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Handbook of Crack Opening Data

T G F Gray

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Handbook of Crack Opening Data

A compendium of equations graphs, computer software and references for opening profiles of cracks in loaded components and structures

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PREFACE

This book deals with idealised representations of cracks, but the incentive to develop the data grew in the first instance from instructive encounters with real cracks. The first of these related to the inspection of a very large iron-ore reclaiming machine, where cracks had been found deep in the interior of the base. This visit made a strong impression, as the reclaimer was kept working as we clambered about inside the hull structure. There was also some nervousness that the machine might at any moment decide to tear itself free from its base and fall over with us inside. However, that apart, it was fascinating to watch the cracks opening and closing in step with the digging cycle.

The mining company involved was keen to find a way of predicting the development of further cracking and this set me thinking about the possibility of getting useful information from simple observations of crack opening. Manipulation of the relevant equations showed that opening should be proportional to the product of stress intensity range and the square root of crack size. As the crack sizes were easy to measure, it was possible to judge the stress intensity range in service and hence the approximate growth rate.

This episode was recalled later in a different context when a non-destructive testing specialist recommended that experiments be carried out to investigate relationships between stress and crack opening. The background in this case was that the response of various ultrasonic inspection systems had been found to depend on the stress in service at the time of inspection. This was creating difficulties in sizing weld defects.

In view of the wide range of possible crack configurations of interest, I suggested that it might be better to examine the problem theoretically. Support for a preliminary study was then found within a programme of research on Structural Integrity Monitoring and most of the results given in this Handbook were developed in that framework. The financial support of the Marine Technology Directorate and several offshore exploration company sponsors of the SIM project is therefore gratefully acknowledged. I am also pleased to thank Dr John Pang, who completed some of the finite element work while an extra-mural student at the National Engineering Laboratory, East Kilbride.

However, the results might not have been published in the present form but for the British Institute of Non-Destructive Testing which organised a Seminar in January 1992 on the theme of 'Combining Fracture Mechanics and NDT'. The meeting attracted a mix of participants from the related activities of fracture mechanics and NDT and this gave me the opportunity to show how crack opening is related quantitatively to stress and crack size. The encouraging response from those present prompted me to consider publishing the results in a form that would be readily accessible to a range of users and Woodhead Publishing was suggested as a channel for specialist information of this kind. The result which you now see is the outcome of subsequent fruitful dialogue with the publishers.

I hope that the users of this book will indeed find the ideas useful and the results accessible.

Thomas G F Gray 23rd May 1992

1 INTRODUCTION

1.1 Basis of the Handbook

The purpose of this Handbook is to provide equations, graphs, computer programs and other information which can be used to calculate crack-face-opening-profiles as a function of configuration and load history. The information was developed in the context of a group of projects concerned with crack detection in steel structures. The response of various non-destructive examination systems to a given crack is known to depend on the separation distance of the crack faces [**1-3**] and it is therefore of interest to know how the crack opening profile depends on the geometry of the cracked body, the applied loading and other factors.

There are also several uses for such information in fracture mechanics; for example in toughness testing and in the application of weight function techniques to the determination of stress intensity factors [**4**].

Various sources and procedures were used to establish the information contained in the Handbook. The basic shape of a line crack which is opened up by normal stress action can be derived from the classical Inglis solution and this was used as a baseline for other geometries. Several numerical results have also been given in the literature for crack-mouth-opening in linear elastic models. However, few exact analytical solutions for complete crack-opening profiles exist. Accordingly, finite element methods and weight function formulations [**5-8**] were used in various cases to 'fill in' profiles between the mouth and the tip of the crack.

There is little experimental information on crack profile and the data given here are essentially theoretical. However, in reference [**9**], the results of an interferometric investigation of crack opening in toughness specimens were compared with the corresponding theoretical determinations in this Handbook. In all cases given here, efforts have been made to cross-validate new information and previously published data.

Emphasis in the Handbook is on simple graphical presentations and formulae which can be readily programmed. The use of a consistent non-dimensional presentation exposes the common features of crack behaviour in different configurations.

Computer programs written in BASIC are available for many of the cases.

1.2 Categorisation of Data

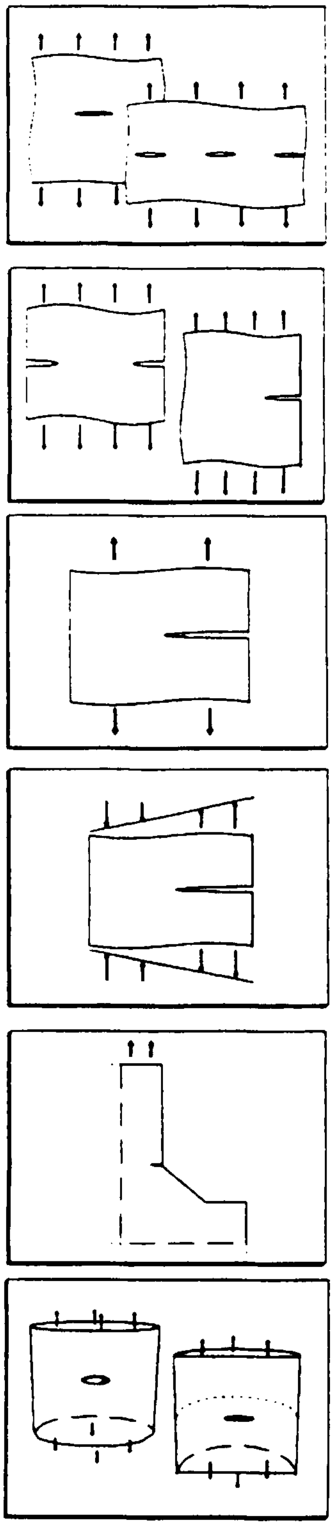
The solutions have been indexed in the Handbook in terms of two types of modelling factor - namely a factor type related to the configuration of the cracked body and a type relating to the constitutive and crack growth aspects.

The term "**configuration**" is used in the sense employed by Parker [10] to include both the geometrical configuration of the cracked body and the pattern of loading relative to it. The loadings are all of the 'Mode I' type in fracture mechanics terminology, ie normal to the crack plane.

Figure 1 shows the main configurations which increase in complexity from top to bottom in the figure. This sequence is followed in the Handbook pages. The classical Inglis/Griffith 2D-infinite plate/internal crack begins the series. The geometrical description becomes more complex, firstly through the addition of free boundaries at finite distance from the crack tip and secondly due to the change from two-dimensional to three-dimensional geometry. (Within each of these categories, further solutions are given which are not necessarily illustrated in Figure 1.)

The loading becomes more complex through the development from uniform stressing, through linearly varying (bending) stress, to non-linear stress gradients such as occur at geometric stress concentrations.

configuration models



2D infinite plate

2D semi-infinite &
double-edge crack

2D finite width
single-edge tension

2D finite width
single-edge bending

2D cruciform
fillet weld

3D geometries

figure 1

The "**constitutive/crack growth**" categorisation divides firstly into stationary-crack and growing-crack models (see Figure 2). Most solutions in fracture mechanics belong to the stationary-crack division, whereby a vanishingly-narrow line crack is considered to open under tensile normal force without lengthening. However, most cracks encountered at the stage of non-destructive examination have been generated by a process of growth which leaves a 'wake' of stretched zones on the flanks of the cracks. The presence of these stretched zones is not included in stationary crack models and, as a consequence, crack-face-opening profiles and the conditions leading to crack closure are incorrectly described by such models. Nevertheless if the load level for first separation of crack faces is identified, relative opening displacements at higher load levels will be given correctly by stationary solutions, which therefore remain useful for growing cracks.

Within the **stationary-crack** division, available solutions have been divided into elastic and elastic/plastic models. Elastic solutions have the advantage that they can be 'superposed', that is, the displacements due to different loadings on the same geometry can be added directly. This facilitates the generation of results for combined tension and bending, for example. Several other combinatory techniques based on superposition are available, for example, the "compounding" method of Cartwright and Rooke [11] which permits the combination of solutions for different geometries. Also there are many 'weight function' techniques which can be used to extend a given solution to describe the effects of alternative loadings.

Elastic/plastic solutions become essential if the nominal stress exceeds about 30% of the yield strength or if crack growth and closure effects are to be properly described. A variety of constitutive laws can be invoked but a simple elastic/perfectly-plastic relation should be sufficient to provide adequate modelling of crack face separation. Hence, the Dugdale model simulation of crack tip plasticity has been used exclusively here, as it provides clear continuity between the stationary elastic models and the growing crack cases. As the Dugdale model is essentially an elastic simulation of an elastic/plastic state, it is also capable of extension via weight function techniques.

Several **growing crack** situations are possible in practice. Firstly, cracks may extend under monotonically increasing load. In such cases, the normal practice is to use stationary crack solutions to describe such features as crack opening. However, the more common case in practice is for incremental cyclic growth, as in fatigue, wherein the stretched zones may be re-compressed to an extent during crack closure. Few solutions exist for this kind of problem (i.e, crack closure models) especially if the load history includes a variable amplitude component. The solution by Newman [12] is used in the Handbook to generate results.

constitutive relation / crack growth models

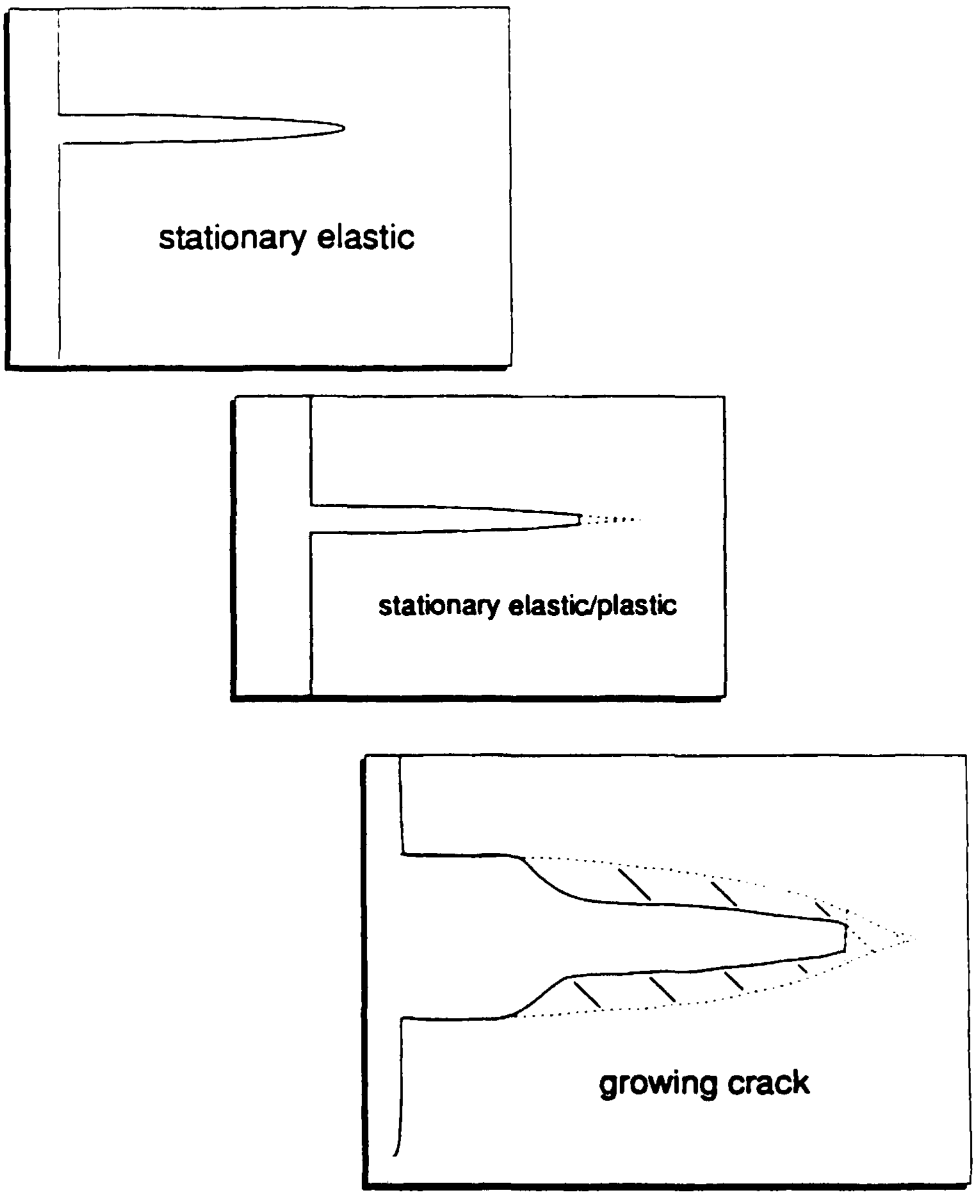


figure 2

1.3 Matrix Index

Data for a particular geometry and modelling category can be located through the general index (page (iii)) or by identifying the category in the matrix-form index opposite, figure 3.

For each solution, background information, references and equations are given on a left-hand page and the corresponding graphs are on the facing right-hand page. Elastic solutions have been given the prefix 'E', elastic/plastic solutions 'P' and growth solutions 'G'. Equations are therefore identified by an appropriate letter prefix and graphs by figure numbers which combine a letter prefix and a geometric category number from 1 to 6 as shown in figure 3.

In some cases, where no specific solutions are given in the Handbook but further information of value is contained in the literature, appropriate references are shown in the matrix index.

Computer Programs

Titles in *ITALIC* refer to computer programs written in BASIC. These were used to generate the data presented in the graphs and may be adapted to other purposes by the user. In general, the programs have been designed to create files of tabulated values, giving crack opening as a function of position along the crack profile. The values may then be listed or exported to standard databases or graph-plotting packages as desired. A non-linear spacing of coordinate values along the crack was generally adopted so that smooth and accurate plotting of the profile would result.

The original programs were written in *Borland Turbobasic Version I* to run on a Personal Computer. If this software is available, relevant program files (with extension **. *BAS*) may be loaded from the disc provided with the Handbook and may also be altered, if desired. The programs create user-named data files (which are given the extension **. *DAT* automatically within the program). Compiled versions of the programs (with extension **. *EXE*) may be run directly from DOS even if *Turbobasic* is not available, but it will not then be possible to make changes to parameters or program flow. No guarantee can be given that the programs will run without modification, or on a given machine, or with the designated software.

Alternatively, fresh programs may be constructed as necessary, using the listings which are included in the Handbook as a guide. In most programs there is a 'reformat' block, whereby the calculated data may be rearranged into a format which is convenient for the plotting software being used.

2 STATIONARY CRACK ELASTIC SOLUTIONS

2.1 2D infinite plate - internal crack

The classical Inglis elastic solution [13] provides the baseline for all the results given in the Handbook. In this case a vanishingly-narrow ellipse opens under normal load to an elliptical shape. Figure E1.1 shows normal displacement in one quadrant. The basic equation for displacement $\mathrm{v}(x)$, where x is measured from mid-point of the crack, is given by

$$\mathrm{v}(x) = 2\frac{\sigma}{E}(a^2 - x^2)^{1/2} \qquad \text{......E1}$$

This formulation implies plane stress. For plane strain, which is in any case more appropriate for a sharp crack in an elastic medium, the Young's Modulus term E should be replaced by $E/(1-\nu^2)$ where ν is Poisson's ratio. (This substitution is also relevant for the 2D solutions on the following pages.)

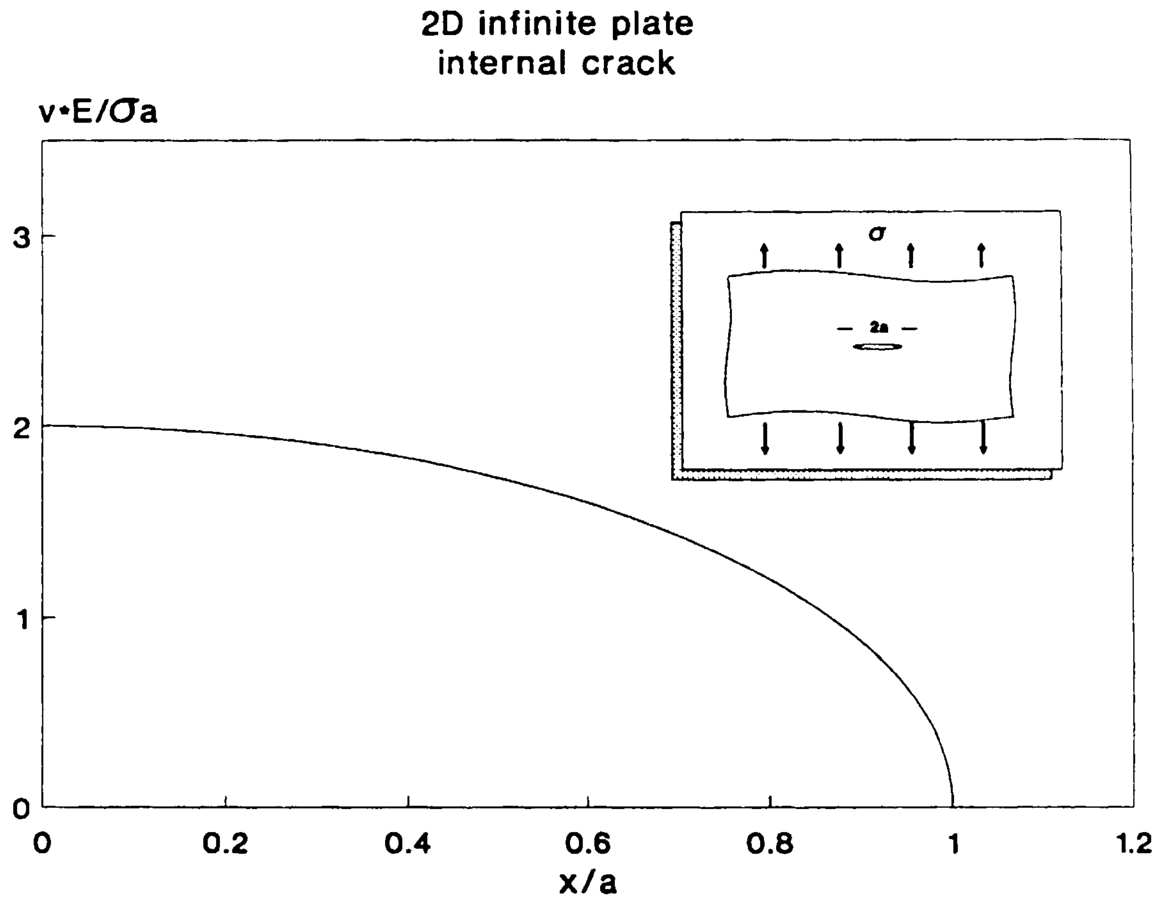
In figure E1.1 the displacement has been non-dimensionalised in terms of distance x/a, giving

$$\mathrm{v}(x/a)\frac{E}{\sigma a} = 2\left[1 - \left(\frac{x}{a}\right)^2\right]^{1/2} \qquad \text{......E2}$$

This form of non-dimensionalisation is used hereinafter. Note also that it is equivalent to

$$\mathrm{v}(x/a)\frac{E}{K_\mathrm{I}}\sqrt{\frac{\pi}{a}}$$

where K_I is the crack-tip stress-intensity-factor.


2D infinite plate
internal crack
v•E/σa
3
2
1
0
0
0.2
0.4
0.6
0.8
1
1.2
x/a
σ
2a
figure E1.1

Near to the crack tip, opening displacement is given alternatively by the Sneddon/Irwin equation [see Parker] as:

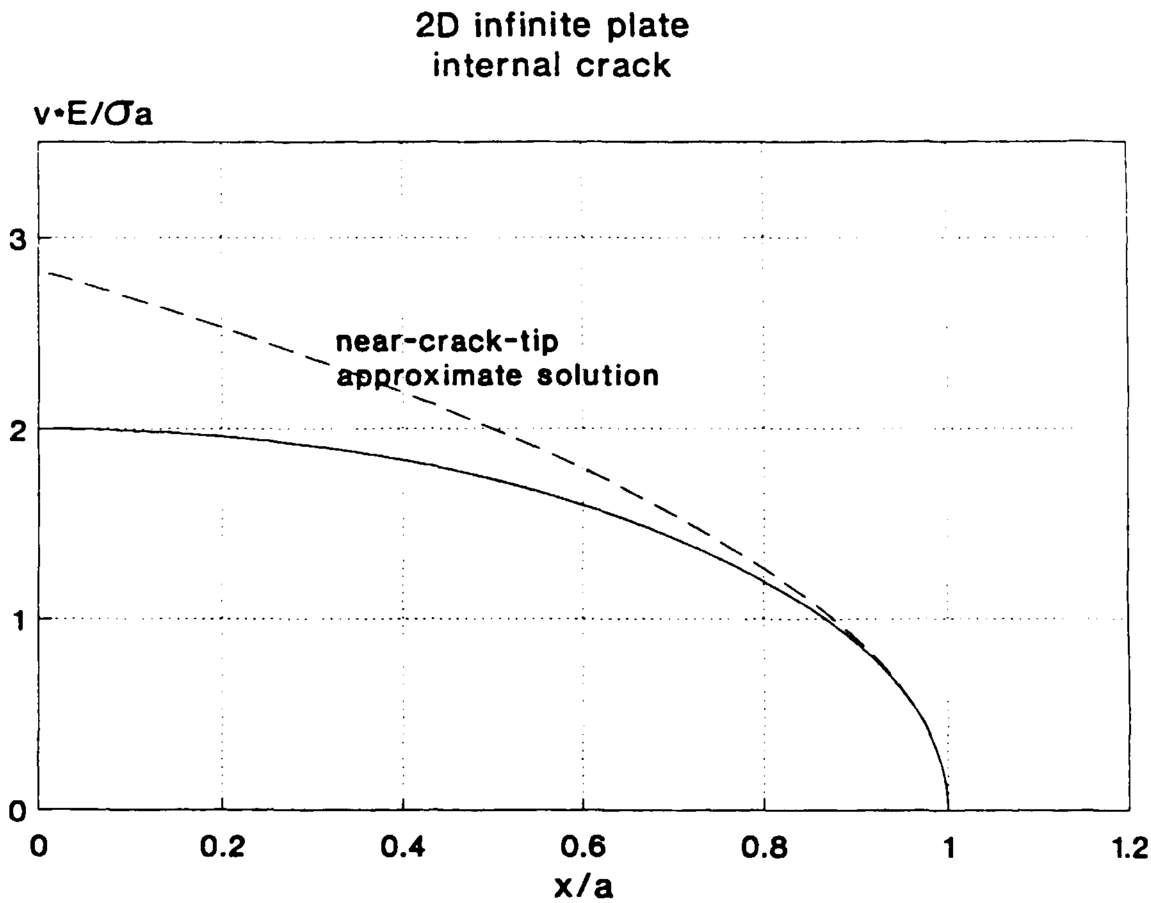
$$v(x/a) = 2\frac{K_I}{E}\left(\frac{2x'}{\pi}\right)^{1/2} \qquad \text{....E3}$$

where x' is measured in the reverse direction from the crack tip. This translates to

$$v(x/a)\frac{E}{\sigma a} = 2\left[2\left(1-\frac{x}{a}\right)\right]^{1/2} \qquad \text{......E4}$$

which can be compared directly with equation E2. This result has been plotted in Figure E1.2 and it is seen to be considerably in error at the crack centre.

