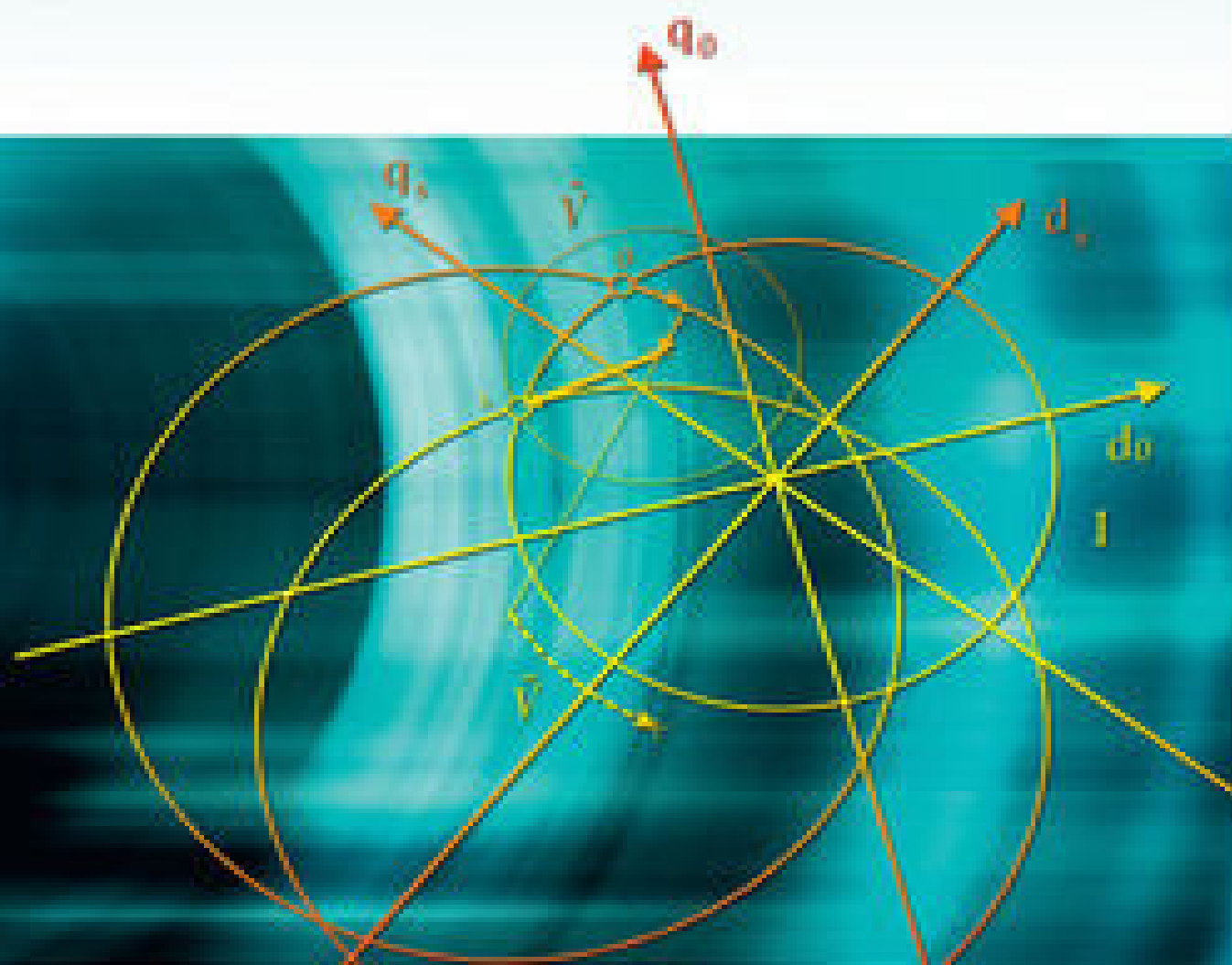


Direct Eigen Control for Induction Machines and Synchronous Motors

Jean Claude Alacoque



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Jean Claude Alacoque
Alstom Transport, France



A John Wiley & Sons, Ltd., Publication

This edition first published 2013
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John Wiley & Sons, Ltd, The Atrium, Southern Gate, Chichester, West Sussex, PO19 8SQ,
United Kingdom

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Library of Congress Cataloging-in-Publication Data

Alacoque, Jean Claude.

Direct eigen control for induction machines and synchronous motors / Jean Claude Alacoque.
pages cm

Includes bibliographical references and index.

ISBN 978-1-119-94270-2 (cloth)

1. Electric motors—Automatic control. 2. Electric machinery, Induction—Automatic control. 3. Control theory. 4. Eigenfunctions. I. Title.

TK2211.A338 2013

621.46—dc23

2012023515

A catalogue record for this book is available from the British Library.

Print ISBN: 9781119942702

Typeset in 10/12pt Times by SPi Publisher Services, Pondicherry, India

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Foreword

There is now a significant number of publications relating to the control of electric motors, particularly AC motors: international scientific papers mainly from the academic world, often collective works, which now constitute a valuable source of reference in terms of adjustable speed drives.

So why an additional book on this well-known topic?

This is not just a book about the subject. It represents the culmination of in-depth thinking from a unique author, an industry expert in the field who is passionate and curious, having spent his entire career in research and development, mainly in the field of railway traction, and who has set a technological challenge of the highest order.

This can be summarized as dealing with the robust discrete-time control of an electrical system used as a static power converter, ensuring all objectives are accurate and dynamic, while respecting a set of technological and industrial constraints.

The author has taken great care to target his method in this very extensive landscape of sometimes very complex control structures for electric motors, justifying precisely the boundaries of his study, particularly in terms of robustness. Based on the latest developments of the ‘direct torque control’ algorithm and the ‘field oriented vector control’ algorithm, this book introduces an original approach to the discrete-time control of electrical systems, through three issues very representative of the constraints encountered with today’s industrial adjustable-speed drives.

This book can be viewed as the indirect result of the decade-long collective works of the author and various research teams from industry and academia.

It is undoubtedly a first reference book, self-contained, dealing with advanced discrete-time control of electric motors. From my point of view, this book is pedagogical, focused on solving several types of industrial problems, highlighting the huge experience of the author in the control of electric motors.

I would like to pay tribute to this unique author who, showing a scientific maturity, tackled the job of writing a book that is both attractive yet deals with a subject that is difficult for communities of experts in both electrical systems and control systems, in both industry and academia.

He has paid great attention, throughout this work, to very carefully and relevantly interpret the different stages in the mathematical development of the subject, making the book approachable by both students and industry experts wishing to evaluate the proposed control laws. I am sure also that professors and lecturers will be able to tap into the proposed approach, to improve it and expand it to other possible areas.

Throughout this book, the author has continued to bear in mind the aim of presenting a unified method, to draw attention to the efficiency and simplicity of this approach and finally to share a certain ‘elegance’ in determining solutions, including through a very original geometric technique.

Also welcome is the author’s willingness to present in detail the complete range of intellectual approaches of R&D, often unpublished, which relates to the drive modeling system, and was completed through the issue of real-time implementation, under many constraints, of an advanced control algorithm in an industrial computer. That this has been done in this book is remarkable.

Finally, this book is meant to be an ‘eigenvector’ of thinking, to apply the same tools to electrical systems other than electric motors, such as flexible alternating current transmission systems (FACTS), in close conjunction with smart grid development, including renewable energy sources.

Prof. Dr Ing. Jean-Luc Thomas

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Foreword

In past decades there have been numerous proposals for efficient linear or non-linear, control approaches in the continuous-time or discrete-time domains, taking into account not only the system itself with its limitations but also the associated actuators and sensors. The work presented in this book is aimed towards the graduate level as well as for young engineers and researchers. The book is self-contained for AC motor modeling and control, and where the prerequisites are:

- an introduction to mathematical analysis at undergraduate level
- an introduction to AC motors and associated power converters at graduate level
- an introduction to the theory of linear systems in continuous-time and discrete-time domains.

The material that is presented in this book is the outcome of several years of industrial research and development based on state-of-the-art control techniques provided by the research community in the field of alternating current motor control on the one hand and the more general field of automatic control on the other.

For applications such as railways requiring high dynamic control response, it is necessary to use discrete-time control methods in order to master the convergence time such as dead-beat (one-sampling-time response) control in the best cases. Moreover, the sampling time could be considered as an additional degree of freedom when the system is driven to its limits. It is necessary to remind the reader that for AC machines, the control inputs are voltage magnitude and position (phase) and could also be the duration of the application of such voltage when one of these first two (magnitude or phase) control inputs is not available.

The author opens a new perspective on control systems by considering this new degree of freedom – the sampling period – which is calculated in real time from one sampling period to the next. The originality of this approach is to let the system itself, according to its limits, decide on the next sampling period suitable for addressing the control objectives.

Moreover, the nature of today's systems (large-scale, interconnected, nonlinear, time-varying), such as smart grids for example, which are integrating predictions and distributed sensors and actuators, requires additional techniques for modeling, stability, protection and control, taking into account the complexity of the whole system (the 'system of systems').

In order to perform such a system of systems control approach, we obviously need to deal with the different timescales either implicit in these systems or imposed by the control hardware infrastructure. This is well-known and applied in the AC power networks primary (seconds timescale) and secondary (minutes timescale) controls, but the concept of having the sampling period as an additional degree of freedom opens a new perspective on the 'control of the future'.

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Acknowledgements

I would like to make a point of sincerely thanking all the individuals and organizations who made this work possible.

I owe a great deal to Raymond Bardot and especially to Prof. Dr Ing. Jean-Luc Thomas, for the endless time and attention given to the second reading of this work, and for corrections that they suggested, as well as to Dr Widad Bouamama for her masterly diagonalization of the input matrix in discrete-time state-space equations.

Particular thanks go to Philippe Bernard, a virtuoso of real-time implementation of microseconds-consuming algorithms. Without his unwavering confidence in the possibility of gaining still more nanoseconds during software execution, the hope and the energy deployed which led us to final algorithms that are usable in real-time, would have been in vain.

This book could never have become reality without the help and support of many friends and colleagues. I want to thank Alstom Transport for their backup during each year of doubt and research towards practical and powerful solutions for controlling traction motors, during a period when the organization of the company was in constant evolution. I am also very grateful to laboratories and teams of Alcatel Alsthom Research, LEEI of ENSEEIHT-INPT in Toulouse, CRAN of ENSEM-INPL in Nancy, and LEG of ENSIEG-INPG in Grenoble for their participation.

I want to thank professors Bernard de Fornel, Claude Iung and Daniel Roye, for the leads and progress in motor control research which they were able to guide and bring about with their competence.

Without the confidence of François Lacôte, head of the corporate technical department in Alstom Transport, the perspicacity, sagacity and permanent questioning of Dr Ing. Benoit Jacquot discoverer of the *discrete predictive reference frame*, the outstanding competence in control stability of Dr Ing. Bertrand Délémontey and the rigorous preliminary work of Prof. Dr Ing. Jean-Luc Thomas on the sampled rotating reference frame, this book would never have been born. To them I express my sincere gratitude.

Special thanks go to Prof. Dr. Ing. Jean-Luc Thomas, Dr. Ing. Abdelkrim Benchaïb and Dr. Ing. Serge Poullain*, to whom this book owes a great deal, in particular for the preliminary work and the fruitfulness of the often impassioned discussions which it generated.

Thanks to Janet Morley for her kind English feedback in spite of her own business commitments.

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Introduction

The applications of control theory for controlling electromechanical actuators (Grellet and Clerc, 1999) have always tried to simultaneously follow, in spite of disturbances, one or several physical variable set-points, and to do so with accuracy, without overshoot or lagging, and with the maximal velocity, compatible with the controlled processes, physical limits resulting from sizing, and the energy cost of the control.

This work proposes a method to develop control laws, to drive electrical actuators, which fulfills these aims as well as possible. The application of this method to electric motors makes it possible to consider its generalization.

1 Formulation of the Motor Control Problem

When one starts designing the process kinematics and the motor control, several important characteristics must be analyzed:

- the required electromagnetic torque rating
- the response time in set-point tracking mode, as well as the response time to any foreseeable disturbances
- physical variable limits.

1.1 *Electromagnetic Torque*

The robot or the table of a machine tool are controlled by position, the automatic subway is controlled by torque and speed, the locomotive is controlled by torque and speed, the rolling mill is controlled by speed or torque according to its position in the roll train.

The kinematic law of mechanics leads to controlling a motor by the torque C_c to overcome a load moment C_r and to accelerate or to slow down an inertia J , thus making it possible to vary its mechanical angular frequency Ω , to reach quickly, and then to maintain, a new speed or a new

position. The position or the speed references are transformed by the control into an acceleration reference, which makes it possible to fix the motor torque set-point by the equation (1).

$$C_c = J \cdot \frac{d\Omega}{dt} - C_r \quad (1)$$

The mechanical inertia J is the rotor inertia, to which it is necessary to add the inertia of the transmission driven by the rotor. The load moment can be made up of dry frictions, viscous or aerodynamic speed dependent ones, and moments directly related to the application and brought back to rotor by the transmission.

Whatever the application, an electric motor is thus controlled initially with its torque. According to the motor type and the regulation mode, to obtain this torque, it is necessary to control the current, the magnetic field and/or the frequency.

1.2 Response Time in Tracking Mode and on Disturbances

Whatever the choice of the actuator type, most industrial applications require short response times and thus high control dynamics compared to a controlled process.

According to the application, the response time during set-point tracking or process disturbances, can be dominating. The control by position of the machine tool table and the robot, or the velocity control of an automatic subway, requires performances of reference tracking, whereas the control of the rolling mill and the locomotive requires an especially fast response to disturbances. However, these two characteristics remain dependent, and one usually requires high dynamics during disturbances of load moments of the table during machining, of the robot at the time of heavy object catching, or of the automatic subway during slope variation or adhesion loss. In the same way, fast set-point tracking is necessary for the rolling mill, locomotive or electric car.

It is noticeable in these examples that the requirement for fast control reaction is not absolute, but on the contrary, has to be related to closed-loop processes. The difference is large between the positioning of a machine tool table which requires a velocity increasing from zero to the maximal speed in a few tens of milliseconds, and a locomotive which, in the best case might take several minutes, or even several tens of minutes.

For the same process, the response time can depend on the process state itself. The arm of a multi-axis robot where the inertia depends on its grip position, must optimize its trajectory according to the target distance, but also to its own variable inertia according to its grip position.

During position or velocity set-point tracking, the influence of the largest time constant is dominating; in general, it is in direct relationship to the inertia of the controlled process itself. The analysis of dynamic requirements of an application should not be limited to this aspect, although it is one of the main sizing criteria of actuators; the kinematic law indeed makes it possible to define the required torque, to obtain the acceleration of the process inertia, with load moments.

However, there are many technological limits that are related to the smallest time constants of the controlled process. These time constants impose a very short response time on the regulation, for an adapted process control.

Thus, the voltage inverter of a locomotive is fed by the DC voltage supply via a second-order passive filter with a series inductance and a parallel capacitor; in general, this filter has

a resonance frequency of few tens of hertz with a high Q-factor, and thus a very low damping to minimize the ohmic losses.

The energy stored in this filter is low compared to the power feeding the locomotive. During repeated pantograph jumps, the power supply is interrupted and the inverter voltage supply can totally disappear in few milliseconds at the rated power.

In the same way, the mechanical drive between the rotor of the electric motor and the wheels presents several natural frequencies, due to the transmission or to axle elasticity, between a few hertz and a few tens of hertz.

Other natural frequencies which appear in the transmission are due to the coupling of the natural frequencies between the transmission and the primary suspensions of the bogie on its axles (a few hertz), or between the transmission and the secondary suspensions of the coach on its bogies (less than 1 hertz).

Sharp variations of the load moment are due to slipping and sliding of wheels on the rails at times of adhesion loss. The coupling between the electric motor torque and the train inertia disappears instantaneously. The motor load moment is reduced to only the transmission inertia of axles and wheels, several orders of magnitude smaller than the train's nominal inertia.

The dynamics of the motor control are thus conditioned, not only by the nominal time of velocity increasing in set-point tracking mode, but also by load moment disturbances which excite electrical and mechanical natural frequencies. A short control response time, or a large bandwidth, is necessary to avoid exciting the fastest phenomena by an exaggerated phase rotation, but also it is essential to damp them, and this requires that all natural frequencies are located within the control bandwidth.

In the case of a locomotive, the response time of the torque control would have to be lower than ten milliseconds to be able to control the fastest phenomena. It is a requirement, but it is not sufficient. The control structure would have to then allow an effective control of all phenomena by measured variables, signal processing, control variable choices, decoupling, limitations,... suitable to ensure an accurate locomotive control during disturbances, right up to the extreme limits of allowed operations.

1.3 Limitations

Any electromechanical device has its own technological limits which become constraints for its control. An electric motor is designed for its maximal torque rating C_M ; according to the application, this maximal torque can depend on the velocity. A motor has its maximal current I_M limited by the sizing of the winding copper section, and its maximal flux Φ_M , limited by the sizing of the steel sheet section.

The inverter used for controlling an electric motor also has its own limits, such as current, voltage and frequency limits, but also the semiconductor temperature limit.

Thus for an induction motor, for instance, the maximal torque according to the number of pole pairs N_p , the magnetizing inductance L_m , the rotor inductance L_r , the stator current limit I_M and the rotor flux Φ_M limit, is given by the cross product of equation (2).

$$\vec{C}_M = N_p \cdot \frac{L_m}{L_r} \cdot \vec{\Phi}_M \times \vec{I}_M \quad (2)$$

Equation (3), informs then us about the maximal allowable angular acceleration.

$$\frac{d\Omega}{dt} = \frac{C_M - C_r}{J} \quad (3)$$

It is an acceleration limit which cannot be exceeded in a torque limitation mode. It is a limit which will, however, be frequently reached, under rated operations, owing to the fact that sizing limits correspond in general to rated operational limits. Starting from a few kilowatts, any oversizing has important repercussions on the volume, the weight and the process cost. These parameters are important, whatever the industrial application type: the volume for tables of machine tools and robots, the weight and volume for distributed electric traction or for the control of plane control surfaces – these affect the cost in all cases.

All limitations must thus be integrated as *constraints* in the development of motor control laws. They should not trigger the operation of equipment safety devices, such inverter blocking or circuit-breaker switching, which destabilize the regulation. This control, under multiple constraints, cannot thus be based on traditional continuous actions of an RST structure type for example (de Larminat, 1996), because of their very strong nonlinearities in the limit vicinity. We will thus prefer a sampled control, very fast compared to process time constants, which makes it possible to instantaneously modify the references when one or more limits could be reached before the next control horizon.

Rather than notice, a posteriori, an overshoot of one or several limits, it is essential with a fast regulation, to *predict* the process behavior to avoid any overshooting. We will thus have to predict the process evolution, using the most accurate motor model, to know a priori the action to be undertaken and to thus avoid exceeding any limits.

For various types of electric motor, there exist several control methods which have been described abundantly by the scientific literature (Leonard, 1996; Canudas de Wit *et al.*, 2000). They have their own limits. It is outside our scope here to describe them in detail and to compare them in order to emphasize their advantages and drawbacks, but we can try to characterize briefly, two important control families – field orientation (field-oriented control, FOC: Vas, 1998) and sliding modes (Bühler, 1986; Utkin, 1992) – at least in their native versions.

2 Field Orientation Controls

Field orientation control (Blaschke, 1972; Chiasson, 2005; Louis, 2010) uses a rotating reference frame with the rotor flux directed according to the d axis, the Park reference frame (Park, 1929), to position the set-point of the stator current vector and to thus regulate the motor flux and torque.

Projection of this set-point current vector on the d axis provides the set-point value of the motor's magnetizing or demagnetizing current, and the projection on the q axis gives the set-point value of the active current, which, combined with the motor flux according to the Lorentz law, produces the required electromagnetic torque.

This control method is very commonly used, but it has two main drawbacks in its basic configuration:

- It controls only the *fundamental* component of the electrical variables, and thus only their steady state or slowly varying modes.

- The same rotating reference frame is used to define measurements and set-points and thus to calculate the control. Actually, a real-time implementation of this kind of control does not take into account the reference frame rotation related to the rotor flux during the required time for the calculation and input vector application.

These two drawbacks require, in practice, an *independent* control voltage, according to d and q axes, to try to minimize errors resulting from the absence of taking account of the reference frame rotation during the calculation and control application. This decoupling is never total, in particular at the time of transient modes caused by set-point modifications or during disturbances.

A control equation discretization, with an expansion limited to the first order (Jacquot, 1995) and a simplified prediction of the position of a new frame to fix set-points and voltage vectors (Jacquot *et al.*, 1995), were then necessary to improve the dynamic behavior (*discrete predictive frame*, DPF). These works took partially into account the reference frame rotation during the computational time.

Several successive developments then made it possible to specify the prediction reference frame position for induction machines (Thomas and Poullain, 2000; Poullain *et al.*, 2003), or for surface-mounted permanent magnet synchronous motors (SMPM-SM) (Benchaïb *et al.*, 2003).

3 Sliding Mode Control Families

A second motor control method is *sliding mode control* (SMC) (Louis, 2010), for instance:

- *direct self-control* (DSC – Depenbrock, 1988; Baader and Depenbrock, 1992)
- *direct torque control* (DTC – Takahashi and Noguchi, 1986; Steimel, 1998).

These control algorithms are discretized and provide high dynamics. Their sampling period is typically about $25\mu\text{s}$. The response time is equal to the calculation time of voltage vector application times, making it possible to maintain the stator flux and the electromagnetic torque of the machine between two predefined limits; it is necessary to add to this time the application durations of the voltage vectors themselves.

With this kind of control algorithm, as soon as one of stator flux or torque limit is crossed, the control calculates, or selects in one look-up table, voltage vectors to force the flux or the torque to return inside their set-point surface. It is thus almost an *a posteriori* control type.

The estimation of the stator flux is based on the stator voltage module and phase, with an approximation which neglects the stator resistance; this highly complicates the motor control at low speed, where the stator voltage is low and where the contribution of ohmic voltage drops can be important and moreover the contribution is directly a function of the stator current.

This control method is able to manage flux and torque limitations, with the accuracy corresponding to the difference between the two regulation limits, and with a time delay corresponding at least to one measure sampling, one control calculation and one voltage vector application period. For motors with a low stator time constant, the time interval necessary for voltage and current measurements, computation and voltage vector applications is often too great to comply accurately with predefined limits. The limits are exceeded with an amplitude

depending directly on motor electrical time constants and on the operation mode; this requires the introduction of a prediction model (Pacas and Weber, 2005) for the motor behavior.

The application time of each voltage vector being one of control variables, the voltage inverter switching frequency is controlled only very indirectly by the difference between two control limits, in other words by the surface boundaries. Both the regulation accuracy and the inverter switching frequency are thus bound by this control process. Prediction models were then used to reduce switching frequencies of DTC (Kley *et al.*, 2008), as well as to reduce ripples of the electromagnetic torque (Escobar *et al.*, 2003).

A high accuracy regulation requires, with this control process in its basic configuration, a short calculation time and thus a high real-time computing power on the one hand, and a high inverter switching frequency inducing high inverter switching losses, on the other hand.

The current distortion ratio due to voltage harmonics can be limited only very indirectly by the amplitude reduction of regulation boundaries, or in other words by a sliding surface reduction. In addition, the smaller the regulation interval is, the more the harmonic spectrum shifts towards high frequencies, the smaller is the emergence of harmonic components from spectrum noise and the higher is the switching frequency. Harmonic spectral distribution is thus also a direct consequence of the choice of the regulation accuracy, and it could be an important problem to solve for railway signaling.

Lastly, in its basic configuration, this control method uses, to decouple the stator flux control from the torque regulation, the six non-null inverter voltage vectors for stator flux (DTC and DSC) and torque (DTC) controls; the two null voltage vectors are used to control the torque (DTC and DSC). Imposed sequences of voltage vectors, calculated by the control with DSC and tabulated with DTC, exclude the use of other sequences of voltage vectors; the control thus loses one degree of freedom that would enable it to define, independently of the torque control, inverter voltage shapes and thus amplitudes of low frequency harmonics, which directly govern torque ripples (Holtz, 1992) (cf. Appendix A).

To improve the performance of this kind of control, a nonlinear prediction model applied to DTC, which became the MPDTC (*model predictive direct torque control*) (Geyer *et al.*, 2009; Papafotiou *et al.*, 2009), allows stator flux, torque and neutral voltage controls, with a significant reduction of overshooting across sliding surface boundaries, as well as with a reduction of more than 50% of the inverter frequency in most operation modes. A prediction model is also used to find the least expensive voltage vector sequences in terms of inverter frequency, among all possible solutions, of which the number depends upon the prediction horizon. During motor operation limitations, the constraint of an inverter frequency reduction is abandoned to make it possible to find an acceptable voltage vector sequence, particularly for a short prediction horizon.

In short, what characterizes this kind of control is the imposition, a priori, of two limits, high and low ones (*hysteresis bounds*), of each controlled variable module, instead of one single accurate set-point per variable; this amounts to simultaneously fixing an inaccurate set-point and the torque ripple amplitude, resulting furthermore from the inverter vector sequence calculated by the control. As this solution is looped, it is not surprising that the fixing of regulation band amplitudes (surface boundaries) narrowly conditions the regulation accuracy, the inverter switching frequency and thus the inverter switching losses, as well as the stator harmonic current spectrum and thus motor iron and copper losses. Solutions, expensive in terms of computational time reduction, must thus be found to improve the necessary compromise between the regulation accuracy and the inverter switching frequency. We have also to

notice that the voltage vector sequence calculated by the regulation does not necessarily comply with constraints on minimal turn-on and turn-off times of inverter semiconductors, which requires altering the sequence before applying it.

4 Objectives of a New Motor Control

An analysis of controls based on the field orientation (FOC), leads us to consider a *predictive* control with a *dead-beat* response, to anticipate the *reference frame rotation* and to avoid *decoupling* the input voltage vector coordinates according to d and q axes.

An analysis of the discontinuous control based on sliding modes (SMC, DTC, DSC), leads us to foresee the need to decouple the motor control from the voltage vector sequence, in order to keep one degree of freedom for the harmonic current optimization. Indeed, this voltage sequence is responsible for the inverter frequency and thus for the inverter losses, but also for the motor current distortion ratio and thus for the torque ripple, iron and copper losses and for the neutral voltage evolution.

The optimal sequences to be applied by the control during one period can be calculated in non-real-time according to mean values of voltage vectors and thus tabulated in various look-up tables, each one optimized according to various motor speed ranges; they can be optimized over the whole speed range by choosing one or more asynchronous or synchronous PWM types. The necessary trade-off for the motor and inverter between conduction and switching losses is thus made at the time of process sizing by the selection of PWM optimized sequences for one application; the only control will be carried out in real-time. A description of calculation methods of optimal sequences, would require a complete work in itself, so it will not be approached within this framework (cf. Appendix A).

To achieve the goals of tracking performance and of response times to disturbances, without overshooting any limit, it is necessary to plan out the torque control:

- discretized, with an exact decoupling between the flux and the active current, to allow independent regulation of these two variables which are linked and in general deeply dependent
- with a dead-beat response in only one period, for dynamics
- with a motor state prediction at the horizon of the end of the control calculation delay, starting from a motor model, to avoid one pure time delay before voltage vector applications
- with a control calculation result providing a mean voltage input vector, defining in an univocal way, at given speed, the optimized voltage vector sequence to apply in order to reach set-points
- with an a priori limitation calculation to allow operations under constraints of limits without changing of control mode.

It will then be possible, by a well-adapted control calculation, to impose the control operation within the limits of each variable (inside surface boundary), and even precisely on one limit, or simultaneously on several limits.

Practically, the discretized linear state-space representation of a linear process allows working out the control equations. Indeed, if the initial motor state at time t_n is represented by

the state-space vector $X(t_n)_0$, the predicted motor state at the next time $(t_n + T)$ will be represented by the predicted state-space vector $X(t_n + T)_p$, after the control vector V application during the time interval T .

With the constant angular frequency ω during the interval T , the predicted state is linked to the initial state and to the control vector by the following discretized linear state-space equation system (Borne *et al.*, 1992):

$$X(t_n + T)_p = F(\omega, T) \cdot X(t_n)_0 + G(\omega, T) \cdot V \quad (5)$$

$F(\omega, T)$ represents the transition matrix, and $G(\omega, T)$ the input matrix. Matrices $F(\omega, T)$ and $G(\omega, T)$ depend upon motor parameters, angular frequency and prediction interval. They constitute a very good modeling of the motor state evolution by the addition of two terms:

- $F(\omega, T) \cdot X(t_n)_0$ which represents the short-circuit motor evolution or *free evolution*
- $G(\omega, T) \cdot V$ which represents the amount of motor evolution due to the control voltage application.

The dead-beat control solution is thus summarized by the voltage vector calculation V from equation (5), according to the initial motor state and to the predicted motor state which we will replace by a set-point vector *complying with limits*, and noted $X(t_n + T)_c$, cf. equation (6).

$$X(t_n + T)_p = X(t_n + T)_c \quad (6)$$

$$X(t_n + T)_c = F(\omega, T) \cdot X(t_n)_0 + G(\omega, T) \cdot V \quad (7)$$

Solution (8) of equation (7), when it exists, thus allows us to reach the set-point vector in only one step, without overshooting limits, whatever the motor type for which matrices are known.

$$\boxed{\vec{V}_s = V \{F, G, X_0, X_c\}} \quad (8)$$

If the solution does not exist, due to constraints related to limitations, the final objective will not be reached within only one control period. It will then be necessary to define an intermediate set-point vector, in the direction and sense of the final set-point vector; the intermediate set-point could then be reached in only one step, by using all the process resources on their limits. A part of the trajectory having been covered, it will be enough to reiterate the operation as many times as necessary to achieve the final objectives. Under constraints, the response will thus be obtained in a few periods. The response time is then only limited by constraints of the process sizing itself.

In general, the $G(\omega, T)$ matrix is not a square one; it is thus not invertible. The solution therefore cannot be a general result.

The solution developed in this work is based on the use of evolution matrix eigenvalues (Rotella and Borne, 1995). They make it possible to obtain an exact solution with control perfectly decoupled from the various physical variables, the solution consisting of rewriting

discretized state-space equations into the eigenvector basis of the evolution matrix; this inversion method will be used in the rest of this work, and will be applied to particular cases of three different motor types but also to a second-order power filter.

For this process, the time delay of the average voltage application is also both the sampling period and the computational period T ; this period is independent of the control method. It thus allows a degree of freedom which can be used to optimize the harmonic contents of the motor current in two different ways which both have practical uses:

- A constant period T can be imposed for a low stator feeding frequency, as long as the stator voltage period is large compared to this period ($\geq 10 \cdot T$); the sampling period is thus asynchronous compared to the stator voltage period, but synchronous compared to PWM.
- Successive calculation periods can be synchronized with the stator voltage periods at high speed, which will make them variable, in particular as function of the motor speed. Analytical solutions of state-space equations are, of course, different in this case. It is the only solution usable when one wishes to apply full voltage to the stator, from a three-phase inverter working in a square-wave mode. The sampling period classically reserved for the measurement sampling at constant period becomes, with square wave PWM, a control variable of the instantaneous angular velocity of the stator flux, and thus a torque control variable. The sampling period is thus synchronous with the stator feeding period, and still synchronous with the PWM.

The inverter control and measurement sampling defining the initial motor state are thus always synchronous. The state-space equation discretization in synchronism with inverter control makes it possible to have a true motor-inverter model. The sampling is synchronous with harmonic contents due to the inverter switching; this choice of synchronism makes it possible to avoid anti-aliasing filters of the switching harmonic spectrum, before the measurement sampling; these filters would reduce measurement bandwidth and thus would reduce the potential control dynamics (Jacquot, 1995). In this way, sampling is carried out on instantaneous current values with the ripples due to the switching harmonics at this time; the useful information is thus not lost, since by this means it is possible to measure the peak current value synchronized with the inverter switching.

The method for generating the mean voltage vector, the solution of the control equation, is not constrained by the regulation process itself. Various sequences of inverter voltage vectors feeding the stator can be easily selected. However, the module and angle of the mean voltage vector over one sampling period are solutions of the control equation.

The control does not have to calculate application times of each voltage vector, which gives many possibilities for optimizing the harmonic content of the applied voltage and thus makes it possible to adapt the spectral contents of the stator current to process limitations (losses, peak currents), as early as the process design.

5 Objectives of this Work

The main objective of this work is to present an exact and general control method of an electromechanical process which allows the fulfilling of all objectives of accuracy and dynamics, while complying with all technological constraints.

The various phases of the control development are as follows:

- writing differential vectorized equations of the electromechanical process behavior
- choice of physical variables to be controlled
- writing state-space linear equations in continuous-time, state-space equation discretization and projecting discretized state-space equations into the eigenvector basis of the evolution matrix
- discretized state-space equation inversion and calculation of the input equation
- control development, real-time implementation, initial state-space prediction, control under constraints and examples.

One might think that the weakness of this method comes from the fact that prediction is based on a nominal model of motor parameters, inevitably slightly different from motor true parameters. It should therefore be remembered that the interest of a model precisely lies in its possibility of predicting the motor state evolution. This prediction cannot be perfectly exact, but it does use all the available knowledge of the process, even if imperfect; this contrasts with control processes based on field orientation (FOC) or based on sliding modes without a prediction model (DSC, DTC) which can only react with a permanent time-delay to motor state evolutions, either by continuous controls and approximate decoupling according to rotating frame axes, or by time-derivative calculations to restore a situation which constantly tends to exceed its limits.

It should, of course, be added that the knowledge which one has of process parameters can be modified in real-time if one has a means of measurement, estimation or adaptation of standard motor parameters. Many known methods exist for adjusting parameters before starting (self-tuning) or during exploitation (adaptation). In a simpler way, one can take account of inductance variations with the current, or of induction machine rotor resistance variations with the motor temperature using a simple rotor thermal model and a history of rms current values.

However, the prediction model cannot predict disturbances of the load moment or of the feeding voltage, nor can it cope with very fast set-point variations. The control will thus have to be able to regulate in a time much shorter than the shortest process time constant, which fixes the maximal computational sampling period.

Simulations and power tests were carried out with the presented control method with a Kalman observer and did not lead, under these conditions, to instabilities, whatever the parametrical drifts that were manually forced on the prediction model or resulting from motor heating. However, the control robustness, with or without an observer, was not studied in an analytical way and still needs to be done, as well as the stability robustness and the performance robustness, to allow taking account of structured (parametric) and non-structured uncertainties; an important analytical study of this type, was made in the case of the DTC (Ortega *et al.* 2001).

As we will see, the model on which the control is based can be coupled with the means of measurement and with the motor state filtering, for its greater accuracy and robustness.

The control method described in this work is applied, with a constant sampling period to:

- non-salient pole induction motors
- surface-mounted permanent magnet synchronous motors
- interior permanent magnet synchronous motors
- a second-order passive filter (inductor–capacitor).

We will not approach the extension of this method to a variable sampling period with motor feeding frequency, nor the extension to *all kinematic sensor* suppression; extensions with a variable sampling period and an SMPM-SM position sensor suppression were made, validated and here briefly referred to, but their exhaustive description would require another volume. This control is generalizable to other processes.

So as not to approach issues that have been brilliantly covered by many authors, several topics are not treated here and are only cited for the comprehension of their use. They are prerequisites for the comprehension of this work:

- Converters allowing variable voltage supply of motors will not be detailed. Their control vector, as well as switching patterns of the voltage applied to the stator, are described in the technical literature, as is the harmonic voltage calculation resulting from it. We will thus use a two-level voltage inverter – a reversible one – allowing a high switching frequency compared to the highest frequency of the voltage fundamental component supplying the motor. The presently most widespread type of inverter is a voltage inverter using insulated gate bipolar transistors (IGBT). The inverter will be controlled by a PWM (Monmasson, 2009); optimization methods of the PWM aimed at decreasing the inverter frequency while preserving acceptable ripple of the motor current and thus of the torque, will not be covered in this work as they would require another volume.
- Electric motor construction, of the stator as well as the rotor, and their winding technology will not be dealt with here either. We refer readers to specialized works and articles.
- Power supplies in all industrial fields, in the transportation domain and in particular in the field of the railways traction have various characteristics. The delivered voltage of power supplies can vary from a few tens of volts to a few tens of kilovolts, and the frequency from zero to several tens of kilohertz. It will be supposed here that the upstream supply, reversible or not, is adapted to feed the three-phase voltage inverter with a DC voltage. The filtering, most usually used upstream of the voltage inverter, is mainly made from one second-order filter with one serial inductor and one parallel capacitor.

1

Induction Machine

The three-phase induction machine with non-salient poles is the most widespread electric motor because of its simple and robust construction; it is perhaps the electrical machine that has the least intuitive operation (Caron and Hautier, 1995). It has been the subject of very many technical publications.

1.1 Electrical Equations and Equivalent Circuits

Starting from the equivalent three-phase electrical circuit of the induction machine without neutral current, let us establish initially the relations between the various electrical variables.

1.1.1 Definitions and Notation

Definitions and notation of motor parameters:

- stator resistance R_s
- rotor resistance R_r
- stator leakage inductance l_s
- rotor leakage inductance l_r
- mutual inductance L_m
- stator inductance $L_s = L_m + l_s$
- rotor inductance $L_r = L_m + l_r$
- stator time constant $\tau_s = \frac{L_s}{R_s}$

- rotor time constant $\tau_r = \frac{L_r}{R_r}$
- pole pair number N_p
- dispersion coefficient $\sigma = 1 - \frac{L_m^2}{L_r \cdot L_s}$

Definitions and notations of mechanical and electrical angular frequencies:

- mechanical angular frequency of the rotor Ω
- *polar* mechanical angular frequency¹ $\omega = N_p \cdot \Omega$
- stator electrical angular frequency ω_s
- rotor electrical angular frequency ω_r
- relative slip $g = \frac{\omega_s - \omega}{\omega_s} = \frac{\omega_r}{\omega_s}$

1.1.2 Equivalent Electrical Circuits

The reduced equivalent electrical circuit for each phase of the balanced three-phase induction machine is that of Figure 1.1.

The directed angular symbol of Figure 1.1 recalls that the coupling between stator and rotor windings is modified with the rotation of the rotor.

This circuit does not show the equivalent resistance of iron losses, in parallel with the mutual inductance; it would represent ohmic losses due to the hysteresis of the magnetic material and to eddy currents in magnetic steel sheets. These losses are in general minimized when designing an electric motor. Resistance values do not take into account the skin effect due to high frequency harmonic currents. Inductances are considered here to be unsaturated. It is nevertheless possible to modify the values of these elements according to the mode of motor feeding, the operation mode and the harmonic content of the voltage inverter output.

The electrical variables defined for this equivalent circuit are variables directly accessible by electrical measurement:

- the phase–neutral instantaneous voltage, per phase:

$$v_{pn}(t) = V \cdot \sin\left(\omega_s \cdot t + p \cdot \frac{2\pi}{3}\right) \quad (1.1)$$

- the instantaneous current in each phase:

$$i_{sp}(t) = I \cdot \sin\left(\omega_s \cdot t + p \cdot \frac{2\pi}{3} - \varphi\right) \quad (1.2)$$

¹ For simplicity, we will refer to the mechanical angular frequency of the motor shaft, multiplied by the pole pair number, as the *polar* mechanical angular frequency, in preference to the name, sometimes used, of the electrical mechanical angular frequency.

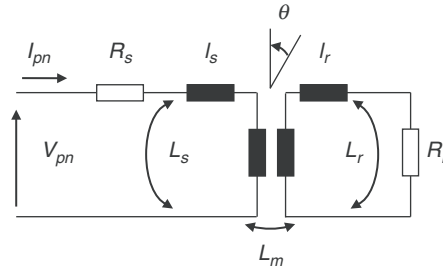


Figure 1.1 Equivalent circuit of one phase of the three-phase induction machine

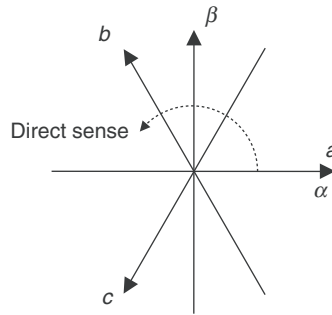


Figure 1.2 Three-phase (a, b, c) and two-phase (α, β) fixed frames

with three phases $p \in \{0; 1; 2\}$, and $\varphi \in \left[0, \frac{\pi}{2}\right]$ is the phase lagging of the current compared to the phase voltage, under traction operation.

The Concordia transformation (Owen, 1999) makes it possible to reduce the three-phase scalar representation in the phase plane, by introducing a vector representation into the orthonormal frame plane of Figure 1.2. The transfer matrix, from the balanced three-phase representation to the two-phase one, which preserves the instantaneous power, is the 2×3 matrix of the Concordia transformation (1.3).

$$\begin{bmatrix} x_\alpha \\ x_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \cdot \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \cdot \begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix} \quad (1.3)$$

The chosen positive sense for measuring angles, as well as for rotation sense and angular velocity, will be always counterclockwise.

After this transformation, the equivalent circuit of the induction machine takes the reduced vector form of Figure 1.3.

The circuits of Figures 1.1 and 1.3 seem identical, but they represent, respectively, just one phase in a three-phase fixed frame, and three phases in a two-phase fixed frame. Represented electrical variables are different and are linked by the Concordia transformation.

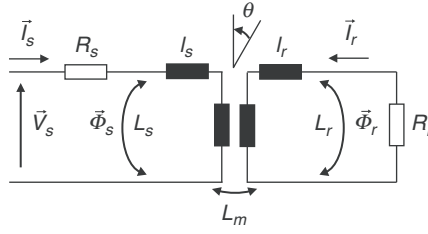


Figure 1.3 Two-phase equivalent circuit of the induction machine

Equations of the system (1.4), are relations between the magnitudes of vectorial variables and maximum values of electrical phase variables, in this transformation.

$$\|\vec{V}_s\| = \sqrt{\frac{3}{2}} \cdot V_{pn} \quad \|\vec{I}_s\| = \sqrt{\frac{3}{2}} \cdot I_{pn} \quad \|\vec{\Phi}\| = \sqrt{\frac{3}{2}} \cdot \Phi_p \quad (1.4)$$

Phase parameter values of the motor remain unchanged.

1.1.3 Differential Equation System

The two meshes of the equivalent electrical circuit of Figure 1.3, make it possible to write a differential vector equation (1.5), and a *partial* time derivative of rotor flux (1.6).

$$\vec{V}_s = R_s \cdot \vec{I}_s + L_s \cdot \frac{d\vec{I}_s}{dt} + L_m \cdot \frac{d\vec{I}_r}{dt} \quad (1.5)$$

$$\frac{\delta \vec{\Phi}_r}{\delta t} = -R_r \cdot \vec{I}_r \quad (1.6)$$

However $\vec{\Phi}_r$ depends on time but also on the coupling between the stator and the rotor, which depends on the angle of the rotor phases compared to the stator phases. The total differential of the rotor flux is thus expressed by the equation (1.7), where the polar mechanical angular frequency $\omega = \frac{d\theta}{dt}$ is introduced.

$$d\vec{\Phi}_r(t, \theta) = \frac{\delta \vec{\Phi}_r}{\delta t} \cdot dt + \frac{\delta \vec{\Phi}_r}{\delta \theta} \cdot d\theta \Rightarrow \frac{d\vec{\Phi}_r}{dt} = \frac{\delta \vec{\Phi}_r}{\delta t} + \omega \cdot \frac{\delta \vec{\Phi}_r}{\delta \theta} \quad (1.7)$$

In addition, currents depend only on the time.

With notations defined previously, the rotor flux is also expressed in terms of the stator and rotor current vectors (1.8).

$$\vec{\Phi}_r = L_m \cdot \vec{I}_s + L_r \cdot \vec{I}_r \quad (1.8)$$

The instantaneous equation (1.8) is differentiated with respect to the time (1.9).

$$\frac{d\vec{\Phi}_r}{dt} = L_m \cdot \frac{d\vec{I}_s}{dt} + L_r \cdot \frac{d\vec{I}_r}{dt} \quad (1.9)$$

Let us eliminate $\vec{I}_r$ between equations (1.6) and (1.8).

$$\frac{\delta \vec{\Phi}_r}{\delta t} = -\frac{R_r}{L_r} \cdot (\vec{\Phi}_r - L_m \cdot \vec{I}_s) \quad (1.10)$$

From the electrical differential equations of the induction machine, we will preserve relations between $\vec{V}_s$, $\vec{I}_s$ and $\vec{\Phi}_r$. The input variable $\vec{V}_s$, as well as the two vectors $\vec{I}_s$ and $\vec{\Phi}_r$, making it possible to define the electromagnetic torque, will be kept. The choice of the measured stator current vector $\vec{I}_s$ and the rotor flux $\vec{\Phi}_r$, which is a non-measurable variable from a measurement made out into the motor air gap, will be justified by the two-phase equivalent circuit, with leakage inductances shifted to the stator of Figure 1.4, in section 1.1.4.

While eliminating $\frac{d\vec{I}_r}{dt}$ between equations (1.5) and (1.9), we obtain the equation (1.11).

$$\vec{V}_s = R_s \cdot \vec{I}_s + L_s \cdot \left(1 - \frac{L_m^2}{L_s \cdot L_r} \right) \cdot \frac{d\vec{I}_s}{dt} + \frac{L_m}{L_r} \cdot \frac{d\vec{\Phi}_r}{dt} \quad (1.11)$$

Let us substitute now $\frac{d\vec{\Phi}_r}{dt}$ by its expression (1.7), in which we replaced the partial time derivative of the rotor flux by the equation (1.10).

$$\vec{V}_s = \left(R_s + R_r \cdot \frac{L_m^2}{L_r^2} \right) \cdot \vec{I}_s + L_s \cdot \left(1 - \frac{L_m^2}{L_s \cdot L_r} \right) \cdot \frac{d\vec{I}_s}{dt} + \omega \cdot \frac{L_m}{L_r} \cdot \frac{\delta \vec{\Phi}_r}{\delta \theta} - \frac{R_r \cdot L_m}{L_r^2} \cdot \vec{\Phi}_r \quad (1.12)$$

By using the dispersion coefficient σ defined in section 1.1.1, we calculate firstly the stator current time derivative:

$$\frac{d\vec{I}_s}{dt} = -\frac{1}{\sigma \cdot L_s} \cdot \left(R_s + R_r \cdot \frac{L_m^2}{L_r^2} \right) \cdot \vec{I}_s + \frac{R_r \cdot L_m}{\sigma \cdot L_s \cdot L_r^2} \cdot \vec{\Phi}_r - \omega \cdot \frac{L_m}{\sigma \cdot L_s \cdot L_r} \cdot \frac{\delta \vec{\Phi}_r}{\delta \theta} + \frac{1}{\sigma \cdot L_s} \cdot \vec{V}_s \quad (1.13)$$

In addition, let us define following parameters to reduce later equations:

$$\boxed{\begin{aligned} R_{sr} &= R_s + R_r \cdot \frac{L_m^2}{L_r^2} \\ \alpha &= \frac{1}{\tau_r} \\ \beta &= \frac{L_m}{\sigma \cdot L_s \cdot L_r} = \frac{1 - \sigma}{\sigma} \cdot \frac{1}{L_m} \\ \lambda &= \sigma \cdot L_s \\ \gamma &= \frac{1}{\sigma} \cdot \left[\frac{1}{\tau_s} + \frac{1 - \sigma}{\tau_r} \right] = \frac{R_{sr}}{\lambda} \end{aligned}} \quad (1.14)$$

Equation (1.13) becomes:

$$\dot{\vec{I}}_s = -\gamma \cdot \vec{I}_s + \alpha \cdot \beta \cdot \vec{\Phi}_r - \omega \cdot \beta \cdot \frac{\delta \vec{\Phi}_r}{\delta \theta} + \frac{1}{\lambda} \cdot \vec{V}_s \quad (1.15)$$

In the same way, the equation (1.7), using the equation (1.10), is modified as follows:

$$\dot{\vec{\Phi}}_r = \alpha \cdot L_m \cdot \vec{I}_s - \alpha \cdot \vec{\Phi}_r + \omega \cdot \frac{\delta \vec{\Phi}_r}{\delta \theta} \quad (1.16)$$

Equations (1.15) and (1.16) we will allow us to establish continuous-time state-space equations of the induction machine; first of all, we will interpret them by reconstituting a new equivalent circuit starting from the obtained expressions.

1.1.4 Interpretation of Electrical Relations

Let us combine (1.15) and (1.16) to eliminate $\frac{\delta \vec{\Phi}_r}{\delta \theta}$:

$$\dot{\vec{I}}_s = -\gamma \cdot \vec{I}_s + \alpha \cdot \beta \cdot L_m \cdot \vec{I}_s - \beta \cdot \dot{\vec{\Phi}}_r + \frac{1}{\lambda} \cdot \vec{V}_s \quad (1.17)$$

then, let us express the stator voltage:

$$\vec{V}_s = \lambda \cdot \dot{\vec{I}}_s + (R_{sr} - \lambda \cdot \alpha \cdot \beta \cdot L_m) \cdot \vec{I}_s + \beta \cdot \lambda \cdot \dot{\vec{\Phi}}_r \quad (1.18)$$

$$\vec{V}_s = \sigma \cdot L_s \cdot \frac{d\vec{I}_s}{dt} + \left(R_s + R_r \cdot \frac{L_m^2}{L_r^2} - \frac{R_r}{L_r} \cdot \frac{L_m^2}{L_r} \right) \cdot \vec{I}_s + \frac{L_m}{L_r} \cdot \dot{\vec{\Phi}}_r \quad (1.19)$$

$$\vec{V}_s = R_s \cdot \vec{I}_s + (\sigma \cdot L_s) \cdot \frac{d\vec{I}_s}{dt} + \frac{d \left(\frac{L_m}{L_r} \cdot \vec{\Phi}_r \right)}{dt} \quad (1.20)$$

Equation (1.20), interpreted as an electrical relation of the stator mesh, shows that all occurs such as if the motor, seen from its stator, would present one resistance R_s , one stator leakage inductance $\lambda = \sigma \cdot L_s$ and one air gap flux $\vec{\Phi}_m$ at the angular frequency ω_s .

$$\vec{\Phi}_m = \frac{L_m}{L_r} \cdot \vec{\Phi}_r \quad (1.21)$$

The fundamental voltage magnitude of the generated back electromotive force (b.e.m.f.) can thus be written:

$$\frac{d\Phi_m}{dt} = \omega_s \cdot \frac{L_m}{L_r} \cdot \Phi_r \quad (1.22)$$

Under these conditions, since the stator inductance is L_s , the equivalent magnetizing inductance is the complement to L_s of the leakage inductance:

$$\lambda_m = (1 - \sigma) \cdot L_s \quad (1.23)$$

In steady state operation, the equivalent resistance of the rotor R_e , can be calculated simply by writing the electromechanical torque, starting from the electrical power in two different ways. The power is calculated by the squared voltage divided by the equivalent resistance; the torque is obtained by dividing the power by the angular frequency. An expression of the torque results from the equivalent circuit of Figure 1.3; the other relation is derived of equation (1.22).

$$\frac{\left(-\frac{d\Phi_m}{dt}\right)^2}{R_e \cdot \omega_s} = \frac{\left(\frac{d\Phi_r}{dt}\right)^2}{R_r \cdot \omega_r} \quad (1.24)$$

We obtain then successively:

$$\frac{\left(\omega_s \cdot \frac{L_m}{L_r} \cdot \Phi_r\right)^2}{R_e \cdot \omega_s} = \frac{(\omega_r \cdot \Phi_r)^2}{R_r \cdot \omega_r} \quad (1.25)$$

$$R_e = \frac{R_r}{\frac{\omega_r}{\omega_s}} \cdot \left(\frac{L_m}{L_r}\right)^2 \quad (1.26)$$

$$R_e = \frac{R_r}{g} \cdot \left(\frac{L_m}{L_r}\right)^2 \quad (1.27)$$

We can now build an equivalent circuit which no longer formally reveals:

- the variable coupling between the stator and the rotor
- the electrical slip created by the differential of electrical angular frequencies between the stator and the rotor
- the secondary magnetizing inductance of the rotating transformer
- the rotor leakage inductance which is never directly measured.

This representation of the equivalent circuit with the rotor leakage inductance shifted to the stator is also the equivalent circuit resulting from the removal of the three-phase transformer between the stator and the rotor; this transformer is a rotating transformer, because of the periodic modification of the coupling between the primary and the secondary by the rotor rotation, which induces in the transformer secondary an electrical frequency slightly different from the primary frequency.

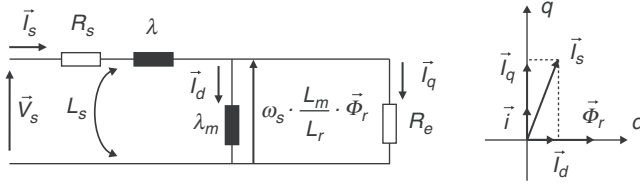


Figure 1.4 Equivalent circuit with shifted rotor leakage inductance

To break up the stator current into two currents, one crossing the magnetizing inductance $\vec{I}_d$, and the other crossing the rotor equivalent resistance $\vec{I}_q$, we can write:

$$\vec{I}_s = \vec{I}_d + \vec{I}_q \quad (1.28)$$

$\vec{I}_d$ is lagging of $\frac{\pi}{2}$ with respect to $\vec{I}_q$.

These two vectorial components of the stator current are in quadrature; they can thus simply be represented in an orthonormal frame (d, q) , by the two coordinates of the stator current vector $\vec{I}_s$. The d axis, having the same direction and the same sense as the current $\vec{I}_d$ and thus as the rotor flux $\vec{\Phi}_r$, rotates with the machine rotor flux involving the q axis in quadrature; the orthonormal frame thus rotates with the rotor flux. This frame is a *rotating frame* (cf. Figure 1.4).

Ultimately, the equivalent electrical circuit for equation systems (1.15) and (1.16) of Figure 1.4, is derived from equation (1.20).

With the selected positive sense for the stator current (receiving sense), the operation represented in Figure 1.4, is a traction operation (positive torque).

This new equivalent circuit makes it possible to highlight the following:

- The transfer of the rotor leakage inductance toward the stator leakage inductance.
- The fundamental current $\vec{I}_d$ in the magnetizing inductance is only responsible for the magnetization of the induction machine, with relations written for the *steady state*:

$$\frac{L_m}{L_r} \cdot \vec{\Phi}_r = \lambda_m \cdot \vec{I}_d = (1 - \sigma) \cdot L_s \cdot \vec{I}_d = \frac{L_m^2}{L_r} \cdot \vec{I}_d \quad (1.29)$$

and thus:

$$\vec{\Phi}_r = L_m \cdot \vec{I}_d \quad (1.30)$$

- The current $\vec{I}_q$ in the equivalent rotor resistance is in lead quadrature compared to the air gap flux vector. In accord with the Lorentz law, the fundamental electrical torque per pole pair is equal to the cross product of the air gap flux vector by the stator current vector. Using Equation (1.21) to reveal the rotor flux vector, and multiplying it by the number of pole pairs, we obtain the total electromagnetic torque:

$$\vec{C} = N_p \cdot \left(\frac{L_m}{L_r} \cdot \vec{\Phi}_r \right) \times \vec{I}_s = N_p \cdot \frac{L_m}{L_r} \cdot \vec{\Phi}_r \times (\vec{I}_d + \vec{I}_q) \quad (1.31)$$

- According to (1.30), vectors $\vec{\Phi}_r$ and $\vec{I}_d$ are collinear, so their cross product is null and the preceding relation is thus reduced:

$$\vec{C} = N_p \cdot \frac{L_m}{L_r} \cdot \vec{\Phi}_r \times \vec{I}_q \quad (1.32)$$

- Vectors $\vec{\Phi}_r$ and $\vec{I}_q$ are in direct quadrature under traction operation. The cross product is calculated then by the scalar relation:

$$C = N_p \cdot \frac{L_m}{L_r} \cdot \Phi_r \cdot I_q \quad (1.33)$$

- The torque is a positive torque when $I_q > 0$, since the cross product (1.32) is then direct, and it is thus a negative braking torque when $I_q < 0$.
- The d axis of the direct orthonormal frame (d, q) is collinear and of the same sense as the rotor flux vector because of (1.30). The $\vec{I}_d$ current creates the air gap flux and the $\vec{I}_q$ current makes the active torque. These two components of the stator current $\vec{I}_s$ in this frame make it possible to regulate independently both the flux, which is one component of the torque, and also the electromagnetic torque itself.
- The only control variable is the stator voltage vector $\vec{V}_s$; the stator current is derived from this voltage vector according to the motor state. The problem posed to the torque control is thus to calculate the voltage vector to regulate independently $\vec{I}_d$ and $\vec{I}_q$. It is the *fundamental problem* of any motor control, which is to be able to independently regulate the magnetizing and active currents, through impedances which vary with speed, with one control voltage having only two degrees of freedom: the two coordinates of the voltage vector in the (α, β) fixed frame.

Some of the relations of this subsection, were established for motor parameters corresponding to the fundamental component of the motor current, itself created by the fundamental component of the control voltage, and therefore at the angular frequency ω_s , by supposing that motor parameters do not vary according to the stator frequency.

The three-phase inverter – which does not work in an analogous way to create a sine wave because of losses which would result from it, but in a switching mode – produces voltage harmonics.

The same relations are applicable to current harmonics created by voltage harmonics at the same frequency, resulting from the voltage inverter switching, but with the required adaptation of motor parameter values to harmonic frequencies, as long as the equivalent circuit of the motor remains *formally* the same. Voltages, currents and fluxes of various harmonic frequencies and fundamental mode are composed then by an instantaneous addition (*superposition theorem*), creating electromagnetic torque components with various frequencies. We will suppose henceforth that the electrical circuit *formally* remains the same, with low frequency harmonics which have the highest amplitude.

1.2 Working out the State-Space Equation System

We now will establish the state-space equations of an induction machine supplied with balanced three-phase by an inverter with two voltage levels.

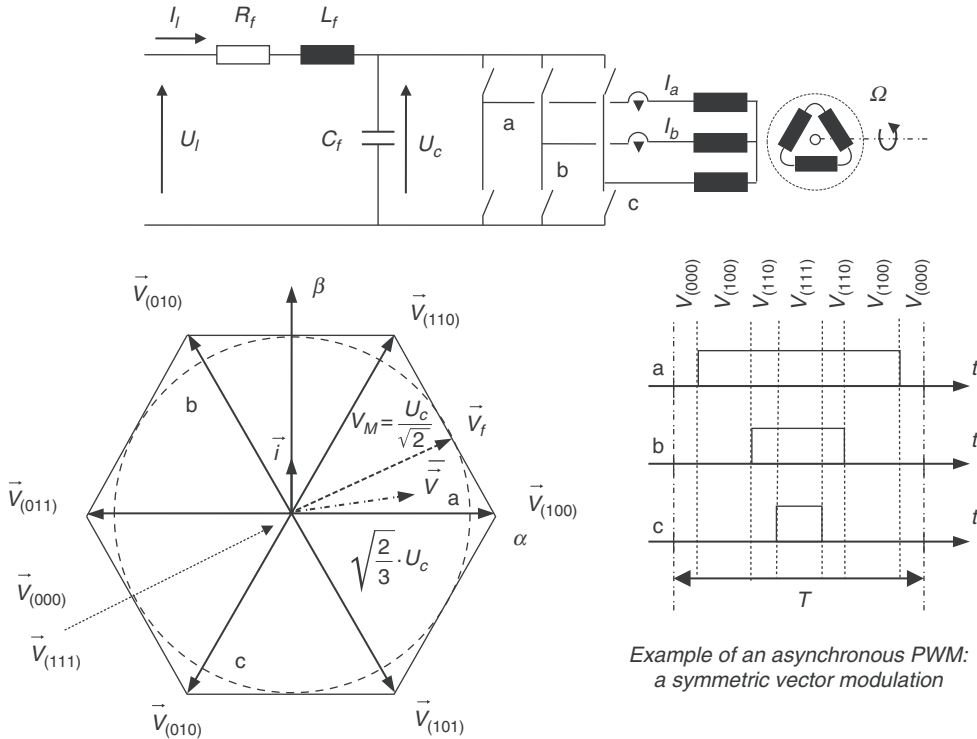


Figure 1.5 Induction motor fed by voltage inverter

The inverter makes it possible to generate in the (α, β) fixed plane, the six voltage vectors, represented in Figure 1.5, as well as the two null vectors, according to the state of the six electronic switches (Louis *et al.*, 2004).

The two electronic switches of each of the three inverter legs, are turned on, each one in its turn; we can thus define the state of one leg according to the potential of the motor phase connected to this leg. According to whether a phase is connected to voltage of the capacitor or to the voltage reference, the state of the switch pair corresponding to one of the phases is thus represented by one or zero respectively; it is like this for each of the three phases, which makes it possible to code the corresponding inverter state by a succession of three binary digits, each one corresponding to the state of A, B and C phases, respectively and also to the state of the two switches of each of the three inverter legs. Thus for instance, $\vec{V}_{(100)}$ is the voltage applied to the motor when the phase A is connected to the capacitor potential, while B and C phases are connected to the reference potential.

The technology of power semiconductors used for electronic switches of inverters was, historically, very varied: thyristors, switching transistors and gate turn-off thyristors (GTO). Now, however, *insulated gate bipolar transistors* (IGBT) are available in a very wide power range. They lead the motor current in the two senses: in the direct sense when they are triggered on, and in the reverse one by their integrated antiparallel diode, when the voltage applied on semiconductors reverses. Each power semiconductor type has its own technological

constraints; for IGBT they are mainly a minimum turn-on time before blocking and a minimum turn-off time once blocked; these unavoidable delays create one dead-time which it is necessary to take into account for control vector impressing.

The magnitude of the six non-null voltage vectors of the inverter measures $\sqrt{2/3}$ times the capacitor's DC voltage, after the Concordia transformation.

Hexagon vertices located at the extremities of the six voltage vectors of the inverter delimit a realizable voltage domain with a three-phase voltage inverter, but without taking into account dead-times. The circle inscribed in the hexagon delimits in its turn a field of the realizable fundamental voltage (first harmonic). The magnitude of the voltage vector $\bar{V}_f$, represents the maximum voltage of the fundamental component; it measures at its maximum $1/\sqrt{2}$ times the DC voltage of the U_c capacitor, if we do not take account of voltage drops or inverter dead-times.

Using various sequences of the six inverter states, we can thus create various vectorial modulations which generate, in their turn, an average voltage vector $\bar{V}$ over one period with a direction (vector argument) and a magnitude adjustable by the choice of the voltage vector sequence, their impressing order and duration.

The example in Figure 1.5 represents one period of a balanced-symmetrical modulation; it presents a minimum switching number during one period, but this modulation produces a common mode voltage because of the null voltage vector use. This common mode voltage can be reduced by half, using other modulation types (Lai, 1999) (cf. appendix A).

The voltage harmonic composition changes, as do the inverter switching losses, according to the choice of the modulation method and the switching period. Thus, the higher the frequency of the pulse width modulation is, compared to frequency of the fundamental voltage to be produced, the lower the distortion ratio of voltage waves supplying the motor is, but the more the frequency of voltage harmonics increases. Ohmic motor losses can be low as long as the skin effect remains negligible, but inverter switching losses increase with frequency.

Sizing of the inverter-motor pair is thus very dependent on the strategy of the inverter control. This strategy can be optimized with the sizing of one specific installation, and thus tabulated according to the motor speed and then to the voltage frequency. However, an analysis of best practice is not a part of this work. Examples of PWM are presented in appendix A.

1.2.1 State-Space Equations in the Fixed Plane

Vector equations (1.15) and (1.16), could now be put into the matrix form of a state-space representation defined in continuous-time:

$$\begin{aligned}\dot{X} &= A \cdot X + B \cdot U \\ Y &= C \cdot X\end{aligned}\tag{1.34}$$

To reduce the algebraic writing, the choice was made here to preserve the usual way of writing the evolution matrix A , input matrix B and output matrix C ; the initial state-space vector X , the time derivative of the state-space vector at the initial time $\dot{X}$, the control vector U and the measurement vector Y . We will also use the same conventions in the rest of this work, except when we want to insist on the vectorial characteristic of a one-column matrix – sometimes simply called vectors – when there is no ambiguity. This algebraic writing does not

make it possible to distinguish matrices from state-space vectors; only the symbols themselves, sanctioned by their use, translate their nature.

This state-space model will enable us to define the behavior of the system using the stator current and the rotor flux of the induction machine, in the (α, β) orthonormal fixed frame related to the motor stator.

$$X = \begin{bmatrix} I_{s\alpha} \\ I_{s\beta} \\ \Phi_{r\alpha} \\ \Phi_{r\beta} \end{bmatrix}, \text{ or } : \vec{X} = \begin{bmatrix} \vec{I}_s \\ \vec{\Phi}_r \end{bmatrix} \quad (1.35)$$

Hereafter, the traditional vector notation will be used for vectors of unspecified size, real or complex, i.e. in this last case, composed of one or several complex vectors referred to one complex orthonormal frame.

When we project equations (1.15) and (1.16) in the (α, β) fixed frame, the coordinates of the vector $\frac{\delta \vec{\Phi}_r}{\delta \theta}$ are derived from the coordinates of the vector $\vec{\Phi}_r$ by a rotation of $\frac{\pi}{2}$ in the direct sense defined as counterclockwise:

$$\left(\frac{\delta \vec{\Phi}_r}{\delta \theta} \right)_\alpha = -\Phi_{r\beta} \quad (1.36)$$

$$\left(\frac{\delta \vec{\Phi}_r}{\delta \theta} \right)_\beta = \Phi_{r\alpha} \quad (1.37)$$

After the projection of the two vector equations (1.15) and (1.16) on the axes α and β :

$$i_{s\alpha} = -\gamma \cdot I_{s\alpha} + \alpha \cdot \beta \cdot \Phi_{r\alpha} + \omega \cdot \beta \cdot \Phi_{r\beta} + \frac{1}{\lambda} \cdot V_{s\alpha} \quad (1.38)$$

$$i_{s\beta} = -\gamma \cdot I_{s\beta} - \omega \cdot \beta \cdot \Phi_{r\alpha} + \alpha \cdot \beta \cdot \Phi_{r\beta} + \frac{1}{\lambda} \cdot V_{s\beta} \quad (1.39)$$

$$\dot{\Phi}_{r\alpha} = \alpha \cdot L_m \cdot I_{s\alpha} - \alpha \cdot \Phi_{r\alpha} - \omega \cdot \Phi_{r\beta} \quad (1.40)$$

$$\dot{\Phi}_{r\beta} = \alpha \cdot L_m \cdot I_{s\beta} + \omega \cdot \Phi_{r\alpha} - \alpha \cdot \Phi_{r\beta} \quad (1.41)$$

For a fixed polar mechanical angular frequency ω , or considered as very slow-varying compared to electromechanical time constants of the motor, this system of four differential equations is linear, thus justifying the choice of the state-space representation for linear systems (1.34).

The two last relations show that the input variable $\vec{V}_s$ does not act directly on the rotor flux. The *evolution* of the rotor flux is dependent on both the stator current and the rotor flux state.

The first two relations show that the evolution of the stator current depends on:

- the control vector
- the current state
- the flux state.

These relations allow us to consider a close coupling between these two physical variables, which will have to be decoupled to be able to control them independently. They also show that the input voltage vector acts on the current and that the current acts in its turn on the flux, following the cause and effect principle.

By gathering the equations into matrix form, we obtain ultimately:

$$\begin{bmatrix} \dot{I}_{s\alpha} \\ \dot{I}_{s\beta} \\ \dot{\Phi}_{r\alpha} \\ \dot{\Phi}_{r\beta} \end{bmatrix} = \begin{bmatrix} -\gamma & 0 & \alpha \cdot \beta & \omega \cdot \beta \\ 0 & -\gamma & -\omega \cdot \beta & \alpha \cdot \beta \\ \alpha \cdot L_m & 0 & -\alpha & -\omega \\ 0 & \alpha \cdot L_m & \omega & -\alpha \end{bmatrix} \begin{bmatrix} I_{s\alpha} \\ I_{s\beta} \\ \Phi_{r\alpha} \\ \Phi_{r\beta} \end{bmatrix} + \begin{bmatrix} \frac{1}{\lambda} & 0 \\ 0 & \frac{1}{\lambda} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_{s\alpha} \\ V_{s\beta} \end{bmatrix} \quad (1.42)$$

This state-space equation system, of the fourth order in the case of an induction machine, is enough to describe completely the motor evolution when the control voltage vector $\vec{V}_s$ supplies it. Notice that the vector equation (1.42) depends on the frame in which it was calculated, even though the general expression (1.34) is not expressed in a particular frame. In the following, we will sometimes need to note temporarily as a subscript, the names of the particular frames in which the state-space variables of a general equation are calculated. See equation (1.113), for instance subscript (d, q) .

It is remarkable that the choice of the state-space variables $\vec{I}_s$ and $\vec{\Phi}_r$ can translate simply, as we saw in section 1.1.4, to the equivalent circuit which gathers leakage stator and rotor inductances at the stator, and, as a consequence, which can also allow the simultaneous representation of active and magnetizing (or reactive) currents of the machine (cf. Figure 1.4).

This property will allow us to:

- reveal very easily with this motor model, the (d, q) rotating frame in which I_d and I_q are projections of the stator current vector $\vec{I}_s$ on the d axis and q axis respectively
- easily translate the torque target of an induction machine into rotor flux and stator current set-points.

Comparing the formalism of (1.42) to equation (1.34), according to parameters of the machine, the evolution and the input matrices are respectively written in the (α, β) fixed frame as:

$$A = \begin{bmatrix} -\gamma & 0 & \alpha \cdot \beta & \omega \cdot \beta \\ 0 & -\gamma & -\omega \cdot \beta & \alpha \cdot \beta \\ \alpha \cdot L_m & 0 & -\alpha & -\omega \\ 0 & \alpha \cdot L_m & \omega & -\alpha \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} \frac{1}{\lambda} & 0 \\ 0 & \frac{1}{\lambda} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (1.43)$$

We can notice a remarkable property of submatrices 2×2 of the evolution matrix and the input matrix: the terms of their diagonals are identical and terms of their antidiagonals are opposite or null. This property is related to the fact that the electrical parameters of the machine are identical, whatever the rotor position. This is the case for non-salient pole induction machines. We will see that it is not the case for the interior permanent magnet synchronous motor (IPM-SM).

It should be noted that stator resistance is taken into account in this state-space model by the parameter γ (cf. equations (1.14)), which will allow, as we will demonstrate it later, to start the motor, to electrically brake it and to control it at very low speed to a stop with very good conditions.

1.2.2 State-Space Equations in the Complex Plane

Choosing the (α, β) fixed frame in a complex plane, with the pure imaginary unit vector $\vec{i}$ directed according to the β axis, we will now show that the system of continuous-time state-space equations of the fourth order of an induction machine, becomes a complex system of the second order, where:

$$\dot{\vec{X}} = \begin{bmatrix} \dot{I}_s \\ \dot{\vec{\Phi}}_r \end{bmatrix} = \begin{bmatrix} \dot{I}_{s\alpha} + i \cdot \dot{I}_{s\beta} \\ \dot{\vec{\Phi}}_{r\alpha} + i \cdot \dot{\vec{\Phi}}_{r\beta} \end{bmatrix}, \quad \vec{X} = \begin{bmatrix} \vec{I}_s \\ \vec{\Phi}_r \end{bmatrix} = \begin{bmatrix} I_{s\alpha} + i \cdot I_{s\beta} \\ \Phi_{r\alpha} + i \cdot \Phi_{r\beta} \end{bmatrix} \quad \text{and} \quad \vec{U} = V_{s\alpha} + i \cdot V_{s\beta} \quad (1.44)$$

and:

$$\begin{bmatrix} \dot{I}_{s\alpha} + i \cdot \dot{I}_{s\beta} \\ \dot{\vec{\Phi}}_{r\alpha} + i \cdot \dot{\vec{\Phi}}_{r\beta} \end{bmatrix} = A \cdot \begin{bmatrix} I_{s\alpha} + i \cdot I_{s\beta} \\ \Phi_{r\alpha} + i \cdot \Phi_{r\beta} \end{bmatrix} + B \cdot (V_{s\alpha} + i \cdot V_{s\beta}) \quad (1.45)$$

The reduction, from the real fourth order to the complex second order, is however only achievable in this case – as we will see it also is with IPM-SM – because the induction machine is a non-salient pole induction machine in this case, which gives this remarkable property to the 2×2 real submatrices of evolution and input matrices.

By multiplying equations (1.39) and (1.41) by the pure imaginary vector of unit magnitude i and by adding obtained equalities with equations (1.38) and (1.40) respectively, we obtain the two following relations:

$$\begin{aligned} \dot{I}_{s\alpha} + i \cdot \dot{I}_{s\beta} &= -\gamma \cdot (I_{s\alpha} + i \cdot I_{s\beta}) + (\alpha \cdot \beta - i \cdot \omega \cdot \beta) \cdot \Phi_{r\alpha} + \dots \\ \dots (\omega \cdot \beta + i \cdot \alpha \cdot \beta) \cdot \Phi_{r\beta} &+ \frac{1}{\lambda} \cdot (V_{s\alpha} + i \cdot V_{s\beta}) \end{aligned} \quad (1.46)$$

$$\dot{\vec{\Phi}}_{r\alpha} + i \cdot \dot{\vec{\Phi}}_{r\beta} = \alpha \cdot L_m \cdot (I_{s\alpha} + i \cdot I_{s\beta}) + (-\alpha + i \cdot \omega) \cdot \Phi_{r\alpha} - (\omega + i \cdot \alpha) \cdot \Phi_{r\beta} \quad (1.47)$$

These two relations are still written:

$$\dot{I}_{s\alpha} + i \cdot \dot{I}_{s\beta} = -\gamma \cdot (I_{s\alpha} + i \cdot I_{s\beta}) + \beta \cdot (\alpha - i \cdot \omega) \cdot (\Phi_{r\alpha} + i \cdot \Phi_{r\beta}) + \frac{1}{\lambda} \cdot (V_{s\alpha} + i \cdot V_{s\beta}) \quad (1.48)$$

$$\dot{\Phi}_{r\alpha} + i \cdot \dot{\Phi}_{r\beta} = \alpha \cdot L_m \cdot (I_{s\alpha} + i \cdot I_{s\beta}) - (\alpha - i \cdot \omega) \cdot (\Phi_{r\alpha} + i \cdot \Phi_{r\beta}) \quad (1.49)$$

which makes it possible to write the system in matrix format:

$$\begin{bmatrix} \dot{I}_{s\alpha} + i \cdot \dot{I}_{s\beta} \\ \dot{\Phi}_{r\alpha} + i \cdot \dot{\Phi}_{r\beta} \end{bmatrix} = \begin{bmatrix} -\gamma & \beta \cdot (\alpha - i \cdot \omega) \\ \alpha \cdot L_m & -(\alpha - i \cdot \omega) \end{bmatrix} \cdot \begin{bmatrix} I_{s\alpha} + i \cdot I_{s\beta} \\ \Phi_{r\alpha} + i \cdot \Phi_{r\beta} \end{bmatrix} + \begin{bmatrix} 1 \\ \frac{1}{\lambda} \\ 0 \end{bmatrix} \cdot (V_{s\alpha} + i \cdot V_{s\beta}) \quad (1.50)$$

and thus:

$$A = \begin{bmatrix} -\gamma & \beta \cdot (\alpha - i \cdot \omega) \\ \alpha \cdot L_m & -(\alpha - i \cdot \omega) \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 \\ \frac{1}{\lambda} \\ 0 \end{bmatrix} \quad (1.51)$$

Finally:

$$\begin{bmatrix} \dot{\vec{I}}_s \\ \dot{\vec{\Phi}}_r \end{bmatrix} = \begin{bmatrix} -\gamma & \beta \cdot (\alpha - i \cdot \omega) \\ \alpha \cdot L_m & -(\alpha - i \cdot \omega) \end{bmatrix} \cdot \begin{bmatrix} \vec{I}_s \\ \vec{\Phi}_r \end{bmatrix} + \begin{bmatrix} 1 \\ \frac{1}{\lambda} \\ 0 \end{bmatrix} \cdot \vec{V}_s \quad (1.52)$$

Note that when the antidiagonal of real submatrices 2×2 carries identically null terms, the corresponding coefficient of evolution and control matrices remains a real one. The complex part of the evolution matrix coefficients comes from the antidiagonal.

The state-space equation system defined in continuous-time is of course always of the fourth degree, but this reduced complex form will enable us to produce the following algebraic calculations.

1.2.3 Complex State-Space Equation Discretization

The state-space equation system defined in discrete-time, which makes it possible to calculate the process state at time $(t_n + T)$, starting from the state at time t_n , is given by the following general matrix expression, if we suppose that ω is constant from t_n to $(t_n + T)$. Indeed, after integration of continuous-time state-space equations (Borne *et al.*, 1992):

$$X(t_n + T) = F \cdot X(t_n) + G \cdot V \quad (1.53)$$

in which:

$$F = e^{A \cdot T} \quad \text{and} \quad G = A^{-1} \cdot (e^{A \cdot T} - I) \cdot B \quad (1.54)$$

F is the transition matrix, G is the input matrix and I is an unit matrix. The mean control vector between t_n and $(t_n + T)$, becomes:

$$V_{t_n \rightarrow (t_n + T)} = \frac{1}{T} \cdot \int_{t_n}^{t_n + T} U(\tau) \cdot d\tau \quad (1.55)$$

This last relation makes it possible to calculate an equivalent constant control vector from t_n to $(t_n + T)$, by temporal integration of the instantaneous voltage vector $U(t)$. This constant control vector can be then used as input vector in equation (1.53), to force the motor state to evolve from the initial state to the final state, between these two instants.

The approximation related to the discretization, lies in the constancy of the polar mechanical angular frequency between the two instants of the integration, which is justified if the period T selected is very small compared to the time necessary to increase the speed of the mechanical inertia.

In addition, equations (1.54) and (1.55), are perfectly exact only if, during integration of the continuous-time state-space equations, one can consider that the voltage vector is constant for the interval of integration and equal to the average vector; we will make this assumption hereafter, owing to the fact that the sampling period must be quite small compared with the electrical time constants in order that the motor can be controllable and that the harmonic distortion remains acceptable. However, it is possible to integrate exactly the state-space equations by breaking up the various sequences of constant voltage vectors of pulse width modulation (PWM) over the time interval.

The transition matrix F gathers all the parameters of the system which make it possible to follow its *free evolution*, i.e. to predetermine (to predict) the new state of the system at the end of the T period, when the control vector V is null. The free evolution of the system is thus, in this case, the motor evolution with balanced three-phase short-circuits. We will note the free evolution vector:

$$X^0(t_n + T) = F \cdot X(t_n) \quad (1.56)$$

which is the free evolution of the system at $(t_n + T)$ starting from the initial state $X(t_n)$ at t_n .

Thanks to the parameters of the system model, which appear in the transition matrix and in the control matrix, equation (1.53) makes it possible to predict, at time t_n , the future state of the system $X(t_n + T)_p$ at time $(t_n + T)$, when a known mean input vector $\vec{V}$ is applied to it, starting from the state of the system $X(t_n)_m$ measured at time t_n :

$$X(t_n + T)_p = F \cdot X(t_n)_m + G \cdot V \quad (1.57)$$

We will make use of this property of prediction to determine the motor state at the end of the computational period of the control vector, for the control algorithm implementation in real-time.

Another use of these discrete relations can be made by replacing the predicted state $X(t_n + T)_p$ at time $(t_n + T)$, by the stator current and rotor flux set-points $X(t_n + T)_c$, to allow calculation of the control vector to be applied to make the system state evolve from the measured state-space $X(t_n)_m$ at time t_n , towards the set-point state-space $X(t_n + T)_c$ at time $(t_n + T)$.

$$X(t_n + T)_c = F \cdot X(t_n)_m + G \cdot V \quad (1.58)$$

There remain now two difficulties to solve, in order to calculate the control vector by inverting this matrix relation:

- variables to be controlled independently are linked as we highlighted it (cf. equations (1.38) to (1.41))

- matrix equation (1.58) is not invertible, mainly owing to the fact that G is not a square matrix (a necessary condition, but not a sufficient one).

To solve these two problems, we will diagonalize the evolution matrix, which will enable us to find a new form for the discretized state-space representation. We will be able then to rewrite the discretized state-space equations within the eigenvector space of the evolution matrix, and to find a mean control vector that will allow reaching the set-points in a single step.

1.2.4 Evolution Matrix Diagonalization

To diagonalize the evolution matrix, it is necessary to start by calculating its eigenvalues, which also will inform us about the free evolution of the motor with three-phase short-circuits.

1.2.4.1 Eigenvalues

The eigenvalue equation giving the eigenvalues μ_i of the evolution matrix A is the relation that sets to zero the determinant of the matrix $(\mu_i \cdot I - A)$:

$$\det(\mu_i \cdot I - A) = 0 \quad \forall i \in \{1; 2\} \quad (1.59)$$

that is to say:

$$\begin{vmatrix} \mu + \gamma & -\beta \cdot (\alpha - i \cdot \omega) \\ -\alpha \cdot L_m & \mu + (\alpha - i \cdot \omega) \end{vmatrix} = 0 \quad (1.60)$$

thus:

$$\mu^2 + (\alpha + \gamma - i \cdot \omega) \cdot \mu + (\gamma - \alpha \cdot \beta \cdot L_m) \cdot (\alpha - i \cdot \omega) = 0 \quad (1.61)$$

Let us note Δ the discriminant of equation (1.61):

$$\Delta = (\alpha + \gamma - i \cdot \omega)^2 - 4 \cdot (\gamma - \alpha \cdot \beta \cdot L_m) \cdot (\alpha - i \cdot \omega) \quad (1.62)$$

that can then be written as:

$$\Delta = (\alpha - \gamma - i \cdot \omega)^2 + 4 \cdot \alpha \cdot \beta \cdot L_m \cdot (\alpha - i \cdot \omega) \quad (1.63)$$

The two eigenvalues are thus expressed by:

$$\mu_1 = -\frac{1}{2} \cdot (\alpha + \gamma - i \cdot \omega - \sqrt{\Delta}) \quad (1.64)$$

$$\mu_2 = -\frac{1}{2} \cdot (\alpha + \gamma - i \cdot \omega + \sqrt{\Delta}) \quad (1.65)$$

The diagonalized evolution matrix A can be written by definition as $D = \begin{bmatrix} \mu_1 & 0 \\ 0 & \mu_2 \end{bmatrix}$; therefore:

$$D = -\frac{1}{2} \begin{bmatrix} \alpha + \gamma - i \cdot \omega - \sqrt{\Delta} & 0 \\ 0 & \alpha + \gamma - i \cdot \omega + \sqrt{\Delta} \end{bmatrix} \quad (1.66)$$

1.2.4.2 Transfer Matrix Algebraic Calculation

Two eigenvectors $\left(\vec{H}_1 = \begin{bmatrix} p_{11} \\ p_{21} \end{bmatrix}, \vec{H}_2 = \begin{bmatrix} p_{12} \\ p_{22} \end{bmatrix} \right)$, corresponding to the two eigenvalues, are one solution, among an infinite number, of the equation:

$$(A - \mu_i \cdot I) \cdot \vec{H}_i = 0 \quad (1.67)$$

Indeed, each eigenvector cannot be unique owing to the fact that the corresponding matrix $(A - \mu_i \cdot I)$ is singular by definition of μ_i , calculated to make its determinant null according to (1.59).

Equation (1.67), applied to the first eigenvector is written:

$$\begin{bmatrix} -\gamma & \beta \cdot (\alpha - i \cdot \omega) \\ \alpha \cdot L_m & -(\alpha - i \cdot \omega) \end{bmatrix} \cdot \begin{bmatrix} p_{11} \\ p_{21} \end{bmatrix} = \mu_1 \cdot \begin{bmatrix} p_{11} \\ p_{21} \end{bmatrix} \quad (1.68)$$

The second line of the matrix equation provides the following relation:

$$\alpha \cdot L_m \cdot p_{11} = (\mu_1 + \alpha - i \cdot \omega) \cdot p_{21} \quad (1.69)$$

which makes it possible to choose, in particular:

$$p_{11} = \frac{2 \cdot (\mu_1 + \alpha - i \cdot \omega)}{2 \cdot \alpha \cdot L_m} = \frac{\alpha - \gamma - i \cdot \omega + \sqrt{\Delta}}{2 \cdot \alpha \cdot L_m} \quad (1.70)$$

$$p_{21} = 1$$

In a similar way, by just using the second line of equation (1.68), to reveal a symmetry, for the second eigenvector:

$$p_{12} = \frac{2 \cdot (\mu_2 + \alpha - i \cdot \omega)}{2 \cdot \alpha \cdot L_m} = \frac{\alpha - \gamma - i \cdot \omega - \sqrt{\Delta}}{2 \cdot \alpha \cdot L_m} \quad (1.71)$$

$$p_{22} = 1$$

The transfer matrix $P = \begin{bmatrix} \vec{H}_1 & \vec{H}_2 \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$, such as by definition $P^{-1} \cdot A \cdot P = D$, can thus be written:

$$P = \begin{bmatrix} \frac{\alpha - \gamma - i \cdot \omega + \sqrt{\Delta}}{2 \cdot \alpha \cdot L_m} & \frac{\alpha - \gamma - i \cdot \omega - \sqrt{\Delta}}{2 \cdot \alpha \cdot L_m} \\ 1 & 1 \end{bmatrix} \quad (1.72)$$

or, by using eigenvalue symbols:

$$P = \frac{1}{\alpha \cdot L_m} \cdot \begin{bmatrix} -(\mu_2 + \gamma) & -(\mu_1 + \gamma) \\ \alpha \cdot L_m & \alpha \cdot L_m \end{bmatrix} \quad (1.73)$$

1.2.4.3 Transfer Matrix Inversion

The inverse of the transfer matrix is calculated simply by transposing its cofactor matrix, divided by its determinant (Rotella and Borne, 1995):

$$P^{-1} = \frac{1}{2\sqrt{\Delta}} \cdot \begin{bmatrix} 1 & -\frac{\alpha - \gamma - i \cdot \omega - \sqrt{\Delta}}{2 \cdot \alpha \cdot L_m} \\ -1 & \frac{\alpha - \gamma - i \cdot \omega + \sqrt{\Delta}}{2 \cdot \alpha \cdot L_m} \end{bmatrix} \quad (1.74)$$

that is to say:

$$P^{-1} = \frac{1}{2\sqrt{\Delta}} \cdot \begin{bmatrix} 2 \cdot \alpha \cdot L_m & -(\alpha - \gamma - i \cdot \omega - \sqrt{\Delta}) \\ -2 \cdot \alpha \cdot L_m & (\alpha - \gamma - i \cdot \omega + \sqrt{\Delta}) \end{bmatrix} \quad (1.75)$$

or, by using eigenvalue symbols to obtain a more reduced form:

$$P^{-1} = \frac{1}{\mu_1 - \mu_2} \cdot \begin{bmatrix} \alpha \cdot L_m & (\mu_1 + \gamma) \\ -\alpha \cdot L_m & -(\mu_2 + \gamma) \end{bmatrix} \quad (1.76)$$

The evolution matrix can be now written from the diagonalized matrix and from the transfer matrix $A = P \cdot D \cdot P^{-1}$.

With notations using eigenvalue symbols, A becomes:

$$A = \frac{1}{\alpha \cdot L_m} \cdot \begin{bmatrix} -(\mu_2 + \gamma) & -(\mu_1 + \gamma) \\ \alpha \cdot L_m & \alpha \cdot L_m \end{bmatrix} \cdot \begin{bmatrix} \mu_1 & 0 \\ 0 & \mu_2 \end{bmatrix} \cdot \frac{1}{\mu_1 - \mu_2} \cdot \begin{bmatrix} \alpha \cdot L_m & (\mu_1 + \gamma) \\ -\alpha \cdot L_m & -(\mu_2 + \gamma) \end{bmatrix} \quad (1.77)$$

$$A = \begin{bmatrix} -\gamma & -\frac{(\mu_1 + \gamma) \cdot (\mu_2 + \gamma)}{\alpha \cdot L_m} \\ \alpha \cdot L_m & \mu_1 + \mu_2 + \gamma \end{bmatrix} \quad (1.78)$$

Equation (1.61), of the second degree in μ , makes it possible to find instantaneously the product and the sum of the two roots using the coefficients of μ^0 and μ^1 respectively, since the coefficient of μ^2 is equal to the unit:

$$\mu_1 \cdot \mu_2 = (\gamma - \alpha \cdot \beta \cdot L_m) \cdot (\alpha - i \cdot \omega) \quad (1.79)$$

$$\mu_1 + \mu_2 = -(\alpha + \gamma - i \cdot \omega) \quad (1.80)$$

Replacing $(\mu_1 \cdot \mu_2)$ and $(\mu_1 + \mu_2)$ by their expression (1.79) and (1.80) in relations $(\mu_1 + \mu_2 + \gamma)$ and $(\mu_1 + \gamma) \cdot (\mu_2 + \gamma)$, we obtain:

$$\mu_1 + \mu_2 + \gamma = -(\alpha - i \cdot \omega) \quad (1.81)$$

$$(\mu_1 + \gamma) \cdot (\mu_2 + \gamma) = -\alpha \cdot \beta \cdot L_m (\alpha - i \cdot \omega) \quad (1.82)$$

That is to say:

$$(\mu_1 + \gamma) \cdot (\mu_2 + \gamma) = \alpha \cdot \beta \cdot L_m \cdot (\mu_1 + \mu_2 + \gamma) \quad (1.83)$$

Matrix (1.78) can thus be written:

$$A = \begin{bmatrix} -\gamma & -\frac{(\mu_1 + \gamma) \cdot (\mu_2 + \gamma)}{\alpha \cdot L_m} \\ \alpha \cdot L_m & \frac{(\mu_1 + \gamma) \cdot (\mu_2 + \gamma)}{\alpha \cdot L_m \cdot \beta} \end{bmatrix} \quad (1.84)$$

Let us use the following reduced variables:

$$\begin{aligned} \xi &= \frac{\gamma}{\alpha \cdot L_m} \\ \xi_0 &= \frac{\mu_1 - \mu_2}{\alpha \cdot L_m} \\ \xi_1 &= \frac{\mu_1 + \gamma}{\alpha \cdot L_m} \\ \xi_2 &= \frac{\mu_2 + \gamma}{\alpha \cdot L_m} \end{aligned} \quad (1.85)$$

to reduce the writing of evolution and transfer matrices:

$$A = \alpha \cdot L_m \cdot \begin{bmatrix} -\xi & -\xi_1 \cdot \xi_2 \\ 1 & \frac{\xi_1 \cdot \xi_2}{\beta} \end{bmatrix}, \quad P = \begin{bmatrix} -\xi_2 & -\xi_1 \\ 1 & 1 \end{bmatrix} \quad \text{and} \quad P^{-1} = \frac{1}{\xi_0} \cdot \begin{bmatrix} 1 & \xi_1 \\ -1 & -\xi_2 \end{bmatrix} \quad (1.86)$$

1.2.5 Projection of State-Space Vectors in the Eigenvector Basis

We obtained various intermediaries of calculation allowing rewriting of the discretized state-space representation, and afterwards to project equations in the eigenvector basis. This operation makes the coordinates of state-space vectors independent, and so we will be able to control them independently.

Let us replace the exponential function of the evolution matrix multiplied by the T period $F = e^{A \cdot T}$, by the exponential function of this same diagonalized matrix $e^{D \cdot T}$ after the multiplication

on the left by the transfer matrix, and on the right by its reverse, according to the well-known relation (Rotella and Borne, 1995).

$$e^{A \cdot T} = P \cdot e^{D \cdot T} \cdot P^{-1} \quad (1.87)$$

Equations (1.53) and (1.54) make it possible then to write:

$$X(t_n + T) = (P \cdot e^{D \cdot T} \cdot P^{-1}) \cdot X(t_n) + A^{-1} \cdot (P \cdot e^{D \cdot T} \cdot P^{-1} - I) \cdot B \cdot V \quad (1.88)$$

Let us project this state-space equation system in the frame related to eigenvectors (eigenvector basis), while multiplying on the left by P^{-1} , which changes the frame of the state-space system.

$$\begin{aligned} [P^{-1} \cdot X(t_n + T)] &= P^{-1} \cdot P \cdot e^{D \cdot T} \cdot [P^{-1} \cdot X(t_n)] + \dots \\ \dots P^{-1} \cdot A^{-1} \cdot (P \cdot e^{D \cdot T} \cdot P^{-1} - P \cdot P^{-1}) \cdot B \cdot V \end{aligned} \quad (1.89)$$

In the expression $(P \cdot e^{D \cdot T} \cdot P^{-1} - P \cdot P^{-1})$, resulting from replacement of the unit matrix by $P \cdot P^{-1}$, it is now possible to factorize P on the left, and P^{-1} on the right:

$$[P^{-1} \cdot X(t_n + T)] = e^{D \cdot T} \cdot [P^{-1} \cdot X(t_n)] + (P^{-1} \cdot A^{-1} \cdot P) \cdot (e^{D \cdot T} - I) \cdot [P^{-1} \cdot B \cdot V] \quad (1.90)$$

Let us notice that (Rotella and Borne, 1995):

$$P^{-1} \cdot A^{-1} \cdot P = (P^{-1} \cdot A \cdot P)^{-1} \quad (1.91)$$

By definition of the transfer matrix:

$$P^{-1} \cdot A \cdot P = D \quad (1.92)$$

The two last relations thus make it possible to write (1.90) in this reduced form:

$$\boxed{P^{-1} \cdot X(t_n + T)_{(\alpha, \beta)} = e^{D \cdot T} \cdot P^{-1} \cdot X(t_n)_{(\alpha, \beta)} + D^{-1} \cdot (e^{D \cdot T} - I) \cdot P^{-1} \cdot B \cdot V_{(\alpha, \beta)}} \quad (1.93)$$

The state-space vector, projected in the eigenvector basis becomes a state-space *eigenvector*² of the induction machine $P^{-1} \cdot X$.

