W. H. Mosley,
R. Hulse and
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REINFORCED CONCRETE DESIGN TO EUROCODE 2 (EC2)

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Preface

The purpose of this book is to provide a straightforward introduction to the principles and methods of design for concrete structures. It is directed primarily at students and young engineers who require an understanding of the basic theory and a concise guide to design procedures. Although the detailed design methods are generally according to European Standards (*EuroCodes*), much of the theory and practice is of a fundamental nature and should, therefore, also be useful to engineers in countries outside Europe.

The search for harmonisation of Technical Standards across the European Community (EC) has led to the development of a series of these *Structural EuroCodes* which are the technical documents intended for adoption throughout all the member states. The use of these common standards is intended to lower trade barriers and enable companies to compete on a more equitable basis throughout the EC. *EuroCode 2* (EC2) deals with the design of concrete structures, which has most recently been covered in the UK by British Standard BS8110 which superseded British Standard CP110 in 1985. Limit state principles established by these British Standards are also adopted by EuroCode 2.

The code drafting committee has also produced a range of supporting documents giving commentary and background explanation for some of the requirements of the code. Further supporting documentation includes, for each separate country, the *National Application Document (NAD)* which includes information specific to the individual member states. Additionally the British Cement Association has produced *The Concise EuroCode for the Design of Concrete Buildings* which contains material that has been distilled from EC2 but is presented in a way that makes it more user-friendly than the main EuroCode and contains only that information which is essential for the design of more everyday concrete structures. It also contains information not included in EC2 such as design charts and design methods drawn from previous British Standards

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such as BS8110 and CP110. In this text, reference is made to both EC2 and the Concise Code.

The presentation of EC2 is quite different from that of BS8110. However the essential feature of EC2 is that the principles of design embodied in the document are almost identical to the principles inherent in the use of BS8110. Hence, although there are some differences in detail, engineers who are used to designing to the existing British Standard should have no difficulty in grasping the essential features of this new code.

Changes in terminology, arising partly from language differences, have resulted in the introduction of a few terms which are unfamiliar to engineers who have worked with BS8110. The most obvious of these is the use of *actions* to describe the loading on structures and the use of the terms *permanent* and *variable actions* to describe dead and imposed loads. Notwithstanding this, UK influence in drafting the document has been very strong and terminology is broadly the same as in existing British Standards. Throughout this text, terminology has been kept as consistently as possible in line with common accepted UK practice and hence, for example, *loads* has been used instead of *actions* in subsequent chapters. Other 'new' terminology will be identified at appropriate points in the text.

The subject matter in this book has been arranged so that chapters 1 to 5 deal mostly with theory and analysis while the subsequent chapters cover the design and detailing of various types of member and structure. In order to include topics that are usually in an undergraduate course, there is a section on earth-retaining structures and also a final chapter on prestressed concrete.

Important equations that have been derived within the text are highlighted by an asterisk adjacent to the equation number and in the Appendix a summary of key equations is given. Where it has been necessary to include material which is not directly provided by the EuroCodes, this has been based on currently accepted UK good practice.

In preparing this book, the principal aim has been to retain the structure and features of the well established book *Reinforced Concrete Design* by Mosley and Bungey (Macmillan Press, 1990) which is based on British Standards and is currently in its 4th edition. By comparing both books it is possible to compare the essential differences between EuroCode 2 and existing British Standards and to contrast the different outcomes when structures are designed to either codes.

It should be emphasised that Codes of Practice are always liable to be revised and readers should ensure that they are using the latest edition of any relevant standard.

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from whom complete copies can be obtained. Extracts from the *Concise EuroCode for the Design of Concrete Structures* are reproduced by permission of the British Cement Association, Century House, Telford Avenue, Crowthorne, Berks., from whom copies of the Concise EuroCode can be obtained.

Finally, the authors would like to thank Mr J. Birch for his assistance with the preparation of parts of the text and Mrs Jamillah Sa'adon for typing part of the manuscript.

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Notation

Notation is generally in accordance with EC2 and the principal symbols are listed below. Other symbols are defined in the text where necessary. The symbols ε for strain and f for stress have been adopted throughout, with the general system of subscripts such that the first subscript refers to the material, c – concrete, s – steel, and the second subscript refers to the type of stress, c – compression, t – tension.

E	modulus of elasticity
F	load (action)
G	permanent load
I	second moment of area
K	prestress loss factor
M	moment or bending moment
N	axial load
Q	variable load
T	torsional moment
V	shear force

a	deflection
b	breadth or width
d	effective depth of tension reinforcement
d'	depth to compression reinforcement
e	eccentricity
h	overall depth of section in plane of bending
i	radius of gyration
k	coefficient
l	length or span
n	ultimate load per unit area
$1/r$	curvature of a beam
s	spacing of shear reinforcement *or* depth of stress block

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t thickness
u punching shear perimeter
x neutral axis depth
z lever arm

A_c concrete cross-sectional area
A_p cross-sectional area of prestressing tendons
A_s cross-sectional area of tension reinforcement
A'_s cross-sectional area of compression reinforcement
$A_{s,req}$ cross-sectional area of tension reinforcement required at the ultimate limit state
$A_{s,prov}$ cross-sectional area of tension reinforcement provided at the ultimate limit state
A_{sw} cross-sectional area of shear reinforcement in the form of links or bent-up bars
E_{cm} secant modulus of elasticity of concrete
E_s modulus of elasticity of reinforcing or prestressing steel
G_k characteristic permanent load
I_c second moment of area of concrete
M_{bal} moment on a column corresponding to the balanced condition
M_{sd} design value of moment
M_u ultimate moment of resistance
N_{bal} axial load on a column corresponding to the balanced condition
N_{sd} design value of axial force
P_0 initial prestress force
Q_k characteristic variable load
T_{Sd} design value of torsional moment
V_{Sd} design value of shear force
W_k characteristic wind load

b_w minimum width of section
f_{ck} characteristic cylinder strength of concrete
f_{cm} mean cylinder strength of concrete
f_{ctm} mean tensile strength of concrete
f_{pk} characteristic yield strength of prestressing steel
f_{yk} characteristic yield strength of reinforcement
g_k characteristic permanent load per unit area
k_1 average compressive stress in the concrete for a rectangular–parabolic stress block
k_2 a factor that relates the depth to the centroid of the rectangular–parabolic stress block and the depth to the neutral axis
l_a lever-arm factor = z/d
l_e effective height of column or wall
q_k characteristic variable load per unit area

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γ_c partial safety factor for concrete strength
γ_f partial safety factor for loads (actions), F
γ_G partial safety factor for permanent loads, G
γ_Q partial safety factor for variable loads, Q
γ_s partial safety factor for steel strength
δ moment redistribution factor
ε strain
σ stress
ϕ bar diameter

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1

Properties of Reinforced Concrete

Reinforced concrete is a strong durable building material that can be formed into many varied shapes and sizes ranging from a simple rectangular column to a slender curved dome or shell. Its utility and versatility are achieved by combining the best features of concrete and steel. Consider some of the widely differing properties of these two materials that are listed below.

	Concrete	*Steel*
strength in tension	poor	good
strength in compression	good	good, but slender bars will buckle
strength in shear	fair	good
durability	good	corrodes if unprotected
fire resistance	good	poor – suffers rapid loss of strength at high temperatures

It can be seen from this list that the materials are more or less complementary. Thus, when they are combined, the steel is able to provide the tensile strength and probably some of the shear strength while the concrete, strong in compression, protects the steel to give durability and fire resistance. This chapter can present only a brief introduction to the basic properties of concrete and its steel reinforcement. For a more comprehensive study, it is recommended that reference should be made to the specialised texts listed in Further Reading at the end of the book.

1.1 Composite Action

The tensile strength of concrete is only about 10 per cent of the compressive strength. Because of this, nearly all reinforced concrete structures

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are designed on the assumption that the concrete does not resist any tensile forces. Reinforcement is designed to carry these tensile forces, which are transferred by bond between the interface of the two materials. If this bond is not adequate, the reinforcing bars will just slip within the concrete and there will not be a composite action. Thus members should be detailed so that the concrete can be well compacted around the reinforcement during construction. In addition, some bars are ribbed or twisted so that there is an extra mechanical grip.

In the analysis and design of the composite reinforced concrete section, it is assumed that there is perfect bond, so that the strain in the reinforcement is identical to the strain in the adjacent concrete. This ensures that there is what is known as 'compatibility of strains' across the cross-section of the member.

The coefficients of thermal expansion for steel and for concrete are of the order of 10×10^{-6} per °C and $7–12 \times 10^{-6}$ per °C respectively. These values are sufficiently close that problems with bond seldom arise from differential expansion between the two materials over normal temperature ranges.

Figure 1.1 illustrates the behaviour of a simply supported beam subjected to bending and shows the position of steel reinforcement to resist the tensile forces, while the compression forces in the top of the beam are carried by the concrete.

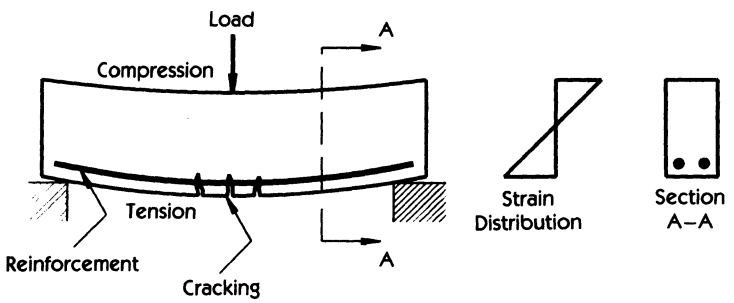

Figure 1.1 *Composite action*

Wherever tension occurs it is likely that cracking of the concrete will take place. This cracking, however, does not detract from the safety of the structure provided there is good reinforcement bond to ensure that the cracks are restrained from opening so that the embedded steel continues to be protected from corrosion.

When the compressive or shearing forces exceed the strength of the concrete, then steel reinforcement must again be provided, but in these cases it is only required to supplement the load-carrying capacity of the

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concrete. For example, compression reinforcement is generally required in a column, where it takes the form of vertical bars spaced near the perimeter. To prevent these bars buckling, steel binders are used to assist the restraint provided by the surrounding concrete.

1.2 Stress–Strain Relations

The loads on a structure cause distortion of its members with resulting stresses and strains in the concrete and the steel reinforcement. To carry out the analysis and design of a member it is necessary to have a knowledge of the relationship between these stresses and strains. This knowledge is particularly important when dealing with reinforced concrete which is a composite material; for in this case the analysis of the stresses on a cross-section of a member must consider the equilibrium of the forces in the concrete and steel, and also the compatibility of the strains across the cross-section.

1.2.1 Concrete

Concrete is a very variable material, having a wide range of strengths and stress–strain curves. A typical curve for concrete in compression is shown in figure 1.2. As the load is applied, the ratio between the stresses and strains is approximately linear at first and the concrete behaves almost as an elastic material with virtually a full recovery of displacement if the load is removed. Eventually, the curve is no longer linear and the concrete behaves more and more as a plastic material. If the load were removed during the plastic range the recovery would no longer be complete and a permanent deformation would remain. The ultimate strain for most structural concretes tends to be approximately 0.0035, irrespective

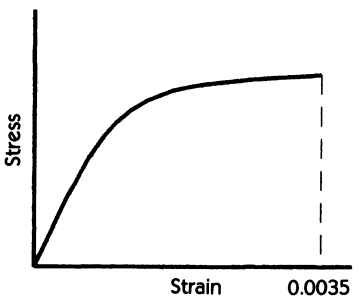

Figure 1.2 *Stress–strain curve for concrete in compression*

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of the strength of the concrete. The precise shape of the curve is very dependent on the length of time the load is applied, a factor which will be further discussed in section 1.4 on creep. Figure 1.2 is typical for a short-term loading.

Concrete generally increases its strength with age. This characteristic is illustrated by the graph in figure 1.3 which shows how the increase is rapid at first, becoming more gradual later. The precise relationship will depend upon the type of cement used. That shown is for the typical variation of an adequately cured concrete made with commonly used class 42.5 Portland cement. Some codes of practice allow the concrete strength used in design to be varied according to the age of the concrete when it supports the design load. EuroCode 2 does not permit the use of strengths greater than the 28-day value in calculations, but the modulus of elasticity may be modified to account for age as shown later.

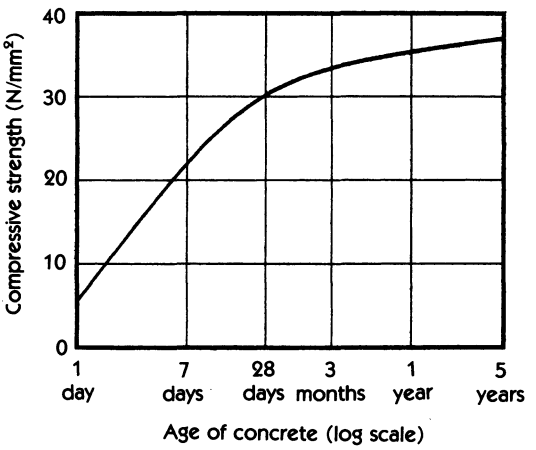

Figure 1.3 *Increase of concrete strength with age: typical curve for a grade 30 concrete made with a class 42.5 Portland cement concrete*

In the United Kingdom, compressive stress has traditionally been measured and expressed in terms of 150 mm cube crushing strength at an age of 28 days. Most other countries use 150 mm diameter cylinders which are 300 mm long. For normal strength concretes, the cylinder strength is, on average, about 0.8 × the cube strength. All design calculations to EuroCode 2 are based on the characteristic cylinder strength f_{ck} as defined in section 2.2.1. Cube strengths may however be used for compliance purposes, with the characteristic strength identified as $f_{ck\,cube}$.

Concretes will normally be specified in terms of these 28 day characteristic strengths, for example strength class C35/45 concrete has a

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characteristic cylinder strength of $35\,N/mm^2$ and a characteristic cube strength of $45\,N/mm^2$. It will be noted that there is some 'rounding off' in these values, which are usually quoted in multiples of $5\,N/mm^2$. The term 'strength class' is often referred to as the 'grade' of concrete and throughout this book the two terms are taken to be synonymous.

Modulus of Elasticity of Concrete

It is seen from the stress–strain curve for concrete that although elastic behaviour may be assumed for stresses below about one-third of the ultimate compressive strength, this relationship is not truly linear. Consequently it is necessary to define precisely what value is to be taken as the modulus of elasticity

$$E = \frac{\text{stress}}{\text{strain}}$$

A number of alternative definitions exist, but the most commonly adopted is $E = E_{cm}$ where E_{cm} is known as the *secant* or *static modulus*. This is measured for a particular concrete by means of a static test in which a cylinder is loaded to just above one-third of the corresponding control cube stress and then cycled back to zero stress. This removes the effect of initial 'bedding in' and minor stress redistributions in the concrete under load. Load is then reapplied and the behaviour will then be almost linear; the average slope of the line up to the specified stress is taken as the value for E_{cm}. The test is described in detail in BS1881 and the result is generally known as the *instantaneous static modulus of elasticity*.

The *dynamic modulus of elasticity*, E_{cq}, is sometimes referred to since this is much easier to measure in the laboratory and there is a fairly well-defined relationship between E_{cm} and E_{cq}. The standard test is based on determining the resonant frequency of a laboratory prism specimen and is also described in BS 1881. It is also possible to obtain a good estimate of E_{cq} from ultrasonic measuring techniques, which may sometimes be used on site to assess the concrete in an actual structure. The standard test for E_{cq} is on an unstressed specimen. It can be seen from figure 1.4 that the value obtained represents the slope of the tangent at zero stress and E_{cq} is therefore higher than E_{cm}. The relationship between the two moduli is given by

$$\text{Static modulus } E_{cm} = (1.25E_{cq} - 19)\,kN/mm^2$$

This equation is sufficiently accurate for normal design purposes.

The actual value of E for a concrete depends on many factors related to the mix, but a general relationship is considered to exist between the modulus of elasticity and the compressive strength. Typical values of E_{cm}

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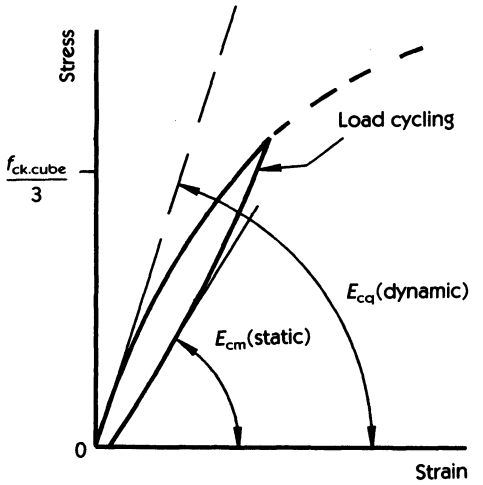

Figure 1.4 *Moduli of elasticity of concrete*

for various concrete classes which are suitable for design are shown in table 1.1. The magnitude of the modulus of elasticity is required when investigating the deflection and cracking of a structure. When considering short-term effects, member stiffnesses will be based on the static modulus E_{cm}, as defined above. If long-term effects are being considered, it can be shown that the effects of creep can be represented by modifying the value of E_{cm} to an effective value $E_{c,ef}$, and this is discussed in section 6.3.2.

Table 1.1 Short-term modulus of elasticity of normal-weight concrete

28 day characteristic strength (N/mm^2)	*Static modulus E_{cm} (kN/mm^2)*
$f_{ck}/f_{ck\ cube}$ *(cylinder/cube)*	*Mean*
20/25	29
25/30	30.5
30/37	32
35/45	33.5
40/50	35
45/55	36
50/60	37

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The elastic modulus at an age other than 28 days may be estimated from this table but using the anticipated strength value at that age. If a typical value of Poisson's ratio is needed, this should be taken as 0.2 for regions which are not subject to tension cracking.

1.2.2 Steel

Figure 1.5 shows typical stress–strain curves for (a) mild steel, and (b) high yield steel. Mild steel behaves as an elastic material, with the strain proportional to the stress up to the yield, at which point there is a sudden increase in strain with no change in stress. After the yield point, mild steel becomes a plastic material and the strain increases rapidly up to the ultimate value. High yield steel, on the other hand, does not have a definite yield point but shows a more gradual change from an elastic to a plastic behaviour. Both materials have a similar slope of the elastic region with $E_s = 200\,\text{kN/mm}^2$ approximately.

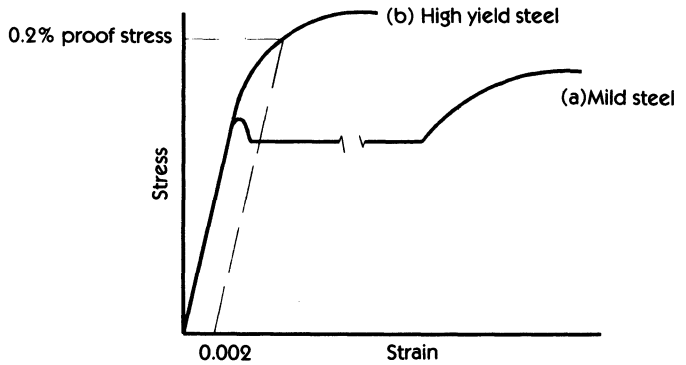

Figure 1.5 *Stress–strain curves for steel*

The specified strength used in design is based on the yield stress for mild steel, whereas for high yield steel the strength is based on a specified proof stress. A 0.2 per cent proof stress is defined in figure 1.5 by the broken line drawn parallel to the linear part of the stress–strain curve.

Removal of the load within the plastic range would result in the stress–strain diagram following a line approximately parallel to the loading portion – see line BC in figure 1.6. The steel will be left with a permanent strain AC, which is known as 'slip'. If the steel is again loaded, the stress–strain diagram will follow the unloading curve until it almost reaches the original stress at B and then it will curve in the

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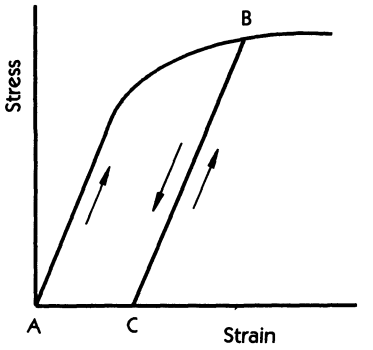

Figure 1.6 *Strain hardening*

direction of the first loading. Thus, the proportional limit for the second loading is higher than for the initial loading. This action is referred to as 'strain hardening' or 'work hardening'.

The deformation of the steel is also dependent on the length of time the load is applied. Under a constant stress the strains will gradually increase - this phenomenon is known as 'creep' or 'relaxation'. The amount of creep that takes place over a period of time depends on the grade of steel and the magnitude of the stress. Creep of the steel is of little significance in normal reinforced concrete work, but it is an important factor in prestressed concrete where the prestressing steel is very highly stressed.

1.3 Shrinkage and Thermal Movement

As concrete hardens there is a reduction in volume. This shrinkage is liable to cause cracking of the concrete, but it also has the beneficial effect of strengthening the bond between the concrete and the steel reinforcement. Shrinkage begins to take place as soon as the concrete is mixed, and is caused initially by the absorption of the water by the concrete and the aggregate. Further shrinkage is caused by evaporation of the water which rises to the concrete surface. During the setting process the hydration of the cement causes a great deal of heat to be generated, and as the concrete cools, further shrinkage takes place as a result of thermal contraction. Even after the concrete has hardened, shrinkage continues as drying out persists over many months, and any subsequent wetting and drying can also cause swelling and shrinkage. Thermal shrinkage may be reduced by restricting the temperature rise during hydration, which may be achieved by the following procedures:

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(1) Use a mix design with a low cement content
(2) Avoid rapid hardening and finely ground cement if possible
(3) Keep aggregates and mixing water cool
(4) Use steel shuttering and cool with a water spray
(5) Strike the shuttering early to allow the heat of hydration to dissipate.

A low water–cement ratio will help to reduce drying shrinkage by keeping to a minimum the volume of moisture that can be lost.

If the change in volume of the concrete is allowed to take place freely without restraint, there will be no stress change within the concrete. Restraint of the shrinkage, on the other hand, will cause tensile strains and stresses. The restraint may be caused externally by fixity with adjoining members or friction against an earth surface, and internally by the action of the steel reinforcement. For a long wall or floor slab, the restraint from adjoining concrete may be reduced by using a system of constructing successive bays instead of alternate bays. This allows the free end of every bay to contract before the next bay is cast.

Day-to-day thermal expansion of the concrete can be greater than the movements caused by shrinkage. Thermal stresses and strains may be controlled by the correct positioning of movement or expansion joints in a structure. For example, the joints should be placed at an abrupt change in cross-section and they should, in general, pass completely through the structure in one plane.

When the tensile stresses caused by shrinkage or thermal movement exceed the strength of the concrete, cracking will occur. To control the crack widths, steel reinforcement must be provided close to the concrete surface; the codes of practice specify minimum quantities of reinforcement in a member for this purpose.

Calculation of Stresses Induced by Shrinkage

(a) Shrinkage Restrained by the Reinforcement

The shrinkage stresses caused by reinforcement in an otherwise unrestrained member may be calculated quite simply. The member shown in figure 1.7 has a free shrinkage strain ε_{cs} if made of plain concrete, but this overall movement is reduced by the inclusion of reinforcement, giving a compressive strain ε_{sc} in the steel and causing an effective tensile strain ε_{ct} in the concrete. Thus

$$\begin{aligned} \varepsilon_{cs} &= \varepsilon_{ct} + \varepsilon_{sc} \\ &= \frac{f_{ct}}{E_{cm}} + \frac{f_{sc}}{E_s} \end{aligned} \tag{1.1}$$

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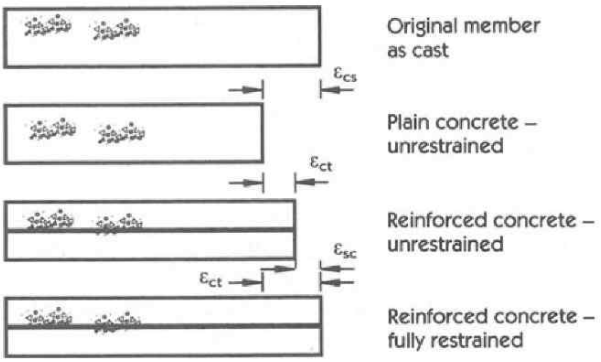

Figure 1.7 *Shrinkage strains*

where f_{ct} is the tensile stress in concrete area A_c and f_{sc} is the compressive stress in steel area A_s.

Equating forces in the concrete and steel for equilibrium gives

$$A_c f_{ct} = A_s f_{sc} \tag{1.2}$$

therefore

$$f_{ct} = \frac{A_s}{A_c} f_{sc}$$

Substituting for f_{ct} in equation 1.1

$$\varepsilon_{cs} = f_{sc}\left(\frac{A_s}{A_c E_{cm}} + \frac{1}{E_s}\right)$$

Thus if

$$\alpha_e = \frac{E_s}{E_{cm}}$$

$$\varepsilon_{cs} = f_{sc}\left(\frac{\alpha_e A_s}{A_c E_s} + \frac{1}{E_s}\right)$$

$$= \frac{f_{sc}}{E_s}\left(\frac{\alpha_e A_s}{A_c} + 1\right)$$

Therefore steel stress

$$f_{sc} = \frac{\varepsilon_{cs} E_s}{1 + \dfrac{\alpha_e A_s}{A_c}} \tag{1.3}$$

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Example 1.1 Calculation of Shrinkage Stresses in Concrete that is Restrained by Reinforcement Only

A member contains 1.0 per cent reinforcement, and the free shrinkage strain ε_{cs} of the concrete is 200×10^{-6}. For steel, $E_s = 200\,\text{kN/mm}^2$ and for concrete $E_{cm} = 15\,\text{kN/mm}^2$. Hence from equation 1.3:

$$\text{stress in reinforcement } f_{sc} = \frac{\varepsilon_{cs} E_s}{1 + \alpha_e \frac{A_s}{A_c}}$$

$$= \frac{200 \times 10^{-6} \times 200 \times 10^3}{1 + \frac{200}{15} \times 0.01}$$

$$= 35.3\,\text{N/mm}^2 \text{ compression}$$

$$\text{stress in concrete } f_{ct} = \frac{A_s}{A_c} f_{sc}$$

$$= 0.01 \times 35.3$$

$$= 0.35\,\text{N/mm}^2 \text{ tension}$$

The stresses produced in members free from external restraint are generally small as in the above example, and can be easily withstood both by the steel and the concrete.

(b) Shrinkage Fully Restrained

If the member is fully restrained, then the steel cannot be in compression since $\varepsilon_{sc} = 0$ and hence $f_{sc} = 0$ (figure 1.7). In this case the tensile strain induced in the concrete ε_{ct} must be equal to the free shrinkage strain ε_{cs}, and the corresponding stress will probably be high enough to cause cracking in immature concrete.

Example 1.2 Calculation of Fully Restrained Shrinkage Stresses

If the member in example 1.1 were fully restrained, the stress in the concrete is given by

$$f_{ct} = \varepsilon_{ct} E_{cm}$$

where

$$\varepsilon_{ct} = \varepsilon_{cs} = 200 \times 10^{-6}$$

then

$$f_{ct} = 200 \times 10^{-6} \times 15 \times 10^3$$

$$= 3.0\,\text{N/mm}^2$$

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When cracking occurs, the uncracked lengths of concrete try to contract so that the embedded steel between cracks is in compression while the steel across the cracks is in tension. This feature is accompanied by localised bond breakdown, adjacent to each crack. The equilibrium of the concrete and reinforcement is shown in figure 1.8 and calculations may be developed to relate crack widths and spacings to properties of the cross-section; this is examined in more detail in chapter 6, which deals with serviceability requirements.

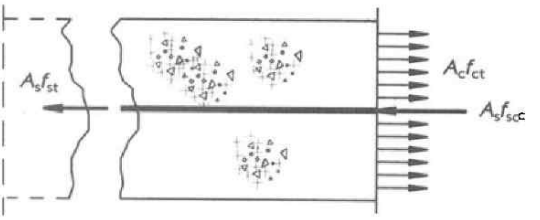

Figure 1.8 *Shrinkage forces adjacent to a crack*

Thermal Movement

As the coefficients of thermal expansion of steel and concrete (α_s and α_c) are similar, differential movement between the steel and concrete will only be very small and is unlikely to cause cracking.

The differential thermal strain due to a temperature change T may be calculated as

$$T(\alpha_c - \alpha_s)$$

and should be added to the shrinkage strain ε_{sh} if significant.

The overall thermal contraction of concrete is, however, frequently effective in producing the first crack in a restrained member, since the required temperature changes could easily occur overnight in a newly cast member, even with good control of the heat generated during the hydration processes.

Example 1.3 Thermal Shrinkage

Find the fall in temperature required to cause cracking in a restrained member if ultimate tensile strength of the concrete $f_{ct,ef} = 2\,N/mm^2$, $E_{cm} = 16\,kN/mm^2$ and $\alpha_c = \alpha_s = 10 \times 10^{-6}$ per °C. Ultimate tensile strain of concrete

$$\varepsilon_{ult} = \frac{f_{ct,ef}}{E_{cm}} = \frac{2}{16 \times 10^3} = 125 \times 10^{-6}$$

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Minimum temperature drop to cause cracking

$$= \frac{\varepsilon_{ult}}{\alpha_c} = \frac{125}{10} = 12.5\,°C$$

It should be noted that full restraint, as assumed in this example, is unlikely to occur in practice; thus the temperature change required to cause cracking is increased. A maximum 'restraint factor' of 0.5 is often used, with lower values where external restraint is likely to be small. The temperature drop required would then be given by the theoretical minimum divided by the 'restraint factor'.

1.4 Creep

Creep is the continuous deformation of a member under sustained load. it is a phenomenon associated with many materials, but it is particularly evident with concrete. The precise behaviour of a particular concrete depends on the aggregates and the mix design, but the general pattern is illustrated by considering a member subjected to axial compression. For such a member, a typical variation of deformation with time is shown by the curve in figure 1.9.

The characteristics of creep are

(1) The final deformation of the member can be three to four times the short-term elastic deformation
(2) The deformation is roughly proportional to the intensity of loading and to the inverse of the concrete strength

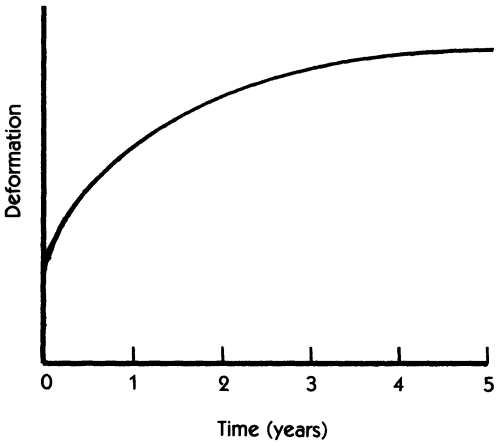

Figure 1.9 *Typical increase of deformation with time for concrete*

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(3) If the load is removed, only the instantaneous elastic deformation will recover – the plastic deformation will not
(4) There is a redistribution of load between the concrete and any steel present.

The redistribution of load is caused by the changes in compressive strains being transferred to the reinforcing steel. Thus the compressive stresses in the steel are increased so that the steel takes a larger proportion of the load.

The effects of creep are particularly important in beams, where the increased deflections may cause the opening of cracks, damage to finishes, and the non-alignment of mechanical equipment. Redistribution of stress between concrete and steel occurs primarily in the uncracked compressive areas and has little effect on the tension reinforcement other than reducing shrinkage stresses in some instances. The provision of reinforcement in the compressive zone of a flexural member, however, often helps to restrain the deflections due to creep.

1.5 Durability

Concrete structures, properly designed and constructed, are long lasting and should require little maintenance. The durability of the concrete is influenced by

(1) the exposure conditions
(2) the concrete quality
(3) the cover to the reinforcement
(4) the width of any cracks.

Concrete can be exposed to a wide range of conditions such as the soil, sea water, stored chemicals or the atmosphere. The severity of the exposure governs the type of concrete mix required and the minimum cover to the reinforcing steel. Whatever the exposure, the concrete mix should be made from impervious and chemically inert aggregates. A dense, well-compacted concrete with a low water–cement ratio is all important and for some soil conditions it is advisable to use a sulfate-resisting cement.

Adequate cover is essential to prevent corrosive agents reaching the reinforcement through cracks and pervious concrete. The thickness of cover required depends on the severity of the exposure and the quality of the concrete (as shown in table 6.2). The cover is also necessary to protect the reinforcement against a rapid rise in temperature and subsequent loss of strength during a fire. EuroCode 2 does not offer guidance, but information concerning this is given in Part 2 of BS8110. Durability requirements with related design calculations to check and control crack widths and depths are described in more detail in chapter 6.

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1.6 Specification of Materials

1.6.1 Concrete

The selection of the type of concrete is frequently governed by the strength required, which in turn depends on the intensity of loading and the form and size of the structural members. For example, in the lower columns of a multi-storey building a higher-strength concrete may be chosen in preference to greatly increasing the size of the column section with a resultant loss in clear floor space.

As indicated in section 1.2.1, the concrete strength is assessed by measuring the crushing strength of cubes or cylinders of concrete made from the mix. These are usually cured, and tested after 28 days according to standard procedures. Concrete of a given strength is identified by its 'class' – a Class 25/30 concrete has a characteristic cylinder crushing strength of 25 N/mm^2 and cube strength of 30 N/mm^2. Table 1.2 shows a list of commonly used classes and also the lowest class appropriate for various types of construction.

Exposure conditions and durability can also affect the choice of the mix design and the class of concrete. A structure subject to corrosive conditions in a chemical plant, for example, would require a denser and higher class of concrete than, say, the interior members of a school or office block. Although Class 42.5 Portland cement would be used in most structures, other cement types can also be used to advantage. Blast-furnace or sulfate-resisting cement may be used to resist chemical attack, low-heat cements in massive sections to reduce the heat of hydration, or

Table 1.2 Strength classes of concrete

Class	*Lowest class for use as specified*
C12/15	
C16/20	Plain concrete
C20/25	
C25/30	Reinforced concrete
C30/37	Prestressed concrete
C35/45	
C40/50	
C45/55	
C50/60	

Note: 'Preferred' classes are underlined.

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rapid-hardening cement when a high early strength is required. In some circumstances it may be useful to replace some of the cement by materials such as Pulverised Fuel Ash or Ground Granulated Blast Furnace Slag which have slowly developing cementitious properties. These will reduce the heat of hydration and may also lead to a smaller pore structure and increased durability. Generally, natural aggregates found locally are preferred; however, manufactured lightweight material may be used when self-weight is important, or a special dense aggregate when radiation shielding is required.

The concrete mix may either be classified as 'designed' or 'prescribed'. A 'designed mix' is one where the contractor is responsible for selecting the mix proportions to achieve the required strength and workability, whereas for a 'prescribed mix' the engineer specifies the mix proportions, and the contractor is responsible only for providing a properly mixed concrete containing the correct constituents in the prescribed proportions. Detailed requirements for mix specification and compliance are given by ENV 206: 1993.

1.6.2 Reinforcing Steel

Table 1.3 lists the characteristic design strengths of several of the more common types of reinforcement currently used in the UK. It is anticipated that in due course Grade 500 (500 N/mm^2 characteristic strength) will replace Grade 250 and Grade 460 reinforcing steel throughout Europe. The nominal size of a bar is the diameter of an equivalent circular area.

Hot-rolled mild-steel bars usually have a smooth surface so that the bond with the concrete is by adhesion only. Mild-steel bars can readily be bent, so they are often used where small radius bends are necessary, such

Table 1.3 Strength of reinforcement

Designation	*Nominal sizes (mm)*	*Specified characteristic strength f_{yk} (N/mm^2)*
Hot-rolled mild steel (BS 4449)	All sizes	250
Hot-rolled high yield (BS 4449)	All sizes	460
Cold-worked high yield (BS 4449)		
Hard-drawn steel wire (BS 4482)	Up to and including 12	460

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as for links in narrow beams or columns, but their availability and usage are becoming less common.

High-yield bars are manufactured either with a ribbed surface or in the form of a twisted square. Ribbed bars are usually described by the British Standards as type 2 bars provided specified requirements are satisfied, and these are the bars most commonly used. For design purposes with EuroCode 2, these may be classified as 'High Bond' with ductility Class H. Square twisted bars have inferior bond characteristics and are usually classified as type 1 bars, although these are more or less obsolete. All deformed bars have an additional mechanical bond with the concrete so that higher ultimate bond stresses may be specified as described in section 5.2. The bending of high-yield bars through a small radius is liable to cause tension cracking of the steel, and to avoid this the radius of the bend should not be less than three times the nominal bar size (see figure 5.10). The ductility of reinforcing steel is also classified for design purposes. All ribbed high yield bars and all Grade 250 mild steel bars may be assumed to be high ductility (Class H).

High-yield steel bars are only slightly more expensive than mild-steel bars. Therefore, because of their significant stress advantage, high-yield bars are the more economical. Nevertheless, mild-steel bars are sometimes preferred in water-retaining structures, where the maximum steel stresses are limited in order to reduce the tensile strains and cracking of the concrete.

Floor slabs, walls, shells and roads may be reinforced with a welded fabric of reinforcement, supplied in rolls and having a square or rectangular mesh. Hard-drawn steel wire is often used for this and can give large economies in the detailing of the reinforcement and also in site labour costs of handling and fixing. Prefabricated reinforcement bar assemblies are also becoming increasingly popular for similar reasons. Welded fabric mesh made of ribbed wire greater than 6 mm diameter may be Class H ductility, but plain or indented wire fabric may be assumed to be normal ductility (Class N).

The cross-sectional areas and perimeters of various sized bars, and the cross-sectional area per unit width of slabs are listed in the Appendix. Reinforcing bars in a member should either be straight or bent to standard shapes. These shapes must be fully dimensioned and listed in a schedule of the reinforcement which is used on site for the bending and fixing of the bars. Standard bar shapes and a method of scheduling are specified in BS4466. The bar types as previously described are commonly identified by the following codes: R for mild steel; Y for high yield deformed steel, type 1; T for high yield deformed steel, type 2; this notation is generally used throughout this book.

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2

Limit State Design

This design of an engineering structure must ensure that (1) under the worst loadings the structure is safe, and (2) during normal working conditions the deformation of the members does not detract from the appearance, durability or performance of the structure. Despite the difficulty in assessing the precise loading and variations in the strength of the concrete and steel, these requirements have to be met. Three basic methods using factors of safety to achieve safe, workable structures have been developed; they are

(1) The permissible stress method in which ultimate strengths of the materials are divided by a factor of safety to provide design stresses which are usually within the elastic range.
(2) The load factor method in which the working loads are multiplied by a factor of safety.
(3) The limit state method which multiplies the working loads by partial factors of safety and also divides the materials' ultimate strengths by further partial factors of safety.

The permissible stress method has proved to be a simple and useful method but it does have some serious inconsistencies. Because it is based on an elastic stress distribution, it is not really applicable to a semi-plastic material such as concrete, nor is it suitable when the deformations are not proportional to the load, as in slender columns. It has also been found to be unsafe when dealing with the stability of structures subject to overturning forces (see example 2.2).

In the load factor method the ultimate strength of the materials should be used in the calculations. As this method does not apply factors of safety to the material stresses, it cannot directly take account of the variability of the materials, and also it cannot be used to calculate the deflections or cracking at working loads.

The limit state method of design overcomes many of the disadvantages of the previous two methods. This is done by applying partial factors of

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safety, both to the loads and to the material strengths, and the magnitude of the factors may be varied so that they may be used either with the plastic conditions in the ultimate state or with the more elastic stress range at working loads. This flexibility is particularly important if full benefits are to be obtained from development of improved concrete and steel properties.

2.1 Limit States

The purpose of design is to achieve acceptable probabilities that a structure will not become unfit for its intended use – that is, that it will not reach a limit state. Thus, any way in which a structure may cease to be fit for use will constitute a limit state and the design aim is to avoid any such condition being reached during the expected life of the structure.

The two principal types of limit state are the ultimate limit state and the serviceability limit state.

(a) Ultimate Limit State

This requires that the structure must be able to withstand, with an adequate factor of safety against collapse, the loads for which it is designed. The possibility of buckling or overturning must also be taken into account, as must the possibility of accidental damage as caused, for example, by an internal explosion.

(b) Serviceability Limit States

Generally the most important serviceability limit states are:

(1) Deflection – the appearance or efficiency of any part of the structure must not be adversely affected by deflections
(2) Cracking – local damage due to cracking and spalling must not affect the appearance, efficiency or durability of the structure
(3) Durability – this must be considered in terms of the proposed life of the structure and its conditions of exposure.

Other limit states that may be reached include:

(4) Excessive vibration – which may cause discomfort or alarm as well as damage
(5) Fatigue – must be considered if cyclic loading is likely
(6) Fire resistance – this must be considered in terms of resistance to collapse, flame penetration and heat transfer
(7) Special circumstances – any special requirements of the structure which are not covered by any of the more common limit states, such as earthquake resistance, must be taken into account.

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The relative importance of each limit state will vary according to the nature of the structure. The usual procedure is to decide which is the crucial limit state for a particular structure and base the design on this, although durability and fire resistance requirements may well influence initial member sizing and concrete grade selection. Checks must also be made to ensure that all other relevant limit states are satisfied by the results produced. Except in special cases, such as water-retaining structures, the ultimate limit state is generally critical for reinforced concrete although subsequent serviceability checks may affect some of the details of the design. Prestressed concrete design, however, is generally based on serviceability conditions with checks on the ultimate limit state.

In assessing a particular limit state for a structure it is necessary to consider all the possible variable parameters such as the loads, material strengths and constructional tolerances.

2.2 Characteristic Material Strengths and Characteristic Loads

2.2.1 Characteristic Material Strengths

The strengths of materials upon which design is based are those strengths below which results are unlikely to fall. These are called 'characteristic' strengths. It is assumed that for a given material, the distribution of strength will be approximately 'normal', so that a frequency distribution curve of a large number of sample results would be of the form shown in

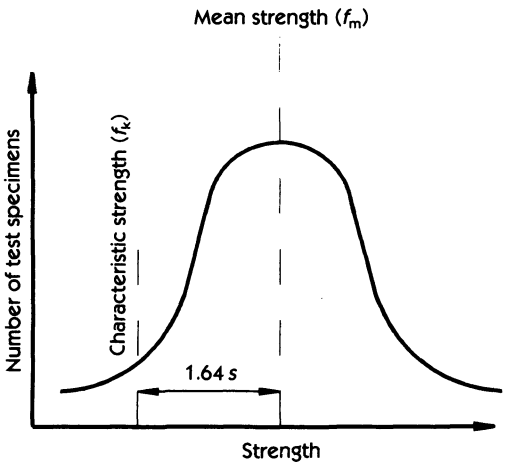

Figure 2.1 *Normal frequency distribution of strengths*

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figure 2.1. The characteristic strength is taken as that value below which it is unlikely that more than 5 per cent of the results will fall. This is given by

$$f_k = f_m - 1.64s$$

where f_k = characteristic strength, f_m = mean strength and s = standard deviation.

The relationship between characteristic and mean values accounts for variations in results of test specimens and will, therefore, reflect the method and control of manufacture, quality of constituents, and nature of the material.

2.2.2 Characteristic Loads

Ideally it should also be possible to assess loads statistically, in which case

characteristic load = mean load ± 1.64 standard deviations

In most cases it is the maximum loading on a structural member that is critical and the upper, positive value given by this expression is used, but the lower, minimum value may apply when considering stability or the behaviour of continuous members.

These characteristic values represent the limits within which at least 90 per cent of values will lie in practice. It is to be expected that not more than 5 per cent of cases will exceed the upper limit and not more than 5 per cent will fall below the lower limit. They are design values which take into account the accuracy with which the loads can be predicted.

Usually, however, there is insufficient statistical data to allow loading to be treated in this way, and in this case the standard loadings, such as those given in BS6399 Design Loads for Buildings, Part 1: Code of Practice for permanent and variable loads, should be used as representing characteristic values.

2.3 Partial Factors of Safety

Other possible variations such as constructional tolerances are allowed for by partial factors of safety applied to the strength of the materials and to the loadings. It should theoretically be possible to derive values for these from a mathematical assessment of the probability of reaching each limit state. Lack of adequate data, however, makes this unrealistic and in practice the values adopted are based on experience and simplified calculations.

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2.3.1 *Partial Factors of Safety for Materials* (γ_m)

$$\text{Design strength} = \frac{\text{characteristic strength } (f_k)}{\text{partial factor of safety } (\gamma_m)}$$

The following factors are considered when selecting a suitable value for γ_m.

(1) The strength of the material in an actual member. This strength will differ from that measured in a carefully prepared test specimen and it is particularly true for concrete where placing, compaction and curing are so important to the strength. Steel, on the other hand, is a relatively consistent material requiring a small partial factor of safety.
(2) The severity of the limit state being considered. Thus, higher values are taken for the ultimate limit state than for the serviceability limit state.

Recommended values for γ_m are given in table 2.1 although it should be noted that for precast factory conditions it may be possible to reduce the value for concrete at the ultimate limit state.

Table 2.1 Partial factors of safety applied to materials (γ_m)

Limit state	*Material*	
	Concrete	*Steel*
Ultimate		
Flexure	1.5	1.15
Shear	1.5	1.15
Bond	1.5	
Serviceability	1.0	1.0

2.3.2 *Partial Factors of Safety for Loads* (γ_f)

Errors and inaccuracies may be due to a number of causes:

(1) design assumptions and inaccuracy of calculation
(2) possible unusual load increases
(3) unforeseen stress redistributions
(4) constructional inaccuracies.

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These cannot be ignored, and are taken into account by applying a partial factor of safety (γ_f) on the loadings, so that

$$\text{design load} = \text{characteristic load} \times \text{partial factor of safety } (\gamma_f)$$

The value of this factor should also take into account the importance of the limit state under consideration and reflect to some extent the accuracy with which different types of loading can be predicted, and the probability of particular load combinations occurring. Recommended values are given in tables 2.2 and 2.3. It should be noted that design errors and construc-

Table 2.2 Load combinations and partial safety factors at the ultimate limit state

Load combination	*Permanent load*		*Variable load*		*Wind*
	Adverse	*Beneficial*	*Adverse*	*Beneficial*	
1. Permanent + variable	1.35	1.35*	1.50	0.0	—
2. Permanent + wind	1.35	1.0	—	—	1.50
3. Permanent + variable + wind	1.35	—	1.35	—	1.35

* (i) For continuous beams with cantilevers, the beneficial effect for the permanent load should be taken as 1.0 for the span adjacent to the cantilever (see figure 7.24).

(ii) Where the results of an analysis are sensitive to the magnitude of the permanent load, the adverse and beneficial partial safety factors of 1.35 and 1.0 respectively should also be considered. This particularly applies to cases involving static equilibrium (see example 2.2).

(iii) The partial safety factor for earth and water pressures should be taken as 1.35.

Table 2.3 Load combinations and partial safety factors at the serviceability limit state

Load combination	*Permanent load*	*Variable load*	*Wind*
1. Permanent + variable	1.0	1.0	—
2. Permanent + wind	1.0	—	1.0
3. Permanent + variable + wind	1.0	0.9	0.9

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tional inaccuracies have similar effects and are thus sensibly grouped together. These factors will account adequately for normal conditions although gross errors in design or construction obviously cannot be catered for.

2.4 Global Factor of Safety

The use of partial factors of safety on materials and loads offers considerable flexibility, which may be used to allow for special conditions such as very high standards of construction and control or, at the other extreme, where structural failure would be particularly disastrous.

The global factor of safety against a particular type of failure may be obtained by multiplying the appropriate partial factors of safety. For instance, a beam failure caused by yielding of tensile reinforcement would have a factor of

$$\gamma_m \times \gamma_f = 1.15 \times 1.35 = 1.55 \qquad \text{for permanent loads only}$$

or

$$1.15 \times 1.5 = 1.72 \qquad \text{for variable loads only}$$

Thus the practical case will have a value between these, depending on the relative loading proportions, and this can be compared with the value of 1.8 which has historically been the order of magnitude used by the load factor method prior to the introduction of Limit State Design.

Similarly, failure by crushing of the concrete in the compression zone has a factor of $1.5 \times 1.5 = 2.25$ due to variable loads only, which reflects the fact that such failure is generally without warning and may be very serious. Thus the basic values of partial factors chosen are such that under normal circumstances the global factor of safety is similar to that used in earlier design methods.

Example 2.1

Determine the cross-sectional area of a mild steel cable which supports a total permanent load of 3.0 kN and a variable load of 2.0 kN as shown in figure 2.2.

The characteristic yield stress of the mild steel is 250 N/mm^2.

Carry out the calculations using:

(1) The load factor method with a load factor = 1.8
(2) A permissible stress design with a factor of safety of 1.8 on the yield stress
(3) A limit state design with the following factors of safety.

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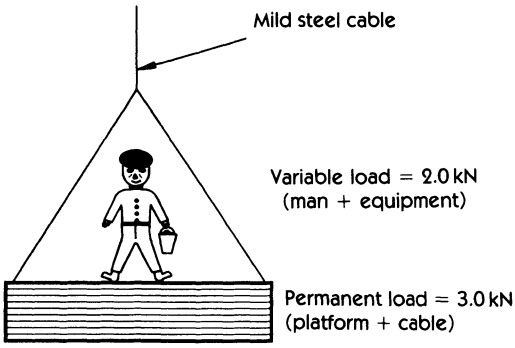

Figure 2.2 *Cable design*

$\gamma_G = 1.35$ for the permanent load, $\gamma_Q = 1.5$ for the variable load, and $\gamma_m = 1.15$ for the steel strength.

(a) Load Factor Method

$$\text{Design load} = \text{load factor (permanent load + variable load)}$$
$$= 1.8(3.0 + 2.0) = 9.0\,\text{kN}$$

$$\text{Required cross-sectional area} = \frac{\text{design load}}{\text{yield stress}}$$
$$= \frac{9.0 \times 10^3}{250} = 36\,\text{mm}^2$$

(b) Permissible Stress Method

$$\text{Design load} = 3.0 + 2.0 = 5.0\text{kN}$$

$$\text{Permissible stress} = \frac{\text{yield stress}}{\text{safety factor}}$$
$$= \frac{250}{1.8} = 139\,\text{N/mm}^2$$

$$\text{Required cross-sectional area} = \frac{\text{design load}}{\text{permissible stress}}$$
$$= \frac{5.0 \times 10^3}{139} = 36\,\text{mm}^2$$

(c) Limit State Method

$$\text{Design load} = \gamma_G \times \text{permanent load} + \gamma_Q \times \text{variable load}$$
$$= 1.35 \times 3.0 + 1.5 \times 2.0 = 7.05\,\text{kN}$$

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$$\text{Design stress} = \frac{\text{characteristic yield stress}}{\gamma_m}$$

$$= \frac{250}{1.15} = 217\,\text{N/mm}^2$$

$$\text{Required cross-sectional area} = \frac{\text{design load}}{\text{design stress}}$$

$$= \frac{7.05 \times 10^3}{217}$$

$$= 32.5\,\text{mm}^2$$

These different design methods all give similar results for the cross-sectional area. Fewer calculations are required for the permissible stress and the load factor methods, so reducing the chances of an arithmetical error. The limit state method provides much better control over the factors of safety, which are applied to each of the variables. For convenience, the partial factors of safety in the example are the same as those recommended in EC2. Probably, in a practical design, higher factors of safety would be preferred for a single supporting cable, in view of the consequences of a failure.

Example 2.2

Figure 2.3 shows a beam supported on foundations at A and B. The loads supported by the beam are its own uniformly distributed permanent weight of 20 kN/m and a 170 kN variable load concentrated at end C.

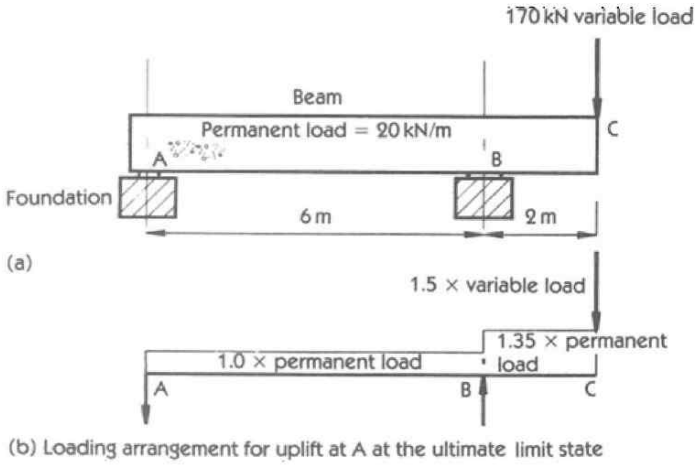

Figure 2.3

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Determine the weight of foundation required at A in order to resist uplift:

(1) by applying a factor of safety of 2.0 to the reaction calculated for the working loads
(2) using a limit state approach with partial factors of safety of γ_G = 1.35 or 1.0 for the permanent load and γ_Q = 1.5 for the variable load.

Investigate the effect on these designs of a 7 per cent increase in the variable load.

(a) Factor of Safety on Uplift = 2.0

Taking moments about B

$$\text{Uplift } R_A = \frac{(170 \times 2 - 20 \times 8 \times 2)}{6.0} = 3.33\,\text{kN}$$

$$\text{Weight of foundation required} = 3.33 \times \text{safety factor}$$
$$= 3.33 \times 2.0 = 6.7\,\text{kN}$$

With a 7 per cent increase in the variable load

$$\text{Uplift } R_A = \frac{(1.07 \times 170 \times 2 - 20 \times 8 \times 2)}{6.0} = 7.3\,\text{kN}$$

Thus with a slight increase in the variable load there is a significant increase in the uplift and the structure becomes unsafe.

(b) Limit State Method – Ultimate Load Pattern

As this example includes a cantilever and also involves the requirement for static equilibrium at A, partial factors of safety of 1.35 and 1.0 were chosen for the permanent load as specified by notes 1 or 2 of table 2.2.

The arrangement of the loads for the maximum uplift at A is shown in figure 2.3b.

$$\text{Design permanent load over BC} = \gamma_G \times 20 \times 2$$
$$= 1.35 \times 20 \times 2 = 54\,\text{kN}$$

$$\text{Design permanent load over AB} = \gamma_G \times 20 \times 6$$
$$= 1.0 \times 20 \times 6 = 120\,\text{kN}$$

$$\text{Design variable load} = \gamma_Q \times 170$$
$$= 1.5 \times 170 = 255\,\text{kN}$$

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Taking moments about B for the ultimate loads

$$\text{Uplift } R_A = \frac{(255 \times 2 + 54 \times 1 - 120 \times 3)}{6.0} = 34\,\text{kN}$$

Therefore weight of foundation required = 34 kN.

A 7 per cent increase in the variable load will not endanger the structure, since the actual uplift will only be 7.3 kN as calculated previously. In fact in this case it would require an increase of 54 per cent in the variable load before the uplift would exceed the weight of a 34 kN foundation.

Parts (a) and (b) of example 2.2 illustrate how the Limit State method of design can ensure a safer result when the stability or strength of a structure is sensitive to a small numerical difference between the effects of two opposing actions of a similar magnitude.

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3

Analysis of the Structure

A reinforced concrete structure is a combination of beams, columns, slabs and walls, rigidly connected together to form a monolithic frame. Each individual member must be capable of resisting the forces acting on it, so that the determination of these forces is an essential part of the design process. The full analysis of a rigid concrete frame is rarely simple; but simplified calculations of adequate precision can often be made if the basic action of the structure is understood.

The analysis must begin with an evaluation of all the loads carried by the structure, including its own weight. Many of the loads are variable in magnitude and position, and all possible critical arrangements of loads must be considered. First the structure itself is rationalised into simplified forms that represent the load-carrying action of the prototype. The forces in each member can then be determined by one of the following methods:

(1) applying moment and shear coefficients
(2) manual calculations
(3) computer methods.

Tabulated coefficients are suitable for use only with simple, regular structures such as equal-span continuous beams carrying uniform loads. Manual calculations are possible for the vast majority of structures, but may be tedious for large or complicated ones. The computer can be an invaluable help in the analysis of even quite small frames, and for some calculations it is almost indispensable. However, the amount of output from a computer analysis is sometimes almost overwhelming; and then the results are most readily interpreted when they are presented diagrammatically by means of a graph plotter or other visual device.

Since the design of a reinforced concrete member is generally based on the ultimate limit state, the analysis is usually performed for loadings corresponding to that state. Prestressed concrete members, however, are normally designed for serviceability loadings, as discussed in chapter 11.

W.H. Mosley et al. *Reinforced Concrete Design to Eurocode 2 (EC2)*
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3.1 Loads

The loads on a structure are divided into two types: permanent loads, and variable (or imposed) loads. Permanent loads are those which are normally constant during the structure's life. Variable loads, on the other hand, are transient and not constant in magnitude, as for example those due to wind or to human occupants. Recommendations for the loadings on buildings are given in the British Standards, numbers BS6399: Part 1. Design loads for Buildings, and CP3: Chapter V: Part 2. Wind loads. Bridge loadings are specified in BS5400: Part 2. Specification for Loads.

A table of values for some useful permanent loads and variable loads is given in the Appendix.

3.1.1 Permanent Loads

Permanent loads include the weight of the structure itself, and all architectural components such as exterior cladding, partitions and ceilings. Equipment and static machinery, when permanent fixtures, are also often considered as part of the permanent load. Once the sizes of all the structural members, and the details of the architectural requirements and permanent fixtures have been established, the permanent loads can be calculated quite accurately; but first of all, preliminary design calculations are generally required to estimate the probable sizes and self-weights of the structural concrete elements.

For most reinforced concretes, a typical value for the self-weight is 24 kN per cubic metre, but a higher density should be taken for heavily reinforced or dense concretes. In the case of a building, the weights of any permanent partitions should be calculated from the architects' drawings. A minimum partition loading equivalent to 1.0 kN per square metre is often specified as a variable load, but this is only adequate for lightweight partitions.

Permanent loads are generally calculated on a slightly conservative basis, so that a member will not need redesigning because of a small change in its dimensions. Over-estimation, however, should be done with care, since the permanent load can often actually reduce some of the forces in parts of the structure as will be seen in the case of the hogging moments in the continuous beam of figure 3.1.

3.1.2 Variable Loads

These loads are more difficult to determine accurately. For many of them, it is only possible to make conservative estimates based on standard

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codes of practice or past experience. Examples of variable loads on buildings are: the weights of its occupants, furniture, or machinery; the pressures of wind, the weight of snow, and of retained earth or water; and the forces caused by thermal expansion or shrinkage of the concrete.

A large building is unlikely to be carrying its full variable load simultaneously on all its floors. For this reason the current British Standard Code of Practice allows a reduction in the total variable floor loads when the columns, walls or foundations are designed, for a building more than two storeys high. Similarly, the variable load may be reduced when designing a beam span which supports a floor area greater then 40 square metres.

Although the wind load is a variable load, it is kept in a separate category when its partial factors of safety are specified, and when the load combinations on the structure are being considered.

3.2 Load Combinations and Patterns

3.2.1 Load Combinations and Patterns for the Ultimate Limit State

Various combinations of the characteristic values of permanent load G_k, variable load Q_k, wind load W_k and their partial factors of safety must be considered for the loading of the structure. The partial factors of safety specified in the code are discussed in chapter 2, and for the ultimate limit state the loading combinations from table 2.2 are as follows.

(1) Permanent and variable load

$$1.35G_k + 1.5Q_k$$

(2) Permanent and wind load

$$1.0G_k + 1.5W_k$$

(3) Permanent, variable and wind load

$$1.35G_k + 1.35Q_k + 1.35W_k$$

The imposed load can usually cover all or any part of the structure and, therefore, should be arranged to cause the most severe stresses. Load combination 1 should also be associated with a minimum beneficial design load applied to such parts of the structure as will give the most unfavourable conditions.

For load combination 1, a three-span continuous beam would have the loading arrangement shown in figure 3.1, in order to cause the maximum sagging moment in the outer spans and the maximum possible hogging moment in the centre span. A study of the deflected shape of the beam would confirm this to be the case.

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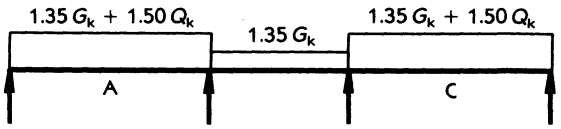


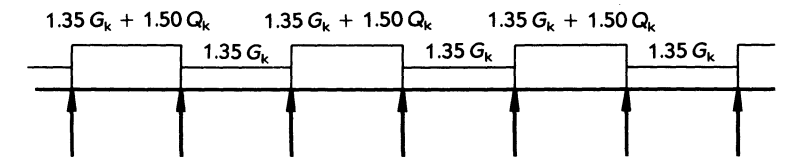
(a) Loading pattern for maximum sagging moment at A and C

(b) Deflected shape

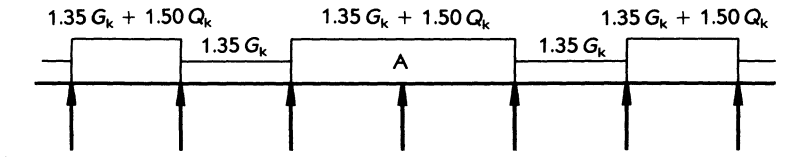
Figure 3.1 *Three-span beam*

Figure 3.2 shows the patterns of vertical loading on a multi-span continuous beam to cause (i) maximum design sagging moments in alternate spans and maximum possible hogging moments in adjacent spans, (ii) maximum design hogging moments at support A, and (iii) the design hogging moment at support A as specified by the EC2 code for simplicity. Thus there is a similar loading pattern for the design hogging moment at each internal support of a continuous beam.

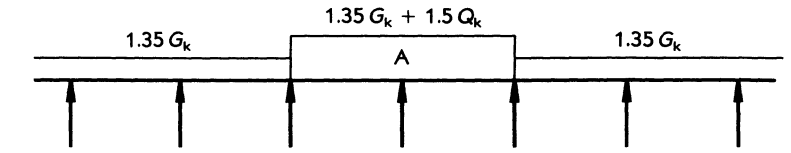
Under load combination 2, permanent and wind load, it is possible



(i) Loading pattern for maximum moments in the spans



(ii) Loading pattern for maximum support moment at A



(iii) Loading for design moment at support according to EC2

Figure 3.2 *Multi-span beam loading patterns. Note that when there is a cantilever span the minimum beneficial load on the span adjacent to the cantilever should be 1.0* G_k *for loading pattern* (i)

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that a critical stability condition may occur if, on certain parts of a structure, the permanent load is taken as $1.35G_k$. An example of this is illustrated in figure 3.3, depicting how the permanent load of the cantilever section increases the overturning moment about support B.

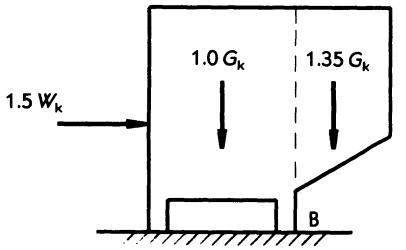

Figure 3.3 *Load combination: permanent plus wind*

3.2.2 Load Combinations for the Serviceability Limit State

A partial factor of safety of $\gamma_f = 1.0$ is usually applied to all vertical load combinations with $\gamma_f = 0.9$ applied to variable and wind loading when acting together at the serviceability limit state, as specified in table 2.3.

In considering deflections, the imposed load should be arranged to give the worst effects. The deflections calculated from the load combinations are the immediate deflections of a structure. Deflection increases due to the creep of the concrete should be based only on the permanent load plus any part of the variable load which is permanently on the structure, this being considered fully in chapter 6, which deals with serviceability requirements.

3.3 Analysis of Beams and Frames

To design a structure it is necessary to know the bending moments, torsional moments, shearing forces and axial forces in each member. An elastic analysis is generally used to determine the distribution of these forces within the structure; but because – to some extent – reinforced concrete is a plastic material, a limited redistribution of the elastic moments is sometimes allowed. A plastic yield-line theory may be used to calculate the moments in concrete slabs. The properties of the materials, such as Young's modulus, which are used in the structural analysis should be those associated with their characteristic strengths. The stiffnesses of the members can be calculated on the basis of any one of the following:

(1) the entire concrete cross-section (ignoring the reinforcement)

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(2) the concrete cross-section plus the transformed area of reinforcement based on the modular ratio
(3) the compression area only of the concrete cross-section, plus the transformed area of reinforcement based on the modular ratio.

The concrete cross-section described in (1) is the simpler to calculate and would normally be chosen.

A structure should be analysed for each of the critical loading conditions which produce the maximum stresses at any particular section. This procedure will be illustrated in the examples for a continuous beam and a building frame. For these structures it is conventional to draw the bending-moment diagram on the tension side of the members.

Sign Conventions

(1) For the moment-distribution analyses anti-clockwise support moments are positive as, for example, in table 3.1 for the fixed end moments (FEM).
(2) For subsequently calculating the moments along the span of a member, moments causing sagging are positive, while moments causing hogging are negative, as illustrated in figure 3.5.

3.3.1 Non-Continuous Beams

One-span, simply supported beams or slabs are statically determinate and the analysis for bending moments and shearing forces is readily performed manually. For the ultimate limit state we need only consider the maximum load of $1.35G_k + 1.5Q_k$ on the span.

Example 3.1 Analysis of a Non-Continuous Beam

The one-span simply supported beam shown in figure 3.4a carries a distributed permanent load including self-weight of 25 kN/m, a permanent concentrated load of 40 kN at mid-span, and a distributed variable load of 10 kN/m.

Figure 3.4 shows the values of ultimate load required in the calculations of the shearing forces and bending moments.

$$\text{Maximum shear force} = \frac{54}{2} + \frac{195}{2} = 124.5\,\text{kN}$$

$$\text{Maximum bending moment} = \frac{54 \times 4}{4} + \frac{195 \times 4}{8} = 151.5\,\text{kN m}$$

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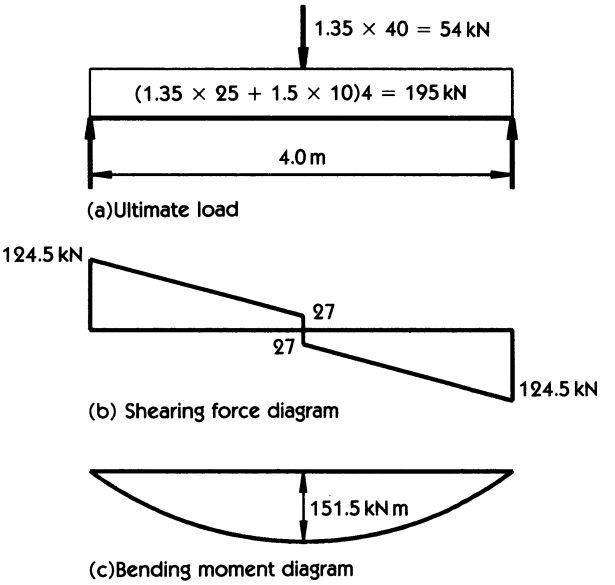

Figure 3.4 *Analysis of one-span beam*

The analysis is completed by drawing the shearing-force and bending-moment diagrams which would later be used in the design and detailing of the shear and bending reinforcement.

3.3.2 Continuous Beams

The methods of analysis for continuous beams may also be applied to continuous slabs which span in one direction. A continuous beam is considered to have no fixity with the supports so that the beam is free to rotate. This assumption is not strictly true for beams framing into columns and for that type of continuous beam it is more accurate to analyse them as part of a frame, as described in section 3.3.3.

A continuous beam should be analysed for the loading arrangements which give the maximum stresses at each section, as described in section 3.2.1 and illustrated in figures 3.1 and 3.2. The analysis to calculate the bending moments can be carried out manually by moment distribution or equivalent methods, but tabulated shear and moment coefficients may be adequate for continuous beams having approximately equal spans and uniformly distributed loads.

For a beam or slab cast monolithically into its supports, the design

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moment at the support can be taken as the moment at the face of the support, such as M'_{AB} in figure 3.5.

Continuous Beams – The General Case

Having determined the moments at the supports by, say, moment distribution, it is necessary to calculate the moments in the spans and also the shear forces on the beam. For a uniformly distributed load, the equations for the shears and the maximum span moments can be derived from the following analysis.

Using the sign convention of figure 3.5 and taking moments about support B:

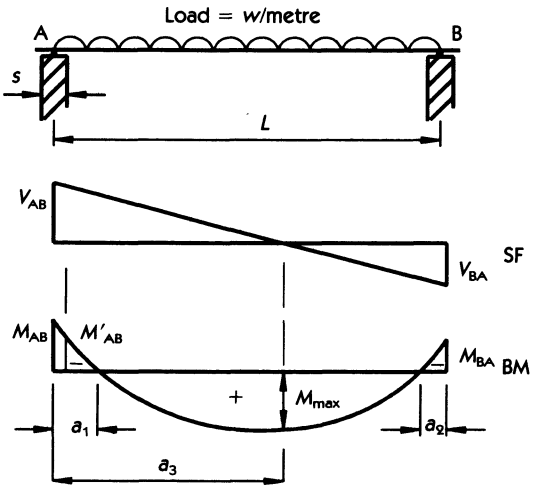

Figure 3.5 *Shears and moments in a beam*

$$V_{AB}L - \frac{wL^2}{2} + M_{AB} - M_{BA} = 0$$

therefore

$$V_{AB} = \frac{wL}{2} - \frac{(M_{AB} - M_{BA})}{L} \tag{3.1}$$

and

$$V_{BA} = wL - V_{AB} \tag{3.2}$$

Maximum span moment M_{max} occurs at zero shear, and distance to zero shear

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$$a_3 = \frac{V_{AB}}{w} \tag{3.3}$$

therefore

$$M_{max} = \frac{V_{AB}^2}{2w} + M_{AB} \tag{3.4}$$

The points of contraflexure occur at $M = 0$, that is

$$V_{AB}x - \frac{wx^2}{2} + M_{AB} = 0$$

where x is the distance from support A. Taking the roots of this equation gives

$$x = \frac{V_{AB} \pm \sqrt{(V_{AB}^2 + 2wM_{AB})}}{w}$$

so that

$$a_1 = \frac{V_{AB} - \sqrt{(V_{AB}^2 + 2wM_{AB})}}{w} \tag{3.5}$$

and

$$a_2 = L - \frac{V_{AB} + \sqrt{(V_{AB}^2 + 2wM_{AB})}}{w} \tag{3.6}$$

A similar analysis can be applied to beams that do not support a uniformly distributed load. In manual calculations it is usually not considered necessary to calculate the distances a_1, a_2 and a_3 which locate the points of contraflexure and maximum moment – a sketch of the bending moment is often adequate – but if a computer is performing the calculations these distances may as well be determined also.

At the face of the support, width s

$$M'_{AB} = M_{AB} - \left(V_{AB} - \frac{ws}{4}\right)\frac{s}{2}$$

Example 3.2 Analysis of a Continuous Beam

The continuous beam shown in figure 3.6 has a constant cross-section and supports a uniformly distributed permanent load including its self-weight of $G_k = 25\,kN/m$ and a variable load $Q_k = 10\,kN/m$.

The critical loading patterns for the ultimate limit state are shown in figure 3.6 where the 'stars' indicate the region of maximum moments, sagging or possible hogging. Table 3.1 is the moment distribution carried out for the first loading arrangement: similar calculations would be

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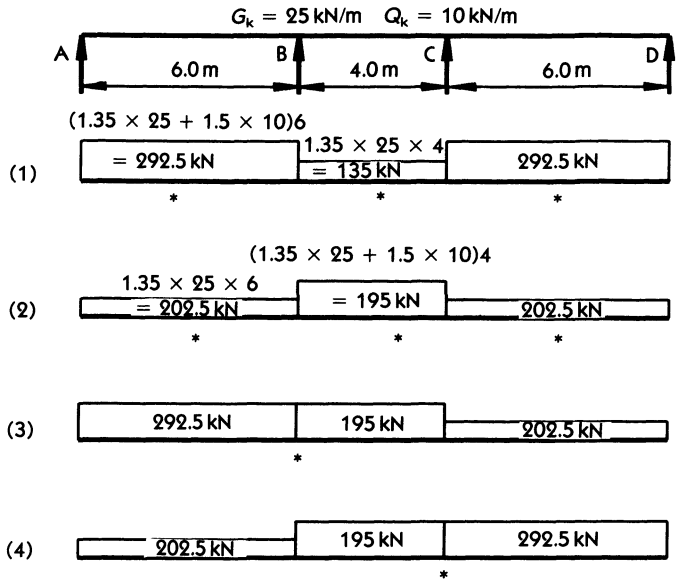

Figure 3.6 *Continuous beam loading patterns*

required for each of the remaining load cases. It should be noted that the reduced stiffness of $\frac{3}{4}I/L$ has been used for the end spans.

The shearing forces, the maximum span bending moments, and their positions along the beam, can be calculated using the formulae previously derived. Thus for the first loading arrangement and span AB, using the sign convention of figure 3.5:

$$\text{Shear } V_{AB} = \frac{\text{load}}{2} - \frac{(M_{AB} - M_{BA})}{L}$$

$$= \frac{292.5}{2} - \frac{132.5}{6.0} = 124.2\,\text{kN}$$

$$V_{BA} = \text{load} - V_{AB} = 292.5 - 124.2 = 168.3\,\text{kN}$$

$$\text{Maximum moment, span AB} = \frac{V_{AB}^2}{2w} + M_{AB}$$

where $w = 292.5/6.0 = 48.75\,\text{kN/m}$. Therefore

$$M_{max} = \frac{124.2^2}{2 \times 48.75} = 158.2\,\text{kN m}$$

$$\text{Distance from A, } a_3 = \frac{V_{AB}}{w} = \frac{124.2}{48.75} = 2.55\,\text{m}$$

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Table 3.1 Moment distribution for the first loading case

	A	*B*		*C*	*D*
Stiffness (k)		$\frac{3}{4}\cdot\frac{I}{L}$		$\frac{I}{L}$	$\frac{3}{4}\cdot\frac{I}{L}$
		$\frac{3}{4}\cdot\frac{1}{6} = 0.125$		$= \frac{1}{4} = 0.25$	$= 0.125$
Distr. factors		$\frac{0.125}{0.125 + 0.25}$		$\frac{0.125}{0.125 + 0.25}$	
		$= 1/3$		$= 2/3$	2/3	1/3	
Load (kN)		292.5		135		292.5	
F.E.M.	0	$\frac{-292.5 \times 6}{8}$		$\pm\frac{135 \times 4}{12}$		$+\frac{292.5 \times 6}{8}$	0
=	0	−219.4		+45.0	−45.0	+219.4	0
Balance		+58.1		+116.3	−116.3	−58.1	
Carry over				−58.1	+58.1		
Balance		+19.4		+38.7	−38.7	−19.4	
Carry over				−19.4	+19.4		
Balance		+6.5		+12.9	−12.9	−6.5	
Carry over				−6.5	+6.5		
Balance		+2.2		+4.3	−4.3	−2.2	
Carry over				−2.2	+2.2		
Balance		+0.7		+1.5	−1.5	−0.7	
M (kN m)	0	−132.5		+132.5	−132.5	+132.5	0

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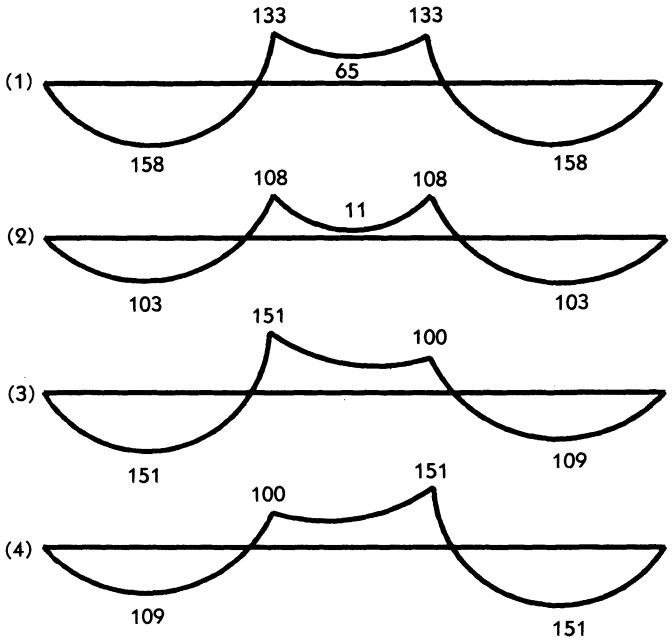

Figure 3.7 *Bending-moment diagrams (kN m)*

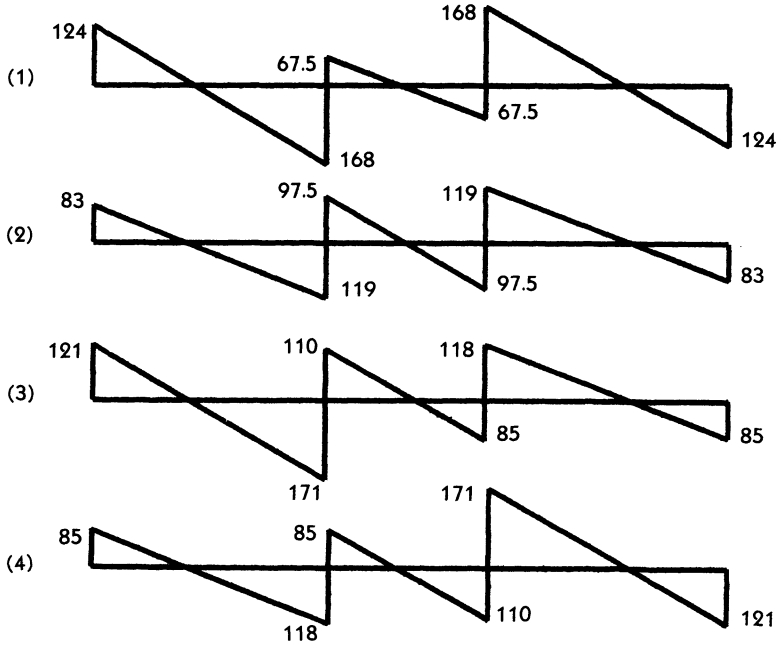

Figure 3.8 *Shearing-force diagrams (kN)*

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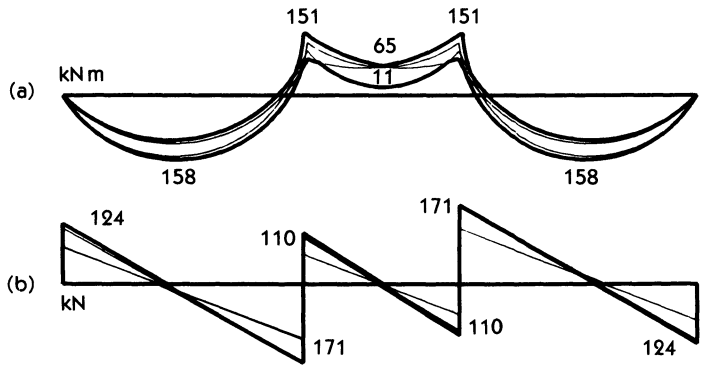

Figure 3.9 *Bending-moment and shearing-force envelopes*

The bending-moment diagrams for each of the loading arrangements are shown in figure 3.7, and the corresponding shearing-force diagrams are shown in figure 3.8. The individual bending-moment diagrams are combined in figure 3.9a to give the bending-moment design envelope. Similarly, figure 3.9b is the shearing-force design envelope. Such envelope diagrams are used in the detailed design of the beams, as described in chapter 7.

In this example, simple supports with no fixity have been assumed for the end supports at A and D. Even so, the sections at A and D should be designed for a hogging moment due to a partial fixity equal to 25 per cent of the maximum moment in the span, that is 158/4 = 39.5 kN m.

Continuous Beams with Approximately Equal Spans and Uniform Loading

The ultimate bending moments and shearing forces in continuous beams of three or more approximately equal spans without cantilevers can be obtained using relevant coefficients provided that the spans differ by no more than 15 per cent of the longest span, that the loading is uniform, and that the characteristic variable load does not exceed the characteristic permanent load. The values of these coefficients are shown in diagrammatic form in figure 3.10 for beams (equivalent simplified values for slabs are given in chapter 8).

The possibility of hogging moments in any of the spans should not be ignored, even if it is not indicated by these coefficients. For example, a beam of three equal spans may have a hogging moment in the centre span if Q_k exceeds $0.45G_k$.

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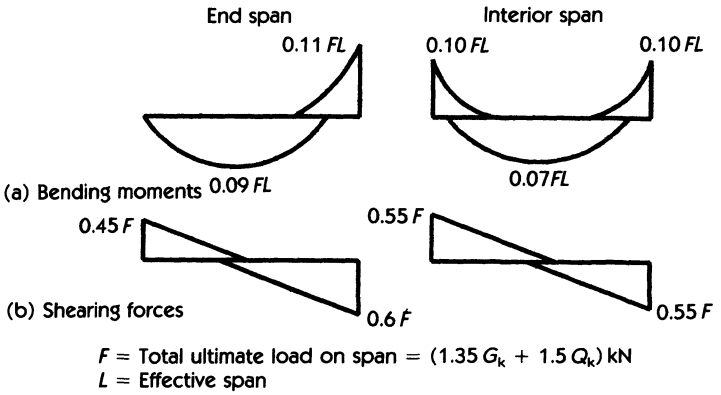

Figure 3.10 *Bending-moment and shearing-force coefficients for beams*

3.3.3 Building Structures and Frames

Building structures can be classified into two main types: (1) braced and (2) unbraced.

A braced structure is one that resists lateral loads such as wind or seismic forces by means of bracing in the form of rigid walls referred to as shear walls; these also include walls framing lift shafts and stairwells. The bracing elements must be capable of transmitting the horizontal loads down to the foundations.

An unbraced structure does not have bracing elements. Horizontal loads are resisted by the frame action of the rigidly connected columns, beams and slabs.

Some structures though, and particularly very high rise buildings, are designed to resist the horizontal load by a combination of frame action and the resistance of a rigid core structure formed by the lift shaft and stairwell.

Both braced and unbraced structures can also be classified as non-sway or sway.

Non-sway structures are those where there is less than a 10 per cent increase in the normal, design moment due to the displacements of the structure. Braced structures with substantial bracing elements may be assumed to be classified as non-sway, except for high rise structures where a second-order analysis may be required.

Sway structures are those where the secondary moments due to displacements are greater than 10 per cent of the normal design moment.

In situ reinforced concrete structures behave as rigid frames, and

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should be analysed as such. They can be analysed as a complete space frame or be divided into a series of plane frames. Bridge-deck types of structures can be analysed as an equivalent grillage, while some form of finite-element analysis can be utilised in solving complicated shear-wall buildings. All these methods lend themselves to solution by the computer, but many frames can be simplified for solution by hand calculations.

The general procedure for a building is to analyse the slabs as continuous members supported by the beams or structural walls. The slabs can be either one-way spanning or two-way spanning. The columns and main beams can be considered as a series of rigid plane frames.

Analysis of Frames for the Vertical Loading

A building frame can be analysed as a complete frame, or it can be simplified into a series of substitute frames for the vertical loading analysis. The frame shown in figure 3.11, for example, can be divided into any of the subframes shown in figure 3.12.

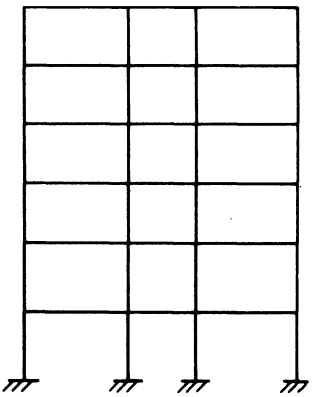

Figure 3.11 *Building frame*

The substitute frame 1 in figure 3.12 consists of one complete floor beam with its connecting columns (which are assumed rigidly fixed at their remote ends). An analysis of this frame will give the bending moments and shearing forces in the beams and columns for the floor level considered.

Substitute frame 2 is a single span combined with its connecting columns and two adjacent spans, all fixed at their remote ends. This frame may be used to determine the bending moments and shearing

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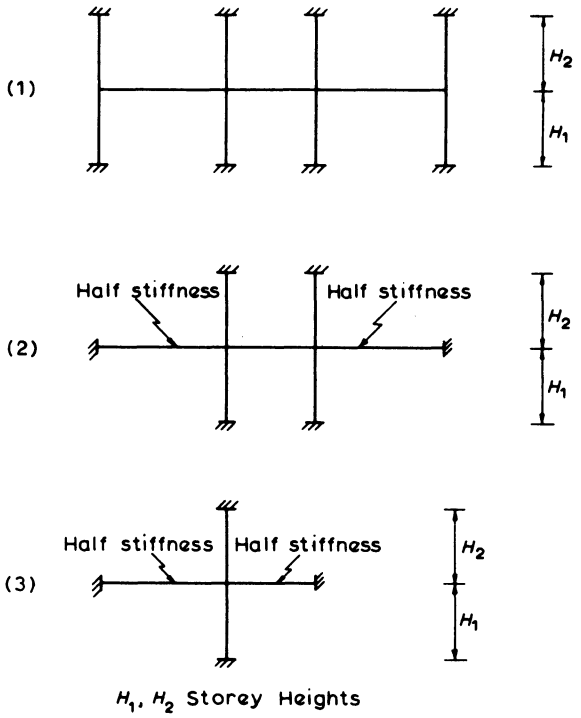

Figure 3.12 *Substitute frames*

forces in the central beam. Provided that the central span is greater then the two adjacent spans, the bending moments in the columns can also be found with this frame.

Substitute frame 3 can be used to find the moments in the columns only. It consists of a single junction, with the remote ends of the members fixed. This type of subframe would be used when beams have been analysed as continuous over simple supports.

In frames 2 and 3, the assumption of fixed ends to the outer beams over-estimates their stiffnesses. These values are, therefore, halved to allow for the flexibility resulting from continuity.

The various critical loading patterns to produce maximum stresses have to be considered. In general these loading patterns for the ultimate limit state are as shown in figure 3.2, except when there is also a cantilever span, in which case the span adjacent to the cantilever should be loaded with $1.0G_k$ for the minimum beneficial loading condition.

When considering the critical loading arrangements for a column, it is sometimes necessary to include the case of maximum moment and

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minimum possible axial load, in order to investigate the possibility of tension failure caused by the bending.

Example 3.3 Analysis of a Substitute Frame

The substitute frame shown in figure 3.13 is part of the complete frame in figure 3.11. The characteristic loads carried by the beams are permanent loads (including self-weight). G_k = 25 kN/m, and variable load, Q_k = 10 kN/m, uniformly distributed along the beam. The analysis of the beam will be carried out by moment distribution: thus the member stiffnesses and their relevant distribution factors are first required.

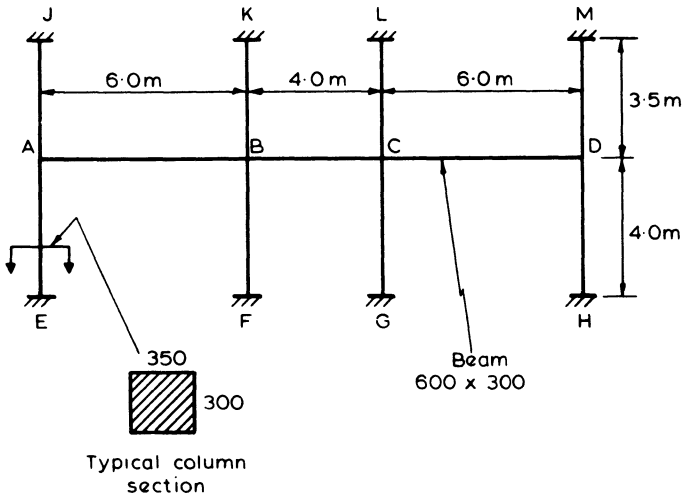

Figure 3.13 *Substitute frame*

Stiffnesses, k

Beam

$$I = \frac{0.3 \times 0.6^3}{12} = 5.4 \times 10^{-3}\,\text{m}^4$$

Spans AB and CD

$$k_{AB} = k_{CD} = \frac{5.4 \times 10^{-3}}{6.0} = 0.9 \times 10^{-3}$$

Span BC

$$k_{BC} = \frac{5.4 \times 10^{-3}}{4.0} = 1.35 \times 10^{-3}$$

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Columns

$$I = \frac{0.3 \times 0.35^3}{12} = 1.07 \times 10^{-3}\,\text{m}^4$$

Upper

$$k_U = \frac{1.07 \times 10^{-3}}{3.5} = 0.31 \times 10^{-3}$$

Lower

$$k_L = \frac{1.07 \times 10^{-3}}{4.0} = 0.27 \times 10^{-3}$$

$$k_U + k_L = (0.31 + 0.27)10^{-3} = 0.58 \times 10^{-3}$$

Distribution Factors

Joints A and D

$$\Sigma k = 0.9 + 0.58 = 1.48$$

$$\text{D.F.}_{AB} = \text{D.F.}_{DC} = \frac{0.9}{1.48} = 0.61$$

$$\text{D.F.}_{cols} = \frac{0.58}{1.48} = 0.39$$

Joints B and C

$$\Sigma k = 0.9 + 1.35 + 0.58 = 2.83$$

$$\text{D.F.}_{BA} = \text{D.F.}_{CD} = \frac{0.9}{2.83} = 0.32$$

$$\text{D.F.}_{BC} = \text{D.F.}_{CB} = \frac{1.35}{2.83} = 0.48$$

$$\text{D.F.}_{cols} = \frac{0.58}{2.83} = 0.20$$

The critical loading patterns for the ultimate limit state are identical to those for the continuous beam in example 3.2, and they are illustrated in figure 3.6. The moment distribution for the first loading arrangement is shown in table 3.2. In the table, the distributions for each upper and lower column have been combined, since this simplifies the layout for the calculations.

The shearing forces and the maximum span moments can be calculated from the formulae of section 3.3.2. For the first loading arrangement and span AB:

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Table 3.2 Moment distribution for the first loading case

	A			*B*				*C*				*D*	
D.F.s	*Cols.* (ΣM)	*AB*		*BA*	*Cols.* (ΣM)	*BC*		*CB*	*Cols.* (ΣM)	*CD*		*DC*	*Cols.* (ΣM)
Load kN	*0.39*	*0.61*	*292.5*	*0.32*	*0.20*	*0.48*	*135*	*0.48*	*0.20*	*0.32*	*292.5*	*0.61*	*0.39*
F.E.M.		+ 146		− 146		+ 45.0		− 45.0		+ 146		− 146	
Bal.	− 56.9	− 89.1		+ 32.3	+ 20.2	+ 48.5		− 48.5	− 20.2	− 32.3		+ 89.1	+ 56.9
C.O.		+ 16.2		− 44.6		− 24.2		+ 24.2		+ 44.6		− 16.2	
Bal.	− 6.3	− 9.9		+ 22.0	+ 13.8	+ 33.0		− 33.0	− 13.8	− 22.0		+ 9.9	+ 6.3
C.O.		+ 11.0		− 5.0		− 16.5		+ 16.5		+ 5.0		− 11.0	
Bal.	− 4.3	− 6.7		+ 6.9	+ 4.3	+ 10.3		− 10.3	− 4.3	− 6.9		+ 6.7	+ 4.3
C.O.		+ 3.4		− 3.4		− 5.2		+ 5.2		+ 3.4		− 3.4	
Bal.	− 1.3	− 2.1		+ 2.8	+ 1.7	+ 4.1		− 4.1	− 1.7	− 2.8		+ 2.1	+ 1.3
M (kN m)	− 68.8	+ 68.8		− 135.0	+ 40.0	+ 95.0		− 95.0	− 40.0	+ 135.0		− 68.8	+ 68.8

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$$\text{Shear } V_{AB} = \frac{\text{load}}{2} - \frac{(M_{AB} - M_{BA})}{L}$$

$$= \frac{292.5}{2} - \frac{(-68.8 + 135.0)}{6.0} = 135\,\text{kN}$$

$$V_{BA} = \text{load} - V_{AB} = 292.5 - 135 = 157\,\text{kN}$$

$$\text{Maximum moment, span AB} = \frac{V_{AB}^2}{2w} + M_{AB}$$

$$= \frac{135^2}{2 \times 48.75} - 68.8$$

$$= 118\,\text{kN m}$$

$$\text{Distance from A, } a_3 = \frac{V_{AB}}{w} = \frac{135}{48.75} = 2.8\,\text{m}$$

Figure 3.14 shows the bending moments in the beams for each loading pattern: figure 3.15 shows the shearing forces. These diagrams have been combined in figure 3.16 to give design envelopes for bending moments and shearing forces.

A comparison of the design envelopes of figure 3.16 and figure 3.9 will

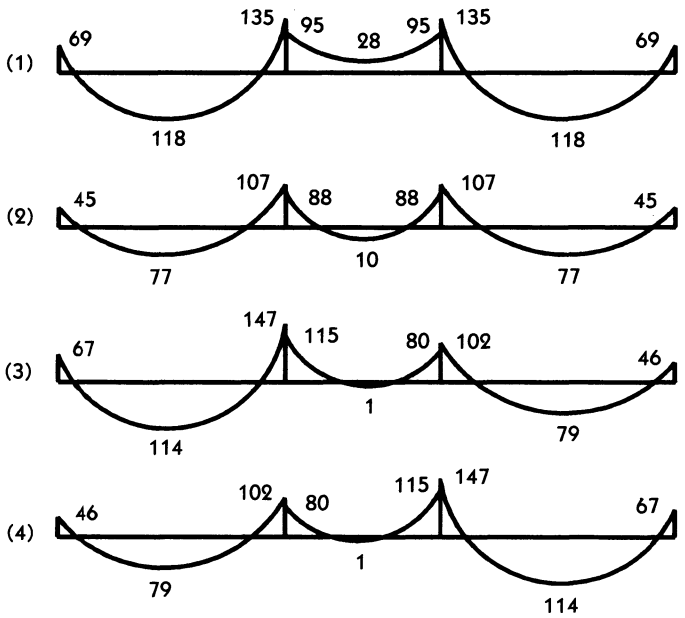

Figure 3.14 *Beam bending-moment diagrams (kN m)*

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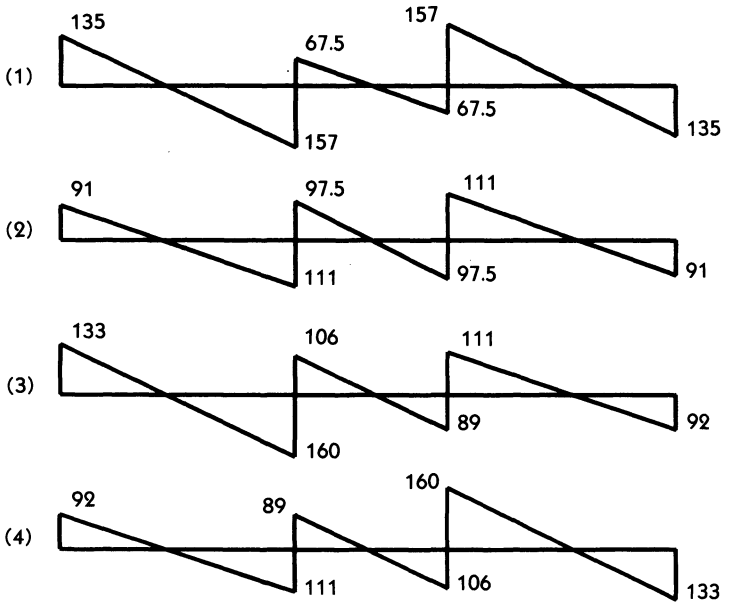

Figure 3.15 *Beam shearing-force diagrams (kN)*

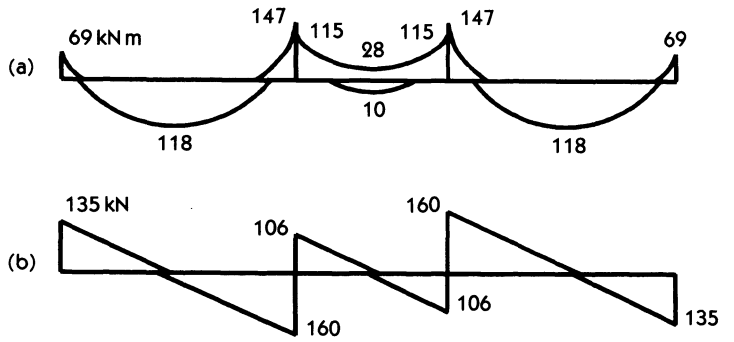

Figure 3.16 *Bending-moment and shearing-force envelopes*

emphasise the advantages of considering the concrete beam as part of a frame, not as a continuous beam as in example 3.2. Not only is the analysis of a subframe more precise, but many moments and shears in the beam are smaller in magnitude.

The moment in each column is given by

$$M_{\text{col}} = \Sigma M_{\text{col}} \times \frac{k_{\text{col}}}{\Sigma k_{\text{cols}}}$$

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Thus, for the first loading arrangement and taking ΣM_{col} from table 3.2 gives

$$\text{column moment } M_{AJ} = 68.8 \times \frac{0.31}{0.58} = 37\,\text{kN m}$$

$$M_{AE} = 68.8 \times \frac{0.27}{0.58} = 32\,\text{kN m}$$

$$M_{BK} = 40 \times \frac{0.31}{0.58} = 21\,\text{kN m}$$

$$M_{BF} = 40 \times \frac{0.27}{0.58} = 19\,\text{kN m}$$

This loading arrangement gives the maximum column moments, as plotted in figure 3.17.

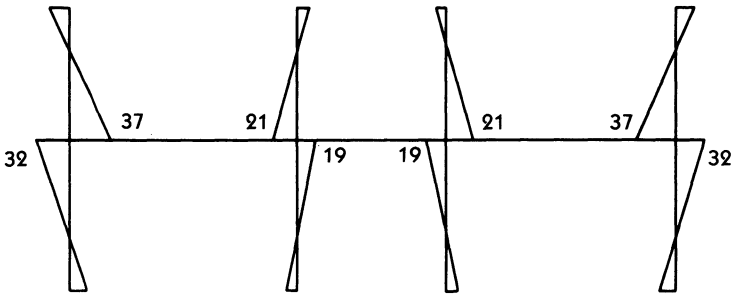

Figure 3.17 *Column bending moments (kN m)*

Example 3.4 Analysis of a Substitute Frame for a Column

The substitute frame for this example, shown in figure 3.18, is taken from the building frame in figure 3.11. The loading to cause maximum column moments is shown in the figure for $G_k = 25\,\text{kN/m}$ and $Q_k = 10\,\text{kN/m}$.

The stiffnesses of these members are identical to those calculated in example 3.3, except that for this type of frame the beam stiffnesses are halved. Thus

$$k_{AB} = \frac{1}{2} \times 0.9 \times 10^{-3} = 0.45 \times 10^{-3}$$

$$k_{BC} = \frac{1}{2} \times 1.35 \times 10^{-3} = 0.675 \times 10^{-3}$$

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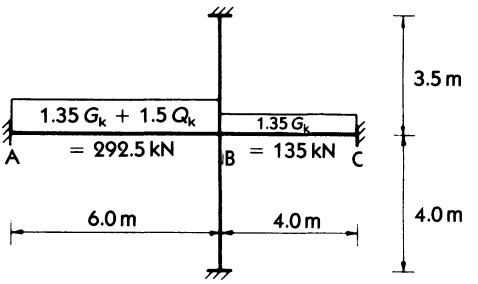

Figure 3.18 *Substitute frame*

$$\text{upper column } k_U = 0.31 \times 10^{-3}$$

$$\text{lower column } k_L = 0.27 \times 10^{-3}$$

$$\Sigma k = (0.45 + 0.675 + 0.31 + 0.27) \times 10^{-3}$$
$$= 1.705 \times 10^{-3}$$

$$\text{fixed-end moment } M_{BA} = 292.5 \times \frac{6}{12} = 146\,\text{kN m}$$

$$\text{fixed-end moment } M_{BC} = 135 \times \frac{4}{12} = 45\,\text{kN m}$$

Column moments are

$$\text{upper column } M_U = (146 - 45) \times \frac{0.31}{1.705} = 18\,\text{kN m}$$

$$\text{lower column } M_L = (146 - 45) \times \frac{0.27}{1.705} = 16\,\text{kN m}$$

The column moments are illustrated in figure 3.19. They should be compared with the corresponding moments for the internal column in figure 3.17.

In examples 3.3 and 3.4 the second moment of area of the beam was calculated as $bh^3/12$ for a rectangular section for simplicity, but where an *in situ* slab forms a flange to the beam, the second moment of area may be calculated for the T-section or L-section.

3.3.4 Lateral Loads on Frames

Lateral loads on a structure may be caused by wind pressures, by retained earth or by seismic forces. In addition, the code specifies a minimum

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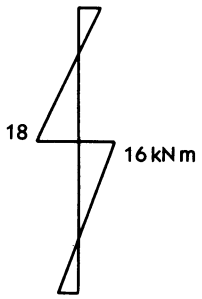

Figure 3.19 *Column moments*

horizontal load that must be resisted by a structure as given by the following equations:

(1) The design horizontal load assumed to act at any floor level, should not be less than

$$H_{min} = V_i \nu \alpha_n$$

where V_i is the total design vertical load above the level considered;
ν is a notional 'out of plumb' angle, in radians, given by $1/(100\sqrt{l}) \geqslant 1/200$, where l is the overall height of the structure, in metres

α_n is an allowance for cases where n vertically continuous elements act together, and is given by

$$\alpha_n = \sqrt{[(1 + 1/n)/2]}$$

(2) In a braced structure, the horizontal elements connecting the vertical elements to the bracing structure should be designed to carry an additional horizontal load equal to

$$H_{fd} = (N_{ba} + N_{bc})\nu/2$$

where N_{ba} and N_{bc} are, respectively, the design vertical loads in the vertical elements above and below the horizontal member considered.

An unbraced frame subjected to wind forces must be analysed for all the three loading combinations described in section 3.2.1. The vertical-loading analysis can be carried out by the methods described previously (see page 43). The analysis for the lateral loads should be kept separate and the forces may be calculated by an elastic analysis or by a simplified approximate method. For preliminary design calculations, and also only for medium-size regular structures, a simplified analysis may well be adequate.

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A suitable approximate analysis is the cantilever method. It assumes that:

(1) points of contraflexure are located at the mid-points of all columns and beams; and
(2) the direct axial loads in the columns are in proportion to their distances from the centre of gravity of the frame. It is also usual to assume that all the columns in a storey are of equal cross-sectional area.

It should be emphasised that these approximate methods may give quite inaccurate results for irregular or high rise structures.

Application of this method is probably best illustrated by an example, as follows.

Example 3.5 Simplified Analysis for Lateral Loads – Cantilever Method

Figure 3.20 shows a building frame subjected to a characteristic wind load of 3.0 kN per metre height of the frame. This load is assumed to be transferred to the frame as a concentrated load at each floor level as indicated in the figure.

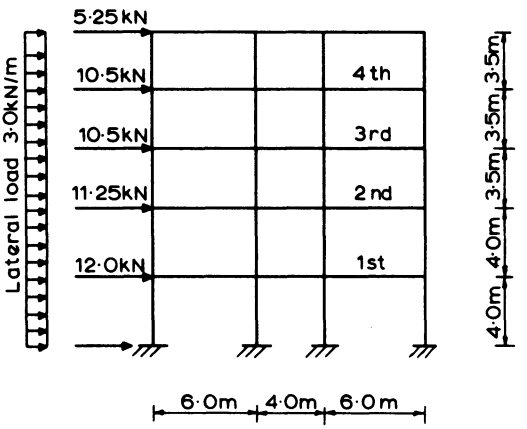

Figure 3.20 *Frame with lateral load*

By inspection, there is tension in the two columns to the left and compression in the columns to the right; and by assumption 2 the axial forces in columns are proportional to their distances from the centre line of the frame. Thus

axial force in exterior column : axial force in interior column
$= 4.0P : 1.0P$

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The analysis of the frame continues by considering a section through the top-storey columns: the removal of the frame below this section gives the remainder shown in figure 3.21a. The forces in this subframe are calculated as follows.

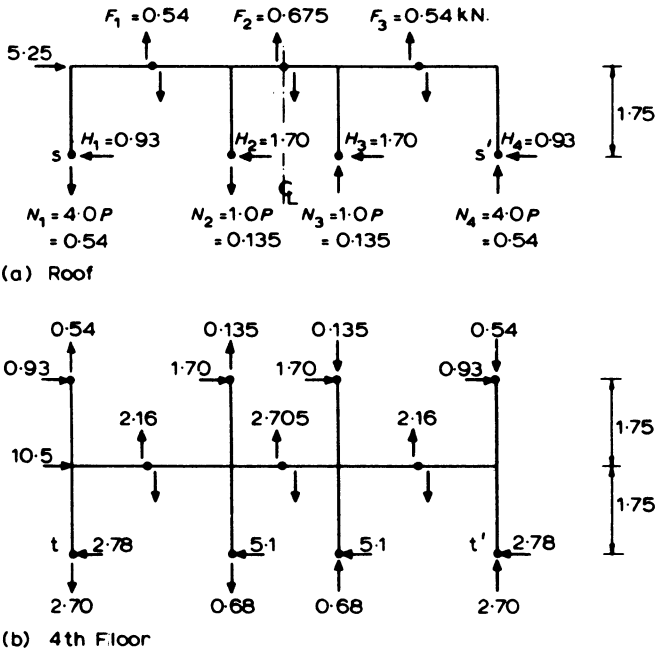

Figure 3.21 *Subframes at the roof and 4th floor*

(a) Axial Forces in the Columns

Taking moments about point s, $\Sigma M_s = 0$, therefore

$$5.25 \times 1.75 + P \times 6.0 - P \times 10.0 - 4P \times 16.0 = 0$$

and therefore

$$P = 0.135\,\text{kN}$$

thus

$$N_1 = -N_4 = 4.0P = 0.54\,\text{kN}$$

$$N_2 = -N_3 = 1.0P = 0.135\,\text{kN}$$

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(b) Vertical Shearing Forces F in the Beams

For each part of the subframe, $\Sigma F = 0$, therefore

$$F_1 = N_1 = 0.54\,\text{kN}$$

$$F_2 = N_1 + N_2 = 0.675\,\text{kN}$$

(c) Horizontal Shearing Forces H in the Columns

Taking moments about the points of contraflexure of each beam, $\Sigma M = 0$, therefore

$$H_1 \times 1.75 - N_1 \times 3.0 = 0$$

$$H_1 = 0.93\,\text{kN}$$

and

$$(H_1 + H_2)1.75 - N_1 \times 8.0 - N_2 \times 2.0 = 0$$

$$H_2 = 1.70\,\text{kN}$$

The calculations of the equivalent forces for the fourth floor (figure 3.21b) follow a similar procedure as follows.

