with load R_L and a source resistance R_S. The *h*-parameters are :
h_{ie} = 1.1 K, $h_{re} = 2.5 \times 10^{-4}$, h_{fe} = 50 and h_{oe} = 25 μS
If R_L = 10K and R_S = 1K, find the various gains and input and output impedances.
Derive the relations used.
[A_i = 40, A_{is} = 20, r_{in} = 1K, A_v = –400, A_{vs} = –200, r_o = 52.5 K, A_p = 16,000, G_p = 42 dB]

9. Calculate the gain of a common-emitter transistor amplifier whose hybrid parameters are:
h_{ie} = 1100 ohms, $h_{re} = 2.5 \times 10^{-4}$; h_{fe} = 50, h_{oe} = 25 μS, R_L = 5K.
Derive any formula used. ***(–213) (Electronic Engg., Indore Univ.)***

10. A *CE* amplifier has h_{ie} = 1.1 K, h_{fe} = 50, $h_{re} = 2.5 \times 10^{-4}$; h_{oe} = 25 μS, $R_S = R_L$ = 500 ohm. Calculate the output impedance and voltage gain.
(Applied Electronics-I, Punjab Univ. Dec.)

11. A junction transistor has the following *h*-parameters h_{ie} = 2000 ohm, $h_{re} = 15 \times 10^{-4}$, h_{fe} = 49, h_{oe} = 50 μS. Determine the current gain, voltage gain, input resistance and output resistance of the *CE* amplifier if the

load resistance is 10K and source resistance is 600 ohm. Derive the expressions used.

(Applied Electronics-1, Punjab May)

12. A *CE* amplifier has $h_{ie} = 1.1$ K, $h_{fe} = 50$, $h_{re} = 2.5 \times 10^{-4}$, $h_{oe} = 25$ μS, $R_S = R_L = 1$K. Calculate the output impedance and voltage gain.

(Applied Electronics-1, Punjab Univ. Dec.)

13. A junction transistor has the following *h*-parameters $h_{ie} = 1$kW, $h_{re} = 1$, $h_{fe} = -50$, $h_{oe} = 25$mA/V. This transistor is connected to a source of internal resistance 600 ohm and a load of 40 kΩ. Calculate the current gain, voltage gain, input resistance and output resistance of the amplifier. Derive the expressions used.

(Applied Electronics-I, Punjab Univ. June)

14. A transistor connected in CE configuration has following *h*-parameters.

$h_{ie} = 1.1$ kΩ $\quad h_{re} = 2.5 \times 10^{-4}$

$h_{fe} = 50$ $\quad h_{oe} = 25$ μ Siemens

and $\quad r_s = r_L = 1$ kΩ

Calculated current gain, input impedance and voltage gain.

(Electronics Engg. Bangalore Univ. 2001)

OBJECTIVE TESTS – 59

1. In an ac amplifier, larger the internal resistance of the ac signal source
(*a*) greater the overall voltage gain
(*b*) greater the input impedance
(*c*) smaller the current gain
(*d*) smaller the circuit voltage gain.

2. The main use of an emitter follower is as
(*a*) power amplifier
(*b*) impedance matching device
(*c*) low-input impedance circuit
(*d*) follower of base signal.

3. An ideal amplifier is one which
(*a*) has infinite voltage gain
(*b*) responds only to signals at its input terminals
(*c*) has positive feeback
(*d*) gives uniform frequency response.

4. The smallest of the four *h*-parameters of a transistor is
(*a*) h_i (*b*) h_r (*c*) h_o (*d*) h_f.

5. The voltage gain of a single-stage *CE* amplifier is increased when
(*a*) its ac load is decreased
(*b*) resistance of signal source is increased
(*c*) emitter resistance R_E is increased.
(*d*) ac load resistance is increased.

6. When emitter bypass capacitor in a *CE* amplifier is removed, its ..... is considerably reduced.
(*a*) input resistance
(*b*) output load resistance
(*c*) emitter current
(*d*) voltage gain

7. Unique features of a *CC* amplifier circuit is that it
(*a*) steps up the impedance level
(*b*) does not increase signal voltage
(*c*) acts as an impedance matching device
(*d*) all of the above.

8. The input impedance h_{11} of a network with output shorted is given by the ratio
(*a*) v_1/i_1 (*b*) v_1/v_2
(*c*) i_2/i_1 (*d*) i_2/v_2

9. The *h*-parameters of a transistor depend on its
(*a*) configuration
(*b*) operating point
(*c*) temperature
(*d*) all of the above

10. The output admittance h_0 of an ideal transistor connected in *CB* configuration is ..... siemens.
(*a*) 0 (*b*) 1/r
(*c*) $1/\beta r_e$ (*d*) –1.

11. A transistor has $h_{fe} = 100$, $h_{ie} = 5.2$ K Ω, and $r_{bb} = 0$. At room temperature, $V_T = 26$ mV, the collector current, $|I_C|$ will be.
(*a*) 10 mA (*b*) 5 mA
(*c*) 1 mA (*d*) 0.5 mA

ANSWERS

1. (*d*) **2.** (*b*) **3.** (*b*) **4.** (*c*) **5.** (*d*) **6.** (*d*) **7.** (*d*) **8.** (*a*) **9.** (*d*) **10.** (*a*) **11.** (*a*)

ROUGH WORK

GO To FIRST

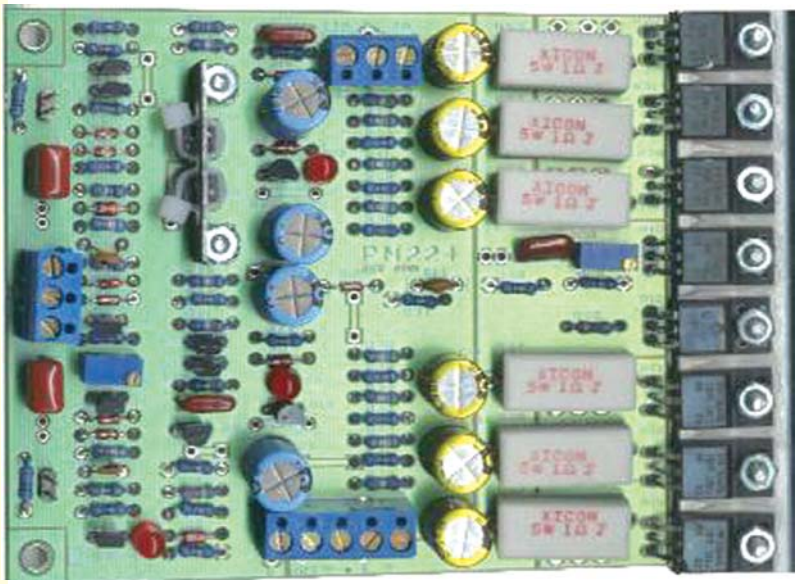
CONTENTS

CHAPTER 60

SINGLE-STAGE TRANSISTOR AMPLIFIERS

Learning Objectives

- Classification of Amplifiers
- Common Base (CB) Amplifier
- Common Emitter (CE) Amplifier
- Common Collector (CC) Amplifier
- Comparison of Amplifier Configurations
- Class-A Amplifier
- Power Rectangle
- Power Efficiency
- Class-B Amplifier
- Maximum Values
- Class-B Push-pull Amplifier
- Crossover Distortion
- Class-B Amplifier
- Class-C Amplifier
- Tuned Amplifier
- Distortion in Amplifiers
- Noise
- The Decibel System
- Value of 1 dB
- Cause of Amplifier Gain Variations
- Miller Effect
- Cut-off Frequencies of Cascaded Amplifiers
- The f_T of a Transistor
- Relation between f_α, f_β and f_T
- Gain-bandwidth Product



Single stage amplifier analysis is of great value in understanding the practical amplifier circuits

CONTENTS

60.1. Classification of Amplifiers

Linear amplifiers are classified according to their mode of operation *i.e.* the way they operate according to a predetermined set of values. Various amplifier descriptions are based on the following factors :

1. **As based on its input**
   (*a*) small-signal amplifier **(*b*)** large-signal amplifier
2. **As based on its output**
   (*a*) voltage amplifier **(*b*)** power amplifier
3. **As based on its frequency response**
   (*a*) audio-frequency (AF) amplifier **(*b*)** intermediate-frequency (IF) amplifier
   (*c*) radio-frequency (RF) amplifier
4. **As based on its biasing conditions**
   (*a*) class-A **(*b*)** class-AB **(*c*)** class-B **(*d*)** class-C
5. **As based on transistor configuration**
   (*a*) common-base (CB) amplifier **(*b*)** common-emitter (CE) amplifier
   (*c*) common-collector (CC) amplifier

The description ***small-signal, class-A, CE, voltage amplifier*** means that input signal is small, biasing condition is class-A, transistor configuration is common-emitter and its output concerns voltage amplification.

We will first take up the basic working of a single-stage amplifier *i.e.* an amplifier having one amplifying element connected in *CB*, *CE* and *CC* configuration.

60.2. Common Base (CB) Amplifier

Both Fig. 60.1 and 60.2 show the circuit of a single-stage *CB* amplifier using *NPN* transistor. As seen, input ac signal is injected into the emitter-base circuit and output is taken from the collector-base circuit. The *E/B* junction is forward-biased by V_{EE} whereas *C/B* junction is reverse-biased by V_{CC}. The *Q*-point or dc working conditions are determined by dc batteries along with resistors R_E and R_C. In other words, values of I_E, I_B and V_{CB} are decided by V_{CC}, V_{EE}, R_E and R_C. The voltage V_{CB} is given by the equation $V_{CB} = V_{CC} - I_C R_C$.

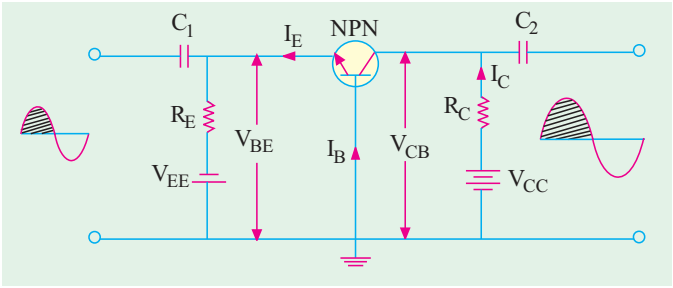


Fig. 60.1

When no signal is applied to the input circuit, the output just ***sits at the Q-point so that there is no output signal.*** Let us now see what happens when we apply an ac signal to the *E/B* junction via a coupling capacitor C_1 (which is assumed to offer no reactance to the signal).

Circuit Operation

When positive half-cycle of the signal is applied, then

1. .forward bias is **decreased** because V_{BE} is already negative with respect to the ground as per biasing rule of Art. 60.3.
2. consequently, I_B is ***decreased.***
3. I_E and hence I_C are ***decreased*** (because they are both nearly β times the base current).
4. the drop $I_C R_C$ *is* ***decreased.***
5. hence, V_{CB} is ***increased*** as seen by the equation given above.

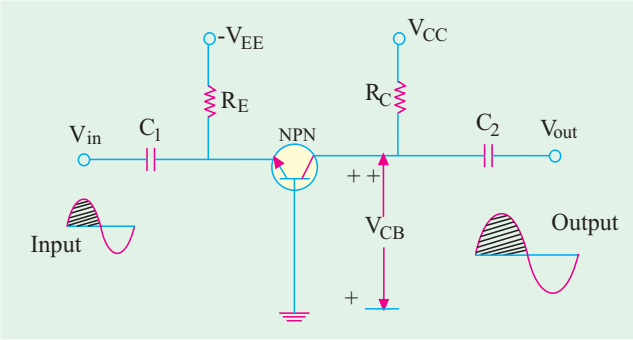


Fig. 60.2

It means that a positive output half-cycle is produced.

Since a ***positive-going*** input signal produces a ***positive-going*** output signal, there is no phase reversal between the two.

Voltage amplification in this circuit is possible by reason of relative input and output circuitry rather than current gain (α) which is always less than unity. The input circuit has low resistance whereas output circuit has very large resistance. Although changes in input and output currents are the same, the ac drop across R_L is very large. Hence, changes in V_{CB} (which is the output voltage) are much larger than changes in input ac signal. Hence, the voltage amplification.

60.3. Various Gains of a CB Amplifier

1. Input Resistance

The ac input resistance of the transistor is given by the emitter junction resistance

$$r_e = \frac{25\,\text{mV}}{I_E} \quad \text{or} \quad \frac{50\,\text{mV}}{I_E}$$

As seen from the ac equivalent circuit (Fig. 10.3)

$$r_{in} = r_e \| R_E$$

2. Output Resistance

$$r_0 = R_C \quad \text{— Fig. 60.2}$$

If a load resistance R_L is connected across output terminals, then

$$r_0 = R_C \| R_L \quad \text{— Fig. 60.3}$$

It is called the output resistance of the stage and is written as $r_{0(stage)}$ or r_0'

3. Current Gain $A_i = \alpha$

4. Voltage Gain $A_v = \frac{r_0}{r_{in}} = \frac{r_0}{r_e}$

5. Power Gain $A_p = A_v . A_i$

The decibel gain is given by $G_p = 10 \log_{10} A_p$ dB

60.4. Characteristics of a CB Amplifier

Common-base amplifier has

1. very low input resistance (30 – 150 Ω),
2. very high output resistance (upto 500 K),
3. a current gain $\alpha < 1$,
4. large voltage gain of about 1500,
5. power gain of upto 30 dB,
6. no phase reversal between input and output voltages.

Uses

One of the important uses of a *CB* amplifier is in matching a low-impedance circuit to a high-impedance circuit.

It also has high stability of collector current with temperature changes.

Example 60.1. *For the single-stage CB amplifier shown in Fig. 60.3 (a), find*
(a) stage input resistance, *(b) stage output resistance,*
(c) current gain, *(d) voltage gain of the stage,*
(e) stage power gain in dB,
Assume $\alpha = 1$. *Neglect* V_{BE} *and use* $r_e = 25\ mV/I_E$
(Basic Electronics, Bombay Univ. 1991)

Solution. The ac equivalent circuit is shown in Fig. 60.3 (*b*).

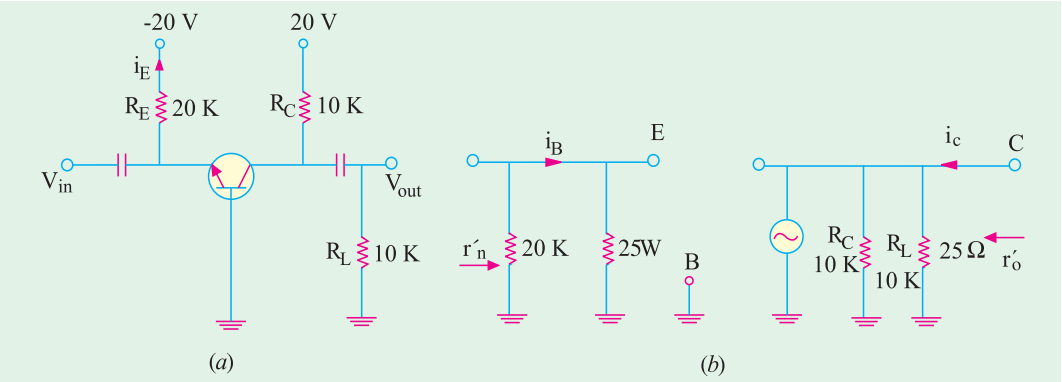


Fig. 60.3

$I_E = 20/20 = 1mA$ $\quad r_e = 25/I_E = 25\ \Omega$
(a) stage $r_{in} = r_e \| R_E = 25\ \Omega \| 20K \cong$ **25 Ω**
(b) stage $r_0 = R_C \| R_L$ $\quad \therefore \quad r_0' = 10K \| 10K =$ **5K**

(c) $A_i = \alpha = 1$ $\quad$ **(d)** $A_v = \dfrac{r_0'}{r_e} = \dfrac{5000}{25} =$ **200**

(e) $A_p = A_v . A_i = 200 \times 1 =$ **200**
$G_p = 10 \log_{10} A_p\ dB = 10 \log_{10} 200 =$ **23 dB**

60.5. Common Emitter (CE) Amplifier

Fig. 60.4 and 60.5 show the circuit of a single-stage *CE* amplifier using an *NPN* transistor. Here, base is the driven element. The input signal is injected into the base-emitter circuit whereas output signal is taken out from the collector-emitter circuit. The *E*/*B* junction is forward-biased by V_{BB} and *C*/*B* junction is reversed-biased by V_{CC} (in fact, same battery V_{CC} can provide dc power for both base and collector as in Fig. 60.5). The *Q*-point or working condition is determined by V_{CC} together with R_B and R_C. The dc equation is (Fig. 60.5).

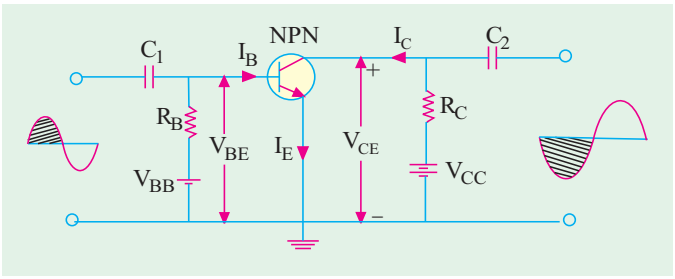


Fig. 60.4

$I_B \cong V_{BB}/R_B$ —neglecting V_{BE}
$I_C = \beta I_B$ and $V_{CE} = V_{CC} - I_C R_C$

Now, let us see what happens when an ac signal is applied at the input terminals of the circuit.

Circuit Operations

When positive half-cycle of the signal is applied (Fig. 60.4)

1. V_{BE} is **increased** because it is already positive w.r.t. the ground as per biasing rule of Art 6.3.
2. it leads to increase in forward bias of base-emitter junction
3. I_B is **increased** somewhat
4. I_C is increased by α times the **increased** in I_B.
5. drop $I_C R_C$ is **increased** considerably and cons- equently.
6. V_{CE} is decreased as seen from the equation given above.

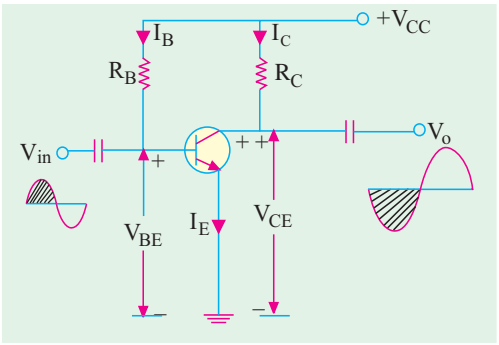


Fig. 60.5

Hence, negative half-cycle of the output is obtained. It means that a positive-going input signal becomes a negative going output signal as shown in Fig. 60.4 and 60.5.

60.6. Various Gains of a CE Amplifier

The ac equivalent of the given circuit (Fig. 60.5) is similar to the one shown in Fig. 60.6 (*b*).

1. Input Resistance

When viewed from base, ac resistance of the emitter junction is β_{re}. As seen, from Fig. 60.6 (*b*), circuit input resistance is

$$r_{in}' = R_B \| \beta r_e \quad \text{—remember } \beta\text{-rule}$$
$$\cong \beta r_e \quad \text{—when } R_B >> \beta r_e$$

It is called input resistance of the stage *i.e.* $r_{in(stage)}$.

2. Output Resistance : $r_0 = R_C$ —Fig. 60.5

However, if a load resistor R_L is connected across the output terminals (Fig. 60.6), then

$$r_0 = R_C \| R_L = r_L \quad \text{— Fig. 60.6 } (b)$$

It is called output resistance of the stage and is written as $r_{o(stage)}$ or r_o'.

3. Current Gain : $A_i = \beta$

4. Voltage Gain : $A_v = \beta . \dfrac{r_0'}{r_{in}'} = \beta . \dfrac{r_0}{\beta r_e} \cong \dfrac{r_0'}{r_e}$ —if $R_B >> \beta r_e$

It is the stage voltage gain,

5. Power Gain : $A_p = A_v . A_i = \beta . \dfrac{r_0'}{r_e}$; $G_p = 10 \log_{10} A_p$ dB

60.7. Characteristics of a CE Amplifier

A *CE* transistor amplifier has the following characteristics :

1. it has moderately low input resistance (1 K to 2 K),
2. its output resistance is moderately large (50 K or so),
3. its current gain (β) is high (50–300),
4. it has very high voltage gain of the order of 1500 or so,
5. it produces very high power gain of the order of 10,000 times or 40 dB,
6. it produces ***phase reversal*** of input signal *i.e.* input and output signals are 180° out of phase with each other.

Uses

Most of the transistor amplifiers are of CE type because of large gains in voltage, current and

power. Moreover, their input and output impedance characteristics are suitable for many applications.

Example. 60.2. *For the single-stage CE amplifier circuit shown in Fig. 60.6 (a), calculate* ***(a)*** r_{in} ***(b)*** r_0 ***(c)*** A_i ***(d)*** A_v *and* ***(e)*** G_p
Take transistor $\beta = 50$. *Neglect* V_{BE} *and take* $r_e = 25\ mV/I_E$

Solution. $I_B = \dfrac{20}{1M} = 20\,\mu A; \qquad I_C = \beta I_B = 50 \times 20 = 1\ mA$

$r_e = 25/1 = 25\ \Omega, \qquad \beta r_e = 50 \times 25 = 1250\ \Omega$

The ac equivalent circuit is shown in Fig. 60.6 (*b*).

(a) $r_{in}' = 1M \parallel 1250\ \Omega \cong$ **1250 Ω**

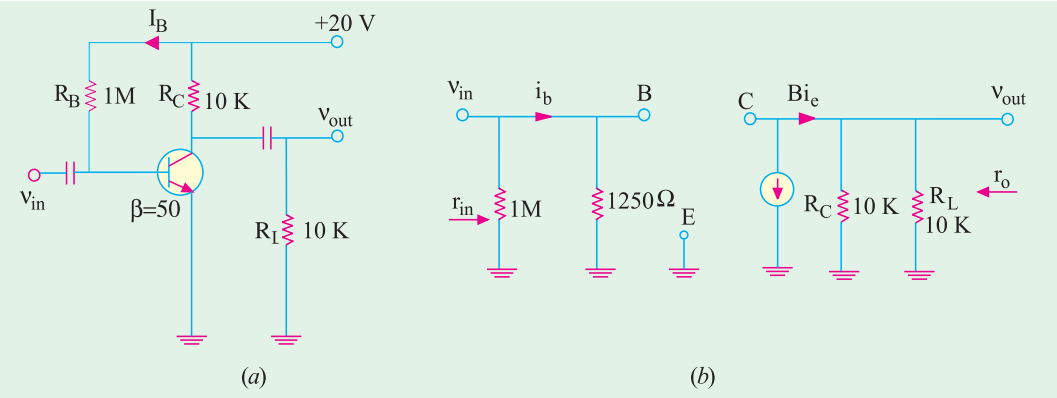


Fig. 60.6

Obviously, it is the input resistance of the stage and not that of the transistor alone.

(b) $r_0' = R_C \parallel R_L = 10K \parallel 10K = 5K$ ***(c)*** $A_i = 50$

(d) $A_v = \dfrac{5K}{25\Omega} = 200$ ***(e)*** $G_p = 10\log_{10}200 =$ **23** dB

60.8. Common Collector (CC) Amplifier

Fig. 60.7 and 60.8 show the circuit of a single-stage *CC* amplifier using an *NPN* transistor. The input signal is injected into the base-collector circuit and output signal is taken out from the emitter-collector circuit. The *E*/*B* junction is forward-biased by V_{EE} and *C*/*B* junction is reverse-biased by V_{CC}. The quiescent values of I_B and I_E are set by V_{CC} and V_{EE} together with R_B and R_E. As seen from Fig. 60.8.

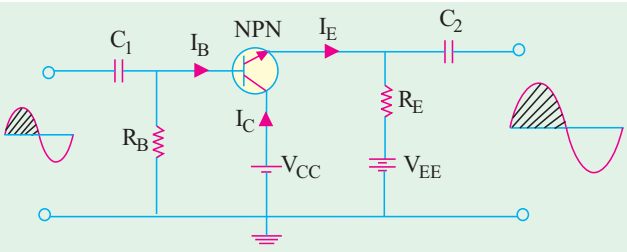


Fig. 60.7

$$I_E = \frac{V_{EE} - V_{BE}}{R_E + R_B/\beta} \cong \frac{V_{EE}}{R_E + R_B/\beta}$$

Let us now see what happens when an ac signal is applied across the input circuit.

Circuit Operation

When positive half-cycle of the signal is applied, then

1. forward bias is **increased** since V_{BE} is positive w.r.t. collector i.e. ground,
2. base current is **increased**,
3. emitter current is **increased**,
4. drop across R_E is **increased**,
5. hence, output voltage (*i.e.* drop across R_E is **increased**.

Consequently, we get positive half-cycle of the output.

It means that a ***positive-going*** input signal results in a ***positive going*** output signal and, consequently, the input and output signals are in phase with each other as shown in Fig. 60.8.

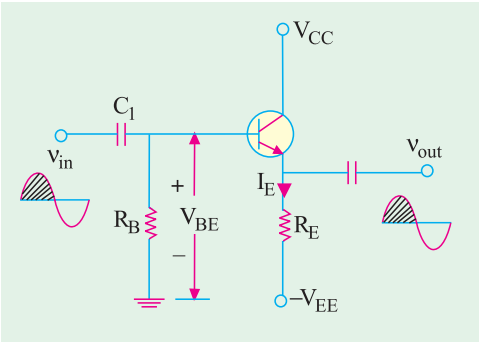


Fig. 60.8

60.9. Various Gains of a CC Amplifier

The ac equivalent circuit of the *CC* amplifier (Fig. 60.8) is given in Fig. 60.9.

1. $r'_{in} = R_B \| \beta (r_e + r_0)$
2. $r'_0 = R_E \| R_L$
3. $A_i = \beta$
4. $A_v = \dfrac{r_0'}{r_0' + r_e}$

Since usually $r_0' >> r_e'$, $A_v \cong \dfrac{r_0'}{r_0'} = 1$

$A_p = A_v . A_i$,

$G_p = 10 \log 10 A_p$ dB

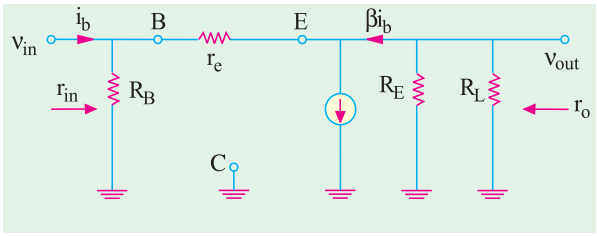


Fig. 60.9

60.10. Characteristics of a CC Amplifier

A *CC* amplifier has the following characteristics :

1. high input impedance (20-500 K),
2. low output impedance (50-1000 Ω),
3. high current gain of (1 + β) *i.e.* 50 – 500,
4. voltage gain of less than 1,
5. power gain of 10 to 20 dB,
6. no phase reversal of the input signal.

60.11. Uses

The CC amplifiers are used for the following purposes :

1. for impedance matching *i.e.* for connecting a circuit having high output impedance to one having low input impedance;
2. for circuit isolation;
3. as a two-way amplifier since it can pass a signal in either direction;
4. for switching circuits.

Example 60.3. *For the CC amplifier circuit of Fig. 60.10 (a), compute*
(i) r_o' ***(ii)*** r_{in}' ***(iii)*** A_v *and* ***(iv)*** A_p
Take transistor $\beta = 100$. *Neglect* V_{BE} *and use* $r_e = 25\ mV/I_E$ **(Electronics-I, M.S. Univ. 1991)**

Solution. $I_E = \dfrac{20}{20} = 1 mA$; $r_c = \dfrac{25}{1} = 25 \Omega$

The ac equivalent circuit is shown in Fig. 60.10 (*b*).

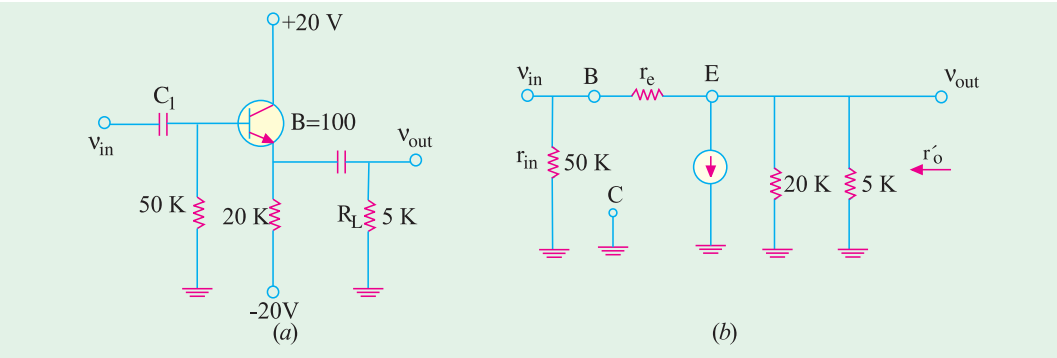


Fig. 60.10

(i) $r_o' = R_E \| R_L = 20K \| 5K = \mathbf{4K}$

(ii) $r_{in}' = R_B \| \beta (r_e + r_0) \cong R_B \| \beta r_0' = \dfrac{50 \times 400}{450} = \mathbf{44.4\ K}$

(iii) $A_v = \dfrac{4k}{4k + 25\Omega}$ ***(iv)*** $A_p = A_v . A_i = 1 \times 100 = 100$

60.12. Comparison of Amplifier Configurations

The basic building blocks of transistor amplifiers are single-stage common-base, common-emitter and common-collecter circuits. The choice of configuration and type of transistor for a given application will depend largely upon the desired input and output impedances, voltage, current and power gains and frequency response. For convenience, these values are tabulated in Table No. 10.1. Here the input and output parameters are resistances and apply only to ***low-frequency conditions.***

Table No. 60.1

	Type of Circuit		
Characteristic	***Common base***	***Common emitter***	***Common collector***
Current gain	nearly unity (α)	high (β)	highest ($1 + \beta$)
Voltage gain	high	very high	nearly unity
Power gain	moderate	highest	lowest
Input impedance	lowest	moderate	highest
Output impedance	highest	moderate	lowest
Phase reversal	No	Yes	No

In each case, the amplifying action depends on low-power input circuits controlling the high-power output circuits.

In the common-base circuit, although the input and output currents are nearly equal, the low-impedance emitter circuit absorbs for less power as compared to that which is available at the high-impedance collector.

The low base current flowing into the common-emitter circuit (where the impedance is a few kilohms) gives rise to much higher collector current flowing out of the high-impedance output circuit.

The common-collector circuit with approximately equal input and output resistances requires a low input current to control much larger output current.

60.13. Amplifier Classification Based on Biasing Conditions

This classification is based on the amount of transistor bias and amplitude of the input signal. It takes into account the portion of the cycle for which the transistor conducts. The three main classifications are :

(*a*) Class-A Amplifier

In this case, the transistor is so biased that output current flows for the full-cycle of the input signal (360°) as shown in Fig. 60.11 (*a*). In other words, the transistor remains *FR*-biased throughout the input cycle. Hence, its conduction angle is 360°.

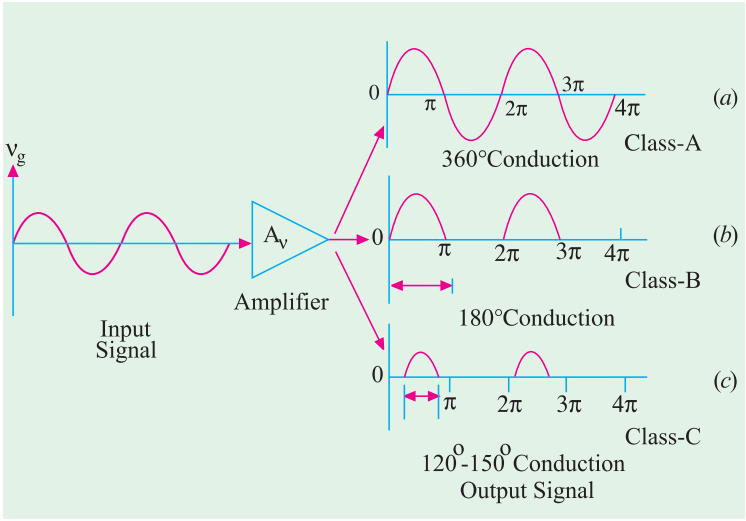


Fig. 60.11

(*b*) Class-B Amplifier

In this case, the transistor bias and the amplitude of input signal are such that output current flows for only half-cycle (180°) of the input signal. It means that transistor stays *FR*-biased for half the input cycle. The transistor conduction angle equals 180°.

(*c*) Class-C Amplifier

In this case, transistor bias and signal amplitude are such that output current flows for appreciably less than half-cycle of the input signal *i.e.* upto 120 ° or 150° angle of conduction as shown in 60.11 (*c*). In other words, transistor remains FR-biased for less than half the cycle.

(*d*) Class-AB Amplifier

The characteristics of such an amplifier lie in-between those of class-A and class-B. Here, biasing conditions are such that output current flows for appreciably more than half but less than the entire cycle *i.e.* current flows for more than 180° but less than 360°.

60.14. Graphic Representation

Since common-emitter is the most versatile and widely-used configuration, we will use CE output characteristic curves to differentiate between main classes of amplifiers.

Fig. 60.12 shows the biasing condition for class-A operation. It is seen that *Q*-point is located at the centre of the load line so that output (collector) current flows for the complete cycle of the input signal (conduction angle of 360°). Because of centred Q-point, the positive and negative swings of the input signal are confined to linear portions of the load line. In this linear region, equal changes in input base current produce equal changes in output (collector) current and voltage. Hence, output is an exact replica of the input. If the amplitude of the input signal is so large as to drive the *Q*-point closer to either cut-off or saturation region (where base current lines are not equally spaced), the output

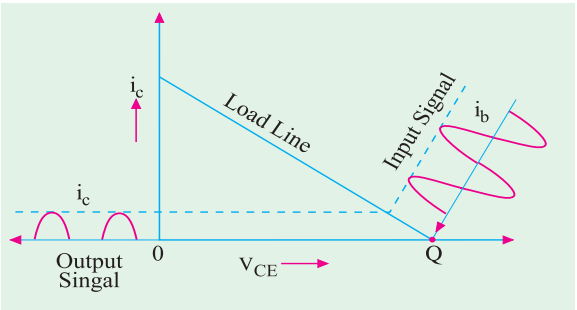


Fig. 60.12

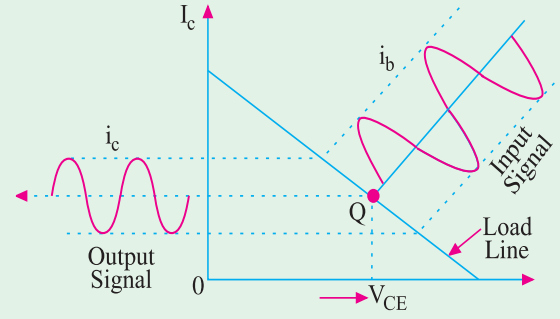


Fig. 60.13

will not be an exact replica of the input *i.e.* output signal will be distorted (Art. 60.30).

Fig. 60.13 shows biasing conditions for class-B operation. In this case, *Q*-point is located at cut-off point or very close to it. It is seen that the output current only flows for positive half-cycles of the input signal. For negative half-cycles, there is no output (collector) current. Obviously, output current flows only for 180° during the positive half-cycles of the input signal.

Fig. 60.14 shows biasing conditions for class-C operation. Here, the transistor is biased well into cut-off. In this case, (collector) current flows only for a part (upto 150°) of the positive half-cycle of the input signal when transistor comes out of the cut-off. During negative half-cycles of the input, the transistor remains deep in cut-off.

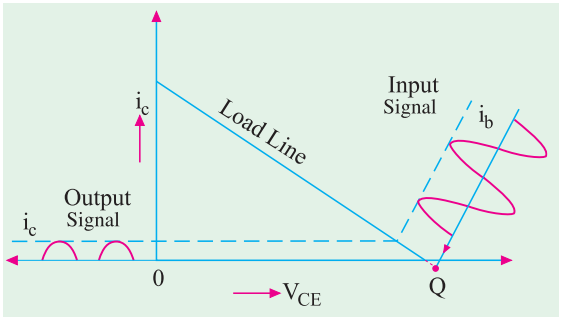


Fig. 60.14

60.15. Class-A Amplifier

It is one which has centred Q-point so that the transistor operates only over the linear region of its load line.

Another way of defining is that it is an amplifier in which the output current flows during the entire cycle of the input signal *i.e.* it has a conduction angle of 360°. In other words, the transistor remains FR-biased throughout the input cycle.

Characteristics

1. Since the transistor operates over the linear portion of the load line, the output waveform is exactly similar to the input waveform. Hence, class-A amplifiers are characterised by a high fidelity of the output. They are used where linearity or freedom from distortion is the prime requisite.
2. Since its operation is restricted only over a small central region of the load line, this amplifier is meant only for amplifying input signals of small amplitude. Large signals, will shift the Q-point into non-linear regions near saturation or cut-off and produce distortion (Art. 60.30).
3. Due to the limitation of the input signal amplitude, ac power output per active device (i.e. transistor) is small.
4. The overall efficiency of the amplifier circuit is

$$= \frac{\text{ac power delivered the load}}{\text{total power delivered by dc supply}} = \frac{\text{average ac power output}}{\text{average dc power input}}$$

5. The collector efficiency of a transistor is defined as

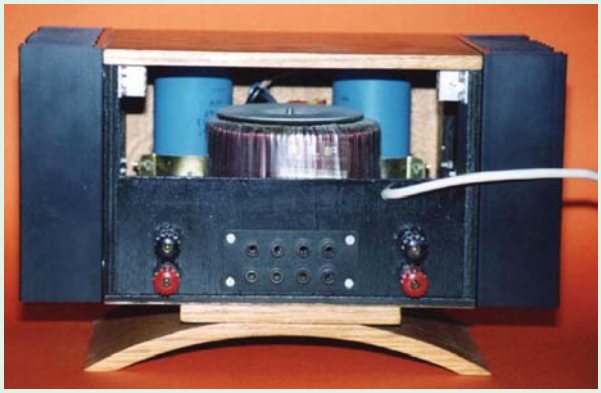
$$= \frac{\text{average ac power output}}{\text{average dc power input to transistor}}$$

The maximum possible collector efficiency of a class – A amplifier with resistive load is 50%

6. In case an output transformer is used, the maximum possible overall efficiency and maximum possible collector efficiency for a class-A amplifier are both 50%.

60.16. Power Distribution in a Class-A Amplifier

Fig. 60.15 (*a*) shows a *CE* connected transistor which forms the active element of a single-stage class-A amplifier. Fig. 60.15 (*b*) shows its output characteristic with a centred *Q*-point. When ac input signal is applied, *Q*-point shifts up and down from its central position. The output current will also increase or decrease from its quiescent (or no-signal) value I_{CQ}. Similarly, collector-emitter voltage V_{CE} will increase or decrease from its quiescent value V_{CEQ}. So long as signal variations are confined to linear region of the load line, average value of collector current is I_{CQ} because positive and negative input signal swings will produce equal changes in I_{CQ}.



Class-A amplifier

Hence, total average dc power drawn by the circuit from collector battery V_{CC} is

$$P_{in(dc)} = V_{CC} . I_{CQ}$$

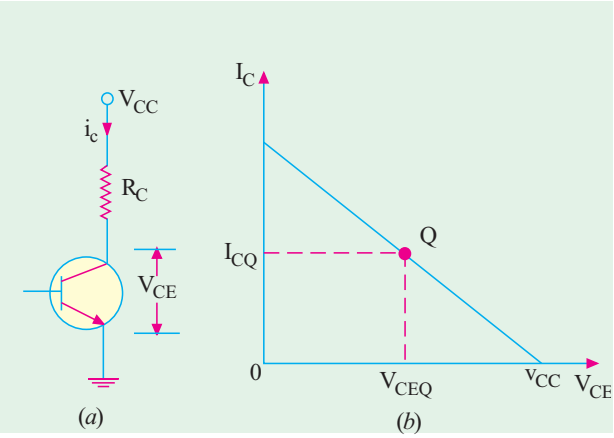


Fig. 60.15

Now, this power goes to supply the following :

(i) heat dissipated by the load resistor *RC* connected to the collector

$P_{RC(dc)} = R_C$

(ii) the balance $P_{tr(dc)}$ is given to the transistor.

It is further subdivided into :

(a) ac power developed across the load resistor which constitutes the ac power output

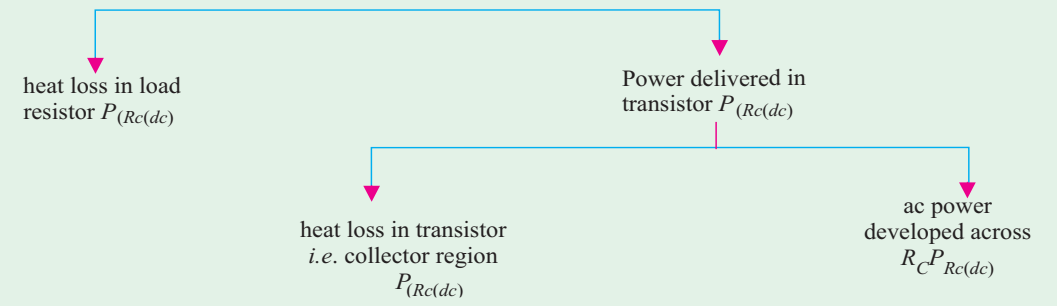
$$P_{o(ac)} = I^2 R_C = \frac{V^2}{R_C} = \frac{V_m^2}{2R_C}$$

where *I* is the rms value of the ac output current through the load, *V* is rms value of ac voltage and V_m is its maximum value.

(b) power dissipated (in the form of heat) by the transistor itself *i.e.* its collector region. It may be called $P_{c(dc)}$.

Since, under zero-signal condition, there is no ac output power, all the power given to the transistor is wasted as heat. Hence, **a transistor dissipates maximum power under zero-signal condition**.

The power flow diagram of the transistor is as follows :



60.17. Power Rectangle

The output or collector characteristic of a *CE*-connected transistor is shown in Fig. 60.16 with centred *Q*-point for class-A operation. When input signal is applied, the *Q*-point shifts to positions Q_1 and Q_2 alternately. The output current varies around its quiescent value from maximum value of $I_{C(max)}$ to minimum value $I_{C(min)}$. Similarly, collector-emitter voltage varies from maximum value of $V_{CE(max)}$ to $V_{CE(min)}$ around its quiescent value of V_{CEQ}.

Let us construct the various rectangles as shown in Fig. 60.16.

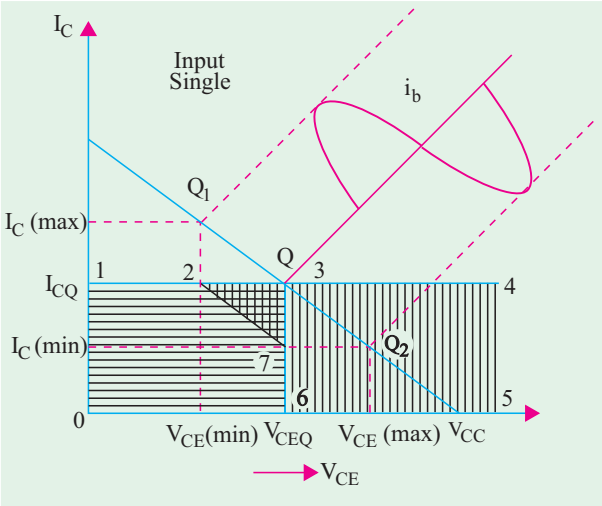


Fig. 60.16

1. rectangle 0-1-4-5
$= V_{CC} \times I_{CQ}$
= total average power supplied to the circuit by V_{CC} battery
$= P_{in(dc)}$

Now, $V_{CC} - V_{CEQ}$
= voltage drop across load resistor R_C

2. rectangle 3-4-5-6 = $(V_{CC} - V_{CEQ})\, I_{CQ}$
= power lost as heat in load resistor $R_C = I_{CQ}^2 R_C$

Now, V_{CEQ} is the voltage drop across the transistor itself *i.e.* potential difference between its collector and emitter as shown in Fig. 60.15 (*b*).

3. rectangle 0-1-3-6 = $V_{CEQ} . I_{CQ}$ = power delivered to transistor = $P_{tr(dc)}$

4. triangle 2-3-7 = ac power across $R_C = P_{o(ac)} = I^2 R_C$

It can also be proved that

$$P_{o(ac)} = \frac{V_{CE(max)} - V_{CE(min)}}{2\sqrt{2}} \times \frac{I_{C(max)} - I_{C(min)}}{2\sqrt{2}}$$

$$= \frac{[V_{CE(max)} - V_{CE(min)}] \times [I_{c(max)} - I_{c(max)}]}{8}$$

5. area 0-1-2-7-6 = power dissipated by the collector region of the transistor
$= P_{c(dc)}$.

Obviously, when there is no input signal, there is no output signal. Hence, $P_{o(ac)} = 0$, which means that area of triangle 2-3-7 is zero. In that case, the transistor collector will have to dissipate

maximum power equal to the area of the rectangle 0-1-3-6. It is called ***worst-case*** condition of the transistor and equals quiescent power.

60.18. Power Efficiency

Fig. 60.17. shows the power rectangles and triangles of a class-A amplifier in a very simple way.

$P_{in(dc)}$ = areas $A + B + C$
$P_{o(ac)}$ = area B
$P_{c(dc)}$ = area C
$P_{Rc(dc)}$ = area A

Amplifier efficiency or circuit efficiency or overall efficiency.

$$\eta_{overall} = \frac{B}{A+B+C} = \frac{P_{o(ac)}}{P_{in(dc)}}$$

The collector efficiency is given by

$$\eta_{overcall} = \frac{B}{B+C} = \frac{P_{o(ac)}}{P_{tr(dc)}}$$

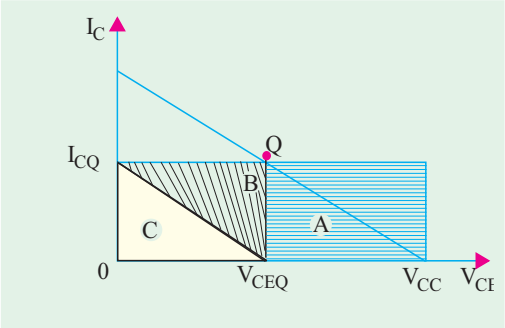


Fig. 60.17

Incidentally, it can be proved that $\eta_{overall} = 25\left[\frac{V_{CE(max)} - V_{CE(min)}}{V_{CC}}\right]$

Now, maximum value of $V_{CE} = V_{CC}$ and minimum value is = 0

$$\therefore \text{ maximum } \eta_{overall} = 25\left[\frac{V_{CC} - 0}{V_{CC}}\right]\% = 25\%$$

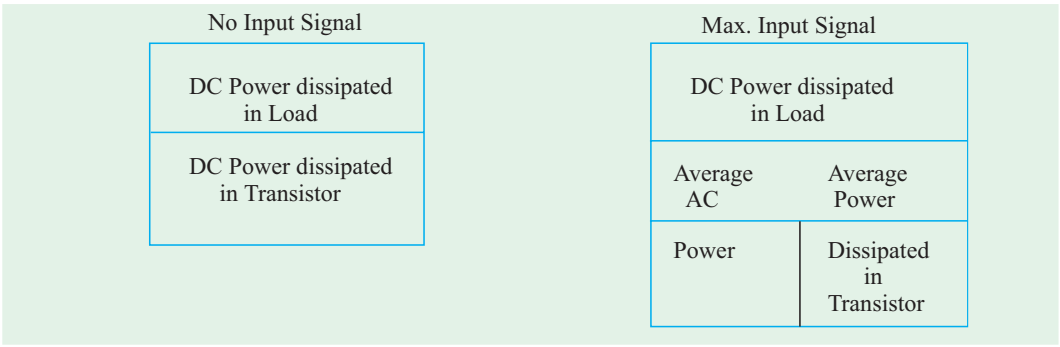


Fig. 60.18

Summary

In a direct-coupled class-A amplifier

1. maximum ac power equals half the power transistor can dissipate (or one-fourth of the total dc input power);
2. maximum ac power the transistor can dissipate is half the dc input power supplied by the battery;
3. the dc input power equals 4 times the ac output in load or twice the maximum power dissipated by the transistor.

The above facts have been illustrated with the help of power square shown in Fig. 60.18.

60.19. Maximum AC Power in Load

Fig. 60.19 (*a*) shows part of a base-driven *CE* amplifier with a collector load of R_C. Fig. 60.19 (*b*) shows its ac load line with centered *Q*-point. For the time being, we will neglect limitations of cut-off and saturation and assume (at least theoretically) that voltage and current swings down to zero are possible. Obviously, under such conditions, we will get the maximum possible ac output power.

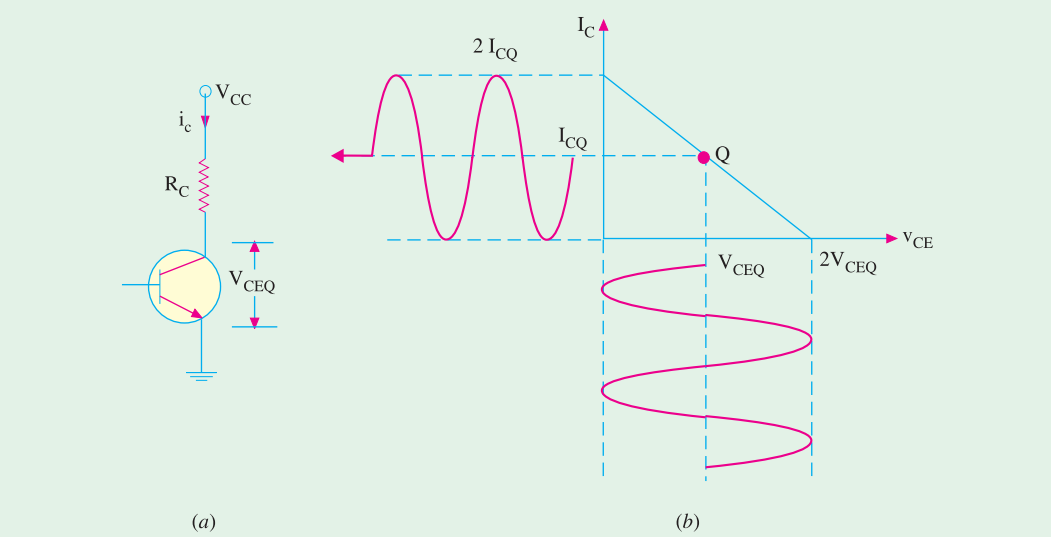


Fig. 60.19

As seen, collector current is a sine wave having peak-to-peak value of $2\,I_{CQ}$ or a maximum value of I_{CQ} or an rms value of $I_{CQ}/\sqrt{2}$. Since it passes through R_C, it delivers an ac power of

$$P_{o(ac)} = (I_{CQ}/\sqrt{2})^2 R_C = 0.5\, I_{CQ}^{\,2} R_C$$

Similarly, ac collector-emitter voltage is a sine wave of peak-to-peak value $2\,V_{CEQ}$ or peak value V_{CEQ} or rms value $V_{CEQ}/\sqrt{2}$. Since this sine wave is applied across R_C, the ac power can also be expressed as

$$P_{o(ac)} = \left(\frac{V_{CEQ}}{\sqrt{2}}\right)^2 / R_C = \frac{0.5\, V_{CEQ}^{\,2}}{R_C}$$

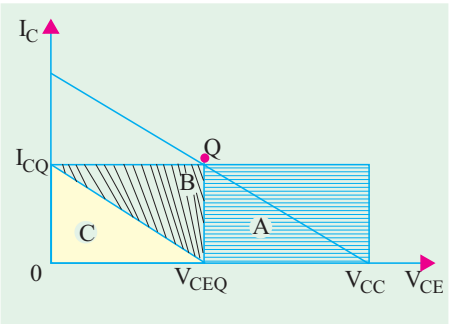


Fig. 60.20

Note. If there had been an unbypassed resistor R_E in the emitter, then ac load R_{ac} or $r_L = R_C + R_E$. Then, we would have used this value rather than R_C alone.

As seen from power rectangle diagram of Fig. 60.20, under maximum ac power condition

area *B* = area *C*

Also, area *A* = area *B* + area *C* = 2B

$$\therefore \quad maximum\,\eta_{overall} = \frac{B}{A+B+C}$$

$$= \frac{B}{2B+2B} = \frac{B}{4B} = 0.25 \quad \text{or} \quad 25\%$$

Example 60.4. *The 2N 1491 power transistor is used in a CE amplifier meant for class-A operation. If its zero-signal power dissipation is 10 W and ac output power is 3.5 W, find*

***(a)** collector efficiency and **(b)** power rating of the transistor*

Solution. (*i*) $\eta_{coll} = \frac{P_{o(ac)}}{P_{tr(dc)}} = \frac{3.5}{10} = 0.35 = \mathbf{35\%}$

(ii) Since zero-signal condition represents worst case condition for the transistor, it means that all the 10 W power is dissipated by it. Hence, transistor power rating is **10 W.**

Example 60.5. *For the class-A, CE amplifier circuit of Fig. 60.21, V_{CEQ} = 10 V and I_{CQ} = 500 mA. If collector i.e. output current varies by ± 250 mA when an input signal is applied at the base, compute*

(i) total dc power taken by the circuit, *(ii) dc power dissipated by the collector load,*
(iii) ac power developed across the load, *(iv) power delivered to the transistor,*
(v) dc power wasted in transistor collector, *(vi) overall efficiency.*
(vii) collector efficiency.

Solution. ***(i)*** $P_{in(dc)} = V_{CC} . I_{CQ} = 20 \times 500$
$= 10^4 \text{ mW} = \mathbf{10\ W}$

(ii) $P_{Rc(dc)} = I_{CQ}^2 R_C = 0.5^2 \times 20 = 5$ **W**

(iii) $P_{o(ac)} = I^2 R_C$

Now, maximum value of output ac current is 250 mA = 0.25A.

Hence, rms value $I = 0.25/\sqrt{2}$ A.

$\therefore \quad P_{o(ac)} = (0.25/\sqrt{2})^2 \times 20 = \mathbf{0.625\ W}$

(iv) $P_{tr(dc)} = 10 - 5 = \mathbf{5\ W}$

(v) $P_{c(dc)} = 5 - 0.625 = \mathbf{4.375\ W}$

(vi) $\eta_{overall} = \frac{0.625}{5} \times 100 = \mathbf{6.25\%}$

(vii) $\eta_{coll} = \frac{0.625}{5} \times 100 = \mathbf{12.5\%}$

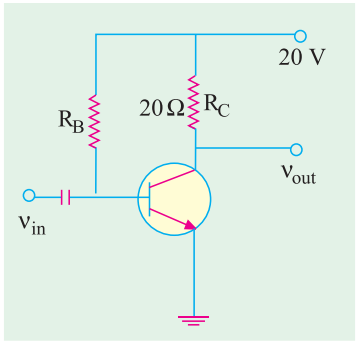


Fig. 60.21

It is seen that due to resistive load in the collector, efficiency of the circuit is very low.

Example 60.6. *For a class-A amplifier, $V_{CE}(max)$ = 25V, $V_{CE}(min)$ = 5V, V_{CC} = 30V. Find its overall efficiency.*

Solution. As seen from Art. 10.18

$$\% \ \eta_{overall} = 25\left[\frac{V_{CE(max)} - V_{CE(min)}}{V_{CC}}\right] = 25\left(\frac{25-5}{30}\right) = \mathbf{16.7}$$

60.20. Transformer-Coupled Class-A Amplifier

The main reason for the poor efficiency of a ***direct-coupled*** class-A amplifier is the large amount of dc power that the resistive load in collector must dissipate. This problem can be solved by using a suitable transformer for coupling the load (say, a speaker) to the amplifier stage as shown in Fig. 60.22. Since the load is not ***directly*** connected to the collector terminal, the dc collector current does not pass through it. In an ideal transformer, primary winding resistance is zero. Hence, dc power loss in the load is zero. In practice, however, there is a small dc resistance of the primary winding which does absorb some power though much less than a direct-coupled load.

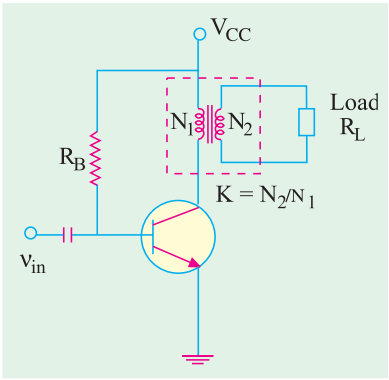


Fig. 60.22

In short, what the transformer does is to substitute ***ac load*** in place of ***ohmic*** or dc load.

The secondary load R_L when referred to primary becomes

$$R_L' = R_L / K^2 = a^2 R_L$$

where K = voltage transformation ratio=$N_2/N_1=V_2/V_1$

a = turns ratio $N_1/N_2 = 1/K$

Since a is usually made much more than unity or K is much less than unity, R_L can be made to look much bigger than what it actually is

In an ideal transformer, there is no primary drop, hence $V_{CC} = V_{CEQ}$. Now, all the power supplied by V_{CC} is delivered to the transistor. Hence, the overall and collector efficiencies become equal.

$$\eta_{overall} = \frac{P_{o(ac)}}{V_{CC}\, I_{CQ}} = \frac{P_{o(ac)}}{V_{CEQ}.I_{CQ}}$$

Power Diagram

The power distribution for a transformer-coupled class-A amplifier is shown in Fig. 60.23. Since load loss is zero, the triangle A of Fig. 60.17 is non-existent. Moreover, the load line of Fig. 60.17 becomes a.c. load line in Fig. 60.23. Consequently, $V_{CEQ} = V_{CC}$. Area of triangle B equals that of triangle C. Also, sum of A and B represents power delivered to the transistor.

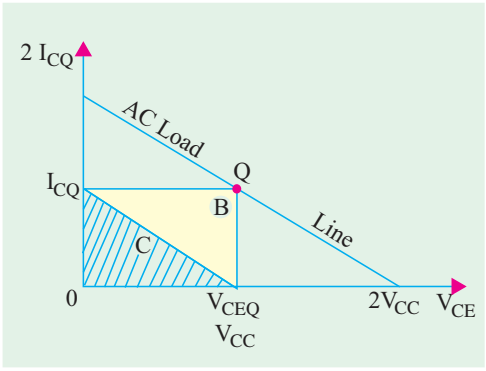


Fig. 60.23

$$\therefore \quad \eta_{overall} = \frac{B}{B+C} = \frac{B}{2B} = 0.5 \text{ or } 50\%$$

It can be proved that maximum possible overall efficiency and the maximum possible collector efficiency of a class-A amplifier using an output transformer are both 50%.

In general, overall efficiency of a transformer-coupled class-A amplifier is given by

$$\%\eta = 50\left[\frac{V_{CE(max)} - V_{CE(min)}}{2V_{CC}}\right]^2 = 50\left[\frac{V_{CE(max)} - V_{CE(min)}}{V_{CE(max)} + V_{CE(min)}}\right]^2$$

Obviously, larger the value of $V_{CE(max)}$ and smaller that of $V_{CE(min)}$ closer the efficiency to the theoretical limit of 50%.

Proof

The conditions for the development of maximum ac power are shown in Fig. 60.24. It must be kept in mind that here

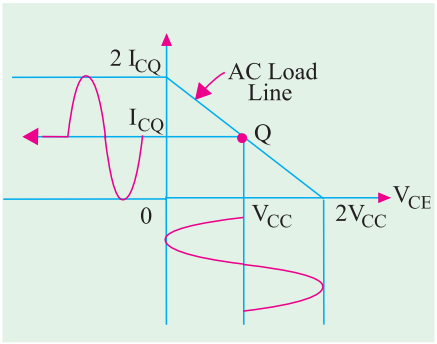


Fig. 60.24

$$V_{CEQ} = V_{CC}$$

Now, average power delivered by the battery

$$P_{in(dc)} = V_{CC}.I_{CQ}$$

Whole of the power is given to the transistor.

rms output voltage $= \frac{V_{CC}}{\sqrt{2}}$

rms output current $= \frac{I_{CQ}}{\sqrt{2}}$

$\therefore$ maximum $P_{o(ac)} = \frac{V_{CC}}{\sqrt{2}} \times \frac{I_{CQ}}{\sqrt{2}} = \frac{1}{2} V_{CC}.I_{CQ}$

= half the input ac power

Hence, maximum value of overall efficiency

$$\eta_{overall} = \frac{P_{o(ac)}}{P_{in(dc)}} = \frac{V_{CC} \cdot I_{CC}/2}{V_{CC} \cdot I_{CQ}} = \frac{1}{2} \text{ or } 0.5 \text{ or } 50\%$$

$$\eta_{coll} = \frac{P_{o(ac)}}{P_{tr(dc)}}$$

Example 60.7. *The optimum load resistance for a certain transistor is 200 Ω. What is the turns ratio (N_1/N_2) of a transformer required to couple an 8-Ω loud-speaker to the transistor ?*

Solution. $R_L' = a^2 R_L$ $\therefore$ $200 = a^2 \times 8$ or $a = 5$

Hence, it should be a 5 : 1 *step-down* transformer.

Example 60.8. *In a transformer-coupled class-A amplifier $V_{CE(max)} = 27$ V and $V_{CE(min)} = 3$V. Compute its overall efficiency.*

Solution. $\eta_{overall} = 50\left(\frac{27-3}{27+3}\right)^2 = 32\%$

Example 60.9. *For the transformer-coupled optimally-biased class-A amplifier shown in Fig. 10.25, find*

***(a)** transformer turns ratio* ***(b)** collector current* ***(c)** transistor power rating*

Solution. As shown in Fig. 60.25, if V_2 is the rms secondary voltage, then

$$P_{load} \text{ or } P_{o(ac)} = \frac{V_2^2}{R_L} = \frac{(V_{2m}/\sqrt{2})^2}{R_L} = \frac{V_{2m}^2}{R_L}$$

where V_{2m} is the maximum value of secondary voltage.

$$\therefore \quad V_{2m} = \sqrt{2RP_{load}} = \sqrt{2 \times 4 \times 4.5} = 6\text{ V}$$

Now, peak, value of primary voltage is

$V_{1m} = a \cdot V_{2m} = 6a$

This also represents the peak value of collector voltage. For optimally-biased circuit, peak value of collector-emitter voltage equals $V_{CC} = 30$V.

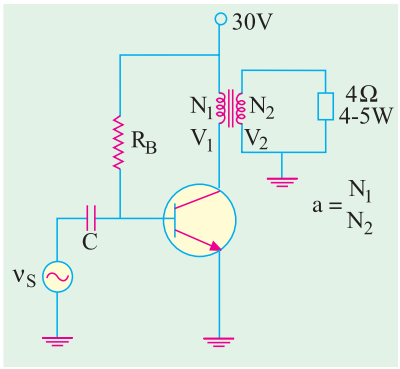


Fig. 60.25

$\therefore$ $6a = 30$ or $a = 5$ $\therefore$ $N_1/N_2 = 5$

(a) Hence, it is a step-down transformer with a turns ratio of **5 : 1**

(b) The overall efficiency is 50%. Hence, for a load power of 4.5 W, total dc input power supply must be 4.5/0.5 = 9 W

$\therefore$ $V_{CC}I_C = P_{in(dc)} = 9$W or $30\, I_C = 9000$ mW $\therefore$ $I_C =$ **300 mA**

(c) Worst case condition for the transistor is when there is no input signal. In that case, transistor has to dissipate the entire input. Therefore, transistor power rating = **9W.**

Example 60.10. *For the optimally-biased transformer -coupled class-A amplifier, maximum collector current change is 100 mA. Find the power transferred to a 4-W speaker load if it is **(a)** directly-coupled **(b)** transformer-coupled to the transistor. Find also the turn ratio of the transformer required.* **(Applied Electronics, Kerala Univ.)**

Solution. (*a*) Direct-coupled Load [Fig. 60.26 (*a*)]

It should be remembered that for an optimally-biased class-A amplifier, peak change in collector-emitter voltage equals $V_{CC} = 10$ V.

RMS value of ac output current I = $(100/\sqrt{2})$ mA

Average power delivered to the speaker

$$= I^2 R_L = \left(\frac{100}{\sqrt{2}} \times 10^{-3}\right)^2 \times 4 = 0.02\text{W} = \textbf{20 mW}$$

(b) Transformer-coupled Load

Now, $\Delta V_{CE} = V_{CC} = 10$ V : $\Delta I_{CQ} = 100$ mA

If R_L' is the secondary load reflected into the primary, then

$$R_L' = \frac{\Delta V_{CE}}{\Delta I_{CQ}} = \frac{10\,\text{V}}{100\,\text{mA}} = 100\,\Omega$$

Now, $a^2 R_L = R_L'$

$\therefore \quad a_2 \times 4 = 100,$

$a = 5$

Hence, it is a step-down transformer with a turns ratio of 5 : 1

Peak value of secondary voltage = 10 × 1/5 = 2 V

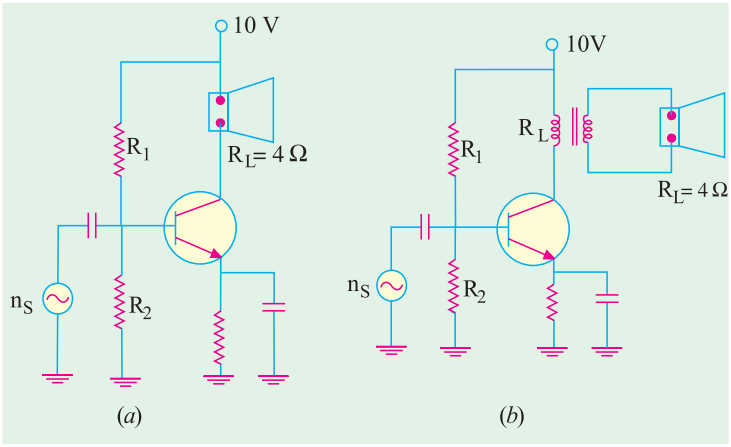


Fig. 60.26

Peak value of secondary current =2 / 4 = 0.5 A

RMS value = 0.5/ $\sqrt{2}$ A

Average power transferred to the speaker = $\left(\frac{0.5}{\sqrt{2}}\right)^2 \times 4 = 0.5$ W = **500 mW**

Obviously, power transferred has increased 500/20 = 25 times *i.e.* $a^2 = (N_1/N_2)^2$ times where *a* is transformer primary-to-secondary turn ratio *i.e.* $a = N_1/N_2$.

Example 60.11. *A class-A amplifier shown in Fig. 60.27 operates from V_{CC} = 20 V, draws a no-signal current of 5A and feeds a load of 40 Ω through a step-up transformer of N_2/N_1 = 3.16. Find*

***(a)** whether the amplifier is properly matched for maximum power transfer,*
***(b)** maximum ac signal power output, **(c)** maximum dc power input,*
***(d)** conversion efficiency at maximum signal input.*

(Electronic Circuits, Gauhati Univ. 1990)

Solution. **(a)** Here, a = N_1/N_2 =1/3.16

$R_L' = a^2 R_L = (1/3.16)^2 \times 40 =$ **4 Ω**

Also, $R_L' = \frac{V_{CC}}{I_{CQ}} = \frac{20}{5} =$ **4Ω**

Since, the two resistance values are the same, the amplifier is properly-matched.

(b) As seen from Art. 60.20, Maximum value of

$$P_{o(ac)} = \frac{1}{2} V_{CC}\, I_{CQ}$$

$$= \frac{1}{2} \times 20 \times 5 = \textbf{50 W}$$

(c) The dc power input

$= V_{CC} I_{CQ} = 20 \times 5 =$ **100 W**

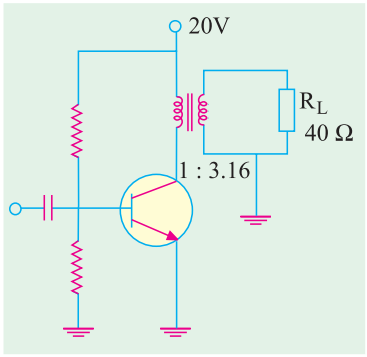


Fig. 60.27

(d) $\eta = \frac{50}{100} \times 100 = 50\%$

60.21. Class-B Amplifier

Biasing condition for class-B operation has been shown both on current transfer characteristic and output characteristic for *CE* configuration in Fig. 60.28.

As seen, the transistor has been biased to cut-off. It remains FR-biased for only half-cycle of the input signal. Hence, its conduction angle is only 180°. It is obvious that with zero signal, its collector current is zero.

Characteristics

1. Since negative half-cycles are totally absent from the output, the signal distortion is high as compared to class-A amplifiers.
2. Since input voltage is large (upto V_{CC}), voltage amplification is reduced.
3. Zero-signal input represents worst condition for class-A amplifiers but best condition for class-B amplifiers.
4. In class-B amplifiers, transistor dissipates more power with increase in signal strength but opposite is the case in class-A amplifiers.
5. Average current in class-B operation is less than in class-A, hence power dissipated is less. Consequently, maximum circuit (or overall) efficiency of a class-B amplifier is 78.5% when peak signal makes $V_{CE(min)} = 0$. In general
$$\eta_{overall} = 78.5\left[1 - \frac{V_{CE(\min)}}{V_{CC}}\right]\%$$
6. The half-sinewave type of collector current contains very pronounced ***even harmonics*** (particularly the second one). These impulses can be rendered useful either by employing push-pull circuit (Art. 60.24) or by using tuned amplifiers (Art. 60.29).

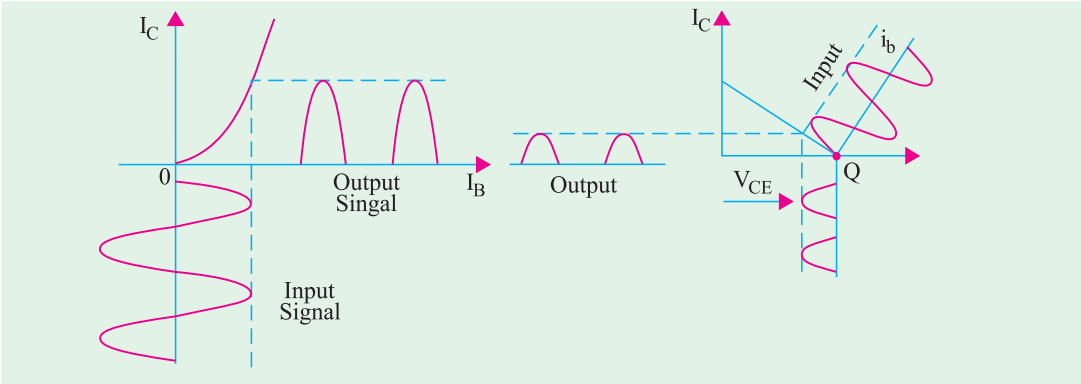


Fig. 60.28

60.22. Power Relations for Class-B Operation

(i) Input DC Power : $P_{in(dc)} = V_{CC} I_{dc}$

where I_{dc} is the average or dc current drawn from the supply.

If $I_{C(max)}$ is the maximum or peak value of collector or output current, then

$$I_{dc} = \frac{I_{C(\max)}}{\pi} \qquad \therefore \quad P_{in(dc)} = V_{CC} \frac{I_{C(\max)}}{\pi}$$

(ii) DC Power Loss in Load : $P_{Rc(dc)} = I_{dc}^2 R_C$ where $I_{dc} = I_{C(max)}/\pi$

(iii) AC Power Output in Load : $P_{o(ac)} = I^2 R_C = \frac{V^2}{R_C} = \frac{V_m^{\,2}}{2R_C}$

where I = rms value of output ac current
V = rms value of output ac voltage
V_m = maximum value of output ac voltage

(iv) DC Power Loss in Collector Region or Transistor

$$P_{c(d)} = P_{in(dc)} - P_{Rc(dc)} - P_{o(ac)}\,; \quad \eta_{overall} = \frac{P_{o(ac)}}{P_{in(dc)}}$$

60.23. Maximum Values

The operation for maximum signal input is shown in Fig. 60.29.

Here, $I_{dc} = \frac{I_{C(\max)}}{\pi} \quad \therefore \quad P_{in(dc)} = V_{CC}.I_{dc} = \frac{V_{CC}.I_{C(\max)}}{\pi}$

RMS value of output or collector current $= \frac{I_{C(max)}}{\sqrt{2}}$

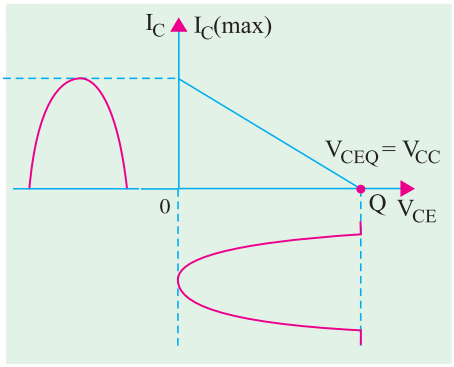


Fig. 60.29

RMS value of output voltage = $\frac{V_{CC}}{\sqrt{2}}$

Hence,ac output power during half-cycle

$$= \frac{1}{2} \times \frac{V_{CC}}{\sqrt{2}} \frac{I_{C(\max)}}{\sqrt{2}}$$

The factor $\frac{1}{2}$ comes in because power is produced during one half-cycle only.

$$P_{o(ac)} = \frac{1}{4}.V_{CC}.I_{C(max)}$$

$$\therefore \quad \eta_{overall} = \frac{P_{o(ac)}}{P_{in(dc)}} = \frac{\pi}{4} = 0.785 \text{ or } \mathbf{78.5\%}$$

Example 60.12. *In a class-B amplifier,* $V_{CE(min)} = 2V$ *and* $V_{CC} = 15$ *V. Find its overall efficiency*

Solution. As seen from Art 60.21, % $\eta = 78.5 \left(1 - \frac{2}{15}\right) = \mathbf{68\%}$

60.24. Class-B Push-Pull Amplifier

It employs two identical transistors operating as a ***single-stage of amplification.*** As shown in Fig. 60.30, the base of the two *CE*-connected transistors have been connected to the opposite ends of the secondary of the input transformer T_1 and collectors to the opposite ends of the primary of the output transformer T_2. For getting a balanced circuit, the two emitters have been returned to the centre tap of T_1 secondary and V_{CC} connected to the centre tap on the primary of T_2. Since zero bias is required for cut-off, the two bases have been earthed. A push-pull amplifier is sometimes referred to as ***balanced amplifier.***

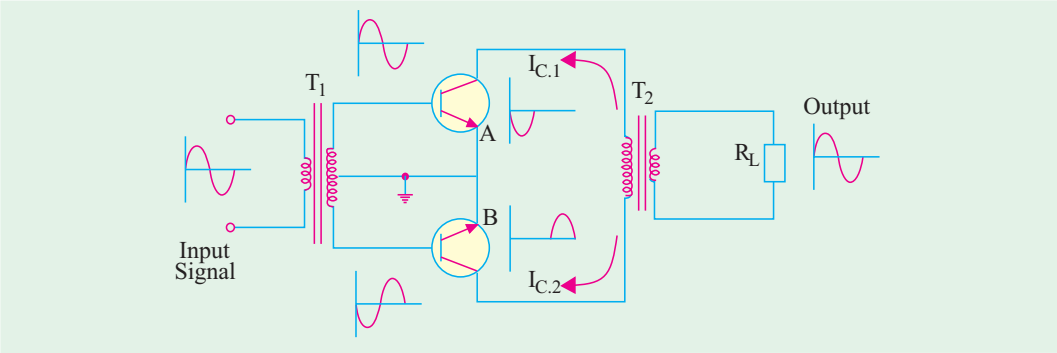


Fig. 60.30

Principle of Operation

It is seen from Fig. 60.30 that transistor *A* and *B* are driven by two input signals which are 180° out of phase with each other. These two signals are produced by T_1.

Transistor *A* takes positive half-cycles of the signal whereas *B* handles negative half-cycles. When the two outputs are combined, an almost undistorted output waveform is produced as seen from Fig. 60.31.

In Fig. 60.31, the transfer characteristic of transistor *B* has been plotted upside down with respect to that of *A* in order to get the combined output.

Detailed operation is as under :

During the positive half-cycle of the signal, *A* is turned ON because its base is driven positive. It draws collector current $I_{C\text{-}1}$ in the upward direction from V_{CC}. Meanwhile, transistor B remains OFF because its base has negative voltage. Hence, $I_{C\text{-}2} = 0$. Obviously, one positive half-cycle of the output signal appears across secondary load R_L of T_2.

During negative half-cycle of the input signal, *B* conducts whereas *A* remains OFF. Hence, $I_{C\text{-}2}$ is taken by B but $I_{C\text{-}1} = 0$. Now, negative output half-cycle is produced across R_L because $I_{C\text{-}2}$ is pulled down through secondary of T_2. It is obvious that in the absence of input signal, neither *A* nor *B* draws any collector current. Hence, there is no drain on V_{CC} battery.

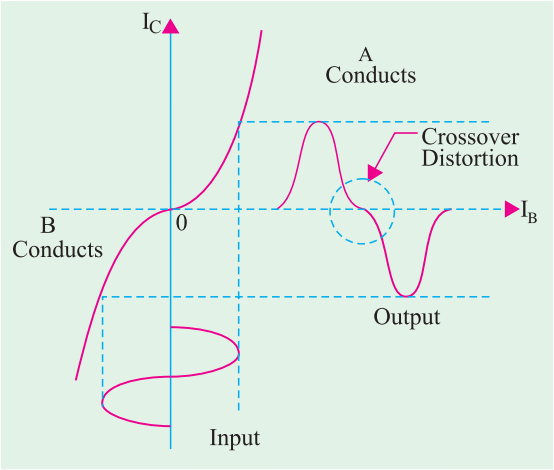


Fig. 60.31

Advantages

1. It has high efficiency, theoretical limit being 78.5%. It is primarily due to the fact that there is ***no power drawn by the circuit under zero-signal condition.***
2. Since in push-pull arrangement, 180° phase difference exists between even-order harmonics produced by each transistor, they cancel out thereby giving an almost distortion-free output. This automatic cancellation of all even-order harmonics from the output current makes class-B push-pull amplifiers highly desirable for ***communication sound equipment.***
3. This double-ended class-B amplifier provides practically four times the power supplied by a single-ended amplifier provided signal load resistance remains the same.

4. The dc components of the two collector currents through the two halves of the primary of T_2 flow in ***opposite directions*** so that net dc magnetisation of the transformer core is almost nil. Hence, distortions due to the effect of dc magnetic saturation of the trans former core are eliminated. Moreover, smaller size core can be used due to very small net core flux.

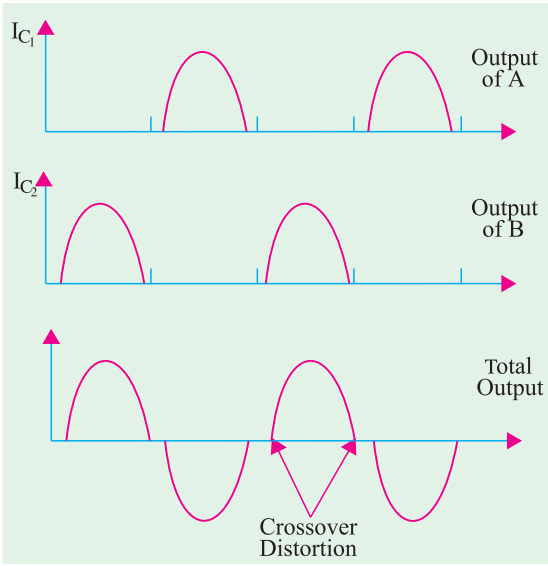


Fig. 60.32

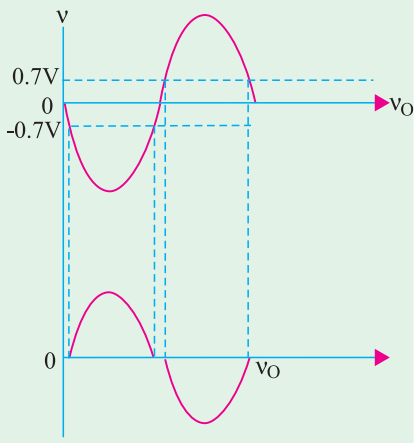


Fig. 60.33

Uses

Class-B push-pull amplifiers are extensively used for audio work in portable record players, as stereo amplifiers and in high-fidelity radio receivers.

60.25. Crossover Distortion

In class-B push-pull operation, there is severe distortion at very low signal level because

(a) bases of the transistors do not turn ON at 0 V but at 0.3 V for Ge and 0.7 V for Si

(b) there is non-linearity in the low-signal area.

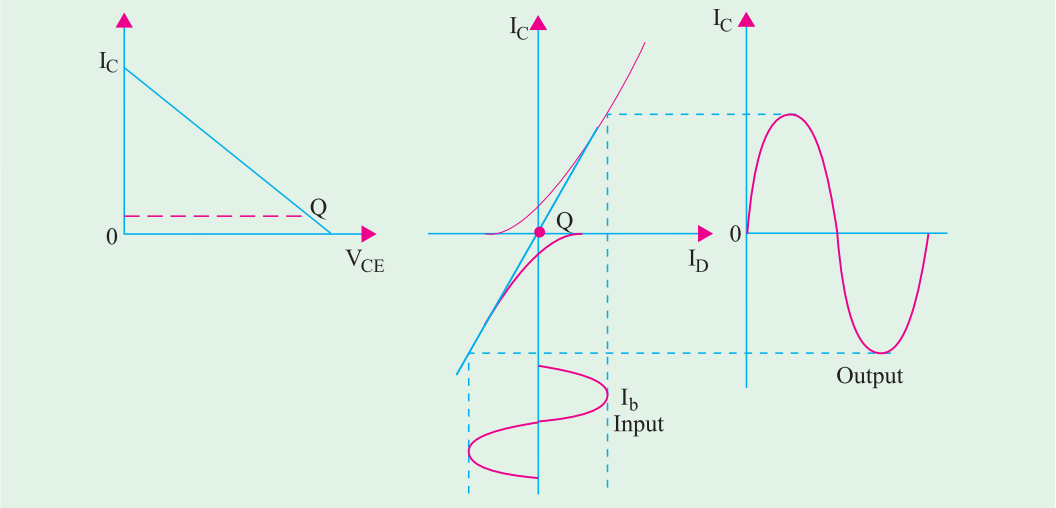


Fig. 60.34

In simple words, crossover distortion occurs as a result of one transistor cutting off before the other begins conducting. The effect is illustrated in Fig. 60.32. For silicon transistors, there is 2 × 0.7 = 1.4 V ***dead zone*** on the input signal within which neither transistor is turned ON and output is zero (Fig. 60.33).

The distortion so introduced is called ***crossover distortion*** because it occurs during the time operation ***crosses over from one transistor to the other in the push-pull amplifier.*** The same was shown earlier in Fig. 60.31 by using transfer characteristics of the two transistors.

Crossover distortion can be eliminated by applying slight forward bias to each emitter diode. It, in effect, means locating the *Q*-point of each transistor slightly above cut-off as shown in Fig. 60.34 so that each one operates for more than one half-cycle. Strictly speaking, it results in class-AB operation because each transistor may operate for about 200° (instead of 180°). However, for all practical purposes, it is still regarded as class-B push-pull operation.

60.26. Power Efficiency of Push-Pull Amplifiers

Consider the circuit shown in Fig. 60.35.

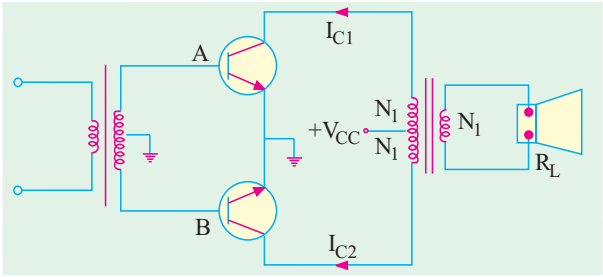


Fig. 60.35

Since the two transistors are identical, $I_{C.1} = I_{C.2} = I_C$

$\therefore \quad I_C = I_m/\pi$

Total dc supply current I_{dc} for the two transistors is

$$I_{dc} = 2I_C = 2\frac{I_m}{\pi}$$

$$\therefore \quad P_{in(dc)} = V_{CC} \times I_{dc}$$

$$= \frac{2I_m}{\pi} V_{CC}$$

For the sake of simplicity, we will assume transformer turns ratio as $(N_1 + N_1) : N_1$ so that for each transistor, turns ratio is $N_1 : N_1$ *i.e.* 1 because each uses half the secondary of the output transformer.

If V_m is the peak voltage at either collector, then peak load voltage is also V_m. The peak current in the load is I_m. Then, ac power delivered to the load is

$$P_{o(dc)} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} = \frac{V_m I_m}{2}$$

The total collector dissipation for the two transistors is

$$2.P_{c(dc)} = P_{in(dc)} - P_{o(dc)} = \frac{2I_m V_{CC}}{\pi} - \frac{V_m I_m}{2} = 2I_m \left(\frac{V_{CC}}{\pi} - \frac{V_m}{4} \right)$$

$$\eta_{overall} = \frac{P_{o(ac)}}{P_{in(dc)}} \times 100 = \frac{V_m I_m / 2}{2I_m / \pi} \times 100 = \frac{\pi}{4} \cdot \frac{V_m}{V_{CC}} \times 100$$

Under ideal condition of maximum power in the load, $V_m = V_{CC}$

$$\therefore \quad \text{maximum } \eta_{overall} = \frac{\pi}{4} \times 100 = 78.5\%$$

Same is the value of collector efficiency.

60.27. Complementary Symmetry Push-Pull Class-B Amplifier

The push-pull amplifier discussed in Art. 60.24 suffers from two disadvantages :

(i) it requires a bulky and expensive output transformer

(*ii*) it requires two out-of- phase input signals which necessitates an input centre-tapped transformer or phase inverter. It makes the driver circuitry quite complicated.

The complementary symmetry amplifier eliminates these two disadvantages while retaining the advantages of push-pull configuration.

As we know, a standard class-B push-pull amplifier requires two power transistors of the ***same type*** with closely-matched parameters. But the chief requirement of a complementary amplifier is a pair of closely-matched but ***oppositely-doped*** power transistors. The term 'complementary' arises from the fact that one transistor is *PNP* type and the other is of *NPN* type. They have ***symmetry i.e.*** both are made with the same material and technology and have the same maximum rating.

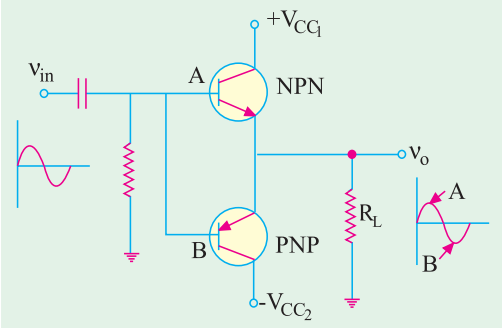


Fig. 60.36

An elementary complementary symmetry class-B push-pull amplifier is diagrammed in Fig. 60.36. The two transistors are complementary to each other and operate as emitter-follower amplifiers. The input is capacitively-coupled whereas output is direct-coupled.

With no input signal, neither transistor conducts and, therefore, current through R_L is zero.

When input signal is positive-going, transistor *A* is biased into conduction whereas *B* is driven into cut-off. When the signal is negative-going. *A* is turned OFF while *B* conducts. Obviously, this circuit is a push-pull amplifier because ***turning one transistor ON turns the other OFF.***

The circuit possesses the essential characteristics of an emitter follower *i.e.* unity voltage gain, no phase inversion and input impedance much higher than output impedance.

The circuit shown in Fig. 60.36 requires two dc supply batteries. Only one dc battery would be enough if ***'totem-pole'*** circuit configuration is used.

Due to the elimination of transformer both the high and low-frequency responses of the circuit are extended apart from reduced cost and weight.

60.28. Class-C Amplifier

In such amplifiers, the active device *i.e.* the transistor is biased much beyond cut-off. Hence,

1. output current flows only during a part of the possible half-cycle of the input signal,
2. there is no output current flow during any part of the negative half-cycle of the input signal,
3. output signal has hardly any resemblance with the input signal. It consists of short pulses only,
4. class-C amplifiers have high circuit efficiency of about 85 to 90%.

Because of his distortion, class-C amplifiers are not used for audio-frequency work. They are used for high-power output at radio frequencies (*i.e.* RF amplifiers) where harmonic distortion can be removed by simple circuits. In reality, they are used as high-frequency power switchers in radio transmitters rather than as amplifiers.

60.29. Tuned Amplifier

The gain of a transistor amplifier depends directly on the value of its load impedance. Such a high impedance can be obtained by using a high-*Q* tuned or resonant *LC* circuit as load (Fig. 60.37). The frequency response curve of the amplifier assumes the same shape as the resonance curve of the tuned circuit. Obviously, only a narrow band of frequencies around the resonant frequency f_o would be amplified well whereas other frequencies would be discriminated against.

Non-linear distortion is eliminated because of high selectivity of the load impedance. Hence, output is nearly sinusodial. With the removal of distortion, high amplifier efficiency can be achieved by operating the transistor in its nonlinear region.

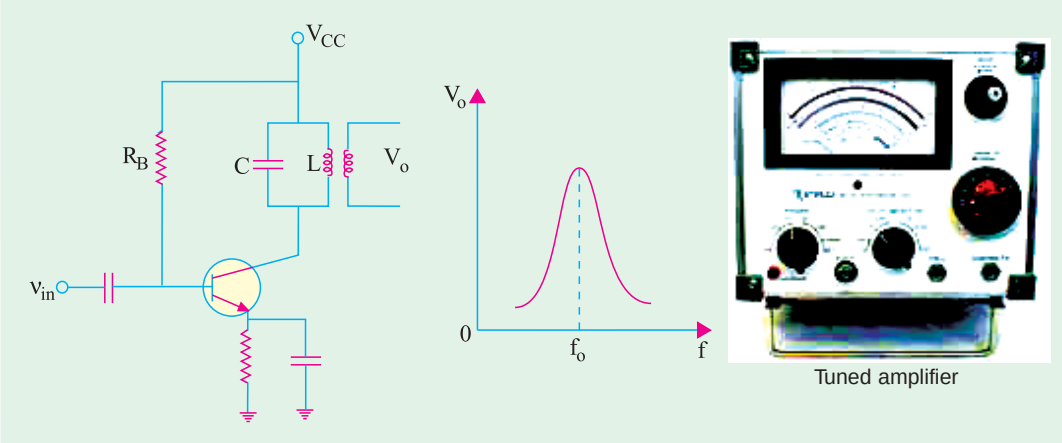


Fig. 60.37

60.30. Distortion in Amplifiers

Amplifiers are supposed to produce an output which does not differ from the input in any respect except amplitude *i.e.* the output is expected to be larger than the input. In actual practice, it is impossible to construct such an ideal amplifier whose output is an exact duplication or replica of the input. The output is always found to differ from the input either in its waveform or frequency content. This difference between the output and input of an amplifier is called distortion.

The amplifier distortions may be divided into two broad categories depending on the region of the characteristic used by the transistor and the associated circuit and device reactances.

(*a*) non-linear distortion

This occurs when transistor operates in the non-linear region of its characteristic.

(*i*) when we visualize the signal in time domain, it is called amplitude distortion or waveform distortion;

(*ii*) when we think of the signal in the frequency domain, it is called harmonic distortion.; This distortion occurs when input signal is of one frequency, say, a pure sine wave.

(*iii*) Intermodulation (IM) distortion—when input signal has more than one frequency (like speech).

Non-linear distortion occurs in the case of large-signal inputs when the active device is driven into the non-linear regions of its characteristic.

(*b*) linear distortion

It occurs even when the active device is ***working on linear part of its characteristic with small-signal inputs.*** It is primarily due to frequency-dependent reactances associated with the circuit or active device itself and occurs when input signal is composite *i.e.* has signals of different frequencies (say, fundamental and its harmonics). However, output contains no frequencies other than those at the input. It may be further subdivided into :

(*i*) ***frequency distortion***—due to unequal amplification of different frequencies present in the input signal;

(*ii*) ***phase or delay distortion***—due to unequal phase shift of various signal components.

60.31. Non-Linear Distortion

Fig. 60.38 gives time-domain view of this distortion when it is called ***amplitude distortion.*** As seen, the positive half-cycle of the input signal has been amplified more than its negative half-cycle. Consequently, waveshape of the output signal differs from that of the input signal. As shown below, it is due to the appearance of new frequencies (called harmonics) at the output which are not present in the input.

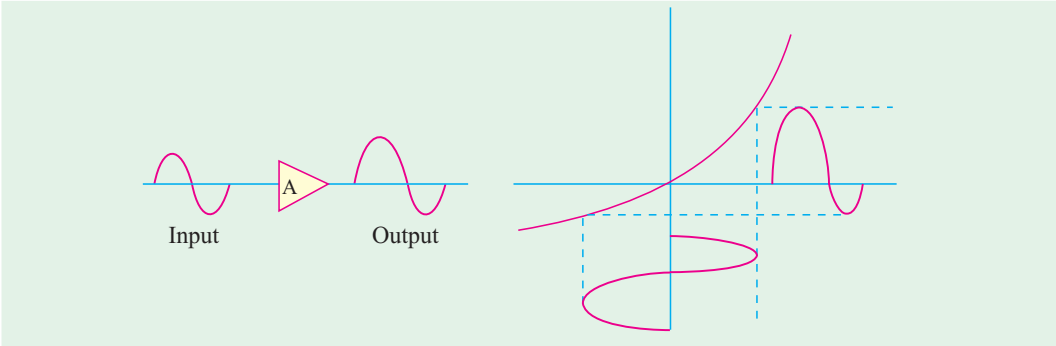


Fig. 60.38

The frequency-domain view of non-linear distortion is shown in Fig. 60.39. The input is a single-frequency (f_1) signal but output signal contains dc component and different harmonics *i.e.* frequencies which are harmonically related to each other. These harmonics are an integral multiple of the input signal frequency. Consequently, the output is distorted—the magnitude of distortion depending on the strength and number of these harmonics.

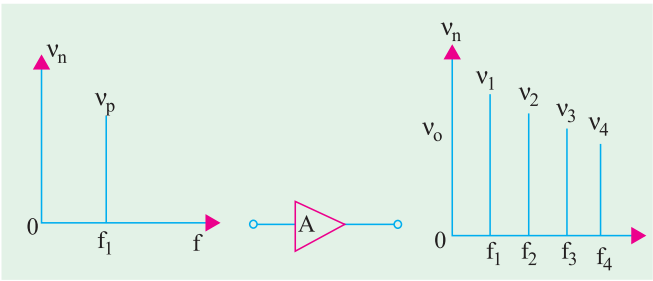


Fig. 60.39

In audio amplifiers used for amplification of speech or music, lesser the harmonic distortion, the better. For speech, harmonic distortion should not exceed 10% otherwise intelligibility will suffer. High-fidelity amplifiers have harmonic and *IM* distortion of less than 1 per cent.

60.32. Intermodulation Distortion

It occurs when input signal (like speech) consists of ***more than one frequency***. It is also a type of non-linear distortion which generates frequency components not harmonically related to the signal frequencies. Suppose, an input signal contains two frequencies f_1 and f_2. The output signal will contain their harmonics *i.e.* f_1, $2f_1$, $3f_1$ etc. and f_2, $2f_2$, $3f_2$ etc. In adddition, there would be components $(f_1 + f_2)$ and $(f_1 - f_2)$ and also the sum and difference of the harmonics. These sum and difference frequencies are called intermodulation (*IM*) frequencies and are quite undesirable in amplifiers because they subtract from original intelligence. Since *IM* frequencies are not harmonically related to the signal, they are easily detected by human ear as noise. Hence, great care is taken to minimize them particularly in hi-fi audio amplifiers.

60.33. Frequency Distortion

It occurs even when the device is working with small-signal inputs over linear region of its characteristic. It is basically due to change in the amplifier gain with frequency *i.e.* the different input signal frequencies are amplified by different amounts.

Suppose the input signal has two frequencies of 50 Hz and 100 Hz. If they are equally amplified, their resultant output wave is as shown in Fig. 60.40.

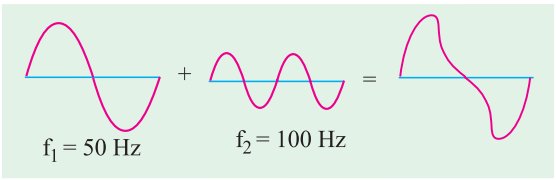


Fig. 60.40

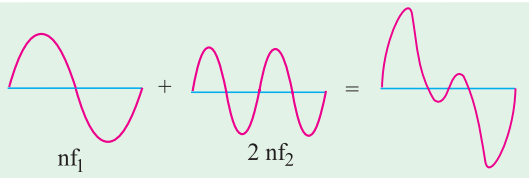


Fig. 60.41

Suppose that the amplifier amplifies the first frequency n times and the second one 2n times. Then their frequency-distorted output would be as shown in Fig. 60.41.

This distortion is due to the various frequency-dependent reactances (both capacitive and inductive) associated either with the circuit or the active device itself. In the case of audio signals the frequency distortion leads to a change in the quality of sound. Hence, in the design of untuned or wide-band amplifiers, special steps are taken to reduce variation of gain with signal frequency.

60.34. Phase or Delay Distortion

Phase distortion is said to take place when phase angles between the component waves of the output are not the same as the corresponding angles of the input. These changes in phase angles are also due to frequency-dependent capacitive and inductive reactances associated with the circuit and the active device of the amplifier.

As seen from Fig. 60.42, there has been a phase shift in the third harmonic. Hence, the resultant wave so obtained is entirely different from the input wave.

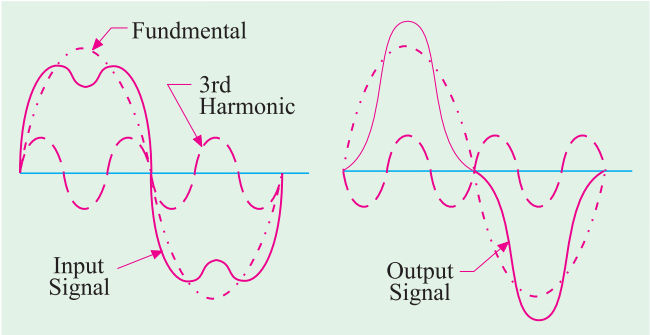


Fig. 60.42

This type of distortion which is due to the non-uniform phase shift of different frequency components, is difficult to eliminate. It should be noted that if all the frequency components in a signal are shifted in phase by an integral multiple of 180°, the resultant output waveform is not changed, although the polarity of the wave may be altered but changes in polarity do not constitute distortion.

Fortunately, the human ear is unable to distinguish phase difference (though eye can) and is thus not sensitive to this distortion. Consequently, phase distortion is of no practical significance in audio amplifiers. But in amplifiers used in television sets (video amplifiers) and other systems where ear is not the final receiver, elimination of phase distortion is important.

60.35. Noise

In general , it may be defined as any kind of unwanted signal not derived from or related to the input signal. Just as distortion is the limiting factor in the amplification of large signals, noise is the limiting factor in the case of small signals.

The signal-to-noise (S/N) ratio should be high for good signal intelligibility. Since all amplifiers contribute some noise of their own to the signal being processed, their output S/N is bound to be less than their input S/N. High quality amplifiers are designed to have an output S/N as close to input S/N as possible. The amplifier performance is measured in terms of noise factor given by

$$\text{Noise factor, } F = \frac{\text{input } S/N}{\text{output } S/N} = \frac{S_i/N_i}{S_0/N_0};$$ obviously, ***F is greater than unity.***

When expressed in decibels, the noise factor is called noise figure (*NF*).

$$\therefore \quad NF = 10 \log_{10} F \, dB$$

If an amplifier could be built which generated no noise of its own, then

(*i*) $S_i/N_i = S_o/N_o$ **(*ii*)** $F = 1$ **(*iii*)** $NF = 0$ dB

If G is the power gain of the amplifier, then output and input signal powers are related by

$$G = \frac{S_0}{S_i} \quad i.e. \quad S_o = G.S_i$$

The output noise $N_o = GN_i + N_A$

where N_i = input noise power

N_A = noise power generated by amplifier itself

Example 60.13. *The signal input to a small-signal amplifier consists of 50 μW of signal power and 0.5 μW of noise power. The amplifier generates an internal noise power of 50 μW and has a gain of 20 dB. For this amplifier, compute*

***(a)** input S/N, **(b)** output S/N, **(c)** noise factor, **(d)** noise figure.*

(Electronics & Commun. Engg. Pune Univ. 1991)

Solution. (*a*) $\frac{S_i}{N_i} = \frac{50}{0.5} = 100$

(*b*) Now, $10 \log_{10} G = 20$ dB or $10 \log_{10} G = 2$ or $G = 10^2 = 100$

$S_o = GS_i = 100 \times 50 = 5000$ μW

$N_o = GN_i + N_A = 100 \times 0.5 + 50 = 100$ μW $\quad \therefore \quad \frac{S_0}{N_0} = \frac{5000}{100} = 50$

(*c*) $F = \frac{S_i / N_i}{S_0 / N_0} = \frac{100}{50} = 2$

(*d*) $N_F = 10 \log_{10} 2$ dB $= 10 \times 0.3 = 3$ dB

60.36. The Decibel System

The decibel system of measurement is widely used in audio, radio, TV and instrument industries for comparing two voltage or power levels. It is based on the established fact that an individual's response to ***seeing or hearing is nonlinear.*** It has been found that the changes in power and audio levels are related logarithmically. For example, when power is increased from 4 W to 16 W, the audio level does not increase 16/4 = 4 times but by a factor of 2 ($\because 4^2 = 16$). Similarly, for a power change from 4 W to 64 W, the audio level changes by a factor of 3 and not by 64/4 = 16. In logarithmic form, the relationship can be written as

increase in audio level = $\log_{10} 64 = 3$.

While comparing two powers, it is common practice to choose the log base of 10. Suppose, we want to compare any two powers P_1 and P_2. The simple method is to take their ratio P_2/P_1 and to state how many times P_2 is bigger or smaller than P_1 which is used as reference power. In the decibel system, we take the log of this ratio.

$\therefore$ power level = $10 \log_{10} R_2/P_1$ bel

Since bel is too large, we use decibel (dB) instead. Remembering that 1 bel = 10 decibel, the power level becomes

$= 10 \log_{10} P_2/P_1$

Similarly, if P_i is the input power of an amplifier and P_o its power output, then power gain of the amplifier in decibels is

$= 10 \log_{10} P_0/P_i$ dB

Also, if the output power of an amplifier changes from P_1 to P_2, the power level change is

$= 10 \log_{10} P_2/P_1$

It is obvious that dB is the unit of power change (*i.e.* increase or decrease) and not of power itself.

Another point worth remembering is that 20 dB is not twice as much power as 10 dB.

Example 60.14. *An amplifier has an input signal of 16 V peak-to-peak and an input impedance of 320 K. It gives an output voltage of 8 V peak-to-peak across a load resistor of 4 W. Calculate the dB power gain of the amplifier.* **(Electronics-I, Gujarat Univ.)**

Solution. $P_i = \frac{V_i^2}{R_i} = \frac{(16/2\sqrt{2})^2}{320\,\text{K}} = 100\,\mu\text{W}$

where V_i = rms value of input voltage = $\frac{V_{i\times p-p}}{2\sqrt{2}}$

If V_o is the rms value of the output voltage, then

$$P_o = \frac{V_o^2}{R_o} = \frac{(8/2\sqrt{2})^2}{4} = 2\,\text{W}$$

$\therefore$ power amplification, $\frac{P_0}{P_i} = \frac{2\,\text{W}}{100\,\mu\,\text{W}} = 20,000$

decibel power gain = $10 \log_{10} 20{,}000 = 10 \times 4.4 =$ **43 dB**

60.37. Other Expressions for Power Gain

Suppose P_o and P_i are the respective output and input powers of an amplifier, R_o and R_i are its output and input resistances and V_0 and V_i the rms values of output and input voltages, then

$$P_o = \frac{V_o^2}{R_o} \quad \text{and} \quad P_i = \frac{V_i^2}{R_i}$$

Power amplification, $A_p = P_o/P_i$. But power gain is given by

$$G_p = 10\log_{10} P_o/P_i = 10\log_{10} \frac{V_0^2/R_o}{V_i^2/R_i} = 10\log_{10} (V_o/V_i)^2 . R_i/R_o$$

$$= \left[\log_{10} (V_o/V_i)^2 + \log_{10} R_i/R_o\right] = 10\log_{10} (V_0/V_i)^2 + 10\log_{10} R_0/R_i$$

$$= 20\log_{10} V_o/V_i + 10\log_{10} R_i/R_o$$

Also $P_o = I_o^2 R_0$ and $P_i = I_i^2 R_i$

$\therefore$ power gain, $G_p = 10\log_{10} P_o/P_i = 10\log_{10} I_o^2 R_o/I_i^2 R_i = 10\left[\log_{10} (I_o/I_i)^2 R_o/R_i\right]$

$$= 10\left[2\log_{10} I_o/I_i + \log_{10} R_o/R_i\right] = 20\log_{10} I_o/I_i + 10\log_{10} R_o/R_i \text{ dB}$$

Of course, if $R_o = R_i$, then $\log_{10} R_o/R_i = \log_{10} 1 = 0$

Hence, in that case, power gain, $G_p = 20\log_{10} I_o/I_i$ dB

60.38. Voltage and Current Levels

Though decibel was initially defined as the unit of power level, it can also be used with voltage and current levels.

Suppose, an amplifier has an input voltage of V_i and gives an output voltage of V_0. Then, its voltage amplification is $A_n = V_o/V_i$ but its decibel voltage gain is

$$G_v = 20 \log_{10} A_v \text{dB} = 20\log 10\ V_0/V_i$$

It should be carefully noted that multiplying factor of 20 has been used and not of 10 as is done for finding power level. In fact, it implies that R_o has been taken equal to R_i in the power gain equation derived in Art 60.37.

Similarly, current amplification is $A_i = I_o/I_i$. However, current gain of the amplifier is

$$G_i = 20 \log_{10} I_o/I_i \text{dB}$$

Example 60.15. *The input and output voltages of a network are 16 V and 8V respectively. If input impedances are equal, find the voltage gain.*

Solution. $G_v = 20 = \log_{10} V_o/V_i = 20 \log_{10} {}^{8/16}$
$= 20\,[\log_{10} 8 - \log_{10} 16] = 20\,(0.90 - 1.2) = 20 \times (-0.3) = -$ **6 dB.**

Alternative Method

When the voltage ratio is less than 1, *its log is negative* which is often difficult to handle. In such cases, it is best to invert the fraction and then make the result negative. The above problem could be solved thus :

$G_v = -20 \log_{10} {}^{16/8} = -20 \times 0.3 -$ **6 dB**

Example 60.16. *A microphone delivers 30 mV to the 300 Ω input of an amplifier. The ac power delivered to an 8-Ω speaker is 18 W. What is the power gain of the amplifier ?*

Solution. We may use any one of the equations given above.

$G_p = 20 \log_{10} V_0/V_i + 10 \log_{10} R_i/R_0$

Now, $P_o = \dfrac{V_o^2}{R_o}$ or $18 = \dfrac{V_o^2}{8}$ $\therefore$ $V_o = \sqrt{8 \times 18} = 12$ V

$\therefore$ $G_p = 20 \log_{10} 12\text{V}/30\text{mV} + 10 \log_{10} 300/8 = 20 \log_{10} 400 + 10 \log_{10} 37.5$
$= 20 \times 2.6 + 10 \times 1.57 =$ **67.7 dB.**

Example 60.17. *The output power of an amplifier is 100 mW when the signal frequency is 5 kHz. When the frequency is increased to 25 kHz, the output power falls to 50 mW. Calculate the dB change in power.* **(Electronics, Gujarat Univ. 1991)**

Solution. The decibel change in power level
$= 10 \log_{10} 50/100 = 10 \log_{10} {}^{1/2} = -10 \log_{10} 2 = -10 \times 0.3 =$ **– 3 dB.**

Example 60.18. *The output voltage of an amplifier is 10 V at 5 kHz and 7.07 V at 25 kHz. What is the decibel change in output power level ?*

Solution. Since changes in voltage are across the same output resistance, the decibel change in power is

$= 20 \log_{10} V_2/V_1 = 20 \log_{10} 7.07/10$ dB $= -20 \log_{10} 10/7.07 = -20 \log_{10} 1.414$ dB
$= -20 \times 0.15 = -3$ dB

It is seen from Ex. 60.17 and Ex. 60.18 that output decibel power falls by a 3 dB when

(i) *absolute value of power falls to half its original value*

or

(ii) *output voltage falls to 0.707 or* $1/\sqrt{2}$ *of its original value.*

Example 60.19. *A certain radio receiver delivers an output power of 3.6 W.*
(i) what would be the decibel gain if power output is increased to 7.2 W ?
(ii) what power output would be required to produce a power gain of 10 dB?

Solution. ***(i)*** $G_p = 10 \log_{10} P_2/P_1 = 10 \log_{10} 7.2/3.6 =$ **3 dB**

(ii) Here, $G_p = 10$, $P_1 = 3.6$ W, $P_2 = ?$

$\therefore$ $10 = 10 \log_{10} P_2/3.6$ $\therefore$ $\log_{10} P_2/3.6 = 1$ or $\dfrac{P_2}{3.6} = 10^1 = 10$

$\therefore$ $P_2 = 3.6 \times 10 =$ **36 W**

60.39. Characteristics of the Decibel System

1. a decibel is a measure of **ratio** and not of **an amount**. It tells us how many times one quantity is greater or lesser with respect to another or the reference quantity. It does not measure the actual (or absolute) voltage or power but only their **changes;**

2. decibel is non-linear *i.e.* 20 dB is not twice as much power or voltage as 10 dB;
3. this log-based system allows a tremendous range of power ratios to be encompassed by using only two-digit numbers. For example
   1 dB = 1.26 : 1 power ratio and 50 dB = 100,000 : 1 power ratio;
4. total dB of a cascaded amplifier can be found by simply adding the stage dBs.

60.40. Value of 1 dB

It can be proved that 1 dB represents the log of two powers which ***have a ratio of 1.26.***

$$1 \text{ dB} = 10 \log_{10} P_2/P_1$$

$$\therefore \quad \log_{10} P_2/P_1 = \frac{1}{10} = 0.1 \quad or \quad \frac{P_2}{P_1} = 10^{0.1} = 1.26$$

Hence, +1 dB represents an increase in power of 26%.

60.41. Zero Decibel Reference Level

By now, it should be clear that decibel does not measure any physical quantity like voltage or power etc., but merely ratio of two physical quantities. For determining the power levels of various powers like P_1, P_2, P_3 etc. it is essential to fix some reference power with which their ratios can be taken. If we fix P_0 as the reference power, then different power levels would be

$$10 \log_{10} P_1/P_o \quad \text{and} \quad 10 \log_{10} P_2/P_o \text{ etc.}$$

Obviously, ***P_o cannot be 0 watt i.e.*** 0 watt cannot be taken as 0 dB because, in that case, power level of P_1 would be

$$= 10 \log_{10} P_1/0 = \infty$$

In fact, any power compared with zero power is infinity. Hence, decibels would not be defined if zero watt is taken as zero reference level.

Following three zero reference levels are in common use :

1. Zero dB refers to 6 mW dissipated in a 500 Ω resistive load.
   The reference voltage value corresponding to 0 dB is
   $P = V^2/R \quad \text{or} \quad 6 \times 10^{-3} = V^2/500 \quad \text{or} \quad V = 1.73 \text{ V}$
2. Zero dB refers to 1 mW dissipated in 600 Ω.
   Here, $1 \times 10^{-3} = V^2/600$; V = 0.774 V
3. Zero dB refers to a 1-mW dissipation.
   It is written as dBm indicating that it uses 1 mW as a reference.
   This reference does not depend on any particular load impedance value.
   Calculations are performed by using the relation.

$$G_p = 10 \log_{10} P_2/0.001 \text{ dB}_{\text{in}}$$

60.42. Variations in Amplifier Gain with Frequency

If the input voltage of an amplifier is kept constant but its frequency is varied, it is found that the amplifier gain

(i) remains practically constant over a sizable range of mid-frequencies,
(ii) decreases at low as well as high frequencies.

A typical frequency- versus-gain curve is shown in Fig. 60.43. While analyzing this curve, three value of frequency are important

(i) mid-frequency range, ***(ii)*** lower cut-off frequency, f_1,
(iii) upper cut-off frequency, f_2.

The lower and upper cut-off frequencies are defined as those frequencies.

(*a*) where voltage gain of the amplifier decreases to 0.707 times the mid-frequency gain *i.e.* to

$$\frac{1}{\sqrt{2}} A_{v(mid)} = 0.707\, A_{v(mid)}$$ or

(*b*) in terms of power, where power amplification of the amplifier decreases to half its value at mid-frequencies

i.e. $A_{p.1}$

or $A_{p.2} = \frac{1}{2} A_{p(mid)}$

(*c*) in terms of decibels, where power gain **falls by 3 dB**.

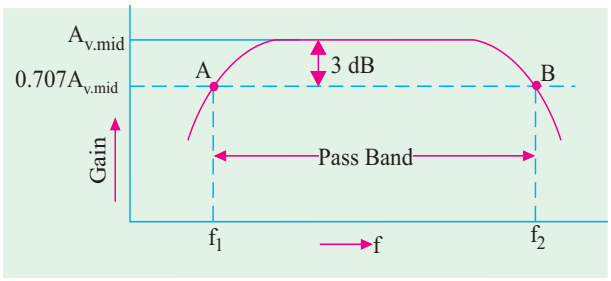


Fig. 60.43

That is why the two cut-off frequencies are referred to as

(*i*) –3 dB frequencies or

(*ii*) down 3 dB frequencies or

(*iii*) 3 dB loss frequencies.

The points A and B in Fig. 60.43 are called 3-dB points or (sometime) as ***minus 3 dB points***. The cut-off frequencies are also called roll-off frequencies because at these frequencies the amplifier gain starts rolling down from its midband value or maximum value.

The frequency span between these two cut-off frequencies is called the passband or bandwidth (BW) of the amplifier.

$$\therefore \quad \Delta f = f_2 - f_1 = \text{band width (BW)} = \text{passband}$$

All frequencies lying between f_1 and f_2 are amplified almost equally.

For maximum bandwidth, the stray capacitances in the amplifier must be kept to the minimum.

60.43. Causes of Amplifier Gain Variations

The primary cause of gain variation in amplifiers is the presence of capacitances, some of which are connected in series along the signal path and some in parallel.

The different types of external capacitances present in an amplifier circuit are :

1. coupling and bypass capacitors–usually of large capacitance value,
2. internal or inter-element capacitances of the transistor [Fig. 60.44 (a)] and stray wiring capacitance [Fig. 60.44 (*b*)].

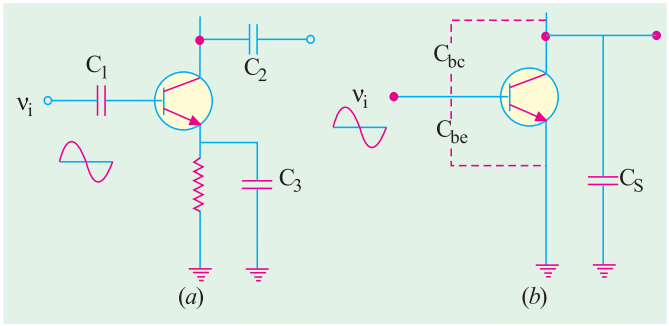


Fig. 60.44

The coupling and bypass capacitors [C_1, C_2 and C_3 in Fig. 60.44 (*a*)] are series-connected whereas interelement capacitances and stray capacitances are parallel-connected [Fig. 60.44 (*b*)] to the signal path. At mid-frequencies, C_1, C_2 and C_3 act almost as 'shorts' but C_{bc} and C_{be} act as 'open'. Hence, their effect on mid-frequencies is negligible.

However, at low frequencies, series-connected capacitors *i.e.* coupling and by-pass capacitors offer relatively large reactance thereby dropping off a large part of the input signal. Hence, amplifier gain starts decreasing as frequency is lowered.

The internal transistor capacitances and stray capacitances are ***(i)*** small and ***(ii)*** parallel-connected to the signal path. At low frequencies, they offer very high reactance and so act as effective 'open'. But with increase in frequency, their reactance keeps decreasing till at very high frequencies, they almost short or shunt the ac signal to ground both at the input and output ends. This explains why amplifier gain starts decreasing at high frequencies.

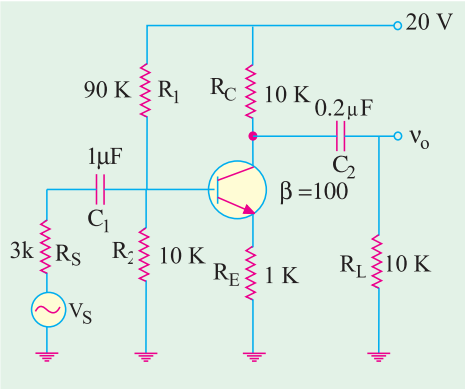


Fig. 60.45

In summary

(i) series-connected coupling and bypass capacitors cause decrease in amplifier gain at low frequencies,

(ii) parallel-connected internal transistor capacitances and stray wiring capacitances cause decrease in amplifier gain at high frequencies.

Example 60.20. *For the RC- coupled circuit of Fig. 60.45, calculate the lower cut-off frequency **(i)** at C_1 **(ii)** at C_2 and **(iii)** for the amplifier.*

Solution. The low-frequency cut-off for each coupling capacitor is given by

$$f_1 = \frac{1}{2\pi C R_{eq.}}$$

where R_{eq} is the resistance 'seen' by the capacitor on its right and left.

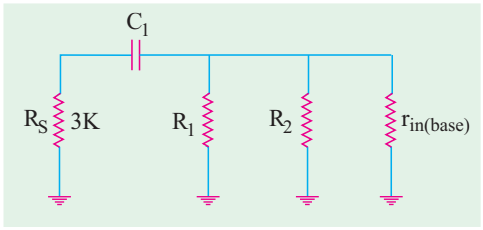


Fig. 60.46

(i) f_1 at C_1

For finding the resistance 'seen' by C_1, the dc and ac sources are shorted out because they have negligible resistance. The circuit becomes as shown in Fig. 60.46.

$$R_{eq} = R_S + R_1 \| R_2 \| r_{in(base)}$$

$$\text{Now, } r_{in(base)} = \beta(r_e + R_E) \cong \beta R_E$$

$$= 100 \text{ K}$$

$$\therefore \quad R_{er} = 3\text{K} + 90\text{K} \| 10\text{K} \| 100\text{K}$$

$$= 3 + 8.26$$

$$= 11.26 \text{ K}$$

$$\therefore \quad f_1 = \frac{1}{2\pi \times 11.26 \times 10^3 \times 1 \times 10^{-6}} = \textbf{40 Hz}$$

(ii) f_1 at C_2

Let us first find the resistance 'seen' by C_2. On one side, it sees R_L = 10 K load resistor to the ground and on the other side, it sees R_C to ground in parallel with the resistance seen looking into transistor collector.

Since *C/B* junction is reverse-biased, its resistance is very high. If we consider it as 'open', the equivalent circuit becomes as shown in Fig. 60.47.

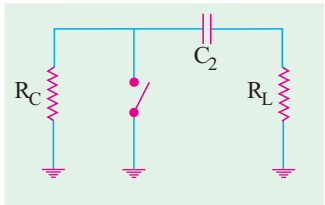


Fig. 60.47

$$\therefore \quad R_{eq} = R_L + R_C = 10 + 10 = 20 \text{ K}$$

$$\therefore \quad f_1 = \frac{1}{2\pi \times 20 \times 10^3 \times 0.2 \times 10^{-6}} = \textbf{Hz}$$

(iii) Since cut-off frequency for C_2 occurs at 40 Hz while cut-off for C_1 occurs way down at 14 Hz, C_2 determines the lower cut-off frequency for the amplifier i.*e.* **14 Hz.**

60.44. Miller Effect

According to this effect, when viewed from input base terminal of the *CE*-connected transistor, the capacitance C_{bc} appears as $(1 + A_v)\, C_{bc}$ *i.e.* it is amplified by a factor of $(1 + A_v)$. In fact, Miller effect takes into account the feedback from the collector to base and *vice-versa* due to C_{bc}.

Proof

When V_i is applied to the transistor's base in Fig. 60.44 (*b*), the change in collector voltage is

$$\Delta V_C = -A_v . V_i$$

The negative sign is due to phase shift inherent in a *CE* amplifier. As seen, V_C is reduced by $(A_v . V_i)$ when base voltage is increased by V_i. Hence, total reduction in collector base voltage is

$$\Delta V_{CB} = V_i + A_v . V_i = V_i (1 + A_v)$$

This also represents the change of voltage across C_{bc} because it is connected across collector and base. Using $Q = CV$, the charge supplied to the input of the circuit is

$$Q = C_{bc} \times \Delta V_{CB} = C_{bc} \times (1 + A_v)\, V_i = (1 + A_v)\, C_{bc} \times V_i$$

Hence, C_{bc} appears as $(1 + A_v)\, C_{bc}$ when looked from the input side of the circuit.

Incidentally, the total input capacitance to the transistor is

$$C_{in} = C_{be} \| (1 + A_v) C_{bc} = C_{be} + (1 + A_v)\, C_{bc}$$

At high frequencies, C_{in} reduces the input impedance of the circuit and affects the frequency response (Ex. 60.22).

Example 60.21. *A CE-connected amplifier has C_{bc} = 4 pF, C_{be} = 10 pF and r_e = 50Ω. If circuit load resistor is 10 K, calculate the value of C_{in}.*

Solution. $A_v \cong \dfrac{R_E}{r_e} = \dfrac{100\,K}{50\,\Omega} = 200$ $\quad \therefore \quad C_{in} = 10 + (1 + 200)4 =$ **814 pF**

Example 60.22. *Calculate the upper cut-off frequency of the CE amplifier shown in Fig. 60.48. Given the input wiring capacitance C_{wi} = 40 pF, C_{bc} = 8 pF, C_b = 10 pF and β = 100.*
(Electronic Engg-I, Osmania Univ.)

Solution. $f_2 = \dfrac{1}{2\pi R_{eq} C_{in}}$

The voltage amplification of the amplifier is

$$A_v \cong \frac{R_C \| R_L}{R_L} = \frac{20\,K \| 20\,K}{400\,\Omega} = 25$$

The total capacitance from base to ground is

$C_{in} = C_{wi} + C_{be} + (1 + A_v)\, C_{bc}$
$= 40 + 10 + (1 + 25) \times 8 = 258$ pF

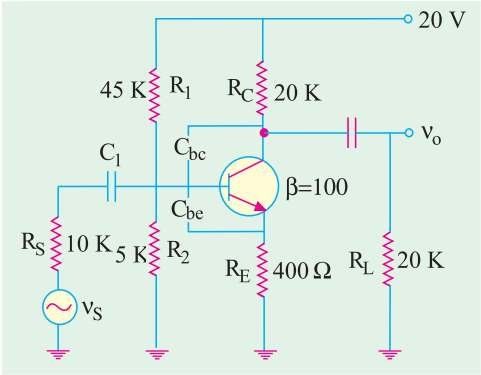


Fig. 60.48

Now, let us determine the resistance 'seen' by C_{in}. If we look to the right, a resistance of $\beta (r_e + R_E) \cong \beta R_E$ is seen while to the left $R_1 \| R_2 \| R_S$ is seen–all in parallel.

$$\therefore \quad R_{eq} = R_1 \| R_2 \| R_S \| \beta R_E$$
$$= 45\ K \| 5K \| 10K \| 40\ K = 2.88\ K$$

$$\therefore \quad f_2 = \frac{1}{2\pi \times 2.88 \times 10^3 \times 258 \times 10^{-12}} = \textbf{214 kHz}$$

60.45. Cut-Off Frequencies of Cascaded Amplifiers

Cascading of stages gives higher amplification but narrow bandwidth because the product of the two remains almost constant. The approximate values of composite lower and upper cut-off frequencies of the cascaded amplifier having n identical stages are

$$f_{1.n} = 1.1\sqrt{n} \times f_1 \qquad \text{per stage} = \frac{f_1 \text{ per stage}}{\sqrt{(2^{1/n} - 1)}}$$

$$f_{2.n} = \frac{f_2 \text{ per stage}}{1.1\sqrt{n}} = \sqrt{(2^{1/n} - 1)} \times f_2 \text{ per stage}$$

The reduced bandwidth of the cascaded amplifier is = $f_{2n} - f_{1.n}$

60.46. Transistor Cut-off Frequencies

Even if no external stray capacitances were present in an amplifier, there would still be an upper limit on its frequency response due to

(i) internal or interelement capacitances of the transistor and

(ii) transit time of charge carriers across the transistor junctions and through the semiconductor material.

This limitation is expressed in terms of

(i) alpha cut-off frequency (f_α) and ***(ii)*** beta cut-off frequency (f_β)

The two are defined below.

60.47. Alpha Cut-off Frequency

It is found that at high frequencies, the value of transistor α begins to fall. This decrease in α is related to the transit time effect of the charge carriers as they move from the emitter to collector.

The alpha cut-off frequency f_α is that high frequency at which the α of a *CB*-connected transistor becomes 0.707 of its low-frequency value (usually 1 kHz).

For example, if value of α at 1 kHz is 0.98, then its value at f_α would be

$$= 0.707 \times 0.98 = 0.693$$

It means that at f_α, the collector current I_C would be only 0.693 of the emitter current rather than 0.98 I_E.

It is found that f_α is

(a) inversely proportional to the square of the base width,

(b) directly proportional to the minority carrier mobility.

In this regard, NPN transistors are superior to *PNP* type because electrons have greater mobility than holes. For decreasing the base transit time, base should be as thin as possible.

It may be noted that alpha cut-off frequency of any given transistor is always greater than its beta cut-off frequency f_β. In fact, $f_\alpha \cong \beta f_\beta$.

60.48. Beta Cut-off Frequency

It is that high frequency at which the β of a *CE*-connected transistor drops to 0.707 of its low-frequency (1 kHz) value.

60.49. The f_T of a Transistor

It is another high-frequency characteristic of a transistor.

It is that high frequency of a *CE*-connected transistor where its β drops to unity *i.e.* $\beta = 1$. This frequency is much larger than f_β but less than f_α. For example, for a typical transistor, their values may be $f_\beta = 6$ MHz, $f_T = 300$ MHz and $f_\alpha = 345$ MHz.

60.50. Relation Between f_a, f_b and f_T

(i) For simple junction transistors $f_\alpha = 1.2\ f_T$ and ***(ii)*** $f_\beta = \dfrac{f_T}{\beta}$, where β refers to its low-frequency value.

60.51. Gain-Bandwidth Product

At its name indicates, it is the product of the gain and bandwidth of an amplifier. It is very useful in comparing the performance capability of various circuits.

For any amplifier, gain-bandwidth product (GBP) is constant and is equal to f_T.

If, for example, β = 1, at a frequency of 6 MHz, then f_T = **6 MHz.**

∴ gain-bandwidth product = **6 MHz.**

Hence, for a given f_T in an amplifier, increased gain may be obtained only at the expense of its bandwidth.

Example 60.23. *A transistor has f_α = 8 MHz and β = 80. When connected as an amplifier, it has stray capacitance of 100 pF at the output terminal. Calculate its upper 3 dB frequency when R_L is* ***(a)*** *10 K and* ***(b)*** *100 K.*

Solution. It should be remembered that stray capacitance would reduce the amplifier gain by 3-dB when

capacitive reactance = output resistance

i.e. $$\frac{1}{2\pi f_S C_S} = R_L \quad \text{or} \quad f_s = \frac{1}{2\pi C_S R_L}$$

First, let us find the value of f_β for comparison with f_s.

$$f_\beta = \frac{f_1}{80} = \frac{8\,\text{MHz}}{80} = 100\,\text{kHZ}$$

(a) **R_L = 10 K**

$$f_2 = \frac{1}{2\pi C_s R_L} = \frac{1}{2\pi \times 100 \times 10^{-12} \times 10 \times 10^3} = 159\,\text{kHz}$$

Obviously, before this frequency is reached, cut-off would have been achieved at f_β = 100 kHz. Hence, $f_2 = f_\beta$ = **100 kHz.**

(b) **R_L=100 K**

In this case, $f_s = \frac{10}{100} \times 159 = 15.9\,\text{kHz}$

In this case, cut-off would be achieved much earlier at 15.9 kHz before f_β is reached

∴ f_2 = **15.9 kHz.**

Tutorial Problems No. 60.1

1. An amplifier raises the power level of its 5-μW input signal by 30 dB. What is the output power ? **[5 mW]**
2. An attenuation network provides an output of 5 μW with an input of 5 mW. Calculate the decibel loss of the network. **[–30 dB]**
3. What is the decibel difference between a 100 kW and 500 kW radio transmitters? **[6.98 dB]**
4. The noise level of a certain tape recording is 30 dB below the signal level. If the signal power is 5 mW, calculate the noise power. **[5 mW]**
5. An amplifier rated at 72-W output is connected to an 8 Ω speaker
   (*a*) what input power is required for full power output if power gain is 30 dB,
   (*b*) what is the input voltage for rated output if amplifier voltage gain is 40 dB.
   [(*a*) 72 μW (b) 240 mV]
6. The characteristics of a certain audio amplifier are such that it gives a voltage amplification of 10 at 100 Hz, 30 at 3 kHz and 60 at 10 kHz. Taking the amplification at 3 kHz as the reference level, calculate the loss or gain in decibels at the other two frequencies. **[–9.54 dB, 6.02 dB]**
7. An amplifier with full power rating of 100 W drives a speaker load of 16 Ω . The hum-level rating of the amplifier is 80 dB below its full-power rating. Calculate

(*i*) hum-level in the load,

(*ii*) voltage produced by the hum across the load. **[(i) 1 μW (ii) 4 mW]**

8. Derive the voltage, current and power gain relationships and then find the voltage gain of the single-stage amplifier shown in Fig. 60.49. An *NPN* transistor that has $h_{ie} = 500\ \Omega$, h_{re} negligible, $h_{fe} = 150$, and $h_{oe} = 50\ \mu S$ is used. Neglect all coupling and stray capacitances.

Comment on the various methods available for biasing the base and discuss their merits. **[– 545]**

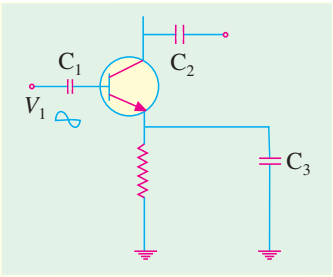


Fig. 60.49

OBJECTIVE TESTS – 60

1. A CB amplifier has very low input resistance because
(*a*) low emitter ac resistance r_e' shunts all other resistances
(*b*) it handles small input signals
(*c*) emitter bulk resistance is small
(*d*) its base is at ac ground.

2. CE amplifier is characterised by
(*a*) low voltage gain
(*b*) moderate power gain
(*c*) signal phase reversal
(*d*) very high output impedance.

3. A CC amplifier has the highest
(*a*) voltage gain (*b*) current gain
(*c*) power gain (*d*) output impedance.

4. In a CC amplifier, voltage gain
(*a*) cannot exceed unity
(*b*) depends on output impedance
(*c*) is dependent on input signal
(*d*) is always constant.

5. In a class-A amplifier, conduction extends over 360° because Q-point is
(*a*) located on load line
(*b*) located near saturation point
(*c*) centred on load line
(*d*) located at or near cut-off point.

6. The circuit efficiency of a class-A ampli- fier can be increased by using
(*a*) low dc power input
(*b*) direct-coupled load
(*c*) low-rating transistor
(*d*) transformer-coupled load.

7. In a class-A amplifier, worst-case condi- tion occurs with
(*a*) zero signal input
(*b*) maximum signal input
(*c*) high load resistance
(*d*) transformer coupling.

8. The output of a class-B amplifier
(*a*) is distortion-free
(*b*) consists of positive half-cycle only
(*c*) is like the output of a full-wave rectifier
(*d*) comprises short-duration current pulses.

9. The maximum overall efficiency of a transformer-coupled class-A amplifier is – per cent.
(*a*) 78.5 (*b*) 25 (*c*) 50 (*d*) 85

10. A transistor audio amplifier is found to have an overall efficiency of 70 per cent. Most probably, it is a ..... amplifier.
(*a*) class-B push-pull
(*b*) single-stage class-C
(*c*) transformer-coupled class-A
(*d*) direct-coupled class-A

11. The main purpose of using transformer coupling in a class-A amplifier is to make it more
(*a*) distortion-free (*b*) bulky
(*c*) costly (*d*) efficient.

12. A class-B push-pull amplifier has the main advantage of being free from
(*a*) any circuit imbalances
(*b*) unwanted noise
(*c*) even-order harmonic distortion
(*d*) dc magnetic saturation effects.

13. Crossover distortion occurs in ..... amplifiers.
(*a*) push-pull (*b*) class-A
(*c*) class-B (*d*) class AB

14. The maximum overall efficiency of a class-B push-pull amplifier cannot exceed – per cent.
(*a*) 100 (*b*) 78.5 (*c*) 50 (*d*) 85

15. The circuit of a class B push-pull amplifier is shown in Fig. 60.50 . If the peak output voltage, V_0 is 16 V, the power drawn from the dc source would be
(*a*) 10 W (*b*) 16 W (*c*) 20 W (*d*) 32 W

16. The dissipation at the collector is zero in the quiescent state and increases with excitation in the case of a
(*a*) class A series-fed amplifier
(*b*) class A transistor coupled amplifier
(*c*) class AB amplifier
(*d*) class B amplifier

17. Class AB operation is often used in power (large signal) amplifiers in order to,

(*a*) get maximum efficiency
(*b*) remove even harmonics
(*c*) overcome cross-over distortion
(*d*) reduce collector dissipation

18. The main use of a class-C amplifier is
(*a*) as an RF amplifier
(*b*) as stereo amplifier
(*c*) in communication sound equipment
(*d*) as distortion generator.

19. If a class C power amplifier has an input signal with frequency of 200 kHz and the width of collector current pulses of 0.1μS, then the duty cycle of the amplifier will be
(*a*) 1% (*b*) 2% (*c*) 10% (*d*) 20%

20. The primary cause of linear distortion in amplifiers is
(*a*) change of gain with frequency
(*b*) unequal phase shift in component frequencies
(*c*) reactances associated with the circuit and active amplifying element
(*d*) inherent limitations of the active de vice.

21. An amplifier is said to suffer from distortion when its output is
(*a*) low
(*b*) different from its input
(*c*) noisy
(*d*) larger than its input.

22. While discussing amplifier performance, noise is defined as any kind of unwanted signal in the output which is
(*a*) unrelated to the input signal
(*b*) derived from the input signal
(*c*) not generated by the amplifier
(*d*) due to associated circuitry.

23. An ideal amplifier has
(*a*) noise figure of less than 1 dB
(*b*) noise factor of unity
(*c*) output S/N more than input S/N
(*d*) noise figure of more than 0 dB.

24. The decibel is a measure of
(*a*) power (*b*) voltage
(*c*) current (*d*) power level.

25. When power output of an amplifier doubles, the increase in its power level is ..... decibels.
(*a*) 2 (*b*) 20 (*c*) 3 (*d*) 10

26. When output power level of a radio recei- ver increases by 3 dB, its absolute power changes by a factor of
(*a*) 2 (*b*) 10 (*c*) 1/2 (*d*) 3.

27. Zero watt cannot be chosen as zero deci- bel level because
(*a*) it is impossible to measure zero watt
(*b*) it is too small
(*c*) every power compared with it would be zero
(*d*) it would be impossible to define a decibel.

28. A minus 3 dB point on the gain versus frequency curve of an amplifier is that point where
(*a*) signal frequency drops to half the mid-band frequency
(*b*) voltage amplification becomes half of its maximum value
(*c*) power falls to half its maximum value
(*d*) upper cut-off frequency becomes twice the lower cut-off frequency

29. The bandwidth of an amplifier may be increased by
(*a*) decreasing the capacitance of its by pass capacitors
(*b*) minimizing its stray capacitances
(*c*) increasing input signal frequency
(*d*) cascading it.

30. Lower cut-off frequency of an amplifier is primarily determined by the
(*a*) interval capacitances of the active device used
(*b*) stray capacitance between its wiring and ground
(*c*) ac β value of its active devices
(*d*) capacitances of coupling and bypass capacitors.

31. The main reason for the variation of amp- lifier gain with frequency is
(*a*) the presence of capacitances, both ex- ternal and internal
(*b*) due to interstage transformers
(*c*) the logarithmic increase in its output power
(*d*) the Miller effect.

32. The gain-bandwidth product of an amp- lifier is given by
(*a*) $f_2 - f_1$ (*b*) $f_\alpha - f_\beta$
(*c*) f_T (*d*) βf_α

33. A Circuit which resonates at 1MHz has *a* of 100. Bandwidth between half-power points is
(*a*) 10 kHz (*b*) 100 kHz
(*c*) 10 Hz (*d*) 100 Hz

(UPSC Engg. Services 2002)
(Hint : BW = F_o/a)

ANSWERS

1. (*a*)	**2.** (*c*)	**3.** (*b*)	**4.** (*a*)	**5.** (*c*)	**6.** (*d*)	**7.** (*a*)	**8.** (*b*)	**9.** (*c*)	**10.** (*a*)	**11.** (*d*)
12. (*c*)	**13.** (*a*)	**14.** (*b*)	**15.** (*b*)	**16.** (*d*)	**17.** (*a*)	**18.** (*a*)	**19.** (*d*)	**20.** (*c*)	**21.** (*b*)	**22.** (*a*)
23. (*b*)	**24.** (*d*)	**25.** (*c*)	**26.** (*a*)	**27.** (*d*)	**28.** (*c*)	**29.** (*b*)	**30.** (*d*)	**31.** (*a*)	**32.** (*c*)	**33.** (*a*)

GO To FIRST

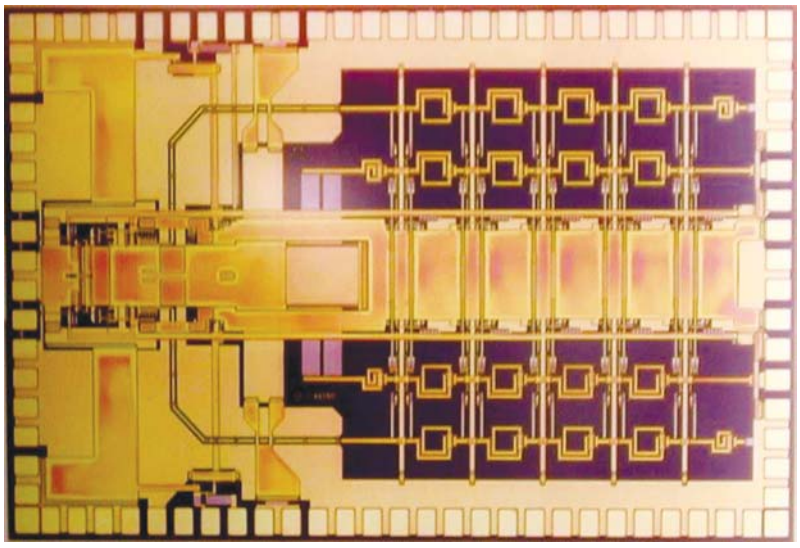
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CHAPTER 61

MULTISTAGE AND FEEDBACK AMPLIFIERS

Learning Objectives

- General
- Amplifier Coupling
- RC-coupled Two-stage Amplifier
- Impedance-coupled Two-stage Amplifier
- Advantages of Impedance Coupling
- Transformer-coupled Two-stage Amplifier
- Advantages of Transformer Coupling
- Frequency Response
- Direct-coupled Two-stage Amplifier Using Similar Transistors
- Direct-coupled Amplifier Using Complementary Symmetry of Two Transistors
- Darlington Pair
- Advantages of Darlington Pair
- Comparison Between Darlington Pair and Emitter Follower
- Special Features of a Differential Amplifier
- Common Code Input
- Differential Amplifier



In a multistage amplifier, a number of single amplifiers are connected an cascade arrangement *i.e.* the output of first stage is connected to the input of second stage.

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61.1. General

Often, the voltage amplification or power gain or frequency response obtained with a single stage of amplification is insufficient to meet the requirements of either a composite electronic circuit or a load device. Hence, two or more single stages of amplification are frequently used to achieve greater voltage or current amplification or both. In such cases, the output of one stage serves as input of the next stage as shown in Fig. 61.1. Such amplifiers may be divided into following two categories :

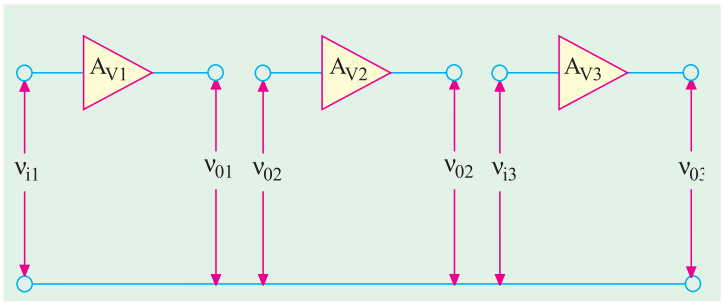


Fig. 61.1

(*a*) Cascaded Amplifiers

In these amplifiers, each stage as well as the type of interstage coupling used are identical.

(*b*) Compound Amplifiers

In these amplifiers, each stage may be different from the other (one may be *CE* and the other may be *CC* stage) and also different types of interstage couplings may be employed.

As stated above, in cascaded amplifiers, the output ac voltage of the first stage becomes the input voltage of the second stage and the ac output of the second stage becomes the input of the third stage and so on. The overall voltage gain of the cascaded amplifier is equal to the ***product*** (not the sum) of the gain of the individual stages.

$$\therefore \qquad A_v = A_{v1} \times A_{v2} \times A_{v3} \times \ldots\ldots\ldots$$

However, when the voltage gain is expressed in decibels (dB), then the overall decibel gain of the multistage amplifier is equal to the ***sum*** of the dB gains of the individual stage *i.e.*

$$G = G_1 + G_2 + G_3 + \ldots\ldots\ldots$$

Similarly, the overall current amplification is given by

$$A_i = A_{i1} \times A_{i2} \times A_{i3} \times \ldots\ldots\ldots$$

The overall power gain is given by

$$A_p = A_v . A_i \qquad \text{and} \qquad G_p = 10 \log_{10} A_P \text{ dB}$$

Suppose in a two-stage cascaded amplifier, first stage has a voltage amplification of 2000 (dB gain of the $20 \log_{10} 2000 = 66$ dB) and second stage has corresponding values of 1000 (60 dB). If the ac output of first stage is fed into the second stage, the overall amplification would theoretically become $= 1000 \times 2000 = 2 \times 10^6$ which corresponds to a dB gain of $(60 + 66) = 126$ *i.e.* $20 \log_{10} 2 \times 10^6 = 20 \times 6.3 = 126$. The above result would be true only when we ***neglect the loading effect of first stage by the second stage.*** It would be approximately true so long as the impedance looking into the input of second stage is much greater than the output impedance of the first stage. Otherwise, the overall gain would be much less.

61.2. Amplifier Coupling

All amplifiers need some ***coupling network***. Even a single-stage amplifier has to be coupled to the input and output devices. In the case of multistage systems, there is ***interstage*** coupling. The type of coupling used determines the characteristics of the cascaded amplifier. In fact, amplifiers are classified according to the coupling network used.

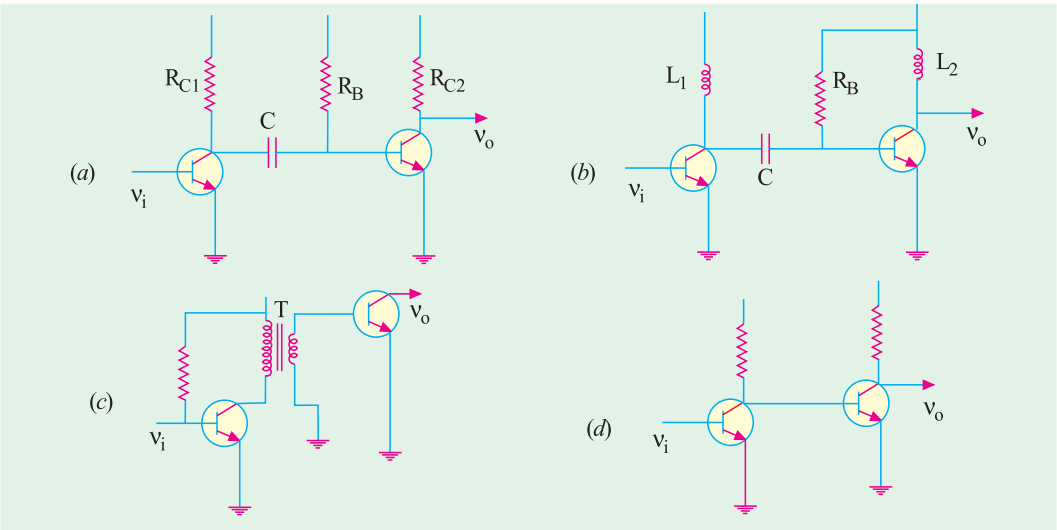


Fig. 61.2

The four basic methods of coupling are :



Modern coupling transformer

1. Resistance-Capacitance (RC) Coupling

It is also known as ***capacitive*** coupling and is shown in Fig. 61.2 (*a*). Amplifiers using this coupling are known as *RC*-coupled amplifiers. Here, *RC* coupling network consists of two resistors R_{C1} and R_{C2} and one capacitor *C*. The connecting link between the two stages is *C*. The function of the *RC*-coupling network is two-fold :

(*a*) to pass ac signal from one stage to the next,

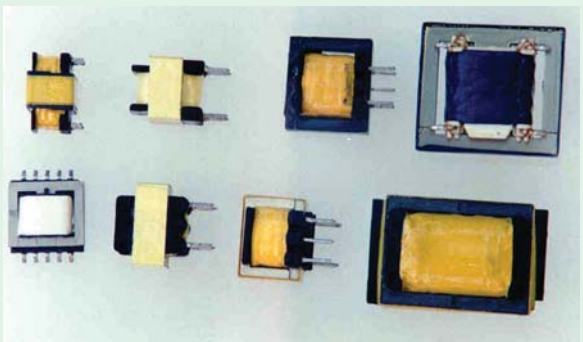
(*b*) to block the passage of dc voltages from one stage to the next.

2. Impedance Coupling or Inductive Coupling

It is also known as choke-capacitance coupling and is shown in Fig. 61.2 (*b*). Amplifiers using this coupling are known as **impedance-coupled** amplifiers. Here, the coupling network consists of L_1, *C* and R_B. The impedance of the coupling coil depends on **(*i*)** its inductance and **(*ii*)** signal frequency.

3. Transformer Coupling

It is shown in Fig. 61.2 (*c*). Since secondary of the coupling transformer conveys the ac component of the signal directly to the base of the second stage, there is ***no need for a coupling capacitor.*** Moreover, the secondary winding also provides a base return path, hence there ***is no need for a base resistance.*** Amplifiers using this coupling are called ***transformer-coupled amplifiers.***



R.C. Coupled two-stage amplifier

4. Direct Coupling

It is shown in Fig. 61.2 (*d*). This coupling is used where it is desirable to connect the load directly in series with the output terminal of the active circuit element. The examples of such load

devices are **(*i*)** headphones **(*ii*)** loud-speakers **(*iii*)** dc meters **(*iv*)** relays and **(*v*)** input circuit of a transistor etc. Of course, direct coupling is permissible only when

(*i*) dc component of the output does not disturb the normal operation of the load device,

(*ii*) device resistance is so low that it does not appreciably reduce the voltage at the electrodes.

61.3. RC-coupled Two-stage Amplifier

Fig. 61.3 shows a two-stage *RC*-coupled amplifier which consists of two single-stage transistor amplifiers using the *CE* configuration. The resistors R_2 and R_3 and capacitor C_2 form the coupling network. R_2 is collector load of Q_1 and R_4 is that of Q_2. Capacitor C_1 couples the input signal whereas C_3 couples out the output signal. R_1 and R_3 provide dc base bias.

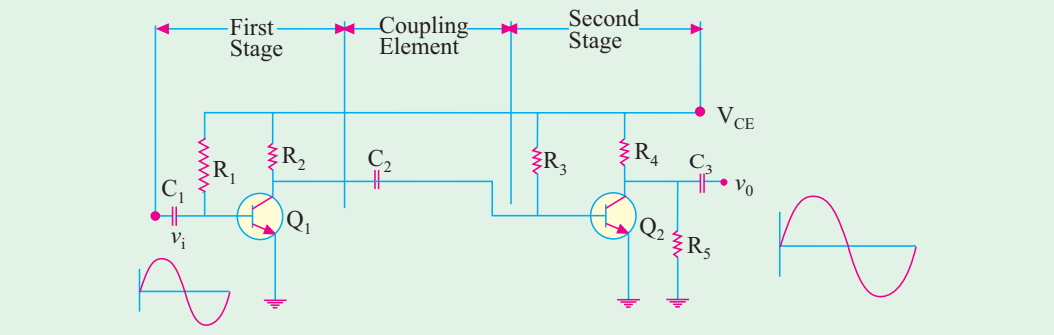


Fig. 61.3

(*i*) Circuit Operation

The brief circuit operation is as under :

1. the input signal n_i is amplified by Q_1. It is phase ***reversed*** (usual with C_E connection);
2. the amplified output of Q_1 appears across R_2;
3. the output of the first stage across R_2 is coupled to the input at R_3 by coupling capacitor C_2. This capacitor is also sometimes referred to as ***blocking capacitor*** because it blocks the passage of dc voltages and currents;

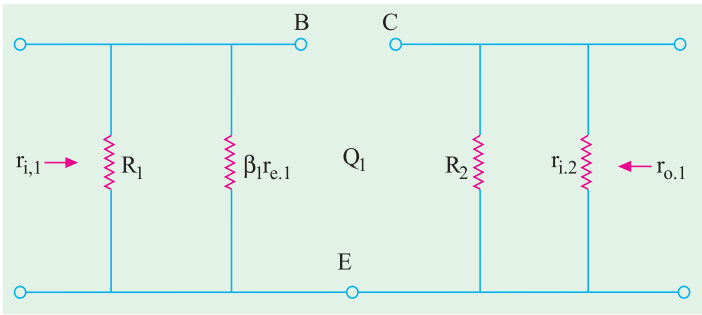


Fig. 61.4

4. the signal at the base of Q_2 is further amplified and ***its phase is again reversed;***
5. the ac output of Q_2 appears across R_4;
6. the output across R_4 is coupled by C_3 to load resistor R_5;
7. the output signal v_0 is the ***twice-amplified replica of the input signal v_i.*** It is in phase with v_i *because it has been reversed twice*.

(*ii*) AC Equivalent Circuit

The ac equivalent circuits for the two stages have been shown separately in Fig. 61.4 and Fig. 61.5 respectively.

If Fig. 61.4, $r_{i.1} = R_1 \parallel \beta_1 . r_{e.1}$

It is the input impedance of the first stage and not $r_{in(base)}$.

The output impedance of the first stage is

$$r_{0.1} = R_2 \| r_{i.2}$$

It is so because the input of the second stage forms a part of the output of the first stage. As seen from Fig. 61.5.

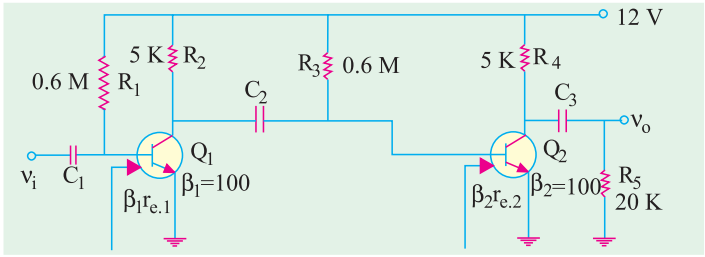


Fig. 61.5

$r_{i.2} = R_3 \| \beta_2 . r_{e2} \cong \beta_2 r_{e.2}$,

where $r_{e.1}$ and $r_{e.2}$ are ac junction resistances of the two transistors and are given by

$$r_{e.1} = \frac{25\,\text{mV}}{I_{E.1}} \text{ or } \frac{50\,\text{mV}}{I_{E.1}} \quad \text{and} \quad r_{e.2} = \frac{25\,\text{mV}}{I_{E.2}} \text{ or } \frac{50\,\text{mV}}{I_{E.2}}$$

The output impedance of Q_2 is $r_{0.2} = R_4 \| R_5$

(*iii*)Voltage Gain

$r_{1.2} = \beta_2 r_{e.2}$ Now $A_{v2} = \beta_2 \dfrac{r_{0.2}}{\beta_2 . r_{1.2}} = \dfrac{r_{0.2}}{r_{e.2}}$

Also, $r_{i.2} = \beta_2 r_{e.2}$ $\therefore$ $A_{v2} = \beta_2 \dfrac{r_{0.2}}{\beta_2 . r_{e.2}} = \dfrac{r_{0.2}}{r_{e.2}}$

The voltage gain of the first stage is also given by a similar equation

$$A_{vi} = \frac{r_{0.1}}{r_{e.1}}$$

Example 61.1. *For the two-stage RC-coupled low-level audio amplifier shown in Fig. 61.6, compute the following :*

(i) r_i ***(ii)*** A_{v1} ***(iii)*** A_{v2} *and* ***(iv)*** A_v *in dB. Neglect* V_{BE} *and take* $r_e = 25\ mV/I_E$.

(Electronic Circuits, Mysore Univ.)

Solution. The input impedance of the cascaded amplifier is

(i) $r_i = R_1 \| \beta_1 . r_{e.1}$

For finding $r_{e.1}$, we need $I_{E.1}$ which approximately equals $I_{C.1}$

Now, $I_{C.1} = \beta . I_{B.1}$

Also, $I_{B1} = 12/R_1 = 12/0.6\ M$
$= 20\ \mu A$

$\therefore$ $I_{C1} = 100 \times 20$
$= 2000\ \mu A = 2\ mA$

$\therefore$ $I_{E.1} = 2\ mA$

$\therefore$ $r_{e.1} = 25/2 = 12.5\ \Omega$;

$\beta_1 r_{e.1} = 100 \times 12.5 = 1250$

$\therefore$ $r_i = R_1 \| \beta_1 . r_{e.1}$
$= 0.6\ M \| 1250\ \Omega$
$\cong$ **1250 Ω**

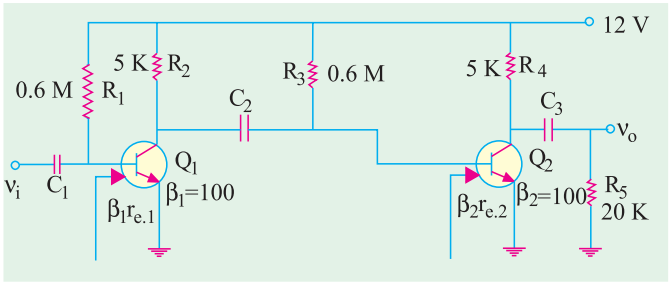


Fig. 61.6

(ii) $A_{v1} = \dfrac{r_{0.1}}{r_{e.1}}$ Now, $r_{0.1} = R_2 \| r_{i.2}$

$r_{i.2} = R_3 \| \beta_2 r_{e.2} = 0.6\ M \| 1250\ \Omega \cong 1250\ \Omega$

$r_{0.1} = 5\ K \| 1250\ \Omega = 1000\ \Omega$; $r_{e.1} = 12.5\ \Omega$ $\therefore$ $A_{v.1} = 1000/12.5 = 80$

(iii) $A_{v2} = \dfrac{r_{0.2}}{r_{e.2}}$

Now, $r_{0.2} = R_4 \| R_5 = 5\text{ K} \| 20\text{ K} = 4\text{ K}$, $I_{E.2} = 2\text{ mA}$ – same as $I_{E.1}$

$r_{e.2} = 25/2 = 12.5\ \Omega$ $\therefore$ $A_{v2} = 4000/12.5 = \mathbf{320}$

(iv) $A_v = A_{v.1} \times A_{v.2} = 80 \times 320 = \mathbf{25{,}600}$

(v) $G_v = 20 \log_{10} A_v\ dB = 20 \log_{10} 25{,}600 = \mathbf{88\ dB}$.

Example 61.2. *For the two-stage RC-coupled amplifier shown in Fig. 61.7 compute the following :*

(i) r_i, *(ii)* $A_{v.1}$, *(iii)* $A_{v.2}$, *(iv)* A_v *in decibels.*

Take $\beta_1 = \beta_2 = 100$. *Neglect* V_{BE} *and use* $r_e = 25\ mV/I_E$.

(Applied Electronics-I, Punjab Univ. 1991)

Solution. ***(i)*** The input impedance of the stage is $r_i = R_1 \| \beta_1 . r_{e.1}$

It should be noted that R_6 does not come into the picture because it has been ac grounded by C_4. However, it would affect the dc emitter current.

$$I_{E.1} = \frac{V_{CC}}{R_6 + R_1/\beta_1} = \frac{25}{10{,}000 + 1.5 \times 10^6/100} = 1\text{mA}$$

$r_{e.1} = 25/1 = 25\ \Omega$; $\beta_1 . r_{e.1} = 100 \times 25 = 2500\ \Omega$

$r_i = 1.5\text{ M} \| 2500\ \Omega \cong \mathbf{2500\ \Omega}$

(ii) $A_{v.1} = \dfrac{r_{0.1}}{r_{e.1}}$

Now, $r_{0.1} = R_2 \| r_{i.2}$ and $r_{i.2} = R_3 \| \beta_2 r_{e.2}$

Now, $I_{E.2} = \dfrac{25}{10{,}000 + 1.5 \times 10^6/100} = 1\text{mA}$

$\therefore$ $r_{e.2} = 25/1 = 25\ \Omega$; $\beta_2 . r_{e.2} = \mathbf{2500\ \Omega}$

$r_{i.2} = 1.5\text{ M} \| 2500\ \Omega \cong 2500\ \Omega$

$r_{0.1} = 5\text{ K} \| 2.5\text{ K} = 1{,}667\ \Omega$

$\therefore$ $A_{v.1} = 1667/25 = \mathbf{66.7}$

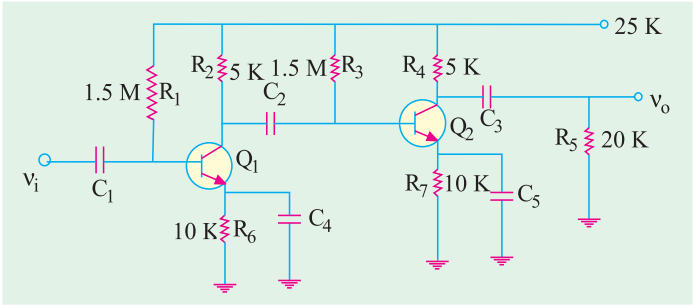


Fig. 61.7

(iii) $A_{v.2} = \dfrac{r_{0.2}}{r_{e.2}}$,

Now, $r_{0.2} = R_4 \| R_5 = 5\text{ K} \| 20\text{ K} = 4\text{ K}$

$\therefore$ $A_{v.2} = 4000/25 = \mathbf{160}$

(iv) $A_v = A_{v.1} \times A_{v.2} = 66.7 \times 160 = \mathbf{10{,}672}$;

$G_v = 20 \log_{10} 10{,}672 = \mathbf{80.3\ dB}$

Example 61.3. *Compute the overall voltage amplification for the two-stage RC-coupled amplifier shown in Fig. 61.8. Express the answer in decibels. Neglect* V_{BE} *and use* $r_e = 50\ mV/I_E$. *Take* $\beta_1 = \beta_2 = 100$. **(Electronics-I, Allahabad Univ. 1990)**

Solution. For finding the overall gain, we will have to find each stage gain.

(i) $A_{v1} = \dfrac{r_{0.1}}{r_{e.1}}$, Now, $r_{0.1} = R_3 \| r_{i.2}$ and $r_{i.2} = R_5 \| R_6 \| \beta_2 . r_{e.2}$

Now, $r_{e.2} = 50/I_{E.2}$

Drop across

$$R_6 = 20 \times \frac{5}{5+45} = 2V$$

It also represents the approximate drop across R_8.

$\therefore\ I_{E2} = 2/1\ K = 2\ mA$

$\therefore\ r_{e.2} = 50/2 = 25\ \Omega.$

$\beta_2.r_{e.2} = 100 \times 25 = 2.5\ K$

$r_{i.2} = 45\ K \parallel 5\ K \parallel 2.5\ K = 1.6\ K;$ $\therefore\ r_{0.1} = 5\ K \parallel 1.6\ K = 1.2\ K$

$\therefore\ r_{e.1} = 50/I_{E.1} = 50/2 = 25\ \Omega$

$\therefore\ A_{v.1} = 1200/25 = 48$

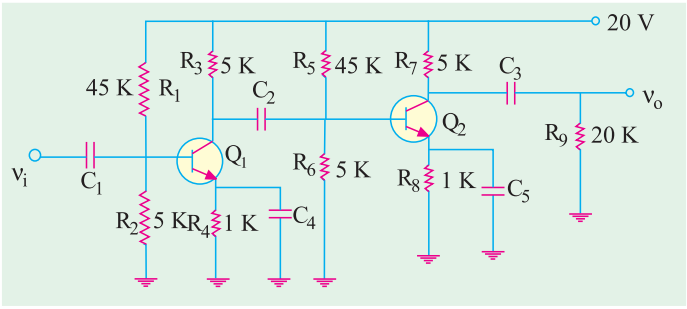


Fig. 61.8

$$A_{v.2} = \frac{r_{o.2}}{r_{c.2}}; \quad r_{0.2} = R_7 \parallel R_9 = 5\,K \parallel 20\,K = 4\,K$$

$\therefore\ A_{v.2} = 4000/25 = 160$ $\therefore\ A_v = 48 \times 160 = 7{,}680$; $G_v = 20 \log_{10} 7{,}680 =$ **77.6 dB**

61.4. Advantages of RC Coupling

1. It requires no expensive or bulky components and no adjustments. Hence, it is small, light and inexpensive.
2. Its overall amplification is higher than that of the other couplings.
3. It has minimum possible nonlinear distortion because it does not use any coils or transformers which might pick up undesirable signals. Hence, there are no magnetic fields to interfere with the signal.
4. As shown in Fig. 61.9, it has a very flat frequency ***versus*** gain curve *i.e.* it gives uniform voltage amplification over a wide range from a few hertz to a few megahertz because resistor values are independent of frequency changes.

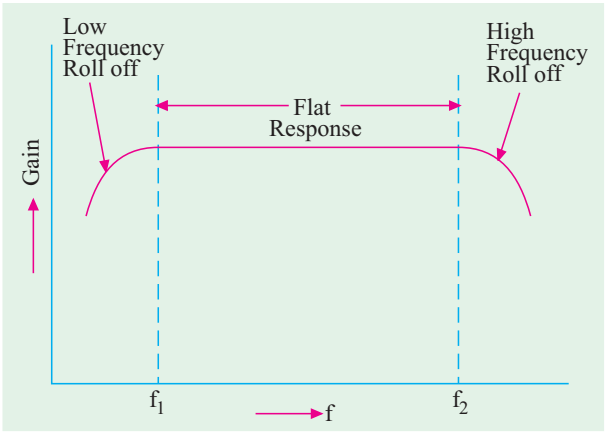


Fig. 61.9

As seen from Fig. 61.9, amplifier gain falls off at very low as well as very high frequencies. At low frequencies, the fall in gain (called ***roll-off***) is due to capacitive reactance of the coupling capacitor between the two stages. The high-frequency roll-off is due to output capacitance of the first stage, input capacitance of the second stage and the stray capacitance.

The only drawback of this coupling is that due to large drop across collector load resistors, the collectors work at relatively small voltages unless higher supply voltage is used to overcome this large drop.

61.5. Impedance-coupled Two Stage Amplifier

The circuit is shown in Fig. 61.10. The coupling network consists of L, C_2 and R_3. The only basic difference between this circuit and the one shown in Fig. 61.3 is that inductor **L *has replaced the resistor R_2.***

(i) AC Equivalent Circuit

The ac equivalent circuit (at mid-frequency) of the cascaded amplifier has been shown in Fig. 61.11. Because of mid-frequency range, effects of all capacitances have been ignored.

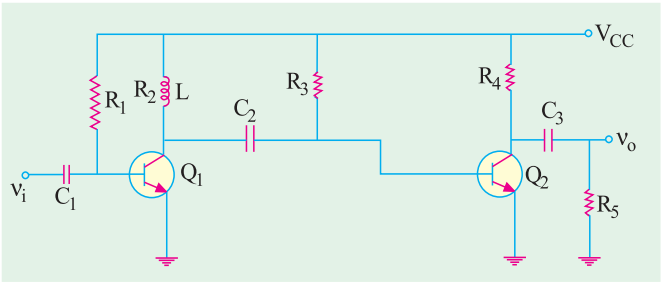


Fig. 61.10

(ii) Circuit Operation

The operation of this circuit is the same as that of the RC-coupled circuit described earlier.

(iii) Voltage Gain

It is given by the product of two stage gains $A_v = A_{v1} \times A_{v2}$

Now, $A_{v.1} = \dfrac{Z_{0.1}}{r_{e.1}}$ where $Z_{0.1} = X_L \| r_{i.2}$ and $r_{i.2} = R_3 \| \beta_2 . r_{e.2}$

In case, $X_L \gg r_{i.2}$, then, $Z_{0.1} \cong r_{i.2}$

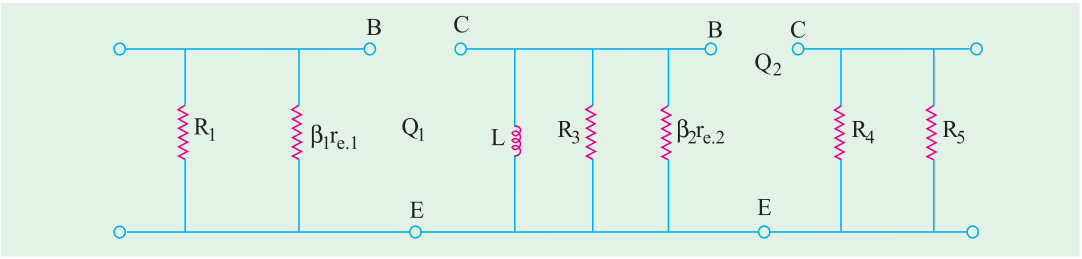


Fig. 61.11

$\therefore \quad A_{v.1} = \dfrac{r_{i.2}}{r_{e.1}}$ and $A_{v.2} = \dfrac{r_{0.2}}{r_{e.2}}$

$\therefore \quad A_v = A_{v.1} \times A_{v.2}$

Example 61.4. *For the impedance-coupled two-stage amplifier shown in Fig. 61.12, compute the values of*

(i) $A_{v.2}$, ***(ii)*** $A_{v.1}$ *at 4 kHz and* ***(iii)*** A_v *in dB.*

Neglect V_{BE} *and use* $r_e = 25\ mV/I_E$. *Take* $\beta_1 = \beta_2 = 100$.

(Industrial Electronics, Calcutta Univ. 1991)

Solution. ***(i)*** $A_{v.2} = \dfrac{r_{0.2}}{r_{e.2}}$

Now, $r_{0.2} = R_4 \| R_5 = 8\ K \| 24\ K = 6\ K$

$I_{B2} = 12/1.2M = 10\ \mu A$, $I_{C.2} = \beta_2 . I_{B.2} = 100 \times 10 = 1000\ \mu A = 1\ mA$

$\therefore \quad I_{E2} \cong 1\ mA$

$\therefore \quad r_{e.2} = 25/1 = 25\ \Omega$

$$A_{V.2} = \frac{6,000}{25} = \mathbf{240}$$

(ii) $A_{v.1} = \dfrac{Z_{0.1}}{r_{e.1}} \cong \dfrac{r_{i.2.}}{r_{e.1}}$

Now, $r_{e.1} = 25\ \Omega$ – equal to $r_{e.2}$

$X_L = 2\pi fL$

$= 2\pi \times 4 \times 10^3 \times 1$

$= 25,130\ \Omega$ $r_{i.2} = R_3$

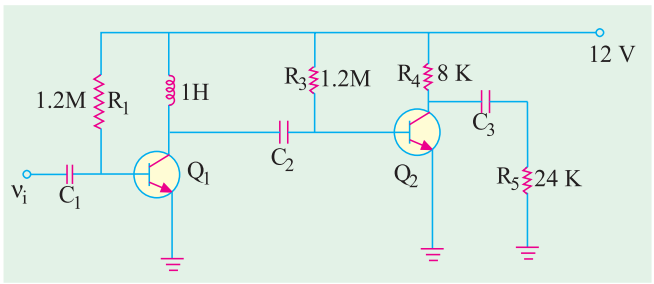


Fig. 61.12

$\| \beta_2.r_{e.2} = 1.2M \| 2500\ \Omega \cong 2500\ \Omega$

Obviously, $X_L \gg r_{i.2}$ thus justifying the above approximation.

$\therefore \quad A_{v.1} = 2500/25 = \mathbf{100}$

(iii) $A_v = 100 \times 240 = 24{,}000$, $G_v = 20 \log_{10} 24{,}000 = \mathbf{87.6\ dB}$

61.6. Advantages of Impedance Coupling

The biggest advantage of this coupling is that there is hardly any dc drop across L so that low collector supply voltages can be used.

However, it has many disadvantages :

1. It is larger, heavier and costlier than *RC* coupling.
2. In order to prevent the magnetic field of the coupling inductor from affecting the signal, the inductor turns are wound on a closed core and are also shielded.
3. Since inductor impedance depends on frequency, the frequency characteristics of this coupling are not as good as those of *BC* coupling. The flat part of the frequency ***versus*** gain curve is small (Fig. 61.13).

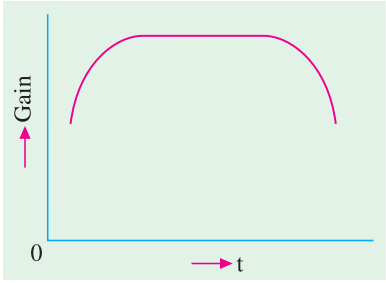


Fig. 61.13

At low frequencies, the gain is low due to large capacitance offered by the coupling capacitor just as in R_C coupled amplifiers. The gain ***increases with frequency*** till it levels off at the middle frequencies of the audio range.

At relatively high frequencies, gain drops of again because of the increased reactance.

Hence, impedance coupling is rarely used beyond audio range.

61.7. Transformer-coupled Two Stage Amplifier

The circuit for such a cascaded amplifier is shown in Fig. 61.14. T_1 is the coupling transformer whereas T_2 is the output transformer. C_1 is the input coupling capacitor whereas C_2, C_3 and C_4 are the bypass capacitors. Resistors R_1 and R_2 as well as R_4 and R_5 form voltage divider circuits whereas R_3 and R_6 are the emitter-stabilizing resistors.

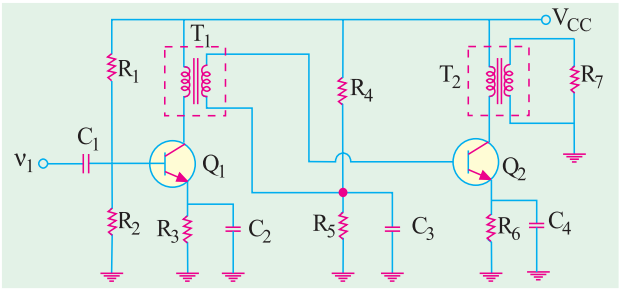


Fig. 61.14

(i) Circuit Operation

When input signal is coupled through C_1 to the base of Q_1, it appears in an amplified form in the primary of T_1. From there, it is passed on to the secondary by magnetic induction. Moreover, T_1 provides dc isolation between the input and output circuits. The secondary of T_1 applies the signal to the base of Q_2 which appears in an amplified form in the primary of T_2.

From there, it is passed on to the secondary by magnetic induction and finally appears across the matched load R_7.

(ii) Voltage Gain

$$A_{v.1} = \frac{r_{0.1}}{r_{e.1}} \qquad \text{Now} \quad r_{0.1} = a^2 r_{i.2} \quad \text{— where } a = N_1 / N_2 \text{ for } T_1$$

$$r_{i.2} = R_4 \| R_5 \| \beta_2.r_{e.2}$$

Similarly, $A_{e.2} = \frac{r_{0.2}}{r_{e.2}}$

where $r_{e.2} = a^2 R_7$

Example 61.5. *For the transformer-coupled two-stage amplifier shown in Fig. 61.15, calculate* ***(i)*** $A_{v.1}$ ***(ii)*** $A_{v.2}$ *and* ***(iii)*** A_v *in dB.*

Neglect V_{BE} *and use* $r_e = 50$ *mV/*I_E. *Take* $\beta_1 = \beta_2 = 50$ *and treat the transformers as ideal ones.*

(Electronics, Indore Univ.)

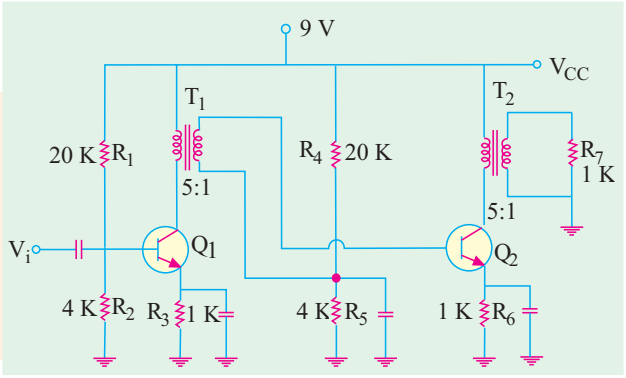


Fig. 61.15

Solution. Here, for each transformer $a = 5$

Drop across $R_2 = 9 \times 4/24 = 1.5$ V

Drop across $R_3 \cong 1.5$ V and $I_{E.1} = 1.5/1 = 1.5$ mA

$r_{e.1} = 50/1.5 = 33.3\ \Omega$

and $r_{e.2} = 33.3\ \Omega$ — same as $r_{e.1}$

β_2. $r_{e.2} = 50 \times 33.3 = 1665\ \Omega$

$r_{i.2} = R_4 \| R_5 \| \beta_2 . r_{e.2}$

$= 20\text{ K} \| 4\text{ K} \| 1665\ \Omega = 1110\ \Omega$

$r_{0.1} = a^2 . r_{i.2} = 5^2 \times 1110 = 27{,}750\ \Omega$

(i) $A_{v.1} = \frac{r_{0.1}}{r_{e.1}} = \frac{27{,}750}{33.3} = $ **830**

(ii) $A_{v.2} = \frac{r_{0.2}}{r_{e.2}} = \frac{25 \times 100}{33.3} = $ **750**

(iii) $A_v = 830 \times 750 = 622{,}500$; $G_v = 20 \log_{10} 622{,}500 =$ **116 dB**

61.8. Advantages of Transformer Coupling

1. The operation of a transformer-coupled system is basically ***more efficient*** because of low dc resistance of the primary connected in the collector circuit,
2. It provides a ***higher*** voltage gain,
3. It provides ***impedance matching*** between stages which is desirable for maximum power transfer. Typically, the input impedance of a transistor stage is less than its output impedance. Hence, secondary impedance of the interstage (or coupling) transformer is typically lower than the primary impedance.

This coupling is effective when the final amplifier output is fed to a low-impedance load. For example, the impedance of a typical loud-speaker varies from 4 Ω to 16 Ω whereas output impedance of a transistor stage is several hundred ohms. Use of an output audio transformer can avoid the bad effects of such a mismatch.

Disadvantages

1. The coupling transformer is costly and bulky particularly when operated at audio frequencies because of its heavy iron core,
2. At radio frequencies, the inductance and winding capacitance present lot of problems,
3. It has poor ***frequency response*** because the transformer is frequency sensitive. Hence, the frequency range of the transformer-coupled amplifiers is limited.
4. It tends to introduce ***'hum'*** in the output,

61.9. Frequency Response

The characteristics of a coupling transformer are that **(*i*)** it introduces inductances in both the input and output circuits **(*ii*)** leakage inductance exists between the primary and secondary windings and **(*iii*)** both windings introduce shunting (distributed) capacitance especially at high frequencies.

A typical gain ***versus*** frequency curve for a transformer-coupled amplifier is shown in Fig. 61.16.

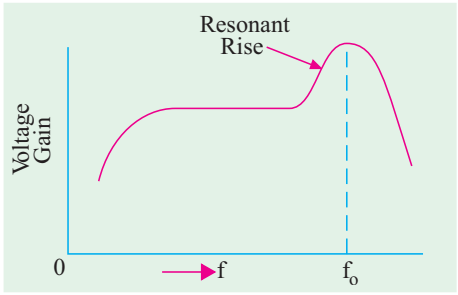


Fig. 61.16

It is seen that

1. There is decrease in gain at low frequencies and
2. Also there is decrease in gain at high frequencies except for the ***resonant rise*** in gain at resonant frequency of the tuned circuit formed by inductance and winding capacitance in the circuit.

The output voltage is equal to ac collector current multiplied by the primary reactance of the coupling transformer. Since at low frequencies, primary reactance is small, the gain is less.

At high frequencies, the distributed capacitance existing between different turns of the winding acts as a ***bypass capacitor*** and so reduces the output voltage and hence the gain. The peak or exaggerated gain occurs due to resonance or tuning effect of inductance and distributed capacitance which ***form a tuned circuit.***

Moreover, there is frequency distortion *i.e.* all frequencies are not amplified equally. In fact, the flat response part of the curve is small as compared to *RC* coupling. However, transformer-coupled amplifiers can be designed to have a flat frequency response curve and excellent fidelity over the entire audio frequency range.

61.10. Applications

Transformer coupling is often employed in the last stage of a multistage amplifier where concerted effort is made to maximise power transfer by perfect impedance matching.

Example 61.6. *In a multistage transformer-coupled amplifier, the output impedance of the first stage is 5 K and the input impedance of the second stage is 1 K. Determine the primary and secondary inductances of the tranformer for perfect impedance matching at f = 2000 Hz. If one turn gives an inductance of 10 μH, find the number of primary and secondary turns.*

(Electronic Engg.-I, Osmania Univ. 1991)

Solution. It should be clearly understood that primary has to match with the output impedance of the first stage and secondary with the input of the second stage.

$\therefore \quad X_{Lp}$ = output of 1st stage or $2\pi f L_p = 5000$

$\therefore \quad L_p = 5000/2\pi \times 2000 =$ **0.4 H**

Also, X_{Ls} = input of 2nd stage

$\therefore \quad 2\pi f_{Ls} = 1000$; $L_s = 1000/2\pi \times 2000 =$ **0.08 H**

Now, inductance of a coil varies as the square of its turns.

$\therefore \quad L \propto N^2 = kN^2$

When, $N = 1$, $L = 10\ \mu H \quad \therefore \quad 10 \times 10^{-6} = k \times l2^2$ or $k = 10^{-5}$

For primary winding $0.4 = 10^{-5} Np^2 \times 5$ or $N_p =$ **632**

For secondary winding, $L_s = kN_s^2$ or $0.08 = 10^{-5} Ns^2$ or $N_s =$ **89**

As seen, it is a nearly 7 : 1 step-down transformer.

61.11. Direct-coupled Two-stage Amplifier Using Similar Transistors

These amplifiers operate without the use of frequency-sensitive components like capacitors, inductors and transformers etc. They are especially suited for amplifying

(*a*) ac signals with frequencies as low as a fraction of a hertz,

(*b*) change in dc voltages.

Fig. 61.17 shows the circuit of such an amplifier which uses two similar transistors each connected in the *CE* mode. Both stages employ direct coupling **(*i*)** collector of Q_1 is connected directly to the base of Q_4 and **(*ii*)** load resistor R_2 is connected to the collector of Q_2. The resistor R_1 establishes the forward bias of Q_1 and also indirectly that of Q_2.

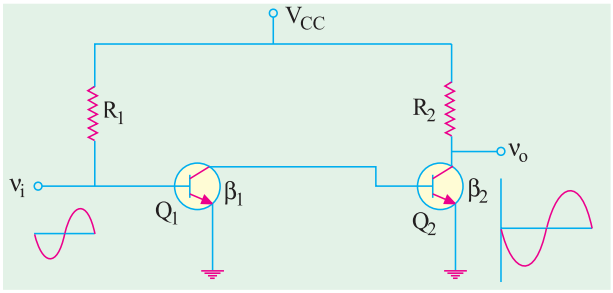


Fig. 61.17

Any signal current at the base of Q_1 is amplified β_1 times and appears at the collector of Q_1 and becomes base signal for Q_2. Hence, it is further amplified β_2 times. Obviously, signal current gain of the amplifier is

$$A_i = \beta_1 \times \beta_2 = \beta^2$$ — if transistors are identical

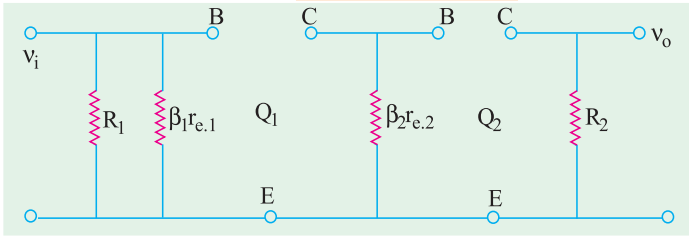


Fig. 61.18

(*i*) AC Equivalent Circuit

The ac equivalent circuit is shown in Fig. 61.18

(*ii*) Voltage Gain

The first stage *i.e.* Q_1 does not produce any voltage gain *i.e.* $A_{v\cdot 1} = 1$ as proved below :

$$A_{v.1} = \frac{r_{0.1}}{r_{e.1}} = \frac{\beta_2 \cdot r_{e.2}}{r_{e.1}} \qquad \text{... (}i\text{)}$$

As seen from Fig. 61.17, $I_{E.2} \cong \beta_2 \, . \, I_{B.2}$

Also $I_{B.2} = I_{C.1} = I_{E.1}$ $\therefore$ $I_{E.2} = \beta_2 . I_{E\cdot 1}$ or $\beta_2 = \dfrac{I_{E.2}}{I_{E.1}}$

Also $r_{e.1} = \dfrac{25}{I_{E.1}}$ and $r_{e.2} = \dfrac{25}{I_{E.2}}$

Substituting all these values in Eq. (*i*) above, we get

$$A_{v.1} = \frac{I_{E.2}}{I_{E.1}} \times \frac{25}{I_{E.2}} \times \frac{I_{E.1}}{25} = 1$$

It proves the statement made above.

Now, $A_{v.2} = \dfrac{r_{0.2}}{r_{e.2}}$ and $A_v = A_{v.1} \times A_{e.2} = A_{v.2}$

(*iii*) Advantages

1. The circuit arrangement is very simple since it uses ***minimum number of components.***
2. It is quite ***inexpensive.***
3. It has the outstanding ability to ***amplify direct current*** (*i.e.* as dc amplifier) and ***low-frequency signals.***

4. It has ***no coupling or by-pass capacitors*** to cause a drop in gain at low frequencies. As seen from Fig. 61.19, the frequency-response curve is flat upto upper cut-off frequency determined by stray wiring capacitance and internal transistor capacitances.

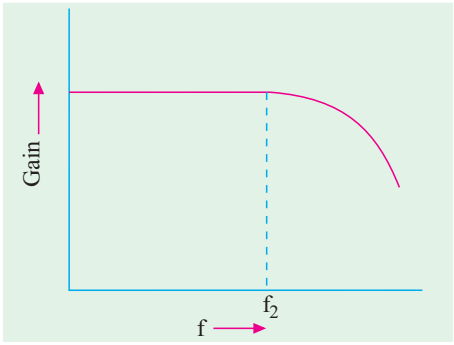


Fig. 61.19

(*iv*) Disadvantages

1. It cannot amplify high-frequency signals.

2. It has poor temperature stability.

It is due to the fact that any variation in base current (due to temperature changes) in one stage is amplified in the following stage (or stages) thereby shifting the Q-point. However, stability can be improved by using emitter-stability resistor (Fig. 61.21).

(*v*) Applications

Some of the applications of direct-coupled amplifiers are in

1. regulator circuits of electronic power supplies,

2. pulse amplifiers **3.** differential amplifiers,

4. computer circuitry, **5.** electronic instruments.

Example 61.7. *For the direct-coupled amplifier of Fig. 61.20, calculate*

(*a*) *current gain ,* **(*b*)** *voltage gain of first stage,*

(*c*) *voltage gain of second stage,* **(*d*)** *overall voltage gain in dB,*

(*e*) *overall power gain in dB,* **(*f*)** *input resistance.*

Neglect V_{BE} *and use* $r_e = 50\ mV/I_E$.

Solution. (*a*) $A_i = \beta_1\beta_2 = 100 \times 50 = \mathbf{5000}$ **(*b*)** $A_{v.1} = 1$ — Art 61.11

(*c*) $A_{v.2} = \dfrac{r_{0.2}}{r_{e.2}}$ Now, $r_{e.2} = \dfrac{50}{I_{E.2}}$

Let us find the value of $I_{E.2}$ starting from the value of $I_{B.1}$. As seen from Fig. 61.20,

$I_{B.1} = 12/1.2M = 10\ \mu A$

$I_{C.1} = \beta_1 . 1_{\beta 1} = 100 \times 10 = 1000\ \mu A$

$I_{E.1} = I_{C.1} = 1000\ \mu A = 1\ mA$

$I_{B.2} = I_{C.1} = 1\ mA$

$I_{C.2} = \beta_2 . I_{C1} = 50 \times 1 = 50\ mA$

$\therefore \quad I_{E.2} = 50\ mA$

and $r_{e.2} = 50/50 = 1\ \Omega$

Also, $r_{e.2} = R_2 = 200\ \Omega$

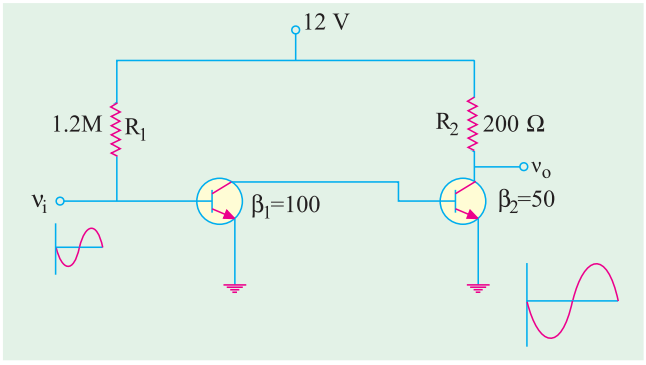


Fig. 61.20

$\therefore \quad A_{v.2} = \dfrac{200}{1} = \mathbf{200}$

(*d*) $A_n = 1 \times 200 = 200$; $G_v = 20 \log_{10} 200 = \mathbf{46\ dB}$

(*e*) $A_p = A_v . A_i = 200 \times 5{,}000 = 10^6$; $Gp = 20 \log_{10} 106 = \mathbf{60\ dB}$

(*f*) $r_i = R_1 \| \beta_1 . r_{e.1}$ — Fig. 61.18

$r_{e.1} = \dfrac{50}{I_{E.1}} = \dfrac{50}{1} = 50\Omega \quad \therefore \quad r_i = 1.2\ M \| (50 \times 100) \cong \mathbf{5\ K}$

Example 61.8. *For the emitter-stabilized direct-coupled amplifier of Fig. 61.21, find* ***(i)*** $A_{v.1}$, ***(ii)*** $A_{v.2}$, ***(iii)*** A_v *and* ***(iv)*** r_i. *Neglect* V_{BE} *and use* $r_e = 50\ mV/I_E$.
(Electronics-I, Mysore Univ. 1992)

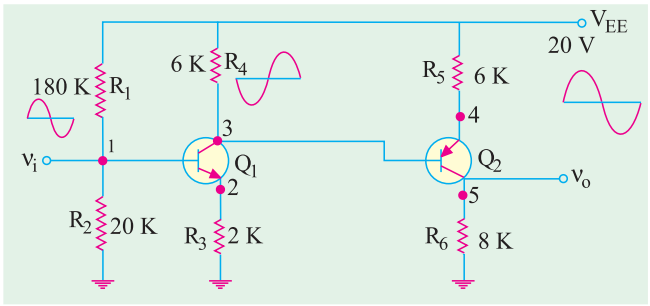


Fig. 61.21

Solution. As seen, emitter resistors R_3 and R_5 have been used to improve temperature stability. The voltage divider $R_1 - R_2$ together with R_3 determines the emitter current of Q_1. The resistor R_4 has dual function ***(i)*** it acts as load resistor for Q_1 and ***(ii)*** it establishes base bias of Q_2.

(i) Since unbypassed resistor R_3 is present

$$A_{v.1} = \frac{r_{0.1}}{r_{e.1} + R_3}$$

As seen from the ac equivalent diagram of Fig. 61.22

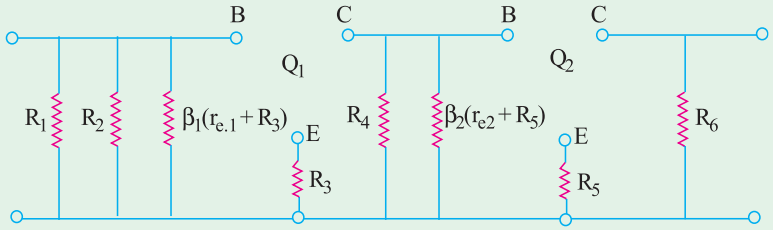


Fig. 61.22

$$r_{0.1} = R_4 \| \beta_2 . (r_{e.2} + R_5)$$

Now $$r_{e.2} = \frac{50}{I_{E.2}}$$

For finding $I_{E.2}$, let us first find $I_{E.1}$

Drop across $$R_2 = 20 \times \frac{20}{180 + 20} = 2\ V$$

Same is the drop across R_3 since V_{BE} has been neglected.

$\therefore$ $$I_{E.1} = 2V/1\ K = 2\ mA$$

Now, $$I_{E.2} = \frac{V_{CC} - I_{E.1}R_4}{R_5} = \frac{20 - 2 \times 8}{2} = 2\ mA$$

$\therefore$ $$r_{e.2} = 50/2 = 25\ \Omega \quad \text{and} \quad \beta_2 (r_{e.2} + R_5) \cong 200\ K$$

$$r_{0.1} = 8\,K \| 200\,K = 7.7\,K_i \; r_{e.1} = \frac{50}{2} = 25\Omega$$

$\therefore$ $$A_{v.1} = \frac{7,700}{(25 + 1000)} = 7.5$$

(ii) $$A_{v.2} = \frac{r_{0.2}}{(r_{e.2} + R_5)}; \; r_{0.2} = R_6 = 6000\Omega \therefore \quad A_{V.2} = \frac{6,000}{(25 + 2000)} \cong 3$$

(iii) $A_v = 7.5 \times 3 =$ **22.5**

(iv) As seen from Fig. 61.22

$$r_i = R_1 \| R_2 \| \beta_1 (r_{e.1} + R_3) = 180\ K \| 20\ K \| 100\ K$$ —neglecting $r_{e.1}$

$$= \mathbf{15.25\ K}$$

Example. 61.9. *In the circuit of Fig. 61.21, find dc voltage at points marked 1, 2, 3 and 4. Neglect* V_{BE}.

If a 1-V dc signal at input 1 changes by 0.01 V, what would be the voltage variation at the output?

Solution. V_1 = drop across R_2 = **2 V**

$V_2 \cong V_1$ = **2 V**; $V_3 = V_{CC} - I_{C.1}$ R_3 = 20 – 2 × 8 = **4 V**

$V_4 \cong V_3$ = **4 V**; $V_5 = V_{CC} - I_{C.2}$ R_6 = 20 – 2 × 6 = **8 V**

$\Delta v_0 = A_v . \Delta v_i$ = 22.5 × 0.01 = 0.225 V = **225 mV.**

61.12. Direct-coupled Amplifier Using Complementary Symmetry of Two Transistors

In this case, an *NPN* transistor is directly-coupled to its complementary *i.e.* a *PNP* transistor. Fig. 61.23 shows a two-stage cascaded amplifier using two complementary transistors connected in CE configuration.

The circuit differs from that shown in Fig. 61.17 in the following three ways :

1. It uses **complementary** transistors rather than **similar** ones,
2. Instead of V_{CC}, V_{EE} power battery has been used,
3. Output is taken directly from terminal of load resistor R_2.

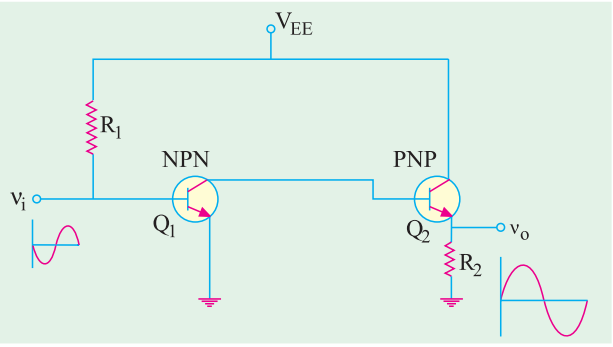


Fig. 61.23

More Practical Circuit

A more practical circuit of the above type is shown in Fig. 61.24. Here, bias current of Q_1 is determined by voltage divider $R_1 - R_2$ and R_3. As before, R_4 performs two functions :

1. It acts as load for Q_1 and **2.** Establishes bias voltage for Q_2

The emitter resistors R_3 and R_6, as usual, meant to improve amplifier stability.

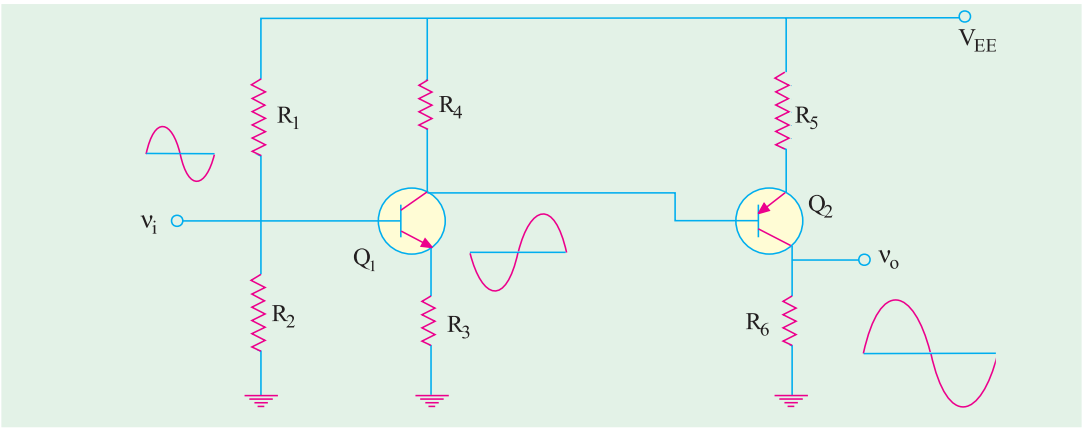


Fig. 61.24

(*i*) Circuit Operation

When a positive-going signal is applied to the base of Q_1, then

1. its base current ***increases,***

2. hence, its collector current increases ($I_C = \beta I_B$),
voltage drop across ***R_4 increases,***
4. consequently, voltage at the collector of Q_1 and the base of Q_2 becomes ***less positive*** or in other words, ***more negative,***
5. hence, a negative-going signal is applied to the base of Q_2.

The negative-going signal applied to the base of Q_2 causes,

1. an ***increase*** in its forward bias (remember, it is a *PNP* transistor),
2. an ***increase*** in collector current
3. an ***increase*** in the voltage developed across R_6,
4. an amplified ***positive going*** output signal at R_6.

Hence, it is seen that a signal applied to the input of a two-stage complementary amplifier appears at the output in an amplified form and ***of the same polarity***.

Voltage Gain

It is the same as for the circuit of Fig. 61.17.

Example 61.10. *For the complementary symmetry circuit of Fig. 61.25, find* ***(a)*** A_v ***(b)*** V_1, V_2, V_3, V_4 *and* V_5. *Neglect* V_{BE} *and assume* R_3, $R_5 \gg r_e$.

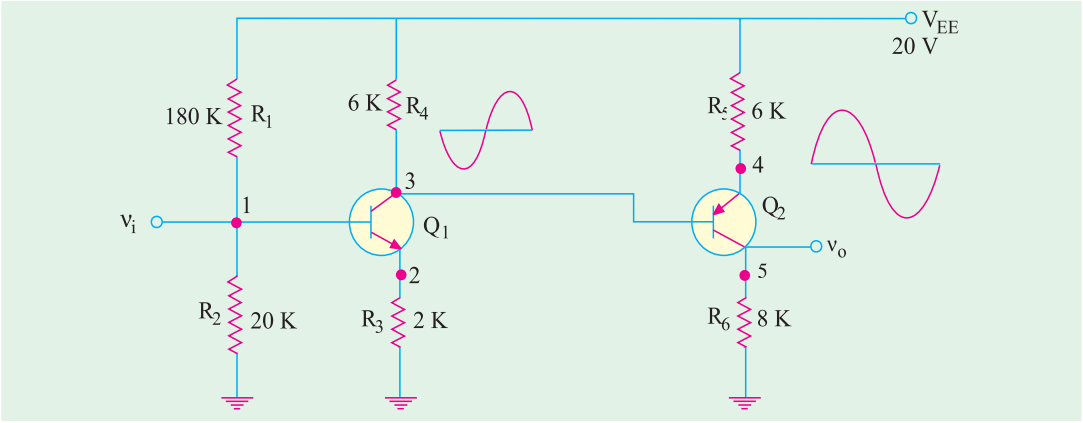


Fig. 61.25

Solution. ***(a)*** The overall voltage gain is

$$A_v = A_{v.1} \times A_{v.2} \cong \frac{R_4}{R_3} \times \frac{R_6}{R_5} = \frac{6}{2} \times \frac{6}{8}$$

(b) $$V_1 = V_{EE} \frac{R_2}{R_1 + R_2} = 2V \qquad \therefore \; V_2 \cong V_1 = \mathbf{2V}$$

$$\therefore \quad I_{E.1} = \frac{2V}{R_3} = \frac{2V}{2K} = 1\text{mA}\ , \qquad I_{C1} \cong I_{E.1} = 1\text{mA}$$

$$V_3 = V_{EE} - I_{C.1} R_3 = 20 - 1 \times 6 = \mathbf{14\ V}, \qquad V_4 \cong V_3 = \mathbf{14\ V}\ ;$$

$$I_{C.2} \frac{20-14}{6K} = 1\text{mA}$$

$$= \frac{20-14}{6K} = 1\text{mA}$$

$$\therefore \quad I_{E.2} \cong I_{C.2} = 1\text{mA}$$

$\therefore \quad V_5 = I_{E.2} \times R_6$

$= 1 \times 8 = \mathbf{8\ V}$

61.13. Darlington Pair

It is the name given to a ***pair of similar*** transistors so connected that emitter of one is directly joined to the base of the other as shown in Fig. 61.26 (*a*). Obviously, the emitter current of Q_1 becomes the base current of Q_2.

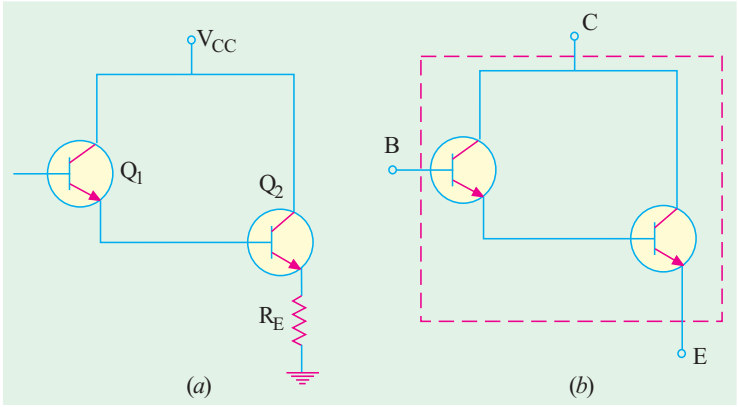


Fig. 61.26

Darlington pairs are commercially mounted in a single package that has only three leads : base, collector and emitter as shown in Fig. 61.26 (*b*). It often forms a ***double CC stage*** in multistage amplifiers. It is so because a Darlington connection can be considered equivalent to two ***cascaded emitter followers.***

Main Characteristics

(*i*) Current Gain

It can be proved that current gain of a Darlington pair is $(1 + \beta_1)(1 + \beta_2) = (1 + \beta)^2 \cong \beta^2$ if the transistors are ***identical*** (*i.e.* $\beta_1 = \beta_2$).

Proof

$$I_{B.2} = I_{E.1} = (1 + \beta_1) I_{B.1} \cong \beta_1 I_{B.1}$$

$$I_{E.2} \cong \beta_2 . I_{B.2} = \beta_1 \beta_2 I_{B.1} \qquad \therefore \qquad A_i = \frac{I_{E.2}}{I_{B.1}} = \beta_1 \beta_2 = \beta^2$$

It means that a Darlington pair behaves like a ***single transistor having a beta*** of β^2.

(*ii*) Input Impedance

In Fig. 61.26 (*a*), the input impedance seen from the base of Q_2 is

$$r_{i.2} = \beta_2 (r_{e.2} + R_E) \cong \beta_2 . R_E$$

Input impedance as seen from the base of Q_1 is

$$r_{i.1} = \beta_1 (r_{e.1} + r_{i.2}) = \beta_1 r_{e.1} + \beta_1 r_{i.2} = \beta_1 r_{e.1} + \beta_1 \beta_2 R_E \cong \beta_1 \beta_2 R_E$$

or $\quad r_{in(base)}$ of $Q_1 = \beta_2 R_E$

Note. If there is a load resistance R_L coupled to the emitter of Q_2, then

$$r_{i.1} - \beta^2 (R_E \| R_L) = \beta^2 . r_E$$

As seen, load impedance R_E has been transformed into $\beta^2 R_E$. Obviously, a Darlington pair is capable of high input impedance. In fact, whenever a load causes a severe loss in voltage gain (loading effect), it is usual to step up load impedance *via* a *FET* stage, a single *CC* stage or Darlington pair when much greater impedance transformation is required.