The BASIC program *ECOINF* may be used to generate data using equations E2 and E4.


2D infinite plate
internal crack
v•E/σa
3
2
1
0
near-crack-tip
approximate solution
0
0.2
0.4
0.6
0.8
1
1.2
x/a
figure E1.2

2.2 2D infinite plate - periodic array

The exact solution for a periodic array is widely used as an approximation for finite width geometries, through the notional insertion of vertical cuts at crack-centre-lines for edge cracks and at mid-ligaments for internal cracks.

The crack face displacement is given by [14] as:-

$$v(x/a)\frac{E}{\sigma a}=2\left[\ln\left\{\cos\left(\frac{\pi x}{2W}\right)+\left[\cos^2\left(\frac{\pi x}{2W}\right)-\cos^2\left(\frac{\pi a}{2W}\right)\right]^{1/2}\right\}-\ln\cos\left(\frac{\pi a}{2W}\right)\right]/\left(\frac{\pi a}{2W}\right) \qquad ..E5$$

It is not immediately obvious that this formulation gives an elliptical opening but this is shown to be the case in Figure E1.3 - for larger values of a/W as well as in the limiting case where $a/W \rightarrow 0$.

The BASIC program *PERIO* generates data for specific incremental values of a/W, which may be changed by the user through alteration of program variables.

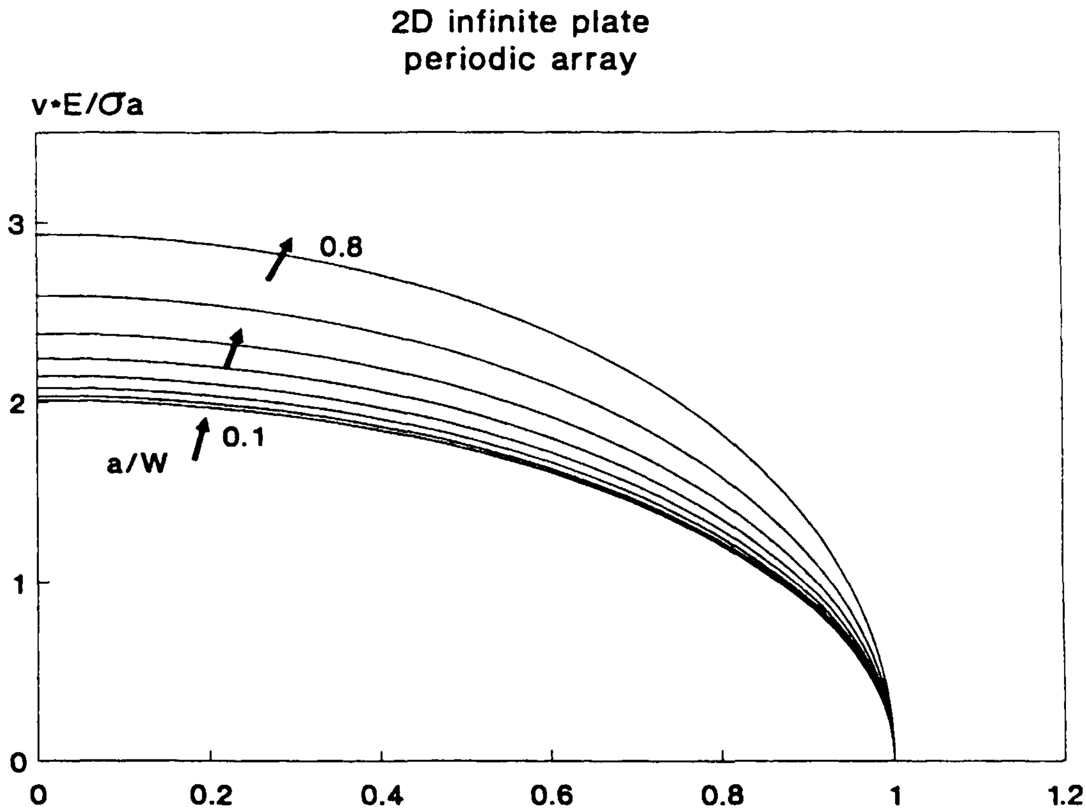
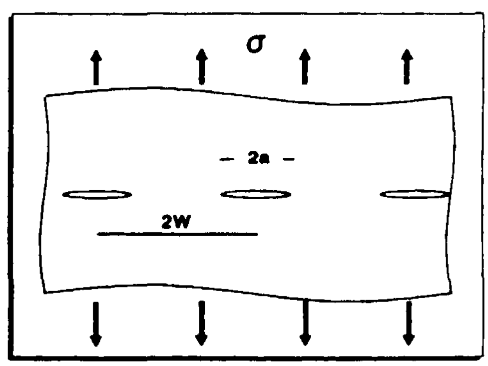

2D infinite plate
periodic array
v•E/σa
0.8
0.1
a/W
0
1
2
3
0.2
0.4
0.6
0.8
1.2
x/a

Figure E1.3


σ
2a
2W

In general, the crack-tip stress intensity can be expressed as

$$K_I = \sigma(\pi a)^{1/2}F(a/W)$$

where F(a/W) is a so-called 'compliance function' or 'configuration factor'. Given that equation E3 is valid for all geometries, it is clear that, near to the crack tip, the displacement will be given by

$$v(x/a)\frac{E}{\sigma a F} = 2\left[2\left(1-\frac{x}{a}\right)\right]^{1/2} \qquad \text{.....E6}$$

(*cf* equation E4). It would therefore be logical to include the factor F(a/W) in the general non-dimensional formulation. For a periodic array,

$$F(a/W) = \left[\left\{\tan\left(\frac{\pi}{2}\frac{a}{W}\right)\right\} / \left(\frac{\pi}{2}\frac{a}{W}\right)\right]^{1/2}$$

Dividing both sides of equation E5 by F(a/W) gives the result in Figure E1.4 and it is clear that this adjustment reduces the non-dimensional openings to a shape which is acceptably close to the base solution, even at the crack centre where the crack-tip approximation is invalid. This form of non-dimensionalisation is used in the remaining finite-width cases in the Handbook.

There are several sources for these factors (also designated 'compliance' functions or 'correction' factors), see for example [14-19].

It should be emphasised however that the crack profile cannot in all cases be generated simply by multiplying the baseline case by an appropriate F(a/W) factor (this will be confirmed in later examples.) Configuration factors generally account for the effects of creating new boundaries and loadings with respect to the baseline case, but only insofar as the stresses and displacements etc in the crack tip region are affected. The fact that different configurations, which generate identical F(a/W) factors, happen to produce identical crack tip mechanical states, does not mean that the crack profile distant from the crack tip will necessarily be the same.

Hence, while the F factor adjustment is used freely in this Handbook, it is not expected to remove all the effects of configuration over the complete crack profile in every case.

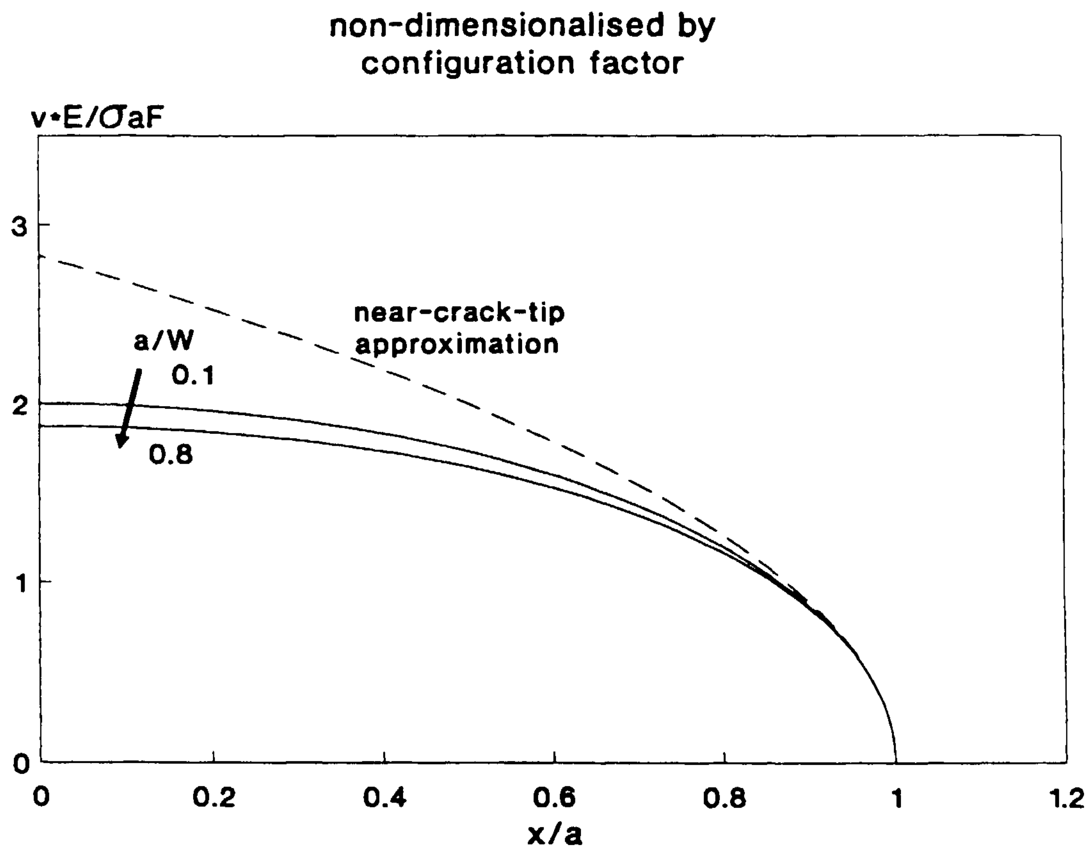
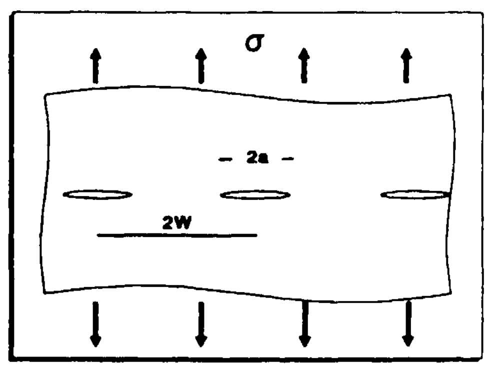

non-dimensionalised by
configuration factor
v•E/σaF
3
2
1
0
near-crack-tip
approximation
a/W
0.1
0.8
0
0.2
0.4
0.6
0.8
1
1.2
x/a

figure E1.4


σ
2a
2W

2.3 2D finite width - double edge crack

No general solutions for crack-face opening-profile are known for finite width geometries but several numerical results for mouth-opening have been given. Results calculated from two of the best-known compendia by Tada **[18]** and Murakami **[19]** are given in Figures E2, (Note, non-dimensional **mouth** opening is shown here against *a/W*). (In the lower Figure E2.2, the F factors corresponding to the respective cited mouth opening solutions were used).

Even when compensation is made for the increased stress intensity caused by releasing the crack centre-line boundary in the periodic crack solution, the mouth-opening is seen to be 30% larger than in the base solution (compare the results of figure E2.2 with the maximum openings shown in figures E1.1 and E1.4). The agreement improves for deeper cracks.

Clarification of the reasons for this increase is given in following pages, through finite element model results and the solution for an edge- crack in a semi-infinite plate.

2D finite width
double edge crack
(mouth opening)

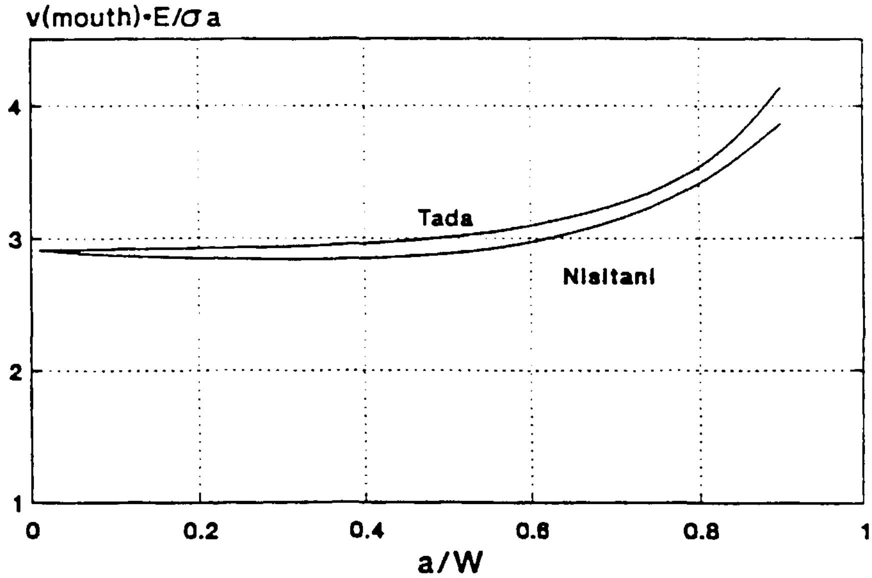
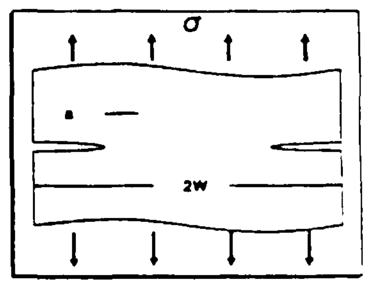


figure E2.1



non-dimensionalised by
configuration factor

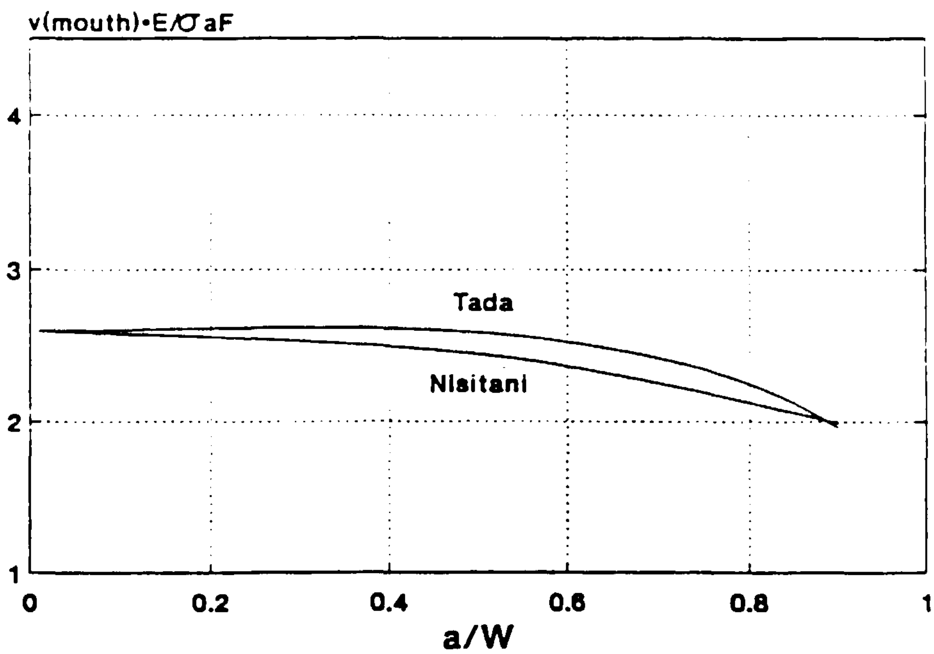


figure E2.2

2.4 2D Finite Element model - double edge crack

Finite element models in the range $0.2 < a/W < 0.5$ were constructed using the ANSYS computer package. Nodal displacements at the crack edges are plotted in Figures E2.3 and E2.4. (In the latter figure, the configuration factors were derived from the FE solution for the sake of consistency.)

These results agree well with Nisitani's solution at the mouth and reduce correctly to the internal-crack elliptical profile at about half the crack length from the tip. The lower figure shows that the profile diverges gradually from the elliptical profile associated with the internal crack case, starting at about half the crack length away from the tip. This is a feature of edge cracks - release of the stress parallel to the crack centre plane causes an increase in mouth opening, as distinct from the general enlargement of the profile from the baseline case.

It is of passing interest that the near-crack-tip approximate solution (equation E3) gives the correct trend in edge-crack profile (compare Figure E2.4 with Figure E1.2) but it over-estimates the mouth opening by about 9%. This near-coincidence can only be regarded as fortuitous but it suggests that equation E4 might form a useful basis for weight-function profiles.

FE model
double-edge crack

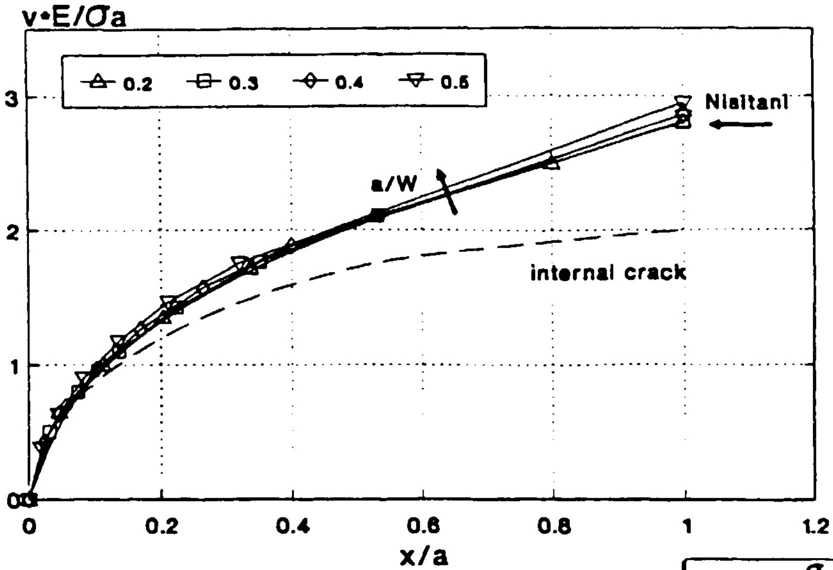
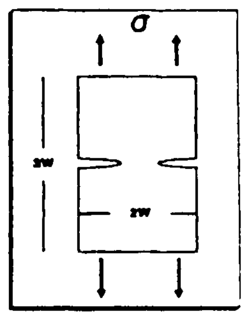


figure E2.3



non-dimensionalised by
configuration factor

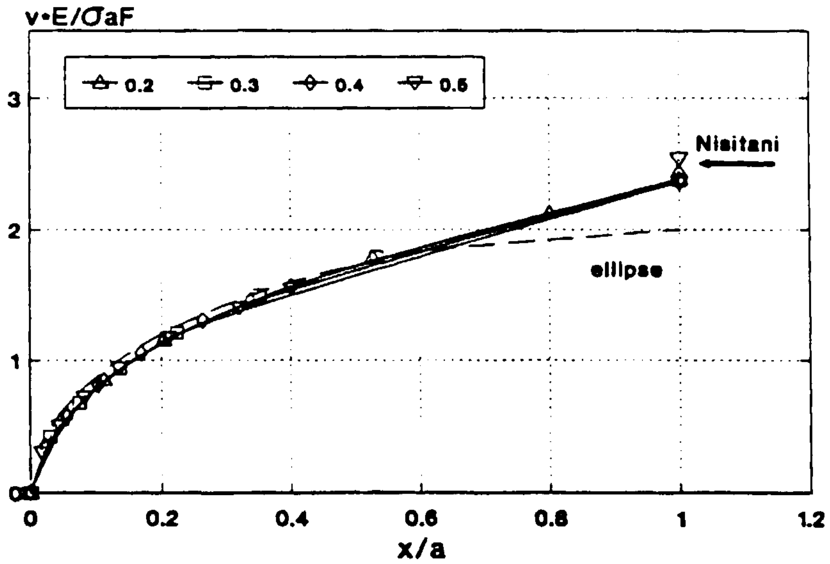


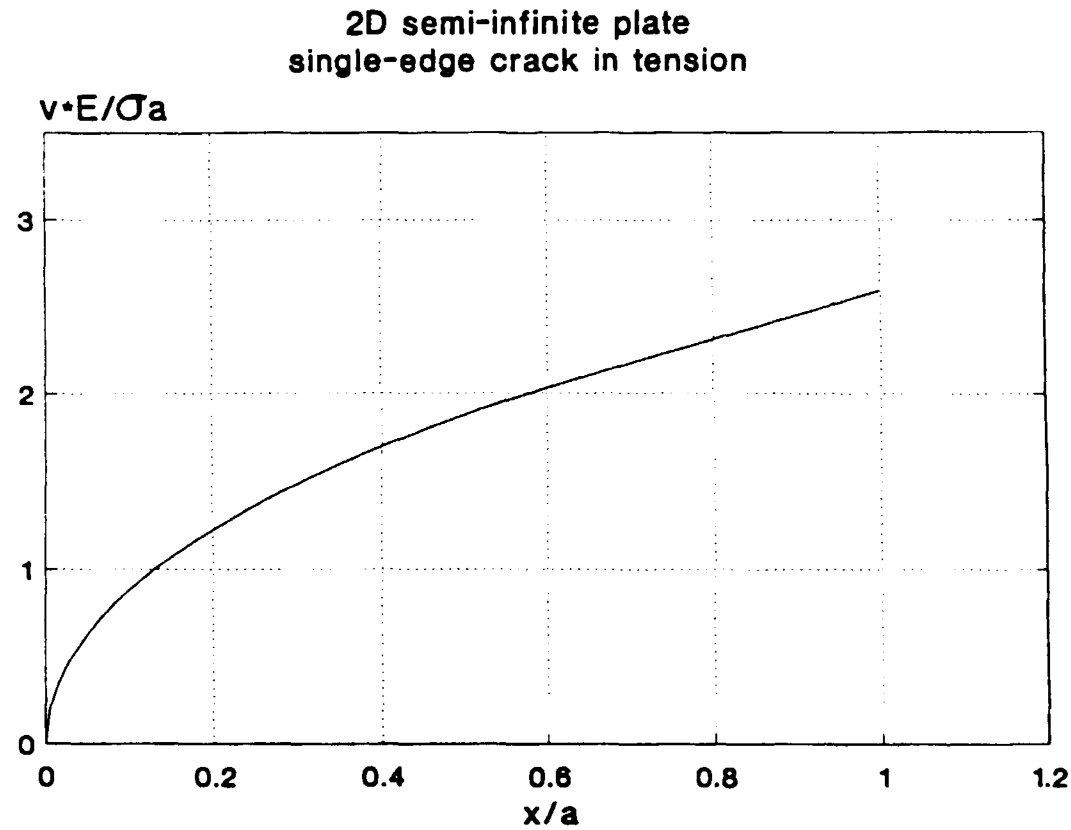
figure E2.4

2.5 2D semi-infinite plate - edge crack in tension

A numerical solution for an edge crack has been given by Wigglesworth [21] for the case where $a/W \rightarrow 0$. This is shown in Figure E2.5 where the characteristic "edge effect" is seen. The mouth opening agrees clearly with Tada's and Nisitani's results. The slight reduction in opening relative to the FE results is probably due to the imposition of finite height ($3W$) in the FE model.