(d) Axial Forces in the Columns

For the frame above section tt′, $\Sigma M_t = 0$, therefore

$$5.25\,(3 \times 1.75) + 10.5 \times 1.75 + P \times 6.0 - P \times 10.0 - 4P \times 16.0 = 0$$

$$P = 0.675\,\text{kN}$$

therefore

$$N_1 = 4.0P = 2.70\,\text{kN}$$

$$N_2 = 1.0P = 0.68\,\text{kN}$$

(e) Beam Shears

$$F_1 = 2.70 - 0.54 = 2.16\,\text{kN}$$

$$F_2 = 2.70 + 0.68 - 0.54 - 0.135 = 2.705\,\text{kN}$$

(f) Column Shears

$$H_1 \times 1.75 + 0.93 \times 1.75 - (2.70 - 0.54)3.0 = 0$$

$$H_1 = 2.78\,\text{kN}$$

$$H_2 = \tfrac{1}{2}(10.5 + 5.25) - 2.78 = 5.1\,\text{kN}$$

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Values calculated for sections taken below the remaining floors are

third floor	$N_1 = 7.03\,\text{kN}$	$N_2 = 1.76\,\text{kN}$
	$F_1 = 4.33\,\text{kN}$	$F_2 = 5.41\,\text{kN}$
	$H_1 = 4.64\,\text{kN}$	$H_2 = 8.49\,\text{kN}$
second floor	$N_1 = 14.14\,\text{kN}$	$N_2 = 3.53\,\text{kN}$
	$F_1 = 7.11\,\text{kN}$	$F_2 = 8.88\,\text{kN}$
	$H_1 = 6.61\,\text{kN}$	$H_2 = 12.14\,\text{kN}$
first floor	$N_1 = 24.37\,\text{kN}$	$N_2 = 6.09\,\text{kN}$
	$F_1 = 10.23\,\text{kN}$	$F_2 = 12.79\,\text{kN}$
	$H_1 = 8.74\,\text{kN}$	$H_2 = 16.01\,\text{kN}$

The bending moments in the beams and columns at their connections can be calculated from these results by the following formulae

$$\text{beams } M_B = F \times \tfrac{1}{2} \text{ beam span}$$

$$\text{columns } M_C = H \times \tfrac{1}{2} \text{ storey height}$$

so at the roof's external connection

$$M_B = 0.54 \times \tfrac{1}{2} \times 6.0$$
$$= 1.6\,\text{kN m}$$
$$M_C = 0.93 \times \tfrac{1}{2} \times 3.5$$
$$= 1.6\,\text{kN m}$$

As a check at each joint, $\Sigma M_B = \Sigma M_C$.

The bending moments due to characteristic wind loads in all the columns and beams of this structure are shown in figure 3.22. It should be noted that in a full analysis of this nature, the ultimate design horizontal load at any floor level should be checked against the minimum value of $H_{min} = V_i \upsilon \alpha_n$.

3.4 Redistribution of Moments

Some method of elastic analysis is generally used to calculate the forces in a concrete structure, despite the fact that the structure does not behave elastically near its ultimate load. The assumption of elastic behaviour is reasonably true for low stress levels; but as a section approaches its ultimate moment of resistance, plastic deformation will occur. This is recognised in EC2, by allowing redistribution of the elastic moments subject to certain limitations.

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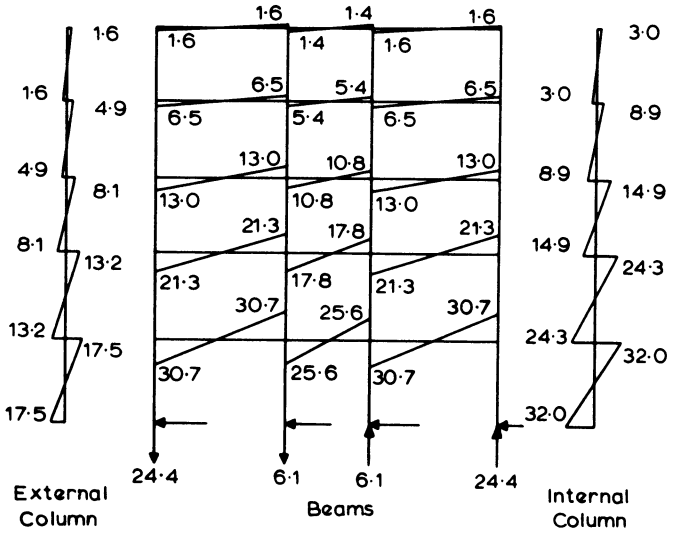

Figure 3.22 *Moments (kN m) and reactions (kN)*

Reinforced concrete behaves in a manner midway between that of steel and concrete. The stress–strain curves for the two materials (figures 1.5 and 1.2) show the elastoplastic behaviour of steel and the plastic behaviour of concrete. The latter will fail at a relatively small compressive strain. The exact behaviour of a reinforced concrete section depends on the relative quantities and the individual properties of the two materials. However, such a section may be considered virtually elastic until the steel yields; and then plastic until the concrete fails in compression. Thus the plastic behaviour is limited by the concrete failure; or more specifically, the concrete failure limits the rotation that may take place at a section in bending. A typical moment–curvature diagram for a reinforced concrete member is shown in figure 3.23.

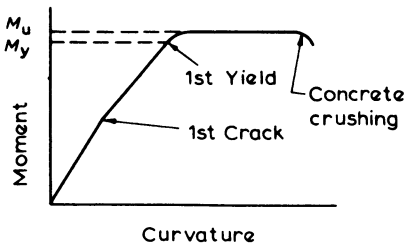

Figure 3.23 *Typical moment/curvature diagram*

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Thus, in an indeterminate structure, once a beam section develops its ultimate moment of resistance M_u, it then behaves as a plastic hinge resisting a constant moment of that value. Further loading must be taken by other parts of the structure, with the changes in moment elsewhere being just the same as if a real hinge existed. Provided rotation of a hinge does not cause crushing of the concrete, further hinges will be formed until a mechanism is produced. This requirement is considered in more detail in chapter 4.

Example 3.6 Moment Redistribution – Single Span Fixed-end Beam

The beam shown in figure 3.24 is subjected to an increasing uniformly distributed load:

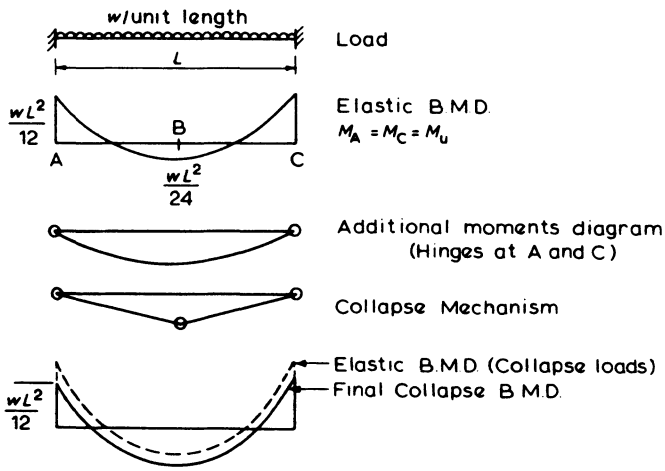

Figure 3.24 *Moment redistribution, one-span beam*

$$\text{Elastic support moment} = \frac{wL^2}{12}$$

$$\text{Elastic span moment} = \frac{wL^2}{24}$$

In the case where the ultimate bending strengths are equal at the span and at the supports, and where adequate rotation is possible, then the additional load w_a, which the member can sustain by plastic behaviour, can be found.

At collapse

$$M_u = \frac{wL^2}{12} = \frac{wL^2}{24} + \text{additional mid-span moment } m_B$$

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where $m_B = (w_a L^2)/8$ as for a simply supported beam with hinges at A and C. Thus

$$\frac{wL^2}{12} = \frac{wL^2}{24} + \frac{w_a L^2}{8}$$

Hence

$$w_a = \frac{w}{3}$$

where w is the load to cause the first plastic hinge; thus the beam may carry a load of $1.33w$ with redistribution.

From the design point of view, the elastic bending-moment diagram can be obtained for the required ultimate loading in the ordinary way. Some of these moments may then be reduced; but this will necessitate increasing others to maintain the static equilibrium of the structure. Usually it is the maximum support moments which are reduced, so economising in reinforcing steel and also reducing congestion at the columns. The requirements for applying moment redistribution are:

(a) Equilibrium between internal and external forces must be maintained, hence it is necessary to recalculate the span bending moments and the shear forces for the load case involved.

(b) At sections of largest moment the depth of neutral axis, x, is limited by

$$x \nless (0.8\delta - 0.35)d \quad \text{for concrete grade } f_{ck} \nless \text{C35/45}$$
$$x \nless (0.8\delta - 0.45)d \quad \text{for concrete grade } f_{ck} > \text{C35/45}$$

where d = the effective depth, and

$$\delta = \frac{\text{moment at section after redistribution}}{\text{moment at section before redistribution}}$$

This rule effectively prevents any reduction of the moments in columns which are primarily compression members with large values of x, and this is dealt with more fully in chapter 4.

(c) The maximum permitted redistribution is:

high ductility steel 30 per cent, that is $\delta \nless 0.7$
low ductility steel 15 per cent, that is $\delta \nless 0.85$

(d) Moment redistribution is not permitted in sway frames.

Example 3.7 Moment Redistribution

In example 3.3 figure 3.14 it is required to reduce the maximum support moment of $M_{BA} = 147\,\text{kN m}$ as much as possible, but without increasing the span moment above the present maximum value of 118 kN m.

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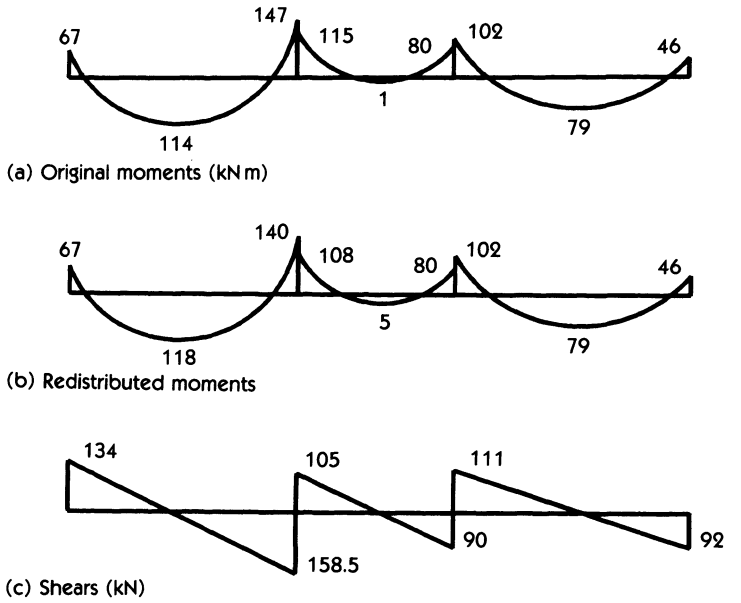

Figure 3.25 *Moments and shears after redistribution*

Figure 3.25a duplicates the original bending-moment diagram (part 3 of figure 3.14) of example 3.3 while figure 3.25b shows the redistributed moments, with the span moment set at 118 kN m. The moment at support B can be calculated, using a rearrangement of equations 3.4 and 3.1. Thus

$$V_{AB} = \sqrt{[(M_{max} - M_{AB})2w]}$$

and

$$M_{BA} = \left(V_{AB} - \frac{wL}{2}\right)L + M_{AB}$$

For span AB, $w = 48.75$ kN/m, therefore

$$V_{AB} = \sqrt{[(118 + 67) \times 2 \times 48.75]} = 134 \text{ kN}$$

$$M_{BA} = \left(134 - \frac{48.75 \times 6.0}{2}\right)6.0 - 67$$

$$= -140 \text{ kN m}$$

and

$$V_{BA} = 292.5 - 134$$

$$= 158.5 \text{ kN}$$

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Reduction in $M_{BA} = 147 - 140$

$$= 7\,\text{kN m}$$

$$= \frac{7 \times 100}{147} = 4.8 \text{ per cent}$$

In order to ensure that the moments in the columns at joint B are not changed by the redistribution, moment M_{BC} must also be reduced by 7 kN m. Therefore

$$M_{BC} = 115 - 7 = 108\,\text{kN m} \quad \text{hogging}$$

For the revised moments in BC:

$$V_{BC} = \frac{(108 - 80)}{4} + \frac{195}{2} = 105\,\text{kN}$$

$$V_{CB} = 195 - 105 = 90\,\text{kN}$$

For span BC:

$$M_{max} = \frac{105^2}{2 \times 48.75} - 108 = 5\,\text{kN m} \quad \text{sagging}$$

Figure 3.25c shows the revised shearing-force diagram to accord with the redistributed moments.

This example illustrates how, with redistribution

(1) the moments at a section of beam can be reduced without exceeding the maximum design moments at other sections
(2) the values of the column moments are not affected; and
(3) the equilibrium between external loads and internal forces is maintained.

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4

Analysis of the Section

A satisfactory and economic design of a concrete structure rarely depends on a complex theoretical analysis. It is achieved more by deciding on a practical overall layout of the structure, careful attention to detail and sound constructional practice. Nevertheless the total design of a structure does depend on the analysis and design of the individual member sections.

Wherever possible the analysis should be kept simple, yet it should be based on the observed and tested behaviour of reinforced concrete members. The manipulation and juggling with equations should never be allowed to obscure the fundamental principles that unite the analysis. The three most important principles are

(1) The stresses and strains are related by the material properties, including the stress–strain curves for concrete and steel
(2) The distribution of strains must be compatible with the distorted shape of the cross-section
(3) The resultant forces developed by the section must balance the applied loads for static equilibrium.

These principles are true irrespective of how the stresses and strains are distributed, or how the member is loaded, or whatever the shape of the cross-section.

This chapter describes and analyses the action of a member section under load. It derives the basic equations used in design and also those equations required for the preparation of design charts. Emphasis has been placed mostly on the analysis associated with the ultimate limit state but the behaviour of the section within the elastic range and the serviceability limit state has also been considered.

Section 4.7 deals with the redistribution of the moments from an elastic analysis of the structure, and the effect it has on the equations derived and the design procedure. It should be noted that EC2 does not give any implicit equations for the analysis or design of sections. The

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equations given in this chapter are developed from the principles of EC2 in a form comparable with the equations formerly given in BS8110.

4.1 Stress–Strain Relations

Short-term stress–strain curves are presented in EC2. These curves are in an idealised form which can be used in the analysis of member sections.

4.1.1 Concrete

The behaviour of structural concrete (figure 4.1) is represented by a parabolic stress–strain relationship, up to a strain ε_0, from which point the strain increases while the stress remains constant. The ultimate design stress is given by

$$\frac{\alpha f_{ck}}{\gamma_c} = \frac{0.85 f_{ck}}{1.5} = 0.567 f_{ck}$$

where the factor of 0.85 allows for the difference between the bending strength and the cylinder crushing strength of the concrete, and $\gamma_c = 1.5$ is the usual partial safety factor for the strength of concrete. The ultimate strain of 0.0035 is typical for all grades of concrete.

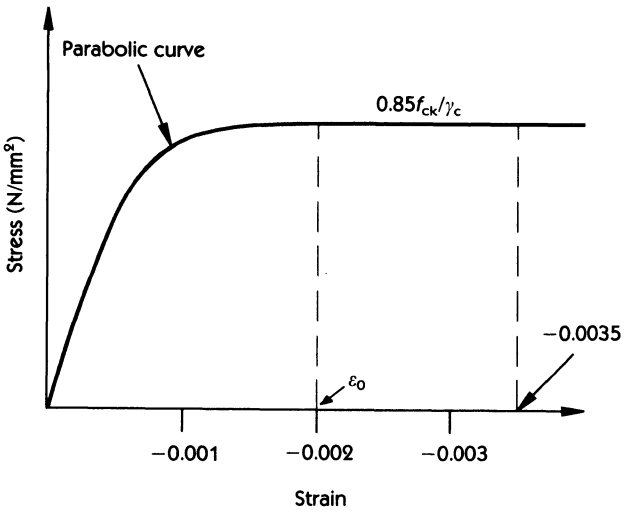

Figure 4.1 *Parabolic–rectangular stress–strain diagram for concrete in compression*

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4.1.2 Reinforcing Steel

The representative short-term design stress–strain curve for reinforcement is given in figure 4.2. The behaviour of the steel is identical in tension and compression, being linear in the elastic range up to the design yield stress of f_{yk}/γ_s where f_{yk} is the characteristic yield stress and γ_s is the partial factor of safety.

Within the elastic range, the relationship between the stress and strain is

$$\begin{aligned} \text{stress} &= \text{elastic modulus} \times \text{strain} \\ &= E_s \times \varepsilon_s \end{aligned} \tag{4.1}$$

so that the design yield strain is

$$\varepsilon_y = \left(\frac{f_{yk}}{\gamma_s}\right) \Big/ E_s$$

At the ultimate limit for $f_{yk} = 460\,\text{N/mm}^2$

$$\begin{aligned} \varepsilon_y &= 460/(1.15 \times 200 \times 10^3) \\ &= 0.002 \end{aligned}$$

and for

$$f_{yk} = 250\,\text{N/mm}^2$$

$$\begin{aligned} \varepsilon_y &= 250/(1.15 \times 200 \times 10^3) \\ &= 0.00109 \end{aligned}$$

It should be noted that EC2 permits the use of an alternative design stress–strain curve to that shown in figure 4.2 with an inclined top branch and the maximum strain limited to 0.01. However the more commonly

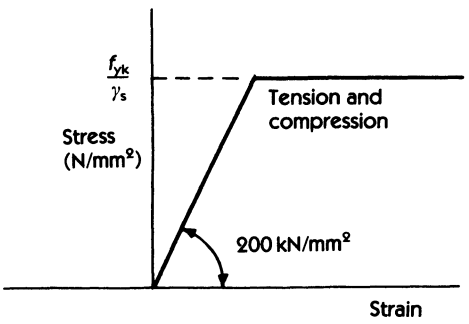

Figure 4.2 *Short-term design stress–strain curve for reinforcement*

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used curve shown in figure 4.2 will be used in this chapter and throughout the text.

4.2 Distribution of Strains and Stresses across a Section

The theory of bending for reinforced concrete assumes that the concrete will crack in the regions of tensile strains and that, after cracking, all the tension is carried by the reinforcement. It is also assumed that plane sections of a structural member remain plane after straining, so that across the section there must be a linear distribution of strains.

Figure 4.3 shows the cross-section of a member subjected to bending, and the resultant strain diagram, together with three different types of stress distribution in the concrete:

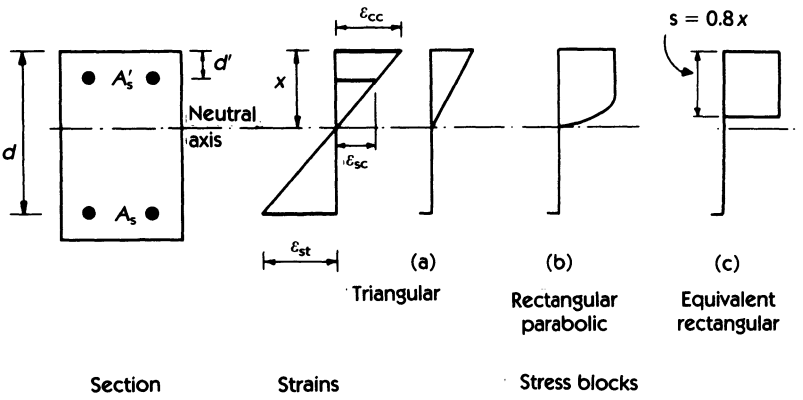

Figure 4.3 *Section with strain diagram and stress blocks*

(1) The triangular stress distribution applies when the stresses are very nearly proportional to the strains, which generally occurs at the loading levels encountered under working conditions and is, therefore, used at the serviceability limit state
(2) The rectangular–parabolic stress block represents the distribution at failure when the compressive strains are within the plastic range and it is associated with the design for the ultimate limit state.
(3) The equivalent rectangular stress block is a simplified alternative to the rectangular–parabolic distribution.

As there is compatibility of strains between the reinforcement and the adjacent concrete, the steel strains ε_{st} in tension and ε_{sc} in compression can be determined from the strain diagram. The relationship between the

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depth of neutral axis (x) and the maximum concrete strain (ε_{cc}) and the steel strains is given by

$$\varepsilon_{st} = \varepsilon_{cc}\left(\frac{d - x}{x}\right) \tag{4.2}$$

and

$$\varepsilon_{sc} = \varepsilon_{cc}\left(\frac{x - d'}{x}\right) \tag{4.3}$$

where d is the effective depth of the beam and d' is the depth of the compression reinforcement.

Having determined the strains, we can evaluate the stresses in the reinforcement from the stress–strain curve of figure 4.2, together with the equations developed in section 4.1.2.

For analysis of a section with known steel strains, the depth of the neutral axis can be determined by rearranging equation 4.2 as

$$x = \frac{d}{1 + \dfrac{\varepsilon_{st}}{\varepsilon_{cc}}} \tag{4.4}$$

At the ultimate limit state the maximum compressive strain in the concrete is taken as

$$\varepsilon_{cc} = 0.0035$$

For steel with $f_{yk} = 460\,\text{N/mm}^2$ the yield strain is 0.002. Inserting these values into equation 4.4:

$$x = \frac{d}{1 + \dfrac{0.002}{0.0035}} = 0.636d$$

Hence, to ensure yielding of the tension steel at the ultimate limit state:

$$x \not> 0.636d$$

At the ultimate limit state it is important that member sections in flexure should be ductile and that failure should occur with the gradual yielding of the tension steel and not by a sudden catastrophic compression failure of the concrete. Also, yielding of the reinforcement enables the formation of plastic hinges so that redistribution of maximum moments can occur, resulting in a safer and more economical structure. To be very certain of the tension steel yielding, the code of practice limits the depth of neutral axis so that:

$$x \not> 0.8(\delta - 0.44)d$$

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for concrete grades less than or equal to C35/45

and

$$x \geqslant 0.8(\delta - 0.56)d$$

for concrete grades greater than C35/45

where

$$\delta = \frac{\text{moment at the section after redistribution}}{\text{moment at the section before redistribution}}$$

Thus with zero moment redistribution and $\delta = 1.0$:

$$x \geqslant 0.45d \quad (f_{ck} \leqslant \text{C35/45})$$

and

$$x \geqslant 0.35d \quad (f_{ck} > \text{C35/45})$$

These limits will normally be adopted for ultimate limit state design, but larger degrees of moment redistribution will require a smaller limit to x to ensure that plastic hinges can form, providing adequate rotation at the critical sections (see section 4.7 and table 4.2). These limiting values of neutral axis depth (x_{bal}) are such that, when reached, it is necessary to change the section design from singly reinforced (tension steel only) to doubly reinforced (compression and tension steel). The corresponding limiting value of moment is referred to as the balanced section moment, M_{bal}.

4.3 Bending and the Equivalent Rectangular Stress Block

For the design of most reinforced concrete structures it is usual to commence the design for the conditions at the ultimate limit state, which is then followed by checks to ensure that the structure is adequate for the serviceability limit state without excessive deflection or cracking of the concrete. For this reason the analysis in this chapter will first consider the simplified rectangular stress block which can be used for the design at the ultimate limit state.

The rectangular stress block as shown in figure 4.4 may be used in preference to the more rigorous rectangular–parabolic stress block. This simplified stress distribution will facilitate the analysis and provide more manageable design equations, in particular when dealing with non-rectangular cross-sections.

It can be seen from the figure that the stress block does not extend to the neutral axis of the section but has a depth $s = 0.8x$. This will result in the centroid of the stress block being $s/2 = 0.40x$ from the top edge of the section, which is very nearly the same location as for the more precise

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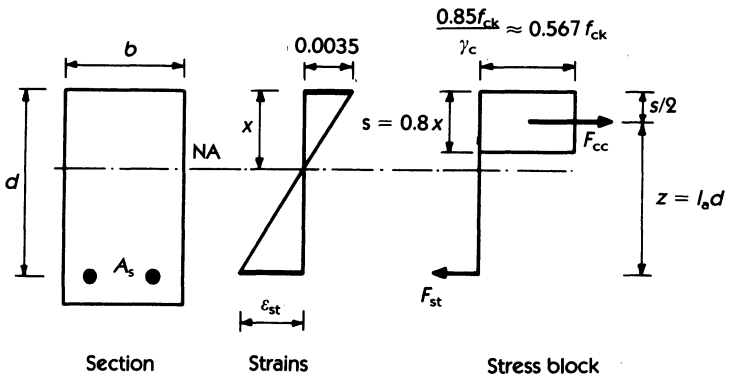

Figure 4.4 *Singly reinforced section with rectangular stress blocks*

rectangular–parabolic stress block; also the areas of the two types of stress block are approximately equal (see section 4.9). Thus the moment of resistance of the section will be similar using calculations based on either of the two stress blocks.

The design equations derived in sections 4.4 to 4.6 are for zero redistribution of moments. When moment redistribution is applied, reference should be made to section 4.7 which describes how to modify the design equations.

4.4 Singly Reinforced Rectangular Section in Bending

Bending of the section will induce a resultant tensile force F_{st} in the reinforcing steel, and a resultant compressive force in the concrete F_{cc} which acts through the centroid of the effective area of concrete in compression, as shown in figure 4.4.

For equilibrium, the ultimate design moment, M, must be balanced by the moment of resistance of the section so that

$$M = F_{cc} \times z = F_{st} z \tag{4.5}$$

where z is the lever arm between the resultant forces F_{cc} and F_{st}.

$$F_{cc} = \text{stress} \times \text{area of action}$$
$$= 0.567 f_{ck} \times bs$$

and
$$z = d - s/2 \tag{4.6}$$

So that substituting in equation 4.5

$$M = 0.567 f_{ck} bs \times z$$

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and replacing s from equation 4.6

$$M = 1.134 f_{ck} b(d - z)z \tag{4.7}$$

Rearranging and substituting $K = M/bd^2 f_{ck}$:

$$(z/d)^2 - (z/d) + K/1.134 = 0$$

Solving this quadratic equation:

$$z = d[0.5 + \sqrt{(0.25 - K/1.134)}] \tag{4.8}*$$

In equation 4.5

$$F_{st} = (f_{yk}/\gamma_s)A_s \qquad \text{with } \gamma_s = 1.15$$
$$= 0.87 f_{yk} A_s$$

Hence

$$A_s = \frac{M}{0.87 f_{yk} \times z} \tag{4.9}*$$

Equations 4.8 and 4.9 can be used to design the area of tension reinforcement in a concrete section to resist an ultimate moment, M.

Equation 4.8 for the lever arm z can be used to set up a table and draw a lever-arm curve as shown in figure 4.5, and the curve may be used to determine the lever arm, z, instead of solving equation 4.8.

The lower limits of $z_{bal} = 0.86d$ and $z_{bal} = 0.82d$ are when the depth of neutral axis equals $0.35d$ and $0.45d$ respectively, which are the maximum values allowed by the code for a singly reinforced section in order to provide a ductile section that will have a gradual tension type failure as already described. The limit of $0.82d$ corresponds to concrete grades not greater than or equal to C35/45 and the limit of $0.86d$ applies to concrete grades greater than C35/45. With $z = 0.82d$ from equation 4.7:

$$M_{bal} = 1.134 f_{ck} b(d - 0.82d) \times 0.82d$$

or

$$M_{bal} = 0.167 f_{ck} bd^2 \tag{4.10}*$$

as marked on the lever-arm diagram.

When

$$\frac{M_{bal}}{bd^2 f_{ck}} = K_{bal} > 0.167 \quad (\leqslant \text{C35/45})$$

compression reinforcement is also required to supplement the moment of resistance of the concrete. Similarly for $z = 0.86d$, the corresponding limit on K_{bal} is 0.136 (see table 4.1 on page 76).

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$K = M/bd^2f_{ck}$	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.136	0.14	0.15	0.160	0.167
$l_a = z/d$	0.954	0.945	0.934	0.924	0.913	0.902	0.891	0.880	0.868	0.861	0.856	0.843	0.830	0.820

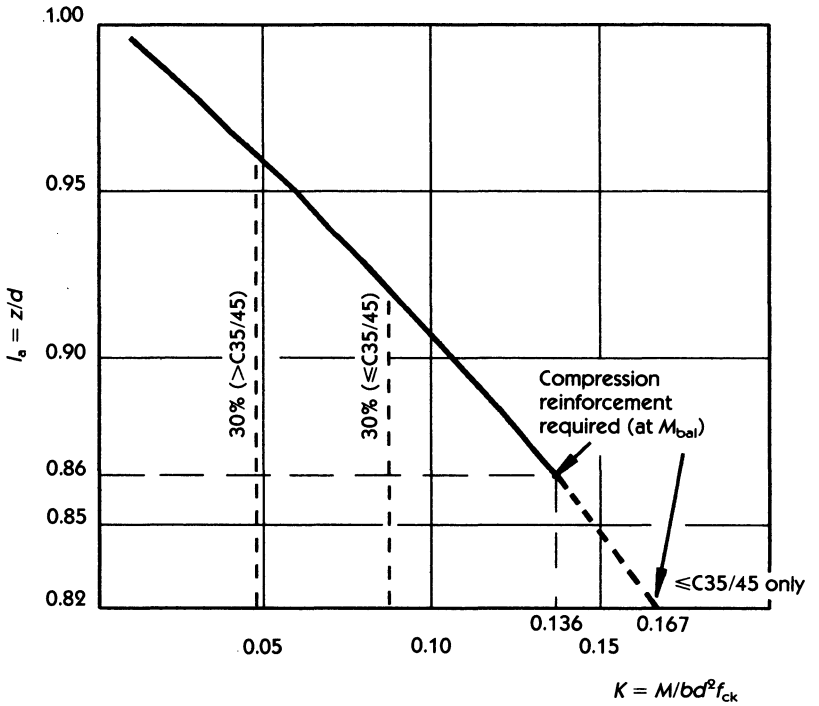

Figure 4.5 *Lever-arm curve. The 30% values on the K-axis mark the limits for singly reinforced sections with moment redistribution applied (see section 4.7)*

Example 4.1 Design of a Singly Reinforced Rectangular Section

The ultimate design moment to be resisted by the section in figure 4.6 is 185 kN m. Determine the area of tension reinforcement (A_s) required given the characteristic material strengths are $f_{yk} = 460\,\text{N/mm}^2$ and $f_{ck} = 25\,\text{N/mm}^2$.

$$K = \frac{M}{bd^2 f_{ck}}$$

$$= \frac{185 \times 10^6}{260 \times 440^2 \times 25} = 0.147$$

$$< 0.167$$

therefore compression steel is not required.

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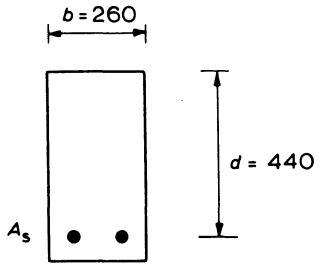

Figure 4.6 *Design example – singly reinforced section*

Lever arm:

$$z = d\left\{0.5 + \sqrt{\left(0.25 - \frac{K}{1.134}\right)}\right\}$$

$$= 440\left\{0.5 + \sqrt{\left(0.25 - \frac{0.147}{1.134}\right)}\right\}$$

$$= 373\,\text{mm}$$

(Or alternatively, the value of $z = l_a d$ could be obtained from the lever-arm diagram, figure 4.5.)

$$A_s = \frac{M}{0.87 f_{yk} z}$$

$$= \frac{185 \times 10^6}{0.87 \times 460 \times 373}$$

$$= 1239\,\text{mm}^2$$

Analysis Equations for a Singly Reinforced Section

The following equations may be used to calculate the moment of resistance of a given section with a known area of steel reinforcement.

For equilibrium of the compressive force in the concrete and the tensile force in the steel in figure 4.4:

$$F_{cc} = F_{st}$$

or

$$0.567 f_{ck} b \times s = 0.87 f_{yk} A_s$$

Therefore depth of stress block is

$$s = \frac{0.87 f_{yk} A_s}{0.567 f_{ck} b} \tag{4.11}$$

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and

$$x = s/0.80$$

Therefore moment of resistance of the section is

$$\begin{aligned} M &= F_{st} \times z \\ &= 0.87 f_{yk} A_s (d - s/2) \\ &= 0.87 f_{yk} A_s \left(d - \frac{0.87 f_{yk} A_s}{1.134 f_{ck} b} \right) \end{aligned} \tag{4.12}$$

These equations assume the tension reinforcement has yielded, which will be the case if $x \not> 0.636d$. If this is not the case, the problem would require solving by trying successive values of x until

$$F_{cc} = F_{st}$$

with the steel strains and hence stresses being determined from equations 4.2 and 4.1, to be used in equation 4.12 instead of $0.87 f_{yk}$.

Example 4.2 Analysis of Singly Reinforced Rectangular Section in Bending

Determine the ultimate moment of resistance of the cross-section shown in figure 4.7 given that the characteristic strengths are $f_{yk} = 460\,\text{N/mm}^2$ for the reinforcement and $f_{ck} = 25\,\text{N/mm}^2$ for the concrete.

For equilibrium of the compressive and tensile forces on the section

$$F_{cc} = F_{st}$$

therefore

$$0.567 f_{ck} b s = 0.87 f_{yk} A_s$$

$$0.567 \times 25 \times 300 \times s = 0.87 \times 460 \times 1470$$

therefore

$$s = 138\,\text{mm}$$

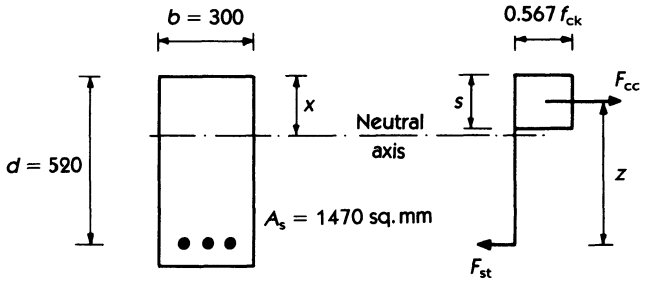

Figure 4.7 *Analysis example – singly reinforced section*

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and

$$x = s/0.8 = 138/0.8$$
$$= 173\,\text{mm}$$

This value of x is less than the value of $0.636d$ derived from section 4.2, and therefore the steel has yielded and $f_{st} = 0.87f_{yk}$ as assumed.

Moment of resistance of the section is

$$M = F_{st} \times z$$
$$= 0.87f_{yk}A_s(d - s/2)$$
$$= 0.87 \times 460 \times 1470(520 - 138/2) \times 10^{-6}$$
$$= 265\,\text{kN m}$$

4.5 Rectangular Section with Compression Reinforcement at the Ultimate Limit State

(a) Derivation of Basic Equations

It should be noted that the equations in this section have been derived for the case of zero moment redistribution. When this is not the case, reference should be made to section 4.7 which deals with the effect of moment redistribution.

From the section dealing with the analysis of a singly reinforced section and for concrete grades not greater than C35/45 when

$$M > 0.167f_{ck}bd^2$$

the design ultimate moment exceeds the moment of resistance of the concrete (M_{bal}) and therefore compression reinforcement is required. For this condition the depth of neutral axis, $x \not> 0.45d$, the maximum value allowed by the code in order to ensure a tension failure with a ductile section.

Therefore

$$z_{bal} = d - s_{bal}/2 = d - 0.8x_{bal}/2$$
$$= d - 0.8 \times 0.45d/2$$
$$= 0.82d$$

For equilibrium of the section in figure 4.8

$$F_{st} = F_{cc} + F_{sc}$$

so that with the reinforcement at yield

$$0.87f_{yk}A_s = 0.567f_{ck}bs + 0.87f_{yk}A'_s$$

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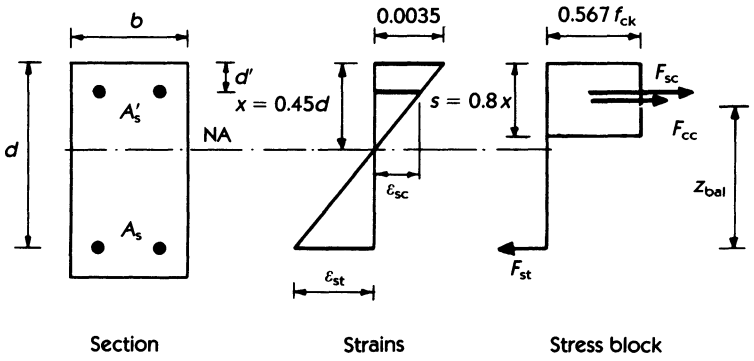

Figure 4.8 *Section with compression reinforcement*

or with

$$s = 0.8 \times 0.45d = 0.36d$$

$$0.87f_{yk}A_s = 0.204f_{ck}bd + 0.87f_{yk}A'_s \tag{4.13}$$

and taking moments about the centroid of the tension steel, A_s

$$\begin{aligned} M &= F_{cc} \times z_{bal} + F_{sc}(d - d') \\ &= 0.204f_{ck}bd \times 0.82d + 0.87f_{yk}A'_s(d - d') \\ &= 0.167f_{ck}bd^2 + 0.87f_{yk}A'_s(d - d') \end{aligned} \tag{4.14}$$

From equation 4.14

$$A'_s = \frac{M - 0.167f_{ck}bd^2}{0.87f_{yk}(d - d')} \tag{4.15}*$$

Multiplying both sides of equation 4.13 by $z = 0.82d$ and rearranging gives

$$A_s = \frac{0.167f_{ck}bd^2}{0.87f_{yk} \times z_{bal}} + A'_s \tag{4.16}*$$

with $z_{bal} = 0.82d$.

Hence the areas of compression steel, A'_s, and tension steel, A_s, can be calculated from equations 4.15 and 4.16.

Substituting $K_{bal} = 0.167$ and $K = M/bd^2f_{ck}$ into these equations would convert them into:

$$A'_s = \frac{(K - K_{bal})f_{ck}bd^2}{0.87f_{yk}(d - d')} \tag{4.17}*$$

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$$A_s = \frac{K_{bal} f_{ck} b d^2}{0.87 f_{yk} z_{bal}} + A'_s \qquad (4.18)^*$$

In this analysis it has been assumed that the compression steel has yielded so that the steel stress $f_{sc} = 0.87 f_{yk}$. From the proportions of the strain distribution diagram:

$$\frac{\varepsilon_{sc}}{x - d'} = \frac{0.0035}{x} \qquad (4.19)$$

so that

$$\frac{x - d'}{x} = \frac{\varepsilon_{sc}}{0.0035}$$

or

$$\frac{d'}{x} = 1 - \frac{\varepsilon_{sc}}{0.0035}$$

At yield with $f_{yk} = 460\,\text{N/mm}^2$, the steel strain $\varepsilon_{sc} = \varepsilon_y = 0.002$. Therefore for yielding of the compression steel

$$\frac{d'}{x} \geqslant 1 - \frac{0.002}{0.0035} \geqslant 0.43 \qquad (4.20)^*$$

or with $x = 0.45d$

$$\frac{d'}{d} \geqslant 0.194 \qquad (4.21)$$

The ratio of d'/d for the yielding of other grades of steel can be determined by using their yield strain in equation 4.19, but for values of f_{yk} less than $460\,\text{N/mm}^2$, the application of equation 4.21 will provide an adequate safe check.

If $d'/d > 0.194$, then it is necessary to calculate the strain ε_{sc} from equation 4.19 and then determine f_{sc} from

$$f_{sc} = E_s \times \varepsilon_{sc} = 200\,000\,\varepsilon_{sc}$$

This value of stress for the compressive steel must then be used in the denominator of equation 4.15 in place of $0.87 f_{yk}$ in order to calculate the area A'_s of compression steel. The area of tension steel is calculated from a modified equation 4.16 such that

$$A_s = \frac{0.167 f_{ck} b d^2}{0.87 f_{yk} z_{bal}} + A'_s \times \frac{f_{sc}}{0.87 f_{yk}}$$

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The above equations apply for the case where the concrete grade is less than or equal to C35/45. For concrete grades greater than C35/45 identical equations, with different constants, can be derived based on the EC2 requirement that for these grades the neutral axis depth, x, should be limited to 0.35 of the effective depth. Table 4.1 shows the corresponding constants for the two different ranges of concrete strength.

Table 4.1 Limiting constant values

	Concrete grade	
	C12/15–C35/45	*>C35/45*
Limiting x_{bal}/d	0.45	0.35
Maximum z (z_{bal})	$0.82d$	$0.86d$
K_{bal} = limiting K	0.167	0.136
Limiting d'/d	0.194	0.151

(b) Design Charts

The equations for the design charts are obtained by taking moments about the neutral axis. Thus

$$M = 0.567f_{ck}0.8x(x - 0.8x/2) + f_{sc}A'_s(x - d') + f_{st}A_s(d - x)$$

This equation and 4.13 may be written in the form

$$f_{st}\frac{A_s}{bd} = 0.454f_{ck}\frac{x}{d} + f_{sc}\frac{A'_s}{bd}$$

$$\frac{M}{bd^2} = 0.454f_{ck}\frac{x^2}{d^2}(1 - 0.40) + f_{sc}\frac{A'_s}{bd}\left(\frac{x}{d} - \frac{d'}{d}\right) + f_{st}\frac{A_s}{bd}\left(1 - \frac{x}{d}\right)$$

For specified ratios of A'_s/bd, x/d and d'/d, the two non-dimensional equations can be solved to give values for A_s/bd and M/bd^2 so that a set of design charts such as the one shown in figure 4.9 may be plotted. Before the equations can be solved, the steel stresses f_{st} and f_{sc} must be calculated for each value of x/d. This is achieved by first determining the relevant strains from the strain diagram (or by applying equations 4.2 and 4.3) and then by evaluating the stresses from the stress–strain curve of figure 4.2. Values of x/d below 0.45 ($f_{ck} \leqslant$ C35/45) or 0.35 ($f_{ck} >$ C35/45) apply when moments are redistributed. It should be noted that neither EC2 nor the Concise Code gives design charts for bending. Hence although

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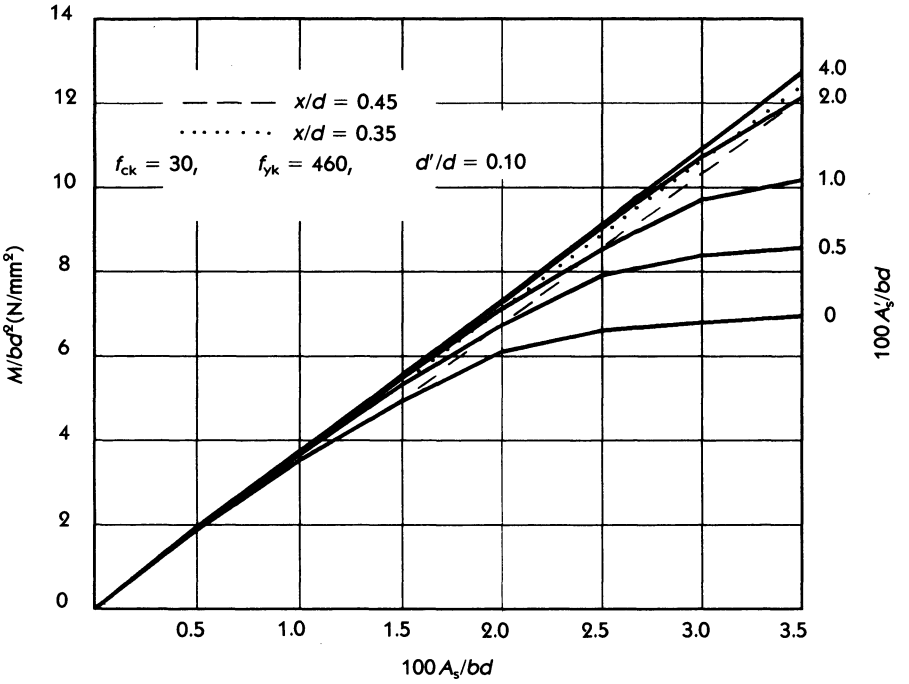

Figure 4.9 *Typical design chart for doubly reinforced beams*

it is possible to derive charts as indicated, it may be simpler to use the equations derived earlier in this chapter or simple computer programs.

Example 4.3 Design of a Rectangular Section with Compression Reinforcement (No Moment Redistribution)

The section shown in figure 4.10 is to resist an ultimate design moment of 285 kN m. The characteristic material strengths are $f_{yk} = 460\,\text{N/mm}^2$ and $f_{ck} = 25\,\text{N/mm}^2$. Determine the areas of reinforcement required.

$$K = \frac{M}{bd^2 f_{ck}}$$

$$= \frac{285 \times 10^6}{260 \times 440^2 \times 25} = 0.226$$

$$> 0.167$$

therefore compression steel is required

$$d'/d = 50/440 = 0.11 < 0.194$$

as in equation 4.21 and the compression steel will have yielded.

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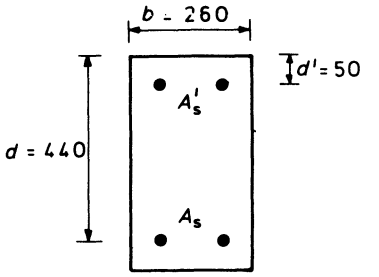

Figure 4.10 *Design example with compression reinforcement, no moment redistribution*

Compression steel:

$$A'_s = \frac{(K - K_{bal})f_{ck}bd^2}{0.87f_{yk}(d - d')}$$
$$= \frac{(0.226 - 0.167)25 \times 260 \times 440^2}{0.87 \times 460(440 - 50)}$$
$$= 476\,\text{mm}^2$$

Tension steel:

$$A_s = \frac{K_{bal}f_{ck}bd^2}{0.87f_{yk}z_{bal}} + A'_s$$
$$= \frac{0.167 \times 25 \times 260 \times 440^2}{0.87 \times 460(0.82 \times 440)} + 476$$
$$= 1455 + 476 = 1931\,\text{mm}^2$$

Example 4.4 Analysis of a Doubly Reinforced Rectangular Section

Determine the ultimate moment of resistance of the cross-section shown in figure 4.11 given that the characteristic strengths are $f_{yk} = 460\,\text{N/mm}^2$ for the reinforcement and $f_{ck} = 25\,\text{N/mm}^2$ for the concrete.

For equilibrium of the tensile and compressive forces on the section:

$$F_{st} = F_{cc} + F_{sc}$$

Assuming initially that the steel stresses f_{st} and f_{sc} are the design yield values, then

$$0.87f_{yk}A_s = 0.567f_{ck}bs + 0.87f_{yk}A'_s$$

Therefore

$$s = \frac{0.87f_{yk}(A_s - A'_s)}{0.567f_{ck}b}$$

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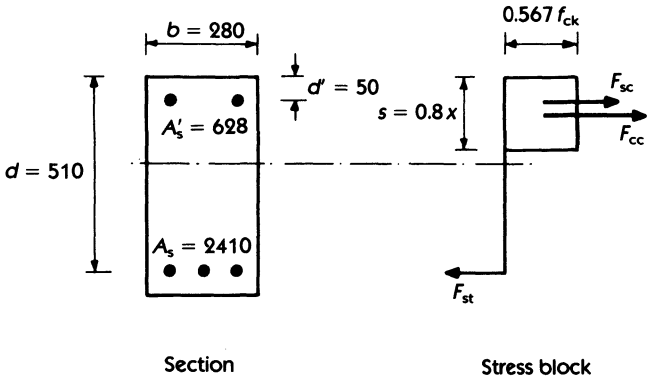

Figure 4.11 *Analysis example, doubly reinforced section*

$$= \frac{0.87 \times 460(2410 - 628)}{0.567 \times 25 \times 280}$$

$$= 180\,\text{mm}$$

$$x = s/0.8 = 225\,\text{mm}$$

$$x/d = 225/510 = 0.44 < 0.636 \text{ (see section 4.2)}$$

so the tension steel will have yielded. Also

$$d'/x = 50/225 = 0.22 < 0.43$$

so the compression steel will also have yielded, as assumed.

Taking moments about the tension steel

$$\begin{aligned} M &= F_{cc}(d - s/2) + F_{sc}(d - d') \\ &= 0.567 f_{ck} bs(d - s/2) + 0.87 f_{yk} A'_s(d - d') \\ &= [0.567 \times 25 \times 280 \times 180(510 - 180/2) \\ &\quad + 0.87 \times 460 \times 620(510 - 50)] \times 10^{-6} \\ &= 414 \times 10^6 \text{ kN m} \end{aligned}$$

If the depth of neutral axis was such that the compressive or tensile steel had not yielded, it would have been necessary to try successive values of x until

$$F_{st} = F_{cc} + F_{sc}$$

balances, with the steel strains and stresses being calculated from equations 4.2, 4.3 and 4.1. The steel stresses at balance would then be used to calculate the moment of resistance.

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4.6 Flanged Section in Bending at the Ultimate Limit State

T-sections and L-sections which have their flanges in compression can both be designed or analysed in a similar manner, and the equations which are derived can be applied to either type of cross-section. As the flanges generally provide a large compressive area, it is usually unnecessary to consider the case where compression steel is required; if it should be required, the design would be based on the principles derived in section 4.6.3.

For the singly reinforced section it is necessary to consider two conditions:

(1) the stress block lies within the compression flange, and
(2) the stress block extends below the flange.

4.6.1 Flanged Section – the Depth of the Stress Block Lies Within the Flange, $s < h_f$ (figure 4.12)

For this depth of stress block, the beam can be considered as an equivalent rectangular section of breadth b_f equal to the flange width. This is because the non-rectangular section below the neutral axis is in tension and is, therefore, considered to be cracked and inactive. Thus $K = M/b_f d^2 f_{ck}$ can be calculated and the lever arm determined from the lever-arm curve of figure 4.5 or equation 4.8. The relation between the lever arm, z, and depth, x, of the neutral axis is given by

$$z = d - 0.5s$$

or

$$s = 2(d - z)$$

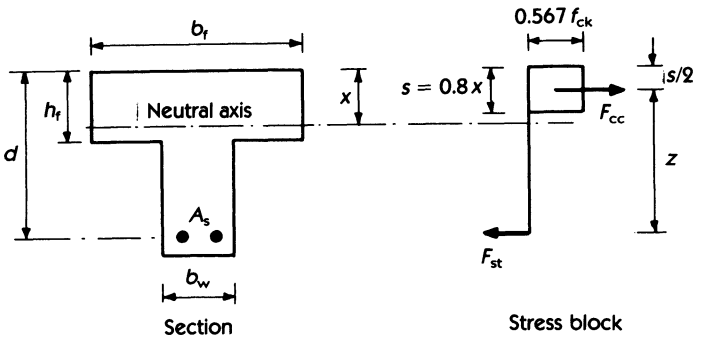

Figure 4.12 *T-section, stress block within the flange, $s < h_f$*

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If s is less then the flange thickness (h_f), the stress block does lie within the flange as assumed and the area of reinforcement is given by

$$A_s = \frac{M}{0.87 f_{yk} z}$$

The design of a T-section beam is described further in section 7.2.3 with a worked example.

Example 4.5 Analysis of a Flanged Section

Determine the ultimate moment of resistance of the T-section shown in figure 4.13. The characteristic material strengths are $f_{yk} = 460\,\text{N/mm}^2$ and $f_{ck} = 25\,\text{N/mm}^2$.

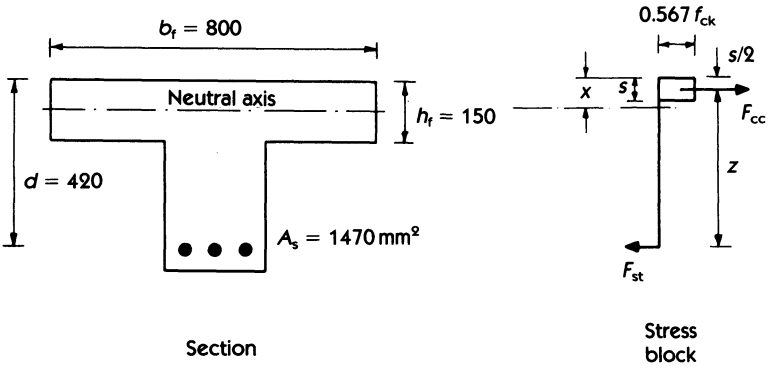

Figure 4.13 *Analysis example of a T-section, $s < h_f$*

Assume initially that the stress block depth lies within the flange and the reinforcement is strained to the yield, so that $f_{st} = 0.87 f_{yk}$.

For no resultant axial force on the sections

$$F_{cc} = F_{st}$$

therefore

$$0.567 f_{ck} b_f s = 0.87 f_{yk} A_s$$

and solving for the depth of stress block

$$s = \frac{0.87 \times 460 \times 1470}{0.567 \times 25 \times 800}$$

$$= 52\,\text{mm}$$

$$x = s/0.8 = 65\,\text{mm}$$

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Hence the stress block does lie within the flange and with this depth of neutral axis the steel will have yielded as assumed.

Lever arm:

$$z = d - s/2$$
$$= 420 - 52/2 = 394\,\text{mm}$$

Taking moments about the centroid of the reinforcement the moment of resistance is

$$M = F_{cc} \times z$$
$$= 0.567 f_{ck} b_f s z$$
$$= 0.567 \times 25 \times 800 \times 52 \times 394 \times 10^{-6}$$
$$= 232\,\text{kN m}$$

If in the analysis it had been found that $s > h_f$, then the procedure would then be similar to that in example 4.7.

4.6.2 *Flanged Section – the Depth of the Stress Block Extends Below the Flange, $s > h_f$*

For the design of a flanged section, the procedure described in section 4.6.1 will check if the depth of the stress block extends below the flange. An alternative procedure is to calculate the moment of resistance, M_f, of the section with $s = h_f$, the depth of the flange (see equation 4.22 of example 4.6 following). Hence if the design moment, M, is such that

$$M > M_f$$

then the stress block must extend below the flange, and

$$s > h_f$$

In this case the design can be carried out by either:

(a) using an exact method to determine the depth of the neutral axis, as in example 4.6 or
(b) designing for the conservative condition of $x = 0.45d$ (or $x = 0.35d$) as described at the end of this section.

Example 4.6 Design of a Flanged Section with the Depth of the Stress Block Below the Flange

The T-section beam shown in figure 4.14 is required to resist an ultimate design moment of 180 kN m. The characteristic material strengths are $f_{yk} = 460\,\text{N/mm}^2$ and $f_{ck} = 25\,\text{N/mm}^2$. Calculate the area of reinforcement required.

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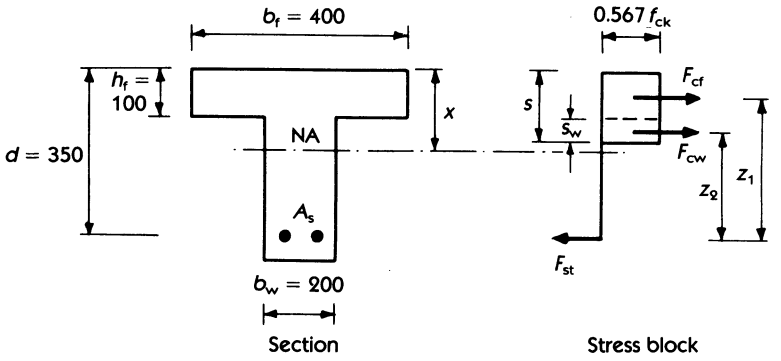

Figure 4.14 *Design example, T-section with* $s > h_f$

In the figure

F_{cf} is the force developed in the flange
F_{cw} is the force developed in the area of web in compression

Moment of resistance, M_f, of the flange is

$$M_f = F_{cf} \times z_1$$

or

$$M_f = 0.567 f_{ck} b_f h_f (d - h_f/2) \qquad (4.22)^*$$
$$= 0.567 \times 25 \times 400 \times 100(350 - 100/2) \times 10^{-6}$$
$$= 170\,\text{kN m} < 180\,\text{kN m}, \text{ the design moment}$$

Therefore, the stress block must extend below the flange.

It is now necessary to determine the depth, s_w, of the web in compression, where $s_w = s - h_f$.

For equilibrium:
Applied moment

$$180 = F_{cf} \times z_1 + F_{cw} \times z_2$$
$$= 170 + 0.567 f_{ck} b_w s_w \times z_2$$
$$= 170 + 0.567 \times 25 \times 200 s_w (250 - s_w/2) \times 10^{-6}$$
$$= 170 + 2835 s_w (250 - s_w/2) \times 10^{-6}$$

This equation can be rearranged into

$$s_w^2 - 500 s_w + 7.05 \times 10^3 = 0$$

Solving this quadratic equation

$$s_w = 15\,\text{mm}$$

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So that the depth of neutral axis

$$x = s/0.8 = (100 + 15)/0.8$$
$$= 144\,\text{mm}$$

As $x < 0.45d$ compression reinforcement is not required.

For the equilibrium of the section

$$F_{st} = F_{cf} + F_{cw}$$

or

$$0.87 f_{yk} A_s = 0.567 f_{ck} b_f h_f + 0.567 f_{ck} b_w s_w$$
$$0.87 \times 460 \times A_s = 0.567 \times 25(400 \times 100 + 200 \times 15)$$

Therefore

$$A_s = \frac{610 \times 10^3}{0.87 \times 460}$$
$$= 1523\,\text{mm}^2$$

Example 4.7 Analysis of a Flanged Section

Determine the ultimate moment of resistance of the T-beam section shown in figure 4.15, given $f_{yk} = 460\,\text{N/mm}^2$ and $f_{ck} = 25\,\text{N/mm}^2$.

The compressive force in the flange is

$$F_{cf} = 0.567 f_{ck} b_f h_f$$
$$= 0.567 \times 25 \times 450 \times 150 \times 10^{-3}$$
$$= 956.8\,\text{kN}$$

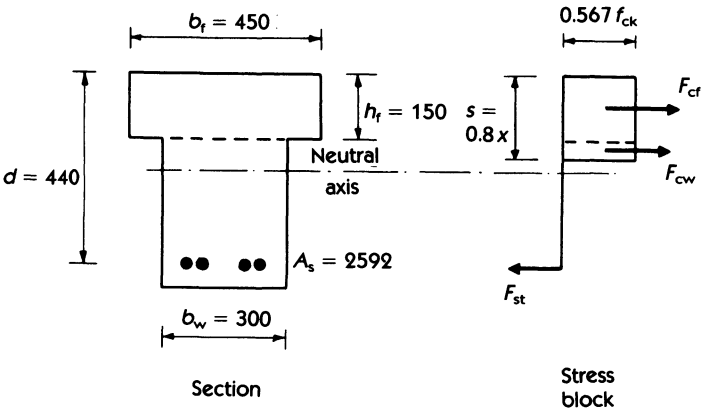

Figure 4.15 *Analysis example of a T-section, $s > h_f$*

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Then tensile force in the reinforcing steel, assuming it has yielded, is

$$F_{st} = 0.87 f_{yk} A_s$$
$$= 0.87 \times 460 \times 2592 \times 10^{-3}$$
$$= 1037.3\,\text{kN}$$

Therefore $F_{st} > F_{cc}$

so that $s > h_f$

and the force in the web is

$$F_{cw} = 0.567 f_{ck} b_w (s - h_f)$$
$$= 0.567 \times 25 \times 300(s - 150) \times 10^{-3}$$
$$= 4.25(s - 150)$$

For equilibrium

$$F_{cw} = F_{st} - F_{cf}$$

or $$4.25(s - 150) = 1037.3 - 956.8$$

Hence

$$s = 169\,\text{mm}$$
$$x = s/0.8 = 211\,\text{mm}$$

With this depth of neutral axis the reinforcement has yielded, as assumed, and

$$F_{cw} = 4.25(169 - 150) = 80.8\,\text{kN}$$

(If $F_{cf} > F_{st}$, then the stress block would not extend beyond the flange and the section would be analysed as in example 4.2 for a rectangular section of dimensions $b_f \times d$.)

Taking moments about the centroid of the reinforcement

$$M = F_{cf}(d - h_f/2) + F_{cw}(d - s/2 - h_f/2)$$
$$= [956.8(440 - 150/2) + 80.8(440 - 169/2 - 150/2)] \times 10^{-3}$$
$$= 372\,\text{kN m}$$

Example 4.8 Design of a Flanged Section with Depth of Neutral Axis $x = 0.45d$

A safe but conservative design for a flanged section with $s > h_f$ can be achieved by setting the depth of neutral axis to $x = 0.45d$ (or for concrete grades greater than C35/45, $x = 0.35d$), the maximum depth allowed in the code. Design equations can be derived for this condition as follows.

Depth of stress block, $s = 0.8x = 0.36d$

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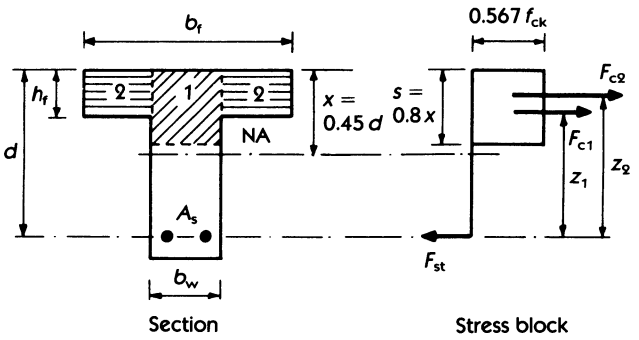

Figure 4.16 *Flanged section with depth of neutral axis x = 0.45d*

Divide the flanged section within the depth of the stress block into areas 1 and 2 as shown in figure 4.16, so that

$$\text{Area 1} = b_w \times s = 0.36 b_w d$$

$$\text{Area 2} = (b_f - b_w) \times h_f$$

and the compression forces developed by these areas are

$$F_{c1} = 0.567 f_{ck} \times 0.36 b_w d = 0.2 f_{ck} b_w d$$

$$F_{c2} = 0.567 f_{ck} h_f (b_f - b_w)$$

Taking moments about F_{c2} at the centroid of the flange

$$M = F_{st}(d - h_f/2) - F_{c1}(s/2 - h_f/2)$$
$$= 0.87 f_{yk} A_s (d - h_f/2) - 0.2 f_{ck} b_w d(0.36d - h_f)/2$$

Therefore

$$A_s = \frac{M + 0.1 f_{ck} b_w d(0.36d - h_f)}{0.87 f_{yk}(d - 0.5 h_f)} \qquad (4.23)^*$$

This equation should not be used when $h_f > 0.36d$.

Applying this equation to example 4.6:

$$A_s = \frac{180 \times 10^6 + 0.1 \times 25 \times 200 \times 350(0.36 \times 350 - 100)}{0.87 \times 460(350 - 100/2)}$$
$$= 1537\,\text{mm}^2 \text{ (compare with } 1523\,\text{mm}^2 \text{ of example 4.6)}$$

For concrete grades greater than C35/45 the corresponding equation is

$$A_s = \frac{M + 0.08 f_{ck} b_w d(0.28d - h_f)}{0.87 f_{yk}(d - 0.5 h_f)}$$

Before using equation 4.23 for calculating A_s, it is necessary to confirm that compression reinforcement is not required. This is achieved by using

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equation 4.24 to check that the moment of resistance of the concrete, M_{bal}, is greater than the design moment, M.

4.6.3 *Flanged Section with Compression Reinforcement*

With $x = 0.45d$ in figure 4.16 and taking moments about A_s, the maximum resistance moment of the concrete is

$$M_{bal} = F_{c1} \times z_1 + F_{c2} \times z_2$$
$$= 0.167 f_{ck} b_w d^2 + 0.567 f_{ck}(b_f - b_w)(d - h_f/2) \quad (4.24)$$

(Note that the value of 0.167 was derived previously for the rectangular section.)

Dividing through by $f_{ck} b_f d^2$

$$\frac{M_{bal}}{f_{ck} b_f d^2} = 0.167 \frac{b_w}{b_f} + 0.567 \frac{h_f}{d}\left(1 - \frac{b_w}{b_f}\right)\left(1 - \frac{h_f}{2d}\right) \quad (4.25)^*$$

If the applied design moment, $M > M_{bal}$ compression reinforcement is required. In which case the areas of steel can be calculated from

$$A'_s = \frac{M - M_{bal}}{0.87 f_{yk}(d - d')} \quad (4.26)$$

and considering the equilibrium of forces on the section

$$F_{st} = F_{c1} + F_{c2} + F_{sc}$$

so that

$$A_s = \frac{0.2 f_{ck} b_w d + 0.567 f_{ck} h_f (b_f - b_w)}{0.87 f_{yk}} + A'_s \quad (4.27)$$

Again, $d'/x \not> 0.43$, otherwise the design compressive steel stress is less than $0.87 f_{yk}$.

Similar equations can be derived for the higher grades of concrete where the depth of neutral axis is limited to $0.35d$. In both cases the equations only apply if no moment redistribution has been applied, otherwise they will need further modifications.

4.7 Moment Redistribution and the Design Equations

The plastic behaviour of reinforced concrete at the ultimate limit state affects the distribution of moments in a structure. To allow for this, the moments derived from an elastic analysis may be redistributed based on the assumption that plastic hinges have formed at the sections with the largest moments. The formation of plastic hinges requires relatively large

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rotations with yielding of the tension reinforcement. To ensure large strains in the tension steel, the code of practice restricts the depth of the neutral axis of a section according to the reduction of the elastic moment so that

$$x_{bal} \geqslant 0.8(\delta - 0.44)d \qquad (4.28)^*$$

for concrete grades less than or equal to C35/45

$$x_{bal} \geqslant 0.8(\delta - 0.56)d$$

for concrete grades greater than C35/45 where d is the effective depth

and $$\delta = \frac{\text{moment at section after redistribution}}{\text{moment at section before redistribution}} \leqslant 1.0$$

So, for the design of a section with compression reinforcement after moment redistribution the depth of neutral axis x will take the maximum value from equation 4.28.

For the case of concrete grades less than or equal to C35/45 the depth of the stress block is

$$s_{bal} = 0.8 \times 0.8(\delta - 0.44)d$$

and the lever arm is

$$z_{bal} = d - \frac{s_{bal}}{2}$$
$$= d - 0.64(\delta - 0.44)d/2 \qquad (4.29)$$

The moment of resistance of the concrete in compression is

$$M_{bal} = F_{cc} \times z_{bal} = 0.567 f_{ck} b s \times z_{bal}$$
$$= 0.567 f_{ck} b \times 0.64(\delta - 0.44)d \times [d - 0.64(\delta - 0.44)d/2]$$

Therefore

$$\frac{M_{bal}}{bd^2 f_{ck}} = 0.567 \times 0.64(\delta - 0.44)[1 - 0.32(\delta - 0.44)]$$
$$= 0.363(\delta - 0.44) - 0.116(\delta - 0.44)^2$$

So that rearranging

$$M_{bal} = K_{bal} bd^2 f_{ck}$$

where $$K_{bal} = 0.363(\delta - 0.44) - 0.116(\delta - 0.44)^2 \qquad (4.30)^*$$

For concrete grades greater than C35/45 the corresponding equation for K_{bal} is

$$K_{bal} = 0.363(\delta - 0.56) - 0.116(\delta - 0.56)^2$$

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When the ultimate design moment is such that

$$M > K_{bal} b d^2 f_{ck}$$

or $$K > K_{bal}$$

then compression steel is required such that

$$A'_s = \frac{(K - K_{bal}) f_{ck} b d^2}{0.87 f_{yk} (d - d')} \quad (4.31)^*$$

and

$$A_s = \frac{K_{bal} f_{ck} b d^2}{0.87 f_{yk} z_{bal}} + A'_s \quad (4.32)^*$$

where $$K_{bal} = \frac{M_{bal}}{b d^2 f_{ck}} \quad (4.33)^*$$

These equations are identical in form to those derived previously for the design of a section with compression reinforcement and no moment redistribution.

Table 4.2 shows the various design factors associated with the moment redistribution. If the value of d'/d for the section exceeds that shown in

Table 4.2 Moment redistribution design factors

Redistribution	δ	x_{bal}/d	z_{bal}/d	K_{bal}	d'/d
Concrete grades less than or equal to C35/45					
0	1.0	0.45	0.821	0.167	0.194
10	0.9	0.37	0.853	0.142	0.158
15[a]	0.85	0.33	0.869	0.129	0.141
20	0.8	0.29	0.885	0.116	0.124
25	0.75	0.25	0.900	0.101	0.107
30[b]	0.70	0.21	0.917	0.087	0.089
Concrete grades greater than C35/45					
0	1.0	0.35	0.859	0.136	0.151
10	0.9	0.27	0.891	0.110	0.117
15[a]	0.85	0.23	0.907	0.096	0.100
20	0.8	0.19	0.923	0.080	0.083
25	0.75	0.15	0.939	0.065	0.065
30[b]	0.70	0.11	0.955	0.049	0.048

[a] Maximum permitted redistribution for normal ductility steel.
[b] Maximum permitted redistribution for high ductility steel.

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the table, the compression steel will not have yielded and the compressive stress will be less than $0.87f_{yk}$. In such cases, the compressive stress f_{sc} will be $E_s\varepsilon_{sc}$ where the strain ε_{sc} is obtained from the proportions of the strain diagram. This value of f_{sc} should replace $0.87f_{yk}$ in equation 4.31, and equation 4.32 becomes

$$A_s = \frac{K_{bal}f_{ck}bd^2}{0.87f_{yk}z_{bal}} + A'_s \times \frac{f_{sc}}{0.87f_{yk}}$$

It should be noted that for a singly reinforced section ($K < K_{bal}$), the lever arm z is calculated from equation 4.8.

For a section requiring compression steel, the lever arm can be calculated from equation 4.29 or by using the equation

$$z_{bal} = d[0.5 + \sqrt{(0.25 - K_{bal}/1.134)}] \qquad (4.34)$$

which is similar to equation 4.8 but with K_{bal} replacing K.

Example 4.9 Design of a Section with Moment Redistribution Applied and $\delta = 0.8$

The section shown in figure 4.17 is subject to an ultimate design moment of 228 kN m. The characteristic material strengths are $f_{yk} = 460\,N/mm^2$ and $f_{ck} = 25\,N/mm^2$. Determine the areas of reinforcement required.

(A) From First Principles

Limiting neutral axis depth, $x_{bal} = 0.8(\delta - 0.44)d = 0.8(0.8 - 0.44)d$
$= 0.29d = 128\,mm$

Stress block depth, $s_{bal} = 0.8x = 0.23d$

Lever arm $z_{bal} = d - s/2 = 0.89d$

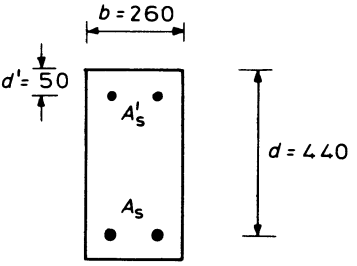

Figure 4.17 *Design example with moment redistribution, $\delta = 0.8$*

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Moment of resistance of the concrete

$$\begin{aligned} M_{bal} = F_{cc} \times z_{bal} &= 0.567 f_{ck} b s_{bal} \times z_{bal} \\ &= 0.567 \times 25 \times 260 \times 0.23 \times 0.89 \times 440^2 \times 10^{-6} \\ &= 146\,\text{kN m} \\ &< 228\,\text{kN m, the applied moment} \end{aligned}$$

therefore compression steel is required.

$$d'/x_{bal} = 50/128 = 0.39 < 0.43$$

therefore the compression steel has yielded.

Compression steel:

$$\begin{aligned} A'_s &= \frac{M - M_{bal}}{0.87 f_{yk}(d - d')} \\ &= \frac{(228 - 146) \times 10^6}{0.87 \times 460(440 - 50)} \\ &= 525\,\text{mm}^2 \end{aligned}$$

Tension steel:

$$\begin{aligned} A_s &= \frac{M_{bal}}{0.87 f_{yk} z_{bal}} + A'_s \\ &= \frac{146 \times 10^6}{0.87 \times 460 \times 0.89 \times 440} + 525 \\ &= 932 + 525 = 1457\,\text{mm}^2 \end{aligned}$$

(B) Alternative Solution Applying Equations Developed in Section 4.7

From equations 4.30 to 4.34:

$$\begin{aligned} K_{bal} &= 0.363(\delta - 0.44) - 0.116(\delta - 0.44)^2 \\ &= 0.363(0.8 - 0.44) - 0.116(0.8 - 0.44)^2 \\ &= 0.116 \end{aligned}$$

$$\begin{aligned} K &= \frac{M}{b d^2 f_{ck}} \\ &= \frac{228 \times 10^6}{260 \times 440^2 \times 25} \\ &= 0.181 > K_{bal} \end{aligned}$$

therefore compression steel is required.

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Compression steel:

$$A'_s = \frac{(K - K_{bal})f_{ck}bd^2}{0.87f_{yk}(d - d')}$$
$$= \frac{(0.181 - 0.116)25 \times 260 \times 440^2}{0.87 \times 460(440 - 50)}$$
$$= 524\,\text{mm}^2$$

Tension steel:

$$z_{bal} = d[0.5 + \sqrt{(0.25 - K_{bal}/1.134)}]$$
$$= d[0.5 + \sqrt{(0.25 - 0.116/1.134)}] = 0.89d$$

$$A_s = \frac{K_{bal}f_{ck}bd^2}{0.87f_{yk}z_{bal}} + A'_s$$
$$= \frac{0.116 \times 25 \times 260 \times 440^2}{0.87 \times 460 \times 0.89 \times 440} + 524$$
$$= 932 + 524 = 1456\,\text{mm}^2$$

4.8 Bending Plus Axial Load at the Ultimate Limit State

The applied axial force may be tensile or compressive. In the analysis that follows, a compressive force is considered. For a tensile load the same basic principles of equilibrium, compatibility of strains, and stress–strain relationships, would apply, but it would be necessary to change the sign of the applied load (N) when we consider the equilibrium of forces on the cross-section. (The area of concrete in compression has not been reduced to allow for the concrete displaced by the compression steel. This could be taken into account by reducing the stress f_{sc} in the compression steel by an amount equal to $0.567f_{ck}$.)

Figure 4.18 represents the cross-section of a member with typical strain and stress distributions for varying positions of the neutral axis. The cross-section is subject to a moment M and an axial compressive force N, and in the figure the direction of the moment is such as to cause compression on the upper part of the section and tension on the lower part. For cases where there is tension in the section (figure 4.18a) the limiting concrete strain is taken as 0.0035 – the value used in the design and analysis of sections for bending. However for cases where there is no tension in the section (figure 4.18b) the limiting strain is taken as a value of 0.002 at the level of 3/7 of the depth of the section.

Let

F_{cc} be the compressive force developed in the concrete and acting through the centroid of the stress block

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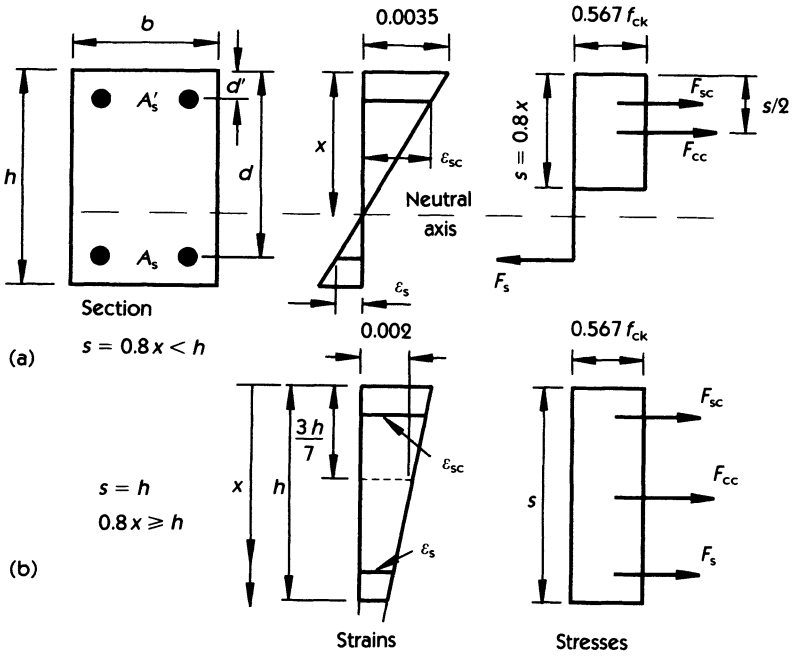

Figure 4.18 *Bending plus axial load with varying position of the neutral axis*

F_{sc} be the compressive force in the reinforcement area A'_s and acting through its centroid

F_s be the tensile or compressive force in the reinforcement area A_s and acting through its centroid.

(i) Basic Equations and Design Charts

The applied force (N) must be balanced by the forces developed within the cross-section, therefore

$$N = F_{cc} + F_{sc} + F_s$$

In this equation, F_s will be negative whenever the position of the neutral axis is such that the reinforcement A_s is in tension, as in figure 4.18a. Substituting into this equation the terms for the stresses and areas

$$N = 0.567 f_{ck} bs + f_{sc} A'_s + f_s A_s \qquad (4.35)^*$$

where f_{sc} is the compressive stress in reinforcement A'_s and f_s is the tensile or compressive stress in reinforcement A_s.

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The design moment M must be balanced by the moment of resistance of the forces developed within the cross-section. Hence, taking moments about the mid-depth of the section

$$M = F_{cc}\left(\frac{h}{2} - \frac{s}{2}\right) + F_{sc}\left(\frac{h}{2} - d'\right) + F_s\left(\frac{h}{2} - d\right)$$

or

$$M = 0.567 f_{ck} bs\left(\frac{h}{2} - \frac{s}{2}\right) + f_{sc} A'_s\left(\frac{h}{2} - d'\right) - f_s A_s\left(d - \frac{h}{2}\right) \qquad (4.36)^*$$

When the depth of neutral axis is such that $0.8x \geqslant h$ as in part (b) of figure 4.18, then the whole concrete section is subject to a uniform compressive stress of $0.567 f_{ck}$. In this case, the concrete provides no contribution to the moment of resistance and the first term on the right side of equation 4.36 disappears.

For a symmetrical arrangement of reinforcement ($A'_s = A_s = A_{sc}/2$ and $d' = h - d$), equations 4.35 and 4.36 can be rewritten in the following form

$$\frac{N}{bhf_{ck}} = \frac{0.567s}{h} + \frac{f_{sc}A_s}{f_{ck}bh} + \frac{f_s A_s}{f_{ck}bh} \qquad (4.37)$$

$$\frac{M}{bh^2 f_{ck}} = \frac{0.567s}{h}\left(0.5 - \frac{s}{2h}\right) + \frac{f_{sc}A_s}{f_{ck}bh}\left(\frac{d}{h} - 0.5\right) - \frac{f_s A_s}{f_{ck}bh}\left(\frac{d}{h} - 0.5\right) \qquad (4.38)$$

In these equations the steel strains, and hence the stresses f_{sc} and f_s, vary with the depth of the neutral axis (x). Thus N/bhf_{ck} and $M/bh^2 f_{ck}$ can be calculated for specified ratios of A_s/bh and x/h so that column design charts for a symmetrical arrangement of reinforcement such as the one shown in figure 4.19 can be plotted.

The direct solution of equations 4.37 and 4.38 for the design of column reinforcement would be very tedious and, therefore, a set of design charts for the usual case of symmetrical sections is available in the Concise Code. Examples showing the design of column steel are given in chapter 9.

(ii) Modes of Failure

The relative magnitude of the moment (M) and the axial load (N) governs whether the section will fail in tension or in compression. With large effective eccentricity ($e = M/N$) a tensile failure is likely, but with a small eccentricity a compressive failure is more likely. The magnitude of

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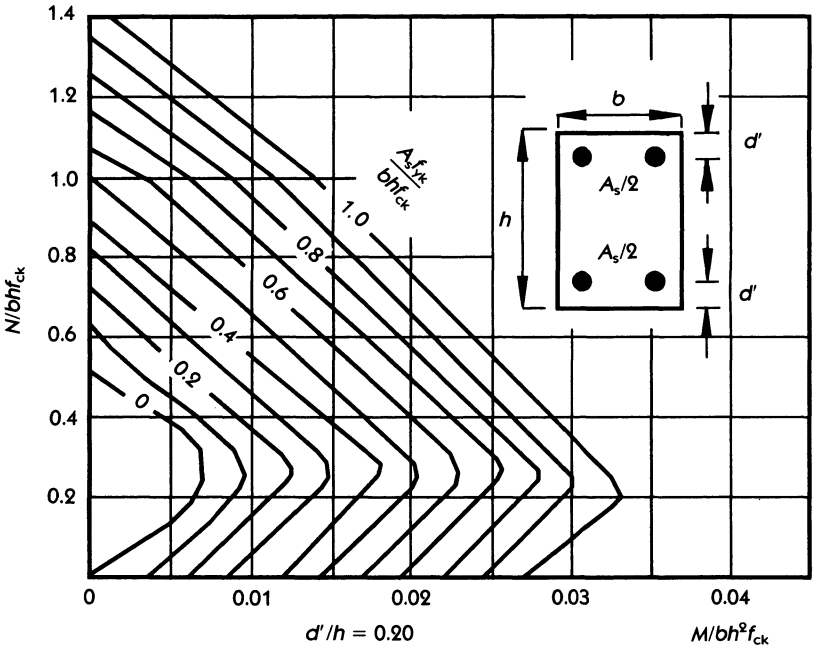

Figure 4.19 *Typical column design chart*

the eccentricity affects the position of the neutral axis and hence the strains and stresses in the reinforcement.

Let

ε_{sc} be the compressive strain in reinforcement A'_s

ε_s be the tensile or compressive strain in reinforcement A_s

ε_y be the tensile yield strain of steel as shown in the stress–strain curve of figure 4.2.

From the linear strain distribution of figure 4.18a

$$\varepsilon_{sc} = 0.0035\left(\frac{x - d'}{x}\right)$$

and (4.39)*

$$\varepsilon_s = 0.0035\left(\frac{d - x}{x}\right)$$

For values of x greater than h, when the neutral axis extends below the section, as shown in figure 4.18b, the steel strains are given by the alternative expressions:

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$$\varepsilon_{sc} = 0.002\frac{7(x - d')}{(7x - 3h)}$$

and

$$\varepsilon_s = 0.002\frac{7(x - d)}{(7x - 3h)}$$

The steel stresses and strains are then related according to the stress–strain curve of figure 4.2.

Consider the following modes of failure of the section as shown on the interaction diagram of figure 4.20.

(a) Tension Failure, $\varepsilon_s > \varepsilon_y$

This type of failure is associated with large eccentricities (e) and small depths of neutral axis (x). Failure begins with yielding of the tensile reinforcement, followed by crushing of the concrete as the tensile strains rapidly increase.

(b) Balanced Failure, $\varepsilon_s = \varepsilon_y$, point b on figure 4.20

When failure occurs with yielding of the tension steel and crushing of the concrete at the same instant it is described as a 'balanced' failure. With $\varepsilon_s = \varepsilon_y$ and from equation 4.39

$$x = x_{bal} = \frac{d}{1 + \dfrac{\varepsilon_y}{0.0035}}$$

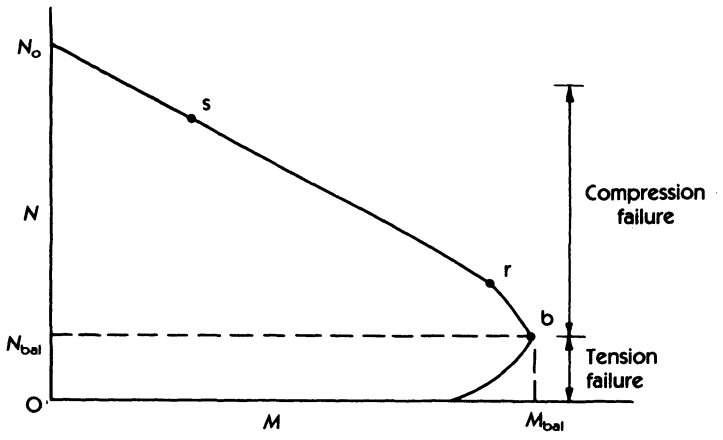

Figure 4.20 *Bending, plus axial load chart with modes of failure*

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For example, substituting the value of $\varepsilon_y = 0.002$ for grade 460 steel

$$x_{bal} = 0.636d$$

Equations 4.35 and 4.36 become

$$N_{bal} = F_{cc} + F_{sc} - F_s$$

and

$$= 0.567 f_{ck} b \times 0.8 x_{bal} + f_{sc} A'_s - 0.87 f_{yk} A_s \qquad (4.40)$$

$$M_{bal} = F_{cc}\left(\frac{h}{2} - \frac{0.8x_{bal}}{2}\right) + F_{sc}\left(\frac{h}{2} - d'\right) + F_s\left(d - \frac{h}{2}\right)$$

where

$$f_{sc} \leqslant 0.87 f_{yk}$$

At point b on the interaction diagram of figure 4.20, $N = N_{bal}$, $M = M_{bal}$ and $f_s = -0.87 f_{yk}$. When the design load $N > N_{bal}$ the section will fail in compression, while if $N < N_{bal}$ there will be an initial tensile failure, with yielding of reinforcement A_s.

(c) Compression Failure

In this case $x > x_{bal}$ and $N > N_{bal}$.

The change in slope at point r in figure 4.20 occurs when

$$\varepsilon_{sc} = \varepsilon_y$$

and from equation 4.39

$$x_r = 0.0035 d'/(0.0035 - \varepsilon_y)$$
$$= 2.33 d' \text{ for grade 460 steel}$$

Point r will occur in the tension failure zone of the interaction diagram if $x_r < x_{bal}$.

When $x < d$

$$f_s \leqslant 0.87 f_{yk} \text{ and tensile}$$

When $x = d$

$$f_s = 0$$

When $x > d$

$$f_s \leqslant 0.87 f_{yk} \text{ and compressive}$$

When x becomes very large and the section approaches a state of uniform axial compression.

$$\varepsilon_s = 0.002 = \varepsilon_y \text{ for grade 460 steel}$$

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At this stage, both layers of steel will have yielded and there will be zero moment of resistance with a symmetrical section, so that

$$N_0 = 0.567 f_{ck} bh + 0.87 f_{yk}(A'_s + A_s)$$

At the stage where the neutral axis coincides with the bottom of the section the strain diagram changes from that shown in figure 4.18a to the alternative strain diagram shown in figure 4.18b. To calculate N and M at this stage, corresponding to point s in figure 4.20, equations 4.35 and 4.36 should be used taking the neutral axis depth equal to the overall section depth, h.

Such M–N interaction diagrams can be constructed for any shape of cross-section which has an axis of symmetry by applying the basic equilibrium and strain compatibility equations with the stress–strain relations, as demonstrated in the following examples. These diagrams can be very useful for design purposes.

Example 4.10 M–N Interaction Diagram for an Unsymmetrical Section

Construct the interaction diagram for the section shown in figure 4.21 with $f_{ck} = 25\,\text{N/mm}^2$ and $f_{yk} = 460\,\text{N/mm}^2$. The bending causes maximum compression on the face adjacent to the steel area A'_s.

For a symmetrical cross-section, taking moments about the centre-line of the concrete section will give $M = 0$ with $N = N_0$ and both areas of steel at the yield stress. This is no longer true for unsymmetrical steel areas as $F_{sc} \neq F_s$ at yield therefore, theoretically, moments should be calculated about an axis referred to as the 'plastic centroid'. The ultimate axial load N_0 acting through the plastic centroid causes a uniform strain across the section with compression yielding of all the reinforcement, and

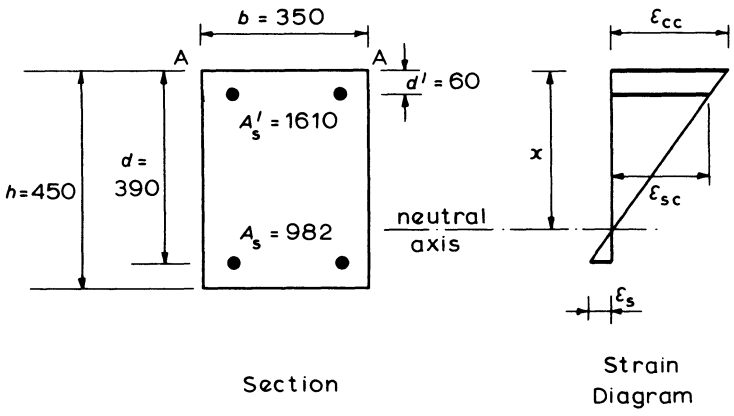

Figure 4.21 *Non-symmetrical section M–N interaction example*

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thus there is zero moment of resistance. With uniform strain the neutral-axis depth, x, is at infinity.

The location of the plastic centroid is determined by taking moments of all the stress resultants about an arbitrary axis such as AA in figure 4.21 so that

$$\bar{x}_p = \frac{\Sigma(F_{cc}h/2 + F_{sc}d' + F_s d)}{\Sigma(F_{cc} + F_{sc} + F_s)}$$

$$= \frac{0.567f_{ck}A_{cc} \times 450/2 + 0.87f_{yk}A'_s \times 60 + 0.87f_{yk}A_s \times 390}{0.567f_{ck}A_{cc} + 0.87f_{yk}A'_s + 0.87f_{yk}A_s}$$

$$= \frac{0.567 \times 25 \times 350 \times 450^2/2 + 0.87 \times 460(1610 \times 60 + 982 \times 390)}{0.567 \times 25 \times 350 \times 450 + 0.87 \times 460(1610 + 982)}$$

$$= 212\,\text{mm from AA}$$

The fundamental equations for calculating points on the interaction diagram with varying depths of neutral axis are

(i) Compatibility of strains (used in table 4.3, columns 2 and 3):

$$\varepsilon_{sc} = 0.0035\left(\frac{x - d'}{x}\right)$$

$$\varepsilon_s = 0.0035\left(\frac{d - x}{x}\right) \qquad (4.41)$$

or when the neutral axis depth extends below the bottom of the section ($x > h$):

$$\varepsilon_{sc} = 0.002\frac{7(x - d')}{(7x - 3h)}$$

$$\varepsilon_s = 0.002\frac{7(x - d)}{(7x - 3h)}$$

(ii) Stress–strain relations for the steel (table 4.3, columns 4 and 5):

$$\varepsilon \geqslant \varepsilon_y = 0.002, \qquad f = 0.87f_{yk}$$

$$\varepsilon < \varepsilon_y \qquad f = E \times \varepsilon \qquad (4.42)$$

(iii) Equilibrium (table 4.3, columns 6 and 7):

$$N = F_{cc} + F_{sc} + F_s$$

or $0.8x < h \qquad N = 0.567f_{ck}b0.8x + f_{sc}A'_s + f_sA_s$

$0.8x \geqslant h \qquad N = 0.567f_{ck}bh + f_{sc}A'_s + f_sA_s$

Taking moments about the plastic centroid

$$0.8x < h \qquad M = F_{cc}(\bar{x}_p - 0.8x/2) + F_{sc}(\bar{x}_p - d') - F_s(d - \bar{x}_p)$$

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$0.8x \geqslant h \qquad M = F_{cc}(\bar{x}_p - h/2) + F_{sc}(\bar{x}_p - d') - F_s(d - \bar{x}_p)$

F_s is negative when f_s is a tensile stress.

These equations have been applied to provide the values in table 4.3 for a range of key values of x. Then the M–N interaction diagram has been plotted in figure 4.22 from the values in table 4.3 as a series of straight lines. Of course, N and M could have been calculated for intermediate values of x to provide a more accurate curve.

Table 4.3 M–N interaction values for example 4.10

(1) x	(2) ε_{sc}	(3) ε_s	(4) f_{sc}	(5) f_s	(6) N	(7) M
$d' = 60$	0	>0.002	0	$-0.87f_{yk}$	−155	114
$2.33d' = 140$	0.002	>0.002	$0.87f_{yk}$	$-0.87f_{yk}$	807	251
$x_{bal} = 0.636d = 248$	>0.002	0.002	$0.87f_{yk}$	$-0.87f_{yk}$	1236	267
$d = 390$	>0.002	0	$0.87f_{yk}$	0	2196	154
$h = 450$	>0.002	0.00047	$0.87f_{yk}$	93.3	2522	99
∞	0.002	0.002	$0.87f_{yk}$	$0.87f_{yk}$	3269	0

Example 4.11 M–N Interaction Diagram for a Non-Rectangular Section

Construct the interaction diagram for the equilateral triangular column section in figure 4.23 with $f_{ck} = 25\,\text{N/mm}^2$ and $f_{yk} = 460\,\text{N/mm}^2$. The bending is about an axis parallel to the side AA and causes maximum compression on the corner adjacent to the steel area A_s'.

For this triangular section, the plastic centroid is at the same location as the geometric centroid, since the moment of F_{sc} equals the moment F_s about this axis when all the bars have yielded in compression.

The fundamental equations for strain compatibility and the steel's stress–strain relations are as presented in example 4.10 and are used again in this example. The equilibrium equations for the triangular section become

$$N = F_{cc} + F_{sc} + F_s$$

or

$0.8x < h \qquad N = 0.567f_{ck}sl/2 + f_{sc}A_s' + f_sA_s$

$0.8x \geqslant h \qquad N = 0.567f_{ck}h \times 400/2 + f_{sc}A_s' + f_sA_s$

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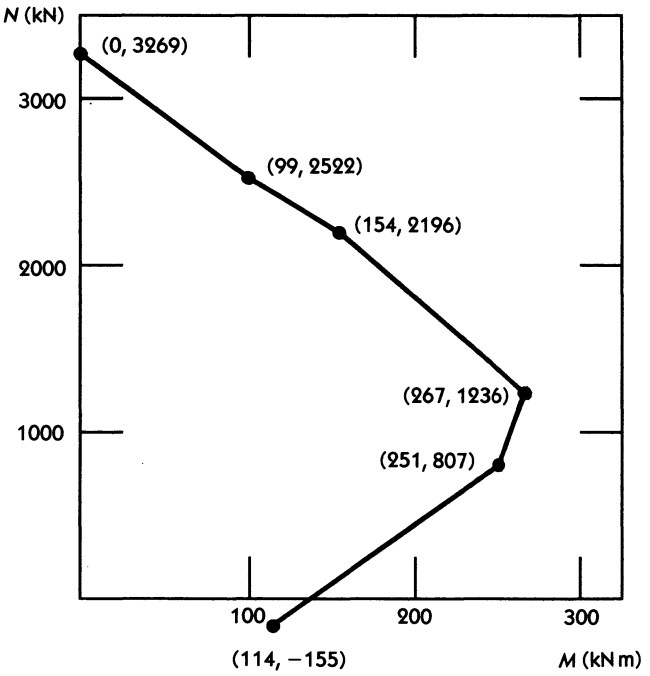

Figure 4.22 *M–N interaction diagram for a non-symmetrical section*

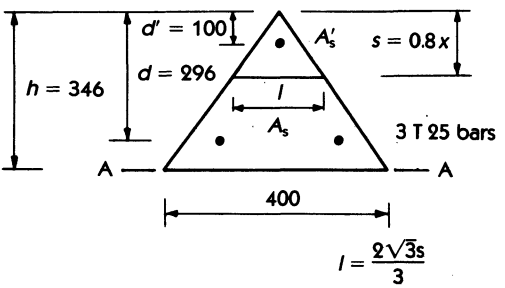

Figure 4.23 *Non-rectangular M–N interaction example*

$0.8x < h$ $\quad M = F_{cc}2(h - 0.8x)/3 + F_{sc}(2h/3 - d') - F_s(d - 2h/3)$

$0.8x \geqslant h$ $\quad M = F_{sc}(2h/3 - d') - F_s(d - 2h/3)$

F_s is negative when f_s is a tensile stress, and from the geometry of figure 4.23 the width of the section at depth $s = 0.8x$ is

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Table 4.4 $M-N$ interaction values for example 4.11

x	ε_{sc}	ε_{st}	f_{sc} (N/mm^2)	f_s (N/mm^2)	N (kN)	M $(kN\,m)$
$d' = 100$	0	>0.002	0	$-0.87f_{yk}$	−340	34.6
$x_{bal} = 0.636d = 188$	0.0016	0.002	328	$-0.87f_{yk}$	−47	68.5
$2.33d' = 233$	0.002	0.00095	$0.87f_{yk}$	−189	295	63.6
$d = 296$	>0.002	0	$0.87f_{yk}$	0	655	50.0
$h = 346$	>0.002	0.00051	$0.87f_{yk}$	101	923	33.6
∞	>0.002	>0.002	$0.87f_{yk}$	$0.87f_{yk}$	1570	0

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$$l = \frac{2}{3}s\sqrt{3}$$

Table 4.4 has been calculated using the fundamental equations with the values of x shown. The interaction diagram is shown constructed in figure 4.24.

With a non-rectangular section, it could be advisable to construct a more accurate interaction diagram using other intermediate values of x. This would certainly be the case with, say, a flanged section where there is sudden change in breadth.

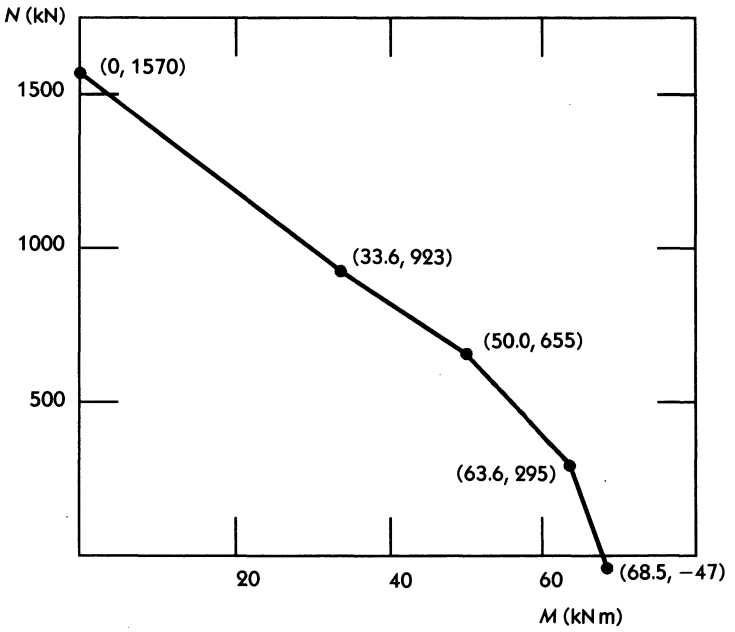

Figure 4.24 *M–N interaction diagram for a non-symmetrical section*

4.9 Rectangular–Parabolic Stress Block

A rectangular–parabolic stress block may be used to provide a more rigorous analysis of the reinforced concrete section. The stress block is similar in shape to the stress–strain curve for concrete in figure 4.1, having a maximum stress of $0.567f_{ck}$ at the ultimate strain of 0.0035.

In figure 4.25

ε_0 = the concrete strain at the end of the parabolic section
w = the distance from the neutral axis to strain ε_0

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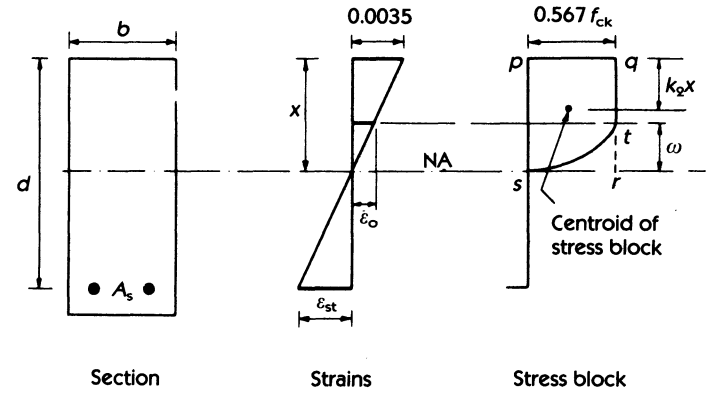

Figure 4.25 *Section in bending with a rectangular–parabolic stress block*

x = depth of the neutral axis
k_1 = the mean concrete stress
k_2x = depth to the centroid of the stress block.

(a) To Determine the Mean Concrete Stress, k_1

From the strain diagram

$$\frac{x}{0.0035} = \frac{w}{\varepsilon_0}$$

therefore

$$w = \frac{x\varepsilon_0}{0.0035}$$

Substituting for $\varepsilon_0 = 0.002$ (see figure 4.1)

$$w = 0.571x \tag{4.43}$$

For the stress block

$$k_1 = \frac{\text{area of stress block}}{x}$$

$$= \frac{\text{area pqrs} - \text{area rst}}{x}$$

Thus, using the area properties of a parabola as shown in figure 4.26, we have

$$k_1 = \frac{0.567f_{ck}x - 0.567f_{ck}w/3}{x}$$

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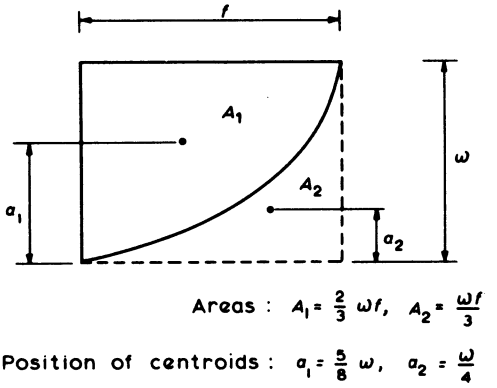

Figure 4.26 *Properties of a parabola*

Substituting for w from equation 4.43 gives

$$k_1 = 0.459 f_{ck} \qquad (4.44)^*$$

(b) To Determine the Depth of the Centroid $k_2 x$

k_2 is determined for a rectangular section by taking area moments of the stress block about the neutral axis – see figures 4.25 and 4.26. Thus

$$(x - k_2 x) = \frac{\text{area pqrs} \times x/2 - \text{area rst} \times w/4}{\text{area of stress block}}$$

$$= \frac{(0.567 f_{ck} x)x/2 - (0.567 f_{ck} w/3)w/4}{k_1 x}$$

$$= \frac{0.567 f_{ck}(x^2/2 - w^2/12)}{k_1 x}$$

Substituting for w from equation 4.43

$$(x - k_2 x) = \frac{0.567 f_{ck} x^2}{k_1 x}\left[0.5 - \frac{0.571^2}{12}\right]$$

hence

$$k_2 = 1 - \frac{0.268 f_{ck}}{k_1} = 1 - \frac{0.268 f_{ck}}{0.459 f_{ck}} = 0.416 \qquad (4.45)^*$$

Once we know the properties of the stress block, the magnitude and position of the resultant compressive force in the concrete can be determined, and hence the moment of resistance of the section calculated using procedures similar to those for the rectangular stress block.

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Comparison of the rectangular–parabolic and the rectangular stress blocks provides

(i) Stress resultant, F_{cc}

rectangular–parabolic: $k_1 bx \approx 0.459 f_{ck} bx$

rectangular: $0.567 f_{ck} \times 0.8bx \approx 0.454 f_{ck} bx$

(ii) Lever arm, z

rectangular–parabolic: $d - k_2 x \approx d - 0.416x$

rectangular: $d - \frac{1}{2} \times 0.8x = d - 0.40x$

So both stress blocks have almost the same moment of resistance, $F_{cc} \times z$, showing it is adequate to use the simpler rectangular stress block for design calculations.

4.10 Triangular Stress Block

The triangular stress block applies to elastic conditions during the serviceability limit state. In practice it is not generally used in design calculations except for liquid-retaining structures, or for the calculations of crack widths and deflections as described in chapter 6. With the triangular stress block, the cross-section can be considered as

(i) cracked in the tension zone, or
(ii) uncracked with the concrete resisting a small amount of tension.

4.10.1 Cracked Section

A cracked section is shown in figure 4.27 with a stress resultant F_{st} acting through the centroid of the steel and F_{cc} acting through the centroid of the triangular stress block.