(*iii*) Voltage Gain

Assuming $r_{e.1} = r_{e.2} = r_e$ we have

$$A_v \cong \frac{R_E}{r_e + R_E} = \frac{1}{1 + \dfrac{r_e}{R_E}} \cong 1$$

—as in an emitter follower

61.14. Advantages of Darlington Pair

1. It can be readly formed from two adjacent transistors in an IC.
2. It has enormous ***impedance transformation capability i.e.*** it can transform a low-impedance load into a high impedance load. Hence, it is used in a high-gain operational amplifier which depends on very high input impedance for its operation as an integrator or summing amplifier in analogue applications.
3. It uses very few components.
4. It provides very high β-value.

61.15. Comparison Between Darlington Pair and Emitter Follower

We will refer to Fig. 9.28 and Fig. 61.26.

1. Input impedance of Darlington pair is $\beta^2 R_E$ whereas that of emitter follower is βR_E (Art. 9.8).
2. Current gain of Darlington pair is β^2 whereas that of emitter follower is β.
3. However, voltage gains of the two are identical.

Example. 61.11. *For the Darlington pair shown in Fig. 61.27, calculate the value of 1.* β, *2.input impedance,3. voltage amplification.*

Assume $\beta_1 = \beta_2 = 100$ *and* $R_L \gg (r_{e.1} + r_{e.2})$.

Solution. The approximate values are as under :

1. β of Darlington pair

$= \beta_1 \times \beta_2$

$= 100 \times 100 =$ **10,000**

2. $r_E = R_E \| R_L$

$= 10\text{ K} \| 500\ \Omega = 475\ \Omega$

$r_{in(base)}$ of $Q_1 \cong \beta^2 r_E$

$= 10{,}000 \times 475 = 4.75\text{ M}$

r_i = input impedance of the pair

$= R_B \| r_{in(base)}$ of Q_1

$= 1\text{ M} \| 4.75\text{ M} = 0.826\text{ M} =$ **826 K**

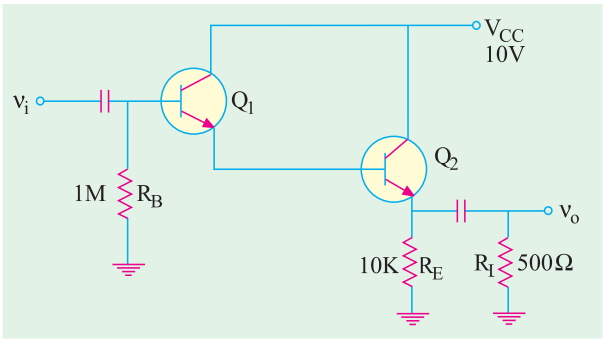


Fig. 61.27

3. $A_v \cong 1$

Example 61.12. *A CE amplifier stage shown in Fig.61.28 is to drive a 100* Ω *load to 10* V_{p-p} *level. An input signal of 1* V_{p-p} *is available. Find out if the stage is overloaded or not. Also, find out if this overloading has been avoided by using a Darlington pair as a buffer between* Q_1 *and the load as shwon in Fig. 61.29. Take* $\beta_1 = \beta_2 = 50$.

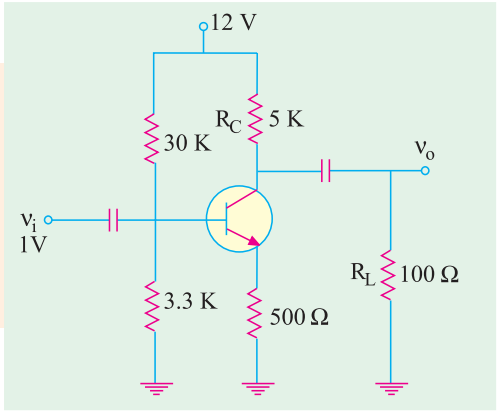


Fig. 61.28

Solution. The approximate voltage gain of Q_1 is given by

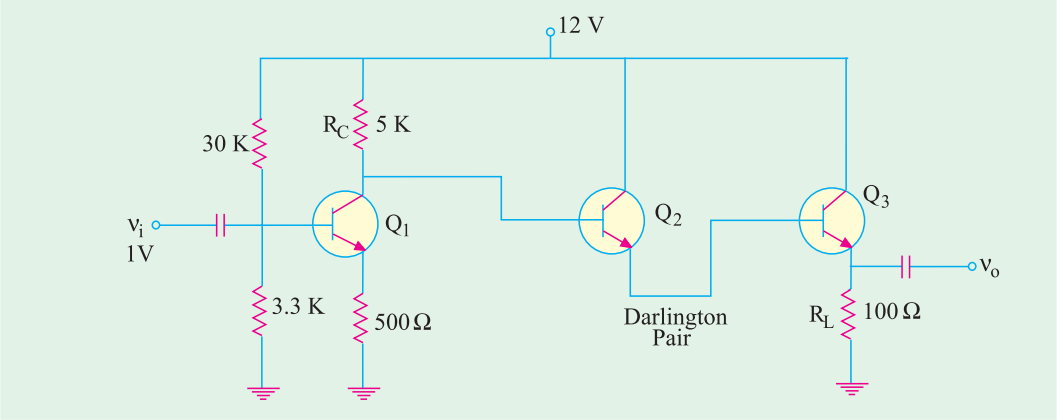


Fig. 61.29

$$A_v = \frac{r_L}{r_E} = \frac{R_C \| R_L}{R_E} = \frac{5\text{K} \| 100\,\Omega}{500\,\Omega} \cong \frac{1}{5}$$

$$\therefore \quad v_0 = 1 \times \frac{1}{5} = 0.2\,\text{mV}$$

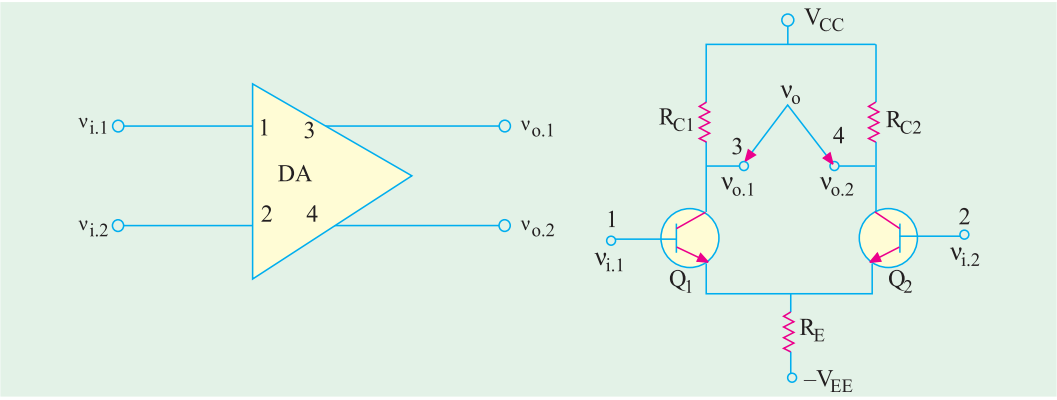


Fig. 61.30

Obviously, ***due to overloading of the stage,*** severe attenuation has occurred, Hence, the amplifier stage cannot, ***by itself,*** drive the heavy load.

By using a Darlington pair, a 100 Ω load has been transformed to

$$\beta^2 R_L = 2500 \times 100 = 250\ \text{K}$$

Now, $r_L = R_C \| \beta^2 R_L = 5\ \text{K} \| 250\ \text{K}$

$$\cong 5K \quad \therefore \ A_v = \frac{5\text{K}}{500\,\Omega} = 10$$

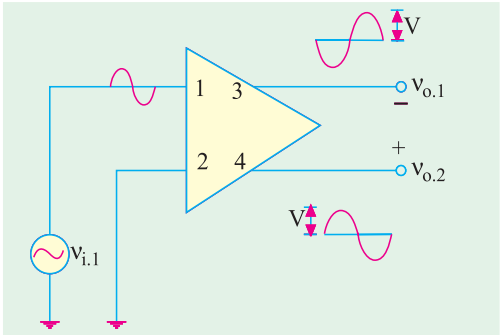


Fig. 61.31

This also represents total voltage gain because voltage gain of Darlington pair is one.

$$\therefore \qquad v_0 = A_v \times v_i = 10 \times 1 = \textbf{10 Vp–p}$$

As seen, now the stage would be able to drive the load since the design goal of 10 V_{P-P} has been met.

61.16. Special Features of a Differential Amplifier

It consists of two basic *CE* amplifiers having their ***emitters directly-coupled to each other.*** Fig. 61.30 shows both the block diagram and the circuit diagram of such an amplifier.

As seen, it has two separate input terminals 1 and 2 and two separate output terminals 3 and 4. Voltages may be applied to either or both input terminals and output may be taken from either or both output terminals.

There are certain specific phase relationships between both input and both output terminals as discussed below.

(*a*) Single-ended Operation

In Fig. 61.31, input signal $v_{i.1}$ is applied to terminal 1 with terminal 2 grounded.

It is seen that an ***amplified*** and ***inverted*** output signal is obtained at terminal 3 (phase inversion of a *CE* amplifier) but an ***equally-amplified*** and ***in-phase*** signal appears across output terminal 4. The differential output voltage has the polarity shown in the figure.

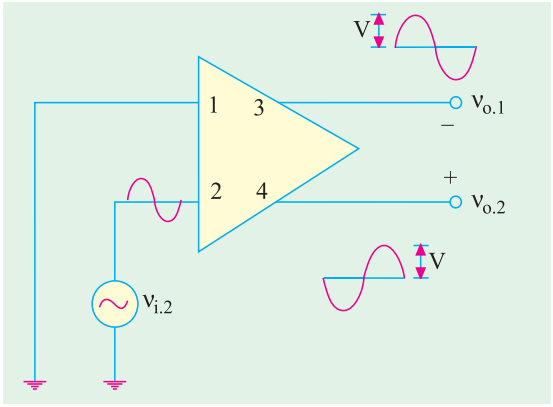
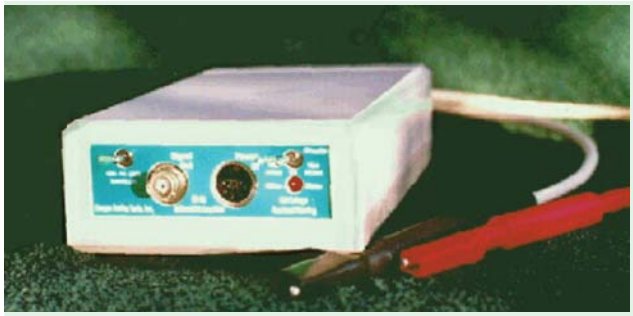


Fig. 61.32



Differential amplifier

As shown in Fig. 61.32, when input signal v_{1-2} is applied to input terminal 2, an amplified and inverted signal appears at output terminal 4 whereas ***equally-amplified*** but ***in-phase*** signal appears at terminal 3.

In summary, we can say that input at any of the two terminals causes outputs at both terminals 3 and 4. The two output are opposite in phase but of equal amplitude.

(*b*) Double-ended Operation

Fig 61.33 illustrates the double ended mode of operation when two input signals of opposite phase are applied to the two input terminals.

Input signal at each input terminal causes signals to appear at both output terminals. The resultant output signals have a peak value of 2 V – twice the value for single-ended operation.

However, if two in-phase and equal signals were applied at the two input terminals, the resultant output signal at each output terminal would be zero as shown in Fig. 61.34. It means that output *between* the collectors would be zero.

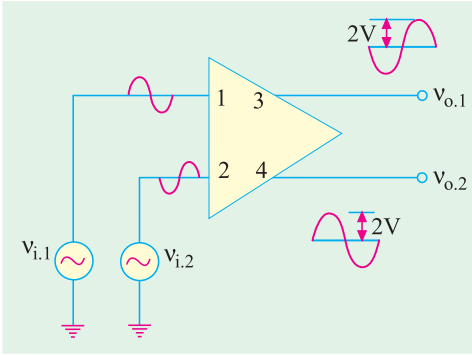


Fig. 61.33

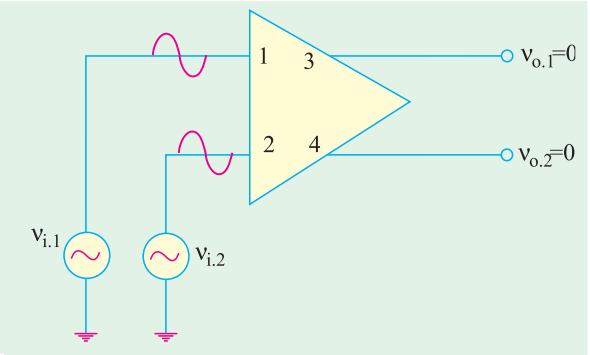


Fig. 61.34

If $v_{i.1}$ and $v_{i.2}$ change by *exactly* the same amount, even then output voltage between terminals 4 and 3 remains zero because of symmetry. Only when $v_{i.1}$ and $v_{i.2}$ differ from each other, we get an output voltage. When $v_{i.1}$ is more positive than $v_{i.2}$, the output terminal 4 is more positive than terminal 3.

(c) Inverting and Non-inverting Inputs

When positive $v_{i.1}$ acts alone, it produces a differential output voltage with terminal 4 positive with respect to terminal 3 as shown in Fig. 61.35. That is why the input terminal 1 is called non-inverting input terminal. How ever, when positive $v_{i.2}$ acts alone, the output voltage is inverted i.e. terminal 3 becomes positive with respect to terminal 4. That is why input terminal 2 is called ***inverting terminal***.

61.17. Common Mode Input

Fig. 61.36 illustrates the common-mode input of a differential amplifier *i.e.* when similar or same input signal is applied to both inputs. If the two halves of the diff-amp are identical, the ac output voltage will be zero. The diff-amp is then said to the perfectly balanced.

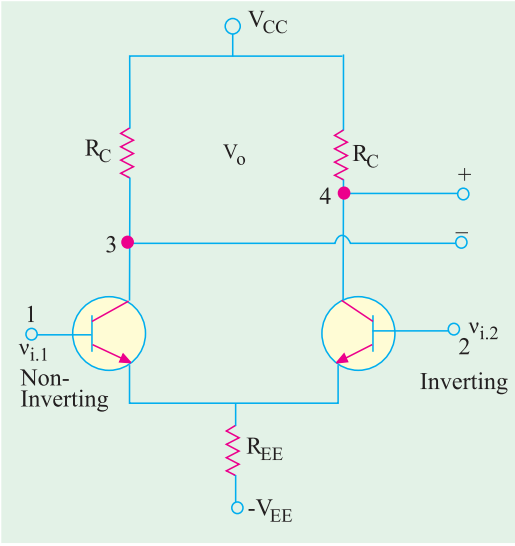


Fig. 61.35

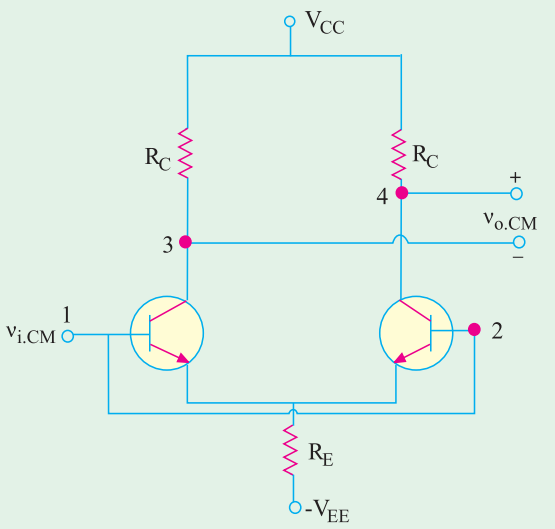


Fig. 61.36

The common-mode rejection ratio (*CMMR*) is defined as

$$CMMR = \frac{A \times v_{i.cm}}{v_{o.cm}}$$

Suppose, $v_{i.cm}$ = 1 V. In a perfectly-balanced and symmetrical diff-amp, the output should be zero. But, in practice, there is always a small output signal because of *non-symmetry*. Suppose, A = 100 and $v_{no.em}$ = 0.01V. Then

$$CMMR = \frac{100 \times 1\text{V}}{0.01\text{V}} = 10,000 = 20 \log_{10} 10,000 = \mathbf{80\ dB}$$

Larger the value of CMMR (*i.e.* smaller the value of $v_{o.cm}$), better the diff-amp. Ideally, a common-mode input should produce zero output voltage. Hence, an ideal diff-amp has a CMMR of ***infinity***.

Example 61.13. *Calculate the approximate output voltage for the diff-amp of Fig. 61.37 which uses only a single-ended non-inverting input of n-1 = 1 mV. Take* $r_e = 25\ mV/I_E$ *and neglect* V_{BE}.

Solution. $I_E \cong V_{EE}/R_E = 12/6 = 2$ mA

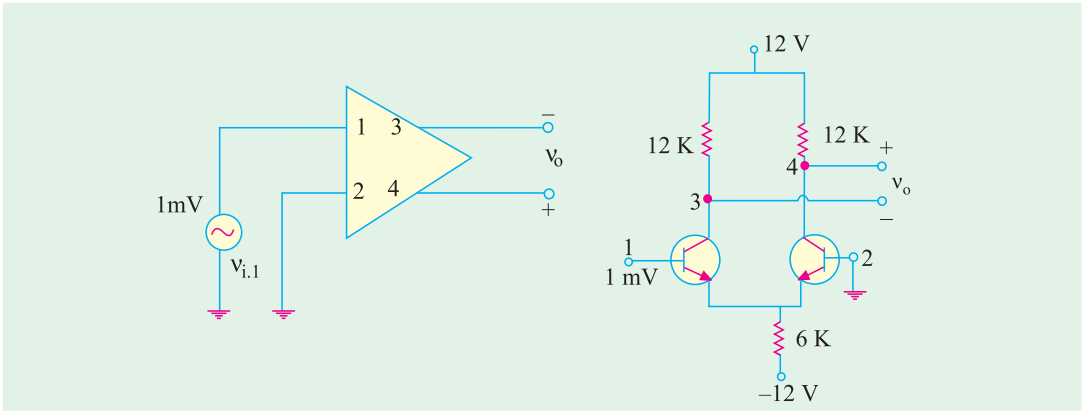


Fig. 61.37

The current is divided equally between the two transistors so that $I_{E.1} = I_{E.2} = 1$ mA.

Voltage amplification of each half is approximately given by $A = R_C/r_e$

Now, $r_e = 25/1 = 25\ \Omega$

$$\therefore \quad A = \frac{12,000}{25} = 480$$

$$\therefore \quad v_0 = A \times v_{i.1} = 480 \times 1$$

$$= \mathbf{480\ mV}$$

The ac output signal is in phase with the input.

Example 61.14. *If a differential input signal of 1 mV is applied to the diff-amp shown in Fig. 61.38, calculate the output voltage. Neglect V_{BE} and take $r_e = 25\ mV/I_E$.*

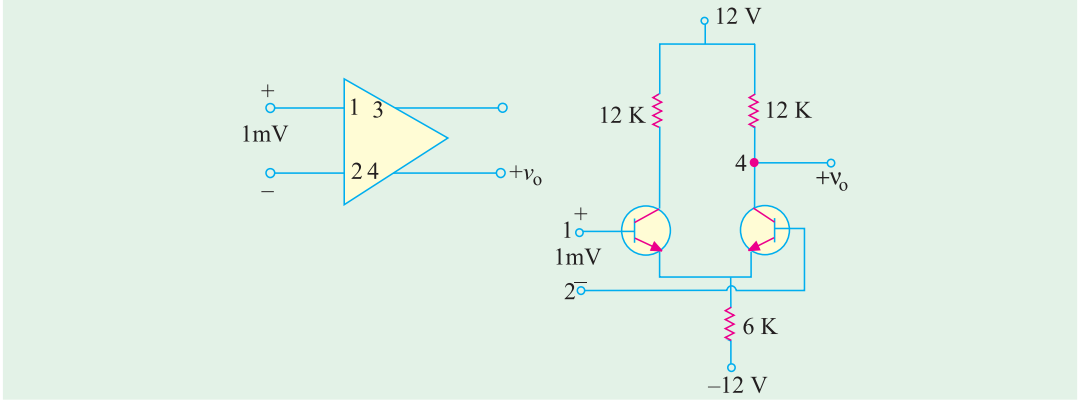


Fig. 61.38

Solution.

Here, $v_{i.1} - v_{i.2} = 1$ mV

As seen from Ex. 61.13,

$A = \mathbf{480}$

$\therefore \quad v_o = A\,(v_{i.1} - v_{i.2}) = 480 \times 1 = \mathbf{480\ mV}$

Example 61.15. *A differential input signal of 1 mV is applied to the diff-amp of Fig. 61.39 when used in single-ended output mode. Calculate the approximate value of output voltage. Neglect V_{BE} and take r_e = 25 mV/I_E.*

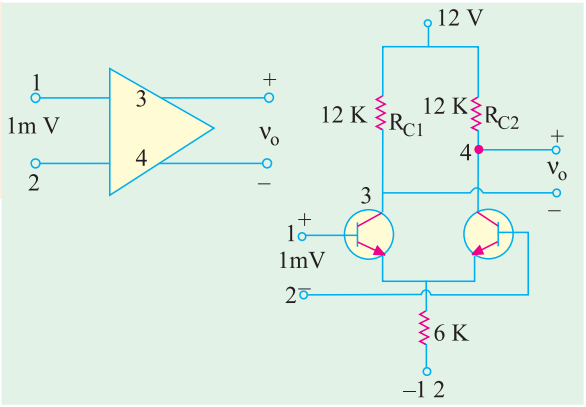


Fig. 61.39

Solution. Here, again 1 mV is the differential input signal *i.e.* $v_{i.1} - v_{i.2}$ = 1 mV

Since single-ended mode is being used,

A = 480/2 = **240**

$$\therefore \quad v_0 = A(v_{i.1} - v_{i.2})$$
$$= 240 \times 1 = \mathbf{240}$$

61.18. Differential Amplifier

Fig. 61.40 shows the circuit of a differential or difference amplifier. As seen

1. it contains two *CE* amplifiers,
2. it uses only resistors and transistors
3. it is directly-coupled (emitter-to-emitter) amplifier,
4. it can accept two inputs by means of T_1 and ground and also T_2 and ground,
5. it can provide two separate outputs by means of T_3 and ground and T_4 and ground,
6. it can provide a single output between T_3 and T_4 *i.e.* differential output.

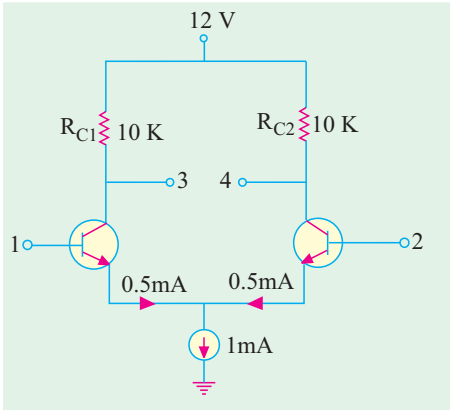


Fig. 61.40

Circuit Operation

We will consider a balanced differential amplifier in which Q_1 and Q_2 are identical and their associated components are matched. In that case, each amplifier stage produces same voltage gain. A = r_0/r_e

In Fig. 61.40. $r_o = R_2$ or R_3

The output voltage between terminals T_3 and T_4 is

$$v_0(T_3 - T_4) = A(v_{i.1} - v_{i.2})$$

where, A = voltage gain of each stage

Advantages

1. ***It uses no frequency-dependent coupling or bypassing capacitors.*** All that it requires is resistors and transistors both of which can be easily integrated on a chip. Hence, it is extensively used in linear IC_5.
2. It can compare any two signals and ***detect any difference.*** Thus, if two signals are fed into its inputs, identical in every respect except that one signal has been slightly distorted, then only the difference between the two signals *i.e.* distortion will be amplified.
3. It gives higher gain than two cascaded stages of ordinary direct coupling.
4. It provides very ***uniform amplification*** of signal from dc upto very high frequencies.

5. It provides isolation between input and output circuits.
6. It is almost a universal choice for amplifying dc.
7. It finds a wide variety of applications such as amplification, mixing, signal generation, amplitude modulation, frequency multiplication and temperature compensation etc.

Example 61.16. *Calculate the single-ended and differential gain of the diff-amp shown in Fig. 61.41. Use* $r_e = 25\ mV/I_E$.

Solution. The current of 1 mA from a constant-current source divides into two equal parts.

$$I_{E.1} = I_{E.2} = 1/2 = 0.5 \text{ mA}$$

$$r_{e.1} = r_{e.2} = \frac{25}{0.5} = 50\,\Omega$$

Hence, single-ended voltage gain is

$$A = \frac{r_0}{r_e} = \frac{R_C}{r_e} = \frac{10\,K}{50\,\Omega} = \mathbf{200}$$

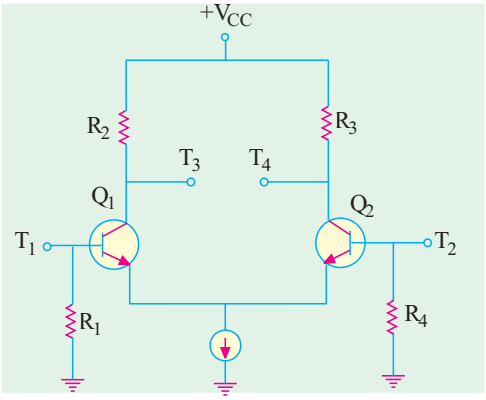


Fig. 61.41

Hence, each stage has a voltage gain of 200.

If we consider differential (double-ended) gain, its value is twice *i.e.* 2 × 200 = **400.**

Example 61.17. *For the diff-amp shown in Fig. 61.41, voltage gain of each stage is 200,* $v_{i.1} = 30\ mV$ *and* $v_{i.2} = 20\ mV$. *Find the voltages between*

(i) T_3 *and ground,* ***(ii)*** T_4 *and ground,* ***(iii)*** T_3 *and* T_4.

What are the polarities of output terminals T_3 *and* T_4 *?*

(Basic Electronics, Bombay Univ.)

Solution. As stated earlier in Art. 61.18.

(i) $v_0(T_3) = A_1 \times v_{i.1} = 200 \times 30 \text{ mV} = \mathbf{6\ V}$

(ii) $v_0(T_4) = A_2 \times v_{i.2} = 200 \times 20 \text{ mV} = \mathbf{4\ V}$

(iii) $v_0(T_3 - T_4) = A(v_{i.1} - v_{i.2}) = 200\ (30 \times 20) \text{ mV} = \mathbf{2\ V}$

Since $v_{i.1} > v_{i.2}$; $i_{c.1}\ R_2 > i_{.2} R_3$, hence T_4 will be positive with respect to T_3.

Example 61.18. *Calculate the over-all voltage gain of the two-stage RC coupled amplifier shown in Fig. 61.42. Neglect* V_{BE} *and take* $\beta_1 = \beta_2 = 100$.

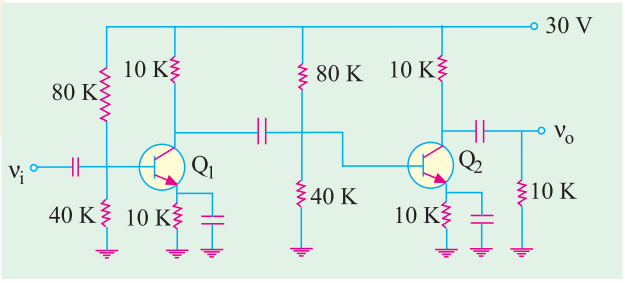


Fig. 61.42

Solution. We will first find gain of Q_2 and then multiply it with that of Q_1 to find the overall gain.

DC voltage from base to ground for Q_2 = drop across 40 K

$$= 30 \times 40/(40 + 80) = 10 \text{ V}$$

Hence, $I_{E.2} \cong 10\ V/10\ K = 1 \text{ mA}$

Assuming silicon transistor

$r_{e.2} = 50/1 \text{ mA} = 50\ \Omega$

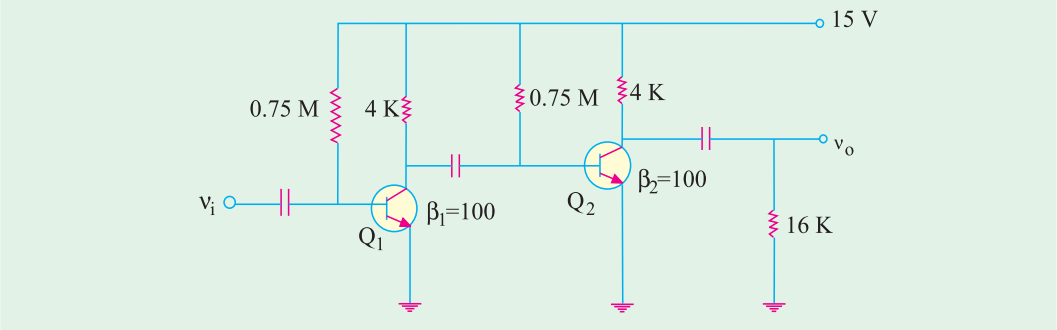


Fig. 61.43

$$\therefore \quad A_{v2} = \frac{r_{L2}}{r_{e2}} = \frac{10\text{K} \parallel 10\text{K}}{50} = 100$$

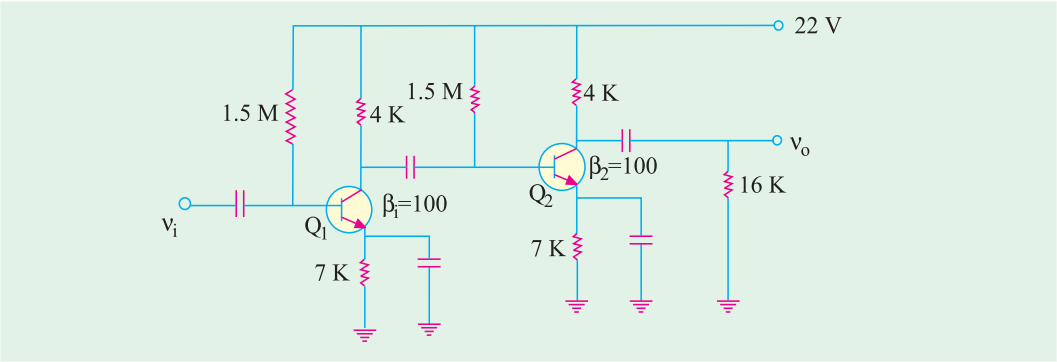


Fig. 61.44

For finding A_{v1}, we must first calculate the ac load resistance r_L as seen by Q_1. It equals the parallel combination of 10 K, 80 K, 40 K and $\beta r_e = 100 \times 50 = 5$ K (because it forms part of the load on Q_1)*.

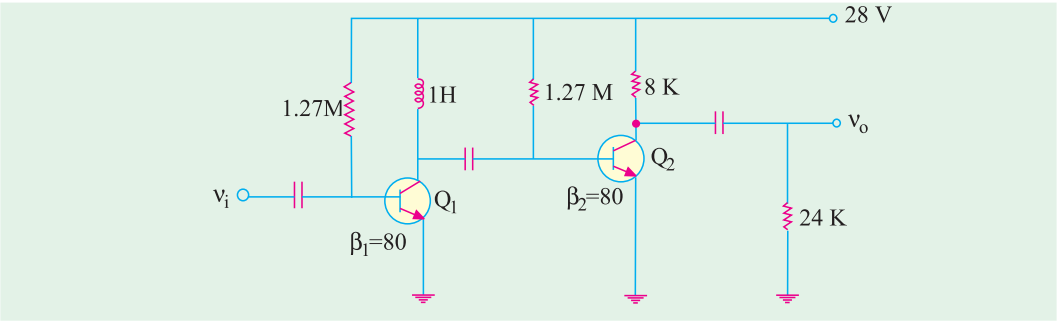


Fig. 61.45

$$\therefore \quad r_{L1} = 10\text{ K} \parallel 80\text{ K} \parallel 40\text{ K} \parallel 5\text{ K} \cong 3\text{ K}$$

$$\therefore \quad A_{v1} = r_{L1}/R_{e1} = 3\text{ K}/50\ \Omega = 60$$

$$\therefore \quad A = A_{v1} \times A_{v2} = 100 \times 60 = \mathbf{6000}$$

* (Emitter resistance does not come into the picture because it has been ac grounded by the capacitor.

Tutorial Problems No. 61.1

1. For the two-stage R_C-coupled amplifier shown in Fig. 61.43 calculate the approximate values of
(*a*) voltage gain for the first stage, (*b*) voltage gain for the second stage,
(*c*) voltage gain for the amplifier, (*d*) ac input resistance of the amplifier.
Neglect V_{BE} and use $r_e = 25\ mV/I_E$ **[(*a*) 76.2 (*b*) 256 (*c*) 19,500 or 85.8 dB (*d*) 1250 Ω]**

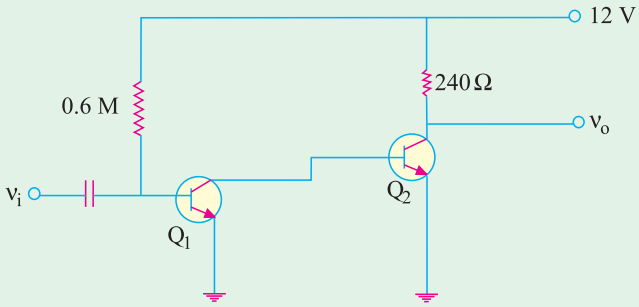


Fig. 61.46

2. For the two-stage *RC*-coupled amplifier using base and emitter bias and shown in Fig. 61.44, calculate approximate values of
(*i*) $A_{v.1}$ (*ii*) $A_{v.2}$ (*iii*) A_v (*iv*) r_i
Neglect V_{BE} and use $r_e = 25\ mV/I_E$ **[(*i*) 61.6 (*ii*) 128 (*iii*) 7,900 or 78 dB (*iv*) 2.6 K]**

3. Fig. 61.45 shows the circuit of a two-stage impedance-coupled amplifier. Compute the approximate values of
(*i*) voltage gain of 1st stage,
(*ii*) voltage gain of the 2nd stage ; both at $f = 2$ kHz,
(*iii*) voltage gain of the amplifier.
Neglect V_{BE} and take $r_e = 25\ mV/I_E$. **[(*i*) 80 (*ii*) 420 (*iii*) 33,600 or 90.5 dB]**

4. For the direct-coupled amplifier shown in Fig. 61.46, calculate the approximate value of
(*i*) A_i, (*ii*) $A_{v.1}$, (*iii*) $A_{v.2}$, (*iv*) A_v, (*v*) A_p and (*vi*) r_4
Neglect V_{BE} and assume $r_e = 25\ mV/I_E$. Also, $\beta_1 = 50$ and $\beta_2 = 25$.
[(*i*) 1250 (*ii*) 1 (*iii*) 240 (*iv*) 240 (*v*) 300,000 or 55 dB (*vi*) 1250 Ω]

OBJECTIVE TESTS – 61

1. The decibel gain of a cascaded amplifier equals the
(*a*) product of individual gains
(*b*) sum of individual gains
(*c*) ratio of stage gains
(*d*) product of voltage and current gains.

2. If two stages of a cascaded amplifier have decibel gains of 60 and 30, then overall gain is ..... dB.
(*a*) 90 (*b*) 1800
(*c*) 2 (*d*) 0.5

3. Cascading two amplifiers will result in
(*a*) reduction in overall gain and increase in overall bandwidth.
(*b*) reduction in overall gain and reduction in overall bandwidth
(*c*) increase in overall gain increase in overall bandwidth
(*d*) increase in overall gain and reduction in overall bandwith

4. The overall bandwidth of two identical voltage amplifiers connected in cascade will
(*a*) remain the same as that of a single stage
(*b*) be worse than that of a single stage
(*c*) be better than that of a single stage
(*d*) be better if stage gain is low and worse if stage gain is high.

5. *RC* coupling is popular in low-level audio amplifiers because it
(*a*) has better low frequency response
(*b*) is inexpensive and needs no adjustments
(*c*) provides an output signal in phase with the input signal
(*d*) needs low voltage battery for collec- tor supply.

6. Frequency response characteristic at a single-stage RC coupled amplifier is shown in Fig. 61.47. The fall in gain at both ends of the characteristic is due to
(*a*) transistor shunt capacitances
(*b*) bypass and coupling capacitances of the circuit
(*c*) transistor shunt capacitances of the lower end and bypass and coupling capacitances at the higher end

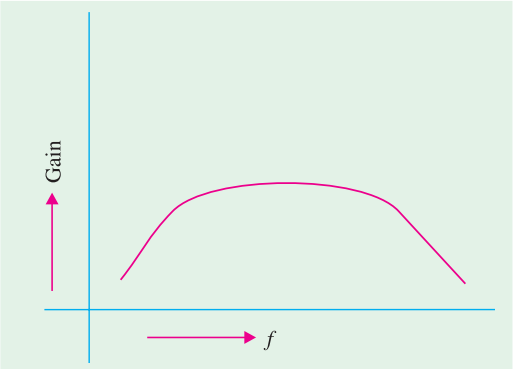


Fig. 61.47

(*d*) transistor shunt capacitances at the higher end and bypass and coupling capacitances at the lower end.

7. The most desirable feature of transformer coupling is its
(*a*) higher voltage gain
(*b*) wide frequency range
(*c*) ability to provide impedance match- ing between stages
(*d*) ability to eliminate hum from the output.

8. A transformer coupled amplifier would give
(*a*) maximum voltage gain
(*b*) impedance matching
(*c*) maximum current gain
(*d*) larger bandwidth

9. In multistage amplifiers, direct coupling is especially suited for amplifying
(*a*) high frequency ac signals
(*b*) changes in dc voltages
(*c*) high-level voltages
(*d*) sinusoidal signals.

10. The outstanding characteristic of a direct-coupled amplifier is its
(*a*) utmost economy
(*b*) temperature stability
(*c*) avoidance of frequency-sensitive components
(*d*) ability to amplify direct current and low-frequency signals.

11. A signal may have frequency components which lie in the range of 0.001 Hz to 10 Hz. Which one of the following types of couplings should be chosen in a multistage amplifier designed to amplify this signal ?
(*a*) *RC* coupling
(*b*) transformer coupling
(*c*) direct coupling
(*d*) double-tuned transformer

12. Darlington pairs are frequently used in linear *ICs* because they
(*a*) do not require any capacitors or inductors
(*b*) have enormous impedance transformation capability
(*c*) can be readily formed from two adjacent transistors
(*d*) resemble emitter followers.

13. When same input signal is applied to both the inputs of an ideal diff-amp, the output
(*a*) is zero
(*b*) depends on its *CMMR*
(*c*) depends on its voltage gain
(*d*) is determined by its symmetry.

14. The common-mode rejection ratio of an ideal diff-amp is
(*a*) zero
(*b*) infinity
(*c*) less than unity
(*d*) greater than unity.

15. One of the advantages of a Darlington pair is that it has enormous ..... transformation capacity.
(*a*) voltage
(*b*) current
(*c*) impedance
(*d*) power

16. A Darlington pair and an emitter follower have the same
(*a*) input impedance
(*b*) current gain
(*c*) voltage gain
(*d*) power gain.

17. Which of the following mode of operation is possible with a differential amplifier ?

(*a*) single-ended input
(*b*) differential input
(*c*) common-mode input
(*d*) all of the above.

18. The gain of a Darlington amplifier pair is determined by ..... the beta values.
(*a*) subtracting
(*b*) adding
(*c*) dividing
(*d*) multiplying

19. The amplifier in which the emitter and collector leads of one transistor are connected to the base and collector leads of a second transistor is called an ..... amplifier.
(*a*) push-pull
(*b*) Darlington
(*c*) differential
(*d*) complementary

ANSWERS

1. (*b*) **2.** (*a*) **3.** (*c*) **4.** (*c*) **5.** (*b*) **6.** (*d*) **7.** (*c*) **8.** (*b*) **9.** (*b*) **10.** (*d*) **11.** (*c*)
12. (*c*) **13.** (*a*) **14.** (*b*) **15.** (*c*) **16.** (*c*) **17.** (*d*) **18.** (*d*) **19.** (*b*)

GO To FIRST

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CHAPTER 62

FEEDBACK AMPLIFIER

Learning Objectives

- Feedback Amplifiers
- Principle of Feedback Amplifiers
- Advantages of Negative Feedback
- Gain Stability
- Decreased Distortion
- Feedback Over Several Stages
- Increased Bandwidth
- Forms of Negative Feedback
- Shunt-derived Series-fed Voltage Feedback
- Current-series Feedback Amplifier
- Voltage-shunt Negative Feedback Amplifier
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- Effect of Negative Feedback on R_{in} and R_{out}
- R_{in} and R_{out} of Inverting Op-amp with Negative Feedback



A feedback amplifier is one in which a fraction of the amplifier output is fed back to the input circuit

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62.1. Feedback Amplifiers

A feedback amplifier is one in which a fraction of the amplifier output is fed back to the input circuit. This partial dependence of amplifier output on its input helps to control the output. A feedback amplifier consists of two parts : an amplifier and a feedback circuit.

(*i*) Positive feedback

If the feedback voltage (or current) is so applied as to increase the input voltage (*i.e.* it is in phase with it), then it is called positive feedback. Other names for it are : ***regenerative* or *direct*** feedback.

Since positive feedback produces excessive distortion, it is seldom used in amplifiers. However, because it increases the power of the original signal, it is used in oscillator circuits.

(*ii*) Negative feedback

If the feedback voltage (or current) is so applied as to reduce the amplifier input (*i.e.* it is 180° out of phase with it), then it is called negative feedback. Other names for it are : ***degenerative*** or ***inverse*** feedback.

Negative feedback is frequently used in amplifier circuits.

62.2. Principle of Feedback Amplifiers

For an ordinary amplifier *i.e.* one without feedback, the voltage gain is given by the ratio of the output voltage V_o and input voltage V_i. As shown in the block diagram of Fig. 62.1, the input voltage V_i is amplified by a factor of A to the value V_o of the output voltage.

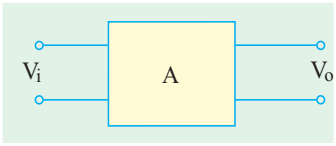


Fig. 62.1

$$\therefore \quad A = V_o/V_i$$

This gain A is often called ***open-loop*** gain.

Suppose a feedback loop is added to the amplifier (Fig. 62.2). If V_o' is the output voltage with feedback, then a fraction β* of this voltage is applied to the input voltage which, therefore, becomes $(V_i \pm \beta V_o')$ depending on whether the feedback voltage is in phase or antiphase with it. Assuming positive feedback, the input voltage will become $(V_i + \beta V_o')$. When amplified A times, it becomes $A(V_i + \beta V_o')$.

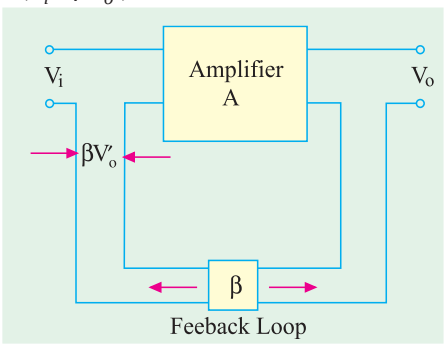


Fig. 62.2

$$\therefore \quad A(V_i + \beta V_o') = V_o'$$

$$\text{or} \quad V_o'(1 - \beta A) = AV_i$$

The amplifier gain A' with feedback is given by

$$A' = \frac{V_o'}{V_i} = \frac{A}{1-\beta A}$$

$$\therefore \quad A' = \frac{A}{1-\beta A} \quad \text{— positive feedback}$$

$$= \frac{A}{1-(-\beta A)} = \frac{A}{1+\beta A}$$

— negative feedback

The term 'βA' is called **feedback factor** whereas β is known as **feedback ratio**. The expression $(1 \pm \beta A)$ is called **loop gain**. The amplifier gain A' with feedback is also referred to as **closed-loop gain** because it is the gain obtained after the feedback loop is closed. The sacrifice factor is defined as $S = A/A'$.

* It may please be noted that it is not the same as the β of a transistor (Art.57.9)

(a) Negative Feedback

The amplifier gain with negative feedback is given by $A' = \dfrac{A}{(1+\beta A)}$

Obviously, $A' < A$ because $|1 + \beta A| > 1$.
Suppose, $A = 90$ and $\beta = 1/10 = 0.1$
Then, gain without feedback is 90 and with negative feedback is

$$A' = \frac{A}{1+\beta A} = \frac{90}{1+0.1\times 90} = 9$$

As seen, negative feedback reduces the amplifier gain. That is why it is called ***degenerative*** feedback. A lot of voltage gain is sacrificed due to negative feedback. When $|\beta A| \gg 1$, then

$$A' \cong \frac{A}{\beta A} \cong \frac{1}{\beta}$$

It means that A' depends only on β. But it is very stable because it is not affected by changes in temperature, device parameters, supply voltage and from the aging of circuit components etc. Since resistors can be selected very precisely with almost zero temperature-coefficient of resistance, it is possible to achieve highly precise and stable gain with negative feedback.

(b) Positive Feedback

The amplifier gain with positive feedback is given by

$$A' = \frac{A}{1-\beta A} \qquad \text{Since } |1-\beta A| < 1,\ A' > A$$

Suppose gain without feedback is 90 and $\beta = 1/100 = 0.01$, then gain with positive feedback is

$$A' = \frac{90}{1-(0.01\times 90)} = 900$$

Since positive feedback increases the amplifier gain. It is called ***regenerative*** feedback. If $\beta A = 1$, then mathematically, the gain becomes infinite which simply means that there is an output without any input! However, electrically speaking, this cannot happen. What actually happens is that the amplifier becomes an oscillator which supplies its own input. In fact, two important and necessary conditions for circuit oscillation are

1. the feedback must be positive, **2.** feedback factor must be unity *i.e.* $\beta A = +1$.

62.3. Advantages of Negative Feedback

The numerous advantages of negative feedback outweigh its only disadvantage of reduced gain. Among the advantages are :

1. higher fidelity *i.e.* more linear operation,
2. highly stabilized gain,
3. increased bandwidth *i.e.* improved frequency response,
4. less amplitude distortion,
5. less harmonic distortion,
6. less frequency distortion,
7. less phase distortion,
8. reduced noise,
9. input and output impedances can be modified as desired.

Example 62.1. *In the series-parallel (SP) feedback amplifier of Fig. 62.3, calculate*
(a) *open-loop gain of the amplifier,* **(b)** *gain of the feedback network,*
(c) *closed-loop gain of the amplifier,* **(d)** *sacrifice factor, S.*
(Applied Electronics-I, Punjab Univ. 1991)

Solution. (a) Since 1 mV goes into the amplifier and 10 V comes out

$$\therefore \quad A = \frac{10\,\text{V}}{1\,\text{mV}} = \mathbf{10{,}000}$$

(*b*) The feedback network is being driven by the output voltage of 10 V.

$\therefore$ Gain of the feedback network

$$= \frac{\text{output}}{\text{input}} = \frac{250\,\text{mV}}{10\,\text{V}} = \mathbf{0.025}$$

(*c*) So far as the feedback amplifier is concerned, input is (250 + 1) = 251 mV and final output is 10 V. Hence, gain with feedback is

$$A' = 10\ \text{V}/251\ \text{mA} = \mathbf{40}$$

(*d*) The sacrifice factor is given by

$$S = \frac{A}{A'} = \frac{10{,}000}{40} = \mathbf{250}$$

By sacrificing so much voltage gain, we have improved many other amplifier quantities. (Art. 62.3)

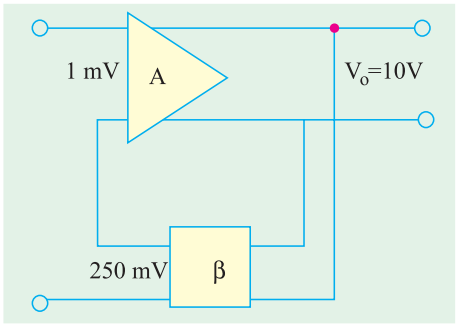


Fig. 62.3

Example 62.2. *Calculate the gain of a negative feedback amplifier whose gain without feedback is 1000 and β = 1/10. To what value should the input voltage be increased in order that the output voltage with feedback equals the output voltage without feedback ?*

Solution. Since $|\beta A| \gg 1$, the closed-loop gain is $A' \cong \frac{1}{\beta} \cong \frac{1}{1/10} = \mathbf{10}$

The new increased input voltage is given by

$V_i' = V_i(1 + \beta A) = 50\ (1 + 0.04 \times 100) = \mathbf{250\ mV}$

Example 62.3. *In a negative-feedback amplifier, A = 100, β = 0.04 and V_i = 50 mV. Find*
(*a*) *gain with feedback,* **(*b*)** *output voltage,*
(*c*) *feedback factor,* **(*d*)** *feedback voltage.* **(Applied Electronics, AMIEE, London)**

Solution. (*a*) $A' = \frac{A}{1+\beta A} = \frac{100}{1 + 0.04 \times 100} = \mathbf{20}$

(*b*) $V_0' = A'\,V_i = 20 \times 50\ \text{mV} = \mathbf{1V}$ **(*c*)** feedback factor $= \beta A = 0.04 \times 100 = \mathbf{4}$

(*d*) Feedback voltage $= \beta V_o' = 0.04 \times 1 = \mathbf{0.04\ V}$

Example 62.4. *An amplifier having a gain of 500 without feedback has an overall negative feedback applied which reduces the gain to 100. Calculate the fraction of output voltage feedback. If due to ageing of components, the gain without feedback falls by 20%, calculate the percentage fall in gain without feedback.* **(Applied Electronics-II, Punjab Univ. 1993)**

Solution. $A' = \frac{A}{1+\beta A} \qquad \therefore \quad 1 + \beta A = \frac{A}{A'}$

$$\therefore \quad \beta = \frac{1}{A'} - \frac{1}{A} = \frac{1}{100} - \frac{1}{500} = \mathbf{0.008}$$

Now, gain without feedback = 80% of 500 = 400

$$\therefore \quad \text{New} \quad A' = \frac{400}{1 + 0.008 \times 400} = 95.3$$

Hence, change in the gain with feedback in the two cases = 100 – 95.3 = 4.7

$$\therefore \quad \text{Percentage fall in gain with feedback is} = \frac{4.7}{100} \times 100 = \mathbf{4.7\%}$$

Example 62.5. *An amplifier with negative feedback has a voltage gain of 100. It is found that without feedback an input signal of 50 mV is required to produce a given output whereas with feedback, the input signal must be 0.6 V for the same output. Calculate the value of voltage gain without feedback and feedback ratio.* **(Bangalore University 2001)**

Solution. $V_o' = AV_i = 100 \times 0.6 = 60$ V and $V_o = AV_i$

Since the output voltage with and without feedback are required to be the same,

$\therefore \quad 60 = A \times 50$ mV, $\quad \therefore \quad A = \dfrac{60}{50\,\text{mV}} = \mathbf{1200}$

The amplifier gain with feedback,

$$A' = \frac{A}{1+\beta A} \quad \text{or} \quad \beta = \frac{A - A'}{AA'} = \frac{1200 - 100}{1200 \times 100} = \mathbf{0.009}$$

62.4. Gain Stability

The gain of an amplifier with negative feedback is given by $A' = \dfrac{A}{1+\beta A}$

Taking logs of both sides, we have $\log_e A' = \log_e A - \log_e(1 + \beta A)$

Differentiating both sides, we get

$$\frac{dA'}{A'} = \frac{dA}{A} - \frac{\beta . dA}{1+\beta A} = dA\left(\frac{1}{A} - \frac{\beta}{1+\beta A}\right) = \frac{1}{1+\beta A}\frac{dA}{A} = \frac{(dA/A)}{1+\beta A}$$

If $\beta A \gg 1$, then the above expression becomes

$$\frac{dA'}{A'} = \frac{1}{\beta A} \cdot \frac{dA}{A}$$

Example 62.6. *An amplifier has an open-loop gain of 400 and a feedback of 0.1. If open-loop gain changes by 20% due to temperature, find the percentage change in closed-loop gain.* **(Electronics-III, Bombay 1991)**

Solution. Here, $A = 400$, $\beta = 0.1$, $dA/A = 20\% = \mathbf{0.2}$

Now, $\dfrac{dA'}{A'} = \dfrac{1}{\beta A} \cdot \dfrac{dA}{A} = \dfrac{1}{0.1 \times 400} \times 20\% = \mathbf{0.5\%}$

It is seen that while the amplifier gain changes by 20%, the feedback gain changes by only 0.5% *i.e.* an improvement of 20/0.5 = 40 times

62.5. Decreased Distortion

Let the harmonic distortion voltage generated within the amplifier change from D to D' when negative feedback is applied to the amplifier.

Suppose $D' = x\,D$ ... **(*i*)**

The fraction of the output distortion voltage which is fedback to the input is

$\beta D' = \beta\, x\, D$

After amplification, it become $\beta\, x\, D_A$ and is antiphase with original distortion voltage D. Hence, the new distortion voltage D' which appears in the output is

$D' = D - \beta\, x\, DA$ ... **(*ii*)**

From **(*i*)** and **(*ii*)**, we get

$xD = D - \beta\, x\, DA$ or $x = \dfrac{1}{1+\beta A}$

Substituting this value of x in Eq. (*i*) above, we have $D' = \dfrac{D}{1+\beta A}$

It is obvious from the above equation that $D' < D$. In fact, negative feedback reduces the amplifier distortion by the amount of loop gain *i.e.* by a factor of $(1 + \beta A)$.

However, it should be noted that improvement in distortion is possible only when the distortion is produced by the ***amplifier itself,*** not when it is already present in the input signal.

62.6. Feedback Over Several Stages

Multistage amplifiers are used to achieve greater voltage or current amplification or both. In such a case, we have a choice of applying negative feedback to improve amplifier performance. Either we apply some feedback across each stage or we can put it in one loop across the whole amplifier.

A multistage amplifier is shown in Fig. 62.4. In Fig. 62.4 (*a*) each stage of the *n*-stage amplifier has a feedback applied to it. Let A and β_1 be the open-loop gain and feedback ratio respectively of each stage and A_1 the overall gain of the amplifier. Fig. 62.4 (*b*) shows the arrangemnent where n amplifiers have been cascaded in order to get a total gain of A^n. Let the overall feedback factor be β_2 and the overall gain A_2. The values of the two gains are given as

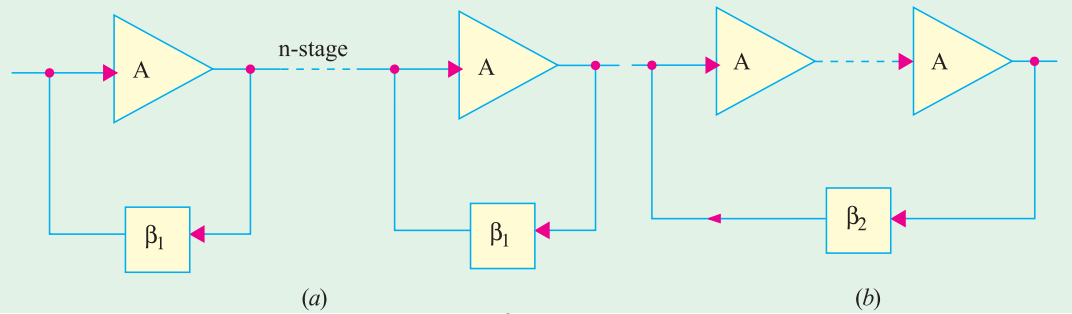


Fig. 62.4

$$A_1 = \left(\frac{A}{1+A\beta_1}\right)^n \qquad \text{and} \qquad A_1 = \frac{A^n}{1+A^n\beta_2} \qquad \text{... (i)}$$

Differentiating the above two expressions, we get

$$\frac{dA_1}{A_1} = \frac{n}{1+A\beta_1}\cdot\frac{dA}{A} \qquad \text{and} \qquad \frac{dA_2}{A_2} = \frac{n}{1+A^n\beta_2}\cdot\frac{dA}{A}$$

For the two circuits to have the same overall gain, $A_1 = A_2$. Hence, from Eqn. (*i*) above, we get $(1 - 1\beta)^n = 1 + A^n \beta_2$

$$\therefore \qquad \frac{dA_2 / A_2}{dA_1 / A_1} = \frac{1}{(1+A\beta)^{n-1}}$$

If $n = 1$, then the denominator in the above equation becomes unity so that fractional gain variations are the same as expected. However, for $n > 1$ and with $(1 + A\beta_1)$ being a normally large quantity, the expression dA_2/A_2 will be less than dA_1/A_1. It means that the overall feedback would appear to be beneficial as far as stabilizing of the gain is concerned.

Example 62.7. *An amplifier with 10% negative feedback has an open-loop gain of 50. If open-loop gain increases by 10%, what is the percentage change in the closed-loop gain ?*

(Applied Electronics-I, Punjab Univ. 1991)

Solution. Let A_1' and A_2' be the closed-loop gains in the two cases and A_1 and A_2 the open-loop gains respectively.

(i) $A_1' = \dfrac{A_1}{1 + \beta A_1} = \dfrac{50}{1+0.1\times 50} = \mathbf{8.33}$

(ii) When open-loop gain changes by 10%, then $A_2 = 50 + 0.1 \times 50 = 55$

$$\therefore \quad A_2' = \frac{A_2}{1+\beta A_2} = \frac{55}{1+0.1\times 55} = 8.46$$

$\therefore$ Percentage change in closed-loop gain is

$$= \frac{A'_2 - A'_1}{A'_1} \times 100 = \frac{8.46 - 8.33}{8.33} \times 100 = \mathbf{1.56\%}$$

Example 62.8. *Write down formulae for **(i)** gain **(ii)** harmonic distortion of a negative feedback amplifier in terms of gain and distortion without feedback and feedback factor. If gain without feedback is 36 dB and harmonic distortion at the normal output level is 10%, what is **(a)** gain and **(b)** distortion when negative feedback is applied, the feedback factor being 16 dB.*

(Electronic Engg. II, Warangal 1991)

Solution. For first part, please refer to Art. 62.6. Distortion ratio is defined as the ratio of the amplitude of the largest harmonic to the amplitude of the fundamental.

$$A_f = A' = \frac{A}{1+\beta A}$$

Now, dB gain $= 20 \log_{10} A$ $\quad\therefore\quad$ $36 = 20 \log_{10} A$, $A = 63$

dB feedback factor $= 20 \log_{10} \beta A$ $\quad$ $16 = 20 \log_{10} \beta A$ or $\quad \beta A = 6.3$

(a) $A_f = A/(1 + \beta A) = 63/(1 + 6.3) =$ **6.63** or **18.72 dB**

(b) $D' = 10$ per cent$/(1 + 6.3) =$ **1.4 per cent**

Example 62.9. *The overall gain of a two-stage amplifier is 150. The second stage has 10% of the output voltage as negative feedback and has –150 as forward gain. Calculate **(a)** gain of the first stage **(b)** the second harmonic distortion, if the second stage introduces 5% second harmonic without feedback. Assume that the first stage does not introduce distortion.*

(Electronics-II, Madras Univ. 1992)

Solution. (a) For second stage $D'_2 = \frac{D_2}{1+\beta A_2} = \frac{0.05}{1+150\times 0.1} =$ **0.31%**

(b) For the second stage, gain with feedback is

$$A'_2 = \frac{A_2}{1+\beta A_2} = \frac{150}{1+150\times 0.1} = 9.38$$

Now, $A_1 \times A'_2 = 150$; $\quad A_1 = 150/9.38 =$ **16**

Example 62.10. *Determine the effective ga-*in of a feedback amplifier having an amplification without feedback of (–200 – j300) if the feedback circuit adds to the input signal, a p.d. which is 0.5 percent of the output p.d. and lags a quarter of a cycle behind it in phase. Explain whether the feedback in this case is positive or negative.* **(Applied Electronics-II, Punjab Univ. 1992)**

Solution. $A = -200 - j300 = 360 \angle -123.7°$

The feedback voltage V_β is 0.5 percent of the output voltage and lags 90° behind it.

$$\therefore \quad V_\beta = \left(\frac{0.5}{100} \angle -90°\right) V_0$$

$$\therefore \quad \beta = \frac{V_\beta}{V_0} = \frac{0.5}{100} \angle -90° = -j0.005$$

$$\therefore \quad \beta A = (-200 - j300)(-j0.005) = -1.5 + j1.0$$

In general, the stage gain with feedback is given by

$$A' = \frac{A}{1-\beta A} = \frac{360\angle -123.7°}{1-(-1.5+j1.0)} = \frac{360\angle -123.7°}{2.69\angle -21.8°} = \mathbf{134\angle -102°}$$

Since both the magnitude and the phase shift of the amplifier are reduced by feedback, the feedback must be negative.

Example 62.11. *An amplifier has a gain of 100 and 5 per cent distortion with an input signal of 1 V. When an input signal of 1 V is applied to the amplifier, calculate*
(i) *output signal voltage,* ***(ii)*** *distortion voltage,* ***(iii)*** *output voltage*

Solution. ***(i)*** Signal output voltage $V_{os} = AV_i = 100 \times 1 = \mathbf{100\ V}$

(ii) Distortion voltage $= DV_o = 0.05 \times 100 = \mathbf{5\ V}$

(iii) Amplifier output voltage $V_o = V_{os} + D = 100 + 5 = \mathbf{105\ V}$

62.7. Increased Bandwidth

The bandwidth of an amplifier without feedback is equal to the separation between the 3 dB frequencies f_1 and f_2.

$$\therefore\ BW = f_2 - f_1$$

where f_1 = lower 3 dB frequency, and f_2 = upper 3 dB frequency.

If A is its gain, the gain-bandwidth product is $A \times BW$.

Now, when negative feedback is applied, the amplifier gain is reduced. Since the gain-bandwidth product has to remain the same in both cases, it is obvious that the bandwidth must increase to compensate for the decrease in gain. It can be proved that with negative feedback, the lower and upper 3 dB frequencies of an amplifier become.

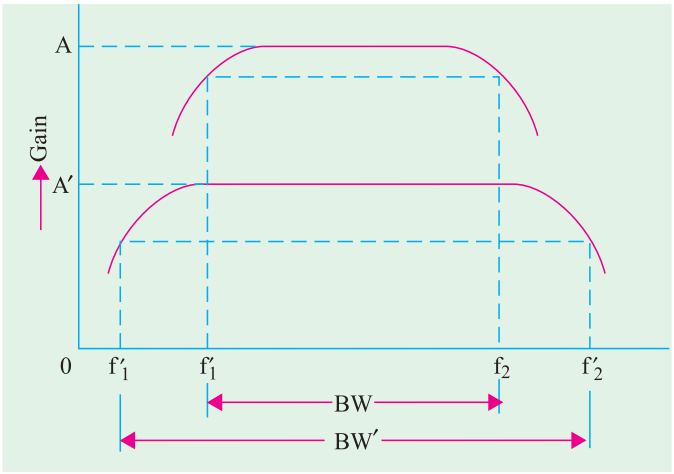


Fig. 62.5

$$(f')_1 = \frac{f_1}{(1+\beta A)} \text{ and } (f)_2 = f_2(1+\beta A)$$

As seen from Fig. 62.5, f_1' has decreased whereas f_2' has increased thereby giving a wider separation or bandwidth. Since gain-bandwidth product is the same in both cases.

$$\therefore\ A \times BW = A' \times BW' \text{ or } A(f_2 - f'_1) = A(f'_2 - f'_1)$$

Example 62.12. *An RC-coupled amplifier has a mid-frequency gain of 200 and a frequency response from 100 Hz to 20 kHz. A negative feedback network with* $\beta = 0.02$ *is incorporated into the amplifier circuit. Determine the new system performance.*

(Electronic Circuits, Mysore Univ. 1990)

Solution.
$$A' = \frac{A}{1+\beta A} = \frac{200}{1+0.02\times 200} = \mathbf{40\ Hz}$$

$$f_1' = \frac{f_1}{1+\beta A} = \frac{100}{1+0.02\times 200} = \mathbf{20\ Hz}$$

$f_2' = f_0(1 + \beta A) = 20(1 + 0.02 \times 200) =$ **100 Hz**

$dW' = f_2' - f_1' \cong$ **100 kHz**

Incidentally, it may be proved that gain-bandwidth product remains constant in both cases.

$$dW = f_2 - f_1 \cong 20 \text{ kHz}$$

$$A \times dW = 200 \times 20 = 4000 \text{ kHz};$$

$$A' \times dW' = 40 \times 100 = 4000 \text{ kHz}$$

As expected, the two are equal.

62.8. Forms of Negative Feedback

The four basic arrangements for using negative feedback are shown in the block diagram of Fig. 62.6. As seen, both voltage and current can be fedback to the input either in series or in parallel. The output voltage provides input in Fig. 62.6 (*a*) and (*b*). However, the input to the feedback network is derived from the output current in Fig. 62.6 (*c*) and (*d*).

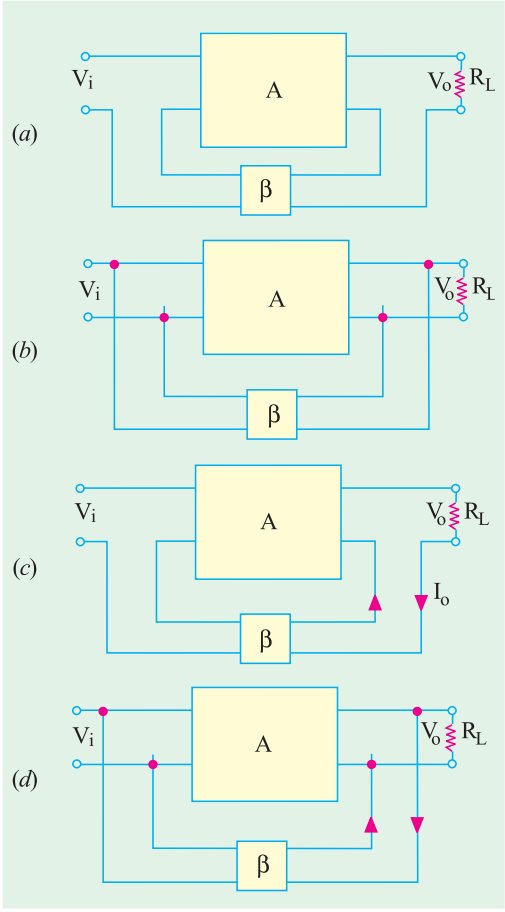


Fig. 62.6

(*a*) Voltage-series Feedback

It is shown in Fig. 62.6 (*a*). It is also called shunt-***derived series-fed feedback.*** The amplifier and feedback circuit are connected series-parallel. Here, a fraction of the output voltage is applied in series with the input voltage via the feedback. As seen, the input to the feedback network is in parallel with the output of the amplifier. Therefore, so far as V_o is concerned, output resistance of the amplifier is reduced by the shunting effect of the input to the feedback network. It can be proved that

$$R'_o = \frac{R_o}{(1+\beta A)}$$

Similarly, V_i sees two circuit elements in series :

(***i***) the input resistance of the amplifier and

(***ii***) output resistance of the feedback network.

Hence, input resistance of the amplifier as a whole is increased due to feedback. It can be proved that

$$R_i' = R_i(1 + \beta A)$$

In fact, ***series feedback always increases the input impedance by a factor of*** $(1 + \beta A)$.



Shunt Voltage

(*b*) Voltage-shunt Feedback

It is shown in Fig. 62.6 (*b*). It is also known as ***shunt-derived shunt-fed feedback*** *i.e.* it is parallel-parallel (*PP*) prototype. Here, a small portion of the output voltage is coupled back to the input voltage parallel (shunt).

Since the feedback network shunts both the output and input of the amplifier, it decreases both its output and input impedances by a factor of $1/(1 + \beta A)$

A shunt feedback ***always decreases input impedance.***

(*c*) Current-series Feedback

It is shown in Fig. 62.6 (*c*). It is also known as ***series-derived series-fed feedback.*** As seen, it is a series-series (*SS*) circuit. Here, a part of the output current is made to feedback a proportional voltage in series with the input. Since it is a series pick-up and a series feedback, both the input and output impedances of the amplifier are increased due to feedback.

(*d*) Current-shunt Feedback

It is shown in Fig. 62.6 (*d*). It is also referred to as ***series-derived shunt-fed feedback.*** It is a parallel-series (*PS*) prototype. Here, the feedback network picks up a part of the output current and develops a feedback voltage in parallel (shunt) with the input voltage. As seen, feedback network shunts the input but is in series with the output. Hence, output resistance of the amplifier is increased whereas its input resistance is decreased by a factor of loop gain.

The effects of negative feedback on amplifier characteristics are summarized below :

Characteristics	*Type of Feedback*			
	Voltage series	Voltage shunt	Current series	Current shunt
Voltage gain	decreases	decreases	decreases	decreases
Bandwidth	increases	increases	increases	increases
Harmonic Distortion	decreases	decreases	decreases	decreases
Noise	decreases	decreases	decreases	decreases
Input Resistance	increases	decreases	increases	decreases
Output Resistance	decreases	decreases	increases	increases

62.9. Shunt-derived Series-fed Voltage Feedback

The basic principle of such a voltage-controlled feedback is illustrated by the block diagram of Fig. 62.7. Here, the feedback voltage is derived from the voltage divider circuit formed of R_1 and R_2.

As seen, the voltage drop across R_1 forms the feedback voltage V_f.

$$\therefore \quad V_f = V_o \frac{R_1}{R_1 + R_2} = \beta V_o$$

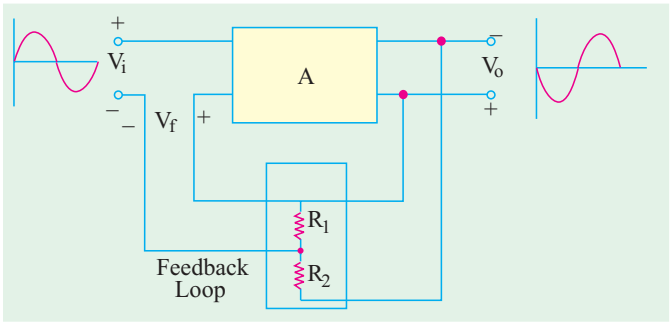


Fig. 62.7

Example 62.13. *In the voltage-controlled negative feedback amplifier of Fig. 62.8, calculate* ***(a)*** *voltage gain without feedback* ***(b)*** *feedback factor* ***(c)*** *voltage gain with feedback. Neglect* V_{BE} *and use* $r_e = 25\ mV/I_E$.

Solution. (*a*) $$A = \frac{r_L}{r_e} = \frac{R_3}{r_e}$$

Now, $I_B = \dfrac{15\,V}{1.5M} = 10\,\mu A$

$I_E = \beta IB = 100 \times 10 = 1\,mA$

$r_e = 25/1 = 25\,\Omega$;

$A = \dfrac{10K}{25\Omega} = \mathbf{400}$

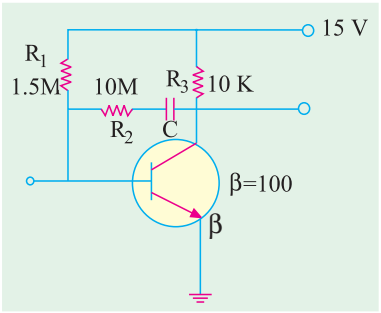


Fig. 62.8

(b) $\beta = \dfrac{R_1}{R_1 + R_2} = \dfrac{1.5 \times 10^6}{(1.5 + 10) \times 10^6} = 0.13$

$\therefore \quad \beta A = 0.13 \times 400 = \mathbf{52}$

(c) $A' = \dfrac{A}{1 + \beta A} = \dfrac{400}{1 + 52} = \mathbf{7.55}$

62.10. Current-series Feedback Amplifier

Fig. 62.9 shows a series-derived series-fed feedback amplifier circuit. Since the emitter resistor is unbypassed, it effectively provides current-series feedback. When I_E passes through R_E, the feedback voltage drop $V_f = I_E R_E$ is developed which is applied in phase opposition to the input voltage V_i. This negative feedback reduces the output voltage V_0. This feedback can, however, be eliminated by either removing or bypassing the emitter resistor.

It can be proved that

$$\beta = \frac{R_E}{R_C} \quad ; \quad A' = \frac{R_C}{r_e + R_E} \quad ; \quad A = \frac{R_C}{r_e}$$

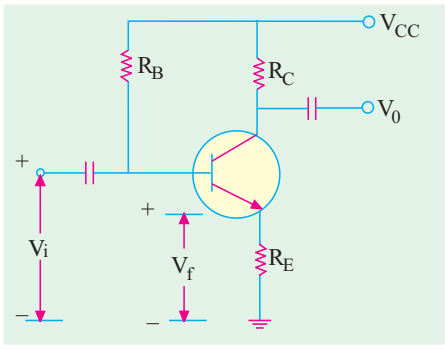


Fig. 62.9

Example 62.14. *For the current-series feedback amplifier of Fig. 62.10, calculate* **(*i*)** *voltage gain without feedback,* **(*ii*)** *feedback factor,* **(*iii*)** *voltage gain with feedback. Neglect V_{BE} and use $r_e = 25\ mV/I_E$.* **(Electronics-I, Madras Univ. 1990)**

Solution. (*i*) $A = \dfrac{R_C}{r_e}$

Now, $I_E = \dfrac{V_{CC}}{R_E + R_B/\beta}$

$= \dfrac{10}{1 + 900/100} = 1\,mA$

$\therefore \quad r_e = 25/I_E = 25\,\Omega$

$\therefore \quad A = \dfrac{10K}{25\Omega} = \mathbf{400}$

(*ii*) $\beta = \dfrac{R_E}{R_C} = \dfrac{1}{10} = 0.1$

$\beta A = 0.1 \times 400 = \mathbf{40}$

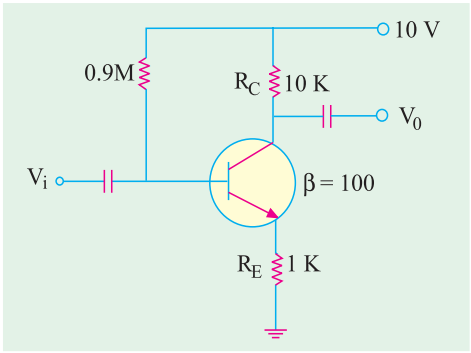


Fig. 62.10

(*iii*) $A' = \frac{R_C}{r_e + R_E} = \frac{10,000}{20+1000} = 9.756$

or $A' = \frac{A}{1+\beta A} = \frac{400}{1+400} = 9.756$

62.11. Voltage-shunt Negative Feedback Amplifier

The circuit of such an amplifier is shown in Fig. 62.11. As seen, a portion of the output voltage is coupled through R_E in parallel with the input signal at the base. This feedback stabilizes the overall gain while decreasing both the input and output resistances. It can be proved that $\beta = R_C/R_F$.

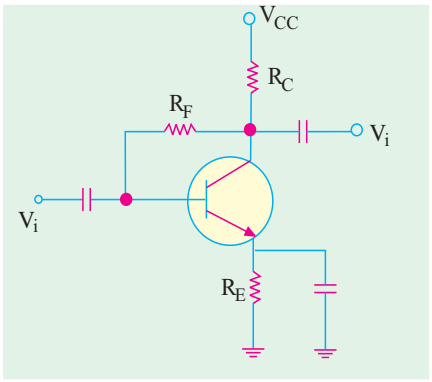


Fig. 62.11

62.12. Current-shunt Negative Feedback Amplifier

The two-stage amplifier employing such a feedback is shown in Fig. 62.12. The feedback circuit (consisting of C_F and R_F) samples the output current and develops a feedback voltage in parallel with the input voltage. The unbypassed emitter resistor of Q_2 provides current sensing.

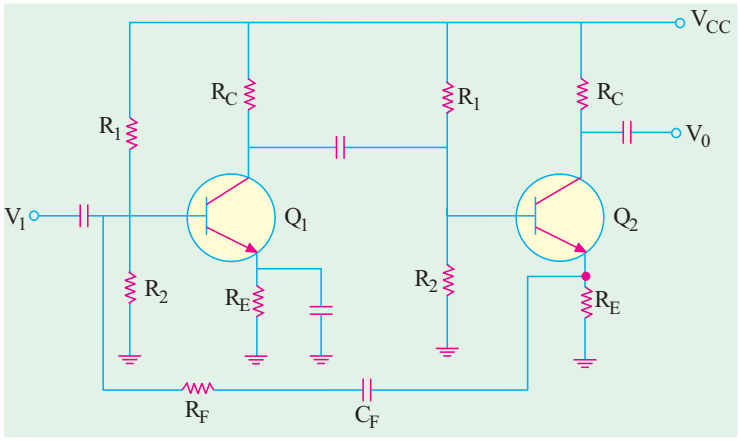


Fig. 62.12

The polarity of the feedback voltage is such that it provides the negative feedback.

Example 62.15. *Calculate A, $r_{in(stage)}$ and $I_{o(stage)}$ of the cascaded amplifier shown in Fig. 62.13 with and without voltage series feedback. The transistor parameters are : h_{fe} = 100,. h_{ie}. = 2 K and h_{oe} = 0.* **(Applied Electronics-I, Punjab Univ. 1992)**

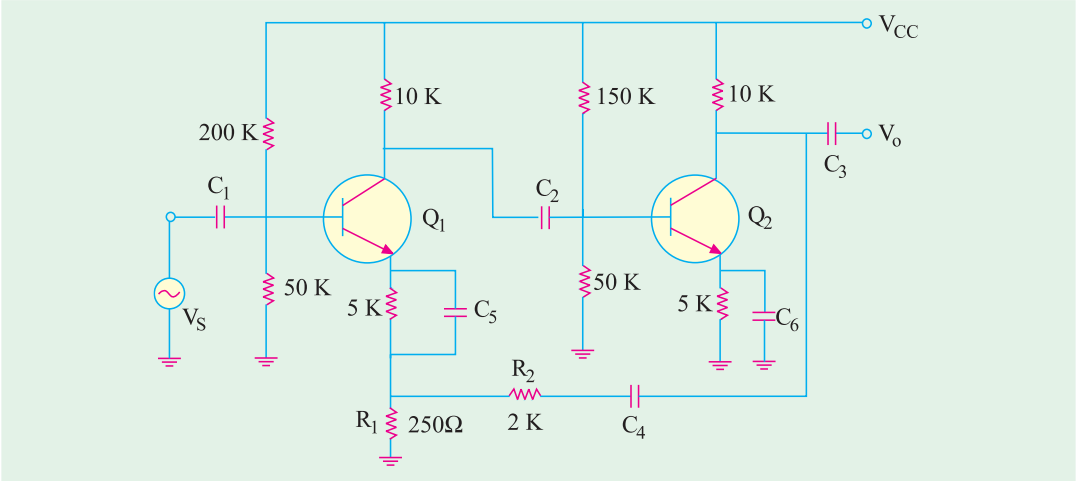


Fig. 62.13

Solution. (*i*) Without Feedback. The $r_{in(base)}$ for Q_1 is = h_{ie}. = 2K. Same is the value for Q_2.

Also, $r_{in(stage)}$ or r_{i-1} for Q_1 = 200 K || 50 K || 2 K = 1.9 K

$r_{0.2}$ or $r_{L.2}$ for Q_2=10 K || (2.0 + 0.25) K = 1.83K

$r_{0.1}$ or $r_{L.1}$ for Q_1 = 10 K ||

150 K // 50 || 2K = 1.6 K

$$\therefore \quad A_{v1} = \frac{h_{fe.1}\, r_{0.1}}{h_{ie}} = \frac{h_{fe.1}\, r_{L.1}}{h_{ie}}$$

$$= \frac{100 \times 1.6}{2} = 80$$

$$A_{v2} = \frac{h_{fe.2} r_{0.2}}{h_{ie}} = \frac{h_{fe.2}\, r_{L2}}{h_{ie}} = \frac{100 \times 1.83}{2} = 92$$

Overall gain, $A_v = A_{v1}.A_{v\cdot 2} = 80 \times 92 =$ **7360**

***(ii)* With Feedback**

The feedback factor, $\beta = \frac{R_1}{R_1 + R_2} = \frac{0.25}{0.25+2.0} = \frac{1}{9}$

$$r_{02f} = \frac{r_{o2}}{1+\beta A} = \frac{1.83}{1 + (1/9) \times 7360} = \mathbf{2.2\ \Omega}$$

$$r_{i.1f} = r_{i-1}(1 + \beta A) = 1.9 \times 819 = \mathbf{1556\ K}$$

$$A_f = \frac{A}{(1 + \beta A)} = \frac{7360}{819} = \mathbf{8.9}$$

Example 62.16. *In the two-stage R_C coupled amplifier (Fig. 62.14) using emitter feedback, find the overall gain. Neglect V_{BE} and take $\beta_1 = \beta_2 = 100$.*

Solution. In this amplifier circuit, voltage gain has been stabilized to some extent with the help of 500 Ω unbypassed emitter resistance. This 500 Ω resistance swamps out r_e.

$$\therefore \quad A_{v.2} = \frac{r_{L.2}}{r_e + r_E} \cong \frac{r_{L.2}}{r_E} = \frac{10K \| 10K}{500\,\Omega} = 10$$

Now, $\beta r_E = 100 \times 500 = 50$ K

$r_{i-2} = 80$ K || 40 K || 50 K

$r_{L.1} = R_{C.1} \| r_{i.2}$

= 10 K || 80 K || 40 K || 50 K = 6.3 K

$$\therefore \quad A_{v.1} = \frac{r_{L1}}{r_E} = \frac{6.3 \times 10^3}{500} = 12.6$$

$\therefore A = 10 \times 12.6 =$ **126**

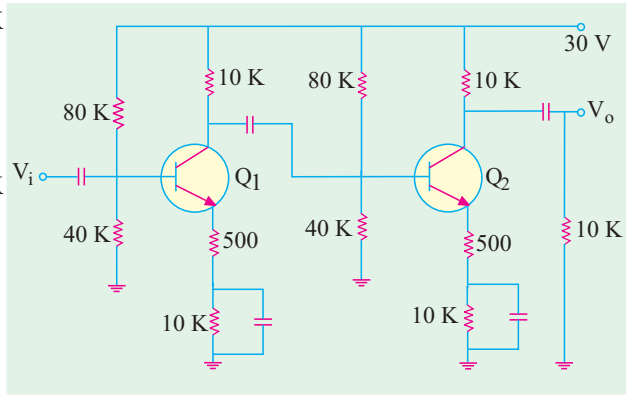


Fig. 62.14

62.13. Noninverting Op-amp With Negative Feedback

The closed-loop noninverting op-amp circuit using negative feedback is shown in Fig. 62.15. The input signal is applied to the noninverting input terminal. The output is applied back to the input terminal through the feedback network formed by R_i and R_f.

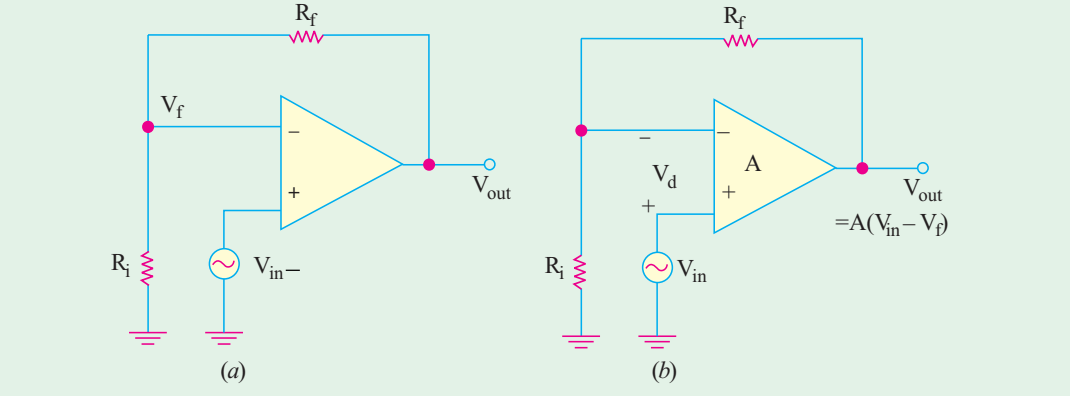


Fig. 62.15

The op-amp acts as both the difference circuit and the open-loop forward gain. The differential input to the op-amp is $(V_{in} - V_f)$. This differential voltage is amplified A times and an output voltage is produced which is given by

$$V_{out} = A_{v1} = A(V_{in} - V_f) \; ; \qquad \text{where } A \text{ is the open-loop gain of the op-amp}$$

Since, $(R_i + R_f)$ acts as voltage divider across V_{out},

$$\therefore \qquad V_f = V_{out} \frac{R_i}{R_i + R_f}$$

Now, $\beta = R_i/(R_i + R_f)$, hence $V_f = \beta V_{out}$

Substituting this value in the above equation, we get

$$V_{out} = A\,(V_{in} - \beta\, V_{out}) \text{ or } V_{out}(1 + \beta A) = AV_{in}$$

Hence, voltage gain A' with negative feedback is

$$A' = \frac{V_{out}}{V_{in}} = \frac{A}{1 + \beta A} = \frac{A}{1 + AR_i\,(R_i + R_f)}$$

If A is so large that 1 can be neglected as compared to βA, the above equation becomes

$$A' = \frac{A}{\beta A} = \frac{1}{\beta} = \frac{R_i + R_f}{R_i}$$

It is seen that closed-loop gain of a noninverting op-amp is essentially independent of the open-loop gain.

Example 62.17. *A certain noninverting op-amp has R_i = 1K, R_f = 99 K and open-loop gain A = 500,000. Determine* ***(i)*** *β,* ***(ii)*** *loop gain,* ***(iii)*** *exact closed-loop gain and* ***(iv)*** *approximate closed-loop gain if it is assumed that open-loop gain A = ∞.*

(Power Electronics, AMIE 1991)

Solution. ***(i)*** $\beta = \dfrac{R_i}{R_i + R_f} = \dfrac{1}{1 + 99} = 0.01$, ***(ii)*** loop gain $= \beta A = 500{,}000 \times 0.01 =$ **5000**

(iii) $A' = \dfrac{A}{1 + \beta A} = \dfrac{500{,}000}{1 + 5000} =$ **9998**

(*iv*) approx. $A' = \frac{1}{\beta} = \frac{1}{0.01} = 100$

It is seen that the gain changes by about 0.02%.

62.14. Effect of Negative Feedback on R_{in} and R_{out}

In the previous calculations, the input impedance of an op-amp was considered to be infinite and its output resistance as zero. We will now consider the effect of a finite input resistance and a non-zero output resistance. Since the two effects are different and their values differ by several order of magnitude, we will focus on each effect individually.

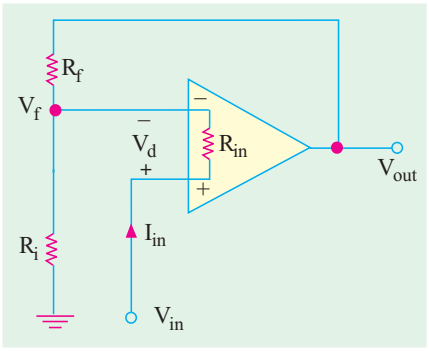


Fig. 62.16

(*a*) R_{in} of Noninverting Op-amp

For this analysis, it would be assumed that a small differential voltage V_d exists between the two inputs of the op-amp as shown in Fig. 62.16. It, in effect, means that neither the input resistance of the op-amp is assumed to be infinite nor its input current zero.

Now, $V_d = V_{in} - V_f$ or $V_{in} = V_d + V_f = V_d + \beta V_{out}$

Also, $V_{out} = A.V_d$ where A is the open-loop gain of the op-amp.

$\therefore$ $V_{in} = V_d + A\beta V_{out} = (1 + \beta A) V_d = (1 + \beta A) I_{in} R_{in}$ $-(\because V_d = I_{in} R_{in})$

where R_{in} is the open-loop impedance of the op-amp (*i.e.* without feedback)

$$\therefore \quad R'_{in} = \frac{V_{in}}{I_{in}} = (1+\beta A) R_{in}$$

where R_{in} is the closed-loop input resistance of the non-inverting op-amp.

It will be seen that the closed-loop input resistance of the non-inverting op-amp is much greater than the input resistance without feedback.

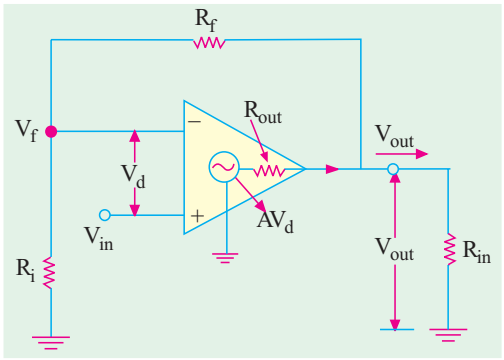


Fig. 62.17

(*b*) R'_{out} of Noninverting Op-amp

An expression for R'_{out} would be developed with the help Fig. 62.17. Using *KVL*, we get

$$V_{out} = AV_d - I_{out} R_{out}$$

Now, $V_d = (V_{in} - V_f)$ and neglecting $I_{out} R_{out}$ as compared to AV_d, we have

$$V_{out} = A(V_{in} - V_f) = A(V_{in} - \beta V_{out})$$

or $AV_{in} = (1 + \beta A) V_{out}$

If, with negative feedback, output resistance of the noninverting op-amp is R'_{out}, then $V_{out} = I_{out}.R'_{out}$.

Substituting this value in the above equation, we get

$$AV_{in} = (1 + \beta A) I_{out} R'_{out} \quad \text{or} \quad \frac{AV_{in}}{I_{out}} = (1 + \beta A) R'_{out}$$

The term on the left is the internal output resistance R_{out} of the op-amp because without feedback, $AV_{in} = V_{out}$.

or $$R_{out} = (1 + \beta A)\, R'_{out} \text{ or } R'_{out} = \frac{R_{out}}{(1+\beta A)}$$

Obviously, output resistance R'_{out} with negative feedback is much less than without feedback (i.e. R_{out}).

Example 62.18. ***(a)*** *Calculate the input and output resistance of the op-amp shown in Fig. 62.18. The data sheet gives : R_{in} = 2M, R_{out} = 75 Ω. and A = 250,000* ***(b)*** *Also, calculate the closed-loop voltage gain with negative feedback.*

(Industrial Electronics, Mysore, Univ. 1992)

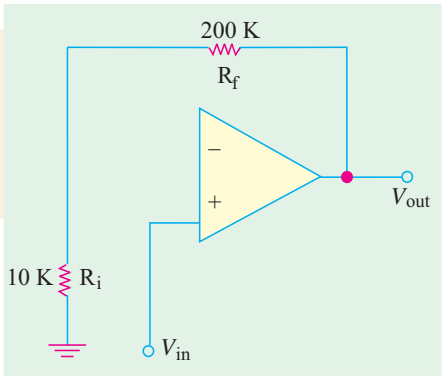


Fig. 62.18

Solution. ***(a)*** The feedback ratio β is given by

$$\beta = \frac{R_i}{R_i + R_f} = \frac{10}{10+200} = \frac{10}{210} = 0.048$$

$$R'_{in} = (1 + \beta A)\, R_{in} = (1 + 250{,}000 \times 0.048) \times 2$$
$$= \mathbf{24{,}002\ M}$$

$$R'_{out} = \frac{R_{out}}{1+\beta A} = \frac{75}{1 + 12000} = \mathbf{0.006\ \Omega}$$

(b) $A' = 1/\beta = 1/0.048 = \mathbf{20.8}$

62.15. R_{in} and R_{out} of Inverting Op-amp with Negative Feedback

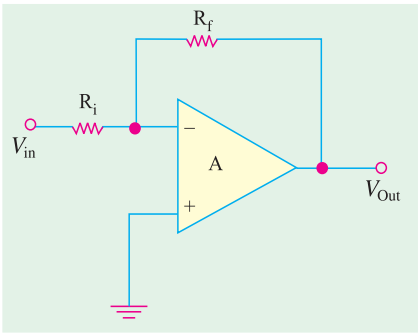


Fig. 62.19

The input resistance R_{in} of the inverting op-amp with negative feedback will be found by using Fig. 62.19. Since both the input signal and the negative feedback are applied to the inverting terminal. Miller's theorem will be applied to this configuration. According to this theorem, the effective input resistance of an amplifier with a feedback resistor from output to input is given by

$$R_{in\,(Miller)} = \frac{R_f}{1 + A} \quad \text{and} \quad R_{out\,(Miller)} = R_f \left(\frac{A}{1 + A}\right)$$

The Miller equivalent of the inverting op-amp is shown in Fig. 62.20 (*a*)

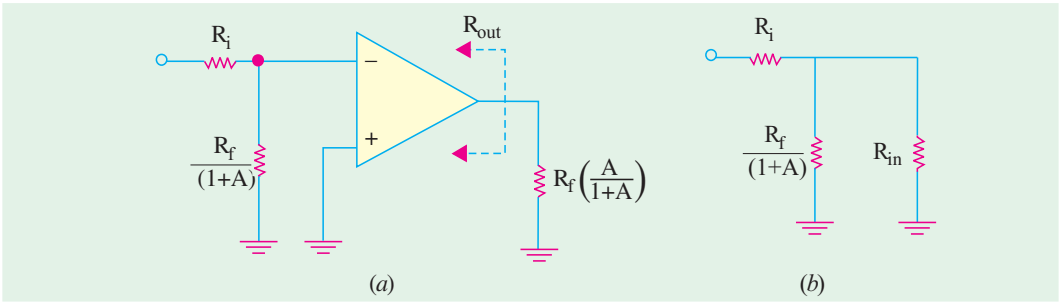


Fig. 62.20

As shown in Fig. 62.20 (*b*), the Miller input resistance appears in parallel with the internal resistance of the op-amp (without feedback) and R_i appears in series with this

$$\therefore \quad R'_{in} = R_i + \frac{R_f}{(1+A)} \parallel R_{in}$$

Typically, the term $R_f(1 + A)$ is much less than R_{in} of an open-loop op-amp. Hence,

$$\frac{R_f}{(1+A)} \| R_{in} \cong \frac{R_f}{1+A}$$

Moreover, $A \gg 1$, hence, $\therefore \quad R'_{in} \cong R_i + \frac{R_f}{A}$

Now, R_i appears in series with (R_f/A) and if $R_i \gg R_f/A$, we have, $R'_{in} \cong R_i$

As seen from Fig. 62.20 (*b*), Miller output resistance appears in parallel with R_{out} of the op-amp.

$$\therefore \quad R'_{out} = R_f \left(\frac{A}{1+A} \right) \| R_{out}$$

Normally, $A \gg 1$ and $R_f \gg R_{out}$ so that R'_{out} simplifies to $R'_{out} = R_{out}$

Example 62.19. *For the inverting op-amp circuit of Fig. 62.21, find* ***(a)*** *input and output resistances and* ***(b)*** *the closed-loop gain. The op-amp has the following parameters :*

$A = 100{,}000, \quad R_{in} = 5\ M\Omega$. *and* $R_{out} = 50\ \Omega$

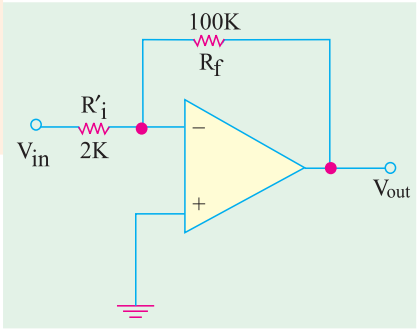


Fig. 62.21

Solution. ***(a)*** $R'_{in} \cong R_i \cong$ **2 k Ω**

$R'_{out} \cong R_{out} =$ **50 Ω**

(b) $A' = \frac{R_f}{R_i} = -\frac{100}{2} =$ **50**

The negative sign indicates the inherent sign inversion in the process.

Tutorial Problems No. 62.1

1. For the series-parallel feedback amplifier shown in Fig. 62.22. Calculate

(*i*) open-loop gain,

(*ii*) gain of feedback loop,

(*iii*) closed-loop gain,

(*iv*) sacrifice factor.

[(*i*) 10^6 (*ii*) 0.025 (*iii*) 40 (*iv*) 25.000]

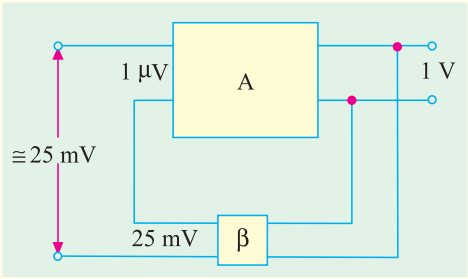


Fig. 62.22

2. A negative-feedback amplifier has the following parameters :

$A = 200$, $\beta = 0.02$ and $V_i = 5$ mV

Compute the following :

(*i*) gain with feedback, (*ii*) output voltage,

(*iii*) feedback factor, (*iv*) feedback voltage.

[(*i*) 40 (*ii*) 200 mV (*iii*) 4 (*iv*) 8 mV]

3. An amplifier has an open-loop gain of 500 and a feedback of 0.1. If open-loop gain changes by 25% due to temperature etc., find the percentage change in closed-loop gain. **[0.5%]**

4. An *RC*-coupled amplifier has a mid-frequency gain of 400 and lower and upper 3-dB frequencies of 100 Hz and 10 kHz. A negative feedback network with $\beta = 0.01$ is incorporated into the amplifier circuit. Calculate

(*i*) gain with feedback,
(*ii*) new bandwidth. **[(*i*) 80 (*ii*) 75 kHz]**

5. In an amplifier with constant signal input of 1 volt, the output falls from 50 to 25 V when feedback is applied. Calculate the fraction of the output which is fed back. If, due to ageing, the amplifier gain fell to 40, find the percentage reduction in stage gain

(*i*) without feedback **(*ii*)** with the feedback connection.

[0.02% (*i*) 20% (*ii*) 11.12%]

6. An amplifier has a gain of 1000 without feedback. Calculate the gain when 0.9 per cent of negative feedback is applied. If, due to ageing, gain without feedback falls to 800, calculate the percentage reduction in gain (*a*) without feedback and (*b*) with feedback. Comment on the significance of the results of (*a*) and (*b*) and state two other advantages of negative feedback.

[100 (*a*) 20% (*b*) 2.44%](*City & Guilds, London*)

7. The open-loop gain of an amplifier is $1000 \angle 70°$ and the feedback factor is $-0.02 \angle 20°$. Calculate the amplifier gain with negative feedback. What is the limiting value of β to make the amplifier unstable ? **[$49.9 \angle -17.1°$; $0.001 \angle -70°$] (*I.E.E. London*)**

8. When voltage feedback is applied to an amplifier of gain 100, the overall stage gain falls to 50. Calculate the fraction of the output voltage fed back. If this fraction is maintained, calculate the value of the amplifier gain required if the overall stage gain is to be 75. **[0.01 ; 300]**

(*City & Guilds, London*)

9. An amplifier having a gain of 100 has 9 per cent voltage negative feedback applied in series with the input signal. Calculate the overall stage with feedback.

If a supply voltage variation causes the gain with feedback to all by 10 percent, determine the percentage change in gain with feedback.

[10; 52.6%] (*City & Guilds, London*)

10. If the gain of an amplifier without feedback is $(800 - j100)$ and the feedback network of $\beta = -1/(40 - j20)$ modifies the output voltage to V_{fb} which is combined in series with the signal voltage, determine the gain of the ampplifier with feeback.

[38.3 – j18.3] (*I.E.R.E., London*)

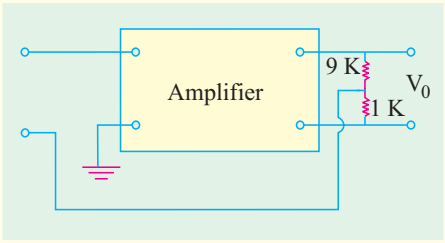


Fig. 62.23

11. Give three reasons for using negative feedback.

In Fig. 62.23, the box represents an amplifier of gain –1000, input impedance 500 kΩ and negligible output impedance.

Calculate the voltage gain and input impedance of the amplifier with feedback.

[– 9.9, 50.5 MΩ]

12. An amplifier with negative feedback has a voltage gain of 100. It is found that without feedback an input signal of 50 mV is required to produce a given output; whereas with feedback, the input signal must be 0.6V for the same output. Calculate the value of voltage gain without feedback and feedback ratio.

(Electronics Engg., Bangalore Univ. 2001)

OBJECTIVE TESTS – 62

1. The advantage of using negative feedback in an amplifier is that its gain can be made practically independent of

(*a*) temperature changes
(*b*) age of components
(*c*) frequency
(*d*) all of the above.

2. Feedback in an amplifier always helps to
(*a*) control its output
(*b*) increase its gain
(*c*) decrease its input impedance
(*d*) stabilize its gain.

3. The only drawback of using negative feedback in amplifiers is that it involves
(*a*) gain sacrifice
(*b*) gain stability
(*c*) temperature sensitivity
(*d*) frequency dependence.

4. Closed-loop gain of a feedback amplifier is the gain obtained when
(*a*) its output terminals are closed
(*b*) negative feedback is applied
(*c*) feedback loop is closed
(*d*) feedback factor exceeds unity.

5. A large sacrifice factor in a negative feedback amplifiers leads to
(*a*) inferior performance
(*b*) increased output impedance
(*c*) characteristics impossible to achieve without feedback
(*d*) precise control over output.

6. Negative feedback in an amplifier
(*a*) lowers its lower 3 dB frequency
(*b*) raises its upper 3 dB frequency
(*c*) increases its bandwidth
(*d*) all of the above.

7. Regarding negative feedback in amplifiers which statement is WRONG ?
(*a*) it widens the separation between 3 dB frequencies
(*b*) it increases the gain-bandwidth product
(*c*) it improves gain stability
(*d*) it reduces distortion.

8. Negative feedback reduces distortion in an amplifier only when it
(*a*) comes as part of input signal
(*b*) is part of its output
(*c*) is generated within the amplifier
(*d*) exceeds a certain safe level.

9. An amplifier with no feedback has a gain-bandwidth product of 4 MHz. Its closed-loop gain is 40. The new band-width is
(*a*) 100 kHz
(*b*) 160 MHz
(*c*) 10 MHz
(*d*) 20 kHz.

10. The shunt-derived series-fed feedback in an amplifier
(*a*) increases its output impedance
(*b*) decreases its output impedance
(*c*) increases its input impedance
(*d*) both (*b*) and (*c*).

11. A feedback amplifier has a closed gain of –200. It should not vary more than 50% despite 25% variation in amplifier gain *A* without feedback. The value of *A* is
(*a*) 800
(*b*) – 800
(*c*) 1000
(*d*) –1000

12. The gain of a negative feedback amplifier is 40 dB. If the attenuation of the feedback path is 50 dB, then the gain of the amplifier without feedback is
(*a*) 78.92
(*b*) 146.32
(*c*) 215.51
(*d*) 317.23

13. In a common emitter amplifier, the unbypassed emitter resistor provides
(*a*) voltage-shunt feedback
(*b*) current-series feedback
(*c*) negative-voltage feedback
(*d*) positive-current feedback

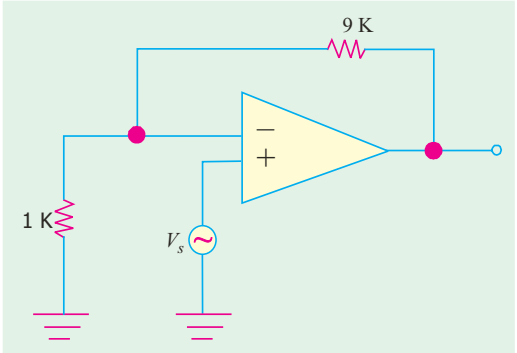


Fig. 62.24

14. The *OP-AMP* circuit shown in Fig. 62.24 has an input impedance of MΩ and an open-loop gain of 10^5. The output impedance seen by the source V_s is

(*a*) $10^{11}\Omega$

(*b*) $10^{10}\ \Omega$

(*c*) 10 kΩ

(*d*) 1 kΩ

15. An *OP-AMP* with an open-loop gain of 10,000, R_{in} = 2 K Ω and R_0 = 500 Ω is used in the non-inverting configuration shown in Fig. 62.25. The output resistance R'_0 is

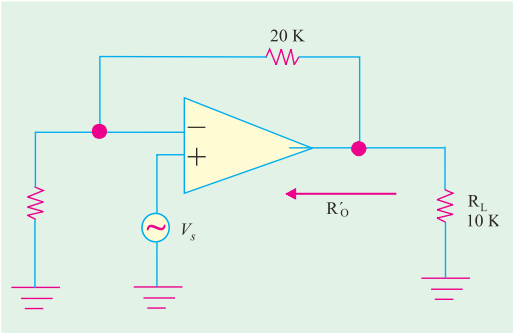


Fig. 62.25

(*a*) 250.5 Ω (*b*) 21 Ω

(*c*) 2 Ω (*d*) 0.998 Ω

16. The feedback used in the circuit shown in Fig. 62.25 can be classified as

(*a*) shunt-series feedback

(*b*) shunt-shunt feedback

(*c*) series-shunt feedback

(*d*) series-series feedback

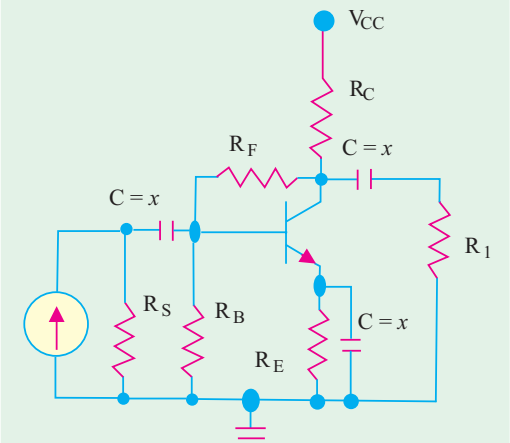


Fig. 62.25

ANSWERS

1. (*d*)	**2.** (*a*)	**3.** (*a*)	**4.** (*c*)	**5.** (*c*)	**6.** (*d*)	**7.** (*b*)	**8.** (*c*)
9. (*a*)	**10.** (*d*)	**11.** (*d*)	**12.** (*b*)	**13.** (*b*)	**14.** (*b*)	**15.** (*d*)	**16.** (*b*)

GO To FIRST

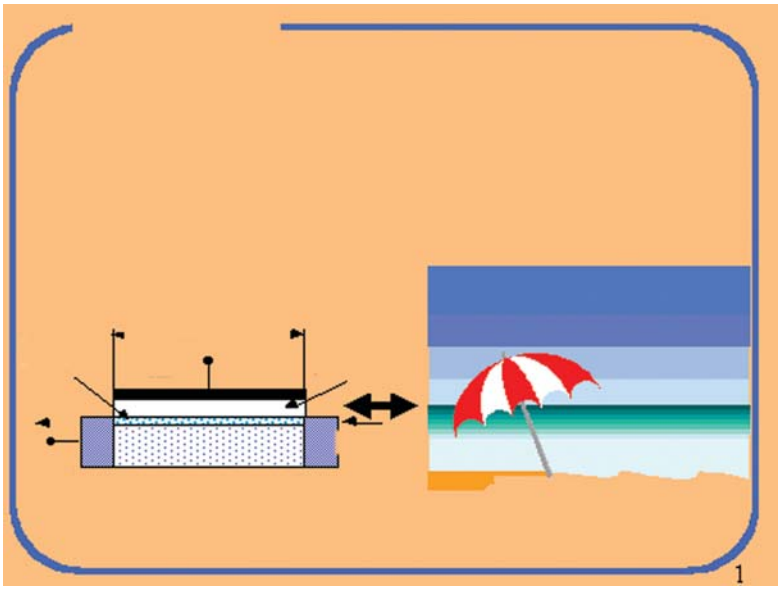
CONTENTS

CHAPTER 63

FIELD EFFECT TRANSISTORS

Learning Objectives

- What is a FET
- Junction FET (JFET)
- Static Characteristics of a JFET
- JFET Drain Characteristic with $V_{GS} = 0$
- Characteristics with External Bias
- Transfer Characteristic
- Small Signal JFET Parameters
- D.C. Baising of a JFET
- DC Load Line
- Common Source JFET Amplifier
- JFET Amplifier Gains
- Advantages of FETs
- MOSFET or IGFET—DE MOSFET
- Schematic Symbols for a DEMOSFET
- Static Characteristics of a DEMOSFET
- Enhancement-only N-Channel MOSFET
- Biasing E-only MOSFET—FET Amplifiers
- FET Applications
- MOSFET Handling



Field Effect Transistor as an Electronic Flute

CONTENTS

63.1. What is a FET ?

The acronym 'FET' stands for **field effect transistor**. It is a three-terminal unipolar solid-state device in which current is controlled ***by an electric field*** as is done in vacuum tubes. Broadly speaking, there are two types of FETs :

(a) junction field effect transistor (JFET)

(b) metal-oxide semiconductor FET (MOSFET)

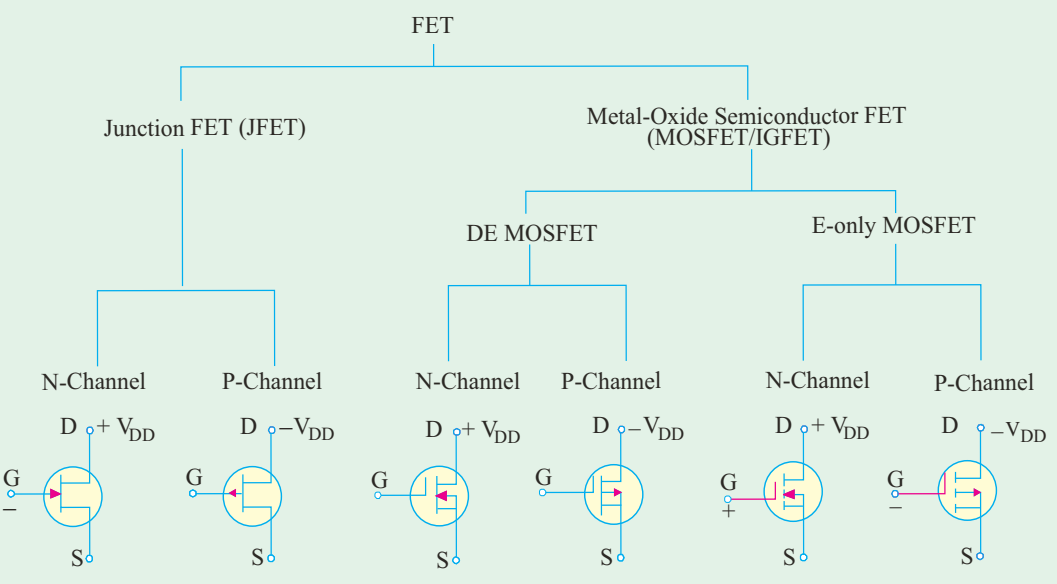
It is also called insulated-gate FET (IGFET). It may be further subdivided into :

(i) depletion-enhancement MOSFET *i.e.* DEMOSFET

(ii) enhancement-only MOSFET *i.e.* E-only MOSFET

Both of these can be either *P*-channel or *N*-channel devices.

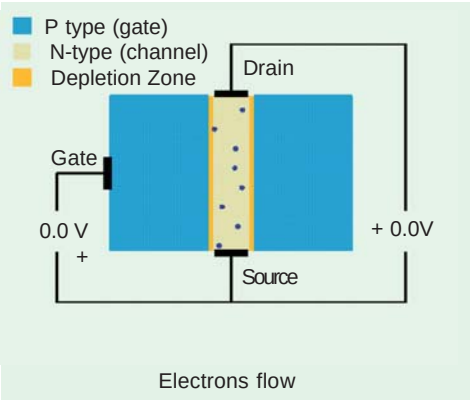
The FET family tree is shown below :



63.2. Junction FET (JFET)

(a) Basic Construction

As shown in Fig. 63.1, it can be fabricated with either an *N*-channel or *P*-channel though *N*-channel is generally preferred. For fabricating an *N*-channel JFET, first a narrow bar of *N*-type semiconductor material is taken and then two *P*-type junctions are diffused on opposite sides of its middle part [Fig. 63.1 (*a*)]. These junctions form two *P*-*N* diodes or **gates** and the area between these gates is called **channel**. The two *P*-regions are internally connected and a single lead is brought out which is called **gate terminal**. Ohmic contacts (direct electrical connections) are made at the two ends of the bar-one lead is called **source terminal** *S* and the other **drain terminal** *D*. When potential difference is established between drain and source, current flows along the length of the 'bar' through the channel located between the two *P*-regions. The current consists of **only majority carriers** which, in the present case, are electrons. *P*-channel JFET is



similar in construction except that it uses *P*-type bar and two *N*-type junctions. The majority carriers are holes which flow through the channel located between the two N-regions or gates.

Following FET notation is worth remembering:

1. **Source.** It is the terminal through which majority carriers enter the bar. Since carriers come from it, it is called the source.
2. **Drain.** It is the terminal through which majority carriers ***leave*** the bar *i.e.* they are drained out from this terminal. The drain-to-source voltage V_{DS} drives the drain current I_D.
3. **Gate.** These are two internally-connected heavily-doped impurity regions which form two *P*-*N* junctions. The gate-source voltage V_{GS} reverse-biases the gates.
4. **Channel.** It is the space between two gates through which majority carriers pass from source-to-drain when V_{DS} is applied.

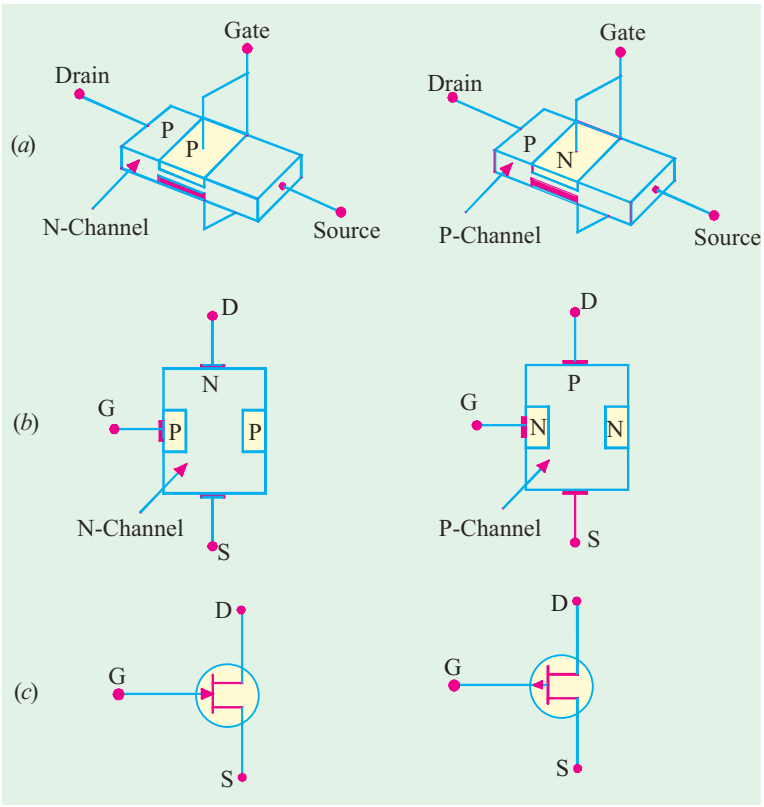


Fig. 63.1

Schematic symbols for *N*-channel and *P*-channel JFET are shown in Fig. 63.1 **(*c*)**. It must be kept in mind that **gate arrow always points to N-type material**.

(*b*) Theory of Operation

While discussing the theory of operation of a JFET, it should be kept in mind that

1. Gates are always reversed-biased. Hence, gate current I_G is practically zero.
2. The source terminal is always connected to that end of the drain supply which provides the necessary charge carriers. In an *N*-channel JFET, source terminal *S* is connected to the negative end of the drain voltage supply (for obtaining electrons). In a *P*-channel JFET, *S* is connected to the positive end of the drain voltage supply for getting holes which flow through the channel.

Let us now consider an ***N-channel*** JFET and discuss its working when either V_{GS} or V_{DS} or both are changed.

(*i*) When $V_{GS} = 0$ and $V_{DS} = 0$

In this case, drain current $I_D = 0$, because $V_{DS} = 0$. The depletion regions around the *P*-*N* junctions are of equal thickness and symmetrical as shown in Fig. 63.2 (*a*)

(*ii*) When $V_{GS} = 0$ and V_{DS} is increased from zero

For this purpose, the JFET is connected to the V_{DD} supply as shown in Fig. 63.2 (*b*). The electrons (which are the majority carriers) flow from *S* to *D* whereas conventional drain current I_D flows through the channel from *D* to *S*. Now, the gate-to-channel bias at any point along the channel is = $|V_{DS}| + |V_{GS}|$ *i.e.* the numerical sum of the two voltages. In the present case, external bias $V_{GS} = 0$.

Hence gate-channel reverse bias is provided by V_{DS} alone. Since the value of V_{DS} keeps decreasing (due to progressive drop along the channel) as we go from D to S, the gate-channel bias also decreases accordingly. It has maximum value in the drain-gate region and minimum in the source-gate region. Hence, depletion regions penetrate more deeply into the channel in the drain-gate region than in the source-gate region. This explains why the depletion regions become wedge shaped when V_{DS} is applied [Fig. 63.2 (*b*)]

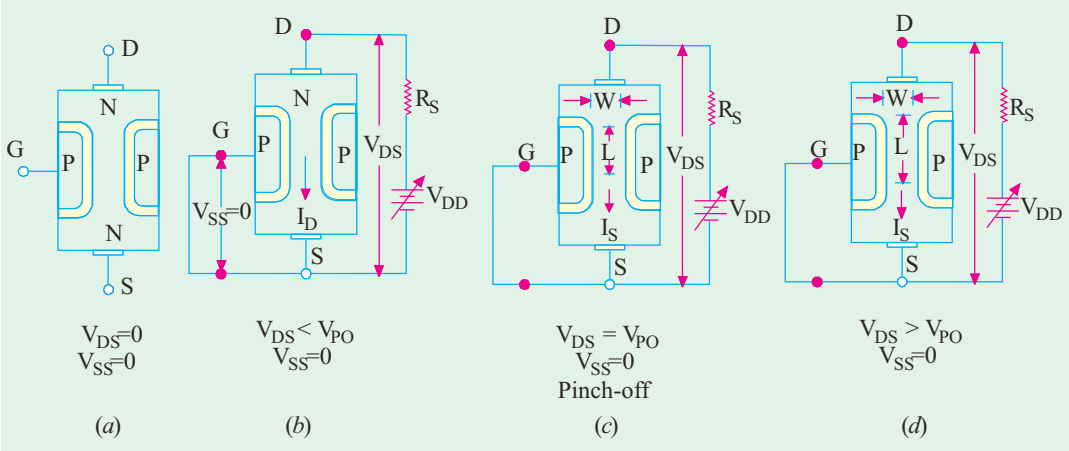


Fig. 63.2

As V_{DS} is gradually increased from zero, I_D increases proportionally as per Ohm's law. It is found that for small initial values of V_{DS}, the N-type channel material acts like a resistor of constant value. It is so because V_{DS} being small, the depletion regions are not large enough to have any significant effect on channel cross-section and, hence, its resistance. Consequently, I_D increases linearly as V_{DS} is increased from zero onwards (Fig. 63.5).

The ohmic relationship between V_{DS} and I_D continues till V_{DS} reaches a certain critical value called ***pinch-off voltage*** V_{PO} when drain current becomes constant at its maximum value called I_{DSS}. The SS in I_{DSS} indicates that the gate is shorted to source to make sure that $V_{GS} = 0$. This current is also known as ***zero-gate-voltage drain current.*** It is seen from Fig. 63.2(*c*) that under pinch-off conditions, separation between the depletion regions near the drain end reaches a minimum value W. It should, however, be carefully noted that ***pinch-off does not mean 'current-off '.*** In fact, I_D is maximum at pinch-off.

When V_{DS} is increased beyond V_{PO}, I_D remains constant at its maximum value I_{DSS} upto a certain point. It is due to the fact that further increase in V_{DS} (beyond V_{PO}) causes more of the channel on the source end to reach the minimum width as shown in Fig. 63.2 (*d*). It means that the channel width does not increase, instead its length L increases. As more of the channel reaches the minimum width, the resistance of the channel increases at the same rate at which V_{DS} increases. In other words, increase in V_{DS} is neutralized by increases in R_{DS}. Consquently, $I_D = (V_{DS} / R_{DS})$ remains unchanged even though V_{DS} is increased. Ultimately, a certain value of V_{DS} (called V_{DSO}) is reached when JFET breaks down and I_D increases to an excessive value as seen from drain characteristic of Fig. 63.5.

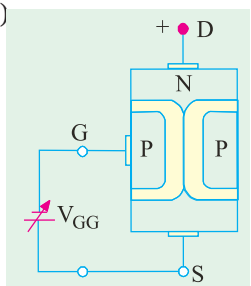


Fig. 63.3

(*iii*) When $V_{DS} = 0$ and V_{GS} is decreased from zero

In this case, as V_{GS} is made more and more negative, the gate reverse bias increases which increases the thickness of the depletion regions. As negative value of V_{GS} is increased, a stage comes when the two depletion regions touch each other as shown in Fig. 63.3. In this condition, the channel is said to be cut-off. This value of V_{GS} which cuts off the channel and

hence the drain current is called $V_{GS(off)}$.*

It may be noted that $V_{GS(off)} = -V_{PO}$ or $|V_{PO}| = |V_{GS(off)}|$. As seen from Fig. 63.6 because $V_{PO} = 4$ V, $V_{GS(off)} = -4$ V. Obviously, their absolute values are equal.

(*iv*) When V_{GS} is negative and V_{DS} is increased

As seen from Fig. 63.6, as V_{GS} is made more and more negative, values of V_P as well as breakdown voltage are decreased.

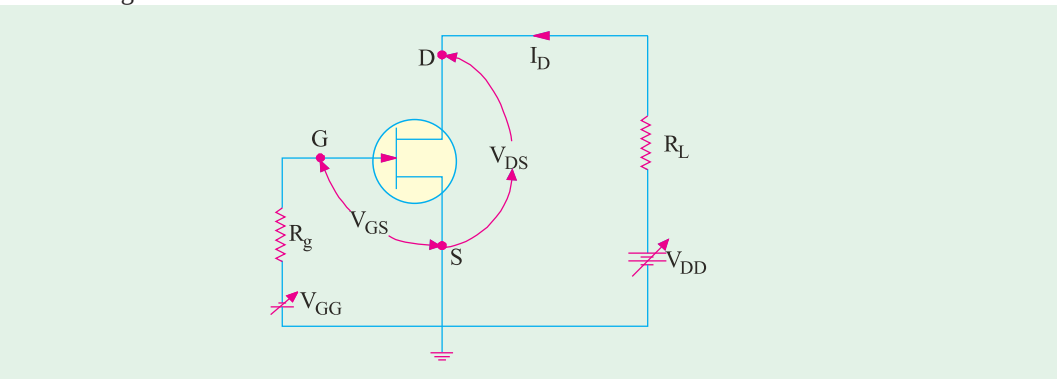


Fig. 63.4

Summary. Summarizing the above, we have that

(*i*) keeping V_{GS} at a fixed value (either zero or negative), as V_{DS} is increased, I_D initially increases till channel pinch-off when it becomes almost constant and finally increases excessively when JFET breaks down under high value of V_{DS}. As V_{GS} is kept fixed at progressively higher negative values, the values of V_P as well as breakdown voltage decrease.

(*ii*) keeping V_{DS} at a fixed value, as V_{GS} is made more and more negative, I_D decreases till it is reduced to zero for a certain value of V_{GS} called $V_{GS(off)}$.

Since gate voltage controls the drain current, JFET is called a ***voltage-controlled*** device. A *P*-channel JFET operates exactly in the same manner as an *N*-channel JFET except that current carriers are holes and polarities of both V_{DD} and V_{GS} are reversed.

Since only one type of majority carrier (either electrons or holes) is used in JFETs, they are called ***unipolar devices*** unlike bipolar junction transistors (*BJTs*) which use both electrons and holes as carriers.

63.3. Static Characteristics of a JFET

We will consider the following two characteristics:

(*i*) drain characteristic

It gives relation between I_D and V_{DS} for different values of V_{GS} (which is called running variable).

(*ii*) transfer characteristic

It gives relation between I_D and V_{GS} for different values of V_{DS}.

We will analyse these characteristics for an *N*-channel JFET connected in the common-source mode as shown in Fig. 63.4. We will first consider the drain characteristic when $V_{GS} = 0$ and then when V_{GS} has any negative value upto $V_{GS(off)}$.

63.4. JFET Drain Characteristic With $V_{GS} = 0$

Such a characteristic is shown in Fig. 63.5 and has been already discussed briefly in Art. 63.2. It can be subdivided into following four regions :

1. Ohmic Region OA

This part of the characteristic is linear indicating that for low values of V_{DS}, current varies directly with voltage following Ohm's Law. It means that JFET behaves ***like an ordinary resistor*** till

* It has negative value for an N-channel JFET but a positive value or a P-channel JFET.

point *A* (called knee) is reached.

2. Curve AB

In this region, I_D increases at reverse square-law rate upto point *B* which is called **pinch-off point**. This progressive decrease in the rate of increase of I_D is caused by the square law increase in the depletion region at each gate upto point *B* where the two regions are closest without touching each other. The drain-to-source voltage V_{DS} corresponding to point *B* is called ***pinch-off voltage Vp**.*** But it is essential to remember that ***"pinch-off "*** does not mean ***"current-off".***

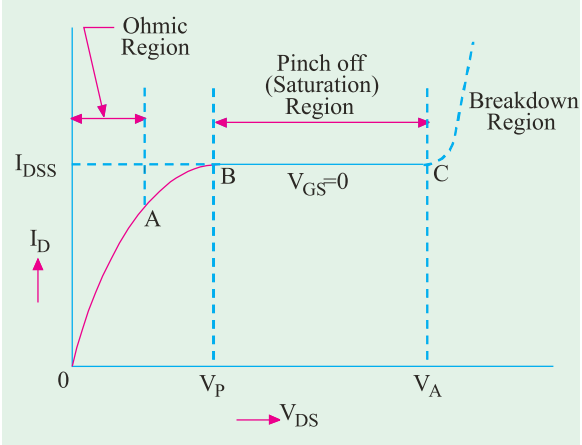


Fig. 63.5

3. Pinch-off Region BC

It is also known as **saturation region** or **'amplified' region**. Here, JFET operates as a constant-current device because I_D is relatively independent of V_{DS}. It is due to the fact that as V_{DS} increases, channel resistance also increases proportionally thereby keeping I_D practically constant at I_{DSS}. It should also be noted that the reverse bias required by the gate-channel junction is supplied entirely by the voltage drop across the channel resistance due to flow of I_{DSS} and none by external bias because $V_{GS} = 0$.

Drain current in this region is given by Shockley's equation

$$I_D = I_{DSS}\left(1-\frac{V_{GS}}{V_P}\right)^2 = I_{DSS}\left(I-\frac{V_{GS}}{V_{GS(off)}}\right)^2$$

It is the normal operating region of the JFET when used as an amplifier.

4. Breakdown Region

If V_{DS} is increased beyond its value corresponding to point *C* (called avalanche breakdown voltage), JFET enters the breakdown region where I_D increases to an excessive value. This happens because the reverse-biased gate-channel *P-N* junction undergoes avalanche breakdown when small changes in V_{DS} produce very large changes in I_D.

It is interesting to note that increasing values of V_{DS} make a JFET behave first as a resistor (ohmic region), then as a constant-current source (pinch-off region) and finally, as a constant-voltage source (breakdown region).

63.5. JFET Characteristics With External Bias

Fig. 63.6 shows a family of I_D versus V_{DS} curves for different values of V_{GS}. It is seen that as the negative gate bias voltage is increased

(i) pinch off voltage is reached at a lower value of I_D than when $V_{GS} = 0$.

(ii) value of V_{DS} for breakdown is decreased.

When an external bias of, say, –1V is applied between the gate and source, the *P-N* junctions become reverse-biased even when $I_D = 0$. Hence,

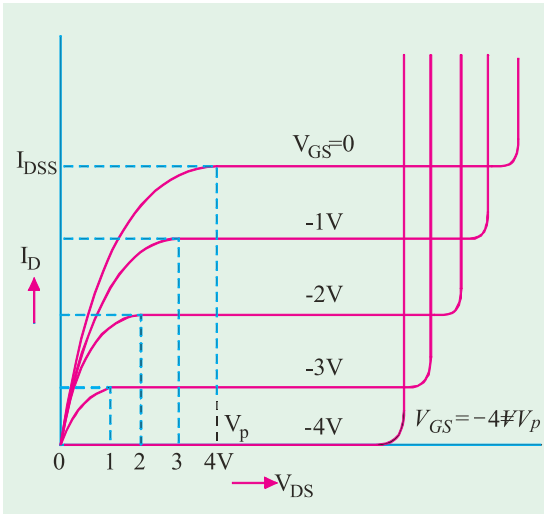


Fig. 63.6

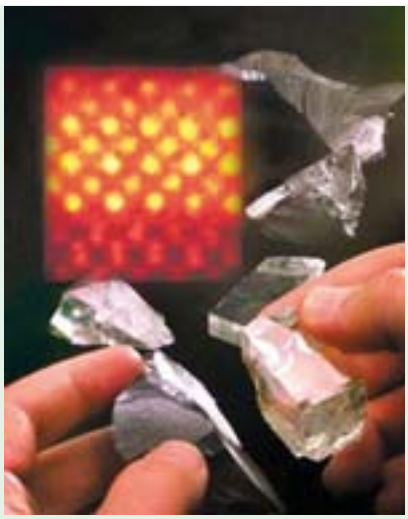
* It is numerically equal to $V_{GS(off)}$ *i.e.* $V_p = / V_{GSC(off)}/$

the depletion regions are already formed which penetrate the channel to a certain extent.

The amount of reverse bias required to be produced by I_D would, obviously, be decreased by 1V. In other words, a smaller voltage drop along the channel (*i.e.* smaller than when $V_{GS} = 0$) will increase the depletion regions to the point where they will pinch off the current. Consequently, V_P is reached at a lower I_D value than when $V_{GS} = 0$.

Now, let us see why value of V_{DS} for breakdown is decreased as the negative gate bias voltage is increased. It is simply due to the fact that V_{GS} keeps adding to the reverse bias at the junction produced by current flow.

It is seen that with $V_{GS} = 0$, I_D saturates at I_{DSS} and the characteristic shows $V_P = 4$V. When an external bias of –1 *V* is applied, gate-channel junctions still require –4 V to achieve pinch-off (remember, $V_{GS} = -V_P$). It means that a 3V drop is now required along the channel instead of the previous 4V. Obviously, this 3V drop can be achieved with a lower value of I_D. Similarly, when V_{GS} is –2V and –3V, pinch-off is achieved with 2 V and 1 V respectively along the channel.



This micrograph shows a mock field effect transistor with a layer of crystalline strontium titanate instead of silicon dioxide as the gate electrode.

These drops of 2 V and 1V are obtained with further-reduced values of I_D. As seen, when $V_{GS} = -4$ V (*i.e.* numerically equal to V_P), no channel drop is required. Hence, I_D is zero.

In general, $V_P = V_{DS(P)} - V_{GS}$ where $V_{DS(P)}$ is the pinch-off value of V_{DS} for a given value of V_{GS}.

63.6. Transfer Characteristic

It is a plot of I_D versus V_{GS} for a constant value of V_{DS} and is shown in Fig. 63.7. It is similar to the transconductance characteristics of a vacuum tube or a transistor. It is seen that when $V_{GS} = 0$, $I_D = I_{DSS}$ and when $I_D = 0$, $V_{GS} = V_P$. The transfer characteristic approximately follows the equation.

$$I_D = I_{DSS}\left(1-\frac{V_{GS}}{V_P}\right)^2 = I_{DSS}\left(1-\frac{V_{GS}}{V_{GS(off)}}\right)^2$$

The above equation can be written as

$$V_{GS} = V_{GS(off)}\left(1-\sqrt{\frac{I_D}{I_{DSS}}}\right)$$

This characteristic can be obtained from the drain characteristics by reading off V_{GS} and I_{DSS} values for different values of V_{DS}.

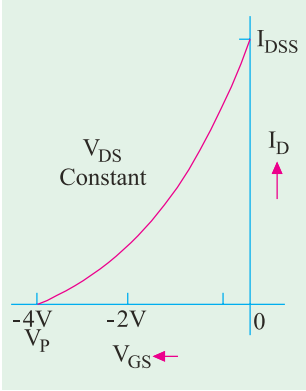


Fig. 63.7

63.7. Small Signal JFET Parameters

The various parameters of a JFET can be obtained from its two characteristics. The main parameters of a JFET when connected in common-source mode are as under :

(*i*) AC Drain Resistance, r_d

It is the ac resistance between drain and source terminals when JFET is operating ***in the pinch-off region.*** It is given by

$$r_d = \frac{\text{change in } V_{DS}}{\text{change in } I_D}\bigg|_{V_{GS}\ \text{constant}} \quad \text{or} \quad r_d = \frac{\Delta V_{DS}}{\Delta I_D}\bigg|_{V_{GS}}$$

An alternative name is ***dynamic drain resistance.*** It is given by the slope of the drain characteristic in the pinch-off region. It is sometimes written as r_{ds} emphasizing the fact that it is the resistance from drain to source. Since r_d is usually the output resistance of a JFET, it may also be expressed as an output admittance y_{os}. Obviously, $y_{os} = 1/r_d$. It has a very high value.

(*ii*) Transconductance, g_m

It is simply the slope of transfer characteristic.

$$g_m = \frac{\text{change in } I_D}{\text{change in } V_{GS}} - V_{DS} \text{ constant or } \quad g_m = \frac{\Delta I_D}{\Delta V_{GS}} \Big| V_{DS}$$

Its unit is Siemens (*S*) earlier called ***mho.*** It is also called ***forward transconductance (g_{fs}) or forward transadmittance y_{fs}.***

The transconductance measured at I_{DSS} is written as g_{mo}.

Mathematical Expression for g_m

The Shockley equation* is $I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$

Differentiating both sides, we have

$$\frac{dI_D}{dI_{DSS}} = 2I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)\left(-\frac{1}{V_P}\right) \quad \text{or} \quad g_m = -\frac{2I_{DSS}}{V_P}\left(1 - \frac{V_{GS}}{V_P}\right)$$

When $V_{GS} = 0$, $g_m = g_{mo}$ $\therefore$ $g_{mo} = -\frac{2I_{DSS}}{V_P}$

From the above two equations, we have $g_m = g_{mo}\left(1 - \frac{V_{DSS}}{V_P}\right) = g_{mo}\sqrt{\frac{I_D}{I_{DSS}}}$

(*iii*) Amplification Factor, μ

It is given by $\mu = \frac{\text{change in } V_{DS}}{\text{change in } V_{GS}} - I_D$ constant or $\mu = \frac{\Delta V_{DS}}{\Delta V_{GS}} \Big| I_D$

It can be proved from above that $\mu = g_m \times r_d = g_{fs} \times r_d$

(*iv*) DC Drain Resistance, R_{DS}

It is also called the static or ohmic resistance of the channel. It is given by

$$R_{DS} = \frac{V_{DS}}{I_D}$$

Example 63.1. *For an N-channel JFET, $I_{DSS} = 8.7$ mA, $V_P = -3$ V, $V_{GS} = -1$ V. Find the values of (i) I_D (ii) g_{mo} (iii) g_m* **(Basic Electronics, Bombay Univ., 1985)**

Solution. (*i*) $I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2 = 8.7\left(1 - \frac{-1\text{ V}}{-3\text{ V}}\right)^2 =$ **3.87 A**

(*ii*) $g_{mo} = \frac{-2I_{DSS}}{V_P} = \frac{-2 \times 8.7}{-3} =$ **5.8 mS**

(*iii*) $g_m = g_{mo}\left(1 - \frac{V_{GS}}{V_P}\right) = 5.8\left(1 - \frac{-1}{-3}\right) =$ **3.87 mS**

* Because of the squared term in the equation, JFET and MOSFET are referred to as square-law devices.

63.8. DC Biasing of a JFET

A JFET may be biased by using either

1. a separate power source V_{GG} as shown in Fig. 63.8 (*a*),
2. some form of self-bias as shown in Fig. 63.8 (*b*),
3. source bias as in Fig. 63.8 (*c*),
4. voltage divider bias as in Fig. 63.8 **(*d*).**

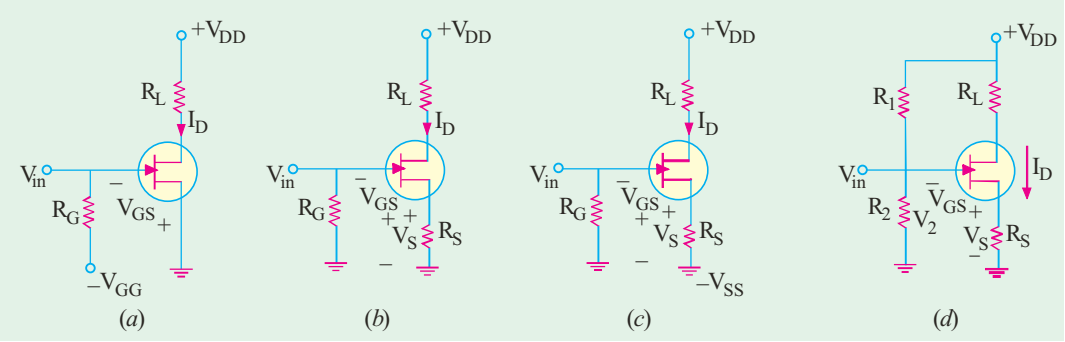


Fig. 63.8

The circuit of Fig. 63.8 (*b*) is called self-bias circuit because the V_{GS} bias is obtained from the flow of JFET's own drawn current I_D through R_S.

$\therefore \qquad V_S = I_D R_S \quad$ and $\quad V_{GS} = - I_D R_S$

The gate is kept at this much negative potential with respect to the ground.

The addition of R_G in Fig. 63.8 (*b*), does not upset this dc bias for the simple reason that no gate current flows through it (the gate leakage current is almost zero). Hence, gate is essentially at ***dc ground***. Without R_G, gate would be kept 'floating' which could collect charge and ultimately cut-off the JFET.

The resistance R_G additionally serves the purpose of avoiding short-circuiting of the ac input voltage, v_{in}. Moreover, in case leakage current is not totally negligible, R_G would provide it an escape route. Otherwise, the leakage current would build up static charge (voltage) at the gate which could change the bias or even destroy the JFET.

Fig. 63.8 (*c*) shows the *source* bias circuit which employs a self-bias resistor R_S to obtain V_{GS}. Here, $V_{SS} = I_D R_S + V_{GS}$ or $V_{GS} = V_{SS} - I_D R_S$.

Fig. 63.8 (*d*) shows the familiar voltage divider bias. In this case, $V_2 = V_{GS} + I_D R_S$ or $V_{GS} = V_2 - I_D R_S$

Since, $\quad V_2 = V_{DD} \dfrac{R_2}{R_1 + R_2}$ $\qquad \therefore \; V_{GS} = V_{DD} \dfrac{R_2}{R_1 + R_2} - I_D R_S$

Example 63.2 . *Find the values of V_{DS} and V_{GS} in Fig. 63.9 for I_D = 4 mA.*
(Applied Electronics-I, Punjab Univ. 1992)

Solution. $\quad V_S = I_D R_S = 4 \times 10^{-3} \times 500 = 2.0$ V

$V_D = V_{DD} - I_D R_L = 12 - 4 \times 1.5 = 6$ V

$\therefore \quad V_{DS} = V_D - V_S = 6 - 2 = 4$ V

Since $\quad V_G = 0, V_{GS} = V_G - V_S = 0 - 2.0 =$ **– 2.0 V**

63.9. DC Load Line

The dc load line for a JFET can be easily drawn by remembering the following two points :

(i) At $\quad I_D = 0, V_{DS} = V_{DD}$

(ii) At $\quad V_{DS} = 0, \quad I_D = \dfrac{V_{DD}}{R_L}$

The *Q*-point is generally situated at the middle point of the load line (for class-A operation) so that

$$V_{DSQ} = \frac{1}{2}V_{DD}$$

$$\text{Also,} \quad I_{DSQ} = \frac{\frac{1}{2}V_{DD}}{R_s + R_L}$$

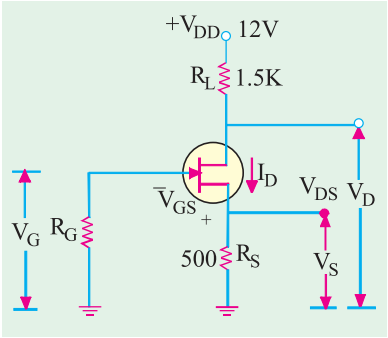


Fig. 63.9

Example 63.3. *For the circuit of Fig. 63.11, find the values of V_{DSQ} and V_{GS} assuming centrally-located Q-point and zero gate current.*

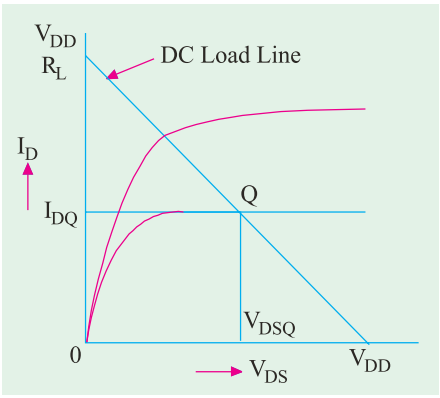


Fig. 63.10

Solution. Since $I_G = 0$, dc circuit is not disturbed.

$$V_{DS} = \frac{1}{2}V_{DD} = \frac{12}{2} = \mathbf{6V}$$

The balance of 6 V drops across series combination of R_L and R_S.

$$\therefore \quad I_D = \frac{6}{150+450} = \mathbf{10\ mA}$$

$$\therefore \quad V_{GS} = -I_D R_S = -10 \times 150 = \mathbf{1.5\ V}$$

It is obvious that gate is 1.5 V negative with respect to the source which is the common point. Incidentally, in common source connection of a JFET, ***gate is the most negative point in the entire circuit.***

Example 63.4. *What values of R_S and R_L are required for the circuit of Fig. 63.12 for setting up an approximate mid-point bias ? The JFET parameters are : $I_{DSS} = 16$ mA, $V_{GS(off)} = -8$ V and $V_D = \frac{1}{2}V_{DD}$.*

Solution. It should be noted that as found from Shockley's equation, for mid-point bias, $I_D \cong I_{DSS}/2$ and $V_{GS} = V_{GS(off)}/4$. Hence, for mid-point bias

$$I_D \cong 16/2 = 8\ \text{mA}$$

$$V_{GS} = \frac{1}{4}V_{GS(off)} = -\frac{8}{4} = -2\text{V}$$

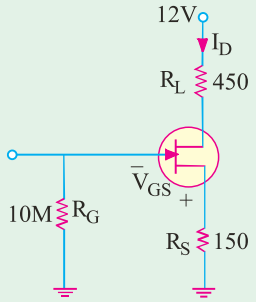


Fig. 63.11

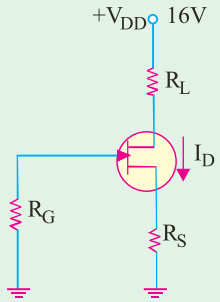


Fig. 63.12

$$R_S = \frac{V_{GS}}{I_D} = \frac{2\text{ V}}{8\,mA} = \mathbf{250\ \Omega}$$

Now, $V_D = V_{DD} - I_D R_L$, $R_L = \dfrac{V_{DD} - V_D}{I_D} = \dfrac{16-8}{8\text{mA}}$

$= \mathbf{1000\ \Omega}$

Example 63.5. *Determine the quiescent value of V_{GS} I_D and V_{DS} for the JFET circuit of Fig. 63.13 given that I_{DSS} = 10 mA, R_S = 5 K and V_P = –5 V.*

(Electronic Devices & Circuits, Pune Univ. 1991)

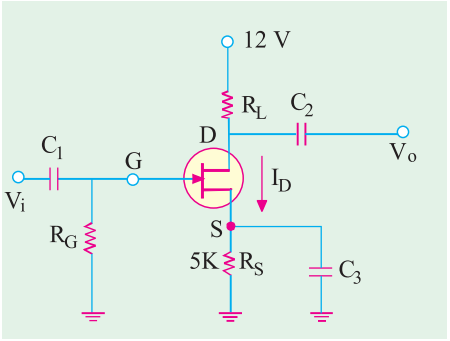


Fig. 63.13

Solution. Since $I_S \cong I_D$, $V_{GS} = -I_D R_S = -5000\ I_D$

Now, $I_D = I_{DSS}\left(1 - \dfrac{V_{GS}}{V_P}\right)^2 = 10 \times 10^{-3}\left(1 - \dfrac{V_{GS}}{-5}\right)^2$

$= 10 \times 10^{-3}(1 + 0.2\ V_{GS})^2$

Substituting this value in the above equation, we get

$V_{GS} = -5000(10 \times 10^{-3})(1 + 0.2\ V_{GS})$

Expanding and rearranging the above, we have

$2V_{GS}^2 + 21V_{GS} + 50 = 0$

$\therefore \quad V_{GS} = -3.65\text{ V} \quad \text{or} \quad -6.85\text{ V}$

Rejecting the higher value because it is more than V_P, we have $V_{GS} = \mathbf{-3.65\ V}$

$\therefore \quad -3.65 = -5000\ I_D \quad \therefore \quad I_D = \mathbf{0.73\ mA}$

$V_D = V_{DD} - I_D R_L = 12 - 0.73 \times 2 = \mathbf{1.54\ V}$

$V_S = I_D R_S = 0.73 \times 5 = 3.65\ V \quad \therefore \quad V_{DS} = V_D - V_S = 10.54 - 3.65 = \mathbf{6.89\ V}$

63.10. Common Source JFET Amplifier

A simple circuit for such an amplifier is shown in Fig. 63.14. Here, R_G serves the purpose of providing leakage path to the gate current, R_S develops gate bias, C_3 provides ac ground to the input signal and R_L acts as drain load.

Working

When ***negative-going*** signal is applied to the input

1. gate ***bias is increased,***
2. depletion regions are ***widened,***
3. channel resistance is ***increased,***
4. ***I_D is decreased,***
5. drop across R_L is ***decreased,***
6. Consequently, a ***positive-going*** signal becomes available at the output through C_2 in Fig. 63.14.

When ***positive-going signal*** is applied at the input, then in a similar way, ***a negative-going signal*** becomes available at the output.

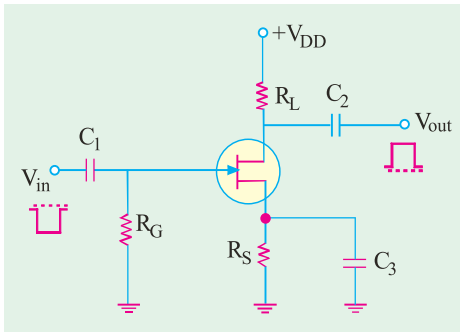


Fig. 63.14

It is seen that there is a ***phase inversion*** between the input signal at the gate and output signal at the drain.

63.11. JFET Amplifier Gains

We will now find the expressions for voltage gain, input resistance, output resistance and input capacitance of a JFET amplifier when connected in different modes. The different capacitance due to *P-N* junction and channel are shown in Fig. 63.15

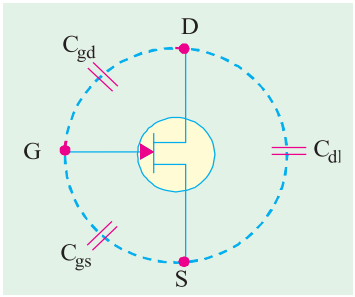


Fig. 63.15

(*a*) Common Source JFET Amplifier (Fig. 63.16)

(*i*) Input Resistance $r'_i = R_G \| R_{GS}$

In an ideal JFET, R_{GS} is infinite because $I_G = 0$. In an actual device, however, R_{GS} is not actually infinite but extremely high (100 M or so) as compared to R_G. Hence, $r_i' \cong R_G$.

(*ii*) Output Resistance $r_o' = r_d \| R_L \cong R_L$ — if $r_d \gg R_L$

(*iii*) Voltage Gain

$$V_o = i_d \times r_d \| R_L \qquad \text{Now,} \quad i_d = -g_m \times v_i$$

$$\therefore \quad V_o = -g_m V_i \times (r_d \| R_L)$$

$$\therefore \quad A_v = \frac{V_o}{V_i} = \frac{-g_m V_i \times (r_d \| R_L)}{V_i} = \frac{-g_m r_d R_L}{r_d + R_L} = \frac{-\mu R_L}{r_d + R_L}$$

Also, $A_v = -g_{fs} \times (r_d \| R_L)$

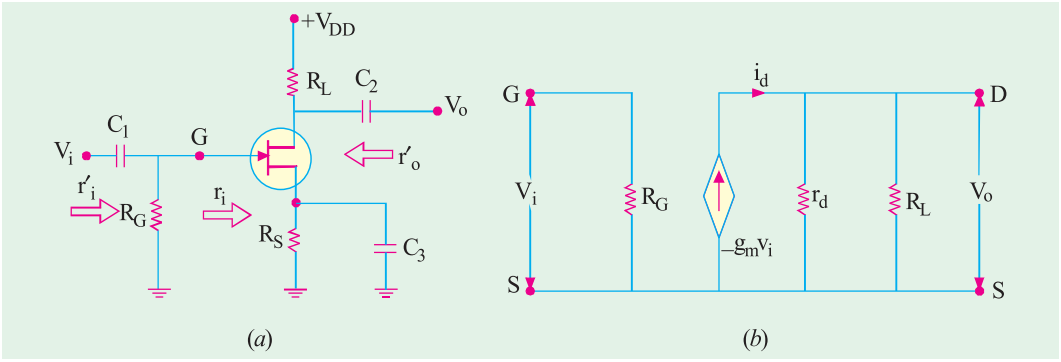


Fig. 63.16

(*iv*) Input Capacitance

The input capacitance is C_{gs} which is increased because of Miller effect.

$$\therefore \quad C_i = C_{gs} + (1 - A_v)\, C_{gd}.$$

It is the large value of C_{gd} which is harmful in high-frequency work.

Example 63.6. *The common-source amplifier of Fig. 63.16 (a) has $r_d = 100$ K, $R_L = 10$ K, $g_m = 3000$ µS, $C_{gs} = 3$ pF and $C_{gd} = 1.5$ pF. Compute its (i) A_v and (ii) C_i.*

Solution. ***(i)*** $$A_v = \frac{-g_m r_d R_L}{r_d + R_L}$$

$$= \frac{-3000 \times 10^{-6} \times 100 \times 10^3 \times 10 \times 10^3}{(100 \times 10^3) + (10 \times 10^3)} = \mathbf{-27.3}$$

(ii) $$C_i = C_{gs} + (1 - A_v)\, C_{gd}$$

$$= 3 + (1 + 27.3) \times 1.5 = \mathbf{45.5\ pF}.$$

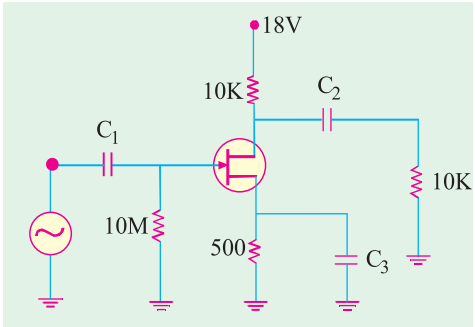


Fig. 63.17

Example 63.7. *The JFET shown in Fig. 63.17 has g_m = 3000 μS and r_{ds} = 100 K. Calculate the voltage gain of the CS amplifier circuit.* **(Basic Electronics, Bombay Univ. 1992)**

Solution. As seen from Art. 63.11 (*a*)

$$A_v = -g_m \times (r_{ds} \| r_L)$$

Now, $r_L = 10\text{ K} \| 10\text{ K} = \mathbf{5\text{ K}}$

$\therefore$ $r_{ds} \| r_L = 100\text{ K} \| 5\text{ K} = \mathbf{4.76\text{ K}}$

$\therefore$ $A_v = -3000 \times 10^{-6} \times 4.76 = \mathbf{-14.3}$

(*b*) Common Drain JFET Amplifier

In the common-drain circuit (also called source follower), the load resistance is in series with the source terminal. There is no drain resistor as shown in Fig. 63.18 (*a*). The input signal is applied to the gate through the capacitor C_1 and the output is taken out from the source *via* C_2. The common-drain equivalent circuit is shown in Fig. 63.18 *(b)*. The current generator is g_m, V_{gs} where $v_{gs} = (V_i - V_o)$. Moreover, $R_G = R_1 \| R_2$.

(*i*) Voltage Gain

$$V_o = i_d \times (r_d \| R_L). \qquad \text{Since,} \quad i_d = g_m V_{gs} = g_m (V_i - V_o),$$

$$\therefore \quad V_o = g_m (V_i - V_o) \times (r_d \| R_L) = g_m (V_i - V_o), \frac{r_d R_L}{r_d + R_L}$$

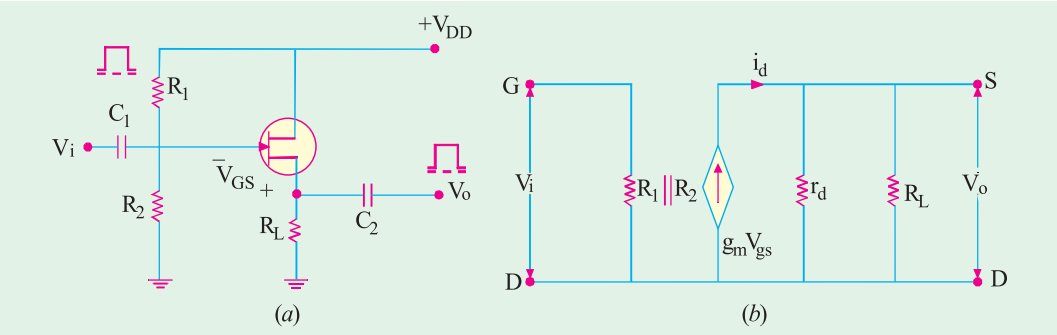


Fig. 63.18

Solving for V_o, we get

$$V_o = g_m V_i \frac{r_d R_L}{r_d + R_L + g_m r_d R_L} \qquad ...(i)$$

$$\therefore \quad A_v = \frac{V_o}{V_i} = \frac{r_d R_L}{r_d + R_L + g_m r_d R_L} \cong 1 \qquad \text{– if } g_m r_d R_L \gg (r_d + R_L)$$

(*ii*) Input Resistance

$r'_{in} = R_1 \| R_2$ – for circuit of Fig. 63.18 (*a*) only.

(*iii*) Output Resistance

In Eq. (*i*) above, $g_m V_i$ is a current which is directly proportional to V_i and $(r_d R_L)/(r_d + R_L + g_m r_d R_L)$ = a resistance, r_o.

$$\therefore \quad r'_o = \frac{r_d R_L}{r_d + R_L (1 + g_m r_d R_L)} = \frac{[r_d / (1 + g_m r_d)] \times R_L}{[r_d / (1 + g_m r_d)] + R_L}$$

$$= \frac{r_d}{1 + g_m r_d} \| R_L \cong \frac{1}{g_m} \| R_L$$

The above result helps us to draw the modified equivalent circuit of Fig. 63.19 for the common-drain amplifier.

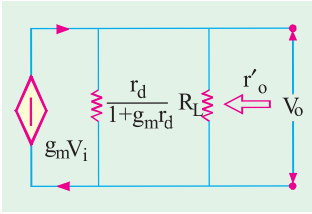


Fig. 63.19

Example 63.8. *The common-drain circuit of Fig. 63.18 (a) uses a JFET having the following parameters :*

$r_d = 100\ K$, $g_m = 3000\ \mu S$ *and* $R_L = 10\ K$

Calculate ***(i)*** A_v *and* ***(ii)*** r'_o.

Solution. ***(i)*** $$A_v = g_m \frac{r_d R_L}{r_d + R_L + g_m r_d R_L}$$

$$= 3000\times10^{-6} \frac{100\times10^3 \times 10\times10^3}{(100\times10^3)+(10\times10^3)+(3000\times10^{-6}\times100\times10^3\times10\times10^3)} = \mathbf{0.965}$$

(ii) $$r'_0 = R_L \,\|\, \frac{r_d}{1+g_m r_d}$$

Now, $$\frac{r_d}{1+g_m r_d} = \frac{100\times10^3}{1+3000\times10^{-6}\times100\times10^3} = \mathbf{330\ \Omega}$$

$$r'_o = 10\ K \,\|\, 330\ \Omega \cong \mathbf{320\ \Omega}$$

(*c*) Common Gate JFET Amplifier

In this amplifier configuration, input signal is applied to the source terminal and output is taken from the drain as shown in Fig. 63.20 (*a*). The gate is grounded, R_L is in series with drain and a source resistance R_S is included in the circuit across which V_i is dropped.

The ac equivalent circuit is shown in Fig. 63.20 (*b*) where current source is connected between the ***drain and the source terminals as always.*** However, since source and drain are the input and output terminals respectively, ($g_m V_{gs}$) appears between the input and the output.

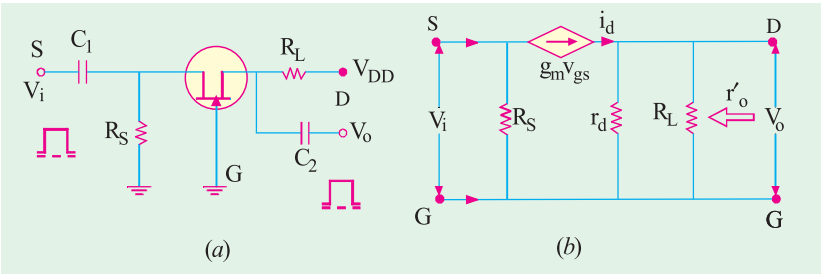


Fig. 63.20

(*i*) Voltage Gain

$$V_o = i_d \times (r_d \,\|\, R_L) = i_d \cdot \frac{r_d R_L}{r_d + R_L}$$

Now, $i_d = g_m V_{gs} = g_m V_i$ $\quad\therefore\quad V_o = g_m V_i \cdot \frac{r_d R_L}{r_d + R_L}$ $\quad\therefore\quad A_v = \frac{V_o}{V_i} = \frac{g_m r_d R_L}{r_d + R_L}$

It is the same as the gain for a common source amplifier except that it is a positive quantity. It means that V_o and V_i are in phase as shown in Fig. 63.20 (*a*).

(*ii*) Input Resistance

If we ignore current through R_S, then as seen from the ac equivalent circuit of Fig. 63.20 (*b*), input current is i_d. Now, $i_d = g_m V_{gs} = g_m V_i$.

The input resistance of the device *i.e.* JFET is $r_i = \frac{V_i}{i_d} = \frac{V_i}{g_m V_i} = \frac{1}{g_m}$

The input resistance of the *circuit* is $r_i' = r_i \| R_S = \dfrac{1}{g_m} \| R_s$

Actually speaking, r_d and R_L are also involved in r_i' but their effect is negligible.

(*iii*) Output Resistance

As seen from the equivalent circuit of Fig. 63.20 (*b*), $r_o' = r_d \| R_L$.

Here, only input capacitance of importance is C_{gs}. Hence, this circuit has low input capacitance as compared to common-source circuit where input capacitance is increased due to Miller effect.*

63.12. Advantages of FETs

FETs combine the many advantages of both BJTs and vacuum tubes. Some of their main advantages are :

1. high input impedance, **2.** small size, **3.** ruggedness, **4.** long life,
5. high frequency response, **6.** low noise,
7. negative temperature coefficient, hence better thermal stability,
8. high power gain,
9. a high immunity to radiations,
10. no offset voltage when used as a switch (or chopper), **11.** square law characteristics.

The only disadvantages are :

1. small gain-bandwidth product,
2. greater susceptibility to damage in handling them.

Tutorial Problems No. 63.1

1. For a particular *N*-channel JFET, $V_{GS(off)} = -4$ V. What would be the value of I_D when $V_{GS} = -6$ V?
[zero]

2. For the *N*-channel JFET shown in Fig. 63.21, $V_P = 8$V and $I_{DSS} = 12$ mA. What would be the value of (*i*) V_{DS} at which pinch-off begins and (*ii*) value of I_D when V_{DS} is above pinch-off but below the breakdown voltage ?
[3V ; 12 mA]

3. The data sheet of a JFET indicates that it has $I_{DSS} = 15$ mA and $V_{GS(off)} = -5$ V. Calculate the value of I_D when V_{GS} is (*i*) 0, (*ii*) –1 V and (*iii*) – 4 V.
[(*i*) 15 mA (*ii*) 9.6 mA (*iii*) 0.6 mA]

4. The data sheet of a JEET gives the following information.
$I_{DSS} = 20$ mA, $V_{GS(off)} = -8$ V. and $g_{mo} = 4000$ µS.
Calculate the value of I_D and g_m for $V_{GS} = -4$ V.
[5 mA ; 2000 µS]

5. For a JEET, I_{DSS} 5 mA and $g_{mo} = 4000$ µS.
Calculate (*i*) $V_{GS(off)}$ and (*ii*) gm at mid-point bias.
[(*i*) –5 V (*ii*) 3000 µS]

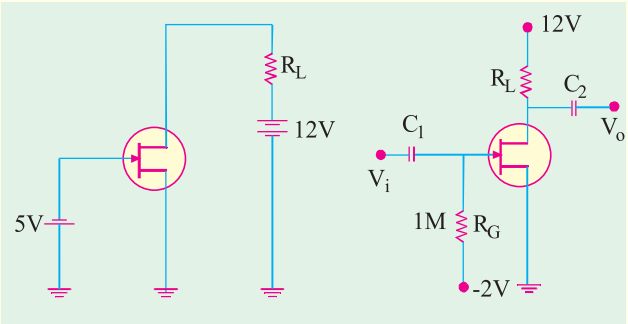


Fig. 63.21

Fig. 63.22

6. At a certain point on the transfer characteristics of an N-channel JFET, following values are read : $I_{DSS} = 8.4$ mA, $V_{GS} = -0.5$ V and $V_P = -3.0$ V.
Calculate (*i*) g_{mo} and (*ii*) g_m at the point.
[(*i*) 5600 µS (*ii*) 4670 µS]

7. For the JFET circuit of Fig. 63.22, $I_{DSS} = 9$ mA and $V_P = -3$ V.
Find the value of R_L for setting the value of V_{DS} at 7 V.
[5 K]
(Hint : $V_{GS} = -2$ V as $I_G = 0$)

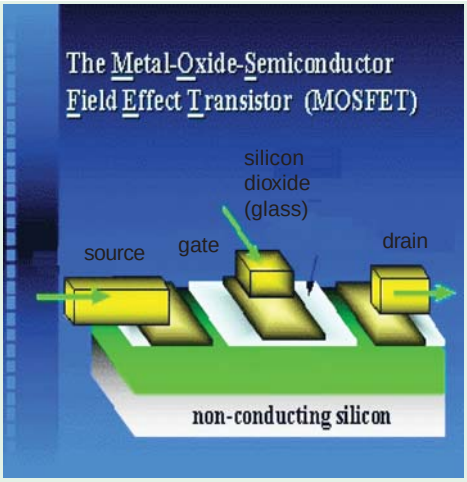
* Miller effect occurs only where output is antiphase with input as *CS* amplifier circuit.

63.13. MOSFET or IGFET

It could be further subdivided as follows :

(*i*) Depletion-enhancement MOSFET or DE MOSFET

This MOSFET is so called because it can be operated in both ***depletion*** mode and ***enhancement*** mode by changing the polarity of V_{GS}. When negative gate-to-source voltage is applied, the *N*-channel DE MOSFET operates in the depletion mode. However, with positive gate voltage, it operates in the enhancement mode. Since a channel exists between drain and source, I_D flows even when $V_{GS} = 0$. That is why DE MOSFET is known as **normally-ON MOSFET.**



Metal-oxide semiconductor field effect transistor

(*ii*) Enhancement-only MOSFET

As its name indicates, this MOSFET operates only in the enhancement mode and has no depletion mode. It works with ***large positive*** gate voltages only. It differs in construction from the DE MOSFET in that structurally ***there exists no channel between the drain and source.*** Hence, it does not conduct when $V_{GS} = 0$. That is why it is called **normally-OFF** MOSFET.

In a DE MOSFET, I_D flows even when $V_{GS} = 0$. It operates in depletion mode with negative values of V_{GS}. As V_{GS} is made more negative, I_D decreases till it ceases when $V_{GS} = V_{GS(off)}$. It works in enhancement mode when V_{GS} is positive as shown in Fig. 63.24 (*b*).

In the case of E-only MOSFET, I_D flows only when V_{GS} exceeds $V_{GS(th)}$ as shown in Fig. 63.30 (*c*).

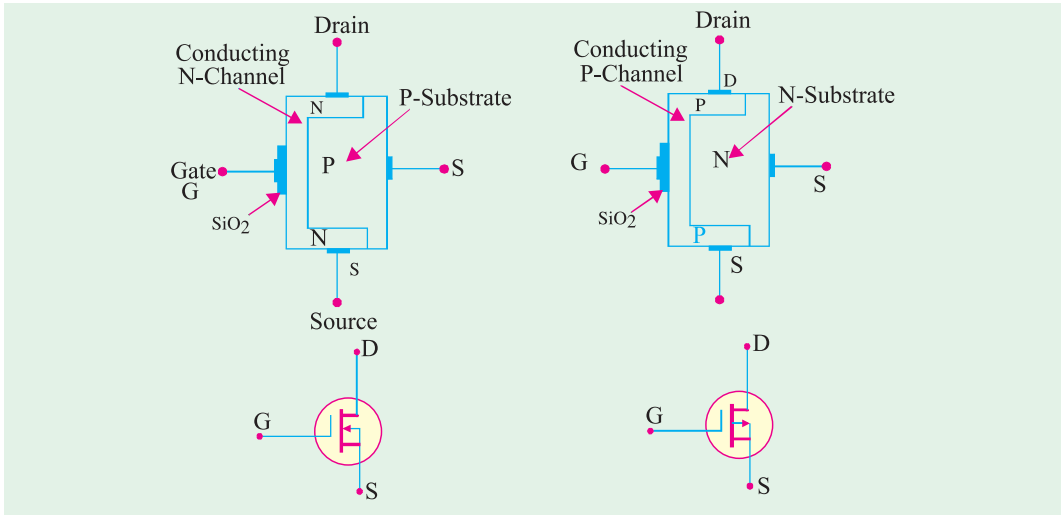


Fig. 63.23

63.14. DE MOSFET

(*a*) Construction

Like JFET, it has source, gate and drain. However, as shown in Fig. 63.23, its gate is insulated from its conducting channel by an ultra-thin metal-oxide insulating film (usually of silicon dioxide SiO_2). Because of this insulating property. MOSFET is alternatively known as insulated-gate field-

effect transistor (IGFET or IGT). Here also, gate voltage controls drain current but main difference between JFET and MOSFET is that, in the latter case, we can apply ***both positive and negative voltages to the gate because it is insulated from the channel.*** Moreover, the gate, SiO_2 insulator and channel from a parallel-plate capacitor. Unlike JFET, a DE MOSFET has only one P-region or N-region called substrate. Normally, it is shorted the ***source internally.*** Fig. 63.23 shows both *P*-channel and *N*-channel DE MOSFETs along with their symbols.

(*b*) Working

(*i*) Depletion Mode of N-channel DE MOSFET

When $V_{GS} = 0$, electrons can flow freely from source to drain through the conducting channel which exists between them. When gate is given negative voltage, it **depletes** the *N*-channel of its

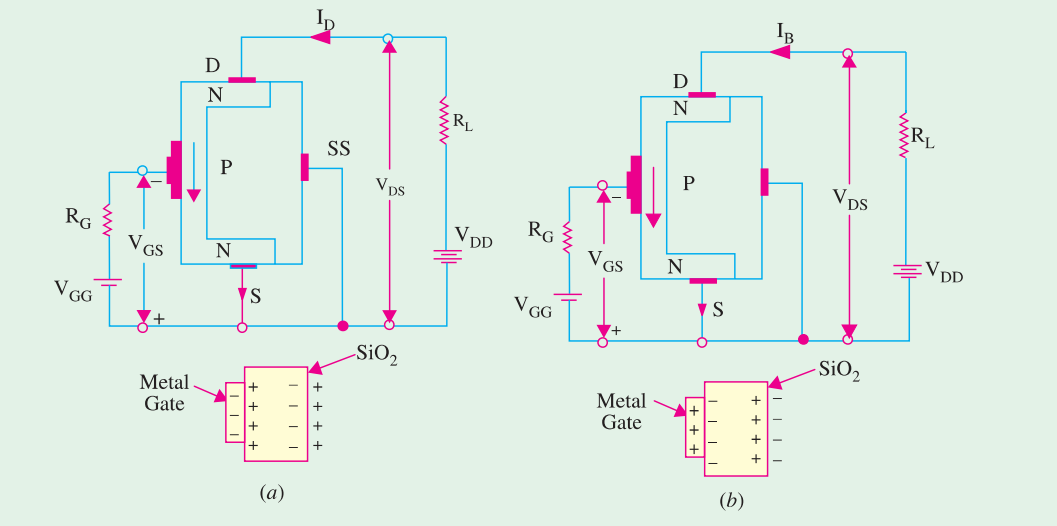


Fig. 63.24

electrons by including positive charge in it as shown in Fig. 63.24 (*a*). Greater the negative voltage on the gate, greater is the reduction in the number of electrons in the channel and, consequently, lesser its conductivity. In fact, too much negative gate voltage called $V_{GS(off)}$ can cut-off the channel. Hence, with negative gate voltage, a DE MOSFET behaves like a JFET.

For obvious reasons, negative-gate operation of a DE MOSFET is called its depletion mode operation.

(*ii*) Enhancement Mode of N-channel DE MOSFET

The circuit connections are shown in Fig. 63.24 (*b*). Again, drain current flows from source to drain even with zero gate bias. When positive voltage is applied to the gate, the input gate capacitor is able to create free electrons in the channel which increases I_D. As seen from the enlarged view of the gate capacitor in Fig. 63.24 (*b*), free electrons are induced in the channel ***by capacitor action.*** These electrons are added to those already existing there. This increased number of electrons ***increases*** or **enhances** the conductivity of the channel. As positive gate voltage is increased, the number of induced electrons is increased, so conductivity of the source-to-drain channel is increased and, consequently, increasing amount of current flows between the terminals. That is why, positive gate operation of a DE MOSFET is known as its **enhancement mode** operation.

Since gate current in both modes is negligibly small, input resistance of a MOSFET is incredibly high varying from $10^{10}\,\Omega$ to $10^{14}\,\Omega$. In fact, MOSFET input current is the leakage current of the capacitor unlike the input current for JFET which is the leakage current of a reverse-biased *P-N* junction.

63.15. Schematic Symbols for a DE MOSFET

Schematic symbol of an *N*-channel normally-ON or DE MOSFET is shown in Fig. 63.25. The gate looks like a metal plate. The arrow is on the substrate and **towards the N-channel.** When *SS* is connected to an external load, we have a 4-terminal device as shown in Fig. 63.25 (*a*) but when it is internally shorted to *S*, we get a 3-terminal device as shown in Fig. 63.25 (*b*). Fig. 63.25 (*c*) shows the symbol for a *P*-channel DE MOSFET.

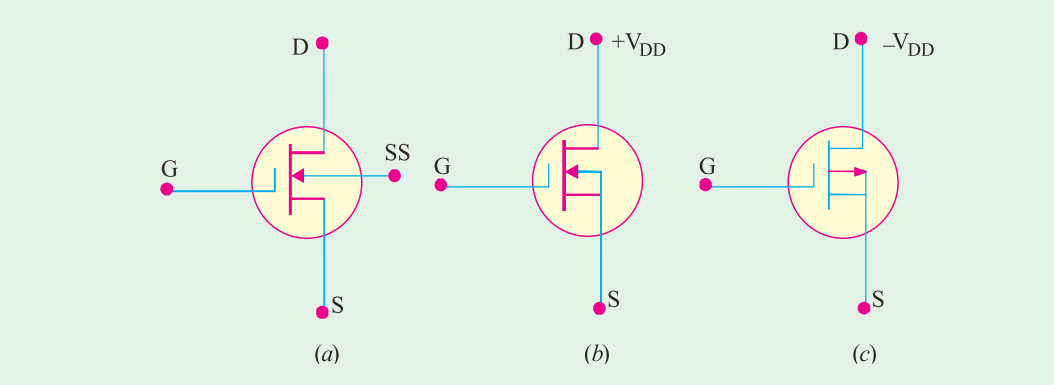


Fig. 63.25

63.16. Static Characteristics of a DE MOSFET

In Fig. 63.26 are shown the drain current and transfer characteristics of a common-source N-channel DE MOSFET for V_{GS} varying from +2 V to $V_{GS(off)}$.

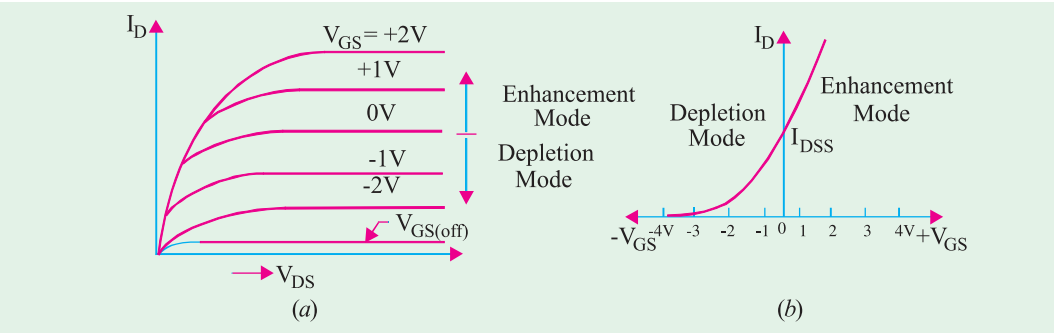


Fig. 63.26

It acts in the enhancement mode when gate is ***positive*** with respect to source and in the depletion mode when gate is ***negative.*** As usual, $V_{GS(off)}$ represents the gate-source voltage which cuts off the source-to-drain current. The transfer characteristic is shown in Fig. 63.26 (*b*). For a given V_{DS}, I_D flows even when $V_{GS} = 0$. However, keeping V_{DS} constant, as V_{GS} is made more negative, I_D keeps decreasing till it becomes zero at $V_{GS} = V_{GS(off)}$. When used in the enhancement mode, I_D increases as V_{DS} is increased positively.

Example 63.9. *For the N-channel zero-biased DE MOSFET circuit of Fig. 63.27, calculate V_{DS} if I_{DSS} = 10 mA and $V_{GS(off)}$ = – 6 V.*

(Electronics-I, Bangalore Univ. 1992)

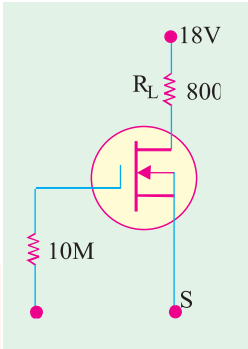


Fig. 63.27

Solution. Since $I_D = I_{DSS}$=10 mA

$\therefore$ $V_{DS} = V_{DD} - I_{DSS} R_L$

$= 18 - 10 \times 10^{-3} \times 800 =$ **10 V**

63.17. Enhancement-only N-Channel MOSFET

This *N*-channel MOSFET (also called NMOS) finds wide application in digital circuitry. As shown in Fig. 63.28 in the NMOS, the *P*-type substrate extends all the way to the metal-oxide layer. Structurally, ***there exists no channel between the source and drain.*** Hence, an NMOS can never operate with a negative gate voltage because it will induce ***positive*** charge in the space between the drain and source which will not allow the passage of electrons between the two. Hence, it operates with ***positive gate voltage only.***

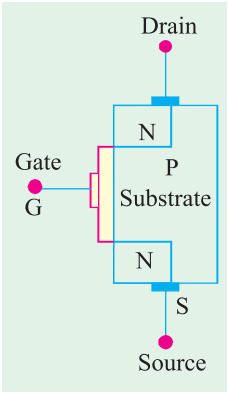


Fig. 63.28

The normal biasing polarities of this *E*-only MOSFET (both *N*-channel and *P*-channel) are shown in Fig. 63.29. With $V_{GS} = 0$, I_D is non-existent even when some positive V_{DD} is applied. It is found that for getting significant amount of drain current, we have to apply sufficiently high positive gate voltage. This voltage is found to produce a thin layer of free electrons very close to the metal-oxide film which stretches all the way from source to drain. The thin layer of *P*-substrate touching the metal-oxide film which provides channel for electrons (and hence acts like *N*-type material) is called ***N*-type *inversion layer* or *virtual*** *N*-channel [Fig. 63.29 (*a*)].

The minimum gate-source voltage which produces this *N*-type inversion layer and hence drain current is called threshold voltage $V_{GS(th)}$ as shown in Fig. 63.30 (*c*). When $V_{GS} < V_{GS(th)}$, $I_D = 0$. Drain current starts only when $V_{GS} > V_{GS(th)}$. For a given V_{DS}, as V_{GS} is increased, virtual channel deepens and I_D increases. The value of I_D is given by $I_D = K(V_{GS} - V_{GS(th)})2$ where K is a constant which depends on the particular MOSFET. Its value can be determined from the data sheet by taking the specified value of I_D called $I_{D(ON)}$ at the given value of V_{GS} and then substituting the values in the above equation. Fig. 63.30 (*a*)

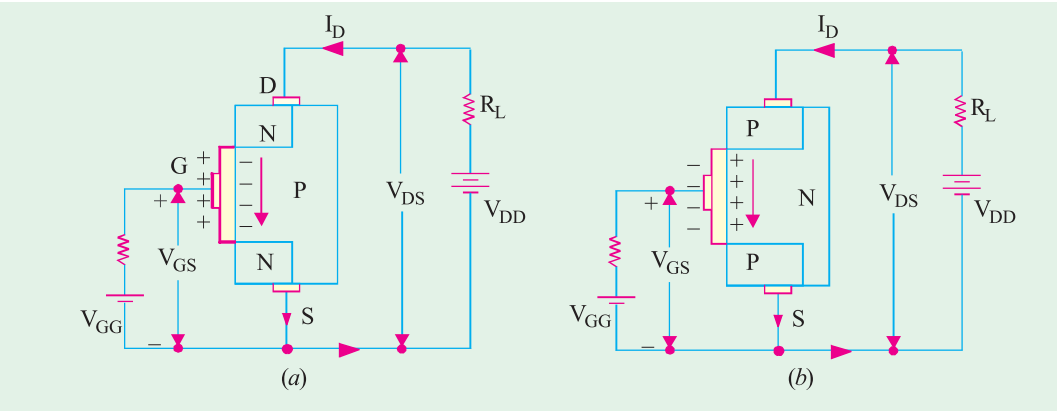


Fig. 63.29

shows the schematic symbol for an *E*-only *N*-channel MOSFET whereas Fig. 63.30 (*b*) shows its typical drain characteristics. As usual, arrow on the substrate points to the *N*-type material and the vertical line (representing-channel) is broken as a reminder of the normally-OFF condition.

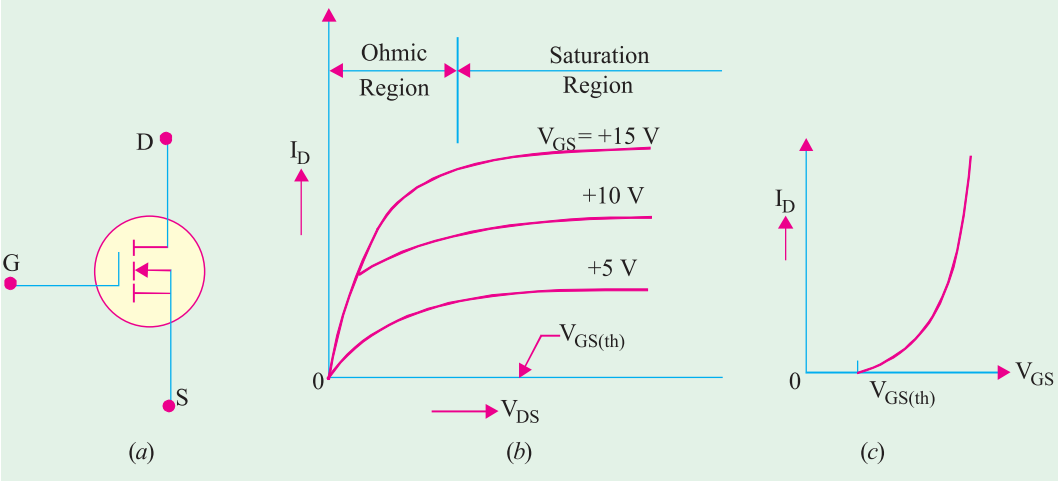


Fig. 63.30

A P-channel E-only MOSFET (PMOS) is constructed like NOMS except that all the *P*– and *N*– regions are interchanged. It operates with ***negative*** gate voltage only.

Differentiating the above-given drain current equation with respect to V_{GS}, we get

$$\frac{dI_D}{dV_{GS}} = g_m = 2K\,(V_{GS} - V_{GS(th)})$$

Transfer Characteristics

It is shown in Fig. 63.30 (*c*). I_D flows only when V_{GS} exceeds threshold voltage $V_{GS(th)}$. This MOSFET does not have an I_{DSS} parameter as do the JFET and DE MOSFET.

63.18. Biasing E-only MOSFET

As stated earlier, enhancement-only MOSFET must have a V_{GS} greater than $V_{GS(th)}$. Fig. 63.31 shows two methods of biasing an *N*-channel E-MOSFET. In either case, the purpose is to make the gate voltage more positive than the source by an amount exceeding $V_{GS(th)}$. Fig. 63.31 (*a*) shows drain-feedback bias whereas Fig. 63.31 (*b*) shows voltage-divider bias.

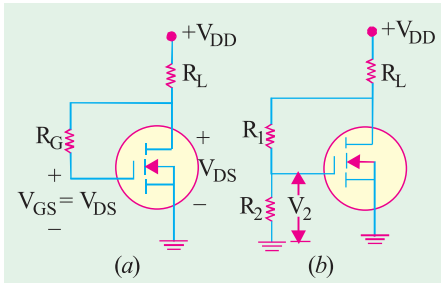


Fig. 63.31

Considering the circuit of Fig. 63.31 (*b*), we have

$$V_{GS} = V_2 = V_{DD}\frac{R_2}{R_1 + R_2}$$

and $$V_{DS} = V_{DD} - I_D R_L$$

where $$I_D = K(V_{GS} - V_{GS(th)})^2.$$

Example 63.10. *The data sheet of the E-MOSFET shown in Fig. 63.32 gives $I_{D(ON)}$ = 4 mA at V_{GS} = 10 V and $V_{GS(th)}$ = 5 V. Calculate V_{GS} and V_{DS} for the circuit.* **(Applied Electronics, Punjab Univ. 1991)**

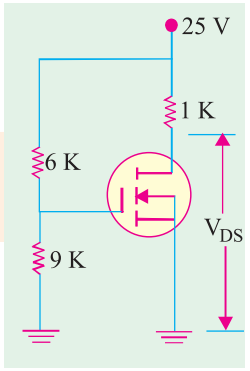


Fig. 63.32

Solution. $V_{GS} = V_2 = 25 \times (9/15) = 15$ V

Let us now find the value of *K*.

$$K = \frac{I_{D(ON)}}{(V_{GS} - V_{GS(th)})^2} = \frac{4}{(10-5)^2} = 0.16\text{mA/V}^2$$

Value of I_D for V_{GS}=15 V is given by

$$I_D = K\,(V_{GS} - V_{GS(th)})^2 = 0.16\,(15-5)^2 = 16\text{ mA}$$

Now, $V_{DS} = V_{DD} - I_D R_L = 25 - 16 \times 1 = \mathbf{9\ V}$

Example 63.11. *An N-channel E-MOSFET has the following parameters:*
$I_{D(ON)} = 4\ mA$ *at* $V_{GS} = 10\ V$ *and* $V_{GS(th)} = 5\ V$
Calculate its drain current for $V_{GS} = 8\ V$.

Solution. $K = \dfrac{I_{D(ON)}}{(V_{GS} - V_{GS(th)})^2} = \dfrac{4\text{mA}}{(10\,\text{V} - 5\text{V})^2} = 0.16\text{mA/V}^2$

Now, using this value of K, I_D can be found thus :

$$I_D = K\,(V_{GS} - V_{GS(th)})^2 = 0.16\,(8-5)^2 = \mathbf{1.44\ mA}$$

63.19. FET Amplifiers

We will consider the DE MOSFET and E-MOSFET separately.

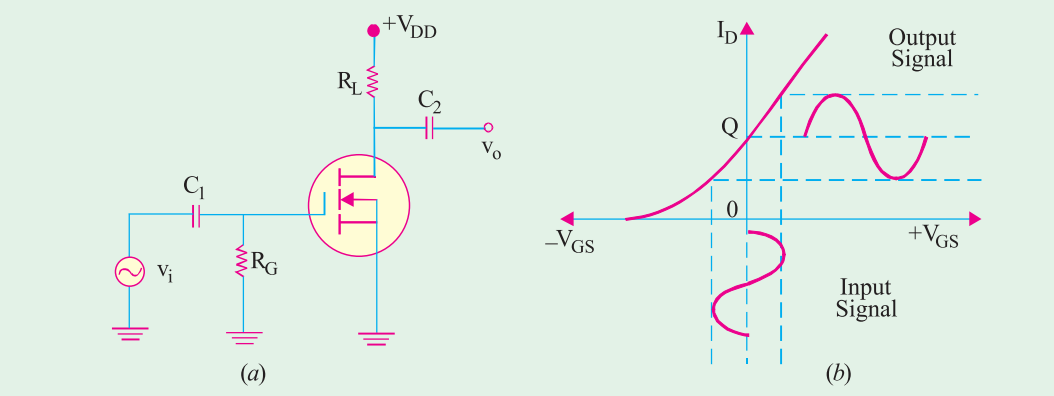


Fig. 63.33

(*i*) DE MOSFET Amplifier

Fig. 63.33 (*a*) shows a zero-biased *N*-channel DE MOSFET with an ac source capacitively-coupled to its gate. Since gate is at 0 volt dc and source is at ground, $V_{GS} = 0$. Fig. 63.33 (*b*) shows the transfer characteristic.

The input ac signal voltage V_i causes V_{GS} to swing above and below its zero value thus producing a swing in I_D as shown in Fig. 63.33 (*b*). The negative swing in V_{GS} produces depletion mode and I_D

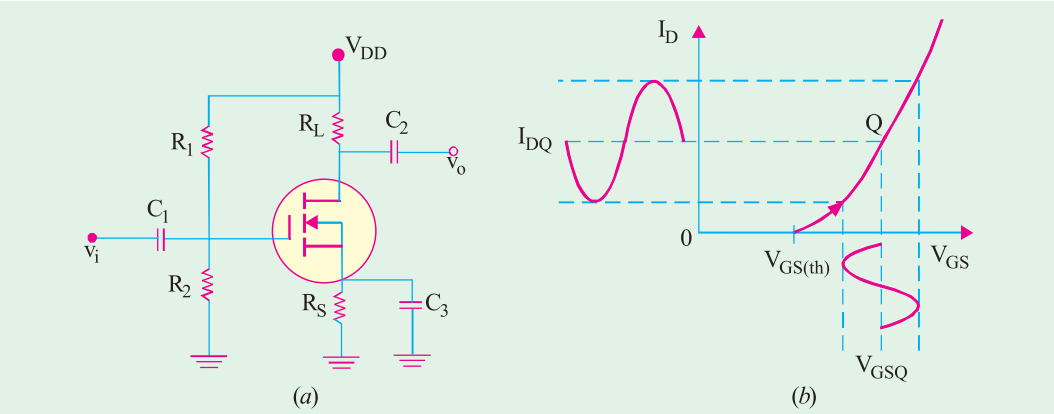


Fig. 63.34

is decreased. A positive swing in V_{GS} produces enhancement mode so that I_D is increased. This leads to a large swing in drop across R_L and can be taken out *via* C_2 as V_o.

(*ii*) E-MOSFET Amplifier

In Fig. 63.34 (*a*) is shown a voltage-divider biased *N*-channel E-MOSFET having an ac signal source coupled to its gate. The gate is biased with a positive voltage such that V_{GS} exceeds $V_{GS(th)}$.

As shown in Fig. 63.34 (*b*), the signal voltage produces a swing in V_{GS} below and above its *Q*-point value. This, in turn, causes a swing in I_D and hence in $I_D R_L$ which gives rise to no.

Example 63.12. *The N-channel E-MOSFET used in the common-source amplifier of Fig. 63.35 has the following parameters :*

$I_{D(ON)} = 4$ mA at $V_{DS} = 10$ V, $V_{GS(th)} = 4$ V and $g_m = 5000$ μS.

Calculate V_{GS}, I_D, V_{GS} *and* v_o. **(Elect. and Electronic Engg., Annamalai Univ. 1992)**

Solution. $V_{GS} = V_{DD}\dfrac{R_2}{R_1+R_2} = 16\times\dfrac{30}{80} = \mathbf{6\ V}$

Now, $K = \dfrac{I_{D(NO)}}{(V_{GS}-V_{GS(th)})^2} = \dfrac{4}{(10-5)^2}$

$= 0.16$ mA/V^2

$\therefore\ I_D = K(V_{GS}-V_{GS(th)})^2 = 0.16(6-4)^2$

$= 0.64$ mA

$\therefore\ V_{DS} = V_{DD} - I_D R_L = 16 - 0.64\times 5 = \mathbf{12.8\ V}$

Now, $A_v = g_m(r_d \| R_L) \cong g_m R_L$

$= 5000\times 10^{-6}\times 5\times 10^3 = 25$

$\therefore\ V_o = A_v v_i = 25\times 100 = 2500$ mV $= \mathbf{2.5\ V}$

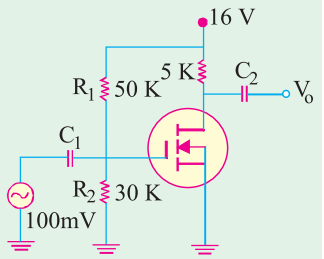


Fig. 63.35

Example 63.13. *The parameters of the enhancement-only NMOS shown in Fig. 63.36 are* $V_{GS(th)} = 2$ *V and* $K = 2\times 10^{-4}$ *A/V*^{2}. *Calculate the values of* I_D *and* V_{DS}.

Solution. Since drain is directly returned to gate, $V_{GS} = V_{DS}$. As seen from the figure

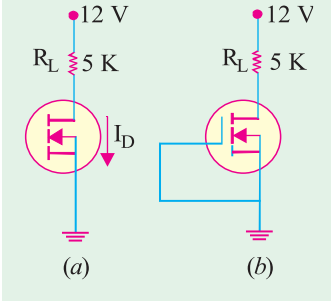


Fig. 63.36

$V_{DS} = V_{DD} - I_D R_L = 12 - 5\times 10^3 I_D$

$\therefore\ I_D = (12 - V_{DS})/5\times 10^3$ A

Now, $I_D = K(V_{GS}-V_{GS(th)})^2 = K(V_{DS}-2)^2$

or $\dfrac{12-V_{DS}}{5\times 10^3} = 2\times 10^{-4}(V_{DS}-2)^2$ or $V_{DS}^2 - 3V_{DS} - 8 = 0$

$\therefore\ V_{DS} = \dfrac{3\pm\sqrt{9+32}}{.2} = \dfrac{3\pm\sqrt{41}}{2} = \mathbf{4.7\ V}$

$\therefore\ I_D = (12-4.7)/5\times 10^3 = \mathbf{1.5\ mA}$

In Fig. 63.36 (*b*), since gate has been directly returned to ground, $V_{GS} = 0$. Hence, $I_D = 0$, because $V_{GS} \ll V_{GS(th)}$. With no current, $V_{DS} = V_{DD} = \mathbf{12\ V}$.

Example 63.14. *A MOSFET has a drain resistance,* R_L *of 44 kΩ and operates at 20 kHz. Calculate the voltage gain of this device as a single stage amplifier. The MOSFET parameters are :*

$g_m = 1.6$ *mA/V,* $r_d = 100$ *k Ω* , $C_{gs} = 3.0$ *pf,* $C_{ds} = 1.0$ *pF and* $C_{gd} = 2.8$ *pF.*

(U.P.S.C. Engg. Services, 1996)

Solution. $A_v = g_m(r_d \| R_L)$

$= 1.6\times 10^{-3}(100\times 10^3 \| 44\times 10^3)$

$= 1.6\times 10^{-3}\times 30.56\times 10^3$

$= \mathbf{48.9}$

63.20. FET Applications

FETs can be used in almost every application in which bipolar transistors can be used. However, they have certain applications which are exclusive to them :

1. As input amplifiers in oscilloscopes, electronic voltmeters and other measuring and testing equipment because their high ***r_{in} reduces loading effect to the minimum.***
2. In logic circuits where it is kept OFF when there is zero input while it is turned ON with very little power input.
3. For mixer operation of *FM* and *TV* receivers.
4. As voltage-variable resistor (*VVR*) in operational amplifiers and tone controls etc.
5. Large-scale integration (*LSI*) and computer memories because of very small size.

63.21. MOSFET Handling

MOSFETs require very careful handling particularly when ***out of circuit.*** In circuit, a MOSFET is as rugged as any other solid-state device of similar construction and size.

It is essential not to permit any stray or static voltage on the gate otherwise the ultra-thin SiO_2 layer between the channel and the gate will get ruptured. Since gate-channel junction looks like a capacitor with extremely high resistance, it requires only a few electrons to produce a high voltage across it. Even ***picking up a*** MOSFET ***by its leads can destroy it.*** Generally, grounding rings are used to short all leads of a MOSFET for avoiding any voltage build up between them. These grounding or shorting rings are removed only after MOSFET is securely wired into the circuit. Sometimes, conducting foam is applied between the leads instead of using shorting rings. As shown in Fig. 13.37, some MOSFETs have back-to-back Zener diodes internally formed to protect them against stray voltages.

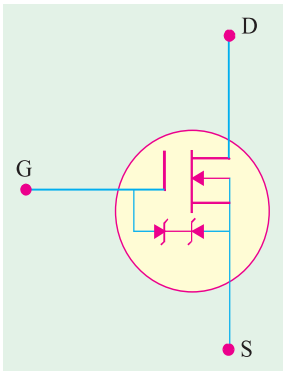


Fig. 63.37

Tutorial Problems No. 63.2

1. For a certain DE MOSFET, $I_{DSS} = 10$ mA and $V_{GS(off)} = -8$V. Calculate I_D when V_{GS} is (*i*) – 4 V and (*ii*) + 4 V. **[(*i*) 2.5 mA (*ii*) 22.5 mA]**
2. The data sheet of a certain zero-biased DE MOSFET gives $I_{DSS} = 15$ mA and $V_{GS(off)} = 5$ V. What is the value of drain current ? **[15 mA]**
3. A certain *E*-only *N*-channel MOSFET has the following parameters :
   $I_{D(on)} = 4$ mA at $V_{GS} = 8$ V and $V_{GS(th)} = 2$ V
   Calculate I_D for $V_{GS} = 6$ V **[1.78 mA]**

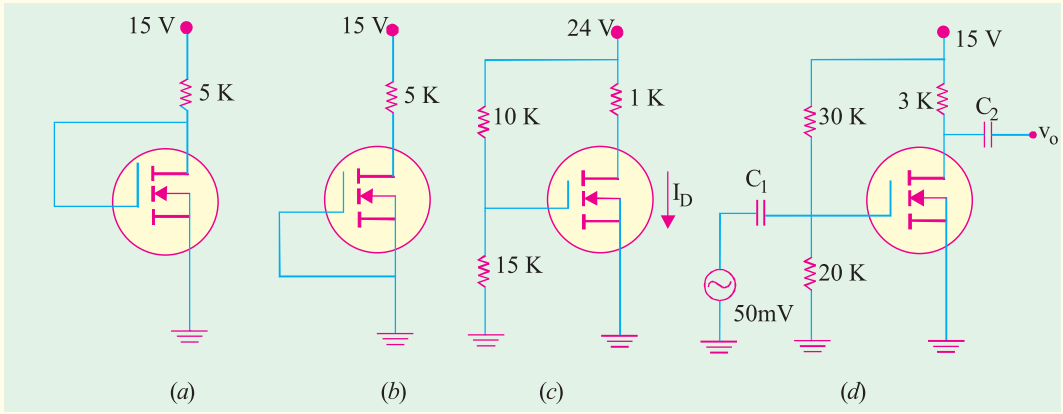


Fig. 63.38

4. The parameters of the *E*-only, *N*-channel MOSFET shown in Fig. 63.38 (*a*) are : $V_{GS(th)} = 2$ V and $K = 0.3$ mA/V^2. Determine the values of I_D and V_{DS}. If the gate is connected as shown in Fig. 63.38 (*b*), what will be the new values of I_D and V_{DS} ? **[(*i*) 2.07 mA (*ii*) 4.63 V ; 0 A, 15 V]**

5. The data sheet for the *E*-only *N*-channel MOSFET of Fig. 63.38 (*c*) gives $I_{D(on)} = 3$ mA at $V_{GS} = 10$ V and $V_{GS(th)} = 5$ V. Calculate the values of V_{GS} and V_{DS}. **[14.4 V ; 13.4 V]**

6. The amplifier circuit of Fig. 63.38 (*d*) used an *E*-only *N*-channel MOSFET having the following parameters : $I_{D(on)} = 5$ mA at $V_{GS} = 10$ V, $V_{GS(th)} = 4$ V, $g_m = 5500$ µS

Calculate V_{GS}, I_D, V_{DS} and v_o. **[6 V ; 0.556 mA ; 12.2 V ; 0.825 V]**

7. A field-effect transistor has a small-signal equivalent circuit with input resistance = 1000 MΩ , forward transfer conductance = 4 m S and output conductance = 100 µS when at the operating point, $V_{DS} = +4$ V, $I_D = 2$ mA, $V_{GS} = -2$ V.

Draw the circuit that you would use for a single-stage voltage amplifier. Describe the use and specify the value of as many components as possible if a 30 V supply was available.

What voltage gain would you expect when the output was unloaded ? Give reasons which might account for not getting this gain exactly.

[Drain load = 12 kΩ, Source Bias Resistor = 1 k Ω, – 21.8]

8. A field-effect transistor is used as a voltage amplifier and with a load resistor of 40 k Ω, a gain of 40 is obtained. If the load resistance is halved, the voltage gain drops to 30. Calculate the output resistance and the mutual conductance of the transistor.

Briefly compare the advantages and limitations of the field-effect transistor with the bipolar transistor. **[20 kΩ, 3 mS]**

OBJECTIVE TESTS – 63

1. A FET consists of a
(*a*) source (*b*) drain
(*c*) gate (*d*) all of the above.

2. FETs have similar properties to
(*a*) *PNP* transistors
(*b*) *NPN* transistors
(*c*) thermonic valves
(*d*) unijunction transistors.

3. For small values of drain-to-source voltage, JFET behaves like a
(*a*) resistor
(*b*) constant-current source
(*c*) constant-voltage source
(*d*) negative resistance

4. In a JFET, the *primary* control on drain current is exerted by
(*a*) channel resistance
(*b*) size of depletion regions
(*c*) voltage drop across channel
(*d*) gate reverse bias.

5. After V_{DS} reaches pinch-off value V_P in a JFET, drain current I_D becomes
(*a*) zero (*b*) low
(*c*) saturated (*d*) reversed.

6. In a JFET, as external bias applied to the gate is increased
(*a*) channel resistance is decreased
(*b*) drain current is increased
(*c*) pinch-off voltage is reached at lower val ues of I_D.
(*d*) size of depletion regions is reduced.

7. In a JFET, drain current is maximum when V_{GS} is
(*a*) zero (*b*) negative
(*c*) positive (*d*) equal to Vp

8. The voltage gain of a given common- source JFET amplifier depends on its
(*a*) input impedance
(*b*) amplification factor
(*c*) dynamic drain resistance
(*d*) drain load resistance.

9. A JFET has the disadvantage of
(*a*) being noisy
(*b*) having small gain-bandwidth product
(*c*) possessing positive temperature coe- fficient
(*d*) having low input impedance.

10. A JFET can be cut-off with the help of
(*a*) V_{GS} (*b*) V_{DS}
(c) V_{DG} (*d*) V_{DD} .

11. The drain source voltage at which drain current becomes nearly constant is called
(*a*) barrier voltage
(*b*) breakdown voltage
(*c*) pick-off voltage
(*d*) pinch-off voltage

12. The transconductance 'g_m' of a JFET is equal to
(*a*) $-\frac{2I_{DSC}}{V_P}$
(*b*) $\frac{2}{|V_P|}\sqrt{I_{DSS}\, I_D}$
(*c*) $-\frac{2I_{DSS}}{V_P}\left(1-\frac{V_{GS}}{V_P}\right)$
(*d*) $\frac{I_{DSS}}{V_P}\left(1-\frac{V_{GS}}{V_P}\right)$

13. An FET source follower circuit has of 2 millimho and of 50 k Ω. If the source resistance R_S is 1k Ω, the output resistance of the amplifier will be
(*a*) 330 Ω (*b*) 450 Ω
(*c*) 500 Ω (*d*) 1 k Ω

14. A DE MOSFET differs from a JFET in the sense that it has no
(*a*) channel (*b*) gate
(*c*) *P-N* junctions (*d*) substrate.

15. For the operation of enhancement only *N*-channel MOSFET, value of gate voltage has to be
(*a*) high positive (*b*) high negative
(*c*) low positive (*d*) zero.

16. The extremely high input impedance of a MOSFET is primarily due to the
(*a*) absence of its channel
(*b*) negative gate-source voltage
(*c*) depletion of current carriers
(*d*) extremely small leakage current of its gate capacitor

17. The main factor which makes a MOSFET likely to break down during normal hand- ling is its
(*a*) very low gate capacitance
(*b*) high leakage current
(*c*) high input resistance
(*d*) both (a) and (c).

18. The main factor which differentiates a DE MOSFET from an *E*-only MOSFET is the absence of
(*a*) insulated gate (*b*) electrons
(*c*) channel (*d*) *P-N* junctions.

19. The polarity of V_{GS} for *E*-only MOSFET is
(*a*) positive
(*b*) negative
(*c*) zero
(*d*) depends on P-or N-channel.