We can now notice that matrix coefficients $e^{D \cdot T}$ and $D^{-1} \cdot (e^{D \cdot T} - I)$ are diagonal matrices allowing a control independence of coordinates of the state-space eigenvector. We will show that these coordinates are two complex linear relations between current and flux vector coordinates.

We now will clarify equation (1.93) in the following section, and we will calculate the control vector.

² The motor state-space vector can be qualified as the state-space *eigenvector*, i.e. the state-space vector of which coordinates are defined in the eigenvector basis, relative to eigenvalues of the evolution matrix of a motor controlled by a voltage inverter.

1.3 Discretized State-Space Equation Inversion

1.3.1 Introduction of the Rotating Frame

In the preceding section we established discretized state-space equations in the eigenvector frame of the evolution matrix. Eigenvalues characterize the motor evolution amplitude when it is short-circuited; eigenvectors characterize the directions of this evolution.

These equations were established in the (α, β) fixed frame compared to the stator, starting from an equivalent circuit seen from the stator, using stator electrical variables $\vec{V}_s$ and $\vec{I}_s$, as well as the rotor flux $\vec{\Phi}_r$. These variables vary with the time at the electrical angular frequency ω_s . It is then possible to represent them in an instantaneous vector form, in the Fresnel (α, β) complex plane, β being the imaginary axis:

Let us consider a *stabilized* and pure sine wave operation mode, at the ω_s electrical angular frequency (cf. Figure 1.6). The three vectors $\vec{V}_s$, $\vec{I}_s$ and $\vec{\Phi}_r$, therefore turn at speed ω_s with a constant magnitude and a constant phase shift if the motor load remains constant. Their projections on the two axes provide an instantaneous value of the corresponding variable on the two phases of the two-phase equivalent circuit, except for a multiplicative constant which, in the case of the Concordia transformation which preserves the instantaneous power, is equal to $\sqrt{2/3}$.

Harmonics can be represented in a similar way, as well as transient operations. The vector composition of harmonic voltage vectors at a given time, has in the general case as resultant vector, a vector with variable amplitude, turning with non-constant angular velocity.

To be able to define the torque produced by the voltage fundamental component and consequently by the current fundamental component and then to calculate the voltage vector to be used to obtain it, it is then convenient to introduce an orthonormal frame turning at the angular velocity of voltage, current and flux fundamental components, therefore with instantaneous angular frequency ω_s . In this frame, magnitudes of fundamental variables are constant in the sine wave stabilized mode.

Let us define the angle ρ at time $(t_n + T)$ of an orthonormal frame (d, q) compared to the (α, β) fixed frame, by:

$$\rho(t_n + T) = \rho_0(t_n) + \theta(T) \quad (1.94)$$

ρ_0 is the value of this angle at time t_n and θ its variation during the time interval T .

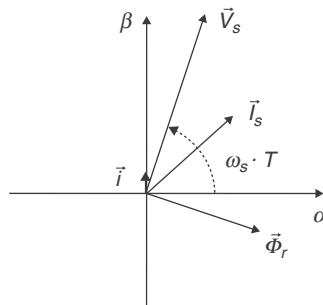


Figure 1.6 Rotating vectors in the fixed frame

The complex scalar of the unit magnitude $e^{-i \cdot \rho}$, which represents a rotation operator of an angle $-\rho$ in the (α, β) complex plane, is also an operator for frame changing of complex vectors from the (α, β) fixed frame, towards the (d, q) orthonormal frame, at time $(t_n + T)$:

$$e^{-i \cdot \rho} = e^{-i(\rho_0 + \theta)} \quad (1.95)$$

Whatever the type of the pulse width modulation used to control the voltage inverter supplying the motor, the voltage harmonics are converted into current harmonics, filtered by the leakage inductances of the induction machine. The resulting equivalent leakage inductance is in general small compared to the stator inductance and thus compared to the magnetizing inductance. According to (1.14) and (1.23):

$$\frac{\lambda}{L_s} = \sigma \quad \text{and} \quad \frac{\lambda_m}{L_s} = 1 - \sigma \quad (1.96)$$

A first-order equivalent to the following ratio is:

$$\frac{\lambda}{\lambda_m} = \frac{\sigma}{1 - \sigma} \cong \sigma \quad (1.97)$$

With a typical value of σ equal to 0.075, the relationship between the leakage inductance and the magnetizing inductance represented in Figure 1.3, would be 0.08.

The harmonic current is thus strongly filtered in the magnetizing inductance and primarily circulates in the equivalent rotor resistance where it creates electromagnetic torque harmonics and losses. The rotor flux is thus well filtered and very near to the fundamental flux. The magnitude of the rotor flux vector is thus practically constant as is its rotation velocity, and this is what justifies the choice of the rotor flux vector to define the phase of the rotating frame in transient operation mode. We saw in the section , that this choice also makes it possible to easily translate the torque set-point into two current set-points I_d and I_q , by the equations (1.30) and (1.33).

This frame is the Park reference frame (Park, 1929), used in particular by field orientation control (FOC).

We will be interested, however, only in the initial and the final position of this reference frame (discrete reference frame), i.e. the rotation velocity of this reference frame can be non-constant between times t_n and $(t_n + T)$; it slightly fluctuates with the rotor flux harmonics. The motor state can be non-stabilized, and the extension of the use of the reference frame to transient operations will enable us to vary the flux and the torque during the sampling period.

1.3.2 State-Space Vector Calculations in the Eigenvector Basis

Let us multiply the two terms of the equality (1.93) between state-space vectors which have their coordinates expressed in the (α, β) complex plane, by equality between complex scalars (1.95).

$$\begin{aligned} e^{-i \cdot \rho} \cdot P^{-1} \cdot X(t_n + T)_{(\alpha, \beta)} &= e^{-i \cdot \rho_0} \cdot e^{-i \cdot \theta} \cdot e^{D \cdot T} \cdot P^{-1} \cdot X(t_n)_{(\alpha, \beta)} + \dots \\ \dots e^{-i(\rho_0 + \theta)} \cdot D^{-1} \cdot (e^{D \cdot T} - I) \cdot P^{-1} \cdot B \cdot V_{(\alpha, \beta)} \end{aligned} \quad (1.98)$$

$$\begin{aligned}
 P^{-1} \cdot \left[e^{-i \cdot \rho} \cdot X(t_n + T) \right]_{(\alpha, \beta)} &= e^{-i \cdot \theta} \cdot e^{D \cdot T} \cdot P^{-1} \cdot \left[e^{-i \cdot \rho_0} \cdot X(t_n) \right]_{(\alpha, \beta)} + \dots \\
 \dots D^{-1} \cdot (e^{D \cdot T} - I) \cdot \left[P^{-1} \cdot B \cdot V_{(\alpha, \beta)} \right] \cdot e^{-i \cdot (\rho_0 + \theta)}
 \end{aligned} \quad (1.99)$$

This last relation now represents the projection of rotating state-space vectors, located in the (d, q) reference frame, in the eigenvector basis of the evolution matrix.

The two state-space vectors $X(t_n + T)_{(\alpha, \beta)}$ and $X(t_n)_{(\alpha, \beta)}$, located by their coordinates in the (α, β) fixed frame, are now expressed in the (d, q) and (d_0, q_0) complex planes at times $(t_n + T)$ and t_n , respectively, by the following relations:

$$\begin{aligned}
 e^{-i \cdot \rho} \cdot X(t_n + T)_{(\alpha, \beta)} &= X(t_n + T)_{(d, q)} \\
 e^{-i \cdot \rho_0} \cdot X(t_n)_{(\alpha, \beta)} &= X(t_n)_{(d_0, q_0)}
 \end{aligned} \quad (1.100)$$

Let us define ρ_0 as the angle of the rotor flux vector $\arg \{ \vec{\Phi}_r(t_n) \}_{(\alpha, \beta)}$, known at time t_n , compared to the (α, β) fixed frame, i.e. such that it cancels the coordinate of the rotor flux according to the q_0 axis. By definition of the angle ρ_0 :

$$\cos(\rho_0) = \frac{\Phi_{r\alpha}(t_n)}{\|\vec{\Phi}_r(t_n)\|} \quad \sin(\rho_0) = \frac{\Phi_{r\beta}(t_n)}{\|\vec{\Phi}_r(t_n)\|} \quad (1.101)$$

and:

$$X(t_n)_{(d_0, q_0)} = \begin{bmatrix} \cos(\rho_0) & \sin(\rho_0) \\ -\sin(\rho_0) & \cos(\rho_0) \end{bmatrix} \cdot X(t_n)_{(\alpha, \beta)} \quad (1.102)$$

$$X(t_n)_{(d_0, q_0)} = \begin{bmatrix} I_{sd_0} \\ I_{sq_0} \\ \Phi_{rd_0} \\ 0 \end{bmatrix} \quad \text{or} \quad X(t_n)_{(d_0, q_0)} = \begin{bmatrix} I_{sd_0} + i \cdot I_{sq_0} \\ \Phi_{rd_0} \end{bmatrix} \quad (1.103)$$

Under these conditions, Φ_{rd_0} is also the flux magnitude at the initial time t_n :

$$\Phi_{rd_0} = \|\vec{\Phi}_r(t_n)\| \quad (1.104)$$

The complex orthonormal reference frame (d_0, q_0) thus has the d_0 axis directed according to the direction and the sense of the rotor flux vector at time t_n .

Assimilating the average angular frequency ω_s of the rotor flux seen from the stator between t_n and $(t_n + T)$, with the *average angular velocity* of the ρ angle variation θ , such that $\frac{\theta}{T} = \omega_s$, the angle of the rotor flux at time $(t_n + T)$, compared to the (α, β) fixed frame, becomes:

$$\rho = \rho_0 + \omega_s \cdot T \quad (1.105)$$

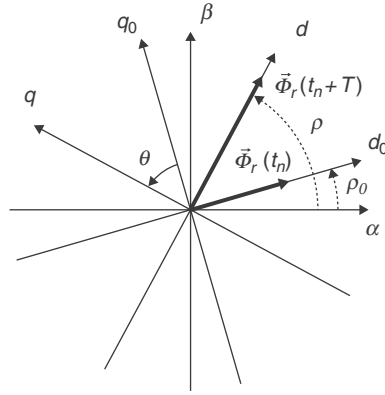


Figure 1.7 Definitions of the initial and the final discrete rotating reference frame

With this new definition, ω_s is no longer related to the concept of the angular frequency of fundamental electrical vectors, a concept critical to be defined out of the steady state or stabilized fundamental mode; ω_s is now related to an *average* angular velocity of the only rotor flux vector between two sampling times.

In the same way, by definition, the complex orthonormal reference frame (d, q) , thus has the d axis directed according to the direction and the sense of the rotor flux at all sampling times, and we can again write:

$$X(t_n + T)_{(d,q)} = \begin{bmatrix} I_{sd} \\ I_{sq} \\ \Phi_{rd} \\ 0 \end{bmatrix} \quad \text{or} \quad X(t_n + T)_{(d,q)} = \begin{bmatrix} I_{sd} + i \cdot I_{sq} \\ \Phi_{rd} \end{bmatrix} \quad (1.106)$$

Under these conditions, Φ_{rd} is also the magnitude of the flux at the final time:

$$\Phi_{rd} = \|\bar{\Phi}_r(t_n + T)\| \quad (1.107)$$

In short, the definition of the angles ρ_0 and ρ , and the equations (1.99) and (1.100) make it possible to write:

$$\begin{aligned} P^{-1} \cdot X(t_n + T)_{(d,q)} &= e^{-i \cdot \omega_s \cdot T} \cdot e^{D \cdot T} \cdot P^{-1} \cdot X(t_n)_{(d_0, q_0)} + \dots \\ &\dots D^{-1} \cdot (e^{D \cdot T} - I) \cdot \left[P^{-1} \cdot B \cdot V_{(\alpha, \beta)} \right] \cdot e^{-i(\rho_0 + \omega_s \cdot T)} \end{aligned} \quad (1.108)$$

The control vector V is always referenced in the (α, β) fixed frame to control the stator voltage of an induction machine with a voltage inverter.

Figure 1.7 simultaneously represents the three orthonormal frames: the stationary one (α, β) , the initial one (d_0, q_0) at time t_n , and the predicted one (d, q) at time $(t_n + T)$.

Notice further that, according to equations (1.51) and (1.86):

$$P^{-1} \cdot B = \frac{1}{\xi_0} \cdot \begin{bmatrix} 1 & \xi_1 \\ -1 & -\xi_2 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{\lambda} \\ 0 \end{bmatrix} \quad (1.109)$$

that is to say:

$$P^{-1} \cdot B = \frac{1}{\lambda \cdot \xi_0} \cdot \begin{bmatrix} 1 \\ -1 \end{bmatrix} \quad (1.110)$$

In addition:

$$D = \begin{bmatrix} \mu_1 & 0 \\ 0 & \mu_2 \end{bmatrix} \Leftrightarrow D^{-1} = \begin{bmatrix} \frac{1}{\mu_1} & 0 \\ 0 & \frac{1}{\mu_2} \end{bmatrix} \quad (1.111)$$

and:

$$e^{D \cdot T} = \begin{bmatrix} e^{\mu_1 \cdot T} & 0 \\ 0 & e^{\mu_2 \cdot T} \end{bmatrix} \quad (1.112)$$

This makes it possible to rewrite equation (1.108):

$$\begin{aligned} \frac{1}{\xi_0} \cdot \begin{bmatrix} 1 & \xi_1 \\ -1 & -\xi_2 \end{bmatrix} \cdot X(t_n + T)_{(d,q)} &= \frac{e^{-i\omega_s \cdot T}}{\xi_0} \cdot \begin{bmatrix} e^{\mu_1 \cdot T} & 0 \\ 0 & e^{\mu_2 \cdot T} \end{bmatrix} \cdot \begin{bmatrix} 1 & \xi_1 \\ -1 & -\xi_2 \end{bmatrix} \cdot X(t_n)_{(d_0,q_0)} + \dots \\ \dots \frac{V_{(\alpha,\beta)} \cdot e^{-i(\rho_0 + \omega_s \cdot T)}}{\lambda \cdot \xi_0} \cdot \begin{bmatrix} \frac{1}{\mu_1} & 0 \\ 0 & \frac{1}{\mu_2} \end{bmatrix} \cdot \begin{bmatrix} e^{\mu_1 \cdot T} - 1 & 0 \\ 0 & e^{\mu_2 \cdot T} - 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -1 \end{bmatrix} \end{aligned} \quad (1.113)$$

$$\begin{aligned} \begin{bmatrix} 1 & \xi_1 \\ 1 & \xi_2 \end{bmatrix} \cdot X(t_n + T)_{(d,q)} &= e^{-i\omega_s \cdot T} \cdot \begin{bmatrix} e^{\mu_1 \cdot T} & 0 \\ 0 & e^{\mu_2 \cdot T} \end{bmatrix} \cdot \begin{bmatrix} 1 & \xi_1 \\ 1 & \xi_2 \end{bmatrix} \cdot X(t_n)_{(d_0,q_0)} + \dots \\ \dots \frac{V_{(\alpha,\beta)}}{\lambda} \cdot e^{-i(\rho_0 + \omega_s \cdot T)} \cdot \begin{bmatrix} \frac{e^{\mu_1 \cdot T} - 1}{\mu_1} \\ \frac{e^{\mu_2 \cdot T} - 1}{\mu_2} \end{bmatrix} \end{aligned} \quad (1.114)$$

Now let us replace the state-space vectors by their complex coordinates from equations (1.103) and (1.106), and the control vector V by the stator voltage vector $\vec{V}_s$ in the (α, β) plane.

$$I_{sd} + i \cdot I_{sq} + \xi_1 \cdot \Phi_{rd} = e^{(\mu_1 - i \cdot \omega_s)T} \cdot (I_{sd0} + i \cdot I_{sq0} + \xi_1 \cdot \Phi_{rd0}) + \frac{\vec{V}_s}{\lambda} \cdot e^{-i(\rho_0 + \omega_s \cdot T)} \cdot \frac{e^{\mu_1 T} - 1}{\mu_1} \quad (1.115)$$

$$I_{sd} + i \cdot I_{sq} + \xi_2 \cdot \Phi_{rd} = e^{(\mu_2 - i \cdot \omega_s)T} \cdot (I_{sd0} + i \cdot I_{sq0} + \xi_2 \cdot \Phi_{rd0}) + \frac{\vec{V}_s}{\lambda} \cdot e^{-i(\rho_0 + \omega_s \cdot T)} \cdot \frac{e^{\mu_2 T} - 1}{\mu_2} \quad (1.116)$$

The projection of rotating vectors $\vec{I}_s$ and $\vec{\Phi}_r$ within the eigenvector space thus had, as logical consequences:

- to create a new state-space vector of real fourth dimension; we will name it, for convenience, a *state-space eigenvector* Ψ , resulting from the vector composition between $\vec{I}_s$ and $\vec{\Phi}_r$, and adapted to the machine
- to separate eigenmodes according to the two coordinates of the state-space eigenvector.

We will thus agree to note the state-space eigenvectors, at the final time $(t_n + T)$ in the (d, q) reference frame:

$$\Psi = \begin{bmatrix} I_{sd} + i \cdot I_{sq} + \xi_1 \cdot \Phi_{rd} \\ I_{sd} + i \cdot I_{sq} + \xi_2 \cdot \Phi_{rd} \end{bmatrix} = \begin{bmatrix} \vec{\Psi}_1 \\ \vec{\Psi}_2 \end{bmatrix}_{(d,q)} \quad (1.117)$$

and at the initial time t_n in (d_0, q_0) :

$$\Psi_0 = \begin{bmatrix} I_{sd0} + i \cdot I_{sq0} + \xi_1 \cdot \Phi_{rd0} \\ I_{sd0} + i \cdot I_{sq0} + \xi_2 \cdot \Phi_{rd0} \end{bmatrix} = \begin{bmatrix} \vec{\Psi}_{10} \\ \vec{\Psi}_{20} \end{bmatrix}_{(d_0, q_0)} \quad (1.118)$$

We can see that the addition to the stator current coordinates of complex terms of the form $\xi_k \cdot \Phi_{rd}$, $k \in \{1; 2\}$ changes the current vector origin in the rotating reference frame, or, what is the same, changes the frame origin for each state-space eigenvector. The coordinates of the frame origin become $-\xi_k \cdot \|\vec{\Phi}_r\|$. ξ_k , being complex numbers, are then interpreted like vectors $\vec{\xi}_k$ in the complex plane for the construction of Figure 1.8, which is not drawn true to scale.

These vectors are independent of the sampling period T , and depend only on ω through the intermediary of the eigenvalues. The directions of new origins compared to the single origin of reference frames (d_0, q_0) and (d, q) depend only on ξ_1 and ξ_2 , so they are identical in the two rotating reference frames and differ only by the magnitude of the rotor flux between t_n and $(t_n + T)$.

Equations (1.115) and (1.116) can now be written in a more concise way with the algebraic writing of state-space eigenvectors:

$$\vec{\Psi}_{1(d,q)} = e^{(\mu_1 - i \cdot \omega_s)T} \cdot \vec{\Psi}_{10(d_0, q_0)} + \vec{V}_{s(\alpha, \beta)} \cdot e^{-i(\rho_0 + \omega_s \cdot T)} \cdot \frac{e^{\mu_1 T} - 1}{\lambda \cdot \mu_1} \quad (1.119)$$

$$\vec{\Psi}_{2(d,q)} = e^{(\mu_2 - i \cdot \omega_s)T} \cdot \vec{\Psi}_{20(d_0, q_0)} + \vec{V}_{s(\alpha, \beta)} \cdot e^{-i(\rho_0 + \omega_s \cdot T)} \cdot \frac{e^{\mu_2 T} - 1}{\lambda \cdot \mu_2} \quad (1.120)$$

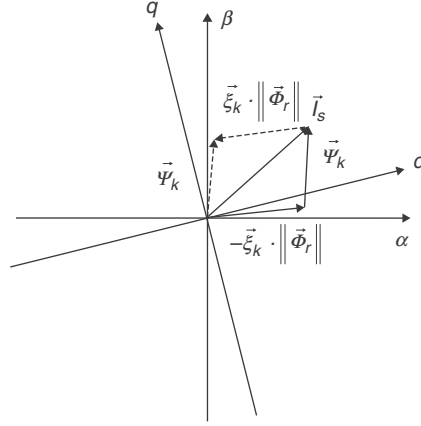


Figure 1.8 Construction of one state-space eigenvector coordinate

or, with a matrix form, while defining:

$$a = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \frac{e^{\mu_1 T} - 1}{\lambda \cdot \mu_1} \\ \frac{e^{\mu_2 T} - 1}{\lambda \cdot \mu_2} \end{bmatrix} \quad (1.121)$$

$$\begin{bmatrix} \vec{\Psi}_1 \\ \vec{\Psi}_2 \end{bmatrix}_{(d,q)} = \begin{bmatrix} e^{(\mu_1 - i\omega_s)T} & 0 \\ 0 & e^{(\mu_2 - i\omega_s)T} \end{bmatrix} \cdot \begin{bmatrix} \vec{\Psi}_{10} \\ \vec{\Psi}_{20} \end{bmatrix}_{(d_0,q_0)} + a \cdot \vec{V}_{s(\alpha,\beta)} \cdot e^{-i(\rho_0 + \omega_s T)} \quad (1.122)$$

The system of complex equations (1.122), is interpretable simply by noticing that:

- $e^{-i\omega_s T} \cdot \vec{\Psi}_{10}$ and $e^{-i\omega_s T} \cdot \vec{\Psi}_{20}$ correspond to modifications of coordinates of vectors $\vec{\Psi}_{10}$ and $\vec{\Psi}_{20}$, constituting the two initial state-space eigenvector coordinates, respectively from the (d_0, q_0) reference frame, towards the (d, q) reference frame
- $e^{\mu_1 T} \cdot [e^{-i\omega_s T} \cdot \vec{\Psi}_{10}]$ and $e^{\mu_2 T} \cdot [e^{-i\omega_s T} \cdot \vec{\Psi}_{20}]$ are free evolutions of complex coordinates of the state-space eigenvector in the (d, q) reference frame, during the period T , when $\vec{V}_s = 0$
- $\vec{V}_s \cdot e^{-i(\rho_0 + \omega_s T)}$ corresponds to modifying the coordinates of the vector $\vec{V}_s$, constant in magnitude and phase during the interval $[t_n, (t_n + T)]$, from the (α, β) fixed frame, to the (d, q) reference frame
- vectors $a_1 \cdot \vec{V}_s \cdot e^{-i\rho}$ and $a_2 \cdot \vec{V}_s \cdot e^{-i\rho}$ represent the evolution in the (d, q) reference frame of currents created by the voltage $\vec{V}_s$ during the interval T . These current vectors, added to the free evolution Ψ_0^0 of vectors Ψ_0 during the time interval T , make it possible to predict the vector Ψ at time $(t_n + T)$ (superposition theorem).

In short, (1.122) reveals simply in the (d, q) reference frame:

- the independence of the two components of the state-space eigenvector compared to the control vector:

$$\begin{bmatrix} \tilde{\Psi}_1 \\ \tilde{\Psi}_2 \end{bmatrix}_{(d,q)} = \begin{bmatrix} e^{\mu_1 \cdot T} & 0 \\ 0 & e^{\mu_2 \cdot T} \end{bmatrix} \cdot \begin{bmatrix} \tilde{\Psi}_{10} \\ \tilde{\Psi}_{20} \end{bmatrix}_{(d,q)} + \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \cdot \vec{V}_{s(d,q)} \quad (1.123)$$

- the free evolution, during a symmetrical short-circuit of the three phases of the stator which corresponds to a null input voltage, each coordinate being only dependent on one of the eigenvalues, by construction of the eigenvector basis:

$$\begin{bmatrix} \tilde{\Psi}_1 \\ \tilde{\Psi}_2 \end{bmatrix}_{(d,q)}^0 = \begin{bmatrix} e^{\mu_1 \cdot T} & 0 \\ 0 & e^{\mu_2 \cdot T} \end{bmatrix} \cdot \begin{bmatrix} \tilde{\Psi}_{10} \\ \tilde{\Psi}_{20} \end{bmatrix}_{(d,q)} \quad (1.124)$$

This relation makes it possible to simply calculate the evolution of current, flux and torque during the three-phase short-circuit

- a steady state of the motor which supposes that the current control:

$$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \cdot \vec{V}_{s(d,q)} \quad (1.125)$$

compensates exactly the free evolution characterized by the diagonalized transition matrix in (d, q) :

$$\begin{bmatrix} e^{\mu_1 \cdot T} & 0 \\ 0 & e^{\mu_2 \cdot T} \end{bmatrix} \quad (1.126)$$

to obtain at the end of the period, the characteristic equality of the steady state, which takes account of the reference frame rotation and the initial state of the following period:

$$\begin{bmatrix} \tilde{\Psi}_1 \\ \tilde{\Psi}_2 \end{bmatrix}_{(d,q)} = \begin{bmatrix} \tilde{\Psi}_{10} \\ \tilde{\Psi}_{20} \end{bmatrix}_{(d,q)} \cdot e^{i \cdot \omega_s \cdot T} \quad (1.127)$$

The input to maintain a steady state defined by the initial state:

$$\begin{bmatrix} \tilde{\Psi}_{10} \\ \tilde{\Psi}_{20} \end{bmatrix}_{(d,q)} \quad (1.128)$$

is thus:

$$a \cdot \vec{V}_{s(d,q)} = \begin{bmatrix} e^{i \cdot \omega_s \cdot T} - e^{\mu_1 \cdot T} & 0 \\ 0 & e^{i \cdot \omega_s \cdot T} - e^{\mu_2 \cdot T} \end{bmatrix} \cdot \begin{bmatrix} \Psi_{10} \\ \Psi_{20} \end{bmatrix}_{(d,q)} \quad (1.129)$$

This relation has the same form in the (d_0, q_0) reference frame, since it is enough to multiply the two terms by $e^{i\omega_s T}$ to project it into the initial reference frame.

1.3.3 Control Calculation – Eigenstate-Space Equation System Inversion

The state-space equation (1.122) of complex second order, will now make it possible to calculate control simply by replacing, in the state-space equation, the predicted state-space vector Ψ by one set-point state-space vector Ψ_c in (d, q) :

$$\Psi = \Psi_c = \begin{bmatrix} \tilde{\Psi}_{1c} \\ \tilde{\Psi}_{2c} \end{bmatrix}_{(d,q)} = \begin{bmatrix} I_{dc} + i \cdot I_{qc} + \xi_1 \cdot \Phi_{rd} \\ I_{dc} + i \cdot I_{qc} + \xi_2 \cdot \Phi_{rd} \end{bmatrix} \quad (1.130)$$

The initial state being known and the sampling period T fixed, it only remains to invert the state-space equation and to calculate two unknowns, the voltage vector $\vec{V}_s$ and the average angular velocity ω_s during the sampling period T , eliminating successively $\vec{V}_s$ and ω_s , between the two complex equations. Let us multiply the two parts of (1.119) and (1.120) by $e^{i\omega_s T}$:

$$\begin{aligned} e^{i\omega_s T} \cdot \tilde{\Psi}_{1c} - e^{\mu_1 T} \cdot \tilde{\Psi}_{10} &= a_1 \cdot \vec{V}_s \cdot e^{-i\rho_0} \\ e^{i\omega_s T} \cdot \tilde{\Psi}_{2c} - e^{\mu_2 T} \cdot \tilde{\Psi}_{20} &= a_2 \cdot \vec{V}_s \cdot e^{-i\rho_0} \end{aligned} \quad (1.131)$$

Let us eliminate $\vec{V}_s$ between the two equations (1.131):

$$e^{i\omega_s T} = \frac{a_1 \cdot e^{\mu_2 T} \cdot \tilde{\Psi}_{20} - a_2 \cdot e^{\mu_1 T} \cdot \tilde{\Psi}_{10}}{a_1 \cdot \tilde{\Psi}_{2c} - a_2 \cdot \tilde{\Psi}_{1c}} \quad (1.132)$$

then let us eliminate $e^{i\omega_s T}$:

$$\vec{V}_s = e^{i\rho_0} \cdot \frac{e^{\mu_2 T} \cdot \tilde{\Psi}_{20} \cdot \tilde{\Psi}_{1c} - e^{\mu_1 T} \cdot \tilde{\Psi}_{10} \cdot \tilde{\Psi}_{2c}}{a_1 \cdot \tilde{\Psi}_{2c} - a_2 \cdot \tilde{\Psi}_{1c}} \quad (1.133)$$

The vector $\vec{V}_s$ is referenced in the (α, β) fixed frame where it will be used by the inverter to generate the stator voltage of the induction machine; vector coordinates of the initial state-space eigenvector Ψ_0 are written in the (d_0, q_0) reference frame, and vector coordinates of the set-point state-space eigenvector Ψ_c , in the (d, q) reference frame. The exact analytical control law is thus written in a very simple way.

The fact that ω_s is one of the unknowns can appear surprising. Actually, it should be remembered that the stator angular frequency variation compared to the rotor angular frequency also creates an electromagnetic torque variation, by modification of the angle between the stator flux and the rotor flux. In transient operation, it is thus necessary to vary this angle during the sampling period T ; this is equivalent to varying the average stator angular frequency which is not regarded as constant, except during a steady state.

The stator angular frequency will thus enable us to fix the phase of the (d, q) reference frame compared to the (d_0, q_0) reference frame, using equation (1.105) $\rho = \rho_0 + \omega_s \cdot T$, and it is in that (d, q) reference frame that we will be able to fix set-point values. The rotating reference frame

is interesting here only in terms of its initial position and its final position, since it does not rotate in a regular way in transient operation.

The two coordinates of the voltage vector and the position of set-point rotating reference frame, represent three control variables which will make it possible to regulate independently, the stator current vector and the rotor flux magnitude, i.e. three physical variables.

Actually, ω_s interests us only because it reveals a link between set-points I_d , I_q and Φ_{rd} in the set-point rotating reference frame, since if we except ω_s , we have only two control variables with stator voltage coordinates for regulating three physical variables on their respective set-point.

An example of the centered-symmetrical vectorial PWM calculation for generating the mean voltage vector (1.133), calculated by the control, is presented in detail in the appendix A.

1.4 Control

The inversion of state-space equations makes it possible to simply discover a new control method. We will analyze the various stages of its realization with an example of a real-time implementation at a constant sampling period.

1.4.1 Constitution of the Set-Point State-Space Vector

Let us eliminate $\vec{V}_s$ between the two coordinates of the equation system (1.122), which will enable us to introduce the three set-point scalars I_{dc} , I_{qc} and $\|\vec{\Phi}_{rc}\|$, in the (d, q) reference frame via $\vec{\Psi}_{1c}$ and $\vec{\Psi}_{2c}$.

$$a_1 \cdot \vec{\Psi}_{2c} - a_2 \cdot \vec{\Psi}_{1c} = e^{-i\omega_s T} \cdot (a_1 \cdot e^{\mu_2 T} \cdot \vec{\Psi}_{20} - a_2 \cdot e^{\mu_1 T} \cdot \vec{\Psi}_{10}) \quad (1.134)$$

$$\begin{aligned} a_1 \cdot (I_{dc} + i \cdot I_{qc} + \xi_2 \cdot \|\vec{\Phi}_{rc}\|) - a_2 \cdot (I_{dc} + i \cdot I_{qc} + \xi_1 \cdot \|\vec{\Phi}_{rc}\|) = \dots \\ \dots e^{-i\omega_s T} \cdot (a_1 \cdot e^{\mu_2 T} \cdot \vec{\Psi}_{20} - a_2 \cdot e^{\mu_1 T} \cdot \vec{\Psi}_{10}) \end{aligned} \quad (1.135)$$

$$I_{dc} + i \cdot I_{qc} + \|\vec{\Phi}_{rc}\| \cdot \left(\frac{a_1 \cdot \xi_2 - a_2 \cdot \xi_1}{a_1 - a_2} \right) = e^{-i\omega_s T} \cdot \left(\frac{a_1 \cdot e^{\mu_2 T} \cdot \vec{\Psi}_{20} - a_2 \cdot e^{\mu_1 T} \cdot \vec{\Psi}_{10}}{a_1 - a_2} \right) \quad (1.136)$$

The complex term:

$$\eta \cdot e^{i\zeta} = \frac{a_1 \cdot e^{\mu_2 T} \cdot \vec{\Psi}_{20} - a_2 \cdot e^{\mu_1 T} \cdot \vec{\Psi}_{10}}{a_1 - a_2} \quad (1.137)$$

is calculable starting from motor parameters, polar mechanical angular frequency and initial state-space Ψ_0 . Let us separate the real and imaginary parts of equation (1.136):

$$I_{dc} + \|\vec{\Phi}_{rc}\| \cdot \text{Re} \left(\frac{a_1 \cdot \xi_2 - a_2 \cdot \xi_1}{a_1 - a_2} \right) = \eta \cdot \cos(\zeta - \omega_s T) \quad (1.138)$$

$$I_{qc} + \|\vec{\Phi}_{rc}\| \cdot \operatorname{Im} \left(\frac{a_1 \cdot \xi_2 - a_2 \cdot \xi_1}{a_1 - a_2} \right) = \eta \cdot \sin(\zeta - \omega_s \cdot T) \quad (1.139)$$

We can now eliminate ω_s between (1.138) and (1.139) to compute I_{dc} :

$$I_{dc} = -\|\vec{\Phi}_{rc}\| \cdot \operatorname{Re} \left(\frac{a_1 \cdot \xi_2 - a_2 \cdot \xi_1}{a_1 - a_2} \right) + \eta \cdot \cos \left\{ \arcsin \left[\frac{I_{qc}}{\eta} + \frac{\|\vec{\Phi}_{rc}\|}{\eta} \cdot \operatorname{Im} \left(\frac{a_1 \cdot \xi_2 - a_2 \cdot \xi_1}{a_1 - a_2} \right) \right] \right\} \quad (1.140)$$

This relation forces us to remove the indeterminacies or multiple solutions related to the arcsine function, and it is not always obvious how to eliminate all foreign solutions.

It is possible to calculate the magnetizing current set-point by raising to second power equations (1.138) and (1.139), and by adding them to eliminate ω_s . After elimination of foreign solutions introduced by squaring, by considering only the case of an induction machine where $I_{dc} \geq 0$, we obtain ultimately:

$$I_{dc} = -\|\vec{\Phi}_{rc}\| \cdot \operatorname{Re} \left(\frac{a_1 \cdot \xi_2 - a_2 \cdot \xi_1}{a_1 - a_2} \right) - \sqrt{\eta^2 - \left[I_{qc} + \|\vec{\Phi}_{rc}\| \cdot \operatorname{Im} \left(\frac{a_1 \cdot \xi_2 - a_2 \cdot \xi_1}{a_1 - a_2} \right) \right]^2} \quad (1.141)$$

This last relation reveals the link between the three index values so that the rotor flux vector will be directed according to the d axis at the end of the time T . It can still be seen that this relation is different from the instantaneous relation in steady state or transient operation (1.30):

$$I_d = \frac{\Phi_r}{L_m} \quad (1.142)$$

It is actually the point of fixing the current set-point I_{dc} at time t_n , such that it makes it possible to obtain the current I_{qc} and the flux magnitude $\|\vec{\Phi}_{rc}\|$ at time $(t_n + T)$, with an average voltage vector during the whole period T . Thus, it is not about a relation describing an instantaneous electrical state, like equation (1.142).

This relation allows a total decoupling between the rotor flux and the electromagnetic torque tracking. An approximate solution appears in (Ortega and Taoutaou, 1996; Taoutaou *et al.*, 1997).

The set-point state-space eigenvector is thus built in the following successive stages:

- Choose of the strategy for fixing the flux set-point, according to equation (1.33). The flux can be always maximum whatever the required torque to avoid delaying the rise of the torque in transient operations, due to the large time constant of the rotor flux; this strategy maximizes the magnetizing current and thus motor and inverter losses. It is not recommended to choose this strategy if we can control, with satisfactory dynamics, the flux independently of the torque, which is our objective. A better strategy is to increase the flux at the same time as the torque set-point, following, for instance, a law proportional to the square root of the torque set-point, or a particular law relating torque and flux set-points, to minimize currents and losses (Ramirez and Canudas de Wit, 1996).

- Calculate the current I_{qc} starting from the torque set-point C_c and the choice of the rotor flux Φ_{rc} , according to selected strategy of the motor control, by the relation:

$$I_{qc} = \frac{C_c}{N_p \cdot \Phi_{rc}} \cdot \frac{L_r}{L_m} \quad (1.143)$$

- Calculate the current I_{dc} starting from equation (1.140) or (1.141) and from the magnitude η of equation (1.137).
- Constitute a set-point state-space eigenvector in (d, q) reference frame with Φ_{rc} , I_{qc} and I_{dc} :

$$\Psi_c = \begin{bmatrix} I_{dc} + i \cdot I_{qc} + \xi_1 \cdot \Phi_{rc} \\ I_{dc} + i \cdot I_{qc} + \xi_2 \cdot \Phi_{rc} \end{bmatrix}_{(d,q)} = \begin{bmatrix} \bar{\Psi}_{1c} \\ \bar{\Psi}_{2c} \end{bmatrix} \quad (1.144)$$

We fix thus the position of the (d, q) reference frame at the end of the time T , since simultaneously we impose $\Phi_{rq} = 0$ and thus indirectly θ and ω_s .

1.4.2 Constitution of the Initial State-Space Vector

The initial state-space vector is known by measurement of the stator current and the estimation of the rotor flux at the initial time t_n . $\vec{I}_s$ is in general obtained starting from current measures in two of the three phases, when there is no neutral current, and then by the Concordia transformation (cf. equation (1.3)). The estimation of the rotor flux $\vec{\Phi}_r$ can be made using an observer (Jacquot, 1995) in the (α, β) fixed frame; an example of real-time realization is provided below (cf. section 1.4.3.4). It is thus necessary to transfer coordinates of these two vectors from the (α, β) fixed frame into the (d_0, q_0) reference frame after having calculated the angle ρ_0 of the reference frame, with respect to the (α, β) fixed frame. Successively:

$$\rho_0 = \arg(\vec{\Phi}_r) \quad (1.145)$$

$$\Phi_{rd0} = \|\vec{\Phi}_r\| \quad (1.146)$$

$$I_{d0} + i \cdot I_{q0} = (I_{s\alpha} + i \cdot I_{s\beta}) \cdot e^{-i \cdot \rho_0} \quad (1.147)$$

Ultimately:

$$\Psi_0 = \begin{bmatrix} I_{d0} + i \cdot I_{q0} + \xi_1 \cdot \Phi_{rd0} \\ I_{d0} + i \cdot I_{q0} + \xi_2 \cdot \Phi_{rd0} \end{bmatrix}_{(d_0, q_0)} = \begin{bmatrix} \bar{\Psi}_{10} \\ \bar{\Psi}_{20} \end{bmatrix} \quad (1.148)$$

1.4.3 Control Process

1.4.3.1 Real-Time Implementation

The analytical control calculation being now complete, it remains as a practical problem to solve to allow a real-time implementation of control algorithms.

Indeed, it is necessary to be able to calculate the average voltage vector to be used between the initial time t_n , the time when the initial state-space eigenvector coordinates are measured and estimated and the time $(t_n + T)$, the time when set-points are reached. This has to be done without any lagging due to the computation time, which could create a time-delay of the voltage vector impressing on the stator, between initial time and the end of a complete period. Under these conditions, the time reserved for various computations is null.

In practice, it is therefore necessary to anticipate the impressing period of the voltage vector by one period reserved for computations. A second problem appears then because the initial state-space eigenvector has to be defined at time t_n to feed the computation and so it can no longer be based on the measurements at this time.

One solution is to make a computation during a first sampling period T , and to impress the voltage vector during the following second period. However, so as not to cause a pure time-delay of time T under these conditions, it is necessary to predict the motor state at the end of the computational period in progress, and thus at the beginning of the impressing period of the voltage vector; it is this prediction of the state-space eigenvector at the horizon t_n , starting from measurements carried out at $(t_n - T)$, at the beginning of the period reserved to computations (Jacquot, 1995), which then will be used to calculate the voltage vector to be applied between t_n and $(t_n + T)$.

It is thus necessary to define a first period for measurements, measure filtering, rotor flux estimation, prediction of the initial state-space eigenvector and computation of the control voltage vector, then a second period for voltage vector impressing.

During this second period, the voltage vector can be generated by a pulse width modulation, issued from an external circuit different from the main microprocessor, for instance a field programmable gate array (FPGA) circuit, thus releasing the processor for succeeding computations.

The sampling period was selected here as being constant as an example and thus it is asynchronous compared to the stator fundamental voltage. It can be selected as being variable to synchronize it with the stator voltage, as is obligatorily when using a square wave PWM, or to lower the amount of voltage harmonics and thus of current harmonics by choosing a PWM type more adapted to high speed. To change the computational period from one constant period generally used at low speed, to one period synchronous with the stator voltage, it is enough, after a synchronization phase, to calculate the control vector by regarding the period as variable and no longer as a constant parameter of equation (1.133).

The length of a constant sampling period is selected in relation to the time necessary for the algorithmic computations and to the duration of the mean voltage vector application which determines the current shape factor, itself in relation to the selected PWM. The sampling period, being identical to the periodicity of the control vector impressing, also conditions the inverter losses.

This structuring of algorithms for their real-time exploitation, leads to imposing one sampling period, identical to the PWM period and synchronous with it, while remaining asynchronous compared to the voltage fundamental component of the motor when the PWM period is constant at low speed, and synchronous compared to the voltage fundamental component of the motor when the PWM period is variable at high speed. This characteristic leads us to sample, in particular, stator phase currents in a synchronous way with the PWM and thus in a synchronous way with current harmonics created by the PWM; this provision makes it possible to remove the anti-aliasing filter of the current harmonic spectrum generated

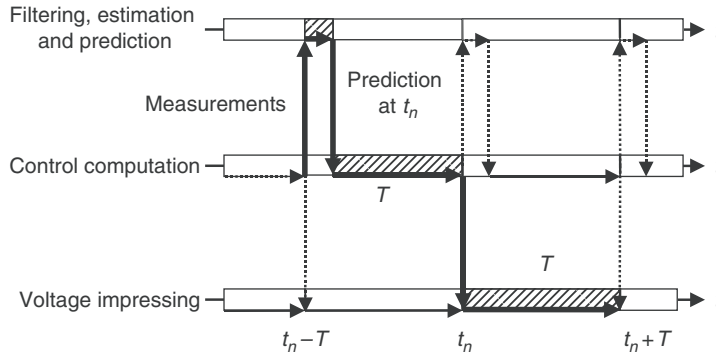


Figure 1.9 Timing diagram of the real-time control process

by the PWM, which is dominating compared to the stochastic spectrum of measurement disturbances, and thus it improves the bandwidth of measurements (Jacquot, 1995). Sampling without an anti-aliasing filter provides, under these conditions, the *instantaneous* value of the current fundamental added to all synchronous current harmonics at the beginning of each PWM period.

In short, the real-time implementation of computational algorithms requests to carry out a prediction of the motor state to constitute the initial state necessary for control vector computation. We can benefit from it, to use it for measure filtering and for observing variables which one cannot measure, by using discretized state-space equations. This relation will thus be used for predicting a state, filtering measurements and estimating non-measurable variables, in a cyclic way; it also was used to calculate the control vector after several transformations.

It should be noticed that set-points which are used in the algorithmic computations, are those which are known, in the example of Figure 1.9, at time $(t_n - T)$. Contrary to the motor state, set-points cannot, in the general case (except with a predefined programme of tracking), be predicted at time t_n ; they will thus be regarded as set-points at time t_n with one pure time delay period. These set-points will thus be reached at time $(t_n + T)$.

To open the loop of the cyclic computational process in real-time, we will define the time origin of the algorithm description.

1.4.3.2 Measure Filtering

The discretized state-space equation (1.57), applied during the period before the measurements:

$$X(t_n - T)_p = F \cdot X(t_n - 2 \cdot T) + G \cdot V \quad (1.149)$$

and which is used to make the prediction of the motor state at time $(t_n - T)$, can be used to filter current measures made at the same time $(t_n - T)$, with the Kalman filter (Kalman, 1982), (Jacquot, 1995). This filtering will be simultaneously used to observe the rotor flux at time $(t_n - T)$ (Jacquot, 1995), if this is not directly accessible by measurement, for example by a Hall effect probe or by a flux measurement coil.

The prediction by Kalman filter of the state-space vector at the horizon t_n , starting from filtered measurements, will then be used to define the initial state-space eigenvector $\Psi_0(t_n)$ for

the computation made between $(t_n - T)$ and t_n , of the control vector to be applied during the following period from t_n to $(t_n + T)$ (Jacquot, 1995).

1.4.3.3 Transition and Input Matrix Calculations

We must then calculate the transition matrix F and the input matrix G of discretized state-space equations of the motor in the (α, β) fixed frame, by comparing equation (1.57) to equation (1.93) of which we have first multiplied its two terms on the left by the transfer matrix P :

$$F = P \cdot e^{D \cdot T} \cdot P^{-1} \quad \text{and} \quad G = P \cdot D^{-1} \cdot (e^{D \cdot T} - I) \cdot P^{-1} \cdot B \quad (1.150)$$

Let us override the various elements by their computed values: P and P^{-1} in (1.86), $e^{D \cdot T}$ in (1.112), D^{-1} in (1.111) and $P^{-1} \cdot B$ in (1.110).

$$F = \frac{1}{\xi_0} \cdot \begin{bmatrix} -\xi_2 & -\xi_1 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} e^{\mu_1 \cdot T} & 0 \\ 0 & e^{\mu_2 \cdot T} \end{bmatrix} \cdot \begin{bmatrix} 1 & \xi_1 \\ -1 & -\xi_2 \end{bmatrix} \quad (1.151)$$

$$G = \frac{1}{\lambda \cdot \xi_0} \cdot \begin{bmatrix} -\xi_2 & -\xi_1 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{e^{\mu_1 \cdot T} - 1}{\mu_1} & 0 \\ 0 & \frac{e^{\mu_2 \cdot T} - 1}{\mu_2} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -1 \end{bmatrix} \quad (1.152)$$

The transition matrix in the (α, β) fixed frame, is thus written:

$$F = \frac{1}{\xi_0} \cdot \begin{bmatrix} \xi_1 \cdot e^{\mu_2 \cdot T} - \xi_2 \cdot e^{\mu_1 \cdot T} & -\xi_1 \cdot \xi_2 \cdot (e^{\mu_1 \cdot T} - e^{\mu_2 \cdot T}) \\ e^{\mu_1 \cdot T} - e^{\mu_2 \cdot T} & \xi_1 \cdot e^{\mu_1 \cdot T} - \xi_2 \cdot e^{\mu_2 \cdot T} \end{bmatrix} \quad (1.153)$$

We can still reduce the writing of the input matrix by using reduced variables $\{a_1, a_2\}$ defined in (1.121):

$$G = \frac{1}{\xi_0} \cdot \begin{bmatrix} a_2 \cdot \xi_1 - a_1 \cdot \xi_2 \\ a_1 - a_2 \end{bmatrix} \quad (1.154)$$

1.4.3.4 Kalman Filter, Observation and Prediction

Ultimately, the prediction used for the current filtering and the rotor flux observation at time $(t_n - T)$, starting from filtered variables at the previous time $(t_n - 2 \cdot T)$ takes the following form in (α, β) :

$$X(t_n - T)_p = F(t_n - 2 \cdot T) \cdot X(t_n - 2 \cdot T)_f + G(t_n - 2 \cdot T) \cdot V_{(t_n - 2 \cdot T) \rightarrow (t_n - T)} \quad (1.155)$$

$$Y(t_n - T)_p = H \cdot X(t_n - T)_p \quad (1.156)$$

with:

$$X = \begin{bmatrix} I_{s\alpha} \\ I_{s\beta} \\ \Phi_{r\alpha} \\ \Phi_{r\beta} \end{bmatrix} \quad \text{and} \quad V = \vec{V}_s = \begin{bmatrix} V_{s\alpha} \\ V_{s\beta} \end{bmatrix} \quad (1.157)$$

Let us choose the stator current vector $Y(t_n - T)_m$ as a measured variable at time $(t_n - T)$, and the rotor flux as the estimated variable; we thus define an output matrix H :

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \text{ (real form), or } H = [1 \quad 0] \text{ (complex form), and } Y = \begin{bmatrix} I_{s\alpha} \\ I_{s\beta} \end{bmatrix} \quad (1.158)$$

Briefly let us recall the various stages of the Kalman filtering at time $(t_n - T)$ (Jacquot, 1995):

- Filter of the stator current $\vec{I}_{sf}$ starting from the current measurement $Y(t_n - T)_m$ and from the predicted state-space vector $X(t_n - T)_p$, and rotor flux observation $\vec{\Phi}_{re}$:

$$X(t_n - T)_f = \begin{bmatrix} \vec{I}_{sf} \\ \vec{\Phi}_{re} \end{bmatrix} \quad (1.159)$$

The filtered state-space vector is calculated by the following relation:

$$X(t_n - T)_f = X(t_n - T)_p + K(t_n - T) \cdot \{Y(t_n - T)_m - Y(t_n - T)_p\} \quad (1.160)$$

- $K(t_n - T)$ is the Kalman gain calculated for the time $(t_n - T)$. The state-space vector, $X(t_n - T)_f$ contains the filtered current measure, but also the observed value of the rotor flux vector, in magnitude and argument, inaccessible simply by a direct measurement
- Calculate the prediction of the state error covariance matrix for t_n , from the one calculated for $(t_n - T)$:

$$P(t_n)_p = F \cdot P(t_n - T) \cdot F^T + Q_0 \quad (1.161)$$

- Q_0 is the state disturbance covariance matrix, considered in this example as constant to simplify the calculation of the matrix of the Kalman gain at time t_n , starting from the prediction of the state error covariance matrix at t_n :

$$K(t_n) = P(t_n)_p \cdot H^T \cdot \{H \cdot P(t_n)_p \cdot H^T + R_0\}^{-1} \quad (1.162)$$

- R_0 is the measurement disturbance covariance matrix, considered also in this example as constant
- Calculate the update of the state error covariance matrix at t_n from the predicted one at t_n :

$$P(t_n) = \{1 - K(t_n) \cdot H\} \cdot P(t_n)_p \quad (1.163)$$

The prediction starting from filtered measurements and from the observation at time $(t_n - T)$, necessary to the calculation of the initial eigenstate-space at time t_n , could thus be made thanks to the following state-space equation, where the control vector impressed during the calculation time, was provided at the end of the previous computational period:

$$X(t_n)_p = F(t_n - T) \cdot X(t_n - T)_f + G(t_n - T) \cdot V_{(t_n - T) \rightarrow t_n} \quad (1.164)$$

This same prediction $X(t_n)_p$ will be used again as a reference variable for a new measure filtering at time t_n , as with $X(t_n - T)_p$ above (1.160), in a cyclic way.

During the computational period of the control, it is in general necessary to make other computations, measurements, estimations, regulations and monitoring necessary to operate controlled process, so that the computation duration necessary for the motor control must be much shorter than the application duration of vector sequences calculated by the regulation.

1.4.3.5 Summary of Measurement, Filtering and Prediction

We will now summarize section 1.4.3 which relates to the real-time implementation, the measurement processing and the filtering, as well as the prediction of the initial state-space vector in Figure 1.10.

These computations are repeated during each PWM period. They start with the current measurement sampling, the input filter voltage sampling and the motor rotation speed sampling in a synchronous way with the beginning of the PWM, here at time $(t_n - T)$. All computations, including the control computation must be finished before the end of the PWM period T .

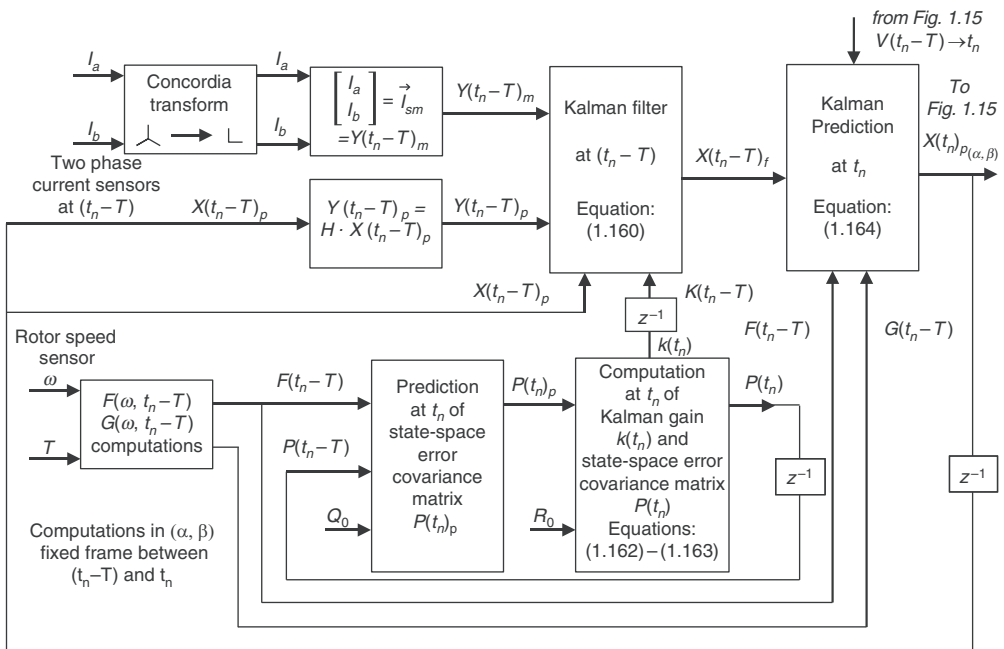


Figure 1.10 Measurements, filtering and prediction

These cyclic computations use some computation results from the previous period which are thus stored to be used one period later. To symbolize this storage, in Figure 1.10 we use the time-delay operator z^{-1} of one computational period.

When the neutral wire (or star point wire) is not connected to the motor, the vector sum of the instantaneous currents of the three phases is null, according to Kirchhoff's law. It is thus enough to measure currents of two phases to derive the current value in the third phase (cf. Figure 1.5) by the following relation.

$$\vec{I}_a + \vec{I}_b + \vec{I}_c = \vec{0} \quad (1.165)$$

The instantaneous current measurement must be made at the beginning of the sampling period with current sensors having a large bandwidth compared to the main harmonic frequencies generated by the inverter, to return the peak current of each of the three phases. For example, for a motor supplied with a fundamental voltage with the maximum frequency of 500Hz, the current sensor bandwidth could be 500kHz.

Current measurements of the three phases are then converted into just one stator current vector in the (α, β) orthonormal fixed frame by the Concordia relation which preserves the power (1.3).

The rotor flux measurement of an induction machine is not generally accessible. To be able to measure the air gap flux – which is actually the vector resultant of the stator flux decreased by the stator leakage flux, and of the rotor flux decreased by the rotor leakage flux – it would be necessary to install one measurement coil or one Hall effect probe.

The Hall effect probes are sensitive to the temperature, fragile and difficult to position with accuracy, but they can measure the constant component of the magnetic field, unlike a coil.

For the voltage e between the wrap terminals of the measurement coil to be sufficient, the flux variation has to be fast, which implies that the motor speed must be sufficiently high. The flux measurement coil cannot measure the air gap flux at low speed. Furthermore its wires and wraps are fragile and difficult to position.

$$e = \frac{d\Phi_e}{dt} \quad (1.166)$$

We will calculate the rotor flux using the Kalman estimator (1.160). It is thus about a flux estimation based on the current measurement $Y(t_n - T)_m$, and on the prediction of the same currents $Y(t_n - T)_p$, starting from the motor state prediction at same time $X(t_n - T)_p$. The motor state prediction is made at the previous sampling period according to equation (1.149). It also requires knowledge of Kalman gain of which we will see the calculation method below.

Practically, the Kalman estimator simultaneously makes it possible to estimate the flux and filter the stator current vector in the same operation, thanks to the discretized model of the motor. The filtered state-space vector $X(t_n - T)_f$ comprises the filtered stator current vector $\vec{I}_{sf}$ and the observed rotor flux vector $\vec{\Phi}_{re}$ (1.159). It is the observed flux which will be used to calculate the control vector (as we will see in Figure 1.15). The Kalman filtering will filter measurement disturbances due to stochastic noises of the measured signal, but also state-space disturbances related to statistical errors of the motor model.

The filtered state-space vector $X(t_n - T)_f$ from current measurements at time $(t_n - T)$, now will make it possible to predict the same state-space vector $X(t_n)_p$ at time t_n corresponding to the end of the computational period in progress, thanks to discretized state-space equations of

the motor (1.164). This predicted state-space vector constitutes the initial state-space vector for control calculating (cf. Figure 1.15).

This computation requires knowledge of the control voltage vector computed during the previous computational period (cf. Figure 1.15) and actually applied to the stator during the present computational period by a device which can be different from the main processor. It requires computation of the discretized evolution matrix and the discretized input matrix which we will now consider.

We start measuring the rotor mechanical angular frequency of the induction machine to convert it into an polar mechanical angular frequency by the multiplication by the pole pair number. The mechanical angular frequency can be estimated or measured by an incremental or absolute speed digital sensor. The choice of a speed sensor must take account of the lowest speed requiring accurate control; this defines the minimum number of pulses of a digital sensor per mechanical revolution of the rotor. To increase the accuracy at low speed, it is also possible to extrapolate the detection of pulse edges of the speed digital sensor and to thus carry out a time estimation of the signal transition, starting from measures of the preceding pulse edges and from a speed time derivative.

Using eigenvalues (1.64) and (1.65), and parameters of the induction machine (1.85) and (1.121), we calculate the transition matrix (1.153) and the input matrix (1.154) at time $(t_n - T)$. We then calculate the prediction at time t_n of the state error covariance matrix $P(t_n)_p$ by equation (1.161), starting from the transition matrix $F(t_n - T)$ and from the state error covariance matrix $P(t_n - T)$ at present time $(t_n - T)$. This last matrix was stored during the previous computation period.

In this computation we use a square matrix which is the state-space disturbance covariance matrix Q_0 , whose size is identical to that of the transition matrix; this matrix can be selected as constant to reduce computations, and its coefficients allow a weighting of state-space filtering for each coordinate of the state-space vectors. These coefficients can be adapted according to the process or according to the process operating mode.

The prediction of the state error covariance matrix $P(t_n)_p$ coupled with the measurement disturbance covariance matrix R_0 , make it possible to calculate the Kalman gain matrix at t_n by equation (1.162), then the state error covariance matrix $P(t_n)$ at time t_n by equation (1.163). These two last computations are stored to feed computations of the Kalman gain and measure filtering during the following period.

In this last computation we use a square matrix which is the measurement disturbance covariance matrix R_0 , of half the dimension of that of the transition matrix; it can be also be selected as constant to reduce computations, and its coefficients allow a weighting of the measure filtering for each coordinate of the current vector. These coefficients can be modified according to the process or to the process operating mode.

Lastly, it should be noted here that these computations can be performed with matrices with real coefficients, or with complex coefficients by using the rotor magnetic isotropy of non-salient pole induction machines, and this allows a halving of the numbers of rows and columns (cf. section 1.2.2) and which reduces the writing of relations.

With complex coefficients, dimensions of matrices are as follows:

$$F : 2 \times 2, G : 2 \times 1, H : 1 \times 2, P : 2 \times 2, Q_0 : 2 \times 2, R_0 : 1 \times 1$$

The real-time implementation of Kalman filter algorithms allows a sequential computation of the control vector during the sampling period with, simultaneously:

- current measure filtering
- rotor flux estimation
- motor state-space vector prediction at time of the end of computations.

Thanks to this provision, the pure time-delay of the control is equal to only one computational period.

1.4.4 Limitations

The control process which has just been described makes it possible to fix the stator current set-point (I_{dc}, I_{qc}) in magnitude and phase, as well as the set-point of the flux magnitude $\|\bar{\Phi}_{rc}\|$, to reach the electromagnetic torque set-point C_c in only one sampling period T , when physical variables are not limited by any constraint of the process sizing.

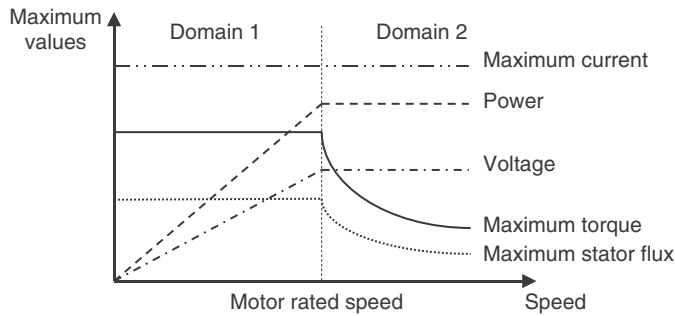


Figure 1.11 Rated speed at maximum torque

However, the feeding voltage of the motor through the inverter is in general limited. The stator current must always be limited in magnitude, either in an instantaneous way to ensure the commutation of the inverter or to restrict the heating of semiconductors, or to limit the temperature of motor windings. The magnetic flux is always limited, either by the voltage available at a given mechanical rotation speed or by the saturation of the magnetic steel sheets of the motor. The sizing of volume, mass, energy and cost of the system defines these limitations.

When the motor speed increases, the back electromotive force (b.e.m.f.) increases and, beyond the rated speed of the motor, the flux of the motor cannot be maintained anymore at its maximum value; for this reason the maximum torque can no longer be maintained beyond this speed without increasing the motor current. This operation beyond the rated speed requires the demagnetizing of the motor, to preserve for example a constant electrical power, and thus there will be a decrease of torque with speed.

Thanks to the motor control in only one sampling period, we now will analyze the way to comply, by anticipation, with all the limits due to the process sizing.

1.4.4.1 Voltage Limitation

V_M is the maximum voltage of the *voltage fundamental component* that the inverter can provide; when the PWM frequency is constant, the relation which connects this voltage to the DC voltage of the input filter U_c is given in a first approximation by the relation: $V_M = \frac{U_c}{\sqrt{2}}$ (cf. Figure 1.5).

If we take account of the dead-time corresponding to the minimum time of non-conduction of the IGBT, the maximum voltage magnitude of the voltage fundamental component is practically reduced to:

$$V_M = \frac{U_c}{\sqrt{2}} \cdot \left(1 - 2 \cdot \frac{t_m}{T} \right) \quad (1.167)$$

With the square wave PWM, the maximum voltage of the *voltage fundamental component* that the voltage inverter can provide at variable frequency is given by $V_M = \frac{\sqrt{6}}{\pi} \cdot U_c$ and the maximum instantaneous voltage corresponding to the magnitude of the six non-null voltage vectors of the voltage inverter is given by: $V_M = \sqrt{\frac{2}{3}} \cdot U_c$ (cf. Figure 1.5).

When we calculate the maximum flux which we can obtain, starting from the fundamental voltage of the inverter with an asynchronous PWM at a constant frequency, we choose the magnitude of the maximum fundamental voltage reduced by dead-times; this magnitude is given by equation (1.167).

The constraint of magnitude limitation of the average voltage vector is expressed then in (d, q) by its coordinates:

$$V_{sd}^2 + V_{sq}^2 \leq V_M^2 \quad (1.168)$$

Equation (1.12) is simplified by using the reduced parameters (1.14):

$$\vec{V}_s = R_{sr} \cdot \vec{I}_s + \sigma \cdot L_s \cdot \frac{d\vec{I}_s}{dt} - \alpha \cdot \frac{L_m}{L_r} \cdot \vec{\Phi}_r + \omega \cdot \frac{L_m}{L_r} \cdot \frac{\delta \vec{\Phi}_r}{\delta \theta} \quad (1.169)$$

Let us project this vector relation into the (d, q) reference frame; one can now write:

$$\frac{d\vec{I}_s}{dt} = \frac{d\vec{I}_s}{d\theta} \cdot \frac{d\theta}{dt} \quad (1.170)$$

Thus:

$$V_{sd} = R_{sr} \cdot I_{sd} - \sigma \cdot L_s \cdot \omega_s \cdot I_{sq} - \alpha \cdot \frac{L_m}{L_r} \cdot \Phi_{rd} \quad (1.171)$$

$$V_{sq} = R_{sr} \cdot I_{sq} + \sigma \cdot L_s \cdot \omega_s \cdot I_{sd} + \omega \cdot \frac{L_m}{L_r} \cdot \Phi_{rd} \quad (1.172)$$

These relations make it possible to express the inequality of fundamental components (1.168):

$$\begin{aligned} & \left(R_{sr} \cdot I_{sd} - \sigma \cdot L_s \cdot \omega_s \cdot I_{sq} - \alpha \cdot \frac{L_m}{L_r} \cdot \Phi_{rd} \right)^2 + \dots \\ & \dots \left(R_{sr} \cdot I_{sq} + \sigma \cdot L_s \cdot \omega_s \cdot I_{sd} + \omega \cdot \frac{L_m}{L_r} \cdot \Phi_{rd} \right)^2 \leq V_M^2 \end{aligned} \quad (1.173)$$

By developing the first term, then by gathering I_{sd} terms on the one hand, and I_{sq} terms on the other hand, we reveal the sum of two squared binomials:

$$(I_{sd} - I_{cd})^2 + (I_{sq} - I_{cq})^2 \leq \frac{V_M^2}{Z_{sr}^2} \quad (1.174)$$

with:

$$I_{cd} = -\frac{L_m}{L_r} \cdot \frac{\Phi_{rd}}{Z_{sr}^2} \cdot (-\alpha \cdot R_{sr} + \omega \cdot \sigma \cdot L_s \cdot \omega_s) \quad (1.175)$$

$$I_{cq} = -\frac{L_m}{L_r} \cdot \frac{\Phi_{rd}}{Z_{sr}^2} \cdot (\omega \cdot R_{sr} + \alpha \cdot \sigma \cdot L_s \cdot \omega_s) \quad (1.176)$$

$$Z_{sr}^2 = R_{sr}^2 + \sigma^2 \cdot L_s^2 \cdot \omega_s^2 \quad (1.177)$$

One meets voltage limits at high speed; we can then assimilate ω into ω_s since the absolute slip is very low compared to these two physical variables beyond the rated motor speed. Let us define the following reduced variables:

$$k_d = \frac{L_m}{L_r} \cdot \frac{\sigma \cdot L_s \cdot \omega^2 - \alpha \cdot R_{sr}}{Z_{sr}^2} \quad (1.178)$$

$$k_q = \frac{L_m}{L_r} \cdot \frac{\omega \cdot (\alpha \cdot \sigma \cdot L_s + R_{sr})}{Z_{sr}^2} \quad (1.179)$$

$$k_c = \frac{L_r}{N_p \cdot L_m} \quad (1.180)$$

$$I_{VM}^2 = \frac{V_M^2}{Z_{sr}^2} \quad (1.181)$$

Inequality (1.174) is reduced to:

$$\boxed{(I_{sd} + k_d \cdot \Phi_{rd})^2 + (I_{sq} + k_q \cdot \Phi_{rq})^2 \leq I_{VM}^2} \quad (1.182)$$

According to this last relation in the (d, q) reference frame, the extremity of the stator current vector must thus remain inside the circle of radius I_{VM} and center:

$$(I_{cd}, I_{cq}) = (-k_d \cdot \Phi_{rd}, -k_q \cdot \Phi_{rq}) \quad (1.183)$$

This circle represents the voltage limit. It is defined with current coordinates, in the (d, q) reference frame, due to the division of the maximum voltage magnitude V_M by the equivalent stator impedance magnitude Z_{sr} . This characteristic will enable us to represent voltage and current limits in the same plane to reveal the intersection of the two domain limits (surface boundaries) of operation.

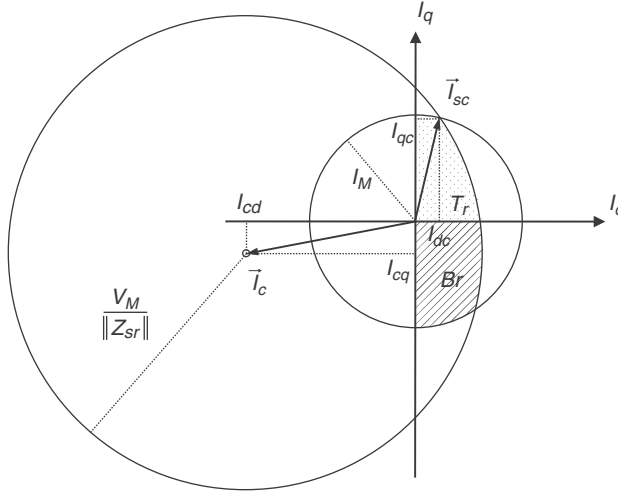


Figure 1.12 Current and voltage limits in the (d, q) rotating reference frame

1.4.4.2 Current Limitation

The stator current limit is a simple magnitude limit, at a definite maximum value for the inverter and the motor, during their sizing:

$$I_s^2 = I_{sd}^2 + I_{sq}^2 \leq I_M^2 \quad (1.184)$$

In the (d, q) plane, the current limit is a circle of radius I_M ; its center is the reference frame origin. We will see that these limits are similar to the limits of the surface-mounted permanent-magnet synchronous motor (SMPM-SM) (Attaianese *et al.*, 2002) and, in general, to the limits of motors which have a magnetic isotropy due to non-salient poles.

1.4.4.3 Operating Area and Limits

In short, the extremity of the reference current vector $\vec{I}_{sc}$ in the (d, q) reference frame must thus be both inside the circle of the voltage limit defined by (1.182), and inside the circle of the current limit defined by (1.184), according to Figure 1.12.

The authorized operation area Tr in a traction mode for $I_{qc} \geq 0$, and the operation area Br in an electrical braking mode for $I_{qc} < 0$, result from the intersection of the circles of voltage and current limits in Figure 1.12, for positive magnetizing currents.

As an example of a double limitation, the extremity of the stator set-point current vector $\vec{I}_{sc}$ represented in the (d, q) reference frame in Figure 1.12 is located simultaneously on circles of voltage and current limitations.

1.4.4.4 Set-Point Limit Algebraic Calculations

1.4.4.4.1 Voltage Limit – Flux Limit

Let us try to express the voltage limit according to the rotor flux and the set-point torque.

For that let us note:

$$Rxia = \text{Re} \left(\frac{a_1 \cdot \xi_2 - a_2 \cdot \xi_1}{a_1 - a_2} \right) \quad \text{and} \quad Ixia = \text{Re} \left(\frac{a_1 \cdot \xi_2 - a_2 \cdot \xi_1}{a_1 - a_2} \right) \quad (1.185)$$

The set-point of the magnetizing current for the control (cf. equation (1.141)) can be written as:

$$I_{dc} = -\|\vec{\Phi}_{rc}\| \cdot Rxia - \sqrt{\eta^2 - \left[I_{qc} + \|\vec{\Phi}_{rc}\| \cdot Ixia \right]^2} \quad (1.186)$$

It is necessary to utilize this current set-point in case of voltage limitation (1.182) and current limitation (1.184). The difficulty which arises then is that the expression of I_{dc} requires an initial knowledge of the flux and the active current set-points, but we do not know if either of the set-points should be limited to comply with the voltage limit and/or the current limit. The problem is thus looped.

For opening the loop for a numerical resolution and to thus avoid iterations, we can find a good approximation of the value of the magnetizing current set-point which takes account of the initial state and of the flux set-point by:

$$I_{dc} \cong -\|\vec{\Phi}_{rc}\| \cdot Rxia - \eta \quad (1.187)$$

This approximation is enough in general. It can be checked a posteriori, after computation of limits; it can be then improved, if necessary, by a second computation. Equation (1.143) will now be useful to calculate the maximum flux with equation (1.180):

$$I_{qc} = \frac{k_c \cdot C_c}{\Phi_{rc}} \quad (1.188)$$

Let us replace, in (1.182), the two current set-points (I_{dc}, I_{qc}) by their respective expression (1.187) and (1.188):

$$\left(-\Phi_{rc} \cdot Rxia - \eta + k_d \cdot \Phi_{rc} \right)^2 + \left(\frac{k_c \cdot C_c}{\Phi_{rc}} + k_q \cdot \Phi_{rc} \right)^2 \leq I_{VM}^2 \quad (1.189)$$

Developing and gathering the powers of Φ_{rc} :

$$\begin{aligned} & \left[(k_d - Rxia)^2 + k_q^2 \right] \cdot \Phi_{rc}^4 - 2 \cdot \eta \cdot (k_d - Rxia) \cdot \Phi_{rc}^3 + \dots \\ & \dots \left(\eta^2 + 2 \cdot k_q \cdot k_c \cdot C_c - I_{VM}^2 \right) \cdot \Phi_{rc}^2 + k_c^2 \cdot C_c^2 \leq 0 \end{aligned} \quad (1.190)$$

To calculate the maximum flux set-point at the voltage limit, for a given set-point torque, we now have to solve a fourth-degree equation of which the odd power coefficients are non-null. The method of the analytical resolution in real-time is well-known, so it is unnecessary to explain it in detail here; it is however useful to recall that foreign roots due to the squaring must be eliminated by likelihood tests.

If the first term of the preceding equation is negative, there is no voltage limitation; we can then choose, for example, the maximum flux of the machine as set-point. In the opposite case, the voltage limitation imposes the computation of the flux set-point limit by equation (1.190), after having eliminated foreign roots which are due to the squaring.

1.4.4.4.2 Voltage Limit and Current Limit

With the flux set-point, the active current is calculated by the traditional equation (1.188). I_{dc} is then calculated with the knowledge of I_{qc} and the flux set-point, by the exact equation (1.186). The checking of the current limitation is then made with the two components of the set-point stator current by (1.184).

If the current limit is exceeded, we replace in equation (1.184), active and magnetizing simplified currents, by their respective expression:

$$\left(-\Phi_{rc} \cdot Rxia - \eta\right)^2 + \frac{k_c^2 \cdot C_c^2}{\Phi_{rc}^2} \leq I_M^2 \quad (1.191)$$

Owing to the fact that an additional constraint has just been added, it is not possible any more to maintain the set-point torque with simultaneous voltage and current limitations, and therefore with a power limitation. It is thus necessary that the torque varies roughly according to the inverse function of the speed. The value of the maximum torque thus becomes one solution of the equation system (1.190) and (1.191), that is to say of the following equations:

$$\begin{aligned} & \left[(k_d - Rxia)^2 + k_q^2 \right] \cdot \Phi_{rc}^4 - 2 \cdot \eta \cdot (k_d - Rxia) \cdot \Phi_{rc}^3 + \dots \\ & \dots \left(\eta^2 + 2 \cdot k_q \cdot k_c \cdot C_c - I_{VM}^2 \right) \cdot \Phi_{rc}^2 + k_c^2 \cdot C_c^2 \leq 0 \\ & Rxia^2 \cdot \Phi_{rc}^4 + 2 \cdot \eta \cdot Rxia \cdot \Phi_{rc}^3 + \left(\eta^2 - I_M^2 \right) \cdot \Phi_{rc}^2 + k_c^2 \cdot C_c^2 \leq 0 \end{aligned} \quad (1.192)$$

There are thus now two unknowns to be calculated using this system of the two equations, i.e. the maximum value of the flux and of the torque set-point.

We start by calculating C_c at the limit according to the flux, by subtracting the two relations for eliminating the squared set-point torque $k_c^2 \cdot C_c^2$:

$$C_c = - \frac{\left(k_d^2 + k_q^2 - 2 \cdot k_d \cdot Rxia \right) \cdot \Phi_{rc}^2 - 2 \cdot \eta \cdot k_d \cdot \Phi_{rc} + \left(I_M^2 - I_{VM}^2 \right)}{2 \cdot k_q \cdot k_c} \quad (1.193)$$

Then, by using the expression of the maximum torque in equation (1.190) and after arranging terms in the descending powers of Φ_{rc} , we obtain, in the limit:

$$\begin{aligned} & \left(k_d^2 + k_q^2 \right) \cdot \left[\left(k_d^2 + k_q^2 \right) + 4 \cdot Rxia \cdot (Rxia - k_d) \right] \cdot \Phi_{rc}^4 + \dots \\ & \dots 4 \cdot \eta \cdot \left(k_d^2 + k_q^2 \right) \cdot \left(2 \cdot Rxia - k_d \right) \cdot \Phi_{rc}^3 + \dots \\ & \dots 2 \cdot \left\{ \left(k_d^2 + k_q^2 \right) \cdot \left(2 \cdot \eta^2 + I_M^2 - I_{VM}^2 \right) \right. \\ & \quad \left. - 2 \cdot \left[k_q^2 \cdot I_M^2 + k_d \cdot Rxia \cdot \left(I_M^2 - I_{VM}^2 \right) \right] \right\} \cdot \Phi_{rc}^2 - \dots \\ & \dots 4 \cdot \eta \cdot k_d \cdot \left(I_M^2 - I_{VM}^2 \right) \cdot \Phi_{rc} + \left(I_M^2 - I_{VM}^2 \right)^2 = 0 \end{aligned} \quad (1.194)$$

This relation is solvable with Φ_{rc} , which then makes it possible to calculate C_c by (1.193), then I_{qc} and I_{dc} by (1.188) and (1.186) respectively.

To calculate the flux in the case of voltage and current limitations, that amounts to privileging the flux realization before that of the torque, since the value of the possible torque is calculated starting from this flux value; this approach is imposed by physics because the torque results from the current and from the flux.

1.4.4.4.3 Current Limit

If the first term of equation (1.190) is negative, there is no voltage limitation. It should, however, be checked to see if there is a current limitation, by (1.191).

If that is the case, we start ensuring first the set-point flux positioning, in general equal to the maximum flux when there is no voltage limitation, while imposing:

$$I_{dc} = -\Phi_{rc} \cdot R_{xia} - \eta \quad (1.195)$$

only if the magnetization current is lower than the maximum current. In the opposite case we choose a magnetization current equal to the maximum current, in particular the case at starting to magnetize the motor. In this case, the torque is null at the very beginning of starting.

Thereafter, the remaining current is used to ensure a torque, lower than the initial set-point torque, since hypothetically the total current is limited:

$$I_{qc} = \text{sgn}(C_c) \cdot \sqrt{I_M^2 - I_{dc}^2} \quad (1.196)$$

A simultaneous progressive set-point of flux and torque makes it possible to avoid a current limitation with null torque at motor starting, and thus to gradually obtain the maximum torque without a dead-time due to installation of the maximum flux. A fast modification of the flux set-point indeed, requires a magnetizing or demagnetizing current set-point of which amplitude directly controls the flux gradient. A progressive variation of the flux set-point thus makes it possible to obtain the set-point torque faster.

1.4.4.4.4 Various Cases of Limitation

Figure 1.13 illustrates the various cases of limitation.

For various cases of limitation, the computation of the two components I_{dc} and I_{qc} of the set-point stator current vector $\vec{I}_{sc}$ in the set-point rotating reference frame can be made using the decision tree of Figure 1.14.

The results obtained in the section 1.4.4.4, make it possible to detect and characterize various cases of voltage, current or torque limitations. According to the diagnosis, established results make it possible to calculate the maximum set-point current vector which allows reaching simultaneously the optimal flux and torque.

Before knowing the possible limitations, we must choose the rotor flux that we want to reach, with the set-point torque necessary for the application. Indeed, according to equation (1.33), the motor torque C is the result of the product $\Phi_r \cdot I_q$. It is thus possible to reach the set-point torque by various paths. The higher the rotor flux is while remaining below the magnetic saturation, the less the current I_q will be for delivering the required torque; however, the higher the flux is, the higher the current I_d will have to be, and that draws the current vector closer to the stator current limit. It is thus necessary to find the optimal trade-off between these three components.

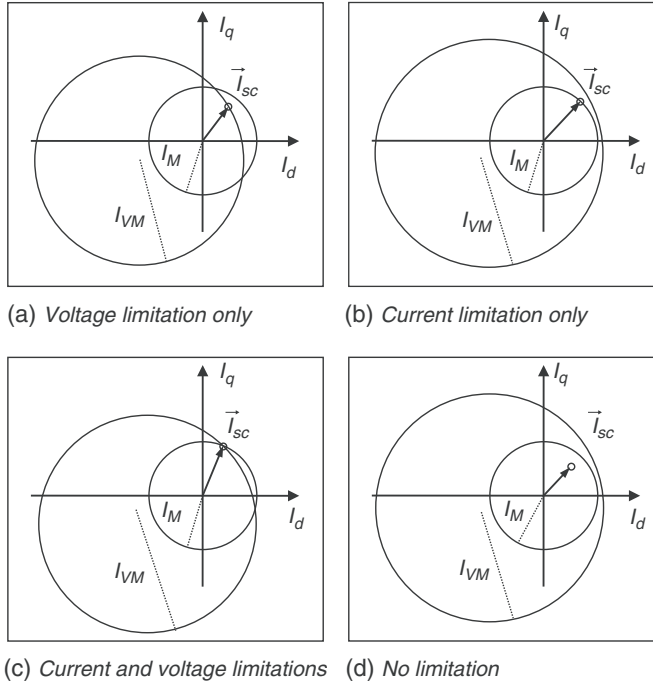


Figure 1.13 Various limitation cases in traction operations

When the required torque is the rated maximum torque of the motor, we will thus approach the flux limit, if the feeding voltage of the motor allows it. For a low set-point torque, an important flux set-point would lead to a high magnetization current and a low active current, which is not the optimum. One can seek an optimal sharing between magnetizing and active currents *in steady state* by using equation (1.142) to replace the rotor flux in equation (1.33).

$$C = N_p \cdot \frac{L_m^2}{L_r} \cdot I_d \cdot I_q \quad (1.197)$$

The sum of two numbers of which their product is constant is minimum when these two numbers are equal. The product $I_d \cdot I_q$ is constant because it depends on the set-point torque. The sum $I_d + I_q$ is thus minimum when $I_d = I_q$. In this case, squared current magnitude $\|\vec{I}_s\|^2$ is also minimum, because squared sum is minimum and product is constant:

$$\|\vec{I}_s\|^2 = I_d^2 + I_q^2 = (I_d + I_q)^2 - 2 \cdot I_d \cdot I_q \quad (1.198)$$

The optimum *in steady state* operation, will thus be reached when $I_d = I_q$. The current magnitude will then be minimum for a given set-point torque.

It should be noted here that this solution is not of interest for continuous controls with low dynamics. Indeed, the time constants of rising of the two components of the stator current

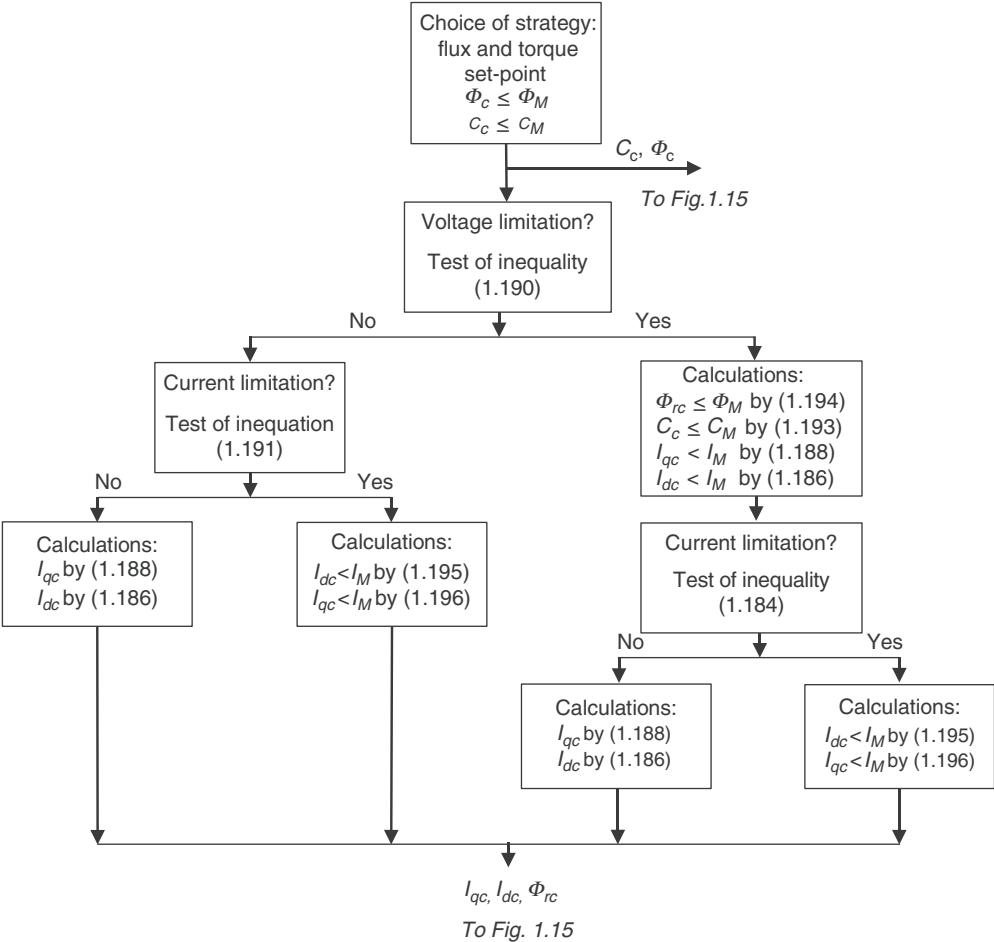


Figure 1.14 Current and voltage limitations

vector are not identical; the active current is settled very quickly in the equivalent resistance R_e of the rotor, whereas the magnetizing current is settled more slowly in the magnetizing inductance λ_m of the equivalent circuit in Figure 1.4. Motor controls with low dynamics, privilege rising up to the maximum flux, so as not to be slow on torque rises in the event of disturbances. This solution especially leads to a low efficiency at low torques.

To fully use this strategy of flux varying in dynamic operations, it is necessary to be able to *dynamically decouple* the torque adjustment from the flux adjustment and thus the adjustment of the two components of the stator current. We will show in section 1.4.5 that the dead-beat control allows this dynamic decoupling.

Whatever the selected strategy, torque and flux set-points are necessary for controlling the motor. If one can only calculate the torque set-point in the whole operation area, one will fix the flux set-point at the maximum flux of the motor below the iron saturation.

The flux set-point and the torque set-point are then used to test inequality (1.190). If this inequality is not satisfied, we will not be able to simultaneously obtain the flux and the torque

set-point, which proves that we have reached a voltage limitation which does not allow us to obtain at the same time the magnetizing and active currents (Figure 1.13a–c).

In this case, we will start by calculating the maximum flux magnitude which we can obtain with the maximum available voltage, at actual motor speed by equation (1.194); equation (1.193) gives the maximum torque corresponding to the maximum flux under the same conditions. We calculate then the I_{qc} necessary to obtain the torque by (1.188), then I_{dc} , to obtain the flux by (1.186).

The computation of the two components of the stator current will then enable us to check if the magnitude of the current vector exceeds the maximum of the current defined by the application, by equation (1.184). If that is the case, it will be enough to again calculate the magnetizing current set-point by the approximate equation (1.194) to ensure the maximum flux, then to again calculate the active current set-point by equation (1.196) to ensure the maximum torque, with voltage and current limitations simultaneously (Figure 1.13c).

Without current limitation, the preceding computation of the two current set-points remains unchanged (Figure 1.13a).

If equation (1.190) does not indicate any voltage limitation (Figure 1.13b or d), it is enough to check whether there is a current limitation by using equation (1.191). If that is the case, (Figure 1.13b) the computation of new coordinates of the stator current starts with the computation of the magnetizing current by (1.195), and continues with the computation of the active current to obtain the maximum torque by equation (1.196).

If there is no voltage (or flux) limitation, nor any current limitation (Figure 1.13d), the two current components I_{qc} and I_{dc} are calculated by (1.188) and (1.186) respectively.

In the event of flux and torque limitation, the strategy which was retained is to initially privilege the rising of the maximum flux compatible with the available feeding voltage then to define the maximum torque corresponding to this flux. This strategy corresponds to the machine sizing such that it is defined up to its limits above the rated motor speed (field 2 of Figure 1.11). We notice in this Figure that above the rated motor speed, the flux and the torque decrease simultaneously in inverse ratio of the speed, which leads to simultaneously reducing the two components of the current vector and thus to obtaining the maximum torque with the minimum current magnitude. In this area, the motor power consumption is appreciably constant.

Other strategies of limitation are possible, for example operations only in the first area of Figure 1.11. The analysis made in section 1.4.4 makes it possible to adapt the control strategies to process constraints.

It is thus shown that it is possible to compute current set-points, during the phase of the computation of the dead-beat control, which comply with all limits. These set-points, and possibly the limits, will be reached at the end of the impressing period of the control voltage vector, thanks to the prediction of the motor state-space evolution.

In the event of limitations of all types, the control process is modified neither in its principle nor in its characteristics; only the way of computing the set-point changes by *anticipating* limitations, which makes it possible to have only one operating mode and thus neither change of gain nor of dynamics will generate control instabilities; this process does not lead to any time-delay for detecting limitations; the control pure time-delay, equal at maximum to one sampling period, is compensated by the prediction based on the motor model.

1.4.4.4.5 Transitory Voltage Limit

In addition, if variations of the flux or torque set-points are very fast compared to the stator and to the rotor time constants of the motor, the calculated voltage vector can *instantaneously* exceed the maximum voltage vector that the inverter could provide.

Indeed, depending on the motor characteristics and the chosen sampling period, it may be that current or flux set-points cannot be reached in only one impressing period of the stator voltage.

This limitation is different from the voltage limitation calculated previously in the stabilized mode and intervenes only on set-point transients of processes requiring very high dynamics.

To rapidly vary the rotor flux, it would indeed be necessary to have a voltage source such that it can impose a current gradient making it possible to reach the current set-points in leakage and magnetizing inductances at the end of only one sampling period.

Sharp variations of set-points could all the more easily be satisfied that the voltage margin – the difference between the feeding voltage and the counter voltage of the motor – is large. At low speed, the motor b.e.m.f. is small. At a speed higher than the rated motor speed, the motor b.e.m.f. has a bigger amplitude and the sudden variations of set-points cannot be rapidly satisfied.

In the opposite case, at high speed, the computation of the voltage vector to be applied by (1.133) will lead to a voltage vector having the correct phase, therefore the correct direction, but of which the magnitude could not be generated by the inverter and will thus be insufficient to achieve the current set-point in only one period. The voltage magnitude will be limited by the inverter. The stator current vector, however, will evolve in the direction of the extremity of the current reference vector, starting from its initial position, without being able to reach it during the first sampling period. The extremity of the stator current vector will progress from one period to the next, in the direction of the extremity of the set-point vector, by using all the available voltage.

Indeed, we saw in section 1.3.2 that the vectors $a_1 \cdot \vec{V}_{s(d,q)}$ and $a_2 \cdot \vec{V}_{s(d,q)}$ represent the evolution in the (d, q) reference frame of currents created by the voltage vector $\vec{V}_{s(d,q)}$ in the interval T . These current vectors, added with the free evolution of the two initial vectors $e^{\mu_i T} \cdot \vec{\Psi}_{i0(d,q)}$, $i \in \{1; 2\}$, during the time interval T , make it possible to predict the two vectors $\vec{\Psi}_{ip}$, $i \in \{1; 2\}$, at time $(t_n + T)$. The two coefficients of the input matrix a in equation (1.121) are complex; they reorient the voltage vector, modifying its magnitude and transforming the control voltage vector into a control current vector, since the denominator of each coefficient is homogeneous with a reactance and that their numerator is without dimension.

When the two predicted vectors $\vec{\Psi}_{ip}$, $i \in \{1; 2\}$, which are the two state-space vector components of the predicted state-space eigenvector, are replaced by the two reference state-space vector components of the set-point state-space eigenvector, it could be that the distance between the two extremities of the free evolution and the set-point vectors, of each vector composing the coordinates of state-space vectors, would be too important compared to the voltage delivered by the inverter, so that the set-point cannot be reached in only one period.