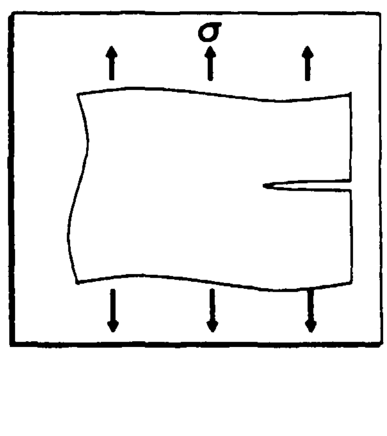
The condition that the edge of the plate must be stress-free at the point where the crack mouth intersects the surface has been shown to imply that the curvature of the crack face must vanish at the edge [22]. This shape should therefore be expected at all surfaces where edge cracks intersect.

Wigglesworth's numerical results can be re-generated from a 13-term series via the BASIC program *WIGOP*.



2D semi-infinite plate
single-edge crack in tension
v*E/σa
3
2
1
0
0
0.2
0.4
0.6
0.8
1
1.2
x/a

figure E2.5



σ

2.6 2D finite width - single-edge-crack in tension

Freeing the central line of symmetry in the double-edge crack model has a strong effect on displacement as a/W increases and the load becomes asymmetric to the uncracked ligament, resulting in a bending action. This is shown in the large mouth openings given in Figure E3.1, where the Tada and Nisitani solutions are seen to be coincident. Incorporation of the configuration factor in Figure E3.2 removes a sizeable proportion of the relative increase in mouth opening, at least up to $a/W = 0.5$.

Numerical solution of single crack geometries with a/W greater than about 0.65 is difficult and there is some disagreement between the configuration factors given by the solution cited here and others given in the literature.

2D finite width
single-edge-crack in tension
(mouth opening)

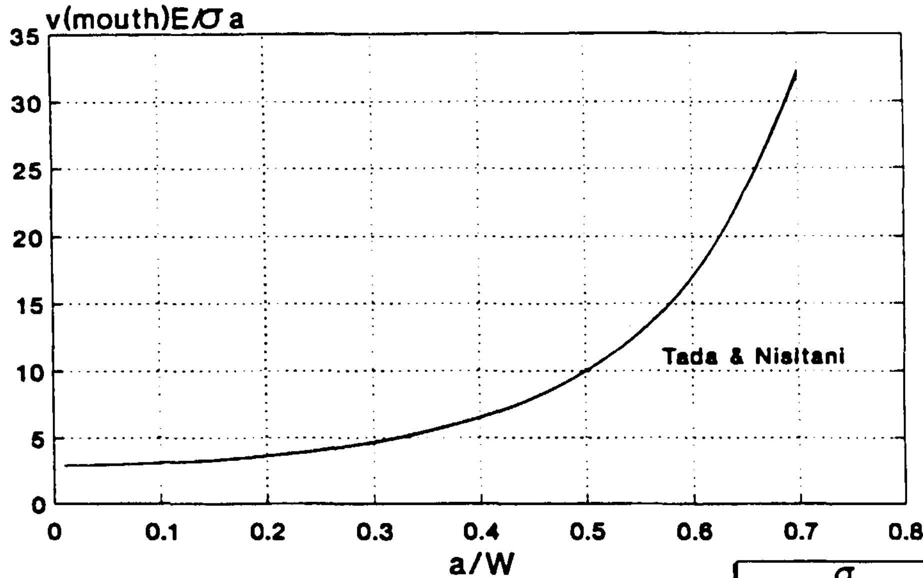
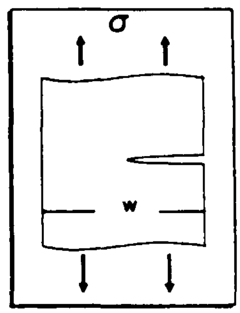


figure E3.1



non-dimensionalised by
configuration factor

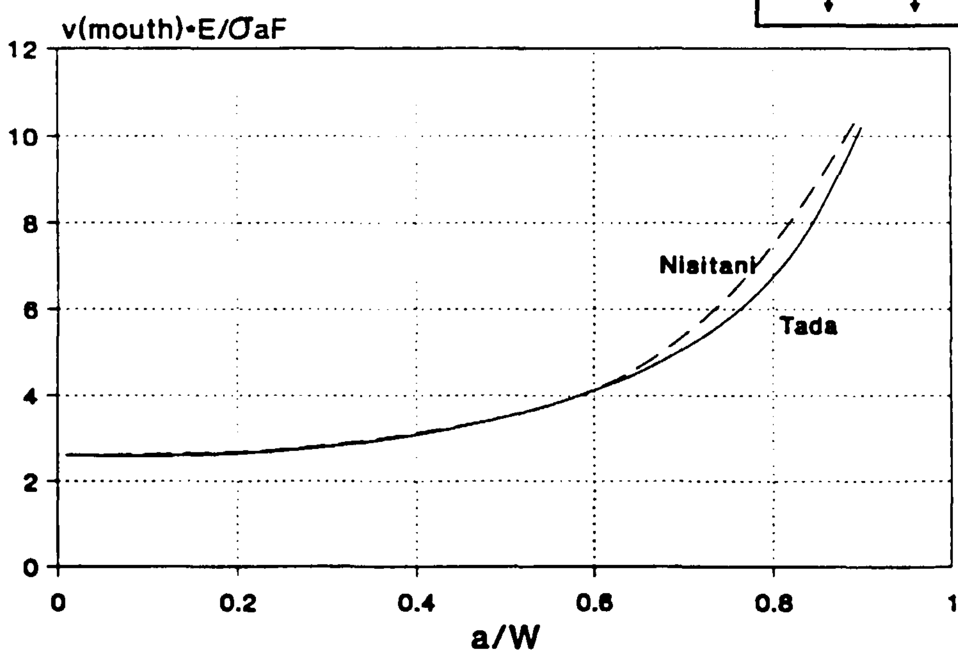


figure E3.2

2.7 2D Finite Element model - single-edge-crack in tension

Nodal displacements are given in Figures E3.3 and E3.4. Nisitani's evaluations exceed the FE results by about 9%, but once again, the lower figure shows that the near-crack tip solution is given correctly by the finite element formulation. Gradual divergence from the elliptical profile occurs away from the crack tip. For $a/W < 0.4$, the point of divergence is similar to the double-edge cases (Figure E2.4) but for higher values, the crack-face outside the influence of the crack-tip stress-field seems to be strongly influenced by bending rotation.

The pattern suggested by comparison of the solutions therefore is that although inclusion of an appropriate configuration factor takes care of the near-crack-tip adequately, the baseline solution has to be increased by an 'edge-effect' factor of about 30% and a 'bending rotation' displacement which increases sharply for $a/W > 0.4$. (Nisitani's solutions could be used to determine an end-point for the rotated face).

FE model
single-edge crack tension

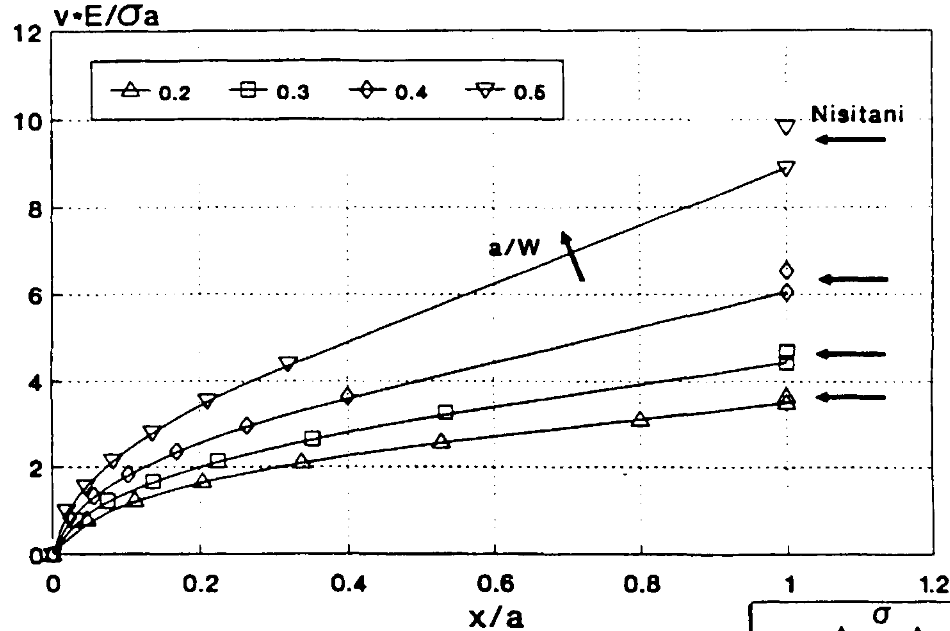
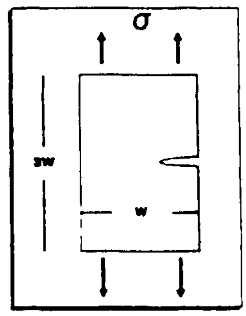


figure E3.3



non-dimensionalised by
configuration factor

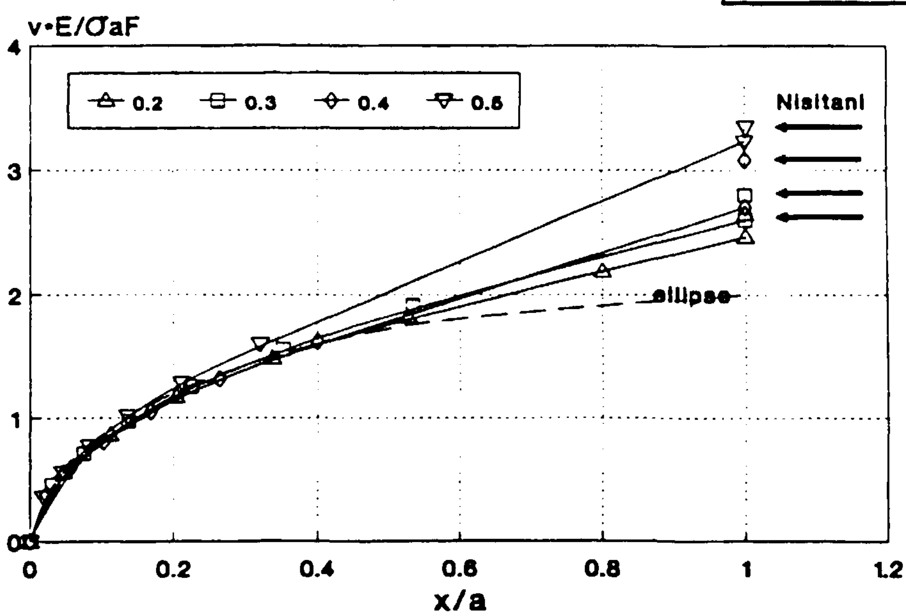


figure E3.4

2.8 2D finite width - single-edge-crack in bending

Application of a bending moment to a single-edge-crack geometry also causes a large mouth opening, as shown in Figure E4.1, although this is not as large in relative terms as for straight tension. (Note that the non-dimensionalising stress used here is the extreme fibre nominal stress on the uncracked geometry.)

Comparison of Figures E4.2 and E3.2 for $a/W \rightarrow 0$ shows that for a given crack tip stress intensity, the edge factor is identical although the nominal stress patterns are quite different.

Opening profile for the common practical case of normal stress and bending combined, can be determined simply by adding the displacements for each case.

2D finite width
single-edge-crack bending
(mouth opening)

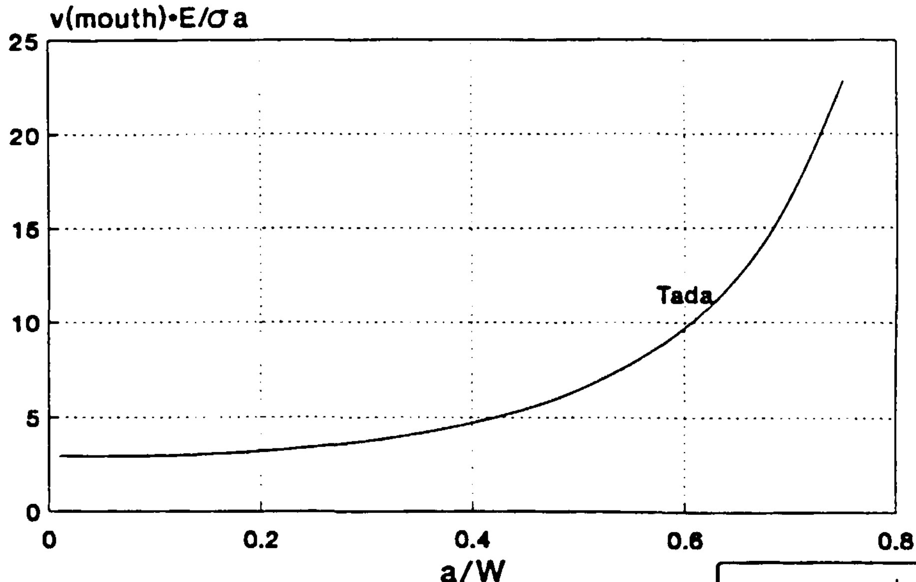
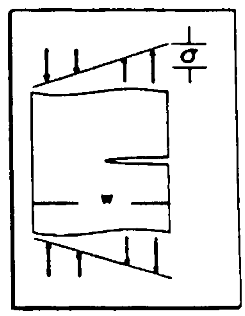


figure E4.1



non-dimensionalised by
configuration factor

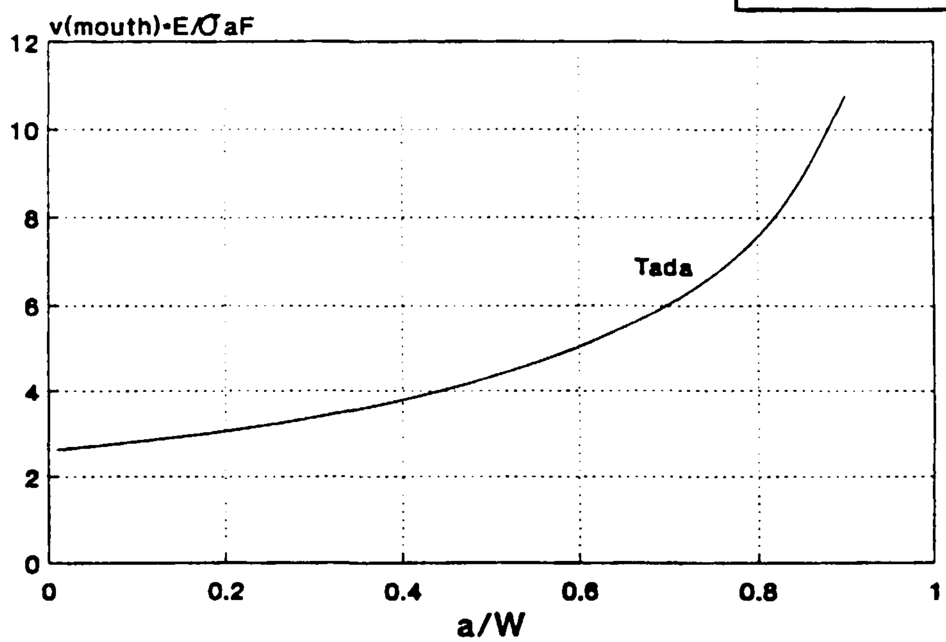


figure E4.2

2.9 2D Finite Element model - single-edge-crack in bending

Tada's mouth opening results are again somewhat larger than the FE results by about 12-15 %. The 'rotation effect' causing divergence from the elliptical profile, is more marked than for the tension case. However, it would appear that Tada's configuration factors are also larger than the finite element results so that the lower non-dimensionalised results are in slightly better agreement. These differences are again probably due to the finite height influence of the FE model.

FE model
single-edge crack bending

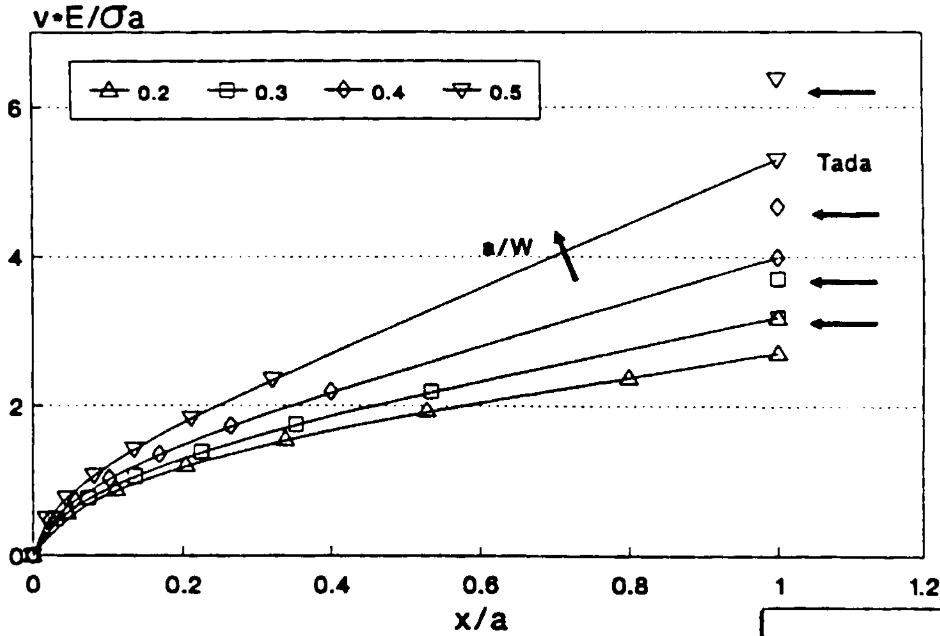
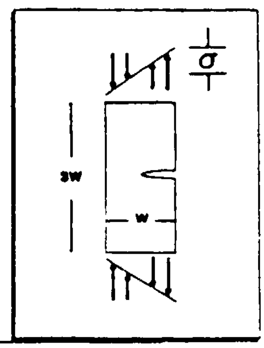


figure E4.3



non-dimensionalised by
configuration factor

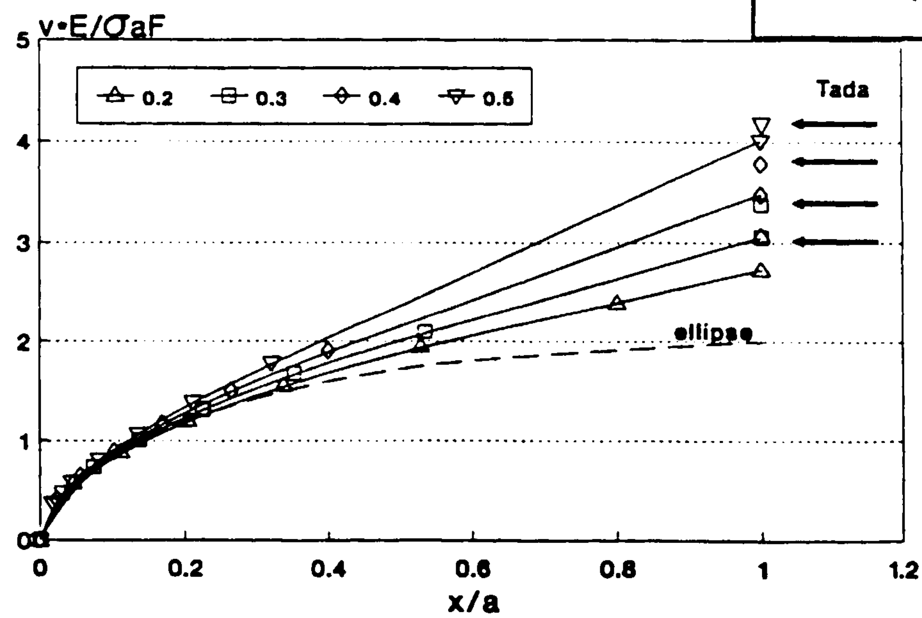


figure E4.4

2.10 2D Finite Element Model - cracked cruciform weld joint [22]

The model weld joint geometry was chosen to represent a configuration which is typical of fatigue test geometries and joints in service. The 45° full-penetration fillet weld has a leg length equal to half the vertical plate thickness and the semi-thickness of the horizontal plate is 26.5 mm.