20. A tranasconductance amplifier has
(*a*) high input impedance and low output impedance
(*b*) low input impedance and high output impedance
(*c*) high input and output impedances
(*d*) low input and output impedances

21. The threshold voltage of an n-channel enhancement mode MOSFET is 0.5 V, when the device is biased at a gate voltage of 3V, pinch-off would occur at a drain voltage of
(*a*) 1.5 V (*b*) 2.5 V
(*c*) 3.5 V (*d*) 4.5 V

22. The zero gate bias channel resistance of a junction field-effect transistor is 750 and the pinch-off voltage is 3V. For a gate bias of 1.5 V and very low drain voltage, the device would behave as a resistance of
(*a*) 320 Ω (*b*) 816 Ω
(*c*) 1000 Ω (*d*) 1270 Ω

23. A MOSFET rated for 15 A, carries a periodic current as shown in Fig. 63.39. the ON state resistance of the MOSFET is 0.15 Ω. The average ON state loss in the MOSFET is

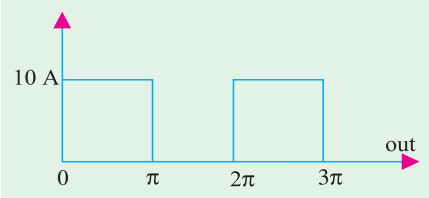


Fig. 63.39

(*a*) 33.8 W (*b*) 15.0 W
(*c*) 7.5 W (*d*) 3.8 W

ANSWERS

1. (*d*) **2.** (*b*) **3.** (*a*) **4.** (*d*) **5.** (*c*) **6.** (*c*) **7.** (*a*) **8.** (*d*) **9.** (*b*) **10.** (*a*) **11.** (*d*)
12. (*c*) **13.** *c* **14.** (*c*) **15.** (*a*) **16.** (*d*). **17.** (*d*) **18.** (*c*) **19.** (*d*) **20.** (*a*) **21.** (*c*) **22.** (*b*)
23. (*c*)

ROUGH WORK

GO To FIRST

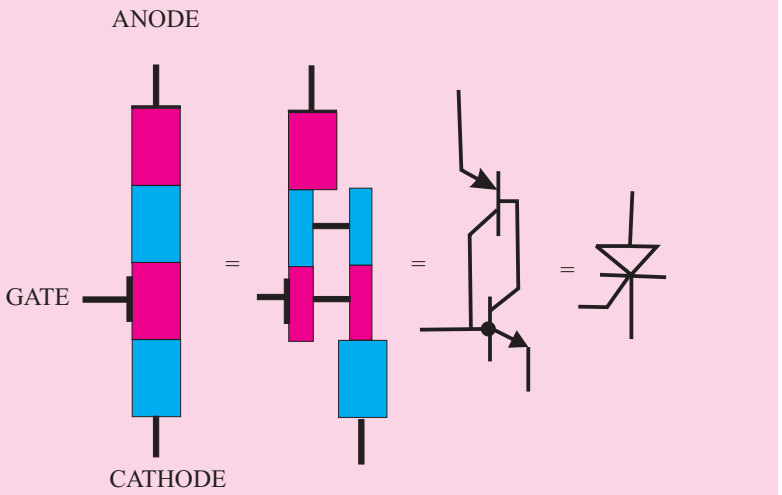
CONTENTS

CHAPTER 64

BREAKDOWN DEVICES

Learning Objectives

- What are Breakdown Devices ?
- Unijunction Transistor
- UJT Relaxation Oscillator
- Programmable UJT(PUT)
- Silicon Controlled Rectifier
- Comparison between Transistors and Thyristors
- Transient Effects in an SCR
- Phase Control
- Theft Alarm
- Emergency Lighting System
- Light Activated SCR (LASCR)
- The Shockley Diode
- Triac
- Diac
- Silicon Controlled Switch (SCS)



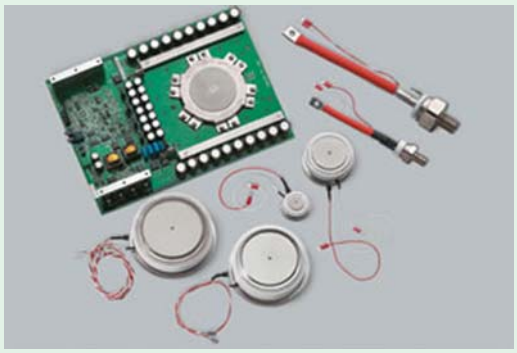
The Silicon Controlled Rectifier, usually referred to as an SCR, is one of the family of semiconductors that includes transistors and diodes

CONTENTS

64.1. What are Breakdown Devices ?

These are solid-state devices whose working depends on the ***phenomenon of avalanche breakdown.*** They are sometimes referred to by the generic name of ***thyristor*** which is a semiconductor switch whose bistable action depends on *P-N-P-N* regenerative feedback. We will discuss the following devices :

1. Unijunction Transistor (UJT).
2. Silicon Controlled Rectifier (SCR).
3. Light Activated SCR (LASCR).
4. Triac (short for 'triode ac').
5. Diac (short for 'diode ac').
6. Silicon Controlled Switch (SCS).



Cat-Transistor

These devices have two or more junctions and can be switched ON or OFF at ***an extremely fast rate.*** They are also referred to as **latching** devices. A latch is a kind of switch which initially once closed, remains closed until someone opens it.

64.2. Unijunction Transistor

Basically, it is a three-terminal silicon diode. As its name indicates, it has only one *P-N* junction. It differs from an ordinary diode in that it has **three** leads and it differs from a FET in that it has **no ability to amplify**. However, it has the ability ***to control a large ac power with a small signal.*** It also exhibits a negative resistance characteristic which makes it useful as an oscillator.

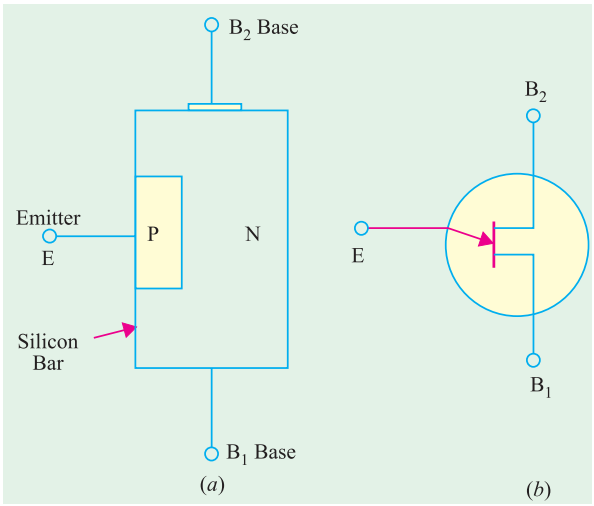


Fig. 64.1

(a) Construction

It consists of a ***lightly-doped silicon*** bar with a heavily-doped *P*-type material alloyed to its one side (closer to B_2) for producing single *P-N* junction. As shown in Fig. 64.1 (*a*), there are three terminals : one emitter, *E* and two bases B_2 and B_1 at the top and bottom of the silicon bar. The emitter leg is drawn at an angle to the vertical and arrow points in the direction of ***conventional*** current when UJT is in the conducting state.

(b) Interbase Resistance (R_{BB})

It is the resistance between B_2 and B_1 *i.e.* it is the total resistance of the silicon bar from one end to the other with emitter terminal open [Fig. 64.2 (*a*)].

From the equivalent circuit of Fig. 64.2 (*b*), it is seen that $R_{BB} = R_{B2} + R_{B1}$

It should also be noted that point *A* is such that $R_{B1} > R_{B2}$. Usually, R_{B1} = 60% of R_{B1}. The resistance R_{B1} has been shown as a variable resistor because its value varies inversely as I_E.

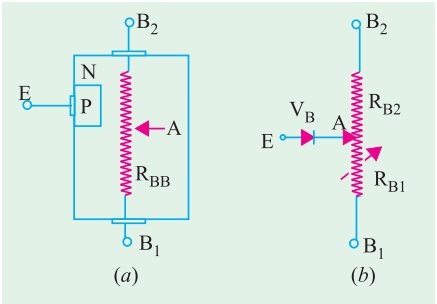


Fig. 64.2

(c) Intrinsic Stand-off Ratio

As seen from Fig. 64.3 (*a*), when a battery of 30 V is applied across $B_2 B_1$, there is a progressive fall of voltage over R_{BB} provided E is open. It is obvious from Fig. 64.3 (*b*) that emitter acts as a voltage-divider tap on fixed resistance R_{BB}.

With emitter open, $I_1 = I_2$, the interbase current is given by Ohm's Law.

$$I_1 = I_2 = V_{BB}/R_{BB}$$

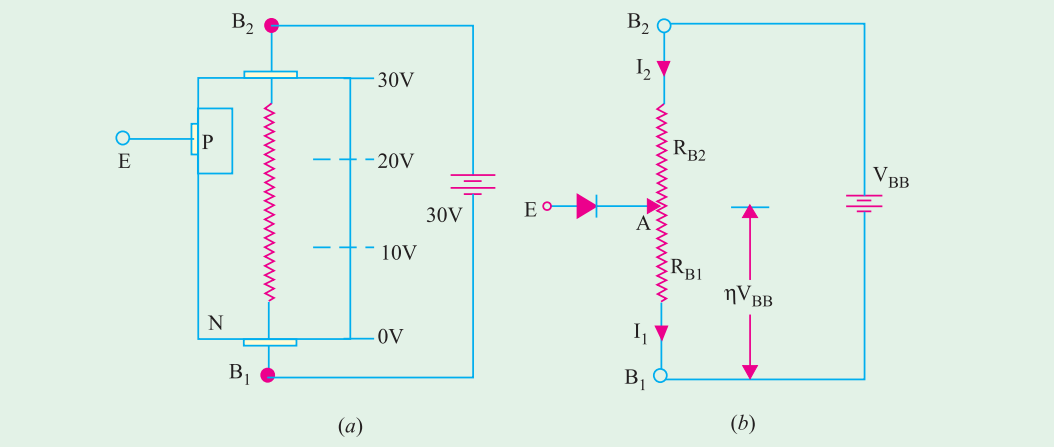


Fig. 64.3

For example, if $V_{BB} = 30$ V and $R_{BB} = 15$ K, $I_1 = I_2 = 2$ mA.

It may be noted that part of V_{BB} is dropped over R_{B2} and part on R_{B1}. Let us call the voltage drop across R_{B1} as V_A. Using simple voltage divider relationship,

$$V_A = V_{BB} \frac{R_{B1}}{R_{B1} + R_{B2}}$$

The voltage division factor is given a special symbol (η) and the name of **'intrinsic stand-off ratio'**.

$$\eta = \frac{R_{B1}}{R_{B1} + R_{B2}} \quad \therefore \quad V_A = \eta V_{BB}$$

The intrinsic stand-off ratio is the property of the UJT and is always less than unity (0.5 to 0.85). If $V_{BB} = 30$ V and $\eta = 0.6$, then potential of point A with respect to point $B_1 = 0.6 \times 30 = 18$ V. The remaining 12 V drop across R_{B2}.

(d) Operation

When V_{BB} is switched on, V_A is developed and ***reverse-biases the junction.*** If V_B is the barrier voltage of the P-N junction, then total reverse bias voltage is

$$= V_A + V_B = \eta V_{BB} + V_B$$

Value of V_B for Si is 0.7 V.

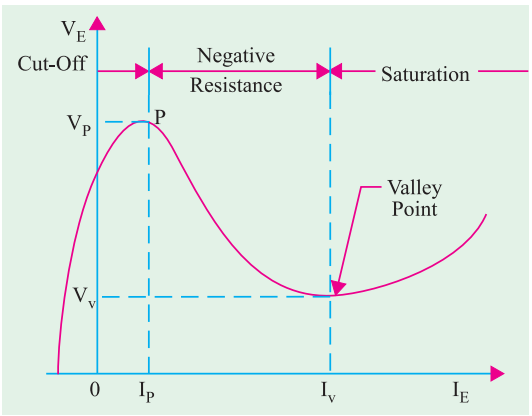


Fig. 64.4

It is obvious that emitter junction will not become forward-biased unless its applied voltage V_E exceeds ($\eta V_{BB} + V_B$). This value of V_E is called ***peak-point voltage*** V_P (Fig. 64.4). When $V_E = V_P$, emitter (peak current), I_P starts to flow through R_{B1} to ground (*i.e.* B_1). The UJT is then said to have been ***fired*** or turned ON. Due to the flow of I_E ($= I_P$) through R_{B1}, number of charge carriers in R_{B1} is increased which ***reduces its resistance.*** As η depends on R_{B1}, ***its value is also decreased.***

Hence, we find that as V_E and hence I_E increases (beyond I_P), R_{B1} decreases, η decreases and V_A decreases. This decrease in V_A causes more emitter current to flow which causes a further reduction in R_{B1}, η and V_A. Obviously, the process is regenerative. V_A as well as V_E quickly drop as I_E increases. Since, V_E decreases when I_E increases, the UJT ***possesses negative resistance.*** Beyond the valley point, UJT is in saturation and V_E increases very little with an increasing I_E.

It is seen that only terminals E and B_1 are the active terminals whereas B_2 is the bias terminal *i.e.* it is meant only for applying external voltage across the UJT.

Generally, UJT is triggered into conduction by applying a suitable positive pulse at its emitter. It can be brought back to OFF state by applying a negative trigger pulse.

(e) Applications

One unique property of UJT is that it can be triggered by (or an output can be taken from) ***any one of its three terminals.*** Once triggered, the emitter current I_E of the UJT increases regeneratively till it reaches a limiting value determined by the external power supply. Because of this particular behaviour, UJT is used in a variety of circuit applications. Some of which are :

1. phase control
2. switching
3. pulse generation,
4. sine wave generator
5. sawtooth generator
6. timing and trigger circuits,
7. voltage or current regulated supplies.

Example 64.1. *A given silicon UJT has an interbase resistance of 10 K. It has R_{B1} = 6 K with I_E = 0. Find*

(a) *UJT current if V_{BB} = 20 V and V_E is less than V_P,*

(b) *η and V_A,* **(c)** *peak point voltage, V_P.*

(Applied Electronics-I, Punjab Univ. 1990)

Solution. (a) Since $V_E < V_P$, $I_E = 0$, because P-N junction is reverse-biased.

$$\therefore \quad I_1 = I_2 = \frac{V_{BB}}{R_{BB}} = \frac{20}{10K} = \mathbf{2mA}$$

(b) $$\eta = \frac{R_{B1}}{R_{BB}} = \frac{6}{10} = \mathbf{0.6;}\ V_A = \eta V_{BB} = 0.6 \times 20 = \mathbf{12V}$$

(c) $$V_P = \eta V_{BB} + V_B = 12 + 0.7 = \mathbf{12.7\ V}$$

64.3. UJT Relaxation Oscillator

The relaxation oscillator shown in Fig. 64.5 consists of a UJT and a capacitor C which is charged through R as V_{BB} is switched on.

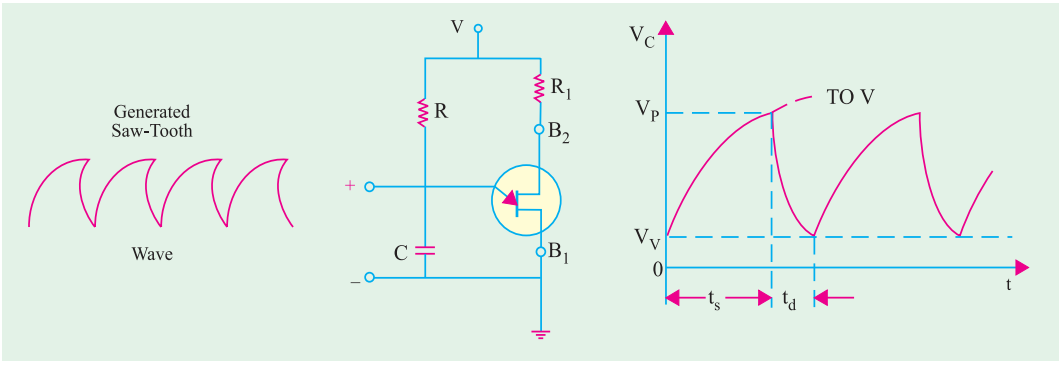


Fig. 64.5

When the capacitor voltage V_C reaches in time t_s the value of V_P, the UJT fires and rapidly discharges C via B_1 till the voltage falls below the minimum value V_V. The device then cuts off and

C starts to charge again. This cycle is repeated continuously thus generating a sawtooth waveform across C.

The inclusion of external resistances R_2 and R_1 in series with B_2 and B_1 (Fig. 64.6) provides ***spike waveforms.*** When the UJT fires, the sudden surge of current through B_1 causes a drop across R_1 which produces positive going spikes. Also, at the time of firing, fall of V_{BB1} causes I_2 to increase rapidly which generates negative going spikes across R_2 as shown in Fig. 64.6.

By switching over to different capacitors, frequency of the output waveform can be changed as desired.

Condition for Turn-ON and Turn-OFF

For satisfactory working of the above oscillator, following two conditions for the turn-on and turn-off of the UJT must be met. To ensure turn-on, R must not limit I_E at peak point to a value less than I_P. It means that

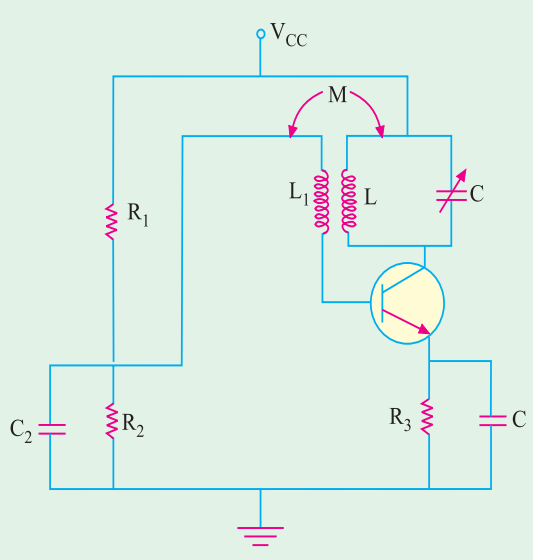


Fig. 64.6

$$V_{BB} - V_P > I_P R \text{ or } R < \frac{V_{BB} - V_P}{I_P}$$

To ensure turn-off of the UJT at valley point, R must be large enough to permit I_E (at valley point) to decrease below the specified value of I_V. In other words, drop across R at valley point must be less than IVR. Hence, condition for turn-off is

$$V_{BB} - V_V < I_V R \text{ or } R > \frac{V_{BB} - V_V}{I_v}$$

Hence, for reliable turn-on and turn-off of the UJT, R must be in the range

$$\frac{V_{BB} - V_P}{I_P} > R > \frac{V_{BB} - V_V}{I_V}$$

It should be noted that charging time constant of the capacitor for voltage V is $T = CR$ whereas discharging time constant is $T_d = CR_{B1}$. The time required to charge upto V_P (called ramp rise time) is $t_s = T \log_e (V - V_V) / (V - V_P)$.

Similarly, time required by the capacitor to discharge from V_P to V_V is $t_d \cong T_d \log_e V_P / V_V$. The frequency of oscillation is given by $f = 1/(t_s + t_d)$.

Example 64.2. *The windshield wiper motor of an automobile is controlled by a UJT with* $\eta = 0.6$. *The capacitor has a value of 50 μF and the charging resistor is a series combination of 50 K resistor and a 500 K potentiometer. Determine the minimum and maximum number of blade strokes per minute possible with this arrangement.*

Solution. The least value of time constant is = $50{,}000 \times 50 \times 10^{-6} = 2.5$ second.

Maximum value of time constant when whole of potentiometer resistance is used is

$$= (50 + 500) \times 10^3 \times 50 \times 10^{-6} = 27.5 \text{ second}$$

Maximum blade strokes per minute= 60/2.5 = 24.

Minimum blade strokes per minute = 60/27.5 = **2.2.**

Example 64.3. *The oscillator circuit shown in Fig. 64.5 uses a UJT with* $R_{BB} = 10$ *K,* $\eta = 0.6$, $V_B = 0.7$ *V,* $V = 50$ *V,* $R_1 = 90$ *K,* $R = 100$ *K and* $C = 0.05$ *μF. When UJT is in conduction,* $R_{B1} = 10\ \Omega$ *and* $V_V = V_B$. *Find* ***(i)*** *ramp rise time,* t_s ***(ii)*** *approximate discharge time,* t_d *and* ***(iii)*** *frequency of oscillation.*

Solution. $V_{BB} = V.R_{BB}/(R_{BB} + R_1) = 50 \times 10/100 = 5$ V

$V_P = \eta V_{BB} + V_B = 0.6 \times 5 + 0.7 = 3.7$ V

$T = CR = 0.05 \times 10^{-6} \times 100 \times 10^3 = 5$ ms

(i) The capacitor charges from $Vc = V_B = 0.7$ V to V_P towards V in the time t_s given by $t_s = T \log_e (V - V_V)/(V - V_P) = 5 \log_e (50 - 0.7)/(50 - 3.7) =$ **0.315 ms.**

(ii) The discharge time, $t_d = CR_{B1} = 0.05 \times 10 = 0.5$ µs. The time taken by C to discharge from V_P to V_V (= 0.7 V) towards 0 volt is

$$T_d \cong t_d \log_e V_P/V_V = 0.5 \log_e 3.7 / 0.7 = \textbf{1.66 µs}$$

(iii) $f = 1/(t_s + t_d) = 1/(0.315 + 0.0016) =$ **3.158 kHz.**

64.4. Programmable UJT (PUT)

Like a SCR, it is also a four-layer or *PNPN* device with a gate G as shown in Fig. 64.7. However, its gate is connected to the *N*-region adjacent to the anode A. This *P-N* junction controls the ON and OFF states of the *PUT*. The gate G is always biased positive with respect to cathode K. When anode voltage exceeds gate voltage by about 0.7 V, the *P-N* junction J_1 becomes forward-biased and the *PUT* turns ON. When the anode voltage falls below this level, the PUT is turned OFF.

As shown in Fig. 64.8 (*a*), gate bias can be adjusted to any bias level with the help of an external voltage divider circuit $R_2 - R_3$. Whenever anode voltage exceeds this **programmable level**, the *PUT* turns ON.

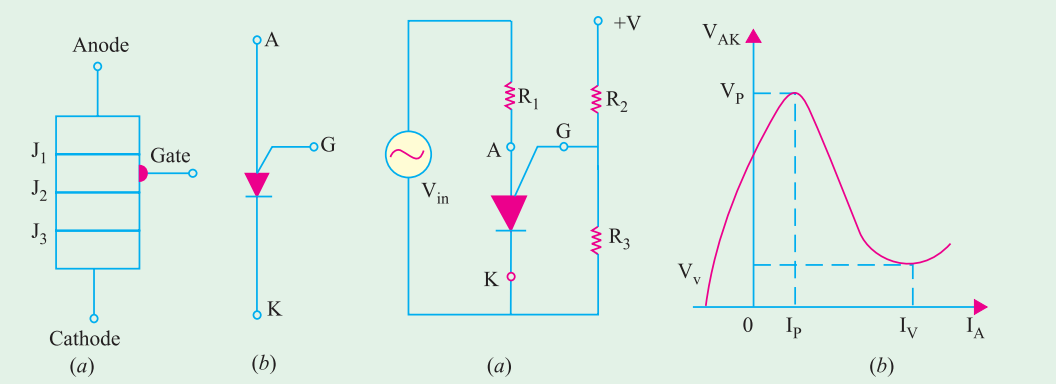


Fig. 64.7 **Fig. 64.8**

Fig. 64.8 (*b*) shows the plot of anode-to- cathode voltage V_{AK} versus anode current I_A. It is similar to the *V/I* characteristic of a *UJT*. Hence, *PUT* replaces *UJT* in many applications, one such application is as relaxation oscillator shown in Fig. 64.9 (*a*).

Since, $R_2 = R_3$, $V_G = 12/2 = 6$ V. When dc voltage is applied, the *PUT* is off but C starts charging towards +12 V through R_1 [Fig. 64.9 (*b*)]. When V_C exceeds (V_G + 0.7 V), the *PUT* turns ON and, at the same time, C starts discharging rapidly through the low ON-resistance of the *PUT* and R_4. Consequently, a voltage spike is developed across R_4 during the discharge. As soon as C discharges, the *PUT* turns OFF and the charging cycle starts all over again as described above.



Silicon controlled rectifier and power diode

64.5. Silicon Controlled Rectifier

It is one of the prominent members of the thyristor family. It is a four-layer or PNPN device. Basically, ***it is a rectifier with a control element.*** In fact, it consists of ***three diodes*** connected back-to-back with a gate connection. It is widely used as a switching device in power control applications. It can control loads by switching current OFF and ON up to many thousand times a second. It can switch ON for variable lengths of time, thereby delivering selected amount of power to the load. Hence, it possesses the advantages of a rheostat and a switch with none of their disadvantages.

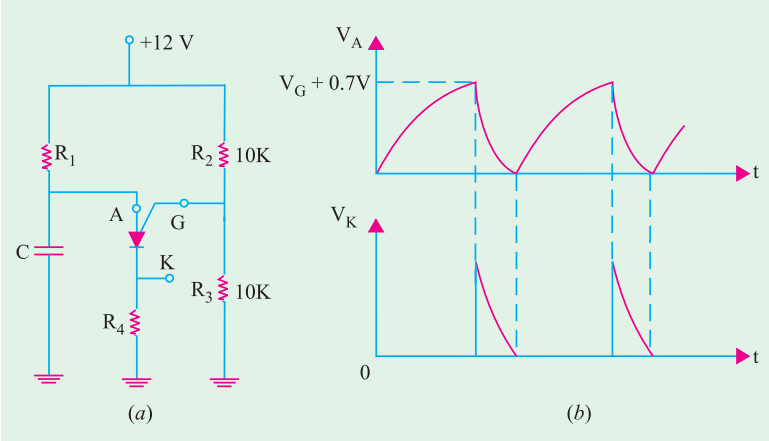


Fig. 64.9

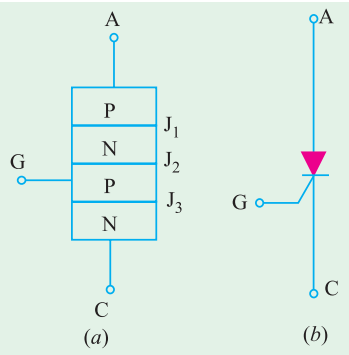


Fig. 64.10

(a) Construction

As shown in Fig. 64.10 (*a*), it is a three terminal four-layer transistor, the layers being alternately of *P*-type and *N*-type silicon. The three junctions are marked J_1, J_2 and J_3 whereas the three terminals are : anode (*A*), cathode (*C*) and gate (*G*) which is connected to the ***inner P***-type layer. The function of the gate is to control the firing of SCR. The schematic symbol is shown in Fig. 64.10 (*b*).

Since, they conduct large currents, junction areas of SCRs are very large. Commonly used stud-mounted units have their anode connected directly to the stud for good heat dissipation whereas larger units are of 'pillow' type in which many units are stacked in series and held in a pressurized clamp.

(b) Biasing

With the polarity of *V* as shown in Fig. 64.11 (*a*), the junctions J_1 and J_3 become forward-biased whereas J_2 is reverse-biased. Hence, no current (except leakage current) can flow through the SCR.

In Fig. 64.11 (*b*), polarity of V has been reversed. It is seen that, now, junctions J_1 and J_3 become reverse-biased and only J_2 is forward-biased. Again, there is no flow of current through the SCR.

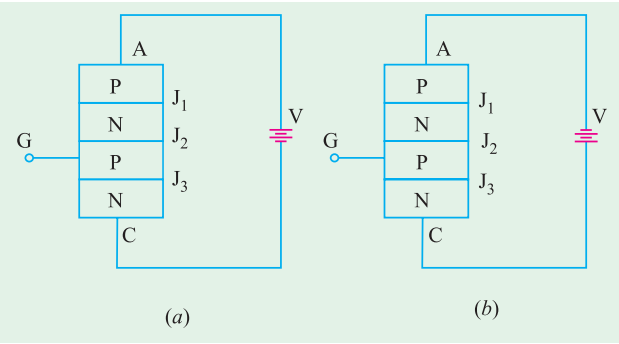


Fig. 64.11

(c) Operation

In Fig. 64.11 (*a*), current flow is blocked due to reverse-biased junction J_2. However, when anode voltage is increased, a certain critical value called forward breakover voltage V_{BO} is reached when J_2 breaks down and SCR switches suddenly to a highly conducting state. Under this condition,

SCR offers very little forward resistance (0.01 Ω – 1.0 Ω) so that voltage across it drops to a low value (about 1 V) as shown in Fig. 64.12 and current is limited only by the power supply and the load resistance. Current keeps flowing indefinitely until the circuit is opened briefly.

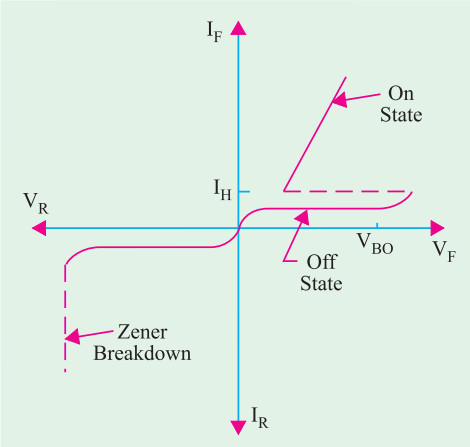


Fig. 64.12

With supply connection as in Fig. 64.11 (*b*), the current through the SCR is blocked by the two reverse-biased junctions J_1 and J_3. When V is increased, a stage comes when Zener breakdown occurs which may destroy the SCR (Fig. 64.12). Hence, it is seen that SCR is a unidirectional device unlike triac ***which is bi-directional.***

(*d*) Two Transistor Analogy

The basic operation of a SCR can be described by using two transistor analogy. For this purpose, SCR is split into two 3-layer transistor structures as shown in Fig. 64.13 (*a*). As seen, transistor Q_1 is a *PNP* transistor whereas Q_2 is an *NPN* device interconnected together. It will also be noted from Fig. 64.13 (*b*) that

(i) collector current of Q_1 is also the base current of Q_2 and

(ii) base current of Q_1 is also the collector current of Q_2.

Suppose that the supply voltage across terminals A and C is such that reverse-biased junction J_2 starts breaking down. Then, current through the device begins to rise. It means that I_{E1} begins to increase.

Then,

1. I_{C1} increases (remember $I_C = \alpha I_E$);
2. since $I_{C1} = I_{B2}$, I_{B2} also increases;
3. hence, I_{C2} increases (remember $I_C = \beta I_B$);
4. now, $I_{C2} = I_{B1}$, hence I_{B1} increases;
5. consequently, both I_{C1} and I_{E1} increase.

As seen, a regenerative action takes place whereby an initial increase in current produces further increase in the same current. Soon, maximum current is reached limited by external resistances. The two transistors are fully turned ON and voltage across the two transistors falls to a very low value. Typical turn-ON times for an SCR are 0.1 to 1.0 μs.

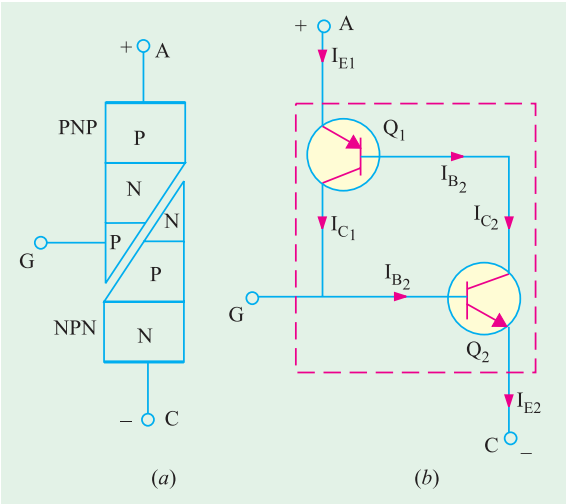


Fig. 64.13

It can be proved that if I_G is the gate current of the SCR and α_1 and α_2, the current gains of the *PNP* and *NPN* transistors respectively, then anode current is given by

$$I_A = \frac{\alpha_2 I_G}{1-(\alpha_1+\alpha_2)}$$

(*e*) Firing and Triggering

Usually, SCR is operated with an anode voltage ***slightly less*** than the forward breakover voltage V_{BO} and is triggered into conduction by a low-power gate pulse. Once switched ON, gate has no further control on the device current. Gate signals can be (*a*) dc firing signals [Fig. 64.14 (*a*)] or (*b*) pulse signals [Fig. 64.14 (*b*)].

In Fig. 64.14 (*a*) with *S* open, SCR does not conduct and the lamp is out. When *S* is closed momentarily, a positive voltage is applied to the gate which forward-biases the centre *P-N* junction. As a result, SCR is pulsed into conduction and the lamp lights up. SCR will remain in the conducting state until the supply voltage is removed or reversed. Fig. 64.14 (*b*) shows triggering by timed pulses obtained from a pulse source.

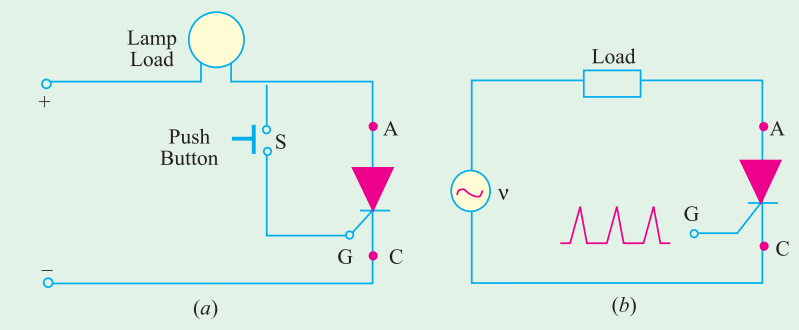


Fig. 64.14

We have discussed above the most common method of SCR triggering *i.e.* gate triggering. However, other available triggering methods are as under :

1. Thermal Triggering

In this case, the temperature of the forward-biased junction is increased till the reverse-biased junction breaks down.

2. Radiation Triggering

Here, triggering is achieved with the help of charge carriers which are produced by the bombardment of the SCR with external high-energy particles like neutrons or protons.

3. Voltage Triggering

In this case, the voltage applied across the anode and cathode of the SCR is increased which decreases the width of the depletion layer at the reverse-biased junction leading to its collapse.

4. *dv/dt* Triggering

In this case, dv / dt is made more than the value of the critical rate of rise of the voltage.

(*f*) Turning OFF

As stated earlier, once 'fired', SCR remains ON ***even when triggering pulse is removed.*** This ability of the SCR to remain ON even when gate current is removed is referred to as **latching**. In fact, SCR belongs to a class of devices known as ***latching devices*****.**

By now, it is clear that an SCR cannot be turned OFF by simply removing the gate pulse. Number of techniques are employed to turn an SCR off. These are :

1. anode current interruption.
2. reversing polarity of anode-cathode voltage as is done each half-cycle by v in Fig. 64.14 (*b*);
3. reducing current through SCR below the holding current I_H (Fig. 64.12). It is also called ***low-current dropout.***

(*g*) Applications

Main application of an SCR is as a ***power control device*****.** It has been shown above that when SCR is OFF, its current is negligible and when it is ON, its voltage is negligible. Consequently, it never dissipates any appreciable amount of power even when controlling substantial amounts of load power. For example, one SCR requires only 150 mA to control a load current of 2500 A. Other common areas of its application include.

1. relay controls,
2. regulated power supplies,
3. static switches,
4. motor controls,
5. inverters,
6. battery chargers,
7. heater controls,
8. phase control.

SCRs have been designed to control powers upto 10 MW with individual ratings as high as 2000 A at 1.8 kV. Its frequency range of application has been extended to about 50 kHz.

Example 64.4. *The two-transistor analogy of an SCR has the following data :*
gain of PNP transistor = 0.4 ; *gain of NPN transistor = 0.5 ;*
gate current = 50 mA.
Calculate the anode current of the device.

Solution. Here, $\alpha_1 = 0.4$; $\alpha_2 = 0.5$ and $I_G = 50\text{ mA} = 0.05\text{ A}$

$$\text{anode current, } I_A = \frac{\alpha_2 I_G}{1-(\alpha_1+\alpha_2)} = \frac{0.5\times 0.05}{1-(0.4+0.5)} = 0.25\text{ A} = \mathbf{250\text{ mA}}$$

64.6. Comparison Between Transistors and Thyristors

Table No. 64.1 gives the comparison between transistors and thyristors.

Table No. 64.1

Sl.No.	Transistors	Thyristors
1.	3-layers, 2-junction devices	4-layer, 2- or more junction devices
2.	fast response	very fast response
3.	high efficiency	very high efficiency
4.	highly reliable	very highly reliable
5.	small voltage drop	very small voltage drop
6.	long life	very long life
7.	small to medium power ratings	very small to very large power ratings
8.	require a continuous flow of current to remaIn in conducting state	require only a small pulse for triggering and thereafter remaining in conducting state.
9.	low power consumption	very low power consumption
10.	low control capability	high control capability
11.	small turn-ON and turn-OFF time	very small turn-ON and turn-OFF time

Example 64.5. *A 250 Ω resistor is connected in series with the gate of an SCR as shown in Fig. 64.15. The gate current required for firing the SCR is 8 mA. Calculate the value of the input voltage V_{in} required for causing the SCR to break down.*
(Basic Electronics, Osmania Univ. 1993)

Solution. The value of V_{in} should be such as to ***(i)*** overcome the barrier voltage of 0.7 *V* and ***(ii)*** cause 8 mA current to flow through 250 Ω resistor.

$$V_{in} = V_{GC} + I_G R = 0.7 + 8\times 10^{-3}\times 250 = \mathbf{2.7\text{ V}}$$

64.7. Transient Effects in an SCR

We will consider the following two effects :

***(i) di/dt* Effect**

This effect is produced due to a high initial rate-of-rise of the anode current when an SCR is just switched ON and results in the formation of a ***local hot spot near the gate connection*** as explained below :

When a trigerring pulse is applied to the gate of an SCR, the holes are injected into the P-region where they crowd together and form an initial conduction zone over a small part of the junction J_2 before spreading the conduct throughout the whole area of junction. If the anode current is allowed to rise very rapidly (as would be the case for resistive or capacitive loads), this high current will be forced to flow through this small conduction zone until the conduction has spread through the entire junction. This may result in local hot-spots in the junction which are likely to damage the SCR permanently.

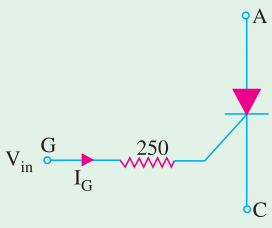


Fig. 64.15

The maximum allowable anode current *di/dt* can be increased (and, hence, turn-on time of an SCR decreased) by using specially-designed gate-connection geometries which result in a more rapid distribution of charge throughout the gate region.

(*ii*) *dv/dt* Effect

It is found that sometimes an SCR unwantedly turns ON by itself during sudden changes of the applied anode potential at a time when there is no gate current applied and the SCR is supposed to be blocking. This false triggering is due to the capacitance possessed by the large-area junction J_2 (Fig. 64.10). When rate-of-rise of the applied anode voltage *dv/dt* is very high, the capacitive charging current may become high enough to initiate switch-on even in the absence of external gate current. False triggerings due to the *dv/dt* are prevented by using a 'snubber circuit'.

64.8. Phase Control

In the phase control circuit of Fig. 64.16, gate triggering current is derived from the supply itself. The variable resistance *R* limits the gate current during positive half-cycles of the supply. If *R* is adjusted to a low value, SCR will trigger almost immediately at the commencement of the positive half-cycle of the input.

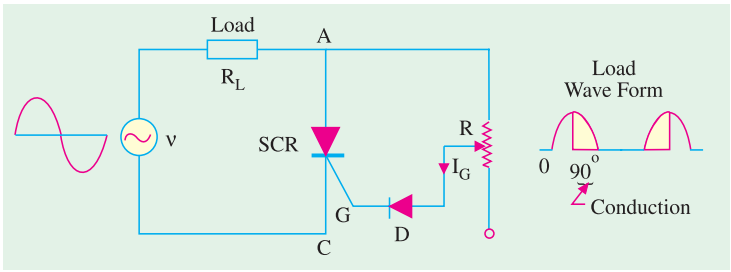


Fig. 64.16

If, on the other hand, *R* is set to a high resistance, SCR may not switch ON until the peak of the positive half-cycle. By adjusting *R* between these two extremes, SCR can be made to switch ON somewhere between the commencement and peak of the positive half-cycle *i.e.* between 0° and 90°.

It is obvious that if I_G is not enough to trigger the SCR at 90°, then the device will not trigger at all because I_G has maximum value then. This operation is sometimes referred to as ***half-wave variable-resistance phase control.*** It is an effective method of controlling the load power.

The purpose of diode *D* is to protect the gate from negative voltage which would otherwise be applied to it during the negative half-cycle of the input.

It is seen from Fig. 64.16 that at the instant of SCR switch-ON, gate current flows through R_L, *R* and *D*. Hence, at that ***instant***

$$v = I_G R_L + I_G R + V_D + V_G \qquad \therefore \quad R = \frac{V - V_D - V_G - I_G R_L}{I_G}$$

Example 64.6. *The circuit of Fig. 64.16 is connected to an ac supply v = 50 sin θ and R_L = 50 Ω. Gate current is 100 μA and V_G = 0.5 V. Determine the range of adjustment of R for the SCR to be triggered between 30° and 90°. Take V_D = 0.7 V.*

Solution. ***(i)*** $\theta = 30°$

Now, $v = 50 \sin \theta = 50 \sin 30° = 25$ V

$$\therefore \quad R = \frac{25 - 0.7 - 0.5 - (100 \times 10^{-6} \times 50)}{100 \times 10^{-6}} = \mathbf{238K}$$

(ii) $\theta = 90°$

$v = 50 \sin 90° = 50$ V

$$R = \frac{50 - 0.7 - 0.5 - 0.005}{100 \times 10^{-6}} = \mathbf{488K}$$

64.9. Theft Alarm

The circuit shown in Fig. 64.17 can be used to protect a car tape deck or a radio receiver from theft. The switch *S* is located at some concealed point in the car and is kept *closed*. Since gate *G* is grounded through the tape deck, the SCR is OFF and the horn is silent. If the tape deck is removed, *G* is no longer grounded. Instead, it gets connected to the car battery through *R*. Consequently, gate current is set up which fires the SCR. As a result, the horn starts blowing and continues to do so until *S* is opened.

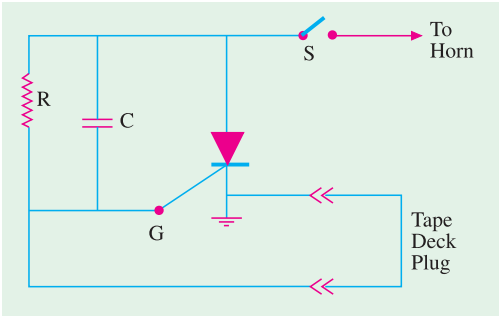


Fig. 64.17

64.10. Emergency Lighting System

SCRs find application in circuits that maintain lighting by using a backup battery in case of ac power failure. Fig. 64.18 shows a centre-tapped full-wave rectifier used for providing power to a low-voltage lamp. So long as ac power is available, the battery is charged *via* diode D_3 and resistor R_1 [Fig. 64.18 (*a*)].

With ac power ON, the capacitor *C* charges to the peak value of the full-wave rectified ac voltage *i.e.* to 12.4 × 1.414 = 17.5 V. Same is the voltage of the SCR cathode K.

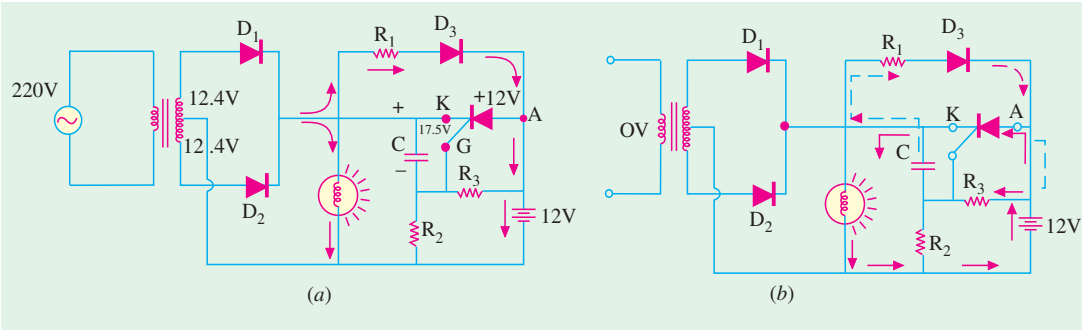


Fig. 64.18

Since voltage of SCR anode *A* is less than that of *K*, the SCR does not conduct. The SCR gate *G* is at a voltage determined by voltage divider R_3– R_2. Under these conditions, the lamp is run by the ac supply and SCR is OFF.

When ac power is interrupted :

(i) the capacitor C discharges through the closed path R_1, D_3 and R_3 shown by dotted arrows;

(ii) the cathode voltage decreases thereby making it less positive than anode;

(iii) this triggers SCR into conduction which allows the battery current to pass through the lamp thus maintaining illumination.

When ac supply is restored, *C* recharges and the SCR turns OFF. The battery starts recharging again.

64.11. Light Activated SCR (LASCR)

It is just an ordinary SCR except that ***it can also be light-triggered.*** Most LASCRs also have a gate terminal for being triggered by an electrical pulse just as a convential SCR. Fig. 64.19 shows the two LASCR symbols used commonly.

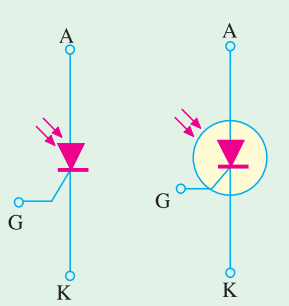


Fig. 64.19

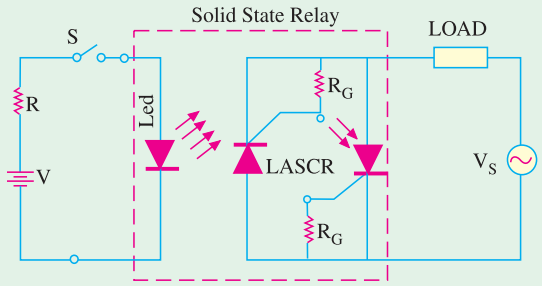
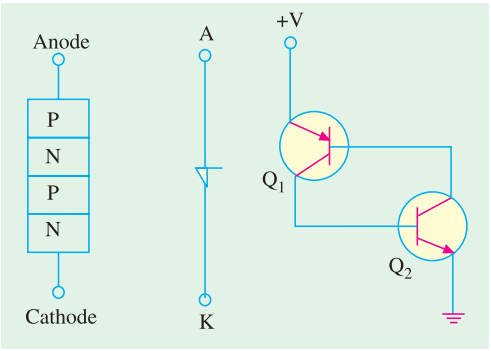


Fig. 64.20

LASCRs are manufactured mostly in relatively low-current ranges and are used for triggering larger SCRs and triacs. They are used in optical light controls, relays, motor control and a variety of computer applications. Some LASCRs have clear windows in their cases so that light sources from other devices can be coupled to them. Many have the light source device encapsulated in the same package so that a relay is formed. Since the relay action does not require direct electrical connection, such relays are often used to couple signals into very high voltage equipment and other dangerous locations. Fig. 64.20 shows the connection of such a solid-state relay. Two LASCRs are connected in reverse parallel in order to obtain conduction in both half-cycles of the applied ac voltage V_S. A single LED is used to trigger both LASCRs. Bias resistors are used to reduce the light sensitivity of the gates and prevent sporadic triggering during off-periods. Usually, all the three active devices and the two bias resistors R_G are encapsulated in the same package.

64.12. The Shockley Diode*

It is a ***two-terminal four-layer*** or *PNPN* device as shown in Fig. 64.21 along with its schematic symbol. It is essentially a low-current ***SCR without a gate***. For switching the diode ON, its anode-to-cathode voltage **(V_{AK})** must be increased to forward switching voltage **(V_S)** which is the equivalent of SCR forward breakover voltage. Like an SCR, it also has a holding current. The *PNPN* structure can be represented by an equivalent circuit consisting of a *PNP* transistor and an *NPN* transistor. One application of the diode is as a relaxation oscillator.



Fvvvig. 64.21

64.13. Triac

It is a ***5-layer bi-directional device*** which can be triggered into conduction by both *positive* and ***negative*** voltages at its anodes and with both ***positive and negative*** triggering pulses at its gate. It behaves like two SCRs ***connected in parallel, upside down with respect to each other.*** That is, the anode of one is tied to the cathode of the other and their gates are directly tied together. Hence, anode and gate voltages applied in either direction will fire a triac because they would fire at least one of the two SCRs which are in opposite directions.

Since a triac responds to both positive and negative voltages at the anode, the concept of cathode used for an SCR is dropped. Instead, the two electrodes are called anodes ***A_1*** and ***A_2***.

* After the name of its inventor William Shockley.

1. Construction

As shown in Fig. 64.22 (*a*), a triac has three terminals A_1, A_2 and G. As seen, gate G is closer to anode A_1. It is clear from Fig. 64.22 (*b*), that a triac is nothing but ***two inverse parallel-connected***

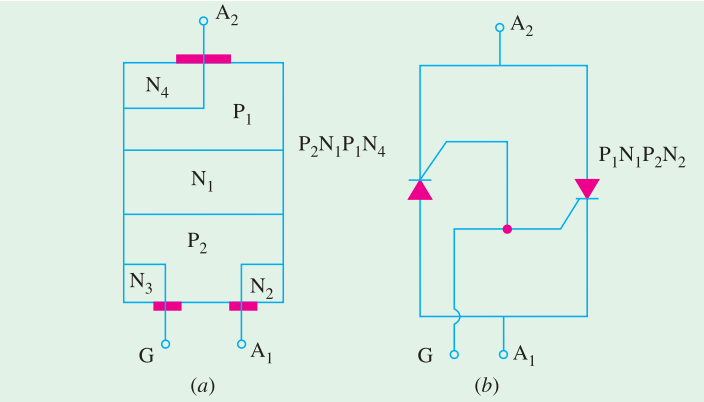


Fig. 64.22

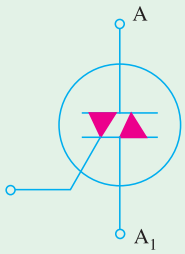


Fig. 64.23

SCRs with a ***common gate terminal.*** As seen, it has six doped regions. Fig. 64.23 shows the schematic symbol which consists of two inverse-connected SCR symbols.

2. Operation

(*a*) When A_2 is Positive

When positive voltage is applied to A_2, path of current flow is P_1-N_1-P_2-N_2. The two junctions P_1-N_1 and P_2 - N_2 are forward-biased whereas N_1-P_2 junction is blocked. The gate can be given either positive or negative voltage to turn ON the triac as explained below.

(*i*) positive gate

A positive gate (with respect to A_1) forward-biases the P_2-N_2 junction and the breakdown occurs as in a normal SCR.

(*ii*) negative gate

A negative gate forward-biases the P_2-N_3 junction and current carriers injected into P_2 turn on the triac.

(*b*) When A_1 is Positive

When positive voltage is applied to anode A_1, path of current flow is P_2-N_1-P_1-N_4. The two junctions P_2-N_1 and P_1-N_4 are forward-biased whereas junction N_1-P_1 is blocked. Conduction can be achieved by giving either positive or negative voltage to G as explained below.

(*i*) positive gate

A positive gate (with respect to A_1) injects current carriers by forward-biasing P_2-N_2 junction and thus initiates conduction.

(*ii*) negative gate

A negative gate injects current carriers by forward-biasing P_2-N_3 junction thereby triggering conduction.

It is seen that there are four triac-triggering modes, two each for the two anodes.

Low-current dropout is the only way to open a triac.

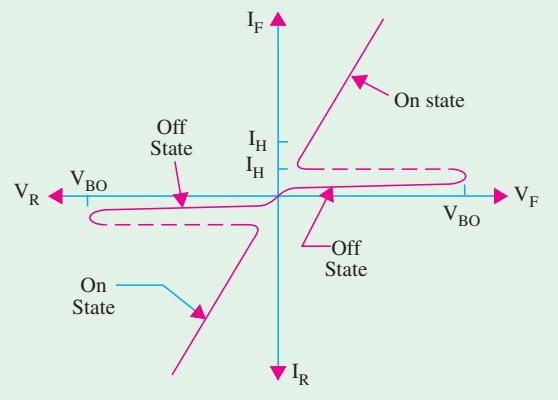


Fig. 64.24

3. V/I Characteristics

Typical characteristics of a triac are shown in Fig. 64.24.

As seen, triac exhibits same forward blocking and forward conducting characteristics as an SCR but *for **either polarity of voltage applied to the main terminal.*** Obviously, a triac has latch current in either direction.

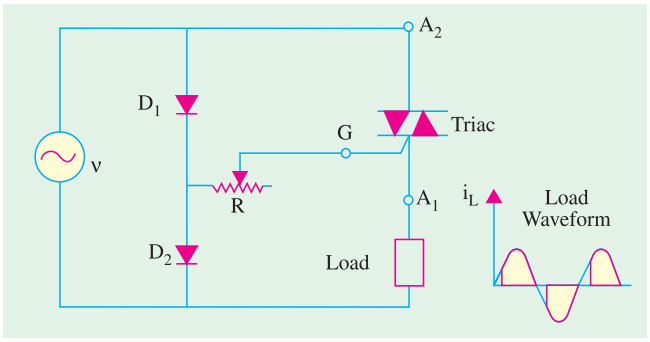


Fig. 64.25

4. Applications

One fundamental application of triac is shown in Fig. 64.25. Here, it is used to control ac power to a load by switching ON and OFF during positive and negative half-cycles of the input ac power.

During positive half-cycle of the input, diode D_1 is forward biased, D_2 is reverse-biased and gate is positive with respect to A_1. By adjusting R, the point at which conduction commences can be varied.

Diac-triac combination for ac load power control is shown in Fig. 64.26. Firing control of diac is achieved by adjusting R.

Other applications of a triac include.

1. as static switch to turn ac power OFF and ON;
2. for minimizing radio interference;
3. for light control;
4. for motor speed control etc.

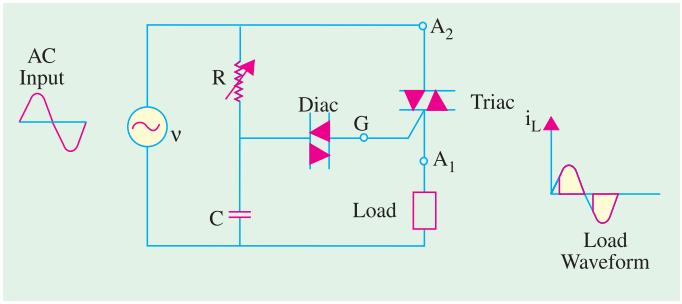


Fig. 64.26

The only disadvantage of triac is that it takes comparatively longer time to recover to OFF state. Hence, its use is limited to ac supply frequencies of upto 400 Hz.

64.14. Diac

To put it simply, a diac is nothing else but a ***triac without its gate terminal*** as shown in Fig.

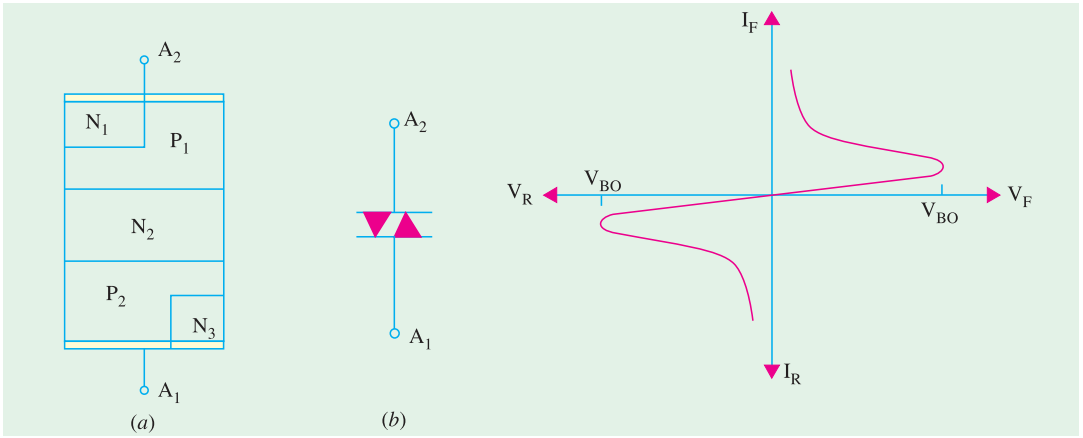


Fig. 64.27

Fig. 64.28

64.27 (*a*). Its equivalent circuit is a pair of inverted four layer diodes. Its schematic symbol is shown in Fig. 64.27 (*b*). As seen, ***it can break down in either direction.***

When anode A_1 is positive, the current path is P_2-N_2-P_1-N_1. Similarly, when A_2 is positive, the current flow path is P_1-N_2-P_2-N_3. Diac is designed to trigger triacs or provide protection against over-voltages.

The operation of a diac can best be explained by imaging it as **two-diodes connected in series**. Voltage applied across it in either direction turns ON one diode, reverse-biasing the other. Hence, it can be switched from OFF to ON state for either polarity of the applied voltage.

The characteristic curve of a typical diac is shown in Fig. 64.28. It resembles the letter Z since diac breaks down in either direction.

As stated above, diac has symmetrical bi-directional switching characteristics. Because of this feature, diacs are frequently used as triggering devices in triac phase control circuits used for light dimming, universal motor speed control and heat control etc.

64.15. Silicon Controlled Switch (SCS)

It is a four-layer, four-terminal *PNPN* device having anode *A*, cathode *C*, anode gate G_1 and cathode gate G_2 as shown in Fig. 64.29. In fact, ***it is a low-current SCR with two gate terminals.*** The two transistor equivalent circuit is shown in Fig. 64.30.

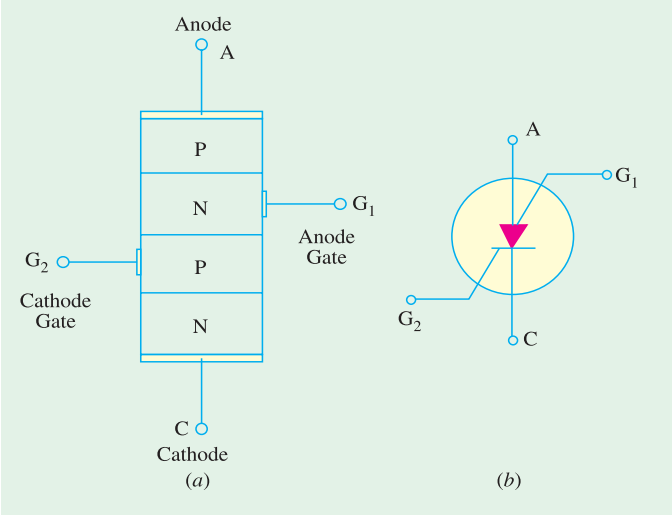


Fig. 64.29

Switching ON and OFF

The device may be switched ON or OFF by a suitable pulse is applied at either gate. As seen from Fig. 64.30, a negative pulse is required at anode gate G_1 to turn the device ON whereas positive pulse is needed to turn it OFF as explained below.

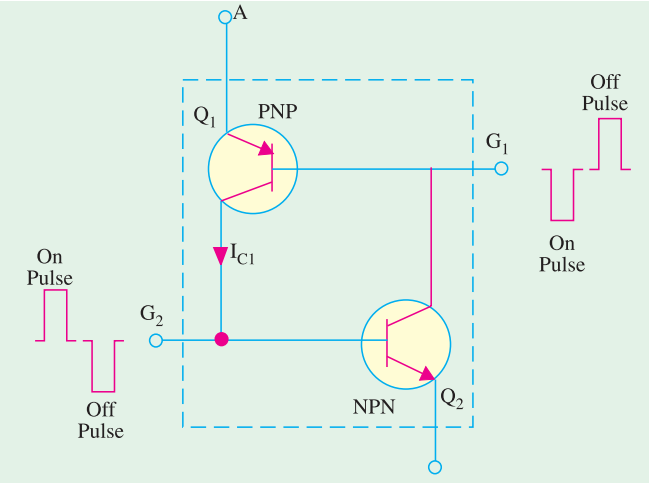


Fig. 64.30

Similarly, at cathode gate G_2, a negative pulse is required to switch the device OFF and a positive pulse to turn it ON.

As seen from Fig. 64.30, when a negative pulse is applied to G_1, it forward-biases Q_1 (being *PNP*) which is turned ON. The resulting heavy collector current I_C, being the base current of Q_2, turns it ON. Hence, SCS is switched ON. A positive pulse at G_1 will reverse bias E/B junction of Q_1 thereby switching the SCS OFF.

V/I Characteristics

The V/I characteristics of an SCS are essentially the same **as those for the SCR** (Fig. 64.13).

As compared to an SCR, an SCS has ***much reduced turn***-OFF ***time***. Moreover, it has higher control and triggering sensitivity and a more predictable firing situation.

Applications

The more common areas of SCS applications are as under :

1. in counters, registers and timing circuits of computers,
2. pulse generators,
3. voltage sensors,
4. oscillators etc.

OBJECTIVE TESTS – 64

1. A unijunction transistor has
(*a*) anode, cathode and a gate
(*b*) two bases and one emitter
(*c*) two anodes and one gate
(*d*) anode, cathode and two gates.

2. Which semiconductor device acts like a diode and two resistors ?
(*a*) SCR (*b*) triac
(*c*) diac (*d*) UJT.

3. A UJT has R_{BB} = 10 K and R_{B2} = 4 K. Its intrinsic stand-off ratio is
(*a*) 0.6
(*b*) 0.4
(*c*) 2.5
(*d*) 5/3.

4. An SCR conducts appreciable current when its ..... with respect to cathode.
(*a*) anode and gate are both negative
(*b*) anode and gate are both positive
(*c*) anode is negative and gate is positive
(*d*) gate is negative and anode is positive.

5. After firing an SCR, the gating pulse is removed. The current in the SCR will
(*a*) remains the same
(*b*) immediately fall to zero
(*c*) rise up
(*d*) rise a little and then fall to zero.

6. An SCR may be turned OFF by
(*a*) interrupting its anode current
(*b*) reversing polarity of its anode- cathode voltage
(*c*) low-current dropout
(*d*) all of the above.

7. A triac behaves like two
(*a*) inverse parallel-connected SCRs with common gate
(*b*) diodes in series
(*c*) four-layer diodes in parallel
(*d*) resistors and one diode.

8. A triac can be triggered into conduction by
(*a*) only positive voltage at either anode
(*b*) positive or negative voltage at either anode
(*c*) positive or negative voltage at gate
(*d*) both (*b*) and (*c*).

9. A diac is equivalent to a
(*a*) pair of SCRs
(*b*) pair of four-layer SCRs
(*c*) diode and two resistors
(*d*) triac with two gates.

10. An SCS has
(*a*) four layers and three terminals
(*b*) three layers and four terminals
(*c*) two anodes and two gates
(*d*) one anode, one cathode and two gates.

11. An SCS may be switched ON by a
(*a*) positive pulse at its anode
(*b*) negative pulse at its cathode
(*c*) positive pulse at its cathode gate G_2
(*d*) positive pulse at its anode gate G_1.

12. The *dv* / *dt* effect in an SCR can result in
(*a*) high rate-of-rise of anode voltage
(*b*) increased junction capacitance
(*c*) false triggering
(*d*) low capacitive charging current.

13. The *di* / *dt* effect in an SCR leads to the formation of
(*a*) local hot spots
(*b*) conduction zone
(*c*) charge spreading zone
(*d*) none of the above.

14. SCR turns OFF from conducting state to blocking state on
(*a*) reducing gate current
(*b*) reversing gate voltage
(*c*) reducing anode current below holding current value
(*d*) applying ac to the gate

15. When a thyristor is negatively biased,
(*a*) all the three junctions are negatively biased
(*b*) outer junctions are positively biased and the inner junction is negatively biased.

(*c*) outer junctions are negatively biased and the inner junction is positively biased
(*d*) the junction near the anode is negatively biased and the one near the cathode is positively biased.

16. A LASCR is just like a conventional SCR except that it
(*a*) cannot carry large current
(*b*) can also be light-triggered
(*c*) has no gate terminal
(*d*) cannot be pulse-triggered.

17. The minimum value of current required to maintain conduction in an SCR is called its ..... current.
(*a*) commutation
(*b*) holding
(*c*) gate trigger
(*d*) breakover

18. Diacs are primarily used as
(*a*) pulse generators
(*b*) triggering devices
(*c*) surge protection devices
(*d*) power thyristors.

ANSWERS

1. *(b)* **2.** *(d)* **3.** *(a)* **4.** *(b)* **5.** *(a)* **6.** *(d)* **7.** *(a)* **8.** *(d)* **9.** *(b)* **10.** *(d)* **11.** *(c)*
12. *(c)* **13.** *(a)* **14.** *(c)* **15.** *(c)* **16.** *(b)* **17.** *(b)* **18.** *(b)*

To FIRST

CONTENTS

CHAPTER 65

SINUSOIDAL AND NON-SINUSOIDAL OSCILLATORS

Learning Objectives

- What is an Oscillator?
- Classification of Oscillators
- Damped and Undamped Oscillations
- Oscillatory Circuit
- Essentials of a Feedback LC Oscillator
- Tuned Base Oscillator
- Tuned Collector Oscillator
- Hartley Oscillator
- FET Hartley Oscillator
- Colpitts Oscillator
- Clapp Oscillator
- FET Colpitts Oscillator
- Crystal Controlled Oscillator
- Transistor Pierce Cystal Oscillator
- FET Pierce Oscillator
- Phase Shift Principle
- RC Phase Shift Oscillator
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- Pulse Definitions
- Basic Requirements of a Sawtooth Generator
- UJT Sawtooth Generator
- Multivibrators (MV)
- Astable Multivibrator
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- Schmitt Trigger
- Transistor Blocking Oscillator



An oscillator is an electronic device used for the purpose of generating a signal. Oscillators are found in computers, wireless receivers and transmitters, and audiofrequency equipment particularly music synthesizers

CONTENTS

65.1. What is an Oscillator ?



Oscillator

An electronic oscillator may be defined in any one of the following four ways :

1. It is a circuit which converts dc energy into ac energy at a very high frequency;
2. It is an electronic source of alternating current or voltage having sine, square or sawtooth or pulse shapes;
3. It is a circuit which generates an ac output signal without requiring any externally applied input signal;
4. It is an unstable amplifier.

These definitions exclude electromechanical alternators producing 50 Hz ac power or other devices which convert mechanical or heat energy into electric energy.

65.2. Comparison Between an Amplifier and an Oscillator

As discussed in Chapter-10, an amplifier produces an output signal whose waveform is similar to the input signal but whose power level is generally high. This additional power is supplied by

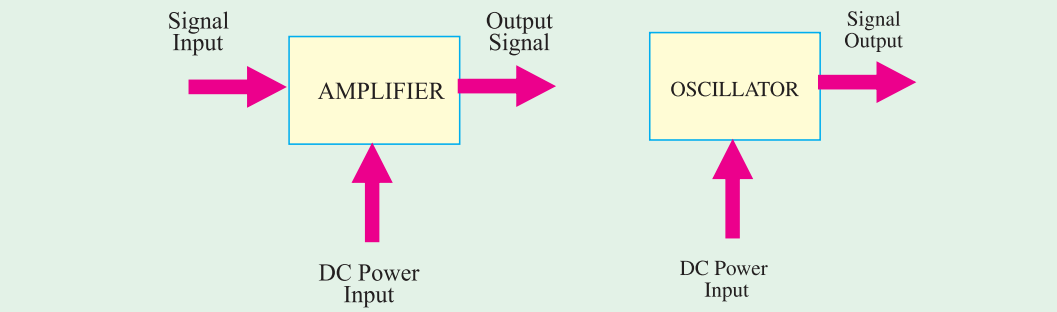


Fig. 65.1

the external dc source. Hence, an amplifier is essentially an energy convertor *i.e.* it takes energy from the dc power source and converts it into ac energy at signal frequency. The process of energy conversion is controlled by the input signal. If there is no input signal, there is no energy conversion and hence there is no output signal.

An oscillator differs from an amplifier in one basic aspect : the oscillator ***does not require an external signal*** either to start or maintain energy conversion process (Fig. 65.1). It keeps producing an output signal so long as the dc power source is connected.

Moreover, the frequency of the output signal is determined by the passive components used in the oscillator and can be varied at will.

65.3. Classification of Oscillators

Electronic oscillators may be broadly divided into following two groups :

(i) Sinusoidal (or harmonic) oscillators—which produce an output having sine waveform;

(ii) Non-sinusoidal (or relaxation) oscillators—they produce an output which has square, rectangular or sawtooth waveform or is of pulse shape.

Sinusoidal oscillators may be further subdivided into :

(a) Tuned-circuits or LC feedback oscillators such as Hartley, Colpitts and Clapp etc.;

(b) *RC* phase-shift oscillators such as Wien-bridge oscillator;

(c) Negative-resistance oscillators such as tunnel diode oscillator;

(d) Crystal oscillators such as Pierce oscillator;

(*e*) Heterodyne or beat-frequency oscillator (BFO).

The active devices (bipolars, FETs or unijunction transistors) in the above mentioned circuits may be biased class-A, B or C. Class-A operation is used in high-quality audio frequency oscillators. However, radio frequency oscillators are usually operated as class-C.

65.4. Damped and Undamped Oscillations

Sinusoidal oscillations produced by oscillators may be (*i*) damped or (*ii*) undamped.

(*i*) Damped Oscillations

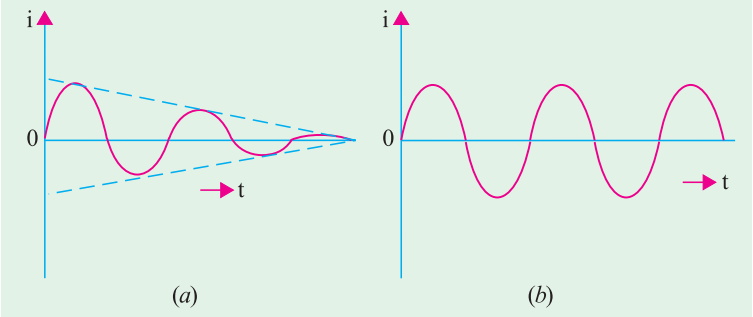


Fig. 65.2

Oscillations whose amplitude keeps decreasing (or decaying) with time are called **damped** or **decaying** oscillations. The waveform of such oscillations is shown in Fig. 65.2 (*a*). These are produced by those oscillator circuits in which I^2R losses take place continuously during each oscillation without any arrangement for compensating the same. Ultimately, the amplitude of the oscillations decays to zero when there is not enough energy to supply circuit losses. However, the frequency or time-period remains constant because ***it is determined by the circuit parameters.***

Sinusoidal oscillators serve a variety of functions in telecommunications and in electronics. The most important application in telecommunication is the use of sine waves as carrier signal in both radio and cable transmissions.

Sine wave signals are also used in frequency response testing of various types of systems and equipment including analogue communication channels, amplifiers and filters and closed-loop control systems.

(*ii*) Undamped Oscillations

Oscillations whose amplitude remains constant *i.e.* does not change with time are called undamped oscillations. These are produced by those oscillator circuits which have no losses or if they have, there is provision for compensating them. The constant-amplitude and constant-frequency sinusoidal waves shown in Fig. 65.2 (*b*) are called ***carrier waves*** and are used in communication transmitters for transmitting low-frequency audio information to far off places.

65.5. The Oscillatory Circuit

It is also called *LC* circuit or tank circuit. The oscillatory circuit (Fig. 65.3) consists of two reactive elements *i.e.* an inductor and a capacitor. Both are capable of storing energy. The capacitor stores energy in its electric field whenever there is potential difference across its plates. Similarly, a coil or an inductor stores energy in its magnetic field whenever current flows through it. Both *L* and *C* are supposed to be loss-free (*i.e.* their *Q*-factors are infinite).

As shown in Fig. 65.3 (*a*), suppose the capacitor has been fully-charged from a dc source. Since *S* is open, it cannot discharge through *L*. Now, let us see what happens when *S* is closed.

1. When *S* is closed [Fig. 65.3 (*b*)] ***electrons*** move from plate *A* to plate *B* through coil *L* as shown by the arrow (or conventional current flows from *B* to *A*). This electron flow reduces the strength of the electric field and hence the amount of energy stored in it.
2. As electronic current starts flowing, the self-induced emf in the coil opposes the current flow. Hence, rate of discharge of electrons is somewhat slowed down.
3. Due to the flow of current, magnetic field is set up which stores the energy given out by

the electric field [Fig. 65.3 (*b*)].

4. As plate *A* loses its electrons by discharge, the electron current has a tendency to die down and will actually reduce to zero when all excess electrons on *A* are driven over to plate *B* so that both plates are reduced to the same potential. At that time, there is no electric field but the magnetic field has maximum value.
5. However, due to self-induction (or electrical inertia) of the coil, more electrons are transferred to plate *B* than are necessary ***to make up the electron deficiency there.*** It means that now plate *B* has more electrons than *A*. Hence, capacitor becomes charged again though in opposite direction as shown in Fig. 65.3 (*c*).
6. The magnetic field *L* collapses and the energy given out by it is stored in the electric field of the capacitor.

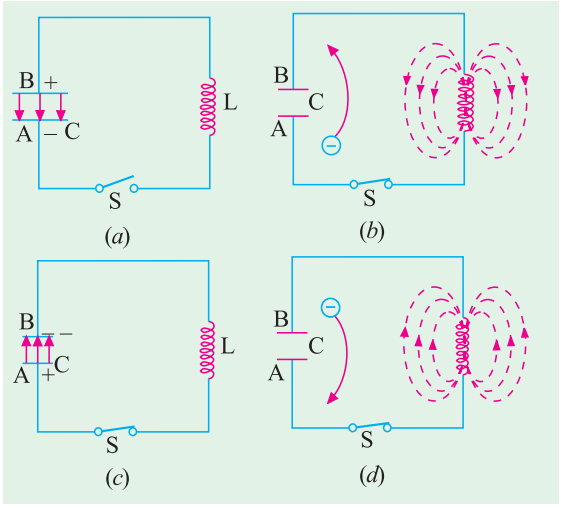


Fig. 65.3

7. After this, the capacitor starts discharging in the opposite direction so that, now, the electrons move from plate *B* to plate *A* [Fig. 65.3 (*d*)]. The electric field starts collapsing whereas magnetic field starts building up again though in the opposite direction. Fig. 65.3 (*d*) shows the condition when the capacitor becomes fully discharged once again.
8. However, these discharging electrons overshoot and again an excess amount of electrons flow to plate *A*, thereby charging the capacitor once more.
9. This sequence of charging and discharging continues. The to and fro motion of electrons between the two plates of the capacitor constitutes an oscillatory current.

It may be also noted that during this process, the electric energy of the capacitor is converted into magnetic energy of the coil and ***vice versa.***

These oscillations of the capacitor discharge are damped because energy is dissipated away gradually so that their amplitude becomes zero after sometime. There are two reasons for the loss of the energy :

(*a*) Some energy is lost in the form of heat produced in the resistance of the coil and connecting wires ;

(*b*) and some energy is lost ***in the form of electromagnetic (EM) waves*** that are radiated out from the circuit through which an oscillatory current is passing.

Both these losses subtract energy from the circuit with the result that circuit current decreases gradually till it becomes zero. The waveform of the oscillatory discharge is similar to that shown in Fig. 65.2 (*a*).

65.6. Frequency of Oscillatory Current

The frequency of time-period of the oscillatory current depends on two factors :

(*a*) Capacitance of the Capacitor

Larger the capacitor, greater the time required for the reversal of the discharge current *i.e.* lower its frequency.

(*b*) Self-inductance of the Coil

Larger the self-inductance, greater the internal effect and hence longer the time required by the current to stop flowing during discharge of the capacitor.

The frequency of this oscillatory discharge current is given by

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{159}{\sqrt{LC}} \text{ kHz}$$

where L = self-inductance in μH and C = capacitance in μF

It may, however, be pointed out here that damped oscillations so produced are not good for radio transmission purpose because of their ***limited range and excessive distortion.*** For good radio transmission, we need undamped oscillations which can be produced if some additional energy is supplied in correct phase and correct direction to the LC circuit for making up the I^2R losses continually occuring in the circuit.

65.7. Frequency Stability of an Oscillator

The ability of an oscillator to maintain a constant frequency of oscillation is called its frequency stability. Following factors affect the frequency stability :

1. Operating Point of the Active Device

The Q-point of the active device (*i.e.* transistor) is so chosen as to confine the circuit operation on the linear portion of its characteristic. Operation on non-linear portion varies the parameters of the transistor which, in turn, affects the frequency stability of the oscillator.

2. Inter-element Capacitances

Any changes in the inter-element capacitances of a transistor particularly the collector- to-emitter capacitance cause changes in the oscillator output frequency, thus affecting its frequency stability. The effect of changes in inter-element capacitances, can be neutralized by adding a swamping capacitor across the offending elements—the added capacitance being made part of the tank circuit.

3. Power Supply

Changes in the dc operating voltages applied to the active device shift the oscillator frequency. This problem can be avoided by using regulated power supply.

4. Temperature Variations

Variations in temperature cause changes in transistor parameters and also change the values of resistors, capacitors and inductors used in the circuit. Since such changes take place slowly, they cause a slow change (called drift) in the oscillator output frequency.

5. Output Load

A change in the output load may cause a change in the Q-factor of the LC tuned circuit thereby affecting the oscillator output frequency.

6. Mechanical Vibrations

Since such vibrations change the values of circuit elements, they result in changes of oscillator frequency. This instability factor can be eliminated by isolating the oscillator from the source of mechanical vibrations.

65.8. Essentials of a Feedback LC Oscillator

The essential components of a feedback LC oscillator shown in Fig. 65.4 are :

1. A resonator which consists of an LC circuit. It is also known as frequency-determining network (FDN) or tank circuit.
2. An amplifier whose function is to amplify the oscillations produced by the resonator.

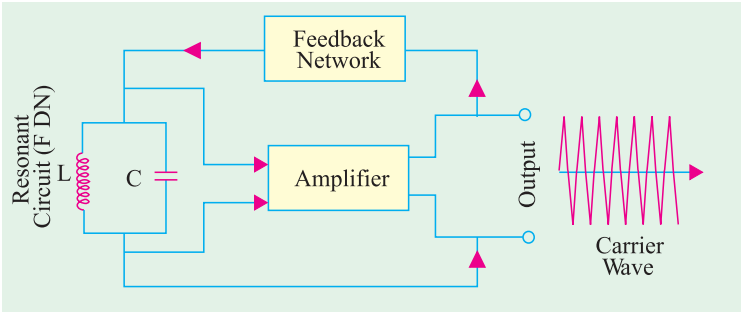


Fig. 65.4

3. A positive feedback network (*PFN*) whose function is to transfer part of the output energy to the resonant *LC* circuit in ***proper phase***. The amount of energy fed back is sufficient to meet I^2R losses in the *LC* circuit.

The essential condition for maintaining oscillations and for finding the value of frequency is

$$\beta A = 1 + j\,0 \quad \text{or} \quad \beta A \angle \phi = 1 \angle 0$$

It means that

(i) The feedback factor or loop gain $|\beta A| = 1$,

(ii) The net phase shift around the loop is 0° (or an integral multiple of 360°). In other words, ***feedback should be positive.***

The above conditions form Barkhausen criterion for maintaining a steady level of oscillation at a specific frequency.

Majority of the oscillators used in radio receivers and transmitters use tuned circuits with positive feedback. Variations in oscillator circuits are due to the different way by which the feedback is applied. Some of the basic circuits are :

1. *Armstrong or Tickler or Tuned-base Oscillator* — it employs inductive feedback from collector to the tuned *LC* circuit in the base of a transistor.

2. *Tuned Collector Oscillator*—it also employs inductive coupling but the *LC* tuned circuit is in the collector circuit.

3. *Hartley Oscillator*—Here feedback is supplied inductively.

4. *Colpitts Oscillator*—Here feedback is supplied capacitively.

5. *Clapp Oscillator*—It is a slight modification of the Colpitts oscillator.

65.9. Tuned Base Oscillator

Such an oscillator using a transistor in *CE* configuration is shown in Fig. 65.5. Resistors R_1, R_2 and R_3 determine the dc bias of the circuit. The parallel R_3—C_2 network in the emitter circuit is a stabilizing circuit to prevent signal degeneration. As usual, C_1 is the dc blocking capacitor. The mutually-coupled coils L_1 and L forming primary and secondary coils of an *RF* transformer provide the required feedback between the collector and base circuits. The amount of feedback depends on the coefficient of coupling between the two coils. The *CE* connected transistor itself provides a phase shift of 180° between its input and output circuits. The transformer provides another 180° phase shift and thus producing a ***total phase shift of 360° which is an essential condition for producing oscillations.***

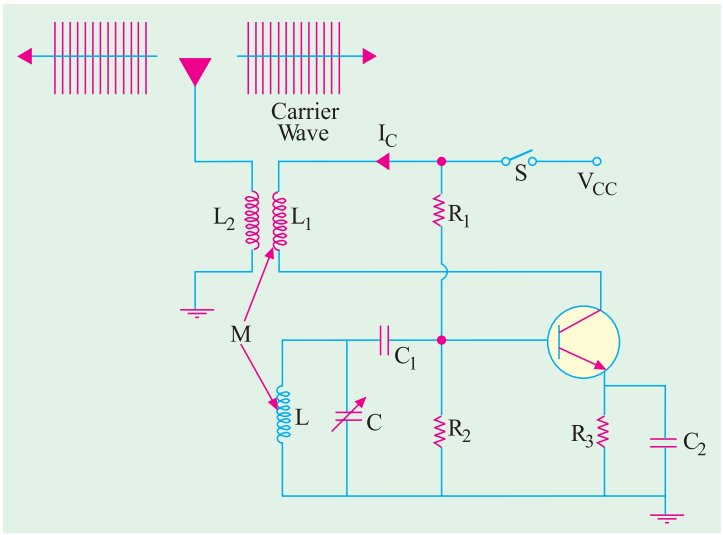


Fig. 65.5

The parallel-tuned *LC* circuit connected between base and emitter is the frequency determining network (*FDN*) *i.e.* it generates the oscillations at its resonant frequency.

Circuit Action

The moment switch *S* is closed, collector current is set up which tends to rise to its quiescent value. This increase in I_C is accompanied by :

1. An expanding magnetic field through L_1 which links with L and
2. An induced *e.m.f.* called feedback voltage in L.

Two immediate reactions of this feedback voltage are:

(i) Increase in emitter-base voltage (and base current) and

(ii) A further increase in collector current I_C.

It is followed by a succession of cycles of

1. An increase in feedback voltage,
2. An increase in emitter-base voltage and
3. An increase in I_C until saturation is reached.

Meanwhile, C gets charged. As soon as I_C ceases to increase, magnetic field of L_1 ceases to expand and thus no longer induces feedback voltage in L. Having been charged to maximum value, C starts to discharge through L. However, decrease in voltage across C causes the following sequence of reactions :

1. A decrease in emitter-base bias and hence in I_B, 2. A decrease in I_C;
3. A collapsing magnetic field in L_1;
4. An induced feedback voltage in L though, this time, in ***opposite*** direction;
5. Further decrease in emitter-base bias and so on till I_C reaches its cut-off value.

During this time, the capacitor having lost its original charge, again becomes fully charged though with ***opposite*** polarity. Transistor being in cut-off, the capacitor will again begin to discharge through L. Since polarity of capacitor charge is opposite to that when transistor was in saturation, the sequence of reactions now will be

1. An increase in emitter-base bias,
2. An increase in I_C,
3. An expanding magnetic field in L_1,
4. An induced feedback voltage in L,
5. A further increase in emitter-base bias and
6. So on till I_C increases to its saturation value.

This cycle of operation keeps repeating so long as enough energy is supplied to meet losses in the LC circuit.

The output can be taken out by means of a third winding L_2 magnetically coupled to L_1. It has approximately the same waveform as collector current.

The frequency of oscillation is equal to the resonant frequency of the LC circuit.

65.10. Tuned Collector Oscillator

Such an oscillator using a transistor in *CE* configuration is shown in Fig. 65.6.

(*i*) Frequency Determining Network (FDN)

It is made up of a variable capacitor C and a coil L which forms primary winding of a step-down transformer. The combination of L and C forms an oscillatory tank circuit to set the frequency of oscillation.

Resistors R_1, R_2 and R_3 are used to dc bias the transistor. Capacitors C_1 and C_2 act to by-pass R_3 and R_2 respectively so that they have no effect on the ac operation of the circuit. Moreover, C_2 provides ac ground for transformer secondary L_1.

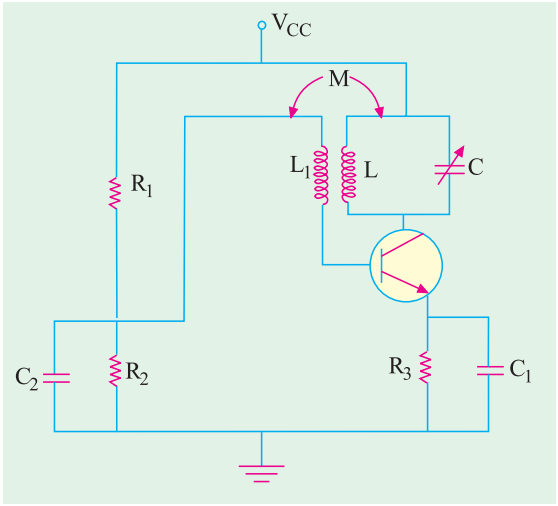


Fig. 65.6

(*ii*) Positive Feedback

Feedback between the collector-emitter circuit and base-emitter circuit is provided by the transformer secondary winding L_1 which is mutually-coupled to L. As far as ac signals are concerned, L_1 is connected to emitter *via* low-reactance capacitors C_2 and C_1.

Since transistor is connected in *CE* configuration, it provides a phase shift of 180° between its

input and output circuits. Another phase shift of 180° is provided by the transformer thus producing a total phase shift of 360° between the output and input voltages resulting in ***positive feedback*** between the two.

(*iii*) Amplifying Action

The transistor amplifier provides sufficient gain for oscillator action to take place.

(*iv*) Working

When the supply is first switched on, a transient current is developed in the tuned LC circuit as the collector current rises to its quiescent value. This transient current initiates natural oscillations in the tank circuit. These natural oscillations induce a small emf into L_1 by mutual induction which causes corresponding variations in base current. These variations in I_B are amplified β times and appear in the collector circuit. Part of this amplified energy is used to meet losses taking place in the oscillatory circuit and ***the balance is radiated out in the form of electromagnetic waves.***

The frequency of oscillatory current is almost equal to the resonant frequency of the tuned circuit.

$$\therefore \qquad f_0 = \frac{1}{2\pi\sqrt{LC}}$$

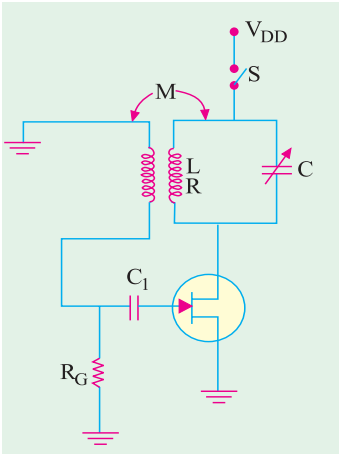


Fig. 65.7

65.11. Tuned Drain Oscillator (FET)

The basic circuit is illustrated in Fig. 65.7. It is similar to the tuned collector oscillator of Fig. 65.6. Because of its high input impedance and high voltage amplification, a FET can be used to construct very simple and efficient oscillator circuit. This frequency of oscillation is given by

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \sqrt{\left(1+\frac{R}{r_d}\right)}$$

where r_d = ac drain resistance

The value of mutual inductance required for maintaining oscillations is $M = \dfrac{r_d RC + L}{\mu}$.

Example 65.1. *A tuned-collector oscillator has a fixed inductance of 100 μH and has to be tunable over the frequency band of 500 kHz to 1500 kHz. Find the range of variable capacitor to be used.* **(Principles of Telecom. Engg. Pune Univ.)**

Solution. Resonant frequency is given by

$$f_0 = 1/2\pi\sqrt{LC} \qquad \text{or} \qquad C = 1/4\pi^2 f_0^2 L$$

where L and C refer to the tank circuit.

When fo = 500 kHz

$C = 1/4\pi^2 \times (500 \times 10^3)^2 \times 100 \times 10^{-6} =$ **1015 pF**

When fo = 1500 kHz

$C = 1015/(1500/500)^2 = 113$ pF

Hence, capacitor range required is **113 – 1015 pF**

Example 65.2. *The resonant circuit of a tuned-collector transistor oscillator has a resonant frequency of 5 MHz. If value of capacitance is increased by 50%, calculate the new resonant frequency.*

Solution. Using the equation for resonant frequency, we have

$$5 \times 10^6 = 1/2\pi\sqrt{LC} \qquad \text{— 1st case}$$

$$f_0 = 1/2\pi\sqrt{L \times 1.5C} \qquad \text{— 2nd case}$$

$$\therefore \quad \frac{f_0}{5\times10^6} = \frac{1}{\sqrt{1.5}} \quad \text{or} \quad f_o = \mathbf{4.08\ MHz}$$

65.12. Hartley Oscillator

In Fig. 65.8 (*a*) is shown a transistor Hartley oscillator using *CE* configuration. Its general principle of operation is similar to the tuned-collector oscillator discussed in Art. 65.10.

It uses a single tapped-coil having two parts marked L_1 and L_2 instead of two separate coils. So far as ac signals are concerned, one side of L_2 is connected to base *via* C_1 and the other to emitter *via* ground and C_3. Similarly, one end of L_1 is connected to collector *via* C_2 and the other to common emitter terminal *via* C_3. In other words, L_1 is in the output circuit *i.e.* collector-emitter circuit whereas L_2 is in the base-emitter circuit *i.e.* input circuit. These two parts are inductively-coupled and form an auto-transformer or a split-tank inductor. Feedback between the output and input circuits is accomplished through autotransformer action which also introduces a phase reversal of 180°. This phase reversal

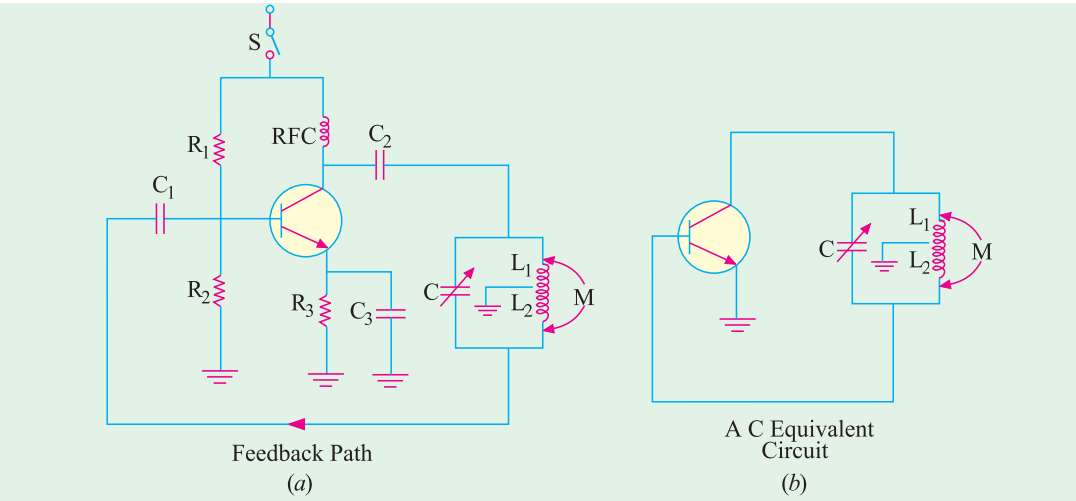


Fig. 65.8



Hartley oscillator

between two voltages occurs because they are taken from ***opposite ends*** of an inductor (L_1 - L_2 combination) with respect to the tap which is tied to common transistor terminal *i.e.* emitter which is ac grounded *via* C_3. Since transistor itself introduces a phase shift of 180°, the total phase shift becomes 360° thereby making the ***feedback positive* or *regenerative*** which is essential for oscillations (Art 65.8). As seen, positive feedback is obtained from the tank circuit and is coupled to the base *via* C_1. The feedback factor is given by the ratio of turns in L_2 and L_1 *i.e.* by N_2/N_1 and its value ranges from 0.1 to 0.5. Fig. 65.8 (*b*) shows the equivalent circuit of Hartley oscillator.

Resistors R_1 and R_2 form a voltage divider for providing the base bias and R_3 is an emitter swamping resistor to add stability to the circuit. Capacitor C_3 provides ac ground thereby preventing any signal degeneration while still providing temperature stabilisation. Radio-frequency choke (RFC) provides dc load for the collector and also keeps ac currents out of the dc supply V_{CC}.

When V_{CC} is first switched on through S, an initial bias is established by R_1 - R_2 and oscillations are produced because of positive feedback from the LC tank circuit (L_1 and L_2 constitute L). The frequency of oscillation is given by

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \qquad \text{where} \quad L = L_1 + L_2 + 2M$$

The output from the tank may be taken out by means of another coil coupled either to L_1 or L_2.

Example 65.3. *Calculate the oscillation frequency for the transistor Hartley oscillator circuit (refer to Fig. 65.8). Given the circuit values: L_{RFC} = 0.5 mH, L_1 = 750 μH, L_2 = 750 μH, M = 150 μH and C = 150 pF.*

Solution. $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$ where $L = L_1 + L_2 + 2M$

$\therefore$ $L = 750\ \mu H + 750\ \mu H + 2 + 150\ \mu H = 1800\ \mu H$

and $f_0 = \dfrac{1}{2\pi\sqrt{1800\mu H \times 150 pF}} = \mathbf{320 kHz}$

Example 65.4. *In an Hartley oscillator if L_1 = 0.1 mH and mutual inductance between the coils equal to 20 μH . Calculate the value of capacitor C of the oscillating circuit to obtain frequency of 4110 kHz.* **(Bangalore University 2001)**

Solution. $L = L_1 + L_2 + 2M = 0.1\ mH + 10\ \mu H + 20\ \mu H = 130\ \mu H$

Now the resonant frequency is given by

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \quad \text{or} \quad C = \frac{1}{4\pi^2 f_0^2 L} = \frac{1}{4\pi^2 \times 4110^2 \times 130\mu H}$$

C = **11.5 pF**

65.13. FET Hartley Oscillator

The basic circuit is shown in Fig. 65.9. R_G is the gate biasing resistor. There is mutual induction between the two parts L_1 and L_2 of the coil.

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

where $L = L_1 + L_2 + 2M$

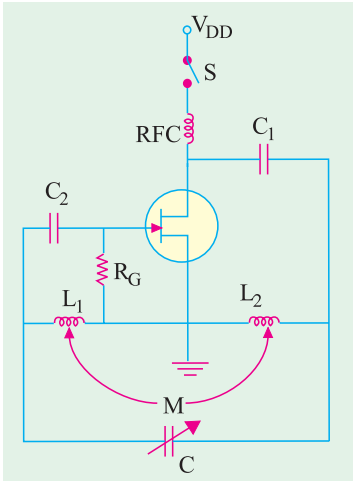


Fig. 65.9

Example 65.5. *Calculate the oscillator frequency for a FET Hartley oscillator (refer to Fig. 65.9), for the following circuit values: C = 250 pF, L_1 = 1.5 mH, L_2 = 1.5 mH, and M = 0.5 mH.*

Solution. $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$ where $L = L_1 + L_2 + 2M$

$\therefore$ $L = 1.5\ mH + 1.5\ mH + 2 \times 0.5\ mH = 4\ mH$

and $f_0 = \dfrac{1}{2\pi\sqrt{4\,mH \times 250\,pF}} = \mathbf{159.1\ kHz}$

65.14. Colpitts Oscillator

This oscillator is essentially the same as Hartley oscillator except for one difference. Colpitts oscillator uses **tapped capacitance** whereas Hartley oscillator uses **tapped inductance*. Fig. 65.10** (*a*)

* As an aid to memory, remember that Hartley begins with letter H for Henry *i.e.* coil and Colpitts begins with *C* for Capacitor.

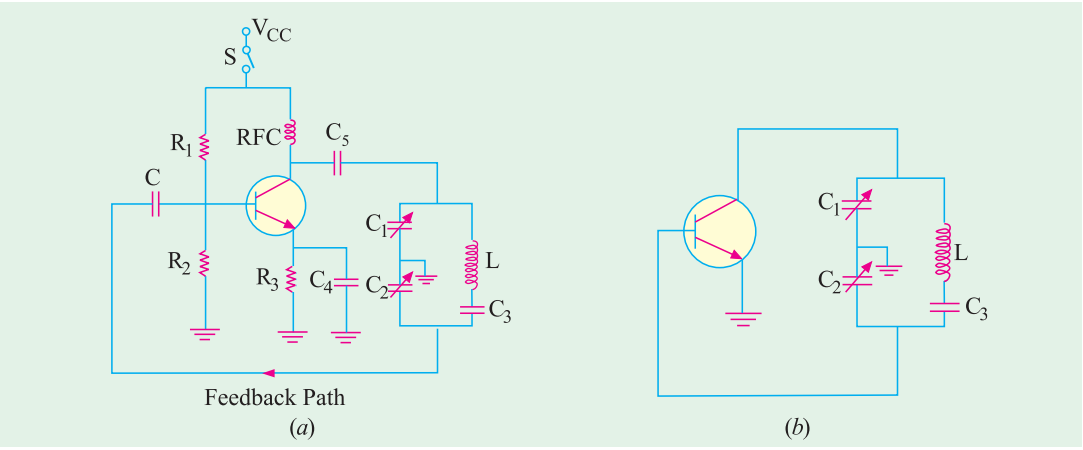


Fig. 65.10

shows the complete circuit with its power source and dc biasing circuit whereas Fig. 65.10 (*b*) shows its ac equivalent circuit. The two series capacitors C_1 and C_2 form the voltage divider used for providing the feedback voltage (the voltage drop across C_2 constitutes the feedback voltage). The feedback factor is C_1/C_2. The minimum value of amplifier gain for maintaining oscillations is

$$A_{v(min)} = \frac{1}{C_1/C_2} = \frac{C_2}{C_1}$$

The tank circuit consists of two ganged capacitors C_1 and C_2 and a single fixed coil. The frequency of oscillation (which does not depend on mutual inductance) is given by

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \qquad \text{where} \quad C = \frac{C_1 C_2}{C_1 + C_2}$$

Transistor itself produces a phase shift of 180°. Another phase shift of 180° is provided by the capacitive feedback thus giving a total phase shift of 360° between the emitter-base and collector-base circuits.

Resistors R_1 and R_2 form a voltage divider across V_{CC} for providing base bias, R_3 is for emitter stabilisation and *RFC* provides the necessary dc load resistance R_C for amplifier action. It also prevents ac signal from entering supply dc V_{CC}. Capacitor C_5 is a bypass capacitor whereas C_4 conveys feedback from the collector-to-base circuit.

When *S* is closed, a sudden surge of collector current **shock-excites** the tank circuit into oscillations which are sustained by the feedback and the amplifying action of the transistor.

Colpitts oscillator is widely used in commercial signal generators upto 1 MHz. Frequency of oscillation is varied by gang-tuning the two capacitors C_1 and C_2.

Example 65.6. *Determine the circuit oscillation frequency for a transistor Colpitts oscillator shown in Fig. 65.10(a). Given,*

$L = 100\ \mu H$, $L_{RFC} = 0.5\ mH$, $C_1 = 0.005\ \mu F$, $C_2 = 0.01\ \mu F$. $C_6 = 10\ \mu F$

Solution. For a transistor Colpitts oscillator, the oscillation frequency,

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \qquad \text{where} \quad C = \frac{C_1 C_2}{C_1 + C_2}$$

$$\therefore \quad C = \frac{0.005\,\mu F \times 0.01\,\mu F}{0.005\,\mu F + 0.01\,\mu F} = 3.3\,nF$$

$$\text{and} \quad f_0 = \frac{1}{2\pi \times \sqrt{100\,\mu H \times 3.3\,nF}} = \mathbf{277\ kHz}$$

Example 65.7. *For the Colpitts oscillator circuit shown in Fig. 65.11, find the values of*

(a) feedback fraction,

(b) minimum gain to sustain oscillations,

(c) emitter resistor R_E.

(Electronics-I, Bangalore Univ.)

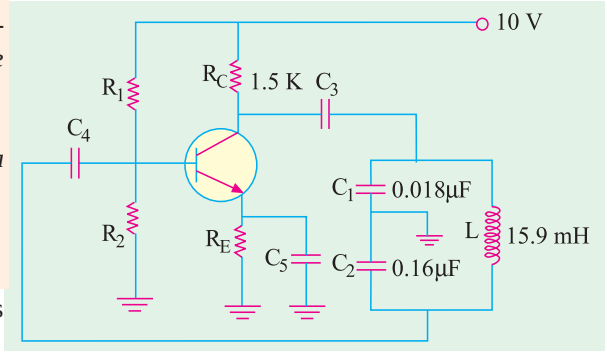


Fig. 65.11

Solution. **(a)** The voltage drop across C_2 is feedback to the input circuit.

feedback fraction

$$= \frac{\text{drop across } C_2}{\text{drop across } C_1} = \frac{IX_2}{IX_1} = \frac{C_1}{C_2} = \frac{0.018}{0.16} = \mathbf{0.11}$$

(b) $A_{v(min)} = \dfrac{1}{\text{feedback fraction}} = \dfrac{1}{0.11} = \mathbf{9}$

(c) Now, $A_v \cong \dfrac{R_C}{R_E}$ $\quad\therefore\quad$ $R_E = \dfrac{R_C}{A_v} = \dfrac{1500}{9} = \mathbf{167\ \Omega}$

It should be noted that the above calculations do not take into account the losses in the coil and the loading effect of the amplifier input impedance.

65.15. Clapp Oscillator

It is a variation of Colpitts oscillator and is shown in Fig. 65.12 (*a*). It differs from Colpitts oscillator in respect of capacitor C_3 only which is joined in series with the tank inductor. Fig. 65.12 (*b*) shows the ac equivalent circuit.

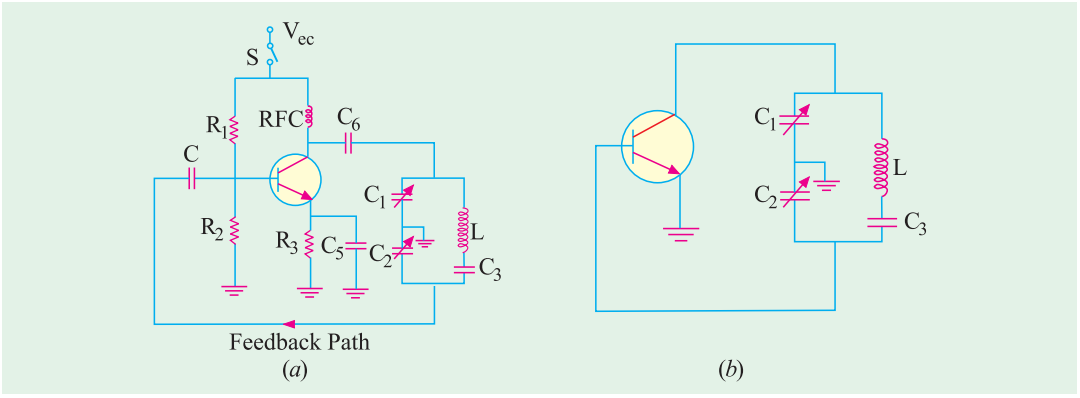


Fig. 65.12

Addition of C_3 ***(i)*** improves frequency stability and ***(ii)*** eliminates the effect of transistor's parameters on the operation of the circuit.

The operation of this circuit is the same as that of the Colpitts oscillator.

The frequency of oscillation is given by

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \quad \text{where} \quad \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

65.16. FET Colpitts Oscillator

The circuit is shown in Fig. 65.13. It is similar to the transistor circuit of Fig. 65.10. Here, R_G is the gate biasing resistor. The radio-frequency coil choke (R_{FC}) performs two functions :

(i) It keeps ac current out of the dc drain supply and

(ii) It provides drain load.

As seen, C_1 is in the input circuit whereas C_2 is in the output circuit.

The frequency of oscillation is given by

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ ; where, } C = \frac{C_1 C_2}{C_1 + C_2}$$

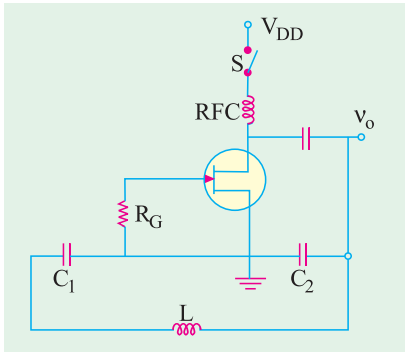


Fig. 65.13

Example 65.8. *Determine the circuit oscillation frequency for the FET Colpitts oscillator (refer to Fig. 65.13), given C_1 = 750 pF, C_2 = 2500 pF and L = 40 μH.*

Solution. For a Colpitts oscillator, the oscillation frequency,

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \qquad \text{where} \quad C = \frac{C_1 C_2}{C_1 + C_2}$$

$$\therefore \qquad C = \frac{750\,\text{pF} \times 2500\,\text{pF}}{750\,\text{pF} + 2500\,\text{pF}} = \mathbf{576.9\ pF}$$

and

$$f_0 = \frac{1}{2\pi \times \sqrt{40\mu H \times 576.9\ pF}} = \mathbf{1.048\ MHz}$$

65.17. Crystals

For an exceptionally high degree of frequency stability, use of crystal oscillators is essential. The crystal generally used is a finely-ground wafer of translucent quartz (or tourmaline) stone held between two metal plates and housed in a package about the size of a postal stamp. The crystal wafers are cut from the crude quartz in two different ways. The method of 'cutting' determines the crystal's natural resonant frequency and its temperature coefficient. When the wafer is cut so that its flat surface are perpendicular to its **electrical axis**, it is called an **X-cut** crystal (Fig. 65.14). But if the wafer is so cut that its flat surfaces are perpendicular to its **mechanical axis**, it is called **Y-cut** crystal.

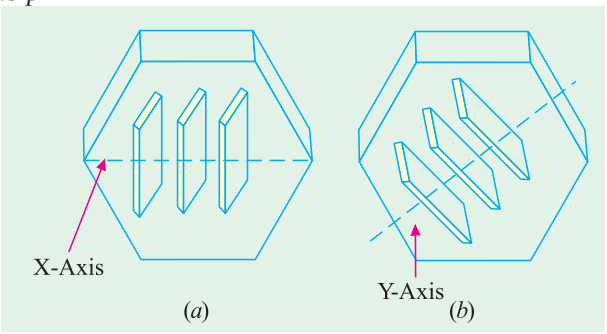


Fig. 65.14

(a) Piezoelectric Effect

The quartz crystal described above has peculiar properties. When mechanical stress is applied across its two opposite faces, **a potential difference is developed across them**. It is called ***piezoelectric effect.*** Conversely, when a potential difference is applied across its two opposite faces, it causes the crystal to ***either expand or contract.*** If an alternating voltage is applied, the crystal wafer is set into vibrations. The frequency of vibration is equal to the resonant frequency of the crystal as determined by its structural characteristics. Where the frequency of the applied ac voltage equals the natural resonant frequency of the crystal, the amplitude of vibration will be maximum. As a general rule, thinner the crystal, ***higher its frequency of vibration.***

(b) Equivalent Electrical Circuit

The electrical equivalent circuit of the crystal is shown in Fig. 65.15 (*b*). It consists of a series RLC_1 circuit in parallel with a capacitor C_2.

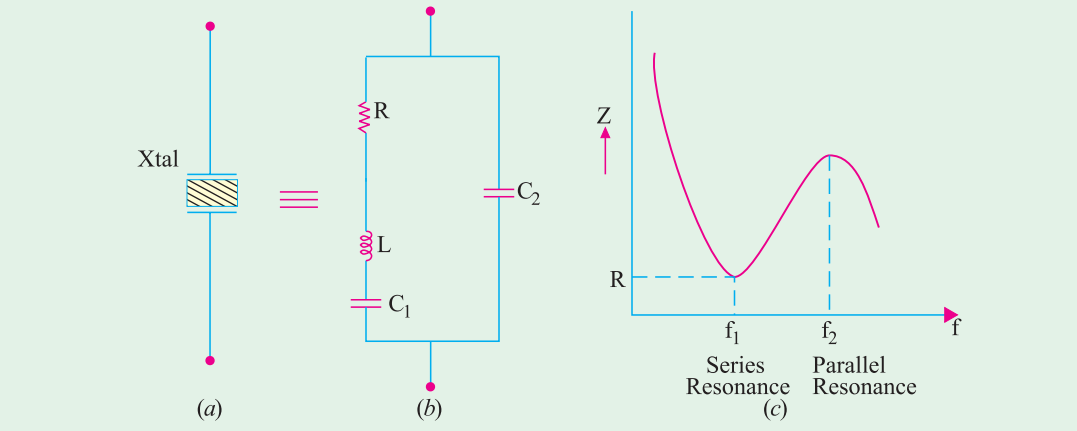


Fig. 65.15

The circuit has two resonant frequencies :

(*i*) one is the lower series resonance frequency f_1 which occurs when $X_L = X_{C1}$. In that case, $Z = R$ as shown in Fig. 65.15 (*c*).

$$f_1 = \frac{1}{2\pi\sqrt{LC_1}}$$

(*ii*) the other is the parallel resonance frequency f_2 which occurs when reactance of the series leg equals the reactances of C_2. At this frequency, the crystal offers very high impedance to the external circuit.

$$f_2 = \frac{1}{2\pi\sqrt{LC}}; \quad \text{where} \quad C = \frac{C_1 C_2}{C_1 + C_2}$$

The impedance versus frequency graph of the crystal is shown in Fig. 65.15 (*c*). Crystals are available at frequencies of 15 kHz and above. However, at frequencies above 100 MHz, they become so small that handling them becomes a problem.

(*c*) Q-factor

The equivalent inductance of a crystal is very high as compared to either its equivalent capacitance or equivalent resistance. Because of high *L*/*R* ratio, the *Q*-factor of a crystal circuit is 20,000 as compared to a maximum of about 1000 for high-quality *LC* circuits. Consequently, greater frequency stability and frequency discrimination are obtained because of extremely high *Q* (upto 10^6) and high *L*/*R* ratio of the series RLC_1 circuit.

(*d*) Temperature Coefficient

Temperature variations affect the resonant frequency of a crystal. The number of cycles change per million cycles for a 1°C change in temperature is called the **temperature coefficient** (TC) of the crystal. It is usually expressed in parts per million (ppm) per °C. For example, a TC of 10 ppm per °C means that frequency variation is 0.001 per cent* per °C change in temperature. It can also be expressed as 10 Hz/MHz/°C. When kept in temperature-controlled ovens, crystal oscillators have frequency stability of about ±1 ppm.

Usually, X-cut crystals have negative TC whereas *Y*-cut crystals have positive TC.

Example 65.9. *A 600 kHz X-cut crystal when calibrated at 50°C has a TC of 20 ppm/°C. What will be its resonant frequency when its temperature is raised to 60°C ?*

Solution. A TC of 20 ppm/°C is the same thing as 20 Hz/MHz/°C. Since temperature rise is 10°C and original frequency is 600 kHz = 0.6 MHz, the total change as calculated by ratio propor-

* Since 10 ppm means a change of 10 in 10^6, hence percentage change is

$$= \frac{10}{10^6} \times 100 = 10^{-3} = 0.001$$

tion is = 20 × 10 × 0.6 = 120 Hz.

Since TC is negative, the new resonance frequency of the crystal is

= 600,000 – 120 = 599,880 = **599.88 kHz**.

Example 65.10. *A certain X-cut quartz crystal resonates at 450 kHz. It has an equivalent inductance of 4.2 H and an equivalent capacitance of 0.0297 pF. If its equivalent resistance is 60 Ω, calculate its Q-factor.*

Solution. $Q = \dfrac{\omega L}{R} = \dfrac{2\pi fL}{R}$

$$= \frac{2\pi \times 450 \times 10^3 \times 4.2}{600} = \mathbf{19{,}790}$$

Example 65.11. *The parameters of a crystal oscillator equivalent circuit are : L_1 = 0.8H; C_1 = 0.08 pF, R = 5 kΩ and C_2 = 1.0 pF. Determine the resonant frequencies f_1 and f_2*

(UPSC Engg. Services 1999)

Solution. $f_1 = \dfrac{1}{2\pi\sqrt{LC_1}} = \dfrac{1}{2\pi\sqrt{0.8 \times 0.08 \times 10^{-12}}} = \dfrac{1}{2\pi \times 0.253 \times 10^{-6}}$

$$= \frac{1}{1.5895 \times 10^{-6}} = 0.629 \times 10^6 \text{ Hz} = \mathbf{629\ kHz}$$

$$f_2 = \frac{1}{2\pi\sqrt{LC}} \text{ where } C = \frac{C_1 C_2}{C_1 + C_2} = \frac{0.08 \times 1}{0.08 + 1} = 0.074\,pF$$

$$\therefore \quad f_2 = \frac{1}{2\pi\sqrt{0.8 \times 074 \times 10^{-12}}} = \frac{1}{2\pi \times 0.0243 \times 10^6}(\text{Hz})$$

$$f_2 = \frac{1}{2\pi\sqrt{0.8 \times 074 \times 10^{-12}}} = \frac{1}{2\pi \times 0.0243 \times 10^6}(\text{Hz})$$

$$= 0.654 \times 10^6 \text{ Hz} = \mathbf{654\ kHz.}$$

65.18. Crystal Controlled Oscillator

Fig. 65.16, shows the use of a crystal to stabilise the frequency of a tuned-collector oscillator which has a crystal (usually quartz) in the feedback circuit.

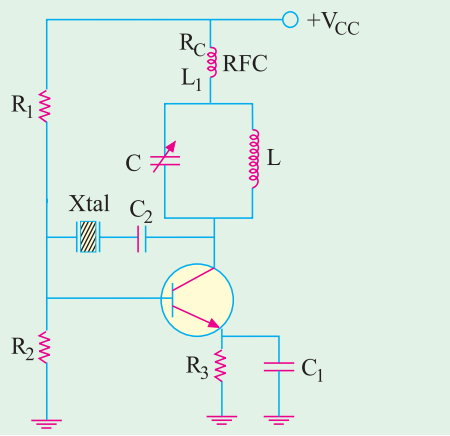
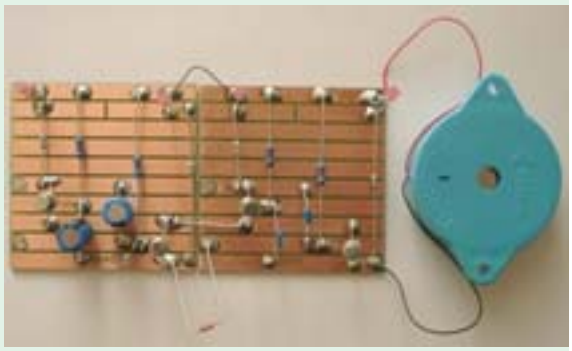


Fig. 65.16



Crystal Controlled oscillator

The *LC* tank circuit has a frequency of oscillation

$$f_0 = 1/2\pi\sqrt{LC}$$

The circuit is adjusted to have a frequency nearabout the desired operating frequency but the exact frequency is set by the crystal and stabilized by the crystal. For example, if natural frequency of vibration of the crystal is 27 MHz, the *LC* circuit is made to resonate at this frequency.

As usual, resistors R_1, R_2 and R_3 provide a volt-

age-divider stabilised dc bias circuit. Capacitor C_1 by-passes R_3 in order to maintain large gain. *RFC* coil L_1 prevents ac signals from entering dc line whereas R_C is the required dc load of the collector. The coupling capacitor C_2 has negligible impedance at the operating frequency but prevents any dc link between collector and base. Due to extreme stability of crystal oscillations, such oscillators are widely used in communication transmitters and receivers where frequency stability is of prime importance.

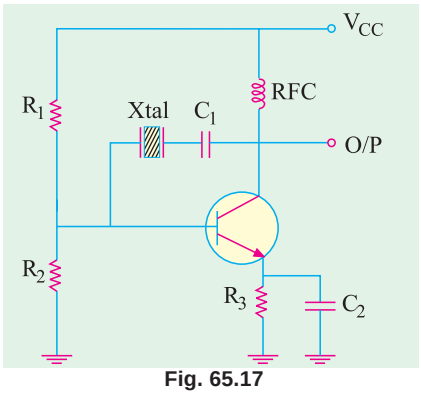


Fig. 65.17

65.19. Transistor Pierce Crystal Oscillator

A typical circuit originally suggested by Pierce is shown in Fig. 65.17. Here, the crystal is excited in the series-resonance mode because it is connected as a series element in the feedback path from collector to the base. Since, in series resonance, crystal impedance is the smallest, the amount of positive feedback is the largest. The crystal not only provides the feedback but also the necessary phase shift.

As usual, R_1, R_2 and R_3 provide a voltage-divider stabilized dc bias circuit. C_2 bypasses R_3 to avoid degeneration. The *RFC* coil provides dc collector load and also prevents any ac signal from entering the dc supply. The coupling capacitor C_1 has negligible reactance at circuit operating frequency but ***blocks any dc flow between collector and base.*** The oscillation frequency equals the series-resonance frequency of the crystal and is given by

$$f_0 = \frac{1}{2\pi\sqrt{LC_1}} \quad \text{— Fig 65.15 } (b)$$

Advantages

1. It is a very simple circuit because no tuned circuit other than the crystal itself is required.
2. Different oscillation frequencies can be obtained by simply replacing one crystal with another. It makes it easy for a radio transmitter to work at different frequencies.
3. Since frequency of oscillation is set by the crystal, it remains unaffected by changes in supply voltage and transistor parameters etc.

65.20. FET Pierce Oscillator

It is shown in Fig. 65.18. Use of FET is desirable because its high input impedance results in light loading of the crystal which

1. results in good stability and
2. does not lower the *Q*-value.

The circuit shown in Fig. 65.18 is essentially a Colpitts oscillator in which

(i) crystal has replaced the inductor and

(ii) inherent FET junction capacitance provide the split capacitance as shown in the figure.

The circuit can be made to operate at different frequencies simply by plugging in different crystals between points *A* and *B* of the circuit.

The circuit oscillation frequency is given by the series-resonance frequency of the crystal (Art. 65.17)

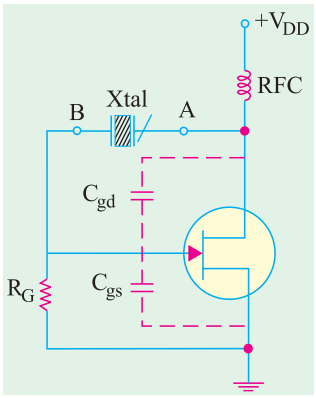


Fig. 65.18

65.21. Phase Shift Principle

Tuned circuits are ***not an essential requirement for oscillation.*** What is essential is that there should be a 180° phase shift around the feedback network* and loop gain should be greater than unity. The 180° phase shift in the feedback signal can be achieved by using a suitable *R-C* network

* Total phase shift required is 360°. However, the balance of 180° is provided by the active device of the amplifier itself.

consisting of three or four *R-C* sections. The sine wave oscillators which use *R-C* feedback network are called **phase-shift oscillators**.

65.22. RC Phase Shift Oscillator

Fig. 65.19, shows a transistor phase-shift oscillator which uses a three-section *R-C* feedback network for producing a total phase shift of 180° (*i.e.* 60° per section) in the signal fed back to the base. Since *CE* amplifier produces a phase reversal of the input signal, total phase shift becomes 360° or 0° which is essential for **regeneration and hence for sustained oscillations.**

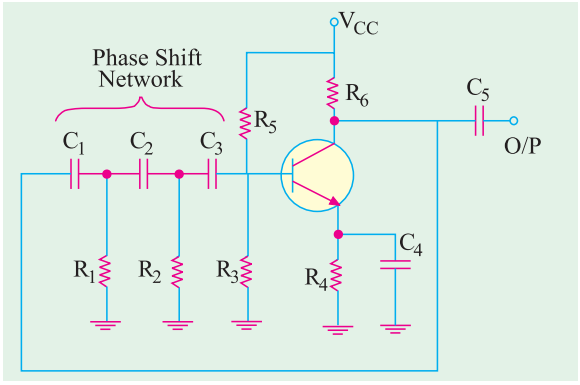


Fig. 65.19

Values of *R* and *C* are so selected that each *RC* section produces a phase advance of 60°. Addition of a fourth section improves oscillator stability. It is found that phase shift of 180° occurs ***only at one frequency*** which becomes the oscillator frequency.

(*a*) Circuit Action

The circuit is set into oscillations by any random or chance variation caused in the base current by

(*i*) noise inherent in a transistor or
(*ii*) minor variation in the voltage of the dc source.

This variation in the base current

1. is amplified in the collector circuit,
2. is then fed back to the *RC* network R_1C_1, R_2C_2 and R_3C_3,
3. is reversed in phase by the *RC* network,
4. is next applied to the base in phase with initial change in base current,
5. and hence is used to sustain cycles of variations in collector current between saturation and cut-off values.

Obviously, the circuit will stop oscillating the moment phase shift differs from 180°.

As is the case with such transistor circuits **(*i*)** voltage divider R_5 - R_3 provides dc emitter-base bias, **(*ii*)** R_6 controls collector voltage and **(*iii*)** R_4, C_4 provide temperature stability and prevent ac signal degeneration. The oscillator output voltage is capacitively coupled to the load by C_5.

(*b*) Frequency of Oscillation

The frequency of oscillation for the three-section *RC* oscillator when the three *R* and *C* components are equal is roughly given by

$$f_0 = \frac{1}{2\pi\sqrt{6}.RC}\text{ Hz} = \frac{0.065}{RC}\text{ Hz}$$

Moreover, it is found that value of β is 1/29. It means that amplifier gain must be more than 29 for oscillator operation.

(*c*) Advantages and Disadvantages

1. Since they do not require any bulky and expensive high-value inductors, such oscillators are well-suited for frequencies below 10 kHz.
2. Since only one frequency can fulfil Barkhausen phase-shift requirement, positive feedback occurs only for one frequency. Hence, pure sine wave output is possible.

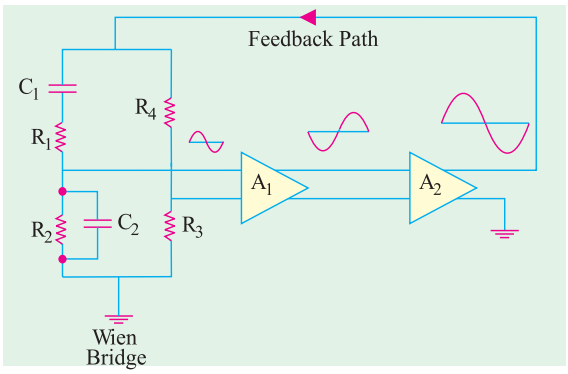


Fig. 65.20

3. It is not suited to variable frequency usage because a large number of capacitors will have to be varied. Moreover, gain adjustment would be necessary every time frequency change is made.
4. It produces a distortion level of nearly 5% in the output signal.
5. It necessitates the use of a high β transistor to overcome losses in the network.

Example 65.12. *It is desired to design a phase-shift oscillator using a BJT and R = 10 kΩ.. Select the value of C for oscillator operation at 1 kHz.*

Solution. $f_0 = \dfrac{0.065}{RC}$ or $C = \dfrac{0.065}{Rf_0} = \dfrac{0.065}{10K \times 1KHz}$

$= 0.0065 \times 10^{-6}$ F $=$ **6.5 nF**

65.23. Wien Bridge Oscillator

It is a low-frequency (5 Hz — 500 kHz), low-distortion, tunable, high-purity sine wave generator, often used in laboratory work. As shown in the block diagrams of Fig. 65.20 and Fig. 65.21, this oscillator uses two *CE*-connected *RC*-coupled transistor amplifiers and one *RC*-bridge (called Wien bridge) network to provide feedback. Here, Q_1 serves as amplifier-oscillator and Q_2 provides phase reversal and additional amplification. The bridge circuit is used to control the phase of the feedback signal at Q_1.

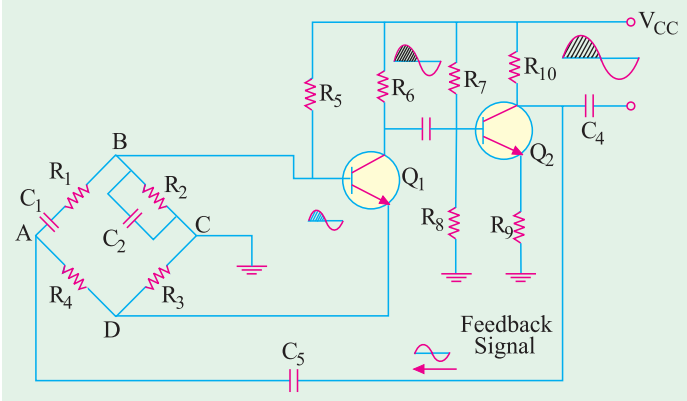


Fig. 65.21

(a) Phase Shift Principle

Any input signal at the base of Q_1 appears in the amplified but phase-reversed form across collector resistor R_6 (Fig. 65.21). It is further inverted by Q_2 in order to provide a total phase reversal of 360° for positive feedback. Obviously, the signal at R_{10} is an amplified replica of the input signal at Q_1 and is of the same phase since it has been inverted twice. We could feed this signal back to the base of Q_1 directly to provide regeneration needed for oscillator operation. But because Q_1 will amplify signals over a wide range of frequencies, direct coupling would result in poor frequency stability. By adding the Wien bridge, oscillator becomes sensitive to a signal of ***only one particular frequency.*** Hence, we get an oscillator of good frequency stability.

(b) Bridge Circuit Principle

It is found that the Wien bridge would become balanced at the signal frequency for which phase shift is exactly 0° (or 360°),

The balance conditions are

$$\frac{R_4}{R_3} = \frac{R_1}{R_2} + \frac{C_2}{C_1} \quad \text{and} \quad \omega_0 = \frac{1}{\sqrt{R_1 C_1 R_2 C_2}} \quad \text{or} \quad f_0 = \frac{1}{2\pi\sqrt{R_1 C_1 R_2 C_2}}$$

$$\text{If } R_1 = R_2 = R \quad \text{and} \quad C_1 = C_2 = C, \quad \text{then} \quad f_0 = \frac{1}{2\pi RC} \quad \text{and} \quad \frac{R_4}{R_3} = 2$$

(c) Circuit Action

Any random change in base current of Q_1 can start oscillations. Suppose, the base current of Q_1 is increased due to some reason. It is equivalent to applying a positive going signal to Q_1. Following sequence of events will take place :

1. An amplified but phase-reversed signal will appear at the collector of Q_1;
2. A still further amplified and twice phase-reversed signal will appear at the collector of Q_2. Having been inverted twice, this output signal will be in phase with the input signal at Q_1;
3. A part of the output signal at Q_2 is fed back to the input points of the bridge circuit (point A-C). A part of this feedback signal is applied to emitter resistor R_3 where it produces degenerative effect. Similarly, a part of the feedback signal is applied across base-bias resistor R_2 where it produces regenerative effect.

At the rated frequency f_o, effect of ***regeneration is made slightly more than that of degeneration*** in order to maintain continuous oscillations.

By replacing R_3 with a thermistor, amplitude stability of the oscillator output voltage can be increased.

(*d*) Advantages

Such a circuit has

1. highly stabilized amplitude and voltage amplification,
2. exceedingly good sine wave output,
3. good frequency stability.

Example 65.13. *Calculate the resonant frequency of a Wien Bridge oscillator (shown in Fig. 65.21) when R = 10 k Ω and C = 2400 pF.*

Solution. $f_0 = \dfrac{1}{2\pi RC} = \dfrac{1}{2\pi \times 10\,K \times 2400\,pF} = \mathbf{6.63\ kHz}$

Example 65.14. *Design RC elements of a Wien Bridge oscillator (shown in Fig. 65.21), for operation at 2.5 kHz.*

Solution. Using equal values of R_1 and R_2, let us select R = 100 kΩ (you can chose any value in kilo ohms). Then from the relation,

$$f_0 = \frac{1}{2\pi RC} \quad \text{we get} \quad C = \frac{1}{2\pi f_0 R} = \frac{1}{2\pi \times 2.5\ \text{kHz} \times 100\,\text{K}}$$

or C = **636 pF**

65.24. Non-sinusoidal Waveforms

Any waveform whose shape is different from that of a standard sine wave is called ***non-sinusoidal waveform.*** Examples are : square, rectangular, sawtooth, triangular waveforms and pulses as shown in Fig. 65.22.

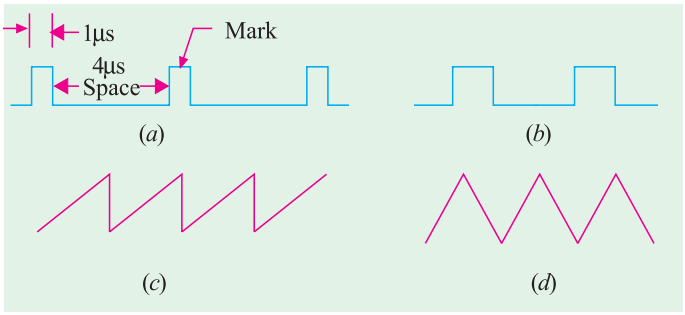


Fig. 65.22

(*a*) Pulses

Fig. 65.22 (*a*) shows a pulse train *i.e.* a stream of pulses at regular intervals. A pulses may, in general, be defined as a voltage or current that changes rapidly from one level of amplitude to another ***i.e.* it is an abrupt discontinuity in voltage or current**. These pulses are extensively used in digital electronics.

1. Mark-to-Space Ratio (MSR)

$$MSR = \frac{\text{pulse width}}{\text{time between pulses}} = \frac{1\,\mu s}{4\,\mu s} = \mathbf{0.25}$$

— Fig. 65.22 (*a*)

Hence, mark-to-space ratio of the pulse shown in Fig. 65.22 (*a*) is 1 : 4.

This name has come from early morse-code transmission systems where a pulse was used to cause a pen to mark the paper.

2. Pulse Repetition Time (PRT)

It may be defined as ***the time between the beginning of one pulse and that of the other.***

As seen from Fig. 65.22 (*a*), *PRT* = 5 μs

3. Pulse Repetition Frequency (PRF)

It is given by ***the number of pulses per second.***

$$PRF = \frac{1}{PRT} = \frac{1}{5\mu s} = \frac{10^6}{5} = 200{,}000\,\text{Hz} = 200\,\text{kHz}$$

Pulse circuits find applications in almost all electronic-based industries. Various types of pulse code modulations are employed in communication systems whereas radars utilize pulses to track targets. Digital computers require circuits that can be switched very rapidly between two states by using appropriate pulses.

(*b*) Square Wave

It is shown in Fig. 65.22 (*b*) and is, in fact, a pulse waveform with a mark-to-space ratio of 1:1

Such square waves or pulses are used

1. for audio frequency note generation,
2. for digital electronic switching as in computers, 3, in radars,
4. as synchronizing pulses in TV,
5. for switching of high-power electronic circuits such as thyristor circuits.

(*c*) Sawtooth Wave

It is shown in Fig. 65.22 (*c*). Such waves are used

1. in the scanning circuits of cathode-ray tubes (CRT),
2. in timing circuits where the time for the wave to proceed from one level to another is measured, such as that produced in an integrating circuit.

(*d*) Triangular Wave

It is shown in Fig. 65.22 (*d*). Such waves are often used

1. in scanning circuits where a uniform left-to-right scan is required as in computer displays,
2. for audio frequency note generation,
3. in timing circuits for electronic applications.

65.25. Classification of Non-sinusoidal Oscillators

Those oscillators which generate waveforms other than sine waveform are called **non-sinusoidal oscillators** or **relaxation oscillators**. Non-sinusoidal waveforms include: square, rectangular, sawtooth and pulse-shaped waves as shown in Fig. 65.22.

A relaxation oscillator may be defined ***as a circuit in which voltage or current changes abruptly from one value to another*** and which continues to oscillate between these two values as long as dc power is supplied to it.

We will consider the following three types of such oscillators :

1. Sawtooth generators **2.** Blocking oscillators **3.** Multivibrators (MV)

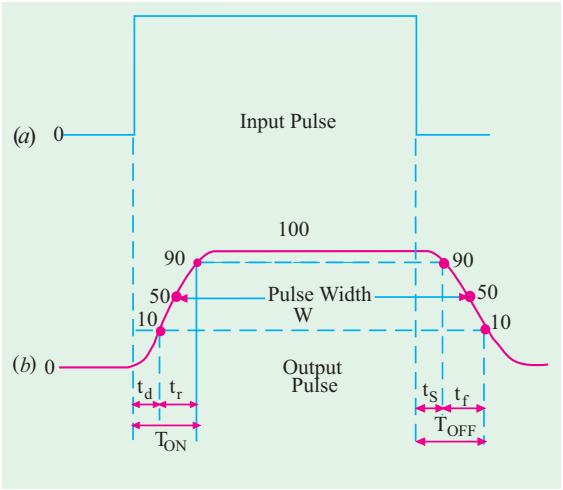


Fig. 65.23

65.26. Pulse Definitions

Due to capacitive effects in a transistor (or to circuit elements external to it), its **output does not directly follow its input**. For example, if we apply a square input pulse to its *E/B* junction, some amount of time lapses be-

fore I_C starts to rise. Similarly, when the input becomes zero, there is some time lapse before I_C starts to decrease. There is always some time delay between the application of input and change in the output. In order to measure how quickly the output changes *i.e.* in order to define the switching (*i.e.* OFF/ON) characteristics of a transistor, we will define the following few terms. It will be assumed that perfect square wave, as shown in Fig. 65.23 (*a*), has been applied at the input. The output wave and the various time delays are shown in Fig. 65.23 (*b*).

1. Time delay, t_d

It is the time interval between the beginning of the input pulse and the time the output voltage (or current) reaches 10 per cent of its maximum value.

It depends on **(*i*)** depletion region capacitances, **(*ii*)** turn-on base current and **(*iii*)** value of transistor β.

2. Rise time, t_r

It is the time taken by the output voltage (or current) to rise from 10% to 90% of its maximum value.

It primarily depends on diffusion capacitance C_D of the transistor (Art. 1.4).

3. Turn-on time T_{ON}

It is equal to the sum of the delay time and rise time *i.e.* $T_{ON} = t_d + t_r$

4. Storage time, T_s

It is the time interval between the end of the input pulse (trailing edge) and the time when output voltage (or current) falls to 90% of its initial maximum value.

It depends on the degree of saturation. Deeper the transistor is driven into saturation, more the stored charge that has to be removed and hence longer the storage time. That is why non-saturated switching is often preferred.

5. Fall time, t_f

It is the time interval during which the output voltage (or current) falls from 90% of its maximum value to 10%.

In simple words, it is the time interval between 90% and 10% levels of the output pulse.

6. Turn-off time, TO_{FF}

It is equal to the sum of storage time and fall time *i.e.* $T_{OFF} = t_s + t_f$

For a fast switching transistor, T_{ON} and T_{OFF} must be of the order of nanoseconds.

7. Pulse Width, *W*

It is the time duration of the output pulse measured between two 50% levels of the rising and falling waveform.

65.27. Basic Requirements of a Sawtooth Generator

The essential requirements of a sawtooth generator are :

1. a dc power source, **2.** a switching device (neon tube, thyratron, thyristor, UJT etc.),
3. a capacitor, **4.** a resistor.

Circuit Action

The *V/I* characteristics of the *RC* circuit play an important role in the operation of such a generator. By restricting the time interval equal to the time constant $\lambda = CR$, only the rising portion of the characteristic (*OA* in Fig. 65.24) which is almost a straight line, is utilized. For periods of time greater than λ , the rising portion of the characteristic is no longer a straight line and hence cannot be utilized.

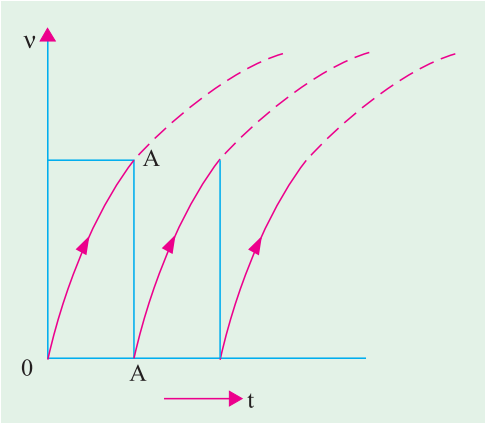


Fig. 65.24

The frequency of the wave is given by the reciprocal of time which elapses between the two waves. In Fig. 65.24, $f = 1/\lambda$.

65.28. UJT Sawtooth Generator

The circuit is shown in Fig. 65.25. It consists of a power source, a unijunction transistor and an *R-C* network.

Circuit Action

When *S* is initially closed, following chain of events takes place :

1. a small current is set up through R_2 and R_1 *via* B_2 and B_1 and an initial reverse bias is established across the E/B_1 junction;
2. at the same time, *C* begins to get charged through R_E and voltage across it increases exponentially with time towards the target voltage *V*;
3. when capacitor voltage equals the emitter firing (or peak point) voltage V_P, E/B_1 junction becomes forward-biased and the emitter goes into the negative region of its characteristic;
4. being forward-biased, E/B_1 junction offers very low resistance. Hence, *C* starts discharging through B_1 and R_1 at a rate determined mainly by E/B_1 junction resistance and R_1;
5. as capacitor voltage approaches zero, the E/B_1 junction again becomes reverse-biased and so stops conducting;
6. we revert to the initial state where *C* begins to charge and the whole cycle of circuit actions is repeated.

The emitter voltage waveform is shown in Fig. 65.25 (*b*). As seen, V_E rises exponentially towards the target voltage *V* but drops to a very low value after it reaches the value V_P due to sudden conduction through E/B_1 junction. Since R_E is large (10 *K* or so) charging rate is comparatively slow but discharge is much quicker since R_1 is very small (50 Ω or less). ***This slow charge and fast discharge produces a sawtooth wave.***

The time required for v_E to rise to V_P is given by

$$T = kR\,C \qquad \text{where} \quad k = log_e^{\,1/(1-\eta)} \text{ and}$$

$$\eta = \text{intrinsic stand-off ratio} = \frac{R_{B1}}{R_{B1} + R_{B2}} \qquad \text{...Art 64.2}$$

Here, R_{B1} and R_{B2} are the internal inter-base resistances.

The frequency of oscillation of the *UJT* or of the output sawtooth wave is

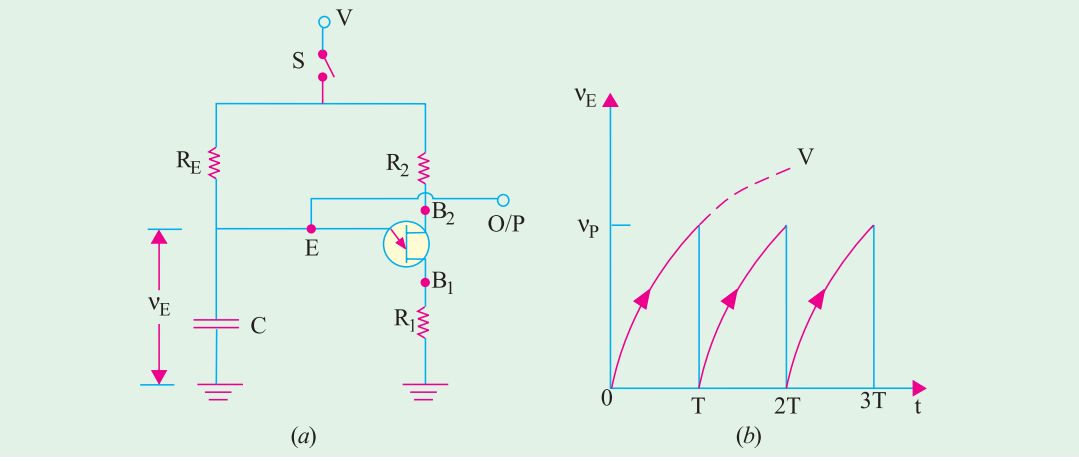


Fig. 65.25

$$f_0 = \frac{1}{T} = \frac{1}{kRC}$$

Its amplitude is determined primarily by applied voltage V and V_P.

Applications

Sawtooth voltage waves are commonly used as

1. sweep voltages at the picture tubes of TV receivers,
2. as sweep voltages of the viewing screens of oscilloscopes and radar equipment.

Example 65.15. *For the UJT oscillator circuit shown in Fig. 65.25, R_E = 10 K, η = 0.75. If the required oscillator frequency is 1 kHz, find the value of C.*

(Components and Devices, Pune Univ.)

Solution. $k = \log_e 1/(1- 0.75) = \log_e 4 = 2.3 \log_{10} 4 \cong 1.4$

Now, $f_0 = \dfrac{1}{kRC}$ $\therefore$ $1 \times 10^3 = \dfrac{1}{1.4 \times 10 \times 10^3 \times C}$ or $C = \mathbf{0.07\ \mu F}$

65.29. Multivibrators (MV)

These devices are very useful as pulse generating, storing and counting circuits. They are basically **two-stage amplifiers with positive feedback from the output of one amplifier to the input of the other**. This feedback (Fig. 65.26) is supplied in such a manner that one transistor is driven to saturation and the other to cut-off. It is followed by new set of conditions in which the saturated transistor is driven to cut-off and the cut-off transistor is driven to saturation.

There are three basic types of *MVs* distinguished by the type of coupling network employed.

1. astable multivibrator (*AVM*),
2. monostable multivibrator (*MMV*),
3. bistable multivibrator (*BMV*).

The first one is the *non-driven type* whereas the other two are the *driven type* (also called triggered oscillators).

1. Astable Multivibrator (AMV)

It is also called ***free-running relaxation oscillator.*** It has no stable state but only two quasi-stable (half-stable) states between which it keeps oscillating continuously of its own accord without any external excitation.

In this circuit, neither of the two transistors reaches a stable state. When one is ON, the other is OFF and they continuously switch back and forth at a rate depending on the *RC* time constant in the circuit. Hence, it oscillates and produces pulses of certain mark-to-space ratio. Moreover, two outputs (180° out of phase with each other) are available.

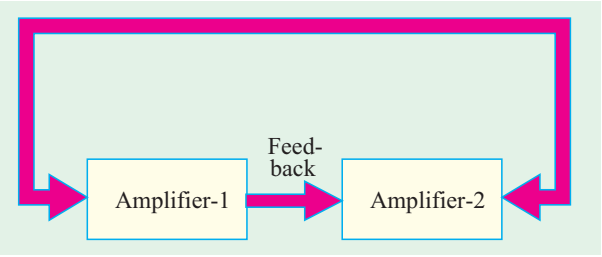


Fig. 65.26

It has two energy-storing elements ***i.e. two capacitors.***

2 . Monostable Multivibrator (MMV)



Multivibrator

It is also called a **single-shot** or **single swing** or a **one-shot** multivibrator. Other names are : ***delay multivibrator and univibrator.*** It has

(i) one absolutely stable (stand-by) state and ***(ii)*** one quasi-stable state.

It can be switched to the quasi-stable state by an external trigger pulse but it returns to the stable condition after a time delay determined by the value of circuit components. It supplies a single output pulse of a desired duration for every input trigger pulse.

It has one energy-storing element *i.e.* **one-capacitor**.

3. Bistable Multivibrator (BMV)

It is also called Eccles-Jordan or flip-flop multivibrator. It has **two absolutely stable states**. It can remain in either of these two states unless an external trigger pulse switches it from one state to the other. Obviously, ***it does not oscillate***. It has **no energy storage element**.

Detailed discrete circuits for the above MVs are discussed below after listing their uses.

65.30. Uses of Multivibrators

Some of their uses are :

1. as frequency dividers,
2. as sawtooth generators,
3. as square wave and pulse generators,
4. as a standard frequency source when synchronized by an external crystal oscillator,
5. for many specialised uses in radar and TV circuits,
6. as memory elements in computers.

65.31. Astable Multivibrator

Fig. 65.27 shows the circuit of a symmetrical collector-coupled AMV using two similar transistors. It, in fact, consists of two *CE* amplifier stages, each providing a feedback to the other. The feedback ratio is unity and positive because of 180° phase shift in each stage. Hence, the circuit oscillates. Because of the very strong feedback signal, the transistors are driven either to saturation or to cut-off (they do not work on the linear region of their characteristics).

The transistor Q_1 is forward-biased by V_{CC} and R_1 whereas Q_2 is forward-biased by V_{CC} and R_2. The collector-emitter voltages of Q_1 and Q_2 are determined respectively by R_{L1} and R_{L2} together with V_{CC}. The output of Q_1 is coupled to the input of Q_2 by C_2 whereas output of Q_2 is coupled to Q_1 by C_1.

Note that it is not essential to draw the coupling leads at 45° to the vertical as shown but it is usually done because it helps to identify the circuit immediately as MV.

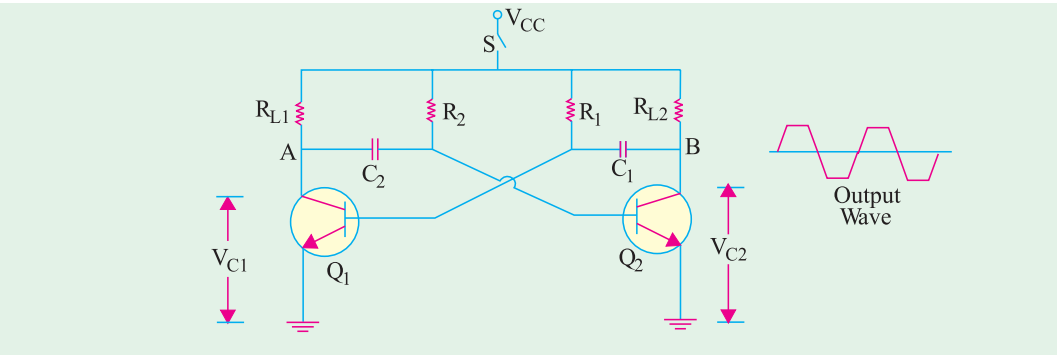


Fig. 65.27

The output can be taken either from point *A* or *B* though these would be phase-reversed with respect to each other as shown in Fig. 65.27.

Circuit Operation

The circuit operation would be easy to understand if it is remembered that due to feedback ***(i)*** when Q_1 is ON, Q_2 is OFF and ***(ii)*** when Q_2 is ON, Q_1 is OFF.

When the power is switched on by closing *S*, one of the transistors will start conducting before the other does (or slightly faster than the other). It is so because characteristics of no two seemingly similar transistors can be exactly alike. Suppose that Q_1 starts conducting before Q_2 does. The feedback system is such that Q_1 will be very rapidly driven to saturation and Q_2 to cut-off.

The following sequence of events will occur :

1. Since Q_1 is in saturation, whole of V_{CC} drops across R_{L1}. Hence, $V_{C1} = 0$ and point A is at zero or ground potential.
2. Since Q_2 is in cut-off *i.e.* it conducts no current, there is no drop across R_{L2}. Hence, point B is at V_{CC}.
3. Since A is at 0 V, C_2 starts to charge through R_2 towards V_{CC}.
4. When voltage across C_2 rises sufficiently (*i.e.* more than 0.7 V), it biases Q_2 in the forward direction so that it starts conducting and is soon driven to saturation.
5. V_{C2} decreases and becomes almost zero when Q_2 gets saturated. The potential of point B decreases from V_{CC} to almost 0 V. This potential decrease (negative swing) is applied to the base of Q_1 through C_1. Consequently, Q_1 is pulled out of saturation and is soon driven to cut-off.
6. Since, now, point B is at 0 V, C_1 starts charging through R_1 towards the target voltage V_{CC}.
7. When voltage of C_1 increases sufficiently, Q_1 becomes forward-biased and starts conducting. In this way, the whole cycle is repeated.

It is seen that the circuit alternates between a state in which Q_1 is ON and Q_2 is OFF and a state in which Q_1 is OFF and Q_2 is ON. The time in each state depends on RC values. Since each transistor is driven alternately into saturation and cut-off the voltage wavefrom at either collector (points A and B in Fig. 65.27) is essentially a square waveform with a peak amplitude equal to V_{CC} (Fig. 65.28).

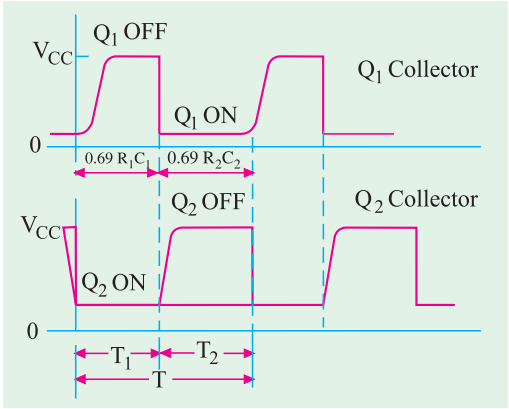


Fig. 65.28

Switching Times

It can be proved that off-time for Q_1 is $T_1 = 0.69 R_1C_1$ and that for Q_2 is $T_2 = 0.69 R_2C_2$.

Hence, total time-period of the wave is

$T = T_1 + T_2 = 0.69 (R_1C_1 + R_2 C_2)$

If $R_1 = R_2 = R$ and $C_1 = C_2 = C$ *i.e.* the two stages are symmetrical, then $T = 1.38\ RC$

Frequency of Oscillation

It is given by the reciprocal of time period,

$$\therefore \quad f = \frac{1}{T} = \frac{1}{1.38\,RC} = \frac{0.7}{RC}$$

Minimum Values of β

To ensure oscillations, the transistors must saturate for which minimum values of β are as under :

$$\beta_1 = \frac{R_1}{R_{L1}} \qquad \text{and} \qquad \beta_2 = \frac{R_2}{R_{L2}}$$

If $R_1 = R_2 = R$ and $R_{L1} = R_{L2} = R_L$. then $\beta_{min} = \dfrac{R}{R_L}$

Example 65.16. *Determine the period and frequency of oscillation for an astable multivibrator with component values: $R_1 = 2$ K, $R_2 = 20$ K, $C_1 = 0.01$ μF and $C_2 = 0.05$ μF.*

Solution. $T_1 = 0.69 \times 2\text{ K} \times 0.01\ \mu\text{F} = 13.8\ \mu\text{s}$

and $T_2 = 0.69 \times 20\ k \times 0.05\ \mu\text{F} = 690\ \mu\text{s}$

$\therefore$ $T = T_1 + T_2 = 13.8\ \mu\text{s} + 690\ \mu\text{s} = 703.8\ \mu\text{s}$

$$\therefore \quad f_0 = \frac{1}{703.8\,\mu\text{s}} = \textbf{1.42 kHz}$$

Example. 65.17. *In the AMV circuit of Fig. 65.27,* $R_1 = R_2 = 10$ *K,* $C_1 = C_2 = 0.01$ *μF and* $R_{L1} = R_{L2} = 1$ *K. Find*

(a) frequency of circuit oscillation, (b) minimum value of transistor β.

(Digital Electronics, Bombay Univ.)

Solution. **(*a*)** $T_1 = T_2 = 0.69 \times 10 \times 10_3 \times 0.01 \times 10^{-6} = 69$ μs

$\therefore \quad T = T_1 + T_2 = 2 \times 69 = 138$ μs $\quad \therefore$

$$f_0 = \frac{1}{138 \times 10^{-6}} = \textbf{7.25 kHz}$$

(*b*) $\beta_{min} = \dfrac{R}{R_L} = \dfrac{10 \times 10^3}{1 \times 10^3} = \textbf{10}$

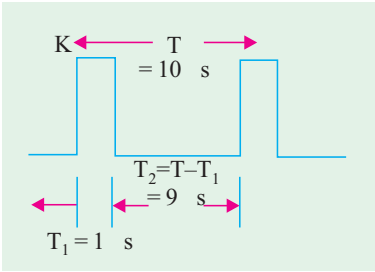


Fig. 65.29

Example 65.18. *Determine the value of capacitors to be used in an astable multivibrator to provide a train of pulses 1*μs *side at a repetition rate of 100 kHz. Given* $R_1 = R_2 = 10$ *K.*

Solution. Fig. 65.29 shows the waveform to be generated by the astable multivibrator. Note that the desired pulse width is actually the time interval $T_1 = 1$ μs and the time period T which equals 10 μs (*i.e.* 1/100 kHz) is the desired repetition time.

Now $\quad T_1 = 0.69\, R_1\, C_1, \quad \therefore \quad C_1 = \dfrac{1\ \mu s}{0.69 \times 10\,K} = \textbf{145pF}$

and $\quad T_2 = 0.69\, R_2\, C_2, \quad \therefore \quad C_2 = \dfrac{9\ \mu s}{0.69 \times 10\,K} = \textbf{1304 pF}$

65.32. Monostable Multivibrator (MMV)

A typical *MMV* circuit is shown in Fig. 65.30. Here, Q_1 is coupled to Q_2 base as in an *AMV* but the other coupling is different. In this multivibrator, a single narrow input trigger pulse produces a single rectangular pulse whose amplitude, pulse width and wave shape depend upon the values of circuit components rather than upon the trigger pulse.

Initial Condition

In the absence of a triggering pulse at C_2 and with *S* closed,

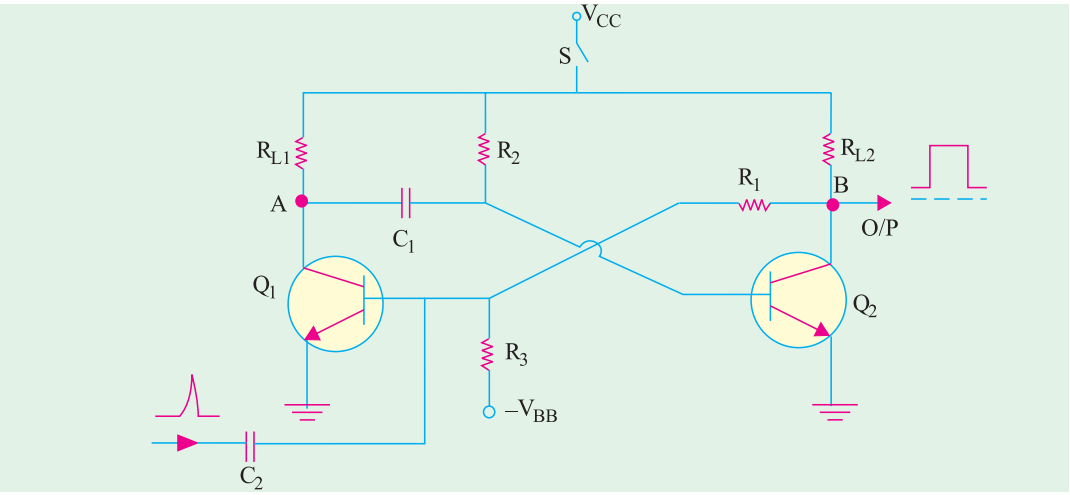


Fig. 65.30

1. V_{CC} provides reverse bias for *C/B* junctions of Q_1 and Q_2 but forward-bias for *E/B* junction of Q_2 only. Hence, Q_2 conducts at saturation.
2. V_{BB} and R_3 reverse bias Q_1 and keep it cut off.
3. C_1 charges to nearly V_{CC} through R_{L1} to ground by the low-resistance path provided by saturated Q_2.

As seen, the initial stable state is represented by

(*i*) Q_2 conducting at saturation and **(*ii*)** Q_1 cut-off.

When Trigger Pulse is Applied

When a trigger pulse is applied to Q_1 through C_2, *MMV* will switch to its opposite unstable state where Q_2 is cut-off and Q_1 conducts at saturation. The chain of circuit actions is as under :

1. If positive trigger pulse is of sufficient amplitude, it will override the reverse bias of the *E/B* junction of Q_1 and give it a forward bias. Hence, Q_1 will start conducting.
2. As Q_1 conducts, its collector voltage falls due to voltage drop across R_{L1}. It means that potential of point *A* falls (negative-going signal). This negative-going voltage is fed to Q_2 *via* C_1 where it decreases its forward bias.
3. As collector current of Q_1 starts decreasing, potential of point *B* increases (positive-going signal) due to lesser drop over R_{L2}. Soon, Q_2 comes out of conduction.
4. The positive-going signal at *B* is fed *via* R_1 to the base of Q_1 where it increases its forward bias further. As Q_1 conducts more, potential of point *A* approaches 0 V.
5. This action is cumulative and ends with Q_1 conducting at saturation and Q_2 cut-off.

Return to Initial Stable State

1. As point A is at almost 0 V, C_1 starts to discharge through saturated Q_1 to ground.
2. As C_1 discharges, the negative potential at the base of Q_2 is decreased. As C_1 discharges further, Q_2 is pulled out of cut-off.
3. As Q_2 conducts further, a negative-going signal from point *B via* R_1 drives Q_1 into cut-off.

Hence, the circuit reverts to its original state with Q_2 conducting at saturation and Q_1 cut-off. It remains in this state till another trigger pulse comes along when the entire cycle repeats itself.

As shown in Fig. 65.30, the output is taken from the collector of Q_2 though it can also be taken from point A of Q_1. The width of this pulse is determined by the time constant of C_1R_2. Since this MV produces one output pulse for every input trigger pulse it receives, it is called ***mono or one-shot multivibrator.***

The width or duration of the pulse is given by $T = 0.69\ C_1R_2$

It is also known as the one-shot period.

Uses

1. The falling part of the output pulse from *MMV* is often used to trigger another pulse generator circuit thus producing a pulse delayed by a time *T* with respect to the input pulse.
2. *MMV* is used for regenerating or rejuvenating old and worn out pulses. Various pulses used in computers and telecommunication systems become somewhat distorted during use. An *MMV* can be used to generate new, clean and sharp pulses from these distorted and used ones.

Example 65.19. *A 20 kHz, 75% duty cycle square (t_p)wave is used to trigger continuously, a monostable multivibrator with a triggered pulse duration of 5μs . What will be the duty cycle of the waveform at output (B) of the monostable multivibrator (refer to Fig. 65.30).*

Solution. Time period of the square wave

$$T = \frac{1}{f} = \frac{1}{20\,\text{kHz}} = 50\,\mu\text{s}$$

Since the duty cycle of the square wave is 75%, therefore the time interval during which the input waveform is at a higher voltage level is, 0.75 × 50 μs = 37.5 μs. Fig. 65.31 (*a*) shows a sketch of the input waveform which is used to trigger the monostable multivibrator.

Now the monostable multivibrator is triggered once each time a new pulse arives. The monostable multivibrator remains triggered only for a duration, t_p = 5 μs. A sketch of the waveform at the output (B) of the monostable multivibrator is as shown in Fig. 65.31(*b*).

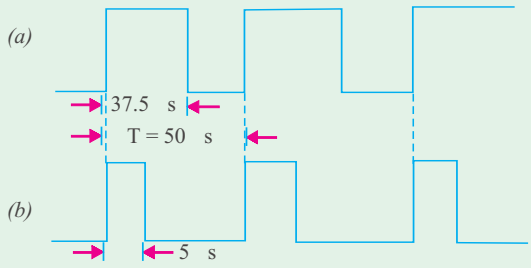


Fig. 65.31

Example 65.20. *A monostable multivibrator is required to convert a 100 kHz, 30% duty cycle square wave to a 100 kHz, 50% duty cycle square wave. Find the values of R_2 and C_2.*

Solution. Fig. 65.32 (*a*) shows a sketch of the input waveform. In order to convert it to a square waveform with 50% duty cycle, we want that the monostable multivibrator must remain triggered for 50% of the time period.

Now time period of 100 kHz square wave,

$$T = \frac{1}{f} = \frac{1}{100\,\text{kHz}} = 10\,\mu\text{s}$$

∴ The duration for which the monostable multivibrator must remain triggered = 50% × 10 μs= 5 μs.

Fig. 65.32 (*b*) shows a sketch of the required waveform at the output of the monostable multivibrator.

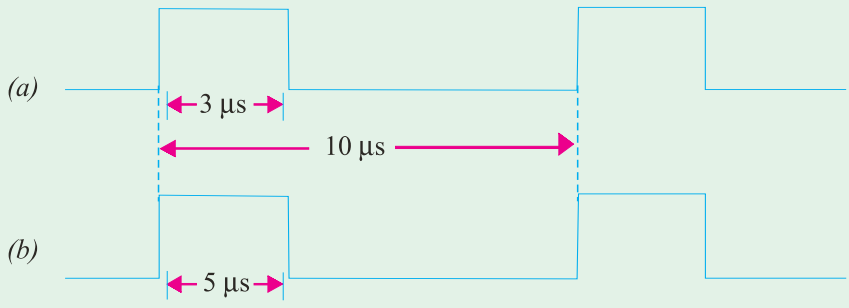


Fig. 65.32

Now width or duration of the pulse,

$T = 0.69\, C_1 R_2$

Let us select *C_1 = 0.001μF, then from the above equation.

* You can chose any value of C which could result in the value of R_2 in kilohms. The reason for this is that we want the current in the circuit to be limited to few milliamperes.

$$R_2 = \frac{T}{0.69C_1} = \frac{5\mu s}{0.69 \times 0.001\mu F} = \mathbf{7.2\ K}$$

65.33. Bistable Multivibrator (BMV)

The basic circuit is shown in Fig. 65.33. As stated earlier, it has **two absolutely stable** states. It can stay ***in one of its two states indefinitely*** (as long as power is supplied) changing to the other state only when it receives a trigger pulse from outside. When it receives another triggering pulse, only then it goes back to its original state. Since one trigger pulse causes the *MV* to 'flip' from one state to another and the next pulse causes it to ***'flop'*** back to its original state, the *BMV* is also popularly known as **'flip-flop'** circuit.

The *BMV* circuit shown in Fig. 65.33 differs from the *AMV* circuit of Fig. 65.27 in the following respects :

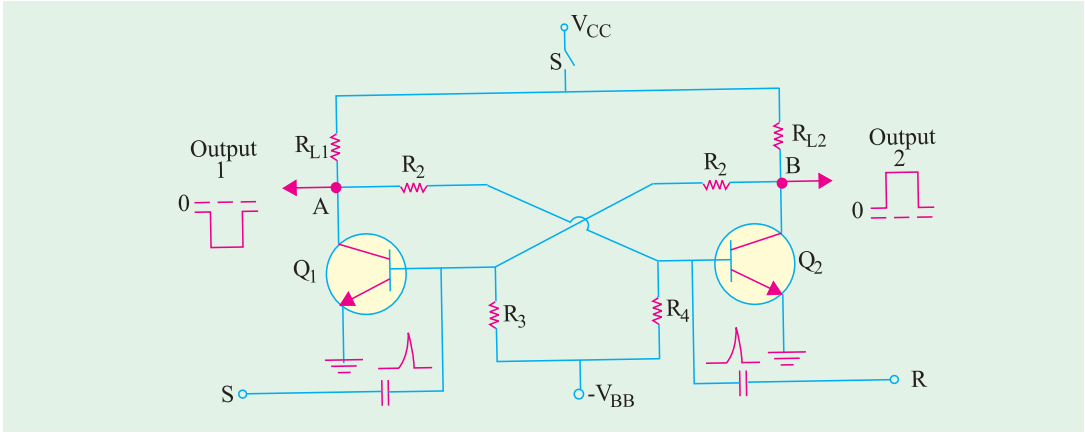


Fig. 65.33

1. the base resistors are not joined to V_{CC} but to a common source $-V_{BB}$,
2. the feedback is coupled through two resistors (not capacitors).

Circuit Action

If Q_1 is conducting, then the fact that point A is at nearly 0 V makes the base of Q_2 negative (by the potential divider $R_2 - R_4$) and holds Q_2 off.

Similarly, with Q_2 OFF, the potential divider from V_{CC} to $-V_{BB}$ (R_{L2}. R_1, R_3) is designed to keep base of Q_1 at about 0.7 V ensuring that Q_1 conducts. It is seen that Q_1 holds Q_2 OFF and Q_3 holds Q_1 ON.

Suppose, now, a positive pulse is applied momentarily to R, it will cause Q_2 to conduct. As collector of Q_2 falls to zero, it cuts Q_1 OFF and, consequently, the *BMV* switches over to its other state.

Similarly, a positive trigger pulse applied to S will switch the *BMV* back to its original state.

Uses

1. in timing circuits as a frequency divider,
2. in counting circuits,
3. in computer memory circuits.

65.34. Schmitt Trigger

The Schmitt trigger (after the name of its invertor) is a **binary circuit** and **closely resembles an MV**. It has two bistable states and the magnitude of the input voltage determines which of the two is possible. It is also called emitter-coupled binary oscillator because positive feedback occurs by coupling through emitter resistor R_E.

The Quiescent Condition

As shown in Fig. 65.34, it consists of two similar transistors Q_1 and Q_2 coupled through R_E. Resistors R_1, R_3 and R_4 form a voltage divider across V_{CC} and $-V_{BB}$ which places a small positive voltage (forward bias) on the base of Q_2. Hence, when power is first switched ON, Q_2 starts conducting. The flow of its current through R_E places a small reverse bias on the base of Q_1, thereby cutting it OFF. Consequently, collector of Q_1 rises to V_{CC}. This positive voltage, coupled to the base of Q_2 through R_3, drives Q_2 into saturation and holds it there.

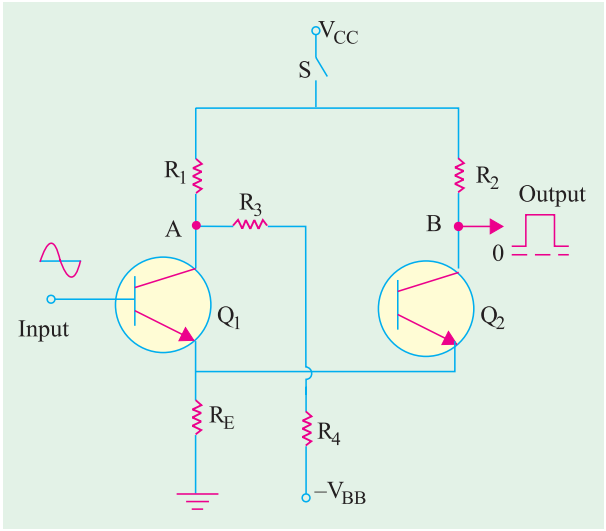


Fig. 65.34

Hence, in the initial static or quiescent condition of the Schmitt trigger,

1. Q_2 is in saturation,
2. Q_1 is cut-off,
3. collector of Q_2 is at 0 V,
4. collector of Q_1 is at V_{CC}.

Circuit Action

Suppose, positive half-cycle of the input ac voltage is applied to the trigger input first. Let us further suppose that this positive voltage is sufficient to overcome the reverse bias on the base of Q_1 placed there by the voltage drop across R_E. Then, the chain of events that follows is as under :

1. Q_1 comes out of cut-off and starts to conduct;
2. as it does so, its collector voltage drops (swings negative);
3. this negative-swinging voltage coupled to the base of Q_2 *via* R_3 reduces its forward bias and hence its emitter current;
4. with reduced emitter current, voltage drop across R_E is reduced;
5. consequently, reverse bias of Q_1 is further lowered and it conducts more heavily;
6. as a result, collector voltage of Q_1 falls further, thereby driving Q_2 still closer to cut-off.

This process is cumulative and ends up with

(a) Q_1 conducting at saturation with its collector voltage almost zero;

(b) Q_2 becoming cut-off with its collector voltage nearly V_{CC}.

Negative Half-cycle of the Input Voltage

Now, when the negative half-cycle of the input voltage is applied

1. Q_1 becomes reverse-biased. Consequently, its collector current falls and collector voltage rises (*i.e.* potential of point A increases towards V_{CC}) ;
2. this positive-swinging voltage is coupled to the base of Q_2 through R_3 and, as a result, Q_2 is driven to saturation;
3. this re-establishes the original conditions of

(a) Q_1 cut off with collector voltage at V_{CC} and

(b) Q_2 at saturation with collector voltage at 0 V.

It completes one cycle. This cycle is repeated as the input voltage rises and falls again. Hence, each cycle of the Schmitt trigger produces a positive-going pulse at its output which is taken out from the collector of Q_2 *i.e.* from point *B* in Fig. 65.34.

Output Pulse Width

It depends on the time during which Q_2 is conducting. It, in turn, depends on the input voltage, within the limits imposed by emitter resistor R_E.

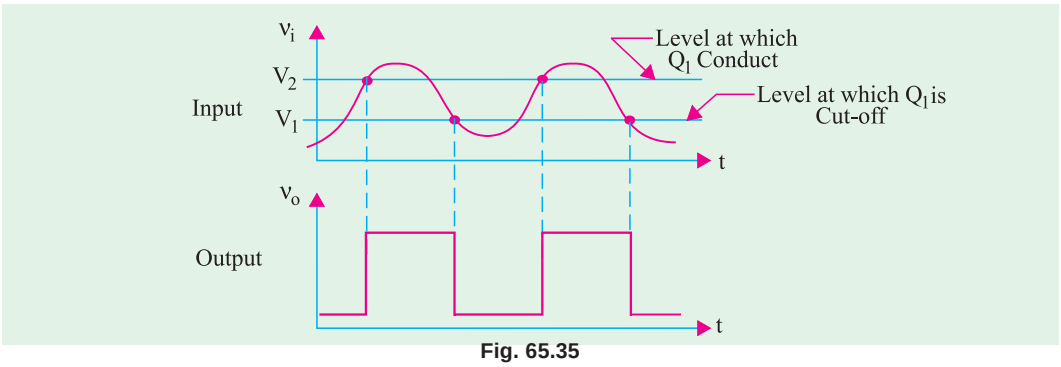


Fig. 65.35

Uses

1. It is frequently used for wave-shaping purposes. As shown in Fig. 65.35, it can convert inputs with any waveshape into output pulses having rectangular or square waveshapes. That is why Schmitt trigger is often called a **'squaring' circuit** or a **'squarer'** circuit.
2. It can reshape worn-out pulses by giving them sharp leading and trailing edges.
3. Since a change of state occurs whenever the input crosses a trigger point, the Schmitt trigger is often used as a **level detector** *i.e.* as a pulse height discriminator.

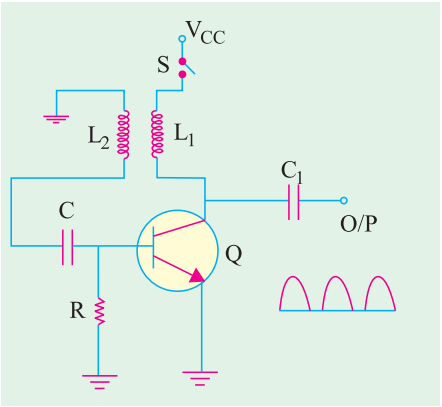


Fig. 65.36

65.35. Transistor Blocking Oscillator

The basic circuit is shown in Fig. 65.36. When *S* is closed, base current rises rapidly due to forward-bias placed by V_{CC} on the *E/B* junction of *Q*. It causes a corresponding increase in its collector current. This rising flow of I_C through L_1 produces an induced *e.m.f.* in L_2. Coupling between L_1 and L_2 is such that lower end of L_2 becomes positive and the grounded end negative.

The positive voltage from L_2 is applied to the base of *Q* through *C*. It further increases the forward-bias of the *E/B* junction which leads to further increase in I_C. Since this process is cumulative, *Q* is quickly driven to saturation. At that point, there is no further increase in I_C and hence, no induced *e.m.f.* in L_2 to be applied to *Q*.

Now, *C* which had been charged earlier, places a negative charge on the base of *Q* which reverse-biases its *E/B* junction and ultimately drives it to cut-off.

The transistor remains at cut-off as *C* now starts to discharge through *R*. When sufficient amount of charge leaks off *C* so that reverse bias of *E/B* junction is removed and forward bias is re-established, *Q* comes out of cut-off and its collector current starts rising once again. Then, the entire cycle of operation is repeated. As shown, the output consists of sharp and narrow pulses.

Tutorial Problems No. 65.1

1. A tuned collector oscillator circuit is tuned to operate at 22 kHz by a variable capacitor set to 2 nF. Find the value of tuned circuit inductance. **(0.026 H)**
2. A tuned collector oscillator operates at 2.2 MHz frequency. At what frequency will it work if its tuned circuit capacitance is reduced by 50% ? **(3.11 MHz)**
3. In a transistorized Hartley oscillator, the tank circuit has the capacitance of 100 pF. The value of inductance between the collector and tapping point is 30 μH and the value of inductance between the tapping point and the transistor base is 100 μH. Determine the frequency of oscillators. Neglect the mutual inductance. **(2.9 MHz)**
4. For the transistor Hartley oscillator circuit shown in Fig. 65.37, find the frequency of operation. Neglect the mutual inductance between the coils. **(73.1 kHz)**

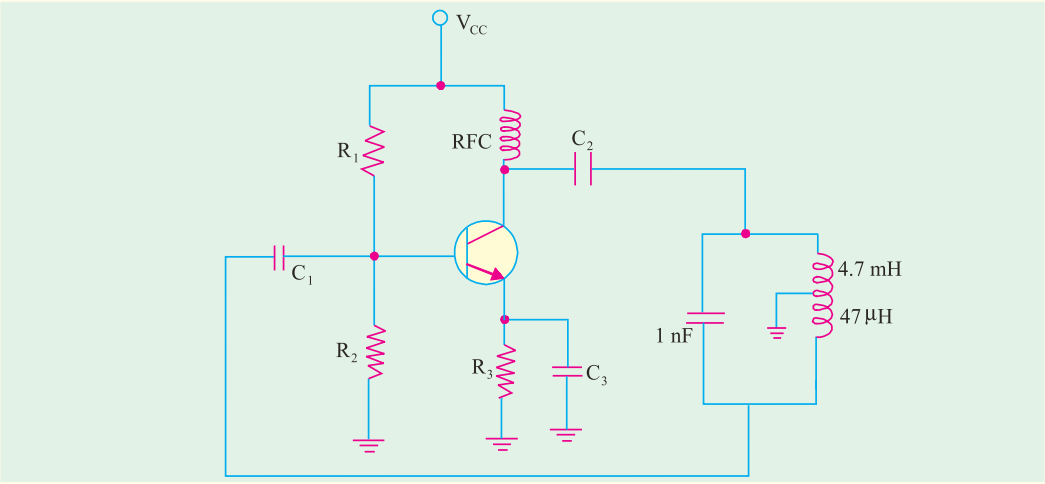


Fig. 65.37

5. A transistor Hartley oscillator is designed with L_1 = 2 mH, L_2 = 20 μH and a variable capacitance. Determine the range of capacitance values if the frequency of operation is varied from 950 kHz to 2050 kHz. **(2.98 pF to 13.9 pF)**
6. In a transistor Colpitts oscillator, C_1 = 0.001μF, C_2 = 0.01 μF and L = 5 μH. Find the required gain for oscillation and the frequency of oscillations. **(0.91 nF and 2.37 MHz)**
7. Determine the frequency of oscillations for the transistor Colpitts oscillator circuit shown in Fig. 65.38. **(24.4 kHz)**
8. A Colpitts oscillator is designed with C_1 = 100 pF, C_2 = 7500 pF. The inductance is variable. Determine the range of inductance values if the frequency of oscillation is to vary between 950 kHz and 2050 kHz. **(61 μH to 284 μH)**

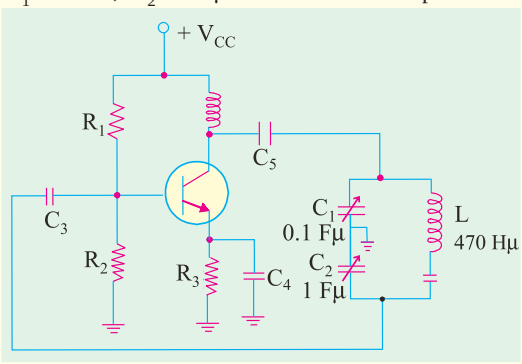


Fig. 65.38

9. The frequency of oscillation of a Colpitts oscillator is given by,

$$f_0 = \frac{1}{2\pi\sqrt{L\left(\frac{C_1C_2}{C_1+C_2}\right)}}$$

where L, C_1 and C_2 are the frequency-determining components. Such a circuit operates at 450 kHz with $C_1 = C_2$. What will be the oscillation frequency if the value of C_2 is doubled.

(389.7 kHz)

10. Calculate the frequency of oscillations for the Clapp oscillator shown in Fig. 65.39. **(734.5 kHz)**

11. A crystal has the following parameters : $L = 0.33$ H, $C_1 = 0.065$ pF, $C_2 = 1$ pF and $R = 5.5$ KΩ. Find the series resonant frequency and Q-factor of the crystal. **(1.09 MHz and 411)**

12. A Wien Bridge oscillator is used for operation at $f_0 = 10$ kHz. The value of R is 100 K, find the value of capacitor, C. Assume $R_1 = R_2 = R$ and $C_1 = C_2 = C$. **(159 pF)**

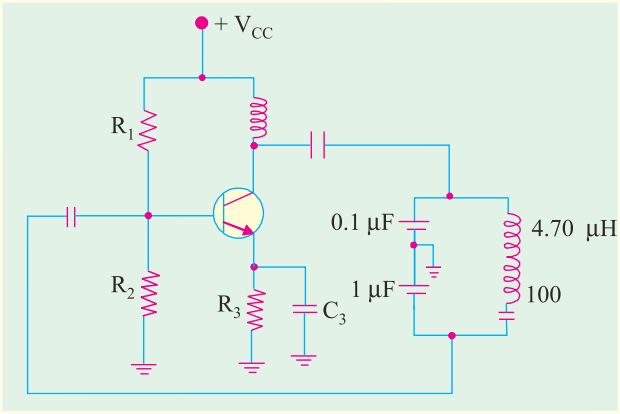


Fig. 65.39

13. *A RC* phase shift oscillator has R = 1kΩ and $C = 0.01$ μF. Calculate the frequency of oscillations ***(Electronics Engg, Annamalai Univ. 2002)***

14. Calculate the frequency and the duty cycle of the output of stable multivibrator and draw the waveform obtained across the capacitor C_T. as shown in Fig. 65.40. ***(Electronics Engg. Bangalore Univ. 2001)***

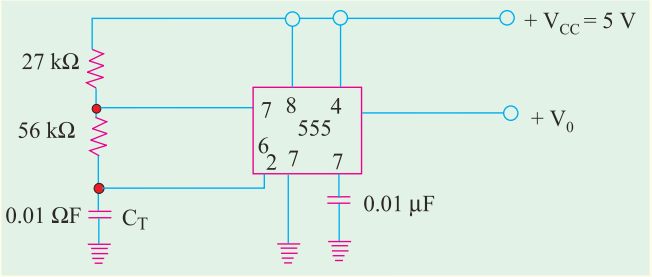


Fig. 65.40

15. In an Hartey oscillator if $L_1 = 0.1$ mH, $L_2 = 10$ μH and mutual inductance between the coils equal to 20 μH. Calculate the value of capacitor C of the oscillatory circuit to obtain frequency of 4110 kHz and also find the condition for sustained oscillations. ***(Electronics Engg: Bangalore Univ. 2001)***

16. Calculate the frequency of oscillation of a colpit oscillator with $C_1 = C_2 = 400$ pF and $L = 2$mH. ***(Electronics Engg., Bangalore Univ. 2002)***

17. In an R-C phase shift oscillator R = 5000 Ω and C = 0.1 MF. Calculate the frequency of oscillations. **(Electronics Engg., Bangalore Univ. 2002**

OBJECTIVE TESTS – 65

1. An electronic oscillator is
(*a*) just like an alternator
(*b*) nothing but an amplifier
(*c*) an amplifier with feedback
(*d*) a converter of ac to dc energy.

2. The frequency of oscillation of an elementary *LC* oscillatory circuit depends on
(*a*) coil resistance
(*b*) coil inductance
(*c*) capacitance
(*d*) both (*b*) and (*c*).

3. For sustaining oscillations in an oscillator
(*a*) feedback factor should be unity
(*b*) phase shift should be 0°
(*c*) feedback should be negative
(*d*) both (*a*) and (*b*).

4. If Barkhausen criterion is not fulfilled by an oscillator circuit, it will
(*a*) stop oscillating
(*b*) produce damped waves continuously
(*c*) become an amplifier
(*d*) produce high-frequency whistles.

5. In a transistor Hartley oscillator
(*a*) inductive feedback is used
(*b*) untapped coil is used
(*c*) entire coil is in the output circuit
(*d*) no capacitor is used

6. A Hartley oscillator is used for generating
(*a*) very low frequency oscillation
(*b*) radio-frequency oscillation
(*c*) microwave oscillation
(*d*) audio-frequency oscillation

7. A Colpitts oscillator uses
(*a*) tapped coil
(*b*) inductive feedback
(*c*) tapped capacitance
(*d*) no tuned *LC* circuit

8. In *RC* phase-shift oscillator circuits.
(*a*) there is no need for feedback
(*b*) feedback factor is less than unity
(*c*) pure sine wave output is possible
(*d*) transistor parameters determine osci- llation frequency.

9. Wien bridge oscillator is most often used whenever
(*a*) wide range of high purity sine waves is to be generated
(*b*) high feedback ratio is needed
(*c*) square output waves are required
(*d*) extremely high resonant frequencies are required.

10. The RC network shown in Fig 65.40 can provide a maximum theoretical phase shift of

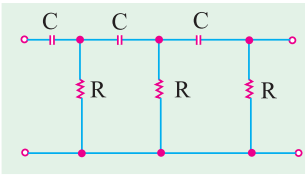


Fig. 65.41

(*a*) 90° (*b*) 180° (*c*) 270° (*d*) 360°
(UPSC Engineering Services 2002)

11. A *CE* amplifier can be converted into oscillator by
(*a*) providing adequate positive feedback
(*b*) phase shifting the output by 180° and feeding this phase-shifted output to the input.
(*c*) using only a series tuned circuit as a load on the amplifier
(*d*) using a negative resistance device as a load on the amplifier
(*e*) all of the above
(UPSC Engineering Service, 2002)

12. The primary advantage of a crystal oscillator is that
(*a*) it can oscillate at any frequency
(*b*) it gives a high output voltage
(*c*) its frequency of oscillation remains almost constant
(*d*) it operates on a very low dc supply voltage.

13. Non-sinusoidal waveforms
(*a*) are departures from sine waveform
(*b*) have low mark-to-space ratio
(*c*) are much easier to generate
(*d*) are unfit for digital operation.

14. A relaxation oscillator is one which
(*a*) has two stable states
(*b*) relaxes indefinitely
(*c*) produces non-sinusoidal output
(*d*) oscillates continuously.

15. A square pulse has a mark-to-space ratio of
(*a*) 1 : 1 (*b*) 1 : 2 (*c*) 2 : 1 (*d*) 1 : 4.

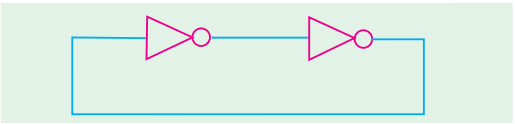
16. Apart from a dc power source, the essential requirements of a sawtooth generator are
(*a*) a resistor (*b*) a capacitor
(*c*) a switching device (*d*) all of the above.

17. Which of the following statement is WRONG ? In a multivibrator
(*a*) output is available continuously
(*b*) feedback between two stages is 100%
(*c*) positive feedback is employed
(*d*) when one transistor is ON, the other is OFF.

18. An MMV circuit
(*a*) has no stable state
(*b*) gives two output pulses for one input trig ger pulse
(*c*) returns to its stand-by states automatically
(d) has no energy-storage element.

19. A BMV circuit
(*a*) has two unstable states
(*b*) has one energy-storage element
(*c*) switches between its two states automatically
(*d*) is not an oscillator.

20. The digital circuit using two inverters shown in Fig. 65.42 will act as
(*a*) a bistable multi-vibrator
(*b*) an astable multi-vibrator
(*c*) a monostable multi-vibrator
(*d*) an oscillator


Fig. 65.42

21. In the Schmitt trigger circuit shown in Fig. 65.43 if $V_{CE(sat)} = 0.1\ V$, the output logic low level (V_{OL}) is
(*a*) 1.25 V
(*b*) 1.35 V
(*c*) 2.50 V
(*d*) 5.00 V

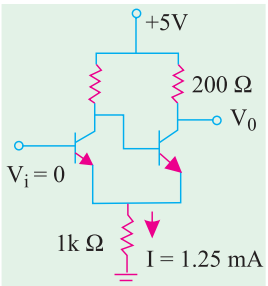


Fig. 65.43

ANSWERS

1. (*c*)	**2.** (*d*)	**3.** (*d*)	**4.** (*a*)	**5.** (*a*)	**6.** (*b*)	**7.** (*c*)	**8.** (*c*)	**9.** (*a*)	**10.** (*b*)	**11.** (*b*)
12. (*c*)	**13.** (*a*)	**14.** (*c*)	**15.** (*a*)	**16.** (*d*)	**17.** (*a*)	**18.** (*c*)	**19.** (*a*)	**20.** (*a*)	**21.** (*b*)	

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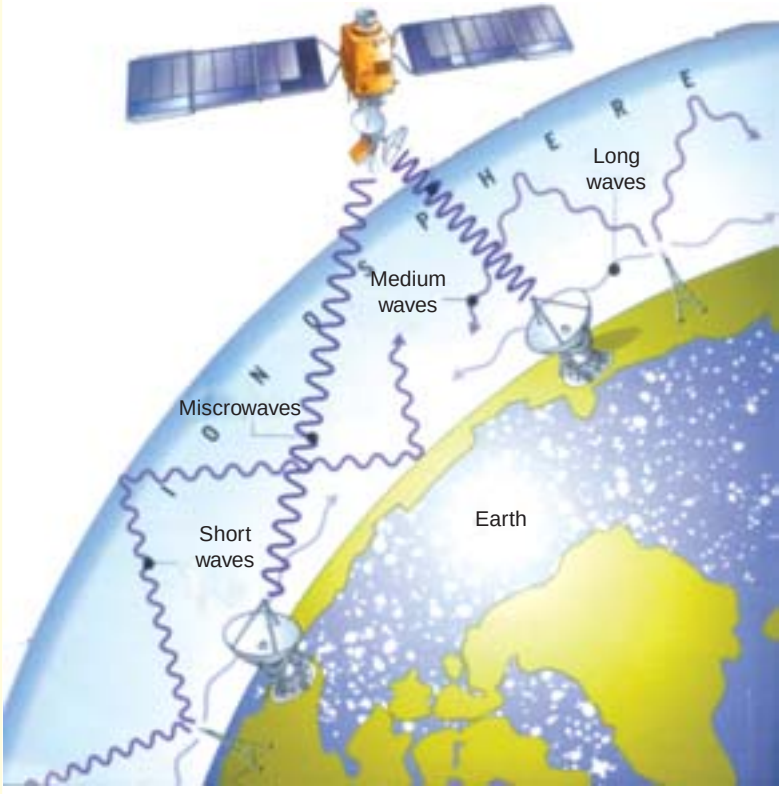
CONTENTS

CHAPTER 66

MODULATION AND DEMODULATION

Learning Objectives

- What is a Carrier Wave?
- Radio Frequency Spectrum
- Sound
- Need for Modulation
- Radio Broadcasting
- Modulation
- Methods of Modulation
- Amplitude Modulation
- Percent Modulation
- Upper and Lower Sidebands
- Mathematical Analysis of a Modulated Carrier Wave
- Power Relation in an AM Wave
- Forms of Amplitude Modulation
- Generation of SSB
- Methods of Amplitude Modulation
- Modulating Amplifier Circuit
- Frequency Modulation
- Modulation Index
- Deviation Ratio
- Percent Modulation
- FM Sidebands
- Modulation Index and Number of Sidebands
- Demodulation or Detection
- Essentials of AM Detection
- Transistor Detectors for AM Signals
- Quadrature Detector
- Frequency Conversion
- Standard Superhet AM Receiver
- FM Receiver
- Comparison between AM and FM
- The Four Fields of FM



The radio waves travel through the air, bounce off a layer of the atmosphere called the ionosphere, or are relayed by satellite

CONTENTS

66.1. Introduction

For successful transmission and reception of intelligence (code, voice, music etc.) by the use of radio waves, two processes are essential :

(*i*) modulation and (*ii*) demodulation.

Speech and music etc. are sent thousands of kilometres away by a radio transmitter. The scene in front of a television camera is also sent many kilometres away to viewers. Similarly, a Moon probe or Venus probe checking its environments, sends the information it gathers millions of kilometres through space to receivers on earth. In all these cases, the carrier is the high-frequency radio wave. The intelligence i.e. sight, sound or other data collected by the probe is impressed on the radio wave and is carried along with it to the destination.

Modulation is the process of **combining the low-frequency signal with a very high-frequency radio wave called carrier wave (CW)**. The resultant wave is called **modulated carrier wave**. This job is done at the transmitting station.

Demodulation is the process of separating or **recovering the signal from the modulated carrier wave**. It is just the opposite of modulation and is performed at the receiving end.

66.2. What is a Carrier Wave?

It is a high-frequency undamped radio wave produced by radio-frequency oscillators (Chapter 65). As seen from Fig. 66.1, the output of these oscillators is first amplified and then passed on to an antenna. This antenna radiates out these high-frequency (electromagnetic) waves into space. These waves have constant amplitude and travel with the velocity of light. They are inaudible *i.e.* by themselves they cannot produce any sound in the loudspeaker of a receiver. As their name shows, their job is to **carry the signal** (audio or video) from transmitting station to the receiving station. The resultant wave is called **modulated** carrier wave.

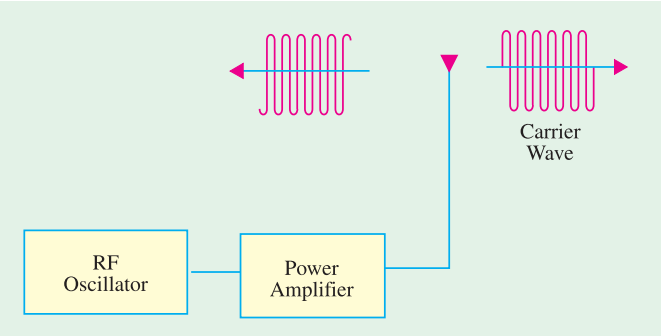


Fig. 66.1

66.3. Radio Frequency Spectrum

Radio frequencies used by different communication systems extend from very low frequencies to extra high frequencies as tabulated below along with their acronym abbreviations.

Table No. 66.1

Frequency	*Designation*	*Abbreviation*	*Uses*
3-30 kHz	very low frequency	VLF	long distance telegraphy broadcasting
30-300 kHz	low frequency	LF	long distance point-to-point service, navigational aids, sound broadcasting and line carrier systems.
300 kHz-3MHz	medium frequency	MF	sound broadcasting, ship-shore services and line carrier systems
3-30 MHz	high frequency	HF	medium and long-distance point-to-point services, sound broadcasting, linear carrier systems.

30-300 MHz	very high frequency	VHF	short-distance communication, TV and
300 MHz-3GHz	ultra high frequency	UHF	sound broadcasting, radar
3-30 GHz	super high frequency	SHF	outer-space radio communication,
30-300 GHz	extra high frequency	EHF	point-to-point microwave communication systems and radar.

Generally, none of the frequencies above 300 GHz is classified as radio waves.

66.4. Sound

It is a sort of disturbance which requires some physical medium for its propagation. Human voice consists of a series of compressions and rarefactions which travel through air with a velocity of about 345 m/s. The frequency range of human voice is from 20-4000 Hz which lies within the audible range of 20 to 20,000 Hz. Variations in human voice can be converted into corresponding variations in electric current with the help of a microphone as shown in Fig.66.2.

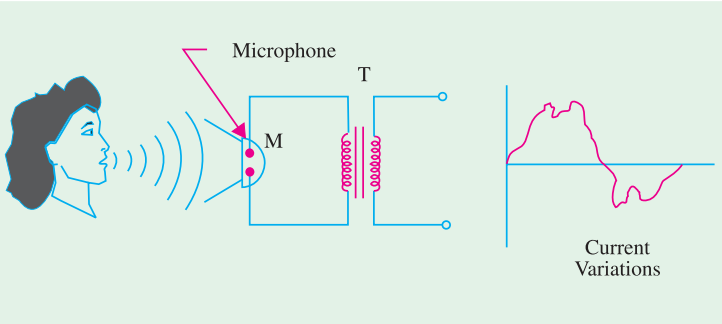


Fig. 66.2

When a sound wave strikes the microphone, it produces AF sound current. The positive half-cycles of sound current are produced by the compressions and negative half cycles by rarefactions.

As seen, human voice does not produce pure sinusoidal current because it does not consist of one frequency alone. It is quite complex and can be analysed to consist of a fundamental (or lowest frequency) and its integral multiple frequencies called **overtones** or **harmonics**.

We are interested in two main characteristics of sound :

(*i*) **Intensity**—It is the energy content of the wave. It depends on its amplitude. In fact, intensity of a wave is directly proportional to the square of its amplitude *i.e.* $I \propto a^2$. Sensation of loudness felt by a listener depends directly on the intensity of the wave falling on his ears.

(*ii*) **Frequency**—It produces the sensation called pitch. Audible sounds have a frequency range from 20 Hz to 20,000 Hz.



Sound wave strikes the microphone and it produces AF sound

Though every sound has complex frequency structure, we will consider only single-frequency sound whose current wave is a pure sine wave as shown in Fig. 66.3. It will be used as the modulating signal when discussing the process of modulation.

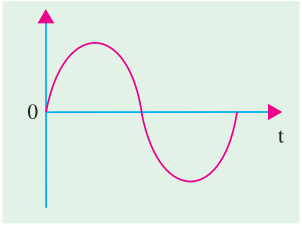


Fig. 66.3

66.5. Need for Modulation

Sometimes, beginners question the necessity of modulation *i.e.* using a carrier wave to carry the low-frequency signal from one place to another. Why not transmit the signals directly and save lot of botheration? Unfortunately, there are three main hurdles in the process of such direct transmission of audio-frequency signals :

1. They have relatively short range,
2. If everybody started transmitting these low-frequency signals directly, mutual interference will render all of them ineffective
3. Size of antennas required for their efficient radiation would be large i.e. about 75 km as explained below.

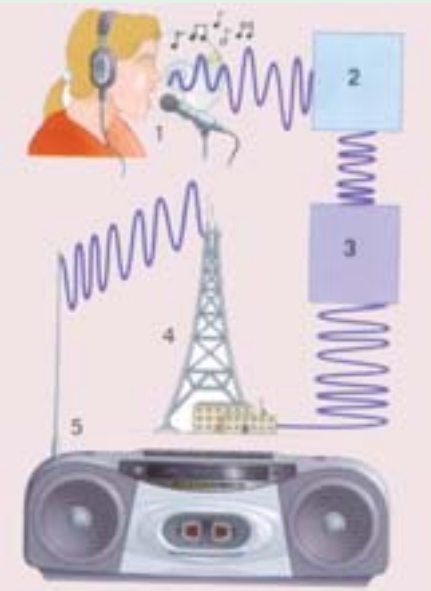
For efficient radiation of a signal, the minimum length of an antenna is one quarter wavelength ($\lambda/4$). The antenna length L is connected with the frequency of the signal wave by the relation $L = 75 \times 10^6/f$ metres. For transmitting an audio signal of $f = 1000$ Hz, $L = 75 \times 10^6/10^3 = 75{,}000$ m = 75 km ! In view of this immense size of antenna length, it is impractical to radiate audio-frequency signals directly into space.

Hence, the solution lies in modulation which enables a low-frequency signal to travel very large distances through space with the help of a high-frequency carrier wave. These carrier waves need reasonably-sized antennas and produce no interference with other transmitters operating in the same area.

66.6. Radio Broadcasting

Let us see how radio broadcasting stations broadcast speech or music etc. from their broadcasting studios.

First, the speech or music which consists of a series of compressions and rarefactions is translated into a tiny varying electric current with the help of a crystal microphone. The frequency of variations of this current lies in the audio-range, hence it is known as audio frequency signal. The audio-frequency signal cannot be radiated out from the antenna directly because transmission at audio-frequencies is not practical. For this purpose, oscillations of very high frequency or radio-frequency are produced with the help of any one of the oscillators discussed in Chapter 15. The electromagnetic waves so produced are of constant amplitude but of extremely high frequency. These waves, which are neither seen nor heard, travel through space with the velocity of light *i.e.* 3×10^8 m/s (approx). The audio-frequency signal which is to be broadcast, is then superimposed on the *RF* waves, which are known as carrier waves (because they carry *A.F.* signal through space to distant places). In a way, the carrier waves can be likened to a horse and the audio-frequency signal to a rider. The process by which *AF* signal or information is impressed on the carrier wave is known as modulation. The horse and rider travel through space. At the receiving end, they strike the receiving aerial and enter the receiver which separates the horse from the rider. The horse *i.e.* carrier wave is returned and the rider *i.e.* audio-frequency signal is converted back into sound. This process by which the *R.F.* waves and *A.F.* waves are separated is known as **detection** or **demodulation** (because it is the reverse of modulation).



Broadcasting through waves

66.7. Modulation

It is the process of combining ***an audio-frequency (AF) signal with a radio frequency (RF) carrier wave.*** The AF signal is also called a ***modulating wave*** and the resultant wave produced is called ***modulated wave***.

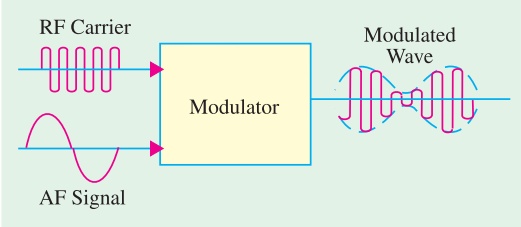


Fig. 66.4

During modulation, some characteristic of the carrier wave is varied in time with the modulating signal and is accomplished by combining the two.

66.8. Methods of Modulation

The mathematical expression for a sinusoidal carrier wave is

$$e = E_C \sin(\omega_c t + \phi) = E_C \sin(2\pi f_c t + \phi)$$

Obviously, the waveform can be varied by any of its following three factors or parameters :

1. E_C — the amplitude, **2.** f_c — the frequency, **3.** ϕ — the phase.

Accordingly, there are three types of sine-wave modulations known as :

1. **Amplitude Modulation (AM)**
   Here, the information or *AF* signal changes the amplitude of the carrier wave without changing its frequency or phase.
2. **Frequency Modulation (FM)**
   In this case, the information signal changes the frequency of the carrier wave without changing its amplitude or phase.
3. **Phase Modulation (PM)**
   Here, the information signal changes the phase of the carrier wave without changing its other two parameters.

66.9. Amplitude Modulation

In this case, the **amplitude** of the carrier wave is varied in proportion to the **instantaneous amplitude** of the information signal or AF signal. Obviously, the amplitude (and hence the intensity) of the carrier wave is changed ***but not its frequency***. Greater the amplitude of the AF signal, greater the fluctuations in the amplitude of the carrier wave.

The process of amplitude modulation is shown graphically in Fig. 66.5. For the sake of simplicity, the *AF* signal has been assumed sinusoidal [Fig. 66.5 (*a*)]. The carrier wave by which it is desired to transmit the *AF* signal is shown in Fig. 66.5 (*b*). The resultant wave called modulated wave is shown in Fig. 66.5 (*c*).

The function of the modulator is to mix these two waves in such a way that (*a*) is transmitted along with (*b*). All stations broadcasting on the standard broadcast band (550-1550 kHz) use *AM* modulation.

If you observe the envelope of the modulated carrier wave, you will realize that it is an exact replica of the *AF* signal wave.

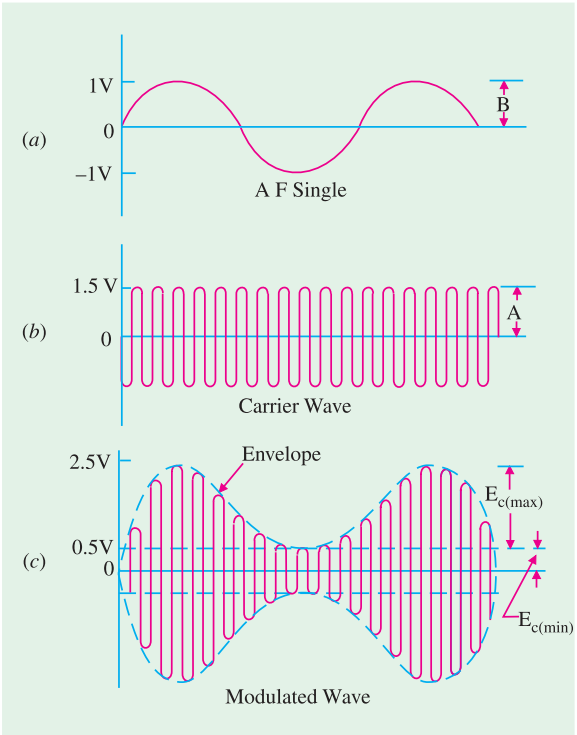


Fig. 66.5

In summary

(i) **fluctuations** in the amplitude of the carrier wave depend on the **signal amplitude**,

(ii) **rate** at which these fluctuations take place depends on the **frequency** of the audio signal.

66.10. Percent Modulation

It indicates the degree to which the AF signal modulates the carrier wave

$$m = \frac{\text{maximum value of signal wave}}{\text{maximum value of carrier wave}} \times 100 = \frac{\text{signal amplitude}}{\text{carrier amplitude}} \times 100$$

$$= \frac{B}{A} \times 100 \quad \text{—Fig. 66.5}$$

The ratio B/A expressed as a fraction is called **modulation index (MI)**

$$m = M.I. \times 100$$

From Fig. 66.5, it is seen that $B = 1$ V and $A = 1.5$ V

$$\therefore \quad m = \frac{1}{1.5} \times 100 = 66.7\%$$

Modulation may also be defined in terms of the values referred to the modulated carrier wave.

$$m = \frac{E_{c(max)} - E_{c(min)}}{E_{c(max)} + E_{c(min)}} \times 100$$

where $E_{c(max)}$ and $E_{c(min)}$ are the maximum and minimum values of the amplitude of the ***modulated carrier wave***.