From equations (1.119) and (1.120), the two conditions:

$$\left\| \vec{\Psi}_{ic} - e^{\mu_i T} \cdot \vec{\Psi}_{i0} \right\|_{(d,q)} \leq \|a_i\| \cdot V_M \quad \forall i \in \{1; 2\} \quad (1.199)$$

must be both verified with V_M representing the maximum voltage delivered by the inverter. However, the two coefficients a_i , depend on:

- the polar mechanical angular frequency via the eigenvalues
- the electromechanical time constants
- and the time by the exponential function of the numerator.

These two coefficients are null for $T=0$, and their numerator tends towards 1 when $T \rightarrow \infty$. It is thus necessary to apply the voltage for a long time because the voltage is low for overcoming the inductive reverse voltages, i.e. the lower the voltage, the longer must be the voltage application time.

We can also conclude from this analysis that only the extremities of set-point vectors located inside a circle of radius $\|a_i\| \cdot V_M$, $i \in \{1; 2\}$, centered on the extremity of the free evolution of the initial vector, could be reached in only one sampling period. To always be satisfied, this condition must be taken into account as soon as we know the inverter and the motor sizing.

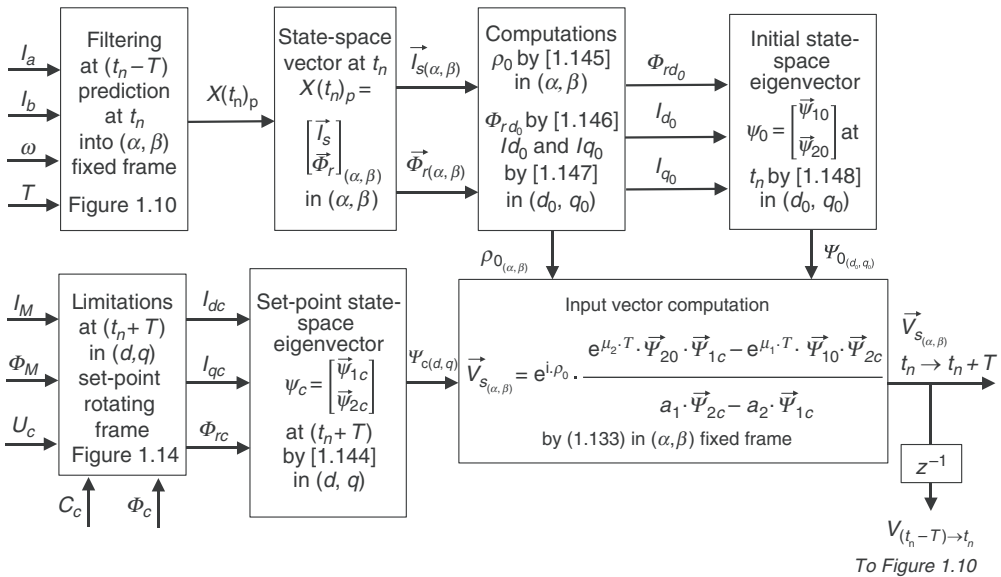
The two control current vectors $a_i \cdot \vec{V}_s$ with $i \in \{1; 2\}$, are represented in Figure 1.18 for a steady state operation, as well as the two circles of the transitory limitation drawn for the maximum available voltage; the two points of Figure 1.18 representing the extremities of the two set-point vectors, the coordinates of the set-point state-space eigenvector for a steady state, are located inside the two circles of the transient voltage limitation.

A geometrical representation of successive limitations of the transitory voltage is made in a simpler case of a synchronous motor in chapter 2, section 2.4.4.4.4.

1.4.4.4.6 Control Vector Computation

A summary of the computation of the control voltage vector is represented in Figure 1.15. We will detail it below.

This computation is repeated at each PWM period. It is carried out after the measure sampling at time $(t_n - T)$, their filtering, the rotor flux estimation and the motor state prediction, and it must be finished before the end of the computational period, i.e. before the prediction at time t_n . These cyclic computations use some computational results of the previous period



Mean voltage vector calculation between t_n and $(t_n - T)$

Figure 1.15 Computation of the mean voltage vector

which are thus stored to be used one period later. To symbolize this storage, we use in Figure 1.15 the operator z^{-1} of one computational period time-delay.

We saw in section 1.4.3.5 (cf. Figure 1.10) how to measure the motor phase currents, then how to filter the values resulting from the measurements, to estimate the rotor flux and finally how to predict the motor state at the end of the computational period; this motor state prediction $X(t_n)_p$ is used to define the initial state in the (α, β) fixed frame for the control computation.

This state-space vector is made from the predicted stator current vector $\vec{I}_{sp}$ and the predicted rotor flux vector $\vec{\Phi}_{rp}$ at time t_n in the (α, β) fixed frame.

$$X(t_n)_p = \begin{bmatrix} \vec{I}_{sp} \\ \vec{\Phi}_{rp} \end{bmatrix}_{(\alpha, \beta)} = \begin{bmatrix} I_{s\alpha} \\ I_{s\beta} \\ \Phi_{r\alpha} \\ \Phi_{r\beta} \end{bmatrix} \quad (1.200)$$

We notice that $\Phi_{r\beta}$ cannot be null, since the rotor flux was estimated in the fixed frame; its two coordinates thus enable us to define the angle ρ_0 of the initial rotating reference frame (d_0, q_0) using equation (1.145) and the flux magnitude Φ_{r0} from equation (1.146); using ρ_0 we calculate the new current vector coordinates in the (d_0, q_0) initial rotating reference frame by (1.147).

The initial state-space vector in the (d_0, q_0) rotating reference frame, becomes:

$$X(t_n)_0 = \begin{bmatrix} \vec{I}_{s0} \\ \vec{\Phi}_{r0} \end{bmatrix}_{(d_0, q_0)} = \begin{bmatrix} I_{d0} \\ I_{q0} \\ \Phi_{d0} \\ 0 \end{bmatrix} \quad (1.201)$$

The rotor flux abscissa in the rotating reference frame becomes null by definition of the rotating reference frame.

It is then enough to calculate the state-space eigenvector Ψ_0 in the (d_0, q_0) reference frame by equation (1.148), with motor parameters defined at the measured polar mechanical angular frequency ξ_1 and ξ_2 (1.85).

In addition, we saw in section 1.4.4.4.4 (cf. Figure 1.14) various cases of voltage, flux, current and torque limitations, and how to calculate set-points of the two coordinates of the stator current vector I_{dc} and I_{qc} as well as the set-point of the rotor flux magnitude Φ_{rc} , by taking into account these limitations and the torque and flux set-points provided by the process; these set-points are used to define a set-point state in the (d, q) set-point rotating reference frame for the control computation.

The set-point state-space vector defines the control goals at time $(t_n + T)$.

$$X(t_n + T)_c = \begin{bmatrix} \vec{I}_{sc} \\ \vec{\Phi}_{rc} \end{bmatrix}_{(d, q)} = \begin{bmatrix} I_{dc} \\ I_{qc} \\ \Phi_{dc} \\ 0 \end{bmatrix}_{(d, q)} \quad (1.202)$$

We notice again here that the rotor flux magnitude becomes the rotor flux vector abscissa in the set-point rotating reference frame, by definition of the (d, q) reference frame.

$$\Phi_{rc} = \|\vec{\Phi}_{rc}\| = \Phi_{dc} \quad (1.203)$$

It is then enough to calculate the state-space eigenvector Ψ_c in (d, q) by equation (1.144) with motor parameters defined at the measured polar mechanical angular frequency ξ_1 and ξ_2 (1.85).

The control voltage vector to be applied during the period following the present computation period from t_n to $(t_n + T)$ is calculated simply by equation (1.133) from Ψ_c in the (d, q) reference frame, from Ψ_0 in the (d_0, q_0) reference frame and from the motor parameters at the angular frequency ω : μ_1 (1.64), μ_2 (1.65), a_1 and a_2 (1.121).

The mean voltage vector is stored (magnitude and angle), to be used on the one hand by the PWM, and on the other hand by the computation of the following period (cf. Figure 1.10).

Flux and torque set-points known at time $(t_n - T)$ are achieved at time $(t_n + T)$, starting from measures made at time $(t_n - T)$, from computations done from $(t_n - T)$ to t_n and from the PWM applied from t_n to $(t_n + T)$. Thanks to the prediction of the motor state at time t_n , and to the computation of the motor state evolution from t_n to $t_n + T$, the control makes it possible to reach the set-point in only one period T , which is the characteristic of a *dead-beat* response. We can, however, notice that between the reading of the torque set-point, which cannot be predicted, and its realization, it runs out two periods of which one period is pure time-delay.

1.4.5 Example of Implementation

To concretize the implementation of this control process, we choose an induction machine having following characteristics in a traction mode:

- $N_p = 2$
 - $R_s = 29.7 \text{ m}\Omega$, $R_r = 25.3 \text{ m}\Omega$
 - $L_s = 9.93 \text{ mH}$, $L'_s = 9.48 \text{ mH}$, $L_r = 9.78 \text{ mH}$
 - $\omega_M = 2 \cdot \pi \cdot 120 \text{ rd/s}$
 - $\Phi_M = 1.33 \text{ Wb}$
 - $C_M = 885 \text{ m.N}$
 - $I_M = 385 \text{ A}$
 - $T = 800 \mu\text{s}$
- (1.204)

Values of the vector magnitude are given here directly in the two-phase fixed frame from the Concordia transformation which preserves the instantaneous power.

The motor rated speed is appreciably less than the half of its maximum speed.

The rated DC feeding voltage of the inverter is 720V.

The motor is supplied according to the equivalent circuit in Figure 1.16, through an inverter.

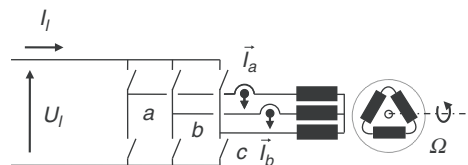


Figure 1.16 Inverter and induction machine

1.4.5.1 Adjustment of Flux and Torque – Limitations in Traction Operation

To allow visualization of all phenomena at various time scales, we chose to force the maximum electromagnetic torque in the motor. The mechanical speed is very quickly increasing from zero to the maximum speed during a timescale less than two seconds, which it would have been difficult to realize in practice. The control and the motor were thus simulated with accuracy. The inertia is not simulated and a constant high speed gradient is imposed.

The starting takes place with null current, null flux and null speed.

The *flux–torque* control strategy that we choose is a strategy with a maximum flux and torque. In addition, in this example, we will impose an electromagnetic torque set-point, only after the maximum flux has been established (priority of the flux over the torque). This strategy makes it possible to visualize, in Figure 1.17, *tracking independence* between the flux and the torque, as well as the minimum delay necessary to totally magnetize the motor with the maximum stator current. It is not the optimal strategy, either for energy consumption (except with a maximum torque), or with respect to the electromagnetic torque availability at starting.

In Figure 1.17 at starting, the stator current increases in only one sampling period of $800\mu\text{s}$ from zero to the maximum current. During a first period of 0.2s approximately, the stator current magnitude remains at its maximum. During this time, the rotor flux increases up to its maximum value. The flux gradient is thus only limited by the available current. During this time, the torque set-point and thus the electromagnetic torque are both null.

After a first period of flux rising, the torque set-point is fixed at a value a little bit below its maximum value that causes a small current decrease. The set-point torque is reached in only one sampling period. We can visualize an instantaneous change of the current phase at the transition time because of the introduction of an active current component and a fast adjustment of the voltage value necessary for a fast current and torque change.

The simulation was made *without* the input filter. Indeed, it is well-known (Délémontey, 1995) that an operation in a traction mode causes an instability of the input filter voltage starting from a low power consumption from the filter and continuing until the divergence, owing to the fact that this filter was dimensioned to have very few losses, and thus a very low damping. This simulation was parameterized to highlight all the dynamics and the compliance with all the process limits of this control. It is obvious that with an input filter, such fast torque variations can also set off instabilities, and thus distort the demonstration.

However, we will see later on how to stabilize the input filter in all circumstances and to preserve high dynamics of this control method in the same conditions, with a regulation process of the input filter voltage, of the same type as that of the motor control, acting in symbiosis.

After 0.8s approximately, the motor rated speed is reached with the maximum available stator voltage. As the speed continues to increase, the flux is thus limited by the available voltage from this instant. It will therefore have to decrease gradually up to the maximum speed.

After about one second, the maximum current is again reached in spite of the current component reduction of the motor magnetization corresponding to the rotor flux reduction. The set-point torque thus cannot be maintained any more beyond this limit; it is limited by the voltage and thus by the flux, but also by the current. The available torque thus decreases gradually with the speed.

This figure makes it possible to visualize current, flux and torque tracking performance of this control method. In practice, we should prefer to gradually magnetize the motor to avoid a pure time-delay for torque increase, a delay due to the motor magnetizing; we should also

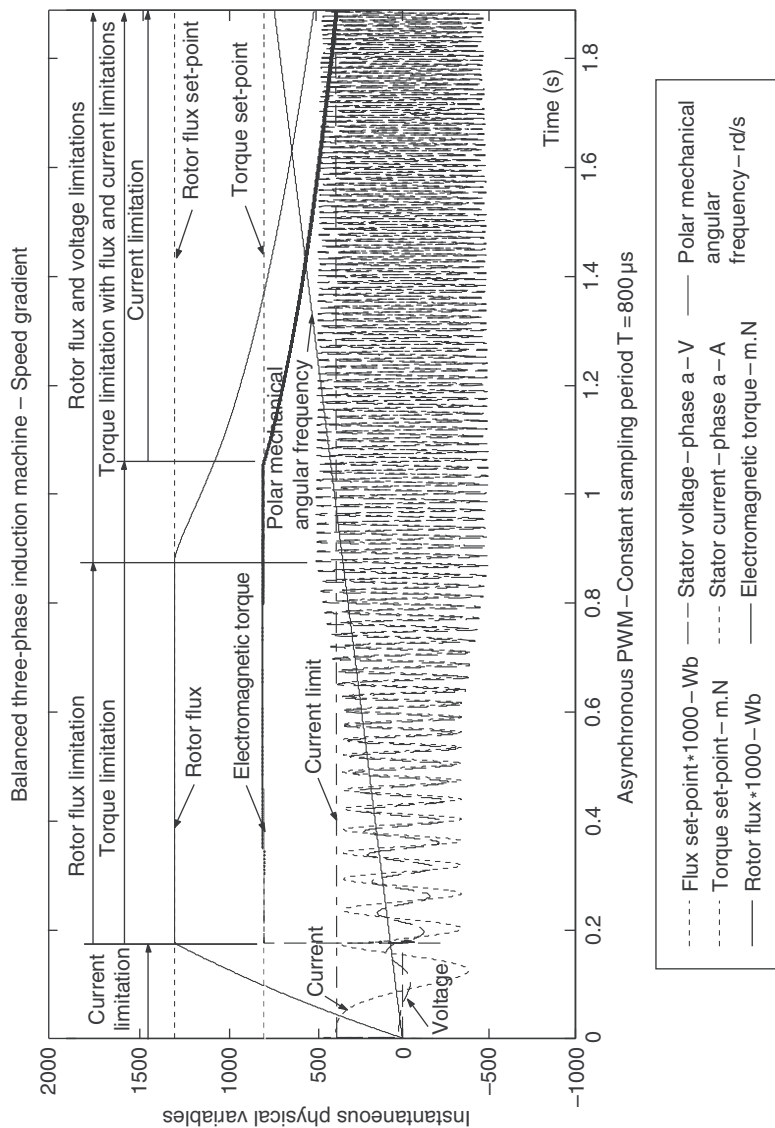


Figure 1.17 Maximum flux and maximum torque

choose to gradually increase the electromagnetic torque to its maximum value, in order not to create a mechanical stress by too high a torque gradient which would result from the application of a torque level from zero to its maximum, as in this example. It is then enough to gradually generate increasing set-points according to process needs.

We do not notice any overshoot of limits, in spite of a set-point variation in only one sampling period. This is achieved owing to the fact that the control is instantaneous (dead-beat response) and that set-points take account of limits by anticipation. It is thus possible, under these conditions, not only to exclude an overshoot of limits, but also to maintain the motor state on each exact operation limit, to avoid any unused margin of operation with regard to these limits. The operation is thus optimized, for example compared to fast variations of the feeding voltage. For the same reason, in order not exceed the inverter peak current, we can add up the instantaneous current set-point and all the forecast harmonics due to the inverter voltage switching.

Since one can take account of all disturbances by measurements (fast variation of the feeding voltage, for example) for calculating set-points and the control voltage vector, the control process allows us, in the case of disturbances, to obtain similar performances to tracking performances, i.e. in only one sampling period and without any overshoot or lagging. Tracking performances or performances during disturbances are identical with this method of control; one can only notice in the worst case, a pure time-delay of one period for taking into account either the new set-point (tracking), or new measurements (disturbances).

Lastly, it is clear that the ripple of the electromagnetic torque increases with speed. This is due to the fact that, within one period of the fundamental voltage component at the maximum speed, the number of sampling periods falls until it reaches a ratio R_T at high speed of:

$$R_T = \frac{2 \cdot \pi}{\omega_M \cdot T} \quad (1.205)$$

In this example, this ratio is less than 10, what degrades the voltage shape factor and thus the current and the torque shape factors. Only the flux is not very sensitive to this ratio; because of its long time constant, all current harmonics are found in the active current, as was explained by the equivalent circuit of Figure 1.4.

1.4.5.2 Adjustment of Flux and Torque – Limitations in Electrical Braking

In regenerative electrical braking mode, rated performances selected here are different from traction operations for the same motor:

$$C_M = 865 \text{ m.N}, I_M = 468 \text{ A} \quad (1.206)$$

with a DC inverter feeding voltage of 850 V.

To allow a visualization of all phenomena at various timescales, we also choose to impose a maximum electromagnetic torque in the regenerative electrical braking mode with a mechanical speed very quickly decreasing from the maximum speed to zero, in a timescale of less than two seconds. The control and the motor were thus very finely simulated.

The start of braking takes place with null current, null flux and maximum speed; it is the most delicate scenario to manage within the limits. When the system was dimensioned to electrically brake, it is rare not to maintain the rotor flux, even with a null torque at high speed, to

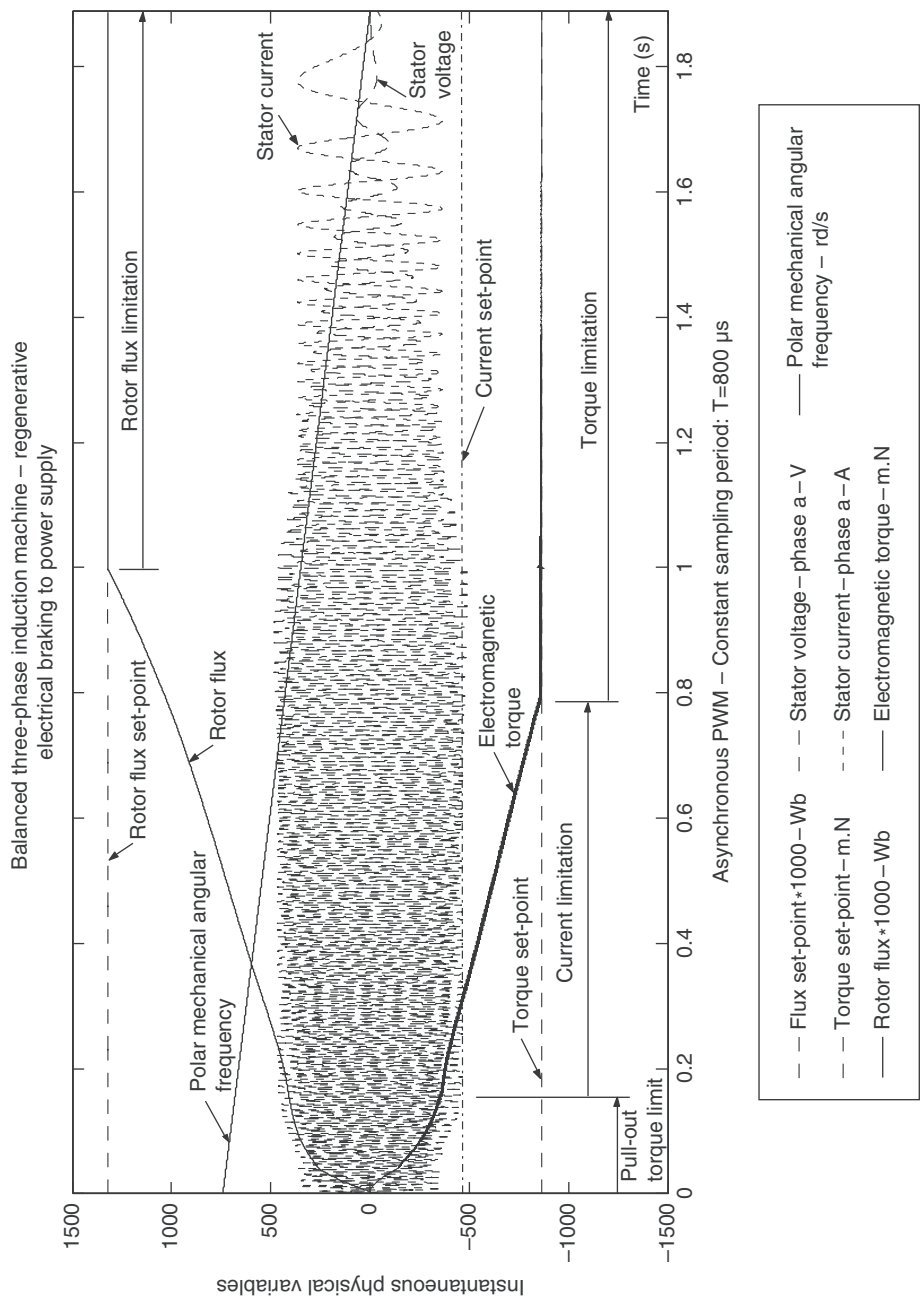


Figure 1.18 Maximum braking torque, starting with null torque

be ready to electrically brake. The scenario with a null flux is, however, requested when the system operation prohibits a flux saving with a null torque for safety reasons or in case of a complete system reset at non-null speed, following an incident.

The *flux–torque* strategy of the control that we choose in the case of braking from null flux is to provide all the possible torque, taking into account the maximum flux gradient, while keeping the priority of the flux compared to the torque. This strategy encounters two distinct limitations:

- There is a torque limitation at high speed which is not due exclusively to current and rotor flux limitations still in phase of growing up, but to the vicinity of the *pull-out torque* of the induction machine when the rotor flux is low. This is the first and the main limitation at high speed. The torque set-point limit was set here at 80% of the pull-out torque under these conditions; this limitation is calculated to permanently follow the flux and the pull-out torque evolutions
- There is also a current limitation, as in the case of the preceding traction operation, but simultaneously with a magnetizing and active current at the beginning of braking, to take into account this new strategy of a simultaneous flux and torque setting up.

In Figure 1.18, for a first period lower than 0.2 s, the torque is limited by the pull-out torque. This increases, however, during this phase, with the squared rotor flux. Then the stator current magnitude reaches the maximum allowable current.

The torque and the flux continue to grow but without reaching a steady state of the rotor flux at any given speed, taking into account the fast deceleration. Indeed, the maximum voltage is reached only slowly, approximately one second after the beginning of the braking. The flux setting thus took approximately one second, owing to the fact that the maximum current was also used to produce immediately the maximum allowable torque. When the maximum torque is reached at about 0.8 s, the gradient of the flux increases; the current magnitude decreases in spite of the increasing magnetizing current, because the active current decreases, controlling the torque, due to the flux increasing.

After approximately one second, the motor rated speed in braking mode is reached with the maximum available stator voltage magnitude. The speed continuing to decrease, the flux is thus limited at its maximum, and the voltage decreases with the speed until the stop. The braking torque is maintained until the stop.

The simulation was made under the same conditions as in traction operation, *without* any feed voltage filtering.

This figure makes it possible to visualize the tracking performances of the current, the flux and the torque of this control process. Whenever possible, one would prefer to preserve the motor magnetization at any moment, including with a null torque, to be able to brake immediately with the maximum allowable torque according to the speed.

The strategy adopted in the braking mode for this simulation is different from that retained for the traction operation; the two strategies highlight the extreme flexibility of the control process for managing set-points on their limits.

1.4.5.3 Free Evolution – Short-Circuit Torque

The free evolution of the state-space eigenvector coordinates is calculable starting from equation (1.124); it is the motor state evolution with three-phase symmetrical short-circuits.

Figure 1.18 was calculated for the motor described in the preceding section with an initial state:

$$X_0 = \begin{bmatrix} \bar{I}_{s0} \\ \bar{\Phi}_{r0} \end{bmatrix}_{(d_0, q_0)} = \begin{bmatrix} 140 + 358 \cdot i \\ 1.33 \end{bmatrix} \quad (1.207)$$

for a constant polar mechanical angular frequency equal to $\omega = 400$ rd/s. The free evolution was calculated every millisecond and the resulting state-space eigenvector:

$$\Psi^0 = \begin{bmatrix} \bar{\Psi}_1 \\ \bar{\Psi}_2 \end{bmatrix}_{(d_0, q_0)}^0 \quad (1.208)$$

was located in the (d_0, q_0) plane. The frame remains fixed during the short-circuit computation. We can write each eigenvalue with its real and imaginary part:

$$\mu = r_p + i \cdot \omega_p \quad (1.209)$$

and then the free evolution takes the following form:

$$\begin{bmatrix} \bar{\Psi}_1 \\ \bar{\Psi}_2 \end{bmatrix}_{(d_0, q_0)}^0 = \begin{bmatrix} e^{r_{p1} \cdot T} \cdot e^{i \cdot \omega_{p1} \cdot T} & 0 \\ 0 & e^{r_{p2} \cdot T} \cdot e^{i \cdot \omega_{p2} \cdot T} \end{bmatrix} \cdot \begin{bmatrix} \bar{\Psi}_{10} \\ \bar{\Psi}_{20} \end{bmatrix}_{(d_0, q_0)} \quad (1.210)$$

The vector extremity trajectory of the first coordinate of the state-space vector $\bar{\Psi}_1^0$ (Psi1 in Figure 1.19), follows a logarithmic spiral curve, due to the exponential function $e^{\mu_1 \cdot T}$ of which the imaginary exponent has a negative real part; this curve is centered on the origin of axes and described in the counterclockwise positive sense of the motor rotation. The convergence towards zero of the vector magnitude comes from the fact that the real part of the first eigenvalue is negative, which is the sign of a dissipative system and thus a stable one.

The vector extremity trajectory of the second coordinate of the state-space vector $\bar{\Psi}_2^0$ (Psi2 in Figure 1.19), follows an almost linear trajectory in the direction of axis origins. The convergence towards zero of the vector magnitude also comes from the fact that the real part of this second eigenvalue is negative; its imaginary part is much smaller than its real part.

Before the short-circuit, the two initial eigenvector coordinates have a magnitude of about 20000A although made up of a current vector which does not exceed 400A (1.204) and of another current vector depending on the rotor flux. In fact, the complex coefficients ξ_k of the rotor flux provide the highest contribution to the vector sum (1.118). It is thus difficult to reproduce on the same Figure both vectors of the vector sum, taking into account the scale difference.

Equation (1.123) projected in the (d_0, q_0) reference frame and applied to a steady state:

$$\begin{bmatrix} \bar{\Psi}_{1c} \\ \bar{\Psi}_{2c} \end{bmatrix}_{(d_0, q_0)} = \begin{bmatrix} \bar{\Psi}_{10} \\ \bar{\Psi}_{20} \end{bmatrix}_{(d_0, q_0)} \cdot e^{i \cdot \omega_s \cdot T} = \begin{bmatrix} e^{\mu_1 \cdot T} & 0 \\ 0 & e^{\mu_2 \cdot T} \end{bmatrix} \cdot \begin{bmatrix} \bar{\Psi}_{10} \\ \bar{\Psi}_{20} \end{bmatrix}_{(d_0, q_0)} + \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \cdot \bar{V}_{s(d_0, q_0)} \quad (1.211)$$

allows us to highlight the action that the control would have if, during the *first* sampling period, it had sought to maintain a steady state characterized by the initial state, or in

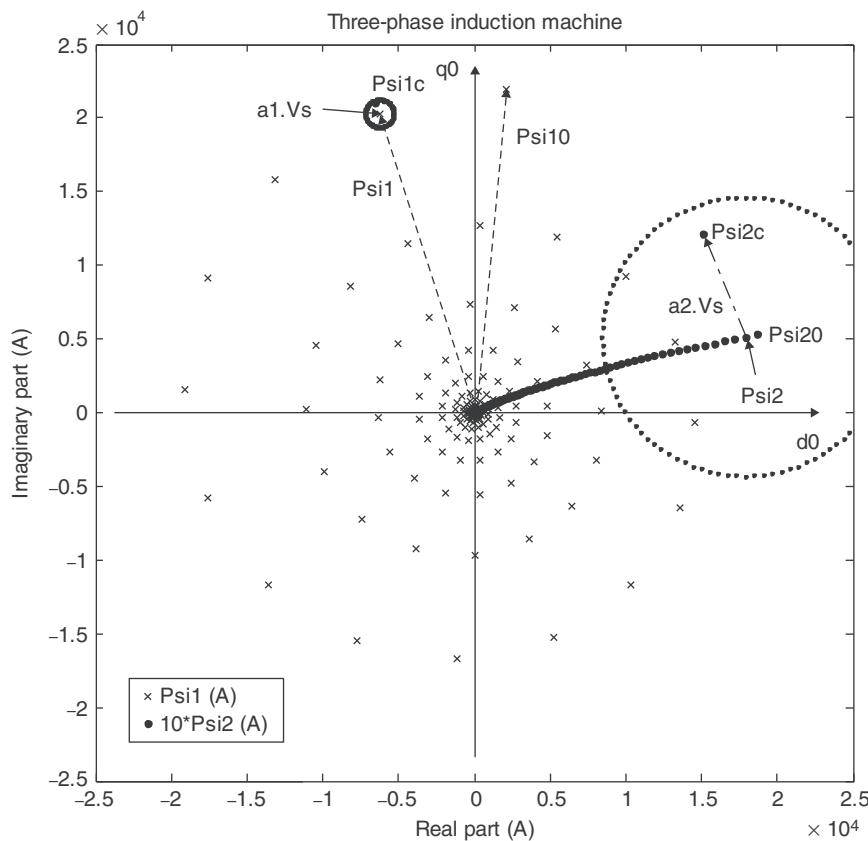


Figure 1.19 Coordinate evolution of the state-space eigenvector Ψ

other words, if it would have compensated exactly the free evolution by the two control current vectors:

$$a_1 \cdot \vec{V}_s \text{ and } a_2 \cdot \vec{V}_s \tag{1.212}$$

The extremities of vector coordinates of the set-point vector corresponding to a steady state were represented in Figure 1.18 by a point at the end of the first sampling period T , as well as the control vector of the largest corresponding current $a_2 \cdot \vec{V}_s$; the control vector $a_1 \cdot \vec{V}_s$, too small to be represented in the same Figure, should be traced between the extremity of the first coordinate of the free evolution state-space vector (cross) and the extremity of the first coordinate of the set-point vector (dot).

At the end of the first period, the circles centered on the free evolution of the two vectorial coordinates of the state-space eigenvector, have a radius corresponding to the two control vector magnitudes (1.212), calculated for the maximum DC inverter supply voltage. If the DC inverter supply voltage decreases and reaches its minimum value, it is possible in particular at high speed, that the steady state operation represented in Figure 1.18, cannot be reached because the circle radius is insufficient to reach the set-point, even transitorily; we then

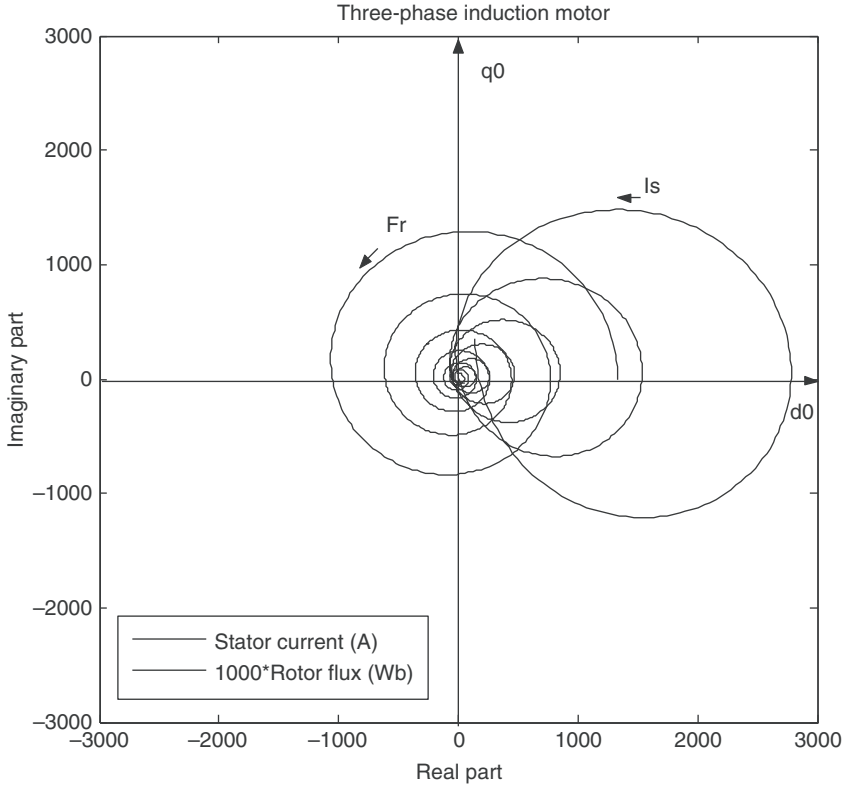


Figure 1.20 Free evolution of the current and of the flux in short-circuit

compute again, first the flux set-point with a voltage limitation at the steady state, then the possible torque so that the transitory voltage limit makes it possible to gradually reach a new steady state.

Figure 1.20 shows the convergence towards zero of the stator current and the rotor flux during a short-circuit under the same conditions, but with one sampling period of $100\mu\text{s}$ to smooth the trajectory representation. The rotor flux decreases gradually whereas the current passes very quickly by a maximum before decreasing gradually with a residual rotor flux. We found the well-known result of the symmetrical short-circuit of an induction machine.

This figure was obtained by writing the state-space eigenvector of the system with the stator current and the rotor flux from equation (1.211) for $\vec{V}_s \equiv 0$:

$$\begin{bmatrix} \vec{I}_s + \xi_1 \cdot \vec{\Phi}_r \\ \vec{I}_s + \xi_2 \cdot \vec{\Phi}_r \end{bmatrix}_{(d_0, q_0)}^0 = \begin{bmatrix} e^{\mu_1 T} & 0 \\ 0 & e^{\mu_2 T} \end{bmatrix} \cdot \begin{bmatrix} \vec{I}_0 + \xi_1 \cdot \vec{\Phi}_0 \\ \vec{I}_0 + \xi_2 \cdot \vec{\Phi}_0 \end{bmatrix}_{(d_0, q_0)} \quad (1.213)$$

then eliminating successively $\vec{\Phi}_r$ and $\vec{I}_s$ between the two complex equations:

$$\vec{I}_s^0 = \vec{I}_0 \cdot \frac{\xi_1 \cdot e^{\mu_2 T} - \xi_2 \cdot e^{\mu_1 T}}{\xi_1 - \xi_2} + \vec{\Phi}_0 \cdot \xi_1 \cdot \xi_2 \cdot \frac{e^{\mu_2 T} - e^{\mu_1 T}}{\xi_1 - \xi_2} \quad (1.214)$$

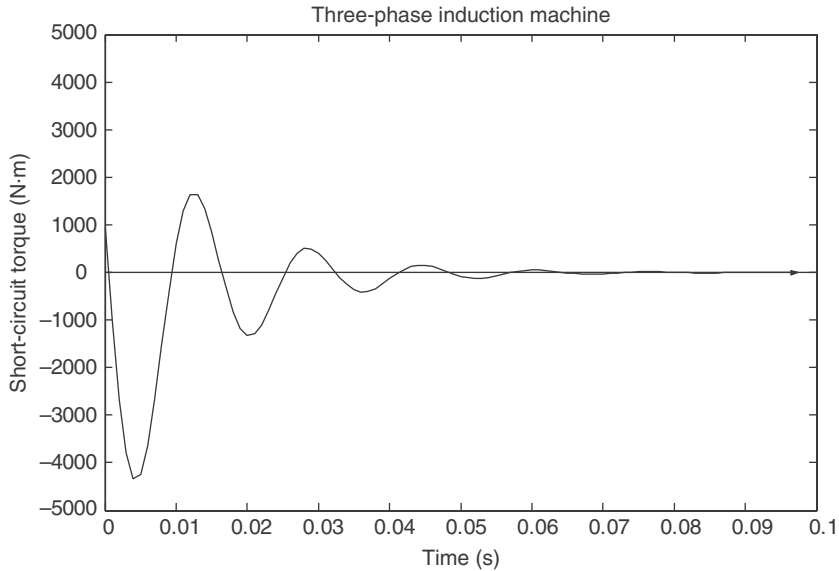


Figure 1.21 Three-phase symmetrical short-circuit torque

$$\vec{\Phi}_r^0 = \vec{I}_0 \cdot \frac{e^{\mu_2 \cdot T} - e^{\mu_1 \cdot T}}{\xi_2 - \xi_1} + \vec{\Phi}_0 \cdot \frac{\xi_2 \cdot e^{\mu_2 \cdot T} - \xi_1 \cdot e^{\mu_1 \cdot T}}{\xi_2 - \xi_1} \quad (1.215)$$

Thus we find, as coefficients of $\vec{I}_0$ and of $\vec{\Phi}_0$, the four coefficients calculated in (1.153) of the transition matrix F . It is indeed logical to find the relation $X^0 = F \cdot X_0$, when we calculate the state-space vectors starting from the state-space eigenvectors.

It should be noted here that the rotor flux does not remain collinear with the d_0 axis owing to the fact that the frame remains fixed.

Figure 1.21 shows the evolution of the electromagnetic torque during the symmetrical short-circuit. We find here a well-known result, which is the relationship between the rated maximum torque before short-circuit and the symmetrical short-circuit torque, close to 5.5 here. This ratio could be even higher in the event of an asymmetrical short-circuit when the motor has two phases in short-circuit to the reference potential for instance, and one phase supplied by the inverter. This study is important for the mechanical sizing of the system, in the event of inverter power switch breakdown. Indeed, to avoid asymmetrical short-circuit in this case and to limit the short-circuit torque at the computed value of Figure 1.21, it is possible to force into short-circuit all the switches of the same inverter level as the broken down switch, generally in short-circuit in the case of power switches.

The short-circuit torque tends towards zero only if the rotor magnetic remnant is negligible.

1.5 Conclusion on the Induction Machine Control

With the regulation method which has just been described, the induction machine can be controlled by the flux and the torque in an uncorrelated way, with very high dynamics corresponding to only one PWM period plus one pure time-delay, in tracking mode as in regulation of disturbances.

This control makes it possible to control the induction machine from full stop until its maximum speed; the maximum speed can be higher than the motor's rated speed, thanks to the limit computations which are used to modify the set-points.

The torque control can then be used to control the motor speed, including at null speed; it can then be used to control the motor in position including at full stop.

It should, however, be noted that the induction machine produces a torque thanks to the rotor current circulating in its resistive squirrel cage. The torque production is thus accompanied by ohmic losses in the rotor, proportional to the square of the rotor current. This motor is thus adapted to hold a position at stop or very low speed, only for a very short time if it is self-cooled (enclosed-ventilated); if the motor is force-ventilated with an external fan and if the rotor is cooled by a heat pipe built into the motor shaft, the torque production at stop or at low speed could have a longer duration.

Synchronous motors, of which we will detail the control in the two following chapters, are however, adapted much better to the torque control at stop or at very low speed, since they have practically no rotor losses. Moreover, synchronous motors are smaller than induction machines, for the same rated electrical power.

On the other hand, the induction machine is more suited to high speeds owing to its simplicity of construction and the strength of its rotor.

2

Surface-Mounted Permanent-Magnet Synchronous Motor

We now will follow the same approach as for induction machines, with the surface-mounted permanent-magnet synchronous motor (SMPM-SM), establishing first the electrical equations. This motor has non-salient poles.

This kind of motor is sometimes called *brushless* by comparison with a DC motor or a universal motor with its brushes and commutator, with which one would have inverted the stator and the rotor; the brushes are done away with by directly supplying the AC current to the three stator phases.

The historical advantage of this kind of motor, used initially with a variable speed for robots and for the table of machine tools, is that one replaces the commutator and the brushes, fragile and also of limited life, with the static commutation of semiconductors such as insulated-gate bipolar transistors (IGBT). The shift of the commutator neutral line of a DC motor was replaced by the phase adjustment of the control voltage vector.

The SMPM-SM does not present an armature reaction because the magnetic permeability of magnets is similar to that of air; it facilitates the control of the air gap flux, starting from measurement of the rotor position by a simple position encoder.

The advantage of this motor type, compared to induction machines, is dual:

- Its physical volume is smaller than that of an induction machine of the same power, owing to the fact that magnets produce the rotor magnetization, instead of the stator current. Because of the magnetization of the magnets, the stator current is lower and thus the copper volume in the stator is less.
- It makes it possible to maintain a torque at low speed, including for use in position controlling, practically without thermal loss in the rotor, unlike the induction machine which must dissipate its rotor losses in proportion to the electromagnetic torque, and which requires forced ventilation at low speed and in particular to control a position at null speed.

2.1 Electrical Equations and Equivalent Circuit

From the equivalent electrical circuit of the balanced three-phase synchronous motor, we will first establish mathematical relations between various electrical variables.

2.1.1 Definitions and Notations

- stator resistor: R_s
- stator inductance: L_s
- inverse inductance: $\beta = \frac{1}{L_s}$
- stator time constant: $\tau_s = \frac{L_s}{R_s}$
- inverse time constant: $\gamma = \frac{R_s}{L_s}$
- rotor magnet flux by pole pair: Φ_a
- number of pole pairs: N_p
- rotor absolute angular position: Ξ .

Definitions and notation for mechanical and electrical angular frequencies:

- rotor mechanical angular frequency: Ω
- *polar*¹ mechanical angular frequency: $\omega = N_p \cdot \Omega$
- stator electrical angular frequency: ω_s .

2.1.2 Equivalent Electrical Circuit

The equivalent electrical circuit, per phase, of the balanced three-phase synchronous motor is shown in Figure 2.1:

This circuit supposes that the electromotive force (e.m.f.) created by the flux of the magnets laid out on the rotor surface and by the mechanical speed of the motor shaft, is not at all influenced by the stator current. In the contrary case, as we will see with the interior permanent magnet synchronous motor (IPM-SM), it is necessary to take into account of the influence of the stator current on the rotor flux in electrical equations.

This absence of coupling between the stator current and the rotor flux for the SMPM-SM is due to the relative permeability near to the unit of the magnets used in this kind of motor. Seen from the stator, the path of the stator magnetic field crosses the air gap and also a part of the magnet's thickness, as if crossing a very important air gap, equivalent to the mechanical air gap added to the thickness of the magnets. Then, the iron of the rotor located under the magnets is

¹ For simplicity, we will name the mechanical angular frequency of the motor shaft multiplied by the number of pole pairs, the *polar* mechanical angular frequency, in preference to the name, sometimes used, of *electrical* mechanical angular frequency.

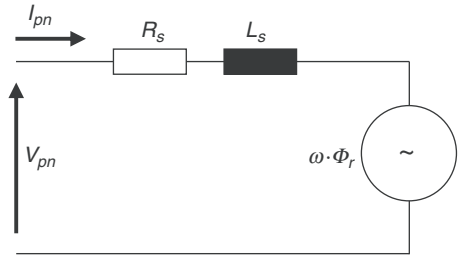


Figure 2.1 Equivalent electrical circuit per phase of the synchronous motor

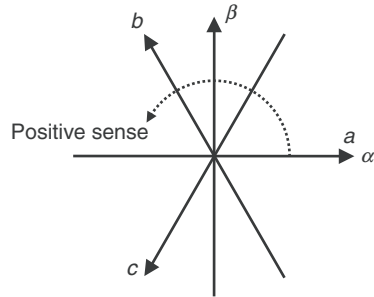


Figure 2.2 Three-phase (a, b, c) and two-phase (α, β) fixed frames

not crossed by lines of the stator magnetic field because of the very high reluctance which this last magnetic path would represent. The rotor, seen from the stator, is magnetically isotropic.

Electrical variables of the equivalent circuit are the variables directly readable by an electrical measurement, as for the induction machine:

- the instantaneous phase-neutral voltage per phase as per equation (1.1)
- the instantaneous current of each stator phase as per equation (1.2)

with $p \in \{0 ; 1 ; 2\}$ for a three-phase supply, and with φ , the phase shift of the current with respect to the voltage phase.

The Concordia transformation (1.3) allows us to simplify the three-phase representation in the phase plane, by introducing a two-phase vector representation of the circuit, in the plane of the fixed orthonormal frame linked to the stator.

We will choose counterclockwise as the positive sense for angular measurements as shown in Figure 2.2.

After transformation, the equivalent circuit of the synchronous motor presents the reduced vectorial form of Figure 2.3:

We have used here the Concordia transform, which preserves instantaneous power. In this case, relations between the magnitude of the vectorial variable and the maximum value of the phase electrical variable, are identical to those calculated for an induction machine in equation (1.4); values of the parameters of the transformation matrix remain unchanged.

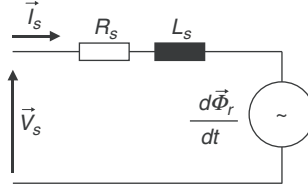


Figure 2.3 Two-phase equivalent circuit of the SMPM-SM

2.1.3 Differential Equation System

The mesh of the circuit (Figure 2.3) makes it possible to write the following vectorial relation:

$$\vec{V}_s = R_s \cdot \vec{I}_s + L_s \cdot \frac{d\vec{I}_s}{dt} + \frac{d\vec{\Phi}_r}{dt} \quad (2.1)$$

However, $\vec{\Phi}_r$ depends on the θ angle of rotor phase compared to stator phase, but its magnitude is constant, equal to Φ_a , and thus independent of time, at least at the timescale of one motor voltage period:

$$d\vec{\Phi}_r(\theta) = \frac{\delta\vec{\Phi}_r}{\delta\theta} \cdot d\theta \quad \text{and} \quad \frac{d\vec{\Phi}_r}{dt} = \frac{\delta\vec{\Phi}_r}{\delta\theta} \cdot \frac{d\theta}{dt} \quad (2.2)$$

We can now write the following system, starting from equations (2.1) and (2.2) by noting that the time derivative angle of phases is equal to the polar mechanical angular frequency:

$$\dot{\vec{I}}_s = -\frac{R_s}{L_s} \cdot \vec{I}_s - \frac{\omega}{L_s} \cdot \frac{\delta\vec{\Phi}_r}{\delta\theta} + \frac{1}{L_s} \cdot \vec{V}_s \quad (2.3)$$

$$\dot{\vec{\Phi}}_r = \omega \cdot \frac{\delta\vec{\Phi}_r}{\delta\theta} \quad (2.4)$$

Let us use reduced variables which were above defined as:

$$\boxed{\dot{\vec{I}}_s = -\gamma \cdot \vec{I}_s - \beta \cdot \omega \cdot \frac{\delta\vec{\Phi}_r}{\delta\theta} + \beta \cdot \vec{V}_s} \quad (2.5)$$

$$\boxed{\dot{\vec{\Phi}}_r = \omega \cdot \frac{\delta\vec{\Phi}_r}{\delta\theta}} \quad (2.6)$$

The choice of electrical variables $\vec{I}_s$ and $\vec{\Phi}_r$ to write equations (2.5) and (2.6), is explained by the Lorentz law, which allows us to express the electromagnetic torque per pole pair from their cross product. We thus obtain the total torque by multiplying the torque per pole pair, by the number of pole pairs:

$$\boxed{\vec{C} = N_p \cdot \vec{\Phi}_r \times \vec{I}_s} \quad (2.7)$$

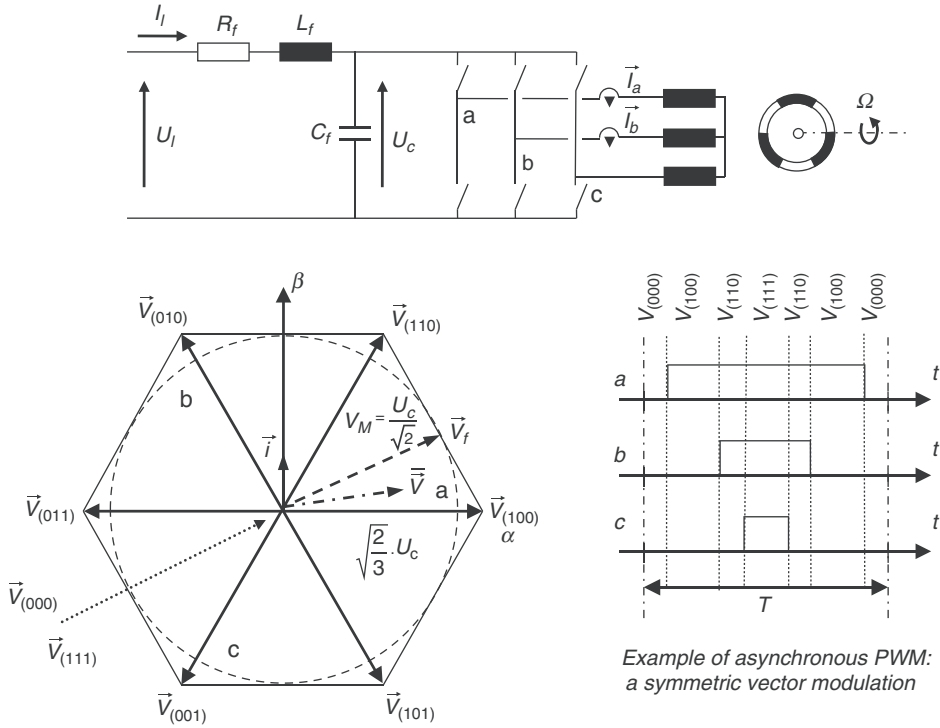


Figure 2.4 Synchronous motor fed by a voltage inverter

In addition, $\vec{I}_s$ is a directly measurable variable, by the measurement of phase currents, and Φ_a can be estimated by the flux magnitude of the magnets and by its position, measured by the rotor position. Of course, it is also possible to estimate or to predict the rotor flux vector, starting from a known position, itself incremented from a known starting position.

2.2 Working out the State-Space Equation System

We will establish state-space equations of the synchronous motor supplied with balanced three-phase voltage by an inverter with two voltage levels, according to the principle shown in Figure 2.4, which was commented on in section 1.2.2, for induction machines:

2.2.1 State-Space Equations in the Fixed Plane

The vector equations (2.5) and (2.6) can now be written in the form of a continuous-time state-space linear equation system, in matrix form:

$$\begin{aligned}\dot{X} &= A \cdot X + B \cdot U \\ Y &= C \cdot X\end{aligned}\tag{2.8}$$

This state-space representation will allow us to define the behavior of the system using the stator current and the rotor flux of the synchronous motor in the fixed frame related to the motor stator.

$$X = \begin{bmatrix} I_{s\alpha} \\ I_{s\beta} \\ \Phi_{r\alpha} \\ \Phi_{r\beta} \end{bmatrix} \quad \text{or} \quad \vec{X} = \begin{bmatrix} \vec{I}_s \\ \vec{\Phi}_r \end{bmatrix} \quad (2.9)$$

Indeed, if we project equations (2.5) and (2.6) in the (α, β) fixed frame, the coordinates of the vector $\frac{\delta \vec{\Phi}_r}{\delta \theta}$ result from a vector rotation of $\frac{\pi}{2}$ in the positive sense, of the coordinates of the vector $\vec{\Phi}_r$:

$$\left(\frac{\delta \vec{\Phi}_r}{\delta \theta} \right)_{\alpha} = -\vec{\Phi}_{r\beta} \quad (2.10)$$

$$\left(\frac{\delta \vec{\Phi}_r}{\delta \theta} \right)_{\beta} = \vec{\Phi}_{r\alpha} \quad (2.11)$$

After projection of these two vector relations on the α and β axes:

$$\dot{I}_{s\alpha} = -\gamma \cdot I_{s\alpha} + \beta \cdot \omega \cdot \Phi_{r\beta} + \beta \cdot V_{s\alpha} \quad (2.12)$$

$$\dot{I}_{s\beta} = -\gamma \cdot I_{s\beta} - \beta \cdot \omega \cdot \Phi_{r\alpha} + \beta \cdot V_{s\beta} \quad (2.13)$$

$$\dot{\Phi}_{r\alpha} = -\omega \cdot \Phi_{r\beta} \quad (2.14)$$

$$\dot{\Phi}_{r\beta} = \omega \cdot \Phi_{r\alpha} \quad (2.15)$$

For one fixed mechanical angular frequency this system of four differential equations is linear, thus justifying the choice of the state-space representation for linear systems (2.8).

The two last relations confirm that neither the input variable $\vec{V}_s$, nor the resulting current $\vec{I}_s$, acts on the rotor flux. The flux evolution is related only to the mechanical rotation speed of rotor poles.

The first two relations show that the evolution of the stator current at a fixed time, depends on:

- the input
- the state of the current itself
- the state of the flux.

By gathering equations in a matrix writing form, we obtain with this choice of variables in the fixed frame:

$$\begin{bmatrix} \dot{I}_{s\alpha} \\ \dot{I}_{s\beta} \\ \dot{\Phi}_{r\alpha} \\ \dot{\Phi}_{r\beta} \end{bmatrix} = \begin{bmatrix} -\gamma & 0 & 0 & \omega \cdot \beta \\ 0 & -\gamma & -\omega \cdot \beta & 0 \\ 0 & 0 & 0 & -\omega \\ 0 & 0 & \omega & 0 \end{bmatrix} \cdot \begin{bmatrix} I_{s\alpha} \\ I_{s\beta} \\ \Phi_{r\alpha} \\ \Phi_{r\beta} \end{bmatrix} + \begin{bmatrix} \beta & 0 \\ 0 & \beta \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} V_{s\alpha} \\ V_{s\beta} \end{bmatrix} \quad (2.16)$$

This fourth-order system, in the case of a synchronous motor, is enough to completely describe the system evolution with the voltage input vector.

By applying this formalism to the first equation (2.8) according to the motor parameters, the evolution and the input matrices are now written:

$$A = \begin{bmatrix} -\gamma & 0 & 0 & \omega \cdot \beta \\ 0 & -\gamma & -\omega \cdot \beta & 0 \\ 0 & 0 & 0 & -\omega \\ 0 & 0 & \omega & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} \beta & 0 \\ 0 & \beta \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (2.17)$$

We can see here a remarkable property of the four 2×2 submatrices of the evolution matrix and of the two submatrices of the input matrix: the terms of diagonals are identical and the terms of the antidiagonals are opposite. This property is related to the fact that electrical parameters of the motor do not depend on the rotor position. This is the case for a synchronous motor with non-salient poles, like an induction machine with non-salient poles; it is not the case of an interior permanent magnet synchronous motor, as we will demonstrate it in chapter 3.

2.2.2 State-Space Equations in the Complex Plane

We now will show that if we choose the two-phase fixed frame in the complex plane, with the imaginary unit vector directed according to the β axis, the system of continuous-time state-space equations becomes a complex system of second order, where:

$$\dot{X} = \begin{bmatrix} \dot{\vec{I}}_s \\ \dot{\vec{\Phi}}_r \end{bmatrix} = \begin{bmatrix} \dot{I}_{s\alpha} + i \cdot \dot{I}_{s\beta} \\ \dot{\Phi}_{r\alpha} + i \cdot \dot{\Phi}_{r\beta} \end{bmatrix}, \quad X = \begin{bmatrix} \vec{I}_s \\ \vec{\Phi}_r \end{bmatrix} = \begin{bmatrix} I_{s\alpha} + i \cdot I_{s\beta} \\ \Phi_{r\alpha} + i \cdot \Phi_{r\beta} \end{bmatrix} \quad \text{and} \quad U = V_{s\alpha} + i \cdot V_{s\beta} \quad (2.18)$$

and:

$$\begin{bmatrix} \dot{I}_{s\alpha} + i \cdot \dot{I}_{s\beta} \\ \dot{\Phi}_{r\alpha} + i \cdot \dot{\Phi}_{r\beta} \end{bmatrix} = A \cdot \begin{bmatrix} I_{s\alpha} + i \cdot I_{s\beta} \\ \Phi_{r\alpha} + i \cdot \Phi_{r\beta} \end{bmatrix} + B \cdot (V_{s\alpha} + i \cdot V_{s\beta}) \quad (2.19)$$

The A and B matrices are complex and of the second order.

The reduction of the real fourth order to the complex second order is only possible – as we saw for an induction machine – because the SMPM-SM is a motor with non-salient poles, which gives this remarkable property to the real 2×2 submatrices of the evolution and input matrices.

By multiplying equations (2.13) and (2.15) by i , and by adding equalities obtained from (2.12) and (2.14) respectively, we obtain the two following relations:

$$\dot{I}_{s\alpha} + i \cdot \dot{I}_{s\beta} = -\gamma \cdot (I_{s\alpha} + i \cdot I_{s\beta}) - i \cdot \omega \cdot \beta \cdot \Phi_{r\alpha} + \omega \cdot \beta \cdot \Phi_{r\beta} + \beta \cdot (V_{s\alpha} + i \cdot V_{s\beta}) \quad (2.20)$$

$$\dot{\Phi}_{r\alpha} + i \cdot \dot{\Phi}_{r\beta} = i \cdot \omega \cdot \Phi_{r\alpha} - \omega \cdot \Phi_{r\beta} \quad (2.21)$$

These two relations can be otherwise written:

$$\dot{I}_{s\alpha} + i \cdot \dot{I}_{s\beta} = -\gamma \cdot (I_{s\alpha} + i \cdot I_{s\beta}) - i \cdot \omega \cdot \beta \cdot (\Phi_{r\alpha} + i \cdot \Phi_{r\beta}) + \beta \cdot (V_{s\alpha} + i \cdot V_{s\beta}) \quad (2.22)$$

$$\dot{\Phi}_{r\alpha} + i \cdot \dot{\Phi}_{r\beta} = i \cdot \omega \cdot (\Phi_{r\alpha} + i \cdot \Phi_{r\beta}) \quad (2.23)$$

That makes it possible to write the system in a matrix form:

$$\begin{bmatrix} \dot{I}_{s\alpha} + i \cdot \dot{I}_{s\beta} \\ \dot{\Phi}_{r\alpha} + i \cdot \dot{\Phi}_{r\beta} \end{bmatrix} = \begin{bmatrix} -\gamma & -i \cdot \omega \cdot \beta \\ 0 & i \cdot \omega \end{bmatrix} \cdot \begin{bmatrix} I_{s\alpha} + i \cdot I_{s\beta} \\ \Phi_{r\alpha} + i \cdot \Phi_{r\beta} \end{bmatrix} + \begin{bmatrix} \beta \\ 0 \end{bmatrix} \cdot (V_{s\alpha} + i \cdot V_{s\beta}) \quad (2.24)$$

and thus:

$$A = \begin{bmatrix} -\gamma & -i \cdot \omega \cdot \beta \\ 0 & i \cdot \omega \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} \beta \\ 0 \end{bmatrix} \quad (2.25)$$

Finally:

$$\boxed{\begin{bmatrix} \dot{\vec{I}}_s \\ \dot{\vec{\Phi}}_r \end{bmatrix} = \begin{bmatrix} -\gamma & -i \cdot \omega \cdot \beta \\ 0 & i \cdot \omega \end{bmatrix} \cdot \begin{bmatrix} \vec{I}_s \\ \vec{\Phi}_r \end{bmatrix} + \begin{bmatrix} \beta \\ 0 \end{bmatrix} \cdot \vec{V}_s} \quad (2.26)$$

It can be seen that when the antidiagonal of real 2×2 submatrices carries identically null terms, the corresponding coefficient of evolution and input matrices remains a real one.

2.2.3 Complex State-Space Equation Discretization

The results of section 1.2.3 are general results which apply in the same way to the synchronous motor; in particular, the discretized form of the state-space equation (1.53):

$$X(t_n + T) = F \cdot X(t_n) + G \cdot V$$

This equation makes it possible to predict the motor state $X(t_n + T)_p$ from an initial known state $X(t_n)_m$, measured or estimated, if the input vector V is known; otherwise, replacing the predicted state-space vector by the set-point vector $X(t_n + T)_c$ according to equations (1.57) and (1.58), the same equation makes it possible to calculate the control vector by inverting the equation system.

We now proceed to diagonalizing the evolution matrix and rewriting equations within the space of eigenvectors. For that, we will have to discretize the continuous-time state-space equations, to find independent set-point vectors in the eigenvector basis, and to derive the control vector from the inversion of the new state-space equations.

We will see that the independence of vector coordinates of the state-space eigenvector lead to characteristic properties for this kind of motor, where the rotor flux is an uncontrollable state-space variable and thus independent of the input voltage and of the stator current.

2.2.4 Evolution Matrix Diagonalization

2.2.4.1 Eigenvalues

The eigenvalue equation giving eigenvalues of the evolution matrix is:

$$\det(\mu_i \cdot I - A) = 0 \quad (2.27)$$

that is to say:

$$\begin{vmatrix} \mu + \gamma & i \cdot \omega \cdot \beta \\ 0 & \mu - i \cdot \omega \end{vmatrix} = 0 \quad (2.28)$$

After calculation of this determinant, the eigenvalue equation becomes:

$$(\mu + \gamma) \cdot (\mu - i \cdot \omega) = 0 \quad (2.29)$$

The two eigenvalues, a real negative one related to the stator time constant, and another one a pure imaginary number related to the rotor rotation, are thus expressed by:

$$\mu_1 = -\gamma = -\frac{1}{\tau_s} \quad (2.30)$$

$$\mu_2 = i \cdot \omega \quad (2.31)$$

The A diagonalized matrix, is written:

$$D = \begin{bmatrix} \mu_1 & 0 \\ 0 & \mu_2 \end{bmatrix}, \text{ and thus: } D = \begin{bmatrix} -\gamma & 0 \\ 0 & i \cdot \omega \end{bmatrix} \quad (2.32)$$

2.2.4.2 Transfer Matrix Calculation

Eigenvectors $\left(\vec{\Pi}_1 = \begin{bmatrix} p_{11} \\ p_{21} \end{bmatrix}, \vec{\Pi}_2 = \begin{bmatrix} p_{12} \\ p_{22} \end{bmatrix} \right)$, corresponding to eigenvalues, are those of the solutions of the equation:

$$(A - \mu_i \cdot I) \cdot \vec{\Pi}_i = 0 \quad (2.33)$$

Indeed, each eigenvector cannot be single, owing to the fact that the corresponding matrix $(A - \mu_i \cdot I)$ is singular by definition of the μ_i , calculated to make its determinant null according to equation (2.27).

Equation (2.33), applied to the coordinates of the eigenvectors is written:

$$\begin{bmatrix} -\gamma & -i \cdot \omega \cdot \beta \\ 0 & i \cdot \omega \end{bmatrix} \cdot \begin{bmatrix} p_{1i} \\ p_{2i} \end{bmatrix} = \mu_i \cdot \begin{bmatrix} p_{1i} \\ p_{2i} \end{bmatrix} \quad (2.34)$$

The first line of the matrix equation (2.34) provides the following relation for the first eigenvector:

$$-(\gamma + \mu_1) \cdot p_{11} = i \cdot \omega \cdot \beta \cdot p_{21} \quad (2.35)$$

which makes it possible to choose, in particular:

$$\begin{aligned} p_{11} &= 1 \\ p_{21} &= 0 \end{aligned} \quad (2.36)$$

In a similar way, by using the first line of equation (2.34), for the second eigenvector:

$$-(\gamma + \mu_2) \cdot p_{12} = i \cdot \omega \cdot \beta \cdot p_{22} \quad (2.37)$$

We can then choose:

$$\begin{aligned} p_{12} &= 1 \\ p_{22} &= -\frac{\gamma + i \cdot \omega}{i \cdot \omega \cdot \beta} \end{aligned} \quad (2.38)$$

The transfer matrix $P = \begin{bmatrix} \vec{\Pi}_1 & \vec{\Pi}_2 \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$, such that $P^{-1} \cdot A \cdot P = D$, can thus be written:

$$P = \begin{bmatrix} 1 & 1 \\ 0 & -\frac{\gamma + i \cdot \omega}{i \cdot \omega \cdot \beta} \end{bmatrix} \quad (2.39)$$

Let us use eigenvalues to find another expression of the matrix:

$$P = \begin{bmatrix} 1 & 1 \\ 0 & -\frac{\mu_2 - \mu_1}{\mu_2 \cdot \beta} \end{bmatrix} \quad (2.40)$$

2.2.4.3 Transfer Matrix Inversion

The inverse of the transfer matrix is calculated simply, starting from its transposed comatrice, divided by its determinant:

$$P^{-1} = \begin{bmatrix} 1 & \frac{\mu_2 \cdot \beta}{\mu_2 - \mu_1} \\ 0 & -\frac{\mu_2 \cdot \beta}{\mu_2 - \mu_1} \end{bmatrix} \quad (2.41)$$

The evolution matrix can be now written from the diagonalized evolution matrix and from the transfer matrix: $A = P \cdot D \cdot P^{-1}$.

With notations using eigenvalues, it becomes:

$$A = \begin{bmatrix} 1 & 1 \\ 0 & -\frac{\mu_2 - \mu_1}{\mu_2 \cdot \beta} \end{bmatrix} \cdot \begin{bmatrix} \mu_1 & 0 \\ 0 & \mu_2 \end{bmatrix} \cdot \begin{bmatrix} 1 & \frac{\mu_2 \cdot \beta}{\mu_2 - \mu_1} \\ 0 & -\frac{\mu_2 \cdot \beta}{\mu_2 - \mu_1} \end{bmatrix} \quad (2.42)$$

$$A = \begin{bmatrix} \mu_1 & -\mu_2 \cdot \beta \\ 0 & \mu_2 \end{bmatrix} \quad (2.43)$$

This result can be derived immediately from equation (2.25).

Let us use the following reduced variable:

$$\xi = i \cdot \frac{\beta \cdot \omega}{\gamma + i \cdot \omega} \quad (2.44)$$

or:

$$\xi = \frac{\mu_2 \cdot \beta}{\mu_2 - \mu_1} \quad (2.45)$$

to simplify the expression of transfer and inverse matrices:

$$P = \begin{bmatrix} 1 & 1 \\ 0 & -\frac{1}{\xi} \end{bmatrix} \quad \text{and} \quad P^{-1} = \begin{bmatrix} 1 & \xi \\ 0 & -\xi \end{bmatrix} \quad (2.46)$$

2.2.5 Projection of State-Space Vectors in the Eigenvector Basis

We now have various intermediate results allowing us to write discretized state-space equations, then to project them in the eigenvector basis.

For that, we can directly use again the general results from the previous chapter, found in the sections 1.2.3 and 1.2.5, results which do not depend on the system described by linear state-space equations.

Thus, the discretized state-space equation (1.53) is written with state-space vectors projected within the space of eigenvectors $P^{-1} \cdot X$, like equation (1.93):

$$\boxed{P^{-1} \cdot X(t_n + T)_{(\alpha, \beta)} = e^{D \cdot T} \cdot P^{-1} \cdot X(t_n)_{(\alpha, \beta)} + D^{-1} \cdot (e^{D \cdot T} - I) \cdot P^{-1} \cdot B \cdot V_{(\alpha, \beta)}} \quad (2.47)$$

2.3 Discretized State-Space Equation Inversion

2.3.1 Introduction of the Rotating Frame

In the same way as in the section 1.3.1, let us define one rotating frame, but in this case, rotating with the speed of the polar mechanical angular frequency, compared to the fixed frame.

Let us define the angle ρ , at time $(t_n + T)$, of the (d, q) orthonormal frame, in the (α, β) fixed frame, by:

$$\rho(t_n + T) = \rho_0(t_n) + \theta(T) \quad (2.48)$$

ρ_0 is the value of this angle at time t_n , and θ its variation during the time interval T .

The complex scalar $e^{-i \cdot \rho}$ has a magnitude equal to the unit; it represents a rotation operator of an angle $-\rho$, in the complex plane (α, β) ; it is also an operator for frame changing of complex vector coordinates, from the (α, β) fixed frame, towards the (d, q) frame, at time $(t_n + T)$:

$$e^{-i \cdot \rho} = e^{-i(\rho_0 + \theta)} \quad (2.49)$$

This is the Park reference frame (Park, 1929). We will be interested, however, only in the initial and final position of this reference frame (*discrete reference frame*).

2.3.2 State-Space Vector Calculations in the Eigenvector Basis

In the same way as in the section 1.3.2 of the chapter 1, for an induction machine, let us multiply the two terms of equality (2.47), of which state-space vectors have their coordinates expressed in the (α, β) complex plane, by the equality between complex scalars (2.49); we then obtain equation (1.99).

By the product on the left of the transfer matrix inverse P^{-1} , this relation represents the projection of rotating state-space vectors located now in (d, q) at time $(t_n + T)$, in the eigenvector basis of the evolution matrix, that is to say:

$$\begin{aligned} \left[P^{-1} \cdot X(t_n + T)_{(d, q)} \right] &= e^{-i \cdot \theta} \cdot e^{D \cdot T} \cdot \left[P^{-1} \cdot X(t_n)_{(d_0, q_0)} \right] + \dots \\ \dots D^{-1} \cdot (e^{D \cdot T} - I) \cdot \left[P^{-1} \cdot B \cdot V_{(\alpha, \beta)} \right] \cdot e^{-i(\rho_0 + \theta)} \end{aligned} \quad (2.50)$$

Let us now define ρ_0 , as the angle of the rotor flux vector $\arg(\vec{\Phi}_r(t_n))$, known at time t_n in the (α, β) fixed frame, such that it cancels the rotor flux coordinate according to the q_0 axis. By definition of the angle ρ_0 :

$$\cos(\rho_0) = \frac{\Phi_{r\alpha}(t_n)}{\|\vec{\Phi}_r(t_n)\|} \quad \sin(\rho_0) = \frac{\Phi_{r\beta}(t_n)}{\|\vec{\Phi}_r(t_n)\|} \quad (2.51)$$

In this particular case:

$$\|\vec{\Phi}_r\| \equiv \Phi_a \quad \forall t \quad (2.52)$$

Thus:

$$X(t_n)_{(d_0, q_0)} = \begin{bmatrix} I_{sd_0} \\ I_{sq_0} \\ \Phi_a \\ 0 \end{bmatrix} \quad \text{or} \quad X(t_n)_{(d_0, q_0)} = \begin{bmatrix} I_{sd_0} + i \cdot I_{sq_0} \\ \Phi_a \end{bmatrix} \quad (2.53)$$

By definition, the (d_0, q_0) orthonormal complex reference frame has its d_0 axis directed according to the direction and the sense of the rotor flux vector at time t_n .

The polar mechanical angular frequency varies very little during the time interval T because of mechanical inertia; it is indeed one of the criteria of choice of the sampling period. We can thus compare the polar mechanical angular frequency ω at the initial time t_n , with the average angular velocity according to the time T of the *polar* mechanical angle ρ , such that $\theta = \rho - \rho_0 = \omega \cdot T$. The rotor flux angle at time $(t_n + T)$ in the (α, β) fixed frame becomes:

$$\rho = \rho_0 + \omega \cdot T \quad (2.54)$$

In the same way, by construction, the (d, q) orthonormal complex reference frame has the d axis directed according to the direction and the sense of the rotor flux at time $(t_n + T)$, and we can again write:

$$X(t_n + T)_{(d, q)} = \begin{bmatrix} I_{sd} \\ I_{sq} \\ \Phi_a \\ 0 \end{bmatrix} \quad \text{or} \quad X(t_n + T)_{(d, q)} = \begin{bmatrix} I_{sd} + i \cdot I_{sq} \\ \Phi_a \end{bmatrix} \quad (2.55)$$

The control vector V is always located in the (α, β) fixed frame relative to the stator of the synchronous motor, to control the voltage inverter.

Figure 2.5 represents simultaneously all orthonormal frames, the (α, β) fixed one, the (d_0, q_0) initial one at time t_n , and the (d, q) predicted one at time $(t_n + T)$.

Notice further that, according to equations (2.25) and (2.46):

$$P^{-1} \cdot B = \begin{bmatrix} 1 & \xi \\ 0 & -\xi \end{bmatrix} \cdot \begin{bmatrix} \beta \\ 0 \end{bmatrix} \quad (2.56)$$

that is to say:

$$P^{-1} \cdot B = \begin{bmatrix} \beta \\ 0 \end{bmatrix} \quad (2.57)$$

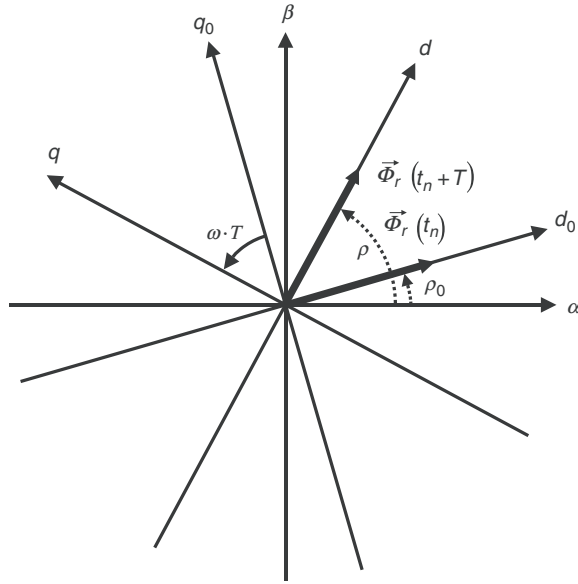


Figure 2.5 Definition of initial and final rotating reference frames

with:

$$D^{-1} = \begin{bmatrix} \frac{1}{\mu_1} & 0 \\ 0 & \frac{1}{\mu_2} \end{bmatrix} \quad (2.58)$$

and:

$$e^{D \cdot T} = \begin{bmatrix} e^{\mu_1 \cdot T} & 0 \\ 0 & e^{\mu_2 \cdot T} \end{bmatrix} \quad (2.59)$$

The preceding results make it possible to rewrite equation (2.50) simply:

$$\begin{aligned} \begin{bmatrix} 1 & \xi \\ 0 & -\xi \end{bmatrix} \cdot X(t_n + T)_{(d,q)} &= e^{-i \cdot \omega T} \cdot \begin{bmatrix} e^{\mu_1 \cdot T} & 0 \\ 0 & e^{\mu_2 \cdot T} \end{bmatrix} \cdot \begin{bmatrix} 1 & \xi \\ 0 & -\xi \end{bmatrix} \cdot X(t_n)_{(d_0,q_0)} + \dots \\ \dots \beta \cdot V_{(\alpha,\beta)} \cdot e^{-i(\rho_0 + \omega T)} \cdot \begin{bmatrix} \frac{1}{\mu_1} & 0 \\ 0 & \frac{1}{\mu_2} \end{bmatrix} \cdot \begin{bmatrix} e^{\mu_1 \cdot T} - 1 & 0 \\ 0 & e^{\mu_2 \cdot T} - 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} \end{aligned} \quad (2.60)$$

$$\begin{aligned}
 \begin{bmatrix} 1 & \xi \\ 0 & -\xi \end{bmatrix} \cdot X(t_n + T)_{(d,q)} &= e^{-i\omega T} \cdot \begin{bmatrix} e^{\mu_1 T} & 0 \\ 0 & e^{\mu_2 T} \end{bmatrix} \cdot \begin{bmatrix} 1 & \xi \\ 0 & -\xi \end{bmatrix} \cdot X(t_n)_{(d_0,q_0)} + \dots \\
 \dots \beta \cdot V_{(\alpha,\beta)} \cdot e^{-i(\rho_0 + \omega T)} \cdot \begin{bmatrix} \frac{e^{\mu_1 T} - 1}{\mu_1} \\ 0 \end{bmatrix} & \quad (2.61)
 \end{aligned}$$

The projection of rotating vectors $\vec{I}_s$ and $\vec{\Phi}_r$ within the space of eigenvectors, thus had as logical consequences:

- to create a new state-space vector, that we will name for convenience a state-space eigenvector Ψ , a real four-dimensional vector, resulting from the vectorial composition between $\vec{I}_s$ and $\vec{\Phi}_r$
- to separate eigenmodes according to the two coordinates of the state-space eigenvector.

By simplifying the two terms of the second row of matrix equation (2.61) by $-\xi$, we will thus see that the state-space eigenvectors, at time $(t_n + T)$ in (d, q) :

$$\Psi = \begin{bmatrix} \vec{I}_s + \xi \cdot \vec{\Phi}_r \\ \vec{\Phi}_r \end{bmatrix} = \begin{bmatrix} \vec{\Psi}_1 \\ \vec{\Psi}_2 \end{bmatrix}_{(d,q)} \quad (2.62)$$

Thus, at the initial time t_n , in (d_0, q_0) :

$$\Psi_0 = \begin{bmatrix} \vec{I}_{s_0} + \xi \cdot \vec{\Phi}_{r_0} \\ \vec{\Phi}_{r_0} \end{bmatrix} = \begin{bmatrix} \vec{\Psi}_{10} \\ \vec{\Psi}_{20} \end{bmatrix}_{(d_0,q_0)} \quad (2.63)$$

Equation (2.61) is now written simply, using the three frames (α, β) , (d_0, q_0) and (d, q) :

$$\Psi_{(d,q)} = \begin{bmatrix} e^{\mu_1 T} & 0 \\ 0 & e^{\mu_2 T} \end{bmatrix} \cdot \left[e^{-i\omega T} \cdot \Psi_{0(d_0,q_0)} \right] + \beta \cdot \left[V_{(\alpha,\beta)} \cdot e^{-i(\rho_0 + \omega T)} \right] \cdot \begin{bmatrix} \frac{e^{\mu_1 T} - 1}{\mu_1} \\ 0 \end{bmatrix} \quad (2.64)$$

It is written even more simply, but only in the final (d, q) reference frame:

$$\Psi_{(d,q)} = \begin{bmatrix} e^{\mu_1 T} & 0 \\ 0 & e^{\mu_2 T} \end{bmatrix} \cdot \Psi_{0(d,q)} + \beta \cdot V_{(d,q)} \cdot \begin{bmatrix} \frac{e^{\mu_1 T} - 1}{\mu_1} \\ 0 \end{bmatrix} \quad (2.65)$$

that is to say still, by extracting the two coordinates:

$$\vec{\Psi}_1 = e^{\mu_1 T} \cdot \vec{\Psi}_{10} + \beta \cdot \frac{e^{\mu_1 T} - 1}{\mu_1} \cdot \vec{V}_s \quad (2.66)$$

$$\vec{\Psi}_2 = e^{\mu_2 \cdot T} \cdot \vec{\Psi}_{20} \quad (2.67)$$

Finally:

$$\vec{\Psi}_1 = e^{-\frac{T}{\tau_s}} \cdot \vec{\Psi}_{10} + \frac{1 - e^{-\frac{T}{\tau_s}}}{R_s} \cdot \vec{V}_s \quad (2.68)$$

$$\vec{\Psi}_2 = e^{i\omega T} \cdot \vec{\Psi}_{20} \quad (2.69)$$

In the (d, q) final plane, these two vector equations are then easily interpreted:

- The first coordinate is homogeneous with a current. The coefficient which transforms the flux into a variable that is homogeneous with a current, is ξ , in particular in equation (2.62). The coefficient which transforms the control voltage into a variable homogeneous with a current is the multiplicative coefficient, inverse of R_s in equality (2.68). The second coordinate is the rotor flux.
- The first coordinate (2.68) of the state-space eigenvector is made up, at the final time, of the sum of the free evolution in the rotating reference frame, simple exponential decay of the magnitude compared to its initial value, and of the control *current* vector, after the transformation of the voltage vector dimension by a coefficient proportional to an admittance.

- in equation (2.68), the larger the sampling period is, the faster the coefficient $\left(1 - e^{-\frac{T}{\tau_s}}\right)$

tends towards 1. On the other hand, the smaller the period T is, the faster the coefficient tends towards zero. This analysis shows that control is all the more effective the longer the voltage vector is applied compared to the stator time constant, which is quite understandable; however, the sampling period having to remain small compared with the stator time constant, modifications of $\vec{\Psi}_1$ with regard to $\vec{\Psi}_{10}$ could be limited by the voltage magnitude within the sampling period.

- the second coordinate (2.69) is simply derived by the rotation from the rotor flux vector, as we would expect with this kind of motor. We thus lose, in this case, two degrees of freedom, which are the control of the two coordinates of rotor flux vector; we gain, however, in control simplicity. This relation simply makes it possible to calculate the angular rotation between the initial and the final reference frame position, by construction of the rotating reference frame.

We can see that the addition with the stator current, of a complex term of the form $\xi \cdot \Phi_a$ changes the origin of the rotating reference frame for a new origin of coordinates $-\xi \cdot \Phi_a$. We find in Figure 2.6, the two ways of building the first vector coordinate of the state-space eigenvector; this remark will be useful later on.

The vector $\vec{\Psi}_1$ is independent of the sampling period T and depends only on ω from the intermediate variable ξ . The direction of the new origin compared to the only origin of the two reference frames (d_0, q_0) and (d, q) depends also only on ξ .

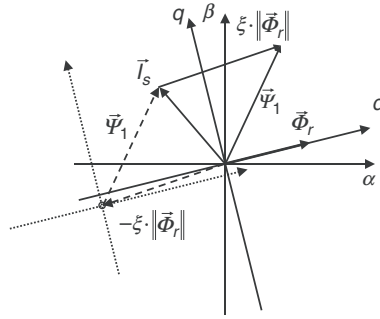


Figure 2.6 Construction of the first coordinate of the state-space eigenvector

Equation (2.68) makes it possible to calculate the free evolution in (d, q) , for $\vec{V}_s = 0$:

$$\vec{\Psi}_1^0 = e^{-\frac{T}{\tau_s}} \cdot \vec{\Psi}_{10} = e^{\mu_1 T} \cdot \vec{\Psi}_{10} \quad (2.70)$$

If we make the T period tend towards infinity the vector $\vec{\Psi}_1$ tends towards zero because of the negative coefficient of the real exponential function. The stator current then tends towards the short-circuit current with the mechanical angular frequency ω :

$$\vec{\Psi}_1^0 \rightarrow \vec{0} \Rightarrow \vec{I}_s \rightarrow \vec{I}_c = -\xi(\omega) \cdot \vec{\Phi}_r \quad (2.71)$$

which provides immediately an interpretation of the short-circuit current vector $-\xi \cdot \vec{\Phi}_r$, as coordinates of the new origin.

It is easy to confirm this interpretation while replacing ξ by its value:

$$\vec{I}_c = -\frac{i \cdot \omega \cdot \vec{\Phi}_r}{R_s + i \cdot L_s \cdot \omega} \quad (2.72)$$

This relation gives the fundamental sinusoidal component of the stator current, created by the rotation of the magnets, into the impedance of the motor in short-circuit:

$$\vec{I}_c = -\frac{1}{\vec{Z}_s} \cdot \frac{d\vec{\Phi}_r}{dt} \quad (2.73)$$

where $\vec{Z}_s$ is the complex stator impedance, at the angular frequency of the current fundamental component ω :

$$\vec{Z}_s = R_s + i \cdot L_s \cdot \omega \quad (2.74)$$

We will notice the negative sign of the *outgoing* stator current in this case.

Matrix equation (2.64) can now be written with the following parameter definition:

$$a = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \beta \cdot \frac{e^{\mu_1 T} - 1}{\mu_1} \\ 0 \end{bmatrix} \quad (2.75)$$

$$\begin{bmatrix} \tilde{\Psi}_1 \\ \tilde{\Psi}_2 \end{bmatrix}_{(d,q)} = \begin{bmatrix} e^{(\mu_1 - i\omega)T} & 0 \\ 0 & e^{(\mu_2 - i\omega)T} \end{bmatrix} \cdot \begin{bmatrix} \tilde{\Psi}_{10} \\ \tilde{\Psi}_{20} \end{bmatrix}_{(d_0, q_0)} + a \cdot \vec{V}_{s(\alpha, \beta)} \cdot e^{-i(\rho_0 + \omega T)} \quad (2.76)$$

The free evolution of the state-space eigenvector is the particular solution for $\vec{V}_s = 0$:

$$\begin{bmatrix} \tilde{\Psi}_1 \\ \tilde{\Psi}_2 \end{bmatrix}_{(d,q)}^0 = \begin{bmatrix} e^{(\mu_1 - i\omega)T} & 0 \\ 0 & e^{(\mu_2 - i\omega)T} \end{bmatrix} \cdot \begin{bmatrix} \tilde{\Psi}_{10} \\ \tilde{\Psi}_{20} \end{bmatrix}_{(d_0, q_0)} \quad (2.77)$$

2.3.3 Control Computation – Eigenstate-Space Equation Inversion

The second-order complex state-space equation (2.76), will now make it possible to calculate the input voltage vector in the (α, β) fixed frame while simply replacing, the predicted state-space vector $\tilde{\Psi}$, by the set-point state-space vector Ψ_c in (d, q) :

$$\Psi = \Psi_c = \begin{bmatrix} \tilde{\Psi}_{1c} \\ \tilde{\Psi}_{2p} \end{bmatrix}_{(d,q)} = \begin{bmatrix} I_{dc} + i \cdot I_{qc} + \xi \cdot \Phi_a \\ \Phi_a \end{bmatrix} \quad (2.78)$$

The second coordinate of the complex state-space eigenvector being not controllable, we cannot transform the prediction into a set-point. It remains the *prediction* of the position for the flux vector and thus the *prediction* of the (d, q) reference frame position. This prediction is directly usable to *estimate* the rotor position and thus to remove the position sensor.

The initial state being known by measurement or estimation in the (d_0, q_0) reference frame, and the period T being fixed, it is enough to invert the state-space equation and to calculate the voltage vector $\vec{V}_s$, starting from the first row of the complex matrix equation (2.76):

$$\tilde{\Psi}_{1c} = e^{(\mu_1 - i\omega)T} \cdot \tilde{\Psi}_{10} + a_1 \cdot \vec{V}_s \cdot e^{-i(\rho_0 + \omega T)} \quad (2.79)$$

that is to say:

$$\vec{V}_s = e^{i\rho_0} \cdot \frac{e^{i\omega T} \cdot \tilde{\Psi}_{1c} - e^{\frac{T}{\tau_s}} \cdot \tilde{\Psi}_{10}}{a_1} \quad (2.80)$$

or:

$$\vec{V}_{s(\alpha, \beta)} = e^{i\rho_0} \cdot \frac{e^{\mu_2 T} \cdot \tilde{\Psi}_{1c} - e^{\mu_1 T} \cdot \tilde{\Psi}_{10}}{a_1} \quad (2.81)$$

with:

$$\vec{\Psi}_{1c} = (I_{dc} + i \cdot I_{qc} + \xi \cdot \Phi_a)_{(d,q)} \quad (2.82)$$

$$\vec{\Psi}_{10} = (I_{d0} + i \cdot I_{q0} + \xi \cdot \Phi_a)_{(d,q_0)} \quad (2.83)$$

$\vec{V}_s$ is expressed in the (α, β) fixed frame, where it will be applied to the motor stator, the first vector coordinate of the initial state-space eigenvector $\vec{\Psi}_{10}$ is expressed in the (d_0, q_0) reference frame, and the first vector coordinate of the set-point state-space eigenvector $\vec{\Psi}_{1c}$ is expressed in the (d, q) reference frame.

Interpretation of the control voltage is simple, if we place all the vectors in the same reference frame, for instance in the (d, q) reference frame where the set-points of stator current coordinates are defined. After multiplication of the two terms by $e^{-i \cdot \omega \cdot T}$, equation (2.81) can be written:

$$a_1 \cdot \vec{V}_{s(\alpha,\beta)} \cdot e^{-i \cdot \rho_0} \cdot e^{-i \cdot \omega T} = e^{-i \cdot \omega T} \cdot e^{\mu_2 \cdot T} \cdot \vec{\Psi}_{1c(d,q)} - e^{\mu_1 \cdot T} \cdot e^{-i \cdot \omega T} \cdot \vec{\Psi}_{10(d_0,q_0)} \quad (2.84)$$

By noting that:

$$\vec{V}_{s(\alpha,\beta)} \cdot e^{-i \cdot \rho_0} \cdot e^{-i \cdot \omega T} = \vec{V}_{s(d,q)} \quad (2.85)$$

$$e^{-i \cdot \omega T} \cdot e^{\mu_2 \cdot T} = e^{-i \cdot \omega T} \cdot e^{i \cdot \omega T} = 1 \quad (2.86)$$

$$e^{-i \cdot \omega T} \cdot \vec{\Psi}_{10(d_0,q_0)} = \vec{\Psi}_{10(d,q)} \quad (2.87)$$

the relation between the control vector and the first coordinate of the state-space eigenvector is then written more simply:

$$a_1 \cdot \vec{V}_{s(d,q)} = \vec{\Psi}_{1c(d,q)} - e^{-\frac{T}{\tau_s}} \cdot \vec{\Psi}_{10(d,q)} \quad (2.88)$$

The vector $\vec{\Psi}_{10(d,q)}$ is the first vector coordinate of the initial state-space eigenvector, represented in the (d, q) reference frame. It should be noted here, that the rotation of the rotor and thus of the rotating reference frame axes between the initial and the final time during the period T , leads to a rotation, in the (d, q) reference frame, of the initial state-space eigenvector of the same angle $\omega \cdot T$, but in the reverse sense (cf. equation (2.87)).

The control voltage vector $\vec{V}_s$ is transformed into a control current vector by the coefficient:

$$a_1 = \frac{1 - e^{-\frac{T}{\tau_s}}}{R_s} \quad (2.89)$$

which is homogeneous with a real admittance. The magnitude of the initial state-space vector decreases in the (d, q) reference frame according to the stator time constant due to the coefficient

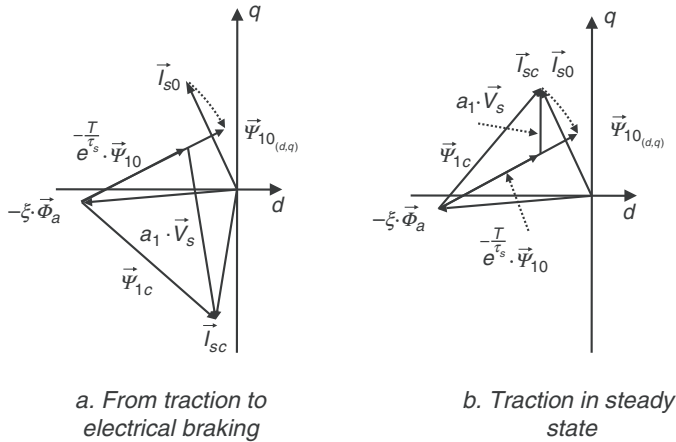


Figure 2.7 Representation of the control current vector in the (d, q) reference frame

$e^{-\frac{T}{\tau_s}}$. The vector $a_1 \cdot \vec{V}_s$ makes it possible to fill in the difference between the initial state-space vector on the one hand, and new set-point state-space vector on the other hand.

Figure 2.7a, represents the case of a switching from a traction set-point ($I_{qc} > 0$), to an electrical braking set-point ($I_{qc} < 0$), which explains the large magnitude of the control vector, as well as the control vector direction. In case of a steady state and a maintained positive torque, the control vector is represented in Figure 2.7b; in this case, the magnitude and the direction of the control current vector, make it just possible to compensate the free evolution.

An example of the calculation of a centered-symmetrical vector PWM making it possible to obtain the average voltage vector (2.81) is presented in detail in appendix A1.

2.4 Control

The inversion of eigenstate-space equations makes it possible to consider a simple control process. We will analyze the various stages of the real-time implementation in an example with a fixed sampling period.

2.4.1 Constitution of the Set-Point State-Space Vector

In equation (2.76), we now will introduce the scalar set-points I_{dc} and I_{qc} , in the first coordinate of the state-space eigenvector, in the (d, q) reference frame.

The component I_{dc} of the real coordinate of the current vector $\vec{\Psi}_{1c}$, which is collinear with the rotor flux, can be selected to regulate the stator flux in the *direction* of the rotor flux and thus to regulate the motor voltage. An adjustment of this magnetizing current is used to reduce the stator b.e.m.f. or e.m.f. due to the magnets and the motor speed, when the motor is used beyond its rated speed, in motor or braking mode respectively. The rated speed is the speed at which the full stator voltage is reached with the rated maximum torque. In this range of adjustment, the I_{dc}

component must be negative, contrary to what is required for an induction machine. We will see how to calculate this component, at the motor voltage limit.

