SOLUTIONS MANUAL

to accompany

ROCKET PROPULSION ELEMENTS, 9th EDITION

By George P. Sutton and Oscar Biblarz

This manual is in part an outgrowth of courses taught by Prof. Biblarz, both at the senior/graduate university level and as short courses. All solutions in this manual were prepared by both authors with some student collaboration. A number of design-type problems are included in several of the book's chapters and solutions to these were largely prepared by George Sutton. The style in this manual is informal.

The solutions given are mostly complete but not all problems are included, particularly in the more applied chapters. For problems which are of a "design nature" more than one answer is possible and expected. A few other problems, as presently stated, need additional information or call for assumptions or value-estimates to get started, otherwise the answers can only be given in parametric form. For the most part, all tabular and unit-conversion information may be found in the book but not necessarily in the same chapter as the problem. There are also several problems that require fundamentals not found explicitly in our book – here other standard references in the field should be consulted before searching for specialty papers from the research literature. In some chapters, this manual contains problems not found in the book which been added (together with their solutions) for the instructor's benefit and are labelled "Extra Problems".

We have attempted to be clear and accurate in the preparation of this manual. Should you discover errors or deficiencies, or wish to make technical comments about specific solutions, please contact Oscar Biblarz by e-mail at: obiblarz@nps.edu or info@oscarbiblarz.com or by regular mail at: Code ME/Bi, Department of Mechanical & Aerospace Engineering, Naval Postgraduate School, Monterey, CA 93943-5146. Any contributions to problems (particularly those presently without a solution) would be most welcome and may appear in subsequent versions of this manual.

It is intended that the distribution of this Solutions Manual for the 9th edition of Rocket Propulsion Elements be limited to professors, instructors, and other qualified teachers who use our book as a text in college courses. It was prepared to be used by individuals knowledgeable in the field and is not meant for student or other interested reader use. Inquiries for obtaining a copy of this manual should go through an appropriate Wiley sales representative. Visit www.wiley.com for more information.

CHAPTER 2

1. A jet of fluid hits a stationary flat plate in the manner shown [see textbook].
(a) If there is 50 kg of fluid flowing per minute at an absolute velocity of 200 m/sec, what will be the force on the plate?

(b) What will this force be when the plate moves in the direction of flow at u **=** 50 km/h? Explain the methodology.

$$\dot{m}(c-u_x)=F_x \quad \text{and} \quad \frac{\dot{m}}{2}u_y-\frac{\dot{m}}{2}u_y=F_y=0$$

$$\dot{m}=\rho A(c-u_x)$$

a) $\dot{m}$ = 50 kg/min, c = 200 m/sec, $u_x = 0$

F_x = (50/60)(200) = 166.7 N on plate

b) $\rho A = \dot{m}/c$ = (50/60)/200 = 0.0042 kg/m

u_x = 50 km/h = 13.9 m/sec

$F_x = 0.0042(200 - 13.9)^2$ = 144.24 N on plate

In Part b), the mass flow rate on the plate has decreased because of the decreased relative velocity. The flow is incompressible.

2. The following data are given for a certain rocket unit: thrust, 8896 N; propellant consumption, 3.867 kg/sec; velocity of vehicle, 400 m/sec; energy content of propellant, 6.911 MJ/kg. Assume 100% combustion efficiency.
Determine **(a)** the effective velocity; **(b)** the kinetic jet energy rate per unit flow of propellant; **(c)** the internal efficiency; **(d)** the propulsive efficiency; **(e)** the overall efficiency; **(f)** the specific impulse; **(g)** the specific propellant consumption.

F = 8896 N, $\dot{m}$ = 3.867 kg/sec, u = 400 m/sec, Q_R = 6.911 MJ/kg, take $\eta_{comb} = 1.0$

a) $c = F/\dot{m}$ = 8896/3.867 = 2,300 m/sec

b) $\dfrac{E_{jet}}{m}=\dfrac{P_{jet}}{\dot{m}}=\dfrac{0.5\dot{m}v_2^{\,2}}{\dot{m}}=0.5c^2 = 2.645\text{x}10^6$ m^2/sec^2 or 2.645 MJ/kg

c) $$\eta_{\text{int}}=\frac{0.5\dot{m}c^2}{\dot{m}Q_R\eta_{comb}}=\frac{c^2}{2Q_R}=0.383$$

d) $$\eta_p=\frac{2u/c}{1+(u/c)^2}=\frac{2(400/2300)}{1+(400/2300)^2}=0.3376$$

e) $\eta=\eta_p\eta_{\text{int}}=12.9\ \%$

OR $$\eta=\frac{Fu}{\dot{m}Q_R+0.5\dot{m}u^2}=13.3\%$$

f) $I_s = \dfrac{F}{mg_0} = \dfrac{8896}{3.867\mathbf{x}9.81} = 234.5$ sec

g) $SFC = 1/I_s = 0.00426$ sec^{-1}

3. A certain rocket has an effective exhaust velocity of 7000 ft/sec; it consumes 280 lbm/sec of propellant mass, each of which liberates 2400 Btu/lbm. The unit operates for 65 sec. Construct a set of curves plotting the propulsive, internal, and overall efficiencies versus the velocity ratio $u/c(0 < u/c < 1.0)$. The rated flight velocity equals 5000 ft/sec. Calculate **(a)** the specific impulse; **(b)** the total impulse; **(c)** the mass of propellants required; **(d)** the volume that the propellants occupy if their average specific gravity is 0.925. Neglect gravity and drag.

[Assume sea-level operation where 1 lbm weights 1 lbf so there is no numerical distinction between the two units.]

c = 7000 ft/se, $\dot{m}$ = 280 lbm/sec, Q_R = 2400 Btu/lbm, t_p = 65 sec, u_{RATED} = 5000 ft/sec.

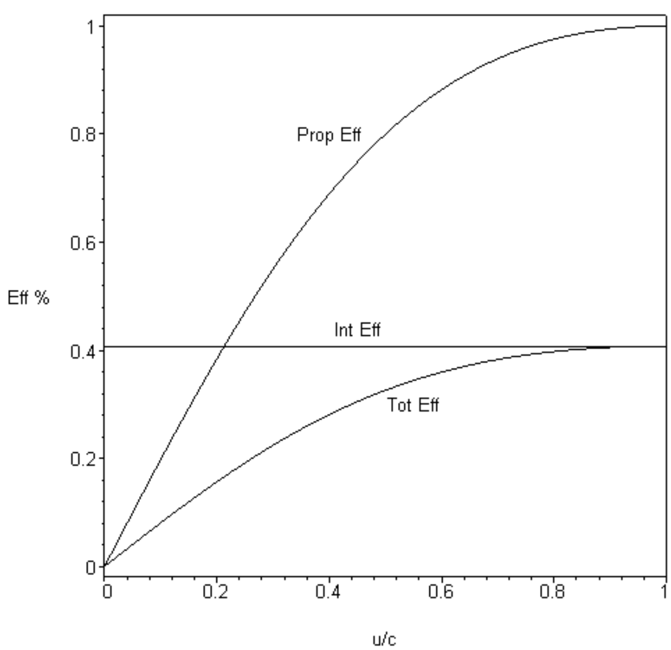
a) $I_s = c/g_0 = 7000/32.174 = 217.5$ sec.

b) $I_t = Ft_p = c\,\dot{m}\,t_p/g_0 = (7000)(280/32.2)(65) = 3.96 \times 10^6$ lbf-sec

c) $w = \dot{w}\,t_p = (280)(65)(32.2/32.2) = 18{,}200$ lbf or 18,200 lbm at sea level

d) SG = (density)/(density of liquid water at standard conditions) = 0.925

$\rho = (0.925)(62.4) = 57.72$ lbm/ft^3 and Volume = 18,200/57.72 = 315.3 ft^3



4. For the rocket in Problem 2, calculate the specific power, assuming a propulsion system dry mass of 80 kg and a duration of 3 min.

DRY MASS = EMPTY MASS (see Fig. 4-1)

Specific Power = $\dfrac{P_{jet}}{m_0} = \dfrac{0.5Fc}{m_f + m_p}$

m_0 = (3.867)(180) + 80 = 776.06 kg

$$\frac{P_{jet}}{m_0} = \frac{0.5(8896)(2300)}{776.06} = 13.18 \text{ kW/kg}$$

5. A Russian rocket engine (RD-110) consists of four thrust chambers supplied by a single turbopump. The exhaust from the turbine of the turbopump is ducted to four vernier chamber nozzles (which can be rotated to provide some control of the flight path). Using the information below, determine the thrust and mass flow rate of the four vernier thrusters.

For the individual thrust chambers (vacuum):

F_c = 73.14 kN, c_c = 2857 m/sec

For the overall engine with verniers (vacuum):

F_{oa} = 297.93 kN, c_{oa} = 2845 m/sec

(a) vernier thrust ; F_v = 297.93 – 4x73.14 = 5.37 kN

(b) vernier mass flow rate: $\dot{m}_{oa} = \dot{m}_c + \dot{m}_v$

so $\dot{m}_v = F_{oa}/c_{oa} - F_c/c_c$ = 297930/2845 - 4x73140/2857 = 2.32 kg/sec

(c) vernier effective exhaust velocity: $c_v = F_v/\dot{m}_v$ = 5370/2.32 = 2315 m/sec

6. A certain rocket engine has a specific impulse of 250 sec. What range of vehicle

velocities (u, in units of ft/sec) would keep the propulsive efficiencies at or greater than 80%. Also, how could rocket–vehicle staging be used to maintain these high propulsive efficiencies for the range of vehicle velocities encountered during launch?

I_s = 250 sec, c = (32.17)(250) = 8,042.5 ft/sec

a) By inspection of Fig. 2-3, $\eta_p \geq 0.8$ for $½ \leq u/c \leq 2$

So, $4{,}021 \leq u \leq 16{,}085$ ft/sec

b) Design upper stages with increasing I_s to keep $u/c \leq 2.0$ within atmospheric flight.

7. For a solid propellant rocket motor with a sea-level thrust of 207,000 lbf, determine: (a) the (constant) propellant mass flow rate $\dot{m}$ and the specific impulse I_s at sea level, (b) the altitude for optimum nozzle expansion as well as the thrust and specific impulse at this optimum condition and (c) at vacuum conditions. The initial total mass of the rocket motor is 50,000 lbm and its propellant mass fraction is 0.90. The residual propellant (called slivers, combustion stops when the chamber pressure falls below a deflagration limit) amounts to 3% of the burnt. The burn time is 50 seconds; the nozzle throat area (A_t) is 164.2 in.^{2} and its area ratio (A_2/A_t) is 10. The chamber pressure (p_1) is 780 psia and the pressure ratio (p_1/p_2) across the nozzle may be taken as 90.0. Neglect any start/stop transients and use the information in Appendix 2.

$m_p = m_m\zeta$ = 50,00x0.9 = 45,000 lbm (propellant loading); usable part = 45,00x0.97 = 43,650 lbm

$\dot{m} = m_p/t$ = 43650/50 = 873.0 lbm/sec

$I_t = Ft$ = 207000x50 = 10,350,000 lb-sec

$I_s = I_t/w$ = 10350000/42650 = 237.1 sec

(b) To determine optimum expansion one needs information from Chapter 3, Fig. 3-4. If k = 1.25 and with the given chamber pressure (780 psia) and area ratio (10), at optimum expansion, $p_2 = p_3$ = 780/90 = 8.666 psia. The nozzle exit area is $A_2 = 10A_t$ = 1642 in^2.

From Eq. 2-13 and the given thrust at sea level we can solve for the *momentum thrust*,

$\dot{m}\, v_2$ = 207,000 – (8.666 – 14.696) = 216,900 lbf = thrust at optimum expansion.

I_s = 216900/873 = 248.45; From Appendix 2, the 8.87 is at approximately 4,200 meters.

(c) For vacuum conditions, p_3 = 0 and Eq. 2-14 applies,

F = 216,900 + 8.67x1642 = 231,000 lbf and I_s = 231000/873 = 295 sec.

8. During the boost phase of the Atlas V, the RD-180 engine operates together with three solid propellant rocket motors (SRBs) for the initial stage. For the remaining thrust time, the RD-180 operates alone. Using the information given in Table 1-3, calculate the *overall effective exhaust velocity* for the vehicle during the initial combined thrust operation.

The individual mass flow rates for each propulsion type may be inferred from Table 1-3.

Main engine: $\dot{m}$ = (3.82x10^3 N)/(337.6 sec) x (9.81 m/sec^2) = 1.15 kg/sec

SRBs: $\dot{m}$ = 3 x 1.878x10^3/279.3 x 9.81 = 1.965 kg/sec

Using Equation 2-25, we find

$$(I_s)_{\mathrm{oa}} = \frac{\Sigma F}{g_0 \Sigma \dot{m}} = \frac{(3.82 + 3\mathrm{x}1.878)\mathrm{x}10^3}{9.81\mathrm{x}(1.15 + 1.965)} = 309 \text{ sec.}$$

9. Using the values given in Table 2–1 for the various propulsion systems, calculate the total impulse for a fixed propellant mass of 20.0 kg.

Use upper values from Table 2-1, and take m_p = 20.0 kg.

$I_t = (m_p g_0) I_s = 196.2\ I_s$ N-sec

Type	I_s (sec)	I_t (N-sec)
Chemical		
Solid	200	3.92 x 10^4
Liquid	410	8.04 x 10^4
Monopropellant	223	4.38 x 10^4
Nuclear Fission	860	1.17 x 10^5
Resistojet	300	5.89 x 10^4
Arcjet-thermal	1200	2.35x10^5

EM, incl. PPT	2500	4.91×10^5
Hall Effect	1700	3.34×10^5
Ion	5000	9.81×10^5
Solar heating	700	1.37×10^5

The total impulse is related to the change of vehicle velocity as is shown in Chapter 4 but in this problem there is an arbitrarily fixed propellant mass which precludes further conclusions.

10. Using the MA-3 rocket engine information given in Example 2-3, calculate the overall specific impulse at sea level and at altitude, and compare these with I_s values for the individual booster engines, the sustainer engine, and the individual vernier engines.

From the results given in Example 2-3,

$(I_s)_{oa}$ = 389300/1637.5 = 238 sec at sea level (SL)

$(I_s)_{oa}$ = 70830/274.76 = 258 sec at altitude

The individual specific impulses follow from their appropriate given information:

Booster (2)	I_s = 247 sec (SL)
Sustainer (1)	I_s = 211 sec (SL) and 259 sec at altitude
Vernier (2)	I_s = 195 sec at altitude

11. Determine the mass ratio **MR** and the mass of propellant used to produce thrust for a solid propellant rocket motor which has an inert mass of 82.0 kg. The motor mass becomes 824.5 kg after loading the propellant. For safety reasons, the igniter is not installed until shortly before motor operation; this igniter has a mass of 5.50 kg of which 3.50 kg is igniter propellant. Upon inspection after firing, the motor is found to have some unburned residual propellant and a motor mass of 106.0 kg.

Mass of propellant loaded:	824.5 – 82.0 = 742.5 kg
Igniter hardware:	5.5 – 3.5 = 2.0 kg
Initial m_0 before operation:	82.0 + 742.5 + 5.5 = 830 kg
Mass ratio:	**MR** = m_f/m_0 = 106/830 = 0.1277 or 1/7.83

The final mass m_f = 106 actually consist of inert mass (82.0 kg) and any residual propellants and the igniter motor hardware (2 kg).

Residual (unused) propellant: 106 – 2 – 82 = 22.0 kg

Propellant used (burned): 742.5 – 22 = 720.5 kg.

CHAPTER 3

1. Certain experimental results indicate that the propellant gases of a liquid oxygen–gasoline reaction have a mean molecular mass of 23.2 kg/kg-mol and a specific heat ratio of 1.22. Compute the specific heat at constant pressure and at constant volume, assuming a perfect-gas relations apply.

M = 23.2 kg/kg-mol $k = c_p/c_v = 1.22$ $J = 1.0$

$$c_p = \frac{kR'}{(k-1)\mathrm{M}} = \frac{(1.22)(8314.3)}{(0.22)(23.2)} = 1{,}987 \text{ J/kg-K}$$

$$c_v = c_p/k = 1{,}629 \text{ J/kg-K}$$

2. The actual conditions for an optimum expansion nozzle operating at sea level are given below. Calculate v_2, T_2, and C_F. The mass flow $\dot{m}$ = 3.7 kg/sec; p_1 = 2.1 MPa; T_1 = 2585 K; M = 18.0 kg/kg-mol; and k = 1.30. [See Fig. 2-1 for notation.]

$\dot{m}$ = 3.7 kg/sec, p_1 = 2.1 MPa, T_1 = 2585 K, M = 18 kg/kg-mol, k = 1.3,

p_3 = 0.1013 MPa

$p_2/p_1 = p_3/p_1 = 0.0482$ at *optimum* conditions so $p_1/p_3 = 20.76$

Using Fig 3-5 with the above information $A_2/A_t \approx 3.3$ and C_F =1.4

Or, from Eqn. 3-25 $A_2/A_t = 3.292$ and from Eq. 3-30 $C_F = 1.394$

Using Eqn. 3-16, $$v_2 = \left[\frac{2.3\text{x}8314\text{x}2585}{0.3\text{x}18}[1-(0.0482)^{2.3/1.3}]\right]^{0.5} = 2{,}281 \text{ m/sec}$$

or 7,482 ft/sec

$$T_2 = T_1(p_2/p_1)^{(k-1)/k} = 2585(0.0482)^{0.3/1.3} = 1284 \text{ K}$$

3. A certain nozzle expands a gas under isentropic conditions. Its chamber or nozzle entry velocity equals 90 m/sec, its final velocity 1500 m/sec. What is the change in enthalpy of the gas? What percentage of error is introduced if the initial velocity is neglected? [See Fig. 2-1 for subscripts notation.]

(a) $h_2 - h_1 = ½(v_1^2 - v_2^2) = -½(1500^2 - 90^2) = -1.12 \times 10^6$ J/kg

(b) $h_2 - h_1 \approx -½v_2^2 = -½(1500^2) = 1.13 \times 10^6$ J/kg

(c) Error = 0.362 %

4. Nitrogen at 500 ◦C (k = 1.38, molecular mass is 28.00 kg/kg-mol) flows at a Mach number of 2.73. What are its local and acoustic velocity?

Nitrogen, T = 500 °C, k = 1.38, M = 28 kg/kg-mol, M = 2.73

a = [1.38 x 8314.3 x (500 + 273)/28]$^{0.5}$ = 562.81 m/sec

$v = Ma$ = 2.73 x 562.81 = 1536.48 m/sec

5. The following data are given for an optimum rocket propulsion system:

Average molecular mass	24 kg/kg-mol
Chamber pressure	2.533 MPa
External pressure	0.090 MPa
Chamber temperature	2900 K
Throat area	0.00050 m2
Specific heat ratio	1.30

Determine **(a)** throat velocity; **(b)** specific volume at throat; **(c)** propellant flow and specific impulse; **(d)** thrust; **(e)** Mach number at throat.

M = 24 kg/kg-mol, $T_0 = T_1$ = 2900 K, $p_0 = p_1$ = 2.533 MPa, A_t = 5 x 10^{-4} m^2, k = 1.3

p_3 = 0.09 MPa = p_2 at the optimum expansion condition.

(a) $v_t = a_t = \sqrt{\dfrac{2kR'}{(k+1)\text{M}}T_1} = \sqrt{\dfrac{2.6\text{x}8314.3}{2.3\text{x}24}2900}$ = 1.066 x 10^3 m/sec

(b) $V_t = \dfrac{RT_1}{p_1}\left[\dfrac{k+1}{2}\right]^{1/(k-1)} = \dfrac{8314.3\text{x}2900}{24\text{x}2.53\text{x}10^6}\left[\dfrac{2.3}{2}\right]^{1/0.3}$ = 0.633 m^3/kg

(c) $\dot{m} = A_t v_t / V_t$ = 5 x 10^{-4} x 1.066 x 10^3/0.632 = 0.843 kg/sec

Since $p_2 = p_3$, c is given by Eq. 3-16, c = 2164 m/sec and I_s = 221 sec

(d) $F = \dot{m}$ c = 0.843 x 2163.57 = 1,824 N

(e) M_t = 1.0 because the nozzle is operating supersonically.

6. Determine the ideal thrust coefficient for Problem 5 by two methods.

i) $C_F = F/(p_1 A_t)$ = 1.83 x 10^3/(2.533 x 10^6 x 5 x 10^{-4}) = 1.44

ii) C_F = Eqn. 3-30 = 1.44

iii) Fig. 3-8 with k = 1.3 and p_1/p_2 = 2.81, C_F = 1.4 [check]

7. A certain ideal rocket with a nozzle area ratio of 2.3 and a throat area of 5 in.^{2} delivers gases at k **=** 1.30 and R **=** 66 ft-lbf/lbm-°R at a chamber pressure of 300 psia and a constant chamber temperature of 5300°R against a back atmospheric pressure of 10 psia. By means of an appropriate valve arrangement, it is possible to throttle the propellant flow to the thrust chamber. Calculate and plot against pressure the following quantities for 300, 200, and 100 psia chamber pressure: **(a)** pressure ratio between chamber and atmosphere; **(b)** effective exhaust velocity for area ratio involved; **(c)** ideal exhaust velocity for optimum and actual area ratio; **(d)** propellant flow; **(e)**

thrust; **(f)** specific impulse; **(g)** exit pressure; **(h)** exit temperature.

A_2/A_t = 2.3, k = 1.3, T_1 = 5300 °R, A_t = 5 in^2, p_3 = 10 psia, R = 66 ft-lbf/lbm-°R

Variable p_1 = 300, 200, and 100 psia [assume nozzle flows full in all cases.]

Isentropic flow: T_2/T_1 = 0.567, p_2/p_1 = 0.085, M_2 = 2.26, [all p_1] but not optimum!

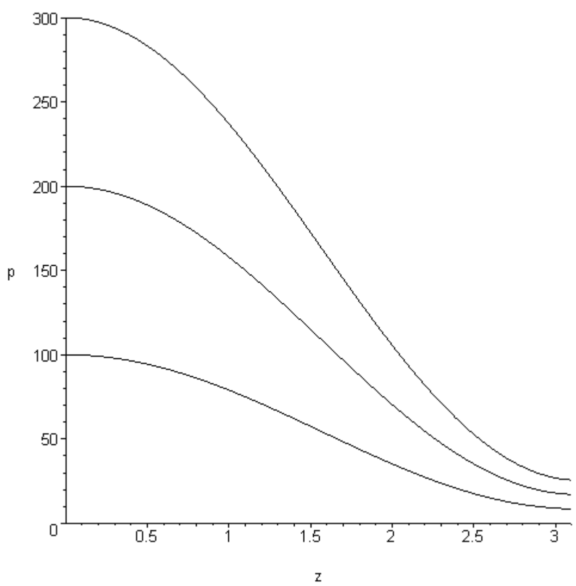
So that A_2 = 11.5 in^2, T_2 = 2990 °R, and v_2 will not change,

$$v_2 = \sqrt{\frac{2kg_0}{k-1}RT_1\left[1-\left(\frac{p_2}{p_1}\right)^{(k-1)/k}\right]} = 6.51 \times 10^3 \text{ ft/sec}$$

From Eq. 3-24, $\dot{m} = 3.20 \times 10^{-2}\, p_1$, from Eq. 2-16, $F = \dot{m}\, c$

where from Eq. 2-15 $c = 6.51 \times 10^3 + 3.7 \times 10^2 \times (p_2 - 10)/\dot{m} = I_s g_0$

p_1 (psia)	p_1/p_3	p_2 (psia)	c (ft/sec)	$\dot{m}$ (lbm/sec)	F (lbf)	T_2 (°R)	v_2 (ft/sec)
300	30	25.5	7110	9.60	2120	2990	6510
200	20	17.0	6920	6.40	1380	2990	6510
100	10	8.5	6340	3.20	630	2990	6510



The coordinate "z" in the plot above represents the nozzle flow axis in arbitrary units. At p_1 of 300 and 200 psia the flow exits underexpanded; at 100 psia it exits overexpanded.

8. For an ideal rocket with a characteristic velocity c^* **=** 1500 m/sec, a nozzle throat diameter of 20 cm, a thrust coefficient of 1.38, and a mass flow rate of 40 kg/sec, compute the chamber pressure, the thrust, and the specific impulse.

$c^* = 1500$ m/sec, $D_t = 20$ cm, $C_F = 1.38$, $\dot{m} = 40$ kg/sec

$c^* = p_1 A_t / \dot{m}$ so $p_1 \approx p_0 = (1500) \times (40)/(\pi/4) \times (0.20)^2 = 1.91$ MPa

$F = C_F A_t p_1$ so $F = (1.38)(\pi/4) \times (0.20)^2 (2.36 \times 10^6) = 8.28 \times 10^4$ N

$I_s = F/\dot{m}\, g_0$ so $I_s = (8.29 \times 10^4)/(40)(9.81) = 211$ sec

9. For the rocket unit given in Example 3–2 compute the exhaust velocity if the nozzle is cut off and the exit area is arbitrarily decreased by 50%. Estimate the losses in kinetic energy and thrust and express them as a percentage of the original kinetic energy and the original thrust. [See Fig. 2-1 for subscript notation.]

The wording "old" = original nozzle and "new" = 50% decrease in exit area. The parameters $p_1 = 2.1$ MPa, $T_1 = 2222$ K, $k = 1.3$ and $A_t = 28.7$ cm^2 remain unchanged by the exit area change. Also, because the flow is chocked at the throat, the flow rate $\dot{m}$ will not change. Others will change according to the isentropic flow relations in Ch. 3.

	A_2/A_t	M_2	p_2/p_1	T_2/T_1	a_2 (m/sec)	$v_2=M_2a_2$ (m/sec)	F (kN)
Old	3.3	2.6	0.0481	0.497	704	1827	1.827
New	1.65	1.94	0.145	0.64	799	1550	1.750

Kinetic Energy Change = $\frac{1}{2}\dot{m}\,(v_2{}^2{}_{\text{new}} - v_2{}^2{}_{\text{old}}) / \frac{1}{2}\,\dot{m}\, v_2{}^2{}_{\text{old}}$ = - 7.20%

Thrust Change = $(F_{\text{new}} - F_{\text{old}})/F_{\text{old}}$ = - 4.21 %

10. What is the maximum velocity if the nozzle in Example 3–2 were designed to expand into a vacuum? If the expansion area ratio was 2000?

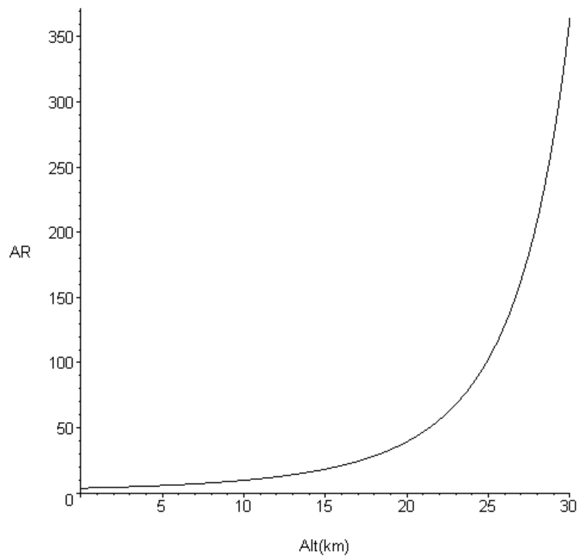
a) For vacuum with $k = 1.3$, $v_{2\max}$ = Eqn. 3-18 = 2,580 m/sec

b) With $A_2/A_t = 2000$, from Eqn. 3-14, $M_2 = 9.698$ and $T_2/T_1 = 0.0665$

$v_2 = M_2 a_2 = 2490$ or 3.5 % less than $v_{2\max}$

11. Construction of a variable-area conventional axisymmetric nozzle has often been considered to make the operation of a rocket thrust chamber take place at the optimum expansion ratio at any altitude. Because of the enormous design difficulties of such a mechanical device, it has never been successfully realized. Assuming that such a mechanism can eventually be constructed, what would have to be the variation of the area ratio with altitude (plot up to50 km) if such a rocket had a chamber pressure of 20 atm? Assume that $k = 1.20$. [The plot below is up to 30 km for greater clarity]

Using Eq. 3-25 at location 2, where $p_2 = p_3$ and the latter is found in Appendix 2,



12. Design a supersonic nozzle to operate at 10 km altitude with an area ratio of 8.0. For the hot gas take T_0 = 3000 K, R = 378 J/kg-K, and k = 1.3. Determine the exit Mach number, exit velocity, and exit temperature, as well as the chamber pressure. If this chamber pressure is doubled, what happens to the thrust and the exit velocity? Assume no change in gas properties. How close to optimum nozzle expansion is this nozzle?

$p_3 = p_2$ = 0.02.615 x 1.013 x 10^5 = 26.49 kPa from Appendix 2 at 10 km.

With k = 1.3 and ϵ = 8, M_2 = 3.39, p_1/p_2 = 80, and T_1/T_2 = 2.73

From Eq. 3-16, v_2 = 2468 m/sec

Double chamber pressure, $p_{1\text{new}}$ = 2 x 80 x 26.49 kPa = 4.24 MPa

$F = C_F p_1 A_t$ where C_F changes from 1.57 to 1.61 (2.5%) and A_t does not change, so that $F_{\text{new}} = 2.05F$ or a 205% increase, $v_{2\text{new}}$ = 3130 m/sec or a 26% increase.

The flow will be underexpanded requiring a bigger ϵ to reach optimum conditions.

13. The German World War II A-4 propulsion system had a sea-level thrust of 25,400 kg and a chamber pressure of 1.5 MPa. If the exit pressure is 0.084 MPa and the exit diameter 740 mm, what is the thrust at 25,000 m?

F_{SL} = 25,400 kgf x 9.81 m/sec^2 = 240,345 N [see units of *Force* in Appendix 1]

$p_1 \approx p_0$ = 1.5 MPa, p_2 = 0.084 MPa (underexpanded), at 25 km p_3 = 0.00255 MPa

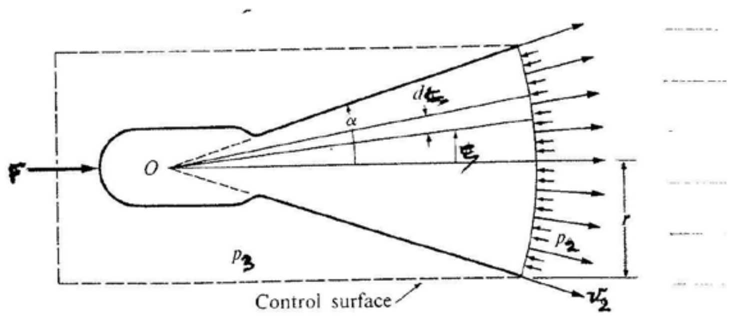
D_2 = 740 mm so A_2 = 0.43 m^2

$\dot{m}\, v_2 = 2.403\text{x}10^5 - 0.430(0.084 - 0.1013)$ x $10^6 = 2.47\text{x}10^5$ N

$F(25\text{ km}) = 2.47 \times 10^5 + 0.430(0.084 - 0.00255) \times 10^6 = 3.294 \times 10^5$ N

14. Derive Eq. 3–34. (*Hint*: Assume that all the mass flow originates at the apex of the cone.) Calculate the nozzle angle correction factor for a conical nozzle whose divergence half angle is 13°.

[See Hill & Peterson, "Mechanics and Thermodynamics of Propulsion, II[nd] Ed., Addison-Wesley , MA (1992), pgs 521/22.]



Spherically symmetric nozzle flow.

In a spherically-symmetric nozzle arrangement, the correction factor λ arises because the flow velocity is not entirely along the axis of the nozzle but acts over a spherical segment. The total half-angle is α, the running variable is ξ.

$$F + (p_2 - p_3)A_2 = \int \rho(\mathrm{v}\cdot\hat{n})\mathrm{v}_x \mathrm{d}A = \rho A_2\, \mathrm{v}_2^2\Big|_{\mathrm{ave}}$$

Take $p_2 = p_3$, $\mathrm{v}_x = \mathrm{v}_2\cos(\xi)$, $\mathrm{d}A = 2\pi R\sin(\xi)(R\mathrm{d}\xi)$, $\dot{m} = \rho\mathrm{v}_2[2\pi R^2(1 - \cos(\alpha))]$

$$\int \rho(\mathrm{v}\cdot\hat{n})\mathrm{v}_x \mathrm{d}A = 2\pi R^2 \rho \mathrm{v}_2^2 \left(\frac{1-\cos^2\alpha}{2}\right)$$

Since $1 - \cos^2(\alpha) = (1 + \cos(\alpha))(1 - \cos(\alpha))$, then $\lambda \equiv F/\dot{m}\,\mathrm{v}_2 = ½(1 + \cos(\alpha))$

For $\alpha = 13°$, $\lambda = 0.987$.

15. In Example 3–2, determine **(a)** the actual thrust; **(b)** the actual exhaust velocity; **(c)** the actual specific impulse; **(d)** the velocity correction factor. Assume that the thrust correction factor is 0.985 and the discharge correction factor is 1.050.

