An Introduction to the LONGITUDINAL STATIC STABILITY OF LOW-SPEED AIRCRAFT

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Foreword

THERE have been two sources of inspiration in writing this book: the teaching of stability and control at Imperial College and the practical stimulus of taking part, in an amateur fashion, in the design and flight testing of gliders. It seemed, on both counts, that there was a demand for a book devoted solely to static longitudinal stability, written less tersely than the original references and trying to include more physical explanation than is usually given in more comprehensive works, where static stability is often only the prelude to an analysis of the dynamics of flight. The standard is roughly that of the second year of a three-year degree course in aeronautics in the U.K., although it includes some additional details. It is also aimed at those concerned with the design and flight testing of low-speed aircraft. For reasons explained in the first chapter, the effects of compressibility and structural distortion have been excluded from most of the theory except that non-linear trim curves are considered in some detail. The reader is assumed to have a knowledge of mathematics which encompasses simple differentiation and integration and to be familiar with the simpler concepts of the mechanics of flight.

My thanks are due to the late Professor H. B. Squire who, despite my qualms, firmly thrust me into the teaching of stability; to successive generations of students at Imperial College, whose ability to detect any obscurity in the minds of their tutors must be unsurpassed; to Mr. J. L. Stollery, who offered valuable criticism on parts of the initial draft; to Messrs. J. A. A. Shaw and M. D. Hodges of the staff of the Royal Aeronautical Society who read the whole manuscript most carefully and suggested many real improvements; to Mrs. S. List, who typed the manuscript; to Mr. F. N. Slingsby and the staff and Design Advisory Panel of

Slingsby Sailplanes who unwittingly provided inspiration and kept me in touch with reality, and who make a very real contribution to the sum of human happiness; and, finally, to the publishers for the help, encouragement and understanding of what is involved in producing a technical work. Errors, omissions and obscurities are solely my responsibility.

Finally, I would suggest that the first seven chapters, with the appropriate parts of Appendix III, would suffice for a first reading.

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CHAPTER 1

The Concept of Static Stability

IN CONSIDERING the stability of a manually controlled aeroplane —or any other vehicle for that matter—the device to be examined is a combination of a machine and a man. To an external observer, its behaviour under the influence of disturbances and when carrying out manœuvres is a consequence of the characteristics of both components, and the most general analysis of its behaviour must take into account both the characteristics of the machine and those of the man who, for this purpose, can be regarded as an exceedingly complex servo-mechanism. Such analyses are indeed carried out, but for vehicles which do not generally extend the man to the limit of his capabilities, a rather simpler approach suffices. Over the years, experience has been accumulated which enables the significant characteristics of the aeroplane to be singled out and defined in quantitative terms[2] so that when it is controlled by a man, he will find it at least satisfactory and safe and, at the best, entirely delightful to fly. Some of these significant characteristics relate to the behaviour of the aeroplane when subjected to disturbances under specified conditions and to the actions required of the pilot when controlling it under certain conditions. As will be seen later, these aspects of the behaviour of the aeroplane are interdependent.

At the beginning of controlled flight, the Wright brothers were generally aware of this situation. They appreciated that a special sort of skill would be involved in flying an aeroplane and in this they differed from Sir George Cayley, who was reputed to have delegated the task of piloting to his coachman. But some of the Wrights' remarks suggest that they were not particularly interested in stability: "The balancing of a gliding or flying machine is very simple in theory. It merely consists in causing the centre of pressure to coincide with the centre of gravity." Indeed, they were inclined to feel that a stable aeroplane would be subject to oscillations. In fact, they were quite correct, but they were hardly to know that the oscillations need not be a source of concern: as a consequence, they must have enhanced their difficulties considerably by flying machines which were sometimes unstable.

It was apparent to early investigators that an aeroplane is free to move in translation and rotation about three axes, and hence requires six equations to describe its motions even when regarded as a rigid body. Writing these equations is a relatively straightforward matter if the aerodynamic forces and moments acting on the aeroplane can be expressed in a suitable manner, and much of this work had been done by the end of the First World War almost in the form in which it is used today, notably by Sir B. Melvill Jones and others at Farnborough, based on Bryan's ideas of 1911.[(4)] However, it is one thing to write such equations and quite another to be able to calculate the aerodynamic forces and moments in a particular case. An appreciable period of development, of assessing which factors were important and of comparing theory with the actual behaviour of aeroplanes, was required before these equations could be applied as a routine matter in aeroplane design. This approach is clearly a comprehensive one: it seeks to describe in some detail the dynamic behaviour of an aeroplane in various circumstances. It will, for example, permit the calculations of the oscillatory motion which occurs when the aeroplane is disturbed in pitch.

One consequence of this body of theory is that it shows that if the aeroplane is symmetrical about a vertical plane through its centre of gravity when flying with wings level, and if the disturbances are small, the behaviour in pitch can be divorced from the other motions. It should be remembered that whilst aeroplanes usually look symmetrical, rotation of the slipstream (in the case of propeller-driven aircraft), gyroscopic engine torques and deflections of the rudder and ailerons all introduce a certain amount of asymmetry. These effects are usually small from the present point of view.

It also emerged that, whilst it is useful to be able to consider the detailed behaviour in pitch, several important features of its behaviour may be related to rather simpler "static" concepts. The type of concept involved is as follows: an aeroplane is flying steadily, in equilibrium, when it is subjected to a small disturbance in pitch. Does the moment acting on it in the disturbed state *tend* to cause it to return to the equilibrium state? If it does, the aeroplane is said to be statically stable under these conditions. Such information, it must be stressed, only indicates a *tendency* to return to an equilibrium state: it does not indicate the motion which occurs as the aeroplane tries to achieve the equilibrium state. If it does so in a "dead beat" fashion, or by way of a damped oscillation, the aeroplane is both dynamically and statically stable. If the tendency to achieve equilibrium leads to an oscillation of increasing amplitude about the equilibrium state, it is statically stable but dynamically unstable. In general, it is desirable that it should be completely stable, both statically and dynamically, so it seems that a study of its static stability only considers part of the problem, and the full equations of motion must be invoked in order to consider the situation as a whole. This indeed is true, but it is also clear that static stability is a prerequisite for complete stability, and is thus quite important in its own right. It is customary at this juncture to draw diagrams showing statically stable, neutrally stable and statically unstable arrangements of a simple type, such as a billiard ball inside a hemispherical bowl, resting on a flat surface, and balanced on top of an inverted hemispherical bowl respectively. No doubt the reader has sufficient imagination to be able to visualize the scene. It is important to appreciate at this stage that whilst such pictures illustrate particular situations involving static stability, they are not strictly analogous to the aeroplane situation. In the case of the billiard ball quoted above, the constraints are such that an out-of-balance configuration can be described by a single equation. In general, an aeroplane is not constrained in this fashion, and under such circumstances an out-of-balance situation in pitch involves three equations relating to the pitching moment and vertical and horizontal components of force. In order to achieve simplicity it is usual to make some assumption about the forces: that they are always in equilibrium or that the vertical forces are in equilibrium and the engine setting is left unaltered during disturbances. These conditions are really equivalent to defining a particular type of disturbance, such that only the out-of-balance moment need be considered in detail. This theme is explained later. It will also be found then that the concepts used to describe the static stability are related to actions required of the pilot in changing the conditions of flight from one equilibrium state to another. He is, in fact, quite aware of the consequences of static stability, or lack of it, although it is not presented to his senses in quite the fashion that the initial concepts might lead one to suppose. Concepts such as "static margin" ultimately emerge as stick-travel and stick-force to change speed from one equilibrium condition to another.

A detailed consideration of the concepts used to define static stability shows that with the restrictions applied in order to keep the mathematics simple and the ideas reasonably general, they do in fact represent a rather simplified statement of even the tendency of the aeroplane to return to its equilibrium state when disturbed. This matter is considered in more detail in Appendix I, which is best kept for a second reading. But a consequence of this approach is that the concepts involved only give a sort of idealized indication of a tendency to seek the equilibrium state. They are best regarded not so much as an indication of the possible behaviour of the aeroplane following a disturbance but strictly as a guide to the pilot's actions mentioned at the end of the previous paragraph.

It may seem that a study of static stability has become rather remote from the behaviour of a real aeroplane under practical conditions. This is partly so: what has happened is that certain rather restricted ideas have been extracted from a large body of more complicated theory. These ideas turn out to be important in themselves because they are a preliminary to more detailed considerations and because they are directly relevant to some real aspects of flying an aeroplane.

A slightly curious historical consequence of this approach is that longitudinal static stability theory was developed after the more general dynamic theory. It became prominent during the later 1930s and was greatly elaborated during the Second World War.[7, 8, 9, 10]

In general, the theory given here is restricted to the simple case in which the speed of the aeroplane is low enough for the effects of compressibility to be negligible. The effects of structural distortion are also neglected. The object of doing so is to attempt to present the subject in such a fashion that it can be understood in terms of physical as well as mathematical concepts. To plunge straight into the more elaborate theory[9] is to risk obscuring the principles in a welter of partial derivatives, so that physical explanation becomes inordinately lengthy. But an extension of the theory given here to the more general situation should represent a fairly straightforward step. It is hoped that such an extension will be made easier by presenting not so much the simpler theory which was developed first in the historical sense, but a simplified version of the more comprehensive later theory which is mathematically more rigorous.

It should not be thought that the criteria developed in the later chapters describing certain actions required of the pilot (e.g. stick-force to change speed, stick-force per g) are the only longitudinal characteristics of an aeroplane which influence its flying qualities as subjectively assessed by skilled pilots. Dynamic effects outside the scope of this book are also involved, often in quite a subtle fashion, so that assessment of a pilot's impressions in terms of aeroplane characteristics may not be easy.

CHAPTER 2

Preliminary Considerations and Definitions

AS THE previous chapter indicates, static stability is concerned with the out-of-balance moment acting on an aeroplane when it is disturbed from an equilibrium state, subject to certain provisions about the forces acting on it. It is therefore necessary to consider the forces and moments acting on an aeroplane, particularly on the wings and tail, examining some of their properties and showing how they are conveniently expressed in mathematical terms.

Properties of Wing Sections

Consider first the simple case of a two-dimensional wing (i.e. a wing extending to infinity at right angles to the plane of the paper in Fig. 2.1), immersed in the flow of a real fluid. In general, considering a portion of area S, a force R will act upon it through a point on the chord-line called the centre of pressure. Definitions of chord-line vary slightly, depending on the way in which the geometry of the wing section is described, but to a sufficient degree of accuracy it can be regarded as the line joining the extremities of the section. It is conventional to resolve R into components at right angles, and two fairly obvious possibilities present themselves: the direction of one component could be either along the chord-line or along the direction of the free stream (i.e. the direction of the flow at a point remote from the wing, where the effects of the wing on the local direction of the flow are negligible).

For most purposes it is usual to employ the latter convention. The component along the free-stream direction is called the drag,

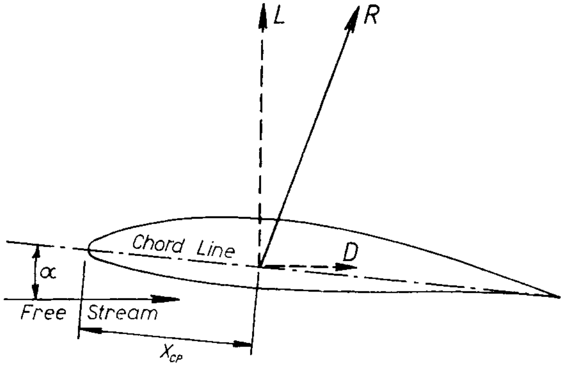

FIG. 2.1. Resolution of the force acting on a two-dimensional aerofoil.

and the component at right angles is called the lift. For a wing of a given chord in a stream of a certain fluid at a given velocity and density, the lift, drag and centre of pressure position will all be functions of the incidence. The incidence, from the geometrical point of view, is the angle between the free-stream direction and the chord line; it will later be found convenient to use a slightly different definition.

In stability calculations it would be somewhat inconvenient to deal with a variable distance defining the location of the centre of

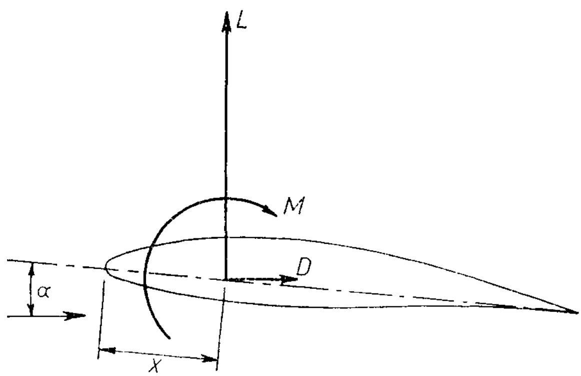

FIG. 2.2. Force components and moment referred to an arbitrary point on the chord line.

pressure. Suppose that a fixed point is chosen at a distance x aft of the leading edge. Then, by elementary statics, the lift and drag forces acting at the centre of pressure x_{CP} aft of the leading edge are equivalent to the same forces acting at the fixed point x aft of the leading edge together with a "pitching" moment M (see Fig. 2.2), such that by taking moments about the c.p.,

$$-M = (L \cos \alpha + D \sin \alpha)(x_{CP} - x). \tag{2.1}$$

If α is sufficiently small to assume

$$\cos \alpha = 1, \qquad \sin \alpha = \alpha,$$

this may be written

$$-M = (L + D\alpha)(x_{CP} - x), \tag{2.2}$$

and since D is usually small compared with L, $D\alpha$ can also be neglected, so finally

$$-M = L(x_{CP} - x). \tag{2.3}$$

The minus sign is inserted because M is conventionally positive in the nose-up sense, clockwise in Fig. 2.2. Under the assumed conditions, M is also a function of α.

Now, in general, L, D and M will be functions of

$$\rho,\ V,\ b,\ c,\ \mu,\ \mathbf{a},\ \alpha. \quad \text{(See list of symbols.)}$$

In the case of the two-dimensional wing under consideration, it is assumed that L, D and M are associated with an arbitrary span b, and the corresponding wing area S is then bc.

It follows from dimensional analysis[13] that if

$$L = f(\rho, V, b, c, \mu, \mathbf{a}, \alpha),$$

then

$$\frac{L}{\frac{1}{2}\rho V^2 S} = F\left(\frac{V}{\mathbf{a}}, \frac{\rho V c}{\mu}, \alpha\right). \tag{2.4}$$

The $\frac{1}{2}$ is inserted in the denominator on the left-hand side because $\frac{1}{2}\rho V^2$ is the dynamic head of the flow. All the terms in (2.4) are now dimensionless.

$\dfrac{L}{\frac{1}{2}\rho V^2 S}$ is called the lift coefficient, symbol C_l;

$\dfrac{V}{\mathbf{a}}$ is called the Mach number, symbol $\mathbf{M}$;

$\dfrac{\rho V c}{\mu}$ is called the Reynolds number, symbol $\mathbf{R}$.

Equation (2.4) may therefore be written:

$$C_l = \frac{L}{\frac{1}{2}\rho V^2 S} = F(\mathbf{M}, \mathbf{R}, \alpha). \tag{2.5}$$

Coefficients of drag and moment may be defined in an analogous fashion, remembering that the moment will have to be divided by $\frac{1}{2}\rho V^2 S \times$ (a length) in order to make it dimensionless. The length used is the chord.

Hence

$$C_d = \frac{D}{\frac{1}{2}\rho V^2 S}, \tag{2.6}$$

$$C_m = \frac{M}{\frac{1}{2}\rho V^2 Sc}, \tag{2.7}$$

and both C_d and C_m will be functions of $\mathbf{M}$, $\mathbf{R}$ and α.

The theory which follows is specifically related to conditions in which speeds are sufficiently low for compressibility effects to be negligible. Under such circumstances, the dependence of the above coefficients on $\mathbf{M}$ ceases to be significant, and they may be regarded as functions of $\mathbf{R}$ and α only. It will also be assumed that $\mathbf{R}$ is reasonably high (although in practice it may be of the order of 10^6 for low-speed aircraft near the stall), at any rate to the extent that the above coefficients vary only slowly with $\mathbf{R}$, so that in the processes of differentiation involved in the theory, partial derivatives with respect to $\mathbf{R}$ need not be included. Under some circumstances, e.g. when considering stability near the stall, it may be necessary to carry out more detailed calculations than are indicated here to take Reynolds number effects into account.

It also follows from the above definitions of the coefficients and from eqn. (2.3) that

$$-C_m = C_l\frac{(x_{CP}-x)}{c}. \tag{2.8}$$

Now according to thin aerofoil theory,[1] if $x/c = \frac{1}{4}$ in eqn. (2.8), the corresponding moment coefficient will be independent of C_l and will depend on the camber. For wing sections with positive camber, as sketched in Fig. 2.2, this constant moment coefficient will be negative and zero for symmetrical sections. Substituting in eqn. (2.8):

$$-C_{m_{c/4}} = C_l\left(\frac{x_{CP}}{c}-\frac{1}{4}\right) = \text{const.} \tag{2.9}$$

Since this expression also applies when $C_l = 0$, it follows that under these circumstances, $x_{CP} = \infty$. In other words, at zero lift, a cambered wing section is subjected to a pure couple.

As would be expected, this situation no longer applies exactly to a real wing section of finite thickness in a real fluid flow. Experiment shows that there still exists a value of x/c such that the corresponding moment coefficient is substantially independent of C_l. This point, for which

$$\frac{\mathrm{d}C_m}{\mathrm{d}C_l} = 0,$$

is called the *aerodynamic centre* of the wing section. The corresponding pitching moment coefficient is denoted by the symbol C_{m_0}. It follows from eqn. (2.8) that the position of the centre of pressure is given by

$$x_{CP} = x_{ac} - \frac{C_{m_0}}{C_l}c. \tag{2.10}$$

The centre of pressure of a section with positive camber therefore moves further forwards as C_l increases. The aerodynamic centre under practical conditions is in general quite close to the theoretical quarter-chord position. For a thick section, a typical position would be 0·27c. It should be emphasized that, for real wing sections, the aerodynamic centre position is only constant when the relevant section characteristics behave in the linear fashion assumed above. For example, the aerodynamic centre position is no longer constant near the stall when the C_l–α curve becomes non-linear. At this juncture it is convenient to summarize the definitions of centre of pressure and aerodynamic centre.

The centre of pressure is implicitly defined above by the condition $M = 0$. Its position, in the absence of compressibility effects, is a function of incidence and Reynolds number.

The aerodynamic centre is defined by the condition

$$\mathrm{d}C_m/\mathrm{d}C_l = 0.$$

For a given section, its position is independent of C_l and hence of α but varies slightly with Reynolds number. The pitching moment coefficient about the aerodynamic centre will be a function of Reynolds number only, and in practice varies very little with **R**.

The characteristics of wing sections may therefore be presented as curves of C_l, C_d and C_{m_0} against α at a series of Reynolds numbers. Typical curves are shown in Fig. 2.3. It will be seen that C_{m_0} is indeed constant, and that over much of the range of values considered, C_l is a linear function of α. Again, this linearity is predicted by thin aerofoil theory, which indicates that the slope of the C_l–α curve should be slightly more than 2π if the incidence is in radians. For actual sections in a low-turbulence wind tunnel, the slope is generally quite close to 2π: for thick low-drag sections it may be 2 or 3 per cent greater. The variation of C_d with α depends on the behaviour of the boundary layer. C_d is substantially constant for many modern aerofoils over a certain range of lift coefficient.

It will also be noted that, for a cambered wing as shown in the diagram, the lift is not zero when the geometric incidence is zero. Zero lift corresponds to a negative geometric incidence. In the theory of static stability it is convenient to measure incidence from the zero-lift datum and the symbol α will in future be used in this context. This is equivalent to adopting as the reference for measuring incidence the *zero-lift line* of the wing, so that α

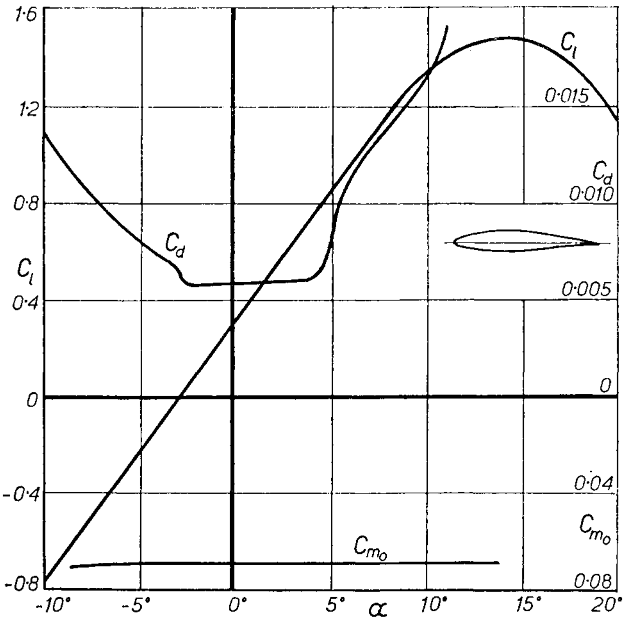

FIG. 2.3. Characteristics of NACA 64_2–215 aerofoil section at a Reynolds number of 3×10^6.

becomes the angle between this line and the free-stream direction (Fig. 2.4). With this convention,

$$C_l = \frac{\partial C_l}{\partial \alpha} \cdot \alpha, \tag{2.11}$$

since $\partial C_l/\partial \alpha$ is constant at a given Reynolds number.

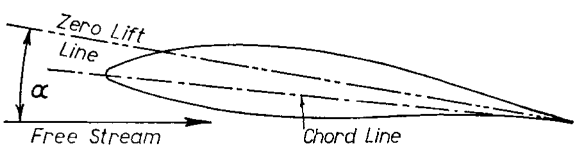

FIG. 2.4. Angle of incidence measured from zero-lift datum.

Wings of Finite Span: Mean Chord and Mean Aerodynamic Centre[11, 12]

All the foregoing relates strictly to wings of infinite span, whereas actual wings are neither infinite nor of constant chord. Moreover, they are often twisted, so that the zero-lift lines at various spanwise positions are not in a single plane, and it is quite usual for the wing section to vary in shape over part or all of the semi-span. The reasons for using such a complicated geometry and the methods of calculating the characteristics of such a wing as functions of incidence are beyond the scope of this book. (Reference 1 describes some of the relevant theories.) For a finite wing, the lift, drag and pitching moment coefficients may be defined in a similar fashion to those given above, but it now

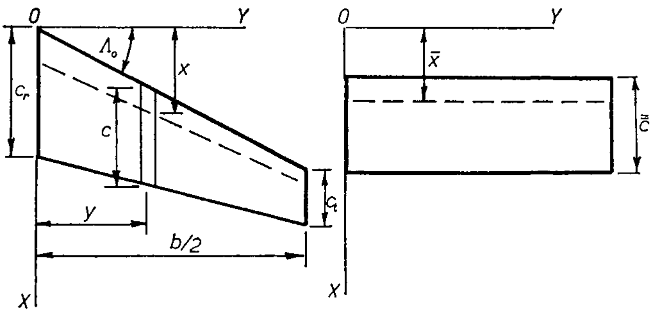

FIG. 2.5. Notation for mean chord calculations.

becomes necessary to define a mean aerodynamic centre at which L and D act, together with an appropriate moment such that its coefficient is independent of the lift coefficient. Moreover, the pitching moment coefficient must be based on a suitable chord.

Given the geometry and local aerodynamic properties of a real wing, an equivalent unswept rectangular wing of the same area is to be defined so that both wings have the same properties so far as lift and pitching moment are concerned.

Figure 2.5 shows, on the left, half of the real wing. It is assumed to be tapered, twisted, swept, and of varying section. For convenience, it is shown as being straight-tapered, with the local aerodynamic centres lying on a straight line, but in general c and x are not necessarily linear functions of y. On the right is shown half of the equivalent untwisted rectangular wing. Both wings are of area S.

If the local lift coefficient of the real wing at any spanwise station y is C_l, and if the overall lift coefficient of the equivalent wing is C_{L_w}, equating lifts and dividing by $\frac{1}{2}\rho V^2$ gives

$$2\int_0^{b/2} C_l c \, dy = C_{L_w} S. \tag{2.12}$$

Since the areas of the real and equivalent wings are the same, it also follows that

$$2\int_0^{b/2} c \, dy = S. \tag{2.13}$$

Now C_l normally varies across the span (i.e. at a given overall C_{L_w} the local C_l is a function of y). In particular, when $C_{L_w} = 0$, C_l is not zero everywhere although the integral on the L.H.S. of eqn. (2.12) would be zero. There might, for example, be a down-load towards the wing tips and an up-load near the roots, which would balance out to give zero total lift. The lift distribution at zero overall lift is termed the "basic" lift distribution and depends primarily on the planform and twist distribution. The corresponding local lift coefficient is given the symbol C_{l_b} and must satisfy the relationship

$$\int_0^{b/2} C_{l_b} c \, dy = 0. \tag{2.14}$$

When the wing is producing an overall lift, this lift may be regarded as the consequence of an "additional" lift distribution superimposed on the basic distribution. The local lift coefficient associated with the additional lift distribution is denoted by C_{l_a}, so that

$$C_l = C_{l_a} + C_{l_b}. \tag{2.15}$$

Substituting in eqn. (2.12) gives

$$2\int_0^{b/2} C_{l_a} c\,\mathrm{d}y + 2\int_0^{b/2} C_{l_b} c\,\mathrm{d}y = C_{L_w} S,$$

but since the second integral is zero (from eqn. (2.14)), the final lift relationship is

$$2\int_0^{b/2} C_{l_a} c\,\mathrm{d}y = C_{L_w} S. \tag{2.16}$$

The additional lift distribution is primarily a function of plan-form and overall lift coefficient, but not of twist distribution.

If the real and equivalent wings are to have the same pitching moments, it follows by taking moments about a convenient datum OY that

$$-2\int_0^{b/2} C_l c x\,\mathrm{d}y + 2\int_0^{b/2} C_{m_0} c^2\,\mathrm{d}y = -C_{L_w}\bar{x}S + C_{M_{0w}}\bar{c}S, \tag{2.17}$$

where $\bar{x}$ is the location of the aerodynamic centre of the equivalent wing and x is the distance of the local aerodynamic centre at a given spanwise station of the real wing, both x and $\bar{x}$ being measured from the same arbitrary datum OY. $\bar{x}$ is said to denote the position of the *Mean Aerodynamic Centre.*

When $C_{l_w} = 0$, the value of C_l in eqn. (2.17) becomes C_{l_b}, and this expression reduces to

$$-2\int_0^{b/2} C_{l_b} c x\,\mathrm{d}y + 2\int_0^{b/2} C_{m_0} c^2\,\mathrm{d}y = C_{M_{0w}}\bar{c}S. \tag{2.18}$$

It is now necessary to assume that x is the same function of y in eqn. (2.17) and (2.18), i.e. that the local aerodynamic centre positions remain the same over the range of values of C_{l_w} of interest. This will be so if each section of the real wing behaves like a two-dimensional aerofoil subject to the linearity of the section characteristics mentioned previously. This assumption may not be valid if the wing has, for example, a low aspect ratio and considerable sweep, but it is a reasonable assumption for the wing geometries usually associated with conventional low-speed aeroplanes.

Equation (2.18) may therefore be subtracted from (2.17), introducing (2.15), whence

$$2\int_0^{b/2} C_{l_a} cx\,\mathrm{d}y = C_{L_w}\bar{x}S. \tag{2.19}$$

This equation defines the location of the mean aerodynamic centre, assuming that x and C_{l_a} are known as functions of y.

Three particular cases are of interest:

(a) The line of aerodynamic centres of the original wing is unswept (i.e. $x = \text{const.}$). It then follows from eqn. (2.19), and (2.16) that $\bar{x} = x$, and is therefore independent of the form of the additional spanwise load distribution.

(b) C_{l_a} is constant across the span. It then follows from eqn. (2.19), (2.16) and (2.13) that

$$\bar{x} = \frac{2\int_0^{b/2} cx\,\mathrm{d}y}{S} = \frac{\int_0^{b/2} cx\,\mathrm{d}y}{\int_0^{b/2} c\,\mathrm{d}y}. \tag{2.20}$$

The significance of $\bar{x}$ as a mean aerodynamic centre is fairly obvious in this case, and it becomes a matter of simple geometry. It is not uncommon to find the mean aerodynamic centre defined as if it always had the location given by eqn. (2.20), although in fact it represents a special case which is not a particularly realistic one.

(c) The *shape* of the additional load distribution curve is constant. This is equivalent to assuming that $C_{l_a} = C_{L_w}.\mathrm{f_a}(y)$, where $\mathrm{f_a}(y)$ is a function depending primarily on the planform of the wing. It then follows from eqn. (2.19), (2.16) and (2.13) that

$$\bar{x} = \frac{2\int_0^{b/2} \mathrm{f_a}(y)cx\,\mathrm{d}y}{S} = \frac{\int_0^{b/2} \mathrm{f_a}(y)cx\,\mathrm{d}y}{\int_0^{b/2} c\,\mathrm{d}y}. \tag{2.21}$$

This situation is often fairly realistic for the usual types of wings relevant to this book. Once $f_a(y)$ is known, it is quite a straightforward matter to calculate $\bar{x}$, generally by graphical integration.

In all these cases, $\bar{x}$ is constant. However, if the shape of the additional spanwise loading distribution is a function of C_{L_w} as well as of y, then $\bar{x}$ becomes a function of C_{L_w} and it is no longer possible to define a fixed point as the mean aerodynamic centre. This situation could occur if the effective section characteristics were non-linear. In the remainder of this book it will be assumed that such is not the case, and that the mean aerodynamic centre can be represented as a fixed point. The function $f_a(y)$ can be obtained from a knowledge of the wing geometry by methods similar to those quoted in Ref. 1.

Now consider eqn. (2.18), which may be regarded as defining the quantity $C_{M_{0w}}\bar{c}$. It is clear that $\bar{c}$ is quite arbitrary so long as a corresponding value of $C_{M_{0w}}$ is taken. In practice it is usual to define $\bar{c}$ by the expression

$$\bar{c} = \frac{2\int\limits_0^{b/2} c^2\,\mathrm{d}y}{S}, \tag{2.22}$$

which is simply a matter of geometry. If, in eqn. (2.18), it is assumed that either there is no basic loading (i.e. $C_{l_b} = 0$ at all values of y) or that $x = \text{const.}$, then the first integral is zero. The adoption of the above expression for $\bar{c}$ is then equivalent to regarding $C_{M_{0w}}$ as a mean pitching moment coefficient such that

$$C_{M_{0w}} = \frac{\int\limits_0^{b/2} C_{m_0} c^2\,\mathrm{d}y}{\int\limits_0^{b/2} c^2\,\mathrm{d}y}. \tag{2.23}$$

This expression gives $C_{M_{0w}}$ an air of reality perhaps rather more convincing than would be the case if some other definition were used for $\bar{\bar{c}}$, but it should be remembered that no condition has been stated which makes it imperative to use eqn. (2.22).

In the more general case, when the first integral of eqn. (2.18) has a value, it is still customary to define $\bar{\bar{c}}$ by eqn. (2.22). This is equivalent to defining $C_{M_{0w}}$ by

$$C_{M_{0w}} = \frac{-\int_0^{b/2} C_{l_b} cx \, dy + \int_0^{b/2} C_{m_0} c^2 \, dy}{\int_0^{b/2} c^2 \, dy}, \tag{2.24}$$

and it now becomes more difficult to visualize $C_{M_{0w}}$ as a mean pitching moment coefficient. Given the geometry of the wing planform, $\bar{\bar{c}}$ is easily obtained from eqn. (2.22). Defined in this way, $\bar{\bar{c}}$ is called the *Mean Aerodynamic Chord* (often abbreviated to M.A.C.), but it should be remembered that its value is not determined by rigorous aerodynamic considerations. To obtain $C_{M_{0w}}$ in the general case requires a knowledge of the spanwise basic load distribution (for example, see Ref. 1). $C_{M_{0w}}$ will be independent of C_{L_w} if x and C_{m_0} are independent of C_{L_w}. Throughout this book it will be assumed that such is the case.