Below the rated speed, this component is in general selected null to minimize the stator current.

We can see, however, that nothing prohibits fixing the magnetizing current set-point at *positive* values; this possibility is very useful transitorily, against a positive surge of the feeding voltage due to an adjustment discontinuity of the inverter voltage.

Thus, the transition from the asynchronous or synchronous PWM to the square wave PWM of the inverter causes a voltage discontinuity applied to the motor which is related to the firing angle limits of semiconductors, during the PWM period immediately preceding the full voltage operation area. These surges are indeed imposed by the semiconductor technology to guarantee the extinction of one of the electronic switches, before lighting of the other one in series in the same inverter leg, or to guarantee a minimum duration of turn-on.

The magnetizing current can thus increase simultaneously and transitorily, so that the created voltage step will be strictly opposed to the voltage surge from the inverter at the same time.

Imaginary coordinate I_{qc} of vector $\vec{\Psi}_{1c}$, perpendicular to the rotor flux vector by definition of the (d, q) reference frame in Figure 2.5, could be simply selected to regulate the electromagnetic torque; according to the cross product of equation (2.7), only this component operates an electromagnetic torque.

The set-point state-space eigenvector is thus created by the following successive stages:

- Calculate the algebraic current I_{qc} from the torque set-point C_c , positive in the traction mode and negative in electrical braking, and from the flux magnitude Φ_a of the magnets:

$$I_{qc} = \frac{C_c}{N_p \cdot \Phi_a} \quad (2.90)$$

- Choose a null current I_{dc} , below the rated speed, to minimize the motor heating and of a negative current above, to limit the motor voltage. We will see how to calculate it further, with voltage limitation according to the upstream feeding voltage of the inverter.
- Constitute the set-point state-space eigenvector in the (d, q) reference frame with Φ_a , I_{qc} and I_{dc} , so that:

$$\Psi_c = \begin{bmatrix} I_{dc} + i \cdot I_{qc} + \xi_1 \cdot \Phi_a \\ \Phi_a \end{bmatrix}_{(d,q)} = \begin{bmatrix} \vec{\Psi}_{1c} \\ \vec{\Psi}_{2c} \end{bmatrix} \quad (2.91)$$

2.4.2 Constitution of the Initial State-Space Vector

The initial state-space vector is known by measurement of the stator current $\vec{I}_s$ and the estimation of the rotor flux vector $\vec{\Phi}_r$ in the (α, β) reference frame.

The estimation of the rotor flux can be made:

- with the knowledge of the flux magnitude of the magnets $\|\vec{\Phi}_r\| = \Phi_a$, possibly corrected according to the motor temperature, itself estimated or measured

- and with the measurement of the position of the rotor $\arg(\vec{\Phi}_{r0}) = N_p \cdot \Xi$, in the (α, β) fixed frame, by an absolute angular position transducer Ξ , for instance a synchro-resolver, an optic sensor or a sensor of any other technology.

We will see that it is also possible to estimate the two coordinates of the rotor flux, for example using a Kalman estimator.

It is then necessary to transfer coordinates from these two vectors in the (d_0, q_0) reference frame, after having calculated the angle of this frame ρ_0 , compared to the (α, β) fixed frame. Successively:

$$\rho_0 = \arg(\vec{\Phi}_{r0}) \quad (2.92)$$

$$\Phi_{rd_0} = \|\vec{\Phi}_r\| = \Phi_a \quad (2.93)$$

$$I_{d0} + i \cdot I_{q0} = (I_{s\alpha} + i \cdot I_{s\beta}) \cdot e^{-i\rho_0} \quad (2.94)$$

Ultimately:

$$\Psi_0 = \begin{bmatrix} I_{d0} + i \cdot I_{q0} + \xi \cdot \Phi_{rd_0} \\ \Phi_{rd_0} \end{bmatrix}_{(d_0, q_0)} = \begin{bmatrix} \tilde{\Psi}_{10} \\ \tilde{\Psi}_{20} \end{bmatrix}_{(d_0, q_0)} \quad (2.95)$$

2.4.3 Control Process

2.4.3.1 Real-Time Implementation

Analytical calculation of the control being now finished, it remains a practical problem to solve for a real-time implementation of control algorithms. In the same way as in section 1.4.3.1, we now will detail the method to be used to optimize the real-time implementation.

Indeed, it is necessary to be able to calculate the average voltage vector to be applied between the initial time when the initial state-space eigenvector is measured and estimated, and the time $(t_n + T)$ when the set-point state-space eigenvector is reached. This has to be done without any lagging due to the computation time; without that, the computation time could cause a time-delay for the voltage vector application on the motor, from the initial time up to the end of a complete period. Under these conditions, the time reserved for various computations would be null.

In practice, it is thus necessary to anticipate the voltage vector impressing period by a period reserved for the computations. A second problem then appears: the initial state-space eigenvector having to be defined at time t_n , to feed the computation, it cannot result any more from measurements at this time.

A solution is to perform a computation during one period, and to apply the voltage vector during the following period. However, in order not to cause a pure time-delay of T under these conditions, it is necessary to predict the motor state at the end of the computational period in progress, and thus at the beginning of the impressing period of the voltage vector; it is this prediction of the initial state-space eigenvector at the horizon t_n , from measurements carried out at $(t_n - T)$, at the beginning of the period reserved to the

computation (Jacquot, 1995), which then will be used to calculate the voltage vector to apply between t_n and $(t_n + T)$.

It is thus necessary to reserve a first period for measuring, measurement filtering, rotor flux estimating, predicting the initial state-space vector and calculating the voltage vector, then to have a second period to impress the voltage vector.

During this second period, the voltage vector can be generated by the PWM, produced by an external circuit different from the computational processor, such as an FPGA or a pure logical circuit, thus reserving the main processor for the other following computations.

Here, the sampling period was selected as constant as an example and thus asynchronous compared to the fundamental motor voltage. It can be selected as a variable period to synchronize it with the voltage period of the motor, as is necessarily the case for square wave generation or to make it possible to reduce the rate of voltage and thus of current harmonics by a choice of the PWM type, better suited to high speed. To switch from one fixed computation period, generally used at low speed, to a computation period synchronized with the motor voltage period, it is enough, after a phase of synchronization, to calculate the control vector by considering the period T as variable, and no longer as a fixed parameter of equation (2.81).

The duration of the sampling period is selected in relation to the time necessary for the computation and to the duration of the average voltage vector application which determines the current shape factor, itself in relation to the selected PWM. The sampling period identical to the periodicity of the control vector application, also governs all the inverter losses.

This structuring of algorithms for their operation in real-time, led to imposing the sampling period identical to the PWM period, and synchronous with it, while remaining asynchronous compared to the fundamental component of the motor voltage when the PWM period is fixed at low speed, and synchronous compared to the fundamental component of the motor voltage when the PWM period is variable at high speed.

In particular, this characteristic leads to sample stator phase currents in synchronism with the PWM and thus in synchronism with harmonic currents created by the PWM; this arrangement makes it possible to remove the anti-aliasing filter of the current harmonic spectrum generated by the PWM, which is preponderant compared to the stochastic spectrum of measurement disturbances, and thus to improve the bandwidth of measurements (Jacquot, 1995). Sampling without the anti-aliasing filter provides, under these conditions, an instantaneous current value of the fundamental component added to all synchronous harmonic currents, at the beginning of each PWM period.

In short, the real-time implementation of the computational algorithms requires us to carry out a prediction of the motor state to construct the initial state, necessary to perform the computation. We can then use the discretized state-space equation to filter the measured values, but also to estimate variables which cannot be measured. The same equation will thus be used for predicting the state, filtering measurements and estimating non-measurable variables, in a cyclic way; it was also used to calculate the control after several transformations.

It should be noted that set-points which are used in the computation are those which are known at time $(t_n - T)$, in the example in Figure 2.8. Contrary to the motor state, set-points cannot, in the general case (except with a predefined tracking program), be predicted at time t_n ; they will thus be regarded as set-points at time t_n , with a pure time-delay of one sampling period. These set-points will thus be reached at time $(t_n + T)$, after two sampling periods.

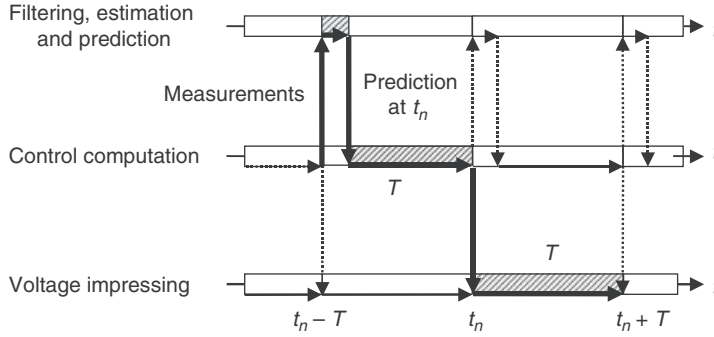


Figure 2.8 Timing diagram of the real-time control process

For opening the algorithmic loop in real-time, we will define the origin of algorithm descriptions at time $(t_n - T)$.

2.4.3.2 Measure Filtering

The discretized state-space equation (1.57), applied during the period preceding measurements:

$$X(t_n - T)_p = F \cdot X(t_n - 2 \cdot T) + G \cdot V \quad (2.96)$$

is used to make the prediction of the motor state at time $(t_n - T)$; it can be used to filter current measures made at time $(t_n - T)$, by a Kalman filter (Kalman, 1982), (Jacquot, 1995). This filtering will be simultaneously used to observe the rotor flux at time $(t_n - T)$, if this is not directly accessible by an absolute measurement from the rotor mechanical position.

A prediction by a Kalman filter of the state-space vector at the horizon t_n , done from filtered measurements, will then be used to define the initial state-space eigenvector $\Psi_0(t_n)$ for the computation between $(t_n - T)$ and t_n of the control vector to be applied at the following period, between t_n and $(t_n + T)$.

2.4.3.3 Transition and Control Matrix Calculations

We must then calculate the transition and the control matrices of the discretized state-space equation of the motor in (α, β) , by linking equations (1.57) and (1.93) which was pre-multiplied on the left by the transfer matrix:

$$F = P \cdot e^{D \cdot T} \cdot P^{-1} \text{ and } G = P \cdot D^{-1} \cdot (e^{D \cdot T} - I) \cdot P^{-1} \cdot B \quad (2.97)$$

Let us replace the various symbols by their computed values, P and P^{-1} , from equation (2.46), $e^{D \cdot T}$ from (2.59), D^{-1} from (2.58) and $P^{-1} \cdot B$ from (2.57).

$$F = \begin{bmatrix} 1 & 1 \\ 0 & -\frac{1}{\xi} \end{bmatrix} \cdot \begin{bmatrix} e^{\mu_1 \cdot T} & 0 \\ 0 & e^{\mu_2 \cdot T} \end{bmatrix} \cdot \begin{bmatrix} 1 & \xi \\ 0 & -\xi \end{bmatrix} \quad (2.98)$$

$$G = \beta \cdot \begin{bmatrix} 1 & 1 \\ 0 & -\frac{1}{\xi} \end{bmatrix} \cdot \begin{bmatrix} \frac{e^{\mu_1 \cdot T} - 1}{\mu_1} & 0 \\ 0 & \frac{e^{\mu_2 \cdot T} - 1}{\mu_2} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad (2.99)$$

The transition matrix in (α, β) is thus written:

$$F = \begin{bmatrix} e^{\mu_1 \cdot T} & \xi \cdot (e^{\mu_1 \cdot T} - e^{\mu_2 \cdot T}) \\ 0 & e^{\mu_2 \cdot T} \end{bmatrix} \quad (2.100)$$

In the same way, for the input matrix in the (α, β) fixed frame:

$$G = \begin{bmatrix} \beta \cdot \frac{e^{\mu_1 \cdot T} - 1}{\mu_1} \\ 0 \end{bmatrix} \quad (2.101)$$

We can further simplify the writing of the input matrix by using the reduced variables $\{a_1, a_2\}$, of which expression was defined by (2.75):

$$G = \begin{bmatrix} a_1 \\ 0 \end{bmatrix} = a \quad (2.102)$$

2.4.3.4 Kalman Filter, Observation and Prediction

Ultimately, the prediction used for current filtering and for the observation of the rotor flux at time $(t_n - T)$, starting from filtered variables at the previous time $(t_n - 2 \cdot T)$ takes the following form in the (α, β) fixed frame:

$$X(t_n - T)_p = F(t_n - 2 \cdot T) \cdot X(t_n - 2 \cdot T)_f + G(t_n - 2 \cdot T) \cdot V_{(t_n - 2 \cdot T) \rightarrow (t_n - T)} \quad (2.103)$$

$$Y(t_n - T)_p = H \cdot X(t_n - T)_p \quad (2.104)$$

with:

$$X = \begin{bmatrix} I_{s\alpha} \\ I_{s\beta} \\ \Phi_{r\alpha} \\ \Phi_{r\beta} \end{bmatrix} \quad \text{and} \quad V = \vec{V}_s = \begin{bmatrix} V_{s\alpha} \\ V_{s\beta} \end{bmatrix} \quad (2.105)$$

Let us choose the stator current vector as a measured variable $Y(t_n - T)_m$ at time $(t_n - T)$ and the rotor flux as an estimated variable; we thus fix the output matrix H :

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \text{ real writing, } H = \begin{bmatrix} 1 & 0 \end{bmatrix} \text{ complex writing, and } Y = \begin{bmatrix} I_{s\alpha} \\ I_{s\beta} \end{bmatrix} \quad (2.106)$$

Let us recall briefly, the main stages of Kalman filtering at time $(t_n - T)$ (Jacquot, 1995):

- Filter the stator current $\bar{I}_{sf}$, starting from the measurement of the current $Y(t_n - T)_m$ and from the predicted state-space vector $X(t_n - T)_p$, simultaneously with the estimation of the rotor flux $\bar{\Phi}_{re}$, if one does not measure the rotor position:

$$X(t_n - T)_f = \begin{bmatrix} \bar{I}_{sf} \\ \bar{\Phi}_{re} \end{bmatrix} \quad (2.107)$$

The filtered state-space vector is calculated by the following relation:

$$X(t_n - T)_f = X(t_n - T)_p + K(t_n - T) \cdot \{Y(t_n - T)_m - Y(t_n - T)_p\} \quad (2.108)$$

- $K(t_n - T)$ is the Kalman gain calculated for the time $(t_n - T)$. The state-space vector $X(t_n - T)_f$ contains the filtered current measure, but also the observed value of the rotor flux vector, in magnitude and argument.

When the position of the rotor flux is measured with a position encoder, it is then possible to replace the argument of the observed rotor flux by the measured angle from the sensor.

In the same way, it is also possible to replace the magnitude of the observed rotor flux by the theoretical magnitude of the magnet flux.

- Calculate a prediction of the state error covariance matrix at t_n , from the one calculated at $(t_n - T)$:

$$P(t_n)_p = F \cdot P(t_n - T) \cdot F^t + Q_0 \quad (2.109)$$

- Q_0 is the state disturbance covariance matrix, regarded in this example as constant for simplicity.
- Calculate the matrix of the Kalman gain at time t_n , starting from the prediction of the state error covariance matrix t_n :

$$K(t_n) = P(t_n)_p \cdot H^t \cdot \{H \cdot P(t_n)_p \cdot H^t + R_0\}^{-1} \quad (2.110)$$

- R_0 is the measurement disturbance covariance matrix, considered also in this example as constant.
- Calculate the update of the state error covariance matrix at t_n , from the predicted one at t_n :

$$P(t_n) = \{1 - K(t_n) \cdot H\} \cdot P(t_n)_p \quad (2.111)$$

From filtered measurements and from the rotor flux observation at time $(t_n - T)$, the prediction necessary to calculate the initial state-space at time t_n , could thus be made, thanks to the following state-space equation, where the voltage applied during the computational time was known at the end of the preceding computational period:

$$X(t_n)_p = F(t_n - T) \cdot X(t_n - T)_f + G(t_n - T) \cdot V_{(t_n - T) \rightarrow t_n} \quad (2.112)$$

This same prediction will be used again for measure filtering and for estimation of the rotor flux at time t_n , as above, in a cyclic way.

In the case of this synchronous motor, the estimation of the rotor flux in magnitude and argument can:

- be replaced by the flux magnitude value of magnets and by the measure of the rotor angular position, and thus be used for a more efficient filtering of the stator current because of the recalibration of the flux estimation
- be kept and used to follow the evolution of the flux magnitude of magnets according to the temperature, and the evolution of the rotor flux argument according to the rotor position, which makes it possible to remove the absolute position encoder of the rotor flux. In this case, however, the measurement of the rotor initial position is necessary in particular before starting (Benchäib *et al.*, 2003). It remains necessary to keep a speed measurement of the rotor, since this information is useful for the calculation of the evolution matrix and thus for all control computations.

During the computational period of the control, it is in general necessary to do other computations, measurements, estimations, regulations and supervisions necessary to the controlled process operation, so that the duration of computations necessary to the motor regulation must be a lot shorter than the application duration of the vector sequence calculated by the regulation.

2.4.3.5 Summary of Measurement, Filtering and Prediction

We now will summarize section 2.4.3, which concerns the real-time implementation, measurements, measure filtering and estimation, as well as the prediction of the initial state-space vector, by the summary in Figure 2.9.

These computations are repeated at each PWM period. They start with synchronous measurements of sampled currents, input filter voltage, rotor position and speed of motor rotation at the beginning of the PWM period, at time $(t_n - T)$. All computations, including the control vector computation must be finished before the end of the PWM period T . These cyclic computations use some computational results from the previous period which are thus stored to be used one period later. To symbolize this storing, we use in Figure 2.9 the time-delay operator z^{-1} of one computational period duration.

Each time the neutral cable is not connected to the motor, the vector sum of instantaneous currents of the three phases, is null, according to Kirchhoff's law. It is thus enough to measure only the currents of two phases, to derive the current value of the third phase (cf. Figure 2.4) by the following relation.

$$\vec{I}_a + \vec{I}_b + \vec{I}_c = \vec{0} \quad (2.113)$$

The instantaneous measurement of currents must be made at the beginning of the sampling period with current sensors having a large bandwidth compared to main harmonic frequencies generated by the inverter, to restore the instantaneous peak current of each of the three phases. For example, for a motor supplied with a fundamental voltage with a maximum harmonic frequency of 500 Hz, the bandwidth of the current sensor could be 500 kHz.

The current measurement of the three phases is then converted into just one stator current vector in the orthonormal fixed frame, by the Concordia transformation which preserves the power (cf. equation (1.3)).

The measurement of the synchronous motor rotor flux is in general not accessible directly. Most *brushless* drives use an absolute rotor position sensor to know the approximate position of the rotor flux and the theoretical flux magnitude value of the magnets to reconstitute the rotor flux vector in its entirety.

This choice presents many disadvantages:

- The measurement of the absolute position by either a synchro-resolver or an optical coder, uses a fragile sensor technology; the alignment of these coders on a high-power motor shaft is hard to adjust and can create mechanical resonances.
- It is necessary to very precisely adjust the sensor reference position on the motor shaft and all the more precisely if the number of motor pole pairs is high; indeed, the more the number of pole pairs, the smaller is the mechanical angle to which it is necessary to discriminate the reference position of pole axes in the fixed frame.
- The angle of the rotor flux can be influenced in a few degrees by the stator current, and a rotor flux position sensor does not take this phenomenon into account.

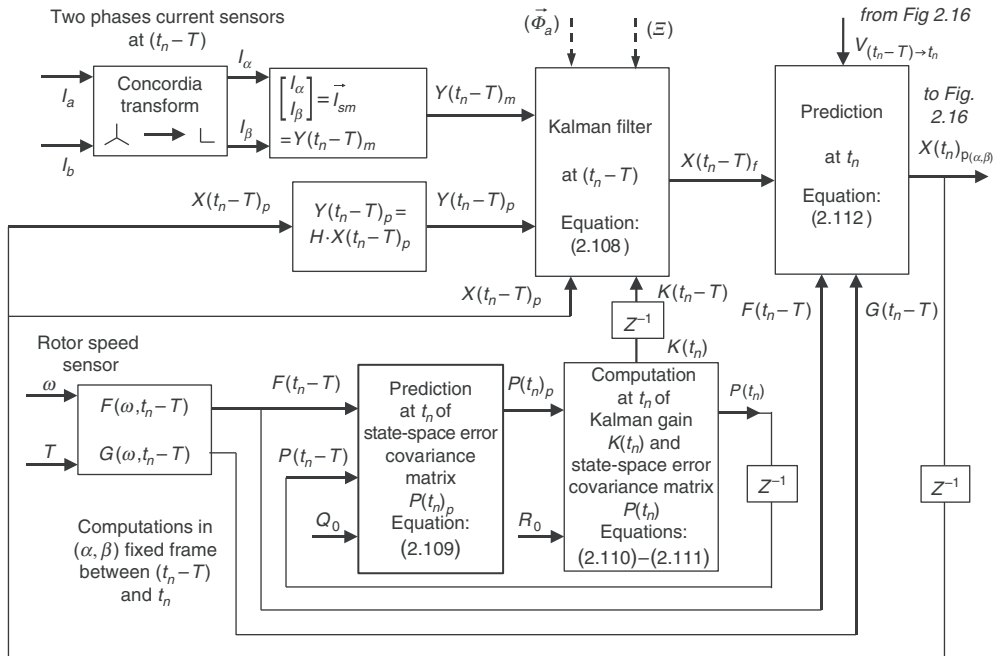


Figure 2.9 Measuring, filtering and prediction

- The magnetic field of the rotor magnets is influenced by the air gap temperature and by the stator current, particularly after motor short-circuits; it is sensitive to ageing when an antagonistic magnetic field comes close to the coercive field of the magnetic material.

We will calculate the rotor flux using a Kalman estimator (2.108). It is thus about a flux estimation based on the one hand on current measurements $Y(t_n - T)_m$, and on the other hand on the measure prediction of these same currents $Y(t_n - T)_p$, starting from the motor state prediction at the same time $X(t_n - T)_p$. The prediction of the motor state is made with the preceding sampling period according to equation (2.96). It also requires the knowledge of the Kalman gain of which we will see the calculation method below.

Practically, a Kalman observation makes it possible to simultaneously filter the stator current vector in the same operation, thanks to the discretized model of the motor. The filtered state-space vector comprises the filtered stator current vector and the estimated rotor flux vector (2.107). The estimated flux will be used to calculate the control (cf. Figure 2.15). The Kalman filter, filters measurement disturbances due to stochastic noises of measured signals, but also state disturbances related to statistical errors of the motor modeling.

The state-space vector $X(t_n - T)_f$ filtered from current measurements at time $(t_n - T)$, will now allow us to predict the same state-space vector $X(t_n)_p$ at time t_n corresponding to the end of the computational period in progress, thanks to discretized state-space equations of the motor (2.112). This predicted state-space vector constitutes the initial state-space vector allowing calculating the control vector (cf. Figure 2.15).

This computation requires the knowledge of the control voltage calculated during the preceding computation period (cf. Figure 2.15) and impressed on the motor during the current computational period by a device which can be different from the computation processor. It requires an analytical calculation of discretized transition and input matrices that we will now discuss.

We start by measuring the mechanical rotor angular frequency of the motor to convert it into an electrical angular frequency by the multiplication by the pole pair number. The mechanical angular frequency can be estimated or measured by a digital speed sensor, in general an incremental one. The speed measurement can also be derived from the time derivative of an absolute rotor position measure, only if this measurement exists. The choice of the speed sensor must take into account the lowest speed measurement requiring accuracy, which defines the minimum pulse repetition frequency of a digital sensor per mechanical rotor revolution. To increase the accuracy at low speed, it is possible to extrapolate the pulse edge detection of the digital speed sensor and thus to carry out an estimation from measurements of the preceding pulses.

Using eigenvalues (2.30) and (2.31), and also motor parameters (2.45), we calculate the transition matrix (2.100) and the input matrix (2.101) at time $(t_n - T)$. We then calculate, at time t_n , the prediction of the state error covariance matrix $P(t_n)_p$ by equation (2.109), from the transition matrix $F(t_n - T)$ and the state error covariance matrix $P(t_n - T)$ at the present time. This last matrix was stored during the computation of the previous period.

One uses in this computation the state disturbance covariance matrix Q_0 , a square matrix of size identical to that of the transition matrix, which can be selected as fixed to reduce computations and of which coefficients allow a weighting of the state filtering for each coordinate of the state-space vector. These coefficients can be adjusted according to the process or according to the process operating mode.

The prediction of the state error covariance matrix $P(t_n)_p$ and of the measurement disturbance covariance matrix R_0 , make it possible to calculate the Kalman gain matrix at t_n by equation (2.110), then the state error covariance matrix $P(t_n)$ at time t_n by equation (2.111). These last two results are stored to feed computations of the Kalman gain and filtering during the following period.

We use in this last computation the disturbance covariance matrix R_0 , a square matrix, a half-dimensional one compared to the transition matrix; it can also be selected with fixed coefficients to reduce the computation delay and to allow a fixed weighting of the measure filtering for each coordinate of the current vector. These coefficients can be adapted according to the process or according to the operating mode of the process.

Lastly, it should be noted here that these matrix computations can be performed with complex coefficients by using the rotor magnetic isotropy of the non-salient poles synchronous motor; this magnetic isotropy allows a reduction by two of the matrix row and column number (cf. section 2.2.2) and simplifies the writing of relations.

With complex coefficients, the matrix dimensions are as follows:

$$F : 2 \times 2, G : 2 \times 1, H : 1 \times 2, P : 2 \times 2, Q_0 : 2 \times 2, R_0 : 1 \times 1$$

The real-time implementation of algorithms of the Kalman filter allows a sequential computation of the control during all the sampling period with, simultaneously:

- measure filtering of currents
- estimate the rotor flux
- predict the motor state-space vector at the instant of the end of computation.

Thanks to this provision, the pure time-delay of the control is equal to just one computational period.

2.4.4 Limitations

The process of the control which has just been described makes it possible to fix the set-point stator current in magnitude and phase, to reach the electromagnetic torque set-point in only one sampling period, without any limitation of physical variables.

However, feeding the motor voltage from an inverter is in general limited. The stator current must always be limited in magnitude, either in an instantaneous way to ensure the inverter commutation and to restrict the heating of semiconductors, or by the motor to limit the temperature of its stator winding. It is the system sizing which defines these limitations.

When the motor speed increases, the b.e.m.f. of the motor increases and, beyond the motor rated speed, the rotor flux can no longer be maintained at its maximum value owing to the fact that the available feeding voltage is maximum; for this reason, the maximum torque can no longer be attained beyond this speed without a current increasing. As the current is limited, the operation beyond the rated speed requests the electrical power to be limited, and thus to limit the torque at a decreasing value with the speed and the flux.

We saw that in case of the synchronous motor, unlike an induction machine, the rotor flux is not a controllable variable. The only possible latitude to operate beyond the motor rated speed is thus to create a stator flux by means of stator inductance, being opposed to the rotor

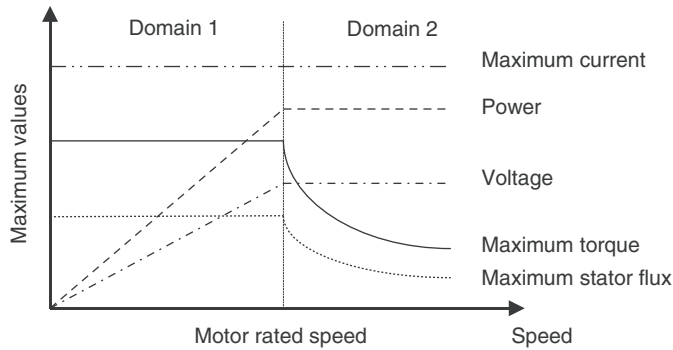


Figure 2.10 Rated motor speed with the maximum torque

flux, which makes it possible to reduce the flux crossing the stator and thus to reduce the b.e.m.f. as the speed increases.

Thanks to the motor control computation during only one sampling period, we now will analyze the best way of not exceeding limitations due to the process sizing.

2.4.4.1 Voltage Limitation

V_M is the maximum value of the *voltage fundamental component*, that the inverter can provide with the Concordia transformation which preserves the instantaneous power; with a fixed frequency PWM, the relation which links this voltage to the input filter voltage U_c is given to a first approximation by the relation: $V_M = \frac{U_c}{\sqrt{2}}$ (cf. Figure 2.4).

If one takes into account the dead-time corresponding to the minimum of the non-turn-on time of the IGBT, the maximum value of the voltage fundamental component is reduced to:

$$V_M = \frac{U_c}{\sqrt{2}} \cdot \left(1 - 2 \cdot \frac{t_m}{T} \right) \quad (2.114)$$

In the square wave operation, the maximum value of the *voltage fundamental component* that the inverter can provide with variable frequency PWM, is given by $V_M = \frac{\sqrt{6}}{\pi} \cdot U_c$ and the *instantaneous* maximum voltage corresponds to the magnitude of the six non-null voltage vectors of the inverter; it is given by $V_M = \sqrt{\frac{2}{3}} \cdot U_c$ (cf. Figure 2.4).

If V_M is the maximum voltage that the inverter can provide, with the Concordia transformation, which preserves the instantaneous power, the relation which links this voltage to the DC voltage of the input filter U_c is:

$$V_M = \sqrt{\frac{2}{3}} \cdot U_c \quad (2.115)$$

When one calculates the maximum flux which one can obtain, starting from the fundamental voltage of the inverter with an asynchronous fixed frequency PWM, one chooses the magnitude of the maximum fundamental voltage corrected by the dead-time; this magnitude is given by equation (2.114).

The stator voltage must thus always be lower than or equal to V_M , a constraint that one expresses in (d, q) , with magnitudes of its two coordinates:

$$V_{sd}^2 + V_{sq}^2 \leq V_M^2 \quad (2.116)$$

We then project equation (2.1), rewritten according to angular partial derivatives:

$$\vec{V}_s = R_s \cdot \vec{I}_s + L_s \cdot \frac{\delta \vec{I}_s}{\delta \theta} \cdot \frac{d\theta}{dt} + \frac{\delta \vec{\Phi}_r}{\delta \theta} \cdot \frac{d\theta}{dt} \quad (2.117)$$

onto the axes of the (d, q) reference frame:

$$V_{sd} = R_s \cdot I_{sd} - L_s \cdot \omega \cdot I_{sq} \quad (2.118)$$

$$V_{sq} = R_s \cdot I_{sq} + L_s \cdot \omega \cdot I_{sd} + \omega \cdot \Phi_a \quad (2.119)$$

These relations make it possible to express the inequality of fundamental components:

$$\left(R_s \cdot I_{sd} - L_s \cdot \omega \cdot I_{sq} \right)^2 + \left(R_s \cdot I_{sq} + L_s \cdot \omega \cdot I_{sd} + \omega \cdot \Phi_a \right)^2 \leq V_M^2 \quad (2.120)$$

The magnitude of the stator impedance, at a frequency ω , derived from the complex impedance $\vec{Z}_s$ in (2.74), is written:

$$Z_s = \sqrt{R_s^2 + L_s^2 \cdot \omega^2} \quad (2.121)$$

and the stator reactance X_s :

$$X_s = L_s \cdot \omega \quad (2.122)$$

We then develop the squared polynomials and by ordering the terms, after having used reduced variables of impedance and reactance, we obtain:

$$Z_s^2 \cdot I_{sd}^2 + 2 \cdot L_s \cdot \omega^2 \cdot \Phi_a \cdot I_{sd} + Z_s^2 \cdot I_{sq}^2 + 2 \cdot R_s \cdot \omega \cdot \Phi_a \cdot I_{sq} + \omega^2 \cdot \Phi_a^2 \leq V_M^2 \quad (2.123)$$

Now let us separate the two coordinates of the stator current in the (d, q) reference frame and look at the squared polynomials of the first degree of these coordinates; after simplifications one obtains:

$$\left(I_{sd} + \frac{X_s \cdot \omega \cdot \Phi_a}{Z_s^2} \right)^2 + \left(I_{sq} + \frac{R_s \cdot \omega \cdot \Phi_a}{Z_s^2} \right)^2 \leq \frac{V_M^2}{Z_s^2} \quad (2.124)$$

With reduced variables:

$$I_{cd} = -\frac{X_s \cdot \omega \cdot \Phi_a}{Z_s^2} \quad (2.125)$$

$$I_{cq} = -\frac{R_s \cdot \omega \cdot \Phi_a}{Z_s^2} \quad (2.126)$$

it becomes possible to simplify the writing and to reveal the equation of a circle of center:

$$(I_{cd}, I_{cq}) = \left(-\frac{X_s \cdot \omega \cdot \Phi_a}{Z_s^2}, -\frac{R_s \cdot \omega \cdot \Phi_a}{Z_s^2} \right) \quad (2.127)$$

and of radius:

$$I_{VM}^2 = \frac{V_M^2}{Z_s^2} \quad (2.128)$$

The coordinates of the circle center are coordinates of the short-circuit current vector; indeed, by setting out again the complex equation (2.72) and separating the real part and the imaginary part, we obtain successively in (d, q) :

$$\vec{I}_c = -\frac{i \cdot \omega \cdot \vec{\Phi}_r}{R_s^2 + L_s^2 \cdot \omega^2} \cdot (R_s - i \cdot L_s \cdot \omega) \quad (2.129)$$

$$\vec{I}_c = -\frac{X_s \cdot \omega \cdot \Phi_a}{Z_s^2} - i \cdot \frac{R_s \cdot \omega \cdot \Phi_a}{Z_s^2} \quad (2.130)$$

We find again by this means, the coordinates of the circle center. Ultimately:

$$\boxed{(I_{sd} - I_{cd})^2 + (I_{sq} - I_{cq})^2 \leq I_{VM}^2} \quad (2.131)$$

According to this last relation, in the (d, q) reference frame, the extremity of the stator current vector must thus remain inside the circle of radius I_{VM} and of center:

$$(I_{cd}, I_{cq}) = (-k_d \cdot \Phi_a, -k_q \cdot \Phi_a) \quad (2.132)$$

with:

$$k_d = \frac{X_s \cdot \omega}{Z_s^2} \quad \text{and} \quad k_q = \frac{R_s \cdot \omega}{Z_s^2} \quad (2.133)$$

This is the circle of voltage limits (Benchair et al., 2008).

Let us notice that coordinates of the circle center, and its radius, depend on the polar mechanical angular frequency.

2.4.4.2 Current Limitation

The stator current limitation is a simple magnitude limitation at the maximum value I_M defined by the sizing of the inverter and of the motor:

$$I_s^2 = I_{sd}^2 + I_{sq}^2 \leq I_M^2 \quad (2.134)$$

In the (d, q) plane, the current limit is a circle of radius I_M and centered on the reference frame origin.

2.4.4.3 Operating Area and Limits

In short, the extremity of the reference current vector $\vec{I}_{sc}$ in the (d, q) reference frame normalized in current must then be, as shown in Figure 2.11:

- inside the circle of voltage limit defined by (2.131)
- inside the circle of current limit defined by (2.134).

The authorized areas Tr for the traction operation $I_{qc} \geq 0$, and Br for the electrical braking $I_{qc} < 0$, result from the intersection of the area bordered by circles of the voltage limit and of the current limit (cf. Figure 2.11), for negative magnetizing currents $I_{dc} \leq 0$ in steady state, but transitorily for positive magnetizing current $I_{dc} > 0$, if necessary.

The extremity of the stator current vector $\vec{I}_{sc}$ represented in the (d, q) reference frame of Figure 2.11 is located simultaneously on the circle of the voltage limit and on the circle of the current limit.

2.4.4.4 Set-Point Limit Calculations

2.4.4.4.1 Voltage Limit

According to torque and rotor flux set-points, the ordinate of the stator current set-point can be expressed by equation (2.90).

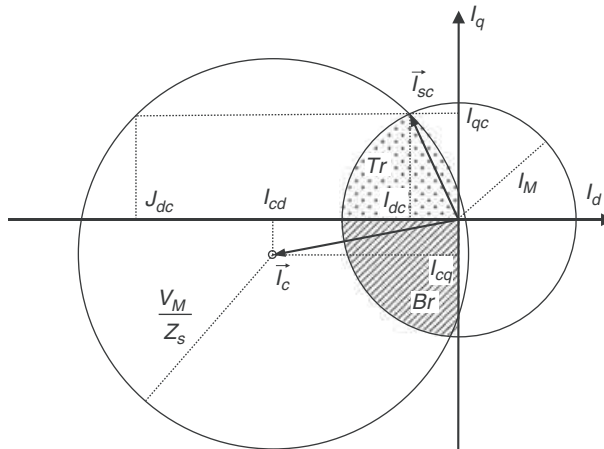


Figure 2.11 Voltage and current limitations

While replacing I_{qc} from equation (2.90), in equation (2.131), which expresses the voltage limit, we can then calculate the abscissa of the stator current set-point in the (d, q) reference frame:

$$(I_{dc} + k_d \cdot \Phi_a)^2 + \left(\frac{C_c}{N_p \cdot \Phi_a} + k_q \cdot \Phi_a \right)^2 \leq I_{VM}^2 \quad (2.135)$$

When, for a null demagnetizing current, the first term of this equation is higher than the second one, there is a voltage limitation:

$$(k_d \cdot \Phi_a)^2 + \left(\frac{C_c}{N_p \cdot \Phi_a} + k_q \cdot \Phi_a \right)^2 > I_{VM}^2 \quad (2.136)$$

If it is the case, it is thus necessary to force a negative current of demagnetization I_{dc} to make the first term, in the limit, equal to I_{VM}^2 :

$$I_{dc} = -k_d \cdot \Phi_a \pm \sqrt{I_{VM}^2 - \left(\frac{C_c}{N_p \cdot \Phi_a} + k_q \cdot \Phi_a \right)^2} \quad (2.137)$$

provided that:

$$\left(\frac{C_c}{N_p \cdot \Phi_a} + k_q \cdot \Phi_a \right)^2 \leq I_{VM}^2 \quad (2.138)$$

There is thus one torque limit, or a limit of the ordinate of the current set-point, so that the voltage limit is not exceeded. This last condition must necessarily be satisfied by the sizing of the motor-inverter unit, compared to the process requiring a torque set-point C_c .

The two roots of equation (2.137) are abscissas I_{dc} and J_{dc} of the two intersections of:

- a line passing by the ordinate I_{qc} , parallel with the d axis,
- with the voltage limitation circle represented in Figure 2.11.

Between the two roots of equation (2.137), we then choose the root which leads to the minimum current, to always minimize the energy and losses:

$$I_{dc} = -k_d \cdot \Phi_a + \sqrt{I_{VM}^2 - \left(\frac{C_c}{N_p \cdot \Phi_a} + k_q \cdot \Phi_a \right)^2} \quad (2.139)$$

2.4.4.4.2 Voltage Limit and Current Limit

The knowledge of the two components of the stator current set-point I_{dc} and I_{qc} in the event of a voltage limitation by equations (2.139) and (2.90) respectively now enables us to check, using the inequality (2.134), that the current limit is not reached.

If the limit of the stator current magnitude is exceeded, the demagnetizing current is linked to the active current which creates the torque, by the relation derived from (2.134), affected by the minus sign to minimize the first term of (2.135):

$$I_{dc} = -\sqrt{I_M^2 - I_{qc}^2} \quad (2.140)$$

This relation is now used with equation (2.131) to express also the voltage limitation with the calculation of I_{qc} , and thus the torque set-point limit, making it possible to satisfy the two limitations simultaneously:

$$\left(-\sqrt{I_M^2 - I_{qc}^2} - I_{cd}\right)^2 + \left(I_{qc} - I_{cq}\right)^2 = I_{VM}^2 \quad (2.141)$$

developing:

$$2 \cdot I_{cd} \cdot \sqrt{I_M^2 - I_{qc}^2} = -I_{cd}^2 - I_{cq}^2 + 2 \cdot I_{cq} \cdot I_{qc} + I_{VM}^2 - I_M^2 \quad (2.142)$$

Let us square the two terms and replace the sum of squared coordinates of the short-circuit current vector, by its squared magnitude I_c^2 .

$$4 \cdot I_{cd}^2 \cdot \left(I_M^2 - I_{qc}^2\right) = \left(2 \cdot I_{cq} \cdot I_{qc} + I_{VM}^2 - I_M^2 - I_c^2\right)^2 \quad (2.143)$$

Defining the M variable as:

$$M = I_M^2 + I_c^2 - I_{VM}^2 \quad (2.144)$$

let us develop and rearrange the terms according to the decreasing powers of I_{qc} :

$$4 \cdot I_c^2 \cdot I_{qc}^2 - 4 \cdot M \cdot I_{cq} \cdot I_{qc} + M^2 - 4 \cdot I_{cd}^2 \cdot I_M^2 = 0 \quad (2.145)$$

solving according to I_{qc} , noticing that $I_{cd}^2 = I_c^2 - I_{cq}^2$:

$$I_{qc} = \frac{I_{cq} \cdot M \pm \sqrt{4 \cdot I_{cd}^2 \cdot I_c^2 \cdot I_M^2 - I_{cd}^2 \cdot M^2}}{2 \cdot I_c^2} \quad (2.146)$$

We can now work out the abscissa of the short-circuit current from the square root, by noticing that:

$$\sqrt{I_{cd}^2} = |I_{cd}| = -I_{cd} \quad \text{and also} \quad \sqrt{I_{cq}^2} = |I_{cq}| = -I_{cq} \quad (2.147)$$

$$I_{qc} = \frac{I_{cq} \cdot M \mp I_{cd} \cdot \sqrt{4 \cdot I_c^2 \cdot I_M^2 - M^2}}{2 \cdot I_c^2} \quad (2.148)$$

Now let us calculate the abscissa of the current set-point using (2.131) and (2.134); in a similar way, we ultimately obtain:

$$I_{dc} = \frac{I_{cd} \cdot M \mp I_{cq} \cdot \sqrt{4 \cdot I_c^2 \cdot I_M^2 - M^2}}{2 \cdot I_c^2} \quad (2.149)$$

We then use the sign of the torque set-point which determines the traction or braking mode, to eliminate the foreign roots due to the squaring.

If the sign of the torque set-point is positive $\text{sgn}(C_c) \geq 0$, then I_{qc} should be positive in equation (2.148) where the two coordinates I_{cd} and I_{cq} are themselves negative. It is then advisable to choose the negative sign for the active component calculation.

Figure 2.11 also shows that the abscissa of the set-point vector is more negative in traction than in braking owing to the fact that the intersections of the two circles are shifted compared to an axis parallel with the q axis, of a positive angle in the counterclockwise direction; this phenomenon is due to the center of the voltage limit circle having a negative ordinate.

It is thus the positive sign which must be selected with the traction operation for the abscissa calculation of the current set-point.

We arrive at the same conclusion by noticing that, with the traction operation, it is necessary to subtract the voltage drop due to the motor impedances from the feeding voltage to calculate the active power and thus the torque; for a given torque, it is thus necessary to demagnetize the motor a little bit more than in braking where the reverse situation occurs.

Ultimately:

$$I_{dc} = \frac{I_{cd} \cdot M + \text{sgn}(C_c) \cdot I_{cq} \cdot \sqrt{4 \cdot I_c^2 \cdot I_M^2 - M^2}}{2 \cdot I_c^2} \quad (2.150)$$

$$I_{qc} = \frac{I_{cq} \cdot M - \text{sgn}(C_c) \cdot I_{cd} \cdot \sqrt{4 \cdot I_c^2 \cdot I_M^2 - M^2}}{2 \cdot I_c^2} \quad (2.151)$$

These results make it possible to calculate the maximum of the realizable torque, from equation (2.7):

$$C_c = N_p \cdot \Phi_a \cdot I_{qc} \quad (2.152)$$

as well as the demagnetizing current necessary to be able to obtain this torque with the polar mechanical angular frequency, when the voltage limit and the current limit are reached simultaneously.

2.4.4.4.3 Current Limit

If the first term of equation (2.131) is negative, there is no voltage limitation. It should, however, be checked to see if there is a current limitation by using (2.134), the equation in which we first adjusted the demagnetizing current to zero in this case, to reserve the current for torque generation. By taking account of the sign of the initial torque set-point:

$$I_{qc} = \text{sgn}(C_c) \cdot I_M \quad (2.153)$$

which makes it possible to calculate the maximum of the allowable torque in this case, by (2.152).

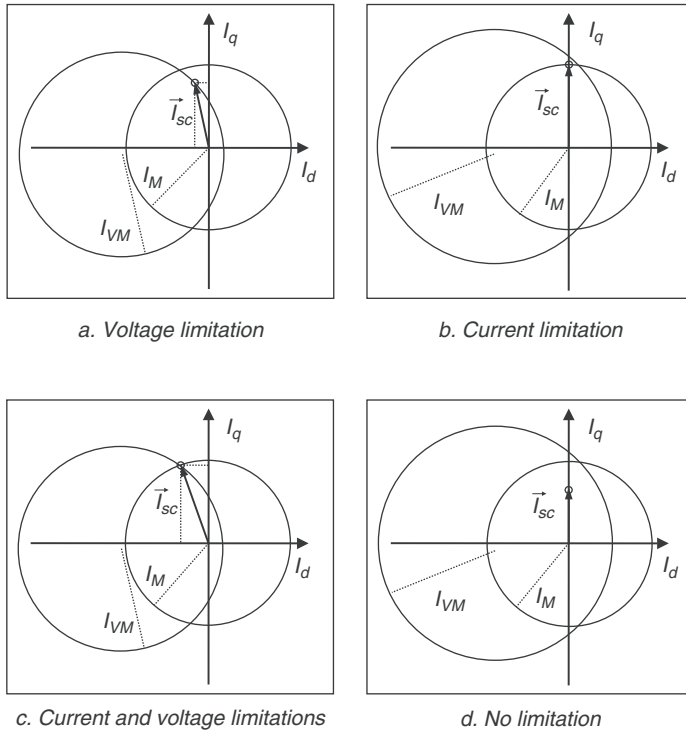


Figure 2.12 Various limitation cases in traction mode

2.4.4.4.4 Various Cases of Limitation

All cases of limitation are represented in Figure 2.12. The circle of the current limit remains fixed whatever the operation mode. The circle of the voltage limit has its radius and its center evolving according to the mechanical speed; the center moves away from the reference frame origin where it stays at null speed, and its radius decreases according to equations (2.127) and (2.128).

For various limitation cases, the calculation of the two components I_{dc} and I_{qc} of the stator current set-point vector $\vec{I}_{sc}$ in the set-point rotating reference frame (d, q) can be made using the decision tree in Figure 2.13.

Results obtained in the section 2.4.4.4 make it possible to detect and characterize the various cases of voltage, current and/or torque limitation. According to the diagnosis, established equations make it possible to calculate the set-point current vector acceptable to reach the optimal torque.

Knowing existing limitations, it is necessary to define the torque set-point which must be used; this must be lower than the maximum torque set-point defined by the process.

The torque of a surface-mounted permanent-magnet synchronous motor depends only on the active stator current component; the torque does not depend on the magnetizing stator current component as in the case of an induction machine which needs this component to create the required rotor flux for the electromagnetic torque.

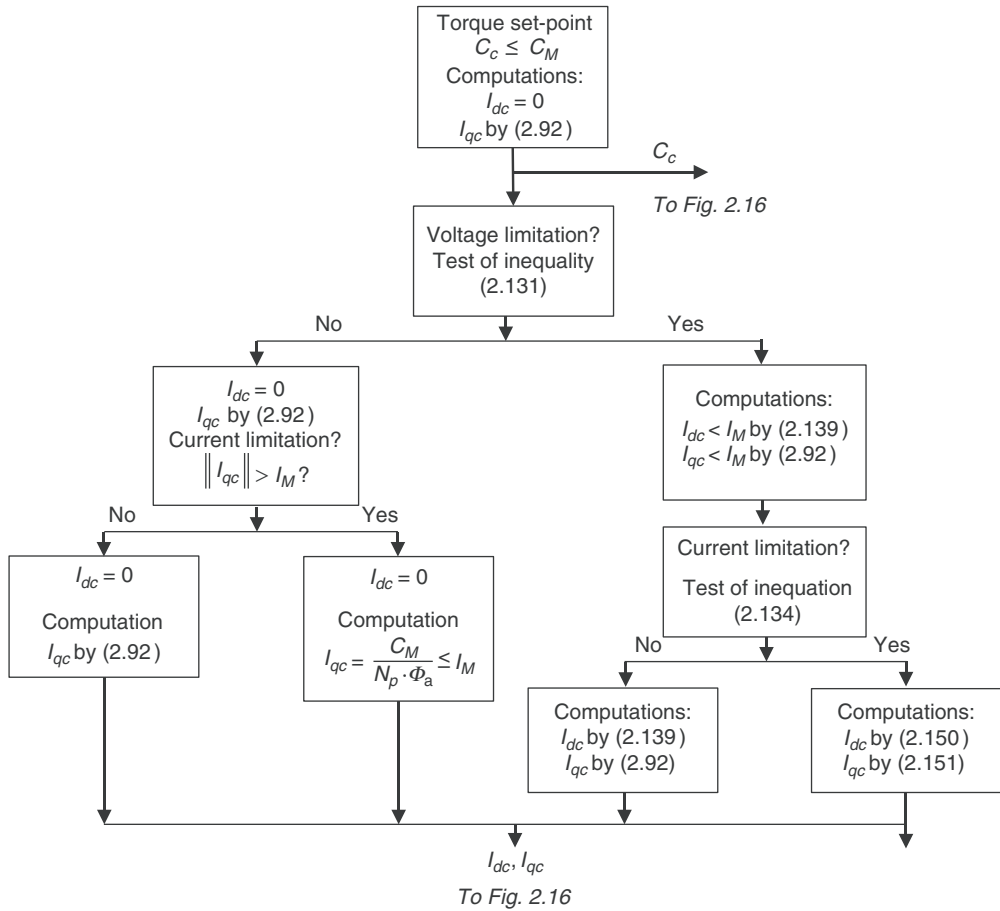


Figure 2.13 Current and voltage limitations

$$C_c = N_p \cdot \Phi_a \cdot I_{qc} \quad (2.154)$$

The reactive component of the stator current will be used to create the stator flux being opposed to the rotor flux, each time the motor feeding voltage reaches its maximum value. To minimize the stator current of the motor, each time the maximum voltage is not reached, we will maintain $I_{dc} = 0$.

The torque set-point is initially used to calculate the active current by equation (2.92). We then check by equation (2.131), with $I_{dc} = 0$, if the voltage limit is reached.

If the voltage limit is exceeded for this pair of current set-points (Figure 2.12a), we must impose a negative magnetizing current in the stator, calculated by equation (2.139). In case of voltage limitation only, the active current is always calculated by equation (2.92) and the torque set-point could be reached.

If the pair of currents calculated by equations (2.139) and (2.92) no longer satisfies inequality (2.134), in the event of a voltage limitation, we thus prove that the current limitation is also reached (Figure 2.12c). In this case, the two coordinates of the set-point current vector I_{dc} and I_{qc} will be calculated by (2.150) and (2.151) respectively. It will thus be necessary to

demagnetize the motor because of the voltage limitation, and moreover the torque set-point will not be reached because of the current limitation.

If equation (2.131) does not point out a voltage limitation (Figure 2.12b or d), it is enough to check if there is a current limitation by equation (2.134). If that is the case, (Figure 2.12b) the computation of new coordinates of the stator current starts by fixing the magnetizing current at zero, and continues by the computation of the active current to obtain the maximum torque from equation (2.92) in which we replace the set-point torque by the maximum torque. If the computation of the active current provides a value higher than the maximum current, one imposes $I_{qc} = I_M$.

If there is neither voltage limitation, nor current limitation (Figure 2.12d), we impose $I_{dc} = 0$ and the current I_{qc} is calculated by (2.92).

The chosen strategy is to minimize the stator current by maintaining the stator magnetizing current at zero, as long as the voltage limit is not reached. This strategy corresponds to the motor sizing such that it is defined below the motor rated speed (domain 1 of Figure 2.10). It is, however, interesting to be able to briefly overexcite the motor at the time of switching to square wave PWM, to create a transitory counter-voltage, cancelling the voltage jump created by the PWM at this switching time.

We can see in this Figure that above the motor rated speed, the torque decreases in an inverse ratio of the speed because of the current limitation. In this area, the motor power consumption is more or less constant. A very different strategy will be necessary for an interior permanent magnet synchronous motor as we will see in chapter 3.

Analysis made in section 2.4.4 makes it possible to adapt the control strategies to process constraints. It is thus shown that it is possible to calculate the current set-point, during the phase of dead-beat control computation, which complies with all limitations.

These set-points, and possibly their limits, will be reached at the end of the impressing period of the control voltage, thanks to the prediction of the motor state evolution.

In the event of all types of limitations, the control process is not modified, either in its principle or in its characteristics; the only way the set-point computations change is by *anticipating* the limitations, which makes it possible to have only one control operating mode and thus no gain or dynamics modification to generate instabilities; this process leads to no time-delay for the detection of limitations; the pure time-delay of the control is equal to a maximum of one sampling period, and is compensated by the prediction based on the motor model.

2.4.4.4.5 Transitory Voltage Limit

In addition, as the set-point modification is very fast compared to the stator time constant of the motor, the calculated voltage vector can instantaneously exceed the maximum of the voltage vector that the inverter can provide.

Indeed, depending on the characteristics of the motor, and on the chosen sampling period, it may be that the current set-point cannot be reached in only one impressing period of the stator voltage.

This limitation is different from the voltage limitation calculated previously and occurs only with set-point transients, for processes needing very great kinematic dynamics compared to inductive current transients.

To rapidly vary the stator current, the voltage source should be able to impose an adequate current gradient in the stator inductance throughout the whole sampling period; that is not always possible.

Fast set-point variations could all the more easily be satisfied, because the voltage reserve, which is the vector difference between the feeding voltage and the motor counter-voltage, will be large. It is often the case at low speed where the motor b.e.m.f. is smaller than above the motor rated speed.

By contrast, at high speed, the computation of the voltage vector from (2.81) will lead to an applied voltage vector having a right phase, therefore a right direction, but the magnitude of which will be transitorily insufficient to reach the current set-point in only one period. Voltage magnitude peaks will be clipped by the inverter. The stator current vector, however, will evolve in the direction of the extremity of the set-point current vector, starting from its initial position, without being able to reach it during the first sampling period. The extremity of the stator current vector will thus progress, step by step at each sampling period, in the direction of the set-point vector extremity, using the available magnitude of the feeding voltage.

A transitory case of voltage limitation is represented in the (d, q) reference frame in Figure 2.14, at the time of switching from traction to electrical braking mode. The preceding motor current set-point $\vec{I}_{s0}$ in the traction mode was limited by the voltage and the current; the current set-point $\vec{I}_{sc}$ of the electrical braking mode corresponds to the full torque at this speed and thus leads to a current limitation. However, in this case, one does not meet simultaneously the voltage limitation since the set-point vector extremity in the braking mode remains inside the voltage limit circle, with a null demagnetizing current. Thus the rated speed in the braking mode was not yet reached at this speed, whereas it is exceeded in the traction mode; indeed, ohmic and inductive voltage drops act in the contrary sense in the traction mode compared to the regenerative braking mode, with respect to the feeding voltage.

Using Figure 2.7, building the control current vector $a_1 \cdot \vec{V}_s$, with $a_1 \in \Re$, it is easy to represent the vector which would be necessary to switch from the initial state to the new braking set-point, in Figure 2.14. However, this vector has its magnitude limited to $a_1 \cdot V_M$. The radius

of the circle centered on the extremity of the vector $e^{-\frac{T}{\tau_s}} \cdot \vec{\Psi}_{10}$, in the (d, q) reference frame, crosses the control current vector which would be necessary in this case. The current set-point would thus not be reached in only one sampling period and an intermediate stage, at least, will be necessary.

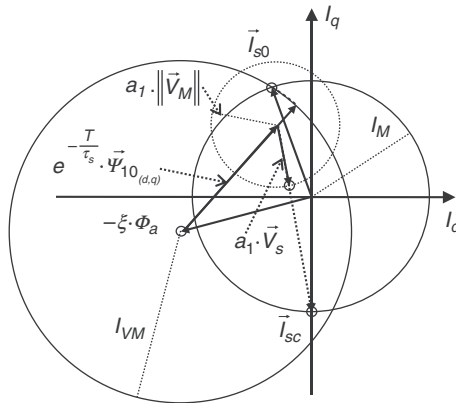


Figure 2.14 First transient voltage limitation

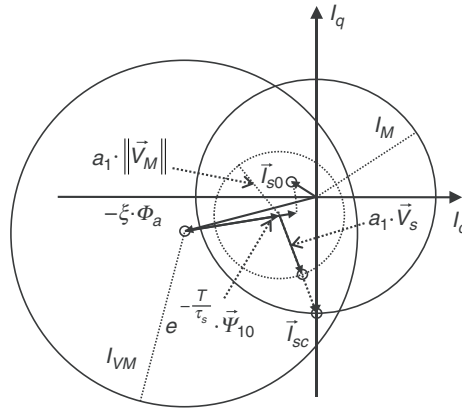


Figure 2.15 Second transient voltage limitation

The second phase is represented in Figure 2.15. In this extreme case it would take three sampling periods to reach the new current set-point. Thus, if the selected sampling period lasts 500 μ s, the switching from the traction to the braking mode would require 1.5 ms, which is in general too fast for the mechanical transmission. In the tracking mode, one would prefer, in most cases, to limit the mechanical jerk, while controlling gradually the set-point gradient.

However, during disturbances it is preferable that the control does not limit kinematic performance; this is very useful with processes where the load moment varies very quickly, such as with people transports, robotics, machine tools and rolling mills; it will be necessary, on the contrary, to adapt the driving torque as quickly as possible to the load moment, as long as this remains within its limits defined by its sizing. Beyond these limits one exceeds sizing margins, which are not always known precisely, and that can lead to degraded operation or to a material destruction.

The expression for a_1 , in (2.89), shows that the product $a_1 \cdot V_M$, for a given value V_M , increases with time, since when $T \rightarrow \infty$:

$$\frac{1 - e^{-\frac{T}{\tau_s}}}{R_s} \cdot V_M \rightarrow \frac{V_M}{R_s} \quad (2.155)$$

However, it should be remembered that when the period increases, the vector:

$$e^{-\frac{T}{\tau_s}} \cdot \vec{\Psi}_{10} \quad (2.156)$$

decreases, making it more difficult to reach the set-point by increasing the required control voltage (2.88). Also, it is not desirable to increase the sampling period with respect to electric time constants, owing to the fact that the shape factor of the stator current becomes worse, creating more harmonic currents to be switched by the inverter and greater losses in the motor to the detriment of the torque fundamental.

With these three types of limitation, the motor control is thus kept within the maximum nominal operation range defined by the process.

One must notice, however, that the control current vector $a_i \cdot \vec{v}_s$ is generated here by an asynchronous PWM to simplify the description of control computations; the angle of this vector can thus be the same as that of the required set-point direction. The magnitude of this control vector is however limited, in accordance with Figure 2.4.

If one must use all the available voltage with square wave PWM, only the six non-null vectors of the inverter are usable (cf. Figure 2.4); control directions are thus limited by the six directions of the inverter voltage vectors in the (α, β) fixed frame, and these directions appear in the (d, q) reference frame, with a rotation $-\rho$ of the angle between these two frames. In this case, the computation of control vectors can no longer be made at a constant period, and the sampling period becomes a control variable; analytical solutions are not presented within the framework of this book.

2.4.4.4.6 Control Vector Computation

A summary of the control voltage computation is presented in Figure 2.16. We will detail it below.

This computation is repeated in each PWM period. It is carried out after measure sampling at time $(t_n - T)$, the filtering, the rotor flux estimation and the state-space prediction, and it must be finished before the end of the computational period which is the instant t_n of the prediction. These cyclic computations use some results of the computation made during the previous period; they are thus stored to be used one period later. To symbolize this storing, we use, in Figure 2.16, the time-delay operator z^{-1} of one computation period.

We saw in section 2.4.3.5 (cf. Figure 2.9) how to measure motor phase currents, then to filter measurements, to estimate the rotor flux and finally to predict the motor state at the end of the computational period; the prediction of the state-space vector $X(t_n)_p$ is used to define the initial state-space vector in the fixed frame (α, β) for the control computation.

This state-space vector is made up of the predicted current vector $\vec{I}_{sp}$ and the predicted rotor flux vector at time t_n , in the (α, β) fixed frame.

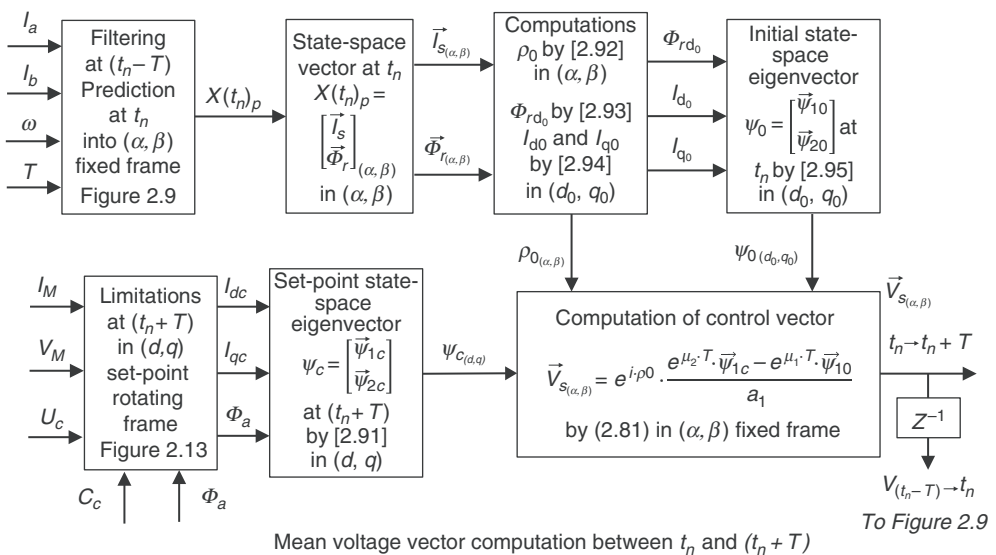


Figure 2.16 Computation of the mean control voltage vector

$$X(t_n)_p = \begin{bmatrix} \bar{I}_{sp} \\ \vec{\Phi}_{rp} \end{bmatrix}_{(\alpha, \beta)} = \begin{bmatrix} I_{s\alpha} \\ I_{s\beta} \\ \Phi_{r\alpha} \\ \Phi_{r\beta} \end{bmatrix} \quad (2.157)$$

It is noted that $\Phi_{r\beta}$ cannot be null since the rotor flux was estimated in the fixed frame; its two coordinates thus enable us to define the angle ρ_0 of the initial rotating reference frame (d_0, q_0) using equation (2.92) and the flux magnitude Φ_{rd_0} from equation (2.93); using ρ_0 , we calculate the new coordinates of the current vector in the initial rotating reference frame (d_0, q_0) from (2.94).

The initial state-space vector in the rotating reference frame (d_0, q_0) , becomes:

$$X(t_n)_0 = \begin{bmatrix} \bar{I}_{s0} \\ \vec{\Phi}_{r0} \end{bmatrix}_{(d_0, q_0)} = \begin{bmatrix} I_{d_0} \\ I_{q_0} \\ \Phi_{d_0} \\ 0 \end{bmatrix} \quad (2.158)$$

The abscissa of the rotor flux in the rotating reference frame becomes null by definition of the rotating reference frame.

If the measurement of the rotor position of the synchronous motor is ensured by a position encoder, it is still possible to replace the estimation of the angle ρ_0 by its measurement, if one can consider this measurement more precise than the estimation. It is however not proven that the position measurement of the motor *shaft* provides a measure more precise than the estimation of the *rotor flux* position in the motor air gap; in general, this is not so.

In the same way, it is still possible to replace the estimation of the flux magnitude of permanent magnets of the rotor by their theoretical flux magnitude, if it were judged that this last value is more precise; however, it is not necessarily so, because the estimation takes account of the evolution of this flux in particular according to the magnet's temperature.

This kind of control makes it possible to remove the position encoder while improving the accuracy of the control and thus of the torque provided with maximum current. One must, however, use a speed sensor, which is not useful with a time derivative of the position encoder angle, if the rotation speed itself is not estimated.

It is then enough to calculate the state-space eigenvector Ψ_0 in the (d_0, q_0) reference frame by equation (2.95) with ξ (2.45) defined with motor parameters at the polar mechanical angular frequency ω .

In addition, we saw in section 2.4.4.4.4 (cf. Figure 2.12) the various cases of voltage, stator flux, current and torque limitations and how to calculate the set-points of the two coordinates of the stator current vector I_{dc} and I_{qc} (cf. Figure 2.13), by taking into account these limitations and the torque set-point provided by the process; set-point currents are used to define a set-point state-space vector in the set-point rotating reference frame (d, q) for the control vector computation.

The set-point state-space vector defines the goal of the current control at time $(t_n + T)$.

$$X(t_n + T)_c = \begin{bmatrix} \vec{I}_{sc} \\ \vec{\Phi}_{rc} \end{bmatrix}_{(d,q)} = \begin{bmatrix} I_{dc} \\ I_{qc} \\ \Phi_a \\ 0 \end{bmatrix}_{(d,q)} \quad (2.159)$$

We can see again here that the rotor flux magnitude becomes the abscissa of the flux vector in the set-point rotating reference frame (d, q) , by definition of this reference frame. The flux magnitude cannot be a set-point, since the rotor flux is not controllable. We replace it either by the theoretical magnitude of flux magnets, or by the magnitude estimated at time $(t_n - T)$:

$$\Phi_{rc} = \|\vec{\Phi}_{rc}\| = \Phi_a \quad \text{or} \quad \Phi_{rc} = \|\vec{\Phi}_{rc}\| = \Phi_e \quad (2.160)$$

It is then enough to calculate the state-space eigenvector Ψ_c in (d, q) by equation (2.91) with motor parameters defined for the polar mechanical angular frequency ω : μ_1 (2.30), μ_2 (2.31) and ξ (2.45).

The control voltage vector to be applied during the sampling period following the present computation period from t_n to $(t_n + T)$ is calculated simply by equation (2.81) with Ψ_c in the (d, q) reference frame, with Ψ_0 in the (d_0, q_0) reference frame and with motor parameters calculated for angular frequency ω : μ_1 (2.30), μ_2 (2.31) and a_1 (2.75).

The magnitude and the angle of the average voltage vector of the control are stored, to be used both by the PWM, and by the computation during the following period (cf. Figure 2.9).

The torque set-point read at time $(t_n - T)$ is reached at time $(t_n + T)$, from measurements made at time $(t_n - T)$, computations performed from time $(t_n - T)$ to t_n , and from the PWM applied from time t_n to $(t_n + T)$. Thanks to the motor state prediction at time t_n , and to the calculation of the motor state evolution from t_n to $(t_n + T)$, the control makes it possible to reach the set-point in only one period T , which is the characteristic of a dead-beat response. We can, however, notice that between the sampling of the torque set-point, which cannot be predicted, and its obtaining, it takes two periods, with one period of pure time-delay.

2.4.5 Example of Implementation

To concretize implementation of this control process, we chose a motor having the following characteristics under traction operation:

- $N_p = 4$
 - $R_s = 31.0 \text{ m}\Omega$
 - $L_s = 0.998 \text{ mH}$
 - $\omega_M = 2 \cdot \pi \cdot 240 \text{ rd/s}$
 - $\Phi_M = 0.517 \text{ Wb}$
 - $C_M = 967 \text{ m.N}$
 - $I_M = 473 \text{ A}$
 - $T = 500 \mu\text{s}$
- (2.161)

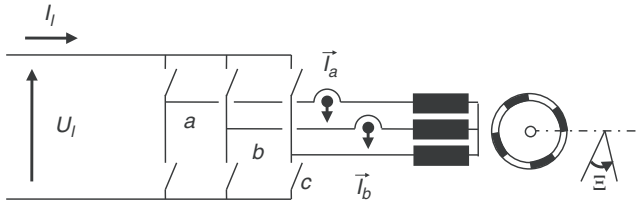


Figure 2.17 Synchronous motor fed by a voltage inverter

The rated motor speed is appreciably less than half of its maximum speed. Magnitudes of electrical vectors are those obtained by the Concordia transformation which preserves the instantaneous power.

The DC supply voltage of the inverter is 720V. The equivalent circuit of the motor feed is represented in Figure 2.17.

2.4.5.1 Adjustment of Torque – Limitations in Traction Operation

To allow the visualization of all phenomena in various timescales, we choose to force the electromagnetic torque to the maximum in the traction operation from starting. The mechanical speed is very quickly increasing from zero to the maximum speed, on a timescale less than 1.5 seconds, which would have been difficult to practically realize. The motor and the control were thus finely simulated.

The starting takes place with null speed, null current and the maximum torque.

In Figure 2.14, during a first period of 0.5s approximately, the torque is maximum; the magnitude of the stator current practically reached its maximum. The maximum torque is reached immediately, as well as the current necessary to its obtaining.

The simulation was made without an input filter. Indeed, it is well-known (Jacquot, 1995) that the traction operation causes an instability of the input filter voltage starting from a low power consumption from the filter, and continuing until the divergence, owing to the fact that this power filter has very few losses. This simulation was parameterized to highlight all the dynamics and also the control method compliance with limitations. It is obvious that with a filter, a very fast torque variation can immediately initiate instability, and then distorts the performance demonstration.

However, we will see later on, in the case of an induction machine, how to stabilize the filter in all circumstances and to preserve high dynamics, by using a regulation process of the input filter voltage of the same type as the motor control process, acting in symbiosis.

After approximately 0.5 s, the rated motor speed is reached with the maximum available magnitude of the stator voltage. The speed continuing to increase, the stator flux must thus be limited by the available voltage from this time. It will thus have to decrease gradually until it reaches the maximum speed. For that, the component of the stator current vector on the axis of the rotor flux, which was hitherto null, grows in the opposite sense to the rotor flux vector, to create a stator flux opposite to the rotor flux and to thus maintain a constant voltage.

At approximately 0.55 s, the magnitude of the stator current, of which the ordinate I_{sq} was hitherto constant to maintain the torque, but of which the negative abscissa I_{sd} grew in absolute value to be opposed to the rotor flux, reached its maximum value. The torque cannot be

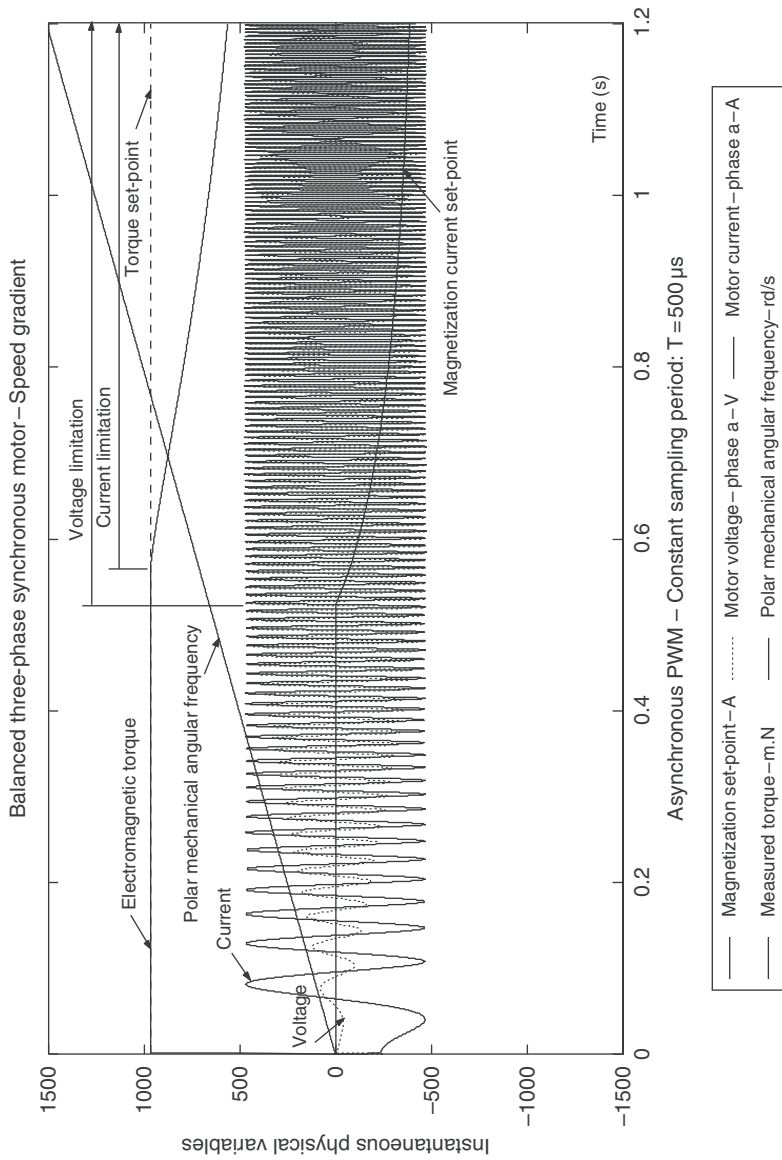


Figure 2.18 Starting with the maximum torque

maintained anymore because of the voltage and current limits. The output torque thus decreases gradually with increasing speed.

Figure 2.18 visualizes the performances of this control method during the torque and the current tracking operation.

We don't see any limit overshoot, in spite of a torque set-point variation in only one sampling period at starting. This comes owing to the fact that the control is instantaneous (dead-beat response) and that the set-points take account of limits by anticipation. It is thus possible, under these conditions, to not only maintain the motor state *on* its rated limits to avoid any overshoot, but also to avoid any unused margin of operation compared to these limits. The operation is also optimized compared to the fast variations of the feeding voltage. For the same reason, we can integrate into the set-point current, the harmonics due to the inverter voltage switching and so not exceed the peak current allowed by the inverter switching.

When one is able to take account of all kinds of disturbances (load moment, feed voltage) by measurements, set-point modifications and control voltage calculations, the control process makes it possible to reach optimum performance during disturbances as during tracking operations, i.e. in only one sampling period, without overshoot or lagging; it should, however, be ensured that the sampling period remains very small with respect to the stator time constant.

The ripple of the electromagnetic torque coming with an increased speed and due to the inverter voltage switching, is not visible here owing to the fact that the maximum speed is relatively low and that the sampling period is small with respect to the motor voltage period.

2.4.5.2 Adjustment of Torque – Limitations in Electrical Braking

In regenerative electrical braking, the rated performances are different for the same motor:

$$C_M = 891 \text{ mN}, I_M = 433 \text{ A}$$

with a DC supply voltage of the inverter of 850V.

To allow the visualization of all phenomena in their various timescales, we choose here also to impose the maximum electromagnetic torque during the regenerative electrical braking operation and a mechanical speed very quickly decreasing from the maximum speed to zero, on a timescale of 1.2 s. The motor and the control were thus simulated.

The start of the braking operation takes place with null current, null electromagnetic torque and at maximum speed.

In Figure 2.19, at the beginning of electrical braking, the maximum electromagnetic torque is reached in 1.5 ms, after one period of initialization, one computational period with null torque control and one impressing period of the calculated voltage.

During a first period of less than 0.5 s, the torque is limited by the magnitude of the stator current. Indeed, the abscissa of the stator current in the (d, q) reference frame is not null to allow the motor demagnetization by the stator, in order to keep the voltage at its maximum. This negative component I_{sd} of which the magnitude decreases with speed, gradually makes it possible for the stator current ordinate I_{sq} to grow, while complying with the current limit; the braking torque can then increase, as the speed decreases.

After this phase, the maximum torque is reached when one leaves the current limitation. The demagnetization current is, however, not yet null, owing to the fact that the voltage limitation is present for about the following 20 ms.

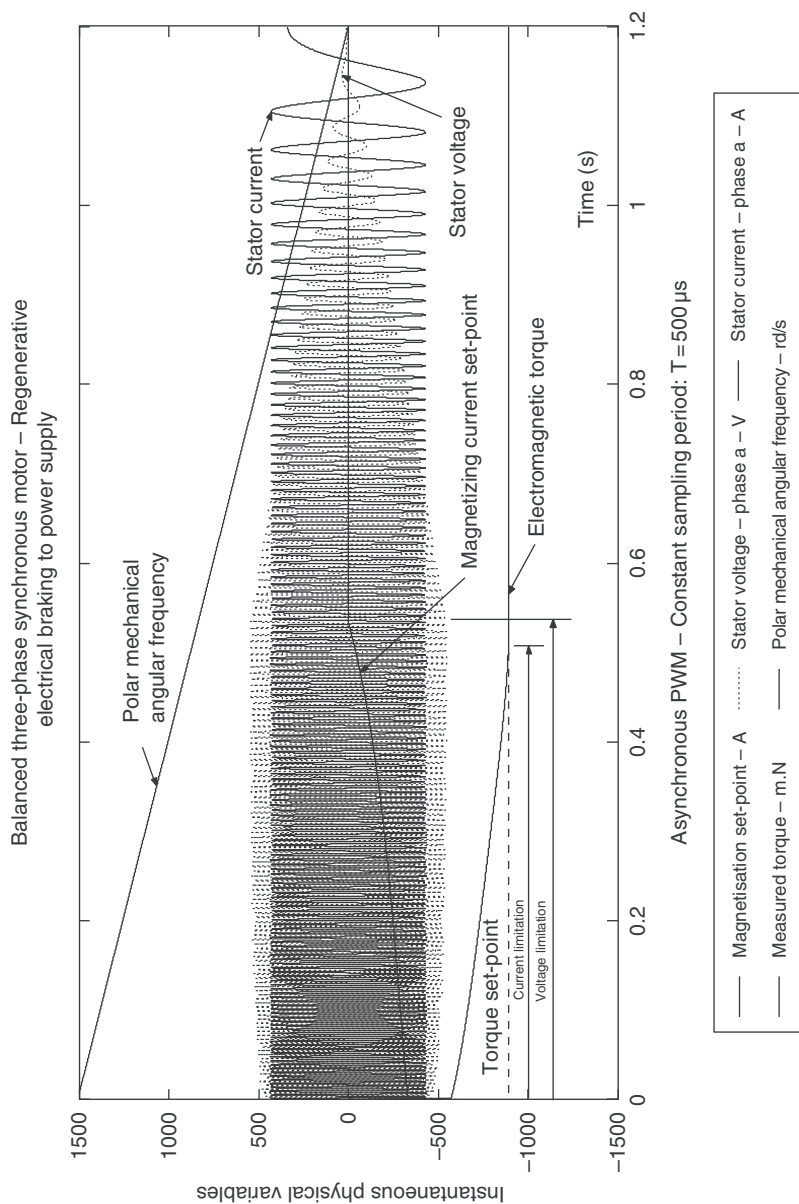


Figure 2.19 Maximal braking torque starting with a null torque

After approximately 0.52 s, the rated motor speed in the braking operation is reached with the maximum available magnitude of the stator voltage, and thus with the maximum magnitude of the phase voltage. The speed continuing to decrease, the voltage decreases with the speed, until stop.

The braking torque is maintained until stop.

The simulation was made under the same conditions as in traction operation, without an input filter.

Figure 2.19 visualizes the performances of this control method, with the current and the torque tracking.

The strategy adopted in the braking mode for this simulation is symmetrical with that obtained for the traction mode; for limiting the stator current, the magnitude of the stator current abscissa is null each time there is no voltage limitation. It is a strategy with the minimum energy. As we will see in chapter 3, it is necessary to proceed in a different way with an interior permanent magnet synchronous motor.

2.4.5.3 Free Evolution – Short-Circuit Torque

The free evolution of the state-space eigenvector is calculable starting from equation (2.77); it is the motor state evolution, with a symmetrical short-circuit of the three phases of the stator, that corresponds to a null input voltage.

$$\begin{bmatrix} \tilde{\Psi}_1 \\ \tilde{\Psi}_2 \end{bmatrix}_{(d,q)}^0 = \begin{bmatrix} e^{(\mu_1 - i\omega)T} & 0 \\ 0 & e^{(\mu_2 - i\omega)T} \end{bmatrix} \cdot \begin{bmatrix} \tilde{\Psi}_{10} \\ \tilde{\Psi}_{20} \end{bmatrix}_{(d_0,q_0)} \quad (2.162)$$

When the initial eigenvector is calculated in the same frame, this relation becomes:

$$\begin{bmatrix} \tilde{\Psi}_1 \\ \tilde{\Psi}_2 \end{bmatrix}_{(d_0,q_0)}^0 = \begin{bmatrix} e^{\mu_1 T} & 0 \\ 0 & e^{\mu_2 T} \end{bmatrix} \cdot \begin{bmatrix} \tilde{\Psi}_{10} \\ \tilde{\Psi}_{20} \end{bmatrix}_{(d_0,q_0)} \quad (2.163)$$

Figure 2.20 was calculated for the motor described at the beginning of section 2.4.5 with the initial state:

$$X_0 = \begin{bmatrix} \tilde{I}_{s0} \\ \tilde{\Phi}_{r0} \end{bmatrix}_{(d_0,q_0)} = \begin{bmatrix} -125.5 + 456 \cdot i \\ 0.517 \end{bmatrix} \quad (2.164)$$

for a polar mechanical angular frequency of 760 rad/s, approximately, above the motor rated speed, so that the coordinate component according to the d_0 axis is not null.

In Figure 2.18, this working point corresponds to a time of approximately 0.6 s. The free evolution calculation was made each millisecond and the results of:

$$\Psi = \begin{bmatrix} \tilde{\Psi}_1 \\ \tilde{\Psi}_2 \end{bmatrix}_{(d_0,q_0)}^0 \quad (2.165)$$

were located in Figure 2.20 in the (d_0, q_0) plane. The frame is maintained fixed during all the short-circuit calculations.

Equation (2.76), entirely transposed in the (d_0, q_0) reference frame to impose a steady state operation:

$$\begin{bmatrix} \vec{\Psi}_{1c} \\ \vec{\Psi}_{2c} \end{bmatrix}_{(d_0, q_0)} = \begin{bmatrix} \vec{\Psi}_{10} \\ \vec{\Psi}_{20} \end{bmatrix}_{(d_0, q_0)} \cdot e^{i\omega T} = \begin{bmatrix} e^{\mu_1 T} & 0 \\ 0 & e^{\mu_2 T} \end{bmatrix} \cdot \begin{bmatrix} \vec{\Psi}_{10} \\ \vec{\Psi}_{20} \end{bmatrix}_{(d_0, q_0)} + \begin{bmatrix} a_1 \\ 0 \end{bmatrix} \cdot \vec{V}_{s(d_0, q_0)} \quad (2.167)$$

highlights the action that the control would have if, during the first sampling period, it had to maintain a steady state or, in other words, if it had to compensate exactly the free evolution by the control vector.

The control vector:

$$a_1 \cdot \vec{V}_s \quad (2.168)$$

maintains the initial current during the operation, while bringing back the free evolution vector $\vec{\Psi}_1^0$ on the extremity of the vector $\vec{\Psi}_{10} \cdot e^{i\omega T}$ to take account of the *set-point* reference frame rotation with the rotor. The vector $\vec{\Psi}_{20}$ is not controllable since $a_2=0$; this is a rotor flux attribute of a surface-mounted permanent magnet synchronous motor.

Figure 2.21 visualizes the convergence towards the short-circuit current of the stator current during the free evolution under the same conditions, but here with a sampling period of 100 μ s instead of 1 ms to have a more precise sight of the trajectory. The magnitude of the current passes very quickly through a maximum before decreasing then to stabilize itself on the short-circuit current which is limited only by the stator impedance.

It is pointless to show here the rotor flux evolution because it remains identical to its representation in Figure 2.20.

We can notice the progressive centering of the current trajectory to the origin, to follow that of the rotor flux at a constant speed, except by a multiplicative factor (stator admittance multiplied by polar mechanical angular frequency) and with a phase shift.

Figure 2.21 was obtained by clarifying the state-space eigenvector system with the stator current and the rotor flux in equation (2.163):

$$\begin{bmatrix} \vec{I}_s + \xi \cdot \vec{\Phi}_r \\ \vec{\Phi}_r \end{bmatrix}_{(d_0, q_0)}^0 = \begin{bmatrix} e^{\mu_1 T} & 0 \\ 0 & e^{\mu_2 T} \end{bmatrix} \cdot \begin{bmatrix} \vec{I}_0 + \xi \cdot \vec{\Phi}_0 \\ \vec{\Phi}_0 \end{bmatrix}_{(d_0, q_0)} \quad (2.169)$$

Let us eliminate $\vec{\Phi}_r$ between the two complex equations:

$$\vec{I}_s = \vec{I}_0 \cdot e^{\mu_1 T} + \vec{\Phi}_0 \cdot \xi \cdot (e^{\mu_1 T} - e^{\mu_2 T}) \quad (2.170)$$

$$\vec{\Phi}_r = e^{\mu_2 T} \cdot \vec{\Phi}_0 \quad (2.171)$$

The coefficients of $\vec{I}_0$ and $\vec{\Phi}_0$ are obviously those of the transition matrix F calculated in (2.100) and applied to the state-space equation describing the free evolution:

$$X^0 = F \cdot X_0 \quad (2.172)$$

$$\begin{bmatrix} \vec{I}_s \\ \vec{\Phi}_r \end{bmatrix}^0 = \begin{bmatrix} e^{\mu_1 \cdot T} & \xi \cdot (e^{\mu_1 \cdot T} - e^{\mu_2 \cdot T}) \\ 0 & e^{\mu_2 \cdot T} \end{bmatrix} \cdot \begin{bmatrix} \vec{I}_0 \\ \vec{\Phi}_0 \end{bmatrix} \tag{2.173}$$

It should be noted here that the rotor flux does not remain collinear with the d_0 axis owing to the fact that the reference frame remains fixed.

Figure 2.22 reproduces the electromagnetic torque evolution during the symmetrical short-circuit. It is interesting to note that the torque, after transients due to the short-circuit, is not null but tends towards a negative residual torque which is the energy lost in the motor winding resistances. At this torque, it is necessary to add dry and viscous friction moments of bearings and the internal aeraulic moment of the motor, to know the residual braking torque on the motor shaft.

Indeed, the short-circuit current calculated from equation (2.72):

$$|\vec{I}_c| = \frac{\omega \cdot \Phi_a}{\sqrt{R_s^2 + L_s^2 \cdot \omega^2}} \tag{2.174}$$

with motor characteristics and the polar mechanical angular frequency is approximately 518A.

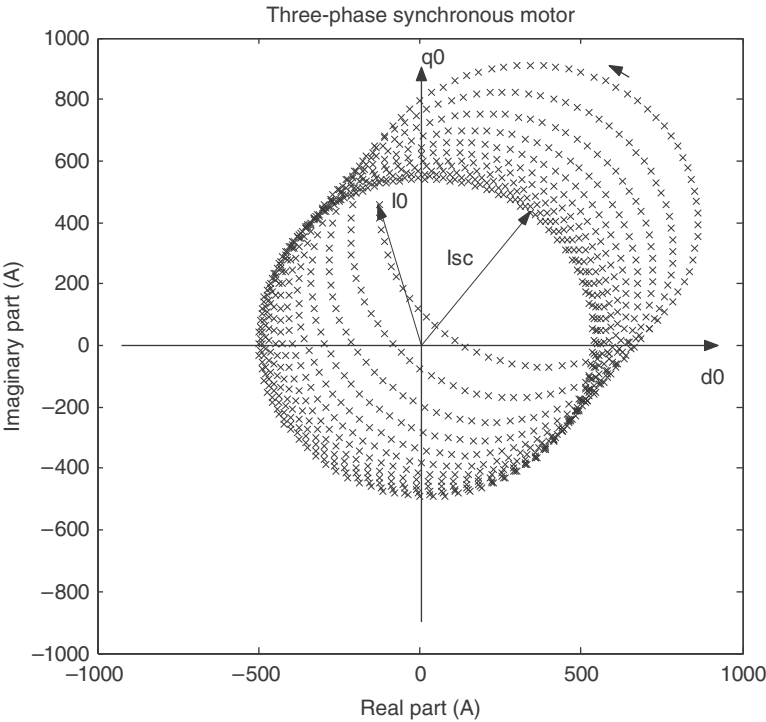


Figure 2.21 Stator short-circuit current evolution

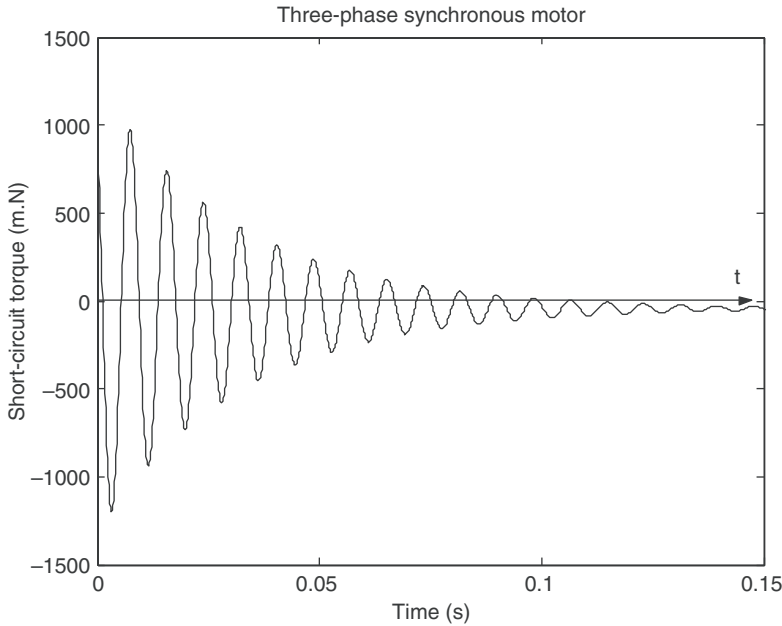


Figure 2.22 Three-phase symmetrical short-circuit torque

The power dissipated in winding resistances is:

$$P = R_s \cdot I_s^2 \quad (2.175)$$

The short-circuit electromagnetic torque, can be calculated by:

$$C = \frac{P}{\omega} \quad (2.176)$$

that is to say about 10 m.N in the case defined in the section 2.4.5.

This study of the transitory short-circuit torque is important for mechanical system sizing, in the event of breakdown in short-circuit of one of the electronic power switches of the inverter; indeed, in the event of a short-circuit of one power switch, it is necessary to instantaneously detect the fault and to force into short-circuit the two other switches of the same leg level, to symmetrize the short-circuit and to limit the resulting torque at the value which has just been calculated.

2.5 Conclusion on SMPM-SM

With the control which has just been described, the synchronous motor can be controlled by its electromagnetic torque with very high dynamics corresponding to only one period of the PWM plus one pure time-delay period, in a tracking mode as well as in a regulating mode with disturbances.

This control makes it possible to control the synchronous motor from the complete stop until its maximum speed; the maximum speed can be higher than the rated motor speed, thanks to limitation calculations which are made to modify the set-points.

The torque control can then be used to bring under control the motor speed, including at null speed; it can also be used to bring under control the motor position including at stop.

The synchronous motor produces a torque thanks to the stator current and the permanent magnet flux. The torque production of is thus accompanied by ohmic losses in the stator proportional to the square of the stator current. This motor is thus adapted to hold a shaft position during stop or at very low speed since losses primarily take place into the stator which is easier to cool by external motor ventilation.

3

Interior Permanent Magnet Synchronous Motor

We now are able to follow the same approach as that we proposed for a surface mounted permanent magnet synchronous motor (SMPM-SM), with an interior permanent magnet synchronous motor (IPM-SM).

We will establish electrical equations for the synchronous motor with permanent magnets set inside magnetic steel rotor sheets. This motor can have a smooth rotor surface, but the rotor magnetic reluctance is not the same, depending on whether the stator magnetic field penetrates in the rotor through the pole of a magnet or between two magnet poles. This motor thus presents an electromagnetic saliency. This magnetic anisotropy is due to the difference between the relative permeability of magnets and that of the magnetic steel sheets of the rotor, but also to notches in the steel sheets which avoid magnetic short-circuits of the permanent magnets themselves. Many and various arrangements of magnets in the rotor have been propounded by technical papers (Moritomo *et al.*, 1996).

Figure 3.1 is one of many examples of a rotor realization of this kind of motor, in this case with two pole pairs. The main field lines of two magnet pairs are represented, as well as the leakage flux of one magnet. The stator leakage flux is shown here for only one stator notch.

Saliency can be a *direct* one if the reluctance is less through the magnet poles (d axis) than that following the axis between magnet poles (q axis), or a *reverse* one in the opposite case. It depends on the magnetic permeability of magnets and on the magnet arrangement within the steel rotor sheets.