For the data of Example 3-2:

a) $\zeta_F = F_a/F_i = 0.985$, $F_a = (0.985)(1827) = 1800$ N

b) $\zeta_d = \dot{m}_a/\dot{m}_i = 1.05$, $\dot{m}_a = (1.050)(1) = 1.05$ kg/sec; $v_2 = F_a/\dot{m}_a = = 1{,}714$ m/sec

c) $I_{sa} = v_2/g_0 = 175$ sec d) $\zeta_v = I_{sa}/I_{si} = 175/186 = 0.939$

16. An ideal rocket has the following characteristics:

Chamber pressure	27.2 atm
Nozzle exit pressure	3 psia
Specific heat ratio	1.20
Average molecular mass	21.0 lbm/lb-mol
Chamber temperature	4200 °F

Determine the critical pressure ratio, the gas velocity at the throat, the expansion area ratio, and the theoretical nozzle exit velocity.

$p_0 \approx p_1 = 27.2$ atm, $k = 1.2$, $\mathfrak{M} = 21$, $p_2 = 3$ psia, $T_0 \approx T_1 = 4660$ °R

Critical pressure when $M = 1.0$ at the throat, $p_t/p_1 = 0.56447$ or $p_1/p_t = 1.77$

$$v_t = a_t = \sqrt{kg_0 \frac{R'}{\mathfrak{M}} \frac{2T_1}{k+1}} = [1.2 \text{ x } 32.2 \text{ x}(1545/21) \text{ x } 0.9091 \text{ x } 4660]^{1/2} = 3{,}470 \text{ ft/sec}$$

$p_2/p_1 = 3/(27.2 \text{ x } 14.7) = 0.007503$, $M_2 \approx 3.55$, $A_2/A_t = 14.785$, $T_2/T_1 \approx 0.45$

$$v_2 = \sqrt{\frac{2kg_0}{k-1} \frac{R'}{\mathfrak{M}} T_1 \left[1 - \left(\frac{p_2}{p_1}\right)^{(k-1)/k}\right]} = 8{,}570 \text{ ft/sec}$$

17. For an ideal rocket with a characteristic velocity c^* of 1220 m/sec, a mass flow rate of 73.0 kg/sec, a thrust coefficient of 1.50, and a nozzle throat area of 0.0248 m^2, compute the effective exhaust velocity, the thrust, the chamber pressure, and the specific impulse.

$c^* = 1220$ m/sec, $\dot{m} = 73$ kg/sec, $C_F = 1.5$, $A_t = 0.0248$ m^2

$F = \dot{m}\, c^* C_F = 1.34 \text{ x } 10^5$ N

$c = F/\dot{m} = 1.83 \text{ x } 10^3$ m/sec

$C_F = F/p_1 A_t$, so $p_1 = F/C_F A_t = 3.59$ MPa and $I_s = c/g_0 = 186.6$ sec.

18. Derive Eqs. 3–24 and 3–25.

The mass flow rate at the throat is $\dot{m} = \rho_t A_t v_t$ where Eq. 3-4 becomes $p_t = \rho_t R T_t$

Since $v_t = a_t$ and using Eqs. 3-20 3-22, and 3-23 and combining above

$\dot{m} = (p_1/RT_1)A_t k^{1/2}[2k/(k+1)]^{(k+1)/2(k-1)} = p_1 A_t/c^*$ Eq. 3-24 or 3-32

At an arbitrary location y, $\dot{m} = \rho_t A_t v_t = \rho_y A_y v_y$ so that $A_t/A_y = \rho_y v_y/\rho_t v_t = p_y T_t v_y / p_t T_y a_t$

Using Eqs. 3-7 and 3-16 (at "t" and an arbitrary y-location) and simplifying yields Eq. 3-25. The inverse of this equation at "location $y = 2$" is a very useful form in rocket calculations. Note further that Eq. 3-26 is not a "local" value like the Mach number.

19. An upper stage of a launch vehicle propulsion unit fails to meet expectations during

sea-level testing. This unit consists of a chamber at 4.052 MPa feeding hot propellant to a supersonic nozzle of area ratio ϵ = 20. The local atmospheric pressure at the design condition is 20 kPa. The propellant has a k = 1.2 and the throat diameter of the nozzle is 9 cm.
(a) Calculate the ideal thrust at the design condition.
(b) Calculate the ideal thrust at the sea-level condition.
(c) State the most likely source of the observed nonideal behavior.

p_1 = 4.052 MPa, p_3 = 20 kPa, ϵ = 20, k = 1.2, d_t = 9 cm

a) Design, $p_2 = p_3$, $A_t = 6.36 \times 10^{-3}$ m^2,

From Eq. 3-30, C_F = 1.72, $F_{OPT} = 4.44 \times 10^4$ N

b) C_F = 1.32, p_2 = 2 kPa, p_3 = 0.1013 MPa

$F_{SL} = 4.44 \times 10^4 + (0.02 - 0.1013) \times 10^6 \times 20 \times 6.36 \times 10^{-3} = 3.41 \times 10^4$ N

c) Likely source: <u>Separation inside the nozzle</u> as per Fig. 3-6, p_1/p_3 = 40 and ϵ = 20.

20. Assuming ideal flow within the propulsion unit:
(a) State all necessary conditions (realistic or not) for
$c^* = c = v_2$
(b) Do the above conditions result in an *optimum* thrust for a given p_1/p_3?
(c) For a launch vehicle designed to operate at some intermediate earth altitude, sketch (in absolute or relative values) how c^*, c, and v_2 would vary with altitude.

a) $c = v_2$ when $p_2 = p_3$, the *Optimum* condition where there is no pressure thrust

$c^* = c$ when C_F = 1.0, which is rarely true

b) YES because $p_2 = p_3$

c) c^* and v_2 do not change with altitude but c changes as C_F [see Fig. 3-9, c =$c^* C_F$].

21. A rocket nozzle has been designed with A_t = 19.2 in.^{2} and A_2 = 267 in.^{2} to operate optimally at p_3 = 4 psia and produce 18,100 lbf of ideal thrust with a chamber pressure of 570 psia. It will use the proven design of a previously built combustion chamber that operates at $T1$ = 6000 °R with k = 1.25 and R = 68.75 ft-lbf/lbm °R, with a c^* *efficiency* of 95%. But test measurements on this thrust system, at the stated pressure conditions, yield a thrust of only 16,300 lbf when the measured flow rate is 2.02 lbm/sec. Find the applicable correction factors (ζ_F , ζ_d , ζ_{CF}) and the actual specific impulse assuming *frozen flow* throughout.

A_t = 19.2 in^2, A_2 = 267 in^2, $p_2 = p_3$ = 4 psia, F_i = 18,100 lbf, p_1 = 570 psi,

T_1 = 6000 °R, k =1.25, R = 68.75 ft-lbf/lbm °R, c^*_{eff} = 0.95 (also ζ_{c^*})

F_a = 16300 lbf, $\dot{m}_a$ = 2.02 lbm/sec

$\zeta_F = F_a/F_i$ = 16300/ 18100 = 0.901

$\dot{m}_i$ = Eqn. 3-20 = (19.2)(570)[(1.25)x(2/2.25)$^{2.25/.25}$/(68.75x32.2x6000)]$^{0.5}$ = 1.98 lbm/sec

$\zeta_d = \dot{m}_a/\dot{m}_i$ = 2.02/1.98 = 1.02

$\zeta_{CF} = \zeta_F/\zeta_d\zeta_{c*}$ = 0.901/(1.02)(0.95) = 0.929

$I_{sa} = F_a/g_0\dot{m}_a$ = 16,300/32.2x2.02 = 251 sec.

22. The reason an optimum thrust coefficient (as shown on Figs. 3-6 and 3-7) exists is that as the nozzle area ratio increases with fixed p_1/p_3 and k, the pressure thrust in Eq. 3-30 changes sign at $p_2 = p_3$. Using $k = 1.3$ and $p_1/p_3 = 50$, show that as p_2/p_1 drops with increasing ϵ, the term 1.964[1 - $(p_2/p_1)^{0.231}]^{0.5}$ increases more slowly than the (negative) term [1/50 - p_2/p_1] ϵ increases (after the peak, where $\epsilon \approx 7$). (Hint: use Eq. 3-25).

One way to show this is as follows:

Looking at Eq. 3-30, as ϵ increases, the first term (the square root) continuously increases because p_2/p_1 is decreasing according to Eq. 3-25 (or as evident from Fig. 3-4). The second term in Eq. 3-30 is positive for $p_2 > p_3$, zero for $p_2 = p_3$, and negative for $p_2 < p_3$. By plotting the suggested curves, after the peak one can show that the change magnitude of the second term with increasing ϵ is always greater than that of the first term and thus the optimum C_F always occurs at $p_2 = p_3$ (and that is what Figs. 3-6 and 3-7 clearly display).

EXTRA PROBLEMS

3A. Some data for the MA-5A multiple liquid propellant engines are shown below. Calculate the mass flow rate and the nozzle area ratio for each for each component. Assume individual flow rates remain constant during rocket operation. The last table entry represents a needed chemical thermodynamics input.

COMPONENT	NO. OF ENGS.	THRUST (lbf) (Each)	SPECIFIC IMPULSE (sec)	p_1/c^* (lbf/in^2/ft/sec)
Booster	2	215,000 (SL) 240,000 (Vac)	264 (SL) 295 (Vac)	0.1287
Sustainer	1	60,000 (SL) 84,000 (Vac)	220 (SL) 309 (Vac)	0.1287
Vernier	2	415 (SL) 500 (Vac)	240 (SL) 290 (Vac)	0.1005

From Eq. 2-13 we can write for each engine

$F_{SL} = \dot{m}\, v_2 + (p_2 - p_{SL})A_2$ and $F_{Vac} = \dot{m}\, v_2 + (p_2\ )A_2$

Subtracting the equations above gives: $F_{Vac} - F_{SL} = p_{SL}A_2$ which can be solved for A_2

Using Eq. 3-32 $p_1 A_t = \dot{m}\, c^*$ and Eq. 2-6 $I_s = F/g_0\, \dot{m}$ we can solve for A_t

Now forming the nozzle area ratio: $\epsilon = A_2/\, A_t = \dfrac{F_{Vac} - F_{SL}}{p_{SL}} \dfrac{p_1}{\dot{m}c^*} = g_0 \dfrac{I_{sVac} - I_{sSL}}{p_{SL}} \left(\dfrac{p_1}{c^*}\right)$

Results for each engine:

COMPONENT	$\dot{m}$ (lbm/sec)	ϵ
Booster	814.4	8.7
Sustainer	272.7	25
Vernier	1.73	11

3B. Using realistic correction factors, design a rocket nozzle to conform to the following conditions:

Chamber pressure	20.4 atm = 2.068 MPa
Atmospheric pressure	1.0 atm
Chamber temperature	2861 K
Mean molecular mass of gases	21.87 kg/kg-mol
Ideal specific impulse	230 sec (at operating conditions)
Specific heat ratio	1.229
Desired thrust	1300 N

Determine the following: actual nozzle throat and exit areas, respective diameters, actual exhaust velocity, and actual specific impulse. Take ζ_F = 0.96 and ζ_v = 0.92.

The theoretical thrust coefficient is found from Eq. 3-30. For optimum conditions, $p_2 = p_3$. By substituting k = 1.229 and p_1/p_2 = 20.4, the thrust coefficient is C_F = 1.405. This value can be checked by interpolation between values of C_F obtained from Figs. 3-6 and 3-7. The throat is found using ζ_F = 0.96 which is based on test data.

$A_t = F/(\ \zeta_F\ C_F p_1) = 1300/(0.96\text{x}1.405\text{x}2.068\text{x}10^6) = 4.66\ \text{cm}^2$

The throat diameter is then 2.43 cm. The area expansion ratio can be determined from Fig. 3-4 or Eq. 3-25 as ϵ = 3.42. The exit area ratio is

A_2 = 4.66x3.42 = 15.9 cm^2

The exit diameter is therefore 4.50 cm. The theoretical exhaust velocity will be equal to

$v_2 = I_s g_0$ = 230x9.81 = 2256 m/sec

By selecting an empirical correction factor such as ζ_v = 0.92 (based on prior related experience), the actual exhaust velocity will equal

$(v_2)_a$ = 2256x0.92 = 2076 m/sec

Because the specific impulse is proportional to the exhaust velocity, its actual value can be found by multiplying the theoretical value of the velocity correction factor ζ_v.

$(I_s)_a$ = 230x0.92 = 212 sec.

[Note that the above quoted correction factors correspond to $\zeta_d = 1.04$ and $\zeta_{c^*} = 0.96$.]

3C. The thrust developed by a thermal rocket propulsion system is 900 lbf when the mass flow rate is 5.20 lbm/sec. The nozzle has a minimum area of 1.0 in^2 and a discharge area of 4.12 in^2. Using the applicable thrust coefficient of 1.50 and a specific heat ratio of 1.30, find the corresponding ideal values of p_1, p_2, p_3 in psia.

F = 900 lbf, $\dot{m}$ = 5.3 lbm/sec, C_F = 1.50, k = 1.30, A_t = 1.0 in^2, A_2 = 4.12 in^2, ϵ = 4.12

$F = C_F A_t/p_1$ → p_1 = 900/1.5x1.0 = 600 psia

From Eq. 3-25 or Fig. 3-4, p_2/p_1 = 3.44x10^{-2} so p_2 = 900x3.44x10^{-2} = 20.6 psia

From Eq. 3-30 or Fig, 3-7, p_1/p_3 = 47.83 so p_3 = 12.51 psia

[As can be seen in Fig. 3-7, this is not an optimum; the flow is underexpanded.]

3C. Take the test data given in Example 3-6 as nominal values and determine the experimental uncertainties for the actual effective exhaust velocity c, the characteristic velocity c^*, and the thrust coefficient C_F given that the uncertainties in the individual measurements are as follows:

Total mass flow rate	± 1.0%
Chamber pressure	± 0.5%
Total thrust	± 0.5%
Throat diameter	± 0.1%

From Figure 3-6 and other 'givens in the example', the nominal value of C_{Fa} = 1.52 and that for throat area can be found using Eq. 3-31 as A_{ta} = 56.24 in^2 (assuming a circular cross section the throat diameter becomes, D_t = 8.46 in). The nominal values of c_a and c^*_a are 8,198 ft/sec and 5,394 ft/sec respectively.

The measured values are represented by:

Flow rate	$\dot{m}_a$ = 208 ± 2.08 lbm/sec
Pressure	p_1 = 620 ± 3.1 psia
Thrust	F_a = 53,000 ± 265 lbf

Diameter $d_t = 8.46 \pm 0.0085$ in.

Now, the uncertainty in c is the same as the uncertainty in the product of c^*C_F by Eq. 3-33 and the same amount applies to their corresponding performance correction factors. Note that the uncertainty in the thrust correction factor ζ_F and in the ζ_d discharge correction factor equal those of their measurement uncertainties (because they are direct ratios) but the uncertainties for the ζ_{c*} (c*-efficiency) and the ζ_{CF} correction factor are derived in nature. For the latter two, the sum of their individual component uncertainties would represent an upper limit or 'the very worst uncertainty'.

For our solution we will use an uncertainty-estimate δ after Kline & McClintock, "Describing Uncertainties in Single Sample Experiments", Mechanical Engineering, Vol. 75, Jan. 1953 [Eq.3-2 in Holman, H.P. and Gadja, J., "Experimental Methods for Engineers", 5th Ed., McGraw-Hill, NY, 1989] which is done as follows:

$\delta_c = g_0[(265/208)^2 + (53000\text{x}2.08/208^2)^2]^{1/2} = 32.2[1.62 + 6.49] = 91.7$ ft/sec or 1.12%

$\delta_{c*} = g_0[(56.24\text{x}3.1/208)^2 + (620\text{x}56.24\text{x}2.08/208^2)^2$
$+(3.14\text{x}620\text{x}8.46\text{x}0.0085/2\text{x}208)^2]^{1/2}$
$= 32.2[0.703 + 2.81 + 0.113]^{1/2} = 61.3$ ft/sec or 1.14%

$\delta_{CF} = [(265/56.24\text{x}620)^2 + (53000\text{x}3.1/56.24\text{x}620^2)^2 +$
$(3.14\text{x}53000\text{x}8.46\text{x}0.0085/2\text{x}620\text{x}56.24^2)^2]^{1/2} = [57.8\text{x}10^{-6} + 57.8\text{x}10^{-6} + 1.3\text{x}10^{-7}]^{1/2} =$
0.0108 or 0.711%

Summarizing:

	Nominal value	δ (%)	Max. δ(%)	Type of meas.
c_a (ft/sec)	8,198	1.12	1.5	derived
c^*_a (ft/sec)	5,394	1.14	1.51	derived
C_{Fa}	1.52	0.711	1.1	derived
F_a (lbf)	53,000	0.5	0.5	direct
$\dot{m}_a$ (lbm/sec)	208	1.0	1.0	direct

So the calculated deviations are about $\pm$ 0.4% below their maximum.
[The relation $\zeta_F \approx \zeta_d \zeta_{c*} \zeta_{C_F}$ therefore applies only within $\pm$ 0.5% at best for in this problem's data; another way to interpret this would say that the correction factors carry an uncertainty between $\pm$ 0.5 and 1.14%. Any experimental indication of much greater uncertainties those for c_a ($\pm$ 1.12%) would reflect either inferior measurements or *noticeably unsteady flow*.]

3D. Non-ideal behavior may be taken into account with a suitable efficiency (η_x) or equivalently with a correction factor (ζ_x). For a certain thermal thruster, the following information applies:

$T_1 = 6200$ °F, $p_1 = 1000$ psia, $k = 1.24$, and M = 23.3 lbm/lb-mol

If the experimental value for c^*_a (or $p_{1a}A_{ta} / \dot{m}_a$) is found to be 1680 m/sec, calculate the c*-efficiency for this thruster. Also, if $\zeta_{CF} = 0.94$, calculate an 'effective exhaust velocity correction factor' (ζ_c).

Because of the mixed units, it is easier to convert to SI-units.

T_1 = 3700 K and from Eq. 3-32,

$$c_i^* = \sqrt{\frac{R'T_1}{k\mathrm{M}}\left[\frac{2}{k+1}\right]^{-(k+1)/(k-1)}} = \sqrt{\frac{8314\times3700}{23.3\times1.24\times(2/2.24)^{2.24.0.24}}} = 1750 \text{ m/sec}$$

So that ζ_{c^*} (c^*-efficiency) = 1680/1750 = 0.96 [this represents the combustion chamber]

From Eq. 3-40, $\zeta_c = \zeta_{c^*}\,\zeta_{CF}$ = 0.96x0.94 = 0.902 [this is the thruster's specific impulse]

CHAPTER 4

1. For a vehicle in gravitationless space, determine the mass ratio necessary to boost the vehicle velocity by **(a)** 1600 m/sec and **(b)** 3400 m/sec; the effective exhaust velocity is 2000 m/sec. If the initial total vehicle mass is 4000 kg, what are the corresponding propellant masses?

$\mathbf{MR} = \exp(-\Delta u/c)$ (a) $\mathbf{MR} = 0.449$ and (b) $\mathbf{MR} = 0.183$
$\zeta = 1 - \mathbf{MR}$ so $m_p = m_0(1 - \mathbf{MR})$ (a) $m_p = 2.20 \times 10^3$ kg (b) $m_p = 3.27 \times 10^3$ kg

2. Determine the burnout velocity and burnout altitude for a dragless projectile with the following parameters for a simplified vertical trajectory: $\overline{c}$ = 2209 m/sec; m_p/m_0 = *0.57; t_p* = 5.0 sec; u_0 = 0; and h_0 = 0. Select a relatively small diameter missile with *L*/*D* of 10 and an average vehicle density of 1200 kg/m³. [*h* is height.]

Eqn. 4-20: $u_p = -\ 2209 \times \ln(1 - 0.57) - 9.81 \times 5 = 1.82 \times 10^3$ m/sec at burnout
Integrating Eq. 4-18 twice without the drag term and with $u_0 = 0$, yields
$h_p = -\ [ct_p/(1/\mathbf{MR} - 1)]\ \ln(1/\mathbf{MR}) + ct_p - \frac{1}{2} g_0 t_p^2 + h_0$
$= -\ [(2209 \times 5)/(1/0.43 - 1)]\ln(1/0.43) + 2205 \times 5 - \frac{1}{2}(9.81 \times 5^2) = 3.80$ km at burnout

3. Assume that the projectile of Problem 2 has a drag coefficient essentially similar to the 0° curve in Fig. 4–4 and redetermine the answers of Problem 2 and the approximate percentage error in u_p and h_p. Use a numerical method.

Additional information on the relevant area *A* or vehicle length *L* is needed. From Table 4-3, the drag is a minor contributor to Eqs. 4-18/19 during vertical, launch trajectories. The drag coefficient peaks early in the ascent trajectory, just beyond Mach = 1, when $u \approx 300$ m/sec. For a V-2 rocket, the velocity decrease due to drag is about 6.5% and the height decrease about 3.7%.

4. A research space vehicle in gravity-free and drag-free outer space launches a smaller spacecraft into a meteor shower region. The 2-kg sensitive instrument package of this spacecraft (25 kg total mass) limits the maximum acceleration to no more than 50 m/sec². It is launched by a solid propellant rocket motor (I_s = 260 sec and ζ = 0.88).
(a) Determine the minimum allowable burn time, assuming steady constant propellant mass flow;
(b) Determine the maximum velocity relative to the launch vehicle.
(c) Solve for (a) and (b) if half of the total impulse is delivered at the previous propellant mass flow rate, with the other half at 20% of this mass flow rate.

$m_0 = m_{pl} + m_{\text{motor}}$ so 25 kg = 2 kg + $m_p/0.88$ thus $m_p = 20.24$ kg
$m_p/m_0 = 0.8096$ and $\mathbf{MR} = 1 - 20.4/25 = 0.1904$
a) $t_{p\max} = (260 \times 9.81 \times 0.8096)/(50 \times 0.1904) = 216.9$ sec
b) $\Delta u = -\ (260 \times 9.81)\ln(0.1904) = 4{,}230.5$ m/sec
c) $\dot{m}_{\text{orig}} = 20.24/216.9 = 0.0933$ kg/sec $I_{t\text{orig}} = cm_p = 260 \times 9.81 \times 20.24 = 51{,}624$ N-sec
½ Impulse at $\dot{m}_{\text{orig}}$: $t_{p1} = 216.9/2 = 108.4$ sec
½ Impulse at $0.2\,\dot{m}_{\text{orig}} = 0.2 \times 0.0933 = 0.01866$ kg/sec
$t_{p2} = \frac{1}{2}(51624)/(0.01866 \times 260 \times 9.81) = 542.33$ sec, so $t_p = 651$ sec "total"
$\Delta u = 4230.5$ m/sec as before because c, m_0, m_f are the same in Eq. 4-7

5. For a satellite cruising in a circular orbit at an altitude of 500 km, determine the period of revolution, the flight speed, and the energy expended to bring a unit mass into this orbit.

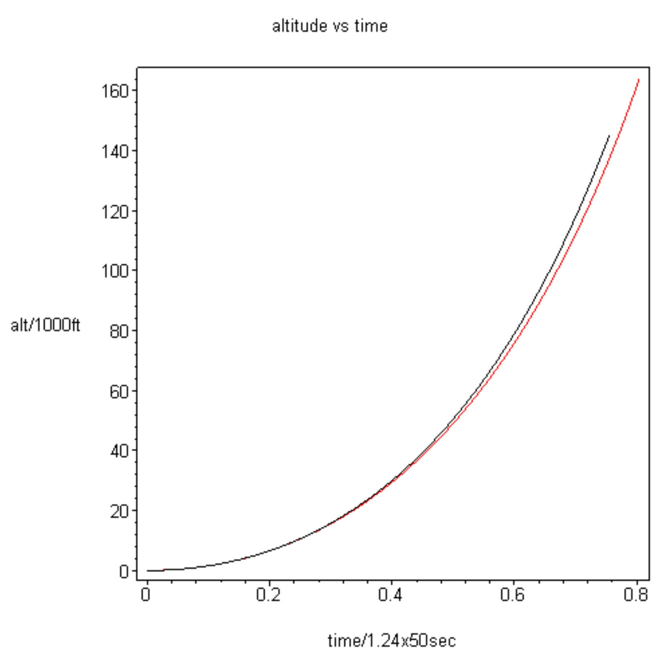
Eq. 4-26: u_s = 6.374 x 106 [9.81 x 10^{-3}/(6374 + 500)]$^{1/2}$ = 7.613 km/sec
Eq. 4-27: τ = 2π(6374 + 500)/7.316 = 5670 sec = 1.58 hr
Eq. 4-28: E = ½(6.374 x 10^6 x 9.81)(6374 + 1000)/(6374 + 500) = 33.52 MJ/kg

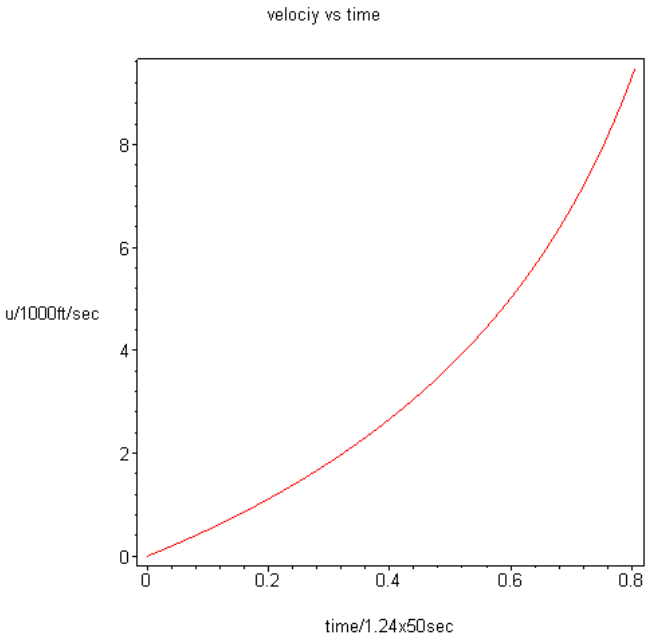
6. A large ballistic rocket vehicle has the following characteristics: propellant mass flow rate: 12 slugs/sec (1 slug **=** 32.2 lbm**=** 14.6 kg); nozzle exit velocity: 7100 ft/sec; nozzle exit pressure: 5 psia (assume no separation); atmospheric pressure: 14.7 psia (sea level); takeoff weight: 12.0 tons (1ton **=** 2000 lbf); burning time: 50 sec; nozzle exit area: 400 in.^{2} Determine **(a)** the sea-level thrust; **(b)** the sea-level effective exhaust velocity; **(c)** the initial thrust-to-weight ratio; **(d)** the initial acceleration; **(e)** the mass inverse ratio m_0/m_f .

a) Eq. 2-13: F = 12 x 7100 + (5 – 14.7)(400) = 81,320 lbf
b) Eq. 2-15 or 2-16: c = 8.13 x 10^4/12 = 6775 ft/sec
c) F/w = 81320/(12 x 2000) = 3.39 (sea level)
d) Newton's Second Law: a_{initial}/g_0 = 81320/(12 x 2000) – 1 = 2.39
e) m_0/m_f = (12 x 2000)/(12 x 2000 – 12 x 50 x 32.2) = 5.13

7. In Problem 6 compute the altitude and missile velocity at the time of power plant cutoff, neglecting the drag of the atmosphere and assuming a simple vertical trajectory.

Shown below is (first) a plot of altitude versus time for the data of Prob. 6. The red curve neglects variations in p_3 and g_0 from their sea level values. The black curve includes these variations (i.e., Eqs. 2-13 and 4-12 where p_3 varies with h as in Appendix 2) and was numerically integrated; there is little difference between the two results. The second plot below is for the missile velocity also as a function of time but neglects g_0 and c variations with elevation (the final values are h_p = 164,000 ft and u_p = 9,470 ft/sec).





8. A spherical satellite has 12 identical monopropellant thrust chambers for attitude control with the following performance characteristics: thrust (each unit): 5 lbf; I_s (steady state or more than 2 sec): 240 sec; I_s (pulsing duration 20 msec): 150 sec; I_s (pulsing duration 100 msec): 200 sec; satellite weight: 3500 lbf; satellite diameter: 8 ft; satellite internal density distribution is essentially uniform; disturbing torques, Y and Z axes: 0.00005 ft-lbf average; disturbing torque, for X axis: 0.001 ft-lbf average; distance between thrust chamber axes: 8 ft; maximum allowable satellite pointing position error: ±1°. Time interval between pulses is 0.030 sec.

(a) What would be the maximum and minimum vehicle angular drift per hour if no correction torque were applied?

(b) What is the frequency of pulsing action (how often does an engine pair operate?) at 20-msec, 100-msec, and 2-sec pulses in order to correct for angular drift? Discuss which pulsing mode is best and which is impractical.

Solution to Problem 8 of Chapter 4

Mass of Satellite $M = 3500$ lbf $= 3500/32.17 = 108.79$ lbm.
The moment of inertia I for a sphere about any axis going through its center is (formula from ME Handbook), where r is the radius of the sphere.

$I = 2Mr^2/5 = 2 \times 108.79 \times 4^2/5 = 696.30$ lbm ft

Part (a)

Given are the disturbing torques T and the maximum allowable deviation angle θ of 1° or 0.01745 radians. The angular acceleration a for the rotation about the Y or Z axis ($T = I/a$)

$a_y = a_z = T/I = 0.0005/696.3 = 7.1808 \times 10^{-7}$ rad/sec^2

For the x-axis the disturbing torque and acceleration are considerably larger.

$a_x = 0.001/696.3 = 1.496 \times 10^{-6}$ rad/sec^2

The time to turn the satellite 1 degree on the Y or Z axes is from $\theta = ½\, a\, t_y^2$

$t_y = t_z = \sqrt{2\,\theta/a} = \sqrt{2 \times 0.01745/(7.1808 \times 10^{-7})} = 220.45$ sec

The angle ξ turned by the satellite in 1 hour, if not corrected

$\xi_y = ½ \times 7.1808 \times 10^{-7}\,(60 \times 60)^2 = 4.65$ radians
$\xi_x = ½ \times 1.439 \times 10^{-6}\,(3600)^2 = 18.65$ radians or almost 3 revolutions

Part (b)

In order to correct for this angular drift of the satellite, pairs of small pulsing thrusters are to be used (a force couple with the diameter of the sphere as the moment arm). The angular speed ω_d induced by the disturbing torque should on the average be equal to the angular speed ω_c induced (in the opposite direction of rotation) by the pulses of the correcting torques from a pair of thrusters. During the correction maneuvers the maximum angle of 1 degree is not to be exceeded. The satellite can be considered to oscillate between rotations caused by the disturbing torque and the periodic application of correcting torques, which can bring the satellite close to its zero angular position. The angular speed ω_d achieved by the disturbing torques is a maximum when the vehicle has been accelerated through 1 degree or for the time of 220.45 sec (Y or Z axis).

$(\omega_d)_y = a\,t = 7.1808 \times 10^{-7} \times 220.45 = 0.0001585$ rad/sec
$(\omega_d)_x = 1.439 \times 10^{-6} \times 110.8 = 0.003189$ rad/sec

The angular speed ω_c caused by a pair of correcting thrusters is obtained from the torque and angular acceleration. The torque of 2 thrusters of 5 lbf each with a moment arm of 8 feet (vehicle diameter) is 40 ft lbf. The angular speed after an accelerating pulse of 20 milliseconds is

$$\omega_c = a\,\Delta t = (T/I) \times 0.020 = (40/696.3) \times 0.020 = 0.001159 \text{ rad/sec-pulse}$$

The number of short correcting pulses in 220.45 sec is about $(\omega_d)/(\omega_c)$ = 0.001585/0.001159 = 1.36 pulses for the Y or Z axis. That means at least 1 pulse per angular excursion of up to 1 degree and sometimes 2 pulses. For the X axis the average number of pulses is 0.003189/0.001159 = 2.75.

Discussion: This control scheme assumes that the control system will be able to sense a smaller angular excursion of less than 1 degree, in order for the control to function properly. The above analysis shows that the number of 20 milliseconds pulses is very small and with a single pulse it will be difficult to control and the satellite will oscillate in rotation up to ± 1° much of the time. The control problem will be simpler and the angular oscillation can be fewer if the pulses would be smaller and more frequent, perhaps 8 to 40 pulses per angular excursion. Pulses of less than 0.020 sec are inefficient, A good solution will be to reduce the thrust to perhaps between 0.1 to 0.5 lbf. At this lower thrust value consideration should be given to lengthen the pulse width somewhat in order to increase the specific impulse and save a little vehicle mass. The pulse widths of 100 milliseconds and 2 sec in the problem statement give angular torques, which are too large and these pulses are therefore impractical.

9. For an ideal multistage launch vehicle with several stages in series, discuss the following:
(a) the effect on the ideal mission velocity if the second and third stages are not started immediately but are each allowed to coast for a short period after shutoff and separation of the expended stage before rocket engine start of the next stage;
(b) the effect on the mission velocity if an engine malfunctions and delivers a few percent less than the intended thrust but for a longer duration and essentially the full total impulse of that stage.

(a) It is assumed to be a multistage space launch vehicle in a vertically upward flight, with a tandem arrangement of the stages (see Fig. 4-15). If there is a time delay between the thrust termination of the lower stage rocket engine and the start of thrust of the upper stage, there will be brief time period of zero thrust (i.e., coasting in space) and the pull of the Earth's gravity will continue to diminish the upward velocity during this time. Therefore, the vehicle's mission will be slightly reduced.

(b) First one needs to determine that the engine malfunction has not resulted in possible secondary malfunctions or failures elsewhere in the vehicle system. For example, there should have been no loss of propellants for the flight of the particular stage, there should have been no change in mixture ratio and no changes in the stage separation procedure. The total thrust (either for a single engine or a cluster of rocket engines) will be somewhat lower but the duration of the firing of the engine(s) will be longer, with no substantial change in total impulse. The mission velocity will usually be essentially the same as that of an equivalent vehicle with healthy engines. The flight duration will be longer and the flight path will be slightly different requiring some real-time analysis and corrections of the vehicle's flight control computer.