The stepped geometry produces a steep non-linear stress gradient which dominates to a depth of about 4 mm from the fillet weld toe. The stress concentration factor associated with the uncracked configuration is effectively infinite, due to the assumption of a sharp transition between the fillet and the plate. In practice, the crack forms typically on a principal tensile stress plane which is not simply transverse to the load, but tends to run at a varying angle with depth.

The solution was run for three different crack sizes - 1.56 mm, 3.13 mm and 6.25 mm, using the *ABAQUS* program. The reference stress used in presentation of the non-dimensional crack profile is in all cases the stress in the 50 mm thick plate and the configuration factors were derived directly from the J integral output of the FE run.

Figure E5.1 gives the nodal displacements for the three cases. Interestingly, the intermediate crack size gives the smallest absolute opening. This can be understood when it is recognised that the smallest crack sits in the highest part of the concentrated stress field and therefore experiences a high stress intensity factor and a large opening; although the largest crack cuts through a region which experiences a lower average stress, it is of sufficient size to experience finite width effects. However, the intermediate size crack lies at a minimum point between these effects and therefore opens least.

These differences are largely removed in the non-dimensionalisation of Figure E5.2 as incorporation of the configuration factor makes the proper allowance for the concentrated stress field. The finite width effect can be judged approximately in terms of the ratio of the crack depth to the vertical plate thickness, giving a/W in the range 0.06 to 0.25. The results for the FE double-edge crack case at $a/W = 0.2$ are included for comparison. The implication therefore is that the crack at the toe of the fillet weld will open as a simple edge crack despite the non-linear stress field.

FE model
cruciform weld

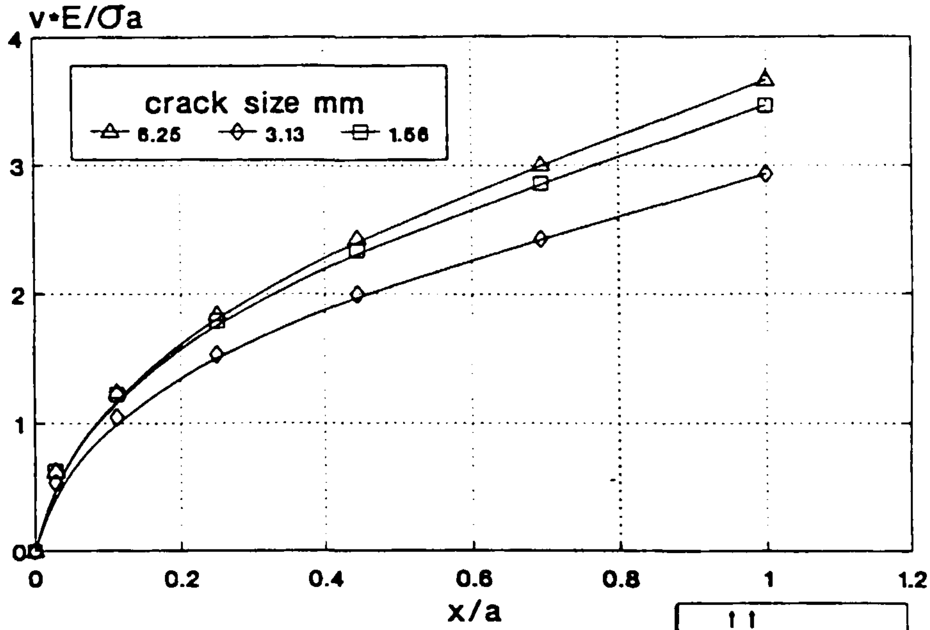
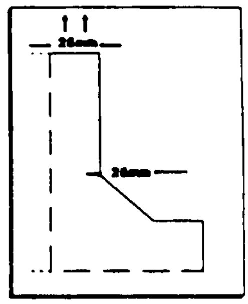


figure E5.1



non-dimensionalised
with DEN comparison

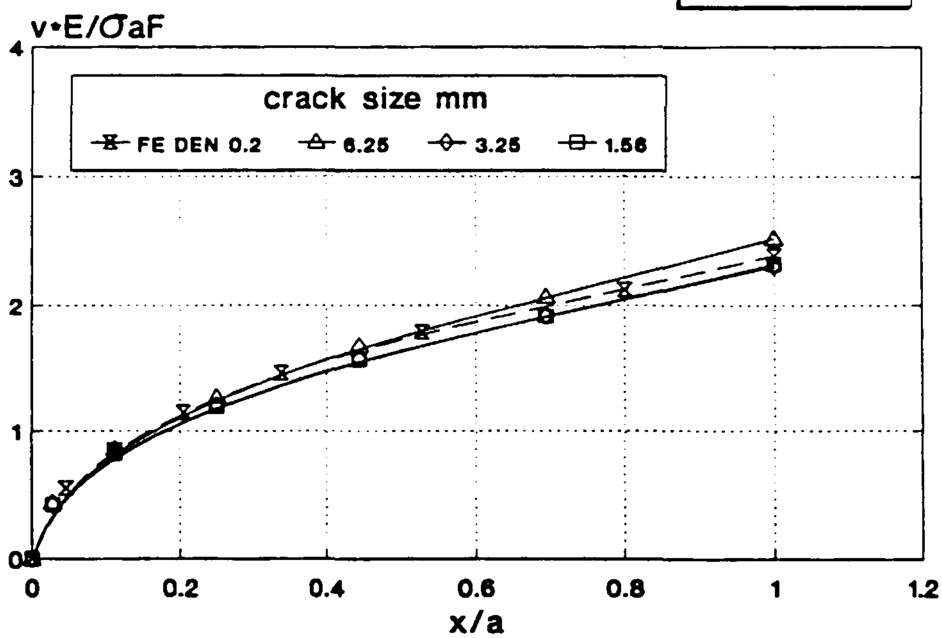


figure E5.2

2.11 3D infinite solid - elliptical planform crack

The crack-face profile of an elliptical planform crack in an infinite solid is ellipsoidal and is given by [23] as

$$v\frac{E}{(1-\nu^2)\,\sigma a}=\frac{2}{E(k)}\left[1-\left(\frac{x}{b}\right)^2-\left(\frac{y}{a}\right)^2\right]^{1/2} \qquad \text{...E7}$$

where *E(k)* is an elliptical integral of the second kind. The coordinate system is shown in Figure E6.1. (The factor $(1-\nu^2)$ has been omitted from E6.1 to facilitate comparison with the baseline solution).

Equation E7 was programmed in BASIC and is given as program *3DECO*.

The value $b/a = 0$ corresponds to a 2D crack and is therefore coincident with the solution E2. The value $b/a = 1$ is the circular or 'penny-shaped' crack and $b/a = 2$ gives a recognisably elliptical planform. The reduction in maximum displacement as b/a increases is entirely due to the *E(k)* term which could, if desired, be included in the non-dimensional displacement formulation as an 'F' factor.

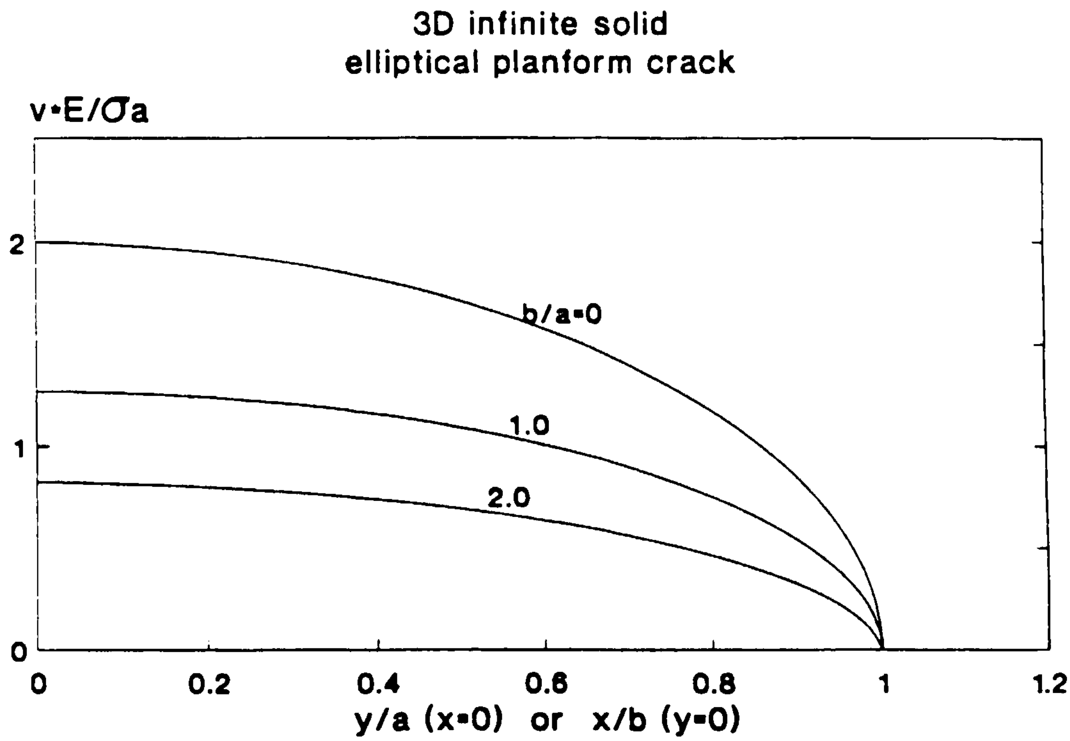
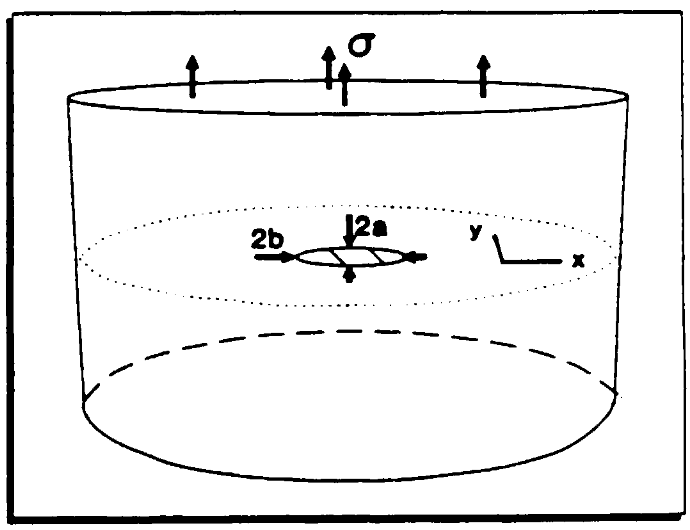

3D infinite solid
elliptical planform crack
v•E/σa
2
1
0
b/a=0
1.0
2.0
0
0.2
0.4
0.6
0.8
1
1.2
y/a (x=0) or x/b (y=0)

figure E6.1


σ
2b
2a
y
x

2.12 3D semi-infinite solid - semi-elliptical surface crack

No exact analytical solutions for the displacement field of a semi-elliptical surface crack have been given and therefore a finite element model was solved using the *ABAQUS* program. The surface crack was placed in a semi-cylindrical solid with aspect ratio and crack size to cylinder radius as shown adjacent to Figure E6.3.

Nodal displacements along the major and minor axes are shown in Figures E6.2 and E6.3. (The configuration factor was derived from the FE output.) The corresponding profiles for an elliptical crack in an infinite solid have been drawn for comparison and an arbitrary elliptical curve has been superimposed on the FE results of Figure E6.2 to show that the surface profile remains elliptical. (The factor $(1-\nu^2)$ has again been removed from the FE results to facilitate comparison.)

The result in Figure E6.3 for the minor axis shows the typical "edge effect" and the mouth opening is only a little greater than the result for a symmetrical edge crack as in Figure E2.4. However it appears that the edge effect applies all along the surface profile, as the enlargement from the infinite solid solution is consistent, resulting in an elliptical surface profile.

The conclusion therefore is that the opening profile of a semi-elliptical surface crack, which is not too large in relation to the containing solid, can be estimated on the assumption that the minor axis profile has the two-dimensional 'edge effect' form and that the surface profile is elliptical. If, in a given situation, the applied stress is known and the corresponding elastic displacement of the surface profile is measured, the depth of the crack (i.e, the aspect ratio) could be estimated.

Mattheck *et al* [24] have also given an approximate solution based on the idea of 2D 'slices' cut in from the flat surface through the semi-elliptical crack profile. This makes use of the F factors of Raju and Newman [25] which are widely quoted for tension and bending of solids with surface cracks. (See also [26].)

3D semi-infinite solid
semi-elliptical planform crack
(surface profile)

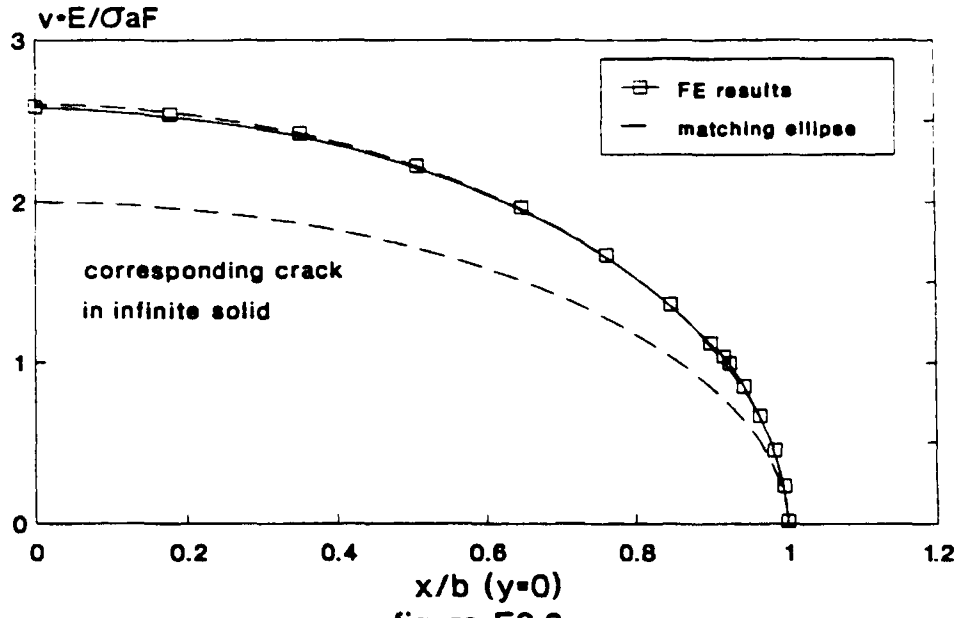
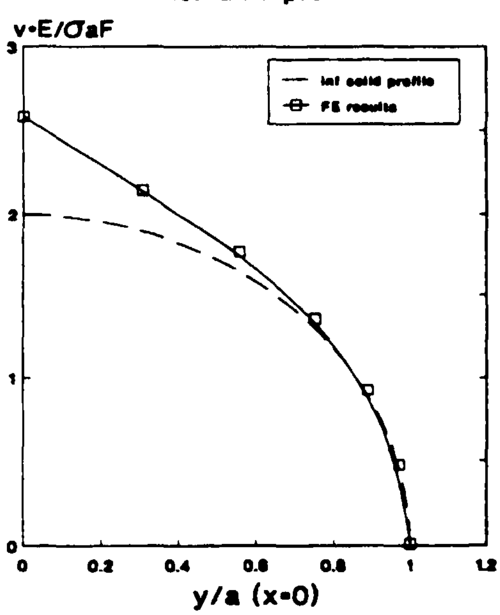


figure E6.2

minor axis profile



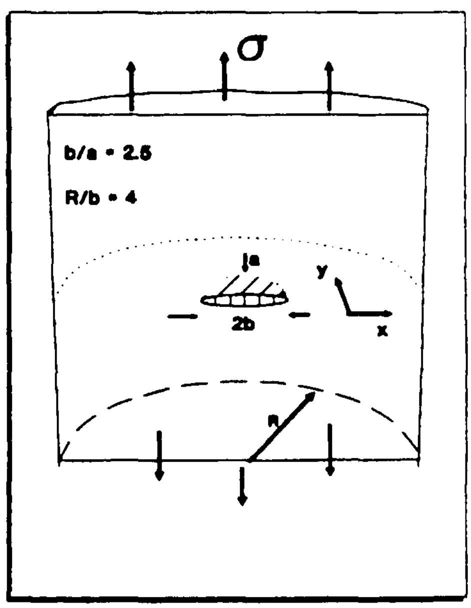


figure E6.3

Stationary Crack Elastic-Plastic Solutions

3 STATIONARY CRACK ELASTIC-PLASTIC SOLUTIONS

3.1 2D infinite plate Dugdale Model - internal crack

The opening profile of a crack with a strip-type plastic zone is given by Parker [10] as:

$$v(x) = \frac{4}{\pi}\frac{\sigma_Y}{E}\left[\frac{a}{2}\ln\left(\frac{t+1}{t-1}\right) - \frac{x}{2}\ln\left(\frac{t+x/a}{t-x/a}\right)\right] \qquad \text{...P1}$$

where $t^2 = \frac{(a+\rho)^2 - x^2}{(a+\rho)^2 - a^2}$ and $\rho = a\left[\sec\left(\frac{\pi}{2}\frac{\sigma}{\sigma_Y}\right) - 1\right]$

ρ being the length of the plastic zone.

For comparison with the foregoing elastic solutions equation P1 can be non-dimensionalised as before to give

$$v(x/a)\frac{E}{\sigma a} = \frac{\sigma_Y}{\sigma}\frac{4}{\pi}\left[\frac{1}{2}\ln\left(\frac{t+1}{t-1}\right) - \frac{1}{2}\frac{x}{a}\ln\left(\frac{t+x/a}{t-x/a}\right)\right] \qquad \text{....P2}$$

The opening profiles for various values of σ/σ_Y are given in Figure P1.1. Comparison of this figure with figure E1.1 shows that the elastic solution is regained as $\sigma/\sigma_Y \to 0$.

Two effects are evident here: first the central opening increases with σ/σ_Y. Secondly, although the profile remains more-or-less elliptical, the major axis of the ellipse extends to a point beyond the true crack tip.

Both of these effects can be treated on an approximate basis by assuming a 'plasticity correction' to the elastic solution, resulting in an increase of K, and by extending the notional, elliptical profile into the plastic zone length. Further investigations are needed to determine exact ratios for such approximations but the present discussion indicates how profile prediction can be readily extended into the elastic/plastic regime via the Dugdale model.

2D infinite plate Dugdale Model
internal crack - elastic non-dimensional

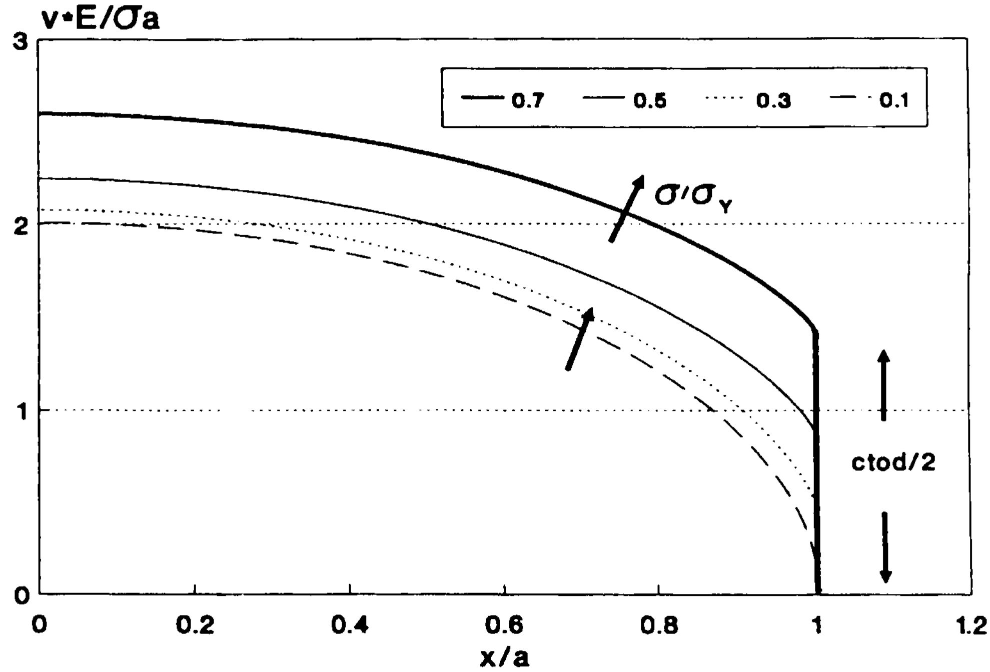
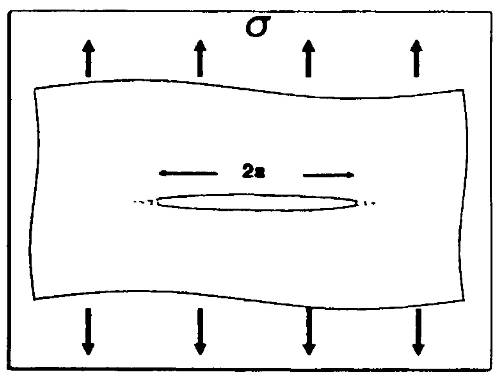


figure P1.1



The powerful non-linear effect of applied stress in elastic/plastic opening is shown more clearly in Figure P1.2 in which displacements are non-dimensionalised with respect to yield strength.