Again, from Fig. 66.5 we see that

$$m = \frac{2.5 - 0.5}{2.5 + 0.5} \times 100$$

$$= \frac{2}{3} \times 100 = 66.7\%$$

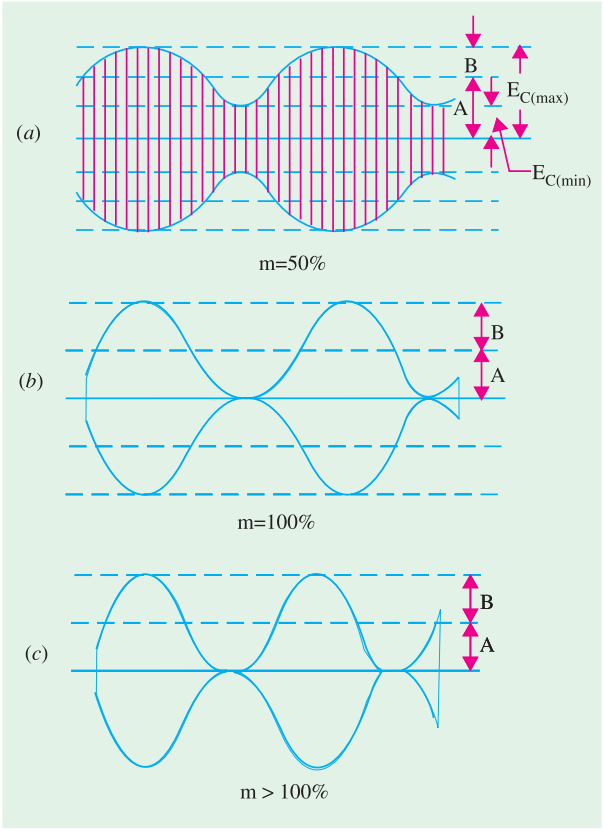


Fig. 66.6

Fig. 66.6 shows a modulated wave with different degrees of modulation. As before, both the signal and carrier ***waves*** are assumed to be sine waves. Smallest value of $m = 0$ *i.e.* when amplitude of the modulating signal is zero. It means that $m = 0$ for an unmodulated carrier wave. Maximum value of $m = 1$ when $B = A$. Value of m can vary anywhere from 0 to 100% without introducing distortion. Maximum undistorted power of a radio transmitter is obtained when $m = 100\%$. If m is less than 100 per cent, power output is reduced though the power content of the carrier is not. Modulation in excess of 100 per cent produces severe distortion and interference (called splatter) in the transmitter output.

Example 66.1 *A modulated carrier wave has maximum and minimum amplitudes of 750 mV and 250 mV. Calculate the value of percentage modulation.*

Solution. The modulated wave is shown in Fig. 66.7.

Here, $E_{c(max)} = 750$ mV and

$E_{c(min)} = 250$ mV

$$\therefore \quad m = \frac{E_{c(max)} - E_{c(min)}}{E_{c(max)} + E_{c(min)}} \times 100$$

$$= \frac{750 - 250}{750 + 250} \times 100 = \mathbf{50\%}$$

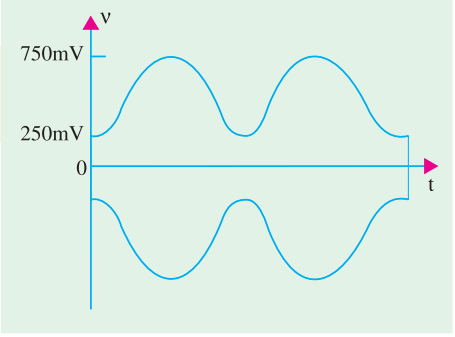


Fig. 66.7

66.11. Upper and Lower Side Frequencies

An unmodulated carrier wave consists of only one single-frequency component of frequency f_c. When it is combined with a modulating signal of frequency fm, heterodyning action takes place.

As a result, two additional frequencies called **side frequencies** are produced. The *AM* wave is found to consist of three frequency components :

1. The original carrier frequency component, f_c .
2. A higher frequency component ($f_c + f_m$). It is called the sum component.
3. A lower frequency component ($f_c - f_m$). It is called the difference component.

The two new frequencies are called the **upper-side frequency** (*USF*) and **lower-side frequency** (*LSF*) respectively and are symmetrically located around the carrier frequency. The modulating frequency remains unchanged but does not appear in the amplifier output because the amplifier's load presents practically zero impedance to this low frequency.

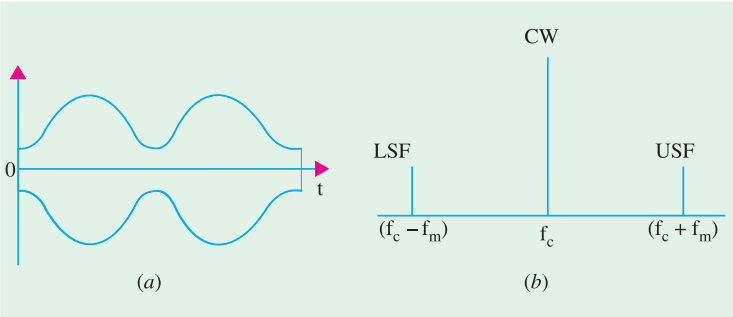


Fig. 66.8

These are shown in time domain in Fig. 66.8 (*a*) and in frequency domain in Fig. 66.8 (*b*). The amplitude of the side frequencies depends on the value of *m*. The amplitude of each side frequency = *mA*/2 where *A* is the amplitude of unmodulated carrier wave.

Example 66.2. *A 10-MHz sinusoidal carrier wave of amplitude 10 mV is modulated by a 5 kHz sinusoidal audio signal wave of amplitude 6 mV. Find the frequency components of the resultant modulated wave and their amplitudes.*

(Electronics & Comm. Engg.; Madras Univ. 1991)

Solution. Here, f_c = 10 MHz and f_m = 5 kHz = 0.005 MHz. The modulated carrier contains the following frequencies :

1. original carrier wave of frequency
   f_c = **10 MHz**
2. USF of frequency
   = 10 + 0.005 = **10.005 MHz**
3. LSF of frequency
   = 10 – 0.005 = **9.995 MHz**

The frequency spectrum is shown in Fig. 66.9

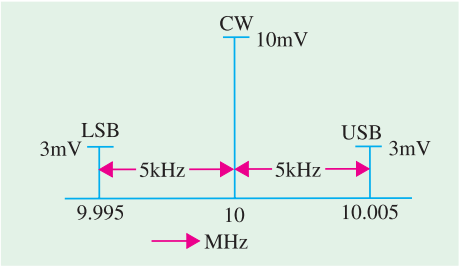


Fig. 66.9

Here, $m = \frac{B}{A} = \frac{6}{10} = 0.6 = 0.6$

Amplitude of *LSF* = *USF* = *mA*/2 = 0.6 × 10/2 = 3 **mV** as shown.

66.12. Upper and Lower Sidebands

In Art 66.11, it was assumed that the modulating signal was composed of one frequency component only. However, in a broadcasting station, the modulating signal is the human voice (or music) which contains waves with a frequency range of 20-4000 Hz. Each of these waves has its own *LSF* and *USF*. When combined together, they give rise to an upper-side **band** (*USB*) and a lower-side **band** (LSB) as shown in Fig. 66.10. The *USB*, in fact, contains all sum components of the signal and carrier frequency whereas *LSB* contains their difference components.

The channel width (or bandwidth) is given by the difference between extreme frequencies i.e. between maximum frequency of *USB* and minimum frequency of *LSB*. As seen,

Channel width = 2 × maximum frequency of modulating signal = $2 \times f_{m\,(max)}$

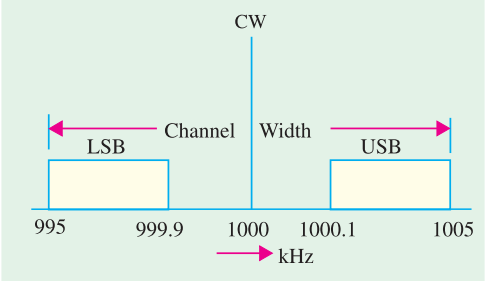


Fig. 66.10

Example 66.3. *An audio signal given by 15 sin 2π (2000 t) amplitude-modulates a sinusoidal carrier wave 60 sin 2π (100,000) t.*

Determine :

(a) *modulation index,* ***(b)*** *percent modulation,* ***(c)*** *frequencies of signal and carrier,*

(d) *frequency spectrum of the modulated wave.*

(Electronics & Telecom Engg., Jadavpur Univ. 1991)

Solution. Here, B = 15 and A = 60

(a) $M.I. = \frac{B}{A} = \frac{15}{60} =$ **0.25**

(b) $m = M.I. \times 100 = 0.25 \times 100 = 25\%$

(c) $f_m =$ **2000 Hz** — by inspection of the given equation

$f_c =$ **100,000 Hz** — by inspection of the given equation

(d) The three frequencies present in the modulated *CW* are

(i) 100,000 Hz = **100 kHz**

(ii) 100,000 + 2000 = 102,000 Hz = **102 kHz**

(iii) 100,000 – 2000 = 98,000 Hz = **98 kHz**

Example 66.4. *A bandwidth of 15 MHz is available for AM transmission. If the maximum audio signal frequency used for modulating the carrier is not to exceed 15 kHz, how many stations can broadcast within this band simultaneously without interfering with each other?*

(Electronics & Telecom Engg.; Pune Univ. 1991)

Solution. *BW* required by each station

$= 2 f_{m(max)} = 2 \times 15 = 30$ kHz

Hence, the number of station which can broadcast within this frequency band without interfering with one another is

$$= \frac{15 \text{ MHz}}{30 \text{ kHz}} = \mathbf{500}$$

Example 66.5. *In a broadcasting studio, a 1000 kHz carrier is modulated by an audio signal of frequency range, 100-5000 Hz. Find* ***(i)*** *width or frequency range of sidebands* ***(ii)*** *maximum and minimum frequencies of USB* ***(iii)*** *maximum and minimum frequencies of LSB and* ***(iv)*** *width of the channel.*

(Electronics & Comm. Engg. IERE, London)

Solution. ***(i)*** Width of sideband = 5000 – 100 = **4900 Hz**

(ii) Max. frequency of USB

= 1000 + 5 = **1005 kHz**

Min. frequency of *USB*

= 1000 + 0.1 = **1000.1 kHz**

(iii) Max. frequency of *LSB*

= 1000 – 0.1 = **999.9 kHz**

Min. frequency of *LSB*

= 1000 – 5 = **995 kHz**

(iv) Width of channel = 1005 – 995 = **10 kHz**

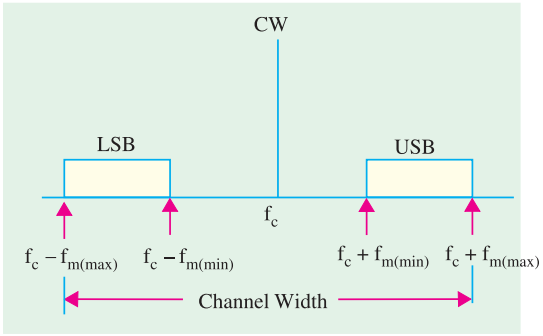


Fig. 66.11

66.13. Mathematical Analysis of a Modulated Carrier Wave

The equation of an unmodulated carrier wave is

$$e_c = E_c \sin 2\pi f_c t = A \sin 2\pi f_c t = A \sin wt$$

where A is the constant amplitude of the carrier wave and $\omega = 2\pi f_c$.

Let the equation of the single-frequency sinusoidal modulating signal be

$$e_m = E_m \sin 2\pi f_m t = B \sin 2\pi f_m t$$
$$= B \sin \pi t \quad \text{— where } p = 2\pi f_m$$

As seen from Fig. 66.12, the amplitude of the modulated carrier wave at any instant is

$$= A + e_m \quad (\therefore A \text{ is constant})$$
$$= (A + B \sin \pi t)$$

Hence, its ***instantaneous*** value is given by

$$e = (A + B \sin pt)\sin \omega t = A \sin \omega t + B \sin \omega t . \sin pt$$

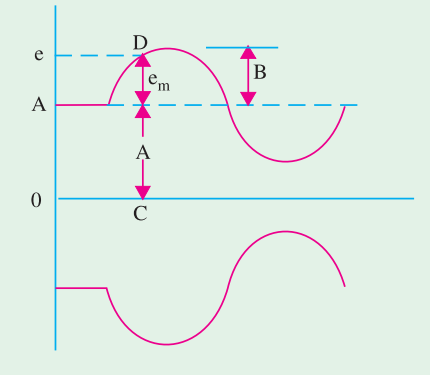


Fig. 66.12

$$= A \sin \omega t + \frac{B}{2} . 2 \sin \omega t . \sin pt \quad = A \sin \omega t + \frac{B}{2}\left[\cos(\omega - p)t - \cos(\omega + p)t\right] \ *$$

$$= A \sin \omega t + \frac{B}{2} \cos (\omega - p)\, t - \cos (\omega + p)t$$

$$= A \sin 2\pi f_c t + \cos 2\pi (f_c f_m)t - \cos 2\pi (f_c + f_m)t$$

As seen from Art. 66.10, $m = B/A$ or $B = mA$

$$\therefore \quad e = A \sin 2\pi f_c t + \cos 2 (f_c - f_m)t - \cos 2 (f_c + f_m)t$$

It is seen that the modulated wave contains three components :

(i)	$A \sin 2\pi f_c t$	— the original carrier wave
(ii)	$\frac{mA}{2} \cos 2\pi (f_c + f_m)t$	— upper side frequency
(iii)	$\frac{mA}{2} \cos 2\pi (f_c - f_m)t$	— lower side frequency

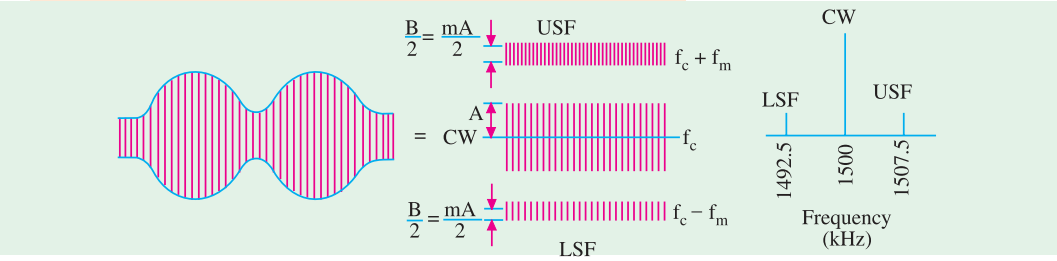


Fig. 66.13

These three frequencies are not a mathematical fiction but they actually exist. In fact, with the help of a narrow band filter, we can separate side frequencies from the carrier wave.

Example 66.6. *The tuned circuit of the oscillator in an AM transmitter uses a 40 mH coil and a 1 nF capacitor. If the carrier wave produced by the oscillator is modulated by audio frequencies upto 10 kHz, calculate the frequency band occupied by the side bands and channel width.*

(Electronics & Telecom. Engg., Calcutta Univ. 1990)

Solution. The resonant frequency is given by

$$f_c = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{40\times10^{-6}\times1\times10^{-9}}} = 796\,\text{kHz}$$

* $2 \sin A . \sin B = \cos (A - B) - \cos (A + B)$

The frequency range occupied by the sidebands is from **786 kHz to 806 kHz**. The channel width

$$= 2 \times 10 = \textbf{20 kHz}$$

66.14. Power Relations in an AM Wave

As discussed in Art. 66.13, a modulated carrier wave consists of the following three components :

1. original carrier wave of amplitude A
2. *USF* wave of amplitude $(mA/2)$
3. *LSF* wave of amplitude $(mA/2)$

Now, power radiated out by a wave through an antenna is proportional to (amplitude)^2.

Carrier power, $P_C \propto A^2 = KA^2$

USB power, $P_{USB} \propto \left(\frac{B}{2}\right)^2 = \frac{KB^2}{4}$; *LSB* power, $P_{LSB} \propto \left(\frac{B}{2}\right)^2 = \frac{KB^2}{4}$

Total sideband power $P_{SB} = 2 \times \frac{KB^2}{4} = \frac{KB^2}{2}$

Total power radiated out from the antenna is $P_T = P_c + P_{SB} = KA^2 + \frac{KB^2}{2}$

Substituting $B = mA$, we get $P_T = KA^2 + \frac{KB^2}{2}(mA)^2 = KA^2\left(1+\frac{m^2}{2}\right)$

Now $P_C = KA^2$

(i) $\therefore \quad P_T = P_C\left(1+\frac{m^2}{2}\right) = P_C\left(\frac{2+m^2}{2}\right)$ **(ii)** $P_C = P_T = P_T\left(\frac{2}{2+m^2}\right)$

(iii) $P_{SB} = P_T - P_C = P_C\left(1+\frac{m^2}{2}\right) - P_C = \frac{m^2}{2}P_C = \left(\frac{m^2}{2+m^2}\right)P_T$

(iv) $P_{USB} = P_{LSB} = \frac{1}{2}P_{SB} = \frac{m^2}{4}P_C = \frac{1}{2}\left(\frac{m^2}{2+m^2}\right)P_T$

Let us consider the case when $m = 1$ or 100%

(i) $P_T = 1.5\,P_C = 1.5 \times$ carrier power **(ii)** $P_C = \frac{2}{3}P_T = \frac{2}{3} \times$ total power radiated

(iii) $P_{USB} = P_{LSB} = \frac{1}{4}P_C = 25\%$ of carrier power

(iv) $P_{SB} = \frac{1}{2}P_C = 50\%$ of carrier power **(v)** $\frac{P_{USB}}{P_T} = \frac{0.25\,P_C}{1.5\,P_C} = \frac{1}{6}$

It means that **single side-band contains 1/6th of the total power radiated out by the transmitter.** That is why single-side band (*SSB*) transmission is more power efficient.

Example 66.7. *The total power content of an AM wave is 1500 W. For a 100 percent modulation, determine*

(i) *power transmitted by carrier,* **(ii)** *power transmitted by each side band.*

(Electronics & Comm. Engg., Kerala Univ. 1991)

Solution. $P_T = 1500$ W, $P_C = ?$ $P_{USB} = ?$ $P_{LSB} = ?$

(i) $P_C = P_T\left(\frac{2}{2+m^2}\right) = P_T\left(\frac{2}{2+1^2}\right) = \frac{2}{3}P_T = \frac{2}{3} \times 1500 = \textbf{1000 W}$

(ii) $P_{USB} = P_{LSB} = \frac{m^2}{4} P_C = \frac{1^2}{4} \times 1000 = \mathbf{250\ W}$

As seen, $P_{USB} = \frac{1}{6} P_T$

Example 66.8. *The total power content of an AM wave is 2.64 kW at a modulation factor of 80%. Determine the power content of*
(i) *carrier,* **(ii)** *each sideband.* **(Electronics & Comm., Roorkee Univ. 1991)**

Solution. (i) $P_C = P_T \left(\frac{2}{2+m^2} \right) = 2.64 \times 10^3 \left(\frac{2}{2+0.8^2} \right) = \mathbf{2000\ W}$

(ii) $P_{USB} = P_{LSB} = \frac{m^2}{4} = \frac{0.8^2}{4} \times 2000 = \mathbf{320\ W}$

Example 66.9. *A transmitter used for radio telephone has an unmodulated carrier power of 10 kW and can be modulated to a maximum of 80 percent by a single-frequency signal before overloading. Find the value to which carrier power can be increased if a 50 percent modulation limit is imposed.* **(Electronics & Comm. Engg. Andhra Univ.)**

Solution. $P_T = P_C \left(\frac{2+m^2}{2} \right) = 10 \left(\frac{2+0.8^2}{2} \right) = \mathbf{13.2\ kW}$

Now, when $m = 0.5$, P_T is still 13.2 kW. Hence, new value of carrier power is given by

$$13.2 = P_C \left(\frac{2+0.5^2}{2} \right); \qquad P_C = \mathbf{11.73\ kW}$$

It is seen that P_C can be increased from 10 kW to 11.73 kW with a total power limit of 13.2 kW and $m = 0.5$

Example 66.10. *A certain transmitter radiates 10 kW of power with the carrier unmodulated and 11.8 kW with the carrier sinusoidally modulated.*
(a) *Find the modulation factor,*
(b) *If another wave modulated to 40% is also transmitted, calculate the radiated power.*

Solution. (a) $11.8 = 10\ (1 + m^2/2)$; $m = 0.6$ or 60%

(b) $P_T = 10 \left(1 + \frac{0.6^2}{2} + \frac{0.4^2}{2} \right) = \mathbf{12.6\ kW}$

66.15. Forms of Amplitude Modulation

As shown in Art. 66.11, **one carrier** and **two sidebands** are produced in AM generation. It is found that it is not necessary to transmit all these signals to enable the receiver to reconstruct the original signal. Accordingly, we may attenuate or altogether remove the carrier or any one of the sidebands without affecting the communication process. The advantages would be

1. less transmitted power and **2.** less bandwidth required

The different suppressed component systems are :

(a) DSB-SC

It stands for ***double-sideband suppressed carrier*** system [Fig. 66.14 (*a*)]. Here, carrier component is suppressed thereby saving enormous amount of power. As seen from Art. 66.14, carrier signal contains 66.7 per cent of the total transmitted power for $m = 1$, Hence, power saving amounts to 66.7% at 100% modulation.

(b) SSB-TC

As shown in Fig. 66.14 (*b*), in this case, one sideband is suppressed but the other sideband and carrier are transmitted. It is called ***single sideband transmitted carrier*** system. For $m = 1$, power saved is 1/6 of the total transmitted power.

(*c*) SSB-SC

This is the **most dramatic suppression of all** because it suppresses one sideband and the carrier and transmits only the remaining sideband as shown in Fig. 66.14 (*c*). In the standard or double-sideband full-carrier (*DSB.FC*) *AM*, carrier conveys **no information but contains maximum power**. Since the two sidebands are exact images of each other, they carry the same audio information. Hence, **all information is available in one sideband only**. Obviously carrier is ***superfluous*** and one side band is ***redundant.*** Hence, one sideband and the carrier can be discarded with no loss of information. The result is *SSB* signal. The advantage of *SSB-SC* system are as follows :

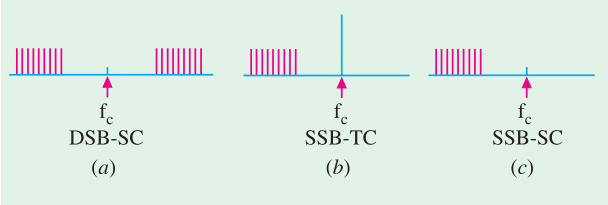


Fig. 66.14

1. Total saving of 83.3% in transmitted power (66.7% due to suppression of carrier wave and 16.6% due to suppression of one sideband). Hence, power is conserved in an SSB transmitter.
2. Bandwidth required is reduced by half *i.e.* 50%. Hence, twice as many channels can be multiplexed in a given frequency range.
3. The size of power supply required is very small. This fact assumes vital importance particularly in a spacecraft.
4. Since the SSB signal has narrower bandwidth, a narrower passband is permissible within the receiver, thereby limiting the noise pick up.

However, the main reason for wide spread use of *DSB-FC* (rather than *SSB-SC*) transmission in broadcasting is the relative simplicity of its modulating equipment.

66.16. Generation of SSB

Three main systems are employed for the generation of *SSB*.

1. Filter method, **2.** Phase cancellation method, **3.** The 'third' method.

All these methods use some form of balanced modulator to suppress the carrier.

Example 66.11. *In an AM wave, calculate the power saving when the carrier and one sideband are suppressed corresponding to*

(*i*) $m = 1$ **(*ii*)** $m = 0.5$ **(Electronics and Comm. Engg., Osmania Univ.)**

Solution. (*i*) When $m = 1$

$$P_T = P_C\left(1+\frac{m^2}{2}\right) = 1.5\,P_C\,; \qquad P_{LSB} = P_{USB} = \frac{m^2}{4}\,P_C = 0.25\,P_C$$

$$\therefore \quad \text{saving} = P_T - P_{USB} = 1.5\,P_c - 0.25\,P_c = 1.25\,P_c$$

$$\text{saving} = \frac{1.25}{1.5P_C} \times 100 = \mathbf{83.3\%}$$

(*ii*) When $m = 0.5$

$$P_T = P_C\left(1+\frac{m^2}{2}\right) = P_C\left(\frac{1+0.5^2}{2}\right) = 1.125P_C$$

$$P_{USB} = P_{LSB} = \frac{m^2}{4}\,P_C = \frac{0.5^2}{4}\,P_C = 0.0625\,P_C$$

$$\therefore \quad \text{saving} = \frac{1.125\,P_c - 0.0625\,P_C}{1.125\,P_C} \times 100 = \mathbf{94.4\%}$$

66.17. Methods of Amplitude Modulation

There are two methods of achieving amplitude modulation :

(i) Amplifier modulation,

(ii) Oscillator modulation.

Block diagram of Fig. 66.15 illustrates the basic idea of amplifier modulation.

Here, carrier and *AF* signal are fed to an amplifier and the result is an *AM* output . The modulation process takes place in the active device used in the amplifier.

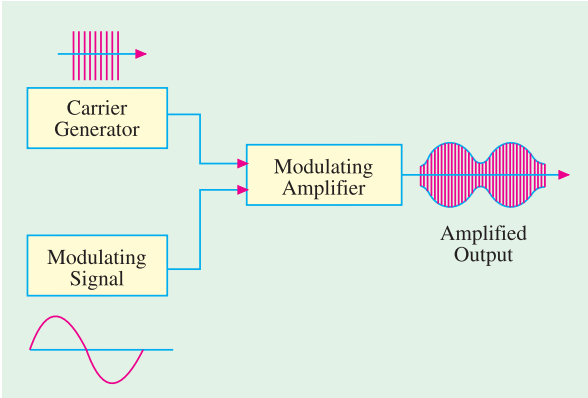


Fig. 66.15

66.18. Block Diagram of an AM Transmitter

Fig. 66.16 shows the block diagram of a typical transmitter. The carrier wave is supplied by a crystal-controlled oscillator at the carrier frequency. It is followed by a tuned buffer amplifier and an *RF* output amplifier.

The source of *AF* signal is a microphone. The audio signal is amplified by a low level audio amplifier and, finally, by a power amplifier. It is then combined with the carrier to produce a modulated carrier wave which is ultimately radiated out in the free space by the transmitter antenna as shown.

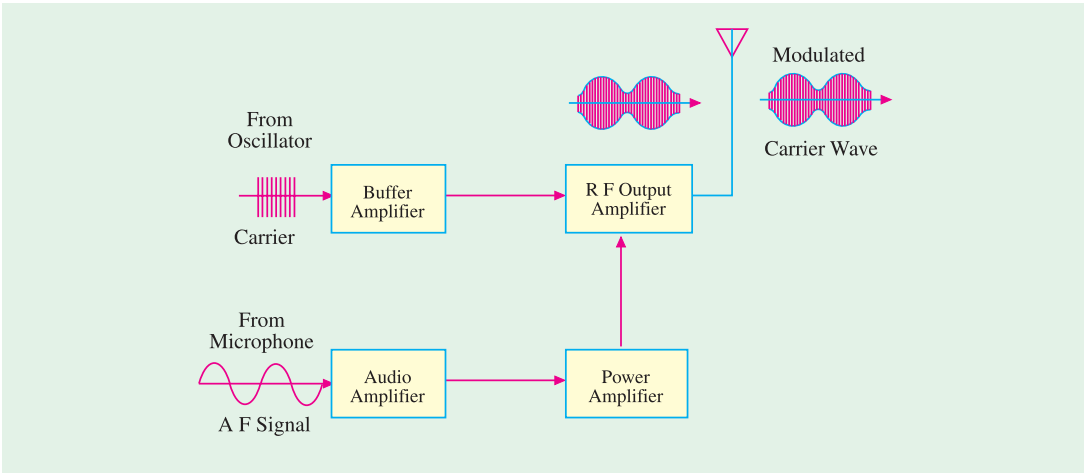


Fig. 66.16

66.19. Modulating Amplifier Circuit

It is shown in Fig. 66.17. The carrier signal from a crystal oscillator is coupled to the base of Q through transformer T_1. The low-frequency modulating signal is also coupled to the base of Q by capacitor C_1. The voltage divider $R_1 - R_2$, as usual, provides proper forward bias for the transistor whereas C_2 and C_3 are bypass capacitors.

The modulating signal applied across R_2 causes variations in the base bias in step with its own variations. The amplitude of the transistor *RF* output changes according to the changes in *AF* modulating signal. The $L_1\ C_4$ circuit is kept tuned to the carrier frequency. The amplitude-modulated

carrier or *RF* current in L_1 induces similar current in L_2 which is connected to an antenna. Finally, the *AM* carrier wave is radiated out in space by the transmitting antenna.

66.20. Frequency Modulation OR CAD Simulation Diagram.

As the name shows, in this modulation, it is only the **frequency of the carrier which is changed and not its amplitude.**

The **amount of change** in frequency is determined by the **amplitude** of the modulating signal whereas **rate of change** is determined by the **frequency** of the modulating signal. As shown in Fig. 16.18, in an *FM* carrier, information (or intelligence) is carried as variations in its frequency. As seen, frequency of the modulated carrier increases as the signal amplitude increases but decreases as the signal amplitude decreases. It is at its highest frequency (point *H*) when the signal amplitude is at its maximum positive value and is at its lowest frequency (point *L*) when signal amplitude has maximum negative value. When signal amplitude is zero, the carrier frequency is at its normal frequency f_0 (also called ***resting or centre*** frequency.)

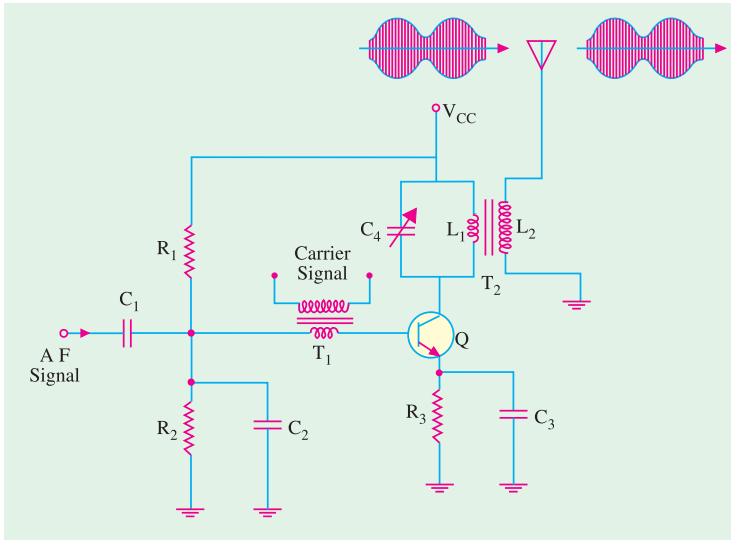


Fig. 66.17

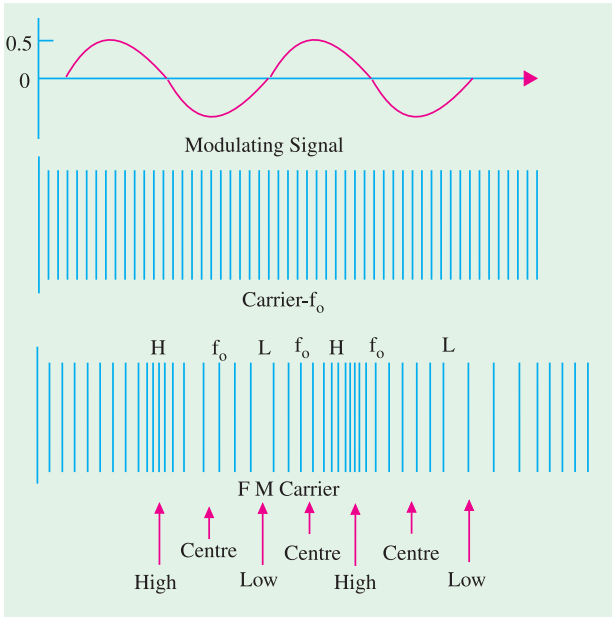


Fig. 66.18

The fact that amount of change in frequency depends on signal amplitude is illustrated in Fig. 66.19 where *R* stands for resting frequency. Here, signal amplitude is almost double of that in Fig. 66.18 ***though its frequency is the same.*** This **louder** signal causes **greater** frequency change in modulated carrier as indicated by ***increased bunching and spreading*** of the waves as compared with relatively weaker signal of Fig. 66.18.

The **rate** at which frequency shift takes place depends on the signal frequency as shown in Fig. 66.20. For example, if the modulating signal is 1 kHz, then the modulated carrier will swing between its maximum frequency and lowest frequency 1000 times per second. If f_m = 2 kHz, the rate of frequency swing would be twice as fast :

In short, we have established two important points about the nature of frequency modulation:

(*i*) The **amount** of frequency deviation (or shift or variation) depends on the **amplitude** (loudness) of the audio signal. ***Louder the sound, greater the frequency deviation and vice-versa.***

However, for the purposes of *FM* broadcasts, it has been internationally agreed to restrict maximum deviation to **75 kHz** on each side of the centre frequency for sounds of maximum loudness. Sounds of lesser loudness are permitted ***proportionately less frequency deviation.***

(*ii*) The **rate** of frequency deviation depends on the **signal frequency**.

66.21. Frequency Deviation and Carrier Swing

The frequency of an *FM* transmitter without signal input is called the **resting frequency** or **centre frequency** (f_0) and is the allotted frequency of the transmitter. In simple words, it is the carrier frequency on which a station is allowed to broadcast. When the signal is applied, the carrier frequency deviates up and down from its resting value f_0.

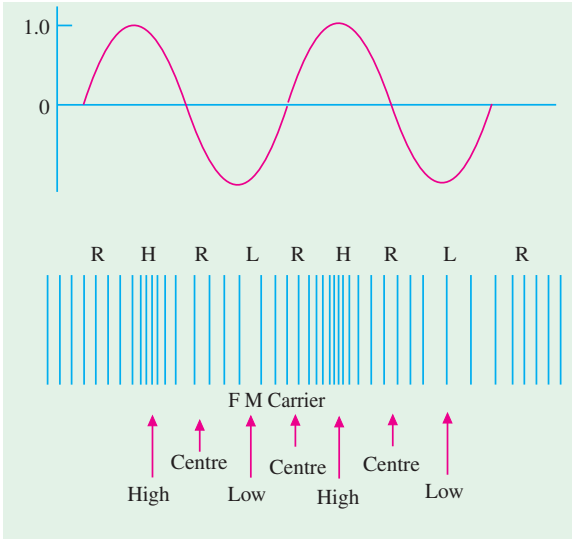


Fig. 66.19

This change or shift either above or below the resting frequency is called frequency deviation (Δ_f)

The total variation in frequency from the lowest to the highest is called carrier swing (CS). Obviously,

carrier swing = 2 × frequency deviation of $CS = 2 \times \Delta_f$

A maximum frequency deviation of 75 kHz is allowed for commercial *FM* broadcast stations in the 88 to 168 MHz *VHF* band. Hence, *FM* channel width is 275 = 150 kHz. Allowing a 25 kHz guard band on either side, the channel width becomes = 2(75 + 25) = 200 kHz (Fig. 66.21). This guard band is meant to prevent interference between adjacent channels. However, a maximum frequency deviation of 25 kHz is allowed in the sound portion of the TV broadcast.

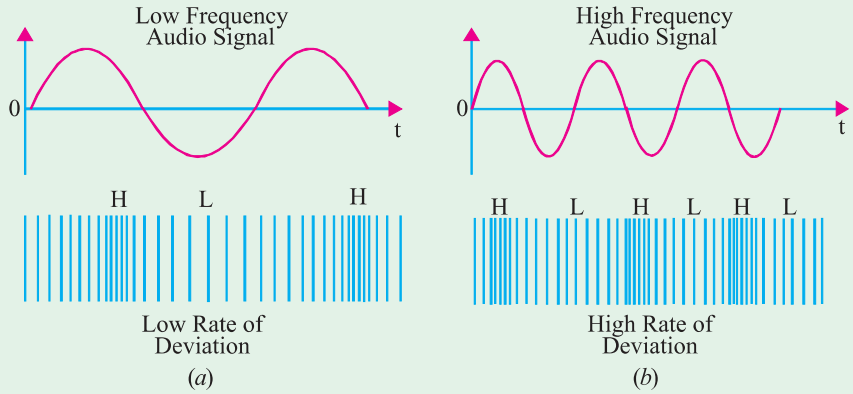


Fig. 66.20

In *FM*, the highest audio frequency transmitted is 15 kHz.

Consider an FM carrier of resting frequency 100 MHz. Since $(\Delta f)_{max}$ = 75 kHz, the carrier frequency can swing from the lowest value of 99.925 MHz to the highest value of 100.075 MHz. Of course, deviations lesser than 75 kHz corresponding to relatively softer sounds are always permissible.

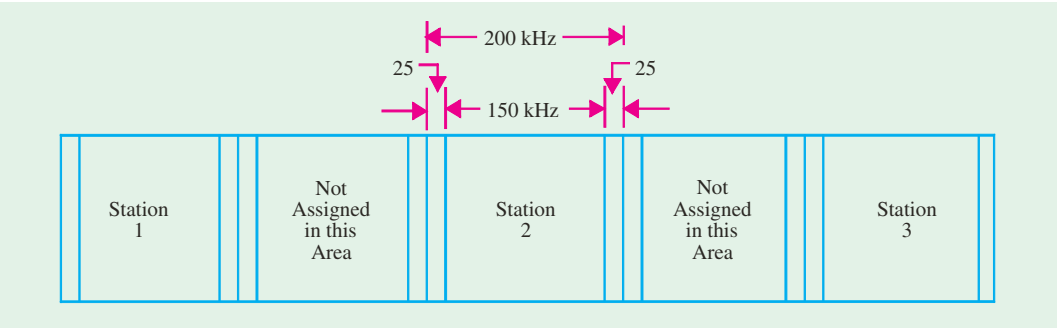


Fig. 66.21

66.22. Modulation Index

It is given by the ratio $m_f = \dfrac{\text{frequency deviation}}{\text{modulation frequency}} = \dfrac{\Delta f}{f_m}$

Unlike amplitude modulation, this **modulation index can be greater than unity**. By knowing the value of m_f, we can calculate the number of significant sidebands and the bandwidth of the *FM* signal.

66.23. Deviation Ratio

It is the worst-case modulation index in which maximum permitted frequency deviation and maximum permitted audio frequency are used.

$$\therefore \quad \text{deviation ratio} = \frac{(\Delta f)}{f_{m(max)}}$$

Now, for *FM* broadcast stations, $(\Delta f)_{max}$ = 75 kHz and maximum permitted frequency of modulating audio signal is 15 kHz.

$$\therefore \quad \text{deviation ratio} = \frac{75\,\text{kHz}}{15\,\text{kHz}} = 5$$

For sound portion of commercial TV deviation ratio = $\dfrac{25\,\text{kHz}}{15\,\text{kHz}} = 1.67$

66.24. Percent Modulation

When applied to *FM*, this term has slightly different meaning than when applied to *AM*. In *FM*, it is given by the ratio of actual frequency deviation to the maximum allowed frequency deviation.

$$m = \frac{(\Delta f)_{actual}}{(\Delta f)_{max}}$$

Obviously, 100% modulation corresponds to the case when actual deviation equals the maximum allowable frequency deviation. If, in some case, actual deviation is 50 kHz, then

$$m = \frac{50}{75} = \frac{2}{3} = 0.667 = \mathbf{66.7\%}$$

Value of $m = 0$ corresponds to zero deviation *i.e.* unmodulated carrier wave. It is seen from the above equation that $m \propto (\Delta f)_{actual}$. It means that when **frequency deviation (*i.e.* signal loudness) is doubled, modulation is doubled**.

Example 66.12. *What is the modulation index of an FM carrier having a carrier swing of 100 kHz and a modulating signal of 5 kHz ?*

Solution. $CS = 2 \times \Delta f$

$$\therefore \quad \Delta f = \frac{CS}{2} = \frac{100}{2} = \mathbf{50\ kHz} \quad \therefore \quad m_f = \frac{\Delta f}{f_m} = \frac{50}{5} = \mathbf{10}$$

Example. 66.13. *An FM transmission has a frequency deviation of 18.75 kHz. Calculate percent modulation if it is broadcast*

(i) *in the 88-108 MHz band* ***(ii)*** *as a portion of a TV broadcast.*

(Elect. and Comm. Engg., Madurai Kamaraj Univ. 1990)

Solution. *(i)* For this transmission band,

$(\Delta f)_{max} =$ 75 kHz $\quad \therefore \quad m = \frac{18.75}{75} \times 100 = \mathbf{25\%}$

(ii) In this case, $(\Delta f)_{max} = 25$ kHz $\quad \therefore \quad m = \frac{18.75}{75} \times 100 = \mathbf{75\%}$

Example 66.14. *An FM signal has a resting frequency of 105 MHz and highest frequency of 105.03 MHz when modulated by a signal of frequency 5 kHz. Determine*

(i) *frequency deviation,* ***(ii)*** *carrier swing,* ***(iii)*** *modulation index,*
(iv) *percent modulation,* ***(v)*** *lowest frequency reached by the FM wave.*

(Electronics and Comm. Engg., Osmania Univ. 1992)

Solution. (*i*) $\Delta f = 105.03 - 105 = 0.03$ MHz = **30 kHz**

(ii) $CS = 2 \times \Delta f = 2 \times 30 = \mathbf{60\ kHz}$ ***(iii)*** $m_f = \frac{30}{5} = \mathbf{6}$

(iv) $m = \frac{30}{5} \times 100 = \mathbf{40\%}$ ***(v)*** ***lowest frequency*** $= 105 - 0.03 = \mathbf{104.97\ kHz}$

66.25. FM Sidebands

In *FM*, when a carrier is modulated, a number of sidebands are formed.* Though theoretically their number is infinite, their strength becomes negligible after a few sidebands. They lie on both sides of the centre frequency spaced fm apart as shown in Fig.66.22. Sidebands at equal distances from f_0 have equal amplitudes. If f_0 is the centre frequency and fm the frequency of the modulating signal, then *FM* carrier contains the following frequencies :

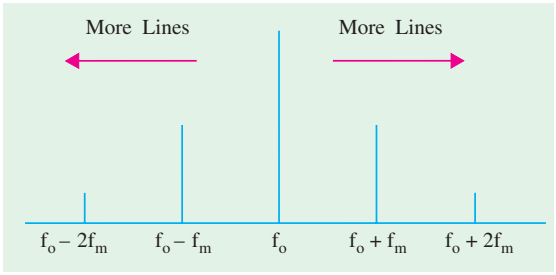


Fig. 66.22

(i) f_0 ***(ii)*** $f_0 \pm f_m$ ***(iii)*** $f_0 \pm 2 f_m$ ***(iv)*** $f_0 \pm 3 f_m$ and so on

The bandwidth occupied by the spectrum is $BW = 2nf_m$ where n is the highest order of the significant sideband.

Another approximate expression for spectrum bandwidth is $BW = 2(1 + m_f)f_m$

Now, $m_f = \frac{\Delta f}{f_m}$, hence $BW = 2(\Delta_f + f_m)$

This expression is based on the assumption that sidebands having amplitudes less than 5% of the unmodulated carrier wave are negligble or when m_f is at least 6.

66.26. Modulation Index and Number of Sidebands

It is found that the number of sidebands

1. depends ***directly*** on the amplitude of the modulating signal,
2. depends ***inversely*** on the frequency of the modulating signal.

* An AM signal has only two side frequencies for each modulating frequency (Art. 16.11)

Since frequency deviation is directly related to the amplitude of the modulating signal, the above two factors can be combined in one factor called **modulation index**.

Hence, number of sidebands depends on $m_f = \Delta f / f_m$

Obviously, the number of pairs of sidebands

(*i*) **increases** as frequency deviation (or amplitude of modulating signal) **increases**.

(*ii*) **increases** as the modulating signal frequency **decreases**.

Table No. 66.2

m_f	0.5	1	2	3	4	5	6	7
n	2	3	4	6	7	8	9	11
$BW = 2nf_m$	$4f_m$	$6f_m$	$8f_m$	$12f_m$	$14f_m$	$16f_m$	$18f_m$	$22f_m$
m_f	8	9	10	11	12	13	14	15
n	12	13	14	15	16	17	18	19
$BW = 2nf_m$	$24f_m$	$26f_m$	$28f_m$	$30f_m$	$32f_m$	$34f_m$	$36f_m$	$38f_m$

Table 66.2 summarizes the relation between the number (n) of sidebands on either side of the carrier and the corresponding bandwidth of the spectrum.

Example 66.15. *A 5 kHz audio signal is used to frequency-modulate a 100 MHz carrier causing a frequency deviation of 20 kHz. Determine*

(*i*) *modulation index* **(*ii*)** *bandwidth of the FM signal.*

Solution. (*i*) $m_f = \dfrac{\Delta f}{f_m} = \dfrac{20}{5} = \mathbf{4}$

As seen from Table 66.2 $BW = 14f_m = 14 \times 5 = \mathbf{70\ kHz}$

Note. We cannot use the alternate expression for BW given in Art. 66.25 above because $m_f < 6$

Example 66.16. *In an FM circuit, the modulation index is 10 and the higest modulation frequency is 20 kHz. What is the approximate bandwidth of the resultant FM signal ?*

(Applied Electronics, Bombay Univ. 1990)

Solution. Since the value of m_f is more than 6, we will use the expression

$BW = 2\,(\Delta_f + f_m)$

Now, $m_f = \dfrac{\Delta f}{f_m}$ or $10 = \dfrac{\Delta f}{20}$ $\therefore$ $\Delta_f = 200$ kHz

$\therefore$ $BW = 2\,(200 + 20) = \mathbf{440\ kHz}$

66.27. Mathematical Expression for FM Wave

The unmodulated carrier is given by $e_c = A \sin 2\pi f_0 t$

The modulating signal frequency is given by $e_m = B \sin 2\pi f_m t$

The modulated carrier frequency f swings around the resting frequency f_0 thus

$$f = f_0 + \Delta_f . \sin 2\pi f_m t$$

Hence, equation for the frequency-modulated wave becomes

$$e = A \sin 2\pi f t = A \sin [2\pi (f_0 + \Delta f . \sin 2\pi f_m t) t]$$

$$= A \sin \left(2\pi f_0 t + \frac{\Delta f}{f_m} + \cos 2\pi f_m t\right) = A \sin (2\pi f_0 t + m_f \cos 2\pi f_m t)$$

66.28. Demodulation or Detection

When the *RF* modulated waves, radiated out from the transmitter antenna, after travelling through space, strike the receiving aerials, they induce very weak *RF* currents and voltages in them. If these high-frequency currents are passed through headphones or loudspeakers, they produce no effect on them because all such sound-producing devices are unable to respond to such high frequencies due to large inertia of their vibrating discs etc. Neither will such *RF* currents produce any effect on human ear because their frequencies are much beyond the audible frequencies (20 to 20,000 Hz approximately). Hence, it is necessary to demodulate them first in order that the sound-producing devices may be actuated by audio-frequency current similar to that used for modulating the carrier wave at the broadcasting station.

This process of ***recovering AF signal from the modulated carrier wave is known as demodulation or detection.***

The demodulation of an *AM* wave involves two operations :

(i) rectification of the modulated wave and

(ii) elimination of the *RF* component of the modulated wave.

However, demodulation of an *FM* wave involves three operations ***(i)*** conversion of frequency changes produced by modulating signal into corresponding amplitude changes, ***(ii)*** rectification of the modulating signal and ***(iii)*** elimination of *RF* component of the modulated wave.

66.29. Essentials of AM Detection

For recovering the *AF* waveform from modulated wave (a mixture of *AF* wave and *RF* carrier), it is essential to find some way of reducing (or better, eliminating) one half of the modulated wave. The result of this elimination (or rectification) would be that the average value of the wave would not be zero because, now, the impulse would be all in one direction as shown in Fig. 66.23. If this new wave is now passed through a headphone shunted by a suitable capacitor, then *AF* wave will pass through the headphone whereas the *RF* wave will be by-passed by the capacitor (because the high inductance of magnet coils of the headphones will offer tremendous impendance to *RF* currents). Hence, two will become separated.

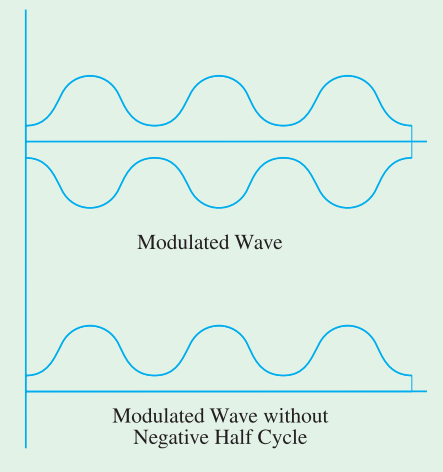


Fig. 66.23

66.30. Diode Detector for AM Signals

Diode detection is also known as **envelope-detection** or **linear detection**. In appearance, it looks like an ordinary half-wave rectifier circuit with capacitor input as shown in Fig. 66.24. It is called **envelope detection** because it recovers the *AF* signal envelope from the composite signal. Similarly, diode detector is called **linear detector** because its output **is *proportional to the voltage of the input signal**.***

Circuit Action

Of the various *RF* voltages induced in the receiver aerial, only those having the ***same*** frequency as the resonant frequency of *LC* circuit are tuned in due to electromagnetic induction between coils L_1 and L. By varying C, the resonant frequency of the *LC* circuit can be varied and hence *RF* signal of any desired frequency can be tuned in. This input signal is rectified by the diode and passed on to the low-pass filter RC_1.

* A detector circuit in which the rectified output is proportional to the square of the input signal voltage is called square-law or non-linear detector.

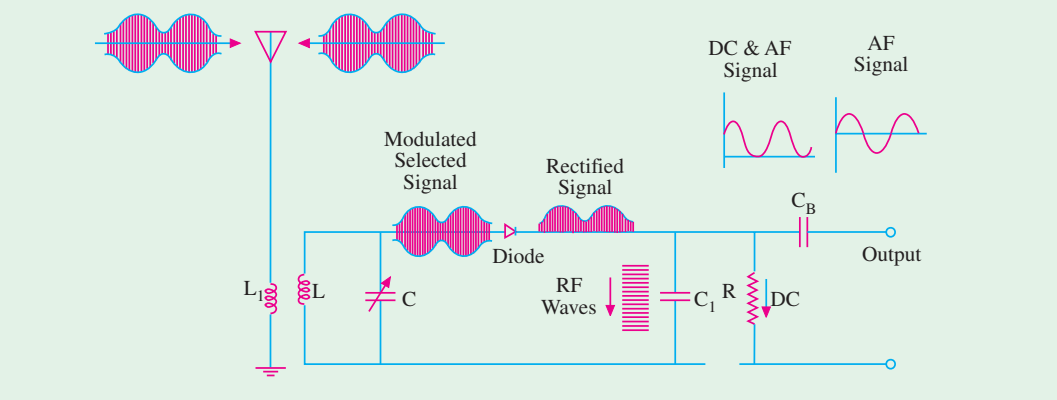


Fig. 66.24

The *RF* carrier wave is filtered out by low-reactance capacitor C_1 which is called *RF* filter capacitor or *RF* by-pass capacitor. The dc component of the remaining signal is shunted out through *R* because it cannot pass through blocking capacitor *CB*. But the low, frequency *AF* signal can easily get through *CB* and becomes available across the output. When passed through a suitable device, say, a headphone, the original sound can be heard.

Advantages

Diode detectors are extensively used in AM broadcast receivers because they have the following advantages :

1. They can handle comparatively large input signals;
2. They can be operated as linear or power detectors;
3. They rectify with negligible distortion and, hence, have good linearity;
4. They are well-adopted for use in simple automatic-gain control circuits.

Disadvantages

However, the disadvantages are that

1. they do not have the ability to amplify the rectified signal by themselves as is done by a transistor detector (Art. 66.31). However, it is not a very serious drawback since signal amplification can be affected both before and after rectification;
2. while conducting, the diode consumes some power which reduces the *Q* of its tuned circuit as well as its gain and selectivity.

66.31. Transistor Detectors for AM Signals

Transistors can be used as detector amplifiers *i.e.* both for rectification and amplification. As shown in Fig. 66.25, the *RF* signal is applied at the base-emitter junction where rectification takes place. The amplification of the recovered signal takes place in the emitter-collector circuit.

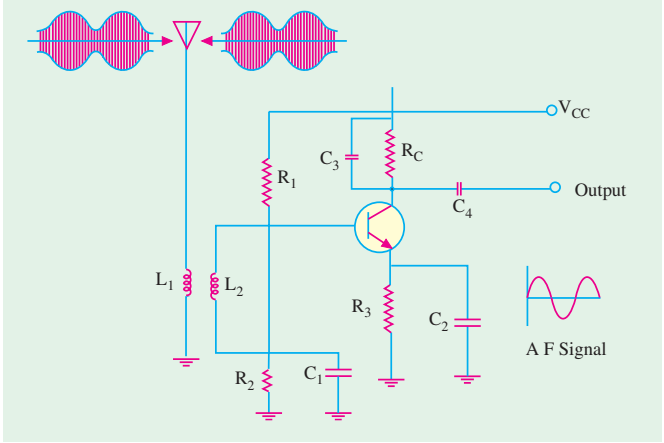


Fig. 66.25

It should be noted that when used as a detector, the transistor has to be biased at cut-off or operated as class-B. As is usual, C_2 ac grounds R_3 whereas voltage divider R_1 — R_2 establishes proper bias which holds the transistor just at cut-off.

Obviously, only positive half-cycles of the modulated *RF* input signal will drive the transistor into conduction, whereas negative half-cycles would be cut-off. Thus, rectification of the input signal takes place in the base-emitter circuit. The small variations in base current produced by the positive half-cycle of the input signal produce large (β time) variations in the collector current. Hence, an amplified version of the rectified signal appears across $R_C - C_3$ combination. C_3 eliminates *RF* component so that only *AF* signal voltage drops across *RC*. The capacitor C_4 permits *AF* signal to pass through but blocks its dc component.

66.32. FM Detection

As discussed earlier, an *FM* carrier signal contains information (or intelligence we wish to convey) in the form of frequency variations above and below the centre frequency of the carrier. For recovering the information, we must first convert the *FM* signal in such a way that it appears as a modulated *RF* voltage across the diode. A simple method of converting frequency variations into voltage variations is to make use of the principle that reactance (of coil or capacitor) varies with frequency. When an *FM* signal is applied to an inductor, the current flowing through it varies in amplitude according to the changes in frequency of the applied signal. Now, changes in frequency of the *FM* signal depend on the amplitude of the modulating *AF* signal. Hence, the current in the inductor varies as per the amplitude of the original modulating signal. In this way, frequency changes in *FM* signal are converted into amplitude changes in current. These changes in current when passed through a resistor produce corresponding changes in voltage.

Hence, we find that, ultimately, frequency variations in *FM* signal are converted into voltage changes. Also, there exists a linear relation between the two – something essential for distortion-less demodulation.

FM demodulation may be carried out with the help of ***(i)*** ratio detector and ***(ii)*** quadrature detector.

66.33. Quadrature Detector

This detector depends on the frequency/phase relationship of a tuned circuit. It uses only one tuned circuit and is becoming increasingly popular in the integrated *FM* strips.

Theory

Let us first consider the general principle. A sinusoidal current is given by the equation

$$i = I_m \sin \theta = I_m \sin \omega t$$

Suppose, it flows through a circuit shown in Fig. 66.27 (*a*). The voltage V_L across the inductor (assumed pure) leads the current *I* by 90°

$$\therefore \qquad V_L = V_L \cos \omega t$$

The voltage V_z across the parallel tuned circuit will be in phase with *I* at resonance. However, at frequencies slightly different (±1%) from the resonant frequency, the phase angle *f* will be given *b*

$$\tan \phi = \frac{yQR}{R} = y\,Q \quad \text{assuming } f_0 / f \cong 1. \qquad \text{— Fig.66.26}$$

Here $$\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} = \frac{f}{f_0} - \frac{f_0}{f} \cong \frac{2\Delta_f}{f_0}$$

where $\Delta f = f - f_0$

f_0 = resonant frequency

f = slightly off-resonance frequency

Hence, equation for V_z is given by $V_z = V_z \sin(\omega t - f)$

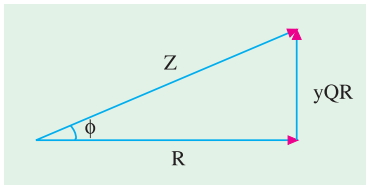


Fig. 66.26

When the two voltages V_L and V_z are applied as inputs to a multiplier, the output voltage V_0 is found to be proportional to their product as shown in Fig. 66.27(*b*).

$$V_0 \propto V_L . V_z \propto \cos \omega t . \sin (\omega t - \phi)$$
$$\propto \sin (2\omega t + f) + \sin \phi$$

A low-pass filter is used to reject the double frequency component sin $(2\omega t + \phi)$ and select only the low-frequency sin ϕ component.

$\therefore \quad V_0 \propto \sin\phi \propto \tan\phi$ — since ϕ is very small

$\propto y\phi \propto y \propto 2.\dfrac{\Delta f}{f_0}$ — where V_m is the modulating *AF* voltage.

Now, $\Delta_f = f - f_0 \propto k\, V_m$

$\therefore \quad V_0 \propto V_m$

It shows that output voltage υ_0 is proportional to the original modulating signal voltage.

66.34. Frequency Conversion

Let us first consider its necessity for radio broadcasting purposes and the basic principle of heterodyning action.

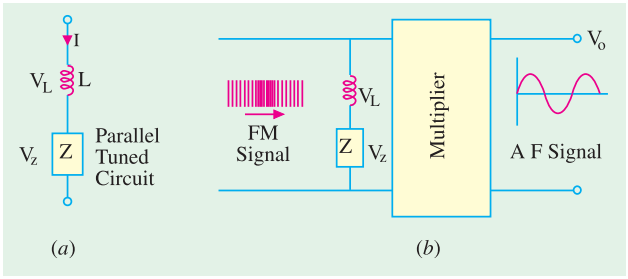


Fig. 66.27

1. Need

It is very difficult to design amplifiers which give uniformly high gain over a wide range of radio frequencies used in commercial broadcast stations. However, it is possible to design amplifiers which can provide high-gain uniform amplification over a narrow band of comparatively lower frequencies called intermediate frequencies (*IF*). Hence, it is necessary to convert the modulated *RF* carrier into modulated *IF* carrier by using a frequency converter.

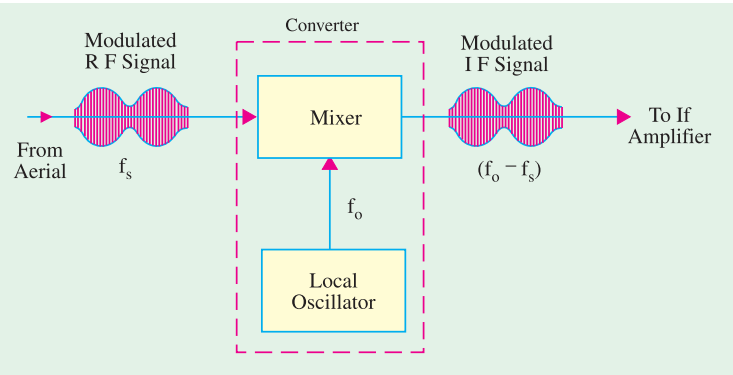


Fig. 66.28

This *IF* signal is then amplified by narrow-band *IF* amplifiers and passed on to the *AM* detectors.

2. Basic Principle

The frequency conversion can be achieved by utilizing the heterodyne principle. For this purpose, the modulated *RF* signal is mixed (in a mixer) with an unmodulated *RF* signal produced by local oscillator as shown in Fig. 66.28.

The oscillator and the mixer may be either two separate devices or may be combined into one device called converter. The process of combining two ac signals of different frequencies in order to obtain a signal of new frequency is called heterodyning action.

3. Heterodyning Action

Suppose the carrier signal of frequency f_s is heterodyned with another signal of frequency f_0, then two additional signals are produced whose frequencies are :

(i) $f_0 + f_s$—the sum component ***(ii)*** $f_0 - f_s$ — the difference component

Usually, the sum frequency is removed by bandpass filtering. The difference frequency (also called ***beat frequency***) is retained and forms the *IF* frequency in *AM* receivers.

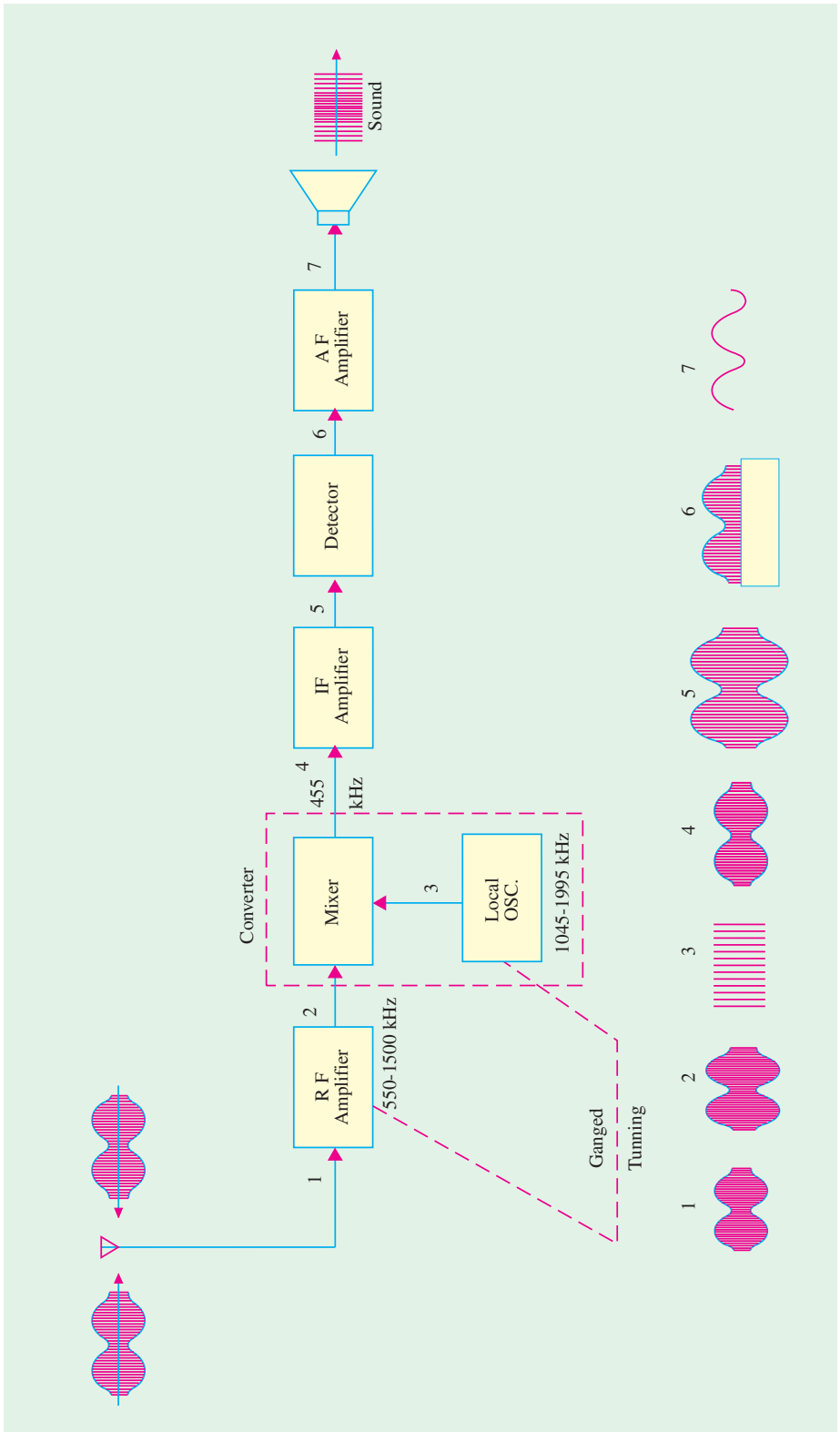

Sound
A F Amplifier
Detector
IF Amplifier
455 kHz
Converter
Mixer
Local OSC.
1045-1995 kHz
R F Amplifier
550-1500 kHz
Ganged Tuning
1
2
3
4
5
6
7

Fig. 66.29

Suppose, an *AM* carrier signal of frequency 1500 kHz is mixed with an unmodulated carrier (produced by local oscillator) of frequency 1955 kHz. Then, following two frequencies are produced

(*i*) 3455 kHz and **(*ii*)** 455 kHz

The **higher frequency** is generally filtered out leaving behind the difference frequency of 455 kHz which forms the *IF* frequency.

66.35. Superhetrodyne AM Receiver

It is also referred to as superhet and is extensively used in modern AM receivers. Its block diagram with signals is shown in Fig. 66.29. The operation of this receiver is as under :

(*i*) Let us assume that the incoming signal frequency is 1500 kHz. It is first amplified by the *R.F.* amplifier.

(*ii*) Next, it enters a mixer circuit which is so designed that it can conveniently combine two radio frequencies—one fed into it by the *R.F.* amplifier and the other by a local oscillator.

(*iii*) The local oscillator is an *RF* oscillator whose frequency of oscillation can be controlled by varying the capacitance of its capacitor. In fact, the tuning capacitor of the oscillator is ganged with the capacitor of the input circuit so that the difference in the frequency of the selected signal and oscillator frequency is always constant. Usually, the difference is maintained at 455 kHz. If signal frequency is 1,500 kHz, then oscillator frequency can be either 1,955 or 1,045 kHz. Let us suppose that it is 1,955 kHz. In fact, local oscillator frequency is always higher than the frequency of the incoming signal.

(*iv*) When two alternating currents of these two different frequencies are combined in the mixer transistors, then phenomenon of beats is produced. In the present case, the beat frequency is 1955-1500 = 455 kHz. Since this frequency is lower than the signal frequency but still above the range of audio frequencies, it is called ***intermediate frequency* (*IF*).**

(*v*) The 455 kHz output of the mixer is then passed on to the *IF* amplifier which is ***fixed-tuned*** to 455 kHz frequency. In practice, one or more stages of *IF* amplification may be used.

(*vi*) The output of *IF* amplifier is demodulated by a detector which provides the audio signal.

(*vii*) This audio signal is amplified by the audio-frequency (*AF*) amplifier whose output is fed to a loud-speaker which reproduces the original sound.

66.36. Standard Superhet AM Receiver

Fig. 66.30 shows the block diagram of a standard superhet *AM* receiver alongwith its seven-transistor circuit diagram. Usually, transistor radios are all-wave radios having two or more bands. With the help of a band switch, the coils and trimmers of any one band can be connected to the circuit at will. For the sake of simplicity, the coils and trimmers of one band only have been shown in Fig. 66.30. The operation of various stages and the functions of various radio components used in the radio circuit of Fig. 66.30 are described below.

1. Frequency Changer or Converter

It is the first stage* in a transistor radio and has two sections—oscillator and mixer. A single silicon transistor Q_1 performs both the above functions. L_4 is the oscillator coil whereas L_2 is the signal input coil. Resistors R_1 and R_2 form a voltage divider for giving base bias to Q_1

The signals intercepted by the aerial are picked up by coil L_1 which is tuned to the transmission frequency of the desired station with the help of ganged capacitor C_1 and its fine-tuning trimmer C_2. This signal is then induced into the secondary winding L_2 which passes it on to the base of Q_1 *via* C_3. The local oscillations produced by L_4 and ganged capacitor C_7 are given to the emitter of Q_1 *via* C_4.

As seen, Q_1 has been used in *CE* configuration for producing oscillations. The positive feedback required for producing oscillations is given from the collector of Q_1 to its emitter through L_3, L_4 and C_4.

* Sometimes, there is an RF stage before it (Fig. 66.29)

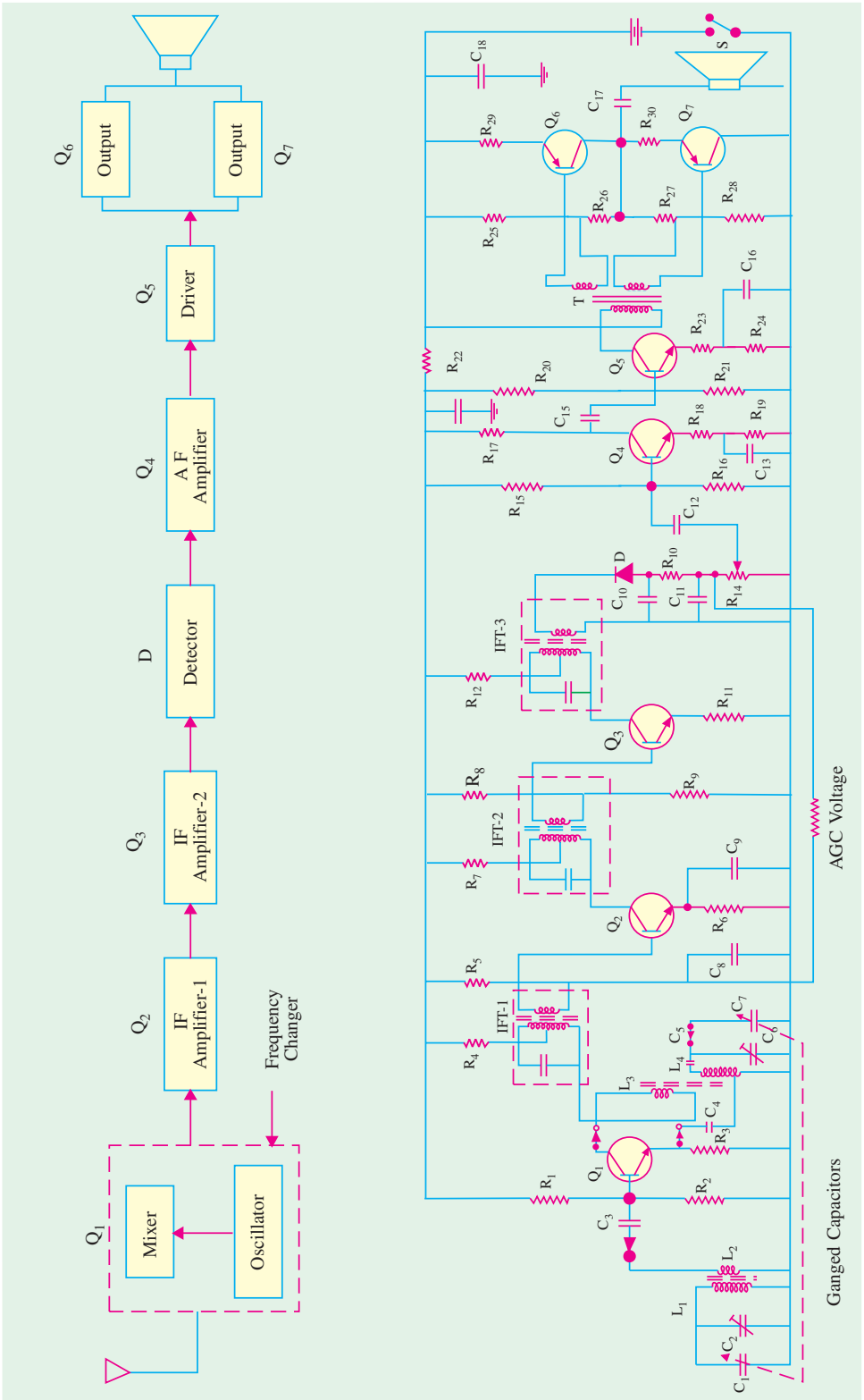

Q6
Output
Output
Q7
Q5
Driver
Q4
A F Amplifier
D
Detector
Q3
IF Amplifier-2
Q2
IF Amplifier-1
Frequency Changer
Q1
Mixer
Oscillator
AGC Voltage
Ganged Capacitors

Fig. 66.30

The transistor Q_1 mixes the incoming signal frequency f_s and oscillation frequency f_0 thereby producing the sum and difference frequencies of $(f_0 + f_s)$ and $(f_0 - f_s)$ respectively. The difference frequency called *IF* frequency is then passed on to the *IF* transformer IFT-1 connected in the collector of Q_1.

2. IF Amplifier

As the name indicates, its function is to amplify the *IF* signal obtained from the frequency changer. Usually, two stages are used to obtain sufficient gain. The gain of the first stage is controlled by the automatic gain control circuit (*AGC*) with the help of dc voltage drop across R_{10}. This dc voltage is obtained from the detector diode. Three fixed-tuned *IF* transformers are used—one at the collector of the frequency changer and the other two for the two *IF* amplifier stages. These transformers are also used as collector load as well as for inter-stage-coupling.

Resistor R_5 gives base bias to Q_2 whereas capacitor C_8 decouples the *AGC* voltage. As usual, R_8 and R_9 form the voltage divider circuit for giving base bias to Q_3.

The amplified *IF* signal obtained from *IFT-2* is further amplified by *IFT-3* of the second *IF* amplifier stage before it is passed on to the detector diode.

3. Detector Circuit

It consists of a diode* which rectifies the *IF* signal. Resistor R_{13}, capacitors C_{10} and C_{11} together filter out *RF*. The audio signal is given to the following audio section through potentiometer R_{14} which is also used as a volume control. Capacitor C_{12} blocks the dc component in the rectified signal. This dc voltage is used for *AGC via* R_{10}. The main purpose of using *AGC* is to minimize the variations in sound due to changes in signal strength (because of fading) and to prevent overloading of second IF stage because of a very strong signal from a close-by transmitter.

4. Audio Frequency (AF) Amplifier

The audio section consists of three stages *i.e.* audio amplifier, driver and output which feeds a loud speaker. The audio-amplifiers amplify the audio signal given to Q_4 through volume control and C_{12}, R_{15} and R_{16} are biasing resistors whereas R_{19} is the emitter resistor with a by-pass capacitor C_{13}. The amplified signal is developed over collector load resistor R_{17}. This amplified signal is capacitively-coupled to the driver stage through C_{14}.

5. Driver

The driver stage further amplifies the audio signal obtained from the previous *AF* amplifier till it is large enough to drive the output stage. Transformer *T* (having two secondary windings) is used for coupling to the output stage. The emitter resistor of Q_5 is split into two parts (R_{23} and R_{24}) with only R_{24} being by-passed by C_{16}. This provides some negative feedback and reduces distortion.

6. Output Stage

The function of this stage is to amplify the audio signal received from the driver stage and hence provide sufficient power to drive the loudspeaker. Class-B push-pull amplifier employing transistors Q_6 and Q_7 is used for economizing battery consumption. Resistors R_{25} and R_{26} provide base bias to Q_6 and R_{27} and R_{28} to Q_7 whereas R_{29} and R_{30} are the emitter resistors of the two respective transistors. The audio signal is given *via* C_{17} to the loudspeaker which converts it into sound similar to the one transmitted by the broadcasting transmitter.

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* Sometimes, transistors are used as detectors.

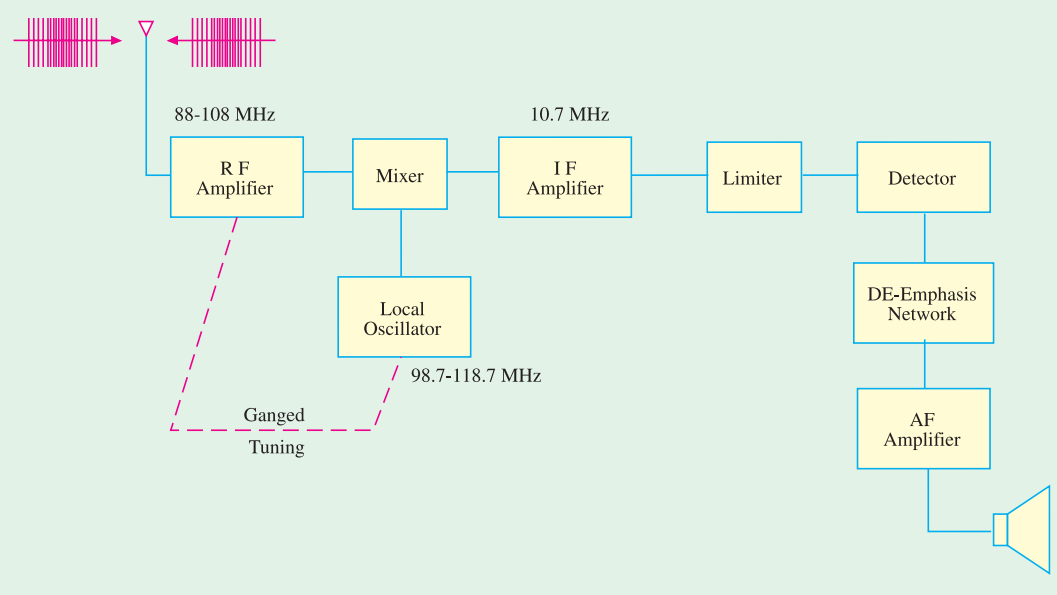


Fig. 66.31

66.37. FM Receiver

The block diagram for such a receiver is shown in Fig. 66.31. These receivers operate in the *VHF* band of 88-108 MHz and have an *IF* of 10.7 MHz with a bandpass of 2 ×75 kHz = 150 kHz.

Like *AM* receivers, such receivers also have a frequency converter and *IF* amplifiers.

The function of the **limiter** is to remove all ***amplitude variations*** (caused by noise) from *IF* signal which might have crept into the *FM* signal. This removal of amplitude variations is necessary for distortionless demodulation. **Limiter is a sort of clipping circuit**.

The de-emphasis network ***reduces the amplitude of high frequencies in the audio signal*** which was earlier increased by the pre-emphasis network at the transmitting station. It serves to re-establish the **tonal balance** of the speech or music etc. lost in the pre-emphasis. Without it, the sound signal would have a heavy treble effect. It is, in fact, a low-pass filter having a time constant of 75 μs.

66.38. Comparison Between AM and FM

Frequency modulation ***(FM)*** has the following advantages as compared to amplitude modulation ***(AM)*** :

1. All transmitted power in ***FM*** is useful whereas in ***AM*** most of it is in carrier which ***serves no useful purpose.***
2. It has high signal-to-noise (S/N) ratio. It is due to two reasons : firstly, there happens to be less noise at ***VHF*** band and secondly, ***FM*** receivers are fitted with amplitude limiters which remove amplitude variations caused by noise.
3. Due to 'guard-band' there is hardly any adjacent-channel interference.
4. Since only transmitter ***frequency*** is modulated in ***FM***, only fraction of a watt of audio power is required to produce 100% modulation as compared to high power required in *AM*.

However, FM has the following disadvantages :

1. It requires **much wider channel** — almost 7 to 15 times as large as needed by AM.
2. It requires complex and expensive transmitting and receiving equipment.
3. Since FM reception is limited to only line of sight, area of reception for FM is much smaller than for *AM*.

66.39. The Four Fields of FM

There are four major areas of application for *FM* transmission :

1. First use is in **FM** broadcast band 88-108 MHz with 200 kHz channels in which commercial *FM* stations broadcast programmes to their listeners.
2. Second use is in TV. Though video signal is amplitude-modulated, sound is transmitted by a separate transmitter which is frequency-modulated.
3. Third use is in the mobile or emergency services which transmit voice frequencies (20-4000 Hz) only.
4. Fourth use is in the amateur bands where again only voice frequencies are transmitted.

Tutorial Problems No. 66.1

1. A carrier wave has an amplitude of 500 mV. A modulating signal causes its amplitude to vary from 200 mV to 800 mV. What is the percentage modulation? **[60%]**
2. A 100.MHz carrier having an amplitude of 50 V is amplitude-modulated by a 5 kHz audio signal having an amplitude of 20 V. Find
   (*i*) modulation index,
   (*ii*) percent modulation,
   (*iii*) components of modulated wave,
   (*iv*) the amplitudes of sidebands.
   [(*i*) 0.4 (*ii*) 40 (*iii*) 100 MHz, 100.005 MHz, 99.995 MHz (*iv*) 10 V]
3. An audio signal given by 30 sin ($2\pi \times 2500t$) is used for modulating a carrier wave given by the equation 60 sin ($2\pi \times 200{,}000t$). Find
   (*i*) percent modulation,
   (*ii*) frequencies of the signal and the carrier,
   (*iii*) frequency spectrum of the modulated wave.
   [(*i*) 50 (*ii*) 2500 Hz , 200,000 Hz (*iii*) 200; 202.5; 197.5 kHz)]
4. How many AM broadcast stations can be accommodated in a 5 MHz bandwidth if each station transmits a signal modulated by an audio signal having a maximum frequency of 5 kHz. **[500]**
5. A 100 kHz bandwidth is to accomodate 5 AM broadcasts simultaneously. What is the maximum modulating frequency permissible for each station? **[5 kHz]**
6. Total power content of an AM signal is 3000 W. For 100 percent modulation, calculate
   (*i*) carrier power (*ii*) power in each sideband **[(*i*) 2 kW (*ii*) 0.5 kW]**
7. The total power content of an AM wave is 1320 W. What is the percent modulation if each sideband contains 160 W ? **[0.8]**
8. An SSB-SC signal contains 2 kW. Find power contained in
   (*i*) carrier (*ii*) sideband.
   [(*i*) 0 (*ii*) 2 kW]
9. As compared to DSB-FC, Calculate the power saving in SSB-SC when percent modulation is (*i*) 1 (*ii*) 0.8 (*iii*) 0.5
   [(*i*) 83.3% (*ii*) 87.87% (*iii*) 94.4%]
10. What is the modulation index of an FM carrier having a carrier swing of 120 kHz and a modulating signal of 10 kHz. **[6]**
11. An FM signal has a resting frequency of 100 MHz and the highest frequency of 100.05 MHz when modulated by an audio signal of 5 kHz. Determine
    (*i*) frequency deviation , (*ii*) carrier swing,
    (*iii*) modulation index, (*iv*) percent modulation.
    [(*i*) 50 kHz (*ii*) 100 kHz (*iii*) 10 (*iv*) 66.7%]
12. A 10 kHz audio signal is used to frequency-modulate a 100 MHz carrier causing a frequency deviation of 50 kHz. Determine
    (*i*) modulation index, (*ii*) bandwidth of FM signal. **[(*i*) 5 (*ii*) 160 kHz)]**

13. A carrier of $1MH_z$ with 400 Watt of its power is A amplitude modulated with a sinusoidal signal of 2500 Hz. The depth of modulation is 75%. Calculate the side band frequencies, the band width, the power in the side bands and the total power in the modulated wave.

[(Electronics Engg; Bangalore Univ. 2004)]

14. The total power content of an AM wave is 2.64 kW at a modulation factor of 80%. Determine the power content of ***(i)*** carrier ***(ii)*** each side band.

[(Electronics Engg. Bangalore Univ. 2002)]

15. A 100MHz carrier wave is frequency modulated by a 10 kHz sinusoidal modulating signal. If the maximum frequency deviation is 50 kHz. Find the modulation index.

[(Electronics Engg. Bangalore Univ., 2002)]

16. A 500 W, 1MHz carrier is amplitude modulated with a sinusoidal signal of 1 kHz. The depth of modulation is 60%. Calculate the band width, power in the side bands and the total power transmitted.

[(Electronics Engg., Bangalore Univ., 2003)]

17. A carrier signal has a peak amplitude of 100V. Modulation index is 40%. Power is developed across a load of 100 Ω. Determine the value of transmitted power.

(Electronics Engg. Bangalore Univ., 2003)

OBJECTIVE TESTS – 66

1. The main purpose of modulation is to
(*a*) combine two waves of different fre- quencies
(*b*) achieve wave-shaping of the carrier wave
(*c*) transmit low-frequency information over long distances efficiently
(*d*) produce sidebands.

2. Demodulation
(*a*) is performed at the transmitting station
(*b*) removes side-bands
(*c*) rectifies modulated signal
(*d*) is opposite of modulation.

3. In amplitude modulation
(*a*) carrier frequency is changed
(*b*) carrier amplitude is changed
(c) three sidebands are produced
(d) fidelity is improved.

4. 100% modulation is produced in AM when carrier
(*a*) frequency equals signal frequency
(*b*) frequency exceeds signal frequency
(*c*) amplitude equals signal amplitude
(*d*) amplitude exceeds signal amplitude.

5. For a given carrier wave, maximum undistorted power is transmitted when value of modulation is
(*a*) 1 (*b*) 0.8
(*c*) 0.5 (*d*) 0.

6. In an AM wave with 100 percent modulation, each sideband carries — of the total transmitted power.
(*a*) one-half (*b*) one-sixth
(*c*) one-third (*d*) two-third

7. Given a carrier frequency of 100 kHz and a modulating frequency of 5 kHz, the bandwidth of AM transmission is____ kHz.
(*a*) 5 (*b*) 200
(*c*) 10 (*d*) 20

8. When modulation of an AM wave is decreased
(*a*) percentage carrier power is decreased
(*b*) percentage carrier power is increased
(*c*) total transmitted power is increased
(*d*) percentage sideband power is unaffe cted.

9. In AM transmission, power content of the carrier is maximum when m equals
(*a*) 0 (*b*) 1
(*c*) 0.8 (*d*) 0.5.

10. In AM transmission with m = 1, suppression of carrier cuts power dissipation by a factor of
(*a*) 6 (*b*) 2
(*c*) 3 (*d*) 4

11. As compared to DSB-FC 100% modulated transmission, power saving in SSB-SC system is _____ per cent.
(*a*) 94.4 (*b*) 50
(*c*) 100 (*d*) 83.3

12. In an AM transmission with 100% modulation, 66.7% of power is saved when _____ is/are suppressed.
(*a*) carrier (*b*) carrier and USB
(*c*) carrier and LSB (*d*) USB and LSB

13. In an AM system, full information can be conveyed by transmitting only
(*a*) the carrier (*b*) the upper sideband
(*c*) the lower sideband (*d*) any one sideband.

14. A 10 M Hz carrier of peak value 10 V is amplifide modulated by a 10 kHz signal of amplitude 6 V. The amplitude of each side band frequency is
(*a*) 3 V (*b*) 4 V
(*c*) 5 V (*d*) 6 V

15. The block diagram of an AM receiver is shown in Fig. 66.32. The missing block and its location are respectively

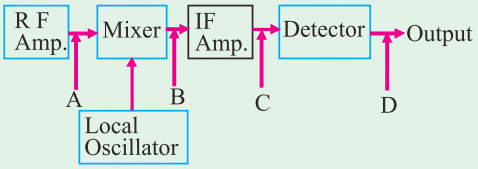


Fig. 66.32

(*a*) Band - pass filter at A
(*b*) High-pass filter at B
(*c*) Band - pass filter at C
(*d*) Low-pass filter (audio amplifier) at D.

16. In FM transmission, amplitude of the modulating signal determines
(*a*) rate of frequency variations
(*b*) amount of frequency shift
(*c*) tonal balance of transmission
(*d*) distance of broadcast.

17. In FM, when frequency deviation is doubled
(*a*) modulation is doubled
(*b*) modulation is halved
(*c*) carrier swing is halved
(*d*) modulation index is decreased.

18. In an FM broadcast in VHF band, channel width is _____
(*a*) 75 (*b*) 25
(*c*) 88 (*d*) 200

19. Modern FM receivers use de-emphasis circuit for
(*a*) reducing high frequency noise
(*b*) reducing the amplitude of high fre- quencies in the audio signal
(*c*) increasing the amplitude of higher modulating frequencies
(*d*) making demodulation easy

20. One of the serious disadvantages of FM transmission is its
(*a*) high static noise
(*b*) limited line-of-sight range
(*c*) expensive equipment
(*d*) adjacent channel interference.

21. In India, the maximum frequency deviation is commercial FM broadcasting signals is about,
(*a*) 5 kHz (*b*) 15 kHz
(*c*) 7.5 kHz (*d*) 200 kHz

22. The correct sequence of subsystems in an FM receiver is
(*a*) mixer, RF amplifier, limiter, IF amplifier, discriminator, audio amplifier
(*b*) RF amplifier, mixer, IF amplifier, limiter, discriminator, audio amplifier
(*c*) RF amplifier, mixer, limiter, discriminator, IF amplifier, audio amplifier
(*d*) mixer, IF amplifier, limiter, audio amplifier, discriminator.

ANSWERS

1. (*c*) **2.** (*d*) **3.** (*b*) **4.** (*c*) **5.** (*a*) **6.** (*b*) **7.** (*c*) **8.** (*b*) **9.** (*a*) **10.** (*c*) **11.** (*d*)
12. (*a*) **13.** (*d*) **14.** (*a*) **15.** (*d*) **16.** (*b*) **17.** (*a*) **18.** (*d*) **19.** (*b*) **20.** (*b*) **21.** (*d*) **22.** (*b*)

GO To FIRST

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CHAPTER 67

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INTEGRATED CIRCUITS



Jack Kilby would justly be considered one of the greatest electrical engineers of all time for one invention; the monolithic integrated circuit, or microchip. He went on to develop the first industrial, commercial and military applications for this integrated circuits- including the first pocket calculator (pocketronic) and computer that used them

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67.1. Introduction

Electronic circuitry has undergone tremendous changes since the invention of a triode by Lee De Forest in 1907. In those days, the active components (like triode) and passive components (like resistors, inductors and capacitors etc.) of the circuits were **separate and distinct units** connected by soldered leads.

With the invention of the transistor in 1948 by W.H. Brattain and I. Bardeen, the electronic circuits became considerably reduced in size. It was due to the fact that a transistor was not only cheaper, more reliable and less power consuming but was also much smaller in size than an electron tube. To take advantage of small transistor size, the passive components too were greatly reduced in size thereby making ***the entire circuit very small.*** Development of printed circuit boards (*PCBs*) further reduced the size of electronic equipment by eliminating bulky wiring and tie points.

In the early 1960s, a new field of **microelectronics** was born primarily to meet the requirements of the Military which wanted to reduce the size of its electronic equipment to approximately one-tenth of its then existing volume. This drive for extreme reduction in the size of electronic circuits has led to the development of microelectronic circuits called **integrated circuits (ICs)** which are so small that their actual construction is done by technicians using high powered microscopes.

67.2. What is an Integrated Circuit ?

To put it very briefly, an integrated circuit (*IC*) is **just a packaged electronic circuit.**

A more detailed definition is as under :

An *IC* is a complete electronic circuit in which both the active and passive components are fabricated on a tiny single chip of silicon.

Active components are those which have the ability to produce gain. Examples are : transistors and *FETs*.

Passive components or devices are those which do not have this ability. Examples are : resistors, capacitors and inductors.

ICs are produced by the same processes as are used for manufacturing individual transistors and diodes etc. In such circuits, different components are isolated from each other by isolation diffusion within the crystal chip and are interconnected by an aluminium layer that serves as wires.

A discrete circuit, on the other hand, is one ***that is built by connecting separate components.*** In this case, each component is produced separately and then all are assembled together to make the electronic circuit.

J.S. Kilby of Texas Instruments was the first person to develop (in 1959) an integrated circuit — a single monolithic silicon chip in which active and passive components were fabricated by successive deposition, etching and diffusions. He was soon followed by Robert Noyce of Fairchild who successfully fabricated a complete *IC* including the interconnections on a single silicon chip. Since then the evolution of this technology is fast-paced.

67.3. Advantages of ICs

As compared to **standard printed circuits** which use discrete components, *ICs* have the following advantages :

1. Extremely small physical size

Often the size is thousands of times smaller than a discrete circuit. The various components and their interconnections are distinguishable only under a powerful microscope.

2. Very small weight

Since many circuit functions can be packed into a small space, complex electronic equipment can be employed in many applications where weight and space are critical, such as in aircraft or space-vehicles.

3. Reduced cost

It is a major advantage of *ICs*. The reduction in cost per unit is due to the fact that many identical circuits can be built simultaneously on a single wafer—this process is called **batch fabrication.** Although the processing steps for the wafer are complex and expensive, the large number of resulting integrated circuits make the ultimate cost of each *IC* fairly low.

4. Extremely high reliability

It is perhaps the **most important** advantage of an *IC* and is due to many factors. Most significant factor is the absence of soldered connections. Another is the need for fewer interconnections—the major cause of circuit failures. Small temperature rise due to low power consumptions of *ICs* also improves their reliability. In fact, an *IC* logic gate has been found to be 100,000 times more reliable than a vacuum tube logic gate and 100 times more reliable than a transistor logic gate.

Obviously, higher reliability means that *ICs* will work for longer periods without giving any trouble—something most desirable from both military and consumer application point of view.

5. Increased response time and speed

Since various components of an *IC* are located close to each other ***in*** or ***on*** a silicon wafer, the time delay of signals is reduced. Moreover, because of the short distances, the chance of stray electrical pickup (called parasitic capacitance) is practically nil. Hence it makes them very suitable for small signal operation and high frequency operation. As a result, the response time or the operating speed of the system is improved.

6. Low power consumption

Because of their small size, *ICs* are more suitable for low power operation than bulky discrete circuits.

7. Easy replacement

ICs are hardly ever repaired because in case of failure, it is more economical to replace them than to repair them.

8. Higher yield

The ***yield*** is the percentage of usable devices. Because of the batch fabrication, the yield is very high. Faulty devices usually occur because of some defect in the silicon wafer or in the fabrication steps. Defects in silicon wafer can occur because of lattice imperfection and strains introduced in crystal growth, cutting and handling of the wafers. Usually such defects are extremely small, but their presence can ruin devices built on or around. Reducing the size of each device greatly increases the chance for a given device to be free of such defects. The same is true for fabrication defects such as the presence of a dust particle on the photolithographic mask.

67.4. Drawbacks of ICs

The integrated circuits suffer from the following drawbacks :

1. coils or inductors cannot be fabricated,
2. *ICs* function at fairly low voltages,
3. they handle only limited amount of power,
4. they are quite delicate and cannot withstand rough handling or excessive heat.

However, the advantages of *ICs* far outweigh their disadvantages or drawbacks.

67.5. Scale of Integration

Level of integration in *ICs* has been increasing ever since they were developed some three and a half decades back. The number of electronic circuits or components that can be fitted into a standard size *IC* has been dramatically increasing with each passing year. In fact, whole **electronic systems** rather than just **a circuit** are incorporated in one package.

An approximate method of classifying the amount of circuit or component density is as follows:

1. SSI—*small scale integration*

In this case, the number of **circuits** contained in one *IC* package is less than 12 (or number of **components** is less than 50).

2. MSI—*medium scale integration*

Here, number of circuits per package is between 13 and 99 (or number of components is between 50 and 5000).

3. LSI—*large scale integration*

In this case, circuit density is between 100 and 9,999 (or component density is between 5000 and 100,000).

4. VLSI—*very large scale integration*

Here the number of circuits per package is between 10,000 to 99,999 (or number of components is between 100,000 – 1,000,000).

5. ULSI—*ultra large scale integration*

In this case, the circuit density is between 100,000 to 999,999 (or component density is between 1,000,000 – 10,000,000).

6. GSI—*Giga scale integration*

Here the number of circuits per package is 1,000,000 or more (or number of components are over 100,000,000).

In summary,

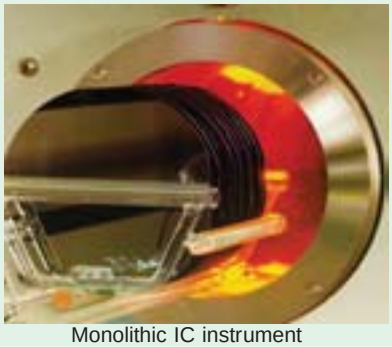
SSI	<	12	circuits per chip
MSI		12 – 99	
LSI		100 – 9,999	
VLSI		10,000 – 99,999	
ULSI		100,000 – 999,999	
GSI	>	1,000,000	

67.6. Classification of ICs by Structure

Structurally speaking, *ICs* can be classified into the following three types :

1. Monolithic Integrated Circuits

The word 'monolithic' means 'single stone' or more appropriately 'a single-solid structure'. In this *IC*, all circuit components (both active and passive) are fabricated inseparably within a single continuous piece of silicon crystalline material called **wafer (or substrate).** All components are **atomically** part of the same chip. Transistors, diodes and other passive components are fabricated at appropriate spots in the substrate using epitaxial diffusion technique.


Monolithic IC instrument

Component interconnections are provided on the surface of the structure and external connecting wires are taken out to

the terminals. It is a complete circuit requiring no 'add ons'.

Despite some of its distinct disadvantages, monolithic *ICs* are in wide use because for mass production, monolithic process has been found to be the most economical.

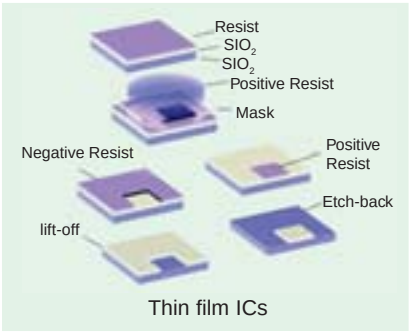
2. Thick and Thin-Film ICs

The essential difference between thick-film and thin-film *ICs* *is not their relative thickness but* the **method** of depositing the film. Both have similar appearance, properties and general characteristics though they both differ in many respects from monolithic *ICs.* These *ICs* are not formed **within** a silicon wafer but **on** the surface of an insulating substrate such as glass or a ceramic material. Moreover, **only passive components** (resistors, capacitors) are formed through thick or thin-film techniques on the insulating surface. The active elements (transistors, diodes) are added externally as **discrete elements** to complete a functional circuit. These discrete active components are frequently produced by using the monolithic process.

As stated above, the primary difference between the thick and thin film techniques is the process used for forming passive components and the metallic conduction pattern.

(*a*) Thin-film ICs

Such circuits are constructed by depositing films (typically 0.1 to 0.5 μm) of conducting material through a mask on the surface of a substrate made of glass or ceramic. Resistors and conductors are formed by varying the width and thickness of the film and by using materials of different resistivity. Capacitors are produced by sandwiching an insulating oxide film between two conducting films. Small inductors can be made by depositing a spiral formation of film. The active components like transistors and diodes etc. are externally added and inter-connected by wire bonds.



Thin film ICs

Following two methods are used to produce thin films :

(*i*) vacuum evaporation

In this method, the vaporised material is deposited through a set of masks on the glass or ceramic substrate contained in vacuum.

(*ii*) cathode sputtering

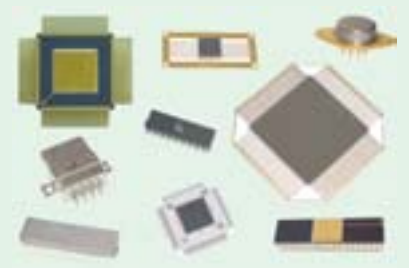
In this method, atoms from a cathode made of the desired film material are deposited on the substrate which is located between the cathode and the anode.

(*b*) Thick-film ICs

Such type of integrated circuits are sometimes referred to as **printed** thin-film circuits. They are so called because silk-screen printing techniques are employed to create the desired circuit pattern on the surface of the substrate. The screens are made of fine stainless steel wire mesh and the 'inks' are pastes (of pulverised glass and aluminium) which have conductive, resistive or dielectric properties. After printing, the circuits are high-temperature fired in a furnace to fuse the films to the insulating substrate. As with thin-film *ICs*, active elements are added externally as discrete components.

3. Hybrid or Multichip ICs

As the name implies, such circuits are formed either by inter-connecting a number of individual chips or by a combination of film and monolithic *IC* techniques. In such *ICs*, active components are first formed within a silicon wafer (using monolithic technique) which is subsequently covered with an insulating layer such as SiO_2. Film techniques are then employed to form passive components on the SiO_2 surface. Connections are made from the film to the monolithic structure through 'windows' cut in the SiO_2 layer.


Hybrid or Multichip ICs

67.7. Comparison Between Different ICs

Each type of *IC* has its own advantages and disadvantages.

Monolithic circuits have ***the advantage of lowest cost and highest reliability.*** However they have the following disadvantages :

1. isolation between components is poorer,
2. range of values of passive components used in the circuits is comparatively small,
3. inductors cannot be fabricated,
4. they afford no flexibility in circuit design because for making any changes in the circuit, **a new set of masks is required.**

The film circuits have the advantage of forming passive components with broader range of values and reduced tolerances as compared to monolithic circuits. Isolation between their components is also better since they are deposited on a substrate that is an insulator. Use of external discrete active components allows greater flexibility in circuit design. These circuits also give better high frequency performance than monolithic circuits.

However, they suffer from the disadvantages of

1. not being able to fabricate active components,
2. comparatively higher cost and 3. larger physical size.

The chief advantage of multichip *ICs* is their greater flexibility but they are **too expensive** for mass production and **have least reliability.** Hence, such circuits are generally used as prototypes for monolithic *ICs*.

The various integrated circuits are arranged in the form of an '*IC*' tree of Fig. 67.1.

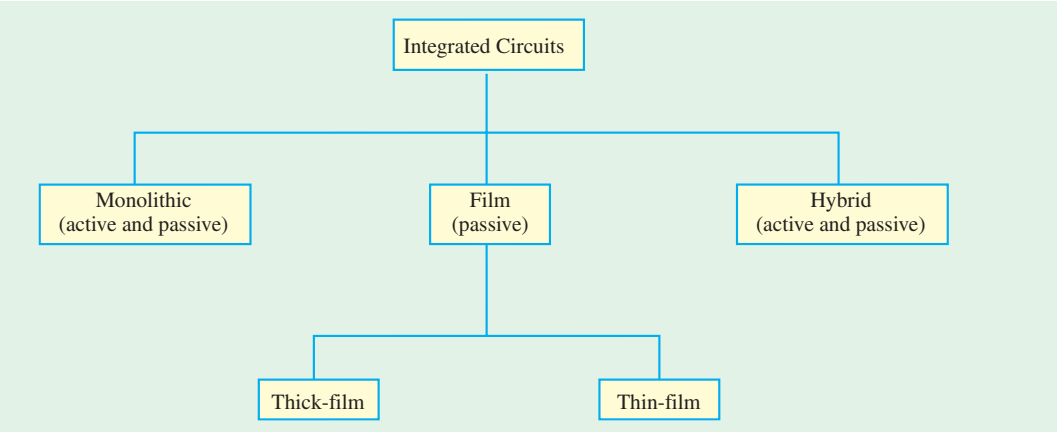


Fig. 67.1

67.8. Classification of ICs By Function

The earlier classification of ICs was based on ***their method of construction.*** However, the integrated circuits can also be classified according to their general function. The two most important categories are :

1. linear and 2. digital

The same fact has been shown in Fig. 67.2.

Examples of linear *ICs* are :

1. *BEL CA*-3020—used as multipurpose wide-band power amplifier.

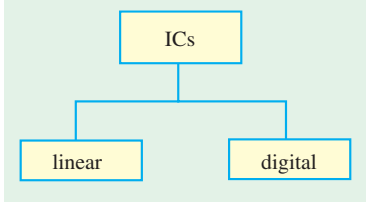


Fig. 67.2

2. *BEL CA*-3065—it is a monolithic *IC* which combines a multistage *IF* amplifier, limiter, an *FM* detector, an electronic attenuator, a Zener diode regulated power supply and an audio amplifier. In fact, this *IC* provides a high performance multistage sub-system of a TV receiver. It is available in 14-pin dual-in-line package.

Both are manufactured by Bharat Electronics Ltd., Bangalore.

3. SSD 710—is a linear *IC* used as Differential Comparator

It is manufactured by Solid State Devices, Syed Abdullah Road, Bombay.

An example of a digital *IC* is

1. *BEL* 7400—is a *TTL* IC gate which provides designer with one of the gating logic necessary to design medium-speed digital control and data processing systems. It is available in 14-pin dual-in-line plastic package.

67.9. Linear Integrated Circuits (LICs)

LICs are also referred to as analog *ICs* because their inputs and outputs can take on a continuous range of values and the outputs are generally proportional to the inputs. As compared to digital *ICs*, *LICs* are used much less. But *LICs* are quickly displacing their discrete circuit counterparts in many applications as their cost becomes competitive. They also possess much higher reliability because so many external connections (major source of circuit failure) are eliminated. *LICs* find wide use in military and industrial applications as well as in consumer products. They are frequently used in



Linear ICs

1. operational amplifiers,
2. small-signal amplifiers,
3. power amplifiers,
4. *RF* and *IF* amplifiers,
5. microwave amplifiers,
6. multipliers,
7. voltage comparators,
8. voltage regulators etc.

Operational amplifier is by far the most versatile form for an *LIC* and is discussed separately.

67.10. Manufacturer's Designation of LICs

Each manufacturer assigns a specific code and type number to the *LICs* produced by him. For example, an internally-compensated op-amp 741 produced by Fairchild is designated as µA 741. Here, µA is the identifying code used by Fairchild. Many other manufacturers produce *LICs* similar to 741 but use their own code while retaining the same type number. For example, Fiarchild's original µA 741 is manufactured by other manufacturers with the following codes :

1. National Semiconductor —— *LM* 741
2. Motorola —— *MC* 1741
3. RCA —— *CA* 3741
4. Texas Instruments —— *SN* 5274

It is seen that the last three digits in each manufacturer's designation are the same *i.e.* 741. All these op-amps have the same specifications. Hence, in practice, the manufacturer's code number is often ignored and all such *LICs* are referred to as 741.

Many *LICs* are available in different classes such as A, B, C, E, S and SC. For example, main classes of 741 are as under :

741	——	Military grade op-amp
741 C	——	Commercial grade op-amp
741 A	——	Improved version of 741
741 E	——	Improved version of 741 C
741 S	——	Military grade op-amp with higher slew rate
741 SE	——	Commercial grade op-amp with higher slew rate

67.11. Digital Integrated Circuits

About 80 per cent of the *IC* market has been captured by digital *ICs* which are mostly utilized by the computer industry. Digital *ICs* lend themselves easily to monolithic integration because a computer ***uses a large number of identical circuits.*** Moreover, such circuits employ relatively few capacitors and values of resistances, voltages and currents are low.

Digital *ICs* contain circuits whose input and output voltages are limited to **two** possible levels—low or high. It is so because ***digital signals are usually binary.*** Sometimes, digital circuits are referred to as switching circuits. Digital *ICs* include circuits such as

1. logic gates **2.** flip-flops **3.** counters **4.** clock-chips
5. calculator chips **6.** memory chips **7.** microprocessors (μP) etc.

67.12. IC Terminology

Some of the common terms used in fabricating integrated circuits are defined below :

1. **Bonding** – attaching the die on the ceramic substrate and then connecting the leads to the package.
2. **Chip** – an extremely small part of a silicon wafer on which *IC* is fabricated. A photograph of the wafer containing hundreds of chips (or dice) and a drawing of a chip are shown in Fig. 67.3. The identical chips, each of which may vary in area from 10 to over 100 mm^2, may contain up to several million devices.

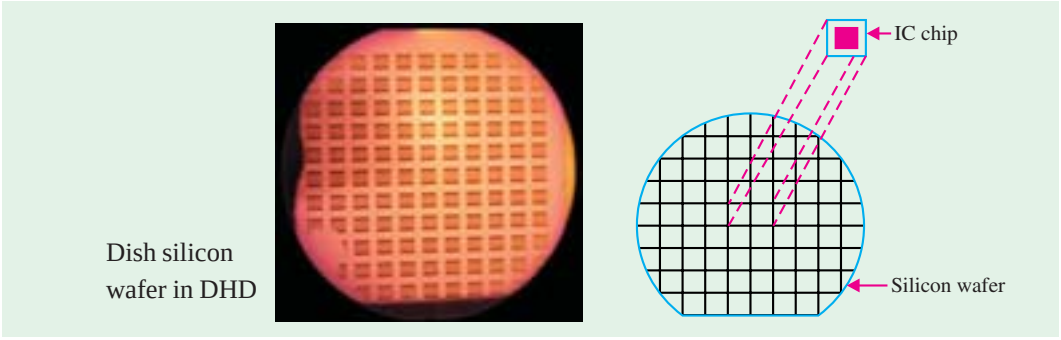


Fig. 67.3

3. **Circuit probing** – testing the electrical performance of each *IC* chip with the help of a microscope and multi-point probe.
4. **Die** – same as a chip.
5. **Diffusion** – a process that consists of the introduction of impurities into selected regions of a wafer to form junctions.
6. **Encapsulation** – putting a cap over the *IC* and sealing it in an inert atmosphere.

7. **Epitaxy** – a process of the controlled growth of a crystalline doped layer of silicon on a single crystal substrate.
8. **Etching** – a process of selective removal of regions of a semiconductor, metal or silicon dioxide.
9. **Mask** – a glass plate with desired pattern for diffusion or metallization. Usually a single mask is not sufficient to fabricate an *IC*.
10. **Metallization** – a process for providing ohmic contacts and interconnections by evaporating aluminium over the chip.
11. **Photolithography** – a process to transfer geometrical pattern from the mask to the surface of the wafer.
12. **Photoresist** – a light-sensitive material that hardens when exposed to ultraviolet light.
13. **Wafer** – a thin disk of semiconductor in which number of *ICs* are fabricated simultaneously.

67.13. Semiconductors Used in Fabrication of ICs and Devices

The fabrication of *ICs* has been based on the use of silicon (Si) as the premier semiconductor. Two other semiconductors used for *IC* fabrication are germanium and gallium arsenide (GaAs). But these semiconductors present special problems for device fabrication as discussed below.

Gallium arsenide has very attractive electrical properties but its crystals have a high density of defects which limit the performance of devices made from it. Moreover gallium arsenide is more difficult to grow in single crystal form. Both silicon and germanium do not suffer from these problems. On the plus side, gallium arsenide has an electron velocity that is larger than silicon. Because of this gallium arsenide devices are faster than silicon devices. Also, gallium arsenide has a lower saturation electric field than silicon. Because of this, the gallium arsenide devices have lower power-delay product. Devices made from substrates of gallium arsenide have lower parasitic capacitances. This property contributes to their speed advantage over the silicon devices. Another advantage of gallium arsenide results from direct band gap which makes it possible to provide certain functions not possible in silicon such as ***coherent* and *incoherent light emission.***

A major advantage of silicon, in addition to its abundant availability in the form of sand, is that it is possible to form a superior stable oxide (SiO_2). This oxide has superb insulating properties and provides an essential and excellent ingredient in the fabrication and protection of devices or *ICs*. On the other hand, germanium oxide is unsuited for device applications. The intrinsic resistivity of germanium is 47 Ω-cm, while that of silicon is 230,000 Ω-cm. The low resistivity of germanium would have precluded the fabrication of rectifying devices with high breakdown voltages. Thus high-voltage rectifying devices and certain infrared sensing devices are practical with silicon. Finally, there is an economic consideration and that is electronic grade germanium is now more costly than silicon. Thus, at present time, silicon remains the major semiconductor in the fabrication of *ICs*.

Some other semiconductors used in semiconductor industry are gallium phosphide (GaP), gallium nitride (GaN), zinc sulphide (ZnS), indium antimonide (InSb), compound of cadmium and selenium (CdSe). GaAs, GaP and GaN are used in high-speed devices and devices requiring emission and absorption of light such as lasers and light emitting diodes (LEDs). ZnS is used as fluorescent material such as those in television screens. In Sb and CdSe are used as light detectors.

67.14. How ICs are Made ?

The *ICs* are manufactured in four distinct stages (refer to Fig. 67.4). These are **(1)** material preparation, **(2)** crystal growing and wafer preparation, **(3)** wafer fabrication and **(4)** testing, bonding and packaging. All these stages are discussed one by one in the following pages.

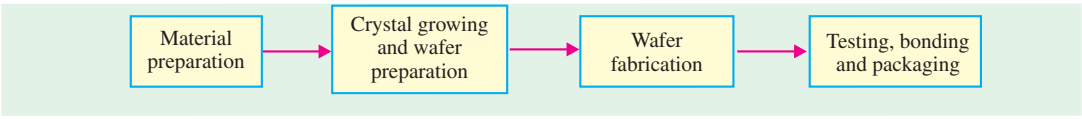


Fig. 67.4

67.15. Material Preparation

Silicon, as an element is not found in nature. However, it is found abundantly in nature in the form of silicon dioxide, which constitutes about 20% of earth's crust. Silicon is commonly found as quartz or sand. A number of processes are required to convert sand into pure silicon with a polycrystalline structure. Fig. 67.5 shows the different processes involved in the preparation of polycrystalline silicon from sand. As seen from this figure, the sand is allowed to react with a gas produced from the burning of carbon (coal, coke and wood chips). This produces silicon with 98% purity. Next silicon is further purified in a reactor to produce electronic-grade polycrystalline silicon.

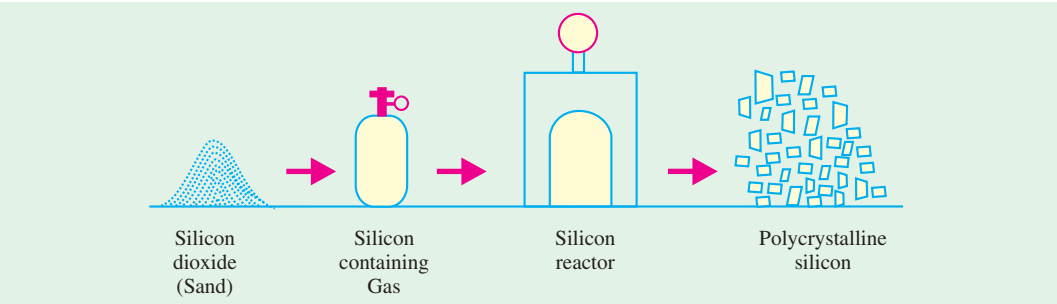


Fig. 67.5

67.16. Crystal Growing and Wafer Preparation

The polycrystalline silicon is composed of many small crystals having random orientation and containing many defects. For silicon to be used in the fabrication of *ICs*, it must be nearly perfect and crystalline in nature. We, therefore, now need to produce single crystals of silicon. This is done by a process called crystal growth. There are two methods to carryout the crystal growth : (***i***) the ***Czochralski*** and (*ii*) the ***flat zone*** process. The Czochralski process prepares virtually all the silicon used for *IC* fabrication. The flat zone process is used to prepare crystals for fabricating high-power, high voltage semiconductor devices.

The Czochralski process. The equipment used for single crystal growth (called puller) is as shown in Fig. 67.6. The puller has three main components : **(*i*)** a furnace which includes quartz crucible, a rotation mechanism (clockwise as shown), and a radio frequency (*RF*) heating element, **(*ii*)** a crystal pulling mechanism which includes a seed holder and a rotation mechanism (counter-clockwise), and **(*iii*)** an ambient control which includes an argon gas source, a flow control and an exhaust system. In addition the puller has a computer system to control process parameters such as temperature, crystal diameter, pull rate and rotation speed.

To grow crystals, the polycrystalline silicon is placed in the crucible. The furnace is heated to a temperature of 1690 K which is slightly greater than the melting point (1685 K) of silicon. A precisely controlled amount of dopant (boron or phosphorus) is added to the melt to make the silicon as *P*-type or *N*-type. A suitable oriented seed crystal (*i.e.*, a small highly perfect crystal) is suspended over the crucible in a seed holder. The seed is inserted into the melt and a small portion of it is

allowed to melt. The seet is rotated and pulled up very slowly, while at the same time, the crucible is rotated in the opposite direction. The molten silicon attaches itself to the seed and it becomes identical to the seed in structure and orientation. As the seed is pulled up, the material that is attached to the seed solidifies (*i.e.*, freezes). Its crystal structure becomes the same as that of the seed and a larger crystal is formed. Thus using this method, cylindrical single crystal bars (called ingots) of silicon are produced.

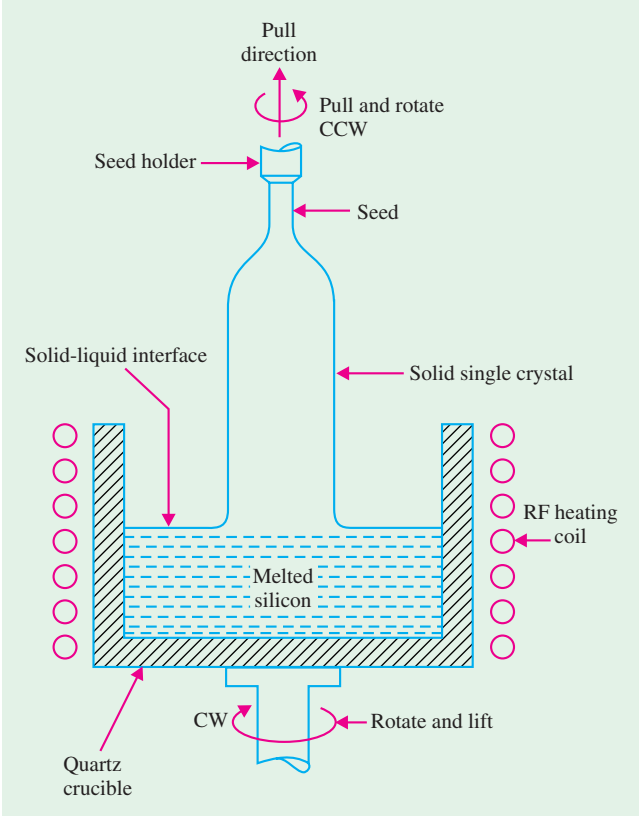


Fig. 67.6

The desired diameter of the silicon ingot is obtained by controlling both the temperature and the pulling speed. In the final step, when the bulk of the melt has been grown, the crystal diameter is decreased until there is a point contact with the melt. The resulting ingot is cooled and is removed to be made into thin discs called ***wafers***. The ingots have diameters as large as 200 mm with the latest ones approaching 300 mm. The ingot length is of the order of 1000 mm.

Wafer preparation : In this stage the ingot surface is grounded throughout to an exact diameter and the top and bottom portions are cut off. After that one or more flat regions are ground along the length of the ingot. These flat regions mark the specific crystal orientation of the ingot and conductivity type (*i.e.*, *P*-type or *N*-type) of silicon material. Refer to Fig. 67.7.

Notice that the ingot is marked with two flats regions : ***(i)*** the larger flat (called primary flat) and ***(ii)*** the smaller flat or secondary flat regions. The primary flat allows a mechanical locator in automatic processing equipment to position the wafer and to orient the devices relative to the crystal in a specific manner. The secondary flat regions are used to identify the orientation and conductivity type of the crystal. The conductivity of the wafer could be either *P*-type or *N*-type and the crystal orientation, {100} or {111} (refer to Art 50.16 to know more about crystal orientation). If there is no secondary flat as shown in Fig. 67.7 (*a*), or in words the primary and the secondary flat are superimposed, the wafer is identified as *P*-type with {111}-crystal orientation. If the secondary flat is 45° with respect to the primary flat as shown in Fig. 67.7 (*b*), it is {111} *N*-type wafer. However, if the secondary flat is 90° with respect to the primary flat as shown in Fig. 67.7 (*c*), it is {100} *P*-type wafer and if the secondary flat is at 180° with respect to the primary flat as shown in Fig. 67.7 (*d*), it is {100} *N*-type wafer.

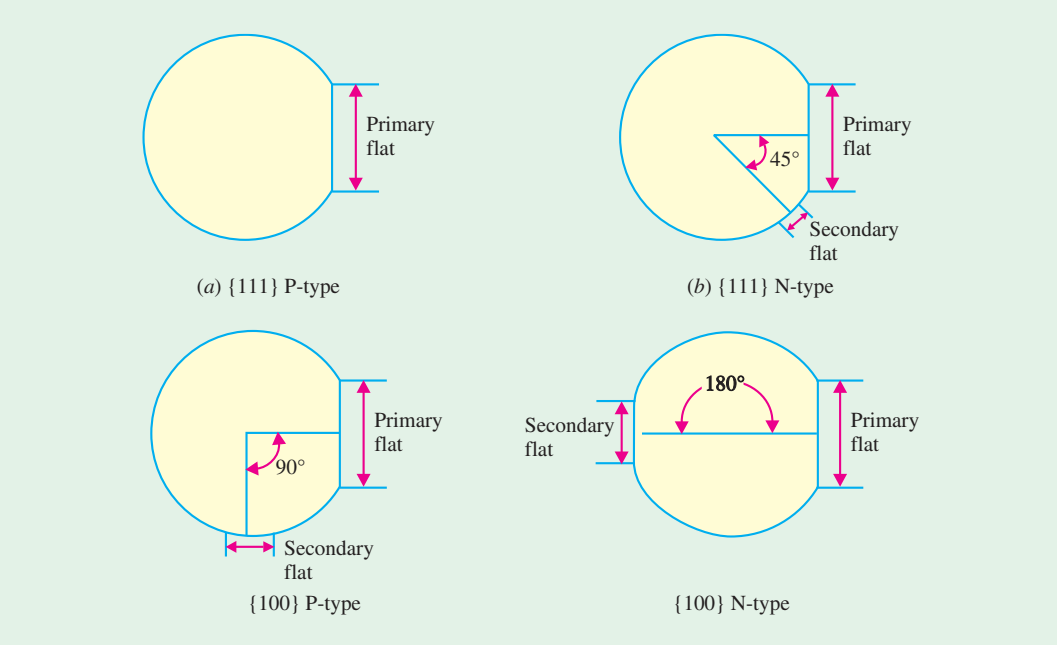


Fig. 67.7

The semiconductor industry uses {111} wafers for fabricating *ICs* with bipolar transistor technology and {100} wafers for metal-oxide semiconductor (MOS) circuits. The choice of wafer conductivity (*i.e.*, *P*-type or *N*-type) depends upon the actual process used for fabricating the *ICs*. Once the orientations are done, the ingot is sliced into wafers by a high-speed ***diamond saw.*** The wafer thickness varies from 0.4 to 1.0 mm.

After slicing, both sides of the wafer are lapped to produce typical flatness uniformity with in 2 μm. The lapped orientation usually leaves the surface and edges of the wafer, damaged and contaminated. These can be removed by a process called chemical etching. The next step is to polish the wafer surface to a mirror-like finish. Finally the wafers are cleaned, rinsed and dried for use in fabrication of *ICs*. It is interesting to note that the final wafer thickness is about one-third less than after the slicing.

67.17. Wafer Fabrication

Following is the category of the processes that are used in the fabrication of *ICs* :

(*i*) oxidation **(*ii*)** etching **(*iii*)** diffusion
(*iv*) ion implantation **(*v*)** photolithography **(*vi*)** epitaxy
(*vii*) metallization and interconnections.

We will study each process separately and apply some of these later to the formation of a diode bipolar transistor and metal oxide semiconductor field effect transistor. The basic fabrication process is called ***planar process,*** *i.e.*, a process in which the introduction of impurities and metallic interconnections is carried out from the top of the wafer. A major advantage of the planar process is that each fabrication step is applied to all identical circuits and each of the many wafers at the same time.

It is important to initially emphasize that the fabrication requires an extremely clean environment in addition to the precise control of temperature and humidity.

67.18. Oxidation

The process of oxidation consists of growing a thin film of silicon dioxide (SiO_2) on the surface of a silicon wafer. Silicon dioxide has several uses :

1. to serve as a mask against implant or diffusion of dopant into silicon,
2. to provide surface passivation,
3. to isolate one device from another,
4. to act as a component in MOS structures.

Fig. 67.8 shows oxide layer grown on the surface of silicon substrate. The commonly used silicon dopants, such as boron, phosphorus, arsenic, and antimony, have very low diffusion coefficients (*i.e.*, they diffuse with great difficulty) in silicon dioxide. Because of this reason, silicon dioxide is used as a shield against infiltration of these dopants. On the other hand, these dopants diffuse easily if the surface is silicon.

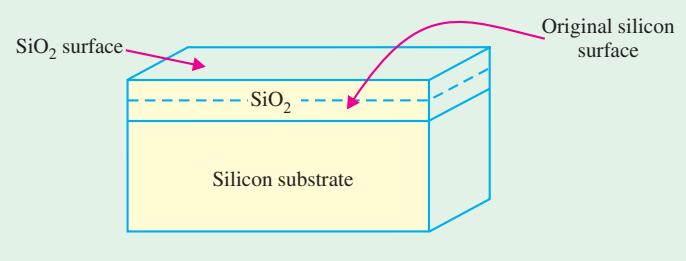


Fig. 67.8

Several techniques have been developed for forming oxide layers. Some of these are thermal oxidation, vapour phase technique [chemical vapour deposition (CVD)], and plasma oxidation. However, thermal oxidation is the more commonly used technique in *IC* processing.

Fig. 67.9 shows a thermal oxidation system. As seen, the oxidation is accomplished by placing the silicon wafers vertically into a quartz boat in a quartz tube. The quartz boat is slowly passed through a resistance heated furnace, in the presence of oxygen, operating at a temperature of about 1000°C. The oxidizing agent may be *dry* by using dry oxygen or be using a mixture of water vapour and oxygen. A computer controls the whole operation in the thermal oxidation system. The operation include regulating the gas flow sequence, automatic insertion and removal of wafers and the furnace temperature.

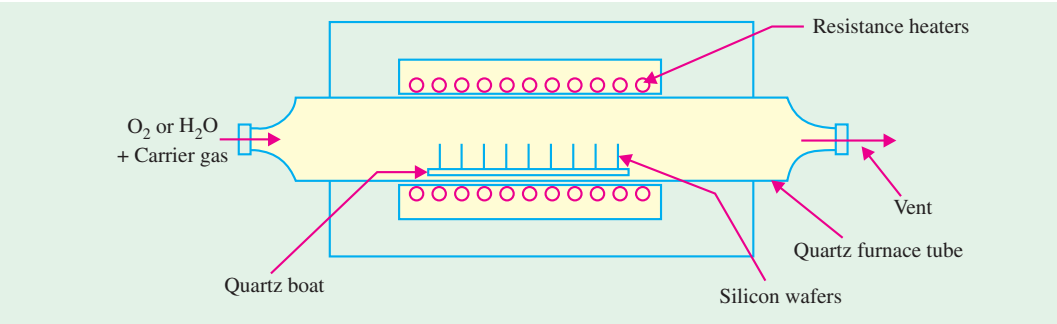


Fig. 67.9

It will be interesting to note that the oxide growth in the dry process is much ***slower*** but it produces an oxidized layer that has ***excellent*** electrical properties. For relatively thin oxides such as gate oxide in a MOSFET (typically 10 nm), dry oxidation is used. However, for thicker oxides (≥ 500 nm) such as field oxides in MOS integrated circuits and for bipolar devices, wet oxidation is used to provide both isolation and passivation.

Once the silicon dioxide layer has been formed on the surface of the wafer, it is selectively removed (etched) from those surfaces where impurities are to be introduced and kept as a shield, for the underlying silicon surface, where no dopants are to be allowed.

Oxide layers are relatively free from defects and provide stable and reliable electrical properties.

67.19. Etching

Etching is the process of selective removal of regions of a semiconductor, metal or silicon dioxide. There are two types of etching : ***wet*** and ***dry***. In wet etching, the wafers are immersed in a chemical solution at a predetermined temperature. In this process, the material to be etched is removed equally in all directions (*i.e.*, etching is isotropic). Because of this, some material is etched from the regions where it is to be left. This becomes a serious problem when dealing with small dimensions.

In dry (or plasma) etching, the wafers are immersed in gaseous plasma created by a radio-frequency electric field applied to a gas such as argon. The gas breaks down and becomes ionized. Electrons are initially released by field emission. These electrons gain kinetic energy from the field, collide with, and transfer energy to gas molecules, which result in generating ions and electrons. The newly generated electrons collide with other gas molecules and the avalanche process continues throughout the gas, forming plasma. The wafer to be etched is placed on an electrode and is subjected to the bombardment of its surface by gas ions. As a result, the transfer of momentum from the ions to the atoms removes atoms at or near the surface to be etched.

Dry etching also called reactive ion etching (RIE) is directional (or anisotropic). In other words, the material is removed only from those regions where it is required. Most modern processes use only dry etching to produce fine line patterns needed for VLSI integrated circuits.

67.20. Diffusion

This process consists of the introduction of impurities into selected regions of a wafer to form junctions. Diffusion occurs in two steps : the ***pre-deposition*** and the ***drive-in*** diffusion. In the pre-deposition step, a high concentration of dopant atoms is introduced at the silicon surface by a vapour that contains the dopant at a temperature of 1000°C. More recently, a more accurate method of pre-deposition known as ***ion implantation*** is used.

Pre-deposition tends to produce, a shallow but heavily doped layer, near the silicon surface. Drive-in is used to drive the impurity atoms deeper into the surface, without adding any more impurities. Fig. 67.10 shows the graph of doping profiles, (*i.e.*, impurity concentration versus depth into the substrate) during the pre-deposition and the drive-in steps of diffusion. The doping profiles indicate that the impurity concentration decreases monotonically from the surface of the substrate. The profiles of the dopant distribution is determined mainly by the temperature and diffusion time. Note that the impurity concentration drops from *OA* (during pre-deposition step) to *OB* at the surface of the substrate after the drive-in step. However the depth into the substrate of impurity has increased from *OC* to *OD* as required.

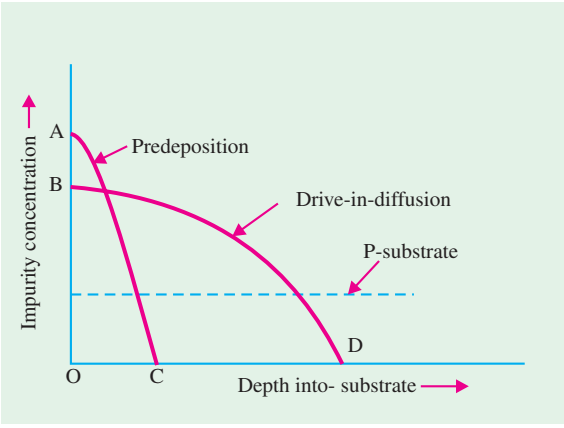


Fig. 67.10

Common dopants are boron for *P*-type layers and phosphorus, antimony and arsenic for *N*-type layers. However, diffusion is rarely performed using pure elements themselves. Rather, compounds of elements are used and impurities may be introduced from either solid, liquid, or gaseous substances.

Fig. 67.11 shows the equipment used in diffusion. The wafers are placed in a quartz boat within a quartz furnace tube. The furnace is heated by resistance heaters surrounding it. To introduce an impurity, phosphorus for example, phosphorus oxychloride ($POCl_3$) is placed in a container either inside the quartz tube in a region of relatively low temperature or in a container outside the furnace at a temperature that maintains its liquid form. For a *P*-type dopant, boron is used. The proper vapour pressure is maintained by a control of the temperature. Nitrogen and oxygen gas are made to pass over the container. These gases react with the silicon, forming a layer on the surface of the wafer that contains silicon, oxygen and phosphorus. At the high temperature of the furnace, phosphorus easily diffuses into the silicon.

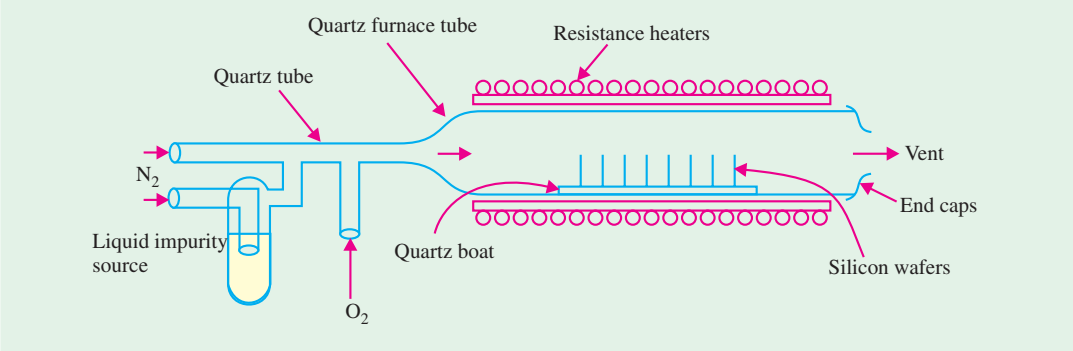


Fig. 67.11

In order that the dopant may be diffused deeper into silicon, the drive-in step follows. This is done at a higher temperature of about 1100°C inside a furnace similar to that used for pre-deposition, except that no dopant is introduced into the furnace. The higher temperature, causes the dopant atoms to move into silicon more quickly. Diffusion depth is controlled by the time and temperature of the drive-in process. By precise control of the time and temperature, accurate junction depths of fraction of a micron can be obtained. Fig. 67.12 shows the diffusion of dopant into silicon.

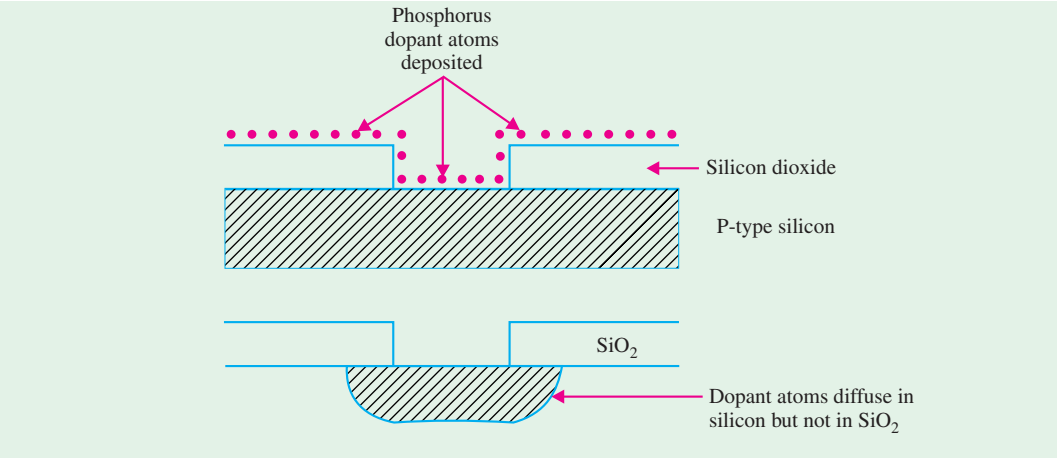


Fig. 67.12

67.21. Ion Implantation

This is a process of introducing dopants into selected areas of the surface of the wafer by bombarding the surface with high-energy ions of the particular dopant.

Fig. 67.13 shows a typical ion implantation equipment. To generate ions, such as those of phosphorus, an arc discharge is made to occur in a gas, such as phosphine (PH_3), that contains the dopant.

The ions are then accelerated in an electric field so that they acquire energy of about 20 keV and are passed through a strong magnetic field. The ions are further accelerated so that their energy reaches several hundred keV or MeV, whereupon they are focused on and strike the surface of the silicon wafer.

As is the case with diffusion, the ion beam is made to penetrate only into selected regions of the wafer by a process of masking (discussed later). On entering the wafer, the ions collide with silicon atoms and lose their energy. The depth of penetration of ions in ion implantation is about 0.1 to 1 μm. The higher the energy of ions and the smaller their mass, the greater is the depth of penetration.

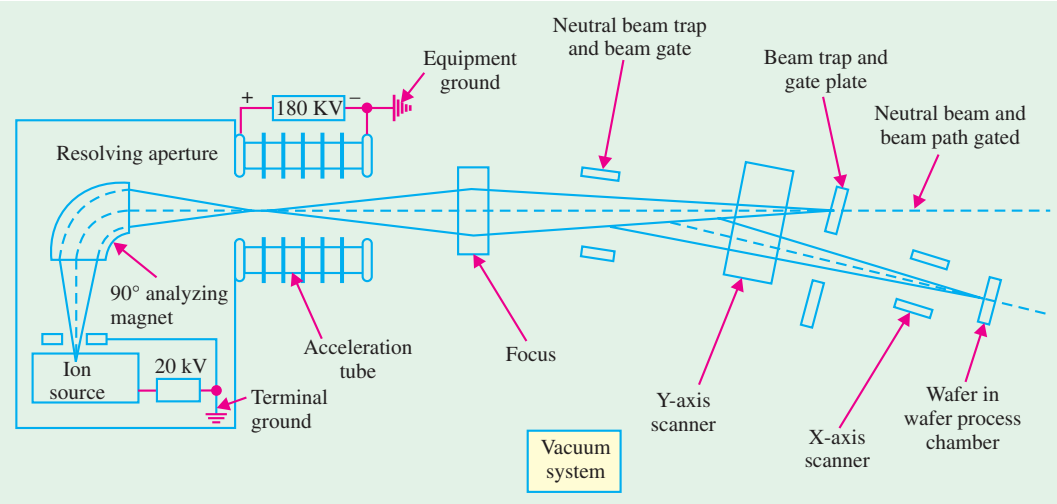


Fig. 67.13

Advantages of ion implantation over diffusion :

1. Doping levels can be precisely controlled since the incident ion beam can be accurately measured as an electric current.
2. The depth of the dopant can be easily regulated by control of the incident ion velocity. It is capable of very shallow penetrations.
3. Extreme purity of the dopant is guaranteed.
4. The doping uniformity across the surface can be accurately controlled.
5. Because the ions enter the solid as a directed beam, there is very little spread of the beam, thus the doping area can be clearly defined.
6. Since ion implantation is carried out at room temperature, wafers do not face temperature stress. In addition photo-resist can be used as mask for masking impurities, thus there is no need to grow thick masking oxides.

Disadvantages of ion implantation over diffusion :

1. The ion implantation may create considerable damage to the crystal structure because of the collisions of the high-energy ions with the silicon. Such damage results in inferior performance of *ICs* made by this process. If the damage is not extensive, the process of ***annealing*** restores the structure.
2. High initial investment and operational cost of the equipment (> US $ 1 million).
3. Uses very toxic gases for some of the dopants such as phosphorus and arsenic.

67.22. Photomask Generation

The whole process of *IC* fabrication consists of identifying selected regions of each circuit of the wafer surface into which identical dopant or metallic interconnections are made, while protecting other regions of the wafer surface. To carry out one of the many fabrication processes, a separate

mask is required for each operation whose function is to expose the selected regions and protect the others. There may be hundreds of identical dies (or *ICs*) on a wafer with each circuit containing hundreds of thousands or millions of devices. Identical steps are carried out simultaneously for each process. For each process, separate mask is needed.

The mask production starts with a drawing using a computer-assisted graphics system with all the information about the drawing stored in digital form. Commands from the computer are prepared that drive a pattern generator, which uses an electron beam to write the particular pattern, for one or several dies, on a glass plate covered with thin chromium film. When the glass plate is prepared, it is called ***reticle***. A mask usually refers to a glass plate that contains a pattern for the whole wafer. The reticle pattern is projected onto the wafer and a ***wafer stepper*** is used that reduces the reticle circuit onto the photo-resist covered wafer that steps across over the surface until the entire array of circuits is built up.

It may be noted that the use of a single mask for all the circuits on a wafer is not feasible for printing very small (< 1 μm) features because of alignment problems. However, the use of single mask is still in use for fabricating simple digital and analog circuits such as light emitting diodes (LEDs).

67.23. Photolithography

It is a process in which the geometrical pattern on the glass plate (called reticle) is transferred to the surface of the wafer. This is done to open identical windows so that the diffusion (or the ion implantation) process may take place in all identical regions of the same *IC* and for all *ICs* on the wafer. As an illustration we assume that the first reticle is used over an oxidized surface as shown in Fig. 67.14 (*a*).

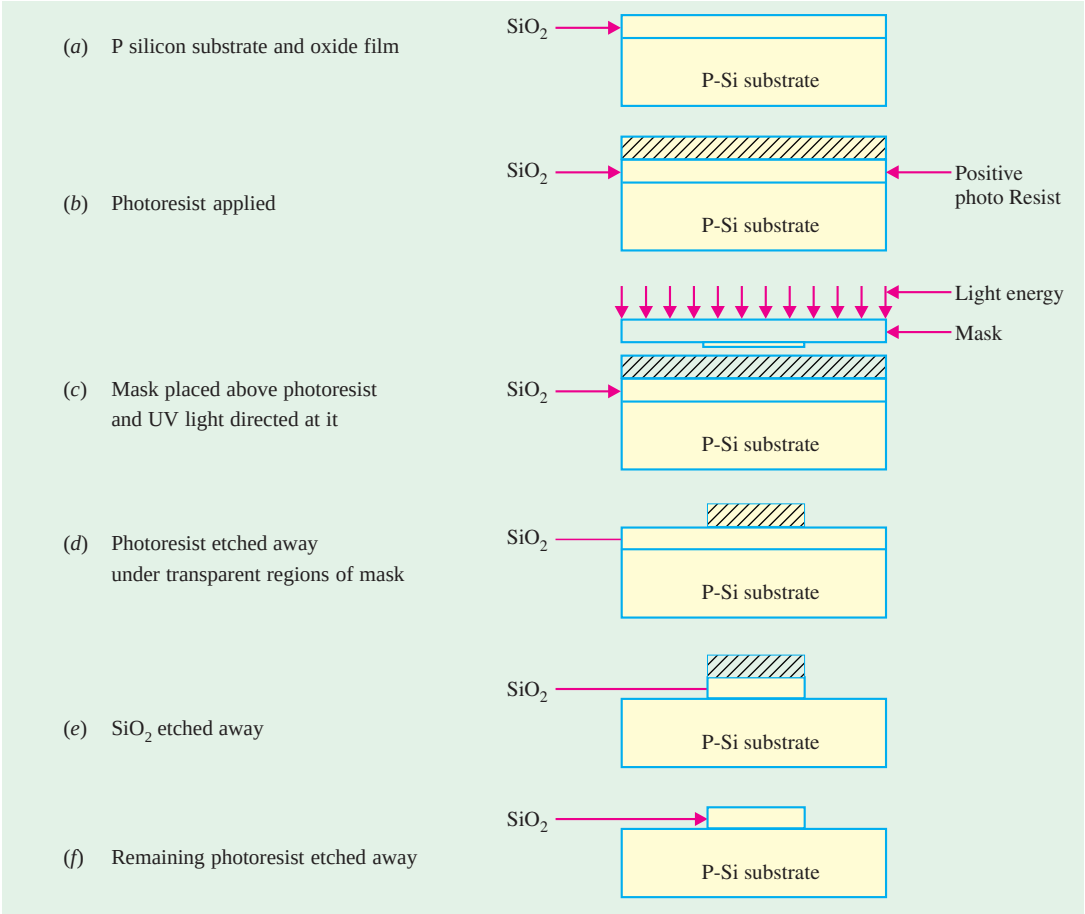
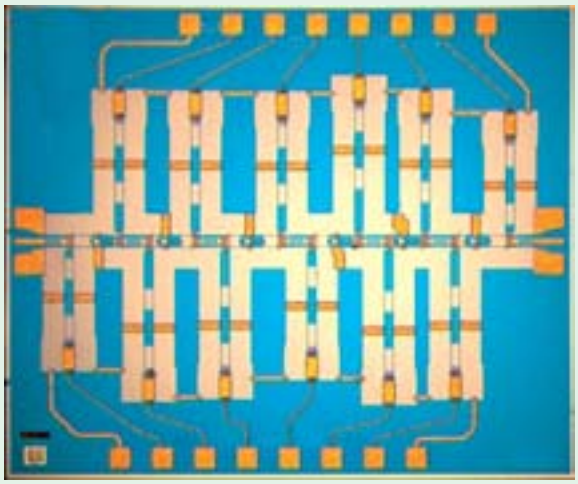


Fig. 67.14

To transfer the patterns, the wafer is coated with light-sensitive material called ***photoresist.*** By applying small amount (about 1 ml) of the photoresist to the wafer surface and spinning the wafer very rapidly, a uniform film of about 1 μm thick is formed over the oxidized surface of the wafer [refer to Fig. 67.14 (*b*)]. After this the following steps are taken to open a window on the surface of the wafer :

1. The wafer is baked at 100°C to solidify the photoresist on the wafer.
2. The glass plate is placed on the wafer and aligned by the computer control.
3. The glass plate is exposed to ultraviolet (UV) light with the transparent parts of the glass plate passing the light on to the wafer. The photoresist under the opaque regions of the glass plate is unaffected [refer to Fig. 67.14 (*c*)].
4. The exposed photoresist is chemically removed by dissolving it in an organic solvent and exposing the silicon dioxide underneath [refer to Fig. 67.14 (*d*)].
5. The exposed silicon dioxide is then etched away using hydrofluoric acid. The hydrofluoric acid dissolves silicon dioxide and not silicon. The regions under the opaque part of the glass plate are still covered by the silicon dioxide and the photoresist [refer to Fig. 67.14 (*e*)].
6. The photoresist under the opaque regions of the glass plate is removed using a proper solvent and the silicon dioxide is exposed [refer to Fig. 67.14 (*f*)].

All surfaces in Fig. 67.14 (*f*) are protected except those covered by silicon only in which diffusion or ion implantation is to take place. It may be noted that the surfaces covered by silicon dioxide do not permit any entry of dopants.



The photolithograph is used to add circuit elements to the wafer. The wafer is coated with layer of a chemical called photoresist.

The photoresist used in the explanation above is called a ***positive photoresist.*** Thus a positive photoresist is that which allows the windows to be opened wherever the UV light passes through the ***transparent*** parts of the mask. On the other hand a photoresist which remains on the surface, when exposed to UV light, and windows are opened under the ***opaque*** parts of the mask, is called a ***negative photoresist.*** The negative photoresist is limited to a resolution of 2 to 3 μm. On the other hand, the positive photoresist sacrifice adhesion and simplicity of development to achieve higher resolution.

The method of using UV light has a practical limit for transferring patterns on the silicon. It can be used for the patterns where the linewidths are above 1 to 2 μm. In order to transfer small patterns (*i.e.*, those with below 1 μm linewidth), very short wavelength radiation such as electron-beam or *X*-rays are used.

67.24. Epitaxy

Epitaxy or **Epitaxial growth** is the process of the ***controlled growth*** of a crystalline doped layer of silicon on a single crystal substrate. The process of diffusion and ion implantation, which

were earlier described, produce a layer at the surface that is of higher doping density than that which existed before the dopant was added. It is not possible by these methods to produce, at the surface a layer of lower concentration than exists there. This can however, be accomplished by the method of epitaxy. In the processes of diffusion and ion implantation, a dopant is driven into a substrate of doped silicon. In epitaxy, a layer of doped silicon is deposited on top of the surface of the substrate. Normally this single crystal layer has different type of doping from that of the substrate.

Epitaxy is used to deposit N or N^+ (*i.e.*, heavily doped *N*-type) silicon, which is impossible to accomplish by diffusion. It is also used in isolation between bipolar transistors where N^- (*i.e.*, lightly doped *N*-type) is deposited on *P*-type layer. It may also be used to improve the surface quality of an *N*-type substrate by depositing *N*-type material over it. A variety of methods are used to grow the epitaxial layer. These methods include vapour-phase epitaxy (VPE), liquid-phase epitaxy (LPE), and molecular beam epitaxy (MPE). The system for growing an epitaxial layer using vapour-phase epitaxy (VPE) is shown in Fig. 67.15.

In this system, silicon wafers are placed in a long boat-shaped crucible made of graphite. The boat is placed in a long cylindrical quartz tube, which has inlets and outlets for the gases. The tube is heated by induction using the heating coils wound around the tube.

All the chemicals that are introduced and that take part in the reactions are in the form of gases, hence the process is known as Chemical Vapour Deposition (CVD). The epitaxial layer is grown from the vapour phase onto the silicon, which is in the solid state. The thickness of the layer varies from 3 to 30 μm and the thickness of the layer and its doping content are controlled to an accuracy of less than 2 percent. The reactions are carried out at a temperature of approximately 1200°C. The high temperature is necessary so that the dopant atoms can acquire a sufficient amount of energy to allow them to move into the crystal to form covalent bonds and become extension of the single crystal. Because the layer is grown on the substrate, epitaxy is a growth technique where the crystal is formed without reaching the melting point of silicon.

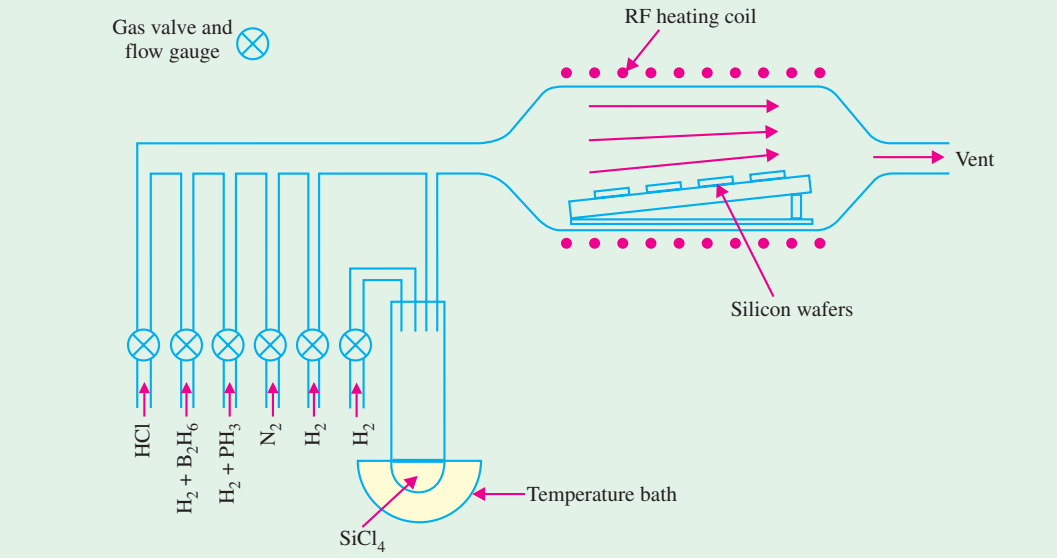


Fig. 67.15

67.25. Metallization and Interconnections

After all the fabrication steps of an *IC* are completed, it becomes necessary to provide metallic interconnections for the *IC* and for external connections to the *IC*. The requirement that must be met by the interconnections is that they have low resistance to minimize both the voltage drops on the lines as well as the capacitances between the lines so as to reduce delay times. The connections must

also make ***ohmic contacts*** to semiconductors in the devices such as *P* and *N* regions of a *PN* junction diode. An ohmic contact is one that exhibits a very low resistance, allowing currents to pass easily in both directions through the contact.

The high conductivity of aluminium makes it the metal of obvious choice, particularly in silicon-based devices. It also has the following advantages :

1. easy to evaporate
2. can be easily etched
3. not expensive,
4. adheres well to silicon dioxide.

There are variety of processes for depositing aluminium on silicon substrates but the following three are important from the subject point of view : **(1)** resistance heating, **(2)** electron beam heating and **(3)** sputtering.



Several cycles of photolithography etching and doping are performed producing multiple layers of circuit elements on the wafer.

Resistance heating : In this process, the source of the heated element and the silicon substrate are located in an evacuated chamber. The source is a small piece of aluminium attached to a coil of tungsten, which serves as a heater [refer to Fig. 67.16 (*a*)]. The heated element with a high melting point remains solid while the aluminium is vaporized. The aluminium atoms travel to the substrate where they condense, depositing an aluminium layer on the surface of the silicon. A photolithographic masking and etching method is used to remove the metal from the regions where it is not wanted. A typical interconnection between two diffused layers is as shown in Fig. 67.16 (*b*).

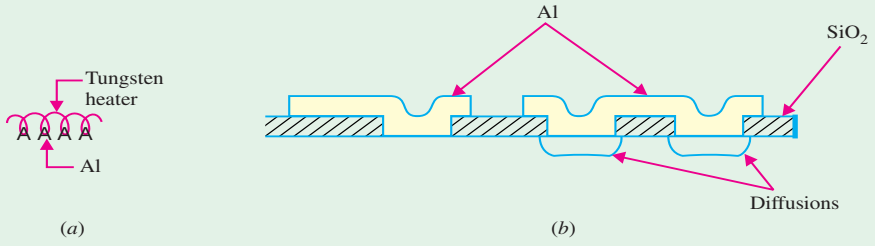


Fig. 67.16

In another method, the evaporation source kept in a boron nitride crucible is heated by radio frequency induction as shown in Fig. 67.17 (*a*). High deposition rates are possible through this process. However, the crucible may contaminate the metal.

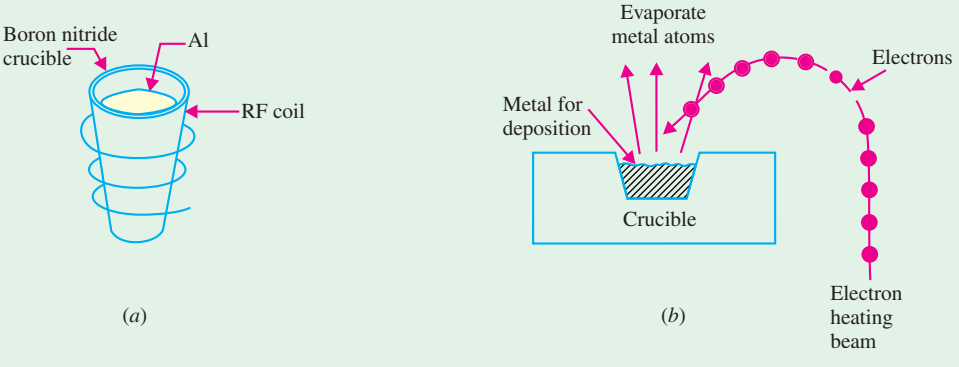


Fig. 67.17

Electron-beam heating : In this process, aluminium in a crucible is placed into a vacuum chamber together with the substrate. The aluminium is subjected to a high intensity electron beam formed by an electron gun, which vaporizes the aluminium as shown in Fig. 67.17 (*b*). This causes the aluminium to travels to the wafer. By the use of mask and photolithography, the aluminium is deposited on the identified regions on the wafer surface.

Sputtering : In this process, the material to be deposited is placed in a container maintained at low pressure in the vicinity of the substrate. The material to be deposited is labelled the cathode or the target, while the anode is the substrate. A DC or a radio-frequency high voltage is applied between anode and cathode. This high voltage ionizes the inert gas in the chamber. The ions are accelerated to the cathode (in this operation the anode is usually grounded) where, by impact with the aluminium target, atoms of aluminium are vaporized. A gas of aluminium atoms is generated and deposited on the surface of the wafer.

Following the deposition of aluminium, the silicon wafers are placed in a furnace to solidify the connections so that low resistance metallic contacts are made.

The interconnections between the elements of an *IC* are made by aluminium lines having a thickness of about 0.5 μm. These are laid on top of the silicon dioxide layer, which covers the surface of the wafer. By using photolithography, openings are made in silicon dioxide so that the aluminium layer is connected to the silicon or to the ohmic contact on the silicon. In very complicated integrated circuits, it is necessary to have two or three vertically stacked layers of interconnections separated by silicon dioxide layers. The interconnecting lines terminate at aluminium pads (called bonding pads) from which connections to the outside are made.

67.26. Testing, Bonding and Packaging

The individual *IC* chip must be connected properly to outside leads and packaged in a way that is convenient for use in a larger circuit or a system. Since the devices are handled individually once they are separated from the wafer, bonding and packaging are expensive processes.

Testing – After the wafer of monolithic circuits has been processed and the final metallization pattern defined, it is placed in a holder under a microscope and is aligned for testing by a machine called ***multiple-point probe.*** The probe contacts the various pads on an individual circuit, and a series of tests are made to verify the electrical properties of the device in a very short time. After all the circuits are tested, the wafer is removed from the testing machine, sawed between the circuits, and broken apart. Then each die that passed the test is picked up and placed in the package.

Bonding – It consists of two steps. In the first step the back of the die is mechanically attached to an appropriate mount medium such as ceramic substrate, multi-layer-ceramic package or metal lead frame. The two common die bonding methods are ***hard solders*** and ***polymers.*** In the second step, the bond pads on the circuit side of the die are connected by wires to the package as shown in Fig. 67.18. The three common schemes of interconnection to the chip bond pads are ***(i) wire bonding, (ii) tape-automated bonding*** (*TAB*), and ***(iii) flip-chip solder bonding.*** Wire bonding is further split into two different processes : ***thermosonic*** and ***thermocompression.*** The further detail of these processes is beyond the scope of this book.

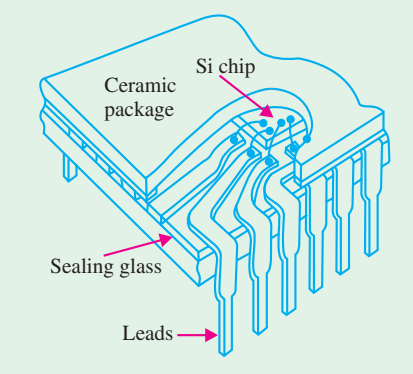


Fig. 67.18

Packaging – The final step in *IC* fabrication is packaging the device in a suitable medium that can protect it from environment of its intended application. In most cases this means the surface of the device must be isolated from the moisture and contaminants and the bonds and other elements must be protected from corrosion and mechanical shocks. The modern *ICs* are mounted in pack-

ages with many output leads. In one version, chip is mounted on a stamped metal lead frame and wire bonding is done between the chip and the leads. The package is formed by applying a ceramic or plastic case and trimming away the unwanted parts of the lead frame.

Fig. 67.19 shows various types of the packaging of *ICs* used these days. As seen from this figure, the packages can be broadly classified into the following two categories :

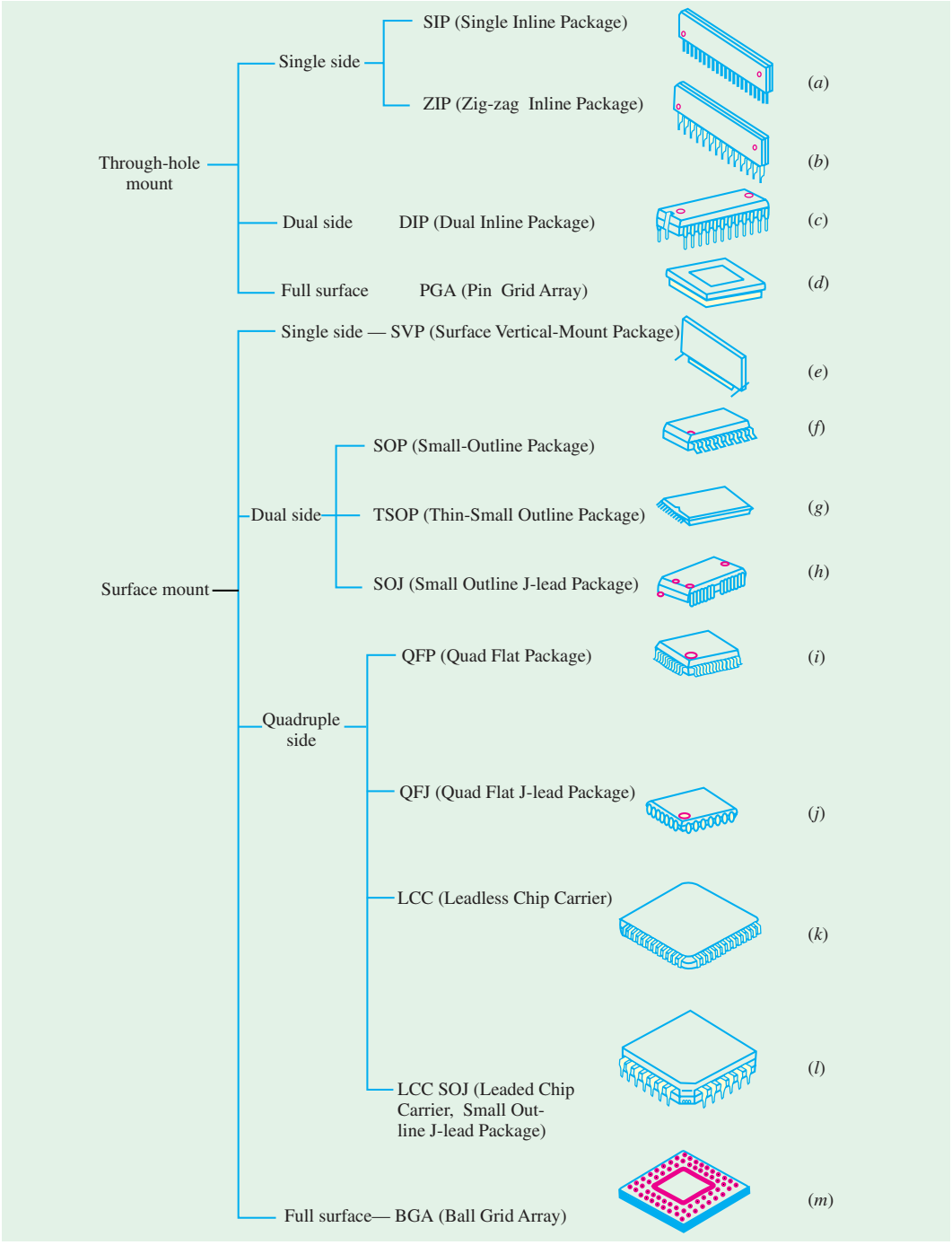


Fig. 67.19

1. ***through-hole mount*** that involve inserting the package pins through holes on the printed circuit board (PCB) before soldering [Fig. 67.19 (*a*) through (*d*)].
2. ***surface mount type*** where the leads do not pass through holes in the PCB. Instead, surface-mounted package leads are aligned to electrical contacts on the PCB, and are connected simultaneously by solder reflow [Fig. 67.19 (*e*) through (*m*)]. As refer to the picture shown in Fig 67.20.

Most packages can be made using ceramic or plastic. The *ICs* are hermetically sealed for protection from the environment. The pins can be on one side (single inline or zigzag pattern of leads), two sides (dual inline package or DIP), or four sides of the package (quad package). Most advanced packages have leads distributed over a large portion of the surface of the package as in through-hole pin grid arrays (PGAs) or surface-mounted ball grid arrays (BGAs).

67.27. Semiconductor Devices and Integrated Circuit Formation

There are literally thousands of different semiconductor device structures. They have been developed to achieve specific performances, either as discrete components or in *ICs*. Accordingly there are many different structures. However, there are basic structures required for each of the major device and circuit types.



Fig. 67.20

1. ***Resistors :*** Most of the resistors in *ICs* are formed by the same processes that are used to form devices (*i.e.*, a sequence of oxidation, masking and doping operation). Fig. 67.21 (*a*) shows a resistor made of a *P*-type region diffused into an *N*-type epitaxial layer. Notice the metallic contacts made at the two ends of the epitaxial layer. The section of the resistor, as dictated by the diffusion, is very nearly rectangular in shape, as shown in Fig. 67.21 (*b*). The value of a resistor (in ohms) is given by :

$$R = \rho . l/a \quad \text{or} \quad \rho.l/w.d$$

where ρ is the resistivity of the layer in ohm-cm, l is the length of the resistive region, and a is the cross-sectional area of the resistive region. The value of a is equals to $w.d$. where w is the width and d is the depth of the resistive region.

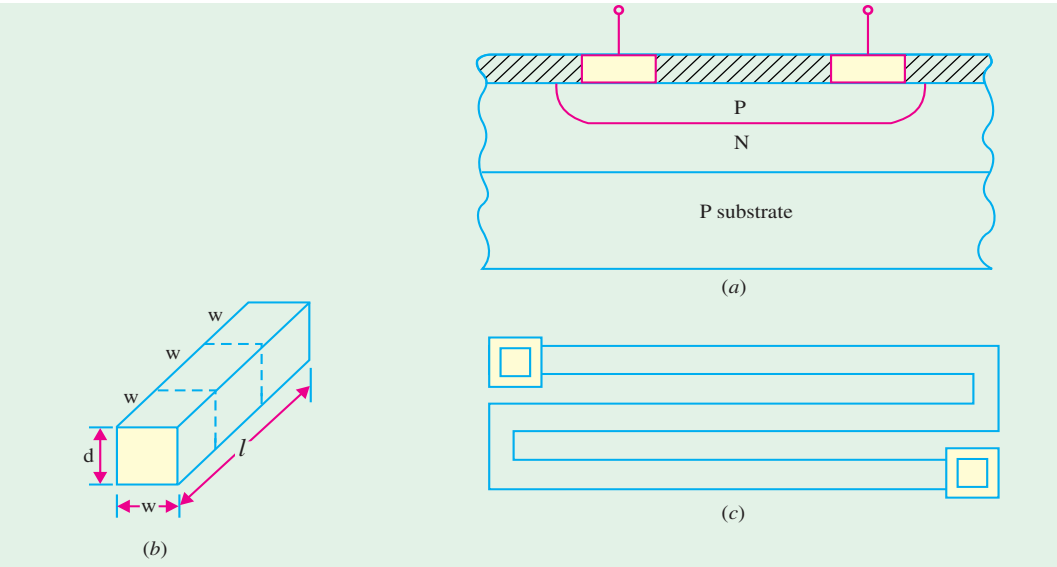


Fig. 67.21

Resistance in monolithic circuits are defined by the term called ***sheet resistance.*** The sheet resistance is the resistance of a square region having $w = l$ and has units of ohms per square. Assuming a sheet resistance of 100 to 200 ohms per square, practical resistors may have values ranging from 100 to several kilohms. Higher resistance values are obtained by using a meander pattern as shown in Fig. 67.21 (*c*). The major problem with the resistors of high values is that they tend to occupy a large area on the chip. For example, a resistor of 50 kΩ uses up an area of the wafer that may be occupied by hundreds of the transistors. Ion-implantation instead of diffusion can be used to make resistors with precise values of resistance.

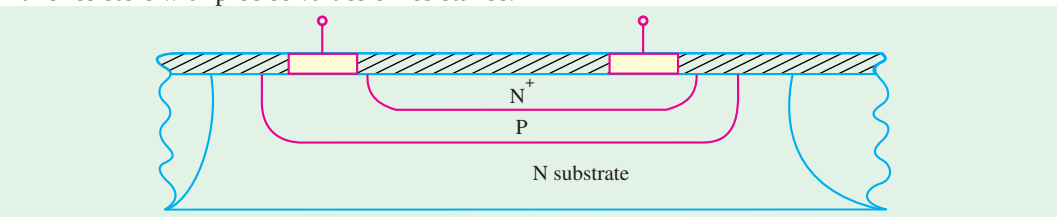


Fig. 67.22

2. *Capacitors :* One type of capacitor in monolithic circuits is made by using the capacitance formed between the *P* and *N* regions of a reverse-biased diode. Such a capacitor is shown in Fig. 67.22. These capacitors are formed by using the same diffusion processes that are used to form devices. Bipolar transistors are made of three regions in which either of the two *PN* junctions may be used as capacitors whereby the breakdown voltage of the capacitor may vary considerably from one to the other. The disadvantage of the junction capacitors is the dependence of the capacitance on the voltage applied to the junction. Capacitors that are voltage-independent can be formed from metal insulator N^+ (*i.e.*, heavily-doped) semiconductor layers as used in MOS structures.

In dense circuits, an oxide/nitride/oxide dielectric sandwich is used. The combination film has a lower dielectric constant, allowing a capacitor area smaller than a conventional silicon dioxide capacitor.

Capacitors can also be fabricated by creating a trench etched vertically into the wafer surface (refer to Fig. 67.23). The trenches are etched either isotropically with wet etching or anisotropically with dry etching techniques. The trench side walls are oxidised and the centre of the trench is filled with deposited polysilicon. The final structure is "wired" from the surface, with the silicon and polysilicon serving as two electrodes and silicon dioxide as a dielectric material.

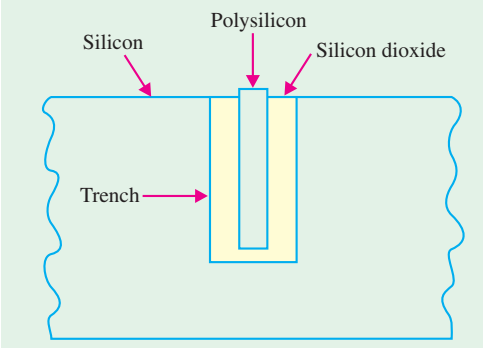


Fig. 67.23

The trenched capacitors are useful when preservation of wafer surface is main criteria. Another alternative to conserve wafer surface area is to build stacked capacitors on the wafer surface. This effort has been driven by the need for small high dielectric capacitors for dynamic random access memory (DRAM) circuits.

3. *Diodes :* In integrated circuits, where all the interconnections and device terminals are made at the surface, a diode is formed in the following two ways :

(*a*) In bipolar circuits from a bipolar transistor by placing a ***short-circuit*** between two of the three terminals of the transistor (*i.e.*, collector to base or emitter to base). Thus there are emitter-base diodes and base-collector diodes.

(*b*) In MOS circuits, most of the diodes are formed with the source-drain doping step.

However, in order to make a discrete *PN* junction diode, we will discuss the various steps involved in the fabrication.

The starting material is a heavily doped *N*-type (N^+) substrate about 150 μm thick. A layer of *N*-type silicon (1 to 5 μm) is grown on the substrate by epitaxy as shown in Fig. 67.24 (*a*). Then a layer of silicon dioxide (SiO_2) is deposited by oxidation as shown in Fig. 67.24 (*b*). Next, the surface is coated with positive photoresist as shown in Fig. 67.24 (*c*). Then a mask is placed on the surface of silicon, aligned and exposed to ultraviolet light (UV) as shown in Fig. 67.24 (*d*). Next, mask is removed, resist is removed and silicon dioxide layer under the exposed resist is etched as shown in Fig. 67.24 (*e*). Then boron is diffused to form P-type region a shown in Fig. 67.24 (*f*). Note that boron diffuses easily in silicon but not in silicon dioxide. Next, a thin aluminium film is deposited over the surface as shown in Fig. 67.24 (*g*). Then the metallized area is covered with resist another mask is used to identify areas where metal is to be preserved. Wafer surface is etched to remove unwanted metal. Resist is then dissolved and we get a structure as shown in Fig. 67.24 (*h*). Finally, contact metal is deposited on the back surface and ohmic contacts are made by heat treatment.

4. *Bipolar Transistor* : Let us consider the fabrication of an NPN bipolar junction transistor (BJT) on a silicon wafer. While our discussion will be focussed on the BJT, it is understood that the surface of the whole wafer is being processed.

The starting material is a lightly doped *P*-type wafer. We will consider a small area on this wafer where we will form a BJT. The base on which the transistor is made is known as the substrate. The function of the substrate is to act as a mechanical support for the device. The reason for the use of a *P*-type substrate for the NPN transistor will be clarified when the term isolation is discussed. The substrate has a resistivity of 3-10 Ω cm with a thickness between 500 and 700 μm for wafers having diameters over 100 mm.

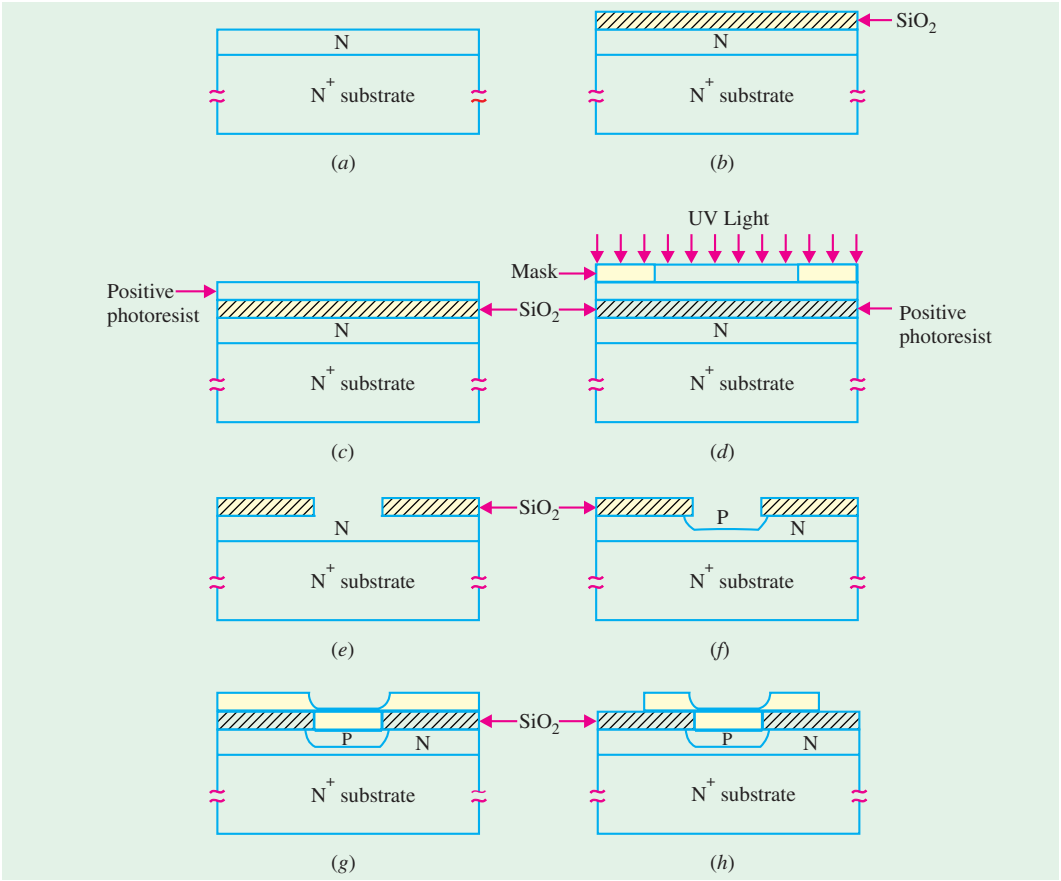


Fig. 67.24

Fig. 67.25 shows the fabrication steps up to, and including the metal contacts to the three regions (*i.e.*, emitter, base and collector).

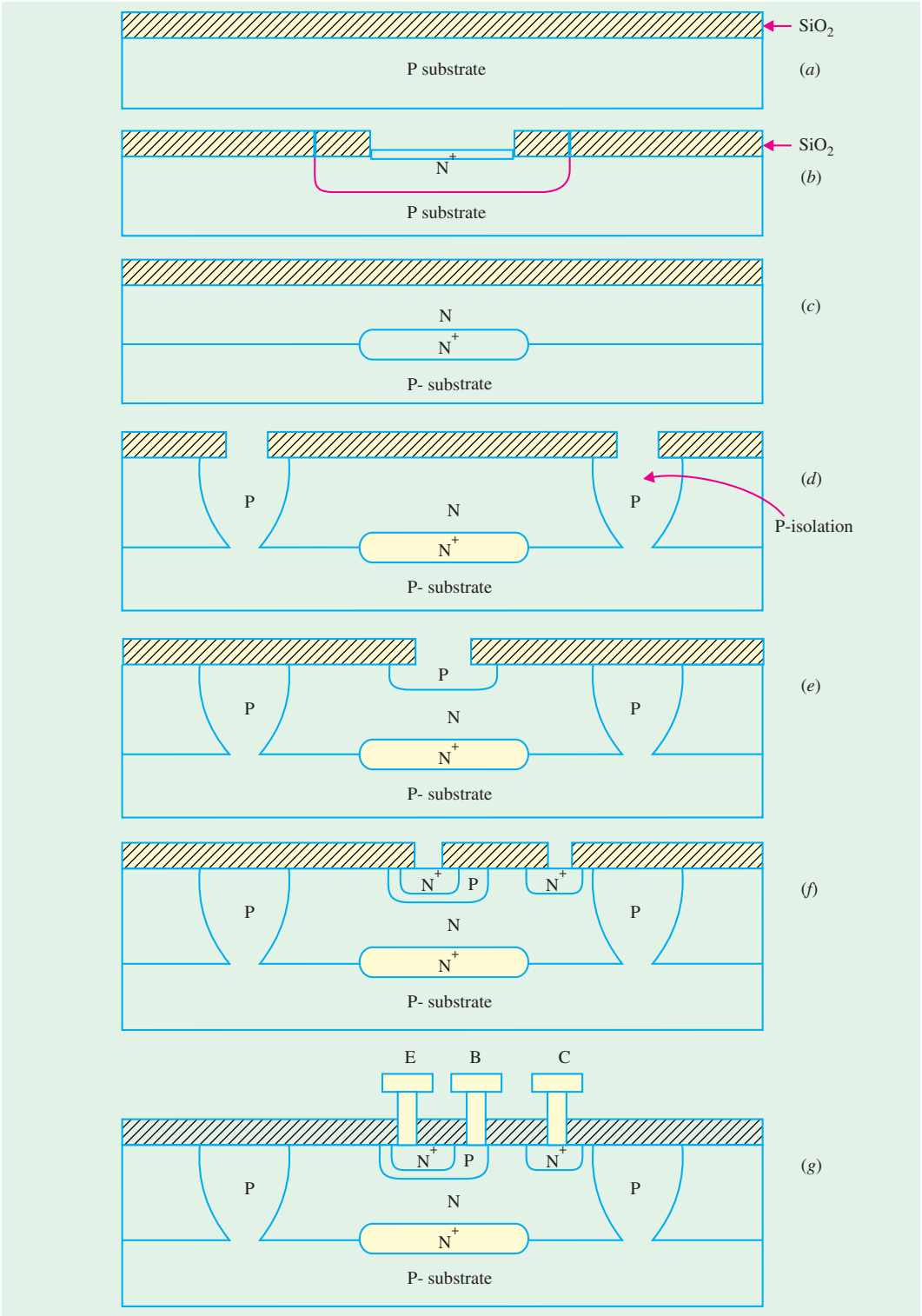


Fig. 67.25

First a layer of silicon dioxide (SiO_2), about 0.5 μm thick, is deposited on the surface of the substrate by thermal oxidation as shown in Fig. 67.25 (*a*). Using the ***first mask*** and the photolithographic process, windows are opened in the oxide for the *buried layer*. The N^+ (*i.e.*, heavily doped *N*-type) buried layer is diffused to a depth of about 3 μm as shown in Fig. 67.25 (*b*). This layer serves to collect the carriers that have crossed the base on their way to the collector terminal. It serves as a sub-collector and is used to reduce the collector ohmic-resistance. After the buried layer is diffused, the wafer is stripped of all oxide to permit the next deposition. It is to be noted that during the subsequent high-temperature processes, the buried layer tends to diffuse out.

The next operation is the deposition of a phosphorus-doped *N*-type ***epitaxial layer*** on the whole wafer. This layer has a resistivity of 0.1 to 1 Ω cm. The thickness of this layer is 0.5 to 5 μm for high-speed digital circuit applications and 10-20 μm for linear analog circuits. A layer of silicon dioxide (SiO_2) about 0.5 to 1 μm thick is grown thermally on the surface of the epitaxial layer as shown in Fig. 67.25 (*c*).

Since the collector of NPN transistor is *N*-type, and so are the collectors of the adjacent transistors, there is an obvious need to isolate the collectors from each other. The ***second mask*** is used to etch windows for this isolation regions, which are formed by the subsequent diffusion of boron extending from the surface down to the substrate as shown Fig. 67.25 (*d*). An *N*-type epitaxial layer separates the isolation regions, thus serving as the tub in which each transistor is formed. The diffusion of the isolation region is followed by oxidation of the wafer surface.

The ***third mask*** is used to open a window for the *P*-type base of the transistor, *P*-type diffusion or ion implantation is driven to form the base to a depth of about 2-3 μm as shown in Fig. 67.25 (*e*). This is followed by the deposition of an oxide layer. The ***fourth mask*** is used to open windows in the oxide for the N^+ emitter and the collector contacts. The phosphorus or arsenic diffusion is driven to a depth of about 2 μm as shown in Fig. 67.25 (*f*). The need for N^+ collector is to form a good ohmic contact. The ohmic contact permits easy current flow in both directions. To form a good ohmic contact to an *N* material, an N^+ region is needed between the metal on the top and the *N* region. Following the N^+ diffusion, an oxidation layer is formed over the entire wafer surface.

The ***fifth mask*** is used to open windows for the formation of metallic contacts to the bipolar transistor terminals. Then an aluminium film 0.5 and 1 μm thick is deposited, by evaporation or sputtering, on the top surface of the *IC* wafer. Assuming that the complete circuits are to be formed on the surface of the wafer, the ***sixth mask*** is used to define the interconnection pattern in the circuits. These interconnections are etched into the metal that has been deposited on the surface, as shown in Fig. 67.25 (*g*).

In order to protect the surface of the wafer from moisture and chemical contamination, a silicon nitride (Si_3N_4), called a ***passivation layer,*** is deposited on the surface. Contacts to the integrated circuits are made on pads that are located on the periphery of the *IC* chip. Since the *IC* chip will be bonded to an *IC* package, connections are to be made from the package leads to the bonding pads on the *IC* chip.

The ***seventh*** mask is used to define the bonding holes over the aluminium pads for external connections. Following the seven masks, the circuits are tested by a computer-controlled system and all faulty chips are identified and marked. The wafer is then sawed into chips, which are bonded on *IC* packages. Gold wires about 25 μm in diameter are used to connect the package leads to the bonding pads on the chip.

Fig. 67.26 shows an additional detail on the buried layer. It indicates the path taken by the carriers on their way from the emitter to the base and to the collector. This path is considerably longer than the path in the discrete BJT shown in Fig. 67.26 (*a*). Because of this the collector series resistance, labelled the ***parasitic resistance,*** is quite large and is of the order of hundreds of ohms. To reduce this resistance, we have placed the low resistivity buried layer in Fig. 67.26 (*b*) into the path of

the carriers which acts as a subcollector. The use of the buried layer reduces the collector resistance by as much as a factor of 20. The result of this is to improve gain-bandwidth product of the transistor by the same factor.

5. *MOSFET :* Let us consider the fabrication of Enhancement mode *N*-channel metal oxide semiconductor field-effect transistor. The starting material is a lightly doped *P*-type silicon substrate having a resistivity of about 5 Ω cm. The doping density is determined by the drain-substrate breakdown voltage of 20-30 V. A thin (about 20 nm) layer of silicon dioxide is formed over the substrate to provide stress relief to the wafer. Next a thin (about 20 nm) layer of silicon nitride (Si_3N_4) is deposited by the chemical vapour deposition (CVD) process on top of the silicon dioxide. The silicon dioxide permits selective oxidation so that a thick oxide (about 500 nm) can be formed in the field region. These operations result in Fig. 67.27 (*a*).

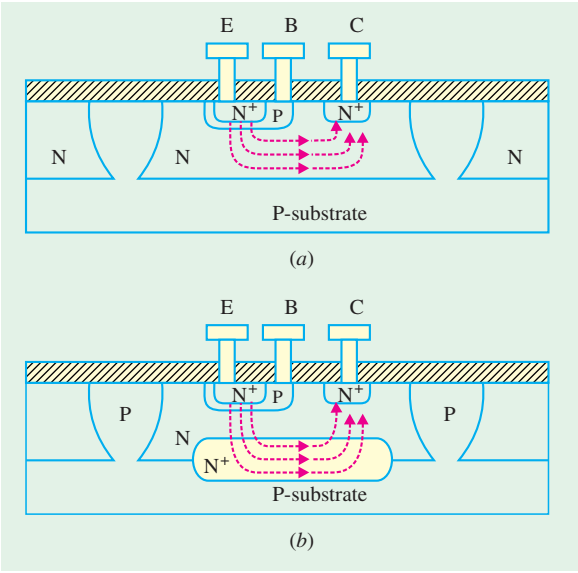


Fig. 67.26

The ***first mask*** defines the field-effect transistor areas, so that the silicon dioxide and the silicon nitride are chemically etched out except where the transistor is formed.

The next step is to diffuse (or ion-implant) boron in the field-regions to form P^+ (*i.e.* heavily doped *P*-type) islands in the substrate. The function of P^+ islands is to help increase the threshold voltage, V_T and prevent the formation of ***"parasitic" transistors*** (or electrical cross talk) between adjacent devices on the wafer. This is followed by the formation of a field oxide layer (about 500 nm thick) over the P^+ implanted region as shown in Fig. 67.27 (*b*). The field oxide layer also helps to increase V_T.

The remaining silicon nitride and silicon dioxide are etched away and an ultra thin (about 5 to 10 nm) layer of SiO_2 is grown over the transistor area (not above the field oxide). This forms the gate-oxide of the transistor. Refer to Fig. 67.27 (*c*). Next, a layer of heavily doped (typically N^+) polysilicon is deposited over the entire wafer surface. This is shown in Fig. 67.27 (*d*).

The ***second*** mask defines the gate region, whereupon the polysilicon is etched away except over the gate as shown in Fig. 67.27 (*e*). The heavily doped polysilicon region above the gate behaves electrically like a metal electrode.

By ion-implantation and using the polysilicon gate and the field oxide, as the mask, the source and drain N^+ regions are formed as shown in Fig. 67.27 (*e*). The N^+ layers literally diffuse a sufficient distance to ensure proper alignment so that the channel length is well defined. The dopants do not penetrate the field oxide. A thin layer of silicon dioxide is then grown, by the CVD process over the wafer.

The ***third mask*** is used to open windows for the metal contacts to the transistor regions as shown in Fig. 67.27 (*f*). The thin layer of silicon dioxide is etched away and aluminium is deposited over the surface of the wafer by evaporation or sputtering. The ***fourth mask*** defines the interconnection pattern that is etched in the aluminium, as shown in Fig. 67.27 (*g*). This is followed by the deposition of a protective passivation layer of phosphosilicate glass (called *P*-glass) over the entire surface of the wafer.

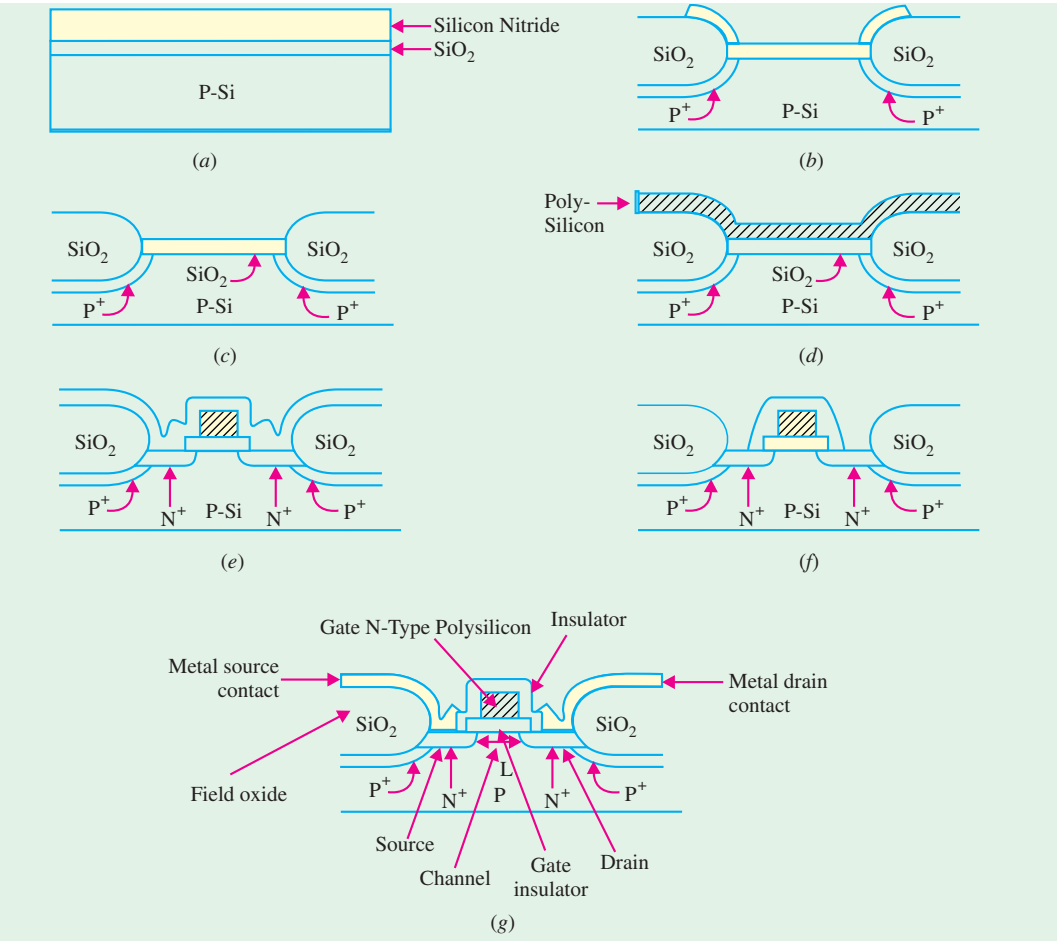


Fig. 67.27

Just as with the BJT, the ***fifth mask*** is used to open windows, so that bonding wires can be connected to the pads on the *IC* chip.

67.28. Comparison of ICs based on MOS and Bipolar Transistor Technology

1. The *ICs* based on MOS are less costly to fabricate as compared to those using bipolar transistor. The reason for this is that MOS devices are self-isolating. Bipolar transistors require tubs to isolate devices from each other on one integrated circuit. Isolation in MOS devices is provided by heavy doping and a thick oxide in the regions between adjacent devices.

2. MOS circuits consume less DC power as compared to bipolar transistor circuits.

3. The MOS transistor has lower value of transconductance (g_m) as compared to the bipolar transistor. This feature makes the bipolar transistor *ICs* superior to MOS circuits for analog circuit applications.

4. For the same channel length and the base width, the limiting cut-off frequency of the MOS transistor is better than a bipolar transistor. Therefore the MOS transistor has a higher bandwidth than the bipolar transistor.

5. The packing density of MOS *ICs* is at least 10 times more than that for bipolar *ICs*. Also, a MOS resistor occupies less than 1% of the area of a conventional diffused resistor. This high packing density makes MOS *ICs* especially suited for LSI, VLSI and ULSI circuits.

The main disadvantage of MOS *ICs* is their slower speed as compared to bipolar *ICs*. Hence they do not compete with bipolar *ICs* in ultrahigh-speed applications.

However, due to their ***(i)*** low cost, ***(ii)*** low power consumption and ***(iii)*** high packing density, MOS *ICs* are widely produced by semiconductor manufacturing industry. The MOS *ICs* are available as calculator chips, memory chips, microprocessors (μP), single-chip computers etc.

67.29. Popular Applications of ICs

There are plenty of electronic products in the market, which used integrated circuits extensively. One of the widely accepted applications is the digital watch, which can display hours, minutes, seconds, day and month. Another popular application is electronic calculator which can perform various functions like addition, subtraction, multiplication and division. There are also advanced scientific calculators that are programmable and can display the graphs.

Modern electronic products such as pocket PC, personal digital assistant (PDA), MP3 players, digital cameras, digital camcorders, mobile phones, digital dictionaries and digital translators, CD (compact disk) players, DVD (digital versatile disk) players etc. also make use of *ICs* extensively. With a Pocket PC, you can read and send e-mail, edit Word, Excel and Outlook files, listen to digital music, read electronic books (eBooks), track your finances, and browse the web. The digital camera shown in Fig. 67.28 can take pictures and store on the floppy diskette or memory stick. A personal digital assistant (PDA) as shown in Fig. 67.29 is a handheld computer which allows the user to organise calendar, contacts or tasks. These little electronic wonders might put office secretary out of a job.

Electronic games have evolved from microelectronics and computer knowhow. Video games that have to be hooked up to home TV have become very popular. Hand held PCs and mobile phones also come with games that require no TV hook-up.



Fig. 67.28
(courtesy Sony Corporation, Japan)



Fig. 67.29
(courtesy : 3Com, USA)

OBJECTIVE TESTS – 67

1. First integrated circuit chip was developed by
(*a*) C.V. Raman
(*b*) W.H. Brattain
(*c*) J.S. Kilby
(*d*) Robert Noyce.

2. An integrated electronic circuit is
(*a*) a complicated circuit
(*b*) an integrating device
(*c*) much costlier than a single transistor
(*d*) fabricated on a tiny silicon chip.

3. Most important advantage of an IC is its
(*a*) easy replacement in case of circuit failure
(*b*) extremely high reliability
(*c*) reduced cost
(*d*) low power consumption.

4. In monolithic ICs, all components are fabricated by ..... process.
(*a*) evaporation
(*b*) sputtering
(*c*) diffusion
(*d*) oxidisation.

5. Monolithic ICs are fabricated within a
(*a*) soft stone
(*b*) single stone
(*c*) silicon layer
(*d*) ceramic base.

6. As compared to monolithic ICs, film ICs have the advantage of
(*a*) better high-frequency response
(*b*) much reduced cost
(*c*) smaller size
(*d*) less flexibility in circuit design.

7. Monolithic technique is ideally suited for fabricating ..... ICs.
(*a*) thin-film
(*b*) digital
(*c*) linear
(*d*) thick-film

8. In the context of IC fabrication, metallisation means
(*a*) connecting metallic wires
(*b*) forming interconnecting conduction pattern and bonding pads
(*c*) depositing SiO_2 layer
(*d*) covering with a metallic cap.

9. Monolithic transistors are formed in the epitaxial *N*-layer
(*a*) in one operation
(*b*) by evaporation process
(*c*) by successive impurity diffusions
(*d*) by oxidisation.

10. The major component of a MOS IC is a/an
(*a*) FET (*b*) MOSFET
(*c*) BJT (*d*) SCR.

11. Processing of MOS ICs is less expensive than bipolar ICs primarily because they
(*a*) use cheaper components
(*b*) need no component isolation
(*c*) require much less diffusion steps
(*d*) have very high packing density.

12. In an integrated circuit, the SiO_2 layer provides
(*a*) electrical connection to the external circuit
(*b*) physical strength
(*c*) isolation
(*d*) conducting path.

13. The foundation on which an IC is built is called an
(*a*) insulator (*b*) base
(*c*) wafer (*d*) plate

14. Monolithic IC construction uses
(*a*) discrete components
(*b*) high-value resistors
(*c*) connecting wires
(*d*) extensive number of components

ANSWERS

1. (*c*) **2.** (*d*) **3.** (*b*) **4.** (*c*) **5.** (*c*) **6.** (*a*) **7.** (*b*) **8.** (*b*) **9.** (*c*) **10.** (*b*) **11.** (*c*) **12.** (*c*) **13.** (*c*) **14.** (*d*)

ROUGH WORK

GO To FIRST

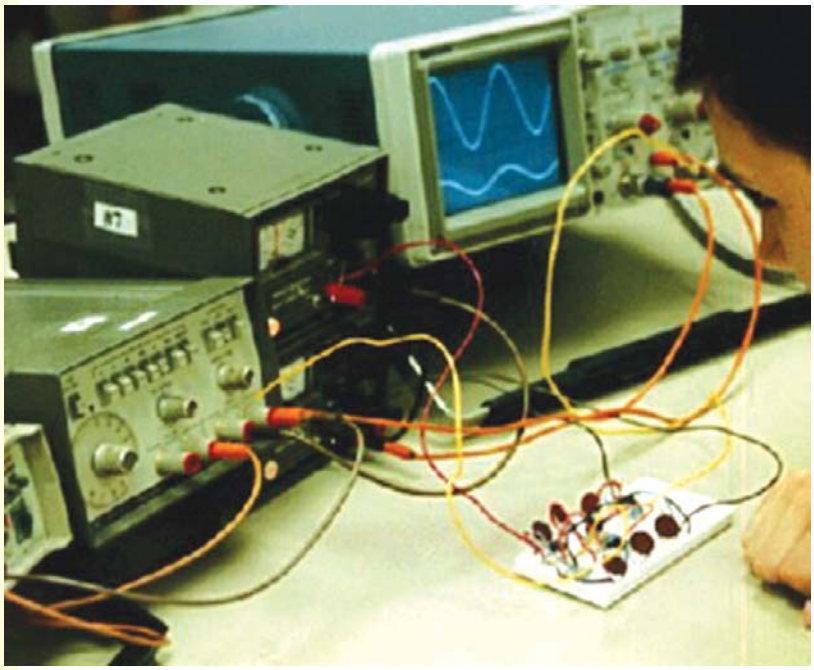
CONTENTS

CHAPTER 68

OP-AMP AND ITS APPLICATIONS

Learning Objectives

- What is an OP-AMP?
- OP-AMP Symbol
- Polarity Conventions
- Ideal Operational Amplifier
- Virtual Ground and Summing Point
- Why V_i is Reduced to Almost Zero?
- OP-AMP Applications
- Linear Amplifier
- Unity Follower
- Adder or Summer
- Subtractor
- Integrator
- Differentiator
- Comparator
- Audio Amplifier
- OP-Amp Based Oscillator Circuits
- OP-Amp Based Wien Bridge Oscillator
- OP-Amp Based Crystal Oscillator
- A Triangular-wave Oscillator
- A Voltage- controlled Sawtooth Oscillator (VCO)
- A Square-wave Relaxation Oscillator
- High-impedance Voltmeter
- Active Filters
- Low-pass Filter
- High-pass Filter
- Band-pass Filter
- Notch Filter



The operational amplifier was designed to perform mathematical operations. Although now superseded by the digital computer, op-amps are a common feature of modern analog electronics

CONTENTS

68.1 What is an OP-AMP ?

It is a very high-gain, high-r_{in} directly-coupled negative-feedback amplifier which can amplify signals having frequency ranging from **0 Hz to a little beyond 1 MHz.** They are made with different internal configurations in linear *ICs*. An *OP-AMP* is so named because it was originally designed to perform mathematical operations like summation, subtraction, multiplication, differentiation and integration etc. in analog computers. Present day usage is much wider in scope but the popular name *OP-AMP* continues.

Typical uses of *OP-AMP* are : scale changing, analog computer operations, in instrumentation and control systems and a great variety of phase-shift and oscillator circuits. The *OP-AMP* is available in three different packages **(*i*)** standard dual-in-line package (*DIL*) **(*ii*)** *TO*-5 case and **(*iii*)** the flat-pack.

Although an *OP-AMP* is a complete amplifier, it is so designed that external components (resistors, capacitors etc.) can be connected to its terminals to change its external characteristics. Hence, it is relatively easy to tailor this amplifier to fit a particular application and it is, in fact, due to this versatility that *OP-AMPs* have become so popular in industry.

An *OP-AMP IC* may contain two dozen transistors, a dozen resistors and one or two capacitors.

Example of OP-AMPs

1. µA 709—is a high-gain operational amplifier constructed on a single silicon chip using planar epitaxial process.

   It is intended for use in dc servo systems, high-impedance analog computers and in low-level instrumentation applications.

   It is manufactured by Semiconductors Limited, Pune.
2. [*LM* 108 – *LM* 208]— Manufactured by Semiconductors Ltd. Bombay,
3. *CA* 741 *CT* and *CA* 741 *T*—these are high-gain operational amplifiers which are intended for use as **(*i*)** comparator, **(*ii*)** integrator, **(*iii*)** differentiator, **(*iv*)** summer, **(*v*)** dc amplifier, **(*vi*)** multivibrator and **(*vii*)** bandpass filter.

   Manufactured by Bharat Electronics Ltd (*BEL*), Bangalore.

68.2. OP-AMP Symbol

Standard triangular symbol for an *OP-AMP* is shown in Fig. 68.1 (*a*) though the one shown in Fig. 68.1 (*b*) is also used often. In Fig. 68.1 (*b*), **the common ground line has been omitted.** It also does not show other necessary connections such as for dc power and feedback etc.

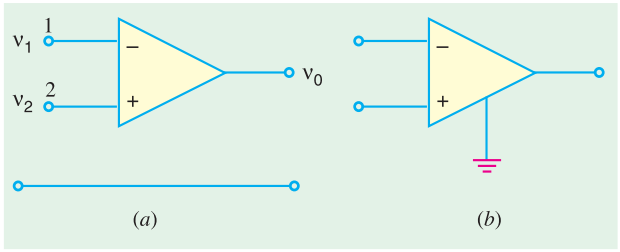


Fig. 68.1

The *OP-AMP's* input can be single-ended or double-ended (or differential input) depending on whether input voltage is applied to one input terminal only or to both. Similarly, amplifier's output can also be either single-ended or double-ended. The most common configuration is ***two input terminals and a single output.***

All OP-AMPs have a minimum of five terminals :

1. inverting input terminal,
2. non-inverting input terminal,
3. output terminal,
4. positive bias supply terminal,
5. negative bias supply terminal.

68.3. Polarity Conventions



In Fig. 68.1 (*b*), the input terminals have been marked with minus (–) and plus (+) signs. These are meant to **indicate the inverting and non-inverting terminals only** [Fig. 68.2]. It simply means that a signal applied at negative input terminal will appear amplified but **phase-inverted at the output terminal** as shown in Fig. 68.2 (*b*). Similarly, signal applied at the positive input terminal will appear amplified and **inphase** at the output. Obviously, these plus and minus polarities **indicate phase reversal only**. It does not mean that voltage v_1 and v_2 in Fig. 68.2 (*a*) are negative and positive respectively. Additionally, it also does not imply that a ***positive input voltage has to be connected to the plus-marked non-inverting terminal 2 and negative input voltage to the negative-marked inverting terminal 1.*** In fact, the amplifier can be used 'either way up' so to speak. It may also be noted that all input and output voltages are referred to a common reference usually the ground shown in Fig. 68.1 (*a*).

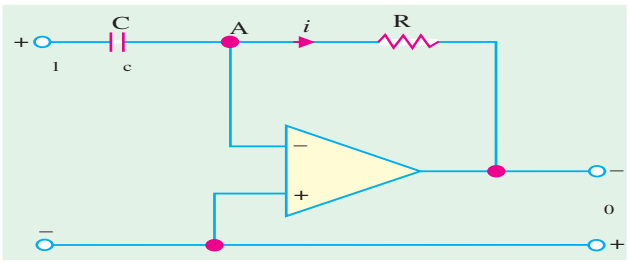


Fig. 68.2

68.4. Ideal Operational Amplifier

When an *OP-AMP* is operated without connecting any resistor or capacitor from its output to any one of its inputs (*i.e.*, without feedback), it is said to be in the **open-loop condition**. The word 'open loop' means that ***feedback path or loop is open.*** The specifications of *OP-AMP* under such condition are called open-loop specifications.

An ideal *OP-AMP* (Fig. 68.3) has the following characteristics :

1. its open-loop gain A_v is ***infinite*** *i.e.*, $A_v = -\infty$
2. its input resistance R_i (measured between inverting and non-inverting terminals) is ***infinite*** *i.e.*, $R_i = \infty$ ohm
3. its output resistance R_0 (seen looking back into output terminals) is ***zero*** *i.e.*, $R_0 = 0\ \Omega$
4. it has ***infinite bandwith*** *i.e.*, it has flat frequency response from dc to infinity.

Though these characteristics cannot be achieved in practice, yet an ideal *OP-AMP* serves as a convenient reference against which real *OP-AMPs* may be evaluated.