10. Given a cylindrically shaped space vehicle *(D* **=** 1.0m, height is 1.0 m, average density is 1.1 g/cm^3) with a flat solar cell panel on an arm (mass of 32 kg, effective moment arm is 1.5 m, effective average area facing normally toward sun is 0.6 m^2) in a set of essentially frictionless bearings and in a low circular orbit at 160 km altitude with sunlight being received, on the average, about 54% of the time:
(a) Compute the maximum solar pressure-caused torque and the angular displacement this would cause during 1 day if not corrected.
(b) Using the data from the atmospheric table in Appendix 2 and an arbitrary average drag coefficient of 1.0 for both the body and the flat plate, compute the drag force.
(c) Using stored high-pressure air at 5000 psi initial pressure as the propellant for attitude control, design an attitude control system to periodically correct for these two disturbances (F, I_s, t , etc.).
State all assumptions.

Dimensions and parameters:

Mass of cylinder $M_c = [(\pi/4)d^2 \text{h}]\,\rho = 0.7854 \times 1.0^2 \times 1.0 \times 1{,}100 = 864.05$ kg

Mass of solar panel M_p is given as 32.0 kg; the arm linking these two parts is small and its mass will be neglected. The total satellite mass is 864 + 32 = 896.0 kg

The center of gravity of the satellite (CG) is at a point where the moment of the cylinder is equal to the moment of the solar panel. The sketch on the next page shows the geometry of the satellite and the moment arms a and b. The distance between the center of the cylinder and the center of the solar panel was given as 1.50 m.

a M_c = b M_p = a 864 = b 32 and a + b = 1.50 m

Solving these two equations gives a = 0.0555 m and b = 1.4445 m identifying the location of the CG of the satellite.

SOLUTION TO 4-10 CONTINUES AT THE END OF THIS SECTION, BECAUSE OF LENGTH

11. Determine the payload for a single-stage vehicle in Example 4–3. Using the data from this example compare your answer with a two-stage and three-stage vehicle.

Eq. 4-7: $\exp(\Delta u/c) = 4.693 = m_0/(m_0 - m_p)$ so $m_p = 3540$ kg
$m_i = 4023$kg and $m_{pl} = 4500 - 4023 = 477$ kg single stage
For two stages, $m_{pl} = 649$ or 678 kg, so single is about 70%.

12. An earth satellite is in an elliptical orbit with the perigee at 600 km altitude and an eccentricity of $e = 0.866$. Determine the parameters of the new satellite trajectory, if a rocket propulsion system is fired in the direction of flight giving an incremental velocity of 200 m/sec when **(a)** fired at apogee, **(b)** fired at perigee, and **(c)** fired at perigee, but in the opposite direction, reducing the velocity. Below, Fig. 4-7 is now 4-8.

Solution to Problem 4-12

It is assumed that there is no drag and that the rocket thrust duration is very short and thus the velocity change can be considered to be instantaneous. First determine the principle parameters of the elliptical flight path as seen in Figure 4-7.

The radius at perigee r_p. consists of the height of the orbit (600 km) plus the mean radius of the earth (6374.2 km); $r_p = 6974.2$ km $= 6.974\text{x } 10^6$ m

There is a right angle triangle in this figure with the hypotenuse a, the side b, and the base of $a^2 - b^2$. By the definition of the eccentricity e, the base of the triangle is equal to ae. In the Pythagoras triangle [$(ae)^2 = a^2 - b^2$]. By inspection of the axis of the ellipse

$r_p = a - ae = a(1\text{-}e)$

With $e = 0.866$ the half-axis a can be determined from this equation as $a = 5.20\text{x}10^7$ m. The minor half axis b is from the Pythagoras equation above.

$b = a\,(1\text{-}\,e^2) = 5.20\text{x } 10^7\,(1 - 0.866^2) = 1.30\text{x}10^7$ m

The radius at apogee $r_a = a + ae = a\,(1+e) = 5.20\text{x}10^7 \text{ X } 1.866 = 9.71 \text{ x } 10^7$ m

From equations 4-30 and 4-31 one can determine the apogee and perigee velocities as $u_a = 742$ m/sec and $u_p = 10{,}330$ m/sec.

Case (a)

The apogee radius r_a will be the same: $9.71\text{x}10^7$ m. The new apogee velocity is $u_a = 747 + 200 = 947$ m/sec. The ellipse will get larger and fatter and the perigee radius will decrease slightly. Equation 4-29, the definition of the ellipse in polar coordinates, will be used in modified form (squared) and solved for a. In this equation at the apogee $R = r_a$, u is u_a, and μ is he gravitational constant.

$$u_a^{\,2} = \mu\,[(2/r_a)\text{–}\,(1/a)] \;= 942^2 = 3.986 \text{ x } 10^{14}\,[(2/9.71\text{x}10^7) - 1/a]$$

Solve for $a = 5.43\text{x}10^7$ m. The eccentricity e can be obtained from the axis of the ellipse and it is

$$r_a = a + ae = a(1{+}e) = 9.71 \text{ x } 10^7 = 5.43\text{x}10^7\,(\,1{+}e\,)$$

$e = 0.786$. Solving for $b = 3.37 \text{ x } 10^7$ m. The perigee radius $r_p = 2a - r_a = 1.15\text{x}10^7$ m. The altitude of the satellite at perigee is now 460 km, which is lower than before the change in velocity.

Case (b)

The perigee radius will remain the same: $r_p = 6.974 \times 10^6$ m. The major axis of the ellipse will be enlarged The new perigee velocity will be $u_p = 10330 + 200 = 10530$ m/sec. Use the same methods as in *Case*(*a*) , but solve for the perigee conditions.

$$u_p^2 = \mu\,[(2/r_p) - 1/a] = 10530^2 = 3{,}986 \times 10^7\,[(2/6.974\text{x}10^7) - 1/a]$$

Solve for the new $a = 1.16\text{x}10^8$ m. The eccentricity $e = 0.939$, $b = 3.99\text{x}10^7$ m, and $r_a = a(1+e) = 2.24\text{x}10^8$ m.

Case (c)

The apogee radius, the major axis and the minor axis will be diminished. Here r_p again stays the same
$r_p = 6974$ km. The new $u_p = 10330 - 200 = 10130$ m/sec. The method are the same as in *Case* (*b*). The results are $a = 3.41 \times 10^7$ m, $b = 2.06 \times 10^7$ m, $e = 0.796$, and $r_a = 6.25 \times 10^7$ m.

13. A sounding rocket (75 kg mass, 0.25 m diameter) is speeding vertically upward at an altitude of 5000 m and a velocity of 700 m/sec. What is the deceleration in multiples of g due to gravity and drag? (Use C_D from Fig. 4–4 and use Appendix 2.)

From Appendix 2, air at 5 km: p = 54 kPa, ρ = 0.763 kg/m3 and T = 256 K.

At u = 700 m/sec, the Mach number is $M = \dfrac{700}{\sqrt{1.4\dfrac{8314}{28}256}} = 2.184$

From Fig. 4-4, at 0° and M = 2.2, $C_D \approx 0.24$ so that the deceleration is *proportional* to

$$C_D(½\rho u^2 A/m_0) = ½(0.24 \times 0.763 \times (700)^2 \times 0.0491)/75 = 29.37 \text{ m/sec}^2$$

Which corresponds to $2.99g_0$'s, which must be contrasted to one-g_0 for gravity itself.

14. Derive Eq. 4–37; state all your assumptions.

See Fig. 4-17. The assumptions listed for this figure are valid and will be used here. The distance travelled to the target is S which can be found by integrating the velocity-time curve of Fig.4-17. There are three regimes in this flight to target and they are:

1. Bottom portion of figure $u_0 t_t$.
2. Powered flight portion $½u_p t_p$
3. Unpowered or free flight $u_p(t_t - t_p)$

Adding the three pieces together, $S = u_0 t_t + ½u_p t_p + u_p(t_t - t_p) = t_t(u_0 + u_p) - ½u_p$

(CONT.)

Solving for the time to target, $t_t = \dfrac{S + 0.5u_p t_p}{u_0 + u_p}$

15. Calculate the change of payload relative to the overall mass (i.e., the payload fraction) for a single stage rocket propelled vehicle when replacing I_s = 292 sec and propellant mass fraction (for the propulsive stage) 0.95 with a new engine of 455 sec and 0.90. Assume that the total mass, the structure and any other fixed masses remain unchanged. Take Δu = 5,000 m/sec.

For a single-stage rocket with a given I_s and a desired Δu, it is more useful to directly calculate and display the payload ratio (m_{pl}/m_0). We can develop an ideal relation for this ratio using a propellant mass fraction of the stage (say, ζ_i corresponding to m_i as per Example 4-3) as follows: $m_0 = m_f + m_p$ and $m_0 = m_{pl} + m_i = m_{pl} + m_p/\zeta_i$.

Eliminating m_p between the two equations and using Eq. 4-7, we write the payload fraction as

$$\frac{m_{pl}}{m_0} = \frac{e^{-\Delta u/c}}{\varsigma_i} - \frac{(1-\varsigma_i)}{\varsigma_i}$$

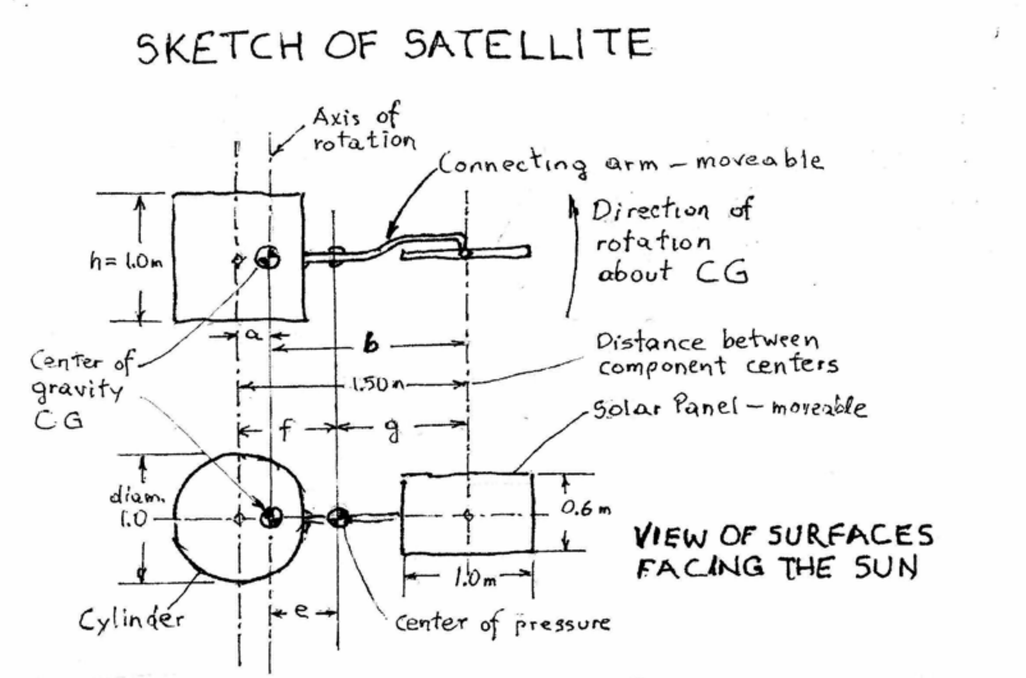
This result has significant consequences such as: not every combination of Δu and I_s is possible in that, for example, abscissa values of $\Delta u/c > 1.85$ with $\zeta_i = 0.85$ would yield negative payload fractions. Moreover, showing the payload fraction explicitly better represents why for a given mission the higher specific impulse of liquid-engine vehicles would be more attractive than solids to a customer that can afford them (where values of $\zeta_i > 0.92$ would not be very realistic even for chemical rockets). Figure 4-3 can be used directly with the given information. Now the solution to this problem becomes simply,

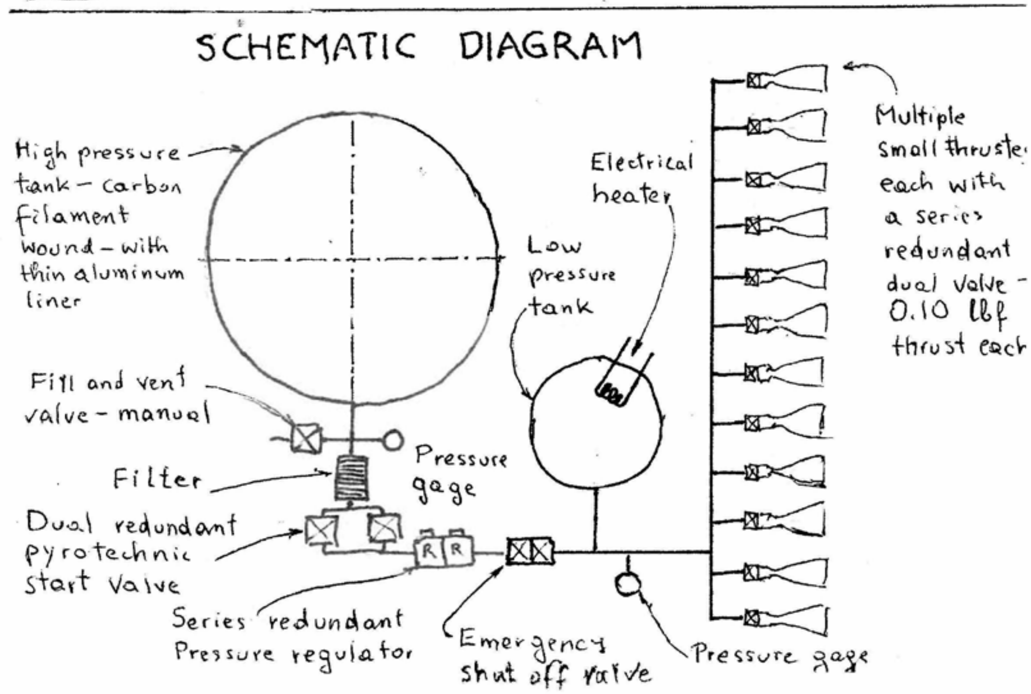
Original case: $\Delta u/c$ = 5000/9.81x292 = 1.75, with ζ_i = 0.95 yielding m_{pl}/m_0 = 0.131

Modified case: $\Delta u/c$ = 5000/9.81x455 = 1.12, with ζ_i = 0.90 yielding m_{pl}/m_0 = 0.251.

Thus, there is a gain in payload fraction even though the propellant mass fraction is less.

SOLUTION TO PROBLEM 4-10 (CONTINUED). Notes: Fig. 4-13 is now 4-14 and 3-6 is now 3-5; 8th Ed. (from page 117, line 17) is now (below Eq. 4-25).





The <u>moment of inertia of the satellite</u> consists of the moment of inertia of the cylinder (I_c) plus the moment of inertia of the solar panel (I_p). Both rotate on an axis, which is parallel to the cylinder axis, but goes through the center of gravity. The moment of inertia of the arm linking these two subassemblies is small and will be neglected. Each of these two moments of inertia of the two parts consists of (1) a moment of inertia where the cylinder or the panel are rotating by themselves about their own axis going through their own geometric center and (2) an extra term of the moment of inertia is added, because the actual axis of rotation goes through the CG of the compete satellite; it is the mass times the square of the distance between the part's center and the axis of rotation.

The <u>moment of inertia of the cylinder</u> I_c rotating about its own cylinder center line is $I_c = M_s\, d^2/4$. An extra term is added when the cylinder rotates about a parallel axis going through the CG of the satellite.

$$I_c = M_c\, \mathrm{d}^2/4 + M_c\, \mathrm{a}^2 = (1/4)\ 864 + 864 \times 0.0555^2 = 218.66 \text{ kg m}^2$$

Here a is the moment arm between the center line of the cylinder and the CG of the satellite, as seen in the sketch. The <u>moment of inertia of the solar panel</u> I_p of mass M_p of 32.0 kg, when rotating about an axis going through the CG of the satellite, is

$$I_p = M_p\, k^2 + M_p\, b^2 = 32 \times 0.330^2 + 32 \times 1.4445^2 = 70.18 \text{ kg m}^2$$

Here k is the radius if gyration of the cylinder (0.330 m) rotating about its own center line and b is shown on the sketch as a moment arm and is 1.445 m. The total moment of inertia of the satellite rotating about an axis going through the center of gravity (CG) is 218.6 + 70.18 = 288.8 kg m^2.

The small solar radiation force caused by the sun's rays is caused by the solar radiation pressure acting upon all surfaces exposed to this radiation. The surface area A consists of the solar cell area on top of the cylinder plus the solar cell area of the solar panel. The satellite geometry, flight maneuvers, and the flight orbits are such that the incoming solar rays are always perpendicular to these solar cell surfaces. The solar radiation pressure p is given by equation 4-32 and shown below. It depends on the incidence angle theta ($\cos\theta = 1.0$) and the specular and diffuse coefficients of reflectivity k_s and k_d; typical values of 0.75 and 0.10 were selected for these coefficients.

$$p = 4.5 \times 10^{-6} \cos\theta\, [(1 - k_s)\cos\theta + 0.67\, k_d]$$
$$= 4.5 \times 10^{-6} \times 1.0\, [(1 - 0.75) \times 1.0 + 0.67 \times 0.10] = 1.4265 \times 10^{-6} \text{ N m}^{-2}$$

The center of pressure can be determined when the moment of the radiation force on the cylinder top surface area A_c is equal to the moment of the radiation force on the solar panel surface area A_p. It is that point where a concentrated single force of equal magnitude as the integrated force of the pressure on all the exposed areas. Such a force would not cause any turning of the satellite. The moment arms f and g are identified in the sketch and can be found from the following two equations:

$p\,A_c\,\mathrm{f} = p\,A_p\,\mathrm{g}$ and $\mathrm{f} + \mathrm{g} = 1.50$ m

The resulting moment arms are f = 0.8608 m and g = 0.6496 m. This identities the location of the center of pressure. The total single force F on the satellite due to solar radiation pressure p is listed next. It is located at the center of pressure.

$F = p\,(A_c + A_p) = 1.4267 \times 10^{-6} \times 1.385 = 1.9764 \times 10^{-6}$ N

The solar radiation pressure also causes a small linear force displacing the satellite from its orbit, but its effect is of no direct concern to this problem or its solution.

Problem 10 continued, part (a); Torque caused by solar pressure and daily angular displacement.

An unbalanced solar pressure distribution will cause the spacecraft to turn about an axis, which goes through the center of gravity (CG) and is parallel to the cylinder axis. The torque turning the satellite is this force caused by solar radiation pressure (considered to be concentrated at the center of pressure) multiplied by the moment arm e, which is the distance between the center of gravity (CG) and the center of pressure. Here

$\mathrm{e} = \mathrm{f} - \mathrm{a} = 0.6496 - 0.0554 = 0.5941$ m.

The torque T can now be found

$T = F\,\mathrm{e} = 1.9764 \times 10^{-6} \times 0.5941 = 1.1741 \times 10^{-6}$ Nm

This torque causes an angular acceleration alpha a expressed in radians/sec^2, which can be determined from $T = I\,a$ and I is the moment of inertia of the satellite. Solve for a

$a = T/I = (1.174 \times 10^{-6})/288{,}8 = 0.44246 \times 10^{-9}$ radians/sec^2

The angle through which the satellite is turned is theta ($\theta = \frac{1}{2}\,a\,t^2$) and the time t here is one day or 24 hours or 86,400 seconds. However this time is reduced to 54 %, because the satellite is in the shadow of the planet earth (no radiation received) during parts of its flight. So the effective time is t = 86,400 sec x 0.540 = 46,656 sec.

The angle traveled in one day is

$\theta = \frac{1}{2}\,a\,t^2 = \frac{1}{2} \times 0.40654 \times 10^{-9} \times 46{,}656^2 = 0.4424$ radians per day or about 25 ^{0}/day.
The attitude control system explained in part (c) is intended to remedy this angle change.

Problem 10, continued, part (b); calculate the drag force and compare it with solar radiation forces. Since an accurate determination of the variable drag is beyond the scope of this book, an order of magnitude simple solution will be considered to be satisfactory.

Equation 4-11 defines the drag. It contains the satellite orbit velocity u, the area A normal to the flight direction, and the air density ρ at an altitude of 160 km (from Appendix 2 this density is 1.233 x 10^{-9} kg/m^3). The velocity of the satellite u in a low circular earth orbit can be obtained from Equation 4-26; here the Earth Gravitational Constant μ is 3.9860 x 10^{14} m^3/sec. The radius of the earth's circular orbit R is the Earth's mean radius R_0 = 6374.2 km (from page 117, line 17) plus the height of the orbit of 160 km making R = 6534.2 km. The orbit velocity is

$$u = \sqrt{\mu/R} = \sqrt{3.986 \times 10^{14}/(6.534 \times 10^6)} = 7810 \text{ m/sec}$$

The area A in equation 4-11 will actually vary during the flight as the satellite is rotated in order to have the solar cells always perpendicular to the sun's rays. It is assumed that the area of the solar cells (previously determined as 1.3855 m^2) and a fixed average drag coefficient C_D of 1.0 give a solution that is within an order of magnitude of a more elaborate and precise solution.. The typical drag will be

$$D = \tfrac{1}{2} C_D \rho A u^2 = 0.50 \times 1.00 \times 1.233 \times 10^{-9} \times 1.3855 \times 7810^2 = 0.521 \text{ N}$$

This compares to a force caused by solar radiation pressure of 1.976 x 10^{-6} N.

The drag will also cause a rotation of the satellite with a much higher torque than that caused by the solar radiation.

Conclusion: The drag force for this low orbit satellite is several orders of magnitude larger than the force caused by the solar pressure. Since the air density and the orbital velocity diminish with increasing altitude, the drag forces will be greatly reduced, but the radiation caused pressures will be about the same. So at altitudes above about 900 km the drag forces will be close to zero.

Problem 10, part (c); Attitude control system

This part of the problem shows only one of a number of possible designs of an attitude control system using compressed air as the propellant. Compared to systems with liquid propellants, the use of compressed air automatically enhances system safety, gives most likely lower costs, potentially high reliability, no toxicity issues, no combustion or high temperature material problems and no explosion or flammability concerns. Here there is a choice of different components, different component arrangements and designs, using experience of successful prior similar systems, or different methods of fabrication, assembly or check-out. The principal disadvantage of gaseous systems is the low performance and the relatively large volume and mass of the propellant tanks.

The thrust level must be high enough to overcome the forces and torques caused by the perturbations or flight disturbances of the vehicle on orbit. A thrust level of 0.10 lbf or 0.448 N is more than adequate for overcoming solar radiation effects. It may also be good enough for also counteracting the drag forces (with two small thrusters pointing in the same direction and operating at the same time), but this was not a part of the problem.

As shown in a simplified schematic diagram a two tank system was selected. A high pressure (5000 psi) tank for air storage and a smaller low pressure tank (about 30 psia) for feeding the individual thrusters through their valves, were selected. With a 50 to 1.0 nozzle exit area ratio in the thrusters, there is the concern that the air will cool as it expands during the flow through the nozzle and become so low in temperature, that the air would become liquid and/or freeze. The freezing point of air is at approximately 60° K. Rough estimates indicate that, when the chamber gas is at ambient temperature, the air just before the nozzle exit, will be close to or below the freezing point. Freezing or liquification of the air would greatly reduce performance, particularly when the freezing occurs part way up the nozzle expansion section.

The gas in the low pressure tank will be electrically heated, typically during the long periods when the satellite does not require maximum power and this tank will be thermally insulated. The tubing going to the thrust chambers and the small valves will also be thermally heated and insulated. Heating will provide a margin for preventing liquefying or freezing the air near the nozzle exit, will increase the specific impulse by about 20 % , and will still allow the use of low cost materials and simple fabrication methods for making the components.

Data about compressed air can be found in Table 7-3 for a storage temperature of 20°C or 293°K and a pressure of 5000 psia and these would be the conditions for the high pressure tank. If the high pressure air would be fed directly at 5000 psia into one of the thrusters with an area ratio of 50 to 1.0 the specific impulse would be 74 seconds, but this would be wasteful of air.

Figure 3-6 shows the optimum pressure ratio for a nozzle exit area ratio of 50 and a specific heat ratio of 1.40. The thrust coefficient is about 1.69 and the optimum pressure ratio p_1/p_2 is approximately 1,350. This pressure ratio is independent of the absolute pressure and the gas temperature. The nozzle exit pressure is

$p_2 = p_1/1350 = 30/1350 = 0.222$ psia.

The expansion from this nozzle exit pressure to the near vacuum of the 160 km altitude takes place outside the nozzle in space and does not contribute to the thrust. A larger nozzle area ratio, say above 200, would theoretically allow a somewhat higher performance (more I_s by up to 3 percent), but in reality it would cause the air to turn into liquid droplets and/or solid particles and cause a major loss in performance..

The pressure regulator maintains constant pressure in the low pressure tank of about 30 psia or 206820 N/m^2. This assures consistent thrust levels for each operation, and this can be important in some flight control systems. Most pressure regulators require a small pressure drop in order to perform its regulating function, but this drop has been neglected in solving this problem. The high pressure tank periodically replaces the expended gas in the low pressure tank.

The heating of the air to about 350°F or 449°K is planned and it actually increases the specific impulse beyond the value given in Table 7-3, which is 74 sec.at 297°K. This increase is in accordance with the square root of the absolute temperature, namely $\sqrt{449/297}$ or a factor of 1.238 to an enhanced specific impulse value of 91.6 sec during steady-state operation. The mass flow decreases by a similar percentage. As a safety feature the air supply from the high pressure tank will be shut off, if the pressure in the low pressure tank exceeds a certain safe value. The low pressure tank, the valves at each thruster and the distribution tubing will be electrically heated and thermally insulated. The electrical power for heating will come from the satellite's electrical system and heating will be done only during those periods, when the satellite itself is not using high power.

The operation of the thrusters will be in a pulsing mode. The valve opens for a short time, arbitrarily chosen as 50 milliseconds, and a small mass of air is supersonically discharged, causing a small impulse to be imparted to the satellite. Then there is a pause of nominally 75 milliseconds and then the next periodic discharge of 50 milliseconds is repeated. See the Section 4.5 on Flight Maneuvers. The satellite's flight control system will select the thruster to be activated and command the number of pulses to be performed. For pure torques a pair of thrusters will be pulsed simultaneously. Pulsing causes a slight decrease in specific impulse, perhaps a couple of percent with gaseous propellants. So $I_s = 0.98 \times 91.6 = 89.8$ sec.

The mass flow $\dot{m} = F/(I_s g_0) = 0.448/(89.8 \times 9.806) = 5.087 \times 10^{-4}$ kg/sec (Equation. 2-5)
A single pulse has a air mass $m = 5.087\ 10^{-4} \times 0.050 = 2.543 \times 10^{-5}$ kg

The satellite high pressure 5,000 psia air tank has an inside diameter of 0.50 m or a radius of 0.25 m. The tank will be spherical and made of wound strong carbon filament with an epoxy resin binder and an internal spherical liner of thin aluminum sheet (0.015 in. thick), which will prevent air leakage through the porous filament material of the tank wall. The density of air from Table 7-3 is 25.5 lb/ft^3 or 403.5 kg/m^3. The mass of stored air

$$M_{\text{air}} = (4\pi/3)\, r^3 \rho = 4.1886 \times 0.253 \times 403.5 = 26.40 \text{ kg}$$

Neglecting the residual gas remaining in the tank, the number of pulses available is approximately $26.40/(2.543 \times 10^{-5}) = 1.0383 \times 10^{6}$ or over 1 million pulses, which seems to be more than needed for correcting only the solar radiation perturbation over a number of years. It provides a further margin. Some of this compressed air can also be used for other purposes, but such use is beyond the scope of this problem.

It requires a minimum of 12 thrusters to obtain translation maneuvers and rotation maneuvers around all axis and in all directions. There will be clusters of 2,3, and/or 4 thrusters located at the periphery of the cylinder. The arrangement or locations of the thrusters have not yet been chosen, because they also depend on certain vehicle parameter, which have not been given. However they will be analogous to those in Figure

4-13 for a spherical satellite. With this arrangement two thrusters will be discharged simultaneously in order to obtain either linear or rotary accelerations.

Thrust chamber geometry: Equation 3-31 shows that the nozzle throat area is $A_t = F/(C_D\, p_1) = 0.10$ lbf/(1.69 x 30 psi) = 1.972 x 10^{-3} in^2. The throat diameter is 0.0393 inches. The nozzle exit area is 50 times larger than A_t and the exit diameter is 0.2778 in., the chamber diameter will be about 2.3 times the nozzle throat diameter and the nozzle converging section and throat region will be well rounded. A full length bell shaped nozzle is visualized with a length of about 0.74 inch from the throat to the exit. A built-in shut-off valve will be fastened to the top of each chamber. It will be solenoid operated and the valve will be heated to about 350^{0}F. This temperature has been selected, because it is not high enough to cause damage to the plastics used in the electrical components and because it allows certain bench tests. An aluminum alloy is chosen for the chamber-nozzle piece. The internal chamber and nozzle surfaces will be highly polished to be very smooth, because this reduces boundary layer friction and turbulence.

END OF SOLUTION TO 4-10

CHAPTER 5

1. Explain the physical or chemical reasons for a maximum value of specific impulse at a particular flow mixture ratio of oxidizer to fuel.

It is a complex balance between (1) the energy released by the combustion process which yields the highest temperature near the stoichiometic mixture ratio and (2) the heating of the gases by this energy up to the combustion temperature. For good performance (high specific impulse) the resulting gas mixture should have a low molecular mass and a high temperature (see Eq. 3-16). Energy is absorbed not only to heat the gases but also in (3) endothermic reactions which dissociate any complex molecules present. If too much energy is absorbed by these endothermic reactions, the combustion temperature might be reduced beyond the reduction of the molecular mass of the mixture thus decreasing the specific impulse. For gases that contain excess fuel (rich in hydrogen and/or carbon monoxide), lower molecular masses exist below the maximum temperature as seen in Fig 5-1 where both I_s and c* display a maximum value but a slightly different fuel-rich conditions.

2. Explain why, in Table 5–8, the relative proportion of monatomic hydrogen and monatomic oxygen changes markedly with different chamber pressures and exit pressures.

High chamber pressures tend to drive chemical reactions into the direction of fewer molecules. Therefore, the monoatomic species of oxygen or hydrogen will recombine into their diatomic form – higher pressures reduce dissociation. At the combustion chamber these molecules will reach equilibrium but at the nozzle exit the much lower pressures will allow some monoatomic species to exist.

3. This chapter contains several charts for the performance of liquid oxygen and RP-1 hydrocarbon fuel. By mistake the next shipment of cryogenic oxidizer contains at least 15% liquid nitrogen. Explain what general trends should be expected in the results of the next test in the performance values, the likely composition of the exhaust gas under chamber and nozzle conditions, and the optimum mixture ratio.

The combustion temperature will be noticeably lower because the nitrogen takes away from the LOX/RP-1combustion energy for vaporization and heating, not adding any chemical energy. The lower temperatures will cause the nozzle flow to be closer to the frozen equilibrium than to the shifting equilibrium conditions. Since the specific impulse for frozen equilibrium is always less than shifting equilibrium, there are two effects that lower the performance.

4. A mixture of perfect gases consists of 3 kg of carbon monoxide and 1.5 kg of nitrogen at a pressure of 0.1 MPa and a temperature of 298.15 K. Using Table 5–1, find **(a)** the effective molecular mass of the mixture, **(b)** its gas constant, **(c)** specific heat ratio, **(d)** partial pressures, and **(e)** density.