For later reference, the crack *tip* displacement is given more simply as:

$$v_{tip}\frac{E}{\sigma_Y a}=\frac{4}{\pi}\ln\sec\left(\frac{\pi}{2}\frac{\sigma}{\sigma_Y}\right) \qquad \text{....P3}$$

and the plastic zone length ρ as:

$$\frac{\rho}{a}=\sec\left(\frac{\pi}{2}\frac{\sigma}{\sigma_Y}\right)-1 \qquad \text{...P4}$$

The curves given were produced using the BASIC program *INFDUG*.

non-dimensionalised
by yield strength

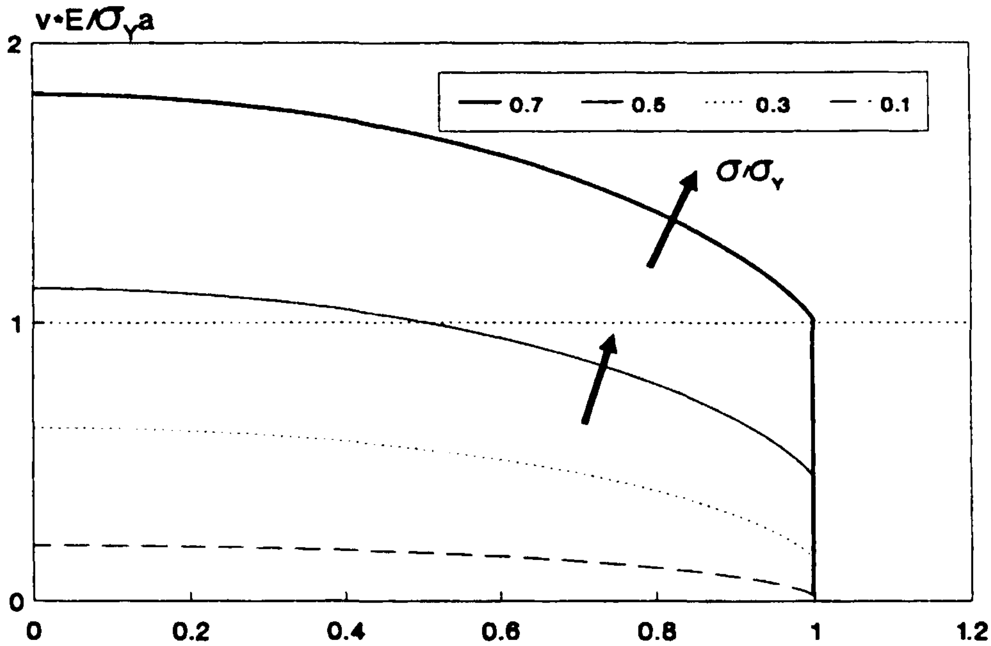
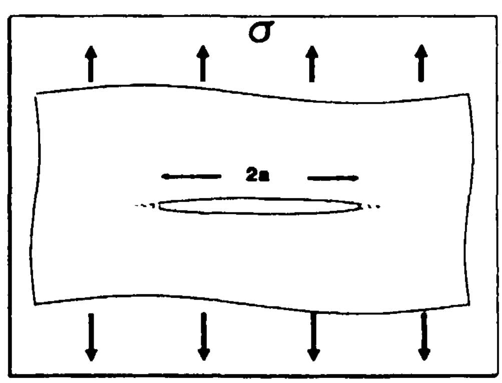


figure P1.2



3.2 2D infinite plate Dugdale Model - unloading

The Dugdale model can also be used to track the unloading behaviour of cracks, this being relevant to crack closure. To apply this technique, the displacement profile for the crack at maximum load is first calculated as before via equation P2, then the displacement reductions due to reduction of stress are calculated and subtracted to give profiles as shown in Figure P1.3.

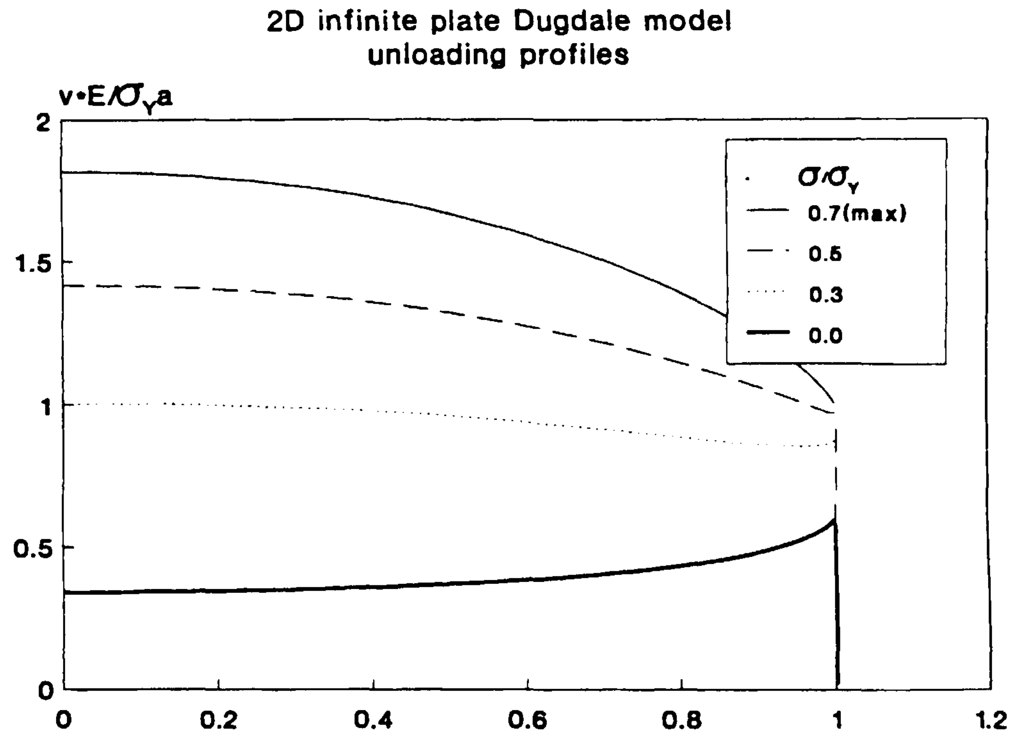
For unloading, the effective yield strength has to be doubled to simulate reversed yielding in the plastic zone. (This neglects Bauschinger effects which are probably quite strong in the case of crack tip plasticity.)

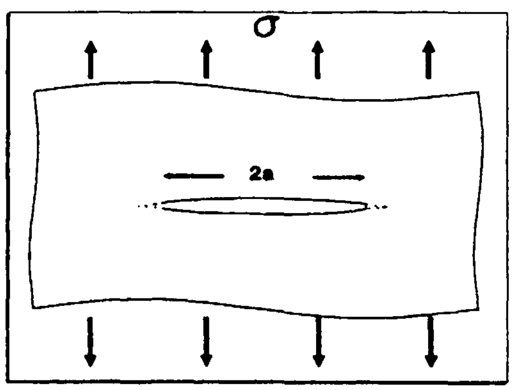
Figure P1.3 shows the crack loaded initially to 70% of yield and unloaded progressively to zero. The initial reduction of CTOD on unloading is small (i.e, for loads reducing to 30% of yield) and most of the crack tip closure occurs between this level and zero load.

Note that at zero load, the crack tip is held open by permanent strain in the reversed plastic zone, so that the tip of the crack is predicted to be more open than the centre. If this actually occurs in practice, it will tend to improve the capability of ultrasonic methods for the accurate sizing of cracks, as the perimeter of the crack will exhibit a clear separation.

An interesting point to observe from this example is that the 'effective' stress range for unloading and reloading is half the actual range (i.e, $\sigma/\sigma_Y = 0.35$) which means that the profile change calculated by an elastic route should be reasonable. Moreover, it will be shown later that flank stretch in crack growth results in closure before the load has reached zero, implying that most of the profile changes shown in Figure P1.3 could be modelled without too much loss of accuracy by assuming the simple elastic model of Figure E1.

These curves were produced by the program *DUGULOD*. The program is easily varied to give results for any maximum load and subsequent unloading stage.


2D infinite plate Dugdale model
unloading profiles
v•E/σ$_Y$a
σ/σ$_Y$
0.7(max)
0.5
0.3
0.0
2
1.5
1
0.5
0
0
0.2
0.4
0.6
0.8
1
1.2
x/a
figure P1.3


σ
2a

3.3 2D finite width Dugdale Model - double-edge crack

No general, closed solutions for crack profiles in *finite* elastic/plastic bodies are available. However it is apparent from Figure P1.1 that knowledge of the CTOD and the plastic zone length provides enough data to estimate profiles where there is extensive crack tip yielding. Even so, there is little enough data for CTOD and plastic zone size ρ and few formulations of crack opening profile. Numerical determinations by Hayes and Williams [27] have been fitted to closed form approximate solutions by Gray [28]. For the double-edge geometries under tension the approximations take the form

$$\frac{\rho}{a} = \left[\sec\left(\frac{\pi}{2}\frac{\sigma}{\sigma_Y}\right) - 1\right]\left[F\left(\frac{a'}{W}\right)\right]^2 S \qquad \text{....P5}$$

$$\text{where } \frac{a'}{W} = \frac{a}{W}\left[1 + 0.2\left(\frac{\sigma}{\sigma_Y}\right)^P\right] / \left[1 - \left(\frac{\sigma}{\sigma_Y}\right)^Q\right] \qquad \text{....P6}$$

$$\text{and } v_{tip}\frac{E}{\sigma_Y W} = \frac{a}{W}\frac{4}{\pi}\ln\sec\left(\frac{\pi}{2}\frac{\sigma}{\sigma_Y}\right)\left[F\left(\frac{\hat{a}}{W}\right)\right]^2 \qquad \text{....P7}$$

$$\text{where } \frac{\hat{a}}{W} = (a + R\rho)/W \qquad \text{....P8}$$

P, Q, R and S are constants in an empirical fit to the finite element data and the functions F(a/W) are the usual elastic configuration factors for the respective geometries. For the double-edge crack case,

$P = 5.1;\ Q = 1.68;\ R = 0.5;\ S = 1$

$$\text{and } F(a/W) = \left\{\tan\left(\frac{\pi}{2}\frac{a}{W}\right) / \left(\frac{\pi}{2}\frac{a}{W}\right) + 0.25\left[1 - \left(\frac{a}{W}\right)^{2.6}\right]^{6.0}\right\}^{1/2} \qquad \text{....P9}$$

2D finite width Dugdale Model
double edge crack (half-ctod)

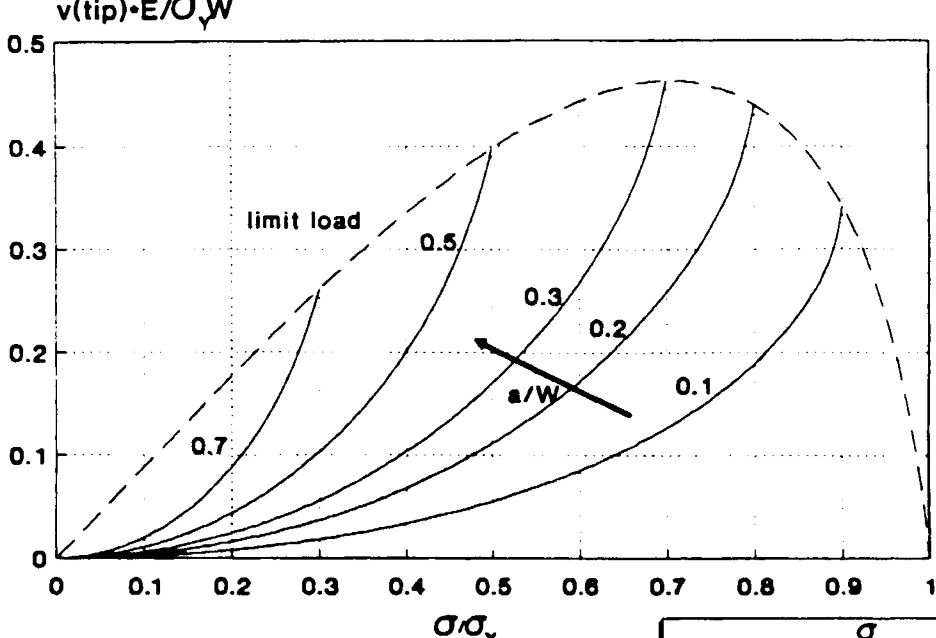
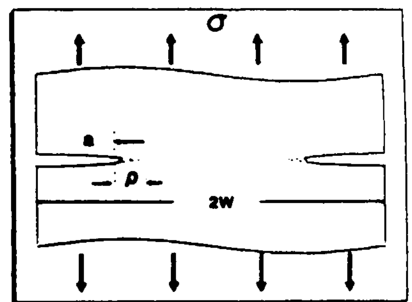


figure P2.1



plastic zone size

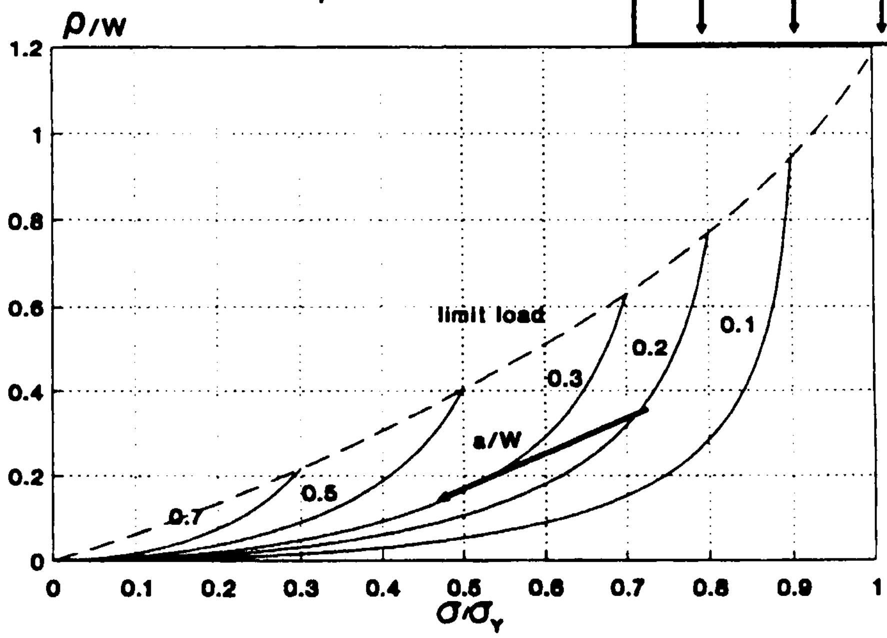


figure P2.2

The equations are essentially empirical, but some points of correspondence with the infinite plate equations are worth noting for clarification.

- Equation P5 is a 'finite-width-adjusted' plastic zone size (cf equation P4) where the 'effective crack size', a' is itself increased as a function of σ/σ_Y,
- Equation P7 is a finite-width-adjusted displacement (cf equation P3) where the effective crack size $\hat{a}$ has been increased by a proportion R of the previously calculated plastic zone size.

Reference [28] can be examined to ascertain quality of fit to the finite element results and the regimes where the equations extrapolate these results.

Equation P7 has been plotted for various values of a/W in Figure P2.1. Note that the Dugdale Model reaches a limit when the plastic zones spread completely across the ligament and touch. This point is reached when

$$\frac{\sigma}{\sigma_Y} = \left(1 - \frac{a}{W}\right) \qquad \text{....P10}$$

The dashed line in Figure P2.1 was drawn by making this substitution for σ/σ_Y in equation P7.

Equation P5 has been plotted in Figure P2.2 where some conservatism in evaluation of the plastic zone size at limit load can also be seen.

Non-dimensional plastic zone sizes and CTOD values can be generated for incremental values of a/W through the program *DENDUG*. The limit load boundary is automatically included in the program flow and one column of the data file includes displacements along this boundary.

Reference [28] also gives solutions for a central internal crack in terms of alternative empirical constants to be used in conjunction with equations P5-8. Reference [29] gives solutions for a circumferential edge crack in a cylindrical solid. This is relevant to the practical case of a crack growing from a circumferential notch or step. The general program *DENDUG* may be used for both of these cases with appropriate substitution of empirical constants P,Q .. etc and the relevant limit load condition.

3.4 2D finite width Dugdale Model - single-edge crack tension

Equations P5-9 are valid for the single-edge crack case with alteration of the empirical constants to:

$P = 0.6;\ Q = 1.6;\ R = 0.47;\ S = 0.82$

The configuration factor used in this case is:

$$\mathrm{F}(a/W) = 1.118 \Big/ \left[1 - 0.7\left(\frac{a}{W}\right)^{1.5} \right]^{3.25} \qquad \text{....P11}$$

The limit load condition is given by:

$$\frac{\sigma}{\sigma_Y} = \left(1 - \frac{a}{W}\right)^2 \qquad \text{....P12}$$

Equation P7 is plotted in Figure P3.1. Note that the typical crack tip displacements are an order-of-magnitude larger than for the symmetrical case, reflecting the strong effect of bending in deep cracks.

Plastic zone sizes and displacements were generated through the program *SENTDUG*.

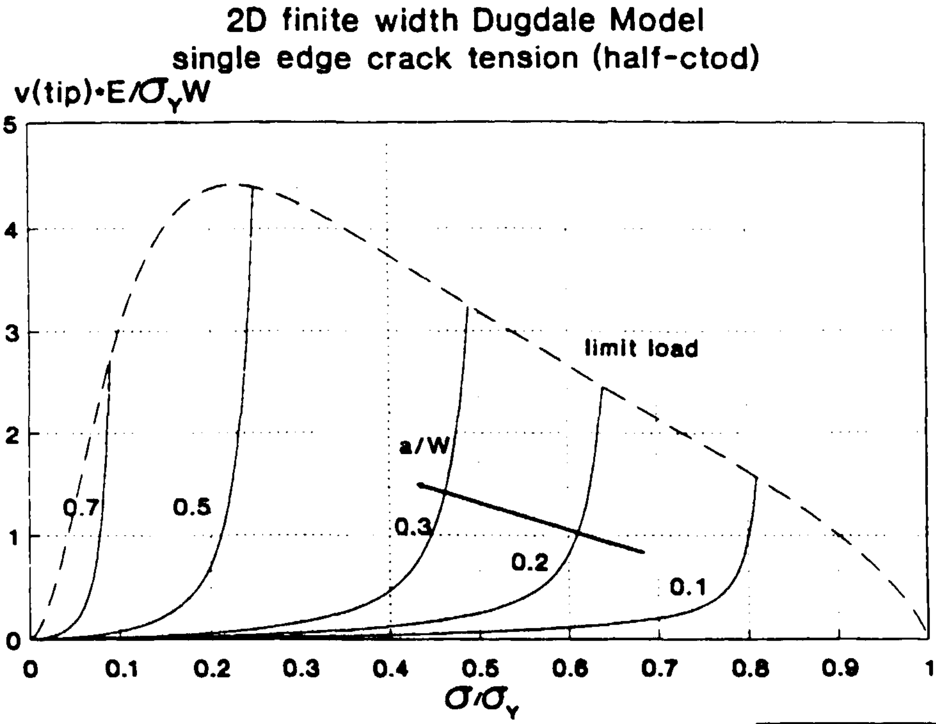
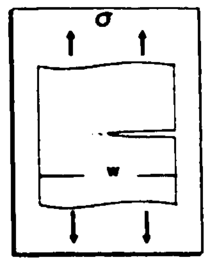


figure P3.1



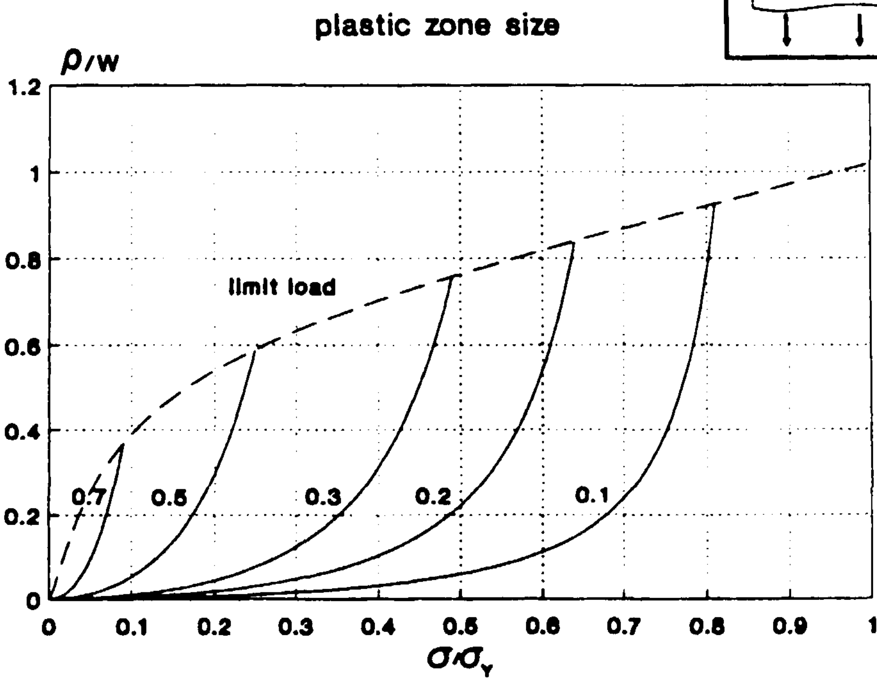


figure P3.2

3.5 2D finite width Dugdale Model - single-edge crack bending

The closed form presentations here of data on bending make use initially of a semi-empirical relation between plastic zone size and stress due to Chell [30] viz;

$$\frac{\pi}{2}\frac{\sigma}{\sigma_Y} = \arccos\left[\left(\frac{a}{a_1}-\frac{a}{W}\right)/\left(1-\frac{a}{W}\right)\right]\frac{F_t(a_1)}{F_b(a_1)}\left(1-\frac{a}{W}\right)^2 \qquad \text{....P13}$$

where $a_1 = (a+\rho)/W$;
F_t is given by equation P11
F_b is given by equation

$$F_b = \left\{1.25/[1-a_1^{1.82}]^{2.57} - \sin\left[\frac{\pi}{2}a_1\right]\right\}^{1/2}$$

The crack tip displacement can then be given by a solution (previously unpublished) as;

$$v_{tip}\frac{E}{\sigma_Y W} = \frac{4}{\pi}\frac{\hat{a}}{W}\ln\left(\frac{\hat{a}}{W}\right)\left[F_b\left(\frac{\hat{a}}{W}\right)\right]^2 \qquad \text{....P14}$$

where $\hat{a} = a + 0.65\rho$

These equations are not so straightforward in use as the earlier Dugdale fits. To find v_{tip} as a function of applied stress, equation P13 must first be solved inversely by varying a_1. The resulting solution value of ρ is then used to evaluate equation P14. These steps are incorporated in program *SENBDUG*.