In case of Figure 3.1, rotor notches introduce a mechanical weakness to centrifugal forces, which limits the peripheral speed of the rotor. This weakness is primarily due to the minimization of magnetic leakage of the rotor magnets, by a reduction of the width of the isthmuses between the motor air gap and the notch housing the magnet.

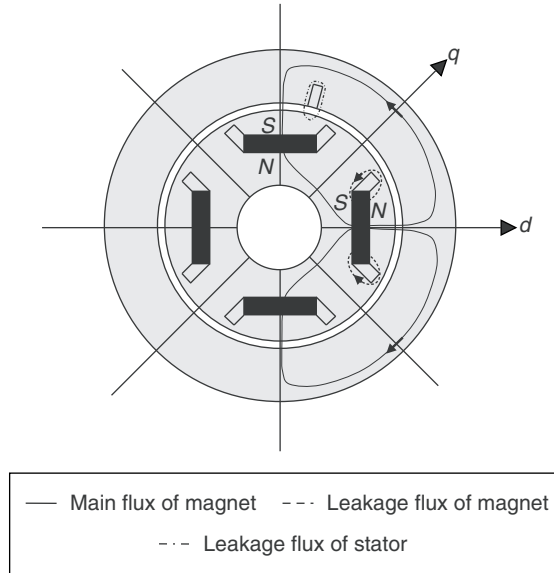


Figure 3.1 Example of an interior permanent magnet synchronous motor

This motor presents an armature reaction owing to the fact that the motor air gap is small, compared to an SMPM-SM; it consists of the air space between stator magnetic steel sheets and those of the rotor. Magnetic steel sheets of the rotor allow the stator magnetic field to loop within the rotor and thus to modify the rotor flux due to the magnets. Parts of the rotor magnetic field loop in the air gap, thus creating the leakage flux of magnets.

3.1 Electrical Equations and Equivalent Circuits

From the equivalent electrical circuit of this type of balanced three-phase synchronous motor, let us initially establish relations between various electrical variables.

3.1.1 Definitions and Notations

- stator resistance: R_s
- d axis stator inductance: L_{sd}
- d axis leakage inductance: L_{fdl}
- d axis magnetizing inductance: L_{md}
- q axis stator inductance: L_{sq}
- q axis leakage inductance: L_{fq}
- q axis magnetizing inductance: L_{mq}
- rotor magnet flux per pole pair: $\vec{\Phi}_a$
- air gap flux: $\vec{\Phi}_e$
- number of pole pairs: N_p
- mechanical angular position of the rotor: Ξ

Definitions and notations of mechanical and electrical angular frequencies:

- mechanical rotor angular frequency: Ω
- *polar*¹ mechanical angular frequency: $\omega = N_p \cdot \Omega$
- stator electrical angular frequency: ω_s .

3.1.2 Equivalent Electrical Circuits

Taking into account the rotor magnetic anisotropy, the equivalent electrical circuit of only one phase of balanced three-phase synchronous motor cannot be established simply, since inductances depend on the axis angle of rotor magnets, compared to axis of each of stator three phases.

We will thus directly describe the motor electrical equivalent circuit in a two-phase circuit by taking electrical axis of poles as the d axis and a direct perpendicular axis between poles in the *electrical plane* as the q axis. There thus is not a single two-phase equivalent circuit, but two circuits to represent electrical operations in each axis (Jahns and Caliskan, 1999):

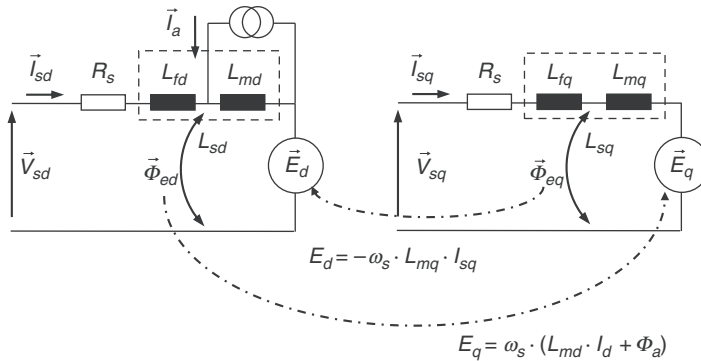


Figure 3.2 Two-phase electrical equivalent circuit of the IPM-SM

In the two circuits, i.e. according to the two axes d and q , the stator inductance in the considered axis, is represented in the form of two inductances, the leakage inductance and the magnetizing inductance, different ones according to the axis.

According to the d axis, the magnetization due to magnets is represented in the form of one magnetizing current $\vec{I}_a$ circulating in the magnetizing inductance L_{md} of the corresponding axis, current created by a fixed equivalent current source. This current source does not exist in the q axis orthogonal with the magnet d axis where the magnet flux is null, as we can see it in Figure 3.1.

The main magnetic flux of inductances of each axis creates in the other axis electromotive forces (e.m.f.) in lead quadrature, because of the flux angular derivative with the rotation speed.

¹ For simplicity, we will name the mechanical angular frequency of the motor shaft multiplied by the number of pole pairs, the *polar* mechanical angular frequency, in preference to the name, sometimes used, of *electrical* mechanical angular frequency.

It is what is represented on both equivalent electrical circuits from the air gap flux $\vec{\Phi}_e$, which results from the composition in the motor air gap, of the rotor flux due to magnets, and of the stator flux due to the stator current.

The Concordia transformation allows, as for an induction machine in the chapter 1, to calculate from a three-phase vectorial representation of voltage and current electrical vectors in the phase plane (a, b, c) , a new vectorial representation of two-phase equivalent circuits in the (α, β) fixed frame.

The transfer matrix from balanced three-phase variables to two-phase is that of the Concordia transform (1.3); it preserves the instantaneous power.

The counterclockwise direction is selected as the positive sense for all angle measurements.

In this case, relations between magnitudes of vector variables and maximum values of phase electrical variables, are given by the equation system (1.4) in the (α, β) fixed frame.

Values of motor parameters remain unchanged in this transformation.

In the case of Figure 3.2, two-phase electrical vectors are calculated by the transformation from the three-phase frame (a, b, c) , to the two-phase frame (α, β) ; they are then projected in the orthonormal frame (d, q) related to the rotor (cf. Figure 3.3).

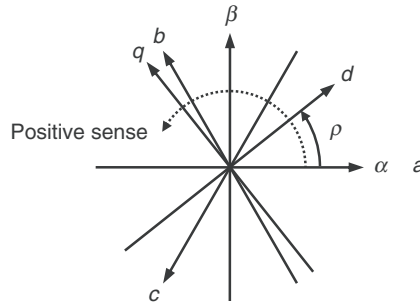


Figure 3.3 Fixed three-phase (a, b, c) , fixed two-phase (α, β) and rotating (d, q) frames

3.1.3 Differential Equation System

Stator electrical equations of a synchronous motor with salient poles have the following differential form in the (α, β) fixed frame (Grellet and Clerc, 1999):

$$\vec{V}_s = R_s \cdot \vec{I}_s + \vec{L}_{sf} \cdot \frac{d\vec{I}_s}{dt} + \frac{d\vec{\Phi}_e}{dt} \quad (3.1)$$

$$\vec{\Phi}_e = \vec{L}_m \cdot \vec{I}_s + \vec{\Phi}_a \quad (3.2)$$

We will notice the vector representation of the magnetizing and leakage inductances in the complex plane to reveal the magnetic difference between the two axes, seen from the stator.

The vector $\vec{\Phi}_a$ is the magnet flux vector of which the *partial* time derivative of the *magnitude* is null, if one does not consider the evolution of the magnetic flux with motor temperature (very slow evolution).

We define here the instantaneous stator angular frequency by the angular derivative of the air gap flux compared to time:

$$\omega_s = \frac{d\theta}{dt} \quad (3.3)$$

The time derivative of the air gap flux angle depends both on the flux angle of magnets and on the electrical angle created by the armature reaction through the magnetizing inductance.

The polar mechanical angular frequency must be regarded as different from the instantaneous stator electrical angular frequency, even if in a steady state operation or as an average value, they are identical for a synchronous motor. Instantaneous values can be different to make the variation of the interior angle of stator and rotor fluxes possible, in particular with each fast variation of the motor torque or the load moment, in the limit of an electrical angle of $\left[0, \frac{\pi}{2}\right]$.

$$\omega_s \neq \omega = N_p \cdot \Omega \quad (3.4)$$

N_p is the number of pole pairs; Ω is the mechanical angular frequency and ω the polar mechanical angular frequency.

We now will project vector equations (3.1) and (3.2) in the (d, q) frame related to the rotor to reveal the inductance values. The d axis being directed in the direction and the sense of the rotor magnet field, $\Phi_{aq} \equiv 0$ by definition and by construction of this rotating frame (Figure 3.3).

We define the ρ angle as the magnet flux angle related to the rotor and compared to the (α, β) fixed frame related to the stator. It is also the angle of the (d, q) frame. By construction:

$$\omega = \frac{d\rho}{dt} \quad (3.5)$$

According to equation (3.2), the air gap flux is the vector sum of the flux due to the magnetizing stator current and the main magnet flux:

$$\vec{\Phi}_e = \vec{L}_m \vec{I}_s + \vec{\Phi}_a = \vec{\Phi}_m + \vec{\Phi}_a \quad (3.6)$$

The total differential of the air gap flux is written in a general way by:

$$d\vec{\Phi}_e = \frac{\delta\vec{\Phi}_e}{\delta t} \cdot dt + \frac{\delta\vec{\Phi}_e}{\delta\theta} \cdot d\theta \Rightarrow \frac{d\vec{\Phi}_e}{dt} = \frac{\delta(\vec{L}_m \cdot \vec{I}_s + \vec{\Phi}_a)}{\delta t} + \frac{\delta(\vec{L}_m \cdot \vec{I}_s + \vec{\Phi}_a)}{\delta\theta} \cdot \frac{d\theta}{dt} \quad (3.7)$$

The magnitude of the magnet flux vector can be regarded as constant over one sampling period of the control. Its differential depends only on the variation of the vector angle ρ , due to the motor rotation.

$$d\vec{\Phi}_a = \frac{\delta\vec{\Phi}_a}{\delta\theta} \cdot d\theta \Rightarrow d\vec{\Phi}_a = \frac{\delta\vec{\Phi}_a}{\delta\rho} \cdot d\rho \quad (3.8)$$

Let us project the time derivative equation (3.7) on the two d - and q -axes:

$$\frac{d\Phi_{ed}}{dt} = L_{md} \cdot \dot{I}_{sd} - \omega_s \cdot L_{mq} \cdot I_{sq} - \omega \cdot \Phi_{aq} \quad (3.9)$$

$$\frac{d\Phi_{eq}}{dt} = L_{mq} \cdot \dot{I}_{sq} + \omega_s \cdot L_{md} \cdot I_{sd} + \omega \cdot \Phi_{ad} \quad (3.10)$$

Now let us project equation (3.1) in the (d, q) frame, and substitute the components of the air gap flux time derivative by preceding relations; let us notice moreover, that $\dot{\Phi}_{aq} \equiv 0$:

$$V_{sd} = R_s \cdot I_{sd} + L_{sd} \cdot \dot{I}_{sd} - \omega_s \cdot L_{mq} \cdot I_{sq} - \omega \cdot \Phi_{aq} \quad (3.11)$$

$$V_{sq} = R_s \cdot I_{sq} + L_{sq} \cdot \dot{I}_{sq} + \omega_s \cdot L_{md} \cdot I_{sd} + \omega \cdot \Phi_{ad} \quad (3.12)$$

where:

$$L_{sd} = L_{fd} + L_{md} \quad \text{and} \quad L_{sq} = L_{fq} + L_{mq} \quad (3.13)$$

The time derivatives of the stator current can now be written starting from equations (3.11) and (3.12):

$$\dot{I}_{sd} = -\frac{R_s}{L_{sd}} \cdot I_{sd} + \omega_s \cdot \frac{L_{mq}}{L_{sd}} \cdot I_{sq} + \omega \cdot \frac{1}{L_{sd}} \cdot \Phi_{aq} + \frac{1}{L_{sd}} \cdot V_{sd} \quad (3.14)$$

$$\dot{I}_{sq} = -\omega_s \cdot \frac{L_{md}}{L_{sq}} \cdot I_{sd} - \frac{R_s}{L_{sq}} \cdot I_{sq} - \omega \cdot \frac{1}{L_{sq}} \cdot \Phi_{ad} + \frac{1}{L_{sq}} \cdot V_{sq} \quad (3.15)$$

The evolution of the magnet flux, is written simply, according to equation (3.8):

$$\dot{\Phi}_{ad} = -\omega \cdot \Phi_{aq} \quad \text{with} \quad \dot{\Phi}_{aq} \equiv 0 \quad (3.16)$$

$$\dot{\Phi}_{aq} = \omega \cdot \Phi_{ad} \quad \text{with} \quad \Phi_{ad} = \vec{\Phi}_a \quad (3.17)$$

The choice of the stator current as a first coordinate of the state-space vector is essential owing to the fact that the current is one measurable quantity, directly from phase currents; moreover, it will be used in the computation of the electromagnetic torque.

The choice of the second coordinate of the state-space vector can be the magnet flux or the air gap flux. These two variables make it possible to express the electromagnetic torque.

Indeed, the Lorentz law expresses the electromagnetic torque directly from the air gap flux, the stator current and the pole pair number:

$$\vec{C} = N_p \cdot \vec{\Phi}_e \times \vec{I}_s \quad (3.18)$$

Let us replace the air gap flux by its expression (3.6) as a function of the stator current in the magnetizing inductance and also of the magnet flux:

$$\vec{C} = N_p \cdot (\vec{L}_m \cdot \vec{I}_s + \vec{\Phi}_a) \times \vec{I}_s \quad (3.19)$$

Let us write the cross product from its coordinates in the (d, q) frame:

$$C = N_p \cdot \left[(L_{md} - L_{mq}) \cdot I_{sd} + \Phi_a \right] \cdot I_{sq} \quad (3.20)$$

This last relation makes it possible to separate the reluctance torque due to the saliency, from the torque due to the magnets.

When one can neglect the pole saliency $L_{md} = L_{mq}$, starting from equation (3.20), we find again the relation giving the torque of a surface-mounted permanent-magnet synchronous motor (SMPM-SM) (chapter 2):

$$\|\vec{C}\| = N_p \cdot \Phi_a \cdot I_{sq} \quad (3.21)$$

Equation (3.20) makes it possible to highlight the fact that a direct saliency $L_{md} > L_{mq}$, leads to an additive reluctance torque for $I_{sd} > 0$. Practically, it is often simpler to build a rotor with a reverse saliency $L_{md} < L_{mq}$, like the rotor represented in Figure 3.1, with magnets with a magnetic permeability smaller than that of steel sheets. In this last case, the reluctance torque is subtractive for $I_{sd} > 0$, but additive for $I_{sd} < 0$. We will see how to use these characteristics.

Owing to the fact that the rotor anisotropy imposes different (but fixed) stator inductances according to the (d, q) frame axes, it seems preferable to preserve these two axes to write the motor model and to thus avoid having to vary with the rotor angle, the motor inductance values in the equations.

Ultimately we will thus choose the following differential equations to define the coordinates of the state-space vector:

$$\begin{aligned} \dot{I}_{sd} &= -\frac{R_s}{L_{sd}} \cdot I_{sd} + \omega_s \cdot \frac{L_{mq}}{L_{sd}} \cdot I_{sq} + \omega \cdot \frac{1}{L_{sd}} \cdot \Phi_{aq} + \frac{1}{L_{sd}} \cdot V_{sd} \\ \dot{I}_{sq} &= -\omega_s \cdot \frac{L_{md}}{L_{sq}} \cdot I_{sd} - \frac{R_s}{L_{sq}} \cdot I_{sq} - \omega \cdot \frac{1}{L_{sq}} \cdot \Phi_{ad} + \frac{1}{L_{sq}} \cdot V_{sq} \\ \dot{\Phi}_{ad} &= -\omega \cdot \Phi_{aq} \\ \dot{\Phi}_{aq} &= \omega \cdot \Phi_{ad} \end{aligned} \quad (3.22)$$

with, moreover, $\Phi_{aq} \equiv 0$.

For known angular frequencies ω and ω_s , this system of four differential equations is linear, which will enable us to use a matrix state-space representation for linear systems, as long as it is possible to consider in particular ω_s constant for one observation period.

3.2 Working out the State-Space Equation System

We now will establish the state-space equations of a balanced three-phase synchronous motor supplied by an inverter with two voltage levels, according to the circuit in Figure 3.4; it is identical to that of an induction machine or an SMPM-SM; it was commented on in section 1.2.

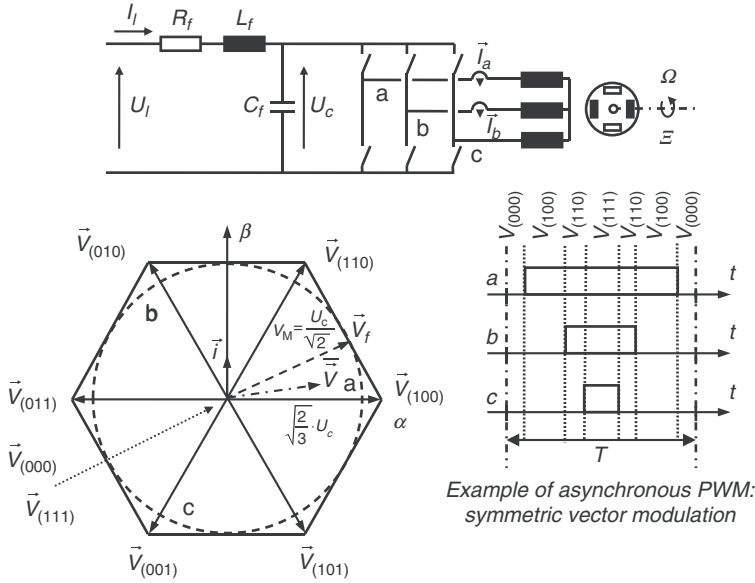


Figure 3.4 Synchronous motor fed by a voltage inverter

3.2.1 State-Space Equations in the Fixed Plane

The linear differential equation system (3.22) can now be written as a matrix state-space representation for linear systems:

$$\begin{aligned}\dot{X} &= A \cdot X + B \cdot U \\ Y &= C \cdot X\end{aligned}\quad (3.23)$$

This state-space model will enable us to define the system behavior using the stator current and the rotor flux of a synchronous machine, directly in the (d, q) orthonormal rotating frame linked to the rotor of the motor. It is indeed less easy to write the state-space equation system in the (α, β) fixed frame than with an induction machine or with an SMPM-SM, because of the rotor magnetic anisotropy seen from the stator; on the other hand by binding the motor model to the rotating frame with the rotor, we obtain one simplified state-space equation system because of the simple modeling of the rotor, but also because of the stator magnetic isotropy seen from the rotor.

$$X = \begin{bmatrix} I_{sd} \\ I_{sq} \\ \Phi_{rd} \\ \Phi_{rq} \end{bmatrix} \quad \text{or} \quad \bar{X} = \begin{bmatrix} \vec{I}_s \\ \vec{\Phi}_r \end{bmatrix}\quad (3.24)$$

The two first equations of (3.22) show that the evolution of the stator current depends on:

- the input vector
- the current state itself
- the magnet flux state.

The two last equations of (3.22) confirm that the evolution of the magnet flux depends only on the motor shaft rotation, contrary to the air gap flux which depends on the stator current (equations (3.9) and (3.10)) and thus indirectly on the input voltage vector (3.22).

By gathering equations in a matrix form, we obtain ultimately:

$$\begin{bmatrix} \dot{I}_{sd} \\ \dot{I}_{sq} \\ \dot{\Phi}_{ad} \\ \dot{\Phi}_{aq} \end{bmatrix} = \begin{bmatrix} -\frac{R_s}{L_{sd}} & \omega_s \cdot \frac{L_{mq}}{L_{sd}} & 0 & \omega \cdot \frac{1}{L_{sd}} \\ -\omega_s \cdot \frac{L_{md}}{L_{sq}} & -\frac{R_s}{L_{sq}} & -\omega \cdot \frac{1}{L_{sq}} & 0 \\ 0 & 0 & 0 & -\omega \\ 0 & 0 & \omega & 0 \end{bmatrix} \cdot \begin{bmatrix} I_{sd} \\ I_{sq} \\ \Phi_{ad} \\ \Phi_{aq} \end{bmatrix} + \begin{bmatrix} \frac{1}{L_{sd}} & 0 \\ 0 & \frac{1}{L_{sq}} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} V_{sd} \\ V_{sq} \end{bmatrix} \quad (3.25)$$

with $\dot{\Phi}_{aq} \equiv 0$.

This real fourth-order system for controlling one IPM-SM, is enough to completely describe the system evolution when it is controlled by a voltage input vector $\vec{V}_s$.

By comparing this formalism to equations (3.23), according to motor parameters, evolution and input matrices, are written respectively:

$$A = \begin{bmatrix} -\frac{R_s}{L_{sd}} & \omega_s \cdot \frac{L_{mq}}{L_{sd}} & 0 & \omega \cdot \frac{1}{L_{sd}} \\ -\omega_s \cdot \frac{L_{md}}{L_{sq}} & -\frac{R_s}{L_{sq}} & -\omega \cdot \frac{1}{L_{sq}} & 0 \\ 0 & 0 & 0 & -\omega \\ 0 & 0 & \omega & 0 \end{bmatrix} \quad B = \begin{bmatrix} \frac{1}{L_{sd}} & 0 \\ 0 & \frac{1}{L_{sq}} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (3.26)$$

We can notice in this case, that the 2×2 submatrices of the evolution matrix and the input matrix, have different values of terms on diagonals but also on antidiagonals, on their two first rows. The property that we highlighted for motors with non-salient poles does not exist anymore, because of the rotor magnetic anisotropy; this first important difference will complicate the formal calculations.

We will also notice that we had to write these matrices in the (d, q) motor rotating frame, to follow this anisotropy and thus to simplify their writing; it is a second important difference compared to smooth rotor motors of which the evolution matrix was written in the (α, β) fixed frame.

3.2.2 State-Space Equations in the Complex Plane

Owing to the fact that the absolute value of parameters of 2×2 submatrices of evolution and control matrices are not equal on either the diagonal or the antidiagonal, it is not possible any more to factorize these coefficients to reveal the two complex coordinates of the current and the flux, as it was in the case of motors with non-salient poles.

The calculation must thus continue in the real plane, with all coefficients, which weighs down somewhat the mathematical formulation.

3.2.3 State-Space Equation Discretization

The formal results of section 1.2.3, are general results which can be applied now in the same way, whatever the chosen frame, in particular the discretized form of the state-space representation (1.53):

$$X(t_n + T) = F \cdot X(t_n) + G \cdot V \quad (3.27)$$

As for an induction machine or an SMPM-SM, we will see that transition and input matrices, F and G , can be calculated starting from specific parameters of this kind of synchronous motor.

Equation (3.27) will thus make it possible to predict the motor state $X(t_n + T)_p$, from one initial state known by measurement or by an estimation $X(t_n)_m$, if the input vector is known; it also makes it possible to fix set-points and to calculate the control vector while replacing the predicted state-space vector by the set-point vector $X(t_n + T)_c$, according to equations (1.57) and (1.58). In the case of a motor with salient poles, the state-space representation will be written in the rotating frame.

We will thus diagonalize the evolution matrix and rewrite equations within the space of eigenvectors. This operation will enable us to discretize continuous-time state-space equations to find independent set-point vectors in this new vector basis, and then to derive the control vector.

3.2.4 Evolution Matrix Diagonalization

3.2.4.1 Eigenvalues

The eigenvalue equation giving eigenvalues μ_i of the matrix A is:

$$\det(\mu \cdot I - A) = 0 \quad (3.28)$$

The matrix I is the unit matrix of the fourth order.

Equation (3.28) can thus still be written:

$$\begin{vmatrix} \mu + \frac{R_s}{L_{sd}} & -\omega_s \cdot \frac{L_{mq}}{L_{sd}} & 0 & -\omega \cdot \frac{1}{L_{sd}} \\ \omega_s \cdot \frac{L_{md}}{L_{sq}} & \mu + \frac{R_s}{L_{sq}} & \omega \cdot \frac{1}{L_{sq}} & 0 \\ 0 & 0 & \mu & \omega \\ 0 & 0 & -\omega & \mu \end{vmatrix} = 0 \quad (3.29)$$

Let us develop initially the determinant from its first column:

$$\left(\mu + \frac{R_s}{L_{sd}} \right) \cdot \begin{vmatrix} \mu + \frac{R_s}{L_{sq}} & \omega \cdot \frac{1}{L_{sq}} & 0 \\ 0 & \mu & \omega \\ 0 & -\omega & \mu \end{vmatrix} - \omega_s \cdot \frac{L_{md}}{L_{sq}} \cdot \begin{vmatrix} -\omega \cdot \frac{L_{mq}}{L_{sd}} & 0 & -\omega \cdot \frac{1}{L_{sd}} \\ 0 & \mu & \omega \\ 0 & -\omega & \mu \end{vmatrix} = 0 \quad (3.30)$$

then the two resulting 3×3 determinants, from their first column; finally, let us calculate the 2×2 determinants by Cramer's rule, which makes it possible to factorize immediately, in the result, the two 2×2 identical determinants.

$$\left\{ \left(\mu + \frac{R_s}{L_{sd}} \right) \cdot \left(\mu + \frac{R_s}{L_{sq}} \right) + \left(\omega_s \cdot \frac{L_{md}}{L_{sq}} \right) \cdot \left(\omega_s \cdot \frac{L_{mq}}{L_{sd}} \right) \right\} \cdot (\mu^2 + \omega^2) = 0 \quad (3.31)$$

The four eigenvalues are roots of the two quadratic equations:

$$\mu^2 + R_s \cdot \frac{L_{sd} + L_{sq}}{L_{sd} \cdot L_{sq}} \cdot \mu + \frac{R_s^2 + \omega_s^2 \cdot L_{md} \cdot L_{mq}}{L_{sd} \cdot L_{sq}} = 0 \quad (3.32)$$

$$\mu^2 + \omega^2 = 0 \quad (3.33)$$

So, for the first equation (3.32) of the second degree:

$$\mu_1 = \frac{-R_s \cdot (L_{sd} + L_{sq}) + \sqrt{R_s^2 \cdot (L_{sd} - L_{sq})^2 - 4 \cdot \omega_s^2 \cdot L_{sd} \cdot L_{sq} \cdot L_{md} \cdot L_{mq}}}{2 \cdot L_{sd} \cdot L_{sq}} \quad (3.34)$$

$$\mu_2 = \frac{-R_s \cdot (L_{sd} + L_{sq}) - \sqrt{R_s^2 \cdot (L_{sd} - L_{sq})^2 - 4 \cdot \omega_s^2 \cdot L_{sd} \cdot L_{sq} \cdot L_{md} \cdot L_{mq}}}{2 \cdot L_{sd} \cdot L_{sq}} \quad (3.35)$$

It will be noted that the roots μ_1 and μ_2 are either real or conjugate-complex, according to the discriminant sign, respectively positive or negative. This remark will be useful for interpreting the final control equation.

In the same way the two last eigenvalues extracted from equation (3.33), are conjugate-complex:

$$\mu_3 = -i \cdot \omega \quad (3.36)$$

$$\mu_4 = i \cdot \omega \quad (3.37)$$

Let us pose:

$$\Delta = \sqrt{R_s^2 \cdot (L_{sd} - L_{sq})^2 - 4 \cdot \omega_s^2 \cdot L_{sd} \cdot L_{sq} \cdot L_{md} \cdot L_{mq}} \quad (3.38)$$

From equations (3.32) and (3.33), we directly derive the sum and the product of roots of the quadratic polynomial:

$$\begin{aligned} \mu_1 + \mu_2 &= -R_s \cdot \frac{L_{sd} + L_{sq}}{L_{sd} \cdot L_{sq}} \\ \mu_1 \cdot \mu_2 &= \frac{R_s^2 + \omega_s^2 \cdot L_{md} \cdot L_{mq}}{L_{sd} \cdot L_{sq}} \\ \mu_3 + \mu_4 &= 0 \\ \mu_3 \cdot \mu_4 &= \omega^2 \end{aligned} \quad (3.39)$$

Sums and products, two by two, of eigenvalues, are real, which confirms the fact, if it were necessary, that when roots are complex they are conjugate-complex two by two.

In addition, their sums being negative or null, they have a real part negative or null, what characterizes a dissipative and stable system when the real part is strictly negative.

From the sum, we derive the following relations, which will be useful to simplify the complete control calculation later on:

$$\mu_1 + \frac{R_s}{L_{sq}} = - \left(\mu_2 + \frac{R_s}{L_{sd}} \right) \quad (3.40)$$

that is to say:

$$L_{sd} \cdot (L_{sq} \cdot \mu_1 + R_s) = -L_{sq} \cdot (L_{sd} \cdot \mu_2 + R_s) \quad (3.41)$$

In a symmetrical way:

$$L_{sq} \cdot (L_{sd} \cdot \mu_1 + R_s) = -L_{sd} \cdot (L_{sq} \cdot \mu_2 + R_s) \quad (3.42)$$

We can also write that the two roots of the quadratic equation (3.31), which are also roots of the first product term of equation (3.31), satisfy this same equation, equalizing to zero the first term in factor:

$$(L_{sd} \cdot \mu_1 + R_s) \cdot (L_{sq} \cdot \mu_1 + R_s) + \omega_s^2 \cdot L_{md} \cdot L_{mq} = 0 \quad (3.43)$$

$$(L_{sd} \cdot \mu_2 + R_s) \cdot (L_{sq} \cdot \mu_2 + R_s) + \omega_s^2 \cdot L_{md} \cdot L_{mq} = 0 \quad (3.44)$$

3.2.4.2 Transfer Matrix Calculation

Eigenvectors are four-dimensional column vectors:

$$\vec{\Pi}_1 = \begin{bmatrix} p_{11} \\ p_{21} \\ p_{31} \\ p_{41} \end{bmatrix}, \quad \vec{\Pi}_2 = \begin{bmatrix} p_{12} \\ p_{22} \\ p_{32} \\ p_{42} \end{bmatrix}, \quad \vec{\Pi}_3 = \begin{bmatrix} p_{13} \\ p_{23} \\ p_{33} \\ p_{43} \end{bmatrix} \quad \text{and} \quad \vec{\Pi}_4 = \begin{bmatrix} p_{14} \\ p_{24} \\ p_{34} \\ p_{44} \end{bmatrix} \quad (3.45)$$

They are one of the possible solutions of the equation:

$$(A - \mu_i \cdot I) \cdot \vec{\Pi}_i = 0 \quad \text{with} \quad i \in \{1; 2; 3; 4\} \quad (3.46)$$

Indeed, each eigenvector cannot be a single solution owing to the fact that the corresponding matrix $(A - \mu_i \cdot I)$ is a singular one by definition of the μ_i , calculated to make its determinant equal to zero, according to (3.28).

The detailed calculation of the four eigenvectors of the evolution matrix of the continuous-time state-space system, is made in appendix B.

From the eigenvector calculation of in appendix B, we reconstitute the transfer matrix as:

$$P = \begin{bmatrix} \vec{\Pi}_1 & \vec{\Pi}_2 & \vec{\Pi}_3 & \vec{\Pi}_4 \end{bmatrix} \quad (3.47)$$

3.2.4.3 Transfer Matrix Inversion

This calculation is essential for the inversion of state-space equations and for the control calculation. The relatively heavy algebraic calculation can be simplified and it will make it possible to obtain an exact and simple analytical expression of the control vector.

3.2.4.3.1 Inversion

The calculation of each coefficient of the reverse transfer matrix is made in appendix C. It is not necessary to recall here all results detailed in appendix.

Ultimately, the inverse matrix can now be written simply as:

$$P^{-1} = \begin{bmatrix} p_{11}^{-1} & p_{12}^{-1} & p_{13}^{-1} & p_{14}^{-1} \\ p_{21}^{-1} & p_{22}^{-1} & p_{23}^{-1} & p_{24}^{-1} \\ p_{31}^{-1} & p_{32}^{-1} & p_{33}^{-1} & p_{34}^{-1} \\ p_{41}^{-1} & p_{42}^{-1} & p_{43}^{-1} & p_{44}^{-1} \end{bmatrix} \quad (3.48)$$

3.2.4.3.2 Reduced Variables

For the control calculation we will need to define reduced variables allowing us to simplify formal calculations of the transfer matrix P , and of the reverse one, P^{-1} (cf. equations (3.47) and (3.48)).

Let us define the following reduced variables:

$$\zeta_{d1} = -\frac{L_{sd} \cdot \mu_1 + R_s}{\omega_s \cdot L_{md}} \quad \zeta_{d2} = -\frac{L_{sd} \cdot \mu_2 + R_s}{\omega_s \cdot L_{md}} \quad (3.49)$$

$$\zeta_{d3} = -\frac{L_{sd} \cdot \mu_3 + R_s}{\omega_s \cdot L_{md}} \quad \zeta_{d4} = -\frac{L_{sd} \cdot \mu_4 + R_s}{\omega_s \cdot L_{md}} \quad (3.50)$$

and also:

$$\zeta_{q3} = -\frac{L_{sq} \cdot \mu_3 + R_s}{\omega_s \cdot L_{mq}} \quad \zeta_{q4} = -\frac{L_{sq} \cdot \mu_4 + R_s}{\omega_s \cdot L_{mq}} \quad (3.51)$$

It will be noticed that ζ_{d1} and ζ_{d2} , on the one hand, ζ_{d3} and ζ_{d4} , on the other hand, and also ζ_{q3} and ζ_{q4} , are respectively conjugated two by two, like the eigenvalues from which they are defined. Indeed, when we consider the conjugate of one of these reduced variables, we can immediately replace the conjugate of the corresponding eigenvalue by the conjugate eigenvalue, which reveals the conjugate reduced variable. For example:

$$\bar{\zeta}_{d1} = -\frac{L_{sd} \cdot \bar{\mu}_1 + R_s}{\omega_s \cdot L_{md}} = -\frac{L_{sd} \cdot \mu_2 + R_s}{\omega_s \cdot L_{md}} = \zeta_{d2} \quad (3.52)$$

From these first definitions, let us create the following new reduced variables:

$$\begin{aligned}\xi_{d1} &= \frac{\omega \cdot (\omega - \mu_1 \cdot \zeta_{d1})}{(\mu_4 - \mu_1) \cdot (\mu_3 - \mu_1)} & \xi_{d2} &= \frac{\omega \cdot (\omega - \mu_2 \cdot \zeta_{d2})}{(\mu_4 - \mu_2) \cdot (\mu_3 - \mu_2)} \\ \xi_{d3} &= \frac{\omega \cdot (\omega - \mu_3 \cdot \zeta_{d3})}{(\mu_3 - \mu_2) \cdot (\mu_3 - \mu_1)} & \xi_{d4} &= \frac{\omega \cdot (\omega - \mu_4 \cdot \zeta_{d4})}{(\mu_4 - \mu_2) \cdot (\mu_4 - \mu_1)}\end{aligned}\quad (3.53)$$

then:

$$\xi_{q3} = \frac{\omega \cdot (\omega - \mu_3 \cdot \zeta_{q3})}{(\mu_3 - \mu_2) \cdot (\mu_3 - \mu_1)} \quad \xi_{q4} = \frac{\omega \cdot (\omega - \mu_4 \cdot \zeta_{q4})}{(\mu_4 - \mu_2) \cdot (\mu_4 - \mu_1)} \quad (3.54)$$

In the same way, we have conjugate-complex variables ξ_{d1} and ξ_{d2} , ξ_{d3} and ξ_{d4} , and then ξ_{q3} and ξ_{q4} .

The following reduced variables are also conjugate-complex, two by two:

$$\xi_{D1} = \frac{\omega \cdot (\mu_1 + \omega \cdot \zeta_{d1})}{(\mu_4 - \mu_1) \cdot (\mu_3 - \mu_1)} \quad \xi_{D2} = \frac{\omega \cdot (\mu_2 + \omega \cdot \zeta_{d2})}{(\mu_4 - \mu_2) \cdot (\mu_3 - \mu_2)} \quad (3.55)$$

$$\xi_{Q3} = \frac{\omega \cdot (\mu_3 + \omega \cdot \zeta_{q3})}{(\mu_3 - \mu_2) \cdot (\mu_3 - \mu_1)} \quad \xi_{Q4} = \frac{\omega \cdot (\mu_4 + \omega \cdot \zeta_{q4})}{(\mu_4 - \mu_2) \cdot (\mu_4 - \mu_1)} \quad (3.56)$$

These previous remarks will allow us to read the two coordinates of the control vector as being real ones.

3.2.5 Projection of State-Space Vectors in the Eigenvector Basis

We wrote various calculation steps allowing us now to project discretized state-space equations into the eigenvector basis.

For that, we can directly reuse the general results found in sections 1.2.3 and 1.2.5, results which do not depend on the system type described by state-space equations.

Thus, the discretized state-space equation (1.53) is written with state-space vectors projected within the eigenvector space $[P^{-1} \cdot X]$, in a way formally identical to equation (1.93):

$$\boxed{P^{-1} \cdot X(t_n + T)_{(\alpha, \beta)} = e^{D \cdot T} \cdot P^{-1} \cdot X(t_n)_{(\alpha, \beta)} + D^{-1} \cdot (e^{D \cdot T} - I) \cdot P^{-1} \cdot B \cdot V_{(\alpha, \beta)}} \quad (3.57)$$

This relation must now be rewritten in the (d, q) frame, for this kind of motor.

3.3 Discretized State-Space Equation Inversion

3.3.1 Rotating Reference Frame

As in section 1.3.1, let us define the frame (d, q) , rotating at the polar mechanical angular frequency ω , compared to the (α, β) fixed frame.

Let us define the angle ρ at time $(t_n + T)$ of the (d, q) orthonormal frame compared to the (α, β) fixed frame, by:

$$\rho(t_n + T) = \rho_0(t_n) + \theta(T) \quad (3.58)$$

ρ_0 is the value of this angle at time t_n and $\theta(T)$ its variation over the time interval T .

The complex scalar $e^{-i \cdot \rho}$ has a magnitude equal of 1; it represents a rotation operator of angle $-\rho$ in the complex plane; it is also an operator used to change complex vector coordinates from the (α, β) fixed frame to the (d, q) orthonormal frame at time $(t_n + T)$.

$$e^{-i \cdot \rho} = e^{-i(\rho_0 + \theta)} \quad (3.59)$$

We will be interested only in the initial position and the final position of this reference frame, i.e. at time t_n and $(t_n + T)$ respectively (*discrete reference frame*).

3.3.2 State-Space Vector Calculations in the Eigenvector Basis

Let us multiply the two terms of equality (3.57), in which state-space vectors have their coordinates written in the (α, β) complex plane, by the equality between complex scalars (3.59).

The multiplication by $e^{-i \cdot \rho}$ of the first term of equation (3.57), which is distributive compared to the matrix product, changes the reference frame of the state-space vector at time $(t_n + T)$, from the (α, β) fixed frame to the (d, q) reference frame.

The multiplication of the second term of equation (3.57) by $e^{-i \cdot \rho_0}$ changes the state-space vector at time t_n from the (α, β) fixed frame, to the (d_0, q_0) initial reference frame.

It results in the following form:

$$\begin{aligned} \left[P^{-1} \cdot X(t_n + T) \right]_{(d,q)} &= e^{-i \cdot \theta} \cdot e^{D \cdot T} \cdot \left[P^{-1} \cdot X(t_n) \right]_{(d_0, q_0)} + \dots \\ \dots D^{-1} \cdot (e^{D \cdot T} - I) \cdot \left[P^{-1} \cdot B \cdot V_{(\alpha, \beta)} \right] \cdot e^{-i(\rho_0 + \theta)} \end{aligned} \quad (3.60)$$

Now let us define ρ_0 as the angle of the rotor flux vector known at time t_n : $\arg(\vec{\Phi}_r(t_n))$, compared to the (α, β) fixed frame, i.e. such that it cancels the rotor flux coordinate according to the q_0 axis. By definition of the angle ρ_0 :

$$\cos(\rho_0) = \frac{\Phi_{r\alpha}(t_n)}{\|\vec{\Phi}_r(t_n)\|} \quad \sin(\rho_0) = \frac{\Phi_{r\beta}(t_n)}{\|\vec{\Phi}_r(t_n)\|} \quad (3.61)$$

In this particular case, the rotor flux, i.e. the magnet flux, can be regarded as constant at a timescale of one period T :

$$\|\vec{\Phi}_r\| \equiv \Phi_a \quad \forall t \in [0, T] \quad (3.62)$$

thus:

$$X(t_n)_{(d_0, q_0)} = \begin{bmatrix} I_{sd_0} \\ I_{sq_0} \\ \Phi_a \\ 0 \end{bmatrix} \quad (3.63)$$

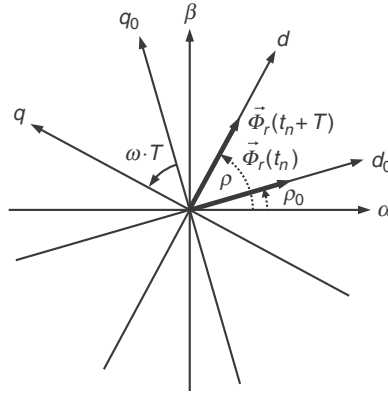


Figure 3.5 Definition of the (d_0, q_0) initial and the (d, q) final rotating frames

The complex orthonormal reference frame (d_0, q_0) has thus the d_0 axis directed according to the direction and the sense of the rotor flux at time t_n .

While assimilating now the polar mechanical angular frequency ω with the *average* angular velocity, such that $\omega = \frac{\theta}{T}$, the angle of the rotor flux at time $(t_n + T)$ compared to the (α, β) fixed frame becomes:

$$\rho = \rho_0 + \omega \cdot T \quad (3.64)$$

By definition, the complex orthonormal reference frame (d, q) has its d axis directed according to the direction and the sense of the rotor flux at this moment, and we can write:

$$X(t_n + T)_{(d, q)} = \begin{bmatrix} I_{sd} \\ I_{sq} \\ \Phi_a \\ 0 \end{bmatrix} \quad (3.65)$$

The control vector V is always located in the (α, β) fixed frame to be able to control the inverter connected to the stator of the synchronous motor.

Figure 3.5 represents, simultaneously, the (α, β) fixed frame, the (d_0, q_0) initial one at time t_n and the (d, q) predicted one at time $(t_n + T)$.

Equation (3.60) can also be written completely in any of the frames (d, q) , (d_0, q_0) or (α, β) at unspecified time. For this kind of motor, we will choose to write state-space vectors in the

(d_0, q_0) reference frame, to simplify the analytical expression. Writing this equation in one of sampled rotating reference frames makes it possible to follow the magnetic anisotropy of the rotor and thus to simplify the equations. Thus by multiplying the two terms of equation (3.60) by $e^{i \cdot \theta}$, we switch the reference frame of state-space vectors:

$$\begin{aligned} \left[P^{-1} \cdot X(t_n + T)_{(d,q)} \cdot e^{i \cdot \theta} \right] &= e^{D \cdot T} \cdot \left[P^{-1} \cdot X(t_n)_{(d_0, q_0)} \right] + \dots \\ \dots D^{-1} \cdot (e^{D \cdot T} - I) \cdot \left[P^{-1} \cdot B \cdot V_{(\alpha, \beta)} \right] \cdot e^{-i \cdot \rho_0} \end{aligned} \quad (3.66)$$

or:

$$\begin{aligned} \left[P^{-1} \cdot X(t_n + T)_{(d_0, q_0)} \right] &= e^{D \cdot T} \cdot \left[P^{-1} \cdot X(t_n)_{(d_0, q_0)} \right] + \dots \\ \dots D^{-1} \cdot (e^{D \cdot T} - I) \cdot \left[P^{-1} \cdot B \cdot V_{(d_0, q_0)} \right] \end{aligned} \quad (3.67)$$

We will notice the following well-known theoretical results:

$$D = \begin{bmatrix} \mu_1 & 0 & 0 & 0 \\ 0 & \mu_2 & 0 & 0 \\ 0 & 0 & \mu_3 & 0 \\ 0 & 0 & 0 & \mu_4 \end{bmatrix} \Leftrightarrow D^{-1} = \begin{bmatrix} \frac{1}{\mu_1} & 0 & 0 & 0 \\ 0 & \frac{1}{\mu_2} & 0 & 0 \\ 0 & 0 & \frac{1}{\mu_3} & 0 \\ 0 & 0 & 0 & \frac{1}{\mu_4} \end{bmatrix} \quad (3.68)$$

and then:

$$e^{D \cdot T} = \begin{bmatrix} e^{\mu_1 \cdot T} & 0 & 0 & 0 \\ 0 & e^{\mu_2 \cdot T} & 0 & 0 \\ 0 & 0 & e^{\mu_3 \cdot T} & 0 \\ 0 & 0 & 0 & e^{\mu_4 \cdot T} \end{bmatrix} \quad (3.69)$$

These two expressions make it possible to calculate matrix coefficients of the state-space equation system projected in the eigenvector basis and now localized in the (d_0, q_0) reference frame:

$$\begin{aligned} P^{-1} \cdot X(t_n + T)_{(d_0, q_0)} &= e^{D \cdot T} \cdot P^{-1} \cdot X(t_n)_{(d_0, q_0)} + \dots \\ \dots D^{-1} \cdot (e^{D \cdot T} - I) \cdot P^{-1} \cdot B \cdot V_{(d_0, q_0)} \end{aligned} \quad (3.70)$$

It is noticed that this expression, which was already established in section 3.2.5, in the (α, β) fixed frame, remains formally identical, if all state-space vectors have their coordinates in the

same frame, as here. We would still get the same formal result in the (d, q) reference frame. This result is obvious when we consider that this relation expresses a vector transformation which is *formally* independent of the frame in which we wish to write its coordinates; only the matrix coefficients and the expressions of vector coordinates depend on the frame.

The coefficient: $D^{-1} \cdot (e^{D \cdot T} - I) \cdot P^{-1} \cdot B$ of V can then be written according to coefficients of the inverse of the transfer matrix noted P_{ij}^{-1} and of the expression of B (cf. equation (3.26)):

$$D^{-1} \cdot (e^{D \cdot T} - I) \cdot P^{-1} \cdot B = \begin{bmatrix} \frac{e^{\mu_1 \cdot T} - 1}{\mu_1} & 0 & 0 & 0 \\ 0 & \frac{e^{\mu_2 \cdot T} - 1}{\mu_2} & 0 & 0 \\ 0 & 0 & \frac{e^{\mu_3 \cdot T} - 1}{\mu_3} & 0 \\ 0 & 0 & 0 & \frac{e^{\mu_4 \cdot T} - 1}{\mu_4} \end{bmatrix} \cdot \begin{bmatrix} \frac{P_{11}^{-1}}{L_{sd}} & \frac{P_{12}^{-1}}{L_{sq}} \\ \frac{P_{21}^{-1}}{L_{sd}} & \frac{P_{22}^{-1}}{L_{sq}} \\ \frac{P_{31}^{-1}}{L_{sd}} & \frac{P_{32}^{-1}}{L_{sq}} \\ \frac{P_{41}^{-1}}{L_{sd}} & \frac{P_{42}^{-1}}{L_{sq}} \end{bmatrix} \quad (3.71)$$

We then replace the matrix coefficients by their expression calculated in appendix C:

$$D^{-1} \cdot (e^{D \cdot T} - I) \cdot P^{-1} \cdot B = \begin{bmatrix} \frac{\omega_s \cdot L_{md}}{L_{sd} \cdot L_{sq}} \cdot \frac{e^{\mu_1 \cdot T} - 1}{\mu_1 \cdot (\mu_2 - \mu_1)} & \frac{L_{sq} \cdot \mu_2 + R_s}{L_{sq}^2} \cdot \frac{e^{\mu_1 \cdot T} - 1}{\mu_1 \cdot (\mu_2 - \mu_1)} \\ \frac{L_{sq} \cdot \mu_2 + R_s}{L_{sd} \cdot L_{sq}} \cdot \frac{e^{\mu_2 \cdot T} - 1}{\mu_2 \cdot (\mu_2 - \mu_1)} & \frac{\omega_s \cdot L_{mq}}{L_{sd} \cdot L_{sq}} \cdot \frac{e^{\mu_2 \cdot T} - 1}{\mu_2 \cdot (\mu_2 - \mu_1)} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (3.72)$$

Now let us introduce again two new reduced variables:

$$e_1 = \frac{e^{\mu_1 \cdot T} - 1}{\mu_1} \quad e_2 = \frac{e^{\mu_2 \cdot T} - 1}{\mu_2} \quad (3.73)$$

which leads to:

$$D^{-1} \cdot (e^{D \cdot T} - I) \cdot P^{-1} \cdot B = \begin{bmatrix} \frac{\omega_s \cdot L_{md}}{L_{sd} \cdot L_{sq}} \cdot \frac{e_1}{\mu_2 - \mu_1} & \frac{L_{sq} \cdot \mu_2 + R_s}{L_{sq}^2} \cdot \frac{e_1}{\mu_2 - \mu_1} \\ \frac{L_{sq} \cdot \mu_2 + R_s}{L_{sd} \cdot L_{sq}} \cdot \frac{e_2}{\mu_2 - \mu_1} & \frac{\omega_s \cdot L_{mq}}{L_{sd} \cdot L_{sq}} \cdot \frac{e_2}{\mu_2 - \mu_1} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (3.74)$$

The projection of rotating vectors $\vec{I}_s$ and $\vec{\Phi}_r$ within the eigenvector space had, as logical consequences:

- to create a new four-dimensional state-space vector, which we will name for convenience a *state-space eigenvector* Ψ , resulting from the vector composition between $\bar{I}_s$ and $\bar{\Phi}_r$
- to separate eigenmodes according to coordinates of the state-space eigenvector.

We now will write the state-space eigenvector in the (d_0, q_0) rotating reference frame by using equations (3.24) and (3.48):

$$P^{-1} \cdot X = \begin{bmatrix} p_{11}^{-1} & p_{12}^{-1} & p_{13}^{-1} & p_{14}^{-1} \\ p_{21}^{-1} & p_{22}^{-1} & p_{23}^{-1} & p_{24}^{-1} \\ p_{31}^{-1} & p_{32}^{-1} & p_{33}^{-1} & p_{34}^{-1} \\ p_{41}^{-1} & p_{42}^{-1} & p_{43}^{-1} & p_{44}^{-1} \end{bmatrix} \cdot \begin{bmatrix} I_{sd} \\ I_{sq} \\ \Phi_{rd} \\ \Phi_{rq} \end{bmatrix} \quad (3.75)$$

$$P^{-1} \cdot X = \begin{bmatrix} p_{11}^{-1} \cdot I_{sd} + p_{12}^{-1} \cdot I_{sq} + p_{13}^{-1} \cdot \Phi_{rd} + p_{14}^{-1} \cdot \Phi_{rq} \\ p_{21}^{-1} \cdot I_{sd} + p_{22}^{-1} \cdot I_{sq} + p_{23}^{-1} \cdot \Phi_{rd} + p_{24}^{-1} \cdot \Phi_{rq} \\ p_{31}^{-1} \cdot I_{sd} + p_{32}^{-1} \cdot I_{sq} + p_{33}^{-1} \cdot \Phi_{rd} + p_{34}^{-1} \cdot \Phi_{rq} \\ p_{41}^{-1} \cdot I_{sd} + p_{42}^{-1} \cdot I_{sq} + p_{43}^{-1} \cdot \Phi_{rd} + p_{44}^{-1} \cdot \Phi_{rq} \end{bmatrix} \quad (3.76)$$

3.3.2.1 Calculation of Third and Fourth Coordinates of the State-Space Equation

We are interested initially in the third and the fourth coordinates of state-space eigenvectors (3.76) of equation (3.70), written in the (d_0, q_0) reference frame:

$$\begin{aligned} & - \frac{\mu_4}{L_{sd} \cdot L_{sq} \cdot (\mu_4 - \mu_3) \cdot (\mu_3 - \mu_2) \cdot (\mu_3 - \mu_1)} \cdot \Phi_{rd} - \dots \\ & \dots \frac{\omega}{L_{sd} \cdot L_{sq} \cdot (\mu_4 - \mu_3) \cdot (\mu_3 - \mu_2) \cdot (\mu_3 - \mu_1)} \cdot \Phi_{rq} = \dots \\ & \dots e^{\mu_3 \cdot T} \cdot \left(\begin{aligned} & - \frac{\mu_4}{L_{sd} \cdot L_{sq} \cdot (\mu_4 - \mu_3) \cdot (\mu_3 - \mu_2) \cdot (\mu_3 - \mu_1)} \cdot \Phi_{rd0} - \dots \\ & \dots \frac{\omega}{L_{sd} \cdot L_{sq} \cdot (\mu_4 - \mu_3) \cdot (\mu_3 - \mu_2) \cdot (\mu_3 - \mu_1)} \cdot \Phi_{rq0} \end{aligned} \right) \end{aligned} \quad (3.77)$$

and:

$$\begin{aligned} & \frac{\mu_3}{L_{sd} \cdot L_{sq} \cdot (\mu_4 - \mu_3) \cdot (\mu_4 - \mu_2) \cdot (\mu_4 - \mu_1)} \cdot \Phi_{rd} + \dots \\ & \dots \frac{\omega}{L_{sd} \cdot L_{sq} \cdot (\mu_4 - \mu_3) \cdot (\mu_4 - \mu_2) \cdot (\mu_4 - \mu_1)} \cdot \Phi_{rq} = \dots \\ & \dots e^{\mu_4 \cdot T} \cdot \left(\begin{aligned} & \frac{\mu_3}{L_{sd} \cdot L_{sq} \cdot (\mu_4 - \mu_3) \cdot (\mu_4 - \mu_2) \cdot (\mu_4 - \mu_1)} \cdot \Phi_{rd0} + \dots \\ & \dots \frac{\omega}{L_{sd} \cdot L_{sq} \cdot (\mu_4 - \mu_3) \cdot (\mu_4 - \mu_2) \cdot (\mu_4 - \mu_1)} \cdot \Phi_{rq0} \end{aligned} \right) \end{aligned} \quad (3.78)$$

To reduce the formal writing, Φ_{rd} now expresses the abscissa value of the rotor flux vector $\vec{\Phi}_r$ at time $(t_n + T)$, in the (d_0, q_0) reference frame at time t_n , and the notation Φ_{rq} expresses the ordinate of the rotor flux vector $\vec{\Phi}_r$ at time $(t_n + T)$, in the (d_0, q_0) reference frame at time t_n . In the same way, the notation Φ_{rd0} expresses the abscissa of the rotor flux vector $\vec{\Phi}_r$ at time t_n , in the (d_0, q_0) reference frame at time t_n , and the notation Φ_{rq0} expresses the ordinate of the rotor flux vector $\vec{\Phi}_r$ at time t_n , in the (d_0, q_0) reference frame at time t_n .

These two expressions can be then simplified:

$$\mu_4 \cdot \Phi_{rd} + \omega \cdot \Phi_{rq} = e^{\mu_3 T} \cdot (\mu_4 \cdot \Phi_{rd0} + \omega \cdot \Phi_{rq0}) \quad (3.79)$$

$$\mu_3 \cdot \Phi_{rd} + \omega \cdot \Phi_{rq} = e^{\mu_4 T} \cdot (\mu_3 \cdot \Phi_{rd0} + \omega \cdot \Phi_{rq0}) \quad (3.80)$$

By replacing eigenvalues by their respective expression (3.36) and (3.37):

$$\mu_3 = -i \cdot \omega \quad (3.81)$$

$$\mu_4 = i \cdot \omega \quad (3.82)$$

While simplifying by ω , we obtain two conjugate-complex equalities, therefore identical ones:

$$i \cdot \Phi_{rd} + \Phi_{rq} = e^{-i \cdot \omega T} \cdot (i \cdot \Phi_{rd0} + \Phi_{rq0}) \quad (3.83)$$

$$-i \cdot \Phi_{rd} + \Phi_{rq} = e^{i \cdot \omega T} \cdot (-i \cdot \Phi_{rd0} + \Phi_{rq0}) \quad (3.84)$$

Let us multiply (3.84) by i :

$$\Phi_{rd} + i \cdot \Phi_{rq} = e^{i \cdot \omega T} \cdot (\Phi_{rd0} + i \cdot \Phi_{rq0}) \quad (3.85)$$

We can conclude from (3.85) that, in the (d_0, q_0) reference frame, the rotor flux vector rotates of an angle of $\theta = \omega \cdot T$, during the time interval T , which is obvious by construction of the rotating reference frame. We will, however, notice by definition of the (d_0, q_0) reference frame, that:

$$\Phi_{rd0} = \Phi_a \quad \text{and} \quad \Phi_{rq0} = 0 \quad (3.86)$$

The new coordinates of the rotor flux vector in this same frame at time $(t_n + T)$, are written simply by a rotation:

$$\boxed{\Phi_{rd} + i \cdot \Phi_{rq} = e^{i \cdot \omega T} \cdot \Phi_a} \quad (3.87)$$

Φ_{rq} is obviously not null in the initial rotating reference frame.

3.3.2.2 Calculation of the First and the Second Coordinate of the State-Space Eigenvector

The calculation of the first and the second coordinates of the state-space vector (3.76) of equation (3.70), written in the (d_0, q_0) reference frame, is carried out in appendix D.

Reduced variables defined in section 3.2.4.3.2, now authorize a simple formulation of these two coordinates from results of appendix D:

$$L_{sd} \cdot I_{sd} + \zeta_{d1} \cdot L_{sq} \cdot I_{sq} + \xi_{d1} \cdot \Phi_{rd} + \xi_{D1} \cdot \Phi_{rq} = \dots \quad (3.88)$$

$$\dots e^{\mu_1 \cdot T} \cdot (L_{sd} \cdot I_{sd0} + \zeta_{d1} \cdot L_{sq} \cdot I_{sq0} + \xi_{d1} \cdot \Phi_a) + e_1 \cdot (V_{sd0} + \zeta_{d1} \cdot V_{sq0})$$

$$L_{sd} \cdot I_{sd} + \zeta_{d2} \cdot L_{sq} \cdot I_{sq} + \xi_{d2} \cdot \Phi_{rd} + \xi_{D2} \cdot \Phi_{rq} = \dots \quad (3.89)$$

$$\dots e^{\mu_2 \cdot T} \cdot (L_{sd} \cdot I_{sd0} + \zeta_{d2} \cdot L_{sq} \cdot I_{sq0} + \xi_{d2} \cdot \Phi_a) + e_2 \cdot (V_{sd0} + \zeta_{d2} \cdot V_{sq0})$$

3.3.3 Control Calculation – Eigenstate-Space Equations Inversion

We saw that the two last coordinates of the state-space vector are coordinates of the rotor flux vector which is not controllable; the calculation of its prediction is summarized by a rotation of a mechanical angle depending only on the rotor speed and on the time.

We will thus be interested only in the first two coordinates of the state-space eigenvector which we calculated above. These complex coordinates have the following general form, in the (d_0, q_0) reference frame:

$$\bar{\Psi}_k = L_{sd} \cdot I_{sd} + \zeta_{dk} \cdot L_{sq} \cdot I_{sq} + \xi_{dk} \cdot \Phi_{rd} + \xi_{Dk} \cdot \Phi_{rq} \quad \text{with } k \in \{1; 2\} \quad (3.90)$$

These eigenvectors have the dimension of a magnetic flux vector.

To calculate the motor control, we have to now replace the prediction of the first two coordinates of the state-space eigenvector of the first term of equation (3.70), by set-point coordinates; only the stator current, which is controllable, can be replaced by the set-point current and we will preserve the prediction of the rotor flux in expression (3.90):

$$\begin{bmatrix} \bar{\Psi}_{1c} \\ \bar{\Psi}_{2c} \end{bmatrix}_{(d_0, q_0)} = \begin{bmatrix} L_{sd} \cdot I_{dc} + \zeta_{d1} \cdot L_{sq} \cdot I_{qc} + \xi_{d1} \cdot \Phi_{dp} + \xi_{D1} \cdot \Phi_{qp} \\ L_{sd} \cdot I_{dc} + \zeta_{d2} \cdot L_{sq} \cdot I_{qc} + \xi_{d2} \cdot \Phi_{dp} + \xi_{D2} \cdot \Phi_{qp} \end{bmatrix}_{(d_0, q_0)} \quad (3.91)$$

Coordinates of the predicted rotor flux vector were calculated by equation (3.87):

$$\begin{bmatrix} \Phi_{dp} + i \cdot \Phi_{qp} \end{bmatrix}_{(d_0, q_0)} = e^{i \cdot \omega T} \cdot \Phi_a \quad (3.92)$$

The first two complex coordinates of the state-space eigenvector are conjugate-complex, owing to the fact that ζ_{d1} and ζ_{d2} , ξ_{d1} and ξ_{d2} , ξ_{D1} and ξ_{D2} are conjugate-complex too.

In the same way, it is necessary to replace the coordinates of the initial state-space eigenvector by a measured or an estimated state:

$$\begin{bmatrix} \bar{\Psi}_{10} \\ \bar{\Psi}_{20} \end{bmatrix}_{(d_0, q_0)} = \begin{bmatrix} L_{sd} \cdot I_{d0} + \zeta_{d1} \cdot L_{sq} \cdot I_{q0} + \xi_{d1} \cdot \Phi_a \\ L_{sd} \cdot I_{d0} + \zeta_{d2} \cdot L_{sq} \cdot I_{q0} + \xi_{d2} \cdot \Phi_a \end{bmatrix}_{(d_0, q_0)} \quad (3.93)$$

Using these results, we can now simplify the writing of equations (3.88) and (3.89) in (d_0, q_0) :

$$\boxed{\bar{\Psi}_{1c(d_0, q_0)} = e^{\mu_1 \cdot T} \cdot \bar{\Psi}_{10(d_0, q_0)} + e_1 \cdot (V_{sd0} + \zeta_{d1} \cdot V_{sq0})} \quad (3.94)$$

$$\boxed{\bar{\Psi}_{2c(d_0, q_0)} = e^{\mu_2 \cdot T} \cdot \bar{\Psi}_{20(d_0, q_0)} + e_2 \cdot (V_{sd_0} + \zeta_{d2} \cdot V_{sq_0})} \quad (3.95)$$

The calculation of the motor control can now be carried out by eliminating one from the two coordinates of the control voltage vector, between the two equations; let us multiply the first equality by $e_2 \cdot \zeta_{d2}$, and the second one by $e_1 \cdot \zeta_{d1}$.

$$e_2 \cdot \zeta_{d2} \cdot \bar{\Psi}_{1c(d_0, q_0)} = e^{\mu_1 \cdot T} \cdot e_2 \cdot \zeta_{d2} \cdot \bar{\Psi}_{10(d_0, q_0)} + e_1 \cdot e_2 \cdot \zeta_{d2} \cdot (V_{sd_0} + \zeta_{d1} \cdot V_{sq_0}) \quad (3.96)$$

$$e_1 \cdot \zeta_{d1} \cdot \bar{\Psi}_{2c(d_0, q_0)} = e^{\mu_2 \cdot T} \cdot e_1 \cdot \zeta_{d1} \cdot \bar{\Psi}_{20(d_0, q_0)} + e_1 \cdot e_2 \cdot \zeta_{d1} \cdot (V_{sd_0} + \zeta_{d2} \cdot V_{sq_0}) \quad (3.97)$$

By subtraction, we obtain:

$$\begin{aligned} e_2 \cdot \zeta_{d2} \cdot \bar{\Psi}_{1c(d_0, q_0)} - e_1 \cdot \zeta_{d1} \cdot \bar{\Psi}_{2c(d_0, q_0)} &= \dots \\ \dots e^{\mu_1 \cdot T} \cdot e_2 \cdot \zeta_{d2} \cdot \bar{\Psi}_{10(d_0, q_0)} - e^{\mu_2 \cdot T} \cdot e_1 \cdot \zeta_{d1} \cdot \bar{\Psi}_{20(d_0, q_0)} &+ e_1 \cdot e_2 \cdot (\zeta_{d2} - \zeta_{d1}) \cdot V_{sd_0} \end{aligned} \quad (3.98)$$

which makes it possible to calculate the abscissa of the control vector, simplifying notations in the (d_0, q_0) reference frame:

$$V_{sd_0} = \frac{(e_2 \cdot \zeta_{d2} \cdot \bar{\Psi}_{1c} - e_1 \cdot \zeta_{d1} \cdot \bar{\Psi}_{2c}) - (e_2 \cdot \zeta_{d2} \cdot e^{\mu_1 \cdot T} \cdot \bar{\Psi}_{10} - e_1 \cdot \zeta_{d1} \cdot e^{\mu_2 \cdot T} \cdot \bar{\Psi}_{20})}{e_1 \cdot e_2 \cdot (\zeta_{d2} - \zeta_{d1})} \quad (3.99)$$

This relation can be also written:

$$\boxed{V_{sd_0} = \frac{e_2 \cdot \zeta_{d2} \cdot (\bar{\Psi}_{1c(d_0, q_0)} - e^{\mu_1 \cdot T} \cdot \bar{\Psi}_{10(d_0, q_0)}) - e_1 \cdot \zeta_{d1} \cdot (\bar{\Psi}_{2c(d_0, q_0)} - e^{\mu_2 \cdot T} \cdot \bar{\Psi}_{20(d_0, q_0)})}{e_1 \cdot e_2 \cdot (\zeta_{d2} - \zeta_{d1})}} \quad (3.100)$$

It will be noted that this is a real equality, even if its calculation is carried out starting from complex intermediate variables. Indeed, it is enough to notice that all variables are conjugate-complex two by two as was previously demonstrated for eigenvalues, reduced variables and state-space eigenvector coordinates; this is thus enough to conjugate the whole expression and to replace conjugate-complex variables by their complex value, simultaneously switching subscripts 1 and 2. The whole expression remains unchanged. V_{sd_0} is thus a real variable, which is necessary since it represents the control vector *abscissa* in the rotating reference frame at the initial time.

In the same way, from equations (3.94) and (3.95):

$$e_2 \cdot \bar{\Psi}_{1c(d_0, q_0)} = e^{\mu_1 \cdot T} \cdot e_2 \cdot \bar{\Psi}_{10(d_0, q_0)} + e_1 \cdot e_2 \cdot (V_{sd_0} + \zeta_{d1} \cdot V_{sq_0}) \quad (3.101)$$

$$e_1 \cdot \bar{\Psi}_{2c(d_0, q_0)} = e^{\mu_2 \cdot T} \cdot e_1 \cdot \bar{\Psi}_{20(d_0, q_0)} + e_1 \cdot e_2 \cdot (V_{sd_0} + \zeta_{d2} \cdot V_{sq_0}) \quad (3.102)$$

by subtraction and simplification of notations:

$$V_{sq0} = - \frac{(e_2 \cdot \vec{\Psi}_{1c} - e_1 \cdot \vec{\Psi}_{2c}) - (e_2 \cdot e^{\mu_1 T} \cdot \vec{\Psi}_{10} - e_1 \cdot e^{\mu_2 T} \cdot \vec{\Psi}_{20})}{e_1 \cdot e_2 \cdot (\zeta_{d2} - \zeta_{d1})} \quad (3.103)$$

We can also write it as:

$$V_{sq0} = - \frac{e_2 \cdot \left(\vec{\Psi}_{1c(d_0, q_0)} - e^{\mu_1 T} \cdot \vec{\Psi}_{10(d_0, q_0)} \right) - e_1 \cdot \left(\vec{\Psi}_{2c(d_0, q_0)} - e^{\mu_2 T} \cdot \vec{\Psi}_{20(d_0, q_0)} \right)}{e_1 \cdot e_2 \cdot (\zeta_{d2} - \zeta_{d1})} \quad (3.104)$$

In the same way, it is a real equality, even if its calculation is carried out starting from complex intermediate variables. This can be shown in a similar way to that presented above.

All coordinates are those of the (d_0, q_0) reference frame, which is logical for vector coordinates of the initial state-space eigenvector, but which is not appropriate for the set-point state-space vector, or for the control voltage vector.

We will see that it will be necessary to establish the coordinates of the set-point current vector in the (d, q) reference frame, then to change its coordinates into the (d_0, q_0) reference frame, in order to calculate the control vector coordinates in the (d_0, q_0) reference frame; in the same way, the control voltage vector will have to be written in the (α, β) fixed frame, to be applied by the inverter in the stator fixed frame.

To obtain the average voltage vector coordinates defined by equations (3.100) and (3.104) in the (α, β) fixed frame, we will change its coordinates in the (d_0, q_0) reference frame, multiplying the complex vector by $e^{i \cdot \rho_0}$: $\vec{V}_{s(\alpha, \beta)} = (V_{sd_0} + i \cdot V_{sq_0}) \cdot e^{i \cdot \rho_0}$.

An example of calculation of a symmetrical vector PWM, is presented in detail in appendix A.

3.4 Control

The inversion of eigenstate-space equations simplifies significantly the control process. We will analyze the various steps of the control design in an example of a real-time implementation with a fixed sampling period.

3.4.1 Constitution of the Set-Point State-Space Vector

To calculate the coordinates of the control vector, with equations (3.100) and (3.104), we will have to calculate $\vec{\Psi}_{1c}$ and $\vec{\Psi}_{2c}$, the first two coordinates of the state-space eigenvector in the (d_0, q_0) reference frame.

Coordinates of the stator current according to the two (d, q) axes:

$$\vec{I}_{sc} = I_{dc} + i \cdot I_{qc} \quad (3.105)$$

depend on the torque set-point, but also on voltage and current limitations.

The real coordinate I_{dc} of the vector $\vec{I}_{sc}$, collinear with the rotor flux, can be selected to regulate the rotor flux in the magnet flux direction and thus to regulate the motor counter-voltage. In general, the adjustment of this magnetizing current is used to oppose a stator flux to that due to magnets, when the motor is operated beyond its rated speed, the speed at which its full voltage is reached with the maximum torque. In this case, the component must be negative in

the range of the motor voltage adjustment beyond this speed. We will see how to calculate this component in the section 3.4.4.4.1 on voltage limitation.

As we will see, for this kind of motor, below its rated speed, this coordinate must be selected non-null to minimize the stator current magnitude, unlike the surface-mounted permanent-magnet synchronous motor, because this component also affects the reluctance torque, according to the saliency type of the motor.

Indeed, the total electromagnetic torque given by equation (3.20), makes it possible to calculate the stator current ordinate, according to the desired torque, but this ordinate component depends also on the abscissa of the same current, still unknown:

$$I_{qc} = \frac{C_c}{N_p \cdot \left[(L_{md} - L_{mq}) \cdot I_{dc} + \Phi_a \right]} \quad (3.106)$$

If $I_{dc} = 0$, the value of I_{qc} results immediately from equation (3.106) when one knows the set-point torque C_c , the number of pole pairs N_p and the permanent magnet flux per pole pair Φ_a . However, the cancellation of the magnetizing current I_{dc} in the pole direction minimizes the denominator of equation (3.106) and increases the active current.

If the motor saliency is a reverse one, i.e. $L_{md} < L_{mq}$, a negative current $I_{dc} < 0$ will make an additive reluctance torque and thus will allow the active current magnitude to be reduced to obtain the same torque, since the denominator of equation (3.106) is then bigger. A similar reasoning would result in imposing a positive current $I_{dc} > 0$ in the case of the direct saliency to make an additive reluctance torque and thus to reduce the active current magnitude to obtain the same torque.

Equation (3.106) is the equation of one hyperbolic function family depending on the set-point torque as parameter, in the (d, q) reference frame; it is the locus of the extremity of the set-point current vector and we can thus minimize its magnitude.

The hyperbola families, shown as full lines in Figure 3.6, represent the case where $L_{md} - L_{mq} < 0$, i.e. where the motor saliency is a reverse one. If the motor saliency is a direct one, the two hyperbolic function families are symmetrical compared to the I_q axis; this case is represented in dotted lines.

For an imposed torque, the minimum set-point current vector is the tangential point between the hyperbola corresponding to the set-point torque and the circle of radius $\|\vec{I}_{sc}\|$ centered on the origin O .

To calculate this point according to the set-point torque, we will calculate the point of the hyperbola minimizing the distance to the frame origin. Let us calculate the squared distance d , by $d^2 = I_d^2 + I_q^2$:

$$d^2 = I_d^2 + \frac{C_c^2}{N_p^2 \cdot \left[(L_{md} - L_{mq}) \cdot I_d + \Phi_a \right]^2} \quad (3.107)$$

Equating to zero the derivative compared to the current abscissa, of the squared distance, for a constant given torque provides the abscissa of the required point:

$$\frac{d(d^2)}{dI_d} = 0 \Rightarrow I_d \cdot \left[(L_{md} - L_{mq}) \cdot I_d + \Phi_a \right]^3 - (L_{md} - L_{mq}) \cdot \frac{C_c^2}{N_p^2} = 0 \quad (3.108)$$

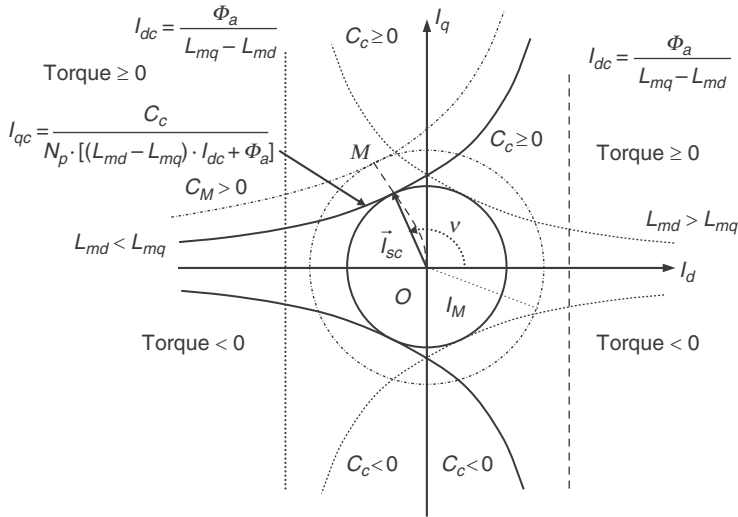


Figure 3.6 Minimum stator current for each set-point torque

that is to say:

$$\begin{aligned}
 I_{dc}^4 + 3 \cdot \frac{\Phi_a}{L_{md} - L_{mq}} \cdot I_{dc}^3 + 3 \cdot \left(\frac{\Phi_a}{L_{md} - L_{mq}} \right)^2 \cdot I_{dc}^2 + \dots \\
 \dots \left(\frac{\Phi_a}{L_{md} - L_{mq}} \right)^3 \cdot I_{dc} - \frac{C_c^2}{N_p^2 \cdot (L_{md} - L_{mq})^2} = 0
 \end{aligned} \quad (3.109)$$

This fourth-degree polynomial makes it possible to calculate initially I_{dc} , then I_{qc} from equation (3.106), when there is neither voltage nor current limitation.

Another solution for finding the coordinates of the tangential point is to use the circular parametric coordinates in the torque equation:

$$\begin{bmatrix} I_d \\ I_q \end{bmatrix} = \|\vec{I}_s\| \cdot \begin{bmatrix} \cos(\nu) \\ \sin(\nu) \end{bmatrix} \Rightarrow C_c = N_p \cdot \left[(L_{md} - L_{mq}) \cdot \|\vec{I}_s\| \cdot \cos(\nu) + \Phi_a \right] \cdot \|\vec{I}_s\| \cdot \sin(\nu) \quad (3.110)$$

We then cancel the derivative of the torque, compared to the ν parameter, from equation (3.110).

The solution of the quadratic equation in $\cos(\nu)$, which results from the equating to zero of the derivative, gives parametric coordinates of the current vector extremity, of which the magnitude is then minimum.

$$\cos(\pm\nu) = \frac{\Phi_a \mp \sqrt{\Phi_a^2 + 8 \cdot \|\vec{I}_s\|^2 \cdot (L_{mq} - L_{md})^2}}{4 \cdot \|\vec{I}_s\| \cdot (L_{mq} - L_{md})} \quad (3.111)$$

This formula makes it possible to highlight very simply, and in a geometrical way, thanks to the parametric representation, the four solutions shown in Figure 3.6, according to the saliency type and to the set-point torque sign.

In Figure 3.6, the locus of tangential points according to the set-point torque, is represented by a curve, plotted with a dashed line starting from the axis origin and leading to the tangential point between the hyperbola of the set-point torque and the current circle with a minimum radius $\|\vec{I}_{sc}\|$. This locus of the vector extremity of the set-point current is drawn here only for the traction operating mode, with a reverse saliency motor type; it is, however, easy to derive from there the three other symmetrical loci both for the braking operation and also the two other operating modes of a direct saliency motor type.

We will see later how to calculate the two coordinates of the stator current to simultaneously satisfy voltage and current limitations, always with the minimum current.

In addition, we saw with equation (3.92) that the rotation of the (d, q) reference frame, and thus of the rotor magnet flux, between the initial instant and the control horizon is $\theta = \omega \cdot T$. Vector flux coordinates, predicted at time $(t_n + T)$ in the (d_0, q_0) reference frame are:

$$\vec{\Phi}_{rp(d_0, q_0)} = (\Phi_{dp} + i \cdot \Phi_{qp})_{(d_0, q_0)} = e^{i \cdot \omega T} \cdot \Phi_a \quad (3.112)$$

We can thus modify the reference frame of current vector coordinates:

$$(I_{dc} + i \cdot I_{qc})_{(d_0, q_0)} = (I_{dc} + i \cdot I_{qc})_{(d, q)} \cdot e^{i \cdot \omega T} \quad (3.113)$$

The two preceding relations make it possible to calculate the first two coordinates of the set-point state-space eigenvector, in the (d_0, q_0) reference frame:

$$\begin{bmatrix} \vec{\Psi}_{1c} \\ \vec{\Psi}_{2c} \end{bmatrix}_{(d_0, q_0)} = \begin{bmatrix} L_{sd} \cdot I_{dc} + \zeta_{d1} \cdot L_{sq} \cdot I_{qc} + \xi_{d1} \cdot \Phi_{dp} + \xi_{D1} \cdot \Phi_{qp} \\ L_{sd} \cdot I_{dc} + \zeta_{d2} \cdot L_{sq} \cdot I_{qc} + \xi_{d2} \cdot \Phi_{dp} + \xi_{D2} \cdot \Phi_{qp} \end{bmatrix}_{(d_0, q_0)} \quad (3.114)$$

starting from reduced variables, depending on motor parameters and on the mechanical speed.

3.4.2 Constitution of the Initial State-Space Vector

The initial state-space eigenvector is calculable from the measurement of the stator current $\vec{I}_s$ and from the estimation of the rotor magnet flux $\vec{\Phi}_a$ in the (d_0, q_0) reference frame.

The estimation of the magnet flux is in general made:

- with the knowledge of the magnet flux magnitude $\|\vec{\Phi}_r\| = \Phi_a$, possibly tabulated according to the motor temperature, itself estimated or measured
- and with the measurement of the rotor electrical position $\arg(\vec{\Phi}_{r0}) = N_p \cdot \Xi$, in the (α, β) fixed frame, by an absolute angular position sensor Ξ , for example by a synchro-resolver.

We will see that it is also possible to estimate these two rotor flux coordinates, for example using a Kalman estimator which can simultaneously filter the measures of the stator current.

It is then necessary to transfer the coordinates of these two vectors from the (α, β) fixed frame to the (d_0, q_0) reference frame, after having calculated the angle ρ_0 of this reference frame, compared to the fixed frame. Successively:

$$\rho_0 = \arg(\vec{\Phi}_{r0}) \Rightarrow I_{d0} + i \cdot I_{q0} = (I_{s\alpha} + i \cdot I_{s\beta}) \cdot e^{-i \cdot \rho_0} \quad (3.115)$$

We will see in section 3.4.4.4.4, using Figure 3.17, how to practically calculate the angle ρ_0 , either starting from the measurement of the motor rotor position or starting from the rotor flux estimation at time t_n .

Ultimately:

$$\begin{bmatrix} \vec{\Psi}_{10} \\ \vec{\Psi}_{20} \end{bmatrix}_{(d_0, q_0)} = \begin{bmatrix} L_{sd} \cdot I_{d0} + \zeta_{d1} \cdot L_{sq} \cdot I_{q0} + \xi_{d1} \cdot \Phi_a \\ L_{sd} \cdot I_{d0} + \zeta_{d2} \cdot L_{sq} \cdot I_{q0} + \xi_{d2} \cdot \Phi_a \end{bmatrix}_{(d_0, q_0)} \quad (3.116)$$

We will see later on that Φ_a can also be estimated using a Kalman filter.

3.4.3 Control Process

The control voltage vector is calculated by its coordinates in the (d_0, q_0) reference frame from equations (3.100) and (3.104). It is impressed on the motor stator by the voltage inverter in the fixed frame. To transmit this information to the PWM, it is thus necessary to change the coordinates of the control voltage vector from the (d_0, q_0) reference frame, towards the (α, β) fixed frame.

$$V_{s\alpha} + i \cdot V_{s\beta} = (V_{sd0} + i \cdot V_{sq0}) \cdot e^{i \cdot \rho_0} \quad (3.117)$$

3.4.3.1 Real-Time Implementation

The analytical control calculation being now finished, it remains as a practical problem to be solved, allowing a real-time implementation of control algorithms.

Indeed, it is necessary to be able to calculate the average voltage vector to be applied between the initial time t_n , at which the initial state-space eigenvector is measured and estimated, and the final time $(t_n + T)$, at which set-points must be reached; the computation duration should not cause any time-delay of the voltage vector application on the motor, from the initial instant to the end of a complete sampling period. Under these conditions, the time reserved for the various computations is null.

In practice, it is thus necessary to precede the voltage vector impressing period by a period reserved for the computation. A second problem then appears: because the initial state-space eigenvector must be defined at time t_n , to feed the computation, it cannot then result from measurements at this time.

The solution is to make a computation during one period T , and to impress the calculated voltage vector during the following one. However, to avoid a pure time-delay of duration T , under these conditions, it is necessary to predict the motor state at the end of the computational period in progress, and thus at the beginning of the impressing period of the voltage vector. It is this prediction of the state-space eigenvector at the horizon t_n , starting from

measurements carried out at $(t_n - T)$, at the beginning of the period reserved to the computation (Jacquot, 1995), which will then be used to calculate the voltage vector to apply between t_n and $(t_n + T)$.

It is thus necessary to keep a first period for measurements, measure filtering, estimation of the rotor flux, prediction of the initial state-space vector and finally for computation of the voltage vector, then a second period to impress the voltage vector.

For this second period, the voltage vector can be generated by one pulse width modulation, from an external circuit different from the main processor – an FPGA for instance or a pure logic one, thus releasing the processor for the other computation during the following period.

The sampling period was selected as constant here, thus it is necessarily an asynchronous sampling period compared to the fundamental voltage of the stator. It can be selected variable to synchronize the period with the motor voltage, as is necessary with square wave operation, or to make it possible to lower the rate of voltage harmonics and thus of current harmonics by a choice more suited to the PWM type at high speed. To switch from the fixed computational period, generally used at low speed, to a synchronous period with the motor voltage, it is enough, after a phase of synchronization, to calculate the control vector by regarding the period T as a variable and no longer as a parameter of equations (3.100) and (3.104).

The sampling period duration is selected in relation to the time necessary first to calculate and then to impress the average voltage vector which determines the current shape factor, itself in relation to the selected PWM type. The sampling period, is identical to the periodicity of the control vector impressing; it also conditions the inverter losses.

This structuring of algorithms for their real-time exploitation led to imposing a sampling period identical to the PWM period and synchronous with it, while remaining asynchronous compared to the motor voltage fundamental component when the PWM period is fixed at low speed and synchronous compared to the voltage fundamental component when the PWM period is variable at high speed. In particular, this characteristic results in sampling the stator phase current in a synchronous way with the PWM and thus in a synchronous way with harmonic currents created by the PWM. This provision makes it possible to remove the anti-aliasing filter of the harmonic current spectrum generated by the PWM, which is dominating compared to the stochastic spectrum of measurement disturbances, and thus to improve the bandwidth of measurements (Jacquot, 1995). Sampling without an anti-aliasing filter provides, under these conditions, an instantaneous fundamental current value added to synchronous harmonic currents at the beginning of each period of the PWM.

In short, the real-time implementation of computational algorithms requires us to carry out a motor state prediction to constitute the initial state-space vector necessary for the control vector computation. We can use it for measure filtering and for observing the non-measurable variables, with discretized state-space equations. This relation will thus be used for predicting a state, filtering measurements and estimating non-measurable variables in a cyclic way; it was also used to calculate the control after several transformations.

It should be noted that the set-points which are used in the computations are known in the example of Figure 3.7, at time $(t_n - T)$. Contrary to the motor state, set-points cannot being predicted at time t_n , in a general way, except in the case of a predefined tracking program; they will thus be regarded as set-points at time t_n with a pure time-delay of one T period. These set-points will thus be reached at time $(t_n + T)$.

For opening the loop of the cyclic process in real-time, we will define the description origin of algorithms at time $(t_n - T)$.

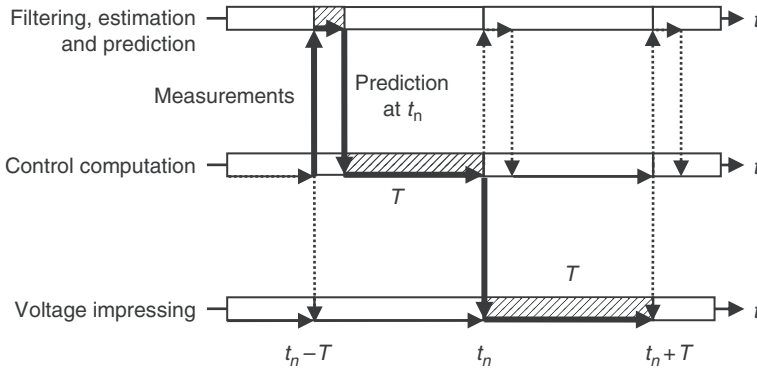


Figure 3.7 Timing diagram of a real-time control process

3.4.3.2 Measure Filtering

The discretized state-space equation (1.57):

$$X(t_n - T)_p = F \cdot X(t_n - 2 \cdot T) + G \cdot V \quad (3.118)$$

applied during the period preceding measurements, is used to make the motor state prediction at time $(t_n - T)$; it can also be used to filter current measures made at time $(t_n - T)$, with a Kalman filter (Kalman, 1982). This filtering will be simultaneously used to observe the rotor flux at time $(t_n - T)$ (Jacquot, 1995), if one does not measure the rotor position and/or if the magnet flux is changing with time or with motor temperature or with the stator current.

The prediction of the state-space vector at the horizon t_n , from filtered measurements by the Kalman filter at $(t_n - T)$ will then be used to define the initial state-space eigenvector $\Psi_0(t_n)$ for the computations done between $(t_n - T)$ and t_n of the voltage vector to be applied during the following period between t_n and $(t_n + T)$ (Jacquot, 1995).

For this motor type, we saw that the evolution, transition and input matrices, in order to present a simplified form, had to be established in the rotating reference frame following the rotor magnetic anisotropy; it is then necessary to know the position of the rotating reference frame since the instant of the current measurement, i.e. at time $(t_n - T)$, to calculate the state-space vector in the (d_m, q_m) rotating reference frame at this time.

When a rotor position encoder exists, it makes it possible to roughly measure the rotor flux position and thus the angle $\rho_m = N_p \cdot \Xi$ of the rotating reference frame compared to the (α, β) fixed frame at time $(t_n - T)$.

When the rotor position is not known from a measurement of the motor shaft position, the angle ρ_m must be estimated from the prediction of the magnet flux position made during the previous period from $(t_n - 2 \cdot T)$ to $(t_n - T)$ (3.118).

Although the solution of the control to apply between t_n and $(t_n + T)$ has been established in the (d_0, q_0) reference frame from the two first coordinates of the initial state-space eigenvector at time t_n , the state-space vector $X(t_n)$ must still be calculated in this reference frame to directly provide elements of computation (3.115) of the first two coordinates of the state-space eigenvector (3.116). We saw in addition in section 3.3.2, how to use measurements or estimations, filtered or not, to calculate the coordinates of $X(t_n)$ in the (d_0, q_0) initial rotating reference frame at time t_n .

Current measures made at $(t_n - T)$ make it possible to know the stator current vector in the (α, β) fixed frame of measurements:

$$Y(t_n - T)_{(\alpha, \beta)} = \begin{bmatrix} I_{s\alpha} \\ I_{s\beta} \end{bmatrix}_m \quad (3.119)$$

The knowledge of the position measurement Ξ , or of the position estimation ρ_m , makes it possible to calculate:

$$Y(t_n - T)_{(d_m, q_m)} = \begin{bmatrix} I_{s\alpha} \\ I_{s\beta} \end{bmatrix} \cdot e^{-i \cdot \rho_m} \quad \text{then} \quad X(t_n - T)_{(d_m, q_m)} = \begin{bmatrix} I_{dm} \\ I_{qm} \\ \Phi_a \\ 0 \end{bmatrix} \quad (3.120)$$

With this expression, discretized state-space equations in the rotating reference frame can be more precisely written:

$$X(t_n)_{p(d_m, q_m)} = F_{dq} \cdot X(t_n - T)_{(d_m, q_m)} + G_{dq} \cdot V_{(d_m, q_m)} \quad (3.121)$$

The predicted state-space vector $X(t_n)_p$ is also calculated in (d_m, q_m) , but it is still necessary to calculate it in (d_0, q_0) to be able to use it directly in the control computation:

$$X(t_n)_{p(d_0, q_0)} = X(t_n)_{p(d_m, q_m)} \cdot e^{-i \cdot \omega T} \quad (3.122)$$

It is preferable to estimate the frame rotation between $(t_n - T)$ and t_n from the prediction, rather than to consider a constant speed between its measurement at $(t_n - T)$ and its use at t_n . Indeed, the prediction of the state-space vector $X(t_n)_{p(d_m, q_m)}$ also enables us to predict the reference frame rotation, calculating the angle of the predicted flux at time t_n in the (d_m, q_m) reference frame by:

$$\begin{aligned} \|\vec{\Phi}_r(t_n)\|^2 &= \Phi_{dm}^2(t_n) + \Phi_{qm}^2(t_n) \\ \cos(\Delta\rho_m) &= \frac{\Phi_{dm}(t_n)}{\|\vec{\Phi}_r(t_n)\|} \quad \sin(\Delta\rho_m) = \frac{\Phi_{qm}(t_n)}{\|\vec{\Phi}_r(t_n)\|} \\ X(t_n)_{p(d_0, q_0)} &= X(t_n)_{p(d_m, q_m)} \cdot e^{-i \cdot \Delta\rho_m} \\ X(t_n)_{p(d_0, q_0)} &= \begin{bmatrix} I_{d0} \\ I_{q0} \\ \Phi_{d0} \\ 0 \end{bmatrix} \end{aligned} \quad (3.123)$$

with $\Phi_{d0} = \|\bar{\Phi}_{re}\|$ if the flux is estimated, or $\Phi_{d0} = \Phi_a$ if it is calculated according to the permanent magnet type and to the iron cross-section.

If the rotor position of the motor is not measured at time $(t_n - T)$, we will see in Figure 3.17 that the reference frame rotation $\Delta\rho_m$ between $(t_n - T)$ and t_n could be used to gradually increment the initial angular position of the (d_m, q_m) reference frame, without risk of divergence because of the flux position filtering by the Kalman filter at $(t_n - T)$. In this case, the initial rotor position will have to be calculated and known before starting (Benchaïb *et al.* 2003).

3.4.3.3 Transition and Input Matrix Calculations

We must then calculate matrices of transition F and of control G of the discretized state-space equation of the motor, from equation (1.57) and from equation (1.93), after multiplication on the left, of the two terms, by the transfer matrix P :

$$F = P \cdot e^{D \cdot T} \cdot P^{-1} \quad \text{and} \quad G = P \cdot D^{-1} \cdot (e^{D \cdot T} - I) \cdot P^{-1} \cdot B \quad (3.124)$$

Let us overwrite the various elements by their computed values P and P^{-1} in (3.47) and (3.48), $e^{D \cdot T}$ in (3.69) and $D^{-1} \cdot (e^{D \cdot T} - I) \cdot P^{-1} \cdot B$ in (3.74).

The detailed calculation of these two matrices is carried out in appendix E.