CO: $m = 3$ kg, $\mathsf{M} = 28.016$, $C_p = 29.142\text{x}10^3$ J/kg-mol K

N_2: $m = 1.5$ kg, $\mathsf{M} = 28.0134$, $C_p = 29.125\text{x}10^3$ J/kg-mol K

$n_{CO} = 3/28.0106 = 0.1071$ mol, $n_{N2} = 1.5/28.0134 = 0.0535$ mol

So $n_{tot} = 0.16065$ mol and $X_{CO} = 0.667$ and $X_{N2} = 0.333$

$\mathsf{M}_{mix} = 0.667\text{x}28.016 + 0.333\text{x}28.0134 = 28.0115$ kg/kg-mol

$R_{mix} = 8314.3/28.0115 = 296.82$ J/kg K

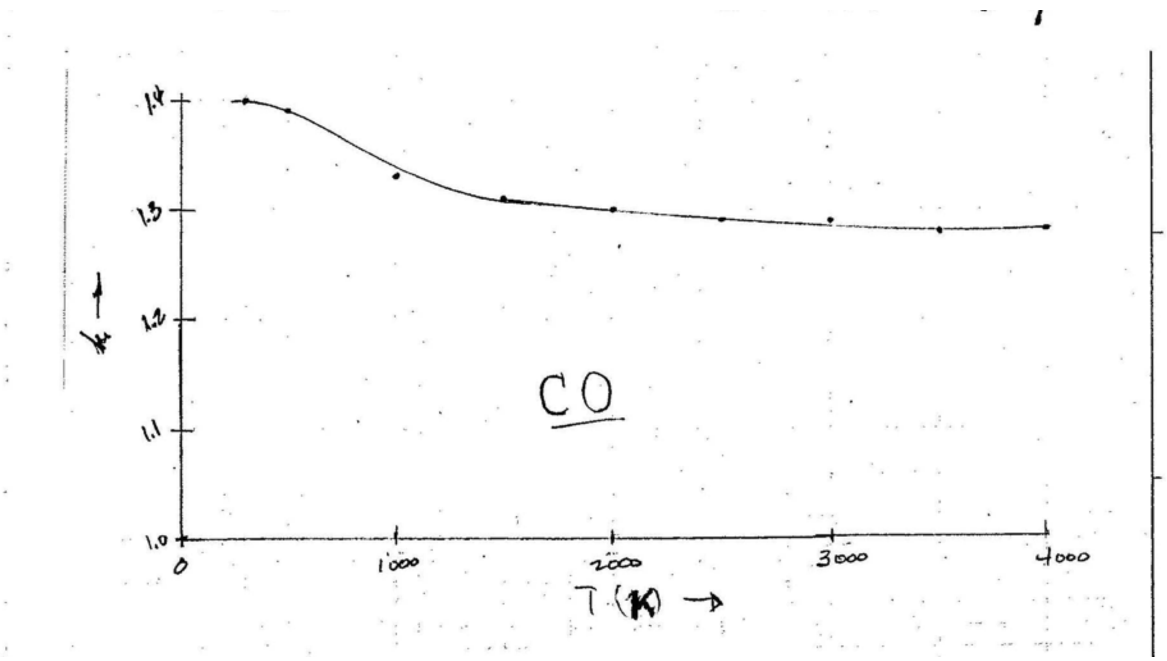
$C_{p\text{mix}} = 0.667\text{x}29.142\text{x}10^3 + 0.333\text{x}29.125\text{x}10^3 = 29.136\text{x}10^3$ J/kg-K

$k_{mix} = 29.136\text{ x}10^3/(29.136\text{x}10^3 – 8.314\text{x}10^3) = 1.40$

$p_{CO} = 0.667\text{x}0.1$ MPa = 66.7 kPa and $p_{N2} = 33.3$ kPa

$\rho = p/RT = 100{,}000$ Pa/(296.82x298.15) = 1.13 kg/m^3

5. Using information from Table 5–2, plot the value of the specific heat ratio for carbon monoxide (CO) as a function of temperature. Notice the trend of this curve; it is typical of the temperature behavior of other diatomic gases.



6. Modify and tabulate two entries in Table 5–5 for operation in the vacuum of space, namely oxygen/hydrogen and nitrogen tetroxide/hydrazine. Assume the data in the table represents the design condition.

Table 5-5, p_1 = 1000 psia, and $p_2 = p_3$ = 14.7 psia. Modify to p_2 = 14.7 psia but $p_3 = 0$.

Eq 3-28, $I_s = I_s)_{\text{opt}} + c^*(p_2/p_1 - p_3/p_1)\epsilon\,/g_0$

O_2/H_2 $\quad I_s$ = 386 sec + 2428(14.7/1000 – 0)ϵ/9.81 = (386 +3.64ϵ) sec

N_2O_4/ Hydrazine $\quad I_s$ = (283 + 2.65ϵ) sec

[For ϵ = 10, I_s = 422 sec and 310 sec respectively.] Area ratio should be 7.92

7. Various experiments have been conducted with a liquid monopropellant called nitromethane (CH_3NO_2), which can be decomposed into gaseous reaction products. Determine the values of T, M, k, $c*$, C_F, and I_s using the water–gas equilibrium conditions. Assume no dissociations and no O_2.

SOLUTION. The chemical reaction for 1 mol of reactant can be described as

$$1.0\ CH_3NO_2 \rightarrow n_{CO}CO + n_{CO_2}CO_2 + n_{H_2}H_2 + n_{H_2O}H_2O + n_{N_2}N_2$$

Neglect other minor products. The mass balances are obtained for each atomic element.

$$\begin{array}{ll} C & 1 = n_{CO} + n_{CO_2} \\ H & 3 = 2n_{H_2} + 2n_{H_2O} \\ O & 2 = n_{CO} + 2n_{CO_2} + n_{H_2O} \\ N & 1 = 2n_{N_2} \quad \text{or} \quad n_{N_2} = 0.5 \end{array}$$

The reaction commonly known as the water–gas reaction is

$$H_2 + CO_2 \rightleftharpoons H_2O + CO$$

Its equilibrium constant K, expressed as molar concentrations, is a function of temperature.

$$K = \frac{n_{H_2O}n_{CO}}{n_{H_2}n_{CO_2}}$$

The five equations above have six unknowns: namely, the five molar concentrations and K, which is a function of temperature. Solving for n_{H_2} and K:

$$(K-1)n_{H_2}^2 - (3 - K/2)n_{H_2} = 2.25$$

K can be obtained from a table of the water–gas reaction as a function of temperature. Try $T = 2500\,K$ and $K = 6.440$ and substitute above.

$$5.440n_{H_2}^2 - 0.220n_{H_2} - 2.25 = 0$$

then

$$\begin{aligned} n_{H_2} &= 0.664 \\ n_{H_2O} &= 1.500 - n_{H_2} = 0.836 \\ n_{CO_2} &= 0.164 \qquad n_{CO} = 0.836 \end{aligned}$$

The heats of formation $\Delta_f H^0$ for the various species are listed in the table below [from the JANAF thermochemical tables (Refs. 5–7 and 5–9)]. The heat of reaction is obtained from Eq. 5–9 in kilojoules per mole. By definition, the heat of formation of H_2 or N_2 is zero. From Eq. 5–9,

$$\begin{aligned} \Delta_r H^0 &= \sum(n\Delta_f H)_{\text{products}} - (\Delta_f H^0)_{\text{reactant}} \\ &= 0.836(-241.8) + 0.164(-393.5) + 0.836(-110.5) - 1.0(-113.1) \\ &= -246\,\text{kJ/mol} \end{aligned}$$

The enthalpy change of the gases going from the reference conditions to the combustion temperature can also be obtained from tables in Refs. 5–7 and 5–8 and is again listed below.

Species	$\Delta_f H^0$	Δh_j^{2500}	Molecular Weight	n_j
N_2	0	74.296	28	0.500
H_2O	−241.826	99.108	18	0.836
H_2	0	70.498	2	0.664
CO	−110.53	74.985	28	0.836
CO_2	−393.522	121.917	44	0.164
CH_3NO_2	−113.1		61	1.000

The gas enthalpy change of the hot gas in the combustion chamber is numerically equal to the heat of formation. Using data from the table,

$$\Delta H_{298}^{2500} = \sum n_j \Delta h_j = 249.5\,\text{kJ/mol}$$

This is not identical to the 246 kJ/mol obtained previously, and therefore a lower temperature is to be tried. After one or two iterations the final combustion temperature of 2470 K will be found where the heat of reaction balances the enthalpy rise. The above-mentioned composition will be approximately the same at the new temperature. The molecular weight can then be obtained from Eq. 5–5:

$$\mathfrak{M} = \frac{\sum n_j \mathfrak{M}}{\sum n_j} = \frac{28 \times 0.5 + 18 \times 0.836 + 2 \times 0.664 + 28 \times 0.836 + 44 \times 0.164}{2 \times (0.836) + 0.664 + 0.164 + 0.500} = 20.3$$

The specific heat varies with temperature, and average specific heat values $\bar{c}_p$ can be obtained from each species by integrating

$$\bar{c}_p = \frac{\int_{298}^{2470} c_p dT}{\int_{298}^{2470} dT}$$

Values of $\bar{c}_p$ can be obtained from tables in Ref. 5–7 and, if not done by computer, the integration can be done graphically. The result is

$$\begin{aligned} \bar{c}_p &= 41{,}440\ \text{kJ/K-kg-mol}/20.3 \\ &= 2040\ \text{kJ/kg-K} \end{aligned}$$

The specific heat ratio is, from Eq. 3–4,

$$k = \frac{C_p}{C_p - R'} = \frac{41{,}440}{41440 - 8314} = 1.25$$

With $\mathfrak{M}$, k, and T_1 now determined, the ideal performance of a nitromethane rocket engine can be established from Eqs. 3–16, 3–30, and 3–32 for $p_1 = 69$ atm and $p_2 = 1.0$ atom. The results are

$$\begin{aligned} c^* &= 1525\,\text{m/sec} \\ C_F &= 1.57 \text{ (from Fig. 3–6)} \\ c &= 1.57 \times 1525 = 2394\,\text{m/sec} \\ I_s &= 2394/9.80 = 244\,\text{sec} \end{aligned}$$

8. The figures in this chapter show several parameters and gas compositions of liquid oxygen burning with RP-1, which is a kerosene-type material. For a mixture ratio of 2.0, use the compositions to verify the molecular mass in the chamber and the specific impulse (frozen equilibrium flow in nozzle) in Fig. 5–1.

Verify the molecular mass and the specific impulse in Fig 5-1 for r = 2.0, k = 1.24, $\mathfrak{M}$ = 21

Fig. 5-2, frozen flow, mol- %: CO_2 = 7.5, H_2O = 27.5, H_2 = 18, CO = 42, H = 2.5, OH = 2

$\mathfrak{M}$ = (0.42)(28) + (0.275)(18) + (0.18)(2) + (0.075)(44) + (0.025)(1) + (0.02)(17) = 20.897

Fig. 5-1 for r = 2.0, k = 1.24, T = 3360 K, I_s = 282 sec

$$I_s = c/g_0 = \left[\frac{1}{9.81}\sqrt{\frac{2\times1.24}{1.24}\frac{8314.3}{20.9}3360\left(1-\left(\frac{14.7}{1000}\right)^{0.24/1.24}\right)} \right] = 2{,}276/9.81 = 283 \text{ sec}$$

CHAPTER 6

1. In an engine with a gas generator engine cycle, the turbopump has to do more work in the pumps, if the thrust chamber operating pressure is raised. This of course requires an increase in turbine gas flow which, when exhausted, adds little to the engine specific impulse. If the chamber pressure is raised too much, the decrease in performance due to an excessive portion of the total propellant flow being sent through the turbine and the increased mass of the turbopump will outweigh the gain in specific impulse that can be attained by increased chamber pressure and also by increased thrust chamber nozzle exit area ratio. Outline in detail a method for determining the optimum chamber pressure where the sea-level performance will be a maximum for a rocket engine that operates in principle like the one shown in Fig. 1–4.

There is more than one way to solve this optimization problem and one of them is discussed below. When there is valid data from earlier successful complete engine or component tests, then the solution will be simpler and more trustworthy that when little or no prior useful data is available. In this example we number the steps.

1. Obtain the best available data on the properties of relevant materials and their fabrication as they relate to this engine. This might include the gas densities and vapor pressures of the propellants, the strength and other physical properties of the metals, their temperature and pressure variation, etc. It should also include obtaining data on prior related liquid propellant rocket engines with the same propellants, with chamber pressures within the range of interest and with the same engine cycle, and their performance, masses of inert parts, reliability, etc. Most rocket propulsion organizations keep data on material properties and prior proven engine components.

2. Estimate the likely minimum and maximum values of the chamber pressure relevant to this problem. For example, if p_1 is too low the thrust chamber will be very large and its dimensions will likely exceed those of the engine compartment of the flight vehicle. If p_1 is too high, this will probably have excessive heat transfer and/or be beyond the pressure range of prior successful rocket tests. To some degree, the determination of the maximum and minimum values of p_1 is arbitrary and based on experience.

3. Select a number of specific chamber pressures (a minimum of 3, but 4 or 5 is better) from the pressure range previously determined.

4. For each of these specific chamber pressures undertake a preliminary design of the engine in enough detail to define key component parameters (thrust chamber, turbopump, gas generators, larger valves, structure, etc.). For each component make a preliminary layout or sketch in enough detail (i.e., wall thickness, materials, sizes, etc.) so as to allow a creditable estimate of the inert mass. Obtain an overall engine mass estimate for each selected chamber pressure from all major component masses together with minor components (bolts, tubes, washers, gaskets, fasteners, etc.).

5. Determine the likely actual performance as a function of chamber pressure and for a given mixture ratio and nozzle area ratio. Often the specific impulse is the chosen performance

parameter, at least initially as outlined in Chapter 3. A more useful vehicle performance parameter considers both the engine specific impulse or effective exhaust velocity and the influence of any increased mass (a change in engine mass affects the vehicle's mass ratio). Some analysts use a terminal flight velocity or a performance parameter, similar to Eq. 4-20. A correction of the performance for going to larger nozzle area ratios particularly at the higher chamber pressures needs to be made. Such a correction depends on relevant design constraints (such as vehicle available space or diameter and limits on engine weights).

6. An alternative approach is to use data from prior proven rocket engines, which are similar to the engine being optimized. Such an engine should preferably use the same propellant with a mixture ratio close to the engine being analyzed, and a chamber pressure within the desired range, see Item 2 above. Rocket propulsion organizations have usually good data on actual performance, actual component mass, etc., on these proven engines. The data of these engines and components has to be modified to the new operating conditions of the selected engine and corrected for such items as chamber pressure, nozzle area ratio, mixture ratio, etc. Both the performance and the inert engine mass have to be corrected. Performance corrections are based on equations or curves available in this book, and some rocket propulsion organizations have developed computer software for correcting actual engine mass on new engine-operating conditions for engines being optimized. In some optimization studies both the new preliminary design layout and an extrapolation of data from prior proven engines is used.

7. The chamber pressure at which the chosen vehicle flight performance parameter is a maximum is the desired optimum pressure.

2. The engine performance data for a turbopump rocket engine system are as follows:

Propellants	Liquid oxygen/kerosene
Engine system specific impulse (steady state)	272 sec
Engine system mixture ratio	2.52
Rated engine system thrust	40,000 N
Oxidizer vapor flow to pressurize oxidizer	0.003% of total oxidizer flow tank
Propellant flow through turbine at rated thrust	2.1% of total propellant flow
Gas generator mixture ratio	0.23
Specific impulse of turbine exhaust	85 sec

Determine performance of the thrust chamber *Is* , *r* , *F* (see Section 11.2).

See also Fig 1-4 and Sections 2.5, 10.8 as well as 11.2.

$$I_{soa} = 272 \text{ sec}, \; I_{sgg} = 85\text{sec}, \; r_{gg} = 0.23, \; r_{oa} = 2.52$$

$$F_{oa} = 40{,}000 \text{ N} = F_c + F_{gg}$$

$$\dot{m}_o)_{tank} = 0.003\ \%\ \dot{m}_o \quad \text{and} \quad \dot{m}_{gg} = 2.1\text{x}10^{-2}\ \dot{m}$$

$$\dot{m} = 40{,}000/(272)(9.81) = 15 \text{ kg/sec} = \dot{m}_c + \dot{m}_{gg}, \; \dot{m}_{gg} = 2.1\text{x}10\text{-}2\,\dot{m} = 0.315 \text{ kg/sec}$$

$$\dot{m}_o = r\dot{m}/(1+r) = 10.74 \text{ kg/sec} = \dot{m}_{oc} + \dot{m}_{ogg} + \dot{m}_o)_{tank}$$

However last term is rather small and usually neglected in preliminary analysis.

$\dot{m}_{ogg}$ = 5.89 x 10^{-2} kg/sec and $\dot{m}_{oc}$ = 10.68 kg/sec

$\dot{m}_f$ = 4.26 kg/sec = $\dot{m}_{fc}$ + $\dot{m}_{fgg}$

$\dot{m}_{fc}$ = 4.009 kg/sec and $\dot{m}_{fgg}$ = 0.252 kg/sec

Combustor and main nozzle,

F_c = 40,000 – F_{gg} = 40000 –(85)(0.315)(9.81) = 39,740 N

I_{sc} = 39740/[(10.68 + 4.0)(9.81)] = 276 sec

r_c = 10.68/4.009 = 2.66

3. For a pulsing rocket engine, assume a simplified parabolic pressure rise of 0.005 sec, a steady-state short period of full chamber pressure, and a parabolic decay of 0.007 sec approximately as shown in the sketch. Plot curves of the following ratios as a function of operating time t from t **=** 0.013 to t **=** 0.200 sec: **(a)** average pressure to ideal steady-state pressure (with zero rise or decay time); **(b)** average *Is* to ideal steady-state *Is* ; **(c)** average *F* to ideal steady-state *F*.

4. For a total impulse of 100 lbf-sec compare the volume and approximate system weights of a pulsed propulsion system using different gaseous propellants, each with a single spherical gas storage tank (at 3500 psi and 0° C). A package of small thrust nozzles with piping, valves, and controls is provided that weighs 3.2 lbf. The gaseous propellants are hydrogen, nitrogen, or argon (see Table 7–3).

I_t = 100 lbf-sec, p_{tank} = 3500 psi, T_{tank} = 0 °C = 492 °R, spherical tanks

$I_t = c m_p / g_0$ for constant specific impulse in Eqs 2-1 and 2-6, EE units.

$c = v_2$ = Eq 3-18 for these comparisons ignore any pressure thrust and residual enthalpy.

$$c = \sqrt{\frac{2kg_0 R' T_0}{(k-1)\mathrm{M}}} = 6.99\times10^3\sqrt{\frac{k}{(k-1)\mathrm{M}}} \ \text{ft/sec}$$

$$\rho = \frac{p}{RT} = \frac{p\mathrm{M}}{R'T} = 0.663\mathrm{M} \ \text{lbm/ft}^3$$

$$m_p = 1.43\times10^{-2} g_0\sqrt{\frac{(k-1)\mathrm{M}}{k}} \ \text{lbm}$$

Gas	W (lbm/lb-mol)	ρ (lbm/ft^3)	m_p (lbm)	Vol (ft^3)
H_2	2.0	1.33	0.348	0.262
N_2	28.0	18.6	1.30	0.0699
Ar	39.9	25.5	1.84	0.0694

The hydrogen propellant has the least propellant mass but the largest tank volume. Thus a hydrogen tank would for the same storage pressure be much heavier. Nitrogen could be optimal here because of its lowest cost.

Note: The mass of the piping should be common to all at these temperatures and pressures. Any calculation of tank wall thickness and thus of tank mass requires further information.

5. Compare several systems for a potential roll control application which requires four thrusters of 1 lbf each to operate for a cumulative duration of 2 min each over a period of several days, which allows a constant gas temperature. Include the following:

Pressurized helium	70 °F temperature
Pressurized nitrogen	70 °F Ambient temperature
Pressurized krypton	70 °F Ambient temperature
Pressurized helium	300 °F (electrically heated)

The pressurized gas is stored at 5000 psi in a single spherical fiber-reinforced plastic tank; use a tensile strength of 150,000 psi and a density of 0.050 lbm/in.3 with a 0.012-in.-thick aluminum inner liner as a seal against leaks. Neglect the gas volume in the pipes, valves, and thrusters, but assume the total hardware mass of these to be about 1.3 lbm. Use Table 7–3. Make estimates of the tank volume and total system weight. Discuss the relative merits of these systems.

6. A sealed propellant tank contains hydrazine. It is stored for long periods of time, and therefore the propellant and the tank will reach thermal equilibrium with the environment. At an ambient temperature of 20 °C and an internal pressure of 1.2 atm the liquid occupies 87% of the tank volume and the helium pressurizing gas occupies 13%. Assume no evaporation of the propellant, no dissolving of the gas in the liquid, and no movement of the tank. Use the hydrazine properties in from Figs. 7–1 and 7–2 and Table 7–1. What will be the approximate volume percentages and the gas pressure at the extreme storage temperatures of 4 and 40 ◦ C?

See Figs. 7-1, 7-2 and Table 7-1. The change of hydrazine density with temperature (between 4 and 40 °C) is very small, perhaps 1% so assume that the helium volume remains constant. The problem states that there will be no evaporation of N_2H_4 so ignore vapor pressure effects.

Temperature		Hydrazine		Helium
°C	K	Specific Gravity	Vapor press (atm.)	Press. (atm.)
4	277.15	1.01	0.001	1.13
20	293.15	1.002	0.09	1.20
40	313.15	0.991	0.5	1.28

The helium pressure changes in the tank changes are proportional to the temperature ($p_1/p_2 = T_1/T_2$). The volume (or volume percentage of He in the tank) will essentially remain unchanged because the hydrazine volume remains unchanged.

7. A liquid hydrogen/liquid oxygen thrust chamber has a constant bipropellant flow rate of 347 kg/sec at a mixture ratio of 6.0. It operates at full thrust for exactly 2 min. The propellant in the vehicle's tanks are initially vented to the atmosphere at the propellant boiling points and are assumed to be of uniform initial temperature at start. Use data from Table 7–1 for the propellant specific gravities. Assuming no losses, find the masses of **(a)** fuel and **(b)** of oxidizer used to produce the thrust for the nominal duration. **(c)** What was the volume of the liquid hydrogen actually used? **(d)** Assuming 4.0% extra fuel mass (for unusable

propellant residual, evaporation, hardware cooling, or venting just prior to start, or propellant consumed inefficiently during startup and shutdown) and a 10% ullage volume (the void space above the liquid in the tank), what will be the volume of the fuel tank? Assume other losses can be neglected.
Answers: **(a)** 5941 kg, **(b)** 35,691 kg, **(c)** 83.83 m^3 **(d)** 95.6 m^3.

8. Prepare dimensioned rough sketches of the four propellant tanks needed for operating a single gimbal-mounted RD 253 engine (Table 11–3) for 80 sec at full thrust and an auxiliary rocket system with a separate pressurized feed system using the same propellants, with two gimbal-mounted small thrust chambers, each of 150 kg thrust, a duty cycle of 12% (fires only 12% of the time), but for a total flight time of 1.00 h. For propellant properties Table 7–1. Describe any assumptions that were made with the propellant budget, the engines, or the vehicle design, as they affect the amount of propellant.

9. Table 11-3 shows that the RD 120 rocket engine can operate at thrusts as low as 85% of full thrust and with a mixture ratio variation of ±10.0%. Assume a 1.0% unavailable residual propellant. The allowance for operational factors, loading uncertainties, off nominal rocket performance, and a contingency is 1.27% for the fuel and 1.15% for the oxidizer.
(a) In a particular flight the average main thrust was 98.0% of nominal and the mixture ratio was off by +2.00% (oxidizer rich). What percent of the total fuel and oxidizer loaded into the vehicle will remain unused at thrust termination?
(b) If we want to run at a fuel-rich mixture in the last 20% of the flight duration (in order to use up all the intended flight propellant), what would the mixture ratio have to be for this last period?
(c) In the worst possible scenario with maximum throttling and extreme mixture ratio excursion (±3.00%, but operating for the nominal duration), what is the largest possible amount of unused oxidizer or unused fuel in the tanks?

EXTRA PROBLEM

6A. A thrust chamber operating with hydrogen-peroxide/hydrogen has an overall propellant flow rate of 147 kg/sec and a mixture ratio of 8.5. The oxidizer is (nearly) pure HTP which decomposes through a catalyst. The chamber operates at a pressure of 13.2 MPa and the nozzle are ratio is 34.0. With the ideal thrust duration as 45 sec,

(a) Assuming no losses or extra propellant needs, find the required mass of fuel and oxidizer.

(b) What will be the corresponding volume of the hydrogen tank if the gas is stored at 0 °C and 3500 psi?

(c) What would be the container volume if the HTP has a specific gravity of 1.46 as stored.

(d) Ignoring here any temperature advantages of the catalytic decomposition of liquid hydrogen peroxide, what is the "mass overhead" for this system compared to using pure oxygen? Give your answer as a mass ratio between pure O_2 and HTP.

The overall reactions are: $2H_2O_2(l) \rightarrow 2H_2O(g) + O_2(g)$ [decomposition]

$$aH_2 + bO_2 \rightarrow \text{Equation 5-19.}$$

For steady flow, $r = \dot{m}_o / \dot{m}_f = m_o / m_f = 8.5$. From Eqs. 6-2 to 6-4,

$\dot{m}_o = 132$ kg/sec and $\dot{m}_f = 15.5$ kg/sec

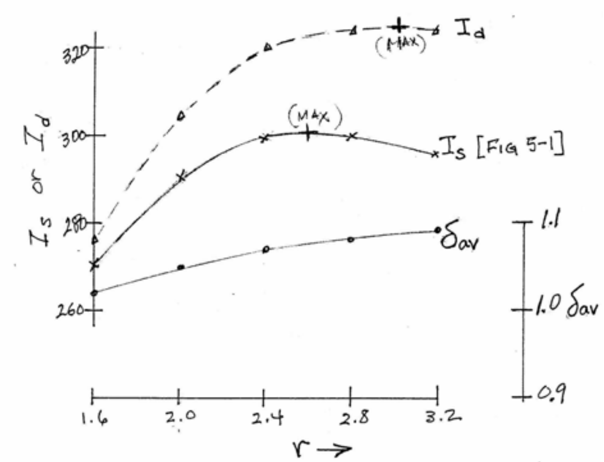
(a) $m_f = 15.5\text{x}45 = 696$ kg and $m_o = 132\text{x}45 = 593$ kg

CHAPTER 7

1. Plot the variation of the *density specific impulse* (product of average specific gravity and specific impulse) with mixture ratio and explain the meaning of the curve. Use the theoretical shifting specific impulse values of Fig. 5–1 and the specific gravities from Fig. 7–1 or Table 7–1 for the liquid oxygen–RP-1 propellant combination.

$$I_d = \delta_{av} I_s \quad \text{where} \qquad \delta_{av} = \frac{\delta_f (1+r)}{(1 + r\delta_f / \delta_o)} = \frac{\delta_o \delta_f (1+r)}{(\delta_o + r\delta_f)}$$

$\delta_o = 1.23$ (77.8 K) and $\delta_f = 0.807$ (298 K); from Fig. 5-1, $1.6 \leq r \leq 3.0$



I_d optimizes at higher r than I_s because the oxidizer has a higher density than the fuel.

2. Prepare a list comparing the relative merits of liquid oxygen and nitrogen tetroxide as rocket engine oxidizers.

Nitrogen Tetroxide (N_2O_4)

Advantages: High density, non-cryogenic and capable of long-term storage, hypergolic with many common fuels. Preferred for small thrusters (suitable for pulsing operations). Lower vapor pressure than LOX (easier to pump & ship over long distances). Easy to work with relative to cryogenic liquids.

Disadvantages: narrow liquid range of temperatures, toxic exhaust. Lower combustion temperatures than LOX.

Liquid Oxygen (LOX)

Advantages: High density, relatively safe (with few precautions), high energy (highest heat of oxidation, except for fluorine), non-toxic exhaust with hydrogen. Preferred for space launch thrusters because of practicality and high performance. Negligible corrosion or health hazards in the short term.

Disadvantages: Cryogenic (needs insulated storage & lines), high vapor pressure, may react explosively at pressures higher than atmospheric.

3. Derive Eq. 7–1 for the average specific gravity.

$$\rho_{\text{av}} = \frac{\text{total mass}}{\text{total volume}} = \frac{m_o + m_f}{V_o + V_f} = \frac{m_f(1 + m_o / m_f)}{V_f(1 + V_o / V_f)}$$

For steady flow, $r = \dot{m}_o / \dot{m}_f = m_o / m_f$

$$\rho_{\text{av}} = \frac{\rho_f(1+r)}{(1 + r\rho_f / \rho_o)} = \frac{\rho_o \rho_f (1+r)}{(\rho_o + r\rho_f)}$$

Note: Eq. 7-1 is in terms of *specific gravities* (δ's) which result from dividing the numerator and denominator in the equation above by the density of water at standard conditions (1.008×10^3 kg/m^3, at 1 atm. and 273.15 K).

4. A rocket engine uses liquid oxygen and RP-1 as propellants at a design mass mixture ratio of 2.40. The pumps used in the feed system are basically constant-volume flow devices. The RP-1 hydrocarbon fuel has a nominal temperature of 298 K and it can vary at about ±25 °C. The liquid oxygen is nominally at its boiling point (90 K), but, after the tank is pressurized, this temperature can increase by 30 K during long time storage. What are the extreme mixture ratios under unfavorable temperature conditions? If this engine has a nominal mass flow rate of 100 kg/sec and a duration of 100 sec, what is the maximum residual propellant mass when the other propellant is fully consumed? Use the curve slopes of Fig. 7–1 to estimate changes in density. Assume that the specific impulse is constant for the relatively small changes in mixture ratio, that small vapor pressure changes have no influence on the pump flow, that there is no evaporation of the oxygen in the tank, and that the engine has no automatic control for mixture ratio. Assume zero residual propellant at the design condition.

At nominal conditions, $\dot{m}_o = \dfrac{\dot{m}r}{r+1}$ = 100 x 2.4/3.4 = 70.59 kg/sec and $\dot{m}_f$ = 29.41 kg/sec

For LOX: 90 K = 162 °R = - 298 °F, ΔT = 30 K yield, T = – 298 + 54 = - 244 °F

For RP-1 fuel: 298 K = 536.4 °R = 75.8 °F, (note that ± 25 K = ± 45 °F)

Total LOX m_o = 70.59 x 100 = 7,059 kg and total fuel m_f = 24.91 x 100 = 2,491 kg.

SG LOX, from Fig. 7-1, δ_o = 1.16 at – 298 °F and 0.96 at -244 °F

SG RP-1, δ_f = 0.820 at 31 °F and 0.800 at 75.8 °F nominal (0.794 at max temp., 120.8 °F)

[On a cold day, the fuel might be at 31 °F, δ_f = 0.820 so ratio of SGs = 0.82/0.80 = 1.025]

<u>Change in mixture ratio: r = 2.40(0.827/1.025) = 1.936 for warm LOX and cold fuel.</u>

At this low mixture ratio, the fuel will be fully consumed before the oxidizer.

The time for the fuel to run out: t = 100sec/1.025 = 97.5 sec.

At this time mass of warmed LOX used is: m_o = 70.59 x (0.96/1.16) x 97.56 = 5,699 kg

LOX remaining and unused = 7005.9 – 5699.4 = 1,306 kg (this is excessive). One remedy would be to pressurize LOX just before take-off so only toward the end of burn-time would LOX have warmed by perhaps 2° or 3°.

If $\rho_{\text{LOX}})_{\text{ave}}$ = 1.15, then LOX used is 70.59 x (1.15/1.16) x 95.56 = 6,827 kg. The residual LOX would 178.5 kg (still high).

<u>On a day when the fuel is heated, but LOX stays at 90 K</u>:

Cold LOX will be available for full duration – 100 sec

Fuel used during 100 sec: 2941x (0.8/0.82) = 2,869 kg

Remaining fuel is 71.7 kg after 100 sec, ave. new mixture ratio = 7005.9/2869.3 = 2.44

5. The vehicle stage propelled by the rocket engine in Problem7- 4 has a design mass ratio m_f/m_0 of 0.50 (see Eq. 4–6). For the specific impulse use a value half way between the shifting and the frozen equilibrium curves of Fig. 5–1. How much will the worst combined changes in propellant temperatures affect the mass ratio and the ideal gravity-free vacuum velocity?

6. (a) What should be the approximate percent ullage volume for nitrogen tetroxide tank when the vehicle is exposed to ambient temperatures between about 50 °F and about 150 °F?
(b) What is maximum tank pressure at 150 °F.
(c) What factors should be considered in part **(b)**?
Answers: **(a)** 15 to 17%; the variation is due to the nonuniform temperature distribution in the tank; **(b)** 6 to 7 atm; **(c)** vapor pressure, nitrogen monoxide content in the oxidizer, chemical reactions with wall materials, or impurities that result in largely insoluble gas products.

7. An insulated, long vertical, vented liquid oxygen tank has been sitting on the sea-level launch stand for a period of time. The surface of the liquid is at atmospheric pressure and is 10.2 m above the closed outlet at the bottom of the tank. What will be the temperature, pressure, and density of the oxygen at the tank outlet, (a) if oxygen is allowed to circulate within the tank and (b) if it is assumed that there is no heat transfer through the tank wall to the liquid oxygen?

The pressure at the bottom of the tank near the tank outlet will be higher than atmospheric pressure because of the head of the liquid due to the height h

$$p = p_{atm} = \rho\, g_0\, h = 1.103 \times 10^5 + (1.14 \times 10^3)(9.806)(10.2) = 0.2163 \text{ MPa}$$

This is about 2 atmospheres.

(a) With cryogenic propellant tanks there always is a very small heat transfer from the atmosphere (at sea level ambient temperature) through a layer of ice, the thermal external insulation and the tank walls into the cold liquid propellant. Moisture from the air condenses on the outside of the cold tank and forms a layer of cold ice (below ^{0}C) on all outer surfaces of the tank. The thickness of the ice layer can increase gradually, but it will reach a limit. The liquid oxygen next to the tank wall is heated slightly and its density is reduced a little. This slightly warmer, lowerdensity liquid oxygen rises up slowly next to the inner tank wall. At the liquid surface inside the tank near its top a small amount of evaporation of liquid oxygen absorbs energy at atmospheric pressure and causes the liquid oxygen on and near the surface to be cooled down causing the liquid to lose a little temperature and causing the liquid density to increase modestly. This denser, cooler liquid oxygen then sinks down slowly in the middle of the tank and this causes a circulation of the liquid oxygen inside the tank. Ultimately the density of the liquid oxygen at the surface can reach 1140 kg/m2 and the boiling point at one atmosphere 90^{0}K may be attained, although circulation can occur with warmer, less dense liquid oxygen, It is not unusual for the oxygen mass to lose 1% per day due to evaporation with a well insulated tank.

The higher pressure near the tank outlet will allow a temperature rise (typically between about 2 to 15^{o}K) and a corresponding density change of 1 to 10 % (obtained from the slopes of the oxygen curves in Fig. 7-2 and 7-1 for 1 and 2 atm) The exact values will depend on the design details (tank geometry, materials, insulation), tank size, the configuration and temperatures of the ice layer, relief valve limits, or the duration of the storage.