For equilibrium of the section

$$F_{cc} = F_{st}$$

or
$$0.5bxf_{cc} = A_s f_{st} \qquad (4.46)^*$$

and the moment of resistance

$$M = F_{cc} \times z = F_{st} \times z$$

or
$$M = 0.5bxf_{cc}(d - x/3) = A_s f_{st}(d - x/3) \qquad (4.47)^*$$

(i) Analysis of a Specified Section

The depth of the neutral axis, x, can be determined by converting the section into an 'equivalent' area of concrete as shown in figure 4.28,

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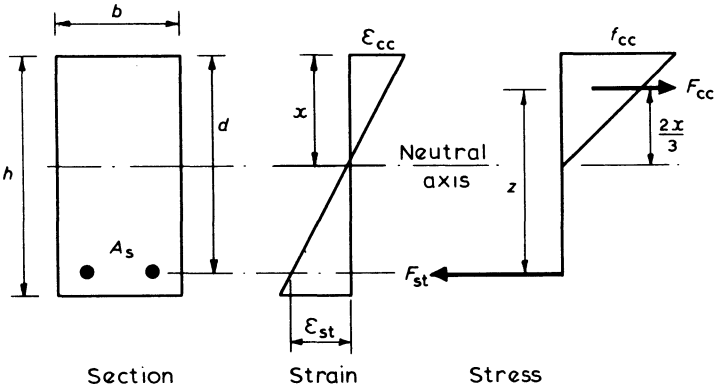

Figure 4.27 *Triangular stress block – cracked section*

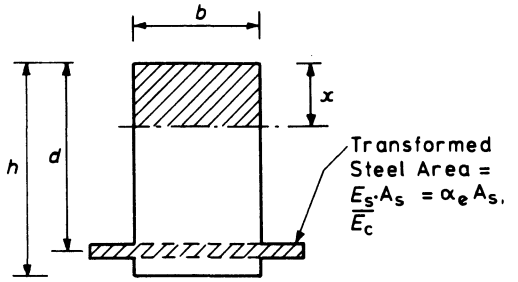

Figure 4.28 *Equivalent transformed section with the concrete cracked*

where $\alpha_e = E_s/E_c$, the modular ratio. Taking area moments about the upper edge:

$$x = \frac{\Sigma(Ax)}{\Sigma A}$$

Therefore

$$x = \frac{bx \times x/2 + \alpha_e A_s d}{bx + \alpha_e A_s}$$

or

$$\frac{1}{2}bx^2 + \alpha_e A_s x - \alpha_e A_s d = 0$$

Solving this quadratic equation gives

$$x = \frac{-\alpha_e A_s \pm \sqrt{[(\alpha_e A_s)^2 + 2b\alpha_e A_s d]}}{b} \qquad (4.48)^*$$

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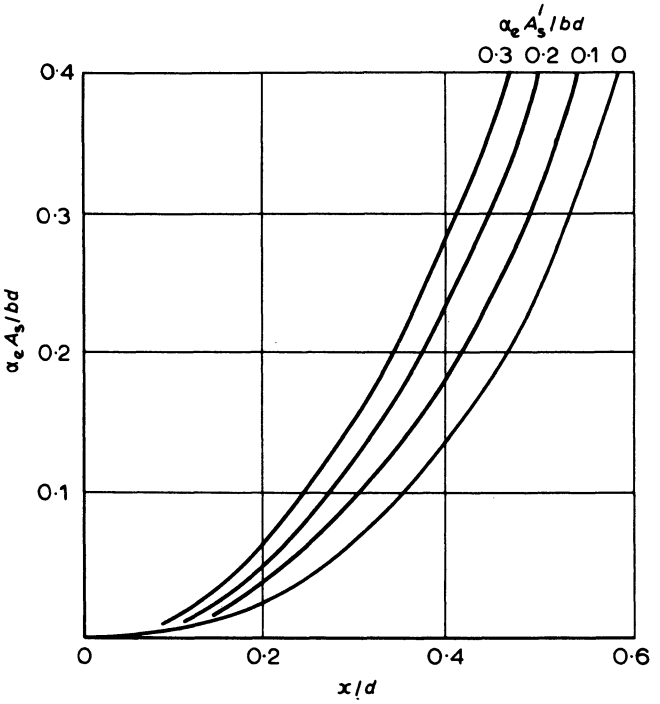

Figure 4.29 *Neutral-axis depths for cracked rectangular section – elastic behaviour*

Equation 4.48 may be solved using a chart such as the one shown in figure 4.29.

Equations 4.46 to 4.48 can be used to analyse a specified reinforced concrete section.

(ii) Design of Steel Area, A_s, with Stresses f_{st} and f_{cc} Specified

The depth of the neutral axis can also be expressed in terms of the strains and stresses of the concrete and steel.

From the linear strain distribution of figure 4.27:

$$\frac{x}{d} = \frac{\varepsilon_{cc}}{\varepsilon_{cc} + \varepsilon_{st}} = \frac{f_{cc}/E_c}{f_{cc}/E_c + f_{st}/E_s}$$

Therefore

$$\frac{x}{d} = \frac{1}{1 + \dfrac{f_{st}}{\alpha_e f_{cc}}} \qquad (4.49)^*$$

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Equations 4.47 and 4.49 may be used to design the area of tension steel required, at a specified stress, in order to resist a given moment.

Example 4.12 Analysis of a Cracked Section using a Triangular Stress Block

For the section shown in figure 4.30, determine the concrete and steel stresses caused by a moment of 120 kN m, assuming a cracked section. Take $E_s/E_c = \alpha_e = 15$.

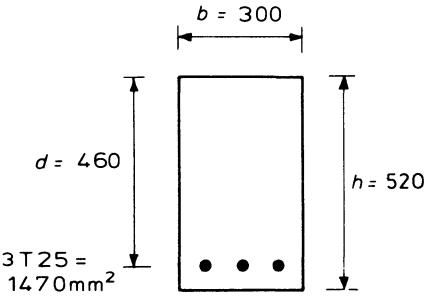

Figure 4.30 *Analysis example with triangular stress block*

$$\alpha_e \frac{A_s}{bd} = \frac{15 \times 1470}{300 \times 460} = 0.16$$

Using the chart of figure 4.29 or equation 4.48 gives $x = 197$ mm.

From equation 4.47

$$M = \frac{1}{2} b x f_{cc} \left(d - \frac{x}{3} \right)$$

therefore

$$120 \times 10^6 = \frac{1}{2} \times 3000 \times 197 \times f_{cc} \left(460 - \frac{197}{3} \right)$$

therefore

$$f_{cc} = 10.3 \text{ N/mm}^2$$

From equation 4.46

$$f_{st} A_s = \frac{1}{2} b x f_{cc}$$

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therefore

$$f_{st} = 300 \times 197 \times \frac{10.3}{2} \times \frac{1}{1470} = 207\,\text{N/mm}^2$$

4.10.2 Triangular Stress Block – Uncracked Section

The concrete may be considered to resist a small amount of tension. In this case a tensile stress resultant F_{ct} acts through the centroid of the triangular stress block in the tension zone as shown in figure 4.31.

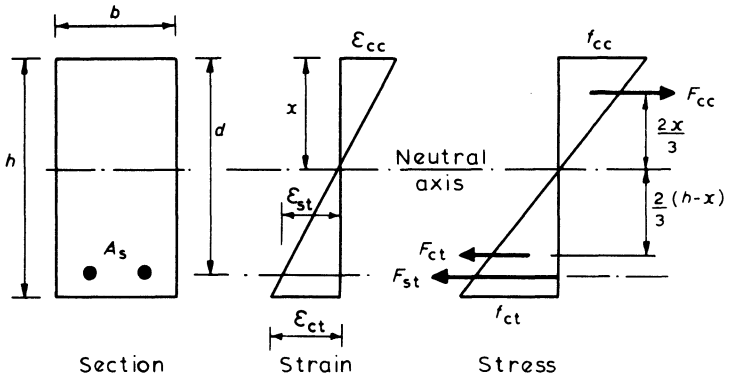

Figure 4.31 *Triangular stress block – uncracked example*

For equilibrium of the section

$$F_{cc} = F_{ct} + F_{st} \qquad (4.50)$$

where $F_{cc} = 0.5bxf_{cc}$

$F_{ct} = 0.5b(h - x)f_{ct}$

and $F_{st} = A_s \times f_{st}$

Taking moments about F_{cc}, the moment of resistance of the section is given by

$$M = F_{st} \times (d - x/3) + F_{ct} \times \left(\frac{2}{3}x + \frac{2}{3}(h - x)\right) \qquad (4.51)^*$$

The depth of the neutral axis, x, can be determined by taking area moments about the upper edge AA of the equivalent concrete section shown in figure 4.32, such that

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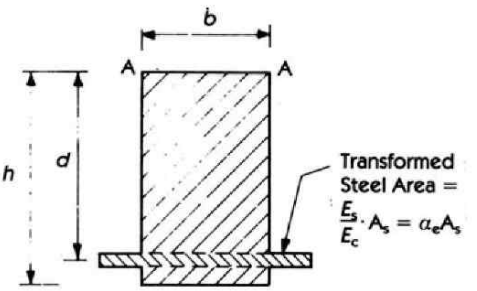

Figure 4.32 *Equivalent transformed section with the concrete uncracked*

$$x = \frac{\Sigma(Ax)}{\Sigma A}$$

$$\alpha_e = \frac{E_s}{E_c} \quad \text{is termed the modular ratio}$$

Therefore

$$x = \frac{bh \times h/2 + \alpha_e A_s \times d}{bh + \alpha_e A_s}$$

$$= \frac{h + 2\alpha_e rd}{2 + 2\alpha_e r} \qquad (4.52)^*$$

where $r = A_s/bh$

From the linear proportions of the strain diagram in figure 4.31:

$$\varepsilon_{cc} = \frac{x}{h - x} \times \varepsilon_{ct}$$

$$\varepsilon_{st} = \frac{d - x}{h - x} \times \varepsilon_{ct} \qquad (4.53)$$

Therefore as stress = $E \times$ strain:

$$f_{ct} = E_c f_{ct}$$

$$f_{cc} = \frac{x}{h - x} \times f_{ct} \qquad (4.54)^*$$

$$f_{st} = \frac{d - x}{h - x} \times \alpha_e f_{ct}$$

Hence if the maximum tensile strain or stress is specified, it is possible to calculate the corresponding concrete compressive and steel tensile stresses from equations 4.54.

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The equations derived can be used to analyse a given cross-section in order to determine the moment of resistance of the uncracked section.

Example 4.13 Analysis of an Uncracked Section

For the section shown in figure 4.30, calculate the serviceability moment of resistance with no cracking of the concrete, given $f_{ct} = 3\,\text{N/mm}^2$, $E_c = 30\,\text{kN/mm}^2$ and $E_s = 200\,\text{kN/mm}^2$.

$$r = \frac{A_s}{bh} = \frac{1470}{300 \times 520} = 0.0094$$

$$\alpha_e = \frac{E_s}{E_c} = \frac{200}{30} = 6.67$$

$$x = \frac{h + 2\alpha_e rd}{2 + \alpha_e r}$$

$$= \frac{520 + 2 \times 6.67 \times 0.0094 \times 460}{2 + 2 \times 6.67 \times 0.0094} = 272\,\text{mm}$$

$$f_{st} = \left(\frac{d - x}{h - x}\right)\alpha_e f_{ct}$$

$$= \frac{(460 - 272)6.67 \times 3}{(520 - 272)} = 15.2\,\text{N/mm}^2$$

$$M = A_s f_{st}\left(d - \frac{x}{3}\right) + \frac{1}{2}b(h - x)f_{ct} \times \left(\frac{2}{3}x + \frac{2}{3}(h - x)\right)$$

$$= 1470 \times 15.2\left(460 - \frac{272}{3}\right)10^{-6} + \frac{1}{2} \times 300(520 - 272) \times 3$$

$$\times\left(\frac{2}{3} \times 272 + \frac{2}{3}(520 - 272)\right)10^{-6}$$

$$= 8.3 + 38.7 = 47\,\text{kN m}$$

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5

Shear, Bond and Torsion

This chapter deals with the theory and derivation of the design equations for shear, bond and torsion. Some of the more practical factors governing the choice and arrangement of the reinforcement are dealt with in the chapters on member design, particularly chapter 7, which contains examples of the design and detailing of shear and torsion reinforcement in beams. Punching shear caused by concentrated loads on slabs is covered in section 8.1.1 of the chapter on slab design.

5.1 Shear

Figure 5.1 represents the distribution of principal stresses across the span of a homogeneous concrete beam. The direction of the principal compressive stresses takes the form of an arch, while the tensile stresses have the curve of a catenary or suspended chain. Towards mid-span, where the shear is low and the bending stresses are dominant, the direction of the stresses tends to be parallel to the beam axis. Near the supports, where the shearing forces are greater, the principal stresses are inclined at a steeper angle, so that the tensile stresses are liable to cause diagonal cracking. If the diagonal tension exceeds the limited tensile strength of the concrete then shear reinforcement must be provided. This reinforcement is either in the form of (1) stirrups, or (2) inclined bars (used in conjunction with stirrups).

The shear in a reinforced concrete beam without shear reinforcement is carried by a combination of three main components. These are

(i) concrete in the compression zone
(ii) dowelling action of tensile reinforcement
(iii) aggregate interlock across flexural cracks.

W.H. Mosley et al. *Reinforced Concrete Design to Eurocode 2 (EC2)*
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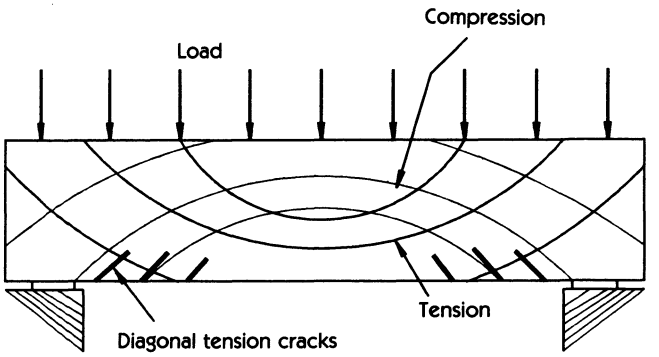

Figure 5.1 *Principal stresses in a beam*

The actual behaviour is complex, and difficult to analyse theoretically, but by applying the results from many experimental investigations, reasonable simplified procedures for analysis and design have been developed.

In EC2 two methods of analysing and designing for shear are presented. The first method, referred to as the *Standard Method*, is the one likely to be most commonly used and is, in fact, the method described in the Concise Code. The second method, the *Variable Strut Inclination Method*, is an alternative approach where it is possible to obtain economies in the amount of shear reinforcement provided, but this economy is balanced by the amount of additional longitudinal reinforcement that must also be provided. Although this alternative method is likely to be the less popular in practice, its use is necessary when considering both shear and torsion acting together. For this reason, both methods are described.

5.1.1 Standard Method of Shear Design

In order to derive simplified equations the action of a reinforced concrete beam in shear is represented, in the Standard Method, by an analogous truss in which the longitudinal reinforcement forms the bottom chord, the stirrups are the vertical members and the concrete acts as the diagonal and top chord compression members as indicated in figure 5.2. In the truss shown, the top and bottom chord are assumed to act at a distance of $0.9d$ apart with the stirrups at the same spacing such that the diagonal compression members are at an angle of 45°, which more or less agrees with experimental observations of the cracking of reinforced concrete beams close to their supports.

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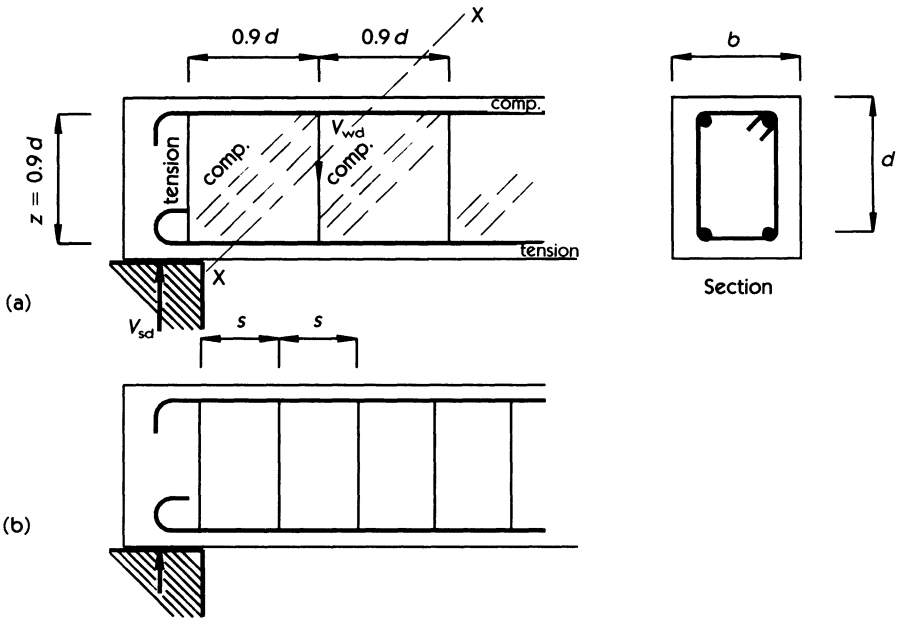

Figure 5.2 *Stirrups and the analogous truss*

In the analogous truss let

A_{sw} = the cross-sectional area of the two legs of the stirrup
f_{ywd} = the design yield strength of the stirrup reinforcement
f_{yk} = the characteristic strength of the stirrup reinforcement
V_{Sd} = the shear force due to the ultimate loads.

Using the method of sections it can be seen at section XX in the figure that at the ultimate limit state the force in the vertical stirrup member (V_{wd}) must equal the shear force (V_{Sd}), that is

$$V_{wd} = V_{Sd} = f_{ywd}A_{sw} = \frac{f_{yk}A_{sw}}{1.15}$$

When the stirrup spacing is less than 0.9 of the effective depth, a series of superimposed equivalent trusses may be considered, so that the force to be resisted by the stirrup is reduced proportionally. Thus if s = the stirrup spacing, the above equation becomes

$$V_{wd}\frac{s}{0.9d} = \frac{f_{yk}\,A_{sw}}{1.15}$$

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or

$$V_{wd} = \frac{1}{1.28}\frac{A_{sw}}{s}df_{yk} \tag{5.1}$$

This is the shear resistance of the vertical reinforcement. Since the concrete is also capable of resisting a limited amount of shear, the total shear resistance of the section (V_{Rd3}) is the addition of the shear resistance of the reinforcement, V_{wd}, and the concrete shear resistance, V_{cd}. That is

$$V_{Rd3} = V_{cd} + V_{wd}$$

In EC2, V_{cd} is also expressed as V_{Rd1}, thus the above equation can also be expressed as

$$V_{Rd3} = V_{Rd1} + V_{wd} \tag{5.2}$$

The concrete shear resistance (V_{Rd1}) is given by an empirical expression

$$V_{Rd1} = [\tau_{Rd}k(1.2 + 40\rho_1)]b_w d \tag{5.3}*$$

where

τ_{Rd} = basic design shear strength given in table 5.1
= $0.035f_{ck}^{2/3}$ (N/mm^2)
k = $(1.6 - d)$ {>1} or 1 where more than 50 per cent of tension reinforcement is curtailed
ρ_1 = $A_{s1}/b_w d$ {<0.02}
A_{s1} = area of longitudinal tension reinforcement extending not less than a full anchorage length plus one effective depth beyond the section considered
b_w = *minimum* width of section over area considered
d = effective depth of section.

Note that in the NAD the maximum value of f_{ck} is recommended to be limited to 40 N/mm^2 (strength class C40/50); hence τ_{Rd} is limited to 0.41 N/mm^2. From equation 5.3 it may be seen that the shear *strength* of the concrete ($V_{Rd1}/b_w d$) increases for shallower sections and for sections with larger percentages of tensile reinforcement. The longitudinal tension bars contribute to the shear resistance by their dowelling action and they also help to prevent shear cracks from commencing at small tension cracks. To be effective, these tension bars should extend not less than one

Table 5.1 Values of τ_{Rd} (N/mm^2) for different concrete grades, f_{ck}

f_{ck}	12	16	20	25	30	35	40	45	50
τ_{Rd}	0.18	0.22	0.26	0.30	0.34	0.37	0.41	0.44	0.48

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effective depth plus an anchorage length beyond the section being considered.

Close to supports, sections have an enhanced shear resistance owing in part to the induced compressive stresses from the concentrated reaction and the steeper angle of the failure plane, which would normally occur at an angle of 30° to the horizontal for an unreinforced section. Within a distance, $x = 2.5d$ from the face of the support, the shear strength, τ_{Rd}, may be enhanced by a factor β when calculating the value of V_{Rd1}. The value of β is given by $2.5d/x$ with a lower and upper limit of 1.0 and 5.0 respectively. This enhancement is useful when designing beams with concentrated loads near to a support, or with corbels and pile caps. As a simplified approach for beams carrying mainly uniformly distributed loads, the critical section for design may be taken at one effective depth from the face of the support and the shear force at this section can be considered to be the critical value in the region of the support without the necessity for any shear enhancement.

Large shearing forces are also liable to cause crushing of the concrete along the direction of the principal compressive stresses and therefore sections close to supports should be checked to determine the maximum allowable shear force, V_{Rd2}, which can be sustained irrespective of any shear reinforcement, given by

$$V_{Rd2} = 0.3\nu f_{ck} b_w d \qquad (5.4)^*$$

where ν is an efficiency factor $= 0.7 - f_{ck}/200$ (≥ 0.5).

The approach to design using the Standard Method is to compare the ultimate shear force, V_{Sd}, at critical sections to the maximum shear capacity V_{Rd2} (equation 5.4). If V_{Sd} is less than V_{Rd2} then the shear reinforcement required can be calculated by equating V_{Rd3} to V_{Sd} and combining equations 5.1 and 5.2 to give

$$\frac{A_{sw}}{s} = \frac{1.28(V_{Sd} - V_{Rd1})}{df_{yk}} \qquad (5.5)^*$$

which, combined with equation 5.3, is the general expression for shear design. Further information on the practical details of shear design and design examples are given in section 7.3 (Design for Shear).

5.1.2 Variable Strut Inclination Method of Shear Design

The approach to shear calculation described above is referred to as the *Standard Method*. An alternative approach, the *Variable Strut Inclination Method*, is given in EC2 where it is assumed that all the shear is resisted by the stirrups with no contribution from the shear resistance of the concrete. The advantage of this alternative approach is that, by choosing

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the inclination of the compressive struts within the analogous truss, it is possible to design for lower amounts of shear reinforcement than is possible using the Standard Method although there is the disadvantage that additional longitudinal reinforcement is then necessary. Consider the analogous truss shown in figure 5.3 which has vertical stirrups acting as transverse ties, longitudinal bottom reinforcement providing a tension chord and the concrete acting as a top compression member and as diagonal compression members which are inclined at a variable angle θ.

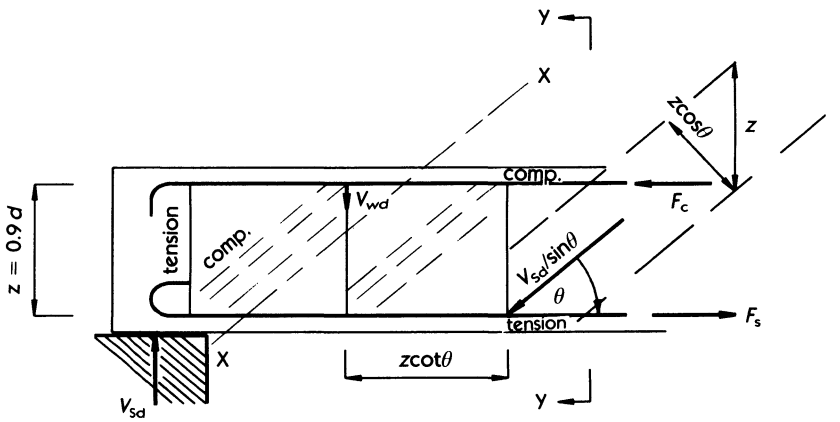

Figure 5.3 *Assumed truss model for the variable strut inclination method*

In this analogous truss, as before, let

A_{sw} = the cross-sectional area of the two legs of the stirrup
f_{ywd} = the design yield strength of the stirrup reinforcement
f_{yk} = the characteristic strength of the stirrup reinforcement
V_{Sd} = the shear force due to the ultimate loads.

Using the method of sections it can be seen at section X–X in the figure that at the ultimate limit state the force in the vertical stirrup member (V_{wd}) must equal the shear force (V_{Sd}), that is

$$V_{wd} = V_{Sd} = f_{ywd}A_{sw} = \frac{f_{yk}A_{sw}}{1.15}$$

In this case the contribution of the concrete to any shear resistance is neglected and the above equation therefore represents the total shear capacity of the section (V_{Rd3}). Hence

$$V_{Rd3} = V_{wd} = V_{Sd} = f_{ywd}A_{sw} = 0.87f_{yk}A_{sw}$$

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If the links are spaced at a distance s apart, then the force in each link is reduced proportionately and is given by

$$V_{Rd3}\frac{s}{z\cot\theta} = 0.87 f_{yk} A_{sw}$$

or

$$V_{Sd} = V_{Rd3} = 0.87\frac{A_{sw}}{s} z f_{yk}\cot\theta \qquad (5.6)^*$$

The shear force applied to the section must also be limited such that excessive compressive stresses do not occur in the diagonal compressive struts, leading possibly to compressive failure of the concrete. Using the method of sections, it can be seen at section YY in figure 5.3 that the vertical component of force in the compressive strut must equal the shear force V_{Sd}. If V_{Sd} is taken as equal to the limiting shear force for compressive failure, V_{Rd2}, it follows that

$$\textit{Force} \text{ in strut} = V_{Rd2}/\sin\theta$$

$$\textit{Area} \text{ of strut} = b_w \times (z\cos\theta)$$

$$\textit{Stress} \text{ in strut} = \frac{\text{Force}}{\text{Area}} = \frac{V_{Rd2}}{b_w z \sin\theta\cos\theta} \leqslant f_{cd}\left(=\frac{f_{ck}}{1.5}\right)$$

where f_{cd} is the design compressive stress in the concrete

or $$V_{Rd2} \leqslant b_w z(f_{ck}/1.5)\sin\theta\cos\theta$$

which can also be expressed as

$$V_{Rd2} \leqslant b_w z f_{ck}/[1.5(\cot\theta + \tan\theta)] \qquad (5.7)$$

In EC2 this equation is modified by the inclusion of an *efficiency factor* (ν) to give

$$V_{Rd2} \leqslant b_w z \nu f_{ck}/[1.5(\cot\theta + \tan\theta)] \qquad (5.8)^*$$

where the efficiency factor takes the value of $(0.7 - f_{ck}/200)$ with a minimum value of 0.50. A further requirement of EC2 is that

$$\frac{A_{sw} f_{ywd}}{b_w s} \leqslant 0.5\nu f_{cd}$$

or expressed in terms of characteristic strengths

$$\frac{A_{sw} f_{yk}}{b_w s} \leqslant 0.38\nu f_{ck} \qquad (5.9)$$

If this alternative method is used then it is necessary to increase the amount of longitudinal reinforcement required above that required for resisting the design ultimate moment. Resolving forces horizontally in the

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section YY shown in figure 5.3, the longitudinal component of the force in the compressive strut is given by

$$\text{Longitudinal force} = (V_{Sd}/\sin\theta) \times \cos\theta$$
$$= V_{Sd}\cot\theta$$

If it is assumed that half of this force is carried by the reinforcement in the tension zone of the beam then the *additional* tensile force (T_d) to be provided in the tensile zone is given by

$$T_d = 0.5V_{Sd}\cot\theta = 0.87f_{yk}A_{sl} \qquad (5.10)^*$$

and *additional* tensile reinforcement area (A_{sl}), working at its full design strength, must be provided to ensure that this force is developed.

Equations 5.6 to 5.10 can be used together to design a section for shear with a value of θ chosen by the designer within limits specified in EC2. From these equations it is obvious that the steel ratio (A_{sw}/s) is a function of the inverse of $\cot\theta$ (equation 5.6); the additional longitudinal steel to be provided varies with $\cot\theta$ (equation 5.10) and the maximum shear force governed by diagonal compression failure is a function of the inverse of ($\cot\theta + \tan\theta$) (equation 5.7).

Figure 5.4 shows the variation of these functions from which it can be seen that as θ is reduced less shear reinforcement is required, but this is compensated for by an increase in the necessary longitudinal reinforcement. At values of θ greater or less than 45° the shear capacity of the section, based on compressive failure in the diagonal struts, is also reduced. The designer is thus free to choose almost any value of θ but for

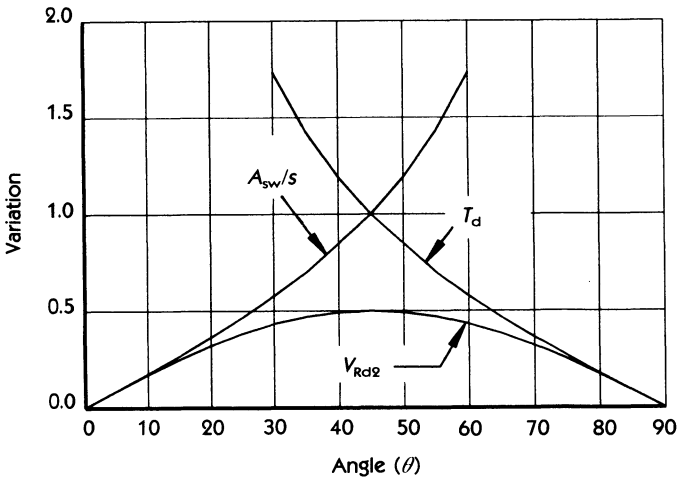

Figure 5.4 *Graph of variation of V_{Rd2}, T_d and A_{sw}/s*

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practical reasons EC2 places a lower and upper limit of 0.4 and 2.5 on the value of $\cot\theta$ when the longitudinal reinforcement is not curtailed within the zone of shear being considered. This corresponds to limiting θ to 68° and 22° respectively. When the longitudinal reinforcement is curtailed the limits on $\cot\theta$ are 0.5 and 2.0. The UK National Application Document further limits $\cot\theta$ to 0.67 and 1.5 in both cases, corresponding to a limit on θ of 56° and 34°.

When using these equations it will normally only be necessary to calculate the amount of shear reinforcement by putting $\cot\theta$ equal to the upper permissible limit in equation 5.6. However when shear stresses are high and diagonal compressive failure is possible then the ultimate shear force, V_{Sd}, should be equated to V_{Rd2} in equation 5.7 and the largest value of $\cot\theta$ determined to satisfy this equation. This value of $\cot\theta$ may then be inserted into equation 5.6, with V_{Rd3} equal to V_{Sd}, and the required amount of shear reinforcement determined.

5.1.2 Bent-up Bars

Standard Method

To resist the shearing forces, longitudinal tension bars may be bent up near to the supports as shown in figure 5.5. The bent-up bars and the concrete in compression are considered to act as an analogous lattice girder and the shear resistance of the bars is determined by taking a section X–X through the girder.

From the geometry of part (a) of the figure, the spacing of the bent-up bars is

$$s = 0.9d(\cot\alpha + \cot\beta)$$

and at the section X–X the shear resistance of a single bent-up bar is

$$V_{wd} = f_{ywd}A_{sw}\sin\alpha = (f_{yk}/1.15)A_{sw}\sin\alpha$$

where A_{sw} is the cross-sectional area of the bent-up bar.

For a multiple system of bent-up bars, as in part (b) of the figure, the shear resistance is increased proportionally to the spacing, s. Hence

$$V_{wd} = \frac{f_{yk}}{1.15}A_{sw}\sin\alpha\, 0.9d\frac{(\cot\alpha + \cot\beta)}{s}$$

If it assumed that the diagonal compressive members in the truss act at $\beta = 45°$ then $\cot\beta = 1$ and the shear resistance of the bent-up bars can be expressed as

$$V_{wd} = \frac{1}{1.28}\frac{A_{sw}}{s}df_{yk}(1 + \cot\alpha)\sin\alpha \qquad (5.11)^*$$

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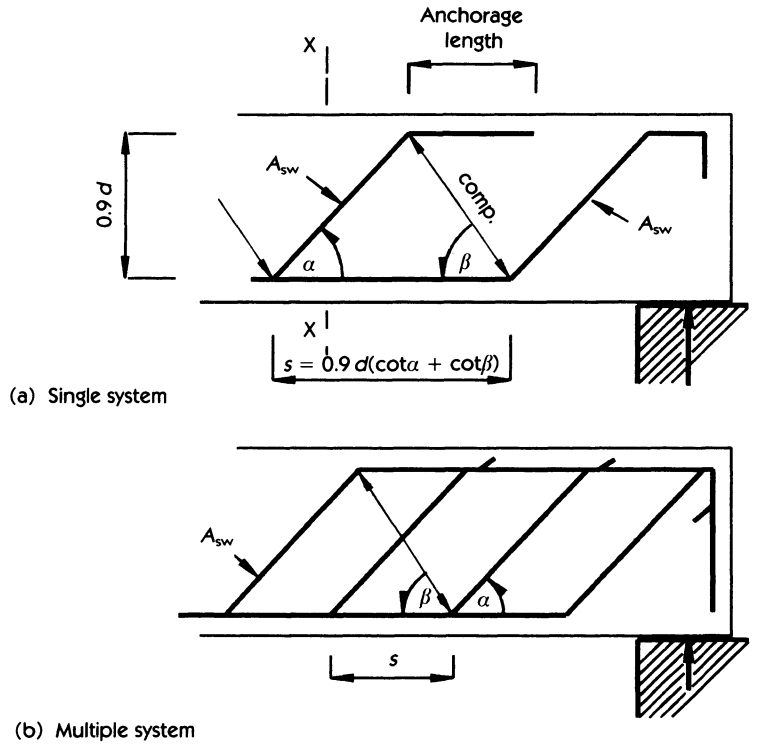

Figure 5.5 *Bent-up bars*

The code requires that the maximum longitudinal spacing of bent-up bars is limited to $0.6d(1 + \cot\alpha)$ and specifies that at least 50 per cent of the required shear reinforcement should be in the form of shear links.

When inclined shear reinforcement is used the maximum allowable shear force, V_{Rd2}, is given by the modified expression

$$V_{Rd2} = 0.3vf_{ck}b_w d(1 + \cot\alpha) \qquad (5.12)^*$$

Variable Strut Inclination Method

When the alternative method is used with inclined shear reinforcement, the previously developed equations for this alternative method can be modified in a similar way to that indicated for the Standard Method to give

$$V_{Sd} = V_{Rd3} = 0.87\frac{A_{sw}}{s}zf_{yk}(\cot\theta + \cot\alpha)\sin\alpha \qquad (5.13)$$

$$V_{Rd2} \leqslant b_w z v f_{ck}/[1.5(\cot\theta + \tan\theta)] \qquad (5.14)$$

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with $$\frac{A_{sw}f_{yk}}{b_w s} \leqslant \frac{0.38 v f_{ck} \sin \alpha}{(1 - \cos \alpha)}$$

and $$T_d = 0.5V_{Sd}(\cot \theta - \cot \alpha) = 0.87 f_{yk} A_{sl} \qquad (5.15)$$

where θ is the assumed inclination of the compressive strut and α is the inclination of the shear reinforcement.

5.1.3 Shear between the Web and Flange of a Flanged Section

The provision of shear links in a flanged beam is identical to that previously described, on the assumption that the web carries all of the shear and that the web width is used as the minimum width of the section in the relevant calculations.

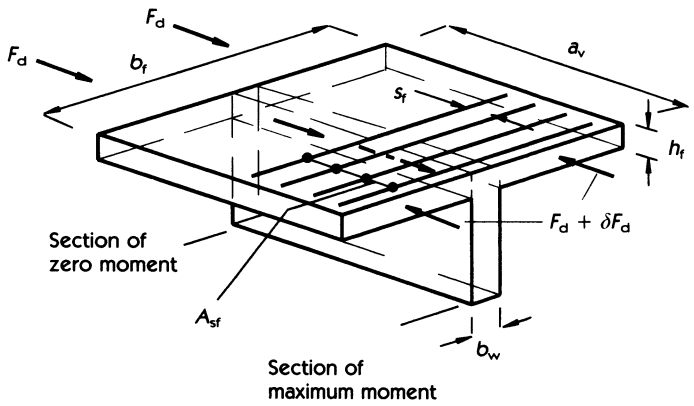

Figure 5.6 *Shear between flange and web*

Longitudinal complementary shear stress also occurs in a flanged section along the interface between the web and flange as shown in figure 5.6. This is allowed for by providing transverse reinforcement over the width of the flange on the assumption that this reinforcement acts as ties combined with compressive struts in the concrete. It is necessary to check the possibility of failure by excessive compression in the struts and to provide sufficient steel area to prevent tensile failure in the ties.

The change in longitudinal force (δF_d) over the distance between points of zero and maximum bending moment (a_v) is divided by that distance to give the shear per unit length (V_{Sd}) along the flange–web interface. The longitudinal force (F) at a section can be determined by an analysis of that section at the ultimate limit state (see section 4.6). The calculated value of V_{Sd} should then be checked such that

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$$V_{Sd} \leq 0.13 f_{ck} h_f$$

and

$$V_{Sd} \leq 2.5 \tau_{Rd} h_f + 0.87 \frac{A_{sf} f_{yk}}{s_f}$$

where the terms are defined in figure 5.6 and τ_{Rd} is determined from table 5.1. When, at the section of maximum moment, it is found that there is tension in the flange, the contribution of the concrete ($2.5\tau_{Rd}h_f$) in the above equations should be ignored (see example 7.5).

Example 5.1 Shear Resistance at a Section

Using the Standard Method determine the shear resistance of the beam shown in figure 5.7 which carries a uniformly distributed load. The characteristic strengths are: $f_{yk} = 250\,N/mm^2$ for the stirrups, $f_{yk} = 460\,N/mm^2$ for the bent-up bars and $f_{ck} = 25\,N/mm^2$ for the concrete.

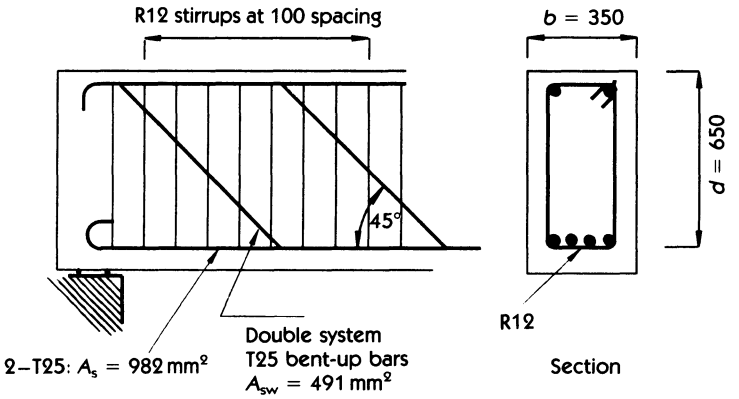

Figure 5.7 *Beam with stirrups and bent-up bars*

Cross-sectional area of a size 12 bar = $113\,mm^2$. Thus $A_{sw}/s = 2 \times 113/100 = 2.26$. The shear resistance of the stirrups is given by equation 5.1:

$$V_{wd} = \frac{1}{1.28} \frac{A_{sw}}{s} d f_{yk}$$
$$= 2.26 \times 650 \times 250 \times 10^{-3}/1.28$$
$$= 286\,kN$$

The shear resistance of the bent-up bars is given by equation 5.11:

$$V_{wd} = \frac{1}{1.28} \frac{A_{sw}}{s} d f_{yk} (1 + \cot \alpha) \sin \alpha$$

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The bent-up bars are inclined at 45° and hence the spacing, s, can be taken as $0.9d$. Hence

$$V_{wd} = 1.23A_{sw}f_{yk}$$
$$= 1.23 \times 491 \times 460 \times 10^{-3}$$
$$= 278\,\text{kN}$$

$$A_{s1}/b_w d = 982/(350 \times 650) = 0.0043$$

From table 5.1: $\tau_{Rd} = 0.30\,\text{N/mm}^2$. The shear resistance of the concrete is given by equation 5.3:

$$V_{Rd1} = [\tau_{Rd}k(1.2 + 40\rho_1)]b_w d$$

where $k = (1.6 - 0.65) = 0.95 < 1$; hence $k = 1$. So

$$V_{Rd1} = [0.30 \times 1.0 \times (1.2 + 40 \times 0.0043)] \times (350 \times 650) \times 10^{-3}$$
$$= 94\,\text{kN}$$

Hence the total shear resistance of the stirrups, bent-up bars and concrete is given by:

$$V_{sd} = 286 + 278 + 94 = 658\,\text{kN}$$

It should be noted that the shear resistance of 286 kN provided by the stirrups is greater than that provided by the bent-up bars as required by EC2. It should also be checked that at the face of the support, the ultimate shear force does not exceed the maximum allowable shear force, V_{Rd2}.

5.2 Anchorage Bond

The reinforcing bar subject to direct tension shown in figure 5.8 must be firmly anchored if it is not to be pulled out of the concrete. Bars subject

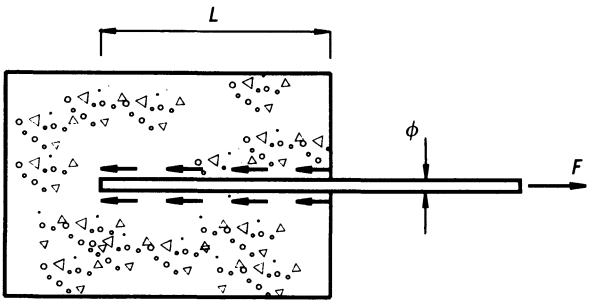

Figure 5.8 *Anchorage bond*

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to forces induced by flexure must be similarly anchored to develop their design stresses. The anchorage depends on the bond between the bar and the concrete, the area of contact and whether or not the bar is located in a region where good bond conditions can be expected. Let

l_b = *basic* anchorage length to prevent pull out
ϕ = bar size or nominal diameter
f_{bd} = ultimate anchorage bond stress
f_s = the direct tensile or compressive stress in the bar.

Considering the forces on the bar:

tensile pull-out force = bar's cross-sectional area × direct stress

$$= \frac{\pi\phi^2}{4} f_s$$

anchorage force = contact area × anchorage bond stress

$$= (l_b \pi\phi) \times f_{bd}$$

therefore

$$(l_b \pi\phi) \times f_{bd} = \frac{\pi\phi^2}{4} f_s$$

hence

$$l_b = \frac{f_s \phi}{4 f_{bd}}$$

and when $f_s = f_{yd}$, the design yield strength of the reinforcement $(= f_{yk}/1.15)$ the anchorage length is given by

$$l_b = (\phi/4)([f_{yk}/1.15]/f_{bd})$$

$$l_b = (f_{yk}/4.6 f_{bd})\phi \qquad (5.16)^*$$

This equation may be used to determine the basic anchorage length of bars which are either in tension or compression. For the calculation of anchorage lengths, design values of ultimate anchorage bond stresses are specified according to whether the bond conditions are good or otherwise. Good bond conditions are considered to be when (a) bars are inclined at an angle of between 45° and 90° to the horizontal or (b) zero to 45° provided that in this second case additional requirements are met. These additional conditions are that bars are

(1) either placed in members whose depth in the direction of concreting does not exceed 250 mm or
(2) embedded in members with a depth greater than 250 mm and are either in the lower half of the member or at least 300 mm from the top surface.

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These conditions are illustrated in figure 5.9. When bond conditions are poor then the specified ultimate bond stresses should be reduced by a factor of 0.7.

Table 5.2 gives the design values of ultimate bond stresses for 'good' conditions. These depend on the grade of concrete and the type of reinforcement and are obtained from the following equations:

$$\text{for plain bars:} \quad f_{bd} = 0.24 f_{ck}^{0.5}$$

$$\text{for high bond bars:} \quad f_{bd} = 1.50 f_{ctk}$$

where f_{ctk} is the characteristic tensile strength of the concrete.

The *basic* anchorage length discussed above must be further modified to give the *minimum* net anchorage length taking into account factors not directly covered by table 5.2. The required minimum anchorage length ($l_{b,net}$) is given by

$$l_{b,net} = \alpha_a l_b A_{s,req}/A_{s,prov} \qquad (5.17)^*$$

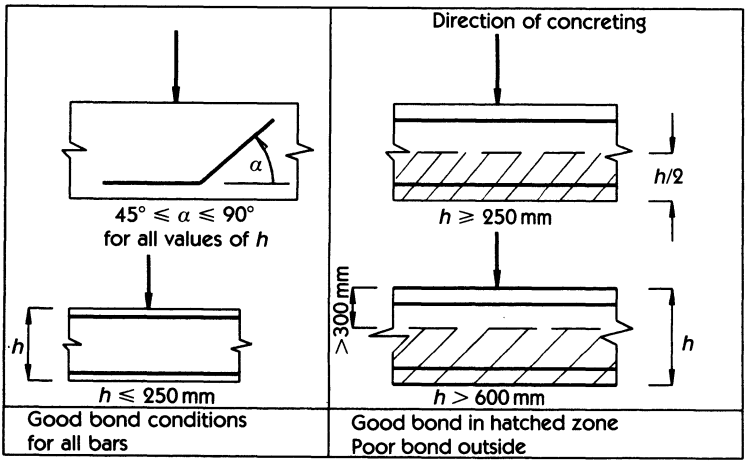

Figure 5.9 *Definition of good bond conditions*

Table 5.2 Design values of bond stresses (N/mm^2)

f_{ck}	*12*	*16*	*20*	*25*	*30*	*35*	*40*	*45*	*50*
Plain bars	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7
High bond bars ≤ 32 mm dia.	1.6	2.0	2.3	2.7	3.0	3.4	3.7	4.0	4.3

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where α_a = 1 for straight bars or 0.7 for curved bars in tension provided that there is cover of at least 3 bar diameters in the region of a hook, bend etc.

$A_{s,req}A_{s,prov}$ = Area of reinforcement required and provided at the section

This minimum value must not be less than

for tension bars: $0.3l_b$

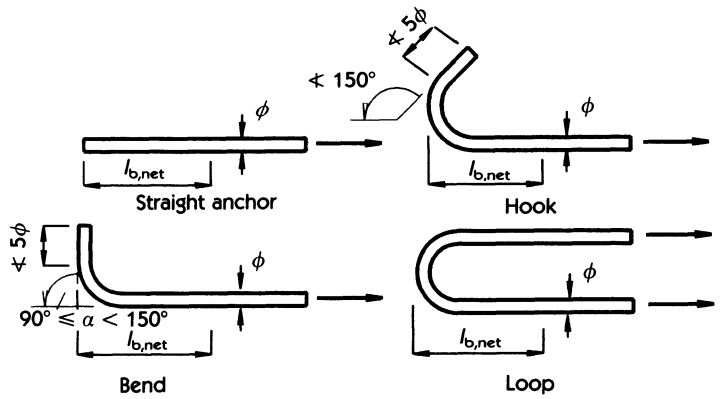
for compression bars: $0.6l_b$

In both these cases the minimum value must also exceed both 10 bar diameters and 100 mm. Anchorages may also be provided by hooks or bends in the reinforcement. Hooks and bends are considered adequate forms of anchorage to the main reinforcement if they satisfy the minimum dimensions shown in figure 5.10. Straight anchorages are not recommended for smooth bars greater than 8 mm diameter and bends and hooks are not recommended for use as compression anchorages.

Example 5.2 Calculation of Anchorage Length

Determine the basic length of tension anchorage required for the 25 mm diameter deformed high yield steel reinforcing bars in the cantilever of figure 5.11. The characteristic material strengths are f_{ck} = 25 N/mm^2 and f_{yk} = 460 N/mm^2. Assume good bond conditions.

From table 5.2 the ultimate anchorage bond stress, f_{bd} = 2.7 Nmm^2.



For plain bars, grade 250, the minimum internal radius of a hook, bend or loop = 2ϕ
For high bond bars, grade 460, the minimum internal radius = 3ϕ or 4ϕ for $\phi \geq$ 20 mm

Figure 5.10 *Anchorage of hooks and bends*

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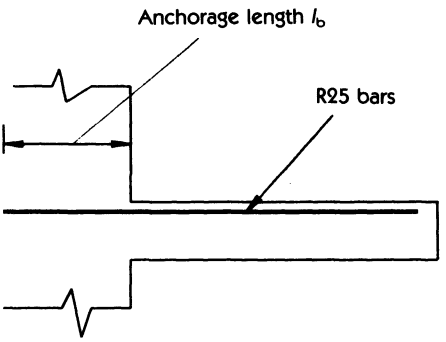

Figure 5.11 *Anchorage for a cantilever beam*

Hence

$$\begin{aligned} l_b &= (f_{yk}/4.6f_{bd})\phi \\ &= (460/(4.6 \times 2.7)) \times 25 \\ &= 926\,\text{mm} \end{aligned}$$

See also table A.5 in the Appendix for tabulated values of anchorage lengths.

5.3 Laps in Reinforcement

Lapping of reinforcement is often necessary to transfer the forces from one bar to another. Laps between bars should be staggered and should not occur in regions of high stress. The length of the lap should be based on the minimum anchorage length modified to take into account factors such as cover, bar spacing etc. (see figure 5.12). The lap length (l_s) required is given by

$$l_s = l_{b,net} \times \alpha_1 \qquad (5.18)^*$$

where α_1 = (i) 1 for compression laps;

(ii) 1 for tension laps with less than 30 per cent lapped bars and cover (b) is greater than 2 diameters *and* spacing between adjacent sets of lapped bars (a) is greater than 6 diameters;

(iii) 1.4 for tension laps with greater than 30 per cent lapped bars;

(iv) 1.4 for tension laps and cover (b) is not greater than 2 diameters *or* spacing (a) between adjacent sets of lapped bars is not greater than 6 diameters;

(v) 2.0 for tension laps where the conditions of (iii) and (iv) above are both satisfied.

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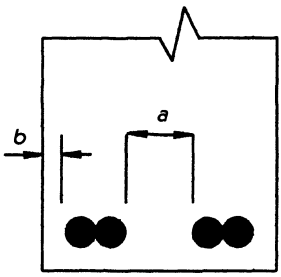

Figure 5.12 *Cover and spacing for lapped bars*

Notwithstanding the above requirements, the absolute minimum lap length is given as

$$l_{s,min} = 0.3\alpha_a\alpha_1 l_b \geqslant 15 \text{ diameters} \geqslant 200\,\text{mm} \tag{5.19}$$

where α_a is as given in the calculation of $l_{b,net}$.

Transverse reinforcement must be provided around laps other than where the lapped bars are less than 16 mm diameter or there is less than 20 per cent lapped bars. In this case minimum transverse reinforcement provided for other purposes such as shear links will be adequate. Otherwise transverse reinforcement must be provided as shown in figure 5.13, having a total area of not less than the area of one spliced bar.

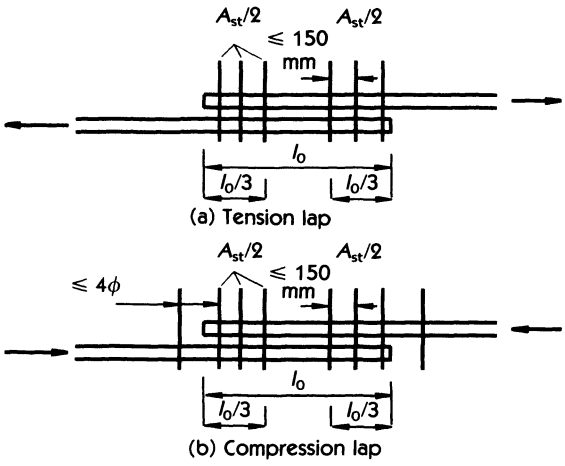

Figure 5.13 *Transverse reinforcement for lapped bars*

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5.4 Analysis of Section Subject to Torsional Moments

5.4.1 Development of Torsional Equations

Torsional moments produce shear stresses which result in principal tensile stresses inclined at approximately 45° to the longitudinal axis of the member. Diagonal cracking occurs when these tensile stresses exceed the tensile strength of the concrete. The cracks will form a spiral around the member as in figure 5.14.

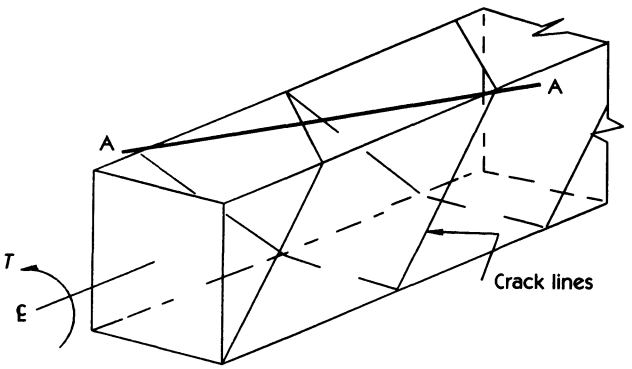

Figure 5.14 *Torsional reinforcement*

Reinforcement in the form of closed links and longitudinal bars will carry forces from increasing torsional moment after cracking, by a truss action with reinforcement acting as tension members and concrete as compressive struts between links. Failure will eventually occur by reinforcement yielding, coupled with crushing of the concrete along line A–A as cracks on the other face open up. It is assumed that once the torsional shear stress on a section exceeds the value to cause cracking, tension reinforcement in the form of closed links must be provided to resist the full torsional moment.

The equations for torsional design are developed from a structural model where it is assumed that the concrete beam in torsion behaves in a similar fashion to a thin walled box section. The box is reinforced with longitudinal bars in each corner with closed loop stirrups as transverse tension ties and the concrete providing diagonal compression struts. It is assumed that the concrete cannot provide any tensile resistance.

Consider figure 5.15a. The applied torque (T) at the far end of the section produces a *shear flow* (q) around the perimeter of the box section at the near end of the diagram. The shear flow is the product of the shear stress (τ) and the thickness of the hollow section. Hence from classical

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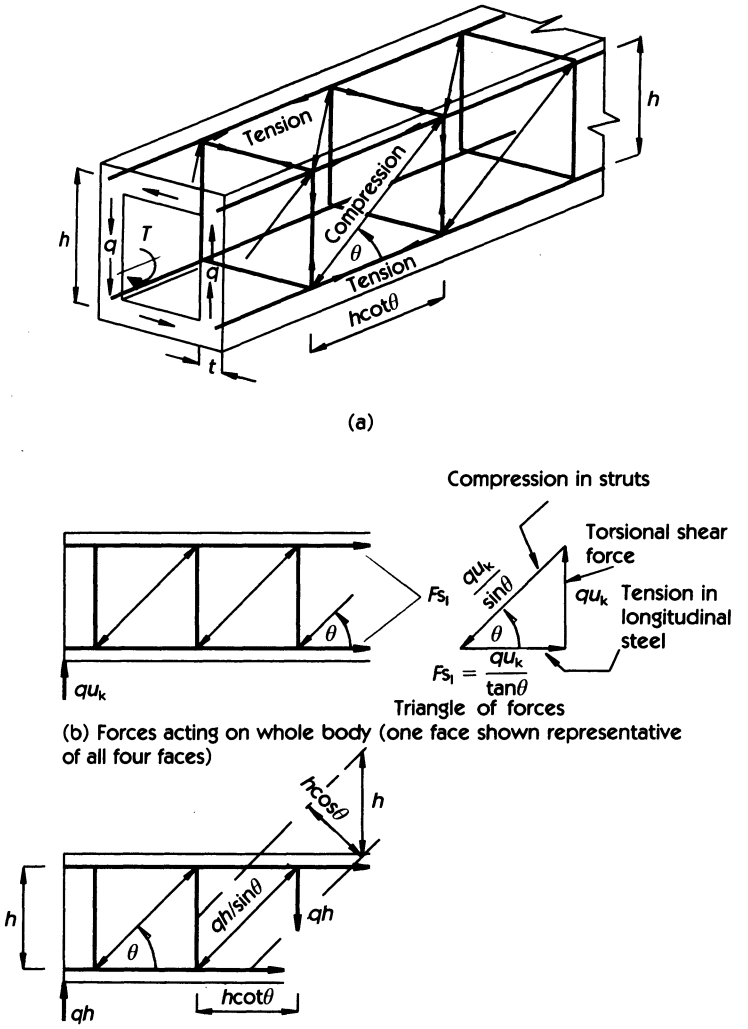

Figure 5.15 *Structural model for torsion*

elastic theory the applied torque can be related to the shear flow by the expression

$$T = 2A_k q$$

hence

$$q = T/(2A_k) \tag{5.20}$$

where A_k is the area enclosed within the centre line of the hollow box section.

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As q is the shear force per unit length of the circumference of the box section, the *force* produced by the shear flow is the product of q and the circumference (u_k) of the area A_k. Hence, if it is assumed that this force is resisted by the truss action of the concrete compressive struts acting at an angle, θ, together with tension in the longitudinal steel, from figure 5.15b the force (F_{s1}) in the longitudinal tension steel is given by

$$F_{s1} = \frac{qu_k \cos\theta}{\sin\theta} = \frac{qu_k}{\tan\theta} = \frac{Tu_k}{2A_k \tan\theta} \tag{5.21}$$

The required area of longitudinal tension steel to resist torsion (A_{s1}), acting at its design strength ($f_{ylk}/1.15$), is therefore given by

$$\frac{A_{s1}f_{y1k}}{1.15} = \frac{Tu_k}{2A_k \tan\theta} = \frac{Tu_k \cot\theta}{2A_k} \tag{5.22}$$

In the above equation the torque, T, is the maximum that can be resisted by the longitudinal reinforcement and is given the symbol T_{Rd2}. Hence

$$\frac{A_{s1}f_{y1k}}{1.15} = \frac{T_{Rd2}u_k \cot\theta}{2A_k} \tag{5.23}*$$

The required cross-sectional area of torsional links can be determined by considering one face of the box section as shown in figure 5.15c. If it is assumed that the area of one leg of a link (A_{sw}) is acting at its design yield strength ($f_{yk}/1.15$) the force in one link is given by

$$A_{sw}f_{yk}/1.15 = q \times h$$

However if the links are spaced at a distance s apart the force in each link is reduced proportionately and is given by

$$\frac{A_{sw}f_{yk}}{1.15} = q \times h \times \frac{s}{h\cot\theta} = \frac{qs}{\cot\theta}$$

$$= \frac{T_{Rd2}s}{2A_k \cot\theta} \tag{5.24}*$$

Equations 5.23 and 5.24 can be used to design a section to resist torsion and an example of their use is given in chapter 7. The calculated amount of reinforcement must be provided in addition to the full bending and shear reinforcement requirements for the ultimate load combinations corresponding to the torsional load case considered. Where longitudinal bending reinforcement is required the additional torsional steel area may either be provided by increasing the size of bars provided, or by additional bars. Torsional links must consist of fully anchored closed links spaced longitudinally no more than $u_k/8$ apart. The longitudinal steel must consist

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of at least one bar in each corner of the section with other bars distributed around the inner periphery of the links at no more than 350 mm centres.

Where the reinforcement is known equations 5.23 and 5.24 can be rearranged for analysis purposes to give T_{Rd2} and θ as follows:

$$T_{Rd2} = 2A_k\left(\frac{A_{sw}}{s}0.87f_{yk}\frac{A_{sl}}{u_k}0.87f_{ylk}\right)^{0.5} \qquad (5.25)^*$$

and

$$\tan^2\theta = \left(\frac{A_{sw}}{s}f_{yk}\right)\Big/\left(\frac{A_{sl}}{u_k}f_{ylk}\right) \qquad (5.26)^*$$

The use of all the above equations assumes that the section is replaced by an equivalent hollow box section. To determine the thickness of the section an equivalent thickness is used, defined as equal to the total area of the cross-section divided by the outer circumference of the section. In the case of an actual hollow section the cross-sectional area would include any inner hollow areas and the calculated thickness should not be taken as greater than the actual wall thickness. In no case should the thickness be taken as less than twice the cover to the longitudinal bars.

When analysing or designing a section it is also necessary to check that excessive compressive stresses do not occur in the diagonal compressive struts, leading possibly to compressive failure of the concrete. With reference to figure 5.15c and taking the limiting torsional moment for strut compressive failure as T_{Rd1}:

$$\textit{Force} \text{ in strut} = (q \times h)/\sin\theta$$

$$\textit{Area} \text{ of strut} = t \times (h\cos\theta)$$

$$\textit{Stress} \text{ in strut} = \text{Force/Area} = \frac{q}{t\sin\theta\cos\theta} \leqslant f_{ck}/1.5$$

where f_{ck} is the characteristic compressive stress in the concrete. As $q = T_{Rd1}/(2A_k)$ then the above equation can be expressed as

$$\frac{T_{Rd1}/(2A_k)}{t\sin\theta\cos\theta} \leqslant f_{ck}/1.5$$

or

$$T_{Rd1} \leqslant 1.33f_{ck}tA_k\sin\theta\cos\theta$$

which can also be expressed as

$$T_{Rd1} \leqslant 1.33f_{ck}tA_k/(\cot\theta + \tan\theta) \qquad (5.27)$$

In EC2 this equation is modified by the inclusion of an *efficiency factor* (ν) to give

$$T_{Rd1} \leqslant 1.33\nu f_{ck}tA_k/(\cot\theta + \tan\theta) \qquad (5.28)^*$$

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where the efficiency factor takes the value of $\{0.7\ (0.7 - f_{ck}/200)\}$ with a minimum value of 0.35 for sections reinforced with closed torsional links around the outer circumference of the section. If closed links are provided in both walls of the actual or equivalent box section, then ν will take the value of $(0.7 - f_{ck}/200)$ with a minimum value of 0.5.

In using the above equations to design for torsion, the designer is free to choose a value of θ which will permit a reduction in link requirements balanced by a corresponding increase in longitudinal steel, as for the *Variable Strut Inclination Method* for shear design. However there are practical limitations on the values of θ that can be used and EC2 recommends that $0.4 \leqslant \cot\theta \leqslant 2.5$ representing limiting values of θ of 68° and 22° respectively. The UK National Application Document further limits $0.67 \leqslant \cot\theta \leqslant 1.5$ corresponding to θ equal to 56° and 34° respectively.

The approach to design for torsion is therefore:

(a) Based on the calculated ultimate torsional moment (T_{Sd}), check the maximum torsional moment that can be carried by the section (T_{Rd1}) which is governed by compression in the concrete struts, as given by equation 5.28:

$$T_{Sd} < T_{Rd1} = 1.33\nu f_{ck} t A_k/(\cot\theta + \tan\theta)$$

(b) Calculate the torsional reinforcement required from equation 5.24:

$$A_{sw}/s = T_{Sd}/(2A_k 0.87 f_{yk} \cot\theta)$$

where A_{sw} is the area of *one leg* of a link.

(c) Calculate the additional longitudinal reinforcement (A_{s1}) from equation 5.23:

$$A_{s1} = (T_{Sd} u_k/2A_k)\cot\theta/(0.87 f_{y1k})$$

Further information on the practical details of design for torsion and a design example is given in chapter 7.

5.4.2 *Torsion in Complex Shapes*

A section consisting of T, L or I shape should be divided into component rectangles and each component is then designed separately to carry a proportion of the total torque (T). The torsion carried by each rectangle (T_i) can be determined elastically by calculating the torsional stiffness of each part according to its *St Venant* torsional stiffness from the expression

$$T_i = \frac{TK_i(h_{min}{}^3 h_{max})_i}{\Sigma(Kh_{min}{}^3 h_{max})} \qquad (5.29)^*$$

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where h_{min} and h_{max} are the minimum and maximum dimension of each section. K is the *St Venants torsional constant* that varies according to the ratio of h_{max}/h_{min}; typical values of which are shown in table 5.3. The subdivision of a shape into its component rectangles should be done in order to maximise the stiffness expression $\Sigma(Kh_{min}{}^3h_{max})$.

Table 5.3 St Venant's Torsional Constant K

h_{max}/h_{min}	K	h_{max}/h_{min}	K
1.0	0.14	3.0	0.26
1.2	0.17	4.0	0.28
1.5	0.20	5.0	0.29
2.0	0.23	10.0	0.31
2.5	0.25	>10	0.33

5.4.3 Torsion Combined with Bending and Shear Stresses

Torsion is seldom present alone, and in most practical cases will be combined with shear and bending stresses.

(a) Shear Stresses

Diagonal cracking will start on the side face of a member where torsion and shear stresses are additive. Figure 5.16 shows a typical ultimate torsion and ultimate shear interaction diagram from which it can be seen

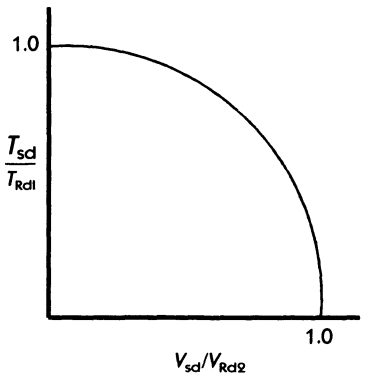

Figure 5.16 *Combined shear and torsion*

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that the beam's resistance to combined shear and torsion is less than that when subject to either effect alone. The effect of combined shear stress and torsional stress may therefore need to be considered and in this case both shear and torsion are calculated on the basis of the same equivalent thin walled section previously described for torsional design.

The recommended simplified approach to design is to ensure that the ultimate shear force (V_{Sd}) and the ultimate torsional moment (T_{Sd}) satisfy the interaction formula

$$[T_{Sd}/T_{Rd1}]^2 + [V_{Sd}/V_{Rd2}]^2 \leqslant 1 \qquad (5.30)^*$$

where

T_{Rd1} = the design torsional resistance (equation 5.28)

V_{Rd2} = the design shear resistance calculated by the *Variable Strut Inclination Method* (equation 5.8).

If this interaction equation is satisfied, the design of the shear and torsional links can be carried out separately providing that the assumed angle of the compressive struts (θ) is the same for both torsional and shear design.

However for a solid rectangular section neither shear nor torsional reinforcement is necessary if

$$T_{Sd} \leqslant V_{Sd} b_w/4.5$$

and

$$V_{Sd}[1 + (4.5T_{Sd})/(V_{Sd}b_w)] \leqslant V_{Rd1} \qquad (5.31)$$

where b_w is the minimum width of the section and all other terms are as previously defined.

It should be noted that when shear and torsion are considered together, the calculation of the shear reinforcement must be based on the *Variable Strut Inclination Method*. It should also be noted that the calculation for A_{sw} for shear (equation 5.6) gives the required cross-sectional area of both legs of a link whereas equation 5.24 for torsion gives the required cross-sectional area of a single leg of a link. This needs to be taken into consideration when determining the total link requirement as in example 7.9. Furthermore, the additional area of longitudinal reinforcement for shear design (equation 5.10) must be provided in the tension zone of the beam, whereas the additional longitudinal reinforcement for torsion (equation 5.23) must be distributed around the inner periphery of the links.

(b) Bending Stresses

When a bending moment is present, diagonal cracks will usually develop from the top of flexural cracks. The flexural cracks themselves only

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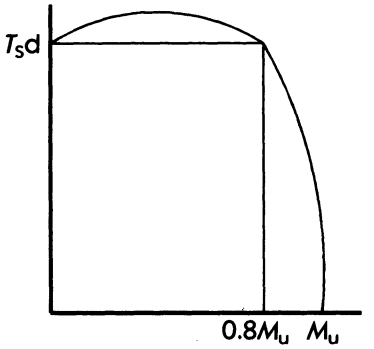

Figure 5.17 *Combined bending and torsion*

slightly reduce the torsional stiffness provided that the diagonal cracks do not develop. The final mode of failure will depend on the distribution and quantity of reinforcement present.

Figure 5.17 shows a typical ultimate moment and ultimate torsion interaction curve for a section. As can be seen, for moments up to approximately 80 per cent of the ultimate moment the section can resist the full ultimate torsional moment. Hence no calculations for torsion are generally necessary for the ultimate limit state of bending of reinforced concrete unless torsion has been included in the original analysis or is required for equilibrium.

When combined flexure and torsion is considered the longitudinal steel for both cases can be determined separately. In the flexural tension zone the longitudinal steel required for both cases can be added. However in the flexural compressive zone no additional torsional longitudinal steel is necessary if the longitudinal force due to torsion is less than the concrete compressive force due to flexure.

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6

Serviceability, Durability and Stability Requirements

The concept of serviceability limit states has been introduced in chapter 2, and for reinforced concrete structures these states are often satisfied by observing empirical rules which affect the detailing only. In some circumstances, however, it may be desired to estimate the behaviour of a member under working conditions, and mathematical methods of estimating deformations and cracking must be used. The design of prestressed concrete is based primarily on the avoidance or limitation of cracking and this is considered separately in chapter 11.

Where the foundations of a structure are in contact with the ground, the pressures developed will influence the amount of settlement that is likely to occur. To ensure that these movements are limited to acceptable values and are similar throughout a structure, the sizes of the foundations necessary are based on the service loads for the structure.

Consideration of durability is necessary to ensure that a structure remains serviceable throughout its lifetime. This requirement will involve aspects of design, such as concrete mix selection and determination of cover to reinforcing bars, as well as selection of suitable materials for the exposure conditions which are expected. Good construction procedures including adequate curing are also essential if reinforced concrete is to be durable.

Simplified rules governing selection of cover, member dimensions and reinforcement detailing are given in sections 6.1 and 6.2, while more rigorous procedures for calculation of actual deflection and crack widths are described in sections 6.3 to 6.5. Durability and fire resistance are discussed in section 6.6.

The stability of a structure under accidental loading, although an ultimate limit state analysis, will usually take the form of a check to ensure that empirical rules, designed to give a minimum reasonable

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resistance against misuse or accident are satisfied. Like serviceability checks, this will often involve detailing of reinforcement and not affect the total quantity provided. Stability requirements are discussed in section 6.7.

6.1 Detailing Requirements

These are to ensure that a structure has satisfactory durability and serviceability performance under normal circumstances. EC2 and the Concise Code recommend simple rules concerning the concrete mix and cover to reinforcement, minimum member dimensions, and limits to reinforcement quantities, spacings and bar diameters which must be taken into account at the member sizing and reinforcement detailing stage. Reinforcement detailing may also be affected by stability considerations as described in section 6.7, as well as rules concerning anchorage and lapping of bars which have been discussed in sections 5.2 and 5.3.

6.1.1 Minimum Concrete Mix and Cover (Exposure Conditions)

These requirements are interrelated and although not fully detailed in EC2, the Concise Code and ENV 206 *Concrete – Performance, Production, Placing and Compliance Criteria: 1992* give much more detailed guidance on minimum combinations of thickness of cover and mix characteristics for various classes of exposure. The mixes are expressed in terms of minimum cement content, maximum free water/cement ratio and corresponding lowest concrete strength class. These requirements are given in tables 6.1 and 6.2. Table 6.1 gives the nine classifications of exposure which then define the cover requirements etc. which must be complied with.

The nominal cover is that to all steel and includes an allowance for fixing tolerances. The actual cover should never be greater than the nominal cover less 5 mm. The nominal cover should not be less than the nominal maximum size of the aggregate nor less than the bar size or the size of a bar of equivalent area where pairs or bundled bars have been used. In the case of slabs in exposure classes 2 to 5 the cover indicated may be reduced by 5 mm.

6.1.2 Minimum Member Dimensions and Cover (Fire Resistance)

In order that a reinforced concrete member is capable of withstanding fire for a specified period of time, it is necessary to ensure the provision of

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Table 6.1 Exposure classes

Exposure class		*Examples of environmental conditions*
1. Dry environment		Interior of habitable buildings and offices
2. Humid environment	(2a) without frost	• Interior of buildings with high humidity • Exterior components • Components in non-aggressive soil and/or water
	(2b) with frost	• Exterior components exposed to frost • Components in non-aggressive soil and/or water and exposed to frost • Interior components; high humidity with frost
3. Humid environment with frost and de-icing salts		• Interior and exterior components exposed to frost and salts
4. Seawater environment	(4a) without frost	• Components completely/partially submerged in seawater or in splash zone • Components in saturated salt air
	(4b) with frost	• Components completely/partially submerged in seawater or in splash zone; exposed to frost • Components in saturated salt air and exposed to frost
5. Aggressive	(5a)	• Slightly aggressive chemical environment • Aggressive industrial atmosphere
	(5b)	• Moderately aggressive chemical environment
	(5c)	• Highly aggressive chemical environment

Note: Classes (5a) to (5c) may occur alone or in combination with other classes.

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Table 6.2 Cover to reinforcement

Exposure class	*Nominal cover (mm)*[i]				
1	20	20	20	20	20
2a	—	35	35	30	30
2b	—	—	35	30	30
3	—	—	40	35	35
4a	—	—	40	35	35
4b	—	—	40	35	35
5a	—	—	35	30	30
5b	—	—	—	35	35
5c	—	—	—	—	45
Max. free water/ cement ratio	0.65	0.60	0.55	0.50	0.45
Min. cement content (kg/m^3)	260	280	300[ii]	300	300
Lowest concrete strength class	C25/30	C30/37	C35/45	C40/50	C45/55

(i) Cover to prestressing steel will normally be 5 mm greater than the values indicated.
(ii) For exposure classes 2b and 5a the minimum cement content may be taken as 280 kg/m^3.

minimum dimensions and nominal cover for various types of concrete member. The detailed requirements of EC2 have not yet been drafted and until they are it is permissible to follow the recommendations of BS8110 which are summarised in tables 6.3 and 6.4.

6.1.3 Maximum Spacing of Reinforcement

Cracking of a concrete member can result from the effect of loading or can arise because of restraint to shrinkage or thermal movement. In addition to providing a minimum area of bonded reinforcement (see section 6.1.5), cracking due to loading is minimised by ensuring that the maximum clear spacings between longitudinal reinforcing bars in beams is limited to that given in table 6.5 for high bond bars. This will ensure that the maximum crack widths in the concrete do not exceed 0.3 mm. It can be seen that the spacing depends on the stress in the reinforcement which should be taken as the stress under the action of the *quasi-permanent* loadings. The *quasi-permanent* loading is taken as the permanent load,

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Table 6.3 Nominal cover for fire resistance

Fire resistance (hours)	*Nominal cover to all reinforcement (mm)*						
	Beams		*Floors*		*Ribs*		*Columns*
	s.s.	*cont.*	*s.s.*	*cont.*	*s.s.*	*cont.*	
0.5	20	20	20	20	20	20	20
1.0	20	20	20	20	20	20	20
1.5	20	20	25	20	35	20	20
2.0	40	30	35	25	45*	35	25
3.0	60*	40	45*	35	55*	45*	25
4.0	70*	50*	55*	45*	65*	55*	25

* Additional measures necessary to reduce risk of spalling.

Table 6.4 Minimum dimensions of RC members for fire resistance (nominal cover requirements satisfied)

Fire resistance (hours)	*Minimum dimensions (mm)*					
	Beam width	*Rib width*	*Floor thickness*	*Exposed column width*	*Wall thicknesses*	
					$\frac{A_s}{A_c} < 0.4\%$	$> 1.0\%$
0.5	200	125	75	150	150	75
1.0	200	125	95	200	150	75
1.5	200	125	110	250	175	100
2.0	200	125	125	300		100
3.0	240	150	150	400		150
4.0	280	175	170	500		180

G_k, plus a proportion of the variable load, Q_k, depending on the type of structure. The calculation of this stress level (f_s) can be complicated and an acceptable approximation is to take f_s as

$$f_s = \frac{f_{yk}}{1.15} \times \frac{G_k + 0.3Q_k}{(1.35G_k + 1.5Q_k)} \frac{1}{\delta} \tag{6.1}$$

where f_{yk} is the characteristic strength of the reinforcement. δ will have a value of 1.0 unless moment redistribution has been carried out, in which

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Table 6.5 Maximum clear bar spacings (mm) for high bond bars in tension caused predominantly by loading

Steel stress (N/mm^2)	*Maximum bar spacing (mm)*	
	Pure flexure	*Pure tension*
160	300	200
200	250	150
240	200	125
280	150	75
320	100	—
360	50	—

case δ is the ratio of the distributed moment to the undistributed moment at the section at the ultimate limit.

These spacing rules do not apply to slabs of overall thickness less than 200 mm. In this case the spacing of both longitudinal and secondary reinforcement should be no greater than three times the overall slab depth or 500 mm, whichever is the lesser.

6.1.4 Minimum Spacing of Reinforcement

To permit concrete flow around reinforcement during construction, the clear distance between bars should not be less than (i) the maximum bar size, (ii) 20 mm, or (iii) the maximum aggregate size plus 5 mm (if the aggregate size exceeds 32 mm), whichever is the greater figure.

6.1.5 Minimum Areas of Reinforcement

For most purposes, thermal and shrinkage cracking can be controlled within acceptable limits by the use of minimum reinforcement quantities. The principal requirements, to be checked at the detailing stage, are as specified in table 6.6. Requirements for shear links are given in sections 7.3 and 9.3.

In addition to the requirements of table 6.6 a minimum steel area, $A_{s,min}$, must be provided in all cases to control cracking. The provision of the minimum steel area ensures that the reinforcement does not yield when the concrete in the tension zone cracks with a sudden transfer of stress to the reinforcement. This could cause the uncontrolled develop-

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Table 6.6 Minimum areas of reinforcement

Tension reinforcement in beams:	$A_{s,min} > 0.6b_t d/f_{yk} > 0.0015b_t d$
(for grade 250 steel	$A_{s,min}/b_t d > 0.0024$
for grade 460 steel	$A_{s,min}/b_t d > 0.0015$)
Tension reinforcement in slabs:	$A_{s,min} > 0.6b_t d/f_{yk} > 0.0015b_t d$
(for grade 250 steel	$A_{s,min}/b_t d > 0.0024$
for grade 460 steel	$A_{s,min}/b_t d > 0.0015$)
Secondary reinforcement > 20 per cent of main reinforcement	
Longitudinal reinforcement in columns:	$A_{s,min} > 0.15N_{Sd}/0.87f_{yk} > 0.003A_c$ where N_{Sd} is the axial compression force
Vertical reinforcement in walls:	$A_{s,min} > 0.004A_c$

Note: b_t is the mean width of the tension zone.

ment of a few wide cracks. Provided that this minimum area is provided, then yield should not occur and cracking will then be distributed throughout the section with a greater number of cracks but of lesser width. $A_{s,min}$ is given by the expression

$$A_{s,min} = k_c k f_{ct.ef} A_{ct}/f_{yk} \tag{6.2}$$

where $A_{s,min}$ = minimum area of reinforcement that must be provided within the tensile zone

A_{ct} = area of concrete within tensile zone defined as that area which is in tension just before the formation of the first crack

$f_{ct.ef}$ = tensile strength of concrete at time of cracking with a suggested minimum of $3\,N/mm^2$ – otherwise obtained from table 6.9 using a concrete strength class appropriate to the anticipated time of cracking

k_c = stress distribution coefficient (1.0 for pure tension, 0.4 for flexure)

k = non-linear stress distribution coefficient – for stresses induced by loading or restraint to shrinkage or early thermal cracking then k should be taken as 0.8 but for rectangular sections greater than 800 mm deep a value of 0.5 may be used; for stresses induced by external effects such as settlement then k should be taken as 1.0.

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6.1.6 *Maximum Areas of Reinforcement*

These are determined largely from the practical need to achieve adequate compaction of the concrete around the reinforcement. The limits specified are as follows:

(a) For a slab or beam, tension or compression reinforcement

$$100A_s/A_c \leqslant 4 \text{ per cent other than at laps}$$

(b) For a column

$$100A_s/A_c \leqslant 8 \text{ per cent including at laps}$$

(c) For a wall, vertical reinforcement

$$100\ A_s/A_c \leqslant 4 \text{ per cent}$$

6.1.7 *Maximum Bar Sizes*

Section 6.1.3 described the limitations on bar spacing to ensure that crack widths due to loading are kept within acceptable limits. When considering load-induced cracking bar diameters may be restricted as indicated in table 6.7 as an alternative to limiting spacing. In calculating the steel stress, the approximation given in equation 6.1 may be used.

When cracking is as a result of restraint to shrinkage or thermal effects then the bar sizes *must* be limited as indicated in table 6.7, but the maximum spacing limits of table 6.5 do not need to be applied. The steel stress to be used in table 6.7 can be calculated from the expression

$$f_s = f_{yk}A_{s,min}/A_{s,prov} \tag{6.3}$$

Table 6.7 Maximum bar diameters

Steel stress (N/mm^2)	*Maximum bar size (mm)*
160	32
200	25
240	20
280	16
320	12
360	10
400	8
450	6

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where $A_{s,prov}$ is the steel area provided at the section under consideration and $A_{s,min}$ is given in equation 6.2.

6.1.8 Side Face and Surface Reinforcement in Beams

In beams over 1 m deep additional reinforcement must be provided in the side faces to control cracking as indicated in figure 6.1a. This reinforcement should be distributed evenly between the main tension steel and the neutral axis and *within* the stirrups. The minimum area of this reinforcement can be calculated from equation 6.2 with k taken as 0.5. In assessing the maximum spacing of this reinforcement from table 6.5 a stress value equal to one-half of that calculated for the main tensile reinforcement may be used and it may be assumed that the side face reinforcement is in pure tension.

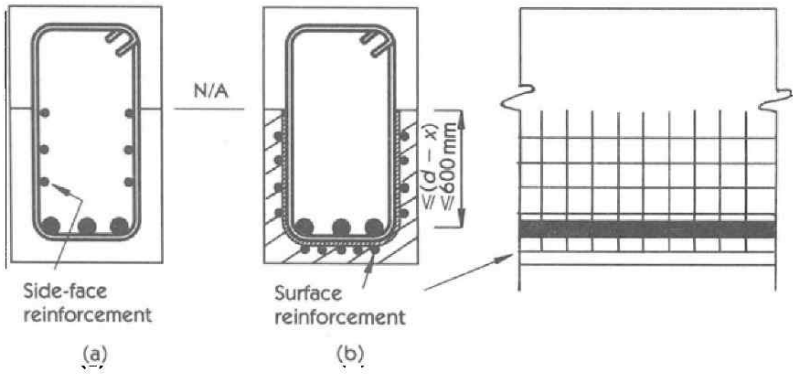

Figure 6.1 *Side-face and surface reinforcement*

In addition to the above requirement, surface reinforcement must be provided where it is necessary to control spalling of the concrete due to fire or where bundled bars or bars greater than 32 mm diameter are used as main reinforcement.

The surface reinforcement should consist of welded mesh or small diameter high bond bars located *outside* the links as indicated in figure 6.1b. Cover to this reinforcement must comply with the requirements of section 6.1.1 and the minimum area of longitudinal surface reinforcement should be 1 per cent of the area of the concrete outside the links and in the tension zone below the neutral axis; shown as the shaded area in figure 6.1. The surface reinforcement bars should be spaced no further than 150 mm apart and if properly anchored can be taken into account as longitudinal bending and shear reinforcement.

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6.2 Span–Effective Depth Ratios

The appearance and function of a reinforced concrete beam or slab may be impaired if the deflection under serviceability loading is excessive. Deflections can be calculated as indicated in section 6.3 but it is more usual to control deflections by placing a limit on the ratio of the span to the effective depth of the beam or slab. EC2 specifies a set of basic span–effective depth ratios, to control deflections to a maximum of span/250, which are given in table 6.8 for rectangular sections and for grade 460 steel. The ratios can also be used for flanged sections except that where the ratio of the width of flange to the width of web exceeds 3 then the basic values should be multiplied by 0.8. For two-way spanning

Table 6.8 Basic span–effective depth ratios (f_{yk} = 460)

	Basic span–effective depth ratio		
Structural system ($100A_{s,prov}/bd$)	*Concrete Highly stressed (>1.5)*	*Concrete Lightly stressed (0.5)*	*Concrete Nominally reinforced (<0.15)*
1. Simply supported beam or one/two-way spanning simply supported slab	15.6	21.7	29.6
2. End span of continuous beam or one-way continuous slab or two-way slab continuous over one long side	20.0	27.8	38.3
3. Interior span of continuous beam or one-way or two-way spanning slab	21.7	30.4	41.7
4. Slab on columns without beams (flat slab) based on longer span	18.3	26.1	35.7
5. Cantilever	6.1	8.7	12.2

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slabs, the check for the basic span–effective depth ratio should be based on the shorter span whereas for flat slabs calculations should be based on the longer span.

The three columns given in table 6.8 correspond to three levels of concrete stress under serviceability conditions: highly stressed when the steel ratio ρ exceeds 1.5 per cent, lightly stressed when ρ equals 0.5 per cent and nominally reinforced for ρ less than or equal to 0.15 per cent. ρ is given by $100A_{s,prov}/bd$ where $A_{s,prov}$ is the area of tension reinforcement actually provided in the section. Interpolation between the values of ρ indicated is permissible. In the case of slabs it is reasonable to assume that they are lightly stressed.

The basic ratios given in table 6.8 are modified in particular cases according to:

(a) For spans longer than 7 m (except flat slabs) and where it is necessary to limit deflections to ensure that finishes are not damaged, the basic values should be multiplied by 7/span.
(b) For flat slabs with spans in excess of 8.5 m, multiply the basic ratios by 8.5/span.
(c) For characteristic steel strengths other than 460 N/mm^2, multiply the basic ratios by $460/f_{yk}$.
(d) Where more tension reinforcement is provided ($A_{s,prov}$) than that calculated ($A_{s,req}$) at the ultimate limit state, multiply the basic ratios by $A_{s,prov}/A_{s,req}$. The span–effective depth ratio should not however exceed that indicated for 'nominally reinforced' concrete.

Example 6.1 Span-Effective Depth Ratio

A rectangular continuous beam spans 10 m. If the breadth is 300 mm, check the acceptability of an effective depth of 600 mm when high yield reinforcement, $f_{yk} = 460\,N/mm^2$, is used. At the ultimate limit state it is determined that 1250 mm^2 of tension steel is needed and 3 No. 25 mm diameter reinforcing bars ($A_{s,prov} = 1470\,mm^2$) are actually provided.

$$\rho = 100A_{s,prov}/bd = (100 \times 1470)/(300 \times 600) = 0.8 \text{ per cent.}$$

Therefore basic span–effective depth ratio (table 6.8, interpolating between $\rho = 21.7$ and $\rho = 30.4$) = 27.8.
To avoid damage to finishes for span greater than 7 m:

$$\text{Modified ratio} = 27.8 \times \frac{7}{10} = 19.5$$

Modification for steel area provided:

$$\text{Modified ratio} = 19.5 \times \frac{1470}{1250} = 22.9$$

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$$\text{Span–effective depth ratio provided} = \frac{10 \times 10^3}{600} = 16.7$$

which is less than the allowable upper limit, thus deflection requirements are likely to be satisfied.

6.3 Calculation of Deflection

The general requirement is that neither the efficiency nor the appearance of a structure is harmed by the deflections that will occur during its life. Deflections must thus be considered at various stages. The limitations necessary to satisfy the requirements will vary considerably according to the nature of the structure and its loadings, but for reinforced concrete the following may be considered as reasonable guides:

(1) the final deflection of a beam, slab or cantilever should not exceed span/250
(2) that part of the deflection which takes place after the application of finishes or fixing of partitions should not exceed span/500 to avoid damage to fixtures and fittings.

The Code suggests that deflections should be calculated under the action of the *quasi-permanent* load combination, assuming this loading to be of long-term duration. Hence the total loading to be taken in the calculation will be the permanent load plus a proportion of the variable load which will typically be 30 per cent of the variable load for office type construction. This is a reasonable assumption as deflection will be affected by long-term effects such as concrete creep, while not all of the variable load is likely to be long term and hence will not contribute to the creep effects.

Lateral deflection must not be ignored, especally on tall slender structures, and limitations in these cases must be judged by the engineer. It is important to realise that there are many factors which may have significant effects on deflections, and are difficult to allow for, thus any calculated values must be regarded as an estimate only. The most important of these factors are:

(1) support restraint must be estimated on the basis of simplified assumptions, which will have varying degrees of accuracy
(2) the precise loading cannot be predicted and errors in permanent loading may have a significant effect
(3) a cracked member will behave differently from one that is uncracked – this may be a problem in lightly reinforced members where the working load may be near to the cracking limit
(4) the effects of floor screeds, finishes and partitions are very difficult

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to assess – frequently these are neglected despite their 'stiffening' effect.

It may be possible to allow for these factors by averaging maximum and minimum estimated effects and, provided that this is done, there are a number of calculation methods available which will give reasonable results. The method adopted by EC2 is based on the calculation of curvature of sections subjected to the appropriate moments with allowance for creep and shrinkage effects where necessary. Deflections are then calculated from these curvatures. A rigorous approach to deflection is to calculate the curvature at intervals along the span and then use numerical integration techniques to estimate the critical deflections, taking into account the fact that some sections along the span will be cracked under load and others, in regions of lesser moment, will be uncracked. Such an approach is rarely justified and the approach adopted below, based on EC2, assumes that it is acceptably accurate to calculate the curvature of the beam or slab based on both the cracked and uncracked sections and then to use an 'average' value in estimating the final deflection using standard deflection formulae or simple numerical integration based on elastic theory.

The procedure for estimating deflections involves the following stages which are illustrated in example 6.2.

6.3.1 Calculation of Curvature

Curvature under the action of the quasi-permanent load should be calculated based on both the *cracked* and *uncracked* section. An estimate of an 'average' value of curvature can then be obtained using the formula:

$$1/r = \zeta(1/r)_{cr} + (1 - \zeta)(1/r)_{uc} \tag{6.4}$$

where

$1/r$ = average curvature

$(1/r)_{uc}$, $(1/r)_{cr}$ = values of curvature calculated for the uncracked case and cracked case respectively

ζ = coefficient given by $1 - \beta_1\beta_2(\sigma_{sr}/\sigma_s)^2$

β_1 = bond coefficient (1 for high bond, 0.5 for plain bars)

β_2 = load duration factor (1 for a single short-term load; 0.5 for sustained loads or cyclic loading)

σ_s = stress in the tension steel for the cracked concrete section

σ_{sr} = stress in the tension steel calculated on the basis of a cracked section *under the loading that will just cause cracking at the section being considered.*

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Table 6.9 Mean tensile strengths of concrete and secant modulus of elasticity

	Strength class						
	20/25	*C25/30*	*C30/37*	*C35/45*	*C40/50*	*C45/55*	*C50/60*
f_{ctm} (N/mm^2)	2.2	2.6	2.9	3.2	3.5	3.8	4.1
E_{cm} (kN/mm^2)	29	30.5	32	33.5	35	36	37

Appropriate values of concrete tensile strength to be used in the calculation of σ_{sr} can be obtained from table 6.9. In calculating ζ, the ratio (σ_{sr}/σ_s) can more conveniently be replaced by (M_{cr}/M) where M_{cr} is the moment that will just cause cracking of the section and M is the design moment for the calculation of curvature and deflection.

In order to calculate the 'average' curvature, separate calculations have to be carried out for both the cracked and the uncracked case:

Uncracked Section

The assumed elastic strain and stress distribution for an uncracked section is shown in figure 6.2.

For a given moment, M, and from elastic bending theory, the curvature of the section, $(1/r)_{uc}$, is given by

$$(1/r)_{uc} = \frac{M}{E_{c.ef} I_{uc}} \tag{6.5}$$

where $E_{c.ef}$ is the effective elastic modulus of the concrete allowing for creep effects and I_{uc} is the second moment of area of the uncracked concrete section.

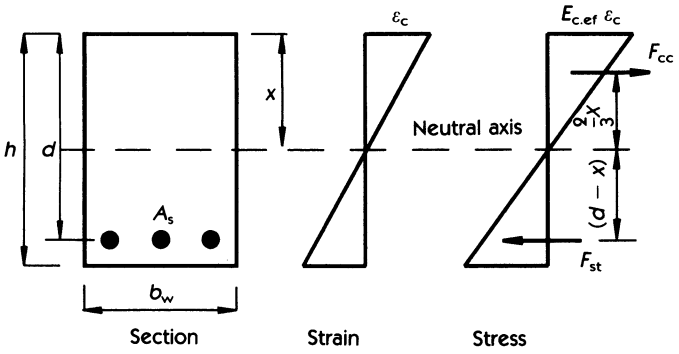

Figure 6.2 *Uncracked section – strain and stress distribution*

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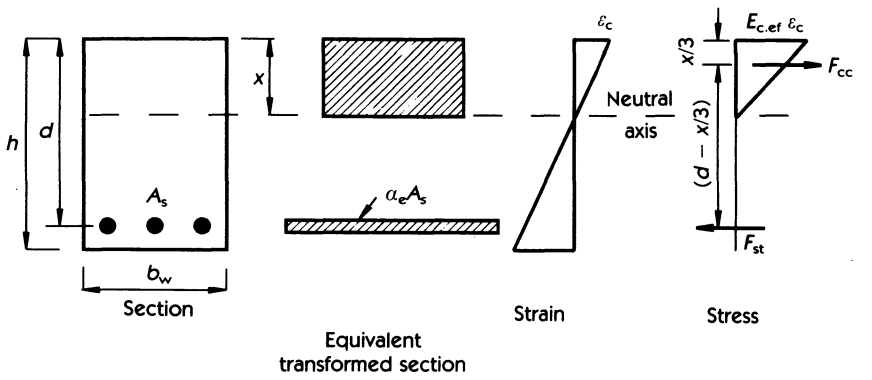

Figure 6.3 *Cracked section – strain and stress distribution*

Cracked Section

The assumed elastic strain and stress distribution for a cracked section is shown in figure 6.3. This is identical to that shown in figures 4.27 and 4.28 of chapter 4 and equation 4.48 or figure 4.29 can be used to determine the neutral-axis depth. Alternatively, moments of area can be taken to establish the neutral-axis depth directly. The second moment of area of the cracked section can then be determined by taking second moments of area about the neutral axis

$$I_{cr} = bx^3/3 + \alpha_e A_s(d - x)^2 \tag{6.6}$$

where α_e is the modular ratio equal to the ratio of the elastic modulus of the reinforcement to that of the concrete.

For a given moment, M, and from elastic bending theory, the curvature of the cracked section, $(1/r)_{cr}$, is therefore given by

$$(1/r)_{cr} = \frac{M}{E_{c.ef} I_{cr}} \tag{6.7}$$

6.3.2 *Creep and Shrinkage Effects*

Creep

The effect of creep will be to increase deflections with time and this should be allowed for in the calculations by using an effective modulus, $E_{c,ef}$, using the equation

$$E_{c.ef} = E_{cm}/(1 + \phi) \tag{6.8}$$

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where ϕ is a creep coefficient equal to the ratio of creep strain to initial elastic strain. The value of ϕ, while being affected by aggregate properties, mix design and curing conditions, is also governed by age at first loading, the duration of load and the section dimensions. Table 6.10 gives long-term values of ϕ as suggested by EC2. The 'notional size' of the section is taken as twice the cross-sectional area divided by the perimeter of the area. An estimate of the elastic modulus of concrete, E_{cm}, can be obtained from table 6.9 or from the expression

$$E_{cm} = 9.5(f_{ck} + 8)^{1/3}\,\text{kN/mm}^2$$

Table 6.10 Final creep coefficient of normal weight concrete

Age at loading (days)	*Notional size ($2A_c/u$) mm*					
	50	*150*	*600*	*50*	*150*	*600*
	Dry atmosphere (inside: 50%RH)			*Humid atmosphere (outside: 80%RH)*		
1	5.4	4.4	3.6	3.5	3.0	2.6
7	3.9	3.2	2.5	2.5	2.1	1.9
28	3.2	2.5	2.0	1.9	1.7	1.5
90	2.6	2.1	1.6	1.6	1.4	1.2
365	2.0	1.6	1.2	1.2	1.0	1.0

Note: In the above table, A_c = cross-sectional area of concrete, u = perimeter of that area.

Shrinkage

The effect of shrinkage of the concrete will be to increase the curvature and hence the deflection of the beam or slab. The curvature due to shrinkage can be calculated using the equation

$$1/r_{cs} = \varepsilon_{cs}\alpha_e S/I \qquad (6.9)$$

where $1/r_{cs}$ = the shrinkage curvature
ε_{cs} = free shrinkage strain
S = first moment of area of reinforcement about the centroid of the section
I = second moment of area of section (cracked or uncracked as appropriate)
α_e = effective modular ratio ($E_s/E_{c.ef}$).

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Shrinkage is influenced by many features of the mix and construction procedures but for most normal weight concretes values of ε_{cs} may be obtained from table 6.11. The 'average' shrinkage curvature can be calculated from equation 6.4 having calculated the curvature based on both the 'cracked' and 'uncracked' sections.

Table 6.11 Final shrinkage strains of normal weight concrete (10^{-6})

Location of member	*Relative humidity (%)*	*Notional size* $(2A_c/u)$ *mm*	
		<150	*600*
Inside	50	600	500
Outside	80	330	280

Note: In the above table, A_c = cross-sectional area of concrete, u = perimeter of that area.

6.3.3 Calculation of Deflection from Curvature

The total curvature can be determined by adding the shrinkage curvature to the calculated curvature due to the quasi-permanent loads, having made allowance for creep effects.

The deflection of the beam or slab can be calculated from the total curvature using elastic bending theory which for small deflections is based on the expression

$$M_x = EI\frac{d^2y}{dx^2} \tag{6.10}$$

where M_x is the bending moment at a section distance x from the origin as shown in figure 6.4. For small deflections the term d^2y/dx^2 approximately equals the curvature which is the reciprocal of the radius of curvature. Double integration of equation 6.10 will yield an expression for deflection. This may be illustrated by considering the case of a pin-ended beam subjected to constant moment M throughout its length, so that $M_x = M$.

$$EI\frac{d^2y}{dx^2} = M \tag{6.11}$$

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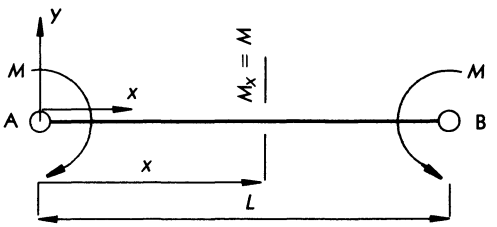

Figure 6.4 *Pin-ended beam subject to a constant moment M*

therefore

$$EI\frac{dy}{dx} = Mx + C$$

but if the slope is zero at mid-span where $x = L/2$, then

$$C = -\frac{ML}{2}$$

and

$$EI\frac{dy}{dx} = Mx - \frac{ML}{2}$$

Integrating again gives

$$EIy = \frac{Mx^2}{2} - \frac{MLx}{2} + D$$

but at support A when $x = 0$, $y = 0$. Hence

$$D = 0$$

thus

$$y = \frac{M}{EI}\left(\frac{x^2}{2} - \frac{Lx}{2}\right) \quad \text{at any section} \tag{6.12}$$

The maximum deflection in this case will occur at mid-span, where $x = L/2$, in which case

$$y_{max} = -\frac{M}{EI}\frac{L^2}{8} \tag{6.13}$$

but since at any uncracked section

$$\frac{M}{EI} = \frac{1}{r}$$

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the maximum deflection may be expressed as

$$y_{max} = -\frac{1}{8}L^2\frac{1}{r}$$

In general, the bending-moment distribution along a member will not be constant, but will be a function of x. The basic form of the result will however be the same, and the deflection may be expressed as

$$\text{maximum deflection } a = KL^2\frac{1}{r_b} \qquad (6.14)^*$$

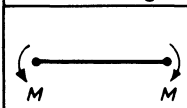
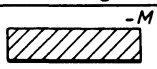
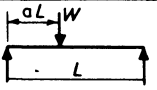
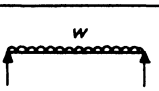
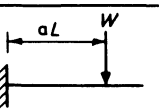
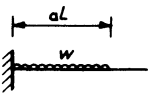
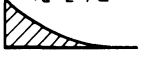
where K = a constant, the value of which depends on the distribution of bending moments in the member

L = the effective span

$\frac{1}{r_b}$ = the mid-span curvature for beams, or the support curvature for cantilevers

Typical values of K are given in table 6.12 for various common shapes of bending-moment diagrams. If the loading is complex, then a value of K must be estimated for the complete load since summing deflections of simpler components will yield incorrect results.

Table 6.12 Typical deflection coefficients

Loading	B.M. Diagram	K
		0·125
		$\frac{4a^2-8a+1}{48a}$ [if a = 1/2 then K = 0·083]
		0·104
		End Deflection $\frac{a(3-a)}{6}$ [if a = 1 then K = 0·33]
		End Deflection $\frac{a(4-a)}{12}$ [if a = 1 then K = 0·25]

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Although the derivation has been on the basis of an uncracked section, the final expression is in a form that will deal with a cracked section simply by the substitution of the appropriate curvature.

Since the expression involves the square of the span, it is important that the true effective span as defined in chapter 7 is used, particularly in the case of cantilevers. Deflections of cantilevers may also be increased by rotation of the supporting member, and this must be taken into account when the supporting structure is fairly flexible.

Example 6.2 Calculation of Deflection

Estimate the long-term deflection of the beam shown in figure 6.5. It spans 9.5 metres and is designed to carry a uniformly distributed load giving rise to a quasi-permanent moment of 200 kN m. It is constructed with grade C25/30 concrete, is made of normal concrete and the construction props are removed at 28 days.

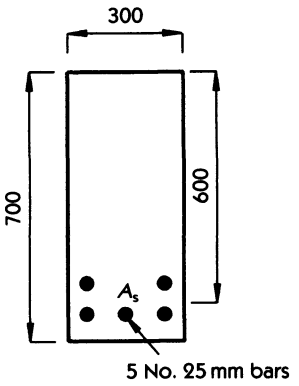

Figure 6.5

(a) Calculate curvature due to uncracked section

From equation 6.5:

$$(1/r)_{uc} = \frac{M}{E_{c.ef} I_{uc}}$$

where from table 6.9, $E_{cm} = 30.5\,\text{kN/mm}^2$. From table 6.10, assuming loading at 28 days with indoor exposure, the creep coefficient $\phi \approx 2.5$ as $(2A_c/u = (2 \times [700 \times 300])/2000 = 210)$ and hence the effective modulus is given by

$$E_{c.ef} = 30.5/(1 + 2.5) = 8.71\,\text{kN/mm}^2$$

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Hence $$(1/r)_{uc} = \frac{200 \times 10^6}{8.71 \times 10^3 \times (300 \times 700^3/12)}$$
$$= 2.68 \times 10^{-6}/\text{mm}$$

Note that in the above calculation I_{uc} has been calculated on the basis of the gross concrete sectional area ignoring the contribution of the reinforcement. A more accurate calculation could have been performed, as in example 4.12 in chapter 4, but such accuracy is not justified and the simpler approach indicated will be sufficiently accurate.

(b) Calculate curvature due to cracked section

To calculate the curvature of the cracked section the I value of the transformed concrete section must be calculated. With reference to figure 6.3 the calculations can be set out as below.

(i) Calculate the neutral axis position:

Taking area moments about the neutral axis:

$$b \times x \times x/2 = \alpha_e A_s(d - x)$$

$$300 \times x^2/2 = \frac{200}{8.71} \times 2450(600 - x)$$

which has the solution

$$x = 322.7\,\text{mm}$$

(ii) Calculate the second moment of area of the cracked section:

$$I_{cr} = bx^3/3 + \alpha_e A_s(d - x)^2$$
$$= \frac{300 \times 322.7^3}{3} + \frac{200}{8.71}2450(600 - 322.7)^2$$
$$= 7686 \times 10^6\,\text{mm}^4$$

(iii) Calculate the curvature of the cracked section:

From equation 6.7

$$(1/r)_{cr} = \frac{M}{E_{c.ef} I_{cr}}$$
$$= \frac{200 \times 10^6}{8.71 \times 10^3 \times 7686 \times 10^6}$$
$$= 2.99 \times 10^{-6}/\text{mm}$$

(c) Calculate the 'average' of the cracked and uncracked curvature

From equation 6.4:

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$$1/r = \zeta(1/r)_{cr} + (1 - \zeta)(1/r)_{uc}$$

where $\zeta = 1 - \beta_1\beta_2(\sigma_{sr}/\sigma_s)^2$
$= 1 - \beta_1\beta_2(M_{cr}/M)^2$

(i) Calculate M_{cr}:

From table 6.9 the cracking strength of the concrete, f_{ctm}, is given as $2.6\,N/mm^2$. Hence from elastic bending theory and considering the uncracked concrete section, the moment that will just cause cracking of the section, M_{cr}, is given by

$$M_{cr} = f_{ctm} \times (b_w h^2/6)$$
$$= 2.6 \times (300 \times 700^2/6) \times 10^{-6}$$
$$= 63.7\,kN\,m$$

(ii) Calculate ζ:

$$\zeta = 1 - \beta_1\beta_2(M_{cr}/M)^2$$
$$= 1 - 1 \times 0.5 \times (63.7/200)^2$$
$$= 0.95$$

(iii) Calculate the 'average' curvature:

$$1/r = \zeta(1/r)_{cr} + (1 - \zeta)(1/r)_{uc}$$
$$= 0.95 \times 2.99 \times 10^{-6} + (1 - 0.95) \times 2.68 \times 10^{-6}$$
$$= 2.97 \times 10^{-6}/mm$$

(d) Calculate Shrinkage Curvature

(i) For the cracked section:

$$1/r_{cs} = \varepsilon_{cs}\alpha_e S/I_{cr}$$

where

$$S = A_s(d - x)$$
$$= 2450(600 - 322.7)$$
$$= 679.3 \times 10^3\,mm^3$$

and from table 6.11, $\varepsilon_{cs} \approx 590 \times 10^{-6}$ as $(2A_c/u = 210$, as in part (a)). Therefore

$$1/r_{cs} = \frac{590 \times 10^{-6}(200/8.71)679.3 \times 10^3}{7686 \times 10^6}$$
$$= 1.20 \times 10^{-6}/mm$$

(ii) For the uncracked section:

$$1/r_{cs} = \varepsilon_{cs}\alpha_e S/I_{uc}$$

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where

$$S = A_s(d - x)$$
$$= 2450(600 - 700/2)$$
$$= 612.5 \times 10^3\,\text{mm}^3$$

Therefore

$$1/r_{cs} = \frac{590 \times 10^{-6}(200/8.71)612.50 \times 10^3}{300 \times 700^3/12}$$
$$= 0.97 \times 10^{-6}/\text{mm}$$

(iii) Calculate the 'average' shrinkage curvature:

$$1/r_{cs} = \zeta(1/r)_{cr} + (1 - \zeta)(1/r)_{uc}$$
$$= 0.95 \times 1.2 \times 10^{-6} + (1 - 0.95) \times 0.97 \times 10^{-6}$$
$$= 1.19 \times 10^{-6}/\text{mm}$$

(e) Calculate Deflection

Curvature due to loading $= 2.97 \times 10^{-6}$/mm
Curvature due to shrinkage $= 1.19 \times 10^{-6}$/mm
Therefore total curvature $= 4.16 \times 10^{-6}$/mm

For a simply supported span subjected to a uniformly distributed load, the maximum mid-span deflection is given by

$$\text{Deflection} = 0.104L^2(1/r)$$
$$= 0.104 \times 9500^2 \times 4.16 \times 10^{-6}$$
$$= 39.0\,\text{mm}$$

This value marginally exceeds the allowable value of span/250 (9500/250 = 38 mm) but, because of the inherent uncertainty of some of the parameters used in the calculations, would probably be considered to be just acceptable.

6.3.4 Basis of Span–Effective Depth Ratios

The calculation of deflections has been shown to be a tedious operation, however, for general use rules based on limiting the span–effective depth ratio of a member are adequate to ensure that the deflections are not excessive. The application of this method is described in section 6.2.