Another mean chord, called the *Geometric Mean Chord* or *Standard Mean Chord*, symbol $\bar{c}$, is sometimes encountered. This is a simple arithmetical mean, defined by

$$\bar{c} = \frac{S}{b} = \frac{2\int_0^{b/2} c \, dy}{2\int_0^{b/2} dy}. \tag{2.25}$$

In the past, this quantity was used as a basis for calculating pitching moments, associated with a corresponding coefficient so as to satisfy eqn. (2.18) with $\bar{c}$ replacing $\bar{\bar{c}}$. It is not now used in this context, although there is no theoretical objection to its use. Throughout the rest of this book, the mean aerodynamic chord $\bar{\bar{c}}$ as defined by eqn. (2.22), will be used as the length for defining pitching moment coefficients. Its location in the fore-and-aft sense is defined by $\bar{x}$, it being assumed that the quarter-chord point of the M.A.C. coincides with the mean aerodynamic centre. For straight-tapered wings with spanwise tips, eqn. (2.22) is equivalent to writing

$$\frac{\bar{c}}{c_r} = \frac{2}{3}\left(\frac{1+\lambda+\lambda^2}{1+\lambda}\right), \tag{2.26}$$

where $\lambda = c_t/c_r$, the taper ratio.

In very restricted circumstances, a simple expression can be used to define the position of the mean aerodynamic chord. If eqn. (2.20) applies and if the local aerodynamic centres lie at the quarter-chord points, then eqn. (2.20) and (2.22) amount to defining the distance of the leading edge of $\bar{c}$ from the apex of the wing by the following expression:

$$\frac{\bar{x}_{LE}}{c_r} = \left(\frac{1+2\lambda}{12}\right) A \tan \Lambda_0, \tag{2.27}$$

where A is the aspect ratio b^2/S, or $b/\bar{c}$.

Other Effects of Finite Span

Since the drag coefficient is not of great relevance in stability calculation (see Chapter 11), it will not be considered in further detail except to say that the drag coefficient of a wing of finite span includes not only the "profile" drag due to the two-dimensional section characteristics, but an "induced" drag which depends on the lift. Ultimately, the induced drag may be regarded as a consequence of the energy loss associated with pushing air downwards in order to produce a lift force in accordance with Newton's second and third laws. A quantitative analysis of the simplest type involves replacing the wing and downwash system by a pattern of vortices. The downwash at the wing itself may be regarded as rotating the lift vector, thus producing a component in the streamwise sense.

The downwash at the wing also reduces the local incidence and hence the local lift coefficients. The simplest case relates to an untwisted wing having an elliptical spanwise lift distribution. The downwash and local lift coefficient are both constant across the span (and hence the planform is also elliptical). The downwash ε_w is then

$$\varepsilon_w = \frac{C_{L_w}}{\pi A}, \tag{2.28}$$

and the aerodynamic incidence is $\alpha - \varepsilon_w$, where α is the angle between the section zero-lift line and the free stream far ahead of the wing. If the two-dimensional lift-curve slope $\partial C_l/\partial\alpha$ is denoted by a_0, then

$$C_{L_w} = a_0(\alpha - \varepsilon_w). \tag{2.29}$$

Combining eqn. (2.28) and (2.29),

$$C_{L_w} = a_0\left(\frac{\pi A}{a_0 + \pi A}\right)\alpha = a_w\alpha, \tag{2.30}$$

so the effective lift-curve slope of the wing becomes

$$[a_0\pi A/(a_0 + \pi A)]$$

instead of a_0 (see Fig. 2.6).

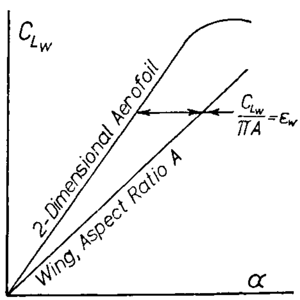

FIG. 2.6. Effect of finite span on lift-curve slope.

Under any other circumstances (e.g. a straight-tapered wing with twist), the downwash varies across the span and these simple relations no longer apply. Detailed calculation is fairly lengthy but for certain planforms, the work is reduced by the use of tabulated data (see Ref. 1). Overall properties of wings of various planforms are given in Ref. 11. In general, the lift-curve slope of a wing is therefore a function of **R** and **M**, since a_0 depends on these quantities, and of the planform and twist distribution.

In most cases, it becomes necessary to consider a wing to which are attached various bodies (fuselage, engine nacelles, etc.). The effect of these bodies is to modify the characteristics of the wing, particularly the lift-curve slope, pitching moment coefficient about the aerodynamic centre and the aerodynamic centre position. (See Chapter 4 and Ref. 11 for data on these effects.)

Exactly similar observations relate to tails since they are small wings, usually of fairly low aspect ratio. Not only is the lift-curve slope of the tail influenced by its own downwash field, but the tail (if of the conventional type) operates in the downwash field due to the wing. Chapter 4 shows how account is taken of the wing downwash.

Properties of Tail Surfaces

A tail which, for most of the following chapters, will be regarded as "conventional", consists of a small wing whose forward portion is attached to the aeroplane. In general, the attachment will be regarded as rigid in that the setting of the tail relative to the aeroplane remains fixed. However, for some purposes, it may be convenient to provide an adjustment to the tail setting. The rear portion of the tail is attached by hinges to the forward portion so as to form a flap, and is called the elevator. It is generally connected directly to the pilot's control column or "stick" in small aeroplanes by a system of levers, push-rods or cables. In larger aeroplanes it is commonly operated via some powered system. The sense in which it is connected to the stick is shown diagrammatically in Fig. 2.7.

There may also be a smaller flap at the trailing edge of the elevator, the "elevator trim tab". This can be moved *relative to the elevator* by means of a separate control in the cockpit. Whereas the stick and elevator rotate in opposite senses, the tab and its control lever rotate in the same sense. It will be appreciated after

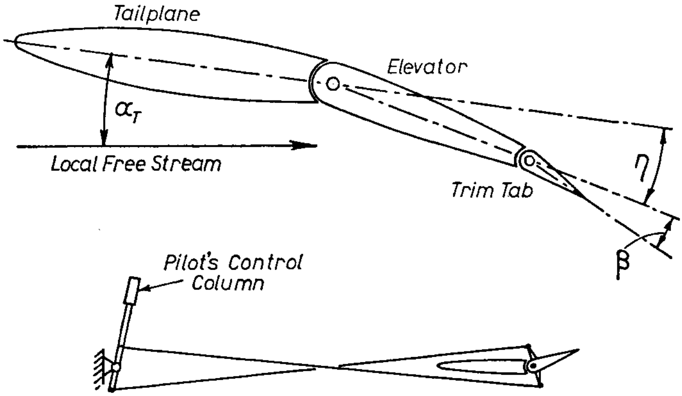

FIG. 2.7. Conventional tail configuration showing positive sense of incidence and control deflections (above) and connection to control column (below).

reading Chapter 6 that connections in these senses provide the pilot with control levers which work in the instinctive senses. Downward deflection of tail control surfaces (i.e. clockwise as drawn) are conventionally positive.

The tail lift coefficient now becomes a function not only of the tail incidence α_T but also of the elevator and tab angles η and β respectively. If the angles are reasonably small, C_{L_T} may be assumed to be a linear function of these angles, as predicted by thin aerofoil theory, i.e.

$$C_{L_T} = a_1\alpha_T + \alpha_2\eta + a_3\beta, \tag{2.31}$$

where

$$a_1 = \frac{\partial C_{L_T}}{\partial \alpha_T}, \qquad a_2 = \frac{\partial C_{L_T}}{\partial \eta}, \qquad \text{and} \quad a_3 = \frac{\partial C_{L_T}}{\partial \beta}. \tag{2.32}$$

a_1 is therefore the tail lift-curve slope with the elevator and tab angles fixed and, as implied above, is exactly analogous to a_w.

a_2 is the slope of the C_{L_T}–η curve at constant incidence and tab setting. It depends on the geometry of the tail and elevator, particularly the ratio of the elevator chord to the total chord, and is affected by features such as any central cut-out to clear the rudder.

a_3 is exactly analogous to a_2. Again it depends on the geometry of the tail, remembering also that tabs rarely extend over the full elevator span.

For a given tail geometry, a_1, a_2 and a_3 are also functions of Reynolds number and Mach number. It is important to remember that whilst eqn. (2.31) indicates that C_{L_T} is linear with η, other angles being constant, the contribution to C_{L_T} due to an elevator displacement in subsonic flow is not just a consequence of the changing surface pressures on the elevator alone. The elevator displacement influences C_{L_T} by varying the surface pressures over the whole tail. So, although a_2 is the coefficient of η in eqn. (2.31), it is in fact a property not only of the elevator but also of the whole assembly. Similar remarks apply to a_1 and a_3. Since positive increments of α_T, η and β all produce increases in C_{L_T}, a_1, a_2 and a_3 are all positive.

The tail lift is related to the tail lift coefficient in the usual way:

$$L_T = C_{L_T}\tfrac{1}{2}\rho V^2 S_T. \tag{2.33}$$

Also of interest is the elevator hinge moment, since under steady conditions it must be balanced by a moment arising from a stick-force applied by the pilot. Again, it is convenient to define a dimensionless coefficient related to this hinge moment, as follows:

$$C_H = \frac{H}{\tfrac{1}{2}\rho V^2 S_\eta c_\eta}, \tag{2.34}$$

and C_H is linearly related to the various angles

$$C_H = b_1\alpha_T + b_2\eta + b_3\beta, \tag{2.35}$$

where

$$b_1 = \frac{\partial C_H}{\partial \alpha_T}, \qquad b_2 = \frac{\partial C_H}{\partial \eta}, \qquad \text{and} \quad b_3 = \frac{\partial C_H}{\partial \beta}, \tag{2.36}$$

and all these partial derivatives are assumed to be constant for small angles, and depend on the tail geometry, Reynolds number and Mach number. It can be imagined that positive increments of any of the angles in eqn. (2.35) for plain controls as shown in Fig. 2.7 will produce changes of hinge moment in the negative (elevator-up) sense. Hence b_1, b_2 and b_3 are normally all negative. However, b_1 and b_2 may be varied appreciably by placing part of the elevator ahead of the hinge line as a "horn balance" (see Fig. 2.8).

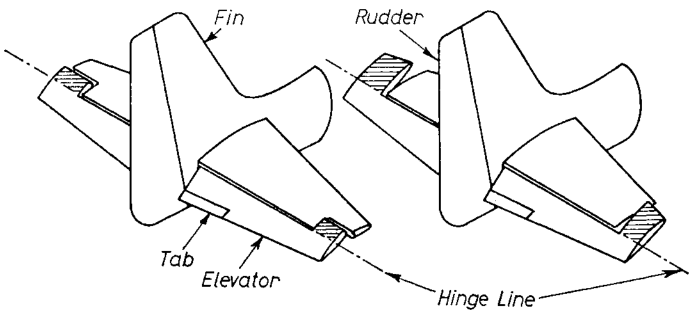

FIG. 2.8. Elevator horn balances: shielded (left) and unshielded (right). The balance areas are shaded and the elevator is shown deflected downwards.

By careful design it is possible to make b_1 positive whilst keeping b_2 negative, with consequences which will be examined in Chapter 5.

The object of fitting a trim tab now becomes apparent. It can be imagined (as will be proved in Chapter 4) that for a given aeroplane in steady flight under given conditions, there is a certain tail lift, and hence a certain C_{L_T}, required to maintain equilibrium. It can also be imagined that α_T is determined by the conditions of flight. In the absence of a trim tab, it follows from eqn. (2.31) that the elevator angle under these conditions is fixed. Hence, omitting the final term of eqn. (2.35), the elevator hinge moment will be determined and, in general, will not be zero. To fly under the desired conditions, the pilot would therefore have to apply a constant stick-force, which would be very tedious. By fitting a tab and introducing an extra variable, β, it becomes possible to satisfy two conditions simultaneously: that C_{L_T} shall have the desired value and $C_H = 0$. In effect, the tab enables the pilot to replace the moment to be supplied by a stick-force by an aerodynamic moment. This matter is examined in detail in Chapter 5.

Curves for finding the *a*'s and *b*'s are given in Ref. 11.

Equivalent Airspeed

Throughout this chapter the quantity $\frac{1}{2}\rho V^2$ has appeared in the definitions of the dimensionless coefficients. It is generally more convenient to define an "equivalent airspeed", V_i such that

$$\tfrac{1}{2}\rho_0 V_i^2 = \tfrac{1}{2}\rho V^2, \tag{2.37}$$

where ρ_0 is the standard sea-level atmospheric density.

Alternatively,

$$V_i = V\sqrt{\frac{\rho}{\rho_0}} = V\sqrt{\sigma}. \tag{2.38}$$

Since σ, the local relative density of the atmosphere, is generally less than unity, the equivalent airspeed V_i is generally less than the true airspeed V. The merit of using the equivalent airspeed is that it combines two variables V and ρ in a single quantity. It follows from eqn. (2.37) that dimensionless coefficients are easily defined in terms of V_i, e.g.

$$C_L = \frac{L}{\frac{1}{2}\rho_0 V_i^2 S}, \tag{2.39}$$

so that for an aeroplane in steady level flight at a given weight, C_L is simply a function of V_i. Equivalent airspeed is of considerable significance to the pilot, since for a given aeroplane for which compressibility effects are negligible, important speeds (e.g. stalling speed, minimum drag speed) really correspond to more or less fixed values of C_L, and hence of V_i. Moreover, in incompressible flow, V_i may be regarded as the reading of a "perfect" airspeed indicator, although in practice there are various sources of discrepancy between the equivalent and indicated airspeeds.

When the dynamics of the aeroplane are important (see Chapter 9), the true airspeed becomes relevant, as it also does if the Mach number is required. But in the following chapters speeds will generally be "equivalent" unless specifically stated otherwise.

CHAPTER 3

Conditions for Static Stability

IN THE most elementary analysis of longitudinal static stability, the following assumptions are made:

1. The aeroplane is in level flight, or at any rate the flight path is inclined at a sufficiently small angle to the horizontal to permit the assumption that the total lift is equal to the weight.
2. The moments about the centre of gravity due to thrust and drag are neglected, and the centre of gravity lies on the mean aerodynamic chord.
3. The components of aerodynamic force acting perpendicular to the mean aerodynamic chord are the same as the lift forces. Since lift is defined as acting perpendicular to the free-stream direction, this is equivalent to assuming small angles of incidence.
4. The effects of slipstream or jet efflux are neglected.
5. The air in which the aeroplane is flying may be regarded as an incompressible fluid, so that aerodynamic force coefficients for an aeroplane of given geometry depend only on the incidence of the wing and the angular settings of the tail and control surfaces. Variations of the coefficients with Reynolds number and Mach number are neglected.
6. The angles mentioned in the previous paragraph are all small, so that the aerodynamic force coefficients are linearly related to them.
7. The air density remains constant.
8. The aeroplane structure is rigid.

Corrections will be made later for some of the effects neglected as a consequence of these assumptions.

The Equilibrium Condition

An aeroplane will continue in steady straight flight only if the resultant force acting on it is zero and the resultant moment about the centre of gravity is zero. Under any other circumstances, the aeroplane will be subjected to the appropriate linear or angular accelerations. The aeroplane is therefore in a state of equilibrium or "trim" when these resultants are zero.

As a consequence of the above assumptions this general statement, which is simply Newton's first law, reduces to the following:

(1) The total lift is equal to the weight,
(2) the thrust is equal to the drag, and
(3) the total aerodynamic pitching moment about the centre of gravity must be zero.

It is clear that only aerodynamic pitching moments about the centre of gravity are involved, since in steady flight under the assumed conditions no other moments can occur. Since the thrust and drag do not contribute to the pitching moment (assumption (2)), they will be omitted from this discussion.

The equilibrium situation therefore reduces to that shown in Fig. 3.1, where

$$L = W, \tag{3.1}$$

$$M_G = 0. \tag{3.2}$$

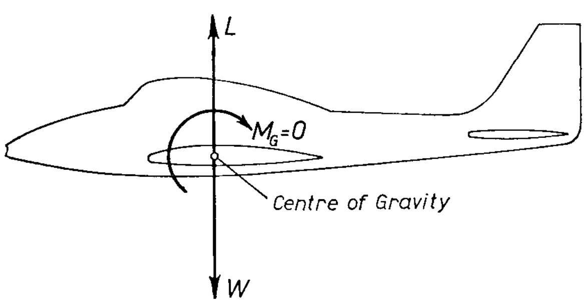

FIG. 3.1. Equilibrium conditions in steady flight.

However, in order to generalize this discussion, it is more convenient to deal in force and moment coefficients rather than in actual forces and moments. As indicated in Chapter 2, the discussion is then no longer restricted to a particular size of aeroplane under specific conditions. Dividing these equations by $\frac{1}{2}\rho V^2 S$ and $\frac{1}{2}\rho V^2 S\bar{c}$ respectively, the equivalent expressions in terms of coefficients are

$$C_L = \frac{L}{\frac{1}{2}\rho V^2 S} = \frac{W}{\frac{1}{2}\rho V^2 S}, \tag{3.3}$$

$$C_{M_G} = \frac{M_G}{\frac{1}{2}\rho V^2 S\bar{c}} = 0. \tag{3.4}$$

The Pitching Moment

For an aeroplane with the elevator and tab held fixed, there will generally only be one value of C_L at which C_{M_G} is zero. At any other value of C_L, the moment coefficient will have some value. Suppose that the aeroplane, with elevator fixed, is supported in a large wind tunnel and its incidence is varied. The corresponding values of lift and pitching moment about the centre of gravity can be measured and the corresponding coefficients obtained. In the wind tunnel, the out-of-balance aerodynamic moment can be resisted by an external mechanical moment so that non-equilibrium conditions can be directly observed in a fashion which is not possible in free flight.

The pitching moment coefficient can now be plotted either as a function of incidence or of lift coefficient. The latter plot is more relevant to considerations of static stability and for a typical aeroplane would have the general shape shown in Fig. 3.2.

It will later be seen (in Appendix I, p. 120) that this discussion about a wind tunnel is not always applicable when compared with a real aeroplane in free flight but it is quite legitimate to consider pitching moments in this fashion provided that the airspeed is so low that the flow can be assumed incompressible.

In Fig. 3.2, C_{M_G} is only zero at the lift coefficient corresponding to point A. At the particular elevator setting considered, the aeroplane would fly in trim at this lift coefficient, and hence at the corresponding equivalent airspeed given by eqn. (3.3).

Suppose now that a disturbance occurs so that the lift coefficient increases to that corresponding to point B. The disturbance might obviously be an increase of incidence due to, say, an up-gust. In

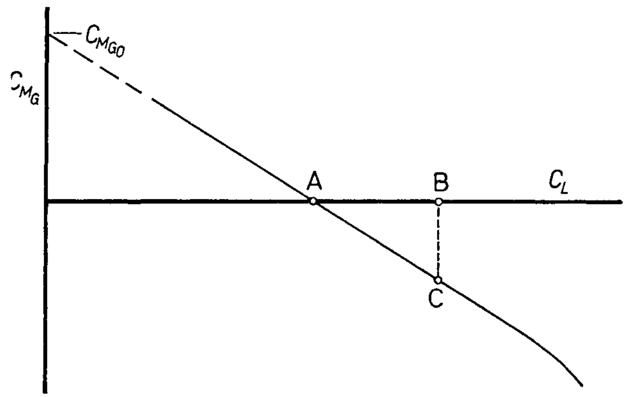

FIG. 3.2. Pitching moment coefficient curve for a statically stable aeroplane.

the most general study of static stability, the precise nature of the disturbance is of some importance. As explained in Appendix I, it is usual to consider a change of lift coefficient subject to the condition that eqn. (3.3) always applies. In other words, an increase in incidence is accompanied by a decrease in forward speed so that the lift always remains equal to the weight. At this stage, such a statement of the nature of the disturbance is not essential to the discussion, and the student need only refer to Appendix I at a later reading.

When the disturbance has caused the lift coefficient to increase to that at B in Fig. 3.2, there is now an out-of-balance moment acting on the aeroplane, the corresponding coefficient being given by BC. In the diagram, this is negative and is therefore in the nose-down sense. Such a moment will tend to cause the incidence (and hence the lift coefficient) to decrease again towards the equilibrium value at point A. The aeroplane is therefore statically stable, in that it *tends* to revert to the equilibrium condition after a disturbance in pitch. These observations only indicate a *tendency* to revert to the equilibrium condition by considering pitching moments assessed under *static* conditions. They do not indicate the nature of the motion which occurs as the aeroplane tries to re-establish the equilibrium condition: since the pitching moments are themselves altered by such motion, an investigation of the dynamic situation is required to describe the motion in detail.

By a similar argument, it is clear that if the lift coefficient is decreased to some value less than that corresponding to point A in Fig. 3.2, the pitching moment will be positive (i.e. nose-up) and again the aeroplane will tend to revert to the equilibrium condition. If the C_{M_G}–C_L curve has the opposite slope, as in Fig. 3.3, the aeroplane will be statically unstable.

It will be in equilibrium at A′, but if the lift coefficient is increased to that corresponding to B′, the out-of-balance pitching moment coefficient becomes B′C′. Since this moment is positive (nose-up) the lift coefficient will tend to increase still further and the aeroplane will tend to diverge from the equilibrium condition.

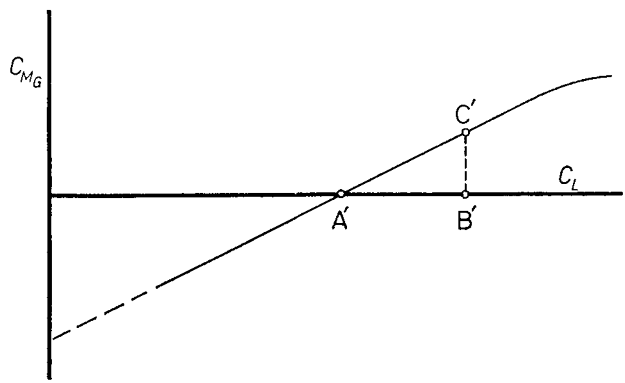

FIG. 3.3. Pitching moment coefficient curve for a statically unstable aeroplane.

Criterion of Static Stability

The condition for static longitudinal stability is therefore that, at the equilibrium condition, the slope of the C_{M_G}–C_L curve should be negative. When assumption (6) above is valid, the C_{M_G}–C_L curve is linear: Fig. 3.3 shows some departure from linearity at high lift coefficients, as would occur in practice near the stall, since the above assumption then ceases to be valid. In practice, further departures from linearity occur due to the flexibility of real aircraft structures.

The whole of the above discussion leads to the conclusions that if the aeroplane is in trim and is statically stable, the C_{M_G}–C_L curve must be such that

$$C_{M_G} = 0$$

and

$$\frac{\mathrm{d}C_{M_G}}{\mathrm{d}C_L} \quad \text{must be negative.}$$

It will be seen from Fig. 3.2, that if the C_{M_G}–C_L curve is extrapolated to zero C_L, the corresponding pitching moment coefficient being $C_{M_{G0}}$ then an alternative statement is that

$$C_{M_{G0}} \quad \text{must be positive}$$

and

$$\frac{\mathrm{d}C_{M_G}}{\mathrm{d}C_L} \quad \text{must be negative}$$

if the aeroplane is in trim at some positive value of C_L and is statically stable.

This discussion suggests, on mainly physical grounds, that a suitable measure of static stability would be

$$\left(-\frac{\mathrm{d}C_{M_G}}{\mathrm{d}C_L}\right).$$

The negative sign is inserted so that the criterion will be positive when the aeroplane possesses positive static stability. An alternative way of looking at this criterion is to regard it as the quotient of a dimensionless quantity describing the restoring moment and a dimensionless quantity describing the magnitude of the disturbance. The change of lift coefficient is not the only way of describing the disturbance but, as explained in Appendix I, there are good grounds for using it in preference to, say, an incidence change.

The above criterion is that generally used in British practice. It is called the "Static Margin".

Possible Aeroplane Configurations

In the previous paragraph it has been shown that if an aeroplane is to be both statically stable and in trim at some positive lift coefficient, both $C_{M_{G0}}$ and the static margin must be positive. In the next chapter it will be shown that the static margin can generally be made positive by arranging the c.g. sufficiently far forward. In practice, the c.g. position cannot generally be located quite as arbitrarily as this statement would suggest, but it still remains true that the primary criterion which permits a configuration to be feasible is that $C_{M_{G0}}$ should be positive.

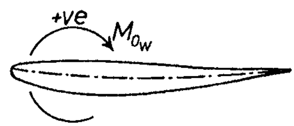

FIG. 3.4. An unswept wing with negative camber, having a positive pitching moment at zero lift.

For straight-winged aeroplanes, an aerofoil with positive camber will give a negative $C_{M_{0w}}$ for the wing alone, and vice versa. It therefore follows that if an unswept tailless aeroplane is to be in trim at some positive lift coefficient, it must have an aerofoil with negative camber.

In general, designers wish to use aerofoils with a positive camber, so that the minimum profile drag coefficient will occur at some chosen positive lift coefficient. Wings with positive-camber sections therefore require some auxiliary surface, or sweep with twist in order to make the overall $C_{M_{G0}}$ positive.

Figures 3.5, 3.6 and 3.7 show three possible configurations. Figure 3.5 is the conventional rear-tail arrangement: with zero

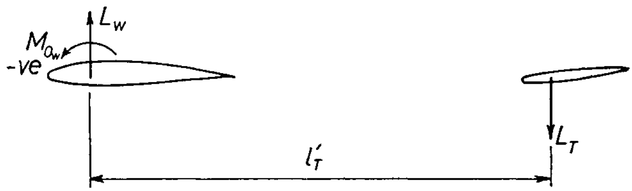

FIG. 3.5. An aeroplane with a conventional tail and wing with positive camber. At zero total lift there must be a down-load on the tail to produce an overall nose-up moment.

FIG. 3.6. A "tail-first" aeroplane. At zero total lift there must be an up-load on the forward "tail" to produce an overall nose-up moment.

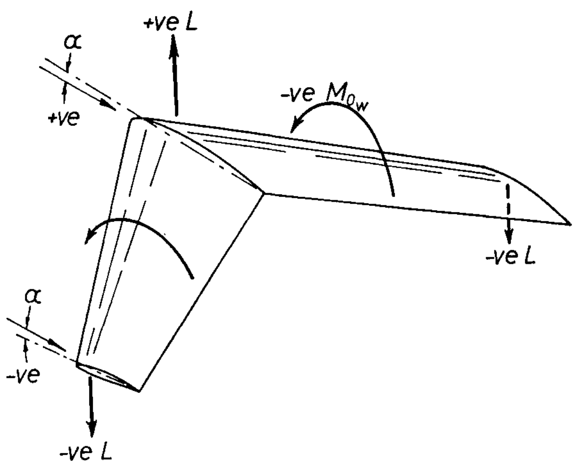

FIG. 3.7. A swept tailless aeroplane. The wing is twisted so that the overall moment is nose-up at zero total lift.

overall lift, the tail is at a negative incidence and produces a downwards lift force. Since only a resultant aerodynamic moment is under consideration, it is convenient to take moments about the aerodynamic centre of the wing. If the overall pitching moment at zero lift is to be positive, then the tail incidence must be such that

$$M_{0_w} - L_T l'_T > 0,$$

or, dividing by $\frac{1}{2}\rho V^2 S\bar{c}$,

$$C_{L_T} \frac{l'_T}{\bar{c}} < C_{M_{0w}}. \tag{3.5}$$

Alternatively, the "tail" may be placed ahead of the wing, in which case it must produce an upwards lift force at zero overall lift (Fig. 3.6).

In Fig. 3.7, a swept-back wing is shown twisted so that the outer, rearmost, parts produce a downwards lift whilst the inner, forward parts produce an equal upwards lift. Again, if sufficient twist is applied, the nose-up pitching moment due to this lift distribution can be caused to exceed the nose-down moment due to the wing-section characteristics.

CHAPTER 4

General Stability Considerations

As THE previous chapter implies, a consideration of the overall balance of forces and moments acting on a conventional aeroplane is facilitated by considering separately those due to the tail and those due to the rest of the aeroplane. Whilst the latter are largely influenced by the wing, other parts such as the fuselage and engine nacelles shift the aerodynamic centre (usually forwards), modify the lift-curve slope (usually an increase) and alter C_{M_0} (usually more negative). It is conventional to use the mean aerodynamic chord of the wing as a reference length, and its leading edge as a datum for certain dimensions, but it should be understood that the suffix *wb* applied to aerodynamic forces and coefficients relates them not only to the wing but to the aeroplane-less-tail.

Subject to the above provisos, and to the assumptions listed on p. 26, the significant forces and moments acting on an aeroplane will be as shown in Fig. 4.1.

The various dimensions are conventionally as shown in the diagram, and are explained in the list of symbols. Note that the positions of the aerodynamic centre of the aeroplane-less-tail and the c.g. are expressed as multiples of the mean aerodynamic chord (M.A.C.) so that h_0 and h are dimensionless.

For a given aeroplane, $h_0\bar{c}$ differs from the quantity $\bar{x}$ considered in Chapter 2 by an amount which depends on the effect of the fuselage, nacelles, etc., and by the shift of datum.

The Overall Pitching Moment

Taking moments about the c.g., the resultant moment M_G is given by:

$$M_G = M_{0_{wb}} + (h-h_0)\bar{c}L_{wb} + M_{0_T} - l_T L_T. \qquad (4.1)$$

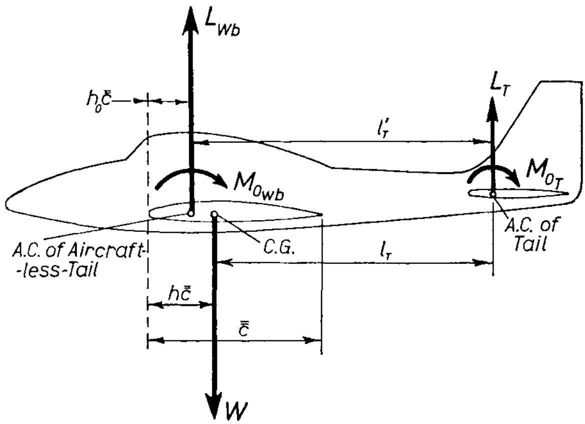

FIG. 4.1. Forces and moments acting on a conventional aeroplane in steady level flight.

Now the moment M_{0_T} is usually very small. It arises as a consequence of "camber" of the tail and since the basic tail section is usually symmetrical, any effective camber is due to elevator deflections. Except near the stall, these are usually small; moreover, the tail area is not very large compared with the wing area. For both these reasons M_{0_T} can be neglected in eqn. (4.1).