3.4.3.3.1 Calculation Results of the Transition Matrix F

Results of the calculation of transition matrix coefficients of appendix B are listed finally below:

$$F_{11} = \frac{\omega_s \cdot L_{md}}{L_{sd}} \cdot \frac{e^{\mu_2 \cdot T} \cdot \zeta_{d1} - e^{\mu_1 \cdot T} \cdot \zeta_{d2}}{\mu_2 - \mu_1} \quad (3.125)$$

$$F_{12} = \frac{\omega_s \cdot L_{mq}}{L_{sd}} \cdot \frac{e^{\mu_2 \cdot T} - e^{\mu_1 \cdot T}}{\mu_2 - \mu_1} \quad (3.126)$$

$$F_{13} = \frac{\omega_s}{L_{sd}} \cdot \left(\frac{L_{md}}{L_{sd}} \cdot \frac{e^{\mu_2 \cdot T} \cdot \zeta_{d1} \cdot \xi_{d2} - e^{\mu_1 \cdot T} \cdot \zeta_{d2} \cdot \xi_{d1}}{\mu_2 - \mu_1} - \frac{L_{mq}}{L_{sq}} \cdot \frac{e^{\mu_4 \cdot T} \cdot \xi_{Q4} - e^{\mu_3 \cdot T} \cdot \xi_{Q3}}{\mu_4 - \mu_3} \right) \quad (3.127)$$

$$F_{14} = \frac{\omega_s}{L_{sd}} \cdot \left(\frac{L_{md}}{L_{sd}} \cdot \frac{e^{\mu_2 \cdot T} \cdot \zeta_{d1} \cdot \xi_{D2} - e^{\mu_1 \cdot T} \cdot \zeta_{d2} \cdot \xi_{D1}}{\mu_2 - \mu_1} + \frac{L_{mq}}{L_{sq}} \cdot \frac{e^{\mu_4 \cdot T} \cdot \xi_{q4} - e^{\mu_3 \cdot T} \cdot \xi_{q3}}{\mu_4 - \mu_3} \right) \quad (3.128)$$

$$F_{21} = \frac{\omega_s}{L_{sq}} \cdot \frac{e^{\mu_1 \cdot T} \cdot L_{md} - e^{\mu_2 \cdot T} \cdot L_{mq}}{\mu_2 - \mu_1} \quad (3.129)$$

$$F_{22} = \frac{\omega_s}{L_{sd}} \cdot \frac{e^{\mu_1 \cdot T} \cdot \zeta_{d1} \cdot L_{md} - e^{\mu_2 \cdot T} \cdot \zeta_{d2} \cdot L_{mq}}{\mu_2 - \mu_1} \quad (3.130)$$

$$F_{23} = \frac{\omega_s}{L_{sd} \cdot L_{sq}} \cdot \left(\frac{e^{\mu_1 \cdot T} \cdot \xi_{d1} \cdot L_{md} - e^{\mu_2 \cdot T} \cdot \xi_{d2} \cdot L_{mq}}{\mu_2 - \mu_1} - L_{md} \cdot \frac{e^{\mu_4 \cdot T} \cdot \xi_{d4} - e^{\mu_3 \cdot T} \cdot \xi_{d3}}{\mu_4 - \mu_3} \right) \quad (3.131)$$

$$F_{24} = \frac{\omega_s}{L_{sd} \cdot L_{sq}} \cdot \left(\frac{e^{\mu_1 \cdot T} \cdot \xi_{D1} \cdot L_{md} - e^{\mu_2 \cdot T} \cdot \xi_{D2} \cdot L_{mq}}{\mu_2 - \mu_1} - L_{md} \cdot \frac{e^{\mu_4 \cdot T} \cdot \xi_{q4} - e^{\mu_3 \cdot T} \cdot \xi_{q3}}{\mu_4 - \mu_3} \right) \quad (3.132)$$

$$F_{31} = 0 \quad (3.133)$$

$$F_{32} = 0 \quad (3.134)$$

$$F_{33} = \frac{e^{\mu_3 \cdot T} \cdot \mu_4 - e^{\mu_4 \cdot T} \cdot \mu_3}{\mu_4 - \mu_3} \quad (3.135)$$

$$F_{34} = \omega \cdot \frac{e^{\mu_3 \cdot T} - e^{\mu_4 \cdot T}}{\mu_4 - \mu_3} \quad (3.136)$$

$$F_{41} = 0 \quad (3.137)$$

$$F_{42} = 0 \quad (3.138)$$

$$F_{43} = \omega \cdot \frac{e^{\mu_4 \cdot T} - e^{\mu_3 \cdot T}}{\mu_4 - \mu_3} \quad (3.139)$$

$$F_{44} = \frac{e^{\mu_4 \cdot T} \cdot \mu_4 - e^{\mu_3 \cdot T} \cdot \mu_3}{\mu_4 - \mu_3} \quad (3.140)$$

$$F = \begin{bmatrix} F_{11} & F_{12} & F_{13} & F_{14} \\ F_{21} & F_{22} & F_{23} & F_{24} \\ F_{31} & F_{32} & F_{33} & F_{34} \\ F_{41} & F_{42} & F_{43} & F_{44} \end{bmatrix}$$

3.4.3.3.2 Calculation Results of the Discretized Input Matrix G

This result is issued from the calculation of discretized input matrix coefficients in appendix B.

$$G = \frac{\omega_s}{L_{sd} \cdot (\mu_2 - \mu_1)} \cdot \begin{bmatrix} \frac{L_{md}}{L_{sd}} \cdot (\zeta_{d1} \cdot e_2 - \zeta_{d2} \cdot e_1) & \frac{L_{mq}}{L_{sq}} \cdot (e_2 - e_1) \\ \frac{1}{L_{sq}} \cdot (L_{md} \cdot e_1 - L_{mq} \cdot e_2) & \frac{1}{L_{sq}} \cdot (\zeta_{d1} \cdot L_{md} \cdot e_1 - \zeta_{d2} \cdot L_{mq} \cdot e_2) \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (3.141)$$

3.4.3.4 Kalman Filter

Ultimately, the prediction used for current filtering and for rotor flux estimation at time $(t_n - T)$, from filtered variables at the previous instant $(t_n - 2 \cdot T)$, take the following form in the *rotating reference frame* for this motor:

$$X(t_n - T)_p = F(t_n - 2 \cdot T) \cdot X(t_n - 2 \cdot T)_f + G(t_n - 2 \cdot T) \cdot V_{(t_n - 2 \cdot T) \rightarrow (t_n - T)} \quad (3.142)$$

$$Y(t_n - T)_p = H \cdot X(t_n - T)_p \quad (3.143)$$

with:

$$X = \begin{bmatrix} I_{sd} \\ I_{sq} \\ \Phi_{rd} \\ \Phi_{rq} \end{bmatrix} \quad \text{and} \quad V = \bar{V}_s = \begin{bmatrix} V_{sd} \\ V_{sq} \end{bmatrix} \quad (3.144)$$

Let us choose the stator current vector as a variable $Y(t_n - T)_m$, measured at time $(t_n - T)$, and the rotor flux as an observed variable; we thus fix the output matrix H .

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad \text{and} \quad Y = \begin{bmatrix} I_{sd} \\ I_{sq} \end{bmatrix} \quad (3.145)$$

The following main steps of the Kalman filtering (Jacquot, 1995) at time $(t_n - T)$ must be carried out in the rotating reference frame related to the rotor of this particular type of synchronous motor:

- The position of the rotating reference frame ρ_m is either measured by a position encoder mechanically linked to the rotor of the motor $\rho_m = N_p \cdot \Xi$, or estimated by a Kalman filtering, thus by a position prediction, incrementing successive positions from an initial one measured by specific electrical means (Benchaiïb *et al.*, 2003).
- From the current measurement in (α, β) , transferred in the (d_m, q_m) rotating reference frame by:

$$Y(t_n - T)_m = Y(t_n - T)_{(\alpha, \beta)} \cdot e^{-i \cdot \rho_m} \quad (3.146)$$

and from the calculation of the state-space vector prediction $X(t_n - T)_p$ in the (d_m, q_m) rotating reference frame during the previous period $(t_n - 2 \cdot T)$, we calculate the difference between the predicted value and the measured value of the current measurement, multiplied by the Kalman gain. This corrective measure of the predicted state-space vector, will make it possible to filter the state-space vector measurement:

$$X(t_n - T)_f = X(t_n - T)_p + K(t_n - T) \cdot \{Y(t_n - T)_m - Y(t_n - T)_p\} \quad (3.147)$$

- The filtered state-space vector is thus immediately written in the rotating reference frame according to the filtered current and to the rotor flux vector observed by its magnitude and argument:

$$X(t_n - T)_f = \begin{bmatrix} \bar{I}_{sf} \\ \bar{\Phi}_{re} \end{bmatrix} \quad (3.148)$$

- $K(t_n - T)$ is the Kalman gain computed for the time $(t_n - T)$ during the previous period from $(t_n - 2 \cdot T)$ to $(t_n - T)$.
- The prediction calculation of the state error covariance matrix $P(t)$ for t_n is made from that calculated for $(t_n - T)$ during the previous period from $(t_n - 2 \cdot T)$ to $(t_n - T)$:

$$P(t_n)_p = F \cdot P(t_n - T) \cdot F^t + Q_0 \quad (3.149)$$

- Q_0 is the state disturbance covariance matrix regarded in this example as constant for simplicity.
- The Kalman gain at time t_n is calculated from the prediction of the state error covariance matrix for t_n :

$$K(t_n) = P(t_n)_p \cdot H^t \cdot \{H \cdot P(t_n)_p \cdot H^t + R_0\}^{-1} \quad (3.150)$$

- R_0 is the measurement disturbance covariance matrix, considered as constant in this example for simplicity.
- The calculation of the state error covariance matrix for t_n is updated from that predicted for t_n :

$$P(t_n) = \{1 - K(t_n) \cdot H\} \cdot P(t_n)_p \quad (3.151)$$

- The prediction from estimated and filtered measures at $(t_n - T)$, done for the calculation of the initial eigenstate at time t_n , could then be made, thanks to the following state-space equation, in the (d_0, q_0) rotating reference frame:

$$X(t_n)_p = F(t_n - T) \cdot X(t_n - T)_f + G(t_n - T) \cdot V_{(t_n - T) \rightarrow t_n} \quad (3.152)$$

This same prediction will be used again as a reference variable for measure filtering at time t_n , as above in a cyclic way.

In the case of synchronous motors, the observation of the rotor flux in magnitude and argument, can be either:

- replaced by the value of the magnet's flux magnitude Φ_a and/or by the measurement of the rotor electrical angular position $\rho_m = N_p \cdot \Xi$, and thus to be used for a more effective filtering of the stator current owing to the estimation adjustment
- kept and used to follow the evolution of the magnet's flux magnitude according to the temperature, and the rotor flux argument according to the rotor position, which makes it possible to remove the rotor flux absolute position encoder. In this case, a measurement of the initial rotor position is necessary in particular when the motor is stopped (Benchai'b *et al.*, 2003).

Equation (3.152) makes it possible to predict the position of the rotor flux vector in the (d_0, q_0) reference frame starting from its position in the (d_m, q_m) reference frame. Using equations (3.123), the rotation $\Delta\rho_m$ of the rotating reference frame between time $(t_n - T)$ and t_n can thus be calculated, as well as the initial state-space vector.

However, it is necessary to preserve a measurement, or an estimation, of the rotor speed, since this information is necessary, in particular, for the computation of evolution matrix coefficients and thus for the whole control computation.

During the computational period of the control, it is in general necessary to carry out other computations, measurements, estimations, regulations and monitoring for the controlled process operation, so that the duration of computations necessary for motor regulation must be a lot shorter than the duration of the impressing of the voltage vector sequences calculated by the control.

3.4.3.5 Summary of Measurement, Filtering and Prediction

We now will summarize section 3.4.3 with regard to the real-time implementation, measure processing and filtering, as well as to the prediction of the initial state-space vector, by the synopsis of Figure 3.8.

These computations are repeated in each PWM period. They start with sampled measurements of currents, input filter voltage, rotor position and/or rotor angular speed; the sampled measurements are synchronized at the beginning of each PWM period, here at time $(t_n - T)$. All computations, including the control computation must be finished before the end of the period T of the PWM. These cyclic computations use part of the computation results of the previous period which are thus stored to be used one period later. To symbolize this memorizing, we use in Figure 3.8 the time-delay operator z^{-1} of one computation period.

When the neutral cable is not connected to the motor, the vector sum of instantaneous currents of the three phases is null, according to Kirchhoff's law. It is thus enough to measure the current of two phases to derive the value of the current in the third phase (cf. Figure 3.4) by the following relation.

$$\vec{I}_a + \vec{I}_b + \vec{I}_c = \vec{0} \quad (3.153)$$

Measurements of instantaneous currents must be made at the beginning of a sampling period by current sensors with a large bandwidth compared to main harmonic frequencies generated by the inverter, in order to obtain the instantaneous peak current of each of the three phases. For example, for a motor supplied by a fundamental voltage with a maximum frequency of 500 Hz, a bandwidth of the current sensor could be from 0 to 500 kHz.

Current measures of the three phases are then converted into one stator current vector with two coordinates in the orthonormal fixed frame by the Concordia transformation which preserves the instantaneous power (cf. equation (1.3)).

The measure of the rotor flux of a synchronous motor is in general not directly accessible. Most synchronous drives use an absolute position sensor of the rotor position to know the approximate rotor flux position, and the theoretical value of the magnet flux magnitude of the rotor flux to reconstitute the vector in its entirety.

This choice presents many disadvantages:

- The measurement of the absolute position is made by a fragile technology of sensor, either one synchroresolver, or one optical coder; the alignment of these small coders on the shaft of one high-power motor, is difficult and can create mechanical resonances.
- It is necessary to very precisely fix the position sensor on the motor shaft and all the more precisely so if the number of motor pole pairs is high; indeed, the higher the number of pole pairs, the more accurately must we be able to discriminate the shaft mechanical angle of the position of each of the pole axes.
- The magnetic field of rotor magnets is influenced by the motor temperature.

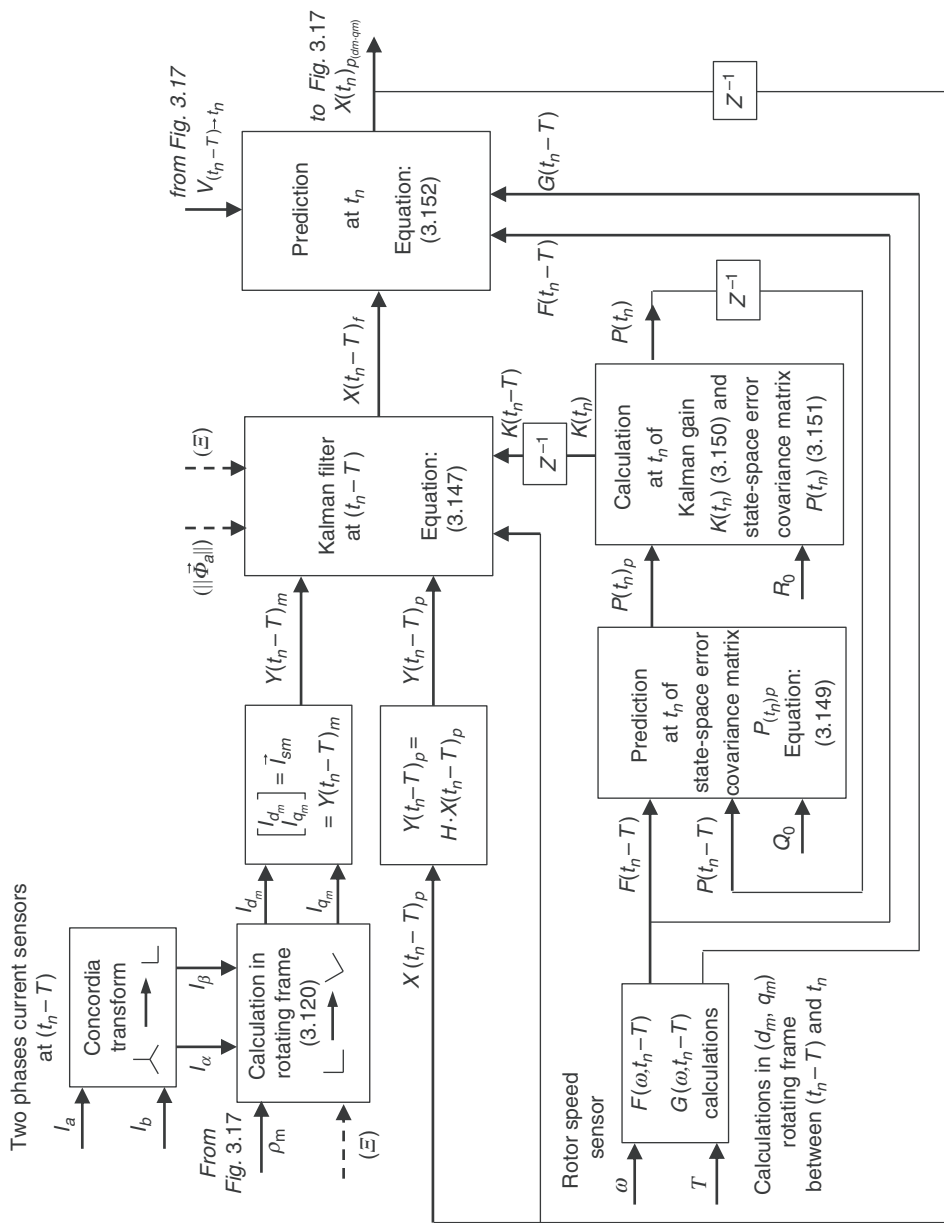


Figure 3.8 Measurements, filtering and prediction

The vector of the rotor flux will thus systematically be estimated from a Kalman filter since this estimation is provided during the current filtering; we will then be able either to preserve the estimation or to replace it by the measured position and/or the magnet's flux magnitude calculated during the process sizing.

The stator current vector measured in the fixed frame is then projected in the (d_m, q_m) rotating reference frame at the time of the measurement. For that, the knowledge of this reference frame angle compared to the fixed frame is necessary (3.120). This angle is either calculated from the prediction, as we will see in Figure 3.17 during the control computation, or measured from the position encoder when it exists.

We now will filter the current vector and estimate the rotor flux using the Kalman filter (3.147). The flux estimation is based on current measures $Y(t_n - T)_m$ and on measure predictions of these same currents $Y(t_n - T)_p$, issued from the prediction of the motor state $X(t_n - T)_p$ at the same time. The motor state prediction is made during the preceding sampling period, according to equation (3.118). It also requires a knowledge of the Kalman gain, of which we will see the computation method below.

Practically, the Kalman observation simultaneously makes it possible to filter the stator current vector in the same operation, thanks to the discretized model of the motor. The filtered state-space vector $X(t_n - T)_f$ includes the filtered stator current vector $\bar{I}_{sf}$ and the estimated rotor flux vector $\bar{\Phi}_{re}$ (3.148). This is the estimated flux which will be used to calculate the control (cf. Figure 3.17). The Kalman filter filters the measure disturbances due to stochastic disturbances of measurement signals, but also the state disturbances related to statistical errors of the motor model.

The filtered state-space vector $X(t_n - T)_f$ starting from current measurements at time $(t_n - T)$, will now allow prediction of the same state-space vector $X(t_n)_p$ at time t_n corresponding to the end of the computational period in progress, thanks to discretized state-space equations of the motor (3.152). This predicted state-space vector constitutes the initial state-space vector allowing calculation of the control vector (cf. Figure 3.17).

This computation requires a knowledge of the control voltage calculated during the preceding computational period (cf. Figure 3.17) and impressed on the motor during the present period by a device which can be different from the computational processor. It requires the computation of discretized transition and input matrices that we will now consider.

We measure the rotor mechanical angular frequency of the motor to convert it into an electrical angular frequency by multiplying by the number of pole pairs. The mechanical angular frequency can be estimated, or measured by an incremental or absolute digital speed sensor. The choice of a digital speed sensor must keep account of the lowest speed requiring accurate regulation, which defines the minimum pulse repetition frequency per rotor mechanical revolution. To increase accuracy at low speed, it is also possible to extrapolate the detection of pulse edges to carry out an estimation from measurement of preceding pulse edges.

Using eigenvalues (3.34) and (3.35), and motor parameters, we calculate the transition matrix, (3.125) to (3.140), and the input matrix (3.141) at time $(t_n - T)$. We then calculate the prediction at time t_n of the state error covariance matrix $P(t_n)_p$ by equation (3.149), from the transition matrix $F(t_n - T)$ and the state error covariance matrix at the present time $P(t_n - T)$. This last matrix was stored during the computation of the previous period.

We use in this computation the state-space disturbance covariance matrix Q_0 ; it is a square 4×4 matrix, identical to the size of the transition matrix; it can be selected as constant to

reduce computations, and its coefficients allow a weighting of the state filtering of each coordinate of the state-space vector. These coefficients can be adapted according to the process or according to the process operating mode.

The prediction of the state error covariance matrix $P(t_n)_p$ and of the measurement disturbance covariance matrix R_0 , makes it possible to then calculate the matrix of the Kalman gain at t_n by equation (3.150), then the state error covariance matrix $P(t_n)$ at time t_n by equation (3.151). These last two computations are stored to feed computations of the Kalman gain and the filtering of the following period.

We use in this last computation the measurement disturbance covariance matrix; its 2×2 size is half the transition matrix size; it can also be selected as constant to reduce computations, and its coefficients allow a weighting of the measure filtering for each coordinate of the current vector. These coefficients can be modified according to the process or to the process operating mode.

Lastly, it should be noted that these computations are performed with matrices with real coefficients because of the rotor magnetic anisotropy of an IPM-SM which does not allow the halving of the matrix size.

With real coefficients, matrix dimensions are as follows:

$$F : 4 \times 4, G : 4 \times 2, H : 2 \times 4, P : 4 \times 4, Q_0 : 4 \times 4, R_0 : 2 \times 2$$

The real-time implementation of Kalman filter algorithms allows a sequential computation of the control throughout each sampling period with, simultaneously:

- the current measure filtering
- the rotor flux estimation
- the motor state-space vector prediction at the end of the computation.

Thanks to this provision, the pure time-delay of the control is equal to only one computational period.

3.4.4 Limitations

The control process which has just been described makes it possible to fix the magnitude and the phase of the stator current set-point to reach the set-point electromagnetic torque in only one sampling period without any limitation of physical variables.

However, the motor voltage fed by the inverter is in general limited. The stator current must always be limited in magnitude, either in an instantaneous way to ensure the inverter commutation and to restrict the heating of the semiconductors, or by the motor to limit the temperature of its stator windings. It is the system sizing which defines these limitations.

As can be seen in Figure 3.9, when the motor speed increases, the back electromotive force (b.e.m.f.) of the motor increases and, beyond the rated motor speed, the rotor flux seen from the stator, cannot be maintained anymore with its maximum value, owing to the fact that the available voltage is maximum; for this reason the maximum torque can no longer be obtained beyond this rated speed without the current increasing. As the current is in general limited, an operation beyond the rated speed requires a limit on the power and the torque at values which decrease with the speed.

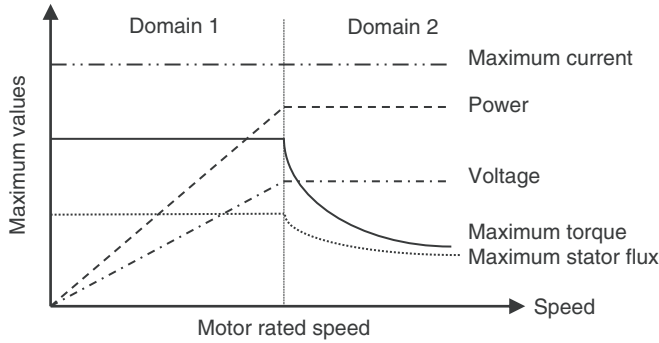


Figure 3.9 Rated speed at maximum torque

With a magnet synchronous motor, unlike an induction machine, the rotor flux is not a controllable variable. The only possible latitude to work beyond the rated motor speed is thus to create a stator flux opposed to the rotor flux to modify the armature reaction by the inductance L_{md} , which makes it possible to reduce the flux and thus to reduce the b.e.m.f. as the speed increases.

The IPM-SM presents another characteristic which is to produce a saliency torque or homopolar torque besides the electromagnetic torque created by permanent magnets. We can highlight that the magnetizing current was not to be null at low speed (cf. Figure 3.6) to allow the decreasing of the stator current magnitude for a given torque set-point; the rotor flux, which is influenced by the stator flux, thus cannot remain constant and equal to the flux of permanent magnets at low speed. The rotor flux thus depends on motor parameters, at low speed as at high speed.

Thanks to the motor control, in only one sampling period, we will now analyze the method for complying, by anticipation, with all limitations due to process sizing.

3.4.4.1 Voltage Limitation

V_M is the maximum value of the *voltage fundamental component* that the inverter can provide, with the Concordia transformation which preserves the instantaneous power; when the PWM is generated at a fixed frequency, the relation which links this voltage to the DC voltage of the input filter U_c is given in a first approximation by the relation $V_M = \frac{U_c}{\sqrt{2}}$ (cf. Figure 3.4).

If we take account of the dead-time t_m corresponding to the minimum non-conduction time of IGBT (turn-on time plus turn-off time), the maximum magnitude of the voltage fundamental component is reduced to:

$$V_M = \frac{U_c}{\sqrt{2}} \cdot \left(1 - 2 \cdot \frac{t_m}{T}\right) \quad (3.154)$$

With square wave PWM, the maximum magnitude of the *voltage fundamental component*, which can be provided by the voltage inverter with a variable frequency PWM, is given by

$V_M = \frac{\sqrt{6}}{\pi} \cdot U_c$ and the maximum instantaneous voltage corresponding to the magnitude of the

six non-null voltage vectors of the voltage inverter is given by $V_M = \sqrt{\frac{2}{3}} \cdot U_c$ (cf. Figure 3.4).

When we calculate the maximum flux, which can be obtained from the fundamental voltage of the inverter with an asynchronous PWM at a fixed frequency, we choose the maximum magnitude of the fundamental voltage corrected by dead-times; this magnitude is given by equation (3.154).

So the stator voltage must always be lower than V_M , a constraint that can be written in the (d, q) reference frame by the sum of squared magnitudes of the two voltage coordinates:

$$V_{sd}^2 + V_{sq}^2 \leq V_M^2 \quad (3.155)$$

We then project equations (3.1) and (3.2) rewritten for current and flux *fundamental* components on the (d, q) reference frame axes. It will be supposed that the average stator angular frequency over one period is roughly equal to the polar mechanical angular frequency $\omega_s \cong \omega$ when the motor state is stabilized; during one period, amplitudes of electrical variables do not depend on the time.

The derivative of the air gap flux given by equation (3.2) and projected in the rotating reference frame is written:

$$\begin{aligned} \frac{d\Phi_{ed}}{dt} &= -\omega \cdot L_{mq} \cdot I_{sq} - \omega \cdot \Phi_{aq} \\ \frac{d\Phi_{eq}}{dt} &= \omega \cdot L_{md} \cdot I_{sd} + \omega \cdot \Phi_{ad} \end{aligned} \quad (3.156)$$

and the fundamental voltage vector of the motor for a stabilized state:

$$V_{sd} = R_s \cdot I_{sd} - L_{sq} \cdot \omega \cdot I_{sq} - \omega \cdot \Phi_{aq} \quad (3.157)$$

$$V_{sq} = R_s \cdot I_{sq} + L_{sd} \cdot \omega \cdot I_{sd} + \omega \cdot \Phi_{ad} \quad (3.158)$$

These relations are then used to define the voltage limit:

$$\left(R_s \cdot I_{sd} - L_{sq} \cdot \omega \cdot I_{sq} \right)^2 + \left(R_s \cdot I_{sq} + L_{sd} \cdot \omega \cdot I_{sd} + \omega \cdot \Phi_a \right)^2 \leq V_M^2 \quad (3.159)$$

since $\Phi_{aq} \equiv 0$ and $\Phi_{ad} = \|\tilde{\Phi}_a\| = \Phi_a$.

In particular, (3.159) is verified in the (d, q) predicted reference frame, where stator current set-points are defined:

$$\left(R_s \cdot I_{dc} - L_{sq} \cdot \omega \cdot I_{qc} \right)^2 + \left(R_s \cdot I_{qc} + L_{sd} \cdot \omega \cdot I_{dc} + \omega \cdot \Phi_a \right)^2 \leq V_M^2 \quad (3.160)$$

First, we will write condition (3.160) in the case where the voltage resistive drops can be neglected. This calculation will be further carried out in the general case, with a stabilized sinusoidal operation.

3.4.4.1.1 Negligible Ohmic Drops

If we can neglect ohmic losses ($R_s \equiv 0$) compared to inductive voltage drops, we can simplify equation (3.160) (Bianchi and Bolognani, 1994)

$$\left(L_{sq} \cdot \omega \cdot I_{qc} \right)^2 + \left(L_{sd} \cdot \omega \cdot I_{dc} + \omega \cdot \Phi_a \right)^2 \leq V_M^2 \quad (3.161)$$

so that we can still write:

$$\frac{\left(I_{dc} + \frac{\Phi_a}{L_{sd}}\right)^2}{L_{sq}^2} + \frac{I_{qc}^2}{L_{sd}^2} \leq \frac{V_M^2}{L_{sd}^2 \cdot L_{sq}^2 \cdot \omega^2} \quad (3.162)$$

The locus of the set-point current vector extremity (I_{dc}, I_{qc}) , complying with the voltage limitation in a stabilized operation, is an ellipse of center:

$$\left(-\frac{\Phi_a}{L_{sd}}, 0\right) \quad (3.163)$$

The abscissa of this center is a constant value, always negative whatever the motor saliency type. Both half axes $a = \frac{V_M}{L_{sd} \cdot \omega}$ and $b = \frac{V_M}{L_{sq} \cdot \omega}$ tend towards zero when the speed increases, but they tend towards infinity when the angular frequency is null, which can be explained by the fact that the resistance is regarded here as null; thus there is no counter-voltage due to motor impedances supplied by the DC current at null speed, and thus the motor current is not limited.

The center of the voltage limitation circle of the SMPM-SM has the same ordinate when we equate to zero the stator resistance in equation (2.126). In addition, for this last type of synchronous motor $L_{sd} = L_{sq}$, which implies $a = b$ and that the ellipse eccentricity is null; we thus find the results (2.125) and (2.126) with a null stator resistance.

We will see in the general case that these results are quite different when the stator resistance is not null.

3.4.4.1.2 General Case of Voltage Limitation

We will study the case where stator resistances are not negligible anymore.

Let us change the reference frame by a simple rotation of the initial (d, q) reference frame through an angle η , as presented in Figure 3.10. Old coordinates of the current reference vector are written according to new coordinates and to the angle η by equation (3.164).

$$\begin{bmatrix} I_{dc} \\ I_{qc} \end{bmatrix} = \begin{bmatrix} \cos(\eta) & -\sin(\eta) \\ \sin(\eta) & \cos(\eta) \end{bmatrix} \cdot \begin{bmatrix} I_{Dc} \\ I_{Qc} \end{bmatrix} \quad (3.164)$$

Let us change the frame of coordinates of inequality (3.162):

$$\begin{aligned} & \left[\{R_s \cdot \cos(\eta) - L_{sq} \cdot \omega \cdot \sin(\eta)\} \cdot I_{Dc} - \{R_s \cdot \sin(\eta) + L_{sq} \cdot \omega \cdot \cos(\eta)\} \cdot I_{Qc} \right]^2 + \dots \\ & \dots \left[\{R_s \cdot \sin(\eta) + L_{sd} \cdot \omega \cdot \cos(\eta)\} \cdot I_{Dc} + \{R_s \cdot \cos(\eta) - L_{sd} \cdot \omega \cdot \sin(\eta)\} \cdot I_{Qc} + \omega \cdot \Phi_a \right]^2 \leq V_M^2 \end{aligned} \quad (3.165)$$

A necessary and sufficient condition, so that (D, Q) axes are axes of one conic, is that the coefficient of the double product $I_{Dc} \cdot I_{Qc}$ was equated to zero:

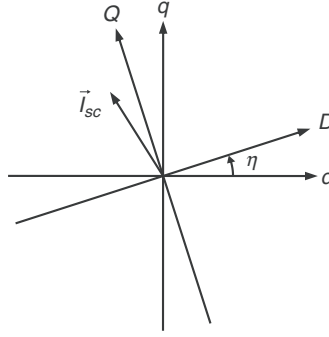


Figure 3.10 Frame change for the voltage limit calculation

$$\begin{aligned} & \{R_s \cdot \cos(\eta) - L_{sq} \cdot \omega \cdot \sin(\eta)\} \cdot \{R_s \cdot \sin(\eta) + L_{sq} \cdot \omega \cdot \cos(\eta)\} - \dots \\ & \dots \{R_s \cdot \sin(\eta) + L_{sd} \cdot \omega \cdot \cos(\eta)\} \cdot \{R_s \cdot \cos(\eta) - L_{sd} \cdot \omega \cdot \sin(\eta)\} = 0 \end{aligned} \quad (3.166)$$

This equation is solved simply with the tangent of the double angle:

$$\operatorname{tg}(2 \cdot \eta) = \frac{2 \cdot R_s}{(L_{sd} + L_{sq}) \cdot \omega} \quad (3.167)$$

The angle η defines the angle of the conic D axis compared to the (d, q) reference frame. We notice that for $R_s \cong 0$, this angle is null, in accordance with equation (3.162). This angle varies with speed; its value is $\frac{\pi}{4}$ at null speed, and it tends towards zero at high speed.

Now we will find the conic center. For that, let us define initially the following reduced variables:

$$\begin{aligned} Z_a &= R_s \cdot \cos(\eta) - L_{sq} \cdot \omega \cdot \sin(\eta) \\ Z_b &= R_s \cdot \sin(\eta) + L_{sq} \cdot \omega \cdot \cos(\eta) \\ Z_c &= R_s \cdot \sin(\eta) + L_{sd} \cdot \omega \cdot \cos(\eta) \\ Z_d &= R_s \cdot \cos(\eta) - L_{sd} \cdot \omega \cdot \sin(\eta) \end{aligned} \quad (3.168)$$

A simplified form is then obtained:

$$\left[Z_a \cdot I_{Dc} - Z_b \cdot I_{Qc} \right]^2 + \left[Z_c \cdot I_{Dc} + Z_d \cdot I_{Qc} + \omega \cdot \Phi_a \right]^2 \leq V_M^2 \quad (3.169)$$

and condition (3.166), becomes:

$$Z_a \cdot Z_b - Z_c \cdot Z_d = 0 \quad (3.170)$$

The development of inequality (3.169), using condition (3.170), makes it possible to cancel crossed terms, and after some simple formal calculations the inequality appears in the following form:

$$\frac{\left(I_{Dc} + \omega \cdot \Phi_a \cdot \frac{Z_c}{Z_a^2 + Z_c^2}\right)^2}{Z_b^2 + Z_d^2} + \frac{\left(I_{Qc} + \omega \cdot \Phi_a \cdot \frac{Z_d}{Z_b^2 + Z_d^2}\right)^2}{Z_a^2 + Z_c^2} \leq \frac{V_M^2}{(Z_b^2 + Z_d^2) \cdot (Z_a^2 + Z_c^2)} \quad (3.171)$$

The conic representing the limit of the second degree inequality (3.169) is thus an ellipse of which we will determine the characteristics.

In the (D, Q) frame, the authorized domain is thus bordered by an ellipse of center:

$$(I_{0D}, I_{0Q}) = \left(-\omega \cdot \Phi_a \cdot \frac{Z_c}{Z_a^2 + Z_c^2}, -\omega \cdot \Phi_a \cdot \frac{Z_d}{Z_b^2 + Z_d^2} \right) \quad (3.172)$$

and of half axes (cf. Figure 3.11):

$$a = \frac{V_M}{\sqrt{Z_a^2 + Z_c^2}} \quad \text{and} \quad b = \frac{V_M}{\sqrt{Z_b^2 + Z_d^2}} \quad (3.173)$$

At high speed, higher than the rated motor speed, it is relatively simple to find an equivalent of the expressions of half axes $a \approx \frac{V_M}{L_{sd} \cdot \omega}$ and $b \approx \frac{V_M}{L_{sq} \cdot \omega}$, since $\eta \approx 0$ according to (3.167), and reactances are somewhat higher than the stator resistance in general beyond this speed. We can conclude that both half axes tend towards 0, and that a is one half of the long axis for a motor with a reverse saliency $L_{sd} < L_{sq}$, but that it is one half of short axis for a motor with a direct saliency.

We also demonstrate, with equivalents at an increased speed, that the coordinates of the ellipse center tend towards $\left(-\frac{\Phi_a}{L_{sd}}, 0\right)$, which is the ellipse center when we regard the stator resistance as null, which is consistent with the fact that the stator resistance becomes negligible compared to the reactances at high speed.

At null speed, it is easy to show, with equations (3.167), (3.168) and (3.172), that the ellipse center is the reference frame origin and that both half axes (3.173) are equal to $\frac{V_M}{R_s}$, which is explained by the fact that the motor, supplied with a DC maximum voltage V_M at stop, limits its stator current only by its resistance.

We notice here that for an SMPM-SM, $L_{sd} = L_{sq} = L_s$, which leads to $Z_a = Z_d$ and $Z_b = Z_c$, and finally to $a = b$. In this case, as we saw in chapter 2, the limit of the current domain when motor voltage is maximum is a circle. We found the center coordinates (equation (2.127)) by using equation (3.172) with, in this case, $Z_a^2 + Z_c^2 = Z_b^2 + Z_d^2 = Z_s^2$, $\eta = 0$, $Z_c = L_s \cdot \omega$ and $Z_d = R_s$.

3.4.4.1.3 Locus of Ellipse Points Tangential to the Torque Hyperbola

In the case of a permanent voltage limitation, it is interesting to find the coordinates of ellipse points for which the torque is maximum.

For that, it is necessary to find coordinates of the intersection of the hyperbola with the ellipse in Figure 3.11, then to cancel the torque derivative compared to coordinates of the locus of these points. It is preferable to convert Cartesian coordinates into parametric coordinates for this operation.

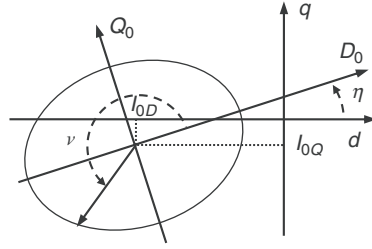


Figure 3.11 Frame for the ellipse of the permanent voltage limit

By taking, as a parameter, the angle ν of polar coordinates of ellipse points in the new frame of origin coordinates (I_{0D}, I_{0Q}) , which are also the coordinates of the ellipse center, the ellipse equation in the (d, q) reference frame becomes:

$$\begin{bmatrix} I_{dc} \\ I_{qc} \end{bmatrix} = \begin{bmatrix} I_{0D} \\ I_{0Q} \end{bmatrix} + \begin{bmatrix} \cos(\eta) & -\sin(\eta) \\ \sin(\eta) & \cos(\eta) \end{bmatrix} \cdot \begin{bmatrix} a \cdot \cos(\nu) \\ b \cdot \sin(\nu) \end{bmatrix} \quad (3.174)$$

Let us define new intermediate variables as follows:

$$\begin{aligned} I_{ac} &= a \cdot \cos(\eta) \\ I_{as} &= a \cdot \sin(\eta) \\ I_{bc} &= b \cdot \cos(\eta) \\ I_{bs} &= b \cdot \sin(\eta) \end{aligned} \quad (3.175)$$

The ellipse parametric equation in the (d, q) reference frame becomes:

$$\begin{bmatrix} I_{dc} \\ I_{qc} \end{bmatrix} = \begin{bmatrix} I_{0D} \\ I_{0Q} \end{bmatrix} + \begin{bmatrix} I_{ac} \cdot \cos(\nu) - I_{bs} \cdot \sin(\nu) \\ I_{as} \cdot \cos(\nu) + I_{bc} \cdot \sin(\nu) \end{bmatrix} \quad (3.176)$$

We can now transfer the ellipse parametric coordinates in the equation of the set-point torque (3.20) to find the torque value obtained at the intersection of the hyperbola with the ellipse, for a given value of the maximum voltage, according to the parameter ν :

$$\begin{aligned} C_c &= N_p \cdot \left[(L_{md} - L_{mq}) \cdot \{ I_{0D} + I_{ac} \cdot \cos(\nu) - I_{bs} \cdot \sin(\nu) \} + \Phi_a \right] \cdot \dots \\ &\dots \{ I_{0Q} + I_{as} \cdot \cos(\nu) + I_{bc} \cdot \sin(\nu) \} \end{aligned} \quad (3.177)$$

Clarifying the product of the two main factors, we obtain:

$$\begin{aligned} \frac{C_c(\nu)}{N_p} &= (L_{md} - L_{mq}) \cdot \dots \\ &\dots \left[I_{0D} \cdot I_{0Q} + (I_{0Q} \cdot I_{ac} + I_{0D} \cdot I_{as}) \cdot \cos(\nu) + (I_{0D} \cdot I_{bc} - I_{0Q} \cdot I_{bs}) \cdot \sin(\nu) + \dots \right] + \dots \\ &\dots \left[\dots (I_{bc} \cdot I_{ac} - I_{as} \cdot I_{bs}) \cdot \sin(\nu) \cdot \cos(\nu) + I_{as} \cdot I_{ac} \cdot \cos^2(\nu) - I_{bc} \cdot I_{bs} \cdot \sin^2(\nu) \right] \\ &\dots \Phi_a \cdot [I_{0Q} + I_{as} \cdot \cos(\nu) + I_{bc} \cdot \sin(\nu)] \end{aligned} \quad (3.178)$$

The maximization of the torque value corresponds to the solution which cancels the partial torque derivative according to the parameter ν :

$$\frac{\delta \left(\frac{C_c(\nu)}{N_p} \right)}{\delta \nu} = 0 \quad (3.179)$$

Let us calculate the partial derivative and equate it to zero:

$$\begin{aligned} & - \left(I_{0Q} \cdot I_{ac} + I_{0D} \cdot I_{as} + \frac{I_{as} \cdot \Phi_a}{L_{md} - L_{mq}} \right) \cdot \sin(\nu) + \dots \\ & \dots \left(I_{0D} \cdot I_{bc} - I_{0Q} \cdot I_{bs} + \frac{I_{bc} \cdot \Phi_a}{L_{md} - L_{mq}} \right) \cdot \cos(\nu) + \dots \\ & \dots (I_{bc} \cdot I_{ac} - I_{as} \cdot I_{bs}) \cdot [\cos^2(\nu) - \sin^2(\nu)] - \dots \\ & \dots 2 \cdot (I_{as} \cdot I_{ac} + I_{bc} \cdot I_{bs}) \cdot \sin(\nu) \cdot \cos(\nu) = 0 \end{aligned} \quad (3.180)$$

Equation (3.180) is solvable with the tangent of the half angle. Let us define the following reduced variables:

$$t = \tan\left(\frac{\nu}{2}\right) \quad (3.181)$$

and:

$$\begin{aligned} A &= I_{ac} \cdot I_{bc} - I_{as} \cdot I_{bs} \\ B &= I_{ac} \cdot I_{as} + I_{bc} \cdot I_{bs} \\ C &= I_{0D} \cdot I_{bc} - I_{0Q} \cdot I_{bs} + \frac{I_{bc} \cdot \Phi_a}{L_{md} - L_{mq}} \\ D &= I_{0Q} \cdot I_{ac} + I_{0D} \cdot I_{as} + \frac{I_{as} \cdot \Phi_a}{L_{md} - L_{mq}} \end{aligned} \quad (3.182)$$

The solutions of t , and thus of ν , are ultimately given by the equation of the fourth degree:

$$(A - C) \cdot t^4 + 2 \cdot (2B - D) \cdot t^3 - 6A \cdot t^2 - 2 \cdot (2B + D) \cdot t + (A + C) = 0 \quad (3.183)$$

Figure 3.12 illustrates the solution in traction operation for a reverse saliency motor and without current limitation. The three other solutions of equation (3.183) correspond to braking operation for a reverse saliency motor, and also to traction and braking operations for a direct saliency motor. Figure 3.6 makes it possible to discriminate easily these four solutions.

Knowledge of ν makes it possible to know the corresponding coordinates of the current vector directly in the (d, q) reference frame, by equation (3.174).

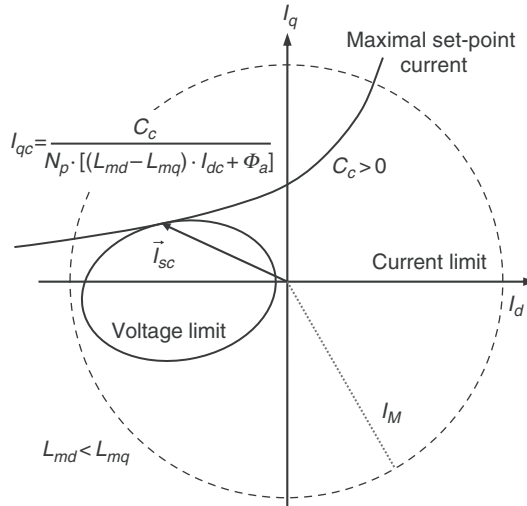


Figure 3.12 Maximum torque with a voltage limitation

3.4.4.2 Current Limitation

The stator current limitation is a magnitude limitation with a definite maximum value for the inverter and the motor, arising from their sizing:

$$I_s^2 = I_{sd}^2 + I_{sq}^2 \leq I_M^2 \quad (3.184)$$

In the (d, q) plane the current limit is a circle of radius I_M and of center the reference frame origin, as represented in Figure 3.13.

On the limit, the intersection of hyperbola families with the circle of the current limit is given by the combination of torque equations (3.106) and of the stator current limit estimated by equation (3.184); we then calculate the set-point current abscissa for a set-point torque corresponding to the maximum current by the relation:

$$I_{dc}^2 + \left[\frac{C_c}{N_p \cdot \{(L_{md} - L_{mq}) \cdot I_{dc} + \Phi_a\}} \right]^2 = I_M^2 \quad (3.185)$$

The four solutions of this equation provide four solutions for the control current vector, two for a positive torque according to the motor type and two symmetrical compared to the I_d axis for a negative torque. However, these solutions do not take account of the optimization of the current magnitude calculated by (3.109), nor of a possible voltage limitation.

We can now visualize, in Figure 3.13, two solutions, one in the particular case of a current limitation with a voltage limitation, the other leading to too high a motor voltage for an available feeding voltage.

The maximum torque that it is possible to reach with just the current limitation can now be calculated by initially calculating the abscissa of the set-point current satisfying at the same time:

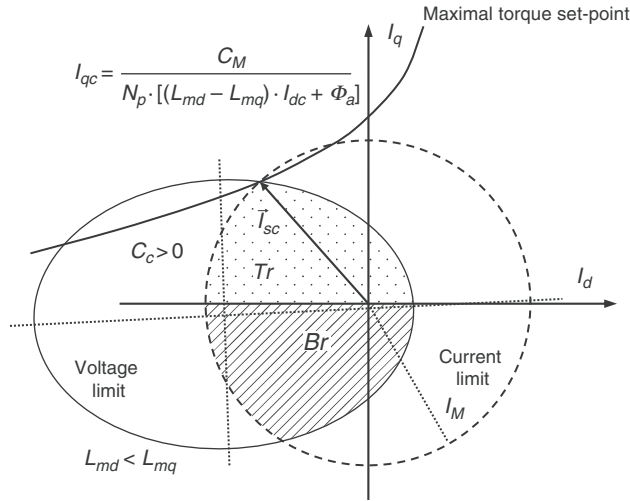


Figure 3.13 Current and voltage limits

- equation (3.185) of the intersection of the circle of which the radius corresponds to the maximum current, with the hyperbola, and
- the equation giving the maximum torque for a fixed current magnitude, with equation (3.109).

Let us eliminate the set-point torque from these two equations:

$$\begin{aligned}
 I_{dc}^4 + 3 \cdot \frac{\Phi_a}{L_{md} - L_{mq}} \cdot I_{dc}^3 + 3 \cdot \left(\frac{\Phi_a}{L_{md} - L_{mq}} \right)^2 \cdot I_{dc}^2 + \dots \\
 \dots \left(\frac{\Phi_a}{L_{md} - L_{mq}} \right)^3 \cdot I_{dc} - \frac{(I_M^2 - I_{dc}^2) \cdot [(L_{md} - L_{mq}) \cdot I_{dc} + \Phi_a]^2}{(L_{md} - L_{mq})^2} = 0
 \end{aligned} \quad (3.186)$$

while developing:

$$\begin{aligned}
 2 \cdot I_{dc}^4 + \frac{5 \cdot \Phi_a}{L_{md} - L_{mq}} \cdot I_{dc}^3 + \left[\frac{4 \cdot \Phi_a^2}{(L_{md} - L_{mq})^2} - I_M^2 \right] \cdot I_{dc}^2 + \dots \\
 \dots \frac{\Phi_a}{(L_{md} - L_{mq})} \cdot \left[\frac{\Phi_a^2}{(L_{md} - L_{mq})^2} - 2 \cdot I_M^2 \right] \cdot I_{dc} - \frac{\Phi_a^2 \cdot I_M^2}{(L_{md} - L_{mq})^2} = 0
 \end{aligned} \quad (3.187)$$

This equation gives the abscissas of the four tangential points between the four hyperbolas of the maximum torque in traction and braking operation, according to both the type of the motor saliency and the circle of the current limit. One of the four points is represented by *M* in Figure 3.6, for traction operation and for the reverse saliency motor.

The ordinates are calculated by equation (3.184), with the following equality:

$$I_{qc}^2 = I_M^2 - I_{dc}^2 \quad (3.188)$$

The maximum torque is estimated by equation (3.106), from solutions of equation (3.187).

$$C_c^2 = N_p^2 \cdot (I_M^2 - I_{dc}^2) \cdot \left[(L_{md} - L_{mq}) \cdot I_{dc} + \Phi_a \right]^2 \quad (3.189)$$

3.4.4.3 Operating Area and Limits

In short, the extremity of the set-point current vector $\vec{I}_{sc}$ in the (d, q) reference frame must be:

- inside the ellipse of the voltage limit defined by (3.171)
- inside the circle of the current limit defined by (3.184), according to Figure 3.13, and
- on the hyperbola defining the set-point torque, limited or not by the voltage and/or the current.

The authorized area, for traction (Tr with $I_{qc} \geq 0$) and electrical braking (Br with $I_{qc} < 0$) operations, results from the intersection of the ellipse of the voltage limit and the circle of the current limit in Figure 3.13, with a negative magnetizing current $I_{dc} \leq 0$ for a reverse saliency motor as represented on this figure, and positive or negative for a direct saliency motor. The separation between traction and braking operation areas is made according to the sign of the set-point current ordinate I_q .

The extremity of the stator current vector $\vec{I}_{sc}$ represented in the (d, q) reference frame in Figure 3.13 is located simultaneously on the ellipse of the voltage limit and on the circle of the current limit. This extremity is on the hyperbola defining the maximum electromagnetic torque that is possible to reach under these limitation conditions, in traction operation and for a motor with a reverse saliency.

3.4.4.4 Set-Point Limit Calculation

In the (I_d, I_q) plane, we thus established equations of the torque hyperbola family, of voltage limitation ellipses, of current limitation circles and of optimal torque trajectory coordinates (I_{dc}, I_{qc}) leading to the minimum current magnitude for a given set-point torque. We analyzed the following situations:

- no limitation and optimization of the current magnitude (Figure 3.6)
- current limitation only (Figure 3.6)
- voltage limitation only (Figure 3.12)
- voltage and current limitations (Figure 3.13).

We also calculated the set-point current in the two first cases. It is now necessary to calculate the set-point currents in the cases of both voltage limitation only and voltage and current limitations.

3.4.4.4.1 Voltage Limit

In the equation of the voltage limit (3.159) we replace the current ordinate by its expression according to the current and the torque abscissa (3.106):

$$\begin{aligned} & \left(R_s \cdot I_{dc} - \frac{L_{sq} \cdot \omega \cdot C_c}{N_p \cdot \{(L_{md} - L_{mq}) \cdot I_{dc} + \Phi_a\}} \right)^2 + \dots \\ & \dots \left(\frac{R_s \cdot C_c}{N_p \cdot \{(L_{md} - L_{mq}) \cdot I_{dc} + \Phi_a\}} + L_{sd} \cdot \omega \cdot I_{dc} + \omega \cdot \Phi_a \right)^2 = V_M^2 \end{aligned} \quad (3.190)$$

After development and arranging powers of I_{dc} in descending order we have:

$$\begin{aligned} & (L_{md} - L_{mq})^2 \cdot (R_s^2 + L_{sd}^2 \cdot \omega^2) \cdot I_{dc}^4 + \dots \\ & \dots 2 \cdot \Phi_a \cdot (L_{md} - L_{mq}) \cdot \{R_s^2 + \omega^2 \cdot L_{sd} \cdot [(L_{md} - L_{mq}) + L_{sd}]\} \cdot I_{dc}^3 + \dots \\ & \dots \left\{ \begin{aligned} & 2 \cdot R_s \cdot C_c / N_p \cdot \omega \cdot (L_{md} - L_{mq}) \cdot (L_{sd} - L_{sq}) + \dots \\ & \dots \omega^2 \cdot \Phi_a^2 \cdot [(L_{md} - L_{mq})^2 + L_{sd}^2 + 4 \cdot (L_{md} - L_{mq}) \cdot L_{sd}] + \dots \end{aligned} \right\} \cdot I_{dc}^2 + \dots \\ & \dots \left\{ \begin{aligned} & \dots R_s^2 \cdot \Phi_a^2 - V_M^2 \cdot (L_{md} - L_{mq})^2 \\ & \dots 2 \cdot \Phi_a \cdot \left\{ \begin{aligned} & R_s \cdot C_c / N_p \cdot \omega \cdot [(L_{md} - L_{mq}) + L_{sd} - L_{sq}] + \dots \\ & \dots \omega^2 \cdot \Phi_a^2 \cdot [(L_{md} - L_{mq}) + L_{sd}] - V_M^2 \cdot (L_{md} - L_{mq}) \end{aligned} \right\} \end{aligned} \right\} \cdot I_{dc} + \dots \\ & \dots (R_s \cdot C_c / N_p + \omega \cdot \Phi_a^2)^2 + \omega^2 \cdot (C_c / N_p)^2 \cdot L_{sq}^2 - V_M^2 \cdot \Phi_a^2 = 0 \end{aligned} \quad (3.191)$$

One possible root of the fourth-degree polynomial (3.191) provides the abscissa of the current set-point, and thus the sought set-point torque, imposing the set-point current ordinate calculated by equation (3.160) (I_{dc}, I_{qc}). One eliminates the three other foreign roots.

It should then be checked that:

$$I_{dc}^2 + I_{qc}^2 \leq I_M^2 \quad (3.192)$$

3.4.4.4.2 Current and Voltage Limitations

If the current limit is reached with a voltage limitation, it is necessary to reduce the set-point torque to satisfy simultaneously the two limitations.

For that, let us use the relation which gives the set-point torque at the maximum current as a function of the abscissa of the set-point current (3.189):

$$(C_c / N_p)^2 = (I_M^2 - I_{dc}^2) \cdot [(L_{md} - L_{mq}) \cdot I_{dc} + \Phi_a]^2 \quad (3.193)$$

and let us replace its squared value in the equation giving the current abscissa in the event of a voltage limitation only (3.191). We will be able to thus preserve only the factors of the term C_c/N_p which is not squared and cannot thus be overridden.

After arranging and factorizing the fourth-degree polynomial in I_{dc} , we obtain a product of two second-degree polynomials as a factor of ω^2 :

$$\begin{aligned} \omega^2 \cdot \left\{ \begin{array}{l} (L_{md} - L_{mq}) \cdot (L_{sd} - L_{sq}) \cdot I_{dc}^2 + \dots \\ \dots \Phi_a \cdot [(L_{md} - L_{mq}) + L_{sd} - L_{sq}] \cdot I_{dc} + \dots \\ \dots \Phi_a^2 \end{array} \right\} \cdot \left\{ \begin{array}{l} (L_{md} - L_{mq}) \cdot (L_{sd} + L_{sq}) \cdot I_{dc}^2 + \dots \\ \dots \Phi_a \cdot [(L_{md} - L_{mq}) + L_{sd} + L_{sq}] \cdot I_{dc} + \dots \\ \dots \Phi_a^2 \end{array} \right\} + \dots \\ \dots \left[I_M^2 \cdot (R_s^2 + \omega^2 \cdot L_{sq}^2) - V_M^2 \right] \cdot [(L_{md} - L_{mq}) \cdot I_{dc} + \Phi_a]^2 + \dots \\ \dots 2 \cdot \omega \cdot R_s \cdot C_c / N_p \cdot \left\{ \begin{array}{l} (L_{md} - L_{mq}) \cdot (L_{sd} - L_{sq}) \cdot I_{dc}^2 + \dots \\ \dots \Phi_a \cdot [(L_{md} - L_{mq}) + L_{sd} - L_{sq}] \cdot I_{dc} + \dots \\ \dots \Phi_a^2 \end{array} \right\} \leq 0 \end{aligned} \quad (3.194)$$

The first second-degree polynomial which is also a factor of C_c/N_p can be written:

$$\left\{ \begin{array}{l} (L_{md} - L_{mq}) \cdot (L_{sd} - L_{sq}) \cdot I_{dc}^2 + \dots \\ \dots \Phi_a \cdot [(L_{md} - L_{mq}) + L_{sd} - L_{sq}] \cdot I_{dc} + \dots \\ \dots \Phi_a^2 \end{array} \right\} = [(L_{md} - L_{mq}) \cdot I_{dc} + \Phi_a] \cdot [(L_{sd} - L_{sq}) \cdot I_{dc} + \Phi_a] \quad (3.195)$$

In the same way, the second polynomial of the second degree, in factor of ω^2 , can be factorized:

$$\left\{ \begin{array}{l} (L_{md} - L_{mq}) \cdot (L_{sd} + L_{sq}) \cdot I_{dc}^2 + \dots \\ \dots \Phi_a \cdot [(L_{md} - L_{mq}) + L_{sd} + L_{sq}] \cdot I_{dc} + \dots \\ \dots \Phi_a^2 \end{array} \right\} = [(L_{md} - L_{mq}) \cdot I_{dc} + \Phi_a] \cdot [(L_{sd} + L_{sq}) \cdot I_{dc} + \Phi_a] \quad (3.196)$$

The double limitation can be written more simply:

$$\begin{aligned} \omega^2 \cdot [(L_{md} - L_{mq}) \cdot I_{dc} + \Phi_a]^2 \cdot [(L_{sd} - L_{sq}) \cdot I_{dc} + \Phi_a] \cdot [(L_{sd} + L_{sq}) \cdot I_{dc} + \Phi_a] + \dots \\ \dots \left[I_M^2 \cdot (R_s^2 + \omega^2 \cdot L_{sq}^2) - V_M^2 \right] \cdot [(L_{md} - L_{mq}) \cdot I_{dc} + \Phi_a]^2 + \dots \\ \dots 2 \cdot \omega \cdot R_s \cdot C_c / N_p \cdot [(L_{md} - L_{mq}) \cdot I_{dc} + \Phi_a] \cdot [(L_{sd} - L_{sq}) \cdot I_{dc} + \Phi_a] \leq 0 \end{aligned} \quad (3.197)$$

We then divide the preceding expression by the polynomial (3.198), strictly positive, in the case of a reverse saliency, and also positive in the case of a direct saliency; indeed, torque due to magnets is in general higher than the reluctance torque. The direction of the inequality thus does not reverse.

$$(L_{md} - L_{mq}) \cdot I_{dc} + \Phi_a \quad (3.198)$$

We thus obtain, in the limit:

$$\begin{aligned} & \left[(L_{md} - L_{mq}) \cdot I_{dc} + \Phi_a \right] \cdot \dots \\ & \dots \left[\omega^2 \cdot \left[(L_{sd} - L_{sq}) \cdot I_{dc} + \Phi_a \right] \cdot \left[(L_{sd} + L_{sq}) \cdot I_{dc} + \Phi_a \right] + \dots \right] = \dots \quad (3.199) \\ & \dots \left[\dots I_M^2 \cdot (R_s^2 + \omega^2 \cdot L_{sq}^2) - V_M^2 \right. \\ & \left. \dots - 2 \cdot \omega \cdot R_s \cdot C_c / N_p \cdot \left[(L_{sd} - L_{sq}) \cdot I_{dc} + \Phi_a \right] \right] \end{aligned}$$

We then raise to the second power, the two terms, and again we replace $(C_c / N_p)^2$ by expression (3.193).

All calculations done, after arranging polynomial terms according to the decreasing powers of I_{dc} :

$$\begin{aligned} & \omega^2 \cdot \left[(L_{sd}^2 - L_{sq}^2) \cdot \omega^2 + 4 \cdot R_s^2 \cdot (L_{sd} - L_{sq})^2 \right] \cdot I_{dc}^4 + \dots \\ & 4 \cdot \omega^2 \cdot \Phi_a \cdot \left[L_{sd} \cdot \omega^2 \cdot (L_{sd}^2 - L_{sq}^2) + 2 \cdot R_s^2 \cdot (L_{sd} - L_{sq}) \right] \cdot I_{dc}^3 + \dots \\ & \dots 2 \cdot \omega^2 \cdot \left\{ 2 \cdot \Phi_a^2 \cdot (R_s^2 + L_{sq}^2 \cdot \omega^2) - 2 \cdot R_s^2 \cdot I_M^2 \cdot (L_{sd} - L_{sq})^2 + \dots \right\} \cdot I_{dc}^2 + \dots \\ & \dots 4 \cdot \omega^2 \cdot \Phi_a \cdot \left\{ L_{sd} \cdot \left[\omega^2 \cdot \Phi_a^2 + I_M^2 \cdot (R_s^2 + L_{sq}^2 \cdot \omega^2) - V_M^2 \right] - 2 \cdot R_s^2 \cdot I_M^2 \cdot (L_{sd} - L_{sq}) \right\} \cdot I_{dc} + \dots \\ & \dots \left[\omega^2 \cdot \Phi_a^2 + I_M^2 \cdot (R_s^2 + L_{sq}^2 \cdot \omega^2) - V_M^2 \right]^2 - 4 \cdot \omega^2 \cdot R_s^2 \cdot I_M^2 \cdot \Phi_a^2 = 0 \quad (3.200) \end{aligned}$$

Thanks to simplifications made before squaring, the resulting polynomial is of fourth degree instead of eighth degree. The resolution of the fourth degree equation by a traditional process provides at least an acceptable root if the problem is *consistent with the process sizing*.

The value of I_{dc} , solution of equation (3.200), makes it possible to calculate the torque set-point limited by the double limitation of voltage and current:

$$(C_M / N_p)^2 = (I_M^2 - I_{dc}^2) \cdot \left[(L_{md} - L_{mq}) \cdot I_{dc} + \Phi_a \right]^2 \quad (3.201)$$

and finally, the ordinate of the current set-point:

$$I_{qc} = \frac{C_M / N_p}{(L_{md} - L_{mq}) \cdot I_{dc} + \Phi_a} \quad (3.202)$$

3.4.4.4.3 Various Configurations Summary

Various configurations of limitations are summarized in Figure 3.14, in the case of the traction operation, and for a reverse saliency motor.

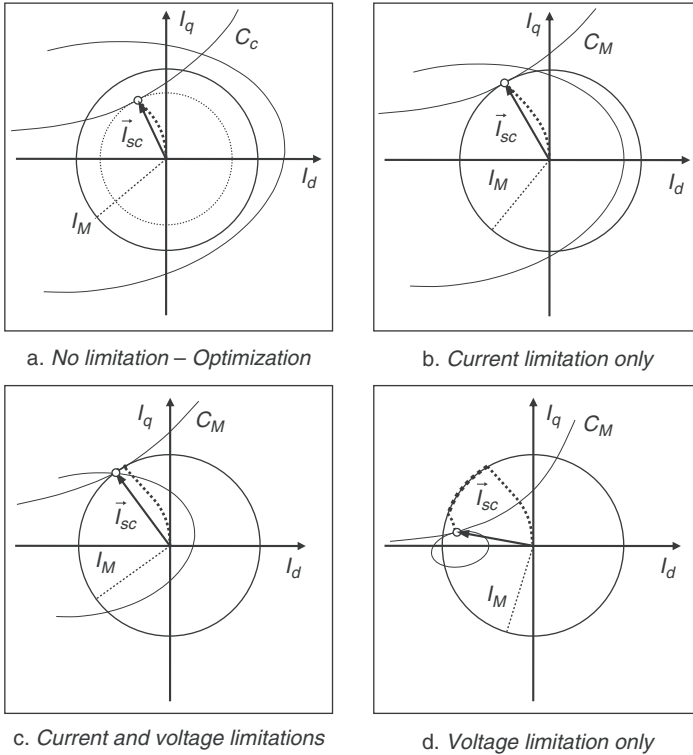


Figure 3.14 Various cases of limitations with a reverse saliency motor

For a motor with a reverse saliency without current or voltage limitation, the minimization of the current magnitude requires a negative magnetizing current to take advantage of the homopolar torque. The reduction of ellipse axis lengths with the polar mechanical angular frequency led to gradually increasing the demagnetizing current to limit the voltage.

For the same motor in the electrical braking operation the configurations are virtually identical to those which we would obtain by a symmetry compared to the I_d axis. The symmetry compared to the I_d axis is limited, however, to the torque hyperbola family and to the limitation circle of the current magnitude. Indeed, the ellipses of the voltage limitation are not modified in braking mode.

In the case of a direct saliency motor without current or voltage limitation the minimization of the current magnitude, requires a positive magnetizing current to take advantage of the

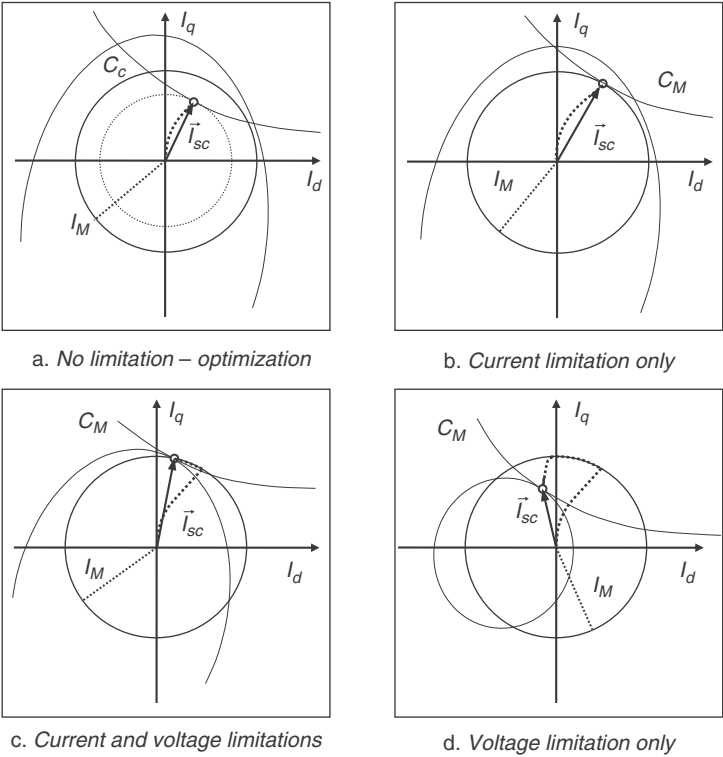


Figure 3.15 Various cases of limitations with a direct saliency motor

homopolar torque. The first two configurations in Figure 3.15 are symmetrical with regard to the I_d axis, compared to the corresponding configurations in Figure 3.14. Symmetry with regard to the I_d axis is limited, however, to the torque hyperbola family and to the circle of the current magnitude limitation. The ellipses of the voltage limitation exchange the minor and major axes, and the reduction of these axes with the polar mechanical angular frequency led to gradually reversing the magnetizing current in order to limit the voltage.

For the same motor in the electrical braking mode configurations are virtually identical to those which one would obtain by a symmetry compared to the I_d axis. The ellipse of the voltage limitation is, however, not modified in braking mode.

For various cases of limitation, the computation of the two components I_{dc} and I_{qc} of the stator set-point current vector $\vec{I}_{sc}$ in the (d, q) set-point rotating reference frame can be made, using the decision tree of Figure 3.16.

Results obtained in section 3.4.1 make it possible to minimize the stator current without limitation; results obtained in section 3.4.4.4, make it possible to detect and characterize the various cases of voltage, current or torque limitations. According to the diagnosis, established formulas make it possible to calculate the acceptable current reference vector which will make it possible to reach the optimal torque.

For knowing the possible limitations, it is necessary to define first the set-point torque of the process C_c ; this must be lower than the maximum set-point torque defined by the process sizing.

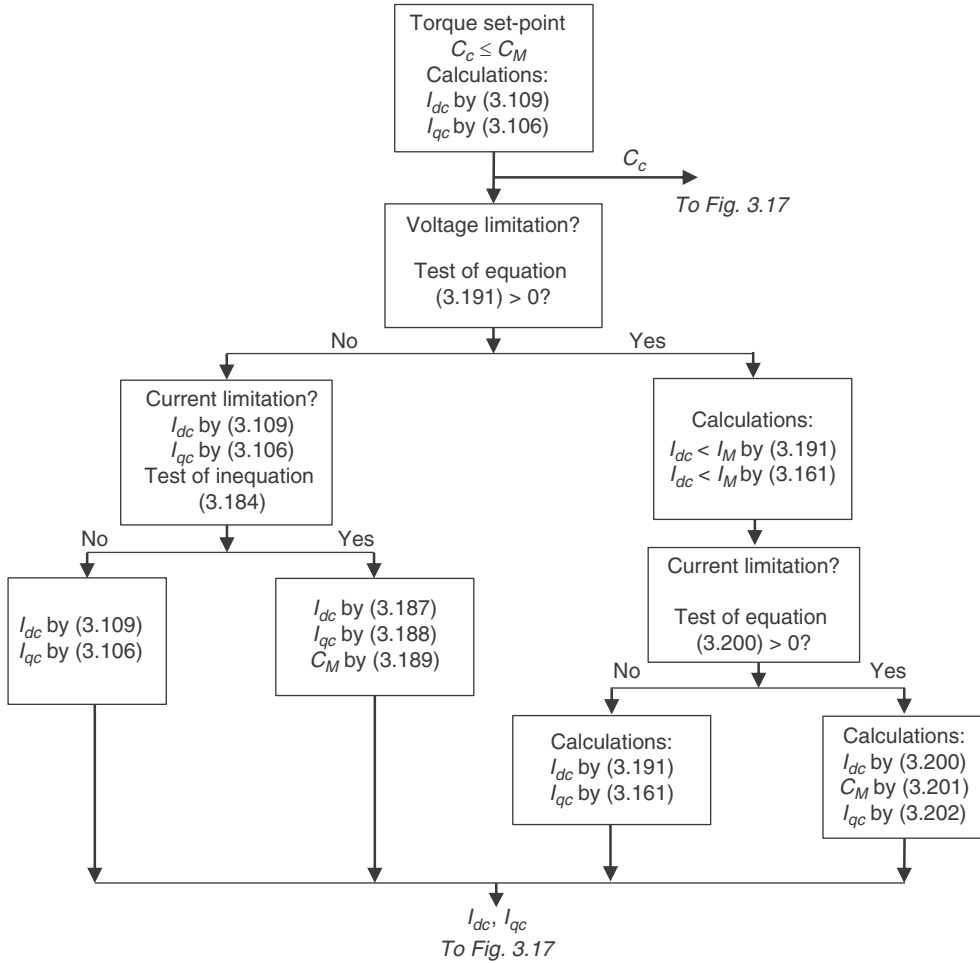


Figure 3.16 Current and voltage limitations

We start by calculating the optimal magnetizing current for the set-point torque by equation (3.109), and then the active current by equation (3.106) since the torque depends at the same time on the magnetizing current and on the active current.

We then check with the previously calculated currents if the first term of equation (3.191) is positive.

In this case, the voltage limit is exceeded for this pair of set-point currents (Figures 3.14d and 3.15d); we must impose a negative or positive magnetizing current, according to the saliency type, calculated by equation (3.191). In the case of a voltage limitation only, the active current is calculated by equation (3.161) and the set-point torque can be reached.

If the pair of currents calculated by equations (3.191) and (3.161) leads so that the first term of equation (3.200) is positive in the case of a voltage limitation, this proves that the current limitation is also reached (Figures 3.14c and 3.15c). In this case, the two coordinates of the stator set-point current vector I_{dc} and I_{qc} will be calculated respectively by (3.200) and (3.202),

after having calculated the maximum torque by (3.201). It will thus be necessary to modify the magnetic state of the motor because of the voltage limitation and moreover the set-point torque will not be reached because of the current limitation.

If the test of equation (3.191) does not indicate any voltage limitation (Figures 3.14a or b and 3.15a or b), it is enough to check whether there is a current limitation by using inequality (3.184) with the pair of currents calculated without limitation (3.109) and (3.106).

In this case (Figures 3.14b and 3.15b), the computation of I_{dc} is carried out using equation (3.187) and that of I_{qc} with equation (3.188), which makes it possible to know the maximum torque under these conditions using equation (3.189).

If there is neither voltage nor current limitation (Figures 3.14a and 3.15a), the initial computation of the two coordinates of the optimal stator current, realized by equations (3.109) and (3.106), is unchanged.

The strategy which was chosen for the computation of the set-point current is to minimize the stator current for a known set-point torque, as long as the voltage or the current limit is not reached.

This strategy corresponds to the optimal sizing of the motor and of the inverter. It uses homopolar torque of this particular motor type which has armature reaction and a rotor magnetic anisotropy. This characteristic requires a current component according to the d axis for generating the homopolar torque, and a component according to the q axis for the electromagnetic torque using the flux of permanent magnets. The current according to the d axis is thus not exclusively a demagnetizing current as for an SMPM-SM.

The analysis made in section 3.4.4 makes it possible to adapt the control strategy to the process sizing constraints. We thus demonstrated that it is possible to calculate the set-point current, during the phase of the dead-beat control computation, which complies with all limits.

This set-point, and possibly one or more limits, will be reached at the end of the voltage impressing period of the control, thanks to the prediction of the motor state evolution.

In the event of whatever limitation, the control process is not modified, either in its principle or characteristics; only the method of set-point computation changes, *anticipating* limits, which makes it possible to keep the same operating mode and thus to avoid any change of gain or dynamics inducing instabilities; this method does not lead to any time-delay of the limit detection; a pure time-delay of the control, equal to a maximum of one sampling period, is compensated for by a prediction based on the motor model.

3.4.4.4.4 Transitory Voltage Limit

In addition, if modifications of set-points are very fast compared to the stator time constant, the calculated voltage vector can instantaneously exceed the maximum voltage vector that the inverter can provide.

Indeed, according to the inductive characteristics of the motor, and according to the chosen sampling period, it may be that the current set-point cannot be reached during only one impressing period of the stator voltage.

This limitation is different from the voltage limitation previously calculated for stabilized modes, and occurs only on fast set-point transients for processes requiring unlimited dynamics.

To swiftly vary the stator current, it would indeed be necessary to have a voltage source that can impose the adequate current gradient into the stator inductance, within just one sampling period.

Taking into account the difference existing between inductances according to the d and q axes, we can expect that the voltage necessary for a fast current variation is more important depending on the axis where the stator inductance is the most important.

Fast modifications of set-points could all the more easily be satisfied, if the reserve of voltage – the difference between the feeding voltage and the motor counter-voltage – is large. It is often the case at low speed, where the motor b.e.m.f. is smaller than it is above the rated motor speed.

In the opposite case, at high speed, the computation of the control voltage vector from equations (3.100) and (3.104), will lead to one voltage vector having the right phase – and thus the right direction – but an insufficient magnitude to reach, within just one period, the current set-point. The voltage magnitude will be shortened by the inverter. However, the stator current vector will progress in the direction of the extremity of the current reference vector, starting from its initial position, without being able to reach it, during the first sampling period. The extremity of the stator current vector will progress from one period to the next, in the direction of the extremity of the set-point vector, using all the available voltage.

Let us calculate the control voltage vector in the (d_0, q_0) reference frame, from equations (3.100) and (3.104).

$$\vec{V}_{s(d_0, q_0)} = \frac{(\zeta_{d2} - i)}{e_1 \cdot (\zeta_{d2} - \zeta_{d1})} \cdot (\vec{\Psi}_{1c} - e^{\mu_1 \cdot T} \cdot \vec{\Psi}_{10}) + \frac{(\zeta_{d1} - i)}{e_2 \cdot (\zeta_{d1} - \zeta_{d2})} \cdot (\vec{\Psi}_{2c} - e^{\mu_2 \cdot T} \cdot \vec{\Psi}_{20}) \quad (3.203)$$

The two complex coefficients $\frac{\zeta_{di} - i}{e_j \cdot (\zeta_{di} - \zeta_{dj})}$ of the two vectors representing a vector difference between each of the two vector coordinates $\vec{\Psi}_{ic}$ of set-point state-space eigenvectors and the free evolution $e^{\mu_i \cdot T} \cdot \vec{\Psi}_{i0}$, have their magnitudes which tend towards infinity as $T \rightarrow 0$, owing to the fact that parameters $e_i \rightarrow 0$, $\forall ij \in \{1; 2\}$, according to the definition (3.73).

For a magnitude of the control voltage $\|\vec{V}_s\|$ limited by the maximum voltage that the inverter can deliver, not all set-points can thus be reached within just one period, and especially so when T is small.

An analysis of the robustness of the dead-beat control should thus be made starting from equation (3.203); it is not carried out in this work.

3.4.4.4.5 Control Computation

A synopsis of the control voltage computation is presented in Figure 3.17. We will detail it below.

This computation is repeated during each PWM period. It is carried out after sampled measurements at time $(t_n - T)$, measure filtering, rotor flux estimation and motor state prediction, and it must be finished before the end of the computational period at time t_n of the prediction. These cyclic computations use various results of computations carried out during the previous period and thus stored to be used one period later. To symbolize this storage, Figure 3.17 uses the time-delay operator z^{-1} of one computational period.

We saw in section 3.4.3.5 (cf. Figure 3.8) how to measure motor phase currents in the (α, β) fixed frame, then how to filter the measures and to estimate the rotor flux in the (d_m, q_m) reference frame, and finally how to predict the motor state at the end of the computational period in the (d_m, q_m) reference frame; this prediction of the motor state will be used to define the initial state in the (d_0, q_0) reference frame for the control computation.

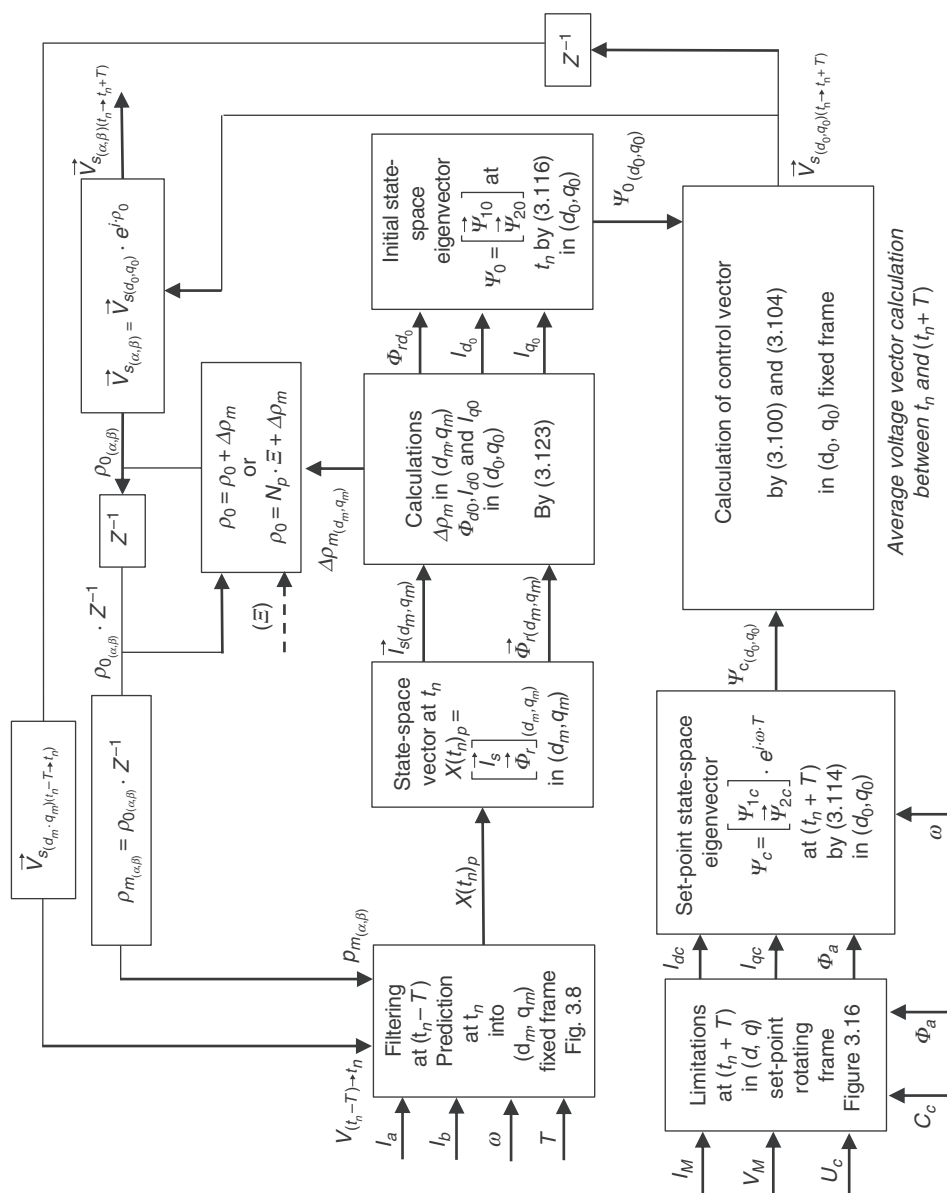


Figure 3.17 Computation of the average control voltage

This state-space vector is set up from the predicted stator current vector $\bar{I}_{sp}$, and from the rotor flux vector $\bar{\Phi}_{rp}$ predicted at time t_n , in the (d_m, q_m) reference frame.

$$X(t_n)_p = \begin{bmatrix} \bar{I}_{sp} \\ \bar{\Phi}_{rp} \end{bmatrix}_{(d_m, q_m)} = \begin{bmatrix} I_{dm} \\ I_{qm} \\ \Phi_{dm} \\ \Phi_{qm} \end{bmatrix} \quad (3.204)$$

It should be noted that Φ_{qm} cannot be null since the rotor flux was estimated in the (d_m, q_m) rotating reference frame at the time of the measurement; it was then predicted in the same rotating reference frame at t_n ; its two coordinates thus enable us to calculate the angular rotation $\Delta\rho_m$ of the initial rotating reference frame (d_m, q_m) , as well as the flux magnitude Φ_{d0} , by the prediction using equation (3.123); using $\Delta\rho_m$ we calculate the new current vector coordinates in the (d_0, q_0) initial rotating reference frame by (3.123).

The initial state-space vector in the (d_0, q_0) rotating reference frame, becomes:

$$X(t_n)_0 = \begin{bmatrix} \bar{I}_{s0} \\ \bar{\Phi}_{r0} \end{bmatrix}_{(d_0, q_0)} = \begin{bmatrix} I_{d0} \\ I_{q0} \\ \Phi_{d0} \\ 0 \end{bmatrix} \quad (3.205)$$

The abscissa of the rotor flux in the rotating reference frame becomes null by definition of the initial rotating reference frame, and $\Phi_{d0} = \Phi_a$ or $\Phi_{d0} = \Phi_{re}$ according to whether the flux estimated by the Kalman estimator was replaced by the theoretical flux of permanent magnets or was estimated.

It is then enough to calculate the state-space eigenvector Ψ_0 in the (d_0, q_0) reference frame by equation (3.116), with motor parameters defined at the measured polar mechanical angular frequency ζ_{d1} and ζ_{d2} (3.49) and ξ_{d1} and ξ_{d2} (3.53).

In addition, we saw in section 3.4.4.4.3 (cf. Figures 3.14 and 3.15) the various cases of voltage, flux, current and torque limitations and how to calculate the two coordinates of the stator set-point current vector, I_{dc} and I_{qc} (cf. Figure 3.16), by taking into account these limitations and the torque set-point provided by the process.

The rotor flux magnitude Φ_{rc} is not controllable; it is replaced by the magnitude of the theoretical flux or by the estimated flux magnitude, and set-points are used to define a set-point state in the (d, q) set-point rotating reference frame for the control computation. The set-point state-space vector defines the control objective in the (d, q) reference frame at time $(t_n + T)$.

$$X(t_n + T)_c = \begin{bmatrix} \bar{I}_{sc} \\ \bar{\Phi}_{rc} \end{bmatrix}_{(d, q)} = \begin{bmatrix} I_{dc} \\ I_{qc} \\ \Phi_{dc} \\ 0 \end{bmatrix}_{(d, q)} \quad (3.206)$$

We notice again here, that the rotor flux magnitude becomes the abscissa of the rotor flux vector in the set-point rotating reference frame, by definition of the (d, q) reference frame.

$$\Phi_{rc} = \|\tilde{\Phi}_{re}\| = \Phi_{dc} \quad \text{or} \quad \Phi_{rc} = \Phi_a = \Phi_{dc} \quad (3.207)$$

It is then enough to calculate the state-space eigenvector Ψ_c no longer in the (d, q) reference frame, but in (d_0, q_0) , by equations (3.112), (3.113) and (3.114), with motor parameters defined at the measured polar mechanical angular frequency ζ_{d1} and ζ_{d2} (3.49), and ξ_{d1} and ξ_{d2} (3.53).

The control voltage vector to be applied during the period following the present computation period, from t_n to $(t_n + T)$, is calculated simply from equations (3.100) and (3.104), from Ψ_c and Ψ_0 in the (d_0, q_0) reference frame and from motor parameters at the ω angular frequency, μ_1 (3.34), μ_2 (3.35), ζ_{d1} and ζ_{d2} (3.49).

The average voltage vector of the control is stored as magnitude and angle (cf. Figure 3.17) to be used by the computation during the following period (cf. Figure 3.8) in the (d_0, q_0) reference frame at time t_n , which becomes the new the (d_m, q_m) measurement frame at time t_n .

To be able to impress the average voltage vector on the motor by the PWM inverter from t_n to $(t_n + T)$, it is still necessary to project this average vector in the (α, β) fixed frame, starting from the knowledge of its coordinates in the (d_0, q_0) reference frame. It is thus necessary to calculate the angle ρ_0 between these two frames.

When the rotor position is known by its measurement Ξ , the angle ρ_0 can be calculated by various methods:

$$\rho_0 = N_p \cdot \Xi + \omega \cdot T \quad \text{or} \quad \rho_0 = N_p \cdot \Xi + \Delta\rho_m \quad (3.208)$$

When the rotor position is not measured, we estimate the angle by successive increments $\omega \cdot T$ or $\Delta\rho_m$, starting from an initial position known from a specific electrical process:

$$\rho_0(t_n) = \rho_0(t_n - T) + \omega \cdot T \quad \text{or} \quad \rho_0(t_n) = \rho_0(t_n - T) + \Delta\rho_m \quad (3.209)$$

This method for estimating the angular evolution of the predicted reference frame compared to the (α, β) fixed frame is like a digital integration, over a long time, of angle increments of the rotating reference frame during one sampling period. However, the filtering, the estimation and the Kalman prediction, based on the motor model make it possible to avoid the divergence of this integration by a permanent self-adjustment. It remains to formally demonstrate the control robustness provided by the observer under usual conditions of operation.

The angle $\rho_0(t_n)$ makes it possible to calculate the average voltage vector in the (α, β) fixed frame (cf. section 3.3.3) where it can be impressed on the motor by the PWM (cf. Figure 3.17).

$$\bar{V}_{s(\alpha, \beta)} = (V_{sd0} + i \cdot V_{sq0}) \cdot e^{i \cdot \rho_0} \quad (3.210)$$

The angle $\rho_0(t_n)$ thus calculated and stored, becomes the angle $\rho_m(t_n)$ of the (d_m, q_m) reference frame at measurement time t_n (cf. Figure 3.8).

The torque set-point read at time $(t_n - T)$ is reached at $(t_n + T)$ from measurements made at $(t_n - T)$, computations done from $(t_n - T)$ to t_n and PWM impressed from t_n to $(t_n + T)$. Thanks to the motor state prediction at t_n , and to computation of the motor state evolution from t_n to $(t_n + T)$, the control makes it possible to reach the set-point vector in only one period, which is

the characteristic of a dead-beat response. However, we can see that between the reading of the set-point, which cannot be predicted, and its obtaining, is a time of two periods, of which one is a pure time-delay period.

3.4.5 Example of Implementation

To concretize the implementation of this control process, we chose a motor, fed according to Figure 3.18, and having the following characteristics in a traction mode:

- $N_p = 6$
 - $R_s = 31.8\text{m}\Omega$
 - $L_{fd} = 0.246\text{mH}$
 - $L_{md} = 0.650\text{mH}$
 - $L_{fq} = 0.246\text{mH}$
 - $L_{mq} = 1.30\text{mH}$
 - $\omega_M = 2\pi \cdot 400 \text{ rd / s}$
 - $\Phi_a = 0.600\text{Wb}$
 - $C_M = 2483\text{m.N}$
 - $I_M = 598\text{A}$
 - $T = 200\mu\text{s}$
- (3.211)

The DC voltage supplying the inverter is 1600V.

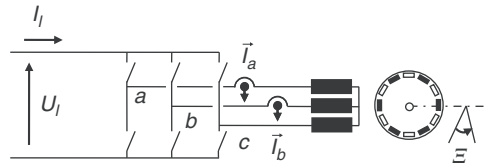


Figure 3.18 Voltage inverter and interior permanent magnet synchronous motor

The electrical vectors are those resulting from the computation of the Concordia transform which preserves the instantaneous power.

The magnetizing inductance according to the d axis is lower than the inductance according to the q axis. The saliency of this motor is a reverse one.

3.4.5.1 Adjustment of Torque – Limitations in Traction Mode

To allow the visualization of all phenomena with various timescales, for a short time we force starting with a null torque then with a maximum electromagnetic torque in traction mode and with a mechanical speed very quickly increasing from zero to the maximum speed on a scale of time of 2 s; it should be difficult to practically realize this speed gradient. The control and the motor were thus simulated for this reason. The mechanical speed is imposed because the mechanical inertia is not simulated.

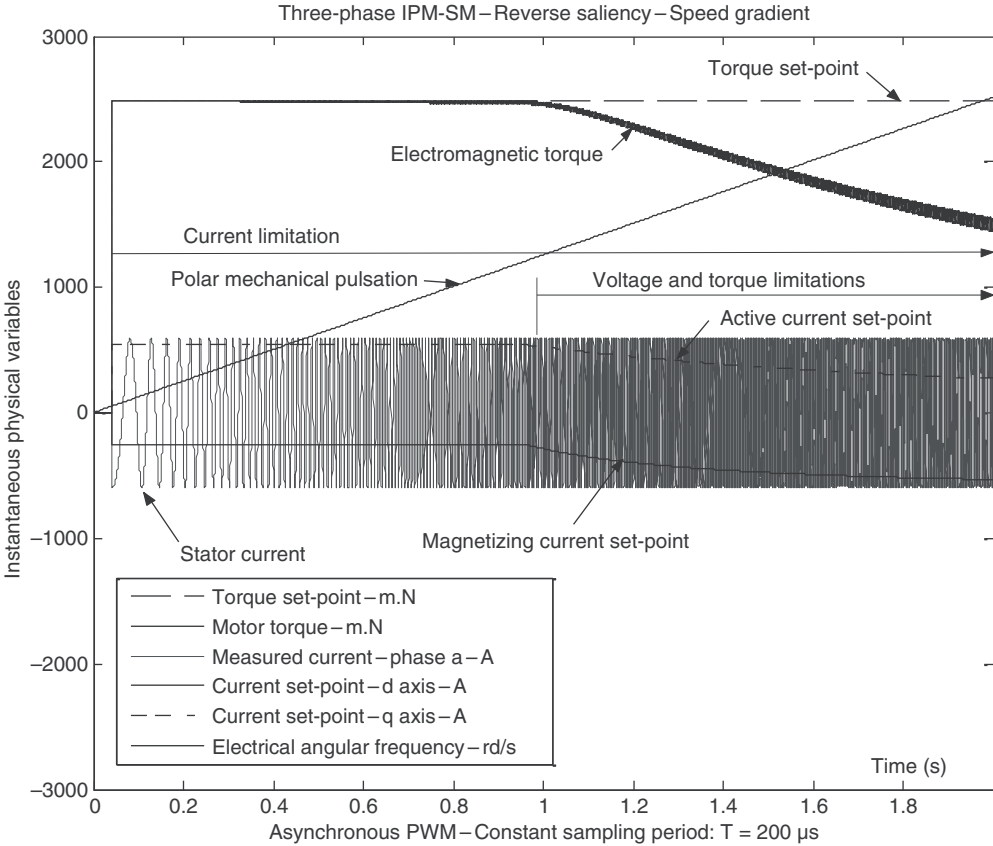


Figure 3.19 Starting with null torque, then maximum torque

The starting takes place with null current, null torque and null speed. At starting with null torque, the current can be null only if the starting is done with a speed lower than the rated motor speed where the maximum voltage is not yet reached; at starting in this condition it is not necessary to demagnetize the motor.