The relationship between the deflection and the span–effective depth ratio of a member can be derived from equation 6.14; thus

$$\text{deflection } a = K\frac{1}{r_b}L^2$$

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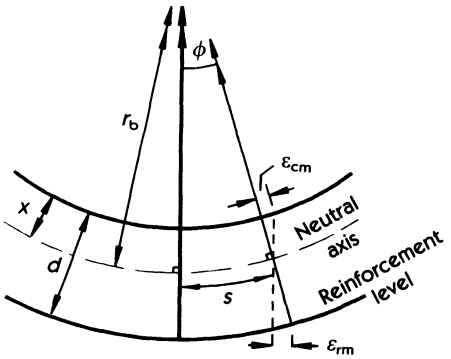

Figure 6.6 *Curvature and strain distribution*

and for small deflections it can be seen from figure 6.6 that for unit length, s

$$\phi = \frac{1}{r_b} = \frac{\varepsilon_{cm} + \varepsilon_{rm}}{d}$$

where ε_{cm} = maximum compressive strain in the concrete
ε_{rm} = tensile strain in the reinforcement
K = a factor which depends on the pattern of loading.

Therefore

$$\frac{\text{span}}{\text{effective depth}} = \frac{L}{d} = \frac{a}{L}\frac{1}{K}\frac{1}{(\varepsilon_{cm} + \varepsilon_{rm})}$$

The strains in the concrete and tensile reinforcement depend on the areas of reinforcement provided and their stresses. Thus for a particular member section and a pattern of loading, it is possible to determine a span–effective depth ratio to satisfy a particular a/L or deflection/span limitation.

The span–effective depth ratios obtained in section 6.2 are based on limiting the total deflection to span/250 for a uniformly distributed loading and are presented for different stress and strain levels depending on whether the concrete is highly or lightly stressed or just nominally reinforced. This in turn depends on the percentage of tension reinforcement in the section. For spans of less than 7 m this should also ensure that the limits of span/500 after application of finishes are met but, for spans over 7 m where avoidance of damage to finishes may be important, the basic ratios of table 6.8 should be factored by 7/span.

For loading patterns other than uniformly distributed a revised ratio is

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given by changing the basic ratio in proportion to the relative values of K, as shown in example 6.3. Similarly, for limiting the deflection to span/β

$$\text{revised ratio} = \text{basic ratio} \times \frac{250}{\beta}$$

In cases where the basic ratio has been modified for spans greater than 7 m, maximum deflections are unlikely to exceed span/500 after construction of partitions and finishes.

When another deflection limit is required, the ratios given should be multiplied by 500α/span where α is the proposed maximum deflection.

Example 6.3 Adjustment of Basic Span to Effective Depth Ratio

Determine the appropriate basic ratio for a cantilever beam supporting a uniform load and a concentrated point load at its tip as shown in figure 6.7. Assume that the concrete is highly stressed.

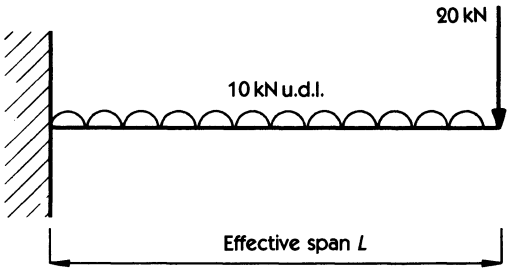

Figure 6.7 *Point load on a cantilever example*

Basic ratio from table 6.8 = 6.1 for a u.d.l.

From table 6.12:

K for cantilever with u.d.l. over full length = 0.25
K for cantilever with point load at tip = 0.33

Thus, for the point load only, adjusted basic ratio equals

$$6.1 \times \frac{0.25}{0.33} = 4.6$$

An adjusted basic ratio to account for both loads can be obtained by factoring the moment due to the point load by the ratio of the K values as follows

$$M_{\text{udl}} = 10 \times L/2 = 5L$$

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$$M_{point} = 20L$$

$$\text{Adjusted basic ratio} = \text{Basic ratio}\left(\frac{M_{udl} + M_{point} \times K_{udl}/K_{point}}{M_{udl} + M_{point}}\right)$$

$$= 6.1\left(\frac{5 + 20 \times 0.25/0.33}{5 + 20}\right)$$

$$= 4.9$$

6.4 Flexural Cracking

Members subject to bending generally exhibit a series of distributed flexural cracks, even at working loads. These cracks are unobtrusive and harmless unless the width becomes excessive, in which case appearance and durability suffer as the reinforcement is exposed to corrosion.

The actual widths of cracks in a reinforced concrete structure will vary between wide limits and cannot be precisely estimated, thus the limiting requirement to be satisfied is that the probability of the maximum width exceeding a satisfactory value is small. For exposure classes 2 to 4 (see table 6.1) the maximum acceptable value suggested by EC2 is 0.3 mm under the action of the *quasi-permanent* combination of loads, although under exposure class 1 this requirement could be relaxed. Other codes of practice may recommend lower values of crack widths for important members and requirements for special cases, such as water-retaining structures, may be even more stringent.

Flexural cracking is generally controlled by providing a minimum area of tension reinforcement (section 6.1.5) and limiting bar spacings (section 6.1.3) or limiting bar sizes (section 6.1.7). If calculations to estimate maximum crack widths are performed, they are based on the *quasi-permanent* combination of loads and an effective modulus of elasticity of the concrete should be used to allow for creep effects.

6.4.1 Mechanism of Flexural Cracking

This can be illustrated by considering the behaviour of a member subject to a uniform moment.

A length of beam as shown in figure 6.8 will initially behave elastically throughout, as the applied moment M is increased. When the limiting tensile strain for the concrete is reached, a crack will form and the adjacent tensile zone will no longer be acted on by direct tension forces. The curvature of the beam, however, causes further direct tension stresses to develop at some distance from the original crack to maintain equilibrium. This in turn causes further cracks to form, and the process continues until

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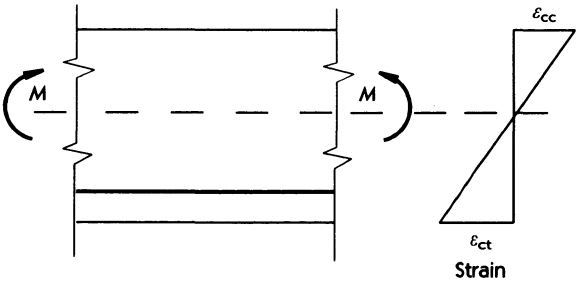

Figure 6.8 *Bending of a length of beam*

the distance between cracks does not permit sufficient tensile stresses to develop to cause further cracking. These initial cracks are called 'primary cracks', and the average spacing in a region of constant moment is largely independent of reinforcement detailing.

As the applied moment is increased beyond this point, the development of cracks is governed to a large extent by the reinforcement. Tensile stresses in the concrete surrounding reinforcing bars are caused by bond as the strain in the reinforcement increases. These stresses increase with distance from the primary cracks and may eventually cause further cracks to form approximately mid-way between the primary cracks. This action may continue with increasing moment until the bond between concrete and steel is incapable of developing sufficient tension in the concrete to cause further cracking in the length between existing cracks. Since the development of the tensile stresses is caused directly by the presence of reinforcing bars, the spacing of cracks will be influenced by the spacing of the reinforcement.

If bars are sufficiently close for their 'zones of influence' to overlap then secondary cracks will join up across the member, while otherwise they will form only adjacent to individual bars. According to EC2 (see section 6.4.2) the average crack spacing in a flexural member depends in part on the efficiency of bond, the diameter of reinforcing bar used and the quantity and location of the reinforcement in relation to the tension face of the section.

6.4.2 Estimation of Crack Widths

If the behaviour of the member in figure 6.9 is examined, it can be seen that the overall extension per unit length at a depth y below the neutral axis is given by

$$\varepsilon_1 = \frac{y}{(d - x)}\varepsilon_s$$

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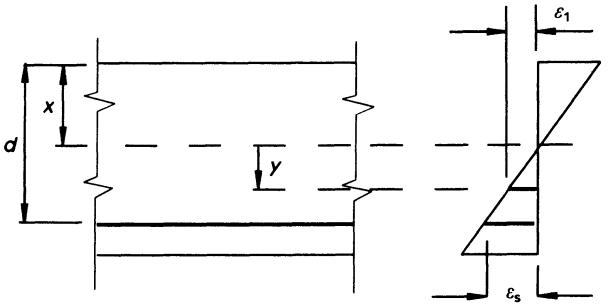

Figure 6.9 *Bending strains*

where ε_s is the average strain in the main reinforcement over the length considered, and may be assumed to be equal to σ_s/E_s where σ_s is the steel stress at the cracked sections. ε_1 is the strain at level y which by definition is the extension over the unit length of the member. Hence, assuming any tensile strain of concrete between cracks as small, since full bond is never developed, the total width of all cracks over this unit length will equate to the extension per unit length, that is

$$\varepsilon_1 = \frac{y}{(d-x)}\frac{\sigma_s}{E_s} = \Sigma w$$

where Σw = the sum of all crack widths at level y.

The actual width of individual cracks will depend on the number of cracks in this unit length, the average being given by unit length/average spacing (s_{rm}). Thus

$$\text{average crack width } w_{av} = \frac{\Sigma w}{\text{av. number of cracks}}$$

$$= \frac{\varepsilon_1}{(1/s_{rm})} = s_{rm}\varepsilon_1$$

The designer is concerned however with the maximum crack width which has an acceptably low probability of being exceeded. For design purposes the design maximum crack width, w_k, can be taken as β times the average crack width, where β is an experimentally determined coefficient. Hence the design crack width at any level defined by y in a member will thus be given by

$$w_k = \beta s_{rm}\varepsilon_1$$

The expression for the design crack width given in EC2 is of the above form and is given as

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$$w_k = \beta s_{rm} \varepsilon_{sm} \qquad (6.15)^*$$

where

w_k = the design crack width
s_{rm} = the average crack spacing
ε_{sm} = the *mean strain* allowing for the effects of tension stiffening of the concrete, shrinkage etc.
β = the coefficient relating average to design crack widths, taken as 1.7 for load-induced cracking.

The mean strain, ε_{sm} will be less than the apparent value ε_1 and is given by the expression

$$\varepsilon_{sm} = \sigma_s / E_s [1 - \beta_1 \beta_2 \{\sigma_{sr}/\sigma_s\}^2] \qquad (6.16)^*$$

where σ_s is the stress in the tension steel calculated using the cracked concrete section and σ_{sr} is the stress in the tension steel calculated on the basis of the cracked section *under the loading that will just cause cracking at the section being considered*. β_1 is a coefficient that accounts for the bond properties of the reinforcement (1 for high bond, 0.5 for plain bars) and β_2 is a factor that accounts for the nature and duration of loading (1 for single short-term load, 0.5 for sustained loads or cyclic loads).

The average crack spacing, s_{rm}, is given by the empirical formula

$$s_{rm} = 50 + 0.25 k_1 k_2 \phi / \rho_r \qquad (6.17)^*$$

where ϕ is the bar size in mm or an average bar size where a mixture of different sizes have been used. k_1 is a coefficient accounting for the bond properties of the reinforcement (0.8 for high bond, 1.6 for plain bars) and k_2 is a coefficient accounting for the nature of the strain distribution which for cracking due to flexure can be taken as 0.5. ρ_r is the effective reinforcement ratio, $A_s/A_{c.ef}$, where A_s is the area of reinforcement within an effective tension area of concrete $A_{c.ef}$, as shown in figure 6.10.

The effective tension area is that area of the concrete cross-section which will crack due to the tension developed in bending. This is the cracking which will be controlled by the presence of an appropriate type, amount and distribution of reinforcement. Generally the effective tension area should be taken as having a depth equal to 2.5 times the distance from the tension face of the concrete to the centroid of the reinforcement, although for slabs the depth of this effective area should be limited to $(h - x)/3$.

Although not directly incorporated into the above formulae, it should be noted that crack widths may vary across the width of the soffit of a beam and are generally likely to be greater at positions mid-way between longitudinal reinforcing bars and at the corners of the beam.

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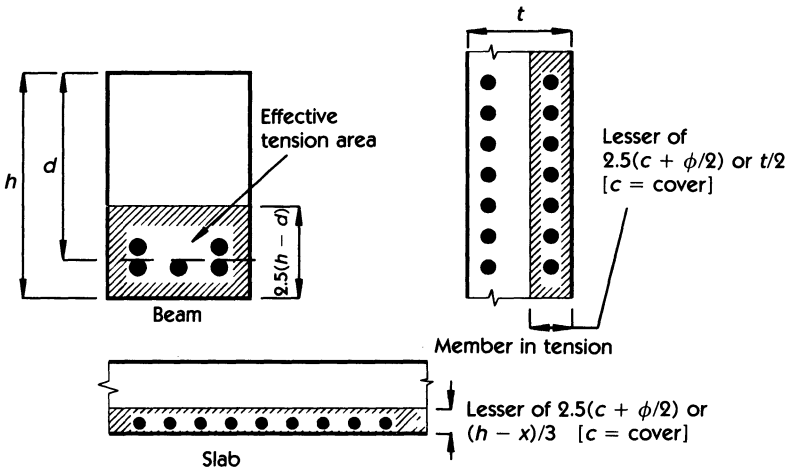

Figure 6.10 *Typical examples of effective concrete tension area*

6.4.3 Analysis of Section to Determine Crack Widths

To use the formula of EC2 it is necessary to carry out an elastic analysis of the cracked concrete section using an effective modulus $E_{c.ef}$, as given in equation 6.8 to allow for creep effects.

The methods discussed in section 4.10.1 should be used to find the neutral axis position, x, and hence the stresses, σ_s and σ_{sr}, in the tensile reinforcement from which ε_{sm} (equation 6.16) can be obtained.

Example 6.4 Calculation of Flexural Crack Widths

Calculate the design flexural crack widths for the beam shown in figure 6.11 when subject to a quasi-permanent moment of 650 kN m. The concrete is grade C25/30 and the reinforcement is high bond with a total cross-sectional area of 3770 mm^2.

(a) Calculate the mean strain ε_{sm}

From table 6.9, E_{cm} = 30.5 kN/mm^2. From table 6.10, assuming loading at 28 days with indoor exposure, the creep coefficient $\phi \approx 2.33$ as ($2A_c/u$ = 2 × [1000 × 400]/2800 = 285) and hence the effective modulus is given by equation 6.8 as

$$E_{c,ef} = 30.5/(1+ 2.33) = 9.16\,\text{kN/mm}^2$$

(i) Calculate the neutral axis depth of the cracked section:

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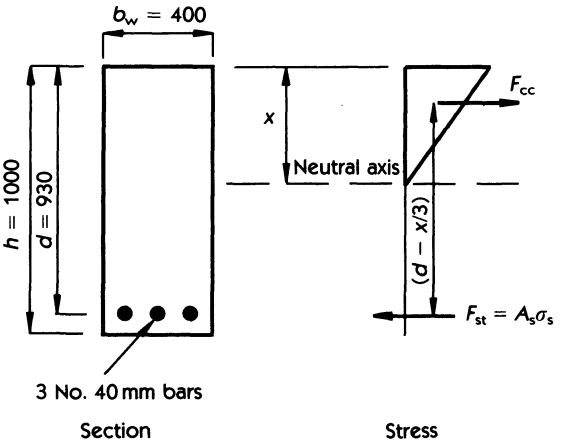

Figure 6.11

Taking moments about the neutral axis:

$$b \times x \times x/2 = \alpha_e A_s(d - x)$$

$$400 \times x^2/2 = \frac{200}{9.16} \times 3770(930 - x)$$

which has the solution $x = 446\,\text{mm}$.

(ii) Calculate the stress in the tension steel, σ_s, by taking moments about the level of the compressive force in the concrete

$$\sigma_s = M/(d - x/3)A_s$$

$$= \frac{650 \times 10^6}{(930 - 446/3)3770}$$

$$= 221\,\text{N/mm}^2$$

(iii) Calculate the stress in the tension reinforcement, σ_{sr}, due to the moment that just causes cracking

From table 6.9, the cracking strength of the concrete, f_{ctm}, is given as $2.6\,\text{N/mm}^2$. Hence from elastic bending theory and considering the uncracked concrete section, the moment that will just cause cracking at the section, M_{cr}, is given by

$$M_{cr} = f_{ctm} \times (b_w h^2/6)$$

$$= 2.6 \times (400 \times 1000^2/6) \times 10^{-6}$$

$$= 173.3\,\text{kN m}$$

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Therefore the stress in the reinforcement, based on the cracked section and the cracking moment, is given by taking moments about the level of the compressive force in the concrete

$$\sigma_{sr} = M/(d - x/3)A_s$$
$$= \frac{173.3 \times 10^6}{(930 - 446/3)3770}$$
$$= 58.83\,\text{N/mm}^2$$

Hence
$$\varepsilon_{sm} = \sigma_s/E_s[1 - \beta_1\beta_2\{\sigma_{sr}/\sigma_s\}^2]$$
$$= \frac{221}{200 \times 10^3}[1 - 1 \times 0.5\{58.83/221\}^2]$$
$$= 1.06 \times 10^{-3}$$

(b) Calculate the average crack spacing

$$s_{rm} = 50 + 0.25k_1k_2\phi/\rho_r$$
$$= 50 + 0.25 \times 0.8 \times 0.5 \times 40/\rho_r$$

where
$$\rho_r = A_s/A_{c.ef} = 3770/(2.5\{1000 - 930\}400)$$
$$= 0.0539$$

Hence
$$s_{rm} = 50 + 0.25 \times 0.8 \times 0.5 \times 40/0.0539$$
$$= 124\,\text{mm}$$

(c) Calculate the design crack width

$$w_k = \beta s_{rm}\varepsilon_{sm}$$
$$= 1.7 \times 124 \times 1.06 \times 10^{-3}$$
$$= 0.22\,\text{mm}$$

This value is within the recommended limit of 0.3 mm.

6.4.4 Control of Crack Widths

It is apparent from the expressions derived above that there are four fundamental ways in which surface crack widths may be reduced:

(i) reduce the stress in the reinforcement (σ_s) which will hence reduce ε_{sm}
(ii) reduce the bar diameters (ϕ) which will reduce bar spacing and have the effect of reducing the crack spacing (s_{rm})
(iii) increase the effective reinforcement ratio (ρ_r)
(iv) use high bond rather than plain bars (β_1 and k_1).

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The use of steel at reduced stresses is generally uneconomical and, although this approach is used in the design of water-retaining structures where cracking must often be avoided altogether, it is generally easier to limit the bar diameters, increase ρ_r or use high bond bars in preference to plain bars.

To increase ρ_r, the effective tension area $A_{c.ef}$ should be made as small as possible. This is best achieved by placing the reinforcement close to the tension face such that the depth of the tension area $\{2.5(h - d)\}$ is made as small as possible recognising, nevertheless, that durability requirements limit the minimum value of cover.

The calculation of the design crack widths indicated above only apply to regions within the effective tension zone. Since cracking can also occur in the side face of a beam it is also good practice to consider the provision of longitudinal steel in the side faces of beams. The critical position for the width of such cracks is likely to be approximately mid-way between the main tension steel and the neutral axis. Recommendations regarding this and requirements for the main reinforcement are given in EC2 and are discussed in section 6.1. If these recommendations are followed, it is not necessary to calculate crack widths except in unusual circumstances. Reinforcement detailing, however, has been shown to have a large effect on flexural cracking, and must in practice be a compromise between the requirements of cracking, durability and constructional ease and costs.

6.5 Thermal and Shrinkage Cracking

Thermal and shrinkage effects, and the stresses developed prior to cracking of the concrete, were discussed in chapter 1. The rules for providing minimum areas of reinforcement and limiting bar sizes to control thermal and shrinkage cracking were discussed in sections 6.1.5, 6.1.7 and 6.1.8. In this section, further consideration will be given to the control of such cracking and the calculations that can be performed, if necessary, to calculate design crack widths.

Consider the concrete section of figure 6.12 which is in a state of stress owing to thermal contraction and concrete shrinkage and the effects of external restraint. After cracking, the equilibrium of concrete adjacent to a crack is as illustrated.

Equating tension and compression forces:

$$A_s f_{st} = A_{ct} f_{ct} - A_{sc} f_{sc}$$

If the condition is considered when steel and concrete simultaneously reach their limiting values in tension, that is $f_{st} = f_{yk}$ and $f_{ct} = f_{ct.ef}$ = the tensile strength of concrete at the time when cracking is expected to develop (usually taken as three days), then

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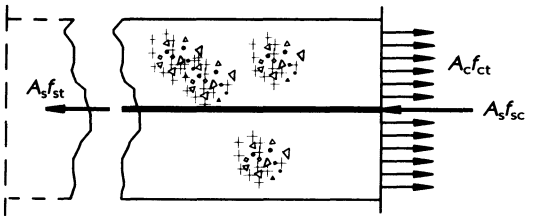

Figure 6.12 *Forces adjacent to a crack*

$$A_s f_{yk} = A_{ct} f_{ct.ef} - A_{sc} f_{sc}$$

The value of f_{sc} can be calculated but is generally very small and may be taken as zero without introducing undue inaccuracy. Hence the critical value of steel area is

$$A_{s,min} = A_{ct} f_{ct.ef} / f_{yk}$$

If the steel area is less than this amount then the steel will yield in tension, resulting in a few wide cracks; however, if it is greater, then more cracks will be formed but of narrower width. In EC2 this formula is modified by the inclusion of a stress distribution coefficient (k_c) taken as 1.0 for pure tension and a further coefficient (k) that accounts for non-linear stress distribution within the section. For thermal and shrinkage effects, k can be taken as 0.8. Hence the recommended minimum steel area required to control thermal and shrinkage cracking is given by

$$\begin{aligned} A_{s,min} &= k_c k A_{ct} f_{ct.ef} / f_{yk} \\ &= 0.8 A_{ct} f_{ct.ef} / f_{yk} \end{aligned} \qquad (6.18)^*$$

For rectangular sections greater than 800 mm deep, k can be reduced to 0.5.

6.5.1 Crack Width Calculation

The expression for the design crack width given in EC2, and discussed in section 6.4.2 for the case of flexural cracking, can be used for the calculation of thermal and shrinkage cracking with some minor modifications. The equation for crack width is given in equation 6.15 by

$$w_k = \beta s_{rm} \varepsilon_{sm}$$

where w_k is the design crack width, s_{rm} is the average crack spacing and ε_{sm} is the mean strain in the section. In the case of thermal and shrinkage cracking, the coefficient β should be taken as 1.7 for sections with a minimum dimension of less than 800 mm or 1.3 for sections with a

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minimum dimension less than 300 mm, with intermediate interpolation between these limits permitted.

For steel areas greater than the minimum required value as given by equation 6.18, and when the total contraction exceeds the ultimate tensile strain for the concrete, the shrinkage and thermal movement will be accommodated by controlled cracking of the concrete. If it is assumed that any tensile strain in the concrete between cracks is small and can be neglected, the mean strain in the section is the sum of the free shrinkage and thermal strains. That is

$$\varepsilon_{sm} = (\varepsilon_{cs} + T\alpha_c) \qquad (6.19)^*$$

where ε_{cs} is the shrinkage strain, T is the fall in temperature from the hydration peak and α_c is the coefficient of thermal expansion of concrete – taken as half the value for mature concrete to allow for creep effects.

The average crack spacing is given by an empirical expression almost identical to equation 6.17

$$s_{rm} = 50 + 0.25(k_1 \times k)k_2\phi/\rho_r$$

where the various terms have been defined previously in section 6.4.2, and k and k_2 take the values of 0.8 and 1.0 respectively. Hence

$$s_{rm} = 50 + 0.2k_1\phi/\rho_r \qquad (6.20)^*$$

In practice, variations in restraint conditions cause large variations within members, and between otherwise similar members, with 'full' restraint seldom occurring. Cracking behaviour depends considerably on the degree and nature of the restraint and temperatures at the time of casting. Calculations of crack widths should therefore be considered as realistic 'estimates' only and engineering judgement may need to be applied in interpreting such results.

Example 6.5 Calculation of Shrinkage and Thermal Crack Widths

A fully restrained section of reinforced concrete wall is 150 mm thick, and drying shrinkage strain of 50 microstrain (ε_{cs}) is anticipated together with a temperature drop (T) of 20 °C after setting. Determine the minimum reinforcement to control cracking and calculate the design crack width and average spacing for a suitable reinforcement arrangement. The following design parameters should be used:

Three-day tensile strength of concrete ($f_{ct.ef}$) = 1.5 N/mm^2
Effective modulus of elasticity of concrete ($E_{c,ef}$) = 10 kN/mm^2
Coefficient of thermal expansion for mature concrete (α_c)
= 12 microstrain/°C
Characteristic yield strength of reinforcement (f_{yk}) = 460 N/mm^2
Modulus of elasticity of reinforcement = 200 kN/mm^2

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Minimum steel area to be provided, from equation 6.18:

$$A_{s,min} = 0.8A_{ct}f_{ct.ef}/f_{yk}$$

The area of concrete within the tensile zone, A_{ct}, can be taken as the full wall thickness multiplied by a one metre length. Hence

$$A_{s,min} = 0.8(150 \times 1000) \times 1.5/460$$
$$= 391\,mm^2/m$$

This could be conveniently provided as 10 mm bars at 300 mm centres in each face of the member (524 mm^2/m). For this reinforcement, the average crack spacing is given by equation 6.20 as

$$s_{rm} = 50 + 0.2k_1\phi/\rho_r$$

where $k_1 = 0.8$ for high bond bars

$\phi = 10\,mm$

$\rho_r = A_s/A_{c.eff} = 524/(150 \times 1000) = 0.0035$

therefore $s_{rm} = 50 + 0.2 \times 0.8 \times 10/0.0035 = 507\,mm$

The mean strain in the section is given by equation 6.19:

$$\varepsilon_{sm} = (\varepsilon_{cs} + T\alpha_c)$$
$$= (50 + 20\{12/2\}) \times 10^{-6}$$
$$= 170 \text{ microstrain}$$

The ultimate tensile strain for the concrete

$$\varepsilon_{ult} = f_{ct.ef}/E_{c,ef} = 1.5/(10\,000) = 150 \text{ microstrain}$$

Therefore the section can be considered as cracked. The design crack width is given as

$$w_k = \beta s_{rm}\varepsilon_{sm}$$
$$= 1.3 \times 507 \times 170 \times 10^{-6}$$
$$= 0.11\,mm$$

6.6 Other Serviceability Requirements

The two principal other serviceability considerations are those of durability and resistance to fire, although occasionally a situation arises in which some other factor may be of importance to ensure the proper performance of a structural member in service. This may include fatigue due to moving loads or machinery, or specific thermal and sound insulation properties. The methods of dealing with such requirements may range from the use

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of reduced working stresses in the materials, to the use of special concretes, for example lightweight aggregates for good thermal resistance.

6.6.1 Durability

Deterioration will generally be associated with water permeating the concrete, and the opportunities for this to occur should be minimised as far as possible by providing good architectural details with adequate drainage and protection to the concrete surface.

Permeability is the principal characteristic of the concrete which affects durability, although in some situations it is necessary to consider also physical and chemical effects which may cause the concrete to decay.

For reinforced concrete, a further important aspect of durability is the degree of protection which is given to the reinforcement. Carbonation by the atmosphere will, in time, destroy the alkalinity of the surface zone concrete, and if this reaches the level of the reinforcement will render the steel vulnerable to corrosion in the presence of moisture and oxygen.

If a concrete is made with a sound inert aggregate, deterioration will not occur in the absence of an external influence. Since concrete is a highly alkaline material, its resistance to other alkalis is good, but it is however very susceptible to attack by acids or substances which easily decompose to produce acids. Concrete made with Portland cement is thus not suitable for use in situations where it comes into contact with such materials, which include beer, milk and fats. Some neutral salts may also attack concrete, the two most notable being calcium chloride and soluble sulfates. These react with a minor constituent of the hydration products in different ways. The chloride must be in concentrated solution, when it has a solvent effect on the concrete in addition to its more widely recognised action in promoting corrosion of the reinforcement. Sulfates need only be present in much smaller quantities to cause internal expansion of the concrete with consequent cracking and strength loss. Sulfates present the most commonly met chemical-attack problem for concrete since they may occur in groundwater and sewage. In such cases cements containing reduced proportions of the vulnerable tricalcium aluminate, such as Sulfate Resisting Portland Cement, should be used. The addition of Pulverised Fuel Ash (Pfa) or ground granulated blast furnace slag (ggbfs) may also be beneficial. Both chlorides and sulfates are present in sea water, and because of this the chemical actions are different, resulting in reduced sulfate damage, although if the concrete is of poor quality, serious damage may occur from reactions of soluble magnesium salts with the hydrated compounds. Well-constructed Portland cement structures have nevertheless been found to endure for many years in sea water.

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The matter of exposure classifications related to environmental conditions is dealt with in detail in ENV 206 together with the provision of appropriate concrete materials. This includes the use of a system of classification of a wide range of chemically agressive environments based on ISO 9690. It should be noted however that this whole area is the subject of considerable debate within Europe at present owing to major differences in established national practices.

Physical attack of the concrete must also be considered. This may come from abrasion or attrition as may be caused by sand or shingle, and by alternate wetting and drying. The latter effect is particularly important in the case of marine structures near the water surface, and causes stresses to develop if the movements produced are restrained. It is also possible for crystal growth to occur from drying out of sea water in cracks and pores, and this may cause further internal stresses, leading to cracking. Alternate freezing and thawing is another major cause of physical damage, particularly in road and runway slabs and other situations where water in pores and cracks can freeze and expand thus leading to spalling. It has been found that the entrainment of a small percentage of air in the concrete in the form of small discrete bubbles offers the most effective protection against this form of attack. Although this reduces the strength of the concrete, it is recommended that between 4.0 and 6.0 per cent by volume of entrained air should be included in concrete subjected to wetting and drying combined with frost.

All these forms of attack may be minimised by the production of a dense, well-compacted and well cured concrete with low permeability, thus restricting damage to the surface zone of the member. Aggregates which are likely to react with the alkali matrix should be avoided (or the alkali levels of the cement carefully limited), as must those which exhibit unusually high shrinkage characteristics. If this is done, then permeability, and hence durability, is affected by

(1) aggregate type and density
(2) water-cement ratio
(3) degree of hydration of cement
(4) degree of compaction.

A low water–cement ratio is necessary to limit the voids due to hydration, which must be well advanced with the assistance of good curing techniques. ENV 206 recommends minimum curing periods taking account of exposure classification, concrete strength development rate, concrete temperature and ambient conditions. Coupled with this is the need for non-porous aggregates which are hard enough to resist any attrition, and for thorough compaction. It is essential that the mix is designed to have adequate workability for the situation in which it is to be used, thus the cement content of the mix must be reasonably high.

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ENV 206 and the Concise Code specify minimum cement contents for various exposure conditions, as well as minimum strength and maximum water cement ratio, related to minimum cover requirements as described in section 6.1.1.

The consequences of thermal effects on durability must not be overlooked, and very high cement contents should only be used in conjunction with a detailed cracking assessment. A cement content of 550 kg/m^3 is often regarded as an upper limit for general use.

Provided that such measures are taken, and that adequate cover of sound concrete is given to the reinforcement, deterioration of reinforced concrete is unlikely. Thus although the surface concrete may be affected, the reinforcing steel will remain protected by an alkaline concrete matrix which has not been carbonated by the atmosphere. Once this cover breaks down and water and possibly chemicals can reach the steel, rusting and consequent expansion lead rapidly to cracking and spalling of the cover concrete and severe damage – visually and sometimes structurally.

6.6.2 Fire Resistance

Depending on the type of structure under consideration, it may be necessary to consider the fire resistance of the individual concrete members. Three conditions must be examined:

(1) effects on structural strength
(2) flame penetration resistance } in the case of dividing members
(3) heat transmission properties } such as walls and slabs

Concrete and steel in the form of reinforcement or prestressing tendons exhibit reduced strength after being subjected to high temperatures. Although concrete has low thermal conductivity, and thus good resistance to temperature rise, the strength begins to drop significantly at temperatures above 300 °C and it has a tendency to spall at high temperatures. The extent of this spalling is governed by the type of aggregate, with siliceous materials being particularly susceptible while calcareous and lightweight aggregate concretes suffer very little. Reinforcement will retain about 50 per cent of its normal strength after reaching about 550 °C, while for prestressing tendons the corresponding temperature is only 400 °C.

Thus as the temperature rises the heat is transferred to the interior of a concrete member, with a thermal gradient established in the concrete. This gradient will be affected by the area and mass of the member in addition to the thermal properties of the concrete, and may lead to expansion and loss of strength. Dependent on the thickness and nature of cover, the steel will rise in temperature and lose strength, this leading to deflections and eventual structural failure of the member if the steel

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temperature becomes excessive. Design must therefore be aimed at providing and maintaining sound cover of concrete as a protection, thus delaying the temperature rise in the steel. The presence of plaster, screeds and other non-combustible finishes assists the cover in protecting the reinforcement and may thus be allowed for in the design.

BS8110 gives tabulated values of minimum dimensions and nominal covers for various types of concrete member which are necessary to permit the member to withstand fire for a specified period of time. Until further guidance is issued by the EuroCode Committee, these values should continue to be used as the basis of providing fire resistance. Although these values, which have been summarised in tables 6.3 and 6.4, do not take into account the influence of aggregate type, they may be considered adequate for most normal purposes. More detailed information concerning design for fire resistance is given in Part 2 of BS 8110 including concrete type, member type and details of finishes. The period that a member is required to survive, both in respect of strength in relation to working loads and the containment of fire, will depend upon the type and usage of the structure – and minimum requirements are generally specified by building regulations. Prestressed concrete beams must be considered separately in view of the increased vulnerability of the prestressing steel.

6.7 Limitation of Damage caused by Accidental Loads

While it would be unreasonable to expect a structure to withstand extremes of accidental loading as may be caused by collision, explosion or similar happening, it is important that resulting damage should not be disproportionate to the cause. It follows therefore that a major structural collapse must not be allowed to be caused by a relatively minor mishap which may have a reasonably high probability of happening in the anticipated lifetime of the structure.

The possibilities of a structure buckling or overturning under the 'design' loads will have been considered as part of the ultimate limit state analysis. However, in some instances a structure will not have an adequate lateral strength even though it has been designed to resist the specified combinations of wind load and vertical load. This could be the case if there is an explosion or a slight earth tremor, since then the lateral loads are proportional to the mass of the structure. Therefore it is recommended that at any floor level, a structure should always be capable of resisting a minimum lateral force as detailed in section 3.3.4.

Damage and possible instability should also be guarded against wherever possible, for example vulnerable load-bearing members should be protected from collision by protective features such as banks or barriers.

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6.7.1 Ties

In addition to these precautions, the general stability and robustness of a building structure can be increased by providing reinforcement acting as ties. These ties should act both vertically between roof and foundations, and horizontally around and across each floor (figure 6.13), and all external vertical load-bearing members should be anchored to the floors and beams. Detailed design information is not given in EC2 and the NAD recommends that the design method of BS8110 may be used as discussed below.

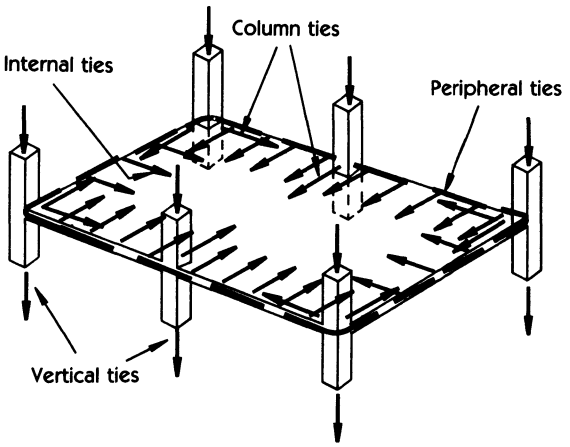

Figure 6.13 *Tie forces*

Vertical Ties

Vertical ties are not generally necessary in structures of less than five storeys, but in higher buildings should be provided by reinforcement, effectively continuous from roof to foundation by means of proper laps, running through all vertical load-bearing members. This steel should be capable of resisting a tensile force equal to the maximum design ultimate load carried by the column or wall from any one storey or the roof. In *in situ* concrete, this requirement is almost invariably satisfied by a normal design, but joint detailing may be affected in precast work.

Horizontal Ties

Horizontal ties should be provided for all buildings, irrespective of height, in three ways

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(1) peripheral ties
(2) internal ties
(3) column and wall ties.

The resistance of these ties when stressed to their characteristic strength is given in terms of a force F_t, where $F_t = 60\,\text{kN}$ or $(20 + 4 \times$ number of storeys in structure) kN, whichever is less. This expression takes into account the increased risk of an accident in a large building and the seriousness of the collapse of a tall structure.

(a) Peripheral Ties

The peripheral tie must be provided, by reinforcement which is effectively continuous, around the perimeter of the building at each floor and roof level. This reinforcement must lie within 1.2 m from the outer edge and at its characteristic stress be capable of resisting a force of at least F_t.

(b) Internal Ties

Internal ties should also be provided at each floor in two perpendicular directions and be anchored at each end either to the peripheral tie or to the continuous column or wall ties.

These ties must be effectively continuous and they may either be spread evenly across a floor, or grouped at beams or walls as convenient. Where walls are used, the tie reinforcement must be concentrated in the bottom 0.5 m.

The resistance required is related to the span and loading. Internal ties must be capable of resisting a force of F_t kN per metre width or $[F_t(g_k + q_k)/7.5]\ L/5$ kN per metre width, if this is greater. In this expression, L is the greatest horizontal distance in the direction of the tie between the centres of vertical load-bearing members, or if smaller, $5 \times$ the clear storey height measured to underside of the beams. The loading $(g_k + q_k)$ kN/m^2 is the average characteristic load on unit area of the floor considered. Internal ties parallel to cross-walls occurring in one direction only, on plan, need only resist the force F_t kN per metre width.

(c) Column and Wall Ties

Column and wall ties must be able to resist a force of at least 3 per cent of the total vertical ultimate load for which the member has been designed. Additionally, the resistance provided must not be less than the smaller of $2F_t$ or $F_t l_0/2.5$ kN where l_0 is the floor to ceiling height in metres. Wall ties are assessed on the basis of the above forces acting per metre length of the wall, while column ties are concentrated within 1 m either side of the column centre line. Particular care should be taken with corner columns to ensure they are tied in two perpendicular directions.

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In considering the structure subjected to accidental loading it is assumed that no other forces are acting, thus reinforcement provided for other purposes may also act as ties. Indeed, peripheral and internal ties may also be considered to be acting as column or wall ties.

As with vertical ties, the provision of horizontal ties for *in situ* construction will seldom affect the amount of reinforcement provided. Detailing of the reinforcement may however be affected, and particular attention must be paid to the manner in which internal ties are anchored to peripheral ties. The requirements for the full anchorage of ties are illustrated in figure 6.14. If these are not met, then the assumed stresses in the ties must be reduced appropriately.

Precast concrete construction however presents a more serious problem since the requirements of tie forces and simple easily constructed joints are not always compatible. Unless the required tie forces can be provided with the bars anchored by hooks and bends in the case of column and wall ties, an analysis of the structure must be performed to assess the remaining stability after a specified degree of structural damage.

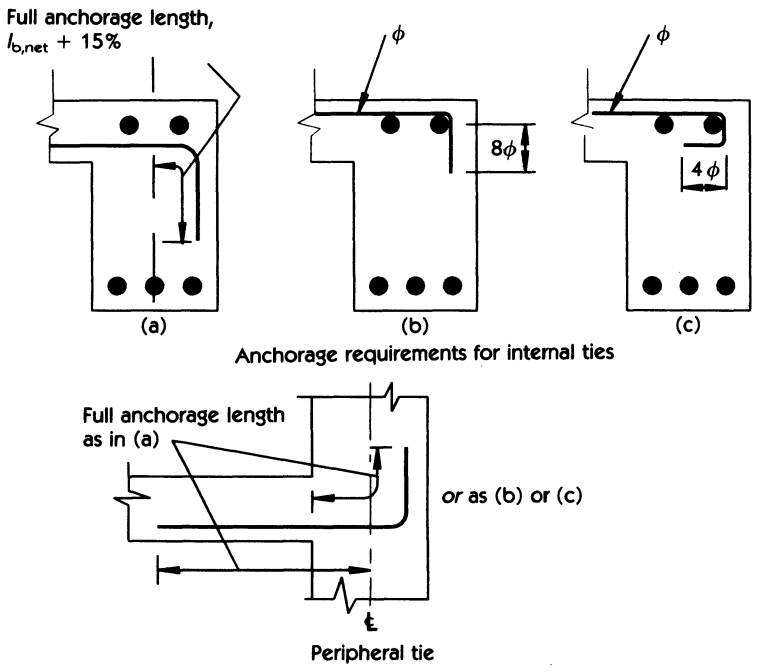

Figure 6.14 *Anchorage of ties*

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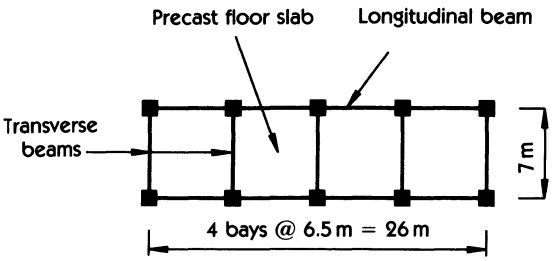

Figure 6.15

Example 6.6 Stability Ties

Calculate the stability ties required in an eight-storey building of plan area shown in figure 6.15:

Clear storey height under beams = 2.9 m

Floor to ceiling height (l_0) = 3.4 m

Characteristic permanent load (g_k) = 6 kN/m^2

Characteristic variable load (q_k) = 3 kN/m^2

Characteristic steel strength (f_{yk}) = 460 N/mm^2

F_t = (20 + 4 × number of storeys)

= 20 + 4 × 8 = 52 kN < 60 kN

(a) Peripheral ties

Force to be resisted = F_t = 52 kN

$$\text{Bar area required} = \frac{52 \times 10^3}{460} = 113\,\text{mm}^2$$

This could be provided by one T12 bar.

(b) Internal ties

$$\text{Force to be resisted} = \frac{F_t(g_k + q_k)}{7.5} \times \frac{L}{5}\,\text{kN per metre}$$

(1) *Transverse direction*

$$\text{Force} = \frac{52(6 + 3)}{7.5} \times \frac{7}{5} = 87.4\,\text{kN/m} > F_t$$

Force per bay = 87.4 × 6.5

= 568.1 kN

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Therefore, bar area required in each transverse interior beam is

$$\frac{568.1 \times 10^3}{460} = 1235\,\text{mm}^2$$

This could be provided by 4T20 bars.

(2) *Longitudinal direction*

$$\text{Force} = \frac{52(6 + 3)}{7.5} \times \frac{6.5}{5} = 81.1\,\text{kN/m} > F_t$$

Therefore force along length of building = $81.1 \times 7 = 567.7\,\text{kN}$, hence bar area required in each longitudinal beam is

$$\frac{567.7 \times 10^3}{2 \times 460} = 617\,\text{mm}^2$$

This could be provided by 2T20 bars.

(3) *Column ties*

Force to be designed for is

$$\left(\frac{l_0}{2.5}\right)F_t = \left(\frac{3.4}{2.5}\right)52 = 70.7\,\text{kN} < 2F_t$$

or 3 per cent of ultimate floor load on a column is

$$8\left[\frac{3}{100}(1.35 \times 6 + 1.5 \times 3) \times 6.5 \times \frac{7}{2}\right] = 69\,\text{kN at ground level}$$

To allow for 3 per cent of column self-weight, take design force to be 72 kN, say, at each floor level.

$$\text{Area of ties required} = \frac{72 \times 10^3}{460} = 156\,\text{mm}^2$$

This would be provided by 1T20 and incorporated with the internal ties.

(c) Vertical ties

Maximum column load from one storey is approximately equal to

$$(1.5 \times 3 + 1.35 \times 6) \times 3.5 \times 6.5 = 286.7\,\text{kN}$$

Therefore bar area required throughout each column is equal to

$$\frac{286.7 \times 10^3}{460} = 623\,\text{mm}^2$$

This would be provided by 4T16 bars.

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6.7.2 Analysis of 'Damaged' Structure

This must be undertaken when a structure has five or more storeys and does not comply with the vertical-tie requirements, or when every precast floor or roof unit does not have sufficient anchorage to resist a force equal to F_t kN per metre width acting in the direction of the span. The analysis must show that each key load-bearing member, its connections, and the horizontal members which provide lateral support, are able to withstand a specified loading from any direction. If this cannot be satisfied, then the analysis must demonstrate that the removal of any single vertical load-bearing element, other than key members, at each storey in turn will not result in collapse of a significant part of the structure.

The minimum loading that may act from any direction on a key member is recommended as $34\,kN/m^2$. The decision as to what loads should be considered acting is left to the engineer, but will generally be on the basis of permanent and realistic variable load estimates, depending on the building usage. This method is attempting therefore to assess quantitatively the effects of exceptional loading such as explosion. The design 'pressure' must thus be regarded as a somewhat arbitrary value.

The 'pressure' method will generally be suitable for application to columns in precast framed structures; however, where precast load-bearing panel construction is being used an approach incorporating the removal of individual elements may be more appropriate. In this case, vertical loadings should be assessed as described, and the structure investigated to determine whether it is able to remain standing by a different structural action. This action may include parts of the damaged structure behaving as a cantilever or a catenary, and it may also be necessary to consider the strength of non-load-bearing partitions or cladding.

Whichever approach is adopted, such analyses are tedious, and the provision of effective tie forces within the structure should be regarded as the preferred solution both from the point of view of design and performance.

Continuity reinforcement and good detailing will greatly enhance the overall fire resistance of a structure with respect to collapse. A fire-damaged structure with reduced member strength may even be likened to a structure subjected to accidental overload, and analysed accordingly.

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7

Design of Reinforced Concrete Beams

Reinforced concrete beam design consists primarily of producing member details which will adequately resist the ultimate bending moments, shear forces and torsional moments. At the same time serviceability requirements must be considered to ensure that the member will behave satisfactorily under working loads. It is difficult to separate these two criteria, hence the design procedure consists of a series of interrelated steps and checks. These steps are shown in detail in the flow chart in figure 7.1, but may be condensed into three basic design stages:

(1) preliminary analysis and member sizing
(2) detailed analysis and design of reinforcement
(3) serviceability calculations.

Much of the material in this chapter depends on the theory and design specification from the previous chapters. The loading and calculation of moments and shear forces should be carried out using the methods described in chapter 3. The equations used for calculating the areas of reinforcement have been derived in chapters 4 and 5.

Full details of servicebility requirements and calculations are given in chapter 6, but it is normal practice to make use of simple rules which are specified in the Code of Practice and are quite adequate for most situations. Typical of these are the span–effective depth ratios to ensure acceptable deflections, and the rules for maximum bar spacings, maximum bar sizes and minimum quantities of reinforcement, which are to limit cracking, as described in chapter 6.

Design and detailing of the bending reinforcement must allow for factors such as anchorage bond between the steel and concrete. The area of the tensile bending reinforcement also affects the subsequent design of the shear and torsion reinforcement. Arrangement of reinforcement is

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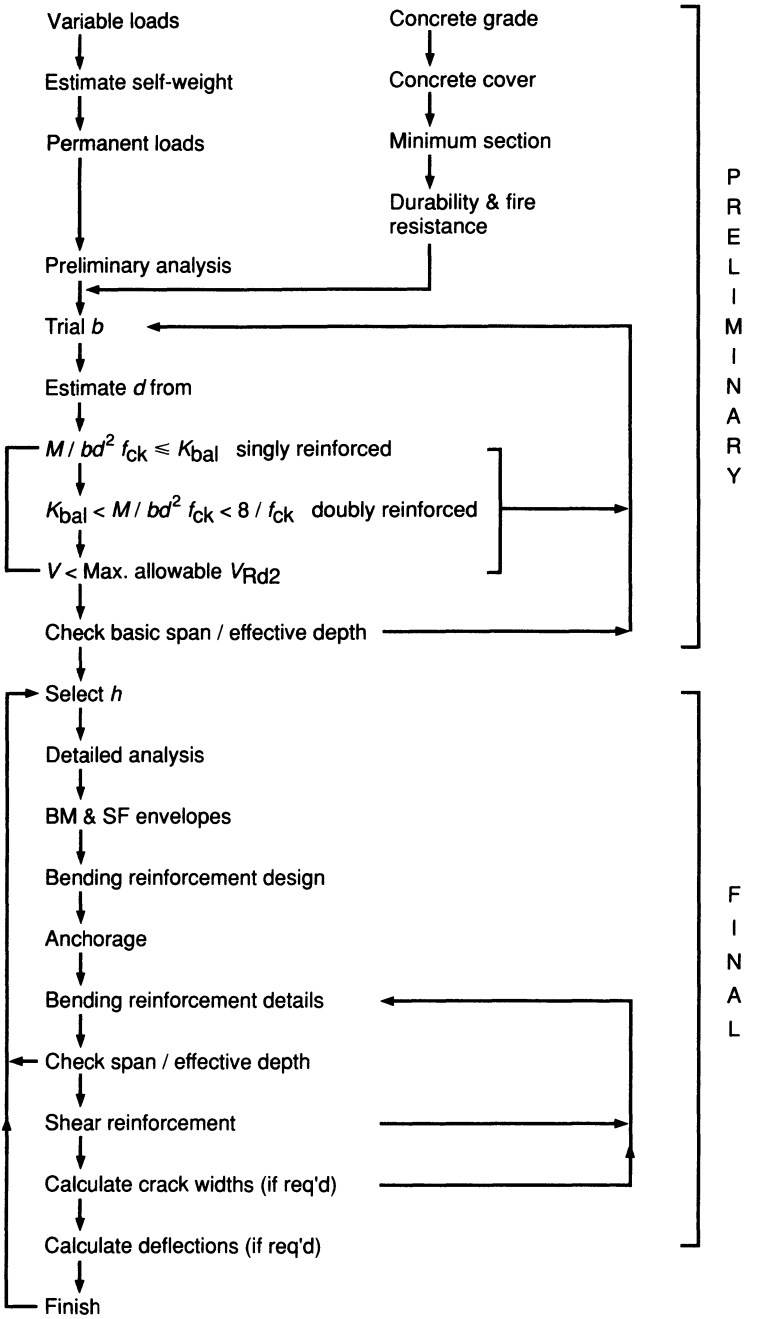

Figure 7.1 *Beam design flowchart*

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constrained both by the requirements of the codes of practice for concrete structures and by practical considerations such as construction tolerances, clearance between bars and available bar sizes and lengths. Many of the requirements for correct detailing are illustrated in the examples which deal with the design of typical beams.

All calculations should be based on the effective span of a beam which is given as follows:

(1) a simply supported beam – the lesser of the distances between the centres of bearings, or the clear distance between supports plus one-third of the overall depth
(2) a continuous beam – the distance between centres of supports
(3) an isolated cantilever – the length to the face of the support.

7.1 Preliminary Analysis and Member Sizing

The layout and size of members are very often controlled by architectural details, and clearances for machinery and equipment. The engineer must either check that the beam sizes are adequate to carry the loading, or alternatively, decide on sizes that are adequate. The preliminary analysis need only provide the maximum moments and shears in order to ascertain reasonable dimensions. Beam dimensions required are

(1) cover to the reinforcement
(2) breadth (b)
(3) effective depth (d)
(4) overall depth (h).

Adequate concrete cover is required to ensure adequate bond and to protect the reinforcement from corrosion and damage. The necessary cover depends on the grade of concrete, the exposure of the beam, and the required fire resistance. Table 6.2 gives the nominal cover which should be provided to all reinforcement, including links. This cover may need to be increased to meet the fire resistance requirements of the Code of Practice.

The strength of a beam is affected considerably more by its depth than its breadth. A suitable breadth may be a third to half of the depth; but it may be much less for a deep beam and at other times wide shallow beams are used to conserve headroom. The beam should not be too narrow; if it is much less than 200 mm wide there may be difficulty in providing adequate side cover and space for the reinforcing bars.

Suitable dimensions for b and d can be decided by a few trial calculations as follows:

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(1) For no compression reinforcement

$$K = M/bd^2f_{ck} \leq K_{bal}$$

where $K_{bal} = 0.167$ for $f_{ck} \leq$ C35/45

or $K_{bal} = 0.136$ for $f_{ck} >$ C35/45.

With compression reinforcement it can be shown that approximately

$$M/bd^2f_{ck} < 8/f_{ck}$$

if the area of bending reinforcement is not to be excessive.

(2) The maximum design shear V_{max} should not be greater than $V_{Rd2} = 0.3\nu f_{ck}b_w d$, where $\nu = 0.7 - f_{ck}/200 \geq 0.5$. To avoid congested shear reinforcement, V_{max} should preferably be somewhat closer to half (or less) of the maximum allowed.

(3) The span–effective depth ratio for spans not exceeding 7 m should be within the basic values given in table 6.8. For spans greater than 7 m, the basic ratios are multiplied by 7/span.

(4) The overall depth of the beam is given by

$$h = d + \text{Cover} + t$$

Where t = estimated distance from the outside of the link to the centre of the tension bars (see figure 7.2). For example, with nominal sized 12 mm links and one layer of 32 mm tension bars, $t = 28$ mm approximately. It will, in fact, be slightly larger than this with deformed bars as they have a larger overall dimension than the nominal bar size.

Example 7.1 Beam Sizing

A concrete lintel with an effective span of 4.0 m supports a 230 mm brick wall as shown in figure 7.3. The loads on the lintel are $G_k = 100$ kN and $Q_k = 40$ kN. Determine suitable dimensions for the lintel if class C25/30 concrete is used.

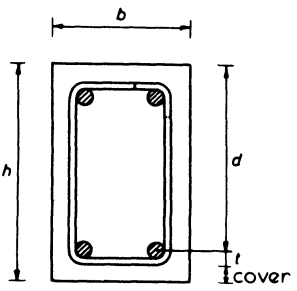

Figure 7.2 *Beam dimensions*

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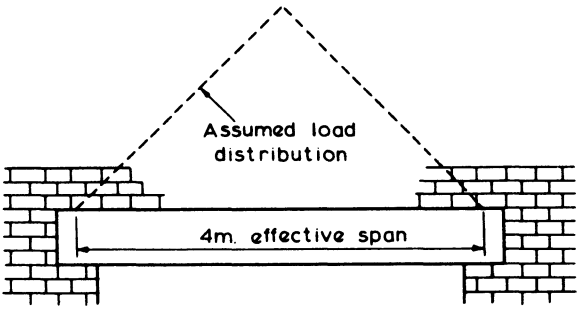

Figure 7.3

The beam breadth b will match the wall thickness so that

$$b = 230\,\text{mm}$$

Allowing, say, 14 kN for the weight of the beam, gives the ultimate load

$$F = 1.35 \times 114 + 1.5 \times 40$$
$$= 214\,\text{kN}$$

Therefore maximum shear

$$V = 107\,\text{kN}$$

Assuming a triangular load distribution for the preliminary analysis, we have

$$M = \frac{F \times \text{span}}{6} = \frac{214 \times 4.0}{6}$$
$$= 143\,\text{kN m}$$

For such a relatively minor beam the case with no compression steel should be considered

$$K = \frac{M}{bd^2 f_{ck}} < K_{bal} = 0.167$$

therefore

$$\frac{143 \times 10^6}{230 \times d^2 \times 25} < 0.167$$

$$d > 386\,\text{mm}$$

For a dry environment exposure the cover = 20 mm (tables 6.1 and 6.2). So for 10 mm links and, say, 32 mm bars

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$$\text{overall depth } h = d + 20 + 10 + 32/2$$
$$= d + 46$$

Therefore make $h = 525\,\text{mm}$ as an integer number of brick courses. So that

$$d = 525 - 46 = 479\,\text{mm}$$

Maximum shear resistance

$$V_{Rd2} = 0.3vf_{ck}bd$$

where $v = 0.7 - f_{ck}/200 = 0.575$

Therefore $V_{Rd2} = 0.3 \times 0.575 \times 25 \times 230 \times 479 \times 10^{-3} = 475\,\text{kN}$

Hence

$$V = 107\,\text{kN} < V_{Rd2} = 475\,\text{kN}$$

$$\text{Basic span–effective depth} = \frac{4000}{479} = 8.35 < 21.7 \quad \text{from table 6.8}$$

A beam size of 230 mm by 525 mm deep would be suitable.

$$\text{Weight of beam} = 0.23 \times 0.525 \times 4.0 \times 24$$
$$= 11.6\,\text{kN}$$

which is sufficiently close to the assumed value.

7.2 Design for Bending

The calculation of main bending reinforcement is performed using the equations and charts derived in chapter 4. In the case of rectangular sections which require only tension steel, the lever-arm curve method is probably the simplest. Where compression steel is required, either design charts or a manual approach with the simplified design formulae may be used. When design charts are not applicable or not available, as in the case of non-rectangular sections, the formulae based on the equivalent rectangular stress block will simplify calculations considerably.

The type of reinforcing steel to be used must be decided initially since this, in conjunction with the chosen concrete grade, will affect the areas required and also influence bond calculations. In most circumstances one of the available types of high-yield bars will be used unless cracking is critical, as for example in water-retaining structures, when mild steel may be preferred. Areas of reinforcement are calculated at the sections with maximum moments, and suitable bar sizes selected. (Tables of bar areas are given in the appendix.) This permits anchorage calculations to be

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performed and details of bar arrangement to be produced, taking into account the guidance given by the codes of practice.

An excessive amount of reinforcement usually indicates that a member is undersized and it may also cause difficulty in fixing the bars and pouring the concrete. Therefore the code stipulates A_s/bh should not exceed 4.0 per cent, except at laps. On the other hand too little reinforcement is also undesirable therefore A_s/bd should not be less than $0.6/f_{yk}$ or 0.15 per cent. Checks may also be required on minimum reinforcement required for crack control.

To avoid excessive deflections it is also necessary to check the span to effective depth ratio as outlined in chapter 6.

7.2.1 *Singly Reinforced Rectangular Section*

A beam section needs reinforcement only in the tensile zone when $K = M/bd^2f_{ck}$ is not greater than K_{bal}

where $K_{bal} = 0.167$ for $f_{ck} \leq$ C35/45

or $K_{bal} = 0.136$ for $f_{ck} >$ C35/45

This is not true if the moments at a section have been reduced as a result of a redistribution of the elastic moments, and in this case reference should be made to equations 7.2 and 7.6 in order to decide whether or not compression steel is necessary.

The singly reinforced section considered is shown in figure 7.4 and it is subjected to a sagging moment M at the ultimate limit state. Using the lever-arm curve, the design calculations for the longitudinal steel can be summarised as follows:

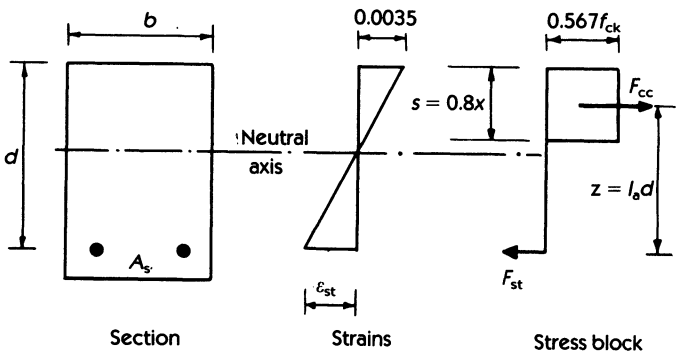

Figure 7.4 *Singly reinforced section*

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(1) Calculate $K = M/bd^2f_{ck} \not> K_{bal}$

(2) Determine the lever-arm, z, from the curve of figure 7.5 or from the equation

$$z = d[0.5 + \sqrt{(0.25 - K/1.134)}] \qquad (7.1)$$

(3) The area of tension steel is given by

$$A_s = \frac{M}{0.87f_{yk}z}$$

(4) Select suitable bar sizes.

(5) Check that the area of steel actually provided is within the limits required by the code, that is

$$100\frac{A_s}{bh} \leqslant 4.0$$

and

$$100\frac{A_s}{bd} \geqslant 60/f_{yk} \quad \text{or} \quad 0.15$$

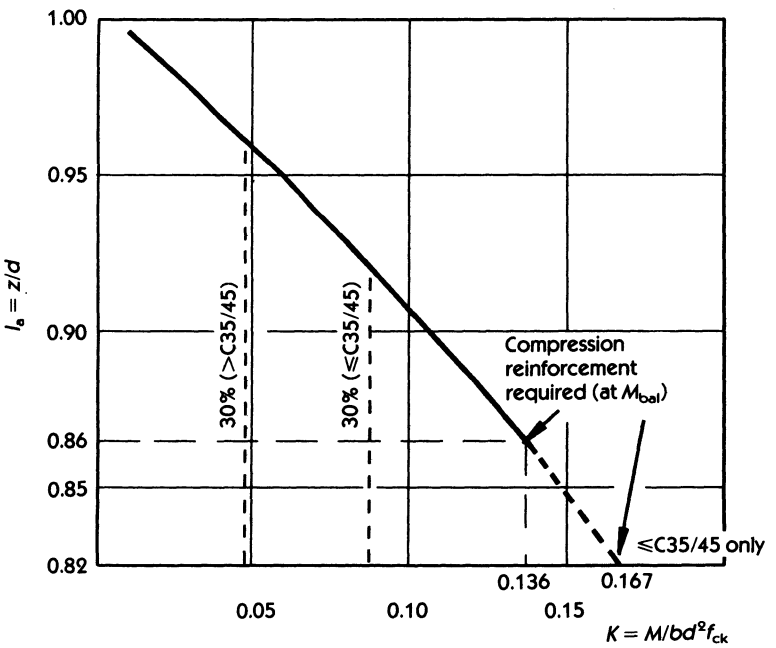

Figure 7.5 *Lever-arm curve. The 30% values on the K-axis mark the limits for singly reinforced sections with moment redistribution applied (see section 4.7)*

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Example 7.2 Design of Tension Reinforcement for a Rectangular Section

The beam section shown in figure 7.6 has characteristic material strengths of $f_{ck} = 25\,N/mm^2$ for the concrete and $f_{yk} = 460\,N/mm^2$ for the steel. The design moment at the ultimate limit state is 165 kN m which causes sagging of the beam

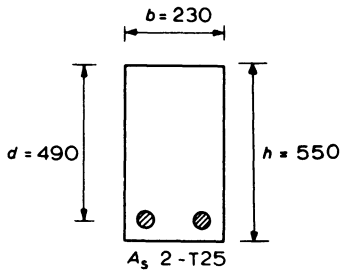

Figure 7.6 *Singly reinforced example*

$$K = \frac{M}{bd^2 f_{ck}} = \frac{165 \times 10^6}{230 \times 490^2 \times 25} = 0.12$$

This is less than $K_{bal} = 0.167$ therefore compression steel is not required.

From the lever-arm curve of figure 7.5 $l_a = 0.88$, therefore

$$\text{lever arm } z = l_a d = 0.88 \times 490 = 431\,mm$$

and

$$A_s = \frac{M}{0.87 f_{yk} z} = \frac{165 \times 10^6}{0.87 \times 460 \times 431} = 957\,mm^2$$

Provide two T25 bars, area = $982\,mm^2$. For the steel provided

$$\frac{100A_s}{bd} = \frac{100 \times 982}{230 \times 490} = 0.87$$

and

$$0.15 < \frac{100A_s}{bd} \quad \text{and} \quad \frac{100A_s}{bh} < 4.0$$

therefore the steel percentage is within the limits specified by the code.

7.2.2 Rectangular Section with Compression Reinforcement

Compression steel is required whenever the concrete in compression is unable, by itself, to develop the necessary moment of resistance. Design

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charts such as the one in figure 4.9 may be used to determine the steel areas but the simplified equations based on the equivalent rectangular stress block are quick to apply.

The maximum moment of resistance that can be developed by the concrete occurs with the neutral axis at the maximum depth allowed by the code of practice. This limiting depth is given as

$$x_{bal} = 0.8(\delta - 0.44)d \quad \text{with} \quad f_{ck} \leqslant \text{C35/45}$$

or

$$x_{bal} = 0.8(\delta - 0.56)d \quad \text{with} \quad f_{ck} > \text{C35/45} \tag{7.2}$$

where

$$\delta = \frac{\text{moment at the section after redistribution}}{\text{moment at the section before redistribution}}$$

With no moment redistribution $x_{bal} = 0.45d$ for $f_{ck} \leqslant$ C35/45 or $x_{bal} = 0.35d$ for $f_{ck} >$ C35/45.

With d'/x greater than 0.43 the stress in the compression steel may be considerably less than the yield, therefore, the design procedure is somewhat different if moment redistribution has been applied.

It should also be noted that, in order to maintain the limitation on the depth of neutral axis as specified in equation 7.2, the areas of reinforcement required and provided should meet the following requirement

$$(A'_{s.prov} - A'_{s.req}) \geqslant (A_{s.prov} - A_{s.req}) \tag{7.3}$$

This is to ensure a gradual tension type failure with yielding of the tension reinforcement as explained in chapter 4.

No Moment Redistribution

If d'/x is not greater than 0.43, as is usually the case (figure 7.7), the proportions of the strain diagram will ensure that the compression steel will have yielded.

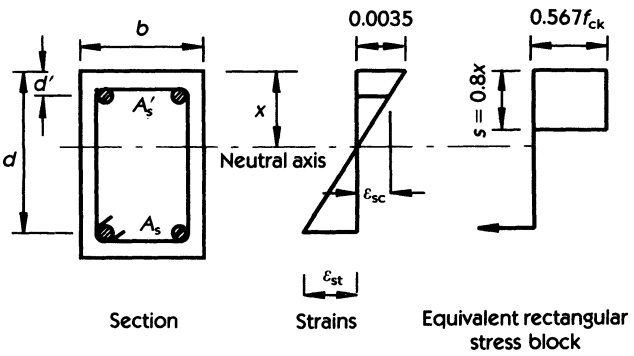

Figure 7.7 *Beam doubly reinforced to resist a sagging moment*

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Compression reinforcement is required if

$$M > K_{bal} f_{ck} b d^2$$

where $K_{bal} = 0.167$ for $f_{ck} \leq$ C35/45

or $K_{bal} = 0.136$ for $f_{ck} >$ C35/45

and the design equations as given in section 4.5 are

(1) Area of compression steel

$$A'_s = \frac{(M - K_{bal} f_{ck} b d^2)}{0.87 f_{yk}(d - d')} \tag{7.4}$$

(2) Area of tension steel

$$A_s = \frac{K_{bal} f_{ck} b d^2}{0.87 f_{yk} z} + A'_s \tag{7.5}$$

with lever arm $z = 0.82d$ with $f_{ck} \leq$ C35/45

or $z = 0.86d$ with $f_{ck} >$ C35/45

If d'/x is greater than 0.43 the stress in the compression steel should be determined as outlined in part (2) of example 7.4.

With Moment Redistribution

The limiting depth of the neutral axis can be calculated from equation 7.2 and compression steel is required if

$$M > 0.567 f_{ck} b s \left(d - \frac{s}{2}\right) \tag{7.6}$$

where s = depth of stress block = $0.8x$.

The design procedure is

(1) Calculate $K = M/bd^2 f_{ck}$

(2) Calculate $K_{bal} = 0.363(\delta - 0.44) - 0.116(\delta - 0.44)^2$ for $f_{ck} \leq$ C35/45

or $K_{bal} = 0.363(\delta - 0.56) - 0.116(\delta - 0.56)^2$ for $f_{ck} >$ C35/45

If $K < K_{bal}$, compression steel is not required so proceed as for a singly reinforced section as in example 7.2.

If $K > K_{bal}$, compression steel is required.

(3) If $d'/x < 0.43$, the compression steel has yielded and $f_{sc} = 0.87 f_{yk}$.

If $d'/x > 0.43$, calculate the steel compressive strain ε_{sc} and hence the stress f_{sc} as in example 7.4.

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(4) Calculate the area of compression steel from

$$A'_s = \frac{(K - K_{bal})f_{ck}bd^2}{0.87f_{yk}(d - d')} \tag{7.7}$$

(5) Calculate the area of tension steel from

$$A_s = \frac{K_{bal}f_{ck}bd^2}{0.87f_{yk}z} + A'_s\frac{f_{sc}}{0.87f_{yk}}$$

where $z = d - 0.8x/2$.

Closed links should be provided to give lateral restraint to the outer layer of compression steel.

Example 7.3 Design of Tension and Compression Reinforcement, No Moment Redistribution

The beam section shown in figure 7.8 has characteristic material strengths of $f_{ck} = 25\,N/mm^2$ and $f_{yk} = 460\,N/mm^2$. The ultimate moment is 165 kN m, causing hogging of the beam:

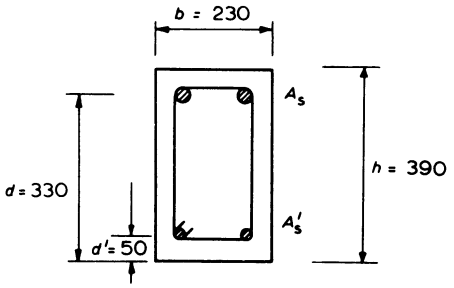

Figure 7.8 *Beam doubly reinforced to resist a hogging moment*

$$\frac{M}{bd^2f_{ck}} = \frac{165 \times 10^6}{230 \times 330^2 \times 25}$$
$$= 0.26 > K_{bal} = 0.167$$

so that compression steel is required, and from equation 7.2

$$x = 0.45d = 0.45 \times 330 = 148\,mm$$
$$d'/x = 50/148 = 0.34 < 0.43$$

therefore

$$f_{sc} = 0.87f_{yk}$$

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From equation 7.4

$$\text{Compression steel } A'_s = \frac{(M - 0.167 f_{ck} bd^2)}{0.87 f_{yk}(d - d')}$$
$$= \frac{(165 \times 10^6 - 0.167 \times 25 \times 230 \times 330^2)}{0.87 \times 460(330 - 50)}$$
$$= 539\,\text{mm}^2$$

And from equation 7.5

$$\text{tension steel } A_s = \frac{0.167 f_{ck} bd^2}{0.87 f_{yk} z} + A'_s$$
$$= \frac{0.167 \times 25 \times 230 \times 330^2}{0.87 \times 460 \times 0.82 \times 330} + 539$$
$$= 1505\,\text{mm}^2$$

Provide two T25 bars for A'_s, area = $982\,\text{mm}^2$ and two T32 bars for A_s, area = $1610\,\text{mm}^2$, so that for the areas of steel required and provided in equation 7.3

$$982 - 539 > 1610 - 1505$$

Also the bar areas provided are within the limits specified by the code.

Example 7.4 Design of Tension and Compression Reinforcement, With 15 per cent Moment Redistribution, $\delta = 0.85$

The beam section shown in figure 7.9 has characteristic material strengths of $f_{ck} = 25\,\text{N/mm}^2$ and $f_{yk} = 460\,\text{N/mm}^2$. The ultimate moment is 370 kN m, causing hogging of the beam.

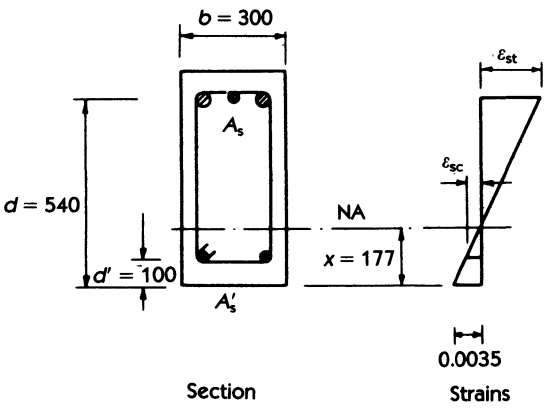

Figure 7.9 *Beam doubly reinforced to resist a hogging moment*

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As the moment reduction factor $\delta = 0.85$, the limiting depth of the neutral axis is

$$x = 0.8(\delta - 0.44)d$$
$$= 0.8(0.85 - 0.44)540 = 177\,\text{mm}$$
$$K = M/bd^2f_{ck} = 370 \times 10^6/(300 \times 540^2 \times 25)$$
$$= 0.169$$
$$K_{bal} = 0.363(\delta - 0.44) - 0.116(\delta - 0.44)^2$$
$$= 0.363(0.85 - 0.44) - 0.116(0.85 - 0.44)^2$$
$$= 0.129$$

$K > K_{bal}$ therefore compression steel is required

$$d'/x = 100/177 = 0.56 > 0.43$$

therefore $f_{sc} < 0.87f_{yk}$

(1) Steel compressive strain $\varepsilon_{sc} = \dfrac{0.0035(x - d')}{x}$

$$= \frac{0.0035(177 - 100)}{177} = 0.00152$$

(2) From the relevant equation of section 4.1.2

$$\text{Steel compressive stress} = E_s\varepsilon_{sc}$$
$$= 200\,000 \times 0.00152$$
$$= 304\,\text{N/mm}^2$$

(3) Compression steel $A'_s = \dfrac{(K - K_{bal})f_{ck}bd^2}{f_{sc}(d - d')}$

$$= \frac{(0.169 - 0.129)25 \times 300 \times 540^2}{304(540 - 100)}$$
$$= 654\,\text{mm}^2$$

(4) Tension steel $A_s = \dfrac{K_{bal}f_{ck}bd^2}{0.87f_{yk}z} + A'_s\dfrac{f_{sc}}{0.87f_{yk}}$

$$= \frac{0.129 \times 25 \times 300 \times 540^2}{0.87 \times 460(540 - 0.8 \times 177/2)}$$
$$+ 654 \times \frac{304}{0.87 \times 460}$$
$$= 1999\,\text{mm}^2$$

Provide two T25 bars for A'_s, area = 982 mm^2 and two T32 plus one T25 bars for A_s, area = 2101 mm^2, which also meet the requirements of equation 7.3.

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These areas lie within the maximum and minimum limits specified by the code. To restrain the compression steel, at least 8 mm links at 300 mm centres should be provided.

7.2.3 T-beams

Figure 7.10 shows sections through a T-beam and an L-beam which may form part of a concrete beam and slab floor. When the beams are resisting sagging moments, part of the slab acts as a compression flange and the members may be designed as T- or L-beams. With hogging moments the slab will be in tension and assumed to be cracked, therefore the beam must then be designed as a rectangular section of width b_w and overall depth h.

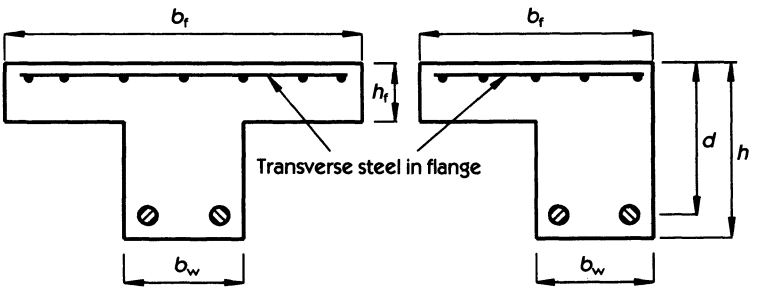

Figure 7.10 *T-beam and L-beam*

When the slab does act as the flange its effective width b_f is defined by empirical rules which are specified as follows:

(1) T-section – the lesser of the actual flange width, or the width of the web plub one-fifth of the distance l_0 between zero moments.
(2) L-section – the lesser of the actual flange width or the width of the web plus one-tenth of the distance l_0 between zero moments.

As a simple rule, the distance l_0 between the points of zero moment for a continuous beam may be taken as

$l_0 = 0.7l$ for an interior span

and $l_0 = 0.85l$ for an end span

where l is the effective span

Since the floor slab acts as a large compression area, the stress block for the T- or L-section usually falls within the slab thickness. For this

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position of the stress block, the section may be designed as an equivalent rectangular section of breadth b_f.

Transverse reinforcement should be placed across the full width of the flange to resist the shear developed between the web and the flange as described in section 5.1.3.

Design Procedure

(1) Calculate $M/b_f d^2 f_{ck}$ and determine l_a from the lever-arm curve of figure 7.5

Lever arm $z = l_a d$ or from equation 7.1

(2) If $s = 2(d - z) < h_f$ the stress block falls within the flange depth, and the design may proceed as for a rectangular section, breadth b_f. On the very few occasions that the neutral axis does fall below the flange, reference should be made to the methods described in section 4.6.2 for a full analysis.

(3) Design transverse steel in the top of the flange to resist shear – see section 5.1.3. This area of transverse steel should not be less than the minimum required in a slab.
Hence $A_s \nless 0.15 h_f \times 1000/100 = 1.5 h_f$ mm^2 per metre length of the span.

Example 7.5 Design of Reinforcement for a T-section

A simply supported beam spanning 6.0 m has the cross-section shown in figure 7.11. The characteristic material strengths are $f_{ck} = 25$ N/mm^2 and $f_{yk} = 460$ N/mm^2 and the design moment at the ultimate limit state is 190 kN m, causing sagging.

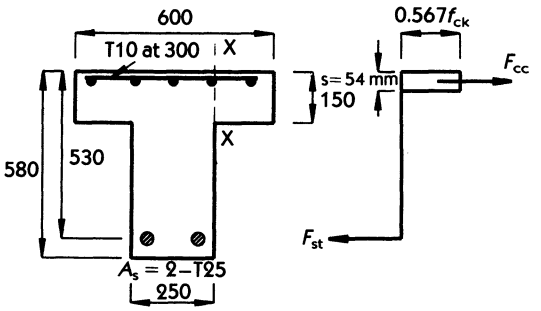

Figure 7.11 *T-beam*

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(a) Longitudinal reinforcement

$$\frac{M}{b_f d^2 f_{ck}} = \frac{190 \times 10^6}{600 \times 530^2 \times 25} = 0.045$$

From the lever-arm curve, figure 7.5, $l_a = 0.95$, therefore

$$\text{lever arm } z = l_a d = 0.95 \times 530 = 503\,\text{mm}$$

$$\text{Depth of stress block } s = 2(d - z) = 2(530 - 503)$$
$$= 54\,\text{mm} < h_f$$

Thus the stress block lies within the flange

$$A_s = \frac{M}{0.87 f_{yk} z} = \frac{190 \times 10^6}{0.87 \times 460 \times 503}$$
$$= 943\,\text{mm}^2$$

Provide two T25 bars, area = $982\,\text{mm}^2$. For these bars

$$\frac{100 A_s}{b_w d} = \frac{100 \times 982}{250 \times 530} = 0.74 \text{ per cent} > \frac{60}{f_{yk}} \quad \text{or} \quad 0.15$$

Thus the steel percentage is greater than the minimum specified by the code.

(b) Transverse steel in the flange (see section 5.1.3)

(1) Calculate design transverse shear V_{Sd}
Compressive force F_{cf} in the flange causing shear at section X–X is given by

$$F_{cf} = 0.567 f_{ck} s(b_f - b_w)/2$$
$$= [0.567 \times 25 \times 54(600 - 250)/2] \times 10^{-3}$$
$$= 134\,\text{kN}$$

$$V_{sd} = \frac{134}{(1/2)\text{span}} = \frac{134}{3.0} = 44.7\,\text{kN/m} = 44.7\,\text{N/mm}$$

(2) Check V_{sd} < Maximum shear resistance V_{Rd2}

$$V_{Rd2} = 0.13 f_{ck} h_f$$
$$= 0.13 \times 25 \times 150$$
$$= 488 > V_{Sd} = 44.7\,\text{N/mm}$$

(3) Design transverse reinforcement

Transverse shear resistance V_{Rd3} = Shear resistance of concrete plus reinforcement

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that is $$V_{Rd3} = 2.5\tau_{Rd}h_f + 0.87\frac{A_{sf}}{s_f}f_{yk} \geq V_{Sd} = 44.7\,\text{N/mm}$$

From table 5.1 $\tau_{rd} = 0.30\,\text{N/mm}^2$

$$\therefore \quad 2.5\tau_{Rd}h_f = 2.5 \times 0.3 \times 150$$
$$= 112.5 > 44.7\,\text{N/mm}$$

So use minimum transverse steel $= 0.15bh_f/100$

hence $$A_s = 1.5h_f = 1.5 \times 150 = 225\,\text{mm}^2/\text{m}$$

Provide T10 bars at 300 mm centres $= 262\,\text{mm}^2/\text{m}$

$$(V_{Rd3} = 112.5 + (262/300) \times 0.87 \times 460 = 462\,\text{N/mm})$$

7.2.4 Anchorage Bond

From section 5.2 the basic anchorage length, l_b, for a reinforcing bar is given by the following equation:

$$l_b = \frac{f_{yk}}{4.6f_{bd}}\phi$$

where f_{bd} is the ultimate bond stress given in table 5.2 and ϕ = the bar size.

The required anchorage length should be measured from the point at which the bar is assumed to be stressed.

The appendix lists the anchorage lengths appropriate to the ultimate stress, $0.87f_{yk}$, for various grades of concrete and steel. The effective anchorage lengths for hooks and bends in reinforcing bars are detailed in figure 5.10.

The anchorage of a bar is more effective in the lower part of a member and this should be considered when detailing the reinforcement. Anchorage requirements are also important when detailing the curtailment of bars as described in the following section.

7.2.5 Curtailment of Bars

As the magnitude of the bending moment on a beam decreases along its length so may the area of bending reinforcement be reduced by curtailing the bars since they are no longer required, as shown in figure 7.12. It should be recognised though that because of the approximations and assumptions made for the loading, the structural analysis and the behaviour of the reinforced concrete, the curtailment cannot be a

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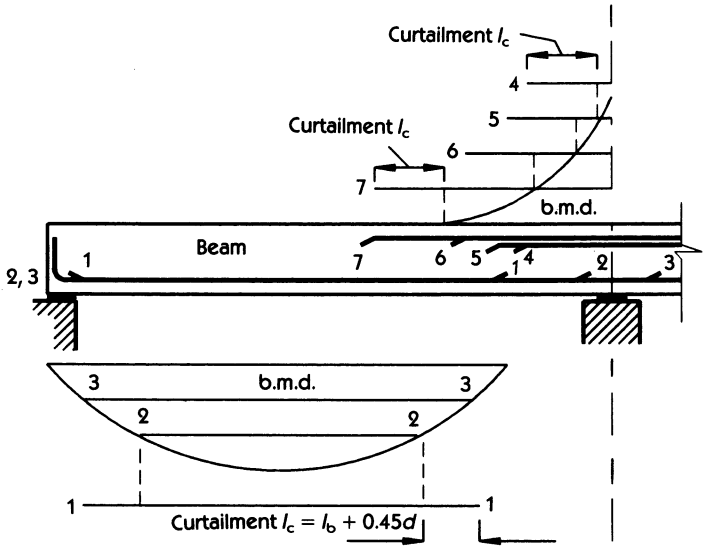

Figure 7.12 *Curtailment of reinforcement*

particularly precise procedure. In addition the curtailment lengths are in many cases superseded by the requirements for serviceability, durability and detailing, such as maximum bar spacing, minimum bars and curtailment beyond the critical sections for shear.

Each curtailed bar should extend a curtailment length beyond the point at which it is no longer needed so that it is well anchored into the concrete. For a straight bar the curtailment length l_c is given by

$$l_c = l_b + 0.45d$$

where the anchorage length $l_b = \dfrac{f_{yk}}{4.6f_{bd}}\phi$

as derived in section 5.2, and the extra distance $0.45d$ is to allow for the tension induced by the shear as implied by the truss analogy for shear design.

The maximum bending moment value on the moment envelope diagram is divided up in proportion to the moment of resistance of each bar $M_i = 0.87f_yA_{si}z_i$ in the order the bars are curtailed. The lever arm z_i of the bar could by calculated by analysing the sections as the bars are curtailed but for manual calculations it is sufficiently accurate to take a value, say, of $z_i = 0.9d_i$ approximately for each bar.

Thus in figure 7.12, for the sagging moment envelope and three T20 bars arranged in one layer, the maximum moment value would be divided

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into three equal parts. The minimum curtailment lengths l_c would then be as shown in the figure.

The hogging moment envelope would be divided proportionately into four parts for each bar taking into account the area A_{si} and effective depth d_i for each bar in the two layers. Bars 4 and 5 in the lower layer are curtailed before bars 6 and 7 in the upper layer which have the greater moment of resistance.

The curtailment of bars should be staggered wherever possible in order to avoid sudden changes in cross-section with resulting stress concentrations and possible cracking. This curtailment can often be achieved while using bars of equal length, as illustrated in figure 7.13.

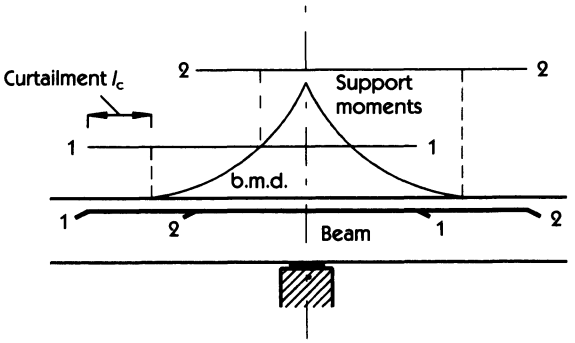

Figure 7.13 *Staggering the curtailment of bars*

When considering the curtailment the following rules must also be applied:

(1) At least one-quarter of the bottom reinforcement should extend to the supports.

(2) The bottom reinforcement at an end support should be anchored into the support as shown in figure 7.14 where

$$l_{bnet} = \alpha_a l_b A_{s,req}/A_{s,prov}$$

as in equation 5.17 of section 5.2.

$A_{s,req}$ may be taken as one-quarter of the bottom reinforcement in the span but not less than that required by condition (3) as follows.

(3) At an end support where there is little or no fixity the bottom steel should be designed to resist a tensile force of $0.5V_{Sd}$ to allow for the tension induced by the shear.

(4) At an end support where there is fixity but it has been analysed as a

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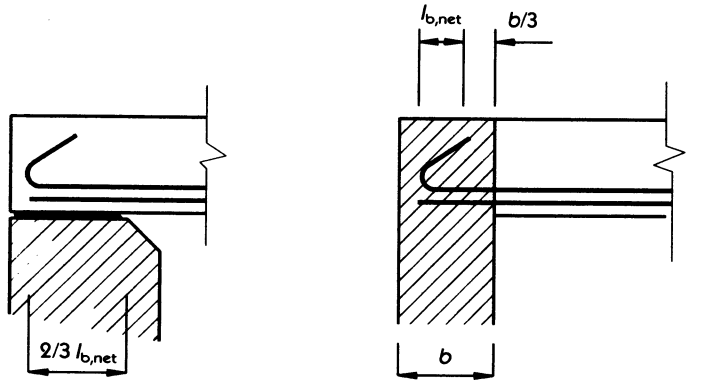

Figure 7.14 *Alternative anchorage lengths at simple end supports*

simple support, top steel should be designed and anchored to resist at least 25 per cent of the maximum span moment.

(5) At internal supports the bottom steel should extend 10 bar diameters ϕ beyond the face of the support. To achieve continuity and resistance to accidental damage, splice bars should be provided across the support with a full anchorage lap on each side.

(6) Where the loads on a beam are substantially uniformly distributed, simplified rules for curtailment may be used. These rules only apply to continuous beams if the characteristic variable load does not exceed the characteristic permanent load and the spans are approximately equal. Figure 7.15 shows the rules in diagrammatic form.

7.2.6 Span–Effective Depth Ratios

As already described in section 7.1, it is necessary to check the span–effective depth ratios to ensure that the deflections are not excessive. This is unlikely to be a problem with beams except perhaps for cantilevers or long span beams. These requirements are fully described and explained in chapter 6, dealing with Serviceability.

7.2.7 Bending-reinforcement Example

The following example describes the calculations for designing the bending reinforcement for a simply supported beam. It brings together many of the items from the previous sections. The shear reinforcement for this beam is designed later in example 7.7.

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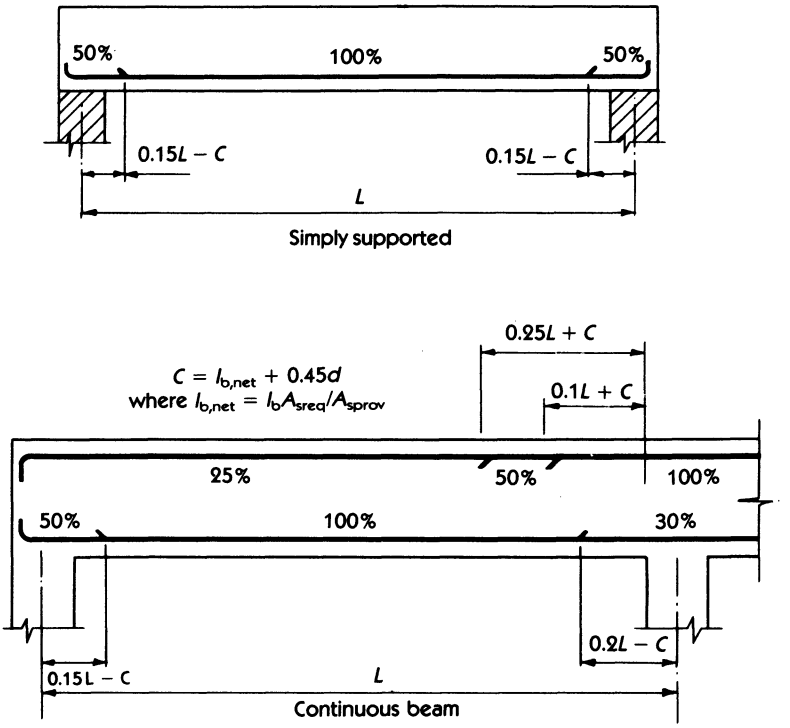

Figure 7.15 *Simplified rules for curtailment of bars in beams*

Example 7.6 Design of a Beam – Bending Reinforcement

The beam shown in figure 7.16 supports the following uniformly distributed loads

permanent load $g_k = 40\,\text{kN/m}$, including self-weight

variable load $q_k = 12\,\text{kN/m}$

The characteristic material strengths are $f_{ck} = 25\,\text{N/mm}^2$ and $f_{yk} = 460\,\text{N/mm}^2$. Effective depth, $d = 550\,\text{mm}$ and breadth, $b = 300\,\text{mm}$.

(a) Analysis

Ultimate load $w_u = (1.35g_k + 1.5q_k)\,\text{kN/metre}$

$= (1.35 \times 40 + 1.5 \times 12) = 72\,\text{kN/metre}$

therefore

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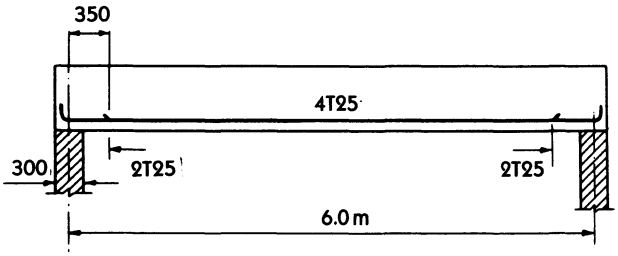

Figure 7.16 *One-span beam-bending reinforcement*

$$\text{maximum design moment } M = \frac{w_u L^2}{8} = \frac{72 \times 6.0^2}{8} = 324\,\text{kN m}$$

(b) Bending Reinforcement

$$K = \frac{M}{bd^2 f_{ck}} = \frac{324 \times 10^6}{300 \times 550^2 \times 25} = 0.143 < K_{bal} = 0.167$$

From the lever-arm curve, figure 7.5, $l_a = 0.85$. Therefore

$$\text{lever arm } z = l_a d = 0.85 \times 550 = 468\,\text{mm}$$

$$A_s = \frac{M}{0.87 f_{yk} z} = \frac{324 \times 10^6}{0.87 \times 460 \times 468} = 1730\,\text{mm}^2$$

Provide four T25 bars, area = $1960\,\text{mm}^2$, $100A_s/bd = 1.19 > 0.15$.

(c) Curtailment at Support

The reinforcement should be anchored over the supports with a bend as shown in figure 7.14. Two bars may be curtailed near to the supports according to the requirements of figure 7.15 for a simply supported beam.

(d) Span–Effective Depth Ratio

The basic permissible ratio for a simply supported beam from table 6.8 = 18, and

$$\text{actual } \frac{\text{span}}{d} = \frac{6000}{550} = 10.9$$

7.3 Design for Shear

The theory and design requirements for shear were covered in chapter 5. There are two methods that can be used:

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(1) Standard Method
(2) Variable Strut Inclination Method.

This section will outline the procedure for the Standard Method.

The shear reinforcement will usually take the form of vertical links or a combination of links and bent-up bars. This reinforcement together with the shear resistance of the concrete V_{Rd1} must resist the design ultimate shear V_{Sd} as given by the shear–force envelope diagram.

7.3.1 Vertical Stirrups or Links

The design procedure is as follows:

(1) Check the shear V_{Sf} at the face of the support so that

$$V_{Sf} \not> V_{Rd2}$$

where V_{Sf} = design shear at the support face

and $V_{Rd2} = 0.3\nu f_{ck} b_w d(1 + \cot\alpha)$

$\nu = 0.7 - f_{ck}/200 \not< 0.5$

α = is the angle between the shear reinforcement and the axis of the beam.

For vertical links $\cot\alpha = 0$ and

$$V_{Rd2} = 0.3\nu f_{ck} b_w d$$

(2) Calculate the shear resistance V_{Rd1} of the concrete from

$$V_{Rd1} = [\tau_{Rd} k(1.2 + 40\rho_l)]b_w d$$

where $\tau = 0.035 f_{ck}^{2/3}$ or as given in table 5.1

$k = 1.6 - d$ with d in metres

when $d \geq 0.6$ m or more than 50 per cent of the bottom reinforcement is curtailed, $k = 1.0$

$\rho_l = A_s/bd$ for the tension reinforcement which extends not less than $d + l_{b.net}$ beyond the section considered, $l_{b.net}$ being the anchorage bond length.

(3) With a mainly uniformly distributed load the critical section is taken as distance d from the face of the support where the design shear is V_{Sd}.

If $V_{Sd} > V_{Rd1}$ shear reinforcement is required and is calculated from

$$\frac{A_{sw}}{s} = \frac{1.28(V_{Sd} - V_{Rd1})}{df_{yk}}$$

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where A_{sw} is the cross-sectional area of the legs of the stirrups ($2 \times \pi\phi^2/4$ for single strirrups)
s is the spacing of the strirrups.

(4) Minimum shear reinforcement must be provided in the central part of the beams span where the design shear is small such that

$$\frac{A_{sw}}{s} = \rho_w b_w$$

where values of ρ_w are given in table 7.1.

Table 7.1 Values of ρ_w for minimum stirrups

Concrete strength class	*Steel grade*	
	250	*460*
C12/15 and C20/25	0.0015	0.0008
C25/30 to C35/45	0.0022	0.0012
C40/50 to C50/60	0.0028	0.0014

(5) Calculate the shear resistance V_{min} of the minimum stirrups plus the concrete from

$$V_{min} = \frac{df_{yk}}{1.28}\frac{A_{sw}}{s} + V_{Rd1}$$

Note that A_{sw}/s is for the minimum links as calculated in (4) and V_{Rd1} should be calculated for the effective area of tension steel at the section considered.

(6) The value of V_{min} should be marked on the shear force envelope to give the extent of the shear links and the minimum links as shown in figure 7.19 of example 7.7.

(7) The maximum spacing of the stirrups or links in the longitudinal and transverse directions should be checked to ensure they meet the following requirements:

The longitudinal spacing of shear links should not exceed the smaller of:

(a) $0.8d$ or 300 mm, if $V_{Sd} \leq (1/5)V_{Rd2}$
(b) $0.6d$ or 300 mm, if $(1/5)V_{Rd2} < V_{Sd} \leq (2/3)V_{Rd2}$
(c) $0.3d$ or 200 mm, if $V_{Sd} > (2/3)V_{Rd2}$.

The transverse spacing of the legs of shear links should not exceed:

(a) d or 800 mm, if $V_{Sd} \leq (1/5)V_{Rd2}$

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(b) $0.6d$ or 300 mm, if $(1/5)V_{Rd2} < V_{Sd} \leq (2/3)V_{Rd2}$
(c) $0.3d$ or 200 mm, if $V_{Sd} > (2/3)V_{Rd2}$.

The minimum spacing of the links is governed by the requirements of placing and compacting the concrete and should not normally be less than about 80 mm.

Types of links or stirrups are shown in figure 7.17. The open links are usually used in the span of the beam with longitudinal steel consisting of top hanger bars and bottom tensile reinforcement. The closed links are used to enclose top and bottom reinforcement such as near to the supports. Multiple links are used when there is a high shear force to be resisted.

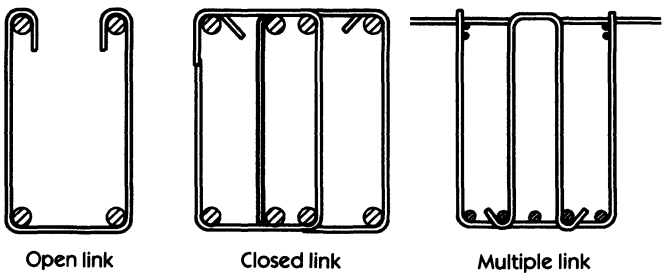

Figure 7.17 *Types of shear link*

The choice of steel type is often governed by the fact that mild steel may be bent to a smaller radius than high-yield steel. This is particularly important in narrow members to allow correct positioning of tension reinforcement as shown in figure 7.18.

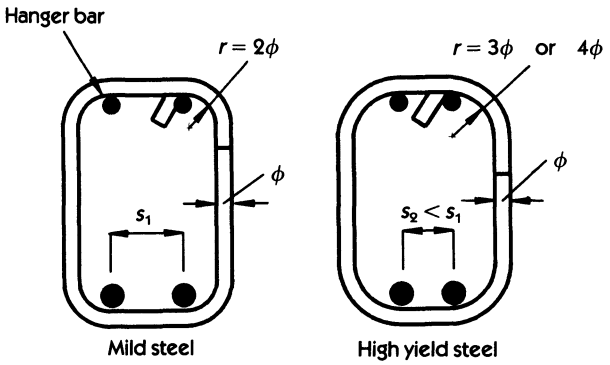

Figure 7.18 *Bending of links*

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The advantages of mild steel links are further increased by the need to provide anchorage for the vertical leg of a stirrup within the compression zone. Although high-yield reinforcement has better bond characteristics, anchorage lengths are greater than for mild steel bars of comparable size if the steel is to act at its full design stress. This factor is of particular importance if 'open' links are to be used.

Example 7.7 Design of Shear Reinforcement for a Beam

Shear reinforcement is to be designed for the one-span beam of example 7.6 as shown in figures 7.16 and 7.19. The characteristic strength of the concrete and the mild steel links is $f_{ck} = 25\,\text{N/mm}^2$ and $f_{yk} = 250\,\text{N/mm}^2$.

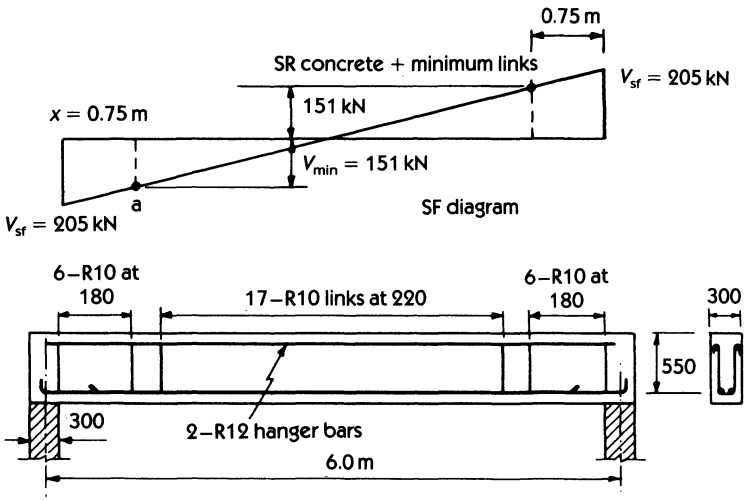

Figure 7.19 *Non-continuous beam-shear reinforcement*

(a) *Check maximum shear*

Total load on span, $F = w_u \times \text{span}$

$= 72.0 \times 6.0 = 432\,\text{kN}$

At face of support

Shear $V_{Sf} = F/2 - w_u \times \text{support width}/2$

$= 432/2 - 72.0 \times 0.15 = 205\,\text{kN}$

Maximum design shear resistance is

$$V_{Rd2} = 0.3vf_{ck}b_w d$$

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where $$v = 0.7 - f_{ck}/200 = 0.7 - 25/200$$
$$= 0.575$$

hence $$V_{Rd2} = 0.3 \times 0.575 \times 25 \times 300 \times 550/10^3$$
$$= 712\,\text{kN} > V_{sf} = 205\,\text{kN}$$

(b) *Shear links*

$$\frac{A_{sw}}{s} = \frac{1.28(V_{Sd} - V_{Rd1})}{df_{yk}}$$

At distance d from face of support, shear V_{Sd} is

$$V_{Sd} = V_{Sf} - w_u d$$
$$= 205 - 72.0 \times 0.55 = 166\,\text{kN}$$

Shear resistance of concrete V_{Rd1} is

$$V_{Rd1} = \tau_{rd} k(1.2 + 40\rho_l) b_w d$$

From table 5.1 $\tau_{Rd} = 0.30$

$$k = 1.6 - d = 1.6 - 0.55 = 1.05$$

Only two 25 mm bars extend a distance d past the critical section. Therefore

$$\rho_l = A_s/b_w d = 982/300 \times 550 = 0.00595$$

hence $$V_{Rd1} = 0.30 \times 1.05(1.2 + 40 \times 0.00595)300 \times 550 \times 10^{-3}$$
$$= 74.7\,\text{kN}$$

So $$\frac{A_{sw}}{s} = \frac{1.28(166 - 74.7) \times 10^3}{550 \times 250} = 0.85$$

Provide R10 links at 180 mm centres

$$\frac{A_{sw}}{s} = \frac{2 \times 78.5}{180} = 0.87 > 0.85$$

(c) *Minimum links*

$$\frac{A_{sw}}{s} = \rho_w b_w$$

From table 7.1 $\rho_w = 0.0022$, therefore

$$\frac{A_{sw}}{s} = 0.0022 \times 300 = 0.66$$

Provide R10 links at 220 centres

$$\frac{A_{sw}}{s} = \frac{2 \times 78.5}{220} = 0.71 > 0.66$$

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Shear resistance of minimum links + concrete, V_{min}, is

$$V_{min} = \frac{df_{yk}}{1.28}\frac{A_{sw}}{s} + V_{Rd1}$$

$$= \frac{550 \times 250}{10^3 \times 1.28} \times 0.71 + 74.7 = 151\,\text{kN}$$

(d) *Extent of shear links*

Shear links are required at each end of the beam from the face of the support to the point a at shear V_{min} as shown on the shear force diagram of figure 7.19.

$$\text{Distance } x = \frac{V_{Sf} - V_{min}}{w_u} = \frac{205 - 151}{72.0}$$

$$= 0.75 \text{ metres from the face of the support}$$

Therefore the number of R10 links at 180 mm centres required at each end of the beam is

$$1 + (x/s) = 1 + (750/180) = 6$$

spaced over a distance of $(6 - 1)180 = 900$ mm

7.3.2 Bent-up Bars

In regions of high shear forces it may be found that the use of links to carry the full force will cause steel congestion and lead to constructional problems. In these situations, consideration should be given to 'bending up' main reinforcement which is no longer required to resist bending forces. At least 50 per cent of the shear resistance provided by the steel should be in the form of links.

For a 'double system' of bent-up bars at 45° and spaced $(0.9d)$ apart, as described in section 5.1.2, the shear resistance (table 7.2) is

$$V_{wd} = 1.23 f_{yk} A_{sw}$$

Table 7.2 Shear resistance in kN of bent-up bars, 'double system'

f_{yk} (N/mm^2)	*Bar size* (ϕ)					
	12	*16*	*20*	*25*	*32*	*40*
250	35	62	96	151	247	387
460	64	114	177	278	455	713

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where A_{sw} is the cross-sectional area of a bent-up bar.

Bent-up bars must be fully anchored past the point at which they are acting as tension members, as indicated in figure 5.5.

7.4 Bar Spacing

There are limitations on the minimum and maximum spacing of the reinforcing bars. In the case of minimum values this is governed by constructional requirements to allow for the access of poker vibrators and the flow of concrete to obtain a well-compacted and dense concrete. The maximum limitations on spacings are to prevent excessive cracking caused by shrinkage of the concrete and thermal expansion and contraction of the member. These serviceability requirements are dealt with in chapter 6.

7.5 Continuous Beams

Beams, slabs and columns of a cast *in situ* structure all act together to form a continuous load-bearing structure. The reinforcement in a continuous beam must be designed and detailed to maintain this continuity by connecting adjacent spans and tying together the beam and its supporting columns. There must also be transverse reinforcement to unite the slab and the beam.

The bending-moment envelope is generally a series of sagging moments in the spans and hogging moments at the supports as in figure 7.20, but occasionally the hogging moments may extend completely over the span. Where the sagging moments occur the beam and slab act together, and the beam can be designed as a T-section. At the supports, the beam must be designed as a rectangular section – this is because the hogging moments cause tension in the slab.

The moment of resistance of the concrete T-beam section is somewhat greater than that of the rectangular concrete section at the supports. Hence it is often advantageous to redistribute the support moments as described in chapter 3. By this means the design support moments can be reduced and the design span moments possibly increased.

Design of the beam follows the procedures and rules set out in the previous sections. Other factors which have to be considered in the detailed design are as follows:

(1) At an exterior column the beam reinforcing bars which resist the design moments must have an anchorage bond length within the column.