As usual, it is more convenient to express the above equation in terms of coefficients. Dividing by $\frac{1}{2}\rho_0 V_i^2 S\bar{c}$ it becomes

$$C_{M_G} = C_{M_{0wb}} + (h - h_0)C_{L_{wb}} - \frac{l_T S_T}{\bar{c}S} C_{L_T}, \tag{4.2}$$

since

$$L_T = C_{L_T} \tfrac{1}{2}\rho_0 V_i^2 S_T. \tag{4.3}$$

The quantity $l_T S_T/\bar{c}S$ is a dimensionless quantity describing the tail size. It is intuitively apparent that the efficacy of the tail will depend not only on its area but on the tail moment arm. Other things being equal, doubling the moment arm and halving the tail area will lead to the same result so far as the contribution of the tail to the aerodynamic moment about the c.g. is concerned. Accordingly, the quantity $l_T S_T$ appears in eqn. (4.2). In the process of obtaining this dimensionless equation, $l_T S_T$, which has the dimensions of a volume, is divided by another "volume" based on the wing dimensions $S\bar{c}$. The quotient is called the "tail volume" coefficient, symbol $\bar{V}$. Also, since $C_{M_{0wb}}$ is the only moment coefficient of significance, the suffix *wb* will now be omitted. The final form of eqn. (4.2) is therefore

$$C_{M_G} = C_{M_0} + (h - h_0)C_{L_{wb}} - \bar{V}C_{L_T}. \qquad (4.4)$$

As explained in the previous chapter, it will become necessary to differentiate this expression with respect to C_L, where C_L relates to the whole aircraft. It is therefore convenient to modify this equation, putting it in terms of C_L rather than $C_{L_{wb}}$.

From the balance of vertical forces

$$W = L = L_{wb} + L_T. \qquad (4.5)$$

So eqn. (4.1) can be written

$$M_G = M_{0_{wb}} + (h - h_0)\bar{c}(L - L_T) - l_T L_T \qquad (4.6)$$

(omitting M_{0_T} as explained above), i.e.

$$\begin{aligned} M_G &= M_{0_{wb}} + (h - h_0)\bar{c}L - [l_T + (h - h_0)\bar{c}]L_T \\ &= M_{0_{wb}} + (h - h_0)\bar{c}L - l'_T L_T. \end{aligned} \qquad (4.7)$$

Again, dividing by $\frac{1}{2}\rho_0 V_i^2 S\bar{c}$, this equation becomes, in coefficient form,

$$C_{M_G} = C_{M_0} + (h - h_0)C_L - \frac{l'_T S_T}{\bar{c}S} C_{L_T}. \qquad (4.8)$$

The quantity $l'_T S_T/\bar{c}S$ is similar in form to the previous tail volume coefficient, but contains l'_T, the distance between the aerodynamic centres of the aeroplane-less-tail and the tail, instead of l_T. It is called the "modified tail volume coefficient", symbol $\bar{V}'$. Equation (4.8) therefore becomes

$$C_{M_G} = C_{M_0} + (h - h_0)C_L - \bar{V}'C_{L_T}. \qquad (4.9)$$

Equations (4.1), (4.4), (4.7), (4.9) apply whether the aeroplane is in trim or not. If it is in trim,

$$M_G = 0 \quad \text{and} \quad C_{M_G} = 0. \qquad (4.10)$$

Inserting this condition in eqn. (4.7) and (4.9) enables the tail lift and tail lift coefficient to be found when the aeroplane is in trim at a given lift coefficient (or equivalent airspeed),

$$L_T = \frac{C_{M_0}\frac{1}{2}\rho_0 V_i^2 S\bar{c}+(h-h_0)\bar{c}L}{l'_T}, \tag{4.11}$$

and

$$C_{L_T} = \frac{C_{M_0}+(h-h_0)C_L}{\bar{V}'}. \tag{4.12}$$

The Stability Criterion

In the previous chapter, it was shown that $\left(-\frac{dC_{M_G}}{dC_L}\right)$ must be positive for positive static stability. This quantity is simply obtained by differentiating eqn. (4.9),

$$\left(-\frac{dC_{M_G}}{dC_L}\right) = (h_0-h)+\bar{V}'\frac{dC_{L_T}}{dC_L}, \tag{4.13}$$

since, by definition,

$$\frac{dC_{M_0}}{dC_L} = 0.$$

An important feature of aeroplane static stability is immediately apparent. If h is less than $h_0+\bar{V}'(dC_{L_T}/dC_L)$, i.e. if the c.g. is sufficiently far forward, the static margin will be positive. This is the basis of the observation of p. 32. The further forward the c.g. (i.e. the less h is made), the greater is the static margin. Physically, it is clear that as the c.g. is moved further forward, the restoring moment about the c.g. due to a small change of tail lift is increased whilst the corresponding disturbing moment due to the change of wing lift will be decreased. Equation (4.13) is merely a mathematical statement of this situation.

It may seem strange at this juncture that the expression for the static margin in eqn. (4.13) takes no account of the condition for trim ($C_{M_G} = 0$), although this condition must always apply in steady flight. In this simple treatment, the trim condition merely defines C_{L_T}. The quantity $\mathrm{d}C_{L_T}/\mathrm{d}C_L$ is the only aerodynamic expression in eqn. (4.13) which might be affected by the trim condition and in incompressible flow it is constant for a given aeroplane, as will be seen later.

It is clear from eqn. (4.13) that the static margin is zero if

$$h = h_N = h_0 + \bar{V}' \frac{\mathrm{d}C_{L_T}}{\mathrm{d}C_L}, \tag{4.14}$$

this particular value of h being given the symbol h_N. If the c.g. were $h_N\bar{c}$ aft of the datum, the static margin would be zero and the aeroplane would have neutral static stability. This point, at $h_N\bar{c}$ aft of the datum, is termed the "neutral point" and this expression is often used loosely as referring to the dimensionless quantity h_N.

With the c.g. at the neutral point, $\mathrm{d}C_{M_G}/\mathrm{d}C_L = 0$, or in general, taking moments about the neutral point, $\mathrm{d}C_{M_{np}}/\mathrm{d}C_L = 0$.

Comparing this situation with the definition of aerodynamic centre for a wing alone, or in combination with a fuselage, it will be seen that the neutral point is in fact the aerodynamic centre of the complete aeroplane. The two expressions are formally interchangeable but, in practice, "neutral point" is reserved for the complete machine whilst "aerodynamic centre" is used for components. Equation (4.14) shows that the effect of the tail is to shift the aerodynamic centre of the whole aeroplane aft of that of the aeroplane-less-tail by an amount $\bar{V}' \dfrac{\mathrm{d}C_{L_T}}{\mathrm{d}C_L}$.

The dimensionless distance between the actual c.g. and the neutral point is $(h_N - h)$. From eqn. (4.13) and (4.14),

$$\left(-\frac{\mathrm{d}C_{M_G}}{\mathrm{d}C_L}\right) = h_N - h. \tag{4.15}$$

Now $(h_N - h)$ is called the "c.g. margin" (also often applied to the dimensional quantity $(h_N - h)\bar{c}$), so that under the conditions postulated in this analysis, the static margin is equal to the c.g. margin. Note that this situation is only true in incompressible flow. Under any other more general conditions, the equation for the static margin does not have the simple form of (4.13), and hence the subsequent conclusions no longer follow. Throughout this book, the expression "static margin" will be used for $\left(-\frac{dC_{M_G}}{dC_L}\right)$ and will generally be treated as being interchangeable with the c.g. margin. Equation (4.15) shows that if the aeroplane is to be statically stable, i.e. if the static margin is to be positive,

$$h < h_N, \tag{4.16}$$

that is to say, the c.g. must be ahead of the neutral point. If $h > h_N$ the static margin is negative and the aeroplane will be statically unstable.

So far, it has only been assumed that a change in C_L is accompanied by a change in C_{L_T}, so that dC_{L_T}/dC_L has a value. In order to consider static stability in more specific terms, this quantity must be considered in more detail.

The Influence of the Tail lift-curve Slope

From eqn. (4.5),

$$L = L_{wb} + L_T,$$

i.e.

$$C_L = C_{L_{wb}} + C_{L_T}\frac{S_T}{S}. \tag{4.17}$$

If the lift-curve slope of the aeroplane without tail (i.e. $dC_{L_{wb}}/d\alpha$) is denoted by a, this becomes

$$C_L = a\alpha + C_{L_T}\frac{S_T}{S}. \tag{4.18}$$

It should be noted that the mean airstream velocity at the tail V' may be different from the forward speed of the aeroplane V. If C_{L_T} were based on V', a factor $(V'/V)^2$ would appear in eqn. (4.17) and would be carried through to many of the subsequent expressions. This quantity is sometimes termed the "tailplane efficiency factor". Modern practice is to base C_{L_T} on V, so that C_{L_T} effectively includes this "tailplane efficiency factor". In a clean modern aeroplane, the main effect which will cause V' to be less than V will be the fuselage boundary layer, which generally encompasses the inboard portions of the tailplane. This effect may be noticeable, particularly if engine cooling air is ejected along the sides of the fuselage. In older types of aeroplane, the tailplane efficiency factor may be substantially less than unity despite the effect of the slipstream, since the airflow about the tail may contain the wakes of sundry wires and struts. It follows that the tailplane should always be located so that it is clear of wakes due to the wing or airbrakes when fitted. In order to consider $\mathrm{d}C_{L_T}/\mathrm{d}C_L$ it may be written in the following form:

$$\frac{\mathrm{d}C_{L_T}}{\mathrm{d}C_L} = \frac{\mathrm{d}C_{L_T}}{\mathrm{d}\alpha_T} \cdot \frac{\mathrm{d}\alpha_T}{\mathrm{d}\alpha} \cdot \frac{\mathrm{d}\alpha}{\mathrm{d}C_L}, \tag{4.19}$$

where α_T is the tail incidence.

Now $\mathrm{d}C_{L_T}/\mathrm{d}\alpha_T$ is simply the tailplane lift-curve slope. This will depend on the aerodynamic properties of the tail, together with the consequences of any constraints which may be applied to the elevator.

From Fig. 4.2 and the definitions of the various angles, the tailplane incidence will be

$$\alpha_T = \alpha - \varepsilon + \eta_T. \tag{4.20}$$

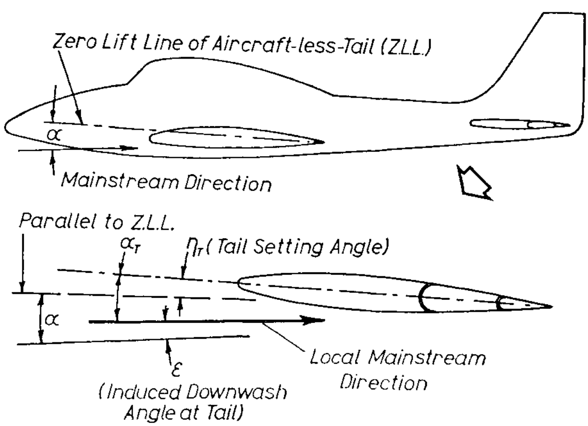

FIG. 4.2. The angles relevant to the calculation of tail incidence.

Simple wing theory suggests that the downwash ε should be proportional to the incidence α, so that eqn. (4.20) may be written

$$\alpha_T = \alpha\left(1 - \frac{d\varepsilon}{d\alpha}\right) + \eta_T, \tag{4.21}$$

where $d\varepsilon/d\alpha$ is assumed to be constant.

This expression assumes that $\varepsilon = 0$ when $\alpha = 0$. In practice this is not always quite so, and hence strictly η_T should be measured not from the zero-lift line of the aeroplane-less-tail, but from the direction of the airflow at the tail at $\alpha = 0$.

For the most conventional arrangement, η_T is fixed, and then

$$\frac{d\alpha_T}{d\alpha} = 1 - \frac{d\varepsilon}{d\alpha}. \tag{4.22}$$

From eqn. (4.18)

$$\alpha = \frac{C_L - C_{L_T}\dfrac{S_T}{S}}{a}, \tag{4.23}$$

and hence

$$\frac{d\alpha}{dC_L} = \frac{1}{a}\left(1 - \frac{S_T}{S}\frac{dC_{L_T}}{dC_L}\right). \tag{4.24}$$

Substituting eqn. (4.22) and (4.24) in (4.19), and introducing a_T for $dC_{L_T}/d\alpha_T$,

$$\frac{dC_{L_T}}{dC_L} = a_T\left(1 - \frac{d\varepsilon}{d\alpha}\right)\frac{1}{a}\left(1 - \frac{S_T}{S}\frac{dC_{L_T}}{dC_L}\right),$$

which leads to

$$\frac{dC_{L_T}}{dC_L} = \frac{1}{\left[1 + \dfrac{S_T}{S}\cdot\dfrac{a_T}{a}\left(1 - \dfrac{d\varepsilon}{d\alpha}\right)\right]}\frac{a_T}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right). \tag{4.25}$$

This equation may be written

$$\frac{dC_{L_T}}{dC_L} = \frac{1}{1+\mathfrak{F}}\frac{a_T}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right), \tag{4.26}$$

where

$$\mathfrak{F} = \frac{S_T}{S}\frac{a_T}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right). \tag{4.27}$$

It is also worth noting that the lift-curve slope for the complete aeroplane may now be obtained from eqn. (4.24) and (4.26) and is simply

$$\frac{\mathrm{d}C_L}{\mathrm{d}\alpha} = (1+\mathfrak{F})a. \tag{4.28}$$

In other words, the lift-curve slope of the complete aeroplane exceeds that of the aeroplane-less-tail by a factor $(1+\mathfrak{F})$. The quantity $\mathfrak{F}$, which denotes the proportional contribution of the tail to the total lift-curve slope, is often omitted, and eqn. (4.26) then becomes

$$\frac{\mathrm{d}C_{L_T}}{\mathrm{d}C_L} \doteq \frac{a_T}{a}\left(1 - \frac{\mathrm{d}\varepsilon}{\mathrm{d}\alpha}\right). \tag{4.29}$$

Since $\mathfrak{F}$ may be as large as 0·1 for machines with large tails, it will be included in the theory which follows.

If eqn. (4.26) is substituted in (4.13), an expression for the static margin is obtained in terms of the tail lift-curve slope

$$\left(-\frac{\mathrm{d}C_{M_G}}{\mathrm{d}C_L}\right) = (h_0 - h) + \frac{\bar{V}'}{1+\mathfrak{F}}\frac{a_T}{a}\left(1 - \frac{\mathrm{d}\varepsilon}{\mathrm{d}\alpha}\right), \tag{4.30}$$

and it follows that the neutral point is given by

$$h_N = h_0 + \frac{\bar{V}'}{1+\mathfrak{F}}\frac{a_T}{a}\left(1 - \frac{\mathrm{d}\varepsilon}{\mathrm{d}\alpha}\right). \tag{4.31}$$

In practice, expressions of this type are frequently used with $\mathfrak{F}$ omitted. For the sake of rigour, it is worth saying that there are two alternative ways of considering the $(1+\mathfrak{F})$ term as it appears in eqn. (4.30) and (4.31). It can either be considered in conjunction with a, in which case $(1+\mathfrak{F})a$ is the lift-curve slope of the complete aeroplane as previously explained, or it can be considered in conjunction with $\bar{V}'$, in which case $\bar{V}'/(1+\mathfrak{F})$ can be regarded as yet another tail volume coefficient in its own right.

The Significance of $\bar{V}'/(1+\mathfrak{F})$

From the foregoing and by reference to Fig. 4.3, it will be seen that the distance l_N from the neutral point to the aerodynamic

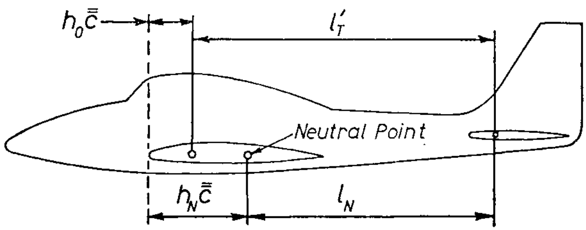

FIG. 4.3. Tail moment arm measured from the neutral point.

centre of the tail is

$$l_T' - (h_N - h_0)\bar{c} = l_T' - \frac{\bar{V}'}{1+\mathfrak{F}}\frac{a_T}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right)\bar{c}. \tag{4.32}$$

Introducing eqn. (4.27), and substituting for $\bar{V}'$,

$$l_N = l_T' - \frac{l_T' S_T}{\bar{c}S}\frac{1}{1+\mathfrak{F}} \cdot \mathfrak{F}\frac{S}{S_T} \cdot \bar{c} = \frac{l_T'}{1+\mathfrak{F}}. \tag{4.33}$$

It is therefore evident that

$$\frac{\bar{V}'}{1+\mathfrak{F}} = \frac{l_N S_T}{\bar{c}S}, \tag{4.34}$$

and therefore represents a tail volume coefficient based on a tail arm measured from the neutral point to the aerodynamic centre of the tail.

Hence, neglecting $\mathfrak{F}$ in eqn. (4.30) and (4.31) is equivalent either to neglecting the effect of the tail on the lift-curve slope of the complete aeroplane, or to neglecting the distance between the aerodynamic centre of the aeroplane-less-tail and the neutral point compared with the tail moment arm.

CHAPTER 5

Stick-fixed and Stick-free Stability

EXPRESSIONS have now been obtained for the static margin and neutral point in terms of the tail lift-curve slope a_T (eqn. (4.30) and (4.31)). So far, the theory has been quite general, and applies in all circumstances in which a_T can be defined. It now becomes necessary to consider a_T in more detail.

If the aeroplane is completely conventional and has a fixed tail-plane and an elevator and trim tab separately controlled by the pilot, it is convenient to consider two limiting conditions.

(a) The "stick-fixed" case. Strictly, this should be termed "elevator fixed", since the assemblage of push rods, control cables, levers, etc., which connect the control column or "stick" to the elevator is rarely very rigid. It is assumed that the elevator and tab are adjusted so that the aeroplane is trimmed at a certain lift coefficient; the static stability is then assessed when the aeroplane is subjected to a small disturbance from the equilibrium state, the settings of the control surfaces being left unchanged.

(b) The "stick-free" case. Strictly, this should be termed "elevator free", since practical elevator circuits are seldom free from friction. It is assumed that the elevator and tab are adjusted so that the aeroplane is trimmed at a certain lift coefficient and the elevator hinge moment is zero. The static stability is then assessed when the aeroplane is subjected to a small disturbance from the equilibrium state and the elevator is left freely floating. In the disturbed condition, it takes up an angle determined by the condition that the hinge moment is always zero. It is also assumed that the tab is always set at the angle appropriate to the equilibrium condition.

It is apparent that both of these cases are somewhat artificial. In a sense, they may be regarded as limiting cases with an infinite variety of more realistic conditions falling between them. But they are in fact related to quite real aspects of the process of controlling the aeroplane, as will be shown later.

Stick-fixed Stability

It follows from the above definition that the static margin is to be found subject to the condition that η and β are both constant.

Since the general expression for the tail lift coefficient is, from eqn. (2.31),

$$C_{L_T} = a_1\alpha_T + \alpha_2\eta + a_3\beta,$$

it follows that, under the prescribed conditions,

$$a_T = \frac{\mathrm{d}C_{L_T}}{\mathrm{d}\alpha_T} = a_1. \tag{5.1}$$

In the stick-fixed case, the static margin is denoted by the symbol K_n, i.e.

$$K_n = \left(-\frac{\mathrm{d}C_{M_G}}{\mathrm{d}C_L}\right)_{\substack{\eta=\text{const.}\\ \beta=\text{const.}}}. \tag{5.2}$$

The value of $\mathfrak{F}$ given by putting $a_T = a_1$ in eqn. (4.27) will be denoted by F, i.e.

$$F = \frac{S_T}{S}\frac{a_1}{a}\left(1 - \frac{\mathrm{d}\varepsilon}{\mathrm{d}\alpha}\right). \tag{5.3}$$

From eqn. (4.30), the expression for the stick-fixed static margin is therefore

$$K_n = (h_0 - h) + \frac{\bar{V}'}{1+F}\frac{a_1}{a}\left(1 - \frac{\mathrm{d}\varepsilon}{\mathrm{d}\alpha}\right), \tag{5.4}$$

and the stick-fixed neutral point, from eqn. (4.31) becomes

$$h_n = h_0 + \frac{\bar{V}'}{1+F}\frac{a_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right), \tag{5.5}$$

where h_n is now the symbol defining the stick-fixed neutral point.

It is customary to replace $\bar{V}'/(1+F)$ by V_T, where, from eqn. (4.34),

$$V_T = \frac{l_n S_T}{\bar{c}S}, \tag{5.6}$$

l_n being the distance between the stick-fixed neutral point and the aerodynamic centre of the tail. As mentioned in the previous chapter, F is frequently neglected.

Equations (5.4) and (5.5) may therefore be written alternatively as follows:

$$K_n = (h_0 - h) + V_T\frac{a_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right), \tag{5.7}$$

$$h_n = h_0 + V_T\frac{a_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right). \tag{5.8}$$

The distance between the c.g. and the stick-fixed neutral point is termed the "stick-fixed c.g. margin" and, as explained in Chapter 4, is equal to the stick-fixed static margin under the conditions of this analysis.

Stick-free Stability

In this case, the static margin is to be found subject to the condition that $C_H = 0$ and β is constant.

From eqn. (2.35) the elevator hinge moment coefficient is

$$C_H = b_1\alpha_T + b_2\eta + b_3\beta.$$

If $C_H = 0$,

$$\eta = -\left(\frac{b_1\alpha_T + b_3\beta}{b_2}\right). \tag{5.9}$$

This equation defines the angle at which the elevator will float freely for given values of α_T and β.

Substituting eqn. (5.9) in (2.31), the tail lift coefficient with the elevator floating freely becomes

$$C_{L_T} = a_1\alpha_T - \frac{a_2}{b_2}(b_1\alpha_T + b_3\beta) + a_3\beta,$$

i.e.

$$C_{L_T} = a_1\left(1 - \frac{a_2 b_1}{a_1 b_2}\right)\alpha_T + a_3\left(1 - \frac{a_2 b_3}{a_3 b_2}\right)\beta. \tag{5.10}$$

Writing

$$\bar{a}_1 = a_1\left(1 - \frac{a_2 b_1}{a_1 b_2}\right), \tag{5.11}$$

and

$$\bar{a}_3 = a_3\left(1 - \frac{a_2 b_3}{a_3 b_2}\right), \tag{5.12}$$

eqn. (5.10) becomes, more neatly,

$$C_{L_T} = \bar{a}_1\alpha_T + \bar{a}_3\beta. \tag{5.13}$$

$\bar{a}_1$ and $\bar{a}_3$ may be regarded as modified values of a_1 and a_3, taking into account the freely floating elevator. It should be noted that with normal values of the various coefficients, $\bar{a}_3$ is negative. In other words, if the tab is given a positive displacement $\delta\beta$ with the elevator floating freely, the increment in tail lift coefficient $a_3\delta\beta$ is numerically less than the decrease $(a_2 b_3/b_2)\delta\beta$ caused by the elevator deflection which results in order to keep $C_H = 0$.

It then follows from eqn. (5.13) that with β constant,

$$a_T = \left(\frac{\partial C_{L_T}}{\partial \alpha_T}\right)_{C_H=0} = \bar{a}_1. \tag{5.14}$$

In the stick-free case, the static margin is denoted by the symbol K_n', i.e.

$$K_n' = \left(-\frac{dC_{M_G}}{dC_L}\right)_{C_H=0}. \tag{5.15}$$

The value of $\mathfrak{F}$ given by putting $a_T = \bar{a}_1$ in eqn. (4.27) will be denoted by $\bar{F}$, i.e.

$$\bar{F} = \frac{S_T}{S}\frac{\bar{a}_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right). \tag{5.16}$$

The stick-free static margin, from eqn. (4.30) is therefore

$$K_n' = (h_0 - h) + \frac{\bar{V}'}{1+\bar{F}}\frac{a_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right). \tag{5.17}$$

The stick-free neutral point is obtained by putting $K_n' = 0$ in eqn. (5.17) and is denoted by the symbol h_n', i.e.

$$h_n' = h_0 + \frac{\bar{V}'}{1+\bar{F}}\frac{a_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right). \tag{5.18}$$

The effective tail volume coefficient $\bar{V}'/(1+\bar{F})$ is usually denoted by $\bar{V}_T$ where, from eqn. (4.34),

$$\bar{V}_T = \frac{l_n' S_T}{\bar{c}S}, \tag{5.19}$$

where l_n' is the distance from the stick-free neutral point to the aerodynamic centre of the tail.

Equations (5.17) and (5.18) may therefore be written more briefly as

$$K_n' = (h_0 - h) + \bar{V}_T \frac{\bar{a}_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right), \tag{5.20}$$

$$h_n' = h_0 + \bar{V}_T \frac{\bar{a}_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right). \tag{5.21}$$

The distance between the c.g. and the stick-free neutral point is called "the stick-free c.g. margin", and is equal to the stick-free static margin under the assumed conditions.

Comparison of Stick-fixed and Stick-free Static Margins

Equation (5.7) was

$$K_n = (h_0 - h) + V_T \frac{a_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right),$$

and (5.20) may be written in the form

$$K_n' = (h_0 - h) + \bar{V}_T \frac{a_1}{a}\left(1 - \frac{a_2 b_1}{a_1 b_2}\right)\left(1 - \frac{d\varepsilon}{d\alpha}\right).$$

If it is assumed that V_T and $\overline{V}_T$ do not differ appreciably from $\overline{V}'$, then the difference between the static margins is approximately

$$K_n - K_n' \doteq \overline{V}' \frac{a_2 b_1}{a_1 b_2}\left(1 - \frac{\mathrm{d}\varepsilon}{\mathrm{d}\alpha}\right). \tag{5.22}$$

With a conventional tail without aerodynamic balance of the elevator, b_1 and b_2 are both negative, all other quantities are positive and $\mathrm{d}\varepsilon/\mathrm{d}\alpha < 1$, so that

$$K_n > K_n'.$$

Hence, in the simplest case, the static stability, stick-fixed, is greater than the static stability, stick-free. This is a consequence of the effective tail lift-curve slope being reduced from a_1, stick-fixed, to $a_1\left(1 - \frac{a_2 b_1}{a_1 b_2}\right)$, stick-free. In physical terms, the tail-plane and elevator behave like a rigid wing in the stick-fixed case. The only significant aerodynamic tail parameter is the overall lift-curve slope a_1, which is primarily determined by the geometry of the complete horizontal tail. To a first order, features such as the ratio of elevator chord to total chord are of no consequence.

In the stick-free case, an increase in tail incidence $\delta\alpha_T$ results in an increment of lift coefficient $a_1\,\delta\alpha_T$ and, since the elevator will float up a little so as to keep $C_H = 0$, there will also be a decrease in tail lift coefficient due to this elevator deflection, of magnitude $(a_2 b_1/b_2)\delta\alpha_T$. The geometry of the elevator now becomes important, since it determines a_2, b_1 and b_2.

It will be noted that, from eqn. (5.8) and (5.18) that the dimensionless distance between the neutral points $(h_n - h_n')$ is also given by the right-hand side of eqn. (5.22).

If the elevator is fitted with suitable horn balances, it is possible to make b_1 positive whilst keeping b_2 negative. Under these circumstances, $K_n' > K_n$ and the aeroplane is more stable in the stick-free condition. Whilst the relevant sections of British Civil Airworthiness Requirements, Sections D and E, almost constitute an invitation to achieve this condition, it is said to make the aeroplane rather unpleasant to fly when it is attained by making b_1 positive, due to the "feed-back" of aerodynamic moments to the control column during flight in rough air.

Pitching Moments

In order to consider the actions required of the pilot in controlling an aeroplane, an expression for the pitching moment coefficient is required in a more explicit form than eqn. (4.9). This was

$$C_{M_G} = C_{M_0} + (h - h_0)C_L - \bar{V}'C_{L_T}.$$

The following expressions have already been obtained

$$C_{L_T} = a_1\alpha_T + a_2\eta + a_3\beta; \tag{2.31}$$

$$\alpha_T = \alpha\left(1 - \frac{d\varepsilon}{d\alpha}\right) + \eta_T; \tag{4.21}$$

$$\alpha = \frac{C_L - \dfrac{S_T}{S}C_{L_T}}{a}. \tag{4.23}$$

Substituting eqn. (4.21) and (4.23) in (2.31)

$$C_{L_T} = \frac{a_1}{a}\left(C_L - \frac{S_T}{S}C_{L_T}\right)\left(1 - \frac{d\varepsilon}{d\alpha}\right) + a_1\eta_T + a_2\eta + a_3\beta,$$

i.e.

$$C_{L_T}\left[1 + \frac{a_1}{a}\frac{S_T}{S}\left(1 - \frac{d\varepsilon}{d\alpha}\right)\right] = \frac{a_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right)C_L + a_1\eta_T + a_2\eta + a_3\beta,$$

or

$$C_{L_T} = \frac{1}{1+F}\left[\frac{a_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right)C_L + a_1\eta_T + a_2\eta + a_3\beta\right]. \tag{5.23}$$

This expression for the tail lift coefficient is quite general and applies under any circumstances in which C_L and the various angles on the right-hand side are known. It is important to remember that it applies to the lift coefficient of a tail forming part of an aeroplane whose overall lift coefficient, including the tail contribution, is C_L.

Hence, from eqn. (4.9) and (5.23)

$$C_{M_G} = C_{M_0} + (h - h_0)C_L - \frac{\bar{V}'}{1+F}\left[\frac{a_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right)C_L + a_1\eta_T + a_2\eta + a_3\beta\right],$$

or

$$C_{M_G} = C_{M_0} + (h - h_0)C_L - \bar{V}_T\left[\frac{a_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right)C_L + a_1\eta_T + a_2\eta + a_3\beta\right]. \quad (5.24)$$

This equation could, of course, be used to obtain the stick-fixed static margin, by differentiating with respect to C_L, and obviously leads to the same result as that given by eqn. (4.5).

Whilst eqn. (5.24) always applies, it is convenient to express it in a somewhat different form in the stick-free case, when the elevator angle is determined by the condition $C_H = 0$. From eqn. (5.13), the tail lift coefficient under these circumstances is

$$C_{L_T} = \bar{a}_1\alpha_T + \bar{a}_3\beta.$$

The expression corresponding to eqn. (5.23) may be written by replacing a_1 and a_3 by $\bar{a}_1$ and $\bar{a}_3$ respectively and omitting the term in η, remembering that F then becomes $\bar{F}$, i.e.

$$C_{L_T} = \frac{1}{1+\bar{F}}\left[\frac{\bar{a}_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right)C_L + \bar{a}_1\eta_T + \bar{a}_3\beta\right], \quad (5.25)$$

and hence the pitching moment coefficient, stick-free, becomes (by analogy with eqn. (5.24))

$$C_{M_G} = C_{M_0} + (h - h_0)C_L - \bar{V}_T\left[\frac{\bar{a}_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right) + \bar{a}_1\eta_T + \bar{a}_3\beta\right]. \quad (5.26)$$

The significance of eqn. (5.24) and (5.26) will be seen in Chapter 6.

CHAPTER 6

The Influence of Static Stability on the Pilot's Actions

Elevator Angles to Trim

In Chapter 3 it was noted that for a stable aeroplane with the elevator fixed, C_{M_G} was a linear function of C_L (see Fig. 6.1) the line having a negative slope. At A, $C_{M_G} = 0$, and hence the aeroplane is in trim at this lift coefficient for the chosen elevator angle. It is now possible to consider the relationship between the elevator angle and the lift coefficient at which the aeroplane will fly in equilibrium.

The equation of the line in Fig. 6.1 is in fact eqn. (5.24) in which all the quantities on the right-hand side are fixed except for

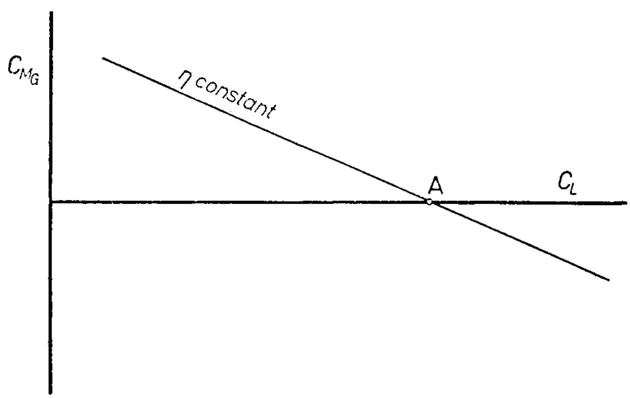

FIG. 6.1. Pitching moment coefficient curve at constant elevator angle.