(b) If ideal conditions are assumed, namely no heat transfer through the walls and only oxygen vapor at atmospheric pressure in the ullage volume, then the wall temperatures and density will be at the atmospheric freezing point (90^{o}K) and 1140 kg/m^3 through out the mass of oxygen. This is not likely to happen. The assumption of no heat transfer is unrealistic. If the vented ullage volume near the top inside of the tank contains some atmospheric air or some of the pressurizing gases at ambient pressure, they can cause a heating of the oxygen near or at the liquid surface and cause a very small amount of oxygen evaporation to again cool the cryogenic liquid.

EXTRA PROBLEM

7A. A rocket propulsion concept called "microwave thermal propulsion" consists of ground based microwaves beamed and focused on the underbelly of a rocket to heat a propellant which is flowing through suitably positioned microwave-absorbent heat exchanger walls. The goal here is to heat the propellant up to a stagnation temperature as high as 2500 K without having to carry the source of heat. Assuming a weak dependence on "*k*" and that the same hardware is used so that $C_F \approx 1.7$ applies to all propellants of interest, calculate and compare the ideal *Is* for the following cold gas propellants: methane, helium, hydrogen and nitrogen.

$T_0 = T_1 = 2500$ K, $C_F \approx 1.7$, and various *k*s (use Table 7-3).

Applying Eq. 3-32 in its multiple forms,

$$I_s = \frac{C_F c^*}{g_0} = \frac{C_F}{g_0}\sqrt{\frac{R'}{\mathrm{M}}\frac{T_1/k}{\left[2/(k+1)\right]^{(k+1)/(k-1)}}} = 7.90x10^2\sqrt{\frac{((k+1)/2)^{(k+1)/(k-1)}}{k\mathrm{M}}}$$

Methane, $k = 1.30$, $\mathrm{M} = 16$, $Is \approx 296$ sec

He, $k = 1.67$, $\mathrm{M} = 4.0$, $Is \approx 544$ sec

H_2, $k = 1.40$, $\mathrm{M} = 2$, $Is \approx 833$ sec

N_2, $k = 1.40$, $\mathrm{M} = 28$, $I_s \approx 218$ sec

[Note above that the difference in I_s comes principally from the difference of molecular mass so k has a weaker influence than M . Additional information from the gas tanks and their storage pressure, as well as heat transfer criteria, would have to be considered before any one propellant can be selected; however, because He is the only one above that does not dissociate at the higher temperatures, it may represent a best choice. The general idea was first proposed in 1924 by the Russian rocket pioneer, K. Tsiolkovki.]

(b) Eq. 3-4 in terms of the density, $\rho = p/RT = 21.2$ kg/m^3 so Vol)$_{H2}$ = 696/21.2 = 32.83 m^3

(c) Vol)$_{HTP}$ = 593/1.46x10^3 = 0.406 m^3

(d) Because 2 moles of HTP are needed to produce each mol of O_2(g) compared to one mol of O_2(g) from an oxygen tank, we may define a *mass overhead* as the ratio of the molar masses of HTP to O_2 in the decomposition equation above: mass overhead = 2x34.016/31.999 = 2.126.

CHAPTER 8

1. How much total heat per second can be absorbed in a thrust chamber with an inside wall surface area of 0.200 m^2 if the coolant is liquid hydrogen and the coolant temperature does not exceed 145 K in the jacket? Coolant flow is 2 kg/sec. What is the average heat transfer rate per second per unit area? Use the data from Table 7–1 and the following:

Heat of vaporization near boiling point		446 kJ/kg
Thermal conductivity	(gas at 21 K)	0.013 W/m-K
	(gas at 194.75 K)	0.128 W/m-K
	(gas at 273.15 K)	0.165 W/m-K

For liquid hydrogen at the BP from Table 7-1, c_p = 1.75 kcal/kg-K = 7.33 kJ/kg-K

$Q_{\text{total}} = Q_{\text{heat-absorbing capacity}} + Q_{\text{vaporization}} = \dot{m}\, c_p\, \Delta T + \dot{m}\, h_v$

= 2 x [7.33 (145 – 20.4) + 446] = 2,726 kJ/ sec (max. heat transfer rate)

$q = Q/A$ = 13,625 kJ/m^2-sec   average heat transfer rate

2. During a static test a certain steel thrust chamber is cooled by water in its cooling jacket. The following data are given for the temperature range and pressure of the coolant:

Average water temperature	100 °F
Thermal conductivity of water	1.07×10^{-4} Btu/sec-ft^2-°F/ft
Gas temperature	4500 °F
Specific gravity of water	1.00 [density = 62.42 lbm/ft^3]
Viscosity of water	2.5×10^{-5} lbf-sec/ft^2
Specific heat of water	1.3 Btu/lb-°F
Cooling passage dimensions	1/4 × 1/2 in.^{2} [not circular]
Water flow through passage	0.585 lbm/sec
Thickness of inner wall	1/8 in.
Heat absorbed	1.3 Btu/in.^{2}-sec
Thermal conductivity of wall material	26 Btu/hr-ft^2-°F/ft

Determine **(a)** the film coefficient of the coolant; **(b)** the wall temperature on the coolant side; **(c)** the wall temperature on the gas side.

a) $\dot{m} = \rho A v$ so v = (0.585 x 144)/(62.42 x 0.25 x 1.25) = 10.8 ft/sec

D = 4 x [cross-sect/wetted-perim] = 4 x (0.25 x ½)/2(0.25 + ½) = 0.333 in = 0.0278 ft

Eq. 8-24, $h_\ell = 0.023 \times (0.585 \times 1.0/ 8.68 \times 10^{-4})(0.0278 \times 10.8 \times 62.42/2.5 \times 10^{-5} \times 32.2)^{-0.2}$

$(2.5 \times 10^{-5} \times 32.2 \times 1.0/1.07 \times 10^{-4})^{-2/3} = 2.70$ Btu/ft^2-sec-°F

b) Eq. 8-18, $T_{w\ell} = q/h_\ell + T_\ell$ = 1.3 x 144/2.70 + 100 = 169 °F

c) Eq. 8-17, $T_{wg} = qt_w/k + T_{w\ell}$ = 1.3 x (1/8) x 12 x 3600/26 + 169 = 439 °F

3. In the example of Problem 8-2 determine the water flow required to decrease the wall temperature on the gas side by 100 °F. What is the percentage increase in coolant velocity? Assume that the various properties of the water and the average water temperature do not change.

4. Determine the absolute and relative reduction in wall temperatures and heat transfer caused by applying insulation in a liquid-cooled rocket chamber with the following data:

Tube wall thickness	0.381 mm
Gas temperature	2760 K
Gas-side wall temperature	1260 K
Heat transfer rate	15 MW/m^2-sec
Liquid-film coefficient	23 kW/m^2-K
Wall material	Stainless steel AISI type 302

A 0.2-mm-thick layer of insulating paint is applied on the gas side; the paint consists mostly of very small magnesia particles. The average conductivity of this magnesia paint is

2.59W/m²-K/m over the temperature range. The stainless steel has an average thermal conductivity of 140 Btu/hr-ft2-◦F/in. and a specific gravity of 7.98

Orig. situation: Eq 8-16, $h_g = q/(T_g - T_{wg}) = 15 \times 10^6/(2760 - 1260) = 1.00 \times 10^4$ W/m^2-K
New situation: extending Eq. 8-15 to two conduction resistances in series:

$$q = \frac{T_g - T_\ell}{1/h_g + t_{w-ins}/k_{ins} + t_w/k_w + 1/h_\ell} = \frac{2760 - 400}{1/10^4 + 2\text{x}10^{-4}/2.59 + 3.81\text{x}10^{-4}/20.12 + 1/2.3\text{x}10^4}$$

= 10 MW/m^2 assuming that the coolant liquid is at 400 K. This is a 33% reduction in heat transfer.
The stainless wall temperature (rocket chamber side) changes from T_{wg} = 1,260 K to $T_{wg} = T_1 - q(1/h_g + t_{\text{w-ins}}/k_{\text{ins}}) = 2760 - 10^7 \text{x} (10^{-4} + 2\text{x}10^{-4}/2.59) = 988$ K or a 22% drop.

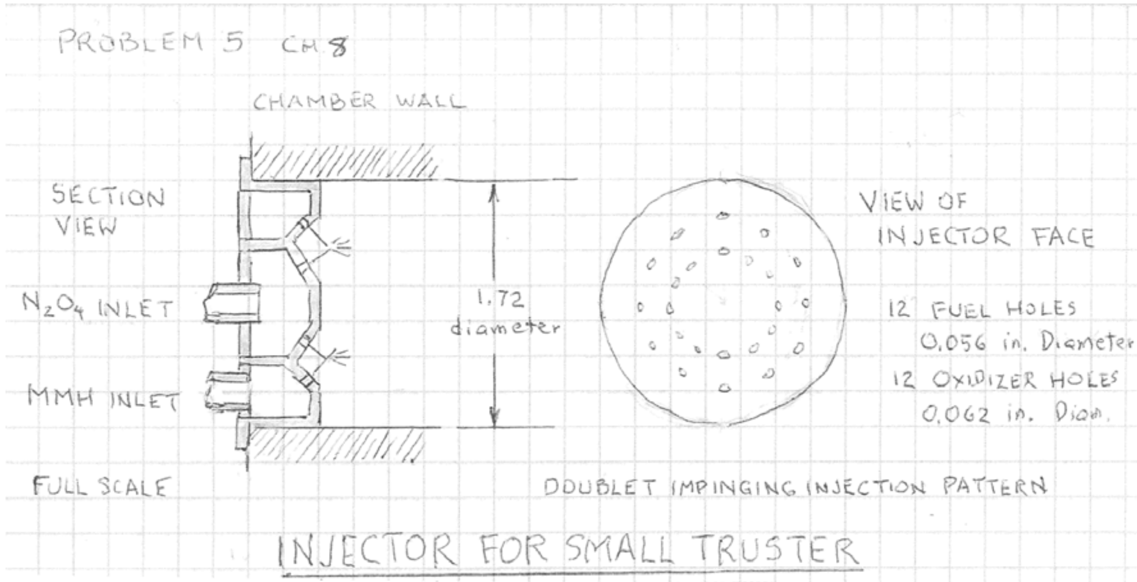
5. A small thruster has the following characteristics:

Propellants	Nitrogen tetroxide and monomethyl hydrazine
Injection individual hole size	Between 0.063 and 0.030 in.
Injection hole pattern	Unlike impinging doublet
Thrust chamber type	Ablative liner with a carbon–carbon nozzle throat insert
Specific gravities	1.446 for oxidizer and 0.876 for fuel
Impingement point	0.25 in. from injector face
Direction of jet momentum	Parallel to chamber axis after impingement

r = 1.65 (fuel rich)	$(I_s)_{\text{actual}}$ = 251 sec
F = 300 lbf	t_b = 25 sec
p_1 = 250 psi	A_1/A_t = 3.0
$(\Delta p)_{\text{inj}}$ = 50.0 psi	$(C_d)_o = (C_d)_f$ = 0.86

Determine the number of oxidizer and fuel injection holes and their angles. Make a sketch to show the symmetric hole pattern and the feed passages in the injector. To protect the wall the outermost holes should all be fuel holes.

See sketch and solution to Problem 8-5 on the next page. The velocity symbol should be a υ instead of v.



With this propellant combination and for a mixture ratio of 1.65 - the volumes of the two propellant tanks and the two volumetric propellant flow rates are equal. This implies that the same identical tanks can be used for both the oxidizer and the fuel. The discharge coefficients' value of 0.82 means that the injection holes have smooth surfaces and rounded entries. We use inches as the dimensional parameters, because this is still common practice in USA. The oxidizer and fuel densities are corrected to lbm/in.^{3}.

$\rho_o = 1.446$ kg/m^3 = 0.05213 lbm/in^3 $\qquad\qquad$ $\rho_o = 0.3157$ lbm/in^3

The total mass flow $\dot{m} = F/c = 300$ lbf/(251 sec x 32.2) = 0.1219 lbm/in^3
From equations 6-2 to 6-4 $\dot{m}_o = \dot{m}\, r/(r+1) = 0.1219 \times 1.65/2.65 = 0.0759$ lbm/sec
$\dot{m}_f = \dot{m}/(r+1) = 0.0460$ lbm/sec

The volume flows $\dot{V} = \dot{m}/c$ are listed below for the densities of $\rho_o = 0.05213$ lbm/in^3. $\qquad$ $\rho_f = .003157$ lbm/in^3
(For comparison water has a density of 0.036 lbm/in^3)
$\dot{V}_o = 0.0759/0.05213 = 1.4559$ in^3/sec
$\dot{V}_f = 0.0460/0.03157 = 1.4568$ in^3/sec $\qquad$ These are the same flow values.

The mass flow through injection holes is (equation 8-2) $\dot{m} = C_d A \sqrt{2\rho\, \Delta p}$

$A_o = \dot{m}/(C_d \sqrt{2\rho\, \Delta p}) = 0.0759$ lbm/(0.86 x$\sqrt{2 \times 0.05213 \text{ lbm in}^{-3} \times 50\text{psi}}$ = 0.0365 in^2
This is the area for all the oxidizer injection holes.
$A_f = 0.0469/(0.86 \times \sqrt{2 \times 0.03157 \times 50}) = 0.0301$ in^2 (for all the fuel injection holes)

[continued next page]

List of typical hole properties (hole diameters correspond to standard drill sizes)

Hole diameter	*Hole area*	*Number of holes (N_2O_4)*	*Number of holes MMH*
0.063 inch	0.00312 in.^{2}	11.698	9.6 Dia. - too large for fuel
0.049 inch	0.001519 in.^{2}	24	19
0.030 inch	0.000701 in.^{2}	52	39

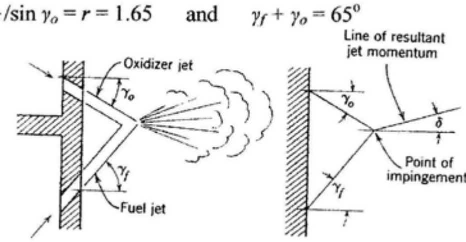
Select 12 holes (smallest number to fabricate, easiest to drill) for both fuel and oxidizer

Drill diameter	*Area of hole*	*Propellant*	*Number of holes for total area*	
0.062 inch	0.003017 in2	Oxidizer	12.1	This is close enough to 12.00
0.056 inch	0.00246 in2	Fuel	12.2	

Next select hole angles γ to get axial resulting momentum parallel to chamber axis. Use Equation 8-7; at $r = 1.65$ the flow velocities of fuel v_f and oxidizer v_o are equal and they can be cancelled out of the equation. Solving these equations for several angles γ:

$v_o \dot{m}_o \sin\gamma_o = v_f \dot{m}_f \sin\gamma_f$ or $\sin\gamma_f / \sin\gamma_o = r = 1.65$ and $\gamma_f + \gamma_o = 65^\circ$

γ_o	γ_f	$\sin\gamma_f/\sin\gamma_o$
20°	45°	2.067
22	43	1.820
23	42	1.712
23.5	41.5	1.662
25	40	1.521
30	35	1.147



Select $\gamma_o = 23.5^\circ$ and $\gamma_f = 41.5^\circ$; 12 holes each. It is shown in the sketch above. The 65 degree included angle has been satisfactory before. For making a sketch of the injector the chamber diameter will be needed. First the nozzle throat diameter is determined from Equations 2-17 and 3-32 (pages 36 and 68) and the characteristic velocity can be found in Table 5-5 on page 184. The throat area of the nozzle is

$A_t = \dot{m}\, c^*/p_1 = (F\, c^*)/(c\, p_1) =$
$= (300 \text{ lbf} \times 1591 \text{ m/sec} \times 39.370 \text{ in./m})/(251 \text{ sec} \times 32.17 \text{ ft/sec}^2 \times 12 \text{ in./ft} \times 250 \text{ psi}) =$
$= 0.7750 \text{ in.}^2$

The throat diameter is 0.9933 inch or almost an inch. The chamber area (using the area ratio $A_c/A_t = 3.0$ given in the problem statement) is 2.325 in^2 or a diameter of 1.720 inch. There is now enough information to make the sketch and it is shown on the next page. It is difficult to accurately drill small holes at an angle and therefore an annular groove is put into the injector so that the drill is at right angle to the surface. The fuel holes should be in a circle on the outside and the oxidizer holes at a smaller diameter on the inside.

6. A large, uncooled, uninsulated, low-carbon-steel thrust chamber burned out in the throat region during a test. The wall (0.375 in. thick) had melted and there were several holes. The test engineer said that he estimated the heat transfer to have been about 15 Btu/in.^{2}. The chamber was repaired and you are responsible for the next test. Someone suggested that a series of water hoses be hooked up to spray plenty of water on the outside of the nozzle wall at the throat region during the next test to prolong the firing duration. The steel's melting point is estimated to be 2550°F. Because of the likely local variation in mixture ratio and possibly imperfect impingement, you anticipate some local gas regions that are oxidizer rich and could start the rapid oxidation of the steel. You therefore decide that 2150°F should be the maximum allowable inner wall temperature. Besides knowing the steel weight density (0.284 lbf/in.^{3}), you have the following data for steel for the temperature range from ambient to 2150°F: the specific heat is 0.143 Btu/lbm-°F and the thermal conductivity is 260 Btu/hr-ft^2-°F/in. Determine the approximate time for running the next test (without burnout) both with and without the water sprays. Justify any assumptions you make. If thewater spray seems to be worth while (getting at least 10% more burning time), make sketches with notes on how the mechanic should arrange for this water flow so it will be most effective.

7. The following conditions are given for a double-walled cooling jacket of a rocket thrust chamber assembly:

Rated chamber pressure	210 psi
Rated jacket pressure	290 psi
Chamber diameter	16.5 in.
Nozzle throat diameter	5.0 in.
Nozzle throat gas pressure	112 psi
Average inner wall temperature at throat region	110°F
Average inner wall temperature at chamber region	800°F
Cooling passage height at chamber and nozzle exit	0.375 in.
Cooling passage height at nozzle throat	0.250 in.
Nozzle exit gas pressure	14.7 psi.
Nozzle exit diameter	10 in.
Wall material	1020 carbon steel
Inner wall thickness	0.08 in.
Safety factor on yield strength	2.5
Cooling fluid	RP-1
Average thermal conductivity of steel	250 Btu/hr-ft^2-°F/in.

Assume other parameters, if needed. Compute the outside diameters and the thickness of the inner and outer walls at the chamber, at the throat, and at the nozzle exit.

8. Determine the hole sizes and the angle setting for a multiple-hole, doublet impinging stream injector that uses alcohol and liquid oxygen as propellants. The resultant momentum should be axial, and the angle between the oxygen and fuel jets $(\gamma_o + \gamma_f)$ should be 60°. Assume the following:

$(C_d)_o$	0.87	Chamber pressure	300 psi
$(C_d)_f$	0.91	Fuel pressure	400 psi
ρ_o	71 lb/ft^3	Oxygen pressure	380 psi
ρ_f	51 lb/ft^3	Number of jet pairs	4
r	1.20	Thrust	250 lbf
	Actual specific impulse 218 sec		

Answers: 0.197 in.; 0.214 in.; 32.2°; 27.7°.

9. Table 11–3 shows that the RD-120 rocket engine can operate down to 85% of full thrust and at a mixture ratio variation of ±10.0%. In a particular static test the average thrust was held at 96% of nominal and the average mixture ratio was 2.0% fuel rich. Assume a 1.0% residual propellant, but neglect other propellant budget allowances. What percentage of the fuel and oxidizer that have been loaded will remain unused at thrust termination? If we want to correct the mixture ratio in the last 20.0% of the test duration and use up all the available propellant, what would be the mixture ratio and propellant flows for this last period?

[Solution found on next page]

Assume that the propellant tanks are always loaded for the full duration of 100 sec at full (100 % thrust) at the design mixture ratio r of 2.60 and a total propellant flow $\dot{m}$ = 242.9 kg/sec. From equations 6-2 to 6-4 (page 191) obtain the oxidizer and fuel useable propellant flows.

$(\dot{m}_o)_1 = \dot{m}\, r/(r+1) = 242.9 \times 2.6/3.6 = 175.42$ kg/sec
$(\dot{m}_f)_1 = \dot{m}/(r+1) = 242.9/3.6 = 67.47$ kg/sec

The useful propellants in the tanks for a period of 100 sec will be approximately 17,542 kg and 6747 kg. The 1% more for trapped or residual propellant has to be included in the tank volumes, but otherwise they are of no concern to this problem. Assume that the start and stop operations are very short and do not use propellant inefficiently. Assume that the performance of the engine (I_s) is essentially the same at 96% thrust and with slight changes in mixture ratio. Since thrust is proportional to propellant mass flow at the same mixture ratio

$(\dot{m}_o)_2 = 0.96 \times 175.42 = 168.4$ kg/sec $\quad (\dot{m}_f)_2 = 0.96 \times 67.47 = 64.77$ kg/sec
The total propellant flow is 233.2 kg/sec

If the fuel rich mixture during this test has 2 % more fuel then

$(\dot{m}_f)_3 = 1.02 \times (\dot{m}_f)_2 = 66.06$ kg/sec or it has 1.29 kg/sec more. If the total flow and thrust are the same (96% of full value) then the oxidizer flow has to be decreased or adjusted by the mass flow increase of the fuel flow and $(\dot{m}_o)_3 = 168.4 - 1.29 = 1.671$ kg/sec. The mixture ratio is 167.1/66.06 = 2.529. The total propellant flow is the same 233.2 kg/sec

The maximum duration at 96% thrust is limited by the fuel flow or 6747/66.06 = 102.13 sec. For the same thrust and total flow the oxidizer flow $(\dot{m}_o)_3$ has to be decreased by the mass flow increase of the fuel flow

$(\dot{m}_o)_3 = 168.4 - 1.29 = 167.1$ kg/sec. The mixture ratio $r = 167.1/66.06 = 2.529$

The oxidizer will flow for a little longer time than the fuel flow, 17542/167.1 = 104.9 sec or 2.87 sec longer. This means about 2.87 x 167.1 = 479.6 kg of oxidizer will not be burnt or used effectively, and if the valves are not closed, it will dribble out of the thrust chamber.

If the mixture ratio is changed for the last 20% or 20.4 sec of the test duration (in order to use up all the oxygen) the mixture ratio will be changed from fuel rich to more oxidizer rich. The duration of the fuel rich (2% extra fuel) operation is 102.13 – 20.4 = 81.7 sec. At this time the mixture ratio will be adjusted. The thrust and total propellant flow remain the same 233.2 kg/sec

The fuel used during this first period of 81.7 sec is 66.06 x 81.7 = 5397.1 kg
The useful remaining fuel in the tank at that time is 6747 – 5397 = 1349.9 kg
The oxidizer used in the first 81.7 sec is 167.1 x 81.7 = 13,652 kg
The oxidizer remaining to be used is 17542 – 13723 = 3889.9 kg
The new mixture ratio for the last period of about 20.4 sec is $r = 3889.9/1349.9 = 2.88$

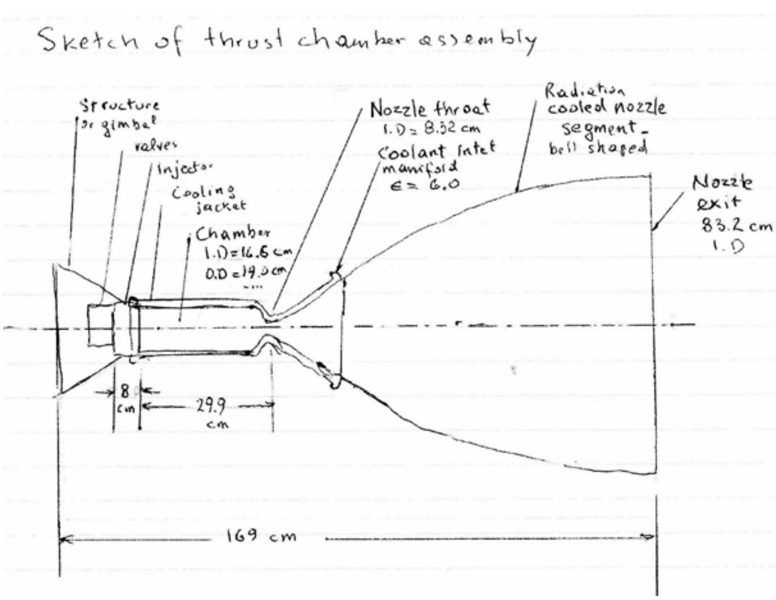
This mixture ratio of 2.88 is just outside of the ±10% proven capability of this RD-120 engine (r is between 2.34 and 2.86). See Table 11-3. An evaluation is needed to determine if the engine would operate satisfactory at the higher mixture ratio. If engine test have to be performed before this analysis is completed, it may be prudent to operate at a mixture ratio of 2.86, which is within the r limits. The unused remaining oxygen would be a very small amount, perhaps 2 or 3 kilograms.

10. Make a simple cross-section sketch approximately to scale of the thrust chamber that was analyzed in Section 8.9. The various dimensions should be close, but need not be accurate. Include or make separate detailed-section sketches of the cooling jacket and the injector. Also compile a table of all the key characteristics, similar to Table 8–1, but include gas generator flows and key materials. Make estimates or assumptions for any key data that is not mentioned in Section 8.9.

This problem is primarily concerned with the thrust chamber of Section 8.9., Example 8-2, which includes a sketch of the thrust chamber and a list of key parameters. However, some engine data and gas generator information is also given.
Application: Rocket propulsion for top upper stage of launch vehicle

ENGINE PARAMETERS

Thrust	50,000 N or 11241 lbf
Propellant	Liquid oxygen and RP-1 or RP-2 (a type of kerosene)
Specific Impulse (engine)	330 sec
Total propellant (useable)	7,478 kg
(This does not include residual propellant or propellant used for non-propulsive purposes)	
Mass flow rate (engine)	15.446 kg/sec ($\dot{m}_o$ = 10.765 kg/sec and $\dot{m}_f$ = 4.680 kg/sec)
Engine cycle	Gas Generator
Single start	Operation in vacuum of space
Vehicle outside diameter	2.00 m
Engine outside diameter	1.90 m
Stage length portion of propellant tanks	4.50 m
Chamber pressure	700 psi
Length of engine	169 cm



THRUST CHAMBER (TC) PARAMETERS

Thrust	50,000 N or 11241 lbf
Mixture ratio	2.45
$(\dot{m})_{TC}$	
$(\dot{m}_o)_{TC}$	10.75 + 4.39 = 15.13 kg/sec
$(\dot{m}_f)_{TC}$	
Length of TC	157 cm (including injector valve)
Max TC outside diameter	84 cm
Milled channel cooling-jacket design	
Chamber gas temp.	3600 K (calculated)
Nozzle throat gas temp.	3214 K (calculated)
Nozzle exit gas temp.	1430 K (calculated)
Nozzle throat I.D.	8.23 cm
Chamber I.D.	16.46 cm
Chamber length	29.9 cm
Nozzle exit area ratio	100:1.0

The structure shown in the sketch above is rigid and provides for a fixed nozzle. Alternatively, a gimbal mounting could have been provided.

A milled chamber design (see Fig. 8-15) was selected because of its high heat transfer rate and the low conductivity of the chosen material, namely stainless steel. A design with 150 channels was selected with a channel cross-section of 0.277x0.188 (in the chamber) and 0.100x0.243 (in the throat region). Stainless steel is the material to be brazed together. Cooling velocity in the chamber is 7 m/sec and in the throat region 15 m/sec. Inner wall thickness = 0.5 mm.

A_{throat}	54.53 cm^2
D_{throat}	8.326 cm (inside diameter)
$D_{\text{nozzle-exit}}$	83.26 cm (inside diameter)
Length of chamber	29.9 cm (including converging section)
Chamber Diam.	$D_c = 2 \times D_{\text{throat}}$ = 16.6 cm (inside)
Chamber Diam.	17.5 cm (approx., excluding external bends)
D_{throat}	8.74 cm (approx., outside diameter)
Injector Thickness	8.0 cm along chamber axis.

NOZZLE (part of thrust chamber)

Nozzle is bell-shaped, shortened to 80% of an equivalent conical nozzle. Length of nozzle in the divergent section is 112 cm (approx., not including nozzle throat radius). Nozzle contour inversion angle is from 34 to 36°. Nozzle contour exit angle with nozzle axis is 7°.

The nozzle is cooled-down up to an area ratio of 6.0 (on the divergent nozzle portion) using fuel coolant in the cooling jacket. Beyond this area ratio, it is radiation cooled.

The outer part (between a nozzle exit area-ratio of 6 to about 20) is made of niobium metal, coated with a disilicate (for oxidation protection) and between an area-ratio of 20 and 100 titanium metal is used. Not shown in the sketch are reinforcing rings to prevent deflection or flutter in the thin cooled nozzle exit section.

INJECTOR (part of thrust chamber)

The selected design is a forged and machined piece of stainless steel with an oxidizer dome and radial fuel supply lines to grooves which contain brazed rings with the injection holes. It is similar to the injector shown in Figs. 8-1 and 8-5, except that it has fewer grooves. Injection takes place through pairs of inclined holes drilled into the brazed rings, and the entrance to each hole is rounded to a smooth entry surface. Pair of holes eject propellant streams that impinge at an angle with each other forming a doublet self-impinging stream pattern similar to the one shown in Fig. 8-5. The outer brazed ring contains the fuel holes for film cooling and the axes of these holes are parallel to the chamber axis.

Discharge coefficient selected = 0.80 for holes with rounded entrances.
Number of holes and Diameter of holes (must be an even number):

Oxidizer	66 holes	2.00 mm diam.
Fuel	66 holes	1.32 mm diam.

In adjacent rings, pairs of oxidizer and pairs of fuel holes must line up on the same radial line. In addition, the outer ring the holes for film cooling of the chamber (which uses 10% of the fuel flow).

Film cooling hole number	100
Film cooling hole size	0.50 mm diam.

Pressure drop across all injectors id 140 psi (approx.).

GAS GENERATOR

$\dot{m}_{gg}$	2.0 % of total flow = 0.3089 kg/sec (with r = 0.55)
$(\dot{m}_o)_{gg}$	0.2928 kg/sec
$(\dot{m}_f)_{gg}$	0.0161 kg/sec

Other information Gas Generators is contained in Section 10.8.

EXTRA PROBLEM

8A. Consider a *somewhat modified* RL-10B2 engine where the hydrogen fuel is to be replaced by methane. Liquid methane-LOX might prove to be a practical bi-propellant combination both as an upper stage during launch (higher density specific impulse than hydrogen-oxygen) and for deep space propulsion (potential of refueling in the outer planetary moons). It is also judged to be somewhat "greener" because of its availability and of its products of combustion (primarily CO_2 and H_2O) – although some noticeable CO is also produced and any unburned methane or its leaks become strongly polluting in the earth's atmosphere. Assuming that the entries on Table 5-5 are relevant, calculate an ideal new nozzle exit pressure and a new specific impulse when methane replaces hydrogen in the RL-10B2 engine, shown in Fig. 8-17. For this new engine, take ϵ and p_1 as found in Table 11-2. Also, compare the density specific impulse values between H_2-LOX and CH_4-LOX.

From the Table 5-5 under frozen flow, k = 1.20 and c^* = 1835 m/sec (these values would need to be modified for different mixture ratio and other applicable pressures).
For this engine, from Table 11-2, ϵ = 385 and p_1 = 4.36 MPa.
From the givens we find (Eq.3-25), $p_2/p_1 = 1.23\text{x}10^{-4}$, and (Eq.3-30) C_F = 2.028 (p_3 =0)
$p_2 = 1.23\text{x}10^{-4}\text{x}4.412\text{x}10^{6} = 536.3\ \text{N/m}^2 \approx 0.536$ kPa
From Eq. 3-32, $I_s = c^* C_F/g_0$ = 1835x2.028/9.81 = 379.3 sec (in vacuum)
The density specific impulse is found from Eqs. 7-1 to 7-3. For this comparison, we may estimate the required tank densities using the frozen-flow values found in Table 5-5

H_2-LOX: δ_{av} = 0.26, I_s = 386 sec so I_d = 0.26x386 = 100.4 sec
CH_4-LOX: δ_{av} = 0.81, I_s = 296 sec so I_d = 0.81x296 = 239.8 sec

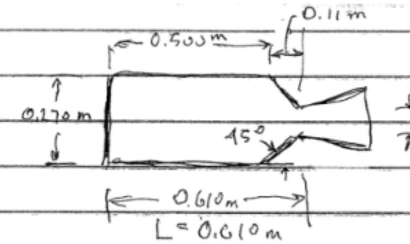
[The modified engine could be of interest for a number of missions because of its substantially higher density specific impulse. A thermochemical analysis needs to be performed to define actual values of c^* and k in the modified design according to the mixture ratio and to the degree of frozen equilibrium (for an oxygen/methane mixture ratio of 3.6 by mass at 4.36 MPa, $c^* \approx$ 1825 m/sec and $k \approx$ 1.13 from equilibrium calculations in the combustion chamber). Data given Table 11-2 are not sufficient to find *correction factors* for this engine design, but designers would aim to retain values over 90%. Regenerative heat transfer calculations with methane would need to be performed to adapt the thermal design.]