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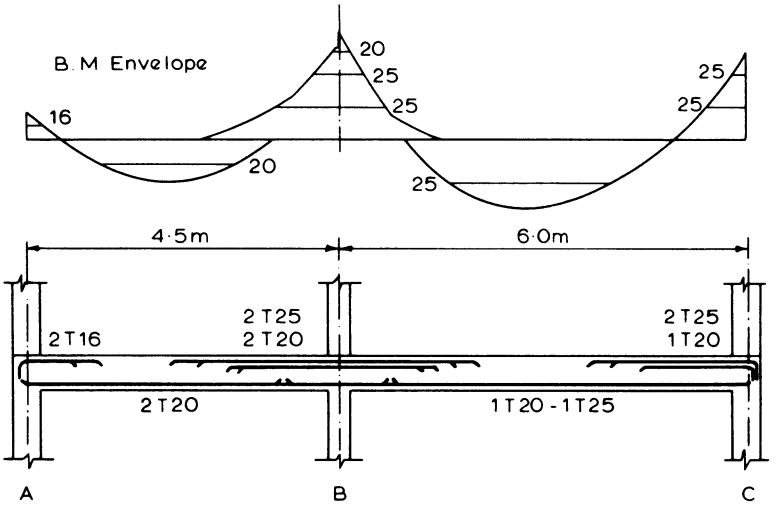

Figure 7.20 *Arrangement of bending reinforcement*

(2) In monolithic construction where a simple support has been assumed in the structural analysis, partial fixity of at least 25 per cent of the span moment should be allowed for in the design.
(3) Reinforcement in the top of the slab must pass over the beam steel and still have the necessary cover. This must be considered when detailing the beam reinforcement and when deciding the effective depth of the beam at the support sections.
(4) The column and beam reinforcement must be carefully detailed so that the bars can pass through the junctions without interference.

Figure 7.20 illustrates a typical arrangement of the bending reinforcement for a two-span continuous beam. The reinforcement has been arranged with reference to the bending-moment envelope and in accordance with the rules for anchorage and curtailment described in sections 7.2.4 and 7.2.5. The bending-moment envelope has been divided into sectors equivalent to the moment of resistance of each reinforcing bar. This establishes the cut-off points beyond which the bars must extend at least a curtailment anchorage length. It should be noted that at the external columns the reinforcement has been bent to give a full anchorage bond length.

The shear-force envelope and the arrangement of the shear reinforcement for the same continuous beam is shown in figure 7.21. On the shear-force envelope the resistance of the concrete plus the minimum stirrups has been marked and this shows the lengths of the beam which need shear reinforcement. When designing the shear reinforcement, reference

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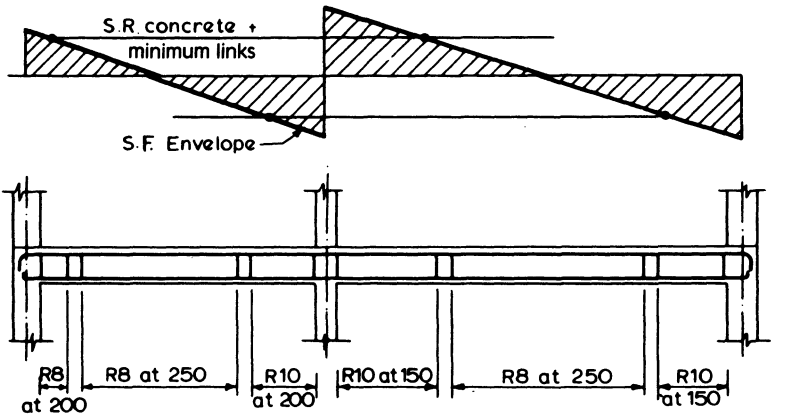

Figure 7.21 *Arrangement of shear reinforcement*

should be made to the arrangement of bending reinforcement to ensure that the longitudinal tension bars used to establish V_{Rd1} extend at least $d + l_{b.net}$ beyond the section being considered.

Example 7.8 Design of a Continuous Beam

The beam is 300 mm wide by 660 mm deep with three equal 5.0 m spans. In the transverse direction, the beams are at 4.0 m centres with a 180 mm thick slab, as shown in figure 7.23.

The variable load q_k on the beam is 50 kN/m and the permanent load g_k, including self-weight, is 85 kN/m.

Characteristic material strengths are $f_{ck} = 25\,N/mm^2$, $f_{yk} = 460\,N/mm^2$ for the longitudinal steel and $f_{yk} = 250\,N/mm^2$ for the links.

For each span

$$\text{ultimate load } w_u = (1.35g_k + 1.5q_k)\,kN/metre$$
$$(1.35 \times 85 + 1.5 \times 50) = 190\,kN/metre$$

Total ultimate load on a span is

$$F = 190 \times 5.0 = 950\,kN$$

As the loading is uniformly distributed, $q_k \not> g_k$, and the spans are equal, the coefficients shown in figure 7.22 have been used to calculate the design moments and shears.

Bending

(a) Mid-span of 1st and 3rd Spans – Design as a T-section

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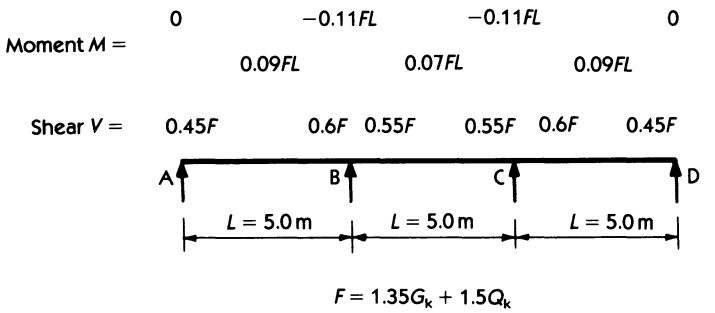

Figure 7.22 *Continuous beam with ultimate bending and shear-force coefficients*

$$\text{Moment } M = 0.09FL = 0.09 \times 950 \times 5 = 428\,\text{kN m}$$

$$\text{Effective width of flange} = b_w + 0.85L/5$$

$$= 300 + \frac{0.85 \times 5000}{5} = 1150\,\text{mm}$$

therefore

$$\frac{M}{bd^2 f_{ck}} = \frac{428 \times 10^6}{1150 \times 600^2 \times 25} = 0.041$$

From the lever-arm curve, $l_a = 0.96$, therefore

$$z = 0.96 \times 600 = 576\,\text{mm}$$

and

$$d - z = 600 - 576 = 24 < h_f/2$$

so that the stress block must lie within the 180 mm thick flange. Therefore

$$A_s = \frac{M}{0.87 f_{yk} z} = \frac{428 \times 10^6}{0.87 \times 460 \times 576} = 1857\,\text{mm}^2$$

Provide four T25 bars, area = 1960 mm^2 (bottom steel).

(b) Interior Supports – Design as a Rectangular Section

$$M = 0.11FL = 0.11 \times 950 \times 5 = 523\,\text{kN m hogging}$$

$$\frac{M}{bd^2 f_{ck}} = \frac{523 \times 10^6}{300 \times 580^2 \times 25} = 0.207 > 0.167$$

Thus, compression steel is required.

$$A_s' = \frac{M - 0.167 f_{ck} bd^2}{0.87 f_{yk}(d - d')}$$

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$$= \frac{523 \times 10^6 - 0.167 \times 25 \times 300 \times 580^2}{0.87 \times 460(580 - 50)} = 479\,\text{mm}^2$$

This area of steel will be provided by extending the span reinforcement beyond the supports.

$$A_s = \frac{0.167 f_{ck} b d^2}{0.87 f_{yk} z} + A'_s$$

$$= \frac{0.167 \times 25 \times 300 \times 580^2}{0.87 \times 460 \times 0.82 \times 580} + 479 = 2693\,\text{mm}^2$$

Provide five T25 plus one T20 bars, area = 2764 mm^2 (top steel). The arrangement of the reinforcement is shown in figure 7.23. At end support A two T25 bars have been provided as top continuity steel to meet the requirement of item (2) in section 7.5.

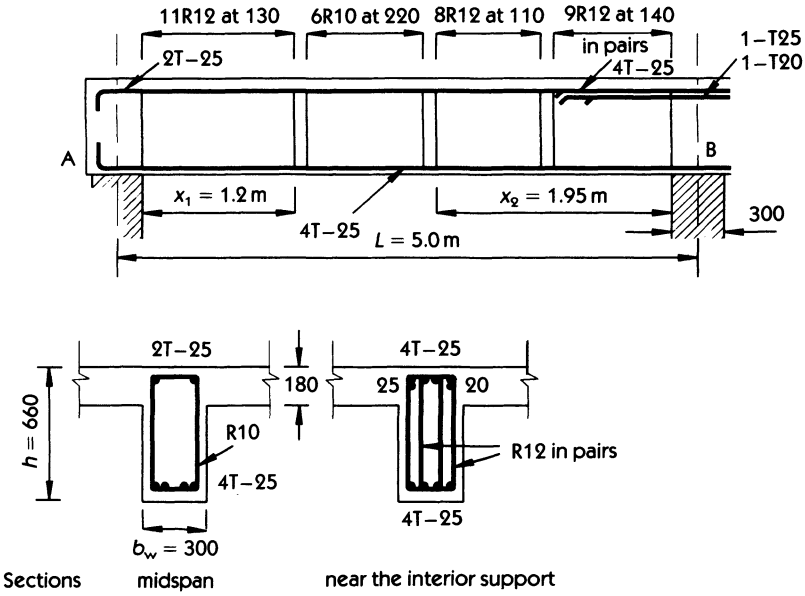

Figure 7.23 *End-span reinforcement details*

(c) Mid-span of 2nd span – Design as a T-section

$$b_f = 300 + 0.7 \times 5000/5 = 1000\,\text{mm}$$

$$M = 0.07FL = 0.07 \times 950 \times 5 = 333\,\text{kN m}$$

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Using the lever-arm curve, it is found that $l_a = 0.96$

$$A_s = \frac{M}{0.87 f_{yk} z} = \frac{333 \times 10^6}{0.87 \times 460(0.96 \times 600)} = 1445\,\text{mm}^2$$

Provide three T25 bars, area = 1470 mm^2 (bottom steel).

Shear

(a) Check maximum shear

Maximum shear at face of support is

$$V_{Sf} = 0.6F - w_u \times \text{support width}/2$$
$$= 0.6 \times 950 - 190 \times 0.15 = 542\,\text{kN}$$

Maximum design shear resistance $V_{Rd2} = 0.3 v f_{ck} bd$

where $v = 0.7 - f_{ck}/200 = 0.7 - 25/200$
$= 0.575$

therefore $V_{Rd2} = 0.3 \times 0.575 \times 25 \times 300 \times 580 \times 10^{-3}$
$= 750\,\text{kN} > V_{Sf} = 542\,\text{kN}$

(b) Shear links at end supports A and D

$$\frac{A_{sw}}{s} = \frac{1.28(V_{Sd} - V_{Rd1})}{d f_{yk}}$$

At distance d from face of support, shear V_{Sd} is

$$V_{Sd} = 0.45F - w_u(d + \text{support width}/2)$$
$$= 0.45 \times 950 - 190(0.6 + 0.15)$$
$$= 285\,\text{kN}$$

Shear resistance of concrete

$$V_{Rd1} = \tau_{Rd} k(1.2 + 40\rho_1) b_w d$$

From table 5.1 $\tau_{Rd} = 0.30$

$$k = 1.6 - d = 1.6 - 0.6 = 1.0$$

For four T25 $A_s = 1960\,\text{mm}^2$

therefore $\rho_1 = A_s/b_w d = 1960/300 \times 600 = 0.0109$

hence $V_{Rd1} = 0.3 \times 1.0(1.2 + 40 \times 0.0109)300 \times 600 \times 10^{-3}$
$= 88.3\,\text{kN}$

and so $\frac{A_{sw}}{s} = \frac{1.28(285 - 88.3) \times 10^3}{600 \times 250} = 1.68$

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Provide R12 links at 130 mm centres, $A_{sw}/s = 1.74$.

(c) Shear links in first and third spans at interior supports B and C

Distance d from the support face

$$\text{Shear} \quad V_{Sd} = 0.6 \times 950 - 190(0.58 + 0.15)$$
$$= 431\,\text{kN}$$

$$\frac{A_{sw}}{s} = \frac{1.28(431 - 88.3)10^3}{580 \times 250} = 3.03$$

Provide R12 links in pairs at 140 mm centres

$$A_{sw}/s = 3.23$$

These R12 double links at 140 mm centres are used for 1.12 metres then the design shear force reduces to

$$V = 0.6 \times 950 - 190(1.12 + 0.15)$$
$$= 329\,\text{kN}$$

It can be shown using the design equation for A_{sw}/s that single R12 links at 110 mm centres as shown on figure 7.23 are now adequate.

(d) Shear links second span, supports B and C

Distance d from the support face

$$\text{Shear} \quad V_{Sd} = 0.55 \times 950 - 190(0.58 + 0.15)$$
$$= 384\,\text{kN}$$

Calculations would show R12 links in pairs at 170 mm would suffice.

(e) Minimum links

$$\frac{A_{sw}}{s} = \rho_w b_w$$

From table 7.1 $\rho_w = 0.0022$

hence $$\frac{A_{sw}}{s} = 0.0022 \times 300 = 0.66$$

Provide R10 links at 220 centres, $A_{sw}/s = 0.71$.
Shear resistance, V_{min}, of minimum links + concrete is

$$V_{min} = \frac{df_{yk}}{1.28}\frac{A_{sw}}{s} + V_{Rd1}$$
$$= \frac{600 \times 250}{10^3 \times 1.28} \times 0.71 + 88.3 = 171\,\text{kN}$$

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(f) Extent of shear links

Links to resist shear are required over a distance from the face of the support to the point on the shear force diagram where the shear can be resisted by V_{min} provided by the minimum links plus the concrete.
For the face of the end support the distance x_1 is

$$x_1 = \frac{V_{Sf} - V_{min}}{w_u} + d$$
$$= \frac{285 - 171}{190} + 0.6 = 1.2\,\text{m}$$

For the interior support of the 1st and 3rd spans

$$x_2 = \frac{431 - 171}{190} + 0.58 = 1.95\,\text{m}$$

Based on these dimensions the links are arranged as shown in figure 7.23.

Similar calculations can be carried out for the arrangement of the links in the middle span.

7.6 Cantilever Beams

The moments, shears and deflections for a cantilever beam are substantially greater than those for an equivalently loaded span that is supported at both its ends. Also the moments in a cantilever can never be redistributed to other parts of the structure – the beam must always be capable of resisting the full static moment. Because of these factors and the problems that often occur with increased deflections due to creep, the design and detailing of a cantilever beam should be done with care.

Loads on a cantilever can cause the adjacent interior span to be subjected to a hogging moment over all or most of its span. The critical loading pattern for this condition should be as shown in figure 7.24.

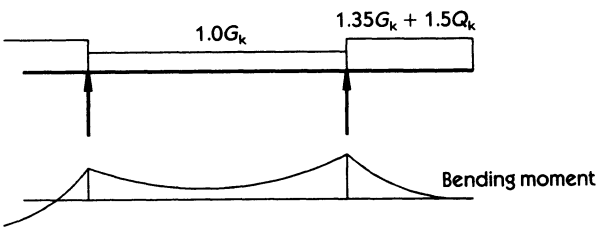

Figure 7.24 *Cantilever loading pattern*

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7.7 Design for Torsion

The theory and design requirements for torsion were covered in section 5.4. The design procedure consists of calculations to determine additional areas of links and longitudinal reinforcement to resist the torsional moment, using an equivalent hollow box section.

Usually it is not necessary to design for torsion in statically indeterminate structures where the torsional forces are often only a secondary effect and the structure can be in equilibrium even if the torsion is neglected. When the equilibrium depends on the torsional resistance the effects of torsion must be considered.

7.7.1 Design Procedure for Torsion Combined with Shear

Notation

T_{Sd} Design torsion moment
T_{Rd1} Maximum torsional moment of resistance based on the compressive strut action
V_{Sd} Design shear force
V_{Rd1} Shear resistance of the concrete
V_{Rd2} Maximum shear resistance based on the web crushing

(1) Check if torsion reinforcement is required. For an approximately rectangular section torsional reinforcement is not required if

(a) $T_{Sd} \leqslant V_{Sd} b_w/4.5$

and (b) $V_{Sd}[1 + (4.5T_{Sd}/V_{Sd}b_w)] \leqslant V_{Rd1}$

where $V_{Rd1} = [\tau_{Rd} k(1.2 + 40\rho_1)]b_w d$

and $\tau = 0.035 f_{ck}^{2/3}$ or as given in table 5.1

$k = (1.6 - d)$ with d in metres

when $d \geqslant 0.6$ m, or if more than 50 per cent of the bottom reinforcement is curtailed $k = 1.0$

$\rho_1 = A_s/bd$ for the tension reinforcement which extends not less than $d + \ell_{b.net}$ beyond the section considered, $\ell_{b.net}$ being the anchorage bond length.

(2) Convert the section into an equivalent hollow box section of thickness t where (see figure 7.25b):

$$t = \frac{\text{Area of the section}}{\text{Perimeter of the section}} = \frac{A}{u}$$

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so that for a rectangular section $b \times h$

$$t = \frac{bh}{2(b + h)}$$

Calculate the area A_k within the centreline of the equivalent hollow box section.

For a rectangular section

$$A_k = (b - t)(h - t)$$

and the perimeter of the centreline is

$$u_k = 2(b + h - 2t)$$

(3) Check that the concrete section is adequate to resist the combined shear and torsion using the interaction condition

$$\left(\frac{T_{Sd}}{T_{Rd1}}\right)^2 + \left(\frac{V_{Sd}}{V_{Rd2}}\right)^2 \leqslant 1.0$$

where $T_{Rd1} = 1.33 v f_{ck} t A_k / (\cot\theta + \tan\theta)$

and $v = 0.7(0.7 - f_{ck}/200) \nless 0.35$

for closed stirrups around the outer periphery of the section, or

$$v = 0.7 - f_{ck}/200 \nless 0.5$$

for closed stirrups in both walls of the actual or equivalent hollow section.

The value of $\cot\theta$ is permitted to be taken between

$$0.67 \leqslant \cot\theta \leqslant 1.5$$

The value taken should be used throughout the design.

The calculation for V_{Rd2} should be based on the Variable Strut Inclination Method (equation 5.8) such that

$$V_{Rd2} = b_w z v f_{ck} / [1.5(\cot\theta + \tan\theta)]$$

$$v = 0.7 - f_{ck}/200 \nless 0.5$$

and $z = 0.9d$

(4) Design for shear using the Variable Strut Inclination Method of section 5.1.2.

(5) Calculate the additional stirrup reinforcement required to resist torsion from the equation

$$\frac{A_{sw}}{s} = \frac{T_{Sd}}{2A_k 0.87 f_{yk} \cot\theta}$$

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The spacing s of the stirrups should not exceed $u_k/8$ and the stirrups should be of the closed type fully anchored by means of laps.

(6) Calculate the area A_{sl} of the additional longitudinal reinforcement from

$$A_{sl} = \frac{T_{Sd} u_k \cot\theta}{2A_k \, 0.87 f_{ylk}}$$

This reinforcement should be arranged so that there is at least one bar at each corner with the other bars distributed equally around the inner periphery of the links spaced at not more than 350 mm centres.

Example 7.9 Design of Torsional Reinforcement

Torsional reinforcement is to be designed for the beam of example 7.7 with an ultimate torsional moment of $T_{Sd} = 48\,\text{kN m}$ in addition to the uniformly distributed loading already considered in the example.

The beam cross-section is shown in figure 7.25a. The steps in the calculations are numbered as outlined in the previous description of the design procedure. Steel grade $f_{yk} = 460\,\text{N/mm}^2$.

(1) Check if torsional reinforcement is required

As $V_{Sd} = 166\,\text{kN} > V_{Rd1} = 74.7\,\text{kN}$ the requirement

$$V_{Sd}[1 + (4.5T_{Sd}/V_{Sd}b_w)] \leqslant V_{Rd1}$$

is not satisfied therefore torsional reinforcement is required.

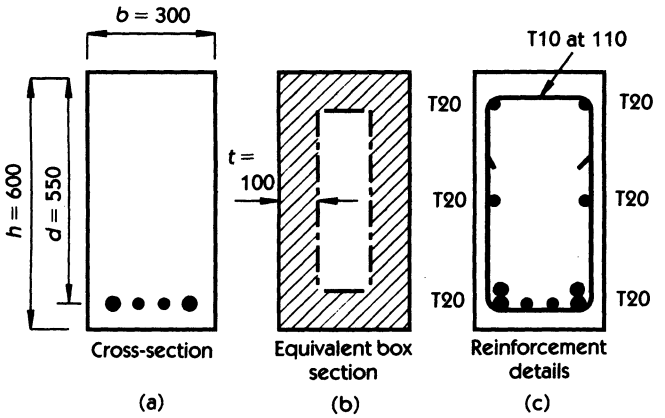

Figure 7.25 *Torsion example*

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(2) Convert to an equivalent hollow box section (see figure 7.25b)

Thickness of box section

$$t = \frac{A}{u} = \frac{600 \times 300}{2(600 + 300)} = 100\,\text{mm}$$

Area within centreline $A_k = (b - t)(h - t)$

$= 500 \times 200$

$= 100 \times 10^3\,\text{mm}^2$

Perimeter of centreline $= 2(b + h - 2t)$

$= 1400\,\text{mm}$

(3) Check if concrete section is adequate

$$\left(\frac{T_{Sd}}{T_{Rd1}}\right)^2 + \left(\frac{V_{Sd}}{V_{Rd2}}\right)^2 \leqslant 1.0$$

$$T_{Rd1} = 1.33 v f_{ck} t A_k / (\cot\theta + \tan\theta)$$

$$v = 0.7(0.7 - f_{ck}/200)$$

$$= 0.7(0.7 - 25/200) = 0.4 > 0.35$$

To minimise link reinforcement take $\cot\theta = 1.5$ so that $(\cot\theta + \tan\theta) = (1.5 + 1/1.5) = 2.17$

therefore $T_{Rd1} = 1.33 \times 0.4 \times 25 \times 100 \times 100 \times 10^3 \times 10^{-6}/2.17$

$= 61.3\,\text{kN m}$

From example 7.7, $V_{Sd} = 166\,\text{kN}$ at distance d from the support

$$V_{Rd2} = b_w z v f_{ck}/1.5(\cot\theta + \tan\theta)$$

$$v = 0.7 - f_{ck}/200 = 0.7 - 25/200$$

$$= 0.575 > 0.5$$

Take $z = 0.9d = 0.9 \times 550 = 495\,\text{mm}$

Hence $V_{Rd2} = (300 \times 495 \times 0.575 \times 25/1.5 \times 2.17)10^{-3}$

$= 656\,\text{kN}$

so $\left(\frac{48}{61.3}\right)^2 + \left(\frac{166}{656}\right)^2 = 0.613 + 0.064$

$= 0.677 < 1.0$

Therefore the concrete section is adequate.

(4) Design for shear using the Variable Strut Inclination Method (see equations 5.6 and 5.10, section 5.1.2)

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$$\frac{A_{sw}}{s} = \frac{V_{Sd}}{0.87 f_{yk} z \cot\theta}$$
$$= \frac{166 \times 10^3}{0.87 \times 460 \times 0.9 \times 550 \times 1.5}$$
$$= 0.56$$

For additional longitudinal steel

$$A_{sl} = \frac{0.5 V_{Sd} \cot\theta}{0.87 f_{yk}}$$
$$= \frac{0.5 \times 166 \times 10^3 \times 1.5}{0.87 \times 460}$$
$$= 311\,\text{mm}^2$$

(5) Calculate the additional link reinforcement required to resist torsion. Note that A_{sw} is for one leg only.

$$\frac{A_{sw}}{s} = \frac{T_{Sd}}{2A_k 0.87 f_{yk} \cot\theta}$$
$$= \frac{48.0 \times 10^6}{2 \times 100 \times 10^3 \times 0.87 \times 460 \times 1.5} = 0.40$$

Therefore for shear plus torsion and based on the area of two legs

$$\frac{A_{sw}}{s} = 0.56 + 2 \times 0.40 = 1.36$$

For T10 stirrups at 110 mm centres $A_{sw}/s = 1.43$.
Spacing $s = 110\,\text{mm} < u_k/8 = 175\,\text{mm}$.

(6) Calculate the area A_{s1} of the additional longitudinal reinforcement

$$A_{s1} = \frac{T_{Sd} u_k \cot\theta}{2A_k\, 0.87 f_{y1k}}$$
$$= \frac{(48 \times 10^6 \times 1400)1.5}{2 \times 100 \times 10^3 \times 0.87 \times 460}$$
$$= 1259\,\text{mm}^2$$

Total additional longitudinal steel = 311 + 1259
= 1570 mm^2

Arrangement of the proposed reinforcement is shown in figure 7.25c.

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8

Design of Reinforced Concrete Slabs

Reinforced concrete slabs are used in floors, roofs and walls of buildings and as the decks of bridges. The floor system of a structure can take many forms such as *in situ* solid slabs, ribbed slabs or precast units. Slabs may span in one direction or in two directions and they may be supported on monolithic concrete beams, steel beams, walls or directly by the structure's columns.

Continuous slabs should in principle be designed to withstand the most unfavourable arrangements of loads, in the same manner as beams. As for beams, bending moment coefficients, as given in table 8.1, may be used for one-way spanning slabs. These coefficients are similar to those in figure 7.22 for continuous beams and are as given in the Concise Code. If these coefficients are used the reinforcement must be of high ductility and the neutral-axis depth, x, should be no greater than 0.25 of the effective depth such that the lever arm, z ($= d - 0.8x/2$), is not less than $0.9d$. In addition, as for beams, table 8.1 should only be used when there are at least three spans that do not differ in length by more than 15 per cent and G_k should be greater than or equal to Q_k.

The moments in slabs spanning in two directions can also be determined using coefficients tabulated in the Concise Code. Slabs which are not rectangular in plan or which support an irregular loading arrangement may be analysed by techniques such as the yield line method or the Hilleborg strip method, as described in section 8.9.

Concrete slabs behave primarily as flexural members and the design is similar to that for beams, although in general it is somewhat simpler because

(1) the breadth of the slab is already fixed and a unit breadth of 1 m is used in the calculations

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(2) the shear stresses are usually low in a slab except when there are heavy concentrated loads, and
(3) compression reinforcement is seldom required.

Table 8.1 Ultimate bending moment and shear force coefficients in one-way spanning slabs

	At outer support	*Near middle of end span*	*At first interior support*	*At middle of interior spans*	*At interior supports*
Moment	0	$0.09Fl$	$0.09Fl$	$0.07Fl$	$0.07Fl$
Shear	$0.45F$	—	$0.6F$	—	$0.55F$

l = effective span.
F = total ultimate load = $1.35G_k + 1.50Q_k$.

8.1 Shear in Slabs

The shear resistance of a solid slab may be calculated by the procedures given in chapter 5. Experimental work has indicated that, compared with beams, shallow slabs fail at slightly higher shear stresses and this is incorporated into the values of the ultimate concrete shear resistance, V_{Rd1}, as given in equation 5.3. Calculations are usually based on a strip of slab 1 m wide.

Since shear stresses in slabs subject to uniformly distributed loads are generally small, shear reinforcement will seldom be required and it would be usual to design the slab such that the design ultimate shear force, V_{Sd}, is less than the shear strength of the unreinforced section, V_{Rd1}. In this case the Code does not require the provision of any shear reinforcement. As for beams, the section should also be checked to ensure that V_{Sd} does not exceed the maximum permissible shear force, V_{Rd2}. If shear reinforcement is required then the methods given in chapter 5 can be used, although practical difficulties concerned with bending and fixing shear reinforcement make it unlikely that shear reinforcement could be provided in slabs less than 200 mm thick.

Localised 'punching' actions due to heavy concentrated loads may, however, cause more critical conditions as shown in the following sections.

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8.1.1 Punching Shear – Analysis

A concentrated load on a slab causes shearing stresses on a section around the load; this effect is referred to as punching shear. The critical surface for checking punching shear is shown as the perimeter in figure 8.1 which is located at $1.5d$ from the loaded area. The maximum force that can be carried by the slab without shear reinforcement (V_{Rd1}) is given by a similar expression to that given in equation 5.3 for normal shear in beams and slabs:

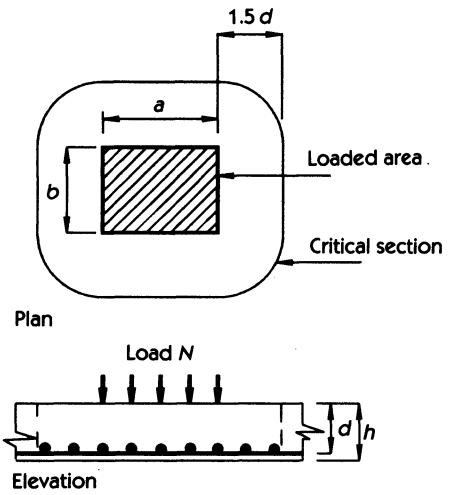

Figure 8.1 *Punching shear*

$$V_{Rd1} = [\tau_{Rd} k(1.2 + 40\rho_1)]du \tag{8.1}$$

where

τ_{Rd} = basic design shear strength given in table 5.1
k = $(1.6 - d)$ $\{>1\}$
ρ_1 = $\sqrt{(\rho_x \rho_y)}$ where ρ_x and ρ_y are the reinforcement ratios, A_s/bd, in the two mutually perpendicular directions (see table A.3 in the Appendix for A_s)
d = effective depth of section
u = length of the punching shear perimeter.

Checks must also be undertaken to ensure that the maximum permissible shear force (V_{Rd2}) is limited to the smaller of $0.9u_1 d\sqrt{f_{ck}}$ or $2V_{Rd1}$ where u_1 is the perimeter of the loaded area.

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Example 8.1 Punching Shear

A slab, 175 mm thick, average effective depth 145 mm is constructed with C25/30 concrete and reinforced with 12 mm bars at 150 mm centres one way (754 mm^2/m) and 10 mm bars at 200 mm centres in the other direction (393 mm^2/m). Determine the maximum ultimate load that can be carried on an area 300 × 400 mm.

From equation 8.1:

$$V_{Rd1} = [\tau_{Rd} k(1.2 + 40\rho_1)]du$$

where

$$\tau_{Rd} = 0.30\,N/mm^2 \text{ (table 5.1; for C25/30)}$$

$$k = (1.6 - 0.145) = 1.455$$

$$\rho_x = 754/(1000 \times 145) = 0.0052$$

$$\rho_y = 393/(1000 \times 145) = 0.0027$$

$$\rho_1 = \sqrt{(\rho_x \times \rho_y)}$$
$$= \sqrt{(0.0052 \times 0.0027)} = 0.0038$$

$$d = 145\,mm$$

$$u = (2a + 2b + 2 \times \pi \times 1.5d)$$
$$= 600 + 800 + 2 \times \pi \times 1.5 \times 145$$
$$= 2767\,mm$$

Hence, maximum ultimate load

$$V_{Rd1} = 0.30 \times 1.455(1.2 + 40 \times 0.0038)145 \times 2767 \times 10^{-3}$$
$$= 236\,kN$$

The maximum permissible shear force based on the face of the loaded area

$$V_{Rd2} = 0.9u_1 d\sqrt{f_{ck}}$$
$$= 0.9(600 + 800) \times 145 \times \sqrt{25} \times 10^{-3}$$
$$= 914\,kN$$

Hence the maximum load that the slab can carry is 236 kN.

8.1.2 Punching Shear – Reinforcement Design

If reinforcement is required to prevent punching shear around the critical zone shown in figure 8.1, it should be positioned within the failure zone and distributed around at least two perimeters. These perimeters should

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be located between the failure perimeter and a perimeter at no greater than $0.5d$ from the loaded area with a maximum spacing between these perimeters of $0.75d$. Provided that the slab is greater than 200 mm thick overall then the amount of reinforcement required is given by one of two equations:

when $V_{Rd1} < V_{Sd} \leqslant 1.6V_{Rd1}$:

$$\Sigma A_{sw} \sin\alpha \geqslant (V_{Sd} - V_{Rd1})/0.87f_{yk} \tag{8.2}$$

or when $1.6V_{Rd1} < V_{Sd} \leqslant 2.0V_{Rd1}$:

$$\Sigma A_{sw} \sin\alpha \geqslant (V_{Sd} - 1.4V_{Rd1})/0.29f_{yk} \tag{8.3}$$

where ΣA_{sw} is the total area of shear reinforcement provided within the critical zone and α is the angle between the shear reinforcement and the plane of the slab such that $\sin\alpha$ is 1 for vertical reinforcement.

Similar procedures must be applied to the regions of flat slabs which are close to supporting columns but allowances must be made for the effect of moment transfer from the columns as described in section 8.6.

Example 8.2 Design of Punching Shear Reinforcement

A 260 mm thick slab of grade 25/30 concrete is reinforced by 12 mm high yield bars at 125 mm centres in each direction. The slab is subject to a dry environment and must be able to carry a localised concentrated ultimate load of 650 kN over a square area of 300 mm side. Determine the shear reinforcement required for $f_{yk} = 250\,N/mm^2$.

For exposure class 1, cover required for a C25/30 concrete is 20 mm, thus average effective depth allowing for 8 mm links is equal to

$$260 - (20 + 8 + 12) = 220\,mm$$

Check maximum permissible shear force at face of loaded area

$$\text{Perimeter } u = 4 \times 300 = 1200\,mm$$

$$V_{Rd2} = 0.9ud\sqrt{f_{ck}} = 0.9 \times 1200 \times 220 \times \sqrt{25} \times 10^{-3}$$
$$= 1188\,kN > V_{Sd} = 650\,kN$$

The calculations for the punching are presented in tabular form in columns (i) to (vi) as shown.

(i) Critical perimeters are checked for punching shear starting $x = 1.5d$ from the face of the loaded area and then successive perimeters $0.75d$ further away.

(ii) Perimeter $u = 2(a + b + x\pi)$
So that for the first perimeter $x = 1.5d$ and

$$u = 2(300 + 300 + 1.5 \times 220\pi) = 3273\,mm$$

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(iii) Shear resistance of concrete $V_{Rd1} = [\tau_{Rd}k(1.2 + 40\rho_1)]du$ where

$$\tau_{Rd} = 0.30\,N/mm^2 \text{ (table 5.1; for C25/30)}$$
$$k = 1.6 - d = 1.6 - 0.22 = 1.38$$
$$\rho_1 = A_s/bd = 905/(1000 \times 220)$$
$$= 0.00411 > 0.002$$

Hence

$$V_{Rd1} = 0.3 \times 1.38(1.2 + 40 \times 0.002)220u \times 10^{-3}$$
$$= 0.1166u$$
$$= 382\,kN \text{ for the first perimeter}$$

(iv) The area A_{sw} of links required is obtained from equation 8.2 or 8.3. For the first perimeter $V_{Sd}/V_{Rd1} = 650/382 = 1.7 > 1.6$
Therefore $1.6\,V_{Rd1} < V_{Sd} < 2.0\,V_{Rd1}$ and

$$\Sigma A_{sw} = (V_{Sd} - 1.4\,V_{Rd1})/0.29f_{yk}$$
$$= (650 - 1.4 \times 382) \times 10^3/(0.29 \times 250)$$
$$= 1589\,mm^2$$

(v) The number of legs required for 8 mm links is

$$n_{req} = A_{sw}/(2\pi\phi^2/4) = A_{sw}/100.6$$

(vi) The links required are distributed proportionally around two perimeters within a failure zone starting at a perimeter $0.5d$ (= 110 mm) from the loaded face and then on perimeters successively $0.75d$ further away. Calculations for the first failure zone would show that the 16 links required would be distributed with 6 on the first and 10 on the second perimeter.
The number of links provided is however governed by the permissible maximum spacing of the legs in the transverse direction, which in this case is $0.6d$ (=132 mm) as given in section 7.3.1. In this example the legs are spaced at 125 mm to match the spacing of the bending reinforcement in the slab, resulting in the numbers shown in column (vi).

(i)	(ii)	(iii)	(iv)	(v)	(vi)
Dist. x	u	V_{Rd1}	A_{sw}	No. of links	
(mm)	(mm)	(kN)	(mm^2)	n_{req}	n_{prov}
$1.5d$	3273	382	1589	16	16
$2.25d$	4310	502	680	7	20
$3.0d$	5347	623	124	2	24
$3.75d$	6384	$744 > V_{Sd}$	0	0	0

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8.2 Span–Effective Depth Ratios

Excessive deflections of slabs will cause damage to the ceiling, floor finishes or other architectural finishes. To avoid this, limits are set on the span–depth ratio. These limits are exactly the same as those for beams as described in section 6.2. As a slab is usually a slender member, the restrictions on the span–depth ratio become more important and this can often control the depth of slab required. In terms of the span–effective depth ratio, the depth of slab is given by

$$\text{minimum effective depth} = \frac{\text{span}}{\text{basic ratio} \times \text{correction factors}}$$

The correction factors account for cases of spans greater than 7 metres and for flat slabs greater than 8.5 metres. The basic ratio may also be corrected to account for grades of steel other than grade 460 and for when more reinforcement is provided than that required for design at the ultimate limit state. (Also see figure A.2 in the Appendix.)

It may normally be assumed that, in assessing the basic ratio, slabs are lightly stressed although a more exact determination can be made from table 6.8 when the percentage of tension reinforcement is known. It can be seen from table 6.8 that the basic ratio can be increased by reducing the stress condition in the concrete. The concrete stress may be reduced by providing an area of tension reinforcement greater than that required to resist the design moment, or alternatively mild steel reinforcement with its lower characteristic strength may be used. It should be noted, however, as indicated elsewhere in this text, that mild steel is not included in prEN 10080 and once this becomes the accepted standard the use of mild steel may be prohibited with consequent implications on the design of slabs.

In the case of two-way spanning slabs, the check on the span–effective depth ratio should be based on the *shorter* span length. This does not apply to flat slabs where the longer span should be checked.

8.3 Reinforcement Details

To resist cracking of the concrete, codes of practice specify details such as the minimum area of reinforcement required in a section and limits to the maximum and minimum spacing of bars. Some of these rules are as follows.

(a) *Minimum Areas of Reinforcement*

$$\text{minimum area} = 0.6b_t d/f_{yk} \geqslant 0.0015b_t d$$

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in both directions, where b_t is the mean width of the section. The minimum reinforcement provision for crack control, as specified in section 6.1.5 may also have to be considered. Secondary reinforcement should not be less than 20 per cent of the main reinforcement.

(b) *Maximum Spacing of Bars*

Bar spacing should not exceed three times the overall depth of slab or 500 mm whichever is the lesser. This applies to both main and secondary reinforcement.

(c) *Reinforcement in the Flange of a T- or L-Beam*

This is described in detail in section 5.1.3.

(d) *Curtailment and Anchorage of Reinforcement*

The general rules for curtailment of bars in flexural members were discussed in section 7.2.5. Simplified rules for curtailment in different types of slab are illustrated in the subsequent sections of this chapter. At a simply supported end, at least half the span reinforcement should be anchored as specified in figure 7.14.

8.4 Solid Slabs Spanning in One Direction

The slabs are designed as if they consist of a series of beams of 1 m breadth. The main steel is in the direction of the span and secondary or distribution steel is required in the transverse direction. The main steel should form the outer layer of reinforcement to give it the maximum lever arm.

The calculations for bending reinforcement follow a similar procedure to that used in beam design. The lever arm curve of figure 4.5 is used to determine the lever arm (z) and the area of tension reinforcement is then given by

$$A_s = \frac{M}{0.87 f_{yk} z}$$

For solid slabs spanning one-way the simplified rules for curtailing bars as shown in figure 8.2 may be used provided that the loads are uniformly distributed. With a continuous slab it is also necessary that the spans are approximately equal. These simplified rules are not given in EC2 but are

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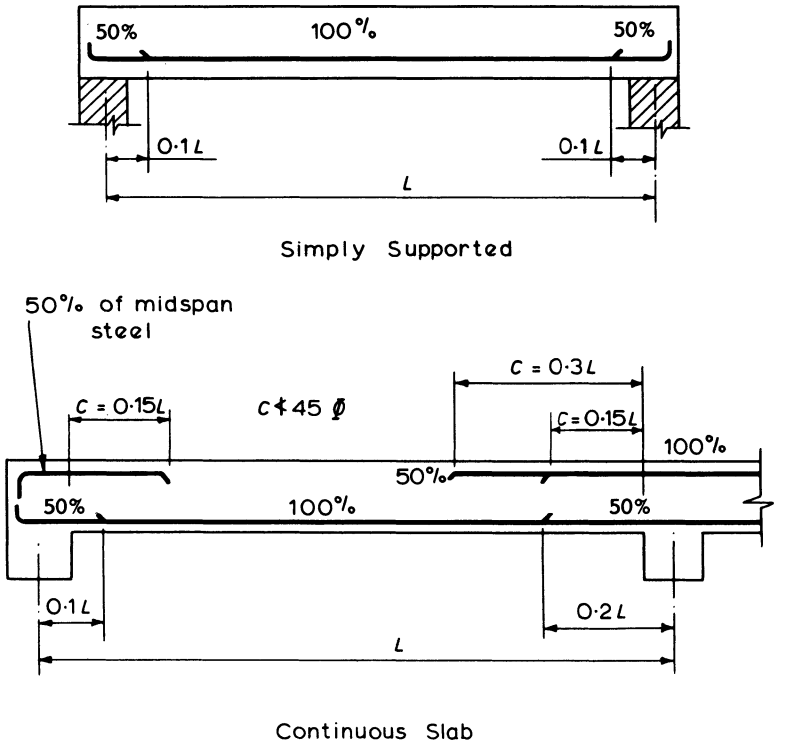

Figure 8.2 *Simplified rules for curtailment of bars in slab spanning in one direction*

recommended in the Concise Code on the basis of proven satisfactory performance established in previous codes of practice.

8.4.1 Single-Span Solid Slabs

The basic span–effective depth ratio for this type of slab is 21.7:1 on the basis that it is 'lightly stressed' and that grade 460 steel is used in the design. A figure as high as 29.6:1 may be used when the slab is 'nominally reinforced' and for a start-point in design a value between these two figures can be estimated and subsequently checked once the main tension reinforcement has been designed.

The effective span of the slab may be taken as the clear distance between the face of the supports plus a distance at both ends taken as the lesser of (i) the distance from the face of the support to its centreline and (b) one third the overall depth of the slab.

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Example 8.3 Design of a Simply Supported Slab

The slab shown in figure 8.3 is to be designed to carry a variable load of $3.0\,kN/m^2$ plus floor finishes and ceiling loads of $1.0\,kN/m^2$. The characteristic material strengths are $f_{ck} = 25\,N/mm^2$ and $f_{yk} = 460\,N/mm^2$. Basic span–effective depth ratio = 21.7 for a lightly stressed slab and 29.6 for a nominally reinforced slab.

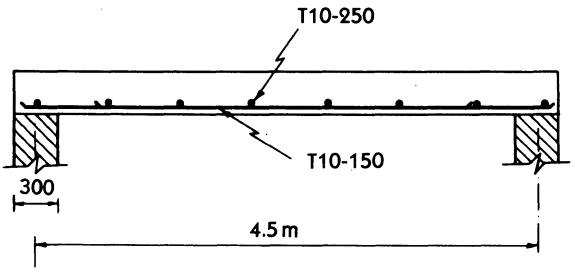

Figure 8.3

For simplicity take the effective span to be 4.5 metres.

(1) Estimate of Slab Depth

Try a basic span-depth ratio mid-way between the limits of 21.7 and 29.6:

$$\text{minimum effective depth} = \frac{\text{span}}{25.7 \times \text{correction factors (c.f.)}}$$

$$= \frac{4500}{25.7 \times \text{c.f.}} = \frac{175}{\text{c.f.}}$$

As high yield steel is being used the correction factor can be taken as unity. Try an effective depth of 175 mm. For a class 1 exposure the cover = 20 mm. Allowing, say, 5 mm as half the bar diameter of the reinforcing bar:

$$\text{overall depth of slab} = 175 + 20 + 5 = 200\,\text{mm}$$

Slab Loading

$$\text{self-weight of slab} = 200 \times 24 \times 10^{-3} = 4.8\,kN/m^2$$

$$\text{total permanent load} = 1.0 + 4.8 = 5.8\,kN/m^2$$

For a 1 m width of slab:

$$\text{ultimate load} = (1.35g_k + 1.5q_k)4.5$$

$$= (1.35 \times 5.8 + 1.5 \times 3.0)4.5 = 55.5\,kN$$

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$$M = 55.5 \times 4.5/8 = 31.2\,\text{kN m}$$

Bending Reinforcement

$$\frac{M}{bd^2 f_{ck}} = \frac{31.2 \times 10^6}{1000 \times 175^2 \times 25} = 0.041$$

From the lever-arm curve of figure 4.5, $l_a = 0.96$. Therefore, lever-arm $z = l_a d = 0.96 \times 175 = 168\,\text{mm}$:

$$A_s = \frac{M}{0.87 f_{yk} z} = \frac{31.2 \times 10^6}{0.87 \times 460 \times 168}$$
$$= 464\,\text{mm}^2/\text{m}$$

Provide T10 bars at 150 mm centres, $A_s = 523\,\text{mm}^2/\text{m}$ (as shown in table A.3 in the Appendix).

Check Span–Effective Depth Ratio

$$\frac{100 A_s}{bd} = \frac{100 \times 523}{1000 \times 175} = 0.299$$

Interpolating from table 6.8 this corresponds to a basic span–effective depth ratio of 26.2. The actual ratio = 4500/175 = 25.7; hence the chosen effective depth is acceptable.

Shear

At the face of the support

$$\text{Shear } V_{Sd} = \frac{55.5}{2}\left(\frac{2.25 - 0.5 \times 0.3}{2.25}\right)$$
$$= 25.9\,\text{kN}$$

$$V_{Rd1} = [\tau_{Rd} k(1.2 + 40\rho_1)]bd$$
$$= [0.3(1.6 - 0.175) \times (1.2 + 40 \times 0.00299)]$$
$$\times 1000 \times 175 \times 10^{-3}$$
$$= 98.7\,\text{kN}$$

As V_{Sd} is less than V_{Rd1} then no shear reinforcement is required.

End Anchorage (figure 7.14)

From the table of anchorage lengths in the Appendix the tension anchorage length = $37\phi = 37 \times 10 = 370\,\text{mm}$. Hence required anchorage

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beyond face of support = 2/3 × 370 = 247 mm. At least half of the tension steel should be anchored in this way.

Distribution Steel

Provide minimum = $0.0015bd$

$= 0.0015 \times 1000 \times 175 = 263\,\text{mm}^2/\text{m}$

Provide T10 at 250 mm centres (314 mm^2/m).

(2) The second part of this example illustrates how a smaller depth of slab is adequate provided it is reinforced with steel of low characteristic strength and therefore a high span–effective depth ratio. Try a thickness of slab, $h = 170$ mm and $d = 145$ mm:

self-weight of slab $= 0.17 \times 24 = 4.08\,\text{kN/m}^2$

total permanent load $= 5.08\,\text{kN/m}^2$

ultimate load $= (1.35g_k + 1.5q_k)$

$= (1.35 \times 5.08 + 1.5 \times 3.0)4.5 = 51.1\,\text{kN}$

Bending Reinforcement

$$M = 51.1 \times \frac{4.5}{8} = 28.8\,\text{kN m}$$

$$\frac{M}{bd^2 f_{ck}} = \frac{28.8 \times 10^6}{1000 \times 145^2 \times 25} = 0.055$$

From the lever-arm curve of figure 4.5, $l_a = 0.95$. Therefore, lever-arm $z = l_a d = 0.95 \times 145 = 138$ mm. Using mild steel:

$$A_s = \frac{M}{0.87 f_{yk} z} = \frac{28.8 \times 10^6}{0.87 \times 250 \times 138}$$

$$= 960\,\text{mm}^2/\text{m}$$

Provide R12 bars at 100 mm centres, $A_s = 1130\,\text{mm}^2/\text{m}$.

Check Span–Effective Depth Ratio

$$\frac{100A_s}{bd} = \frac{100 \times 1130}{1000 \times 145} = 0.779$$

Interpolating from table 6.8 this corresponds to a basic span–effective depth ratio of 20.0. This can now be modified to account for the steel grade and the amount of tension steel provided:

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$$\text{Limiting} \quad \frac{\text{Span}}{\text{Eff. depth}} = \text{basic ratio} \times \frac{460}{f_{yk}} \times \frac{A_{s.prov}}{A_{s.req}}$$

$$= 20.0 \times \frac{460}{250} \times \frac{1130}{960}$$

$$= 43.3$$

$$\text{Actual} \quad \frac{\text{Span}}{\text{Eff. depth}} = \frac{45\,000}{145} = 31.0$$

Therefore $d = 145$ mm is adequate.

8.4.2 Continuous Solid Slab Spanning in One Direction

For a continuous slab, bottom reinforcement is required within the span and top reinforcement over the supports. The effective span is the distance between the centreline of the supports and the basic span–effective depth ratio of an interior span varies from 30.4 for 'lightly stressed' to 41.7 for 'nominally reinforced' where grade 460 steel is used. The corresponding limits for an end span are 27.8 and 38.3 respectively.

If the conditions given on page 227 are met, the bending moment and shear force coefficients given in table 8.1 may be used.

Example 8.4 Design of a Continuous Solid Slab

The four-span slab shown in figure 8.4 supports a variable load of 3.0 kN/m^2 plus floor finishes and a ceiling load of 1.0 kN/m^2. The characteristic material strengths are $f_{ck} = 25$ N/mm^2 and $f_{yk} = 460$ N/mm^2.

Estimate of Slab Depth

As the end span is more critical than the interior spans, try a basic span–effective depth ratio mid-way between the end-span limits of 27.8 and 38.3:

$$\text{minimum effective depth} = \frac{\text{span}}{33.0 \times \text{correction factors}}$$

$$= \frac{4500}{33.0 \times \text{c.f.}} = \frac{136}{\text{c.f.}}$$

As high yield steel is being used the correction factor can be taken as unity. Try an effective depth of 140 mm. For a class 1 exposure the cover = 20 mm. Allowing, say, 5 mm as half the bar diameter of the reinforcing bar:

$$\text{overall depth of slab} = 140 + 20 + 5 = 165 \text{ mm}$$

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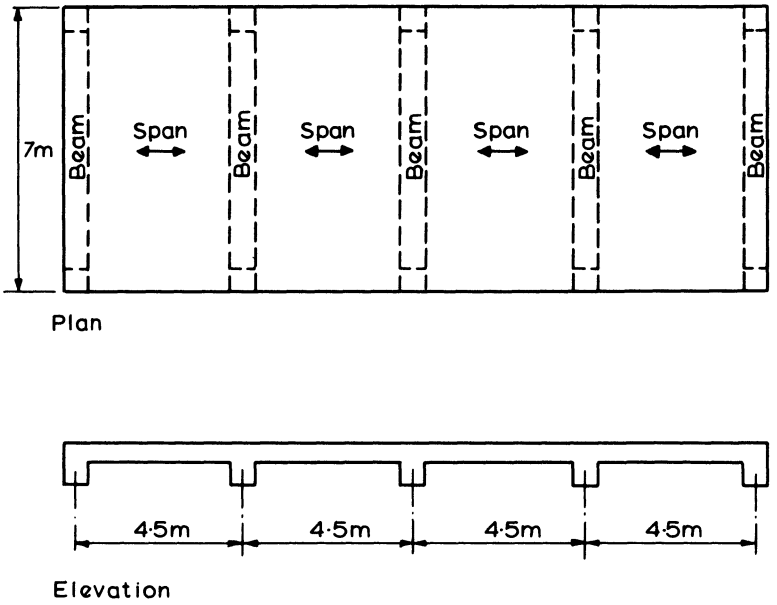

Figure 8.4 *Continuous slab*

Slab Loading

$$\text{self-weight of slab} = 165 \times 24 \times 10^{-3} = 3.96\,\text{kN/m}^2$$

$$\text{total permanent load} = 1.0 + 3.96 = 4.96\,\text{kN/m}^2$$

For a 1 m width of slab

$$\text{ultimate load, } F = (1.35g_k + 1.5q_k)4.5$$
$$= (1.35 \times 4.96 + 1.5 \times 3.0)4.5 = 50.38\,\text{kN}$$

Using the coefficients of table 8.1 the moment at the middle of the end span is given by

$$M = 0.09Fl = 0.09 \times 50.38 \times 4.5 = 20.40\,\text{kN m}$$

Bending Reinforcement

$$\frac{M}{bd^2f_{ck}} = \frac{20.40 \times 10^6}{1000 \times 140^2 \times 25} = 0.0416$$

From the lever-arm curve of figure 4.5, $l_a = 0.96$. Therefore, lever-arm $z = l_a d = 0.96 \times 140 = 134\,\text{mm}$:

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$$A_s = \frac{M}{0.87 f_{yk} z} = \frac{20.40 \times 10^6}{0.87 \times 460 \times 134}$$
$$= 380\,\text{mm}^2/\text{m}$$

Provide T10 bars at 200 mm centres, $A_s = 393\,\text{mm}^2/\text{m}$.

Check Span–Effective Depth Ratio

$$\frac{100A_s}{bd} = \frac{100 \times 393}{1000 \times 140} = 0.281$$

Interpolating from table 6.8 this corresponds to a basic span–effective depth ratio of 34.4. The actual ratio = 4500/140 = 32.1; hence the chosen effective depth is acceptable.

Similar calculations for the supports and the interior span give the steel areas shown in figure 8.5.

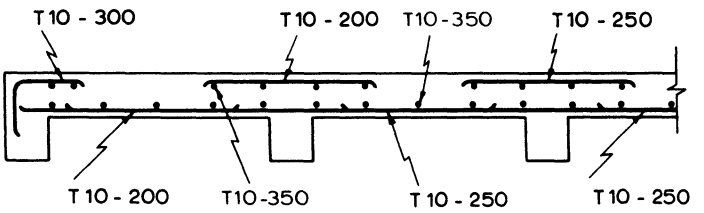

Figure 8.5

At the end supports there is a monolithic connection between the slab and the beam, therefore top steel should be provided to resist any negative moment. The moment to be designed for is a minimum of 25 per cent of the span moment, that is 5.1 kN m. In fact, to provide a minimum of 0.15 per cent of steel T10 bars at 300 mm centres have been specified. The layout of the reinforcement in figure 8.5 is according to the simplified rules for curtailment of bars in slabs as illustrated in figure 8.2.

$$\text{Transverse reinforcement} = 0.0015bd$$
$$= 0.0015 \times 1000 \times 140$$
$$= 210\,\text{mm}^2/\text{m}$$

Provide T10 at 350 mm centres top and bottom, wherever there is main reinforcement (224 mm^2/m).

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8.5 Solid Slabs Spanning in Two Directions

When a slab is supported on all four of its sides it effectively spans in both directions, and it is sometimes more economical to design the slab on this basis. The amount of bending in each direction will depend on the ratio of the two spans and the conditions of restraint at each support.

If the slab is square and the restraints are similar along the four sides then the load will span equally in both directions. If the slab is rectangular then more than one-half of the load will be carried in the stiffer, shorter direction and less in the longer direction. If one span is much longer than the other, a large proportion of the load will be carried in the short direction and the slab may as well be designed as spanning in only one direction.

Moments in each direction of span are generally calculated using coefficients which are tabulated in the Concise Code. Areas of reinforcement to resist the moments are determined independently for each direction of span. The slab is reinforced with bars in both directions parallel to the spans with the steel for the shorter span placed furthest from the neutral axis to give it the greater effective depth.

The span–effective depth ratios are based on the shorter span and the percentage of reinforcement in that direction.

With a uniformly distributed load the loads on the supporting beams may generally be apportioned as shown in figure 8.6.

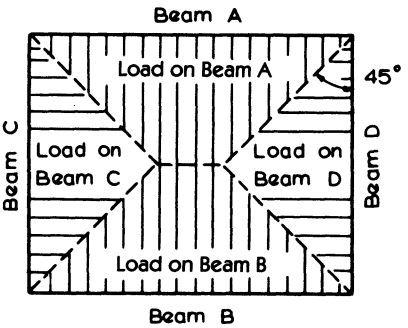

Figure 8.6 *Loads carried by supporting beams*

8.5.1 Simply Supported Slab Spanning in Two Directions

A slab simply supported on its four sides will deflect about both axes under load and the corners will tend to lift and curl up from the supports, causing torsional moments. When no provision has been made to prevent

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Table 8.2 Bending-moment coefficients for slabs spanning in two directions at right angles, simply supported on four sides

l_y/l_x	1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0
α_{sx}	0.062	0.074	0.084	0.093	0.099	0.104	0.113	0.118
α_{sy}	0.062	0.061	0.059	0.055	0.051	0.046	0.037	0.029

this lifting or to resist the torsion then the moment coefficients of table 8.2 may be used and the maximum moments are given by

$$M_{sx} = \alpha_{sx} n l_x^2 \quad \text{in direction of span } l_x$$

and

$$M_{sy} = \alpha_{sy} n l_x^2 \quad \text{in direction of span } l_y$$

where M_{sx} and M_{sy} are the moments at mid-span on strips of unit width with spans l_x and l_y respectively, and

$n = (1.35g_k + 1.5q_k)$, that is the total ultimate load per unit area
l_y = the length of the longer side
l_x = the length of the shorter side

and α_{sx} and α_{sy} are the moment coefficients from table 8.2.

The area of reinforcement in directions l_x and l_y respectively are

$$A_{sx} = \frac{M_{sx}}{0.87 f_{yk} z} \quad \text{per metre width}$$

and

$$A_{sy} = \frac{M_{sy}}{0.87 f_{yk} z} \quad \text{per metre width}$$

The slab should be reinforced uniformly across the full width, in each direction.

The effective depth d used in calculating A_{sy} should be less than that for A_{sx} because of the different depths of the two layers of reinforcement.

At least 40 per cent of the mid-span reinforcement should extend to the supports and the remaining 60 per cent should extend to within $0.1l_x$ or $0.1l_y$ of the appropriate support.

It should be noted that the above method is not specifically mentioned in either EC2 or the Concise Code. However, as the method was deemed acceptable in BS8110, its continued use should be an acceptable method of analysing this type of slab.

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Example 8.5 Design the Reinforcement for a Simply Supported Slab 220 mm Thick and Spanning in Two Directions

The effective span in each direction is 4.5 m and 6.3 m and the slab supports a variable load of 10 kN/m^2. The characteristic material strengths are $f_{ck} = 25\,\text{N/mm}^2$ and $f_{yk} = 460\,\text{N/mm}^2$.

$$l_y/l_x = 6.3/4.5 = 1.4$$

From table 8.2, $\alpha_{sx} = 0.099$ and $\alpha_{sy} = 0.051$.

$$\text{Self-weight of slab} = 220 \times 24 \times 10^{-3} = 5.3\,\text{kN/m}^2$$

$$\text{ultimate load } n = 1.35g_k + 1.5q_k$$
$$= 1.35 \times 5.3 + 1.5 \times 10.0 = 22.16\,\text{kN/m}^2$$

Bending – Short Span

With class 1 exposure conditions take $d = 185\,\text{mm}$.

$$M_{sx} = \alpha_{sx} n l_x^2 = 0.099 \times 22.16 \times 4.5^2$$
$$= 44.4\,\text{kN m}$$

$$\frac{M_{sx}}{bd^2 f_{ck}} = \frac{44.4 \times 10^6}{1000 \times 185^2 \times 25} = 0.052$$

From the lever-arm curve, figure 4.5, $l_a = 0.95$. Therefore

$$\text{lever-arm } z = 0.95 \times 185 = 176\,\text{mm}$$

and

$$A_s = \frac{M_{sx}}{0.87 f_{yk} z} = \frac{44.4 \times 10^6}{0.87 \times 460 \times 176}$$
$$= 630\,\text{mm}^2/\text{m}$$

Provide T12 at 175 mm centres, $A_s = 646\,\text{mm}^2/\text{m}$

Span–Effective Depth Ratio

$$\frac{100A_s}{bd} = \frac{100 \times 646}{1000 \times 185} = 0.349$$

From table 6.8, this corresponds to a basic span–effective depth ratio of 25.1:

$$\text{basic} \quad \frac{\text{span}}{\text{effective depth}} = 25.1$$

$$\text{actual} \quad \frac{\text{span}}{\text{effective depth}} = \frac{4500}{185} = 24.3$$

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Thus $d = 185\,\text{mm}$ is adequate.

Bending – Long Span

$$M_{sy} = \alpha_{sy} n l_x^2 = 0.051 \times 22.16 \times 4.5^2$$
$$= 22.9\,\text{kN m}$$

Since the reinforcement for this span will have a reduced effective depth, take $z = 176 - 12 = 164\,\text{mm}$. Therefore

$$A_s = \frac{M_{sy}}{0.87 f_{yk} z} = \frac{22.9 \times 10^6}{0.87 \times 460 \times 164}$$
$$= 349\,\text{mm}^2/\text{m}$$

Provide T10 at 200 mm centres, $A_s = 393\,\text{mm}^2/\text{m}$

$$\frac{100 A_s}{bd} = \frac{100 \times 393}{1000 \times 164} = 0.24$$

which is greater than 0.15, the minimum for transverse high yield steel.

The arrangement of the reinforcement is shown in figure 8.7.

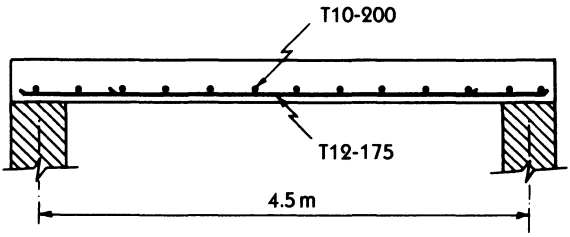

Figure 8.7 *Simply supported slab spanning in two directions*

8.5.2 Restrained Slab Spanning in Two Directions

When the slabs have fixity at the supports and reinforcement is added to resist torsion and to prevent the corners of the slab from lifting then the maximum moments per unit width are given by

$$M_{sx} = \beta_{sx} n l_x^2 \quad \text{in direction of span } l_x$$

and

$$M_{sy} = \beta_{sy} n l_x^2 \quad \text{in direction of span } l_y$$

where β_{sx} and β_{sy} are the moment coefficients given in table A.2 of the

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Concise Code for the specified end conditions, and $n = (1.35g_k + 1.5q_k)$, the total ultimate load per unit area.

The slab is divided into middle and edge strips as shown in figure 8.8 and reinforcement is required in the middle strips to resist M_{sx} and M_{sy}. The arrangement this reinforcement should take is illustrated in figure 8.2. In the edge strips only nominal reinforcement is necessary, such that $100A_s/bd = 0.15$ for high yield steel or 0.24 for mild steel.

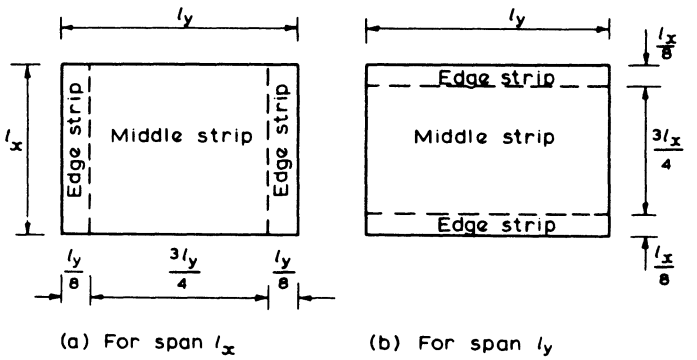

Figure 8.8 *Division of slab into middle and edge strips*

In addition, torsion reinforcement is provided at discontinuous corners and it should:

(1) consist of top and bottom mats, each having bars in both directions of span
(2) extend from the edges a minimum distance $l_x/5$
(3) at a corner where the slab is discontinuous in both directions have an area of steel in each of the four layers equal to three-quarters of the area required for the maximum mid-span moment.
(4) at a corner where the slab is discontinuous in one direction only, have an area of torsion reinforcement only half of that specified in rule 3.

Torsion reinforcement is not, however, necessary at any corner where the slab is continuous in both directions.

Where $l_y/l_x > 2$, the slabs should be designed as spanning in one direction only.

Shear force coefficients are also given in the Concise Code for cases where torsion corner reinforcement is provided, and these are based on a simplified distribution of load to supporting beams which may be used in preference to the distribution shown in figure 8.6. It should be noted that

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the coefficients for both shear and moments can only be used if high ductility reinforcement is specified and the ratio x/d is limited to 0.25.

Example 8.6 Moments in a Continuous Two-way Slab

The panel considered is an edge panel, as shown in figure 8.9 and the uniformly distributed load, $n = (1.35g_k + 1.5q_k) = 10\,\text{kN/m}^2$.

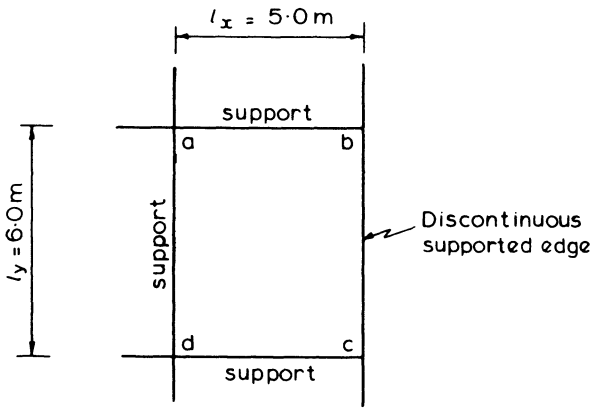

Figure 8.9 *Continuous panel spanning in two directions*

The moment coefficients are taken from case 3 of table A.2 of the Concise Code.

$$\frac{l_y}{l_x} = \frac{6.0}{5.0} = 1.2$$

Positive moments at mid-span

$$M_{sx} = \beta_{sx} n l_x^2 = 0.042 \times 10 \times 5^2$$
$$= 10.5\,\text{kN m in direction } l_x$$

$$M_{sy} = \beta_{sy} n l_x^2 = 0.028 \times 10 \times 5^2$$
$$= 7.0\,\text{kN m in direction } l_y$$

Negative moments

$$\text{Support ad, } M_x = 0.056 \times 10 \times 5^2 = 14\,\text{kN m}$$
$$\text{Supports ab and dc, } M_y = 0.037 \times 10 \times 5^2 = 9.3\,\text{kN m}$$

The moments calculated are for a metre width of slab.

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The design of reinforcement to resist these moments would follow the usual procedure. Torsion reinforcement, according to rule 4 is required at corners b and c. A check would also be required on the span–effective depth ratio of the slab.

8.6 Flat Slab Floors

A flat slab floor is a reinforced concrete slab supported directly by concrete columns without the use of intermediary beams. The slab may be of constant thickness throughout or in the area of the column it may be thickened as a drop panel. The column may also be of constant section or it may be flared to form a column head or capital. These various forms of construction are illustrated in figure 8.10.

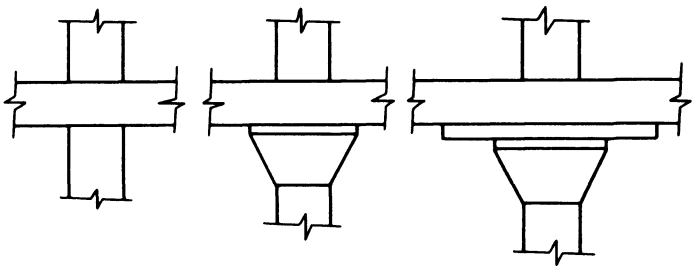

Figure 8.10 *Drop panels and column heads*

The drop panels are effective in reducing the shearing stresses where the column is liable to punch through the slab, and they also provide an increased moment of resistance where the negative moments are greatest.

The flat slab floor has many advantages over the beam and slab floor. The simplified formwork and the reduced storey heights make it more economical. Windows can extend up to the underside of the slab, and there are no beams to obstruct the light and the circulation of air. The absence of sharp corners gives greater fire resistance as there is less danger of the concrete spalling and exposing the reinforcement. Deflection requirements will generally govern slab thicknesses which should not be less than 125 mm.

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The analysis of a flat slab structure may be carried out by dividing the structure into a series of equivalent frames. The moments in these frames may be determined by

(a) a method of frame analysis such as moment distribution, or the stiffness method on a computer
(b) a simplified method using the moment and shear coefficients of table 8.1 subject to the following requirements:
    (i) the lateral stability is not dependent on the slab-column connections
    (ii) the conditions for using table 8.1 described on page 227 are satisfied
    (iii) there are at least three rows of panels of approximately equal span in the direction being considered.

Interior panels of the flat slab should be divided as shown in figure 8.11 into column and middle strips. Drop panels should be ignored if their smaller dimension is less than the 1/3 of the smaller panel dimension l_x. If a panel is not square, strip widths in both directions are based on l_x.

Moments determined from a structural analysis or the coefficients of table 8.1 are distributed between the strips as shown in table 8.3.

Reinforcement designed to resist these slab moments may be detailed according to the simplified rules for slabs, and satisfying normal spacing

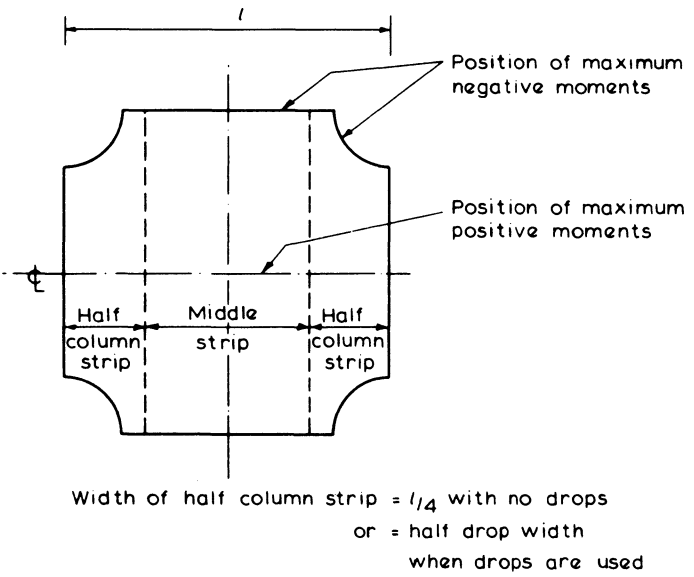

Figure 8.11 *Flat slab divided into strips*

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Table 8.3 Division of moments between strips

	Column strip	*Middle strip*
Negative moment at edge column	100% but not more than $0.18b_e d^2 f_{ck}$	0
Negative moment at internal column	75%	25%
Positive moment in span	55%	45%

b_e = width of edge strip.

limits. This should be spread across the respective strip but, in solid slabs without drops, steel to resist negative moments in column strips should have two-thirds of the area located in the central 1/2 strip width. If the column strip is narrower because of drops, the moments resisted by the column and middle strips should be adjusted proportionally as illustrated in example 8.7.

Column moments can be calculated from the analysis of the equivalent frame. Particular care is needed over the transfer of moments to edge columns. This is to ensure that there is adequate moment capacity within the slab adjacent to the column since moments will only be able to be transferred to the edge column by a strip of slab considerably narrower than the normal internal panel column strip width. As seen in table 8.3, a limit is placed on the negative moment transferred to an edge column. If exceeded the moment should be limited to this value and the positive moment increased to maintain equilibrium.

The reinforcement for a flat slab should generally be arranged according to the rules illustrated in figure 8.2.

Important features in the design of the slabs are the calculations for punching shear at the head of the columns and at the change in depth of the slab, if drop panels are used. The design for shear should take the procedure described in the previous section on punching shear except that EC2 requires that the design shear force be increased above the calculated value by 15 per cent for internal columns, up to 40 per cent for edge columns and 50 per cent for corner columns, to allow for the effects of moment transfer. In this respect it can be advantageous to use mild steel in the design, as the resulting higher percentages of reinforcement will allow a correspondingly higher ultimate concrete shear force.

In considering punching shear, EC2 places additional requirements on the amount and distribution of reinforcement around column heads to ensure that full punching shear capacity is developed. Column strips, as

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indicated in figure 8.12, must be reinforced to resist the minimum design moments indicated in table 8.4. The reinforcement provided to resist the normal design loads must be sufficient to ensure that these minimum moment requirements are also met.

The usual basic span–effective depth ratios may be used but where the greater span exceeds 8.5 m the basic ratio should be multiplied by 8.5/span. For flat slabs the span–effective depth calculation should be based on the longer span.

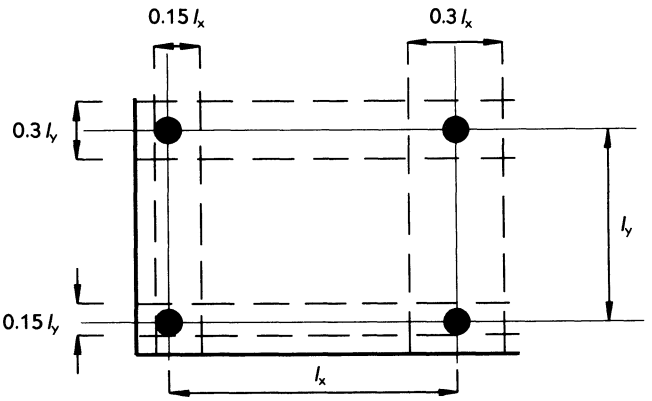

Figure 8.12 *Column strips to be reinforced*

Table 8.4 Minimum moments for punching shear resistance

Case	*Strip width*	*Total hogging moment*	*Total sagging moment*
Internal column	$0.3l_p$	$0.0375V_{Sd}l_p$	—
Edge column perpendicular to edge	$0.3l_p$	$0.0375V_{Sd}l_p$	$0.0375V_{Sd}l_p$
Edge column parallel to edge	$0.15l_p$	$0.0375V_{Sd}l_p$	—
Corner Column	$0.15l_p$	$0.075V_{Sd}l_p$	$0.075V_{Sd}l_p$

For moments in x direction $l_p = l_y$.
For moments in y direction $l_p = l_x$.
V_{Sd} = effective design punching shear force.

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Reference should be made to codes of practice for further detailed information describing the requirements for the analysis and design of flat slabs.

Example 8.7 Design of a Flat Slab

The columns are at 6.5 m centres in each direction and the slab supports a variable load of 5 kN/m^2. The characteristic material strengths are f_{ck} = 25 N/mm^2 and f_{yk} = 460 N/mm^2 for the reinforcement.

It is decided to use a floor slab as shown in figure 8.13 with 250 mm overall depth of slab, and drop panels 2.5 m square by 100 mm deep. The column heads are to be made 1.4 m diameter.

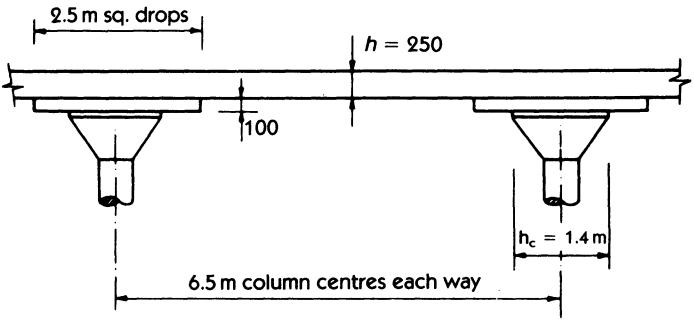

Figure 8.13

Permanent load

Weight of slab = $0.25 \times 24 \times 6.5^2 = 253.5$ kN

Weight of drop = $0.1 \times 24 \times 2.5^2 = 15.0$ kN

Total = 268.5 kN

Variable load

Total = $5 \times 6.5^2 = 211.3$ kN

Therefore

ultimate load on the floor, $F = 1.35 \times 268.5 + 1.5 \times 211.3$
= 680 kN per panel

and equivalent distributed load, $n = \dfrac{680}{6.5^2} = 16.1$ kN/m^2

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$$\text{The effective span, } L = \text{clear span} - 2h_c/3$$
$$= 6.5 - 2 \times 1.4/3$$
$$= 5.6\,\text{m}$$

A concrete cover of 20 mm has been allowed, and where there are two equal layers of reinforcement the effective depth has been taken as the mean depth of the two layers in calculating the reinforcement areas.

The drop dimension is greater than one-third of the panel dimension, therefore the column strip is taken as the width of the drop panel (2.5 m).

Bending Reinforcement

Since the variable load is less than the permanent load, from table 8.1:

(1) Centre of interior span

$$\text{Positive moment} = 0.07Fl$$
$$= 0.07 \times 680 \times 5.6 = 267\,\text{kN m}$$

The width of the middle strip is $(6.5 - 2.5) = 4$ m which is greater than half the panel dimension, therefore the proportion of this moment taken by the middle strip is given by

$$0.45 \times \frac{4}{6.5/2} = 0.55$$

Thus middle strip positive moment $= 0.55 \times 267 = 147$ kN m.
The column strip positive moment $= (1 - 0.55) \times 267 = 120$ kN m.

(a) For the middle strip

$$\frac{M}{bd^2 f_{ck}} = \frac{147 \times 10^6}{4000 \times 210^2 \times 25} = 0.033$$

From the lever-arm curve, figure 4.5, $l_a = 0.97$, therefore

$$A_s = \frac{M}{0.87 f_{yk} l_a d} = \frac{147 \times 10^6}{0.87 \times 460 \times 0.97 \times 210}$$
$$= 1803\,\text{mm}^2 \text{ bottom steel}$$

Thus provide nine T16 bars ($A_s = 1810\,\text{mm}^2$) each way in the span, distributed evenly across the 4 m width of the middle strip (spacing = 500 mm = maximum allowable for a slab).

(b) The column strip moments will require $1472\,\text{mm}^2$ bottom steel which can be provided as eight T16 bars ($A_s = 1610\,\text{mm}^2$) in the span distributed evenly across the 2.5 m width of the column strip.

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(2) Interior support

$$\text{Negative moment} = -0.07Fl$$
$$= -0.07 \times 680 \times 5.6 = 267\,\text{kN m}$$

and this is also divided into

$$\text{middle strip} = 0.25 \times \frac{4 \times 267}{6.5/2} = 0.31 \times 267$$
$$= 83\,\text{kN m}$$

$$\text{and column strip} = 0.69 \times 267 = 184\,\text{kN m}$$

(a) For the middle strip

$$\frac{M}{bd^2 f_{ck}} = \frac{83 \times 10^6}{4000 \times 210^2 \times 25} = 0.019$$

From the lever-arm curve, figure 4.5, $l_a = 0.98$, therefore

$$A_s = \frac{M}{0.87 f_{yk} l_a d} = \frac{83 \times 10^6}{0.87 \times 460 \times 0.98 \times 210} = 1008\,\text{mm}^2$$

Provide nine evenly spaced T12 bars as top steel ($A_s = 1020\,\text{mm}^2$).

(b) For the column strip

$$\text{Design moment} = 184\,\text{kN m} = 184/2.5 = 74\,\text{kN m/m}$$

This should be checked against the minimum moment requirement for punching shear as given in table 8.4.

$$\text{Minimum moment} = 0.0375\, V_{Sd} l_p = 0.0375 \times 754 \times 6.5$$
$$= 184\,\text{kN m}$$

which should be provided over a strip of width $0.3 l_p = 0.3 \times 6.5 = 1.95\,\text{m}$. The value of V_{Sd} in this calculation is calculated in the *punching shear* calculations below.

Equivalent moment/unit width = 184/1.95 = 94.4 kN m/m.

Hence design for a moment of 94.4 kN m/m

$$\frac{M}{bd^2 f_{ck}} = \frac{94.4 \times 10^6}{1000 \times 310^2 \times 25} = 0.039$$

From the lever-arm curve, figure 4.5, $l_a = 0.96$, therefore

$$A_s = \frac{M}{0.87 f_{yk} l_a d} = \frac{94.4 \times 10^6}{0.87 \times 460 \times 0.96 \times 310} = 793\,\text{mm}^2/\text{m}$$

Provide T16 bars as top steel at 200 mm centres ($A_s = 1010\,\text{mm}^2/\text{m}$). This is equivalent to fourteen bars over the full 2.5 m width of the column

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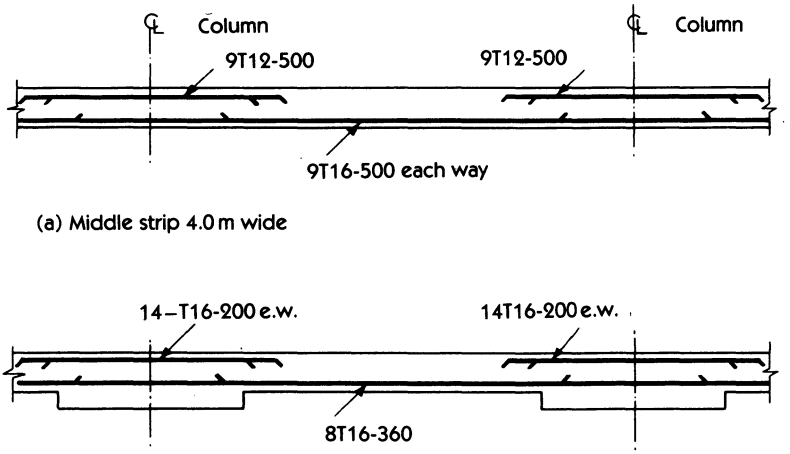

Figure 8.14 *Details of bending reinforcement*

strip. The bending reinforcement requirements are summarised in figure 8.14.

Punching Shear

(1) At the column head

$$\text{perimeter } u = \pi \times \text{diameter of column head}$$
$$= \pi \times 1400 = 4398\,\text{mm}$$

$$\text{shear force } V_{\text{Sd}} = F - \frac{\pi}{4}1.4^2 n = 680 - \frac{\pi}{4}1.4^2 \times 16.1$$
$$= 655.2\,\text{kN}$$

To allow for the effects of moment transfer, V is increased by 15 per cent, thus

$$V_{\text{Sd.eff}} = 1.15 \times 656.2 = 754\,\text{kN}$$

The maximum permissible shear force, V_{Rd2}, is given by

$$V_{\text{Rd2}} = 0.9u_1 d\sqrt{f_{\text{ck}}} = 0.9 \times 4398 \times 310 \times \sqrt{25} \times 10^{-3} = 6135\,\text{kN}$$

$V_{\text{Sd.eff}}$ is less than V_{Rd2}.

(2) The critical section for shear is 1.5 × effective depth from the face

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of the column head, that is, a section of diameter $1.4 + 2 \times 1.5 \times 0.310 = 2.33$ m.

Thus the length of the perimeter $u = \pi \times 2330$

$= 7320$ mm

Ultimate shear force, $V_{Sd} = 680 - \frac{\pi}{4} \times 2.33^2 \times 16.1$

$= 611$ kN

$V_{Sd.eff} = 1.15 \times 611 = 703$ kN

$$V_{Rd1} = [\tau_{Rd} k(1.2 + 40\rho_1)]du$$

where

$\tau_{Rd} = 0.30$ N/mm^2 (table 5.1, for C25/30)

$k = (1.6 - 0.310) = 1.29$

$\rho_x = 1010/(1000 \times 310) = 0.0033$(T16 at 200 mm centres))

$\rho_y = 1010/(1000 \times 310) = 0.0033$

$\rho_1 = \sqrt{(\rho_x \times \rho_y)}$

$= 0.0033$

$d = 310$ mm

$u = 7320$ mm

therefore

$$V_{Rd1} = [0.3 \times 1.29 \times (1.2 + 40 \times 0.0033)] \times 310 \times 7320 \times 10^{-3}$$

$= 1170$ kN

As $V_{Sd.eff}$ is less than V_{Rd1} the section is adequate.

(3) At the dropped panel the critical section is $1.5 \times 210 = 315$ mm from the panel with a perimeter given by

$$u = (2a + 2b + 2\pi \times 1.5d)$$

$$= (4 \times 2500 + 2\pi \times 315) = 11\,979 \text{ mm}$$

The area within the perimeter is given by

$$= (2.5 + 3d)^2 - (4 - \pi)(1.5 \times 0.210)^2$$

$$= (2.5 + 3 \times 0.210)^2 - (4 - \pi)(0.315)^2$$

$$= 9.711 \text{ m}^2$$

Ultimate shear force, $V_{Sd} = 680 - 9.711 \times 16.1$

$= 524$ kN

$V_{Sd.eff} = 1.15 \times 524 = 603$ kN

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$$V_{Rd1} = [\tau_{Rd} k(1.2 + 40\rho_1)]du$$

Hence

$$V_{Rd1} = \left[0.3 \times (1.6 - 0.210) \times \left(1.2 + 40 \times \frac{1010}{\{1000 \times 210\}}\right)\right] \times 210 \times 11\,979 \times 10^{-3}$$
$$= 1461\,\text{kN}$$

As $V_{Sd.eff}$ is less than V_{Rd1} the section is adequate.

Span–Effective Depth Ratios

At the centre of the span

$$\frac{100A_s}{bd} = \frac{100 \times 1810}{4000 \times 210} = 0.22$$

Interpolating from table 6.8 the basic span–effective depth ratio for flat slab construction (case 4) is 33.70.

actual span–effective depth ratio = 6500/210 = 30.95

Hence the slab effective depth is acceptable. To take care of stability requirements, extra reinforcement may be necessary in the column strips to act as a tie between each pair of columns – see section 6.7.

8.7 Ribbed and Hollow Block Floors

Cross-sections through a ribbed and hollow block floor slab are shown in figure 8.15 The ribbed floor is formed using temporary or permanent shuttering while the hollow block floor is generally constructed with blocks

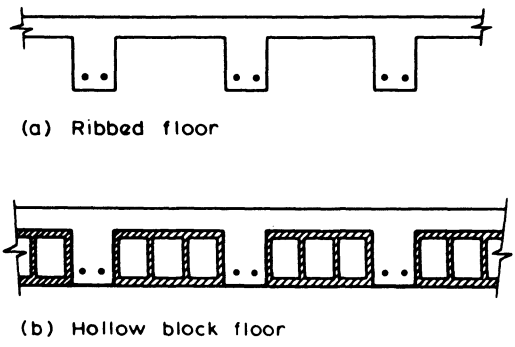

Figure 8.15 *Sections through ribbed and hollow block floors*

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made of clay tile or with concrete containing a light-weight aggregate. If the blocks are suitably manufactured and have an adequate strength they can be considered to contribute to the strength of the slab in the design calculations, but in many designs no such allowance is made.

The principal advantage of these floors is the reduction in weight achieved by removing part of the concrete below the neutral axis and, in the case of the hollow block floor, replacing it with a lighter form of construction. Ribbed and hollow block floors are economical for buildings where there are long spans, over about 5 m, and light or moderate live loads, such as in hospital wards or apartment buildings. They would not be suitable for structures having a heavy loading, such as warehouses and garages.

Near to the supports the hollow blocks are stopped off and the slab is made solid. This is done to achieve a greater shear strength, and if the slab is supported by a monolithic concrete beam the solid section acts as the flange of a T-section. The ribs should be checked for shear at their junction with the solid slab. It is good practice to stagger the joints of the hollow blocks in adjacent rows so that, as they are stopped off, there is no abrupt change in cross-section extending across the slab. The slabs are usually made solid under partitions and concentrated loads.

During construction the hollow tiles should be well soaked in water prior to placing the concrete, otherwise shrinkage cracking of the top concrete flange is liable to occur.

The thickness of the concrete flange should not be less than:

(1) 40 mm or one-tenth of the clear distance between ribs, whichever is the greater, for slabs with permanent blocks
(2) 50 mm or one-tenth of the clear distance between ribs, whichever is the greater, for slabs without permanent blocks.

The rib width will be governed by cover, bar-spacing and fire resistance (section 6.1). The ribs should be spaced no further apart than 1.5 m and their depth below the flange should not be greater than four times their width. Transverse ribs should be provided at spacings no greater than ten times the overall slab depth.

Provided that the above dimensional requirements are met, ribbed slabs can be treated as solid slabs and the design requirements can be based on those of a solid slab.

Ribbed slabs will be designed for shear using the approach described previously with b_w taken as the breadth of the rib. Although no specific guidance is given in either EC2 or the Concise Code, previous practice suggests that, where hollow blocks are used, the rib width may be increased by the wall thickness of the block on one side of the rib.

Span–effective depth ratios will be based on the shorter span with the basic values given in table 6.8 multiplied by 0.8 where the ratio of the

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flange width to the rib width exceeds 3. Again, no specific guidance is given in the Code but previous practice suggests that the thickness of the rib width may include the thickness of the two adjacent block-walls.

At least 50 per cent of the tensile reinforcement in the span should continue to the supports and be anchored. In some instances the slabs are supported by steel beams and are designed as simply supported even though the topping is continuous. Reinforcement should be provided over the supports to prevent cracking in these cases. This top steel should be determined on the basis of 25 per cent of the mid-span moment and should extend at least 0.15 of the clear span into the adjoining span.

A light reinforcing mesh in the topping flange can give added strength and durability to the slab, particularly if there are concentrated or moving loads, or if cracking due to shrinkage or thermal movement is likely. The minimum area of reinforcement required to control shrinkage and thermal cracking can be calculated, as given in chapter 6, but established practice suggests that an area of mesh equivalent to 0.12 per cent of the topping flange will be adequate.

Example 8.8 Design of a Ribbed Floor

The ribbed floor is constructed with permanent fibreglass moulds; it is continuous over several spans of 5.0 m. The characteristic material strengths are $f_{ck} = 25\,\text{N/mm}^2$ and $f_{yk} = 460\,\text{N/mm}^2$.

An effective section as shown in figure 8.16 is to be tried. The characteristic permanent load including self-weight and finishes is $4.5\,\text{kN/m}^2$ and the characteristic variable load is $2.5\,\text{kN/m}^2$.

The calculations are for an interior span for which the moments and shears can be determined from the coefficients in table 8.1.

Considering a 0.4 m width of floor as supported by each rib:

$$\begin{aligned}\text{Ultimate load} &= 0.4(1.35g_k + 1.5q_k)\\ &= 0.4(1.35 \times 4.5 + 1.5 \times 2.5)\\ &= 3.93\,\text{kN/m}\end{aligned}$$

Ultimate load on the span, $F = 3.93 \times 5.0 = 19.65\,\text{kN}$

Bending

(1) At mid-span design as a T section:

$$M = 0.07Fl = 0.07 \times 19.65 \times 5.0 = 6.88\,\text{kN m}$$

$$\frac{M}{bd^2 f_{ck}} = \frac{6.88 \times 10^6}{400 \times 160^2 \times 25} = 0.027$$

From the lever-arm curve, figure 4.5, $l_a = 0.98$. Thus the neutral-axis depth lies within the flange and

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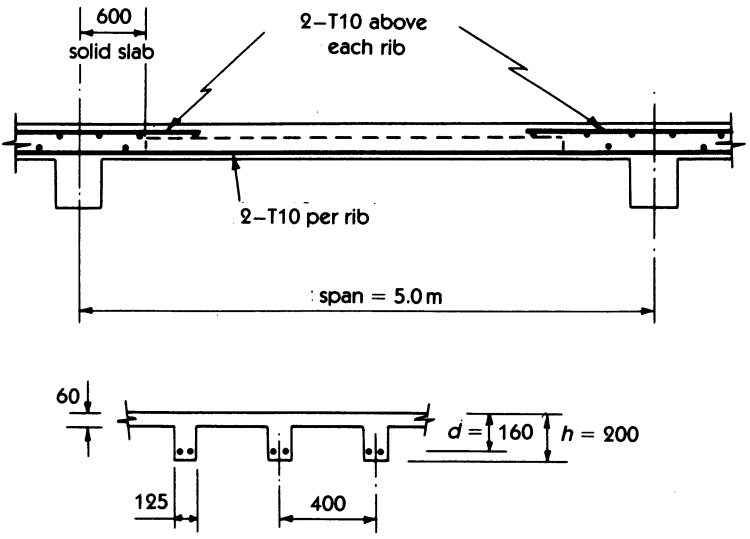

Figure 8.16

$$A_s = \frac{M}{0.87 f_{yk} l_a d} = \frac{6.88 \times 10^6}{0.87 \times 460 \times 0.98 \times 160} = 110\,\text{mm}^2$$

Provide two T10 bars in the ribs, $A_s = 157\,\text{mm}^2$.

(2) At a support design as a rectangular section for the solid slab:

$M = 0.07Fl = 0.07 \times 19.65 \times 5.0 = 6.88\,\text{kN m}$ as in (1)

and

$A_s = 110\,\text{mm}^2$ as at mid-span

Provide two T10 bars in each 0.4 m width of slab, $A_s = 157\,\text{mm}^2$.

(3) At the section where the ribs terminate: this occurs 0.6 m from the centreline of the support and the moment may be hogging so that 125 mm ribs must provide the concrete area to resist the design moment. The maximum moment of resistance of the concrete ribs is

$$M = 0.167 f_{ck} b d^2 = 0.167 \times 25 \times 125 \times 160^2 \times 10^{-6}$$
$$= 13.36\,\text{kN m}$$

which must be greater than the moment at this section, therefore compression steel is not required.

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Span–Effective Depth Ratio

At the centre of the span

$$\frac{100A_s}{bd} = \frac{100 \times 157}{400 \times 160} = 0.245$$

Interpolating from table 6.8 the basic span–effective depth ratio is 38.63. Allowing for the actual steel area provided:

$$\begin{aligned}\text{limiting span–effective depth ratio} &= 38.63 \times A_{s.prov}/A_{s.req} \\ &= 38.63 \times 157/110 \\ &= 55.14\end{aligned}$$

This exceeds the limiting value of 41.7 for a 'nominally reinforced' section and hence the limiting span–depth ratio should be taken as 41.7 (see table 6.8).

For a T-section with a flange width greater than three times the web width this should be multiplied by 0.8 to give a limiting ratio of $0.8 \times 41.7 = 33.36$.

$$\text{actual span–effective depth ratio} = 5000/160 = 31.3$$

Thus $d = 160\,\text{mm}$ is adequate.

Shear

Maximum shear in the rib 0.6 m from the support centreline

$$V_{Sd} = 0.5F - 0.6 \times 3.93 = 0.5 \times 19.65 - 0.6 \times 3.93 = 7.47\,\text{kN}$$

$$\rho_1 = \frac{A_s}{bd} = \frac{157}{125 \times 160} = 0.0079$$

$$\begin{aligned}V_{Rd1} &= [\tau_{Rd}k(1.2 + 40\rho_1)]bd \\ &= [0.3(1.6 - 0.160) \times (1.2 + 40 \times 0.0079) \times 125 \times 160 \\ &\quad \times 10^{-3} \\ &= 13.10\,\text{kN}\end{aligned}$$

As V_{Sd} is less than V_{Rd1} then no shear reinforcement is required provided that the bars in the ribs are securely located during construction.

8.8 Stair Slabs

The usual form of stairs can be classified into two types: (1) those spanning horizontally in the transverse direction, and (2) those spanning longitudinally.

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8.8.1 Stairs Spanning Horizontally

Stairs of this type may be supported on both sides or they may be cantilevered from a supporting wall.

Figure 8.17 shows a stair supported on one side by a wall and on the other by a stringer beam. Each step is usually designed as having a breadth b and an effective depth of $d = D/2$ as shown in the figure; a more rigorous analysis of the section is rarely justified. Distribution steel in the longitudinal direction is placed above the main reinforcement.

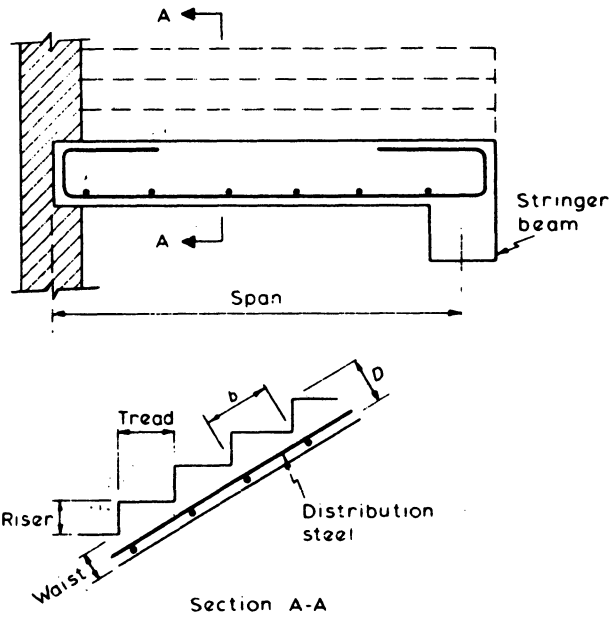

Figure 8.17 *Stairs spanning horizontally*

Details of a cantilevered stair are shown in figure 8.18. The effective depth of the member is taken as the mean effective depth of the section and the main reinforcement must be placed in the top of the stairs and anchored into the support. A light mesh of reinforcement is placed in the bottom face to resist shrinkage cracking.

8.8.2 Stair Slab Spanning Longitudinally

The stair slab may span into landings which span at right angles to the stairs as in figure 8.19 or it may span between supporting beams as in figure 8.20 of the example.

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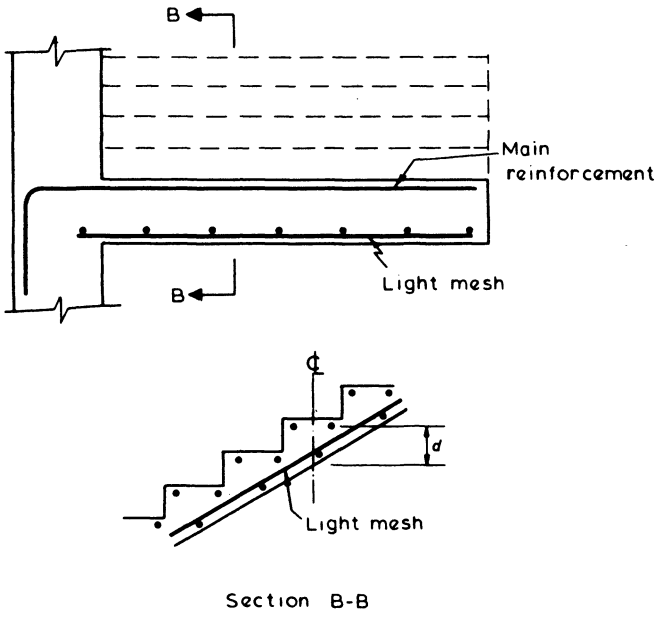

Figure 8.18 *Cantilevered stairs*

The permanent load is calculated along the slope length of the stairs but the variable load is based on the plan area. Loads common to two spans which intersect at right angles and surround an open well may be assumed to be divided equally between the spans. The effective span (l) is measured horizontally between the centres of the supports and the thickness of the waist (h) is taken as the slab thickness.

Stair slabs which are continuous and constructed monolithically with their supporting slabs or beams can be designed for a bending moment of say $Fl/10$, where F is the total ultimate load. But in many instances the stairs are precast or constructed after the main structure, pockets with dowels being left in the supporting beams to receive the stairs, and with no appreciable end restraint the design moment should be $Fl/8$.

Example 8.9 Design of a Stair Slab

The stairs are of the type shown in figure 8.20 spanning longitudinally and set into pockets in the two supporting beams. The effective span is 3 m and the rise of the stairs is 1.5 m with 260 mm treads and 150 mm risers. The variable load is 3.0 kN/m^2 and the characteristic material strengths are f_{ck} = 25 N/mm^2 and f_{yk} = 460 N/mm^2.

Try a 140 mm thick waist, effective depth, d = 115 mm. This would give an initial estimate of the span–effective depth ratio of 26.1 (3000/115) which, from table 6.8, lies approximately half-way between the basic

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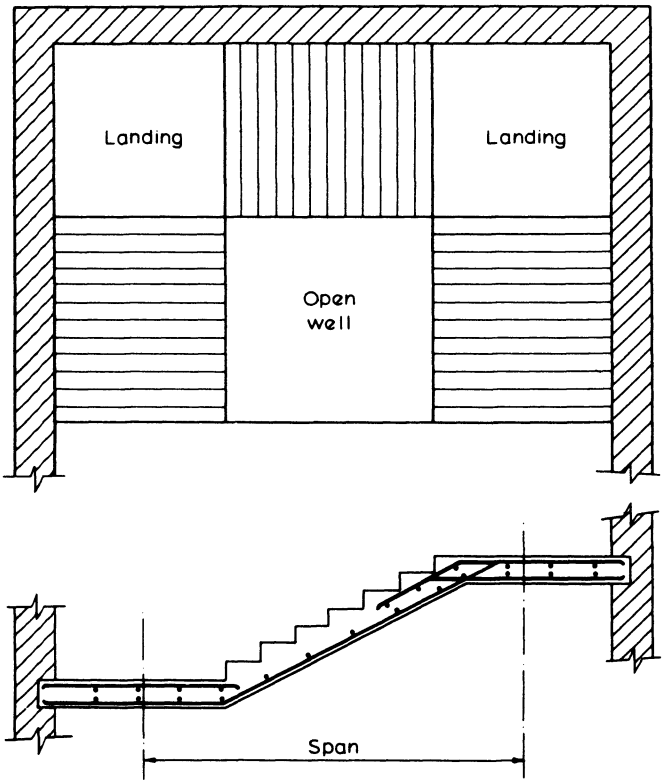

Figure 8.19 *Stairs spanning into landings*

values for a 'lightly stressed' and 'nominally reinforced' simply supported slab.

$$\text{Slope length of stairs} = \sqrt{(3^2 + 5^2)} = 3.35\,\text{m}$$

Consider a 1 m width of stairs:

$$\text{weight of waist plus steps} = (0.14 \times 3.35 + 0.26 \times 1.5/2)24$$
$$= 15.94\,\text{kN}$$

$$\text{Variable load} = 3.0 \times 3 = 9.0\,\text{kN}$$

$$\text{Ultimate load, } F = 1.35 \times 15.94 + 1.5 \times 9.0 = 35.02\,\text{kN m}$$

With no effective end restraint:

$$M = \frac{Fl}{8} = \frac{35.02 \times 3.0}{8} = 13.13\,\text{kN m}$$

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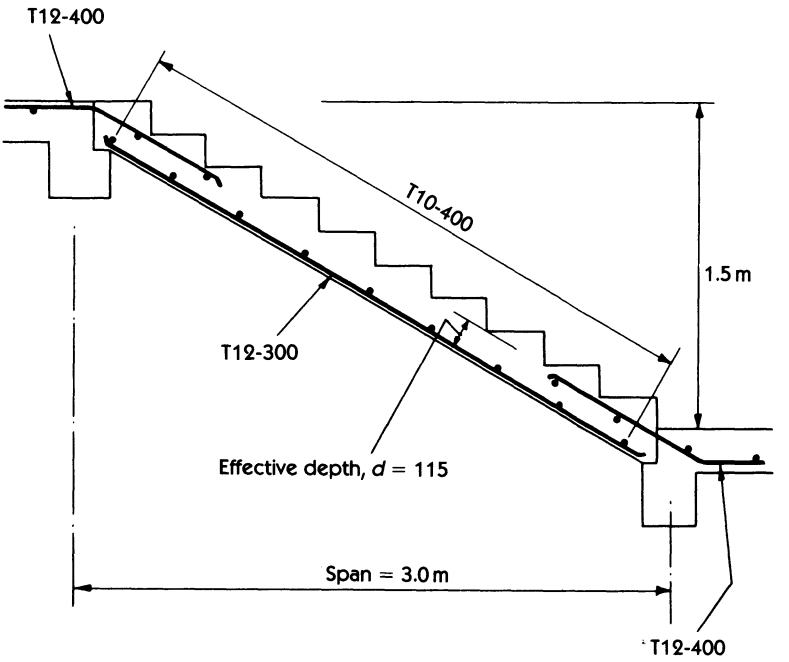

Figure 8.20 *Stairs supported by beams*

Bending Reinforcement

$$\frac{M}{bd^2 f_{ck}} = \frac{13.13 \times 10^6}{1000 \times 115^2 \times 25} = 0.04$$

From the lever-arm curve, figure 4.5, $l_a = 0.96$, therefore

$$A_s = \frac{M}{0.87 f_{yk} l_a d} = \frac{13.13 \times 10^6}{0.87 \times 460 \times 0.96 \times 115} = 297\,\text{mm}^2/\text{m}$$

Provide T12 bars at 300 mm centres, area = 377 mm^2/m.

Span–Effective Depth Ratio

At the centre of the span

$$\frac{100 A_s}{bd} = \frac{100 \times 377}{1000 \times 115} = 0.33$$

Interpolating from table 6.8 the basic span–effective depth ratio for a simply supported span is 25.54. Allowing for the actual steel area provided:

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$$\text{limiting span–effective depth ratio} = 25.54 \times A_{s.prov}/A_{s.req}$$
$$= 25.54 \times 377/297$$
$$= 32.42$$

This exceeds the limiting value of 29.6 for a 'nominally reinforced' section and hence the limiting span–depth ratio should be taken as 29.6 (see table 6.8).

$$\text{actual span–effective depth ratio} = 3000/115 = 26.09$$

Hence the slab effective depth is acceptable.

Secondary Reinforcement

$$\text{Transverse distribution steel} = \frac{0.15bd}{100} = \frac{0.15 \times 1000 \times 115}{100}$$
$$= 173\,\text{mm}^2/\text{m}$$

Provide T10 bars at 400 mm centres, area = 196 mm^2/m.

Continuity bars at the top and bottom of the span should be provided and, whereas about 50 per cent of the main steel would be reasonable, the maximum spacing is limited to $3h$ = 420 mm ($\leqslant$500 mm). Hence provide, say, T12 bars at 400 mm centres as continuity steel.

8.9 Yield Line and Strip Methods

For cases which are more complex as a result of shape, support conditions, the presence of openings, or loading conditions it may be worth while adopting an ultimate analysis method. The two principal approaches are the yield line method, which is particularly suitable for slabs with a complex shape or concentrated loading, and the strip method which is valuable where the slab contains openings.

These methods have been the subject of research in recent years, and are well documented although they are of a relatively specialised nature. A brief introduction is included here to illustrate the general principles and features of the methods, which are particularly valuable in assisting an understanding of failure mechanisms.

8.9.1 Yield Line Method

The capacity of reinforced concrete to sustain plastic deformation has been described in section 3.4. For an under-reinforced section the capacity to develop curvatures between the first yield of reinforcement and failure

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due to crushing of concrete is considerable. For a slab which is subjected to increasing load, cracking and reinforcement yield will first occur in the most highly stressed zone. This will then act as a plastic hinge as subsequent loads are distributed to other regions of the slab. Cracks will develop to form a pattern of 'yield lines' until a mechanism is formed and collapse is indicated by increasing deflections under constant load. To ensure that adequate plastic deformation can take place the Code specifies that slabs designed by the yield line method must be reinforced with high ductility steel and the ratio x/d should not exceed 0.25.