C_L, and η has the chosen value. This equation may be written in a slightly different way, by introducing eqn. (5.7), whence

$$C_{M_G} = C_{M_0} - C_L K_n - V_T(a_1 \eta_T + a_2 \eta + a_3 \beta). \tag{6.1}$$

Lines representing the variation of C_{M_G} with C_L could therefore be plotted for various values of η (see Fig. 6.2), and the aeroplane would be in trim at the lift coefficients corresponding to points A, B and C for elevator angles η_1, η_2 and η_3 respectively. From eqn. (6.1) and Fig. 6.2, it is clear that η_1, η_2 η_3 represent increasing values of η, i.e. deflections increasing in the downward sense.

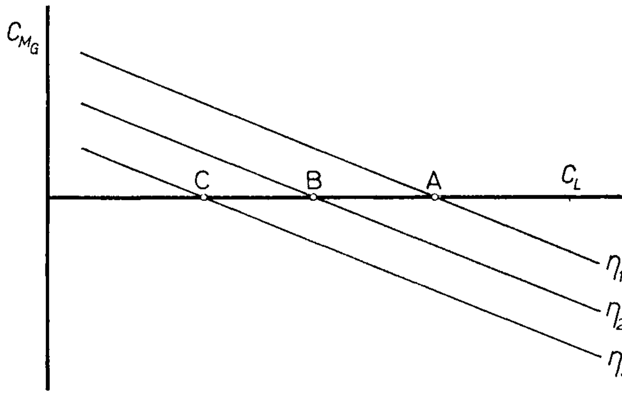

FIG. 6.2. Pitching moment coefficient curves for three values of the elevator angle.

When the aeroplane is in trim, $C_{M_G} = 0$, and the elevator angle to trim at a certain lift coefficient, η_{trim}, can be found from eqn. (6.1)

$$\eta_{trim} = \frac{1}{a_2}\left(\frac{C_{M_0} - C_L K_n}{V_T} - (a_1 \eta_T + a_3 \beta)\right). \tag{6.2}$$

For a given aeroplane, this equation also defines the total elevator travel to be provided to enable the aeroplane to be trimmed over the ranges of lift coefficient and static margin envisaged. In practice, rather more elevator travel would be provided on account of non-linear effects, particularly at high lift coefficients, and to permit the pilot to make corrections following disturbances when flying under extreme conditions.

If, from Fig. 6.2, elevator angles to trim were plotted against corresponding lift coefficients (Fig. 6.3), a line would be obtained whose equation is (6.2). It should be noted that for a statically

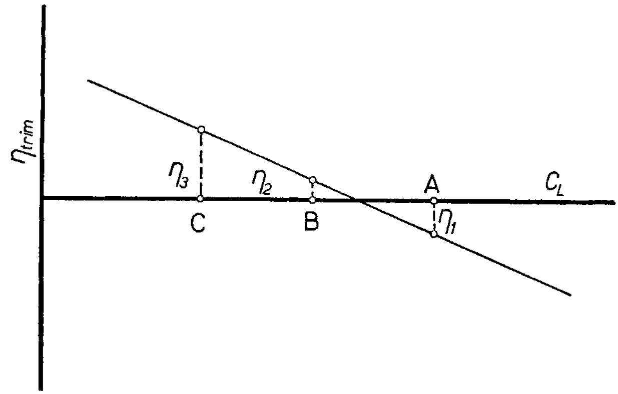

FIG. 6.3. Elevator angles-to-trim as a function of lift coefficient.

stable aeroplane ($K_n > 0$), the slope of this line is negative and is

$$\frac{d\eta_{trim}}{dC_L} = -\frac{K_n}{a_2 V_T}. \tag{6.3}$$

It also follows from eqn. (6.2) that if the η_{trim}–C_L curve is extrapolated to $C_L = 0$, the corresponding elevator angle, η_0, is independent of K_n, i.e.

$$\eta_0 = \frac{1}{a_2}\left[\frac{C_{M_0}}{V_T} - (a_1\eta_T + a_3\beta)\right]. \tag{6.4}$$

Hence if "trim curves" such as Fig. 6.3 are drawn for different values of K_n their slopes will be proportional to K_n and they will pass through the same point when extrapolated to $C_L = 0$. This feature of trim curves is often of practical value when plotting results obtained from flight tests. (See Fig. 6.4.)

In the foregoing theory, it has been assumed that the tab angle β is constant. In practice, as the pilot changes the speed from one value to another, he will normally adjust the tab angle so that the stick-force is always zero at the trimmed speed. Under such circumstances, β in eqn. (6.2) will also be a function of C_L, and eqn. (6.3) will be slightly modified as a consequence. The effect is not large, and will be considered in more detail in Chapter 7.

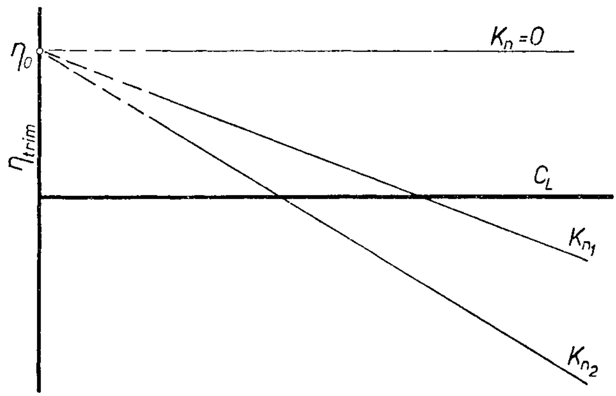

FIG. 6.4. Curves of elevator angle-to-trim as a function of lift coefficient at two values of the static margin.

In obtaining eqn. (6.3) it has been supposed that the aeroplane behaves strictly in accordance with the initial assumptions and that in particular the trim curves are linear and the static margin is constant at a given c.g. position. In fact, it is not necessary to make such assumptions and it is useful to prove this relationship under more general conditions.

Suppose that the aeroplane is initially in trim at a lift coefficient C_L. Since the pitching moment must be zero, eqn. (4.9) will become

$$C_{M_G} = 0 = C_{M_0} + (h - h_0)C_L - \bar{V}'C_{L_T}. \qquad (6.5)$$

If the lift coefficient is now altered by δC_L, the elevator angle being kept constant, an out-of-balance pitching moment coefficient δC_{M_G} will result. Now, in eqn. (6.5), the only quantities which can strictly be regarded as constant are h and $\bar{V}'$, since they are simply matters of geometry. Hence the out-of-balance pitching moment at the new lift coefficient will be

$$\delta C_{M_G} = (C_{M_0}+\delta C_{M_0})+[h-(h_0+\delta h_0)][C_L+\delta C_L] \\ -\bar{V}'(C_{L_T}+\delta C_{L_T}). \qquad (6.6)$$

In this expression, δC_{L_T} is a change in tail lift coefficient arising as a consequence of a change in the aeroplane lift coefficient at a constant elevator angle.

If, at the new lift coefficient, the out-of-balance pitching moment is again made zero by a further change of the tail lift coefficient ΔC_{L_T}, obtained by moving the elevator by a small amount, then the new expression corresponding to eqn. (6.6) is

$$0 = (C_{M_0}+\delta C_{M_0})+[h-(h_0+\delta h_0)][C_L+\delta C_L] \\ -\bar{V}'(C_{L_T}+\delta C_{L_T}+\Delta C_{L_T}). \qquad (6.7)$$

Subtracting (6.7) from (6.6),

$$\delta C_{M_G} = \bar{V}'\Delta C_{L_T}. \qquad (6.8)$$

In other words, the change of pitching moment due to a small change in lift coefficient at constant control angle can be balanced out by applying a small control deflection at the new lift coefficient.

From eqn. (5.23), under these conditions,

$$\Delta C_{L_T} = \left(\frac{\partial C_{L_T}}{\partial \eta}\right)\delta\eta_{trim} = \frac{a_2}{1+F}\,\delta\eta_{trim}. \qquad (6.9)$$

a_2 and F need not be strictly constant, provided that the values taken in eqn. (6.9) are those appropriate to the prevailing conditions. (In this case, to the appropriate lift coefficient.)

Equations (6.8) and (6.9) give

$$\delta C_{M_G} = \frac{\bar{V}'}{1+F}\,a_2\,\delta\eta_{trim},$$

i.e.

$$K_n = -\frac{dC_{M_G}}{dC_L} = -V_T a_2 \frac{d\eta_{trim}}{dC_L}, \qquad (6.10)$$

which is the result given by eqn. (6.3).

The generality of this result is important, since it relies only on the definition of static margin and on taking a_2 and V_T appropriate to the prevailing conditions of flight. This result is essential to the analysis of trim curves obtained in flight, which are rarely straight lines in practice.

The consequences of eqn. (6.10) are noticed by the pilot as a variation of stick position with speed. This is shown in the following analysis.

Write

$$\frac{\mathrm{d}\eta_{trim}}{\mathrm{d}V_i} = \frac{\mathrm{d}\eta_{trim}}{\mathrm{d}C_L} \cdot \frac{\mathrm{d}C_L}{\mathrm{d}V_i}. \tag{6.11}$$

For an aeroplane in steady level flight at an equivalent airspeed V_i,

$$C_L = \frac{W}{\frac{1}{2}\rho_0 V_i^2 S}, \tag{6.12}$$

and hence

$$\frac{\mathrm{d}C_L}{\mathrm{d}V_i} = -\frac{4W}{\rho_0 V_i^3 S}. \tag{6.13}$$

Substituting (6.10) and (6.13) in (6.11)

$$\frac{\mathrm{d}\eta_{trim}}{\mathrm{d}V_i} = \frac{K_n}{V_T a_2} \cdot \frac{4W}{\rho_0 V_i^3 S} = \left(\frac{4W}{V_T a_2 \rho_0 S}\right)\frac{K_n}{V_i^3}. \tag{6.14}$$

Let the stick-elevator gearing be m_e radians of elevator travel per foot travel of the hand-grip, then

$$x_s = \frac{\eta}{m_e}, \tag{6.15}$$

x_s, the stick displacement, being taken as positive away from the pilot. The slope of the stick position–equivalent airspeed curve is therefore

$$\frac{\mathrm{d}x_s}{\mathrm{d}V_i} = \frac{1}{m_e}\left(\frac{4W}{V_T a_2 \rho_0 S}\right) \cdot \frac{K_n}{V_i^3}. \tag{6.16}$$

Again, this result is generally true, provided that the appropriate values of V_T, a_2 and K_n are taken.

In the simple case, when these quantities are all constant, the variation of stick position throughout the speed range can be found from eqn. (6.2), (6.12) and (6.15)

$$x_s = \frac{1}{a_2 m_e}\left[\frac{C_{M_0}}{V_T} - (a_1\eta_T + a_3\beta) - \frac{K_n}{V_T}\cdot\frac{W}{\frac{1}{2}\rho_0 V_i^2 S}\right]. \quad (6.17)$$

As one would expect, differentiating this expression leads to the same result as eqn. (6.16) and letting V_i tend to infinity (which corresponds to $C_L \rightarrow 0$) leads to a value corresponding to η_0 given by eqn. (6.4).

Bearing in mind the signs of the various quantities in eqn. (6.17), the variation of stick position with airspeed will be as shown in Fig. 6.5.

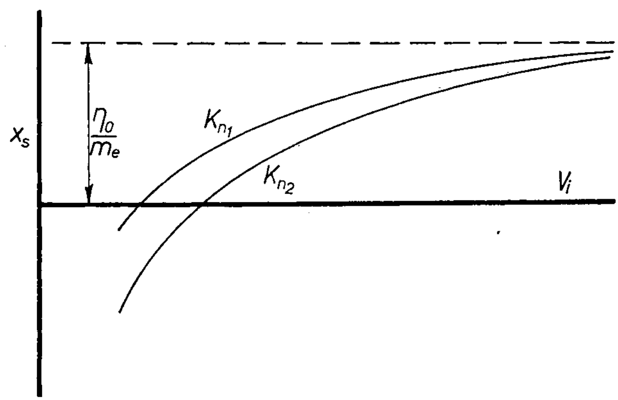

FIG. 6.5. Stick position-to-trim as a function of equivalent airspeed for two values of the static margin.

It should be remembered that all the above theory relates only to steady stick positions under equilibrium conditions: it does not indicate how the stick should be moved in order to *initiate* a change of speed. Figure 6.5 shows that as an aeroplane, statically stable stick-fixed, is flown at a series of equilibrium conditions corresponding to progressively increasing airspeeds, it will be found that at each speed the stick position is further forwards.

Whilst this situation seems entirely logical, the pilot is not usually very conscious of it except in extreme circumstances; he is more inclined to notice variations of stick-force with speed, and stick movements and forces to initiate a change of speed. In fact, it is entirely possible, and permitted by British Civil Airworthiness Requirements, to fly an aeroplane which is slightly unstable stick-fixed provided that stick-forces occur in the expected sense. This matter will be examined in the next sections of this chapter.

Finally, it may seem rather paradoxical that the variation of elevator angle with speed is related to the "stick-fixed" stability. The explanation lies in the sentence following eqn. (6.8).

Tab Angles to Trim

The theory in the previous section assumes that the tab setting β remains fixed as the aeroplane is flown at different speeds. Under these circumstances, the stick-force will only be zero at one particular speed: at lower speeds, the pilot will have to exert a pull-force, at higher speeds a push.

Now the function of the trim-tab is to enable the pilot to make the stick force zero at any desired speed. Instead of applying a moment manually to balance the elevator hinge moment, he applies an aerodynamic moment. In considering the variation of tab angle with speed to keep $C_H = 0$, it should be remembered that the elevator is always floating freely and hence it would be expected that the relationship between tab angle and airspeed (or C_L) would depend on the stick-free stability.

Consider the variation of pitching moment coefficient with C_L at a given value of β, the elevator being free. This is described by eqn. (5.26)

$$C_{M_G} = C_{M_0} + (h - h_0)C_L - \bar{V}_T\left[\frac{\bar{a}_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right)C_L + \bar{a}_1\eta_T + \bar{a}_3\beta\right].$$

This may be written somewhat differently by introducing eqn. (5.20)

$$K_n' = (h_0 - h) + \bar{V}_T\frac{\bar{a}_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right),$$

whence

$$C_{M_G} = C_{M_0} - C_L K_n' - \bar{V}_T(\bar{a}_1 \eta_T + \bar{a}_3 \beta), \tag{6.18}$$

which is analogous to eqn. (6.1) in the stick-fixed case.

For a given value of β, this expression gives a C_{M_G}–C_L curve as shown in Fig. 6.6.

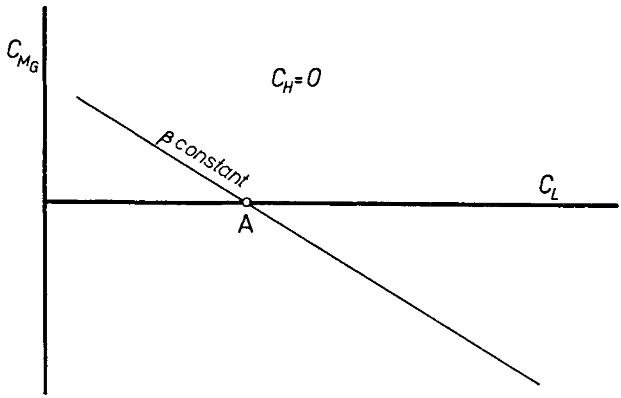

FIG. 6.6. Pitching moment coefficient curve with elevator free at constant tab angle.

At all points on this curve, C_H must be zero since this condition is implicit in eqn. (6.18), and at A, C_{M_G} will also be zero. At the value of C_L corresponding to A, the aeroplane will be in trim with zero stick-force.

Similar lines can be drawn for other values of β (see Fig. 6.7) and the aeroplane will be in trim (i.e. $C_{M_G} = 0$) at the lift coefficients corresponding to points A, B and C for tab angles β_1, β_2 and β_3 respectively. From eqn. (6.18), it is clear that β_1, β_2 and β_3 represent increasing values of β, i.e. deflection increasing in the downward sense.

The tab angle to trim at a certain C_L can be found by putting $C_{M_G} = 0$ in eqn. (6.18), whence

$$\beta_{trim} = \frac{1}{\bar{a}_3}\left[\frac{C_{M_0} - C_L K_n'}{\bar{V}_T} - \bar{a}_1 \eta_T\right]. \tag{6.19}$$

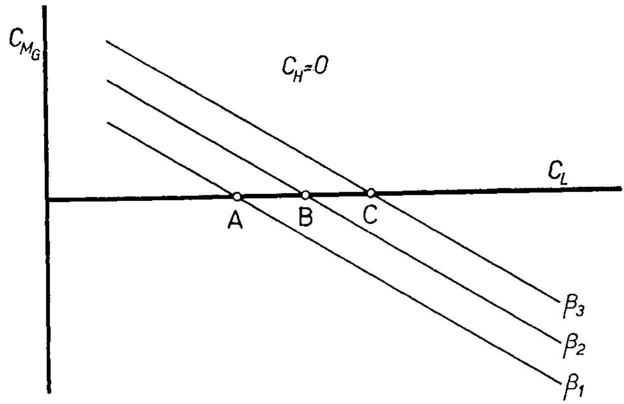

FIG. 6.7. Pitching moment coefficient curves with elevator free at three values of the tab angle.

If, from Fig. 6.7 tab angles to trim are plotted against corresponding lift coefficients, a line is obtained whose equation is (6.19).

It should be noted that for an aeroplane which is statically stable stick-free, the slope of this line is positive and is

$$\frac{d\beta_{trim}}{dC_L} = -\frac{K_n'}{\bar{a}_3 \bar{V}_T}. \tag{6.20}$$

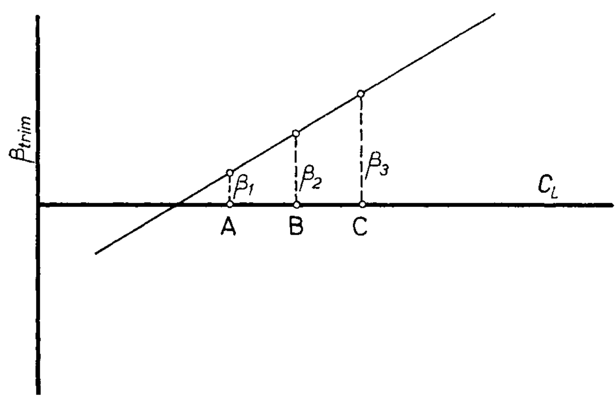

FIG. 6.8. Tab angles-to-trim as a function of lift coefficient.

$\bar{a}_3$ (see eqn. (5.12)) is normally negative: it represents the variation of tail lift coefficient with tab angle, the elevator being free, and the major contribution arises from the elevator deflection in the negative sense which is consequential upon a positive tab deflection.

It also follows from eqn. (6.19) that if the β_{trim}–C_L curve is extrapolated to $C_L = 0$, the corresponding tab angle, β_0, is independent of K_n'

$$\beta_0 = \frac{1}{\bar{a}_3}\left[\frac{C_{M_0}}{\bar{V}_T} - \bar{a}_1\eta_T\right]. \tag{6.21}$$

Hence, if "trim curves" such as Fig. 6.8 are drawn for different values of K_n', their slopes will be proportional to K_n' and they will pass through the same point when extrapolated to $C_L = 0$ (Fig. 6.9).

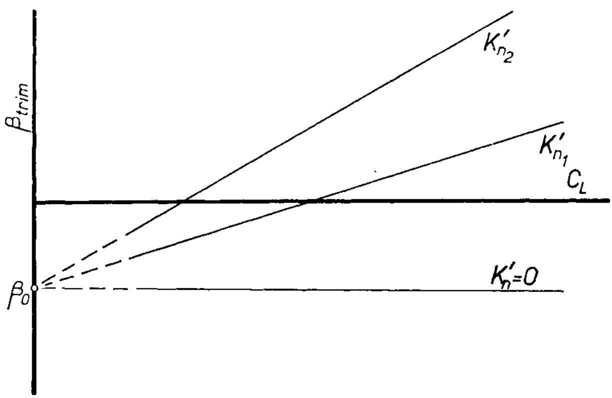

FIG. 6.9. Curves of tab angle-to-trim as a function of lift coefficient for two values of the stick-free static margin.

It will be seen that the relationship between the tab angle–lift coefficient curves and the stick-fixed static margins is very similar to that between the elevator angle–lift coefficient curves and the stick-fixed static margins although the constants involved are naturally different.

Again, the relationship expressed in eqn. (6.20) is not only true in the simple linear situation assumed in the above theory but in the more general case. A discussion similar to that leading to eqn. (6.8) applies, and so once more

$$\delta C_{M_G} = \bar{V}'\Delta C_{L_T}, \tag{6.22}$$

where δC_{M_G} is the pitching moment coefficient which occurs when the lift coefficient of the aeroplane is altered from an initial trimmed value C_L to $C_L+\delta C_L$, now with the elevator free, and ΔC_{L_T} is the change in tail lift coefficient which must be applied to trim the aeroplane at $C_L+\delta C_L$. The term ΔC_{L_T} now results from a change in tab angle with the elevator free.

ΔC_{L_T} may be written $(\partial C_{L_T}/\partial\beta)\delta\beta_{trim}$, where the partial derivative implies constant lift coefficient. Under these conditions, its value will be obtained from eqn. (5.25), so that

$$\Delta C_{L_T} = \frac{1}{1+\bar{F}}\bar{a}_3\,\delta\beta_{trim}. \tag{6.23}$$

$\bar{a}_3$ and $\bar{F}$ need not be strictly constant provided that the values taken in eqn. (6.23) are appropriate to the prevailing conditions. Equations (6.22) and (6.23) give

$$\delta C_{M_G} = \frac{\bar{V}'}{1+\bar{F}}\bar{a}_3\,\delta\beta_{trim},$$

i.e.

$$K_n' = -\frac{\mathrm{d}C_{M_G}}{\mathrm{d}C_L} = -\bar{V}_T\bar{a}_3\frac{\mathrm{d}\beta_{trim}}{\mathrm{d}C_L}, \tag{6.24}$$

which is the result given by eqn. (6.20).

If desired, an expression analogous to eqn. (6.14) can be obtained, giving the variation of $\mathrm{d}\beta_{trim}/\mathrm{d}V_i$ with equivalent airspeed.

Stick-force to Change Speed

The above analysis assumes that when the pilot wishes to change speed by a small amount, he makes a small adjustment to the tab angle so that $C_H = 0$ in both the initial and the final states. However, he could produce a small change in speed by moving the elevator the appropriate amount and holding the hinge moment which would then occur by applying a force to the stick instead of cancelling out this moment by use of the trim tab. One would expect that the small hinge moment would be simply related to the trimmer movement required to cancel it out. The pilot's impression of the "heaviness" of the elevator is largely related to the stick-force required to change speed by a small amount from an initially trimmed condition.

The various conditions are therefore as follows.

Initially: Aeroplane lift coefficient: C_L
Tail lift coefficient: C_{L_T}
Elevator angle: η
Tab angle: β

$$C_{M_G} = 0, \qquad C_H = 0.$$

Finally: Aeroplane lift coefficient: $C_L + \delta C_L$
Tail lift coefficient: $C_{L_T} + \delta C_{L_T}$

$$C_{M_G} = 0,$$

and either (a) Elevator angle: $\eta + \delta\eta$
Tab angle: $\beta + \delta\beta$

$$C_H = 0,$$

or (b) Elevator angle: $\eta + \delta\eta + \Delta\eta$
Tab angle: β
and the pilot applies a stick-force to balance a hinge moment δC_H.

The elevator angles in cases (a) and (b) are slightly different in order to satisfy the condition that the tail lift coefficients shall be the same in both cases despite the different tab angles. The values of α_T are, of course, the same.

Hence

$$\begin{aligned} C_{L_T} + \delta C_{L_T} &= a_1 \alpha_T + a_2(\eta + \delta\eta) + a_3(\beta + \delta\beta) \\ &= a_1 \alpha_T + a_2(\eta + \delta\eta + \Delta\eta) + a_3 \beta, \end{aligned}$$

i.e.

$$a_3 \delta\beta = a_2 \Delta\eta, \tag{6.25}$$

and the hinge moment coefficients are:

(a) $$0 = b_1 \alpha_T + b_2(\eta + \delta\eta) + b_3(\beta + \delta\beta);$$

(b) $$\delta C_H = b_1 \alpha_T + b_2(\eta + \delta\eta + \Delta\eta) + b_3 \beta;$$

i.e.

$$\delta C_H = b_2 \Delta\eta - b_3 \delta\beta. \tag{6.26}$$

Eliminating $\Delta\eta$,

$$\delta C_H = \left(\frac{b_2 a_3 - a_2 b_3}{a_2}\right) \delta\beta$$

$$= \frac{b_2}{a_2} \bar{a}_3 \delta\beta, \tag{6.27}$$

or

$$\frac{d\beta_{trim}}{dC_L} = \frac{a_2}{b_2 \bar{a}_3} \left(\frac{dC_H}{dC_L}\right)_0. \tag{6.28}$$

where the suffix "0" signifies an initially trimmed condition with zero stick-force.

Combining eqn. (6.24) and (6.28)

$$\left(\frac{dC_H}{dC_L}\right)_0 = -\frac{b_2}{a_2} \frac{K_n'}{\bar{V}_T}. \tag{6.29}$$

As would be expected, the C_H–C_L gradient under initially trimmed conditions is proportional to the stick-free static margin.

Now the elevator hinge moment is given by

$$H = C_H \tfrac{1}{2} \rho_0 V_i^2 S_\eta c_\eta, \tag{6.30}$$

and if the stick-elevator gearing is as defined by eqn. (6.15), it follows by considering the work done in displacing the stick that $P_e x_s = -H\eta$, and hence

$$P_e = -m_e H, \tag{6.31}$$

where P_e is the force to be applied to the stick by the pilot, a positive value representing a push.

The pilot is interested in the stick-force/speed gradient at an initially trimmed condition. From eqn. (6.30) and (6.31), in general

$$\frac{dP_e}{dV_i} = -m_e \tfrac{1}{2} \rho_0 S_\eta c_\eta \left[2 C_H V_i + V_i^2 \frac{dC_H}{dV_i}\right],$$

but, in the initially trimmed condition with zero stick-force, $C_H = 0$ and hence

$$\left(\frac{\mathrm{d}P_e}{\mathrm{d}V_i}\right)_0 = -\, m_e \tfrac{1}{2}\rho_0 V_{i_0}^2 S_\eta c_\eta \left(\frac{\mathrm{d}C_H}{\mathrm{d}V_i}\right)_0,$$

where V_{i_0} is the equivalent airspeed at which $C_H = 0$.

Introducing eqn. (6.13), this may finally be put in the form

$$\left(\frac{\mathrm{d}P_e}{\mathrm{d}V_i}\right)_0 = m_e \tfrac{1}{2}\rho_0 V_{i_0}^2 S_\eta c_\eta \left(\frac{4W}{\rho_0 V_{i_0}^3 S}\right)\left(\frac{\mathrm{d}C_H}{\mathrm{d}C_L}\right)_0$$

$$= \frac{2m_e S_\eta c_\eta W}{V_{i_0} S}\left(\frac{\mathrm{d}C_H}{\mathrm{d}C_L}\right)_0. \tag{6.32}$$

Eliminating $(\mathrm{d}C_H/\mathrm{d}C_L)_0$ from eqn. (6.32) and (6.29) the stick-force/speed gradient is finally

$$\left(\frac{\mathrm{d}P_e}{\mathrm{d}V_i}\right)_0 = -\frac{b_2}{a_2} \cdot \frac{2m_e S_\eta c_\eta W}{S\bar{V}_T} \cdot \frac{K_n'}{V_{i_0}}. \tag{6.33}$$

It is important to stress that this expression represents the stick-force/speed gradient at an initially trimmed condition with zero stick-force: it does not apply if the pilot is already applying an appreciable stick-force.

Some light aeroplanes and gliders are not fitted with a trim tab, and it is then useful to know how the stick-force varies with speed over the whole range. The result obtained will also apply to an aeroplane in which the tab setting is left constant, so that the stick-force is only zero at one particular speed. The result cannot be obtained by integrating eqn. (6.33), because this result only applies when the stick-force is zero.

Equation (6.29) applies throughout the speed range, whether the elevator hinge moment is zero or not. As written, it seems to apply only when $C_H = 0$, but this is really a consequence of deriving it via considerations of tab angle. Taking eqn. (6.8) as the starting point, it is clear that the subsequent argument (in particular eqn. (6.26)) is not invalidated if any hinge moment coefficient C_H is superimposed on the small changes under consideration.

Equation (6.29) can therefore be written quite generally as

$$\frac{dC_H}{dC_L} = -\frac{b_2}{a_2}\frac{K_n'}{\bar{V}_T},$$

and this can be integrated to give

$$C_H = -\frac{b_2}{a_2}\frac{K_n'}{\bar{V}_T}(C_L + C_{L_0}), \tag{6.34}$$

where C_{L_0} is a constant and is in fact the value of C_L at which $C_H = 0$. Substituting for C_H and C_L from eqn. (6.30) and (6.12), and introducing (6.31), with a little rearrangement, this becomes

$$P_e = \frac{b_2}{a_2}\frac{K_n'}{\bar{V}_T}\frac{W}{S} m_e S_\eta c_\eta \left(1 - \frac{V_i^2}{V_{i_0}^2}\right), \tag{6.35}$$

where, again, V_{i_0} is the equivalent airspeed at which $P_e = 0$. The stick-force/speed gradient for a *constant* trimmer setting is obtained by differentiating eqn. (6.35)

$$\frac{dP_e}{dV_i} = -\frac{b_2}{a_2}\frac{K_n'}{\bar{V}_T}\frac{W}{S} m_e S_\eta c_\eta \left(\frac{2V_i}{V_{i_0}^2}\right). \tag{6.36}$$

Putting $V_i = V_{i_0}$ naturally gives the same result as eqn. (6.33). Hence, at a fixed trimmer setting the stick-force/speed gradient is proportional to the equivalent airspeed. If the trimmer is adjusted so that $C_H = 0$ at a series of speeds, the stick-force/speed gradient at a particular speed will be inversely proportional to the equivalent airspeed. This situation is illustrated by Fig. 6.10.

An alternative form of eqn. (6.35) is

$$C_{P_e} = \frac{P_e}{\frac{1}{2}\rho_0 V_i^2 S_\eta} = \frac{b_2}{a_2}\frac{K_n'}{\bar{V}_T} m_e c_\eta (C_L - C_{L_0}). \tag{6.37}$$

For a given value of C_{L_0}, plots of stick-force coefficient C_{P_e} against lift coefficient are therefore straight lines whose gradients are proportional to the stick-free static margin. They are, in fact, exactly analogous to tab angle–lift coefficient trim curves. If extrapolated to zero-lift coefficient, the intercept on the C_{P_e} axis will be proportional to $K_n' C_{L_0}$. If lines are plotted for different values of K_n', they will only pass through the same point at $C_L = 0$

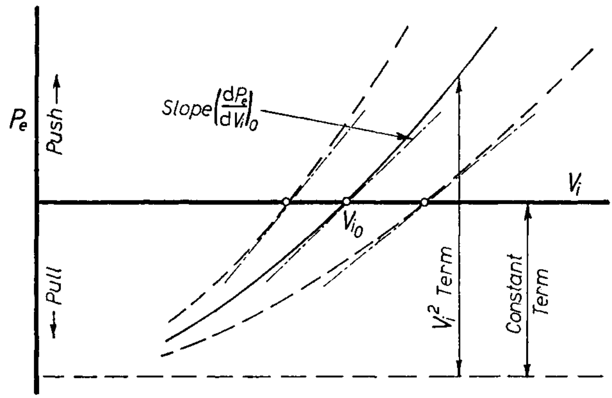

FIG. 6.10. Curves of stick-force as a function of equivalent airspeed for a given stick-free static margin. Each curve corresponds to a fixed tab angle.

if $K_n'C_{L_0}$ is the same for each line. It follows from eqn. (6.19) that this condition is equivalent to requiring the tab setting to be the same under all conditions. If the aeroplane is not fitted with a tab, this condition is effectively satisfied automatically, in that the values of C_{L_0} corresponding to the different c.g. positions giving the required values of K_n' will automatically adjust themselves so that $K_n'C_{L_0}$ is constant.