Following additional points are worth noting :

1. infinte input resistance means that input current $i = 0$ as indicated in Fig. 68.3. It means that an ideal *OP-AMP* is a voltage-controlled device.
2. $R_0 = 0$ means that v_0 is not dependent on the load resistance connected across the output.
3. though for an ideal *OP-AMP* $A_v = \infty$, for an actual one, it is extremely high *i.e.*, about 10^6. However, it does not mean that 1 V signal will be amplified to 10^6 V at the output. Actually, the maximum value of v_0 is limited by the basis supply voltage, typically ± 15 V. With $A_v = 10^6$ and $v_0 = 15\ V_2$ the maximum value of input voltage is limited to $15/10^6 = 15\ \mu V$. Though 1 μV in the *OP-AMP*, can certainly become 1 V.

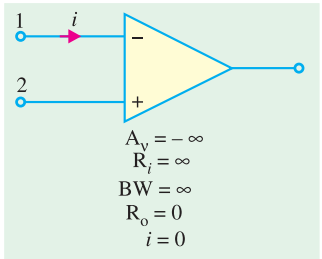


Fig. 68.3

68.5. Virtual Ground and Summing Point

In Fig. 68.4 is shown an *OP-AMP* which *employs* ***negative feedback*** with the help of resistor R_f which feeds a portion of the output to the input.

Since input and feedback currents are algebraically added at point *A*, it is called the **summing point**.

The concept of **virtual** ground arises from the fact that input voltage v_i at the inverting terminal of the *OP-AMP* is forced to such a small value that, for all practical purposes, it may be assumed to be zero. Hence, point *A* is essentially at ground voltage and is referred to as *virtual ground*. Obviously, ***it is not the actual ground,*** which, as seen from Fig. 68.4, is situated below.

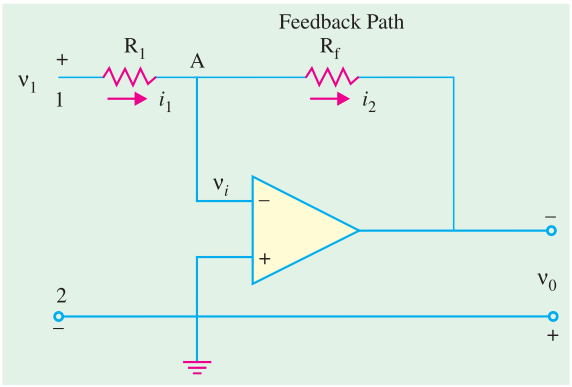


Fig. 68.4

68.6. Why V_i is Reduced to Almost Zero ?

When v_1 is applied, point *A* attains some positive potential and at the same time v_0 is brought into existence. Due to negative feedback, some fraction of the output voltage is fed back to point *A* antiphase with the voltage already existing there (due to v_1).

The algebraic sum of the two voltages is almost zero so that $v_i \cong 0$. Obviously, v_i will become exactly zero when **negative feedback voltage at A is exactly equal to the positive voltage produced by v_1 at A.**

Another point worth considering is that there exists a virtual short between the two terminals of the *OP-AMP* because $v_i = 0$. It is virtual because no current flows (remember $i = 0$) despite the existence of this short.

68.7. OP-AMP Applications

We will consider the following applications :

1. as scalar or linear (*i.e.*, small-signal) constant-gain amplifier both inverting and non-inverting,
2. as unity follower,
3. Adder or Summer
4. Subtractor,
5. Integrator
6. Differentiator
7. Comparator.

Now, we will discuss the above circuits one by one assuming an ideal *OP-AMP*.

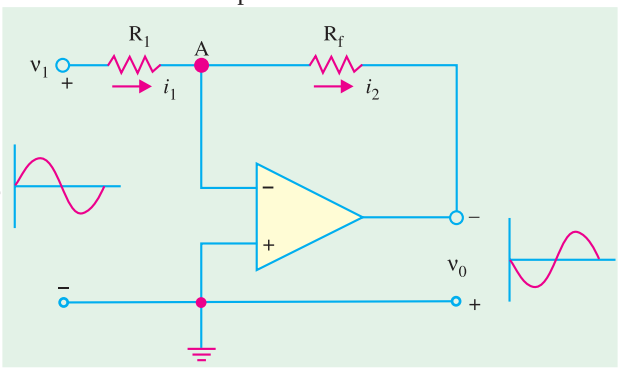


Fig. 68.5

68.8. Linear Amplifier

We will consider the functioning of an *OP-AMP* as constant-gain amplifier both in the inverting and non-inverting configurations.

(*a*) Inverting Amplifier or Negative Scale.

As shown in Fig. 68.5, noninverting

terminal has been grounded, whereas R_1 connects the input signal v_1 to the inverting input. A feedback resistor R_f has been connected from the output to the inverting input.

Gain

Since point A is at ground potential*, $i_1 = \frac{v_{in}}{R_1} = \frac{v_1}{R_1}$

$$i_2 = \frac{-v_0}{R_f} \quad \text{Please note –ve sign}$$

Using *KCL* (Art. 2.2) for point A,

$$i_1 + (-i_2) = 0 \quad \text{or} \quad \frac{v_1}{R_1} + \frac{v_0}{R_f} = 0 \quad \text{or} \quad \frac{v_0}{R_f} = -\frac{v_1}{R_1} \quad \text{or} \quad \frac{v_0}{v_1} = -\frac{R_f}{R_1}$$

$$\therefore \quad A_v = -\frac{R_f}{R_1} \quad \text{or} \quad A_v = -K \quad \text{Also, } v_0 = -Kv_{in}$$

It is seen from above, that closed-loop gain of the inverting amplifier depends on the ratio of the two external resistors R_1 and R_f and is independent of the amplifier parameters.

It is also seen that the *OP-AMP* works as a negative scaler. It scales the input *i.e.*, it multiplies the input by a minus constant factor K.

(*b*) Non-inverting Amplifier or Positive Scaler

This circuit is used when there is need for an output which is equal to the input multiplied by a positive constant. Such a positive scaler circuit which uses negative feedback but provides an output that equals the input multiplied by a positive constant is shown in Fig. 68.6.

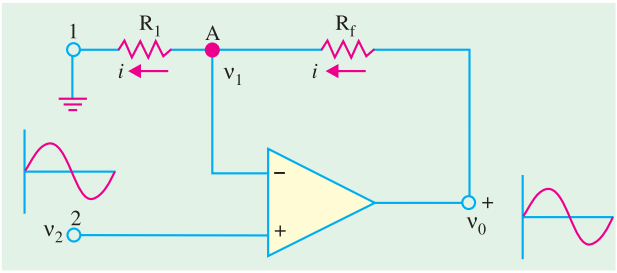


Fig. 68.6

Since input voltage v_2 is applied to the non-inverting terminal, the circuits is also called **non-inverting amplifier.**

Here, polarity of v_0 is the same as that v_2 *i.e.*, both are positive.

Gain

Because of virtual short between the two *OP-AMP* terminals, voltage across R_1 is the input voltage v_2. Also, v_0 is applied across the series combination of R_1 and R_f.

$$\therefore \quad v_{in} = v_2 = iR_1, v_0 = i(R_1 + R_f)$$

$$\therefore \quad A_v = \frac{v_0}{v_{in}} = \frac{i(R_1 + R_f)}{iR_1} \quad \text{or} \quad A_v = \frac{R_1 + R_f}{R_1} = \left(1 + \frac{R_f}{R_1}\right)$$

Alternative Derivation

As shown in Fig. 68.7, let the currents through the two resistors be i_1 and i_2.

The voltage across R_1 is v_2 and that across R_f is $(v_0 - v_2)$.

$$\therefore \quad i_1 = \frac{v_2}{R_1} \quad \text{and} \quad i_2 = \frac{v_0 - v_2}{R_f}$$

Applying *KCL* to junction A, we have

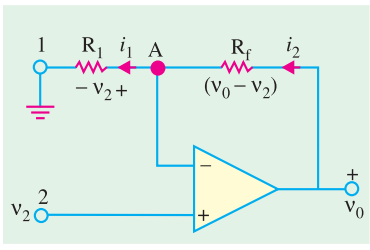


Fig. 68.7

* If not, then $i_1 = \frac{v_1 - v_i}{R_1}$ and $i_2 = \frac{v_0 - v_1}{R_f}$

$$(-i_1) + i_2 = 0 \quad \text{or} \quad \frac{v_2}{R_1} + \frac{(v_0 - v_2)}{R_f} = 0$$

$$\therefore \quad \frac{v_0}{R_f} = v_2\left(\frac{1}{R_1} + \frac{1}{R_f}\right) = v_2\,\frac{R_1 + R_f}{R_1 R_f}$$

$$\therefore \quad \frac{v_0}{v_2} = \frac{R_1 + R_f}{R_1} \quad \text{or} \quad A_v = 1 + \frac{R_f}{R_1} \quad \text{—as before}$$

Example 68.1. *For the inverting amplifier of Fig. 68.5, R_1 = 1 K and R_f = 1 M. Assuming an ideal OP-AMP amplifier, determine the following circuit values :*

(a) *voltage gain,* ***(b)*** *input resistance,* ***(c)*** *output resistance*

Solution. It should be noted that we will be calculating values of the circuit and not *for* the *OP-AMP proper*.

(a) $$A_v = -\frac{R_f}{R_1} = -\frac{1000\ K}{1\ K} = \mathbf{-1000}$$

(b) Because of virtual ground at A, $R_{in} = R_1 = \mathbf{1\ K}$

(c) Output resistance of the circuit equal the output resistance of the *OP-AMP i.e.*, **zero ohm**.

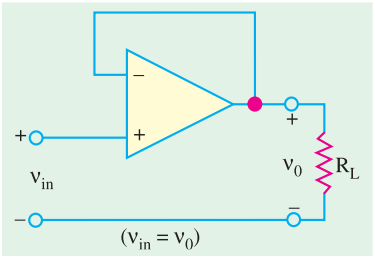


Fig. 68.8

68.9. Unity Follower

It provides a gain of unity without any phase reversal. It is very much similar to the emitter follower (Art 68.8) except that its gain is very much closer to being exactly unity.

This circuit (Fig. 68.8) is useful as a buffer or isolation amplifier because it allows, input voltage v_{in} to be transferred as output voltage v_0 while at the same time preventing load resistance R_L from loading down the input source. It is due to the fact that its $R_i = \infty$ and $R_0 = 0$.

In fact, circuit of Fig. 68.8 can be obtained from that of Fig. 68.6 by putting

$$R_1 = R_f = 0$$

68.10. Adder to Summer

The adder circuit provides an output voltage proportional to or equal to the algebraic sum of two or more input voltages each multiplied by a constant gain factor. It is basically similar to a scaler (Fig. 68.5) except that it has more than one input. Fig. 68.9 shows a three-input inverting adder circuit. As seen, ***the output voltage is phase-inverted.***

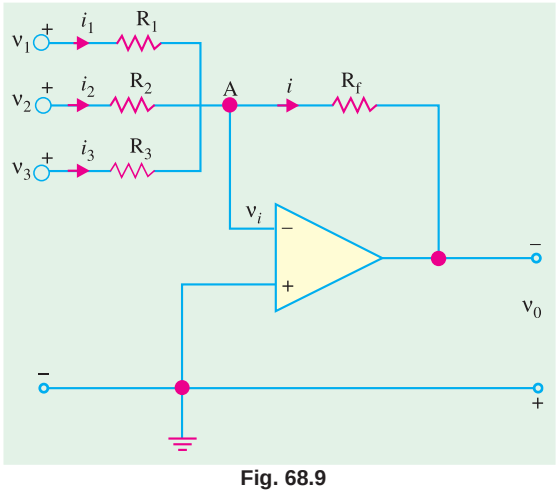


Fig. 68.9

Calculations

As before, we will treat point A as virtual ground

$$i_1 = \frac{v_1}{R_1} \quad \text{and} \quad i_2 = \frac{v_2}{R_2}$$

$$i_3 = \frac{v_3}{R_3} \quad \text{and} \quad i = -\frac{v_0}{R_f}$$

Applying *KCI* to point A, we have

$$i_1 + i_2 + i_3 + (-i) = 0$$

or $$\frac{v_1}{R_1}+\frac{v_2}{R_2}+\frac{v_3}{R_3}-\left(\frac{-v_0}{R_f}\right)=0$$

$$\therefore \quad v_0=-\left(\frac{R_f}{R_1}v_1+\frac{R_f}{R_2}v_2+\frac{R_f}{R_3}v_3\right)$$

or $$v_0=-(K_1v_1+K_2v_2+K_3v_3)$$

The overall negative sign is unavoidable because we are using the inverting input terminal.

If $R_1 = R_2 = R_3 = R$, then

$$v_0 = -\frac{R_f}{R}(v_1+v_2+v_3) = -K(v_1+v_2+v_3)$$

Hence, output voltage is proportional to ***(not equal to)*** the **algebraic sum** of the three input voltages.

If $R_f = R$, then ouput exactly equals the sum of inputs. However, if $R_f = R/3$

then $$v_0 = -\frac{R/3}{R}(v_1+v_2+v_3) = -\frac{1}{3}(v_1+v_2+v_3)$$

Obviously, the output is equal ***to the average of the three inputs.***

68.11. Subtractor

The function of a subtractor is to provide an output proportional to or equal to the difference of two input signals. As shown in Fig. 68.10 we have to apply the inputs at the inverting as well as non-inverting terminals.

Calculations

According to Superposition Theorem (Art. 2.17) $v_0 = v_0' + v_0''$

where v_0' is the output produced by v_1 and v_0'' is that produced by v_2.

Now, $$v_0' = -\frac{R_f}{R_1}\cdot v_1 \qquad \text{...Art 67.37 } (a)$$

$$v_0'' = \left(1+\frac{R_f}{R_1}\right)v_2 \qquad \text{...Art 67.37 } (b)$$

$$\therefore \quad v_0 = \left(1+\frac{R_f}{R_1}\right)v_2 - \frac{R_f}{R_1}\cdot v_1$$

Since $R_f \gg R_1$ and $R_f/R_1 \gg 1$, hence

$$v_0 \cong \frac{R_f}{R_1}(v_2-v_1) = K(v_2-v_1)$$

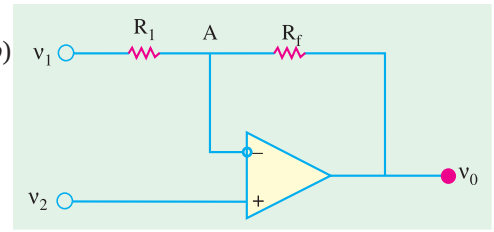


Fig. 68.10

Further, If $R_f = R_1$, then

$$v_0 = (v_2 - v_1) = \text{difference of the two input voltages}$$

Obviously, if $R_f \neq R_1$, then a scale factor is introduced.

Example 68.2. *Find the output voltages of an OP-AMP inverting adder for the following sets of input voltages and resistors. In all cases, R_f = 1 M.*

$$v_1 = -3\,V, v_2 = +3V, v_3 = +2V; R_1 = 250\,K, R_2 = 500\,K, R_3 = 1\,M$$

[Electronic Engg. Nagpur Univ. 1991]

Solution. $$v_0 = -(K_1v_1 + K_2v_2 + K_3v_3)$$

$$K_1 = \frac{R_f}{R_1} = \frac{1000\,K}{250\,K} = 4,\ K_2 = \frac{1000}{500} = 2,\ K_3 = \frac{1M}{1M} = 1$$

$$\therefore \qquad v_0 = -[(4 \times -3) + (2 \times 3) + (1 \times 2)] = \mathbf{+ 4V}$$

Example 68.3. *In the subtractor circuit of Fig. 68.10, $R_1 = 5$ K, $R_f = 10$ K, $v_1 = 4$ V and $v_2 = 5$ V. Find the value of output voltage.*

Solution. $$v_0 = \left(1 + \frac{R_f}{R_1}\right) v_1 - \frac{R_f}{R_1} v_2 = \left(1 + \frac{10}{5}\right) 4 - \frac{10}{5} \times 5 = \mathbf{+ 2V}$$

Example 68.4. *Design an OP-AMP circuit that will produce an output equal to $-(4\ v_1 + v_2 + 0{\cdot}1\ v_3)$. Write an expression for the output and sketch its output waveform when $v_1 = 2 \sin \omega t$, $v_2 = + 5$ V dc and $v_3 = -100$ V dc.* **[Banglore University 2001]**

Solution. $$v_0 = -\left[\frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2 + \frac{R_f}{R_3} v_3\right] \qquad \text{...(1)}$$

and also $$v_0 = -(4 v_1 + v_2 + 0{\cdot}1 v_3) \qquad \text{...(2)}$$

Comparing equations **(1)** and **(2)**, we find,

$$\frac{R_f}{R_1} = 4, \quad \frac{R_f}{R_2} = 1, \quad \frac{R_f}{R_3} = 0{\cdot}1$$

Therefore if we assume $R_f = 100\ K$, then $R_1 = 25\ K$, $R_2 = 100\ K$ and $R_3 = 10\ K$. With there values of R_1, R_2 and R_3, the *OP-AMP* circuit is as shown in Fig. 68.11(*a*)

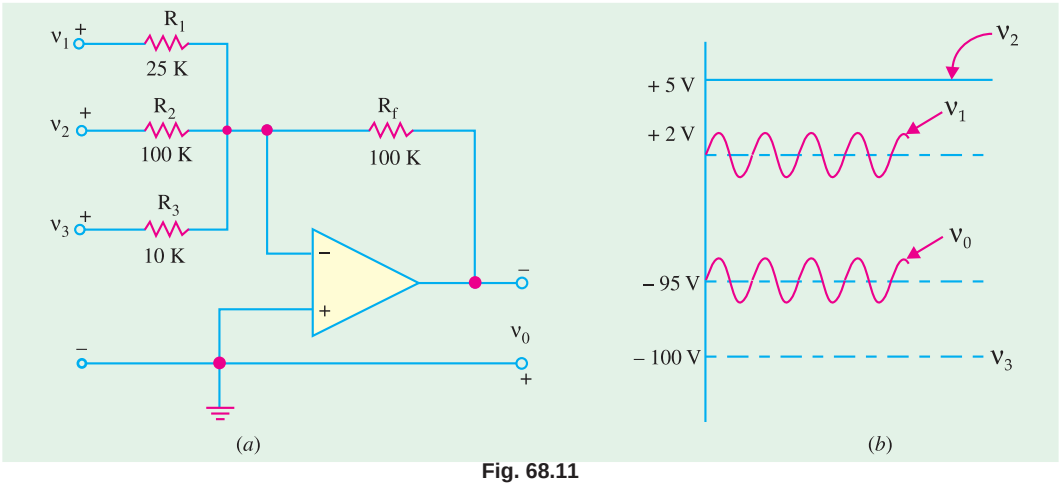


Fig. 68.11

With the given values of $v_1 = 2 \sin \omega t$, $v_2 = + 5$V, $v_3 = -100$ V dc, the output voltage, $v_0 = 2 \sin \omega t + 5 - 100 = 2 \sin \omega t - 95$ V. The waveform of the output voltage is sketched as shown in Fig. 68.11 (*b*).

68.12. Integrator

The function of an integrator is to provide an output voltage which is proportional to the integral of the input voltage.

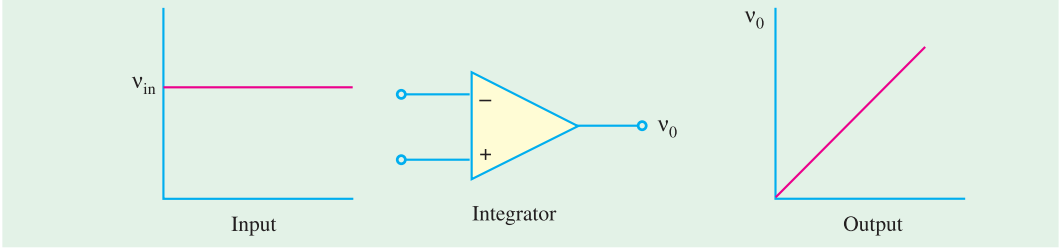


Fig. 68.12

A simple example of integration is shown in Fig. 68.12 where input is dc level and its integral is ***a linearly-increasing ramp output.*** The actual integration circuit is shown in Fig. 68.13. This circuit is similar to the scaler circuit of Fig. 68.5 except that **the feedback component is a capacitor C instead of a resistor R_f.**

Calculations

As before, point A will be treated as virtual ground.

$$i_1 = \frac{v_1}{R}\ ;\ i_2 = -\frac{v_0}{X_C} = -\frac{v_0}{1/j\omega C} = -\frac{v_0}{1/sC} = -s\,C\,v_0$$

where $s = j\,\omega$ in the Laplace notation.

Now $i_1 = i_2$ ...Art. 68.26 (*a*)

$$\therefore \quad \frac{v_1}{R} = -s\,Cv_0$$

$$\therefore \quad \frac{v_0}{v_{in}} = \frac{v_0}{v_1} = -\frac{1}{s\,CR} \qquad ...(i)$$

$$\therefore \quad A_v = -\frac{1}{s\,CR}$$

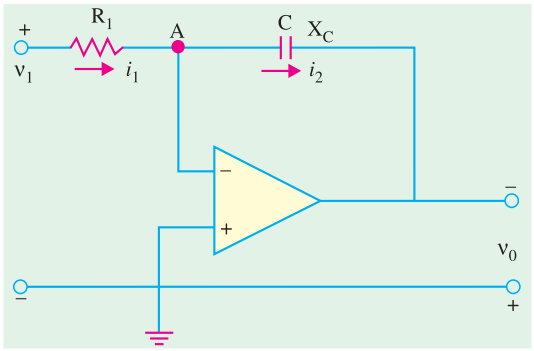


Fig. 68.13

Now, the expression of Eq. (*i*) can be written in time domain as

$$v_0(t) = -\frac{1}{40\,\pi}(\cos 2000\,\pi t - 1)$$

It is seen from above that output (right-hand side expression) is an integral of the input, with an inversion and a scale factor of $1/CR$.

This ability to integrate a given signal enables an analog computer solve differential equations and to set up a wide variety of electrical circuit analogs of physical system operation. For example, let

$R = 1$ M and $C = 1$ μF. Then

$$\text{scale factor} = -\frac{1}{CR} = -\frac{1}{10^6 \times 10^{-6}} = -1$$

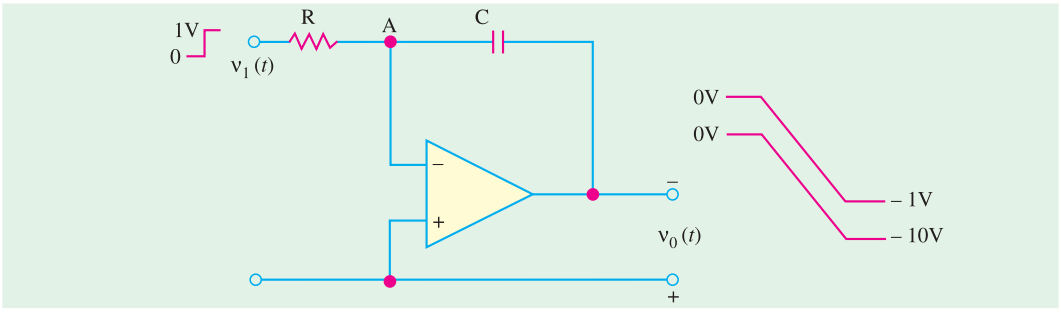


Fig. 68.14

As shown in Fig. 68.14 the input is a step voltage, whereas output is a ramp (or linearly-changing voltages) with a scale multiplier of –1. However, when $R = 100$ K, then

$$\text{scale factor} = -\frac{1}{10^5 \times 10^{-6}} = -10$$

$$\therefore \quad v_0(t) = -10\int v_1(t)\,.\,dt$$

It is also shown in Fig. 68.14. Of course, we can integrate more than one input as shown below in Fig. 68.15. With multiple inputs, the output is given by

$$v_0(t) = -\left[K_1 \int v_1(t)\,dt + K_2 \int v_2(t)\,dt + k_3 \int v_3(t)\,dt\right]$$

where $K_1 = \frac{1}{CR_1}$, $K_2 = \frac{1}{CR_2}$ and $K_3 = \frac{1}{CR_3}$

Fig. 68.15 (*a*) shows a summing integrator as used in an analog computer. It shows all the three resistors and the capacitor. The analog computer representation of Fig. 68.15 (*b*) indicates only the scale factor for each input.

Example 68.5. *A 5-mV, 1-kHz sinusoidal signal is applied to the input of an OP-AMP integrator of Fig. 64.37 for which R = 100 K and C = 1 μF. Find the output voltage.*

[Electronic & Comm. Engg. Kurukshetra Univ. 1990]

Solution. Scale factor $= -\frac{1}{CR} = \frac{1}{10^5 \times 10^{-6}} = -10$

The equation for the sinusoidal voltage is

$$v_1 = 5 \sin 2\pi ft = 5 \sin 2000\,\pi t$$

Obviously, it has been assumed that at $t = 0$, $v_1 = 0$

$$\therefore \quad v_0(t) = -10 \int_0^t 5 \sin 2000\,\pi t = -50 \left| \frac{-\cos 2000\,\pi t}{2000} \right|_0^t$$

$$= -\frac{1}{40\,\pi}(\cos 2000\,\pi t - 1)$$

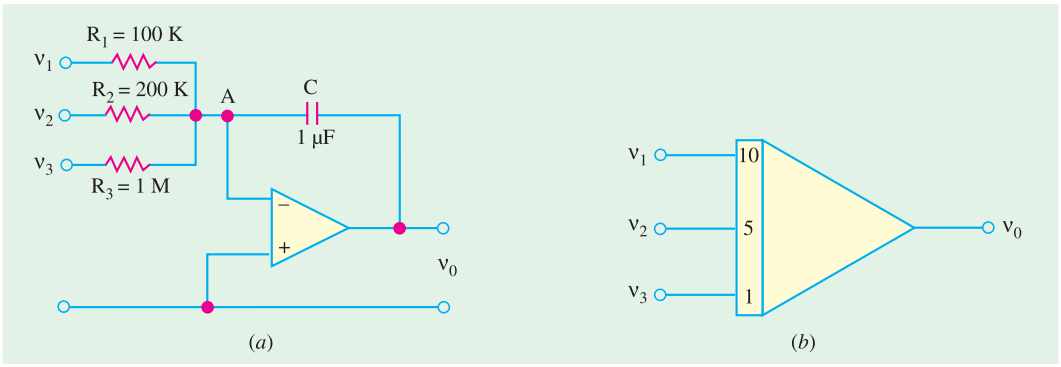


Fig. 68.15

68.13. Differentiator

Its function is to provide an output voltage which is ***proportional to the rate of the change of the input voltage.*** It is an inverse mathematical operation to that of an integrator. As shown in Fig. 68.16, when we feed a differentiator with linearly-increasing ramp input, we get a constant dc output.

Circuit

Differentiator circuit can be obtained by interchanging the resistor and capacitor of the integrator circuit of Fig. 68.13.

Let i = rate of change of charge $= \frac{dq}{dt}$

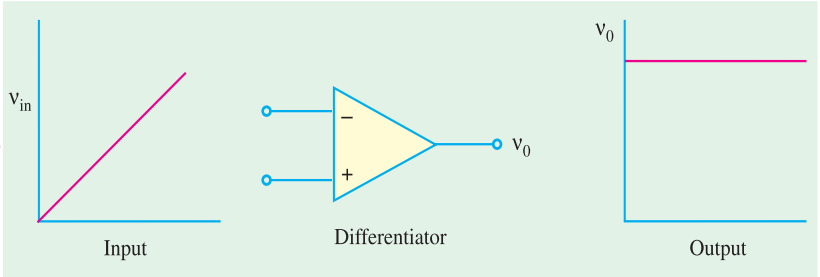


Fig. 68.16

Now, $q = Cv_c$

$$\therefore \quad i = \frac{d}{dt}(Cv_c) = C\frac{dv_c}{dt}$$

Taking point A as virtual ground

$$v_0 = -iR = -\left(C \cdot \frac{dv_c}{dt}\right)R = -CR \cdot \frac{dv_c}{dt}$$

As seen, output voltage is proportional to the derivate of the input voltage, the constant of proportionality (*i.e.*, scale factor) being $(-RC)$.

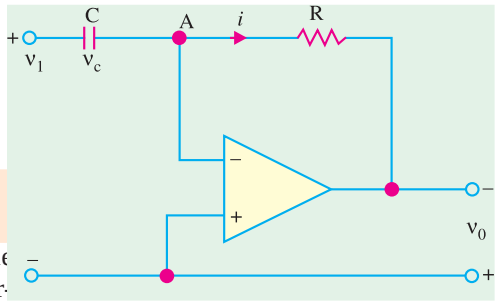


Fig. 68.17

Example 68.6. *The input to the differentiator circuit of Fig. 68.17 is a sinusoidal voltage of peak value of 5 mV and frequency 1 kHz. Find out the output if R = 1000 K and C = 1 μF.*

Solution. The equation of the input voltage is

$$v_1 = 5 \sin 2\pi \times 1000\, t = 5 \sin 2000\, \pi t \text{ mV}$$

scale factor $= CR = 10^{-6} \times 10^5 = 0{\cdot}1$

$$v_0 = 0{\cdot}1 \frac{d}{dt}(5 \sin 2000\, \pi t) = (0{\cdot}5 \times 2000\, \pi) \cos 2000\, \pi t = 1000\, \pi \cos 2000\, \pi t \text{ mV}$$

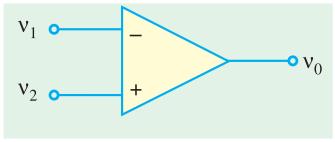


Fig. 68.18

As seen, output is a cosinusoidal voltage of frequency 1 kHz and peak value 1000 π mV.

68.14. Comparator

It is a circuit which compares two signals or voltage levels. The circuit is shown in Fig. 68.18 and (like that of the unity follower) is the simplest because it needs no additional external components.

If v_1 and v_2 are equal, then v_0 should idealy be zero. Even if v_1 differs from v_2 by a very small amount, v_0 is large because of amplifier's high gain. Hence, circuit of Fig. 68.18 can detect very small changes which is another way of saying that it compares two signals,

68.15. Audio Amplifier

As a matter of fact, in most communication receivers, the final output stage is the audio amplifier. The ideal audio amplifier will have the following characteristics :

1. High gain
2. Minimum distortion in the audio frequency range (*i.e.*, 20 Hz to 20 kHz range).
3. High input resistance (or impedance).
4. Low output resistance (or impedance) to provide optimum coupling to the speaker.

The use of *OP-AMP* is an audio amplifier will fullfill the requirements listed above very nicely. An *OP-AMP* audio amplifier is shown in Fig. 68.19.

Note that the *OP-AMP* is supplied only for $+V$ volt power supply, the $-V$ terminal is grounded. Because of this, the output will be between the limits of $(+V - 1)$ volts and +1 volt approximately. Also notice the use of a coupling capacitor C_2 between the *OP-AMP* and speaker. This capacitor is necessary to reference the speaker signal around ground. The capacitor C_s, is included in the V_{CC} line to prevent any transient current caused by the operation of *OP-AMP* from being coupled back to Q_1 through the power supply. The high gain requirements is accomplished by the combination of two amplifier stages. The high R_m/Low R_{out} of the audio amplifier is accomplished by the *OP-AMP* itself, as the low distortion characteristic.

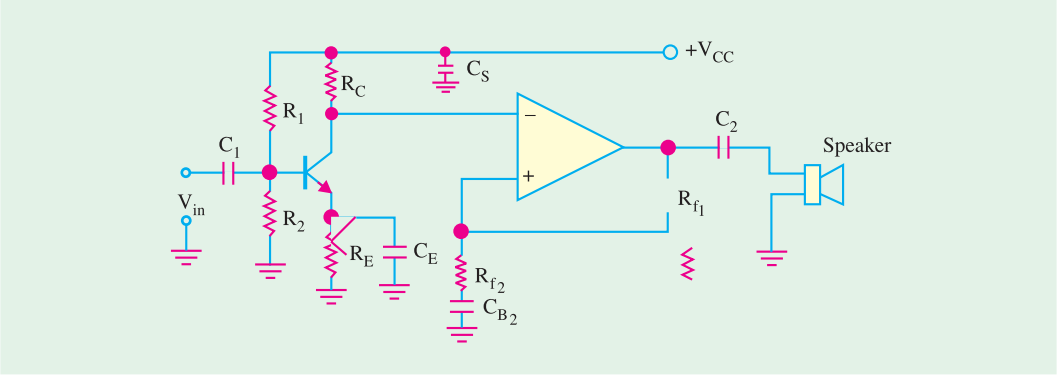


Fig. 68.19

68.16. OP-AMP Based Oscillator Circuits

We have already discussed about sinusoidal oscillators in Chapter 15. There we defined the oscillator as a circuit that produces an output waveform without any external signal source. The only input to an oscillator is the dc power supply. As such, the oscillator can be viewed as being a signal generator. We have also discussed in the same chapter, about the different types of oscillator circuits (like Wien Bridge oscillator, Colpitts Oscillator and Crystal Oscillator) using bipolar junction and field-effect transistor. Now we still study these oscillator circuits using *OP-AMP*.

68.17. OP-AMP Based Wien Bridge Oscillator

Fig. 68.20 shows the basic version of a Wien Bridge oscillator. The circuits uses an *OP-AMP* and *RC* bridge circuit. Note the basic bridge connection carefully. Resistors R_1, R_2 and capacitors $C_1$and C_2 form the frequency adjustment elements while resistors R_3 and R_4 form part of the feedback path. The *OP-AMP* output is connected as the bridge input as points '*A*' and '*C*'. The bridge circuit output at points '*B*' and '*D*' is the input to the *OP-AMP* shows an alternative way of connecting the *RC* bridge circuit to the *OP-AMP*. In a typical Wien Bridge oscillator, $R_1 = R_2 = R$, and $C_1 = C_2 = C$. This means that the two *RC* circuits will have the same cut-off frequency. The cut-off frequency is given by $(1/2\ \pi RC)$.

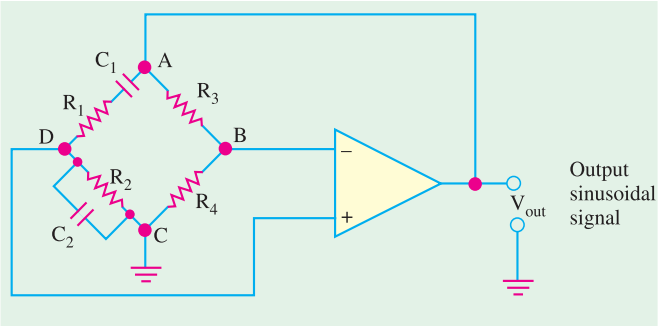


Fig. 68.20

Example 68.7. *Fig. 68.21 shows the circuit of a Wien-Bridge Oscillator using OP-AMP as an amplifier. Notice the components R_1, C_1, R_2, C_2, R_3 and R_4 are connected in the bridge configuration in the same way as shown in Fig. 68.22.*

Calculate the frequency of the Wien Bridge oscillator.

Solution. The frequency of oscillations,

$$f_0 = \frac{1}{2\pi RC} = \frac{1}{2\pi \times 51\,K \times 0{\cdot}001\mu F} = \mathbf{3{\cdot}12\ kHz}$$

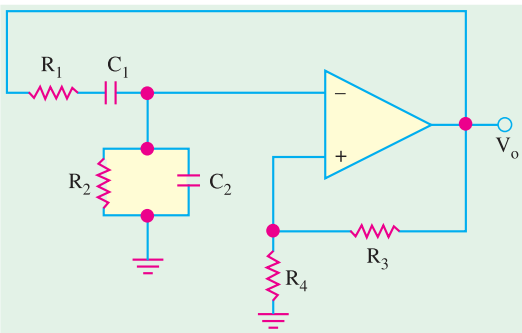


Fig. 68.21

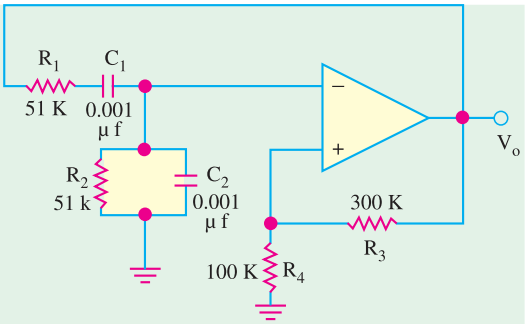


Fig. 68.22

68.18. OP-AMP Based Colpitts Oscillator

An *OP-AMP* based Colpitts oscillator is as shown in Fig. 68.23. Here the *OP-AMP* provides the basic amplification needed while a *LC* feedback network of Colpitts configuration sets the oscillator frequency.

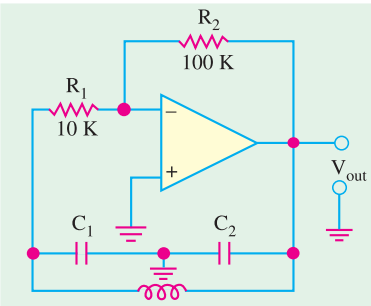


Fig. 68.23

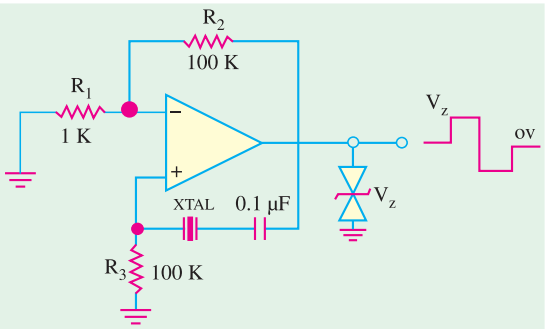


Fig. 68.24

68.19. OP-AMP Based Crystal Oscillator

An *OP-AMP* can be used in a crystal oscillator as shown in Fig. 68.24. The crystal is connected in the series resonant path and operates at the crystal series resonant frequency. The present circuit has high gain so that an output square-wave results are shown in the figure. A pair of Zener diodes is shown at the output to provide output amplitude at exactly the Zener voltage (V_Z).

68.20. A Triangular-Wave Oscillator

Fig. 68.25 shows an *OP-AMP* circuit to generate a triangular-wave from a square-wave. The circuit makes use of two *OP-AMP* : one of them is used as a comparator and the other as an integrator. The operation of the circuit is as given below :

To begin with, let us assume that the output voltage of the comparator is at its maximum negative level. This output is connected to the inverting input

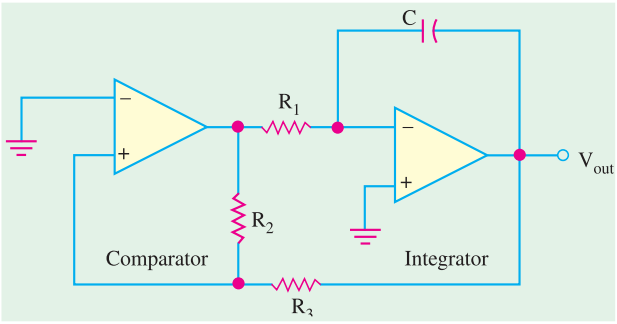


Fig. 68.25

of the integrator through resistor, R_1. This produces a positive-going ramp on the output of the integrator. When the ramp voltage reaches the upper trigger point (UTP), the comparator switches to its maximum positive level. This positive level causes the integrator ramp to change to a negative going direction. The ramp continues in this direction until the lower trigger point (LTP) of the comparator is reached. At this point, the comparator output switches back to its maximum negative level and the cycle repeats. This action is shown in Fig. 68.26.

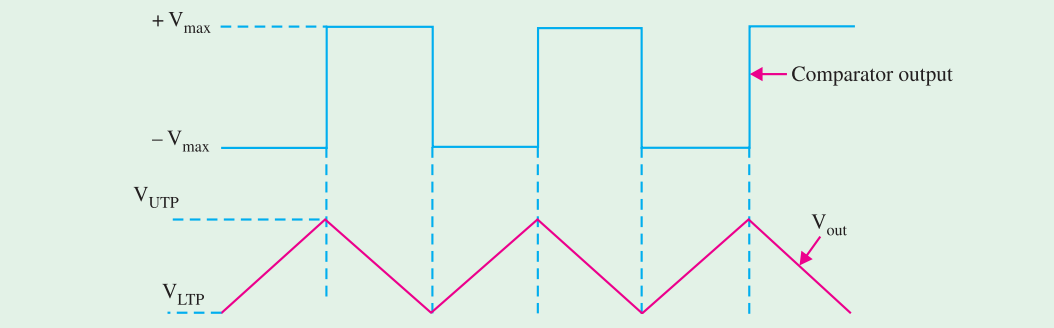


Fig. 68.26

Since the comparator produces square-wave output, therefore, circuit shown in Fig. 68.25 can be used as both a triangular-wave oscillator and a square-wave oscillator. Devices of this type are commonly known as function generator because they produce more than one output function. The output amplitude of the square-wave is set by the output swing of the comparator. While the output amplitude of the triangular-wave is set by the resistors R_2 and R_3 by establishing the UTP and LTP voltages according to the following formulas :

$$V_{UTP} = + V_{max}\left(\frac{R_3}{R_2}\right)$$

$$V_{LTP} = - V_{max}\left(\frac{R_3}{R_2}\right)$$

It may be noted that the comparator output levels, $+ V_{max}$ are equal. The frequency of both waveforms depend on the $R_1 C$ time constant as well as the amplitude-setting resistors R_2 and R_3. By varying R_1, the frequency of oscillation can be adjusted without changing the output amplitude.

$$f = \frac{1}{4 R_1 C}\left(\frac{R_2}{R_3}\right)$$

68.21. A Voltage-Controlled Sawtooth Oscillator (VCO)

The voltage-controlled oscillator (VCO) is an oscillator whose frequency can be changed by a variable dc control voltage. The VCOs can be either sinusoidal or nonsinusoidal. One way to build a voltage-controlled sawtooth oscillator is shown in Fig. 68.27 (*a*). This circuit makes use of an *OP-AMP* integrator that uses switching device called programmable unijunction transistor (abbreviated as PUT) in parallel with the feedback capacitor to terminate each ramp at a prescribed level and effectively "reset" the circuit.

The programmable unijunction transistor (PUT) is a three terminal device *i.e.*, it has an anode, a cathode and a gate terminal. The gate is always biased positively with respect to the cathode. When the anode voltage exceeds the gate voltage by approximately 0·7 V, the PUT turns on and acts as a forward biased diode. When the anode voltage falls below this level, the PUT turns off. Also, the value of current must be above the holding value to maintain conduction.

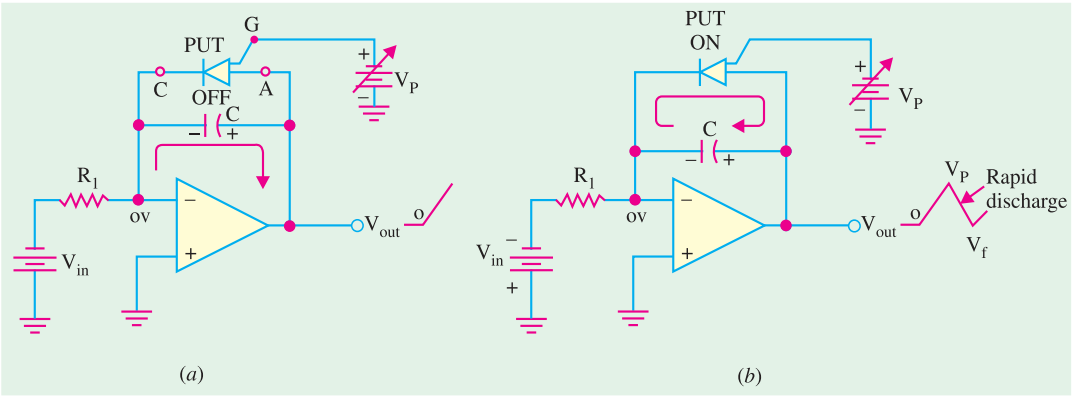


Fig. 68.27

The operation of the circuit may be explained as below. The negative dc input voltage, $-V_{IN}$, produces a positive-going ramp on the output. During the time that ramp is increasing the circuit acts as a regular integrator. When the ramp voltage (*i.e.*, voltage at PUT anode) exceeds the gate voltage by 0·7 V, the PUT turns on. This forces the capacitor to discharge rapidly as shown in Fig. 68.27 (*b*). However, the capacitor does not discharge completely to zero because of the PUT's forward voltage, V_F. Discharging of the capacitor continues until the current through PUT drops below the holding value. At this point, the PUT turn off and capacitor begins to charge again, thus generating a new output ramp. The cycle repeats and the resulting output is a repetitive sawtooth waveform as shown in the figure. It may be noted that the sawtooth amplitude and period can be adjusted by varying the PUT gate voltage.

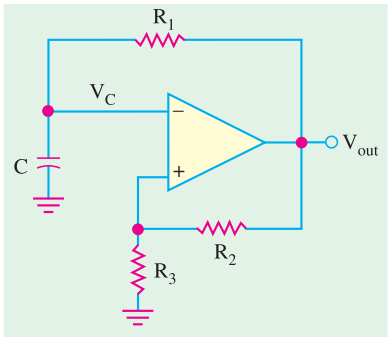


Fig. 68.28

The frequency of smooth curve is given by the relation :

$$f = \frac{|V_{IN}|}{R_1 C}\left(\frac{1}{V_P - V_F}\right)$$

It is evident from the above equation that the frequency depends upon the time constant "$R_1 C$" of the integrator and the peak voltage set by the PUT. The time period of the sawtooth wave is the reciprocal of the frequency, *i.e.*,

$$T = \frac{V_P - V_F}{|V_{IN}|/R_1 C}$$

68.22. A Square-wave Relaxation Oscillator

Fig. 1.28 shows the circuit of a basic relaxation oscillator. Its operation depends upon charging and discharging of a capacitor. Notice that the *OP-AMP*'s inverting input is the capacitor voltage and the noninverting input is a portion of the output fed-back through resistors R_2 and R_3.

When the circuit is first turned on, the capacitor is uncharged. Because of this, the inverting input is at 0 V. This makes the output a positive maximum, and capacitor begins to charge towards V_{out} through R_1. When the capacitor voltage reaches a value equal to the feedback voltage on the noninverting input, the switches to the maximum negative stage. At this point, the capacitor begins to discharge from $+V_f$ toward $-V_f$. When the capacitor voltage reaches $-V_f$ the *OP-AMP* switches back to the maximum positive state. This action continues to repeat and a square wave input is obtained as

shown in Fig. 68.29.

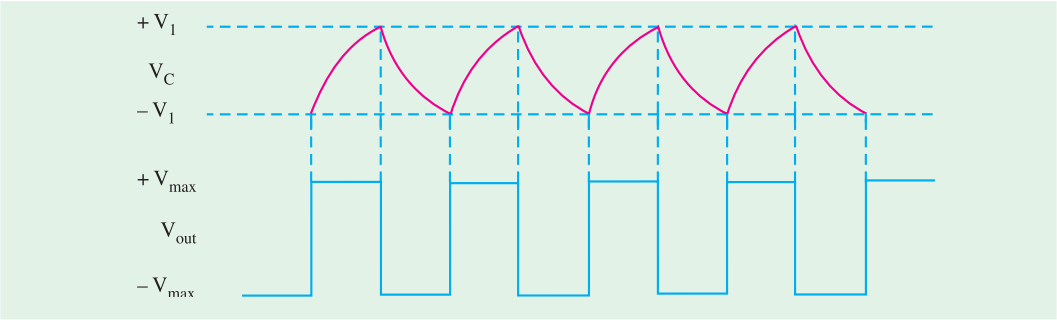


Fig. 68.29

68.23. High-impedance Voltmeter

Fig. 68.30 shows the circuit of a high impedance voltmeter. In such a circuit, the closed loop-gain depends on the internal resistance of the meter R_M. The input voltage will be amplified and the output voltage will cause a proportional current to flow through the meter. By adding a small series potentiometer in the feedback loop, the meter can be calibrated to provide a more accurate reading.

The high input impedance of the *OP-AMP* reduces the circuit loading that is caused by the use of the meter. Although this type of circuit would cause some circuit loading, it would be much more accurate than a VOM (Volt-Ohm-Meter) with an input impedance of 20 K/V.

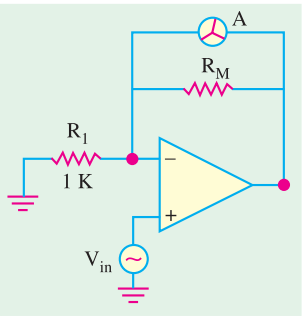


Fig. 68.30

68.24. Active Filters

We have already discussed in Chapter 10, about tuned amplifiers. Such amplifiers are designed to amplify only those frequencies that are within certain range. As long as the input signal is within the specified range, it will be amplified. If it goes outside of this frequency range, amplification will be drastically reduced. The tuned amplifier circuits using *OP-AMP* are generally referred to as active filters. Such circuits do not require the use of inductors. The frequency response of the circuit is determined by resistor and capacitor values.

A filter circuit can be constructed using passive components like resistors and capacitors. But an active filter, in addition to the passive components makes use of an *OP-AMP* as an amplifier. The amplifier in the active filter circuit may provide voltage amplification and signal isolation or buffering.

There are four major types of filters namely, low-pass, filter-high-pass filter, and band-pass filter and band-stop or notch filter. All these four types of filters are discussed one by one in the following pages.

68.25. Low-Pass Filter

A filter that provides a constant output from dc up to a cut-off frequency (f_{OH}) and then passes no signal above that frequency is called an ideal low-pass filter. The ideal response of a low-pass filter is as shown in Fig. 68.31 (*a*). Notice that the response shows that the filter has a constant output (indicated by a horizontal line AB) from dc or zero frequency up to a cutt-off frequency (f_{OH}). And beyond f_{OH}, the output is zero as indicated by the vertical line 'BC' in the figure.

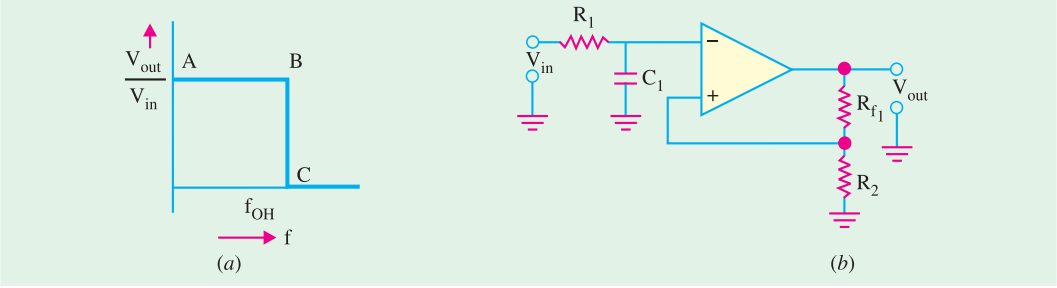


Fig. 68.31

Fig. 68.31 (*b*) shows the circuit of a low-pass active filer using a single resistor and capacitor. Such a ricuit is also referred to as first-order (or single-pole) low-pass filter. It is called first-order because it makes use of a single resistor and a capacitor. The response of such a first-order low pass filter is as shown in Fig. 68.32. Notice that the response below the cut-off frequency (f_{OH}) shows a constant gain (indicated by a horizontal line 'AB')

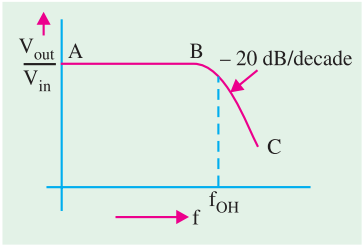


Fig. 68.32

However, beyond the cut-off frequency, the gain does not reduce immediately to zero as expected in Fig. 68.31 (*a*) but reduces with a slope of 20 dB/decade (means that the output voltage reduces by a factor of 100 when the fequency increases by a factor of 10). The voltage gain for a low-pass filter below the cut-off frequency (f_{OH}) is given by the relation.

$$A_v = 1 + \frac{R_3}{R_1}$$

And the cut-off frequency is determined by the relation

$$f_{OH} = \frac{1}{2\pi R_1 C_1}$$

It is possible to connect two sections of the filter together as shown in Fig. 68.33 (*a*). Such a circuit is called second-order (or two-pole) low pass filter. Fig. 68.33 configuration of the second-order low-pass filter.

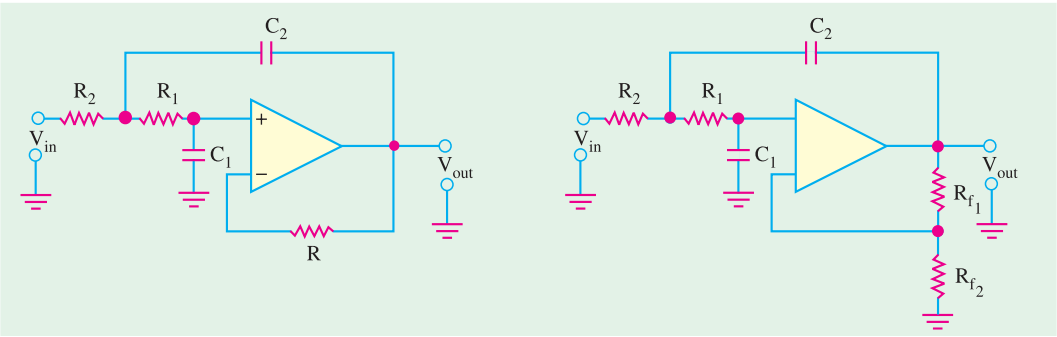


Fig. 68.33

Each circuit shown in Fig. 68.33 has two RC circuits, $R_1 - C_1$ and $R_2 - C_2$. As the operating frequency increases beyond f_2, each circuit will be dropping the closed-loop gain by 20 dB, giving a total roll-off rate of 40 dB/decade when operated above f_2. The cut-off frequency for each of the circuit is given by,

$$f_2 = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}}$$

68.26. High-pass Filter

As a matter of a fact, there is very little difference between the high-pass filter and the low-pass filter. Fig. 68.34 (*a*) shows the circuit of a first order (or single-pole) high-pass filter and Fig. 68.34 (*b*), the circuit of a second-order (or two-pole) high-pass filter. Notice that the only thing that has changed is the position of the capacitors and resistors. The value of cut-off frequencies f_1 and f_2 is obtained by using the same equations we used for low-pass filter.

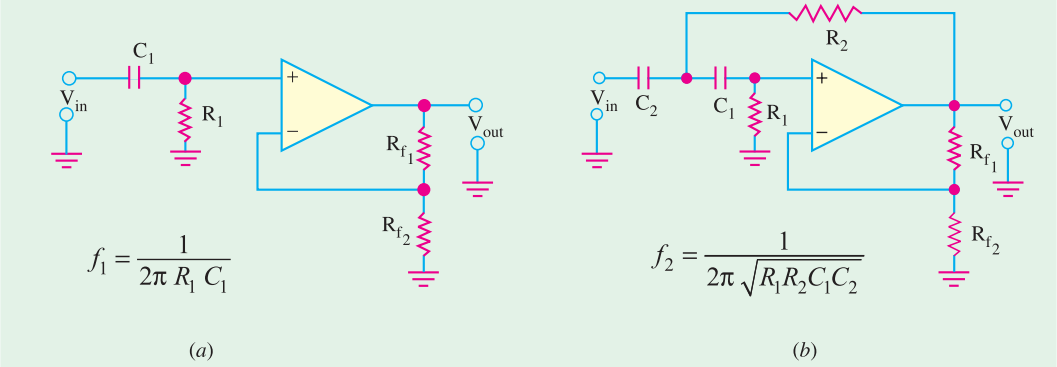


Fig. 68.34

Fig. 68.35 shows the gain versus frequency response of a high-pass filter. Notice that the solid line indicates the ideal response while the dashed line, corresponds to the actual response of the filter circuit. The ideal curve indicates that the filter has a zero output for the frequencies below f_{OL} (indicated by the line 'AB'). And beyond f_{OL}, it has a constant output. The actual response curve may correspond to the roll-off gain by 20 dB/decade for first order to 40 dB/decade for second-order low-pass filters.

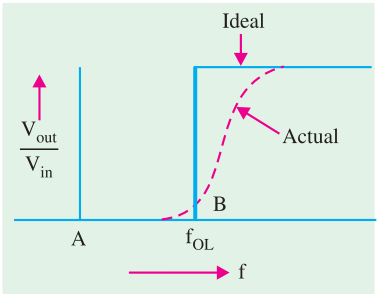


Fig. 68.35

68.27. Band-pass Filters

A band-pass filter is the one that is designed to pass all frequencies within its bandwith. A simple way to construct a band-pass filter is to cascade a low-pass filters and a high-pass filter as shown in Fig. 68.36. The first stage of the band-pass filter will pass all frequencies that are below its cutt-off value, f_2. All the frequencies passed by the first stage will head into the second stage. This stage will pass all frequencies above its value of f_1. The result of this circuit action is as shown in Fig. 68.37. Note that the only frequencies that all will pass through the amplifier are those that fall within the pass band of both amplifiers. The values of f_1 and f_2 can be obtained by using the relations, $1/2\pi R_1 C_1$ and, $1/2\pi R_2 C_2$. Then bandwith,

$$BW = f_2 - f_1$$

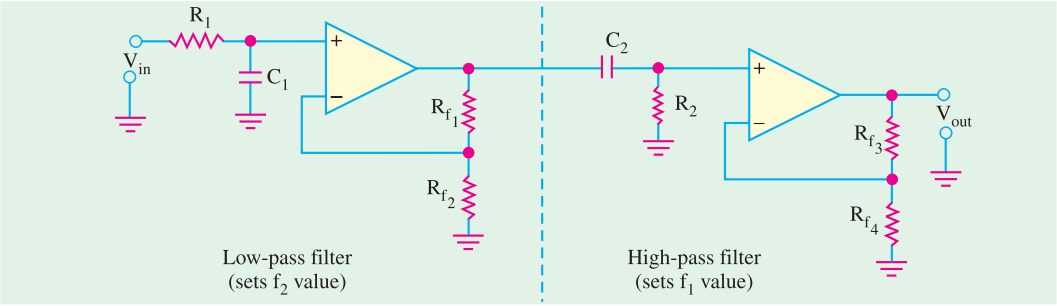


Fig. 68.36

And the centre frequency,

$$f_0 = f_1 \cdot f_2$$

The Quality-factor (or Q-factor) of the band-pass filter circuit.

$$Q = \frac{f_0}{BW}$$

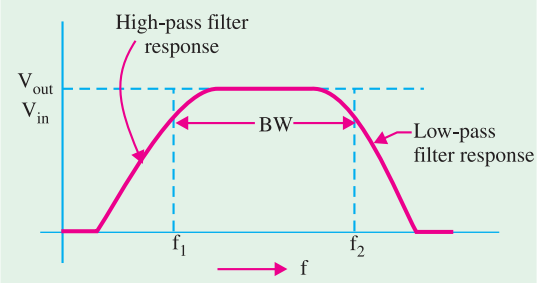


Fig. 68.37

68.28. Notch Filter

The notch filter is designed to block all frequencies that fall within its bandwith Fig. 68.38 (*a*) shows a block diagram and 68.38 (*b*), the gain versus frequency response curve of a multistage notch filter.

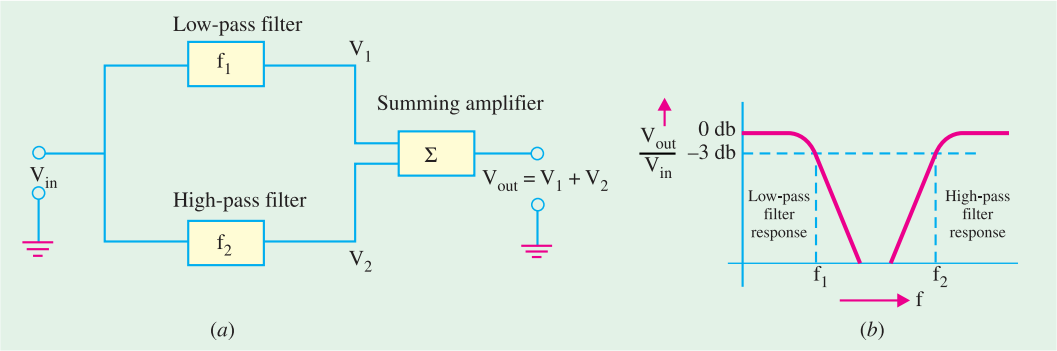


Fig. 68.38

The block diagram shows that the circuit is made up of a high-pass filter, a low-pass filter and a summing amplifier. The summing amplifier produces an output that is equal to a sum of the filter output voltages. The circuit is designed in such a way so that the cut-off frequency, f_1 (which is set by a low-pass filter) is lower in value than the cut-off frequency, f_2 (which is set by high-pass filter). The gap between the values of f_1 and f_2 is the bandwidth of the filter.

When the circuit input frequency is lower than f_1, the input signal will pass through low-pass filter to the summing amplifier. Since the input frequency is below the cut-off frequency of the high-pass filter, v_2 will be zero. Thus the output from the summing amplifier will equal the output from the low-pass filter. When the circuit input frequency is higher than f_2, the input signal will pass through the high-pass filter to the summing amplifier. Since the input frequency is above the cut-off frequency of the low-pass filter, v_1 will zero. Now the summing amplifier output will equal the output from the high-pass filter.

It is evident from the above discussion that frequencies below f_1 and those above f_2, have been passed by the notch filter. But when the circuit frequency between f_1 and f_2, neither of the filters will produce an output. Thus v_1 and v_2 will be both zero and the output from the summing amplifier will also be zero.

The frequency analysis of the notch filter is identical to the band-pass filter. Fisrt, determine the cut-off frequencies of the low-pass and the high-pass filters. Then using these calculated values, determine the bandwidth, center frequency and Q values of the circuit.

Tutorial Problems No. 68

1. Determine the maximum allowable value of v_{in} for the circuit shown in Fig. 68.39. Assume that the gain of the amplifier is 200. **(40 mV)**

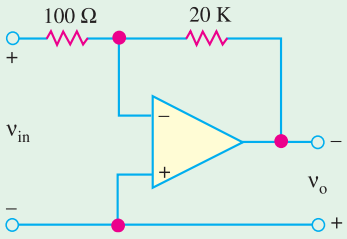


Fig. 68.39

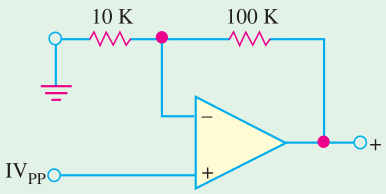


Fig. 68.40

2. For the non-inverting amplifier shown in Fig. 68.40, find the voltage gain of the circuit. **(11)**
3. Determine the value of output voltage for the summing circuit shown in Fig. 68.41. **(3·52 V)**

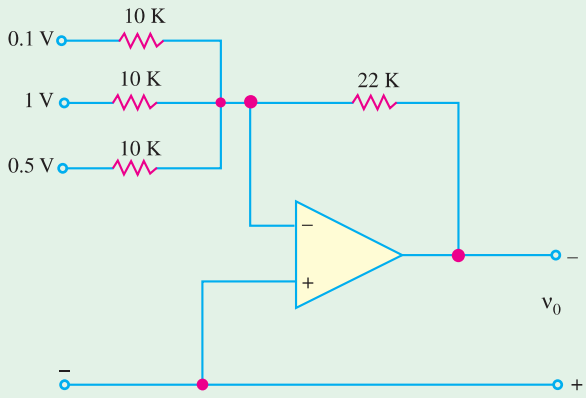


Fig. 68.41

OBJECTIVE TESTS – 68

1. An OP-AMP can be classified as ..... amplifier.
   (*a*) linear
   (*b*) low-r_{in}
   (*c*) positive feedback
   (*d*) *RC*-coupled.
2. An ideal OP-AMP has
   (*a*) infinite A_v
   (*b*) infinite R_i
   (*c*) zero R_0
   (*d*) all the above.
3. OP-AMP have become very popular in industry mainly because
   (*a*) they are dirt cheap
   (*b*) their external characteristics can be changed to suit any application
   (*c*) of their extremely small size
   (*d*) they are available in different packages.
4. Since input resistance of an ideal OP-AMP is infinte
   (*a*) its output resistance is zero
   (*b*) its output voltage becomes independent of load resistance
   (*c*) its input current is zero
   (*d*) it becomes a current-controlled device.
5. The gain of an actual OP-AMP is around
   (*a*) 1,000,000 (*b*) 1000
   (*c*) 100 (*d*) 10,000
6. When an input voltage of 1 V is applied to an OP-AMP having $A_v = 10^6$ and bias supply of + 15 V, the output voltage available is
   (*a*) 15×10^6 V (*b*) 10^6 V
   (*c*) 15 μV (*d*) 15 V.

7. An inverting amplifier has $R_f = 2$ M and $R_1 = 2$ K, Its scale factor is
(*a*) 1000 (*b*) −1000
(*c*) 10^{-3} (*d*) -10^{-3}

8. In an inverting amplifier, the two input terminals of an ideal OP-AMP are at the same potential because
(*a*) the two input terminals are directly shorted internally
(*b*) the input impedance of the OP-AMP is infinity
(*c*) common-mode rejection ratio is infinity
(*d*) the open-loop gain of the OP-AMP is infinity.

9. The open-loop gain of an operational amplifier is 10^5. An input signal of 1 mV is applied to the inverting input with the non-inverting input connected to the ground. The supply voltages are ±10 V. The output of the amplifier will be
(*a*) + 100 V
(*b*) – 100 V
(*c*) + 10 V (approximately)
(*d*) – 10 V (approximately)

10. The output voltage of the circuit shown in Fig. 68.42 is

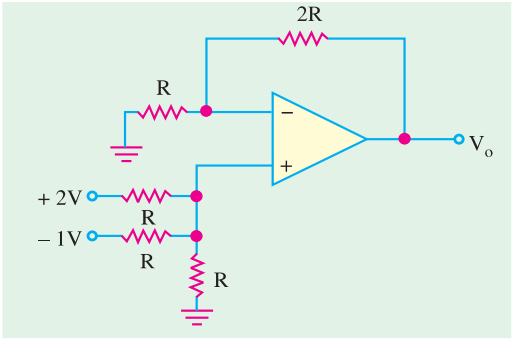


Fig. 68.42

(*a*) 1·0 V (*b*) 1·5 V
(*c*) 2·0 V (*d*) 2·5 V

11. In the circuit shown in Fig. 68.43, the value of output, v_0 is
(*a*) + 3 V (*b*) – 3 V
(*c*) – 7 V (*d*) + 7 V

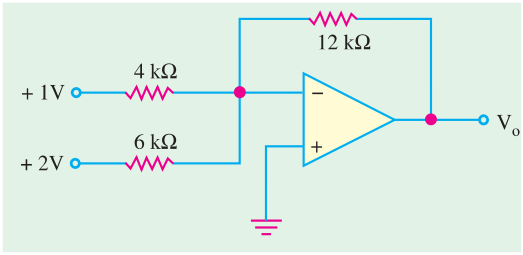


Fig. 68.43

12. When in a negative scaler, both R_1 and R_f are reduced to zero, the circuit functions as
(*a*) integrator (*b*) subtractor
(*c*) comparator (*d*) unity follower.

13. The two input terminals of an OP-AMP are known as
(*a*) positive and negative
(*b*) differential and non-differential
(*c*) inverting and non-inverting
(*d*) high and low.

14. The purpose of comparator is to
(*a*) amplify an input voltage
(*b*) detect the occurrence of a changing input voltage
(*c*) maintain a constant output when the dc input voltage changes.
(*d*) produce a change in input voltage when an input voltage equals the reference voltage.

15. The OP-AMP comparator circuit uses
(*a*) positive feedback
(*b*) negative beedback
(*c*) regenerative feedback
(*d*) no feedback

16. The feedback path in an OP-AMP integrator consists of
(*a*) a resistor
(*b*) a capacitor
(*c*) a resistor and a capacitor in series
(*d*) a resistor and a capacitor in parallel.

17. The feedback path in an OP-AMP differentiator consists of
(*a*) a resistor
(*b*) a capacitor
(*c*) a resistor and a capacitor in series
(*d*) a resistor and a capacitor in parallel.

ANSWERS

1. *(a)* **2.** *(d)* **3.** *(b)* **4.** *(c)* **5.** *(a)* **6.** *(d)* **7.** *(b)* **8.** *(b)* **9.** *(b)* **10.** *(c)* **11.** *(d)* **12.** *(d)*
13. *(c)* **14.** *(d)* **15.** *(d)* **16.** *(b)* **17.** *(a)*

GO To FIRST

CONTENTS

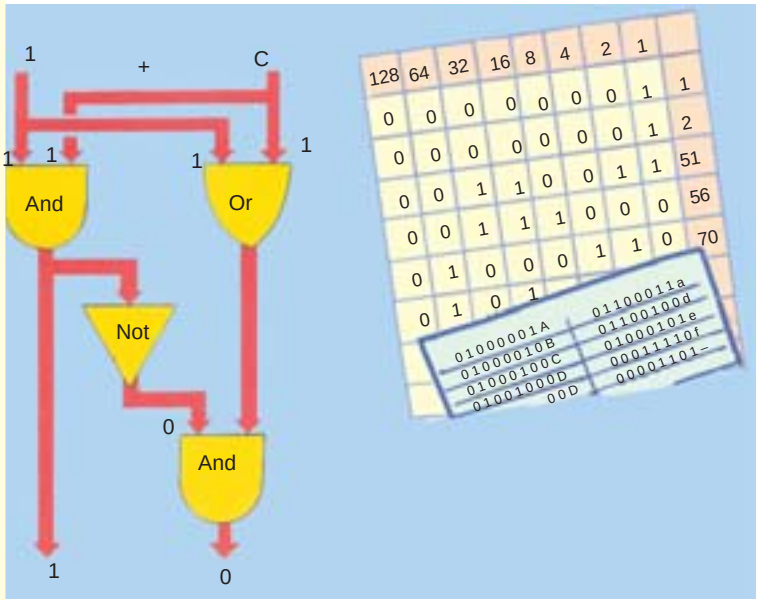
CHAPTER 69

NUMBER SYSTEMS AND CODES

Learning Objectives

- Number Systems
- The Decimal Number System
- Binary Number System
- Binary to Decimal Conversion
- Binary Fractions
- Double-Dadd Method
- Decimal to Binary Conversion
- Shifting the Place Point
- Binary Operations
- Binary Addition
- Binary Subtraction
- Complement of a Number
- 1's Complemental Subtraction
- 2's Complemental Subtraction
- Binary Multiplication
- Binary Division
- Shifting a Number to Left or Right
- Octal Number System
- Octal to Decimal Conversion
- Decimal to Octal Conversion
- Binary to Octal Conversion
- Octal to Binary Conversion
- Binary to Hexadecimal Conversion
- Decimal to Hexadecimal Conversion
- Hexadecimal to Decimal Conversion
- Digital Coding
- Binary Coded Decimal (BCD) Code
- Octal Coding
- Hexadecimal Coding
- Excess-3 Code
- Gray Code
- Excess-3 Gray Code
- ASCII Code

CONTENTS



Logic gates use switches that control the flow of an electrical current. '1' is 'true' and '0' is 'false'. The columns in the binary system wave the values 1, 2, 4, 8, 16, 32 and so no.

69.1. Number Systems

The number systems are used quite frequently in the field of digital electronics and computers. However the type of number system used in computers could be different at different stages of the usage. For example, when a user key-in some data into the computer, s(he), will do it using decimal number system *i.e.* the system we all have used for several years for doing arithmetic problems. But when the information goes inside the computer, it needs to be converted to a form suitable for processing data by the digital circuitry. Similarly when the data has to be displayed on the monitor for the user, it has to be again in the decimal number system. Hence the conversion from one number system to another one is an important topic to be understood.

There are four systems of arithmetic which are often used in digital circuits. These systems are:

1. **Decimal**—it has a base (or radix) of 10 *i.e.* it uses 10 different symbols to represent numbers.
2. **Binary**—it has a base of 2 *i.e.* it uses only two different symbols.
3. **Octal**—it has a base of 8 *i.e.* it uses eight different symbols.
4. **Hexadecimal**—it has a base of 16 *i.e.* it uses sixteen different symbols.

All these systems use the same type of **positional notation** except that

— decimal system uses powers of 10
— binary system uses power of 2
— octal system uses powers of 8
— hexadecimal system uses powers of 16.

Decimal numbers are used to represent quantities which are outside the digital system. Binary system is extensively used by digital systems like digital computers which operate on binary information. Octal system has certain advantages in digital work because it requires less circuitry to get information into and out of a digital system. Moreover, it is easier to read, record and print out octal numbers than binary numbers. Hexadecimal number system is particularly suited for microcomputers.

69.2. The Decimal Number System

We will briefly recount some important characteristics of this more-familiar system before taking up other systems. This system has a base of 10 and is a **position-value system** (meaning that value of a digit depends on its **position**). It has following characteristics :

(*i*) Base or Radix

It is defined as ***the number of different digits which can occur in each position in the number system.***

The decimal number system has a base of 10 meaning that it contains ten unique symbols (or digits). These are : 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. Any one of these may be used in each position of the number.

Incidentally, it may be noted that we call it a decimal (10's) system although ***it does not have a distinct symbol of 10.*** As is well-known, it expresses 10 and any number above 10 as a combination of its ten unique symbols.

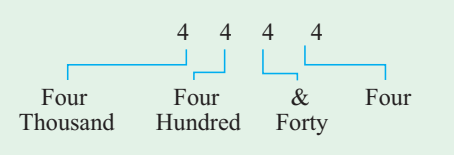


Fig. 69.1

(*ii*) Position Value

The absolute value of each digit is fixed but its **position value** (or place value or weight) is determined by its ***position*** in the overall number. For example, position value of 3 in 3000 is not the same as in 300. Also, position value of each 4 in the number 4444 is different as shown in Fig. 69.1.

Similarly, the number 2573 can be broken down as follows :

$2573 = 2 \times 10^3 + 5 \times 10^2 + 7 \times 10^1 + 3 \times 10^0$

It will be noted that in this number, 3 is the **least significant digit** (*LSD*) whereas 2 is the **most significant digit** (*MSD*).

Again, the number 2573.469 can be written as

$2573.469 = 2 \times 10^3 + 5 \times 10^2 + 7 \times 10^1 + 3 \times 10^0 + 4 \times 10^{-1} + 6 \times 10^{-2} + 9 \times 10^{-3}$

It is seen that position values are found by raising the base of the number system (*i.e.* 10 in this case) to the power of the position. Also, powers ***are numbered to the left of the decimal point starting with 0 and to the right of the decimal point starting with –1.***

69.3. Binary Number System

Like decimal number (or denary) system, it has a radix and it also uses the same type of position value system.

(*i*) Radix

Its base or radix is **two** because it uses only two digits 0 and 1 (the word 'binary digit' is contracted to **bit**). All binary numbers consist of a string of 0s and 1s. Examples are 10, 101 and 1011 which are read as one-zero, one-zero-one and one-zero-one-one to avoid confusion with decimal numbers. Another way to avoid confusion is to add a subscript of 10 for decimal numbers and of 2 for binary numbers as illustrated below.

10_{10}, 101_{10}, 5742_{10} —decimal number and10_2, 101_2, 110001_2 — binary numbers.



Binary numbers represent all values within computers

It is seen that the subscript itself is in decimal. It may be noted that binary numbers need ***more places for counting because their base is small***

(*ii*) Position Value

Like the decimal system, binary system is also positionally-weighted. However, in this case, the position value of each bit corresponds to some power of 2. In each binary number, the value increases in powers of 2 starting with 0 to the left of the binary point and decreases to the right of the binary point starting with power of –1. The position value (or weight) of each bit alongwith a 7-bit binary number 1101.011 is shown in Fig. 69.2.

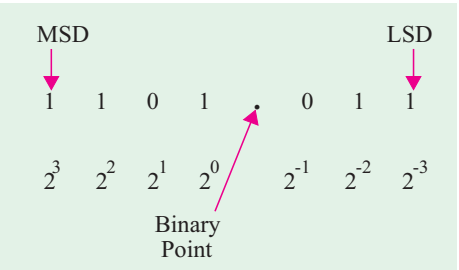


Fig. 69.2

As seen, the fourth bit to the left of binary point **carries the maximum weight** (*i.e.* it has the highest value) and is called most significant digit (*MSD*). Similarly, the third bit to the right of the binary point is called least significant digit (*LSD*). The decimal equivalent of the binary number may be found as under

$1101.011_2 = (1 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0) + (0 \times 2^{-1}) + (1 \times 2^{-2}) + (1 \times 2^{-3})$

$= 8 + 4 + 0 + 1 + 0 + \frac{1}{4} + \frac{1}{8} = 13.375_{10}$

As stated earlier, position values of different bits are given by **ascending powers of 2** to the **left** of binary point and by **descending power** of 2 to the **right** of binary point. The different digit positions of a given binary number have the following decimal weight (Fig. 69.3)

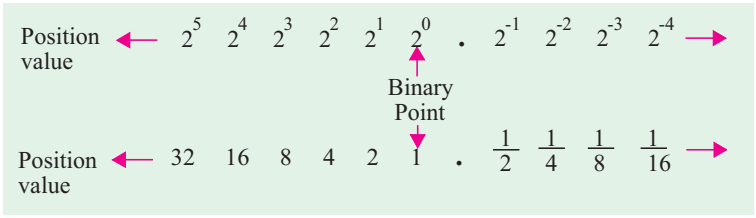


Fig. 69.3

Binary numbers are used extensively by all digital systems primarily due to the nature of electronics itself. The bit 1 may be represented by a saturated (fully-conducting) transistor, a light turned ON, a relay energised or a magnet magnetised in a particular direction. The bit 0, on the other hand, can be represented as a cut-off transistor, a light turned OFF, a relay de-energised or a magnet magnetised in the opposite direction. In such cases, there are only two values which a device can assume.

69.4. Binary to Decimal Conversion

Following procedure should be adopted for converting a given binary integer (whole number) into its equivalent decimal number :

Step 1. Write the binary number *i.e.* all its bits in a row.

Step 2. Directly under the bits, write 1, 2, 4, 8, 16, ...starting from *right to left*.

Step 3. Cross out the decimal weights which lie under 0 bits.

Step 4. Add the remaining weights to get the decimal equivalent.

Example 69.1. *Convert 11001_2 to its equivalent decimal number.*

Solution. The four steps involved in the conversion are as under

Step 1.	1	1	0	0	1
Step 2.	16	8	4	2	1
Step 3.	16	8	~~4~~	~~2~~	1
Step 4.	16 + 8 + 1 = 25		∴		$11001_2 = 25_{10}$

It is seen that the number contains 1 sixteen, one eight, 0 four's, 0 two's and 1 one. Certain decimal and binary equivalent numbers are tabulated below in Table No. 69.1

Table No. 69.1

Decimal	*Binary*	*Decimal*	*Binary*	*Decimal*	*Binary*
1	1	11	1011	21	10101
2	10	12	1100	22	10110
3	11	13	1101	23	10111
4	100	14	1110	24	11000
5	101	15	1111	25	11001
6	110	16	10000	26	11010
7	111	17	10001	27	11011
8	1000	18	10010	28	11100
9	1001	19	10011	29	11101
10	1010	20	10100	30	11110

69.5. Binary Fractions

Here, procedure is the same as for binary integers except that the following weights are used for different bit positions .

●		2^{-1}	2^{-2}	2^{-2}	2^{-4}	→
↑		$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	→

Binary Point

Example 69.2. *Convert the binary fraction 0.101 into its decimal equivalent.*

Solution. The following four steps will be used for this purpose.

Step 1. 0 ● 1 0 1

Step 2. $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{8}$

Step 3. $\frac{1}{2}$ ~~$\frac{1}{4}$~~ $\frac{1}{8}$

Step 4. $\frac{1}{2} + \frac{1}{8} = 0.625$

$\therefore$ $0.101_2 = 0.625_{10}$

Example 69.3. *Find the decimal equivalent of the 6-bit binary number 101.101_2.*

Solution.

1	0	1	1 ●	0	1	
4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	
4	~~2~~	1	$\frac{1}{2}$	~~$\frac{1}{4}$~~	$\frac{1}{8}$	$= 5 + \frac{1}{2} + \frac{1}{8} = 5.625$

$\therefore$ $101.101_2 = 5.625_{10}$

69.6. Double-Dadd Method

This method of converting binary integers into decimal equivalents is much simpler and quicker than the method given in Art. 69.4 especially in the case of large numbers. Following three steps are involved :

1. Double the first bit to the extreme left and add this doubled value to the next bit on the right.
2. Double the sum obtained and add the doubled value to the next bit.
3. Continue step 2 until the last bit has been added to the previously-doubled sum.

The conversion of 11001_2 is shown in Fig. 69.4. It is seen that $11001_2 = 25_{10}$

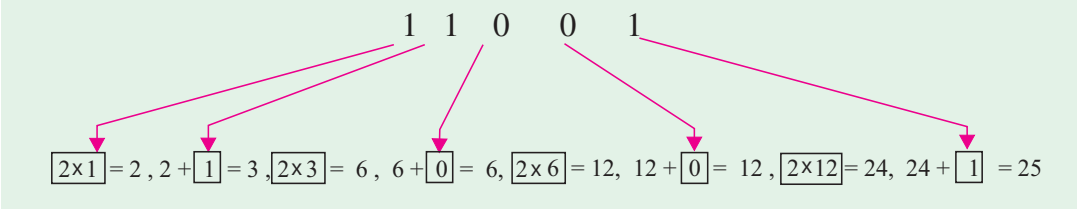


Fig. 69.4

Using double-dadd method, let us convert 111010_2 into its binary equivalent.

1. $2 \times 1 = 2$, add next bit 1 so that $2 + 1 = 3$
2. $2 \times 3 = 6$, add next bit 1 so that $6 + 1 = 7$
3. $2 \times 7 = 14$, add next bit 0 so that $14 + 0 = 14$
4. $2 \times 14 = 28$, add next bit 1 so that $28 + 1 = 29$

5. $2 \times 29 = 58$, add next bit 0 so that $58+0 = 58$

$\therefore \quad 111010_2 = 58_{10}$

69.7. Decimal to Binary Conversion

(*a*) Integers

Such conversion can be achieved by using the so-called **double-dabble method.** It is also known as **divide-by-two** method. In this method, we progressively divide the given decimal number by 2 and write down the remainders after each division. These remainders taken in the *reverse* order (*i.e.* from bottom-to-top) form the required binary number. As an example, let us convert 25_{10} into its binary equivalent.

$25 \div 2 = 12$ + remainder of 1 — TOP

$12 \div 2 = 6$ + remainder of 0

$6 \div 2 = 3$ + remainder of 0

$3 \div 2 = 1$ + remainder of 1

$1 \div 2 = 0$ + remainder of 1

$\therefore \quad 25_{10} = 11001_2$ — BOTTOM

The above process may be simplified as under :

Successive Divisions		*Remainders*
2) 2 5		
2) 1 2	———	1
2) 6	———	0
2) 3	———	0
2) 1	———	1
2) 0	———	1

Reading the remainders from bottom to top, we get $25_{10} = 11001_2$

It may also be put in the following form :

$25 \div 2 = 12 + 1$

$12 \div 2 = 6 + 0$

$6 \div 2 = 3 + 0$

$3 \div 2 = 1 + 1$

$1 \div 2 = 0 + 1$

$\therefore$ decimal 25 = 1 1 0 0 1 binary

(*b*) Fractions

In this case, **Multiply-by-two** rule is used *i.e.* we multiply each bit by 2 and record the carry in the integer position. These carries taken in the **forward** (top-to-bottom)direction gives the required binary fraction.

Let us convert 0.8125_{10} into its binary equivalent.

$0.8125 \times 2 = 1.625 = 0.625$ with a carry of 1

$0.625 \times 2 = 1.25 = 0.25$ with a carry of 1

$0.25 \times 2 = 0.5 = 0.5$ with a carry* of 0

$0.5 \times 2 = 1.0 = 0.0$ with a carry of 1

$\therefore \quad 0.8125_{10} = 0.1101_2$

* It is so because there is 0 at the interger position *i.e.* to the left of the decimal point.

Please note that ***we have to add the binary point from our side.*** Let us now convert 0.77_{10} into its binary equivalent.

$0.77 \times 2 = 1.54 = 0.54$ with a carry of 1
$0.54 \times 2 = 1.08 = 0.08$ with a carry of 1
$0.08 \times 2 = 0.16 = 0.16$ with a carry of 0
$0.16 \times 2 = 0.32 = 0.32$ with a carry of 0
$0.32 \times 2 = 0.64 = 0.64$ with a carry of 0
$0.64 \times 2 = 1.28 = 0.28$ with a carry of 1

We may stop here but the answer would be approximate. $\therefore 0.77_{10} \cong .110001_2$

Example 69.4. *Convert 25.625_{10} into its binary equivalent.*

Solution. We will do the conversion in two steps ***(i)*** first for the integer and ***(ii)*** then for the fraction.

(*a*) Integer

$25 \div 2 = 12 + 1$
$12 \div 2 = 6 + 0$
$6 \div 2 = 3 + 0$
$3 \div 2 = 1 + 1$
$1 \div 2 = 0 + 1$
$\therefore \quad 25_{10} = 11001_2$

(*b*) fraction

$0.625 \times 2 = 1.25 = 0.25 + 1$
$0.25 \times 2 = 0.5 = 0.5 + 0$
$0.5 \times 2 = 1.0 = 0.0 + 1$
$\therefore \quad 0.625_{10} = 0.101_2$

Considering the complete number, we have $25.625_{10} = 11001.101_2$

Obviously, binary system needs more bits to express the same number than decimal system.

69.8. Shifting the Place Point

In a decimal number if the decimal point is moved one place to the right, **the number is multiplied by 10**. For example, when decimal point in 7.86 is shifted one place to the right, it becomes 78.6 *i.e.* it increases the value of the number 10 times. Moving the decimal point one place to the left reduces its value to one-tenth.

In binary numbers, shifting the binary point by one place multiplies or divides the number by 2. For example, 111.0_2 is equal to 7_{10} but 1110.0_2 is 14_{10}. As seen, 7 is doubled to 14 by moving the binary point one place to the right.

Similarly, 11.1_2 is $(2 + 1 + \frac{1}{2}) = 3.5_{10}$. Hence, 111.0_2 is halved to 3.5_{10} by moving its binary point one place to the left.

69.9. Binary Operations

We will now consider the following four binary operations :

1. addition **2.** subtraction **3.** multiplication **4.** division

Addition is the most important of these four operations. In fact, by using **'complements'**, subtraction can be reduced to addition. Most digital computers subtract by complements. It leads to reduction in hardware because only adding type of circuits are required. Similarly, multiplication is nothing but repeated addition and, finally, division is nothing but repeated subtraction.

69.10. Binary Addition

Addition is simply the manipulation of numbers for combining physical quantities. For example, in the decimal number system, 2 + 3 = 5 means the combination of • • with • • • to give a total of • • • • •. Addition of binary numbers is similar to the decimal addition.

Following points will help in understanding the rules of binary addition.

1. When 'nothing' is combined with 'nothing', we get nothing.
   Binary representation of the above statement is : 0 + 0 = 0
2. When nothing is combined with •, we get •.
   In binary language 0 + 1 = 1
3. Combining • with nothing, gives •.
   The binary equivalent is 1 + 0 = 1
4. When we combine • with •, we get • •.
   The binary representation of the above is 1 + 1 = 10

It should be noted that the above sum is not 'ten' but 'one-zero' *i.e.* it represents • • and not • • • • • • • • • •. In other words, it is 10_2 which represents decimal 2. ***It is not decimal ten.***

The last rule is often written as 1 + 1 = 0 with a carry of 1

The above rules for binary addition can be summarized as under :

0 + 0 = 0 0 + 1 = 1
1 + 0 = 1 1 + 1 = 0 with a carry of 1 or $=10_2$

It is worth noting that 'carry-overs' are performed in the same manner as in decimal arithmetic. The rules of binary addition could also be expressed in the form of a table as shown below

	0	1
0	0	1
1	1	10_2

or

	0	1
0	0	1
1	1	0

with a carry 1

As an illustration, let us add 101 and 110.

```
  1 0 1     — first column      1 + 0 = 1
+ 1 1 0     — second column     0 + 1 = 1
1 0 1 1     — third column      1 + 1 = 10
```

(*i.e.* 0 with carry 1)

Similarly,

```
    1 carry
  1 1 1     — 1st column        1 + 0 = 1
+ 1 1 0     — 2nd column        1 + 1 = 0   with carry 1
1 1 0 1     — 3rd column        1 + 1 + carry of 1
                                = 10 + 1= 11₂
```

Let us consider one more example :

```
                ←           ←           ←
    1 carry     1 1 carry   1 1 carry   1 1 carry
  1 0 1 1     1 0 1 1     1 0 1 1     1 0 1 1
+ 1 0 0 1   + 1 0 0 1   + 1 0 0 1   + 1 0 0 1
    0           0 0         1 0 0       1 0 1 0 0
```

Hence, we find from the above examples that the only two possible combinations with a carry are :

(a) 1 + 1 = sum of 0 with a carry of 1.
It is binary 10 *i.e.* 10_2 which equals *decimal* 2.

(b) 1 + 1 + carry of 1 = a sum of 1 with a carry of 1.
It equals binary 11 *i.e.* 11_2 or decimal 3.

Example 69.5. *Add* 110011_2 *to* 101101_2 **(Digital Electronics, Bombay Univ. April)**

Solution.

$$\begin{array}{r} 1\,1\,0\,0\,1\,1 \\ 1\,0\,1\,1\,0\,1 \\ \hline 1\,1\,0\,0\,0\,0\,0 \end{array}$$

1. **first column :** 1 + 1 = 0 with a carry of 1. Hence, we put down zero there and carry 1 to the second column.
2. **second column :** 1 + 0 = 1. But combined with carry 1 from first column, it gives 1 + 1 = 0 and a carry of 1. Hence, we put down 0 there and carry 1 further to the third column.
3. **third column :** 0 + 1 = 1. Again, when it is combined with carry of 1 from the second column, we get 1 + 1 = 0 with a carry of 1. Hence, again, we put down 0 and carry 1 to the fourth column.
4. **fourth column :** Here, 0 + 1 = 1. When combined with carry 1 from third column, we get 1 + 1 = 0 with a carry of 1. Hence, we put down 0 there and carry 1 to the fifth column.
5. **fifth column :** Here, 1 + 0 = 1 When combined with the carry 1 from fourth column, we get 1 + 1 = 0 with a carry of 1. Hence, we put down 0 there and carry 1 to the sixth column.
6. **sixth column :** Here, it is a case of 1 + 1 + carry of 1 = 11_2 as stated earlier in (*b*) above.

69.11. Binary Subtraction

It is also performed in a manner similar to that used in decimal subtraction. Because binary system has only two digits, binary subtraction requires more borrowing operations than decimal subtraction. The four rules for binary subtraction are as under:

1. 0 – 0 = 0, 2. 1 – 0 = 1, 3. 1 – 1 = 0,
4. 0 – 1 = 1 with a borrow of 1 from the next column of the minuend

or 10 – 1 = 1

The last result represents • • – • – • which makes sense.

While using Rule 4, it should be borne in mind that borrow reduces the remaining minuend by 1. It means that a borrow will cause a 1 in the next column to the left in the minuend to become 0. If the next column also happens to contain 0, it is changed to a 1 and the succeeding 0s in the minuend are changed to 1s until a 1 is found which is then changed to a 0.

Example 1

Let us subtract 0101_2 from 1110_2. The various steps are explained below :

$$\begin{array}{r} \not{0}\ 1 \text{ borrow} \\ 1\,1\,1\,0 \\ -\,0\,1\,0\,1 \\ \hline 1 \end{array} \qquad \begin{array}{r} \not{0}\ 1 \\ 1\,1\,1\,0 \\ -\,0\,1\,0\,1 \\ \hline 0\,1 \end{array} \qquad \begin{array}{r} \not{0}\ 1 \\ 1\,1\,1\,0 \\ -\,0\,1\,0\,1 \\ \hline 0\,0\,1 \end{array} \qquad \begin{array}{r} \not{0}\ 1 \\ 1\,1\,1\,0 \\ -\,0\,1\,0\,1 \\ \hline 1\,0\,0\,1 \end{array}$$

Explanation

1. In the first column, since we cannot subtract 1 from 0, we borrow 1 from the next column to the **left**. Hence, we put down 1 in the answer and change the 1 of the next left column to a 0.
2. We apply Rule 1 to next column *i.e.* 0 – 0 = 0
3. We apply Rule 3 to the 3rd column *i.e.* 1 – 1 = 0
4. Finally, we apply Rule 2 to the last *i.e.* fourth column it. 1 – 0 = 1

   As a check, it may be noted that talking in terms of decimal numbers, we have subtracted 5 from 14. Obviously, the answer has to be 9 (1001_2).

Example 2.

Let us now try subtracting 0001_2 from 1000_2.

Step 1	*Step 2*	*Step 3*	*Step 4*
1	1 1	0 1 1	0 1 1
1 0 Ø 0	1 Ø Ø 1	1 Ø Ø 1	1 Ø Ø 1
− 0 0 0 1	− 0 0 0 1	− 0 0 0 1	− 0 0 0 1
1	1	1	0 1 1 1

Since there happened to be a 0 in the second column, it was changed to 1. Again, there was a 0 in the third column, so it was also changed to a 1. Finally, we met a 1 in the forth column which was changed to a 0 and the final answer was written down as shown above.

Example 69.6. *Subtract 0111_2 from 1001_2* **(Digital Computations, Punjab Univ. 1991)**

Solution.

1 0 0 1	1st column	:	1 – 1 = 0
− 0 1 1 1	2nd column	:	0 – 1 = 1 with a borrow of 1
0 0 1 0	3rd column	:	1 (after borrow) – 1 = 0
	4th column	:	0 (after borrow) – 0 = 0

Example 69.7. *Subtract 01011_2 from 10110_2* **(Computer Technology, Pune Univ.)**

Solution. Step-wise the solution is as under :

Step 1	*Step 2*	*Step 3*	*Step 4*	*Step 5*
0	0 0	0 0	0 0 0	0 0 0
1 0 1 1 0	1 0 1 1 0	1 0 1 1 0	1 0 1 1 0	1 0 1 1 0
− 0 1 0 1 1	− 0 1 0 1 1	− 0 1 0 1 1	− 0 1 0 1 1	− 0 1 0 1 1
1	1 1	0 1 1	1 0 1 1	0 1 0 1 1

69.12. Complement of a Number

In digital work, two types of complements of a binary number are used for ***complemental subtraction*** :

(*a*) 1's complement

The 1's complement of a binary number is obtained by changing its each 0 into a 1 and each 1 into a 0. It is also called ***radix-minus-one complement.*** For example, 1's complement of 100_2 is 011_2 and of 1110_2 is 0001_2.

(*b*) 2's complement

The 2's complement of a binary number is obtained by adding 1 to its 1's complement.

2's complement = 1's complement + 1

It is also known **as true** complement. Suppose we are asked to find 2's complement of 1011_2. Its 1's complement is 0100_2. Next, add 1 to get 0101_2. Hence, 2's complement of 1011_2 is 0101_2.

The complement method of subtraction reduces **subtraction to an addition process.**

This method is popular in digital computers because

1. only adder ciruits are needed thus simplifying the circuitry,
2. it is easy with digital circuits to get the complements.

69.13. 1's Complemental Subtraction

In this method, instead of subtracting a number, we add its 1's complement to the minuend. The last carry (whether 0 or 1) is then added to get the final answer. The rules for subtraction by 1's complement are as under :

1. compute the 1's complement of the subtrahend by changing all its 1s to 0s and all its 0s to 1s.
2. add this complement to the minuend
3. perform the end-around carry of the last 1 or 0
4. if there is no end-around carry (*i.e.* 0 carry), then the answer must be recomplemented and a negative sign attached to it.
5. if the end-around carry is 1, no recomplementing is necessary.

Suppose we want to subtract 101_2 from 111_2. The procedure is as under :

```
  1 1 1
+ 0 1 0      ← 1's complement of subtrahend 101
-------
  1 0 0 1
      1      ← end-round carry
-------
  0 1 0
```

As seen, we have removed from the addition sum the 1 carry in the last position and added it onto the remainder. It is called ***end-around carry***.

Let us now subtract 1101_2 from 1010_2.

```
  1 0 1 0
+ 0 0 1 0      ← 1's complement of 1101
---------
  1 1 0 0
  NO CARRY
```

As seen, there is no end-around carry in this case. Hence, as per Rule 4 given above, answer must be recomplemented to get 0011 and a negative sign attached to it. Therefore, the final answer becomes—0011.

Finally, consider the complemental subtraction of 1110_2 from 0110_2.

```
  0 1 1 0
+ 0 0 0 1      ← 1's complement of 1110₂
---------
  0 1 1 1
  NO CARRY
```

As seen, there is no carry. However, we may add an extra 0 from our side to make it a 0 carry as shown below.

```
  0 1 1 0
+ 0 0 0 1      ← 1's complement of subtrahend
---------
  0 0 1 1 1
        0      ← end-around carry
---------
  0 0 1 1 1
```

After recomplementing, it becomes 1000. When negative sign is attached, the final answer becomes -1000_2.

Example 69.8. *Using 1's complemental method, subtract 01101_2 from 11011_2.*

Solution.

```
  1 1 0 1 1
+ 1 0 0 1 0      ← 1's complement of subtrahend
-----------
  1 0 1 1 0 1
          1      ← end-around carry
-----------
  0 1 1 1 0
```

Since end-around carry is 1, we take the final answer as it is (Rule 5).

Example 69.9. *Use 1's complement to subtract 11011_2 from 01101_2.*
(Computer Science, Allahabad Univ.)

Solution.
$$\begin{array}{rl} 0\,1\,1\,0\,1 & \\ +\,0\,0\,1\,0\,0 & \leftarrow \text{1's complement of } 11011_2 \\ \hline 1\,0\,0\,0\,1 & \rightarrow -01110 \end{array}$$
NO CARRY

Since there is no final carry, we recomplement the answer and attach a minus sign to get the final answer—01110_2.

69.14. 2's Complemental Subtraction

In this case, the procedure is as under :

1. find the 2's complement of the subtrahend,
2. add this complement to the minuend,
3. drop the final carry,
4. if the carry is 1, the answer is positive and needs no recomplementing,
5. if there is no carry, recomplement the answer and attach minus sign.

Example 69.10. *Using 2's complement, subtract 1010_2 from 1101_2.*

Solution. The 1's complement of 1010 is 0101. The 2's complement is 0101 + 1 = 0110. We will add it to 1101.

$$\begin{array}{rl} 1\,1\,0\,1 & \\ +\;0\,1\,1\,0 & \leftarrow \text{2's complement of } 1010_2 \\ \hline 1\,0\,0\,1\,1 & \end{array}$$
DROP

The final answer is 0011_2.

Example 69.11. *Use 2's complement to subtract 1101_2 from 1010_2.*
(Digital Computations, Punjab Univ. 1992)

Solution. The 1's complement of 1101 is 0010. The 2's complement is 0011.

$$\begin{array}{rl} 1\,0\,1\,0 & \\ +\;0\,0\,1\,1 & \leftarrow \text{2's complement of } 1101_2. \\ \hline 1\,1\,0\,1 & \end{array}$$
NO CARRY

In this case, there is no carry. Hence, we have to recomplement the answer. For this purpose, we first subtract 1 from it to get 1100. Next, we recomplement it to get 0011. After attaching the minus sign, the final answer becomes -0011_2.

Talking in terms of decimal numbers, we have subtracted 13 from 10. Obviously the answer is –3.

69.15. Binary Multiplication

The procedure for this multiplication is the same as for decimal multiplication though it is comparatively much easier. The four simple rules are as under:

1. $0 \times 0 = 0$,
2. $0 \times 1 = 0$,
3. $1 \times 0 = 0$,
4. $1 \times 1 = 1$.

The rules of binary multiplication could be summarized in the form of a table as shown.

$$\begin{array}{r} 0\,1 \\ \hline 0\,0\,0 \\ \hline 1\,0\,1 \end{array}$$

As in the decimal system, the procedure is

1. copy the multiplicand when multiplier digit is 1 but not when it is 0
2. shift as in decimal multiplication
3. add the resulting binary numbers according to the rules of binary addition.

Example 69.12. *Multiply 111_2 by 101_2 using binary multiplication method.*
(Electronics-1, Indore Univ. 1991)

Solution.

```
  1 1 1
× 1 0 1
-------
  1 1 1
0 0 0          — shift left, no add
1 1 1          — shift left and add
-------------
1 0 0 1 1 1
```

Example 69.13. *Multiply 1101_2 by 1100_2.* **(Digital Electronics, Bombay Univ. 1990)**

Solution.

```
    1 1 0 1
  × 1 1 0 0
  ---------
    0 0 0 0
  0 0 0 0
1 1 0 1
1 1 0 1
---------------
1 0 0 1 1 1 0 0
```

Example 69.14. *Multiply 1111 by 0111_2.*

Solution. This example has been included for the specific purpose of explaining how to handle the addition if multiplication results in ***columns with more than two 1s***.

1. Result of the first column is 1.
2. In the second column, addition of $1 + 1 = 10_2$. Hence, we put down 0 there and carry 1 to the third column.
3. In the third column, $1 + 1 + 1 + 1 = 100_2$ (decimal 4). We keep one 0 there, put the second 0 in fourth column and pass on 1 to the fifth column.
4. In the fourth column, $1 + 1 + 1 + 0 = 11$, (decimal 3). Hence, one 1 is kept there and the other 1 is passed on to the fifth column.
5. In the fifth column, $1 + 1 + 1 + 1 = 100_2$ (decimal 4). Again, one 0 is retained there, second 0 is passed on to the sixth column and 1 to the seventh column.
6. In the sixth column, $1 + 0 = 1$.
7. The seventh column already has 1 given by the addition of the fifth column.

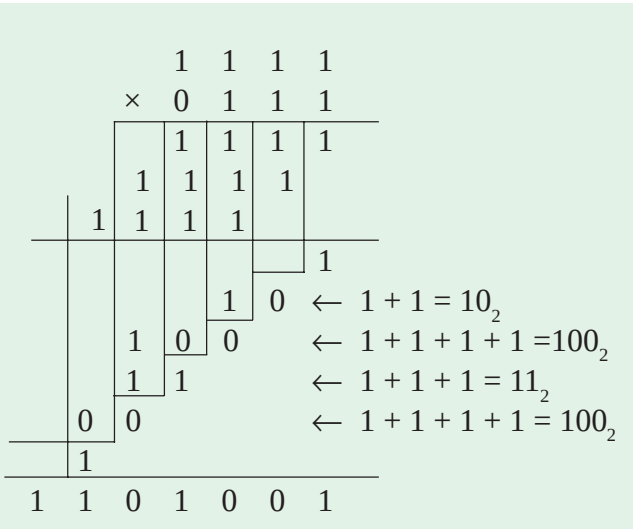


Fig. 69.5

69.16. Binary Division

It is similar to the division in the decimal system. As in that system, here also division by 0 is meaningless. Rules are :

1. $0 \div 1 = 0$ or $\frac{0}{1} = 0$, **2.** $1 \div 1 = 1$ or $\frac{1}{1} = 1$.

Example 69.15. *Carry out the binary division 11001 ÷ 101*

Solution.

```
          1 0 1
      ---------
1 0 1 ) 1 1 0 0 1
        1 0 1
        -----
          1 0
          1 0 1
          -----
            1 0 0
            1 0 1
            -----
              0 0 0
```

After we bring down the next 0 bit, the number 10 so formed is not divisible by 101. Hence, we put a 0 in the quotient. Therefore, the answer is 101_2 (decimal 5).

Incidentally, it may be noted that the dividend 25_{10} and divisor is 5_{10} so that the result of division, as expected, is 5_{10}.

Example 69.16. *Divide 11011_2 by 100_2* **(Digital Computations, Punjab Univ. 1990)**

Solution.

```
          1 1 0 . 1 1
      -------------
1 0 0 ) 1 1 0 1 1
        1 0 0
        -----
          1 0 1
          1 0 0
          -----
            1 1
            1 1 0
            1 0 0
            -----
              1 0 0
              1 0 0
              -----
              0 0 0
```

```
        6.75
     -------
4 ) 27
    24
    --
    30
    28
    --
    20
    20
    --
    00
```

69.17. Shifting a Number to Left or Right

Shifting binary numbers one step to the left or right corresponds respectively to multiplication or division by decimal 2.

When binary number 101100_2 (44_{10}) is shifted one step **to the left**, it becomes 1011000_2 which is 88_{10} *i.e.* it is doubled. If the given number is shifted one step **to the right**, it becomes 10110_2 which is 22_{10}. Obviously, the number is halved.

69.18. Representation of Binary Numbers as Electrical Signals

As seen from above, any binary number can be represented as a string of 0s and 1s. However, it is fine for paper and pencil calculations only. Practical problem is how to apply the desired binary information to logic circuits in digital computers. For that purpose, two types of electrical signals are selected to represent 1 and 0. Since speed and accuracy are of primary importance in digital circuits, the two electrical signals chosen to represent 1 and 0 must meet very rigid requirements.

1. they must be suitable for use in high-speed circuitry,
2. the signals should be very easy to tell apart,
3. they must be hard to confuse with each other.

The second and third statements may look alike but they, in fact, are not so. It is found that all transistor circuits distort, to some extent, the electrical signals that pass through them. Sometime, these distorted signals can look confusingly alike. Hence, this effect of distortion or degradation has to be kept in mind while selecting the two signals.

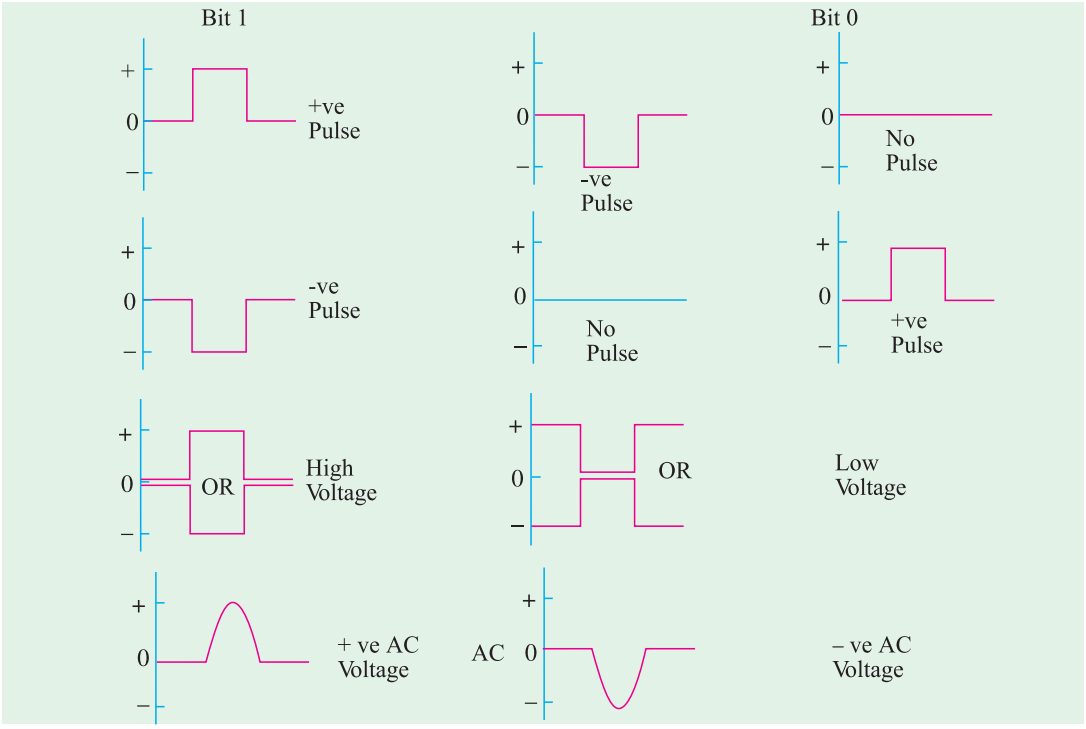


Fig. 69.6

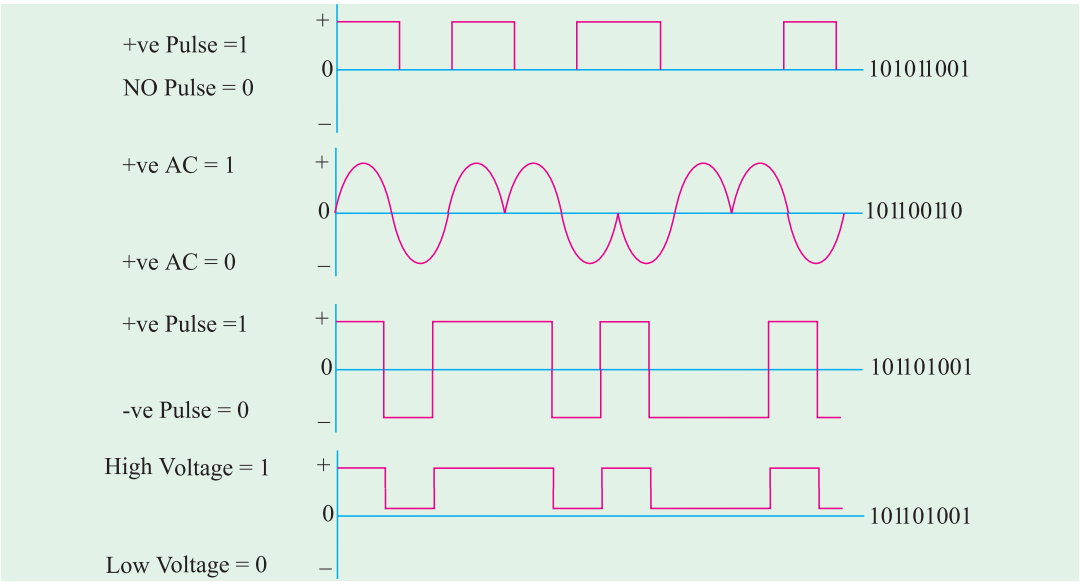


Fig. 69.7

In Fig. 69.6 are shown several signal pairs that meet the above requirements. It will be noted that it is impossible to distort a positive pulse (representing 1) to look like the no pulse or negative pulse (representing 0).

Fig. 69.7 shows how signal pairs can be used to represent different binary numbers.

69.19. Octal Number System

(*i*) Radix or Base

It has a base of 8 which means that it has eight distinct counting digits :

0, 1, 2, 3, 4, 5, 6, and 7

These digits 0 through 7, have exactly the same physical meaning as in decimal system.

For counting beyond 7, **2-digit combinations are formed taking the second digit followed by the first, then the second followed by the second and so on.** Hence, after 7, the next octal number is 10 (second digit followed by first), then 11 (second digit followed by second) and so on. Hence, different octal numbers are :

0,	1,	2,	3,	4,	5,	6,	7,
10,	11,	12,	13,	14,	15,	16,	17,
20,	21,	22,	23,	24,	25,	26,	27,
30,	31,	32,	...	...	...	...	...

(*ii*) Position Value

The position value (or weight) for each digit is given by different powers of 8 as shown below:

$$\leftarrow \quad 8^3 \quad 8^2 \quad 8^1 \quad 8^0 \quad \cdot \quad 8^{-1} \quad 8^{-2} \quad 8^{-3} \quad \rightarrow$$

$$\uparrow$$

octal point

For example, decimal equivalent of octal 352 is

$$\begin{array}{cccccc} 3 & 5 & 2 & \cdot & 0 & \\ 8^2 & 8^1 & 8^0 & & & \\ 64 & 8 & 1 & = & 3\times 64 + 5\times 8 + 2\times 1 = 234_{10} & \end{array}$$

or $$352_8 = 3\times 8^2 + 5\times 8^1 + 2\times 8^0 = 192 + 40 + 2 = 234_{10}$$

Similarly, decimal equivalent of octal 127.24 is

$$127.24_8 = 1\times 8^2 + 2\times 8^1 + 7\times 8^0 + 2\times 8^{-1} + 4\times 8^{-2}$$

$$= 64 + 16 + 7 + \frac{2}{8} + \frac{4}{64} = 87.3125_{10}$$

69.20. Octal to Decimal Conversion

Procedure is exactly the same as given in Art. 68.4 except that we will use digit of 8 rather than 2.

Suppose, we want to convert octal 206.104_8 into its decimal equivalent number. The procedure is as under :

$$\begin{array}{ccccccc} 2 & 0 & 6 & \cdot & 1 & 0 & 4 \\ 8^2 & 8^1 & 8^0 & & 8^{-1} & 8^{-2} & 8^{-3} \end{array}$$

$$\therefore \quad 206.104_8 = 2\times 8^2 + 6\times 8^0 + \frac{1}{8} + \frac{4}{8^3} = 128 + 6 + \frac{1}{8} + \frac{1}{128} = \left(134\,\frac{17}{128}\right)_{10}$$

69.21. Decimal to Octal Conversion

The **double-dabble** method (Art 69.7) is used with 8 acting as the **multiplying** factor for **integers** and the **dividing** factor for **fractions.**

Let us see how we can convert 175_{10} into its octal equivalent.

$175 \div 8 = 21$ with 7 remainder
$21 \div 8 = 2$ with 5 remainder
$2 \div 8 = 0$ with 2 remainder

Taking the remainders in the **reverse order**, we get 257_8. $\therefore \quad 175_{10} = 257_8$

Let us now take decimal fraction 0.15. Its octal equivalent can be found as under:

$0.15 \times 8 = 1.20 = 0.20$ with a carry of 1
$0.20 \times 8 = 1.60 = 0.60$ with a carry of 1
$0.60 \times 8 = 4.80 = 0.80$ with a carry of 4

$\therefore \quad 0.15_{10} \cong 114_8$

As seen, here carries have been taken in the **forward direction** *i.e.* from top to bottom.

Using positional notation, the first few octal numbers and their decimal equivalents are shown in Table 69.2.

Table No. 69.2

Octal	*Decimal*	*Octal*	*Decimal*	*Octal*	*Decimal*
0	0	12	10	24	20
1	1	13	11	25	21
2	2	14	12	26	22
3	3	15	13	27	23
4	4	16	14	30	24
5	5	17	15	31	25
6	6	20	16	32	26
7	7	21	17	33	27
10	8	22	18	34	28
11	9	23	19	35	29

69.22. Binary to Octal Conversion

The simplest procedure is to use **binary-triplet** method. In this method, the given binary number is arranged into groups of 3 bits starting from the octal point and then each group is converted to its equivalent octal number. Of course, where necessary, extra 0s can be added **in front** (*i.e.* left end) of the binary number to complete groups of three.

Suppose, we want to convert 101011_2 into its octal equivalent. Converting the bits into groups of three, we have $\therefore$

101 011

Now, 101_2 is 5 octal and 011 is 3 octal.

$\therefore$ 101 011
↓ ↓
5 3 $\quad \therefore \quad 101\ 011_2 = 53_8$

Now, take 111110111_2. We will first split it into groups of **three bits** (space is left between the groups for easy reading). Then, each group is given its octal number as shown below.

111 110 111
↓ ↓ ↓
7 6 7 $\quad \therefore \quad 111\ 110\ 111_2 = 767_8$

Finally, take the example of a mixed binary number 10101.11_2. Here, we will have to add one 0 in front of the integral part as well as to the fractional part

$$\begin{array}{ccc} 010 & 101 & 110 \\ \downarrow & \downarrow & \downarrow \\ 2 & 5 & 6 \end{array} \qquad \therefore \quad 10101.11_2 = 25.6_8$$

The equivalence between binary triplets and octal numbers is given in Table 69.3.

Table No. 69.3

Binary	*Octal*	*Binary*	*Octal*
000	0	1010	12
001	1	1011	13
010	2	1100	14
011	3	1101	15
100	4	1110	16
101	5	1111	17
110	6	10000	20
111	7	10001	21
1000	10	10010	22
1001	11	10011	23

69.23. Octal to Binary Conversion

The procedure for this conversion is just the opposite of that given in Art 69.22. Here, each digit of the given octal number is converted into its equivalent binary triplet. For example, to change 75_8 into its binary equivalent, proceed as under:

$$\begin{array}{cc} 7 & 5 \\ \downarrow & \downarrow \\ 111 & 101 \end{array} \qquad \therefore \quad 75_8 = 111\ 101_2$$

Similarly, 74.562_8 can be converted into binary equivalent as under:

$$\begin{array}{ccccccc} 7 & 4 & \cdot & 5 & 6 & 2 & \\ \downarrow & \downarrow & & \downarrow & \downarrow & \downarrow & \\ 111 & 100 & & 101 & 110 & 010 & \end{array} \qquad \therefore \quad 74.562_8 = 111\ 100.101\ 110\ 010_2$$

Incidentally, it may be noted that number of digits in octal numbers is **one-third of that in equivalent binary numbers**. In the present case, it is five versus fifteen.

69.24. Usefulness of Octal Number System

We have already discussed the octal number system and conversion from the binary and decimal numbers to octal and vice versa. The ease with which conversions can be made between octal and binary makes the octal system attractive as a "shorthand" means of expressing large binary numbers, In computer work, binary number with up to 64 bits are not uncommon. These binary numbers, as we shall see, do not always represent a numerical quantity but are often some type of code that conveys non numerical information. In computers, binary numbers might represent :

1. actual numerical data
2. numbers corresponding to a location called (address) in memory,
3. an instruction code

4. a code representing alphabetic and other non numerical characters,
5. group of bits representing the status of devices internal or external to the computer

When dealing with a large quantity of binary numbers of many bits, it is convenient and more efficient for us to write the numbers in octal rather than binary. However keep in mind that the digital circuits and systems work strictly in binary. We use octal numbers only as a convenience for the operators of the system.

69.25. Hexadecimal Number System

The characteristics of this system are as under :

1. it has a base of 16. Hence, it uses sixteen distinct counting digits 0 through 9 and A through F as detailed below :
   0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F
2. place value (or weight) for each digit is in **ascending powers** of 16 for integers and **descending powers** of 16 for fractions.

The chief use of this system is in connection with **byte-organised machines.** It is used for specifying addresses of different binary numbers stored in computer memory.

69.26. How to Count Beyond F in Hex Number System ?

As usual, we resort to **2-digit combinations.** After reaching **F**, we take the second digit followed by the first digit, then second followed by second, then second followed by third and so on. The first few 'hex' numbers and their decimal equivalents are given in Table 69.4.

Table No. 69.4

Hexadecimal	*Decimal*	*Hexadecimal*	*Decimal*	*Hexadecimal*	*Decimal*
0	0	B	11	16	22
1	1	C	12	17	23
2	2	D	13	18	24
3	3	E	14	19	25
4	4	F	15	1A	26
5	5	10	16	1B	27
6	6	11	17	1C	28
7	7	12	18	1D	29
8	8	13	19	1E	30
9	9	14	20	1F	31
A	10	15	21	20	32
				21	33

69.27. Binary to Hexadecimal Conversion

The simple method is to split the given binary number into **4-bit groups** (supplying 0s from our own side if necessary) and then give each group its 'hex' value as found from Table 69.5.

Table No. 69.5

Binary	*Hex.*	*Binary*	*Hex.*	*Binary*	*Hex.*
0000	0	0110	6	1100	C
0001	1	0111	7	1101	D
0010	2	1000	8	1110	E
0011	3	1001	9	1111	F
0100	4	1010	A	10000	10
0101	5	1011	B	10001	11

Let us see how we would convert 10001 100_2 into its hexadecimal equivalent number. We will first split the given binary number into 4-bit groups and then give each group its proper value from Table 69.5.

$$\begin{array}{cc} 1000 & 1100 \\ \downarrow & \downarrow \\ 8 & C \end{array} \qquad \therefore \quad 1000\ 1100_2 = 8C_{16}$$

Let us now consider 1011010111_2. Following the above procedure, we have

$$\begin{array}{ccc} 0010 & 1101 & 0111 \\ \downarrow & \downarrow & \downarrow \\ 2 & D & 7 \end{array} \qquad \therefore \quad 1011010111_2 = 2D7_{16}$$

It is seen that two 0s have been added to complete the 4-bit groups.

69.28. Hexadecimal to Binary Conversion

Here, the procedure is just the reverse of that given in Art. 69.27. Each hexadecimal digit is converted into its equivalent 4-bit binary.

Suppose, we want to convert $23A_{16}$ into its binary equivalent. It can be done as given below:

$$\begin{array}{ccc} 2 & 3 & A \\ \downarrow & \downarrow & \downarrow \\ 0010 & 0011 & 1010 \end{array} \qquad \therefore \quad 23A_{16} = 0010\ 0011\ 1010_2$$

Incidentally, hex numbers contain **one-fourth** the number of bits contained in the equivalent binary number. Optionally, we could drop off the two 0s in front of the binary equivalent.

69.29. Decimal to Hexadecimal Conversion

Two methods are available for such a conversion. One is to go from decimal to binary and then to hexadecimal. The other method called **hex dabble** method is similar to the double-dabble (or divide-by-two) method of Art 69.7 except that we use 16 (instead of 2) for successive divisions. As an example let us convert decimal 1983 into hexadecimal by consulting Table No. 69.4 for remainders.

Hence, $1983_{10} = 7BF_{16}$

$$\begin{aligned} 1983 \div 16 &= 123 + 15 \rightarrow F \\ 123 \div 16 &= 7 + 11 \rightarrow B \\ 7 \div 16 &= 0 + 7 \rightarrow 7 \end{aligned}$$



69.30. Hexadecimal to Decimal Conversion

Two methods are available for such a conversion. One is to convert from hexadecimal to binary and then to decimal. The other direct method is as follows :

Instead of using powers of 2, use power of 16 for the weights. Then, sum up the products of hexadecimal digits and their weights to get the decimal equivalent. As an example, let us convert F6D9 to decimal.

$$\begin{aligned} F6D9 &= F(16^3) + 6(16^2) + D(16^1) + 9(16^0) = 15 \times 16^3 + 6 \times 16^2 + 13 \times 16^1 + 9 \times 16^0 \\ &= 61{,}440 + 1536 + 208 + 9 = 63{,}193_{10} \end{aligned}$$

Example 69.17. *Find the binary, octal and hexadecimal equivalents of the following decimal numbers* ***(i)*** *32* ***(ii)*** *256* ***(iii)*** *51.* **(Digital Computations, Punjab Univ. May 1990)**

Solution. (*i*) Decimal number 32

As seen from Art : 10–7

32	16	8	4	2	1
1	Ø	Ø	Ø	Ø	Ø

$\therefore \quad 32_{10} = 100000_2$

For octal conversion :

$$\begin{array}{cc} 100 & 000 \\ \downarrow & \downarrow \\ 4 & 0 \end{array}$$

$\therefore \quad 32_{10} = 40_8$

For hexadecimal conversion

$$\begin{array}{cc} 0010 & 0000 \\ \downarrow & \downarrow \\ 2 & 0 \end{array} \qquad \therefore \quad 32_{10} = 20_{16}$$

As will be seen, the binary number has been divided into two 4-bit groups for which purpose two 0s have been added to the left.

(ii) In the same way, it can be found that $256_{10} = 100000000_8 = 100_{16}$

(iii) Also $51_{10} = 110011_2 = 63_8 = 33_{16}$

Example 69.18. *Convert the following numbers to decimal*

(i) $(11010)_2$ ***(ii)*** $(AB60)_{16}$ ***(iii)*** $(777)_8$

(Digital Computations, Punjab Univ. 1992)

Solution. *(i)* For binary to decimal conversion, we will follow the procedure given in Art. 69.4.

$$\begin{array}{ccccc} 1 & 1 & 0 & 1 & 0 \\ 16 & 8 & 4 & 2 & 1 \\ 16 & 8 & 4 & 2 & 1 \end{array}$$

$\therefore \quad 11010_2 = 16 + 8 + 2 = 26_{10}$

(ii) Following the procedure given in Art. 69.30, we have,

$$AB60_{16} = A\,(16^3) + B\,(16^2) + 6\,(16^1) + 0\,(16^0) = 10 \times 16^3 + 11 \times 16^2 + 6 \times 16^1 + 0 \times 16^0$$
$$= 43{,}872_{10}$$

(iii) As per the procedure given in Art. 69.20, $777_8 = 7(8^2) + 7(8^1) + 7(8^0) = 511_{10}$.

Example 69.19. *A computer is transmitting the following groups of bytes (each consisting of 8-bits) to some output device. Give the equivalent octal and hexadecimal listings.*

1000	*1100*	*0011*	*1010*
0010	*1110*	*1001*	*0101*
0101	*1111*	*1011*	*0110*
0111	*1011*	*0101*	*1011*

Solution.

Binary		*Octal*			*Hexadecimal*	
1000	1100	10	001	100	1000	1100
		2	1	4	8	C
0010	1110	00	101	110	0010	1110
		0	5	6	2	E
0101	1111	01	011	111	0101	1111
		1	3	7	5	F
0111	1011	01	111	011	0111	1011
		1	7	3	7	B
0011	1010	00	111	010	0011	1010
		0	7	2	3	A
1001	0101	10	010	101	1001	0101
		2	2	5	9	5
1011	0110	10	110	110	1011	0110
		2	6	6	B	6
0101	1011	01	011	011	0101	1011
		1	3	3	5	B

Hence, the groups of given memory bytes when expressed in different number systems become as under :

Binary	*Octal*	*Hexadecimal*
10001100	214	8C
00101110	056	2E
0101111	137	5F
01111011	173	7B
00111010	072	3A
10010101	225	95
10110110	266	B6
01011011	133	5B

It is clear from above that so far as the computer operator (or programmer) is concerned, it is much easier to handle this data when expressed in octal or hexadecimal system than in the binary system. For example, it is much easier and less error-prone to write the hexadecimal 8C than the binary 10001100 or 6AF than 011010101111. Of course, when the need arises, the operator can easily convert from octal or hexadecimal to binary.

69.31. Digital Coding

In digital logic circuits, each number or piece of information is defined by an equivalent combination of binary digits. A complete group of these combinations which represents numbers, letters or symbols is called a ***digital code.***

Codes have been used for security reasons so that others may not be able to read the message even if it is intercepted. In modern digital equipment, codes are used to represent and process numerical information. The choice of a code depends on the function or purpose it has to serve. Some codes are suitable where arithmetic operations are performed whereas others have high efficiency *i.e.* they give more information using fewer bits.

In certain applications, use of one code or the other simplifies and reduces the circuitry required to process the information. By limiting the switching circuitry, reliability of the digital system is increased. Of continuing importance are other codes which allow for error detection or correction. These codes enable the computers to determine whether the information that was coded and transmitted is received correctly and, if there is an error, to correct it. Since coding itself is a detailed subject, only few of the more familiar codes will be discussed.

69.32. Binary Coded Decimal (BCD) Code

It is a binary code in which *each* decimal digit is represented by a group of four bits. Since the right-to-left weighting of the 4-bit positions is 8-4-2-1, it is also called an 8421 code. It is a weighted numerical code. As said above, here each decimal digit from 0 through 9 requires a 4-bit binary-coded number. For example, the decimal number 35 in *BCD* code is 0101. The coding of ten decimal digits is given in Table No. 69.6. Lest you think that *BCD* code is the same thing as binary numbers, consider the following.

In the binary system, ten is represented by 1010 but in *BCD* code, it is 0001 0000. Seventeen in binary is 10001 but in *BCD* code, it is 0001 0111. See the difference ! Actually, the confu-

Table No. 69.6

Decimal	*BCD*
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

sion is due to the fact that the first nine numbers in *BCD* and binary are exactly similar (Table No. 69.6). After that, they become quite different (Table No. 69.7).

Table No. 69.7

Decimal	*BCD*			
26			0010	0110
59			0101	1001
673		0110	0111	0011
2498	0010	0100	1001	1000

It should be realized that with four-bits, sixteen num bers (2^4) can be represented although in the *BCD* code only ten of these are used. The following six combinations are *invalid* in the *BCD* code : 1010, 1011, 1100, 1101, 1110 and 1111.

The main advantage of *BCD* code is that it can be read and recognised easily although special adders are needed for arithmetic operations.

Any decimal number can be expressed in *BCD* code by replacing each decimal digit by the appropriate 4-bit combination. Conversely, a *BCD* number can be easily converted into a decimal number by dividing the coded number into groups of four bits (starting with *LSB*) and then writing down the decimal digit represented by each four-bit group.

Example 69.20. *Write the decimal number 369 in BCD code.*

Solution. For writing this 3-digit number in *BCD*, the value of each digit must be replaced by its 4-bit equivalent from the *BCD* code. From Table No. 69.6, we get

3 = 0011, 6 = 0110, 9 = 1001

$\therefore \quad 369_{10} = 001101101001_{BCD}$

Example 69.21. *Typically digital thermometers use BCD to drive their digital displays. How many BCD bits are required to drive a 3-digit thermometer display? What 12 bits are sent to display for a temperature of 157 degrees.*

Solution. There are 12 *BCD* bits required to drive a 3-digit thermometer display because each *BCD* digit is represented by a group of four bits.

In order to display a temperature of 157 degrees, we know that we have to send 12-bits. These bits can be determined by replacing each decimal digit by its equivalent four bit binary. Thus,

$$\begin{array}{ccc} 1 & 5 & 7 \\ \downarrow & \downarrow & \downarrow \\ 0001 & 0101 & 0111 \end{array}$$

$\therefore \quad 157_{10} = 000101010111_{BCD}.$

Example 69.22. *Find the equivalent decimal value for the BCD code number 0001010001110101.* **(Applied Electronics, A.M.I.E.E., London)**

Solution. Starting from the *LSB*, the given number can be divided into groups of four bits as 0001 0100 0111 0101. As seen from Table No. 69.6,

0001 = 1, 0100 = 4,
0111 = 7 and 0101 = 5

Hence, $0001010001110101_{BCD} = 1475_{10}$

Example 69.23. **(*a*)** *Convert the hexadecimal number F8E6 to the corresponding decimal number.*

(*b*) *Convert the decimal number 2479 to the corresponding hexadecimal number.*

(*c*) *Encode the following decimal numbers into 8421 BCD numbers* ***(i)*** *59* ***(ii)*** *39 and* ***(iii)*** *584.*

(*d*) *Decode the following 8421 BCD numbers* ***(i)*** *0101* ***(ii)*** *0111.*

(Digital Computations, Punjab Univ. 1990)

Solution. ***(a)*** $F8E6 = F(16^3) + 8(16^2) + E(16^1) + 6(16^0)$

$= 15 \times 16^3 + 8 \times 16^2 + 14 \times 16^1 + 6 \times 16^0 =$ **63,71810**

(b) We would use the hex-dabble method explained in Art 69.29

$2479 \div 16 = 154 + 15 \rightarrow F$

$154 \div 16 = 9 + 10 \rightarrow A$

$\therefore \quad 2479_{10} = 9AF_{16} \quad 9 \div 16 = 0 + 9 \rightarrow 9$

(c) As seen from Table No. 69.6

5	9	3	9	5	8	4
↓	↓	↓	↓	↓	↓	↓
0101	1001	0011	1001	0101	1000	0100

(d) Again, consulting Table No. 69.6, we have

0101	0111
↓	↓
5	7

69.33. Octal Coding

It involves grouping the bits in *three's*. For example, $(1756)_8 = (001\ \ 111\ \ 101\ \ 110)_2 = (001111101110)_2$. Similarly, the 24-bit number stored in the computer memory such as 101 010 011 100 010 111 000 110 can be read in the octal as

101	010	011	100	010	111	000	110
↓	↓	↓	↓	↓	↓	↓	↓
5	2	3	4	2	7	0	6

Apart from ease of recognition and conversion to binary, one important feature of the octal code is that its numbers are straight binary numbers which can be manipulated mathematically. For example, octal 25 expressed in octal code is 010 101 which can be read as binary 010101

$(25)_8 = 2 \times 8^1 + 5 \times 8^0\ (21)_{10}$

$(010101)_2 = 0 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = (21)_{10}$

You might recall that in *BCD* code, the resulting number is always a 4-bit group and a special adder is needed to convert it into decimal. In octal coding, 3-bit grouping is used but the resulting binary number can be considered a single number in natural binary form.

69.34. Hexadecimal Coding

The advantage of this coding is that four bits are expressed by a single character. However, the disadvantage is that new symbols have to be used to represent the values from 1010 to 1111 binary. As seen from Table No. 69.5, the binary number 1010 0101 is hex number A5. Similarly, hexadecimal number *C7* is $(11000111)_2$. To prove that the resulting binary number is the same as the hexadecimal value, consider the following example:

$(3D)_{16} = 3 \times 16^1 + D \times 16^0 = 3 \times 16 + 13 \times 1 = (61)_{10}$

$(3D)_{16} = (00111101)_2 = 1 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = (61)_{10}$

Table No. 69.8

Decimal	*XS–3*			
3				0110
26			0101	1001
629		1001	0101	1100
3274	0110	0101	1010	0111

69.35. Excess-3 Code

It is an unweighted code and is a modified form of *BCD*. It is widely used to represent numerical data in digital equipment. It is abbreviated as XS-3. As its name implies, each coded number in XS-3 is ***three larger*** than in *BCD* code. For example, six is written as 1001. As compared to *BCD*, the XS-3 has poorer recognition but it is more desirable for arithmetic operations. A few numbers using Excess-3 code are given in Table 69.8.

69.36. Gray Code

It is an unweighted code for numbers 0 through 9 and is largely used in mechanical switching systems. As seen from Table No. 69.9, only a single bit changes between each successive word. Because of this, the amount of switching is minimized and the reliability of the switching system is improved.

69.37. Excess-3 Gray Code

It is shown in Table No. 69.9 and is the original gray code shifted by three binary combinations. It exhibits the same properties as the Gray Code.

Table No. 69.9

Decimal	*Gray*	*XS-3 Gray*
0	0000	0010
1	0001	0110
2	0011	0111
3	0010	0100
4	0110	0100
5	0111	1100
6	0101	1101
7	0100	1111
8	1100	1110
9	1101	1010

Example 69.24. *Express the number 43_{10} in XS-3 code.*

Solution. Let us first represent each decimal digit by its 4-bit XS-3 code.

$4 = 0111, \quad 3 = 0110 \quad \therefore \quad 43_{10} = 01110110_{XS-3}$

Example 69.25. *The number 0110 1001 is expressed in XS-3 code. What is its decimal value ?*

Solution. Starting from least significant bit (*LSB*), the given number is first separated into groups of four and then each group is replaced by its equivalent value *i.e.* actual value decreased by 3.

$0110 = 6 - 3 = 3; \; 1001 = 9 - 3 = 6$

$\therefore \quad 01101001_{XS-3} = (36)_{10}$

69.38. Other Codes

Some of the other codes which are presently popular are given below:

(*a*) 4-bit codes

The different 4-bit weighted *BCD* codes for decimal numbers 0 through 9 in use are : 5421, 2*421, 7421, 7421, etc. and are tabulated below:

Table No. 69.10

Decimal	*5421*	*2*421*	*7421*	*74$\bar{2}\bar{1}$*
0	0000	0000	0000	0000
1	0001	0001	0001	0111
2	0010	0010	0010	0110
3	0011	0011	0011	0101
4	0100	0100	0100	0100
5	1000	1011	0101	1010
6	1001	1100	0110	1001
7	1010	1101	1000	1000
8	1011	1110	1001	1111
9	1100	1111	1010	1110

(*b*) 5-bit Codes

(*i*) 2-out-of-5 codes is an unweighted *BCD* code and allows easy error detection. It has been used in communications and telephone operation.

(*ii*) 51111 Code is a weighted *BCD* code and is much easier to operate with electronic circuitry.

(*iii*) Shift-counter (Johnson) Code is an unweighted *BCD* code and because of its pattern is easily operated on with electronic circuitry.

(*c*) 7-bit Biquinary Code— it uses a group of seven bits to represent decimal numbers and has code features which provide easy error detection and ease of operation.

(*d*) Ring-counter Code— it is also called 10-bit code because it uses a group of 10 bits to represent a decimal number. Though it requires as many as 10 positions, the ease of error detection with the code and of operating electronic circuits to implement the code make it quite attractive.

(*e*) Alphanumeric Code— In addition to numerical data, a computer must be able to handle non-numerical information used in input/output (I/O) processing. In other words, a computer should recognize codes that represent letters of the alphabet, punctuation marks, and other special characters as well as numbers. These codes are called alphanumeric codes. A complete alphanumeric code would include the 26 lowercase letters, 26 uppercase letters, 10 numeric digits, 7 punctuation marks, and anywhere from 20 to 40 other characters, such as +, –, /, #, $, “, and so on. We can say that an alphanumeric code represents all of the various characters and functions that are found on a computer keyboard. The most widely used alphanumeric code is the American standard code for Information Interchange (ASCII). Another similarly I/O-oriented code is EBCDIC (Extended Binary Coded Decimal Interchange Code).

69.39. ASCII Code

The ASCII code (Pronounced “askee) is a seven-bit code, and so it has 2^7 (=128) possible code groups. This is more than enough to represent all of the standard keyboard characters as well as control functions such as the (RETURN) and LINEFEED) functions. Table No. 69.11 shows a partial listing of the ASCII code. In addition to the binary code group for each character, the table gives the octal and hexadecimal equivalents. The complete list of the ASCII code is given in the Appendix.

Table No. 69.11

Character	*7-Bit ASCII*	*Octal*	*Hex*	*Character*	*7-Bit ASCII*	*Octal*	*Hex*
A	100 0001	101	41	Y	101 1001	131	59
B	100 0010	102	42	Z	101 1010	132	5A
C	100 0011	103	43	0	011 0000	060	30
D	100 0100	104	44	1	011 0001	061	31
E	100 0101	105	45	2	011 0010	062	32
F	100 0110	106	46	3	011 0011	063	33
G	100 0111	107	47	4	011 0100	064	34
H	100 1000	110	48	5	011 0101	065	35
I	100 1001	111	49	6	011 0110	066	36
J	100 1010	112	4A	7	011 0111	067	37
K	100 1011	113	4B	8	011 1000	070	38
L	100 1100	114	4C	9	011 1001	071	39
M	100 1101	115	4D	blank	010 0000	040	20
N	100 1110	116	4E	.	010 1110	056	2E
O	100 1111	117	4F	(	010 1000	050	28
P	101 0000	120	50	+	010 1011	053	2B
Q	101 0001	121	51	$	010 0100	044	24
R	101 0010	122	52	*	010 1010	052	2A

S	101 0011	123	53	)	010 1001	051	29
T	101 0100	124	54	—	010 1101	055	2D
U	101 0101	125	55	/	010 1111	057	2F
V	101 0110	126	56	,	010 1100	054	2C
W	101 0111	127	57	=	011 1101	075	3D
X	101 1000	130	58	<RETURN	000 1101	015	0D
				<LINEFEED>	000 1010	012	0A

Example. 69.26. *The following is a message encoded in ASCII code. What is the message ?*

1001000 1000101 1001100 1001100 1001111

Solution. Convert each seven-bit code to its equivalent hexadecimal number. The resulting values are:

48 45 4C 4C 4F

Now locate these hexadecimal values in Table No. 19.11 and determine the character represented by each. The results are:

"H E L L O".

Tutorial Problems No. 69.1

1. Find the decimal equivalents of the following binary numbers:
(*a*) 101 (*b*) 1001 (*c*) 10.011 **[(*a*) 510 (*b*) 910 (*c*) 3.037510]**

2. What are the decimal equivalents of the following binary numbers ?
(*a*) 1111 (*b*) 10100 (*c*) 11011001 (*d*) 10011001
[(*a*) 1510 (*b*) 2010 (*c*) 10910 (*d*) 15310]

3. Express the following binary numbers into their equivalent decimal numbers :
(*a*) 11.01 (*b*) 101.11 (*c*) 110.01 **[(*a*) 3.2510 (*b*) 5.7510 (*c*) 6.2510]**

4. Convert the following decimal numbers into their binary equivalents:
(*a*) 25 (*b*) 125 (*c*) 0.85
[(*a*) 110012 (*b*) 11111012 (*c*) 0.1101102]

5. What are the binary equivalents of the following decimal numbers ?
(*a*) 27 (*b*) 92 (*c*) 64
[(*a*) 110112 (*b*) 10111002 (*c*) 10000002]

6. Perform the following binary additions:
(*a*) 1011 + 1001 (*b*) 1011 + 110 (*c*) 101101 + 1101101 (*d*) 1011.01 + 1001.11
(*e*) 0.0011 + 0.1110 (*f*) 1111 + 111 + 1111
[(*a*) 101002 (*b*) 100012 (*c*) 11000112 (*d*) 101012 (*e*) 1.00012 (*f*) 11010012]

7. Add the following binary numbers 1011 and 1001
[10100](*Digital Computations, Punjab Univ. Dec.*)

8. Perform the following binary subtractions :
(*a*) 1101 – 1011 (*b*) 111 – 101 (*c*) 1000 – 11 (*d*) 101011 – 10010
[(*a*) 00102 (*b*) 0102 (*c*) 010012 (*d*) 0110012]

9. Carry out the following subtractions using binary number system:
(*a*) $64_{10} - 32_{10}$ (*b*) $128_{10} - 64_{10}$ (*c*) $93.5_{10} - 42.75_{10}$ (*d*) $7_{10} - 11_{10}$
[(*a*) 1000002 (*b*) 10000002 (*c*) 110010.112 (*d*) Ä1002]

10. Subtract the following binary numbers: 100011 – 111010
[–10111](*Digital Computations, Punjab Univ., Dec. 1984*)

11. Find the 1's complements of the following binary numbers:
(*a*) 01101 (*b*) 1101 (*c*) 1001 (*d*) 1010
[(*a*) 100102 (*b*) 00102 (*c*) 01102 (*d*) 01012]

12. What are 2's complements of the following binary numbers ?
(*a*) 1011 (*b*) 11011 (*c*) 11011.01 (*d*) 10011.11
[(*a*) 01012 (*b*) 001012 (*c*) 00100.112 (*d*) 01100.012]

13. Use the 1's and 2's complements of the following binary subtractions :
(*i*) 1111 – 1011 (*ii*) 110011 – 100101 (*ii*) 100011 – 111010
[(*i*) 0100 (*ii*) 1110 (*iii*) –10111] (*Digital Computations, Punjab Univ. Dec.*)

14. Multiply the following binary numbers:
(*a*) 1100 × 101 (*b*) 10101 × 101 (*c*) 10111 × 101 (*d*) 1110 × 111
[(*a*) 1111002 (*b*) 11010012 (*c*) 11100112 (*d*) 11000102]

15. Perform the following binary divisions :
(*a*) 11011 ÷ 100 (*b*) 1110011 ÷ 101 (*c*) 1100010 ÷ 111
[(*a*) 110.112 (*b*) 101112 (*c*) 11102]

16. Convert the following binary numbers into their octal equivalents :
(*a*) 1001 (*b*) 11011 (*c*) 10 101 111 (*d*) 1101.0110111
(*e*) 11 111 011 110 101
[(*a*) 118 (*b*) 338 (*c*) 258 (*d*) 15.3348 (*e*) 373658]

17. Convert the undergiven octal numbers into their binary equivalents:
(*a*) 13 (*b*) 11 (*c*) 713 (*d*) 3674
[(*a*) 0112 (*b*) 10012 (*c*) 111 001 0112 (*d*) 11 110 111 1002]

18. Convert the following numbers :
(*a*) 357_8 to decimal (*b*) 6421_8 to decimal (*c*) 1359_{10} to octal (*d*) 7777_{10} to octal
[(*a*) 23910 (*b*) 334510 (*c*) 25178 (*d*) 171418]

19. Convert the following real numbers to the binary numbers:
(*i*) 12.0 (*ii*) 25.0 (*iii*) 0.125
[(*i*) 1100 (*ii*) 11001 (*iii*) 0.001] (*Digital Computations, Punjab Univ.*)

20. Convert the following binary numbers into their equivalent hexadecimal numbers:
(*a*) 1101 0111 (*b*) 1010 0110 (*c*) 1001 1110 (*d*) 1100 1111
[(*a*) D716 (*b*) A616 (*c*) 9E16 (*d*) CF16]

21. Convert the following decimal numbers to binary numbers by converting them to octal, then to binary.
(*i*) 850 (*ii*) 7563
[(*a*) 1101010010 (*ii*) 1110110001011] (*Digital Computations, Punjab Univ. Dec.*)

22. Convert the following numbers to decimal
(*i*) $(11010)_2$ (*ii*) $(777)_8$
[(*i*) 26 (*ii*) 511] (*Digital Computations, Punjab Univ. June*)

23. What are the binary equivalents of the following hexadecimal numbers ?
(*a*) 9F (*b*) A2 (*c*) ED (*d*) 27
[(*a*) 1001 11112 (*b*) 1010 00102 (*c*) 1110 11012 (*d*) 0011 01112]

24. Convert the decimal number 4397 and the octal number 2735 to hexadecimal numbers.
[(*i*) 5DD (*ii*) 12D] (*Power & Digital Electronics, Punjab Univ. Dec. 1984*)

25. Convert the following decimal numbers into *BCD* code
(*a*) 18 (*ii*) 92 (*iii*) 321 and (*iv*) 4721
[(*i*) 0001 1000 (*ii*) 1001 0010 (*iii*) 0011 0010 0001 (*iv*) 0100011100100001]

26. Find the decimal values for the BCD-coded numbers
(*i*) 0110 1000 (*ii*) 0111 0100 1001 and (*iii*) 1000 0100 0111 0110]
[(*i*) 68 (*ii*) 749 (*iii*) 8476]

27. Convert the following hexadecimal numbers into binary numbers:
(*i*) E 5 (*ii*) B4D (*iii*) 7AF4
(*Digital Computations, Punjab Univ. May*)
[(*i*) 11100101 (*ii*) 101101001101 (*iii*) 0111 1010 1111 0100]

28. Encode the following decimal numbers into 8421 numbers.
(*i*) 45 (*ii*) 732 (*iii*) 94,685
(*Digital Computations, Punjab Univ. May*)
[(*i*) 0100 0101 (*ii*) 111 0011 0010 (*iii*) 001 0100 0110 1000 0101]

29. Decode the following 8421 *BCD* numbers :
(*i*) 0011 1000 0111 (*ii*) 1001 0110 0111 1000 0111 0011
[(*i*) 387 (ii) 967,873] (*Digital Computations, Punjab Univ. Dec.*)

30. Decode the following 8421 *BCD* numbers :
(*i*) 0011 (*ii*) 0111 (*iii*) 0101 (*iv*) 0110
(*v*) 1000 **[(*i*) 3 (*ii*) 7 (*iii*) 5 (*iv*) 6 (*v*) 8] (*Digital Computations, Punjab Univ. June*)**

31. Convert the following binary numbers into octal code :
(*i*) 100110111 and (*ii*) 101010001 **[(*i*) 4678 (*ii*) 5218]**

32. Convert the following decimal numbers into XS-3 code:
(*i*) 35 (*ii*) 159 (*iii*) 340
[(*i*) 01101000 (*ii*) 010010001100 (*iii*) 011001110011]

33. What is the decimal value of the following numbers expressed in Excess-3 code ?
(*i*) 10010110 and (*ii*) 100101011100 **[(*i*) 63 (*ii*) 629]**

OBJECTIVE TESTS – 69

1. The digital systems usually operate on .....system.
(*a*) binary (*b*) decimal
(*c*) octal (*d*) hexadecimal.

2. The binary system uses powers of .....for positional values.
(a) 2 (*b*) 10
(*c*) 8 (*d*) 16

3. After counting 0, 1, 10, 11, the next binary number is
(*a*) 12 (*b*) 100
(*c*) 101 (*d*) 110.

4. The number 1000_2 is equivalent to decimal number
(*a*) one thousand
(*b*) eight (*c*) four
(*d*) sixteen.

5. In binary numbers, shifting the binary point one place to the right.
(*a*) multiplies by 2 (*b*) divides by 2
(*c*) decreases by 10 (*d*) increases by 10.

6. The binary addition 1 + 1 + 1 gives
(a) 111 (*b*) 10
(*c*) 110 (*d*) 11

7. The cumulative addition of the four binary bits (1 + 1 + 1 + 1) gives
(*a*) 1111 (*b*) 111
(*c*) 100 (*d*) 1001

8. The result of binary subtraction (100 – 011) is
(*a*) –111 (*b*) 111
(*c*) 011 (*d*) 001.

9. The 2's complement of 1000_2 is
(*a*) 0111 (*b*) 0101
(*c*) 1000 (*d*) 0001

10. The chief reason why digital computers use complemental subtraction is that it
(*a*) simplifies their circuitry
(*b*) is a very simple process
(*c*) can handle negative numbers easily
(*d*) avoids direct subtraction.

11. The result of binary multiplication $111_2 \times 10_2$ is
(*a*) 1101 (*b*) 0110
(*c*) 1001 (*d*) 1110

12. The binary division $11000_2 \div 100_2$ gives
(a) 110 (*b*) 1100
(*c*) 11 (*d*) 101

13. The number 12_8 is equivalent to decimal
(*a*) 12 (*b*) 20
(*c*) 10 (*d*) 4

14. The number 100101_2 is equivalent to octal
(*a*) 54 (*b*) 45
(*c*) 37 (*d*) 25.

15. The number 17_8 is equivalent to binary
(*a*) 111 (*b*) 1110
(*c*) 10000 (*d*) 1111.

16. Which of the following is NOT an octal number ?
(*a*) 19 (*b*) 77
(*c*) 15 (*d*) 101

17. Hexadecimal number system is used as a shorthand language for representing .....numbers.
(*a*) decimal (*b*) binary
(*c*) octal (*d*) large

18. The binary equivalent of A_{16} is
(*a*) 1010 (*b*) 1011
(*c*) 1000 (*d*) 1110.

19. *BCD* code is
(*a*) non-weighted
(*b*) the same thing as binary numbers
(c) a binary code
(*d*) an alphanumeric code.

20. Which of the following 4-bit combinations is/are invalid in the *BCD* code ?
(*a*) 1010 (*b*) 0010
(*c*) 0101 (*d*) 1000.

21. Octal coding involves grouping the bits in
(*a*) 5's (*b*) 7's
(*c*) 4's (*d*) 3's.

22. In Excess-3 code each coded number is .....than in BCD code.
(*a*) four larger (*b*) three smaller
(*c*) three larger (*d*) much larger.

23. Which numbering system uses numbers and letters as symbols ?
(*a*) decimal (*b*) binary
(*c*) octal (*d*) hexadecimal

24. To convert a whole decimal number into a hexadecimal equivalent, one should divide the decimal value by.....
(*a*) 2 (*b*) 8
(*c*) 10 (*d*) 16.

ANSWERS

1. (*a*) **2.** (*a*) **3.** (*b*) **4.** (*b*) **5.** (*a*) **6.** (*d*) **7.** (*b*) **8.** (*d*) **9.** (*c*) **10.** (*a*)
11. (*d*) **12.** (*a*) **13.** (*c*) **14.** (*b*) **15.** (*d*) **16.** (*a*) **17.** (*a*) **18.** (*a*) **19.** (*c*) **20.** (*a*)
21. (*d*) **22.** (*c*) **23.** (*d*) **24.** (*a*)

GO To FIRST

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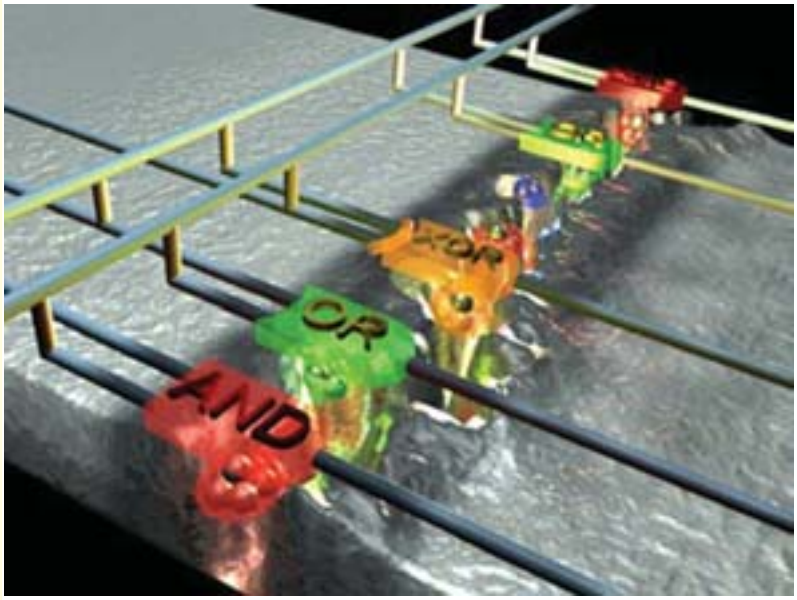
CHAPTER 70

Learning Objectives

- Definition of a Logic Gate
- Positive and Negative Logic
- The OR Gate
- Equivalent Relay Circuit of an OR Gate
- Diode OR Gate
- Transistor OR Gate
- OR Gate Symbolizes Logic Addition
- Three Input OR Gate
- Exclusive OR Gate
- The AND Gate
- Equivalent Relay Circuit of an AND Gate
- Diode AND Gate
- Transistor AND Circuit
- AND Gate Symbolizes Logic Multiplication
- The NOT Gate
- The NOT Operation
- Bubbled Gates
- The NOR Gate
- The NAND Gate
- The XNOR Gate
- Logic Gate at Glance
- Digital Signals Applied to Logic Gates
- Sequential Logic Circuits
- Adders and Subtractors
- Half Adder
- Full Adder
- Parallel Binary Adder
- Half Subtractor
- Full Subtractor

CONTENTS

LOGIC GATES



A logic gate is a circuit that has one or more input signals but only one output signal. All logic gates can be analysed by constructing a truth table

70.1. Definition of a Logic Gate

A logic gate is an electronic circuit **which makes logic decisions**. It has one output and one or more inputs. The output signal appears only for certain combinations of input signals. Logic gates are the basic building blocks from which most of the digital systems are built up. They implement the hardware logic function based on the logical algebra developed by George Boole which is called Boolean algebra in his honour. A unique characteristic of the Boolean algebra is that variables used in it **can assume only one of the two values** *i.e.* either 0 or 1. Hence, every variable is either a 0 or a 1.

These gates are available today in the form of various IC families. The most popular families are: transistor-transistor logic (*TTL*), emitter-coupled logic (*ECL*), metal-oxide-semiconductor (*MOS*) and complementary metal-oxide-semiconductor (*CMOS*).

In this chapter, we will consider the *OR*, *AND*, *NOT*, *NOR*, *NAND*, exclusive *OR* (*XOR*) and exclusive *NOR* (*XNOR*) gates along with their truth tables.

70.2. Positive and Negative Logic

In computing systems, the number symbols 0 and 1 represent two possible states of a circuit or device. It makes no difference if these two states are referred to as *ON* and *OFF*, *CLOSED* and *OPEN*, *HIGH* and *LOW PLUS* and *MINUS* or *TRUE* and *FALSE* depending on the circumstances. Main point is that **they must be symbolized by two opposite conditions**.

In positive logic, a 1 represents

1. an *ON* circuit **2.** a *CLOSED* switch **3.** a *HIGH* voltage

4. a *PLUS* sign **5.** a *TRUE* statement

Consequently, a 0 represents

1. an *OFF* circuit **2.** an *OPEN* switch **3.** a *LOW* voltage **4.** a *MINUS* sign **5.** a *FALSE* statement.

In negative logic, just opposite conditions prevail.

Suppose, a digital system has two voltage levels of 0V and 5V. If we say that symbol 1 stands for 5V and symbol 0 for 0V, then we have positive logic system. If, on other hand, we decide that a 1 should represent 0 V and 0 should represent 5V, then we will get negative logic system.

Main point is that in ***positive logic*, the more positive** of the two voltage levels represents the 1 while in negative logic, **the more negative** voltage represents the 1. Moreover, it is not essential that a 0 has to be represented by 0V although in some cases the two may coincide. Suppose, in a circuit, the two voltage levels are 2V and 10V. Then for positive logic, the 1 represents 10V and the 0 represents 2V (*i.e.* lesser of the two voltages). If the voltage levels are – 2V and – 8V, then, in positive logic, the 1 represents – 2V and the 0 represents – 8V (*i.e.* lesser of the two voltages).

Unless stated otherwise, we will be using only ***positive logic*** in this chapter.

70.3. The OR Gate

The electronic symbol for a two-input *OR* gate is shown in Fig. 70.1 (*a*) and its equivalent switching circuit in Fig. 70.1 (*b*). The two inputs have been marked as *A* and *B* and the output as *X*. It is worth reminding the reader that as per Boolean algebra, the three variables *A*, *B* and *X* can have only one of the two values *i.e.* either 0 or 1.

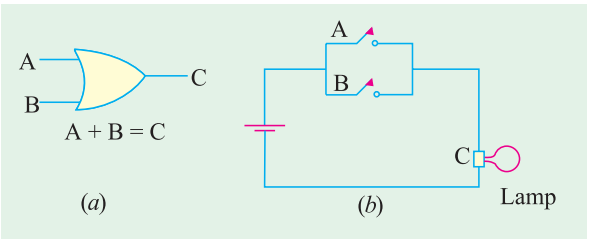


Fig. 70.1

Logic Operation

The *OR* gate has an **output of 1 when either A or B or both are 1**.

In other words, it is an **any-or-all** gate because an output occurs when any or all the inputs are present.

As seen from Fig. 70.1 (*b*), the lamp will light up (logic 1) when either switch *A* or *B* or both are closed.

Obviously, the output would be 0 **if and only if both its inputs are 0**. In terms of the switching conditions, it means that lamp would be *OFF* (logic 0) only when both switches *A* and *B* are *OFF*.

The *OR* gate represents the Boolean equation $A + B = X$

The meaning of this equation is that *X* is true when either *A* is true or *B* is true or both are true. Alternatively, it means that output *X* is 1 when either *A* or *B* or both are 1.

Table 70.1

A	B	X
0	0	1
0	1	0
1	0	0
1	1	1

Fig. 70.2

The above logic operation of the *OR* gate can be summarised with the help of the truth table given in Fig. 70.2. A truth table may be defined as a table which ***gives the output state for all possible input combinations.*** The *OR* Table 70.1 gives outputs for all possible *A B* inputs of 00, 01, 10 and 11.

We may interpret the truth table as follows:

When both inputs are 0 (switches are *OPEN*), output *X* is 0 (lamp is *OFF*). When *A* is in logic state 0 (switch *A* is *OPEN*) but *B* is in logic state 1 (switch *B* is *CLOSED*), the output *X* is logic state 1 (lamp is *ON*). Lamp would be also *ON* when *A* is *CLOSED* and *B* is *OPEN*. Of course, lamp would be *ON* when both switched are *CLOSED*. It is so because an *OR* gate is equivalent to a **parallel *circuit in its logic function.***

Another point worth remembering is that the above *OR* gate is called **inclusive** *OR* gate because **it includes the case when both inputs are true**.

70.4. Equivalent Relay Circuit of an OR Gate

In Fig. 70.3, the relay contacts have been wired ***in parallel.*** When +5V is applied to *A*, relay K_1 is energised and pulls *M* down thereby closing the contact. Hence, supply voltage of +5V appears at the output *X*.

Similarly, when +5V are applied to input *B*, K_2 is energised and pulls *N* down thereby bringing *X* in contact with *S*. Of course, when both *A* and *B* are at +5V, *X* is at +5V. Incidentally, when inputs at *A* and *B* are 0, *X* is also 0.

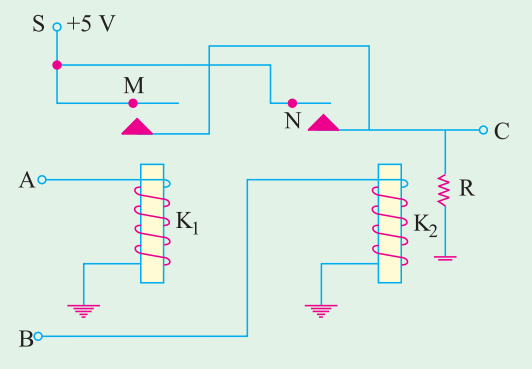


Fig. 70.3

70.5. Diode OR Gate

Fig. 70.4. shows the diode *OR* gate consisting of two ideal diodes D_1 and D_2 connected in parallel across the output *X*.

1. When *A* is at +5V, D_1 is forward-biased and hence conducts. The circuit current flows *via R* dropping 5V across it. In this way, point *X* achieves potential of +5 V.
2. When + 5V is applied to *B*, D_2 conducts causing point *X* to go to +5 V.
3. When both *A* and *B* are +5V, the drop across *R* is 5V because voltages of *A* and *B* ***are in parallel.***

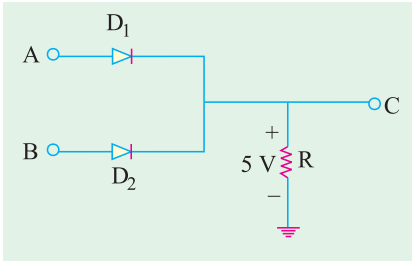


Fig. 70.4

Again, point X is driven to +5 V.

4. Obviously, when there is no voltage either at A or B, output X remains 0.

70.6. Transistor OR Gate

Fig. 70.5 illustrates a possible transistor *OR* gate consisting of three interconnected transistors Q_1, Q_2, and Q_3 supplied from a common supply V_{cc} = +5 V.

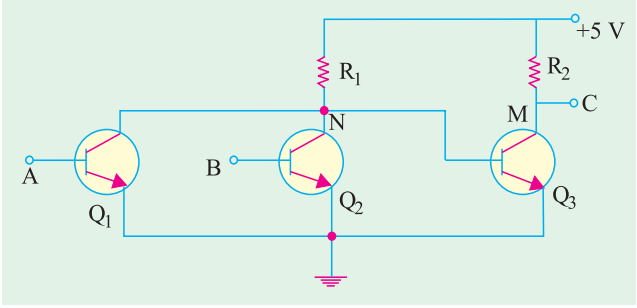


Fig. 70.5

1. When +5 V is applied to A, Q_1 is forward-biased and so it conducts. Assuming that Q_1 is saturated, entire V_{cc} = 5 V drops across R_1 thus causing N to go to ground. This, in turn, cuts off Q_3 thereby causing X to go to V_{cc} *i.e.* + 5V.

2. When +5 V is applied to B, Q_2 conducts thereby driving N to ground *i.e.* 0V. With no forward bias on its base, Q_3 is cut-off thus driving X again to V_{cc} *i.e.* +5 V.

3. If both inputs A and B are grounded, Q_1 and Q_2 are cut-off driving N to +5 V. As a result, Q_3 becomes forward-biased and conducts fully. In that case, entire V_{cc} drops across R_2 driving M and hence X to ground.

70.7. OR Gate Symbolizes Logic Addition

According to Boolean algebra, *OR* gate performs **logical addition**. Its truth table can be written as given below:

$$0 + 0 = 0$$
$$0 + 1 = 1$$
$$1 + 0 = 1$$
$$1 + 1 = 1$$

It must be clearly understood that '+' sign in Boolean algebra **does not stand for the addition** as understood in the ordinary or numerical algebra. In symbolic logic, the '+' sign indicates **OR** operation whose rules are given above. In logic algebra, $A + B = X$ means that if A is true *OR* B is true, then X will be true. ***It does* not *mean here that sum of A and B equals X.*** The other symbols used for '+' sign are U and V. Hence, the above equation could also be written as $A \cup B = X$ or $A \vee B = X$.

The meaning of the last three logic additions is that output is 1 when either input A or B or both are 1. The first addition implies that output is 0 ***only when both inputs are 0.***

The meaning of the '+' sign often becomes clear from the context as shown below:

$1 + 1 = 2$ — decimal addition
$1 + 1 = 10$ — binary addition
$1 + 1 = 1$ — *OR* addition

We can put the above *OR* laws in more general terms

$$A + 1 = 1$$
$$A + 0 = A$$
$$A + A = A \quad \text{— not } 2A$$

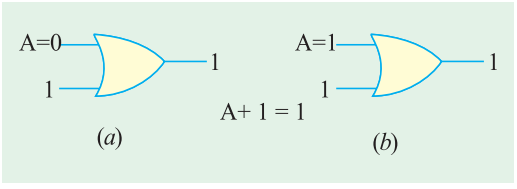


Fig. 70.6

(*i*) A + 1 = 1

As we know, A can have two values: 0 or 1. When A is 0, then we have 0 + 1 = 1 as shown in Fig. 70.6 (*a*).

When A = 1, then the above expression becomes : 1 + 1 = 1 as shown in Fig. 70.6 (*b*), Hence, we find that **irrespective of the value of A.**

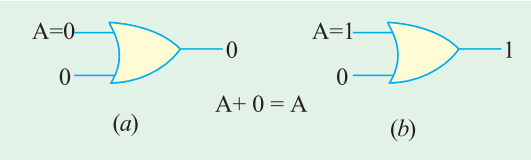


Fig. 70.7

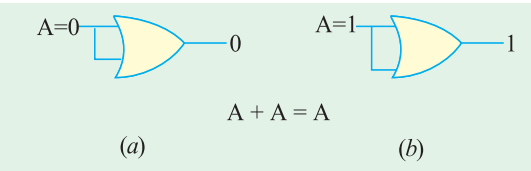


Fig. 70.8

$A + 1 = 1$

(*ii*) A + 0 = A

If $A = 0$, then $0 + 0 = 0$ *i.e.* output is 0 which is correct and is shown in Fig. 70. 7 (*a*). The output is what the value of A is.

As shown in Fig. 70.7 (*b*), when $A = 1$, output is 1 because $1 + 0 = 1$. Again, output is what the value of A is.

(*iii*) A + A = A

With A set to 0, the output is 0 because $0 + 0 = 0$ as shown in Fig. 70.8 (*a*).

With A set to 1, the output is 1 because $1 + 1 = 1$ as shown in Fig. 70.8 (*b*). Obviously, the output in both cases is A.

70.8. Three Input OR Gate

The electronic symbol for a 3-input (fan-in of 3) ***inclusive OR*** gate is shown in Fig. 70.9. As is usual in logic algebra, the inputs A, B, C as well as the output X can have only one of the two values *i.e.* 0 or 1.

Truth Table

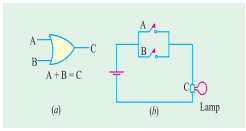


Fig. 70.9

It is shown in Table 70.2. Following points are worth noting:

1. The number of rows in the table is $2^3 = 8$ *i.e.* there are eight ways of combining the three inputs. In general, the number of horizontal rows is 2^n where n is the number of inputs.
2. In first column A, logic values alternate between 0 and 1 every four rows twice.
3. The second input column B alternates between 0 and 1 every two rows four times.
4. The third input column C alternates between 0 and 1 every other row eight times.

The truth table symbolizes the Boolean equation $A + B + C = X$ which means that output X is 1 when ***either A or B or C is 1 or all are 1.*** Alternatively, X is true when either A or B or C is true or all are true.

Table No. 70.2

A	B	C	X
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

vvhh

Its electronic symbol is shown in Fig. 70.10 (*a*) and its equivalent switching circuit in Fig. 70.10 (*b*).

In this gate, output is 1 if its either input **but not both**, is 1. In other words, it has an output 1 **when its inputs are different**. The output is 0 only when inputs **are the same**.

To put it a bit differently, this logic gate has output 0 **when inputs are either all 0 or all 1.**

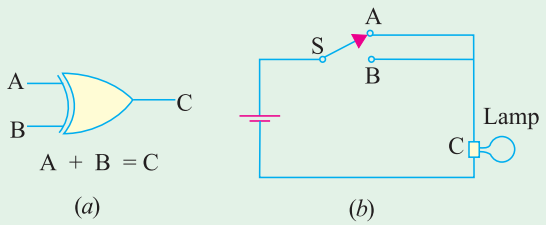


Fig. 70.10

A	B	C
0	0	0
0	1	1
1	0	1
1	1	0

Fig. 70.11

This gate works on the Boolean equation $A \oplus B = X$

The circle around plus (+) sign is worth noting.

The circuit is also called an ***inequality comparator*** or detector because it produces an output only **when the two inputs are different**.

Explanation

The ***inclusive*** *OR* gate exemplifies the everyday usage of the word *OR* which stands for one or the other or both. Take the following statement:

To qualify for a competition, you might have to subscribe to a magaine OR belong to a club.

Obviously, there is no bar on your doing both. But now take **exclusive** statement:

You can be rich OR you can be poor.

Obviously, you cannot be both at the same time.

The change-over switching ***circuit*** of Fig. 70.10 (*b*) simulates the ***exclusive OR (XOR)*** gate. Switch positions *A* and *B* will individually light up the lamp but a combination of *A* and *B* is not possible.

The truth table for a 2-input *XOR* gate is given in Table No.70.3. It is instructive to compare it with that for an *inclusive OR* gate (Table 70.1).

70.10. The AND Gate

The electronic (or logic) symbol for a 2-input *AND* gate is shown in Fig. 70.12 (*a*) and its equivalent switching circuit in Fig. 70.12 (*b*). It is worth reminding the readers once again that the three variables *A*, *B*, *C* can have a **value of either 0 or 1**.

Logic Operation

1. The *AND* gate gives an output only **when all its inputs are present**.
2. The *AND* gate has a 1 output when both *A* **and** *B* are 1. Hence, this gate is an **all-or-nothing** gate whose output occurs only when all its inputs are present.
3. In True/False terminology, the output of an *AND* gate will be **true** only if **all its inputs are true**. Its output would be false if **any *of its inputs is false.***

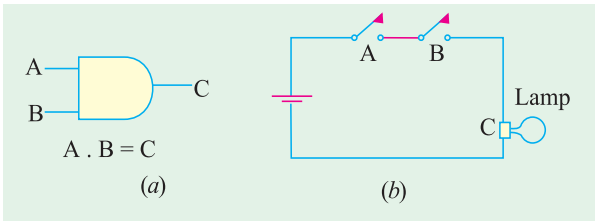


Fig. 70.12

The *AND* gate works on the Boolean algebra

$$A \times B = X \quad \text{or} \quad A \,.\, B = X \quad \text{or} \quad AB = X$$

It is a **logical** multiplication and is different from the **arithmetic** multiplication. Often the sign '×' is replaced by a dot which itself is generally omitted as shown above. The logical meaning of the above equation is that

1. output X is 1 only when both A *and* B are 1.
2. output X is true only when both A *and* B are true.

Table 70.4

A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

Fig. 70.13

Table 70.5

A	B	C	X
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

ABC = X

Fig. 70.14

As seen from Fig. 70.12 (*b*), the lamp would be *ON* when both switches A and B are closed. Even when one switch is open, the lamp would be *OFF*. Obviously, an *AND* gate is equivalent to a **series switching circuit.**

Truth Table Fig. 70.13 shows truth table for a 2-input *AND* gate and Fig. 70.14 gives the same for a 3-input *AND* gate.

As seen, X is at logic 1 only when *all* inputs are at logic 1, not otherwise. The procedure for writing down the first three columns is the same as explained in Art. 70.8 earlier.

70.11. Equivalent Relay Circuit of an AND Gate

The *AND* gate can be physically realized with the help of relay circuit shown in Fig. 70.15. Here, the two relay contacts have been **wired in series**.

When +5 V is applied to both input circuits, relays K_1 and K_2 are energized thereby pulling M and N downwards which brings C in contact with the supply point S. Hence, X goes to +5 V.

It is obvious that energizing only ***one relay*** will not make X go to + 5 V.

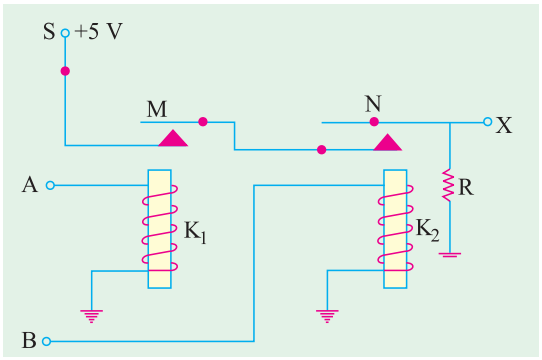


Fig. 70.15

70.12. Diode AND Gate

It is shown in Fig. 70.16. Its logical operation is as under :

1. When A is at 0 V, diode D_1 conducts and the supply voltage of +5 V drops across R. Consequently, point N and hence point X are driven to 0 V. Therefore, the output C is 0.
2. Similarly, when B is at 0 V, D_2 conducts thereby driving N and hence X to ground.
3. Obviously, when both A and B are at 0 V, both diodes conduct and, again, the output X is 0.
4. There is no supply current and hence no drop across R ***only when both A and B are at +5 V.*** Only in that case, the output X goes to supply voltage of +5 V.

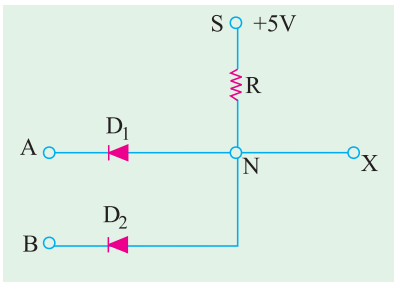


Fig. 70.16

70.13. Transistor AND Circuit

It is shown in Fig. 70.17. When both A and B are at +5 V, the two transistors Q_1 and Q_2 conduct. The current so produced drops the supply voltage of +5 V across R_1 thereby driving point N and hence base of Q_3 to ground or 0V. This cuts off Q_3 so that X goes to supply voltage of +5 V.

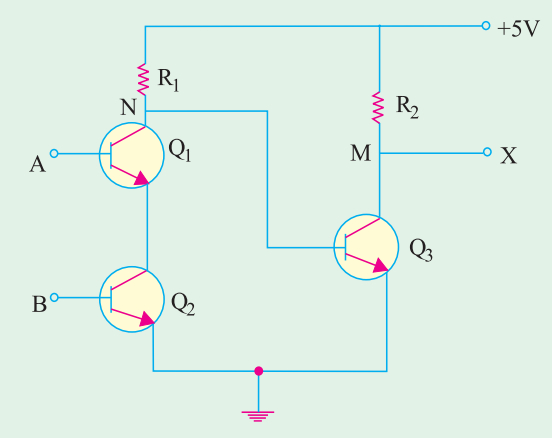


Fig. 70.17

Obivously, there is an output at X only when there is an input at A *and* B.

If either A or B is at 0 V, then Q_1 or Q_2 will be cut off and no drop will take place across R_1. Hence, point N will go to supply voltage of +5V. Consequently, Q_3 will conduct and whole of supply voltage will be dropped across R_2. As a result, point M and hence output X will go to 0 V.

70.14. AND Gate Symbolizes Logic Multiplication

According to Boolean algebra, the *AND* gate performs logical multiplication on its inputs as given below:

$0.0 = 0$
$0.1 = 0$
$1.0 = 0$
$1.1 = 1$

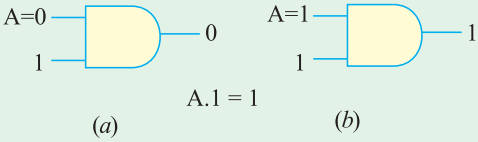


Fig. 70.18

In general, we can put the laws of Boolean multiplication in the following form:

$A.1 = A$ $\quad A.0 = 0$

$AA = A$ — not A^2

The above indentities can be verified by giving values of 0 and 1 to A.

1. A.1 = A When $A = 0$

then $0.1 = 0$ —Fig. 70.18 (*a*)

When $A = 1$

then $1.1 = 1$ —Fig. 70.18 (*b*)

It is seen that in each case, output has the same value as that of A.

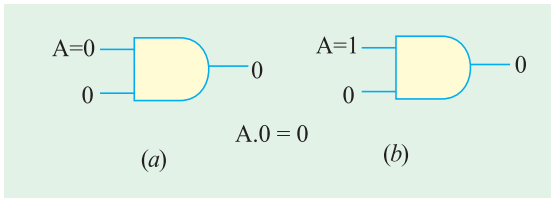


Fig. 70.19

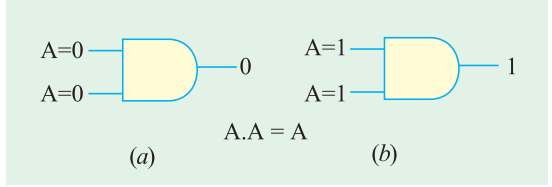


Fig. 70.20

2. A.0 = 0

When $A = 0$

then $0.0 = 0$ —Fig. 70.19 (*a*)

When $A = 1$

then $1.0 = 0$ — Fig. 70.19 (*b*)

It is seen that output is always 0 ***whatever the value of A.***

3. A.A = A

When $A = 0$, then $0.0 = 0$ — Fig 70.20 (*a*)

When $A = 1$, then $1.1 = 1$ — Fig. 70.20 (*b*)

It is seen that output always ***takes on the value of A.***

70.15. The NOT Gate

It is so called because *its output is NOT the* ***same as its input.*** It is also called an ***inverter*** because it inverts the input signal. It has **one** input and **one** output as shown in Fig. 70.21 (*a*). All it does is to invert (or complement) the input as seen from its truth table of Fig. 70.21 (*b*).

The schematic symbol for inversion is a small circle as shown in Fig. 70.21 (*a*). The logical symbol for inversion or negation or complementation is a bar over the function to indicate the opposite state.

Sometimes, a prime is also used as A'. For example, $\overline{A}$ means not-A. Similarly, $\overline{(A + B)}$ means the complement of $(A + B)$.

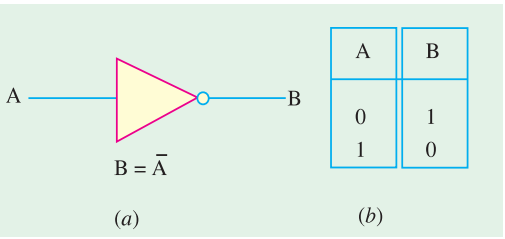


Fig. 70.21

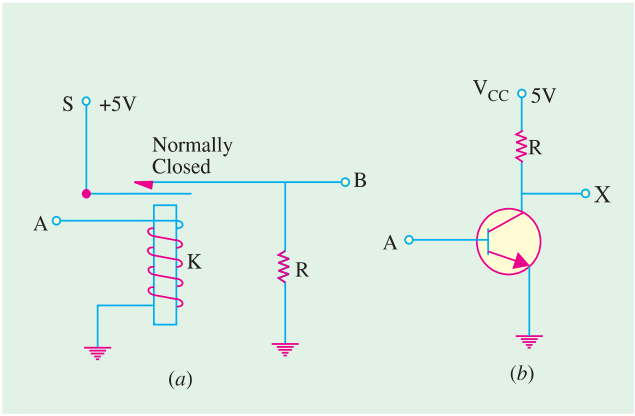


Fig. 70.22

70.16. Equivalent Circuits for a NOT Gate

The relay circuit of Fig. 70.22 (*a*) is a physical realization of the complementation operation of the Boolean algebra. When +5 V is applied to input A, K is energised and opens the ***normally-closed*** contact thereby driving output X to 0 V. Of course, when A is at 0V, X has the supply voltage of +5 V applied to it because the relay contact is normally closed.

In the equivalent transistor circuit of Fig. 70.22 (*b*) when +5V is applied to A, the transistor will be fully turned *ON*, drawing maximum ***collector*** current. Hence, whole of $V_{CC} = 5$ V will drop across R thereby sending X to 0 V. With 0 V applied at A, the transistor will be cut *OFF* and the output X, therefore, will go to V_{CC} *i.e.* + 5 V. Obviously, in each case, **output is the opposite of input**.

70.17. The NOT Operation

It is a ***complementation*** operation and its symbol is an **overbar**. It can be defined as under:

As stated earlier, $\overline{0}$ means taking the negation or complement of 0 which is 1.

$$\overline{0} = 1$$

$$\overline{1} = 0$$

It should also be noted that complement of a value can be taken repeatedly. For example,

$$\overline{\overline{1}} = \overline{0} = 1 \quad \text{or} \quad \overline{\overline{0}} = \overline{1} = 0$$

As seen double complementation gives the original value as shown in Fig. 70.23.

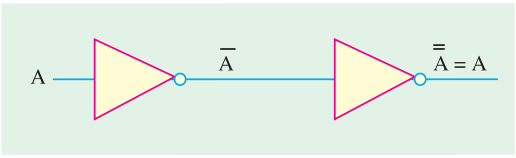


Fig. 70.23

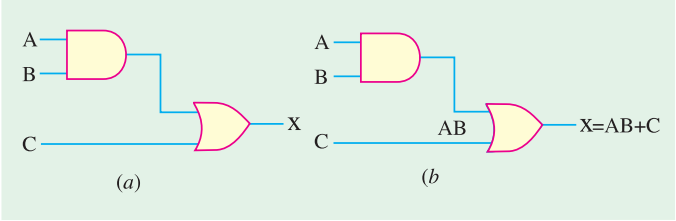


Fig. 70.24

Example 70.1. *Find the Boolean equation for the output X of Fig. 70.24 (a). Evaluate X when* ***(i)*** *A = 0, B = 1, C = 1* ***(ii)*** *A = 1, B = 1, C = 1.* **[Computer Engg., Pune Univ. 1991]**

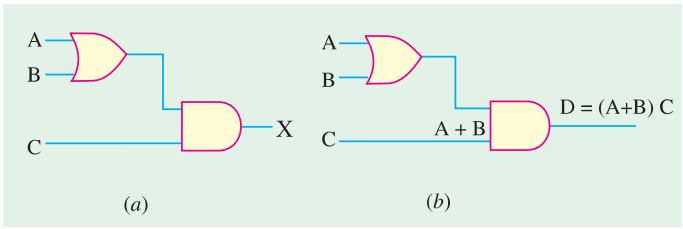


Fig. 70.25

Solution. The output of the *AND* gate is *AB*. It then becomes one of the inputs for the 2 input *OR* gate. When *AB* is ORed with *C*, we get (*AB* + C).

$\therefore \quad X = AB + C$ —Fig. 70.24 (*b*)

(i) $X = 0.1 + 1 = 0 + 1 = 1$ —Art 70.7

(ii) $X = 1.1 + 1 = 1 + 1 = \mathbf{1}$

Example 70.2. *Find the Boolean expression for the output of Fig. 70.25 (a) and evaluate it when* ***(i)*** *A = 0, B = 1, C = 1,* ***(ii)*** *A = 1, B = 1, C = 0.*

Solution. The output of the *OR* gate is (*A* + *B*). Afterwards, it becomes the input of the *AND* gate. When *AND*ed with *C*, it becomes (*A* + *B*).*C*.

$\therefore \quad X = (A + B).C$ —Fig. 70.25 (*b*)

(i) $X = (0 + 1).1 = 1.1 = 1$

(ii) $X = (1 + 1).0 = 1.0 = 0$

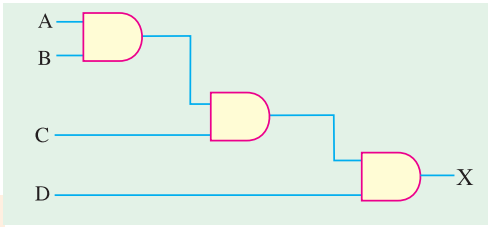


Fig. 70.26

Example 70.3. *Find the Boolean expression for the output of Fig. 70.26 and compute its value when A = B = C = 1 and X = 0.*

(Digital Computations, Punjab Univ. 1990)

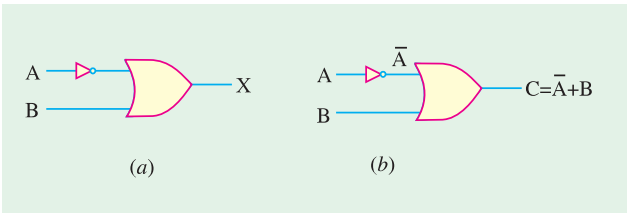


Fig. 70.27

Solution. The circuit is made up of three *AND* gates. Obviously, it is equivalent to a single 4-input *AND* gate *i.e.* an *AND* gate with a fan-in of four.

Output of the first gate is *AB*, that of the second is *ABC* and that of the third is *ABCD*. Hence, final output is *X* = *ABCD*.

Substituting the given values, we get

$X = 1.1.1.0 = 1.1.0 = 1.0 = \mathbf{0}$

Example 70.4. *Find the Boolean expression for the output X of Fig. 70.27 (a) and compute its value when*

(i) $A = 0, B = 1$

(ii) $A = 1, B = 0$

Solution. As seen, one of the inputs to the *OR* gate is inverted *i.e. A* becomes $\bar{A}$ as shown in Fig. 70.27 (*b*). Hence, output is given by $X = \bar{A} + B$

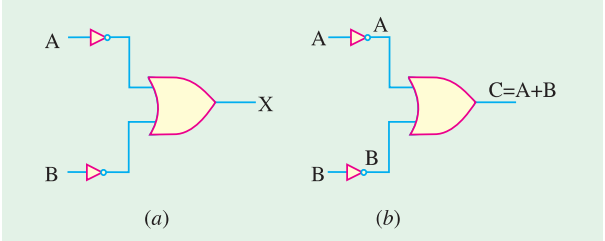


Fig. 70.28

(i) $X = \bar{0} + 1 = 1 + 1 = \mathbf{1}$ **(ii)** $X = \bar{1} + 0 = 0 + 0 = \mathbf{0}$

Example 70.5. *What is the Boolean expression for the output X of Fig. 70.28 (a) ? Compute the value of X when*

(i) $A = 0, B = 0$ *(ii)* $A = 1, B = 1$

Solution. As seen, in this case, both inputs to the OR gate have been inverted. Hence as shown in Fig. 70. 28 (*b*), the inputs become $\bar{A}$ and $\bar{B}$. Therefore, Boolean expression for the output becomes $X = \bar{A} + \bar{B}$.

(i) $X = \bar{0} + \bar{0} = 1 + 1 = \mathbf{1}$

(ii) $X = \bar{1} + \bar{1} = 0 + 0 = \mathbf{0}$

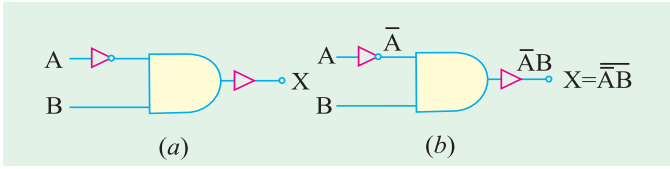


Fig. 70.29

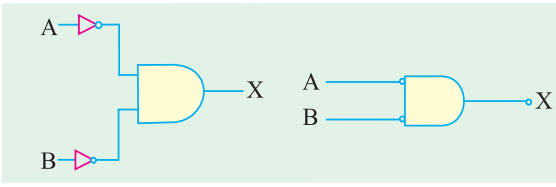


Fig. 70.30

Example 70.6. *Write down the Boolean equation for the output X of Fig. 70.29 (a). Compute its value when*

(i) $A = 0, B = 0$

(ii) $A = 0, B = 1$

(iii) $A = 1, B = 0$

(iv) $A = 1, B = 1$

Solution. As seen, inputs to the *AND* gate are A and B [Fig. 70.29 (*a*)]. The output of the *AND* gate is *A.B.* However, this output is inverted by the second inverter connected in the output. Hence, final output equation is

$X = \overline{A\,B}$

(i) $X = \overline{\bar{0}.0} = \overline{1.0} = \bar{0} = \mathbf{1}$

(ii) $X = \overline{\bar{0}.1} = \overline{1.1} = \bar{1} = \mathbf{0}$

(iii) $X = \overline{\bar{1}.0} = \overline{0.0} = \bar{0} = \mathbf{1}$

(iv) $X = \overline{\bar{1}.1} = \overline{0.1} = \bar{0} = \mathbf{1}$

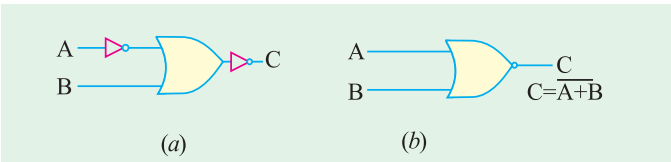


Fig. 70.31

While evaluating expressions of the above type, you must remember the following two points :

1. take the *NOT i.e.* inversion of the individual term first.
2. When a *NOT* or ***inversion*** **is** applied to more than one term (like 1.0), you should work out the *OR* (or *AND*) operation first and then take the *NOT* of the result so obtained.

70.18. Bubbled Gates

A bubbled gate is one whose inputs are *NOT*ed or inverted *i.e.* it is a negated gate. Fig. 70.30 (*a*) shows an *AND* gate whose both input are inverted.

In practice, instead of this logic symbol, the one shown in Fig. 70.30 (*b*) is widely used. As seen, the inverter triangles have been eliminated and the two small circles or bubbles have been moved to the inputs of the gate. Such a gate is called a ***bubbled AND*** gate, the bubbles acting as a reminder of the inversion or complementation that takes place before *AND*ing the inputs.

It would be shown later that a bubbled *AND* gate is equivalent to a *NOR* gate.

Table 70.6

A	B	X
0	0	1
0	1	0
1	0	0
1	1	0

Fig. 70.32

Similarly, a bubbled *OR* gate is equivalent to a *AND* gate.

70.19. The NOR Gate

In fact, it is a *NOT-OR* gate. It can be made out of an *OR* gate by connecting an inverter in its output as shown in Fig. 70.31 (*a*).

The output equation is given by

$$X = \overline{A + B}$$

A *NOR* function is just the reverse of the *OR* function.

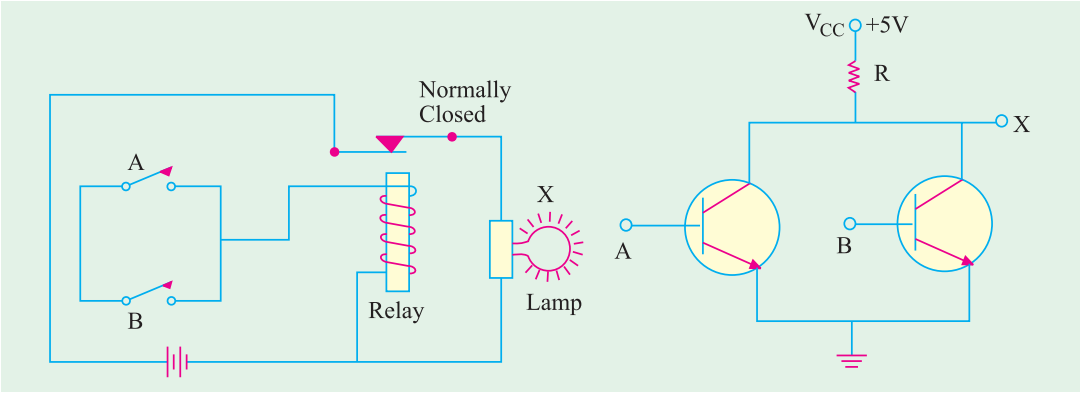


Fig. 70.33 Fig. 70.34

Logic Operation

A *NOR* gate will have an output of 1 **only when all its inputs are 0**. Obviously, if any input is **1**, the output will be 0. Alternatively, in a *NOR* gate, output is **true** only **when all inputs are false**.

The truth table for a 2-input *NOR* gate is shown in Fig. 70.32. It will be observed that the output *X* is just the reverse of that shown in Fig. 70.2.

The equivalent relay circuit for a *NOR* gate is shown in Fig. 70.33.

It is seen that the lamp glows under 00 input condition only but not under 01, 10, 11 input conditions.

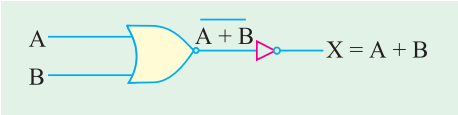


Fig. 70.35

The transistor equivalent of the *NOR* gate is shown in Fig. 70.34. As seen, output *X* is 1 only when both transistors are cut-off *i.e.* when $A = 0$ and $B = 0$. For any other input condition like 01, 10 and 11, one or both transistors saturate forcing point *X* to go to ground.

70.20. NOR Gate is a Universal Gate

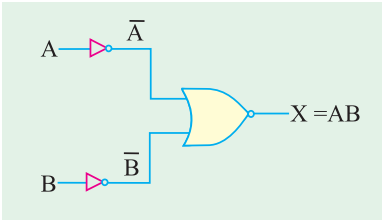


Fig. 70.36

It is interesting to note that a *NOR* gate can be used to realize the basic logic functions : *OR*, *AND* and *NOT*. That is why it is often referred to as a ***universal*** gate. Examples are given below:

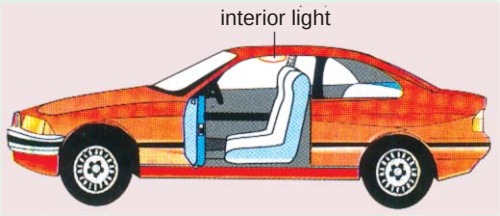
1. As OR Gate

As shown in Fig. 70.35, the output from *NOR* gate is $A + B$. By using another inverter in the output, the final output is inverted and is given by $X = A + B$ which is the logic function of a normal *OR* gate.

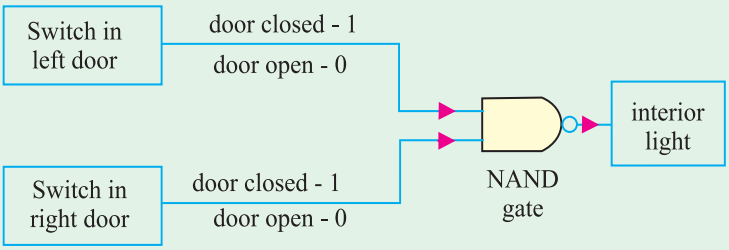
2. As AND Gate

Here, two inverters have been used, one for each input (Fig. 70.36). The inputs have, thus, been **inverted before they are applied** to the *NOR* gate. The output is $\overline{A} + \overline{B}$ which can be proved (with the help of De Morgan's theorem) to be equal to *AB*.

Incidentally, we could have used a bubbled *NOR* gate for the above purpose.



3. As NOT Gate



The NAND gate is used to design an interior lighting system of a car such that the light is switched off only when both doors are shut

The two inputs have been tied together as shown in Fig. 70.37 (*a*). The output is $\overline{A} + \overline{A}$ which can be proved to be equal to *A* with the help of De Morgan's theorem. Instead of the first symbol, the second symbol shown in Fig. 70.37 (*b*) is widely used where only single input has been used.

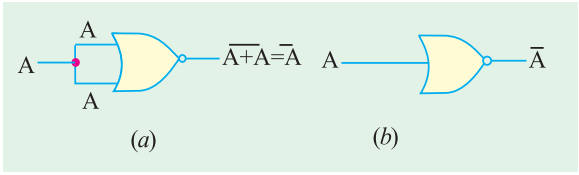


Fig. 70.37

Here is a different way of making *OR* and *AND* gates. Fig. 70.38 (*a*) shows how we can use *NOR* gates to produce an *OR* gate. Similarly, Fig. 70.38 (*b*) shows the formation of an *AND* gate from three *NOR* gates. Knowledge of De Morgan's theorem is needed to understand their logic operation.

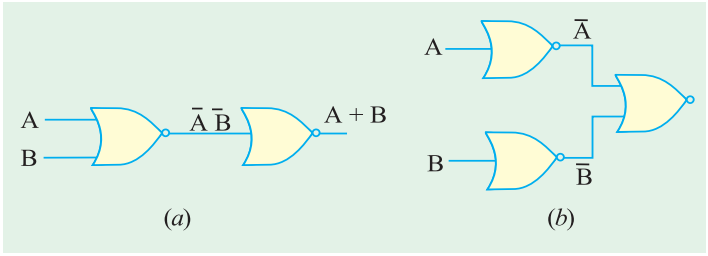


Fig. 70.38

70.21. The NAND Gate

It is, in fact, a *NOT-AND* gate. It can be obtained by connecting a NOT gate in the output of an AND gate as shown in Fig. 70.39. Its output is given by the Boolean equation.

This gate gives an output of 1 if its ***both inputs are* not 1**. In other words, it gives an output 1 if **either A or B or both are 0**.

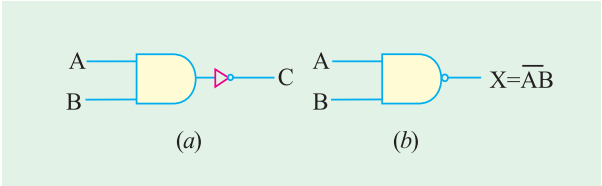


Fig. 70.39

The truth table for a 2-input *NAND* gate is given in Fig. 70.40. It is just the opposite of the truth for *AND* gate. It is so because *NAND* gate performs reverse function of an *AND* gate.

The equivalent relay circuit of a *NAND* gate is shown in Fig. 70.41.

It is seen that lamp glows under input conditions of 00, 01, 10 but not under 11 input condition

A	B	X
0	0	1
0	1	1
1	0	1
1	1	0

Fig. 70.40

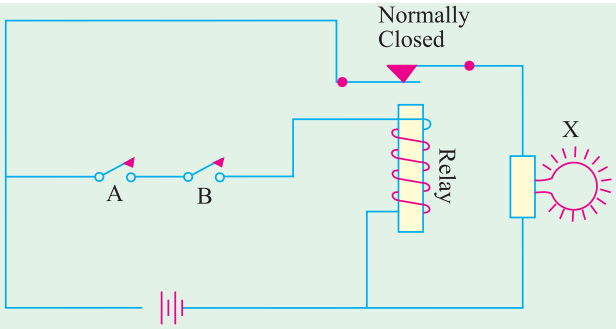


Fig. 70.41

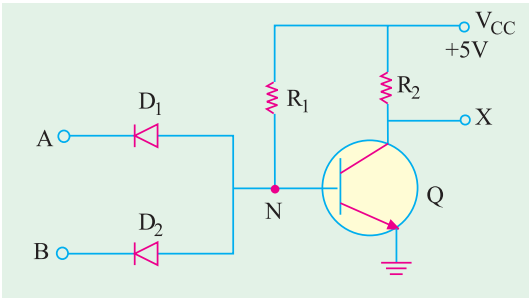


Fig. 70.42

when both switches A and B are ON. The diode-transistor equivalent of a $NAND$ gate is shown in Fig. 70.42.

It is seen that point N would be driven to ground when either D_1 or D_2 or both D_1 and D_2 conduct. It represents input conditions of 10, 01 and 11*. Under such conditions, Q is cut off and hence X goes to V_{CC} meaning logic 1 state. Only time X is 0 is when $A = 1$ and $B = 1$ (*i.e.* input voltages at A and B are at +5V) so that N is +5 V and Q is saturated.

70.22. NAND Gate is a Universal Gate

$NAND$ gate is also called universal gate because it can perform all the three logic functions of an OR gate, AND gate and inverter as shown below.

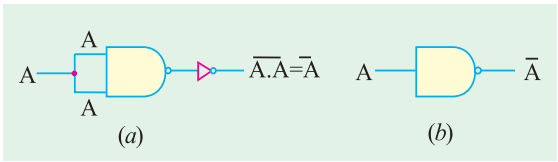


Fig. 70.43

As shown in Fig. 70.43 (*a*), a NOT gate can be made out of a $NAND$ gate by connecting its two inputs together. When a $NAND$ gate is used as a NOT gate, the logic symbol of Fig. 70.43 (*b*) is employed instead.

The use of two $NAND$ gates to produce an AND gate is shown in Fig. 70.44 (*a*).

Similarly, Fig. 70.44 (*b*) shows how OR gate can be made out of three $NAND$ gates. The OR function may not be clear from the figure because we need De Morgan's theorem to prove that $\overline{A}\,\overline{B} = A + B$.

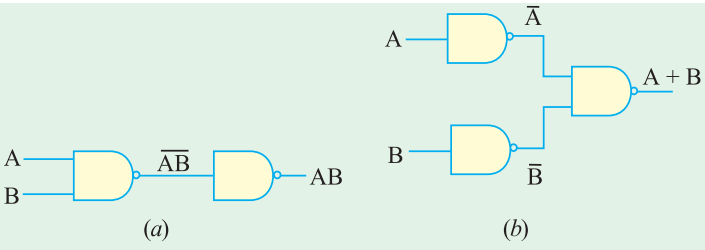
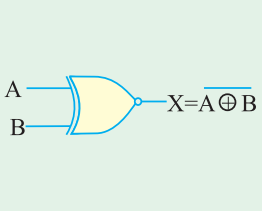


Fig. 70.44

* In this case, $V_A = V_B = 0$ V

70.23. The XNOR Gate

It is known as a not-*XOR* gate *i.e.* $\overline{XOR}$ gate. Its logic symbol and truth table are shown in Fig. 70.45.



A	B	X
0	0	1
0	1	0
1	0	0
1	1	1

Fig. 70.45

Its logic function and truth table are ***just the reverse of those for XOR gate*** (Art 70.9).

This gate has an output 1 if **its both inputs are either 0 or 1**. In other words, for getting an output, its both inputs should be ***at the* same** *logic level* of either 0 or 1. Obviously, it produces **no** output if its two inputs are at the **opposite** logic level.

70.24. Logic Gates at a Glance

In Fig. 70.46 is shown the summary of all the 2-output logic gates considered so far along with their truth tables.

Following points should prove helpful when writing these truth tables:

1. In first column *A*, logic values alternate between 0 and 1 every two rows
2. In second column *B*, logic values alternate every other row
3. Column *X* is filled up as per the logic function it performs

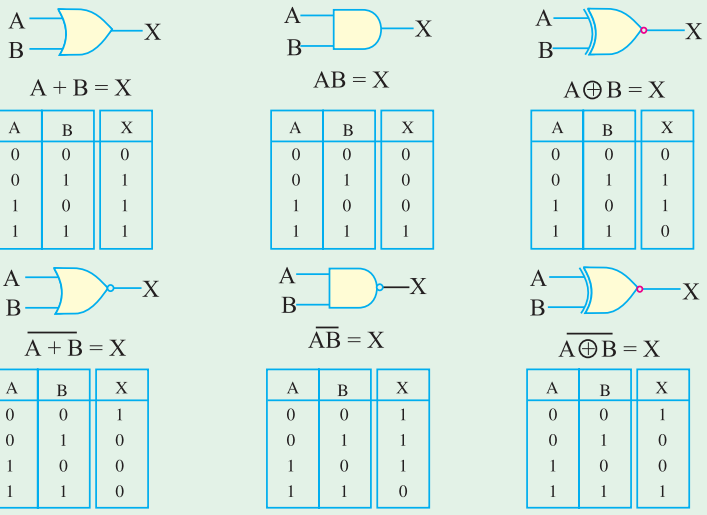


Fig. 70.46

4. Truth tables for *NOR*, *NAND* and *XNOR* (or $\overline{XOR}$) gates are ***just the opposite*** of those for *OR*, *AND* and *XOR* gates.