It is assumed that a pattern of yield lines can be superimposed on the slab, which will cause a collapse mechanism, and that the regions between yield lines remain rigid and uncracked. Figure 8.21 shows the yield line mechanism which will occur for the simple case of a fixed ended slab spanning in one direction with a uniform load. Rotation along the yield lines will occur at a constant moment equal to the ultimate moment of resistance of the section, and will absorb energy. This can be equated to the energy expended by the applied load undergoing a compatible displacement and is known as the virtual work method.

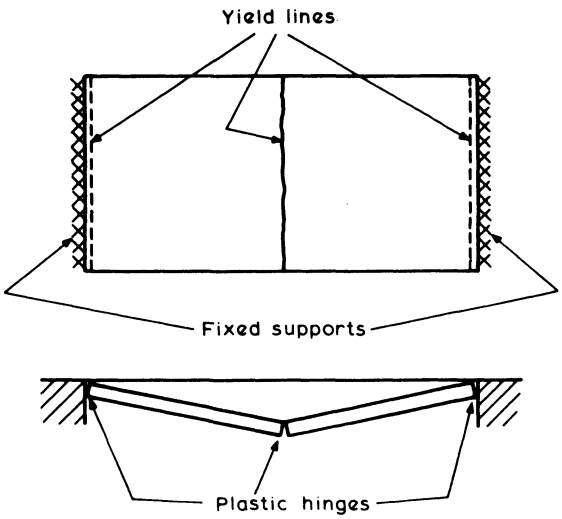

Figure 8.21

Considerable care must be taken over the selection of likely yield line patterns, since the method will give an 'upper bound' solution, that is, either a correct or unsafe solution. Yield lines will form at right angles to bending moments which have reached the ultimate moment of resistance of the slab, and the following rules may be helpful:

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(1) Yield lines are usually straight and end at a slab boundary.
(2) Yield lines will lie along axes of rotation, or pass through their points of intersection.
(3) Axes of rotation lie along supported edges, pass over columns or cut unsupported edges.

In simple cases the alternative patterns to be considered will be readily determined on the basis of common sense, while for more complex cases differential calculus may be used. The danger of missing the critical layout of yield lines, and thus obtaining an incorrect solution, means that the method can only be used with confidence by experienced designers.

A number of typical patterns are shown in figure 8.22.

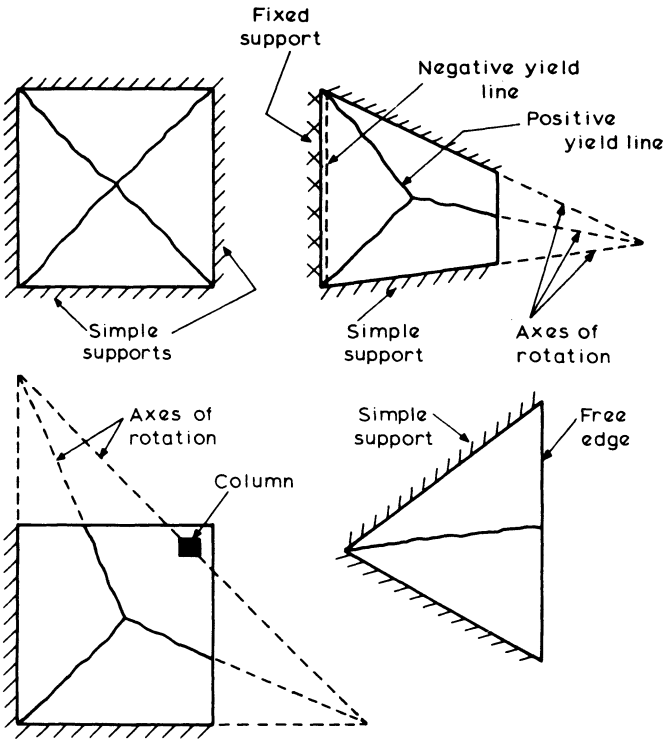

Figure 8.22

A yield line caused by a sagging moment is generally referred to as a 'positive' yield line and represented by a full line, while a hogging moment causing cracking on the top surface of the slab causes a 'negative' yield line shown by a broken line.

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The basic approach of the method is illustrated for the simple case of a one-way spanning slab in example 8.10.

Example 8.10 Simply Supported, One-way Spanning Rectangular Slab

The slab shown in figure 8.23 is subjected to a uniformly distributed load w per unit area. Longitudinal reinforcement is provided as indicated giving a uniform ultimate moment of resistance m per unit width.

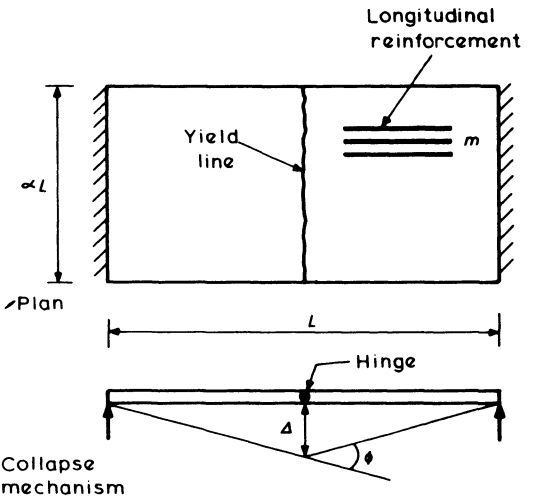

Figure 8.23

The maximum moment will occur at midspan and a positive yield line can thus be superimposed as shown. If this is considered to be subject to a small displacement Δ, then

$$\text{external work done} = \text{area} \times \text{load} \times \text{average distance moved for each rigid half of the slab}$$

$$= \left(aL \times \frac{L}{2}\right) \times w \times \frac{\Delta}{2}$$

therefore

$$\text{total} = \frac{1}{2}aL^2w\Delta$$

Internal energy absorbed by rotation along the yield line is

$$\text{moment} \times \text{rotation} \times \text{length} = m\phi aL$$

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where $\phi \approx 2\left(\frac{\Delta}{0.5L}\right) = \frac{4\Delta}{L}$

hence internal energy $= 4m\alpha\Delta$

Thus equating internal energy absorbed with external work done

$$4m\alpha\Delta = \frac{1}{2}\alpha L^2 w\Delta$$

or

$$m = \frac{wL^2}{8} \quad \text{as anticipated}$$

Since the displacement Δ is eliminated, this will generally be set to unity in calculations of this type.

In the simple case of example 8.10, the yield line crossed the reinforcement at right angles and transverse steel was not involved in bending calculations. Generally, a yield line will lie at an angle θ to the orthogonal to the main reinforcement and will thus also cross transverse steel. The ultimate moment of resistance developed is not easy to define, but Johansen's stepped yield criteria is the most popular approach. This assumes that an inclined yield line consists of a number of steps, each orthogonal to a reinforcing bar as shown in figure 8.24.

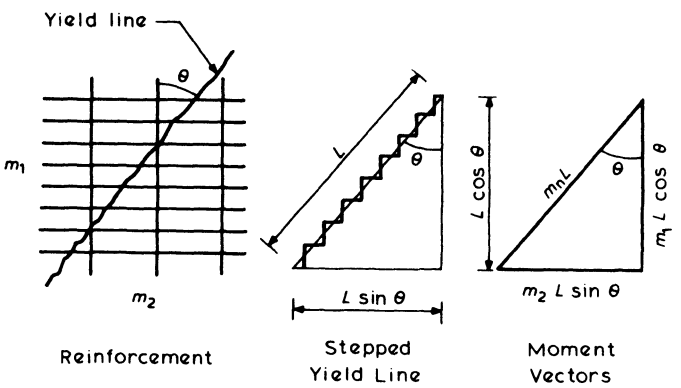

Figure 8.24

If the ultimate moments of resistance provided by main and transverse steel are m_1 and m_2 per unit width, it follows that for equilibrium of the vectors shown, the ultimate moment of resistance normal to the yield line m_n per unit length is given by

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$$m_n L = m_1 L \cos\theta \times \cos\theta + m_2 L \sin\theta \times \sin\theta$$

hence $$m_n = m_1 \cos^2\theta + m_2 \sin^2\theta$$

In the extreme cases of $\theta = 0$, this reduces to $m_n = m_1$, and when $m_1 = m_2 = m$, then $m_n = m$ for any value of θ. This latter case of an orthotropically reinforced slab (reinforcement mutually perpendicular) with equal moments of resistance is said to be isotropically reinforced.

When applying this approach to complex situations it is often difficult to calculate the lengths and rotations of the yield lines, and a simple vector notation can be used. The total moment component m_n can be resolved vectorially in the x and y directions and since internal energy dissipation along a yield line is given by moment × rotation × length it follows that the energy dissipated by rotation of yield lines bounding any rigid area is given by

$$m_x l_x \phi_x + m_y l_y \phi_y$$

where m_x and m_y are yield moments in directions x and y, l_x and l_y are projections of yield lines along each axis, and ϕ_x and ϕ_y are rotations about the axes. This is illustrated in example 8.11.

Example 8.11 Slab Simply Supported on Three Sides

The slab shown in figure 8.25 supports a uniformly distributed load (u.d.l.) of w per unit area.

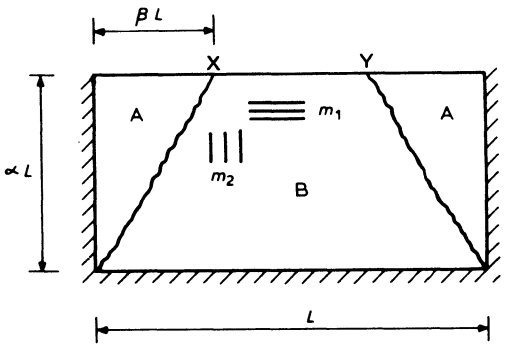

Figure 8.25

Internal energy absorbed (E) for unit displacement at points X and Y
Area A

$$E_A = m_x l_x \phi_x + m_y l_y \phi_y$$

where $\phi_x = 0$; hence

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$$E_A = m_1 \alpha L \times \frac{1}{\beta L} = m_1 \frac{\alpha}{\beta}$$

Area B

$$E_B = m_x l_x \phi_x + m_y l_y \phi_y$$

where $\phi_y = 0$; hence

$$E_B = 2m_2 \beta L \times \frac{1}{\alpha L} = 2m_2 \frac{\beta}{\alpha}$$

hence total for all rigid areas is

$$2E_A + E_B = \frac{2}{\alpha\beta}(m_1\alpha^2 + m_2\beta^2)$$

External work done can also be calculated for each region separately

$$W_A = \frac{1}{2}(\alpha L \times \beta L)w \times \frac{1}{3} = \frac{1}{6} w\alpha\beta L^2$$

$$W_B = \left[\frac{1}{6} w\alpha\beta L^2 + \alpha L\left(\frac{L}{2} - \beta L\right)w \times \frac{1}{2}\right] \times 2$$

therefore total $= 2W_A + W_B$

$$= \frac{1}{6}\alpha(3 - 2\beta)L^2 w$$

Hence equating internal and external work, the maximum u.d.l. that the slab can sustain is given by

$$w_{max} = \frac{2}{\alpha\beta}(m_1\alpha^2 + m_2\beta^2) \times \frac{6}{\alpha(3 - 2\beta)L^2}$$

$$= \frac{12(m_1\alpha^2 + m_2\beta^2)}{\alpha^2 L^2(3\beta - 2\beta^2)}$$

It is clear that the result will vary according to the value of β. The maximum value of w may be obtained by trial and error using several values of β, or alternatively, by differentiation, let $m_2 = \mu m_1$, then

$$w = \frac{12m_1(\alpha^2 + \mu\beta^2)}{\alpha^2 L^2(3\beta - 2\beta^2)}$$

and

$$\frac{d(m_1/w)}{d\beta} = 0 \text{ will give the critical value of } \beta$$

hence

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$$3\mu\beta^2 + 4\alpha^2\beta - 3\alpha^2 = 0$$

and

$$\beta = \frac{\alpha^2}{\mu}\left[\pm\sqrt{\left(\frac{4}{9} + \frac{\mu}{\alpha^2}\right)} - \frac{2}{3}\right]$$

A negative value is impossible, hence the critical value of β for use in the analysis is given by the positive root.

8.9.2 Hillerborg Strip Method

This is based on the 'lower bound' concept of plastic theory which suggests that if a stress distribution throughout a structure can be found which satisfies all equilibrium conditions without violating yield criteria, then the structure is safe for the corresponding system of external loads. Although safe, the structure will not necessarily be serviceable or economic, hence considerable skill is required on the part of the engineer in selecting a suitable distribution of bending moments on which the design can be based. Detailed analysis of a slab designed on this basis is not necessary, but the designer's structural sense and 'feel' for the way loads are transmitted to the supports are of prime importance.

Although this method for design of slabs was proposed by Hillerborg in the 1950s, developments by Wood and Armer in the 1960s have produced its currently used form. The method can be applied to slabs of any shape, and assumes that at failure the load will be carried by bending in either the x or y direction separately with no twisting action. Hence the title of 'strip method'.

Considering a rectangular slab simply supported on four sides and carrying a uniformly distributed load, the load may be expected to be distributed to the supports in the manner shown in figure 8.26.

Judgement will be required to determine the angle α, but it can be seen that if $\alpha = 90°$ the slab will be assumed to be one-way spanning and, although safe, is unlikely to be serviceable because of cracking near the supports along the y-axis. Hillerborg suggests that for such a slab, α should be 45°. The load diagram causing bending moments along typical strips spanning each direction are also shown. It will be seen that the alternative pattern, suggested by Wood and Armer in figure 8.27 will simplify the design, and in this case five strips in each direction may be conveniently used as shown. Each of these will be designed in bending for its particular loading, as if it were one-way spanning using the methods of section 8.4. Reinforcement will be arranged uniformly across each strip, to produce an overall pattern of reinforcement bands in two directions. Support reactions can also be obtained very simply from each strip.

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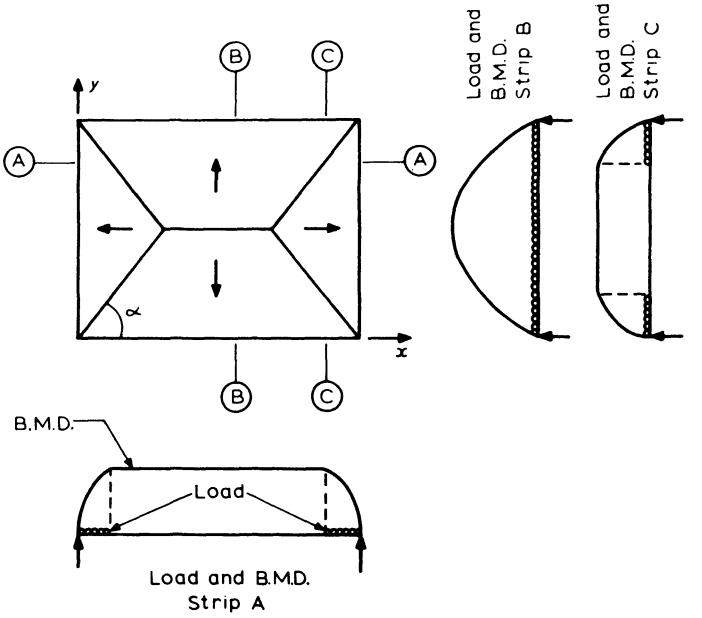

Figure 8.26

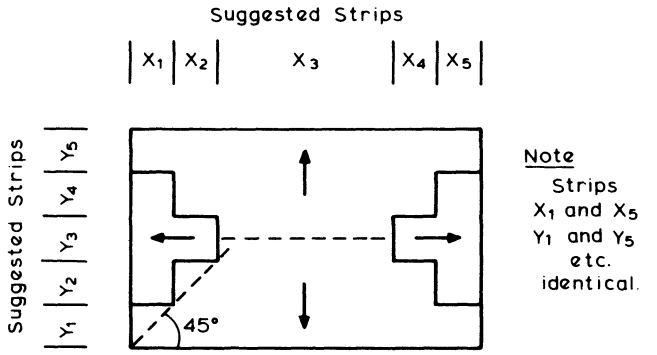

Figure 8.27

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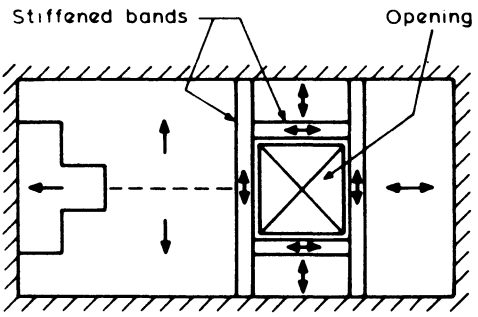

Figure 8.28

The approach is particularly suitable for slabs with openings, in which case strengthened bands can be provided round the openings with the remainder of the slab divided into strips as appropriate. A typical pattern of this type is shown in figure 8.28.

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9

Column Design

The columns in a structure carry the loads from the beams and slabs down to the foundations, and therefore they are primarily compression members, although they may also have to resist bending forces due to the continuity of the structure. The analysis of a section subjected to an axial load plus bending is dealt with in chapter 4, where it is noted that a direct solution of the equations which determine the areas of reinforcement can be very laborious and impractical. Therefore, design charts or some form of electronic computer are often employed to facilitate the routine design of column sections.

Design of columns is governed by the ultimate limit state; deflections and cracking during service conditions are not usually a problem, but nevertheless correct detailing of the reinforcement and adequate cover are important (figure 9.1 illustrates the stages of design).

Many of the principles used in this chapter for the design of a column can also be applied in a similar manner to other types of members which also resist an axial load plus a bending moment.

9.1 Loading and Moments

The loading arrangements and the analysis of a structural frame have been described with examples in chapter 3. In the analysis it was necessary to classify the structure into one of the following types:

(1) braced – where the lateral loads are resisted by shear walls or other forms of bracing capable of transmitting all horizontal loading to the foundations, and
(2) unbraced – where horizontal loads are resisted by the frame action of rigidly connected columns, beams and slabs.

With a braced structure the axial forces and moments in the columns are caused by the permanent and variable load only, whereas with an

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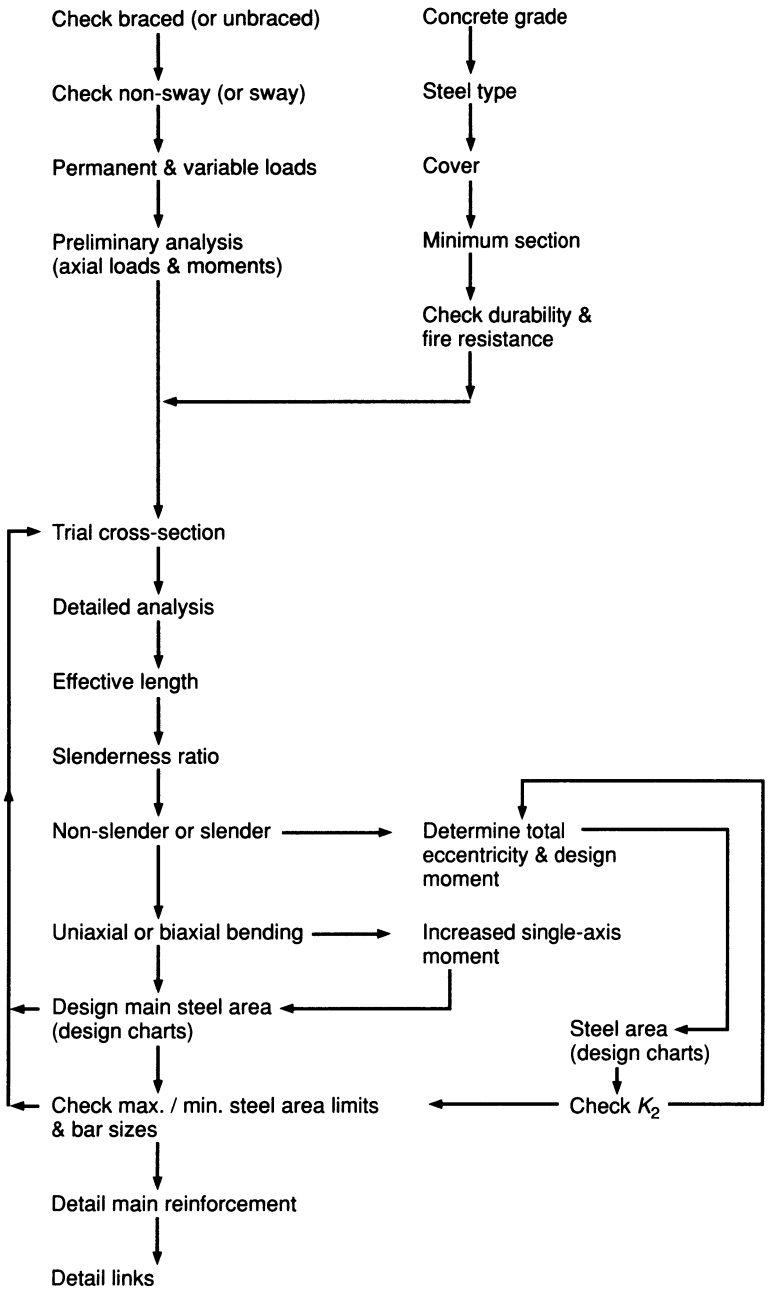

Figure 9.1 *Column design flowchart*

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unbraced structure the loading arrangements which include the effects of the lateral loads must also be considered. Both braced and unbraced structures can be classified as sway or non-sway. A structure is non-sway if there is less than a 10 per cent increase in the normal design moment due to the displacement of the structure. Substantially braced structures can normally be considered to be non-sway. EC2 gives further guidance concerning the classification of unbraced structures. In this chapter only braced non-sway structures will be considered.

For a braced structure the critical arrangement of the ultimate load is usually that which causes the largest moment in the column, together with a large axial load. As an example, figure 9.2 shows a building frame with the critical loading arrangement for the design of its centre column at the first-floor level and also the left-hand column at all floor levels. When the moments in columns are large and particularly with unbraced columns, it may also be necessary to check the case of maximum moment combined with the minimum axial load.

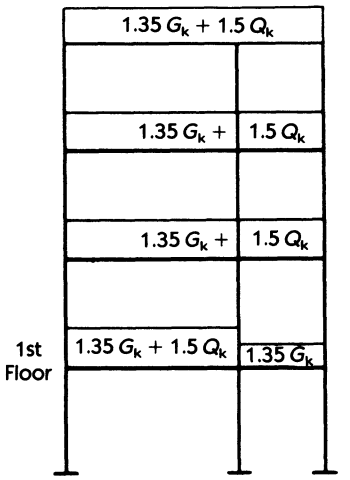

Figure 9.2 *A critical loading arrangement*

In the case of braced frames, the axial column forces due to the vertical loading may be calculated as though the beams and slabs are simply supported, provided that the spans on either side of the column differ by no more than 30 per cent and there is not a cantilever span.

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9.2 Column Classifications and Failure Modes

(1) *Slenderness Ratio of a Column*

The slenderness ratio λ of a column bent about an axis is given by

$$\lambda = \ell_0/i = \ell_0/\sqrt{(I/A)} \tag{9.1}$$

where ℓ_0 is the effective height of the column
i is the radius of gyration about the axis considered
I is the second moment of area at the section about the axis
A is the cross-sectional area of the column.

(2) *Effective Height ℓ_0 of a Column*

$$\ell_0 = \beta \ell_{col} \tag{9.2}$$

where ℓ_{col} is the height of the column between the centres of end restraints.
β is a coefficient from figure 9.3 as a function of the stiffnesses at the end connections as defined in the figure.

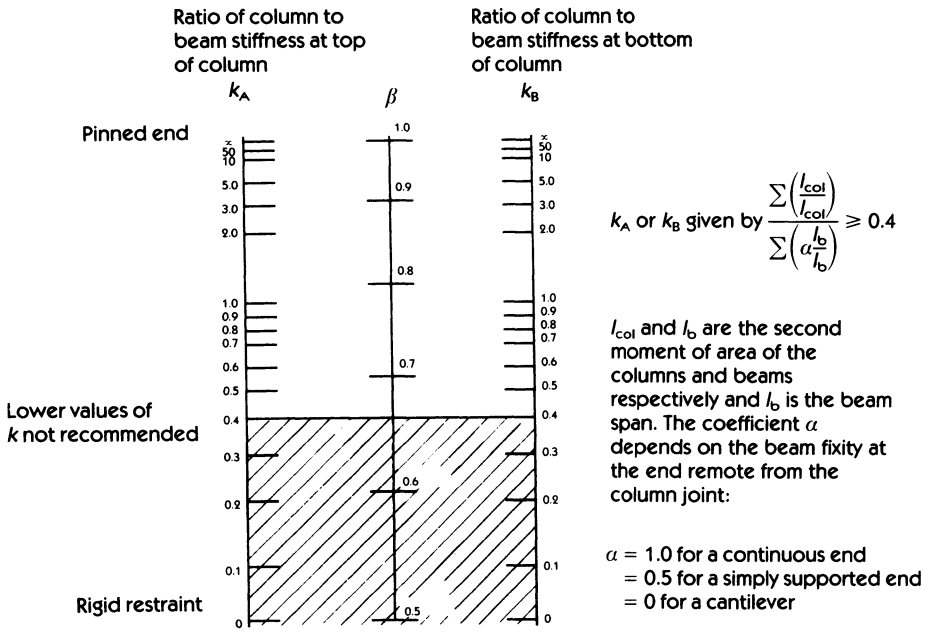

Figure 9.3 *Nomogram for calculation of effective height of a braced column*

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(3) *Short or Slender Column*

The column is short if about both axes of bending the slenderness ratio $\lambda \leqslant 25$ or $15/\sqrt{v_u}$ whichever is the greater

where $v_u = N_{sd}/(A_c f_{cd})$
N_{sd} is the design ultimate axial load
A_c is the area of the cross-section
f_{cd} is the design concrete strength $= f_{ck}/1.5$.

Example 9.1 Short or Slender Column

Determine if the column shown in figure 9.4 is short or slender. The concrete strength $f_{ck} = 25\,\text{N/mm}^2$.

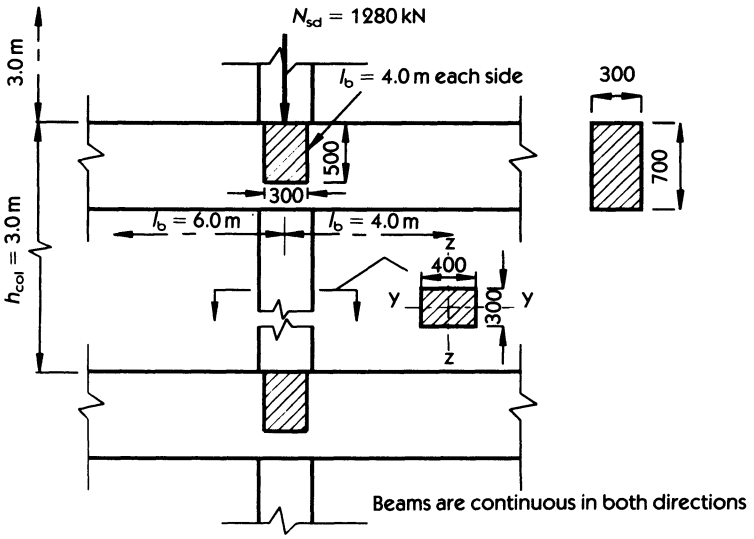

Figure 9.4 *Column end support details*

It can be seen that the column will have the highest slenderness ratio for bending about axes YY where $h = 300\,\text{mm}$ and also the end restraints are the less stiff 300 × 500 beams.

Effective Column Height ℓ_0

$$I_{col} = 400 \times 300^3/12 = 900 \times 10^6\,\text{mm}^4$$

$$I_{beam} = 300 \times 500^3/12 = 3125 \times 10^6\,\text{mm}^4$$

$\alpha = 1.0$ as the beams are continuous (figure 9.3)

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$$k_A = k_B = \frac{\Sigma(I_{col}/\ell_{col})}{\Sigma(\alpha I_{beam}/\ell_{beam})} = \frac{2 \times 900 \times 10^6/3.0 \times 10^3}{2 \times 1 \times 3125 \times 10^6/4.0 \times 10^3}$$
$$= 0.38$$

From figure 9.3 take $\beta = 0.66$, therefore

Effective column height $\ell_0 = 0.66 \times \ell_{col} = 0.66 \times 3.0 = 1.98\,m$

Slenderness Ratio λ

$$\text{Radius of gyration, } i = \sqrt{\left(\frac{I_{col}}{A_{col}}\right)} = \sqrt{\left(\frac{bh^3/12}{bh}\right)} = \frac{h}{3.46} = 86.6\,mm$$

Slenderness ratio $\lambda = \ell_0/i = 1.98 \times 10^3/86.6 = 22.9 < 25$

Hence the column is short.

$(N_{sd}/(A_c f_{cd}) = 1280 \times 10^3/(400 \times 300 \times 25/1.5) = 0.64$ thus $15/\sqrt{0.64} = 18.75 < 25$ which is the greater value.)

(4) Failure Modes

Short columns usually fail by crushing but a slender column is liable to fail by buckling. The end moments on a slender column cause it to deflect sideways and thus bring into play an additional moment Ne_{add} as illustrated in figure 9.5. The moment Ne_{add} causes a further lateral deflection and if the axial load (N) exceeds a critical value this deflection, and the additional moment become self-propagating until the column buckles. Euler derived the critical load for a pin-ended strut as

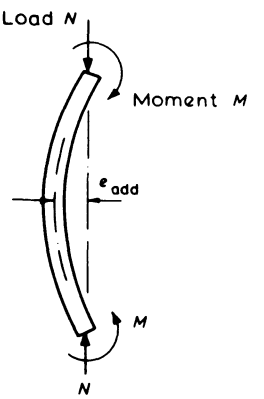

Figure 9.5 *Slender column with lateral deflection*

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$$N_{crit} = \frac{\pi^2 EI}{l^2}$$

The crushing load N_{ud} of a truly axially loaded column may be taken as

$$N_{ud} = 0.567 f_{ck} A_c + 0.87 A_s f_{yk}$$

where A_c is the area of the concrete and A_s is the area of the longitudinal steel.

Values of N_{crit}/N_{ud} and l/i have been calculated and plotted in figure 9.6 for a typical column cross-section.

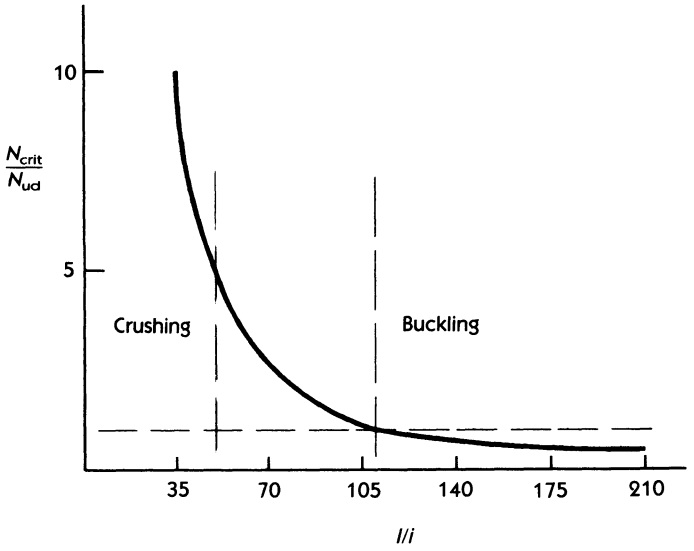

Figure 9.6 *Column failure modes*

The ratio of N_{crit}/N_{ud} in the figure determines the type of failure of the column. With l/i less than, say, 50 the load will probably cause crushing, N_{ud} is much less than N_{crit}, the load that causes buckling – and therefore a buckling failure will not occur. This is not true with higher values of l/i and so a buckling failure is possible, depending on such factors as the initial curvature of the column and the actual eccentricity of the load. When l/i is greater than 110 then N_{crit} is less than N_{ud} and in this case a buckling failure will occur for the column considered.

The mode of failure of a column can be one of the following:

(1) Material failure with negligible lateral deflection, which usually occurs with short columns but can also occur when there are large end moments on a column with an intermediate slenderness ratio.

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(2) Material failure intensified by the lateral deflection and the additional moment. This type of failure is typical of intermediate columns.
(3) Instability failure which occurs with slender columns and is liable to be preceded by excessive deflections.

9.3 Reinforcement Details

The rules governing the minimum and maximum amounts of reinforcement in a load bearing column are as follows.

Longitudinal Steel

(1) A minimum of four bars is required in a rectangular column (one bar in each corner) and six bars in a circular column. Bar diameter should not be less than 12 mm.
(2) The minimum area of steel is given by

$$A_s = \frac{0.15N_{sd}}{0.87f_{yk}} \not< 0.003A_c$$

(3) The maximum area of steel, even at laps, is given by

$$\frac{A_s}{A_c} \not> 0.008$$

where A_s is the total area of longitudinal steel and A_c is the cross-sectional area of the column.

Links

(1) Minimum size = 1/4 × size of the largest compression bar but not less than 6 mm.
(2) Maximum spacing should not exceed the lesser of 12 × size of the smallest compression bar or the least lateral dimension of the column or 300 mm. This spacing should be reduced by a factor of 0.67
    (a) for a distance equal to the larger lateral dimension of the column above and below a beam or a slab, and
    (b) at lapped joints of longitudinal bars >20 mm diameter.
(3) Where the direction of the longitudinal reinforcement changes, the spacing of the links should be calculated, while taking account of the lateral forces involved.
(4) Every longitudinal bar placed in a corner should be held by transverse reinforcement.
(5) A maximum of 5 bars in or close to each corner can be secured against buckling by any one set of transverse reinforcement.

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Although links are popular in the United Kingdom, helical reinforcement is popular in the USA and provides added strength in addition to added protection against seismic loading. Sizing and spacing of helical reinforcement should be similar to links.

Figure 9.7 shows possible arrangements of reinforcing bars at the junction of two columns and a floor. In figure 9.7a the reinforcement in the lower column is cranked so that it will fit within the smaller column above. The crank in the reinforcement should, if possible, commence above the soffit of the beam so that the moment of resistance of the column is not reduced. For the same reason, the bars in the upper column should be the ones cranked when both columns are of the same size as in figure 9.7b. Links should be provided at the points where the bars are cranked in order to resist buckling due to horizontal components of force in the inclined lengths of bar. Separate dowel bars as in figure 9.7c may also be used to provide continuity between the two lengths of column. The column-beam junction should be detailed so that there is adequate space for both the column steel and the beam steel. Careful attention to detail on this point will greatly assist the fixing of the steel during construction.

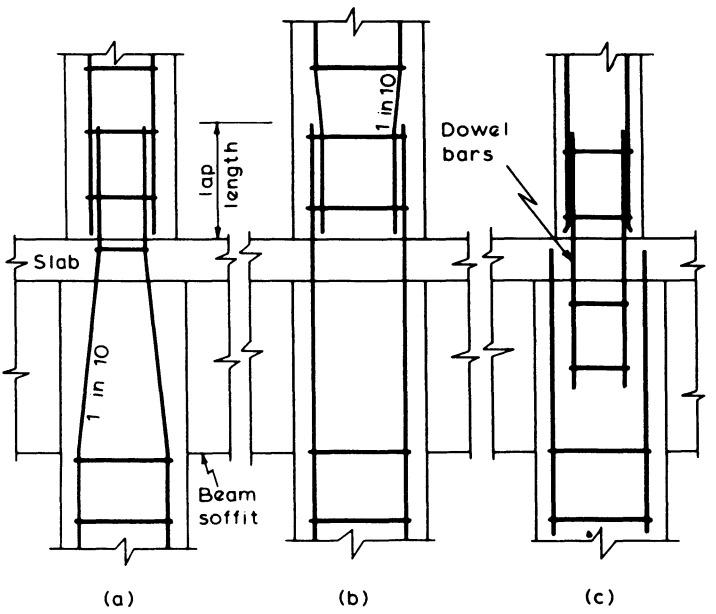

Figure 9.7 *Details of splices in column reinforcement*

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9.4 Short Columns Resisting Moments and Axial Forces

The area of longitudinal steel for these columns is determined by:

(1) using design charts or constructing M–N interaction diagrams as in chapter 4
(2) a solution of the basic design equations, or
(3) an approximate method.

Design charts are usually used for columns having a rectangular or circular cross-section and a symmetrical arrangement of reinforcement, but interaction diagrams can be constructed for any arrangement of cross-section as illustrated in examples 4.10 and 4.11. The basic equations or the approximate method can be used when an unsymmetrical arrangement of reinforcement is required, or when the cross-section is non-rectangular as described in section 9.5.

Whichever design method is used, a column should not be designed for a moment less than $N_{sd} \times e_{min}$, where e_{min} has the lesser value of $h/20$ or 20 mm. This is to allow for tolerances in construction. The dimension h is the overall size of the column cross-section in the plane of bending.

9.4.1 Design Charts and Interaction Diagrams

The design of a section subjected to bending plus axial load should be in accordance with the principles described in section 4.8, which deals with the analysis of the cross-section. The basic equations derived for a rectangular section as shown in figure 9.8 and with a rectangular stress block are

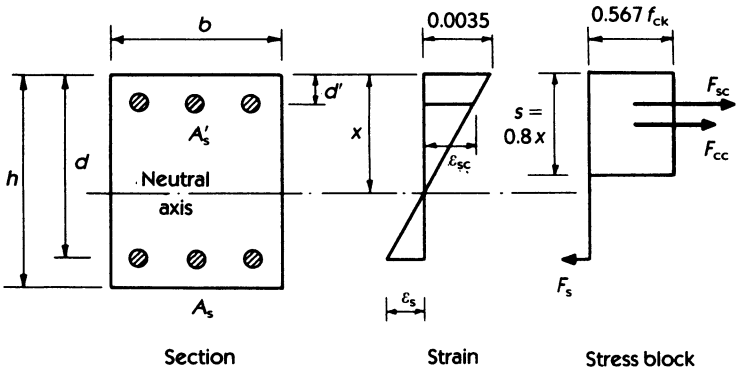

Figure 9.8 *Column section*

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$$N_{sd} = F_{cc} + F_{sc} + F_s \qquad (9.3)$$
$$= 0.567 f_{ck} bs + f_{sc} A'_s + f_s A_s$$

$$M_{sd} = F_{cc}\left(\frac{h}{2} - \frac{s}{2}\right) + F_{sc}\left(\frac{h}{2} - d'\right) - F_s\left(d - \frac{h}{2}\right) \qquad (9.4)$$

N_{sd} = design ultimate axial load
M_{sd} = design ultimate moment
s = the depth of the stress block = $0.8x$
A'_s = the area of longitudinal reinforcement in the more highly compressed face
A_s = the area of reinforcement in the other face
f_{sc} = the stress in reinforcement A'_s
f_s = the stress in reinforcement A_s, negative when tensile.

These equations are not suitable for direct solution and the design of a column with symmetrical reinforcement in each face is best carried out using design charts as illustrated in figure 9.9. A fuller set of these charts can be found in the Appendix based on the Concise EuroCode.

Example 9.2 Column Design Using Design Charts

Figure 9.10 shows a frame of a heavily loaded industrial structure for which the centre columns along line PQ are to be designed in this example. The frames at 4 m centres, are braced against lateral forces, and support the following floor loads:

permanent load $g_k = 10\,\text{kN/m}^2$

variable load $q_k = 15\,\text{kN/m}^2$

Characteristic material strengths are $f_{ck} = 25\,\text{N/mm}^2$ for the concrete and $f_{yk} = 460\,\text{N/mm}^2$ for the steel.

Maximum ultimate load at each floor $= 4.0(1.35g_k + 1.5q_k)$ per metre length of beam
$= 4(1.35 \times 10 + 1.5 \times 15)$
$= 144\,\text{kN/m}$

Minimum ultimate load at each floor $= 4.0 \times 1.35g_k$
$= 4.0 \times 13.5 \times 10$
$= 54\,\text{kN}$ per metre length of beam

Consider first the design of the centre column at the underside (u.s.) of the first floor. The critical arrangement of load which will cause the maximum moment in the column is shown in figure 9.11a.

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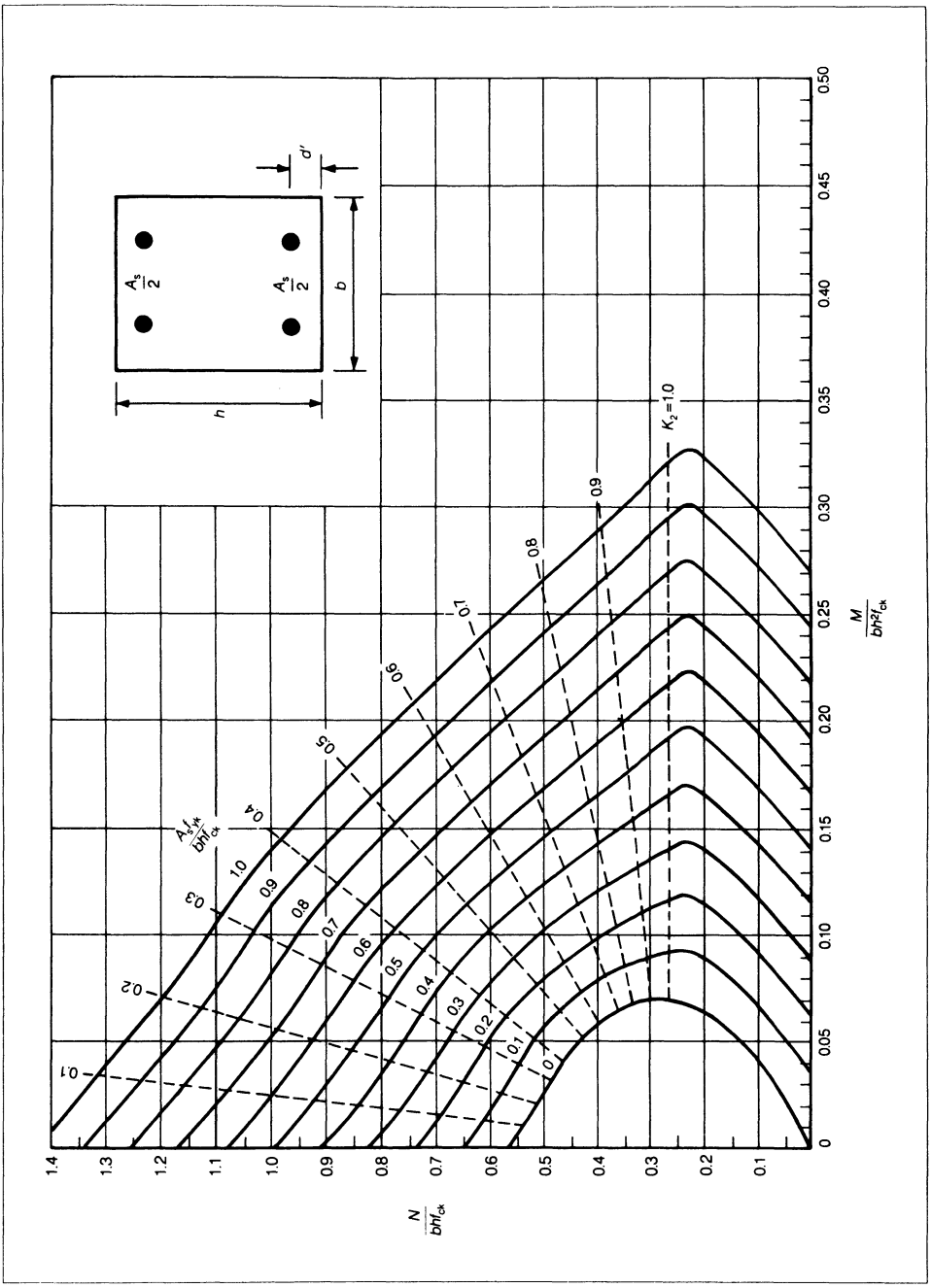

Figure 9.9 *Rectangular column (d'/h = 0.20)*

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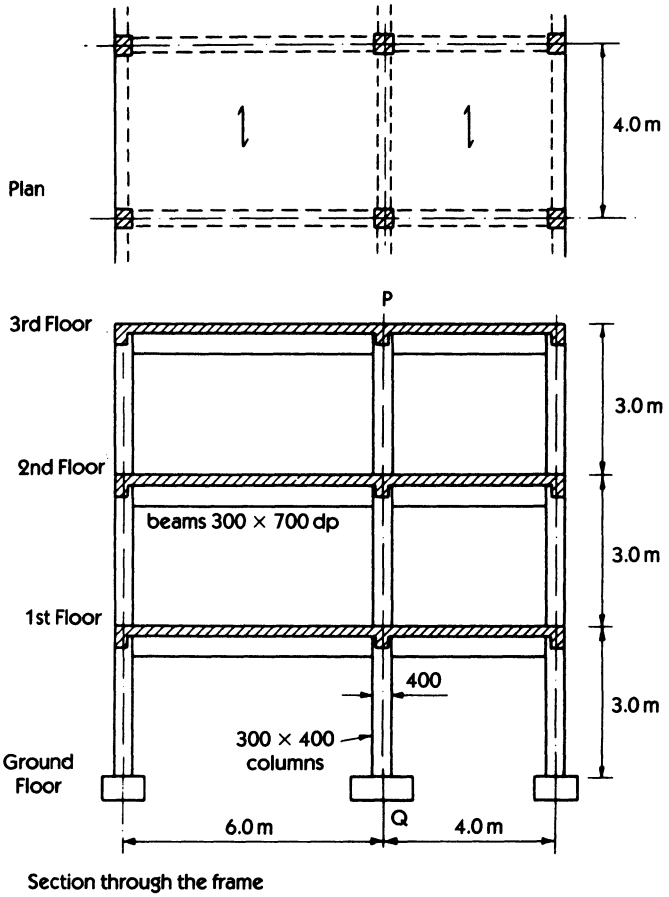

Figure 9.10 *Columns in an industrial structure*

Column loads

second and third floors = 2 × 144 × 10/2 = 1440 kN
first floor = 144 × 6/2 + 54 × 4/2 = 540
Column self-weight, say 2 × 14 = 28
N_{sd} = 2008 kN

Similar arrangements of load will give the axial load in the column at the underside (u.s.) and top side (t.s.) of each floor level and these values of N_{sd} are shown in table 9.1.

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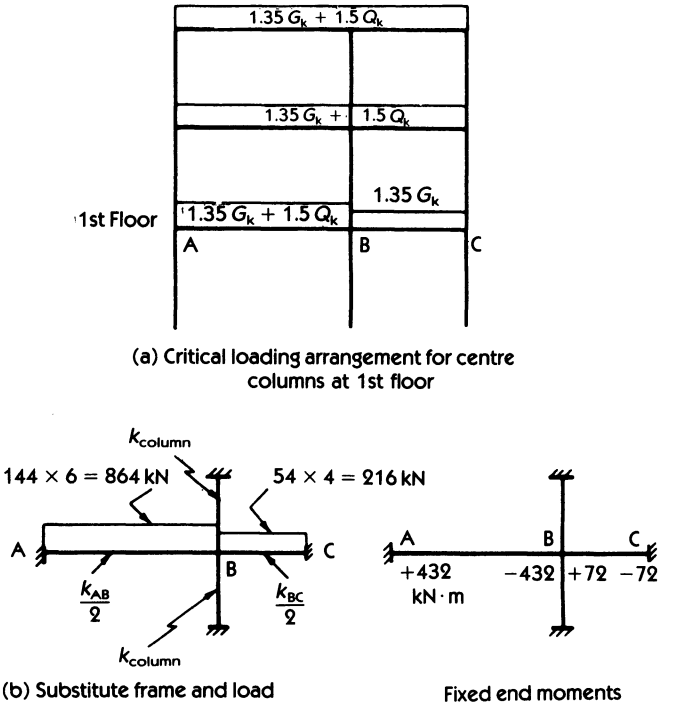

Figure 9.11

Table 9.1

Floor	N_{sd} *(kN)*	M_{sd} *(kN m)*	$\frac{N_{sd}}{bhf_{ck}}$	$\frac{M_{sd}}{bh^2 f_{ck}}$	$\frac{A_s f_{yk}}{bhf_{ck}}$	A_s *(mm^2)*
3rd u.s.	540	82.6	0.18	0.07	0	360
2nd t.s.	734	68.4	0.24	0.06	0	360
	+ 540					
2nd u.s.	1274	68.4	0.42	0.06	0	360
1st t.s.	1468	68.4	0.49	0.06	0.10	652
	+ 540					
1st u.s.	2008	68.4	0.67	0.06	0.30	1956

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Column Moments

The loading arrangement and the substitute frame for determining the column moments at the first and second floors are shown in figure 9.11b. Member stiffnesses are

$$\frac{k_{AB}}{2} = \frac{1}{2} \times \frac{bh^3}{12L_{AB}} = \frac{1}{2} \times \frac{0.3 \times 0.7^3}{12 \times 6} = 0.71 \times 10^{-3}$$

$$\frac{k_{BC}}{2} = \frac{1}{2} \times \frac{0.3 \times 0.7^3}{12 \times 4} = 1.07 \times 10^{-3}$$

$$k_{col} = \frac{0.3 \times 0.4^3}{12 \times 3.0} = 0.53 \times 10^{-3}$$

therefore

$$\Sigma k = (0.71 + 1.07 + 2 \times 0.53)10^{-3} = 2.84 \times 10^{-3}$$

and

$$\text{distribution factor for the column} = \frac{k_{col}}{\Sigma k} = \frac{0.53}{2.84} = 0.19$$

Fixed end moments at B are

$$\text{F.E.M.}_{BA} = \frac{144 \times 6^2}{12} = 432\,\text{kN m}$$

$$\text{F.E.M.}_{BC} = \frac{54 \times 4^2}{12} = 72\,\text{kN m}$$

Thus

$$\text{column moment } M_{sd} = 0.19(432 - 72) = 68.4\,\text{kN m}$$

At the 3rd floor

$$\Sigma k = (0.71 + 1.07 + 0.53)10^{-3} = 2.31 \times 10^{-3}$$

and

$$\text{column moment } M_{Sd} = \frac{0.53}{2.31}(432 - 72) = 82.6\,\text{kN m}$$

The areas of reinforcement in table 9.1 are determined by using the design chart of figure 9.9. Sections through the column are shown in figure 9.12.

Cover for the reinforcement is taken as 50 mm and $d'/h = 80/400 = 0.2$. The minimum area of reinforcement allowed in the section is given by

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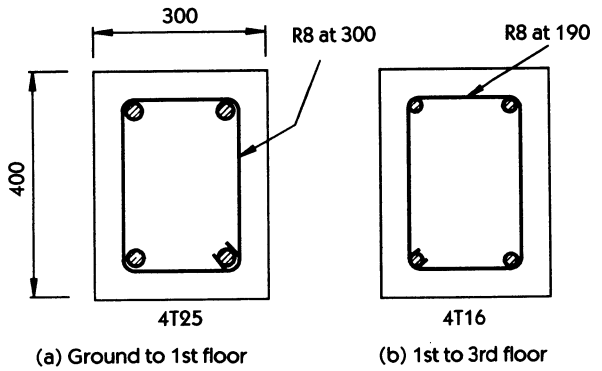

Note that link spacing is reduced to 0.67 × these values for 400 mm above and below each floor level and at laps below 1st floor level

Figure 9.12 *Column sections in design example*

$$A_s = 0.003bh = 0.003 \times 300 \times 400 = 360\,\text{mm}^2$$

and the maximum area is

$$A_s = 0.08 \times 300 \times 400 = 9600\,\text{mm}^2$$

and the reinforcement provided is within these limits.

Although the EuroCode permits the use of 12 mm main steel, 16 mm bars have been used to ensure adequate rigidity of the reinforcing cage.

A smaller column section could have been used above the first floor but this would have involved changes in formwork and possibly also increased areas of reinforcement.

9.4.2 Design Equations for a Non-symmetrical Section

The symmetrical arrangement of the reinforcement with $A'_s = A_s$ is justifiable for the columns of a building where the axial loads are the dominant forces and where any moments due to the wind can be acting in either direction. But some members are required to resist axial forces combined with large bending moments so that it is not economical to have equal areas of steel in both faces, and in these cases the usual design charts cannot be applied. A rigorous design for a rectangular section as shown in figure 9.13 involves the following iterative procedure:

(1) Select a depth of neutral axis, x (for this design method where the moments are relatively large, x would generally be less than h).
(2) Determine the steel strains ε_{sc} and ε_s from the strain distribution.

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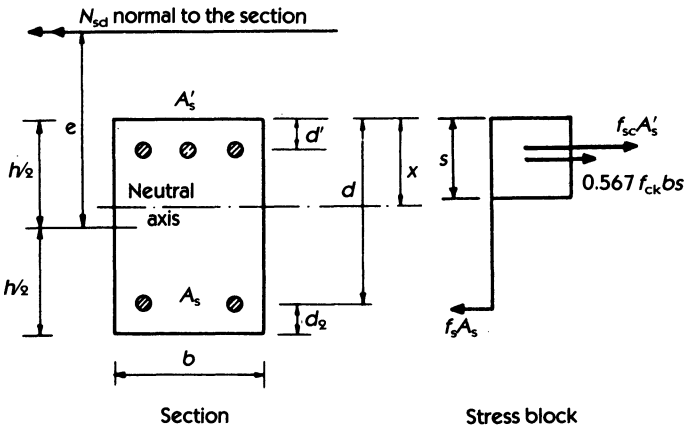

Figure 9.13 *Column with an unsymmetrical arrangement of reinforcement*

(3) Determine the steel stresses f_{sc} and f_s from the equations relating to the stress–strain curve for the reinforcing bars (see section 4.1.2).

(4) Taking moments about the centroid of A_s

$$N_{sd}\left(e + \frac{h}{2} - d_2\right) = 0.567 f_{ck} bs(d - s/2) + f_{sc}A'_s(d - d')$$

where $s = 0.8x$ (9.5)

This equation can be solved to give a value for A'_s.

(5) A_s is then determined from the equilibrium of the axial forces, that is

$$N_{sd} = 0.567 f_{ck} bs + f_{sc}A'_s + f_sA_s \quad (9.6)$$

(6) Further values of x may be selected and steps (1) to (5) repeated until a minimum value for $A'_s + A_s$ is obtained.

The term f_{sc} in the equations may be modified to $(f_{sc} - 0.567 f_{ck})$ to allow for the area of concrete displaced by the reinforcement A'_s. Stress f_s has a negative sign whenever it is tensile.

Example 9.3 Column Section with an Unsymmetrical Arrangement of Reinforcement

The column section shown in figure 9.14 resists an axial load of 1100 kN and a moment of 230 kN m at the ultimate limit state. Determine the areas of reinforcement required if the characteristic material strengths are $f_{yk} = 460\,\text{N/mm}^2$ and $f_{ck} = 25\,\text{N/mm}^2$.

(1) Select a depth of neutral axis, $x = 190$ mm.

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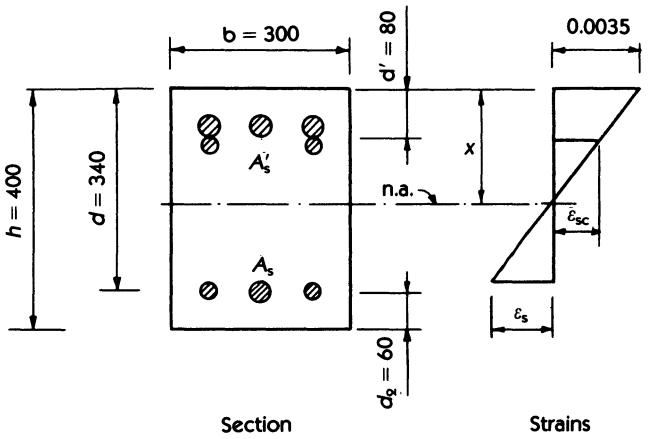

Figure 9.14 *Unsymmetrical column design example*

(2) From the strain diagram

$$\text{steel strain } \varepsilon_{sc} = \frac{0.0035}{x}(x - d')$$

$$= \frac{0.0035}{190}(190 - 80) = 0.00203$$

and

$$\text{steel strain } \varepsilon_s = \frac{0.0035}{x}(d - x)$$

$$= \frac{0.0035}{190}(340 - 190) = 0.00276$$

(3) From the stress–strain curve and the relevant equations of section 4.1.2

yield strain, $\varepsilon_y = 0.002$ for grade 460 steel

Both ε_{sc} and $\varepsilon_s > 0.002$

therefore $f_{sc} = 460/1.15 = 400\,\text{N/mm}^2$

and $f_s = 400\,\text{N/mm}^2$, tension.

(4) In equation 9.5

$$N_{sd}\left(e + \frac{h}{2} - d_2\right) = 0.567 f_{ck} bs(d - s/2) + f_{sc} A'_s(d - d')$$

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$$e = \frac{M_{sd}}{N_{sd}} = \frac{230 \times 10^6}{1100 \times 10^3} = 209\,\text{mm}$$

$$s = 0.8x = 0.8 \times 190$$
$$= 152\,\text{mm}$$

To allow for the area of concrete displaced

$$f_{sc} \text{ becomes } 400 - 0.567 f_{ck} = 400 - 0.567 \times 25$$
$$= 386\,\text{N/mm}^2$$

and from equation 9.5

$$A'_s = \frac{1100 \times 10^3(209 + 140) - 0.567 \times 25 \times 300 \times 152(340 - 152/2)}{386(340 - 80)}$$
$$= 2125\,\text{mm}^2$$

(5) From equation 9.6

$$N_{sd} = 0.567 f_{ck} bs + f_{sc} A'_s + f_s A_s$$

$$A_s = \frac{(0.567 \times 25 \times 300 \times 152) + (386 \times 2125) - (1100 \times 10^3)}{400}$$
$$= 917\,\text{mm}^2$$

Thus

$$A'_s + A_s = 3042\,\text{mm}^2 \quad \text{for} \quad x = 190\,\text{mm}$$

(6) Values of $A'_s + A_s$ calculated for other depths of neutral axis, x, are plotted in figure 9.15. From this figure the minimum area of reinforcement required occurs with $x \approx 215$ mm. Using this depth of neutral axis, steps 2 to 5 are repeated giving

$$\varepsilon_{sc} = 0.00219, \quad \varepsilon_s = 0.00203$$

$$f_{sc} = f_{yk}/\gamma_m = 400\,\text{N/mm}^2 \quad \text{and} \quad f_s = 400\,\text{N/mm}^2 \text{ tension}$$

so that

$$A'_s = 1975\,\text{mm}^2 \quad \text{and} \quad A_s = 984\,\text{mm}^2$$

(Alternatively separate values of A'_s and A_s as calculated for each value of x could also have been plotted against x and their values read from the graph at $x = 215$ mm.) This area would be provided with

$$A'_s = \text{three T25 plus two T20 bars} = 2098\,\text{mm}^2$$

and

$$A_s = \text{one T25 plus two T20 bars} = 1119\,\text{mm}^2$$

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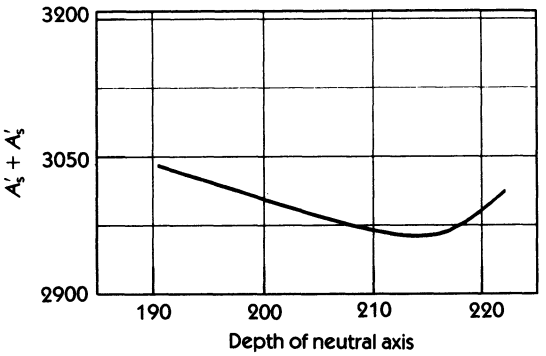

Figure 9.15 *Design chart for unsymmetrical column example*

With a symmetrical arrangement of reinforcement the area from the design chart of figure 9.9 would be $A'_s + A_s = 3230\,\text{mm}^2$ or 9 per cent greater than the area with an unsymmetrical arrangement, and including no allowance for the area of concrete displaced by the steel.

These types of iterative calculations are readily programmed for solution by a small computer, which could find the optimum steel areas without the necessity of plotting a graph.

9.4.3 Simplified Design Method

As an alternative to the previous rigorous method of design an approximate method may be used when the eccentricity of loading, e is not less than $(h/2 - d_2)$.

The moment M_{sd} and the axial force N_{sd} are replaced by an increased moment M_a where

$$M_a = M_{sd} + N_{sd}\left(\frac{h}{2} - d_2\right) \tag{9.7}$$

plus a compressive force N_{sd} acting through the tensile steel A_s as shown in figure 9.16. Hence the design of the reinforcement is carried out in two parts.

(1) The member is designed as a doubly reinforced section to resist M_a acting by itself. The equations for calculating the areas of reinforcement to resist M_a for a C12/15 to C35/45 concrete are given in section 4.5 as

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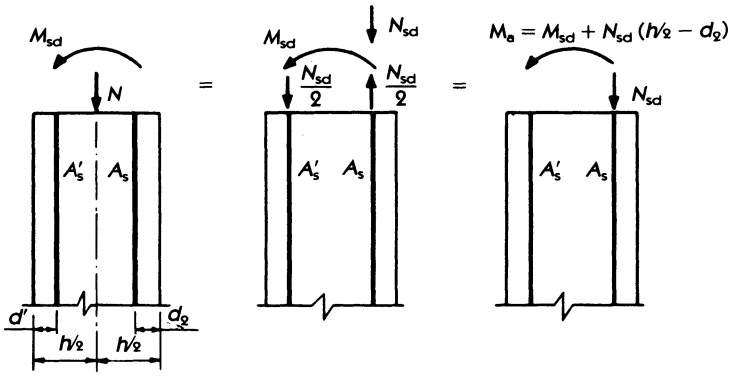

Figure 9.16 *Simplified design method*

$$M_a = 0.167 f_{ck} b d^2 + 0.87 f_{yk} A'_s (d - d') \tag{9.8}$$

$$0.87 f_y A_s = 0.204 f_{ck} b d + 0.87 f_{yk} A'_s \tag{9.9}$$

(2) The area of A_s calculated in the first part is reduced by the amount $N_{sd}/0.87 f_{yk}$.

This preliminary design method is probably most useful for non-rectangular column sections as shown in example 9.5, but the procedure is first demonstrated with a rectangular cross-section in the following example.

Example 9.4 Column Design by the Simplified Method

Calculate the area of steel required in the 300 × 400 column of figure 9.14. $N_{sd} = 1100\,\text{kN}$, $M_{sd} = 230\,\text{kN m}$, $f_{ck} = 25\,\text{N/mm}^2$ and $f_{yk} = 460\,\text{N/mm}^2$.

$$\text{Eccentricity } e = \frac{230 \times 10^6}{1100 \times 10^3} = 209\,\text{mm} > \left(\frac{h}{2} - d_2\right)$$

(1) Increased moment

$$M_a = M_{sd} + N_{sd}\left(\frac{h}{2} - d_2\right)$$

$$= 230 + 1100(200 - 60)10^{-3} = 384\,\text{kN m}$$

The area of steel to resist this moment can be calculated using the formulae 9.8 and 9.9 for the design of a beam with compressive reinforcement, that is

$$M_a = 0.167 f_{ck} b d^2 + 0.87 f_{yk} A'_s (d - d')$$

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and

$$0.87f_{yk}A_s = 0.204f_{ck}bd + 0.87f_{yk}A'_s$$

therefore

$$384 \times 10^6 = 0.167 \times 25 \times 300 \times 340^2 + 0.87 \times 460A'_s(340 - 80)$$

so that

$$A'_s = 2300\,\text{mm}^2$$

and

$$0.87 \times 460A_s = 0.204 \times 25 \times 300 \times 340 + 0.87 \times 460 \times 2300$$

$$A_s = 3600\,\text{mm}^2$$

(2) Reducing this area by $N_{sd}/0.87f_{yk}$

$$A_s = 3600 - \frac{1100 \times 10^3}{0.87 \times 460}$$

$$= 852\,\text{mm}^2$$

This compares with $A'_s = 1975\,\text{mm}^2$ and $A_s = 984\,\text{mm}^2$ with the design method of example 9.3. (To give a truer comparison the stress in the compressive reinforcement should have been modified to allow for the area of concrete displaced, as was done in example 9.3.)

9.5 Non-Rectangular Sections

Design charts are not usually available for columns of other than a rectangular or a circular cross-section. Therefore the design of a non-rectangular section entails either (1) an iterative solution of design equations, (2) a simplified form of design, or (3) construction of M–N interaction diagrams.

9.5.1 Design Equations

For a non-rectangular section it is much simpler to consider the equivalent rectangular stress-block. Determination of the reinforcement areas follows the same procedure as described for a rectangular column in section 9.4.2, namely

(1) Select a depth of neutral axis.
(2) Determine the corresponding steel strains.
(3) Determine the steel stresses.
(4) Take moments about A_s so that with reference to figure 9.17:

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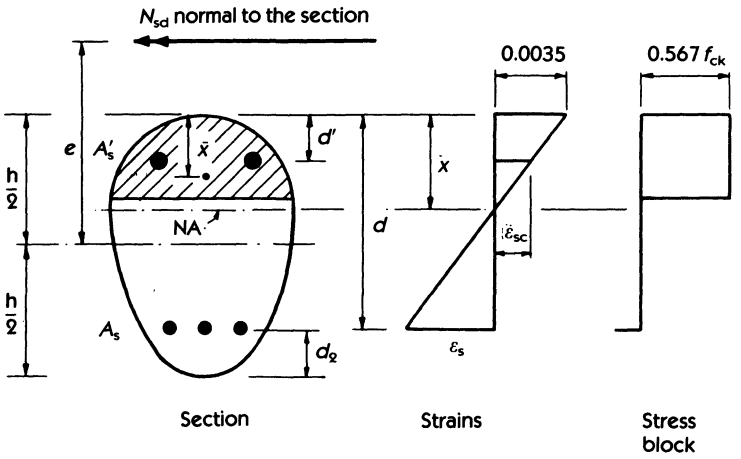

Figure 9.17 *Non-rectangular column section*

$$N_{sd}\left(e + \frac{h}{2} - d_2\right) = 0.567 f_{ck} A_{cc}(d - \bar{x}) + f_{sc} A'_s (d - d')$$

Solve this equation to give A'_s.

(5) For no resultant force on the section

$$N_{sd} = 0.567 f_{ck} A_{cc} + f_{sc} A'_s + f_s A_s$$

Solve this equation to give A_s.

(6) Repeat the previous steps for different values of x to find a minimum $(A'_s + A_s)$.

In steps (4) and (5)

A_{cc} is the area of concrete in compression shown shaded
$\bar{x}$ is the distance from the centroid of A_{cc} to the extreme fibre in compression
f_s the stress in reinforcement A_s, negative if tensile.

The calculation for a particular cross-section would be very similar to that described in example 9.3 except when using the design equations it would be necessary to determine A_{cc} and $\bar{x}$ for each position of the neutral axis.

9.5.2 Simplified Preliminary Design Method

The procedure is similar to that described for a column with a rectangular section as described in section 9.4.3 and figure 9.16.

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The column is designed to resist a moment M_a only, where

$$M_a = M_{sd} + N_{sd}\left(\frac{h}{2} - d_2\right) \tag{9.10}$$

The steel area required to resist this moment can be calculated from

$$M_a = 0.567 f_{ck} A_{cc}(d - \bar{x}) + 0.87 f_{yk} A'_s(d - d') \tag{9.11}$$

and

$$0.87 f_{yk} A_s = 0.567 f_{ck} A_{cc} + 0.87 f_{yk} A'_s \tag{9.12}$$

where A_{cc} is the area of concrete in compression with $x = 0.45d$ for a C12/15 to C35/45 concrete, and $\bar{x}$ is the distance from the centroid of A_{cc} to the extreme fibre in compression.

The area of tension reinforcement, A_s as given by equation 9.12 is then reduced by an amount equal to $N_{sd}/0.87 f_{yk}$.

This method should not be used if the eccentricity, e, is less than $(h/2 - d_2)$.

9.5.3 M–N Interaction Diagram

These diagrams can be constructed using the method described in section 4.8 with examples 4.10 and 4.11. They are particularly useful for a column in a multi-storey building where the moments and associated axial forces change at each storey. The diagrams can be constructed after carrying out the approximate design procedure in section 9.5.2 to obtain suitable arrangements of reinforcing bars.

Example 9.5 Design of a Non-rectangular Column Section

Design the reinforcement for the non-rectangular section shown in figure 9.18 given $M_{sd} = 320\,\text{kN m}$, $N_{sd} = 1200\,\text{kN}$ at the ultimate limit state and the characteristic material strengths are $f_{ck} = 25\,\text{N/mm}^2$ and $f_{yk} = 460\,\text{N/mm}^2$.

$$e = \frac{M_{sd}}{N_{sd}} = \frac{320 \times 10^6}{1200 \times 10^3} = 267\,\text{mm} > \left(\frac{h}{2} - d_2\right)$$

$$\text{Increased moment } M_a = M_{sd} + N_{sd}\left(\frac{h}{2} - d_2\right)$$

$$= 320 + 1200(200 - 80)10^{-3}$$

$$= 464\,\text{kN m}$$

With $x = 0.45d = 144\,\text{mm}$, $s = 0.8x = 115\,\text{mm}$ and the width (b_1) of the section at the limit of the stress block

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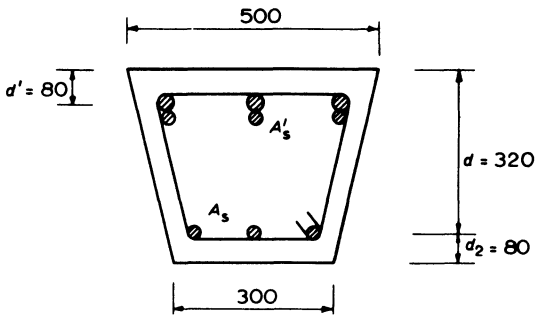

Figure 9.18

$$b_1 = 300 + \frac{200(400 - 115)}{400} = 443\,\text{mm}$$

$$A_{cc} = \frac{x(b + b_1)}{2}$$

$$= \frac{115(500 + 443)}{2} = 54\,223\,\text{mm}^2$$

The depth of the centroid of the trapezium is given by

$$\bar{x} = \frac{s(b + 2b_1)}{3(b + b_1)}$$

$$= 115\frac{(500 + 2 \times 443)}{3(500 + 443)} = 56.3\,\text{mm}$$

Therefore substituting in equation 9.11

$$464 \times 10^6 = 0.567 \times 25 \times 54\,223(320 - 56.3) + 0.87 \times 460A'_s(320 - 80)$$

hence

$$A'_s = 2721\,\text{mm}^2$$

Provide three T32 plus two T16 bars, area = 2812 mm^2.

From equation 9.12

$$0.87f_{yk}A_s = 0.567 \times 25 \times 54\,223 + 0.87 \times 460 \times 2721$$

therefore

$$A_s = 4642\,\text{mm}^2$$

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Reducing A_s by $N_{sd}/0.87f_{yk}$ gives

$$A_s = 4642 - \frac{1200 \times 10^3}{0.87 \times 460}$$
$$= 1643\,\text{mm}^2$$

Provide one T16 plus two T32 bars, area = $1811\,\text{mm}^2$.

The total area of reinforcement provided = $4364\,\text{mm}^2$ which is less than the 8 per cent allowed.

An M–N interaction diagram could now be constructed for this steel arrangement, as in section 4.8, to provide a more rigorous design.

9.6 Biaxial Bending of Short Columns

For most columns, biaxial bending will not govern the design. The loading patterns necessary to cause biaxial bending in a building's internal and edge columns will not usually cause large moments in both directions. Corner columns may have to resist significant bending about both axes, but the axial loads are usually small and a design similar to the adjacent edge columns is generally adequate.

A design for biaxial bending based on a rigorous analysis of the cross-section and the strain and stress distributions would be according to the fundamental principles of chapter 4. For members with a rectangular cross-section, separate checks in the two principal planes are permissible if the ratio of the corresponding eccentricities satisfies one of the following conditions:

either $$\frac{e_z}{h} \Big/ \frac{e_y}{b} \leqslant 0.2$$

or $$\frac{e_y}{b} \Big/ \frac{e_z}{h} \leqslant 0.2$$

where e_y and e_z are the first-order eccentricities in the directions of the section dimensions b and h respectively. Where these conditions are not fulfilled, the Concise EuroCode recommends that the column be designed using the principles of BS 8110.

This method specifies that a column subjected to an ultimate load N_{sd} and moments M_z and M_y in the direction of the ZZ and YY axes respectively (see figure 9.19) may be designed for single axis bending but with an increased moment and subject to the following conditions:

(a) if $$\frac{M_z}{h'} \geqslant \frac{M_y}{b'}$$

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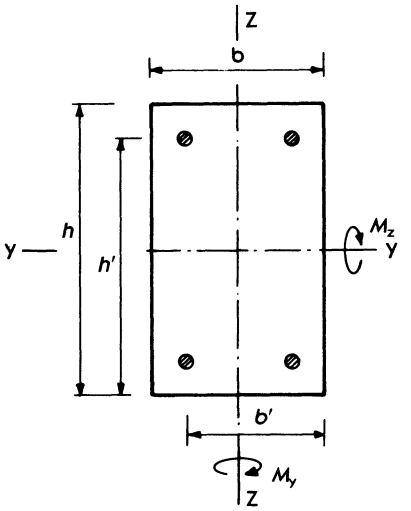

Figure 9.19 *Section with biaxial bending*

then increased single axis design moment is

$$M'_z = M_z + \beta \frac{h'}{b'} \times M_y$$

(b) if $\dfrac{M_z}{h'} < \dfrac{M_y}{b'}$

then increased single axis design moment is

$$M'_y = M_y + \beta \frac{b'}{h'} \times M_z$$

The dimensions h' and b' are defined in figure 9.19 and the coefficient β is specified in table 9.2

Example 9.6 Design of a Column for Biaxial Bending

The column section shown in figure 9.20 is to be designed to resist an ultimate axial load of 1200 kN plus moments of $M_z = 75$ kN m and $M_y = 80$ kN m. The characteristic material strengths are $f_{ck} = 25\,\text{N/mm}^2$ and $f_{yk} = 460\,\text{Nmm}^2$.

$$e_z = \frac{M_z}{N_{sd}} = \frac{75 \times 10^6}{1200 \times 10^3} = 62.5$$

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Table 9.2 Values of coefficient β for biaxial bending

$\frac{N_{sd}}{bhf_{ck}}$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	≥0.75
β	1.00	0.91	0.81	0.72	0.63	0.53	0.44	0.35	0.3

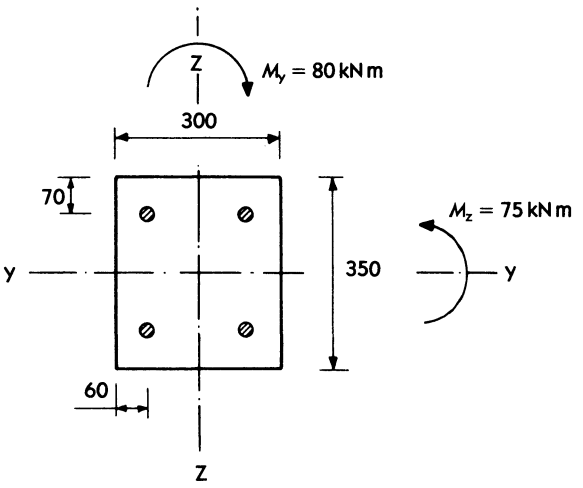

Figure 9.20 *Biaxial bending example*

$$e_y = \frac{M_y}{N_{sd}} = \frac{80 \times 10^6}{1200 \times 10^3} = 66.7$$

thus

$$\frac{e_z}{h} \Big/ \frac{e_y}{b} = \frac{62.5}{350} \Big/ \frac{66.7}{300} = 0.8 > 0.2$$

and

$$\frac{e_y}{b} \Big/ \frac{e_z}{h} = \frac{66.7}{300} \Big/ \frac{62.5}{350} = 1.24 > 0.2$$

Hence the column must be designed for biaxial bending.

$$\frac{M_z}{h'} = \frac{75}{(350 - 70)} = 0.268$$

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$$\frac{M_y}{b'} = \frac{80}{(300 - 60)} = 0.333$$

$$M_z/h' < M_y/b'$$

therefore increased single axis design moment is

$$M'_y = M_y + \beta \frac{b'}{h'} \times M_z$$

$$N_{sd}/bhf_{ck} = 1200 \times 10^3/(300 \times 350 \times 25) = 0.46$$

From table 9.2, $\beta = 0.57$

$$M'_y = 80 + 0.57 \times \frac{240}{280} \times 75 = 116.6\,\text{kN m}$$

thus

$$\frac{M_{sd}}{bh^2 f_{ck}} = \frac{116.6 \times 10^6}{350 \times (300)^2 \times 25} = 0.15$$

From the design chart of figure 9.9

$$\frac{A_s f_{yk}}{bhf_{ck}} = 0.47$$

Therefore required $A_s = 2682\,\text{mm}^2$
So provide four T32 bars.

9.7 Design of Slender Columns

As specified in section 9.2, a column is classified as slender if the slenderness ratio about either axis is

$$\lambda > \lambda_{min} = 25 \quad \text{or} \quad 15/\sqrt{\nu_u}$$

There is a general restriction on the maximum slenderness ratio of columns with circular or rectangular sections of

$$\lambda_{max} \not> 140$$

If $\lambda < \lambda_{min}$ then the column may be classified as short and the slenderness effect may be neglected. A further limiting value of λ is given by

$$\lambda_{crit} = 25(2 - e_{01}/e_{02})$$

where e_{01} and e_{02} are the eccentricities of the axial loads at the ends of the member with e_{01} taken as less than or equal to e_{02}. If $\lambda_{min} \leqslant \lambda \leqslant \lambda_{crit}$ then second-order effects can be neglected and the section should be designed

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with a minimum design moment of $N_{sd}h/20$ even though it may have been classified as slender.

A slender column with $\lambda > \lambda_{crit}$ must be designed for an additional moment caused by its curvature at ultimate conditions. The expressions given in EC2 for the additional moments were derived by studying the moment/curvature behaviour for a member subject to bending plus axial load. The equations for calculating the design moments are only applicable to columns of a rectangular or circular section with symmetrical reinforcement and where the first-order eccentricity is greater than $0.1h$.

A slender column should be designed for an ultimate axial load (N_{sd}) plus an increased moment given by

$$M_{sd} = N_{sd}e_{tot}$$

where $e_{tot} = e_0 + e_a + e_2$

e_0 is an equivalent first-order eccentricity

e_a is an accidental eccentricity which accounts for geometric imperfections in the column

e_2 is the second-order eccentricity.

The equivalent eccentricity e_0 is given by the greater of

$$0.6e_{02} + 0.4e_{01}$$

or

$$0.4e_{02}$$

where e_{01} and e_{02} are the first-order eccentricities at the two ends of the column as described above, and $|e_{02}|$ is greater than $|e_{01}|$.

The accidental eccentricity is given by the equation

$$e_a = \nu\frac{\ell_0}{2}$$

where ℓ_0 is the effective column height about the axis considered and

$$\nu = \frac{1}{100\sqrt{\ell}} \nless \frac{1}{200}$$

where ℓ is the total height of the structure in metres.

The second-order eccentricity e_2 is an estimate of the deflection of the column at failure and is given by the equation

$$e_2 = K_1\frac{\ell_0^2}{10}\left(\frac{2K_2\varepsilon_{yd}}{0.9d}\right)$$

where $K_1 = \frac{\lambda}{20} - 0.75$ for $15 \leqslant \lambda \leqslant 35$

and $K_1 = 1$ for $\lambda > 35$.

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In most practical cases, where $\lambda > 35$, this may be simplified to

$$e_2 = \frac{K_2 \ell_0^2 f_{yk}}{1\,035\,000d}$$

The coefficient K_2 is a reduction factor to allow for the fact that the deflection must be less when there is a large proportion of the column section in compression. The value for K_2 is given by the equation

$$K_2 = \frac{N_{ud} - N_{sd}}{N_{ud} - N_{bal}} \leqslant 1.0 \tag{9.13}$$

where N_{ud} is the ultimate axial load such that

$$N_{ud} = 0.567 f_{ck} A_c + 0.87 f_{yk} A_{sc}$$

and N_{bal} is the axial load at balanced failure defined in section 4.8 and may be taken as $N_{bal} = 0.267 f_{ck} A_c$ for symmetrical reinforcement.

In order to calculate K_2, the area A_s of the column reinforcement must be known and hence a trial-and-error approach is necessary, taking an initial conservative value of $K_2 = 1.0$. Values of K_2 are also marked on the column design charts as shown in figure 9.9 and in the Appendix.

Example 9.7 Design of a Slender Column

A non-sway column of 300 × 450 cross-section resists at the ultimate limit state an axial load of 1700 kN and end moments of 70 kN m and 10 kN m causing double curvature about the minor axis YY as shown in figure 9.21. The column's effective heights are $\ell_{ez} = 6.75$ m and $\ell_{ey} = 8.0$ m and the characteristic material strengths $f_{ck} = 25\,\text{N/mm}^2$ and $f_{yk} = 460\,\text{N/mm}^2$.

Eccentricities are

$$e_{01} = \frac{M_1}{N_{sd}} = \frac{10 \times 10^3}{1700} = 5.9\,\text{mm}$$

$$e_{02} = \frac{M_2}{N_{sd}} = \frac{-70 \times 10^3}{1700} = -41.2\,\text{mm}$$

where e_{02} is negative since the column is bent in double curvature.

Limiting slenderness ratio

$$\lambda_{crit} = 25\left(2 - \frac{e_{01}}{e_{02}}\right)$$

$$= 25\left(2 - \frac{5.9}{-41.2}\right)$$

$$= 53.57$$

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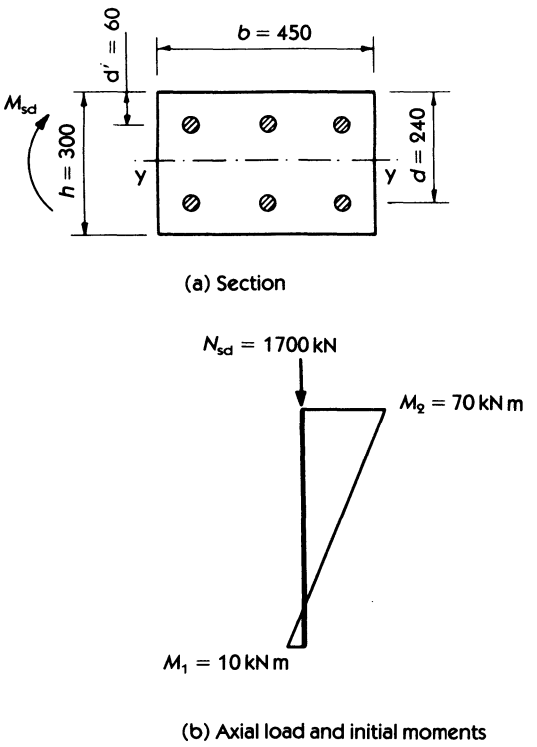

Figure 9.21 *Slender column example*

Actual slenderness ratios are

$$\lambda_z = \frac{\ell_{ez}}{i_z} = \frac{6.75}{0.3} \times 3.46 = 77.85 > 53.57$$

$$\lambda_y = \frac{\ell_{ey}}{i_y} = \frac{8.0}{0.45} \times 3.46 = 61.55 > 53.57$$

Therefore the column is slender, and λ_z is critical.

Equivalent eccentricity = $0.6e_{02} + 0.4e_{01} \geqslant 0.4e_{02}$

$$0.6e_{02} + 0.4e_{01} = 0.6 \times 41.2 + 0.4 \times (-5.9) = 22.35\,\text{mm}$$

$$0.4e_{02} = 0.4 \times 41.2 = 16.47\,\text{mm}$$

Therefore the equivalent eccentricity $e_e = 22.35\,\text{mm}$

Taking v as 1/200 the accidental eccentricity is

$$e_a = v\frac{\ell_{ez}}{2} = \frac{1}{200} \times \frac{6750}{2} = 16.88\,\text{mm}$$

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Since $\lambda > 35$, the second-order eccentricity is

$$e_2 = \frac{K_2 \ell_{ez}^2 f_{yk}}{1\,035\,000 d}$$
$$= \frac{1 \times 6750^2 \times 460}{1\,035\,000 \times 240}$$
$$= 84.38\,\text{mm}$$

with $K_2 = 1.0$ for the initial value.

For the first iteration the total eccentricity is

$$e_{tot} = e_0 + e_a + e_2$$
$$= 22.35 + 16.88 + 84.38 = 123.61\,\text{mm}$$

and the total moment is

$$M_t = N_{sd} e_{tot} = 1700 \times 123.61 \times 10^{-3} = 210\,\text{kN m}$$

$$\frac{N_{sd}}{bhf_{ck}} = \frac{1700 \times 10^3}{450 \times 300 \times 25} = 0.504$$

$$\frac{M_t}{bh^2 f_{ck}} = \frac{210 \times 10^6}{450 \times 300^2 \times 25} = 0.208$$

From the design chart of figure 9.9

$$\frac{A_s f_{yk}}{bhf_{ck}} = 0.77 \quad \text{and} \quad K_2 = 0.76$$

This new value of K_2 is used to calculate e_2 and hence M_t for the second iteration. The design chart is again used to determine $A_s f_{yk}/bhf_{ck}$ and a new value of K_2 as shown in table 9.3. The iterations are continued until the value of K_2 in columns (1) and (5) of the table are in reasonable agreement, which in this design occurs after two iterations. So that the steel area required is

Table 9.3

(1) K_2	*(2)* M_t	*(3)* $M/bh^2 f_{ck}$	*(4)* $\frac{A_s f_{yk}}{bhf_{ck}}$	*(5)* K_2
1.0	210	0.208	0.77	0.76
0.76	176	0.174	0.6	0.71

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$$A_s = \frac{0.6bhf_{ck}}{f_{yk}} = \frac{0.6 \times 450 \times 300 \times 25}{460} = 4400\,\text{mm}^2$$

and $K_2 = 0.71$.

As a check on the final value of K_2 interpolated from the design chart:

$$\begin{aligned} N_{bal} &= 0.267 f_{ck} A_c \\ &= 0.267 \times 25 \times 300 \times 450 \times 10^{-3} \\ &= 901\,\text{kN} \end{aligned}$$

$$\begin{aligned} N_{ud} &= 0.567 f_{ck} A_c + 0.87 f_{yk} A_s \\ &= (0.567 \times 25 \times 300 \times 450 + 0.87 \times 460 \times 4400)10^{-3} \\ &= 3674\,\text{kN} \end{aligned}$$

$$K_2 = \frac{N_{ud} - N_{sd}}{N_{ud} - N_{bal}} = \frac{3674 - 1700}{3674 - 901} = 0.71$$

which agrees with the final value in column 5 of the table.

9.8 Walls

Walls may take the form of non-structural dividing elements in which case their thickness will often reflect sound insulation and fire resistance requirements. Nominal reinforcement will be used to control cracking in such cases. More commonly, reinforced concrete walls will form part of a structural frame and will be designed for vertical and horizontal forces and moments obtained by normal analysis methods. In this situation a wall is defined as being a vertical load-bearing member whose length is not less than four times its thickness.

Where several walls are connected monolithically so that they behave as a unit, they are described as a wall system. Sometimes horizontal forces on a structure are resisted by more than one wall or system of walls, in which case the distribution of forces between the walls or systems will be assumed to be in proportion to their stiffnesses.

It is normal practice to consider a wall as a series of vertical strips when designing vertical reinforcement. Each strip is then designed as a column subject to the appropriate vertical load and transverse moments at its top and bottom. Slenderness effects must be considered where necessary, as for columns. If a wall is subject predominantly to lateral bending, the design and detailing will be undertaken as if it were a slab, but the wall thickness will usually be governed by slenderness limitations, fire resistance requirements and construction practicalities.

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Reinforcement Detailing

For a wall designed either as a series of columns or as a slab, the area of vertical reinforcement should lie between $0.004A_c$ and $0.04A_c$ and this will normally be equally divided between each face. Bar spacing along the length of the wall should not exceed the lesser of 300 mm or twice the wall thickness.

Horizontal bars should have a diameter of not less than one-quarter of the vertical bars, and with a total area of not less than one-half of the vertical bars. The horizontal bars should lie between the vertical bars and the concrete surface, with a spacing which is not greater than 300 mm.

If the area of vertical steel exceeds $0.02A_c$, then the bars should be enclosed by stirrups designed according to the rules for columns.

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10

Foundations and Retaining Walls

The design of foundations and retaining walls is based on the fundamental principles and techiques of analysis which have been discussed in previous chapters. The analysis and design of both forms of structure are presented in this chapter. The reader should however be aware that some of the design values used, which have been extracted from Part 1 of EC2, may not be the design parameters to be used when the definitive EuroCode documents for foundations are eventually published.

A building is generally composed of a superstructure above the ground and a substructure which forms the foundations below ground. The foundations transfer and spread the loads from a structure's columns and walls into the ground. The safe bearing capacity of the soil must not be exceeded otherwise excessive settlement may occur, resulting in damage to the building and its service facilities, such as the water or gas mains. Foundation failure can also affect the overall stability of a structure so that it is liable to slide, to lift vertically or even overturn.

The earth under the foundations is the most variable of all the materials that are considered in the design and construction of an engineering structure. Under one small building the soil may vary from a soft clay to a dense rock. Also the nature and properties of the soil will change with the seasons and the weather. For example Keuper Marl, a relatively common soil, is hard like a rock when dry but when wet it can change into an almost liquid state.

It is important to have an engineering survey made of the soil under a proposed structure so that variations in the strata and the soil properties can be determined. Drill holes or trial pits should be sunk, *in situ* tests such as the penetration test performed and samples of the soil taken to be tested in the laboratory. From the information gained it is possible to recommend safe earth bearing pressures and, if necessary, calculate

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Table 10.1 Typical allowable bearing values

Rock or soil	*Typical bearing value (kN/m^2)*
Massive igneous bedrock	10 000
Sandstone	2000 to 4000
Shales and mudstone	600 to 2000
Gravel, sand and gravel, compact	600
Medium dense sand	100 to 300
Loose fine sand	less than 100
Hard clay	300 to 600
Medium clay	100 to 300
Soft clay	less than 75

possible settlements of the structure. Representative values of the safe bearing pressures for typical soils are listed in table 10.1.

In the design of foundations, the areas of the bases in contact with the ground should be such that the safe bearing pressures will not be exceeded. Settlement takes place during the working life of the structure, therefore the design loading to be considered when calculating the base areas should be those that apply to the serviceability limit state, and typical values that can be taken are

(1) permanent plus variable load $= 1.0G_k + 1.0Q_k$
(2) permanent plus wind load $= 1.0G_k + 1.0W_k$
(3) permanent plus variable plus wind load $= 1.0G_k + 0.9Q_k + 0.9W_k$

These partial factors of safety are suggested as it is highly unlikely that the maximun imposed load and the worst wind load will occur simultaneously.

Where the foundations are subject to a vertical and a horizontal load the following rule can be applied.

$$\frac{V}{P_v} + \frac{H}{P_h} < 1.0$$

where V = the vertical load
H = the horizontal load
P_v = the allowable vertical load
P_h = the allowable horizontal load.

The allowable horizontal load would take account of the passive resistance of the ground in contact with the vertical face of the foundation plus the friction and cohesion along the base.

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The calculations to determine the structural strength of the foundations, that is the thickness of the bases and the areas of reinforcement, should be based on the loadings and the resultant ground pressures corresponding to the ultimate limit state.

With some structures, such as the type shown in figure 10.1, it may be necessary to check the possibility of uplift to the foundations and the stability of the structure when it is subjected to lateral loads. To ensure adequate safety, the stability calculations should also be for the loading arrangements associated with the ultimate limit state. The critical loading arrangement is usually the combination of maximum lateral load with minimum permanent load and no variable load, that is $1.5W_k + 1.0G_k$. Minimum permanent load can sometimes occur during erection when many of the interior finishes and fixtures may not have been installed.

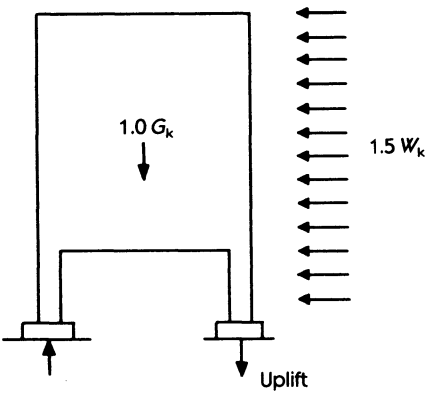

Figure 10.1 *Uplift on footing*

For most designs a linear distribution of soil pressure across the base of the footing is assumed as shown in figure 10.2a. This assumption must be based on the soil acting as an elastic material and the footing having infinite rigidity. In fact, not only do most soils exhibit some plastic behaviour and all footings have a finite stiffness, but also the distribution of soil pressure varies with time. The actual distribution of bearing pressure at any moment may take the form shown in figure 10.2b or c, depending on the type of soil and the stiffness of the base and the structure. But as the behaviour of foundations involves many uncertainties regarding the action of the ground and the loading, it is usually unrealistic to consider an analysis which is too sophisticated.

Foundations should be constructed so that the underside of the bases are below frost level. As the concrete is subjected to more severe

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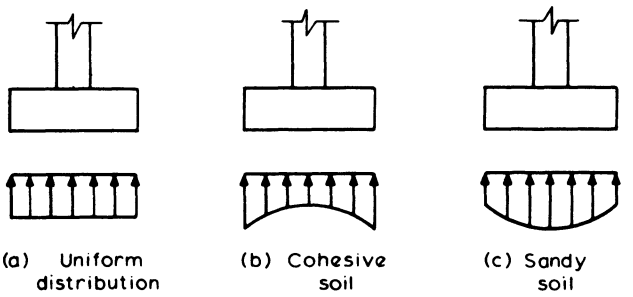

Figure 10.2 *Pressure distributions under footings*

exposure conditions a larger nominal cover to the reinforcement is required. It is recommended that the minimum cover should be not less than 75 mm when the concrete is cast against the ground, or less than 50 mm when the concrete is cast against a layer of blinding concrete. A concrete grade of at least $f_{ck} = 30\,N/mm^2$ (that is, C30/37) is required to meet the durability requirements, see tables 6.1 and 6.2.

10.1 Pad Footings

The footing for a single column may be made square in plan, but where there is a large moment acting about one axis it may be more economical to have a rectangular base.

Assuming there is a linear distribution the bearing pressures across the base will take one of the three forms shown in figure 10.3, according to the relative magnitudes of the axial load N and the moment M acting on the base.

(1) In figure 10.3a there is no moment and the pressure is uniform

$$p = \frac{N}{BD} \qquad (10.1)^*$$

(2) With a moment M acting as shown, the pressures are given by the equation for axial load plus bending. This is provided there is positive contact between the base and the ground along the complete length D of the footing, as in figure 10.3b so that

$$p = \frac{N}{BD} \pm \frac{My}{I}$$

where I is the second moment of area of the base about the axis of bending and y is the distance from the axis to where the pressure is

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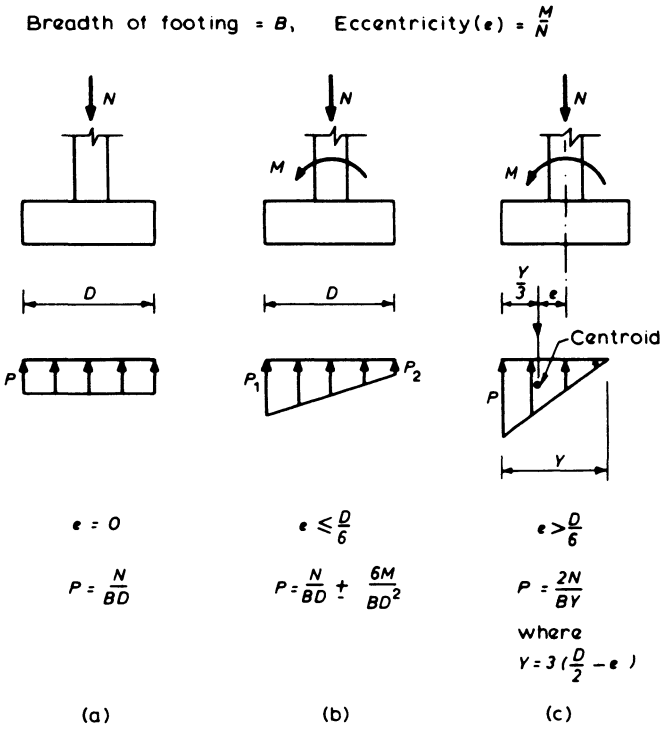

Figure 10.3 *Pad footing – pressure distributions*

being calculated. Substituting for $I = BD^3/12$ and $y = D/2$, the maximum pressure is

$$p_1 = \frac{N}{BD} + \frac{6M}{BD^2} \qquad (10.2)^*$$

and the minimum pressure is

$$p_2 = \frac{N}{BD} - \frac{6M}{BD^2} \qquad (10.3)^*$$

There is positive contact along the base if p_2 from equation 10.3 is positive. When pressure p_2 just equals zero

$$\frac{N}{BD} - \frac{6M}{BD^2} = 0$$

or

$$\frac{M}{N} = \frac{D}{6}$$

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So that for p_2 to always be positive, M/N – or the effective eccentricity, e – must never be greater than $D/6$. In these cases the eccentricity of loading is said to lie within the 'middle third' of the base.

(3) When the eccentricity, e is greater than $D/6$ there is no longer a positive pressure along the length D and the pressure diagram is triangular as shown in figure 10.3c. Balancing the downward load and the upward pressures

$$\frac{1}{2}pBY = N$$

therefore

$$\text{maximum pressure } p = \frac{2N}{BY}$$

where Y is the length of positive contact. The centroid of the pressure diagram must coincide with the eccentricity of loading in order for the load and reaction to be equal and opposite. Thus

$$\frac{Y}{3} = \frac{D}{2} - e$$

or

$$Y = 3\left(\frac{D}{2} - e\right)$$

therefore in this case of $e > D/6$

$$\text{maximum pressure } p = \frac{2N}{3B(D/2 - e)} \qquad (10.4)^*$$

A typical arrangement of the reinforcement in a pad footing is shown in figure 10.4. With a square base the reinforcement to resist bending should be distributed uniformly across the full width of the footing. For a rectangular base the reinforcement in the short direction should be distributed with a closer spacing in the region under and near the column, to allow for the fact that the transverse moments must be greater nearer the column. If the footing should be subjected to a large overturning moment so that there is only partial bearing, or if there is a resultant uplift force, then reinforcement may also be required in the top face.

Dowels or starter bars should extend from the footing into the column in order to provide continuity to the reinforcement. These dowels should be embedded into the footing and extend into the columns a full lap length. Sometimes a 75 mm length of the column is constructed in the same concrete pour as the footing so as to form a 'kicker' or support for

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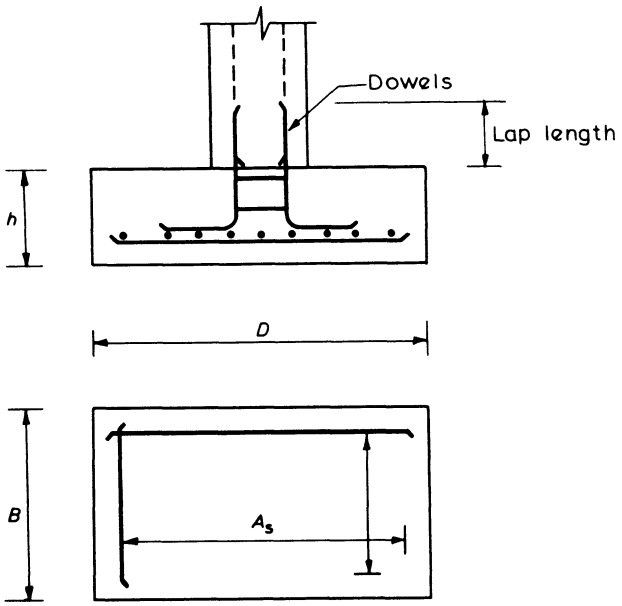

Figure 10.4 *Pad footing reinforcement details*

the column shutters. In these cases the dowel lap length should be measured from the top of the kicker.

The critical sections through the base for checking shear, punching shear and bending are shown in figure 10.5. The shearing force and bending moments are caused by the ultimate loads from the column and the weight of the base should not be included in these calculations.

The thickness of the base is often governed by the requirements for shear resistance.

The principal steps in the design calculations are as follows:

(1) Calculate the plan size of the footing using the permissible bearing pressure and the critical loading arrangement for the serviceability limit state.
(2) Calculate the bearing pressures associated with the critical loading arrangement at the ultimate limit state.
(3) Assume a suitable value for the thickness (h) and effective depth (d). Check that the shear force at the column face is less than $0.9ud\sqrt{f_{ck}}$ where u is the perimeter of the column.
(4) Check the thickness for punching shear, assuming a probable value for the basic design shear stress, τ_{Rd}, from table 5.1.
(5) Determine the reinforcement required to resist bending.

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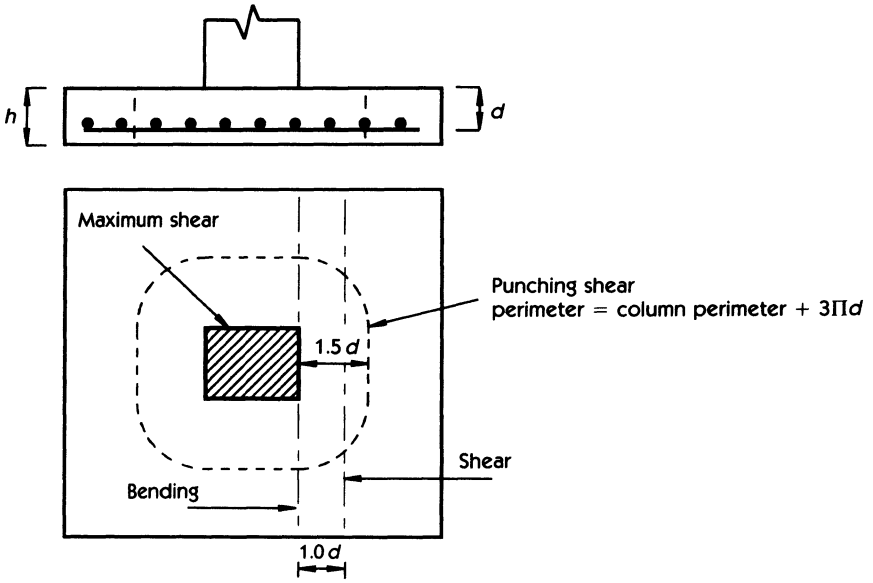

Figure 10.5 *Critical sections for design*

(6) Make a final check of the punching shear, having established τ_{Rd} precisely.
(7) Check the shear force at the critical sections.
(8) Where applicable, foundations and structure should be checked for overall stability at the ultimate limit state.

Reinforcement to resist bending in the bottom of the base should extend at least a full tension anchorage length beyond the critical section for bending.

Example 10.1 Design of a Pad Footing

The footing (figure 10.6) is required to resist characteristic axial loads of 1000 kN permanent and 350 kN variable from a 400 mm square column. The safe bearing pressure on the soil is 200 kN/m^2 and the characteristic material strengths are f_{ck} = 30 N/mm^2 and f_{yk} = 460 N/mm^2.

Assume a footing weight of 150 kN so that the total permanent load is 1150 kN.

(1) For the Serviceability Limit State

$$\text{Total design axial load} = 1.0G_k + 1.0Q_k$$
$$= 1150 + 350 = 1500\,\text{kN}$$

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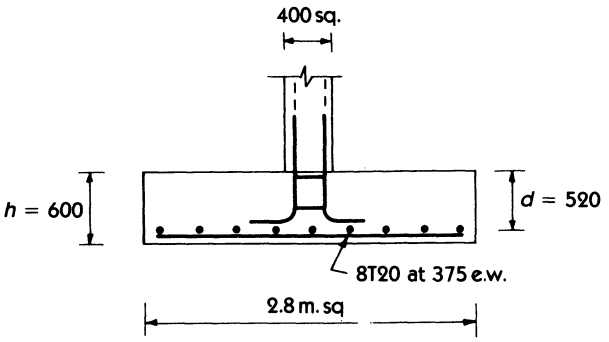

Figure 10.6 *Pad footing example*

$$\text{Required base area} = \frac{1500}{200} = 7.5\,\text{m}^2$$

Provide a base 2.8 m square = 7.8 m^2.

(2) For the Ultimate Limit State

$$\begin{aligned}\text{Column design axial load, } N_{sd} &= 1.35G_k + 1.5Q_k \\ &= 1.35 \times 1000 + 1.5 \times 350 \\ &= 1875\,\text{kN}\end{aligned}$$

$$\text{Earth pressure} = \frac{1875}{2.8^2} = 239\,\text{kN/m}^2$$

(3) Assume a 600 mm thick footing and with the footing constructed on a blinding layer of concrete the minimum cover is taken as 50 mm. Therefore take mean effective depth d = 520 mm.

At the column face

$$\begin{aligned}\text{Maximum shear resistance } V_{Rd2} &= 0.9ud\sqrt{f_{ck}} \\ &= 0.9 \times 1600 \times 520 \times \sqrt{30} \times 10^{-3} \\ &= 4100\,\text{kN}\ (>N_{sd} = 1875\,\text{kN})\end{aligned}$$

(4) Punching Shear – see figure 10.5

$$\begin{aligned}\text{Critical perimeter} &= \text{column perimeter} + 3\pi d \\ &= 4 \times 400 + 3\pi \times 520 = 6500\,\text{mm}\end{aligned}$$

$$\begin{aligned}\text{Area within perimeter} &= (400 + 3d)^2 - (4 - \pi)(1.5d)^2 \\ &= (400 + 1560)^2 - (4 - \pi)780^2 \\ &= 3.32 \times 10^6\,\text{mm}^2\end{aligned}$$

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therefore

$$\text{Punching shear force } V_{Sd} = 239\,(2.8^2 - 3.32)$$
$$= 1080\,\text{kN}$$

$$\text{Punching shear stress } v_{Sd} = \frac{V_{Sd}}{\text{Perimeter} \times d}$$
$$= \frac{1080 \times 10^3}{6500 \times 520} = 0.32\,\text{N/mm}^2$$

From table 5.1 this ultimate shear stress is not excessive, therefore $h = 600\,\text{mm}$ will be suitable.