These features of the relationship between stick-force and speed may become important when flight-testing certain types of aircraft, as explained in Chapter 7.

In practice, plots of C_{P_e} against C_L may not be quite linear but, for the same reasons which apply to tab and elevator angle curves, the slope at a given C_L is still proportional to the stick-free static margin at that C_L.

Finally, it is important to stress once more that the preceding discussion of tab angles and stick-forces relates entirely to equilibrium conditions, although expressions such as "stick-force to change speed" may seem to imply some connection with the unsteady process of initiating a speed change. This is not so: the gradients given by expressions such as (6.24), (6.29), (6.33) and (6.36) all apply to curves which represent a series of steady flight conditions. For example, "stick-force to change speed" is only intended to indicate the difference in stick-force required to achieve equilibrium when the aeroplane is flown first at one steady speed and then at some slightly different steady speed. The process whereby the alteration in speed takes place is not of concern in this context.

CHAPTER 7

Flight Tests to Measure Static Stability

Stick-fixed Stability

In essence, the determination of the stick-fixed neutral point consists of finding the c.g. position for which the slope of the η_{trim}–C_L curve becomes zero. In general, it is not usually necessary to achieve this actual condition in flight, since it can be found by extrapolation from flight tests at more forward c.g. positions. Throughout this chapter, it will be assumed that the aeroplane is in level, or nearly level, flight.

A test is conducted by flying the aeroplane at a series of steady speeds, at a given c.g. position. At each speed, the elevator angle required to trim the aircraft is observed, and is plotted against the lift coefficient corresponding to the observed speed. A curve of η_{trim} against C_L is therefore obtained for the chosen c.g. position. The test is then repeated for another c.g. position. If the aeroplane is fitted with a trim tab, it is usual to combine this test with the determination of stick-free neutral point by adjusting the tab to give zero stick-force at each speed, and observing both the tab and elevator angles. Since the theory in the previous chapter, relating the slope of the η_{trim}–C_L curve to the static margin, assumes constant tab angle, the observed elevator angles must be corrected to allow for the variation of β_{trim} with C_L.

Suppose that the observed values of the elevator and tab angles to trim at a given C_L are η_{obs} and β_{obs} respectively. Then, if the corrected elevator angle corresponding to $\beta = 0$ is η_{corr}, it is related to the observed values by the condition that the tail lift must be the same under both the observed and corrected conditions, i.e.

$$a_2\eta_{obs}+a_3\beta_{obs} = a_2\eta_{corr},$$

or

$$\eta_{corr} = \eta_{obs} + \frac{a_3}{a_2}\beta_{obs}. \tag{7.1}$$

In order to apply the correction; a_3/a_2 must therefore be determined. At a given speed and c.g. position, the tab is moved as far in one direction as is conveniently possible, with the pilot applying a stick-force to resist the out-of-balance hinge moment. The elevator and tab angles are observed, η_1 and β_1, say. The tab is then moved as far as is conveniently possible in the opposite direction at the same speed, again with the pilot applying a suitable stick-force. The elevator and tab angles, η_2 and β_2 respectively, are again observed. Once more, these angles are related by the condition that the tail lift must be the same in both cases and therefore

$$a_2\eta_1+a_3\beta_1 = a_2\eta_2+a_3\beta_2,$$

and hence

$$\frac{a_3}{a_2} = \frac{\eta_1-\eta_2}{\beta_2-\beta_1}. \tag{7.2}$$

Under the simplest conditions corresponding to the linear theory of the previous chapter, the curves of the corrected values of η_{trim} against lift coefficient will be straight lines passing through the same point when extrapolated to $C_L = 0$ (see Fig. 7.1).

The slopes of these lines are plotted against c.g. position h (see Fig. 7.2), and it follows from eqn. (6.3) that a straight line through the plotted points is given by

$$\frac{d\eta_{trim}}{dC_L} = -\frac{K_n}{a_2 V_T}.$$

If the line is extrapolated to the h axis, the intercept corresponds to $K_n = 0$, and hence to $h = h_n$. This intercept therefore gives the stick-fixed neutral point. It may also be of interest to note that the slope of this line is $-1/a_2 V_T$.

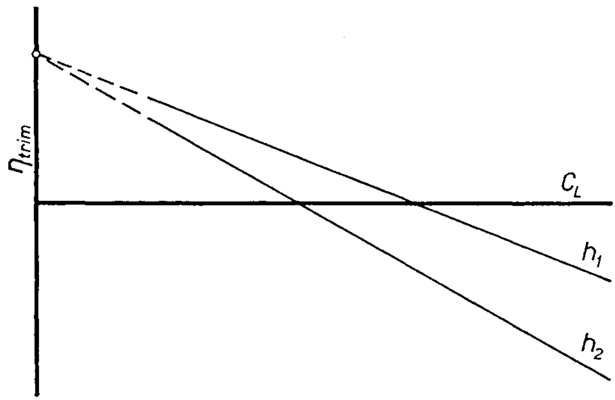

FIG. 7.1. Curves of elevator angle-to-trim as a function of lift coefficient for two c.g. positions.

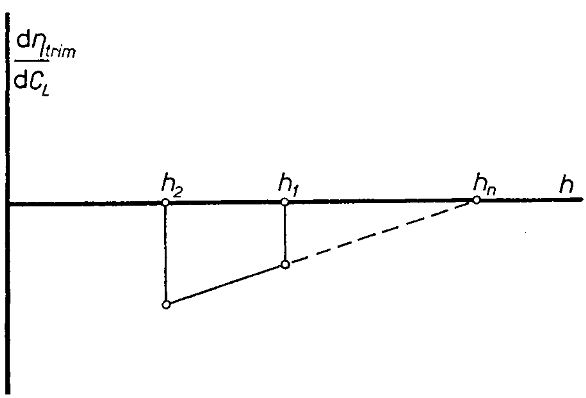

FIG. 7.2. Extrapolation of the slopes of the elevator angle-to-trim curves to determine the stick-fixed neutral point.

As was noted in the previous chapter, the curves of corrected η_{trim} against C_L are rarely straight lines in practice, but eqn. (6.3) still applies at a given C_L. Given trim curves at two c.g. positions as in Fig. 7.3, the slopes of the two curves are measured at a given value of C_L.

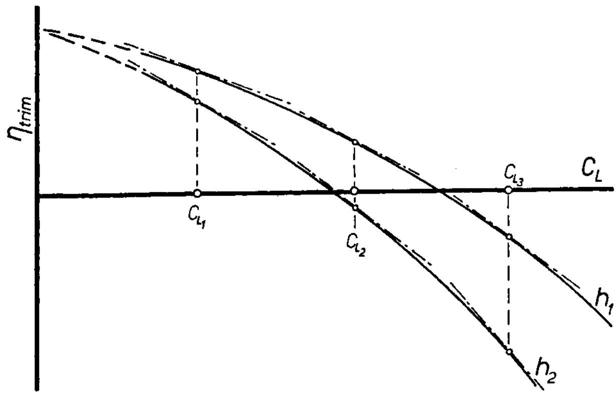

FIG. 7.3. Non-linear elevator angle-to-trim curves. The slopes of the curves are to be obtained at various values of the lift coefficient.

By plotting these slopes against the corresponding values of h and extrapolating to the h axis, the stick-fixed neutral point position is obtained for this value of C_L. When the η_{trim}–C_L curves are non-linear, the procedure described above gives the neutral point position because the static margin remains a linear function of h if all other quantities are fixed. However, the static margins can only be deduced by further analysis beyond the scope of this book. Under these conditions, the distance between the c.g. and the neutral point, which is defined as the c.g. margin, is no longer equal to the static margin defined as $-(dC_{M_G}/dC_L)$. Similar remarks apply to the stick-free analysis considered below.

The procedure may be repeated for other values of the lift coefficient, and finally h_n may be obtained as a function of C_L (see Figs. 7.4 and 7.5).

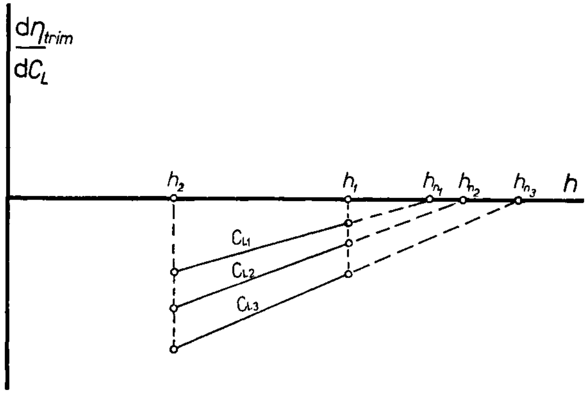

FIG. 7.4. Slopes of the elevator angle-to-trim curves plotted as a function of c.g. position for the chosen lift coefficients. Extrapolation to the h axis gives the neutral point position for each value of C_L.

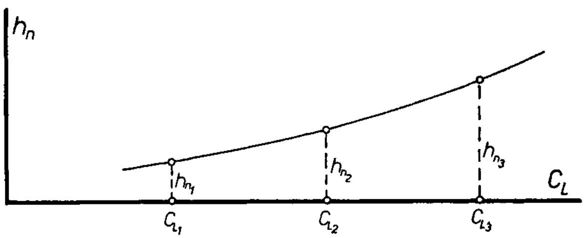

FIG. 7.5. Stick-fixed neutral point position as a function of lift coefficient.

The effects of Mach number have not been explicitly mentioned previously, but it is worth noting here that if the effects of compressibility are appreciable, the relationship between Mach number and C_L should be made the same at both c.g. positions. h_n is then obtained as a function of C_L and of the corresponding Mach numbers. If the whole process is repeated at a series of different heights, giving different relationships between C_L and **M**, the dependence of h_n on both C_L and **M** can be established.

Stick-free Stability

The procedure described above also enables curves of β_{trim} against C_L to be plotted for two c.g. positions. Again, simple theory suggests that straight lines should be obtained passing through the same point when extrapolated to $C_L = 0$. If the slopes of these lines are plotted against c.g. position h it follows from eqn. (6.20) that a straight line through the plotted points is given by

$$\frac{d\beta_{trim}}{dC_L} = -\frac{K_n'}{\bar{a}_3 \bar{V}_T}.$$

If the line is extrapolated to the h axis, the intercept corresponds to $K_n' = 0$ and hence to $h = h_n'$. This intercept therefore gives the

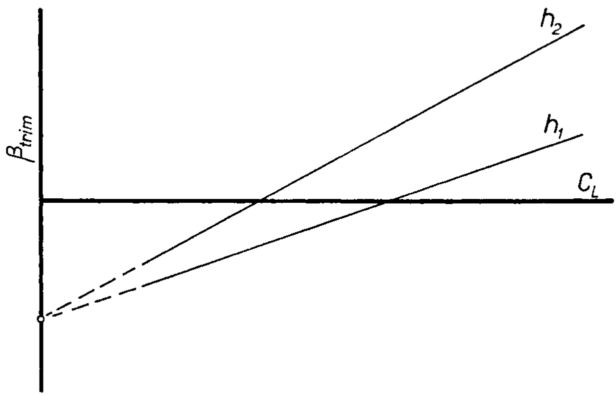

FIG. 7.6. Curves of tab angle-to-trim as a function of lift coefficient for two c.g. positions.

stick-free neutral point, and the gradient of the line is $-1/\bar{a}_3\bar{V}_T$ where $\bar{a}_3$ is normally negative (see Figs. 7.6 and 7.7).

As in the case of the elevator angle curves, the β_{trim}–C_L curves are rarely straight in practice, but eqn. (6.20) still applies at a given C_L. By following a procedure similar to that described above, h_n' can be obtained as a function of C_L (see Figs. 7.8, 7.9 and 7.10).

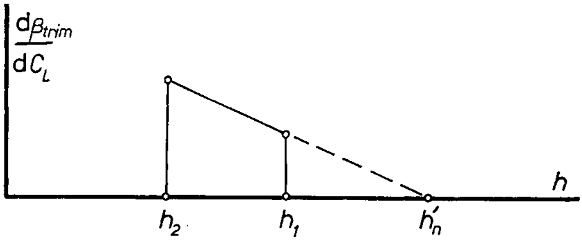

FIG. 7.7. Extrapolation of the slopes of the tab angle-to-trim curves to determine the stick-free neutral point.

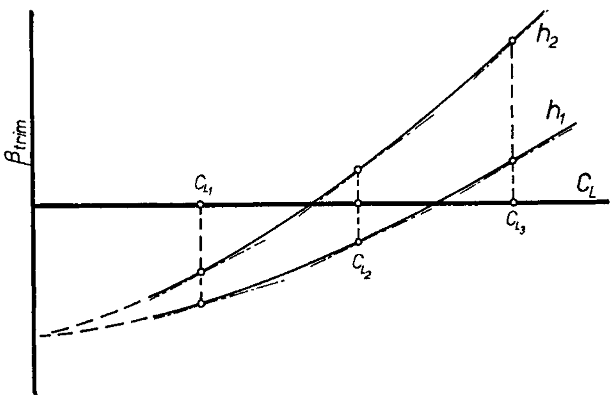

FIG. 7.8. Non-linear tab angle-to-trim curves. The slopes of the curves are to be determined at various values of the lift coefficient.

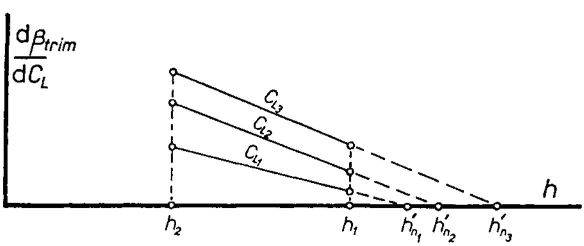

FIG. 7.9. Slopes of the tab angle-to-trim curves plotted as a function of c.g. position for the chosen lift coefficients. Extrapolation to the h axis gives the neutral point position for each value of C_L.

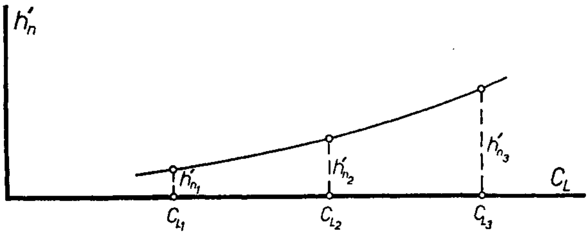

FIG. 7.10. Stick-free neutral point position as a function of lift coefficient.

In the most general case, this procedure can again be elaborated to give h'_n as a function of both C_L and **M**.

The above explanations assume that elevator and tab angles can be measured to a reasonable degree of accuracy. It is usual to do so by attaching a "Desynn" transmitter to the controls by means of a simple linkage and observing the appropriate indicators in the cockpit, having carried out a suitable calibration. It might be thought that it would be sufficient to observe the positions of the stick and trimmer control lever in the cockpit, particularly since their movements come to the pilot's notice rather than the control angles. Measurements of stick position relative to, say, the instrument panel are simple to perform and avoid the complication of installing the Desynn circuit. In practice, measurement of stick position may be reasonably satisfactory if the aircraft can be trimmed to give zero stick-force at each speed. If tests are conducted with an out-of-balance elevator hinge moment which has to be held by a stick-force (see below), any stretch in the elevator system gives greater stick movements for given elevator deflections than those corresponding to a static calibration. The slopes of the elevator trim curves are therefore somewhat exaggerated, by an amount which depends primarily on the stick-free static margin. The deduced position of the stick-fixed neutral point is therefore likely to be rather inaccurate. Moreover, any measurements of stick position, whatever the detailed technique of the tests, are unlikely to give consistent results from one day to another due to the differential effects of temperature changes on the fuselage structure and the elevator circuit. Such effects are particularly marked when the stick is connected to the elevator by means of cables. It is therefore very desirable to observe elevator angle rather than stick position and the installation of the appropriate equipment is usually not too difficult even in small aircraft or gliders.

Measurement of tab angle is likely to be much more difficult. In small aircraft, it is usually impracticable to mount a transmitter at the tab itself, and since tab actuating circuits are generally subject to noticeable amounts of stretch and backlash, measurements taken elsewhere in the circuit are rarely reliable. It is generally more satisfactory to deduce the stick-free neutral point position from measurements of stick-force.

Measurements of Stick-force

In principle such measurements may be made in two ways: either by trimming the aircraft by means of the tab to give zero stick-force at a series of given speeds and observing the stick-force required to produce a change of speed of, say, 10 knots or by fixing the trim tab at one setting and measuring the stick-force required to hold the aircraft at a series of steady speeds.

The first method effectively gives the stick-force/speed gradient directly, as in eqn. (6.33), but since the forces involved are likely to be small they are difficult to measure accurately. Friction in the elevator circuit tends to introduce relatively large errors, particularly in gliders where there is no engine vibration. It is generally more satisfactory to use the second method, plotting curves of C_{P_e} against C_L (see eqn. (6.37)), and remembering that the tab-setting must be the same at each c.g. position. If the aircraft is not fitted with a tab, this procedure is obviously the only one which can be used. Again, the slopes of these curves are plotted against h at given values of C_L and linear extrapolation to the h axis gives the stick-free neutral points corresponding to the appropriate values of C_L. The construction is exactly analogous to that relating to tab angles as described above.

A consequence of the way in which stick-force varies with speed is that qualitative tests to assess the stick-free stability may be misleading once the stick-free static margin is no longer constant for a given c.g. position. If it is a function of C_L (or V_i at a given weight) eqns. (6.34)–(6.37) no longer apply, since they all assume constant K_n'. Only the general form of eqn. (6.29) remains applicable. It has previously been shown (see p. 66 and p. 68) that, if the stick-force is not initially zero, then

$$\frac{dP_e}{dV_i} = -m_e \tfrac{1}{2}\rho_0 S_\eta c_\eta \left[2C_H V_i + V_i^2 \frac{dC_H}{dV_i}\right]$$

$$= -m_e \tfrac{1}{2}\rho_0 S_\eta c_\eta \left[2C_H V_i - \frac{4W}{\rho_0 V_i S}\frac{dC_H}{dC_L}\right]. \tag{7.3}$$

and

$$\frac{dC_H}{dC_L} = -\frac{b_2}{a_2}\frac{K_n'}{\bar{V}_T} \quad \text{in all circumstances.} \tag{7.4}$$

Now, for a stable aeroplane, dP_e/dV_i is positive (i.e. push to go faster). It therefore follows that if the term in square brackets in eqn. (7.3) is negative, the aeroplane will feel as if it is stable, stick-free. Now if K_n' becomes negative, dC_H/dC_L becomes positive, but if C_H is already negative (i.e. if the pilot is pushing on the stick) and of sufficient magnitude, the term in square brackets may remain negative. In other words, the increase in stick-force due to the V_i^2 effect outweights the decrease due to the negative static margin. It therefore follows that neither the slope of the stick-force/speed curve, nor the sign of the force, is necessarily any guide to the sign of the static margin: an aeroplane which feels stable at a certain speed V_{i_1} in an out-out-trim condition (and which will tend to revert to the trimmed speed V_{i_0} if the stick is released) may suffer a reversal of the stick-force/speed slope when it is trimmed at speed V_{i_1}. It therefore follows that the stick-free stability can only be assessed by plotting dimensionless curves of C_{P_e} against C_L.

Whatever method is used, friction in the elevator circuit tends to introduce errors into measurements of stick-force. The second method described above has the advantage that the force over much of the speed range is likely to be relatively large, so that the percentage error is reduced. Since a large number of points can be obtained at a given tab setting, it is relatively easy to draw a reasonable mean curve.

Finally, the whole of this chapter has been written as if measurements at two c.g. positions provide enough information to determine neutral points, etc., as theory suggests. In practice, it is prudent to obtain confirmatory data by tests at a third c.g. position.

CHAPTER 8

The Effect of a Weight or Spring in the Elevator Circuit

SO FAR, it has been assumed that the elevator hinge moment is produced solely by aerodynamic means: this implies that the c.g. of the elevator lies on the hinge-line, so that there is no weight moment. It is often desirable that this condition should be fulfilled in order to avoid flutter, and weights are attached to the elevator, forward of the hinge-line, in order to balance the weight of the control surface. However, full mass-balance of this type may not be required as an anti-flutter measure on slow-speed aircraft (e.g. British Civil Airworthiness Requirements do not call for it on gliders having a Design Dive Speed less than 130 knots), and there may then be an out-of-balance weight moment to be added to the aerodynamic moment. It will be shown that such a moment acting in the positive (i.e. elevator-down) sense moves the stick-free neutral point aft and it may therefore be desirable to introduce such a moment deliberately. Alternatively, a mechanical moment may be introduced by connecting a spring to the elevator circuit (see Fig. 8.1).

So far as the stick-free static stability is concerned, the method of producing the mechanical moment is of no consequence. In the analysis which follows, it will be assumed that the moment is independent of elevator deflection, as would occur if it were due to a long spring or a weight connected by a suitable linkage. It is clear that such arrangements do not affect the stick-fixed stability, since hinge moments are of no consequence in the stick-fixed case.

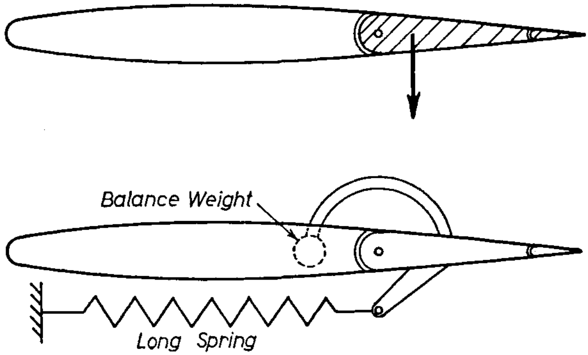

FIG. 8.1. Mechanical elevator moments due to an unbalanced weight moment (above) and a spring (below).

Under the assumed conditions, the total elevator hinge moment becomes

$$H_{tot} = H_s + C_H \tfrac{1}{2}\rho_0 V_i^2 S_\eta c_\eta, \tag{8.1}$$

where H_s is the mechanical moment, and the aerodynamic moment is given by the second term in the usual way.

In coefficient form

$$C_{H_{tot}} = \frac{H_s}{\tfrac{1}{2}\rho_0 V_i^2 S_\eta c_\eta} + C_H. \tag{8.2}$$

As explained in Chapter 3, static margins are always found subject to the condition that the vertical forces are in equilibrium. In the analysis given in Chapter 5, it is not in fact necessary to invoke this condition but it is convenient to do so here in order to express H_s in a suitable dimensionless form. Under these conditions, in steady level flight,

$$\tfrac{1}{2}\rho_0 V_i^2 = \frac{W}{C_L S}. \tag{8.3}$$

Substituting eqn. (8.3) and (2.35) into (8.2),

$$C_{H_{tot}} = \left[\frac{H_s}{W c_\eta} \cdot \frac{S}{S_\eta}\right] C_L + b_1 \alpha_T + b_2 \eta + b_3 \beta. \tag{8.4}$$

The quantity in brackets is a constant for a given aeroplane at a given weight, and will therefore be given the symbol v.

If the aircraft is in trim with zero stick-force, $C_{H_{tot}} = 0$, and hence

$$\eta = -\left(\frac{vC_L + b_1\alpha_T + b_3\beta}{b_2}\right), \tag{8.5}$$

which differs from eqn. (5.9) by the term $-vC_L/b_2$.

Since the expression for C_{L_T} contains a term $a_2\eta$, it follows that eqn. (5.10) (giving C_{L_T} with the elevator free) must be modified by adding $-a_2 vC_L/b_2$, i.e.

$$C_{L_T} = a_1\left(1 - \frac{a_2 b_1}{a_1 b_2}\right)\alpha_T + a_3\left(1 - \frac{a_2 b_3}{a_3 b_2}\right)\beta - \frac{a_2 vC_L}{b_2}$$

$$= \bar{a}_1\alpha_T + \bar{a}_3\beta - \frac{a_2 vC_L}{b_2}. \tag{8.6}$$

This additional term is carried through the subsequent calculations, and the expression corresponding to eqn. (5.25), giving C_{L_T} as a function of C_L, becomes

$$C_{L_T}(1+\bar{F}) = \frac{\bar{a}_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right)C_L + \bar{a}_1\eta_T + \bar{a}_3\beta - \frac{a_2 vC_L}{b_2}. \tag{8.7}$$

Differentiating this expression with respect to C_L and substituting in eqn. (4.13), the stick-free static margin becomes

$$K_n' = (h_0 - h) + \bar{V}_T\left[\frac{\bar{a}_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right) - \frac{a_2}{b_2}v\right]. \tag{8.8}$$

Since, in general, a_2 is positive and b_2 is negative, $-a_2 v/b_2$ has the same sign as v. Hence a positive H_s (i.e. elevator-down) increases the static margin stick-free by $(-a_2 v\bar{V}_T/b_2)$, i.e. the stick-free neutral point is moved aft by this amount. Conversely, a negative H_s (elevator-up), moves the neutral point forwards.

It also follows that the equation for the total pitching moment, equivalent to eqn. (5.26), is

$$C_{M_G} \doteq C_{M_0} + (h - h_0)C_L$$

$$-\bar{V}_T\left[\frac{\bar{a}_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right)C_L + \bar{a}_1\eta_T + \bar{a}_3\beta - \frac{a_2 vC_L}{b_2}\right]. \tag{8.9}$$

Combining eqn. (8.8) and (8.9) gives

$$C_{M_G} = C_{M_0} - C_L K_n' - \bar{V}_T(\bar{a}_1 \eta_T + \bar{a}_3 \beta), \tag{8.10}$$

which is identical with eqn. (6.18). It therefore follows that in the most simple case, following the argument which results from eqn. (6.18), the relationship between tab angle to trim and C_L remains unchanged, and the stick-free neutral point position can be deduced from β_{trim}–C_L curves whether there is a mechanical elevator moment or not. It should be noted that in one respect, eqn. (8.10) is not exactly analogous to (6.18), in that $\bar{V}_T$ in (8.10) is still a tail volume coefficient based on the tail moment arm from the neutral point position *without a mechanical moment present* to the aerodynamic centre of the tail and not on a tail arm measured from the new neutral point.

A rearward shift of the neutral point can only arise because the effect of a positive mechanical moment is to increase the effective lift-curve slope of the tail with the elevator free. This can be seen from eqn. (8.6), (4.21) and (4.23), by eliminating α and C_L and differentiating with respect to α_T. The effective lift-curve slope is then

$$\frac{\partial C_{L_T}}{\partial \alpha_T} = \frac{\bar{a}_1 - \dfrac{a}{1 - \dfrac{d\varepsilon}{d\alpha}} \cdot \dfrac{a_2}{b_2} v}{1 + \dfrac{S_T}{S} \dfrac{a_2 v}{b_2}}, \tag{8.11}$$

but it must be remembered that this expression is subject to the condition that $L = W$ for the whole aeroplane. This result would not be obtained if the lift-curve slope of the tail were measured in a wind tunnel at constant speed; under these conditions, $\bar{a}_1$ would apply whether a constant mechanical moment was applied or not.

The influence of the mechanical moment may be explained in physical terms by comparing the effects at the tails of similar aeroplanes flying under the same conditions, (a) with a positive spring in the elevator circuit and (b) without a spring. In both cases, the tail lift coefficients will be the same and the total hinge moments will be zero. It follows that the tab of aeroplane (a) must be deflected by $\Delta\beta$ relative to the tab of aeroplane (b), to supply an aerodynamic moment which balances the mechanical moment. There will also be a small difference of elevator angle $\Delta\eta$ in order to maintain the same tail lift coefficient despite the different tab angles, i.e.

$$a_2\Delta\eta + a_3\Delta\beta = 0, \tag{8.12}$$

and

$$H_s = \Delta C_H \tfrac{1}{2}\rho_0 V_{i_1}^2 S_\eta c_\eta = [b_2\Delta\eta + b_3\Delta\beta]\tfrac{1}{2}\rho_0 V_{i_1}^2 S_\eta c_\eta. \tag{8.13}$$

Combining (8.12) and (8.13)

$$H_s = \left[b_3 - \frac{b_2 a_3}{a_2}\right]\Delta\beta . \tfrac{1}{2}\rho_0 V_{i_1}^2 S_\eta c_\eta. \tag{8.14}$$

Suppose the lift coefficient of both aeroplanes is increased and hence, since $L = W$, the equivalent airspeed is decreased. In both cases the elevator will tend to float upwards somewhat, due to the increase in tail incidence. But in aeroplane (a), the aerodynamic moment given by the right-hand side of eqn. (8.14) will decrease, due to the decreased airspeed. Since H_s remains constant, there will now be a resultant positive moment which is not present in aeroplane (b). The elevator in aeroplane (a) will therefore adopt a small positive deflection $\delta\eta$ relative to that of aeroplane (b), so that the aerodynamic moment again balances the mechanical moment

$$H_s = \left[\left(b_3 - \frac{b_2 a_3}{a_2}\right)\Delta\beta + b_2\delta\eta\right]\tfrac{1}{2}\rho_0 V_{i_2}^2 S_\eta c_\eta, \tag{8.15}$$

where V_{i2} is the new equivalent airspeed, less than V_{i_1}.

Hence, the constant mechanical moment decreases the upfloat of the elevator by $\delta\eta$, and the change of tail lift coefficient of aeroplane (a) is greater than that of aeroplane (b) by $a_2\delta\eta$. The nose-down moment about the c.g. of aeroplane (a) is therefore greater than that of aeroplane (b), and the stick-free stability has therefore been increased as a consequence of the mechanical moment.

The change in stick-free static margin due to a mechanical moment applied to the elevator circuit appears, from the point of view of the pilot, as a change in stick-force/speed gradient. Indeed, it might be more logical to suggest that the direct effect of the mechanical moment is to change the stick-force/speed gradient, which thus produces an effective change in stick-free static margin.

The effect on the stick-force at a given speed may be seen by introducing into eqn. (6.35) the effective change of stick-free static margin from eqn. (8.4) and (8.8). From these latter equations,

$$\Delta K_n' = -\frac{a_2}{b_2}\bar{V}_T\left[\frac{H_s S}{W c_\eta S_\eta}\right]. \tag{8.16}$$

The corresponding change in stick-force at an equivalent airspeed V_i is then

$$\Delta P_e = \frac{b_2}{a_2}\frac{\Delta K_n'}{\bar{V}_T}\frac{W}{S}\, m_e S_\eta c_\eta\left(1-\frac{V_i^2}{V_{i_0}^2}\right) \quad \text{from (6.35)}$$

$$= -m_e H_s\left(1-\frac{V_i^2}{V_{i_0}^2}\right). \tag{8.17}$$

When it is remembered that $(-m_e H_s)$ is the stick-force to be applied to move the elevator when the aeroplane is stationary, the simplicity of this result becomes apparent, suggesting that there must be a simple physical explanation.

Consider, as before, an aeroplane flying in trim with zero elevator hinge moment at V_{i_0} without a mechanical moment applied. Compare it with the corresponding case in which a positive mechanical moment is present. Then to restore the resultant hinge moment to zero an additional tab deflection $\Delta\beta$ must be applied, thus balancing the mechanical moment by an aerodynamic moment. At V_{i_0} it follows from eqn. (8.14) that the balance of the additional moments may be represented by an expression of the form

$$H_s - \text{const.} \times \Delta\beta V_{i_0}^2 = 0. \tag{8.18}$$

At any other speed V_i, the mechanical moment will remain the same, but the aerodynamic moment due to the tab deflection $\Delta\beta$

will be proportional to V_i^2. There will therefore be an out-of-balance hinge moment given by

$$\Delta H_s = H_s - \text{const.} \times \Delta\beta V_i^2$$
$$= H_s - H_s\left(\frac{V_i^2}{V_{i_0}^2}\right), \tag{8.19}$$

on introducing eqn. (8.18).