Figures P4.1 and P4.2 were drawn up by varying ρ between zero and an upper limit set by the limit load condition

$$\frac{\sigma}{\sigma_Y} = 2\left(1-\frac{a}{W}\right)^2 \qquad \text{....P15}$$

For values of $a/W > 0.5$, an upper limit of $a_1/W = 0.8$ was set to avoid instability in the calculation.

2D finite width Dugdale Model
single edge crack bending (half-ctod)

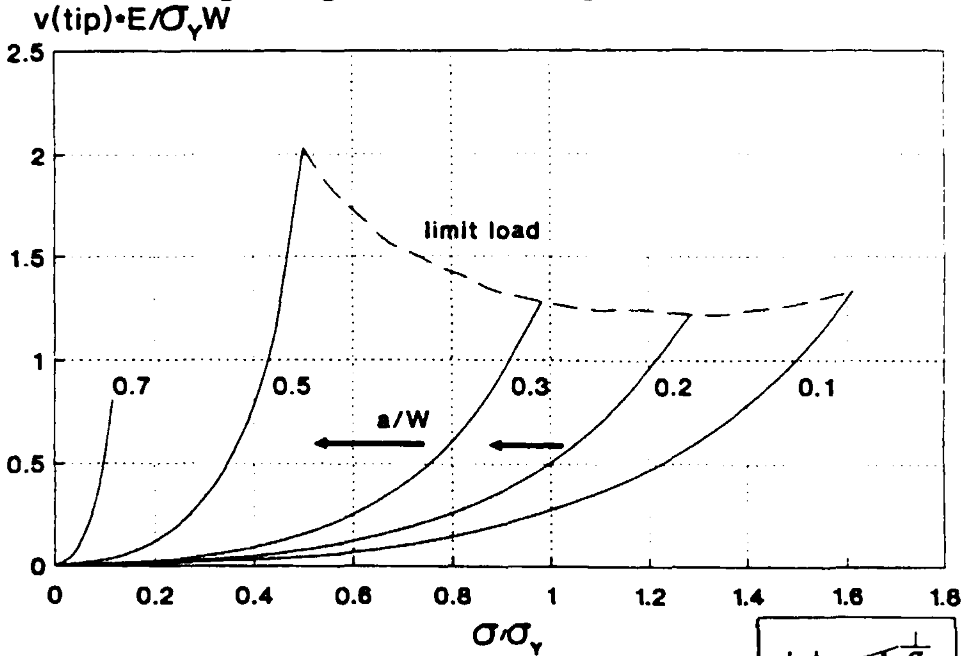
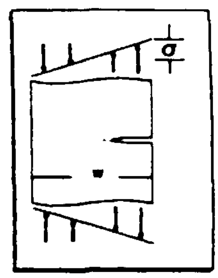


figure P4.1



plastic zone size

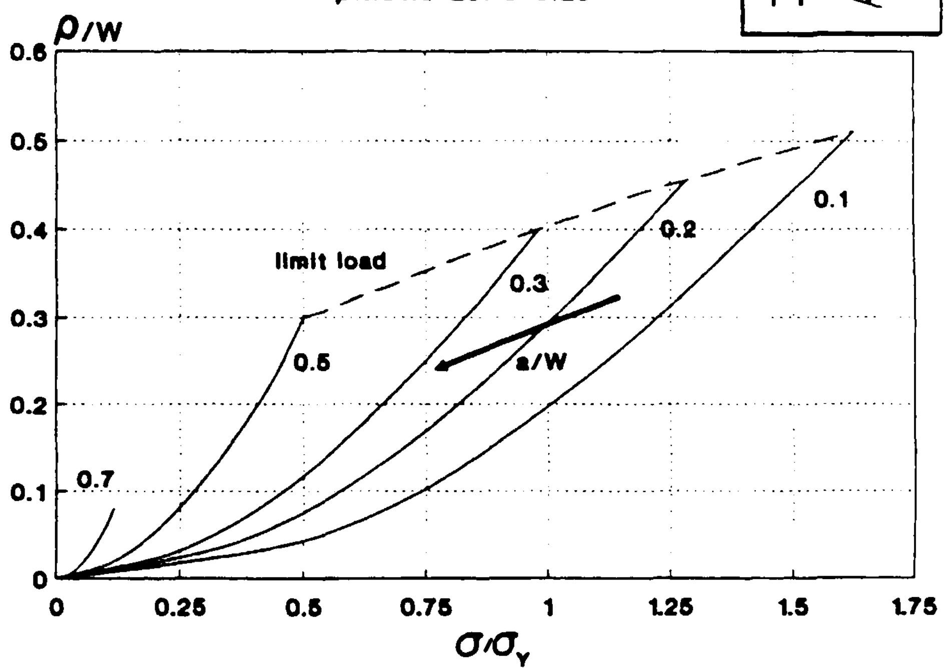


figure P4.2

Growing Crack Constant-Amplitude Loading

4 GROWING CRACK CONSTANT-AMPLITUDE LOADING

4.1 Dugdale-type growth model - internal crack

The final unloading profile shown in Figure P1.3 can be taken as the starting point for consideration of a growth model. When the crack propagates beyond the end of the blunt tip shown in this figure, a considerable thickness of stretched material will be added to each crack flank. The new crack faces will thus make contact during the unloading phase, before the load is completely released. Calculation of the stretched zone thickness is complicated by plastic compression of stretched zones as the crack propagates, especially if the external load cycle is partly compressive, but the general effect is as described.

Several models for the closure problem have been given but the model of Newman [12] is particularly convenient for the present purpose. Newman's equations have been incorporated in the program *NEWMANOP*, used to plot the curves opposite.

Figure G1.1 identifies the point of crack opening and closure during a constant amplitude cycle as shown in the accompanying nomenclature key. (The stress levels for opening during loading and closing during unloading are reckoned to be slightly different. However the differences are probably insignificant.) The term σ_{op} is taken here to be the first point when the crack faces become completely separated on loading and also when they first touch on unloading.

The figure shows how the opening stress depends on the stress ratio R which characterises the stress cycle and on the maximum stress in the cycle, expressed in terms of the yield strength. The dashed 'locus' line represents the case where $\sigma_{op} = \sigma_{min}$, i.e, where the crack is open throughout the cycle.

For most practical cases, it can be seen that the crack is at least partially closed for up to half of the load cycle, even when the loading is always in tension (R positive).

Figure G1.1 has been drawn up for the assumption of plane stress triaxial constraint in the crack tip plastic zone. Other constraint assumptions are possible as discussed by Newman and alternative diagrams for these assumptions can be readily drawn by changing coefficients in the formulation.

Other models for crack closure and the effects on growth rate are given in refs [31,32].

Dugdale-type growth model
opening/closure stress

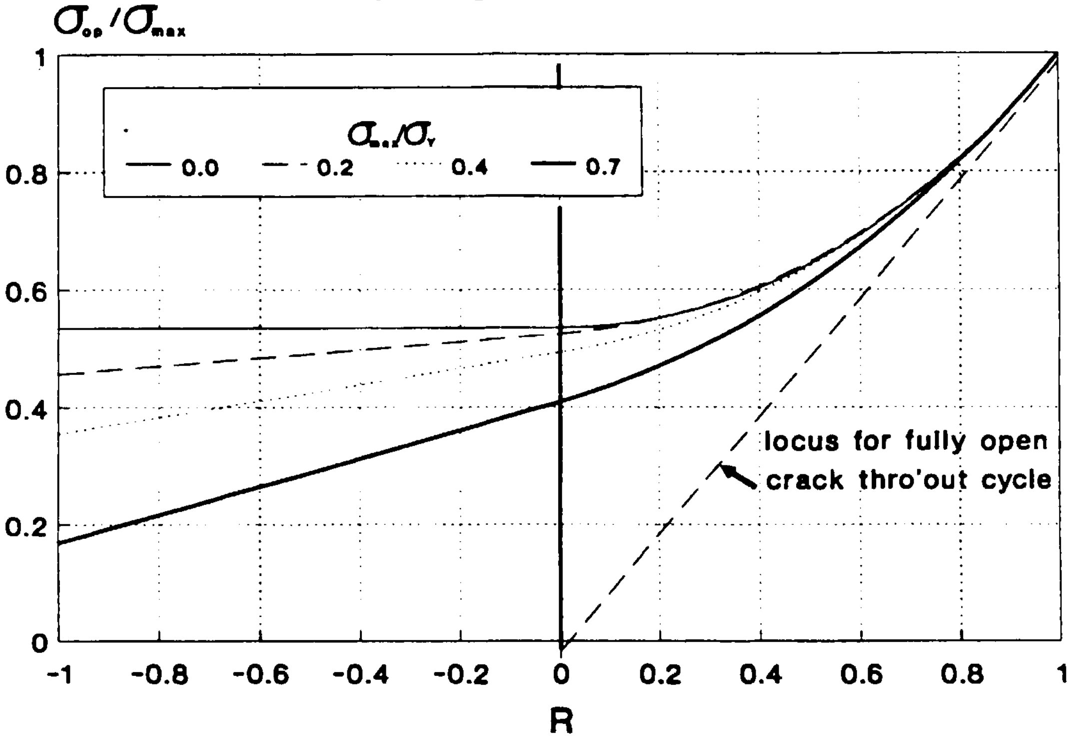
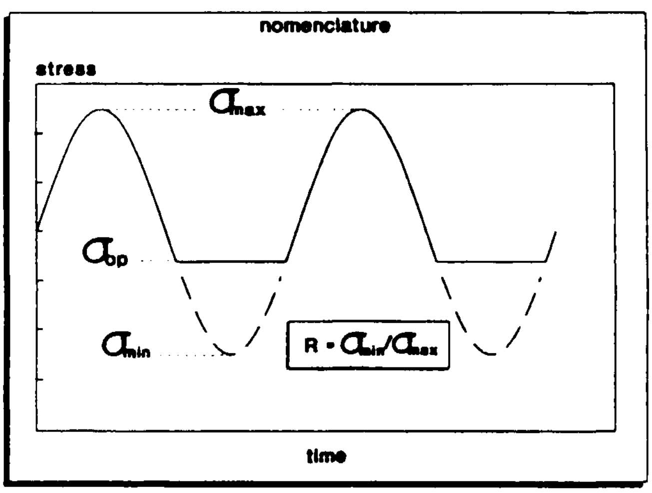
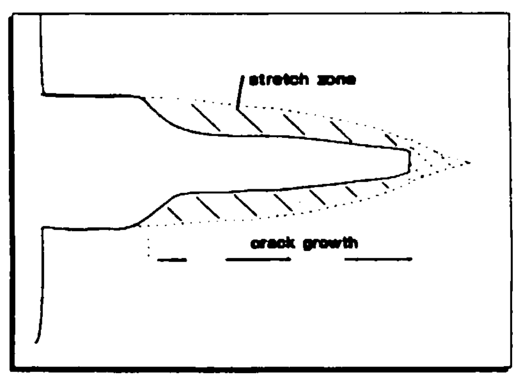


figure G1.1





4.2 Dugdale unloading model with stretch zones - Internal crack

The formulation underlying Figure G1.1 has been used with the stationary crack unloading model used for Figure P1.3 to draw simulated crack profiles for a growing crack. The required sequence of calculations is incorporated in the program *GROCRACK*.

If the closing stress for a given Dugdale crack is known (for example, from the Newman model as developed in the program *NEWMANOP*) then the thickness of the associated stretch zone can be determined by scanning the stationary unloading profile at the closing stress level in order to find the minimum gap at that stage of unloading. In the case shown in Figure G1.2 ($\sigma_{max}/\sigma_Y = 0.7$; R = 0 and a plane stress assumption is used) the opening/closing stress ratio σ_{op}/σ_Y is found to be 0.17.

The resulting stretched zone thickness is then determined and subtracted from the profiles shown in Figure P1.3 from which the successive unloading profiles are drawn as in Figure G1.2. (The key figure identifies the load levels corresponding to the profiles in the upper figure).

At the closure stress level, the crack faces are seen to be just touching at a point approximately 70% from the mid-point of the internal crack. Further closure takes place as the load is reduced to zero but this process cannot be modelled so readily as it becomes a contact mechanics problem.

The software program used to draw up this figure will accept any values of σ_{max}/σ_Y, and R can be altered to reflect other crack tip plasticity constraints.

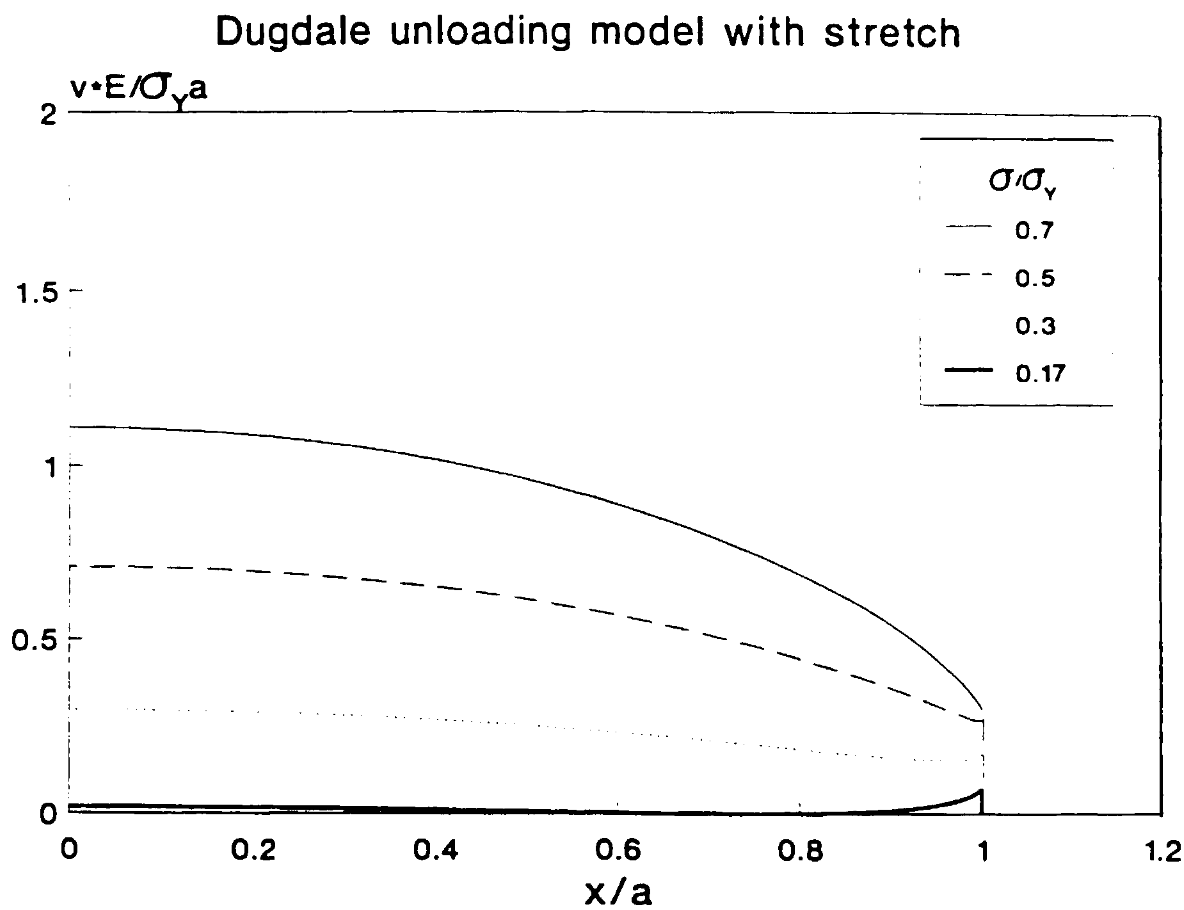
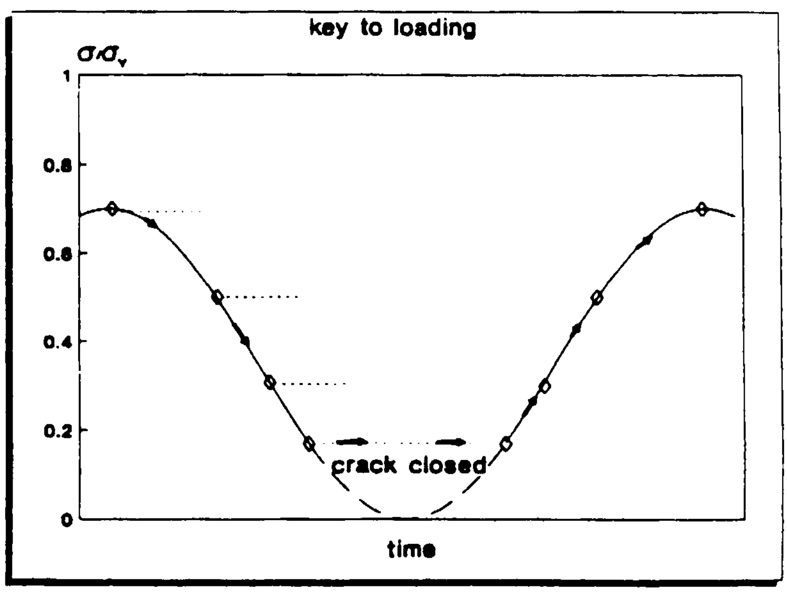

Dugdale unloading model with stretch
v*E/σYa
2
1.5
1
0.5
0
0
0.2
0.4
0.6
0.8
1
1.2
σ/σY
0.7
0.5
0.3
0.17
x/a

figure G1.2


key to loading
σ/σY
1
0.8
0.6
0.4
0.2
0
crack closed
time

5 COMPUTER SOFTWARE

5.1 Programs available

Elastic

ECOINF evaluates opening profile of crack in an infinite plate under normal stress. Also gives near-crack-tip approximation (*vapp*).

PERIO evaluates opening profiles for a periodic array of cracks, using equation E5. If required, input values of *a/W* can be changed in 'calculation 1' block (see figure E1.3). Non-dimensional form is listed but can be easily changed to dimensional.

WIGOP evaluates single edge crack opening in semi-infinite plate using series function due to Wigglesworth. Program first prints out series constants.

3DECO evaluates profile of elliptical planform crack of any aspect ratio and at any cross section parallel to a major axis. Based on Equation E7 (see figure E6.1).

Elastic/plastic*

INFDUG draws profiles for Dugdale cracks in an infinite plate using equation P1. Allows for 10 user-defined input values of σ/σ_Y. Results are non-dimensionalised in terms of σ (Figure P1.1) or with respect to σ_Y (Figure P1.2).

DUGULOD draws infinite plate Dugdale Model profiles for unloading a stationary crack from a selected maximum stress level (Figure P1.3). Intermediate unloading levels are also requested.

---

* In all Dugdale Model formulations, openings under alternative constraint (plane stress/plane strain) may be calculated by substituting a different factor for yield stress elevation σ_Y.

DENDUG evaluates plastic zone sizes and crack tip displacements for DENT geometry Dugdale Model. Five user-defined values of *a/W* are specified within the 'initialise:' block. Can be used for other geometries given suitable alteration of coefficients in equations P5-(Figure P2, P3). These coefficients are also defined in the 'initialise:' block and the 'F factor' should also be changed appropriate to the geometry. Limit load end-points are incorporated.

SENTDUG as above for single edge crack in tension. Note altered limit load criterion.

SENBDUG evaluates stress levels and tip displacements corresponding to plastic zone sizes for single-edge crack Dugdale Model subject to bending (Figure P4). Hayes and Williams data are included for comparison with the empirical equations.

Growing Cracks

NEWMANOP determines crack opening stress via [12]. User-defined input values of R and σ_{max}/σ_Y (Figure G1.1).

GROCRACK includes crack flank stretching in Dugdale program. Input maximum stress in cycle and desired intermediate stress value for profile. Checks value to ensure it is equal to or greater than opening stress level (Figure G1.2).