Example 70.7. *An electrical signal is expressed as 101011. Explain its meaning. If this signal is applied to a NOT gate, what would be the output signal ?*

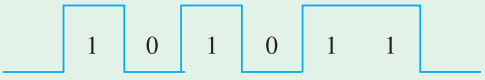


Fig. 70.47

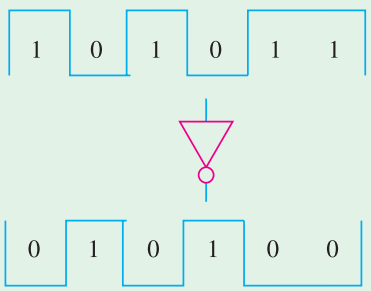


Fig. 70.48

Solution. The signal represents binary number 101011_2. It is electrically represented as a train of pulses. Taking positive logic, 1 will represent high voltage and 0 will represent low (or zero) voltage as shown in Fig. 70.47.

When such a signal is applied to a *NOT* gate, it would be inverted or complemented as shown in Fig. 70.48.

The *NOT* output will represent the binary number 010100_2.

Example 70.8. *Two electrical signals represented by A = 101101 and B = 110101 are applied to a 2-input AND gate. Sketch the output signal and the binary number it represents.*

Solution. The pulse trains corresponding to *A* and *B* are shown in Fig. 70.49.

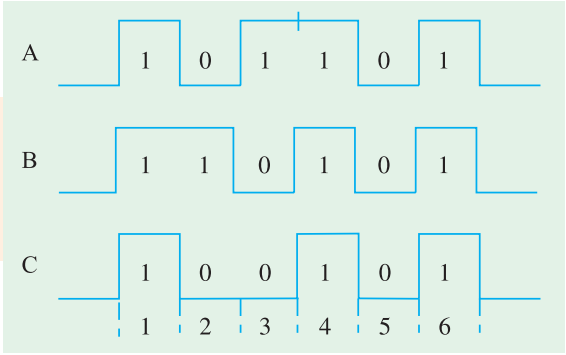


Fig. 70.49

Remember that in an *AND* gate, *C* is 1 only when both *A* and *B* are 1. It is an ***all-or-nothing gate***. The output can be found in different time intervals as under :

1. 1st interval : $1 + 1 = 1$
2. 2nd interval : $0 + 1 = 0$
3. 3rd interval : $1 + 0 = 0$
4. 4th interval : $1 + 1 = 1$
5. 5th interval : $0 + 0 = 0$
6. 6th interval : $1 + 1 = 1$

Hence, ouput of the *AND* gate is 100101_2. It is sketched in Fig. 70.49.

Example 70.9. *Convert the Boolean expression (AB + C) into a logic circuit using different logic gates.* **(Computer Engg. Pune Univ. 1992)**

Solution. In such cases, it is best to start with the ***output and work towards the input***. As seen, *C* has been *OR*ed with *AB*. Hence, the output gate must be a 2-input *OR* gate as shown in Fig. 70.50 (*a*).

Now, term *AB* is an *AND* function. Hence, we need an *AND* gate with inputs *A* and *B*. The complete logic circuit is shown in Fig. 70.50 (*b*).

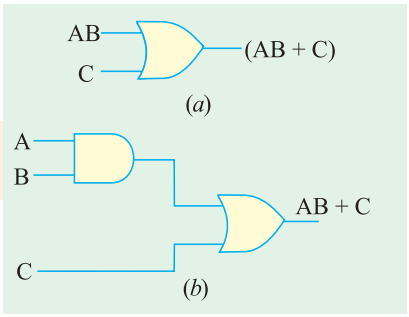


Fig. 70.50

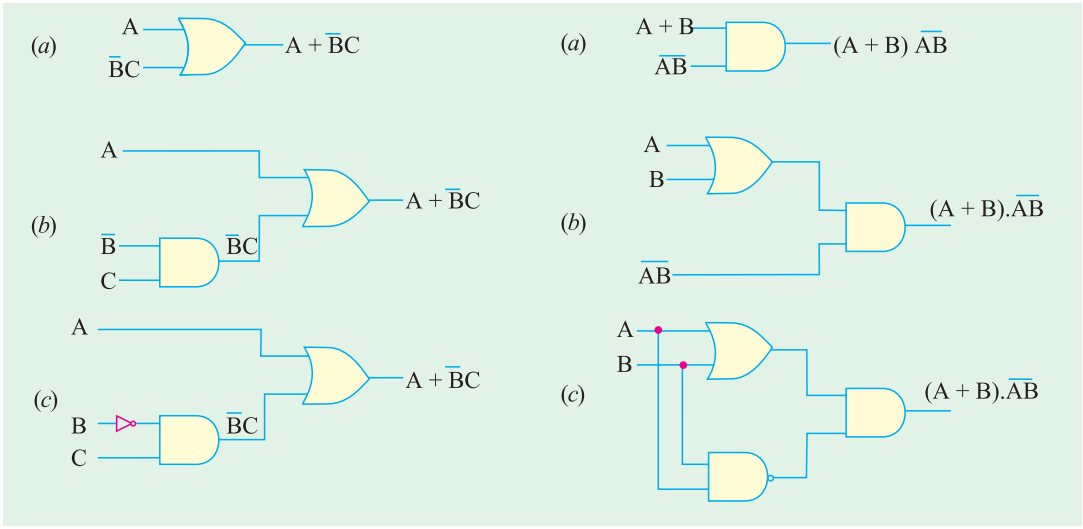
Example 70.10. *Design logic hardware based on the Boolean expression* $(A + \overline{B}C)$.

Solution. We will work from ***output to input.*** It is seen that the last gate is a 2-input *OR* gate with inputs of *A* and $\overline{B}C$. It is shown in Fig. 70.51 (*a*).

Since $\overline{B}$ has been *AND*ed with *C*, it requires an *AND* gate as shown in Fig. 70.51 (*b*). For inversion of *B*, a *NOT* gate has been used as shown in Fig. 70.51 (*c*).

Example 70.11. *Design a logic circuit whose output is given by the Boolean expression* $(A + B).\overline{AB}$. **(Computer Science, Allahabad Univ. 1992)**

Solution. Working from output to input, we find that the output gate has to be a 2-input *AND* gate with inputs of $(A + B)$ and $\overline{AB}$. The first step of the circuit design is shown in Fig. 70.52 (*a*). It is also seen that the input to the entire circuit consists of *A* and *B* only.



Fi.g 70.51

Fig. 70.52

The input of $(A + B)$ has been obtained with the help of an *OR* gate as shown in Fig. 70.52 (*b*).

Finally, a *NAND* gate is connected **in parallel** with the *OR* gate for getting its inputs of *A* and *B* and thereafter for supplying an output of $\overline{AB}$. The complete circuit is shown in Fig. 70.52 (*c*).

70.25. Digital Signals Applied to Logic Gates

A binary digital signal applied to a logic gate is nothing else but the application of a time sequence of 1's and 0's. The response of *AND* and *OR* gates to various periodic digital signals is shown in Fig. 70.53.

Fig. 70.54 shows how two waveforms can be *OR*ed by using two input gates.

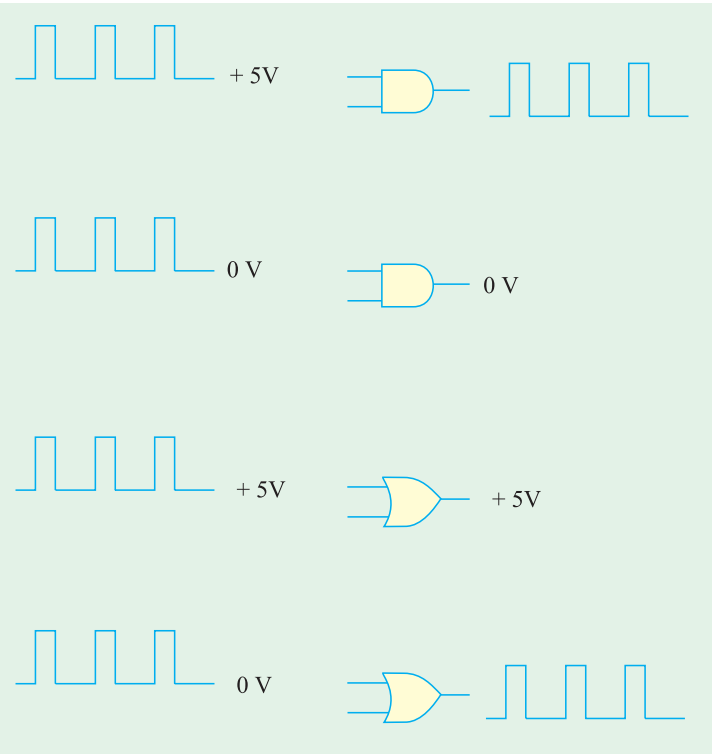


Fig. 70.53

70.26. Applications of Logic Gates

Range of application of the logic gates is very wide but the main headings would include.

1. to build more complex devices like binary counters etc.,
2. for decision making in automatic control of machines and various industrial processes,
3. in calculators and computers,
4. in digital measuring techniques,

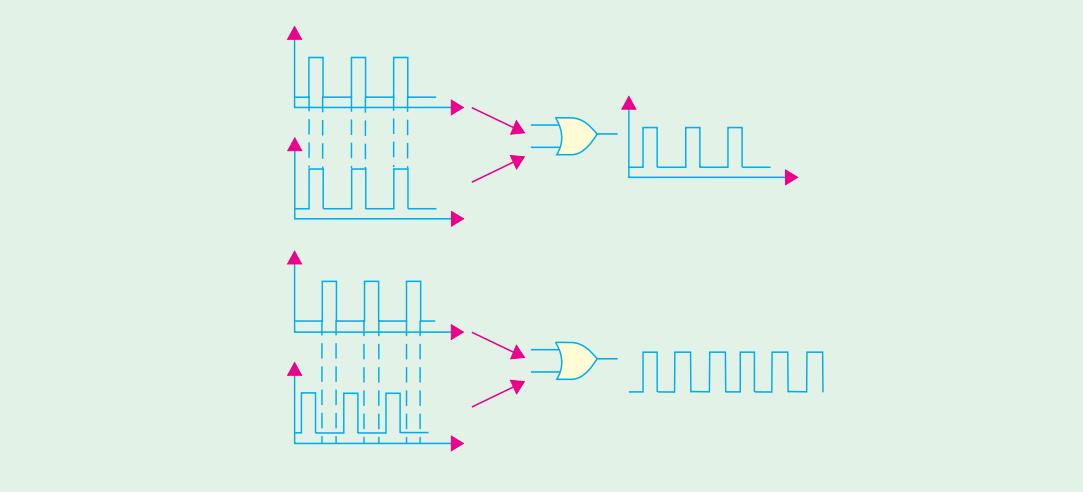


Fig. 70.54

5. in digital processing of communications,
6. in musical instruments, games and domestic appliances etc.

Sometimes it is more convenient to show the digital signals or pulse waveforms without the *x*- and *y*-axes. The examples below will follow this practice.

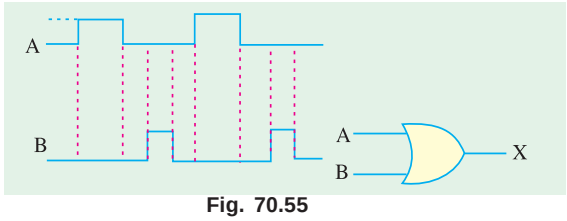


Fig. 70.55

Example 70.12. *Fig. 70.55 shows an OR gate with two input waveforms. What is the resulting output waveform ?*

Solution. Remember that the output of an OR gate is 1 when either or both inputs are 1. Therefore, we can sketch the output wave form, *C* as shown in Fig. 70.56.

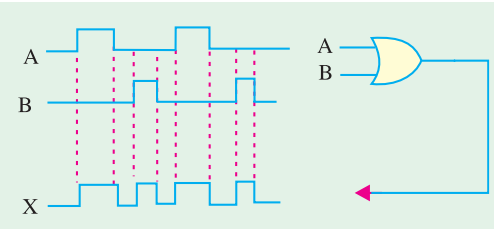


Fig. 70.56

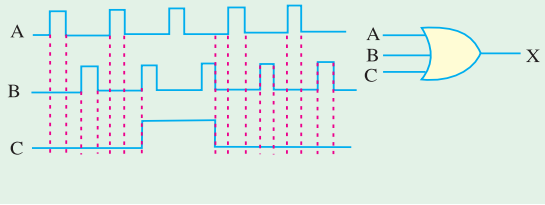


Fig. 70.57

Example 70.13. *For a three-input OR gate shown in Fig. 70.57 determine the output waveform.*

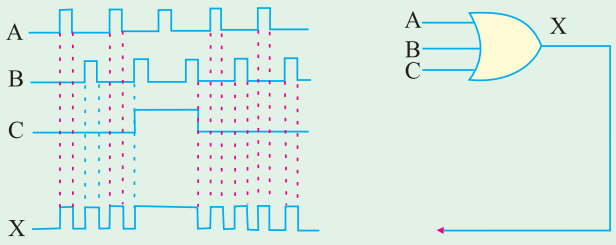


Fig. 70.58

Solution. Remember the output of a three-input *OR* gate is 1 when one or more of the inputs are

1. Therefore, we can sketch the output waveform, *D* as shown in Fig. 70.58.

Example 70.14. *Fig. 70.59 shows a 2-input AND gate with waveforms A and B. Sketch the resulting output waveform.*

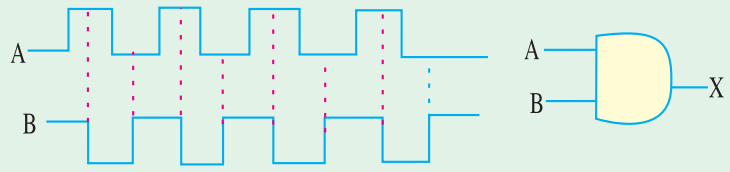


Fig. 70.59

Solution. Remember the *AND* gate produces an output 1 only when all its inputs are present. Thus the output of *AND* gate is 1 when both *A* and *B* are 1. Its output is 0 when any of its inputs is 0. Using this concept, we can sketch the output waveform as shown in Fig. 70.60.

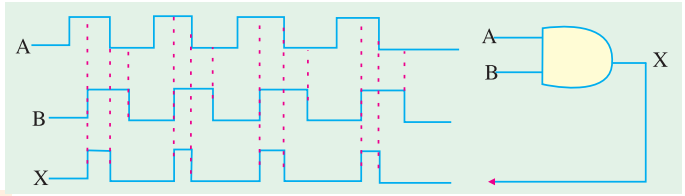


Fig. 70.60

Example 70.15. *For a 3-input AND gate with waveforms A, B and C at its inputs as shown in Fig. 70.61, determine the resulting output waveform.*

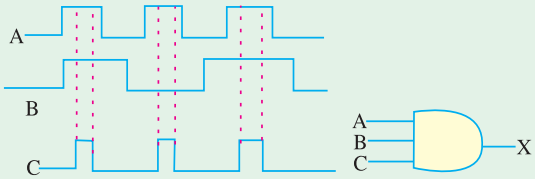


Fig. 70.61

Solution. A 3-input AND gate produces an output 1 only when all the three inputs *A*, *B* and *C* are 1. Its output is 0 when any one of three inputs is 0. Using this concept, the resulting output waveform is as shown in Fig. 70.62.

Example 70.16. *Fig. 70.63 shows the two waveforms applied to the NOR gate inputs. Sketch the resulting output waveform.*

Solution. Remember, a *NOR* gate will have an output of 1 only when all its inputs are 0. Obviously, if any input is 1, the output will be 0. Using this concept, we can sketch the output waveform as shown in Fig. 70.64.

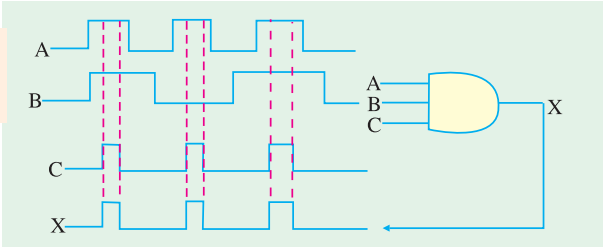


Fig. 70.62

Example 70.17. *Sketch the output waveform for a 3-input NOR gate shown in Fig. 70.65. Showing the proper time relationship to the inputs.*

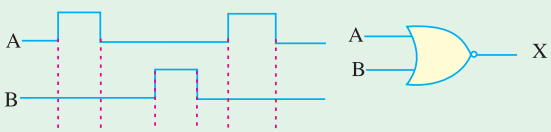


Fig. 70.63

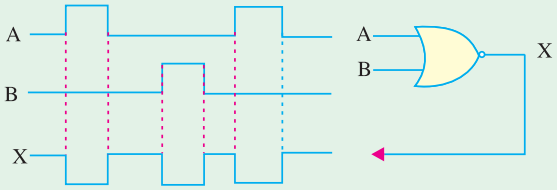


Fig. 70.64

Solution. Remember, a 3-input *NOR* gate will have an output of 1 only when all the three inputs *A*, *B* and *C* are 0. Obviously if any one of the three inputs *A*, *B* and *C* or all are 1, the output will be 0.

Using this concept, we can sketch the output waveform as shown in Fig. 70.66.

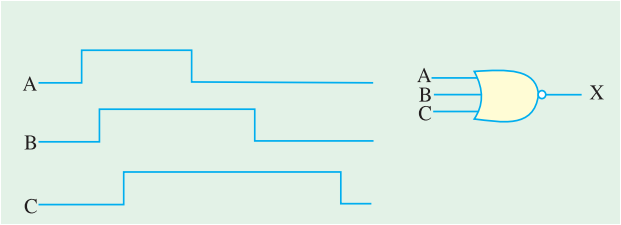


Fig. 70.65

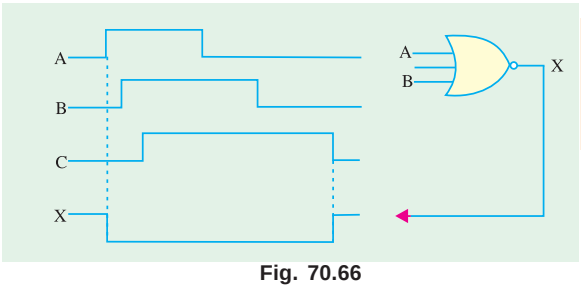


Fig. 70.66

Example 70.18. *Fig. 70.67 shows the two waveforms A and B applied to the NAND gate inputs. Determine the resulting output waveform.*

Solution. Remember, the output of *NAND* gate is 1 if either *A* or *B* or both are 0. Using this concept, we can sketch the output waveform as shown in Fig. 70.68.

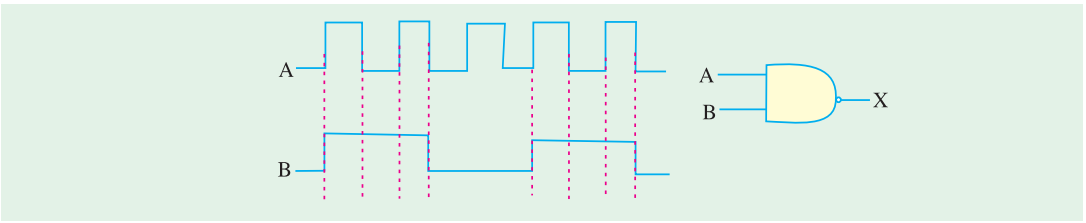


Fig. 70.67

Example 70.19. *Sketch the output waveform for a 3-input NAND gate shown in Fig. 70.69*

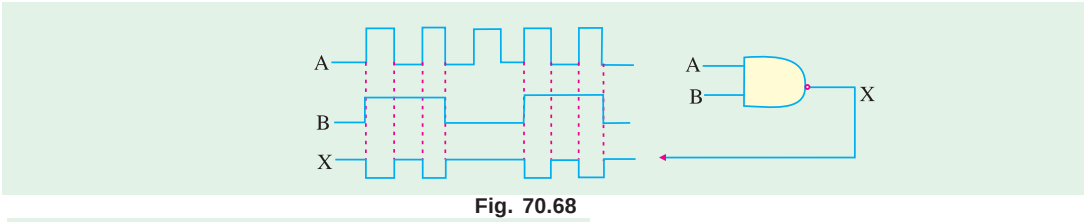


Fig. 70.68

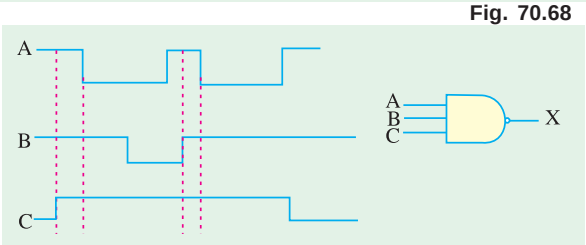


Fig. 70.69

Solution. Remember, the output of a 3-input NAND gate is 1 only if either any one of the inputs A, B and C or all are 0. Using this concept, we can sketch the output waveform as shown in Fig. 70.70.

Example 70.20. *The waveforms A and B are applied as an input to the XOR and XNOR gate as shown in Fig. 70.71. Determine the output waveforms of these logic gates.*

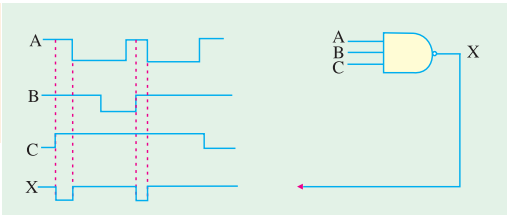


Fig. 70.70

Solution. In exclusive–*OR* (*XOR*) gate, output is 1 if its either input but not both, is 1. In other words, it has an output 1 when its inputs are different. The output is 0 only when inputs are the same. Using this concept, we can sketch the *XOR* output waveform as shown in Fig. 70.72.

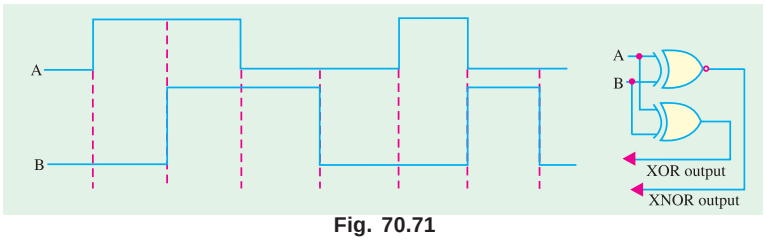


Fig. 70.71

In *XNOR* gate, output is 1 if its both inputs are either 0 or 1. It produces 0 output if its two inputs are at the opposite logic level. Using this concept, we can sketch the *XNOR* output waveform as shown in Fig. 70.72

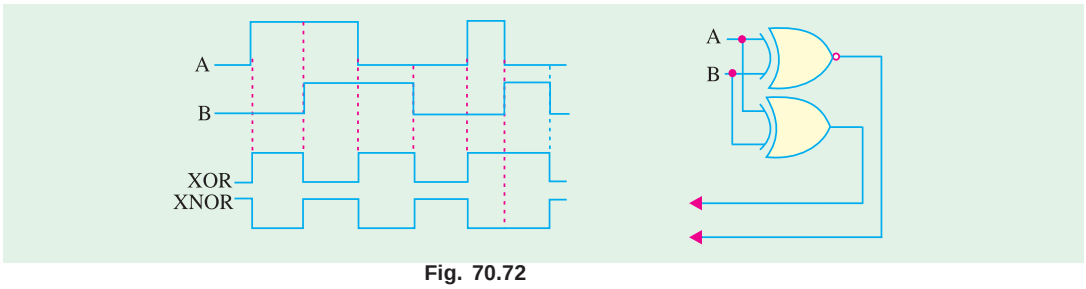


Fig. 70.72

70.27. Combinational Logic Circuit

It is a circuit built from various logic gate combinations. The circuit possesses a set of inputs, a ***memoryless*** logic network to operate on the inputs and a set of outputs as shown in Fig. 70.73 (*a*). The output from a combinational logic circuit depends solely on the ***present*** input values and not on the previous ones. Moreover, output combinational networks are used to make logical decisions and control the operation of different circuits in digital electronic systems. For a given set of input conditions, the output of such a circuit is the same. Consequently, a truth table can fully describe the

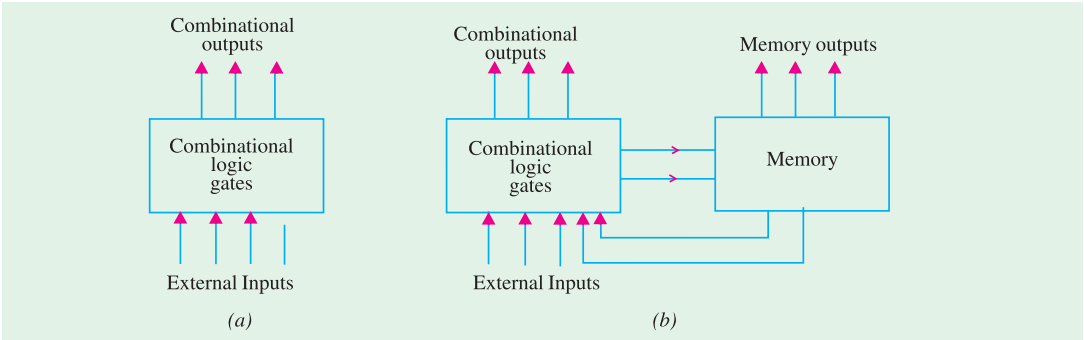


Fig. 70.73

operation of such a circuit. Examples of such a circuit are : decoders, adders, multiplexer and demultiplexers etc.

70.28. Sequential Logic Circuits

Such circuits have inputs, logic network, outputs and a memory as shown in Fig. 70.73 (*b*). Their present output depends not only on their present inputs but also on the previous logic states of the outputs.

Examples of such circuits are a variety of latches and flip-flops. Sequential logic circuits may be either synchronous or asynchronous. The synchronous sequential circuits are built to operate at a clocked rate whereas asynchronous ones are without clocking.

70.29. Adders and Subtractors

The logical gates discussed so far can be used for performing arithmetical functions like addition, subtraction, multiplication and division in electronic calculators and digital instruments. In the central processing unit (*CPU*) of a computer, these arithmetic functions are carried out by the arithmetic and logic unit (*ALU*). The logic functions used generally are *XOR*, *OR* and *AND*. We will consider the following:

1. Half Adder—it is 1-bit adder and carries out binary addition with the help of *XOR* and *AND* gates. It has **two** inputs and **two** outputs.
2. Full Adder—It has **three** inputs and can add three bits at a time. It is made up of ***two half adders and one OR gate.***
   These adders can also perform subtraction by the method of 1's and 2's complements.
3. Half Subtractor—it uses one *XOR* and one *AND* gate.
4. Full Subtractor—it employs two half subtractors and one *OR* gate.

70.30. Half Adder

It can add 2 binary digits **at a time** and produce a 2-bit data *i.e.* sum and carry according to the binary addition rules (Art. 70.10).

Block Diagram

It is shown in Fig. 70.74. As can be seen, it has two inputs for applying the two binary digits to be added. As is well known, binary addition of two bits always produces 2-bit output data *i.e.* one *SUM* and one *CARRY*. For example, (1 + 1) gives a sum of 0 and a carry of 1. Also, (0 + 0) gives the sum 0 and carry 0. That is why the adder has two outputs : one for *SUM* and the other for *CARRY*.

Table 70.7

Input		Output	
A	*B*	*S*	*C*
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

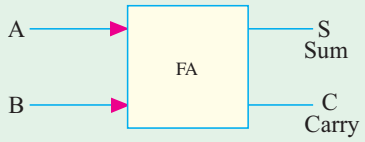


Fig. 70.74

Truth Table 70.7 lists the two columns of input, one of *SUM* and one of *CARRY*. The *SUM* output has the same logic pattern as when *A* is *XOR*ed with *B*. In fact, ***to add it to XOR.*** Also, the *CARRY* output has the same logic pattern as when *A* is *AND*ed with *B*. That is why a half-adder can be formed from a combination of one *XOR* gate and one *AND* gate as shown in Fig. 70.75.

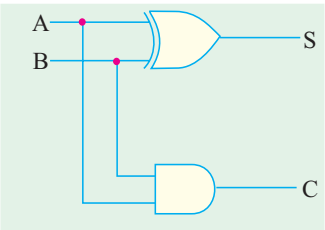
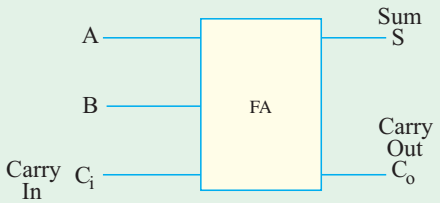


Fig. 70.75

The circuit is called ***half-adder*** because it cannot accept a carry-in from previous additions. For that purpose, we need a 3-*input* adder called full-adder.

Incidentally, the logical equations for the *SUM* and *CARRY* are S = A ⊕ B and $C = A.B$



Input			Output	
A	B	C_i	S	C_o
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Fig. 70.76

70.31. Full Adder

As shown in the block diagram of Fig. 70.76, it has ***three inputs and two outputs.*** It can add 3 digits (or bits) **at a time**. The bits *A* and *B* which are to be added come from the two registers and the third input comes from the carry generated by the previous addition. It produces two outputs; *SUM* and *CARRY-OUT*.

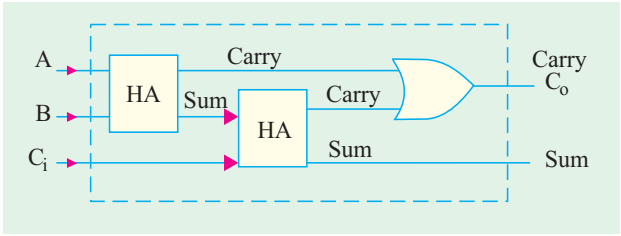


Fig. 70.77

The truth table 70.9 gives all possible input/output relationships for the full adder. *A* and *B* are the inputs from the respective digits of the registers to be added and C_i is the input for any carry generated by the previous stage. The *SUM* output gives binary addition of *A*, *B* and C_i. The other output generates the carry C_0 to be added to the next stage.

The full-adder can be constructed from two half-adders and one *OR* gate (Fig. 70.77).

Working. Let us illustrate, with the help of two examples, how this full adder adds three bits.

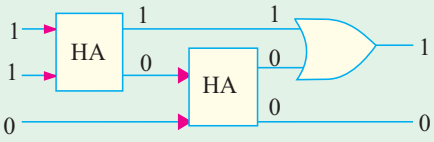


Fig. 70.78

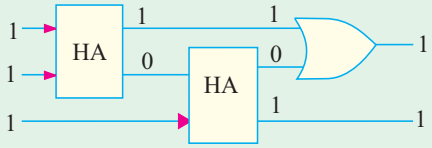


Fig. 70.79

(*i*) A = 1, B = 1, Ci = 0

The full adder with these three inputs is shown in Fig. 70.78. First half adder gives a sum of 0 and a carry of 1. The second *HA* gives a sum of 0 with a carry of 0. The final output is : *SUM* 0, *CARRY* **1.** As we know from the rules of binary addition, $1 + 1 + 0 = 10_2$ (*i.e* decimal 2).

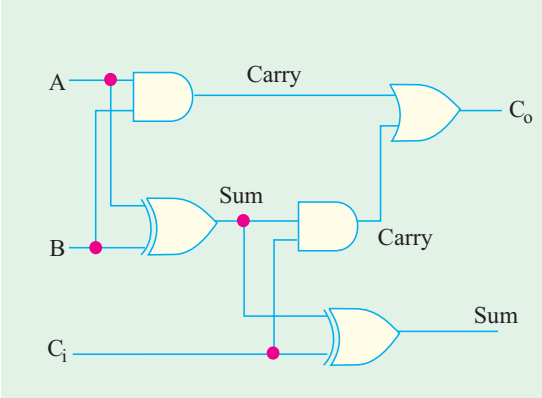


Fig. 70.80

(ii) **A = 1, B =1, C = 1**

As detailed in Fig. 70.79, we get a final *SUM* 1 with a *CARRY* 1. The result conforms to the binary addition : $1 + 1 + 1 = 11_2$ (*i.e.* decimal 3)

Detailed Circuit

Fig. 70.80 shows more circuit details of a full adder. The two half adders have been replaced by their *XOR* and *AND* gates. The final carry is given by the *OR* gate and final sum by the *XOR* gate of the second adder.

70.32. Parallel Binary Adder

For adding two 4-bit numbers, we need 4 full adders *connected in parallel* as shown in Fig. 70.81. The two numbers being added are $A_3\,A_2\,A_1\,A_0$ and $B_3\,B_2\,B_1\,B_0$ and their sum is $S_4\,S_3\,S_2\,S_1\,S_0$.

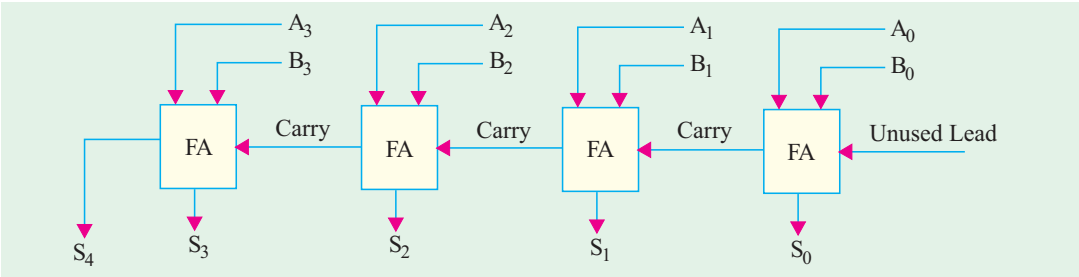


Fig. 70.81

The first adder could be a half adder though we may use a full adder but leave its *CARRY*-IN lead unconnected. As seen, different bits are fed to the four adders from two parallel registers which hold these bits. The final *SUM* appears as a 5-digit display.

Operation

The actual operation may be better understood with the help of the diagram of Fig. 70.82. Suppose, we want to add the following two 4-bit numbers.

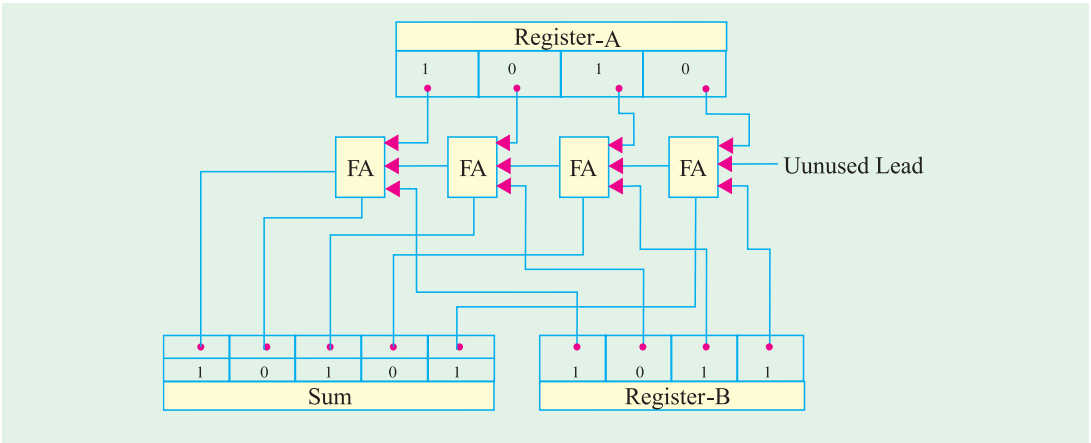


Fig. 70.82

$$\begin{array}{r} 1010 \\ +1011 \\ \hline 10101 \end{array} \qquad \begin{array}{r} 10 \\ +11 \\ \hline 21 \end{array}$$

The first adder performs 0 + 1 binary addition, giving a sum of 1 and a carry of 0. The two bits 0 and 1 are supplied ***simultaneously*** from the two registers A and B. The sum 1 appears on the display panel and carry 0 is passed on to the next full adder.

The next adder adds 1 + 1 +0 carry = sum 1 with a carry 1. The third adder performs 0 + 0 + 1 carry = 1 with carry 0. The fourth adder adds 1 + 1 + 0 carry = sum 0 with carry 1 both of which appear on the display unit. Hence, the final addition of the two numbers appears as 10101.

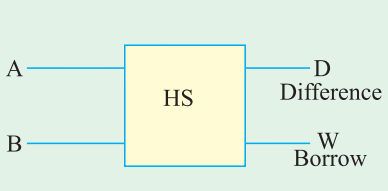


Table 70.10

Input		Output	
A	B	W	D
0	0	0	0
0	1	1	1
1	0	0	1
1	1	0	0

Fig. 70.83

It may be noted that the largest binary numbers that can be added by this parallel adder are 1111 and 1111 which give a sum of 11110_2 (decimal 30). To increase its capacity, more full adders may be connected at the left end of Fig. 70.82. For example, for adding 6-bit numbers, we will have to add two more adders thus making a total of **six**.

70.33. Half Subtractor

It can subtract only two binary digits ***at a time*** and produce an output of a difference and a borrow. As shown in the block diagram of Fig. 70.83, it has two inputs and two outputs.

The operation of a half subtractor is based on the rules of binary subtraction illustrated in the truth Table 70.10 for all possible input/output combinations. The difference output in the fourth column has the same logic pattern as when A is XORed with B (same was the case for SUM in Art. 70.30). Hence, we can use an XOR gate to get the difference of two bits. The borrow output in the third column can be obtained by ANDing $\overline{A}$ with B.

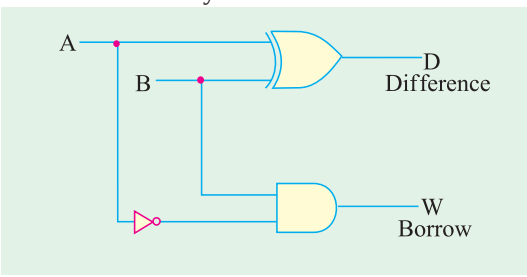


Fig. 70.84

The circuit for a half subtractor is shown in Fig. 70.84.

As mentioned earlier, the logical equations for the difference and borrow are given by

$D = A \oplus B$ and $W = \overline{A}\,B$

70.34. Full Subtractor

As shown in the block diagram of Fig. 70.85, it has three inputs and two outputs. As explained above, half subtractor can handle only 2 bits at a time and can be used for the least significant column of a subtraction problem. A full subtractor can, however, take care of higher-order columns.

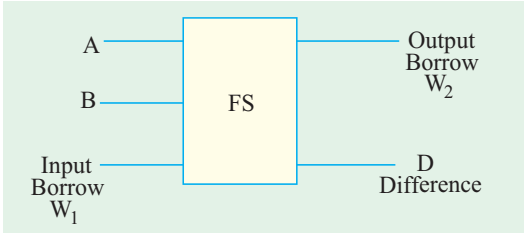


Fig. 70.85

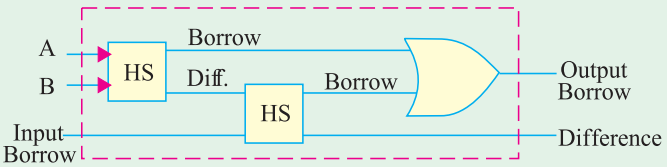


Fig. 70.86

As shown in Fig. 70.86, a full subtractor consists of two half subtractors and one *OR* gate.

It may be remarked here that by cascading 4 full subtractors, we can directly subtract 4-bit numbers *i.e.* we can subtract $B_3\ B_2\ B_1\ B_0$ from $A_3 A_2 A_1 A_0$.

Tutorial Problems No. 70.1

1. Find the Boolean equation for the output of the logic circuit shown in Fig. 70.87. What would be the output if $A = 1$, $B = 0$, $C = 1$, $D = 1$ ***[X = AB + CD; 1]***

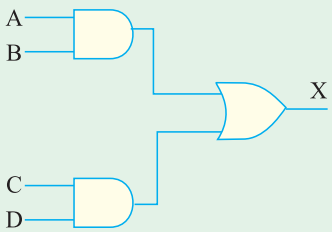


Fig. 70.87

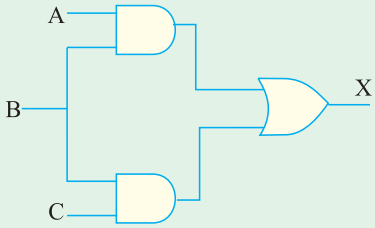


Fig. 70.88

2. What is the output equation of the logic circuit shown in Fig. 70.88 ? Evaluate the output if $A = 1$, $B = 1$, $C = 0$. ***[X= AB + (B + C); 1]***

3. After finding the Boolean equation for the circuit shown in Fig. 70.89, compute the output if $A = 1$, $B = 0$, $C = 1$, $D = 0$. ***[X = (A + B) (C + D) ; 1]***

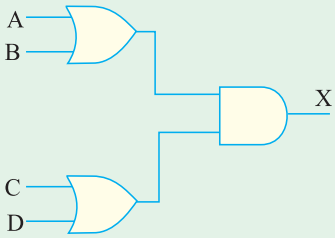


Fig. 70.89

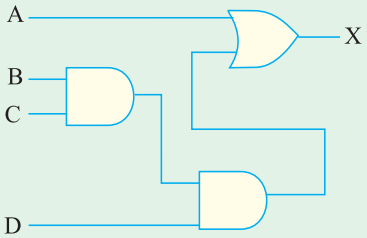


Fig. 70.90

4. Translate the hardware shown in Fig. 70.90 into a Boolean expression. Compute the value of the output if $A = 0$, $B = 1$, $C = 0$, $D = 1$. ***[X = A + BCD ; 0]***

5. What is the Boolean expression for the logic diagram shown in Fig. 70.91? Evaluate its output if $A = 1$, $B = 1$, and $C = 1$. ***[X= AB + C; 1]***

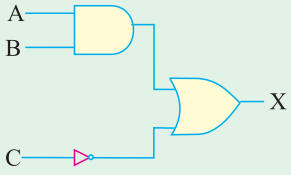


Fig. 70.91

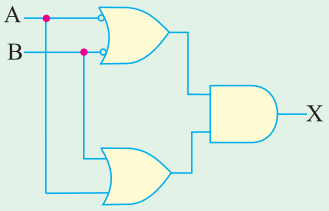


Fig. 70.92

6. Find Boolean expression for the logic circuit of Fig. 70.92. What is the output if $A = 1$, $B = 1$?

[$X = (\overline{A} + \overline{B}).\ (A + B)\ ;\ 0$]

7. Give the logic functions performed by the circuits shown in Fig. 70.93 (*a*), (*b*) and (*c*).

[(*a*) $X = \overline{AB}.\overline{(C + D)}$ (*b*) $X = \overline{AB.C}$ (*c*) $\overline{(A + B)CD}$]

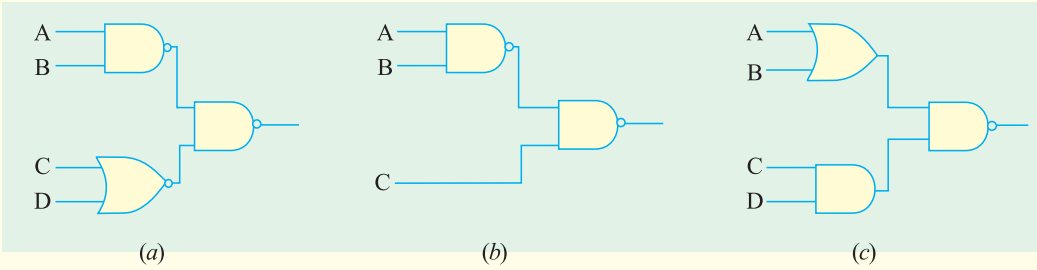


Fig. 70.93

8. State the logic functions of the circuits shown in Fig. 70.94 (*a*), (*b*) and (*c*).

[(*a*) (A + B).(CD) (*b*) AB + CD (*c*) (A + B). C.D.E]

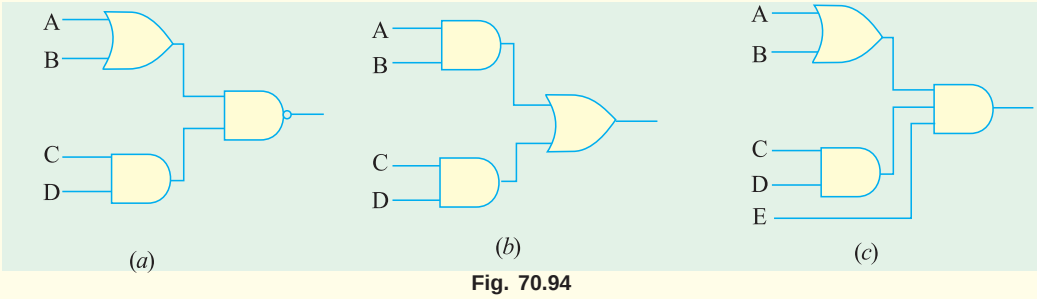


Fig. 70.94

9. State the logic functions performed by the circuits of Fig. 70.95 (*a*), (*b*) and (*c*).

[(*a*) $\overline{AB}.\overline{(C + D)}$ (*b*) $\overline{AB + C}$ (*c*) A + B]

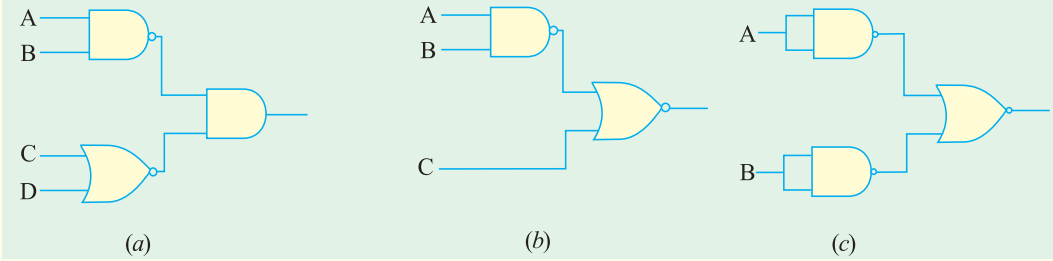


Fig. 70.95

10. Write the logic functions for the circuits shown in Fig. 70.96 (*a*), (*b*) and (*c*).

[(*a*) A(B + C) + CD (*b*) $\overline{A + B.A.C}$ (*c*) $A\overline{B} + \overline{C}D$]

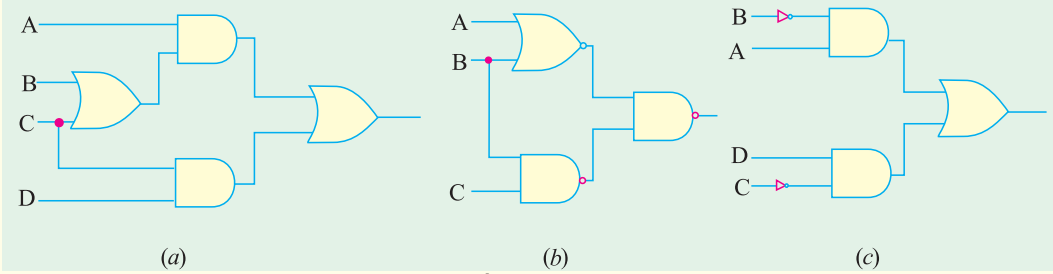


Fig. 70.96

11. What would be the output signal if two signals $A = 101011_2$ and $B = 110101_2$ are applied to the inputs of an *AND* gate? **[100 001]2**

12. What would be the output signal if two input binary signals given by $A = 100101$ and $B = 110110$ are applied to (*a*) *OR* gate (*b*) *NAND* gate and (*c*) *XNOR* gate?

[(*a*) 1101112 (*b*) 0110112 (*c*) 1011002]

13. Sketch the output waveform at *C*-for a 2-input *OR* gate shown in Fig. 70.97 and with the given *A* and *B* input waveforms.

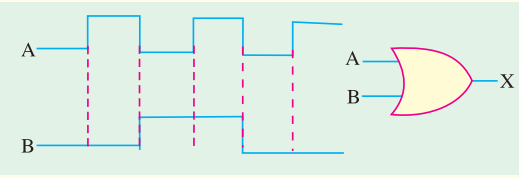


Fig. 70.97

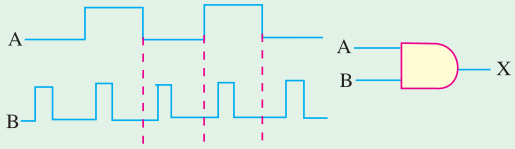


Fig. 70.98

14. Sketch the output waveform of a 2-input *AND* gate shown in Fig. 70.98 with the input waveforms *A* and *B*.

15. Sketch the output waveform at *D* for a 3-input *AND* gate shown in Fig. 70.99, with the given *A*, *B* and *C* input waveforms.

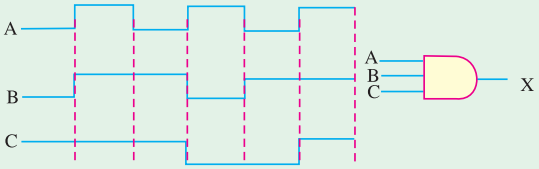


Fig. 70.99

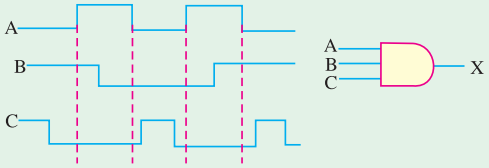


Fig. 70.100

16. Sketch the output waveform at *D* for a 3-input *AND* gate shown in Fig. 70.100, with the given *A*, *B* and *C* input waveforms.

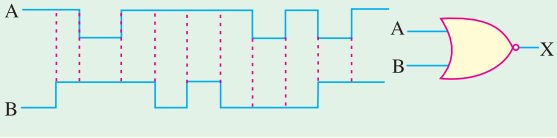


Fig. 70.101

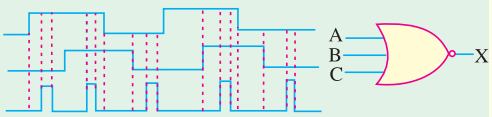


Fig. 70.102

17. Sketch the output waveform at *C* for a 2-input *NOR* gate shown in Fig. 70.101, with the given *A* and *B* input waveforms.

18. Sketch the output waveform at *D* for a 3-input *NOR* gate shown in Fig. 70.102, with the given *A*, *B* and *C* input waveforms.

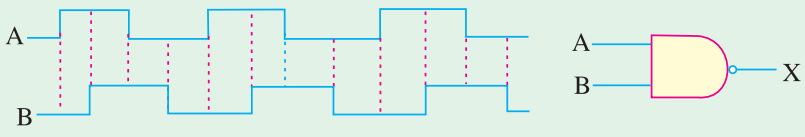


Fig. 70.103

19. Sketch the output waveform at *C* for a 2-input *NAND* gate shown in Fig. 70.103, with the given *A* and *B* input waveforms.

20. Sketch the output waveform at *D* for a 3-input *NAND* gate shown in Fig. 70.104, with the given *A*, *B* and *C* input waveforms.

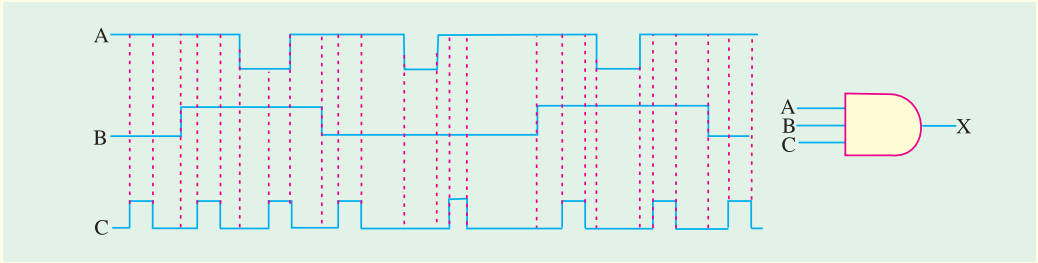


Fig. 70.104

OBJECTIVE TESTS – 70

1. A logic gate is an electronic circuit which
(*a*) makes logic decision
(*b*) allows electron flow only in one direction
(*c*) works on binary algebra
(*d*) alternates between 0 and 1 values.

2. In positive logic, logic state 1 corresponds to
(*a*) positive voltage
(*b*) higher voltage level
(*c*) zero voltage level
(*d*) lower voltage level.

3. In negative logic, the logic state 1 corresponds to
(*a*) negative voltage
(*b*) zero voltage
(*c*) more negative voltage
(*d*) lower voltage level.

4. The voltage levels of a negaive logic system
(*a*) must necessarily be negative
(*b*) may be negative orpositive
(*c*) must necessarily be positive
(*d*) must necessarily be 0V and –5V

5. The output of a 2-input *OR* gate is zero only when its
(*a*) both inputs are 0
(*b*) either input is 1
(*c*) both inputs are 1
(*d*) either input is 0.

6. An *XOR* gate produces an output only when its two inputs are
(*a*) high (*b*) low
(*c*) different (*d*) same.

7. An AND gate
(*a*) implements logic addition
(*b*) is equivalent to a series switching circuit
(*c*) is an any-or-all gate
(*d*) is equivalent to a parallel switching circuit.

8. When an input electrical signal A = 10100 is applied to a NOT gate, its output signal is
(*a*) 01011 (*b*) 10101
(*c*) 10100 (*d*) 00101.

9. The only function of a NOT gate is to
(*a*) stop a signal
(*b*) recomplement a signal
(*c*) invert an input signal
(*d*) act as a universal gate.

10. A NOR gate is ON only when all its inputs are
(*a*) ON (*b*) positive
(*c*) high (*d*) OFF.

11. For getting an output from an XNOR gate, its both inputs must be
(*a*) high (*b*) low
(*c*) at the same logic level
(*d*) at the opposite logic levels.

12. In a certain 2-input logic gate, when $A = 0$, $B = 0$, then $C = 1$ and when $A = 0$, $B = 1$, then again $C = 1$. It must be ..... gate.
(*a*) XOR (*b*) AND
(*c*) NAND (*d*) NOR

13. The logic symbol shown in Fig. 70.105 represents
(*a*) single-output AND gate
(*b*) NAND gate
(*c*) NAND gate used as NOT gate
(*d*) NOR gate.

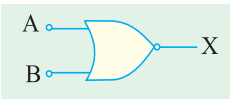


Fig. 70.105

14. The output from the logic gate shown in Fig. 70.106 will be available when inputs ..... are present.

(*a*) *A* and *C*

(*b*) *B* and *C*

(*c*) *A*, *B* and *C*

(*d*) *A* and *B*

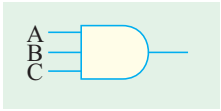


Fig. 70.106

15. To get an output 1 from circuit of Fig. 70.107, the input must be *A B C*

(*a*) 0 1 0 (*b*) 1 0 0

(*c*) 1 0 1 (*d*) 1 1 0

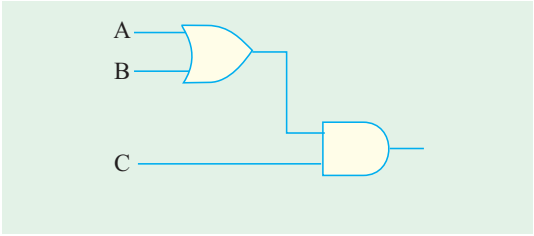


Fig. 70.107

16. Which of the following logic gates in Fig. 70.108 will have an output of 1 ?

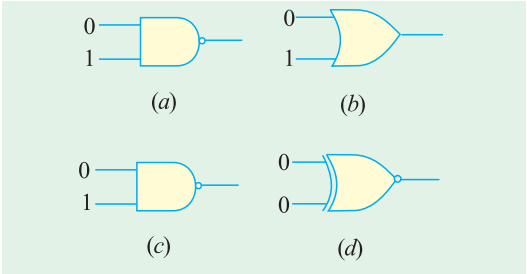


Fig. 70.108

17. A half-adder can be constructed from

(*a*) two XNOR gates only

(*b*) one XOR and one OR gate with their outputs connected in parallel

(*c*) one XOR and one OR gate with their inputs connected in parallel

(*d*) one XOR gate and one AND gate

18. The digital equivalent of an electric series circuit is the ..... gate.

(*a*) NOR (*b*) NAND

(*c*) OR (*d*) AND

19. Which of the following represents analog data ?

(*a*) ON and OFF states (*b*) 0 and 1

(*c*) 0V and 5V (*d*) 1.5, 3.2, 4 and 5V

20. The logic gate which produces a 0 or low-level output when one or both of the inputs are 1 is called ..... gate.

(*a*) AND (*b*) OR

(*c*) NOR (*d*) NAND

21. The output X of the gated network shown in Fig. 70.109 is

(*a*) $\overline{AB} \cdot \overline{CD} \cdot \overline{EF}$

(*b*) $\overline{AB} + \overline{CD} + \overline{EF}$

(*c*) AB + CD + EF

(*d*) (A + B) (C + D) (E + F)

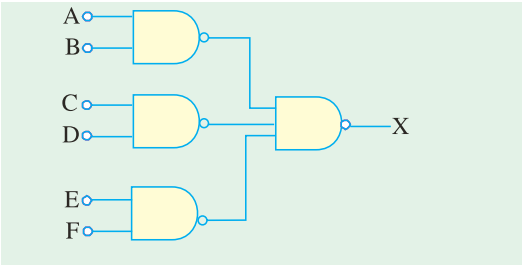


Fig. 70.109

22. The digital circuit shown in Fig 70.110 generates a modified clock pulse at the output. Choose the correct output waveform from the options given below. ***(GATE; 2004)***

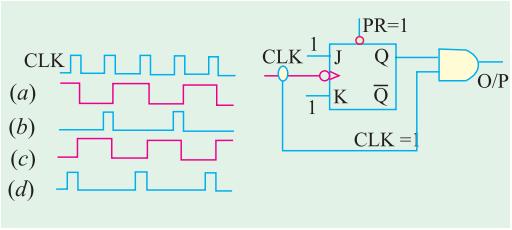


Fig. 70.110

ANSWERS

1. (*a*) **2.** (*b*) **3.** (*d*) **4.** (*b*) **5.** (*a*) **6.** (*c*) **7.** (*b*) **8.** (*a*) **9.** (*c*) **10.** (*d*)
11. (*c*) **12.** (*c*) **13.** (*d*) **14.** (*c*) **15.** (*c*) **16.** (*d*) **17.** (*c*) **18.** (*d*) **19.** (*d*) **20.** (*c*)
21. (*c*) **22.** (*d*)

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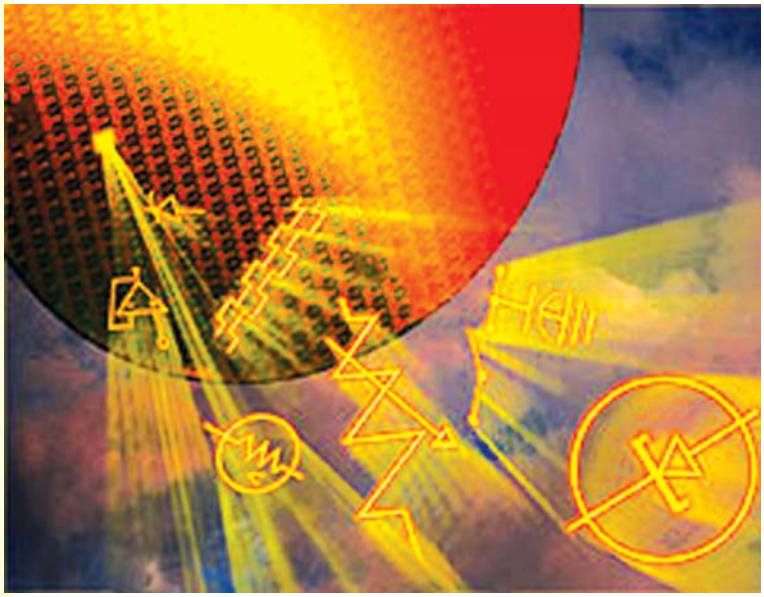
CHAPTER 71

Learning Objectives

- Unique Feature of Boolean Algebra
- Laws of Boolean Algebra
- Equivalent Switching Circuits
- DeMorgan's Theorem's
- The Sum-of-Products (SOP) Form
- The Standard SOP Form
- The Standard POS Form
- The Karnaugh Map
- The Four-variable Karnaugh Map
- Square Adjacency in Karnaugh Map
- Mapping Directly on Karnaugh Map from a Truth Table
- "Don't Care" Conditions
- Main Logic Families
- Saturated and Non-saturated Logic Circuits
- DC supply voltage
- Noise Immunity
- Noise Margin-Power Dissipation
- Power Dissipation versus Frequency
- Propagation Delay
- Speed-Power Product
- RTL Circuit
- DTL Circuit
- TTL Circuit
- TTL Sub-families
- ECL Circuit
- MOS family

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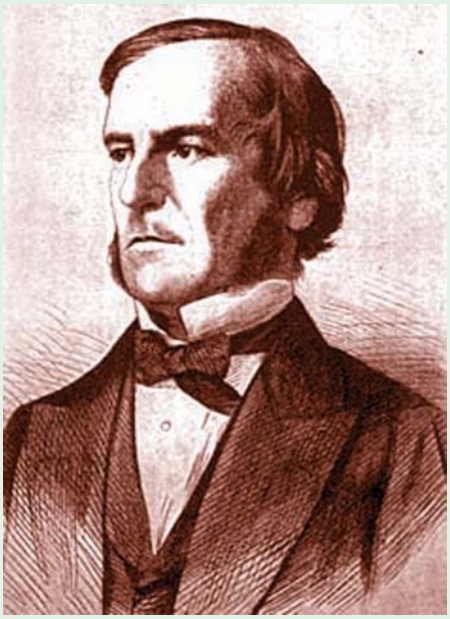
BOOLEAN ALGEBRA AND LOGIC FAMILIES



DNA Logic gates have been developed and are the first step towards creating a computer that has a structure of that of electronic PC

71.1. Introduction

Boolean algebra, named after its pioneer George Boole (1815-1864) is the algebra of logic presently applied to the opertion of computer devices. The rules of this algebra are based on human reasoning. It originated from the study of how we reason, what lines of reasoning are valid and what constitutes proof etc.


George Boole (1815–1864)

Starting with his investigation of the laws of thought, Boole developed in 1854 a mathematical system of logic in which he expressed truth functions *as* ***symbols*** and then manipulated these symbols to arrive at a conclusion. His new system was not the **ordinary numerical algebra** we know from our high school days but a totally new system called **logic algebra.** For example, in Boolean algebra $A + A = A$ and not $2A$ as is the case in ordinary algebra.

Boolean algebra remained in the realm of philosophy till 1938 when Claude *E*. Shannon used it to solve relay logic problems. As we know, all thinking and logic is concerned with finding answers to binary or two-valued questions like : is it good or bad, right or wrong, true or false etc. This binary nature of logic is exactly like the binary working of relay and switching circuits where relay is either energised or not, light is ON or OFF or pulse is present or not. Because of its very logical nature, Boolean algebra is ideal for the design and analysis of ligic circuits used in computers. Morever, it provides an economical and straight forward way of describing computer circuitry and complicated switching circuits. As compared to other mathematical tools of analysis and design, Boolean algebra has the advantages of simplicity, speed and accuracy.

71.2. Unique Feature of Boolean Algebra

As we know, the different variables used in ordinary algebra can have ***any value*** including plus and minus values. There is no restriction on the value they can assume. For example, in the equation $2x + 3y = z$, the variables x, y and z can take on any value available in the entire field of real numbers.

However, the variables used in Boolean algebra have a unique property *i.e.* they can assume ***only one of the two possible values of 0 and 1.*** Each of the variable used in a logical or Boolean equation can assume only the value 0 or 1. For example, in the logical equation $A + B = C$, **each** of the three variables A, B and C can have only the values of either 0 or 1. ***This point must be clearly taken note of by the reader for easy understanding of the laws of Boolean algebra.***

71.3. Laws of Boolean Algebra

As started earlier, Boolean algebra is a system of Mathematics based on ***logic.*** It has its own set of fundamental laws which are necessary for manipulating different Boolean expressions.

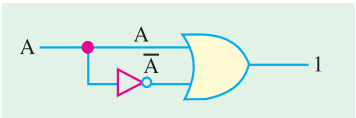


Fig. 71.1

1. OR Laws

These four laws have already been discussed in the previous chapter. These are

Law 1. $A + 0 = A$ Law 3. $A + A = A$

Law 2. $A + 1 = 1$ Law 4. $A + \overline{A} = 1$

The expression given in Law 4 can be understood with the help of Fig. 71.1. Consider the following two possibilities :

(*i*) When $A = 0, \bar{A} = 1$ $\therefore$ $A + \bar{A} = 0 + 1 = 1$

(*ii*) When $A = 1, \bar{A} = 0$ $\therefore$ $A + \bar{A} = 1 + 0 = 1$

2. AND Laws

Law 5. $A \,.\, 0 = 0$ Law 7. $A \,.\, A = A$

Law 6. $A \,.\, 1 = A$ Law 8. $A \,.\, \bar{A} = 0$

The expression for Law 8 can be easily understood with the help of the logic circuit of Fig. 71.2. Consider the following two possibilities :

(*i*) When $A = 0, \bar{A} = 1$ $\therefore$ $A \,.\, \bar{A} = 0.1 = 0$

(*ii*) When $A = 1, \bar{A} = 0$ $\therefore$ $A \,.\, \bar{A} = 1.0 = 0$

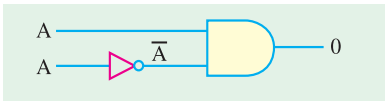


Fig. 71.2

3. Laws of Complementation

Law 9. $\bar{0} = 1$ Law 10. $\bar{1} = 0$

Law 11. if $A = 0$, then $\bar{A} = 1$ Law 12. if $A = 1$, then $\bar{A} = 0$

Law 13. $\bar{\bar{A}} = A$

4. Commutative Laws

These laws allow ***change in the position*** of variables in *OR* and *AND* expressions.

Law 14. $A + B = B + A$ Law 15. $A \,.\, B = B \,.\, A$

These two laws express the fact that the order in which a combination of terms is performed does not affect the final result of the combination.

5. Associative Laws

These laws allow removal of brackets from logical expression and regrouping of variables.

Law 16. $A + (B + C) = (A + B) + C$

Law 17. $(A + B) + (C + D) = A + B + C + D$ Law 18. $A \,.\, (B \,.\, C) = (A \,.\, B) \,.\, C$

6. Distributive Laws

These laws permit factoring or multiplying out of an expression.

Law 19. $A\,(B + C) = AB + AC$ Law 20. $A + BC = (A + B)\,(A + C)$

Law 21. $A + \bar{A} \,.\, B = A + B$

7. Absorptive Laws

These enable us to reduce a complicated logic expression to a simpler form by absorbing some of the terms into existing terms.

Law 22. $A + AB = A$ Law 23. $A \,.\, (A + B) = A$

Law 24. $A \,.\, (\bar{A} + B) = AB$

The above laws can be used to prove any given Boolean identiry and also for simplifying complicated expressions.

71.4. Equivalent Switching Circuits

The equivalent circuits to illustrate some of the *OR* and *AND* laws given above are shown in Fig. 71.3.

(*i*) Fig. 71.3 (*a*) illustrates $A + 1 = 1$. Here, lower switch is permanently closed representing 1. Hence, value of the *OR* function is 1 (*i.e.* it is *ON*) whatever the value of A.

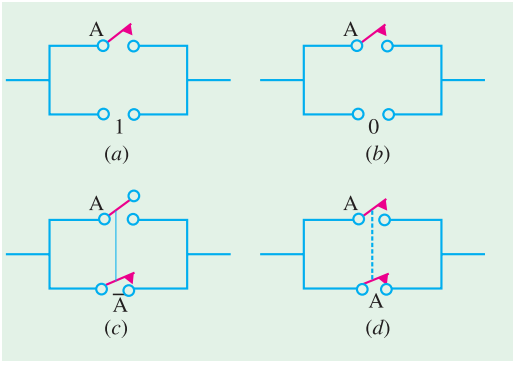


Fig. 71.3

(*ii*) Fig. 71.3 (*b*) represents $A + 0 = A$. Here, function value is determined by A alone.

(*iii*) In Fig. 71.3 (*c*) when A opens, A closes and *vice versa*. Obviously, whatever the position of A, the circuit would always be *ON* proving that $A + \bar{A} = 1$.

(*iv*) Fig. 71.3 (*d*) proves $A + A = A$. It shows that final result depends on the value of A alone. If $A = 0$, the two switches are open, hence circuit is *OFF*. If $A = 1$, both switches are closed. Hence, circuit is *ON*.

(*v*) In Fig. 71.4 (*a*), the circuit is permanently *OFF* irrespective of the value of A. It is due to ***permanent*** open (0) in the circuit. Hence, it proves $A \,.\, 0 = 0$.

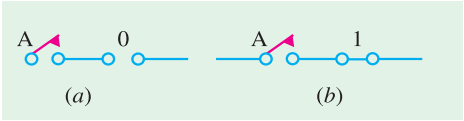


Fig. 71.4

(*vi*) Fig. 71.4 (*b*) shows that circuit conditions will depend solely on the position of the switch. If $A = 1$, circuit is *ON* (1) and when $A = 0$, circuit is *OFF* (0). It is all due to the presence of a ***permanent*** short (1) in the series circuit. Hence, everything depends on A.

Example 71.1. *Prove the following Boolean identity* : $AC + ABC = AC$

Solution. Taking the left hand side expression as y, we get

$$y = AC + ABC = AC(1 + B)$$

Now $1 + B = 1$ —Law 2

$\therefore$ $y = AC \,.\, 1 = AC$ —Law 6

$\therefore$ $AC + ABC = AC$

Example 71.2. *Determine the logic expression for the output Y, from the truth table shown in Fig. 71.5. Simplify and sketch the logic circuit for the simplified expression.*

Solution. There are two 1*s* in the output column of the given truth table. The corresponding binary values are 001 and 101. These values are converted into product terms as follows :

$$001 \rightarrow \bar{A}\ \bar{B}\ C \quad \text{and} \quad 101 \rightarrow A\ \bar{B}\ C.$$

Inputs			Output	
A	B	C	Y	
0	0	0	0	
0	0	1	1	$\rightarrow \bar{A}\ \bar{B}\ C$
0	1	0	0	
0	1	1	0	
1	0	0	0	
1	0	1	1	$\rightarrow A\ \bar{B}\ C$
1	1	0	0	
1	1	1	0	

Fig. 71.5

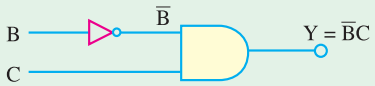


Fig. 71.6

The resulting expression for the output,

$$Y = \bar{A}\ \bar{B}\ C + A\ \bar{B}\ C$$
$$= (\bar{A} + A)\ \bar{B}\ C \quad \text{...(Law 19)}$$
$$= 1.\ \bar{B}\ C \quad \text{...(Law 4)}$$
$$= \bar{B}\ C \quad \text{...(Law 6)}$$

Fig. 71.6 shows the logic circuit to implement the simplified logic expression for the output. As seen it is formed by ANDing the variables $\bar{B}$ and $\bar{C}$. The $\bar{B}$ can be obtained by inverting B.

Example 71.3. *Prove the following Boolean identity* : $(A + B)(A + C) = A + BC$

Solution. Putting the left hand side expression equal to y, we get

$$\begin{aligned} Y &= (A+B)(A+C) \\ &= AA+AC+AB+BC && \text{—Law 19} \\ &= A+AC+AB+BC && \text{—Law 7} \\ &= A+AB+AC+BC \\ &= A(1+B)+AC+BC && \text{—Law 19} \\ &= A+AC+BC && \text{—Law 2} \\ &= A(1+C)+BC && \text{—Law 19} \\ &= A+BC && \text{—Law 2} \end{aligned}$$

$\therefore \quad (A+B)(A+C) = A+BC$

Example 71.4. *Prove the following identity* : $A+\bar{A}B = A+B$

Solution. Let the left-hand side expression be put equal to Y.

$$\begin{aligned} Y &= A+\bar{A}B \quad = A.1+\bar{A}B && \text{—Law 6} \\ &= A(1+B)+\bar{A}B && \text{—Law 12} \\ &= A.1+AB+\bar{A}B && \text{—Law 19} \\ &= A+BA+B\bar{A} && \text{—Law 6 and 15} \\ &= A+B(A+\bar{A}) && \text{—Law 19} \\ &= A+B.1 && \text{—Law 4} \\ &= A+B && \text{—Law 6} \end{aligned}$$

$\therefore \quad A+\bar{A}B = A+B$

Example 71.5. *Prove the following Boolean identity* : $(A+B)(A+\bar{B})(\bar{A}+C) = AC$

(Digital Computations, Punjab Univ. 1990)

Solution. Left the left-hand side expressionbe represented by Y.

$$\begin{aligned} \therefore \quad Y &= (A+B)(A+\bar{B})(\bar{A}+C) = (AA+A\bar{B}+BA+B\bar{B})(\bar{A}+C) \\ &= (A+AB+A\bar{B})(\bar{A}+C) = [A(1+B)+A\bar{B}](\bar{A}+C) && \because B\bar{B}=0 \\ &= (A+A\bar{B})(\bar{A}+C) = A(1+\bar{B})(\bar{A}+C) \\ &= (A.1)(\bar{A}+C) = A(\bar{A}+C) = A\bar{A}+AC = AC && \because A\bar{A}=0 \end{aligned}$$

Example 71.6. *Prove the following Boolean identity* :

$$ABC+A\bar{B}C+AB\bar{C} = A+(B+C)$$

(Digital Electronic Systems-I, Kurukshetra Univ. 1991)

Solution. Equating the left-hand side expression to Y, we have

$$\begin{aligned} Y &= ABC+A\bar{B}C+AB\bar{C} = AC(B+\bar{B})+AB\bar{C} \\ &= AC+AB\bar{C} && \text{—Law 4} \\ &= A(C+B\bar{C}) \\ &= A(C+B) && \text{—Law 21} \\ &= A(B+C) && \text{—Law 14} \end{aligned}$$

Hence, it proves the given identity.

Example 71.7. *Simplify the following Boolean Expression* :

$$AB\bar{C}+A\bar{B}\bar{C}+\bar{A}BC+ABC+A\bar{B}C$$

(Digital Computations, Punjab Univ. 1992)

Solution. Bringing together those terms which have *two* common letters, we get

$$Y = AB\bar{C}+ABC+A\bar{B}\bar{C}+A\bar{B}C+\bar{A}BC$$

$= AB(\bar{C} + C) + A\bar{B}(\bar{C} + C) + \bar{A}BC$

$= AB + A\bar{B} + \bar{A}BC$ —Law 4

$= A(B + \bar{B}) + \bar{A}BC$ —Law 4

$= A + \bar{A}BC = A + BC$ —Law 21

Example 71.8. *Simplify the following expression and show the minimum gate implementation.*

$$Y = A.B.\bar{C}.\bar{D} + \bar{A}.B\bar{C}.\bar{D} + B.\bar{C}.D$$

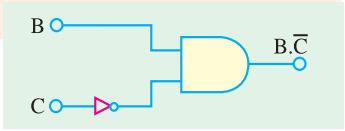


Fig. 71.7

Solution. As seen from *OR* and *AND* laws of Art 67.3, $A + \bar{A} = 1$ and $A \cdot 1 = A$

$\therefore \quad Y = B\bar{C}.\bar{D}(A + \bar{A}) + B\bar{C}.D = B\bar{C}.\bar{D}.1 + B\bar{C}.D.$

$= B.\bar{C}.\bar{D} + B.\bar{C}.D = B.\bar{C}.(D + \bar{D}) = B.\bar{C}.1 = B.\bar{C}.$

Minimum gate implementation is shown by the circuit of Fig. 71.7

Example 71.9. *Simplify the following Boolean expression and draw the logic circuits for the simplified expressions.*

(a) $Y = \bar{A}BC + A\bar{B}C + ABC + B\bar{C}$ ***(b)*** $Y = \bar{B}(A + C) + C(\bar{A} + B) + AC$

Solution. (*a*) $Y = \bar{A}BC + A\bar{B}C + ABC + B\bar{C} = BC(A + \bar{A}) + AC(\bar{B} + B) + B\bar{C}$

$= BC + AC + B\bar{C} = B(C + \bar{C}) + AC = AC + B$

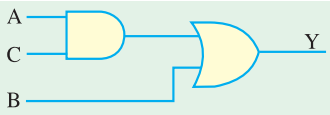


Fig. 71.8

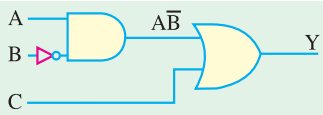


Fig. 71.9

(*b*) $Y = \bar{B}(A + C) + C(\bar{A} + B) + AC = A\bar{B} + \bar{B}C + \bar{A}C + BC + AC$

$= A\bar{B} + C(\bar{B} + \bar{A} + B + A) = A\bar{B} + C.1 = A\bar{B} + C$

Example 71.10. *Using truth table, prove that* $A + \bar{A}B = A + B$ *and illustrate the equivalence with the help of a switching circuit.*

Solution. Since there are only two variables *A* and *B*, their number of possible combination is $2^2 = 4$ in terms of 0 and 1.

As seen, $\bar{A}$ is the negative of A. In the fourth column of Table 71.1, $\bar{A}$ has been *AND*ed with *B*. In the fifth column, *A* has been *OR*ed with $\bar{A}B$. The values in last column have been obtained by *OR*ing *A* with *B*. By comparing results of column 5 to 6, the equivalence between the two statements can be proved.

Table No. 71.1

A	B	$\bar{A}$	$\bar{A}B$	$A + \bar{A}B$	$A + B$
0	0	1	0	0	0
0	1	1	1	1	1
1	0	0	0	1	1
1	1	0	0	1	1

Switching circuit of Fig. 71.10 (*a*) represents $(A + \bar{A}B)$. In this circuit, when *A* is open, $\bar{A}$ is closed and *vice versa*. It can be shown that his circuit becomes closed with either *A* or *B* is closed.

(*i*) when *A* is closed, then $\bar{A}$ opens. Circuit is completed *via* the upper branch.

(*ii*) keeping *A* open, when we close *B*, the circuit again becomes closed *via* lower branch because $\bar{A}$ is already closed (due to *A* being open).

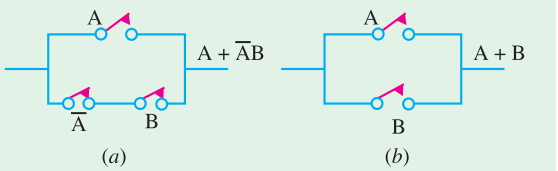


Fig. 71.10

Hence, all that we have to do for closing the circuit of Fig. 71.10 (*a*) is to close either switch *A* or *B*. It is exactly what circuit of Fig. 71.10 (*b*) does.

Example 71.11. *Simplify the expression* : $(AB + C)(AB + D)$

Solution. Let $Y = (AB+C)(AB+D)$

$= ABAB + ABD + ABC + CD$ —Law 19

$= AABB + ABD + ABC + CD$

$= AB + ABD + ABC + CD$ —Law 7

$= AB(1 + D) + ABC + CD = AB + ABC + CD$ —Law 2

$= AB(1 + C) + CD = AB + CD$

$\therefore \quad (AB + C)(AB + D) = AB + CD$

71.5. DE Morgan's Theorem

These two theorems (or rules) are a great help in *simplifying complicated logical expressions.* The theorems can be started as under :

Law 25. $\overline{A + B} = \bar{A} \cdot \bar{B}$ Law 26. $\overline{A \cdot B} = \bar{A} + \bar{B}$

The first statement says that ***the complement of a sum equals the product of complements.*** The second statement says that ***the complement of a product equals the sum of the complements.*** In fact, it allows transformation from a sum-of-products from to a product-of-sum from.

As seen from the above two laws, the procedure required for taking out an expression from under a *NOT* sign is as follows :

1. ***complement the given expression i.e., remove the overall NOT sign***
2. ***change all and ANDs to ORs and all the ORs to ANDs.***
3. ***complement or negate all individual variables.***

As an illustration, take the following example

$\overline{A + BC} = A + BC$ —step 1

$= A(B + C)$ —step 2

$= \bar{A}(\bar{B} + \bar{C})$ —step 3

Next, consider this example,

$\overline{(\bar{A} + B + \bar{C})(\bar{A} + B + C)} = (\bar{A} + B + \bar{C})(\bar{A} + B + C)$ —step 1

$= \bar{A}B\bar{C} + \bar{A}BC$ —step 2

$= \bar{\bar{A}}\bar{B}\bar{\bar{C}} + \bar{\bar{A}}\bar{B}\bar{C}$ —step 3

$= A\bar{B}C + A\bar{B}\bar{C}$

This process is called **demorganization.** It should, however, be noted that opposite procedure would be followed in bringing an expression under the *NOT* sign. Let us bring the expression $\bar{A} + \bar{B} + \bar{C}$ under the *NOT* sign.

$\bar{A} + \bar{B} + \bar{C} = \bar{\bar{A}} + \bar{\bar{B}} + \bar{\bar{C}}$ —step 3

$= A + B + C$

$= ABC$ —step 2

$= \overline{ABC}$ —step 1

Fig. 71.11 shows the circuits to illustrate De Morgan's theorems.

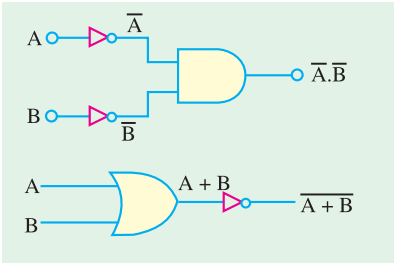


Fig. 71.11

As seen, basic logic function can be either an *OR* gate or an *AND* gate.

Example 71.12. *Demorganize the expression :* $\overline{(A+B)(C+D)}$

Solution. The procedure is as explained above.

$$\overline{(A+B)(C+D)} = (A+B)(C+D) \quad \text{—step 1}$$
$$= (AB)+(CD) \quad \text{—step 2}$$
$$= \overline{AB} + \overline{CD} \quad \text{—step 3}$$



De Morgan (1806–1871)

Example 71.13. *Simplify each of the following expressions using De Morgan's theorems :*

(a) $\overline{A\,\overline{\overline{(B+\overline{C})}D}}$

(b) $\overline{(M+\overline{N})(\overline{M}+N)}$ **(c)** $\overline{\overline{AB}\,\overline{C}\,D}$

Solution. Please note that there is more than one way of simplifying the expressions given in part (*a*), (*b*) and (*c*).

(a)
$$\overline{A\,\overline{\overline{(B+\overline{C})}D}} = A\,\overline{(B+\overline{C})}\,D \quad \text{... step 1}$$
$$= A+\overline{(B+\overline{C})}+D \quad \text{... step 2}$$
$$= \overline{A}+\overline{\overline{B+\overline{C}}}+\overline{D} \quad \text{... step 3}$$
$$= \overline{A}+B+\overline{C}+\overline{D} \quad \text{...}(\because\ \overline{\overline{B+\overline{C}}} = B+\overline{C})$$

(b)
$$\overline{(M+\overline{N})(\overline{M}+N)} = (M+\overline{N})(\overline{M}+N) \quad \text{... step 1}$$
$$= (M\overline{N})+(\overline{M}N) \quad \text{... step 2}$$
$$= (\overline{M}\,\overline{\overline{N}})+\overline{\overline{M}}\,\overline{N} \quad \text{... step 3}$$
$$= \overline{M}N+M\overline{N} \quad \text{... step 4}$$

(c)
$$\overline{\overline{\overline{AB}\,\overline{C}\,D}} = \overline{\overline{AB}\,\overline{C}}\,D \quad \text{... step 1}$$
$$= \overline{\overline{AB}\,\overline{C}}+D \quad \text{... step 2}$$
$$= \overline{\overline{\overline{AB}\,\overline{C}}}+\overline{D} \quad \text{... step 3}$$
$$= \overline{AB}\,\overline{C}+\overline{D} \quad \text{...}(\because\ \overline{\overline{AB}\,\overline{C}} = \overline{AB}\,\overline{C})$$

The term $\overline{AB}\,\overline{C}$ can be simplified further by De Morganising the term $\overline{AB}$ as $\overline{A}+\overline{B}$. Thus

$$\overline{\overline{\overline{AB}\,\overline{C}}\,D} = \overline{AB}\,\overline{C}+\overline{D} = (\overline{A}+\overline{B})\,\overline{C}+\overline{D} = \overline{A}\,\overline{C}+\overline{B}\,\overline{C}+\overline{D}$$

Example 71.14. *Find switching circuits for the following logic expressions :*

(i) $A\,.\,(B+C)$ **(ii)** $A\,\overline{B}+CD$ **(iii)** $(\overline{A}\,B+AC)\,\overline{C}$

(Industrial Electronics, City & Guilds, London)

Solution. (*i*) *A* . (*B* + *C*)

Here, switches *B* and *C* have been *OR*ed *i.e.* connected in parallel. This parallel circuit is connected in series with switch *A* because (*B* + *C*) has been *AND* ed with *A*. hence, the circuit becomes as shown in Fig. 71.12 (*a*). As seen, it is a series-parallel circuit.

(ii) $A\overline{B} + CD$

Here, $A\overline{B}$ has been *OR*ed with CD. We can easily make out that a 2-brached circuit is needed for this logic expression. One branch contains switch A in series with $\overline{B}$ (with one contact point shows raised to indicate negation) and the other branch contains two series-connected switches C and D as show in Fig. 71.12 (*b*).

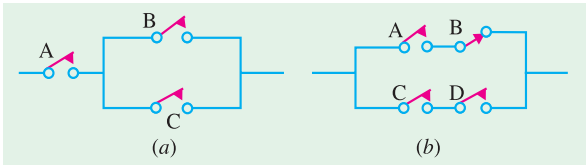


Fig. 71.12

(iii) $(\overline{A}B + AC)\,\overline{C}$

From the look of it, we can make out that it consists of a series-parallel circuit as shown in Fig. 71.13. $\overline{A}\,B$ has been *OR*ed with AC *i.e.* $\overline{A}\,B$ and AC are in parallel. Of course, $\overline{A}$ and B are in series in one branch where as A and C are in series in the other branch. Both these parallel branches are in series with $\overline{C}$.

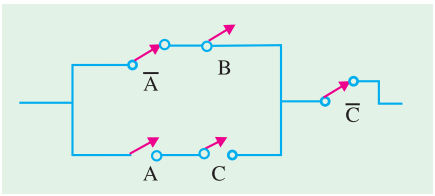


Fig. 71.13

Example 71.15. *Prove that 3-input NAND gate of Fig. 71.14 (a) is equivalent to the bubbled AND gate of Fig. 71.14 (b).*

Solution. The output of *NAND* gate is $\overline{ABC}$ and that of bubbled *OR* gate is $\overline{A} + \overline{B} + \overline{C}$. We have to show that the above two expressions are equivalent.

De Morgan's theorem can be used to prove the above equivalence,

$\overline{ABC} = ABC$ —step 1

$= A + B + C$ —step 2

$= \overline{A} + \overline{B} + \overline{C}$ —step 3

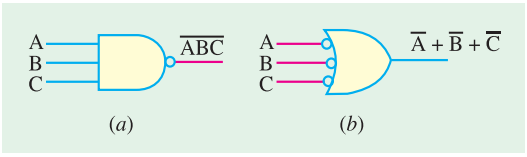


Fig. 71.14

71.6. Duals

Basic duality underlies all Boolean algebra. ***Each expression has its dual which is as true as the original expression.*** For getting the dual of a given Boolean expression, the procedure is to convert

1. all 1*s* to 0*s* and all 0*s* to 1*s*.
2. all *ANDs* to *ORs* and all *ORs* to *ANDs*.

The dual so obtained ***is also found to be true.***

Some of the Boolean relations and their duals are given in Table 71.2.

Table No. 71.2

Relation		*Dual Relation*	
$A \cdot 0$	$= 0$	$A + 1$	$= 1$
$A \cdot A$	$= A$	$A + A$	$= A$
$A \cdot \overline{A}$	$= 0$	$A + \overline{A}$	$= 1$
$A \cdot 1$	$= A$	$A + 0$	$= A$
$A \cdot (A + B)$	$= A$	$A + AB$	$= 1$
$A \cdot (\overline{A} + B)$	$= AB$	$A + \overline{A}B$	$= A + B$

Example 71.16. *Design a logic circuit whose output is HIGH* **only** *when a majority of the inputs A, B and C are HIGH.*

Solution. Since there are three inputs, A, B and C, therefore whenever two or more than two (*i.e.* a majority) inputs are HIGH, the output is HIGH. This situation can be represented in the form of a truth table as shown in Fig. 71.15.

A	B	C	Y	
0	0	0	0	
0	0	1	0	
0	1	0	0	
0	1	1	1	$\rightarrow \overline{A}BC$
1	0	0	0	
1	0	1	1	$\rightarrow A\overline{B}C$
1	1	0	1	$\rightarrow AB\overline{C}$
1	1	1	1	$\rightarrow ABC$

Fig. 71.15

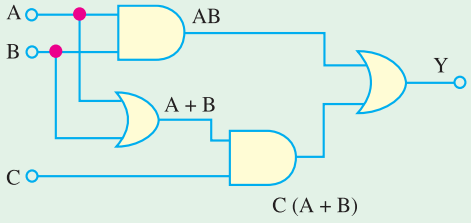


Fig. 71.16

There are four 1*s* in the output column of the truth table. The corresponding binary values are 011, 101, 110 and 111 respectively. Converting these values into product terms and summing up all the terms, we get

$$Y = AB\overline{C} + A\overline{B}C + \overline{A}BC + ABC$$

Adding the term ABC two times from our side in the Boolean expression for the output,

$$Y = AB\overline{C} + A\overline{B}C + \overline{A}BC + ABC + ABC + ABC \qquad (\because ABC + ABC + ABC = ABC)$$

Bringing together those terms which have two common letters, we get,

$$\begin{aligned} Y &= AB\overline{C} + ABC + A\overline{B}C + ABC + \overline{A}BC + ABC \\ &= AB(\overline{C} + C) + AC(\overline{B} + B) + BC(\overline{A} + A) \\ &= AB + AC + BC \qquad (\because \overline{C} + C = \overline{B} + B = \overline{A} + A = 1) \\ &= AB + C(A + B) \end{aligned}$$

The logic circuit that produces the output $Y = AB + C(A + B)$ is as shown in Fig. 71.16.

Alternatively :

You can also arrange the equation,

$$Y = AB + AC + BC$$

as $$Y = A(B + C) + BC \qquad \ldots (i)$$

or $$Y = (A + C)B + AC \qquad \ldots (ii)$$

If you implement equation **(*i*)** or **(*ii*)** using AND and OR logic gates, the number of logic gates is used will still be the same (4).

Example 71.17. *Determine the Boolean expression for the logic circuit shown in Fig. 71.17. Simplify the Boolean expression using Boolean Laws and De Morgan's theorem. Redraw the logic circuit using the simplified Boolean expression.*

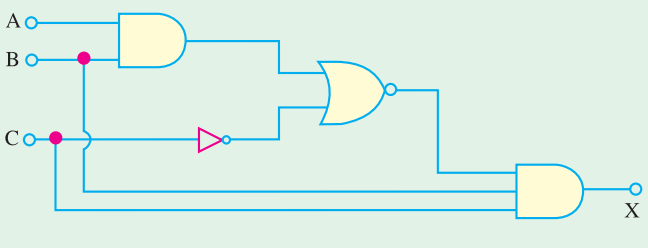


Fig. 71.17

Solution. The output of the given circuit can be obtained by determining the output of each logic gate while working from left to right.

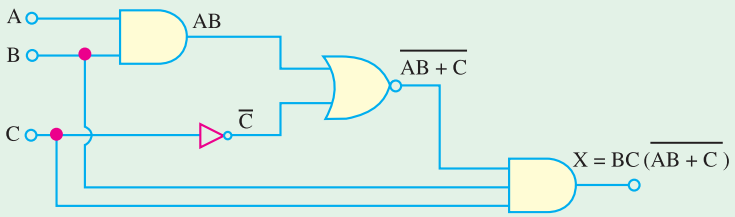


Fig. 71.18

As seen from the logic circuit shown in Fig. 71.18. The output of the circuit is,

$$X = BC\ \overline{(AB + \overline{C})}$$

The output, X can be simplified by De Morganizing the term $\overline{(AB + \overline{C})}$ as shown below.

$$\begin{aligned}
BC\ \overline{(AB + \overline{C})} &= BC\,(A\,B + \overline{C}) && \text{... step 1}\\
&= BC\,(A + B)\,.\,\overline{C} && \text{... step 2}\\
&= BC\ (\overline{A} + \overline{B})\,.\,\overline{\overline{C}} && \text{... step 3}\\
&= BC\ (\overline{A} + \overline{B})\,C && \text{... Law 13}\\
&= BC\ (\overline{A} + \overline{B}) && \text{... Law 7}\\
&= \overline{A}\,BC + BC\ \overline{B}\\
&= \overline{A}\,BC + 0 = && \text{... Law 8}\\
&= \overline{A}\,BC && \text{... Law 1}
\end{aligned}$$

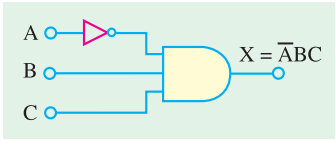


Fig. 71.19

The logic circuit with a simplified Boolean expression $X = \overline{A}\,BC$ is as shown in Fig. 71.19.

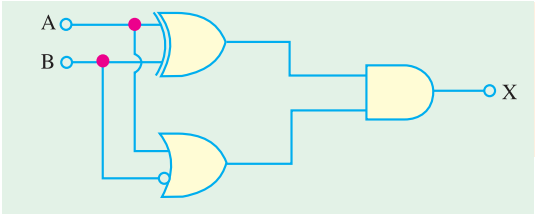


Fig. 71.20

Example 71.18. *Determine the output X of a logic circuit shown in Fig. 71.20. Simplify the output expression using Boolean Laws and theorems. Redraw the logic circuit with the simplified expression.*

Solution. The output of the given logic circuit can be obtained by determining the output of each logic gate while working from left to right.

As seen from Fig. 71.21, the output,

$$\begin{aligned}
X &= (\overline{A}B + A\overline{B})\,(A + \overline{B})\\
&= \overline{A}\,BA + A\,\overline{B}\,A + \overline{A}\,B\,\overline{B} + A\,\overline{B}\,\overline{B}\\
&= 0 + A\,\overline{B}\,A + 0 + A\,\overline{B}\,\overline{B} && \text{... Law 8}\\
&= A\,\overline{B} + A\,\overline{B} && \text{... Law and Law 1}\\
&= A\,\overline{B} && \text{... Law 3}
\end{aligned}$$

Using the simplified Boolean expression, the logic circuit is as shown in Fig. 71.22.

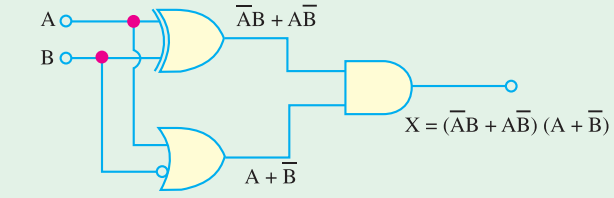


Fig. 71.21

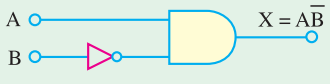


Fig. 71.22

Example 71.19. *Consider the logic circuit shown in Fig. 71.23. Determine the Boolean expression at the circuit output, simplify it. From the simplified Boolean expression, find which logic gate is redundant in the given logic circuit.*

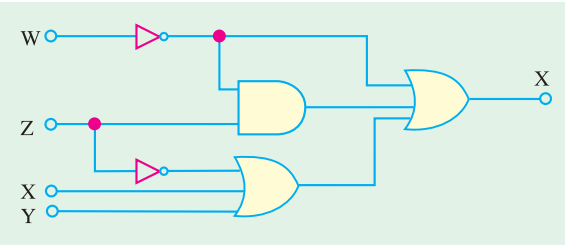


Fig. 71.23

Solution. As seen from Fig. 71.24, the logic circuit output,

$$X = \overline{W} + \overline{W}Z + XY\overline{Z}$$

The expression can be simplified as follows :

$$= \overline{W} + \overline{W}Z + XY\overline{Z}$$

$$= \overline{W}(1 + Z) + XY\overline{Z} \quad \text{—Law 19}$$

$$= \overline{W} + XY\overline{Z} \quad \text{—Law 2}$$

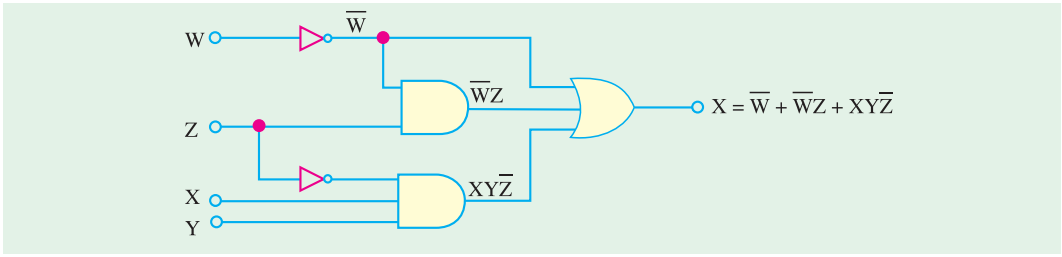


Fig. 71.24

From the simplified Boolean expression of the output $X = \overline{W} + XY\overline{Z}$, and the actual output $\overline{W} + \overline{W}Z + XY\overline{Z}$, we find that the two-input *AND* gate (producing the term $\overline{W}Z$) is redundant.

Example 71.20. *Determine the output of the logic circuit shown in Fig. 71.25. Simplify the output Boolean expression and sketch the logic circuit.*

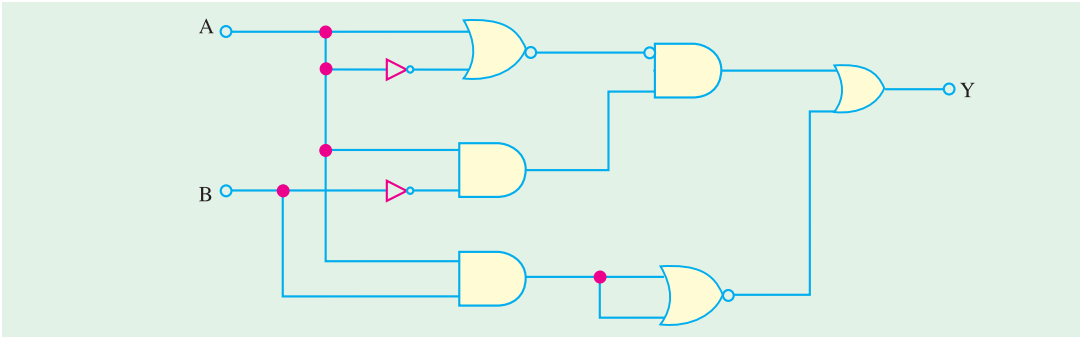


Fig. 71.25

Solution. The output of the circuit can be obtained by determining the output of each logic gate while working from left to right.

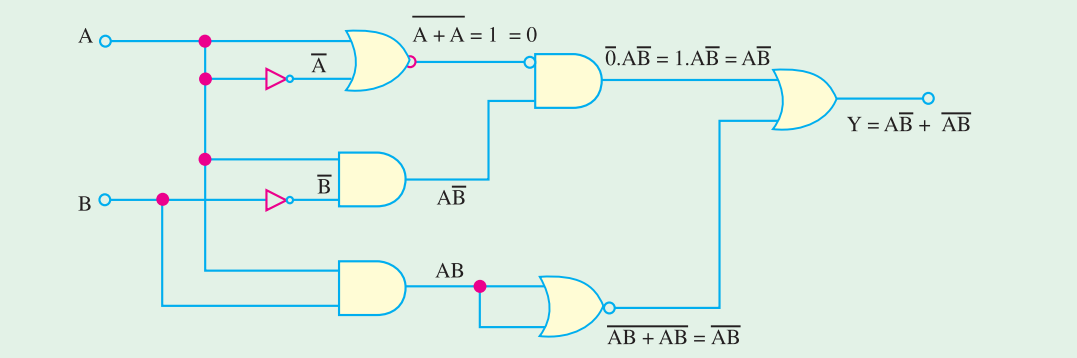


Fig. 71.26

As seen from the circuit shown in Fig. 71.26, we find that the output,

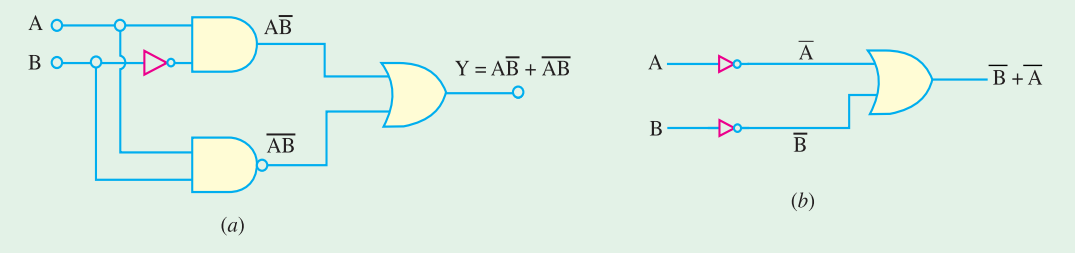


Fig. 71.27

$$y = A\overline{B} + \overline{AB}$$

The sketch of a logic circuit for the simplified Boolean expression is as shown in Fig. 71.27.

Alternatively :

$$\begin{aligned} y &= A\overline{B} + \overline{A}\overline{B} \\ &= A\overline{B} + (\overline{A} + \overline{B}) \\ &= (A\overline{B} + \overline{B}) + \overline{A} \\ &= (A+1)\overline{B} + \overline{A} \\ &= \overline{B} + \overline{A} \end{aligned}$$

The logic circuit to implement this logic equation is as shown in Fig. 71.27(*b*). Notice the difference in terms of the number and type of logic gates used in the circuits shown in Fig. 71.27. So when you are simplifying and designing logic circuits, it is always possible to have more than one solution.

The circuit shown in Fig. 71.27 shows that it makes use of one inverter (or NOT gate), one AND gate, one NAND gate and one OR gate. In other words, there are four different types of logic gates. However the logic circuit of Fig. 71.27 (*b*) makes use of only three logic gates (two inverters and one OR gate).

71.7. Standard Forms of Boolean Expressions

All Boolean expressions, regardless of their form, can be converted into either of the two following standard forms :

1. Sum-of-products (*SOP*) form and **2.** Product-of-sums (*POS*) form.

The standardization of Boolean expressions makes their evaluation, simplification and implementation much more systematic and easier. Now we shall discuss these two standard forms in more detail.

71.8. The Sum-of-Products (SOP) Form

A product term is a term consisting of the product (or Boolean multiplication) of literals (*i.e.* variables or their complements). When we add two or more product terms, the resulting expression is called, sum-of-products (*SOP*) expression. Some examples of sum-of-products expressions are $A\overline{B} + \overline{A}BC$, $\overline{A}BC + A\overline{C}D + A\overline{B}CD$, $\overline{A}B + AC + \overline{A}\,\overline{B}\,C$

Sometimes, it is convenient to define the set of variables contained in the expression (in either complemented or uncomplemented form) as a ***domain***. For example, domain of the expression $A\,\overline{B}$ + AB is the set of variables A and B. Similarly the domain of the expression $\overline{A}\,BC + ABC + A\,\overline{B}\,CD$ is the set of variables A, B, C and D.

Any logic expression can be changed into *SOP* form by applying Boolean algebra laws and theorems. For example, the expression $A\ (BC + D)$ can be converted to *SOP* form by applying the distributive law :

$$A\,(BC + D) \ = ABC + AD$$

71.9. The Standard SOP Form

So far, we have seen *SOP* (sum-of-products) expressions in which some of the product terms do not contain all the variables in the domain of the expression. For example, the expression, $\overline{A}\,BC + A\,\overline{C}\,D\ + A\,\overline{B}\,CD$ has a domain made up of the variables A, B, C and D. But notice that the first two terms contains only three variables, *i.e.* D or $\overline{D}$ is missing from the first term and B or $\overline{B}$ is missing from the second term.

A standard *SOP* expression is defined as an expression in which all the variables in the domain apper in each product term. For example $\overline{A}\,BCD + AB\,\overline{C}\,D + A\,\overline{B}\,CD$ is a standard *SOP* expression. Standard *SOP* expressions are important in constructing truth tables and in karnaugh map simplification method.

It is very straightforward to convert non-standard product term to a standard *SOP* using Boolean algebra. Each product term in the *SOP* expression that does not contain all the variables in the domain has to be expanded to standard form to include all the variables in the domain and their complements. As stated below, a nonstandard *SOP* expression is converted into standard form using Boolean algebra law 4 : $A\ +\ \overline{A}\ = 1$ *i.e.* a variable added to its complement equals 1.

Step 1. Multiply each nonstandard product term by a term made up of the sum of a missing variable and its complement. This results in two product terms. It is possible because we know that we can multiply anything by 1 without changing its value.

Step 2. Repeat step 1 until all resulting product terms contain all variables in the domain in either complemented or uncomplemented form. Note that in converting a product term to a standard form, the number of product terms is doubled for each missing variable.

For example, suppose we want to convert the Boolean expression $\overline{A}\,BC + A\,\overline{C}\,D + A\,\overline{B}\,CD$ to a standard *SOP* form. Then following the above procedure we proceed as below :

$$\overline{A}\,BC\,(D + \overline{D}\,) + A\,(B + \overline{B}\,)\,\overline{C}\,D + A\,\overline{B}\,CD$$
$$= \overline{A}\,BCD + \overline{A}\,BC\,\overline{D}\ + AB\,\overline{C}\,D + A\,\overline{B}\ \overline{C}\,\mathrm{D} + A\,\overline{B}\,CD$$

The expression given above is a standard *SOP* expression.

71.10. The Product-of-sums (POS) Form

The sum term is a term consisting of the sum (or Boolean addition) of literals (*i.e.* variables or their complements). When we multiply two or more sum terms, the resulting expression is called product-of-sums (*POS*). Some examples of *POS* form are (A + $\overline{B}$) ($\overline{A}$ + B + C), ($\overline{A}$ + B + C) (A + $\overline{C}$ + D) (A + $\overline{B}$ + C + D) and ($\overline{A}$ + $\overline{B}$) (*A* + *C*) ($\overline{A}$ + $\overline{B}$ + *C*) .

It may be carefully noted that a *POS* expression, can contain a single-variable term as in $\overline{A}$ (*A* + $\overline{B}$ + *C*) (*B* + $\overline{C}$ + $\overline{D}$). In *POS* expression, a single overbar cannot extend over more than one variable, although more than one variable in a term can have an overbar.

71.11. The Standard POS Form

So far, we have seen *POS* (product-of-sums) expressions in which some of the sum terms do not contain all the variables in the domain of the expression. For example, the expression, ($\overline{A}$ + *B* + *C*), (*A* + $\overline{C}$ + *D*) (*A* + $\overline{B}$ + *C* + *D*) has a domain made up of variables, *A*, *B*, *C* and *D*. Notice that the complete set of variables in the domain is not represented in the first two terms of the expression, *i.e.* *D* or $\overline{D}$ is missing in the first term and *B* or $\overline{B}$ is missing in the second term.

A standard *POS* expression is defined as an expression in which all the variables in the domain appear in each sum term. For example ($\overline{A}$ + *B* + *C* + *D*) (*A* + *B* + $\overline{C}$ + *D*) (*A* + $\overline{B}$ + *C* + *D*) is a standard *POS* form. Any nonstandard *POS* expression can be converted to a standard form using Boolean algebra.

Each sum term in an *POS* expression that does not contain all the variables in the domain can be expanded to standard form to include all variables in the domain and their complements. As stated below : a nonstandard *POS* expression is converted into a standard form using Boolean algebra Law 8 : *A* . $\overline{A}$ = 0, *i.e.* a variable multiplied by its complemented equals 0.

Step 1. Add to each nonstandard product term a term made up of the ptoduct of a missing variable and its complement. This results in two sum terms. This is possible because we know that we can add 0 to anything without changing its value.

Step 2. Apply law 20, *i.e.* *A* + *BC* = (*A* + *B*) (*A* + *C*)

Step 3. Repeat Step 1 until all resulting sum terms contain all variables in the domain in either complemented or uncomplemented form.

For example we want to convert the Boolean expression,

($\overline{A}$ + *B* + *C*) (*A* + $\overline{C}$ + *D*) (*A* + $\overline{B}$ + *C* + *D*)

into a standard *POS* form. Then following the above prodcedure, we proceed as below :

($\overline{A}$ + *B* + *C* + *D* $\overline{D}$) (*A* + *B* $\overline{B}$ + $\overline{C}$ + *D*) (*A* + $\overline{B}$ + *C* + *D*)

= ($\overline{A}$ + *B* + *C* + *D*) ($\overline{A}$ + *B* + *C* + $\overline{D}$) (*A* + *B* + $\overline{C}$ + *D*) (*A* + $\overline{B}$ + $\overline{C}$ + *D*) (*A* + $\overline{B}$ + *C* + *D*)

The expression given above is a standard *POS* expression

71.12. The Karnaugh Map

The Karnaugh map (or simply a *K*-map) is similar to a truth table because it presents all the possible values of input variables and the resulting output for each value. However, instead of being organised into columns and rows like a truth table, the Karnaugh map is an array of squares (or cells) in which each square represents a binary value of the input variables. The squares are arranged in a way so that simplification of given expression is simply a matter of grouping the squares. Karnaugh maps can be used for expression with two-three, four, and five variable Karnaugh maps to illustrate the principles. Karnaugh map with five-variables is beyond the scope of this book. For higher number of

variables, a Quine-McClusky method can be used. This method is also beyond the scope of this book.

The number of squares in a Karnaugh map is equal to the total number of possible input variable combinations (as is the number of rows in a truth table). For two variables, the number of square is $2^2 = 4$, for three variables, the number of squares is $2^3 = 8$ and for four variables, the number of squares is $2^4 = 16$.

71.13. The Two-variable Karnaugh Map

Fig. 71.28 (*a*) shows a two-variable Karnaugh map. As seen, it is an array of four squares. In this cass, A and B are used for two variables although any other two letters could be used. The binary values of A (*i.e.* 0 and 1) are indicated along the left side as $\overline{A}$ and A (notice the sequence) and the binary values of B are indicated across the top as $\overline{B}$ and B. The value of a given square is the value of A at the left in the same row combined with the value of B at the top in the same column. For example, a square in the upper left corner has a value of $\overline{A}\,\overline{B}$ and a square in the lower right corner has a value of $A\,B$. Fig. 71.28 (*b*) shows the standard product terms represented by each square in the Karnaugh map.

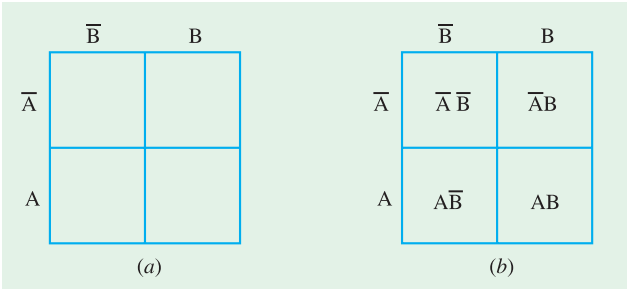


Fig. 71.28

71.14. The Three-variable Karnaugh Map

Fig. 71.29 (*a*) shows a three-variable Karnaugh map. As seen it is an array of eight squares. In this case, A, B and C are used for the variables although any other three letters could be used. The value of A and B are along the left side (notice the sequence carefully) and the values of C are across the top.

The value of a given square is the values of A and B at the left in the same row combined with the value of C at the top in the same column. For example, a square in the upper left corner has a value of $\overline{A}\,\overline{B}\,\overline{C}$ and a square in the bottom right corner has a value of $A\,\overline{B}\,C$. Fig. 71.29 (*b*) shows the product terms that are represented by each square in the Karnaugh map.

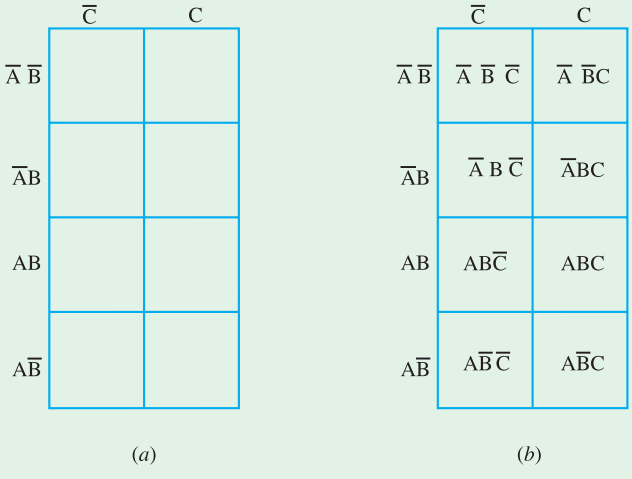


Fig. 71.29

71.15. The Four-variable Karnaugh Map

Fig. 71.30 (*a*) shows a four-variable Karnaugh map. As seen, it is an array of sixteen squares. In this case *A*, *B*, *C* and *D* are used for the variables. The values of *A* and *B* are along the left side, and the values of *C* and *D* are across the top. The sequence of the variable values may be noted carefully.

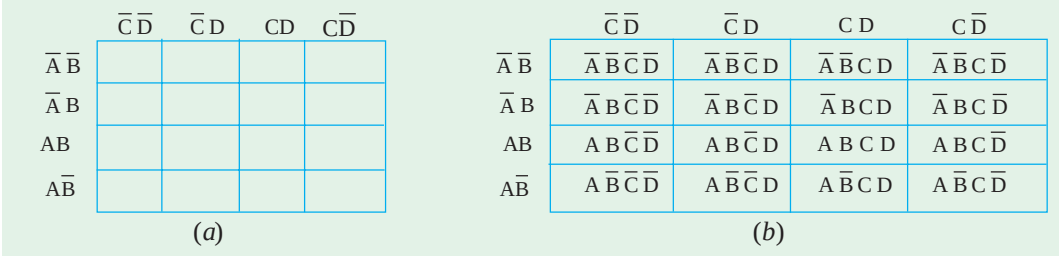


Fig. 71.30

The value of a given square is the values of *A* and *B* at the left in the same row combined with the values of *C* and *D* at the top in the same column. For example, a square in the upper right corner has a value $\overline{A}\,\overline{B}\,C\,\overline{D}$ and a square in the lower left corner has a value $A\,\overline{B}\,C\,\overline{D}$. Fig. 71.30 (*b*) shows the standard product terms that are represented by each square in the four-variable Karnaugh map.

71.16. Square Adjacency in Karnaugh Map

We have already discussed a two-variable Karnaugh map, a three-variable Karnaugh map and a four-variable Karnaugh map. Now we shall discuss the concept of square adjacency in a Karnaugh map.

It will be interesting to know that the squares in a Karnaugh map are arranged in such a way that there is only a single-variable change between adjacent squares. Adjacency is defined as a single-variable change. It means the squares that differ by only one variable are adjacent. For example, in a three-variable Karnaugh map shown in Fig. 71.29 (*b*), the $\overline{A}\,B\,\overline{C}$ square is adjacent to $\overline{A}\,\overline{B}\,\overline{C}$ square, the $\overline{A}BC$ square and the $AB\overline{C}$ square. It may be carefully noted that square with values that differ by more than one variable are not adjacent. For example, the $\overline{A}\,B\,\overline{C}$ square is not adjacent to the $\overline{A}\,\overline{B}\,C$ square, the ABC square, the $A\,\overline{B}\,\overline{C}$ square or the $A\,\overline{B}\,C$ square. In other words, each square is adjacent to the squares that are immediately next to it on any of its four sides. However, a square is not adjacent to the squares that diagonally touch any of its corners.

It may also be noted that squares in the top row are adjacent to the corresponding squares in the bottom row and squares in the outerleft column are adjacent to the corresponding squares in the outer right column. This is called "**wrap around**" adjacency because we can think of the map as wrapping around from top to bottom to form a cylinder or from left to right to form a cylinder. Fig. 71.31 (*a*) and (*b*) shows the square adjacencies with a three-variable and a four-variable Karnaugh maps respectively.

Notice the square adjacencies in a four variable Karnaugh map :

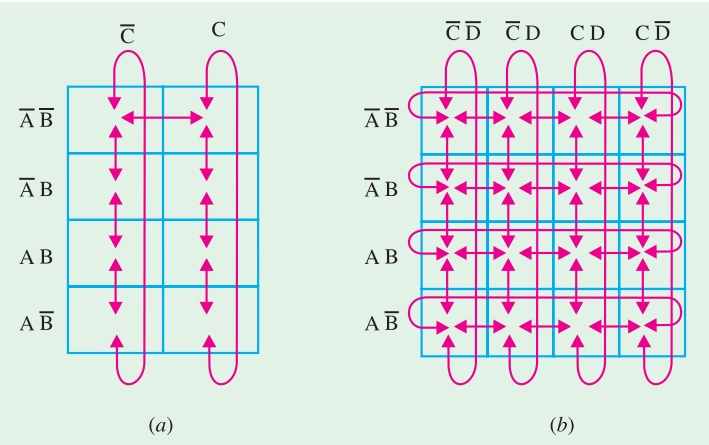


Fig. 71.31

Here for example, the square $\overline{A}\,\overline{B}\,\overline{C}\,\overline{D}$ is adjacent to $\overline{A}\,\overline{B}\,\overline{C}\,D$ square, $\overline{A}\,B\,\overline{C}\,\overline{D}$ square, $A\,\overline{B}\,\overline{C}\,\overline{D}$ square and $A\,\overline{B}C\,\overline{D}$ square. Similarly $\overline{A}\,B\,\overline{C}\,D$ square is adjacent to $\overline{A}\,\overline{B}\,\overline{C}\,D$ square, $\overline{A}\,B\,\overline{C}\,\overline{D}$ square, $\overline{A}BCD$ square and $AB\overline{C}D$ square.

71.17. Mapping a Standard SOP Expression on the Karnaugh Map

Consider a *SOP* (sum-of-products) expression. $\overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,B\,\overline{C} + A\overline{B}C + A\,\overline{B}\,\overline{C}$. In order to map this expression on the Karnaugh map, we need a three variable Karnaugh-map because the given expression has three variables *A*, *B*, and *C*. Then select the first product term $\overline{A}\,\overline{B}\,\overline{C}$ and enter 1 in the corresponding square (*i.e.* the first row and the first column) as shown in Fig. 71.32.

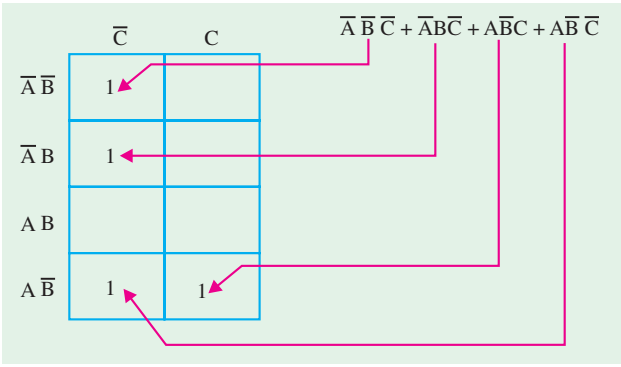


Fig. 71.32

Similarly, for the second product term, $\overline{A}\,B\,\overline{C}$ place a 1 in the second row and first column. Repeat this process for the other two product terms, *i.e.* $A\,\overline{B}\,C$ and $A\,\overline{B}\,\overline{C}$. The squares that do not have 1 are the squares for which the expression is 0. Usually when working with sum-of-products expressions, the 0*s* are left off the map.

Example 71.21. *Map the following SOP expression on the Karnaugh map :* $\overline{A}\,\overline{B}\,C + \overline{A}BC + AB\overline{C} + ABC$.

Solution. Sketch a three variable Karnaugh map as shown in Fig. 71.33. Select the first product term $\overline{A}\,\overline{B}\,C$ and enter 1 in the corresponding square. Similarly enter 1 for the other product terms in the given *SOP* expression. Check that number of 1*s* in the Karnaugh map is equal to the number of product terms in the given *SOP* expression.

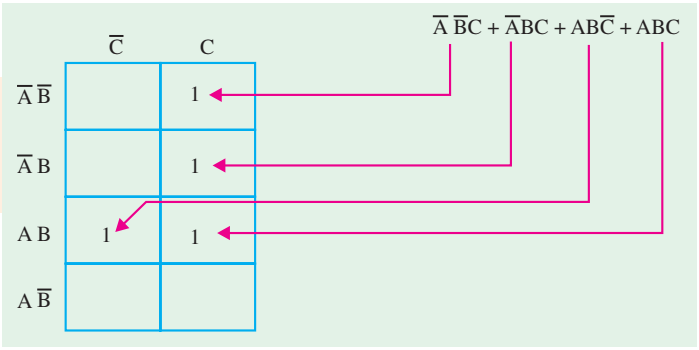


Fig. 71.33

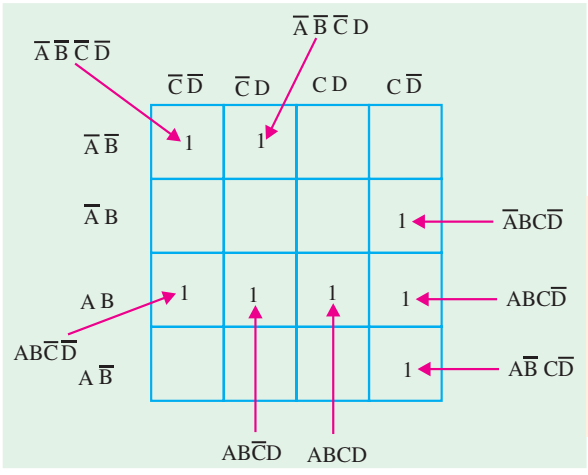


Fig. 71.34

Example 71.22. *Map the following standard sum-of-products (SOP) expression on a Karnaugh map :*

$$\overline{A}\,B\,C\,\overline{D} + AB\,C\,\overline{D} + A\,B\,\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + A\,B\,\overline{C}D + \overline{A}\,\overline{B}\,\overline{C}\,D + A\,\overline{B}C\,\overline{D} + ABCD$$

Solution. Sketch a four variable Karnaugh map. Select the first product term $\overline{A}\,B\,C\,\overline{D}$ from the given *SOP* expression and enter 1 in the corresponding square as shown in Fig. 71.34. Similarly enter 1 for the other product terms in the given *SOP* expression. Check that number of 1*s* in the Karnaugh Map is equal to the number of product terms in the given *SOP* expression.

71.18. Mapping a Nonstandard SOP Expression on the Karnaugh Map

A nonstandard sum-of-products (*SOP*) expression is a one that has product terms with one or more missing variables. In such a case, Boolean expression must first be converted to a standard form by a procedure explained in Art. 71.9.

Let us consider an example to illustrate the procedure for mapping a nonstandard *SOP* expression on the Karnaugh map. Suppose we have the *SOP* expression :

$$\overline{A} + A\overline{B} + AB\overline{C}$$

As seen, this expression is obviously not in standard form because each product term does not have three variables. The first term is missing two variables, the second term is missing one variable and the third term is standard. In order to convert the given nonstandard *SOP* expression to a standard form, we multiply the first product term by $B + \overline{B}$ and $C + \overline{C}$, and the second term by $C + \overline{C}$. Expanding the resulting expression, we get,

$$\overline{A}\,(B + \overline{B})\,(C + \overline{C}) + A\overline{B}\,(C + \overline{C}) + AB\overline{C}$$

$$= \overline{A}BC + \overline{A}\,\overline{B}\,C + \overline{A}\,B\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C} + A\overline{B}C + A\,\overline{B}\,\overline{C} + AB\overline{C}$$

Rearranging the expression for our convenience, we get

$$\overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,C + \overline{A}\,B\,\overline{C} + \overline{A}BC + AB\overline{C} + A\,\overline{B}\,\overline{C} + A\overline{B}C$$

This expression can be mapped on a three-variable Karnaugh map as shown in Fig. 71.35.

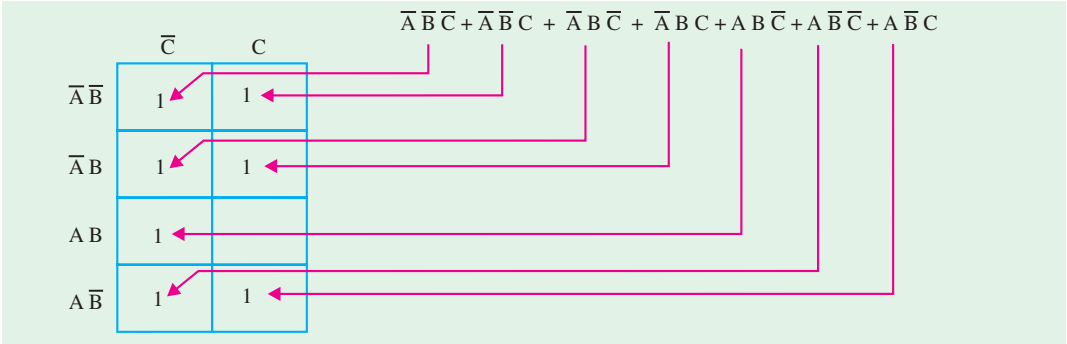


Fig. 71.35

71.19. Simplification of Boolean Expression Using Karnaugh Map

The process that results in an expression containing the minimum number of possible terms with the minimum number of variables is called **simplification** or **minimization.** After the *SOP* (or the Boolean) expression has been mapped on the Karnaugh map, there are three steps in the process of obtaining a minimum *SOP* expression. The three steps are : **(*a*)** grouping the 1*s*, **(*b*)** determining the product term for each group and **(*c*)** summing the resulting product terms.

(*a*) *Grouping the 1s* : We can group the 1*s* on the Karnaugh map according to the following rules by enclosing those adjacent squares containing 1*s*. The objective is to maximize the size of the groups and to minimize the number of groups.

1. A group must contain either 1, 2, 4, 8 or 16 squares. In the case of two-variable Karnaugh map, 4 squares is the maximum group, for three-variable map, 8 squares are the maximum group and so on.
2. Each square in the group must be adjacent to one or more squares in that same group but all squares in the same group do not have to be adjacent to each other.
3. Always include the largest possible number of 1*s* in a group in accordance with rule 1.
4. Each 1 on the Karnaugh map must be included in at least one group. The 1*s* already in a group can be included in another group as long as the overlapping groups include non-common 1*s*.

(*b*) *Determining the Product Term for each group* : Following are the rules that are applied to find the minimum product terms and the minimum sum-of-products expression :

1. ***Group the squares that have 1s* :** Each group of squares containing 1*s* creates one product term composed of all variables that occur in only one form (either uncomplemented or complemented) within the group. Variables that occur both uncomplemented and complemented within the group are eliminated. These are known as contradictory variables.
2. In order to determine the minimum product term for each group, we need to look at the standard methodology for three-variable and four-variable Karnaugh map respectively.
   - **(*i*) *For a three-variable K-map* :** **(1)** for 1-square, group we get a three-variable product term, **(2)** for a 2-square group, we get a two-variable product term, **(3)** for a 4-square product term, we get a one-variable product term.
   - **(*ii*) *For a four-variable K-map* : (1)** For a 1-square group, we get a four-variable product term, **(2)** for a 2-square group, we get a three-variable product term, **(3)** for a 4-square group, we get a two-variable product term and **(4)** for a 8-square group, we get a one-variable product term.

(*c*) *Summing the resulting product terms* : When all the minimum product terms are derived from the Karnaugh map, these are summed to form the minimum sum-of-products expression.

Note : In some cases, there may be more than one way to group the 1*s* to form the product terms. Whatever be the way, the minimal expression must have the same number of product terms and each product term, the same number of Boolean variables.

The examples given below will help you to understand and apply the simplification of the *SOP* expression using Karnaugh map.

Example 71.23. *Simplify the following Boolean expression using the Karnaugh mapping technique* :

$$X = \overline{A}\,B + \overline{A}\,\overline{B}C + A\,B\,\overline{C} + A\,\overline{B}\,\overline{C}$$

Solution. The first step is to map the given Boolean expression on the Karnaugh map. Notice that there are three variables *A*, *B* and *C* in the Boolean expression, therefore we need a three-variable Karnaugh map.

The Boolean expression to be mapped is,

$$X = \overline{A}\,B + \overline{A}\,\overline{B}C + A\,B\,\overline{C} + A\,\overline{B}\,\overline{C}$$

Note that the given Boolean expression is a nonstandard *SOP* expression because the first product term $\overline{A}\,B$ has the variable *C* missing in it. This can be converted into a standard *SOP* form by modifying the expression as below.

$$\begin{aligned} X &= \overline{A}\,B.1 + \overline{A}\,\overline{B}C + A\,B\,\overline{C} + A\,\overline{B}\,\overline{C} \\ &= \overline{A}\,B\,(C + \overline{C}) + \overline{A}\,\overline{B}C + A\,B\,\overline{C} + A\,\overline{B}\,\overline{C} && \ldots(C + \overline{C} = 1) \\ &= \overline{A}\,BC + \overline{A}\,B\,\overline{C} + \overline{A}\,\overline{B}C + A\,B\,\overline{C} + A\,\overline{B}\,\overline{C} && \ldots(1) \end{aligned}$$

Equation (1) can be mapped on the Karnaugh map as shown in Fig. 71.36. In order to simplify the given expression, the 1*s* can be grouped together as shown by the loop around the 1*s*. The four 1*s* in the first column are grouped together and the term we get is $\overline{C}$. This is because of the fact that the squares within this group contain both A and $\overline{A}$ and B and $\overline{B}$, so these variables are eliminated. Similarly, the two 1*s* in the second row are grouped together and the term we get is $\overline{A}B$. This is because of the fact that squares in this group contain both C and $\overline{C}$ which is eliminated. Summing up the two-product terms, the simplified expression is,

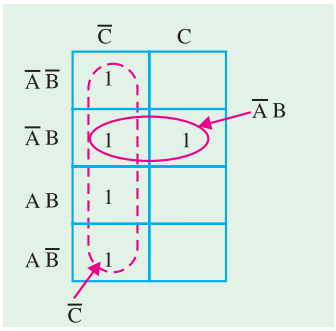


Fig. 71.36

$$X = \overline{A}B + \overline{C}.$$

Example 71.24. *Simplify the following SOP expression using the Karnaugh mapping procedure :*

$$X = \overline{A}\,B\overline{C}\,D + A\,\overline{B}\,\overline{C}\,D + \overline{A}\,\overline{B}\,C\,D + A\,B\,\overline{C}\,D + A\,B\,\overline{C}\,\overline{D} + ABCD$$

Solution. First of all, notice that the given *SOP* expression is already in the standard from *i.e.* all the product terms in the given expression have all the four variables *A*, *B*, *C* and *D*.

Next sketch a four-variable Karnaugh map. Select the first product term ($\overline{A}\,B\overline{C}\,D$) from the given expression and enter 1 in the corresponding square as shown in Fig. 71.37. Similarly enter 1 for the other product terms in the given *SOP* expression to complete the mapping. In order to simplify the given *SOP* expression, the 1*s* can be grouped together as shown by the loop around the 1*s*. The four 1*s* in the second column are grouped together and the product term we get is $\overline{C}\,D$. This is because of the fact that squares within this group contain both A and $\overline{A}$ and B and $\overline{B}$, so these variables are eliminated.

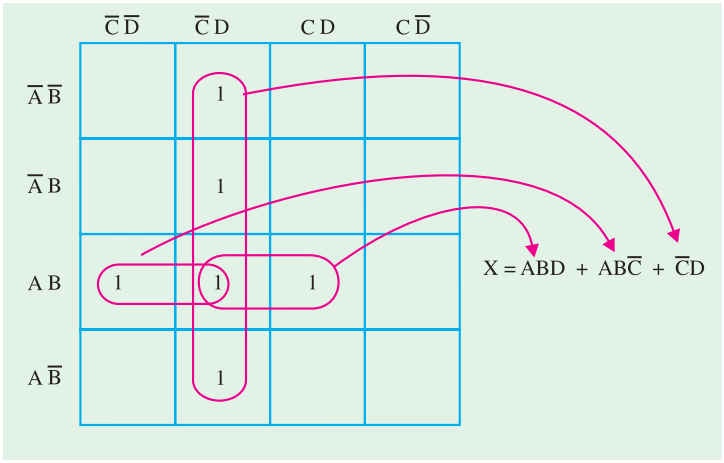


Fig. 71.37

The two 1*s* in the first and second columns can be grouped together. This group contains both D and $\overline{D}$, so this variable is eliminated and the resulting product term is $AB\overline{C}$. Similarly the two 1*s* in the second and third columns can be grouped together. This group contains both C and $\overline{C}$, so this variable is eliminated and the resulting product term is ABD.

The resulting minimal or simplified *SOP* expression is obtained by summing up the three product terms $\overline{C}\,D$, $A\,B\,\overline{C}$ and ABD as shown below :

$$X = ABD + AB\overline{C} + \overline{C}\,D.$$

Example 71.25. *Simplify the following SOP expression using the Karnaugh mapping technique.*

$$X = B\overline{C}\,\overline{D} + \overline{A}\,B\,\overline{C}\,D + AB\overline{C}D + \overline{A}BCD + ABCD$$

Solution. First of all, notice that the given *SOP* expression is in the nonstandard *SOP* form because the first product term ($B\overline{C}\,\overline{D}$) has a variable A or $\overline{A}$ missing in it. Let us convert the given *SOP* expression into a standard *SOP* form as shown below :

$$X = 1.B\overline{C}\,\overline{D} + \overline{A}\,B\,\overline{C}\,D + AB\overline{C}D + \overline{A}BCD\,ABCD$$
$$= (\overline{A} + A)\,B\,\overline{C}\,\overline{D} + \overline{A}\,B\,\overline{C}\,D + AB\overline{C}D + \overline{A}BCD + ABCD \quad ...(\because \overline{A} + A = 1)$$
$$= \overline{A}B\,\overline{C}\,\overline{D} + A\,B\,\overline{C}\,\overline{D} + \overline{A}B\overline{C}D + AB\overline{C}D + \overline{A}BCD + ABCD$$

This expression can be mapped on to the four-variable Karnaugh by entering 1 for each product term in the corresponding square as shown in Fig. 71.38.

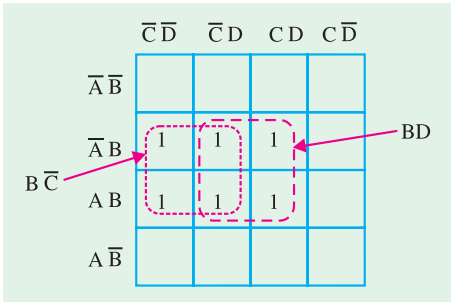


Fig. 71.38

In order to simplify the *SOP* expression, the 1*s* can be grouped together as shown by the loop around the 1*s*. The four 1*s* looped together form the first and second cloumns, contain both A and $\overline{A}$ and D and $\overline{D}$, so these variables are eliminated and the resulting product term is $B\overline{C}$. Similarly, the four 1*s* looped together form the second and the third column contain both A and $\overline{A}$ and C and $\overline{A}$, so these variables are eliminated and the resulting product term is BD. The resulting simplified *SOP* expression is the sum of the product terms $B\overline{C}$ and BD, *i.e.*,

$$X = B\overline{C} + BD$$

Example 71.26. *Fig. 71.39 shows a Karnaugh map of a sum-of-products (SOP) function. Determine the simplified SOP function.* **(UPSC Civil Services 2000)**

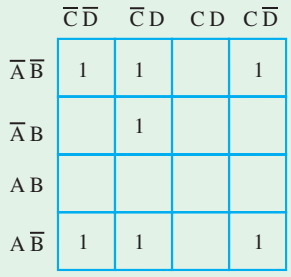


Fig. 71.39

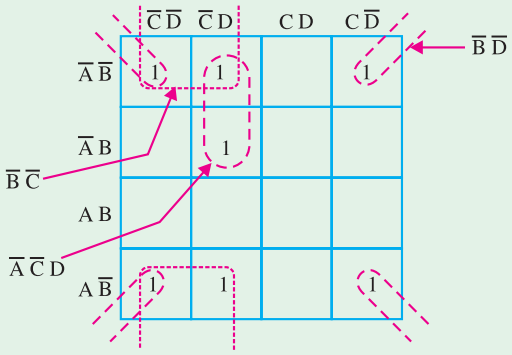


Fig. 71.40

Solution. The grouping of 1*s* is as shown in the Fig. 71.40. Notice the "wrap around" four-square group that includes the 1*s* on four corners of the Karnaugh map. This group produces a product term $\overline{B}\,\overline{D}$. This is determined by observing that the group contains both A and $\overline{A}$ and C and $\overline{C}$, so these variables are eliminated.

Another group of four with "wrap-around" adjacency is formed form the top and the bottom rows of the Karnaugh map. This group overlaps with the previous group and produces a product term $\overline{B}\,\overline{C}$. This is determined by observing that this group contains both A and $\overline{A}$ and D and $\overline{D}$, so these variables are eliminated.

The remaining 1 is absorbed in a overlapping group of two squares. This group produces a three-variable term $\overline{A}C\overline{D}$. This is determined by observing that this group contains both B and $\overline{B}$, so this variable is eliminated. This resulting simplified *SOP* function is the sum of the product terms $\overline{B}\,\overline{D}$ + $\overline{B}\,\overline{C}$ and $\overline{A}C\overline{D}$, *i.e.*

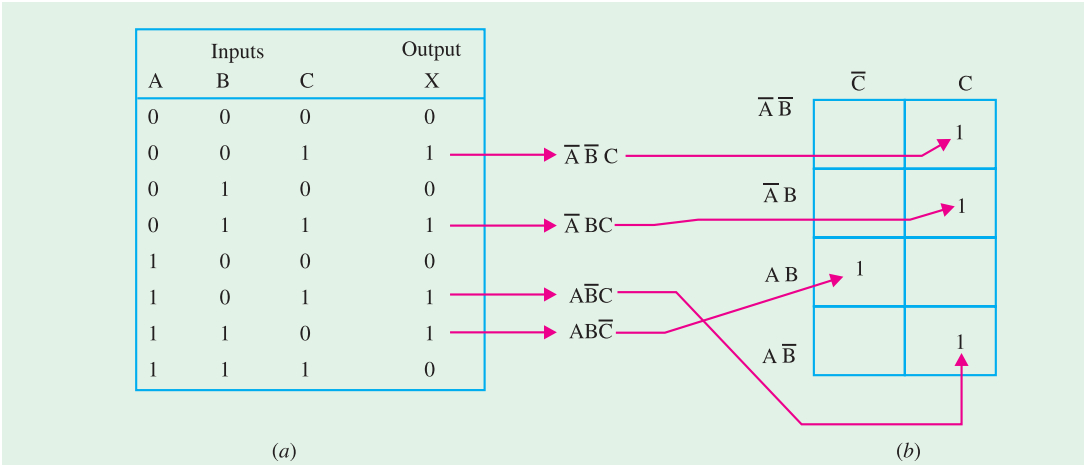
$$X = \overline{B}\,\overline{D} + \overline{B}\,\overline{C} + \overline{A}C\overline{D}$$

71.20. Mapping Directly on Karnaugh Map from a Truth Table

It is possible to map directly on Karnaugh map from a truth table. Recall that a truth table gives the output of a Boolean expression for all possible input variable combinations. Let us illustrate direct mapping through an example of a Boolean expression and its truth table representation.

Let $X = \overline{A}\,\overline{B}C + \overline{A}BC + A\overline{B}C + AB\overline{C}$. Then its truth table can be indicated as shown in Fig. 71.41 (*a*) and its Karnaugh mapping is shown in Fig. 71.41 (*b*). Notice in the truth table that the output X is 1 for four different input variable combinations.

It is evident from the Fig. 71.41 (*a*) and (*b*) that truth table and Karnaugh map are simply different ways to represent a logic function.



Inputs			Output
A	B	C	X
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

Fig. 71.41

Example 71.27. *Implement the following Boolean expression using minimum number of 3-input NAND gates.*

$$f(A, B, C, D) = \Sigma(1, 2, 3, 4, 7, 9, 10, 12)$$ **(UPSC Engg. Services 1991)**

Solution. The given Boolean function indicates that its output is 1 corresponding to the terms indicated within the expression *i.e.*, 1, 2, 3, 4, 7, 9, 10 and 12. This is shown in Fig. 71.42 (*a*). We can map these values directly on to the four-variable Karnaugh map as shown in Fig. 71.42 (*b*). In order to simplify the Boolean expression represented on the Karnaugh map, group the 1*s* as shown in the Fig. 71.42 (*b*). The group of two squares in the first coloum produces a product term $B\,\overline{C}\,\overline{D}$. This is determined by observing that the group contains both A and $\overline{A}$, so this variable is eliminated. Another group of two squares in the second column produces term $\overline{B}\,CD$. The variable A is eliminated because the group contains both A and $\overline{A}$.

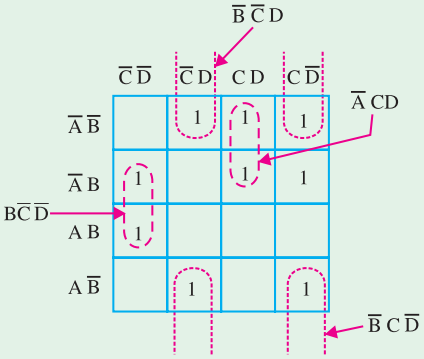
Another group of two squares in the third column produces the term $\overline{A}\,CD$. The variable B is eliminated because the group contains both B and $\overline{B}$. Still another group of two squares in the fourth column produces the term $\overline{B}C\,\overline{D}$. The variable A is eliminated because the group contains both A and $\overline{A}$. Thus the resulting simplified expression,

$$f(A, B, C, D) = B\overline{C}\,\overline{D} + \overline{B}\overline{C}\,D + \overline{A}C\,D + \overline{B}C\,\overline{D}$$

This can be implemented using 3-input NAND gate as shown in Fig. 71.43.

Decimal Number	Inputs A	B	C	D	Output f	
0	0	0	0	0	0	
1	0	0	0	1	1	$\bar{A}\,\bar{B}\,\bar{C}\,D$
2	0	0	1	0	1	$\bar{A}\,\bar{B}\,C\,\bar{D}$
3	0	0	1	1	1	$\bar{A}\,\bar{B}\,C\,D$
4	0	1	0	0	1	$\bar{A}\,B\,\bar{C}\,\bar{D}$
5	0	1	0	1	0	
6	0	1	1	0	0	
7	0	1	1	1	1	$\bar{A}\,B\,C\,D$
8	1	0	0	0	0	
9	1	0	0	1	1	$A\,\bar{B}\,\bar{C}\,D$
10	1	0	1	0	1	$A\,\bar{B}\,C\,\bar{D}$
11	1	0	1	1	0	
12	1	1	0	0	1	$A\,B\,\bar{C}\,\bar{D}$
13	1	1	0	1	0	
14	1	1	1	0	0	
15	1	1	1	1	0	

(*a*)



(*b*)

Fig. 71.42

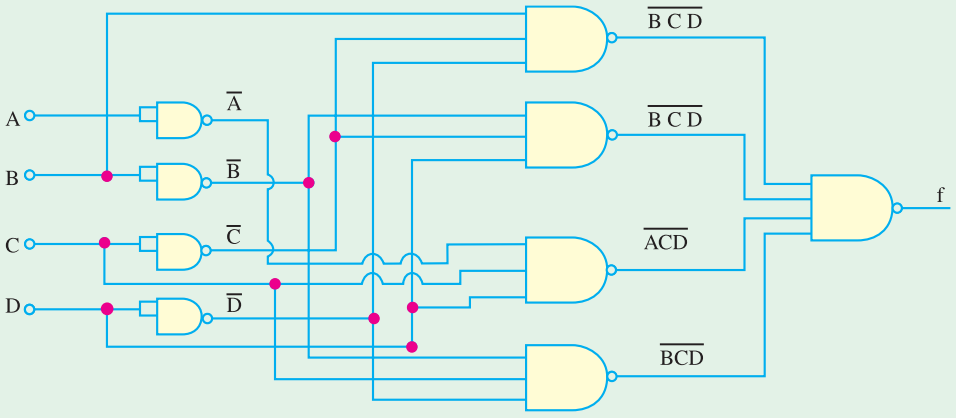


Fig. 71.43

71.21. "Don't Care" conditions

In digital systems design sometimes a situation arises in which some input variable conditions are not allowed. For example in a *BCD* (binary coded decimal) code, there are six invalid combinations : 1010, 1011, 1100, 1101, 1110 and 1111. Since these unallowed states will never occur in an application involving the *BCD* code, they can be treated as "don't care" terms with respect to their effect on the output. That is, for these "don't care" terms either 1 or 0 may be assigned to the output.

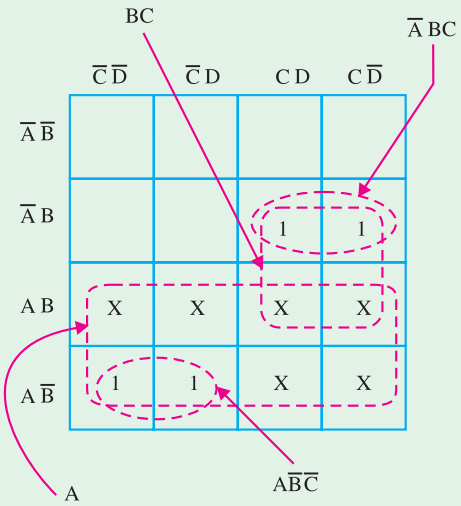
Now, we shall discuss as how the "don't care" terms can be used to advantage on the Karnaugh map for simplifying the Logic equations.

Consider for example, a combinational circuit which products a '1' output corresponding to a

BCD input equal and greater than 6. The output is 0 corresponding to a *BCD* input less than 6. The truth table for this situation is as shown in Fig. 71.44 (*a*).

Inputs				Output	
A	*B*	*C*	*D*	*X*	
0	0	0	0	0	
0	0	0	1	0	
0	0	1	0	0	
0	0	1	1	0	
0	1	0	0	0	
0	1	0	1	0	
0	1	1	0	1	→ $\overline{A}BC\overline{D}$
0	1	1	1	1	→ $\overline{A}BCD$
1	0	0	0	1	→ $A\overline{B}\overline{C}\overline{D}$
1	0	0	1	1	→ $A\overline{B}\overline{C}D$
1	0	1	0	*X*	
1	0	1	1	*X*	
1	1	0	0	*X*	
1	1	0	1	*X*	
1	1	1	0	*X*	
1	1	1	1	*X*	

(*a*)



(*b*)

Fig. 71.44

We know that we can place 1*s* directly from the truth table on the Karnaugh map. Similarly we can place *X* for the "don't care" enteries directly on the Karnaugh map as shown in Fig. 71.44 (*b*). When grouping the 1*s*, *X*s can be treated as 1*s* make a larger grouping or as 0*s*, if they cannot be used to advantage Recall the larger the group of 1*s* the simpler the resulting term will be. Taking advantage of the "don't care" and using them as 1*s*, the resulting expression for the output is $A + BC$. However, if the "don't cares" are not used as 1*s*, the resulting expression is $A\overline{B}\,\overline{C} + \overline{A}BC$. Thus we can see the advantage of using "dont care" terms to get the simplest logic expression.

	$\overline{C}\,\overline{D}$	$\overline{C}$ D	C D	C $\overline{D}$
$\overline{A}\,\overline{B}$	1		X	X
$\overline{A}$ B		1	X	1
A B		X	1	
A $\overline{B}$	1	X		X

Fig. 71.45

Example 71.28. *Consider the Karnaugh map shown in Fig. 71.45. Determine the logic function represented by the map and simplify it in the minimal form.* **(UPSC Engg. Services 1997)**

Solution. We know that when grouping the 1*s*, X*s* can be treated as 1*s* to make a larger grouping or as 0*s* if they cannot be used to advantage. Recall, the larger the group of 1*s*, the simpler the resulting term will be. Taking advantage of the *X*s and using them as 1*s*, the grouping of 1*s* and *X*s is as shown in Fig. 71.46.

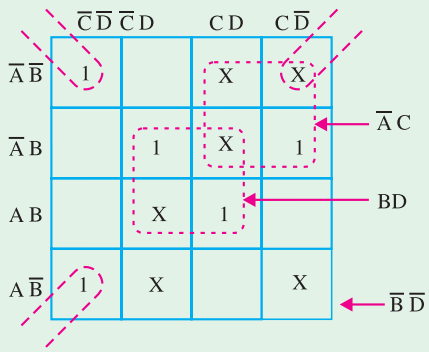


Fig. 71.46

Notice the "wrap-around" four-square group that includes the 1*s* and *X*s on fours corners of the Karnaugh map. This group produces a product term $\overline{B}\,\overline{D}$. This is determined by observing that the group contains both A and $\overline{A}$ and C and $\overline{C}$, so these variables are eliminated. Another group of four squares containing 1*s* and *X*s around the centre of Karnaugh map is formed.

This group produces a product term BD. This is determined by observing that the group contains both A and $\overline{A}$ and C and $\overline{C}$, so these variables are eliminated.

Another group of four containing 1*s* and *X*s is formed near the top right corner of Karnaugh map. This group overlaps with the previous group and produces a product term $\overline{A}\,C$. This is determined by observing that the group contains both B and $\overline{B}$ and D and $\overline{D}$, so these variables are eliminated.

The resulting simplified logic function is the sum of the three product terms : $\overline{B}\,\overline{D}$, BD and $\overline{A}\,C$, *i.e.*,

$$X = \overline{B}\,\overline{D} + BD + \overline{A}\,C$$

71.22. Main Logic Families

Most digital systems are designed by combining various logic functions discussed in Chapter 19. All these logic circuits are available in *IC* modules and are divided into many 'families'. Each family is classified by abbreviations which indicate the type of logic circuit used. For example, *RTL* means resistor-transistor logic. We will discuss the following seven transistor logic families although the first two are, at present, of historic interest only.

1. **Resistance-transistor logic (*RTL*) :** it was the first family group of logic circuits to be developed and packaged in *IC* form in *early* 1960*s*;
2. **Diode-transistor logic (*DLT*) :** It followed *RTL* in *late* 1960*s*;
3. **Transistor-transistor logic (*TTL*) *OR* (T^2L) :** was introduced in the early 1970 *s*;
4. **Schottky *TTL* :** was introduced to improve the speed of *TTL*;
5. **Emitter-coupled logic (*ECL*) :** It is fastest logic line currently available;
6. **Integrated-injection logic (I^2L) :** It is one of the latest of the bipolar types of logic;
7. **Complementary metal-oxide semiconductor (*CMOS*) :** It has the lowest power dissipation of the currently-available logic circuits.

The various logic families discussed above posses different characteristics as detailed below.

71.23. Saturated and Non-saturated Logic Circuits

Those logic circuits in which transistors are driven into saturation are called **saturated** logic circuits or simply **saturated logic.** Those circuits which avoid saturation of their transistors are designed **non-saturated logic.**

The disadvantage of saturated logic is the delay that occurs when the transistors is brought out of saturation. When a transistor is saturated, its base is flooded with carriers. Even when base voltage is switched off, the base remains flooded for some time till all carriers leave it. The time required by the carriers to leave the base is called **saturation delay time** (t_s). Obviously, saturated logic circuits have low switching speeds whereas non-saturated type are much faster. *TTL* is the example of a saturated logic whereas *ECL* represents a non-saturated logic.

71.24. Basic Operating Characteristics and Parameters of Logic Families

When we work with digital ICs from different logic families, we should be familiar with, not only their logical operation but also with the basic operational properties. Following are the important basic operational properties important from the subject point of view.

1. *DC* supply voltage.
2. *TTL* and *CMOS* logic levels
3. Noise immunity
4. Noise margin.
5. Power dissipation.
6. Propagation delay.
7. Speed-power product.
8. Loading and fan-out.

Now we will describe all the above operational characteristics one by one in the following pages.

71.25. DC Supply Voltage

The standard value of the dc supply voltage for *TTL* (*i.e.*, transistor-transistor logic) and CMOS (*i.e.*, complementary metal-oxide semiconductor) device is + 5*V*. For simplicity, the dc supply voltage is usually omitted from the logic circuits. But in practice, it is connected to the V_{CC} or V_{DD} pin of an *IC*

package and the ground is connected to the *GND* pin of an *IC* package. Both the voltage and ground are distributed internally to all the logic gates with the package as shown in Fig. 71.47 (*a*). The connections for the single logic gate are as shown in Fig. 71.47 (*b*).

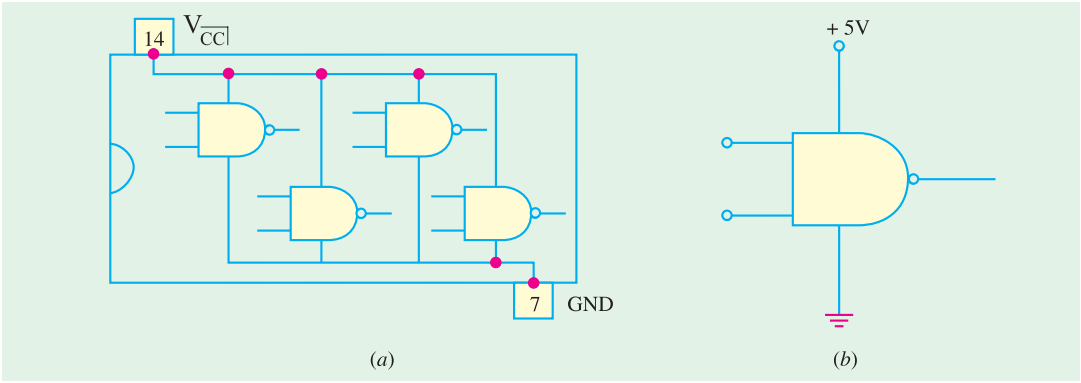


Fig. 71.47

71.26. TTL and CMOS Logic Levels

For TTL circuits, the range of input voltages, that can represent a valid *LOW* (logic 0) and a valid *HIGH* (logic 1) is as shown in Fig. 71.48 (*a*). As seen from this diagram, the range of input voltages that can represent a valid LOW (logic 0) is from 0 to 0.8 V. The LOW input voltage is indicated by the symbol V_{IL}. The lower limit for V_{IL} is represented by $V_{IL(min)}$ and the higher limit for V_{IL}, by $V_{IL(max)}$. The range of input voltages that can represent a valid HIGH (logic 1) is from 2 V to V_{CC} (usually 5 V). The HIGH input voltage is indicated by symbol V_{IH}. The lower limit for V_{IH} is represented by $V_{IH(min)}$ and the higher limit for V_{IH} by $V_{IH(max)}$. Note that the range of values between 0.8 V and 2 V is called ***indeterminate.*** This means that when a input voltage is in this range, it can be interpreted as a HIGH or LOW by the logic circuit. Therefore TTL logic gates cannot be operated reliably when input voltages are in this range.

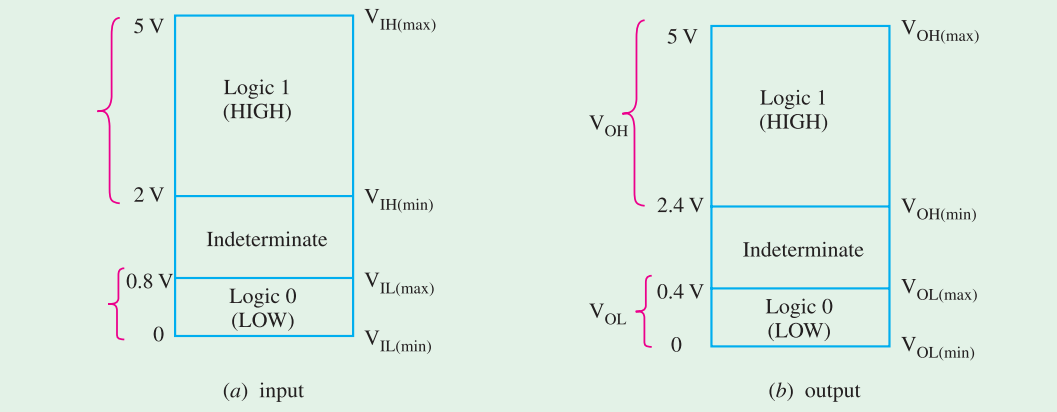


Fig. 71.48

Fig. 71.48 (*b*) shows the range of *TTL* output voltages that can represent valid HIGH (logic 1) and valid *LOW* (logic 0). As seen the range for logic 1 output ifs from 2.4 V to 5 V and logic 0 output is from 0 to 0.4 V. Note again that the range of values between 0.4 V and 2.4 V is indeterminate. Also note that the output voltage for logic 1 is indicated by the symbol V_{OH}. The lower limit for V_{OH} is represented by $V_{OH(min)}$ and the higher limit for V_{OH} by $V_{OH(max)}$. Similarly the output voltage for logic 0 is indicated by the symbol V_{OL}. The lower limit for V_{OL} is represented by $V_{OL(min)}$ and the higher limit for V_{OL} by $V_{OL(max)}$.

It may be noted from Fig. 71.48 (*a*) and (*b*) that the minimum HIGH output voltage, $V_{OH\,(min)}$ is greater than the minimum HIGH in put voltage $V_{IH\,(min)}$. On the other hand, the maximum LOW output voltage, $V_{OL\,(min)}$ is less than the maximum LOW input voltage, $V_{IL(max)}$.

The input and output voltages for a device from HCMOS (*i.e.* High-speed CMOS) logic family for V_{DD} = 5 V as shown in Fig. 71.49 (*a*) and (*b*) respectively.

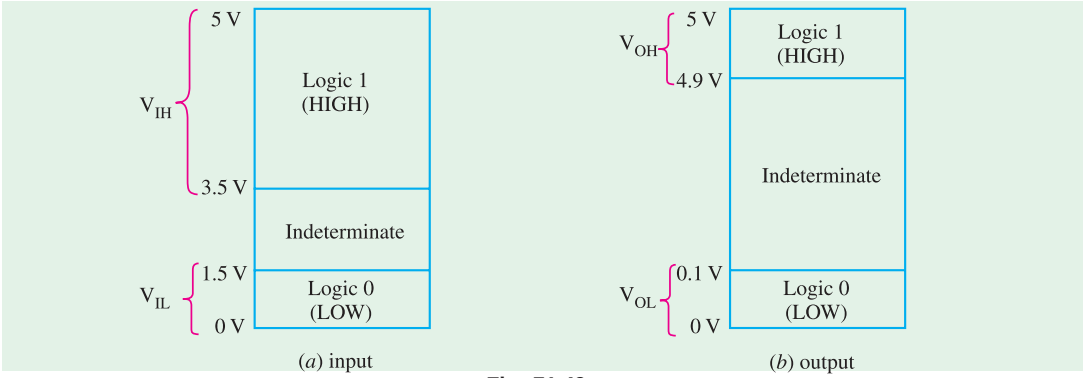


Fig. 71.49

Notice that the input and output voltage values in the HCMOS device are different from that of a TTL device. In a HCMOS device, $V_{IL(max)}$ is 1.5 V (its value is 0.8 V in TTL), $V_{IH\,(min)}$ is 3.5 V (its value is 2 V in TTL), $V_{OL\,(max)}$ is 0.1 V (its value is 0.4 V is TTL), $V_{OH\,(min)}$ is 4.9 V (its value is 2.4 V in TTL).

71.27. Noise Immunity

The noise immunity of a logic circuit refers to the circuit's ability to tolerate noise without causing a false change in its output voltage. The noise voltage is produced by stray electric and magnetic fields on the connecting wires between logic circuits. Sometimes, too much noise voltage cause the voltage at the input of the logic circuit to drop below $V_{IH\,(min)}$ or rise above $V_{IL\,(max)}$. This could produce unpredictable operation in a logic circuit.

71.28. Noise Margin

A quantitative measure of a circuit's noise immunity is called noise margin. It is expressed in volts. There are two values of noise margin specified for a given logic circuit as described below.

1. The high level noise margin (V_{NH})
2. The low level noise margin (V_{NL})

These parameters are shown in Fig. 71.50 and are given by the equations,

$$V_{NH} = V_{OH\,(min)} - V_{IH\,(min)}$$

$$V_{NL} = V_{IL\,(max)} - V_{OL\,(max)}$$

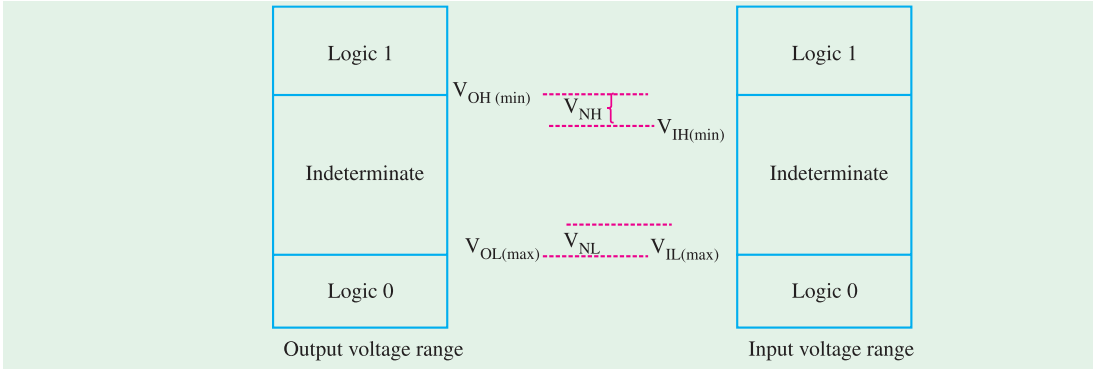


Fig. 71.50

Example 71.29. *Table shows the input/output voltage specifications for the standard TT family.*

Parameter	*Min (V)*	*Typical (V)*	*Max (V)*
V_{OH}	2.4	3.4	—
V_{OL}	—	0.2	0.4
V_{IH}	2.0	—	—
V_{IL}	—	—	0.8

Using these values, find ***(a)*** *the maximum value of noise spike that can be tolerated when a HIGH output is driving an input,* ***(b)*** *the maximum value of noise spike when a LOW output is driving an input.*

Solution. The maximum value of the noise spike, when driven by a HIGH output,

$$V_{NH} = V_{OH\,(\min)} - V_{IH\,(\min)} = 2.4 - 2.0 = 0.4\ \text{V}$$

and the maximum value of the noise spike, when driven by a LOW output,

$$V_{NL} = V_{IL\,(\max)} - V_{OL\,(\max)} = 0.8 - 0.4 = 0.4\ \text{V}$$

It is observed that a TTL gate is immunte to 0.4 V of poise for both the HIGH and LOW input states.

71.29. Power Dissipation

As a matter of fact, all logic gates draw current from the dc supply voltage for its normal operation. When the logic gate is in the HIGH output state, it draws an amount of current, I_{CCH}, as shown in Fig. 71.51 (*a*) and when in LOW output state, it draws an amount of current I_{CCL} as shown in Fig. 71.51 (*b*).

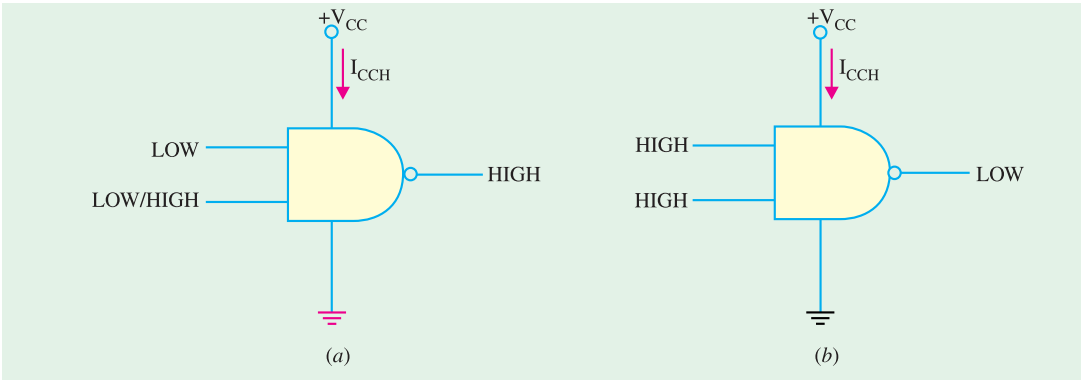


Fig. 71.51

The power dissipation of a logic gate is given by the product of the dc supply votage (V_{CC}) and the amount of current drawn from the suppy (*i.e.*, I_{CCH} or I_{CCL}). Thus power dissipation is given by :

$$P_D = V_{CC} \cdot I_{CCH} \text{ or } V_{CC} \cdot I_{CCL}$$

For example, if I_{CCH} is 2.5 *m A* when V_{CC} *is* 5 V, the power dissipation,

$$P_D = V_{CC} \cdot I_{CCH} = 5\ \text{V} \times 2.5\ \text{mA} = 12.5\ \text{mW}.$$

Usually, the logic gate operates with the inputs, which keep on changing with time (*i.e.*, the input is pulsed). Accordingly the output of a logic gate also switches back and forth between HIGH and LOW. Because of this, the amount of current drawn from the dc supply also varies between I_{CCH} and I_{CCL}. In such a situation, we calculate the average power dissipation. The average power dissipation depends upon the duty cycle and is usually specified for a duty cycle of 50%, the output is HIGH half the time and LOW the other half. Therefore average supply current is :

$$I_{CC} = \frac{I_{CCH} + I_{CCL}}{2}$$

and the average power dissipation is,

$$P_D = V_{CC} . I_{CC}$$

Example 71.30. *A TTL logic gate draws 2 mA when its output is HIGH and 3.5 mA when its output is LOW. Calculate the average power dissipation if the supply voltage is 5 V and the logic gate is operated on 50% duty cycle.*

Solution. The average supply current,

$$I_{CC} = \frac{I_{CCH} + I_{CCL}}{2} = \frac{2\text{mA} + 3.5\text{mA}}{2} = 2.75\text{ mA}$$

∴ The average power dissipation,

$$P_D = V_{CC} . I_{CC} = 5\,r \times 2.75\text{ mA} = \mathbf{13.7\ mW}$$

71.30. Power Dissipation versus Frequency

Fig. 71.52 shows a graph of power dissipation versus frequency for a *TTL* and a *CMOS* logic gate. As seen from this graph, the power dissipation in a *TTL* circuit is essentially constant over its range of operating frequencies. However, the power dissipation in a *CMOS* circuit is frequency dependent. That is, it is extremely low under zero frequency (or dc) conditions. But the power dissipation increases as the frequency increases. For example, the power dissipation of a typical TTL logic gate is a constant 2 mW. On the other hand, power dissipation of typical *CMOS* logic gate is 0.0025 mW under static (or dc) conditions and 0.17 mW at 100 kHz.

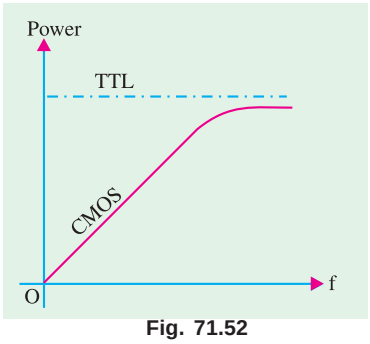


Fig. 71.52

71.31. Propagation Delay

When a signal passes (*i.e.*, propagates) through a logic circuit, it always experiences a finite time delay as shown in Fig. 71.53. It shows that the change in output level occurs after a short time, (called the propagation delay time), later than the change in input level that caused it. There are two propagation delay times specified for logic gates.

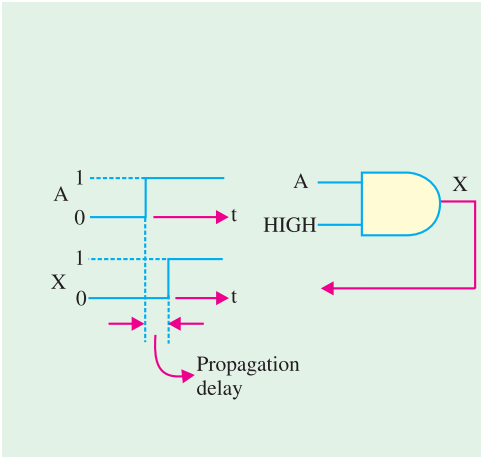


Fig. 71.53

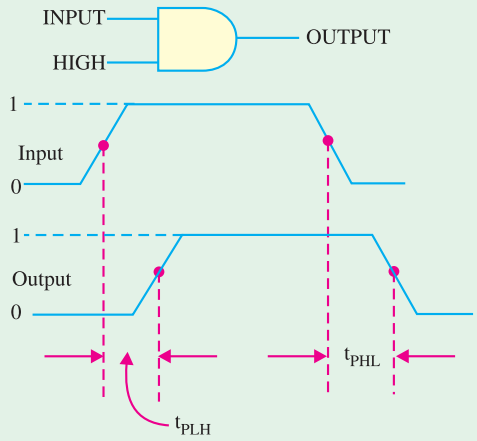


Fig. 71.54

1. t_{PLH} – It is the time interval between a designated point on the input pulse and the corresponding point on the output pulse when the output is changing from LOW to HIGH (or 0 to 1) as shown in Fig. 71.54.

2. t_{PHL} – It is the time interval between a designated point on the input pulse and the corresponding pont on the output pulse when the output is changing from HIGH to LOW (or 1 to 0) as shown in Fig. 71.54.

It may be noted that the propagation delay times are indicated in Fig. 71.54 with 50% points on the pulse edges used as references.

The propagation delay time of a logic gate limits its maximum operating frequency. The greater the propagation delay time of a logic gate, the lower is its maximum operating frequency. This means a high speed logic gate is a one that has a small propagation delay time. For example, a logic gate with a delay time of 2 ns is faster than a logic gate that has a delay time of 10 ns.

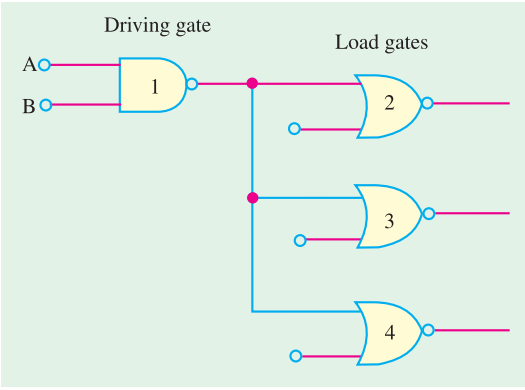


Fig. 71.55

71.32. Speed-Power Product

It provides a basis for comparison of logic circuits when the propagation delay and power dissipation are important considerations in the selection of the type of logic family to be used in certain application. The speed power product is exdpressed in picojule (pJ). It may be noted carefully that the lower the speed-product, the better is the value. Typically the *CMOS* family has much lower value of speed-power product as compared to the *TTL* logic family. For example, a typical *CMOS* logic family has a speed-power product of 1.5 pJ at 100 Hz while a typical *TTL* has speed-power product of 20 pJ.

71.33. Loading and Fan-out

When the output of any logic gate is connected to one or more inputs of other logic gates, a load on the driving gate is created. This is shown in Fig. 71.55. Here the output of a logic gate (labeled as 1) is connected to the inputs of 3 other logic gates (labeled as 2, 3, and 4). Note that the logic gate labeled 1 is called a ***driving gate*** while the logic gates labeled as 2, 3 and 4 are called ***load gates***.

In any logic family, there is a limit to the number of load gate inputs that a given logic gate can drive. This limit is called the ***fan-out*** of the logic gate. Now we will study the loading and fan-out in *TTL* and *CMOS* logic families in more detail.

Loading and fan-out in TTL family : A *TTL* gate that acts as a driving gate ***sources*** (*i.e.*, supplies) current to a load gate input in the logic HIGH state and *sinks* (*i.e.*, receives) current from the load gate in the logic LOW state. Fig. 71.56 ***(a)*** illustrates the current sourcing and ***(b)*** shows current sinking in the logic gates.

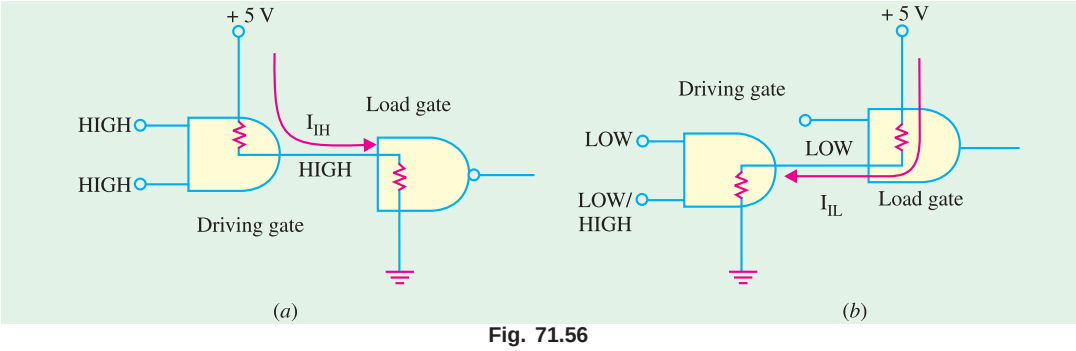


Fig. 71.56

Note that I_{IH} is the current supplied by the driving gate to the load gate when the input of the load gate is HIGH similarly. I_{IL} is the current received by the driving gate from the load gate when the input of the load gate is *LOW*.

As more and more number of load gates are connected to the driving gate, the loading on the driving gate increases. The total source current increases with each load gate input that is added as illustrated in Fig. 71.57. As the source current increase, the internal voltage drop of the driving gate increases V_{OH}. This causes the voltage drop V_{OH} to decreases. If a large number of load gate inputs are connected, drops below $V_{OH\,(min)}$. As a result of this, HIGH level noise margin is reduced, thus affecting the specified operating characteristics of the gate. Moreover, as the total source current increases, the power dissipation of the driving gate increases.

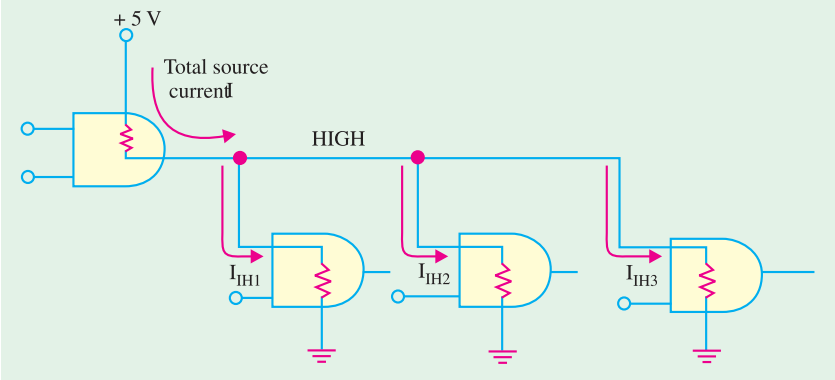


Fig. 71.57

The maximum number of load gate inputs that can be connected without affecting the specified operational characteristics of the driving gate is called ***fan-out.*** Its value is important for designing logic circuits. For example, the standard *TTL* has a fan-out of 10. One input of the same logic family as the driving gate is referred to as a ***unit load.*** This we can also say that a standard *TTL* has a fan-out of 10 unit loads.

The total sink current also increases with each load gate input that is added as shown in Fig. 71.58. As this current increases, the internal voltage drop of the driving gate increases, causing V_{OL} to increase. If a large number of loads are added, V_{OL} exceeds $V_{OL\,(max)}$ and the LOW-level margin is reduced.

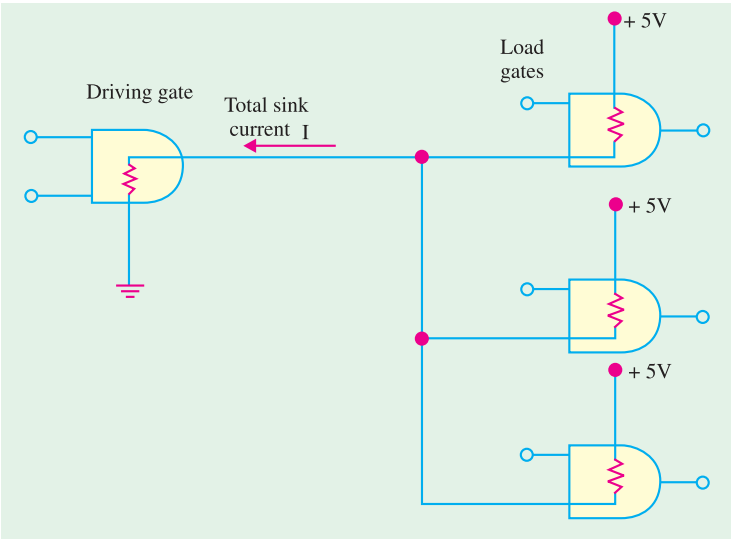


Fig. 71.58

As a matter of fact, in *TTL*, the current sinking capability (in the *LOW* state) is the limiting factor in determining the fan-out.

Loading and fan-out in CMOS: Loading in *CMOS* logic family differs from that in *TTL*. It is because of the fact that the field-effect transistors used in *CMOS* logic family present a predominantly capacitive load to the driving gate as shown in Fig. 71.59. In this case, the limitations are the charging and discharging times associated with the output resistance of the driving gate and the input capacitance of the load gates.

When the output of the driving gate is HIGH, the input capacitance of the load gate is charging through the output resistance of the driving gate. When the output of the driving gate is LOW, the

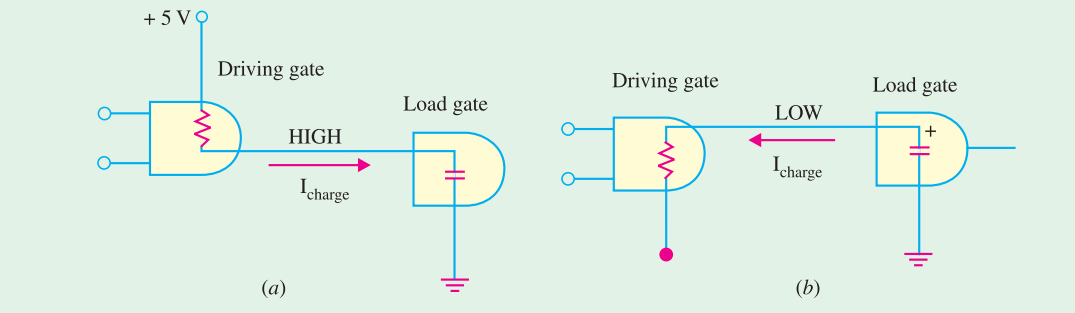


Fig. 71.59

capacitance is discharging. When more and more number of load gate inputs are added to the driving gate output, the total capacitance increases because the input capacitances effectively appear in parallel. This increase in capacitance increases the charging and discharging times. As a result, the maximum operating frequency of the logic gate is reduced. Thus fan-out in a *CMOS* logic family depends upon the operating frequency. The smaller the number of load gate inputs, the greater the maximum operating frequency.

71.34. RTL Circuit

It is a ***saturated* logic.** It uses only transistors and resistors as circuit elements and also resistances in the input to each base. This family is based on the *NOR* circuit shown in Fig. 71.60. All other members of the family are made up of *NOR* cells or variations on them.

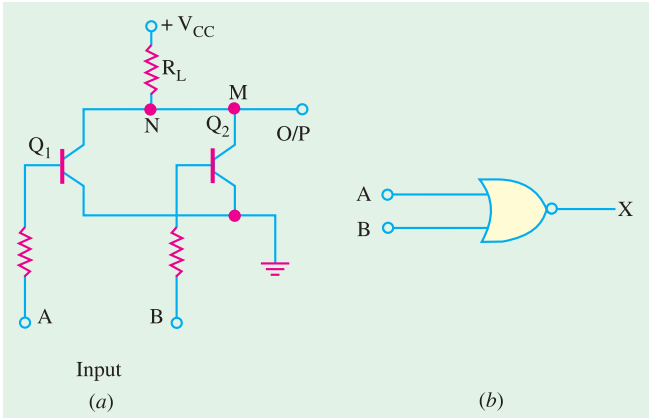


Fig. 71.60

Circuit Operation

We will assume ideal transistors. When both inputs *A* and *B* are 0 V (or logic 0) both transistors are turned *OFF*, hence point *M* goes to + V_{CC} so that output is logic 1.

If either or both input terminals are at + V_{CC} *i.e.* are high (or logic 1), one or both transistors would be fully turned *ON* (*i.e.* saturate) thereby reducing the voltage of point *N* to almost 0 V. Hence, output would be at logic 0.

It is seen that the output is at logic 1 only when ***both inputs are at logic 0*** — the *NOR* logic functions as shown in Fig. 71.60 (*b*).

The *RTL* family has the following characteristics :

1. relatively slow speed,
2. low fan-out of 6 and a fan-in of 4,
3. poor noise immunity,
4. expensive since resistors are required to be fabricated,
5. cannot operate at speed above 4 MHz.

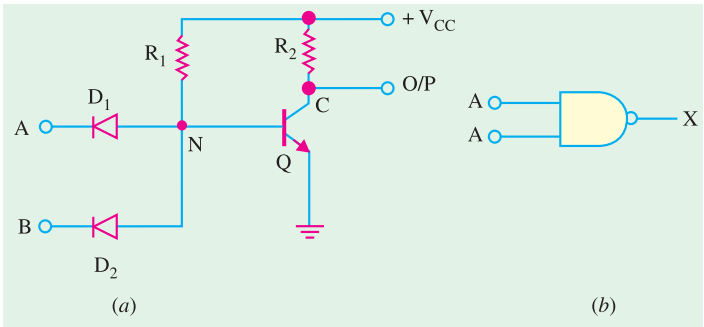


Fig. 71.61

71.35. DTL Circuit

It is a ***saturated*** logic because transistors between cut-off and saturation. It was the next family to be introduced after *RTL*. It consists of diodes, resistors and transistors. The basic gate of this family performs *NAND* function. As shown in Fig. 71.61 (*a*) the circuit basically consists of a diode *AND* gate followed by a transistor inverter which leads to a *NAND* gate.

Circuit Operation

1. When both D_1 and D_2 have positive voltage applied to them (logic 1), neither conducts and Q is turned *ON* by the current provided by V_{CC} through R_1. Since Q becomes saturated, point C is brought to 0V (logic 0). Hence output goes logic 0.
2. If either or both inputs are at 0V (logic 0), the associated diode will conduct driving point N to ground *i.e.* 0V. Since there is no base voltage for Q, it will be cut *OFF* thereby driving point C and hence output to V_{CC} *i.e.* logic 1.

It is seen that output is low (a logic 0) only when all inputs are high—the condition for a *NAND* gate.

The *DTL* family is characterised by

1. relatively lower speed,
2. Comparatively better noise immunity,
3. propagation delay of 30 ns,
4. a fan-out of 5.

71.36. TTL Circuit

It is a ***saturated* logic.** It is the most widely used circuit line since early 1970*s* because of its speed, good fan-out and easy interface with other digital circuitry. The unique feature of this circuit is that it used ***multiple-emitter transistor*** at input which replaces the input diodes of the *DTL*. Th number of emitters is equal to the number of inputs of the logic circuit (limited to 8). Since a multi-emitter transistor is small in area than the diodes it replaces, the yield from a wafer is increased. Moreover, smaller area resultes in ***lower capacitance*** to the substrate, thereby wide selection of circuit modules ranging from simple gates and flip-flops in *SSI* circuit series through various registers in computers in *MSI* circuit series to micro-processor bit-slice chips in the *LSI* series.

Basic Circuit

The basic circuit of the *TTL* familyis the *NAND* gate cell shown in Fig. 71.62. However, at present *NOR*, *OR* and *AND* gate configurations have also been added to the series.

Circuit Operation

1. If both inptus A and B are high (logic 1), E/B junction of Q_1 is *reverse-biased* so that it has no emitter current. Hence Q_1 is *OFF*. However, its C/B junction is ***forward-baised*** supplying base current to Q_2 from V_{CC} *via* R_1. As a result transistor Q_2 is turned fully *ON* (*i.e.* it becomes saturated) driving point N to OV. Hence, output is a logic 0.
2. When either or both inputs are at $0V$ (logic 0), the associated E/B junction becomes forward-biased. The value of R_1 is so selected as to ensure that Q is turned fully *ON*. The voltage at point M falls to $0V$ with the result the base current for Q_2 is reduced to zero. Hence, Q_2 is cut *OFF* driving point N and the output to logic 1.

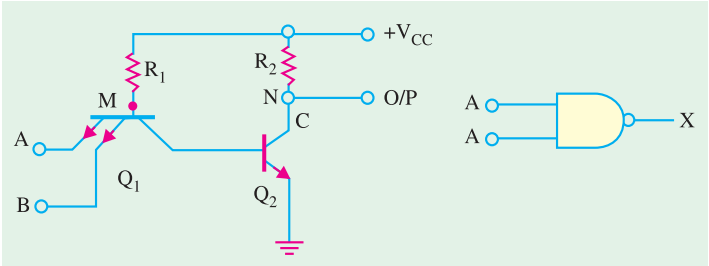


Fig. 71.62

Totem Pole Output

The basic circuited of Fig. 71.62 is never used in practice. Its modified version with an added output stage is in common use (Fig. 71.63). This extra output stage is often known as *totem-pole* stage because the three components Q_3, Q_4 and D are stacked one on top of the other in the manner of a totem-pole. The circuit action is as follows :

1. When Input is High

In this case, the two input terminals have ***positive*** voltage (logic 1). the E/B junction is reverse-biased because of which there is no emitter current. Hence, Q_1 is *OFF*. Since C/B junction of Q_1 is forward-biased, base current of Q_2 flows from V_{CC} through R_1. Hence, Q is turned *ON*. As a result, potential of point N falls so much that Q_2 is turned *OFF*. At the same time Q_4 is turned *ON* by the voltage drop across R_3. Now, when Q_4 is *ON*, its collector potential (*i.e.* potential of point C) is nearly that of its emitter. Hence, output is low *i.e.* at logic 0.

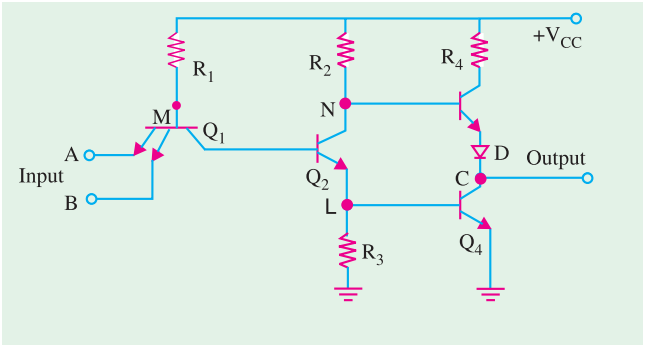


Fig. 71.63

In short, when inputs are at logic 1, Q_1 is *OFF*, Q_2 is *ON*, Q_3 is *OFF* and Q_4 is *ON* because of which output because logic 0.

2. When Input is Low

If any of the two inputs or both are low (logic 0), Q_1 turns *ON* and potential of its collector (point M) falls. Hence, Q_2 is turned *OFF*, grounding its emitter and the base of Q_4 so that Q_4 is also turned *OFF*.

Since N is at V_{CC}, it turns Q_3 *ON*. The potential of point C is V_{CC} minus drop in R_4, Q_3 and D. Since these drops do not amount to much, output is at logic 1.

It may be noted that when ***even-numbered transistors are ON, the odd-numbered ones are OFF and vice-versa.***

The function of diode D in Fig. 71.63 is to prevent both Q_3 and Q_4 from being turned *ON* simultaneously. If both were to be *On* at the same time, they would offer low immedance to teh suppy which will draw excessive current and produce large noise 'spikes' in the output. It may also be noted that the addition of a pair of totem pole transistor increases the operating speed and output current capability of this circuit. The standard *TTL*-family has

1. greater speed then *DTL*,

2. less noise immunity (0.4 *V*),

3. average propagation delay per gate of 9 *ns*,

4. average power dissipation of 10 mW,

5. a fan-out of 10 meaning one output can drive 10 other *TTL* inputs,

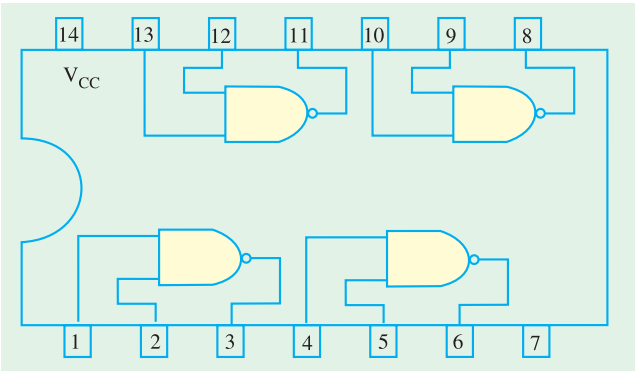


Fig. 71.64

Fig. 71.64 shows a pin-out of a *BEL* 7400 *IC* referred to as 'quad (quadrupole). 2-input *NAND* gate chip'. As seen, there are four separate

2-input *NAND* gates of which any or all may be used at any point in time. It is manufactured by Bharat Electronics LTD. (BEL) Bangalore, India.

71.37. TTL Sub-families

TTL has several ***sub-families*** having different speed and lower dissipation charcteristics as detailed below :

1. **74L00** series—the letter *L* standing for low power consumption. It has an average power dissipation of 1 mW per gate but an average propagation delay of 33 ns.
2. **74H00** series—the letter *H* standing for higher speed. It has a propagation delay of 6 ns but averge power dissipation of 23 mW/gate.
3. **74S00**—the letter *S* representing Schottky. It has the highest speed because its average propagation delay is just 3 ns per gate. However, its average power dissipation is 23 mW/gate.
4. **74LS00**—It is called low-power Schottky *TTL*. It has an average propagation delay of 9.5 ns and an average power dissipationof 2 mW.
5. **74AS00** series—The letter *A* representing Advanced and *S* stnading for schottky. It is called advanced schottky *TTL* series. It is the fastest *TTL* series.
6. **74ALS00** series—It is called Advanced Low-power Schottky *TTL* series. The 74*ALS* series has the lower speed-power product and the lowest gate power dissipation of all the *TTL* series.
7. **74F00** series —The letter *F* standing for fast. This logic family uses a new *IC* fabrication technique to reduce interdevice capacitances to achieve reduced propagation delays. It has a propagation delay of 3 ns and a power consumption of 6 mW.

Table 71.3 summarizes the important characteristics of the each of the *TTL* sub-families.

Table 71.3

Characteristic	*TTL* sub-family					
	74	74S	74LS	74AS	74ALS	74F
Propagation delay (ns)	9	3	9.5	1.7	4	3
Power Dissipation (mW)	10	20	2	8	1.2	6
Speed-power product (pJ)	90	60	19	13.6	4.8	18
Fan-out	10	20	20	40	20	33

71.38. ECL Circuit

The *ECL* also called current-mode logic (*CML*), has the **highest speed** of any of the currently-available logic circuits. It is primarily due to the fact that transistors never operate fully saturated or cut-off. That is why *ECL* is known as ***non-saturated* logic.** The latest *ECL* series has propagation delay time varying from 0.1ns to 0.8 ns. However, power dissipation is increased since one transistor is always in the active region.

Another feature of *ECL* is that it provides two outputs which are always complement of each other (Fig. 71.65). Itis so because the circuit operation is based on a differential amplifier.

This family is particularly suited to monolithic fabrication techniques because logic levels are function of resistor ratios.

Circuit Operation

The basic circuit shown in Fig. 71.65 is combined *OR/NOR* circuit and is operated from a $V_{EE} = -5.2\ V$ supply. A built-in constant-current source provides current to the emitters. Strictly speaking,

logic 1 is represented by – 0.9 *V* (less negative) and logic 0 by – 1.75 *V* (more negative). Please note that it is a *positive* logic. In negative logic, the functions would be *AND/NAND*. A reference voltage of – 1.29 *V* is applied to the base of Q_3 from a built-in temperature-compensated reference voltage source (*RVS*).

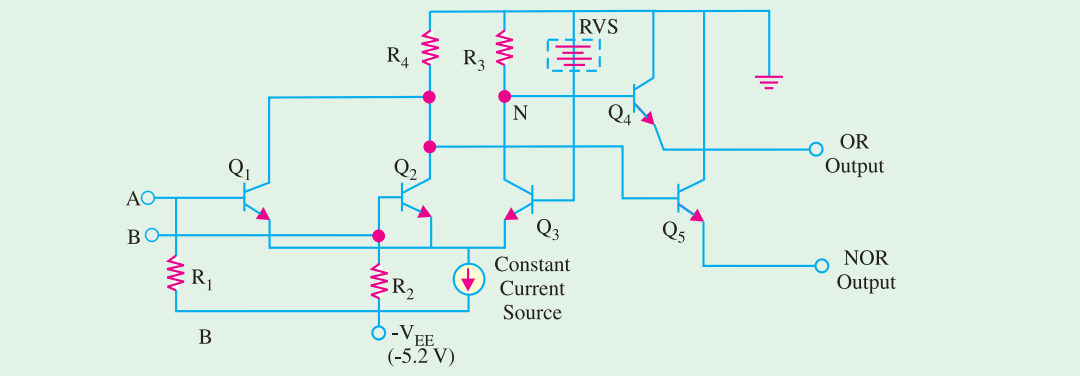


Fig. 71.65

1. When both inptus are logical 0 *i.e.* –1.75 V

In this case, base potential of Q_3 is less negative (or more positive) than the base potential of either Q_1 or Q_2. Hence, Q_3 conducts whilst Q_1 and Q_2 do not. Only enough base current is drawn by Q_3 from RVS so as to remain out of saturation. The collector current of Q_3 develops a voltage of – 1.0 V across R_3 which makes Q_4 to conduct. Transistor Q_4 gives an output voltage at the emitter of about –1.0 – 0.7 = –1.7 V which represents logic 0. Since collector potentials of Q_1 and Q_2 are nearly zero (because they are cut off), the output voltage at the emitter of Q_5 is 0 – 0.7 = –0.7 V which is a logic 1. Obviously, the two outputs are complements of each other.

2. When either input A or B is at logical 1 *i.e.* –0.9 V

In that case, the associated transistor (either Q_1 or Q_2) is turned *ON* while Q_3 is turned *OFF*. The collector potentials of Q_1/Q_2 and are opposite of the previous case. Hence, now Q_5 output is at logical 1 and Q_4 output is at logical 0.

Typical characteristics of an *ECL* family are :

1. propagation delay time per gate of 0.3 ns (meaning extremely fast speed),
2. power dissipation of 25 mW,
3. fan-out of 25 to 50,
4. noise margin from about 0.2 to 0.25 V.

It will be interesting to know that *ECL* family of *ICs* does not include a wide range of general purpose logic devices as do the *TTL* and *CMOS* logic families. *ECL* does include complex, special purpose *ICs* used in applications such as high-speed data transmission, high-speed memories and high-speed arithmetic units. The relatively low noise margins and high power drain of *ECL* are disadvantages compared with *TTL* and *CMOS*. Another drawback is its negative power supply voltage and logic levels which are not compatible with those of other logic families. This makes it difficult to use *ECL* devices in conjunction with *TTL* and/or *CMOS ICs*. Special level-shifting (also called interface) circuits must be connected between *ECL* devices and the *TTL* (or *CMOS*) devices on both input and output.

71.39. I^2L Circuit

It is the latest entry into the bipolar ***saurated*** logic field. It uses no biasing and loading resistors at all! Resistors require lot of power and space on an *IC* chip. Hence, their elimination results in ***higher density circuits operating at much reduce power.*** Because of its **high speed** and **less power dissipa-**

tion, it is used in large computers. Such circuits are also used where high packing density is of prime consideration as in digital wrist watches. I^2L chips are capable of microwatt power dissipation yet can provide high currents when necessary to drive *LED* displays. Another feature of I^2L (integrated injection logic) is that it is easy to fabricate.

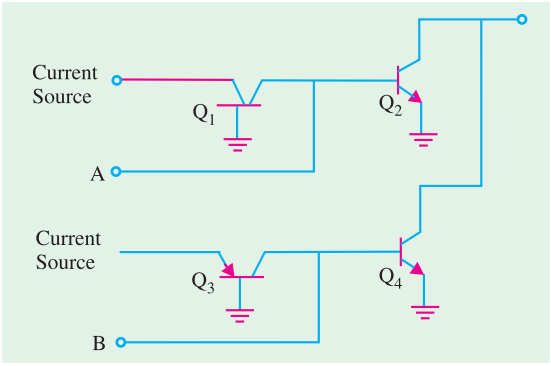


Fig. 71.66

Circuit Operation

I^2L *NOR* logic circuit is shown in Fig. 71.66. Here, transistors Q_1 and Q_2 act as current sources to the bases of Q_3 and Q_4 respectively.

If *A* input goes low, the current to the base of Q_2 will be shorted to ground which will result in Q_2 being turned *OFF*. The input *B* controls Q_4 in a similar way.

If *A* is high, base current flowing to Q_2 will turn in *ON*, making output *C* low. Same would be the case when *B* is high.

It is obvious that output would be high only when ***both inputs*** *A* and *B* are low.

The output would be low when either *A* or *B* or both are high *i.e. NOR* logic function.

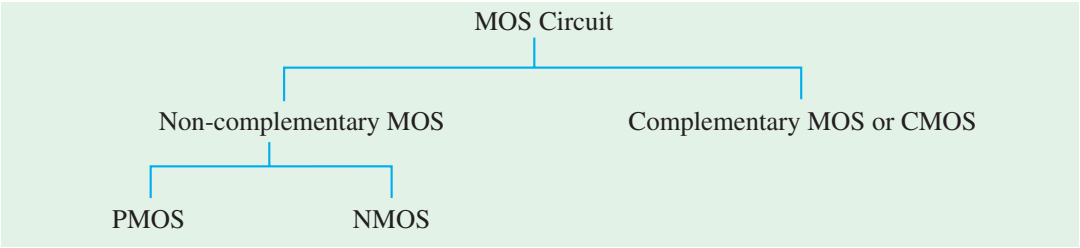
71.40. MOS Family

It does not use bipolar transistors but just Enhancement-only *MOSFETs* (Art. 71.1). There are two kinds of digital *MOS* circuits.

(a) one which uses *MOSFETs* of one polarity either all of *N-type* called *NMOS* or all of *P*-type called *PMOS* but not both on the same chip,

(b) the other which employs both *N*-type and *P*-type *MOSFETSs* on the same chip. It is called comlementary *MOS* (*CMOS*).

The *MOS* family may be subdivided as under :



Since a *FET* requires small area, it is possible to fabricate a large number of *MOS* ciruits on a single small chip. Gating arrays with thousands of gates and flip-flops are manufactured in standard containers and are often used in *IC* memories and microprocessors.

Followings are same of the advantages of *MOS ICs* over the bipolar *ICs* (i.e. *TTL*, *ECL* etc.)

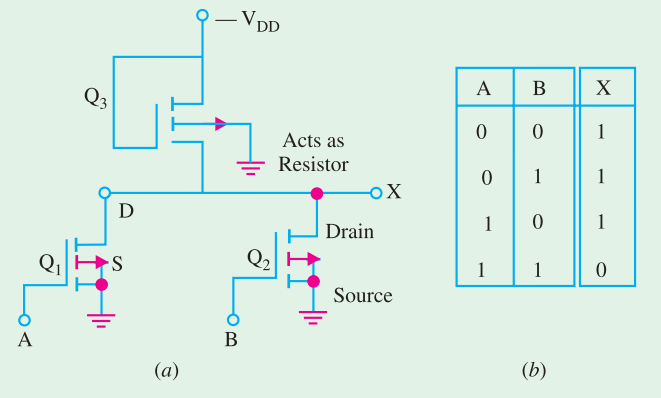
1. The *MOS IC* is relatively simple and inexpensive to fabricate.
2. The *MOS* device size is small and it consumers less power. Because of the small size, the *MOS ICs* can accommodate a much larger number of circuit elements on a single chip than bipolar *IC* in the area of large scale integration. This makes them especially well-suited for complex *ICs* such as microprocessor, memory chips etc.
3. *MOS* digital *ICs* normally do not use the *IC* resistor elements that take up so much of the chip area or bipolar *ICs*.

The continuous improvement in *MOS IC* technology has led to the device that are faster than *TTL* devices. Consequently *MOS* devices (especially *CMOS*) have become dominant in the *SSI* (small-scale integration) and *MSI* (medium-scale integration) market.

The major disadvantage of *MOS* devices is that they are susceptible to static-electricity damage. Although we can minimize it by adopting proper handling procedures, yet *TTL* is still more durable for laboratory experimentation. As a result, we are likely to see *TTL* devices used in education as long as they are available.

71.41. PMOS Circuit

A *PMOS NAND* gate is shown in Fig. 71.67 (*a*). As seen, there are no resistors in the circuit. The gate consists of two *E* only *MOSEFETs* Q_1 and Q_2 as logic elements and the third one Q_3 as load resistors. When $-V_{DD}$ (say −12 V) is applied, *MOSFETs* will be turned *ON* and when 0 V is applied they would be turned *OFF*. Hence, with positive logic, 0 V would be 1 and – 12 V would be 0 since 1 is assigned to the most positive voltage.



A	B	X
0	0	1
0	1	1
1	0	1
1	1	0

Fig. 71.67

Circuit Operation

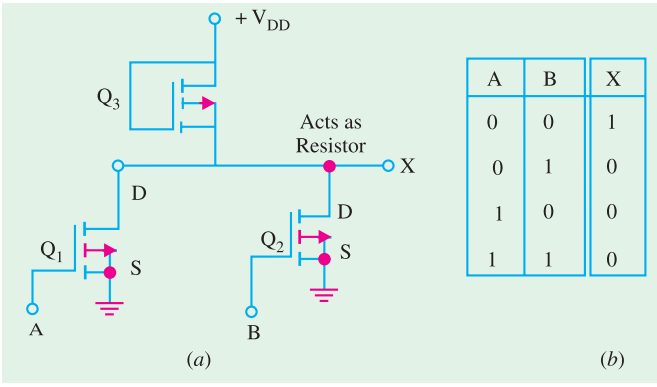
1. If any of the two inputs *A* or *B* is at logic 0 (*i.e.* – 12 V), the concerned *MOSFET* would turn *ON* thereby offering a low resistance from drain to source thus causing the output to be nearly 0*V i.e.* a logic 1.
2. The output can be at – 12 V *i.e.* logic 0 only when both inputs *A* and *B* are at 0 V *i.e.* logic 1.