(5) Bending Reinforcement – see figure 10.7a

At the column face which is the critical section

$$M_{sd} = (239 \times 2.8 \times 1.2) \times \frac{1.2}{2}$$
$$= 482\,\text{kN m}$$

For the concrete

$$M_{bal} = 0.167 f_{ck} b d^2$$
$$= 0.167 \times 30 \times 2800 \times 520^2 \times 10^{-6}$$
$$= 3793\,\text{kN m} > 482$$

$$A_s = \frac{M_{sd}}{0.87 f_{yk} z}$$

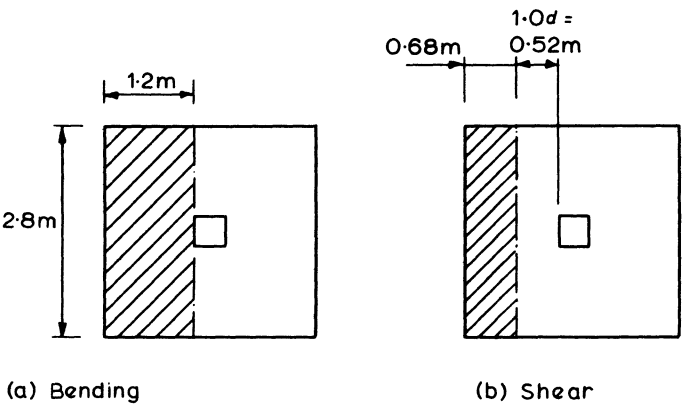

Figure 10.7 *Critical sections*

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From the lever-arm curve, figure 4.5, $l_a = 0.98$. Therefore

$$A_s = \frac{482 \times 10^6}{0.87 \times 460(0.98 \times 520)}$$
$$= 2363\,\text{mm}^2$$

Provide eight T20 bars at 375 mm centres, $A_s = 2510\,\text{mm}^2$. Therefore

$$\frac{100A_s}{bd} = \frac{100 \times 2510}{2800 \times 520} = 0.172 > 0.15$$

that is, the minimum steel area requirement is satisfied.

However for crack control purposes the minimum steel area must be further checked together with the maximum bar size or bar spacing. From equation 6.2:

Minimum steel area

$$A_s = k_c k f_{ct.ef} A_{ct}/f_{yk}$$
$$= k_c k f_{ct.ef}(bh/2)/f_{yk}$$

If k is taken as 1.0, assuming that cracking is likely to be caused by foundation settlement then

$$A_s = k_c \times 1.0 \times f_{ct.ef} \times (2800 \times 600/2)/f_{yk}$$
$$= 0.4 \times 1.0 \times 3.0 \times (2800 \times 300)/460$$
$$= 2191\,\text{mm}^2$$

Hence eight T20 bars at 375 mm centres, $A_s = 2510\,\text{mm}^2$, are adequate.

Maximum bar size

The steel stress should be calculated under the action of the quasi-permanent loading which can be estimated from equation 6.1 as follows:

$$f_s = \frac{f_{yk}(G_k + 0.3Q_k)}{1.15(1.35G_k + 1.5Q_k)}$$
$$= \frac{460(1000 + 0.3 \times 350)}{1.15(1.35 \times 1000 + 1.5 \times 350)}$$
$$= 236\,\text{N/mm}^2$$

Therefore from table 6.7 the maximum allowable bar size is 20 mm. Hence minimum area and bar size requirements as specified by the code for the purposes of crack control are met.

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(6) Final Check of Punching Shear

From table 5.1, for $f_{ck} = 30$, $\tau_{Rd} = 0.34\,N/mm^2$. Therefore

Shear resistance of concrete, $V_{Rd1} = [\tau_{Rd}k(1.2 + 40\rho)]ud$

where $k = 1.6 - d = 1.6 - 0.52 = 1.08$ and $\rho = A_s/bd = 0.00172$. Hence

$$V_{Rd1} = [0.34 \times 1.08 \times (1.2 + 40 \times 0.00172)]6500 \times 520 \times 10^{-3}$$
$$= 1575\,kN\ (>\text{punching shear } V_{sd} = 1080\,kN)$$

(7) Maximum Shear Force – see figure 10.7b

At the critical section for shear, $1.0d$ from the column face:

$$\text{Design shear } V_{Sd} = 239 \times 2.8 \times 0.68$$
$$= 455\,kN$$

$$V_{Rd1} = [\tau_{Rd}k(1.2 + 40\rho)]b_w d$$
$$= [0.34 \times 1.08 \times (1.2 + 40 \times 0.00172)]2800 \times 520 \times 10^{-3}$$
$$= 678\,kN\ (>V_{sd} = 455\,kN)$$

Instead of assuming a footing weight of 150 kN at the start of this example it is possible to allow for the weight of the footing by using a net safe bearing pressure p_{net} where

$$p_{net} = 200 - h \times \text{unit weight of concrete}$$
$$= 200 - 0.6 \times 24 = 185.6\,N/m^2$$

Therefore

$$\text{Required base area} = \frac{1.0 \times \text{column load}}{p_{net}} = \frac{1000 + 350}{185.6}$$
$$= 7.3\,m^2$$

It should be noted that the self-weight of the footing or its effect must be included in the calculations at serviceability for determining the area of the base but at the ultimate limit state the self-weight should not be included.

10.2 Combined Footings

Where two columns are close together it is sometimes necessary or convenient to combine their footings to form a continuous base. The dimensions of the footing should be chosen so that the resultant load passes through the centroid of the base area. This may be assumed to give a uniform bearing pressure under the footing and help to prevent dif-

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ferential settlement. For most structures the ratios of permanent and variable loads carried by each column are similar so that if the resultant passes through the centroid for the serviceability limit state then this will also be true – or very nearly – at the ultimate limit state, and hence in these cases a uniform pressure distribution may be considered for both limit states.

The shape of the footing may be rectangular or trapezoidal as shown in figure 10.8. The trapezoidal base has the disadvantage of detailing and cutting varying lengths of reinforcing bars; it is used where there is a large variation in the loads carried by the two columns and there are limitations on the length of the footing. Sometimes in order to strengthen the base and economise on concrete a beam is incorporated between the two columns so that the base is designed as an inverted T-section.

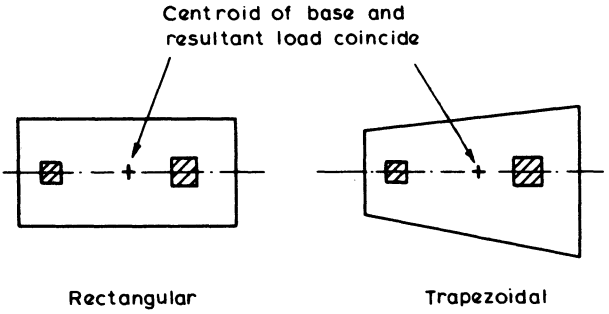

Figure 10.8 *Combined bases*

The proportions of the footing depend on many factors. If it is too long, there will be large longitudinal moments on the lengths projecting beyond the columns, whereas a short base will have a larger span moment between the columns and the greater width will cause large transverse moments. The thickness of the footing must be such that the shear stresses are not excessive.

Example 10.2 Design of a Combined Footing

The footing supports two columns 300 mm square and 400 mm square with characteristic permanent and variable loads as shown in figure 10.9. The safe bearing pressure is 300 kN/m^2 and the characteristic material strengths are f_{ck} = 30 N/mm^2 and f_{yk} = 460 N/mm^2. Assume a base thickness h = 800 mm.

(1) Base area

Net safe bearing pressure $p_{net} = 300 - 24h = 300 - 24 \times 0.8$
$= 280.8\,\text{kN/m}^2$

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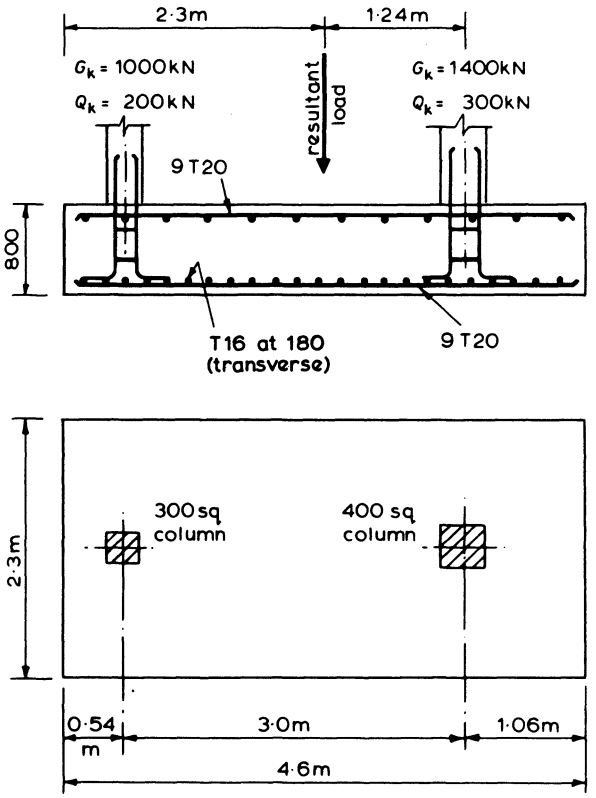

Figure 10.9 *Combined footing example*

Total load = 1000 + 200 + 1400 + 300 = 2900 kN

$$\text{Area of base required} = \frac{2900}{280.8} = 10.3\,\text{m}^2$$

Provide a rectangular base, 4.6 m × 2.3 m, area = 10.58 m^2.

(2) Resultant of column loads and centroid of base: taking moments about the centre line of the 400 mm square column

$$\bar{x} = \frac{1200 \times 3}{1200 + 1700} = 1.24\,\text{m}$$

The base is centred on this position of the resultant of the column loads as shown in figure 10.9.

(3) Bearing pressure at the ultimate limit state:

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$$\text{column loads} = 1.35 \times 1000 + 1.5 \times 200 + 1.35 \times 1400 + 1.5 \times 300$$
$$= 1650 + 2340 = 3990\,\text{kN}$$

therefore

$$\text{earth pressure} = \frac{3990}{4.6 \times 2.3} = 377\,\text{kN/m}^2$$

(4) Assuming $d = 740\,\text{mm}$ for the longitudinal bars and with a mean $d = 730\,\text{mm}$ for punching shear calculations:
At the column face

$$\text{Maximum shear resistance, } V_{Rd2} = 0.9ud\sqrt{f_{ck}}$$

For 300 mm square column

$$= 0.9 \times 1200 \times 730 \times \sqrt{30} \times 10^{-3}$$
$$= 4318\,\text{kN} \; (> N_{sd} = 1650\,\text{kN})$$

For 400 mm square column

$$= 0.9 \times 1600 \times 730 \times \sqrt{30} \times 10^{-3}$$
$$= 5757\,\text{kN} \; (> N_{sd} = 2340\,\text{kN})$$

(5) Longitudinal moments and shear forces: the shear-force and bending-moment diagrams at the ultimate limit state and for a net upward pressure of $377\,\text{kN/m}^2$ are shown in figure 10.10.

(6) Longitudinal bending
(i) Maximum moment is at mid-span between the columns

$$A_s = \frac{M_{sd}}{0.87 f_{yk} z} = \frac{679 \times 10^6}{0.87 \times 460 \times 0.98 \times 740}$$
$$= 2340\,\text{mm}^2$$

$$\text{Minimum } A_s = 0.15bd/100 = 0.15 \times 2300 \times 730/100$$
$$= 2519\,\text{mm}^2$$

Provide nine T20 at 270 mm centres, area $= 2830\,\text{mm}^2$, top and bottom to meet the minimum area requirements.

(7) Transverse bending

$$M_{sd} = 377 \times \frac{1.15^2}{2} = 249\,\text{kN m/m}$$

$$A_s = \frac{M_{sd}}{0.87 f_{yk} z} = \frac{249 \times 10^6}{0.87 \times 460 \times 0.98 \times 720}$$
$$= 882\,\text{mm}^2/\text{m}$$

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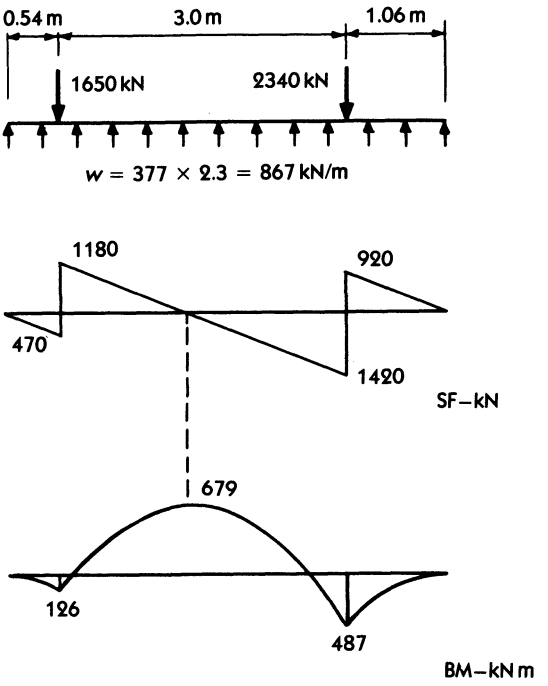

Figure 10.10 *Shear-force and bending-moment diagrams*

But

$$\text{minimum } A_s = \frac{0.15bd}{100} = \frac{0.15 \times 1000 \times 730}{100}$$
$$= 1095\,\text{mm}^2/\text{m}$$

Provide T16 bars at 180 mm centres, area = 1117 mm^2 per metre.

The transverse reinforcement should be placed at closer centres under the columns to allow for greater moments in those regions. For the purposes of crack control, the minimum steel area together with the maximum bar size or maximum bar spacing should also be checked as in example 10.1.

(8) Shear: punching shear cannot be checked, since the critical perimeter $1.5d$ from the column face lies outside the base area. The critical section for shear is taken $1.0d$ from the column face. Therefore with $d = 730$ mm

$$\text{Design shear } V_{Sd} = 1420 - 377 \times 2.3\,(0.73 + 0.2)$$
$$= 614\,\text{kN}$$

$$\text{Shear resistance } V_{Rd1} = [\tau_{Rd}k(1.2 + 40\rho)]b_w d$$

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From table 5.1, $\tau_{Rd} = 0.34\,N/mm^2$, $k = 1.0$ and $\rho = A_s/bd = 0.0017$

Therefore

$$V_{Rd1} = [0.34 \times 1.0\,(1.2 + 40 \times 0.0017)2300 \times 730 \times 10^{-3}$$
$$= 724\,kN\ (>V_{Sd} = 614\,kN)$$

Therefore shear reinforcement is not required.

10.3 Strap Footings

Strap footings, as shown in figure 10.11, are used where the base for an exterior column must not project beyond the property line. A strap beam is constructed between the exterior footing and the adjacent interior footing – the purpose of the strap is to restrain the overturning force due to the eccentric load on the exterior footing.

The base areas of the footings are proportioned so that the bearing pressures are uniform and equal under both bases. Thus it is necessary that the resultant of the loads on the two footings should pass through the centroid of the areas of the two bases. The strap beam between the footings should not bear against the soil, hence the ground directly under the beam should be loosened and left uncompacted.

To achieve suitable sizes for the footings several trial designs may be necessary. With reference to figure 10.11 the principal steps in the design are as follows:

(1) Choose a trial width D for the rectangular outer footing and assume weights W_1 and W_2 for the footings and W_s for the strap beam.

(2) Take moments about the centre line of the inner column in order to determine the reaction R_1 under the outer footing. The loadings should be those required for the serviceability limit state. Thus

$$(R_1 - W_1)\left(L + f - \frac{D}{2}\right) - N_1 L - W_s \frac{L}{2} = 0 \qquad (10.5)$$

and solve for R_1. The width B of the outer footing is then given by

$$B = \frac{R_1}{pD}$$

where p is the safe bearing pressure.

(3) Equate the vertical loads and reactions to determine the reaction R_2 under the inner footing. Thus

$$R_1 + R_2 - (N_1 + N_2 + W_1 + W_2 + W_s) = 0 \qquad (10.6)$$

and solve for R_2. The size S of the square inner footing is then given by

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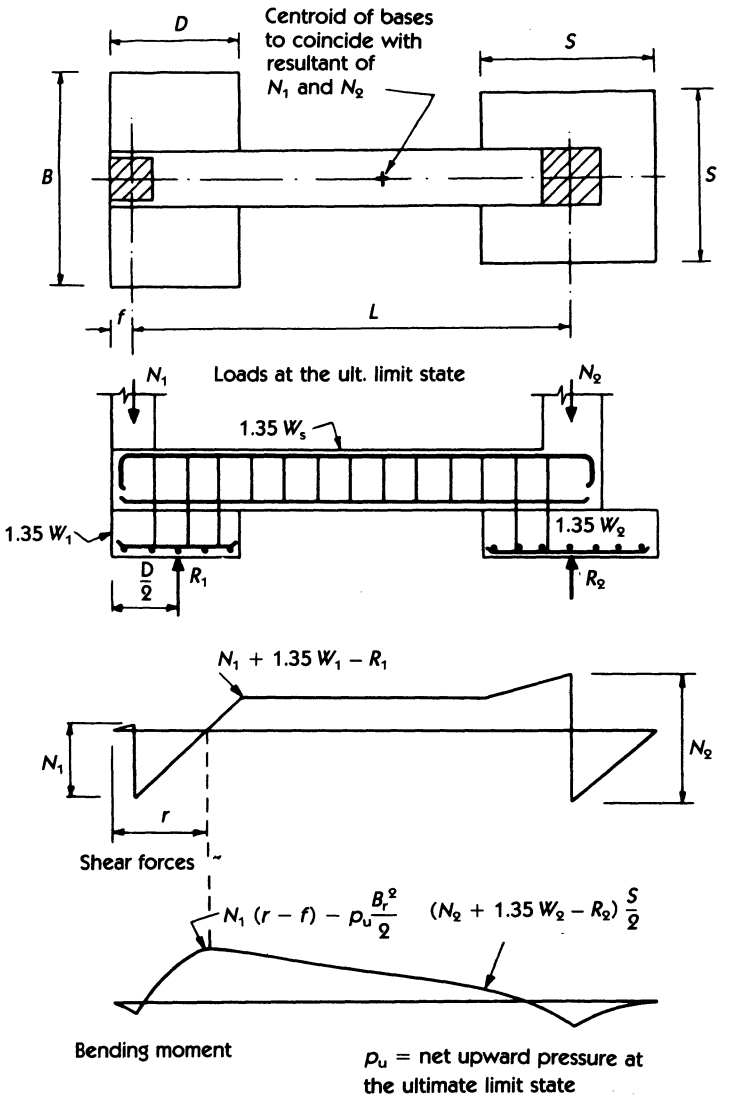

Figure 10.11 *Strap footing with shearing force and bending moments for the strap beam*

$$S = \sqrt{\frac{R_2}{p}}$$

(4) Check that the resultant of all the loads on the footings passes through the centroid of the areas of the two bases. If the resultant is too far away from the centroid then steps (1) to (4) must be repeated until there is adequate agreement.

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(5) Apply the loading associated with the ultimate limit state. Accordingly, revise equations 10.5 and 10.6 to determine the new values for R_1 and R_2. Hence calculate the bearing pressure p_u for this limit state. It may be assumed that the bearing pressures for this case are also equal and uniform, provided the ratios of dead load to imposed load are similar for both columns.
(6) Design the inner footing as a square base with bending in both directions.
(7) Design the outer footing as a base with bending in one direction and supported by the strap beam.
(8) Design the strap beam. The maximum bending moment on the beam occurs at the point of zero shear as shown in figure 10.11. The shear on the beam is virtually constant, the slight decrease being caused by the beam's self-weight. The stirrups should be placed at a constant spacing but they should extend into the footings over the supports so as to give a monolithic foundation. The main tension steel is required at the top of the beam but reinforcement should also be provided in the bottom of the beam so as to cater for any differential settlement or downward loads on the beam.

10.4 Strip Footings

Strip footings are used under walls or under a line of closely spaced columns. Even were it possible to have individual bases, it is often simpler and more economic to excavate and construct the formwork for a continuous base.

On a sloping site the foundations should be constructed on a horizontal bearing and stepped where necessary. At the steps the footings should be lapped as shown in figure 10.12.

The footings are analysed and designed as an inverted continuous beam subjected to the ground bearing pressures. With a thick rigid footing and a firm soil, a linear distribution of bearing pressure is considered. If the columns are equally spaced and equally loaded the pressure is uniformly distributed but if the loading is not symmetrical then the base

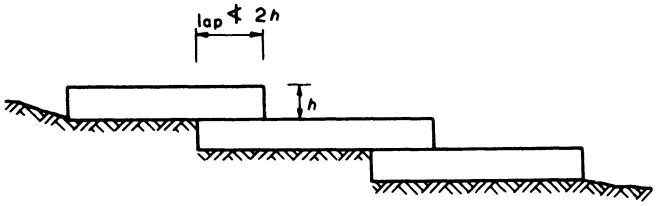

Figure 10.12 *Stepped footing on a sloping site*

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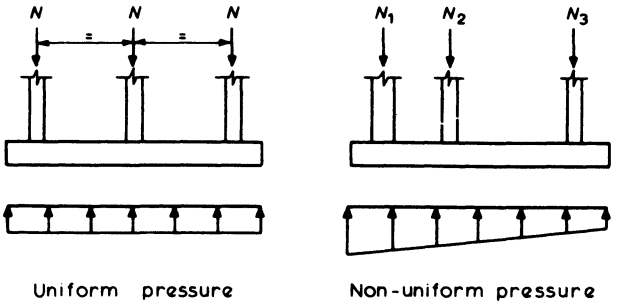

Figure 10.13 *Linear pressure distribution under a rigid strip footing*

is subjected to an eccentric load and the bearing pressure varies as shown in figure 10.13.

The bearing pressures will not be linear when the footing is not very rigid and the soil is soft and compressible. In these cases the bending-moment diagram would be quite unlike that for a continuous beam with firmly held supports and the moments could be quite large, particularly if the loading is unsymmetrical. For a large foundation it may be necessary to have a more detailed investigation of the soil pressures under the base in order to determine the bending moments and shearing forces.

Reinforcement is required in the bottom of the base to resist the transverse bending moments in addition to the reinforcement required for the longitudinal bending. Footings which support heavily loaded columns often require stirrups and bent-up bars to resist the shearing forces.

Example 10.3 Design of a Strip Footing

Design a strip footing to carry 400 mm square columns equally spaced at 3.5 m centres. On each column the characteristic loads are 1000 kN permanent and 350 kN variable. The safe bearing pressure is 200 kN/m^2 and the characteristic material strengths are $f_{ck} = 30\,\text{N/mm}^2$ and $f_{yk} = 460\,\text{N/mm}^2$.

(1) Try a thickness of footing = 800 with $d = 740$ mm for the longitudinal reinforcement.

$$\text{Net bearing pressure, } p_{net} = 200 - 24h = 200 - 24 \times 0.8$$
$$= 180.8\,\text{kN/m}^2$$

$$\text{Width of footing required} = \frac{1000 + 350}{180.8 \times 3.5} = 2.13\,\text{m}$$

Provide a strip footing 2.2 m wide.

At the ultimate limit state

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column load, $N_{sd} = 1.35 \times 1000 + 1.5 \times 350 = 1875\,\text{kN}$

$$\text{bearing pressure} = \frac{1875}{2.2 \times 3.5}$$
$$= 244\,\text{kN/m}^2$$

(2) *Punching shear* at the column face

Maximum shear resistance, $V_{Rd2} = 0.9ud\sqrt{f_{ck}}$
$= 0.9 \times 1600 \times 740 \times \sqrt{30} \times 10^{-3}$
$= 5836\,\text{kN}\ (>N_{sd} = 1875\,\text{kN})$

By inspection, the normal shear on a section at the column face will be significantly less severe.

(3) *Longitudinal reinforcement*

Using the moment and shear coefficients for an equal-span continuous beam (figure 3.10), for an interior span

moment at the columns $= 244 \times 2.2 \times 3.5^2 \times 0.10$
$= 658\,\text{kN m}$

therefore

$$A_s = \frac{658 \times 10^6}{0.87 \times 460 \times 0.98 \times 740} = 2267\,\text{mm}^2$$

$$\text{Minimum } A_s = \frac{0.15bd}{100} = 0.15 \times 2200 \times \frac{740}{100} = 2442\,\text{mm}^2$$

Provide eight T20 at 300 mm centres, area $= 2510\,\text{mm}^2$, bottom steel. In the span

$M_{sd} = 244 \times 2.2 \times 3.5^2 \times 0.07$
$= 460\,\text{kN m}$

therefore

$$A_s = \frac{460 \times 10^6}{0.87 \times 460 \times 0.98 \times 740} = 1585\,\text{mm}^2$$

Provide eight T20 bars at 300 mm centres, area $= 2510\,\text{mm}^2$, top steel (figure 10.14) ($100A_s/bd = 0.154$).

(4) *Transverse reinforcement*

$$M_{sd} = 244 \times \frac{1.1^2}{2} = 148\,\text{kN m/m}$$

$$A_s = \frac{148 \times 10^6}{0.87 \times 460 \times 0.98 \times 720} = 524\,\text{mm}^2\text{/m}$$

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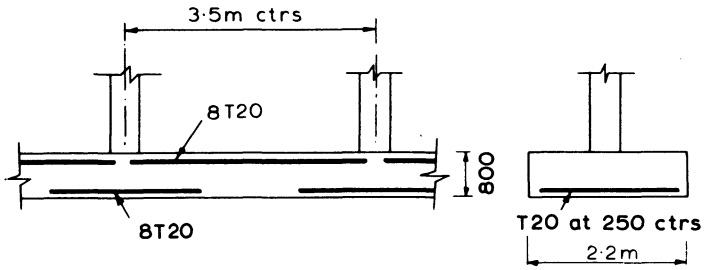

Figure 10.14 *Strip footing with bending reinforcement*

$$\text{Minimum } A_s = \frac{0.15bd}{100} = 0.15 \times 1000 \times \frac{740}{100} = 1110\,\text{mm}^2$$

Provide T20 bars at 250 mm centres, area = 1260 mm^2/m, bottom steel.

(5) *Normal shear* will govern as the punching shear perimeter is outside the footing.

1.0d from column face

$$V_{Sd} = 244 \times 2.2(3.5 \times 0.55 - 0.74 - 0.2) = 529\,\text{kN}$$

(The coefficient of 0.55 is from figure 3.10)

From table 5.1, for $f_{ck} = 30$, $\tau_{Rd} = 0.34\,\text{N/mm}^2$. Therefore

Shear resistance of concrete, V_{Rd1}

$$= [\tau_{Rd}k(1.2 + 40\rho)]b_w d$$
$$= [0.34 \times 1.0 \times (1.2 + 40 \times 0.00154)]2200 \times 740 \times 10^{-3}$$
$$= 698\,\text{kN}\ (>V_{sd} = 529\,\text{kN})$$

10.5 Raft Foundations

A raft foundation transmits the loads to the ground by means of a reinforced concrete slab that is continuous over the base of the structure. The raft is able to span over any areas of weaker soil and it spreads the loads over a wide area. Heavily loaded structures are often provided with one continuous base in preference to many closely spaced, separate footings. Also where settlement is a problem, because of mining subsidence, it is common practice to use a raft foundation in conjunction with a more flexible superstructure.

The simplest type of raft is a flat slab of uniform thickness supporting the columns. Where punching shears are large the columns may be provided with a pedestal at their base as shown in figure 10.15. The

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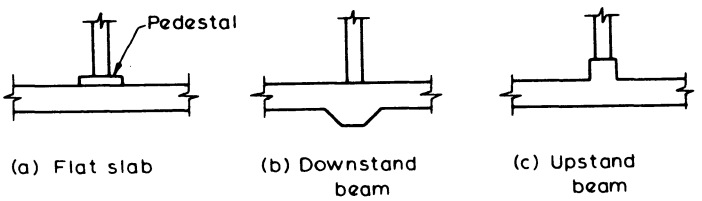

Figure 10.15 *Raft foundations*

pedestal serves a similar function to the drop panel in a flat slab floor. Other, more heavily loaded rafts require the foundation to be strengthened by beams to form a ribbed construction. The beams may be downstanding, projecting below the slab or they may be upstanding as shown in the figure. Downstanding beams have the disadvantage of disturbing the ground below the slab and the excavated trenches are often a nuisance during construction, while upstanding beams interrupt the clear floor area above the slab. To overcome this a second slab is sometimes cast on top of the beams, so forming a cellular raft.

Rafts having a uniform slab, and without strengthening beams, are generally analysed and designed as an inverted flat slab floor subjected to the earth bearing pressures. With regular column spacing and equal column loading, the coefficients tabulated in section 8.6 for flat slab floors are used to calculate the bending moments in the raft. The slab must be checked for punching shear around the columns and around pedestals, if they are used.

A raft with strengthening beams is designed as an inverted beam and slab floor. The slab is designed to span in two directions where there are supporting beams on all four sides. The beams are often subjected to high shearing forces which need to be resisted by a combination of stirrups and bent-up bars.

Raft foundations which are below the level of the water table as in figure 10.16 should be checked to ensure that they are able to resist the uplift forces due to the hydrostatic pressure. This may be critical during construction before the weight of the superstructure is in place, and it may be necessary to provide extra weight to the raft and lower the water table by pumping. An alternative method is to anchor the slab down with short tension piles.

10.6 Piled Foundations

Piles are used where the soil conditions are poor and it is uneconomical, or not possible, to provide adequate spread foundations. The piles must

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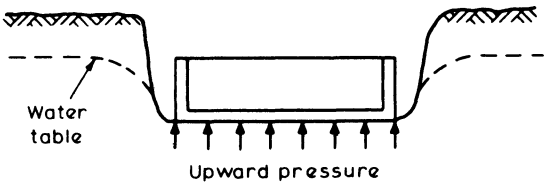

Figure 10.16 *Raft foundation subject to uplift*

extend down to firm soil so that the load is carried by either (1) end bearing, (2) friction, or (3) a combination of both end bearing and friction. Concrete piles may be precast and driven into the ground, or they may be the cast *in situ* type which are bored or excavated.

A soils survey of a proposed site should be carried out to determine the depth to firm soil and the properties of the soil. This information will provide a guide to the lengths of pile required and the probable safe load capacity of the piles. On a large contract the safe loads are often determined from full-scale load tests on typical piles or groups of piles. With driven piles the safe load can be calculated from equations which relate the resistance of the pile to the measured set per blow and the driving force.

The load-carrying capacity of a group of piles is not necessarily a multiple of that for a single pile – it is often considerably less. For a large group of closely spaced friction piles the reduction can be of the order of one-third. In contrast, the load capacity of a group of end bearing piles on a thick stratum of rock or compact sand or gravel is substantially the sum total of the resistance of each individual pile. Figure 10.17 shows the bulbs of pressure under piles and illustrates why the settlement of a group of piles is dependent on the soil properties at a greater depth.

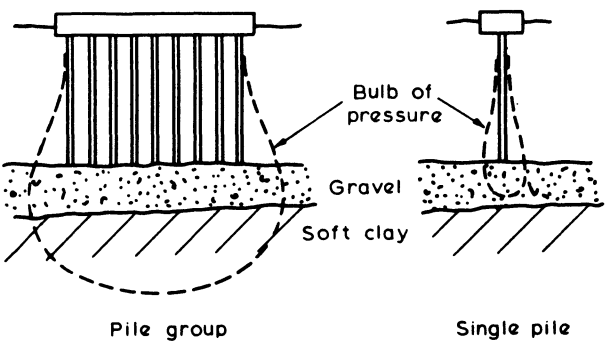

Figure 10.17 *Bulbs of pressure*

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The minimum spacing of piles, centre to centre, should not be less that (1) the pile perimeter – for friction piles, or (2) twice the least width of the pile – for end bearing piles. Bored piles are sometimes enlarged at their base so that they have a larger bearing area or a greater resistance to uplift.

A pile is designed as a short column unless it is slender and the surrounding soil is too weak to provide restraint. Precast piles must also be designed to resist the bending moments caused by lifting and stacking, and the head of the pile must be reinforced to withstand the impact of the driving hammer.

It is very difficult if not impossible to determine the true distribution of load of a pile group, therefore, in general, it is more realistic to use methods that are simple but logical. A vertical load on a group of vertical piles with an axis of symmetry is considered to be distributed according to the following equation, which is similar in form to that for an eccentric load on a pad foundation:

$$P_n = \frac{N}{n} \pm \frac{Ne_{xx}}{I_{xx}} y_n \pm \frac{Ne_{yy}}{I_{yy}} x_n$$

where P_n is the axial load on an individual pile
N is the vertical load on the pile group
n is the number of piles
e_{xx} and e_{yy} are the eccentricities of the load N about the centroidal axes XX and YY of the pile group
I_{xx} and I_{yy} are the second moments of area of the pile group about axes XX and YY
x_n and y_n are the distances of the individual pile from axes YY and XX, respectively.

Example 10.4 Loads in a Pile Group

Determine the distribution between the individual piles of a 1000 kN vertical load acting at position A of the group of vertical piles shown in figure 10.18. Centroid of the pile group: taking moments about line T–T

$$\bar{y} = \frac{\Sigma y}{n} = \frac{2.0 + 2.0 + 3.0 + 3.0}{6} = 1.67\,\text{m}$$

where n is the number of piles. Therefore the eccentricities of the load about the XX and YY centroidal axis are

$$e_{xx} = 2.0 - 1.67 = 0.33\,\text{m}$$

and

$$e_{yy} = 0.2\,\text{m}$$

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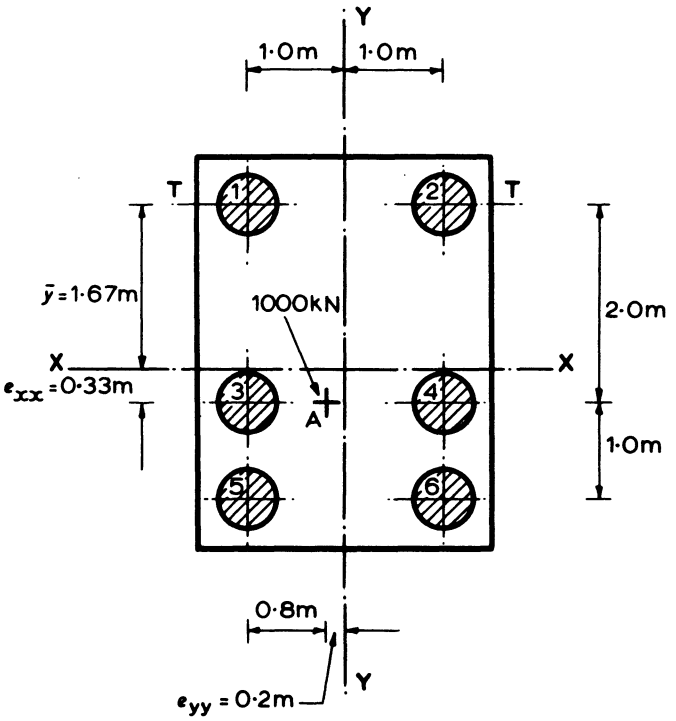

Figure 10.18

$$I_{xx} = \Sigma y_n^2 \text{ with respect to the centroidal axis } XX$$
$$= 2 \times 1.67^2 + 2 \times 0.33^2 + 2 \times 1.33^2 = 9.33$$

similarly

$$I_{yy} = \Sigma x_n^2 = 3 \times 1.0^2 + 3 \times 1.0^2 = 6.0$$

Therefore

$$P_n = \frac{N}{n} \pm \frac{Ne_{xx}}{I_{xx}} y_n \pm \frac{Ne_{yy}}{I_{yy}} x_n$$
$$= \frac{1000}{6} \pm \frac{1000 \times 0.33}{9.33} y_n \pm \frac{1000 \times 0.2}{6.0} x_n$$
$$= 166.7 \pm 35.4 y_n \pm 33.3 x_n$$

Therefore, substituting for y_n and x_n

$$P_1 = 166.7 - 35.4 \times 1.67 + 33.3 \times 1.0 = 140.9\,\text{kN}$$
$$P_2 = 166.7 - 35.4 \times 1.67 - 33.3 \times 1.0 = 74.3\,\text{kN}$$

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$$P_3 = 166.7 + 35.4 \times 0.33 + 33.3 \times 1.0 = 211.7\,\text{kN}$$
$$P_4 = 166.7 + 35.4 \times 0.33 - 33.3 \times 1.0 = 145.1\,\text{kN}$$
$$P_5 = 166.7 + 35.4 \times 1.33 + 33.3 \times 1.0 = 247.1\,\text{kN}$$
$$P_6 = 166.7 + 35.4 \times 1.33 - 33.3 \times 1.0 = 180.5\,\text{kN}$$
$$\text{Total} \qquad 999.6 \approx 1000\,\text{kN}$$

When a pile group is unsymmetrical about both co-ordinate axes it is necessary to consider the theory of bending about the principal axes which is dealt with in most textbooks on strength of materials. In this case the formulae for the pile loads are

$$P_n = \frac{N}{n} \pm Ay_n \pm Bx_n$$

where

$$A = \frac{N(e_{xx}\Sigma x_n^2 - e_{yy}\Sigma x_n y_n)}{\Sigma x_n^2 \Sigma y_n^2 - (\Sigma x_n y_n)^2}$$

and

$$B = \frac{N(e_{yy}\Sigma y_n^2 - e_{xx}\Sigma x_n y_n)}{\Sigma x_n^2 \Sigma y_n^2 - (\Sigma x_n y_n)^2}$$

Note that e_{xx} is the eccentricity about the XX axis, while e_{yy} is the eccentricity about the YY axis, as in figure 10.18.

Piled foundations are sometimes required to resist horizontal forces in addition to the vertical loads. If the horizontal forces are small they can often be resisted by the passive pressure of the soil against vertical piles, otherwise if the forces are not small then raking piles must be provided as shown in figure 10.19a.

To determine the load in each pile either a static method or an elastic method is available. The static method is simply a graphical analysis using Bow's notation as illustrated in figure 10.19b. This method assumes that the piles are pinned at their ends so that the induced loads are axial. The elastic method takes into account the displacements and rotations of the piles which may be considered pinned or fixed at their ends. The pile foundation is analysed in a similar manner to a plane frame or space frame and available computer programs are commonly used.

The pile cap must be rigid and capable of transferring the column loads to the piles. It should have sufficient thickness for anchorage of the column dowels and the pile reinforcement, and it must be checked for punching shear, diagonal shear, bending and bond. Piles are rarely positioned at the exact locations shown on the drawings, therefore this must be allowed for when designing and detailing the pile cap.

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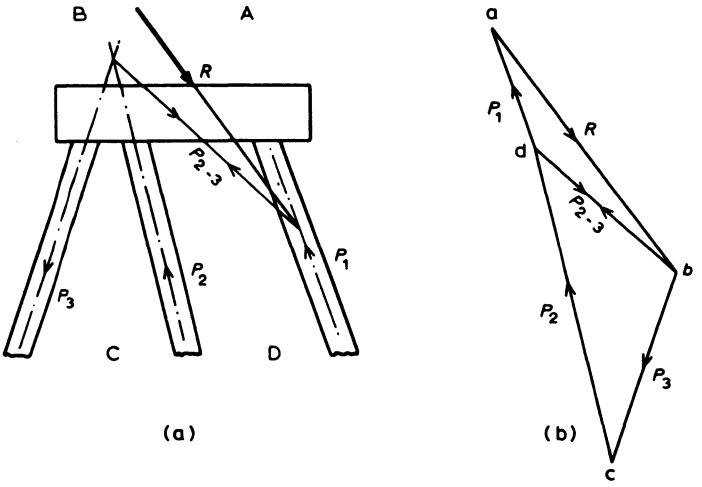

Figure 10.19 *Forces in raking piles*

10.7 Retaining Walls

Such walls are usually required to resist a combination of earth and hydrostatic loadings. The fundamental requirement is that the wall is capable of holding the retained material in place without undue movement arising from deflection, overturning or sliding.

10.7.1 Types of Retaining Wall

Concrete retaining walls may be considered in terms of three basic categories: (1) gravity, (2) counterfort, and (3) cantilever. Within these groups many common variations exist, for example cantilever walls may have additional supporting ties into the retained material.

The structural action of each type is fundamentally different, but the techniques used in analysis, design and detailing are those normally used for concrete structures.

(i) Gravity Walls

These are usually constructed of mass concrete, with reinforcement included in the faces to restrict thermal and shrinkage cracking. As illustrated in figure 10.20, reliance is placed on self-weight to satisfy stability requirements, both in respect of overturning and sliding.

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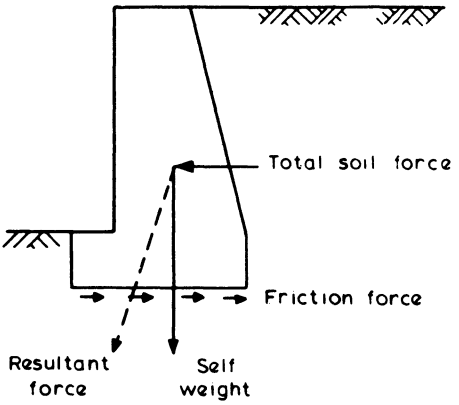

Figure 10.20 *Gravity wall*

It is generally taken as a requirement that under working conditions the resultant of the self-weight and overturning forces must lie within the middle third at the interface of the base and soil. This ensures that uplift is avoided at this interface, as described in section 10.1. Friction effects which resist sliding are thus maintained across the entire base.

Bending, shear, and deflections of such walls are usually insignificant in view of the large effective depth of the section. Distribution steel to control thermal cracking is necessary, however, and great care must be taken to reduce hydration temperatures by mix design, construction procedure and curing techniques.

(ii) Counterfort Walls

This type of construction will probably be used where the overall height of wall is too large to be constructed economically either in mass concrete or as a cantilever.

The basis of design of counterfort walls is that the earth pressures act on a thin wall which spans horizontally between the massive counterforts (figure 10.21). These must be sufficiently large to provide the necessary permanent load for stability requirements, possibly with the aid of the weight of backfill on an enlarged base. The counterforts must be designed with reinforcement to act as cantilevers to resist the considerable bending moments that are concentrated at these points.

The spacing of counterforts will be governed by the above factors, coupled with the need to maintain a satisfactory span–depth ratio on the wall slab, which must be designed for bending as a continuous slab. The advantage of this form of construction is that the volume of concrete involved is considerably reduced, thereby removing many of the problems

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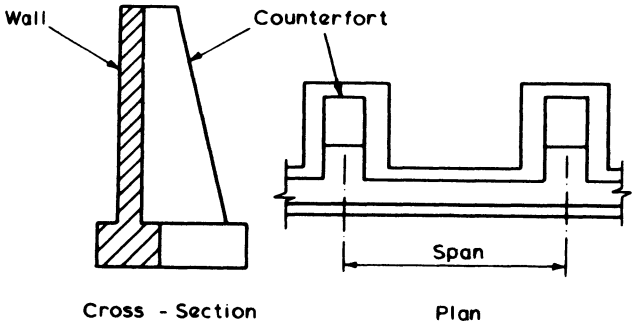

Figure 10.21 *Counterfort wall*

of large pours, and reducing the quantities of excavation. Balanced against this must be considered the generally increased shuttering complication and the probable need for increased reinforcement.

(iii) Cantilever Walls

These are designed as vertical cantilevers spanning from a large rigid base which often relies on the weight of backfill on the base to provide stability. Two forms of this construction are illustrated in figure 10.22. In both cases, stability calculations follow similar procedures to those for gravity walls to ensure that the resultant force lies within the middle third of the base and that overturning and sliding requirements are met.

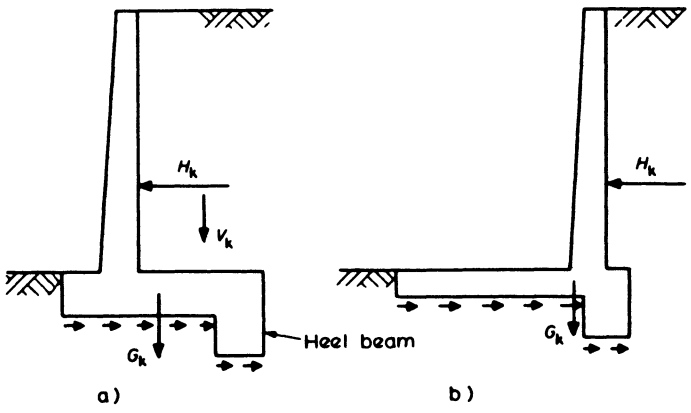

Figure 10.22 *Cantilever walls*

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10.7.2 Analysis and Design

The design of retaining walls may be split into three fundamental stages: (1) Stability analysis – ultimate limit state, (2) Bearing pressure analysis – serviceability limit state, and (3) Member design and detailing – ultimate and serviceability limit states.

(i) Stability Analysis

Under the action of the loads corresponding to the ultimate limit state, a retaining wall must be stable in terms of resistance to overturning and sliding. This is demonstrated by the simple case of a gravity wall as shown in figure 10.23.

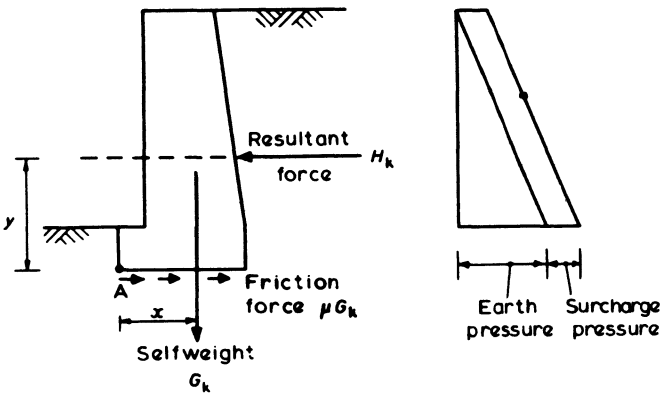

Figure 10.23 *Forces and pressures on a gravity wall*

The critical conditions for stability are when a maximum horizontal force acts with a minimum vertical load. To guard against a stability failure, it is usual to apply conservative factors of safety to the force and loads. The values given in table 2.2 are appropriate to strength calculations but a value of $\gamma_f = 1.5$ or higher should be used for stability calculations. For overturning, a minimum value of 2.0 would be realistic.

If this force is predominantly hydrostatic and well defined, a factor 1.35 may be used. A partial factor of safety of $\gamma_G = 1.0$ is usually applied to the permanent load G_k.

For resistance to overturning, moments would normally be taken about the toe of the base, point A on figure 10.23, thus the requirement is that

$$1.0G_k x \geq \gamma_f H_k y$$

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Resistance to sliding is provided by friction between the underside of the base and the ground, and thus is also related to total self-weight G_k. Resistance provided by the passive earth pressure on the front face of the base may make some contribution, but since this material is often backfilled against the face, this resistance cannot be guaranteed and is usually ignored. Thus, if the coefficient of friction between base and soil is μ, the total friction force will be given by μG_k for the length of wall of weight G_k; and the requirement is that

$$1.0\mu G_k \geqslant \gamma_f H_k$$

where H_k is the horizontal force on this length of wall.

If this criterion is not met, a heel beam may be used, and the force due to the passive earth pressure over the face area of the heel may be included in resisting the sliding force. The partial load factor γ_f on the heel beam force should be taken as 1.0 to give the worst condition. To ensure the proper action of a heel beam, the front face must be cast directly against sound, undisturbed material, and it is important that this is not overlooked during construction.

In considering cantilever walls, a considerable amount of backfill is often placed on top of the base, and this is taken into account in the stability analysis. The forces acting in this case are shown in figure 10.24. In addition to G_k and H_k there is an additional vertical load V_k due to the

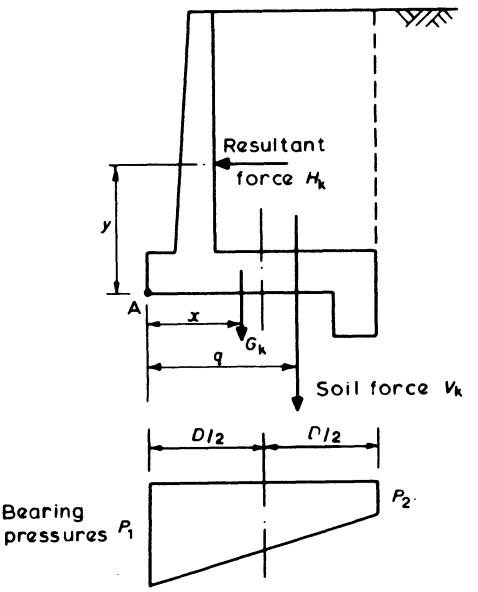

Figure 10.24 *Forces on a cantilever wall*

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material above the base acting a distance q from the toe. The worst condition for stability will be when this is at a minimum; therefore a partial load factor $\gamma_f = 1.0$ is appropriate. The stability requirements then become

$$1.0G_k x + 1.0V_k q \geqslant \gamma_f H_k y \quad \text{for overturning} \tag{10.7}$$

$$\mu(1.0G_k + 1.0V_k) \geqslant \gamma_f H_k \quad \text{for sliding} \tag{10.8}$$

When a heel beam is provided the additional passive resistance of the earth must be included in equation 10.8.

Stability analysis, as described here, will normally suffice. However, if there is doubt about the foundation material in the region of the wall or the reliability of loading values, it may be necessary to perform a full slip-circle analysis, using techniques common to soil mechanics, or to use increased factors of safety.

(ii) Bearing Pressure Analysis

As with foundations, the bearing pressures underneath retaining walls are assessed on the basis of the serviceability limit state when determining the size of base that is required. The analysis will be similar to that discussed in section 10.1 with the foundation being subject to the combined effects of an eccentric vertical load, coupled with an overturning moment.

Considering a unit length of the cantilever wall (figure 10.24) the resultant moment about the centroidal axis of the base is

$$M = \gamma_{f1} H_k y + \gamma_{f2} G_k (D/2 - x) + \gamma_{f3} V_k (D/2 - q) \tag{10.9}$$

and the vertical load is

$$N = \gamma_{f2} G_k + \gamma_{f3} V_k \tag{10.10}$$

where in this case for the serviceability limit state the partial factors of safety are

$$\gamma_{f1} = \gamma_{f2} = \gamma_{f3} = 1.0$$

The distribution of bearing pressures will be as shown in the figure, provided the effective eccentricity lies within the 'middle third' of the base, that is

$$\frac{M}{N} \leqslant \frac{D}{6}$$

The maximum bearing pressure is then given by

$$p_1 = \frac{N}{D} + \frac{M}{I} \times \frac{D}{2}$$

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where $I = D^3/12$. Therefore

$$p_1 = \frac{N}{D} + \frac{6M}{D^2} \tag{10.11}$$

and

$$p_2 = \frac{N}{D} - \frac{6M}{D^2} \tag{10.12}$$

(iii) Member Design and Detailing

As with foundations, the design of bending and shear reinforcement is based on an analysis of the loads for the ultimate limit state, with the corresponding bearing pressures. Gravity walls will seldom require bending or shear steel, while the walls in counterfort and cantilever construction will be designed as slabs. The design of counterforts will generally be similar to that of a cantilever beam unless they are massive.

With a cantilever-type retaining wall the stem is designed to resist the moment caused by the force $\gamma_f H_f$, with $\gamma_f = 1.35$ or larger, depending on how accurately the load may be predicted. For preliminary sizing, the thickness of the wall may be taken as 80 mm per metre depth of backfill.

The thickness of the base is usually of the same order as that of the stem. The heel and toe must be designed to resist the moments due to the upward earth bearing pressures and the downward weight of the soil and base. The soil bearing pressures are calculated from equations 10.9 to 10.12, provided the resultant of the horizontal and vertical forces lies within the 'middle third'. Should the resultant lie outside the 'middle third', then the bearing pressures should be calculated using equation 10.4. The partial factors of safety γ_{f1}, γ_{f2} and γ_{f3} should be taken to provide a combination which gives the critical design condition.

Reinforcement detailing must follow the general rules for slabs and beams as appropriate. Particular care must be given to the detailing of reinforcement to limit shrinkage and thermal cracking. Gravity walls are particularly vulnerable because of the large concrete pours that are generally involved.

Restraints to thermal and shrinkage movement should be reduced to a minimum; however, this is counteracted in the construction of bases by the need for good friction between the base and soil; thus a sliding layer is not possible. Reinforcement in the bases must thus be adequate to control the cracking caused by a high degree of restraint. Long walls restrained by rigid bases are particularly susceptible to cracking during thermal movement due to loss of hydration heat, and detailing must attempt to distribute these cracks to ensure acceptable widths. Complete vertical movement joints must be provided. These joints will often

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incorporate a shear key to prevent differential movement of adjacent sections of wall, and waterbars and sealers should be used.

The back faces of retaining walls will usually be subject to hydrostatic forces from groundwater. This may be reduced by the provision of a drainage path at the face of the wall. It is usual practice to provide such a drain by a layer of rubble or porous blocks as shown in figure 10.25, with pipes to remove the water, often through to the front of the wall. In addition to reducing the hydrostatic pressure on the wall, the likelihood of leakage through the wall is reduced, and water is also less likely to reach and damage the soil beneath the foundations of the wall.

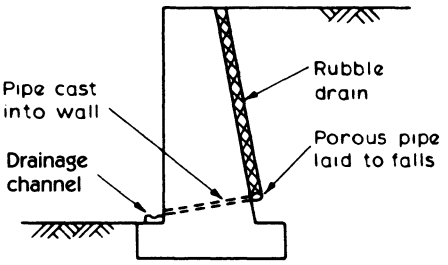

Figure 10.25 *Drainage layer*

Example 10.5 Design of a Retaining Wall

The cantilever retaining wall shown in figure 10.26 supports a granular material of saturated density 2000 kg/m^3, and the allowable bearing pressure is 110 kN/m^2. It is required to:

(1) check the stability of the wall
(2) determine the actual bearing pressures, and
(3) design the bending reinforcement using high-yield steel, f_{yk} = 460 N/mm^2 and concrete class C30/37.

(a) Stability

Horizontal force: it is assumed that the coefficient of active pressure K_a = 0.33, which is a typical value for a granular material. So the earth pressure is given by

$$p = K_a \rho g h$$

where ρ is the density of the backfill and h is the depth considered. Thus, at the base

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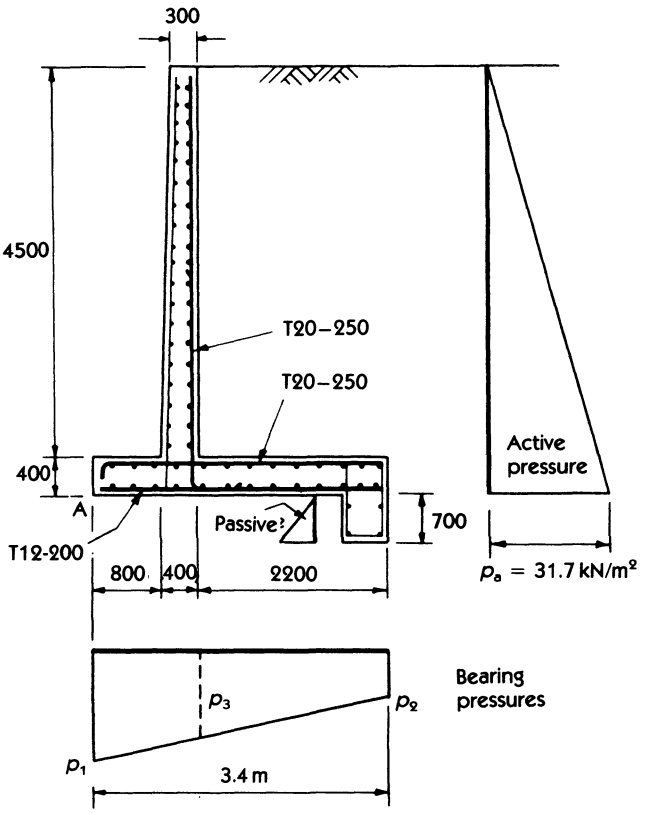

Figure 10.26

$$p = 0.33 \times 2000 \times 10^{-3} \times 9.81 \times 4.9$$
$$= 31.7\,\text{kN/m}^2$$

Therefore horizontal force on 1 m length of wall is

$$H_k = 0.5ph = 0.5 \times 31.7 \times 4.9 = 77.7\,\text{kN}$$

Vertical loads

wall	$= \frac{1}{2}(0.4 + 0.3) \times 4.5 \times 24$	$= 37.8\,\text{kN}$
base	$= 0.4 \times 3.4 \times 24$	$= 32.6$
earth	$= 2.2 \times 4.5 \times 2000 \times 10^{-3} \times 9.81$	$= 194.2$
	Total	$= 264.6\,\text{kN}$

For stability calculations a partial factor of safety of 1.5 is used for the lateral loadings, while 1.35 will be used for strength calculations.

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(i) Sliding: from equation 10.8 it is necessary that

$$\mu(1.0G_k + 1.0V_k) \geqslant \gamma_f H_k \quad \text{for no heel beam}$$

Assuming a value of coefficient of friction $\mu = 0.40$

$$\text{frictional resisting force} = 0.40 \times 1.0 \times 264.6 = 105.8\,\text{kN}$$

$$\text{sliding force} = 1.5 \times 77.7 = 116.6\,\text{kN}$$

Since the sliding force exceeds the frictional force, resistance must also be provided by the passive earth pressure acting against the heel beam and this force is given by

$$H_p = \gamma_f \times 0.5 K_p \rho g a^2$$

where K_p is the coefficient of passive pressure, assumed to be 3.0 for this granular material and a is the depth of the heel. Therefore

$$H_p = 1.0 \times 0.5 \times 3.0 \times 2000 \times 10^{-3} \times 9.81 \times 0.7^2$$
$$= 14.4\,\text{kN}$$

Therefore total resisting force is

$$105.8 + 14.4 = 120.2\,\text{kN}$$

which exceeds the sliding force.

(ii) Overturning: taking moments about point A at the edge of the toe, at the ultimate limit state

$$\text{overturning moment} = \gamma_f H_k h/3 = 1.5 \times 77.7 \times 4.9/3$$
$$= 190\,\text{kN m}$$

$$\text{restraining moment} = 1.0(37.8 \times 1.0 + 32.6 \times 1.7 + 194.2 \times 2.3)$$
$$= 540\,\text{kN m}$$

Thus the criterion for overturning is satisfied.

(b) Bearing Pressures at Service

From equations 10.11 and 10.12 the bearing pressures are given by

$$p = \frac{N}{D} \pm \frac{6M}{D^2}$$

where M is the moment about the base centreline. Therefore

$$M = 77.7 \times 4.9/3 + 37.8(1.7 - 1.0) + 194.2(1.7 - 2.3)$$
$$= 126.9 + 26.5 - 116.5 = 36.9\,\text{kN m}$$

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Therefore

$$\text{maximum bearing pressure } p_1 = \frac{264.6}{3.4} + \frac{6 \times 36.9}{3.4^2}$$
$$= 77.8 + 19.2 = 97\,\text{kN/m}^2$$

which is less than the allowable.

(c) Bending Reinforcement

(i) Wall

$$\text{Horizontal force} = \gamma_f 0.5 K_a \rho g h^2$$
$$= 1.35 \times 0.5 \times 0.33 \times 2000 \times 10^{-3} \times 9.81 \times 4.5^2$$
$$= 88.50\,\text{kN}$$

considering the effective span, the maximum moment is

$$M_{sd} = 88.5 \times (4.5/3) = 132.8\,\text{kN m}$$

$$\frac{M_{sd}}{bd^2 f_{ck}} = \frac{132.8 \times 10^6}{1000 \times 330^2 \times 30} = 0.041$$

for which $l_a = 0.96$ (figure 4.5). Therefore

$$A_s = \frac{132.8 \times 10^6}{0.96 \times 330 \times 0.87 \times 460} = 1047\,\text{mm}^2/\text{m}$$

Provide T20 bars at 250 mm centres.

(ii) Base: the bearing pressures at the ultimate limit state are obtained from equations 10.9 to 10.12. The critical partial factors of safety are

$$\gamma_{f1} = 1.35 \quad \text{and} \quad \gamma_{f2} = \gamma_{f3} = 1.0$$

Using the figures from part (b) of this example, the moment about the base centreline is

$$M_{sd} = \gamma_{f1} \times 126.9 + \gamma_{f2} \times 26.5 - \gamma_{f3} \times 116.5 = 81.3\,\text{kN m}$$

and

$$N_{sd} = \gamma_{f2}(37.8 + 32.6) + \gamma_{f3} \times 194.2 = 264.6\,\text{kN}$$

Therefore

$$\text{pressure } p_1 = \frac{264.6}{3.4} + \frac{6 \times 81.3}{3.4^2} = 78 + 42 = 120\,\text{kN/m}^2$$

$$p_2 = 78 - 42 = 36\,\text{kN/m}^2$$

and in figure 10.26

$$p_3 = 36 + (120 - 36)2.2/3.4 = 90\,\text{kN/m}^2$$

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Heel: taking moments about the stem centreline for the vertical loads and the bearing pressures

$$M_{sd} = \gamma_{f2} \times 32.6 \times 1.3 \times \frac{2.2}{3.4} + \gamma_{f3} \times 194.2 \times 1.3 - 36 \times 2.2 \times 1.3 - (90 - 36) \times \frac{2.2}{2} \times 0.93 = 122\,\text{kN m}$$

therefore

$$A_s = \frac{122 \times 10^6}{0.87 \times 460 \times 0.97 \times 330} = 952\,\text{mm}^2/\text{m}$$

Provide T20 bars at 250 mm centres, top steel.

Toe: taking moments about the stem centreline

$$M_{sd} \approx \gamma_{f2} \times 32.6 \times 0.6 \times \frac{0.8}{3.4} - \gamma_{f3} \times 120 \times 0.8 \times 0.6$$

$$= 53\,\text{kN m}$$

(In fact for this wall the design moment for the toe would be marginally higher with $\gamma_{f2} = 1.35$ and $\gamma_{f3} = 1.35$ throughout.)

$$A_s = \frac{53 \times 10^6}{0.87 \times 460 \times 0.98 \times 330} = 410\,\text{mm}^2/\text{m}$$

The minimum area for this, and for longitudinal distribution steel which is also required in the wall and the base is

$$A_s = 0.0015 \times 1000 \times 330 = 495\,\text{mm}^2/\text{m}$$

Thus, provide T12 bars at 200 mm centres, bottom and distribution steel.

Also steel should be provided in the compression face of the wall in order to prevent cracking – say, T10 bars at 200 mm centres each way.

Bending reinforcement is required in the heel beam to resist the moment due to the passive earth pressure. This reinforcement would probably be in the form of closed links.

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11

Prestressed Concrete

The analysis and design of prestressed concrete is a specialised field which cannot possibly be covered comprehensively in one chapter. This chapter concentrates therefore on the basic principles of prestressing, and the analysis and design of statically determinate members in bending for the serviceability and ultimate limit states.

A fundamental aim of prestressed concrete is to limit tensile stresses, and hence flexural cracking, in the concrete under working conditions. Design is therefore based initially on the requirements of the serviceability limit state. Subsequently considered are ultimate limit state criteria for bending and shear. In addition to the concrete stresses under working loads, deflections must be checked, and attention must also be paid to the construction stage when the prestress force is first applied to the immature concrete. This stage is known as the *transfer condition*.

Design of prestressed concrete may therefore be summarised as

(1) design for serviceability – cracking
(2) check stresses at transfer
(3) check deflections
(4) check ultimate limit state – bending
(5) design shear reinforcement for ultimate limit state.

The stages are illustrated by the flowchart in figure 11.1.

When considering the basic design of a concrete section subject to prestress, the stress distribution due to the prestress must be combined with the stresses from the loading conditions to ensure that permissible stress limits are satisfied. Many analytical approaches have been developed to deal with this problem; however, it is considered that the method presented offers many advantages of simplicity and ease of manipulation in design.

W.H. Mosley et al. *Reinforced Concrete Design to Eurocode 2 (EC2)*
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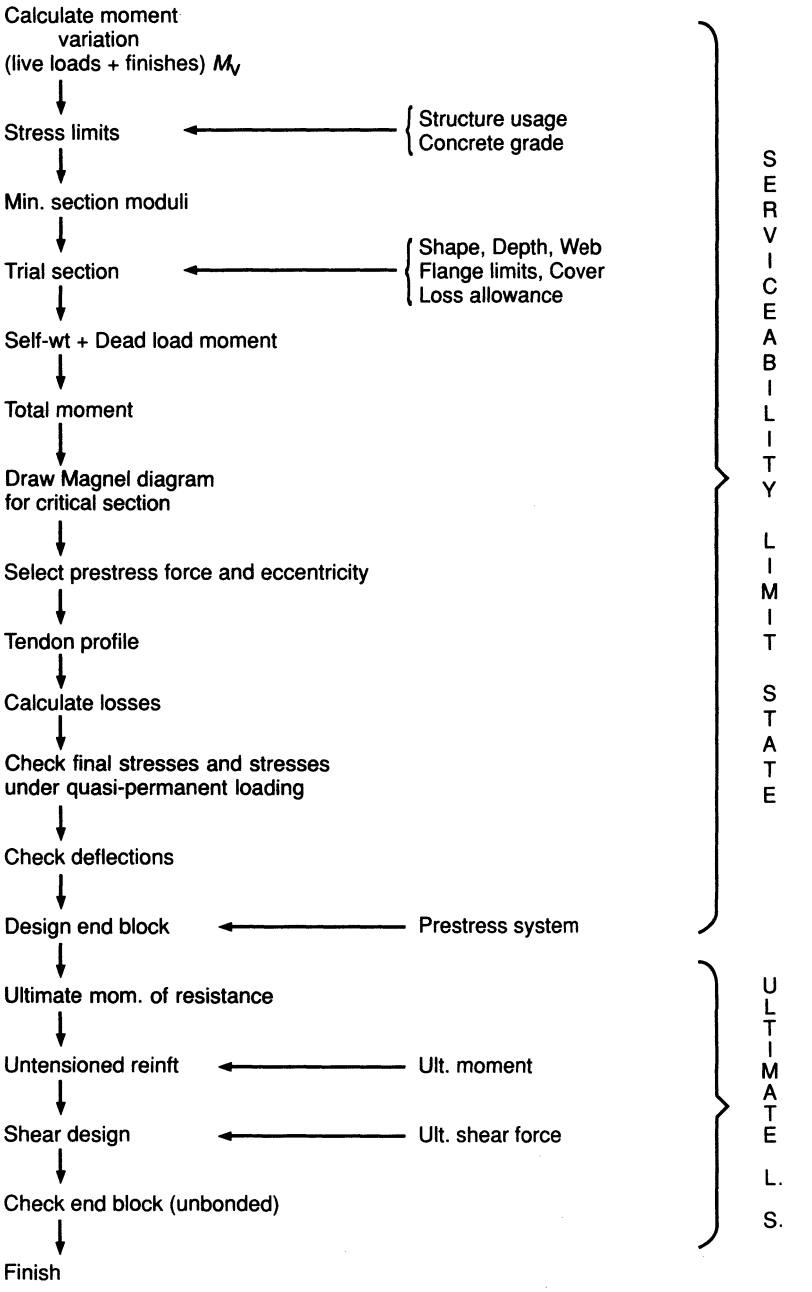

Figure 11.1 *Prestressed concrete design flowchart*

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11.1 Principles of Prestressing

In the design of a reinforced concrete beam subjected to bending it is accepted that the concrete in the tensile zone is cracked, and that all the tensile resistance is provided by the reinforcement. The stress that may be permitted in the reinforcement is limited by the need to keep the cracks in the concrete to acceptable widths under working conditions, thus there is no advantage to be gained from the use of very high strength steels which are available. The design is therefore uneconomic in two respects: (1) dead weight includes 'useless' concrete in tensile zone, and (2) economic use of steel resources is not possible.

'Prestressing' means the artificial creation of stresses in a structure before loading, so that the stresses which then exist under load are more favourable than would otherwise be the case. Since concrete is strong in compression the material in a beam will be used most efficiently if it can be maintained in a state of compression throughout. Provision of a longitudinal compressive force acting on a concrete beam may therefore overcome both of the disadvantages of reinforced concrete cited above. Not only is the concrete fully utilised, but also the need for conventional tension reinforcement is removed. The compressive force is usually provided by tensioned steel wires or strands which are anchored against the concrete and, since the stress in this steel is not an important factor in the behaviour of the beam but merely a means of applying the appropriate force, full advantage may be taken of very high strength steels.

The way in which the stresses due to bending and an applied compressive force may be combined are demonstrated in figure 11.2 for the case of an axially applied force acting over the length of a beam. The stress distribution at any section will equal the sum of the compression and bending stresses if it is assumed that the concrete behaves elastically. Thus it is possible to determine the applied force so that the combined stresses are always compressive.

By applying the compressive force eccentrically on the concrete cross-section, a further stress distribution, due to the bending effects of the couple thus created, is added to those shown in figure 11.2. This effect is illustrated in figure 11.3 and offers further advantages when attempting to produce working stresses within required limits.

Early attempts to achieve this effect were hampered both by the limited steel strengths available and by shrinkage and creep of the concrete under sustained compression, coupled with relaxation of the steel. These meant that the steel lost a large part of its initial pretension and as a result residual stresses were so small as to be useless. It is now possible, however, to produce stronger concretes which have good creep properties, and very high strength steels which can be stressed up to a high percentage of their 0.2 per cent proof stress are also available. For example, hard-

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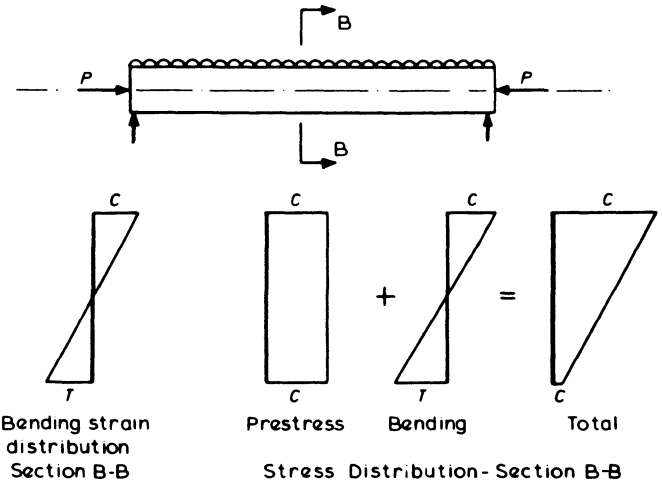

Figure 11.2 *Effects of axial prestress*

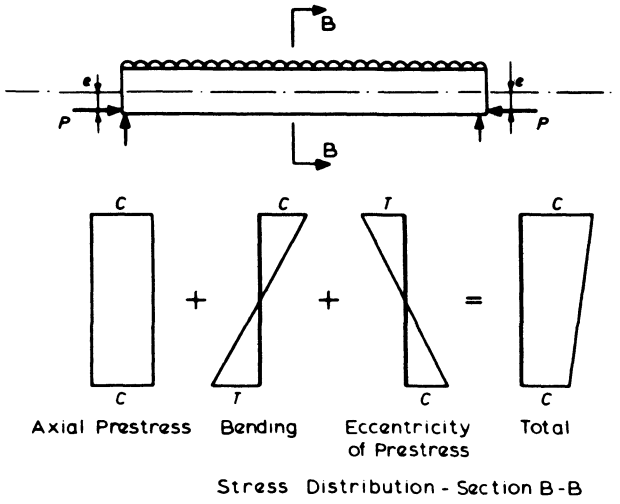

Figure 11.3 *Effects of eccentric prestress*

drawn wires may carry stresses up to about six times those possible in mild steel. This not only results in savings of steel quantity, but also the effects of shrinkage and creep become relatively smaller and may typically amount to the loss of only about 25 per cent of the initial applied force. Thus, modern materials mean that the prestressing of concrete is a prac-

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tical proposition, with the forces being provided by steel passing through the beam and anchored at each end while under high tensile load.

11.2 Methods of Prestressing

Two basic techniques are commonly employed in the construction of prestressed concrete, their chief difference being whether the steel tensioning process is performed before or after the hardening of the concrete. The choice of method will be governed largely by the type and size of member coupled with the need for precast or *in situ* construction.

11.2.1 Pretensioning

In this method the steel wires or strands are stretched to the required tension and anchored to the ends of the moulds for the concrete. The concrete is cast around the tensioned steel, and when it has reached sufficient strength, the anchors are released and the force in the steel is transferred to the concrete by bond. In addition to long-term losses due to creep, shrinkage and relaxation, an immediate drop in prestress force occurs due to elastic shortening of the concrete. These features are illustrated in figure 11.4.

Because of the dependence on bond, the tendons for this form of construction generally consist of small diameter wires or small strands which have good bond characteristics. Anchorage near the ends of these

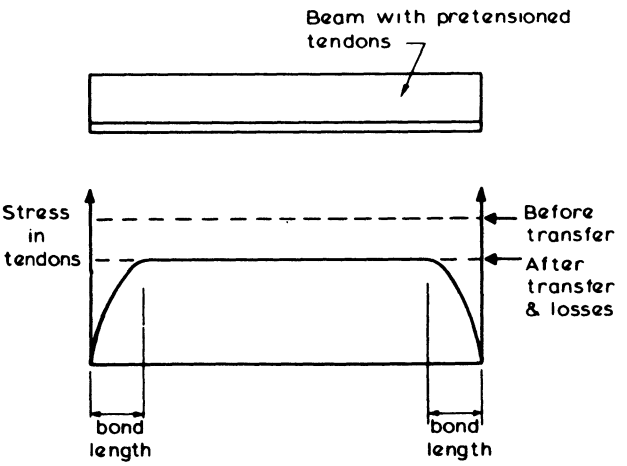

Figure 11.4 *Tendon stresses – pretensioning*

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wires is often enhanced by the provision of small indentations in the surface of the wire.

The method is ideally suited for factory production where large numbers of identical units can be economically made under controlled conditions, a development of this being the 'long-line' system where several units can be cast at once – end to end – and the tendons merely cut between each unit after release of the anchorages. An advantage of factory production of prestressed units is that specialised curing techniques such as steam curing can be employed to increase the rate of hardening of the concrete and to enable earlier 'transfer' of the stress to the concrete. This is particularly important where re-use of moulds is required, but it is essential that under no circumstances must calcium chloride be used as an accelerator because of its severe corrosion action on small diameter steel wires.

One major limitation of this approach is that tendons must be straight, which may cause difficulties when attempting to produce acceptable final stress levels throughout the length of a member. It may therefore be necessary to reduce either the prestress force or eccentricity of force near the ends of a member, in which case tendons must either be 'debonded' or 'deflected':

(1) Debonding consists of applying a wrapping or coating to the steel to prevent bond developing with the surrounding concrete. Treating some of the wires in this way over part of their length allows the magnitude of effective prestress force to be varied along the length of a member.

(2) Deflecting tendons is a more complex operation and is usually restricted to large members, such as bridge beams, where the individual members may be required to form part of a continuous structure in conjunction with *in situ* concrete slabs and beams. A typical arrangement for deflecting tendons is shown in figure 11.5, but it must be appreciated that substantial ancillary equipment is required to provide the necessary reactions.

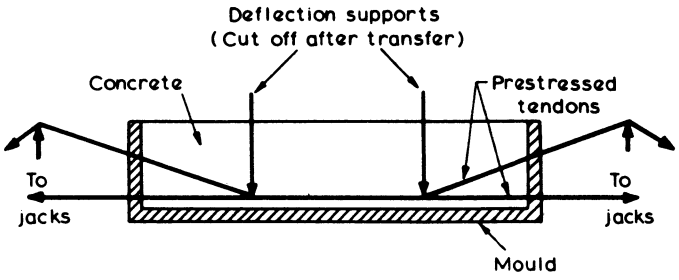

Figure 11.5 *Tendon deflection*

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11.2.2 Post-tensioning

This method, which is the most suitable for *in situ* construction, involves the stressing against the hardened concrete of tendons or steel bars which are not bonded to the concrete. The tendons are passed through a flexible sheathing, which is cast into the concrete in the correct position. They are tensioned by jacking against the concrete, and anchored mechanically by means of steel thrust plates or anchorage blocks at each end of the member. Alternatively, steel bars threaded at their ends may be tensioned against bearing plates by means of tightening nuts. It is of course usually necessary to wait a considerable time between casting and stressing to permit the concrete to gain sufficient strength under *in situ* conditions.

The use of tendons consisting of a number of strands passing through flexible sheathing offers considerable advantages in that curved tendon profiles may be obtained. A post-tensioned structural member may be constructed from an assembly of separate pre-cast units which are constrained to act together by means of tensioned cables which are often curved as illustrated in figure 11.6. Alternatively, the member may be cast as one unit in the normal way but a light cage of untensioned reinforcing steel is necessary to hold the ducts in their correct position during concreting.

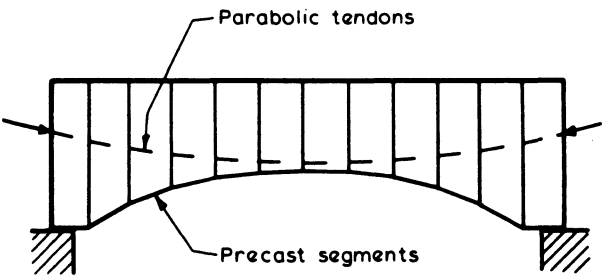

Figure 11.6 *Post-tensioned segmental construction*

After stressing, the remaining space in the ducts may be left empty ('unbonded' construction), or more usually will be filled with grout under high pressure ('bonded' construction). Although this grout assists in transmitting forces between the steel and concrete under live loads, and improves the ultimate strength of the member, the principal use is to protect the highly stressed strands from corrosion. The bonding of the highly stressed steel with the surrounding concrete beam also greatly assists demolition, since the beam may then safely be 'chopped-up' into small lengths without releasing the energy stored in the steel.

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11.3 Analysis of Concrete Section under Working Loads

Since the object of prestressing is to maintain favourable stress conditions in a concrete member under load, the 'working load' for the member must be considered in terms of both maximum and minimum values. Thus at any section, the stresses produced by the prestress force must be considered in conjunction with the stresses caused by maximum and minimum values of applied moment.

Unlike reinforced concrete, the primary analysis of prestressed concrete is based on service conditions, and on the assumption that stresses in the concrete are limited to values which will correspond to elastic behaviour. In this section, the following assumptions are made in analysis:

(1) Plane sections remain plane.
(2) Stress–strain relationships are linear.
(3) Bending occurs about a principal axis.
(4) The prestressing force is the value remaining after all losses have occurred.
(5) Changes in tendon stress due to applied loads on the member have negligible effect on the behaviour of the member.
(6) Section properties are generally based on the gross concrete cross-section.

The stress in the steel is unimportant in the analysis of the concrete section under working conditions, it being the force provided by the steel that is considered in the analysis.

The sign conventions and notations used for the analysis are indicated in figure 11.7.

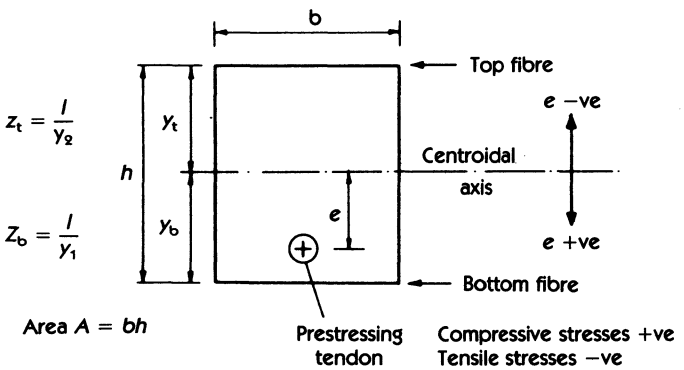

Figure 11.7 *Sign convention and notation*

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11.3.1 Member Subjected to Axial Prestress Force

If section BB of the member shown in figure 11.8 is subjected to moments ranging between M_{max} and M_{min}, the net stresses at the outer fibres of the beam are given by

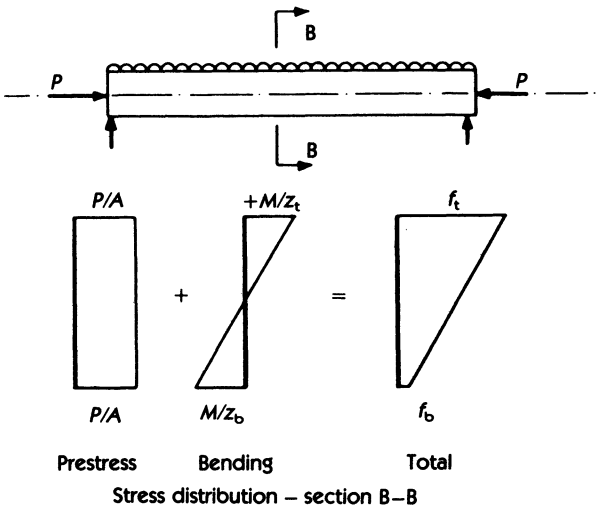

Figure 11.8 *Stresses in member with axial prestress force*

$$\text{under } M_{max} \begin{cases} f_t = \dfrac{P}{A} + \dfrac{M_{max}}{z_t} & \text{at the top} \quad (11.1) \\ f_b = \dfrac{P}{A} - \dfrac{M_{max}}{z_b} & \text{at the bottom} \quad (11.2) \end{cases}$$

$$\text{under } M_{min} \begin{cases} f_t = \dfrac{P}{A} + \dfrac{M_{min}}{z_t} & \text{at the top} \quad (11.3) \\ f_b = \dfrac{P}{A} - \dfrac{M_{min}}{z_b} & \text{at the bottom} \quad (11.4) \end{cases}$$

where z_b and z_t are the elastic section moduli and P is the final prestress force.

The critical condition for tension in the beam is given by equation 11.2 which for no tension, that is $f_b = 0$, becomes

$$\frac{P}{A} = \frac{M_{max}}{z_b}$$

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or

$$P = \frac{M_{max}A}{z_b} = \text{minimum prestress force required}$$

For this value of prestress force, substitution in the other equations will yield the stresses in the beam under maximum load and also under minimum load. Similarly the stresses immediately after prestressing, before losses have occurred, may be calculated if the value of losses is known.

For example, the maximum stress in the top of the member is given by equation 11.1

$$f_t = \frac{P}{A} + \frac{M_{max}}{z_t}$$

where

$$P = \frac{M_{max}A}{z_b}$$

therefore

$$f_t = \frac{P}{A} + \frac{P}{A}\frac{z_b}{z_t} = \frac{P}{A}\frac{(z_b + z_t)}{z_t}$$

It can be seen from the stress distributions in figure 11.8 that the top fibre is generally in considerable compression, while the bottom fibre is generally at lower stresses. Much better use of the concrete could be made if the stresses at both top and bottom can be caused to vary over the full range of permissible stresses for the two extreme loading conditions. This may be achieved by providing the force at an eccentricity e from the centroid.

11.3.2 Member Subjected to Eccentric Prestress Force

The stress distributions will be similar to those in section 11.3.1 but with the addition of the term $\pm Pe/z$ due to the eccentricity e of the prestressing force. For the position shown in figure 11.9, e will have a positive value. So that

under M_{max}

$$f_t = \frac{P}{A} + \frac{M_{max}}{z_t} - \frac{Pe}{z_t} \quad \text{at the top} \tag{11.5}$$

$$f_b = \frac{P}{A} - \frac{M_{max}}{z_b} + \frac{Pe}{z_b} \quad \text{at the bottom} \tag{11.6}$$

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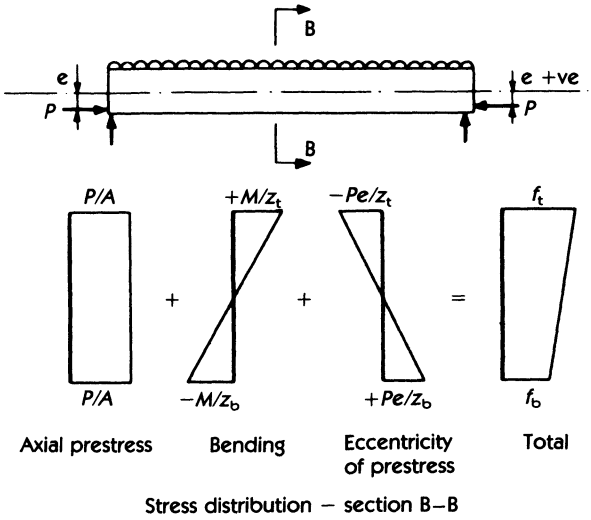

Figure 11.9 *Stresses in member with eccentric prestress force*

$$\text{under } M_{\min} \begin{cases} f_t = \dfrac{P}{A} + \dfrac{M_{\min}}{z_t} - \dfrac{Pe}{z_t} & \text{at the top} \qquad (11.7) \\ f_b = \dfrac{P}{A} - \dfrac{M_{\min}}{z_b} + \dfrac{Pe}{z_b} & \text{at the bottom} \qquad (11.8) \end{cases}$$

Note that, as the prestressing force lies below the neutral axis, it has the effect of causing hogging moments in the section.

The critical condition for no tension in the bottom of the beam is again given by equation 11.6, which becomes

$$\frac{P}{A} - \frac{M_{\max}}{z_b} + \frac{Pe}{z_b} = 0$$

or

$$P = \frac{M_{\max}}{\left(\dfrac{z_b}{A} + e\right)} = \text{minimum prestress force required for no tension in bottom fibre}$$

Thus for a given value of prestress force P, the beam may carry a maximum moment of

$$M_{\max} = P\left(\frac{z_b}{A} + e\right)$$

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When compared with $M_{max} = Pz_b/A$ for an axial prestress force it indicates an increase in moment carrying capacity of Pe.

The maximum stress in the top of the beam is given by equation 11.5 as

$$f_t = \frac{P}{A} + \frac{M_{max}}{z_t} - \frac{Pe}{z_t}$$

where

$$M_{max} = \frac{Pz_b}{A} + Pe$$

thus

$$f_t = \frac{P}{A} + \frac{Pz_b}{Az_t} + \frac{Pe}{z_t} - \frac{Pe}{z_t}$$

$$= \frac{P}{A}\left(\frac{z_b + z_t}{z_t}\right)$$

which is that same as that obtained in section 11.3.1 for an axially prestressed member. Thus the advantages of an eccentric prestress force with respect to the maximum moment-carrying capacity of a beam are apparent.

If the stress distributions of figure 11.9 are further examined, it can be seen that the differences in the net stress diagrams for the extreme loading cases are solely due to the differences between the applied moment terms M_{max} and M_{min}. It follows that by increasing the range of the stresses by the use of an eccentric prestress force the range of applied moments that the beam can carry is also increased. The minimum moment M_{min} that can be resisted is generally governed by the need to avoid tension in the top of the beam, as indicated by equation 11.7.

In the design of prestressed beams it is important that the minimum moment condition is not overlooked, especially when straight tendons are employed, as stresses near the ends of beams where moments are small may often exceed those at sections nearer mid-span. This feature is illustrated by the results obtained in example 11.1.

Example 11.1 Calculation of Prestress Force and Stresses

A rectangular beam 300 × 150 mm is simply supported over a 4 m span, and supports a variable load of 10 kN/m. If a straight tendon is provided at an eccentricity of 65 mm below the centroid of the section, find the minimum prestress force necessary for no tension under variable load at mid-span. Calculate the corresponding stresses under self-weight only at mid-span and at the ends of the member.

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(a) Beam Properties

$$\text{Self-weight} = 0.15 \times 0.3 \times 24 = 1.08\,\text{kN/m}$$

$$\text{Area} = 45 \times 10^3\,\text{mm}^2$$

$$\text{Section moduli } z_t = z_b = z = \frac{bh^2}{6} = \frac{150 \times 300^2}{6} = 2.25 \times 10^6\,\text{mm}^3$$

(b) Loadings (Mid-span)

$$M_{max} = \frac{(10 + 1.08) \times 4^2}{8} = 22.2\,\text{kN m}$$

$$M_{min} = \frac{1.08 \times 4^2}{8} = 2.2\,\text{kN m}$$

(c) Calculate Minimum Prestress Force

For no tension at the bottom under M_{max}

$$\frac{P}{A} - \frac{M_{max}}{z} + \frac{Pe}{z} = 0$$

where

$$e = 65\,\text{mm}$$

hence

$$P = \frac{M_{max}}{\left(\frac{z}{A} + e\right)} = \frac{22.2 \times 10^6 \times 10^{-3}}{\frac{2.25 \times 10^6}{45 \times 10^3} + 65}$$

$$= 193\,\text{kN}$$

(d) Calculate Stresses at Mid-span under M_{min}

$$\text{Stress at top } f_t = \frac{P}{A} + \frac{M_{min}}{z} - \frac{Pe}{z}$$

where

$$\frac{P}{A} = \frac{193 \times 10^3}{45 \times 10^3} = 4.3\,\text{N/mm}^2$$

$$\frac{M_{min}}{z} = \frac{2.2 \times 10^6}{2.25 \times 10^6} = 1.0\,\text{N/mm}^2$$

$$\frac{Pe}{z} = \frac{193 \times 10^3 \times 65}{2.25 \times 10^6} = 5.6\,\text{N/mm}^2$$

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Hence

$$\text{stress at top } f_t = 4.3 + 1.0 - 5.6 = -0.3\,\text{N/mm}^2 \text{ (tension)}$$

and

$$\text{stress at bottom } f_b = \frac{P}{A} - \frac{M_{min}}{z} + \frac{Pe}{z}$$
$$= 4.30 - 1.0 + 5.6 = +8.9\,\text{N/mm}^2$$

This calculation shows that with minimum load it is possible for the beam to hog with tensile stresses in the top fibres. This is particularly so at the initial transfer of the prestress force to the unloaded beam

(e) Calculate Stresses at Ends

In this situation $M = 0$. Hence

$$f_t = \frac{P}{A} - \frac{Pe}{z} = 4.3 - 5.6 = -1.3\,\text{N/mm}^2$$

and

$$f_b = \frac{P}{A} + \frac{Pe}{z} = 4.3 + 5.6 = 9.9\,\text{N/mm}^2$$

11.4 Design for the Serviceability Limit State

The design of a prestressed concrete member is based on maintaining the concrete stresses within specified limits at all stages of the life of the member. Hence the primary design is based on the serviceability limit state, with the concrete stress limits based on the acceptable degree of flexural cracking, the necessity to prevent excessive creep and the need to ensure that excessive compression does not result in longitudinal and micro-cracking.

Guidance regarding the allowable concrete compressive stress in bending is given in EC 2 as limited to:

(i) $0.6f_{ck}$ under the action of characteristic loads
and (ii) $0.45f_{ck}$ under the action of the *quasi-permanent* loads.

The *quasi-permanent* loads are the permanent and prestressing load, $G_k + P_{m,t}$, plus a proportion of the characteristic variable imposed load. This proportion is taken as 0.2 for dwellings, 0.3 for offices and stores, 0.6 for parking areas and 0.0 for snow and wind loading.

If the tensile stress in the concrete is limited to the values of f_{ctm} given in table 6.9 then all stresses can be calculated on the assumption that the

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section is uncracked and the gross concrete section is resisting bending. If this is not the case then calculations may have to be based on a cracked section. Limited cracking is permissible depending on whether the beam is pre or post-tensioned and the appropriate exposure class and generally crack widths should be limited to 0.2 mm under the action of the *frequent* loading combination taken as the permanent characteristic and prestressing load, $G_k+P_{m,t}$, plus a proportion of the characteristic variable imposed load. This proportion is taken as 0.4 for dwellings, 0.6 for offices and stores, 0.7 for parking areas and 0.2 for snow and wind loading.

At initial transfer of prestress to the concrete, the prestress force will be considerably higher than the 'long-term' value as a result of subsequent losses which are due to a number of causes including elastic shortening, creep and shrinkage of the concrete member. Estimation of losses is described in section 11.4.7. Since these losses commence immediately, the condition at transfer represents a transitory stage in the life of a member and further consideration should be given to limiting both compressive and tensile stresses at this stage. In addition, the concrete, at this stage, is usually relatively immature and not at full strength and hence transfer is a critical stage which should be considered carefully. No clear guidance on transfer stress conditions is given in EC2 other than a recognition that minimum concrete strengths at transfer must be specified. However practice based on previous codes is to limit the compressive stress at transfer to one-half of the actual transfer strength (based on the cube strength) and the tensile stress to $1\,N/mm^2$ for sections designed not to be in tension in service. Where limited flexural stress under service loads is permitted, some limited tensile stress is permitted at transfer.

The choice of whether to permit cracking to take place or not will depend on a number of factors which include conditions of exposure and the nature of loading. If a member consists of precast segments with mortar joints, or if it is essential that cracking should not occur, then it will be designed to be in compression under all load conditions. However a more efficient use of materials can be made if the tensile strength of the concrete, f_{ctm}, given in table 6.9 is utilised. Provided these stresses are not exceeded then the section can be designed, based on the gross uncracked section.

Unless the section is designed to be fully in compression under the characteristic loads, a minimum amount of bonded reinforcement should be provided to control cracking. This is calculated in an identical manner to the minimum requirement for reinforced concrete (see section 6.1.5) with the allowance that up to 50 per cent of the prestressing tendons can be counted towards this minimum area within a 300 mm square around the tendon.

The design of prestressing requirements is based on the manipulation of the four basic expressions given in section 11.3.2 describing the stress

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distribution across the concrete section. These are used in conjunction with permissible stresses appropriate to the type of member and covering the following conditions:

(1) initial transfer of prestress force with the associated loading (often just the beam's self-weight)
(2) at service, after prestress losses, with minimum and maximum characteristic loading
(3) at service with the quasi-permanent loading.

The loadings must encompass the full range that the member will encounter during its life, and the minimum values will thus be governed by the construction techniques used. The partial factors of safety applied to these loads will be those for serviceability limit state, that is 1.0 for both permanent and variable loads. The quasi-permanent loading situation is considered with only a proportion of the characteristic variable load acting.

For a beam with a cantilever span or a continuous beam it is necessary to consider the loading patterns of the variable loads at service in order to determine the minimum and maximum moments for load condition (2) above. For a single span, simply supported beam usually it is the minimum moment at transfer and the maximum moment at service which will govern, as shown in figure 11.10.

From figure 11.10 the governing equations for a single span beam are

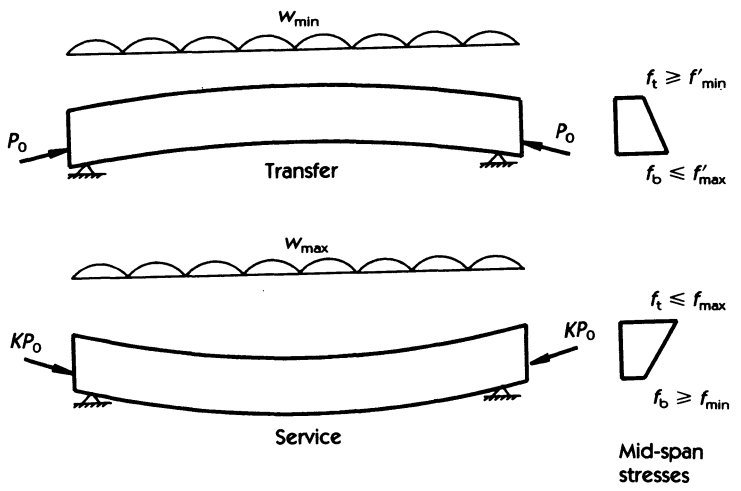

Figure 11.10 *Prestressed beam at transfer and service*

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At transfer:

$$\frac{P_0}{A} - \frac{P_0 e}{z_t} + \frac{M_{min}}{z_t} = f'_t \geq f'_{min} \qquad (11.9)^*$$

$$\frac{P_0}{A} + \frac{P_0 e}{z_b} - \frac{M_{min}}{z_b} = f'_b \leq f'_{max} \qquad (11.10)^*$$

At service:

$$\frac{KP_0}{A} - \frac{KP_0 e}{z_t} + \frac{M_{max}}{z_t} = f_t \leq f_{max} \qquad (11.11)^*$$

$$\frac{KP_0}{A} + \frac{KP_0 e}{z_b} - \frac{M_{max}}{z_b} = f_b \geq f_{min} \qquad (11.12)^*$$

where f'_{max}, f'_{min}, f_{max}, and f_{min}, are the appropriate permissible stresses at transfer and serviceability conditions. P_0 is the prestressing force at transfer and K is a loss factor that accounts for the prestress losses. For example, for 20 per cent losses, $K = 0.8$.

11.4.1 Determination of Minimum Section Properties

The two pairs of expressions can be combined as follows.
11.9 and 11.11:

$$(M_{max} - KM_{min}) \leq (f_{max} - Kf'_{min})z_t \qquad (11.13)$$

11.10 and 11.12:

$$(M_{max} - KM_{min}) \leq (Kf'_{max} - f_{min})z_b \qquad (11.14)$$

Hence, if $(M_{max} - KM_{min})$ is written as M_v, the moment variation

$$z_t \geq \frac{M_v}{(f_{max} - Kf'_{min})} \qquad (11.15)^*$$

and

$$z_b \geq \frac{M_v}{(Kf'_{max} - f_{min})} \qquad (11.16)^*$$

In equations 11.15 and 11.16 for z_t and z_b it can be assumed with sufficient accuracy, for preliminary sizing, that

$$M_v \approx M_{max} - M_{min}$$

M_{max} will depend on both the variable and permanent (self-weight) load and M_{min} will depend on the permanent (self-weight) load only, so that in effect the calculations for M_v become independent of the self-weight of the beam.

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These minimum values of section moduli must be satisfied by the chosen section in order that a prestress force and eccentricity exist which will permit the stress limits to be met, but to ensure that practical considerations are met the chosen section must have a margin above the minimum values of section moduli calculated above. The equations for minimum moduli depend on the *difference* between the maximum and minimum moment values. The maximum moment on the section has not directly been included in these figures, thus it is possible that the resulting prestress force may not be economic or practicable. However, it is found in the majority of cases that if a section is chosen which satisfies these minimum requirements, coupled with any other specified requirements regarding the shape of the section, then a satisfactory design is usually possible. The ratio of acceptable span–depth for a prestressed beam cannot be categorised on the basis of deflections as easily as for reinforced concrete. In the absence of any other criteria, the following formulae may be used as a guide and will generally produce reasonably conservative designs for post-tensioned members:

$$\text{span} \leqslant 36\,\text{m} \qquad h = \frac{\text{span}}{25} + 0.1\,\text{m}$$

$$\text{span} \geqslant 36\,\text{m} \qquad h = \frac{\text{span}}{20}\,\text{m}$$

In the case of short-span members it may be possible to use very much greater span–depth ratios quite satisfactorily, although the resulting prestress forces may become very high.

Other factors which must be considered at this stage include the slenderness ratio of beams, where the same criteria apply as for reinforced concrete, and the possibility of web and flange splitting in flanged members.

Example 11.2 Selection of Cross-section

Select a rectangular section for a post-tensioned beam to carry, in addition to its own self-weight, a uniformly distributed load of 3 kN/m over a simply supported span of 10 m. The member is to be designed with a concrete strength class C40/50 without lateral support. Assume 20 per cent loss of prestress ($K = 0.8$).

Design concrete stresses:

At service:

$$f_{max} = 0.6 f_{ck} = 0.6 \times 40 = 24\,\text{N/mm}^2;\ f_{min} = 0.0\,\text{N/mm}^2$$

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At transfer:

$$f'_{max} = 16\,N/mm^2 \approx 0.5 \text{ strength at transfer; } f'_{min} = -1.0\,N/mm^2$$

$$M_v = 3.0 \times 10^2/8 = 37.5\,kN\,m$$

From equations 11.15 and 11.16:

$$z_t \geqslant \frac{M_v}{(f_{max} - Kf'_{min})} = \frac{37.5 \times 10^6}{(24 - 0.8\{-1\})} = 1.5 \times 10^6\,mm^3$$

$$z_b \geqslant \frac{M_v}{(Kf'_{max} - f_{min})} = \frac{37.5 \times 10^6}{(0.8 \times 16 - 0.0)} = 2.93 \times 10^6\,mm^3$$

To prevent lateral buckling, EC2 specifies maximum span/breadth = 50. Take $b = 200\,mm$ which meets the span/breadth requirement. Hence

$$z = 200h^2/6 \geqslant 2.93 \times 10^6$$

therefore

$$h \geqslant \sqrt{(2.93 \times 10^6 \times 6/200)} = 297\,mm$$

The minimum depth of beam is therefore 297 mm and to allow a margin in subsequent detailed design a depth of 350 mm would be appropriate as a first attempt.

11.4.2 Design of Prestress Force

The inequalities of equations 11.9 to 11.12 may be rearranged to give expressions for the minimum required prestress force for a given eccentricity:

$$P_0 \leqslant \frac{(z_t f_{max} - M_{max})}{K(z_t/A - e)} \tag{11.17}$$

$$P_0 \geqslant \frac{(z_t f'_{min} - M_{min})}{(z_t/A - e)} \tag{11.18}$$

$$P_0 \geqslant \frac{(z_b f_{min} + M_{max})}{K(z_b/A + e)} \tag{11.19}$$

$$P_0 \leqslant \frac{(z_b f'_{max} + M_{min})}{(z_b/A + e)} \tag{11.20}$$

Note that in equations 11.17 and 11.18 it is possible that the denominator term, $(z_t/A - e)$, might be negative if $e > z_t/A$. In this case, the sense of the inequality would have to change as the effect of dividing an inequality by a negative number is to change its sense.

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These equations give a range within which the prestress force must lie to ensure that the allowable stress conditions are met at all stages in the life of the member. In the case of a simply supported beam, the design prestress force will generally be based on the minimum value which satisfies these equations at the critical section for bending in the member.

Although a range of values of permissible prestress force can be found, this makes no allowance for the fact that the corresponding eccentricity must lie within the beam. It is therefore necessary to consider the effect of limiting the eccentricity to a maximum practical value for the section under consideration. The effect of this limitation will be most severe when considering the maximum moments acting on the section, that is, the inequalities of equations 11.11 and 11.12.

If the limiting value for maximum eccentricity, e_{max}, depends on cover requirements, equation 11.11 becomes

$$M_{max} \leqslant f_{max} z_t - KP_0\left(\frac{z_t}{A} - e_{max}\right) \tag{11.21}$$

and equation 11.12 becomes

$$M_{max} \leqslant KP_0\left(\frac{z_b}{A} + e_{max}\right) - f_{min} z_b \tag{11.22}$$

These represent linear relationships between M_{max} and P_0. For the case of a beam subject to sagging moments, e_{max} will generally be positive in value, thus equation 11.22 is of positive slope and represents a lower limit to P_0. It can be shown also that for most practical cases $[(z_t/A) - e_{max}] < 0$, thus equation 11.21 is similarly a lower limit of positive, though smaller slope.

Figure 11.11 represents the general form of these expressions, and it

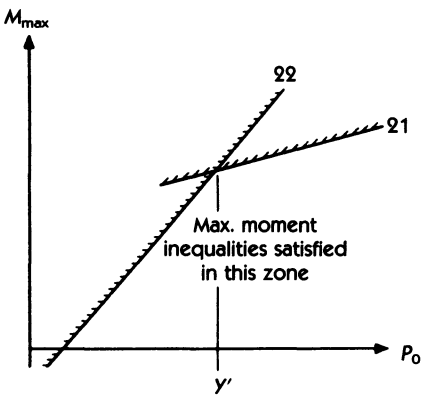

Figure 11.11 *Maximum moment and prestress force relationship*

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can be seen clearly that providing a prestress force in excess of Y' produces only small benefits of additional moment capacity. The value of Y' is given by the intersection of these two expressions, when

$$KP_0\left(\frac{z_b}{A} + e_{max}\right) - f_{min}z_b = f_{max}z_t - KP_0\left(\frac{z_t}{A} - e_{max}\right)$$

thus

$$P_0 = \frac{f_{max}z_t + f_{min}z_b}{K\left(\frac{z_b + z_t}{A}\right)} \tag{11.23}$$

Thus the value of prestress force $P_0 = Y'$ may be conveniently considered as a maximum economic value beyond which any increase in prestress force would be matched by a diminishing rate of increase in moment carrying capacity. If a force larger than this limit is required for a given section it may be more economic to increase the size of the section.

Example 11.3 Calculation of Prestress Force

The 10 metre span beam in example 11.2 was determined to have a breadth of 200 mm and a depth of 350 mm ($z_b = z_t = 4.08 \times 10^6\,mm^3$). Determine the minimum initial prestress force required for an assumed maximum eccentricity of 75 mm.

From example 11.2:

$$f'_{max} = 16\,N/mm^2 \qquad f_{max} = 24\,N/mm^2$$

$$f'_{min} = -1.0\,N/mm^2 \qquad f_{min} = 0.0\,N/mm^2$$

$$\text{Self-weight of beam} = 0.2 \times 0.35 \times 24 = 1.68\,kN/m$$

$$M_{min} = 1.68 \times 10^2/8 = 21\,kN\,m$$

$$M_{max} = 3.0 \times 10^2/8 + 21 = 58.5\,kN\,m$$

(a) From equation 11.17:

$$P_0 \leqslant \frac{(z_t f_{max} - M_{max})}{K(z_t/A - e_{max})}$$

$$\leqslant \frac{(4.08 \times 10^6 \times 24 - 58.5 \times 10^6)}{0.8(4.08 \times 10^6/70\,000 - 75)} \times 10^{-3}$$

and allowing for the division by the negative denominator:

$$P_0 \geqslant -2950\,kN$$

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Similarly from equations 11.18 to 11.20:

$$P_0 \leqslant 1505\,\text{kN}$$

$$P_0 \geqslant 548\,\text{kN}$$

$$P_0 \leqslant 647\,\text{kN}$$

The minimum value of prestress force is therefore 548 kN with an upper limit of 647 kN.