Combining eqn. (8.19) and (6.31) leads to exactly the same result as eqn. (8.17). All these effects are superimposed on the usual changes of stick-force due to change of speed, arising from purely aerodynamic effects. Putting this more briefly, the elevator is subjected to a positive constant mechanical moment, together with a negative aerodynamic moment which varies as V_i^2. At some speed V_{i_0}, they are assumed to cancel, but at other speeds they will not, in accordance with eqn. (8.19).

In this chapter, only the effects of constant mechanical moments are discussed in detail. Other arrangements are quite feasible and may have advantages in certain circumstances. For example, a mechanical moment may be applied by a constant rate spring, so that it is proportional to the elevator deflection from some datum value. By varying this datum value, this arrangement can also be used as a mechanical trimming device. A somewhat primitive example of such a mechanism is to be found on the "Tiger Moth". An analysis of such an arrangement shows that, at a given c.g. position, the stick-free static margin becomes a function of speed, becoming greater as the speed increases. Because the simple analysis of the earlier chapters leads to a fixed position for the stick-free neutral point position of a given aeroplane, it is easy to assume that this represents a desirable state of affairs; but this is not necessarily so.

Although not strictly relevant to this chapter, it may also be of interest to mention the matter of the stick-elevator gearing. In Chapter 6, it is assumed to be linear, largely for the sake of simplicity. In practice, this is usually very nearly true over the relatively small range of elevator deflections used in normal flight, but it may be found convenient to introduce marked nonlinearity into the mechanism connecting the control column to the elevator when large elevator deflections are involved, as when landing.

Whilst the effect of a given mechanical moment on the stick-free static margin is the same, whether it is produced by a weight or a spring, the effect on the stick-force required to perform a pull-out manœuvre *is* influenced by the method of applying the moment, as shown in the next chapter.

The application of a mechanical moment to the elevator circuit represents one method of artificially increasing the stick-free stability without altering the external aerodynamic features of the aeroplane. In Chapter 10, it will be shown that by adopting a somewhat different tail arrangement, the stick-free neutral point can be adjusted over much wider limits than might be thought from considering conventional expressions such as eqn. (5.22).

CHAPTER 9

Manœuvrability in Pitch

ALL THE preceding analysis has been concerned with steady flight under equilibrium conditions. It is now of interest to consider the stick movements and forces required to produce a rate of pitch, because this analysis represents a simple extension of the previous theory. Moreover, it indicates the actions required of the pilot to initiate a change of equilibrium condition.

In analysing such a pitching manœuvre, it is usual to consider the simple case in which the rate of pitch and forward speed (and hence the radial acceleration) are constant. This may represent a considerable idealization compared with a real pull-out, in which all these quantities will vary in a fashion which depends on the precise actions of the pilot. Moreover, it assumes a prescribed variation of thrust which is unlikely to occur in practice. It is probably fair to say that the assumed conditions can be achieved to a fair degree of approximation in a fast aeroplane, in which quite low rates of pitch produce appreciable normal accelerations, thus giving the pilot plenty of time to establish the required conditions. In slower aircraft, the rate of pitch to produce a given normal acceleration is much higher and a pull-out manœuvre tends to be rather transitory, so that conditions are changing continuously. Even so, although the "ideal" manœuvre is not necessarily very realistic in the case of slow aeroplanes, the analysis given here and the concepts which emerge (such as stick-force per *g* and manœuvre margin) are a useful guide to the "feel" of the aeroplane when manœuvring in pitch and form a basis of comparison.

In addition to the assumptions of Chapter 3, the following are also made:

(1) The forward speed and angular velocity are substantially constant. The flight path is therefore circular in the vertical plane.

(2) The radial component of gravity is substantially constant. This means that the analysis is limited to that part of the flight path which is inclined at a small angle to the horizontal. In the interests of simplicity, the equations given below relate to the instant at which the flight path is truly horizontal, at the bottom of the pull-out. A more general analysis is given in Ref. 9, but it only differs in including some cos γ terms, where γ is the inclination of the flight path to the horizontal.

Stick-travel per *g*

Conditions are therefore as shown in Fig. 9.1 where q is the rate of pitch.

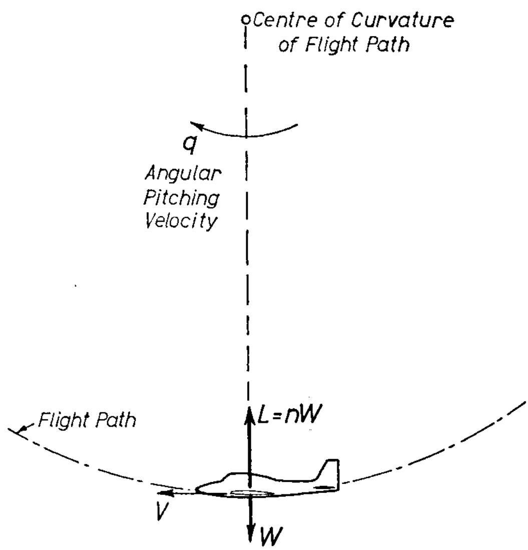

FIG. 9.1. An aeroplane performing a pull-out manœuvre.

At the instant considered, let the load factor be n, where n is defined as

$$n = \frac{L}{W}. \tag{9.1}$$

Defined in this way, n is also the reading of an accelerometer in the aircraft reading in g units, g being the acceleration due to gravity.

The equation of motion in the radial direction is

$$L - W = \frac{W}{g} Vq, \tag{9.2}$$

and hence from (9.1) and (9.2),

$$n - 1 = \frac{Vq}{g},$$

i.e. the radial acceleration

$$Vq = (n-1)g. \tag{9.3}$$

This is obviously the radial acceleration imposed upon the aeroplane by the curvature of the flight path. It should be noted that in some analyses (e.g. Ref. 9), the symbol n is used to define the radial acceleration, i.e. $Vq = ng$, and in such usage, n differs from the load factor by unity. The definition of eqn. (9.1) is used here in order to be consistent with the terminology used for stressing purposes.

In steady level flight at a true speed V, let

$$W = C_{L_0} \tfrac{1}{2}\rho V^2 S, \tag{9.4}$$

and whilst performing a pull-out at the same speed and height,

$$L = nW = C_{L_1} \tfrac{1}{2}\rho V^2 S, \tag{9.5}$$

so that from eqn. (9.1)

$$C_{L_1} = nC_{L_0}. \tag{9.6}$$

Since the aeroplane is travelling on a curved path, the tailplane incidence will be greater than that corresponding to straight flight at C_{L_1} by an amount $\Delta\alpha_T$ (see Fig. 9.2), where

$$\Delta\alpha_T = \frac{q l'_T}{V}. \tag{9.7}$$

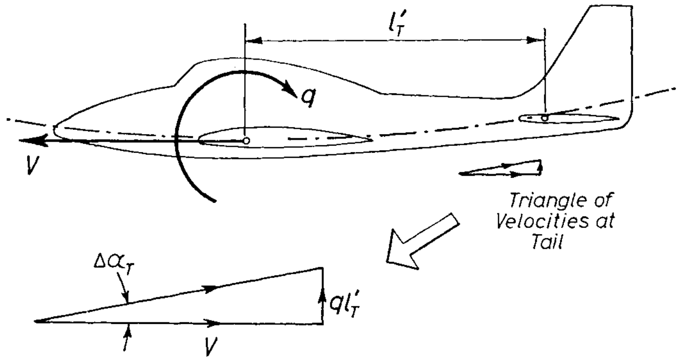

FIG. 9.2. The change of tail incidence due to the rate of pitch in a pull-out manœuvre.

Introducing eqn. (9.3) and (9.4)

$$\Delta\alpha_T = \frac{(n-1)gl'_T}{V^2} = \frac{(n-1)C_{L_0}}{2}\left[\frac{g\rho Sl'_T}{W}\right]. \tag{9.8}$$

Now W/gSl'_T is the mass of the aeroplane divided by a quantity having the dimensions of a volume, based on the size of the aircraft. It is, in effect, a sort of aircraft density.

If μ_1 is defined by

$$\mu_1 = \frac{W}{g\rho Sl'_T}, \tag{9.9}$$

then μ_1 is the ratio of the "aircraft density" to the local air density, and maybe regarded as a "relative density" for the aircraft at this particular height. (The symbol μ is given the suffix '1' to distinguish this relative density based on the tail moment arm from an analogous quantity based on the span which occurs in lateral stability theory.)

So, from eqn. (9.8) and (9.9)

$$\Delta\alpha_T = \frac{(n-1)C_{L_0}}{2\mu_1}. \tag{9.10}$$

For an aircraft in steady level flight at C_{L_0}, the pitching moment, from eqn. (5.24) is

$$C_{M_G} = C_{M_0}+(h-h_0)C_{L_0} - V_T\left[\frac{a_1}{a}\left(1-\frac{\mathrm{d}\varepsilon}{\mathrm{d}\alpha}\right)C_{L_0}+a_1\eta_T+a_2\eta+a_3\beta\right] = 0. \quad (9.11)$$

If it is assumed that the lift relationships for the wing and tail are unaffected by the rate of pitch, the corresponding equation applying at the bottom of the pull-out will differ from (9.11) only in the following respects:

(a) The lift coefficient is now C_{L_1}.
(b) There will be an additional term $a_1\Delta\alpha_T$ inside the square brackets due to the change in tail incidence.
(c) The elevator will have an additional deflection $\Delta\eta$ in order to perform the manœuvre.

Since the resultant pitching moment will again be zero,

$$C_{M_G} = C_{M_0}+(h-h_0)nC_{L_0} - V_T\left[\frac{a_1}{a}\left(1-\frac{\mathrm{d}\varepsilon}{\mathrm{d}\alpha}\right)nC_{L_0}+a_1\eta_T+a_1\Delta\alpha_T + a_2(\eta+\Delta\eta)+a_3\beta\right] = 0. \quad (9.12)$$

Subtracting eqn. (9.11) from (9.12) and introducing (9.10)

$$0 = (n-1)C_{L_0}(h-h_0) - V_T\left[(n-1)C_{L_0}\frac{a_1}{a}\left(1-\frac{\mathrm{d}\varepsilon}{\mathrm{d}\alpha}\right)+a_1\frac{(n-1)C_{L_0}}{2\mu_1}+a_2\Delta\eta\right], \quad (9.13)$$

i.e.

$$\frac{V_T a_2\Delta\eta}{(n-1)C_{L_0}} = (h-h_0)-V_T\left[\frac{a_1}{a}\left(1-\frac{\mathrm{d}\varepsilon}{\mathrm{d}\alpha}\right)+\frac{a_1}{2\mu_1}\right]. \quad (9.14)$$

This equation gives the elevator movement $\Delta\eta$ required to produce the normal acceleration $(n-1)g$. It will be seen that there is a value of h for which $\Delta\eta$ is zero, i.e. if the c.g. is sufficiently far aft, no change in elevator angle is required to produce the radial acceleration. This c.g. position is denoted by h_m, where from eqn. (9.14)

$$h_m = h_0 + V_T\left[\frac{a_1}{a}\left(1-\frac{d\varepsilon}{d\alpha}\right)+\frac{a_1}{2\mu_1}\right]. \tag{9.15}$$

This c.g. position (in dimensional units $h_m\bar{c}$) is termed the "manœuvre point, stick-fixed" by analogy with the neutral point stick-fixed. For any other c.g. position

$$-\frac{V_T a_2 \Delta\eta}{(n-1)C_{L_0}} = h_m - h = H_m, \quad \text{say.} \tag{9.16}$$

H_m is termed the "manœuvre margin, stick-fixed". It follows that if the c.g. is ahead of the manœuvre point, stick-fixed, $\Delta\eta$ is negative, i.e. an upward deflection of the elevator is required to produce an upward normal acceleration.

The "stick-travel per g" is then

$$Q_1 = \frac{x_s}{(n-1)} = \frac{\Delta\eta}{m_e(n-1)} \quad \text{(from eqn. (6.15))}$$

$$= -\frac{C_{L_0}H_m}{m_e V_T a_2} \tag{9.17}$$

$$= -\frac{W}{\tfrac{1}{2}\rho_0 V_i^2 S}\cdot\frac{H_m}{m_e V_T a_2}. \tag{9.18}$$

It is therefore apparent that, other things being equal, the stick-travel per g is inversely proportional to V_i^2.

From eqn. (5.8) the stick-fixed neutral point position is

$$h_n = h_0 + V_T\left[\frac{a_1}{a}\left(1-\frac{d\varepsilon}{d\alpha}\right)\right].$$

Comparing this expression with (9.15), it will be seen that the manœuvre point, stick-fixed, is aft of the neutral point, stick-fixed, by an amount $V_T a_1/2\mu_1$.

Since μ_1 increases as the height increases (due to the decreasing air density), this quantity decreases with increasing altitude. The stick-fixed manœuvre point therefore moves forward as the height increases and the stick-travel per g becomes correspondingly less. Also, for a given wing loading, μ_1 decreases as the aircraft becomes larger, and hence, in dimensionless terms, the manœuvre point moves further aft of the neutral point.

As the c.g. of a given aeroplane is moved further aft, the gradient of the elevator angle-to-trim curve reverses when the c.g. is aft of the stick-fixed neutral point, but the "stick movement per g" only reverses when the c.g. is aft of the stick-fixed manœuvre point.

Flight with negative stick-fixed stability may be quite practicable, since the pilot is not usually very conscious of the variation of trimmed stick position with speed at a series of steady speeds, but it would be virtually impossible to fly the aeroplane with a negative stick-fixed manœuvre margin since, having initiated the application of a normal acceleration, the stick would have to be moved in the "wrong" sense to prevent the manœuvre getting out of hand.

Stick-force per g.

The simplest way of finding the stick-force per g is to suppose that a pull-out manœuvre is performed by a movement of the tab, and then to find the equivalent stick-force.

By analogy with the previous theory, but using eqn. (5.26) for the pitching moment with elevator free, the tab movement $\Delta\beta$ required to perform a pull-out with zero stick-force will be,

$$\frac{\bar{V}_T \bar{a}_3 \Delta\beta}{(n-1)C_{L_0}} = (h-h_0) - \bar{V}_T\left[\frac{\bar{a}_1}{a}\left(1-\frac{d\varepsilon}{d\alpha}\right)+\frac{\bar{a}_1}{2\mu_1}\right]. \qquad (9.19)$$

This equation is exactly analogous to eqn. (9.14).

From eqn. (6.27), the hinge moment coefficient at constant tab angle which is equivalent to a tab movement $\Delta\beta$ at zero hinge moment is given by

$$\Delta C_H = \frac{b_2}{a_2}\bar{a}_3\Delta\beta,$$

and hence from eqn. (9.19) and (6.27)

$$\frac{a_2 \bar{V}_T \Delta C_H}{b_2(n-1)C_{L_0}} = (h-h_0) - \bar{V}_T\left[\frac{\bar{a}_1}{a}\left(1-\frac{d\varepsilon}{d\alpha}\right)+\frac{\bar{a}_1}{2\mu_1}\right]. \qquad (9.20)$$

ΔC_H becomes zero when the c.g. position corresponds to h'_m where

$$h'_m = h_0 + \overline{V}_T\left[\frac{\bar{a}_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right) + \frac{\bar{a}_1}{2\mu_1}\right]. \tag{9.21}$$

By analogy with the previous notation, h'_m is termed the "manœuvre point stick-free". For any other c.g. position

$$-\frac{a_2 \overline{V}_T \Delta C_H}{b_2(n-1)C_{L_0}} = h'_m - h = H'_m, \text{ say.} \tag{9.22}$$

H'_m is termed the "manœuvre margin stick-free". If H'_m is positive and a_2 and b_2 have the usual signs, an upward normal acceleration causes a positive aerodynamic moment to act on the elevator. The pilot therefore has to apply a pull-force to the stick, conventionally taken as negative.

The stick-force per g will be

$$\frac{P_e}{n-1} = -m_e \tfrac{1}{2}\rho V^2 S_\eta c_\eta \frac{\Delta C_H}{n-1}. \tag{9.23}$$

Substituting eqn. (9.22) and (9.4) in (9.23)

$$\frac{P_e}{n-1} = \frac{m_e b_2 W S_\eta c_\eta H'_m}{a_2 \overline{V}_T S}. \tag{9.24}$$

It follows that the stick-force per g is independent of speed. This arises because the hinge moment will be proportional to (elevator angle per g) $\times V_i^2$ and the elevator angle per g is itself inversely proportional to V_i^2. The manœuvre point, stick-free, is aft of the neutral point, stick-free, by an amount

$$\overline{V}_T \bar{a}_1 / 2\mu_1, \tag{9.25}$$

which varies with height and aircraft size in much the same fashion as the corresponding stick-fixed quantity. As the c.g. of a given aeroplane is moved further aft, the gradient of the curve of stick-force to change speed from an initially trimmed condition against speed reverses when the c.g. is aft of the stick-free neutral point, but the stick-force per g only reverses when the c.g. is aft of the stick-free manœuvre point. Although it is possible to fly an aeroplane which is unstable stick-free, it is very unpleasant and potentially dangerous; British Civil Airworthiness Requirements state, in effect, that the aeroplane must always be stable stick-free. This automatically ensures that the stick-force per g is in the correct sense.

Effect of Weights and Springs in the Elevator Circuit

Mechanical moments applied to the elevator do not affect the stick movement per g, since this quantity is not influenced by hinge moments.

The stick-force per g is proportional to the hinge moment required to apply a given normal acceleration and may therefore be affected by non-aerodynamic moments. If such a moment is applied to the elevator by means of a spring (in order to modify the stick-free static stability) an additional tab deflection will be required in level flight at C_{L_0} in order to provide an aerodynamic moment to balance the spring moment.

There will also be a corresponding small adjustment to the elevator angle to cancel out the change of tail lift due to the tab deflection.

When this situation is compared with that prevailing in a pull-out at C_{L_1} and the same forward speed and the same tab angle, it must be remembered that the spring is assumed to be long, so that the spring moment is unaffected by the elevator deflection. Assuming that the mass of the spring is negligible, the spring moment will also be unaffected by the normal acceleration. During the pull-out, the elevator movement is as given by the above theory, and the hinge moment which results is solely the aerodynamic hinge moment already obtained. In effect the aerodynamic hinge moment due to the initial tab deflection will balance the spring moment both in level flight and in the pull-out. The effect of the spring is thus to alter the "datum" elevator angle at C_{L_0} slightly, as a secondary consequence of the tab deflection required to balance out the spring moment, but the *change* in elevator angle between steady flight at C_{L_0} and a pull-out at C_{L_1} is not affected, and hence the stick-force per g is unaltered by inserting a spring in the system. The stick-free manœuvre point is still given by eqn. (9.21) but the expression

(9.25) only relates to the distance between the manœuvre point and the neutral point in the absence of a spring, since a spring *does* affect the position of the stick-free neutral point.

If one now considers the effect of an elevator hinge moment applied by means of a weight, an argument similar to that given above applies with the sole difference that the moment due to the weight *is* affected by normal accelerations. In both the level flight, and pull-out conditions, a moment H_s is balanced by an aerodynamic moment due to a tab deflection. But, in addition to the aerodynamic hinge moment given by eqn. (9.20), there is an additional "inertia" hinge moment $(n-1)H_s$. The stick-force per g is therefore increased by

$$-\frac{m_e(n-1)H_s}{(n-1)} = -m_e H_s. \tag{9.26}$$

From eqn. (9.24), this is equivalent to a rearward shift of the stick-free manœuvre point $\delta h'_m$, where

$$\delta h'_m = -\frac{H_s a_2 \bar{V}_T S}{b_2 W S_\eta c_\eta} = -\frac{a_2 v \bar{V}_T}{b_2}. \tag{9.27}$$

The effect of a positive mechanical moment H_s provided by a weight or a spring on the stick-free neutral point and manœuvre point may therefore be summarized as follows:

Change of	Moment H_s applied by	
	weight	spring
Stick-free neutral point	$-\dfrac{a_2\bar{V}_T S H_s}{b_2 W S_\eta c_\eta}$	$-\dfrac{a_2\bar{V}_T S H_s}{b_2 W S_\eta c_\eta}$
Stick-free manœuvre point	$-\dfrac{a_2\bar{V}_T S H_s}{b_2 W S_\eta c_\eta}$	zero

A positive weight moment therefore shifts both the stick-free neutral point and the stick-free manœuvre point aft by the same amount, so that the distance between them continues to be given by expression (9.25).

By suitable use of both weights and springs, it is possible to vary independently the positions of the stick-free neutral point and manœuvre point. For example, if it is desired to reduce the stick-force per g without altering the stick-force to change speed, a negative weight moment and an equal and opposite spring moment could be applied to the elevator.

Stick-force in Turns

If an aeroplane is performing a steady level turn with an angular velocity Ω about a vertical axis (see Fig. 9.3) then Ω will have a component about the y-axis of the aeroplane given by

$$q = \Omega \sin \phi. \tag{9.28}$$

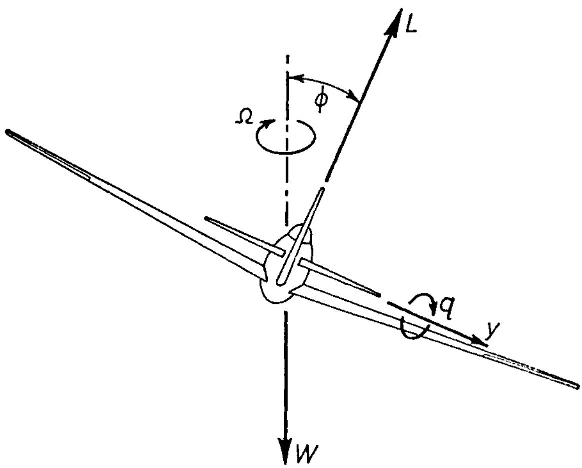

FIG. 9.3. An aeroplane performing a steady level turn.

One would expect that, in order to apply this rate of pitch, some stick movement and stick-force would be required. The analysis which follows shows that the stick movement and stick-force required to perform a steady level turn at a certain load factor and forward speed are not the same as those required to perform a pull-out at the same load factor and forward speed. The reason is simply that the rates of pitch associated with these two manœuvres are not the same and hence the increment of tail incidence due to rate of pitch in a turn is not that given by eqn. (9.8).

Referring again to Fig. 9.3, and resolving forces horizontally and vertically:

$$L \sin \phi = \frac{W}{g} V\Omega, \tag{9.29}$$

and

$$L \cos \phi = W,$$

or

$$n = \frac{L}{W} = \sec \phi. \tag{9.30}$$

Hence,

$$\begin{aligned} Vq &= V\Omega \sin \phi, && \text{from eqn. (9.28)} \\ &= \frac{gL}{W} \sin^2 \phi, && \text{from eqn. (9.29)} \\ &= gn\left(1 - \frac{1}{n^2}\right). && \text{from eqn. (9.30)} \end{aligned}$$

So finally,

$$Vq = (n-1)\left(\frac{n+1}{n}\right)g. \tag{9.31}$$

This equation corresponds to (9.3) of the previous theory. It therefore follows that the previous theory applies except that the terms relating to the change in tailplane incidence due to the rate of pitch must be multiplied by $(n-1)/n$. Denoting the elevator travel required to perform a steady level turn by $\Delta\eta_{st}$, the expression corresponding to eqn. (9.14) is,

$$\frac{V_T a_2 \Delta\eta_{st}}{(n-1)C_{L_0}} = (h-h_0) - V_T\left[\frac{a_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right) + \frac{a_1}{2\mu_1}\left(\frac{n+1}{n}\right)\right]. \tag{9.32}$$

Denoting the elevator deflection given by eqn. (9.14) by $\Delta\eta_{po}$, it follows that

$$\frac{\Delta\eta_{po}-\Delta\eta_{st}}{n-1}=\frac{1}{n}\left(\frac{C_{L_0}a_1}{2\mu_1 a_2}\right). \tag{9.33}$$

Remembering that both angles on the L.H.S. of this equation are negative, this expression shows that more upward elevator deflection is required to perform a turn compared with a pull-out at the same load factor and forward speed.

Similar considerations apply to the stick-force. It exceeds that required to perform a pull-out at the same load factor in accordance with the following expression:

$$\frac{P_{e_{po}}-P_{e_{st}}}{n-1}=-\frac{m_e W S_\eta c_\eta \bar{a}_1 b_2}{2nSa_2\mu_1}. \tag{9.34}$$

The stick-force per g in a turn is therefore no longer independent of the load factor.

It will also be seen from expressions such as eqn. (9.32) that if $n \to 1$, then the apparent manœuvre points in gentle turns are aft of those corresponding to the pull-out case by $V_T a_1/2\mu_1$ and $\bar{V}_T \bar{a}_1/2\mu_1$ stick-fixed and stick-free respectively, but tend towards the pull-out values as n increases (i.e. as the turn becomes tighter). This situation is to be expected since a very tight turn is almost entirely a pitching manœuvre. It becomes clear from the foregoing that determination of manœuvre points by measurements made in turns require careful analysis of the experimental results.

This analysis of turning flight involves several implicit assumptions:

(a) That aerodynamic pitching moments are unaffected by the rate of yaw.

(b) That gyroscopic pitching moments due to the engine in the presence of a rate of yaw may be neglected. In practice, this effect is usually small and allowance may be made by taking the means of readings obtained in turns in opposite directions.

(c) That the turns are really level. If the aeroplane is climbing or descending whilst turning, so that the flight path is a helix (often loosely, and inaccurately, termed "spiral turns"), an additional inertia pitching moment arises.

Static Margins, Manœuvre Margins, and Flying an Aeroplane

It is now useful to reiterate the practical consequences of the theory presented in Chapters 6 and 9. In the former, it was shown that the static margins, stick-fixed and stick-free, were respectively related to the variation of stick position and stick-force (or tab angle) with speed, always under steady flight conditions. The unsteady process of changing from one steady condition to another was specifically excluded. In this chapter it has been shown that the manœuvre margins, stick-fixed and stick-free, are respectively related to the stick deflection and stick-force required to apply a certain rate of pitch. The manœuvre margin theory, although rather an idealization of what happens in a real pitching manœuvre, *is* related to the process of initiating a change of equilibrium conditions. It does not, of course, describe in detail the transient motions which occur if a sudden elevator deflection is applied in steady flight: such a description involves a proper analysis of the dynamic behaviour of the aeroplane.

By way of illustration, consider an aeroplane with a negative stick-fixed static margin, but a positive stick-fixed manœuvre margin, a condition which is not at all unlikely. Suppose that, when flying at 120 knots, the pilot observes the stick position. He then wishes to reduce speed to 100 knots, and initiates the change by a nose-up pitching manœuvre. Since the stick-fixed manœuvre margin is positive, he will have to apply an initial stick-movement in the "correct" sense, i.e. backwards. Suppose that when the new equilibrium condition has been established, he again observes the stick position. Since the stick-fixed static margin is negative, the stick position at 100 knots will be forward of that corresponding to 120 knots, so that the change in equilibrium stick position will be in the "wrong" sense. The additional forward movement of the stick will have occurred in the course of stopping the pitching manœuvre and re-establishing the new steady condition. As indicated elsewhere, the pilot is not particularly sensitive to changes of stick position corresponding to successive equilibrium states, but he is acutely aware of the relationship between stick deflection and rate of pitch, so that the situation envisaged above can be acceptable.

CHAPTER 10

Moving Tailplanes

Trimming by Adjusting the Tail Setting

From eqn. (5.26), the total pitching moment coefficient with elevator free was

$$C_{M_G} = C_{M_0} + (h - h_0)C_L - \bar{V}_T\left[\frac{\bar{a}_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right)C_L + \bar{a}_1\eta_T + \bar{a}_3\beta\right].$$

In the ensuing discussion on stick-free stability, it was assumed that C_{M_G} was made zero, in order to trim the aircraft, by adjusting the tab angle β. It is apparent from this expression that η_T, the angle of the tailplane relative to the zero-lift line of the wing-plus-fuselage, could also be varied to make C_{M_G} zero at different values of C_L, and the trim tab would then become superfluous. Under such circumstances, the stick-free static margin is related to the tailplane angle to trim, $\eta_{T_{trim}}$, instead of the tab angle to trim. The theory follows exactly the same arguments as those relating to tab angles in Chapter 6, except that β_{trim} is replaced by $\eta_{T_{trim}}$ and $\bar{a}_3$ by $\bar{a}_1$. For example, eqn. (6.20), is replaced by:

$$\frac{d\eta_{T_{trim}}}{dC_L} = -\frac{K_n'}{\bar{a}_1\bar{V}_T}. \tag{10.1}$$

Tailplanes which are adjustable for trimming purposes are much in favour for transonic and supersonic aeroplanes, since they obviate the difficulties associated with large changes of tab effectiveness at high Mach numbers. On low-speed aeroplanes, the only aerodynamic advantage is that the adjustable tail plus elevator may have slightly less profile drag under certain conditions of flight than the fixed tail/elevator/adjustable tab arrangement. On the other hand, the mechanism required to provide adjustment of the tail setting will tend to be heavier than that associated with a tab.

All-moving Tails

High-speed aeroplanes frequently use all-moving tails, where the angle η_T is directly controlled by the movements of the pilot's control column. If the tailplane is moved via an irreversible power-control system, stick-free stability is no longer of significance, and suitable stick-forces can be arranged by artificial means. The object of such an arrangement is to avoid the difficulties associated with large changes of elevator effectiveness at high Mach numbers.

It may also be useful to use an all-moving tail on low-speed aircraft since it is possible to devise an arrangement which gives great flexibility in choosing a stick-free neutral point position. On physical grounds, it may be shown that a simple surface attached to the fuselage by a pivot and directly geared to the stick, is unsatisfactory. Suppose that the tail is of symmetrical section and

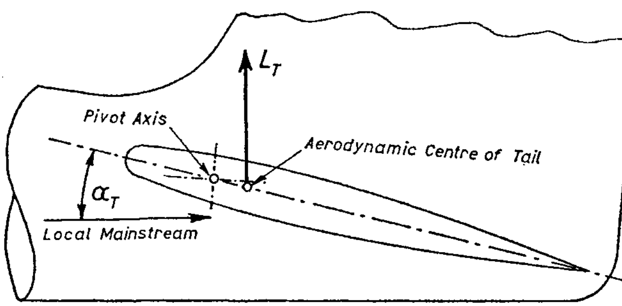

FIG. 10.1. An all-moving "slab" tail with the pivot ahead of the aerodynamic centre.

is pivoted at a point forward of its aerodynamic centre (see Fig. 10.1). If the tail is at some incidence α_T to the local airflow, the only moment about the pivot point if the stick is free will be due to the tail lift. This acts in such a sense that the tail is "stable" and will therefore tend to set itself so that the moment is zero, and hence α_T is zero. In other words, it tends to trail along the direction of the local airflow. It follows that, whatever the aircraft lift coefficient may be, the tail load in the stick-free case is always zero. The tail therefore makes no contribution to the stick-free stability of the aeroplane, and the stick-free neutral point therefore coincides with the aerodynamic centre of the aircraft-less-tail. The aeroplane will therefore only be stable, stick-free, if the c.g. is ahead of this aerodynamic centre, and further consideration would show that only in this configuration would the stick-forces be in the "correct" sense.