5.2 Program listings

ECOINF

'Elastic crack opening

```
initialise:
  N=100
  DIM x#(1:N+1),v#(1:N+1)
  dim vapp#(1:N+1)
```

```
Printout:
  INPUT "Do you want a printout of crack profile data (Y/N)"; Y$
  If Y$="Y" or Y$="N" then
    Goto Datafile
  Else
    PRINT "Try again"
    Goto Printout
  End if
```

```
Datafile:
  INPUT "File name for data"; D$
  D$=D$+".DAT"
```

```
Ellipse:
  A=1
  x#(1)=0
  v#(1)=2
  vapp#(1)=2.828
  For I=1 to N
    x#(I+1)=1-1/I^0.6
    v#(I+1)=2*Sqr(1-(x#(I+1))^2)
    vapp#(I+1)=2*sqr(2)*sqr(1-x#(I+1))
    Next I
```

```
  PRINT
  PRINT "Crack profile (Ev/Sa):"
  PRINT "x","v"
  For I=1 to N+1 step 5
    PRINT USING "#.####      "; x#(I), v#(I), vapp#(I)
  Next I
```

```
'printout instructions

  If Y$="N" then goto Senddata:
  LPRINT
  LPRINT "Crack profile (Ev/Sa):"
  LPRINT "x","v"
  For I=1 to N+1 step 20
    LPRINT USING "#.####      "; x#(I), v#(I)
  Next I
```

```
Senddata:
  Open D$ for output as #1
  PRINT #1, "Elastic crack opening":PRINT
  For I=1 to N+1
    PRINT #1, USING " +#.####    ";x#(I), v#(I)
     next I
  Close #1
```

```
PRINT "Press ESC to return to main menu"

End
```

PERIO

'Elastic crack opening based on periodic array of cracks

```
initialise:
  Clear
  CLS
  N=100
  DIM x#(1:N+1), v#(1:8,1:N+1), Plot#(1:9,1:N+1)
  PI#=4*ATN(1.0)
```

```
Printout:
  INPUT "Do you want a printout of crack profile data (Y/N)"; Y$
  If Y$="Y" or Y$="N" then
    Goto Datafile
  Else
    PRINT "Try again"
    Goto Printout
  End if
```

```
Datafile:
  INPUT "File name for data"; D$
  D$=D$+".DAT"
```

```
Calculation1:
'calculates crack profile for a/w=0.1 to 0.8
  For C=1 to 8
    aw#=C/10
    A=1
    x#(1)=A
    v#(C,1)=0
    For I=1 to N
      x#(I+1)=A-(I/N)^2
      v#(C,I+1)=4/PI#/aw#*(Log(Cos(PI#/2*x#(I+1)*aw#)+(Cos(PI#/2*x#(I+1)*_
            aw#)^2-Cos(PI#/2*aw#)^2)^0.5)-Log(Cos(PI#/2*aw#)))
      v#(C,I+1)=v#(C,I+1)/Sqr(Tan(PI#/2*aw#)/(PI#/2*aw#))
    Next I
  Next C
```

```
Reformat:
  For I=1 to N+1
    Plot#(1,I)=x#(I)
    For C=1 to 8
      Plot#(C+1,I)=v#(C,I)
    Next C
  Next I

  PRINT
  PRINT "Crack profiles:"
  PRINT
  PRINT "x","","a/w"
  PRINT "     =0.1  =0.2  =0.3  =0.4  =0.5  =0.6  =0.7  =0.8"
  PRINT "-----------------------------------------------------"
  For I=1 to N+1 step 20
    For T=1 to 9
      PRINT USING "+#.### "; Plot#(T,I),
    Next T
  PRINT ""
  Next I
```

```
'printout instructions

  If Y$="N" then goto Senddata:
  LPRINT
  LPRINT "Crack profiles based on periodic crack array:"
  LPRINT "x","","a/w"
  LPRINT "     =0.1  =0.2  =0.3  =0.4  =0.5  =0.6  =0.7  =0.8"
  LPRINT "-----------------------------------------------------"
  For I=1 to N+1 step 20
    For T=1 to 9
      LPRINT USING "+#.### "; Plot#(T,I),
    Next T
  LPRINT ""
  Next I
```

```
Senddata:
  Open D$ for output as #1
  PRINT #1, "Elastic crack opening based on array of cracks":PRINT
  For I=1 to N+1
    For T=1 to 9
      PRINT #1, USING " +#.####  "; Plot#(T,I),
    Next T
  PRINT #1,""
  Next I
  Close #1
```

```
PRINT "Press ESC to return to main menu"

End
```

WIGOP

'calculates wigglesworth single edge notch data

```
initialise:
print "initialising"
 clear
 cls
 n=100
 dim plot(1:2,1:n+1)
 dim a(1:13)
 for i=1 to 13
     read a(i)
     print i,a(i)
 next i
```

```
calculate:

print "calculating"
 for I=0 to n
  x=1-(I/n)^0.5
  y=0
   for j=1 to 13
    y=y+a(j)*x^(j-1+0.5)
   next j
  y=sqr(8)*y
  plot(1,I+1)=x:plot(2,I+1)=y
 next I
```

```
prnt:

print "printing"
  print "x","y"
  j=0
  for I=1 to n+1
   j=j+1 : if j<>10 goto 20
      print plot(1,I),plot(2,I)
     j=0
20  next I
```

```
stordat:
 input "file name for data ";d$
 d$=d$+".dat"
 input "title for data ";q$
 open d$ for output as #1
 print #1,q$
 for I=1 to n+1
   print #1,using " +##.### ";plot(1,I),plot(2,I)
 next I
close #1
end
```

```
 data 1,-0.143719,0.019965,0.019665,0.011856
 data 0.006254,0.002993,0.001256,0.00039,-0.00001
 data -0.000172,-0.000213,-0.000212
```

3DECO

'3-D (Linear Elastic) Crack profile

```
Initialise:
  Clear
  CLS
  N=100
  D=50
  DIM f#(0:D), phi#(0:D), x#(1:N+1), v#(1:N+1)
  PI#=4*ATN(1.0)
```

```
Printout:
  INPUT "Do you want a printout of crack profile data (Y/N)"; Y$
  If Y$="Y" or Y$="N" then
    Goto datafile
  Else
    PRINT "Try again"
    Goto Printout
  End if
```

```
Datafile:
  INPUT "File name for data"; D$
  D$=D$+".dat"
```

```
Calculation1:
'calculate value of E(k)
  INPUT "Enter value of b/a for the crack: ", R#
  k2#=1-R#^2
  For I=0 to D
    phi#(I)=PI#/2/D*I
    f#(I)=(1-k2#*Sin(phi#(I))^2)^0.5
  Next I
  S1#=0
  S2#=0
  For I=1 to D-1 step 2
    S1#=S1#+f#(I)
  Next I
  For I=2 to D-2 step 2
    S2#=S2#+f#(I)
  Next I
  Ek#=1/3*PI#/2/D*(f#(0)+f#(D)+4*S1#+2*S2#)

  PRINT "Hence E(k)="; Ek#
```

```
'printout instructions

  If Y$="N" then goto calculation2
  LPRINT "Chosen b/a="; R#
  LPRINT "Hence E(k)="; Ek#
```

```
Calculation2:
'calculates Ev/Sa versus x/a for chosen value of y/b
  A#=1.0
  INPUT "Enter value of y/b at which profile is required: ", y#

  x#(1)=A#*Sqr(1-y#^2)
  v#(1)=0.0
  For I=1 to N
    x#(I+1)=x#(1)*(1-(I/N)^2)
    v#(I+1)=2/Ek#*Sqr((1-x#(I)^2-y#^2))
  Next I
  PRINT
  PRINT "Crack profile at y/b="; y#
  PRINT "x", "v"
  For I=1 to N+1 step 20
    PRINT USING " #.####      "; x#(I), v#(I)
  Next I
```

```
'printout instructions

  If Y$="N" then goto Senddata
  LPRINT
  LPRINT "Datafile= "; D$
  LPRINT "Crack profile at y/b="; y#
  LPRINT "x", "v"
  For I=1 to N+1 step 20
    LPRINT USING " #.####      "; x#(I), v#(I)
  Next I
  LPRINT "-----------------------"
```

```
Senddata:
  Open D$ for output as #1
  PRINT #1, "3-D Elastic crack opening for b/a=" R# ", at y/b=" y# :PRINT
  For I=1 to N+1
    PRINT #1, USING " #.#### "; x#(I), v#(I)
  Next I
  Close #1
```

```
PRINT "Press ESC to return to main menu"

End
```

INFDUG

'Dugdale loading model
'Calculates variation of Ev/Sya and Ev/Sa with x/a and exports to datafile.
'Note that in this program: x is used for x/a
'   vload " " Ev/Sya
'   vnonD " " Ev/Sa

```
initialize:
  Clear
  CLS
  N=100
  C=0
  DIM x#(1:N+1), vload#(1:10,1:N+1), vnonD#(1:10,1:N+1), S#(1:10)
  PI#=4*ATN(1.0)
```

```
Printout:
  INPUT "Do you want a printout of crack profile data (Y/N)"; Y$
  If Y$="Y" or Y$="N" then
    Goto Datafile
  Else
    PRINT "Try again"
    Goto Printout
  End if
```

```
Datafile:
  Input "File name for data"; D$
  D$=D$+".dat"
```

```
Calculation1:
'calculates v at each point on loading
  C=C+1
  A#=1
  x#(1)=A#
  If C=1 then
    INPUT "Enter Smax/Sy: ", S#(C)
  Elseif C<=10 then
    PRINT "Enter intermediate S/Sy #"C-1": ";
    INPUT, S#(C)
  Elseif C>10 then
    PRINT "Too many intermediate S/Sy's. You need to increase the"
    PRINT "range of S#(C) and vload#(C,I) in the initialisation."
Saveorabort:
    INPUT "Do you want to save the data so far, or abort (S/A) ", P$
    If P$="S" then
      Goto Choice
    Elseif P$="A" then
      Goto Finish
    Else
      PRINT "Try again"
      Goto Saveorabort
    End if

  End if
  R#=1/Cos(S#(C)*PI#/2)-1
  vload#(C,1)=4/PI#*Log(R#+1)
```

```
  For I=1 to N
    x#(I+1)=A#-(I/N)^2
    x1#=x#(I+1)
    T#=((1+R#)^2-x1#^2)/((1+R#)^2-1)
    T#=Sqr(T#)
    vload#(C,I+1)=4/PI#*(Log((T#+1)/(T#-1))/2-x1#/2*Log((T#+x1#)/_
         (T#-x1#)))
  Next I
  If C=1 then
    PRINT "Crack profile at Smax/Sy=" S#(C) ":"
  Else
    PRINT "Crack profile at S/Sy=" S#(C) ":"  End if
  PRINT "x", "v"
  For I=1 to N+1 step 20
    PRINT USING "#.####        "; x#(I), vload#(C,I)
  Next I
```

```
'printout instructions:

  If Y$="N" goto Calculation2
  LPRINT
  If C=1 then
    LPRINT "Crack profile at Smax/Sy=" S#(C) ":"
  Else
    LPRINT "Crack profile at S/Sy=" S#(C) ":"
  End if
  LPRINT "x", "v"
  For I=1 to N+1 step 20
    LPRINT USING "#.####        "; x#(I), vload#(C,I)
  Next I
```

```
Calculation2:
'calculates v non-dimensionalised with respect S

  For I=1 to N+1
    vnonD#(C,I)=vload#(C,I)/S#(C)
  Next I
  PRINT "Crack profile non-dimensionalised w.r.t. S:
  PRINT "x", "v"
  For I=1 to N+1 step 20
    PRINT USING "#.####        "; x#(I), vnonD#(C,I)
  Next I
```

```
'printout instructions:

  If Y$="N" goto More
  LPRINT
  LPRINT "Crack profile non-dimensionalised w.r.t. S:
  LPRINT "x", "v"
  For I=1 to N+1 step 20
    LPRINT USING "#.####        "; x#(I), vnonD#(C,I)
  Next I
'-----------------------------------
```

```
More:
  INPUT "Do you want another, intermediate, S/Sy (Y/N)? ", M$
  If M$="Y" then
    Goto Calculation1
  Elseif M$="N" then
    Goto Choice
  Else
    PRINT "Try again"
    Goto Choice
  End if
```

```
Choice:
  INPUT "Do you want v(w.r.t. Sy) or v(w.r.t. S) sent to file (vSy/vS)"; Z$
  If Z$="vSy" then
    Goto Reformat
  Elseif Z$="vS" then
    Goto NonD
  Else
    PRINT "Try again"
    Goto Choice
  End if
```

```
NonD:
  For B=1 to C
    For I=1 to N+1
      vload#(B,I)=vnonD#(B,I)
    Next I
  Next B
```

```
Reformat:
'reformat data for sending to datafile
  DIM Plot#(C+1,1:N+1)
  For I=1 to N+1
    Plot#(1,I)=x#(I)
    For B=1 to C
      Plot#(B+1,I)=vload#(B,I)
    Next B
  Next I
```

```
Senddata:
  Open D$ for output as #1
  PRINT #1, "Dugdale loading model":PRINT
  For I=1 to N+1
  For H=1 to C+1
  PRINT #1, USING " #.#### "; Plot#(H,I),
  Next H
  PRINT #1,""
  Next I
  Close #1
```

```
Finish:
  PRINT "Press ESC to return to main menu"

End
```

DUGULOD

```
'Dugdale model with unloading
'Calculates variation of Ev/Sya with x/a at Smax/Sy and Smin/Sy,
'and at intermediate values of S/Sy on the unloading range.
'Note that in this program:  x is used for x/a
'                            vload "  " Ev/Sya at Smax/Sy
'                            vnet  "  "   "  " Smin/Sy
```

```
initialize:
  Clear
  CLS
  N=100
  C=0
  DIM x#(1:N+1), vload#(1:N+1), vunload#(1:10,1:N+1), vnet#(1:10,1:N+1)
  DIM S5#(1:10)
  PI#=4*ATN(1.0)
```

```
Printout:
  INPUT "Do you want a printout of crack profile data (Y/N)"; Y$
  If Y$="Y" or Y$="N" then
    Goto Datafile
  Else
    PRINT "Try again"
    Goto Printout
  End if
```

```
Datafile:
  Input "File name for data"; D$
  D$=D$+".dat"
```

```
Calculation1:
'calculates v at Smax/Sy
  A#=1
  x#(1)=A#
  INPUT "Enter Smax/Sy: ", S#
  R#=1/Cos(S#*PI#/2)-1
  vload#(1)=4/PI#*Log(R#+1)
  For I=1 to N
    x#(I+1)=A#-(I/N)^2
    x1#=x#(I+1)
    T#=((1+R#)^2-x1#^2)/((1+R#)^2-1)
    T#=Sqr(T#)
    vload#(I+1)=4/PI#*(Log((T#+1)/(T#-1))/2-x1#/2*Log((T#+x1#)/_
        (T#-x1#)))
  Next I

  PRINT "Crack profile at Smax/Sy:
  PRINT "x", "v"
  For I=1 to N+1 step 20
    PRINT USING "#.####      "; x#(I), vload#(I)
  Next I
```

```
'printout instructions:

  If Y$="N" goto Calculation3
  LPRINT
  LPRINT "Crack profile at Smax/Sy:
  LPRINT "x", "v"
  For I=1 to N+1 step 20
    LPRINT USING "#.####       "; x#(I), vload#(I)
  Next I
```

```
Calculation3:
'calculates crack profile at Smin/Sy and at each intermediate
'value of S/Sy on the unloading range
    C=C+1
    If C=1 then
      INPUT "Enter Smin/Sy: ", S5#(C)
    Elseif C<=10 then
      PRINT "Enter intermediate S/Sy #"C-1": ";
      INPUT, S5#(C)
    Elseif C>10 then
      PRINT "Too many intermediate S/Sy's. You need to increase the range"
      PRINT "of S#(C), vunload#(C,I) and vnet#(C,I) in the initialisation"
Saveorabort:
      INPUT "Do you want to save the data so far, or abort (S/A)? ", P$
      If P$="S" then
        Goto Reformat
      Elseif P$="A" then
        Goto Finish
      Else
        PRINT "Try again"
        Goto Saveorabort
      End if
    End if

    S2#=S5#(C)
    Gosub Unloadprofile
    PRINT
    If C=1 then
      PRINT "Crack profile at Smin/Sy=" S5#(C) ":"
    Else
      PRINT "Crack profile at S/Sy=" S5#(C) ":"
    End if
    PRINT "x", "v"
    For I=1 to N+1 step 20
      PRINT USING "#.####       "; x#(I) vnet#(C,I)
    Next I
```

```
'printout instructions

  If Y$="N" then goto More
  LPRINT
  If C=1 then
    LPRINT "crack profile at Smin/Sy=" S5#(C) ":"
  Else
    PRINT "Crack profile at S/Sy=" S5#(C) ":"
  End if
  LPRINT "x", "v"
  For I=1 to N+1 step 20
    LPRINT USING "#.####       "; x#(I) vnet#(C,I)
  Next I
```

```
More:
  INPUT "Do you want another, intermediate, S/Sy (Y/N)? ", M$
  If M$="Y" then
    Goto Calculation3
  Elseif M$="N" then
    Goto Reformat
  Else
    PRINT "Try again"
    Goto More
  End if
```

```
Reformat:
'reformat data for sending to datafile
  DIM Plot#(1:C+2,1:N+1)
  For I=1 to N+1
    Plot#(1,I)=x#(I)
    Plot#(2,I)=vload#(I)
    For B=1 to C
      Plot#(B+2,I)=vnet#(B,I)
    Next B
  Next I
```

```
Senddata:
  Open D$ for output as #1
  PRINT #1, "Dugdale unloading model":PRINT
  For I=1 to N+1
  For H=1 to C+2
    PRINT #1, USING " #.####  "; Plot#(H,I),
  Next H
  PRINT #1,""
  Next I
  Close #1
```

```
Finish:
  PRINT "Press ESC to return to main menu"

  End
```

```
Unloadprofile:
    S3#=(S#-S2#)*PI#/4
    R#=1/Cos(S3#)
    vunload#(C,1)=8/PI#*Log(R#)
    For I=1 to N
      x1#=x#(I+1)
      T#=(R#^2-x1#^2)/(R#^2-1)
      T#=Sqr(T#)
      vunload#(C,I+1)=8/PI#*(Log((T#+1)/(T#-1))/2-x1#/2*Log((T#+x1#)/(T#-x1#)))
    Next I
    For I=1 to N+1
      vnet#(C,I)=vload#(I)-vunload#(C,I)
    Next I
  Return
```

DENDUG

'Dugdale Model to give CTOD via TGFG empirical fits
'double edge notch tension

```
clear
GOSUB initialise
GOSUB calculation1
print "end of a/W calculations"
GOSUB calculation2
print
print "end of limit load calculations"
gosub reformat
print "end of reformat"
gosub results
goto saveorabort
999 end
```

```
initialise:

  cls
  n%=200
  dim Y#(1:n%+1,1:6),X#(1:n%+1),plot#(1:n%+1,1:7)
  pi#=4*atn(1.0)
 'empirical constants for model
  P=5.1:Q=1.68:Q1=0.5:Q2=0:S1=1
 'empirical constants for F factor
  F1=1:F2=2.6:F3=6
 'a/W ratios
  data 0.1,0.2,0.3,0.5,0.7
return
```

```
'calculation of CTOD for various a/W and varying S/Sy
calculation1:
   for J%=1 to 5
     read A
     flag1=0
     L#=1-A  'limit load

print "limit load = ",L#
     Y#(1,J%)=0
print " A =    ";int(10*A)/10
        for I%=1 to n%
          if flag1=1 then 10  'limit load check flag
          S# = I%/n%+0.000001
        if S# >= L# then 20
          gosub ctodcalc
          Y#(I%+1,J%)=D#: goto 30
 20       flag1 = 1
          gosub ctodcalc
          Y#(I%+1,J%)=D#: goto 30
 10        Y#(I%+1,J%)=Y#(I%,J%)
 30       next I%
print "A,s/sy,D  ";A,S#,D#
 delay 1
    next J%
return
```

```
'calculates ctod for limit load boundary

calculation2:
 Y#(1,6)=0:X#(1)=0
  for I%=1 to n%-1
   S#=I%/n%
   A=1-S#
   gosub ctodcalc
   Y#(I%+1,6)=D#
   X#(I%+1)=S#
  next I%
 X#(n%+1)=1:Y#(n%+1,6)=0
return
```

```
'reformats data for transmission to draw file
reformat:
 for I%=1 to n%+1
  plot#(I%,1)=X#(I%)
   for J%=1 to 6
    plot#(I%,J%+1)=Y#(I%,J%)
   next J%
 next I%
return
```

```
'prints results from plotarray
results:
 print "S/Sy","","a/W"
 print "        0.1    0.2    0.3    0.5    0.7     limit load"
 print
 for I%=1 to n%+1 step 10
  for J%=1 to 7
   print using " +#.###  ";plot#(I%,J%),
  next J%
 print
 next I%
return
```

```
'subr to calculate CTOD
ctodcalc:
 SP#=S#*pi#/2
 A0#=A*(1+0.2*S#^P)/(1-S#^Q)
 gosub fcalc
 RO#=A*(1/cos(SP#)-1)*F#*S1
 'space for mods
 '
 '
 '
 '
 A0#=A+Q1*RO#
 gosub fcalc
  D#=4/pi#*log(1/cos(SP#))*F#*A
return
```

```
'calculation of F factor
fcalc:
 AP#=A0#*pi#/2
 F#=tan(AP#)/AP#+(1-A0#^F2)^F3/4
return
```

```
'send to data file
senddata:
  open D$ for output as #1
   print #1,"double edge notch 0.1<a/W<0.7"
   for I%=1 to n%+1
      for J%=1 to 7
         print #1,using "  +##.###  ";plot#(I%,J%),
      next J%
   print #1,""
   next I%
 close #1
 return
```

```
saveorabort:

  input "save(s) or abort(a) ";Q$
  if Q$="s" then
  goto hgpltflname
  elseif Q$="a" then
  goto 999
  else
  goto saveorabort
```

```
hgpltflname:

  input "give file name for data";D$
  D$=D$+".DAT"
  gosub senddata
end if
```