This is the *NAND* function as shown by the truth table of Fig. 71.67 (*b*).

Incidentally, the positive logic *NAND* gate of Fig. 71.67 (*a*) would be the ***negative logic NOR*** gate since the two are identical.

71.42. NMOS Circuit

In Fig. 71.68 (*a*) is shown a two-input *NOR* gate circuit consisting of two *MOSFETs* Q_1 and Q_2 acting as logic elements and Q_3 as a load resistor. For positive logic, 0*V* will be logic 0 and positive voltge ($+V_{DD}$) will be logic 1.



A	B	X
0	0	1
0	1	0
1	0	0
1	1	0

Fig. 71.68

Circuit Operation

1. If any of the inputs *A* or *B* is at logic 1, the corresponding *MOSFET* will conduct causing the output to go low *i.e.* logic 0.
2. If both inpouts *A* and *B* are at logic 0, then both *MOSFETs* will be *OFF* driving the output to logic 1.

This is *NOR* function as shown by the trugh table of Fig. 71.68 (*b*).

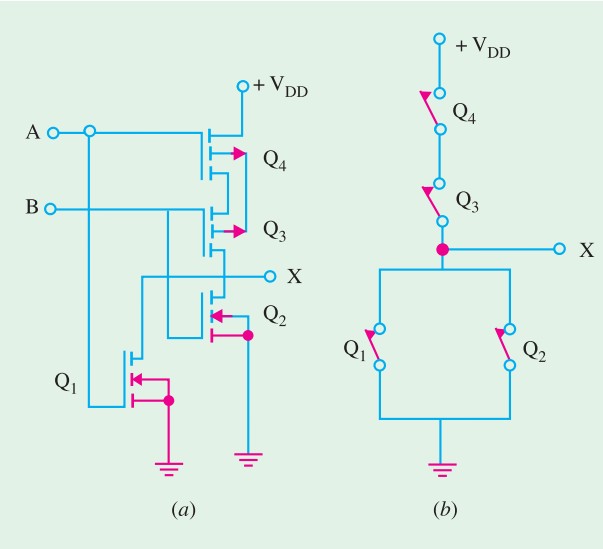


Fig. 71.69

71.43. CMOS Circuit

CMOS logic circuits use both *PMOS* and *NMOS* devices in the same circuit. It gives the advnatage of ***drastic decrease in power dissipation*** (12 nW per gate) and ***increase in speed of operation*****.** In fact, it has the **lowest power dissipation** amongest different logic families. It has very high packing density *i.e.* larger number of circuits can be placed on a single chip. As a result, it is extensively used in *VLSI* circuits such as on-chip computers and memory systems. The recent silicon-on-saphire *MOS* (*SOSMOS*) is 2 to 4 times faster than the standard *CMOS*. Hence, they are being widely used for everything from electronic watches and calculators to micro processors.

Fig. 71.69 (*a*) shows a *CMOS NOR* circuit which has two *N*-channel *MOSFETs* Q_1 and Q_2 and tow *P*-channel *MOSFETs* Q_3 and Q_4. The two inputs *A* and *B* switch between + V_{DD} (logic 1) and ground (logic 0).

Circuit Operation

1. Let $A = B$ = logic *i.e.* have positive voltage. In that case, Q_1 and Q_2 are *ON* (closed switches) but Q_3 and Q_4 are *OFF* and act as open switches as shown in Fig. 71.79 (*b*). Henc, output *C* is logic 0.
2. If either *A* or *b* is at logic 1, then the associated *N*-channel *MOSFET* (Q_1 or Q_2) is turned *ON* but the associated *P*-channel *MOSFET* (Q_3 or Q_4) is turned *OFF*. Since either Q_3 or Q_4 would be *OFF* [Fig. 71.69 (*b*)], output *C* would be at logic 0.
3. When both *A* and *B* are at logic 0, Q_3 and Q_4 would be *ON* but Q_1 and Q_2 would be *OFF*—just the opposite of that in Fig. 71.69 (*b*). Hence output *C* would be at logic 1 (remember, voltage across an open equals the supply voltage—Art. 1.21.

A	B	C
0	0	1
0	1	0
1	0	0
1	1	0

Fig. 71.70

The above logic represents a *NOR* function as shown in the truth table of Fig. 71.70. It would be observed from Fig. 71.69 that in each combination of *A* and *B*, there is at least ***one open switch*** between + V_{DD} and ground. Hence, the gate draws ***only leakage current from supply for any static state.*** However, when the gate switches from one level to another, some power is consumed becasue two *MOSFETs* are ***partly ON*** at the same time. Because of this reason, power dissipated by *CMOS* circuit *is a* ***function of input signal frequency.*** Higher the frequency, greater the power dissipation.

71.44. CMOS Sub-families and their characteristics

The *CMOS* family of *ICs* competes directly with *TTL* in the small and medium-scale integration. As *CMOS* technology has produced better and better performance characteristics, it has gradually taken over the field that has been dominated by *TTL* for so long. As a matter of fact, *TTL* devices will be around for a long time, but more and more new equipment is using *CMOS* logic circuits. These days, the *CMOS ICs* provide all the logic-functions that are available in *TTL* Some special-purpose functions provided in *CMOS ICs* are not provided even by *TTL*. Before we look at the various *CMOS*

subfamilies, let us define few terms that are used when *ICs* from different families or series are to be used together or as replacements for one another.

(*a*) Pin-to-pin Compatible : Two *ICs* are said to be pin-to-pin compatible when three pin configurations are the same. For example, pin 14 on both *ICs* is V_{CC} supply. Pin 7 on both is ground etc.

(*b*) Functionally Equivalent : Two *ICs* are said to be functionally equivalent when the logic function they perform are exactly the same. For example, both contain four two-input *AND* gates, or two-input *NAND* gates etc.

(*c*) Electrically compatible : Two *ICs* are said to be electricaly compatible when they can be connected directly to each other without taking any special measures to ensure proper operation.

Let us now study the different *CMOS* subfamilies.

1. **5000/14000 series :** The *CMOS* 4000 series or the improved 4000 *B* is the oldest series and was first introduced by *RCA*. This series is functionally equivalent to 14000 series from Motorola. The *CMOS* devices in 4000/14000 series have a very low power dissipation and can operate over a wide range of supply voltage (*i.e.* from 3 to 15 V). These devices are very slow as compared to *TTL* and other *CMOS* subfamilies. The 40H00 series was designed to be faster than 4000 series. It did overcome some of the speed limitations, but it is still much slower than *LSTTL* series. The 4000 series have very low output current capability. The devices in 4000 series are not pin-compatible or electrically compatible with any *TTL* series.
2. **74C00 series :** This is pin-compatible as well as functionally equivalent to *TTL* devices having the same device number. For example, 74C04 is a HEX inverter that has the same pin configuration as the *TTL* 7404 HEX inverter *IC*. The performance characteristics of 74C00 series are the same as those of 4000 series.
3. **74HC/HCT00 series :** The letter *H/HC* is standing for high-speed *CMOS* series. This series has a speed which is 10 times faster than of 74*LS*00 series devices. It has also a higher current capability than that of 74*C*00 series. The 74*HC/HCT00* series *ICs* are pin-compatible with and functionally equivalent to *TTL ICs* with the same device number. This series has become the most widely used series.
4. **74AC/ACT 00 series :** This series is referred as advanced *CMOS* logic. It is functionally equivalent to the *TTL* series but not pin-compatible.
5. **74AHC00 series :** This series is referred to as advanced high-speed *CMOS* logic. It is faster, and haslower-power dissipation. The devices in this series are 3 times faster and can be used as direct replacements for *HC* series devices.
6. **74-BiCMOS series :** These days, the *IC* manufacturers have developed a new logic series-called *BiCMOS* logic. This series combines the best features of bipolar and *CMOS* logic *i.e.* low power characteristics of *CMOS* and high speed characteristics of bipolar circuits. *BiCMOS ICs* are available only in those functions that are used in microprocessor interfacing and memory applications such as latches, buffers, drivers and transceivers.
7. **74-Low Voltage series :** A new series of logic using a nominal supply voltage of 3.3 V has been developed to meet the extremely low power design requirements of battery powered and hand held devices. These *ICs* are being designed into the circuits of notebook computers, mobile radios, hand-held video games, telecommunication equipment, personal digital assistant (*PDA*) and high performance work station computers.

   Table 71.4 shows some of the common Low-voltage families identified by the suffixes as indicated :

Table 71.4

Suffixes	*Low-voltage series*
LV	Low-voltage *HCMOS*
LVC	Low-voltage *CMOS*
LVT	Low-voltage technology
ALVC	Advanced Low-voltage *CMOS*
HLL	High speed Low-power Low-voltage

The power consumption of *CMOS* logic *ICs* decreases approximately with the square of the supply voltage. On the other hand, the propagation delay increases slightly at this reduced voltage. However the speed is restored and even increased by using finer geometry and sub-micron (<1μm) *CMOS* technology that is tailored for low-power and low-voltage applications.

8. **74 AHC/AHCT series :** This is an enchanced version of 74 *HC/HCT*00 series. It provides superior speed and low power consumption. The 74*AHC* series devices have half the static power consumption, one-third the propagation delay, high output drive current, and can operate at V_{CC} of 3.3 to 5 V.

Table 71.5 shows a comparison of the important characteristics of the *CMOS* logic families. It is evident from this table that the devices from 74*AHC*/*AHCT* series has the lowest propagation delay and the smallest value of power dissipation.

Table 71.5

S. No.	*Characteristic*	*CMOS logic family*			
		4000 B	*74 HC/HCT*	*74 BiCMOS*	*74 AHC/AHCT*
1.	Propagation delay (ns)	50	8	2.9	3.7
2.	Power dissipation static (mW)	0.001	0.0025	0.0003 – 7.5	0.000009
3.	Speed-power product (pw-s)	105	15	0.00087 to 22	—

Tutorial Problems No. 71.1

1. Prove the following identities :

   (*i*) $AB + A\overline{B} = A$ (*ii*) $A + A\overline{B} = A + B$ (*iii*) $(A + B)(A + \overline{B}) = A$

2. Simplify the following Boolean expressions :

   (*i*) $A\,\overline{A}\,C$ (*ii*) $ABCD + ABD$ (*iii*) $ABCD + A\,\overline{B}\,C\,D$

   [(*i*) 0, (*ii*) *ABD*, (*iii*) *ACD*]

3. Simplify the following Boolean functions :

   (*i*) $A + \overline{A}\,B + AB$ (*ii*) $\overline{A}\,B + \overline{A}\,B + AB\,\overline{A}\,B$ (*iii*) $(AB + C)(AB + D)$

   (*iv*) $A + \overline{B}\,C\,(A + B\,\overline{C})$ (*v*) $A[B + C\,(\overline{AB + AC})]$ (*vi*) $(A + B).(A + C)$

   [(*i*) $A + B$ (*ii*) 1 (*iii*) $AB + CD$ (*iv*) A (*v*) AB (*vi*) $A + B.C$ (*vii*) $\overline{B}$]

4. Simplify the following Boolean expressions :

(*i*) $A\,\overline{B}\,C + A\,B\,\overline{C} + A\,B\,C$ (*ii*) $\overline{A}\,\overline{C} + B\,\overline{C} + \overline{A}\,B\,C + ABC$

(*iii*) $AD + A\,\overline{B}\,\overline{C} + \overline{B}\,C\,\overline{D} + \overline{A}\,C\,D + \overline{A}\,B\,C\,\overline{D}$

(*iv*) $A\,\overline{B}\,\overline{D} + A\,B\,\overline{C}\,\overline{D} + A\,B\,C\,\overline{D} + \overline{A}\,B\,C\,\overline{D}$

[(*i*) $A\,(B + C)$ (*ii*) $B + \overline{A}\,\overline{C}$ (*iii*) $AD + \overline{A}\,\overline{B} + \overline{A}\,\overline{C}$ (*iv*) $A\,\overline{B}\,\overline{D} + A\,\overline{D} + C\,\overline{D}$]

5. Use truth tables to verify the following identities :

(*i*) $\overline{A} + B = \overline{A}\,.\,B$ (*ii*) $A + BC = (A + B)\,(A + C)$ (*iii*) $A\,(\overline{A} + \overline{B}) = AB$

6. Write the logic equation for the switching circuit of Fig. 71.71. **[$A\,(B + C + D + E$]**

7. Write down the logical expression which describes the working of the circuit of Fig. 71.72.

[$AB\,(CD)$]

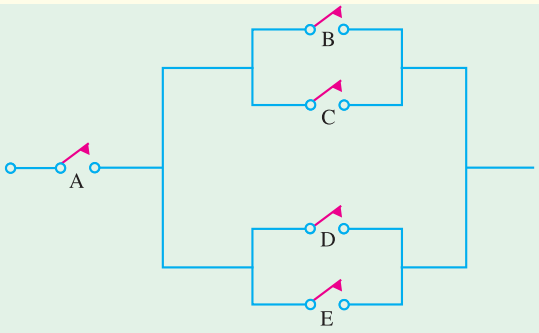


Fig. 71.71

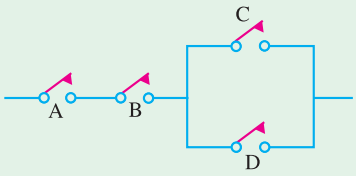


Fig. 71.72

8. Simplify the following Boolean expressions :

(*i*) $y = AB\overline{C} + A\overline{B}\,\overline{C} + A\overline{B}\,\overline{C} + \overline{A}\,\overline{C}\,\overline{B} + A\overline{B}$

(*ii*) $y = AB\overline{C}\,\overline{D} + AB\overline{C}D + ABC + ABD + CD$ **[(*i*) $A\overline{C} + \overline{B}$ (*ii*) $AB + CD$]**

9. Design a logic circuit whose output is HIGH only when majority of inputs *A*, *B* and *C* are LOW.

10. Determine the Boolean expression for the logic circuit shown in Fig. 71.73. Simplify the Boolean expression using Boolean Laws and De Morgan's theorem. Redraw the logic circuit using the simplified Boolean expression.

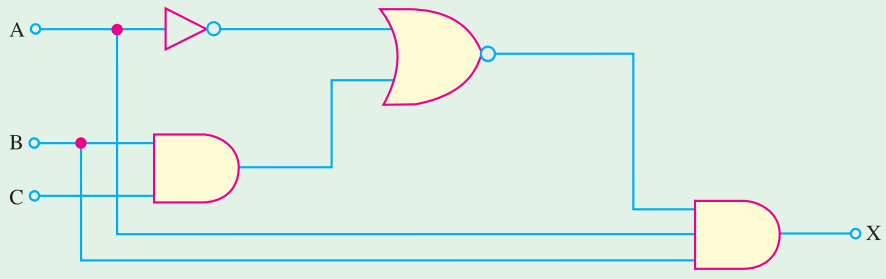


Fig. 71.73

11. Determine the output, *X* of a logic circuit shown in Fig. 71.74. Simplify the output expression using Boolean Laws and theorems. Redraw the logic circuit with the simplified expression.

($\overline{B}\overline{C}D + AC$)

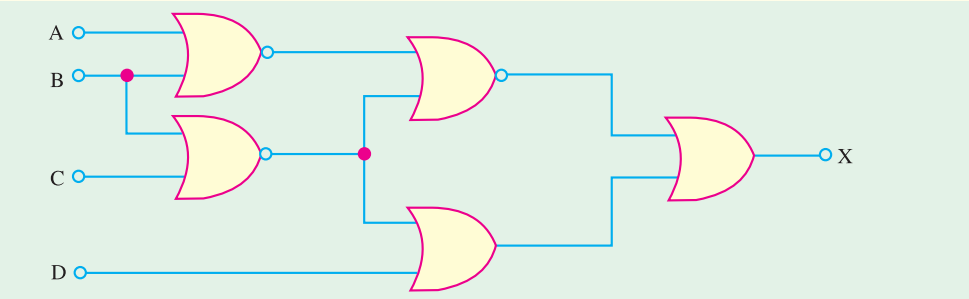


Fig. 71.74

12. Consider the logic circuit shown in Fig. 71.75. Determine the Boolean expression at the circuit output X, simplify it. Using the simplified expression, redraw the logic circuit.

***A* ($\overline{B}$ + *C*)**

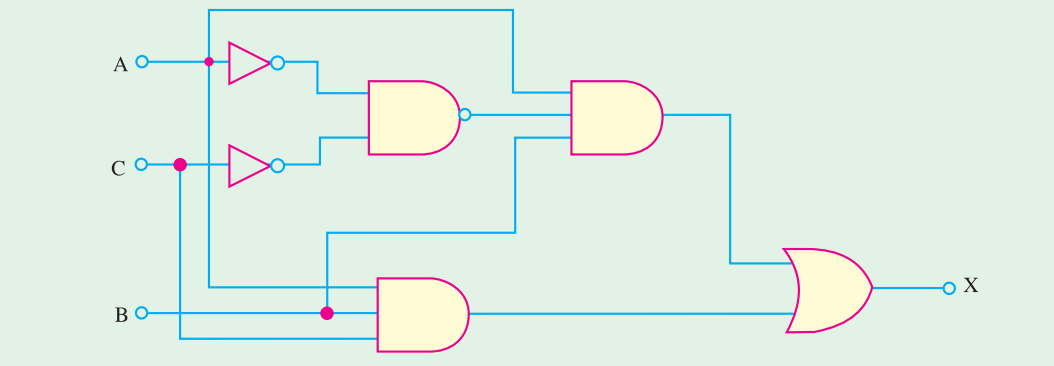


Fig. 71.75

13. Fig. 71.76 (*a*) shows a three-variable Karnaugh map. Group the 1*s* and hence obtain the minimized Boolean expression. ***(AB + BC + $\overline{A}$ $\overline{B}$ $\overline{C}$)***

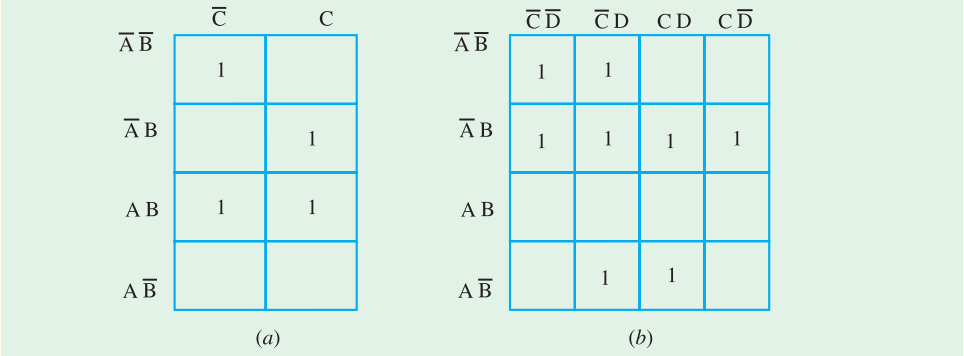


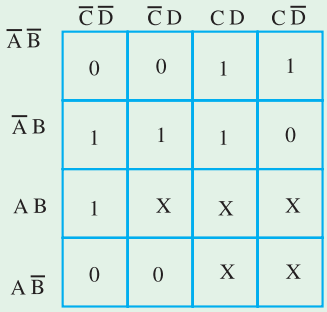
Fig. 71.76

14. Fig. 71.76 (*b*) shows a four-variable Kamaugh-map. Group the 1*s* and hence obtain the minimized Boolean expression. ***($\overline{A}$ B + $\overline{A}$ $\overline{C}$ + A$\overline{B}$D)***

15. A truth table has output 1*s* for these inputs : $ABCD = 0011$, $ABCD = 0110$, $ABCD = 1000$ and $ABCD = 1100$, and 0*s* for the other inputs. Draw the Karnaugh map and find the simplified Boolean equation for the truth table. ***(A$\overline{C}$ $\overline{D}$ + $\overline{A}$ $\overline{B}$ CD + $\overline{A}$BC$\overline{D}$)***

16. Determine the minimized expression for the Karnaugh map shown in Fig. 71.77 (*a*).

(B$\overline{C}$ + BD + $\overline{B}$C)



	$\overline{C}\,\overline{D}$	$\overline{C}D$	CD	$C\overline{D}$
$\overline{A}\,\overline{B}$	0	0	1	1
$\overline{A}B$	1	1	1	0
AB	1	X	X	X
$A\overline{B}$	0	0	X	X

(*a*)

	$\overline{C}\,\overline{D}$	$\overline{C}D$	CD	$C\overline{D}$
$\overline{A}\,\overline{B}$	1	1		1
$\overline{A}B$	1	1		1
AB				1
$A\overline{B}$				

(*b*)

Fig. 71.77

17. Determine the simplified Boolean expression for the Karnaugh map shown in Fig. 71.77 (*b*).

(UPSC Engg. Services 1995) $(\overline{A}\,\overline{C} + \overline{A}\,\overline{D} + ABC)$

OBJECTIVE TESTS – 71

1. Boolean algebra is essential based on
(*a*) symbols (*b*) logic
(*c*) truth (*d*) numbers

2. The first person who used Boolean algebra for the design of relay switching circuits was
(*a*) Aristotle (*b*) Boole
(*c*) Shannon (*d*) Ramanujam

3. Different variables used in Boolean algebra can hvae values of
(*a*) 0 or 1 (*b*) low or high
(*c*) true or false (*d*) On or OFF.

4. According to the algebra of logic, $(A + \overline{A})$ equals
(*a*) A (*b*) 1
(*c*) 0 (*d*) $\overline{AA}$.

5. According to the absorptive Laws of Boolean algebra , expressioin $(A + AB)$ equals
(*a*) A (*b*) B
(*c*) AB (*d*) A

6. When we demorganize $\overline{A}\,\overline{B}$, we get
(*a*) $\overline{AB}$ (*b*) $\overline{A} + \overline{B}$
(*c*) $\overline{A + B}$ (*d*) $\overline{AB}$.

7. The dual of the statement $(A + 1) = 1$ is
(*a*) $A \,.\, 1 = A$ (*b*) $A \,.\, 0 = 0$
(*c*) $A + A = A$ (*d*) $A \,.\, A = 1$.

8. The expression $\overline{ABC}$ can be simplified to
(*a*) $\overline{A} \,.\, B \,.\, \overline{C}$ (*b*) $AB + BC + CA$
(*c*) $AB + \overline{C}$ (*d*) $\overline{A} + \overline{B} + \overline{C}$.

9. While obtaining minimal sum-of-products expression,
(*a*) all don't cares are ignored
(*b*) all don't cares are treated as logic 1*s*
(*c*) all don't cares are treated as logic 0*s*
(*d*) only such don't cares that aid minimization are treated as logic 1*s*.

10. In a saturated bipolar logic circuit, transistor operate
(*a*) in deep cut-off (*b*) over active region
(*c*) in saturation
(*d*) just short of saturation

11. Saturated logic circuits have inherently
(*a*) short saturation delay time
(*b*) low switching speed
(*c*) higher power dissipation
(*d*) lower noise immunity.

12. Noise margin is expressed in
(*a*) decibel (*b*) watt
(*c*) volt (*d*) phon

13. DTL family employs
(*a*) resistors and transistors
(*b*) diode and resistor
(*c*) diode and transistors
(*d*) diodes, resistors and transistors.

14. The chief advantage of Schotty *TTL* logic family is its least
(*a*) power dissipation (*b*) propagation delay
(*c*) fan-in (*d*) noise immunity.

15. The main advantage claimed for *ECL* family oflogic gates is its
(*a*) very large fan-in
(*b*) use of negative power supply voltage
(*c*) extremely low propagation times
(*d*) least power dissipation.

16. Special feature of an I^2L logic circuit is that it
(*a*) uses ony high-value resistors

(*b*) dissipates negligible power
(*c*) is a bipolar saturated logic
(*d*) is easy to fabricate
(*e*) uses no biasing and loading resistors.

17. A unique advantages feature of *CMOS* logic family is its
(*a*) use of *NMOS* circuits
(*b*) power dissipation in nanowatt range
(*c*) speed
(*d*) dependence on frequency for power dissipation.

18. CMOS circuits are extensively used for on-chip computers mainly because of their extremely
(*a*) low power dissipation
(*b*) large packing density
(*c*) high noise immunity
(*d*) low cost.

19. The main advantage of a CMOS logic family over the TTL family is its
(*a*) much reduced power
(*b*) increased speed of operation
(*d*) extremely low cost
(*d*) series base resistor

20. CMOS logic family uses only
(*a*) MOSFETs and resistors
(*b*) NMOS circuits
(*c*) MOSFETs (*d*) bipolar transistors.

21. Power is drawn by a CMOS circuit only when
(*a*) its output is high (*b*) its output is low
(*c*) it switches logic levels
(*d*) in static state.

22. The most obvious identifying feature of a *TTL* gate is its
(*a*) large fan-out (*b*) high power dissipation
(*c*) interconnected transistors
(*d*) multiemetter input transistor.

23. In digital circuits shottky transistors are preferred over normal transistor because of their
(*a*) lower propagation delay
(*b*) higher propagation delay
(*c*) lower power dissipation
(*d*) higher power dissipation

24. A unique operting feature of *ECL* circuit is its
(*a*) very high speed
(*b*) high power dissipation
(*c*) series base resistor
(*d*) compatibility with other logic series.

25. The fan-in a logic gate refers to the number of
(*a*) input devices that can be connected
(*b*) input terminals (*c*) output terminals
(*d*) circuits output can drive

26. Which of the following statements regarding *ICs* is not correct,
(*a*) *ECL* has the least propagation delay
(*b*) *TTL* has the largest fan out
(*c*) *CMOS* has the biggest noise margin
(*d*) *TTL* has the lowest power consumption.

27. The Boolean equation for *NOR* gate is
(*a*) $\overline{AB} = C$ (*b*) $\overline{A} + \overline{B} = C$
(*c*) $A + B = C$ (*d*) $\overline{A + B} = C$.

28. The Boolean equation for a NAND gate is
(*a*) $\overline{A + B} = C$ (*b*) $\overline{\overline{AB}} = C$
(*c*) $A + B = C$ (*d*) $\overline{A} + \overline{B} = C$

29. The logic function of an invertor is
(*a*) $B = A$ (*b*) $B = \overline{A}$
(*c*) $B = \overline{\overline{A}}$ (*d*) None of these

30. The simplified form of the Boolean expression $Y = (\overline{A}.BC + D)\ (\overline{A}.D + \overline{B}.\overline{C})$ can be written as
(*a*) $\overline{A}.D + \overline{B}.\overline{C}.D$ (*b*) $AD + B.\overline{C}.D$
(*c*) $(\overline{A} + D)(\overline{B}.C + \overline{D})$ (*d*) $A.\overline{D} + BC.\overline{D}$

(GATE; 2004)

ANSWERS

1. (*b*) **2.** (*c*) **3.** (*a*) **4.** (*b*) **5.** (*a*) **6.** (*c*) **7.** (*b*) **8.** (*d*) **9.** (*d*) **10.** (*c*) **11.** (*b*) **12.** (*c*)
13. (*d*) **14.** (*b*) **15.** (*c*) **16.** (*e*) **17.** (*b*) **18.** (*b*) **19.** (*a*) **20.** (*c*) **21.** (*c*) **22.** (*d*) **23.** (*a*) **24.** (*a*)
25. (*b*) **26.** (*d*) **27.** (*d*) **28.** (*d*) **29.** (*b*) **30.** (*a*)

GO To FIRST

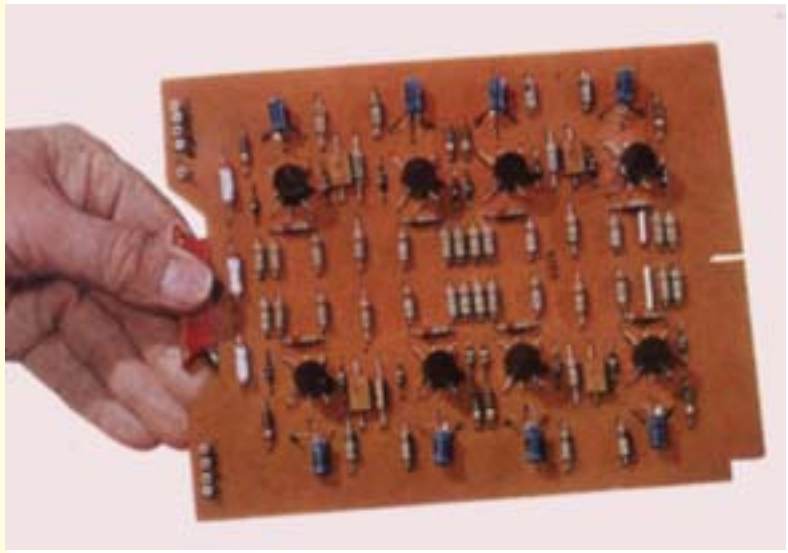
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CHAPTER 72

FLIP-FLOPS AND RELATED DEVICES

Learning Objectives

- Flip-Flop (FF)
- Latch
- NAND Gate Latch
- NOR Gate Latch
- Clocked Signals
- Some Main Ideas Common to Clocked Flip-Flops
- Clocked S-C Flip-Flop
- Clocked J-K Flip-Flop
- Clocked D Flip-Flop
- Parallel Transfer of Data Using D-Flip-Flops
- D Latch (Transparent Latch)
- Clocked J-K Flip-Flop with Asynchronous Inputs
- Flip-Flop Timing Parameters
- IC Flip-Flop Timing Values
- Applications of Flip-Flop
- Flip-Flop Synchronization
- Parallel Data Transfer
- Frequency Division
- Counting
- Schmitt Trigger Devices
- Types of one-shots
- Retriggerable One-Shot
- Actual One-Shot Devices
- Clock Generator Circuits
- Schmitt Trigger Oscillator
- 555 Timer Used as an Astable Multivibrator



A flip-flop is made up of logic gates. The flip-flpos are extensively used as a memory cell in static random access memory of a computer

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72.1. Introduction

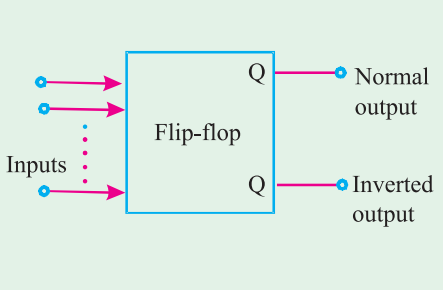
The output logic levels of combinational logic circuits at any instant of time are dependent on the logic levels present at the inputs at that time. Any prior input-logic level conditions have no effect on the present outputs. This is due to the fact that combinational logic circuits have no ***memory.*** However, it will be interesting to know that most digital systems are made up of both combinational logic circuits and memory elements.

The most important memory element is the flip-flop (abbreviated as FF). The FF is made up of an assembly of logic gates. It may be noted that even though a logic gate, by itself, has no storage capability, but several such logic gates can be connected together in ways that permit information to be stored. Several different gate arrangements are used to produce these flip-flops.

The flip-flops are used extensively as a memory cell in static random access memory (SRAM) of a computer.

72.2. Flip-Flop (FF)

We have already discussed in the last article that flip-flop (abbreviated as FF) is made up of logic gates and it permits information to be stored in it. Fig. 72.1 (*a*) shows a general type of symbol used for a flip-flop.



(*a*) Flip-flop symbol

Output states

$Q = 1, \overline{Q} = 0$: It is called HIGH or 1 State; also called SET state

$Q = 1, \overline{Q} = 0$: It is called LOW or 0 state; also called CLEAR or RESET state

(*b*) Operating states of Flip-flop

Fig. 72.1

As seen from this diagram, the flip-flop has two outputs labeled as Q and $\overline{Q}$ that are inverse (or complement) of each other. The Q output is called the ***normal*** flip-flop output and $\overline{Q}$ is the ***inverted*** flip-flop output. It may be carefully noted that whenever we refer to the state of a flip-flop, we are referring to the state of its ***normal*** (Q) output. For instance if we say flip-flop is in the HIGH state, we mean that $Q = 1$. On the other hand if we say flip-flop is in LOW state, we mean that $Q = 0$. It is automatically understood that the $\overline{Q}$ state will always be ***inverse*** of Q, *i. e.*, if $Q = 1$, $\overline{Q} = 0$, and if $Q = 0$, $\overline{Q} = 1$.

Fig. 72.1 (*b*) shows the two possible operating states of a flip-flop. One possible state is $Q = 1$, $\overline{Q} = 0$ and the other $Q = 0$, $\overline{Q} = 1$. The state $Q = 1$, $\overline{Q} = 0$ is called HIGH state or 1 state. It is also called *SET* state. Whenever, the inputs to a flip-flop cause it to go to the $Q = 1$ state, we call this ***setting*** the flip-flop or the flip-flop is set.

In a similar way, the state $Q = 0$, $\overline{Q} = 1$ is called LOW state or 0 state. It is also called *RESET* or CLEAR state. Whenever, the inputs to a flip-flop cause it to go to the $Q = 0$ state, we call this ***resetting*** **or** ***clearing*** the flip-flop. In the later part of the chapter we will see that many flip-flops will have a *SET* input and/or RESET (CLEAR) input that is used to drive the flip-flop into a specific output state.

It may be noted from the flip-flop symbol (Fig. 72.1 (*a*)) that a flip-flop can have one or more inputs. These inputs can be used to switch the flip-flop back and forth between its two possible output states.

It will be shown later that most flip-flop inputs need only to be momentarily activated (or pulsed) in order to cause a chage in the output state. The flip-flop output will remain in the new state even after the input pulse is over. This is flip-flop's memory characteristic.

It will be interesting to know that a flip-flop is also known as a latch and a bistable multivibrator. The term ***"latch"*** is used for certain types of flip-flops that will be described later. The term bistable multivibrator is the more technical name but flip-flop is the one used more frequently among the engineers and technologists.

72.3. Latch

A latch is the most basic type of flip-flop circuit. It can be constructed using NAND or NOR gates. Accordingly the latches are of two types :

1. NAND gate latch and

2. NOR gate latch

Both these types of latch are discussed one by one in the following pages.

72.4. NAND Gate Latch

Construction. Fig. 72.2 shows a latch constructed from NAND gates. It is called NAND gate latch or simply NAND latch. As seen from this diagram, the two NAND gates are cross-coupled. This means output of NAND-1 is connected to one of the inputs of NAND-2. Similarly, the output of NAND-2 is connected to one of the inputs of NAND-1. The NAND gate latch outputs are labeled as Q and $\overline{Q}$ respectively. Under normal conditions, these outputs will always be the inverse (or complement) of each other. The latch has two inputs namely SET and CLEAR. The SET input sets Q output of the latch to 1 state while the CLEAR input sets the Q output to the 0 state.

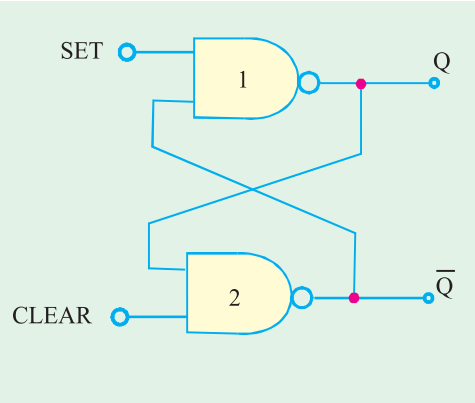


Fig. 72.2. NAND gate latch.

Operation. Let us now understand the operation of NAND gate latch. Normally, the SET and CLEAR inputs of the latch are resting in the HIGH state. Whenever we want to change the latch outputs, one of the two inputs will be pulsed LOW. Let us begin our study by showing that there are two equally likely output states when SET = CLEAR = 1. One possibility is shown in Fig. 72.3 (*a*) where we have $Q = 0$ and $\overline{Q} = 1$. With $Q = 0$, the inputs to the NAND-2 are 0 and 1, which produces $\overline{Q} = 1$. The 1 from $\overline{Q}$ causes NAND-1 to have 1 at both inputs to produce a 0 output at Q. Thus we find that a LOW at the NAND-1 output produces a HIGH at NAND-2 output which in turn keeps the NAND-1 output LOW.

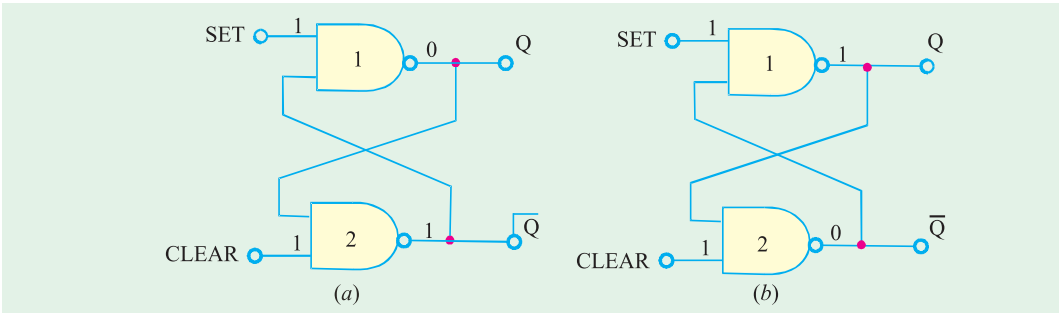


Fig. 72.3. Illustrating the two possible output states of NAND gate latch.

Fig. 72.3 (*b*) shows the second possibility. Here $Q = 1$ and $\overline{Q} = 0$. As seen from this figure, the HIGH from NAND-1 produces a LOW at the NAND-2 output, which in turn keeps the NAND-1 output HIGH. Thus there are two possible output states when SET = CLEAR = 1. However which one of these two states exist will depend on what has occurred previously at the inputs. This is discussed below in more detail where we will take up three situations : **(1)** setting the latch, **(2)** clearing the latch and simultaneous setting and clearing the latch.

1. Setting the Latch. We have already mentioned above that if SET = CLEAR = 1, the output Q can be in two possible states *i.e.* $Q = 0$ or $Q = 1$. Now we will study as what happens when the SET input is momentarily pulsed LOW while CLEAR is kept HIGH.

Fig. 72.4 (*a*) shows what happens when $Q = 0$ prior to the occurrence of the pulse. As SET input is pulsed LOW at time 't_0', Q will go HIGH. This will force $\overline{Q}$ to go LOW so that NAND-1 has now two LOW-inputs. Thus when SET returns to 1 state at t_1, the NAND-1 output remains HIGH which in turn keeps the NAND-2 output LOW.

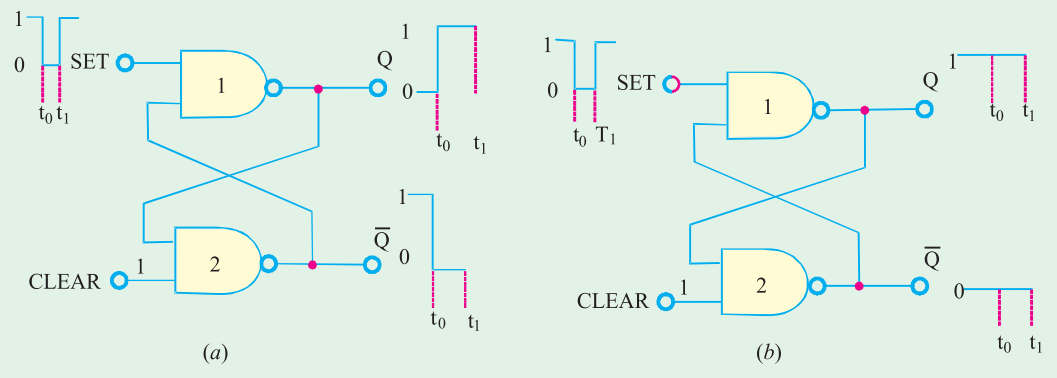


Fig. 72.4. Setting the NAND gate latch

Fig. 72.4 (*b*) shows what happens when $Q = 1$ and $\overline{Q} = 0$. Prior to the application of the SET pulse. Since $\overline{Q} = 0$ is already keeping the NAND-1 output HIGH, the LOW pulse at SET will not change anything. Thus when SET returns HIGH, the latch outputs are still in the $Q = 1$, $\overline{Q} = 0$ state (i.e no change in states).

2. Clearing the latch. Now we will study what happens when CLEAR input is pulsed LOW while SET is kept HIGH. Fig. 72.5 (*a*) shows the situation when $Q = 0$ and $\overline{Q} = 1$ prior to the application of the pulse. Since $Q = 0$ is already keeping the NAND-2 output HIGH, the LOW pulse at CLEAR will not have any effect. When CLEAR returns HIGH, the latch outputs are still $Q = 0$ and $\overline{Q} = 1$.

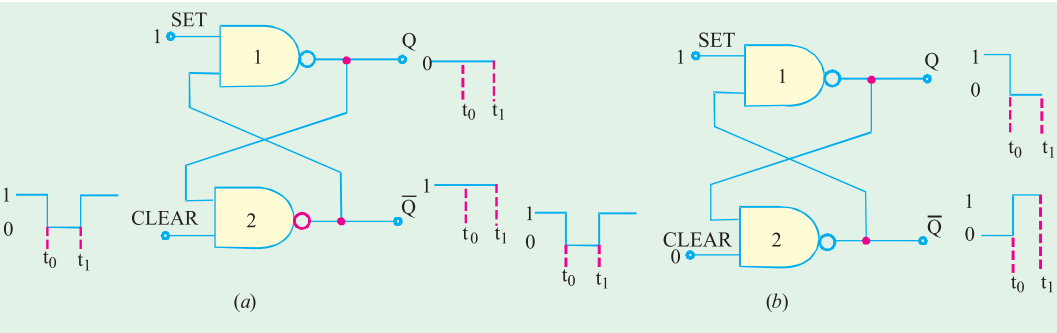


Fig 72.5. Clearing the NAND gate latch.

Fig 72.5 (*b*) shows the situation where $Q = 1$ prior to the occurrence of the CLEAR pulse. As CLEAR is pulsed LOW AT t_0, $\overline{Q}$ will go HIGH and this HIGH forces Q to go LOW because NAND-2 now has two LOW inputs. Thus when CLEAR returns HIGH at t_1, the NAND-2 output remains HIGH. This in turn keeps the NAND-1 output LOW.

3. Simultaneous setting and clearing. If the SET and CLEAR inputs are simultaneously pulsed LOW, this will produce HIGH levels at both NAND outputs so that $Q = \overline{Q} = 1$. This is an undesirable condition as the two outputs are supposed to be inverse of each other. Moreover, when the SET and CLEAR inputs return HIGH, the resulting output state will depend on which input returns HIGH first. Simultaneous transitions back to 1 state will produce unpredictable results. Because of these reasons the SET = CLEAR = 0 condition is normally not used for the NAND latch and is considered as invalid condition.

Inputs		*Output*
SET	*CLEAR*	
1	1	No change
0	1	$Q = 1$
1	0	$Q = 0$
0	0	$Q = \overline{Q} = 1$ (Invalid)

Fig. 72.6. Truth table for NAND gate latch.

The operation of NAND gate latch may be summarized in the form of a truth table as shown in Fig 72.6. Each line of the truth table is described as below :

1. **SET = CLEAR = 1.** This condition is the normal resting state of the latch.The Q and $\overline{Q}$ outputs will remain in the same state in which they were prior to this input condition.
2. **SET = 0, CLEAR = 1.** This condition will always cause the output to go to $Q = 1$ state where it will remain even after SET returns HIGH. This is called setting the latch.
3. **SET = 1, CLEAR = 0.** This condition will always produce $Q = 0$. The output will remain in this state even after CLEAR returns HIGH. This is called clearing or resetting the latch.
4. **SET = CLEAR = 0.** This condition tries to set and clear the latch simultaneously. It produces invalid results and should not be used.

72.5. Alternative Representations of NAND Gate Latch

We have already discussed in the last article about the NAND gate latch operation. It was mentioned that both SET and CLEAR inputs are active-LOW. Further the SET input will set $Q = 1$ when SET goes LOW. On the other hand, the CLEAR input will clear $Q = 0$ when CLEAR goes LOW. Because of this reason NAND gate latch is often drawn using the alternative representation for each NAND gate as shown in Fig 72.7 (*a*). (Recall from Art 3-41 that a NAND gate is equivalent to a bubbled or gate). The bubbles on the inputs of OR gate as well as labelling of the signals as $\overline{\text{SET}}$ and $\overline{\text{CLEAR}}$ indicate the active-LOW status of these inputs.

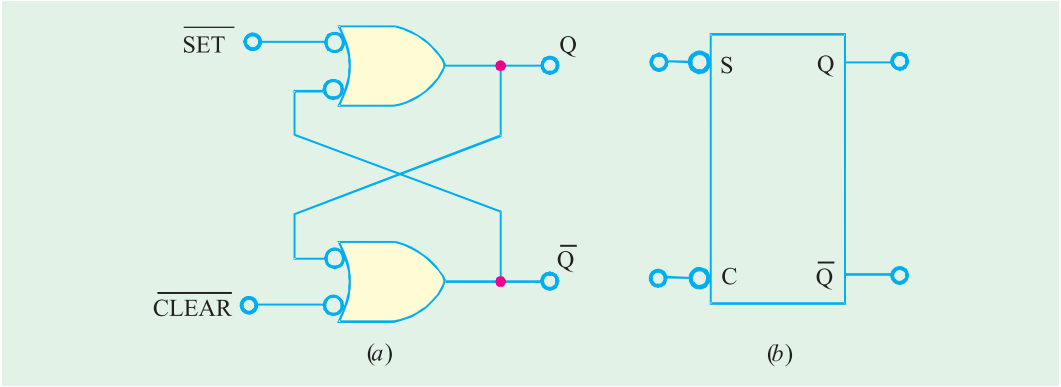


Fig. 72.7. Alternative representations of NAND gate latch.

Fig. 72.7 (*b*) shows a block representation. The *S* and *C* labels represent the SET and CLEAR inputs. While the bubbles at *S* and *C* inputs indicate the active LOW nature of these inputs. So remember whenever we use this block symbol, it represents a NAND gate latch.

Note. The action of ***Clearing*** a flip-flop or a latch is also called ***resetting.*** Both these terms (*i.e.* clearing or resetting) is used interchangeably in the field of digital electronics. Thus a SET-CLEAR latch can also be refered to as SET-RESET latch (or simply *S* - *R* latch).

Example 72.1. *The waveforms shown in Fig* 72.8 (*a*) *are applied to the inputs of the NAND latch shown in Fig.* 72.8 (*b*). *Assume that initially Q* = 0 *and determine the Q – waveform.*

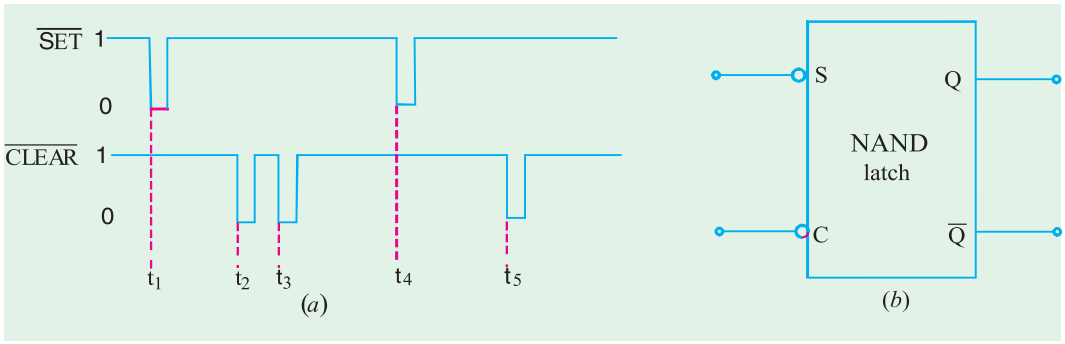


Fig. 72.8

Solution Given :

In order to determine the waveform at *Q*-output, refer to the truth table shown in Fig. 72.6 and the input waveforms. Notice that prior to $t = t_1$, $\overline{SET}$ = $\overline{CLEAR}$ = 1 and *Q* = 0, At $t = t_1$, $\overline{SET}$ goes 0 and $\overline{CLEAR}$ stays at 1. Referring to the truth table in Fig. 72-6, we find that if $\overline{SET}$ = 0, $\overline{CLEAR}$ = 1, *Q* = 1. Therefore the *Q* – output is indicated as 1 at $t = t_1$ in Fig. 72.9.

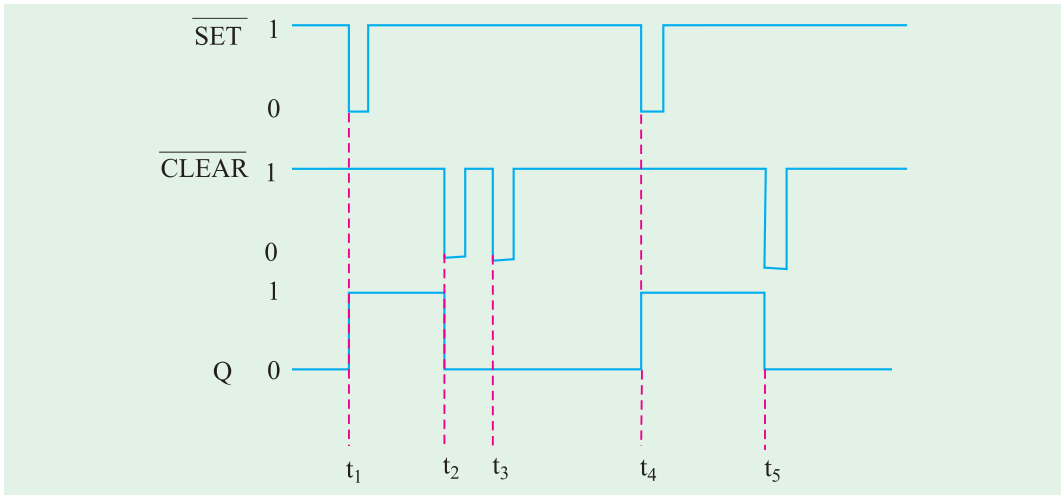


Fig. 72.9

At $t = t_2$, $\overline{SET}$ = 1, and $\overline{CLEAR}$ goes 0. Again referring to the truth table shown in Fig. 72.6, we find that *Q* = 0, therefore we sketch the *Q* output as 0 in Fig. 72.9. Similarly, at $t = t_3$, $\overline{SET}$ = 1, and $\overline{CLEAR}$ = 0. Since there is no change in inputs, the *Q*-output also remains at 0 as indicated in Fig. 72.9. At $t = t_4$, $\overline{SET}$ goes 0, and $\overline{CLEAR}$ is 1, therefore the output goes 1. At $t = t_5$, $\overline{SET}$ = 1 and $\overline{CLEAR}$ goes 0, therefore output goes 0. The complete *Q*-output waveform is shown in Fig. 72.9.

Example 72.2. *If the* $\overline{SET}$ *and* $\overline{CLEAR}$ *waveforms shown in Fig* 72.10 (*a*) *are applied to the inputs of NAND latch shown in Fig.* 72.10 (*b*) *determine the waveform that will be observed on the Q output. Assume that initially Q* = 0.

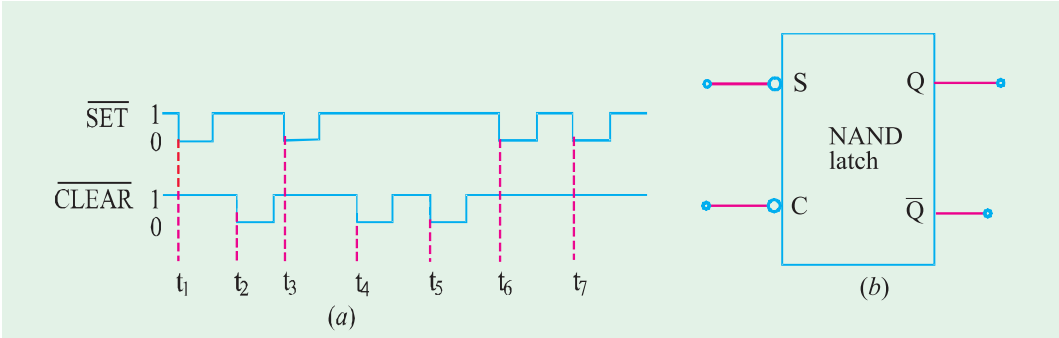


Fig. 72.10

Solution :

In order to determine, the Q-output, let us refer to the truth table shown in Fig. 72 .6 and the input waveforms. At $t = t_1$, $\overline{SET}$ goes from 1 to 0, $\overline{CLEAR}$ = 1, so the output Q goes from 0 to 1 as shown in Fig 72.11. At $t = t_2$, $\overline{SET}$ = 1, $\overline{CLEAR}$ goes from 1 to 0, so does the Q-output (*i.e*. it also goes from 1 to 0). At $t = t_3$, $\overline{SET}$ goes from 1 to 0, $\overline{CLEAR}$ = 1, therefore Q-output goes 1, At $t = t_4$, SET =1, $\overline{CLEAR}$ goes to 0, therefore Q-output goes 0 and so on. The complete waveform at Q-output is shown in Fig. 72.11.

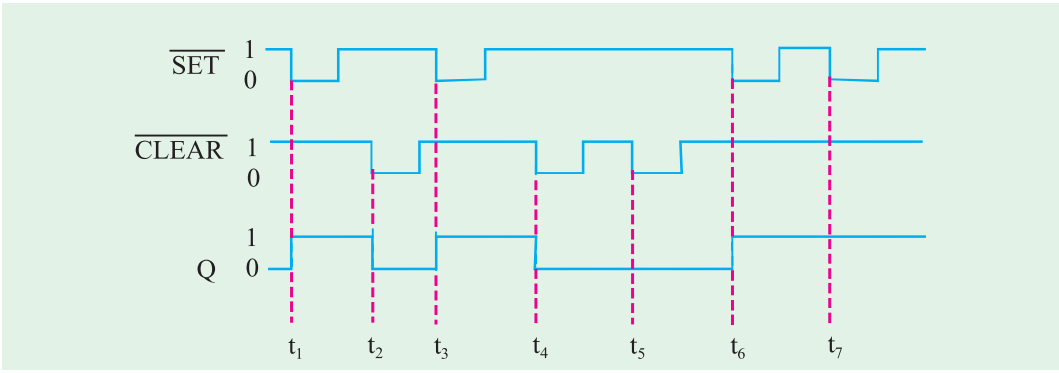


Fig. 72.11

72.6. Application of NAND Latch to Debounce a Mechanical Switch

Consider a mechanical switch with contact points *A* and *B* connected to V_{cc} (+ 5V) supply and ground respectively as shown in the Fig. 72.12 (*a*). It has been found experimentally that when the switch moves from contact position *A* to *B*, it produces several output voltage transitions as shown in Fig. 72.12 (*b*). It is due to a phenomenon called ***switch bounce*** *i.e*. the switch makes and breaks contact with contact *B* several times before coming to rest on contact *B*. In a similar manner, when the switch moves from contact position *B* to *A*, it again produces several output voltage transitions before coming to rest on contact *A*.

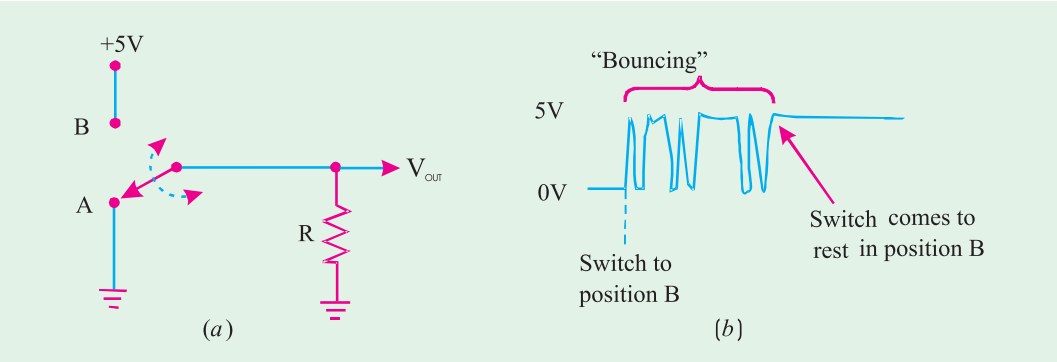


Fig. 72.12. Illustrating Switch Contact bounce

As a matter of fact, the multiple transitions on the output signal generally last only for few milliseconds. But such transitions are unacceptable in many applications. A NAND latch can be used as shown in Fig. 72.13 (*a*) to eliminate the switch bounce.

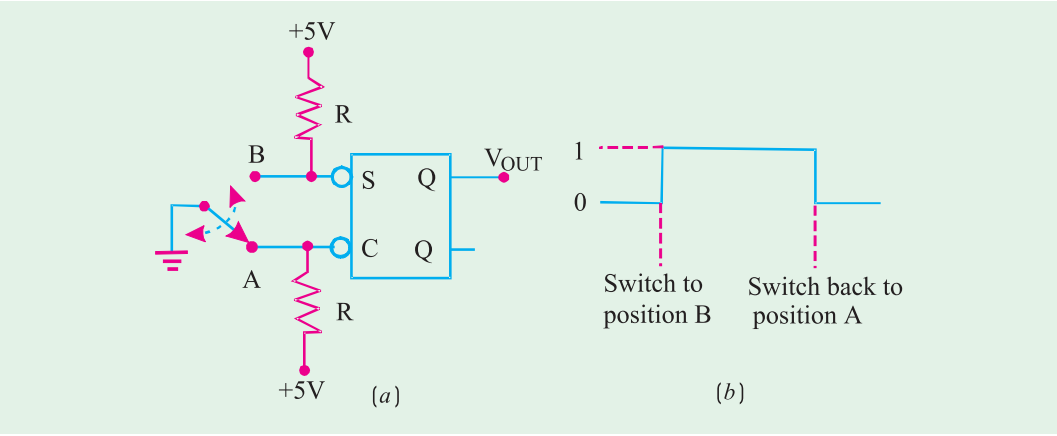


Fig. 72.13. Application of NAND latch to eliminate switch contact bounce.

The operation of the switch debouncing circuit may be understood as follows : Let us assume that initially the switch is resting in position *A* so that $\overline{\text{CLEAR}}$ input is LOW and $Q = 0$. When the switch is moved to position *B*, $\overline{\text{CLEAR}}$ will go HIGH and a LOW will appear on the $\overline{\text{SET}}$ input as the switch first makes contact. This will set $Q = 1$ within few nanoseconds. Now if the switch bounces off contact *B*, $\overline{\text{SET}}$ and $\overline{\text{CLEAR}}$ will both be HIGH and *Q* will not be affected, *i.e.* it will stay HIGH. Thus nothing will happen at *Q* as the switch bounces on and off contact *B* before finally coming to rest in position *B* as shown in Fig. 72.13 (*b*).

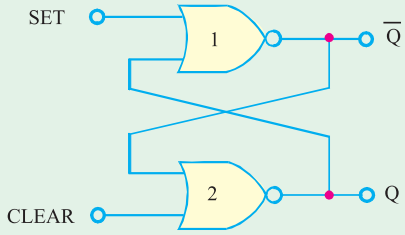
In a similar manner, when the switch is moved from position *B* back to position *A*, it will place a LOW on the $\overline{\text{CLEAR}}$ input as it first makes contact. This clears *Q* to the LOW state. It will remain there ever if the switch bounces on and off contact *A* several times before coming to rest.

It is evident from the above discussion that the output at *Q* will consists of single transition each time the switch is moved from one position to the other.

72.7. NOR Gate Latch

Fig. 72.14 (*a*) shows a latch constructed from two cross-coupled NOR gates. It is called NOR gate latch or simply the NOR latch. As seen in the figure, the arrangement is similar to the

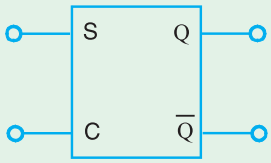
NAND gate latch (shown in Fig 72.2 on page 2637) except that the Q and $\overline{Q}$ outputs have reversed positions.



(*a*) Construction

Inputs		*Output*
SET	*CLEAR*	
0	0	No change
1	0	$Q = 1$
0	1	$Q = 0$
1	1	*Invalid

(*b*) Truth table



(*c*) Logic symbol

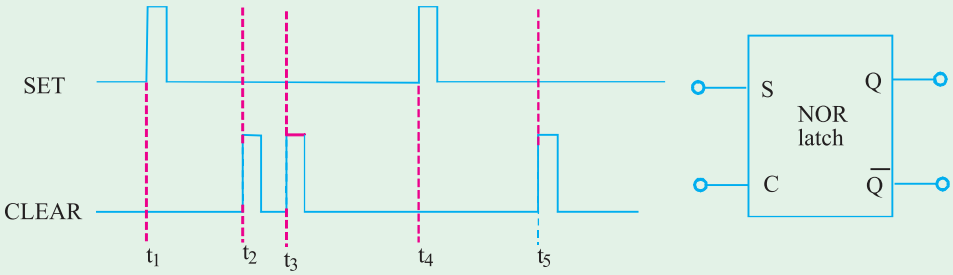
Fig. 72.14. NOR gate latch.

The detailed operation of the NOR latch can be understood exactly in the same manner as for the NAND latch. The results are given in the truth table as shown in Fig 72.14 (*b*) and are summarized as follows :

1. **SET = CLEAR = 0.** This condition is the normal resting state for the NOR latch. It has no effect on the output state. In other words, Q and $\overline{Q}$ will remain in the same state in which they were prior to this input condition.
2. **SET = 1, CLEAR = 0.** This condition will always cause the output to go to $Q = 1$ state where it will remain even after SET returns to 0.
3. **SET = 0, CLEAR = 1.** This condition will always cause the output to go to $Q = 0$ state where it will remain even after CLEAR returns to 0.
4. **SET = 1, CLEAR = 1.** This condition tries to set and clear the latch simultaneously. It produces invalid results and should not be used.

It may be carefully noted that the NOR latch operates exactly in the same manner as the NAND latch. However, the SET and CLEAR inputs are active-HIGH rather than active-LOW. Moreover the normal resting state is SET = CLEAR = 0. Further the output Q will be set HIGH by a HIGH pulse on the SET input and it will be cleared LOW by a HIGH pulse on the CLEAR input. Fig 72.14 (*c*) shows the logic symbol for the NOR latch. Notice that there are no bubbles on the S and C inputs (unlike NAND latch) which indicates that these inputs are active-HIGH.

Example 72.3. *The waveforms shown in Fig.* 72.15 (*a*) *are applied to the inputs of the NOR latch shown in Fig.* 72.15 (*b*). *Assume that initially,* $Q = 0$ *and determine the Q-waveform.*



(*a*) (*b*)

Fig. 72.15

Solution :

In order to determine the Q-waveform, we will refer to the truth table of the NOR latch (shown in

Fig. 72.14 (*b*) page 2643) and the SET and CLEAR waveforms of Fig. 72.15 (*a*). At $t = t_1$, SET goes 1 and CLEAR is 0, therefore *Q*-output also goes 1. AT $t = t_2$, SET = 0 and CLEAR goes 1, therefore the *Q*-output goes 0. At $t = t_3$, SET = 0 and CLEAR goes 1 again, therefore the Q-output remains 0. At $t = t_4$, SET goes 1 and CLEAR is 0, therefore *Q*-output goes 1. At $t = t_5$, SET = 1 and CLEAR goes 1, therefore *Q*-output goes 0. The complete sketch of the *Q*-waveform along with SET and CLEAR waveforms is as shown in Fig. 72.16.

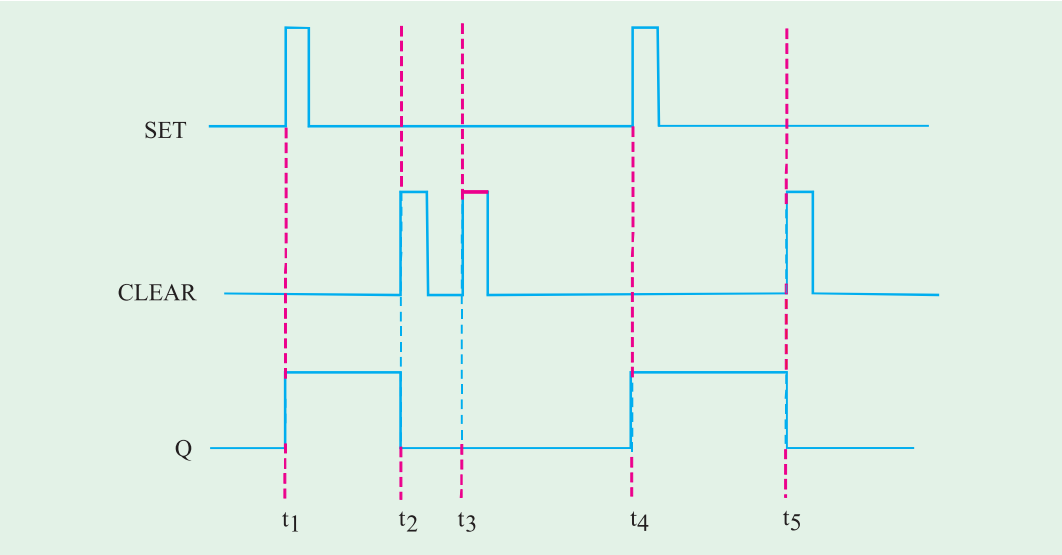


Fig. 72.16

Example 72.4. *If the SET and CLEAR waveforms shown in Fig.* 72.16. (*a*) *are applied to the inputs of NOR latch shown in Fig.* 72.17. (*b*), *determine the waveform that will be observed on the Q-output. Assume that initially Q* = 0.

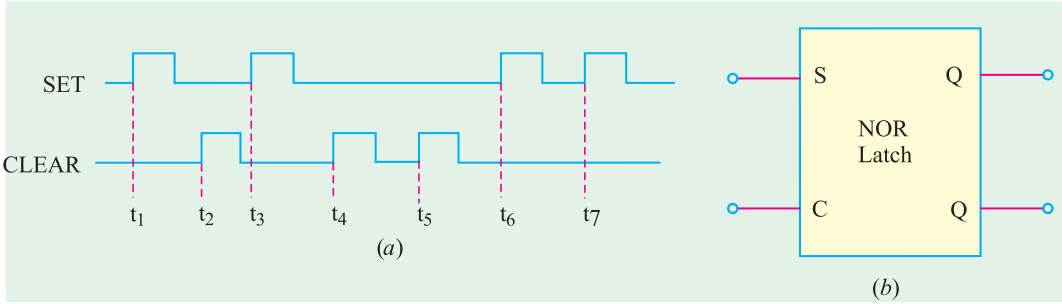


Fig. 72.17

Solution :

In order to determine the Q-waveform, we will refer to the truth table of the NOR latch (shown in Fig. 72.14 (*b*) page 2643) and the SET and CLEAR waveforms of Fig. 72.17. At $t = t_1$, SET goes 1 and CLEAR = 0, therefore *Q* goes 1. At $t = t_2$, SET = 0 and CLEAR goes 1, therefore *Q* goes 0. At $t = t_3$, SET = 1 again, and CLEAR = 0, therefore *Q* goes 1 again. At $t = t_4$, SET = 0. and CLEAR = 1, therefore *Q* goes 0 and so on. A complete sketch of *Q* waveform along with input waveforms is shown in Fig. 72.18.

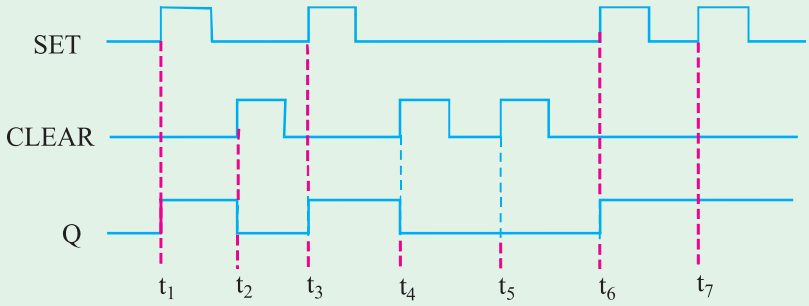


Fig. 72.18

72.8. Clocked Signals

Strictly speaking, the digital systems are of the following two types :

1. Asynchronous systems. In these systems, the outputs of logic circuits can change state any time one or more of the inputs change. Generally, an asynchronous system is more difficult to design and troubleshoot than a synchronous system.

2. Synchronous systems. In these systems, the exact time at which the output can change states are determined by a signal commonly called a ***clock.***

The clock signal is generally a rectangular pulse train as shown in Fig. 72.19 (*a*) or a square wave as shown in Fig. 72.19 (*b*). The clock signal is distributed to all parts of the digital system and most of the system outputs can change state only when the clock makes a transition. The transitions are more commonly referred to as ***edges*** and are pointed out in Fig. 72.19.

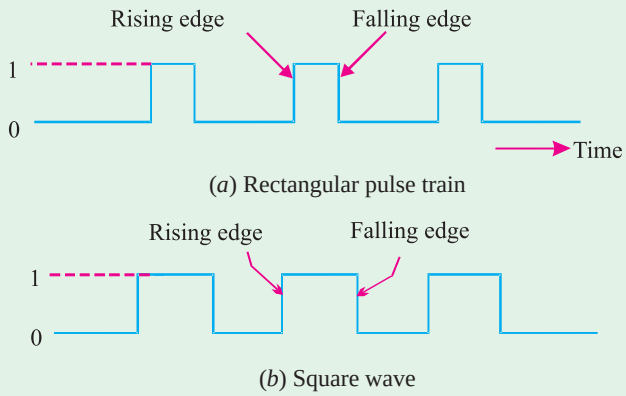


Fig. 72.19

It may be noted that when the clock changes from 0 to 1, this is called rising edge or ***positive-going transition*** (PGT). On the other hand, when the clock changes from 1 to 0, this is called falling edge or ***negative going transition*** (NGT).

As a matter of fact, most digital systems are principally synchronous (although there are always some asynchronous parts). It is due to the fact that synchronous circuits are easier to design and troubleshoot. The synchronization is accomplished through the use of ***clocked flip-flops*** that are designed to change states on one or the other of the clock's transitions.

72.9. Some Main Ideas Common to Clocked Flip-Flops

There are several types of flip-flops that are used in a wide range of applications in the field of digital electronics. Before we begin our study of the different clocked flip-flops, let us describe the main ideas that are common to all of them.

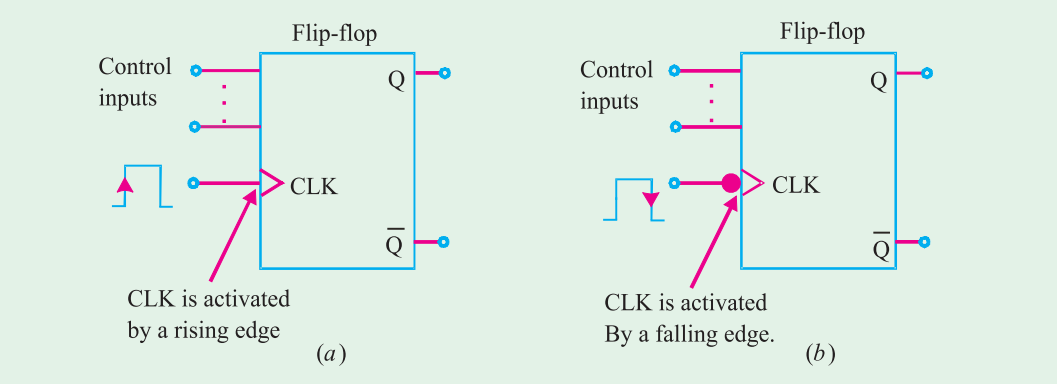


Fig 72.20. Clocked Flip-Flops.

Fig. 72.20. (*a*) and (*b*) shows the logic symbols for a typical clocked flip-flop. As seen, a clocked flip-flop has a clock input and some control inputs. There are described below in more detail.

CLK Input. Clocked flip-flops have a ***clock*** input that is labeled as CLK, CK or CP. However, we will normally use CLK as shown in Fig. 72.20 (*a*) and (*b*). In most clocked flip-flops, the CLK input is ***edge-triggered.*** This means the CLK input is activated by a signal transition. The edge-triggered activation is indicated by the presence of a small triangle on the CLK input. This contrasts with the latches, which are level-triggered.

Fig. 72.20 (*a*) shows a flip-flop with a small triangle on its CLK input to indicate that the input is activated only when a rising edge occurs. It may be noted that no other part of the input pulse will have an effect on the CLK input.

Fig 72.20 (*b*) shows a flip-flop symbol which has a bubble (a small circle) as well as a triangle on its CLK input. This signifies that CLK input is activated ***only*** when a falling edge occurs. Again note that no other part of the input pulse will have an effect on the CLK input.

Control Inputs. Clocked flip-flops also have one or more ***control inputs*** that can have various names depending on their operation. The control inputs will have no effect on *Q* until the active clock transition occurs. In other words, their effect is synchronized with the signal applied to CLK. Because of this reason, the control inputs are refered to as ***synchronized control inputs.***

72.10. Setup and Hold Times in Clocked Flip-Flops

Strictly speaking, there are two timing requirements that must be met if a clocked flip-flop is to respond relialably to its control inputs when the active CLK edge occurs. These two requirements are : **(1)** set up time, t_S and **(2)** hold time, t_H . Both these requirements are discussed below.

1. Set up time, t_s. It is the time interval immediately preceeding the active edge of the CLK signal during which the control input must be maintained at the proper level. The situation is illustrated in Fig 72.21 (*a*) for a flip-flop that triggers on the rising edge. Usually, the IC manufacturers specify the minimum allowable set up time, t_S (min). If this time requirement is not met, the flip-flop may not respond reliably when the clock edge occurs.

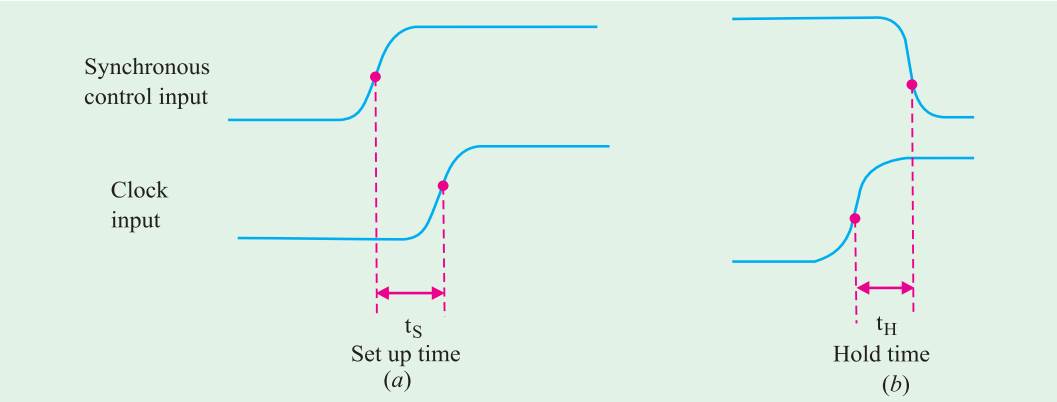


Fig. 72.21. Illustrating the set-up and hold time requirements, for a flip-flop that triggers on the rising edge.

2. Hold time, t_H. It is the time interval immediately following the active edge of the CLK signal during which the synchronous control input must be maintained at the proper level. Refer to Fig. 72.21 (*b*) usually the IC manufacturers specify the minimum acceptable value of hold time, t_H (min). If this requirement is not met, the flip-flop will not trigger reliably.

It is evident from the above discussion that in order to ensure that a clocked flip-flop will respond properly when the active clock edge occurs, the control inputs must be stable (*i.e.* unchanging) for at least a time interval.

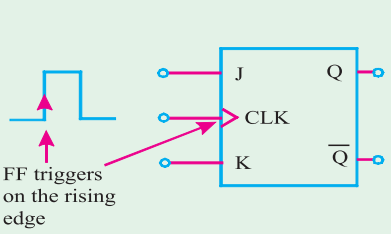
1. t_S (min) prior to the active clock edge.
2. t_H (min) after the active clock edge.

A typical value of t_S (min) is in the range of 5 to 50 ns whereas hold times are generally from 0 to 10 ns. It may be carefully noted that the set up and hold times are measured between the 50 per cent points on the edges.

The set up and hold time requirements are extremely important in synchronous systems. This is due to the reason that there will be many situations where the synchronous control inputs to a flip-flop are changing at approximately the same time as the CLK input.

72.11. Clocked *S* - *C* Flip-Flop

Fig. 72.22 (*a*) shows the logic symbol for a clocked *S* - *C* flip-flop that is triggered by the rising edge of the clock signal. In other words this flip-flop can change output states only when a signal applied to its clock input makes a transition from 0 to 1. The *S* and *C* inputs control the state of the flip-flop in the same manner as discussed earlier for the NOR latch. But it may be noted that the flip-flop does not respond to these inputs until the occurrence of the rising edge of the clock signal.



(*a*) Symbol

Inputs			*Output*
S	*C*	*CLK*	*Q*
0	0	↑	Q_0 (no change)
1	0	↑	1
0	1	↑	0
1	1	↑	Ambiguous

(*b*) Truth table

Fig. 72.22. Clocked *S*-*C* Flip-Flop.

Fig. 72.22 (*b*) shows the truth table for a clocked *S-C* flip-flop that is triggered by the rising edge of the clock signal. The truth table indicates how the flip-flop output will respond to the rising edge at the CLK input for the various combinations of *S* and *C* inputs. The up arrow (↑) in the truth table indicates that the rising edge is required at CLK input. The label Q_0 indicates the level at *Q* prior to the rising edge. This nomenclature is used quite often by IC manufacturers in their IC data sheets/ manuals.

The operation of *S-C* flip-flop may be understood with the help of input and output waveforms shown in Fig 72.23. Assuming that the setup and hold time requirements are being met in all cases, the waveforms can be analysed as follows :

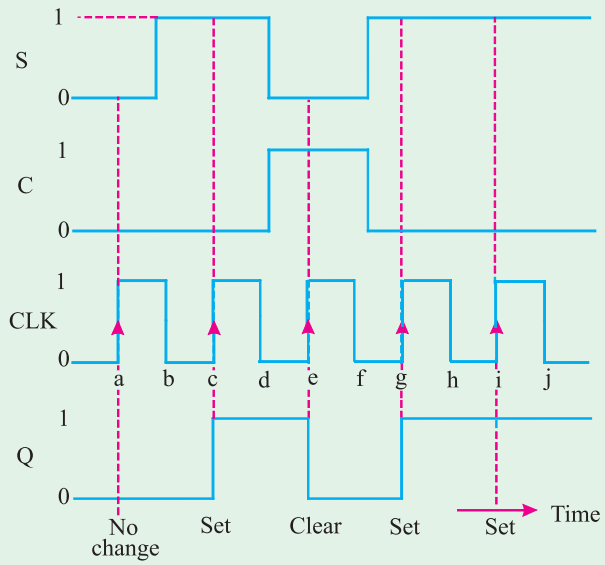
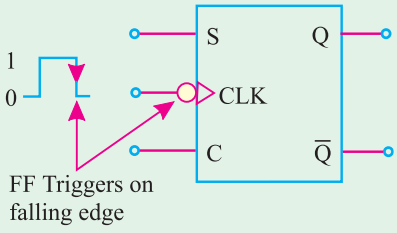


Fig. 72.23. Typical input and output waveforms of a *S-C* flip-flop.

1. Notice that initially all the inputs (S, C and CLK) are 0 and the *Q* output is assumed to be zero, *i.e.* $Q_0 = 0$.
2. When the rising edge of the first clock pulse occurs (refer to point '*a*'), both the *S* and *C* inputs are 0, so the flip-flop output is not affected and remains in the $Q = 0$ state (*i.e.* $Q = Q_0 = 0$).
3. At the occurrence of the rising edge of the second clock pulse (refer to point '*c*'), the *S* input is now HIGH, with *C* input still LOW. This causes the flip-flop output to set to 1 state.
4. At the occurrence of the rising edge of the third clock pulse (refer to point '*e*'), the *S* input is LOW with *C* input HIGH. This causes the flip-flop output to clear to 0 state.
5. The rising edge of the fourth clock pulse sets the flip-flop output to the $Q = 1$ state (refer to point '*g*') because $S = 1$ and $C = 0$.
6. The fifth clock pulse also finds that $S = 1$ and $C = 0$ at the rising edge (refer to point '*i*'). This situation will produce HIGH output. But since *Q* is already HIGH, so it remains in that state.

It may be carefully noted from the waveforms shown in Fig. 72.23, that the flip-flop is not affected by the falling edge of the clock pulses. It may also be noted that *S* and *C* input levels have no effect on the flip-flop except upon the occurrence of a rising edge of the clock signal.



Inputs			Output
S	C	CLK	Q
0	0	↓	Q_0 (no change)
1	0	↓	1
0	1	↓	0
1	1	↓	Ambiguous

(*a*) Symbol (*b*) Truth table

Fig. 72.24. Falling-edge triggered S-C flip-flop.

Fig. 72.24 (*a*) shows the symbol and 72.24 (*b*) the truth table for a clocked *S* - *C* flip-flop that triggers on the falling edge of the CLK input. Notice the presence of a small circle and a triangle on the CLK input of the flip-flop. These indicate that this flip-flop will trigger only when CLK input goes from 1 to 0. This flip-flop operates in the same way as the rising-edge flip-flop except that the output can change states only on the falling edge of the clock pulses (refer to points *b, d, f, h* and *j* in Fig. 72.23). In actual practice both the rising-edge and falling-edge triggered flip-flops are used in digital systems.

Example 72.5. *Fig.* 72.25 (*a*) *shows the SET and CLEAR waveforms applied at the inputs of the clocked S-C flip-flop shown in Fig.* 72.25 (*b*)

Sketch the waveforms at Q and $\overline{Q}$ *outputs of the flip-flop. Assume that initially Q* = 0 *and* $\overline{Q}$ = 1.

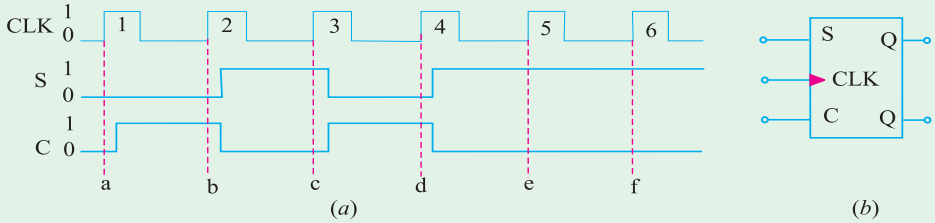


Fig. 72.25

Solution. Given :

In order to determine the *Q*-output, we will refer to the waveforms applied at the flip-flop inputs shown in Fig. 72.25 (*a*) and its truth table shown in Fig. 72.24 (*b*). At point '*a*', $S = C = 0$, therefore $Q_0 = 0$, At point '*b*', $S = 0$ and $C = 1$, therefore *Q* remains 0. At point '*c*', $S = 1$ and $C = 0$, therefore $Q = 1$. At point '*d*', $S = 0$ and $C = 1$, therefore $Q = 0$. At point '*e*', $S = 1$, $C = 0$, therefore $Q = 1$. At point '*f*', *S* and *C* inputs remain the same, therefore *Q* also remains 1. The sketch of *Q*-output along with the CLK, *S* and *C* waveforms is as shown in Fig. 72.26. The $\overline{Q}$ -output waveform is determined by inverting the *Q*-waveform.

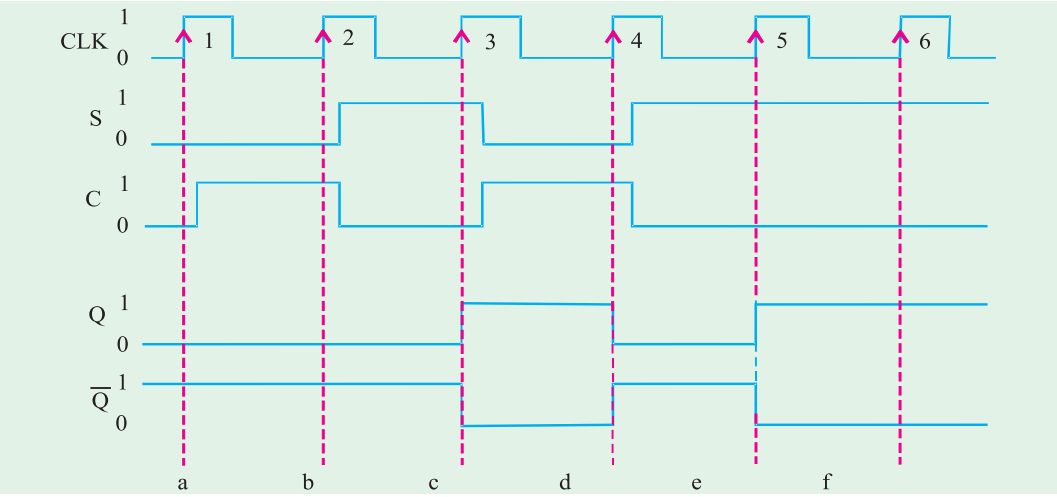


Fig. 72.26

72.12. Internal Circuitry of an Edge-triggered S-C Flip-Flop

Fig . 72.26 shows a simplified version of an internal circuitry of an edge triggered *S-C* flip-flop. Notice that the circuit contains the following three sections :

1. A basic NAND latch
2. A pulse steering circuit and
3. An edge-detector circuit

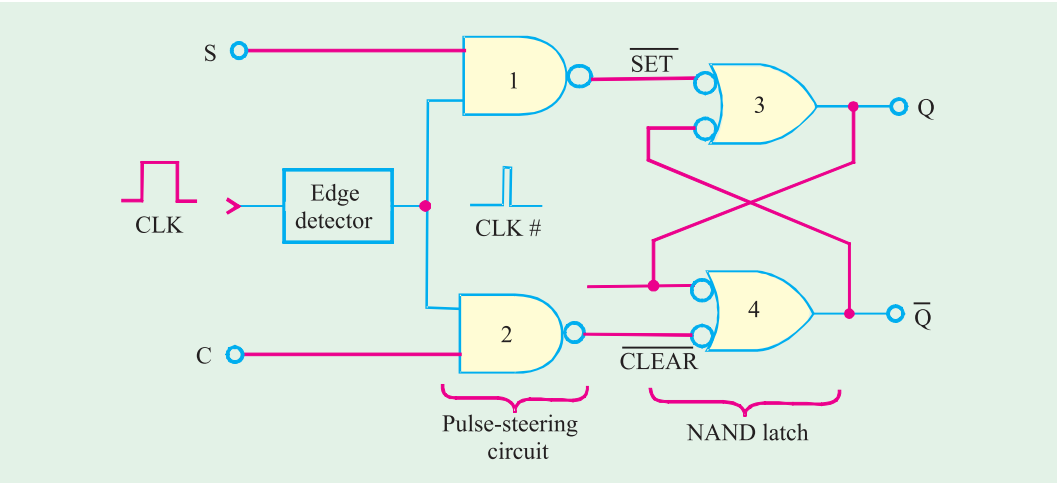


Fig. 72.27. Internal circuitry of an edge-triggered S-C flip-flop.

As seen from the Fig. 72.27, the edge detector produces a narrow positive-going spike (CLK #). The pulse-steering circuit "steers" (or moves) the spike through to the SET and CLEAR input of the latch in accordance with the levels present at *S* and *C* inputs. For example with $S = 1$ and $C = 0$, the spike is inverted and passed through NAND gate-1 to produce a LOW pulse at the SET input of the latch that sets $Q = 1$. With $S = 0$ and $C = 1$, the spike is inverted and passed through NAND gate-2 to produce a LOW pulse at the CLEAR input of the latch that resets $Q = 0$.

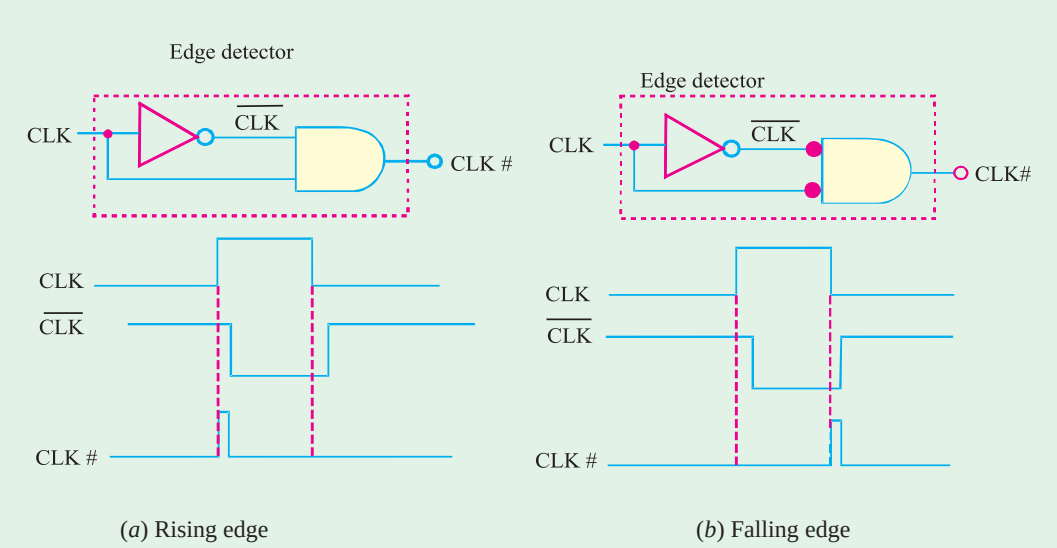


Fig. 72.28. Implementation of edge-detector circuits used in edge triggered flip-flops.

Fig. 72.28 shows the implementation of edge-detector circuits used in edge-triggered flip-flops. Thus Fig. 72.28 (*a*) shows how the spike is generated for edge-triggered flip-flops that trigger on the rising edge of the CLK pulse. As seen from this figure, the INVERTER produces an output with a delay of a few nanoseconds. Because of this the transitions of $\overline{CLK}$ occur a little bit after that of CLK. Next the AND gate produces an output spike that is HIGH only for the few nanoseconds when both CLK and $\overline{CLK}$ are HIGH. As a result of this, we get a narrow pulse at edge detector output (CLK #) which occurs on the rising edge of the CLK.

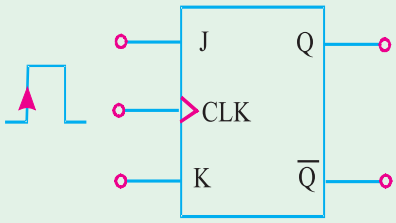
Similarly Fig 72.28(*b*) produces CLK # on the falling edge of CLK for flip-flops that are to trigger on the falling edge.

It may be noted that since the CLK # signal is HIGH for only a few nanoseconds, the *Q* output is affected by the levels at *S* and *C* only for a short time during and after the occurrence of the active edge of CLK. This gives the flip-flop its edge-triggered property.

72.13. Clocked *J-K* Flip-Flop

Fig. 72.28 (*a*) shows the symbol and (*b*) the truth table for a clocked *J-K* flip-flop that is triggered by the rising edge of the clock signal. The *J* and *K* inputs control the state of the flip-flop in the same manner as the *S* and *C* inputs do for the *S-C* flip-flop. However there is one major difference-the $J = K = 1$ condition in *J-K* flip-flop does not result in an ambiguous output unlike $S = C = 1$ in *S-C* flip-flop for, $J = K = 1$ condition, the J-K flip-flop will always go to its opposite state upon the rising edge of the clock signal. This is called ***trigger mode.*** In this mode, if both *J* and *K* are left HIGH, the flip-flop will change states (i.e. toggle) for each rising edge of the clock.

The operation of *J-K* flip-flop for each combination of *J* and *K* is summarized in Fig. 72.29 (*b*) Notice that the truth table is the same except for $J = K = 1$ condition. This condition results in $Q = \overline{Q}_0$ which means that the new value of *Q* will be the inverse of the value it had prior to the rising edge of the clock pulse. It is called toggle operation.



(*a*) Symbol

Inputs			*Output*
J	*K*	*CLK*	*Q*
0	0	↑	Q_0 (no change)
1	0	↑	1
0	1	↑	0
1	1	↑	Q_0 (toggles)

(*b*) Truth table

Fig. 72.29. Rising edge triggered J-K flip-flop.

In order to understand the operation of J-K flip-flop, let us consider the *J*, *K* and CLK waveforms as shown in Fig. 72.30. Assume that set up and hold time requirements are met. The operation may be explained as below.

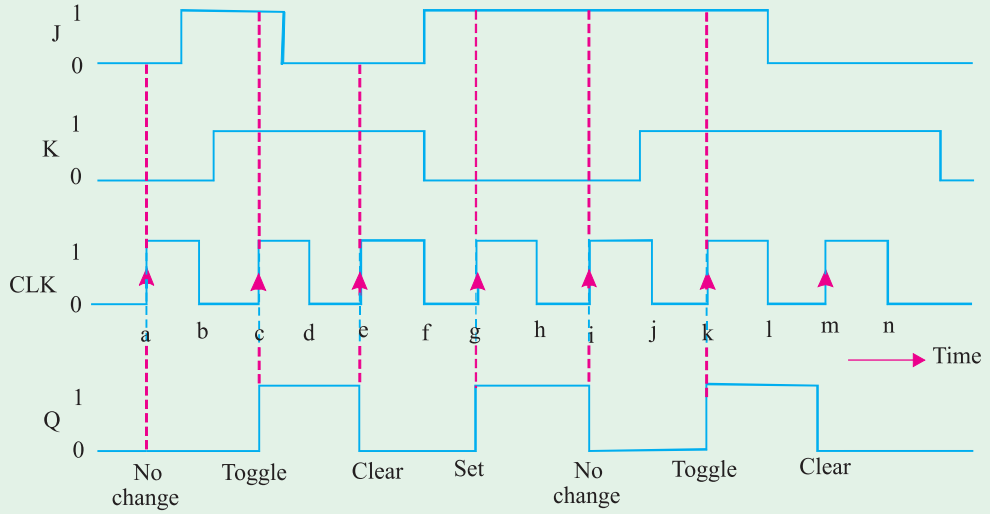
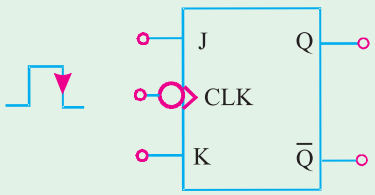


Fig. 72.30

1. Initially all the inputs are 0, and the *Q* output is also assumed to be 0, *i.e.* $Q_0 = 0$.
2. When the rising edge of the first clock pulse occurs (refer to point *a*), the $J = K = 0$ condition exists. Thus the flip-flop does not change its output state, *i.e.* $Q = Q_0 = 0$.
3. When the rising edge of the second clock pulse occurs (refer to point *c*), the $J = K = 1$ condition exists. Thus the flip-flop toggles to its opposite state *i.e.* $Q = \overline{Q}_0 = \overline{0} = 1$.
4. When the rising edge of the third clock pulse occurs (refer to point *e*), $J = 0$ and $K = 1$ condition exists. Thus the flip-flop is cleared to the $Q = 0$ state.
5. When the rising edge of the fourth clock pulse occurs (refer to point *g*), $J = 1$ and $K = 0$ condition exists. This condition sets the output *Q* to 1 state.
6. When the rising edge of fifth clock pulse occurs (refer to point *i*), $J = 1$ and $K = 0$ condition exists. This is the condition that sets the output *Q* to 1 state. However since *Q* is already 1, so it will remain there. Hence no change in the output state.
7. When the rising edge of six clock pulse occurs (refer to point *k*), $J = K = 1$ condition exists. This condition causes the flip-flop to toggle to its opposite state.

8. When the rising edge of seventh clock pulse occurs (refer to point m), $J = 0$ and $K = 1$ condition exists. This condition causes the flop-flop to clear to $Q = 0$ state.

It may be noted from the waveforms that the flip-flop is not affected by the falling edge of the clock pulses. Also the *J* and *K* input levels have no effect except the occurrence of the rising edge of the clock signal. The *J* and *K* inputs by themselves cannot cause the flip-flop to change states.



(*a*) Symbol

Inputs			*Output*
J	*K*	*CLK*	*Q*
0	0	↓	Q_0 (no change)
1	0	↓	1
0	1	↓	0
1	1	↓	Q_0 (toggles)

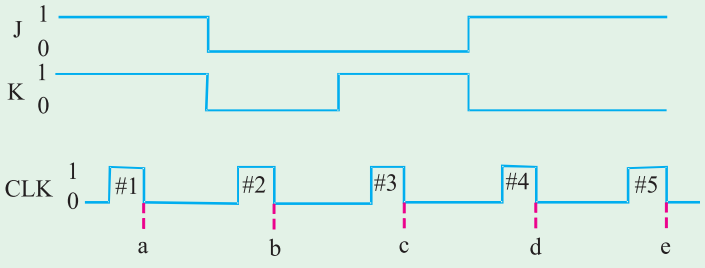
(*b*) Truth table

Fig. 72.31. Falling edge triggered J-K flip-flop.

Fig. 72.31 shows the symbol and 72.31 (*b*) the truth table for a clocked *J-K* flip-flop that triggers on the falling edge of the clock pulse. The small circle on the CLK input indicates that the flip-flop will trigger when the CLK input goes from 1 to 0. This flip-flop operates in the same way as the rising edge of the flip-flop of Fig. 72.27 except that the output can change states only on the falling edge of CLK signal (*i.e.* points *b, d, f, h, j,* l and *n*). As a matter of fact, both polarities of edge-triggered *J-K* flip-flops are in common usage in the field of digital electronics.

Strictly speaking, the *J-K* flip-flop is much more versatile than the *S-C* flip-flop because it has no ambiguous states. The $J = K = 1$ condition, which produces the toggling operation, finds extensive use in all types of binary counters.

Example 72.6. Fig 72.32 (*a*) *shows the waveforms applied at J, K and CLK inputs of the clocked J-K flip-flop shown in Fig.* 72.32 (*b*). *Sketch the Q output waveform Assume* $Q = 0$ *initially.*



(*a*) (*b*)

Fig. 72.32

Solution :

Notice that flip-flop shown in Fig. 72.32 (*b*) is a clocked *J-K* flip-flop. Also notice the presence of a small circle at the CLK input of the flip-flop. This indicates that the flip-flop will trigger coresponding to the falling edge of the clock pulse. So you need to identify the states of *J* and *K* inputs

coresponding to the falling edge points of the CLK pulse waveform *i.e.* points *a, b, c, d* and *e* shown in Fig. 72.32.

Using the truth table of clocked *J-K* flip-flop shown in Fig. 72.31, and the given waveforms the *Q*-waveform sketch is shown in Fig. 72.33. On the arrival of first CLK pulse at point '*a*' $J = K = 1$, the a output does not change, *i.e.* it stays at $Q = 1$ level. On the arrival of third CLK pulse *J-K* flip-flop will toggle, i.e., its Q-output changes from 0 to 1. On the arrival of second CLK pulse, at point '*b*', $J = K = 0$ the *Q*-at point '*c*' $J = 0$, $K = 1$, the Q output goes 0. On the arrival of fourth CLK pulse, at point '*d*' $J = 1$, $K = 0$, the *Q*-output goes 1. On the arrival of fifth CLK pulse at point '*e*' $J = 1$, $K = 0$, the *Q*-output remains at 1 level.

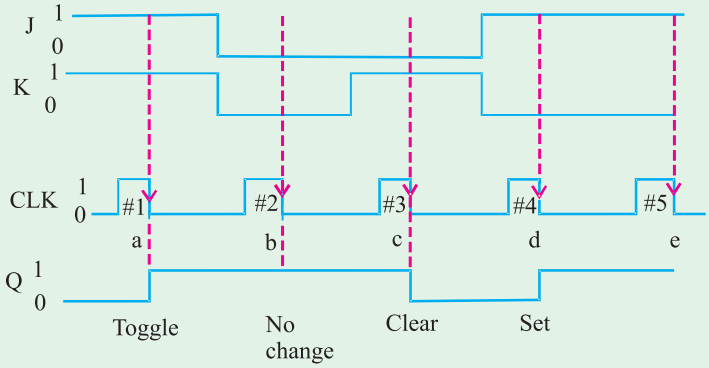


Fig. 72.33

Example 72.7. *What will be the output waveform Q of a J-K flip-flop if the following waveforms are applied at the input? Assume the flip-flop triggers at the falling edge of clock pulse.*

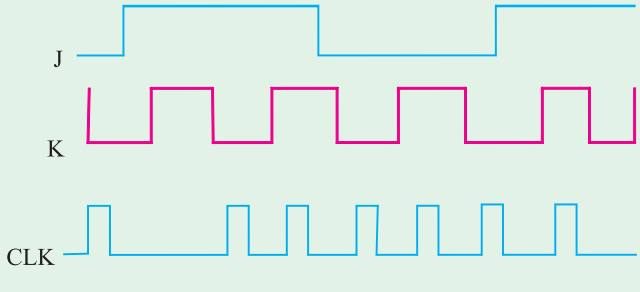


Fig. 72.34

(Grad. IETE Dec. 1996)

Solution :

Recall the truth table of a clocked *J-K* flip-flop, the flip-flop triggers corresponding to the logic levels at the *J* and K inputs. If $J = K = 1$, the flip-flop output remains in its previous state. If $J = 1$, $K = 0$ the flip-flop output goes 1, if $J = 0$, $K = 1$, the flip-flop goes LOW. However, if $J = K = 1$, the flip-flop output toggles. Keeping it in mind, we can sketch the *Q*-output waveform as shownin Fig. 72.35.

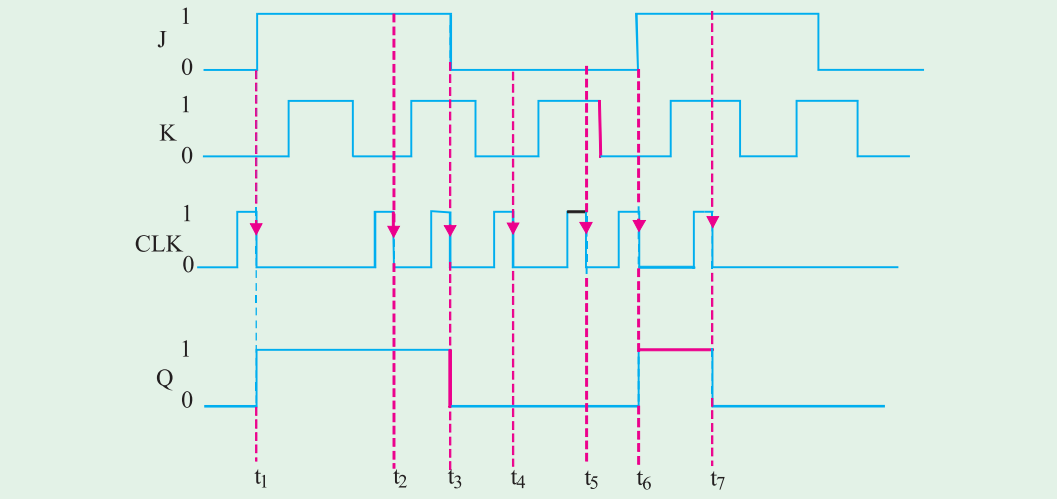


Fig. 72.35

Notice that at t_1, $J = 1, K = 0$ therefore the Q–output goes 1. At t_2, $J = 1, K = 0$ so the Q–output remains 1. At t_3, J goes 0, $K = 1$, therefore the Q-output goes 0. At t_5, J is still 0, K is still 1, therefore the Q-output stays 0. At t_6, J goes 1, K is 0, so the Q-output goes 1. At t_6 both J and K are 1, therefore the Q-output toggles and goes 0, as shown in the Fig. 72.35.

72.14. Internal Circuitry of an Edge-triggered *J-K* Flip-Flop

Fig. 72.36 shows a simplified version of the internal circuitry of an edge-triggered *J-K* flip-flop. As seen, the flip-flop contains the same three sections as the *S-C* triggered flip-flop (refer to Fig 72.27 page 2646), *i.e.*,

1. A basic NAND gate latch
2. A pulse steering circuit and
3. An edge detector circuit

The only difference between the internal circuitry of edge-triggered *J-K*. flip-flop and that of *S-C* flip flop is that in *J-K* flip-flop circuit, Q and $\overline{Q}$ outputs are fed back to the pulse-steering NAND gates.

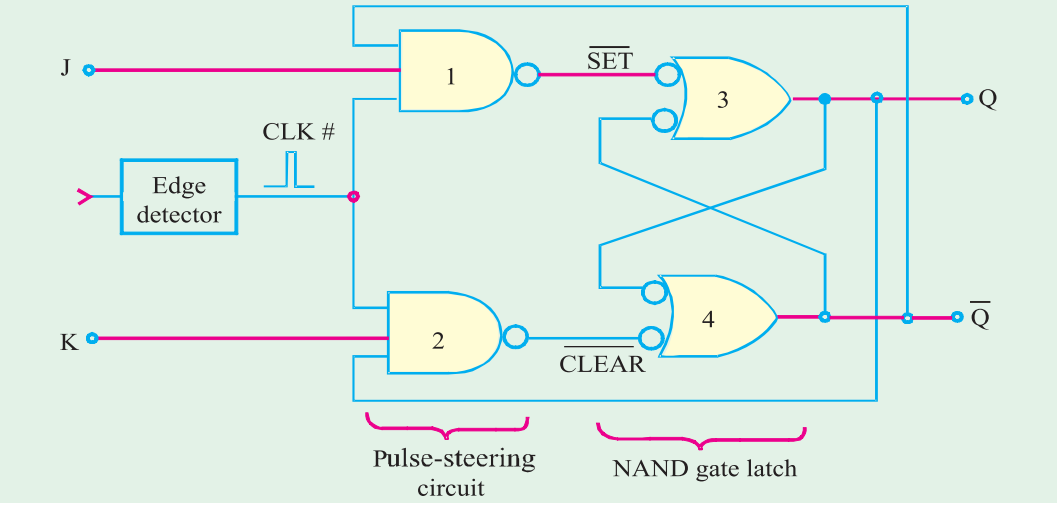


Fig 72.36. A simplified version of an internal circuitry of an edge-triggered *J-K* flip-flop.

It is because of this feedback connection that J-K flip-flop gives the toggle operation for $J = K = 1$ condition.

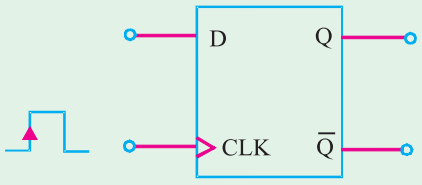
Let us examine the toggle condition in more detail. Assume that $J = K = 1$ and that $Q = 0$ when a CLK pulse occurs. With $Q = 0$ and $\overline{Q} = 1$, NAND gate 1 will steer CLK # (inverted) to $\overline{\text{SET}}$ input of the NAND gate latch to produce $Q = 1$.

If we assume $Q = 1$. When a CLK pulse occurs, NAND gate 2 will steer CLK # (inverted) to the $\overline{\text{CLEAR}}$ input of the latch to produce $Q = 0$. It is evident from the above discussion that Q always ends up in the opposite state.

Note. It may be carefully noted that in order for toggle operation to work, the CLK # pulse must be very narrow. It must return to 0 before the Q and $\overline{Q}$ outputs toggle to their new states. Otherwise the new states of Q and $\overline{Q}$ will cause the CLK # pulse to toggle the latch outputs again.

72.15. Clocked *D* Flip-Flop

Fig. 72.37 (*a*) shows the symbol and 72.37 (*b*) the truth table for a clocked *D* flip-flop that triggers on the rising edge of the clock pulse. Notice that this flip-flop has only one synchronous control input, *D* which stands for data.



(*a*) Symbol

Inputs		Output
D	*CLK*	*Q*
0	↑	Q
1	↑	1

(*b*) Truth table

Fig. 72.37. Clocked D flip-flop.

The operation of the clocked *D* flip-flop is very simple. The output *Q* will go to the same that is present on the D input when the rising edge occurs at the CLK. In other words, the level present at D will be stored in the flip-flop at the instant the rising edge occurs.

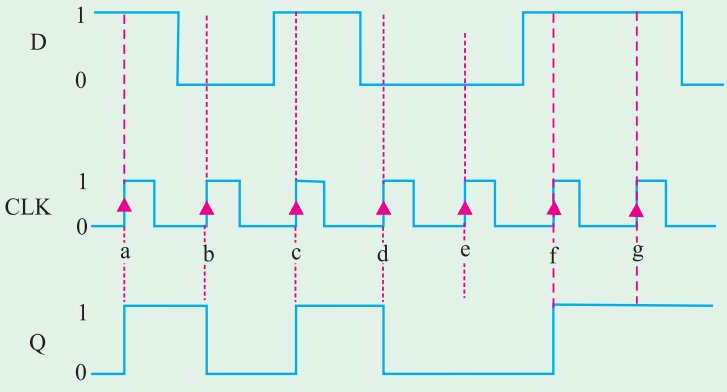


Fig. 72.38. Input and output waveforms to illustrate the operation of clocked *D* flip-flop.

In order to understand the operation of flip-flop in more detail, consider the waveforms at *D* and CLK input as shown in Fig. 72.38. Assume that *Q* is initially 0.

1. When the first rising edge of the CLK pulse occurs (refer to point '*a*'), the *D* input is 1, therefore *Q* will go to 1 state. Even though the *D* input level changes between the points '*a*' and '*b*', it has no effect on *Q*. The output *Q* is storing the 1 that was on *D* at point '*a*'.
2. When the second rising edge of the CLK pulse occurs (refer to point '*b*'), *Q* goes to 0 state since *D* is 0 at that time. The output *Q* stores this 0 value until the rising edge of the third CLK pulse (at point '*c*') causes *Q* to go to 1 since *D* is 1 at that time.
3. In a similar manner, the *Q* output takes on the levels present at *D* when the rising edges occur at points '*d*', '*e*', '*f*' and '*g*'. Notice that *Q* stays 0 at point '*e*' because *D* is still 0.

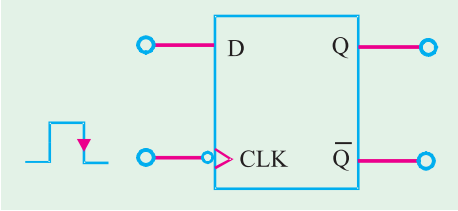


Fig. 72.39. Symbol for a falling-edge triggered *D* flip-flop.

A falling-edge triggered *D* flip-flop operates in the same way as the rising edge triggered *D* flip-flop. However, the difference is that *Q* will take on the value of *D* when a falling edge occurs at the CLK.

A falling-edge triggered *D* flip-flop operates in the same manner as the *D* flip-flop discussed above except that *Q* will take on the value of *D* when a falling edge occurs at the CLK input. The symbol for D flip-flop that triggers on the falling edge has a bubble on the CLK input as shown in Fig. 72.39.

IC 7474 is an example of clocked *D* flip-flop. It contains two rising edge triggered D flip-flops. The pin configuration and some other specification can be found in the data sheet of the device.

72.16. Implementation of *D* Flip-Flop from a *J-K* Flip-Flop

An edge-triggered *D* flip-flop can be obtained easily by adding a single INVERTER to an edge-triggered *J-K* flip-flop as shown in Fig. 72.40. A similar approach can be used to convert a *S-C* flip-flop to a *D* flip-flop.

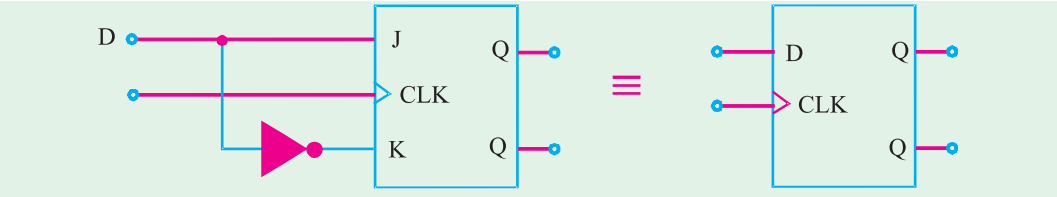


Fig. 72.40. Implementation of *D* flip-flop from a *J-K* flip-flop.

Example 72.8. *Fig* 72.41 *shows a D flip-flop with the input and clock waveforms applied at their respective inputs. Determine the Q* (*or output*) *waveform.*

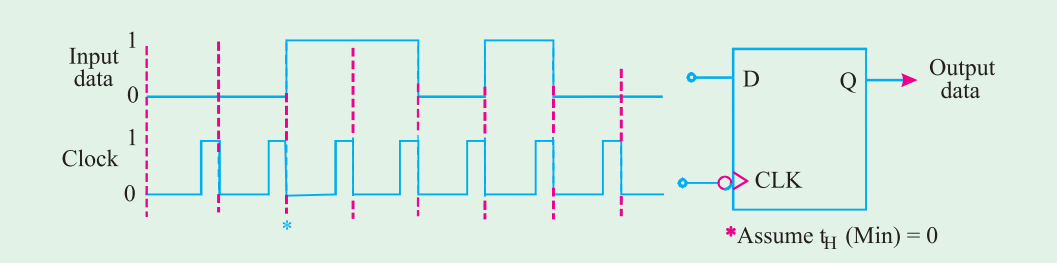


Fig. 72.41

Solution :

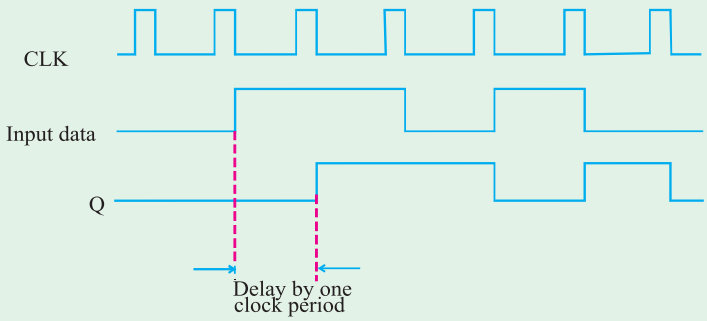


Fig. 72.42

Note. The output (Q) is delayed from the input by one clock period. This is an advantage as a D-Flip-flop is used sometimes to delay a binary waveform so that the binary information appears at the output a certain amount of time after it appears at the D input.

It is also possible to delay the input by two clock periods. This can be achieved by connecting Q to the D input of a second flip-flop and connect the clock signal to the second flip-flop. The output of the second flip-flop will be delayed by 2 clock periods from the input data.

Example 72.9. *An edge-triggered D Flip-flop can be made to operate in the toggle mode by connecting it as shown in Fig.* 72.43. *Assume* $Q = 0$ *initially and determine the* Q (*output*) *waveform.*

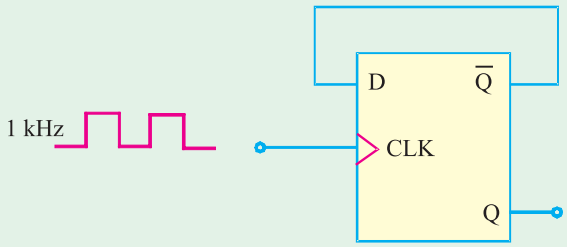


Fig. 72.43

Solution : The clock is 1 kHz waveform. The output waveform can be obtained from the input as shown in Fig. 72.44.

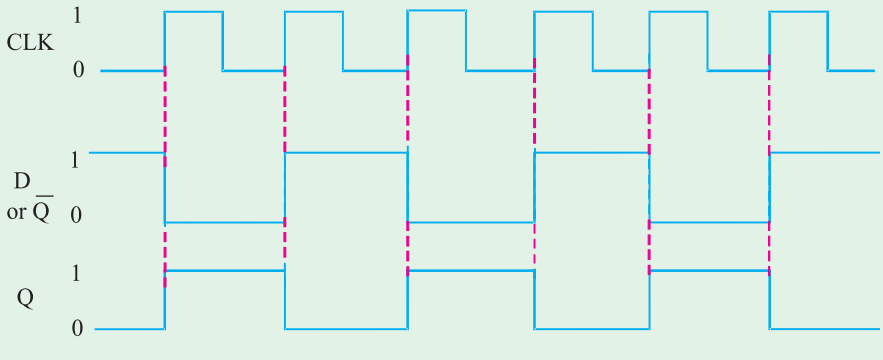


Fig. 72.44

Notice that the output is a square wave of half-the frequency of input *i.e.* 7200 kHz.

Note. A *D*- flip-flop with $\overline{Q}$ tied to its *D*–input as shown in Fig 72.43 is also known as *T* flip-flop where *T* stands for Toggle (or trigger).

72.17. Parallel Transfer of Data Using *D*-Flip-Flops

We have already discussed in Art 72.15 that the *Q*-output of a *D* flip-flop is the same as the *D* input. Let us now study the usefulness of this flip-flop.

Fig. 72.45 shows an application of *D* flip-flop used for parallel transfer of binary data from *X*, *Y*, *Z* – the three outputs of a combinational logic circuit to the outputs Q_1, Q_2, and Q_3 of the *D* flip-flops for storage. The transfer occurs upon application of TRANSFER pulse to the common CLK inputs. The flip-flops can store these values for subsequent processing.

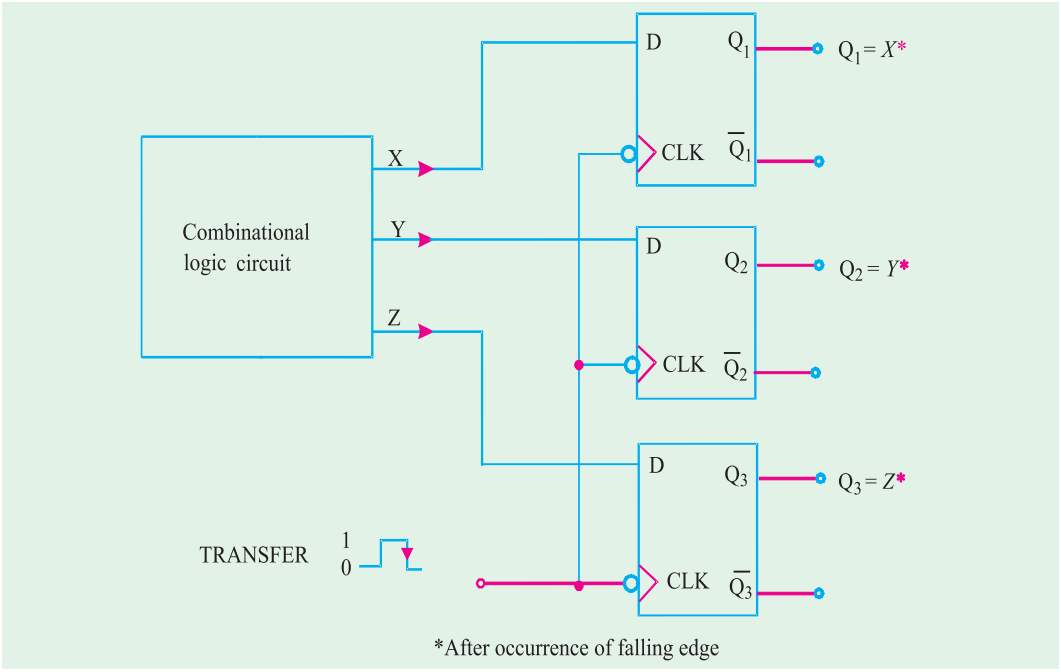
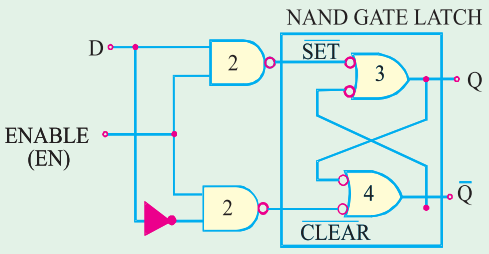


Fig. 72.45. Illustrating parallel transfer of data using *D* flip-flops.

72.18. *D* Latch (Transparent Latch)

We have already discussed in Art. 72.15 about an edge – triggered *D* flip-flop. Such a flip-flop uses an edge-detector circuit to ensure that the output will respond to the *D* input only when the active transition of the clock occurs. If this edge detector circuit is not used, the resultant circuit operates somewhat differently. It is called a *D* latch and has the arrangement shown in Fig 72.46 (*a*).



(*a*) Circuit arrangement

Inputs *EN*	*D*	*Output* *Q*
0	X	Q_0 (no change)
1	0	0
1	1	1

(*b*) Truth table

Fig. 72.46. D latch.

As seen from Fig. 72.46. (*a*), the circuit contains a NAND gate latch, and the steering NAND gates without the edge-detector circuit. The common input to the steering gates is called an enable input (abbreviated as EN) rather than a clock input because its effect on Q and $\overline{Q}$ outputs is not restricted to occurring only on its transitions. The operation of the *D* latch may be explained as follows:

1. When EN is LOW, whatever be the value of *D* input, it is inhibited from affecting the NAND gate latch. It is due to the reason that the outputs of both the steering gates will be held HIGH. Thus the Q and $\overline{Q}$ outputs will stay at whatever level they had just before EN went LOW. In other words, the outputs are "latched" to their current level and cannot change while EN is LOW even if *D* changes. This is represented as Q_0 in the truth table. Refer to Fig. 72.46 (*b*).

2. When EN is HIGH, the *D* input will produce a LOW at either $\overline{\text{SET}}$ or the $\overline{\text{CLEAR}}$ inputs of the NAND gate latch to cause Q to become the same level as *D*. If *D* changes while EN is HIGH, Q will follow the change exactly. In other words, while EN = 1, the Q output will look exactly like *D*. Because of this reason, the *D* latch in this mode is called "transparent".

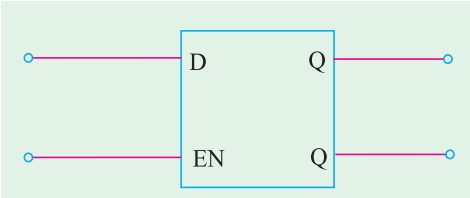


Fig. 72.47. Logic symbol for a D-latchs.

The operation is summarized in the truth table shown in Fig. 72.46 (*b*). The logic symbol for the *D*-latch is shown in Fig. 72.47. It may be noted that even though the EN input operates in the same way as CLK input of an edge-triggered flip-flop, there is no small triangle on the EN input. This is because the small triangle symbol is used strictly for inputs that can cause an output change only when an edge occurs. So remember the *D* latch is not an edge-triggered device.

IC 7475 is an example of *D*-latch. It contains four transparent *D*-latches.

Example 72.10. *Fig.* 72.48 *shows the waveforms applied at the D and EN inputs of a D latch. Sketch the output waveform at Q-output Assume initiually* $Q = 0$

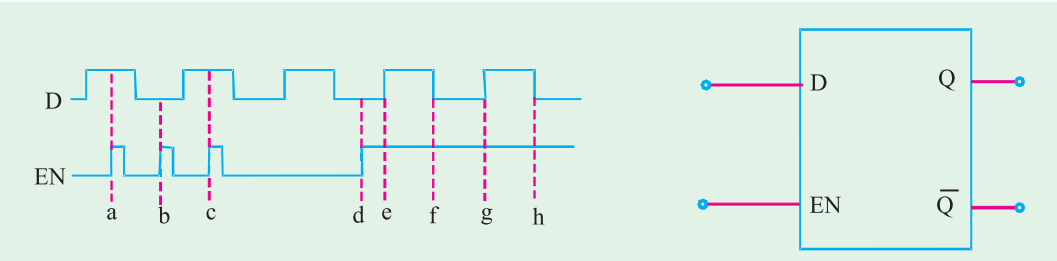


Fig. 72.48

Solution.

Using the truth table of *D*-latch shown in Fig. 72.46, and the input waveforms, the output waveform at Q can be obtained and is sketched as shown in Fig. 72.49. Notice that at point '*a*', EN goes 1, and *D* is 1, therefore a output goes 1. At point '*b*' EN is 1, but $D = 0$, therefore Q goes 0, At point '*c*', EN is 1, therefore Q goes 1 and so on.

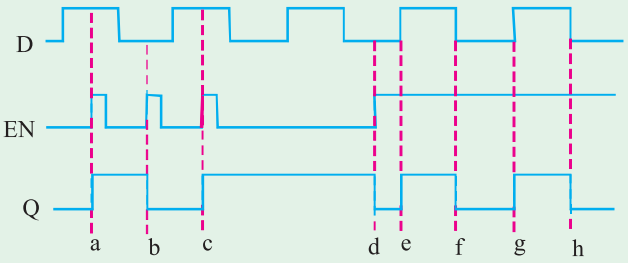
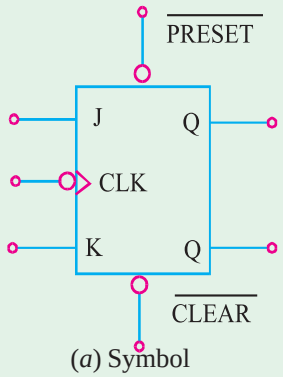


Fig. 72.49

72.19. Clocked *J-K* Flip-Flop with Asynchronous Inputs

Strictly speaking for the clocked flip-flops, the *S, C, J, K* and *D* inputs discussed in the previous articles are refered to as ***control inputs.*** These inputs are also known as ***synchronous inputs.*** These are called synchronous because their effect on the flip-flop output is synchronized with the CLK (*i.e.* clock) input. As discussed earlier, the synchronous control inputs must be used in conjunction with a signal to trigger the flip-flop.

As a matter of fact, most clocked flip-flops also have one-or more ***asynchronous*** inputs. These inputs operate independently of the synchronous inputs and clock input. The asynchronous inputs of a flip-flop can be used to set its output to 1 state or clear to the 0 state at any time regardless of the conditions at the other inputs. In other words, the asynchronous inputs can be used to ***override*** all the other inputs in order to place the flip-flop output in one state or the other. Because of this reason, the asynchronous inputs are also called ***override inputs.***



(*a*) Symbol

Inputs		*Output / Flip-flop response*
$\overline{PRESET}$	$\overline{CLEAR}$	
1	1	Clocked operation*
0	1	Q = 1 (regardless of CLK)
1	0	Q = 0 (regardless of CLK)
0	0	Not used

(*b*) Truth table

Fig . 72.50

Fig. 72.50 shows a clocked J-K flip-flop with two asynchronous inputs namely (1) $\overline{\text{PRESET}}$ and (2) $\overline{\text{CLEAR}}$. Both these inputs are active-LOW. This is indicated by the bubbles on the flip-flop symbol. Fig 72.50 (*b*) shows the truth table for the clocked *J-K* flip-flop with asynchronous inputs. Let us study the various cases given in the truth table :

1. **$\overline{\text{PRESET}}$ = $\overline{\text{CLEAR}}$ = 1.** This condition indicates that both the asynchronous inputs are inactive and the flip-flop is free to respond to the *J, K* and CLK inputs. In other words, the flip-flop operates as a normal clocked flip-flop.
2. **$\overline{\text{PRESET}}$ = 0, $\overline{\text{CLEAR}}$ = 1.** This condition indicates that the asynchronous input, $\overline{\text{PRESET}}$ is activated. Because of this, the output, *Q* is immediately set to 1, no matter what conditions are present at the *J, K* and CLK inputs. It may be carefully noted that the CLK input cannot affect the flip-flop while $\overline{\text{PRESET}}$ = 0.

3. **$\overline{\text{PRESET}}$ = 1, $\overline{\text{CLEAR}}$ = 0.** This condition indicates that the asynchronous input, $\overline{\text{CLEAR}}$ is activated. Because of this, the output, Q is immediately set to 0, no matter what conditions are present at the J, K and CLK inputs. It may be carefully noted that the CLK input cannot affect the flip-flop while $\overline{\text{CLEAR}}$ = 0.
4. **$\overline{\text{PRESET}}$ = $\overline{\text{CLEAR}}$ = 0.** This condition should not be used because it can produce an ambiguous response.

It will be interesting to know that the $\overline{\text{PRESET}}$ and $\overline{\text{CLEAR}}$ inputs respond to dc levels. This means that if a constant 0 is held on the $\overline{\text{PRESET}}$ input, the flip-flop will remain in the Q = 1 state regardless of what is occurring at the other inputs. Similarly, a constant 0 at the $\overline{\text{CLEAR}}$ input holds the flip-flop in Q = 0 state. Thus the asynchronous inputs can be used to hold the flip-flop in a particular state for any desired interval of time. However, in actual practice, the asynchronous inputs are used to set or clear the flip-flop to the desired state by application of a momentary pulse.

Many clocked flip-flops that are commercially available as ICs, will have both $\overline{\text{PRESET}}$ and $\overline{\text{CLEAR}}$. However some ICs will have only $\overline{\text{CLEAR}}$ input. Some other ICs will have asynchronous inputs that are active-HIGH. For these ICs, the flip-flop symbol would not have a bubble on the asynchronous inputs.

ICs 7476 and 74LS76 are popular *J-K* flip-flops with asynchronous inputs preset and clear. The preset is designated by $\overline{S}_D$ and clear by $\overline{R}_D$. The IC 7476 is a rising edge triggered while 74LS76 is a falling edge triggered device. Each package contains two *J-K* flip-flops.

Example 72.11. Fig. 72.51 (*a*) *shows the logic symbol for a J-K flip-flop that responds to the falling edge on its clock pulse and has active-LOW asynchronous inputs. The J and K inputs are tied HIGH.*

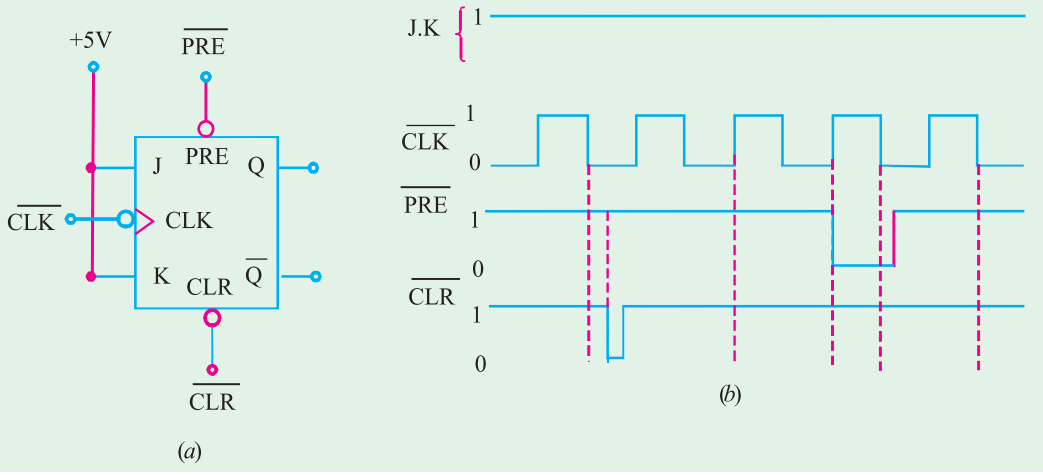


Fig. 72.51

Determine the Q-output in response to the waveforms shown in Fig. 72.51 (*b*). *Assume that initially* Q = 0.

Solution. Notice that the *J* and *K* are tied to +5V. The waveforms shown in Fig. 72.51 (*b*) are applied at the inputs of the *J-K* flip flop.

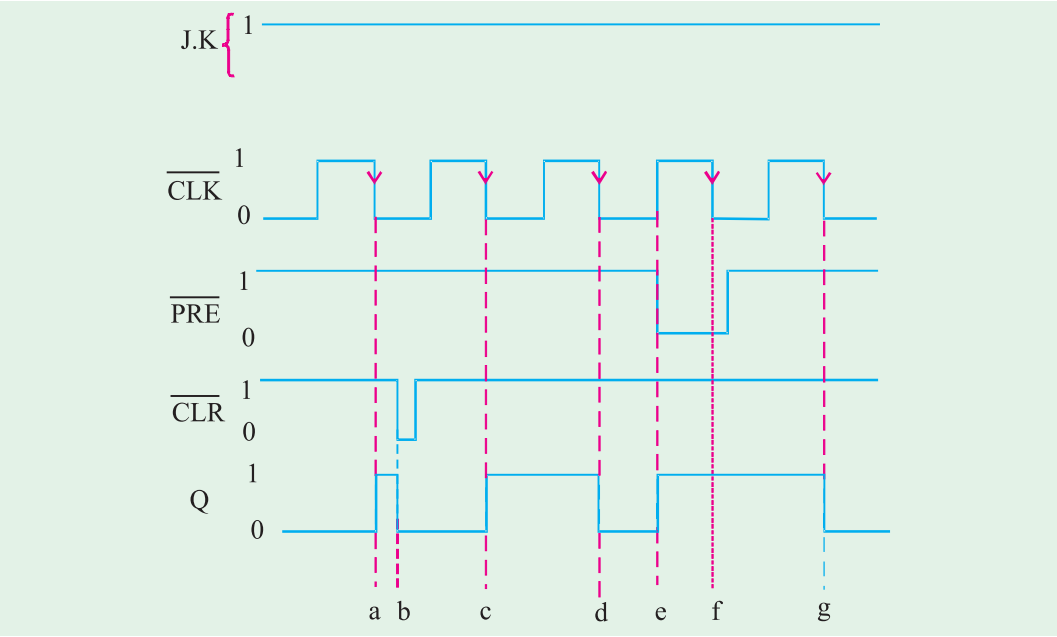


Fig. 72.52

In order to sketch the *Q*-output waveform, recall the truth table of *J-K* flip-flop indicated in Fig. 72.31 (*b*) and Fig. 72.50 (*b*). Notice that when $\overline{PRE}=1$ and $\overline{CLR}=1$, the *J-K* flip-flop will trigger at the falling edge of the clock pulse. Thus at point '*a*', $\overline{PRE}=\overline{CLR}=1$, $J=K=1$, therefore the *Q*-output toggles *i.e.* it goes from 0 to 1. At point '*b*' $\overline{CLR}$ goes 0, therefore *Q* goes 0. At point '*c*', $\overline{PRE}=\overline{CLR}=1$, therefore under normal operation *J-K* flip-flop toggles *i.e.* *Q* goes from 0 to 1 again. At point '*d*' there is no change in $\overline{PRE}$ or $\overline{CLR}$ inputs. So the *Q*-output toggles again, *i.e.* *Q* goes 0. At point '*e*' $\overline{PRE}$ goes 0, therefore *Q*-output goes 1. At point '*g*', $\overline{PRE}=\overline{CLR}=1$, and *Q* toggles to 0. The complete *Q*-output waveform is shown in Fig. 72.52 along with the input waveforms.

Example 72.12. *Determine the Q-output for the JK flip-flop shown in Fig.* 72.53, *Assume that* $Q = 0$ *initially and remember that the asynchronous inputs override all other inputs.*

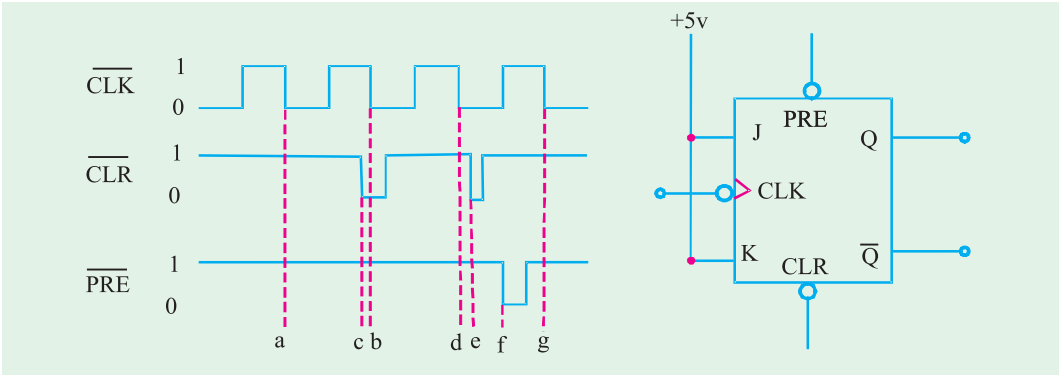


Fig. 72.53

Solution :

The waveform at the *Q*-output can be obtained by recalling the truth-table of a *JK* flip-flop. Shown in Fig. 72.31 (*b*) and 72.50 (*b*).

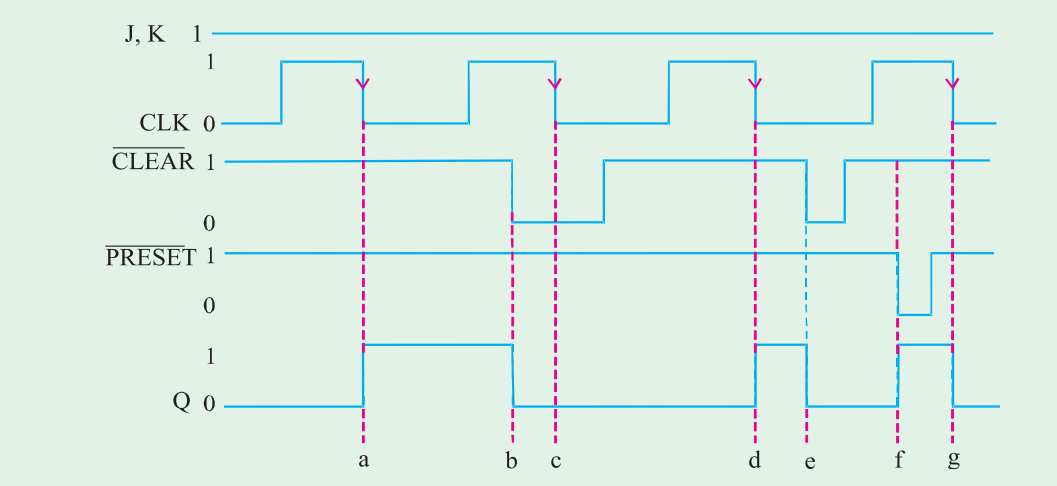


Fig. 72.54

Notice that at point '*a*', $J = K = 1$, $\overline{\text{PRESET}} = \overline{\text{CLEAR}} = 1$, the falling edge of CLK pulse toggles the *J-K* flip-flop *Q* output to 1. At point '*b*' $\overline{\text{CLEAR}}$ goes 0, therefore irrespective of the other inputs, *Q*-output goes 0. At point '*c*' the ouput remains 0 because $\overline{\text{CLEAR}}$ is still 0. At point '*d*', $\overline{\text{PRESET}} = \overline{\text{CLEAR}} = 1$, $J = K = 1$, the *J-K* flip-flop output toggles on the arrival of falling edge of CLK, *i.e. Q* goes 1. At point '*e*', $\overline{\text{CLEAR}}$ goes 0 again, therefore *Q*-output goes 0. At point '*f*', $\overline{\text{PRESET}}$ goes 0, therefore *Q*-output goes 1. Finally at point '*g*', $\overline{\text{CLEAR}} = \overline{\text{PRESET}} = 1$, $J = K = 1$, the output toggles on the arrival of falling edge of CLK pulse, *i.e. Q* goes 0.

The complete sketch of *Q*-output waveform along with other input waveforms is shown in Fig. 72.54.

Example 72.13. *In the circuit of Fig.* 72.55, *inputs A, B and C are all initially LOW. Output Y is supposed to go HIGH only when A, B and C goes HIGH in a certain sequence.*

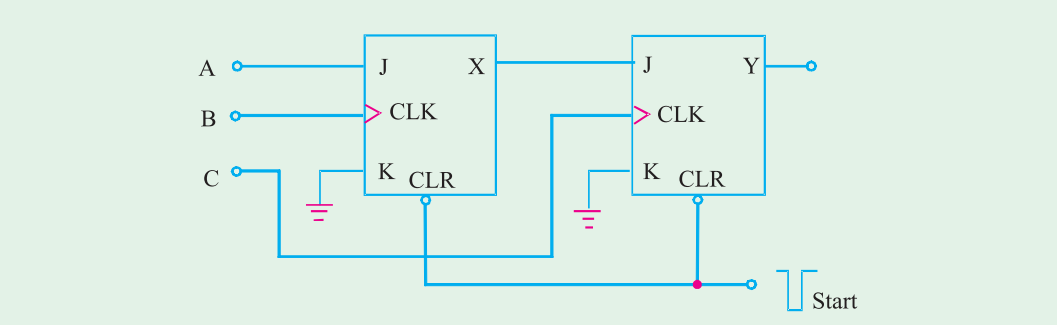


Fig. 72.55

(a) Determine the sequence that will make Y go HIGH.

(b) Explain why the START pulse is needed.

Solution.

(a) Sequence at the inputs (A, B and C)

We know that *Y* can go HIGH only when *C* goes HIGH while *X* is already HIGH.

X can go HIGH only if *B* goes HIGH while *A* is HIGH.

Thus the correct sequence that makes Y go HIGH is A, B, C.

(b) Need for START pulse

We know that the outputs X and Y need to be cleared to 0 before applying the A, B, C signals. To clear the outputs, we need a negative going START pulse at CLR input. (notice that CLR input of JK flip-flop is active LOW).

72.20. Alternative Designations for Asynchronous Inputs

We have already discussed in the last article about the asynchronous clocked *J-K* flip-flop. These flip-flops are available as ICs and have two asynchronous inputs namely $\overline{\text{PRESET}}$ and $\overline{\text{CLEAR}}$ (or PRESET and CLEAR). IC manufacturers have not agreed on what nomenclature is used for these asynchronous inputs. The most common designations are $\overline{\text{PRE}}$ (short for $\overline{\text{PRESET}}$) and $\overline{\text{CLR}}$ (short for $\overline{\text{CLEAR}}$). The designations $\overline{S}_{\text{D}}$ (direct $\overline{\text{SET}}$) and $\overline{R}_{\text{D}}$ (direct $\overline{\text{RESET}}$) are also used. However we will use the labels $\overline{\text{PRE}}$ and $\overline{\text{CLR}}$ to represent asynchronous inputs. Whenever these asynchronous inputs are active–HIGH, we will not use the overbar to indicate their active-HIGH status *i.e.* PRE and CLR.

72.21. Flip-Flop Timing Parameters

The performance, operating requirements and limitations of flip-flops are specified by several timing parameters found on the data sheet for the device refer to Appendix A. Generally the timing parameters are applicable to all CMOS and TTL flip-flops.

1. Setup and Hold Times. These have already been discussed in Art 72.10 (page 2637). They represent requirements that must be met for reliable flip-flop triggering. The manufacturer's IC data sheet will always specify the minimum values of t_S and t_H. For example, IC 7474 has set up time of 20 ns and hold time of 5 ns. On the other hand 74HC112 has set up time of 25ns and zero hold time.

2. Propagation delay. It is the interval of time required after an input signal has been applied for the resulting output change to occur. Fig. 72.56 (*a*) illustrates the propagation delays that occur in response to a rising edge of the CLK input when *Q*-output changes from 0 to 1. Similarly Fig. 72.56 (*b*) illustrates the propagation delay that occur in response to the rising edge of the CLK input when *Q*-output changes from 1 to 0. It may be noted that these delays are measured between the 50% points (between logic 0 and logic 1 voltage levels) on the input and output waveforms. The same type of delays occur in response to the signals on a flip-flops asynchronous inputs (*i.e.* PRESET and CLEAR). The manufacturer's data sheets usually specify propagation delays in response to all inputs and they usually specify the maximum values for t_{PLH} and t_{PHL}. Notice that t_{PLH} is the delay going from logic LOW to HIGH level whereas t_{PHL} is the delay going from logic HIGH to LOW level.

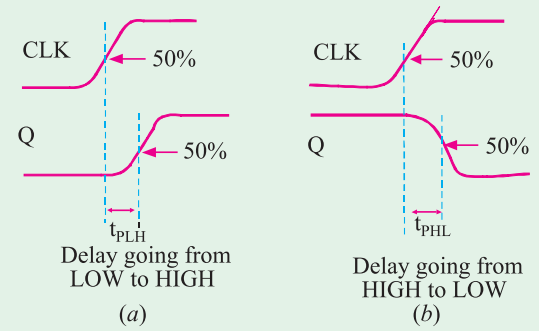


Fig. 72.56

3. Maximum Clock Frequency (f_{max}). This is the highest frequency that may be applied to the CLK input of a flip-flop and still have it triggered reliably. The value of f_{max} limit will vary from flip-flop to flip-flop even with flip-flops having the same device number. For example, f_{max} for IC7474 is 15 MHz, for 74LS112, it is 30 MHz, for 74C74, it is 5 MHz and for 74HC112, it is 20 MHz.

4. Clock Pulse HIGH and LOW Times. The manufacturer will also specify the minimum time duration that the CLK signal must remain LOW before it goes HIGH {represented as $t_w(L)$} and the minimum time that CLK must be kept HIGH before it returns LOW (represented as t_w (H)}. These times are illustrated in Fig. 72.57 (*a*) Failure to meet these minimum time requirements can result in unreliable triggering. It may be noted that these time values are measured between the halfway (50%) points on the signal transitions.



(*a*) Clock Pulse HIGH-LOW times (*b*) Asynchronous active pulse width

Fig. 72.57

5. Asynchronous Active Pulse Width. It is the minimum time duration that a PRESET or CLEAR input must be kept in its active state in order to set or clear the flip-flop reliably. Fig. 72.57 (b) shows $t_w(L)$ for active-LOW asynchronous inputs.

6. Clock Transition Times. Strictly speaking, the rise and fall times of a clock waveform should be kept very short for reliable triggering. If the clock signal takes too long to change from one logic level to the other, the flip-flop may trigger erratically or not at all. IC manufacturers usually do not list a maximum transition time requirement for each flip-flop integrated circuit. Instead, it is usually given as a general requirement for all ICs within a given logic family. For example, the transition times should generally be ≤ 50 ns for TTL devices and ≤ 200 ns for CMOS. These requirements will vary among the different manufacturers and among the various subfamilies within the broad TTL and CMOS logic-families.

72.22. IC Flip-Flop Timing Values

We have already discussed the various flip-flop timing parameters in the last article. Table 72.1. lists all these timing values for the various flip-flops such as 7474, 74 LS112, 74C74 and 74 HC112. Notice that 7474 is a dual edge triggered *D* flip-flop (Standard TTL) device whereas 74LS74 is a dual edge-triggered *J-K* flip-flop (low-power schottky TTL) device. Similarly 74C74 is a dual edge-triggered D flip-flop (metal-gate CMOS) device whereas 74HC112 is a dual edge-triggered *J-K* flip-flop (high-speed CMOS) device.

Table 72.1. IC flip-flop timing values

Parameter	*TTL*		*CMOS*	
(Times in ns)	*7474*	*74LS112*	*74C74*	*74HC112*
t_S	20	20	60	25
t_H	5	0	0	0
t_{PHL} from *CLK* to *Q*	40	24	200	31
t_{PLH} from *CLK* to *Q*	25	16	200	31
t_{PHL} from $\overline{CLR}$ to *Q*	40	24	225	41
t_{PLH} from $\overline{PRE}$ to *Q*	25	16	225	41
t_W(L) *CLK* LOW time	37	15	100	25
t_W(H) *CLK* HIGH time	30	20	100	25
t_W(L) at $\overline{PRE}$ or $\overline{CLR}$	30	15	60	25
f_{Max} in MHz	15	30	5	20

Following are some of the important points which may be noted carefully from Table 72.1.

1. The 74HC series of CMOS devices has timing values that are comparable to those of TTL devices. This can be observed by reading the values below the 7474 column and the values below 74HC112 column.
2. The 74C series devices are much slower than the 74HC series. This can be observed by comparing all the minimum timing requirements below the 74C74 and 74HC112 column.
3. All the flip-flops have nonzero set up time requirement.
4. All of the flip-flops have very low hold-time (t_H) requirements *e.g.* The IC7474 has t_H = 5 ns whereas IC74LS112, 74C74 and 74HC112 has t_H = 0.

Example 72.14. *The data sheet of a certain flip-flop specifies that the minimum HIGH time $t_W(H)$ for the clock pulse is* 30 *ns and the minimum LOW time $t_W(L)$ is 37 ns. What is the maximum operating frequency*?

Solution. Given : $t_W(H)$ = 30 ns = 30 × 10^{-6} s and $t_W(L)$ = 37 ns = 37 × 10^{-6} s.

We know that the minimum time period,

$$t_{min} = t_w(H) + t_w(L)$$
$$= \left(30\times10^{-6} + 37\times10^{-6}\right) = 47\times10^{-6}\text{ s}$$

and the maximum operating frequency,

$$f_{max} = \frac{1}{t_{min}} = \frac{1}{47\times10^{-6}_{ns}} = 21.276\times10^{6}\ Hz$$

= 21.276 MHz.

Example 72.15. *Using the flip-flop timing values shown in Table* 72.2, *determine the followings* :

Table 72.2.

Parameter	*TTL*		*CMOS*	
(time in ns)	*7474*	*74LS112*	*74C74*	*74HC112*
t_{PHL} from *CLK* to *Q*	40	24	200	31
t_{PLH} from *CLK* to *Q*	25	16	200	31
t_W (L) at $\overline{PRE}$ or $\overline{CLR}$	30	15	60	25

(a) *Assume that initially Q = 0. How long it can take for Q to go HIGH when a rising edge appears at the CLK input of* IC7474?

(b) *Assume that initially Q = 1. How long it can take for Q to go LOW in response to the* $\overline{\text{CLR}}$ *input of an* IC 74HC112?

(c) *What is the narrowest pulse that should be applied to the* $\overline{\text{CLR}}$ *input of the* IC 74LS112 *flip-flop to clear Q reliably?*

Solution.

(a) The rising edge will cause the *Q* output to go from LOW to HIGH. The delay from CLK to *Q* is listed as t_{PLH} = 25 ns for IC7474.

(b) For the IC 74HC112, the time required for *Q* to go from HIGH to LOW in response to $\overline{\text{CLR}}$ input is listed as t_{PHL} = 31 ns.

(c) For the IC 74HC112, the narrowest pulse at the $\overline{\text{CLR}}$ input is listed as $t_w(L)$ = *15* ns.

72.23. Potential Timing Problems in Flip-Flop Circuits

As a matter of fact, in many digital circuits, the output of one flip-flop is connected either directly or through logic gates to the input of another flip-flop and both flip-flops are triggered by the same clock signal. This situation leads to a potential timing problem. In order to understand this problem, consider a situation shown in Fig. 72.58. Here the output of flip-flop (1) Q_1 is connected to the *J* input of flip-flop (2). Notice that both the flip-flops are clocked by the same signal at their CLK inputs.

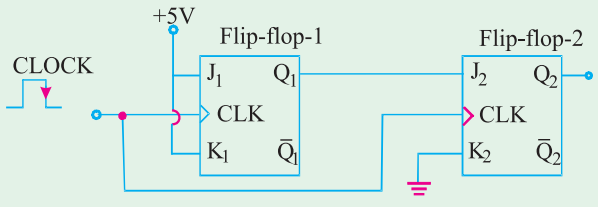


Fig. 72.58

Let us understand the potential timing problem now. Since Q_1 will change on the rising edge of the clock pulse, the J_2 input will be changing as it receives the same rising edge. This could lead to an unpredictable response at Q_2.

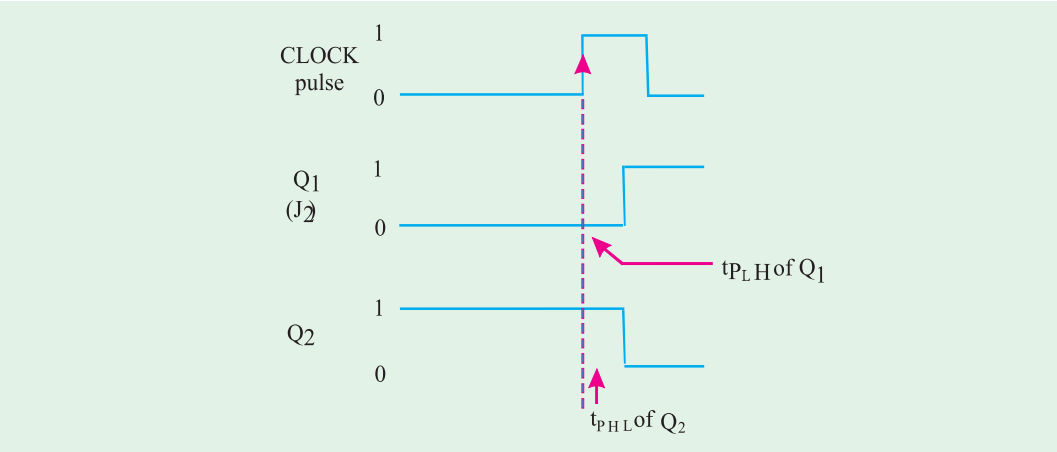


Fig. 72.59

Assume that initially $Q_1 = 0$ and $Q_2 = 1$. Further the flip-flop (1) has $J_1 = K_1 = +5V$ (*i.e* HIGH) and flip-flop (2) has $J_2 = Q_1$ and $K_2 = 0$ prior to the occurrence of rising edge of the clock pulse. When the rising edge occurs, Q_1 will toggle to the HIGH state but it will not actually go HIGH until after its propagation delay, t_{PLH} as shown in Fig. 72.59. The same rising edge will reliably clock Q_2 to the LOW state provided that t_{PLH} is greater than Q_2's hold time requirement, t_H as shown in the figure. If this condition is not met, the response of Q_2 will be unpredictable.

Since all edge-triggered flip-flops have hold time (t_H) requirement that is 5 ns or less. Therefore for those flip-flops, situations like that shown in Fig. 72.59 will not be a problem.

Thus in all of the flip-flop circuits that you will study in this book, we will assume that flip-flop's hold time requirement is short enough to respond reliably according to the following rule:

The flip-flop output will go to a state determined by the logic levels present at its synchronous control inputs just prior to the active clock edge.

Applying the above stated rule to the circuit shown in Fig. 72.59, it says that the output of flip-flop (2), Q_2 will go to a state determined by $J_2 = 1$ and $K_2 = 0$ condition that is present just prior to the occurrence of the rising edge of the clock pulse. The fact that J_2 is changing in response to the same rising edge has no effect.

72.24. Applications of Flip-Flop

Although there is a wide variety of applications of edge-triggered (clocked) flip-flops, yet the following are important from the subject point of view :

1. Flip-flop synchronization
2. Data storage and transfer
3. Serial data transfer-shift registers
4. Frequency division
5. Counting

All these applications are discussed one by one in the following pages.

72.25. Flip-Flop Synchronization

Strictly speaking, most digital systems are principally synchronous in their operation. It means, most of the signals will change states in synchronism with the clock transitions. However, in many cases, there will be an external signal that is not synchronised to the clock. In other words such an external signal is an asynchronous signal. Such signals often occur as a result of a human operator actuating an input switch at some random time relative to the clock signal. This randomness can

produce unpredictable and undesirable results. Now we will study how a flip-flop can be used to synchronize the effect of an asynchronous input.

Fig. 72.60 shows a situation where a signal *X* is generated from a debounced switch that is actuated by an operator. The switch output A goes high when the operator actuates the switch and goes LOW when the operator releases the switch. The switch output *X* is used as an input to control the passage of the control signal through the AND gate so that clock pulses appear at output *Y* only as long as *X* is HIGH.

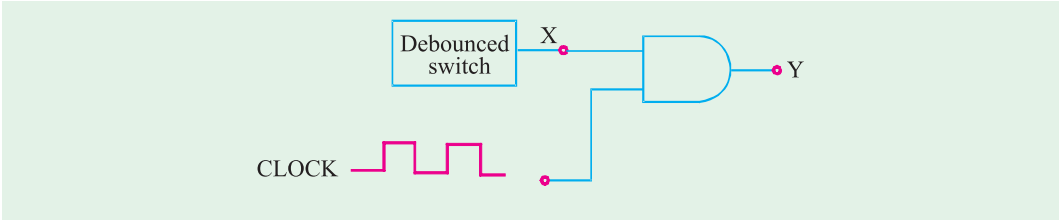


Fig. 72.60

The problem with the circuit shown in Fig. 72.60 is that signal *X* is asynchronous. That is it can change states at any time relative to the clock signal because the exact times when the operator actuates or releases the switch are essentially ***random.*** This can produce partial clock pulses at output *Y* if either transition of *X* occurs while the clock signal is HIGH. It is shown in Fig. 72.61. This type of output is often not acceptable. So a method for preventing the appearance of partial pulses at *Y* must be developed. One possible solution is shown in Fig. 72.61.

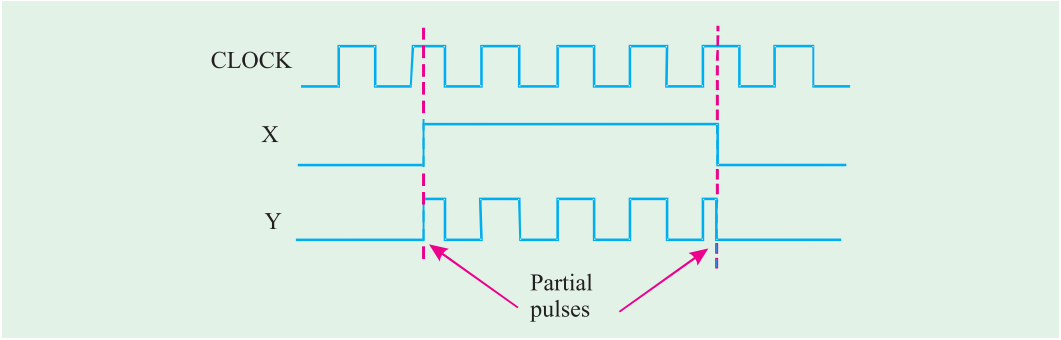


Fig. 72.61

As seen in Fig. 72.62, the *X* signal is connected to the *D* input of a flip-flop. The flip-flop is clocked by the falling edge of the clock signal. Thus when *X* goes HIGH, *Q* will not go HIGH until the next falling edge of the clock at time t_1. This HIGH at *Q* will enable the AND gate to pass subsequent complete clock pulses to *Y* as shown in Fig. 72.61.

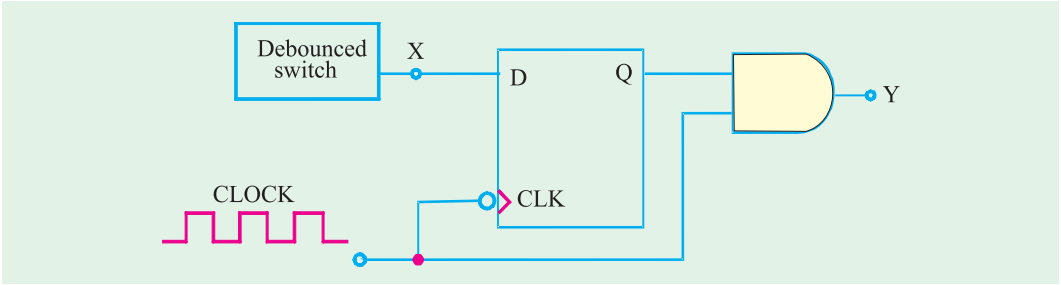


Fig. 72.62

Similarly, when *X* goes LOW, *Q* will not go LOW until the next falling edge of the clock at t_2. Thus the AND gate will not inhibit clock pulses until clock pulse that ends at t_2 has passed through to *Y*. Therefore, output *Y* contains only complete pulses (see Fig. 72.63).

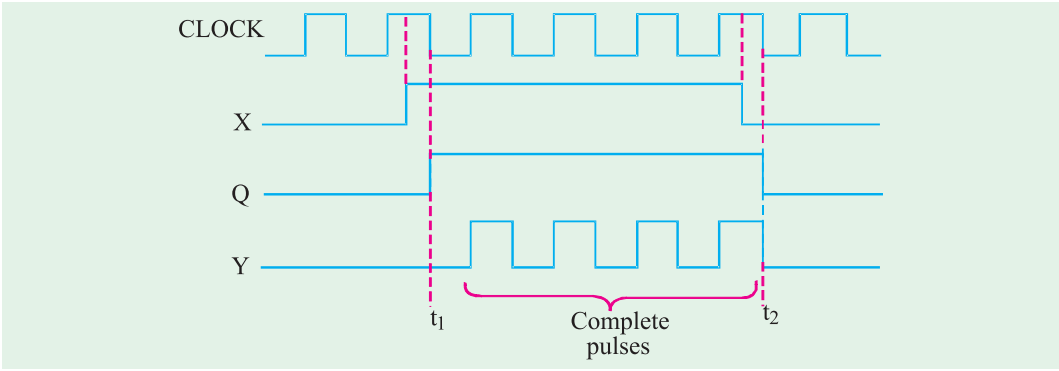


Fig. 72.63

72.26. Data Storage and Transfer

The most common use of flip-flops in the field of digital electronics, is for the storage of data or information. The data may represent numerical values (*e.g.* binary numbers, BCD -binary coded decimal numbers or any data that have been encoded in binary. These data are generally stored in groups of flip-flops called ***registers.*** The operation most often performed on data that are stored in a flip-flop or a register is the ***data transfer*** operation. This operation involves the transfer of data from one flip-flop (or register) to another. The data transfer is of two types: **(*a*)** synchronous and **(*b*)** asynchronous.

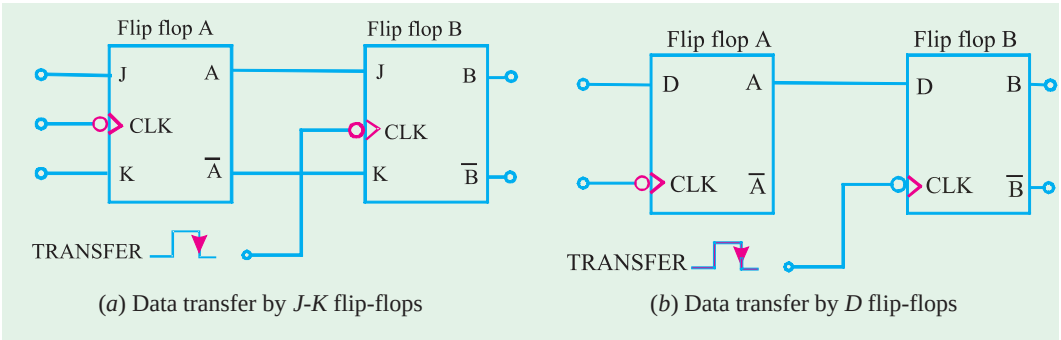


Fig. 72.64

Fig. 72.64 (*a*) and (*b*) shows how synchronous data transfer operation can be accomplished between two flip-flops using *J-K* and *D* flip-flops respectively. In each case, the logic value that is currently stored in flip-flop *A* is transferred to flip-flop *B* upon the falling edge of the TRANSFER pulse. Thus after this falling edge, the *B* output will be the same as the *A* output.

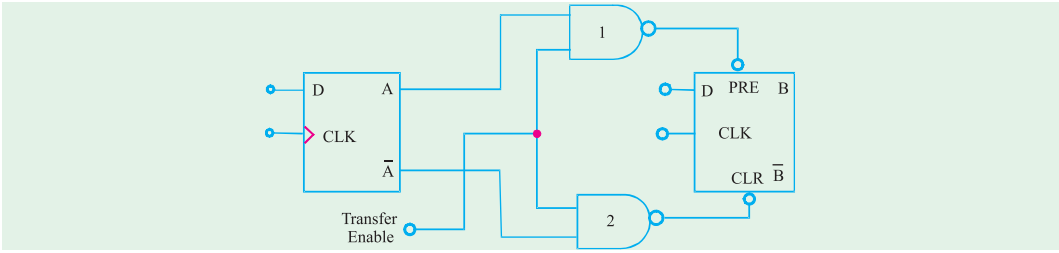


Fig. 72.65. Illustrating asynchronous data transfer operation.

The data transfer illustrated in Fig. 72.64 is called synchronous transfer because the synchronous control and CLK inputs of a flip-flop are used to perform the transfer operation. Fig. 72.64 shows the data transfer that, can be obtained using the asynchronous inputs of a flip-flop. This method called asynchronous transfer can be accomplished using the PRESET and CLEAR inputs of any flip-flop. Notice that flip-flop outputs A and $\overline{A}$ are connected to PRESET and CLEAR through two-input NAND gates 1 and 2 respectively.

In this method, the asynchronous inputs (*i.e.* PRESET and CLEAR) respond to LOW levels. The operation of the circuit may be explained as follows :

When the ***Transfer Enable*** line is held LOW, the two NAND outputs are kept HIGH, with no effect on the flip-flop outputs. But when the ***Transfer Enable*** line is made HIGH, one of the NAND outputs will go LOW depending on the state of A and $\overline{A}$ outputs. This LOW will either set or clear the flip-flop B.

72.27. Parallel Data Transfer

We have already discussed in the last article that flip-flops can be used for data storage as well as for transfer of data from one flip-flop to another. This idea can be further extended for transfer of data from one group of flip-flops (called register A) to another group of flip-flops (called register B). Register A consists of four flip-flops A_1, A_2, A_3 and A_4. Similarly register B also consists of four flip-flops labelled as B_1, B_2, B_3 and B_4 upon application of falling edge of a TRANSFER pulse, the logic level stored in A_1 is transferred to B_1, A_2 is transferred to B_2, A_3 is transferred to B_3 and A_4 is transferred to B_4. It may be noted that the transfer of contents from register A to B occurs at the same time. Because of this reason it is called synchronous transfer or parallel data transfer.

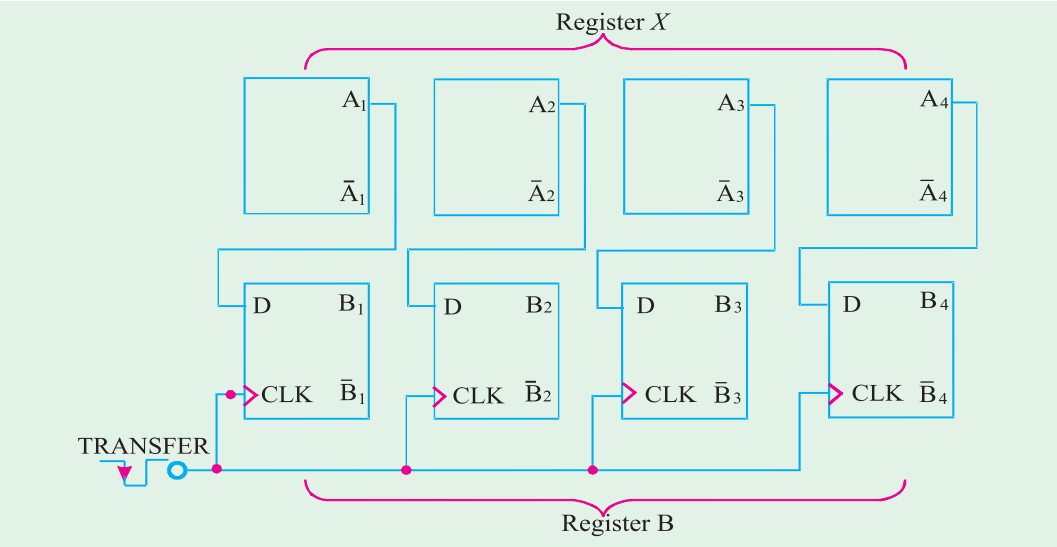


Fig. 72.66. Illustrating parallel data transfer from register X into register Y

It may be carefully noted that parallel data transfer does not change the contents of the register that is the source of data. For example in Fig. 72-66 if $A_1\,A_2\,A_3\,A_4 = 1011$ and $B_1\,B_2\,B_3\,B_4 = 0100$ prior to the occurrence of the TRANSFER pulse, then both registers will be holding 1011 after the TRANSFER pulse. Another point is that the group of four flip-flops indicated above is an example of a basic register used for data storage. In digital systems, data are normally stored in groups of bits (usually 8, 16, 32, 64 …) that represent numbers, codes or other information.

Example 72.16. ***(a)*** *Draw a circuit diagram for the synchronous parallel transfer of data from one three-bit register to another using J-K Flip-flops.* ***(b)*** *Repeat for asynchronous parallel transfer.*

Solution. (*a*) Fig. 72.67 shows the circuit diagram for the synchronous parallel transfer of data from one three-bit register X to another register *Y* using *J-K* flip-flops. Notice that the contents of X_1, X_2, and X_3 are transferred simultaneously into Y_1, Y_2, and Y_3. It may be carefully noted that the circuit allows the parallel transfer of data from the normal outputs as well as the complemented outputs of the flip-flops constituting register *X* .

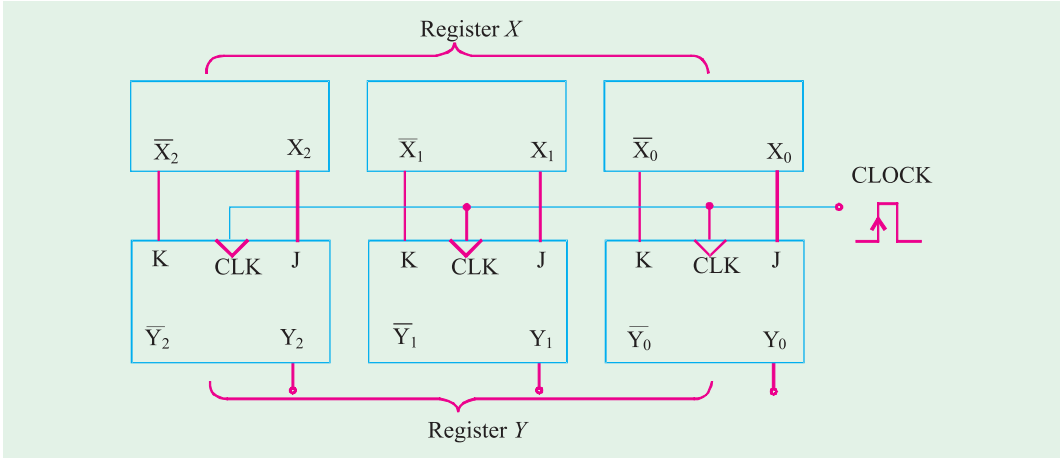


Fig. 72.67. Synchronous parallel transfer of data from one three-bit register to another

(*b*) Fig. 72-68 shows the circuit diagram for the asynchronous parallel data transfer from one three-bit register to another using *J-K* flip-flops. The circuit allows data transfer from either normal outputs X_0, X_1, and X_2 or the complemented outputs $\overline{X}_1$, $\overline{X}_2$ and $\overline{X}_3$.

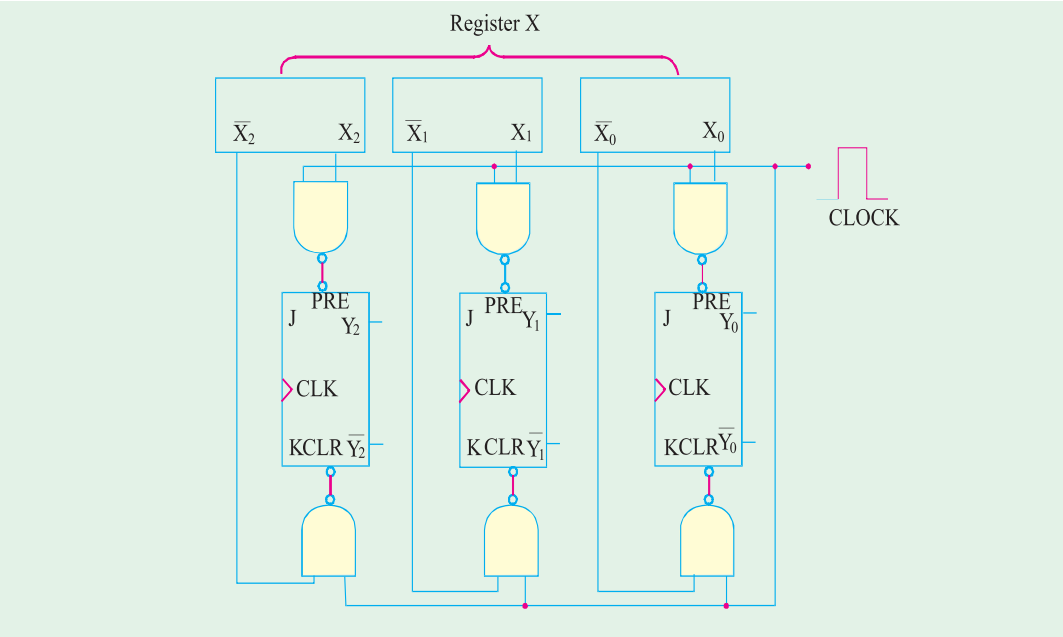


Fig. 72.68. Asynchronous parallel transfer of data from one three-bit register to another.

72.28. Serial Data Transfer : Shift Registers

We have already discussed in the last article that a ***register*** is basically a group of flip-flops to store data. But there is no interconnection between the flip-flops. A ***shift register*** is a group of flip-flops arranged in such a way that the binary numbers stored in the flip-flops are shifted from one flip-flop to the next for every clock pulse. Fig. 72.69 shows a four-bit shift register using *J-K* flip-flops. Notice that the J and K inputs of flip-flops '3' are fed by DATA IN waveform. The *J* and K inputs of flip-flop '2' are fed by the X_3 and $\overline{X}_3$. Similarly *J* and *K* inputs of flip-flop '1' are fed by the X_2 and $\overline{X}_2$ and *J* and *K* inputs of flip-flop '0' are fed by the X_1 and $\overline{X}_1$. The CLK input of all the four flip-flops are connected together to a common input which receives shift pulses. Incidently notice that all the flip-flops trigger on the falling edge of the CLK pulse.

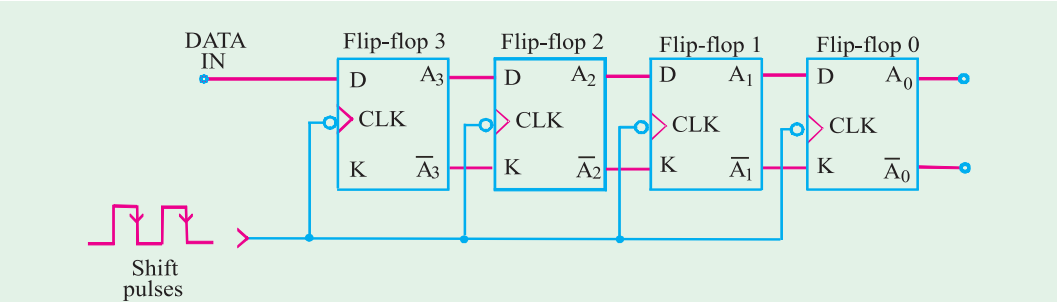


Fig. 72.69. Four-bit shift register.

The operation of the shift register may be understood from the explanation given below. Let us assume that initially all the flip-flops are in the 0 state and input of flip-flop '3' is fed by the DATA IN waveform as shown in Fig. 72.70. The other waveforms shown in Fig. 72.70 indicate how the input data are shifted from left to right from flip-flop to flip-flop as shift pulses are applied. When the first falling edge occurs at t_1, each of the flip-flop outputs A_2, A_1 and A_0 will have D = 0 condition present at its inputs because of the state of the flip-flop on its left. The flip-flop '3' will have D = 1, because of DATA-IN. Thus at t_1 only A_3 will go HIGH while all the other flip-flop outputs remain LOW. When the second falling edge occurs at t_2, flip-flop '3' will have D = 0 because of DATA-IN. The Flip-flop '2' will have D = 1 because of current HIGH at A_3. Flip-flops '1' and '0' will still have D = 0. Thus at t_2, only flip-flop '2' output A_2 will go HIGH, flip-flop output A_3 will go LOW, and A_2 and A_0 will remain LOW.

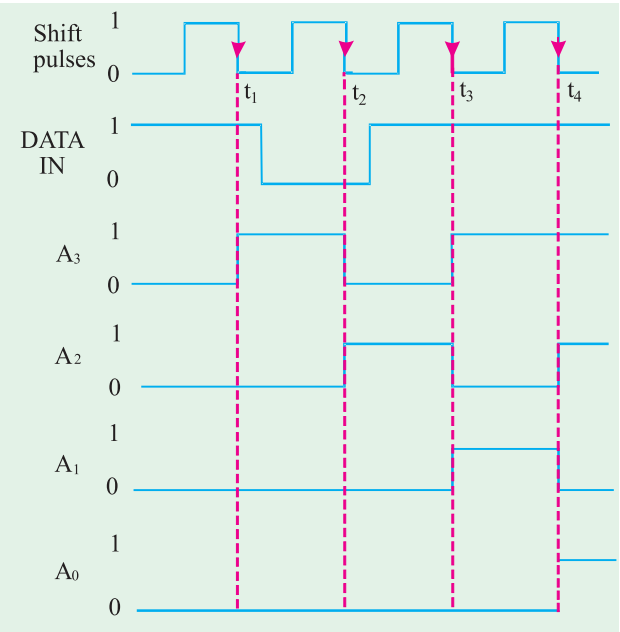


Fig. 72.70. Illustrating serial data transfer.

A similar reasoning can be used to determine how the waveforms change at t_3 and t_4. It may be carefully noted that on each falling edge of the shift pulses, each flip-flop output takes on the level

that was present at the output of the flip-flop on its left just ***prior*** to the falling edge. Of course, flip-flop '3' output X_3 takes on the level that was present at DATA IN just prior to the falling edge.

Note. In the shift register arrangement discussed above, it is necessary that the flip-flops have a very small hold time requirement. This is because of the fact that there are times when the *J* and *K* inputs are changing at about the same time as the CLK edge. For example, the flip-flop '3' output, X_3 changes from 1 to 0 in response to the falling edge at t_2, causing the *J* and *K* inputs of flip-flop '2' output X_2 to change while its CLK input is changing. Actually because of the propagation delay of flip-flop '3', the *J* and *K* inputs of flip-flop '2' won't change for a short time after the falling edge. Because of this reason, a shift register should be implemented using edge-triggered flip-flops that have a t_H value less than one CLK-to-output propagation delay. This later requirement is easily satisfied by most modern edge triggered flip-flops.

72.29. Serial Data Transfer Between Registers

We have already discussed in the last article that a shift register is a group of flip-flops arranged in such a manner that the binary numbers stored in the flip-flops are shifted from one flip-flop to the next after every clock pulse. Now we will study as how data can be serially transferred (or shifted) from one register to the other.

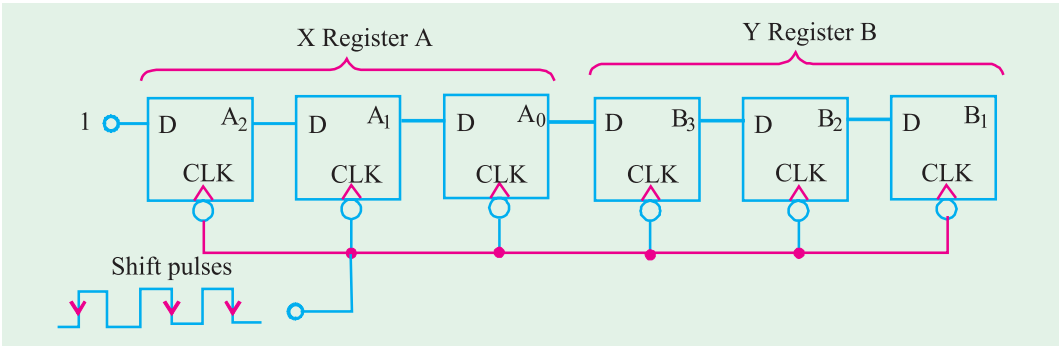


Fig. 72.71

Fig. 72.71 shows three-bit shift registers connected in such a manner that the contents of the A-register will be serially transferred (shifted) into register *B*. Notice how A_0, output of last flip-flop of register *A* is connected to the *D*-input of the first flip-flop of register *B*. The CLK input of all the flip-flops is connected together at one point where the shift pulses are applied.

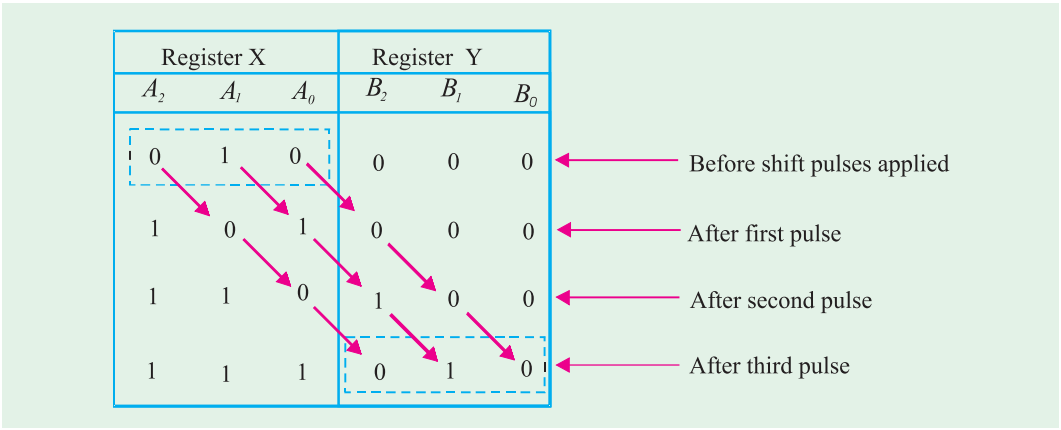


Fig. 72.72

As the shift pulses are applied, the data transfer takes place as follows:

$$A_2 \rightarrow A_1 \rightarrow A_0 \rightarrow B_2 \rightarrow B_1 \rightarrow B_0$$

The flip-flop output A_2 will go to a state determined by its D input. For discussion purpose let us suppose that D is held HIGH, so that A_2 will go HIGH on the first pulse and will remain there. Further let us assume that before any shift pulses are applied, the contents of A register are 010. (*i.e.* $A_2 = 0$, $A_1 = 1$, $A_0 = 0$) and the B register is at 000. As the shift pulse are applied one after the another, the states of each flip-flop output change are indicated in table as shown in Fig. 72.72. Following are some of the points which may be carefully noted from this table.

1. On the falling edge of each clock pulse, each flip-flop takes on the value that was stored in the flip-flop on its left prior to the occurrence of the pulse.
2. After first pulse, A_2 is set to 1, the 0 that was initially in A_2 is in A_1, the 1 that was initially in A_1 is in A_0, the 0 that was initially in A_0 is in B_2, the 0 that was initially in B_2 is in B_1, the 0 that was initially in B_1 is in B_0. The 0 that was initially in Y_0 is lost.
3. After second pulse, A_2 is still at 1 (because D is HIGH), the 1 that was previously in A_2 is in A_1, the 0 that was previously in A_1 is in A_0, the 1 that was previously in A_0 is in B_2, the 0 that was previously in B_2 is in B_1 and 0 that was previously in B_1 is in B_0.
4. After third pulse, A_2 is still at 1 (because D is held HIGH), the 1 that was previously in A_2 is in A_1, the 1 that was previous in A_1 is in A_0, the 0 that was previously in A_0 is in B_2, the 1 that was previously in B_2 is in B_1, the 0 that was previously in B_1 is in B_0.

It is evident from the above discussion that after these shift pulses, the 010 stored in the A register has now been shifted to B register. The register A is now at 111. Notice that it has lost its original data.

Notes.

1. The flip-flops shown in Fig. 72.71 can also be connected easily so that data shifts from right to left. As a matter of fact, there is no general advantage of shifting in one direction over the other. The direction chosen by the digital system designer depends upon the nature of application.
2. In serial data transfer (refer to Fig. 72.71) of N bits of data requires N clock pulses. This means three bits of data required three pulses, four bits requires four pulses and so on. However in parallel transfer, all the data is transferred simultaneously upon the occurrence of a single transfer pulse (refer to Fig. 72.66). In other words it does not matter how many bits are being transferred. It is obvious that parallel transfer is much faster than serial transfer using shift registers. However, parallel transfer requires more interconnections between the sending register (A) and receiving register (B) than does the serial transfer. This is an important consideration when the sending and receiving registers are at a distance from each other. The choice of either parallel or serial transmissions depends on the particular system application and specifications. In actual practice, a combination of two types is used to take advantage of the speed of parallel transfer and economy and simplicity of serial transfer.

72.30. Frequency Division

Many applications require frequency division. For example a quartz (or digital) watch makes use of a quartz crystal to generate a very stable oscillator frequency. This frequency is usually 1MHz or more. In order to advance the "seconds" display once every second, the oscillator frequency is divided by a value that will produce a very stable and accurate 1Hz output frequency. The frequency division can be achieved by using flip-flops.

Fig. 72.73 (*a*) shows a single *J-K* flip-flop connected to toggle (*i.e.* $J = K = 1$). When the clock pulses are applied at its CLK input, the Q output is a square wave with one-half the frequency of the clock input as shown in Fig. 72.73 (*b*). Thus a single flip-flop can be used as a divide-by-2 device. As

seen from the figure, the flip-flop changes state on each falling edge of the clock pulse. This results in an output that changes at half the frequency of the clock waveform.

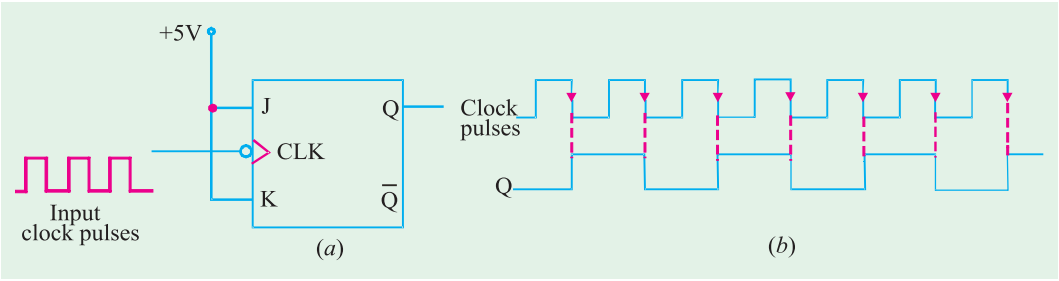


Fig. 72.73

Further division of the clock frequency can be achieved by using the output of one flip-flop as the clock input to the second flip-flop as shown in Fig. 72.74. Notice that the frequency of Q_0 output is divided by 2 by flip-flop 1. Therefore the Q_1 output is one-fourth the frequency of the original clock input. It may be carefully noted that propagation delays are not shown on the timing diagrams.

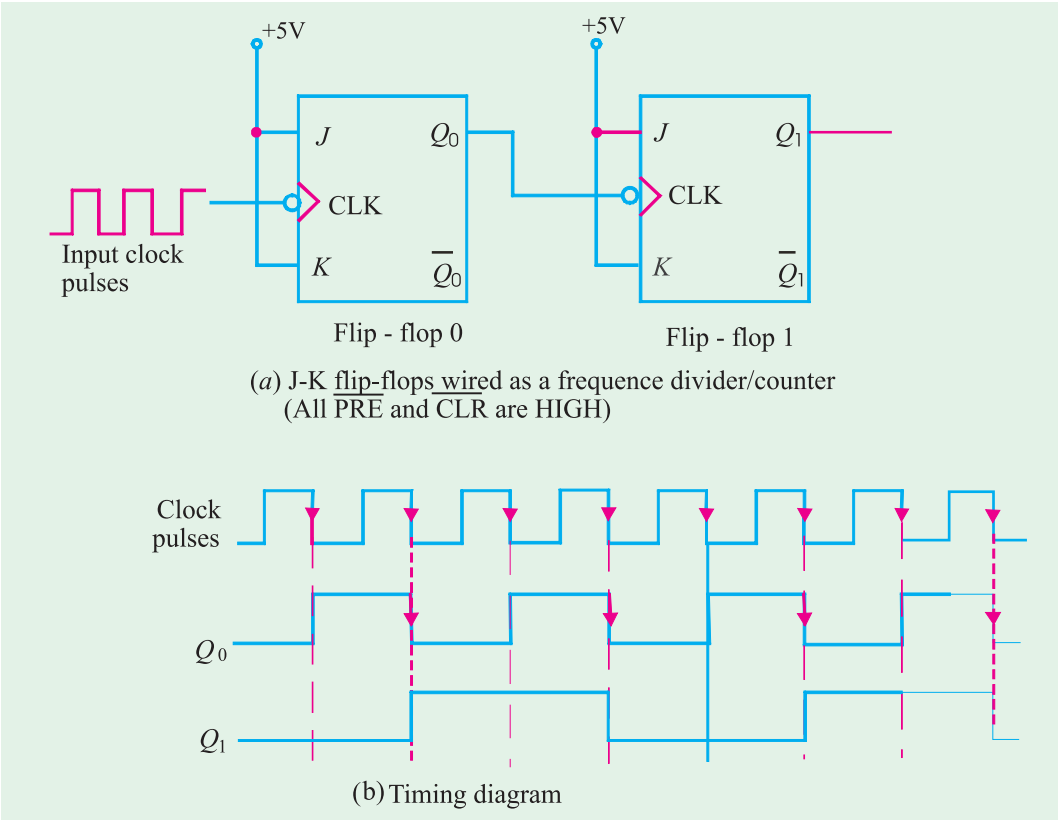


Fig. 72.74

By connecting the flip-flops in the way described above, a frequency division of 2^N is achieved, where N is the number of flip-flops. For example, three flip-flops divide the clock frequency by 2^3= 8, four flip-flops divide the clock frequency by 2^4=16 and so on.

Example 72.17. *How many flip-flops are required to divide a frequency by thirty-two.*

Solution. We know that a single flip-flop can divide the input frequency by 2. Two flip-flops

connected together can divide the frequency by 4. In gereal a frequency division of 2^N can be achieved by using N number of flip-flops. Thus,

$$2^N = 32 \Rightarrow N = 5 \quad \textbf{Ans.}$$

Example 72.18. *Fig. 72.74 shows three J-K flip-flops wired as frequency divider.*

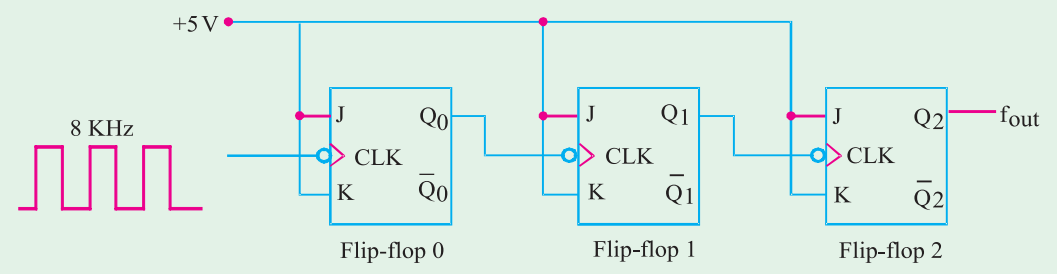


Fig. 72.75

Sketch the frequency waveform at Q_2 output when an 8 kHz square wave input is applied at CLK input of the flip-flop 'D'.

Solution. Note that each flip-flop is connected to toggle (*i.e.* $J = K = 1$). Since these are falling edge triggered flip-flops, the outputs change on the falling edge of clock pulse as shown in Fig. 72.76. There is one output pulse at Q_2 for every eight input pulses, so the output frequency is $f_{out} = 8\,\text{kHz} / 8 = 1\,\text{kHz}$. Fig 72.76 shows the waveforms at Q_0 and Q_1 outputs also.

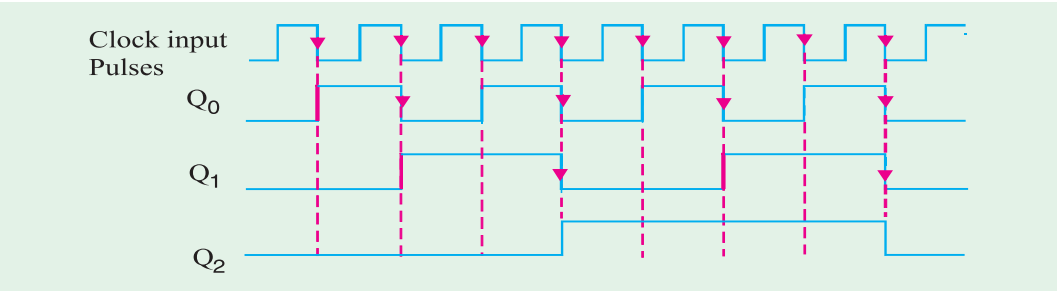


Fig. 72.76

72.31. Counting

Another important application of flip-flops is in digital counters. These are covered in more detail in chapter 7 on counters and Registers Fig 72.77 illustrates the concept of a 2-bit counter. Here two *J-K* flip-flops are wired to toggle (*i.e.* $J = K = 1$). Both the flip-flops are initially RESET. Flip-flop 0 toggles on the falling edge of each clock pulse. The *Q*-output of flip-flop 0 clocks flip-flop 1, so that each time Q_0 makes a HIGH-to-LOW transition, flip-flop 1 toggles. The resulting Q_0 and Q_1 waveforms are shown in the figure.

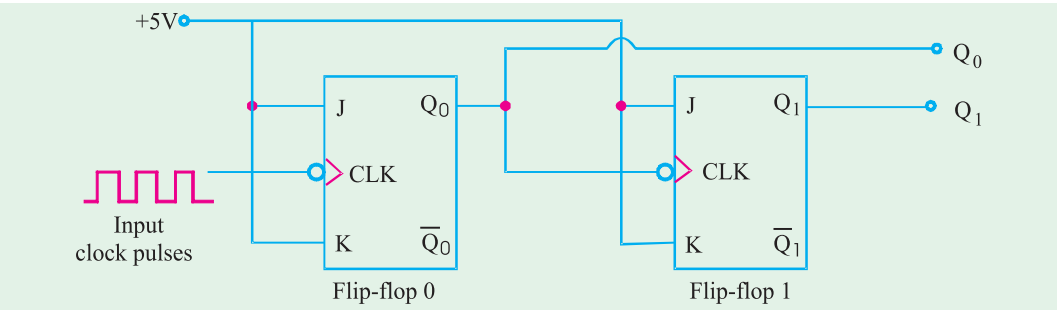


Fig. 72.77. J-K flip-flops wired as a 2-bit counter

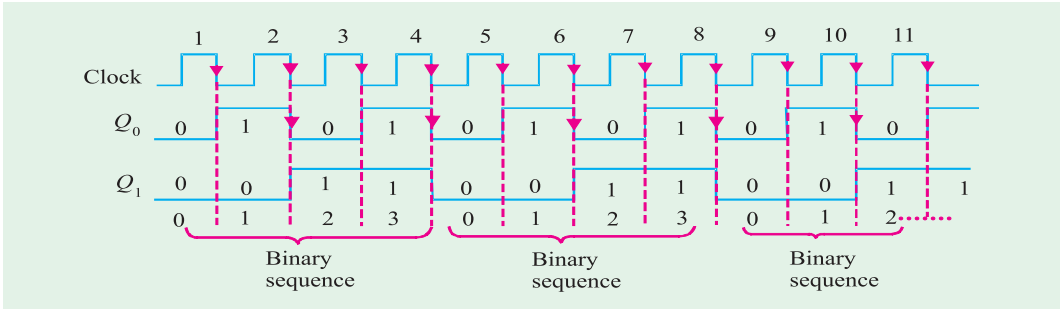


Fig. 72.78

Examine the sequence of Q_0 and Q_1 in Fig. 72.77. Prior to clock pulse 1, $Q_0 = 0$ and $Q_1 = 0$, after clock pulse 1, $Q_0 = 1$ and $Q_1 = 0$, after clock pulse 2, $Q_0 = 0$ and $Q_1 = 0$, and after clock pulse 3, $Q_0 = 1$ and $Q_1 = 1$. If we take Q_0 as the least significant bit, a two bit sequence is produced as the flip-flops are clocked. This binary sequence repeats every four clock pulses as shown in the timing diagram of Fig. 72.78. Thus flip-flops are counting in sequence from 0 to 3 (*i.e.* 00, 01, 10 and 11) and then recycling back to 0 to begin the counting sequence again.

Example 72.19. *Fig. 72.80 shows a J-K flip-flops wired as a 3-bit counter.*

Sketch the output waveforms at Q_0, Q_1 and Q_2. Also show the binary sequence represented by these waveforms.

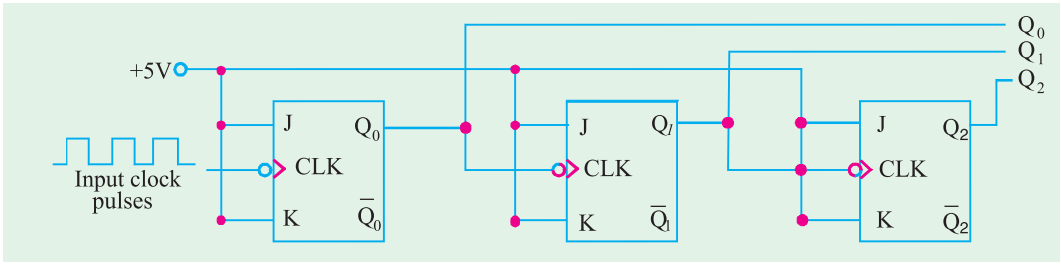


Fig. 72.79

Solution. Fig. 72.80 shows the output waveforms at Q_0, Q_1 and Q_2. Notice that the outputs change on the falling edge of the clock pulses. The outputs go through the binary sequence 000, 001, 010, 011, 100, 101, 110 and 111 as indicated.

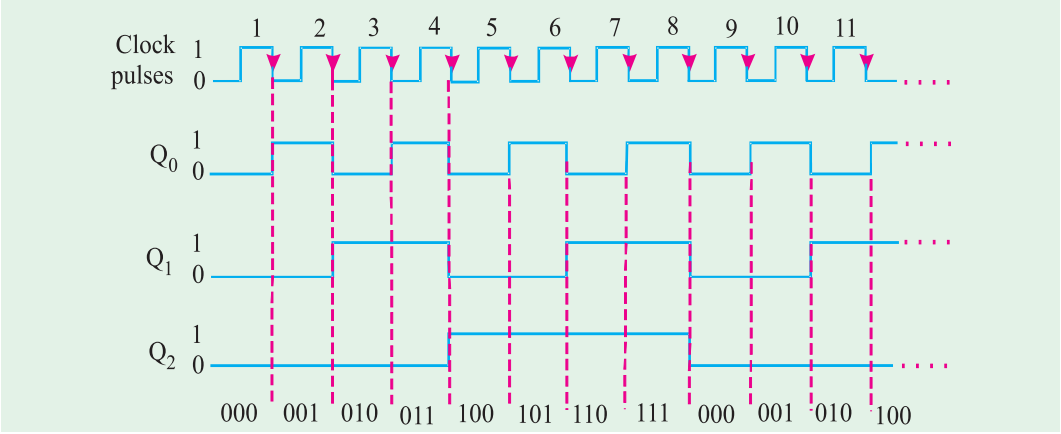
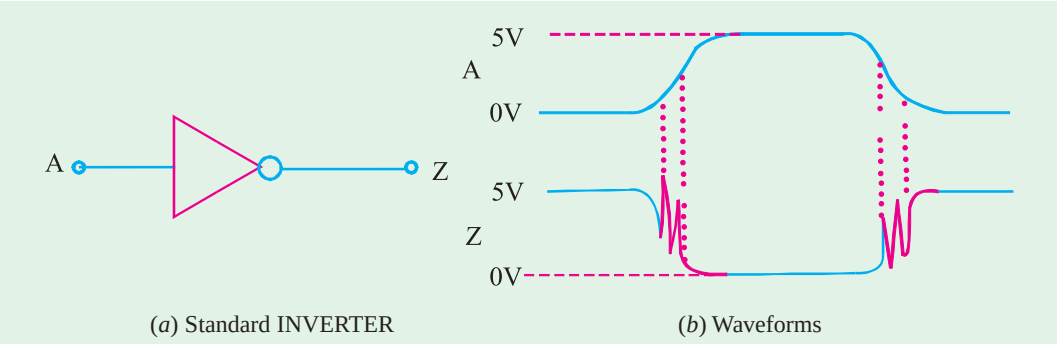


Fig. 72.80

72.32. Schmitt-Trigger Devices

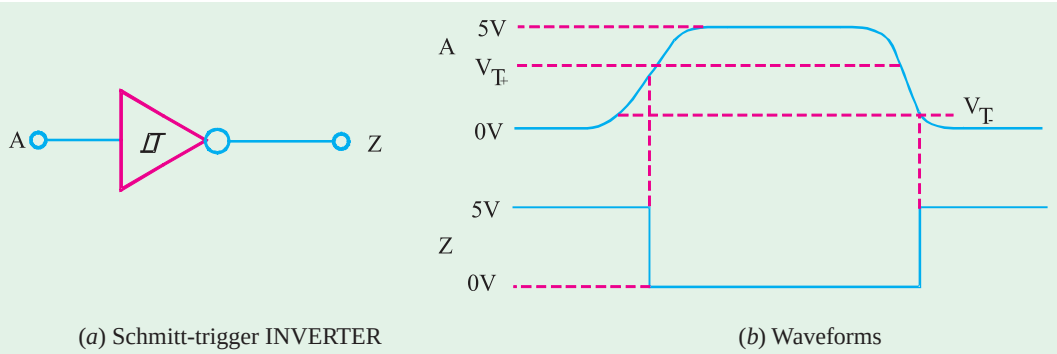
Strictly speaking, a Schmitt –trigger circuit is not classified as a flip-flop but it does exhibit a type of memory characteristic that makes it useful in certain special situations. One of those situations is shown in Fig. 72.81 (*a*). Here a standard INVERTER is being driven by a logic input that has relatively slow transition times. When these transition times exceed the maximum allowed values, the outputs of logic gates and INVERTERS may produce oscillations as the input signal posses through the indeterminate range (refer to Fig. 72.81 (*b*)). The same input conditions can also produce erratic triggering of flip-flops.



(*a*) Standard INVERTER (*b*) Waveforms

Fig. 72.81

A device that has a Schmitt-trigger type of input is designed to accept slow changing signals and produce an output that has oscillation free transitions. The output will generally have a very rapid transition times (typically 10ns) that are independent of the input signal characteristics. Fig. 72.82 (*a*) shows the symbol for a Schmitt-trigger INVERTER and (*b*) shows its response to a slow-changing input.



(*a*) Schmitt-trigger INVERTER (*b*) Waveforms

Fig. 72.82

As seen from Fig. 72.82 (*b*), we find that the output does not change from HIGH to LOW until the input exceeds the rising-edge threshold voltage, V_{T+}. Once the output goes LOW, it will remain there even when the input drops back below V_{T+} (this is its memory characteristic) until it drops all the way down below the falling edge threshold voltage, V_{T-}. The values of the two threshold voltages will vary from one logic family to another. But V_{T-} will always be less than V_{T+}.

It is evident from the above discussion that schmitt trigger is a special device that is used to transform slowly changing waveforms into sharply defined, jitter-free output singals. They are useful for changing clock edges that may have slow rise and fall times into straight vertical edges.