CHAPTER 9

1. For a particular liquid propellant thrust chamber the following data are given:

Chamber pressure	68 MPa
Chamber shape	Cylindrical
Internal chamber diameter	0.270 m
Length of cylindrical section	0.500 m
Nozzle convergent section angle	45°
Throat diameter and radius of wall curvature	0.050 m
Injector face	Flat
Average chamber gas temperature	2800 K
Average chamber gas molecular weight	20 kg/kg-mol
Specific heat ratio	1.20

Assume the gas composition and temperature to be uniform in the cylindrical chamber section. State any other assumptions that may be needed. Determine the approximate resonance frequencies in the first longitudinal mode, radial mode, and tangential mode.



Assume converging cone of nozzle is at 45°

Geometry (simplified) of thrust chamber

Axial distance of cone $= \dfrac{0.270 - 0.050}{2} = 0.110$ m

Length - injector face to nozzle throat = 0.500 + 0.110 = 0.610 m

Gas constant $R = R'/\mathfrak{M} = 8314.3 \text{ J/(kg mol) K} / 20$ kg mol

$R = 415.7$ J/°K

Velocity of sound a (eqn. 3-10)

$a = \sqrt{kRT} = \sqrt{1.20 \times 415.7 \times 2800} = 1182$ m/sec

Frequency of longitudinal oscillation a/L

$= 1182/0.610 = 1937.7$ cycles sec

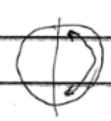
With inclined conical converging configuration assume reflection at throat

With the open throat area and the 45° converging section the oscillation will be weak in longitudinal direction

Radial vibrations

Radial distance is $.270/2 = 0.135$ m

$f = 1182/0.135 = 8755$ cps = frequency



Tangential mode

distance is $\pi D/2 = 3.1419 \times 0.270/2 = d = 0.4241$ m

frequency $= 1182/0.4241 = 2788$ cps

2. Explain how the three frequencies from Problem 1 will change with combustion temperature, chamber pressure, chamber length, chamber diameter, and throat diameter.

Refer to Eq. 9-1.
> An increase in T_1 in the combustion chamber increases the acoustic velocity a_1 and in turn raises the frequencies of all modes of vibration (see Eq. 3-10).
> The chamber pressure affects the gas film coefficient h_g, which is proportional to the gas density raised to the 0.8 power (Eq. 8-21), and this density increases with gas pressure. An increase of chamber pressure will thus cause an increase of heat transfer and in turn raise the gas temperature. This will cause increases in frequency.
> Increasing the chamber length will reduce the longitudinal combustion vibration frequency, assuming the gas conditions are unchanged. It should not affect the transverse vibration frequencies. A longer chamber also tends to give a more stable boundary layer.
> Increasing the chamber diameter [without changing the gas conditions (T_1, p_1, r, ρ_1, etc.)] will reduce the transverse vibration frequencies (radial and tangential modes). It should not affect the longitudinal vibration mode.

3. Discuss why heat transfer increase during combustion instability?

Thin boundary layers always form under steady (non-oscillating) flows that thicken along the surface downstream direction. The layer of hot gas very close to the wall is cooler than the remainder in the boundary layer because of the ongoing heat transfer (see Figure 3-15); the chamber and nozzle walls are cooled on the outside by a cooling jacket or by radiation. These walls, therefore, are not exposed to the full combustion temperature.

Combustion vibration and gas pressure oscillations can destroy any thin laminar and/or induce a turbulent boundary layer. This not only greatly increasing the gas-side film coefficients but also more directly expose the walls to the higher temperatures. This causes major and rapid increases in the heat flow to the wall.

4. Prepare a list of steps for undertaking a series of tests to validate the stability of a new pressure-fed liquid bipropellant rocket engine. State the assumptions.

1. Assumptions: The rocket engine hardware is in good condition and available for testing, the intended performance parameters of the engine are known, the propellant is 100% pure and the pressurizing gas is available, the test facility is able to accept the engine for testing, and any special fixture, features or instrumentation (e.g., high frequency pressure sensors) are all available. Trained personnel are available. (This paragraph applies to most any kind of testing and is not specific top stability testing).
2. From prior tests of this rocket engine or from similar engines obtain details of previous combustion instability incidences and the likely causes and effects.
3. From prior tests of this rocket engine or from similar or modified versions, obtain the history of prior successful test data on performance, effects on hardware, maintenance efforts and repairs/replacements of components. This may save doing some of the same tests.

4. Select the method for triggering the instability and the criteria of what constitutes an instability (explosive change of mass and method of delivery) as mentioned in Chapter 9.
5. Consider using an Engine Health Monitoring System as discussed in Sections 11.5. and 21.3.
6. Prepare a preliminary test plan. Decide on the purpose for each test, the desired measurements, test durations, nominal operating conditions, the data recordings to be obtained, and the number of tests.
7. Often some combustion stability tests are run using a thick-walled, uncooled heavy-duty thrust chamber with the same internal geometry and the same injector and igniter for very short run times. This set up can be reused and is less costly than using flight vehicle cooled thrust chambers which are easily damaged even for short duration testing. It is also much easier to put a side-injection port (for the explosive trigger charge) into heavy, uncooled thrust chamber walls than on flight vehicle chambers.
8. Determine the range of needed operating conditions for the engine tests. This can include limits (+ and -) of (a) liquid propellant temperatures, (b) thrust, (c) mixture ratio, (d) start times to reach full thrust and other parameters, and (e) throttling.
9. With guidance from the propulsion requirements and data from the first 6 items above, select a series of instability tests and a sequence in which they are too be performed. This is a test plan.
10. Review all plans with the help of experienced personnel and make changes as necessary.

5. Estimate the resonant frequency of a set of each of nine cavities similar to Fig. 9–7. Here the chamber diameter $D = 0.200$ m, the slot width is 1.0 mm, and the width and height of the cavity are each 20.0 mm. The walls separating the individual cavities are 10.0 mm thick. Assume $L = 4.00$ mm, $\Delta L = 2.00$ mm, and $a = 1050$ m/sec.

Answer: Approximately 3,138 cycles/sec.

CHAPTER 10

1. A rocket engine with two TPs delivers the fuel, namely UDMH, at a pump discharge pressure of 555 psia, a flow of 10.2 lbm/sec, at 3860 rpm, and a fuel temperature of 68 °F. Using UDMH properties from Table 7-1 and its efficiency from Table 10-2,determine the following:
(a) The fuel pump power for these nominal conditions.
(b) When the fuel flow is reduced to 70% of nominal, what will be the approximate power level, discharge pressure, and shaft speed? Assume that the oxidizer pump is also reduced by 70% and so is the gas flow to the turbines, but the gas temperature is unchanged.
(c) If the anticipated temperature variation of the propellants is at −40◦F and on another day +120◦F, escribe how this variation will this affect the power level, shaft speed, and the discharge pressure of the fuel pump?

The pump output power is the product of the volume flow rate Q (ft^3/sec or m^3/sec) and the pressure rise in the pump, discharge pressure minus suction pressure (lbf/in^2 or N/m^2). From Fig. 7-1 and Table 7-1 for UDMH the average specific gravity is 0.786 so the density is 786 kg/m^3 at standard conditions. Assume a 25 psi suction pressure.

Q = 10.2x(0.4536)/786 = 0.00589 m^3/ sec.

Pressure rise Δp= 555 - 25 = 530 psi

(a) Power (liquid) = $Q\Delta p$ = 0.00589x530x(6894.8) = 21.524 kJ/sec = 21.524 kW
(b) From Eq. 10-8, the fuel flow is proportional to the shaft speed N which will also be at 70% of nominal decreasing to 2702 rpm. Pump power is proportional to N^3 so it will decrease by $(0.7)^3$ or 0.343 from nominal to 7.38 kJ/sec. The pressure rise (pump head) is proportional to N^2 so it will become 530x0.49 = 259.7 psi.
(c) From Fig. 7-1, the specific gravities of UDMH are 0.86 at – 40 °F and 0.73 at + 120 °F. This translates into a 9.4% increase at the low temperature and a 7.1 % decrease at the high temperature. Fuel flows will change by a similar percentage (proportionately) becoming more than nominal at – 40 °F and less than nominal at +120 °F. Here we need to assume that the liquid oxidizer's specific gravity will also be enhanced or reduced. Thus, the power level, shaft speed, and discharge pressure will experience similar changes (except for their magnitudes). These arguments show that it is feasible to implement flow control of gas generators which will maintain a nearly constant rocket engine thrust, irrespective of propellant temperature.

2. What are the specific speeds of the four SSME pumps? (See the data given in Table 10–1.)

Sample calculation for the LPFTP (low pressure fuel turbopump)
Eq 10-9; $N_s = N(Q_e)^{1/2}/(g_0\Delta H_e)^{3/4}$
N = pump speed, Q_e = 70.4 kg/sec liquid-hydrogen at optimum efficiency,
Inlet Head = 0.9 MPa, Outlet or Discharge Head = 2.09 MPa, so pump pressure rise = 2.09 – 0.9 = 1.19 MPa = ΔH
When expressed as head increase in "meters",

ΔH = (pressure)/(density) = 1.19x10^6/(71.2x9.81) = 1700 m

$N_s = N$x$(70.4)^{1/2}/(9.81\text{x}1700)^{3/4}$ = 12,300N (metric)

This corresponds to "near axial" flow type impeller, see Table 10-2

3. Compute the turbine power output for a gas consisting of 64% by weight of H_2O and 36% by weight of O_2, if the turbine inlet is at 30 atm and 658 K with the outlet at 1.4 atm and with 1.23 kg flowing each second. The turbine efficiency is 37%.

4. Compare the pump discharge gauge pressures and the required pump powers for five different pumps using water, gasoline, alcohol, liquid oxygen, and diluted nitric acid. The respective specific gravities are 1.00, 0.720, 0.810, 1.14, and 1.37. Each pump delivers 100 gal/min, a head of 1000 ft, and arbitrarily has a pump efficiency of 84%.
Answers: 433, 312, 350, 494, and 594 psi; 30.0, 21.6, 24.3, 34.2, and 41.1 hp.

Q = 100 gal/min = 0.223 ft^3/sec, ΔH = 1000 ft, and $\eta_p = 0.84$
so according to Eq. 10-16, pump power $P_p = \rho Q \Delta H / \eta_p = 265\rho$ ft^4/sec
Water: P_p = 62.4 x 265/550 = 30.1 hp; LOX: P_p = 34.3 hp
Gauge pressure, $p_g = \rho \Delta H$; Water: p_g =62.4 x 1000/144 = 433 psi; LOX: p_g = 494 psi
The other liquids follow same calculations with indicated SG's.

5. The following data are given on a liquid propellant rocket engine:

Thrust	40,200 lbf
Thrust chamber specific impulse	210.2 sec
Fuel	Gasoline (specific gravity 0.74)
Oxidizer	Red fuming nitric acid (sp. gr.1.57)
Thrust chamber mixture ratio	3.25
Turbine efficiency	58%
Required pump power	580 hp
Power to auxiliaries mounted on turbopump gear case	50 hp
Gas generator mixture ratio	0.39
Turbine exhaust pressure	37 psia
Turbine exhaust nozzle area ratio	1.4
Enthalpy available for conversion in turbine per unit of gas	180 Btu/lb
Specific heat ratio of turbine exhaust gas	1.3

Determine the engine system mixture ratio and the system specific impulse.
Answers: 3.07 and 208 sec.

CHAPTER 11

1. Estimate the mass and volume of nitrogen gas required to pressurize an N_2O_4−MMH feed system for a 4500 N thrust chamber of 25 sec duration (ζ_F = 0.92, the ideal, I_s = 285 sec at 1000 psi or 6894 N/m² and expansion to 1 atm). The chamber pressure is 20 atm (absolute) and the mixture ratio is 1.65. The propellant tank pressure is 30 atm, and the initial gas tank pressure is 150 atm. Allow for 3% excess propellant and 50% excess gas to allow some nitrogen to dissolve in the propellant. The nitrogen regulator requires that the gas tank pressure does not fall below 29 atm.

Assume initial temperature = 20 °C = 293 K, actual I_s = 285 x 0.92 = 262.2 sec
From table 7-1, for N_2O_4 SG = 1.447 (293 K) and for MMH SG = 0.8788 (293 K)
(Note values for t_p and regulator lock-up pressure have been changed in this problem.)
Eq. 2-16: $\dot{m} = F/c = F/I_s g_0$ = 4500/262.2 x 9.81 = 1.75 kg/sec
For t_p = 50 sec, m_p = 1.75 x 50 x 1.03 = 90.14 kg (with 3% excess propellant)
For r = 1.65, Eq. 6-3, m_o = 1.65 x 90.14/2.65 = 56.1 kg and m_f = 90.14 – 56.1 = 34.0 kg
Volume of oxidizer = 56.1/1447 = 0.0388 m³ and Volume of fuel = 34/878.8 = 0.0388 m³
Gas volume for expulsion of prop at 30 atm, Vol = 2 x 0.0388 + 10% ullage = 0.0854 m³
From Table 5-5, propellant properties, k = 1.22 and with $\mathfrak{M}$ = 22 so that R = 377 J/kg-K
Mass of nitrogen gas with a 50% excess, Eq. 6-7 for an isothermal process,

$$m_0)\text{gas} = \frac{p_p V_p}{RT_0}\left(\frac{1}{1-p_g/p_0}\right) = \frac{30\text{x}0.1013\text{x}10^6\text{x}0.0854\text{x}(1.5)}{377\text{x}293\text{x}(1-30/150)} = 4.41\,\text{kg}$$

At 150 atm, Vol of N_2 = $(RT_0/p_0)m_0$ = [(8314/28) x 293/(150 x101300)] 4.41 = 0.0252 m³
The gas tank has nearly the same volume as that of each propellant tank but its inert mass is much larger.

2. A rocket engine operating on a gas generator engine cycle has the following data as obtained from tests:

Engine thrust	100,100 N
Engine specific impulse	250.0 sec
Gas generator flow	3.00% of total propellant flow
Specific impulse of turbine exhaust flowing through a low area ratio nozzle	100.2 sec

Determine the specific impulse and thrust of the single thrust chamber.

3. This problem concerns the various potential propellant loss/utilization categories. The first section of this chapter identifies most of them. This engine has a turbopump feed system with a single TP, a single fixed-thrust chamber (no thrust vector control), no auxiliary small thrusters, storable propellants, good priming of the pumps prior to start, and a gas generator engine cycle.
Prepare a list of propellant utilization/loss categories for which propellant has to be provided. Which of these categories will have a little more propellant or a little less propellant, if the engine is operated with either warm propellant (perhaps 30 to 35 °C) and alternatively in a cold space environment with propellants at −25 °C? Give brief reasons, such as "higher vapor pressure will be more likely to cause pump cavitation." [Assume that the propellant does not freeze or boil during typical ambient temperature changes and either that one or the other or both of the bipropellants are heated or cooled during storage.]

Propellant utilization categories listed on Sect. 11.1. apply to this problem, in particular items 1, 2, 6, 7, 8, 9 and 11. However there are other categories not listed:

1. The air density can change with solar activity and weather. It is possible for the air density at altitude to change by a factor of 2. This directly changes the drag of a vehicle and drag reduces thrust. Enough propellant has to be available to compensate for likely drag increases. Also, an improperly built vehicle may have more skin drag or form drag (see Chp. 4 for drag concepts), e.g., a crooked nose.
2. If the combustion in the thrust chamber is, for some reason, inefficient, it will require more propellant to achieve the desired thrust. This will depend on the thrust control methods if there is such control. Otherwise the thrust and combustion temperature will be below normal or design value.
3. Similarly, if the combustion in the Gas Generator is inefficient (and the gas does not reach the desired temperature) then the gas generator requires more propellant flow to drive the turbines and pumps to deliver the design propellant flows and pressures. This also is a function of the method of Gas Generator control.
4. Unexpected minor mixture-ratio changes usually have little effect on I_s or F.
5. Problem 11-4 (below) discusses the change of thrust and propellant flow with the liquid propellant's initial temperature being supplied to the engine.
6. The amount of residual propellant (with relatively large tank wall surfaces) can increase or decrease with propellant temperature. Warm propellants have lower viscosity and tend to form thinner of residual propellant on the wall. Colder propellants do not flow as readily, are stickier and tend to form somewhat thicker layers resulting in larger totals of residual propellant.
7. Many liquid storable propellants are pressurized in their tanks by inert gases so there must usually be an allocation for this purpose.
8. Some propellants exhibit major changes in vapor pressure with ambient or storage temperature changes; at warmer temperatures these propellants may become difficult to pump. Other propellants do not have any significant vapor pressure changes with temperature and cause no problems.
9. The internal energy of a propellant increases slightly with temperature increases and this may result in a slight improvement in specific impulse. It is well known that heating the propellant in a cooling jacket (typically 100 to 300 °C) can yield a small increase in performance but there is an associated loss of pressure.
10. As the ambient temperature of a stored propellant is reduced, the specific gravity of the two propellants will likely increase. This is turn means a somewhat higher mass flow and higher thrust. If, for the given change in ambient temperature, the respective changes in specific gravities of the propellant are proportional to each other, then there will be little or no change in mixture ratio.
11. If the respective changes in specific gravities are not the same and are not proportional (one propellant becoming denser more rapidly than the other during the temperature change), then the mixture ratio will also change with temperature.
12. If the engine is supplied with oxidizer which is warmer than usual and with a fuel that is colder than usual, then there will a noticeable change in mixture ratio and the amount of residual propellant will be increased.

4. What happens to the thrust and the total propellant flow if an engine (calibrated at 20 °C) is supplied with propellants at high or at low storage temperature?

In general, a warmer propellant causes a slight reduction of the flow and/or thrust and a colder propellant causes a slight rise, but there is a partially compensating effect (as mentioned in Item 9 in the answer to Prob. 11-3 above). Warmer propellants give a little more energy to the combustion reaction.

As the temperature drops the propellant density will increase, thus for a fixed volumetric flow the mass flow rate and the thrust go up slightly. This trend applies to both pressurized or turbopump fed systems. If the temperature during storage increases, the densities with be reduced together with the mass flow rate and thrust.

CHAPTER 12

1. What is the ratio of the burning area to the nozzle throat area for a solid propellant motor with these characteristics? Also, calculate the temperature coefficient (a) and the temperature sensitivity of pressure (π_K).

Propellant specific gravity	1.71
Chamber pressure	14 MPa
Burning rate	38 mm/sec
Temperature sensitivity σ_P	0.007 /K
Specific heat ratio	1.27
Chamber gas temperature	2220 K
Molecular mass	23 kg/kg-mol
Burning rate exponent n	0.3

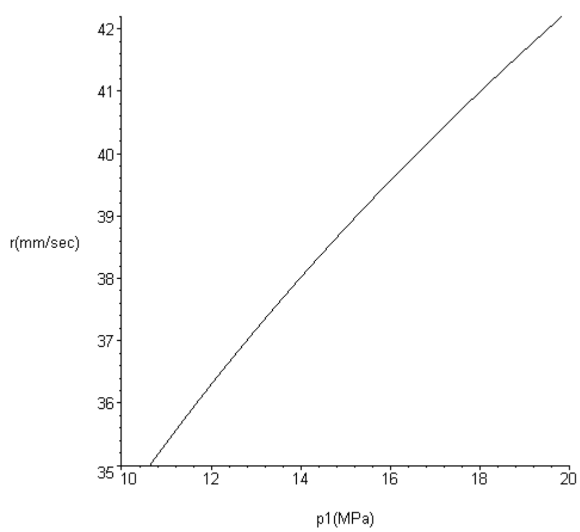
Solving Eq. 12-4 for K and introducing Eq. 3-32 for c*,

$$K = \frac{14\text{x}10^6}{1.71\text{x}10^3\text{x}38\text{x}10^{-3}}\sqrt{\frac{1.27\left(\dfrac{2}{2.27}\right)^{2.27/0.27}}{\dfrac{8314.3}{23}2220}} = 159$$

Using Eq. 12-5, $a = r/p_1{}^n = 38 \text{ x } 10^{-3}/(14 \text{ x } 10^6)^{0.3} = 2.73 \text{ x } 10^{-4}$ (MKS units)

Using Eq. 12-14, $\pi_K = \sigma_p/(1 - n) = 0.007/(1 - 0.3) = 0.01/\text{K}$

2. Plot the burning rate against chamber pressure for the motor in Problem 1 using Eq. 12–5 between chamber pressures of 11 and 20 MPa.



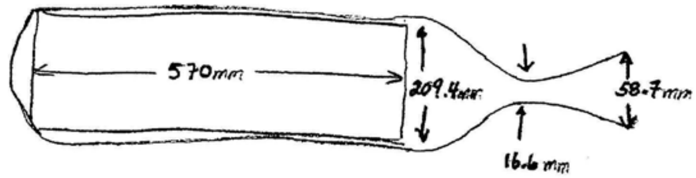
3. What would the area ratio A_b/A_t in Problem 12-1 be if the pressure were increased by 10%? (Use curve from Problem 12-2.)

$K = 159$ at $p_1 = 14$ MPa, new $p_1 = 15.4$ MPa so new $r = 38 \text{ x } (15.4/14)^{0.3} = 39.1$ mm/sec

New $K = (159)(38/39.1)(15.4/14) = 170$

4. Design a simple rocket motor for the conditions given in Problems 12-1 and 2 for a thrust of 5000 N and a duration of 15 sec. Determine principal dimensions and approximate weight.

From Figs. 3-4, 3-6 and 3-7 $C_F \approx 1.65$ and $A_2/A_t \approx 12.5$

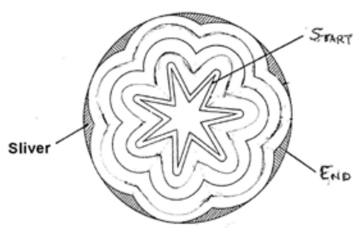


Eq. 3-31, $A_t = F/C_F p_1$ = 5000/(1.65 x 14 x 106) = 2.165 cm^2
$A_b = K A_t$ = 159.17 x 2.16 x 10^{-4} = 0.03445 m^2 = 344.5 cm^2
Eq. 12-1, $\dot{m}$ = 0.03445 x 0.038 x 1710 = 2.387 kg/sec
For t_p = 15 sec, m_p = 15 x 2.387 = 33.58 kg and *Vol*. = 33.58/1710 = 0.01964 m^3
Make it an "end burner" as shown in Fig. 12-18, and for $\zeta \approx 0.9$, m_0 = 37.31kg.

5. For the Orbus-6 rocket motor described in Table 12–3 determine the total impulse-to-weight ratio, the thrust-to-weight ratio, and the acceleration at start and burnout if the vehicle inert mass and the payload come to about 6000 lbm. Use burn time from Table 12–3 and assume $g \approx 32.2$ ft/sec^2.

I_t/w = 1,738,000/(6,515 + 6,000) = 139 sec
F/w = 17,175//(6,515 + 6,000) = 1.37
Initial a = 17,175g_0/(6,515 + 6,000) = 1.37g_0
Final a = 17,175g_0/(513 + 6,000) = 2.64 g_0

6. For a cylindrical grain with two slots the burning progresses in finite time intervals approximately as shown by the successive burn surface contours in the book's drawing. Draw a similar set of progressive burning surfaces for any one configuration shown in Figure 12–16, and draw an approximate thrust–time curve from these plots, indicating the locations where slivers will remain. Assume the propellant has a low value of n and thus the motor experiences little change in burning rate with chamber pressure.



Star-grain successive burning surface contours. This configuration represents "neutral burning" and the pressure- or thrust-time history is shown in Fig. 12-15. Adapted from H. Koelle, Handbook of Astronautical Engineering, McGraw-Hill, 1961.

7. Discuss the significance of the web fraction, the volumetric loading ratio, and the L/D ratio in terms of vehicle performance and design influence.

Web fraction b_f and *volumetric loading fraction* V_f are defined in Sec. 12.3. For L/D see Table 12-1 (under Typical Characteristics) and Fig.12-16, Table 12-4 and accompanying text.

8. Equations 12–8 and 12–9 express the influence of temperature on the burning of a solid propellant. Describe how a set of tests should be set up and what should be measured in order to determine these coefficients over a range of operating conditions.

Relevant information here is given near and below Eqns. 12-8 and 12- 9.

9. What would be the likely changes in burning rate (at 1000 psi), action time, average thrust, action-time-average chamber pressure, burn time, and action-time total impulse for the three rocket motors described in Table 12-3 if each were operated with the rocket motor at a storage temperature lower than that given in the table? Describe what results qualitatively.

p_1 and F would decrease according to Eqs. 12-10 and 3-31 (neglecting changes in C_F)
r would decrease because p_1 and a would have decreased, Eqs. 12-10 and 12-12.
t_b would increase because the product $Ft_b \approx$ constant.
$I_s = c^*C_F/g_0$ would remain approximately constant if T_1 and k don't change.
I_t would remain unchanged since the energy content of the propellant is unchanged.

10. A newly designed case-bonded rocket motor with a simple end-burning grain failed and exploded on its first test. The motor worked well for about 20% of its burn time, when the record showed a rapid rise in chamber pressure. It was well conditioned at room temperature before firing, and the inspection records did not show any flaws or voids in the grain. Make a list of possible causes for this failure and suggestions on what to do in each case to avoid a repetition of the failure.

i) The development of a "conical" burning surface as depicted on Fig. 12-18 is a likely candidate. A doubling of A_b is quite possible with the larger case diameters and that would cause substantial increases of p_1 which then leads to failure.
ii) Other possibilities relate to propellant batch, manufacturing details and to vibration.

11. For the AP-CMDB (30%, 150 μm) propellant shown in Fig. 12–7 as the solid dots, find the chamber pressure that would result from an increase of $\sigma_P \Delta T_b = 0.3$ (this would correspond to a 30 °F change with $\sigma_P = 0.01$/°F). Take the reference values as 28 atm and 70 °F. Values of $n(p_1)$ at various pressure ranges from Fig. 12–7 may be taken as:

n(5–10 atm) = 0.38, n(10–30 atm) = 0.54, n(30–100 atm) = 0.58

At 28 atm, Eqs. 12-10 & 12-14 $\ln p_1 = \ln(28) + (0.3)/(1 - 0.54) = 3.98$ or $p_1 = 53.8$ atm
Now using Eq. 12-16, with the n value for the latter pressure

$$\frac{(1-0.58)}{(1-0.54)}\ln p_1 = \ln(28) + \frac{0.3}{(1-0.54)} = 3.98 \quad \text{and} \quad p_1 = 78.6 \text{ atm}$$

Since this new pressure is below 100 atm this calculation is sufficient. Note, however, that the chamber pressure that results is some 25 atm higher than in the constant n case.

12. What will be the percent change in nominal values of A_t, r, I_s, T_0, t_b, A_b/A_t and the nozzle throat heat transfer rate, if the Orbus-6 rocket motor listed in Table 12–3 is to be downgraded in thrust for a particular flight by 15% by substituting a new nozzle with a larger nozzle throat area but the same nozzle exit area? The propellants, grain, insulation, and igniter will be the same.

Take $n \approx 0.4$ and neglect the small changes in C_F.
Eqn. 3-31, $F = C_F A_t p_1$ so a 15% decrease is F translates into a 15% decrease in $A_t p_1$
From Eq. 12-6, $A_t p_1^{(1-n)} = A_b c^* a \rho_b \approx$ const, combining $F/p_1^{\,n} \approx$ const
A 15% decrease in F yields $p_{1new} = p_1 (0.85)^{(1/0.4)} = 0.667 p_1$ or a decrease of 33.4%.
Thus $A_{tnew} = (0.85/0.667)A_t = 1.28A_t$, a 28% increase and $K = A_b/A_t$ decreases by 21.6%.
For the same quantity of propellant, $Ft_b \approx$ const so t_b would increase by 15%.
$I_s = c^*C_F/g_0$ would remain approximately constant if T_1 and k don't change appreciably.
r would decrease and according to Eq. 12-5, $r_{new} = 0.85r$.
T_0 should stay approximately constant and the heat transfer rate at the throat should decrease, see Eq.8-22.

13. What would be the new values of I_t, I_s, p_1, F, t_b, and r for the first stage of the Minuteman rocket motor described in Table 12–3, if the motor were fired at sea level with the grain temperature 20 ◦F hotter than the data shown. Use only data from this table.

Use "burn time" in Table 12-3. I_t = 10,240,00 lbf-sec and I_s = 254 sec remain unchanged.
π_K = 0.00102/°F, $t_{b\text{nom}}$ = 52.6 sec, n = 0.21, F_{nom} = 194,600 lbf, $p_{1\text{nom}}$= 780 lbf/in^2
r = 0.349 in/sec at 1000 psia, so r_{nom} = 0.349(780/1000)$^{0.21}$ = 0.331 in /sec at 780 psia
New values: from Eq. 12-10, $p_1 = p_{1\text{nom}}\exp(\pi_K \Delta T)$ = 780exp(0.00102 x 20) = 796 psia
$F \approx F_{\text{nom}}(p_1/\,p_{1\text{nom}})$ = 194,600(796/780) = 1.99 x 10^5 lbf
$t_b = t_{b\text{nom}}(F_{\text{nom}}/\,F)$ = 52.6(1.946/1.99) = 51.5 sec
$\sigma_p = \pi_K(1 - n)$ = 8.06 x 10^{-4} and integrating Eq. 12-12 $a = a_{\text{nom}} \exp(\sigma_p$
$\Delta T)$
r = 0.331 x [exp(8.06 x 10^{-4} x 20)] x (796/780)$^{0.21}$ = 0.338 in/sec

14. Calculate K-ratio of the burning area to the nozzle throat area in two separate ways for the STARTM 27 motor using *only* the data found in Table 12–3. Refer to Figure 12–13 for several of the definitions. Compare the two values of K and comment.

a) From Table 12-3, A_b = 1378 in^2 and A_t = 5.9 in^2, so K = 1378/5.9 = $\underline{234}$
b) Using Eq. 12-4 and the data from Table 12-3, p_1 = 552 psia,
r = 0.280(552/1000)$^{0.28}$ = 0.237 in/sec, c^* = 5180 ft/sec, ρ_b = 0.0641 lbm/in^3
K = (552) x (32.2)/[(0.237) x (0.0641) x (5180)] = $\underline{226}$
They compare rather well (234 vice 236) considering the variability in the data.

15. We may take K as constant when the chamber contour lengths remain within ± 15% during propellant burning (see Fig. 12-5). Calculate the variation in thrust that a ± 8% change would represent in a rocket motor with a fixed nozzle geometry and n = 0.5. What would be the corresponding changes in thrust and mass-flow rate. State your assumptions noting that in the real world some nozzle erosion and other effects may occur.

For A_b changes of ± 8%, Eqs. 12-4 and 12-5 give the resulting variation in p_1
$p_1 = \left[K\rho_b a c^*\right]^{1/(1-n)}$ so that $p_{1\text{new}} = p_{1\text{old}}\left[1.0 \pm .08\right]^{1/(1-n)}$
For + 8%, $p_{1\text{new}} = 1.17 p_{1\text{old}}$ and for – 8%, $p_{1\text{new}} = 0.846 p_{1\text{old}}$
With no changes in thermophysical variables in the gas, from Eq. 3-24
$\dot{m}_{\text{new}} = \dot{m}_{\text{old}}(p_{1\text{new}}/p_{1\text{old}})$ that is the flow rates change in proportion to the pressure,
For + 8%, $\dot{m}_{\text{new}} = 1.17\,\dot{m}_{\text{old}}$ and for – 8%, $\dot{m}_{\text{new}} = 0.846\,\dot{m}_{\text{old}}$
Under the above assumptions, c^* and C_F would not change so the thrust would also follow the pressure changes (unless there is substantial nozzle erosion).
From Eq. 3-33, $F = C_F \dot{m}\ c^*$.

CHAPTER 13

1. Ideally the solid oxidizer particles in a propellant can be considered spheres of uniform size. Three sizes of particles are available: coarse at 500 μm, medium at 50 μm, and fine at 5 μm, all at a specific gravity of 1.95, and a viscoelastic fuel binder at a specific gravity of 1.01. Assume that these materials can be mixed and vibrated so that the solid particles will touch each other, there are no voids in the binder, and the particles occupy a minimum of space similar to the sketch of the cross section shown here. It is desired to put 94 wt % of oxidizer into the propellant mix, for this will give maximum performance. **(a)** Determine the maximum weight percentage of oxidizer if only coarse crystals are used or if only medium-sized crystals are used. **(b)** Determine the maximum weight of oxidizer if both coarse and fine crystals are used, with the fine crystals filling the voids between the coarse particles. What is the optimum relative proportion of coarse and fine particles to give a maximum of oxidizer? **(c)** Same as part **(b),** but use coarse and medium crystals only. Is this better and, if so, why? **(d)** Using all three sizes, what is the ideal weight mixture ratio and what is the maximum oxidizer content possible and the theoretical maximum specific gravity of the propellant? (*Hint*: The centers of four adjacent coarse crystals form a tetrahedron whose side length is equal to the diameter.) [See problem drawing in the book.]

2. Suggest one or two specific applications (intercontinental missile, anti-aircraft, space launch vehicle upper stage, etc.) for each of the propellant categories listed in Table 13–2 and explain why it was selected when compared to other propellants.