Placing the pivot point aft of the aerodynamic centre of the tail is obviously impracticable since, on releasing the stick, the moment due to the tail lift acts on the tail in the "unstable" sense, causing it to assume the largest possible incidence. A limiting case occurs when the pivot point coincides with the aerodynamic centre. The moment about the pivot point is then always zero: the stick-free stability is not a very meaningful expression, since the tail can assume any arbitrary incidence, but since the stick-force is zero under all circumstances it appears to the pilot that the stick-free stability is zero and the stick-force per g is also zero. Such an arrangement has been used in the past on some types of glider, but would not now be considered acceptable. In such circumstances, it is possible to provide artificial feel by restraining the elevator with suitable springs, but the effect is, of course, different from that provided by an aerodynamic moment from the pilot's point of view. With such spring restraint, and with the aerodynamic moment always zero, the free tail will always trail at the same attitude relative to the fuselage. The stick-fixed and stick-free neutral points will then coincide.

None of the above arrangements are really satisfactory because in general the stick-force depends only on the tail incidence and not on some angle defining the position of a control surface in relation to an aircraft datum, such as elevator angle. This situation may be resolved by providing a geared tab which rotates in the same sense as the tailplane. A schematic arrangement is shown in Fig. 10.2, where the axis of rotation of the tailplane is assumed to coincide with the aerodynamic centre of the basic symmetrical tail section.

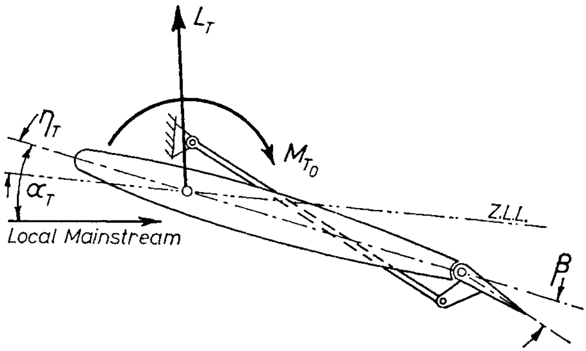

FIG. 10.2. An all-moving tail fitted with a geared tab, with the pivot at the aerodynamic centre.

If, in this configuration, the tail lift is assumed to act through the axis of rotation, the only moment about this axis will be due to the tab. Suppose that the moment coefficient about the aerodynamic centre of the basic symmetrical section is proportional to the tab angle:

$$C_{M_{T0}} = c_3\beta, \tag{10.2}$$

where c_3 will be negative.

Suppose also that, by virtue of tab gearing, the tab angle is a linear function of tailplane angle η_T:

$$\beta = \beta_0 + k\eta_T, \tag{10.3}$$

where β_0 is a datum setting adjustable for trimming purposes and k is a positive gear ratio.

On releasing the stick, the tail will adopt an attitude such that $C_{M_{T0}} = 0$ and hence $\beta = 0$. From eqn. (10.3), this corresponds to

$$\eta_T = -\frac{\beta_0}{k}, \tag{10.4}$$

and hence, for a given value of β_0, the tail always assumes a constant angle relative to the wing-plus-fuselage zero-lift line. It immediately follows that, even with the stick free, the tail behaves as if it were fixed relative to the fuselage. The stick-fixed and stick-free neutral points therefore coincide. This result is independent of the size of the tab, represented above by the magnitude of c_3, but in practice the tab size would be determined by the required stick-force characteristics.

The above arrangement, which lends itself to simple physical discussion, is a particular case of the configuration shown in Fig. 10.3, where the axis of rotation of the tail is x_T aft of the aerodynamic centre of the basic symmetrical section.

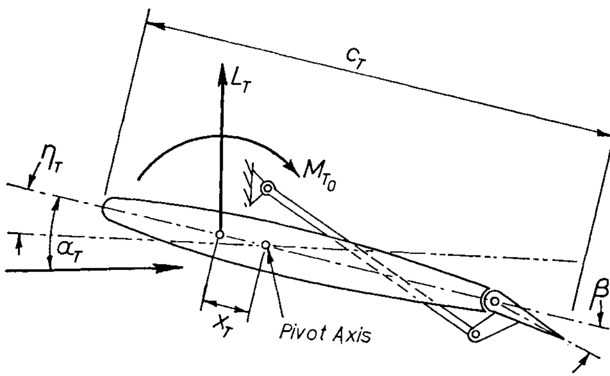

FIG. 10.3. An all-moving tail fitted with a geared tab, with the pivot aft of the aerodynamic centre.

Equations (10.2) and (10.3) still apply, but there is now an additional contribution to the hinge moment about the axis of rotation due to the tail lift, so that

$$C_{H_T} = C_{M_{T0}} + C_{L_T} \frac{x_T}{c_T}. \tag{10.5}$$

Equation (2.24) becomes simply

$$C_{L_T} = a_1 \alpha_T + a_3 \beta. \tag{10.6}$$

Introducing eqn. (4.21) and (4.23) it follows that

$$C_{L_T} = \frac{1}{1+F}\left[\frac{a_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right)C_L + (a_1 + ka_3)\eta_T + a_3\beta_0\right]. \tag{10.7}$$

The stick-fixed stability is only affected by the tail size and lift-curve slope a_1, and not by hinge moment considerations. The stick-fixed neutral point position is therefore given by eqn. (5.8). In considering the variation of tailplane angle to trim with C_L eqn. (10.7) should be compared with (5.23). η_T now becomes the significant variable instead of η and hence (a_1+ka_3) replaces a_2 in the subsequent calculations. Hence, from eqn. (6.3),

$$\frac{d\eta_{T_{trim}}}{dC_L} = -\frac{K_n}{(a_1+ka_3)\bar{V}_T}. \tag{10.8}$$

In the stick-free case, putting $C_{H_T} = 0$ in eqn. (10.5) and eliminating β and η_T from eqn. (10.2), (10.3) and (10.7), the tail-lift coefficient is given by:

$$C_{L_T} = \frac{1}{1+\bar{F}}\left[\frac{\bar{a}_1}{a}\left(1-\frac{d\varepsilon}{d\alpha}\right)C_L - \frac{\bar{a}_1}{k}\beta_0\right], \tag{10.9}$$

where

$$\bar{a}_1 = a_1\left[\frac{kc_3}{kc_3+(a_1+ka_3)\dfrac{x_T}{c_T}}\right], \tag{10.10}$$

and

$$\bar{F} = \frac{\bar{a}_1}{a}\frac{S_T}{S}\left(1-\frac{d\varepsilon}{d\alpha}\right). \tag{10.11}$$

Comparing eqn. (10.9) with (5.25), it is clear that the effective tail-lift-curve slope, stick-free, is $\bar{a}_1$.

The expression for the neutral point, stick-free, by analogy with eqn. (5.21) is now

$$h_n' = h_0 + \bar{\bar{V}}_T\frac{\bar{a}_1}{a}\left(1-\frac{d\varepsilon}{d\alpha}\right), \tag{10.12}$$

where

$$\bar{\bar{V}}_T = \frac{\bar{V}'}{1+\bar{F}}. \tag{10.13}$$

Several interesting conclusions may be drawn from these expressions.

(1) If the tailplane axis of rotation coincides with the aerodynamic centre of the basic section, then from eqn. (10.10), $\bar{a}_1 = a_1$. The tail lift-curve slope, stick-free, is the same as that stick-fixed.

(2) If no tab is fitted, $kC_3 = 0$ and then $\bar{a}_1 = 0$ from eqn. (10.10). The tail then makes no contribution to the static stability, stick-free.

(1) and (2) have previously been deduced on physical grounds.

(3) If $x_T < 0$ (i.e. axis ahead of basic a.c.), $\bar{a}_1 < a_1$ remembering that c_3 is negative in eqn. (10.10). Hence the stick-free neutral point will be forward of the stick-fixed neutral point.

(4) If $x_T > 0$ (i.e. axis aft of basic a.c.) $\bar{a}_1 > a_1$, and the stick-free neutral point will be aft of the stick-fixed neutral point.

In particular, as

$$\frac{x_T}{c_T} \to \frac{kc_3}{a_1 + ka_3}, \tag{10.14}$$

then

$$\bar{a}_1 \to \infty.$$

This condition sets a rearward limit to the position of the axis of rotation.

(5) It is of interest to consider the location of the neutral point when $\bar{a}_1 \to \infty$.

From (10.11), as $\bar{a}_1 \to \infty$, $\bar{F} \to \infty$.

But eqn. (10.12) may be written

$$h_n' = h_0 + \frac{\bar{V}'}{1+\bar{F}} \cdot \frac{\bar{F} S_T}{S}. \tag{10.15}$$

So, as $\bar{a}_1 \to \infty$,

$$h_n' \to h_0 + \bar{V}' \frac{S_T}{S},$$

i.e.

$$h_n' \to h_0 + \frac{l_T'}{\bar{c}}. \tag{10.16}$$

Hence, in this limiting case, the stick-free neutral point approaches the aerodynamic centre of the basic symmetrical tail.

For a given tail volume coefficient $\bar{V}'$ and lift-curve slope a_1, the stick-fixed neutral point is fixed. By a suitable choice of tab size, tab gear ratio, and position of the axis of rotation of the tail, the stick-free neutral point position can be varied within very wide limits (in principle, h_n' can lie anywhere between h_0 and $h_0 + l_T'/\bar{c}$), and at the same time satisfactory values of other quantities such as stick-force to change speed may be achieved. The merits of this configuration are apparent.

CHAPTER 11

Some Miscellaneous Effects

IN ALL the previous theory, it has been assumed that moments about the c.g. due to thrust and drag are negligibly small and consequently it is superfluous to insert any condition relating to the longitudinal forces. Other effects induced by the power plant have also been neglected. In practice, some or all of the following influences may be present:

(a) If the resultant drag force does not act through the c.g., it produces a pitching moment.
(b) Similarly for the thrust.
(c) If the axis of a propeller is inclined to the mainstream, a component of force normal to the axis is produced. A tractor propeller therefore has a de-stabilizing effect, rather as if a small wing surface had been placed at the nose.(5)
(d) The slipstream of a propeller influences the wing and tail lift.
(e) A jet near the tail may have an appreciable effect on the local downwash.
(f) A jet engine at incidence produces a momentum change giving a force component normal to the thrust line(6) which may have a moment about the c.g.
(g) Various other effects due to rotation of the slipstream.

A consequence of such effects is that flight tests must be carefully conducted if they are to give results which are really applicable to some desired condition. For example, in the determination of the stick-fixed neutral point under cruise conditions, it is necessary to find the gradients of the η–C_L curves at the appropriate lift coefficient, at two c.g. positions, and at the engine setting appropriate to the cruise. In order to obtain the trim curves in the vicinity of the desired lift coefficient, the aircraft must be flown at a few different speeds bracketing the cruising speed. If the engine setting is left unchanged, the thrust will only equal the drag at the cruising speed and at other speeds a climb or descent will result. Provided that the angle of inclination of the flight path is small, this will not matter. The result obtained in this way may be somewhat different from that obtained by adjusting the engine setting at each speed so that the aeroplane is always in level flight, and the former situation is the more relevant.

The whole of this chapter is concerned with corrections to the theory developed in Chapters 4 and 5. Since these corrections do not depend on considerations of elevator hinge moment, they apply equally to both stick-fixed and stick-free conditions.

Most of the effects listed above are examined in more detail in Refs. 3, 6 and 9, but it is of interest to consider briefly the influence of thrust and drag moments to indicate how the engine setting can influence the apparent stability.

Moments Due to Thrust and Drag

Suppose that the thrust and drag forces act at distances z_p and z_d below the c.g. respectively (see Fig. 11.1).

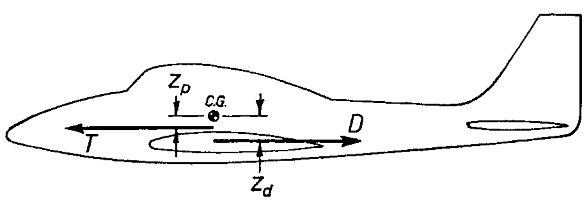

FIG. 11.1. An aeroplane with the thrust and drag forces acting below the c.g.

Then an additional moment ΔM occurs, compared with the case in which the effect of these forces is neglected, where

$$\Delta M = Tz_p - Dz_d. \tag{11.1}$$

When the aeroplane is in trim, this additional moment will give rise to an appropriate increment in the tail lift.

If the thrust is always equal to the drag, the above equation may be written in coefficient form:

$$\Delta C_M = C_D\left(\frac{z_p - z_d}{\bar{c}}\right). \tag{11.2}$$

The corresponding contribution to the static margin will be

$$-\frac{\mathrm{d}}{\mathrm{d}C_L}(\Delta C_M) = \frac{\mathrm{d}C_D}{\mathrm{d}C_L}\left(\frac{z_d - z_p}{\bar{c}}\right). \tag{11.3}$$

Now if the total drag coefficient may be written

$$C_D = C_{D_0} + \frac{kC_L^2}{\pi A}, \tag{11.4}$$

it follows that

$$\frac{\mathrm{d}C_D}{\mathrm{d}C_L} = \frac{2kC_L}{\pi A}. \tag{11.5}$$

Hence, from (11.3) and (11.5)

$$-\frac{\mathrm{d}}{\mathrm{d}C_L}(\Delta C_M) = \frac{2kC_L}{\pi A}\left(\frac{z_d - z_p}{\bar{c}}\right). \tag{11.6}$$

With the arrangement shown in Fig. 11.1, there is therefore a positive contribution to the static margin, compared with the case in which the thrust and drag forces are omitted.

In gliding flight, the thrust term is absent and then

$$\left[-\frac{\mathrm{d}}{\mathrm{d}C_L}(\Delta C_M)\right]_{T=0} = \frac{2kC_L}{\pi A}\frac{z_d}{\bar{c}}. \tag{11.7}$$

Comparing (11.6) and (11.7), there is an apparent loss of stability in level flight compared with gliding flight at the same lift coefficient.

However, it has already been observed that a more realistic situation corresponds to constant engine setting. For a turbo-jet aeroplane, this would correspond roughly to constant thrust.

Assuming also that $L = W$, the expression corresponding to eqn. (11.3) becomes

$$\left[-\frac{\mathrm{d}}{\mathrm{d}C_L}(\Delta C_M)\right]_{T=\text{const.}} = \frac{z_d}{\bar{c}}\frac{\mathrm{d}C_D}{\mathrm{d}C_L} - \frac{z_p}{\bar{c}}\frac{\mathrm{d}}{\mathrm{d}C_L}\left(\frac{T}{\frac{1}{2}\rho V^2 S}\right)$$

$$= \frac{z_d}{\bar{c}}\frac{\mathrm{d}C_D}{\mathrm{d}C_L} - \frac{z_p}{\bar{c}}\frac{\mathrm{d}}{\mathrm{d}C_L}\left(\frac{T}{W}C_L\right)$$

$$= \frac{z_d}{\bar{c}}\frac{\mathrm{d}C_D}{\mathrm{d}C_L} - \frac{z_p}{\bar{c}}\frac{T}{W}. \tag{11.8}$$

At the equilibrium condition

$$\frac{T}{W} = \frac{D}{L} = \frac{C_D}{C_L}. \tag{11.9}$$

So, from eqn. (11.8), (11.5), (11.9) and (11.4):

$$\left[-\frac{\mathrm{d}}{\mathrm{d}C_L}(\Delta C_M)\right]_{T=\text{const.}}$$

$$= \frac{z_d}{\bar{c}}\frac{2kC_L}{\pi A} - \frac{z_p}{\bar{c}}\left(\frac{C_{D_0}}{C_L} + \frac{kC_L}{\pi A}\right) \tag{11.10}$$

$$= \frac{2kC_L}{\pi A}\left(\frac{z_d - z_p}{\bar{c}}\right) - \frac{z_p}{\bar{c}}\left(\frac{C_{D_0}}{C_L} - \frac{kC_L}{\pi A}\right) \tag{11.11}$$

It follows from eqn. (11.10) that, at constant engine setting, the aeroplane is less stable than in the gliding case.

At speeds above the minimum drag speed:

$$C_{D_0} > \frac{kC_L^2}{\pi A},$$

and hence from eqn. (11.11), at constant engine setting the aeroplane is less stable than when the engine setting is varied so that the thrust is always equal to the drag.

To summarize, for an aeroplane giving the configuration shown in Fig. 11.1, flying at a speed higher than the minimum drag speed, the following conditions correspond to decreasing static stability, in order:

(1) Gliding flight ($T = 0$).
(2) Powered flight with $T = D$.
(3) Powered flight with $T = \text{const.}$

A similar analysis applied to a piston-engined aeroplane with a constant speed propeller at constant engine setting, so that the thrust power (TV) is approximately constant, leads to an equation similar to (11.10), except that all terms in z_p are multiplied by 1·5. The order of conditions corresponding to decreasing stability is still as stated above, but the relevant minimum speed is now the minimum power speed. This may be shown by differentiating the second term in the first line of eqn. (11.8) subject to the conditions $TV = \text{const.}$ and $W = C_L \frac{1}{2} \rho V^2 S$.

Effect of the c.g. Height

In the theory developed in Chapter 3 and subsequently, it was assumed that the c.g. of the aeroplane lay on the mean chord line. The effect of a displacement of the c.g. above the drag and thrust

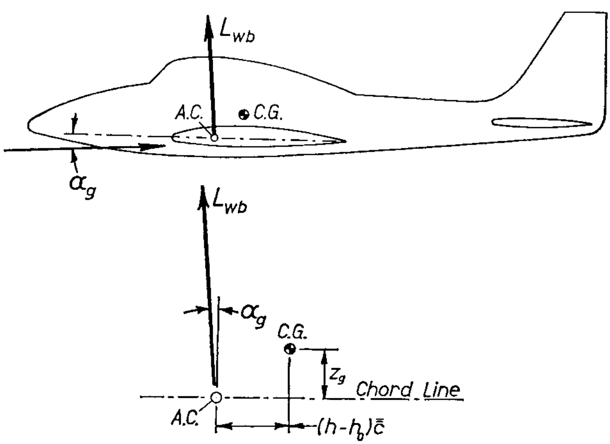

FIG. 11.2. Notation to consider the effect of the height of the c.g. above the chord line.

vectors has been considered above, but a further effect arises from the moment of the lift about the c.g. Assuming that the c.g. is located as shown in Fig. 11.2 by coordinates taken along and perpendicular to the chord line, and that the geometric incidence (i.e. the angle between the chord line and the free stream) is α_g, then the lift will act perpendicular to the free-stream direction.

The moment of the lift of the aeroplane-less-tail about the c.g. will therefore be

$$L_{wb}[(h-h_0)\bar{c}\cos\alpha_g + z_g \sin\alpha_g].$$

Assuming that

$$L_{wb} \doteq L, \qquad \cos\alpha_g \doteq 1, \qquad \text{and} \quad \sin\alpha_g \doteq \alpha_g,$$

this becomes

$$L[(h-h_0)\bar{c} + z_g\alpha_g].$$

Comparing this value with the corresponding quantity occurring in eqn. (4.7), it is clear that there is an additional moment δM_G due to the z_g term

$$\delta M_G = L z_g \alpha_g, \qquad \text{or} \quad \delta C_{M_G} = C_L \frac{z_g}{\bar{c}} \alpha_g. \tag{11.12}$$

If

$$\alpha_g = \alpha - \alpha_0,$$

where, as usual, α is the incidence measured from the zero-lift line and assuming that, neglecting the tail lift, $C_L = a\alpha$ then eqn. (11.12) becomes

$$\delta C_{M_G} = C_L \frac{z_g}{\bar{c}} \left(\frac{C_L}{a} - \alpha_0\right), \tag{11.13}$$

and the corresponding contribution to the static margin is therefore

$$\delta K_n = -\left(\frac{2C_L}{a} - \alpha_0\right)\frac{z_g}{\bar{c}}. \tag{11.14}$$

It therefore follows that, as C_L increases, the static margin decreases for the geometry shown in Fig. 11.2. A consequence of eqn. (11.14) is that the elevator angle to trim curves will become slightly parabolic. This effect may not be negligible: the change in static margin when C_L is varied by ΔC_L is

$$\Delta K_n = -\frac{2\Delta C_L}{a}\frac{z_g}{\bar{c}}. \tag{11.15}$$

If a is of the order of 6, the shift of neutral point for $\Delta C_L = 1{\cdot}0$ is about $-\frac{1}{3}z_g/\bar{c}$. This could be of the order of 0·05, probably an appreciable proportion of the static margins.

APPENDIX I

The Criterion of Static Stability

In the simplified theory of longitudinal static stability, as developed in this book, it does not really matter whether $(-\mathrm{d}C_M/\mathrm{d}C_L)$ or $(-\mathrm{d}C_M/\mathrm{d}\alpha)$ is used as the criterion of stability, since for a rigid aeroplane in incompressible flow these quantities are simply related by the constant lift-curve slope. Either criterion leads to the same neutral point locations, for example. But it is useful to adopt a criterion which is also valid and meaningful under the more general conditions of a non-rigid aeroplane in compressible flow.

In the general case, the aeroplane is subjected to some small disturbance which produces an out-of-balance pitching moment. To obtain the magnitude of this moment, it is necessary to define the nature of the disturbance. Following Gates and Lyon,[9] the classical British approach is to impose the condition that the vertical forces acting on the aeroplane are always in equilibrium. Hence, for an aeroplane in initially horizontal flight, the lift must always equal the weight and it therefore follows that a change of lift coefficient must be accompanied by a simultaneous change of forward speed. Under such conditions, the expression for the out-of-balance pitching moment becomes quite complicated: it will include partial derivatives of all the relevant aerodynamic coefficients with respect to speed (or Mach number), whereas in the simplified theory presented in this book, these Mach number derivatives are taken to be zero. The out-of-balance moments may be imagined as applying either to a rather special type of disturbance or, more convincingly, to conditions which change so slowly that the vertical forces remain in equilibrium. The latter condition is more obviously compatible with the concept of static stability. At this juncture, it may be argued that such a concept is rather artificial: it will not necessarily be a guide to the behaviour of an aeroplane when subjected to real disturbances. This is indeed so, but to assess such behaviour it would be necessary to consider the dynamics of the aeroplane motion in detail. However, if the criterion $(-\mathrm{d}C_M/\mathrm{d}C_L)$ at constant lift is considered, it is found that, even in the general case, it is related to the change of stick position or the stick-force required to change speed in steady flight by a small amount from some initial trimmed speed. It should be re-emphasized that in all discussions of static stability, stick positions and stick-forces relate to steady conditions. The pilot's actions to *initiate* a change in conditions are not considered in this context. The general analysis is similar to that presented in Chapter 6, except that quantities such as a_2 in eqn. (6.10) now become functions of Mach number. The static margin, defined as $(-\mathrm{d}C_M/\mathrm{d}C_L)$ at constant lift is therefore related to a real aspect of the aeroplane's behaviour in flight, whether the general or the more simplified conditions are considered.

If the quantity $(-\mathrm{d}C_M/\mathrm{d}\alpha)$ is now considered, subject again to the condition that the vertical forces remain in equilibrium, it will also be related to the changes of stick position and stick-force with speed, always in steady flight, in the simplified case. But in the general case, Ref. 9 shows that this will no longer be so. Under certain circumstances $\mathrm{d}C_M/\mathrm{d}C_L$ and $\mathrm{d}C_M/\mathrm{d}\alpha$ could have opposite signs. The quantity $(-\mathrm{d}C_M/\mathrm{d}\alpha)$ therefore has no general relevance either to the behaviour of the aeroplane when disturbed from an equilibrium condition nor to the pilot's actions.

Reference 6 argues the case for adopting $(-\partial C_M/\partial\alpha)$ as the criterion for static stability, with the implied condition that the speed and hence the Mach number is constant. This may be regarded as a measure of the out-of-balance moment with respect to small disturbances in incidence at constant forward speed, the vertical forces no longer being in equilibrium. It is the sort of quantity one could easily measure in a wind tunnel running at constant speed. At first sight, this might appear to be a fairly realistic criterion until one considers the dynamics of aeroplane motions involving such a disturbance. One type of motion which is relevant is the steady pull-out at constant speed, treated in an elementary fashion in Chapter 9, from which it will be seen that the actual restoring moment (to be balanced by the application of an elevator deflection) includes a term depending on the rate of pitch. One of the natural modes of motion of an aeroplane in pitch is a rapid, well-damped oscillation which approximates to an incidence variation at constant forward speed. Again, the pitching moments which arise depend on both $\partial C_M/\partial \alpha$ and a rate-of-pitch term, so that the criterion of stability in this mode is in fact the manœuvre margin and not $\partial C_M/\partial \alpha$ alone. The latter quantity cannot therefore be interpreted as having a direct connection with either the behaviour of the aeroplane after a disturbance, or the actions of the pilot in flying it: it is best regarded as a contribution to the manœuvre margin.

Further discussion of static margin involves a consideration of the dynamic behaviour of an aeroplane (see Ref. 9) rather beyond the scope of the present book. It can be inferred from the foregoing that if dangerous instabilities are to be avoided, the stick-fixed and stick-free manœuvre margins must certainly be positive. If, at the same time, the static margins are negative, the aeroplane may not be dangerous: it will tend to depart from a trimmed condition, but usually fairly slowly. The stick-fixed static margin, being related to the change of stick position with speed in steady flight, is often a matter of little concern to the pilot. The effects of a negative stick-free static margin are immediately apparent: if the speed is increased above a trimmed value, the pilot has to resist a tendency for the speed to increase still further by applying a pull-force to the stick. This is distinctly unpleasant, but not necessarily dangerous immediately, if both manœuvre margins are positive; but it would not be tolerable for any length of time. All these considerations are reflected in British Civil Airworthiness Requirements, which permit a slightly negative stick-fixed static margin, but require the stick-free static margin and the manœuvre margins to be positive.

As Chapter 11 indicates, it is generally necessary to specify some condition relating to the horizontal forces when defining static margins, since effects due to the power-plant may be significant. If it is desired to find the neutral points at, say, the cruise condition, then tests should be conducted at the appropriate engine setting and the aeroplane will only be in level flight at one speed. This may be regarded as a realistic case, since the engine setting will normally be left unchanged when the aeroplane is subjected to small disturbances from the equilibrium condition. If, on the other hand, it is desired to assess the change in elevator angle corresponding to a change in speed under level flight conditions, the engine setting must be varied so that the thrust always equals the drag. Even if the effective static margins deduced under the latter conditions are slightly negative, the aeroplane will not necessarily show a tendency to depart from the initial trimmed conditions when the engine setting is kept constant.

Strictly, the only physical interpretation to be attached to static margins relates to the elevator movement and stick-force changes when the speed is changed from one steady value to another subject to specified engine setting conditions.

Finally, it should be noted that Ref. 9 is largely concerned with total derivatives with respect to C_R (e.g. $-\mathrm{d}C_M/\mathrm{d}C_R$), where C_R is the coefficient of the resultant aerodynamic force acting on the aircraft and is defined by

$$C_R = \sqrt{(C_L^2 + C_D^2)}$$

The analysis of Gates and Lyon is therefore strictly only applicable to steady gliding flight, in which the resultant aerodynamic force is equal to the weight of the aircraft. In the present case it has been assumed that the aeroplane is in steady horizontal flight, so that the drag is balanced by the thrust. The resultant aerodynamic force then becomes the lift, and under these circumstances, $C_R = C_L$. It is also worth saying that in considering the static stability of gliders it is usual to assume that the angle of glide is so flat (i.e. $C_D^2 \ll C_L^2$) that C_L and C_R are interchangeable from the present point of view.

APPENDIX II

References and Suggestions for Further Reading

In the list given below, R. & M. relates to the Reports and Memoranda of the Aeronautical Research Council, published by Her Majesty's Stationery Office. Reference 3 encompasses all the R. & Ms mentioned below, and contains extensive lists of further references. All the books mentioned deal with dynamic stability, mostly treating static longitudinal stability rather briefly. Works of transatlantic origin (e.g. Ref. 6) are written mostly in American notation, which is somewhat different from that used in the U.K. In some cases, the initial concepts are also somewhat different. The Royal Aeronautical Society Data Sheets represent the most convenient source of information on the properties of wings, tails, controls, etc. The more important information is reproduced in Refs. 3 and 6.

1. Abbott, I. H. and von Doenhoff, A. E. *Theory of Wing Sections*. Dover, New York, 1959.
2. *Agard Flight Test Manual*. Vol. II. *Stability and Control*. Pergamon, Oxford, 1959.
3. Babister, A. W. *Aircraft Stability and Control*. Pergamon, Oxford, 1961.
4. Bryan, G. H. *Stability in Aviation*. Macmillan, London, 1911.
5. Duncan, W. J. *The Principles of the Stability and Control of Aircraft*. Cambridge University Press, 1952.
6. Etkin, B. *Dynamics of Flight. Stability and Control*. Wiley, New York, 1959.
7. Francis, R. H. and Pringle, G. E. *Notes on Longitudinal Stability at Low Speeds*. R. & M. 1833, H.M S.O., London, 1938.
8. Gates, S. B. *An Analysis of Static Longitudinal Stability in Relation to Trim and Control Force*. R. & M. 2132, H.M.S.O., London, 1939.

9. Gates, S. B. and Lyon, H. M. *A Continuation of Longitudinal Stability and Control Analysis*. Part I. *General Theory*. R. & M. 2027, H.M.S.O., London, 1944.
10. *Ibid.* Part II. *Interpretation of Flight Tests*. R. & M. 2028, H.M.S.O., London, 1944.
11. Royal Aeronautical Society Data Sheets. *Aerodynamics*, Vols. I, II, III and IV.
12. Yates, A. H. Notes on the mean aerodynamic chord and the mean aerodynamic centre of a wing, *J. Roy. Aero. Soc.*, June 1952, pp. 461–474.
13. Duncan, W. J. *Physical Similarity and Dimensional Analysis* (Chapter 2). Edward Arnold, 1953. (Treats fully the subject of dimensional analysis.)

APPENDIX III

An Example of Stability Calculations

THE ULTIMATE aim of a body of engineering theory is to facilitate the design of a useful machine. In this case, it should enable a designer to choose a suitable size of tail, elevator and tab, and settle in principle other associated features of an aeroplane. It is very difficult to present a realistic example of such a design process: the number of requirements to be fulfilled is large and the solution is rarely a closed one, but more usually evolves from a series of successive approximations. It is therefore much easier to present an analysis rather than a synthesis, which explains why most examination questions are analytical. The aim of the analysis below is to demonstrate how the appropriate equations are applied and to give some idea of orders of magnitude, but the author is well aware of the academic nature of such an exercise.

This example relates to a semi-mythical glider having the following characteristics:

Gross wing area, S	173 ft^2
Empty weight	580 lb
Maximum all-up weight	830 lb
Design Dive Speed, E.A.S.	120 knots
Mean Aerodynamic Chord, $\bar{c}$	2·91 ft
Distance from aerodynamic centre of aircraft-less-tail to a.c. of tail, l'_T	12·96 ft
c.g. of unladen aircraft aft of datum	1·94 ft
c.g. of pilot-plus-parachute forward of datum	1·90 ft

Aerodynamic centre of aircraft-less-tail, aft of datum	0·67 ft
Slope of C_L–α curve of aircraft-less-tail, per radian	5·61
C_{M_0} of aircraft-less-tail about a.c.	−0·116
$d\varepsilon/d\alpha$ at tail	0·19
Gross area of horizontal tail, S_T	27·8 ft²
Elevator area, S_η	11·8 ft²
Mean elevator chord, c_η	1·34 ft
Elevator/stick gearing, m_e, radians/ft	1·11
a_1 per radian	3·55
a_2 per radian	2·36
b_1 per radian	−0·327
b_2 per radian	−0·653
η_T	−8·2°

The datum is the leading edge of the M.A.C. The only load is assumed to be the pilot-plus-parachute (W_p), for which the extreme values are taken as 150 lb and 250 lb.