There are several ICs with Schmitt-trigger inputs. The 7414, 74LS14 and 74HC14 are hex INVERTER ICs with Schmitt-trigger inputs. On the other hand 7413, 74LS13 and 74HC13 are dual four-input NANDs with Schmitt-trigger inputs.

72.33. One-Shot (Monostable Multivibrator)

A one-shot (abbreviated as *OS*) is also referred to as monostable multivibrator. It is a digital circuit that is somewhat related to a flip flop. For instance, like a flip-flop, the one-shot has two inputs, Q and $\overline{Q}$ which are inverse (or complement) of each other. However, unlike the flip-flop, the one-shot has only one stable output state (normally, $Q = 0$ and $\overline{Q} = 1$). The one-shot remains in this stable state until it is triggered by an input signal. Once triggered, the one-shot outputs switch to the opposite state ($Q = 1, \overline{Q} = 0$). This state is called quasi-stable (i.e. unstable) state. The one-shot remains in this quasi-stable state for a fixed period of time, t_p, which is determined by the RC time constant. The RC time constant in turn is determined by the values of external components connected to one-shot. After a time, t_p, the one-shot outputs return to their resting state until triggered again.

Fig. 72.83 (*a*) shows the logic symbol for one-shot. As seen from this figure, it has one input labeled as trigger input (T), a normal output (Q), and an inverted output ($\overline{Q}$). It needs two external components: resistor (R_T) and a capacitor (C_T) for its operation. The values of R_T and C_T determines the exact value of time period (t_p) for which the one-shot switches to its quasi stable state. The value of tp can vary from several nanoseconds to several tens of seconds. Fig. 72.83 (*b*) shows the stable state (where $Q = 0$, $\overline{Q} = 1$) and the quasi-stable state (where $Q = 1$ and $\overline{Q} = 0$).

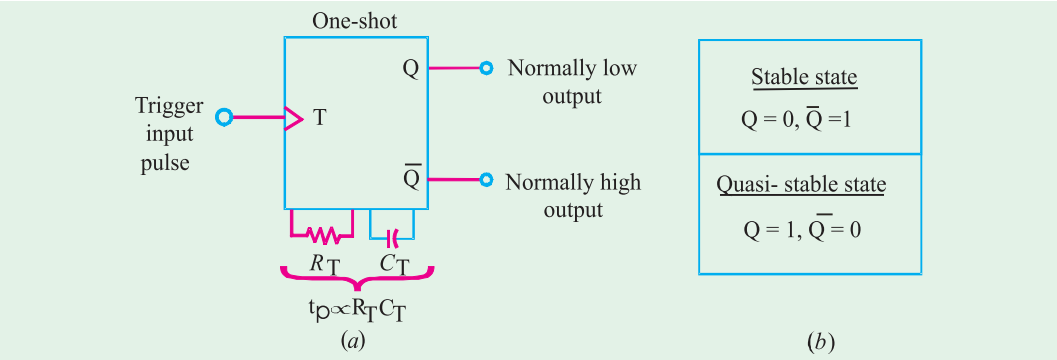


Fig. 72.83. One-shot symbol and Output states.

72.34. Types of One-Shots

Depending upon the operation, the one-shot is of the following two types :

1. Non-retriggerable one-shot and
2. Retriggerable one-shot

Both the type of one-shots are available in the IC form. Now we shall study both these one-shots one by one in the following pages.

72.35. Non-Retriggerable One-Shot

We have already discussed in the last article that one-shot (or a monostable multivibrator) is a device with only one stable state. It is normally in its stable state and will change to its quasi-stable state only when triggered. Once it is triggered, the one-shot remains in its quasi-stable state for a fixed

period of time and then automatically returns to its stable state. The time that the device remains in the quasi-stable state determines the pulse width (t_p) of the output.

Fig. 72.84 (*a*) shows the waveforms of a trigger input signal *T* and output of one-shot. Notice that the rising edge of the trigger signal will trigger the one-shot to its quasi-stable state. The one-shot will remain in the quasi-stable state for a time, t_p after which it automatically returns to its stable state.

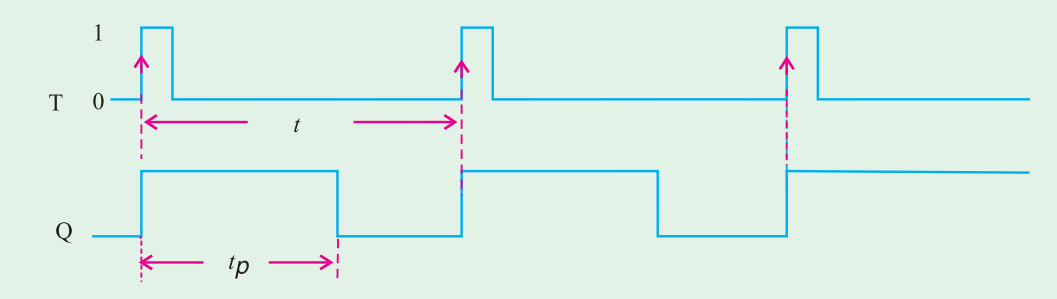


Fig. 72.84

It may be noted from the diagram shown above that one-shot is being triggered at intervals greater than its pulse width (*i.e.* $t > t_p$).

Now we will consider a situation where the one-shot is being triggered at intervals less than its pulse width (*i.e.* $t < t_p$). Refer to Fig. 72.85. Notice that the rising edges of the T signal at points '*b*' has no effect on the one-shot because it has already been trigged to the quasi-stable state by the rising edge at point '*a*'. The one-shot must return to the stable state before it can be triggered. The same argument is valid for the rising edge at point '*d*'. It has no effect on the one –shot because it has already been trigged by the rising edge at point '*c*'.

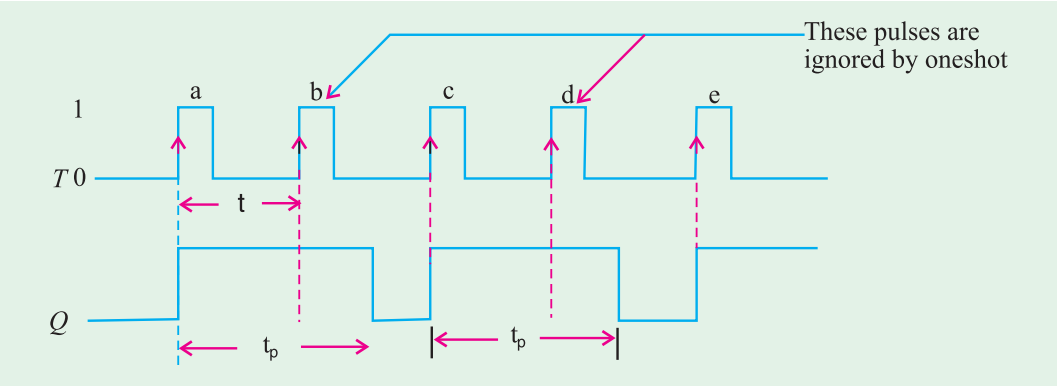


Fig. 72.85

It may be carefully noted that one-shot output pulse duration (t_p) is always the same regardless of the input pulses. The pulse duration t_p, depends only on the values of external components R_T and C_T and the internal one-shot circuitry. In a typical monoshot, the value of t_p is given by the relation.

$$t_p = 0.7\, R_T C_T$$

Example 72.20. *Fig.* 72.84 *shows three non-retriggerable one-shots connected in a timing chain that produces three sequential output pulses. Note the* "1" *in front of the pulse on each one-shot symbol to indicate non-retriggerable operation.*

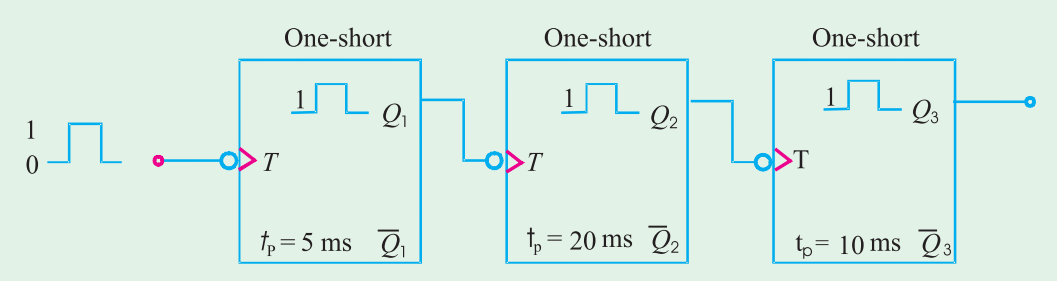


Fig. 72.86

Draw a timing diagram showing the relationship between the input pulse and the three one-shot outputs. Assume an input pulse duration of 10 *ms.*

Solution. Notice that all the one-shots trigger at the falling edge of the input signal. To begin with, the first one-shot triggers at the falling edge of the input signal and the second one-shot triggers at the falling edge of Q_1. The third one-shot triggers at the falling edge of Q_2.

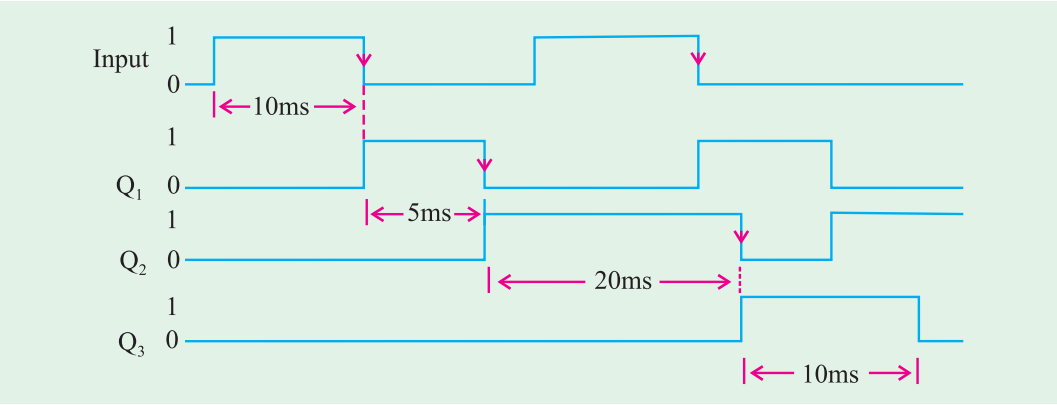


Fig. 72.87

Fig. 72.87 shows a sketch of the timing waveforms at Q_1, Q_2 and Q_3 along with the input trigger waveform.

Example 72.21. *Sketch the waveforms at Q and* $\overline{Q}$ *outputs of a one-shot which is being triggered by a signal shown in Fig.* 72.88. *Assume that one-shot triggers at the rising edge of the trigger signal and has a* $t_p = 1.5$ *ms.*

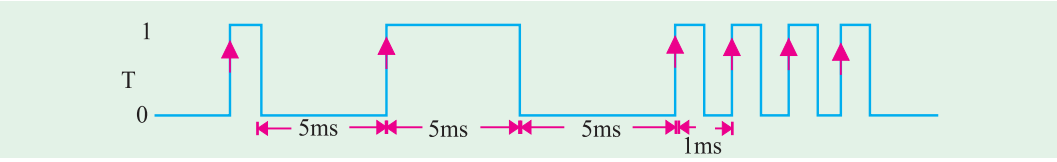


Fig. 72.88

Solution. Fig. 72.89 shows the waveforms at the one-shot outputs Q and $\overline{Q}$ in response to the trigger signal (T). Notice that the rising edges at points *a*, *b*, *c* and *e* of the *T* signal, will trigger one-shot to its quasi-stable state. The one-shot will remain in the quasi-stable state for a time, t_p after which it automatically returns to its stable state.

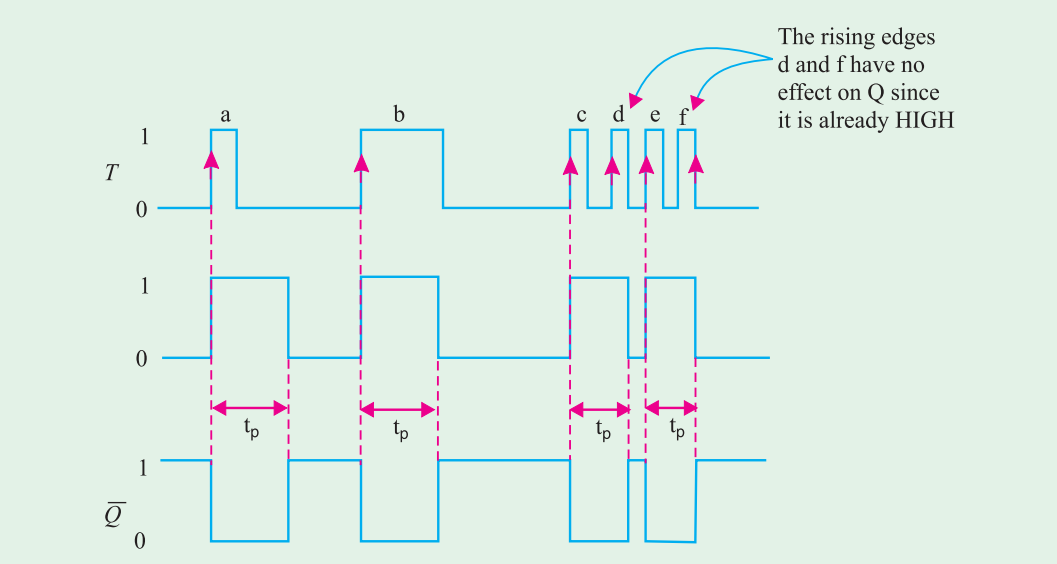


Fig. 72.89

The rising edges of the *T* signal at points '*d*' and '*f*' have no effect on the one-shot because it has already been triggered to the quasi-stable state. The one-shot must return to the stable state before it can be triggered.

72.36. Retriggerable One-Shot

This one-shot usually operates in the same manner as the non-retriggerable one-shot except for one-major difference. The major difference is that retriggerable one-shot can be retriggered while it is in the quasi-stable state, and it will begin a new t_p interval. Fig. 72.90 compares the response of both types of one-shot using a t_p of 2 ms interval.

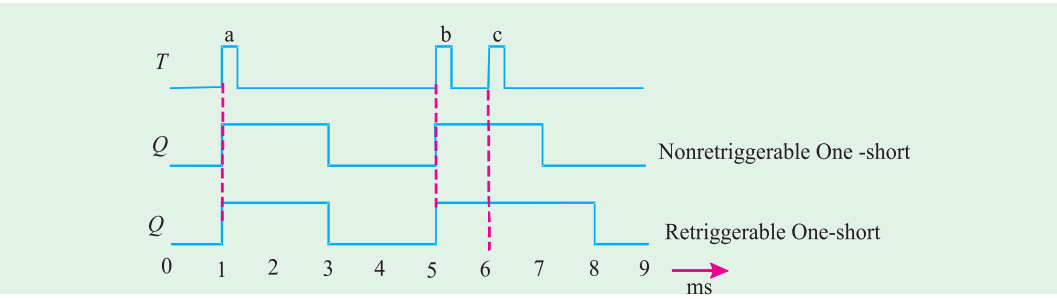


Fig. 72.90

As seen from the diagram, both types of one-shot respond to the first trigger pulse indicated as '*a*' at t = 1 ms by going HIGH for 2 ms and then returning LOW. The second trigger pulse indicated as '*b*' at t = 5 ms triggers both one – shots to the HIGH state. The third trigger pulse indicated as '*c*', at t = 6 ms has no effect on the non-retriggerable one-shot because it is already in the quasi-stable state. However, this trigger pulse will retrigger the retriggerable one-shot to begin a new t_p = 2 ms internal. Thus it will stay HIGH for 2 ms after this third trigger pulse.

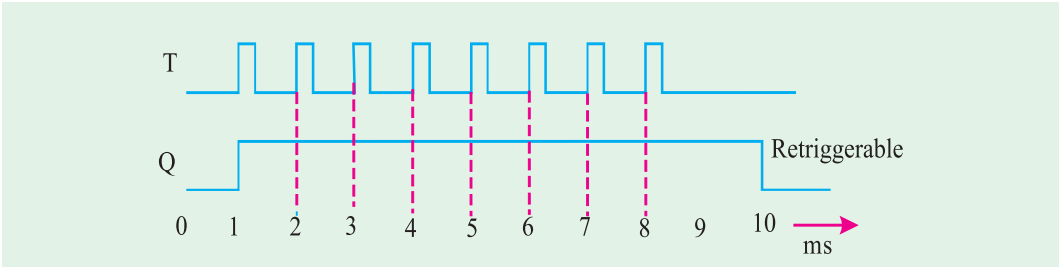


Fig. 72.91

It is evident from the discussion above that a retriggerable one-shot begins a new t_p interval each time a trigger pulse is applied, regardless of the current state of its *Q*-output. As a matter of fact, trigger pulses can be applied at a rate fast enough that one-shot will always be retriggered before the end of the t_p interval and *Q*-output will remain HIGH. Such a situation is shown in Fig. 72.91. As seen from this diagram, eight pulses are applied every 1 ms. The *Q*-output does not return LOW until 2 ms after the last trigger pulse.

Example 72.22. *A retriggerable one-shot can be used as a pulse-frequency detector that detects when the frequency of a pulse input is below a predetermined value. A simple example of this application is shown in Fig.* 72.92. *The operation begins by momentarily closing switch SW*1.

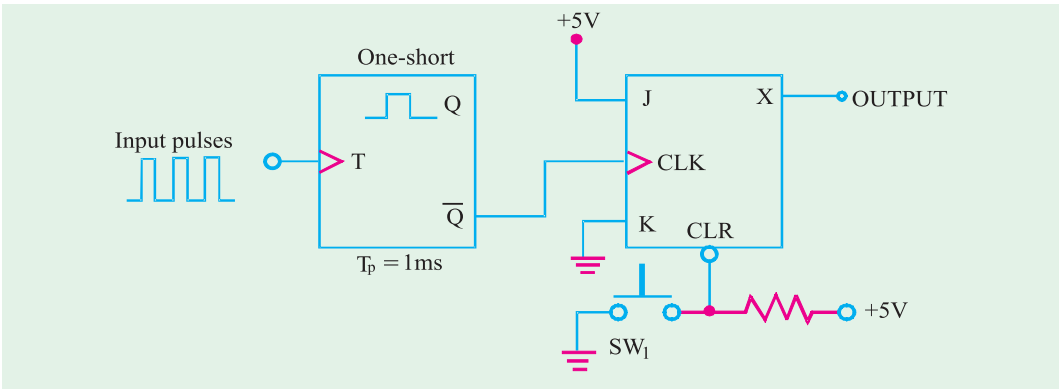


Fig. 72.92

(*a*) *Describe how the circuit responds to input frequencies above* 1*kHz.* (*b*) *Describe how the circuit responds to input frequencies below* 1*kHz.* (*c*) *How would you modify the circuit to detect when the input frequency drops below* 50 kHz?

Solution.

(*a*) Circuit response to input frequencies above 1kHz. Closing the switch SW1, clears *X* to Zero. Since the one-shot has t_p = 1 ms, the one-shot will be retriggered before the end of the t_p interval for frequencies greater than 1 kHz. Thus $\overline{Q}$ will stay LOW. As a result of this, there is no clock input to *JK* flip-flop and hence *X* will remain LOW.

(*b*) Circuit response to input frequencies below 1kHz. If the input frequency falls below 1 kHz, the $\overline{Q}$ will return HIGH before the one-shot is triggered again. This positive-going-trigger will clock *JK* flip-flop and *X* will change to HIGH.

(*c*) Modification of the circuit to detect frequencies below 50 kHz. In order to detect frequencies below 50 kHz, we will have to change t_p of the one-shot to 1/50 kHz = 20 ms.

72.37. Actual One-Shot Devices

There are several one-shot ICs that are available commercially in both the non-retriggerable and retriggerable versions. Table 72.3 shows some of the most popular one-short ICs available commercially. As seen from the table 72.3 we find that 74121 and 74221 ICs are non-retriggerable one-shots. Where as 74122 and 74123 ICs retriggerable one-shots.

Table 72.3. Actual one-shot devices

Device	*Description*
74121	A *single non-retriggerable* one-shot IC in standard TTL series.
74221	A *dual non-retriggerable* one-shot IC in standard TTL series.
74LS221	A *dual non-retriggerable* one-shot IC in Low-power SchottkyTTL series
74HC221	A *dual non-retriggerable* one-shot IC in high-speed CMOS series
74122	A *single retriggerable* one-shot IC in standard TTL series
74LS122	A single retriggerable one-shot in IC low-power Schottky TTL series.
74123	A dual retriggerable one-shot IC in standard TTL series
74LS123	A dual retriggerable one-shot IC in lower-power Schottky TTL series
74HC123	A dual retriggerable one-shot IC in high-speed CMOS series.

Fig. 72.93 shows the logic symbol for the 74121 non-retriggerable one-shot IC. As seen from the diagram, the 74121 IC contains internal logic gates to allow inputs $\overline{A}_1$, $\overline{A}_2$, and B to trigger the device in a variety of ways. The B input is a Schmitt-Trigger type of input that is allowed to have slow transition times and still reliably trigger the one-shot. The pins labelled R_{EXT}, R_{EXT} /C_{EXT} are used to connect an external resistor and capacitor to achieve the desired output pulse duration.

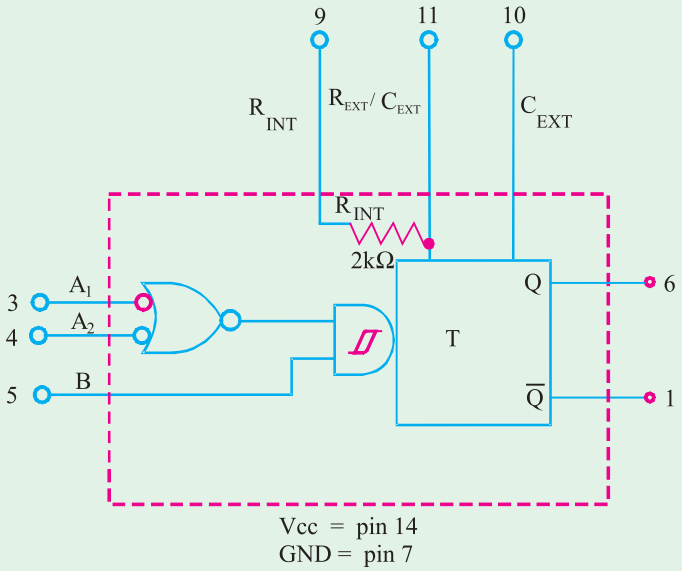


Fig. 72.93. Logic symbol of 74121.

A typical pulse width of 30 ns is produced when no external components are used and the internal timing registor (2 k Ω) is connected to V_{CC}. The pulse width can be set anywhere between 30 ns and 28 ns by the use of external components, (*i.e.* R_{EXT} and C_{EXT}). Mathematically the pulse width.

$$t_p = 0.7\ R\ C_{EXT} \quad ...(i)$$

where R is either R_{INT} or R_{EXT}. Notice that if R is in kilohms (kΩ) and C_{EXT} is in picofarads (PF), the output pulse width (t_p) is in nanoseconds. Further if R is in kilohms (kΩ) and C_{EXT} is in microfarads (μF), the output pulse width (t_p) is in microseconds (μS).

Table 72.4 shows the truth table of 74121. Notice that for the first four rows of the table, the Q-output is LOW.

Table 72.4

Inputs			Output	
A_1	$\overline{A}_2$	B	Q	$\overline{Q}$
L	X	H	L	L
X	L	H	Ll	Ll
X	X	L	L	L
H	H	X	L	L
H	↓	H	⎍	⎎
↓	H	H	⎍	⎎
↓	↓	H	⎍	⎎
L	X	↑	⎍	⎎
X	L	↑	⎍	⎎

H = High voltage level
L = Low voltage level
X = Don't case
↑ = Rising edge
↓ = Falling edge

It is because of the fact that no trigger pulse is applie at any one of the three inputs. In the next five lines of the truth table, notice that we have a trigger pulse at one of the inputs $(A_1\ A_2$ or $B)$ and the other two inputs are connected LOW or HIGH.

Fig. 72.94 (a) shows the logic symbol and 72.94 (b) the truth table for 74123 a dual retriggerable one = shot. As seen from the diagram, the 74123 IC contains two retrigger able one-shots. Each retriggerable one-shot contains $\overline{A}_1$, B_1 and R_D to trigger the device in a variety of ways.

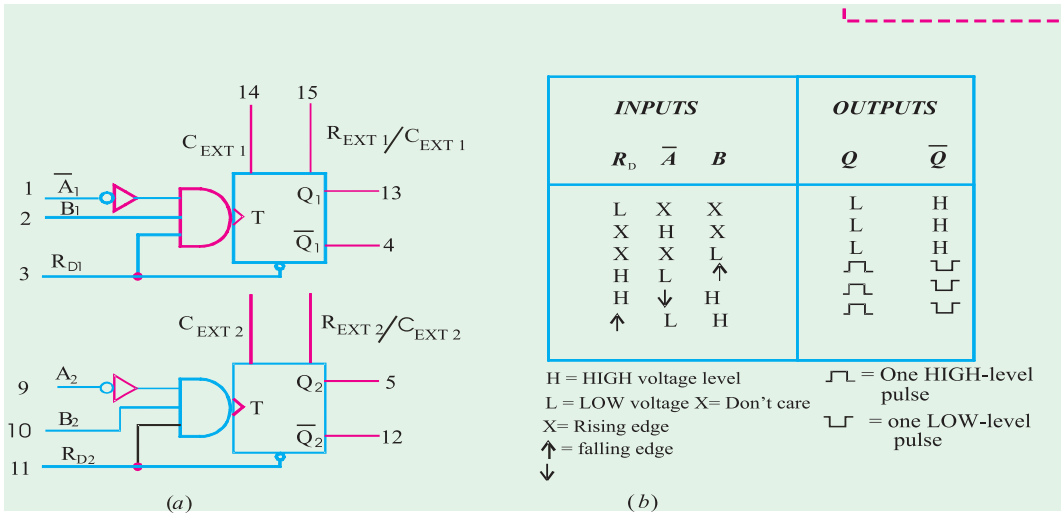


Fig. 72.94

A minimum pulse width of approximately 45 ns is obtained with no external components. Wider pulse widths are achieved by using external components. A general formula for calculating the values of these components for a specified pulse width (t_p), provided $C_{EXT} > 1000$ pF,

$$t_p = 0.28\ \mathrm{R}\ C_{\mathrm{EXT}}\left(1 + \frac{0.7}{\mathrm{R}}\right) \quad \text{.... } (ii)$$

where 0.28 is a constant determined by a particular type of one-shot, R is either the internal or external resistor, C_{EXT} is in pF and t_p is in ns.

Notes.

1. While selecting the value of R_{EXT}, care should be taken that its value must be in kilohms ($k\Omega$) so that the circuit current is in milli amperes (mA). If the resistor value is chosen in ohms (Ω), the circuit current will be in amperes which the IC will not be able to handle it. On the other hand, if the value of R_{EXT} is in megohms ($M\Omega$), the current will be in micro amperes (μA) which may be too small for internal operation of 74121. Moreover, resistances with values in the range of megohms are susceptible to electrostatic noise.
2. For a desired pulse width t_w, we can determine the values of external components in two ways.
   - ***(i)*** Arbitrarily select the value of capacitor, C_{EXT} and then using equation $t_W = 0.7\ R_{EXT}\ C_{EXT}$, determine the value of $R_{EXT.}$ be in microfarads (μF).
   - ***(ii)*** Arbitrarily select the value of R_{EXT} and then using the equation, $t_W = 0.7\ R_{EXT}\ C_{EXT}$, determine the value of C_{EXT}.
   - ***(iii)*** If t_w is required to be in nanoseconds, choose R_{EXT} in kilohms ($k\Omega$) so that C_{EXT} is in picofarads. However if the desired t_w is required to be in milliseconds, still select R_{EXT} is in kilohms ($k\Omega$), but C_{EXT} will now be in microfareds (μF). Care should be taken while selecting the component values in the sense that the components are availably only in certain standard values.
3. While selecting the value of C_{EXT}, care should be taken that it is much larger than any stray capacitance that might be encountered in a typical electronic circuit. Values of capacitance less than 100 pF (0.0001 μF) may be unsuitable because it is not uncommon for there to be 50 pF of stray capacitance between traces in a printed-circuit board.

Example 72.23. *A certain digital circuit requires a one-shot with a pulse width of 10* ms. *Determine the values of external components to be connected with an* IC *74121 to produce the desired pulse width. Also show the connections of the external components to 74121.*

Solution.

Let R_{EXT} = The value of external resistor and

C_{EXT} = The value of external capacitor.

Then we know that the pulse width for a 74121 IC (t_p),

$$10 \times 10^{-3} = 0.7\ R_{EXT}\ C_{EXT} \qquad \text{... (i)}$$

Since the desired pulse width is in milli seconds, let us arbitrarily select $C_{EXT} = 1$ μF. Then from equations (*i*).

$$10 \times 10^{-3} = 0.7 \times R_{EXT} \times (1 \times 10^{-6})$$

$$\therefore \quad R_{EXT} = \frac{10 \times 10^{-6}}{0.7\,(1 \times 10^{-6})} = 14.28 \times 10^{-3}\ \Omega = 14.28\ k\Omega$$

In order to select a resistor value of 14.28 $k\Omega$, we can use a 10 $k\Omega$ fixed resistor with a 5 $k\Omega$ potentiometer.

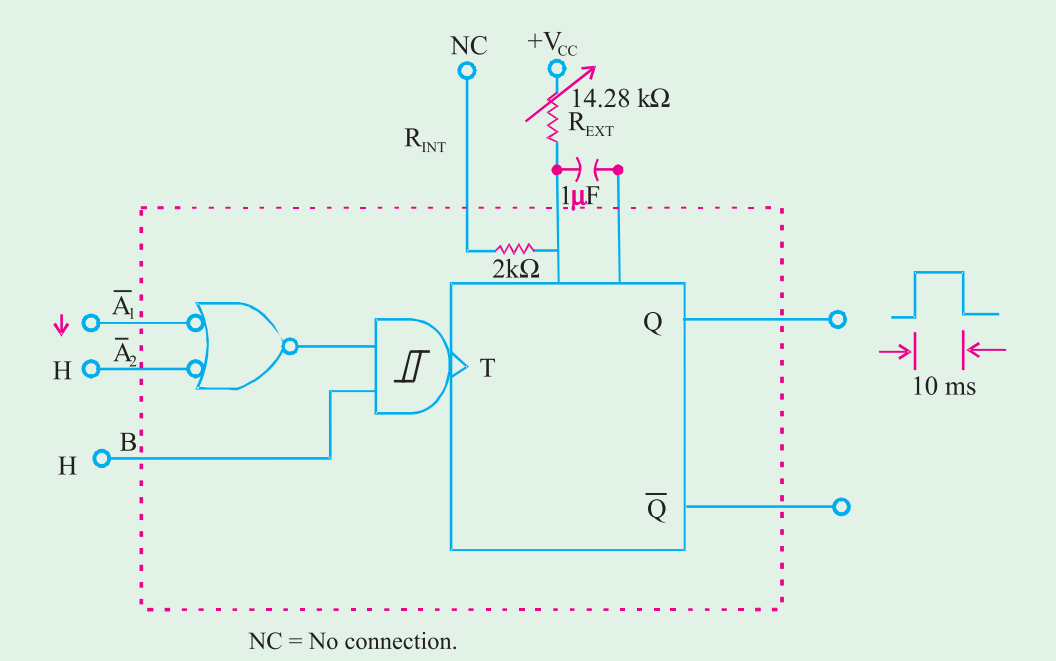


Fig. 72.95

Fig. 72.95 shows a 14.28 k Ω resistor with 1 μF capacitor connected to the 74121 IC for producing a desired pulse width of 10 ms. Notice the settings on $\overline{A}_1$, $\overline{A}_2$ and *B* inputs of 74121 as well.

Example 72.24. *Design a circuit using a* 74121 *to convert a* 50 *kHz,* 80% *duty cycle square wave to a* 50 *kHz,* 50% *duty cycle square wave.*

Solution.

Fig. 72.96 shows the given square wave. Notice that it has a time period of 1/50 kHz (= 20 μs) and has a pulse width of 80% of 20 μS (= 16 μs).

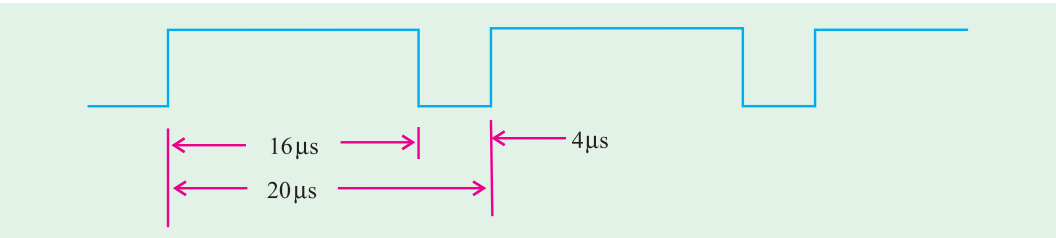


Fig. 72.96

Now we have to stretch the 4 μs negative pulse to 10 μs to make the duty cycle 50 %. If we use the falling edge on the negative pulse to trigger $\overline{A}_1$ input to a 74121 and set the output pulse width (t_p) to 10 μs, we should have the solution as shown in 72.97 (*b*). The output will be taken from $\overline{Q}$ because it provides a negative pulse when triggered.

Let R_{EXT} = The value of external resister and

C_{EXT} = The value of external capacitor,

Then the pulse width (t_p)

$$10 \times 10^{-6} = 0.7\, R_{EXT} C_{EXT}$$

Let us arbitrarily select C_{EXT} = 0.001 μ*F* (equal to 1000pf); then

$$10\ \mu s = 0.7\, R_{EXT}$$

$$R_{EXT} = \frac{10 \times 10^{-6}}{0.7 \times 0.001 \times 10^{-6}}$$

or

$$= 14.4\,k\Omega$$

In order to select a resistor of 14.4 k Ω, we can choose a fixed resistor of 10 k Ω with a 5 k Ω potentiometer.

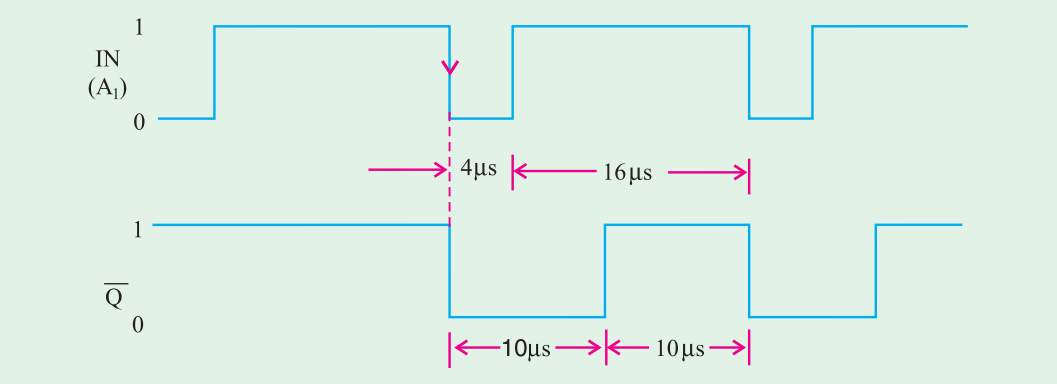


Fig. 72.97

Fig. 72.98 shows the circuit connections for producing the desired waveform. Notice that the input waveform is applied at $\overline{A}_1$ input while $\overline{A}_2$ and B are tied HIGH. The output is obtained from $\overline{Q}$.

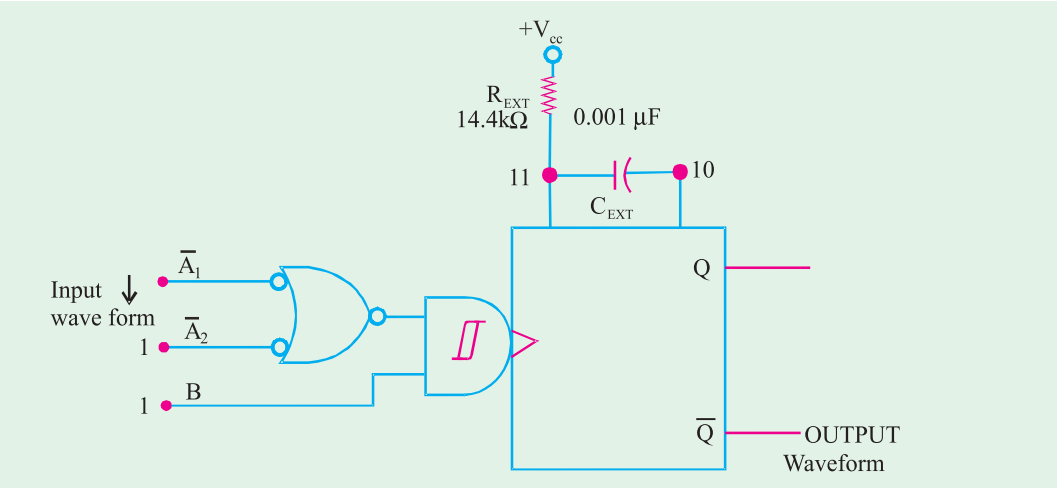


Fig. 72.98

Example 72.25. *Determine the value of R_{EXT} and C_{EXT} that will produce a pulse width of* 50 µS *when connected to a* 74123.

Solution.

Let R_{EXT} = The value of external resistor and

C_{EXT} = The value of external capacitor

Then we know that the pulse width for a 74123 IC (t_p),

$$50 \times 10^{-6} = 0.28\, R_{EXT}\, C_{EXT} \left(1 + \frac{0.7}{R_{EXT}}\right) \quad \text{... (i)}$$

Let us arbitrarily select C_{EXT} = 0.01 μF (recall that C_{EXT} has to be greater than 1000 pF), then, from equation (*i*),

$$50 \times 10^{-6} = 0.28\ R_{EXT}\ (0.01 \times 10^{-6}) \left(1 + \frac{0.7}{R_{EXT}}\right)$$

$$= 0.28\ (R_{EXT} + 0.7)\ (0.01 \times 10^{-6})$$

$$\therefore \qquad R_{EXT} + 0.7 = \frac{50 \times 10^{-6}}{0.28 \times (0.01 \times 10^{-6})} = 17857\ \Omega$$

or $$R_{EXT} = 17857 - 0.7 = 17856.3\ \Omega = 17.9\ \text{k}\ \Omega$$

Thus we can use a fixed resistor of 15 k and 5 k potentiometer to select $R_{EXT} = 17.9\ k\ \Omega$.

72.38. Clock Generator Circuits

We have already discussed in Art. 72.2 that flip-flop has two stable states due to which they are known as bistable multivibrators. Further in Art. 72.33 we have discussed that one-shot has one stable. state due to which they are referred to as monostable multivibrators. Now we shall discuss a third type of multivibrator which has no stable states. Such a multivibrator is called an astable or free-running multivibrator. An astable multivibrator switches back and forth (*i.e.* oscillates) between two unstable states without any external triggering.

There are several types of astable multivibrators that are in common use but the following three are important from the subject point of view;

1. Schmitt-Trigger oscillator
2. 555 Timer used as astable oscillator.
3. Crystal-controlled clock generator.

Now we shall discuss all the three oscillators one by one in the following pages.

72.39. Schmitt-Trigger Oscillator

Fig. 72.99 shows a Schmitt-trigger INVERTER connected as an oscillator. The signal at V_{OOT} is an approximate square wave with a frequency that depends upon the values of *R* and *C* values. The relationship between the frequency and RC values is shown in Fig. 72.100 for three different Schmitt-trigger INVERTERS.

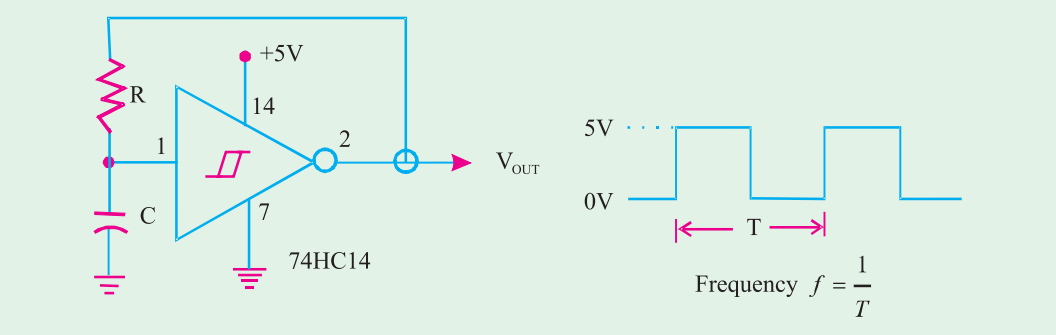


Fig. 72.99

As seen from this diagram, for 7414 Schmitt-trigger INVERTER, the frequency is given by, f = 0.8/RC. Notice that for this IC, the value of $R \leq 500\ \Omega$. For 74LS14 IC, the frequency of the output square wave is again 0.8/RC but the value of $R \leq 2\ k\Omega$. On the other hand for 74HC14, the frequency of the output square wave is 1.2/RC, where the value of *R* must be $10\ \text{M}\Omega$.

IC	Frequency	Remarks
7414	≈ 0.8/RC	(R ≤ 500 Ω)
74LS14	≈ 0.8/RC	(R ≤ 2 kΩ)
74HC14	≈ 1.2/RC	(R ≤ 10 MΩ)

Table 72.5

72.40. 555 Timer Used as an Astable Multivibrator

The 555 IC is a very popular, general purpose timer IC. It can be connected as a one-shot or as an astable multivibrator or as a free running oscillator. Fig. 72.100 shows how external components can be connected to a 555 so that it operates as a free-running oscillator. The output of the oscillator is a repetitive rectangular waveform as shown in Fig. 72.100 (*b*).

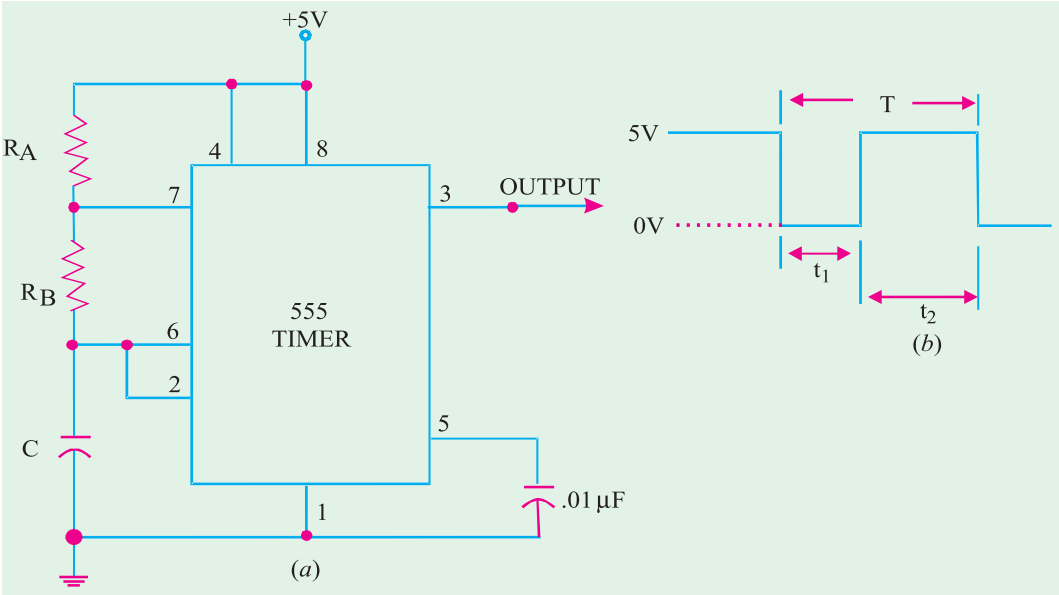


Fig. 72.100. 555 timer used as an astable multivibrator.

As seen from the diagram, the output switches between two logic levels with the time intervals ($t_1$and t_2) at each level. These two time intervals are determined by the *R* and *C* values.

The time interval t_1 is given by the equation,

$$t_1 = 0.7\, R_B C \qquad \text{... (i)}$$

and the time interval,

$$t_2 = 0.7\, (R_A + R_B)\, C \qquad \text{... (ii)}$$

$\therefore$ The time period, of the output waveform

$$T = t_1 + t_2$$

Substituting the values of t_1 and t_2 from equations (*i*) and (*ii*), the time period,

$$\text{or} \qquad T = 0.7\, R_B C + 0.7\, (R_A + R_B)\, C$$
$$= 0.7\, (R_A + 2\, R_B)\, C$$

The frequency of the output waveform,

$$f = \frac{1}{T} = \frac{1}{0.7\, (R_A + 2\, R_B)\, C}$$

$$= \frac{1.44}{(R_A + 2R_B)C}$$

Further, the duty cycle of the output waveform,

$$\text{duty cycle} = \frac{t_2}{T} \times 100\ \%$$

It may be noted from equations (*i*) & (*ii*) that t_1 and t_2 cannot be equal unless R_A is made zero. This cannot be done without producing excess current through the device. This means, it is impossible to produce an output waveform with a perfect 50% duty cycle. However, it is possible to get very close to 50% duty cycle (*i.e.* $t_1 \approx t_2$) by selecting,

$$R_B \gg R_A$$

It may be carefully noted that the value of R_A must be greater than 1k Ω and the value of C must be greater than 500 pF.

Example 72.26. *Fig.* 72.101 *shows the circuit of a* 555 *timer used as an astable multivibrator.*

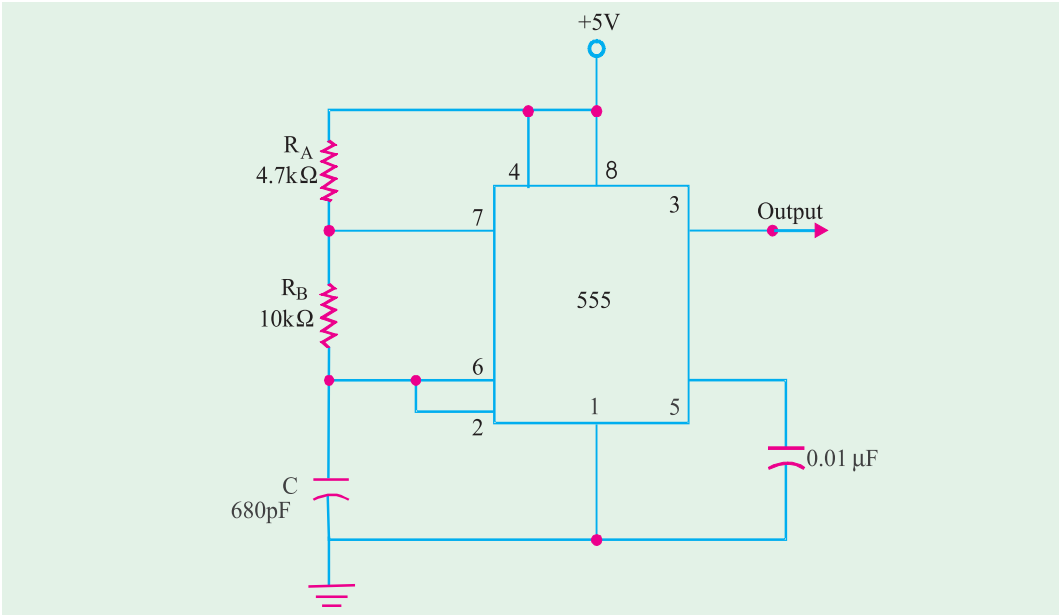


Fig. 72.101

Determine the values t_1, t_2, frequency and duty cycle of the output waveform.

Solution.

$$t_1 = 0.7\ R_B\ C \times 0.7 \times (10 \times 10^3) \times (680 \times 10^{-12})$$
$$= 4.76 \times 10^{-6}\ s = 4.76\ \mu s$$
$$t_2 = 0.7\ (R_A + R_B)\ C$$
$$= 0.7 \times ((4.7 \times 10^3) + (10 \times 10^3)) \times (680 \times 10^{-12})$$
$$= 6.997 \times 10^{-6}\ s \approx 6.997\ \mu s$$
$$T = t_1 + t_2 = 4.76 + 6.997 = 11.757\ \mu s$$
$$f = \frac{1}{T} = \frac{1}{11.757 \times 10^{-6}} = 8.5 \times 10^4 = 85\ \text{kHz}$$

$$\text{Duty cycle} = \frac{t_2}{T} \times 100\% = \frac{6.997}{11.757} = 100\% = 59.5\ \%$$

Tutorial Problems No. 72.1

1. The waveforms of Fig 72.100 are applied to the inputs of a NAND gate latch shown in Fig.72.102. Assume that initially $Q = 0$, and determine the waveform at Q-output.

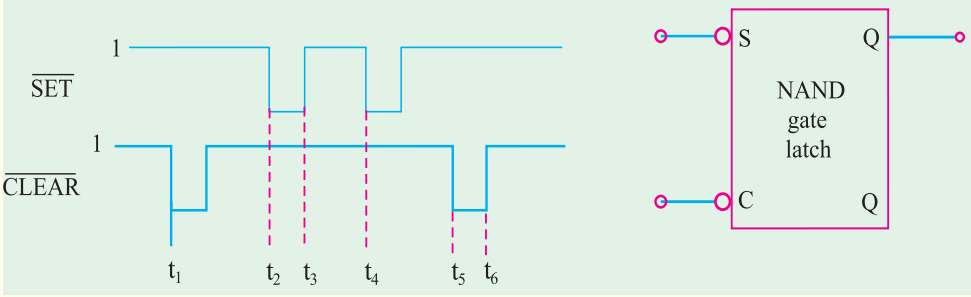


Fig. 72-102

2. The waveforms of Fig. 72.103 (*a*) are applied to the inputs of a NOR gate latch shown in Fig. 72.103 (*b*). Assume that initially $Q = 0$ and determine the waveform at Q output.

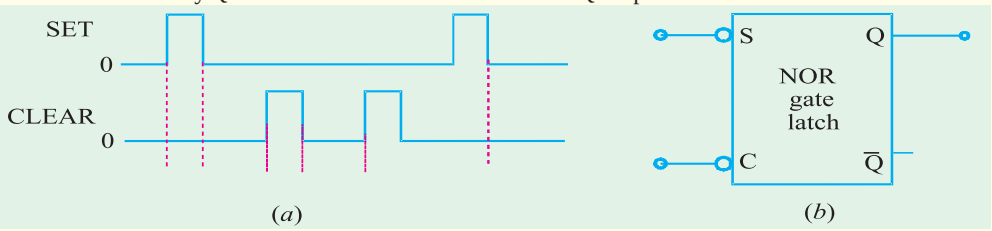


Fig. 72.103

3. Apply the *J-K* and CLK waveforms to a flip-flop shown in Fig. 72.104. Assume that $Q = 0$ initially. Sketch the output waveform.

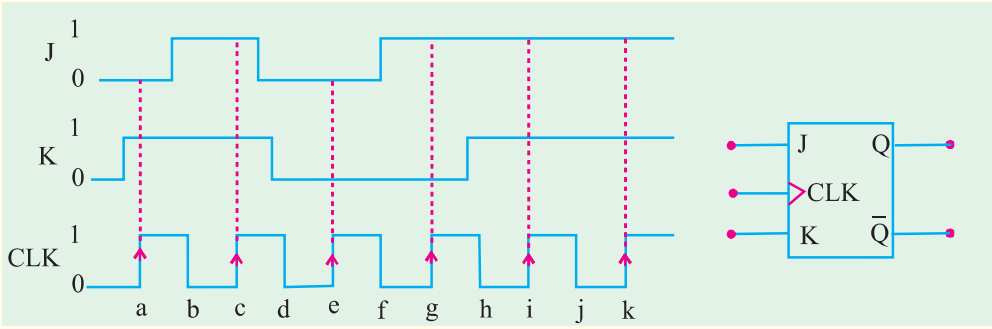


Fig. 72.104

4. Determine the output waveform for a positive-edge triggered *D* flip-flop for the *J*, *K* and CLK waveforms as shown in Fig. 72.105.

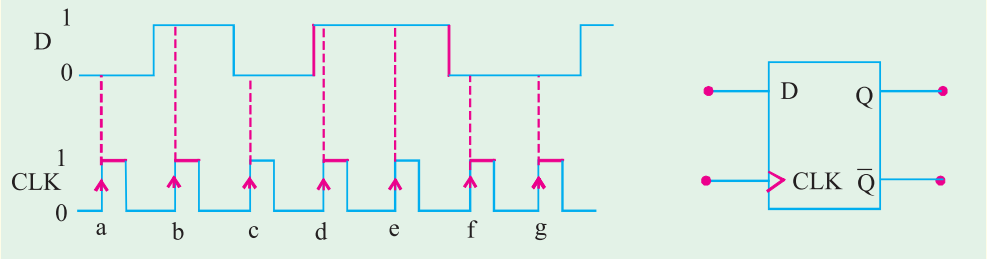


Fig. 72.105

5. Determine the *Q* waveform for a *D* latch with ENABLE (EN) and DATA (*D*) input waveforms shown in Fig. 72.106. Assume $Q = 0$ initially.

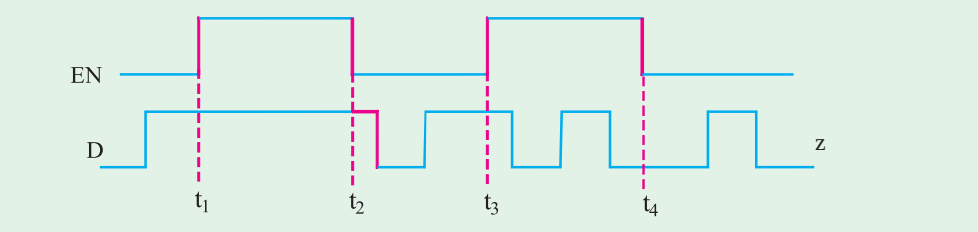


Fig. 72.106

6. Fig. 72.107 (*a*) shows the symbol for a *J-K* flip-flop that responds to a NGT (*i.e.* negative going trigger) on its clock input and has asynchronous inputs. Sketch the output waveform in response to the $\overline{CLK}$, $\overline{PRE}$ and $\overline{CLR}$ waveforms as shown in Fig. 72.107 (*b*).

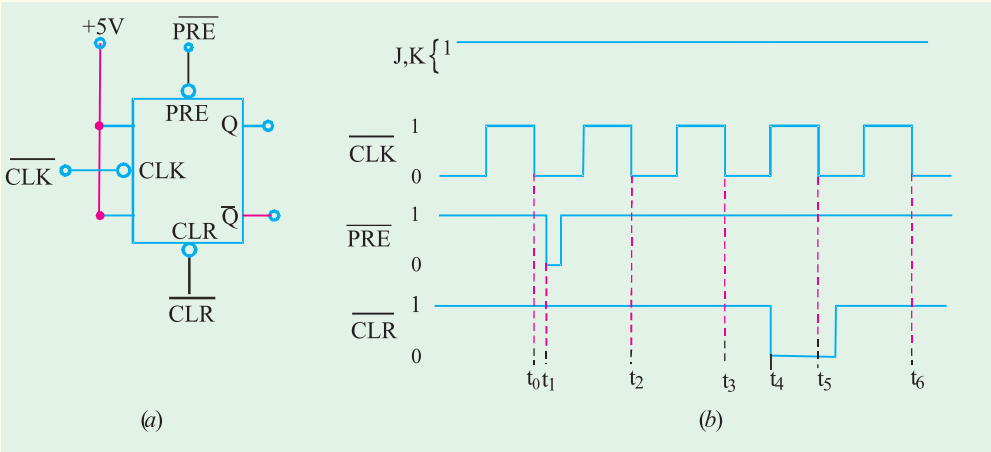


Fig. 72.107

7. Determine the *Q* output for a negative edge triggered *J-K* flip-flop for the input waveforms shown in Fig. 72.106. Assume $t_H = 0$ and that $Q = 0$ initially.

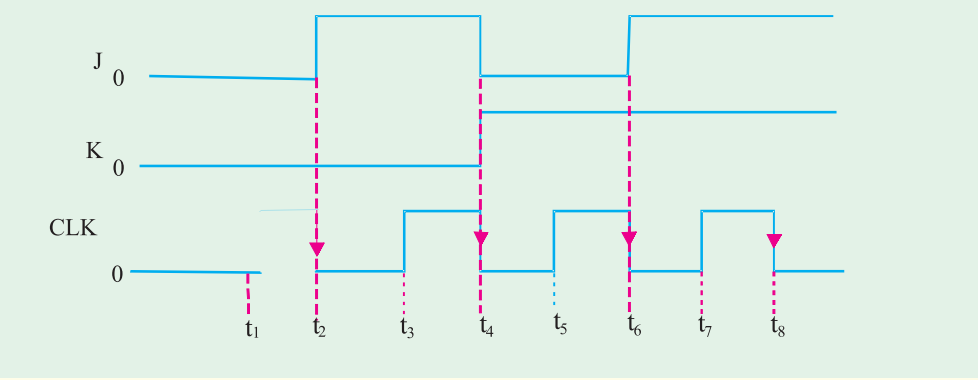


Fig. 72.108

8. A 20-kHz clock signal is applied to a *J-K* flip-flop with $J = K = 1$. Sketch the output waveform and determine its frequency. (**Ans** 10 kHz)

9. In a J-K flip-flop, $J = K = 1$. A 1 MHz square wave is applied at its clock input. It has a propagation delay of 50 ns. Draw the input square wave and the output waveform expected at Q. Show the propagation delay time.

10. *A D* flip-flop has following datasheet information :

Set up-time = 5 ns; Hold time = 10 ns; Propagation time = 15 ns

How far ahead of the triggering clock edge must the data be applied.

11. Draw the block diagram of a 4-bit shift register using *D* flip-flops. If initially all the flip-flop outputs are in zero state, prepare the truth table when the input sequence is 1, 1, 0, 1, 0. Draw the above shift register using *J-K* flip-flops only.

12. Consider a 4-bit shift register using J-K flip-flops. Assume that initially, $Q_0 = 1$, $Q_1 = 1$, $Q_2 = 1$ and $Q_3 = 1$. Sketch the output of each flip-flop if an input sequence 101101 is applied to D_0 synchronously with the clock (Q_0 is LSB).

13. Calculate the frequency and the duty cycle of the 555 astable multivibrator output for $C = 0.001\ \mu F$, $R_A = 2.2\ k\Omega$ and $R_B = 100\ k\Omega$. (**Ans** 50.5%)

OBJECTIVE TESTS – 72

1. If a NAND latch has a 1 on the SET input and a 0 on the CLEAR input, then the SET input goes to 0, the latch will be:

(*a*) HIGH (*b*) LOW

(*c*) Invalid (*d*) None of these

2. The invalid state of a NAND latch occurs when

(*a*) $S = 1$, $C = 0$

(*b*) $S = 0$, $C = 1$

(*c*) $S = 1$, $C = 1$

(*d*) $S = 0$, $C = 0$

3. The invalid state of a NOR latch occurs when

(*a*) $S = 1$, $C = 0$

(*b*) $S = 0$, $C = 1$

(*c*) $S = 1$, $C = 1$

(*d*) $S = 0$, $C = 0$

4. Like a latch, the flip-flop belongs to a category of logic circuits known as

(*a*) monostable multivibrators

(*b*) astable multivibrators

(*c*) bistable multivibrators

(*d*) none of these

5. Which of the following flip-flops is used as a latch?

(*a*) *J-K* flip-flop

(*b*) *S-C* flip-flop

(*c*) *D* flip-flop

(*d*) *T* flip-flop

6. A flip-flop which can have an uncertain output state is :

(*a*) *J-K* flip-flop

(*b*) *S-C* flip-flop

(*c*) *D* flip-flop

(*d*) *T* flip-flop

7. The purpose of a clock input to a flip-flop is to

(*a*) clear the device

(*b*) set the device

(*c*) always cause the output to change the states

(*d*) cause the output to assume a state dependent on the controlling (*S - C, J-K* or *D*) inputs.

8. A feature that distinguishes the *J-K* flip-flop from an *S-C* flip-flop is the

(*a*) toggle condition

(*b*) preset input

(*c*) type of clock

(*d*) clear input.

9. A J-K flip-flop is in the toggle condition when

(*a*) $J = 1$, $K = 0$

(*b*) $J = 1$, $K = 1$

(*c*) $J = 0$, $K = 0$

(*d*) $J = 0$, $K = 1$

10. A *J-K* flip-flop with $J = 1$ and $K = 1$ has a 10 kHz clock input. The Q = output is,

(*a*) constantly LOW

(*b*) constantly HIGH

(*c*) a 5 kHz square wave

(*d*) a 10 kHz square wave

11. For an edge-triggered *D* flip-flop,

(*a*) a change in the state of the flip-flop can occur only at a clock pulse edge

(*b*) the state that flip-flop goes to depends on the *D* input

(*c*) the output follows the input at each clock pulse

(*d*) all of these answers

12. The flip-flop shown in Fig. 72.109 logically behaves as

(*a*) a *D* flip-flop (*b*) *a J-K* flip-flop

(*c*) a *T* flip-flop (*d*) an *R-S* flip-flop

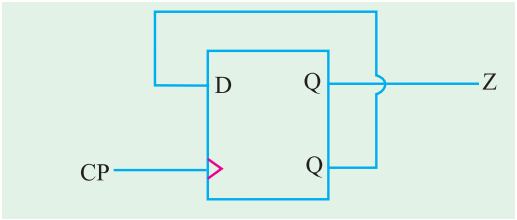


Fig. 72.109

(Grad. I.E.T.E. June, 1997)

13. A *J-K* flip-flop is a device to

(*a*) divide the frequency by 2

(*b*) divide the frequency by 4

(*c*) generate waveform of same frequency as that of the input

(*d*) cannot be used for frequency division.

14. A 1 µs pulse can be converted into a 1 ms pulse by using

(*a*) a monostable multivibrator

(*b*) an astable multivibrator

(*c*) a bistable multivibrator

(*d*) a *J-K* flip-flop

(U.P.S.C. Engg. Services, 1999)

15. The following is not a sequential circuit

(*a*) *J-K* flip-flop (*b*) counter

(*c*) full-adder (*d*) shift register

(Dip. I.E.T.E., Dec. 1996)

16. A one = shot is a type of

(*a*) monostable multivibrator

(*b*) astable multivibrator

(*c*) bistable multivibrator

(*d*) timer

17. An astable multivibrator

(*a*) requires a periodic trigger input

(*b*) has one stable states

(*c*) is an oscillator

(*d*) produces a non-periodic pulse output.

18. A 1 msec pulse can be converted to a 10 msec pulse by using:

(*a*) an astable multivibrator

(*b*) a monostable multivibrator

(*c*) a bistable multivibrator

(*d*) a J-K flip-flop

(U.P.S.C. Engg. Services 1990)

19. A retriggerable one shot is one which

(*a*) can be triggered only once

(*b*) has two quassi-stable states

(*c*) cannot be triggered until full pulse has been outputted.

(*d*) is capable of being triggered while the output is being generated.

20. The output of the circuit shown in Fig. 72.110 will be

(*a*) delayed pulses

(*b*) squarewaves

(*c*) trianglular waves

(*d*) trapezoidal waves.

(U.P.S.C Engg. Services 1997)

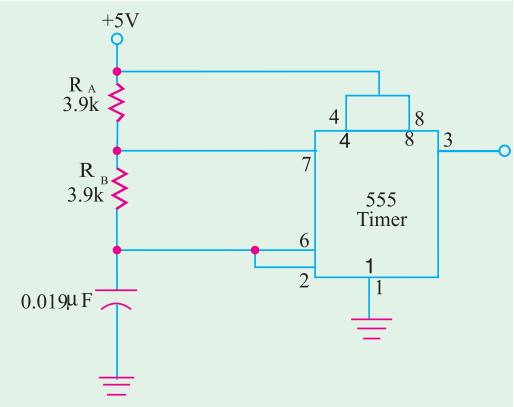


Fig. 72.110

21. The input waveform V_i and the output waveform V_o of a Schmitt NAND are shown in Fig 72.111. The duty cycle of the output waveform is

(*a*) 100 % (*b*) 85.5 %
(*c*) 72.2 % (*d*) 25 %

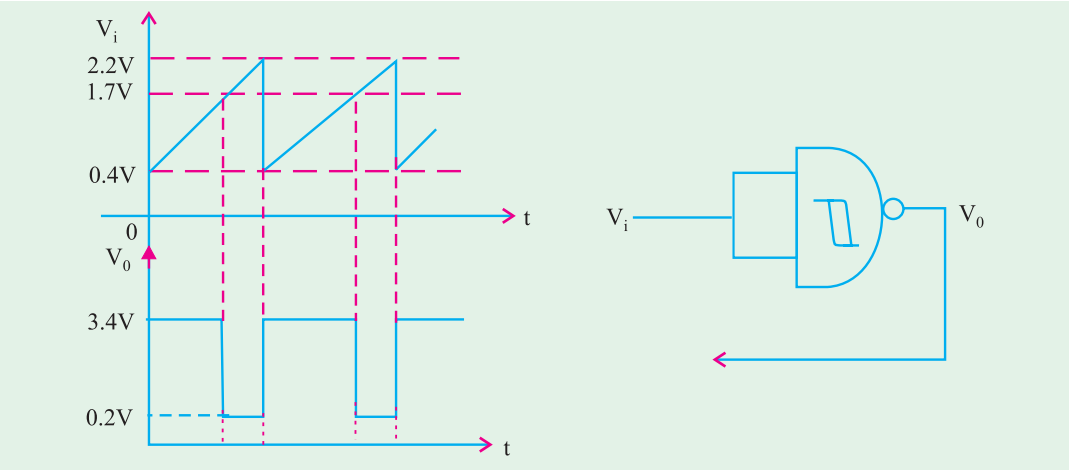


Fig. 72.111

(U.P.S.C Engg. Services. 1999)

22. Which of the following circuit is used for production of delays?

(*a*) astable multivibrator
(*b*) bistable multivibrator
(*c*) monostable multivibrator
(*d*) schmitt-trigger

23. A Schmitt-trigger can be used as a

(*a*) comparator only
(*b*) square-wave generator only
(*c*) flip-flop only
(*d*) comparator, square wave generator or flip-flop.

24. A 555 IC timer can be used to operate as

(*a*) as monostable multivibrator
(*b*) an astable multivibrator
(*c*) a voltage controlled oscillator
(*d*) all of the above

ANSWERS

1. (*a*)	**2.** (*d*)	**3.** (*c*)	**4.** (*c*)	**5.** (*b*)	**6.** (*b*)
7. (*d*)	**8.** (*a*)	**9.** (*b*)	**10.** (*c*)	**11.** (*d*)	**12.** (*c*)
13. (*a*)	**14.** (*a*)	**15.** (*c*)	**16.** (*a*)	**17.** (*c*)	**18.** (*b*)
19. (*d*)	**20.** (*b*)	**21.** (*c*)	**22.** (*c*)	**23.** (*a*)	**24.** (*d*)

GO To FIRST

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CHAPTER 73

ELECTRONIC INSTRUMENTS

Learning Objectives

- Introduction
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- Variation of Basic Meter Movement
- Converting Basic Meter to DC Ammeter
- Multirange Meter
- Measurement of Current
- Loading Effect of a Voltmeter
- Ohmmeter
- The Multimeter
- Rectifier Type AC Meter
- Electronic Voltmeters
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- Cathode Ray Tube (CRT)
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- Dual Trace CRO—Dual Beam CRO
- Lissajous Figures
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The Digital Voltmeter

73.1. Introduction

Electronic instrumentation is such an interesting field that it combines elements of technologies ranging from the nineteenth to the twenty first centuries. Modern computer-based instrumentation is now evident in every reasonably equipped laboratory and workshop and in catalogs and advertisements of all of the manufacturers. Yet at the root of many space-age instruments is circuitry, such as the wheatstone bridge that is found in nineteenth-century textbooks. Although newer techniques are still in widespread use in new as well as old instruments. In this chapter on electronic instruments you will find both types discussed.

The scientific and technological progress of any nation depends on its ability to measure, calculate and finally, estimate the unknown. Also, the success of an engineer or technician is judged by his ability to measure precisely and to correctly interpret the circuit performance. There are three ways of making such measurements :

(*a*) by mechanical means–like measuring gas pressure by Bourdon pressure gauge.

(*b*) by electrical means–like measuring potential difference with an electrical voltmeter.

(*c*) by electronic means–which is a very sensitive way of detecting the measured quantity because of amplification provided by the active electron device.

The electronic instruments generally have higher sensitivity, faster response and greater flexibility than mechanical or electrical instruments in indicating, recording and, where required, in controlling the measured quantity.

73.2. Analog and Digital Instruments

The deflection type instruments with a scale and movable pointer are called ***analog*** instruments. The deflectiion of the pointer is a function of (and, hence, analogous to) the value of the electrical quantity being measured.

Digital instruments are those which use logic circuits and techniques to obtain a measurement and then display it in numerical-reading (digital) form. The digital readouts employ either *LED* displays or liquid crystal displays (*LCD*).

Some of the advantages of digital instruments over analog instruments are as under :

1. easy readability

2. greater accuracy

3. better resolution

4. automatic polarity and zeroing

73.3. Functions of Instruments

Functionally, different instruments may be divided into the following three categories :

1. Indicating instruments

These are the instruments which indicate the instantaneous value of quantity being measured, at the time it is being measured. The indication is in the form of pointer deflection (analog instruments) or digital readout (digital instruments). Ammeters and voltmeters are examples of such instruments.

2. Recording instruments

Such instruments provide a graphic record of the variations in the quantity being measured over a selected period of time. Many of these instruments are electromechanical devices which use paper charts and mechanical writing instruments such as an inked pen or stylus.

Electronic recording instruments are of two types :

(*a*) null type–which operate on a comparison basis.

(*b*) galvanometer type–which operate on deflection type.

3. Controlling instruments

These are widely used in industrial processes. Their function is to control the quantity being measured with the help of information feed back to them by monitoring devices. This class forms the basis of automatic control systems (automation) which are extensively employed in science and industry.

73.4. Electronic Versus Electrical Instruments

Both electrical and electronic instruments measure electrical quantities like voltage and current etc. Purely electrical instruments do not have any built-in amplifying device to increase the amplitude of the quantity being measured. The common dc voltmeter based on moving-coil meter movement is clearly an electrical instrument.

The electronic instruments always include in their make-up some active electron device such as vacuum tube, semiconductor diode or an integrated circuit etc.

As seen, the main distinguishing factor between the two types of instruments is the presence of an electron device in the electronic instruments. Of course, movement of electrons is common to both types, their main difference being that control of electron movement is more effective in electronic instruments than in electrical instruments.

Although electronic instruments are usually more expensive than their electrical counterparts, they offer following advantages for measurements purposes :

1. since electronic instruments can amplify the input signal, they possess very high sinsitivity *i.e.* they are capable of measuring extremely small (low-amplitude) signals,
2. because of high sensitivity, their input impedance is increased which means less loading effect when making measurements,
3. they have greater speed *i.e.* faster response and flexibility,
4. they can monitor remote signals.

73.5. Essentials of an Electronic Instrument

As shown Fig. 73.1, an electronic instrument is made up of the following three elements :

1. Transducer

It is the first sensing element and is required only when measuring a non-electrical quantity, say, temperature or pressure. Its function is to convert the non-electrical physical quantity into an electrical signal.

Of course, a transducer is not required if the quantity being measured is already in the electrical form.

2. Signal Modifier

It is the second element and its function is to make the incoming signal suitable for application to the indicating device.

For example, the signal may need amplification before it can be properly displayed. Other types of signal modifiers are : voltage dividers for reducing the amount of signal applied to the indicating device or wave shaping circuits such as filters, rectifiers or chopper etc.

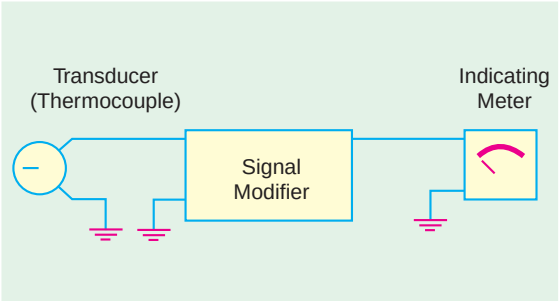


Fig. 73.1

3. Indicating Device

For general purpose instruments like voltmeters, ammeters or ohm meters, the indicating device is usually a deflection type meter as shown in Fig. 73.1. In digital readout instruments, the indicating device is of digitial design.

73.6. Measurement Standards

All instruments, whether electrical or electronic, are calibrated at the time of manufacture against a measurement standard.

1. International Standards

These are defined by international agreement and are maintained at the international Bureau of Weights and Measurements in Paris.

2. Primary Standards

These are maintained at national standards laboratories in each country. They are not available for use outside these laboratories. Their principal function is to calibrate and verify the secondary standards used in industry.

3. Secondary Standards

These are the basic reference standards used by industrial laboratories and are maintained by the particular industry to which they belong. They are periodically sent to national laboratory for calibration and verification against primary standards.

4. Working Standards

These are the main tools of a measurement laboratory and are used to check and calibrate the instrument used in the laboratory.

73.7. The Basic Meter Movement

It is also called D' Arsonval meter movement or a permanent-magnet moving-coil (PMMC) meter movement. Since it is widely used in electronic instruments, it is worthwhile to discuss its construction and principle of operation.

1. Construction

As shown in Fig. 73.2, it consists of a permanent horse-shoe magnet with soft iron pole pieces attached to it. Between the two pole-pieces is situated a cylinder-shaped soft iron core around which moves a coil of fine wire wound on a light metal frame. The metal frame is mounted in jewel bearings

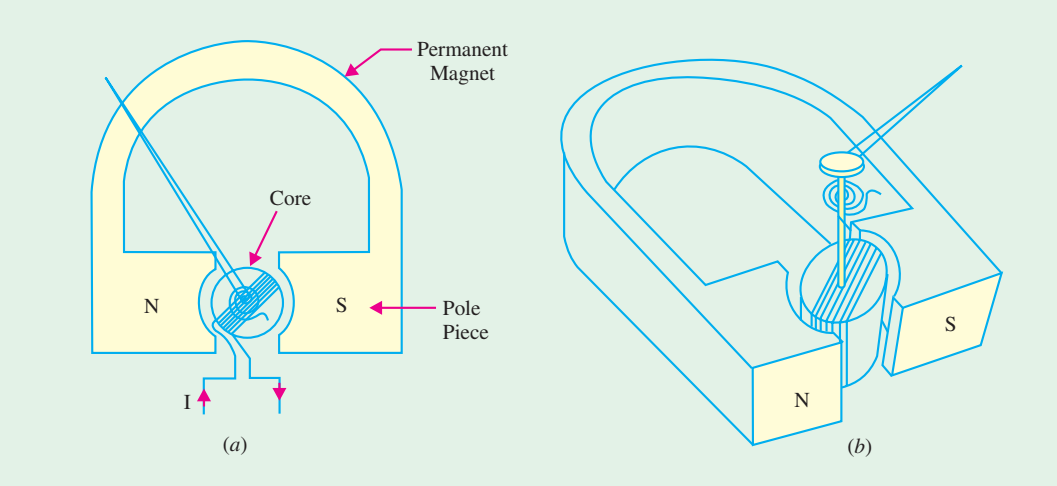


Fig. 73.2

so that it can rotate freely. A light pointer attached to the moving coil moves up-scale as the coil rotates when current is passed through it. The rotating coil is prevented from continuous rotation by a spring which provides restoring torque.

The moving coil movement described above is being increasingly replaced by tautband move-

ment in which the moving coil and the pointer are suspended between bands of spring metal so that the restoring force is torsional. The bands perform two functions ***(i)*** they support the coil and ***(ii)*** they provide restoring torque thereby eliminating the pivots and jewels used with coil spring movement.

As compared to pivoted movement, the taut-band has the advantages of

1. greater sensitivity *i.e.* small full-scale deflection current
2. ruggedness,
3. minimal friction,
4. easy to manufacture.

2. Principle of Operation

This meter movement works on the ***motor*** principle and is a current-responding device. The deflection of the pointer is directly proportional to the amount of current passing through the coil.

When direct current flows through the coil, the magnetic field so produced reacts with the field of the permanent magnet. The resultant force turns the coil alongwith its pointer. The amount of deflection is directly proportional to the amount of current in the coil. Hence, their scale is linear. With correct polarity, the pointer reads up-scale to the right whereas incorrect polarity forces the pointer off-scale to the left.

73.8. Characteristics of Moving Coil Meter Movement

We will discuss the following three characteristics :

(i) full-scale deflection current (I_m),
(ii) internal resistance of the coil (R_m),
(iii) sensitivity (S).

1. Full-scale Deflection Current (I_m)

It is the current needed to deflect the pointer all the way to the right to the last mark on the calibrated scale. Typical values of I_m for D' Arsonval movement vary from 2 μA to 30 mA.

It should be noted that for smaller currents, the number of turns in the moving coil has to be ***more*** so that the magnetic field produced by the coil is strong enough to react with the field of the permanent magnet for producing reasonable deflectin of the pointer. Fine wire has to be used for reducing the weight of the moving coil but it increases its resistance. Heavy currents need thick wire but lesser number of turns so that resistance of the moving coil is comparatively less. The schematic symbol is shown in Fig. 73.3.

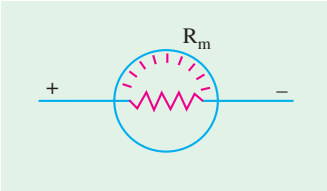


Fig. 73.3

2. Internal Resistance (R_m)

It is the dc ohmic resistance of the wire of the moving coil. A movement with smaller I_m has higher R_m and *vice versa.* Typical values of R_m range from 1.2 Ω for a 30 mA movement to 2 kΩ for a 50 μA movement.

3. Sensitivity (S)

It is also known as ***current sensitivity*** **or** ***sensitivity factor.*** It is given by the reciprocal of full-scale deflection current I_m.

$$\therefore \qquad S = \frac{1}{I_m} \text{ ohm/volt.}$$

For example, the sensitivity of a 50-μA meter movement is

$$S = \frac{1}{50\,\mu A} = \frac{1}{50 \times 10^{-6}} \ \Omega/\text{V} = 20{,}000\ \Omega/\text{V} = 20\ \text{k}\Omega/\text{V}$$

The above figure shows that a full-scale deflection of 50 μA is produced whenever 20,000 Ω of resistance is present in the meter circuit ***for each volt of applied voltage.*** It also represents the ohms-per-volt rating of the meter. The sensitivity of a meter movement depends on the strength of the permanent magnet and number of turns in the coil. Larger the number of turns, smaller the amount of current required to produce full-scale deflection and, hence, higher the sensitivity. A high current sensitivity means a high quality meter movement. It also determines the lowest range that can be covered when the meter movement is modified as an ammeter (Art 73.10) or voltmeter (Art 73.12)

73.9. Variations of Basic Meter Movement

The basic moving-coil system discussed in Art 73.7 can be converted into an instrument to measure dc as well as ac quantities like current, voltage and resistance etc. Without any modification, it can carry a maximum current of I_m can withstand a maximum dc voltage $v = I_m R_m$.

1. DC instruments

(*a*) it can be made into a dc ammeter, milliammeter or microammeter by adding a suitable shunt resistor R_{sh} in parallel with it as shown in Fig. 73.4 (*a*),

(*b*) it can be changed into a dc voltmeter by connecting a multiplier resistor R_{mult} in series with it as shown in Fig. 73.4 (*b*),

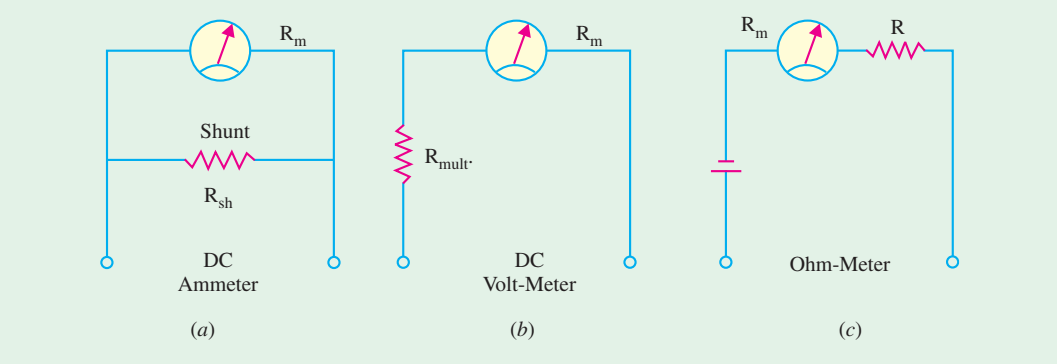


Fig. 73.4

(*c*) it can be converted into an ohmmeter with the help of a battery and series resistor R as shown in Fig. 73.4 (*c*).

2. AC Instruments

(*a*) it can be changed into an ac audio-frequency ammeter or voltmeter by simply adding an extra rectifier as shown in Fig. 73.5 (*a*).

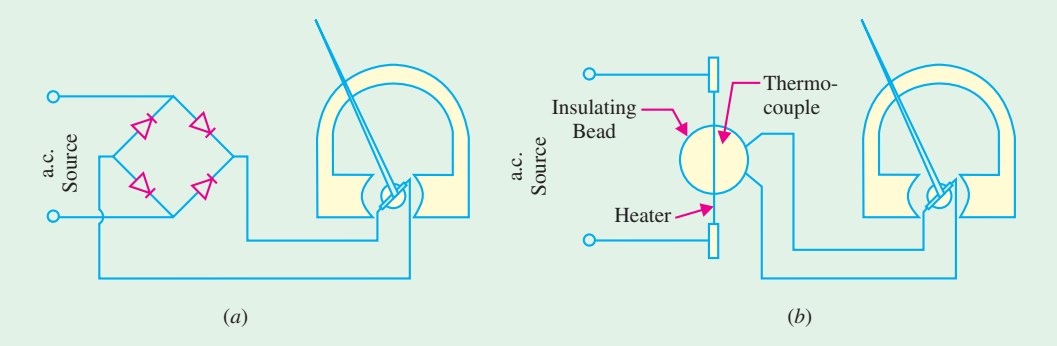
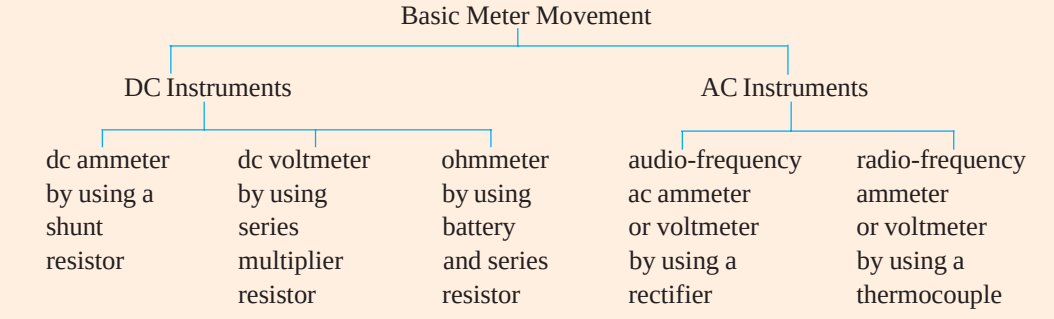


Fig. 73.5

(*b*) it can be converted into a radio frequency ammeter or voltmeter by adding a thermocouple as shown in Fig. 73.5 (*b*).

The above modifications of the basic meter movement have been tabulated below :



73.10. Converting Basic Meter to DC Ammeter

As stated earlier and again shown in Fig. 73.6 (*a*), the basic meter movement can carry a maximum current of I_m *i.e.* its full-scale deflection current. However, its current range can be increased (*i.e.* multiplied) to any value by connecting a low resistance (called shunt resistance R_{sh}) in parallel with it as shown in Fig. 73.6 (*b*). The shunted meter works as an ammeter ***with an extended range.***

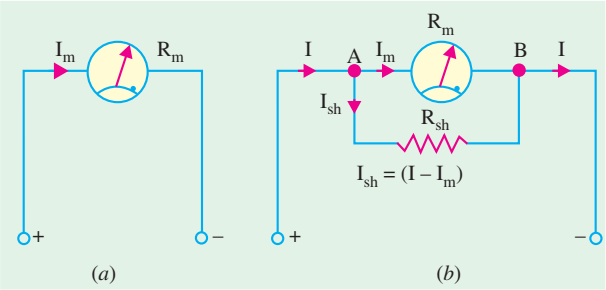


Fig. 73.6

Suppose, we want to measure a line current of I with the help of this meter movement. Obviously, the value of R_{sh} should be such as to shunt or bypass a current of $(I - I_m)$. As seen, range extension is from I_m to I. The ratio $I/I_m = n$ is known as the ***multiplying power or multiplying factor*** of the shunt. It means that a shunt allows the meter to measure current I which is n times larger than I_m.

Value of R_{sh}

In Fig. 73.6 (*b*), voltage across the meter and the shunt is the same because they are joined in parallel.

$$\therefore \quad I_m R_m = I_{sh} \,.\, R_{sh} = (I - I_m)\, R_{sh}$$

$$\therefore \quad R_{sh} = \frac{I_m}{(I - I_m)} \,.\, R_m = \frac{1}{(I / I_m - 1)} \,.\, R_m \quad \therefore \quad R_{sh} = \frac{R_m}{(n-1)}$$

Hence, n is the multiplying factor of the shunt. It is seen that larger the value of n *i.e.* greater the range extension required, smaller the shunt resistance needed. Incidentally, it may be noted that the resistance of the ***shunted*** meter is

$$= R_m \,||\, R_{sh} = \frac{R_m\, R_{sh}}{R_m + R_{sh}}$$

It is much less than either R_m or R_{sh}

Example 73.1. *It is required to convert a 5-mA meter with 20 Ω internal resistance into a 5-A ammeter. Calculate*

(*a*) *the value of shunt resistance required*

(*b*) *multiplying factor of the shunt.*

Solution. Here, $I = 5A$, $I_{sh} = 5\text{ mA} = 0.005\,A$, $R_m = 20\,\Omega$

(a) $$R_{sh} = \frac{I_m}{(I - I_{sh})} . R_m = \frac{0.005}{(5 - 0.005)} \times 20 = \mathbf{0.02\ \Omega\ (approx)}$$

(b) $$n = \frac{I}{I_{sh}} = \frac{5}{0.005} = \mathbf{1000}$$

Note. $$R_{sh} = \frac{R_m}{(n-1)} = \frac{20}{(1000-1)} = \frac{20}{999} = 0.02\,\Omega$$ —as found above

Fig. 73.7 shows such an ammeter connected in a load circuit.

73.11. Multirange Meter

The shunt resistance discussed above gives only a single range ammeter. By using universal shunt (also called Ayrton shunt), we can obtain a multirange ammeter as shown in Fig. 73.8.

It is seen that by changing the switch position from *A* to *B* to *C* and finally to *D*, the current range can be extended as desired.

1. Switch at A

Here, the meter is unshunted and so can read up to its full-scale deflection current of 1 mA only.

2. Switch at B

In this case, R_1 shunts the meter and extends its range to 10 mA *i.e.* increases it ten times.

Since $n = 10$ $\quad\therefore\quad R_1 = \frac{R_m}{(n-1)} \cong \frac{100\,\Omega}{(10-1)} = 11.11\,\Omega$

3. Switch at C

Here, R_2 shunts R_m and extends meter range from 1 mA to 0.1 A *i.e.* to 100 mA. Obviously,

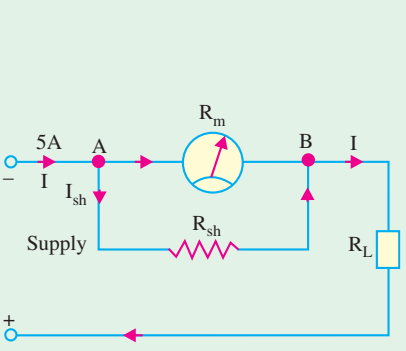


Fig. 73.7

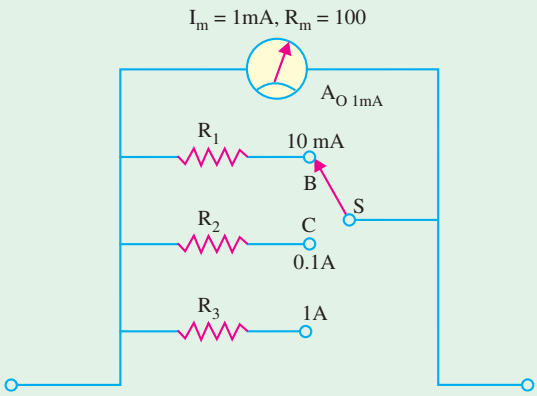


Fig. 73.8

$$n = \frac{100}{1} = 100 \quad\therefore\quad R_2 = \frac{100\,\Omega}{(100-1)} = 1.01\,\Omega$$

4. Switch at D

In this case, R_3 shunts R_m and extends the current range of the meter from 1 mA to 1.0 A *i.e.* 1.0 A to 1000 mA. Hence $n = 1000/1 = 1000$.

$$\therefore \quad R_3 = \frac{100\,\Omega}{(1000-1)} = 0.1001\,\Omega$$

Incidentally, it may be noted that greater the range extension, smaller the shunt resistance.

Alternative Method

An alternative circuit for range extension is shown in Fig. 73.9. It is called 'add on' method of shunting the meter because resistances can be added one after another for changing the range. Unlike in Fig. 73.8, there is no possibility of the meter being in the circuit without any shunt.

As seen, the universal, shunt consists of three resistances R_1, R_2 and R_3. How they are connected as a shunt is determined by the switch position. When S is at position A, the combination $(R_1 + R_2 + R_3)$ becomes connected across R_m. When S is at position B, $(R_2 + R_3)$ become connected in parallel across $(R_1 + R_m)$ and so on.

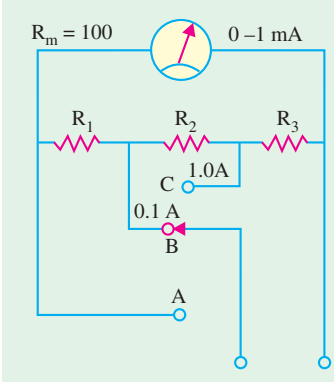


Fig. 73.9

1. Switch at A

In this case, multiplying factor n = 10 mA/1 mA = 10

$$\therefore \quad (R_1 + R_2 + R_3) = \frac{R_m}{(n-1)} = \frac{100}{9}\ \Omega \qquad ...(i)$$

2. Switch at B

Here, $(R_2 + R_3)$ become in parallel with $(R_1 + R_m)$ or $(R_1 + 100)$. Also, n = 100/1 = 100

$$\therefore \quad R_2 + R_3 = \frac{R_1 + 100}{99} \qquad ...(ii)$$

3. Switch at C

In this position, R_3 is in parallel with $(R_1 + R_2 + 100)$ and n = 1000/1 = 1000

$$\therefore \quad R_3 = \frac{R_1 + R_2 + 100}{999} \qquad ...(iii)$$

Solving for R_1, R_2 and R_3 from Eq. (i), (ii) and (iii) we have

$$R_1 = 10\ \Omega,\ R_2 = 1\ \Omega \text{ and } R_3\ 1/9\ \Omega$$

73.12. Measurement of Current

While measuring current flowing in a circuit, following two points must be kept in mind :

1. The current meter must be connected in series with the circuit where current is to be measured (Fig. 73.7). The full circuit current cannot flow through the meter unless it is made a series component.
2. The dc meter must be connected with the correct polarity for the pointer to read up-scale to the right. Reversed polarity deflects the pointer down-scale to the left forcing it against the stop which can sometime bend the pointer.

73.13. Converting Basic Meter to DC Voltmeter

The basic meter movement can measure a maximum voltage of $I_m R_m$ which is very small [Fig. 73.10 (a)]. However, its voltage range can be extended to any value by connecting a large resistance in series with it as shown in Fig. 73.10 (b). The series resistance is also called ***multiplier resistance*** because it multiplies the voltage reading capability of the meter many times. It is usually connected inside the voltmeter case.

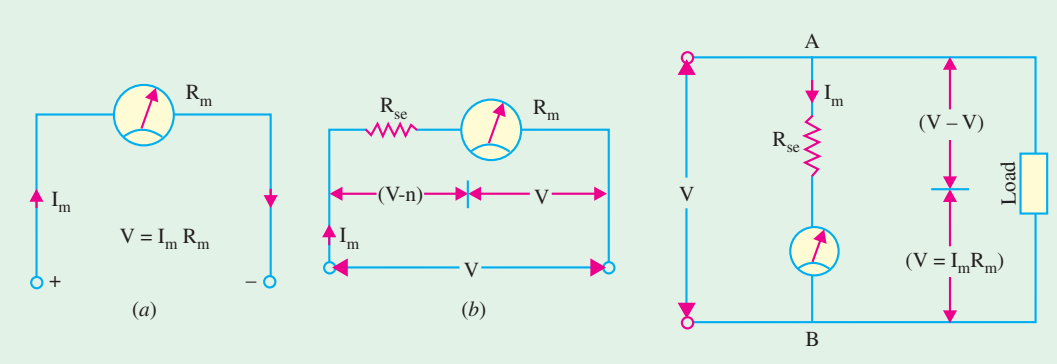


Fig. 73.10 Fig. 73.11

But it should be noted that the voltmeter is connected in parallel with the load across which the voltage is to be measured (Fig. 73.11).

Value of R_{sc}

Suppose, it is desired to extend the voltage range of the meter from v to V. The ratio V/v is known as the ***voltage multiplication.*** As seen from Fig. 73.11, drop across R_{se} is $(V - v)$ and current through it is the same as meter current *i.e.* I_m

$$\therefore \quad I_m R_{se} = (V - v) \qquad \text{...(i)}$$

$$\therefore \quad R_{se} = \frac{V - v}{I_m} = \frac{V - I_m R_m}{I_m} = \frac{V}{I_m} - R_m$$

The voltage multiplication (m) can be found from Eq. (i) above,

Dividing both sides by v, we get

$$\frac{I_m R_{se}}{v} = V_v - 1$$

$$\therefore \quad \frac{V}{v} = 1 + \frac{I_m R_{se}}{v} = 1 + \frac{I_m R_{se}}{I_m R_m}$$

$$\therefore \quad V_v = \left(1 + \frac{R_{se}}{R_m}\right) \quad \therefore \quad m = \left(1 + \frac{R_{se}}{R_m}\right)$$

It is seen that for a given meter, higher the series resistance, greater the voltage range extension.

Example 73.2. *A 50-μA meter movement with an internal resistance of kΩ is to be used as dc voltmeter of range 50 V. Calculate the*

(*a*) *multiplier resistance required and* **(*b*)** *voltage multiplication.*

Solution. (*a*)

$$R_{se} = \frac{V}{I_m} - R_m$$

$$= \frac{50}{50 \times 10^{-6}} - 1000$$

$$= 10^6 - 1000 = 999{,}000\ \Omega = \mathbf{999\ k\Omega}$$

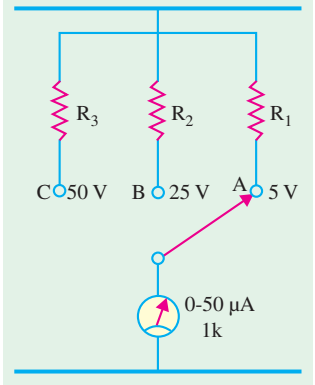


Fig. 73.12

73.14. Multirange DC Voltmeter

A multirange voltmeter with ranges of 0–5 V. 0–25 V and 0–50 V is shown in Fig. 73.12. Different values of resistors R_1, R_2 and R_3 can be found in the same way as in Art. 73.11. It would be found that for the meter movement shown in figure

$$R_1 = 99\ \text{K}, R_2 = 499\ \text{K}, R_3 = 999\ \text{K}$$

It is seen that higher the voltage range greater the multiplier resistance required (in almost the same proportion as the ranges).

73.15. Loading Effect of a Voltmeter

When the voltmeter resistance is not high as compared to the resistance of the circuit across which it is connected, the measured voltage becomes less. The decrease in voltage may be negligible or it may be appreciable depending on the sensitivity (ohms-per-volt rating) and input resistance of the voltmeter. It is called voltmeter ***loading effect*** because the voltmeter loads down the circuit across which it is connected. Since input resistance of electronic voltmeter is very high (10 M Ω or more), loading is not a problem in their case.

Consider the circuit shown in Fig. 73.13 in which two 15-K resistors are connected in series across a 100-V dc source. The drop across each is 50V. Now, suppose, that a 30-K voltmeter is connected across R_2 to measure voltage drop across it. Due to loading effect of the voltmeter, the reading is reduced from 50V to 40V as explained below. As seen from Fig. 73.13 (*b*), combined resistance of R_2 and voltmeter is 15 K || 30 K = 10 K.

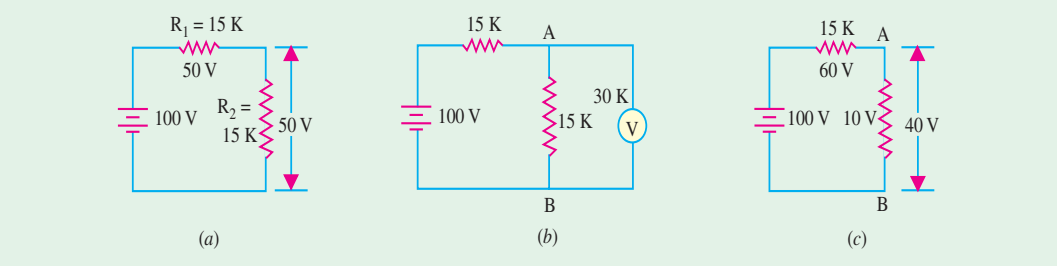


Fig. 73.13

$$\text{drop across 10 K} = \frac{10}{10+15} \times 100 = 40V$$

Loading effect can be minimized by using a voltmeter whose resistance is as high as possible as compared to that of the circuit across which it is connected.

Correction Formula

The loading effect can be neutralized by using the following formula :

$$V_{corr} = V_{means} + \frac{R_1 R_2}{R_v (R_1 + R_2)} \cdot V_{means}$$

where
V_{corr} = corrected voltage reading
V_{meas} = measured voltage reading
R_v = voltmeter resistance
R_1, R_2 = voltage dividing resistances in the circuit

In the above case,

$$V_{corr} = 40 + \frac{15 \times 15}{30\ (15+15)} \times 40 = 40 + 10 = \mathbf{50\ V}$$

73.16. Ohmmeter

The basic meter movement can be used to measure resistance it is combined with a battery and a current-limiting resistance as shown in Fig. 73.14 (*a*). In that case, it is known as an ohmmeter.

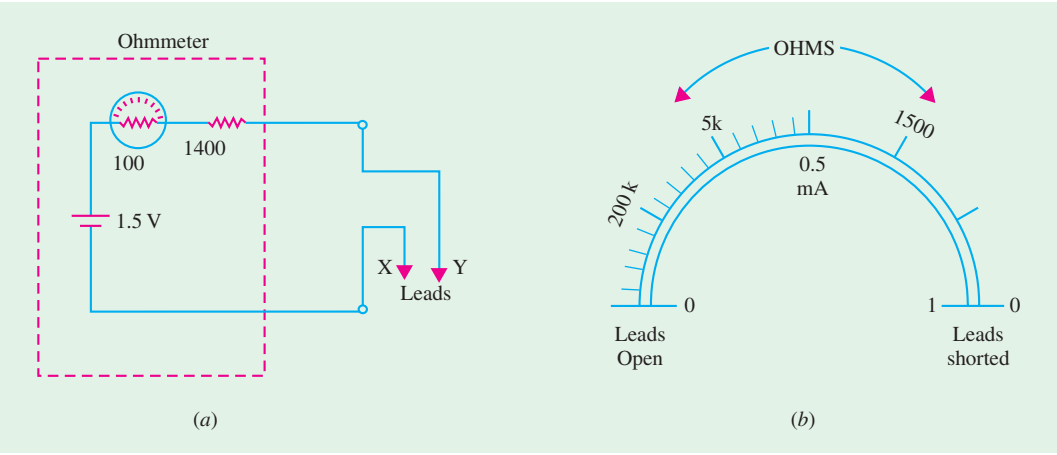


Fig. 73.14

For measuring resistance, the ohm-meter leads *X*-*Y* are connected across the unknown resistance after switching off the power in the circuit under test. Only in that case, the ohmmeter battery can provide current for the meter movement. Since the amount of current depends on the amount of external resistance, the meter scale can be calibrated in ohms (instead of mA).

When the leads *X*-*Y* are shorted, meter current is $1.5V/(100 + 1400)\ \Omega = 1$ mA. The meter shows full-scale deflection to the right. The ohmmeter reading corresponds to 0 Ω because external resistance is zero. When leads *X*-*Y* are open *i.e.* do not touch each other, meter current is zero. Hence, it corresponds to infinite resistance on the ohmmeter scale.

Following points about the ohmmeter are worth noting :

1. the resistance scale is non-linear *i.e.* it is expanded at the right near zero ohm and crowded at the left near infinite ohm. This nonlinearity is due to the reciprocal function $I = V/R$.;
2. the ohmmeter reads up-scale regardless of the polarity of the leads because direction of current is determined by the internal battery;
3. at half-scale deflection, external resistance equals the internal resistance of the ohmmeter.
4. the test leads should be shorted and 'ZERO OHMS' control adjusted to bring the pointer to zero on each range.



Fig. 73.15. Digital micro-ohmmeter



Fig. 73.16. Digital milli-ohmmeter

Fig. 73.15 shows a digital micro-ohmmeter having a range of 1μΩ – 2 kΩ with 3½ digit, 7-segment *LED* display. It has a basic accuracy of ± 0.2% ± 1 digit and is based on a design using *MOS LSI ICs* and glass epoxy *PCB*.

Fig. 73.16 shows a battery-operated portable digital milli-ohmmeter having a measurement range of 200 mΩ – 2 kΩ with an accuracy of ± 0.5% ± 1 digit. It has a 3½ digit 7-segment *LED* display.

73.17. The Multimeter

It is extensively used in cable industry, motor industry, transformer and switchgear industry. It is also called volt-ohm-milliammeter (VOM). It is a general purpose instrument having the necessary circuitry and switching arrangement for measuring ac/dc voltage or ac/dc current or resistance. It is simple, compact and portable because the only power it uses is the battery for the ohm-meter.



Fig. 73.17. A digital multimeter

Multimeters may be of analog type (Fig. 73.17) or digital type (Fig. 73.18). The analog type is of the pointer and scale type *i.e.* it uses the basic *D'* Arsoval meter movement. However digital multimeters (*DMMs*) are becoming increasingly popular because of their easy readability, numerical display and imporved accuracy.

Fig. 73.17 shows the photograph of an analog multimeter designed primarily for electrical, electronic, radio and TV engineers and technicians. It sells under the brand name of Motwane Multimeter 8 X Mark-III.

It is a 5-function, 30-range meter which measures high ac/dc voltages from 0 to 2.5 kV and ac/dc currents from 0 to 10 A. Its three resistance ranges cover from 0 to 20 MΩ. It is reputed for its excellent reliability, operational simplicity and easy portability.



Fig. 73.18
Courtesy : Fluke Corporation

Fig. 73.18 depicts a digital multimeter which can measure dc voltage upto 1000 V, ac voltages upto 750 V, ac/dc currents from 15 μA to 10 A and resistances from 0 Ω to 100 MΩ. It has a 5 digit multifunction vacuum fluorescent display allowing the user to measure two different parameters of the same signal from one test connection. The user can also view both measurements at the same time.



Fig. 73.19. Courtesy : Fluke Corporation

In Fig. 73.19 is shown a hand-held autoranging, digital multimeter (DMM) having high contrast, 4 digit *LCD* read-out. It has been designed for speed, accuracy and reliability.

73.18. Rectifier Type AC Meter

The D' Arsonval meter movement can be used for measuring alternating quantities provided a rectifier is added to the measuring circuit. A similar rectifier arrangement is found as part of *AC VOLTS* function in multimeters (Art. 73.17). Such as meters are more widely used than either (costly but more accurate) dynamometer type or more delicate thermal and hotwire type.

(*a*) With Half-Wave Rectifier

The circuit of an ac Voltmeter using half-wave diode recifier is shown in Fig. 73.20. Here, a half-wave rectifier has been combined in series with a dc meter movement.

When used as a dc voltmeter (*i.e.* without rectifier) it would have (say, for example) a range of $10V$. However, if an ac voltages of rms value $10V$ is applied across input terminals AB, it would read $4.5\,V$.

It is so because an *ac* voltage of rms value 10 V has a peak value of $10 \times \sqrt{2} = 14\ V$ and an average value of $0.636 \times 14 = 9V$. Since in the half-wave rectified output, one half-cycle is absent, the average for the full cycle is $9/2 = 4.5\ V$. The meter movement will, therefore read $4.5\ V$ *i.e.* 45% of the dc value. It may also be noted that ac sensitivity of a half-wave ac meter is only 45 per cent of the dc sensitivity.

(*b*) With Full-Wave Rectifier

The circuit is shown in Fig. 73.21. In this case, the meter reading would be 90% of rms input voltage *i.e.* 90% of the dc value.

73.19. Electronic Voltmeters

A *VOM* can be used to measure voltages but it lacks both sensitivity and high input resistances. Moreover, its input resistance is different for each range. The electronic voltmeter (*EVM*), on the other hand, has input resistance ranging from 10 $M\,\Omega$ to 100 $M\Omega$, thus producing less loading of the circuit under test than the *VOM*. Another advantage of *EVM* is that its input resitance remains constant over all ranges.

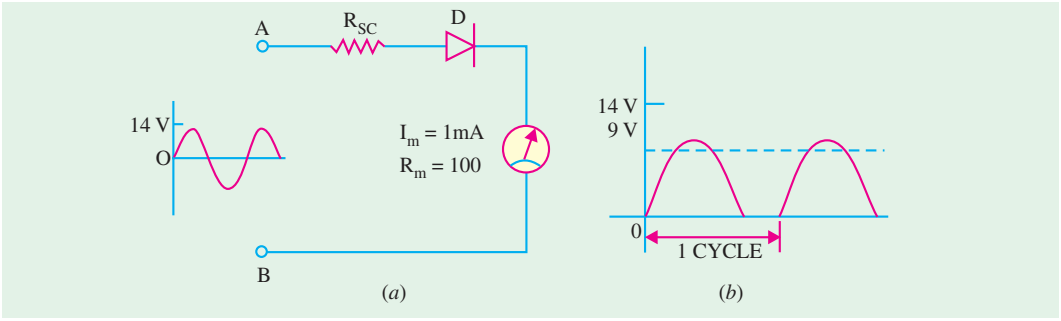


Fig. 73.20

Two types of voltmeters are in use today **(*i*)** analog and **(*ii*)** digital. However, a distinction must be made between a digital instrument and an instrument with digital readout. A digital instrument is one which uses internal circuitry of digital design. A digital readout instrument is one whose measuring circuitry is of analog design but the indicating device is of digital design.

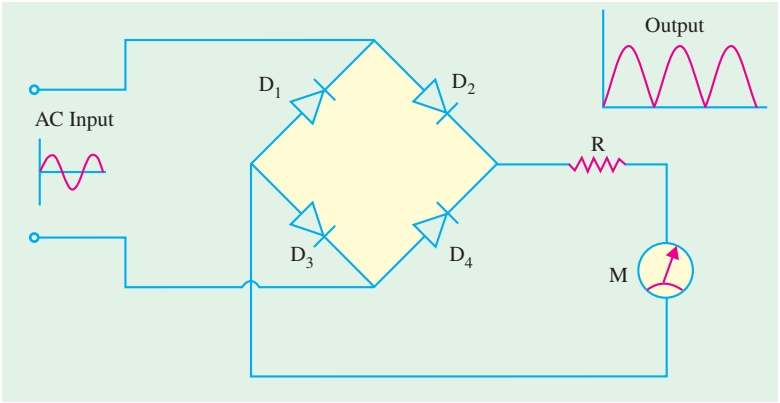


Fig. 73.21

The electronic voltmeters go by a variety of nemes reflecting the technology used.

(*i*) vacuum-tube voltmeter (*VTVM*)–it uses vacuum tubes with deflection meter movement,

(*ii*) digital voltmeters like transistor voltmeter (*TVM*) and *FET* voltmeter (*FETVM*).

73.20. Direct Current *FET VM*

The schematic diagram of a *FET VM* using difference amplifier is shown in Fig. 73.22. The two *FETs* are identical so that increase in the current of one *FET* is offset by corresponding decrease in the source current of the other. The two *FETs* form the lower arms of the balanced bridge circuit whereas the two drain resistors R_D form the upper arms. The meter movement is connected across the drain terminals of the *FETs*.

The circuit is balanced under zero-input-voltage condition provided the two *FETs* are identical. In that case, there would be no current through *M*. Zero-Adjust potentiometer is used to get null deflection in case there is a small current through *M* under zero-signal condition.

Full-scale calibration is adjusted with the help of variable resistor *R*.

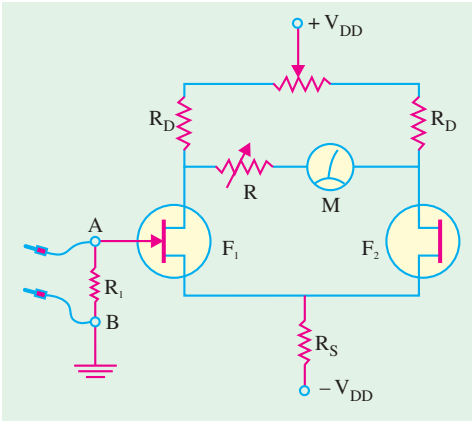


Fig. 73.22

When positive voltage is applied to the gate of F_1, some current flows through *M*. The magnitude of this current is found to be proportional to the voltage being measured. Hence, meter is calibrated in volts to indicate input voltage.

73.21. Electronic Voltmeter for Alternating Currents

The block diagram of such an *EVM* for ac measurements is shown in Fig. 73.23 where voltage divider allows range selection. The amplifier provides the necessary gain to establish voltmeter sensitivity as well as high input impedance. The negative feed-back ensures stability and accurate overall gain.

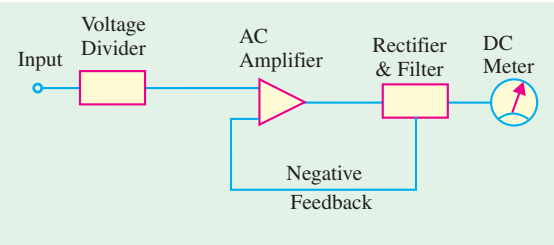


Fig. 73.23

73.22. The Digital Voltmeter (DVM)

Such a voltmeter displays measurements of dc or ac voltages as discrete numerals instead of pointer deflections on a continuous scale as in analog instruments. As compared to other voltmeters, a *DVM* offers the advantages of :

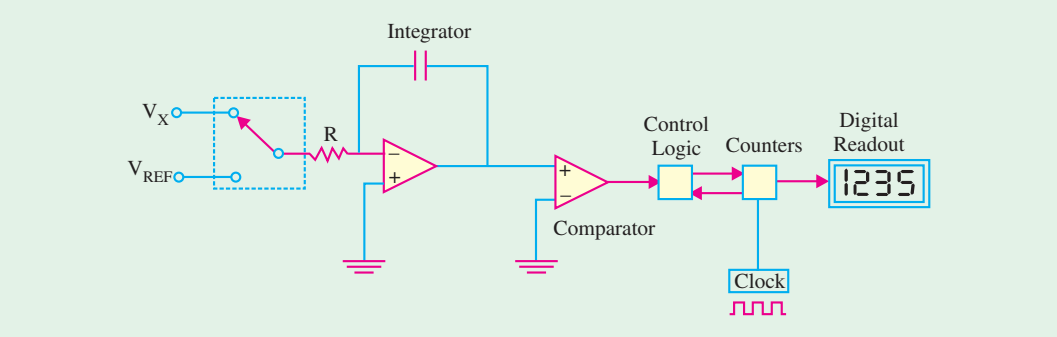


Fig. 73.24

1. greater, speed, **2.** higher accuracy and resolution, **3.** no parallax, **4.** reduced human error, **5.** compatibility with other digital equipment for further processing and recording.

With the development and perfection of IC modules, the size and power requirement of *DVMs* have reduced to a level where they can compete with conventional analog instrument both in price and portability.

The block diagram of a *DVM* based on dual-slope technique is shown in Fig. 73.24. The dual-slope analog-digital (*A - D*) converter consists of five basic blocks : an Op-Amp used as an integrator, a level comparator, a basic clock (for generating timing pulses), a set of decimal counters and a block of logic circuitry.

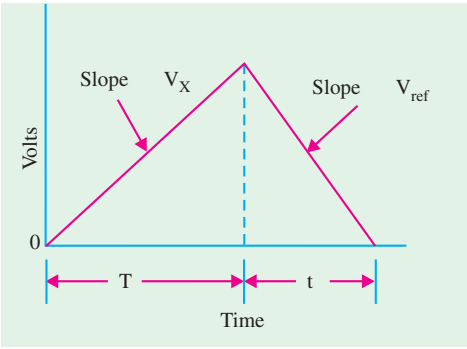


Fig. 73.25

The unknown voltage V_x is applied through switch *S* to the integrator for a known period of time *T* as shown in Fig. 73.25. This period is determined by counting the clock frequency in decimal counters. During time period *T*, *C* is charged at a rate proportional to V_x.

At the end of time interval *T*, *S* is shifted to the reference voltage V_{ref} of ***opposite*** polarity. The capacitor charge begins to decrease with time and results in a down-ward linear ramp voltage. During the second period a known voltage (*i.e.* V_{ref} is observed for an unkown time (*t*). This unknown time *t* is determined by counting timing pulses from the clock until the voltage across the capacitor reaches its basic reference value (reference may be ground or any other basic reference level). From similar triangles of Fig. 73.25.

$$\frac{V_x}{T} = \frac{V_{ref}}{t} \qquad \therefore \quad V_x = \frac{V_{ref}}{t} \times V_{ref}$$

The count after *t* which is proportional to the input voltage V_x is displayed as the measured voltage.

By using appropriate signal conditioners, currents, resistances and ac voltages can be measured by the same instrument.

DVMs are often used in data processing systems or data logging systems. In such systems, a number of analog input signals are scanned sequentially by an electronic system and then each signal is converted into an equivalent digital value by the *A/D* converter in the *DVM*. The digital value is then transmitted to a printer alongwith the information about the input line from which the signal has been derived. The whole data is then printed out. In this way, a large number of intput signals can be automatically scanned or processed and their values either printed or logged.



Fig. 73.26. Digital Voltmeter

Fig. 73.26 shows a portable digital dc micro-voltmeter (Agronic-112). It has a measurement range of 1 μV - 1000 V with an accuracy of ± 0.2% ± 1 digit. It uses latest *MOS LSI ICs* and glass epoxy *PCB*. It has 3½ digit, 7-segment *LED* display and is widely-used by the testing and servicing departments of industries, research laboratories, educational institutions and service centres.

73.23. Cathode-Ray Oscilloscope (*CRO*)

It is generally referred to as oscilloscope or scope and is the basic tool of an electronic engineer and technician as voltmeter, ammeter and wattmeter are those of an electrical engineer or electrician.

The *CRO* provides a two-dimensional visual display of the signal waveshape on a screen thereby allowing an electronic engineer to 'see' the signal in various parts of the circuit. It, in effect, gives the electronic engineer an eye to 'see' what is happening inside the circuit itself. It is only by 'seeing' the signal waveforms that he/she can correct errors, understand mistakes in the circuit design and thus make suitable adjustments.

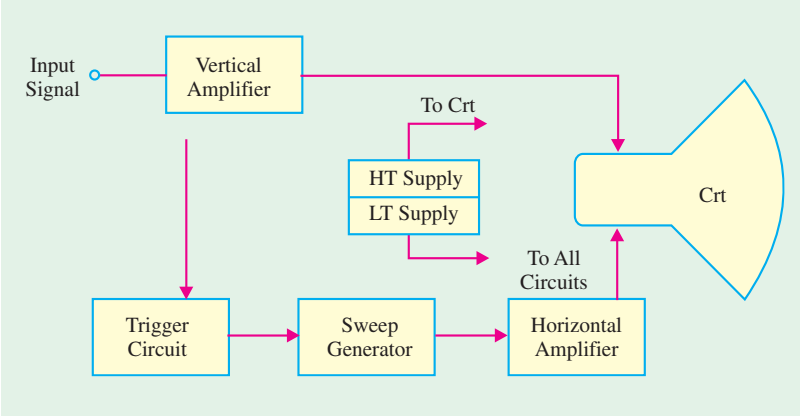


Fig. 73.27

An oscilloscope can display and also measure many electrical quantities like ac/dc voltage, time, phase relationships, frequency and a wide range of waveform characteristics like rise-time, fall-time and overshoot etc. Non-electrical quantities like pressure, strain, temperature and acceleration etc. can also be measured by using different transducers to first convert them into an equivalent voltage.

As seen from the block diagram of an oscilloscope (Fig. 73.27), it consists of the following major sub-systems :

1. **Cathode Ray Tube (*CRT*)**–it displays the quantity being measured.
2. **Vertical amplifier**–it amplifies the signal waveform to be viewed.
3. **Horizontal amplifier**–it is fed with a sawtooth voltage which is then applied to the *X*-plates.
4. **Sweep generator**–produces sawtooth voltage waveform used for horizontal deflection of the electron beam.
5. **Trigger circuit**–produces trigger pulses to start horizontal sweep.
6. **High and low-voltage power supply.**

The operating controls of a basic oscilloscope are shown in Fig. 73.28.

The different **terminals** provide.

1. horizontal amplifier input,
2. vertical amplifier input,
3. sync. input,
4. Z-axis input,
5. external sweep input.

As seen, different **controls** permit adjustment of

1. **Intensity**–for correct brightness of the trace on the screen,
2. **Focus**–for sharp focus of the trace.
3. **Horizontal centering**–for moving the pattern right and left on the screen.
4. **Vertical centering**–for moving the pattern up and down on the screen.
5. **Horizontal gain (also Time/div or Time/cm)**–for adjusting pattern width.
6. **Vertical gain (also volt/div or volt/cm)**–for adjusting pattern height.
7. **Sweep frequency**–for selecting number of cycles in the pattern.
8. **Sync. voltage amplitude**–for locking the pattern.

The different **switches** permit selection of :

1. sweep type,
2. sweep range,
3. sync. type

A *CRO* can operate upto 500 MHz, can allow viewing of signals within a time span of a few nanoseconds and can provide a number of waveform displays simultaneously on the screen. It also has the ability to hold the displays for a short or long time (of many hours) so that the original signal may be compared with one coming on later.

73.24. Cathode Ray Tube (CRT)

It is the 'heart' of an oscilloscope and is very similar to the picture tube in a television set.


A photography of Cathode ray tube

Construction

The cross-sectional view of a general-purpose electrostatic deflection *CRT* is shwon in Fig. 73.29. Its four major components are :

1. an electron gun for producing a stream of electrons,
2. focussing and accelerating anodes-for producing a narrow and sharply-focussed beam of electrons,
3. horizontal and vertical deflecting plates-for controlling the path of the beam,
4. an evacuated glass envelope with a phosphorescent screen which produces bright spot when struck by a high-velocity electron beam.

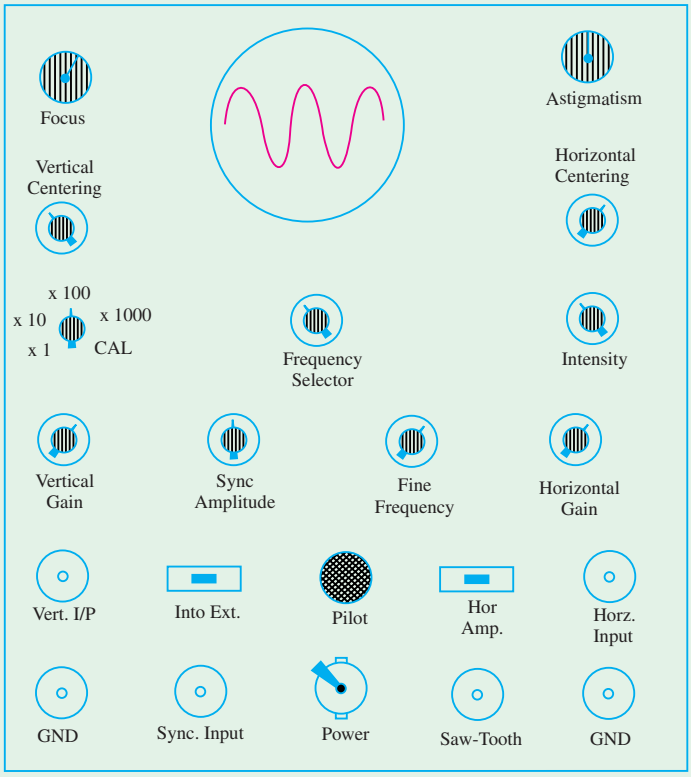


Fig. 73.28

As shown, a *CRT* is a self-contained unit like any electron tube with a base through which leads are brought out for different pins.

1. Electron Gun Assembly

The electron gun assembly consists of an indirectly-heated cathode *K*, a control grid *G*, a pre-accelerator anode A_1, focussing anode A_2 and an accelerating anode A_3. The sole function of the

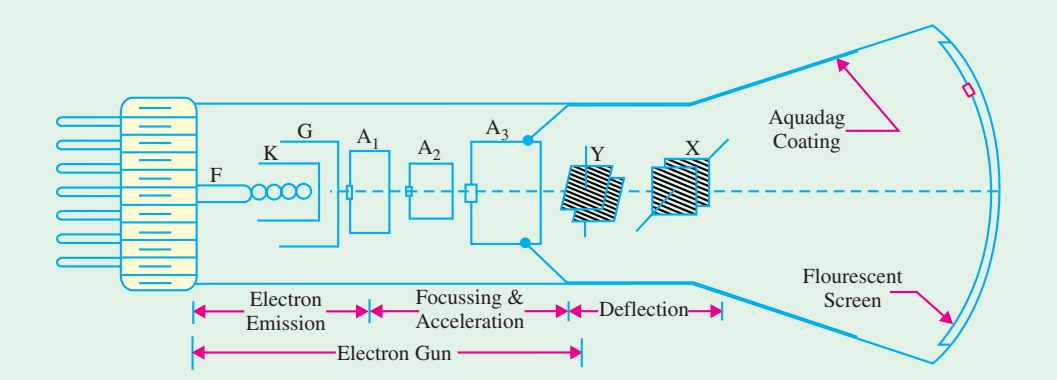


Fig. 73.29

electrons gun assembly is to provide a focussed beam of electrons which is accelerated towards the flourescent screen. The electrons are given off by thermionic emission from the cathode. The control grid is a metallic cylinder with a small aperture in line with the cathode and kept at a negative potential with respect to *K*. The number of electrons allowed to pass through the grid aperture (and, hence, the beam current) depends on the amount of the control grid bias. Since the intensity (or brightness) of the spot *S* on the screen depends on the strength of beam current, the knob controlling the grid bias is called the ***intensity control.***

The anodes A_1 and A_3, which are both at positive potential with respect to *K*, operate to accelerate the electron beam (Fig. 73.30). The cylindrical focussing anode A_2, being at negative potential, repels electrons from all sides and compresses them into a fine beam. The knob controlling the potential of A_2 provides the ***focus control.***

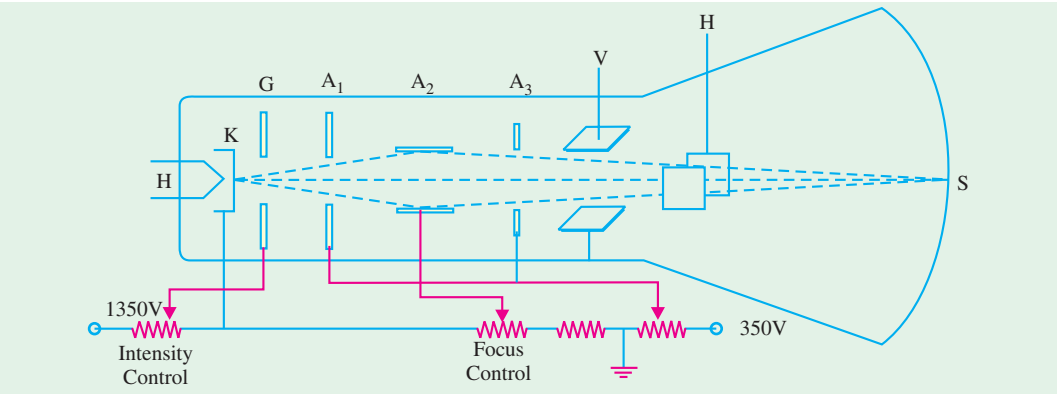


Fig. 73.30

2. Deflecting Plates

Two sets of deflecting plates are used for deflecting the thin pencil-like electronic beam both in the vertical and horizontal directions. The first set marked *Y* (nearer to the gun) is for vertical deflection and *X*-set is for horizontal deflection. When no potential is applied across the plates, beam passes between both sets of plates undeflected and produces a bright spot at the centre of the screen.

If upper Y-plate is given a positive potential, the beam is deflected upwards depending on the value of the applied potential. Similarly, the beam (and hence the spot) deflects downwards when lower Y-plate is made positive. However, if an ***alternating*** voltage is applied across the Y-plates, the spot keeps moving up and down thereby producing a vertical luminous trace on the screen due to persistance of vision. The maximum displacement of the spot from its central position is equal to the amplitude of the applied voltage.

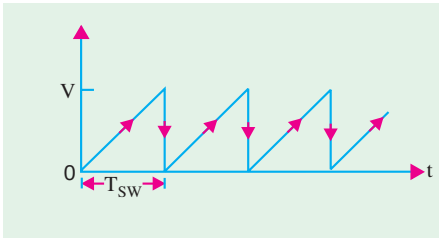


Fig. 73.31

The screen spot is deflected horizontally if similar voltages are applied to the X-plates. The dc potentials on the Y-and X-plates are adjustable by means of ***centring controls***.

It must be remembered that the signal to be displayed on the screen is always applied across the Y-plates. The voltage applied across X-plates is a ramp voltage *i.e.* a voltage which increases linearly with time. It has a sawtooth wave-form as shown in Fig. 73.31. It is also called horizontal time-base or sweep voltage. It has a sweep time of T_{sw}.

3. Glass Envelope

It is funnel-shaped having a phosphor-coated screen at its flared end. It is highly-evacuated in order to permit the electron beam to traverse the tube easily. The inside of the flared part of the tube is coated with a conducting graphite layer called Aquadag which is maintained at the same potential as A_3. This layer performs two functions ***(i)*** it accelerates the electron beam after it passes between the deflecting plates and ***(ii)*** collects the electrons produces by secondary emission when electron beam strikes the screen. Hence, it prevents the formation of negative charge on the screen.

The screen itself is coated with a thin layer of a flourescent material called phosphor. When struck by high-energy electrons, it glows. In other words, it absorbs the kinetic energy of the electrons and converts it into light-the process being known as ***flourescence***. That is why the screen is called ***flourescent screen***. The colour of the emitted light depends on the type of phosphor used.

73.25. Deflection Sensitivity of a *CRT*

Fig. 73.32 shows the upward deflection of an electron beam when it passes between the vertical or Y-plates of a *CRT*. The beam deflects upwards because the upper Y-plate has been made positive with respect to the lower plate. Reversing the polarity of the **applied** voltage would, obviously, cause the beam to deflect downwards.

The vertical deflection of the beam is

$$y = \frac{1}{2} \cdot D \cdot \frac{l}{D} \cdot \frac{V_d}{V_A}$$

where V_A is the accelerating voltage applied to the electrons which make up the electron beam.

The deflection sensitivity of a *CRT* is definition as the vertical deflection of the beam on the screen per unit deflecting voltage.

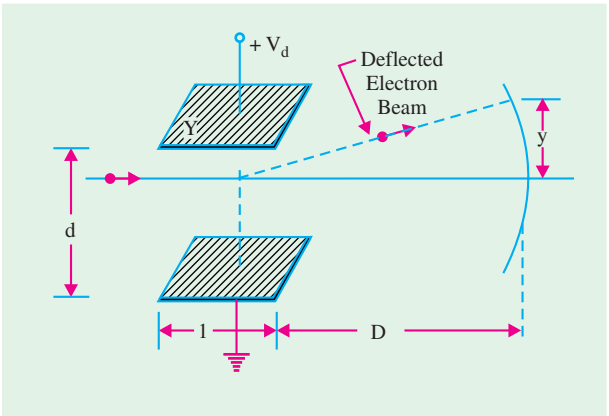


Fig. 73.32

$$\therefore \qquad S = \frac{y}{V_d}$$

Using the above equation, we get $S = \dfrac{lD}{2dV_A}$

The deflection factor which is defined as the reciprocal of deflection sensitivity is given by $G = 1/S$.

Substituting the value of S from above

$$G = 2 . \frac{d}{l} . \frac{V_A}{D} \text{ volt/metre}$$

73.26. Normal Operation of a *CRO*

The signal to be viewed or displayed on the screen is applied across the Y-plates of a *CRT*. But to see its waveform or pattern, it is essential to spread it out horizontally from left to right. It is achieved by applying a sawtooth voltage wave (produced by a time base generator) to X-plates. Under these conditions, the electron beam would move uniformly from left to right thereby graphic vertical variations of the input signal versus time. Due to repetitive tracing of the viewed waveform, we get a continuous display because of persistence of vision. However, for getting a stable stationary display on the screen, it is essential to synchronize the horizontal sweeping of the beam (sync) with the input signal across Y-plates. The signal will be properly synced only when its frequency equals the sweep-generator frequency.

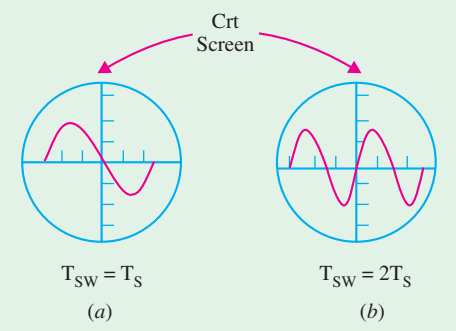


Fig. 73.33

In general, for proper synchronization of time-base with the signal, the condition is

$$T_{sw} = n\, T_s$$

where T_s the time-period of the signal and n is an integer.

If $n = 1$, then $T_{sw} = T_s$ *i.e.* time-periods of the sweep voltage and input signal voltage are equal, then one cycle of the signal would be displayed as shown in Fig. 73.33 (*a*).

On the other hand, if T_{sw} is twice T_s, then two cycles of the signal voltage would be displayed as shown in Fig. 73.33 (*b*). Obviously, three full cycles of the input voltage would be spread out on the screen when $T_{sw} = 3\, T_s$.

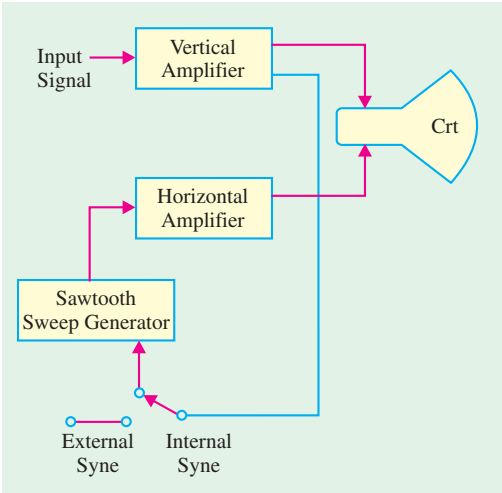


Fig. 73.34

Internal Synchronization

The periodic sawtooth voltage which is applied to X-plates for horizontal sweep (or scan) of the beam across the screen is usually provided by the unijunction relaxation oscillator. When the sawtooth voltage falls abruptly to zero, the beam experience no horizontal deflection and hence flies back almost instantly to the original (central) position.

The usual method of synchronizing the input signal is to use a portion of the input signal to trigger the sweep generator so that the frequency of the sweep signal is locked or synchronized to the input signal. It is called internal sync. because the synchronization is obtained by internal wiring connection as shown in the block the diagram of Fig. 73.34.

73.27. Triggered and Non-Triggered Scopes

Oscilloscopes may be classified into two basic types :

1. triggered sweep type.
2. recurrent sweep (free-running) type.

Triggered oscilloscopes, being more sophisticated, are generally used in industrial laboratories and plants, in engineering and technical school laboratories and in all those applications which require study of low- and high-frequency waveforms, for accurate measurement of time and timing relationships etc.

A non-triggered oscilloscope is generally used in servicing work where a certain amount of waveform error can be tolerated and bandwidth requirements are limited to a few MHz.



Fig. 73.35. A non-triggered oscilloscope

The sweep (or ramp) generator which produces sawtooth voltage for X-deflection plates is prresent in both types of scopes. In non-triggered oscilloscopes, this generator runs continuously (recurrent sweep) and the control and calibration of the sweep is based on the repetition freqeuncy of the sweep. For producing a stable stationary display, the sweep frequency has to be forced into synchronization with the input signal on the Y-plates. This is done by manually adjusting the free-running sweep freqeuncy to a value very close to signal frequency (or some submultiple of it) and then depending on the internal sync signal (derived from the input) for locking the sweep generator into exact step. Unfortunately, this method is limited to the display of signals which have contant frequency and amplitude. Hence voice or music signals from a microphone cannot be displayed on this scope because it has to be readjusted for each for new change in frequency. Moreover, a free-running or recurrent time base cannot display less than one complete cycle of the input signal on the scope screen. On the other hand, triggered time base can be adjusted to pick out a small part of a waveform which can then be expended horizontally for evaluation of waveform details.

The triggered oscilloscope is provided with a triggered (or driven) sweep. Here the input signal is caused to generate pulses that trigger the sweep thereby ensuring that the sweep is necessarily in step with the trigger that drives it. Hence, screen display remains stable in spite of variations in the frequency or amplitude of the input signal. It means that there is automatic mode of triggering in such scopes. Consequently, input signals of very short duration can be displayed for the simple reason that sweep is initiated by a trigger pulse derived from the waveform under observation.

Fig. 73.35 illustrates a triggered oscilloscope having a bandwidth 0-6 MHz, vertical sensitivity of 10 mV/div and horizontal sweep rate varying from 0.2 μs/div to 0.1 s/div.

73.28. Dual Trace CRO

Such oscilloscopes are used extensively by industrial firms and research laboratories. They produce a dual-trace display by means of electronic switching of two separate input signals. As shown in the block diagram of Fig. 73.36, there are two vertical input circuits marked channel *A* and *B* with identical pre-amplifiers. The outputs of the these preamplifiers are fed to an electronic switch which alternately connects them to the main vertical amplifier of the oscilloscope. In this type of scope, ther is only one electron beam. The electron switch is also capable of selecting a variety of display modes.

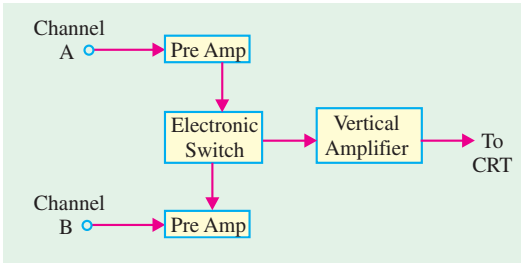


Fig. 73.36

In Fig. 73.37 is shown a dual trace oscilloscope (type *VOS*-26) which has a band-width of 5-15 MHz, vertical sensitivity of 10 mV/cm to 30 V/cm and calibrated sweep speed from 0.3 μs/cm to 10 ms/cm. It is a very sensitive yet simple oscilloscope which assures long life and easy maintenance.

73.29. Dual Beam CRO

Such a *CRO* has two sets of vertical deflection plates and has two electronic beams which produce two separate traces on the scope screen by using the same set of horizontal deflection plates. This scope makes it possible to observe two time-related wave forms at different points *i.e.* the electronic circuit.

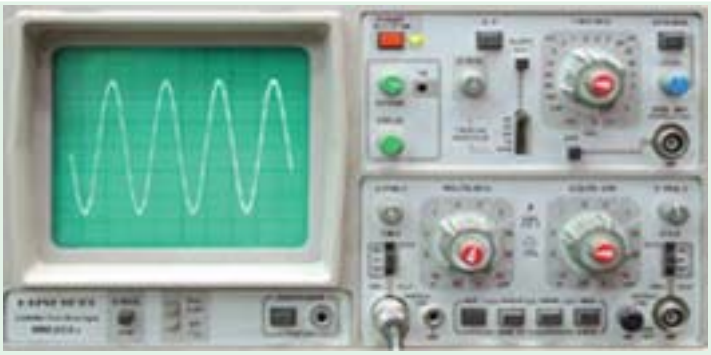


Fig. 73.37. Dual Beam CRO

Such a scope does not have the same number of display modes as the dual-trace scope yet it is ideally suited for different input signals.

73.30. Storage Oscilloscope

It can retain a *CRT* display for 10 to 150 hours after the pattern is first produced on the screen. It uses the phenomenon of secondary electron emission to build up and store electrostatic charges on the surface of an insulated target. Such oscilloscopes are especially useful.



Fig. 73.38. 4 channel storage oscilloscope

1. For real-time observation of events that occur only once.
2. For displaying the waveform of a very low-frequency signal.

Fig. 73.38 shows a 4 channel storage oscilloscope with 400 MHz bandwidth. It has a standard floppy disk drive which makes the saving of screen images or data to a disk, simple. The disk can then be inserted into your personal computer (*PC*) for importing to desk top publishing or spreadsheet programs. The storage oscilloscopes finds their application in biophysics/biomedical research, audio system measurement and analysis, power supply and power-related design, electrophysical and electromechanical system design etc.

73.31. Sampling CRO

It is specifically meant to observe very high frequency repetitive electric signals by using the sampling technique. Such high-frequency signals cannot be viewed by conventional oscilloscopes because its frequency range is limited by the gain-bandwidth product of its vertical amplifier. The sampling technique 'slows down' the signal frequency many thousands of times thereby making it easier to view it on the screen.

The oscilloscope shown in Fig. 73.38 is a sampling cathode ray oscilloscope. Its sample rate is 100 M samples per second.

73.32. Digital Readout CRO

It provides digital readout of the signal information such as voltage or time etc. in addition to the conventional *CRT* display. It consists of a high-speed laboratory *CRO* and an electronic counter, both contained in one cabinet.

73.33. Handheld Battery Operated Oscilloscope

Fig. 73.39 shows a handheld battery operated oscilloscope Model THS 720 P manufactured by Tektronix Corporation. It has built in 3-3/4 digit digital multimeter (DMM) with *data logger and power analyser. It has a bandwidth of over 100 MHz and the sampling rate is as high as 500 M samples per second. The oscilloscope and power analyser can operate simultaneously and independently on the same or separate signals. This type of an oscilloscope is extremely useful for electric/power electronics measurements. Examples of such measurements are : **(1)** testing and verifying correct operation of motors **(2)** checking transformer efficiency, **(3)** verifying power supply performance, **(4)** measuring the effect of neutral current etc.



Fig. 73.39. Courtesy : Tektronix Corporation

73.34. Lissajous Figures

Lissajous figures (or patterns) are named in honour of the French scientist who first obtained them geometrically and optically. They illustrate one of the earliest uses to whch the *CRO* was put.

Lissajous patterns are formed when two sine waves are applied simultaneously to the vertical and horizontal deflecting plates of a *CRO*. The two sine waves may be obtained from two audio oscillators as shown in Fig. 73.40. Obviously, in this case, ***a sine wave sweeps a sine-wave input signal.*** The shape of the Lissajous pattern depends on

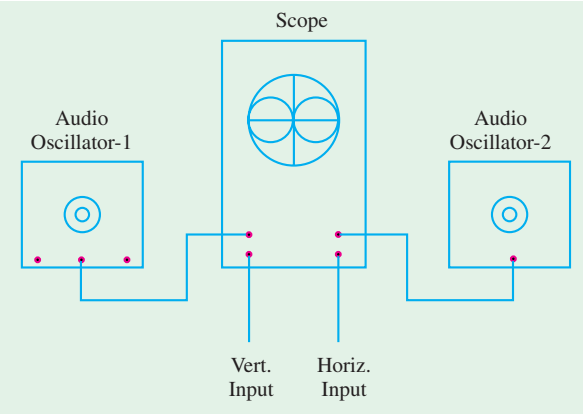


Fig. 73.40

* Data logger is a system which can acquire data and store it in a memory.

the frequency and phase relationship of the two sine waves.

Two sine waves of the ***same*** frequency and amplitude may produce a straight line, an ellipse or a circle depending on their phase difference (73.41).

In general, the shape of Lissajous figures depends on **(*i*)** amplitude, **(*ii*)** phase difference and **(*iii*)** ratio of frequency of the two waves.

Lissajous figures are used for **(*i*)** determining an unknown frequency by comparing it with a known frequency (*ii*) checking audio oscillator with a known-frequency signal and **(*iii*)**checking audio amplifiers and feedback networks for phase shift.

73.35. Frequency Determination with Lissajous Figures

The unknown signal is applied across one set of deflecting plates and a known signal across the other. By studying the resultant Lissajous pattern, unknown frequency can be found.

Depend on the frequency ratio, the various patterns obtained are shown in Fig. 73.42. The ratio of the two frequencies is given by

$$\frac{f_H}{f_v} = \frac{\text{No. of points of horizontal tangency}}{\text{No. of points of vertical tangency}}$$

In Fig. 73.41 (*a*), there is one point of tangency along the horizontal as well as vertical axis. Hence, $f_H = f_v$ *i.e.* the signals have the same frequency. In Fig. 73.42 (*e*) f_H/f_v = 3/2. In other words $f_H = 1.5\, f_v$.

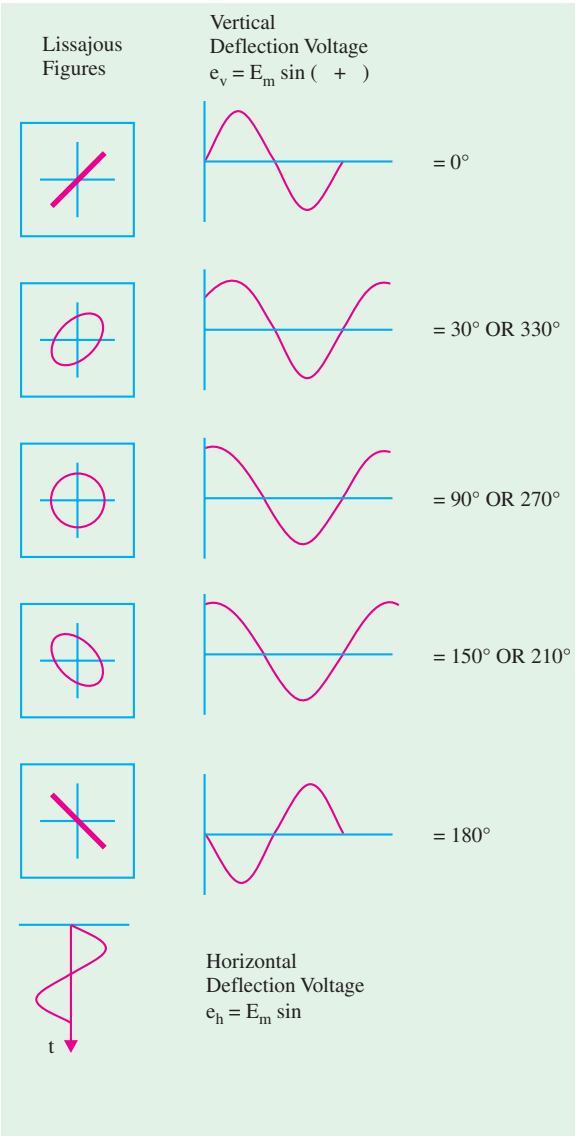


Fig. 73.41

It should be noted in passing that this method of frequency determination has limitations and is being discarded gradually because low-cost digital frequency counters are becoming increasingly available in the market Fig. 73.43. The two main limitatioins of this method are as under :

(*i*) the numerator and denominator of the frequency ratio must be whole numbers,

(*ii*) the maximum ratio of frequencies that can be used is 10 : 1. Beyond that, the Lissajous patterns become too complex to analyse.

Fig. 73.43 shows a 10-digit digital frequency counter Model No. PM 6685 manufactured by Fluke Corporation. This can measure frequencies from 10 Hz to 300 MHz. This is an ideal instrument for *R* and *D* laboratories, testing, servicing and even outside the lab environment such as in base station transmitters of large telecommunication networks like *GSM*.

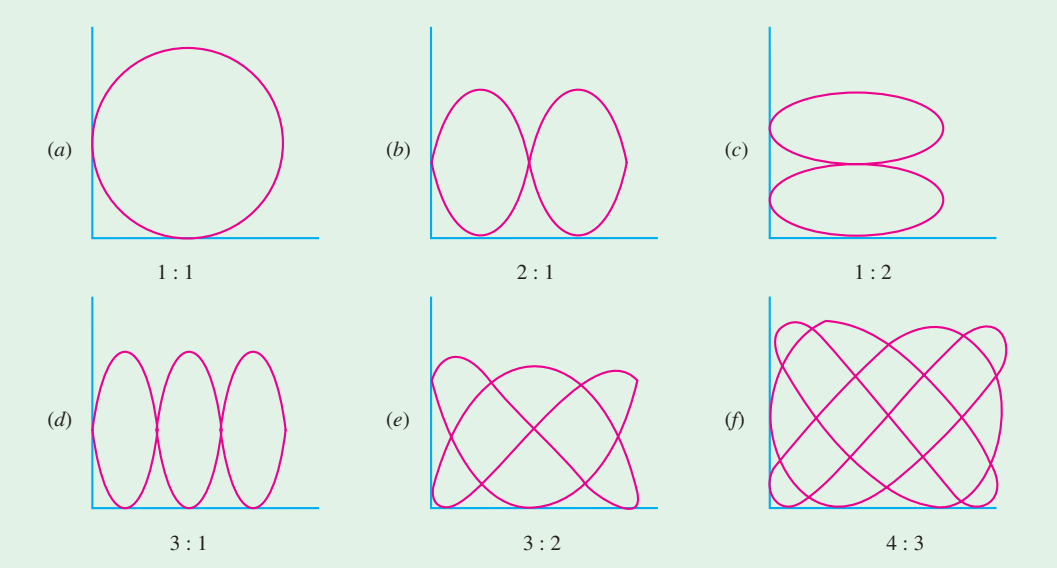


Fig. 73.42

73.36. Applications of a CRO



Fig. 73.43
(Courtesy : Fluke Corporation)

As stated earlier, no other instrument in electronic industry is as versatile as a *CRO*. In fact, a modern oscilloscope is the most useful single piece of electronic equipment that not only removes guess work from technical troubleshooting but makes it possible to determine the trouble quickly. Some of its uses are as under :

(*a*) In Radio Work

1. to trace and measure a signal throughout the *RF*, *IF* and *AF* channels of radio and television receivers.
2. it provides the only effective way of adjusting *FM* receivers, broadband high-frequency *RF* amplifiers and automatic frequency control circuits;
3. to test *AF* circuits for different types of distortions and other spurious oscillations;
4. to give visual display of waveshapes such as sine waves, square waves and their many different combinations;
5. to trace transistor curves
6. to visually show the composite synchronized TV signal
7. to display the response of tuned circuits etc.

(*b*) Scientific and Engineering Applications

1. measurement of ac/dc voltages,
2. finding B/H curves for hysteresis loop,
3. for engine pressure analysis,
4. for study of stress, strain, torque, acceleration etc.,
5. frequency and phase determination by using Lissajous figures,
6. radiation patterns of antenna,
7. amplifier gain,

8. modulation percentage,
9. complex waveform as a short-cut for Fourier analysis,
10. standing waves in transmission lines etc.

73.37. The Q Meter

This instrument is designed to measure some of the electrical properties of coils and capacitors by measuring the *Q*-value of an *R-L-C* circuit.

1. Construction

As shown in Fig. 73.44, it essentially consists of

(i) a frequency-calibrated continuously-variable *RF* oscillator,

(ii) a calibrated variable capacitor *C*,

(iii) *VTVM* which is calibrated to read *Q* directly.

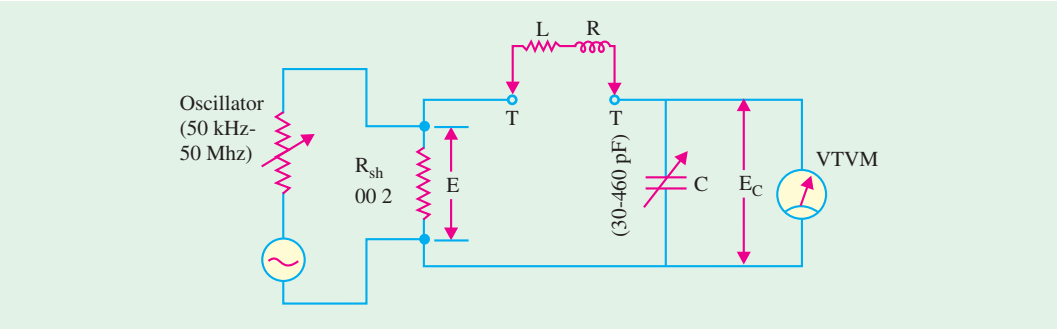


Fig. 73.44

2. Principle of Operation

The basic principle used in *Q*-meter is the resonant rise of the voltage across the capacitor in an *R.L.C* circuit. The condition for series resonance (Art. 14.10) is

$$X_L = X_C \quad \text{and} \quad E = IR$$

The value of circuit *Q* is

$$Q = \frac{X_L}{R} = \frac{X_C}{R} = \frac{E_C}{E}$$

If the applied voltage *E* is constant, then $Q \propto E_C$. Hence, by measuring voltage drop across *C* under resonant conditions, *Q* can be found. Alternatively, *VTVM* can be calibrated directly in terms of *Q* (rather than voltage).

As seen from Fig. 73.44, the oscillator delivers current to a very small (0.02 Ω) shunt resistance R_{sh} thereby developing a voltage *E* across it. It becomes the applied voltage for the *RLC* circuit and corresponds to the source voltage *E* of Fig. 73.45.

It is measured by a thermocouple meter marked 'Multiply *Q* by'. The voltage E_C across variable *C* is measured by the *VTVM*. The value of *Q*-factor is given by $Q = E_C/E$.

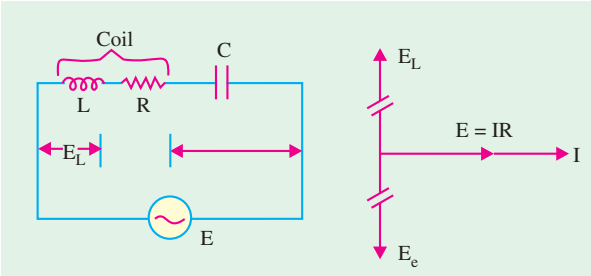


Fig. 73.45

For making measurement, the unknown coil is connected to the test terminals *TT* of the instrument and the circuit is tuned to resonance (Fig. 73.44)

(i) either by setting the oscillator to a given frequency and varying *C*,

(*ii*) or by keeping value of C constant and adjusting the oscillator frequency.

The reading on the *VTVM* must be multiplied by the index setting of the 'Multiply Q by' meter in order to obtain the actual value of Q.

3. Applications

Some of the specialized uses of this instrument are to measure

1. Q of a coil,

2. inductance and capacitance,

3. distributed capacitance of a coil,

4. Q and *p.f.* of a dielectric material,

5. mutual inductance of coupled circuits,

6. coefficient of coupling,

7. critical coupling,

8. reactance and effective resistance of an inductor at operating frequency,

9. bandwidth of a tuned circuit etc.

The above list does not exhaust the number of its possible applications. It has been very truly said that if ever an *RF* problem exists, a Q meter can always provide the answer.

73.38. Logic Analysers

The oscilloscope is probably the best tool for the development of analog or digital system design. It can be used to examine the waveforms and determine the voltage and rise time of the analog or digital signals. But the oscilloscope has two limitations especially when used in digital system design : **(1)** high speed random pulses can not be observed easily **(2)** oscilloscope cannot monitor a few signal lines simultaneously. For example, in a commonly available oscilloscope, the maximum number of inputs are four.

For these reasons, the logic analysers has been developed. It operates on a slightly different principle than that of an oscilloscope. Because there are many signal lines in a digital system (such as a microprocessor based system), the data is changing rapidly on each line, a logic analyser must take a ***snap shot*** of the activities on the lines and store the logic state of each signal in memory for each cycle of the system clock. The conditions under which the snapshot is taken are determined by triggering circuits, which can respond to various combinations of events.

The logic analyser enables the activity of many digital signal points to be recorded simultaneously and then examined in detail. The information is recorded with respect to a clock signal to determine whether they are HIGH or LOW with respect to a defined threshold voltage. This information is stored in memory and is then available for detailed analysis via the logic analyser's display. The clock signal can be internally or externally generated.

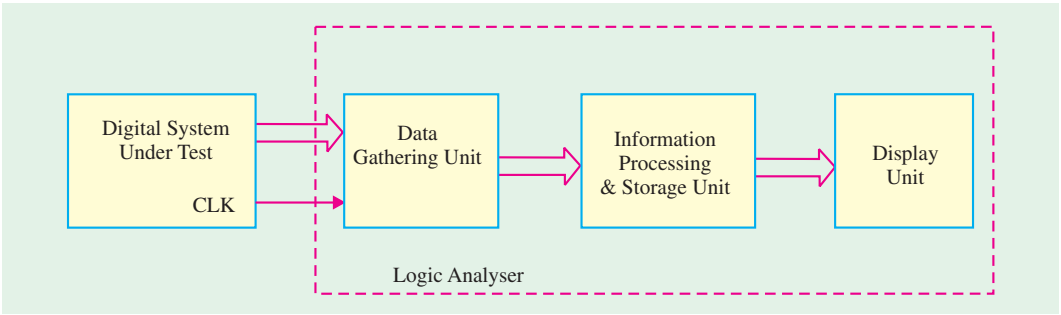


Fig. 73.46

Fig. 73.46 shows a block diagram of a typical logic analyser. It has a data gathering unit, information processing and storage unit and a display unit. The data gathering unit has (1) a pod slots for carrying data from the digital system under test to the logic analyser and (2) a key pad. The key pad is used to enter commands and set up the parameters that the logic anayser will use. The display unit is a cathode ray tube (*CRT*) that displays the command menu for the operator and also displays the output data.

Applications

- hardware/software debugging
- parametric/mixed signal testing
- hardware simulation and stimulus-response testing
- complex debugging with deep memory.



Fig. 73.47. Logic analysers

Fig. 73.47 shows a family of logic analysers, available from Tektronix corporation. Each logic analyser has at least 34 channels, 4-channel digitizing oscilloscope, off-line analysis capability for viewing data and creating setups on a separate PC.

Applications.

Logic analyser is a very powerful tool in the field of microprocessor based system development. Some of its major applications in this area are :

1. Hardware debug and verification.

2. Processor/bus debug and verification.

3. Embedded software integration, debug and verification.

73.39. Spectrum Analysers

The spectrum analyser is an instrument that brings together a superhetrodyne radio receiver with a swept frequency local oscillator and an oscilloscope to present a display of amplitude versus frequency.

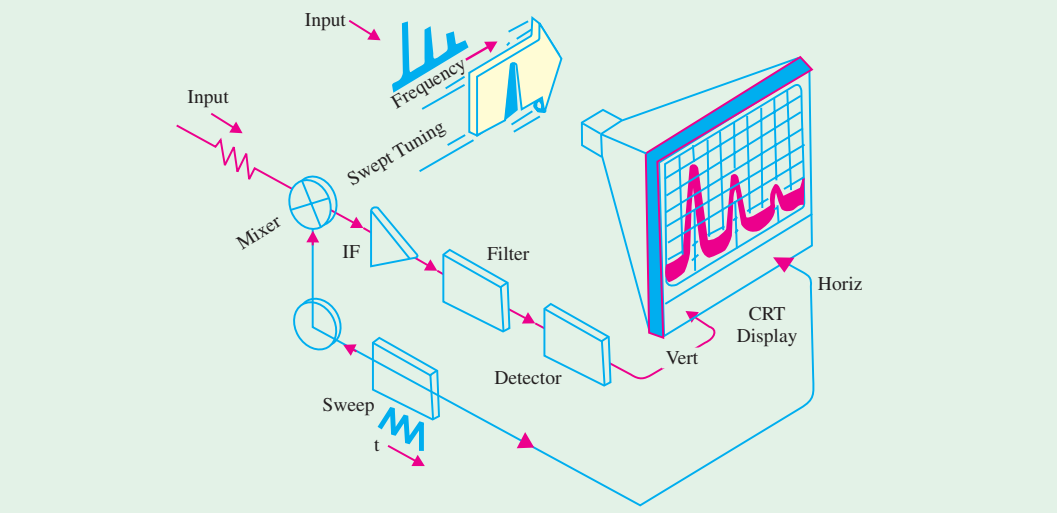


Fig. 73.48

(Courtesy Hewlett Packard)

Fig. 73.48 shows a simple block diagram of a spectrum analyser. As seen, the spectrum analyser is actually a superhetrodyne receiver in which local oscillator is a sweep generator. A low frequency saw-tooth wave is applied to both the sweep oscillator and the horizontal deflection plates of the *CRT*, producing a horizontal deflection that is a function of frequency. The lowest frequency is represented by left side of the trace while the highest frequency is represented by the right side of the trace. The sweep is from left to right.

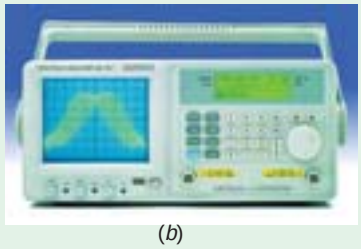
The input signal is mixed with local oscillator to produce the *IF* (*i.e.* intermediate-frequency or difference) signal. The bandwidth of the *IF* amplifier is relatively narrow. so the output signal at the detector will have a strength that is proportional to the frequency that the local oscillator is converting to the *IF* at that instant. The display will then contain "poles" that represent the amplitudes of the various input frequency components.

There are several spectrum analysers available in the market manufactured by the companies like Rhode & Schwarz (Tektronix), Hewlett Packard (now called Agilent Technologies), and so on. Fig. 73.49 shows two commercially available spectrum analyses.

The spectrum analyser shown in Fig. 73.49 (*a*) is Model *FSE* 30 manufactured by Rhode & Schwarz but marked by Tektronix Corporation.



(*a*)



(*b*)

Fig. 73.49
(Courtesy : Rhode & Schwarz and Tektronics Corporation)

It has a frequency range from 20 Hz to 76.5 GHz, a bandwidth from 1 Hz to 10 MHz. Another spectrum analyser Model No. 3066 shown in Fig. 73.49 (*b*) is a real-time instrument. It has a frequency range from *DC* to 5 MHz, a bandwidth from *DC* to 3 GHz. The real-time spectrum analyser take a very different approach compared to traditional sweeping spectrum analysers. Rather than acquiring one frequency step at a time, the real time spectrum analyser captures a block of frequencies all at once.

It is possible to use computers to do spectrum analysis of the signals. There is a variety of software available over the internet from several companies. Some softwares can be downloaded free of cost from the companies web-sites.

Applications

The spectrum analyser is used to :

1. check the spectral purity of signal sources.
2. evaluate local electromagnetic interference (*EMI*) problems.
3. do site surveys prior to installing radio receiving or transmitting equipment.
4. test transmitters.
5. analyse signatures.

73.40. Signal Generators

A signal generator is an instrument that provides a controlled output waveform or signal for use in **testing, aligning** or in **measurements** on other circuits or equipment. The signal generators can be classified into the following categories :

1. Audio generators
2. Function generators
3. Pulse generators
4. *RF* generators
5. Frequency synthesizers
6. Other signal generators.

73.41. Audio Generators

The audio generators covers the frequency range 20 Hz to 20 kHz, although few models produce signals up to 100 kHz. Audio generators always produce ***pure sine*** waves and most also produce square waves. They uses a 600 Ω output impedance and produce output levels from –40 dB mW to + 4 dB mW.

Two methods of frequency selection are typically used in audio signal generators. ***continuous*** and ***step***. On the continuous type of a dial, we turn a knob to the desired frequency. Many such audio

generators have a scale that reads 20 to 200 (or alternatively 2 to 20) and a *range* selector switch determines whether the output frequencies will be 20 to 200 Hz, 200 to 2000 Hz or 2000 to 20,000 Hz. In a step-tuned generator, these controls are replaced by a ***rotary*** or ***pushbutton*** switch bank. As many as four decode switches might be used, although three is a more common number. These will be marked 0 through 100, 0 through 10 and 0.1 through 1.0 in decade steps. A multiplier switch determines whether the actual frequency will be X1, X10, X100 or X1000 the frequency indicated on the selector switches.



Audio signal generator

Fig. 73.50 shows a block diagram of an audio signal generator. The audio oscillator section is usually an *RC* phase-shift oscillator (or a Wien Bridge oscillator) circuit. A power amplifier stage provides buffering between the load and the oscillator and it develops the output signal amplitude. The ac voltmeter at the output is strictly optional, but in some models it is used with a ***level control*** to set precisely the input signal to the attenuator. Not all quality audio signal generators use this feature. So the lack of an ac output meter is not, in itself, indication of quality. In some models, an audio digital frequency counter is used ahead of the attenuator to provide digital display of the output frequency.

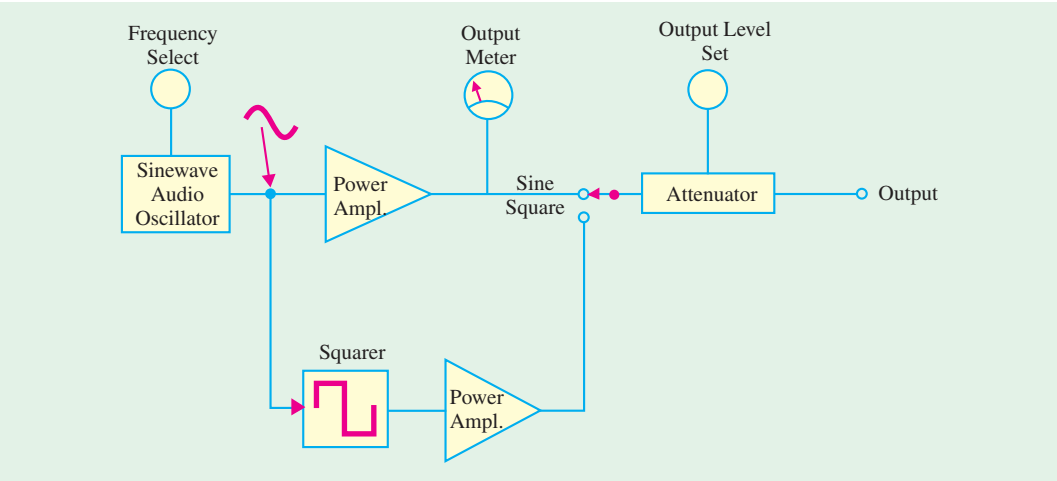


Fig. 73.50

Applications

The audio generators are basically used to test the amplitude and frequency response of audio amplifiers.

73.42. Function Generators

These generators typically, cover at least the same frequency range as audio signal generators (*i.e.* 20 Hz to 20 kHz) but most modern designs have extended frequency ranges. A very common frequency range for function generators is 0.01 Hz to 3 MHz.

The major difference between a function generator and an audio generator is in the number of output waveforms. The audio signal generator produces only sine waves and square waves. While almost all function generators produce these basic waveforms plus triangular waves. Besides this, some function generators also produce sawtooth, pulse and non-symmetrical square waves. Fig. 73.51 shows the controls of a typical function generator.



Fig. 73.51

Fig. 73.52 shows a simple block diagram of a function generator. The major parts of a function generator are schmitt trigger, integrator, sine-wave converter and an attenuator. The schmitt trigger converts a slowly varying input signal to a square wave signal. This square wave signal is available at the output as well as it is also connected to the integrator as an input through a potentiometer (R). The potentiometer is used to adjust the frequency of the output signal. The frequency range is adjusted by selecting the appropriate capacitor connected in the integrator circuit.

The sine-wave converter is a six-level (or more) diode-resistor loading circuit.

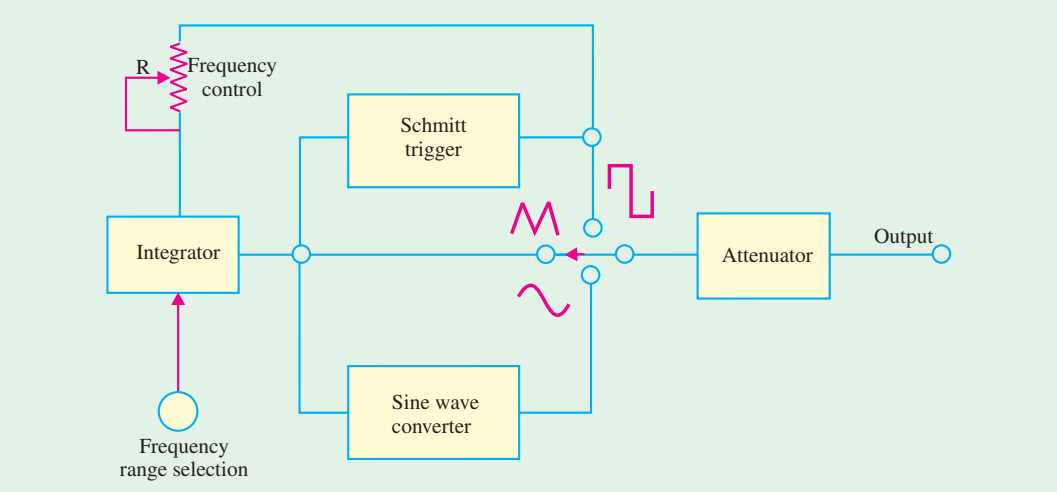


Fig. 73.52

Let us see how a simple diode-resistor circuit shown in Fig. 73.53 (*a*) is used to convert a triangular wave into a square wave.

Note that if diodes D_1 and D_2 and resistors R_3 and R_4 were not present in the circuit, of Fig. 73.53 (*a*), R_1 and R_2 would simply behave as a voltage divider. In this case the output from the circuit would be an attenuated version of the triangular wave :

$$V_0 = V_i \frac{R_2}{R_1 + R_2}$$

With diodes D_1 and R_3 in the circuit, R_1 and R_2 still behave as a simple voltage divider until the voltage drop across R_2, VR_2 exceeds + V_1. At this point D_1 becomes forward biased, and R_3 is effectively in parallel with R_2. Now,

$$V_0 = V_i \frac{R_2 \parallel R_3}{R_1 + R_2 \parallel R_3}$$

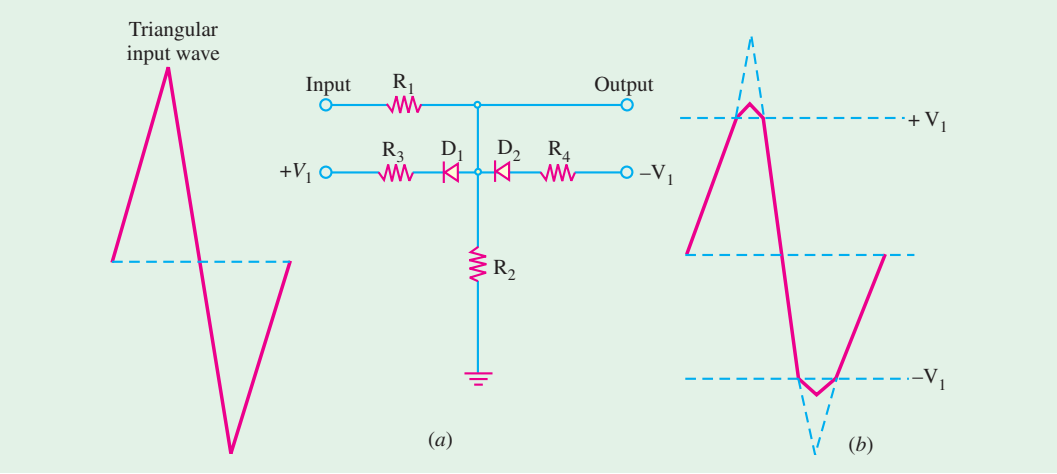


Fig. 73.53

Output voltage levels above + V_1 are attenuated to a greater extent than levels below + V_1. Consequently, the output voltage rises less steeply than without D_1 and R_3 in the circuit (refer to Fig. 73.53 (*b*). When the output falls below + V_1, the diode D_1 is reverse biased. As a result of this, R_3 is no longer in parallel with R_2, and the attenuation is once again $R_2/(R_1 + R_2)$. Similarly during the negative half cycle of the input, the output voltage,

$$V_0 = V_i \frac{R_2}{R_1 + R_2}$$

until V_0 goes below – V_1. Then D_2 becomes forward biased, putting R_4 in parallel with R_2 and making,

$$V_0 = V_i \frac{R_2 \parallel R_4}{R_1 + R_2 \parallel R_4}$$

With $R_3 = R_4$, the negative half-cycle of the output is similar in shape to the positive half-cycle. If we employ six or more diodes, all connected via resistors to different bias voltage levels (refer to Fig. 73.54 (*a*), a good sine-wave approximation can be achieved. With six diodes, the three positive bias voltage levels and three negative bias voltage levels, the slope of the output wave changes three times during each quarter cycle. Assuming correctly selected bias voltages and resistor values, the output wave shape is as shown in Fig. 73.54 (*b*).

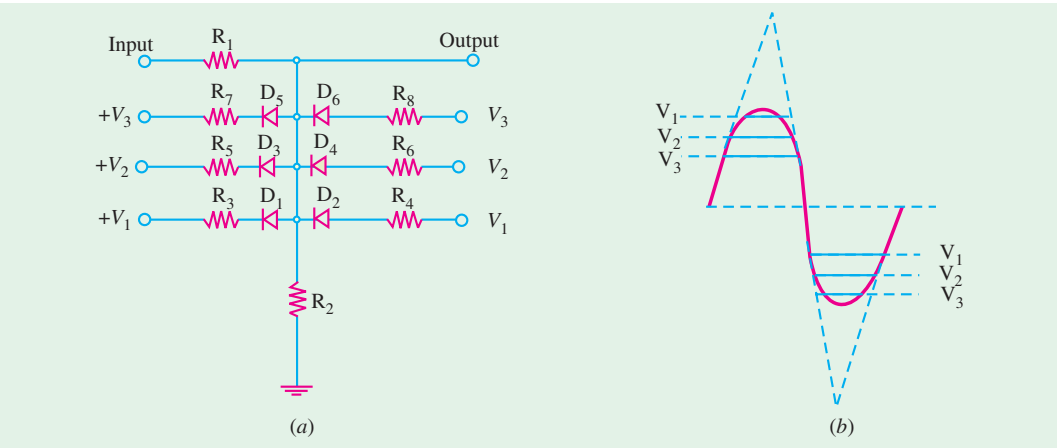


Fig. 73.54

Applications

The function generator is an essential equipment for an electronic laboratory to generate signals to test a variety of analog and digital system during the design phase as well as to trouble shoot such systems.



Fig. 73.55
(Courtesy : Tektronix Corporation)

Fig. 73.55 shows the picture of a Tektronix function generators Model No. CFG 253. This model has a frequency range form 0.03 Hz to 3 MHz. In addition to sine, square and triangular waves it can also produce *TTL* signals. Another function generator from Tektronix (Model No. CFG280 has a wide frequency range form 0.01 Hz to 11 MHz. It has a built in frequency counter with a range from 1 Hz to 100 MHz.

Applications

The function generators can be employed in a variety of applications in the area of product design, training, manufacturing production test, field repair, bench calibration and repair, laboratory and research, and education. Mainly they are used for testing amplifiers, filter and digital circuits.

73.43. Pulse Generators

Fig. 73.56 shows the block diagram of a pulse generator. As seen, an astable multivibrator generates square waves. This is used to trigger monostable multivibrator (*i.e.* one-shot). The pulse repetition rate is set by the square-wave frequency. The one-shot triggers on the leading edge of the square-wave and produces one output pulse for each input cycle. The duration of each output pulse is set by the on-time of the one-shot. It may be very short or may approach the period of the square wave. The attenuator facilities output amplitude control and dc level shifting.

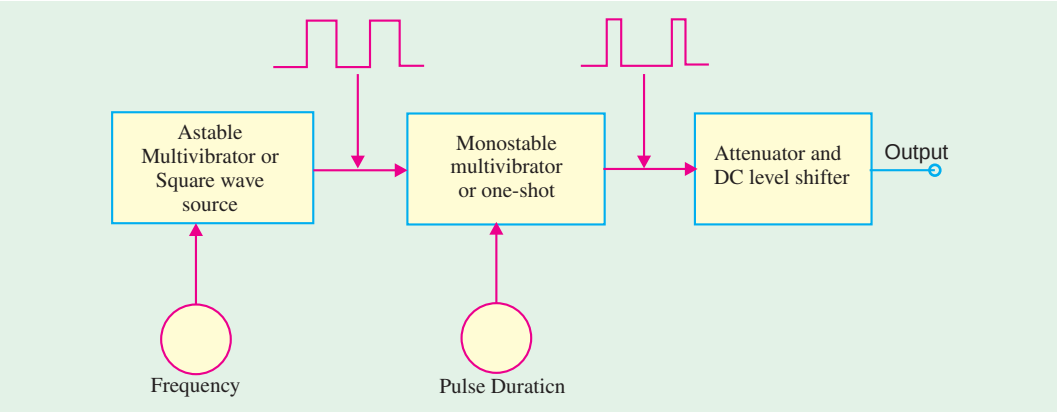


Fig. 73.56

A typical pulse generator will allow the user to select the repetition rate, duration, amplitude and number of output pulses to be output in a given burst. The most common frequency range is from 1 Hz to 50 MHz. The pulse width is adjustable from 10 ns to over 10 ms and the output is variable from 3 mV to 30 V.



Fig. 73.57. (Courtesy Fluke Corporation)

Fig. 73.57 shows a pulse generator from Fluke Corporation Model No. PM5786. This instrument has a frequency range from 1 Hz to 125 MHz. The output pulse width can

be varied from 8 ns to 100 ms. The instrument can also be used to generate pulse bursts. The output voltage level can be adjusted up to 10 V.

Applications

The pulse generators are used extensively to test :

1. Memory circuits
2. Shift registers
3. Counters
4. Other digital components, subsystems and systems.

73.44. RF Generators

A radio frequency (*RF*) signal generator has a sinusoidal output with a frequency somewhere in the range of 100 kHz to 40 GHz region. Fig. 73.58 shows the block diagram of an *RF* generator. As soon, the instrument consists of an *RF* oscillator, an amplifier, a calibrated attenuator and an output level meter.

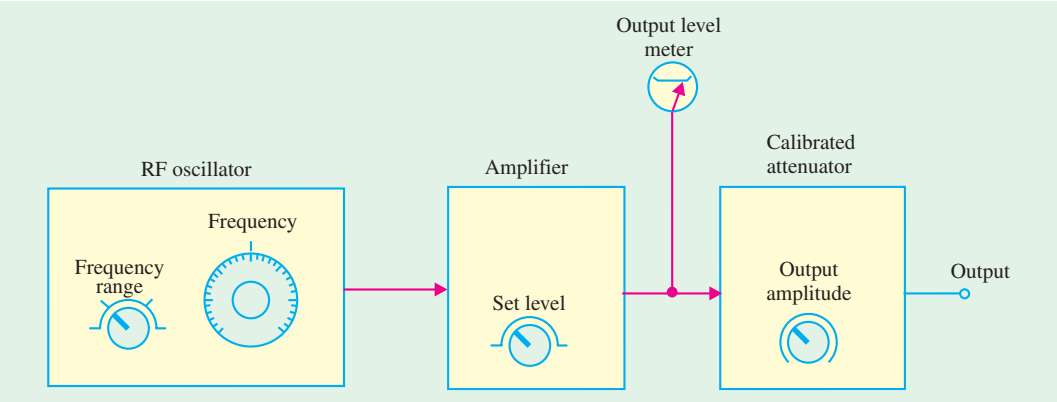


Fig. 73.58

The *RF* oscillator has a continuous frequency control and a frequency range switch, to set the output to any desired frequency. The amplifier includes an output amplitude control. This allows the voltage applied to the attenuator to be set to a calibration point on the output level meter. The output level must always be ***reset*** to this calibration point everytime the frequency is changed. This is necessary to ensure that the output voltage levels are correct, as indicated on the calibrated attenuator.

The oscillator circuit used in an *RF* generator is usually either a Hartley Oscillator or Colpitts oscillator. Most RF signal generators include facilities for amplitude modulation and frequency modulation of the output. Switches are provided on the front panel to allow the user to select no modulation as well as internal or external *AM* or *FM* modulation. It may be noted that each section of the RF generator is ***shielded*** by enclosing it in a metal box. The whole system is then completely shielded. The purpose of shielding is **(1)** to prevent RF interference between the components and **(2)** to prevent the emission of RF energy from any point except the output terminals. As a matter of fact, even the power line is decoupled by means of RF chokes and capacitors to prevent RF emission from it.



Fig. 73.59
(Courtesy Fluke Corporation)

Fig. 73.59 (*a*) shows an analog RF generator Model No. SML01 manufactured by Tektronics Corporation. This

instrument is a general purpose signal generator and is available at low cost. It has a wide frequency range from 9 kHz to 3.3 GHz. Another RF generator Model No. SMP04 shown in Fig. 73.59 (*b*) from Tektronix Corporation is a high precision signal source. It has a wide frequency range from 0.01 GHz to 40 GHz. This instrument can produce, AM–, FM–, phase– and pulse modulated signals as well.

Applications

The RF signal generators are widely used in the area of radar and communication, research and development laboratories, education and training, electromagnetic interference (*EMI*) testing and material testing. Their main applications are :

1. To perform variety of tests on radio transmitters and receivers.

2. To test the amplitude and frequency response of RF amplifiers during the design phase.

73.45. Frequency Synthesizer

It is another type of RF generator that uses ***phase-locked loop*** (*PLL*) to generate output frequencies over a wide range. The most common range is from 1 MHz to 160 MHz. Fig. 73.60 shows a simple block diagram of a frequency synthesizer. As seen, the major components of the frequency synthesizer are : voltage controlled oscillator (*VCO*), divide-by-N counter, phase detector, crystal oscillator, low-pass filter and a square-wave circuit.

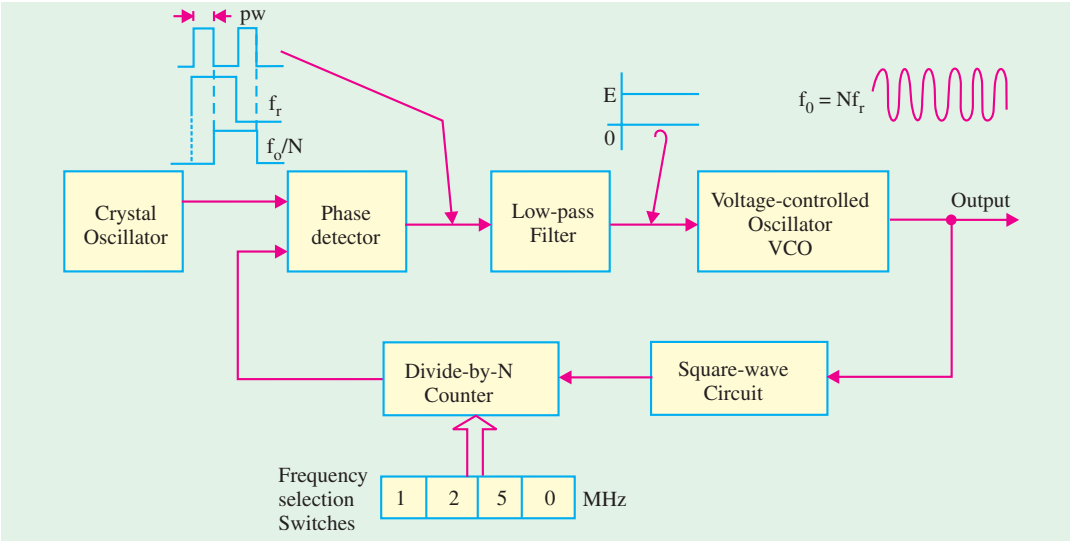


Fig. 73.60

The output of the crystal oscillator (a reference frequency, *fr*), is fed into one input of a phase detector. The other input of a phase detector has another square-wave applied to it as shown in the figure. The frequencies of these two square waves is identical but there is a phase difference (ϕ) between them. The output of the phase detector is a pulse waveform with pulse width controlled by the phase difference. The output of the phase detector is applied to the low-pass filter which converts it into a dc voltage, *E*. The dc voltage, *E* is used as the control voltage for the *VCO* and it determines the output frequency of *VCO*. The output of *VCO* is fed to a circuit that converts it into a square wave for triggering a digital divide-by-N counter. The divide-by-N counter divides the *VCO* frequency by a number set by a bank of switches. These switches may be push buttons with digital readouts or they may be thumb-wheel type which indicate their position numerically. The switches are connected in such a way that the displayed number is the factor *N* by which the output frequency is divided before being applied to the phase detector. The switches allow the user to obtain frequency which is any integer multiple of the crystal oscillator frequency.

Applications

The frequency synthesizer is used in almost same areas as the RF signal generators.

73.46. Other Signal Generators

There are some signal generators that do not fit well in various pres-etabilished categories. Some of these signal generators are as discussed below :

1. RF Markers : These devices are usually crystal controlled and have a fixed output frequency for use as a reference. These are used to calibrate TV signals.

2. Digitally Programmable Test Oscillators : These instruments can have extremely wide frequency range although some versions have much narrow range also. The set frequency can be programmed through the front panel keypad or via a computer interface input such as IEEE-488 general purpose interface bus (commonly known as GPIB).

3. Arbitrary Waveform Generators : These instruments allow the user to design and generate virtually any desired waveform. The arbitrary waveform generator is quite useful to perform a variety of tests on communication equipment. For example, a modulated signal that varies over the entire bandwidth and amplitude range of the equipment shown in Fig. 73.61 could be created for testing purpose. Noise could also be superimposed upon the signal and gaps might be introduced between waveform bursts, to investigate the response of the system. Once such a waveform has been designed, it could be stored in the instrument memory and can be recalled repeatedly for production testing.

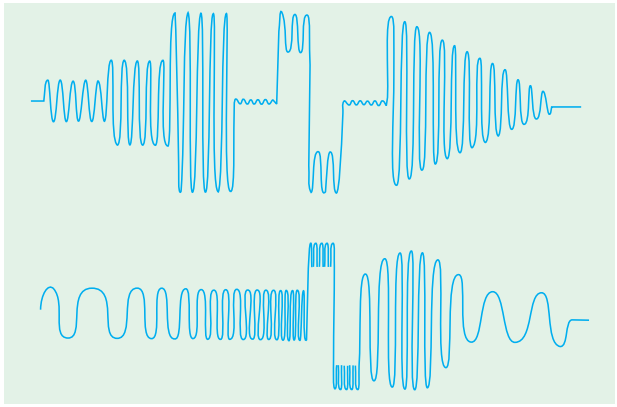


Fig. 73.61

Fig. 73.62 shows an arbitrary waveform generator from Agilent Technologies Model No. 33250A. This instrument can generate real world signals up to 80 MHz. It has a capability to display the waveforms in colour. It can be used as a function generator/pulse generator as well. The instrument has a GPIB/LAN interfaces.

Applications

The arbitary waveform generators are used extensively in the following areas :

1. Communications design and test for producing (*a*) arbitrary IF based signals and (*b*) standard waveforms for communication.
2. Mixed signal design and test.
3. Disk drive read/write design and test.
4. Real word simulations.
5. High-speed low filter data and clock pulse generation.



Fig. 73.62 (Courtesy : Agilent Technologies)

73.47. IEEE-488 General Purpose Interface Bus (GPIB) Instruments

These days automatic test equipment (*ATE*) is one of the leading methods for testing electronic equipment in factory production and troubleshooting situations. The basic method is to use a programmable digital computer to control a bank of test instruments. The bank of instruments can be configured for a special purpose or for general use. For example, we could select a particular line-up of equipment needed to test a broadcast audio console and provide a computer program to do a

variety of measurements such as gain, frequency response, total harmonic distortion etc. The other possibility could be to do a generalized test set. This method is adopted by number of industries who have many electronic devices or systems to test. There is a main bank of electronic test equipment adapters to make the devices (or systems) under test interconnect with the system and a computer program to do a variety of measurements such as gain, frequency response, total harmonic distortion etc. The other possibility could be to do a generalized test set. This method is adopted by number of industries who have many electronic devices or systems to test. There is a main bank of electronic test equipment adapters to make the devices (or systems) under test interconnect with the system and a computer program for each type of equipment. Such an approach reduces the test equipment cost drastically.

The Institution of Electrical and Electronic Engineers (*IEEE*) has laid out a specification titled *IEEE* standard Digital Interface for programmable instrumentation or *IEEE*-488. This specification provides details for a standard interface between a computer and instruments. The *IEEE*-488 bus or General purpose interface bus (*GPIB*) is a tool that is based on the *IEEE* specifications. The Hewlett-Packard interface bus (*HPIB*) is a proprietary version of the *IEEE*-488 bus.

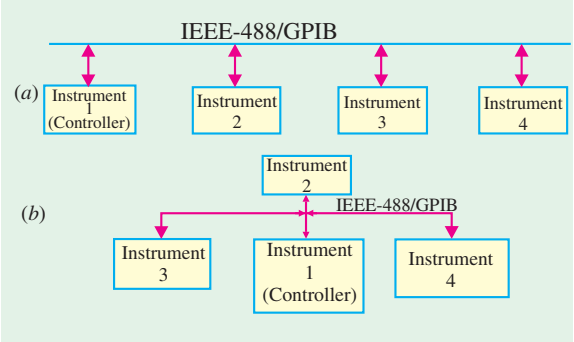


Fig. 73.63

The digital signals on the *IEEE*-488 bus are generally similar to *TTL* (transistor-transistor logic), *i.e.* a logic LOW is less than 0.8 V and a logic *HIGH* is greater than 2.0 V. The digital signals can be connected to the instruments through a multiconductor cable up to 20 metres in length provided that an instrument load is placed every 2 metres. Most *IEEE*-488/*GPIB* systems operate unrestricted to 250 kilobytes per second or faster with some restrictions.

There are two basic configurations for the *IEEE*-488/*GPIB* system : **(1)** linear and **(2)** star. In the linear configuration shown in Fig. 73.63 (*a*), a tap-off to the next instrument is taken from the previous one in series. On the other hand, in star configuration shown in Fig. 73.63 (*b*), the instruments are connected from a central point.

Fig. 73.64 shows the basic structure of *IEEE*-488/*GPIB* system. The figure indicates four different devices (*i.e.* computer, frequency counter, signal generator and digital multimeter) connected to the bus. The *IEEE*-488/*GPIB* system itself consists of three major buses : **(1)** general interface management (*GIM*) bus. **(2)** data *I*/*O* (*DIO*) bus and **(3)** data byte transfer (*DBT*) bus.

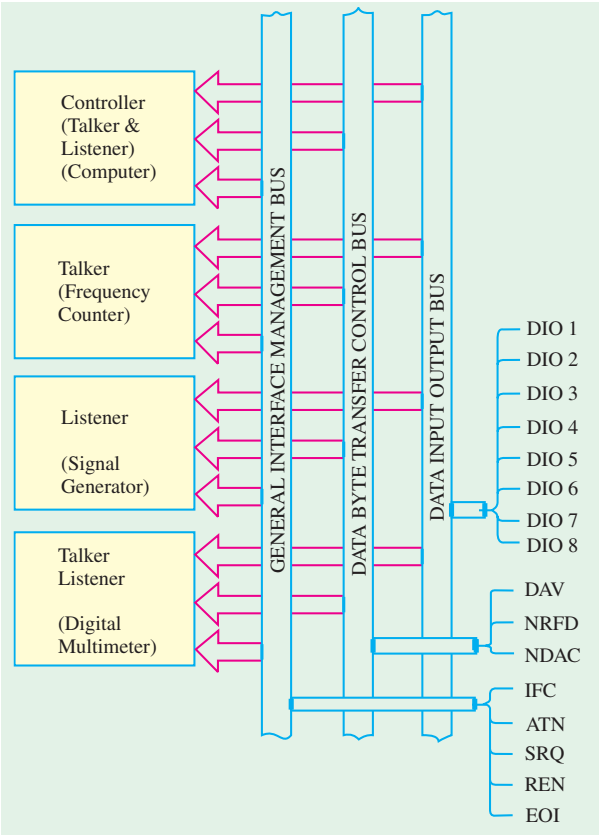


Fig. 73.64

1. General Interface Management (*GIM*) bus : This bus coordinates the whole system and ensures an ordely flow of data over the data input/output (*DIO*) bus. This bus has a number of signals explained below.

***IFC* (Interface Clear Signal) :** This signal is used by the controller to place all devices in a predefined quiescent or standby condition.

***ATN* (Attention Signal) :** This signal is used by the computer/controller to let the system know how data on the *DIO* bus lines are to be interpreted and which device is to respond to the data.

***SRQ* (Service Request Signal) :** This signal is used by a device on the system to ask the controller for attention. It is basically an interupt request signal.

***REN* (Remote Enable Signal) :** This signal is used by the controller to select between two alternate sources of device programming data.

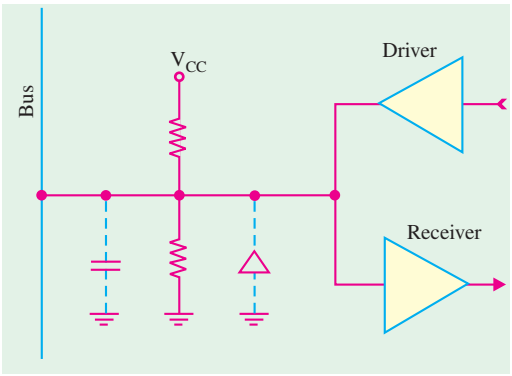


Fig. 73.65

***EOI* (End or Identify Signal) :** This signal is used by talkers for the following two purposes : (1) it will follow the end of a multiple byte sequence of data in order to indicate that the data are now finished. (2) It is also used in conjunction with the *ATN* signal for polling the system.

2. Data *I/O* (*DIO*) bus : This bus is a bidirectional 8-bit data bus that carries data, interface messages and device-dependent messages between the controller, talkers and listeners. This bus sends asynchronously in byte-serial format.

3. Data Byte Transfer (*DBT*) bus: This bus controls the sending of data along the *DIO* bus. There are three signals on this bus as explained below :

***DAV* (Data Valid Signal) :** This signal indicates the availability and validity of data on the line. If the measurement is not finished, for example, the *DAV* signal will be false.

***NRFD* (Not Ready For Data Signal) :** This signal lets the controller know whether or not the specific device addressed is in a condition to receive data.

***NDAC* (Not Data Accepted Signal) :** This signal is used to indicate to the controller whether or not the device accepted the data sent to it over the *DIO* bus.

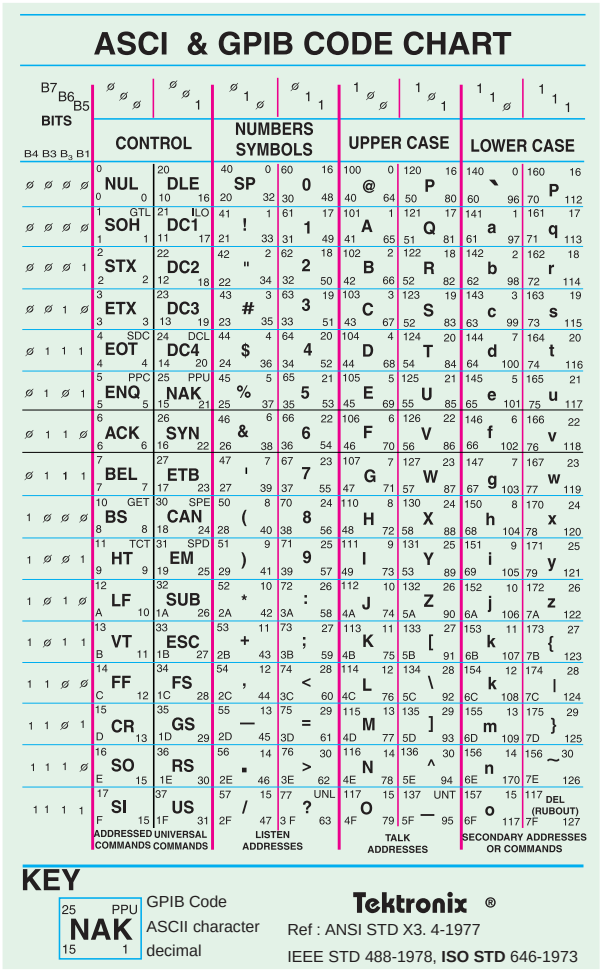


Fig. 73.66

Each signal line in the bus has a circuit similar to the one shown in Fig. 73.65. As seen from this circuit, each signal line has a pull-up and pull-down resistors, receiver, and driver-circuits. Besides this each signal line has also a shunt protection diode and a stray capacitance.

Fig. 73.66 shows the 7-bit binary signals used in the *IEEE*-488/*GPIB* system for *ASCII* and

GPIB message codes.

The signals defined for the three buses in the *IEEE*-488/*GPIB* systems are implemented as conductors in a system interface cable. Each *IEEE*-488/*GPIB* compatible instrument will have a female 36-pin Amphenol-style connector on the rear panel. The pin-out definitions are given in Table 73.1.

The devices connected to *IEEE*-488/*GPIB* system (*i.e.* computer, frequency counter, signal generator and digital multimeter) are categorised as controller, listener and/or talker.

Table 73.1

Pin No	*Signal Lin*	*Pin No*	*Signal Line*
1	*DIO* 1	13	*DIO* 5
2	*DIO* 2	14	*DIO* 6
3	*DIO* 3	15	*DIO* 7
4	*DIO* 4	16	*DIO* 8
5	*EOI*	17	*REN*
6	*DAV*	18	Ground (6)
7	*NRFD*	19	Ground (7)
8	*NDAC*	20	Ground (8)
9	*IFC*	21	(Ground 9)
10	*SRQ*	22	(Ground 10)
11	*ATN*	23	(Ground 11)
12	Shield	24	Digital Ground

1. Controller : Its function is to communicate device addresses and other inface buses to instruments in the system.

2. Listener : Its function is to receive commands from other instruments (usually the controller) when the correct address is placed on the bus. The listener acts on the message received but does not send back any data to the controller. The signal generator shown in Fig. 73.22 is an example of a listener.

3. Talker : Its function is to respond to the message sent to it by the controller. The frequency counter shown in Fig. 73.62 is an example of a talker.



Fig. 73.67

There is also a combination device that accepts commands from the controller to set up ranges, tasks etc. and then returns data back over the *DIO* bus to the controller. The digital multimeter shown in Fig. 73.62 is an example of this category.

The *IEEE*-488 was introduced to the electronic industry in 1977. Since then it has evolved to *IEEE*-488.1 in 1987 and further to *IEEE*-488.2 in 1990. At present the system allows the control upto 14 instruments and it has data transfer rate greater than 1 M bytes/s.

73.48. VXI bus

The *VXI* bus is another fast growing platform for instrumentation systems. It was introduced in 1987 and since then it has experienced tremendous growth and acceptance around the world. *VXI* uses a mainframe chassis with a maximum of 13 slots to hold ***modular instruments* on *plug-in boards.*** Fig. 73.67 shows an example of system using *VXI* instruments. The *VXI* backplane includes

the 32-bit *VME* data bus, high performance instrumentation buses for precision timing and synchronization between instrument components, standarized initialization and resource management to ease configuration.

OBJECTIVE TESTS – 73

1. Digital instruments are those which
(*a*) have numerical readout
(*b*) use *LED* or *LCD* displays
(*c*) have a circuitry of digital design
(*d*) use deflectioin type meter movement.

2. The main difference between the electronic and electrical instruments is that an electronic instrument contains
(*a*) an electronic device (*b*) a transducer
(*c*) a digital readout (*d*) electrons.

3. The essential elements of an electronic instrument are
(*a*) transducer (*b*) signal conditioner
(*c*) indicating device (*d*) all of the above.

4. The current sensitivity of a meter is expressed in
(*a*) ampere (*b*) ohm/ampere
(*c*) ohm/volt (*d*) ampere/division.

5. The basic meter movement can be converted into an ohmmeter by connecting a .....with it.
(*a*) high resistance in series
(*b*) low resistance in parallel
(*c*) battery in series
(*d*) battery and a variable resistance in series

6. The D' Arsonval meter movement can be converted into an audio-frequency ac ammeter by adding a ..... to it.
(*a*) thermocouple (*b*) rectifier
(*c*) chopper (*d*) transducer.

7. In a linear meter, half-scale deflection occurs when there is ... per cent of the rated current through its coil
(*a*) 100 (*b*) 25
(*c*) 50 (*d*) 75

8. A 0-1 mA meter has a sensitivity of
(*a*) 1 k Ω/V (*b*) 1 mA
(*c*) 1 k Ω (*d*) 1000 A.

9. A moving coil instrument has a resistance of 10 Ω and takes 40 mA to produce full-scale deflection. The shunt resistance required to convert this instrument for use as an ammeter of range 0 to 2 A is
(*a*) 0.1021 Ω (*b*) 0.2041 Ω
(*c*) 0.2561 Ω (*d*) 0.4210 Ω

10. A moving coil ammeter has a fixed shunt of 0.02 ohm resistance. If the coil resistance of the meter is 1000 Ω, a potential difference of 500 mV is required across it for full-scale deflection. Under this condition, the current in the shunt would be
(*a*) 2.5 A (*b*) 25 A
(*c*) 0.25 A (*d*) 0.025 A

11. It is desired to convert a 0-1000 μA meter movement, with an internal resistance of 100 Ω into a 0-100 mA meter. The required value of shunt resistance is about
(*a*) 1 Ω (*b*) 10 Ω
(*c*) 99 Ω (*d*) 100 Ω

12. Loading effect is principally caused by... instruments
(*a*) high resistance (*b*) low-sensitivity
(*c*) high-sensitivity (*d*) high-range

13. A multimeter is used to measure
(*a*) resistance (*b*) current
(*c*) voltage (*d*) all of the above

14. A sinusoidal voltage of rms value 10 V is applied to a D' Arsonval movement connected in series with a half-wave rectifier. It will show a reading of... volt
(*a*) 9 (*b*) 4.5
(*c*) 10 (*d*) 7.7

15. A *VTVM* produces negligible loading effect on a circuit under test primarily because
(*a*) it virtually drawn no current from the circuit
(*b*) of its very high internal resistance
(*c*) it uses high vacuum tubes
(*d*) it is a null deflection instrument.

16. In a 3½ digit voltmeter, the largest number that can be read is
(*a*) 0999 (*b*) 1999
(*c*) 4999 (*d*) 9999

17. A 3½ digit voltmeter having a resolution of 100 mV can be used to measure maximum voltage of
(*a*) 100 V (*b*) 200 V
(*c*) 1000 V (*d*) 5000 V

18. The signal to be observed on the screen of an oscilloscope is applied
(*a*) across its *X*-plates (*b*) across its *Y*-plates
(*c*) to the horizontal amplifier
(*d*) to the trigger circuit.

19. When a 30 V dc is applied to the vertical deflection plates of a cathode ray tube, the bright

spot moves 1 cm away from the centre. If 30 V (rms) ac is applied, then the movement of the spot will be nearly

(*a*) 1 cm (*b*) 1.5 cm
(*c*) 2 cm (*d*) 3 cm

20. Production of a steady stationary display of a signal waveform on the scope screen is due to

(*a*) persistence of vision
(*b*) flourescent material of the screen
(*c*) proper sync. between the signal and the sweep generator
(*d*) electrostatic focussing of the electron beam.

21. Two sinusoidal signals of frequency f and $3f$ are applied at x and y inputs respectively to an oscilloscope. Which one of the following patterns can be observed on the screen ?

(*a*)  (*b*) 
(*c*)  (*d*) 

22. The X- and Y-inputs of a *CRO* are respectively $V \sin wt$ and $-V \sin wt$. The resulting Lissajous pattern will be

(*a*) a straight line (*b*) a circle
(*c*) an elipse (*d*) a figure of eight

23. The deflection sensitivity of a *CRT* depends inversely on the

(*a*) length of the vertical deflecting plates
(*b*) distance between screen and deflecting plates
(*c*) deflecting voltage
(*d*) separation between Y-plates.

24. Two complete signal cycles would be displayed on the screen of a scope when time-period of the sweep generator is ..... the signal time period.

(*a*) half (*b*) twice
(*c*) equal (*d*) thrice

25. A non-triggered oscilloscope is one which

(*a*) has no sweep generator
(*b*) cannot produce a stable stationary screen display
(*c*) has a continuously running time-base genera tor
(*d*) can display a portion of the input signal wave form.

26. A dual-trace *CRO* has

(*a*) one electron gun (*b*) two electron guns
(*c*) one electron gun and one two-pole switch
(*d*) two electron guns and one two-pole switch

27. The operation a Q-meter is based on

(*a*) self-induction (*b*) series resonance
(*c*) mutual induction (*d*) eddy currents.

28. The resolution of a logic analyser is

(*a*) maximum number of input channels
(*b*) the minimum duration of the glitch it can capture
(*c*) its internal clock period
(*d*) the minimum amplitude of the input signal it can display

29. A spectrum analyser can be described as

(*a*) voltage selective frequency meter
(*b*) current selective frequency meter
(*c*) frequency selective voltmeter
(*d*) None of these

ANSWERS

1. (*c*)	**2.** (*a*)	**3.** (*d*)	**4.** (*c*)	**5.** (*d*)	**6.** (*b*)	**7.** (*c*)	**8.** (*a*)	**9.** (*b*)	**10.** (*b*)
11. (*a*)	**12.** (*b*)	**13.** (*d*)	**14.** (*b*)	**15.** (*b*)	**16.** (*a*)	**17.** (*c*)	**18.** (*a*)	**19.** (*a*)	**20.** (*c*)
21. (*b*)	**22.** (*a*)	**23.** (*d*)	**24.** (*b*)	**25.** (*c*)	**26.** (*c*)	**27.** (*b*)	**28.** (*a*)	**29.** (*d*)	

GO To FIRST