3. Prepare a detailed outline of a procedure to be followed by a crew operating a propellant mixer. This 1-m^3 vertical solid propellant mixer has two rotating blades, a mixing bowl, a vacuum pump system to allow mix operations under vacuum, feed chutes or pipes with valves to supply the ingredients, and variable-speed electric motor drive, a provision for removing some propellant for laboratory samples, and a double-wall jacket around the mixing bowl to allow heating or cooling. It is known that the composite propellant properties are affected by mix time, small deviations from the exact composition, the temperature of the mix, the mechanical energy added by the blades, the blade speed, and the sequence in which the ingredients are added. It is also known that bad propellant would be produced if there are leaks that destroy the vacuum, if the bowl, mixing blades, feed chutes, and so on, are not clean but contain deposits of old propellant on their walls, if they are not mixed at 80 °C, or if the viscosity of the mix becomes excessive. The sequence of loading ingredients shall be: (1) prepolymer binder, (2) plasticizer, (3) minor liquid additives, (4) solid consisting of first powdered aluminum and thereafter mixed bimodal AP crystals, and (5) finally the polymerizing agent or crosslinker. Refer to Fig. 13–11. Samples of the final liquid mix are taken to check viscosity and density. Please list all the sequential steps that the crew should undertake before, during, and after the mixing operation. Each major step in this outline of the procedure for mixing composite propellants has a purpose or objective that should be stated. Any specific data for each step (e.g., mass, duration, temperature, blade speed, etc.) is not required to be listed; however, if known, it can be mentioned. Mention all instruments (e.g., thermometers, wattmeter, etc.) that the crew should have and identify those that they must monitor closely. Assume that all ingredients were found to be of the desired composition, purity, and quality.

Answer: Outline of Procedure for Propellant Mix Operation

Below is type of answer that would be sought. The sequence, controls, safety features, and operations are somewhat different in each mix facility. The list is not inclusive or complete.

1. Unlock remote control room and separate multistory mixer building. Turn on power, lights, utilities, set room temperature, and if available the humidity control system.
2. Check out all the equipment, tools (wrenches, propellant sampling equipment), sensors, etc. Make sure that all switches and controls are in their "deactivated position".
3. Inspect mixer machinery, loading and weighing equipment ; see that mixer blades and bowl are clean, contain no caked-propellant residue from a prior operation, no residual ingredients in tanks, pipes, chutes, supply machinery, or weighing scales; check for debris.
4. Activate external signal lights, flags, horns, and/or other warning devices to alert other personnel to stay away from the mixer area during the potentially hazardous operations. Only a minimum number of personnel will be allowed

within the facility during the subsequent operations. Notify security and environmental personnel of the impending operation.
5. Bring in the ingredients that are needed. It is usually required that hazardous materials not be stored in the mixer area. The ingredients are mentioned below.
6. If not previously performed elsewhere, weight and load a specified amount of each ingredient into their respective receptacle. Liquid ingredients are to be loaded into elevated tanks (large mixers, such as 1.0 cubic meter capacity, these are found on the floor above the mixer) so that gravity can be used to later feed the ingredients into the mixing bowl. Liquids, such as the binder or monomer, the plasticizer, or the curing agent are each supplied through pipes with electrically actuated shut-off valves. The solid ingredients (aluminum powder, premixed blended oxidizer crystals, etc.) are store in containers (at an elevation above the mixer) and are supplied through chutes (with a rubber hose squeezed valve) into the mixing bowl. For safety reasons oxidizer crystal blending is usually done in a separate building and the blended mixture is then supplied to the mixing facility.
7. Put the master key into the master control switch of the mixer. Only the authorized mix crew leader will have such a key. Open mix bowl and check for cleanliness and possible debris. Opening is done by removing a cover or more likely by slowly lowering the bowl.
8. Close and secure the mixing bowl. Turn on and test the vacuum system and check for leaks. Connect and turn on the flow of water to the jacket around the mix bowl. In some propellant, the bowl is heated by hot water to allow the reaction at elevated temperatures and in other propellants exothermic curing reactions require cooling or heat removal by the water flow. The bowl temperature should be stabilized at the desired value.
9. Start and continue to monitor the vacuum pressure, water inlet and outlet as well as bowl temperatures through the remainder of the process. The various instruments should have been calibrated before the operation.
10. Start the rotation of the mixing blades. They operate like a kitchen mixer. Monitor the rotational speed and the torque or power applied by the electric motor throughout the remainder of the mixing process. Adjust speed to desired value.
11. Admit the prepolymer or binder liquid by opening the proper valve. Close the valve after the binder is inside the bowl. Continue blade rotation.
12. Admit plasticizer and then minor ingredients into the mix bowl. Open and thereafter close the appropriate valves. Run mixer for the predetermined time that assures a uniform mix.
13. Admit the powder aluminum by opening the chute. Then close off the chute.
14. Continue to operate mixer for the predetermined time.
15. Gradually admit the blended oxidizer crystal into the mixer while it is rotating. Mixing with the oxidizer is the most hazardous part of the mix process. Close the oxidizer chute when done.
16. After a proper mix has been achieved admit the curing or cross-linker agent. The liquid will then gradually become more viscous, as the polymers are cured and become harder. The energy input (from the mixing blades to the batch propellant) will increase here. Depending on the propellant, this initial curing process can last an hour or several days. Continue mixing for the predetermined time. Many mixer bowls are equipped with sensors to detect any possible self-ignition or conflagration within the mixer, particularly during this hazardous process step. If such a combustion is sensed, then the mixing bowl is quickly opened to vented the gases that would be created. A fire extinguishing system would also then be activated.
17. Take samples during the curing operation. In many mixing stations the operation is temporarily stopped, while a crew member goes into the mixing building to withdraw a small sample of the mixed viscous liquid from a small vat at the bottom or on the side of the bowl. Typically a viscosity test is performed in situ with the material from the sample. Often a sample is also sent to the laboratory where other tests are performed such as a density test and a firing test to determine the burning rate of the viscous liquid using a special apparatus. Often a sample is allowed to cure fully cure and is tested on a later date. For record purposes, many organizations keep a sample of each mixing batch for possible later evaluation.
18. The mixing procedures are defined during a prior development effort. Mix-time periods, mix blade speeds, temperatures and the sequence for adding materials has been carefully worked out then. If any of the monitored gage readings or the test results from the sample tests give data that is outside the expected tolerances, then the operator must take corrective action. It is often difficult to meet the simultaneous requirements of density, viscosity, burn rate (wet) and an experienced mixer operator will usually know what corrective action to take. This can include such steps as changing the speed of the mixing blades (by changing the input of mechanical energy), changing the jacket temperature, or by adding a small amount of one of the ingredients.
19. When the propellant batch has reached its intended properties (very viscous, just barely able to flow freely), then the mix operation is terminated. Blade rotation is stopped. The vacuum is disconnected and air is admitted to the bowl. The seal between the bowl and its top, which contains the blades, drive motors and certain instruments, is broken and it is possible to look inside the bowl. In some installations, the bowl is lowered automatically exposing the wet blades.
20. In some mixers, the finished mix is loaded into a transfer vessel which then transports it to the casting facilities. The transfer vessel is placed underneath the mixing bowl and a valve is opened at the bottom of the bowl and the mixed ingredients are then transferred to the vessel. In some operations and with some types of rocket motors the casting of the liquid propellant is done directly from the mixing bowl either by moving it to a casting location or by placing rocket

motor hardware directly under the bowl. The curing process will continue during the casting process and also after the rocket motor has been cast.
21. After the bowl is emptied, the flow of water to the bowl's jacket can usually be discontinued. Various sensors can also be turned off.
22. After the propellant batch has left the mix building, an elaborate manual cleaning process is started. All residual propellant that still clings to the surfaces of the blades and the bowl has to be removed and the surfaces have to be cleaned. This is also a hazardous process. Plastic spatulas (non-sparking) and plastic tools are used for the removal of propellant residue. Propellant scraps are sent to a burn facility for disposal. Residues in the liquid containers, solid ingredient chutes, weighing containers and scales are also cleaned. Cleaning materials and tools are put back. The floors are then washed to remove any spilled ingredients or spilled propellant.
23. The warning signals are turned off. The boss and the security department are then notified that the operation was safely completed.
24. Throughout the operation meticulous records are kept of all important steps, measured parameters, and the operator who performed each sub-operation. These can be very useful if there is a problem in subsequent operations or during the firing of the rocket motor.
25. A final check-out and inspection is performed of all equipment, instruments, tools, and facilities prior to closing them for the night or prior to the next mixing operation. The building is left secured and locked.

4. Determine the longitudinal growth of a 24-in.-long freestanding grain with a linear thermal coefficient of expansion of 7.5×10^{-5}/°F for temperature limits of −40 to 140 °F

Growth = (7.5×10^{-5} in./in.-°F)(24 in.)(140 °F – (-40 °F)) = 0.324 in.

5. The following data are given for an internally burning solid propellant grain with inhibited end faces and a small initial port area:

Length	40 in.
Port area	27 in.^{2}
Propellant weight	240 lbf
Initial pressure at front end of chamber	1608 psi
Initial pressure at nozzle end of chamber	1412 psi
Propellant density	0.060 lbm/in.^{3}
Vehicle acceleration	$21.2g_0$

Determine the initial forces on the propellant supports produced by pressure differential And by the vehicle acceleration.

Inhibitor = see Section 13-6, *port area* = gas-flow area inside grain cavity.
Pressure differential force: $(p_{1\text{front end}} - p_{1\text{nozzle end}})A$ = 5,292 to 24,892 lbf.
A is as low as 27 in^2 and as high as 127 in^2 (the cross sectional area of a cylinder that contains 240 lb of propellant and has A_p = 27 in^2), depending on end faces configuration.
Propellant acceleration inertial force: ma = (240/32.2) x $21.2g_0$ = 5,090 lbf

6. A solid propellant rocket motor with an end-burning grain has a thrust of 4700 N and a duration of 14 sec. Four different burning rate propellants are available, all with approximately the same performance and the same specific gravity, but different AP mix and sizes and different burning rate enhancing ingredients. They are 5.0, 7.0, 10, and 13 mm/sec. The preferred L/D is 2.60, but values of 2.2 to 3.5 are acceptable. The impulse-to-initial weight ratio is 96 at an L/D of 2.5. Assume optimum nozzle expansion at sea level. Chamber pressure is 6.894 MPa or 1000 psia and the operating temperature is 20°C or 68°F. Determine grain geometry, propellant mass, hardware mass, and initial mass.

Unfortunately, this problem cannot be worked for an end-burner grain configuration with the listed L/D ratios.

7. For the rocket in Problem 13-6 determine the approximate chamber pressure, thrust, and duration at 245 and 328 K. Assume the temperature sensitivity (at a constant value of A_b/A_t) of 0.01%/K does not change with temperature.

8. A fuel-rich solid propellant for a gas generator drives a turbine of a liquid propellant turbopump. Determine its mass flow rate. The following data are given:

Chamber pressure	p_1 = 5MPa
Combustion temperature	T_1 = 1500K
Specific heat ratio	k = 1.25
Required pump input power	970 kW
Turbine outlet pressure	0 psi [or 1.0 atm abs]
Turbine efficiency	65%
Molecular mass of gas	22 kg/kg-mol
Pressure drop between gas generator and turbine nozzle inlet	0.10 MPa

Windage and bearing friction is 10 kW. Neglect start transients.

From Eqs. 3-6 and 10-18, $P_T = [\dot{m}\,\eta_T T_1 Rk/(k\text{-}1)][1 - (p_2/p_1)^{(k\text{-}1)/k}] = 970 + 10 = 980$ kW
Since p_2 =0.1013 MPa, $\dot{m}$ = 9.80 x 10^5/[0.65 x 1500 x 8314.3 x 1.25/((0.25) x 22)]x (1 – (0.1013/5.0)(0.25/1.25) = 0.288 kg/sec

9. The propellant for this (see Pb. 13-8) gas generator has these characteristics:

Burn rate at standard conditions	4.0 mm/sec
Burn time	110 sec
Chamber pressure	5.1 MPa
Pressure exponent *n*	0.55
Propellant specific gravity	1.47

Determine the size of an end-burning cylindrical grain.
Answer: Single end-burning grain 27.2 cm in diameter and 37.3 cm long, or two endburning opposed grains (each 19.6 cm diameter × 31.9 cm long) in a single chamber with ignition of both grains in the middle of the case.

The relevant burn rate is, Eq. 12-5, $r = 4.0\,(5.1/6.895)^{0.55} = 0.339$ cm/sec
For this cylindrical propellant shape, $m_p = \rho_b(A_b L) = (rA_b\,\rho_b\,)t_b$
so $L = rt_b = 0.339 \times 110 = 37.3$ cm long (without allowances for A_b-changes with time).
Using the resulting $\dot{m}$ = 0.288 kg/sec from Prob. 13-8, $A_b = \dot{m}\,t_b/\rho_b L$ and propellant diameter = $((4/\pi)A_b)^{1/2} = 27.1$ cm

EXTRA PROBLEM

13A. Information on the solid-state oxidizer *ammonium nitrate* is found several places in the book, on Section 13.4. under *inorganic oxidizers*, as well as in Tables 5-6 and 13-1, and in Figure 13-3. Various fuels are involved in the given data so it can only conform within certain margins in the following exercises. Using ideal rocket assumptions verify the applicability of these data by answering: (i) What oxidizer concentration (percent purity) would make the c^*-values given in Table 13-3 conform to those given in Tables 5-6 and 13-1? (ii) Would the values of $(C_F)_{\text{opt}}$ be equivalent between all the above sources? (iii) What do these results say about the shown I_s values of oxidizer concentration in (i)? Assume that the k-value given in Table 5-6 applies to all other data.

AN-oxidizer, various sources (pick concentrations so $c^* \approx$ constant)
(i) Fig. 13-3, AN-concentration ≈ 85%, $T_0 \approx 1500$ K, $\mathfrak{M} \approx 20$, $I_s \approx 190$ sec
Table 13-1, AN/Polymer, $T_0 \approx 1550$ K, $I_s \approx$ 180-190 sec
Table 5-6, AN, $T_0 = 1282$ K, $\mathfrak{M} = 20.1$, $I_s \approx 192$ sec, $k = 1.26$
(ii) If and when k and p_1/p_2 are the same, $(C_F)_{\text{opt}}$ would remain unchanged.
(iii) From Eq. 3-32, $I_s = c^* C_F/g_0$, so if c* remains unchanged the quoted values of I_s would apply as long as the oxidizer concentration is around 85%.
[For cases when p_3 is the same, non-optimum thrust coefficient conditions would apply as well but note as in all cases that I_s is always a strong function of C_F. The values of c^* as obtained from thermochemical analyses depend on propellant concentrations and flame temperatures so item (i) becomes a critical input.]

CHAPTER 14

1. **(a)** Calculate the length of a T-burner to give a first natural oscillation of 2000 Hz using a propellant that has a combustion temperature of 2410 K, a specific heat ratio of 1.25, a molecular mass of 25 kg/kg-mol, and a burning rate of 10.0 mm/sec at a pressure of 68 atm. The T-burner is connected to a large surge tank and prepressurized with nitrogen gas to 68 atm. The propellant disks are 20mm thick. Make a sketch to indicate the T-burner dimensions, including the disks.
(b) If the target frequencies are reached when the propellant is 50% burned, what will be the frequency at propellant burnout?

At the stated conditions, the speed of sound (Eq. 3-10) in the propellant gas is

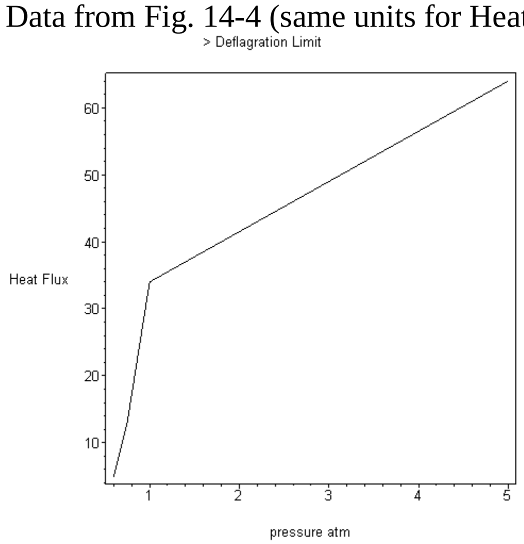
$a = \sqrt{\dfrac{kR'T}{\mathrm{M}}}$ = $(1.25 \times 8324.3 \times 2410/25)^{1/2} = 10^3$ m/sec

Half of the fundamental or the first natural cycle occupies = 1000/(2 x 2000) = 0.250 m
(a) The propellant disks are 20 mm thick each (see Fig. 14-7) and that natural frequency is reached when the disks are 50% burned, so 0.250m = L - 2 x 0.020/2 or L = 0.270 m.
(b) At burnout, the full length of the cavity is available, so that the frequency becomes f = 1000/(2 x 0.27) = 1852 Hz.

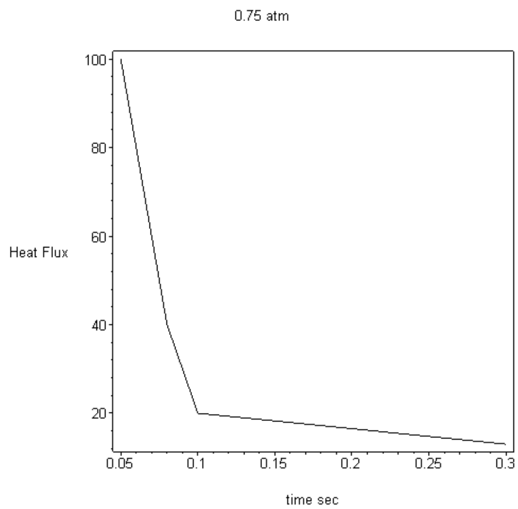
2. An igniter is needed for a rocket motor similar to one shown in Fig. 12–1. Igniters have been designed by various oversimplified design rules such as Fig. 14–3. The motor has an initial internal grain cavity volume of 0.055 m^3 and an initial burning surface of 0.72 m^2. The proposed igniter propellant has these characteristics: combustion temperature 2500K and an energy release of about 40 J/kg-sec. Calculate the minimum required igniter propellant mass **(a)** if the cavity has to be pressurized to about 2 atm (ignore heat losses); **(b)** if only 6% of the igniter gas energy is absorbed at the burning surface, and it requires about 20 cal/cm2-sec to ignite in about 0.13 sec.

3. Using the data from Fig. 14–4, plot the total heat flux absorbed per unit area versus pressure to achieve ignition with the energy needed to ignite being just above the deflagration limit. Then, for 0.75 atm, plot the total energy needed versus ignition time. Give an interpretation of the results and trend for each of the two curves.

Data from Fig. 14-4 (same units for Heat Flux, cal/cm^2-sec):



The heat flux increases with increasing pressure. Note that the knee in the curve is artificial because of scarcity of data, but that the slope decreases markedly with increasing pressure.



The heat flux decreases with time for a fixed pressure. Note that the knee in the curve is artificial because of scarcity of data, but that the slope decreases markedly with increasing time.

CHAPTER 15

1 In Figs. 14-3 and 14-4 it can be seen that higher case pressures and higher heat transfer rates promote faster ignition. One way to promote this more rapid ignition is for the nozzle to remain plugged until a certain minimum initial pressure has been reached, at which time the nozzle plug will be ejected. Analyze the time saving achieved by such a device, assuming that the igniter gas evolution follows Eqs. 12-3 and 12-5. Under what circumstances is this an effective method? State any relevant assumptions about cavity volume, propellant density, and so on.

Combining Eqs. 12-3 and 12-5 with 3-4 ($V_1 \approx$ constant is purely the volume) yields,

$$\frac{dp_1}{dt} = \left(\frac{A_b \rho_b a R T_1}{V_1}\right) p_1^n - \left(\frac{A_t R T_1}{c^* V_1}\right) p_1 = c_1 p_1^n - c_2 p_1 \qquad \text{where } c_1 \text{ and } c_2 \text{ are constants.}$$

For the steady state, $dp_1/dt = 0$ so that, $p_{1s} = [c_1/c_2]^{1/(1-n)} = [A_b \rho_b a c^*/A_t]^{1/(1-n)}$

a) When the nozzle is "plugged" the equation becomes $\frac{dp_1}{dt} = \left(\frac{A_b \rho_b a R T_1}{V_1}\right) p_1^n = c_1 p_1^n$

Which solves as, p_{01} = pressure at $t = 0$, $p_1(t) = [(1-n)c_1 t + p_{01}^{(1-n)}]^{1/(1-n)}$

And the time to achieve a pressure $p_{1s} = [c_1/c_2]^{1/(1-n)}$ will be:

$$c_1 t_s = [c_1/c_2 - p_{01}^{(1-n)}]/(1-n)$$

Using $y \equiv [c_1/c_2]^{1/(1-n)} p_1$ and $\tau \equiv c_2 t$ and $y_0 \equiv [c_1/c_2]^{1/(1-n)} p_{01}$

We get the dimensionless time as, $\tau_s = [1 - y_0^{1-n}]/(1-n)$

b) The original equation integrates to, using the newly defined variables y and τ,

$$y(\tau) = e^{-\tau}\,[e^{(1-n)\tau} + (y_0)^{(1-n)} - 1]^{1/(1-n)}$$

Since the variable $y(\tau)$ approaches its steady-state value of 1.0 asymptotically, we may define the value of τ_s when y_s = 99% (or $p_1/p_{1s} = 0.99$).

c) For a value $y_0 = 0.1$ (or $p_{01} = p_{1s}/10$) and $n = 0.5$, $\tau_s = 1.37$ plugged and 11 unplugged, or about ten times faster. For $y_0 = 0.01$ and $n = 0.5$, $\tau_s = 1.8$ plugged and 12 unplugged.

d) There is a definite time advantage to plugging the nozzle but there must be a safe and reliable mechanism to eject or otherwise dispose of the plug for steady-state operation.

2. Compare a bare simple cylindrical case with hemispheric ends (ignore nozzle entry or igniter flanges) for an alloy steel metal and two reinforced fiber (glass and carbon)-wound filament case. Use the properties in Table 15-2 and thin shell structure theory.

Given:

Length of cylindrical portion	370 mm
Outside cylinder diameter	200 mm
Internal pressure	6 MPa
Web fraction	0.52
Insulator thickness (average)	
for metal case	1.2 mm
for reinforced plastic case	3.0 mm
Volumetric propellant loading	88%
Propellant specific gravity	1.80
Specific impulse (actual)	248 sec
Nozzle igniter and mounting provisions	0.20 kg

Calculate and compare the theoretical propulsion system flight velocity (without payload) in a gravity-free vacuum for these three cases.

3. The following data are given for a case that can be made of either alloy steel or fiber-reinforced plastic.

Type	Metal	Reinforced Plastic
Material	D6aC	Organic filament composite (Kevlar)

Physical properties	See Table 15–2	See Table 15–2
Poisson ratio	0.27	0.38
Coefficient of therm. exp., m/m-K $\times 10^{-6}$	8	45
Outside diameter (m)	0.30	0.30
Length of cyl. section (m)	0.48	0.48
Hemispherical ends		
Nozzle flange diameter (m)	0.16	0.16
Average temp. rise of case material during operation (°F)	55	45

Stating your assumptions, determine the growth in diameter and length of the case due to pressurization, heating, and the combined growth and interpret the results.

4. A high-pressure helium gas tank at 8000 psi maximum storage pressure and 1.5 ft internal diameter is proposed. Use a safety factor of 1.5 on the ultimate strength. The following candidate materials are to be considered:
Kevlar fibers in an epoxy matrix (see Table 15–2)
Carbon fibers in an epoxy matrix
Heat-treated welded titanium alloy with an ultimate strength of 150,000 psi and a weight density of 0.165 lb/in.^{3}
Determine the dimensions and sea-level weights of these three tanks and discuss their relative merits. To contain the high-pressure gas in a composite material that is porous, it is also necessary to include a thin metal inner liner (such as 0.016-in.-thick aluminum) to prevent loss of gas; this liner will not really carry structural loads, but its weight and volume need to be considered.

5. Make a simple sketch and determine the mass or sea-level weight of a rocket motor case that is made of alloy steel and is cylindrical with hemispherical ends. State any assumptions you make about the method of attachment of the nozzle assembly and the igniter at the forward end.

Outer case and vehicle diameters	20.0 in.
Length of cylinder portion of case	19.30 in.
Ultimate tensile strength	172,000 psi
Yield strength	151,300 psi
Safety factor on ultimate strength	1.65
Safety factor on yield strength	1.40
Nozzle bolt circle diameter	12.0 in.
Igniter case diameter (forward end)	3.00 in.
Chamber pressure, maximum	1520 psi

6. Design a solid propellant rocket motor with insulation and liner. Use the AP/Al-HTPB propellant from Table 12–3 for Orbus 6. The average thrust is 3600 lbf and the average burn time is 25.0 sec. State all the assumptions and rules used in your solution and give your reasons for them. Make simple sketches of a cross section and a half section with overall dimensions (length and diameter), and determine the approximate loaded propellant mass.

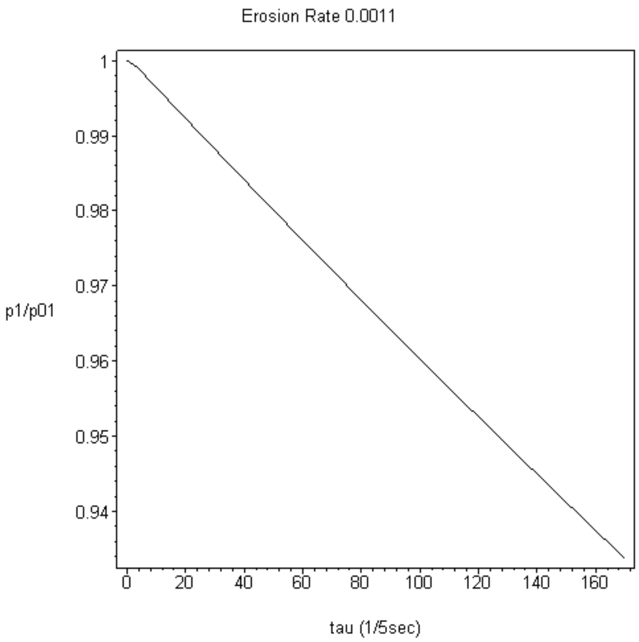
7. The STAR 27 rocket motor (Fig. 12–1 and Table 12–3) has an average erosion rate of 0.0011 in./sec. **(a)** Determine the change in nozzle area, thrust, chamber pressure, burn time, and mass flow at cutoff. **(b)** Also determine those same parameters for a condition when, somehow, a poor grade of ITE material was used that had three times the usual erosion rate. Comment on the difference and acceptability.

Combining Eqs. 12-3 and 12-5 with 3-4 (where V_1 is purely the volume) yields,

$$\frac{dp_1}{dt} = \left(\frac{A_b \rho_b a R T_1}{V_1}\right) p_1^n - \left(\frac{A_t R T_1}{c^* V_1}\right) p_1 = c_1 p_1^n - c_3 (1 + 2\frac{\alpha t}{R_0})^2 p_1$$

From data, $R_{0t} \approx 2.89$ in and $A_{0t} = 26.2$ in^2 = 0.182 ft^2; $V_{1ave} \approx 0.5$ ft^3.
This yields $c_3 \approx 5.0$/sec from Table 12-3.

Defining new variables, $y \equiv p_1/p_{1s}$ and $\tau \equiv c_3 t = 5t$, where $p_{1s} = [A_{b0}\rho_b a\, c^*/A_t]^{1/(1-n)}$ yields the following ODE: $dy/d\tau \approx y^n - y - (3 \times 10^{-4}\tau)y$ with $y(0) = 1.0$
The value of the constant above for the second case (b) is multiplied by a factor of 3.
The differential equation above is solved numerically using MAPLE software,



(a) At the end of the 34 sec, $\tau = 170$, the pressure p_1 has decreased by 6.6% by the action of the increasing A_t due to erosion, where A_t has increased by 5.3%.

Neglecting changes to C_F, $F = C_F A_t p_1$ so F has decreased by about 1.7% to 0.9831 F. The new burn time should be slightly more and the new mass flow rate should be slightly less than nominal, both magnitudes under 2 %.

(b) With the lower grade ITE material, at the end of the 34 sec, the pressure p_1 has decreased by 17.8% by the action of the increasing A_t due to erosion, whereas A_t has increased by 14.7%. So F has decreased by about 5.8% to 0.942F. The new burn time should be somewhat more and the new mass flow rate should be somewhat less than nominal, both magnitudes under 6%.

This is not surprising because the mass flow rate has decreased and this means that the burn time must have increased, but the latter calculation requires that an integral over time equal the total impulse I_t. Case (b) produces much more noticeable changes. Note however that as soon as the port increases sufficiently the erosive burning is likely to stop.

CHAPTER 16

1. Consider a classical hybrid motor with a single solid fuel grain stored in a single cylindrical enclosure. The combustion port radius is R and the length L. For a fuel burning rate given by Eq. 16–2, $\dot{r} = a(G_o)^n$, with $n = 0.7$:
(a) What happens to the thrust with time for a constant oxidizer mass flow rate during the burn?
(b) What happens to the mixture ratio if the oxidizer rate is decreased?

a) $F = \dot{m}\, c^* C_F = (\dot{m}_o + \dot{m}_f) I_s\, g_0$ and $\dot{m}_f = \rho A_b \dot{r}$ and $G_o = \dot{m}_o/\pi R^2$
Eq. 16-10: $\dot{r} = a(\dot{m}_o/\pi R^2)^n$ so that $\dot{m}_f = \rho(2\pi RL)\, a(\dot{m}_o/\pi R^2)^n$ proportional to R^{1-2n}
For $n = 0.7$, $\dot{m}_f \sim R^{-0.4}$ so $\dot{m}_f$ decreases as R increases with burn time.
If T_1, k, $\mathfrak{M}$, and C_F remain unchanged, then both $\dot{m}$ and F will DECREASE.
b) Eqs. 6-1 and 16-11, $r = \dot{m}_o/\dot{m}_f \sim \dot{m}_o^{\,0.3} R^{0.4}$ so for any given R, the mixture ratio decreases. If the dynamic behavior can be represented by Eq. 16-15, then a more complex behavior ensues and more information is needed.

2. (a) Write Eq. 16–1 in terms of a *Reynolds number* based on the free-stream gas flow and the axial location x. What does this say about the Reynolds number dependence of surface regression rate?
(b) It is know that the blowing coefficient is itself roughly proportional to $x^{0.2}$. What additional assumptions might be necessary to transform Eq. 16-1 into 16-2?

a) $\dot{r} = 0.036(\mathrm{Re}x)^{0.8} x^{-1} \mu \rho_f^{-1} \beta^{0.23}$ which is an "0.8-power" dependence on Re.
b) Take $x^{-0.2}\beta^{0.23} \approx$ constant, G proportional to G_o, and $\mu^{0.2}\rho_f^{-1} \approx$ constant. In Eq. 16-2, the constant a, therefore, encompasses several diverse parameters.

3. Calculate the ideal density-specific impulse (in units of kg-sec/m^3) for the propellant combination LOX/HTPB at an oxidizer-to-fuel ratio of 2.3 at a pressure ratio across the nozzle of 1000/14.7 and an optimum expansion. Use the information in Tables 16–1, 2 and a fuel density of 920 kg/m^3. Compare your result with the value of LOX/RP-1 bipropellant listed in Table 5–5.

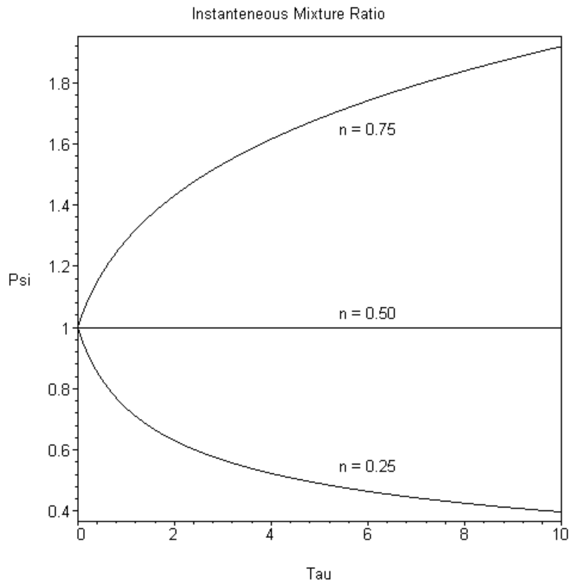
Eq. 7-2: $\rho_{\mathrm{av}} = \dfrac{\rho_o \rho_f (1+r)}{\rho_f r + \rho_o} = \dfrac{1149\mathrm{x}920(1+2.3)}{920\mathrm{x}2.3+1149} = 1.07 \times 10^3$ kg/m^3
From Table 16-2, $c^* = 1790$ m/sec and $k = 1.14$, and from Fig 3-6 $C_F \approx 1.63$
So that from Eq. 3-32, $I_s = c^* C_F/g_0 = 1790 \times 1.63/9.81 = 297$ sec
Eq 7-3 in terms of the density: $I_d = \rho_{\mathrm{av}} I_s = 1.07 \times 10^3 \times 297 = 3.17\times 10^5$ kg-sec/m^3
From Table 5-5, LOX/RP-1, $r = 2.56$, SG)$_{\mathrm{av}} = 1.01$, $I_s = 300$ sec (frozen)
so $I_d = \rho_{\mathrm{av}} I_s = 1.01 \times (1.008 \times 10^3) \times 300 = 3.05 \times 10^5$ kg-sec/m^3
So that the performance of the hybrid unit is quite comparable to LOX/RP-1.