Laden c.g. positions

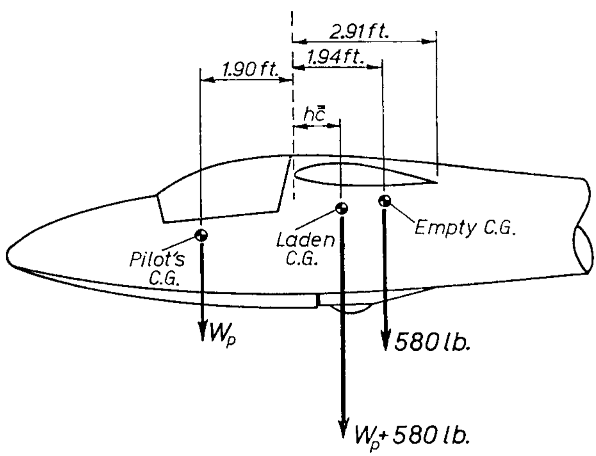

FIG. III.1. Centre of gravity positions.

Taking moments about the datum:

$$(580+W_p)h\bar{c} = 580\times 1{\cdot}94-1{\cdot}90\, W_p,$$

i.e.,

$$h\bar{c} = \frac{1126-1{\cdot}90\, W_p}{580+W_p}.$$

Hence, if

$$W_p = 150\text{ lb}, \qquad h\bar{c} = 1{\cdot}15\text{ ft},$$

and

$$h = 1{\cdot}15/2{\cdot}91 = 0{\cdot}396\text{ (c.g. aft)}.$$

If

$$W_p = 250\text{ lb}, \qquad h\bar{c} = 0{\cdot}785\text{ ft},$$

and

$$h = 0{\cdot}785/2{\cdot}91 = 0{\cdot}270\text{ (c.g. forward)}.$$

Also,

$$h_0 = 0{\cdot}67/2{\cdot}91 = 0{\cdot}23$$

(Note that this figure is less than a typical value for the wing alone (say, 0·27) due to the effect of the fuselage.)

Tail Loads

$$L_T = \frac{C_{M_0}\tfrac{1}{2}\rho_0 V_i^2 S\bar{c}+(h-h_0)\bar{c}L}{l'_T} \tag{4.11}$$

and

$$L = W.$$

ρ_0, the standard sea-level air density $= 0{\cdot}002378$ slugs/ft^3. Hence

$$L_T = \frac{-0{\cdot}116\times 0{\cdot}00119\times 173\times 2{\cdot}91\, V_i^2+(h\bar{c}-0{\cdot}67)W}{12{\cdot}96}$$

If

$$W = 580+150 = 730\text{ lb}, \qquad h\bar{c} = 1{\cdot}15\text{ ft},$$

from the previous calculation, and then

$$L_T = -0{\cdot}00535\, V_i^2+27{\cdot}1\text{ lb}.$$

If

$$W = 830\text{ lb}, \qquad h\bar{c} = 0{\cdot}785\text{ ft},$$

and then

$$L_T = -0{\cdot}00535\, V_i^2+7{\cdot}37\text{ lb}.$$

TABLE 1

V_i (knots)	V_i (ft/sec)	$-0{\cdot}00535\ V_i^2$ (lb)	L_T (lb)	
			c.g. aft	c.g. fwd.
40	67·7	− 24·5	+ 2·6	− 17·1
60	101·6	− 55·1	− 28·0	− 47·7
80	135·0	− 97·7	− 70·6	− 90·3
100	169·0	− 153	− 125·9	− 145·6
120	203·0	− 220	− 192·9	− 212·6

At high speeds, the C_{M_0} term predominates, and the large downward tail loads are worth noting. Since they are about 25 per cent of the all-up weight, it would have been rather unrealistic to assume that the wing lift is equal to the weight under such conditions. Incidentally, the worst stressing case, corresponding to a high load factor with a pitching acceleration, gives a much higher figure.

Also

$$C_{L_T} = \frac{L_T}{\frac{1}{2}\rho_0 V_i^2 S_T} = \frac{L_T}{0{\cdot}00119 \times 27{\cdot}8\, V_i^2}$$

$$= \frac{30 L_T}{V_i^2}\ (V_i \text{ in ft/sec})$$

The tail-lift coefficients may therefore be derived from Table 1 and are as follows:

TABLE 2

V_i (knots)	V_i (ft/sec)	C_{LT}	
		c.g. aft	c.g. fwd.
40	67·7	+0·0171	−0·1125
60	101·6	−0·0818	−0·1392
80	135·0	−0·1168	−0·1495
100	169·0	−0·1327	−0·1534
120	203·0	−0·1412	−0·1557

The range of tail lift coefficients, it will be noted, is quite small.

Tail incidence and elevator angles to trim

$$C_L = \frac{W}{\frac{1}{2}\rho_0 V_i^2 S} = \frac{W}{0{\cdot}00119 \times 173 \times V_i^2} = \frac{4{\cdot}8\,W}{V_i^2}. \tag{III.1}$$

$$C_{L_T} = \frac{1}{1+F}\left[\frac{a_1}{a}\left(1-\frac{\mathrm{d}\varepsilon}{\mathrm{d}\alpha}\right)C_L + a_1\eta_T + a_2\eta + a_3\beta\right], \tag{5.23}$$

where

$$F = \frac{a_1}{a}\frac{S_T}{S}\left(1-\frac{\mathrm{d}\varepsilon}{\mathrm{d}\alpha}\right) = \frac{3{\cdot}55}{5{\cdot}61}\cdot\frac{27{\cdot}8}{173}(1-0{\cdot}19) = 0{\cdot}0825,$$

and

$$\eta_T = -8{\cdot}2° = -0{\cdot}1434 \text{ radians.}$$

Hence, for zero tab angle,

$$1{\cdot}0825\,C_{L_T} = \frac{3{\cdot}55}{5{\cdot}61} \times 0{\cdot}81\,C_L - 3{\cdot}55 \times 0{\cdot}1434 + 2{\cdot}36\,\eta.$$

Whence

$$\eta = 0{\cdot}458\,C_{L_T} - 0{\cdot}217\,C_L + 0{\cdot}216 \text{ radians.} \tag{III.2}$$

Also, from eqn. (2.31),

$$\alpha_T = \frac{C_{L_T}}{a_1} - \frac{a_2}{a_1}\eta \qquad (\text{if } \beta = 0).$$

i.e.

$$\alpha_T = \frac{C_{L_T}}{3{\cdot}55} - \frac{2{\cdot}36}{3{\cdot}55}\eta$$

$$= 0{\cdot}282\,C_{L_T} - 0{\cdot}665\,\eta \tag{III.3}$$

For each value of V_i, C_L can be found from eqn. (III.1), using the appropriate W.

From these values of C_L and the corresponding values of C_{L_T} from Table 2, elevator angles can be calculated from eqn. (III.2), and then tail incidences from eqn. (III.3).

TABLE 3

Elevator angles and tail incidences, c.g. forward ($W = 830$ *lb*)

V_i (ft/sec)	C_L	$0{\cdot}217\ C_L$	$0{\cdot}458\ C_{LT}$	η (rads.)	$\eta°$	$0{\cdot}282\ C_{LT}$	$0{\cdot}665\ \eta$	α_T (rads.)	α_T^0
67·7	0·880	0·191	− 0·0515	− 0·0265	− 1·51	− 0·0317	− 0·0176	− 0·0141	− 0·81
101·6	0·390	0·085	− 0·0639	+ 0·0671	+ 3·84	− 0·0393	+ 0·0446	− 0·0839	− 4·82
135·0	0·221	0·048	− 0·0685	+ 0·1005	+ 5·75	− 0·0422	+ 0·0668	− 0·1090	− 6·25
169·0	0·141	0·031	− 0·0703	+ 0·1147	+ 6·55	− 0·0432	+ 0·0762	− 0·1194	− 6·84
203·0	0·098	0·021	− 0·0712	+ 0·12338	+ 7·08	− 0·0438	+ 0·0821	− 0·1259	− 7·22

Similarly, the values corresponding to the aft c.g. position ($W = 730$ lb) may be found.

TABLE 4

Elevator angles and tail incidences, c.g. aft

V_i (ft/sec)	C_L	η (rads.)	η^0	α_T (rads.)	$\alpha_T{}^0$
67·7	0·777	+0·0558	3·20	−0·0323	−1·85
101·6	0·344	+0·1035	5·93	−0·0918	−5·25
135·0	0·195	+0·1205	6·91	−0·1129	−6·47
169·0	0·124	+0·1284	7·36	−0·1227	−7·02
203·0	0·086	+0·1322	7·58	−0·1278	−7·33

Figure III.2 shows elevator angle plotted against lift coefficient. As one would expect, the lines intersect at $C_L = 0$.

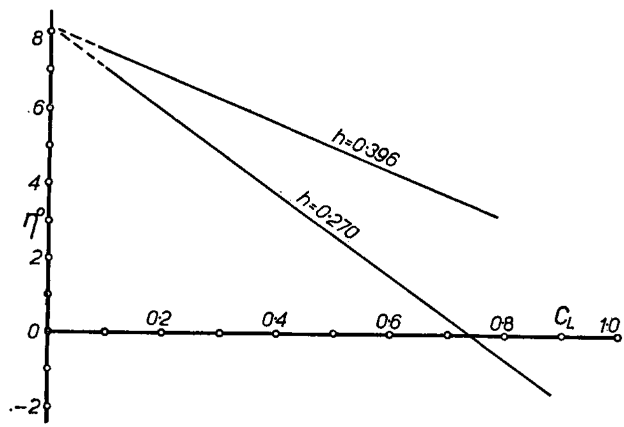

FIG. III.2. Example of elevator angles to trim.

It is also of interest to consider the effects of tail incidence and elevator angle as speed is increased. Knowing that in a stable aeroplane the pilot moves the stick forwards as the speed increases, thus moving the elevator downwards, one tends to visualize the effect of the downwards elevator movement being that of lifting the tail and depressing the nose. Hence the aircraft acquires more speed. But in fact, the tail lift and tail lift coefficient both become more negative as speed is increased. The effect of the increasing downward elevator deflection is indeed to make these quantities more positive, but this effect is out-weighted by the decreasing tail incidence.

Stick-fixed neutral point

$$h_n = h_0 + V_T \frac{a_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right), \tag{5.8}$$

where

$$V_T = \bar{V}'/(1+F) = \bar{V}'/1{\cdot}0825,$$

and

$$\bar{V}' = S_T l'_T / S\bar{c} = 27{\cdot}8 \times 12{\cdot}96/173 \times 2{\cdot}91 = 0{\cdot}715,$$

so that

$$V_T = 0{\cdot}715/1{\cdot}0825 = 0{\cdot}66.$$

Therefore

$$h_n = \frac{0{\cdot}67}{2{\cdot}91} + 0{\cdot}66 \times \frac{3{\cdot}55}{5{\cdot}61}(1 - 0{\cdot}19)$$

$$= 0{\cdot}23 + 0{\cdot}338 = 0{\cdot}568,$$

or

$$h_n \bar{c} = 0{\cdot}568 \times 2{\cdot}91 = 1{\cdot}65 \text{ ft},$$

i.e. the stick-fixed neutral point is 1·65 ft aft of the leading edge of the M.A.C.

Stick-free neutral point

$$h'_n = h_0 + \bar{V}_T \frac{\bar{a}_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right) \tag{5.21}$$

and

$$\frac{\bar{a}_1}{a_1} = 1 - \frac{a_2 b_1}{a_1 b_2} \tag{5.11}$$

$$= 1 - \frac{2{\cdot}36 \times 0{\cdot}327}{3{\cdot}55 \times 0{\cdot}653} = 1 - 0{\cdot}333$$

$$= 0{\cdot}667,$$

i.e. the tail lift-curve slope is reduced by one-third on freeing the stick.

$$\bar{F} = \frac{\bar{a}_1}{a_1} F \qquad \text{(5.3) and (5.16)}$$

$$= 0{\cdot}667 \times 0{\cdot}0825 = 0{\cdot}055.$$

Hence

$$\bar{V}_T = \frac{\bar{V}'}{1+\bar{F}} = \frac{0{\cdot}715}{1{\cdot}055} = 0{\cdot}677.$$

Therefore

$$h_n' = 0{\cdot}23 + 0{\cdot}677 \times \frac{3{\cdot}55}{5{\cdot}61} \times 0{\cdot}667(1-0{\cdot}19)$$

$$= 0{\cdot}23 + 0{\cdot}231 = 0{\cdot}461,$$

or

$$h_n'\bar{c} = 0{\cdot}461 \times 2{\cdot}91 = 1{\cdot}34 \text{ ft},$$

i.e. the stick-free neutral point is 1·34 ft aft of the leading edge of the M.A.C.

Since, in this simple case, the static margins and c.g. margins are the same (i.e. $K_n = h_n - h$; $K_n' = h_n' - h$), they can be found from the values of h already obtained.

TABLE 5

Load (lb)	h	h_n	K_n	h_n'	K_n'
250	0·270	0·568	0·298	0·461	0·191
150	0·396	0·568	0·172	0·461	0·065

It is worth noting that the stick-free static margin, c.g. forward, is three times the value corresponding to c.g. aft. This represents rather a large variation, and it will be difficult to obtain pleasant stick-force characteristics under both conditions.

Stick-force to change speed

The slope of the stick-force/equivalent airspeed curve at a trimmed speed is given by

$$\left(\frac{dP_e}{dV_i}\right)_0 = -\frac{b_2}{a_2} \cdot \frac{2m_e S_\eta c_\eta W}{S\overline{V}_T} \cdot \frac{K_n'}{V_{i0}} \tag{6.33}$$

$$= \frac{0{\cdot}653 \times 2 \times 1{\cdot}11 \times 11{\cdot}8 \times 1{\cdot}34}{2{\cdot}36 \times 173 \times 0{\cdot}677} \cdot \frac{WK_n'}{V_{i0}}$$

$$= 0{\cdot}0828 \frac{WK_n'}{V_{i0}}.$$

Inserting values of K_n' from Table 5, the stick-force/speed gradient may be obtained at trimmed speeds of, say, 40 and 100 knots.

TABLE 6

Load (lb)	W (lb)	K_n'	$(dP_e/dV_i)_0$, (lb/knot)	
			40 knots	100 knots
150	730	0·065	0·098	0·039
250	830	0·191	0·328	0·131

At a given equivalent airspeed, the stick-force to change speed by a given amount for the heavy pilot is roughly three times that for the light pilot. This represents rather a large variation with pilot weight, which can only be reduced by shifting the stick-free neutral point aft.

Effect of a mechanical elevator moment

Increment of static margin due to a mechanical moment H_s, from (8.8) and (8.4) is

$$\delta K_n' = -\overline{V}_T \frac{a_2 H_s S}{b_2 W S_\eta c_\eta}.$$

Suppose

$$H_s = +1{\cdot}5 \text{ lb-ft.}$$

Then

$$\delta K_n' = 0{\cdot}677 \times \frac{2{\cdot}36 \times 1{\cdot}5 \times 173}{0{\cdot}653 \times 11{\cdot}8 \times 1{\cdot}34} \times \frac{1}{W} = \frac{40{\cdot}1}{W}.$$

The new values of K_n' are then obtained by adding the appropriate $\delta K_n'$ to the figures in Table 6, and finding $(\mathrm{d}P_e/\mathrm{d}V_i)_0$ as above.

TABLE 7

Load (lb)	W (lb)	$\delta K_n'$	New K_n'	$(\mathrm{d}P_e/\mathrm{d}V_i)_0$, (lb/knot)	
				40 knots	100 knots
150	730	0·055	0·120	0·183	0·074
250	830	0·048	0·239	0·410	0·164

The ratio of the stick-force/speed gradients has now been reduced from about 3 : 1 to a little over 2 : 1 for the two c.g. positions considered.

The actual gradients are rather large: the worst case gives 0·41 lb/knot, which a glider pilot would interpret as a very heavy elevator control. (The c.g. aft figures would be considered more normal.) If one wished to "lighten" the elevator with the least structural alteration to the machine, the simplest solution would be to reduce the numerical value of b_2, either by horn balances or a geared tab (see Fig. III.3).

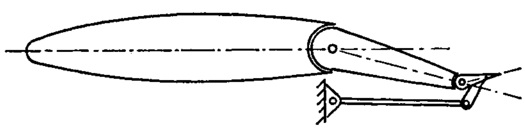

FIG. III.3. Geared tab.

The tab is arranged so that it deflects in the opposite sense to the elevator.

If

$$\beta = \beta_0 + k\eta,$$

where β_0 and k have much the same significance as in Chapter 10, but k is now negative, and

$$C_H = b_1\alpha_T + b_2\eta + b_3\beta \qquad (2.35)$$

then

$$C_H = b_1\alpha_T + (b_2 + kb_3)\eta + b_3\beta_0.$$

The effective b_2 is now $(b_2 - kb_3)$ and, taking the signs of the various quantities into account, the desired result can be achieved by adjusting the magnitude of k.

In practice, in a similar situation, the final remedy was to reduce the elevator chord as a fraction of the total tail chord. This had the effect of reducing the numerical values of b_2, S_η and c_η in eqn. (6.33). In the denominator, a_2 was of course also reduced, but the overall effect was an appreciable lightening of the elevator. A change of this sort alters all the previous calculations: the loss in stability on freeing the stick is reduced, and hence K_n' is increased somewhat, leading to a smaller variation of stick-force to change speed with c.g. position. In altering the tail geometry in this way, it is important to ensure that the elevator power (in effect, $V_T a_2 \eta$, where η corresponds to the maximum deflection) is sufficient for manuœvres such as rounding-out for landing.

Stick-force per g

It is also of interest to calculate this figure, using the original data. From eqn. (9.25), the stick-free manœuvre margin is given by

$$H_m' = K_n' + \bar{V}_T \frac{\bar{a}_1}{2\mu_1},$$

where

$$\mu_1 = \frac{W}{g\rho S l_T'}. \qquad (9.9)$$

At sea level,

$$\mu_1 = \frac{W}{32{\cdot}2 \times 0{\cdot}00238 \times 173 \times 12{\cdot}96}$$
$$= W/171{\cdot}6$$

Hence

$$\overline{V}_T \frac{\bar{a}_1}{2\mu_1} = \frac{0{\cdot}677 \times 3{\cdot}55 \times 0{\cdot}667 \times 171{\cdot}6}{2W}$$
$$= \frac{137{\cdot}3}{W}.$$

From this expression and the values of K_n' from Table 5, H_m' may be found for each c.g. position.

TABLE 8

W (lb)	K_n'	$\overline{V}_T \frac{a_1}{2\mu_1}$	H_m'
730	0·065	0·188	0·253
830	0·191	0·164	0·355

Note that the figures in the third column are large compared with those which would apply to a powered aircraft, because μ_1 is small for gliders (about one-sixth that of a typical fighter). It is also important to note that since these figures depend on the weight, the manœuvre point is also a function of the weight (unlike the neutral points).

In the absence of a mechanical elevator moment:

$$\frac{P_e}{n-1} = \frac{m_e b_2 W S_\eta c_\eta H_m'}{a_2 \overline{V}_T S} \tag{9.24}$$
$$= -\frac{1{\cdot}11 \times 0{\cdot}653 \times 11{\cdot}8 \times 1{\cdot}34\, WH_m'}{2{\cdot}36 \times 0{\cdot}677 \times 173}$$
$$= -0{\cdot}0415\, WH_m'.$$

Using the values of H_m' from Table 8, the stick-forces per g are therefore as follows:

TABLE 9

W (lb)	H_m'	$\dfrac{P_e}{n-1}$ (lb/g)
730	0·253	−7·65
830	0·355	−12·25

Since a mechanical moment due to a spring does not affect the stick-force per g, these figures would also apply in the presence of such a moment.

If the mechanical moment were provided by a weight, the above figures would be increased by $-m_e H_s$ lb (eqn. (9.26)), i.e. by

$$-1{\cdot}11 \times 1{\cdot}5 = -1{\cdot}67 \text{ lb}/g.$$

They would therefore become $-9{\cdot}32$ and $-13{\cdot}92$ lb/g corresponding to all-up weights of 730 and 830 lb respectively. Again, these figures are rather high for a glider, by a factor of about two. For much the same reasons which apply to the considerations of stick-force to change speed, a reduction of elevator chord as a fraction of the total tail chord was found to have a beneficial effect.

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List of Symbols

A	Aspect ratio, b^2/S.
a	Local speed of sound.
a	Lift-curve slope of aeroplane-less-tail, $dC_{L_{wb}}/d\alpha$.
a_0	Lift-curve slope of an aerofoil section, $dC_l/d\alpha$.
a_1	$\partial C_{L_T}/\partial \alpha_T$ — See eqn. (2.32).
a_2	$\partial C_{L_T}/\partial \eta$ — See eqn. (2.32).
a_3	$\partial C_{L_T}/\partial \beta$ — See eqn. (2.32).
$\bar{a}_1$	$\partial C_{L_T}/\partial \alpha_T$ — With elevator free, i.e. $C_H = 0$. See eqn. (5.11) and (5.12).
$\bar{a}_3$	$\partial C_{L_T}/\partial \beta$ — With elevator free, i.e. $C_H = 0$. See eqn. (5.11) and (5.12).
$\bar{\bar{a}}_1$	Effective tail lift-curve slope for an all-moving tail with geared tab, stick-free. See eqn. (10.10).
a_T	Generalized tail lift-curve slope, $dC_{L_T}/d\alpha_T$.
a_w	Lift-curve slope of a three-dimensional wing, $dC_{L_w}/d\alpha$.
b	Wing span.
b_1	$\partial C_H/\partial \alpha_T$ — See eqn. (2.36).
b_2	$\partial C_H/\partial \eta$ — See eqn. (2.36).
b_3	$\partial C_H/\partial \beta$ — See eqn. (2.36).
C_d	Drag coefficient of a two-dimensional wing section or the local value at a given section of a finite wing.
C_{D_w}	Drag coefficient of a three-dimensional wing.
C_D	Drag coefficient of a complete aircraft.
C_{D_0}	C_D at $C_L = 0$.
C_H	Elevator hinge moment coefficient, $H/\frac{1}{2}\rho V^2 S_\eta c_\eta$.
$C_{H_{tot}}$	Total elevator hinge moment coefficient in the presence of a mechanical hinge moment, $H_{tot}/\frac{1}{2}\rho V^2 S_\eta c_\eta$.
C_l	Lift coefficient of a two-dimensional wing section or the local value at a given section of a finite wing.
C_{l_a}	Local lift coefficient associated with the additional lift distribution.

C_{l_b} Local lift coefficient associated with the basic lift distribution.

C_L Lift coefficient of a complete aeroplane.

C_{L_w} Lift coefficient of a three-dimensional wing.

C_{L_0} Lift coefficient of a complete aeroplane at V_{i_0} (Chapter 6), or lift coefficient in level flight at a given speed and height (Chapter 9).

C_{L_1} Lift coefficient of a complete aeroplane in a pull-out at a given speed and height (Chapter 9).

$C_{L_{wb}}$ Lift coefficient of aeroplane-less-tail.

C_{L_T} Tail lift coefficient, $L_T/\frac{1}{2}\rho V^2 S$.

C_m Pitching moment coefficient of a two-dimensional wing section, or the local value at a given section of a finite wing.

$C_{m_{c/4}}$ C_m about the quarter-chord point.

C_{m_0} C_m about the aerodynamic centre.

C_{M_0} Pitching moment coefficient of the aeroplane-less-tail about its aerodynamic centre. (Replaces $C_{M_{0wb}}$ after eqn. (4.2)).

$C_{M_{0w}}$ Pitching moment coefficient of a three-dimensional wing about its aerodynamic centre.

$C_{M_{0wb}}$ Pitching moment coefficient of the aeroplane-less-tail about its aerodynamic centre. (See also C_{M_0}.)

C_{M_G} Pitching moment coefficient of a complete aeroplane about its centre of gravity.

$C_{M_{G0}}$ C_{M_G} at $C_L = 0$.

$C_{M_{np}}$ Pitching moment coefficient of a complete aeroplane about its neutral point.

$C_{M_{T0}}$ Pitching moment coefficient of the tail about the aerodynamic centre of the basic symmetrical section.

C_R Resultant force coefficient, $\sqrt{(C_L^2 + C_D^2)}$.

c Chord of a two-dimensional wing section, or local chord of a three-dimensional wing.

$\bar{c}$ Geometric Mean Chord, or Standard Mean Chord (eqn. (2.25)).

$\bar{\bar{c}}$ Mean Aerodynamic Chord (M.A.C.) (eqn. (2.22)).

c_r Wing root chord.
c_t Wing tip chord.
c_T Mean aerodynamic chord of a tail. (Chapter 10.)
c_η Elevator mean chord.
c_3 $\mathrm{d}C_{M_{T0}}/\mathrm{d}\beta$ (eqn. (10.2)).
D Drag force.
D_w Drag of a three-dimensional wing.
$\mathfrak{F}$ $\dfrac{S_T}{S}\dfrac{a_T}{a}\left(1 - \dfrac{\mathrm{d}\varepsilon}{\mathrm{d}\alpha}\right)$ (eqn. (4.27)).
F Value of $\mathfrak{F}$ when $a_T = a_1$ (eqn. (5.3)).
$\bar{F}$ Value of $\mathfrak{F}$ when $a_T = \bar{a}_1$ (eqn. (5.16)).
$\bar{\bar{F}}$ Value of $\mathfrak{F}$ when $a_T = \bar{\bar{a}}_1$ (eqn. (10.11)).
g Acceleration due to gravity.
H Aerodynamic elevator hinge moment.
H_s Elevator hinge moment due to a bob-weight or spring.
H_{tot} Total elevator hinge moment.
H_m Manœuvre margin, stick-fixed (eqn. 9.16)).
H'_m Manœuvre margin, stick-free (eqn. (9.22)).
h Centre of gravity position (dimensionless), aft of leading edge of M.A.C.
h_0 Position of aerodynamic centre of aeroplane-less-tail (dimensionless), aft of leading edge of M.A.C.
h_m Position of stick-fixed manœuvre point (dimensionless), aft of leading edge of M.A.C. (eqn. (9.15)).
h'_m Position of stick-free manœuvre point (dimensionless), aft of leading edge of M.A.C. (eqn. (9.21)).
h_N Position of the generalized neutral point (dimensionless) aft of leading edge of M.A.C. (eqns. (4.14) and (4.31)).
h_n As h_N, but stick-fixed (eqns. (5.5) and (5.8)).
h'_n As h_N, but stick-free (eqns. (5.18) and (5.21)).
K_n Static margin, stick-fixed (eqns. (5.2), (5.4) and (5.7)).
K'_n Static margin, stick-free (eqns. (5.15), (5.17) and (5.20)).
k Tab/tailplane gear ratio (Chapter 10), or induced drag factor (Chapter 11).
L Lift force (usually on complete aeroplane).
L_T Tail lift.

L_w Lift on a three-dimensional wing.

L_{wb} Lift on aeroplane-less-tail.

l_T Distance from c.g. of aeroplane of aerodynamic centre of tail.

l'_T Distance from aerodynamic centre of aeroplane-less-tail to aerodynamic centre of tail.

l_N Distance from the generalized neutral point to the aerodynamic centre of tail (eqn. (4.32)).

l_n Distance from the stick-fixed neutral point to the aerodynamic centre of tail (eqn. (5.6)).

l'_n Distance from the stick-free neutral point to the aerodynamic centre of tail (eqn. (5.19)).

$\mathbf{M}$ Mach number, V/a.

M Pitching moment.

M_{0_w} Pitching moment of a three-dimensional wing about its mean aerodynamic centre.

$M_{0_{wb}}$ Pitching moment of the aeroplane-less-tail about its aerodynamic centre.

M_G Pitching moment of a complete aeroplane about its centre of gravity.

M_{0_T} Pitching moment of the tail about its aerodynamic centre.

M_{T_0} Pitching moment of the tail about the aerodynamic centre of the basic symmetrical section (applied to moving tails with geared tabs. Chapter 10).

m_e Elevator/stick gear ratio, radians per ft.

n Load factor, L/W.

P_e Stick-force applied by the pilot, positive in the push-forward sense.

Q_1 Stick-travel per g (eqn. (9.17)).

q Rate of pitch.

$\mathbf{R}$ Reynolds number.

R Resultant aerodynamic force.

S Wing area (gross).

S_T Tail area (including control surfaces and tabs).

S_η Elevator area.

T Thrust.

V True airspeed.
V_i Equivalent airspeed.
V' Local airspeed at tail.
$\bar{V}$ Tail volume coefficient, $l_T S_T/\bar{c}S$.
$\bar{V}'$ Modified tail volume coefficient, $l'_T S_T/\bar{c}S$.
V_T Effective tail volume coefficient, stick-fixed, $\bar{V}'/(1+F)$.
$\bar{V}_T$ Effective tail volume coefficient, stick-free, $\bar{V}'/(1+\bar{F})$.
$\bar{\bar{V}}_T$ Effective tail volume coefficient, stick-free, for an all-moving tail, $\bar{V}'/(1+\bar{\bar{F}})$.
W All-up weight of an aeroplane.
W_p Weight of pilot-plus-parachute (Appendix III).
x Distance aft of leading edge (two-dimensional wing sections).
x_{CP} Distance of the centre of pressure aft of the leading edge (two-dimensional wing sections).
x_{ac} Distance of the aerodynamic centre aft of the leading edge (two-dimensional wing sections).
x_s Stick-travel at pilot's hand-grip, positive forwards.
x Position of local aerodynamic centre at a given spanwise location on a three-dimensional wing relative to an arbitrary datum (pp. 13–19).
$\bar{x}$ Position of the mean aerodynamic centre of a three-dimensional wing relative to the same datum.
$\bar{x}_{LE}$ Position of the leading edge of the M.A.C. aft of the apex of a three-dimensional wing.
x_T Location of the pivot axis of a moving tail aft of the aerodynamic centre of the basic symmetrical section.
y Lateral distance from the plane of symmetry of an aeroplane or three-dimensional wing, positive to starboard.
z_d Distance of the line of action of the drag force below the c.g. of the aeroplane.
z_g Distance of the mean chord line of the wing below the c.g. of the aeroplane.
z_p Distance of the line of action of the thrust below the c.g. of the aeroplane.

α Incidence. (Generally the angle between the zero-lift-line of the wing or aeroplane-less-tail and the direction of the free stream.)

α_0 Angle between the zero-lift-line and the chord-line (see Chapter 11).

α_g Geometric incidence (i.e. angle between chord-line and free stream direction).

α_T Tail incidence.

β Tab angle.

β_{trim} Tab angle at which the aeroplane is in trim with zero stick-force (i.e. $C_{M_G} = 0$; $C_H = 0$).

β_0 Tab angle to trim at $C_L = 0$ (Chapter 6), or tab datum setting (Chapter 10).

γ Angle of inclination of the flight path to the horizontal.

ε Induced downwash angle at the tail (due to the wing).

ε_w Induced downwash angle at the wing (three-dimensional wings).

η Elevator angle.

η_T Tail setting angle.

η_{trim} Elevator angle at which the aeroplane is in trim (i.e. $C_{M_G} = 0$).

η_0 Elevator angle to trim at $C_L = 0$.

Λ_0 Angle of sweep of wing leading edge.

λ Taper ratio, c_t/c_r.

μ Viscosity of air.

μ_1 Relative density parameter (eqn. (9.9)).

v Dimensionless mechanical hinge moment (eqn. (8.4)).

ρ Air density.

ρ_0 Standard sea-level air density.

σ Density ratio, ρ/ρ_0.

ϕ Angle of bank.

Ω Rate of turn about a vertical axis.

Notes. For the sake of brevity, quantities such as a_0 and a_w are referred to as "lift-curve slopes" throughout the text. As the context implies, these quantities are dimensionless and should strictly be defined as the slopes of the curves of lift coefficient against incidence.

In Chapter 9, suffix *po* relates to conditions in a pull-out and suffix *st* relates to conditions in a steady level turn.