In Figure 3.19, for a first period of approximately 40ms, the torque is null. Maximal torque is reached immediately after, following the torque set-point; at the same time, the current necessary to obtain the set-point torque is reached, and at this point the current is maximum. In this figure, the maximum electromagnetic torque is reached in 400 μs (one control computational period at null torque, and one impressing period of the calculated voltage) at low speed and at the beginning of starting. It is obvious that the use of all control dynamics in the tracking mode can only be used under conditions requiring it specifically, and it necessitates that mechanics were dimensioned accordingly; during sharp load moment variations, control dynamics allow a better speed regulation particularly with low inertia, or a better position regulation if that is the aim.

The simulated reverse saliency motor requires a negative magnetization current to produce a positive reluctance torque and to thus optimize the current magnitude, making it possible to

obtain the torque set-point. It is one of the main differences from an SMPM-SM, for which, during this phase, the optimal magnetizing current is null up to the rated motor speed.

The simulation was made without an input filter. Indeed, it is well-known (Jacquot, 1995) that the traction operation causes an instability of the input filter voltage starting from a low power consumption from the filter, which continues until the divergence, owing to the fact that this filter has very few losses. This simulation was parameterized to highlight the dynamics and the compliance with physical variable limits of this control process.

It is obvious that with a filter, such fast torque variations can set off this instability, and distort the demonstration. However, we will see later on how to stabilize the filter in all circumstances and to preserve the high dynamics with a regulation process of the input filter voltage of the same type as the process of the motor control, acting together in symbiosis.

After approximately 0.96 s, the motor rated speed is reached with the maximum stator fundamental voltage. The speed continuing to increase, the stator flux must thus be limited by the available voltage starting from this time. The flux will have to therefore decrease gradually up to the maximum speed. For that, the stator current component according to the rotor flux axis grows in an opposite sense to the rotor flux vector, to modify the rotor flux and thus to maintain a constant voltage; at the same time, the active current must then decrease to maintain the current magnitude at its maximum; that causes a torque decrease. The maximum electrical power remains constant.

Figure 3.19 makes it possible to visualize the performances in a tracking mode of the torque and the current, of this control process.

We do not notice any limit overshooting, in spite of the sharp variation of the torque set-point, in just one sampling period. This is due to the fact that the control is instantaneous (dead-beat control) and that the set-points take account of limits. It is thus possible under these conditions, not only to exclude the limit overshoot, but also to maintain the motor state on its limits of use, and thus to avoid any overshoot and any unused margin of operation compared to these limits. The motor operation is thus optimized, for example compared to fast variations of the feeding voltage. For the same reason, we can integrate into the set-point current the harmonics due to the switching of the inverter voltage, depending on the function of the PWM type, in order not to exceed the rated peak current of the inverter switches.

Insofar as we can take account, by measurement or estimation, of disturbances of various kinds (e.g. load moment, feeding voltage) for set-point elaboration and for control voltage computation, the control process makes it possible to maintain all performances during disturbances, in a similar way for performances during tracking mode, i.e. in only one sampling period and without overshoot or lagging.

The electromagnetic torque ripple at high speed, due to inverter voltage switching, is visible here owing to the fact that maximum speed is relatively high and that the relationship between the electrical period of the fundamental voltage and the sampling period, decreases up to a ratio of 12.5 in this case.

3.4.5.2 Adjustment of Torque – Limitations in Electrical Braking

In electrical regenerative braking mode, the rated performances are identical for the same motor.

To allow the visualization of all phenomena with various timescales, we chose in the regenerative braking mode to briefly impose a null torque at the maximum speed for approximately 40 ms, then a maximum electromagnetic torque with a mechanical speed very quickly decreasing from maximum speed to zero, in a time of 2 s. The control and the motor were thus simulated.

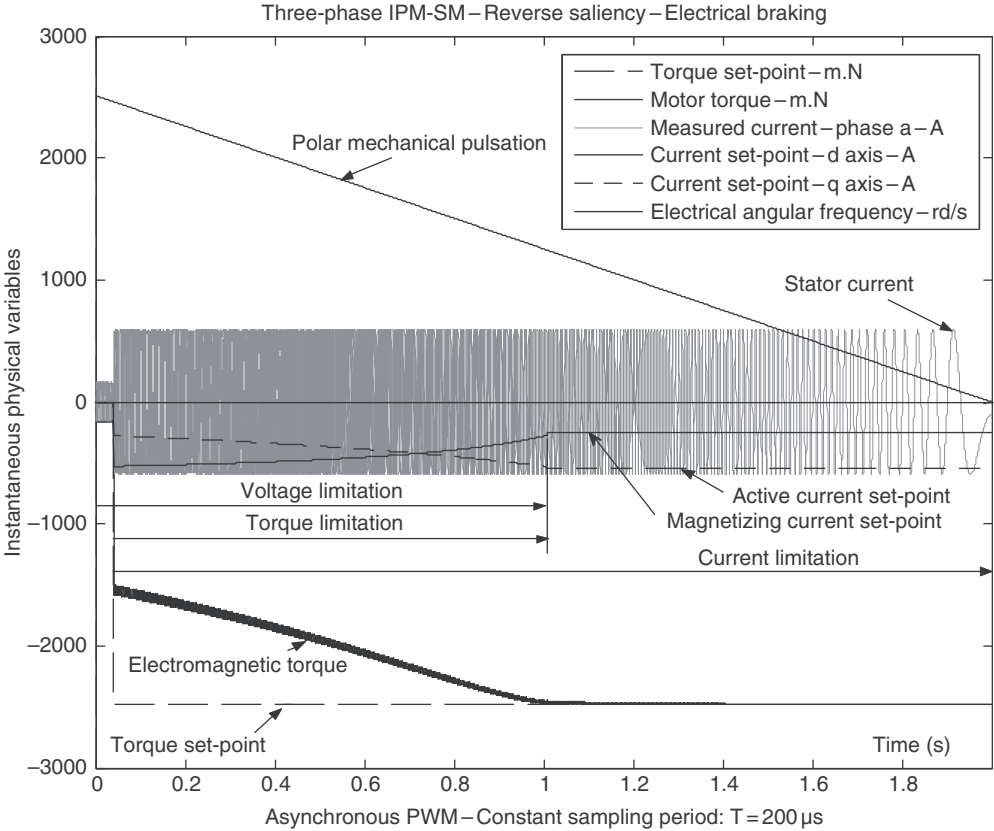


Figure 3.20 Braking with null torque, then maximum torque

Braking cannot start with null current, even without any torque, owing to the fact that with a higher speed than the rated motor speed in braking mode, the maximum voltage would be exceeded and one could not then control the stator regenerative current towards a source with a constant voltage. It is therefore necessary to demagnetize the motor with a current according to the d axis, creating in the rotor a magnetic field opposed to that of the magnets. The current according to the q axis is null during this time, and the torque is thus null, which makes it possible to verify the perfect decoupling between the two current axes allowing the magnetization and the torque to be separately regulated.

After the first 40ms, we suddenly impose the maximum electromagnetic torque. The magnitude of the stator current immediately reaches the maximum current; the demagnetization current makes it possible to maintain an acceptable voltage for regeneration, taking into account voltage drops in the motor, and the active current is calculated for limiting the current magnitude to the maximum allowable current. In the same way as in traction mode, the maximum active current thus depends on the speed via the maximum voltage and on the demagnetizing current which results from it; the demagnetizing current limits the braking torque at high speed.

In Figure 3.20, the maximum electromagnetic torque is reached in 400 μs (one control computation period with null torque, plus one impressing period of the calculated voltage) at

the maximum speed at the beginning of the electrical braking. It is obvious that the use of the whole dynamics of the control in tracking mode can be used only under conditions requesting it specifically and provided that the mechanics were dimensioned accordingly.

During a first period of less than 1 s, the torque is limited by the stator current magnitude. Indeed, the abscissa of the stator current in the (d, q) reference frame is not null but negative to allow for motor demagnetization in order to limit the voltage to the maximum voltage. The magnitude of the negative component $|I_{sd}|$ decreasing gradually with the motor speed, makes it possible for the stator current ordinate magnitude $|I_{sq}|$ to grow, complying with the current limit, and allowing an increasing braking torque as the speed decreases.

After this phase, the maximum torque is always reached with the current limit. However, the current of the demagnetization is not null so that the motor can generate a reluctance torque of the same sense as the magnet torque with this kind of motor. This demagnetizing current is controlled to maximize the produced torque, with as small a stator current as possible. It then remains constant until the stop, as the set-point torque is constant.

After approximately 1 s, the rated motor speed in the braking mode is reached with the stator fundamental voltage at its maximum. The speed continuing to decrease, the voltage decreases with the speed, until the stop.

The braking torque is maintained until the stop.

The simulation was made without an input filter.

Figure 3.20 makes it possible to visualize performances in tracking mode of the current and the torque of this control process, which minimizes the stator current magnitude for a given torque.

The strategy adopted in braking mode for this simulation is symmetrical to that chosen for the traction operation. To limit the stator current magnitude, the abscissa of the stator current is optimized each time there is no current limitation. It is a strategy with minimum energy.

3.4.5.3 Free Evolution – Short-Circuit Torque

In the particular case of an IPM-SM, and because of the rotor magnetic anisotropy, the state-space eigenvector has four complex coordinates.

By contrast, the state-space eigenvectors of an induction machine with non-salient poles, as well as the state-space eigenvectors of an SMPM-SM, have only two complex coordinates, which represent four real coordinates.

The *free evolution* of the state-space eigenvector is calculable from equation (3.70), for $V=0$; it is the motor state evolution, with a symmetrical short-circuit of the three phases of the motor, and it corresponds to a null voltage control. The computation of the free evolution will be carried out in one fixed (d_0, q_0) reference frame. Thus at time $(t_0 + T)$, replacing $P^{-1} \cdot X$ by the state-space eigenvector $\bar{\Psi}$, the free evolution becomes:

$$\begin{bmatrix} \bar{\Psi}_1 \\ \bar{\Psi}_2 \\ \bar{\Psi}_3 \\ \bar{\Psi}_4 \end{bmatrix}_{(d_0, q_0)}^0 = e^{D \cdot T} \cdot \begin{bmatrix} \bar{\Psi}_{10} \\ \bar{\Psi}_{20} \\ \bar{\Psi}_{30} \\ \bar{\Psi}_{40} \end{bmatrix}_{(d_0, q_0)} \quad (3.212)$$

or even:

$$\begin{bmatrix} \bar{\Psi}_1 \\ \bar{\Psi}_2 \\ \bar{\Psi}_3 \\ \bar{\Psi}_4 \end{bmatrix}_{(d_0, q_0)}^0 = \begin{bmatrix} e^{\mu_1 \cdot T} & 0 & 0 & 0 \\ 0 & e^{\mu_2 \cdot T} & 0 & 0 \\ 0 & 0 & e^{\mu_3 \cdot T} & 0 \\ 0 & 0 & 0 & e^{\mu_4 \cdot T} \end{bmatrix} \cdot \begin{bmatrix} \bar{\Psi}_{10} \\ \bar{\Psi}_{20} \\ \bar{\Psi}_{30} \\ \bar{\Psi}_{40} \end{bmatrix}_{(d_0, q_0)} \quad (3.213)$$

Equations (3.83) and (3.84) express the coordinates $\bar{\Psi}_3$ and $\bar{\Psi}_4$ of the state-space eigenvector. These two bound relations, because they are conjugate-complex to each other due to the two conjugate-complex eigenvalues μ_3 and μ_4 , give an account of a simple rotation of the rotor flux vector at the polar mechanical angular frequency, with a constant amplitude (equation (3.87)); the complex coordinate of the state-space eigenvector of an SMPM-SM in Figure 2.20, evolves in the same way. The trajectory of $\bar{\Psi}_3$ and $\bar{\Psi}_4$ is summarized in equation (3.213) by two circles of radius Φ_a , superimposed but described in the reverse sense. It is thus unnecessary to trace them.

The computation of the first two coordinates is detailed in equations (3.94) and (3.95) for $\bar{V}_s = 0$, or by their extraction from (3.213):

$$\begin{aligned} \bar{\Psi}_1 &= e^{\mu_1 \cdot T} \cdot \bar{\Psi}_{10} \\ \bar{\Psi}_2 &= e^{\mu_2 \cdot T} \cdot \bar{\Psi}_{20} \end{aligned} \quad (3.214)$$

Figure 3.21 was calculated with the motor described in the preceding section with the following initial state:

$$X_0 = \begin{bmatrix} \bar{I}_{s0} \\ \bar{\Phi}_{r0} \end{bmatrix}_{(d, q)} = \begin{bmatrix} -250.7 + 542.4 \cdot i \\ 0.6 \end{bmatrix} \quad (3.215)$$

at a polar mechanical angular frequency of approximately 1.1 rd/s, below the rated motor speed, with the maximum current and an optimal demagnetization to pull out the maximum torque from the current magnitude. In Figure 3.19, this stage of operation corresponds to a time of approximately 0.96 s. The first two coordinates of equations (3.213) were calculated every 100 μ s, and the results were drawn in the (d_0, q_0) plane. The frame is maintained stationary during all short-circuit computations.

Trajectories of extremities of the vectors corresponding to the first two coordinates of the state-space vector follow logarithmic spiral curves; vector magnitudes are modulated in amplitude by the non-symmetry of rotor characteristics compared to the two axes in electrical quadrature; we will see the same phenomenon for the short-circuit current and for the torque.

Trajectories in the complex plane are symmetrical compared to the d_0 axis, owing to the fact that the first two coordinates of the state-space eigenvector are conjugate-complex to each other.

Figure 3.22 visualizes the convergence towards the short-circuit current of the stator current under the same conditions as previously. The current rises very quickly to a maximum, before decreasing then to stabilize itself at the short-circuit current value; the short-circuit current magnitude is limited only by the stator impedance.

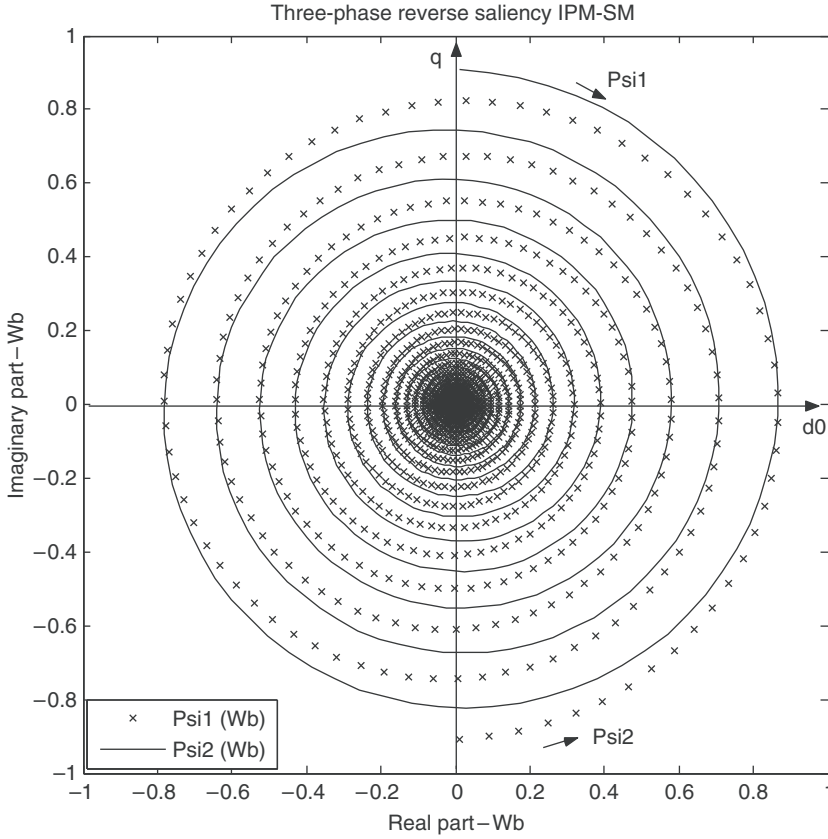


Figure 3.21 Free evolution of the first two coordinates of the state-space eigenvector Ψ

The current magnitude established in the short-circuit is not smoothly decreasing as for a non-salient pole motor. The current is directly influenced by the stator differential impedance seen by the two rotor axes in quadrature.

Figure 3.22 was obtained by calculating the state-space vector system in one fixed (d_0, q_0) reference frame, for $\vec{V}_s \equiv 0$, using equation (3.121).

$$X(t_0 + T)_{p(d_0, q_0)} = F_{dq}(T) \cdot X(t_0)_{0(d_0, q_0)} \quad (3.216)$$

and that, thanks to the calculation of the 16 components of F in the rotating reference frame related to the rotor (section 3.4.3.3.1), with:

$$X_p = \begin{bmatrix} I_{sd} \\ I_{sq} \\ \Phi_{rd} \\ \Phi_{rq} \end{bmatrix} \quad \text{and} \quad X_0 = \begin{bmatrix} I_{d0} \\ I_{q0} \\ \Phi_a \\ 0 \end{bmatrix} \quad (3.217)$$

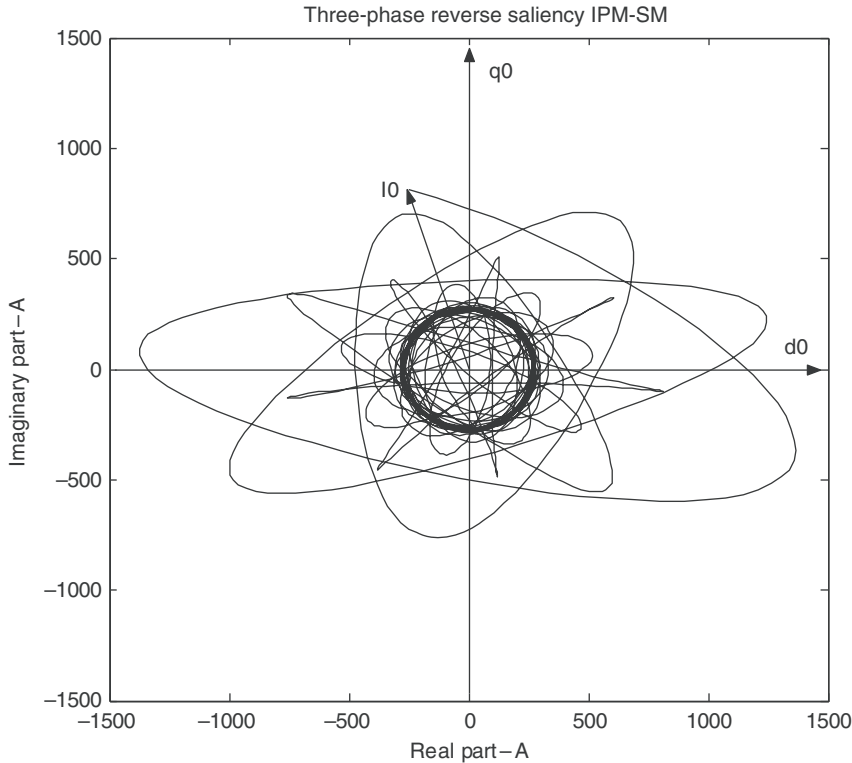


Figure 3.22 Evolution in short-circuit of the stator current

Coordinates are obtained in the (d_0, q_0) reference frame. It should be noted that the rotor flux does not remain collinear with the d_0 axis, owing to the fact that the reference frame remains stationary.

If we follow with the rotor axes, the evolution of the short-circuit current, we find again the convergence of the short-circuit current towards a fixed point with a negative abscissa (cf. Figure 3.23).

Figure 3.24 reproduces the evolution of the electromagnetic torque during the symmetrical short-circuits. The torque was calculated in the rotating reference frame because of the rotor saliency by equation (3.20):

$$C = N_p \cdot \left[(L_{md} - L_{mq}) \cdot I_{sd} + \Phi_a \right] \cdot I_{sq} \tag{3.218}$$

It is also possible to calculate the electromagnetic torque by the Lorentz law, but in this case not by the cross product of the magnet flux multiplied by the stator current; it is indeed necessary to replace the magnet flux by the air gap flux (3.18).

The short-circuit torque decreases as a single damped sine wave, modulated in amplitude by the rotor asymmetry and by the generation of a periodic reluctance torque.

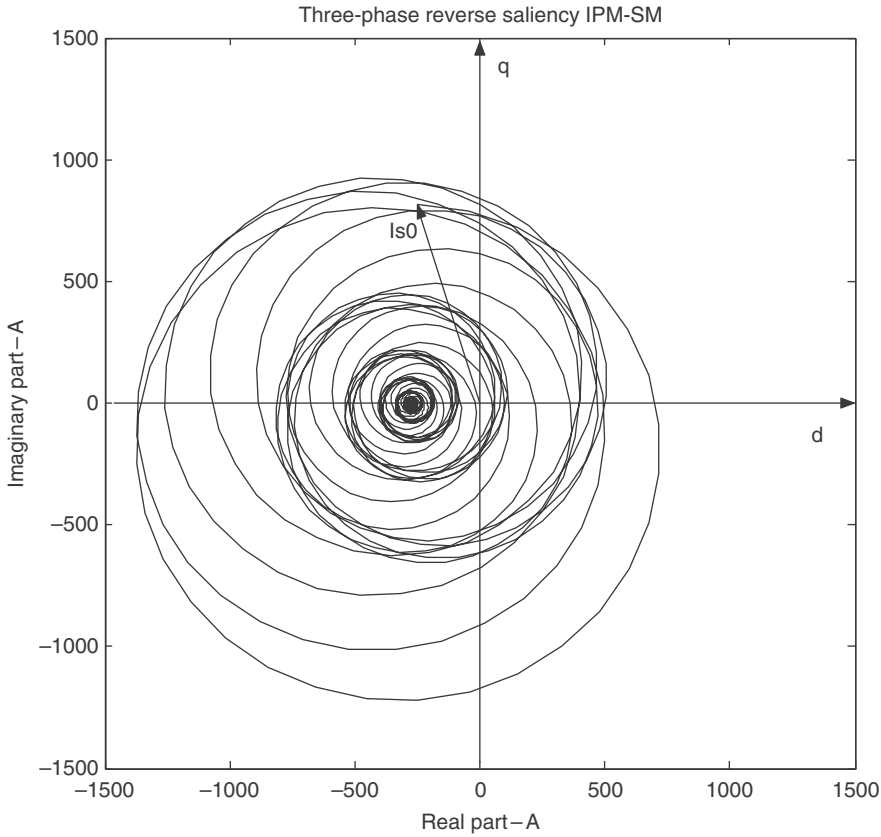


Figure 3.23 Short-circuit current evolution in the (d, q) reference frame

It is interesting to note that the braking torque, after the transient due to the short-circuit, is not completely null but tends towards a negative residual electromagnetic torque which corresponds to the energy lost in the motor winding resistances.

This study is important for the system mechanical sizing in the event of a breakdown in short-circuit of an inverter electronic power switch, in general followed by forcing the short-circuit of the two other switches of the same level, to symmetrize the short-circuit and to limit the resulting torque (Welchko *et al.*, 2003).

It should also be noted that when we impose an inductance equality according to the two perpendicular axes, we obtain the same results as for an SMPM-SM. The interior magnet synchronous motor thus appears as a generalization of a permanent magnet synchronous motor, which makes it possible to take into account all constructional asymmetries of this last kind of motor.

Numerical simulations were made with a synchronous motor with reverse saliency, but the computations could have proceeded in a perfectly symmetrical way for a motor with a direct saliency motor.

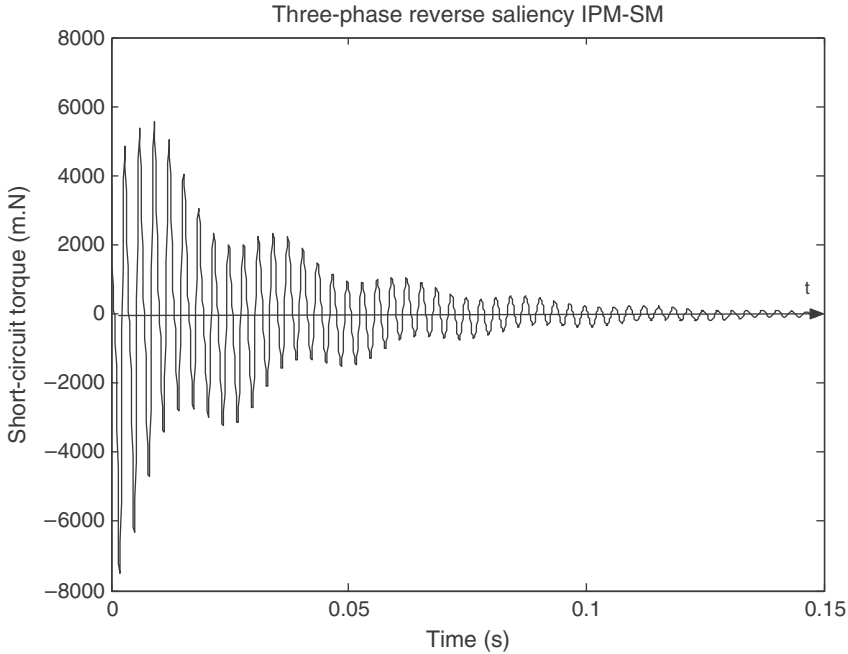


Figure 3.24 Short-circuit torque free evolution

3.5 Conclusions on the IPM-SM

With the control process which has just been described, the synchronous motor can be controlled independently by the torque produced by the magnets or by the homopolar torque, with very high dynamics corresponding to only one PWM period plus one pure time-delay, in a set-point tracking mode as in a disturbance regulation mode.

The control decoupling according to the two axes makes it possible to optimize the way in which the entire torque is produced. This total decoupling between the two axes was also possible with an induction machine, separating the flux control from the torque control.

This control makes it possible to control the synchronous motor from the complete stop up to the maximum speed; the maximum speed can be higher than the rated motor speed, thanks to the calculation of voltage and current limitations, which are used to modify current and torque set-points.

The torque control can further be used to control the motor speed, *null speed included*; it can also be used to control the motor position, *stop included*.

The IPM-SM produces a torque thanks to the stator current and the permanent magnet flux (magnet torque), but also with the stator current and the pole saliency (homopolar torque). The production of the torque is thus accompanied by stator ohmic losses, proportional to the stator current squared. This motor is thus adapted to hold a position at stop or at very low speed since losses primarily take place in the stator which is easier to cool by external ventilation of the motor.

4

Inverter Supply – LC Filter

We will follow the same approach as for the motors, applying the method to the inductance–capacitor power filter; in general this type of filter is used in series with the voltage inverter supply.

This filter makes it possible to prevent supply voltage variations being transmitted immediately to the inverter; it also filters consequences of the DC supply voltage switching by the inverter. It is a passive low-pass filter of the second order; its cut-off frequency must be lower than the frequencies to be filtered, in particular frequencies resulting from the rectification of the AC feeding voltage; the electromagnetic energy reserve of the LC filter is directly dependent on the duration of disturbances to be damped.

This kind of filter must have an excellent efficiency at the power which goes through it. For this, the inductance's resistance has to be minimized. This results in the damping of this filter being very low and thus its Q-factor is high.

Any extraction of active power from the filter, higher than a small threshold, causes divergent oscillations at the natural filter frequency (Jacquot, 1995; Mosskull, 2005).

To avoid this phenomenon, it is thus necessary to damp these oscillations in an active way by using the only available control variable, which is the current feeding inverter (Délémontey, 1995; Délémontey *et al.*, 1995).

We will now study its modeling.

4.1 Electrical Equations and Equivalent Circuit

4.1.1 Definitions and Notations

The filter inductance is often built with a magnetic core in order to reduce its volume and to channel the magnetic field to prevent eddy currents in metallic walls of electrical cubicles.

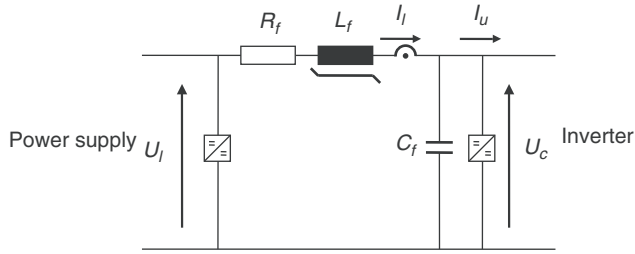


Figure 4.1 Reduced equivalent circuit of the input filter and sensors

The inductance value can thus vary more or less according to the current which flows through it and to the sizing of the magnetic material section.

The capacitor often has a parasitic resistance and inductance in series. These elements are in general negligible, all the more because capacitors with a very low serial resistance and inductance are in general added in parallel directly on the inverter, closest to each inverter leg.

We will write:

- filter inductance resistance: R_f
- filter inductance depending on current: L_f
- filter capacitance: C_f
- supply voltage: U_l
- line current: I_l
- capacitor voltage: U_c
- useful current: I_u

We will neglect the parallel parasitic capacitance of the inductance, as well as the serial resistance and inductance of the capacitor, considering that the sizing and the choice of the technology of these two elements were optimized for this kind of application.

4.1.2 Equivalent Electrical Circuit

The LC filter is inserted between the power supply and the inverter (cf. Figure 4.1).

The supply can be made with DC or AC voltage. If the latter, the supply can be single-phase or three-phase; it is rectified to provide to the inverter with a DC voltage on which voltage harmonics of the rectification are superimposed. These harmonics are partly filtered by the LC filter; its cut-off frequency is lower than the fundamental harmonic depending on the rectification type and on the supply network frequency.

This supply can be entirely reversible, i.e. it provides to the inverter, but also it accepts from the inverter, an active and reactive power.

The inverters are very often reversible, thus making it possible to return towards the supply when it accepts it, an electrical power which in case of electric motors supplied with an inverter, allows regenerative braking.

Whenever the inverter is reversible, but the supply is not, for example in case of a simple diode bridge as a Graetz bridge for a three-phase supply, then for electrical braking it is possible

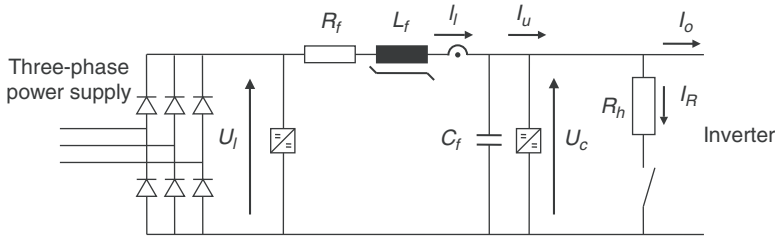


Figure 4.2 Example of an inverter feeding with a non-reversing power supply

to insert between the filter and the inverter, in parallel, a power resistor or a braking rheostat switched by a semiconductor having its own independent capacitor voltage regulation (cf. Figure 4.2). In this case, the electrical braking is a rheostatic braking which is *dissipative*.

When the inverter and the source are both reversible, thanks to an independent regulation of the average capacitor voltage, it becomes possible to control the energy recovered by the supply compared to that dissipated in the rheostat, according to the recovery capability of the supply network at a given time. The two types of electrical brakes are then known as combined.

In this case, the useful current I_u is the current in the braking resistor I_R , algebraically added with the inverter current I_o , according to the selected positive sense of these currents (cf. Figure 4.2).

4.1.3 Differential Equation System

The voltage drop across the inductance terminals due to the line current is simply:

$$U_l - U_c = L_f \cdot \frac{dI_l}{dt} + R_f \cdot I_l \quad (4.1)$$

A second relation can be found by expressing Kirchhoff's law at the main node:

$$I_l - I_u = C_f \cdot \frac{dU_c}{dt} \quad (4.2)$$

These two differential equations are linear, which will make it possible to use the state-space representation for linear systems.

4.2 Working out the State-Space Equation System

This type of filter is a system of the second order, and thus has two degrees of freedom. The state-space vector is thus a vector with two dimensions. The line current and capacitor voltage variables defining the filter state can be selected as the two coordinates of the filter state-space vector:

$$X = \begin{bmatrix} I_l \\ U_c \end{bmatrix} \quad (4.3)$$

These two variables are physical variables and thus in general easy to measure; they are used as input in control and safety system devices. If in a process one cannot measure these values it is necessary to estimate them, and it can be difficult to realize an estimation in the long term for a system with energy accumulation.

System control variables are the useful current and the line voltage. The line voltage can be controllable if there exists an upstream power control device of this voltage, such as a chopper for a DC voltage supply or a controlled bridge for an AC supply. In general, the line voltage is not directly adjustable; it then enters into the model as a measured variable and not as a calculated control variable:

$$V = \begin{bmatrix} I_u \\ U_l \end{bmatrix} \quad (4.4)$$

4.2.1 State-Space Equations in a Fixed Frame

The state-space equation system can thus be written directly, starting from linear differential equations (4.1) and (4.2):

$$\begin{aligned} \frac{dI_l}{dt} &= -\frac{R_f}{L_f} \cdot I_l - \frac{1}{L_f} \cdot U_c + \frac{1}{L_f} \cdot U_l \\ \frac{dU_c}{dt} &= \frac{1}{C_f} \cdot I_l - \frac{1}{C_f} \cdot I_u \end{aligned} \quad (4.5)$$

Continuous-time state-space equations are derived from it, in matrix form:

$$\begin{bmatrix} \frac{dI_l}{dt} \\ \frac{dU_c}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R_f}{L_f} & -\frac{1}{L_f} \\ \frac{1}{C_f} & 0 \end{bmatrix} \cdot \begin{bmatrix} I_l \\ U_c \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{L_f} \\ -\frac{1}{C_f} & 0 \end{bmatrix} \cdot \begin{bmatrix} I_u \\ U_l \end{bmatrix} \quad (4.6)$$

While comparing, with the usual notation of the continuous-time state-space representation of linear systems:

$$\begin{aligned} \dot{X} &= A \cdot X + B \cdot V \\ Y &= C \cdot X \end{aligned} \quad (4.7)$$

the expression of the evolution matrix is obtained:

$$A = \begin{bmatrix} -\frac{R_f}{L_f} & -\frac{1}{L_f} \\ \frac{1}{C_f} & 0 \end{bmatrix} \quad (4.8)$$

with the input matrix expression:

$$B = \begin{bmatrix} 0 & \frac{1}{L_f} \\ -\frac{1}{C_f} & 0 \end{bmatrix} \quad (4.9)$$

and with the output matrix, which is a second-order unit matrix since its measured variables are state variables:

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (4.10)$$

4.2.2 State-Space Equations in the Complex Plane

The evolution and input matrices do not have the symmetry properties that we highlighted in the case of motors with a magnetically isotropic rotor, such as the non-salient pole induction machine or the surface-mounted permanent-magnet synchronous motor.

It is thus not possible to reduce the order of these matrices by using the complex plane; we therefore do not have to deal with complex algebra in the calculations which follow.

4.2.3 State-Space Equation Discretization

The discrete-time state-space representation is given here also from results of section 1.2.3, equation (1.53):

$$X(t_n + T) = F \cdot X(t_n) + G \cdot V \quad (4.11)$$

This equation allows us to predict the motor state $X(t_n + T)_p$, starting from the measured initial state $X(t_n)_m$ if the input V is known, or to fix set-points and to calculate the control by replacing the predicted state $X(t_n + T)_p$ by the set-point vector $X(t_n + T)_c$, according to relations previously established in (1.57) and (1.58).

We now will diagonalize the evolution matrix to discretize the state-space equations, then rewrite the discretized equations within the eigenvector basis, in order to allow an independent control of the two filter states. We will calculate eigenvalues of the evolution matrix to find independent set-point vectors in the new eigenvector basis, and to derive the control vector from it.

4.2.4 Evolution Matrix Diagonalization

4.2.4.1 Eigenvalues

The eigenvalue equation of the evolution matrix is the determinant of the matrix $[\mu \cdot I - A]$ equated to zero, I being the second-order unit matrix:

$$\det \begin{bmatrix} \mu + \frac{R_f}{L_f} & \frac{1}{L_f} \\ -\frac{1}{C_f} & \mu \end{bmatrix} = 0 \quad (4.12)$$

from where we obtain the eigenvalue equation:

$$\mu^2 + \mu \cdot \frac{R_f}{L_f} + \frac{1}{L_f \cdot C_f} = 0 \quad (4.13)$$

The eigenvalues of the evolution matrix are the roots of the eigenvalue equation:

$$\mu_1 = \frac{-R_f \cdot C_f + \sqrt{R_f^2 \cdot C_f^2 - 4 \cdot L_f \cdot C_f}}{2 \cdot L_f \cdot C_f} \quad (4.14)$$

$$\mu_2 = \frac{-R_f \cdot C_f - \sqrt{R_f^2 \cdot C_f^2 - 4 \cdot L_f \cdot C_f}}{2 \cdot L_f \cdot C_f} \quad (4.15)$$

It is interesting to note that these eigenvalues are constant insofar as the inductance does not vary with the current, and that they depend only on the current if the inductance is current dependent.

These roots verify the eigenvalue equation:

$$\left(\mu_i + \frac{R_f}{L_f} \right) \cdot \mu_i + \frac{1}{L_f \cdot C_f} = 0 \quad \forall i \in \{1; 2\} \quad (4.16)$$

The eigenvalue equation (4.13) also provides directly the sum and the product of eigenvalues as significant relations:

$$\mu_1 + \mu_2 = -\frac{R_f}{L_f} \quad (4.17)$$

$$\mu_1 \cdot \mu_2 = \frac{1}{L_f \cdot C_f} \quad (4.18)$$

The double root $\mu_1 = \mu_2$ is obtained by cancellation of the *discriminant* and then, for a critical damping of the filter:

$$R_c = 2 \cdot \sqrt{\frac{L_f}{C_f}} \quad (4.19)$$

where R_c is the critical damping resistance.

If $R_f=0$, then $\mu_i = \pm i \cdot \omega$ with $\omega = \frac{1}{\sqrt{L_f \cdot C_f}}$. One recognizes the conjugate-complex poles,

and the natural angular frequency of an undamped filter.

If $R_f \neq 0$, the poles are conjugate-complex with a negative real part and if the resistance is sufficiently high, above the critical damping, the poles are negative real.

We have found again the well-known results.

4.2.4.2 Transfer Matrix Calculation

Each of the two eigenvectors is calculated by the relation:

$$(\mu_i \cdot I - A) \cdot \Pi_i = 0 \quad \forall i \in \{1; 2\} \quad (4.20)$$

that is to say:

$$\begin{bmatrix} \mu_i + \frac{R_f}{L_f} & \frac{1}{L_f} \\ -\frac{1}{C_f} & \mu_i \end{bmatrix} \cdot \begin{bmatrix} \pi_{1i} \\ \pi_{2i} \end{bmatrix} = 0 \quad (4.21)$$

Equation (4.21) is then written in the form of an equation system:

$$\left(\mu_i + \frac{R_f}{L_f} \right) \cdot \pi_{1i} + \frac{1}{L_f} \cdot \pi_{2i} = 0 \quad (4.22)$$

$$-\frac{1}{C_f} \cdot \pi_{1i} + \mu_i \cdot \pi_{2i} = 0 \quad (4.23)$$

From the second system equation we derive:

$$\pi_{1i} = \mu_i \cdot C_f \cdot \pi_{2i} \quad (4.24)$$

This equation (4.24) makes it possible to rewrite the first equation:

$$\left[\left(\mu_i + \frac{R_f}{L_f} \right) \cdot \mu_i + \frac{1}{L_f \cdot C_f} \right] \cdot \pi_{2i} = 0 \quad (4.25)$$

This equation is always verified for the two eigenvalues, whatever the π_{2i} value, owing to the fact that the first factor is identically equal to zero according to (4.16).

Thus let us choose arbitrarily $\pi_{21} = -1$ and $\pi_{22} = 1$. We thus derive according to (4.24):

$$\pi_{11} = -\mu_1 \cdot C_f \quad \text{and} \quad \pi_{12} = \mu_2 \cdot C_f \quad (4.26)$$

The transfer matrix is constituted by the eigenvectors:

$$P = \begin{bmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{bmatrix} \quad (4.27)$$

that is to say:

$$P = \begin{bmatrix} -\mu_1 \cdot C_f & \mu_2 \cdot C_f \\ -1 & 1 \end{bmatrix} \quad (4.28)$$

4.2.4.3 Transfer Matrix Inversion

The inverse of the transfer matrix is thus very simple to calculate with the determinant of the transfer matrix $(\mu_2 - \mu_1) \cdot C_f$:

$$P^{-1} = \frac{1}{(\mu_2 - \mu_1) \cdot C_f} \cdot \begin{bmatrix} 1 & -\mu_2 \cdot C_f \\ 1 & -\mu_1 \cdot C_f \end{bmatrix} \quad (4.29)$$

4.3 Discretized State-Space Equation Inversion

4.3.1 Evolution Matrix Diagonalization

One can now write the evolution matrix:

$$A = P \cdot D \cdot P^{-1} \quad (4.30)$$

The diagonalized matrix of the evolution matrix uses its eigenvalues as coefficients of its diagonal:

$$D = \begin{bmatrix} \mu_1 & 0 \\ 0 & \mu_2 \end{bmatrix} \quad (4.31)$$

Its inverse is derived simply:

$$D^{-1} = \begin{bmatrix} \frac{1}{\mu_1} & 0 \\ 0 & \frac{1}{\mu_2} \end{bmatrix} \quad (4.32)$$

4.3.2 State-Space Equation Discretization

Discretized state-space equations are obtained by integration from the initial time t_n , upto the end of the sampling period T , as we saw in section 1.2.3:

$$X(t_n + T) = F \cdot X(t_n) + G \cdot V_{t_n \rightarrow t_n + T} \quad (4.33)$$

with:

$$F = e^{A \cdot T} \quad \text{and} \quad G = A^{-1} \cdot (e^{A \cdot T} - I) \cdot B \quad (4.34)$$

If $X(t_n)$ represents the state-space vector at initial time X_0 , $X(t_n + T)$ then represents the prediction of the state-space vector X_p at the horizon $(t_n + T)$, when the control vector is known.

We use the transfer matrix and the diagonal matrix of the evolution matrix to calculate the transition matrix:

$$F = e^{A \cdot T} \Leftrightarrow F = P \cdot e^{D \cdot T} \cdot P^{-1} \quad (4.35)$$

as well as the input matrix:

$$G = A^{-1} \cdot (e^{A \cdot T} - I) \cdot B \Leftrightarrow G = A^{-1} \cdot P \cdot (e^{D \cdot T} - I) \cdot P^{-1} \cdot B \quad (4.36)$$

as we showed in section 1.2.5.

The discretized state-space equation system can then be written:

$$X_p = P \cdot e^{D \cdot T} \cdot P^{-1} \cdot X_0 + A^{-1} \cdot P \cdot (e^{D \cdot T} - I) \cdot P^{-1} \cdot B \cdot V \quad (4.37)$$

4.3.3 State-Space Vector Calculations in the Eigenvector Basis

It is now enough to project the discretized state-space equation system in the eigenvector basis multiplying on the left (4.37) by the inverse transfer matrix, and to thus separate the state-space eigenvectors.

$$P^{-1} \cdot X_p = e^{D \cdot T} \cdot P^{-1} \cdot X_0 + P^{-1} \cdot A^{-1} \cdot P \cdot (e^{D \cdot T} - I) \cdot P^{-1} \cdot B \cdot V \quad (4.38)$$

By noticing that:

$$P^{-1} \cdot A^{-1} \cdot P = (P^{-1} \cdot A \cdot P)^{-1} = D^{-1} \quad (4.39)$$

the vector relation is reduced to:

$$\boxed{P^{-1} \cdot X_p = e^{D \cdot T} \cdot P^{-1} \cdot X_0 + D^{-1} \cdot (e^{D \cdot T} - I) \cdot P^{-1} \cdot B \cdot V} \quad (4.40)$$

To obtain the final representation of the state-space equation, let us multiply the two terms of the equation by the constant $(\mu_2 - \mu_1) \cdot C_f$:

$$\begin{aligned} (\mu_2 - \mu_1) \cdot C_f \cdot P^{-1} \cdot X_p &= (\mu_2 - \mu_1) \cdot C_f \cdot e^{D \cdot T} \cdot P^{-1} \cdot X_0 + \dots \\ &\dots (\mu_2 - \mu_1) \cdot C_f \cdot D^{-1} \cdot (e^{D \cdot T} - I) \cdot P^{-1} \cdot B \cdot V \end{aligned} \quad (4.41)$$

State-space eigenvectors are now defined by:

$$\tilde{\Psi} = (\mu_2 - \mu_1) \cdot C_f \cdot [P^{-1} \cdot X] \quad (4.42)$$

By using the expression of P^{-1} (4.29):

$$\tilde{\Psi} = \begin{bmatrix} 1 & -\mu_2 \cdot C_f \\ 1 & -\mu_1 \cdot C_f \end{bmatrix} \cdot X \quad (4.43)$$

By definition of the state-space vector X (cf. equation (4.3)):

$$\tilde{\Psi} = \begin{bmatrix} 1 & -\mu_2 \cdot C_f \\ 1 & -\mu_1 \cdot C_f \end{bmatrix} \cdot \begin{bmatrix} I_l \\ U_c \end{bmatrix} \quad (4.44)$$

Its coordinates are thus:

$$\tilde{\Psi} = \begin{bmatrix} \Psi_1 \\ \Psi_2 \end{bmatrix} = \begin{bmatrix} I_l - \mu_2 \cdot C_f \cdot U_c \\ I_l - \mu_1 \cdot C_f \cdot U_c \end{bmatrix} \quad (4.45)$$

The matrix state-space equation can thus be written with this new definition by noting in addition that:

$$e^{D \cdot T} = \begin{bmatrix} e^{\mu_1 \cdot T} & 0 \\ 0 & e^{\mu_2 \cdot T} \end{bmatrix} \quad (4.46)$$

$$\tilde{\Psi}_p = e^{D \cdot T} \cdot \tilde{\Psi}_0 + \begin{bmatrix} \frac{e^{\mu_1 \cdot T} - 1}{\mu_1} & 0 \\ 0 & \frac{e^{\mu_2 \cdot T} - 1}{\mu_2} \end{bmatrix} \cdot \begin{bmatrix} 1 & -\mu_2 \cdot C_f \\ 1 & -\mu_1 \cdot C_f \end{bmatrix} \cdot \begin{bmatrix} 0 & \frac{1}{L_f} \\ -\frac{1}{C_f} & 0 \end{bmatrix} \cdot \begin{bmatrix} I_u \\ U_l \end{bmatrix} \quad (4.47)$$

$$\tilde{\Psi}_p = e^{D \cdot T} \cdot \tilde{\Psi}_0 + \begin{bmatrix} \frac{e^{\mu_1 \cdot T} - 1}{\mu_1} & 0 \\ 0 & \frac{e^{\mu_2 \cdot T} - 1}{\mu_2} \end{bmatrix} \cdot \begin{bmatrix} \mu_2 \cdot I_u + \frac{1}{L_f} \cdot U_l \\ \mu_1 \cdot I_u + \frac{1}{L_f} \cdot U_l \end{bmatrix} \quad (4.48)$$

We saw in section 4.2 with equation (4.4), that the control vector was constituted by the utilized current I_u and by the line voltage U_l which can be only measured or estimated but not controlled in the general case, and thus which is an unknown input of the system.

The control eigenvector $\tilde{V}$ is then defined by:

$$\tilde{V} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \mu_2 \cdot I_u + \frac{1}{L_f} \cdot U_l \\ \mu_1 \cdot I_u + \frac{1}{L_f} \cdot U_l \end{bmatrix} \quad (4.49)$$

The state-space equation system can now be written in a reduced way:

$$\bar{\Psi}_p = e^{D \cdot T} \cdot \bar{\Psi}_0 + \begin{bmatrix} \frac{e^{\mu_1 \cdot T} - 1}{\mu_1} & 0 \\ 0 & \frac{e^{\mu_2 \cdot T} - 1}{\mu_2} \end{bmatrix} \cdot \bar{V} \quad (4.50)$$

Let us note:

$$\boxed{\begin{aligned} a_1 &= \frac{e^{\mu_1 \cdot T} - 1}{\mu_1} \\ a_2 &= \frac{e^{\mu_2 \cdot T} - 1}{\mu_2} \end{aligned}} \quad (4.51)$$

By using equation (4.45) we obtain:

$$\boxed{\begin{aligned} \Psi_{1p} &= e^{\mu_1 \cdot T} \cdot \Psi_{10} + a_1 \cdot V_1 \\ \Psi_{2p} &= e^{\mu_2 \cdot T} \cdot \Psi_{20} + a_2 \cdot V_2 \end{aligned}} \quad (4.52)$$

4.4 Control

Analysis of the eigenstate-space equations highlights two state-space variables I_l and U_c , and two control variables of the input filter I_u and U_l . Theoretically, the two states are thus controllable individually thanks to the two control variables.

In practice, the control variable U_l is measured and, in the general case, unusable for the control. It can, however, be a control variable of the filter if the DC supply voltage is stabilized or controlled by a device which can moreover be reversible. We will only treat the most restrictive case where the upstream voltage control is not carried out or is not accessible.

As a result, with only one control variable, we can control only one of the two states at a time; we can alternately control the two states, but only through a state-transition graph, and we then carry out a *hybrid* control. The control of I_u will have to be made, as we saw, through the control of the inverter and possibly of the rheostat.

We will thus approach the control of each of the two states separately.

4.4.1 Constitution of the Set-Point State-Space Vector

We saw in the preceding section that the prediction could be expressed according to the control vector $\bar{V}$ and to the initial state $\bar{\Psi}_0$, by equation (4.50):

$$\bar{\Psi}_p = e^{D \cdot T} \cdot \bar{\Psi}_0 + \begin{bmatrix} a_1 & 0 \\ 0 & a_2 \end{bmatrix} \cdot \bar{V} \quad (4.53)$$

To control the system, we can now replace the prediction by the set-point state-space vector $\bar{\Psi}_c$, according to (4.45):

$$\bar{\Psi}_c = \begin{bmatrix} \Psi_{1c} \\ \Psi_{2c} \end{bmatrix} = \begin{bmatrix} I_{lc} - \mu_2 \cdot C_f \cdot U_{cc} \\ I_{lc} - \mu_1 \cdot C_f \cdot U_{cc} \end{bmatrix} \quad (4.54)$$

When one of the two states I_l or U_c is controlled, the other one will have to be predicted, in the case where we have only one control variable I_u .

4.4.2 Constitution of the Initial State-Space Vector

To control the system, we can now replace the initial state-space eigenvector $\bar{\Psi}_0$ by a measurement vector, according to (4.45):

$$\bar{\Psi}_0 = \begin{bmatrix} \Psi_{10} \\ \Psi_{20} \end{bmatrix}_m = \begin{bmatrix} I_{l0} - \mu_2 \cdot C_f \cdot U_{c0} \\ I_{l0} - \mu_1 \cdot C_f \cdot U_{c0} \end{bmatrix}_m \quad (4.55)$$

The line voltage used in the control vector can be either measured, or estimated. According to (4.49):

$$\bar{V} = \begin{bmatrix} \mu_2 \cdot I_u + \frac{1}{L_f} \cdot U_{l0} \\ \mu_1 \cdot I_u + \frac{1}{L_f} \cdot U_{l0} \end{bmatrix} \quad (4.56)$$

4.4.3 Inversion – Line Current Control by the Useful Current

This kind of control can be chosen when the inductance of the input filter is saturable and if one does not wish to create or to leave the magnetic saturation in order to avoid system nonlinearities. This strategy is also practicable to limit the supply current, and to thus avoid a circuit breaker opening of the energy source, during the limiting of the line overvoltages, or due to too large voltage variations of the filter capacitor.

In this case, we define in general a set-point line current I_{lc} ; the control objective is to calculate I_u , to reach and/or maintain the set-point current in just one period, if it is possible, that is, if no other limitation is encountered, and in a smallest possible number of periods complying with other limitations, if a limitation is encountered.

Within the framework of this strategy, the second state-space variable cannot be simultaneously *controlled*. It can only be *predicted*, since it is related to the line current control by the single control variable which is the useful current. It is thus this predicted variable which will create dynamic limitations via the limitations of amplitude variations accepted by the system, on both sides of the nominal voltage.

Discretized state-space equations are thus written in this precise case:

$$I_{lc} - \mu_2 \cdot C_f \cdot U_{cp} = e^{\mu_1 \cdot T} \cdot (I_{l0} - \mu_2 \cdot C_f \cdot U_{c0}) + a_1 \cdot \left(\mu_2 \cdot I_u + \frac{1}{L_f} \cdot U_{l0} \right) \quad (4.57)$$

$$I_{lc} - \mu_1 \cdot C_f \cdot U_{cp} = e^{\mu_2 \cdot T} \cdot (I_{l0} - \mu_1 \cdot C_f \cdot U_{c0}) + a_2 \cdot \left(\mu_1 \cdot I_u + \frac{1}{L_f} \cdot U_{l0} \right) \quad (4.58)$$

In this system of two equations, the known variables are:

- the set-point line current I_{lc}
- the line current measured at the initial time I_{l0}
- the capacitor voltage measured at the initial time U_{c0}
- the line voltage measured at the beginning of the computation U_{l0} . We assume, due to lack of a better estimation, that this voltage remains constant during the computational period, a period which is small compared to the electromechanical time constants, by definition of a sampling period.

Unknown variables are:

- the capacitor voltage predicted at the end of the period U_{cp}
- the control variable which is the average useful current I_u during the period.

We thus have two equations with two unknown variables. The problem is then solvable within the framework of limitation constraints.

Let us start by computing U_{cp} according to the other variables including I_{lc} , and eliminating I_u between (4.57) and (4.58):

$$U_{cp} = \frac{(a_1 \cdot \mu_2 - a_2 \cdot \mu_1) \cdot I_{lc} - (a_1 \cdot \mu_2 \cdot e^{\mu_2 \cdot T} - a_2 \cdot \mu_1 \cdot e^{\mu_1 \cdot T}) \cdot I_{l0}}{(a_1 - a_2) \cdot \mu_1 \cdot \mu_2 \cdot C_f} + \dots \\ \dots \frac{a_1 \cdot a_2 \cdot (\mu_1 - \mu_2) \cdot U_{l0} - (a_2 \cdot e^{\mu_1 \cdot T} - a_1 \cdot e^{\mu_2 \cdot T}) \cdot U_{c0}}{a_1 - a_2} \quad (4.59)$$

If the prediction of the capacitor voltage is acceptable for the system, i.e. inside the voltage interval $U_{cp} \in [U_{cmin}, U_{cmax}]$ defined for nominal operations, the set-point I_{lc} can be maintained, taking into account the initial conditions and the line voltage. Reaching this current set-point will be done in just one period.

If the set-point cannot be reached because of the capacitor voltage limits, or when there is too important a set-point modification, the preceding equation thus makes it possible to predict a capacitor voltage outside the nominal voltage range. In this case, the current set-point is temporarily abandoned for an intermediate set-point calculated starting from the same relation but replacing the predicted voltage by one of the voltage limits corresponding to the maximum or the minimum voltage limit according to the case $U_{cm} \in \{U_{cmin}; U_{cmax}\}$.

This limited intermediate set-point is thus:

$$I_{lm} = \frac{(a_1 \cdot \mu_2 \cdot e^{\mu_2 \cdot T} - a_2 \cdot \mu_1 \cdot e^{\mu_1 \cdot T}) \cdot I_{l0}}{a_1 \cdot \mu_2 - a_2 \cdot \mu_1} - \dots \\ \dots \frac{\mu_1 \cdot \mu_2 \cdot C_f \cdot [a_1 \cdot a_2 \cdot (\mu_1 - \mu_2) \cdot U_{l0} - (a_2 \cdot e^{\mu_1 \cdot T} - a_1 \cdot e^{\mu_2 \cdot T}) \cdot U_{c0} - (a_1 - a_2) \cdot U_{cm}]}{a_1 \cdot \mu_2 - a_2 \cdot \mu_1} \quad (4.60)$$

This calculation, like the preceding calculations, depends on the inductance value. We will use the value of the inductance calculated from the current measured at the initial time I_{i0} to avoid an iterative computation, which is justified insofar as the computational period is small compared to the filter time constant, by definition.

Whether the set-point is limited or not, it is now advisable to calculate the control current to obtain in just one period, the line current set-point, modified or not, by eliminating now U_{cp} , between (4.57) and (4.58):

$$I_u = \frac{(\mu_1 - \mu_2) \cdot I_{lc} - (\mu_1 \cdot e^{\mu_1 \cdot T} - \mu_2 \cdot e^{\mu_2 \cdot T}) \cdot I_{i0} - \dots}{\mu_1 \cdot \mu_2 \cdot (a_1 - a_2)} \quad (4.61)$$

$$\dots \frac{(a_1 \cdot \mu_1 - a_2 \cdot \mu_2) \cdot \frac{1}{L_f} \cdot U_{i0} - \mu_1 \cdot \mu_2 \cdot (e^{\mu_1 \cdot T} - e^{\mu_2 \cdot T}) \cdot C_f \cdot U_{c0}}{\mu_1 \cdot \mu_2 \cdot (a_1 - a_2)}$$

By noticing that, according to (4.51) and (4.18):

$$a_1 \cdot \mu_1 - a_2 \cdot \mu_2 = e^{\mu_1 \cdot T} - e^{\mu_2 \cdot T} \quad \text{and} \quad \mu_1 \cdot \mu_2 = \frac{1}{L_f \cdot C_f} \quad (4.62)$$

the preceding relation is reduced to:

$$I_u = \frac{(\mu_1 - \mu_2) \cdot I_{lc} - (\mu_1 \cdot e^{\mu_1 \cdot T} - \mu_2 \cdot e^{\mu_2 \cdot T}) \cdot I_{i0} + \mu_1 \cdot \mu_2 \cdot C_f \cdot (e^{\mu_1 \cdot T} - e^{\mu_2 \cdot T}) \cdot (U_{c0} - U_{i0})}{\mu_1 \cdot \mu_2 \cdot (a_1 - a_2)} \quad (4.63)$$

It is useful to check here that this control does not exceed the useful current limit. This useful current can be a positive one in traction operation and/or when one can use a braking resistor, but it can be negative in braking when the supply is reversible, if need be by decreasing the current reverted into the capacitor, by the use of a braking resistor.

If the control current I_u exceeds its limits related to the system sizing, the control current is then limited to the interval $I_u \in [I_{u_{\min}}, I_{u_{\max}}]$ and the set-point is not reached in only one period but in several.

4.4.4 Inversion – Capacitor Voltage Control by the Useful Current

This kind of control can be chosen to stabilize the filter capacitor voltage under nominal operations. It is the most frequent use.

In this case, starting from the measurement of the capacitor voltage, we can calculate an average capacitor voltage over one time interval longer than the filter time constant, for example $10 \cdot \sqrt{L_f \cdot C_f}$, to take account of medium-term fluctuations of the supply DC voltage.

We then use this value as a capacitor voltage set-point U_{cc} . The voltage regulation of this capacitor makes it possible to damp the oscillations which occur on the capacitor voltage at filter's natural frequency, in particular when $I_u > 0$, or during disturbances.

For some applications, in the case of a supply switching off, one may have to impose a minimum capacitor voltage during a limited time to avoid a complete stop of the system; to achieve this, we have to force the electrical braking, when it is available, to direct energy into the filter. The minimum voltage is then used as a braking set-point as soon as the loss of energy source is detected. Thus, with electrical vehicles for public transportation, on an insulated section of the supply line, the energy converter providing the supply of traction auxiliaries and supplied by the traction power filter, should not stop, to avoid losing the light and the ventilation.

The capacitor voltage set-point could be reached during just one period if no limit is reached, in particular a line or a useful current limit.

Within the framework of this strategy, the first coordinate of the state-space vector cannot be simultaneously *controlled*. It can only be *predicted*, since it is related to the capacitor voltage control by the only control variable which is the useful current. It is thus this predicted and not controlled variable which can create the limitation of dynamics via the system limitations.

The discretized state-space equations are thus written in this particular case:

$$I_{lp} - \mu_2 \cdot C_f \cdot U_{cc} = e^{\mu_1 T} \cdot (I_{l0} - \mu_2 \cdot C_f \cdot U_{c0}) + a_1 \cdot \left(\mu_2 \cdot I_u + \frac{1}{L_f} \cdot U_{l0} \right) \quad (4.64)$$

$$I_{lp} - \mu_1 \cdot C_f \cdot U_{cc} = e^{\mu_2 T} \cdot (I_{l0} - \mu_1 \cdot C_f \cdot U_{c0}) + a_2 \cdot \left(\mu_1 \cdot I_u + \frac{1}{L_f} \cdot U_{l0} \right) \quad (4.65)$$

In this system of two equations, the known variables are:

- the capacitor voltage set-point U_{cc}
- the line current measured at the initial time I_{l0}
- the capacitor voltage measured at the initial time U_{c0}
- the line voltage U_{l0} measured at the beginning of the computation. We assume, due to a lack of a better estimation, that this voltage will remain constant during the computation period, a period which is small compared to the electromechanical time constants, by definition of a sampling period.

The unknown variables are:

- the supply current, predicted at the end of the period I_{lp}
- the average useful current during the period I_u .

We now have two equations with two unknowns, so the problem is solvable within the framework of limit constraints.

We thus start by calculating the line current prediction, eliminating the second unknown, the useful current I_u , which is also the control variable in all cases.

$$I_{lp} = \frac{\mu_1 \cdot \mu_2 \cdot C_f \cdot (a_2 - a_1) \cdot U_{cc} - (a_1 \cdot \mu_2 \cdot e^{\mu_2 T} - a_2 \cdot \mu_1 \cdot e^{\mu_1 T}) \cdot I_{l0}}{a_2 \cdot \mu_1 - a_1 \cdot \mu_2} + \dots \quad (4.66)$$

$$\dots \frac{\mu_1 \cdot \mu_2 \cdot C_f \cdot [a_1 \cdot a_2 \cdot (\mu_1 - \mu_2) \cdot U_{l0} + (a_1 \cdot e^{\mu_2 T} - a_2 \cdot e^{\mu_1 T}) \cdot U_{c0}]}{a_2 \cdot \mu_1 - a_1 \cdot \mu_2}$$

If the set-point U_{cc} can only be reached with a predicted line current I_{lp} higher than the maximum supply current $I_{l_{\max}}$, which must be lower than the breaker release current of the supply, the capacitor voltage set-point must be computed again to provide an the intermediate set-point.

In the same way, if the set-point can only be reached with a predicted line current lower than the minimum line current of the supply, which can for example be the desaturation current of a line saturable inductance, the capacitor voltage set-point must also be computed again.

$I_{lm} \in \{I_{l_{\min}}; I_{l_{\max}}\}$ is the current limit; the intermediate voltage set-point, limited by the line current, can then be calculated by:

$$U_{cm} = \frac{(a_2 \cdot \mu_1 - a_1 \cdot \mu_2) \cdot I_{lm} - (a_1 \cdot \mu_2 \cdot e^{\mu_2 \cdot T} - a_2 \cdot \mu_1 \cdot e^{\mu_1 \cdot T}) \cdot I_{l0}}{(a_2 - a_1) \cdot \mu_1 \cdot \mu_2 \cdot C_f} - \dots \quad (4.67)$$

$$\dots \frac{a_1 \cdot a_2 \cdot (\mu_1 - \mu_2) \cdot U_{l0} + (a_1 \cdot e^{\mu_2 \cdot T} - a_2 \cdot e^{\mu_1 \cdot T}) \cdot U_{c0}}{a_2 - a_1}$$

Whether the set-point should be limited or not, it is now advisable to calculate the useful current to reach in one period, the capacitor voltage set-point, modified or not, eliminating I_{lp} between (4.64) and (4.65):

$$I_u = \frac{(e^{\mu_1 \cdot T} - e^{\mu_2 \cdot T}) \cdot I_{l0}}{a_2 \cdot \mu_1 - a_1 \cdot \mu_2} + \dots \quad (4.68)$$

$$\dots \frac{C_f \cdot [\mu_1 \cdot \mu_2 \cdot (a_1 - a_2) \cdot U_{l0} - (\mu_1 - \mu_2) \cdot U_{cc} - (\mu_2 \cdot e^{\mu_1 \cdot T} - \mu_1 \cdot e^{\mu_2 \cdot T}) \cdot U_{c0}]}{a_2 \cdot \mu_1 - a_1 \cdot \mu_2}$$

We will now check that this control does not exceed useful current limits. This current can be positive in a traction operation or when one can use a braking resistor, and negative in braking.

If the control current I_u leaves its limits related to the system sizing, the control current is then reduced to be within the interval $I_u \in [I_{u_{\min}}, I_{u_{\max}}]$ and the set-point is not reached in just one period but in several.

4.4.5 General Case – Control by the Useful Current

The two preceding sections enabled us to invert the state-space equations and simultaneously to treat various cases of limitation.

In the general case, it can be necessary, for example, to stabilize the capacitor voltage inside a voltage range, while limiting the line current, and then to control the line current into limits, while limiting the capacitor voltage.

Whatever the application, the stabilization of the capacitor voltage must be reached quickly during fast variations of the supply DC voltage or of the load current.

In addition, some applications must be supplied by various different types of feeding voltage. To minimize the volume and the weight of electromechanical installations, we have

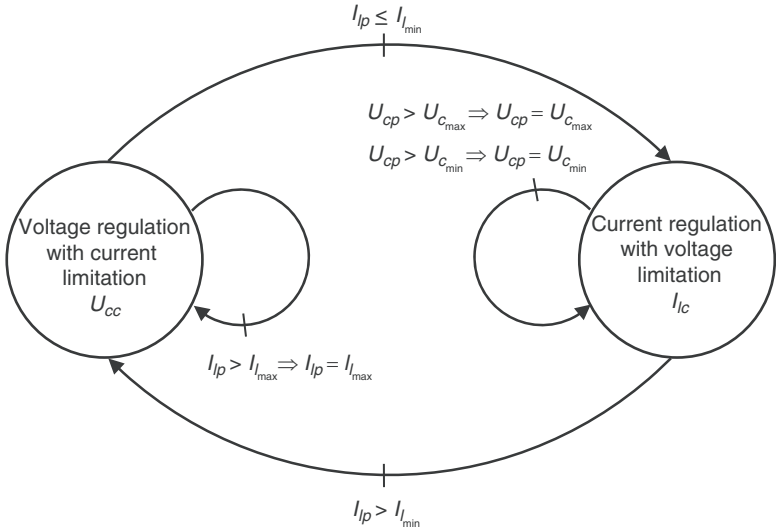


Figure 4.3 Example of a hybrid regulation with $I_{l_{min}} > 0$

to use, in place of the input filter inductance, the magnetizing inductance of a single-phase transformer with a high leakage inductance, not in use with the DC supply. The DC current in transformer windings, saturates the magnetic circuit and radically changes the inductance value, although this remains sufficient for the filtering of the DC current.

In this case, a lower limitation of the line current, for example with the condition $I_{l_{min}} > 0$, keeps the filter inductance in saturation and thus in a linear control area, without an abrupt inductance variation according to the line current.

Under these conditions, we can organize a state-transition diagram in the way represented in Figure 4.3, to meet these multiple targets.

It would be also possible to treat in a similar way, the computations for maintaining a minimum capacitor voltage in the event of a supply line insulation during a short time.

The control method of the useful current depends of course on the process which is using the supply filtering. The control variable of the input filter I_u is added algebraically to the current requested by the inverter of the controlled system and to the current requested by the braking resistor which has its own independent regulation. Limits of the potential variations of the current required by the system and by the braking resistor around nominal operations, thus define the control constraints but also the constraints of the whole system sizing.

The fast variations of the useful current, which make it possible to stabilize the voltage of an input filter, are in general of small amplitude compared to the nominal useful current.

It is not possible to study here, in a general way, the robustness of these hybrid controls, whatever the context of limitations, with or without regulation of the line voltage. Each application constitutes a particular case, and this analysis must be made as early as the system sizing stage; the computation of the LC filter and the fixing of voltage and current limits, directly govern the final system stability, compared to its dynamics of tracking or compared to its dynamics with disturbances, as we will see in the following example.

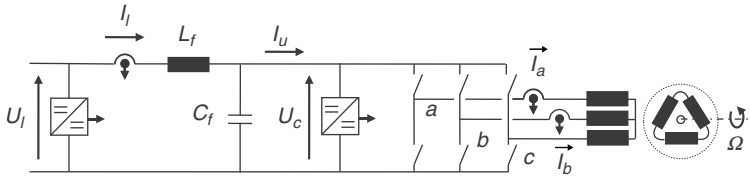


Figure 4.4 Induction machine – inverter and LC filter

Thus, if dynamics of tracking require it, it is pointless to try to fully counter in just one period the disturbances that a brutal set-point modification has itself created on the filter.

4.4.6 Example of Implementation

Let us again look at the example of an induction machine control given in section 1.4.5. The control simulation was produced in Figure 1.16, under traction operation, without LC filter between the supply and the inverter to avoid filter oscillations at its natural frequency.

We can now take again the same example only adding an input LC filter (cf. Figure 4.4), whose characteristics are the following ones:

- line inductance: $L_f = 2.8 \text{ mH}$
- resistance of the line inductance: $R_f = 25 \text{ m}\Omega$
- capacitance of the capacitor: $C_f = 18 \text{ mF}$.

4.4.6.1 Lack of Capacitor Voltage Stabilization

For the first test in this configuration, we will simply add an input filter without any regulation to provide stabilization of the capacitor voltage; this will highlight the structural instability of this configuration under traction operation (Jacquot, 1995).

We can see in Figure 4.5 that the oscillations are never limited and that divergence starts very quickly. The capacitor voltage oscillates, as does the line current.

We chose here to impose a null torque as soon as the instantaneous capacitor voltage exceeds 1,100V for a line voltage of 720V, and to maintain a null torque until the end of the imposed speed gradient. The criterion for stopping could also have been the magnitude of the supply current which is in general limited. It is also possible to combine these two criteria.

The second choice that was made was to maintain a rotor flux compatible with the speed, which does not make it possible to completely cancel the stator current, but which makes it possible to show again the complete independence between the flux and the torque control. It can be seen here that, in spite of maintaining the flux, the return to stability is gradual. Only a positive active current (traction operation) can spontaneously cause this instability beyond a small power (Jacquot, 1995).

It can also be seen that, in spite of the huge capacitor voltage fluctuations, the control tries to maintain the maximum electromagnetic torque before the request for stop, and it succeeds in doing so. Voltage and current limitations of the motor control, make it possible to preserve the traction operation within pre-established limits, and torque variations are drastically limited, taking into account the precarious conditions of the voltage supply. This property is

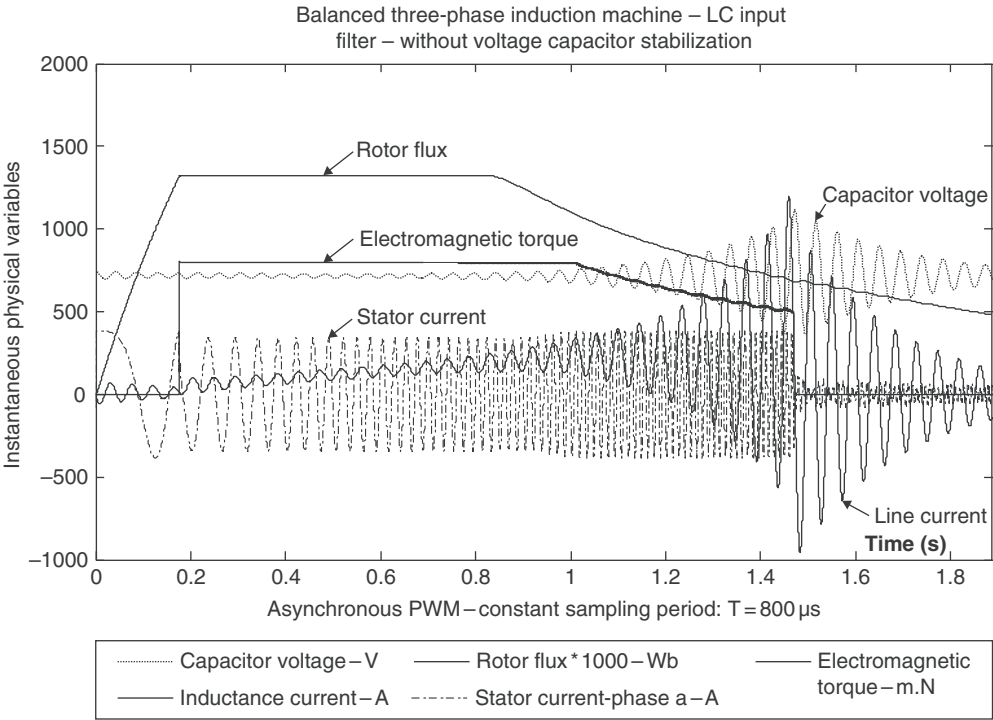


Figure 4.5 Traction operation with a LC filter, *without* voltage capacitor stabilization

related to the high dynamics of this type of control but also to the prediction of the motor behavior thanks to a motor model, and to the measurement of variations of the DC voltage supply of the inverter in the control computation.

4.4.6.2 Capacitor Voltage Stabilization

Figure 4.5 shows that variations of the capacitor voltage are due to two distinct phenomena:

- the fast useful current variation at starting, for increasing the rotor flux with a null average line current, and to the torque set-point variation when the average line current increases with speed; they are damped oscillations when the consumed active power is low
- beyond a consumed active power limit (Jacquot, 1995), oscillations are not damped anymore and the filter becomes unstable.

To contain the first phenomenon with the control which was developed in section 4.4.4, it would be necessary to counter directly and instantaneously the phenomenon which created the damped oscillations and consequently to break the dynamics of the set-point tracking; that is contrary to the required goal. In this case, it is advisable not to compensate *instantaneously* for the voltage variations, in order to preserve acceptable control dynamics, but rather to compensate for them over several sampling periods ($800 \mu s$), but in a time less than the natural filter period ($44.6 ms$). We will choose here to compensate oscillations over about 20 sampling periods.

To eliminate the instability starting from an active power threshold, it is enough to actively compensate for the capacitor voltage oscillations by useful current variations. We can thus use a differential control here.

For this particular application, where the two causes of oscillation come from the useful current, we will thus use a differential control over about 20 sampling periods.

The differential control results easily from equation (4.68) by differentiation:

$$\delta I_u = \frac{-C_f \cdot (\mu_1 - \mu_2) \cdot \delta U_{cc}}{a_2 \cdot \mu_1 - a_1 \cdot \mu_2} \quad (4.69)$$

The capacitor voltage evolves around its average voltage; the capacitor average voltage depends on the resistance of the input filter inductance and on the line current which depends itself on the torque requested by the motor and on the motor rotation speed. It is necessary of course to avoid cancelling the difference between the filter average voltage U_{cm} and the line voltage, because that would result in cancelling the useful current. The differential voltage to be cancelled can thus be expressed simply as the difference between the average capacitor voltage without voltage harmonics at the filter natural frequency U_{cm} and the measured instantaneous capacitor voltage U_{c0} :

$$\delta U_{cc} = U_{cm} - U_{c0} \quad (4.70)$$

Lastly, for example, we will limit the variation of the useful current set-point, over one sampling period at the 20th of this value, in order to not modify the dynamics of the set-point tracking too much. Ultimately, the control over about 20 steps becomes:

$$\delta I_u = \frac{1}{20} \cdot \frac{-C_f \cdot (\mu_1 - \mu_2)}{a_2 \cdot \mu_1 - a_1 \cdot \mu_2} \cdot (U_{cm} - U_{c0}) \quad (4.71)$$

This variation of the useful current set-point is added algebraically to the q axis current set-point defining the motor torque, making it possible to obtain the torque set-point.

The result is now that of Figure 4.6.

The instability of the capacitor voltage at high power has completely disappeared.

The damped voltage oscillation within about the first about 200ms sequence persists; it comes from the sharp variation of the current, making it possible to set up the flux of the induction machine. This oscillation is not compensated by the differential control, which was limited voluntarily at a fraction of the *active* current set-point, which, during this first phase, is null since there is only a magnetizing current; the suppression of the useful current control when the motor torque set-point is null, makes it possible to avoid commutating from a traction operation to a generator operation at low speed at the natural filter frequency; the filter stabilization is blocked when the torque set-point is null.

After the first phase of 200ms, the application of a torque level should make the damped oscillations start again. The stabilization control quickly cancels them to the detriment of the dynamics of the tracking control, since these two actions oppose one another. We thus see appearing the necessary torque modifications for controlling the capacitor voltage, when the braking rheostat cannot be used to increase the control current without increasing the motor torque. If the rheostat can be used, we can limit the motor torque at the maximum motor torque set-point with the rheostat, which then provides the differential of the positive current;

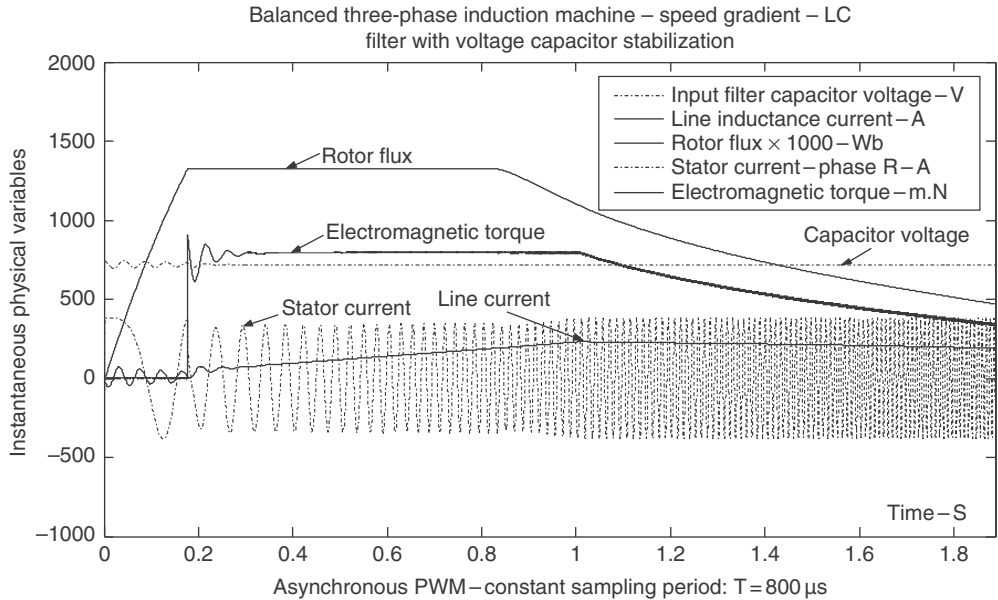


Figure 4.6 Traction operation with LC input filter and voltage stabilization

however, it is not possible to avoid torque gaps, taking into account the lack of control variable compared to the number of degrees of freedom of the system for this application.

To avoid, or limit, the high frequency torque distortion during tracking, it is necessary to adapt the sizing of the electrical filter to the required dynamics, which has important consequences in terms of weight, volume and price of this device. Furthermore, as it is possible to know whether the motor torque set-point changed quickly and/or the motor power is above the power threshold, the control can be dissociated into two controls adapted to these two control modes, operating separately by adding their control currents.

One can compare the result of Figure 4.6, with the result which one could get under the same conditions with a traditional continuous-time control (Délémontey *et al.*, 1995). The results are similar, but this control method, which allows various control aims, is more general.

In case of a line voltage variation, the problem would arise in different terms again, but it would be solvable by control equations.

4.5 Conclusions on Power LC Filter Stabilization

We demonstrated using this application, that the dead-beat control based on a model of the input filter, allows us to obtain high dynamics whether in tracking or in regulation on disturbance mode, even with an undamped system and even if one does not have the same number of control variables as the system state-space.

We had to resort to a hybrid control to achieve the targets of the line current and the capacitor voltage control with the only control variable being the line active current; this control variable is a variable linked to the motor torque control variable.

We also showed that during capacitor voltage regulation, the damping of disturbances that are related to sharp torque set-point variations cannot be made with all the dynamics authorized

by the dead-beat control because of not being opposed to the dynamics of the motor torque control itself.

To preserve dynamics in tracking of the motor torque set-point in a traction operation, it is necessary to lower the dynamics of the control of input filter oscillations at the time of sharp torque set-point variations, without preventing the filter oscillation control above the motor critical power. A trade-off was found in traction operation but dynamics in tracking of the torque set-point, were maximum with a response in a single sampling period. This trade-off is much simpler to find, each time the dynamics of the motor torque control required by the process is lower.

Despite all these extreme conditions, the dead-beat control based on a model made it possible to solve the delicate problem of the stabilization of a second-order power filter, what consolidates this method applicable to many processes.

This method could be the solution of the future, for stabilizing the switched controls of a hybrid control.

5

Conclusion

When one approaches the control of electric motors, which it is currently referred to as *variable speed*, in the industrial field of the robotics or the public transport, the engineer is immediately confronted with many problems of different types: economic, technological, of robustness and of automatic control, accuracy, dynamics, equipment integrity, reliability, maintainability, availability, people safety and electromagnetic compatibility.

Solutions of the problems posed by these various practical requirements with objectives so essentially different, require various a fundamental knowledge of electronics, electro-mechanics, data processing, mathematics, control theory and mechanics.

An engineer is well prepared to use these various theoretical tools. Nevertheless, it is obvious that all main technical choices made *upstream*, fundamentally condition the *downstream* developed solutions. The aim of the development of technical applications is to make compatible various elements of the solution to be detailed downstream, with the constraining targets of the upstream development already specified.

All designers have, at one time or another, faced a difficult technical problem, even impossible to solve completely without modifying some choices made at an earlier stage of the project which did not seem to have, a priori, a constraining effect on the realization.

Another important cause of time-delay and of cost, is still the late discovery of consequences depending, in an indirect way, on one or more of the specifications.

Thus, the structure of continuous-time controls conditions the dynamic and the stability performances of the two control modes: tracking and regulation. Dynamics condition in its turn the limit controls and safety device operation.

With a traditional continuous-time control, when one regulates the nominal dynamics with an acceptable overshoot in disturbance or in tracking mode, one is never sure not to find one or more degraded mode of operations which result in exceeding the sizing limits.

A pantograph separation from an overhead line in the event of frost, often makes it possible to test the dynamics of a continuous-time control, as well as the limits of the sizing, and the equipment integrity by the same occurrence.

What actions would it be appropriate to make, under these extreme conditions with continuous-time control? To briefly block the inverter, once, twice, three times? ...bringing with it the risk of finally destabilizing the continuous-time controls? ...then to open the circuit-breaker?

It is more secure and reliable, under extreme conditions to continue to control in degraded mode, on the limits imposed by the sizing, without utilizing the safety devices, but this aim requires very swift control.

However, if this objective has not been registered as early as the original performance specifications, the solution is too complex to implement downstream in the process development.

The trade-off between the equipment integrity and its availability is delicate in extreme situations; solutions which are developed must make it possible to work continuously under these extreme conditions, which are not as rare as one might think when one exclusively approaches the nominal operating conditions at the very beginning of the development.

The control has huge consequences on the sizing of electromechanical parts and of the kinematic chain. There are numerous examples where, at the end of the development, there are important imperfections to be solved by various adjustments, none of which are simple and many of which are not completely effective.

At the end of the process development, during the commissioning and even after, during duty operations, there are often a lot of problems to be solved with pantograph jumps, with wheel slipping and sliding of the railway traction, or the need to reduce the inverter or motor losses. The choice of electromechanical and control structures adapted to all the targets of the performance specification is thus foundational. It is essential to make these choices with a full knowledge of all the technical interdependences.

The dead-beat control of electric motors makes it possible to formalize a solution which brings together an accurate control computation under constraints (limitations) and the dynamics which are only limited by the electrical and the mechanical sizing.

In tracking mode, it is of course possible not to use the whole available dynamics of this kind of control, by adapting the set-point gradient to the process sizing. On disturbances, the dynamics can never damage the process if they are used to damp the mechanical or electrical oscillations due to natural frequencies and, at least, to avoid amplifying them.

The control method explained here, makes it possible to achieve all these targets without any instability, insofar as the control model is quite representative of the controlled process, and the measures are sampled in due time compared to the control calculation feeding. Practical achievements show that the dead-beat control is stable under these conditions with an observer. It remains to demonstrate in a theoretical way, the robustness of this approach.

The electrical power required by the control is just what is required by the application in the event of the modification of the set-point, and just what is necessary for stabilizing the process in the event of disturbances. It is up to the users of this control method to limit the instantaneous power, according to their own criteria, by modifying the set-point gradients and the electrical variable limits. By this means, one can carry out the best tradeoff between operations made at the nominal sizing and safe operations in a degraded mode.

The continuous-time state-space equations defining the model of the motor and the eigenvectors of the motor evolution matrix makes it possible to obtain exactly the discrete-time

state-space equations in the eigenvector basis. This algebraic formalism decouples the linked motor variables and makes their respective control independent.

The solution of the state-space equations is the average vector of the control voltage vector. This average vector can be generated by any type of PWM complying with the constraint of a very small PWM period compared to electromechanical time constants. The optimization of PWM types, used by a specific application, makes it possible to minimize inverter and motor losses. It can be carried out during process sizing and not in real-time. The PWM generation is performed in real-time by means of the average voltage vector calculated by the control.

All theoretical tools used in this book are well known, in particular in control theory. Their association makes it possible to formalize a control solution, by decoupling the variables to be controlled.

The control method of electric motors detailed in this book solves effectively, quickly and perfectly all technical problems of the controls under constraints (limitations), thanks to the dynamics, the accuracy, the variable decoupling and the conservation of the independence of the inverter's PWM control.

A most interesting characteristic in terms of reducing the delays before the equipment commissioning is the total absence of adjustments and tuning. Adjustments are replaced by the introduction of the motor parameters into the evolution matrix; these parameters are always known and provided by the motor manufacturer. Covariance matrices of state-space and measurement errors can be constant; they make it possible to choose variables to be filtered, to balance their filtering and to limit the measurement errors due to stochastic disturbances.

A mathematical study of the robustness has to be carrying out with a Kalman filter and estimator. Under these conditions, however, all simulations and tests made with large variations of the control model parameters compared to the motor parameters, have proved the advantage of this totally new control approach.

The up-to-date technology of power semiconductors and microprocessors makes it possible to implement the mathematical solutions presented in this work. Its algebraic formulation leads to a very simple control, executable in real-time.

The aims of the motor control detailed in Section 4 of the Introduction, are achieved and even exceeded. This control method developed for three different types of motors, allows a large freedom for designing kinematic chains by decoupling functional constraints of the initial performance specification during the process development. It will be the control of the future for all systems which can be modeled to predict their evolution, and to discriminate the control of each physical variable. So it could be the ultimate motor control.

Appendix A

Calculation of Vector PWM

The average voltage vector making it possible to obtain, at the end of the T sampling period, the set-point state-space vector, was calculated starting from a known initial state, for three different types of three-phase motors:

- a non-salient pole induction machine, equation (1.133) in section 1.3.3, in the (α, β) fixed frame
- a surface-mounted permanent-magnet synchronous motor, equation (2.81) in section 2.3.3, in the (α, β) fixed frame
- an interior permanent magnet synchronous motor (IPM-SM), equations (3.100) and (3.104) in section 3.3.3, in the (d_0, q_0) initial rotating reference frame; in this last case, it is enough to change the vector coordinates to express them in the (α, β) complex plane.

In all cases, the average control vector is thus expressed in a complex fixed frame, by its two coordinates:

$$\bar{\vec{V}}_s = V_{s\alpha} + i \cdot V_{s\beta} \quad (\text{A.1})$$

We now will approach, using an example, the realization of this voltage vector by the voltage inverter. To minimize the conduction losses regulating the average voltage over one sample period, the voltage inverter switches the supply voltage from zero to the line voltage, as shown in section 1.2; it thus remains to calculate the pulse width modulation (PWM) which will allow the average control vector to be generated.

A.1 PWM Types

There are many types of PWM for a voltage inverter, which we can classify first of all, by their frequency characteristics:

- constant frequency PWM – this is the most commonly used form of PWM used in this work. We define in this case, the fixed period T of the PWM, which can, however, change step by step, according to the frequency of the voltage fundamental component feeding the motor, to optimize the harmonic amplitudes. The fundamental PWM frequency is then asynchronous compared to the voltage fundamental component of the motor, but always higher than the motor frequency, by a factor of ten or more.
- PWM with a synchronous frequency compared to the voltage fundamental component of the motor – this solution is, of course, compatible with the control method with a dead-beat response presented in this work, but its presentation would have required significant additional developments because, in this case, the PWM period, necessarily related (at one sixth of the voltage period for symmetry reasons) to the frequency of the voltage fundamental component of the motor feeding, becomes one control variable.
- variable frequency PWM – we meet this kind of PWM for example with sliding mode or dead-band controls. The frequency of the PWM varies in particular according to the regulation band interval and also to the motor state. Frequency is thus mainly variable and asynchronous compared to the motor voltage. The harmonic spectrum is thus structurally variable and spreads out towards high frequencies.

Within each PWM class defined by the characteristics of its frequency, there exist various specific solutions for generating the PWM. They are known as *triangular wave based PWM*, symmetrical or not, *vector centered-symmetrical* or not, and synchronous ones with a variable switching number, modifiable by step.

Each type of PWM makes it possible to optimize some criteria of the current shape, such as the current peak, or the harmonic content, or the harmonic frequencies, or the harmonic distortion, or the shape factor, or the common mode voltage content (Lai, 1999).

We will begin by analyzing the generation of the symmetrical asynchronous vector PWM at a constant frequency, as an illustration, and we will then present other useful examples of PWM.

A.2 Working out the Control Voltage Vector

According to the magnitude and the angle of the average voltage vector to be generated, it is always possible to tabulate, for each type of PWM and for each average voltage level, a sequence of voltage vectors to be applied to the motor's three phases, with their application duration during the PWM period. Practically, this process can be implemented in a programmable integrated circuit of an FPGA type (*field-programmable gate array*), which frees the processor from the computation of this task.

In the particular case of the PWM presented in sections 1.2, 2.2 and 3.2, it is also convenient to directly calculate the semiconductor turn-on times, from the knowledge of the control vector (A.1).

Starting from coordinates $V_{s\alpha}$ and $V_{s\beta}$, we determine initially in which 60° angular sector the average voltage vector should be, from the six possible sectors.

For the average voltage vector $\vec{V}$, represented in Figure A.1, the angular sector is the first sector between the $\vec{V}_{(100)}$ and $\vec{V}_{(110)}$ vectors.

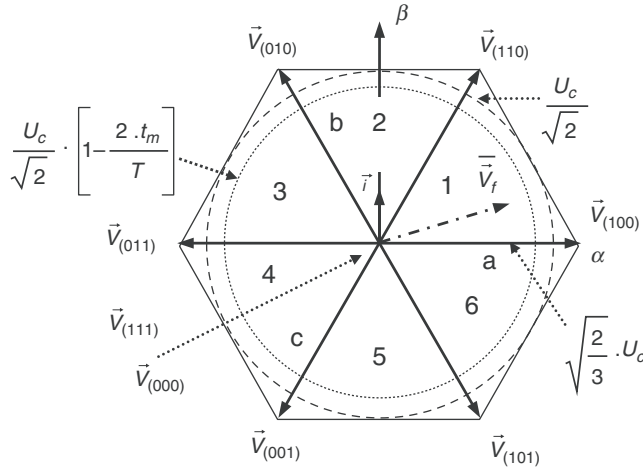


Figure A.1 Angular sector determination

Note the two vectors $\vec{V}_b$ and $\vec{V}_e$ defining the limits of a 60° angular sector found in the counterclockwise direction; their coordinates are known and are expressed simply according to the sine and the cosine of each vector angle multiple of 60° and by the constant vector magnitude equal to $\sqrt{\frac{2}{3}} \cdot U_c$:

$$\vec{V}_b = V_{b\alpha} + i \cdot V_{b\beta} \quad \text{and} \quad \vec{V}_e = V_{e\alpha} + i \cdot V_{e\beta} \quad (\text{A.2})$$

The barycentre of null voltage vectors $\vec{0}$ and of the $\vec{V}_b$ and $\vec{V}_e$ vectors, multiplied by time coefficients, which are the durations of the application of each vector, is the average voltage vector of the control:

$$\vec{\bar{V}}_s = \frac{t_0 \cdot \vec{0} + t_b \cdot \vec{V}_b + t_e \cdot \vec{V}_e + t_1 \cdot \vec{0} + t_e \cdot \vec{V}_e + t_b \cdot \vec{V}_b + t_0 \cdot \vec{0}}{2 \cdot t_0 + 2 \cdot t_b + 2 \cdot t_e + t_1} \quad (\text{A.3})$$

Because of symmetry compared to the middle of the PWM period, chosen here as an example, the elementary application times of the vectors are identical on the two sides of one half period (cf. Figure A.2).