SENTDUG

'Dugdale Model to give CTOD via TGFG empirical fits
'single edge notch tension

```
clear
GOSUB initialise
GOSUB calculation1
print "end of a/W calculations"
GOSUB calculation2
print
print "end of limit load calculations"
gosub reformat
print "end of reformat"
gosub results
goto saveorabort
999 end
```

```
initialise:
  cls
  n%=200
  dim Y#(1:n%+1,1:6),X#(1:n%+1),plot#(1:n%+1,1:7)
  pi#=4*atn(1.0)
 'empirical constants for model
  P=0.6:Q=1.6:Q1=0.47:Q2=0:S1=0.82
 'empirical constants for F factor
  F1=1.25:F2=1.5:F3=6.5
 'a/W ratios
  data 0.1,0.2,0.3,0.5,0.7
return
```

```
'calculation of CTOD for various a/W and varying S/Sy
calculation1:
   for J%=1 to 5
     read A
     flag1=0
     L#=(1-A)*(1-A)  'limit load
print "limit load = ",L#
     Y#(1,J%)=0
print " A =     ";int(10*A)/10
        for I%=1 to n%
          if flag1=1 then 10  'limit load check flag
          S# = I%/n%+0.000001
        if S# >= L# then 20
          gosub ctodcalc
          Y#(I%+1,J%)=D#: goto 30
 20       flag1 = 1
          gosub ctodcalc
          Y#(I%+1,J%)=D#: goto 30
 10        Y#(I%+1,J%)=Y#(I%,J%)
 30      next I%
print "A,s/sy,D   ";A,S#,D#
 delay 1
   next J%
return
```

```
 'calculates ctod for limit load boundary
 calculation2:
   Y#(1,6)=0:X#(1)=0
    for I%=1 to n%-1
      S#=I%/n%
      A=1-sqr(S#)
      gosub ctodcalc
      Y#(I%+1,6)=D#
      X#(I%+1)=S#
    next I%
   X#(n%+1)=1:Y#(n%+1,6)=0
 return
```

```
'reformats data for transmission to draw file
reformat:
  for I%=1 to n%+1
    plot#(I%,1)=X#(I%)
      for J%=1 to 6
        plot#(I%,J%+1)=Y#(I%,J%)
      next J%
  next I%
return
```

```
'prints results from plotarray
results:
  print "S/Sy","","a/W"
  print "        0.1     0.2     0.3     0.5     0.7     limit load"
  print
  for I%=1 to n%+1 step 10
   for J%=1 to 7
    print using " +#.###   ";plot#(I%,J%),
   next J%
  print
  next I%
return
```

```
'subr to calculate CTOD
ctodcalc:
 SP#=S#*pi#/2
 A0#=A*(1+0.2*S#^P)/(1-S#^Q)
 gosub fcalc
 RO#=A*(1/cos(SP#)-1)*F#*S1
 'space for mods
 '
 '
 '
 '
 A0#=A+Q1*RO#
 gosub fcalc
  D#=4/pi#*log(1/cos(SP#))*F#*A
 return
```

```
'calculation of F factor
fcalc:
  F#=F1/(1-0.7*A0#^F2)^F3
return
```

```
'send to data file
senddata:
   open D$ for output as #1
     print #1,"single edge notch 0.1<a/W<0.7"
     for I%=1 to n%+1
        for J%=1 to 7
          print #1,using "  +##.###  ";plot#(I%,J%),
        next J%
     print #1,""
     next I%
 close #1
 return
```

```
saveorabort:
  input "save(s) or abort(a) ";Q$
  if Q$="s" then
  goto hgpltflname
  elseif Q$="a" then
  goto 999
  else
  goto saveorabort
```

```
hgpltflname:

  input "give file name for data";D$
  D$=D$+".DAT"
  gosub senddata
end if
```

SENBDUG

'Dugdale Model to give CTOD via TGFG empirical fits
'single edge notch bending

```
clear
10 pi = 3.141592654
20 def FNacs(c) = atn( sqr(1 / c^2  - 1) )
30 gosub initialise
40 gosub a
42 gosub fmthwdata
55 print "successful end "
gosub results
60 gosub savedata
print "data saved"
65 end
```

```
initialise :
70 p = .6
80 q = 1.6
90 r = .65
100 n% = 100
110 gosub hwdata
  dim plotro!(1:175,1:8),plotd!(1:175,1:8)
  dim ro!(1:101),d!(1:101),s!(1:101)
120 return
```

```
hwdata :
130 dim x(60), y(60), z(60)
150 for i = 1 to 60
160     read z(i), y(i), x(i)
180 next i
190 i = 1
200 for j = 1 to 5
210     read n1,a5
220     for k = 1 to n1
230     x(i) = x(i) * 4/pi
240     z(i) = a5 * (1/z(i) - 1)
250     i = i + 1
260     next k
270 next j
280 i = 0
290 return
```

```
a :      'calculates plastic zone size s, as a function of (a1 - a)

   r%=0  'row number for harvard output file
   i1%=0 'end of curve row index for harvard output file
300 for j% = 1 to 5
     read a
print " a = ";a
delay 2
310    a2 = (.8 - a)/n%
320    a1 = a
     q2=0
print "a      s/sy      ro      ctod/2":print
330    for i% = 1 to n%
         if q2=1 then b2
340    f4 = 1.25/(1 - .7 *a1^1.5)^6.5
350    f5 = 1.25/(1 - a1^1.82)^2.57 - sin(pi/2*a1)
355    s# = (a/a1-a)/(1-a)
360    s#= FNacs(s#) * sqr(f4/f5) * 2/pi * (1-a)^2
370        if s#<2*(1-a)^2 then b1
        q2=1 :n1%=i%
print "limit load at s/sy = ";int(1000*s#)/1000; "a/W  =  ";a
delay 5
        goto b2
```

```
b1:
          ro#=a1-a
          a3=a+r*ro#
          f6=1.25/(1-a3^1.82)^2.57-sin(pi/2*a3)
          d#=4/pi*a3*log(a3/a)*f6
        s!(i%)=s# : ro!(i%)=a1-a :  d!(i%)=d#
380    a1 = a1 + a2

b2:

390    next i%

print using " #.### ";a,s#,a1-a,d#
print "any key to go on";: input q$
print "calc for this j% finished now reformatting"
     k%=i1%+1:l%=1:gosub reformat
```

```
b5:
     if k%+4-i1%<n1% then b4   'continue selecting every 4th
     k%=i1%+n1% : l%=n1% :i1%=k%
     gosub reformat : goto b6  'last point selected for this a
```

```
b4:
     k%=k%+4 : l%=l%+4
     gosub reformat : goto b5 'check for last value
```

```
b6:
400 next j%
   xvals%=r%
410 return
```

```
reformat:
r%=r%+1
        plotro!(r%,1)=s!(l%) : plotd!(r%,1)=s!(l%)
        if j%=1 then r1
              for m%=1 to j%-1    'fills in zero cols before value
                plotro!(r%,m%+2)=0 : plotd!(r%,j%+1)=0
              next m%
r1:
        plotro!(r%,j%+1)= ro!(l%)  : plotd!(r%,j%+1)=d!(l%)
              for m%=j%+2 to 8   'fills in zeroes to end of table
                plotro!(r%,m%)=0 :plotd!(r%,m%)=0
              next m%
 return
```

```
results:
print "enter results"
        t%=0
    for  i%=1 to xvals%+60
              t%=t%+1
            for j%=1 to 7
               print using "  +#.###  ";plotro!(i%,j%),
            next j%
        print
          if t% < 20  then cont
          print
          t%=0
          delay 5
cont:
        next i%
return
```

```
savedata:
  Q$="rodata"
    gosub flname
    open D$ for output as #1
    print #1,title$
    for  i%=1 to xvals%+60
            for j%=1 to 7
               print #1,using "  +##.###  ";plotro!(i%,j%),
            next j%
        print #1,""
        next i%
    close #1

  Q$="ctoddata"
   gosub flname
   open D$ for output as #1
   print #1,title$
   for i%=1 to xvals%+60
           for j%=1 to 7
              print #1,using "  +##.###  ";plotd!(i%,j%),
           next j%
    print #1,""
    next i%
   close #1
 return
```

```
flname:
  print "give filename for ";Q$;"  ";
  input D$: D$=D$+".DAT"
  print "give title for data ";
  input title$
return
```

```
fmthwdata:
      print "format h & w data"
      i=0
      for i%=xvals%+1 to xvals%+60
        i=i+1
        plotro!(i%,1)=y(i):plotd!(i%,1)=y(i)
          for j%=2 to 6
            plotro!(i%,j)=0:plotd!(i%,j)=0
          next j%
        plotro!(i%,7)=z(i):plotd!(i%,7)=x(i)
      next i%

 return
```

```
rem data from hayes & williams
data .6667 , .5058 , .0743
data .5714 , .6274 , .1227
data .5 , .7201 , .1723
data .4444 , .7977 , .2248
data .4 , .866 , .2812
data .3636 , .9284 , .3428
data .3333 , .9869 , .4108
data .3077 , 1.043 , .4866
data .2857 , 1.097 , .5717
data .2667 , 1.15 , .6676
data .25 , 1.202 , .7771
data .2353 , 1.254 , .9002
data .2222 , 1.304 , 1.04
data .2105 , 1.357 , 1.207
data .2 , 1.405 , 1.38
data .1905 , 1.465 , 1.634
data .1818 , 1.501 , 1.789
data .1739 , 1.554 , 2.168
data .1667 , 1.584 , 2.168
data .16 , 1.783 , 3.896
data .8 , .3729 , .0793
data .7273 , .4741 , .1348
data .6667 , .555 , .1939
data .6154 , .6249 , .2581
data .5714 , .6878 , .3283
data .5333 , .7459 , .406
data .5 , .8009 , .4929
data .4706 , .8533 , .5899
data .4444 , .9038 , .6992
data .4211 , .9543 , .8266
```

```
data .4 , 1.001, .9613
data .381 , 1.055 , 1.148
data .3636 , 1.091 , 1.282
data .3478 , 1.14 , 1.517
data .3333 , 1.17 , 1.602
data .32 , 1.327 , 2.505
data .8571 , .2911 , .0818
data .8 , .3738 , .1416
data .75 , .4412 , .2069
data .7059 , .5004 , .2739
data .6667 , .554 , .36
data .6316 , .605 , .4527
data .6 , .6512 , .5527
data .5714 , .7012 , .6832
data .5455 , .738 , .7908
data .5217 , .7824 , .9574
data .5 , .8135 , 1.046
data .48 , .9328 , 1.734
data .8889 , .227 , .0836
data .8421 , .2938 , .1477
data .8 , .3473 , .2174
data .7619 , .3981 , .3028
data .7273 , .4381 , .3841
data .6957 , .4802 , .4945
data .6667 , .5126 , .5752
data .64 , .6006 , .8688
data .9091 , .171 , .0838
data .8696 , .2231 , .1531
data .8333 , .2614 , .2167
data .8 , .3262 , .3625
rem numbers of data points
data 20 , .1 , 16 , .2 , 12 , .3 , 8 , .4 , 4 , .5
data 0.1,0.2,0.3,0.5,0.7
```

NEWMANOP

'Newman opening stress

```
Clear
CLS
N=50
DIM R#(1:N+1),So#(0:8,1:N+1), S#(8), Plot#(10,N+1)
PI#=4*ATN(1.0)
```

```
Printout:
  INPUT "Do you want a printout of stress data (Y/N): "; Y$
  If Y$="Y" or Y$="N" then
    Goto Datafile
  Else
    PRINT "Try again"
    Goto Printout
  End if
```

```
Datafile:
  Input "File name for data "; D$
  D$=D$+".dat"
```

```
Calculation1:
  Input "Enter constraint (1-3): ",A0#
  For S=0 to 8
    S#(S)=S/10
    For I=1 to N+1
      R#(I)=-1.0+(I-1)*2/N
      C0#=Cos(S#(S))^(1/A0#)
      C0#=(0.825-0.34*A0#+0.05*A0#^2)*C0#
      C1#=(0.415-0.071*A0#)*S#(S)
      C3#=2*C0#+C1#-1
      C2#=1-C0#-C1#-C3#
      So#(S,I)=C0#+C1#*R#(I)
      If R#(I)>=0 then So#(S,I)=So#(S,I)+C2#*R#(I)^2+C3#*R#(I)^3
    Next I
  Next S
```

```
'Note that So# = [So/Smax]   not [So/Sy]
```

```
Reformat:
  For I=1 to N+1
    Plot#(1,I)=R#(I)
    For S=0 to 8
      Plot#(S+2,I)=So#(S,I)
    Next S
  Next I
```

```
PRINT "Newman opening stress"
PRINT
PRINT "R","","Smax/Sy"
PRINT "    =0.0  =0.1  =0.2  =0.3  =0.4  =0.5  =0.6  =0.7  =0.8"
PRINT "------------------------------------------------------------"
For I=1 to N+1 step 10
  For T=1 to 10
    PRINT USING "+#.### "; Plot#(T,I),
  Next T
PRINT ""
Next I
```

```
'printout instructions

 If Y$="N" then goto Senddata
 LPRINT "Newman opening stress"
 LPRINT
 LPRINT "R","","Smax/Sy"
 LPRINT "    =0.0  =0.1  =0.2  =0.3  =0.4  =0.5  =0.6  =0.7  =0.8"
 LPRINT "------------------------------------------------------------"
 For I=1 to N+1 step 10
   For T=1 to 10
     LPRINT USING "+#.### "; Plot#(T,I),
   Next T
 LPRINT ""
 Next I
'--------------------------------
```

```
Senddata:
 Open D$ for output as #1
 PRINT #1, "Newman opening stress":PRINT
 For I=1 to N+1
   For T=1 to 10
     PRINT #1, USING " +#.####  "; Plot#(T,I),
   Next T
 PRINT #1,""
 Next I
 Close #1
```

```
PRINT "Press ESC to return to main menu"

End
```

GROCRACK

```
'Dugdale model with unload and stretch
'Calculates variation of Ev/Sya (including stretch) at Smax/Sy, So/Sy and
'at chosen intermediate values of S/Sy.
'Note that in this program:  x is used for x/a
'                     vload "  " Ev/Sya at Smax/Sy
'                     vopen "  "  "  " So/Sy
'                     vnet  "  "  "  " intermediate S/Sy's
```

```
initialize:
  Clear
  CLS
  N=100
  DIM x#(1:N+1), vload#(1:N+1), vunload#(1:N+1)
  DIM vopen#(1:N+1),v#(1:N+1)
  PI#=4*ATN(1.0)
```

```
Printout:
  INPUT "Do you want a printout of crack profile data (Y/N): "; Y$
  If Y$="Y" or Y$="N" then
    Goto Datafile
  Else
    PRINT "Try again"
    Goto Printout
  End if
```

```
Datafile:
  Input "File name for data "; D$
  D$=D$+".dat"
```

```
Calculation1:
'calculates v at Smax/Sy
  A#=1
  x#(1)=A#
  INPUT "Enter Smax/Sy: ", S#
  R#=1/Cos(S#*PI#/2)-1
  vload#(1)=4/PI#*Log(R#+1)
  For I=1 to N
    x#(I+1)=A#-(I/N)^2
    x1#=x#(I+1)
    T#=((1+R#)^2-x1#^2)/((1+R#)^2-1)
    T#=Sqr(T#)
    vload#(I+1)=4/PI#*(Log((T#+1)/(T#-1))/2-x1#/2*Log((T#+x1#)/_
        (T#-x1#)))
  Next I

  PRINT "Crack profile at Smax/Sy, without stretch:"
  PRINT "x", "v"
  For I=1 to N+1 step 20
    PRINT USING "#.####      "; x#(I), vload#(I)
  Next I
```

```
'printout instructions:

 If Y$="N" goto Calculation2
 LPRINT
 LPRINT "Crack profile at Smax/Sy, without stretch:"
 LPRINT "x", "v"
 For I=1 to N+1 step 20
   LPRINT USING "#.####       "; x#(I), vload#(I)
 Next I
```

```
Calculation2:

'calculates Newman opening stress So
 Input "Enter R: ",R0#
 Input "Enter constraint (1-3): ",A0#
 C0#=Cos(S#*PI#/2)^(1/A0#)
 C0#=(0.825-0.34*A0#+0.05*A0#^2)*C0#
 C1#=(0.415-0.071*A0#)*S#
 C3#=2*C0#+C1#-1
 C2#=1-C0#-C1#-C3#
 So#=C0#+C1#*R0#
   If R0#>=0 then So#=So#+C2#*R0^2+C3#*R0^3
```

```
'Note that So# = [So/Smax]   not [So/Sy]
```

```
 PRINT  "So/Smax= " So# ",  So/Sy= " So#*S#
'-----------------------------------
'printout instructions
 If Y$="N" then goto Calculation3
 LPRINT "Chosen R= " R0# ",  Chosen constraint= " A0#
 LPRINT
 LPRINT  "So/Smax= " So# ",  So/Sy= " So#*S#
'-----------------------------------
```

```
Calculation3:
'calculates profile at opening stress level
 S2#=So#*S#
 Gosub Unloadprofile
 For I=1 to N+1
   vopen#(I)=v#(I)
 Next I

   'Thus the crack face profile at opening
   'stress is given by vopen#

 PRINT "Crack profile at opening stress:"
 PRINT "x", "v"
 For I=1 to N+1 step 20
   PRINT USING "#.####       "; x#(I), vopen#(I)
 Next I
```

```
'printout instructions

  If Y$="N" then goto Calculation4
  LPRINT
  LPRINT "Crack profile at opening stress:"
  LPRINT "x", "v"
  For I=1 to N+1 step 20
    LPRINT USING "#.####      "; x#(I), vopen#(I)
  Next I
```

```
Calculation4:
'finds stretch by finding the minimum gap

  D1#=vopen#(1)
  x0#=A#
  For I=2 to N+1
    If vopen#(I)<=D1# then D1#=vopen#(I) : x0#=x#(I)
  Next I

    'Thus stretch=D1#,
    'and the faces just touch at x=x0#

  PRINT "Stretch= " D1#_
      ", and faces just touch at x= " x0#

  For I=1 to N+1
    vopen#(I)=vopen#(I)-D1#
    vload#(I)=vload#(I)-D1#
  Next I

  PRINT "Thus, with stretch, crack profile at max and opening stresses:"
  PRINT "x", "vmax", "vopen"
  For I=1 to N+1 step 20
    PRINT USING "#.####      "; x#(I), vload#(I), vopen#(I)
  Next I
```

```
'printout instructions

  If Y$="N" then goto Calculation5
  LPRINT
  LPRINT  "Stretch= " D1#_
      ", and faces just touch at x= " x0#
  LPRINT "Thus, with stretch, crack profile at max and opening stresses:"
  LPRINT "x", "vmax", "vopen"
  For I=1 to N+1 step 20
    LPRINT USING "#.####      "; x#(I), vload#(I), vopen#(I)
  Next I
```

```
Calculation5:
'beginning of loops to calculate intermediate profiles

  PRINT "How many intermediate values of S/Sy are you wanting"
  Input "to plot "; P
  If P=0 goto senddata
  DIM S5#(P),vnet#(P,I)
  For H=1 to P
1
    PRINT "Enter intermediate S/Sy #"H": ";
    Input, S5#(H)
    If S5#(H)<=So#*S# then Print "S/Sy is less than the opening stress"_
    : Goto 1
  Next H
```

```
Calculation6:
'calculates profile at each S/Sy
  For H=1 to P
    S2#=S5#(H)
    Gosub Unloadprofile
    For I=1 to N+1
      vnet#(H,I)=v#(I)
    Next I
  Next H
```

```
  For H=1 to P
    PRINT
    PRINT "crack profile with stretch at S/Sy= " S5#(H) ":"
    PRINT "x", "v"
    For I=1 to N+1 step 20
      PRINT USING "#.####        "; x#(I), vnet#(H,I)
    Next I
  Next H
```

```
'printout instructions

  If Y$="N" then goto Reformat
  LPRINT
  For H=1 to P
    LPRINT
    LPRINT "crack profile with stretch at S/Sy= " S5#(H) ":"
    LPRINT "x", "v"
    For I=1 to N+1 step 20
      LPRINT USING "#.####        "; x#(I) vnet#(H,I)
    Next I
  Next H
```

```
Reformat:
'reformat data for sending to datafile
  DIM Plot#(P+3,1:N+1)
  For I=1 to N+1
    Plot#(1,I)=x#(I)
    Plot#(2,I)=vload#(I)
    Plot#(3,I)=vopen#(I)
    For H=1 to P
      Plot#(H+3,I)=vnet#(H,I)
    Next H
  Next I
```

```
Senddata:
  Open D$ for output as #1
  PRINT #1, "Dugdale model with stretch":PRINT
  For I=1 to N+1
  For H=1 to P+3
    PRINT #1, USING " #.#### "; Plot#(H,I),
  Next H
  PRINT #1,""
  Next I
  Close #1
```

```
PRINT "Press ESC to return to main menu"

End
```

```
Unloadprofile:
    S3#=(S#-S2#)*PI#/4
    R#=1/Cos(S3#)
    vunload#(1)=8/PI#*Log(R#)
    For I=1 to N
      x1#=x#(I+1)
      T#=(R#^2-x1#^2)/(R#^2-1)
      T#=Sqr(T#)
      vunload#(I+1)=8/PI#*(Log((T#+1)/(T#-1))/2-x1#/2*Log((T#+x1#)/(T#-x1#)))
    Next I
    For I=1 to N+1
      v#(I)=vload#(I)-vunload#(I)
    Next I
  Return
```

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