4. Plot the instantaneous mixture ratio given by Eq. 16–15 as a function of the index n and time. Use values of n greater and less than 1/2 (at n = 1/2 the time dependence vanishes). In order to only focus on the effects of n, work with the modified variables ψ (related to mixture ratio) and τ (related to time) as follows:

$$\psi = [(2n+1)\tau + 1]^{(2n-1)/(2n+1)}$$

Both ψ and τ are dimensionless and defined in the problem statement as

$$\psi \equiv \dot{m}_o/\dot{m}_f (2\rho_f La/R_i(\dot{m}_o/\pi NR_i)^{1-n} \quad \text{and} \quad \tau \equiv a(\dot{m}_o/\pi NR^2_i)^n\, t/R_i$$



As apparent in the figure above, the mixture ratio is independent of time when $n = 0.5$. It increases with time for $n > 0.5$ and deceases for $n < 0.5$. The dimensionless variables generalize the curves but the trends should be representative.

5. A classical hybrid propulsion system has the following characteristics:

Propellants	LOX/HTPB
Nominal burn time	20 sec
Initial chamber pressure	600 psia
Initial mixture ratio	2.0
Initial nozzle exit area ratio	10.0
Fuel grain outside diameter	12 in
Grain geometry	Single cylinder case bonded
Initial HTPB temperature	65°F

Determine initial and final thrust, specific impulse, propellant masses, and useful propellant.

For this problem, additional information is needed otherwise some answers must be left in terms of the mass flow rate of the oxidizer, $\dot{m}_o$. (See the details in Example 16-1.)

From Table 16-2, $c^* = 5912$ ft/sec and $k = 1.152$. For a nozzle area ratio of 10, Figure 3-4 indicates an expansion pressure ratio, $p_1/p_2 \approx 60$. If we assume an optimum expansion, then from Fig. 3-5, $C_F \approx 1.63$ and $c \approx 5912 \times 1.63 = 9{,}640$ ft/sec so $I_s = 299$ sec.

CHAPTER 17

1. The characteristic velocity $v_c = \sqrt{2t_p \alpha \eta}$ is used to achieve a dimensionless representation

of flight performance analysis. Derive Eq. 17–38 without any tankage

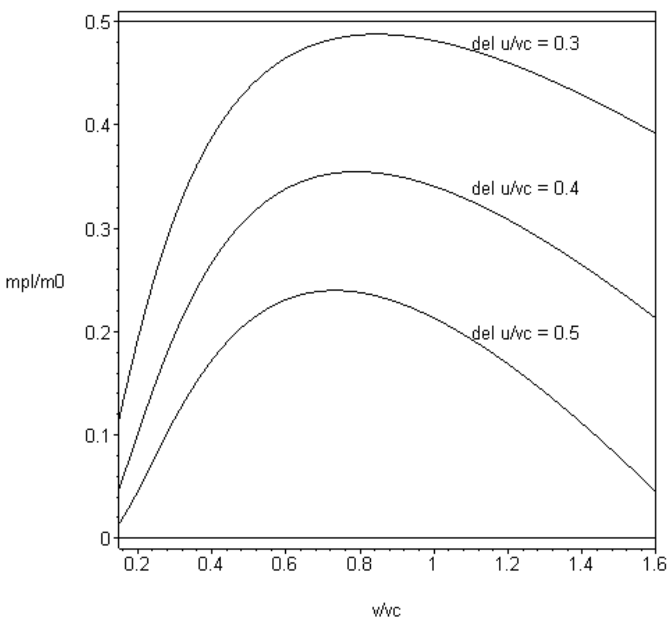
fraction allowance. Also, plot the payload fraction against v/v_c for several values

of $\Delta u/v_c$. Discuss your results with respect to optimum performance.

Starting from Eq. 17-7 solve for $e^{\Delta u/v} = \left[\dfrac{m_0/m_{pl} + m_0/m_{pl}(v/v_c)^2}{1 + m_0/m_{pl}(v/v_c)^2}\right]$

But $\Delta u/v = (\Delta u/v_c)/(v/v_c)$ so that taking anti-logs on both sides,

$$(\Delta u/v_c) = (v/v_c)\ln\left[\frac{1+(v/v_c)^2}{m_{pl}/m_0 + (v/v_c)^2}\right]$$



This clearly shows the existence of an *optimum specific impulse*. See Fig. 17-13.

2. For the special case of zero payload, determine the maximized values of

$\Delta u/v_c$, v/v_c , m_p/m_0, and m_{pp}/m_0 in terms of this characteristic velocity.

$m_{pl} = 0$ in Eq. 17-38, let $y \equiv \Delta u/v_c$ and $x \equiv v/v_c$

With $\varphi = 0$, $y = x\ln[1 + 1/x^2]$ and $dy/dx = 0$ at maximum

So $\ln[1 + 1/x^2] = 2/(x^2 + 1)$ or $x = 0.505$ at the maximum

Thus $v/v_c = 0.505$ and $\Delta u/v_c = 0.505\ln[1 + 1/(0.505)^2] = 0.805$

From Eqs. 17-4 & 17-6, $m_0 = m_p + m_{pp} = m_p[1 + (v/v_c)^2]$ so $m_p/m_0 = 0.7968$

and $m_{pp}/m_0 = 0.7968(0.505)^2 = 0.2032$

3. For a space mission with an incremental vehicle velocity of 85,000 ft/sec and a specific power of $\alpha = 100$ W/kg, determine the optimum values of I_s and t_p for two maximum payload fractions, namely 0.35 and 0.55. Take the thruster efficiency as 100% and $\varphi = 0$.

$\Delta u = 85{,}000$ ft/sec, $\alpha = 100$ W/kg, $\eta = 1.0$ and $\varphi = 0$.

For $m_{pl}/m_0 = 0.35$ and 0.55 determine optimum values using Fig. 17-3

$\underline{m_{pl}/m_0 = 0.35}$, optima: $v/v_c = 0.782$ and $\Delta u/v_c = 0.404$

$v_c = \Delta u/0.404 = 2.10\text{x}10^5$ ft/sec $= 6.41\text{x}10^4$ m/sec

$t_p^* = v_c^2/2\alpha\eta = (6.41\text{x}10^4)^2/200 = 2.06\text{x}10^7$ sec $= 238$ days

$I_s = v/g_0 = (0.782)(6.41\text{x}10^4)/9.81 = 5{,}110$ sec

$\underline{m_{pl}/m_0 = 0.55}$, optima: $v/v_c = 0.864$ and $\Delta u/v_c = 0.257$

$v_c = \Delta u/0.257 = 1.01\text{x}10^5$ m/sec

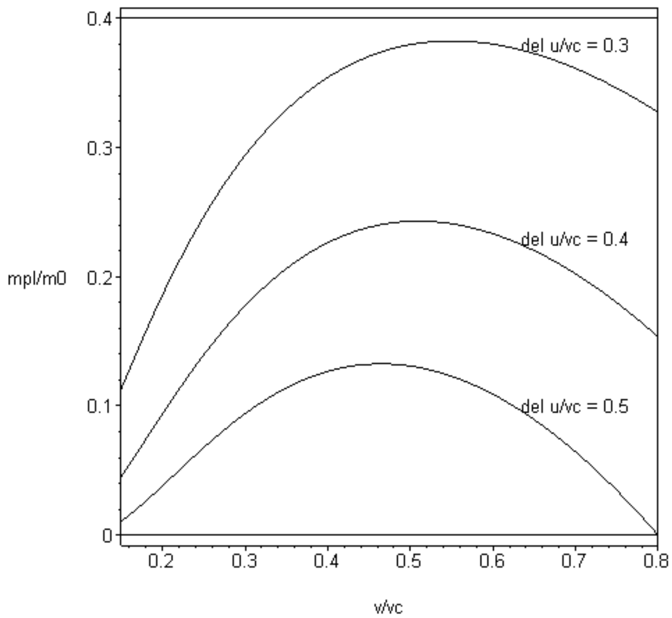
$t_p^* = v_c^2/2\alpha\eta = (1.01\text{x}10^5)^2/200 = 5.10\text{x}10^7$ sec $= 590$ days

$I_s = v/g_0 = (0.864)(1.01\text{x}10^5)/9.81 = 8{,}900$ sec

4. Derive Eq. 17–7 using $m_{pp} = m_p\,[(v/v_c)^2 + (v/v_c)^3]$ instead of Eq. 17–6 (use $\varphi = 0$); this form penalizes the high I_s and/or short t_p missions. Plot and compare to the results shown on Fig. 17–3.

From Eq. 17-4, $m_{pl}/m_0 = 1 - m_p/m_0\,[1 + (v/v_c)^2 + (v/v_c)^3]$ and $m_p/m_0 = 1 - m_f/m_0$

But from Eq. 4-7, $m_f/m_0 = e^{-\Delta u/v}$ so $m_{pl}/m_0 = e^{-\Delta u/v} - (1 - e^{-\Delta u/v})[(v/v_c)^2 + (v/v_c)^3]$



Max. values of m_{pl}/m_0 are lower and with a correspondingly lower v/v_c than in Pb. 17-1.

5. An ion thruster uses heavy positively charged particles with a charge-to-mass ratio of 500 coulombs per kilogram, producing a specific impulse of 3000 sec. **(a)** What acceleration voltage would be required for this specific impulse? **(b)** If the accelerator spacing is 6 mm, what would be the diameter of an ion beam producing 0.5 N of thrust at this accelerator voltage?

Ion thruster, e/μ = 500 coul/kg, I_s = 3000 sec, F = 0.5 N, d = 6 mm, (assume $v_i \approx 0$)

(a) From Eq 17-15, $V = (1/2)(\mu/e)v^2 = (3000 \text{ x } 9.81)^2/(2 \text{ x } 500) = 8.66 \text{ x } 10^5$ volts

(b) From Eq 17-24, $D = (F/6.18 \text{ x } 10^{-12})^{0.5}(d/V_{acc}) = (0.5/6.18)^{0.5}(6/8.66) \text{ x } 10^{-2}$

$= 1.97$ mm

6. An argon ion thruster has the following characteristics and operating conditions:

Voltage across ionizer **=** 400 V. Voltage across accelerator **=** 3 x 10^4 V

Diameter of ion source **=** 5 cm. Accelerator electrode spacing **=** 1.2 cm

Calculate the mass flow rate of the propellant, the thrust, and the thruster overall efficiency (including ionizer and accelerator). Assume singly charged ions.

(Assume $v_i \approx 0$).

$$j = \frac{4\varepsilon_0}{9}\sqrt{\frac{2e}{\mu}}\frac{V_{acc}^{3/2}}{d^2} = 3.09 \text{ x } 10^2 \text{ A/m}^2 \text{ and } I = jA = 6.06 \text{ A}$$

$$\dot{m} = \frac{\mu}{e} I = 2.56 \text{ x } 10^{-7} \text{ kg/sec}$$

$$F = 6.18 \text{ x } 10^{-12}\, V_{acc}{}^2(D/d)^2 = 9.65 \text{ x } 10^{-2} \text{ N}$$

$$\eta = IV_{acc}/I\Sigma V = 3 \text{ x } 10^4/(3 \text{ x } 10^4 + 400) = 98.7\%$$

7. For a given power source of 300 kW electrical output, a propellant mass of 6000 lbm, α **=** 450 W/kg, and a payload of 4000 lbm, determine the thrust, ideal velocity increment, and duration of powered flight for the following three cases:

(a) Arcjet: I_s **=** 500 sec η_t **=** 0.35

(b) Ion engine: I_s **=** 3000 sec η_t **=** 0.75

(c) Hall engine: I_s **=** 1500 sec η_t **=** 0.50

P_e = 3 x 10^5 W, m_p = 6000 lbm, α = 400 W/kg, and m_{pl} = 4000 lbm,

From Eqs 17-2 & 3, $F = 2P_e\eta/g_0I_s = 6.116\text{x}10^4\ \eta/I_s$

From Eqs 17-4 & 5, $m_0 = m_{pl} + m_p + P_e/\alpha$ = 2721 + 1814.4 + 666.67 = 5,203 kg

From Eq. 4-6, $\Delta u = (I_sg_0)\ln(m_0/m_f) = 7.263I_s$

From Eqs 17-2 & 3, $t_p = m_p(I_sg_0)^2/(2\eta P_e) = 0.436I_s{}^2/\eta$

ARCJET: I_s = 500 sec, η = 0.35

F = 42.85 N, Δu = 3,632 m/sec, t_p = 3.114 x 10^5 sec = 3.6 days

ION THRUSTER : I_s = 3000 sec, η = 0.75

F = 15.29 N, Δu = 21,790 m/sec, t_p = 5.232 x 10^6 sec = 60.5 days

HALL THRUSTER: I_s = 1500 sec, η = 0.5

F = 20.38 N, Δu = 10,894 m/sec, t_p = 1.962 x 10^6 sec = 22.71days

8. A formulation for the rail-accelerator exit velocity that allows for a simple estimate of the accelerator length is shown below; these equations relate the accelerator distance to the velocity implicitly through the acceleration time t . Considering a flow at a constant plasma of density ρ_m (which does not choke), solve Newton's second law first for the speed *v(t)* and then for the distance *x(t)* and show that

$v(t) = (E_y/B_z)[1 - e^{-t/\tau}] + v(0)e^{-t/\tau}$

$x(t) = (E_y/B_z)[t + \tau e^{-t/\tau} - \tau] + x(0)$

where $\tau = \rho_m/\sigma B^2_z$ and has units of seconds. For this simplified plasma model of an MPD accelerator, calculate the distance needed to accelerate the plasma from rest up to $v = 0.01(E/B)$ and the time involved. Take $\sigma = 100$ mho/m, $B_z = 10^{-3}$ tesla (Wb/m^2), $\rho_m = 10^{-3}$ kg/m^3, and $E_y = 1000$ V/m.

For constant density plasma flow we may write Newton's second law as follows

$$\tilde{F} = \rho_m \frac{dv}{dt} = \sigma B_z^2 (E_y / B_z - v)$$
$$\int \frac{dv}{E_y / B_z - v} = \frac{\sigma B_z^2}{\rho_m} \int dt = \int d\,\frac{t}{\tau}$$
$$v(t) = E_y / B_z (1 - \exp(t/\tau)) + v(0)\exp(t/\tau)$$

Now, $v = dx/dt$ so the distance $x(t)$ may be found by a second integration as

$$x(t) = (E_y/B_z)[t + \tau\exp(-t/\tau) - \tau] - x(0)$$

For this problem, $E_y/B_z = 1000/10^{-3} = 10^6$ m/sec, $\tau = 10^{-3}/(100)(10^{-3})^2 = 10$ sec

$10^{-2}\, E_y/B_z = E_y/B_z\, [1 - \exp(-t/10)]$ so $t = 0.100503$ sec

$x = 10^6 \text{ x } [0.100503 - 10 \text{ x } 0.99 - 10] = 503$ m.

[Note the importance of *significant figures* in the above numerical result.]

9. Assume that a materials breakthrough makes it possible to increase the operating temperature in the plenum chamber of an *electrothermal engine* from 3000 to 4000 K. Nitrogen gas is the propellant which is available from tanks at 250 K. Neglecting dissociation, and taking $\alpha = 200$ W/kg and $\dot{m} = 3 \times 10^{-4}$ kg/sec, calculate the old and new Δu corresponding to the two temperatures. Operating or thrust time is 10 days, payload mass is 1000 kg, and $k = 1.3$ for the hot diatomic molecule.

$$P_e = \dot{m} c_p \Delta T = \dot{m}\left(\frac{k}{k-1}\frac{R'}{\mathrm{M}}\right)\left(T_{hot} - T_{cold}\right) = 3 \text{ x } 10^{-4}(1.3/0.3)(8314.3/28)(T_{hot} - 250)$$

P_e = 1.061 kW at 3000 K (old) and 1.447 kW at 4000 K (new)

m_p = 3x10^{-4} x (10 x 24 x 3600) = 259 kg

$m_{pp} = P_e/\alpha$ = 5.305 kg (old) and 7.235 kg (new)

$m_0 = m_p + m_{pp} + m_{pl}$ = 1,264.3 kg (old) and 1,266.2 kg (new)

$$v = \sqrt{\frac{2P_e}{\dot{m}}} = 2{,}660 \text{ m/sec (old) and } 3{,}105 \text{ m/sec (new)}$$

$\Delta u = v \ln(m_0/m_f)$ = 610 m/sec (old) and 711 m/sec (new)

10. An arcjet delivers 0.26 N of thrust. Calculate the vehicle velocity increase under gravitationless, dragless flight for a 28-day thrust duration with a payload mass of 100 kg. Take thruster efficiency as 50%, specific impulse as 2600 sec, and specific power as 200 W/kg. This is not an optimum payload fraction; estimate an I_s which would maximize the payload fraction with all other factors remaining the same.

a) $m_p = (F/g_0 I_s)t_p$ = (1.02 x 10-5)(28 x 24 x 3600) = 24.69 kg

$P_e = ½(F/g_0 I_s)v^2/\eta_t$ = 6.64 kW and $m_{pp} = P_e/\alpha$ = 33.2 kg

$m_0 = m_{pl} + m_p + m_{pp}$ = 157.89 kg and m_f = 133.2 kg

$\Delta u = v \ln(m_0/m_f)$ = 4.34 x 10^3 m/sec

b) m_0/m_{pl} = 1.58 or m_{pl}/m_0 = 0.633

From Fig. 17-3, $v/v_c\ (opt) \approx 0.9$ [assuming that tankage fraction φ = 0]

$$I_s(opt) = I_s^* \approx \frac{0.9}{9.81}\sqrt{2\alpha t_p \eta_t} = 2{,}020 \text{ sec}$$

So, decrease I_s to about 2020 sec to *optimize* payload fraction. Verify with Eq. 17-9.

11. A *patent application* describes an electrostatic thruster that accelerates *electrons* as the propellant. The inventor points out that the space–charge limited thrust is independent of the propellant mass and that electrons are very easy to produce (by cathode surface emission) and much easier to accelerate than atomic ions. Show using the basic relationships for electrostatic thrusters given in this chapter that electron acceleration is impractical for any such thruster. Assume that the required thrust is 10^{-5} N per accelerator hole, that there are several thousand holes of "*aspect ratio*" D/d **=** 1.0 in the accelerator, and that the neutralizer operates with protons (which have a mass 1836 times that of the electron).

a) $V_{acc} = [F/6.18\text{x}10^{-12}]^{1/2} = 1.27 \text{ x } 10^3$ Volts (Eq. 17-24 with D/d =1.0)

b) Electron current per hole:

$$I = Aj = \left(\frac{\pi}{4}D^2\right)\left(\frac{4\varepsilon_0}{9}\sqrt{\frac{2e}{\mu}}\frac{V_{acc}^{3/2}}{d^2}\right) = \frac{8.85\text{x}10^{-12}\pi}{9}\sqrt{\frac{2\text{x}1.6\text{x}10^{-19}}{9.11\text{x}10^{-31}}}\left(1.27\text{x}10^3\right)^{3/2}\left(\frac{D}{d}\right)^2$$

$I = 8.31 \text{ x } 10^{-3}$ Amps/hole (which is a very substantial amount of current).

c) Conservation of linear momentum in neutralizer region (idealized case):

$\mu v_{electron} = (\mu + m_{proton})v_{mix}$ or $v_{mix} \approx v_{electron}/1822$

$v_{electron} = \sqrt{2eV_{acc}/\mu} = 2.11 \text{ x } 10^7$ m/sec (non-relativistic)

$v_{mix} \approx 2.11 \text{ x } 10^7/1822 = 1.16 \text{ x } 10^4$ m/sec or I_s = 1,182 sec (exit beam value)

d) Vast amounts of momentum have been lost in accelerating the protons, and beam losses during mixing would likely be catastrophic. Also, sourcing such a large proton neutralizing current will be quite difficult in practice. So perhaps this inventor should give consideration to neutralizing with *positrons* (pending several technological breakthroughs). All of this should be revisited for the case when more energetic/massive relativistic electrons are present.

12. For each of the three thrusters in Example 17-4, calculate the thrust F and input power P_e that would apply for a payload mass m_{pl} of 100 kg. What would result if the spacecraft power supply is limited to 30 kW but mission time could extend up to 100 days?

Using Eqs. 17-3 to 17-6 we can arrive at the following,

$$F = \frac{m_0 - m_{pl}}{\dfrac{t_p}{g_0 I_s} + \dfrac{g_0 I_s}{2\alpha\eta_t}} \quad \text{and} \quad P = \frac{g_0 I_s}{2\eta_t}F$$

	I_s (sec)	α (W/kg)	η_t	t_p (sec)	m_0 (kg)	F (N)	P_e (kW)
Arcjet	800	295	0.32	$1.296\text{x}10^6$	255.8	0.754	9.24
Hall	1983	366	0.57	$0.588\text{x}10^6$	233.1	1.73	29.6
Ion	2800	278	0.50	$0.864\text{x}10^6$	294.1	1.49	40.9

The power supply being limited to 30 kW will only affect the Ion engine. According to Fig. 17-13, longer mission times drive results toward a more optimum specific impulse but at the expense of a lower payload fraction.

EXTRA PROBLEM

17A. An ion thruster using argon is being designed to lift nano-satellites into higher orbits. This thruster design is rated 70% efficient at I_s = 4000 sec and 80% efficient at I_s = 6000 sec. When φ = 0.1 and α = 100 W/kg such propulsion system should be able to deliver a Δu = 15,000 m/sec during a 230-day thrusting schedule. For each specific impulse:

a) Calculate the thrust and the required accelerator voltage for an accelerator unit with a 5000-hole electrode aperture grid of aspect ratio (D/d) = 2.0.

b) Calculate the required power-input Pe.

c) Calculate the total vehicle mass and the payload mass. (Assume $v_i \approx 0$ and $v = c$).

$t_p = 1.5\text{x}10^4$ sec, Δu = 15,000 m/sec, φ = 0.1 and α = 100 W/kg, $\mathfrak{M}$ = 39.948 kg/kg-mol

For each accelerator hole, from Eqs 2-15 and 17-15, $V_{acc} = (I_s g_0/13800)^2\text{x}39.948 = 2.02\text{x}10^{-5}\boldsymbol{I_s}^{\mathbf{2}}$

For 5000 holes, from Eq. 17-24, $F_{TOT} = 5000\text{x}6.18\text{x}10^{-12}\text{x}V_{acc}{}^2\text{x}(2.0)^2 = 1.24\text{x}10^{-7}V_{acc}{}^2$

The power input is found from Eq. 17-3, $P_e = Fg_0I_s/2\eta_t = 4.91FI_s/\eta_t$

For the payload ratio use Fig. 17-13 with the appropriate $(\Delta u/v_c, v/v_c)$

From Eqs 17-4 1nd 17-6, $\boldsymbol{m_0 = m_{pl} + m_p(1.1 + v/v_c{}^2)}$ **and from Eq. 2-14** $\boldsymbol{m_p = Ft_p/v}$

I_s = 4000 sec, η_t = 70%	I_s = 6000 sec, η_t = 80%
V_{acc} = 323 V	V_{acc} = 720 V
F_{TOT} = 12.9mN	F_{TOT} = 64. 3 mN
P_e = 0.34 kW	P_e = 2.37 kW
m_{pl}/m_0 = 0.45 with (0.291, 0.744)	m_{pl}/m_0 = 0.48 with (0.273, 1.04)
m_0 =16.6 kg and m_{pl} = 10.8 kg	m_0 =19.0 kg and m_{pl} = 9.85 kg

[These results show that operation at the lower specific impulse is more attractive. At 4000 sec there is a payload gain of one kilogram with 2.4 kg less total mass and seven times less power required. The highly non-linear nature for getting at the individual numbers is one reason why higher specific impulses become detrimental even when the thruster efficiencies are better.]

CHAPTER 18

When solving problems in thrust vector control, information regarding the *center of mass* of the vehicle and either the *centroidal radius of gyration* or the appropriate *mass moment of inertia* should be deducible from the geometry of the rocket vehicle or should be given explicitly. Many elementary books in dynamics carry useful tables in this regard (see, for example, J. L. Meriam, L. G. Kraige, and J. N. Bolton *Engineering Mechanics: Dynamics*, 8th edition, John Wiley and Sons, Inc., Hoboken, NJ, 2015). Note also that in most chemical propulsion systems, as the propellant is consumed the *center of mass* moves along the vehicle axis during operation.

1. A single-stage weather sounding rocket has a takeoff mass of 1020 kg, a sea-level initial acceleration of 2.00 g_0, carries 799 kg of useful propellant, has an average specific gravity of 1.20, a burn duration of 42 sec, a vehicle body shaped like a cylinder with an *L/D* ratio of 5.00 with a nose cone having a half angle of 12°. Assume the center of mass does not change during the flight. The vehicle tumbled (rotated in an uncontrolled manner) during the flight and failed to reach its objective. Subsequent evaluation of the design and assembly processes showed that the maximum possible thrust misalignment of the main thrust chamber was 1.05° with a maximum lateral off-set *d* of 1.85 mm. Assembly records show it was 0.7° and 1.1 mm for this vehicle. Since the propellant flow rate was essentially constant, the thrust at altitude cutoff was 16.0% larger thanat takeoff. Determine the maximum torque applied by the main thrust chamber at start and at cutoff, without operating any auxiliary propulsion system. Then determine the approximate maximum angle through which the vehicle will rotate during powered flight, assuming no drag. Discuss the results.

Unfortunately there is missing information needed to calculate the moment of inertia and center of mass of combined cylinder/nose-cone. Vector notation below:

Thrust at takeoff = 30,019 N, thrust at burnout = 34,822 N (from Eqn. 4-21)
$\boldsymbol{T} = \boldsymbol{r} \times \boldsymbol{F}$, where $T = I\alpha = \dot{H} = I\ddot{\theta}$; here H = angular momentum (kg-m^2/sec), I = mass moment of inertia (kg-m^2), α = angular acceleration (rad/sec^2), and θ = angular displacement (rad).

2. A propulsion system with a thrust of 400,000 N is expected to have a maximum thrust misalignment θ of $\pm$ 0.50° and a horizontal off-set *d* of the thrust vector of 0.125 in. as shown in the following sketch. One of four small reaction control thrust chambers will be used to counteract the disturbing torque. What should be its maximum thrust level and best orientation? Distance of vernier gimbal to center of mass (CM) is 7 m. [See sketch in textbook]

Given F = 40,000 N, worst angular misalignment 0.5°, worst horizontal offset d = 0.125 in., D = 1m,
$\sin\theta = \tan\theta = 0.008726$ so moment arm = 6x0.008726 + 0.125x0.00254 = 0.055 m
Turning moment = 40,000x0.055 = 22,000 N-m

When the thrust direction of the vernier is at right angles to the vehicle axis then the adjustment thrust is minimal. Vector notation below:

$$22{,}000 = \boldsymbol{F}_{\mathbf{v}} \times \boldsymbol{\ell} = F \times (6+1) \quad \text{so that } F_v = 3143 \text{ N or } 714 \text{ lbf}$$

If the vernier or adjustment thrust is along the surface edge of vehicle, then ℓ = 0.5 m and F_v = 44,000 N which would be prohibitive

CHAPTER 20

1. List at least two parameters that are likely to increase total radiation emission from plumes, and explain how they accomplish this. For example, increasing the thrust increases the radiating mass and size of the plume.

Parameters that increase radiative emission:
1) Increase of thrust enlarges plume and increases volume of radiating gas mass.
2) Adding small particles to a hot plume. Solid particles emit a broad-band spectrum of radiation frequencies (UV, visible and IR). Emission intensity usually peaks in the IR region but the visible region is sufficiently strong to be measured.
3) Add a substance that is a strong radiant emitter, like sodium. Even in small quantities, it colors the plume a bright yellow.
4) At low altitudes the plume mixes with the air at the plume boundary. The oxygen in the air may cause further combustion in fuel-rich exhaust plumes thus enlarging the radiating mass.
5) Multiple nozzles can give a larger body of hot gas than a single nozzle. At the interface between plumes the radiation is enhanced by impingement.

2. Look up the term *chemiluminescence* in a technical dictionary or chemical encyclopedia; provide a definition and explain how it can affect plume radiation.

CHEMILUMINESCENSE is radiation emission in the visible range cause by chemical reactions or by the formation of new compounds, and not necessarily accompanied by heat (not of thermal origin, i.e., not given by Eq. 8-26). In the colorless plume of LOX-LH_2 (major gases in exhaust are H_2O, H_2, and O_2), chemiluminescent reactions make the shock waves visible. It is the 'minor species' such as H, OH, and O^- that emit chemiluminescense with some radiation in the visible.

3. Describe the changes of total radiation intensity from a plume of a vertically ascending rocket vehicle launched at sea level as seen by an observer also at sea level, but at a distance of 50 km away from the launch location.

In the mixing layer (see Fig. 20-1) atmospheric oxygen mixes and burns with the fuel-rich plume gases creating strong radiant emissions. As altitude is gained, the plume spreads resulting in increased radiation. At altitudes beyond about 40 km, air densities are very low and there is very little oxygen to burn with the fuel rich exhaust plume so that temperatures in the mixing layer diminish and so does the radiant emission (see Fig. 20-3). Above about 80 km, there is essentially no atmosphere and the mixing layer disappears so that the plume radiation diminishes being only that from the intrinsic core (which is at a lower temperature).

4. A rocket vehicle flies horizontally at 25 km altitude. Its plume radiation emission is measured by an instrument in the gondola of a balloon 50 km away, but at the same altitude of 25 km. At this location the air has a very low density and plume radiation absorption by the air is small and may be neglected. Describe any corrections that need to be applied to the measurements in order to determine the plume's radiant emission.

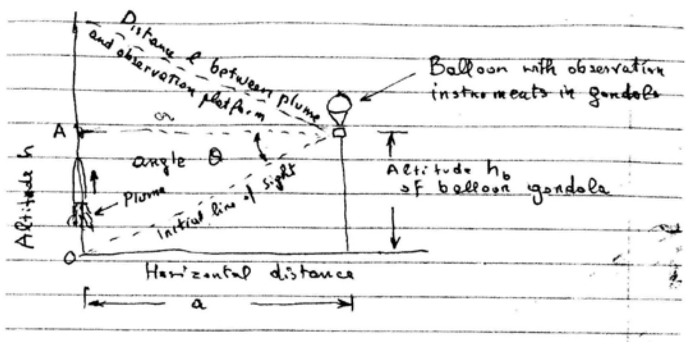
As long as the thrust and exhaust gas temperatures remain essentially constant, there will be no change in plume size or radiation emission. Three adjustments need to be made to the measured radiation: (1) Intensity diminishes as the square of the distance between the plume and the observation station (considered to be stationary); (2) the radiation intensity also diminishes with the cosine of the angle between the plume centerline and the gondola; (3) in addition to distance, the energy measured depends on the instrument's aperture, color sensitivity and field of view. The total radiation from the plume is always much larger than that intercepted by the instrument. .

EXTRA PROBLEM

20A. If a high-altitude plume is seen from a high-altitude balloon, its apparent radiation intensity diminishes with the square of the distance between the plume and the observation platform and as the cosine of the angle of the flight path tangent with the line to the observation station. Establish your own trajectory and its relative location to the observation station. For a plume of an ascending launch vehicle, make a rough estimate of the change in the relative intensity received by the observing sensor during flight. Neglect atmospheric absorption of plume radiation and assume that the intensity of emitted radiation stays constant.

To further simplify the problem make the following assumptions:
(a) The vehicle flight is entirely vertically up.
(b) The balloon does not move during the vehicle's powered flight.

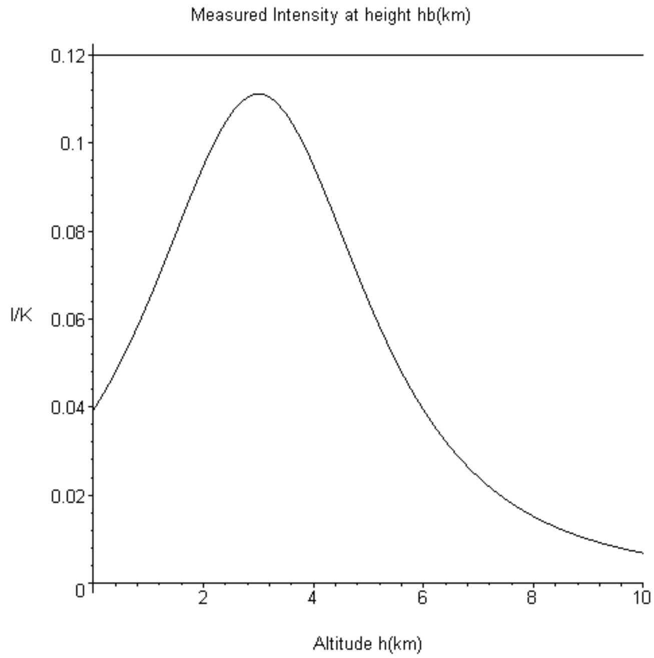
See sketch below:



The measured intensity will vary as cos(θ) and inversely with the square of the distance ℓ between the plume and the observation instrument. K is a constant.

$$I = K\cos(\theta)/\ell^2 = K\frac{\cos(\theta)}{\sqrt{a^2+(h_b^2-h^2)}} = \frac{Ka}{[a^2+(h_b^2-h^2)]^{\frac{3}{2}}}$$

At launch (at point O), θ equals the angle whose cos(θ) equals the altitude of the balloon h_b divided by the distance ℓ. The observed intensity is a maximum when the vehicle reaches elevation A (where cos(θ) = 1 and $h = h_b$) where the distance ℓ is a minimum. As the vehicle gains further altitude the value of the cosine diminishes and the length ℓ increases. A plot of the plume intensity versus altitude would look like:



It is assumed above that a = 3 km and h_b = 3 km. The intensity at the ground is I/K = 0.0303. The maximum intensity of the plume occurs when $h = h_b$ = 3 km, namely, I/K = 0.111. As h increases further the intensity goes to zero as indicated in the above figure.