The voltage vectors which are used to generate the PWM represented in Figure A.2, correspond to the case of the generation of an average voltage vector represented in Figure A.1.

We notice that in all cases, for this kind of PWM:

$$T = 2 \cdot t_0 + 2 \cdot t_b + 2 \cdot t_e + 2 \cdot t_1 \quad (\text{A.4})$$

We can thus reduce equation (A.3), while also eliminating all null vectors:

$$\vec{\bar{V}}_s = \frac{2 \cdot t_b \cdot \vec{V}_b + 2 \cdot t_e \cdot \vec{V}_e}{T} \quad (\text{A.5})$$

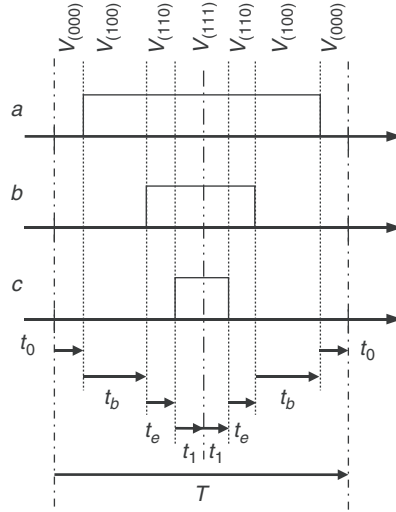


Figure A.2 Symmetrical vector modulation

Let us define:

$$T_b = 2 \cdot t_b \quad \text{and} \quad T_e = 2 \cdot t_e \quad (\text{A.6})$$

They are the total application durations of each of the two non-null vectors.

Let us now express the vectors by their complex coordinates in equation (A.5):

$$T \cdot (V_{s\alpha} + i \cdot V_{s\beta}) = T_b \cdot (V_{b\alpha} + i \cdot V_{b\beta}) + T_e \cdot (V_{e\alpha} + i \cdot V_{e\beta}) \quad (\text{A.7})$$

By separating real and imaginary parts, we obtain a system of two equations with two unknowns T_b and T_e , solvable:

$$\begin{aligned} T_b \cdot V_{b\alpha} + T_e \cdot V_{e\alpha} &= T \cdot V_{s\alpha} \\ T_b \cdot V_{b\beta} + T_e \cdot V_{e\beta} &= T \cdot V_{s\beta} \end{aligned} \quad (\text{A.8})$$

The coefficients: $V_{b\alpha}$, $V_{b\beta}$, $V_{e\alpha}$, $V_{e\beta}$, $V_{s\alpha}$, $V_{s\beta}$ and T are known.

Ultimately:

$$T_b = T \cdot \frac{V_{s\alpha} \cdot V_{e\beta} - V_{s\beta} \cdot V_{e\alpha}}{V_{b\alpha} \cdot V_{e\beta} - V_{b\beta} \cdot V_{e\alpha}} = 2 \cdot t_b \quad (\text{A.9})$$

$$T_e = T \cdot \frac{V_{s\alpha} \cdot V_{b\beta} - V_{s\beta} \cdot V_{b\alpha}}{V_{e\alpha} \cdot V_{b\beta} - V_{e\beta} \cdot V_{b\alpha}} = 2 \cdot t_e \quad (\text{A.10})$$

The application duration of the two null vectors is derived from the preceding calculations, for example at equality of the application duration of the two null vectors, since there is an additional degree of freedom to distribute application durations between the two null vectors:

$$T_0 = \frac{T - (T_b + T_e)}{2} = 2 \cdot t_0 \quad (\text{A.11})$$

$$T_1 = \frac{T - (T_b + T_e)}{2} = 2 \cdot t_1 \quad (\text{A.12})$$

In practice, it will be noticed that neither T_0 , nor T_1 can be null. At least, they must be equal to the inverter dead-time, the sum of the minimum turn-on time and the minimum turn-off time of the inverter.

The maximum average voltage applicable by the inverter is thus limited by these dead-times (cf. Figure A.1):

$$\bar{V}_M = \frac{U_c}{\sqrt{2}} \cdot \left(1 - \frac{2 \cdot t_m}{T} \right) \quad (\text{A.13})$$

A.3 Other Examples of Vector PWM

A.3.1 Unsymmetrical Vector PWM

The preceding example supposes that the vector PWM is centered. Another solution consists of doubling the PWM computation frequency and thus the average voltage vector computation frequency, while preserving the same sequence of application of the voltage vectors as for a centered one in order not to modify the inverter frequency simultaneously (cf. Figure A.3).

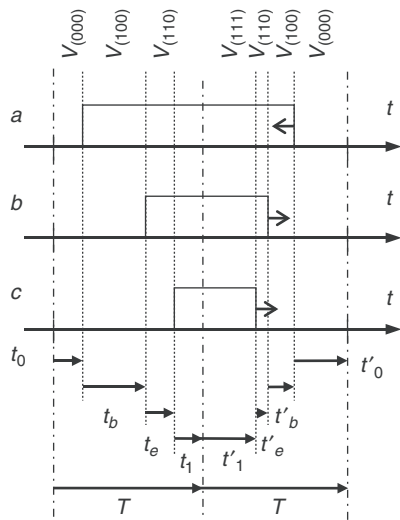


Figure A.3 Unsymmetrical vector modulation

To describe this PWM, it is necessary to draw two complete periods because the pattern is symmetrically reproduced within one period of a $2 \cdot T$ duration, with the two computations of

application times. The interest is to reduce the harmonic level generated, compared to the preceding PWM type, mainly owing to the fact that the computational period is smaller, but with a constant frequency of the power inverter switching; the produced voltage better approximates a sine wave, reducing harmonic amplitudes but with an increase in frequency without increasing the inverter losses.

The multiplication by two of the computation frequency makes it possible to modify the switching patterns of each inverter leg twice, in an independent way, to translate each time, one of the two computation results.

The example of Figure A.3 highlights, from one period to another:

- the magnitude reduction of the voltage vector to be applied, since the application duration of the two null voltage vectors increases in this case
- the angular reduction of the average voltage vector in the (α, β) fixed frame, since the application duration of the vector $V_{(110)}$ decreases compared to that of the vector $V_{(100)}$, still in the configuration of Figure A.1.

A.3.2 Symmetrical Triangular Wave Based PWM

This kind of PWM defines switching angles of inverter legs thanks to a modulating sine wave representing the voltage fundamental component to be applied to the motor and a high frequency carrier wave, here symmetrical.

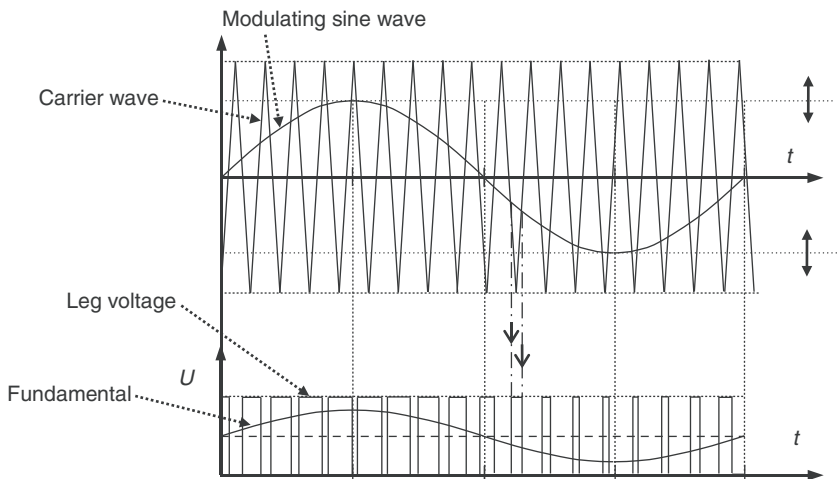


Figure A.4 Symmetric triangular wave based PWM

Figure A.4 is a PWM example of only one of the voltage inverter legs. The amplitude of the voltage fundamental component of a leg is controlled by the amplitude of the modulating sine wave. The three leg voltages are shifted by a $\frac{2\pi}{3}$ angle.

We can also add a third harmonic to the modulating sine wave, to increase the voltage application time and thus the flux magnitude; the harmonic currents of the third order, generated by

the deformation of the modulating sine wave, are compensated and cancelled in a three-phase motor and thus do not affect the torque harmonics.

This modulation principle is not really practicable for generating a quickly varying average voltage. It is more suitable for the classical slow-varying control methods where one can still define one frequency of the motor voltage fundamental component; this is not the case for a dead-beat response.

A.3.3 Synchronous PWM

If we want to reduce the harmonic distortion of the electromagnetic torque, without increasing the frequency of the inverter switching when the motor speed increases, we are led to synchronize the PWM with the leg voltage fundamental component.

Figure A.5 represents a kind of synchronous PWM with three degrees of freedom represented by the three angles, α_1 , α_2 and α_3 . These three angles define the pattern of a $\frac{\pi}{2}$ PWM segment for the first leg; the three other voltage segments of the same inverter leg are then obtained by symmetry, firstly compared to an axis parallel to the voltage magnitude axis at $\frac{\pi}{2}$, later by symmetry compared to the time axis above the π abscissa. The other leg voltage patterns are obtained by two successive phase shifts of a $\frac{2\pi}{3}$ angle.

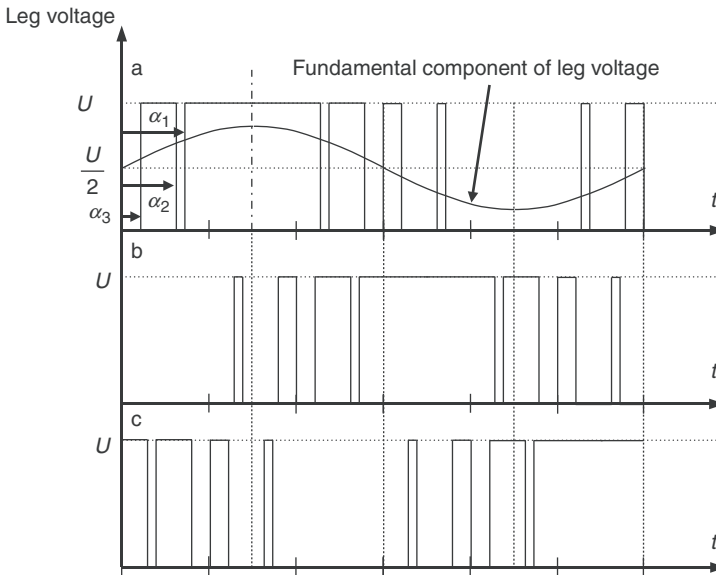


Figure A.5 Synchronous PWM with three degrees of freedom

In this case, the three degrees of freedom make it possible to regulate independently the average voltage of a $\frac{\pi}{2}$ segment, and the levels of the fifth and the seventh harmonic for example, since the even harmonics do not exist with this PWM pattern and the odd harmonic multiples of 3 (3, 9, 15, 21, etc.) are eliminated in the phase currents of the three-phase motors.

As a synchronous PWM feeds one three-phase motor, it is more balanced to choose a sampling period equal to one sixth of the period of the voltage fundamental component. It is then possible to modify the amplitude and the frequency of the fundamental component by one sixth of the period, after it has been verified that the sampling period remains short, in particular compared to the electromechanical time constants at the lowest motor voltage frequency of the use of this synchronous PWM pattern.

It is of course possible to create various types of a synchronous PWM with several degrees of freedom. As the number of degrees of freedom increases and the more the voltage generated by the voltage inverter approaches the sine wave, the more the inverter switching frequency increases; this increases the inverter switching losses. A trade-off must be thus made, and the number of the angles defining the synchronous PWM must decrease as the motor speed increases.

An important synchronous PWM is square wave modulation, but only where the six non-null vectors of the inverter are used successively; this makes it possible to reach the maximum deliverable voltage. However, it requires us to choose one control voltage vector among the six, and to calculate the application duration of each of the six inverter voltage vectors, the duration depending on the motor state and on the process set-points under the voltage and current constraints. This computation is not approached in this work.

A.4 Sampled Shape of the Voltage and Current Waves

If one applies a constant voltage during the sampling period, one gets the result presented in Figure A.6, resulting from a simulation of an induction machine at high speed and in a steady state.

The torque harmonics produce a torque ripple; the current harmonics deform the current fundamental component of the motor phase.

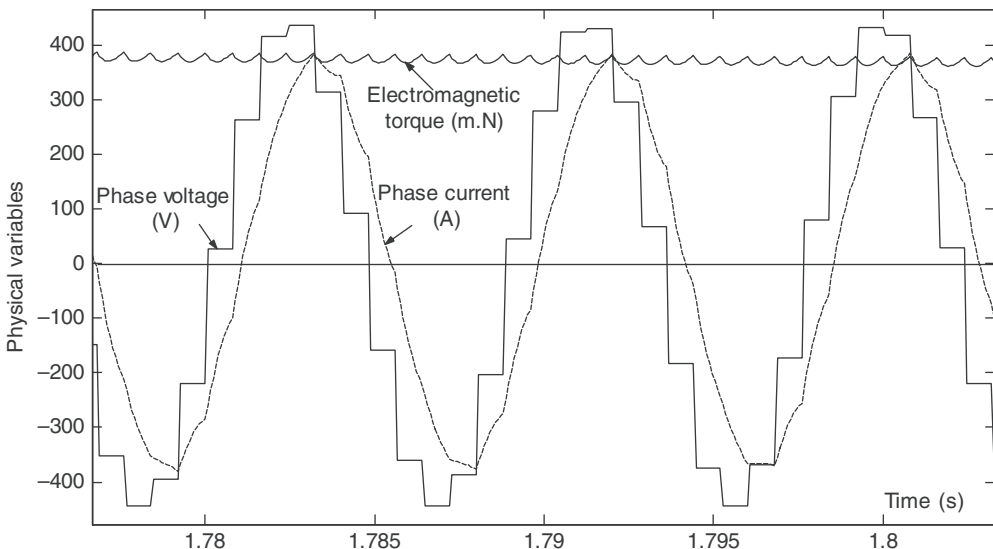


Figure A.6 Voltage, current and torque wave shapes

Appendix B

Transfer Matrix Calculation

To be able to calculate the transfer matrix in the case of an interior permanent magnet synchronous motor (IPM-SM), we will initially calculate the eigenvectors associated with the eigenvalues of the evolution matrix.

The four row eigenvectors $\vec{\Pi}_1$, $\vec{\Pi}_2$, $\vec{\Pi}_3$ and $\vec{\Pi}_4$, constitute the 4×4 transfer matrix:

$$P = \begin{bmatrix} \vec{\Pi}_1 & \vec{\Pi}_2 & \vec{\Pi}_3 & \vec{\Pi}_4 \end{bmatrix} \quad (\text{B.1})$$

B.1 First Eigenvector Calculation

Equation (3.46), applied to the first eigenvector, is written:

$$\begin{bmatrix} -\frac{R_s}{L_{sd}} & \omega_s \cdot \frac{L_{mq}}{L_{sd}} & 0 & \omega \cdot \frac{1}{L_{sd}} \\ -\omega_s \cdot \frac{L_{md}}{L_{sq}} & -\frac{R_s}{L_{sq}} & -\omega \cdot \frac{1}{L_{sq}} & 0 \\ 0 & 0 & 0 & -\omega \\ 0 & 0 & \omega & 0 \end{bmatrix} \cdot \begin{bmatrix} p_{11} \\ p_{21} \\ p_{31} \\ p_{41} \end{bmatrix} = \mu_1 \cdot \begin{bmatrix} p_{11} \\ p_{21} \\ p_{31} \\ p_{41} \end{bmatrix} \quad (\text{B.2})$$

Developing the matrix relation, we obtain the following equation system:

$$\begin{aligned}
 \left(\mu_1 + \frac{R_s}{L_{sd}} \right) \cdot p_{11} - \omega_s \cdot \frac{L_{mq}}{L_{sd}} \cdot p_{21} - \omega \cdot \frac{1}{L_{sd}} \cdot p_{41} &= 0 \\
 \omega_s \cdot \frac{L_{md}}{L_{sq}} \cdot p_{11} + \left(\mu_1 + \frac{R_s}{L_{sq}} \right) \cdot p_{21} + \omega \cdot \frac{1}{L_{sq}} \cdot p_{31} &= 0 \\
 \mu_1 \cdot p_{31} + \omega \cdot p_{41} &= 0 \\
 -\omega \cdot p_{31} + \mu_1 \cdot p_{41} &= 0
 \end{aligned} \tag{B.3}$$

The last two system equations are solved by the elimination of p_{31} , one of the two unknowns:

$$(\mu_1^2 + \omega^2) \cdot p_{41} = 0 \tag{B.4}$$

However $\mu_1^2 + \omega^2 \neq 0$, since $\mu_1 \neq 0 \forall \omega$ (cf. equation (3.34)). The only possible solution is thus:

$$p_{41} = 0 \Leftrightarrow p_{31} = 0 \tag{B.5}$$

It thus remains to solve the system of the first two equations which becomes:

$$\begin{aligned}
 \left(\mu_1 + \frac{R_s}{L_{sd}} \right) \cdot p_{11} - \omega_s \cdot \frac{L_{mq}}{L_{sd}} \cdot p_{21} &= 0 \\
 \omega_s \cdot \frac{L_{md}}{L_{sq}} \cdot p_{11} + \left(\mu_1 + \frac{R_s}{L_{sq}} \right) \cdot p_{21} &= 0
 \end{aligned} \tag{B.6}$$

These two equations are not independent, since the equation system determinant is null by definition of the eigenvalue equation (3.28).

When we eliminate one of the eigenvector unknown coordinates between the two equations (B.6), we indeed find a quadratic polynomial, a factor of the eigenvalue equation (3.31), which makes it possible to calculate the eigenvalues and which is thus identically null.

$$\left\{ \left(\mu_1 + \frac{R_s}{L_{sd}} \right) \cdot \left(\mu_1 + \frac{R_s}{L_{sq}} \right) + \left(\omega_s \cdot \frac{L_{md}}{L_{sq}} \right) \cdot \left(\omega_s \cdot \frac{L_{mq}}{L_{sd}} \right) \right\} \cdot p_{21} \equiv 0 \tag{B.7}$$

The factor of p_{21} is identically null for the first eigenvalue, which means that p_{21} can have whatever value we choose. A symmetrical reasoning can be made with μ_2 and p_{11} . The two coordinates can thus have any value, but linked by one of the two equations (B.6). They are defined except for a multiplicative constant, what fully defines a vector basis.

We can thus solve the second equation while choosing for example:

$$p_{21} = 1 \quad \text{and} \quad p_{11} = -\frac{L_{sq} \cdot \mu_1 + R_s}{\omega_s \cdot L_{md}} \tag{B.8}$$

Ultimately:

$$\vec{\Pi}_1 = \begin{bmatrix} \frac{L_{sq} \cdot \mu_1 + R_s}{\omega_s \cdot L_{md}} \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad (\text{B.9})$$

B.2 Second Eigenvector Calculation

We proceed in the same way for the second eigenvector.

$$\begin{bmatrix} -\frac{R_s}{L_{sd}} & \omega_s \cdot \frac{L_{mq}}{L_{sd}} & 0 & \omega_s \cdot \frac{1}{L_{sd}} \\ -\omega_s \cdot \frac{L_{md}}{L_{sq}} & -\frac{R_s}{L_{sq}} & -\omega_s \cdot \frac{1}{L_{sq}} & 0 \\ 0 & 0 & 0 & -\omega \\ 0 & 0 & \omega & 0 \end{bmatrix} \cdot \begin{bmatrix} p_{12} \\ p_{22} \\ p_{32} \\ p_{42} \end{bmatrix} = \mu_2 \cdot \begin{bmatrix} p_{12} \\ p_{22} \\ p_{32} \\ p_{42} \end{bmatrix} \quad (\text{B.10})$$

that is to say:

$$\begin{aligned} \left(\mu_2 + \frac{R_s}{L_{sd}} \right) \cdot p_{12} - \omega_s \cdot \frac{L_{mq}}{L_{sd}} \cdot p_{22} - \omega_s \cdot \frac{1}{L_{sd}} \cdot p_{42} &= 0 \\ \omega_s \cdot \frac{L_{md}}{L_{sq}} \cdot p_{12} + \left(\mu_2 + \frac{R_s}{L_{sq}} \right) \cdot p_{22} + \omega_s \cdot \frac{1}{L_{sq}} \cdot p_{32} &= 0 \\ \mu_2 \cdot p_{32} + \omega \cdot p_{42} &= 0 \\ -\omega \cdot p_{32} + \mu_2 \cdot p_{42} &= 0 \end{aligned} \quad (\text{B.11})$$

We solve the two last equations of the system in the same way:

$$(\mu_2^2 + \omega^2) \cdot p_{42} = 0 \quad \Rightarrow \quad p_{42} = 0 \Leftrightarrow p_{32} = 0 \quad (\text{B.12})$$

It thus remains to solve the system of the first two equations which becomes:

$$\begin{aligned} \left(\mu_2 + \frac{R_s}{L_{sd}} \right) \cdot p_{12} - \omega_s \cdot \frac{L_{mq}}{L_{sd}} \cdot p_{22} &= 0 \\ \omega_s \cdot \frac{L_{md}}{L_{sq}} \cdot p_{12} + \left(\mu_2 + \frac{R_s}{L_{sq}} \right) \cdot p_{22} &= 0 \end{aligned} \quad (\text{B.13})$$

We can thus solve the first equation while choosing for example:

$$p_{12} = 1 \quad (\text{B.14})$$

$$p_{22} = \frac{L_{sd} \cdot \mu_2 + R_s}{\omega_s \cdot L_{mq}} \quad (\text{B.15})$$

Ultimately:

$$\vec{\Pi}_2 = \begin{bmatrix} 1 \\ \frac{L_{sd} \cdot \mu_2 + R_s}{\omega_s \cdot L_{mq}} \\ 0 \\ 0 \end{bmatrix} \quad (\text{B.16})$$

B.3 Third Eigenvector Calculation

For the third eigenvector, the calculation process is similar.

$$\begin{bmatrix} -\frac{R_s}{L_{sd}} & \omega_s \cdot \frac{L_{mq}}{L_{sd}} & 0 & \omega \cdot \frac{1}{L_{sd}} \\ -\omega_s \cdot \frac{L_{md}}{L_{sq}} & -\frac{R_s}{L_{sq}} & -\omega \cdot \frac{1}{L_{sq}} & 0 \\ 0 & 0 & 0 & -\omega \\ 0 & 0 & \omega & 0 \end{bmatrix} \cdot \begin{bmatrix} p_{13} \\ p_{23} \\ p_{33} \\ p_{43} \end{bmatrix} = \mu_3 \cdot \begin{bmatrix} p_{13} \\ p_{23} \\ p_{33} \\ p_{43} \end{bmatrix} \quad (\text{B.17})$$

that is to say:

$$\begin{aligned} \left(\mu_3 + \frac{R_s}{L_{sd}} \right) \cdot p_{13} - \omega_s \cdot \frac{L_{mq}}{L_{sd}} \cdot p_{23} - \omega \cdot \frac{1}{L_{sd}} \cdot p_{43} &= 0 \\ \omega_s \cdot \frac{L_{md}}{L_{sq}} \cdot p_{13} + \left(\mu_3 + \frac{R_s}{L_{sq}} \right) \cdot p_{23} + \omega \cdot \frac{1}{L_{sq}} \cdot p_{33} &= 0 \\ \mu_3 \cdot p_{33} + \omega \cdot p_{43} &= 0 \\ -\omega \cdot p_{33} + \mu_3 \cdot p_{43} &= 0 \end{aligned} \quad (\text{B.18})$$

The two last equations of the system are solved by:

$$(\mu_3^2 + \omega^2) \cdot p_{43} = 0 \quad \text{and} \quad (\mu_3^2 + \omega^2) \cdot p_{33} = 0 \quad (\text{B.19})$$

In this particular case $\mu_3^2 + \omega^2 = 0$, since $\mu_3 = -i \cdot \omega$ (3.36). The coordinates p_{33} and p_{43} can thus be unspecified.

We have now to solve the system of the first two equations:

$$\begin{aligned} \left(\mu_3 + \frac{R_s}{L_{sd}} \right) \cdot p_{13} - \omega_s \cdot \frac{L_{mq}}{L_{sd}} \cdot p_{23} - \omega \cdot \frac{1}{L_{sd}} \cdot p_{43} &= 0 \\ \omega_s \cdot \frac{L_{md}}{L_{sq}} \cdot p_{13} + \left(\mu_3 + \frac{R_s}{L_{sq}} \right) \cdot p_{23} + \omega \cdot \frac{1}{L_{sq}} \cdot p_{33} &= 0 \end{aligned} \quad (\text{B.20})$$

Let us replace $-\omega \cdot p_{43}$ in the first two lines of system (B.20), by the relation $\mu_3 \cdot p_{33}$ drawn from the third line of (B.18):

$$\begin{aligned} (L_{sd} \cdot \mu_3 + R_s) \cdot p_{13} - \omega_s \cdot L_{mq} \cdot p_{23} + \mu_3 \cdot p_{33} &= 0 \\ \omega_s \cdot L_{md} \cdot p_{13} + (L_{sq} \cdot \mu_3 + R_s) \cdot p_{23} + \omega \cdot p_{33} &= 0 \end{aligned} \quad (\text{B.21})$$

Eliminating p_{33} between the two equations of (B.21):

$$\frac{p_{13}}{p_{23}} = \frac{\mu_3 \cdot (L_{sq} \cdot \mu_3 + R_s) + \omega \cdot \omega_s \cdot L_{mq}}{\omega \cdot (L_{sd} \cdot \mu_3 + R_s) - \mu_3 \cdot \omega_s \cdot L_{md}} \quad (\text{B.22})$$

We choose for example:

$$\begin{aligned} p_{13} &= \mu_3 \cdot (L_{sq} \cdot \mu_3 + R_s) + \omega \cdot \omega_s \cdot L_{mq} \\ p_{23} &= \omega \cdot (L_{sd} \cdot \mu_3 + R_s) - \mu_3 \cdot \omega_s \cdot L_{md} \end{aligned} \quad (\text{B.23})$$

Thus:

$$\begin{aligned} p_{33} &= -\frac{\omega_s \cdot L_{md} \cdot [\mu_3 \cdot (L_{sq} \cdot \mu_3 + R_s) + \omega \cdot \omega_s \cdot L_{mq}]}{\omega} + \dots \\ &\dots \frac{(L_{sq} \cdot \mu_3 + R_s) \cdot [\omega \cdot (L_{sd} \cdot \mu_3 + R_s) - \mu_3 \cdot \omega_s \cdot L_{md}]}{\omega} \end{aligned} \quad (\text{B.24})$$

$$p_{33} = -L_{sd} \cdot L_{sq} \cdot \left(\mu_3^2 + 2 \cdot \mu_3 \cdot \frac{R_s}{2} \cdot \frac{L_{sd} + L_{sq}}{L_{sd} \cdot L_{sq}} + 4 \cdot \frac{R_s^2 + \omega_s^2 \cdot L_{md} \cdot L_{mq}}{4 \cdot L_{sd} \cdot L_{sq}} \right) \quad (\text{B.25})$$

We then produce a quadratic polynomial:

$$p_{33} = -L_{sd} \cdot L_{sq} \cdot \left[\left(\mu_3 + \frac{R_s}{2} \cdot \frac{L_{sd} + L_{sq}}{L_{sd} \cdot L_{sq}} \right)^2 - \dots \right. \\ \left. \dots \frac{R_s^2 \cdot (L_{sd} - L_{sq})^2 - 4 \cdot \omega_s^2 \cdot L_{md} \cdot L_{mq} \cdot L_{sd} \cdot L_{sq}}{4 \cdot L_{sd}^2 \cdot L_{sq}^2} \right] \quad (\text{B.26})$$

Let us use the reduced variable (3.38):

$$\Delta^2 = R_s^2 \cdot (L_{sd} - L_{sq})^2 - 4 \cdot \omega_s^2 \cdot L_{sd} \cdot L_{sq} \cdot L_{md} \cdot L_{mq} \quad (\text{B.27})$$

We then replace the difference of the two squared terms by a product of their sum and their difference:

$$p_{33} = -L_{sd} \cdot L_{sq} \cdot \left(\mu_3 - \frac{-R_s \cdot (L_{sd} + L_{sq}) + \Delta}{2 \cdot L_{sd} \cdot L_{sq}} \right) \cdot \left(\mu_3 - \frac{-R_s \cdot (L_{sd} + L_{sq}) - \Delta}{2 \cdot L_{sd} \cdot L_{sq}} \right) \quad (\text{B.28})$$

Ultimately, we thus find a reduced form:

$$p_{33} = -L_{sd} \cdot L_{sq} \cdot (\mu_3 - \mu_1) \cdot (\mu_3 - \mu_2) \quad (\text{B.29})$$

We derive from that, according to the third relation of (B.18):

$$p_{43} = L_{sd} \cdot L_{sq} \cdot \frac{\mu_3 \cdot (\mu_3 - \mu_1) \cdot (\mu_3 - \mu_2)}{\omega} \quad (\text{B.30})$$

Ultimately:

$$\vec{\Pi}_3 = \begin{bmatrix} \mu_3 \cdot (L_{sq} \cdot \mu_3 + R_s) + \omega \cdot \omega_s \cdot L_{mq} \\ \omega \cdot (L_{sd} \cdot \mu_3 + R_s) - \mu_3 \cdot \omega_s \cdot L_{md} \\ -L_{sd} \cdot L_{sq} \cdot (\mu_3 - \mu_1) \cdot (\mu_3 - \mu_2) \\ L_{sd} \cdot L_{sq} \cdot \frac{\mu_3 \cdot (\mu_3 - \mu_1) \cdot (\mu_3 - \mu_2)}{\omega} \end{bmatrix} \quad (\text{B.31})$$

B.4 Fourth Eigenvector Calculation

The calculation is similar for the fourth eigenvector.

$$\begin{bmatrix} -\frac{R_s}{L_{sd}} & \omega_s \cdot \frac{L_{mq}}{L_{sd}} & 0 & \omega \cdot \frac{1}{L_{sd}} \\ -\omega_s \cdot \frac{L_{md}}{L_{sq}} & -\frac{R_s}{L_{sq}} & -\omega \cdot \frac{1}{L_{sq}} & 0 \\ 0 & 0 & 0 & -\omega \\ 0 & 0 & \omega & 0 \end{bmatrix} \cdot \begin{bmatrix} p_{14} \\ p_{24} \\ p_{34} \\ p_{44} \end{bmatrix} = \mu_4 \cdot \begin{bmatrix} p_{14} \\ p_{24} \\ p_{34} \\ p_{44} \end{bmatrix} \quad (\text{B.32})$$

that is to say:

$$\begin{aligned}
 \left(\mu_4 + \frac{R_s}{L_{sd}} \right) \cdot p_{14} - \omega_s \cdot \frac{L_{mq}}{L_{sd}} \cdot p_{24} - \omega \cdot \frac{1}{L_{sd}} \cdot p_{44} &= 0 \\
 \omega_s \cdot \frac{L_{md}}{L_{sq}} \cdot p_{14} + \left(\mu_4 + \frac{R_s}{L_{sq}} \right) \cdot p_{24} + \omega \cdot \frac{1}{L_{sq}} \cdot p_{34} &= 0 \\
 \mu_4 \cdot p_{34} + \omega \cdot p_{44} &= 0 \\
 -\omega \cdot p_{34} + \mu_4 \cdot p_{44} &= 0
 \end{aligned} \tag{B.33}$$

The two last equations of the system are solved, as previously, by:

$$(\mu_4^2 + \omega^2) \cdot p_{34} = 0 \quad \text{and} \quad (\mu_4^2 + \omega^2) \cdot p_{44} = 0 \tag{B.34}$$

In this case too: $(\mu_4^2 + \omega^2) \equiv 0$. The coordinates p_{34} and p_{44} can have any value. It thus remains to solve the system of the first two equations:

$$\begin{aligned}
 \left(\mu_4 + \frac{R_s}{L_{sd}} \right) \cdot p_{14} - \omega_s \cdot \frac{L_{mq}}{L_{sd}} \cdot p_{24} - \omega \cdot \frac{1}{L_{sd}} \cdot p_{44} &= 0 \\
 \omega_s \cdot \frac{L_{md}}{L_{sq}} \cdot p_{14} + \left(\mu_4 + \frac{R_s}{L_{sq}} \right) \cdot p_{24} + \omega \cdot \frac{1}{L_{sq}} \cdot p_{34} &= 0
 \end{aligned} \tag{B.35}$$

These last relations are *formally* identical to the system (B.20) by simply replacing μ_3 by μ_4 . We can thus conclude that the final vector of the transfer matrix is written:

$$\vec{\vec{\Pi}}_4 = \begin{bmatrix} \mu_4 \cdot (L_{sq} \cdot \mu_4 + R_s) + \omega \cdot \omega_s \cdot L_{mq} \\ \omega \cdot (L_{sd} \cdot \mu_4 + R_s) - \mu_4 \cdot \omega_s \cdot L_{md} \\ -L_{sd} \cdot L_{sq} \cdot (\mu_4 - \mu_1) \cdot (\mu_4 - \mu_2) \\ L_{sd} \cdot L_{sq} \cdot \frac{\mu_4 \cdot (\mu_4 - \mu_1) \cdot (\mu_4 - \mu_2)}{\omega} \end{bmatrix} \tag{B.36}$$

B.5 Transfer Matrix Calculation

It simply remains to constitute the transfer matrix starting from the eigenvectors of the evolution matrix:

$$P = [\vec{\vec{\Pi}}_1 \quad \vec{\vec{\Pi}}_2 \quad \vec{\vec{\Pi}}_3 \quad \vec{\vec{\Pi}}_4] \tag{B.37}$$

Appendix C

Transfer Matrix Inversion

To invert one matrix P , the various steps are as follows:

- calculate its determinant $\det(P)$
- calculate the cofactor p_{ij} of each element, starting from the determinant of the corresponding minor matrix $P_{(ij)}$ of P , i.e. the matrix P , in which one removes the row i and the column j :

$$c_{ij} = (-1)^{i+j} \cdot \det(P_{(ij)}) \quad (\text{C.1})$$

- constitute the cofactor matrix, or the matrix of the cofactors:

$$\text{com}(P) = [c_{ij}] \quad (\text{C.2})$$

- calculate the transpose cofactor matrix $\text{com}(P)^T$, adjugate of P
- calculate the inverse of P by:

$$P^{-1} = \frac{\text{com}(P)^T}{\det(P)} \quad (\text{C.3})$$

These algebraic calculations are heavy, rather than complex. We will, however, carry them out for the IPM-SM, for inverting the transfer matrix P (equation (B.37)); this result is indeed necessary to calculate the control vector.

The following calculations will not detail the intermediate calculation of the coefficients c_{ij} of the adjugate of the P matrix, since it is enough to:

- calculate the 3×3 determinants of the minor matrices of P , using Cramer's rules
- give to the determinants, the corresponding sign $(-1)^{i+j}$
- invert the subscripts i and j to create the coefficients cp_{ij} , of the adjugate matrix:

$$cp_{ij} = c_{ji} \quad (C.4)$$

To calculate the inverse matrix P^{-1} , we will thus calculate firstly the determinant of the transfer matrix: $\det(P)$.

If P_{ij}^{-1} gives the inverse matrix coefficients, we then will calculate each coefficient of the adjugate matrix:

$$\det(P) \cdot p_{ij}^{-1} = cp_{ij} = c_{ji} \quad (C.5)$$

The inverse matrix coefficients P_{ij}^{-1} will be derived by the division of cp_{ij} by the determinant of P .

C.1 Transfer Matrix Determinant Calculation

While developing $\det(P)$ from its first column, the transfer matrix determinant is calculated easily:

$$\begin{aligned} \det(P) = & L_{sd}^2 \cdot L_{sq}^2 \cdot (\mu_4 - \mu_2) \cdot (\mu_4 - \mu_1) \cdot (\mu_3 - \mu_2) \cdot (\mu_3 - \mu_1) \cdot (\mu_4 - \mu_3) \cdot \dots \\ & \dots \frac{(L_{sq} \cdot \mu_1 + R_s) \cdot (L_{sd} \cdot \mu_2 + R_s) + \omega_s^2 \cdot L_{md} \cdot L_{mq}}{\omega \cdot \omega_s^2 \cdot L_{md} \cdot L_{mq}} \end{aligned} \quad (C.6)$$

The last factor of the numerator can be written by using equation (3.44), the solution of the eigenvalue equation, applied to the eigenvalue μ_2 :

$$\omega_s^2 \cdot L_{md} \cdot L_{mq} = -(L_{sd} \cdot \mu_2 + R_s) \cdot (L_{sq} \cdot \mu_2 + R_s) \quad (C.7)$$

This expression makes it possible to factorize $(L_{sd} \cdot \mu_2 + R_s)$ in the expression of the determinant and in various terms of the inverse matrix which we will then calculate.

$$\boxed{\det(P) = -\frac{L_{sd}^2 \cdot L_{sq}^3 \cdot (L_{sd} \cdot \mu_2 + R_s)}{\omega \cdot \omega_s^2 \cdot L_{md} \cdot L_{mq}} \cdot \dots \cdot (\mu_4 - \mu_3) \cdot (\mu_4 - \mu_2) \cdot (\mu_4 - \mu_1) \cdot (\mu_3 - \mu_2) \cdot (\mu_3 - \mu_1) \cdot (\mu_2 - \mu_1)} \quad (C.8)$$

We can now calculate each coefficient of P^{-1} , the transfer matrix inverse.

C.2 First Row, First Column

$$\begin{aligned} \det(P) \cdot p_{11}^{-1} = & -\frac{L_{sd}^2 \cdot L_{sq}^2 \cdot (L_{sd} \cdot \mu_2 + R_s)}{\omega \cdot \omega_s \cdot L_{mq}} \cdot \dots \\ & \dots (\mu_4 - \mu_3) \cdot (\mu_4 - \mu_2) \cdot (\mu_4 - \mu_1) \cdot (\mu_3 - \mu_2) \cdot (\mu_3 - \mu_1) \end{aligned} \quad (C.9)$$

$$p_{11}^{-1} = \frac{\omega_s \cdot L_{md}}{L_{sq} \cdot (\mu_2 - \mu_1)} \quad (C.10)$$

C.3 First Row, Second Column

$$\det(P) \cdot p_{12}^{-1} = \frac{L_{sd}^2 \cdot L_{sq}^2}{\omega} \cdot (\mu_4 - \mu_3) \cdot (\mu_4 - \mu_2) \cdot (\mu_4 - \mu_1) \cdot (\mu_3 - \mu_2) \cdot (\mu_3 - \mu_1) \quad (C.11)$$

$$p_{12}^{-1} = \frac{L_{sq} \cdot \mu_2 + R_s}{L_{sq} \cdot (\mu_2 - \mu_1)} \quad (C.12)$$

C.4 First Row, Third Column

$$\begin{aligned} \det(P) \cdot p_{13}^{-1} &= \frac{L_{sd} \cdot L_{sq} \cdot (\mu_4 - \mu_3) \cdot (\mu_4 - \mu_2) \cdot (\mu_3 - \mu_2)}{\omega \cdot \omega_s \cdot L_{mq}} \dots \\ &\dots \left[\omega \cdot \omega_s \cdot L_{mq} \cdot L_{sd} \cdot (\mu_4 + \mu_3 - \mu_1) - L_{sq} \cdot \mu_3 \cdot \mu_4 \cdot (L_{sd} \cdot \mu_2 + R_s) \right] \end{aligned} \quad (C.13)$$

We can notice here that $\mu_3 + \mu_4 = 0$ and $\mu_3 \cdot \mu_4 = \omega^2$; moreover, equation (C.7), makes it possible to reduce:

$$\begin{aligned} \det(P) \cdot p_{13}^{-1} &= - \frac{L_{sd} \cdot L_{sq} \cdot (L_{sd} \cdot \mu_2 + R_s) \cdot (\mu_4 - \mu_3) \cdot (\mu_4 - \mu_2) \cdot (\mu_3 - \mu_2)}{\omega_s^2 \cdot L_{md} \cdot L_{mq}} \dots \\ &\dots \left[\omega \cdot \omega_s \cdot L_{md} \cdot L_{sq} - L_{sd} \cdot \mu_1 \cdot (L_{sq} \cdot \mu_2 + R_s) \right] \end{aligned} \quad (C.14)$$

$$p_{13}^{-1} = \frac{\omega \cdot \left[\omega_s \cdot L_{md} \cdot L_{sq} - L_{sd} \cdot \mu_1 \cdot (L_{sq} \cdot \mu_2 + R_s) \right]}{L_{sd} \cdot L_{sq}^2 \cdot (\mu_4 - \mu_1) \cdot (\mu_3 - \mu_1) \cdot (\mu_2 - \mu_1)} \quad (C.15)$$

C.5 First Row, Fourth Column

$$\begin{aligned} \det(P) \cdot p_{14}^{-1} &= - \frac{L_{sd} \cdot L_{sq} \cdot (L_{sd} \cdot \mu_2 + R_s) \cdot (\mu_4 - \mu_3) \cdot (\mu_4 - \mu_2) \cdot (\mu_3 - \mu_2)}{\omega_s^2 \cdot L_{md} \cdot L_{mq}} \dots \\ &\dots \left[\omega \cdot L_{sd} \cdot (L_{sq} \cdot \mu_2 + R_s) + \omega_s \cdot L_{md} \cdot L_{sq} \cdot \mu_1 \right] \end{aligned} \quad (C.16)$$

$$p_{14}^{-1} = \frac{\omega \cdot \left[\omega_s \cdot L_{md} \cdot L_{sq} \cdot \mu_1 + \omega \cdot L_{sd} \cdot (L_{sq} \cdot \mu_2 + R_s) \right]}{L_{sd} \cdot L_{sq}^2 \cdot (\mu_4 - \mu_1) \cdot (\mu_3 - \mu_1) \cdot (\mu_2 - \mu_1)} \quad (C.17)$$

C.6 Second Row, First Column

$$\det(P) \cdot p_{21}^{-1} = \frac{L_{sd}^2 \cdot L_{sq}^2}{\omega} \cdot (\mu_4 - \mu_3) \cdot (\mu_4 - \mu_2) \cdot (\mu_4 - \mu_1) \cdot (\mu_3 - \mu_2) \cdot (\mu_3 - \mu_1) \quad (C.18)$$

$$p_{21}^{-1} = \frac{L_{sq} \cdot \mu_2 + R_s}{L_{sq} \cdot (\mu_2 - \mu_1)} \quad (C.19)$$

C.7 Second Row, Second Column

$$\det(P) \cdot p_{22}^{-1} = \frac{L_{sd}^2 \cdot L_{sq}^2 \cdot (L_{sq} \cdot \mu_1 + R_s)}{\omega \cdot \omega_s \cdot L_{md}} \cdot \dots \quad (C.20)$$

$$\dots (\mu_4 - \mu_3) \cdot (\mu_4 - \mu_2) \cdot (\mu_4 - \mu_1) \cdot (\mu_3 - \mu_2) \cdot (\mu_3 - \mu_1)$$

$$p_{22}^{-1} = \frac{(L_{sq} \cdot \mu_1 + R_s) \cdot (L_{sq} \cdot \mu_2 + R_s)}{\omega_s \cdot L_{md} \cdot L_{sq} \cdot (\mu_2 - \mu_1)} \quad (C.21)$$

By using equations (3.44) and (3.41), one can reduce this to:

$$p_{22}^{-1} = \frac{\omega_s \cdot L_{mq}}{L_{sd} \cdot (\mu_2 - \mu_1)} \quad (C.22)$$

C.8 Second Row, Third Column

$$\det(p) \cdot P_{23}^{-1} = \frac{L_{sd} \cdot L_{sq} \cdot (\mu_4 - \mu_3) \cdot (\mu_4 - \mu_1) \cdot (\mu_3 - \mu_1)}{\omega \cdot \omega_s \cdot L_{md}} \cdot \dots \quad (C.23)$$

$$\dots \left[\omega_s \cdot L_{md} \cdot L_{sq} \cdot \mu_3 \cdot \mu_4 - \omega \cdot L_{sd} \cdot \mu_2 \cdot (L_{sq} \cdot \mu_1 + R_s) \right]$$

$$p_{23}^{-1} = \frac{(L_{sq} \cdot \mu_2 + R_s) \cdot \left[\omega_s \cdot L_{md} \cdot L_{sq} \cdot \mu_3 \cdot \mu_4 - \omega \cdot L_{sd} \cdot \mu_2 \cdot (L_{sq} \cdot \mu_1 + R_s) \right]}{\omega_s \cdot L_{md} \cdot L_{sd} \cdot L_{sq}^2 \cdot (\mu_4 - \mu_2) \cdot (\mu_3 - \mu_2) \cdot (\mu_2 - \mu_1)} \quad (C.24)$$

This new expression can be now written using equations (3.43) and (3.42):

$$p_{23}^{-1} = - \frac{\omega \cdot \left[\omega_s \cdot L_{mq} \cdot L_{sd} \cdot \mu_2 + \omega \cdot L_{sq} \cdot (L_{sd} \cdot \mu_1 + R_s) \right]}{L_{sd}^2 \cdot L_{sq} \cdot (\mu_4 - \mu_2) \cdot (\mu_3 - \mu_2) \cdot (\mu_2 - \mu_1)} \quad (C.25)$$

C.9 Second Row, Fourth Column

$$\det(P) \cdot p_{24}^{-1} = \frac{L_{sd} \cdot L_{sq} \cdot (\mu_4 - \mu_3) \cdot (\mu_4 - \mu_1) \cdot (\mu_3 - \mu_1)}{\omega_s \cdot L_{md}} \dots \quad (C.26)$$

$$\dots \left[\omega_s \cdot L_{md} \cdot L_{sq} \cdot \mu_2 + \omega \cdot L_{sd} \cdot (L_{sq} \cdot \mu_1 + R_s) \right]$$

$$p_{24}^{-1} = \frac{\omega \cdot (L_{sq} \cdot \mu_2 + R_s) \cdot \left[\omega_s \cdot L_{md} \cdot L_{sq} \cdot \mu_2 + \omega \cdot L_{sd} \cdot (L_{sq} \cdot \mu_1 + R_s) \right]}{\omega_s \cdot L_{md} \cdot L_{sd} \cdot L_{sq}^2 \cdot (\mu_4 - \mu_2) \cdot (\mu_3 - \mu_2) \cdot (\mu_2 - \mu_1)} \quad (C.27)$$

This new expression can be written, again using equations (3.43) and (3.42):

$$p_{24}^{-1} = \frac{\omega \cdot \left[\omega \cdot \omega_s \cdot L_{mq} \cdot L_{sd} - \mu_2 \cdot L_{sq} \cdot (L_{sd} \cdot \mu_1 + R_s) \right]}{L_{sd}^2 \cdot L_{sq} \cdot (\mu_4 - \mu_2) \cdot (\mu_3 - \mu_2) \cdot (\mu_2 - \mu_1)} \quad (C.28)$$

C.10 Third Row, First Column

$$p_{31}^{-1} = 0 \quad (C.29)$$

C.11 Third Row, Second Column

$$p_{32}^{-1} = 0 \quad (C.30)$$

C.12 Third Row, Third Column

$$\det(P) \cdot p_{33}^{-1} = \frac{L_{sd} \cdot L_{sq}^2 \cdot \mu_4 \cdot (L_{sd} \cdot \mu_2 + R_s)}{\omega \cdot \omega_s^2 \cdot L_{md} \cdot L_{mq}} \cdot (\mu_4 - \mu_2) \cdot (\mu_4 - \mu_1) \cdot (\mu_2 - \mu_1) \quad (C.31)$$

$$p_{33}^{-1} = - \frac{\mu_4}{L_{sd} \cdot L_{sq} \cdot (\mu_4 - \mu_3) \cdot (\mu_3 - \mu_2) \cdot (\mu_3 - \mu_1)} \quad (C.32)$$

C.13 Third Row, Fourth Column

$$\det(P) \cdot p_{34}^{-1} = \frac{L_{sd} \cdot L_{sq}^2 \cdot (L_{sd} \cdot \mu_2 + R_s)}{\omega_s^2 \cdot L_{md} \cdot L_{mq}} \cdot (\mu_4 - \mu_2) \cdot (\mu_4 - \mu_1) \cdot (\mu_2 - \mu_1) \quad (C.33)$$

$$p_{34}^{-1} = -\frac{\omega}{L_{sd} \cdot L_{sq} \cdot (\mu_4 - \mu_3) \cdot (\mu_3 - \mu_2) \cdot (\mu_3 - \mu_1)} \quad (C.34)$$

C.14 Fourth Row, First Column

$$p_{41}^{-1} = 0 \quad (C.35)$$

C.15 Fourth Row, Second Column

$$p_{42}^{-1} = 0 \quad (C.36)$$

C.16 Fourth Row, Third Column

$$\det(P) \cdot p_{43}^{-1} = -\frac{L_{sd} \cdot L_{sq}^2 \cdot \mu_3 \cdot (L_{sd} \cdot \mu_2 + R_s)}{\omega \cdot \omega_s^2 \cdot L_{md} \cdot L_{mq}} \cdot \dots \quad (C.37)$$

$$\dots (\mu_3 - \mu_2) \cdot (\mu_3 - \mu_1) \cdot (\mu_2 - \mu_1)$$

$$p_{43}^{-1} = \frac{\mu_3}{L_{sd} \cdot L_{sq} \cdot (\mu_4 - \mu_3) \cdot (\mu_4 - \mu_2) \cdot (\mu_4 - \mu_1)} \quad (C.38)$$

C.17 Fourth Row, Fourth Column

$$\det(P) \cdot p_{44}^{-1} = -\frac{L_{sd} \cdot L_{sq}^2 \cdot (L_{sd} \cdot \mu_2 + R_s)}{\omega_s^2 \cdot L_{md} \cdot L_{mq}} \cdot (\mu_3 - \mu_2) \cdot (\mu_3 - \mu_1) \cdot (\mu_2 - \mu_1) \quad (C.39)$$

$$p_{44}^{-1} = \frac{\omega}{L_{sd} \cdot L_{sq} \cdot (\mu_4 - \mu_3) \cdot (\mu_4 - \mu_2) \cdot (\mu_4 - \mu_1)} \quad (C.40)$$

C.18 Inverse Transfer Matrix Calculation

Ultimately, the transfer matrix inverse can now be expressed completely by the coefficients calculated above:

$$P^{-1} = \begin{bmatrix} p_{11}^{-1} & p_{12}^{-1} & p_{13}^{-1} & p_{14}^{-1} \\ p_{21}^{-1} & p_{22}^{-1} & p_{23}^{-1} & p_{24}^{-1} \\ 0 & 0 & p_{33}^{-1} & p_{34}^{-1} \\ 0 & 0 & p_{43}^{-1} & p_{44}^{-1} \end{bmatrix} \quad (C.41)$$

Appendix D

State-Space Eigenvector Calculation

The third and fourth coordinates of the state-space eigenvector of IPM-SM, was calculated in section 3.3.2.1.

We now have to calculate the first two coordinates of the state-space equation system:

$$P^{-1} \cdot X(t_n + T) = e^{D \cdot T} \cdot P^{-1} \cdot X(t_n) + D^{-1} \cdot (e^{D \cdot T} - I) \cdot P^{-1} \cdot B \cdot V \quad (D.1)$$

This system is calculated in section 3.2.5 (cf. equation (3.57)).

The state-space eigenvectors (cf. equation (3.76)) will be calculated from coefficients of the inverse of the matrix P (Appendix C):

$$P^{-1} \cdot X = \begin{bmatrix} p_{11}^{-1} \cdot I_{sd} + p_{12}^{-1} \cdot I_{sq} + p_{13}^{-1} \cdot \Phi_{rd} + p_{14}^{-1} \cdot \Phi_{rq} \\ p_{21}^{-1} \cdot I_{sd} + p_{22}^{-1} \cdot I_{sq} + p_{23}^{-1} \cdot \Phi_{rd} + p_{24}^{-1} \cdot \Phi_{rq} \\ p_{31}^{-1} \cdot I_{sd} + p_{32}^{-1} \cdot I_{sq} + p_{33}^{-1} \cdot \Phi_{rd} + p_{34}^{-1} \cdot \Phi_{rq} \\ p_{41}^{-1} \cdot I_{sd} + p_{42}^{-1} \cdot I_{sq} + p_{43}^{-1} \cdot \Phi_{rd} + p_{44}^{-1} \cdot \Phi_{rq} \end{bmatrix} \quad (D.2)$$

The input matrix of state-space equations was calculated (cf. equation (3.72)) in section 3.3.2:

$$D^{-1} \cdot (e^{D \cdot T} - I) \cdot P^{-1} \cdot B = \begin{bmatrix} \frac{\omega_s \cdot L_{md}}{L_{sd} \cdot L_{sq}} \cdot \frac{e^{\mu_1 \cdot T} - 1}{\mu_1 \cdot (\mu_2 - \mu_1)} & \frac{L_{sq} \cdot \mu_2 + R_s}{L_{sq}^2} \cdot \frac{e^{\mu_1 \cdot T} - 1}{\mu_1 \cdot (\mu_2 - \mu_1)} \\ \frac{L_{sq} \cdot \mu_2 + R_s}{L_{sd} \cdot L_{sq}} \cdot \frac{e^{\mu_2 \cdot T} - 1}{\mu_2 \cdot (\mu_2 - \mu_1)} & \frac{\omega_s \cdot L_{mq}}{L_{sd} \cdot L_{sq}} \cdot \frac{e^{\mu_2 \cdot T} - 1}{\mu_2 \cdot (\mu_2 - \mu_1)} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (D.3)$$

as well as the exponential function of the diagonal matrix D , multiplied by the sampling period T (cf. equation (3.69)):

$$e^{D \cdot T} = \begin{bmatrix} e^{\mu_1 \cdot T} & 0 & 0 & 0 \\ 0 & e^{\mu_2 \cdot T} & 0 & 0 \\ 0 & 0 & e^{\mu_3 \cdot T} & 0 \\ 0 & 0 & 0 & e^{\mu_4 \cdot T} \end{bmatrix} \quad (D.4)$$

Let us calculate the first coordinate of the state-space vector (D.2) in equation (D.1), written in the (d_0, q_0) reference frame:

$$\begin{aligned} & \frac{\omega_s \cdot L_{md}}{L_{sq} \cdot (\mu_2 - \mu_1)} \cdot I_{sd} + \frac{L_{sq} \cdot \mu_2 + R_s}{L_{sq} \cdot (\mu_2 - \mu_1)} \cdot I_{sq} + \frac{\omega \cdot [\omega_s \cdot L_{md} \cdot L_{sq} - L_{sd} \cdot \mu_1 \cdot (L_{sq} \cdot \mu_2 + R_s)]}{L_{sd} \cdot L_{sq}^2 \cdot (\mu_4 - \mu_1) \cdot (\mu_3 - \mu_1) \cdot (\mu_2 - \mu_1)} \cdot \Phi_{rd} + \dots \\ & \dots \frac{\omega \cdot [\omega_s \cdot L_{md} \cdot L_{sq} \cdot \mu_1 + \omega \cdot L_{sd} \cdot (L_{sq} \cdot \mu_2 + R_s)]}{L_{sd} \cdot L_{sq}^2 \cdot (\mu_4 - \mu_1) \cdot (\mu_3 - \mu_1) \cdot (\mu_2 - \mu_1)} \cdot \Phi_{rq} = e^{\mu_1 \cdot T} \cdot \dots \\ & \dots \left(\frac{\omega_s \cdot L_{md}}{L_{sq} \cdot (\mu_2 - \mu_1)} \cdot I_{sd_0} + \frac{L_{sq} \cdot \mu_2 + R_s}{L_{sq} \cdot (\mu_2 - \mu_1)} \cdot I_{sq_0} + \frac{\omega \cdot [\omega_s \cdot L_{md} \cdot L_{sq} - L_{sd} \cdot \mu_1 \cdot (L_{sq} \cdot \mu_2 + R_s)]}{L_{sd} \cdot L_{sq}^2 \cdot (\mu_4 - \mu_1) \cdot (\mu_3 - \mu_1) \cdot (\mu_2 - \mu_1)} \cdot \Phi_a \right) + \dots \\ & \dots \frac{\omega_s \cdot L_{md}}{L_{sd} \cdot L_{sq}} \cdot \frac{e^{\mu_1 \cdot T} - 1}{\mu_1 \cdot (\mu_2 - \mu_1)} \cdot V_{sd_0} + \frac{L_{sq} \cdot \mu_2 + R_s}{L_{sq}^2} \cdot \frac{e^{\mu_1 \cdot T} - 1}{\mu_1 \cdot (\mu_2 - \mu_1)} \cdot V_{sq_0} \end{aligned} \quad (D.5)$$

For the second coordinate, we obtain, in the same way:

$$\begin{aligned} & \frac{L_{sq} \cdot \mu_2 + R_s}{L_{sq} \cdot (\mu_2 - \mu_1)} \cdot I_{sd} + \frac{\omega_s \cdot L_{mq}}{L_{sd} \cdot (\mu_2 - \mu_1)} \cdot I_{sq} - \frac{\omega \cdot [\omega_s \cdot L_{mq} \cdot L_{sd} \cdot \mu_2 + \omega \cdot L_{sq} \cdot (L_{sd} \cdot \mu_1 + R_s)]}{L_{sd}^2 \cdot L_{sq} \cdot (\mu_4 - \mu_2) \cdot (\mu_3 - \mu_2) \cdot (\mu_2 - \mu_1)} \cdot \Phi_{rd} + \dots \\ & \dots \frac{\omega \cdot [\omega_s \cdot L_{mq} \cdot L_{sd} - L_{sq} \cdot \mu_2 \cdot (L_{sd} \cdot \mu_1 + R_s)]}{L_{sd}^2 \cdot L_{sq} \cdot (\mu_4 - \mu_2) \cdot (\mu_3 - \mu_2) \cdot (\mu_2 - \mu_1)} \cdot \Phi_{rq} = e^{\mu_2 \cdot T} \cdot \dots \\ & \dots \left(\frac{L_{sq} \cdot \mu_2 + R_s}{L_{sq} \cdot (\mu_2 - \mu_1)} \cdot I_{sd_0} + \frac{\omega_s \cdot L_{mq}}{L_{sd} \cdot (\mu_2 - \mu_1)} \cdot I_{sq_0} - \frac{\omega \cdot [\omega_s \cdot L_{mq} \cdot L_{sd} \cdot \mu_2 + \omega \cdot L_{sq} \cdot (L_{sd} \cdot \mu_1 + R_s)]}{L_{sd}^2 \cdot L_{sq} \cdot (\mu_4 - \mu_2) \cdot (\mu_3 - \mu_2) \cdot (\mu_2 - \mu_1)} \cdot \Phi_a \right) + \dots \\ & \dots \frac{L_{sq} \cdot \mu_2 + R_s}{L_{sd} \cdot L_{sq}} \cdot \frac{e^{\mu_2 \cdot T} - 1}{\mu_2 \cdot (\mu_2 - \mu_1)} \cdot V_{sd_0} + \frac{\omega_s \cdot L_{mq}}{L_{sd} \cdot L_{sq}} \cdot \frac{e^{\mu_2 \cdot T} - 1}{\mu_2 \cdot (\mu_2 - \mu_1)} \cdot V_{sq_0} \end{aligned} \quad (D.6)$$

For the current vector, I_{sd} expresses the abscissa of the stator current vector $\vec{I}_s$ at time $(t_n + T)$, in the (d_0, q_0) reference frame at time t_n ; the notation I_{sq} expresses the ordinate of the current vector $\vec{I}_s$ at time $(t_n + T)$, in the (d_0, q_0) reference frame, at time t_n . The notation I_{sd0} expresses the abscissa of the current vector $\vec{I}_s$ at time t_n , in the (d_0, q_0) reference frame, at time t_n ; the notation I_{sq0} expresses the ordinate of the current vector $\vec{I}_s$ at time t_n , in the (d_0, q_0) reference frame, at time t_n .

Let us use equation (3.42) to eliminate μ_2 in equation (D.5) and to eliminate μ_1 in equation (D.6). We will then multiply the two terms of (D.5) by $L_{sq} \cdot (\mu_2 - \mu_1)$ and the two terms of equation (D.6) by $L_{sd} \cdot (\mu_2 - \mu_1)$.

$$\begin{aligned}
 & \omega_s \cdot L_{md} \cdot I_{sd} - \frac{L_{sq}}{L_{sd}} \cdot (L_{sd} \cdot \mu_1 + R_s) \cdot I_{sq} + \frac{\omega \cdot [\omega \cdot \omega_s \cdot L_{md} + \mu_1 \cdot (L_{sd} \cdot \mu_1 + R_s)]}{L_{sd} \cdot (\mu_4 - \mu_1) \cdot (\mu_3 - \mu_1)} \cdot \Phi_{rd} + \dots \\
 & \dots \frac{\omega \cdot [\omega_s \cdot L_{md} \cdot \mu_1 - \omega \cdot (L_{sd} \cdot \mu_1 + R_s)]}{L_{sd} \cdot (\mu_4 - \mu_1) \cdot (\mu_3 - \mu_1)} \cdot \Phi_{rq} = e^{\mu_1 \cdot T} \cdot \dots \\
 & \dots \left(\omega_s \cdot L_{md} \cdot I_{sd0} - \frac{L_{sq}}{L_{sd}} \cdot (L_{sd} \cdot \mu_1 + R_s) \cdot I_{sq0} + \frac{\omega \cdot [\omega \cdot \omega_s \cdot L_{md} + \mu_1 \cdot (L_{sd} \cdot \mu_1 + R_s)]}{L_{sd} \cdot (\mu_4 - \mu_1) \cdot (\mu_3 - \mu_1)} \cdot \Phi_a \right) + \dots \\
 & \dots \frac{e^{\mu_1 \cdot T} - 1}{\mu_1 \cdot L_{sd}} \cdot [\omega_s \cdot L_{md} \cdot V_{sd0} - (L_{sd} \cdot \mu_1 + R_s) \cdot V_{sq0}]
 \end{aligned} \tag{D.7}$$

In the same way:

$$\begin{aligned}
 & \frac{L_{sd}}{L_{sq}} \cdot (L_{sq} \cdot \mu_2 + R_s) \cdot I_{sd} + \omega_s \cdot L_{mq} \cdot I_{sq} - \frac{\omega \cdot [\omega_s \cdot L_{mq} \cdot \mu_2 - \omega \cdot (L_{sq} \cdot \mu_2 + R_s)]}{L_{sq} \cdot (\mu_4 - \mu_2) \cdot (\mu_3 - \mu_2)} \cdot \Phi_{rd} + \dots \\
 & \dots \frac{\omega \cdot [\omega \cdot \omega_s \cdot L_{mq} + \mu_2 \cdot (L_{sq} \cdot \mu_2 + R_s)]}{L_{sq} \cdot (\mu_4 - \mu_2) \cdot (\mu_3 - \mu_2)} \cdot \Phi_{rq} = e^{\mu_2 \cdot T} \cdot \dots \\
 & \dots \left(\frac{L_{sd}}{L_{sq}} \cdot (L_{sq} \cdot \mu_2 + R_s) \cdot I_{sd0} + \omega_s \cdot L_{mq} \cdot I_{sq0} - \frac{\omega \cdot [\omega_s \cdot L_{mq} \cdot \mu_2 - \omega \cdot (L_{sq} \cdot \mu_2 + R_s)]}{L_{sq} \cdot (\mu_4 - \mu_2) \cdot (\mu_3 - \mu_2)} \cdot \Phi_a \right) + \dots \\
 & \dots \frac{e^{\mu_2 \cdot T} - 1}{\mu_2 \cdot L_{sq}} \cdot [(L_{sq} \cdot \mu_2 + R_s) \cdot V_{sd0} + \omega_s \cdot L_{mq} \cdot V_{sq0}]
 \end{aligned} \tag{D.8}$$

Lastly, after multiplication of equation (D.7) by $\frac{L_{sd}}{\omega_s \cdot L_{md}}$:

$$\begin{aligned}
 & L_{sd} \cdot I_{sd} - \frac{L_{sd} \cdot \mu_1 + R_s}{\omega_s \cdot L_{md}} \cdot L_{sq} \cdot I_{sq} + \frac{\omega \cdot \left[\omega + \mu_1 \cdot \frac{L_{sd} \cdot \mu_1 + R_s}{\omega_s \cdot L_{md}} \right]}{(\mu_4 - \mu_1) \cdot (\mu_3 - \mu_1)} \cdot \Phi_{rd} + \dots \\
 & \dots \frac{\omega \cdot \left[\mu_1 - \omega \cdot \frac{L_{sd} \cdot \mu_1 + R_s}{\omega_s \cdot L_{md}} \right]}{(\mu_4 - \mu_1) \cdot (\mu_3 - \mu_1)} \cdot \Phi_{rq} = e^{\mu_1 \cdot T} \cdot \dots \\
 & \dots \left(L_{sd} \cdot I_{sd0} - \frac{L_{sd} \cdot \mu_1 + R_s}{\omega_s \cdot L_{md}} \cdot L_{sq} \cdot I_{sq0} + \frac{\omega \cdot \left[\omega + \mu_1 \cdot \frac{L_{sd} \cdot \mu_1 + R_s}{\omega_s \cdot L_{md}} \right]}{(\mu_4 - \mu_1) \cdot (\mu_3 - \mu_1)} \cdot \Phi_a \right) + \dots \\
 & \dots \frac{e^{\mu_1 \cdot T} - 1}{\mu_1} \cdot \left[V_{sd0} - \frac{L_{sd} \cdot \mu_1 + R_s}{\omega_s \cdot L_{md}} \cdot V_{sq0} \right]
 \end{aligned} \tag{D.9}$$

After multiplication of equation (D.8) by $\frac{L_{sq}}{L_{sq} \cdot \mu_2 + R_s}$, then by replacement of $\frac{1}{L_{sq} \cdot \mu_2 + R_s}$ from its expression drawn from (3.44):

$$\frac{1}{L_{sq} \cdot \mu_2 + R_s} = -\frac{L_{sd} \cdot \mu_2 + R_s}{\omega_s^2 \cdot L_{md} \cdot L_{mq}} \tag{D.10}$$

$$\begin{aligned}
 & L_{sd} \cdot I_{sd} - \frac{L_{sd} \cdot \mu_2 + R_s}{\omega_s \cdot L_{md}} \cdot L_{sq} \cdot I_{sq} + \frac{\omega \cdot \left[\omega + \mu_2 \cdot \frac{L_{sd} \cdot \mu_2 + R_s}{\omega_s \cdot L_{md}} \right]}{(\mu_4 - \mu_2) \cdot (\mu_3 - \mu_2)} \cdot \Phi_{rd} + \dots \\
 & \dots \frac{\omega \cdot \left[\mu_2 - \omega \cdot \frac{L_{sd} \cdot \mu_2 + R_s}{\omega_s \cdot L_{md}} \right]}{(\mu_4 - \mu_2) \cdot (\mu_3 - \mu_2)} \cdot \Phi_{rq} = e^{\mu_2 \cdot T} \cdot \dots \\
 & \dots \left(L_{sd} \cdot I_{sd0} - \frac{L_{sd} \cdot \mu_2 + R_s}{\omega_s \cdot L_{md}} \cdot L_{sq} \cdot I_{sq0} + \frac{\omega \cdot \left[\omega + \mu_2 \cdot \frac{L_{sd} \cdot \mu_2 + R_s}{\omega_s \cdot L_{md}} \right]}{(\mu_4 - \mu_2) \cdot (\mu_3 - \mu_2)} \cdot \Phi_a \right) + \dots \\
 & \dots \frac{e^{\mu_2 \cdot T} - 1}{\mu_2} \cdot \left[V_{sd0} - \frac{L_{sd} \cdot \mu_2 + R_s}{\omega_s \cdot L_{md}} \cdot V_{sq0} \right]
 \end{aligned} \tag{D.11}$$

In equations (D.9) and (D.11), there now appears the reduced variables defined in equations (3.49), (3.53), (3.55) and (3.73), which give us a reduced formulation in (d_0, q_0) , for the first two coordinates:

$$\boxed{\begin{aligned} &L_{sd} \cdot I_{sd} + \zeta_{d1} \cdot L_{sq} \cdot I_{sq} + \xi_{d1} \cdot \Phi_{rd} + \xi_{D1} \cdot \Phi_{rq} = \dots \\ &\dots e^{\mu_1 \cdot T} \cdot \left(L_{sd} \cdot I_{sd_0} + \zeta_{d1} \cdot L_{sq} \cdot I_{sq_0} + \xi_{d1} \cdot \Phi_a \right) + e_1 \cdot \left(V_{sd_0} + \zeta_{d1} \cdot V_{sq_0} \right) \end{aligned}} \quad (\text{D.12})$$

$$\boxed{\begin{aligned} &L_{sd} \cdot I_{sd} + \zeta_{d2} \cdot L_{sq} \cdot I_{sq} + \xi_{d2} \cdot \Phi_{rd} + \xi_{D2} \cdot \Phi_{rq} = \dots \\ &\dots e^{\mu_2 \cdot T} \cdot \left(L_{sd} \cdot I_{sd_0} + \zeta_{d2} \cdot L_{sq} \cdot I_{sq_0} + \xi_{d2} \cdot \Phi_a \right) + e_2 \cdot \left(V_{sd_0} + \zeta_{d2} \cdot V_{sq_0} \right) \end{aligned}} \quad (\text{D.13})$$

Appendix E

F and G Matrix Calculations

The transition and the input matrix of the discretized state-space equations for the IPM-SM, must be calculated in the (d, q) reference frame, which turns with the rotor, following the rotor magnetic anisotropy, to allow to:

- filter measurements made in the (α, β) fixed frame: the two-phase currents and the position of the rotor, generally
- predict an initial state-space during the control computation.

These two matrices F and G (cf. equations (3.124)) are calculated, from the exponential function of the diagonalized evolution matrix D multiplied by the sampling period T (cf. equation (3.69)), from the input matrix B of the continuous-time state-space equations, and from the transfer matrix and its inverse, calculated in Appendix B and C respectively:

$$F = P \cdot e^{D \cdot T} \cdot P^{-1} \quad \text{and} \quad G = P \cdot D^{-1} \cdot (e^{D \cdot T} - I) \cdot P^{-1} \cdot B \quad (\text{E.1})$$

In addition, the expression of $D^{-1} \cdot (e^{D \cdot T} - I) \cdot P^{-1} \cdot B$, was already calculated in (3.72).

E.1 Transition Matrix Calculation

We will start by calculating $e^{D \cdot T} \cdot P^{-1}$, by simply creating the product of the two matrices $e^{D \cdot T}$ by P^{-1} , and then we will multiply the result by the matrix P on the left.

$$e^{DT} \cdot P^{-1} = \begin{bmatrix} e^{\mu_1 T} \cdot \frac{\omega_s \cdot L_{md}}{L_{sq} \cdot (\mu_2 - \mu_1)} & e^{\mu_1 T} \cdot \frac{L_{sq} \cdot \mu_2 + R_s}{L_{sq} \cdot (\mu_2 - \mu_1)} \\ e^{\mu_2 T} \cdot \frac{L_{sq} \cdot \mu_2 + R_s}{L_{sq} \cdot (\mu_2 - \mu_1)} & e^{\mu_2 T} \cdot \frac{\omega_s \cdot L_{mq}}{L_{sd} \cdot (\mu_2 - \mu_1)} \dots \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} e^{\mu_1 T} \cdot \frac{\omega \cdot \left[\omega_s \cdot L_{md} \cdot L_{sq} - L_{sd} \cdot \mu_1 \cdot (L_{sq} \cdot \mu_2 + R_s) \right]}{L_{sd} \cdot L_{sq}^2 \cdot (\mu_4 - \mu_1) \cdot (\mu_3 - \mu_1) \cdot (\mu_2 - \mu_1)} & e^{\mu_1 T} \cdot \frac{\omega \cdot \left[\omega_s \cdot L_{md} \cdot L_{sq} \cdot \mu_1 + \omega \cdot L_{sd} \cdot (L_{sq} \cdot \mu_2 + R_s) \right]}{L_{sd} \cdot L_{sq}^2 \cdot (\mu_4 - \mu_1) \cdot (\mu_3 - \mu_1) \cdot (\mu_2 - \mu_1)} \\ -e^{\mu_2 T} \cdot \frac{\omega \cdot \left[\omega_s \cdot L_{mq} \cdot L_{sd} \cdot \mu_2 + \omega \cdot L_{sq} \cdot (L_{sd} \cdot \mu_1 + R_s) \right]}{L_{sd}^2 \cdot L_{sq} \cdot (\mu_4 - \mu_2) \cdot (\mu_3 - \mu_2) \cdot (\mu_2 - \mu_1)} & e^{\mu_2 T} \cdot \frac{\omega \cdot \left[\omega \cdot \omega_s \cdot L_{mq} \cdot L_{sd} - L_{sq} \cdot \mu_2 \cdot (L_{sd} \cdot \mu_1 + R_s) \right]}{L_{sd}^2 \cdot L_{sq} \cdot (\mu_4 - \mu_2) \cdot (\mu_3 - \mu_2) \cdot (\mu_2 - \mu_1)} \\ \dots & \dots \\ -e^{\mu_3 T} \cdot \frac{\mu_4}{L_{sd} \cdot L_{sq} \cdot (\mu_4 - \mu_3) \cdot (\mu_3 - \mu_2) \cdot (\mu_3 - \mu_1)} & -e^{\mu_3 T} \cdot \frac{\omega}{L_{sd} \cdot L_{sq} \cdot (\mu_4 - \mu_3) \cdot (\mu_3 - \mu_2) \cdot (\mu_3 - \mu_1)} \\ e^{\mu_4 T} \cdot \frac{\mu_3}{L_{sd} \cdot L_{sq} \cdot (\mu_4 - \mu_3) \cdot (\mu_4 - \mu_2) \cdot (\mu_4 - \mu_1)} & e^{\mu_4 T} \cdot \frac{\omega}{L_{sd} \cdot L_{sq} \cdot (\mu_4 - \mu_3) \cdot (\mu_4 - \mu_2) \cdot (\mu_4 - \mu_1)} \end{bmatrix} \quad (E.2)$$

To reduce the first row of the produced matrix, we then use equations (3.42) and (3.44) and the reduced variable ζ_{d1} , defined in (3.49). A new relation is thus obtained:

$$L_{sq} \cdot \mu_2 + R_s = \zeta_{d1} \cdot \omega_s \cdot L_{md} \cdot \frac{L_{sq}}{L_{sd}} \quad (E.3)$$

To reduce the second row of the produced matrix, we again use equation (3.44), to reveal the reduced variable ζ_{d2} , defined by (3.49), as well as the new form of ζ_{d1} extracted from the preceding equation (E.3). We can thus write successively:

$$\omega_s \cdot L_{mq} = - \left(L_{sq} \cdot \mu_2 + R_s \right) \cdot \frac{L_{sd} \cdot \mu_2 + R_s}{\omega_s \cdot L_{md}} \quad (E.4)$$

$$\omega_s \cdot L_{mq} = \left(L_{sq} \cdot \mu_2 + R_s \right) \cdot \zeta_{d2} \quad (E.5)$$

$$\omega_s \cdot L_{mq} = \zeta_{d1} \cdot \zeta_{d2} \cdot \omega_s \cdot L_{md} \cdot \frac{L_{sq}}{L_{sd}} \quad (E.6)$$

A first step of the matrix simplification resulting from the product, by introduction of these reduced variables, is the following:

$$e^{D \cdot T} \cdot P^{-1} = \begin{bmatrix} e^{\mu_1 \cdot T} \cdot \frac{\omega_s \cdot L_{md}}{L_{sq} \cdot (\mu_2 - \mu_1)} & e^{\mu_1 \cdot T} \cdot \frac{\zeta_{d1} \cdot \omega_s \cdot L_{md}}{L_{sd} \cdot (\mu_2 - \mu_1)} \\ e^{\mu_2 \cdot T} \cdot \frac{\zeta_{d1} \cdot \omega_s \cdot L_{md}}{L_{sd} \cdot (\mu_2 - \mu_1)} & e^{\mu_2 \cdot T} \cdot \frac{\zeta_{d1} \cdot \zeta_{d2} \cdot \omega_s \cdot L_{md} \cdot L_{sq}}{L_{sd}^2 \cdot (\mu_2 - \mu_1)} \dots \\ 0 & 0 \\ 0 & 0 \\ e^{\mu_1 \cdot T} \cdot \frac{\omega \cdot \omega_s \cdot L_{md} \cdot (\omega - \mu_1 \cdot \zeta_{d1})}{L_{sd} \cdot L_{sq} \cdot (\mu_4 - \mu_1) \cdot (\mu_3 - \mu_1) \cdot (\mu_2 - \mu_1)} & e^{\mu_1 \cdot T} \cdot \frac{\omega \cdot \omega_s \cdot L_{md} \cdot (\mu_1 + \omega \cdot \zeta_{d1})}{L_{sd} \cdot L_{sq} \cdot (\mu_4 - \mu_1) \cdot (\mu_3 - \mu_1) \cdot (\mu_2 - \mu_1)} \\ e^{\mu_2 \cdot T} \cdot \frac{\omega \cdot \zeta_{d1} \cdot \omega_s \cdot L_{md} \cdot (\omega - \mu_2 \cdot \zeta_{d2})}{L_{sd}^2 \cdot (\mu_4 - \mu_2) \cdot (\mu_3 - \mu_2) \cdot (\mu_2 - \mu_1)} & e^{\mu_2 \cdot T} \cdot \frac{\omega \cdot \zeta_{d1} \cdot \omega_s \cdot L_{md} \cdot (\mu_2 + \omega \cdot \zeta_{d2})}{L_{sd}^2 \cdot (\mu_4 - \mu_2) \cdot (\mu_3 - \mu_2) \cdot (\mu_2 - \mu_1)} \\ \dots & \dots \\ -e^{\mu_3 \cdot T} \cdot \frac{\mu_4}{L_{sd} \cdot L_{sq} \cdot (\mu_4 - \mu_3) \cdot (\mu_3 - \mu_2) \cdot (\mu_3 - \mu_1)} & -e^{\mu_3 \cdot T} \cdot \frac{\omega}{L_{sd} \cdot L_{sq} \cdot (\mu_4 - \mu_3) \cdot (\mu_3 - \mu_2) \cdot (\mu_3 - \mu_1)} \\ e^{\mu_4 \cdot T} \cdot \frac{\mu_3}{L_{sd} \cdot L_{sq} \cdot (\mu_4 - \mu_3) \cdot (\mu_4 - \mu_2) \cdot (\mu_4 - \mu_1)} & e^{\mu_4 \cdot T} \cdot \frac{\omega}{L_{sd} \cdot L_{sq} \cdot (\mu_4 - \mu_3) \cdot (\mu_4 - \mu_2) \cdot (\mu_4 - \mu_1)} \end{bmatrix} \quad (E.7)$$

We can now write this matrix with reduced variables, using the preceding relations and equations (3.53) and (3.55):

$$e^{D \cdot T} \cdot P^{-1} = \begin{bmatrix} e^{\mu_1 \cdot T} \cdot \frac{\omega_s \cdot L_{md}}{L_{sq} \cdot (\mu_2 - \mu_1)} & e^{\mu_1 \cdot T} \cdot \frac{\zeta_{d1} \cdot \omega_s \cdot L_{md}}{L_{sd} \cdot (\mu_2 - \mu_1)} \\ e^{\mu_2 \cdot T} \cdot \frac{\zeta_{d1} \cdot \omega_s \cdot L_{md}}{L_{sd} \cdot (\mu_2 - \mu_1)} & e^{\mu_2 \cdot T} \cdot \frac{\zeta_{d1} \cdot \zeta_{d2} \cdot \omega_s \cdot L_{md} \cdot L_{sq}}{L_{sd}^2 \cdot (\mu_2 - \mu_1)} \dots \\ 0 & 0 \\ 0 & 0 \\ e^{\mu_1 \cdot T} \cdot \frac{\omega_s \cdot L_{md} \cdot \zeta_{d1}}{L_{sd} \cdot L_{sq} \cdot (\mu_2 - \mu_1)} & e^{\mu_1 \cdot T} \cdot \frac{\omega_s \cdot L_{md} \cdot \zeta_{D1}}{L_{sd} \cdot L_{sq} \cdot (\mu_2 - \mu_1)} \\ e^{\mu_2 \cdot T} \cdot \frac{\zeta_{d1} \cdot \omega_s \cdot L_{md} \cdot \zeta_{d2}}{L_{sd}^2 \cdot (\mu_2 - \mu_1)} & e^{\mu_2 \cdot T} \cdot \frac{\zeta_{d1} \cdot \omega_s \cdot L_{md} \cdot \zeta_{D2}}{L_{sd}^2 \cdot (\mu_2 - \mu_1)} \\ \dots & \dots \\ -e^{\mu_3 \cdot T} \cdot \frac{\mu_4}{L_{sd} \cdot L_{sq} \cdot (\mu_4 - \mu_3) \cdot (\mu_3 - \mu_2) \cdot (\mu_3 - \mu_1)} & -e^{\mu_3 \cdot T} \cdot \frac{\omega}{L_{sd} \cdot L_{sq} \cdot (\mu_4 - \mu_3) \cdot (\mu_3 - \mu_2) \cdot (\mu_3 - \mu_1)} \\ e^{\mu_4 \cdot T} \cdot \frac{\mu_3}{L_{sd} \cdot L_{sq} \cdot (\mu_4 - \mu_3) \cdot (\mu_4 - \mu_2) \cdot (\mu_4 - \mu_1)} & e^{\mu_4 \cdot T} \cdot \frac{\omega}{L_{sd} \cdot L_{sq} \cdot (\mu_4 - \mu_3) \cdot (\mu_4 - \mu_2) \cdot (\mu_4 - \mu_1)} \end{bmatrix} \quad (E.8)$$

From equation (3.41), now let us reveal the reduced variable ζ_{d2} , defined in (3.49):

$$\frac{L_{sq} \cdot \mu_1 + R_s}{\omega_s \cdot L_{md}} = \frac{L_{sq}}{L_{sd}} \cdot \zeta_{d2} \quad (\text{E.9})$$

This new relation will make it possible to slightly modify the expression of the transfer matrix calculated in the Appendix B.

$$P = \begin{bmatrix} -\frac{L_{sq}}{L_{sd}} \cdot \zeta_{d2} & 1 & \mu_3 \cdot (L_{sq} \cdot \mu_3 + R_s) + \omega \cdot \omega_s \cdot L_{mq} & \mu_4 \cdot (L_{sq} \cdot \mu_4 + R_s) + \omega \cdot \omega_s \cdot L_{mq} \\ 1 & -\zeta_{d2} & \omega \cdot (L_{sd} \cdot \mu_3 + R_s) - \mu_3 \cdot \omega_s \cdot L_{md} & \omega \cdot (L_{sd} \cdot \mu_4 + R_s) - \mu_4 \cdot \omega_s \cdot L_{md} \\ 0 & 0 & -L_{sd} \cdot L_{sq} \cdot (\mu_3 - \mu_1) \cdot (\mu_3 - \mu_2) & -L_{sd} \cdot L_{sq} \cdot (\mu_4 - \mu_1) \cdot (\mu_4 - \mu_2) \\ 0 & 0 & L_{sd} \cdot L_{sq} \cdot \frac{\mu_3 \cdot (\mu_3 - \mu_1) \cdot (\mu_3 - \mu_2)}{\omega} & L_{sd} \cdot L_{sq} \cdot \frac{\mu_4 \cdot (\mu_4 - \mu_1) \cdot (\mu_4 - \mu_2)}{\omega} \end{bmatrix} \quad (\text{E.10})$$

We now have to multiply on the left, the expression (E.8), by the expression of P defined in (E.10). We will thus make the calculation term by term successively, reducing whenever possible the result by use of equations (3.41) to (3.44).

$$F_{11} = \frac{\omega_s \cdot L_{md}}{L_{sd}} \cdot \frac{e^{\mu_2 \cdot T} \cdot \zeta_{d1} - e^{\mu_1 \cdot T} \cdot \zeta_{d2}}{\mu_2 - \mu_1} \quad (\text{E.11})$$

$$F_{12} = \frac{\omega_s \cdot L_{mq}}{L_{sd}} \cdot \frac{e^{\mu_2 \cdot T} - e^{\mu_1 \cdot T}}{\mu_2 - \mu_1} \quad (\text{E.12})$$

The following third term of the first row uses the intermediate variables of equations (3.56), as well as the sum and the product of the eigenvalues μ_3 and μ_4 calculated in (3.39):

$$F_{13} = \frac{\omega_s}{L_{sd}} \cdot \left(\frac{L_{md}}{L_{sd}} \cdot \frac{e^{\mu_2 \cdot T} \cdot \zeta_{d1} \cdot \zeta_{d2} - e^{\mu_1 \cdot T} \cdot \zeta_{d2} \cdot \zeta_{d1}}{\mu_2 - \mu_1} - \frac{L_{mq}}{L_{sq}} \cdot \frac{e^{\mu_4 \cdot T} \cdot \zeta_{Q4} - e^{\mu_3 \cdot T} \cdot \zeta_{Q3}}{\mu_4 - \mu_3} \right) \quad (\text{E.13})$$

In the same way, by using this time the reduced variables defined by (3.54) and (3.55):

$$F_{14} = \frac{\omega_s}{L_{sd}} \cdot \left(\frac{L_{md}}{L_{sd}} \cdot \frac{e^{\mu_2 \cdot T} \cdot \zeta_{d1} \cdot \zeta_{D2} - e^{\mu_1 \cdot T} \cdot \zeta_{d2} \cdot \zeta_{D1}}{\mu_2 - \mu_1} + \frac{L_{mq}}{L_{sq}} \cdot \frac{e^{\mu_4 \cdot T} \cdot \zeta_{q4} - e^{\mu_3 \cdot T} \cdot \zeta_{q3}}{\mu_4 - \mu_3} \right) \quad (\text{E.14})$$

Then:

$$F_{21} = \frac{\omega_s}{L_{sq}} \cdot \frac{e^{\mu_1 \cdot T} \cdot L_{md} - e^{\mu_2 \cdot T} \cdot L_{mq}}{\mu_2 - \mu_1} \quad (\text{E.15})$$

and:

$$F_{22} = \frac{\omega_s}{L_{sd}} \cdot \frac{e^{\mu_1 \cdot T} \cdot \zeta_{d1} \cdot L_{md} - e^{\mu_2 \cdot T} \cdot \zeta_{d2} \cdot L_{mq}}{\mu_2 - \mu_1} \quad (\text{E.16})$$

The following third term of the second row uses intermediate variables of equation (3.53):

$$F_{23} = \frac{\omega_s}{L_{sd} \cdot L_{sq}} \cdot \left(\frac{e^{\mu_1 \cdot T} \cdot \xi_{d1} \cdot L_{md} - e^{\mu_2 \cdot T} \cdot \xi_{d2} \cdot L_{mq} - L_{md} \cdot \frac{e^{\mu_4 \cdot T} \cdot \xi_{d4} - e^{\mu_3 \cdot T} \cdot \xi_{d3}}{\mu_4 - \mu_3}}{\mu_2 - \mu_1} \right) \quad (E.17)$$

Then, successively:

$$F_{24} = \frac{\omega_s}{L_{sd} \cdot L_{sq}} \cdot \left(\frac{e^{\mu_1 \cdot T} \cdot \xi_{D1} \cdot L_{md} - e^{\mu_2 \cdot T} \cdot \xi_{D2} \cdot L_{mq} - L_{md} \cdot \frac{e^{\mu_4 \cdot T} \cdot \xi_{q4} - e^{\mu_3 \cdot T} \cdot \xi_{q3}}{\mu_4 - \mu_3}}{\mu_2 - \mu_1} \right) \quad (E.18)$$

$$F_{31} = 0 \quad (E.19)$$

$$F_{32} = 0 \quad (E.20)$$

$$F_{33} = \frac{e^{\mu_3 \cdot T} \cdot \mu_4 - e^{\mu_4 \cdot T} \cdot \mu_3}{\mu_4 - \mu_3} \quad (E.21)$$

$$F_{34} = \omega \cdot \frac{e^{\mu_3 \cdot T} - e^{\mu_4 \cdot T}}{\mu_4 - \mu_3} \quad (E.22)$$

$$F_{41} = 0 \quad (E.23)$$

$$F_{42} = 0 \quad (E.24)$$

$$F_{43} = \omega \cdot \frac{e^{\mu_4 \cdot T} - e^{\mu_3 \cdot T}}{\mu_4 - \mu_3} \quad (E.25)$$

$$F_{44} = \frac{e^{\mu_4 \cdot T} \cdot \mu_4 - e^{\mu_3 \cdot T} \cdot \mu_3}{\mu_4 - \mu_3} \quad (E.26)$$

We can now calculate the input matrix of the discretized state-space equations in the (d, q) reference frame.

E.2 Discretized Input Matrix Calculation

To obtain G , we have now to multiply on the left the expression (3.72), by the expression of P calculated in Appendix B. We will thus make, in the same way, the calculation term by term, reducing whenever possible the result by use of equations (3.41) to (3.44).

$$G_{11} = \frac{\omega_s \cdot L_{md}}{L_{sd}^2 \cdot \mu_1 \cdot \mu_2} \cdot \frac{\mu_1 \cdot \xi_{d1} \cdot (e^{\mu_2 \cdot T} - 1) - \mu_2 \cdot \xi_{d2} \cdot (e^{\mu_1 \cdot T} - 1)}{\mu_2 - \mu_1} \quad (E.27)$$

By using the reduced variables (3.73), we obtain:

$$G_{11} = \frac{\omega_s \cdot L_{md}}{L_{sd}^2} \cdot \frac{\zeta_{d1} \cdot e_2 - \zeta_{d2} \cdot e_1}{\mu_2 - \mu_1} \quad (E.28)$$

To simplify the calculation of the second term of the first row, we will use equation (E.6)

$$G_{12} = \frac{\omega_s \cdot L_{mq}}{L_{sd} \cdot L_{sq} \cdot \mu_1 \cdot \mu_2} \cdot \frac{\mu_1 \cdot (e^{\mu_2 \cdot T} - 1) - \mu_2 \cdot (e^{\mu_1 \cdot T} - 1)}{\mu_2 - \mu_1} \quad (E.29)$$

$$G_{12} = \frac{\omega_s \cdot L_{mq}}{L_{sd} \cdot L_{sq}} \cdot \frac{e_2 - e_1}{\mu_2 - \mu_1} \quad (E.30)$$

Then successively:

$$G_{21} = \frac{\omega_s}{L_{sd} \cdot L_{sq} \cdot \mu_1 \cdot \mu_2} \cdot \frac{L_{md} \cdot \mu_1 \cdot (e^{\mu_2 \cdot T} - 1) - L_{mq} \cdot \mu_2 \cdot (e^{\mu_1 \cdot T} - 1)}{\mu_2 - \mu_1} \quad (E.31)$$

$$G_{21} = \frac{\omega_s}{L_{sd} \cdot L_{sq}} \cdot \frac{L_{md} \cdot e_1 - L_{mq} \cdot e_2}{\mu_2 - \mu_1} \quad (E.32)$$

Ultimately:

$$G_{22} = \frac{\omega_s}{L_{sd} \cdot L_{sq} \cdot \mu_1 \cdot \mu_2} \cdot \frac{\zeta_{d1} \cdot L_{md} \cdot \mu_2 \cdot (e^{\mu_1 \cdot T} - 1) - \zeta_{d2} \cdot L_{mq} \cdot \mu_1 \cdot (e^{\mu_2 \cdot T} - 1)}{\mu_2 - \mu_1} \quad (E.33)$$

$$G_{22} = \frac{\omega_s}{L_{sd} \cdot L_{sq}} \cdot \frac{\zeta_{d1} \cdot L_{md} \cdot e_1 - \zeta_{d2} \cdot L_{mq} \cdot e_2}{\mu_2 - \mu_1} \quad (E.34)$$

The coefficients of the third and the fourth row are null.

These various calculations make it possible to write the discretized input matrix G in extenso:

$$G = \frac{\omega_s}{L_{sd}} \cdot \left[\begin{array}{cc} \frac{L_{md}}{L_{sd}} \cdot \frac{\zeta_{d1} \cdot e_2 - \zeta_{d2} \cdot e_1}{\mu_2 - \mu_1} & \frac{L_{mq}}{L_{sq}} \cdot \frac{e_2 - e_1}{\mu_2 - \mu_1} \\ \frac{1}{L_{sq}} \cdot \frac{L_{md} \cdot e_1 - L_{mq} \cdot e_2}{\mu_2 - \mu_1} & \frac{1}{L_{sq}} \cdot \frac{\zeta_{d1} \cdot L_{md} \cdot e_1 - \zeta_{d2} \cdot L_{mq} \cdot e_2}{\mu_2 - \mu_1} \\ 0 & 0 \\ 0 & 0 \end{array} \right] \quad (E.35)$$

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