(b) Check the upper economic limit to prestress force
From equation 11.23

$$P_0 \leqslant \frac{f_{max}z_t + f_{min}z_b}{K\left(\frac{z_b + z_t}{A}\right)} = \frac{24zA}{2Kz}$$

$$\leqslant 12A/K$$

$$\leqslant 12 \times (350 \times 200) \times 10^{-3}/0.8$$

$$\leqslant 1050\,\text{kN}$$

Since this is higher than the upper limit already established from equation 11.20, a design with an initial prestressing force between 548 kN and 647 kN will be acceptable.

11.4.3 Stresses under the Quasi-Permanent Loading

The calculation in example 11.3 is based on the characteristic loads. Once a value of prestress force lying between the minimum and upper limit value is chosen, the compressive stress at the top of the section under the quasi-permanent loads should also be calculated and compared with the lesser allowable value of $0.45f_{ck}$. If this proves to be critical then the section may have to be redesigned taking the quasi-permanent load condition as more critical than the characteristic load condition.

Example 11.4 Stress under Quasi-Permanent Loads

For the previous example, using minimum prestress force of 548 kN, check the stress condition under the quasi-permanent loading condition. Assume that the 3 kN/m imposed load consists of a permanent load of 2 kN/m as finishes and 1.0 kN/m variable load. Take 30 per cent of the variable load contributing to the quasi-permanent load.

From the previous example:

Moment due to self-weight = 21 kN m

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Moment due to finishes $= 2 \times 10^2/8 = 25.0\,\text{kN m}$

Moment due to variable load $= 1 \times 10^2/8 = 12.5\,\text{kN m}$

Quasi-permanent moment $= 21 + 25.0 + (0.3 \times 12.5)$
$= 49.75\,\text{kN m}$

Stress at top of section given by:

$$f_t = \frac{KP_0}{A} - \frac{KP_0 e}{z_t} + \frac{M}{z_t}$$

$$= \frac{0.8 \times 548 \times 10^3}{70\,000} - \frac{0.8 \times 548 \times 10^3 \times 75}{4.08 \times 10^6} + \frac{49.75 \times 10^6}{4.08 \times 10^6}$$

$$= 6.26 - 8.06 + 12.19$$

$$= 10.39\,\text{N/mm}^2$$

Allowable compressive stress $= 0.45 f_{ck} = 0.45 \times 40 = 18\,\text{N/mm}^2$

Hence the maximum compressive stress is less than the allowable figure.

11.4.4 Magnel Diagram Construction

Equations 11.17 to 11.20 can be used to determine a range of possible values of prestress force for a given or assumed eccentricity. For different assumed values of eccentricity, further limits on the prestress force can be determined in an identical manner although the calculations would be tedious and repetitive. In addition, it is possible to assume values of eccentricity for which there is no solution for the prestress force as the upper and lower limits could overlap.

A much more useful approach to design can be developed if the equations are treated graphically as follows.

Equations 11.9 to 11.12 can be rearranged into the following form:

$$\frac{1}{P_0} \geqslant \frac{K(1/A - e/z_t)}{(f_{max} - M_{max}/z_t)} \tag{11.24}$$

$$\frac{1}{P_0} \geqslant \frac{(1/A - e/z_t)}{(f'_{min} - M_{min}/z_t)} \tag{11.25}$$

$$\frac{1}{P_0} \leqslant \frac{K(1/A + e/z_b)}{(f_{min} + M_{max}/z_b)} \tag{11.26}$$

$$\frac{1}{P_0} \geqslant \frac{(1/A + e/z_b)}{(f'_{max} + M_{min}/z_b)} \tag{11.27}$$

These equations now express linear relationships between $1/P_0$ and e. Note that in equation 11.25 the sense of the inequality has been reversed

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to account for the fact that the denominator is negative (f'_{min} is negative according to the chosen sign convention). The relationships can be plotted as shown in figure 11.12 a and b, and the area of the graph to one side of each line, as defined by the inequality can be eliminated resulting in an area of graph within which any combination of force and eccentricity will simultaneously satisfy all four inequalities and hence will provide a satisfactory design. The lines marked 1 to 4 correspond to equations 11.24 to 11.27 respectively. This form of construction is known as a *Magnel diagram*.

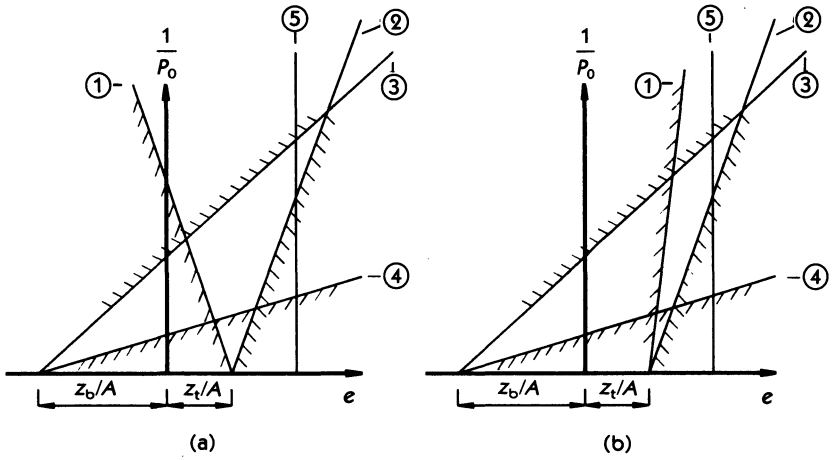

Figure 11.12 *Magnel diagram construction*

The additional line, 5, shown on the diagram corresponds to a possible physical limitation of the maximum eccentricity allowing for the overall depth of section, cover to the prestressing tendons, provision of shear links and so on. Two separate figures are shown as it is possible for line 1, derived from equation 11.24 to have either a positive or negative slope depending on whether f_{max} is greater or less than M_{max}/z_t.

The Magnel diagram is a powerful design tool as it covers all possible solutions of the inequality equations and enables a range of prestress force and eccentricity values to be investigated. Values of minimum and maximum prestress force can be readily read from the diagram as can intermediate values where the range of possible eccentricities for a chosen force can be easily determined. The diagram also shows that the minimum prestress force (largest value of $1/P_0$) corresponds to the maximum eccentricity and as the eccentricity is reduced the prestress force must be increased to compensate.

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Example 11.5 Construction of Magnel Diagram

Construct the Magnel diagram for the beam given in example 11.2 and determine the minimum and maximum possible values of prestress force. Assume a maximum possible eccentricity of 125 mm allowing for cover etc. to the tendons.

From the previous examples:

$$f'_{max} = 16\,N/mm^2 \qquad f_{max} = 24\,N/mm^2$$
$$f'_{min} = -1.0\,N/mm^2 \qquad f_{min} = 0.0\,N/mm^2$$
$$M_{min} = 21\,kN\,m \qquad M_{max} = 58.5\,kN\,m$$
$$K = 0.8 \qquad z_b = z_t = 4.08 \times 10^6\,mm^3$$
$$A = 70\,000\,mm^2$$

From equation 11.24:

$$\frac{1}{P_0} \geqslant \frac{K(1/A - e/z_t)}{(f_{max} - M_{max}/z_t)}$$
$$\geqslant 0.8\left(\frac{1}{70\,000} - \frac{e}{4.08 \times 10^6}\right) \times 10^3 \Big/ \left(24 - \frac{58.5 \times 10^6}{4.08 \times 10^6}\right)$$

which can be rearranged to give

$$\frac{10^6}{P_0} \geqslant 1183 - 20.4e$$

and similarly from the other three inequalities, equations 11.25 to 11.27:

$$\frac{10^6}{P_0} \geqslant -2324 + 39.9e$$

$$\frac{10^6}{P_0} \leqslant 797 + 13.7e$$

$$\frac{10^6}{P_0} \geqslant 676 + 11.6e$$

These inequalities are plotted on the Magnel diagram in figure 11.13 and the zone bounded by the four lines defines an area in which all possible design solutions lie. The line of maximum possible eccentricity is also plotted but, as it lies outside the zone bounded by the four inequalities, does not place any restriction on the possible solutions.

From figure 11.13 it can be seen that the maximum and minimum values of prestress force are given by:

Maximum $10^6/P_0 = 2430$; hence minimum $P_0 = 412\,kN$ ($e = 120\,mm$)
Minimum $10^6/P_0 = 850$; hence maximum $P_0 = 1175\,kN$ ($e = 15\,mm$)

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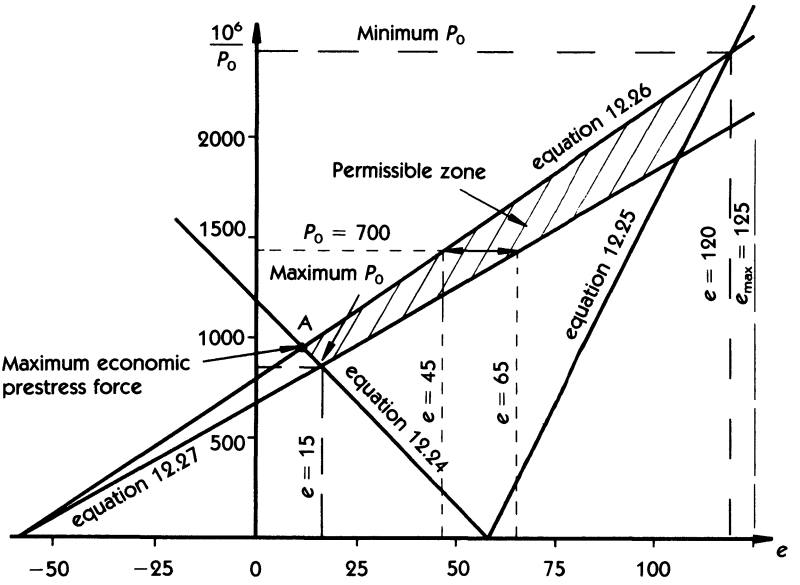

Figure 11.13 *Magnel diagram for example 12.5*

The intersection of the two lines at position A on the diagram corresponds to a value of $P_0 = 1050\,\text{kN}$, established in example 11.3 as the maximum economical value of prestress force for this section (see equation 11.23). Hence the intersection of these two lines should be taken as the maximum prestress force and, as can be seen, this information can be readily determined from the diagram without the need for further calculation.

The Magnel diagram can now be used to investigate other possible solutions for the design prestressing force and eccentricity. For a fixed value of prestress force (and hence fixed value of $1/P_0$) the corresponding range of permissible eccentricity can be read directly from the diagram. Alternatively, if the eccentricity is fixed, the diagram can be used to investigate the range of possible prestress force for the given eccentricity.

11.4.5 Design of Tendon Profiles

Having obtained a value of prestress force which will permit all stress conditions to be satisfied at the critical section, it is necessary to determine the eccentricity at which this force must be provided, not only at the critical section but also throughout the length of the member.

At any section along the member, e is the only unknown term in the four expressions 11.9 to 11.12 and these will yield two upper and two

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lower limits which must all be simultaneously satisfied. This requirement must be met at all sections throughout the member and will reflect both variations of moment, prestress force, and section properties along the member.

The design expressions can be rewritten as

At transfer:

$$e \leqslant \left[\frac{z_t}{A} - \frac{f'_{min} z_t}{P_0}\right] + \frac{M_{min}}{P_0} \tag{11.28}$$

$$e \leqslant \left[-\frac{z_b}{A} + \frac{f'_{max} z_b}{P_0}\right] + \frac{M_{min}}{P_0} \tag{11.29}$$

At service:

$$e \geqslant \left[\frac{z_t}{A} - \frac{f_{max} z_t}{KP_0}\right] + \frac{M_{max}}{KP_0} \tag{11.30}$$

$$e \geqslant \left[-\frac{z_b}{A} + \frac{f_{min} z_b}{KP_0}\right] + \frac{M_{max}}{KP_0} \tag{11.31}$$

Although it is relatively simple to evaluate all four expressions, it can be shown that the governing expressions are either (11.28) and (11.29) at transfer and (11.31) at service. The moments M_{max} and M_{min} are those relating to the section being considered.

For a member of constant cross-section, if minor changes in prestress force along the length are neglected, the terms in brackets in the above expressions are constants. Therefore the zone within which the centroid of prestress force must lie is governed by the shape of the bending-moment envelopes, as shown in figure 11.14. In the case of uniform loading, these are parabolic, hence the usual practice is to provide parabolic tendon profiles if a straight profile will not fit within this zone.

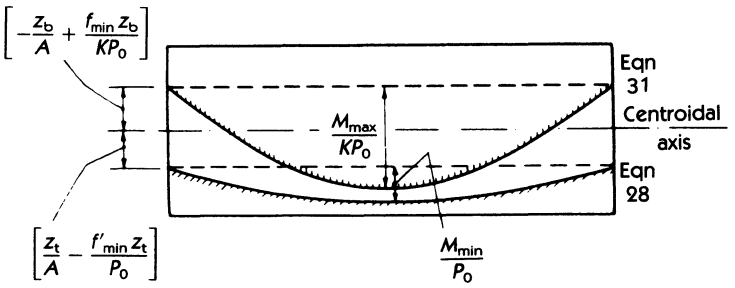

Figure 11.14 *Cable zone limits*

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At the critical section, the zone is generally narrow and reduces to zero if the value of prestress force is taken as the minimum value from the Magnel diagram. At sections away from the critical section, the zone becomes increasingly wide as the moments reduce and the prestress force provided is increasingly greater than the minimum required.

Example 11.6 Calculation of Cable Zone

Determine the cable zone limits at mid-span and ends of the member designed in examples 11.2 to 11.5 for a constant initial prestress force of 700 kN. Data for this question are given in the previous examples.

(a) Ends of beam

Limits to cable eccentricity are given by equation 11.29 which, at the end section can be readily shown, for this example, to be more critical than equation 11.28:

$$e \leqslant \left[-\frac{z_b}{A} + \frac{f'_{max} z_b}{P_0} \right] + \frac{M_{min}}{P_0}$$

and equation 11.31

$$e \geqslant \left[-\frac{z_b}{A} + \frac{f_{min} z_b}{KP_0} \right] + \frac{M_{max}}{KP_0}$$

As there are no moments due to external loading at the end of a simply supported beam equation 11.29 becomes

$$\begin{aligned} e &\leqslant \left[-\frac{4.08 \times 10^6}{(350 \times 200)} + \frac{16 \times 4.08 \times 10^6}{700 \times 10^3} \right] + 0 \\ &\leqslant -58.28 + 93.25 \\ &\leqslant 35 \text{ mm} \end{aligned}$$

Similarly equation 11.31 becomes

$$\begin{aligned} e &\geqslant \left[\frac{-4.08 \times 10^6}{(350 \times 200)} + 0 \right] + 0 \\ &\geqslant -58.29 \text{ mm} \end{aligned}$$

At the ends of the beam where the moments are zero, and for $z_t = z_b$, the inequality expressions can apply with the tendon eccentricities above or below the neutral axis (e positive or negative), so that e must lie within the range ± 35 mm.

(b) Mid-span

Equation 11.28 becomes

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$$e \leqslant \left[\frac{4.08 \times 10^6}{(350 \times 200)} - \frac{(-1)4.08 \times 10^6}{700 \times 10^3}\right] + \frac{21 \times 10^6}{700 \times 10^3}$$
$$\leqslant 64.1 + 30$$
$$\leqslant 94.1\,\text{mm}$$

Equation 11.29 might be more critical than equation 11.28 and should be also checked. From equation 11.29:

$$e \leqslant \left[-\frac{4.08 \times 10^6}{(350 \times 200)} + \frac{16 \times 4.08 \times 10^6}{700 \times 10^3}\right] + \frac{21 \times 10^6}{700 \times 10^3}$$
$$\leqslant -58.3 + 93.3 + 30$$
$$\leqslant 65\,\text{mm}$$

Hence equation 11.29 is critical and the eccentricity must be less than 65 mm.

Equation 11.31 gives

$$e \geqslant \left[-\frac{4.08 \times 10^6}{(350 \times 200)} + 0\right] + \frac{58.5 \times 10^6}{0.8 \times 700 \times 10^3}$$
$$\geqslant -58.3 + 104.5$$
$$\geqslant 46.2\,\text{mm}$$

Hence at mid-span the resultant of the tendon force must lie at an eccentricity in the range of 65 to 46.2 mm.

Provided that the tendons can be arranged so that their resultant force lies within the calculated limits then the design will be acceptable.

If a Magnel diagram for the stress condition at mid-span had been drawn, as in example 11.5, then the eccentricity range could have been determined directly from the diagram without further calculation. For tendons with a combined prestress force at transfer of $P_0 = 700\,\text{kN}$ ($10^6/P_0 = 1428$), plotting this value on the diagram of figure 11.13 will give the range of possible eccentricity between 45 mm and 65 mm.

11.4.6 Width of Cable Zone

From the Magnel diagram of figure 11.12 it can be seen that for any chosen value of prestress force there is an eccentricity range within which the resultant tendon force must lie. As the force approaches a value corresponding to the top and bottom limits of the diagram, the width of the available cable zone diminishes until at the very extremities the upper and lower limits of eccentricity coincide, giving zero width of cable zone.

Practically, therefore, a prestress force will be chosen which has a value between the upper and lower limits of permissible prestress force

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while, at the same time, ensuring that, for the chosen force, a reasonable width of cable zone exists. The prestressing cables must also satisfy requirements of cover, minimum spacing between tendons, available size of tendons and so on. A number of alternative tendon combinations and configurations are likely to be tried so that all requirements are simultaneously met. The advantage of the Magnel diagram is that a range of alternatives can be quickly considered without the necessity for any further calculation, as illustrated at the end of example 11.6.

11.4.7 Prestress Losses

From the moment that the prestressing force is first applied to the concrete member, losses of this force will take place because of the following causes:

(1) Elastic shortening of the concrete.
(2) Creep of the concrete under sustained compression.
(3) Relaxation of the prestressing steel under sustained tension.
(4) Shrinkage of the concrete.

These losses will occur whichever form of construction is used, although the effects of elastic shortening will generally be much reduced when post-tensioning is used. This is because stressing is a sequential procedure, and not instantaneous as with pre-tensioning. Creep and shrinkage losses depend to a large extent on the properties of the concrete with particular reference to the maturity at the time of stressing. In pre-tensioning, where the concrete is usually relatively immature at transfer, these losses may therefore be expected to be higher than in post-tensioning.

In addition to losses from these causes, which will generally total between 20 to 30 per cent of the initial prestress force at transfer, further losses occur in post-tensioned concrete during the stressing procedure. These are due to friction between the strands and the duct, especially where curved profiles are used, and to mechanical anchorage slip during the stressing operation. Both these factors depend on the actual system of ducts, anchorages and stressing equipment that are used.

Thus although the basic losses are generally highest in pre-tensioned members, in some instances overall losses in post-tensioned members may be of similar magnitude.

Elastic Shortening

The concrete will immediately shorten elastically when subjected to compression, and the steel will generally shorten by a similar amount

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(as in pre-tensioning) with a corresponding loss of prestress force. To calculate this it is necessary to obtain the compressive strain at the level of the steel.

If the transfer force is P_0, and the force after elastic losses is P' then

$$P' = P_0 - \text{loss in force}$$

and the corresponding stress in the concrete at the level of the tendon

$$\sigma_{cp} = \frac{P'}{A} + \frac{(P'e) \times e}{I} + \sigma_{cg}$$

where σ_{cg} is the stress due to self-weight and any other permanent load which for self-weight only will be relatively small when averaged over the length of the member and may thus be neglected. Hence

$$\sigma_{cp} = \frac{P'}{A}\left(1 + \frac{e^2 A}{I}\right)$$

and concrete strain $= \sigma_{cp}/E_{cm}$, thus reduction in steel strain $= \sigma_{cp}/E_{cm}$ and

$$\text{reduction in steel stress} = \left(\frac{\sigma_{cp}}{E_{cm}}\right)E_s = \alpha_e \sigma_{cp}$$

thus with A_p = area of tendons

$$\text{loss in prestress force} = \alpha_e \sigma_{cp} A_p$$

$$= \alpha_e \frac{A_p}{A} P'\left(1 + \frac{e^2 A}{I}\right)$$

hence

$$P' = P_0 - \alpha_e \frac{A_p}{A} P'\left(1 + \frac{e^2 A}{I}\right)$$

so that

$$\text{remaining prestress force } P' = \frac{P_0}{1 + \alpha_e \dfrac{A_p}{A}\left(1 + \dfrac{e^2 A}{I}\right)}$$

In pre-tensioned construction this full loss will be present; however when post-tensioning the effect will only apply to previously tensioned cables and although a detailed calculation could be undertaken it is normally adequate to assume 50 per cent of the above losses. In this case the remaining prestress force is

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$$P' = \frac{P_0}{1 + 0.5\alpha_e \frac{A_p}{A}\left(1 + \frac{e^2 A}{I}\right)}$$

and it is this value which applies to subsequent loss calculations. In calculating α_e, E_{cm} may be taken from table 6.9 where f_{ck} should be taken as the transfer strength of the concrete.

Creep of Concrete

The sustained compressive stress on the concrete will also cause a long-term shortening due to creep, which will similarly reduce the prestress force. As above, it is the stress in the concrete at the level of the steel which is important, that is

$$\sigma_{cp} = \frac{P'}{A}\left(1 + \frac{e^2 A}{I}\right)$$

and

$$\text{loss of steel stress} = E_s \sigma_{cp} \times \text{specific creep strain}$$

then

$$\text{loss of prestress force} = E_s \frac{A_p}{A} P'\left(1 + \frac{e^2 A}{I}\right) \times \text{specific creep strain}$$

The value of specific creep used in this calculation will be influenced by the factors discussed in section 6.3.2, and may be obtained from the values of the final creep coefficient $\phi(\infty, t_0)$ given in table 11.1 using the relationship

$$\text{specific creep strain} = \frac{\phi(\infty, t_0)}{E_{cm}} \Big/ \text{N/mm}^2$$

Table 11.1 (on page 382) may be used where the concrete stress does not exceed $0.45 f_{ck}$ at transfer.

Relaxation of Steel

Despite developments in prestressing steel manufacture which have taken place in recent years, relaxation of the wire or strand under sustained tension may still be expected to be a significant factor. The precise value will depend upon whether pre-tensioning or post-tensioning is used and the characteristics of the steel type. Factors allowing for method of construction and the characteristics of the prestressing strand are given in EC2 which should be applied to 1000 hour relaxation values provided by the manufacturer. The amount of relaxation will also depend upon

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Table 11.1 Final creep coefficient of normal weight concrete

Age at loading (t_0 days)	*Dry conditions (inside: r.h. = 50%)*			*Humid conditions (outside: r.h. = 80%)*		
	Notional size ($2A_c/u$) mm			*Notional size ($2A_c/u$) mm*		
	50	*150*	*600*	*50*	*150*	*600*
1	5.4	4.4	3.6	3.5	3.0	2.6
7	3.9	3.2	2.5	2.5	2.1	1.9
28	3.2	2.5	2.0	1.9	1.7	1.5
90	2.6	2.1	1.6	1.6	1.4	1.2
365	2.0	1.6	1.2	1.2	1.0	1.0

A_c is the cross-sectional area of the concrete, u is the perimeter of that area.

the initial tendon load relative to its breaking load. In most practical situations the transfer steel stress is about 70 per cent of the characteristic strength and relaxation losses are likely to be approximately 4–10 per cent of the tendon load remaining after transfer.

Shrinkage of Concrete

This is based on empirical figures for shrinkage/unit length of concrete ($\varepsilon_s(t, t_0)$) for particular curing conditions and transfer maturity as discussed in chapter 6. Typical values range from 280×10^{-6} for UK outdoor exposure to 600×10^{-6} for indoor exposure, depending on the notional size of the member. More detailed guidance in unusual circumstances may be obtained from section 6.3.2.

The loss in steel stress is thus given by $\varepsilon_s(t, t_0)E_s$, hence

$$\text{loss in prestress force} = \varepsilon_s(t, t_0)E_sA_p$$

Friction in Ducts (Post-tensioning only)

When a post-tensioned cable is stressed, it will move relative to the duct and other cables within the duct and friction will tend to resist this movement hence reducing the effective prestress force at positions remote from the jacking point. This effect may be divided into unintentional profile variations, and those due to designed curvature of ducts:

(a) 'Wobble' effects in straight ducts will usually be present. If P_0 = jack force, and P_x = cable force at distance x from jack then it is generally estimated that

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$$P_x = P_0 e^{-\mu k x}$$

where e = base of napierian logs (2.718) and k = unintentional angular displacement per unit length generally in the range of 0.005 to 0.01 radians/metre.

(b) Duct curvature will generally cause greater prestress force losses, and is given by

$$P_x = P_0 e^{-\mu\theta}$$

where μ = coefficient of friction (typically 0.17 for cold drawn wire and 0.19 for strand) and θ is the sum of the angular displacements over the distance x. If the duct curvature is not constant, the profile must be subdivided into sections, each assumed to have constant curvature, in which case P_0 is taken as the force at the jacking end of the section and x the length of the segment. P_x, the force at the end remote from the jack then becomes P_0 for the next section and so on.

The above effects may be combined to produce an effective prestress force diagram for a member. If friction losses are high, it may be worth while to jack simultaneously from both ends, in which case the two diagrams may be superimposed, maintaining symmetry of prestress force relative to the length of the member.

Code Formula for Time-Dependent Losses

Time-dependent losses due to creep, shrinkage and relaxation can be calculated separately, as indicated above, or the combined formula, as given in EC2 can be used to determine the variation of stress, $\delta\sigma_{p,c+s+r}$ at location x and at time t, where

$$\delta\sigma_{p,c+s+r} = \frac{\varepsilon_s(t, t_0)E_s + \delta\sigma_{pr} + \alpha_e\phi(t, t_0)(\sigma_{cg} + \sigma_{cpo})}{1 + \alpha\dfrac{A_p}{A}\left[\left(1 + \dfrac{A}{I}e^2\right)\left(1 + 0.8\phi(t, t_0)\right)\right]} \qquad (11.32)$$

where σ_{cpo} is the initial stress in the concrete adjacent to the tendons due to prestress, $\delta\sigma_{pr}$ is the variation of stress in the tendons due to relaxation and the other terms in the formula are as previously defined.

Example 11.7 Estimation of Prestress Losses at Mid-span

A post-tensioned beam shown in figure 11.15 is stressed by two tendons with a parabolic profile and having a total cross-sectional area A_p = 7500 mm^2. The total initial prestress force is P_0 = 10 500 kN and the total characteristic strength is P_{pk} = 14 000 kN.

Assume the following data for estimating losses:

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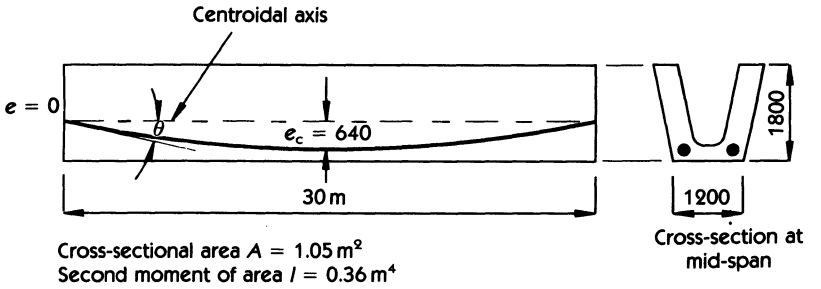

Figure 11.15 *Post-tensioned beam*

Coefficient of friction $\mu = 0.19$; wobble factor $k = 0.01$/metre.

Elastic modulus E_{cm} (transfer) $= 32\,\text{kN/mm}^2$; $E_s = 200\,\text{kN/mm}^2$

Creep coefficient $\phi(\infty, t_0) = 1.6$

Shrinkage strain $\varepsilon(t, t_0) = 330 \times 10^{-6}$

The tendon supplier specifies class 2 strands with a 1000 hour relaxation loss of 2.5 per cent at 70 per cent of the characteristic strength.

(1) Friction

The equation of the parabola is $y = Cx^2$ and with the origin at mid-span when $x = 15000$, $y = 640$, so that $C = 640/15000^2 = 2.844 \times 10^{-6}$.

The gradient θ at the ends is given by

$$\theta = dy/dx = 2Cx = 2 \times 2.844 \times 10^{-6} \times 15000$$
$$= 0.0853 \text{ radians}$$

At mid-span

$$\text{loss } \Delta P(x) = P_0(1 - e^{-\mu(\theta+kx)})$$
$$= P_0(1 - e^{-0.19(0.0853+0.01\times15)})$$
$$= 0.044P_0 = 460\,\text{kN} = 4.4 \text{ per cent}$$

(2) Elastic shortening for post-tensioned construction

$$P' = \frac{P_0}{1 + 0.5\alpha\frac{A_p}{A}\left(1 + e^2\frac{A}{I}\right)}$$

Take the average eccentricity for the parabolic tendon as $\frac{5}{8}e_c = \frac{5}{8} \times 640 = 400\,\text{mm}$ and $\alpha = E_s/E_{cm} = 200/32 = 6.25$.

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$$P' = \frac{P_0}{1 + 0.5 \times 6.25 \times \dfrac{7.5 \times 10^3}{1.05 \times 10^6}\left(1 + 400^2 \dfrac{1.05 \times 10^6}{0.36 \times 10^{12}}\right)}$$

$$= 0.968P_0 = 10\,160\,\text{kN}$$

loss $\Delta P = 10\,500 - 10\,160 = 340\,\text{kN} = 3.2$ per cent

Total short-term losses $= 460 + 340 = 800\,\text{kN}$

$P' = P_0 -$ short-term losses $= 10\,500 - 800 = 9700\,\text{kN}$

(3) Creep

$$\text{loss } \Delta P = \phi \frac{E_s A_p}{E_{cm} A}\left(1 + e^2 \frac{A}{I}\right)P'$$

$$= 1.6 \frac{200 \times 10^3}{32 \times 10^3} \times \frac{7.5 \times 10^3}{1.05 \times 10^6}\left(1 + 400^2 \frac{1.05 \times 10^6}{0.36 \times 10^{12}}\right)9700$$

$$= 1016\,\text{kN}\ (= 9.7 \text{ per cent of } P_0)$$

(4) Shrinkage

$$\text{loss } \Delta P = \varepsilon_s(t, t_0) E_s A_p = 330 \times 10^{-6} \times 200 \times 7.5 \times 10^3$$

$$= 495\,\text{kN}\ (= 4.7 \text{ per cent of } P_0)$$

(5) Relaxation

$P' \approx 70$ per cent of the characteristic strength P_0
Long-term relaxation loss factor = 1.5 for class 2 strand from table 8.4 of the Concise EuroCode

$$\text{loss } \Delta P = (1.5 \times 2.5/100)P' = 0.0375 \times 9700$$

$$= 364\,\text{kN}\ (= 3.5 \text{ per cent of } P_0)$$

$$\text{Total estimated losses} = 800 + 1016 + 495 + 364 = 2675$$

$$= 25 \text{ per cent of } P_0.$$

11.4.8 Calculation of Deflection

The anticipated deflection of a prestressed member must always be checked since span–effective depth ratios are not specified in the code for prestressed concrete members. The deflection due to the eccentric prestress force must be evaluated and added to that from the normal permanent and variable load on the member. In the majority of cases, particularly where the member is designed to be uncracked under full load, a simple linear elastic analysis based on the gross concrete section will be sufficient to give a reasonable and realistic estimate of deflections.

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Where the member is designed such that, under the characteristic loads, the tensile strength exceeds the cracking strength of the concrete, f_{ctm}, it may be necessary to base the calculation of deflection on the cracked concrete section and reference should be made to the Code for the method of dealing with this situation.

The basic requirements which should generally be satisfied in respect of deflections are similar to those of a reinforced concrete beam (section 6.3) which are:

(1) Deflection under the action of the *quasi-permanent load* $\leqslant$ span/250 measured below the level of the supports
(2) Span/500 maximum movement after other elements, which are susceptible to damage by movement, are applied.

The evaluation of deflections due to prestress loading can be obtained by double integration of the expression

$$M_x = Pe_x = \frac{EI\mathrm{d}^2y}{\mathrm{d}x^2}$$

over the length of the member, although this calculation can prove tedious for complex tendon profiles.

The simple case of straight tendons in a uniform member however, yields $M = -Pe$ = a constant, which is the situation evaluated in section 6.3.3 to yield a maximum mid-span deflection of $ML^2/8EI = -PeL^2/8EI$. If the cables lie below the centroidal axis, e is positive, and the deflection due to prestress is then negative, that is upwards.

Another common case of a symmetrical parabolic tendon profile in a beam of constant section can also be evaluated quite simply by considering the bending-moment distribution in terms of an equivalent uniformly distributed load.

For the beam in figure 11.16 the moment due to prestress loading at any section is $M_x = -Pe_x$ but since e_x is parabolic, the prestress loading may be likened to a uniformly distributed load w_e on a simply supported beam; then mid-span moment

$$M = \frac{w_e L^2}{8} = -Pe_c$$

thus

$$w_e = \frac{-8Pe_c}{L^2}$$

But since the mid-span deflection due to a uniformly distributed load w over a span L is given by

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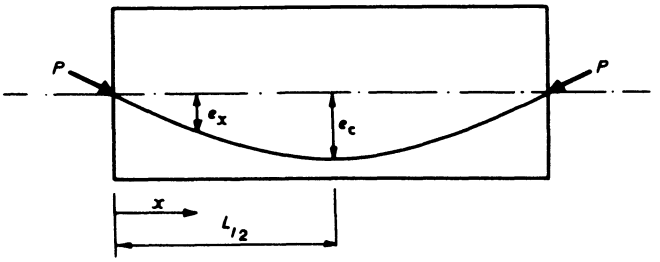

Figure 11.16 *Parabolic tendon profile*

$$y = \frac{5}{384}\frac{wL^4}{EI}$$

the deflection due to w_e is

$$y = -\frac{5}{48}\frac{(Pe_c)L^2}{EI}$$

If the prestress force does not lie at the centroid of the section at the ends of the beam, but at an eccentricity e_0 as shown in figure 11.17, the expression for deflection must be modified. It can be shown that the deflection is the same as that caused by a force P acting at a constant eccentricity e_0 throughout the length of the member, plus a force P following a parabolic profile with mid-span eccentricity e'_c as shown in figure 11.17.

The mid-span deflection thus becomes

$$y = \frac{(Pe_0)L^2}{8EI} - \frac{5}{48}\frac{(Pe'_c)L^2}{EI}$$

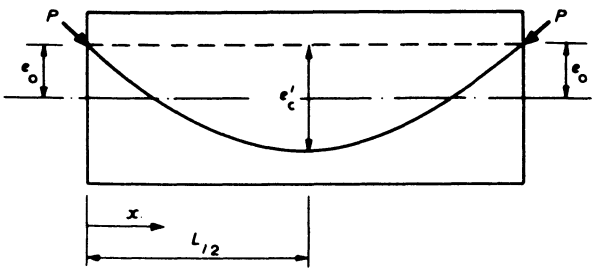

Figure 11.17 *Parabolic tendon profile eccentric at ends of beam*

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Deflections due to more complex tendon profiles are most conveniently estimated on the basis of coefficients which can be evaluated for commonly occurring arrangements. These are on the basis $y = (KL^2)/EI$ where K incorporates the variations of curvature due to prestress along the member length.

There are three principal stages in the life of a prestressed member at which deflections may be critical and may require to be assessed:

(1) At transfer – a check of actual deflection at transfer for comparison with estimated values is a useful guide that a prestressed beam has been correctly constructed.
(2) Under dead load, before application of finishes – deflections must be evaluated to permit subsequent movement and possible damage to be estimated.
(3) Long term under full quasi-permanent load – deflections are required, both to determine the subsequent movement and also to assess the appearance of the final structure.

Short-term deflections will be based on materials properties associated with characteristic strengths ($\gamma_m = 1$) and with actual loading ($\gamma_f = 1$). Long-term assessment however must not only take into account loss in prestress force, but also the effects of creep both on the applied loading and the prestress loading components of the deflection. Creep is allowed for by using an effective modulus of elasticity for the concrete, as discussed in section 6.3.2.

Thus if $E_c(t_0)$ is the instantaneous value, the effective value after creep is given by

$$E_{c,\text{eff}} = \frac{E_c(t_0)}{1 + \phi(t, t_0)}$$

where the value of $\phi(t, t_0)$, the creep coefficient, can be obtained from table 11.1.

It can be shown in some instances that when net upward deflections occur, these often increase because of creep, thus the most critical downward deflection may well be before creep losses occur, while the most critical upward deflection may be long term. This further complicates a procedure which already has many uncertainties as discussed in chapter 6; thus deflections must always be regarded as estimates only.

Example 11.8 Calculation of Deflection

Estimate transfer and long-term deflections for a 200 × 350 mm beam of 10 m span. The prestressing tendon has a parabolic profile with mid-span eccentricity = 75 mm and the end eccentricity = 0 at both ends. The initial prestress force at transfer, P_0, is 548 kN and there are 20 per cent

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losses. The imposed load consists of 2.0 kN/m finishes and 1.0 kN/m variable load. $E_{cm} = 35\,kN/mm^2$ and the creep factor $\phi(t, t_0) = 2.1$.

$$\text{Self-weight} = 0.2 \times 0.35 \times 24 = 1.68\,kN/m$$

$$I = \frac{bh^3}{12} = \frac{200 \times 350^3}{12} = 715 \times 10^6\,mm^4$$

(a) At transfer

$$\text{Deflection } y_a = \frac{5}{384}\frac{w_{min}L^4}{E_{cm}I} - \frac{5}{48}\frac{(P_0 e_c)L^2}{E_{cm}I}$$

$$= \frac{5}{384}\frac{1.68 \times 10^4 \times 10^{12}}{35 \times 10^3 \times 715 \times 10^6} - \frac{5}{48}\frac{548 \times 10^3 \times 75 \times 10^2 \times 10^6}{35 \times 10^3 \times 715 \times 10^6}$$

$$= 8.8 - 17.1$$

$$= -8\,mm \text{ (upwards)}$$

(b) At application of finishes

Assume that only a small proportion of prestress losses have occurred:

$$\text{Weight of finishes} = 2.0\,kN/m$$

therefore

$$y_b = y_a - \frac{5 \times 2.0 \times 10^4 \times 10^{12}}{384 \times 35 \times 10^3 \times 715 \times 10^6}$$

$$= -8 + 10\,mm = 2\,mm \text{ (downwards)}$$

(c) In the long term due to the quasi-permanent load plus prestress force after losses

Assuming 30 per cent of the variable load contributes to the quasi-permanent load:

$$\text{Quasi-permanent load} = \text{self-weight} + \text{finishes} + 0.3 \times \text{variable load}$$

$$= 1.68 + 2.0 + 0.3 \times 1.0 = 3.98\,kN/m$$

$$\text{Prestress forces after losses} = 0.8P_0 = 0.8 \times 548 = 438\,kN$$

$$E_{c,eff} = \frac{E_{cm}}{(1 + \phi(t, t_0))} = \frac{35}{(1 + 2.1)} = 11.3\,kN/mm^2$$

$$y_c = \frac{5}{384}\frac{3.98 \times 10^4 \times 10^{12}}{11.3 \times 10^3 \times 715 \times 10^6} - \frac{5}{48}\frac{438 \times 10^3 \times 75 \times 10^2 \times 10^6}{11.3 \times 10^3 \times 715 \times 10^6}$$

$$= 64.1 - 42.4 = 22\,mm \text{ (downwards)} \leq \text{span}/250 = 40\,mm$$

therefore satisfactory

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(d) Movement after application of finishes

$$y_d = y_c - y_b = 22 - 2 = 20\,\text{mm} \leqslant \text{span}/500 = 20\,\text{mm (satisfactory)}$$

11.4.9 End Blocks

In pre-tensioned members, the prestress force is transferred to the concrete by bond over a definite length at each end of the member. The transfer of stress to the concrete is thus gradual. In post-tensioned members however, the force is concentrated over a small area at the end faces of the member, and this leads to high tensile forces at right angles to the direction of the compression force. This effect will extend some distance from the end of the member until the compression has distributed itself across the full concrete cross-section. This region is known as the 'end block' and must be heavily reinforced by steel to resist the bursting tension forces. End block reinforcement will generally consist of closed links which surround the anchorages, and the quantities provided are usually obtained from empirical or simplified design methods.

Typical 'flow lines' of compressive stress are shown in figure 11.18, from which it can be seen that whatever type of anchorage is used, the required distribution can be expected to have been attained at a distance from the loaded face equal to the lateral dimension of the member. This is relatively independent of the anchorage type.

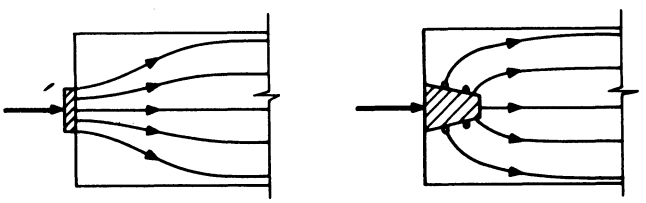

Figure 11.18

In designing the end block it is necessary to check that the bearing stress behind the anchorage plate due to the prestressing force does not exceed the limiting stress, f_{Rdu}, given by

$$f_{Rdu} = 0.67 f_{ck}(A_{ct}/A_{co})^{0.5} \leqslant 2.2 f_{ck}$$

where

A_{co} is the loaded area of the anchorage plate

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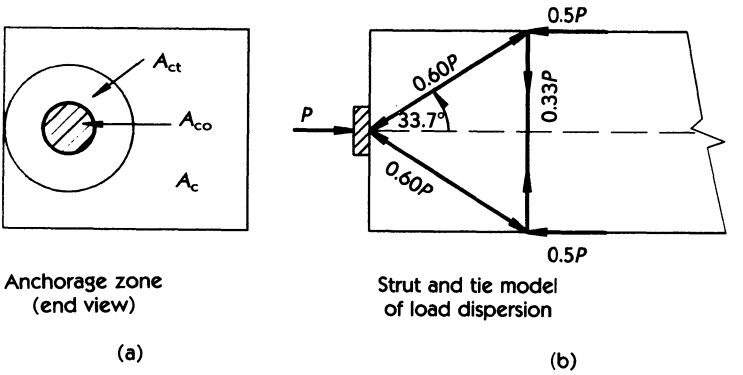

Figure 11.19

A_{ct} is the maximum area, having the same shape as A_{co} which can be inscribed in the total area A_c, as shown in figure 11.19a.

The lateral tensile bursting forces can be established by the use of a statically determinate strut and tie model where it is assumed that the load is carried by a truss consisting of concrete struts and links of reinforcement acting as steel ties. EC2 suggests that in determining the geometry of this truss the prestressing force can be assumed to disperse at an angle of 33.7° to the longitudinal axis of the beam as shown in figure 11.19b. The compressive stresses in the assumed struts should not exceed $0.4f_{ck}$ and the reinforcement is designed to act at a design strength of $0.87f_{yk}$. This reinforcement, in the form of closed links, is then distributed over a length of the end-block equal to the greater lateral dimension of the block, this length being the the length over which it is assumed that the lateral tensile stresses are acting. A minimum reinforcement ratio of 0.15 per cent should be provided on either side of the end block in both directions.

Example 11.9 Design of End Block Reinforcement

The beam in figure 11.20 is stressed by four identical 100 mm diameter conical anchorages located as shown, with a jacking force of 300 kN applied to each. The area may be subdivided into four equal end zones of 200 × 150 mm each. Determine the reinforcement required around the anchorages: $f_{ck} = 40\,\text{N/mm}^2$, $f_{yk} = 250\,\text{N/mm}^2$.

Consider one anchor:

(a) Check bearing stress under the anchor

$$\text{Actual bearing stress} = \frac{\text{Prestressing force}}{\text{Loaded area}}$$

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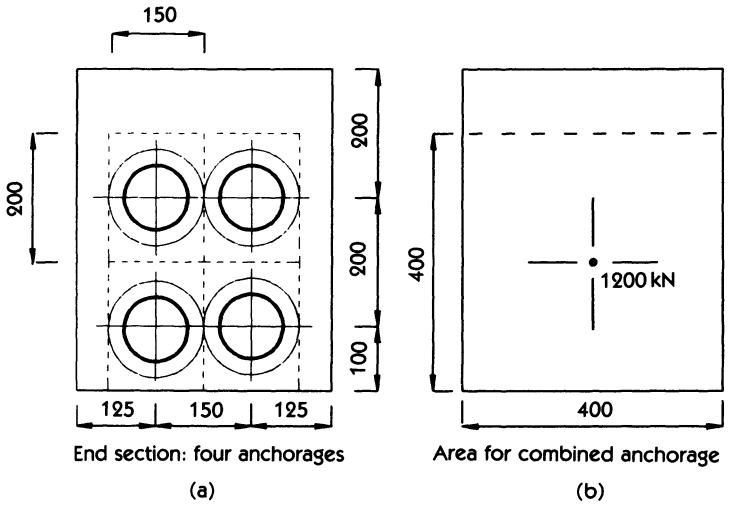

Figure 11.20

$$= \frac{300 \times 10^3}{\pi \times 100^2/4}$$
$$= 38.2\,\text{N/mm}^2$$

Allowable bearing stress

$$f_{\text{Rdu}} = 0.67 f_{\text{ck}} (A_{\text{ct}}/A_{\text{co}})^{0.5}$$
$$= 0.67 \times 40 \left(\frac{\pi \times 150^2/4}{\pi \times 100^2/4}\right)^{0.5}$$
$$= 40.2\,\text{N/mm}^2 \quad (>38.2)$$

(b) Reinforcement

From figure 11.19b, the tensile force in the tie of the equivalent truss is given by

$$T = 0.33 \times 300 = 100\,\text{kN}$$

Area of tensile steel required

$$A_s = \frac{100 \times 10^3}{0.87 \times 250}$$
$$= 460\,\text{mm}^2$$

This can be provided by three 10 mm closed links (471 mm^2) at, say, 50, 125 and 200 mm from the end face; that is, distributed over a length equal to the largest dimension of the anchorage block (200 mm).

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(c) Check compressive stress in the struts

$$\text{Allowable compressive stress} = 0.4 f_{ck}$$
$$= 0.4 \times 40 = 16\,\text{N/mm}^2$$

$$\text{Actual stress in strut} = \frac{\text{Force in strut}}{\text{Cross-sectional area}}$$
$$= \frac{0.60 \times 300 \times 10^3}{(200 \times 150 \times \cos 33.7°)}$$
$$= 7.21\,\text{N/mm}^2$$

The effect of the combined anchorage can be considered by considering the total prestress force of 1200 kN acting on an effective end block of 400 × 400 mm.

The tensile force in the tie of the equivalent truss is given by

$$T = 0.33 \times 1200 = 400\,\text{kN}$$

Area of tensile steel required

$$A_s = \frac{400 \times 10^3}{0.87 \times 250}$$
$$= 1839\,\text{mm}^2$$

This can be provided by nine 12 mm closed links (2040 mm^2) distributed over a length equal to the largest dimension of the anchorage block, that is, 400 mm.

11.5 Analysis and Design at the Ultimate Limit State

After a prestressed member has been designed to satisfy serviceability requirements, a check must be carried out to ensure that the ultimate moment of resistance and shear resistance are adequate to satisfy the requirements of the ultimate limit state. The partial factors of safety on loads and materials for this analysis are the normal values for the ultimate limit state which are given in chapter 2.

11.5.1 Analysis of the Section

As the loads on a prestressed member increase above the working values, cracking occurs and the prestressing steel begins to behave as conventional reinforcement. The behaviour of the member at ultimate is exactly the same as that of an ordinary reinforced concrete member except that the initial strain in the steel must be taken into account in the calculations.

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The section may easily be analysed by the use of the equivalent rectangular stress block described in chapter 4.

Although illustrated by a simple example this method of analysis may be applied to a cross-section of any shape which may have any arrangement of bonded prestressing wires or tendons. Use is made of the stress–strain curve for the prestressing steel as shown in figure 11.21, to calculate tension forces in each layer of steel. The total steel strain is that due to bending added to the initial strain in the steel resulting from prestress. For a series of assumed neutral axis positions, the total tension capacity is compared with the compressive force developed by a uniform stress of $0.567f_{ck}$, and when reasonable agreement is obtained, the moment of resistance can be evaluated.

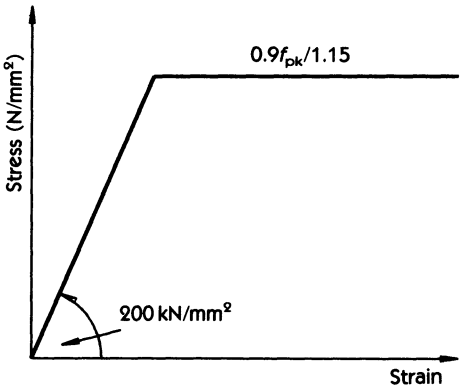

Figure 11.21 *Stress–strain curve for prestressing steel*

Example 11.10 Calculation of Ultimate Moment of Resistance

The section of a pretensioned beam shown in figure 11.22 is stressed by ten 5 mm wires of characteristic strength $f_{pk} = 1470\,N/mm^2$. If these wires are initially stressed to $1000\,N/mm^2$ and 30 per cent losses are anticipated, estimate the ultimate moment of resistance of the section if grade C35/45 concrete is used. The stress–strain curve for the prestressing wire is shown in figure 11.23.

$$\text{Area of 5 mm wire} = \pi \times 5^2/4 = 19.6\,mm^2$$

$$\text{Stress in steel after losses} = 1000 \times 0.7 = 700\,N/mm^2$$

therefore

$$\text{strain in steel after losses} = \frac{f_s}{E_s} = \frac{700}{200 \times 10^3} = 0.0035$$

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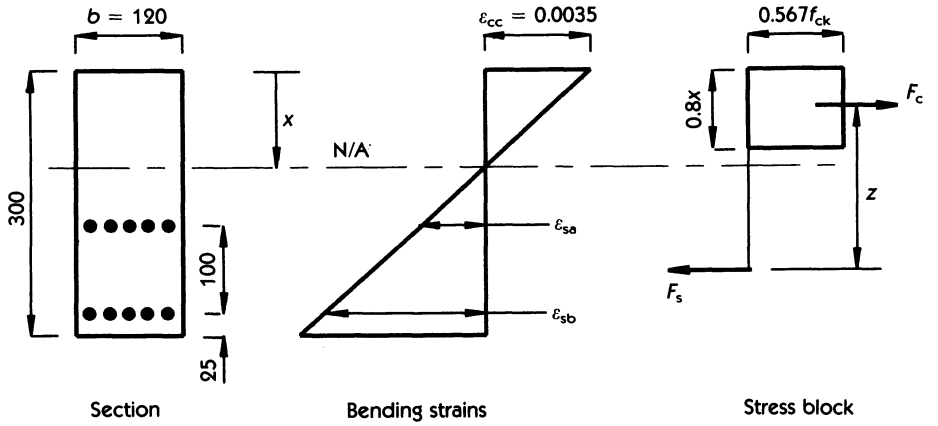

Figure 11.22

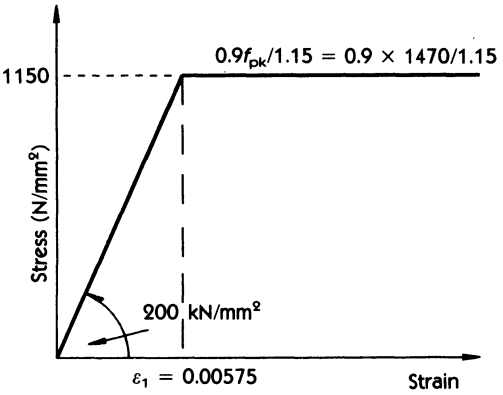

Figure 11.23 *Stress–strain curve for prestressing wire*

which is less than ε_1, the yield strain.

A depth x of neutral axis must be found for which the compressive force F_c in the concrete is balanced by the tensile force F_s in the steel. Then the ultimate moment of resistance is given by

$$M_u = F_c z = F_s z \tag{11.33}$$

where z is the lever arm between F_c and F_s.

As a first attempt try x = 130 mm, approximately equal to $0.5d$.

(a) Steel Strains

Final steel strain, ε_s = prestress strain + bending strain, ε_s'

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(In calculating ε'_s, the initial concrete strain due to prestress can be ignored without undue error.)

Top layer

$$\varepsilon_{sa} = 0.0035 + \varepsilon'_{sa}$$

therefore
$$\varepsilon_{sa} = 0.0035 + \frac{(175 - x)}{x}\varepsilon_{cc} \tag{11.34}$$
$$= 0.0035 + \frac{(175 - 130)}{130}0.0035$$
$$= 0.0047$$

Bottom layer

$$\varepsilon_{sb} = 0.0035 + \varepsilon'_{sb}$$
$$= 0.0035 + \frac{(275 - x)}{x}\varepsilon_{cc} \tag{11.35}$$
$$= 0.0035 + \frac{(275 - 130)}{130}0.0035$$
$$= 0.0074$$

(b) Steel Stresses

From the stress–strain curve the corresponding steel stresses are

Top layer

$$f_{sa} = \varepsilon_{sa} \times E_s = 0.0047 \times 200 \times 10^3 = 940\,\text{N/mm}^2 \tag{11.36}$$

and

$$f_{sb} = 1150\,\text{N/mm}^2$$

as the strain in the bottom steel exceeds the yield strain.

(c) Forces in Steel and Concrete

Steel tensile force $F_s = \Sigma f_s A_s = (f_{sa} + f_{sb})5 \times 19.6$ (11.37)
$$= (940 + 1150)98$$
$$= 205 \times 10^3\,\text{N}$$

With a rectangular stress block

concrete compressive force $F_c = 0.567 f_{ck} b \times 0.8x$ (11.38)
$$= 0.567 \times 35 \times 120 \times 0.8 \times 130$$
$$= 248 \times 10^3\,\text{N}$$

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Table 11.2

x (mm)	Strains		Stresses		Forces	
	ε_{sa}	ε_{sb}	f_{sa}	f_{sb}	F_s	F_c
	$(\times 10^3)$		(N/mm^2)		(kN)	
130	4.7	7.4	940	1150	205	248
110	5.6	8.8	1114	1150	222	210
120	5.1	8.0	1021	1150	213	229
114	5.4	8.4	1075	1150	218	217

The force F_c in the concrete is larger than the force F_s in the steel, therefore a smaller depth of neutral axis must be tried.

Table 11.2 shows the results of calculations for further trial depths of neutral axis. For $x = 110$, F_c became smaller than F_s, therefore $x = 120$ and 114 were tried and it was then found that $F_s = F_c$.

In terms of the tensile force in the steel, the ultimate moment of resistance of the section is given by

$$M_u = F_s z = \Sigma[f_s A_s(d - 0.40x)] \qquad (11.39)$$
$$= 5 \times 19.6[1075(175 - 0.40 \times 114) + 1150(275 - 0.40 \times 114)]$$
$$= 39.5 \times 10^6 \text{ N mm}$$

If x had been incorrectly chosen as 130 mm then using equation 11.39 M_u would equal 36.5 kN m, or in terms of the concrete

$$M_u = 0.567 f_{ck} b \times 0.8xz$$
$$\approx 0.567 \times 35 \times 120 \times 0.8 \times 130(225 - 0.4 \times 130) \times 10^{-6}$$
$$\approx 43 \text{ kN m}$$

Comparing the average of these two values of M_u (= 39.8 kN m) with the correct answer, it can be seen that a slight error in the position of the neutral axis does not have any significant effect on the calculated moment of resistance.

11.5.2 Design of Additional Reinforcement

If it is found that the ultimate limit state requirements are not met, additional untensioned or partially tensioned steel may be added to increase the ultimate moment of resistance.

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Example 11.11 Design of Untensioned Reinforcement

Design untensioned high yield reinforcement (f_{yk} = 460 N/mm^2) for the rectangular beam section shown in figure 11.24 which is stressed by five 5 mm wires, if the ultimate moment of resistance is to exceed 40 kN m for grade C40/50 concrete. The characteristic strength of tensioned steel, f_{pk} = 1670 N/mm^2.

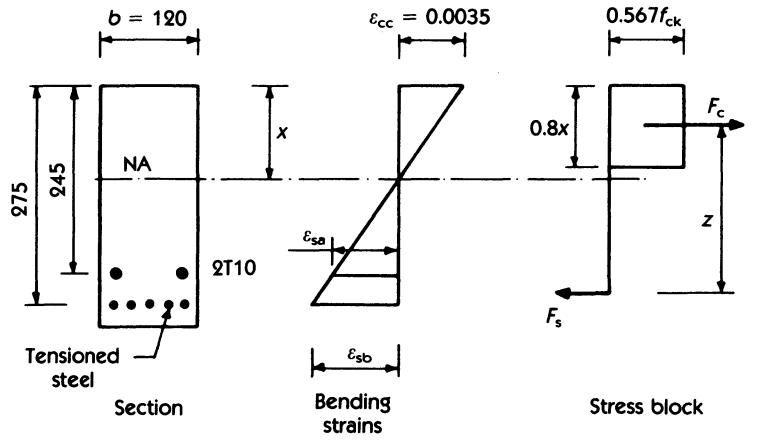

Figure 11.24

(a) Check Ultimate Moment of Resistance

$$\text{Maximum tensile force if prestressing steel yielded} = 5 \times 19.6 \times \frac{(0.9 \times 1670)}{1.15} \times 10^{-3}$$

$$= 128\,\text{kN}$$

$$\text{Concrete compressive area to balance} = \frac{128 \times 10^3}{0.567 \times 40} = 0.8 \times 120x$$

thus, neutral axis depth x = 59 mm.

Assuming prestrain as calculated in example 11.10

$$\text{total steel strain} = \text{prestrain} + \text{bending strain}$$

$$= 0.0035 + \frac{(d - x)}{x} \times 0.0035$$

$$= 0.0035 + \frac{216}{59} \times 0.0035$$

$$= 0.0163\ (> \text{yield})$$

$$\text{Lever arm} = 275 - 0.40 \times 59 = 251\,\text{mm}$$

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Hence

$$\text{ultimate moment of resistance} = 251 \times 128 \times 10^{-3} = 32.1\,\text{kN m}$$

Untensioned steel is therefore required to permit the beam to support an ultimate moment of 40 kN m.

Additional moment of capacity to be provided = 40 − 32.1 = 7.9 kN m
Effective depth of additional steel = 245 mm

then

$$\text{lever arm to additional steel} \approx 220\,\text{mm}$$

then

$$\text{additional tension force required} = \frac{7900}{220} = 35.9\,\text{kN}$$

thus

$$\text{estimated area of untensioned steel required at its yield stress} = \frac{35\,900}{460 \times 0.87} = 90\,\text{mm}^2$$

Try two 10 mm diameter bars (157 mm^2).

(b) Check Steel Strain

If additional steel has yielded, force in two T10 bars = $157 \times 460 \times 10^{-3}/1.15$ = 62.8 kN, therefore

$$\text{total tensile force if all the steel has yielded} = 128 + 62.8 = 190.8\,\text{kN}$$

Thus

$$\text{depth of neutral axis at ultimate} = \frac{190.8 \times 10^3}{0.567 \times 40 \times 120 \times 0.8} = 88\,\text{mm}$$

Therefore

$$\text{prestressing steel strain } \varepsilon_{sb} = \frac{275 - 88}{88} \times 0.0035 + 0.0035 = 0.0109\ (> \text{yield})$$

and

$$\text{untensioned steel strain } \varepsilon_{sa} = \frac{245 - 88}{88} \times 0.0035 = 0.0062$$

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This value is greater than the yield strain of 0.002 from section 4.1.2.

(c) Check Ultimate Moment of Resistance

Taking moments about the centre of compression

$$\begin{aligned} M_u &= 128(275 - 0.40x) + 62.8(245 - 0.40x) \\ &= [128(275 - 0.40 \times 88) + 62.8(245 - 0.40 \times 88)]10^{-3} \\ &= 43.9\,\text{kN m} \end{aligned}$$

If it had been found in (b) that either the prestressing steel or untensioned steel had not yielded, then a trial-and-error approach similar to example 11.10 would have been necessary.

11.5.3 Shear

Shear in prestressed concrete is considered at the ultimate limit state. Design for shear therefore involves the most severe loading conditions, with the usual partial factors of safety on loading for the ultimate limit state being incorporated.

The action of a member in resisting shear is similar to that for reinforced concrete, but with the additional effects of the compression due to the prestress force. This will increase the shear resistance considerably and this is taken into account in EC2 by enhancing the equation for the shear capacity (V_{Rd1}) of the section without shear reinforcement. With a few slight modifications, the Code gives an identical approach to shear design in prestressed sections as is used in reinforced concrete sections as outlined in chapter 5.

In calculating the design shear force, V_{Sd}, it is permissible to take into account the vertical component of force in any inclined tendons which will tend to act in a direction that resists shear, thus enhancing the shear capacity of the section. The approach to shear design is indicated below and comparison should be made with the *Standard Method* given in chapter 5 for reinforced concrete.

Shear Strength without Shear Reinforcement

The concrete shear strength (V_{Rd1}) is given by the empirical expression:

$$V_{Rd1} = [\tau_{Rd}k(1.2 + 40\rho_1) + 0.15\sigma_{cp}]b_w d \qquad (11.40)$$

where

τ_{Rd} = basic design shear strength given in table 5.1
k = $(1.6 - d)$ $\{>1\}$ or 1 where more than 50 per cent of tension reinforcement is curtailed; d is in metres

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$\rho_1 = A_{s1}/b_w d$ {<0.02}
A_{s1} = area of longitudinal tension reinforcement
b_w = *minimum* width of section over area considered
d = effective depth of section
σ_{cp} = axial stress in section due to prestress (KP_0/A).

Upper Limit to Shear Force

The maximum design shear force that a section can carry (V_{Rd2}) irrespective of any shear reinforcement is given by

$$V_{Rd2} = 0.3vf_{ck}b_w d \tag{11.41}$$

where v is an efficiency factor = $0.7 - f_{ck}/200$ (≥ 0.5)

If σ_{cp} exceeds $0.27f_{ck}$, the above expression is reduced to

$$V_{Rd2.red} = 1.67V_{Rd2}(1 - 1.5\sigma_{cp}/f_{ck}) \tag{11.42}$$

If the web of the section contains grouted ducts with a diameter greater than one-eighth of the web thickness then, in the calculation of V_{Rd2}, the web thickness should be reduced by one-half of the sum of the duct diameters measured at the most unfavourable section of the web. If the design shear force exceeds V_{Rd2} or $V_{Rd2.red}$, then it will be necessary to increase the section size.

Shear Reinforcement

As in reinforced concrete, shear reinforcement must be provided to resist the additional shear force that cannot be resisted by the concrete alone. If the shear resistance of any shear reinforcement is given by V_{wd}, then the total shear resistance of the section, V_{Rd3}, is given by

$$V_{Rd3} = V_{Rd1} + V_{wd} \geq V_{Sd}; \text{ the design shear} \tag{11.43}$$

The shear resistance of the shear links is given by equation 5.1 as

$$V_{wd} = \frac{1}{1.28}\frac{A_{sw}}{s}df_{yk}$$

Hence combining the above two equations for design purposes:

$$\frac{A_{sw}}{s} = 1.28(V_{Sd} - V_{Rd1})/(df_{yk}) \tag{11.44}$$

Minimum areas of shear reinforcement should be provided, as indicated in table 7.1. These minimum areas are identical to the requirements for reinforced concrete.

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Design Procedure

The design procedure necessitates calculating the ultimate shear force, V_{Sd}, at intervals along the length of the beam. At each interval, V_{Sd} should be compared with the limiting value, V_{Rd2}, and if V_{Sd} exceeds V_{Rd2} then a larger section size should be considered.

If this check proves adequate then the amount of shear reinforcement can be determined at any cross-section using equation 11.44 where, if inclined or curved tendons are used, the contribution of these in resisting shear can be taken into account by increasing the shear capacity, V_{Rd1}, by an amount equal to the vertical component of prestress at the section being considered.

Example 11.12 Design of Shear Reinforcement

The beam cross-section in figure 11.25 is constant over a 30 m span with a parabolic tendon profile and an eccentricity varying between 300 mm at the ends and 750 mm at mid-span, measured below the neutral axis in both cases. The beam supports an ultimate uniformly distributed load of 43 kN/m and is of strength class C35/45 concrete.

Given data: Prestress force after losses = 2590 kN

$I = 145\,106 \times 10^6\,\text{mm}^4$

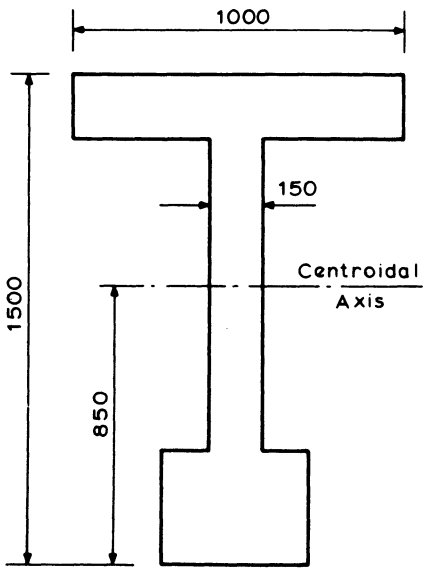

Figure 11.25

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$$A = 500 \times 10^3\,\text{mm}^2$$

$$A_p = 3450\,\text{mm}^2$$

$$f_{yk} = 250\,\text{N/mm}^2 \text{ for the shear links}$$

The design calculations will be shown in full at the support section and will then be tabulated at other cross-sections.

(a) Upper limit to shear force

From equation 11.41:

$$\begin{aligned} V_{Rd2} &= 0.3(0.7 - f_{ck}/200)f_{ck}b_w d \\ &= 0.3(0.7 - 35/200) \times 35 \times 150 \times 950 \times 10^{-3} \\ &= 786\,\text{kN} \end{aligned}$$

$$KP_0/A = (2590 \times 10^3)/(500 \times 10^3) = 5.18\,\text{N/mm}^2$$

Checking for $V_{Rd2.red}$ of equation 11.42:

$$0.27f_{ck} = 0.27 \times 35 = 9.45\,\text{N/mm}^2$$

Hence, as $KP_0/A < 0.27f_{ck}$ then $V_{Rd2} = 786\,\text{kN}$.

At the end of the beam the design shear force is $(43 \times 30/2) = 645\,\text{kN}$ and hence the upper limit to the shear force is not exceeded.

(b) Shear Resistance of the Section

The effective shear resistance of the section is the sum of the shear resistance of the concrete, V_{Rd1}, and the vertical shear resistance of the inclined tendons.

(i) Shear resistance of the concrete

From equation 11.40:

$$V_{Rd1} = [\tau_{Rd}k(1.2 + 40\rho_1) + 0.15\sigma_{cp}]b_w d$$

where

$\tau_{Rd} = 0.37\,\text{N/mm}^2$ from table 5.1

$d = 1.5 - 0.85 + e = 0.95\,\text{m}$ at the supports

$\therefore k = (1.6 - d) = (1.6 - 0.95) = 0.65 < 1$; therefore $k = 1$

$\rho_1 = A_{s1}/b_w d = 3450/(150 \times 950) = 0.0242 > 0.02$ therefore $\rho_1 = 0.02$

$\sigma_{cp} = KP_0/A = 5.18\,\text{N/mm}^2$

Therefore

$$\begin{aligned} V_{Rd1} &= [0.37 \times 1(1.2 + 40\{0.02\}) + 0.15 \times 5.18] \times 150 \times 950 \times 10^{-3} \\ &= 216\,\text{kN} \end{aligned}$$

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Table 11.3 Shear calculations at 3 m intervals

x	d	V_{Rd2}	A_{s1}/bd	V_{Rd1}	V_t	$V_{Rd1} + V_t$	V_{Sd}	$V_{Sd} - (V_{Rd1} + V_t)$	A_{sw}/s	A_{sw}/s *(min.)*
0	1400	1158	0.016	307	0	307	0	—	—	0.33
3	1382	1143	0.017	304	31	335	129	—	—	0.33
6	1328	1098	0.017	294	62	356	258	—	—	0.33
9	1238	1024	0.019	278	93	371	387	16	—	0.33
12	1112	919	0.020	253	124	377	516	139	0.64	—
15	950	786	0.020	216	155	371	645	274	1.48	—

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(ii) Shear Resistance of the Inclined Tendons

The vertical component of prestress force is $P\sin\beta$ where β = tendon slope. The tendon profile is $y = Cx^2$ with the origin of the cable profile taken at mid-span; hence at $x = 15000$, $y = 750 - 300 = 450$ and

$$450 = C \times 15000^2$$

$$C = 2.0 \times 10^{-6}$$

Therefore the tendon profile is $y = 2.0 \times 10^{-6}x^2$ and

$$\text{tendon slope} = \frac{dy}{dx} = 2Cx$$

$$\text{at end } \frac{dy}{dx} = 2 \times 2.0 \times 10^{-6} \times 15000 = 0.060 = \tan\beta$$

$$\text{hence } \beta = 3.43° \text{ and } \sin\beta \approx \tan\beta = 0.06$$

Therefore vertical component, V_t, of prestress force at the supports is

$$V_t = 2590\sin\beta = 2590 \times 0.06 = 155\,\text{kN}$$

and the total shear capacity is

$$V_{Rd1} + V_t = 216 + 155 = 371\,\text{kN at the supports}$$

(c) Shear Reinforcement

From equation 11.44 the required amount of shear reinforcement is given by

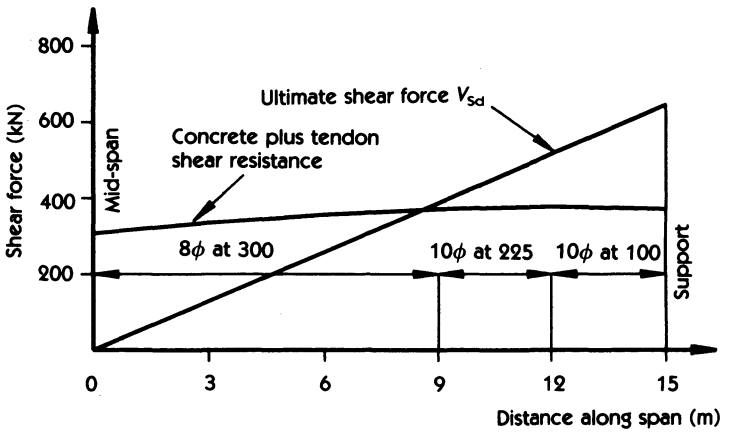

Figure 11.26

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$$\frac{A_{sw}}{s} = 1.28(V_{Sd} - V_{Rd1})/(df_{yk})$$

$$= 1.28(645 - 371) \times 10^3/(950 \times 250)$$

$$= 1.48$$

(Minimum $A_{sw}/s = 0.0022b_w = 0.0022 \times 150 = 0.33$)

Therefore provide 10 mm links at 100 mm centres ($A_{sw}/s = 1.57$).

These calculations can be repeated at other cross-sections and are tabulated in table 11.3 from which it can be seen that from mid-span to a section 9 m from mid-span nominal shear reinforcement is required, and in the outer 6 m of the span fully designed shear reinforcement is required. This can be provided as (figure 11.26) 10 mm links at 100 mm centres in the outer 3 metres ($A_{sw}/s = 1.57$) changing to 10 mm links at 225 mm centres ($A_{sw}/s = 0.698$) between 3 and 6 m from the end of the beam and then to 8 mm at 300 centres ($A_{sw}/s = 0.335$) throughout the rest of the span.

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Appendix

Typical Weights and Live Loads

$$1\,\text{lb} = 0.454\,\text{kg} = 4.448\,\text{N force}$$
$$1\,\text{lb/ft}^2 = 4.88\,\text{kg/m}^2 = 47.9\,\text{N/m}^2$$
$$1\,\text{lb/ft}^3 = 16.02\,\text{kg/m}^3 = 157\,\text{N/m}^3$$

Weights

	kN/m^3
Aluminium, cast	26
Asphalt paving	23
Bricks, common	19
Bricks, pressed	22
Clay, dry	19–22
Clay, wet	21–25
Concrete, reinforced	24
Glass, plate	27
Lead	112
Oak	9.5
Pine, white	5
Sand, dry	16–19
Sand, wet	18–21
Steel	77
Water	9.81

	kN/m^2
Brick wall, 115 mm thick	2.6
Gypsum plaster, 25 mm thick	0.5
Glazing, single	0.3

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Floor and roof loads	kN/m^2
Classrooms	3.0
Dance halls	5.0
Flats and houses	1.5
Garages, passenger cars	2.5
Gymnasiums	5.0
Hospital wards	2.0
Hotel bedrooms	2.0
Offices for general use	2.5
Flat roofs, with access	1.5
Flat roofs, no access	0.75

Bar Areas and Perimeters

Table A.1 Sectional areas of groups of bars (mm^2)

Bar size (mm)	*Number of bars*									
	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
6	28.3	56.6	84.9	113	142	170	198	226	255	283
8	50.3	101	151	201	252	302	352	402	453	503
10	78.5	157	236	314	393	471	550	628	707	785
12	113	226	339	452	566	679	792	905	1 020	1 130
16	201	402	603	804	1010	1210	1410	1610	1 810	2 010
20	314	628	943	1260	1570	1890	2200	2510	2 830	3 140
25	491	982	1470	1960	2450	2950	3440	3 930	4 420	4 910
32	804	1610	2410	3220	4020	4830	5630	6 430	7 240	8 040
40	1260	2510	3770	5030	6280	7540	8800	10 100	11 300	12 600

Table A.2 Perimeters and weights of bars

Bar size (mm)	6	8	10	12	16	20	25	32	40
Perimeter (mm)	18.85	25.1	31.4	37.7	50.2	62.8	78.5	100.5	125.6
Weight (kg/m)	0.222	0.395	0.616	0.888	1.579	2.466	3.854	6.313	9.864

Bar weight based on a density of 7850 kg/m^3.

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Table A.3 Sectional areas per metre width for various bar spacings (mm^2)

Bar size (mm)	*Spacing of bars*								
	50	*75*	*100*	*125*	*150*	*175*	*200*	*250*	*300*
6	566	377	283	226	189	162	142	113	94.3
8	1 010	671	503	402	335	287	252	201	168
10	1 570	1 050	785	628	523	449	393	314	262
12	2 260	1 510	1 130	905	754	646	566	452	377
16	4 020	2680	2 010	1 610	1340	1150	1010	804	670
20	6 280	4190	3 140	2 510	2090	1800	1570	1260	1050
25	9 820	6 550	4 910	3 930	3270	2810	2450	1960	1640
32	16 100	10 700	8 040	6 430	5360	4600	4020	3220	2680
40	25 100	16 800	12 600	10 100	8380	7180	6280	5030	4190

Shear Reinforcement

Table A.4 A_{sw}/s for varying stirrup diameter and spacing

Stirrup diameter (mm)	*Stirrup spacing (mm)*										
	85	*90*	*100*	*125*	*150*	*175*	*200*	*225*	*250*	*275*	*300*
8	1.183	1.118	1.006	0.805	0.671	0.575	0.503	0.447	0.402	0.366	0.335
10	1.847	1.744	1.57	1.256	1.047	0.897	0.785	0.698	0.628	0.571	0.523
12	2.659	2.511	2.26	1.808	1.507	1.291	1.13	1.004	0.904	0.822	0.753
16	4.729	4.467	4.02	3.216	2.68	2.297	2.01	1.787	1.608	1.462	1.34

Note: A_{sw} is based on the cross-sectional area of two legs of the stirrup.

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Anchorage and Lap Requirements

Table A.5 Anchorage and lap lengths (length $L = K_A \times$ bar size) for good bond conditions[1,2,3]

	K_A *for concrete strength,* f_{ck} *(N/mm²)*				
	20	*25*	*30*	*35*	*40*
Straight bars					
Anchorage in tension and compression	44 (50)	37 (46)	34 (42)	30 (39)	27 (37)
Curved bars					
Anchorage in tension and compression	31 (35)	26 (32)	24 (30)	21 (28)	19 (26)
Compression and tension laps[4]	44 (50)	37 (46)	34 (42)	30 (39)	27 (37)
Tension laps[5]	62 (70)	52 (64)	48 (60)	42 (56)	38 (52)
Tension laps[6]	88 (100)	74 (92)	68 (84)	60 (78)	54 (74)

Notes:

1. The figures given in the table refer to deformed type 2 bars (f_{yk} = 460 N/mm²). The figure in brackets refer to plain bars (f_{yk} = 250 N/mm²).
2. For poor bond conditions divide the figures by 0.7.
3. For type 2 bars greater than 32 mm divide the figures by [(132 − Φ)/100] where Φ is the bar size.
4. These figures apply when there are less than 30 per cent of the bars lapped at the section *and* the clear spacing between bars is greater than 6Φ *and* the side cover to the outer bars is greater than 2Φ.
5. These figures apply when there are more than 30 per cent of the bars lapped at the section *or* the clear spacing between bars is less than 6Φ *or* the side cover to the outer bars is less than 2Φ.
6. These figures apply when there are more than 30 per cent of the bars lapped at the section *and* either the clear spacing between bars is greater than 6Φ *or* the side cover to the outer bars is greater than 2Φ.

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Maximum and Minimum Areas of Reinforcement

Table A.6 Maximum areas of reinforcement

For a slab or beam, tension or compression reinforcement $100A_s/A_c \leq 4$ per cent, other than at laps
For a column $100A_s/A_c \leq 8$ per cent, including at laps
For a wall, vertical reinforcement $100A_s/A_c \leq 4$ per cent

Table A.7 Minimum areas of reinforcement

Tension reinforcement in beams:	$A_{s,min} > 0.6b_t d/f_{yk} > 0.0015b_t d$ (for grade 250 steel $A_{s,min}/b_t d > 0.0024$ for grade 460 steel $A_{s,min}/b_t d > 0.0015$)
Tension reinforcement in slabs:	$A_{s,min} > 0.6b_t d/f_{yk} > 0.0015b_t d$ (for grade 250 steel $A_{s,min}/b_t d > 0.0024$ for grade 460 steel $A_{s,min}/b_t d > 0.0015$)
Secondary reinforcement	>20 per cent of main reinforcement
Longitudinal reinforcement in columns:	$A_{s,min} > 0.15N_{Sd}/0.87f_{yk} > 0.003A_c$ where N_{Sd} is the axial compression force
Vertical reinforcement in walls:	$A_{s,min} > 0.004A_c$

Note: b_t is the mean width of the tension zone.

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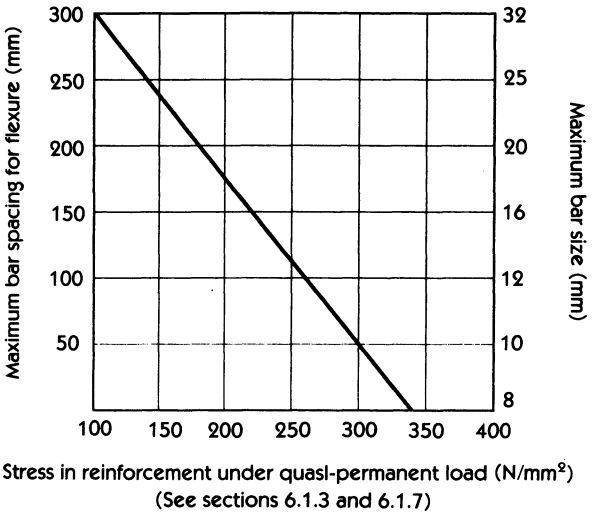

Figure A.1 *Maximum bar sizes and spacing for crack control*

Summary of Basic Design Equations for the Design of Reinforced Concrete

(a) Design for Bending

(see chapters 4 and 7)

For a singly reinforced section:

$$A_s = \frac{M}{0.87 f_{yk} z}$$

$$z = d\{0.5 + (0.25 - K/1.134)^{1/2}\}$$

$$K = M/bd^2 f_{ck}$$

For a doubly reinforced section ($K > K_{bal}$) – see figure A.3:

$$A'_s = \frac{(K - K_{bal}) f_{ck} b d^2}{0.87 f_{yk}(d - d')}$$

$$A_s = \frac{K_{bal} f_{ck} b d^2}{0.87 f_{yk} z_{bal}} + A'_s$$

for concrete grades C12/15 to C35/45

$$z_{bal} = 0.82d; \quad K_{bal} = 0.167$$

for concrete grades greater than C35/45

$$z_{bal} = 0.86d; \quad K_{bal} = 0.136$$

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1 = Simply supported beam or one-way or two-way simply supported slab
2 = End span of continuous beam or one-way continuous slab or two-way spanning continuous over one long edge
3 = Interior span of beam or one-way or two-way spanning slab
4 = Slab supported on columns without beams (flat slab) based on longer span
5 = Cantilever

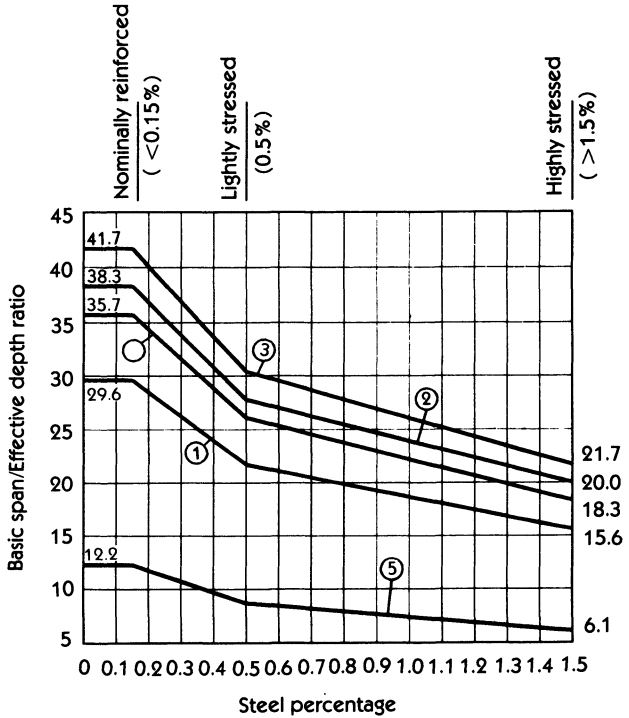

Figure A.2 *Basic span/effective depth ratios* ($f_{yk} = 460\,N/mm^2$)

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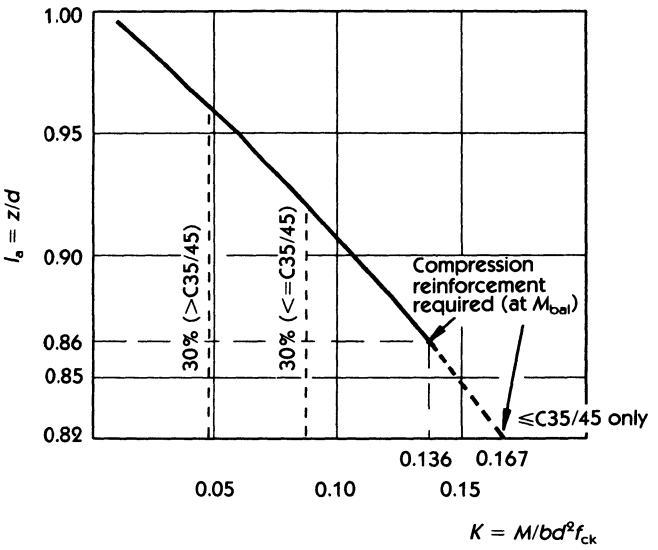

Figure A.3 *Lever-arm curve. The 30% values on the K-axis mark the limits for singly reinforced sections with moment redistribution applied (see section 4.7)*

When moment redistribution has been applied then the above equations must be modified – see table 4.2.

(b) Design for Shear (Standard Method)
(see chapters 5 and 7)

$$V_{Rd1} = [\tau_{Rd}k(1.2 + 40\rho_1)]b_w d$$

$$\rho_1 = A_{sl}/b_w d < 0.02$$

k = $(1.6 - d)$ $\{>1\}$ or 1 where more than 50 per cent of tension reinforcement is curtailed

$$V_{Sd} < V_{Rd2} = 0.3vf_{ck}b_w d$$

$$\frac{A_{sw}}{s} = \frac{1.28(V_{Sd} - V_{Rd1})}{df_{yk}}$$

Table A.8 Values of τ_{Rd} (N/mm^2) for different concrete grades, f_{ck}

f_{ck}	12	16	20	25	30	35	40	45	50
τ_{Rd}	0.18	0.22	0.26	0.30	0.34	0.37	0.41	0.44	0.48

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(c) Design for Torsion
(see chapters 5 and 7)

$$T_{Sd} < T_{Rd1} = 1.33vf_{ck}tA_k/(\cot\theta + \tan\theta)$$

$$A_{sw}/s = T_{Sd}/(2A_k \times 0.87f_{yk} \times \cot\theta)$$

$$A_{s1} = (T_{Sd}u_k/2A_k)\cot\theta/(0.87f_{y1k})$$

A_{sw} is the cross-sectional area of a single leg of a link.

(d) Design for Punching Shear in Slabs
(see chapter 8)

$$V_{Sd} < 0.9u_1d(f_{ck})^{1/2}$$

$$V_{Rd1} = [\tau_{Rd}k(1.2 + 40\rho_1)]b_wu$$

When $V_{Rd1} < V_{sd} \leqslant 1.6V_{Rd1}$:

$$\Sigma A_{sw}\sin\alpha > (V_{Sd} - V_{Rd1})/0.87f_{yk}$$

When $1.6V_{Rd1} < V_{sd} \leqslant 2.0V_{Rd1}$:

$$\Sigma A_{sw}\sin\alpha < (V_{Sd} - 1.4V_{Rd1})/0.29f_{yk}$$

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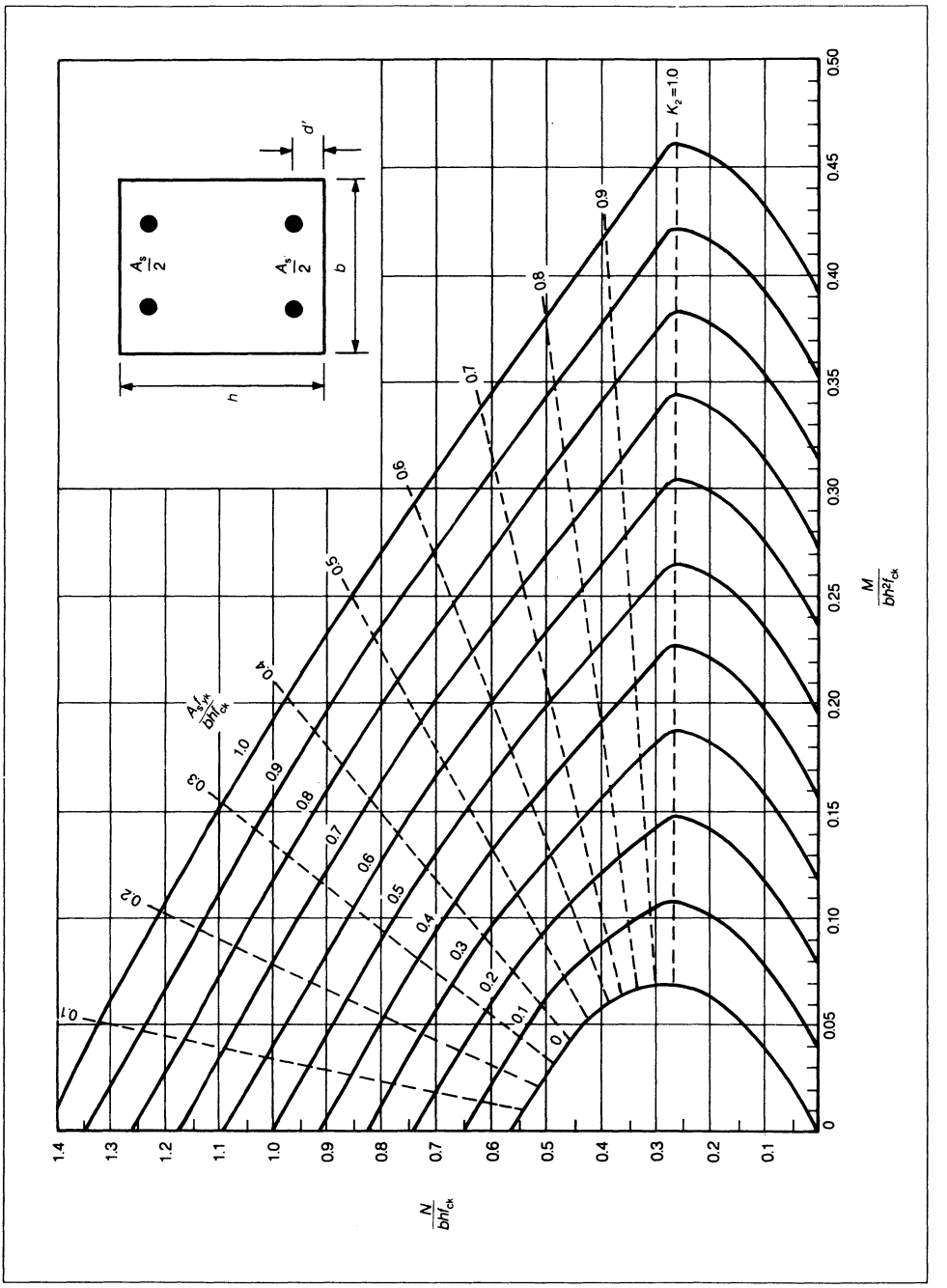

Figure A.4 *Rectangular columns (d'/h = 0.05) [Reproduced with permission of British Cement Association]*

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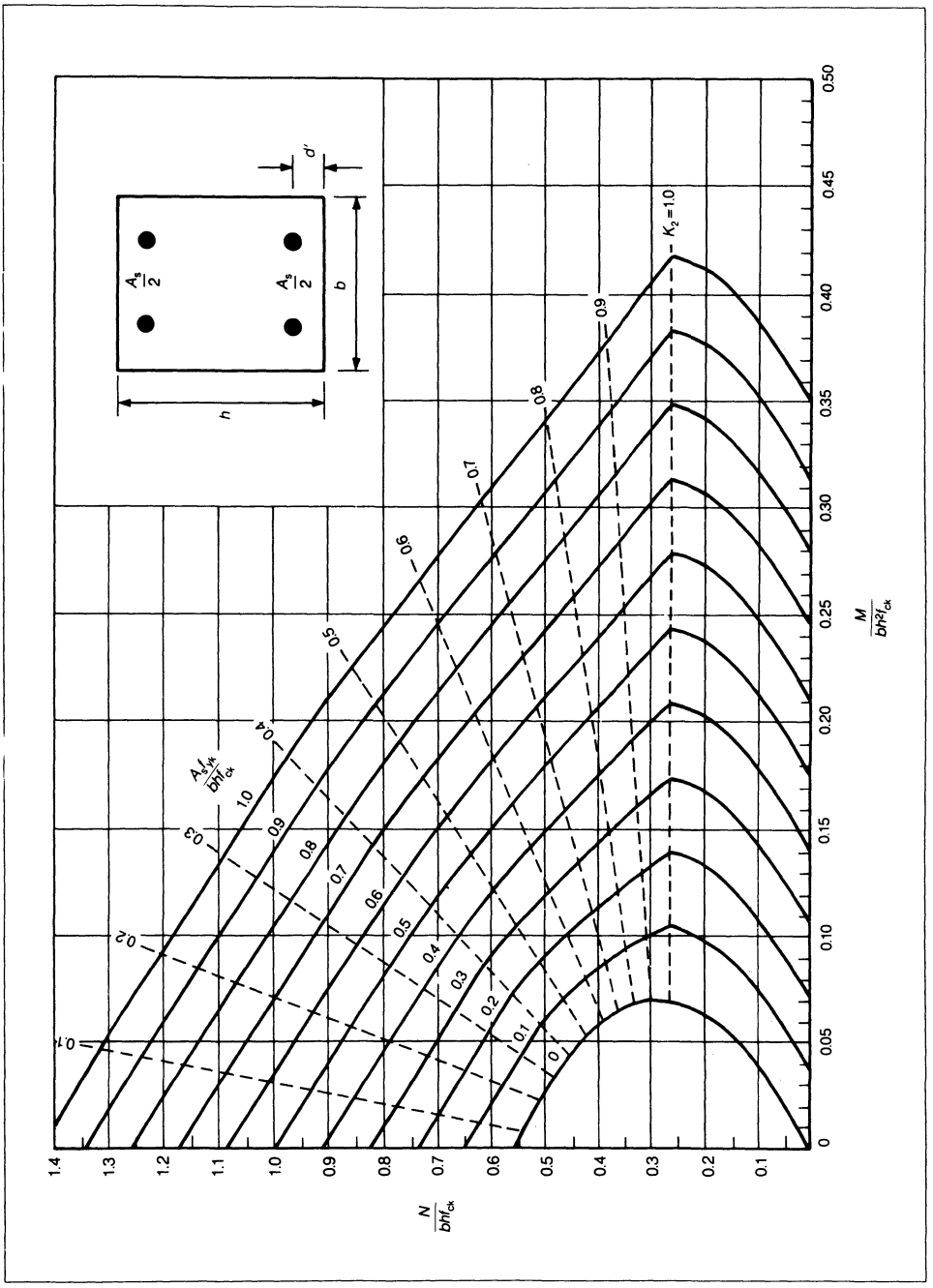

Figure A.5 *Rectangular columns (d'/h = 0.10) [Reproduced with permission of British Cement Association]*

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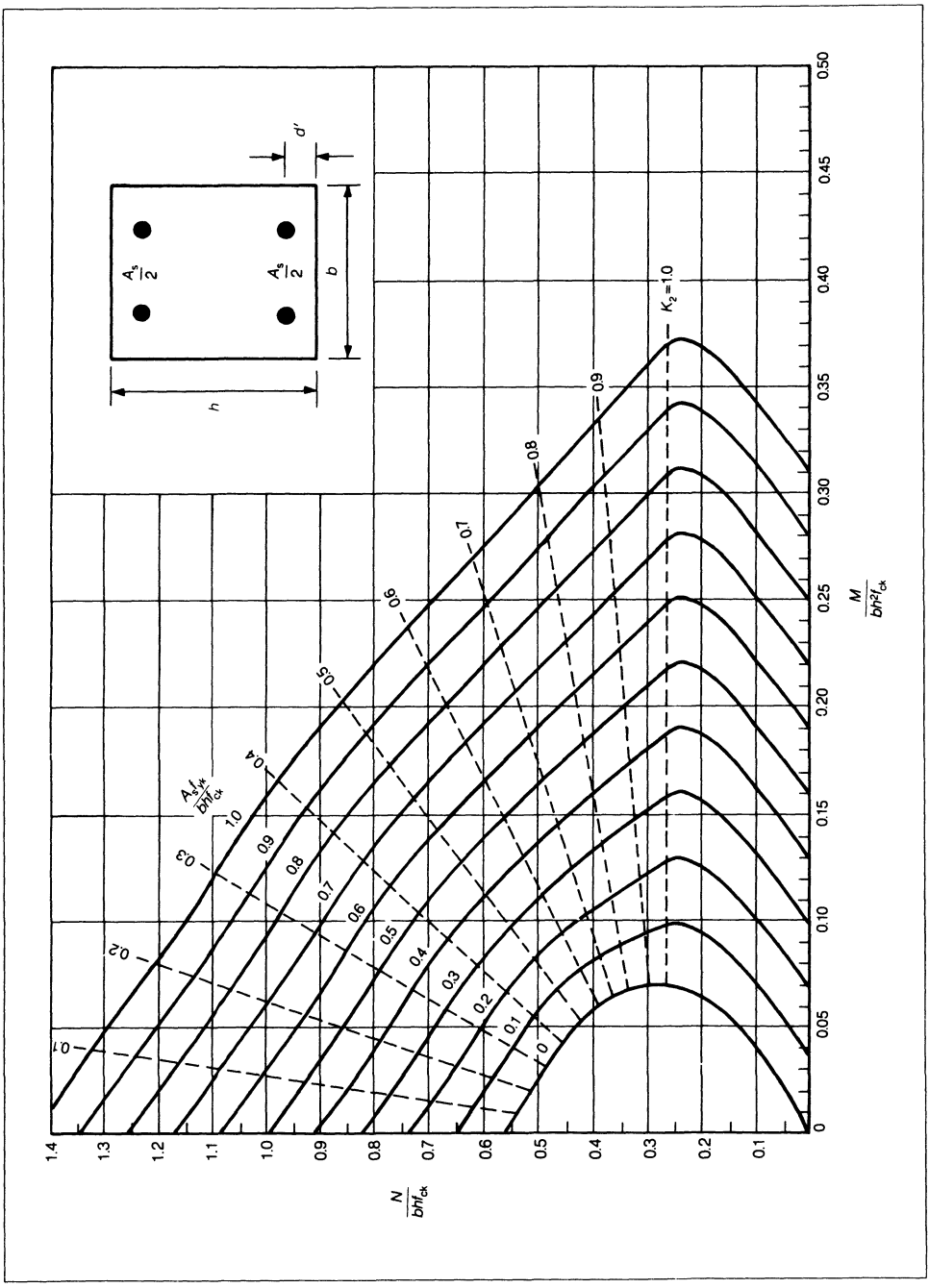

Figure A.6 *Rectangular columns (d'/h = 0.15) [Reproduced with permission of British Cement Association]*

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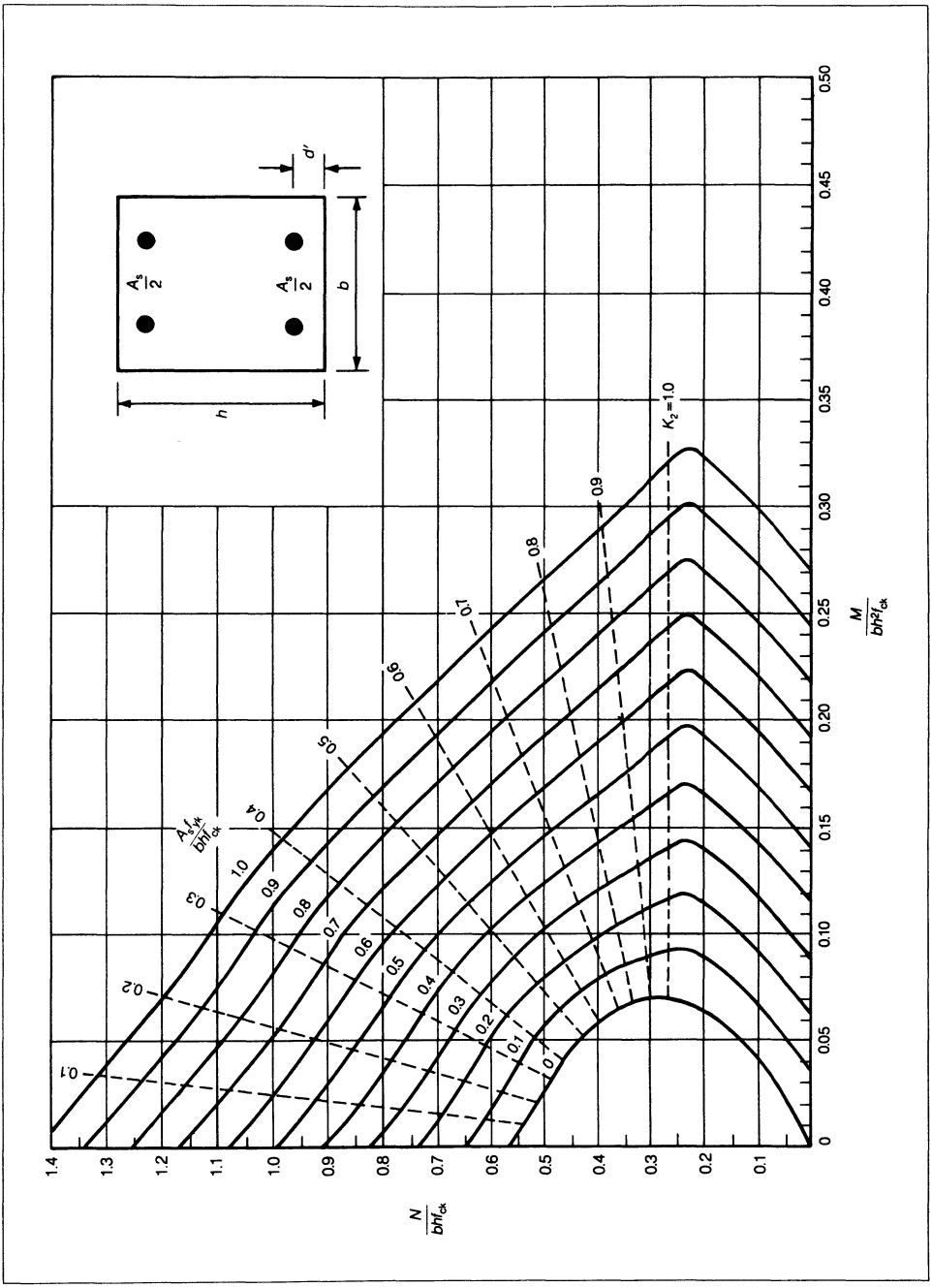

Figure A.7 *Rectangular columns (d'/h = 0.20) [Reproduced with permission of British Cement Association]*

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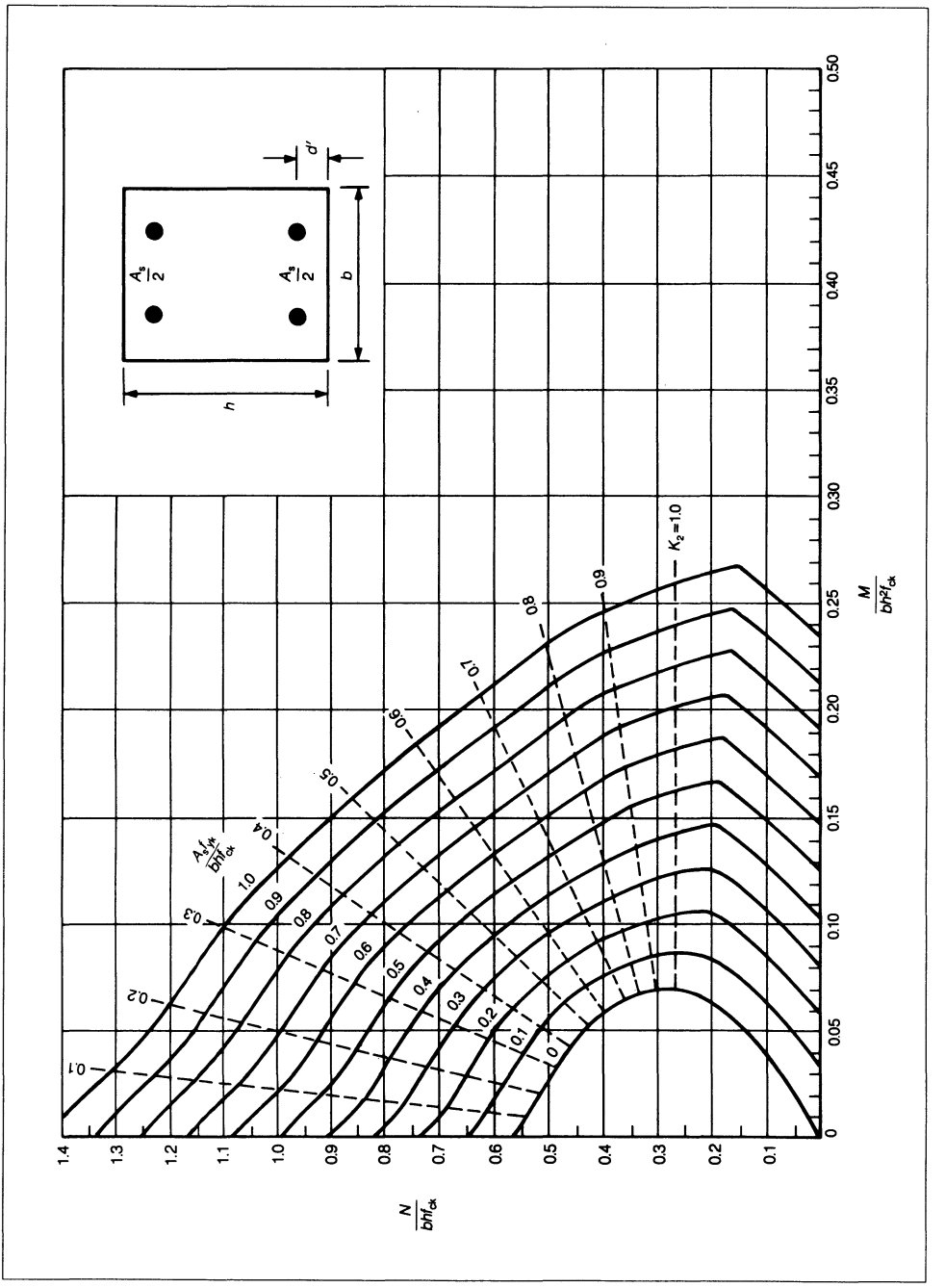

Figure A.8 *Rectangular columns (d'/h = 0.25) [Reproduced with permission of British Cement Association]*

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Further Reading

(a) British Standards and EuroCodes

BS1881	*Methods of testing concrete*
BS4449	*Specification for carbon steel bars for the reinforcement of concrete*
BS4466	*Specification for bending dimensions and scheduling of reinforcement for concrete*
BS4482	*Cold reduced steel wire for the reinforcement of concrete*
BS4483	*Steel fabric for the reinforcement of concrete*
BS5057	*Concrete admixtures*
BS5896	*Specification for high tensile steel wire and strand for the prestressing of concrete*
BS6399	*Design loading for buildings*
BS8110	*Structural use of concrete, Parts 1, 2 and 3*
CP3	*Code of basic data for the design of buildings* *Chapter V Loading* *Part 2 Wind Loads*
CP8004	*Foundations*
DD ENV 1992-1-1	*EuroCode 2; Design of concrete structures. Part 1*
DD ENV206	*Concrete – performance, production, placing and compliance criteria*
Draft prEN 10080	*Steel for the reinforcement of concrete*

(b) Textbook and Other Publications

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